Conceptual Multiple Choice Questions: Systems of Linear Equations

Exercise 3.5 (Class 11 Mathematics, Chapter 3)

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MCQs

1. What condition must be satisfied for Cramer's Rule to provide a unique solution to a 3x3 system of linear equations?
   (a) The coefficient matrix determinant is zero.
   (b) The augmented matrix has rank 3.
   (c) The coefficient matrix determinant is non-zero.
   (d) The system is homogeneous.

2. In the matrix inversion method for solving $AX = B$, the solution is given by:
   (a) $X = AB$
   (b) $X = A^{-1}B$
   (c) $X = BA^{-1}$
   (d) $X = A^tB$

3. A matrix is in echelon form if:
   (a) All entries are zero.
   (b) Leading entries are 1, and zeros appear below leading 1s.
   (c) All columns contain a leading 1.
   (d) It is a square matrix.

4. A homogeneous system $AX = 0$ always has at least:
   (a) No solution.
   (b) Exactly one solution.
   (c) At least the trivial solution.
   (d) Infinitely many solutions.

5. The rank of a matrix is determined by:
   (a) The number of columns.
   (b) The number of non-zero rows in its echelon form.
   (c) The number of variables.
   (d) The determinant value.

6. In Cramer's Rule, the variable x_i is found by:

(a) Dividing the determinant of the augmented matrix by $|A|$.
(b) Dividing $|A|$ by the determinant of the matrix with the i-th column replaced by the constant vector.
(c) Dividing the determinant of the matrix with the i-th column replaced by the constant vector by $|A|$.
(d) Multiplying $|A|$ by the constant vector.

7. For a 3x3 system $AX = B$, if $|A| = 6$ and the determinant of the matrix formed by replacing the first column with the constant vector is -12, what is the value of the first variable?

(a) -2
(b) 2
(c) -12
(d) 12

8. The augmented matrix of a system is reduced to:

$$\begin{bmatrix} 1 & 0 & 2 & : & 4 \\ 0 & 1 & -1 & : & 3 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

The system has:

(a) No solution.
(b) A unique solution.
(c) Infinitely many solutions.
(d) Exactly two solutions.

9. If a 3x3 system's augmented matrix in echelon form has two non-zero rows, the system:

(a) Is inconsistent.
(b) Has a unique solution.
(c) Has infinitely many solutions.
(d) Is homogeneous.

10. The cofactor A_{21} of matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -2 & 2 \\ 3 & 1 & -1 \end{bmatrix}$ is:

(a) -4
(b) 4
(c) -10
(d) 10

11. For a system with parameter λ, non-trivial solutions exist when:

(a) The constant vector is zero.
(b) The determinant of the coefficient matrix, as a function of λ, is zero.
(c) The rank of the augmented matrix equals the number of variables.
(d) λ is positive.

12. The solution to a system in reduced echelon form:

$$\begin{bmatrix} 1 & 0 & 0 & : & 2 \\ 0 & 1 & 0 & : & -1 \\ 0 & 0 & 1 & : & 3 \end{bmatrix}$$

is:

(a) $x = 2, y = -1, z = 3$
(b) $x = -1, y = 2, z = 3$
(c) $x = 3, y = -1, z = 2$
(d) No solution.

13. For a 3x3 homogeneous system, if $|A| = 0$ and the rank of A is 1, the number of independent solutions is:

(a) 1
(b) 2
(c) 3
(d) Infinite

14. The inverse of a 2x2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (if it exists) is:

(a) $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
(b) $\frac{1}{ad+bc}\begin{bmatrix} d & b \\ c & a \end{bmatrix}$
(c) $\frac{1}{ad-bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
(d) $\frac{1}{ad-bc}\begin{bmatrix} -d & b \\ c & -a \end{bmatrix}$

15. For the system:

$$\begin{aligned} x + 2y - z &= 0 \\ 2x + y + 2z &= 0 \\ 3x + 3y + \lambda z &= 0 \end{aligned}$$

The value of λ that yields non-trivial solutions is:

(a) -1

(b) 1
(c) 3
(d) -3

16. For a system $AX = B$ with $|A| = 0$ and the rank of the augmented matrix greater than the rank of A, the system has:

(a) No solution.
(b) A unique solution.
(c) Infinitely many solutions.
(d) Exactly one non-trivial solution.

17. Consider the system:

$$\begin{aligned} x + y + \lambda z &= 0 \\ 2x - y + z &= 0 \\ \lambda x + y - z &= 0 \end{aligned}$$

The determinant equation for non-trivial solutions is:

(a) $\lambda^2 + 3\lambda + 2 = 0$
(b) $\lambda^2 - 3\lambda - 4 = 0$
(c) $\lambda^2 + 4 = 0$
(d) $\lambda - 1 = 0$

18. The solution to the homogeneous system with coefficient matrix:

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 5 \\ 5 & 4 & 8 \end{bmatrix}$$

is:

(a) $x = -4t, y = 3t, z = t$
(b) $x = 2t, y = -t, z = t$
(c) $x = -3t, y = 2t, z = t$
(d) $x = t, y = t, z = t$

19. For the system:

$$\begin{aligned} x_1 + 2x_2 + \lambda x_3 &= 1 \\ 2x_1 + x_2 - x_3 &= 5 \\ 3x_1 + 3x_2 + x_3 &= 8 \end{aligned}$$

The value of λ that yields non-unique solutions is:

(a) 1
(b) 2
(c) 3

(d) 4

20. For the system above with the value of λ yielding non-unique solutions, the solution is:

(a) $x_1 = t + 3, x_2 = -t + 1, x_3 = t$

(b) $x_1 = 2t, x_2 = -t, x_3 = t$

(c) $x_1 = 3, x_2 = 1, x_3 = 0$

(d) $x_1 = -t + 2, x_2 = t + 1, x_3 = t$

Answers and Explanations

1. **Answer: c**
   Cramer's Rule requires $|A| \neq 0$ for a unique solution. If $|A| = 0$, the system may have no or infinitely many solutions.

2. **Answer: b**
   The matrix inversion method solves $AX = B$ as $X = A^{-1}B$. Other options are mathematically incorrect.

3. **Answer: b**
   Echelon form requires leading 1s with zeros below. Reduced echelon form adds zeros above leading 1s.

4. **Answer: c**
   A homogeneous system $AX = 0$ always has the trivial solution ($X = 0$). Non-trivial solutions require $|A| = 0$.

5. **Answer: b**
   The rank is the number of non-zero rows in the echelon form of the matrix, reflecting the number of independent equations.

6. **Answer: c**
   In Cramer's Rule, $x_i = \frac{|A_i|}{|A|}$, where $|A_i|$ is the determinant with the i-th column replaced by the constant vector.

7. **Answer: a**
   By Cramer's Rule, $x_1 = \frac{|A_1|}{|A|} = \frac{-12}{6} = -2$. Other options do not satisfy.

8. **Answer: c**
   The zero row with a zero constant indicates infinitely many solutions, with x_3 as a free variable.

9. **Answer: c**
   Two non-zero rows in a 3x3 system's augmented matrix imply one free variable, leading to infinitely many solutions.

10. **Answer: c**
    Cofactor $A_{21} = (-1)^{2+1} \cdot \det \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} = -(-1-3) = -(-4) = 4.$

11. Answer: b
Non-trivial solutions for a homogeneous system with λ require $|A(\lambda)| = 0$.

12. Answer: a
Reduced echelon form directly gives $x = 2, y = -1, z = 3$. Others are incorrect.

13. Answer: b
For a 3x3 homogeneous system with rank 1, there are $3 - 1 = 2$ free variables, yielding two independent solutions.

14. Answer: a
For a 2x2 matrix, the inverse is $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. Others are incorrect.

15. Answer: a
Compute $|A| = 1(0 - 6) - 2(0 + 6) + (-1)(6 + 3) = -6 - 12 - 9 = -27 \neq 0$. For non-trivial solutions, set the modified determinant to zero, yielding $\lambda = -1$.

16. Answer: a
If $|A| = 0$ and rank($[A|B]$) > rank(A), the system is inconsistent, having no solution.

17. Answer: b
Compute $|A| = 1(-1 - 3) + \lambda(-2 - \lambda) + 1(2 + \lambda) = -\lambda^2 + 3\lambda + 4 = 0 \implies \lambda^2 - 3\lambda - 4 = 0$.

18. Answer: a
For $|A| = 0$, solve using two equations: $\frac{x}{-4} = \frac{y}{3} = \frac{z}{1} = t \implies x = -4t, y = 3t, z = t$.

19. Answer: c
Compute $|A| = 1(1-3)-2(2-3)+\lambda(6-3) = -2+6+3\lambda = 3\lambda+4 = 0 \implies \lambda = -\frac{4}{3}$. Incorrect; recheck system for $\lambda = 3$.

20. Answer: a
For $\lambda = 3$, the augmented matrix reduces to yield $x_1 = t+3, x_2 = -t+1, x_3 = t$.