Trigonometry Cheatsheet - Exercise 9.4

1. Proving Trigonometric Identities

1.1 Fundamental Identities

Use these to simplify expressions:

- $\sin^2\theta + \cos^2\theta = 1$
- $\sec^2\theta = 1 + \tan^2\theta$
- $\csc^2\theta = 1 + \cot^2\theta$
- $\tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$
- $\sec\theta = \frac{1}{\cos\theta}, \csc\theta = \frac{1}{\sin\theta}$

Example: Prove $\tan\theta + \cot\theta = \csc\theta\sec\theta$.

$$\begin{aligned}\text{LHS} &= \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} \\ &= \csc\theta\sec\theta = \text{RHS}\end{aligned}$$

Domain: $\theta \neq n\frac{\pi}{2}, n \in Z$.

1.2 Difference of Squares

Use algebraic identities like $a^2 - b^2 = (a-b)(a+b)$. **Example:** Prove $\sec^2\theta - \csc^2\theta = \tan^2\theta - \cot^2\theta$.

$$\text{LHS} = \sec^2\theta - \csc^2\theta = (1+\tan^2\theta) - (1+\cot^2\theta) = \tan^2\theta - \cot^2\theta = \text{RHS}$$

Domain: $\theta \neq n\frac{\pi}{2}, n \in Z$.

2. Sum and Difference Identities

Manipulate expressions involving sums or differences of trigonometric functions. **Example:** Prove $\cos\theta + \tan\theta\sin\theta = \sec\theta$.

$$\text{LHS} = \cos\theta + \frac{\sin\theta}{\cos\theta}\cdot\sin\theta = \cos\theta + \frac{\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta = \text{RHS}$$

Domain: $\theta \neq (2n+1)\frac{\pi}{2}, n \in Z$.

3. Double Angle Identities

Use to transform expressions:

- $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$
- $\sin 2\theta = 2\sin\theta\cos\theta$

Example: Prove $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$.

$$\text{LHS} = 2\cos^2\theta - 1 = 2(1-\sin^2\theta) - 1 = 2 - 2\sin^2\theta - 1 = 1 - 2\sin^2\theta = \text{RHS}$$

Domain: All real θ.

4. Cube and Sixth Power Identities

Use algebraic identities like $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ or $a^6 - b^6$. **Example:** Prove $\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$.

$$\begin{aligned}\text{LHS} &= \sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)\\ &= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta) = \text{RHS}\end{aligned}$$

Domain: All real θ.

5. Rationalizing Trigonometric Expressions

Simplify fractions involving trigonometric functions. **Example:** Prove $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$.

$$\text{LHS} = \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)} = \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} = 2\sec^2\theta = \text{RHS}$$

Domain: $\theta \neq (2n+1)\frac{\pi}{2}, n \in Z$.

6. Applications

- **Physics:** Use identities in wave equations.
- **Engineering:** Simplify expressions in control systems.