Trigonometric Functions Cheatsheet - Exercise 11.1

1. Domain and Range of Trigonometric Functions

Function	Domain	Range
$y = \sin x$	R	$-1 \leq y \leq 1$
$y = \cos x$	R	$-1 \leq y \leq 1$
$y = \tan x$	$x \neq (2n+1)\frac{\pi}{2}, n \in Z$	R
$y = \cot x$	$x \neq n\pi, n \in Z$	R
$y = \sec x$	$x \neq (2n+1)\frac{\pi}{2}, n \in Z$	$y \geq 1$ or $y \leq -1$
$y = \csc x$	$x \neq n\pi, n \in Z$	$y \geq 1$ or $y \leq -1$

2. Periodicity

2.1 Definition

The period P is the smallest positive number such that $f(x + P) = f(x)$.

2.2 Standard Periods

$$\begin{aligned} \sin x &: 2\pi & \tan x &: \pi \\ \cos x &: 2\pi & \cot x &: \pi \\ \sec x &: 2\pi & \csc x &: 2\pi \end{aligned}$$

2.3 Period of Modified Functions

For $f(kx)$, where $k \neq 0$: - Period of $\sin kx, \cos kx, \sec kx, \csc kx = \frac{2\pi}{|k|}$. - Period of $\tan kx$, $\cot kx = \frac{\pi}{|k|}$. - Constant coefficients (e.g., $3\sin x$) do not affect the period.

Example: Find the period of $\sin 3x$.

$$\sin 3x = \sin(3x + 2\pi) = \sin 3\left(x + \frac{2\pi}{3}\right) \implies \text{Period} = \frac{2\pi}{3}$$

3. Solving Period Problems

Steps: 1. Identify the trigonometric function (sin, cos, etc.). 2. Determine the standard period (2π or π). 3. Find the coefficient k in the argument kx. 4. Compute the period using $\frac{\text{Standard Period}}{|k|}$. 5. Ignore constant coefficients outside the function (e.g., $3\sin x$).

Example: Find the period of $\cot \frac{x}{2}$.

$$\cot \frac{x}{2} = \cot\left(\frac{x}{2} + \pi\right) = \cot \frac{1}{2}(x + 2\pi) \implies \text{Period} = \frac{\pi}{\frac{1}{2}} = 2\pi$$

4. Common Examples

$$\sin\frac{x}{3} : \text{Period} = \frac{2\pi}{\frac{1}{3}} = 6\pi$$

$$\cos 2x : \text{Period} = \frac{2\pi}{2} = \pi$$

$$\tan 4x : \text{Period} = \frac{\pi}{4}$$

$$\sec 9x : \text{Period} = \frac{2\pi}{9}$$

$$3\cos\frac{x}{5} : \text{Period} = \frac{2\pi}{\frac{1}{5}} = 10\pi$$

5. Tips and Tricks

- Coefficients outside the function (e.g., $3\sin x$) affect amplitude, not period.

- For fractional coefficients (e.g., $\sin\frac{x}{k}$), period is standard period multiplied by k.

- Verify by substituting $x + P$ into the function to confirm $f(x + P) = f(x)$.

6. Applications

- **Physics:** Periodicity models wave motion and oscillations.

- **Engineering:** Used in signal processing and circuit analysis.

- **Graphing:** Periods determine the length of one complete cycle on a graph.