Trigonometry Cheatsheet - Exercise 9.1

1. Angle Measurement

1.1 Sexagesimal System

The sexagesimal system measures angles in degrees (°), minutes ('), and seconds ("):

- $1^\circ = 60'$ (1 degree = 60 minutes)
- $1' = 60''$ (1 minute = 60 seconds)

For example, $45^\circ 30' 15''$ represents 45 degrees, 30 minutes, and 15 seconds. To convert to decimal degrees:

$$45^\circ + \frac{30}{60} + \frac{15}{3600} = 45 + 0.5 + 0.00417 = 45.50417^\circ$$

1.2 Radian Measure

A radian is the angle subtended at the center of a circle by an arc equal in length to the radius. Key facts:

- π radians $= 180^\circ$
- 1 radian $\approx 57.296^\circ$
- $1^\circ \approx 0.0175$ radians

Example: Convert 60° to radians.

$$60^\circ \times \frac{\pi}{180} = \frac{\pi}{3} \text{ radians}$$

1.3 Conversion Between Degrees and Radians

- **Degrees to Radians:** Multiply by $\frac{\pi}{180}$
- **Radians to Degrees:** Multiply by $\frac{180}{\pi}$

Example: Convert $\frac{3\pi}{4}$ radians to degrees.

$$\frac{3\pi}{4} \times \frac{180}{\pi} = \frac{3 \times 180}{4} = 135^\circ$$

2. Types of Angles

2.1 Coterminal Angles

Coterminal angles share the same initial and terminal sides, differing by multiples of $360°$ or 2π radians.

- Formula: $\theta + 360°k$ or $\theta + 2\pi k$ (where k is an integer)

Example: Find two coterminal angles for $45°$.

$$45° + 360° = 405° \quad \text{and} \quad 45° - 360° = -315°$$

2.2 General Angles

General angles are expressed as $\theta+2k\pi$ (in radians) or $\theta+360°k$ (in degrees), representing all coterminal angles.

2.3 Standard Position

An angle is in standard position when its vertex is at the origin and its initial side lies along the positive x-axis.

2.4 Quadrantal Angles

Angles with terminal sides on the x- or y-axis, e.g., $0°, 90°, 180°, 270°, 360°$.

2.5 Allied Angles

Angles related to a given angle θ by multiples of $90°$, such as $90° \pm \theta, 180° \pm \theta$.

2.6 Reference Angles

The acute angle between the terminal side of an angle and the x-axis. **Example:** Find the reference angle for $210°$ (in Quadrant III).

$$210° - 180° = 30°$$

3. Arc Length and Sector Area

3.1 Arc Length

For a circle of radius r and central angle θ (in radians), the arc length is:

$$l = r\theta$$

Example: Calculate the arc length with $r = 6$ cm and $\theta = \frac{\pi}{2}$ radians.

$$l = 6 \times \frac{\pi}{2} = 3\pi \approx 9.42 \text{ cm}$$

3.2 Area of a Sector

The area of a sector with radius r and central angle θ (in radians) is:

$$A = \frac{1}{2}r^2\theta$$

Example: Find the sector area with $r = 5$ cm and $\theta = \frac{\pi}{3}$ radians.

$$A = \frac{1}{2} \times 5^2 \times \frac{\pi}{3} = \frac{1}{2} \times 25 \times \frac{\pi}{3} = \frac{25\pi}{6} \approx 13.09 \text{ cm}^2$$

4. Applications

4.1 Clock Angles

The angle between clock hands depends on their positions (hour hand moves 0.5° per minute, minute hand moves 6° per minute). **Example:** Angle at 4:30.

$$\text{Hour hand} = 4 \times 30^\circ + 30 \times 0.5^\circ = 120^\circ + 15^\circ = 135^\circ$$

$$\text{Minute hand} = 30 \times 6^\circ = 180^\circ$$

$$\text{Angle} = |180^\circ - 135^\circ| = 45^\circ$$

4.2 Pendulum Motion

Arc length applies to pendulum swings. **Example:** A 15 cm pendulum swings through 30°. Find the arc length.

$$\theta = 30^\circ \times \frac{\pi}{180} = \frac{\pi}{6} \text{ radians}$$

$$l = 15 \times \frac{\pi}{6} = \frac{15\pi}{6} = \frac{5\pi}{2} \approx 7.85 \text{ cm}$$

4.3 Circular Motion

Distance traveled by a point on a rotating object. **Example:** A wheel of radius 2 m rotates through 1.5 radians. Find the distance.

$$l = 2 \times 1.5 = 3 \text{ m}$$