

Tugas #3

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No.
 Date

1.) $S = \{0, a, b, c\}$

+	0	a	b	c	x	0	a	b	c
0	0	a	b	c	0	0	0	0	0
a	a	0	c	b	a	0	a	b	c
b	b	c	0	a	b	0	a	b	c
c	c	b	a	0	c	0	0	0	0

$(S, +, x)$ ring? Ring komutatif?

Pembuktian:

Closure:

- Jika dilihat pada tabel operasi kedua (perkalian), semua elemen pada setiap sel merupakan anggota dari himp. S .

Jadi, terbukti closure

Abelian:

- elemen identitas:

$$0 + a = a + 0 = a$$

- elemen invers:

$$a + a = 0 ; b + b = 0 ; c + c = 0$$

- Komutatif

$$a + b = b + a$$

Jika dilihat pada tabel operasi pertama (+). Memenuhi semua syarat di atas, Jadi, terbukti Abelian

Asosiatif:

$$a(bc) = (ab)c$$

$$\downarrow \quad \downarrow$$

$$a(a) = (b)c$$

$$\downarrow \quad \downarrow$$

$$c = c$$

Pada tabel operasi kedua (x).

Terbukti Asosiatif

Distributif:

$$a(b+c) = ab + ac$$

$$; (a+b)c = ac + bc$$

$$a(a) = b + c$$

$$(c)c = c + c$$

$$a = a$$

$$0 = 0$$

Proses tersebut sesuai

estudee

36 lines (6mm spaced)

dan tabel (+ & x). Jadi, terbukti distributif

Dari syarat tersebut, $(S, +, \times)$ merupakan Ring.

Cek, untuk ring komutatif:

$$ab = ba$$

$$\downarrow \quad \downarrow$$

$$b \neq a \rightarrow \text{tidak memenuhi.}$$

Jadi, $(S, +, \times)$ adalah RING (memenuhi aksioma ring),

tetapi RING ini TIDAK KOMUTATIF karena $ab \neq ba$.

2) Tentukan gcd dua polinomial $(x^3 - 2x + 1)$
dan $(x^2 - x - 2)$ pd $\mathbb{GF}(5)$.

Jawab.

$$\begin{aligned} \gcd(x^3 - 2x + 1, x^2 - x - 2) &= \gcd(x^2 - x - 2, x^3 - 2x + 1 \text{ rem } x^2 - x - 2) \\ &= \gcd(x^2 - x - 2, x + 3) \end{aligned}$$

$$\begin{array}{r} x+1 \\ x^2-x-2 \overline{) x^3-2x+1} \\ \underline{x^3-x^2-2x} \\ x^2+1 \\ \underline{x^2-x-2} \\ x+3 \end{array}$$

$$\begin{aligned} &= \gcd(x+3, x^2 - x - 2 \text{ rem } x+3) \\ &= \gcd(x+3, 0) \end{aligned}$$

$$\begin{array}{r} x+3 \\ x+3 \overline{) x^2-x-2} \\ \underline{x^2+3x} \\ -4x-2 \\ \underline{-4x+12} \\ 10 \end{array}$$

$10 \rightarrow$ Karena $\mathbb{GF}(5)$
maka $10 = 0$

$$\begin{aligned} &\gcd(x+3, 0) \\ &= x+3 \end{aligned}$$

Hasil : $x+3$

3) Tentukan gcd dua polinomial $(x^4 + 8x^3 + 7x + 8)$ dan $(2x^3 + 9x^2 + 10x + 1)$ pada $\mathbb{Z}_6[x]$.

Jawab

$$\gcd(x^4 + 8x^3 + 7x + 8, 2x^3 + 9x^2 + 10x + 1)$$

$$= \gcd(2x^3 + 9x^2 + 10x + 1, x^4 + 8x^3 + 7x + 8 \bmod 2x^3 + 9x^2 + 10x + 1)$$

$$\begin{array}{r} 2x^3 + 9x^2 + 10x + 1 \quad \overline{) \quad x^4 + 8x^3 + 7x + 8} \\ \underline{2x^3 + 16x^2 + 5x + 2} \\ 9x^3 + 6x^2 + x + 8 \end{array}$$

$$= \gcd(2x^3 + 9x^2 + 10x + 1, 9x^3 + 6x^2 + x + 8)$$

$$= \gcd(9x^3 + 6x^2 + x + 8, 2x^3 + 9x^2 + 10x + 1 \bmod 9x^3 + 6x^2 + x + 8)$$

$$\begin{array}{r} 9x^3 + 6x^2 + x + 8 \quad \overline{) \quad 2x^3 + 9x^2 + 10x + 1} \\ \underline{2x^3 + 5x^2 + 10x + 3} \\ 4x^2 + 9 \end{array}$$

$$= \gcd(9x^3 + 6x^2 + x + 8, 4x^2 + 9)$$

$$= \gcd(4x^2 + 9, 9x^3 + 6x^2 + x + 8 \bmod 4x^2 + 9)$$

$$\begin{array}{r} 4x^2 + 9 \quad \overline{) \quad 9x^3 + 6x^2 + x + 8} \\ \underline{9x^3 + x} \\ 6x^2 + 8 \end{array}$$

$$= \gcd(4x^2 + 9, 6x^2 + 8) = \gcd(6x^2 + 8, 4x^2 + 9 \bmod 6x^2 + 8)$$

$$\begin{array}{r} 6x^2 + 8 \quad \overline{) \quad 4x^2 + 9} \\ \underline{4x^2 + 9} \\ 0 \end{array}$$

$$= \gcd(6x^2 + 8, 0) = 6x^2 + 8 //$$

kog. terbalik = 6 $\rightarrow 6^{-1} = 2$, maka:

$$(6x^2 + 8) \cdot 2 = 12x^2 + 16 \equiv x^2 + 5 \pmod{6}$$

Data Penjumlahan

+	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
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6	6	7	4	5	2	3	0	1	14	15	12	13	10	11	8	9	22	23	20	21	18	19	16	17	30	31	28	29	26	27	24	25
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31	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

Data Perkalian

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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