Conceptual Multiple Choice Questions: Synthetic Division and Quadratic Equations (Exercise 4.5)

Class 11 Mathematics (Chapter 4)

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MCQs

1. The remainder when $x^2 + 3x + 7$ is divided by $x + 1$ is:

(a) 5
(b) -5
(c) 7
(d) 0

2. The remainder when $x^3 - x^2 + 5x + 4$ is divided by $x - 2$ is:

(a) 18
(b) 4
(c) -18
(d) 0

3. The remainder when $3x^4 + 4x^3 + x - 5$ is divided by $x + 1$ is:

(a) -7
(b) 7
(c) -5
(d) 0

4. The remainder when $x^3 - 2x^2 + 3x + 3$ is divided by $x - 3$ is:

(a) 21
(b) -21
(c) 3
(d) 0

5. The remainder when $2x^3 - 3x^2 + x - 8$ is divided by $x - 1$ is:

(a) -8
(b) 8
(c) -6
(d) 0

6. For $x^n + a^n$ with odd n, the remainder when divided by $x + a$ is:

(a) 0

(b) a^n

(c) $-a^n$

(d) 1

7. The value of k when $x^4 + 2x^3 + kx^2 + 3$ divided by $x - 2$ has remainder 1 is:

(a) $-\frac{17}{2}$

(b) $\frac{17}{2}$

(c) -17

(d) 17

8. The value of k when $x^3 + 2x^2 + kx + 4$ divided by $x - 2$ has remainder 14 is:

(a) -3

(b) 3

(c) -14

(d) 14

9. Using synthetic division, the quotient when $x^3 - 7x + 6$ is divided by $x - 2$ is:

(a) $x^2 + 2x - 3$

(b) $x^2 - 2x + 3$

(c) $x^2 + 3x - 1$

(d) $x^2 - 3x + 1$

10. The complete factorization of $x^3 - 7x + 6$ given $x = 2$ is a root is:

(a) $(x - 2)(x + 3)(x - 1)$

(b) $(x - 2)(x - 3)(x + 1)$

(c) $(x + 2)(x + 3)(x - 1)$

(d) $(x + 2)(x - 3)(x + 1)$

11. The quotient when $x^3 - 28x - 48$ is divided by $x + 4$ is:

(a) $x^2 - 4x - 12$

(b) $x^2 + 4x + 12$

(c) $x^2 - 6x + 8$

(d) $x^2 + 6x - 8$

12. The complete factorization of $x^3 - 28x - 48$ given $x = -4$ is a root is:

(a) $(x + 4)(x - 6)(x + 2)$

(b) $(x + 4)(x + 6)(x - 2)$

(c) $(x - 4)(x - 6)(x + 2)$

(d) $(x - 4)(x + 6)(x - 2)$

13. The quotient when $2x^4 + 7x^3 - 4x^2 - 27x - 18$ is divided by $x - 2$ is:

(a) $2x^3 + 11x^2 + 18x + 9$
(b) $2x^3 - 11x^2 + 18x - 9$
(c) $2x^3 + 11x^2 - 18x + 9$
(d) $2x^3 - 11x^2 - 18x - 9$

14. The complete factorization of $2x^4 + 7x^3 - 4x^2 - 27x - 18$ given roots $x = 2, -3$ is:

(a) $(x - 2)(x + 3)(2x + 3)(x + 1)$
(b) $(x - 2)(x - 3)(2x + 3)(x + 1)$
(c) $(x + 2)(x + 3)(2x - 3)(x - 1)$
(d) $(x + 2)(x - 3)(2x - 3)(x - 1)$

15. The values of p, q if $x + 1, x - 2$ are factors of $x^3 + px^2 + qx + 6$ are:

(a) $p = -4, q = 1$
(b) $p = 4, q = -1$
(c) $p = -4, q = -1$
(d) $p = 4, q = 1$

16. The values of a, b if $-2, 2$ are roots of $x^3 - 4x^2 + ax + b$ are:

(a) $a = -4, b = 16$
(b) $a = 4, b = -16$
(c) $a = -4, b = -16$
(d) $a = 4, b = 16$

17. For a quadratic $ax^2 + bx + c = 0$, the sum of roots is:

(a) $-\frac{b}{a}$
(b) $\frac{b}{a}$
(c) $-\frac{c}{a}$
(d) $\frac{c}{a}$

18. For a quadratic $ax^2 + bx + c = 0$, the product of roots is:

(a) $\frac{c}{a}$
(b) $-\frac{c}{a}$
(c) $\frac{b}{a}$
(d) $-\frac{b}{a}$

19. The quadratic equation with roots $3, -2$ is:

(a) $x^2 - x - 6 = 0$

(b) $x^2 + x - 6 = 0$

(c) $x^2 - x + 6 = 0$

(d) $x^2 + x + 6 = 0$

20. If $x - 1$ is a factor of $x^3 - kx^2 + 2x - 2$, the value of k is:

(a) 1

(b) -1

(c) 2

(d) -2

Answers and Explanations

1. **Answer: a**
Explanation: By the Remainder Theorem, the remainder when a polynomial $f(x) = x^2 + 3x + 7$ is divided by $x + 1$ is $f(-1)$. Compute $f(-1) = (-1)^2 + 3(-1) + 7 = 1 - 3 + 7 = 5$. Option (a) is correct because it matches the calculated remainder. Options (b), (c), and (d) are incorrect as they do not equal 5.

2. **Answer: a**
Explanation: For $f(x) = x^3 - x^2 + 5x + 4$ divided by $x - 2$, the remainder is $f(2)$. Calculate $f(2) = 2^3 - 2^2 + 5 \cdot 2 + 4 = 8 - 4 + 10 + 4 = 18$. Option (a) is correct. Options (b), (c), and (d) are incorrect because they do not match the value 18.

3. **Answer: a**
Explanation: For $f(x) = 3x^4 + 4x^3 + x - 5$ divided by $x + 1$, the remainder is $f(-1)$. Compute $f(-1) = 3(-1)^4 + 4(-1)^3 + (-1) - 5 = 3 \cdot 1 + 4 \cdot (-1) - 1 - 5 = 3 - 4 - 1 - 5 = -7$. Option (a) is correct. Options (b), (c), and (d) do not equal -7.

4. **Answer: a**
Explanation: For $f(x) = x^3 - 2x^2 + 3x + 3$ divided by $x - 3$, the remainder is $f(3)$. Calculate $f(3) = 3^3 - 2 \cdot 3^2 + 3 \cdot 3 + 3 = 27 - 18 + 9 + 3 = 21$. Option (a) is correct. Options (b), (c), and (d) are incorrect as they do not equal 21.

5. **Answer: c**
Explanation: For $f(x) = 2x^3 - 3x^2 + x - 8$ divided by $x - 1$, the remainder is $f(1)$. Compute $f(1) = 2 \cdot 1^3 - 3 \cdot 1^2 + 1 - 8 = 2 - 3 + 1 - 8 = -6$. Option (c) is correct. Options (a), (b), and (d) do not match -6.

6. **Answer: a**
Explanation: For $f(x) = x^n + a^n$ divided by $x + a$, the remainder is $f(-a)$. Since n is odd, $f(-a) = (-a)^n + a^n = -a^n + a^n = 0$. Option (a) is correct because the remainder is 0, indicating $x + a$ is a factor. Options (b), (c), and (d) are incorrect as they do not yield 0.

7. **Answer: a**
Explanation: For $f(x) = x^4 + 2x^3 + kx^2 + 3$ divided by $x - 2$, the remainder is $f(2) = 1$. Compute $f(2) = 2^4 + 2 \cdot 2^3 + k \cdot 2^2 + 3 = 16 + 16 + 4k + 3 = 35 + 4k$. Set $35 + 4k = 1 \implies 4k = -34 \implies k = -\frac{34}{4} = -\frac{17}{2}$. Option (a) is correct. Other options do not satisfy the equation.

8. **Answer: a**
Explanation: For $f(x) = x^3 + 2x^2 + kx + 4$ divided by $x - 2$, the remainder is $f(2) = 14$. Compute $f(2) = 2^3 + 2 \cdot 2^2 + k \cdot 2 + 4 = 8 + 8 + 2k + 4 = 20 + 2k$. Set $20 + 2k = 14 \implies 2k = -6 \implies k = -3$. Option (a) is correct. Other options do not yield 14.

9. **Answer: a**
Explanation: Perform synthetic division on $f(x) = x^3 + 0x^2 - 7x + 6$ by $x - 2$:

2	1	0	-7	6
		2	4	-6
	1	2	-3	0

The quotient is $x^2 + 2x - 3$. Option (a) is correct. Other options do not match the quotient coefficients.

10. **Answer: a**
Explanation: Given $x = 2$ is a root, synthetic division yields quotient $x^2 + 2x - 3$. Factorize: $x^2 + 2x - 3 = (x + 3)(x - 1)$. Thus, $f(x) = (x - 2)(x + 3)(x - 1)$. Option (a) is correct. Other options include incorrect factors.

11. **Answer: a**
Explanation: Perform synthetic division on $f(x) = x^3 + 0x^2 - 28x - 48$ by $x + 4$ ($x = -4$):

-4	1	0	-28	-48
		-4	16	48
	1	-4	-12	0

The quotient is $x^2 - 4x - 12$. Option (a) is correct. Other options do not match.

12. **Answer: a**
Explanation: Given $x = -4$ is a root, the quotient is $x^2 - 4x - 12 = (x - 6)(x + 2)$. Thus, $f(x) = (x + 4)(x - 6)(x + 2)$. Option (a) is correct. Other options include incorrect factors.

13. **Answer: a**
Explanation: Perform synthetic division on $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$ by $x - 2$:

2	2	7	-4	-27	-18
		4	22	36	18
	2	11	18	9	0

The quotient is $2x^3 + 11x^2 + 18x + 9$. Option (a) is correct. Other options have incorrect coefficients.

14. **Answer: a**
Explanation: Given roots $x = 2, -3$, synthetic division by $x - 2$ and $x + 3$ yields quotient $2x^2 + 5x + 3 = (2x + 3)(x + 1)$. Thus, $f(x) = (x - 2)(x + 3)(2x + 3)(x + 1)$. Option (a) is correct. Option (b) incorrectly uses $x - 3$.

15. **Answer: a**
Explanation: Since $x + 1, x - 2$ are factors, $f(-1) = 0$ and $f(2) = 0$. Perform synthetic division: For $x = -1$: Coefficients $1, p, q, 6$ yield remainder $p - q + 5 = 0$. For $x = 2$: Coefficients yield remainder $p+q+3 = 0$. Solve: $p-q+5 = 0$, $p + q + 3 = 0 \implies 2p + 8 = 0 \implies p = -4, q = 1$. Option (a) is correct.

16. **Answer: a**
Explanation: Since $-2, 2$ are roots, $f(-2) = 0$, $f(2) = 0$. Compute: $f(-2) = -8 - 16 - 2a + b = 0 \implies -24 - 2a + b = 0$. $f(2) = 8 - 16 + 2a + b = 0 \implies -8 + 2a + b = 0$. Solve: Add equations to get $2b - 32 = 0 \implies b = 16$. Then, $-24 - 2a + 16 = 0 \implies a = -4$. Option (a) is correct.

17. **Answer: a**
Explanation: For a quadratic $ax^2 + bx + c = 0$, the sum of roots is $-\frac{b}{a}$. This follows from the quadratic formula, where roots α, β satisfy $\alpha + \beta = -\frac{b}{a}$. Option (a) is correct. Others are incorrect formulas.

18. **Answer: a**
Explanation: The product of roots for $ax^2 + bx + c = 0$ is $\frac{c}{a}$. From the quadratic formula, $\alpha\beta = \frac{c}{a}$. Option (a) is correct. Others do not represent the product.

19. **Answer: a**
Explanation: For roots $3, -2$, sum = $3 + (-2) = 1$, product = $3 \cdot (-2) = -6$. The quadratic is $x^2 - (sum)x + product = x^2 - x - 6 = 0$. Option (a) is correct. Others yield incorrect sums or products.

20. **Answer: a**
Explanation: If $x-1$ is a factor, $f(1) = 0$ for $f(x) = x^3 - kx^2 + 2x - 2$. Compute $f(1) = 1 - k + 2 - 2 = 1 - k = 0 \implies k = 1$. Option (a) is correct. Other values of k do not yield a remainder of 0.