Binomial Theorem Cheatsheet - Class 11 Mathematics

Prepared for Entry Test Preparation

1. Binomial Theorem

For any positive integer n, the expansion of $(a+x)^n$ is:

$$(a+x)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} x^r$$

- Number of terms: $n+1$. - General term: $T_{r+1} = \binom{n}{r} a^{n-r} x^r$. - Binomial coefficients: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.

2. Sums of Binomial Coefficients

- Total sum: $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n$. - Even coefficients: $\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots = 2^{n-1}$. - Odd coefficients: $\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots = 2^{n-1}$. - Weighted sum: $\binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \cdots + \frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1}-1}{n+1}$.

3. Middle Term

- If n is even: Middle term is $\left(\frac{n}{2}+1\right)^{\text{th}}$ term. - If n is odd: Middle terms are $\left(\frac{n+1}{2}\right)^{\text{th}}$ and $\left(\frac{n+3}{2}\right)^{\text{th}}$ terms.

4. Key Applications

- Expand expressions like $(a+x)^n$ or $(a-x)^n$ (Q.1). - Approximate numerical values (e.g., $(0.97)^3$) (Q.2). - Simplify sums of expansions (e.g., $(a+\sqrt{2}x)^4 + (a-\sqrt{2}x)^4$) (Q.3). - Expand trinomials in ascending/descending powers (Q.4, Q.5). - Find specific terms or coefficients (Q.6–9). - Find middle terms (Q.10) or terms from the end (Q.11).

Examples

1. **Expansion (Q.1(i))**: Expand $(a+2b)^5$.

$$(a+2b)^5 = \binom{5}{0}a^5 + \binom{5}{1}a^4(2b) + \binom{5}{2}a^3(2b)^2 + \binom{5}{3}a^2(2b)^3 + \binom{5}{4}a(2b)^4 + \binom{5}{5}(2b)^5$$
$$= a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5.$$

2. **Numerical Approximation (Q.2(i))**: Compute $(0.97)^3 = (1-0.03)^3$.

$$(1-0.03)^3 = 1 - 3(0.03) + 3(0.03)^2 - (0.03)^3 = 1 - 0.09 + 0.0027 - 0.000027 = 0.912673.$$

3. **Sum of Expansions (Q.3(i))**: Simplify $(a+\sqrt{2}x)^4 + (a-\sqrt{2}x)^4$.

$$= 2a^4 + 12a^2(2x^2) + 2(4x^4) = 2a^4 + 24a^2x^2 + 8x^4.$$

4. **Term with Specific Power (Q.6(i))**: Find term with x^4 in $(3-2x)^7$.

$$T_{r+1} = \binom{7}{r}3^{7-r}(-2x)^r = \binom{7}{r}3^{7-r}(-2)^r x^r.$$

For x^4, set $r=4$:

$$T_5 = \binom{7}{4}3^3(-2)^4x^4 = 35\cdot 27\cdot 16\cdot x^4 = 15120x^4.$$

5. **Middle Term (Q.10(i))**: Find middle term in $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}$.

$$n=12 \text{ (even)},\ \text{middle term is } \left(\frac{12}{2}+1\right)^{\text{th}} = 7^{\text{th}},\ \text{so } r=6.$$

$$T_7 = \binom{12}{6}\left(\frac{1}{x}\right)^{12-6}\left(-\frac{x^2}{2}\right)^6 = 924\cdot\frac{1}{x^6}\cdot\frac{x^{12}}{64} = \frac{231}{16}x^6.$$

5. Tips for Solving

- For negative terms, include $(-1)^r$ in the general term.
- For fractional or radical terms, simplify exponents carefully.
- For numerical approximations, express as $(1\pm x)^n$ with small x.
- For sums of expansions, add/subtract to cancel odd-powered terms.
- For specific terms, set the exponent of the variable to the desired power and solve for r.