Trigonometry Cheatsheet - Exercise 9.2

1. Signs of Trigonometric Functions

1.1 Quadrant Rules

Each quadrant determines the signs of trigonometric functions based on the terminal arm's position:

- **Quadrant I (0° to 90°):** All functions ($\sin, \cos, \tan, \csc, \sec, \cot$) are positive.
- **Quadrant II (90° to 180°):** sin and csc are positive; others are negative.
- **Quadrant III (180° to 270°):** tan and cot are positive; others are negative.
- **Quadrant IV (270° to 360°):** cos and sec are positive; others are negative.

Example: Determine the sign of $\sin 160°$.

- $160°$ is in Quadrant II, where sin is positive.
- Thus, $\sin 160°$ is positive.

1.2 Negative Angles

For negative angles, use the following identities:

- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$
- $\csc(-\theta) = -\csc\theta$
- $\sec(-\theta) = \sec\theta$
- $\cot(-\theta) = -\cot\theta$

Example: Evaluate $\cos(-75°)$.

$$\cos(-75°) = \cos 75° \text{ (positive, as cos is even)}$$

2. Quadrant Identification

To determine the quadrant of an angle based on the signs of two trigonometric functions, analyze their signs:

- **Example:** If $\sin\theta < 0$ and $\cos\theta > 0$, the angle is in Quadrant IV (sin is negative, cos is positive).

Steps:

1. Identify the signs of the given functions.
2. Match them to the quadrant where both conditions are satisfied.

Example: If $\cot\theta > 0$ and $\sin\theta < 0$, find the quadrant.

- $\cot\theta > 0$ in Quadrants I and III.
- $\sin\theta < 0$ in Quadrants III and IV.
- Intersection: Quadrant III.

3. Finding Trigonometric Functions

Given one trigonometric function and the quadrant, find others using identities:

- **Pythagorean Identities:**

$$\sin^2\theta + \cos^2\theta = 1, \quad \sec^2\theta = 1 + \tan^2\theta, \quad \csc^2\theta = 1 + \cot^2\theta$$

- **Ratio Identities:**

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}$$

Example: If $\sin\theta = \frac{12}{13}$ in Quadrant I, find others.

- $\csc\theta = \frac{1}{\sin\theta} = \frac{13}{12}$
- $\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{144}{169}} = \frac{5}{13}$
- $\sec\theta = \frac{1}{\cos\theta} = \frac{13}{5}$
- $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$
- $\cot\theta = \frac{1}{\tan\theta} = \frac{5}{12}$

4. Algebraic Expressions

Evaluate expressions involving trigonometric functions by computing each function's value. **Example:** If $\cot\theta = \frac{5}{2}$ in Quadrant I, find $\frac{3\sin\theta+4\cos\theta}{\cos\theta-\sin\theta}$.

- $\tan\theta = \frac{2}{5}$
- $\csc\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+\frac{25}{4}} = \frac{\sqrt{29}}{2} \Rightarrow \sin\theta = \frac{2}{\sqrt{29}}$
- $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{4}{29}} = \frac{5}{\sqrt{29}}$
- Expression: $\frac{3\cdot\frac{2}{\sqrt{29}}+4\cdot\frac{5}{\sqrt{29}}}{\frac{5}{\sqrt{29}}-\frac{2}{\sqrt{29}}} = \frac{\frac{6+20}{\sqrt{29}}}{\frac{3}{\sqrt{29}}} = \frac{26}{3}$

5. Applications

- **Angle Analysis:** Determine quadrants for engineering or physics problems.
- **Trigonometric Calculations:** Solve for angles in navigation or mechanics using derived functions.