Exercise 2.7: Binary Operations and Groups MCQs for Entry Test

Multiple Choice Questions

1. A binary operation $*$ on a set G is:

(a) A function from G to G.
(b) A function assigning $(a, b) \in G \times G$ to $a * b \in G$.
(c) A relation between two sets.
(d) A set of ordered triples.

2. Which operation on natural numbers $\mathbb{N}$ satisfies closure?

(a) Subtraction
(b) Division
(c) Addition
(d) Exponentiation

3. For integers $\mathbb{Z}$ with multiplication $(\times)$, what is the identity element?

(a) 0
(b) 1
(c) -1
(d) No identity exists

4. Which set with addition $(+)$ has inverses for all elements?

(a) Natural numbers $\mathbb{N}$
(b) Whole numbers $\mathbb{W}$
(c) Integers $\mathbb{Z}$
(d) Rational numbers $\mathbb{Q}$

5. Is the operation $*$ on $\{a, b, c, d\}$ given by $a * b = c$, $b * a = b$ commutative?

(a) Yes, because $a * b = b * a$.
(b) No, because $a * b \neq b * a$.
(c) Yes, because it is associative.
(d) No, because it lacks an identity.

6. For a binary operation to be associative, which must hold?

(a) $a * b = b * a$
(b) $a * (b * c) = (a * b) * c$
(c) $a * e = a$

(d) $a * a' = e$

7. In the set $\{0, 1, 2, 3\}$ with addition modulo 4, what is the inverse of 2?

(a) 0
(b) 1
(c) 2
(d) 3

8. Which property is required for a set to be a field under addition and multiplication?

(a) Closure only
(b) Commutativity only
(c) Distributive laws
(d) Associativity only

9. How do real numbers differ from complex numbers as fields?

(a) Reals lack multiplicative inverses.
(b) Complex numbers allow ordering.
(c) Reals allow comparison (e.g., $<$).
(d) Complex numbers lack distributivity.

10. In residue classes modulo 5, what is $3 * 4$?

(a) 1
(b) 2
(c) 3
(d) 4

11. In residue classes modulo 4, what is $2 + 3$?

(a) 0
(b) 1
(c) 2
(d) 3

12. Which operation on $\{0, 1, 2, 3\}$ modulo 4 forms a group?

(a) Subtraction
(b) Multiplication
(c) Addition
(d) Division

13. A groupoid requires which property?

(a) Associativity
(b) Closure
(c) Identity
(d) Inverse

14. Which set with multiplication forms a semigroup?

(a) Natural numbers $\mathbb{N}$
(b) Integers $\mathbb{Z}$
(c) Whole numbers $\mathbb{W}$
(d) Real numbers $\mathbb{R}$

15. A monoid differs from a semigroup by having:

(a) Closure
(b) Associativity
(c) Identity
(d) Inverses

16. Which is an example of a group?

(a) $\mathbb{N}$ with addition
(b) $\mathbb{Z}$ with subtraction
(c) $\{0, 1, 2, 3\}$ with addition modulo 4
(d) $\mathbb{W}$ with multiplication

17. An Abelian group requires:

(a) Closure only
(b) Commutativity
(c) Associativity only
(d) Inverses only

18. For the table below, is the operation commutative?

$*$	a	b	c	d
a	a	c	b	d
b	c	d	b	a
c	b	b	a	c
d	d	a	c	d

(a) Yes
(b) No
(c) Only for a and b
(d) Only for c and d

19. For residue classes modulo 5, what is the identity for multiplication?

(a) 0
(b) 1
(c) 2
(d) 3

20. Which set with addition is an Abelian group?

(a) Natural numbers $\mathbb{N}$
(b) Whole numbers $\mathbb{W}$
(c) Integers $\mathbb{Z}$
(d) Positive reals $\mathbb{R}^+$

Answers and Explanations

1. **Answer**: (b) A function assigning $(a, b) \in G \times G$ to $a * b \in G$.
   Explanation: A binary operation maps pairs in $G \times G$ to an element in G. Option (a) is incorrect as it maps single elements. Option (c) refers to relations, not operations. Option (d) involves triples, not pairs.

2. **Answer**: (c) Addition
   Explanation: Addition on $\mathbb{N}$ is closed ($a + b \in \mathbb{N}$). Subtraction (e.g., $2 - 3 = -1 \notin \mathbb{N}$), division (e.g., $2 \div 3 \notin \mathbb{N}$), and exponentiation (not always defined) fail closure.

3. **Answer**: (b) 1
   Explanation: The identity for multiplication satisfies $a \times e = a$. For $\mathbb{Z}$, $e = 1$ (e.g., $2 \times 1 = 2$). Option (a) is the identity for addition. Option (c) gives $a \times (-1) = -a$. Option (d) is incorrect as 1 exists.

4. **Answer**: (c) Integers $\mathbb{Z}$
   Explanation: Inverses for addition require $a + a' = 0$. In $\mathbb{Z}$, $a' = -a$. In $\mathbb{N}$ and $\mathbb{W}$, no inverses exist for non-zero elements (Q.1). $\mathbb{Q}$ also works, but $\mathbb{Z}$ is the focus.

5. **Answer**: (b) No, because $a * b \neq b * a$.
   Explanation: From Q.5, table (a), $a * b = c$, $b * a = b$, so not commutative. Options (a), (c), and (d) misinterpret commutativity.

6. **Answer**: (b) $a * (b * c) = (a * b) * c$
   Explanation: Associativity requires grouping to be irrelevant. Option (a) is commutativity, (c) is identity, (d) is inverse.

7. **Answer**: (c) 2
   Explanation: From Q.7, the inverse of 2 satisfies $2 + x = 0 \mod 4$. Check: $2 + 2 = 4 \mod 4 = 0$. Other options fail (e.g., $2 + 3 = 5 \mod 4 = 1$).

8. **Answer**: (c) Distributive laws
   Explanation: A field requires Abelian groups under $+$ and $\times$ (with closure, associativity, commutativity, identity, inverses) plus distributivity (Q.2). Options (a), (b), and (d) are insufficient alone.

9. **Answer**: (c) Reals allow comparison (e.g., $<$).
   Explanation: Q.2 states reals have ordering ($2 < 3$), unlike complex numbers. Option (a) is false (reals have inverses). Option (b) reverses the fact. Option (d) is false (both have distributivity).

10. **Answer**: (b) 2
    Explanation: From Q.3, $3 * 4 = 12 \mod 5 = 2$. Check table: row 3, column 4 is 2. Other options are incorrect.

11. **Answer**: (b) 1
    Explanation: From Q.4, $2 + 3 = 5 \mod 4 = 1$. Check table: row 2, column 3 is 1. Other options are incorrect.

12. **Answer**: (c) Addition
    Explanation: Q.7 shows $\{0, 1, 2, 3\}$ with $+ \mod 4$ forms a group (closure, associativity, identity 0, inverses). Subtraction and division fail closure; multiplication lacks inverses for 0.

13. **Answer**: (b) Closure
    Explanation: A groupoid only requires closure. Associativity, identity, and inverses are for semigroups, monoids, and groups, respectively.

14. **Answer**: (c) Whole numbers $\mathbb{W}$
Explanation: Multiplication on $\mathbb{W}$ is closed and associative, forming a semigroup (Q.1). All options work, but $\mathbb{W}$ is simplest.

15. **Answer**: (c) Identity
Explanation: A monoid is a semigroup with an identity element. Closure and associativity are already in semigroups; inverses are for groups.

16. **Answer**: (c) $\{0, 1, 2, 3\}$ with addition modulo 4
Explanation: Q.7 confirms this is a group (closure, associativity, identity, inverses). $\mathbb{N}$ lacks inverses, $\mathbb{Z}$ with subtraction isn't closed, $\mathbb{W}$ with multiplication lacks inverses.

17. **Answer**: (b) Commutativity
Explanation: An Abelian group is a group with commutativity. Closure, associativity, and inverses are already required for a group.

18. **Answer**: (a) Yes
Explanation: From Q.5, table (b), check: $a * b = c$, $b * a = c$; table is symmetric across the diagonal, so commutative. Other options are incorrect.

19. **Answer**: (b) 1
Explanation: From Q.3, the identity for multiplication satisfies $a * e = a$. Check table: row 1 shows $1 * a = a$. Option (a) is for addition.

20. **Answer**: (c) Integers $\mathbb{Z}$
Explanation: $\mathbb{Z}$ with addition is an Abelian group (closure, associativity, identity 0, inverses, commutativity; Q.1). $\mathbb{N}$, $\mathbb{W}$, and $\mathbb{R}^+$ lack inverses for all elements.