Oblique Triangle Area Cheatsheet
Exercise 12.7

1 Oblique Triangle Fundamentals

1.1 Definition and Notation

An oblique triangle has no right angle ($\alpha + \beta + \gamma \neq 90°$). In $\triangle ABC$:

- Angles: α (at A), β (at B), γ (at C).
- Sides: a (opposite α), b (opposite β), c (opposite γ).

1.2 Key Formulas

- **Angle Sum**: $\alpha + \beta + \gamma = 180°$.
- **Case I: Two Sides and Included Angle**:

$$\Delta = \frac{1}{2}bc\sin\alpha = \frac{1}{2}ab\sin\gamma = \frac{1}{2}ac\sin\beta$$

- **Case II: One Side and Two Angles**:

$$\Delta = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha} = \frac{b^2\sin\alpha\sin\gamma}{2\sin\beta} = \frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$$

- **Case III: Three Sides (Heron's Formula)**:

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)}, \quad S = \frac{a+b+c}{2}$$

2 Finding Triangle Area

2.1 Steps: Two Sides and Included Angle

1. Identify sides and included angle (e.g., a, b, γ).
2. Use formula: $\Delta = \frac{1}{2}ab\sin\gamma$.
3. Compute sine of the angle using tables/calculator.
4. Report area in square units, typically to two decimal places.

2.2 Steps: One Side and Two Angles

1. Find third angle: $\gamma = 180° - \alpha - \beta$.
2. Use formula: $\Delta = \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}$ (or equivalent).
3. Compute sines using tables/calculator.
4. Report area in square units.

2.3 Steps: Three Sides (Heron's Formula)

1. Calculate semi-perimeter: $S = \frac{a+b+c}{2}$.
2. Compute differences: $S - a, S - b, S - c$.
3. Use formula: $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$.
4. Report area, exact or to two decimal places.

2.4 Example: Two Sides and Included Angle

Given: $a = 200, b = 120, \gamma = 150°$.

$$\Delta = \frac{1}{2} \cdot 200 \cdot 120 \cdot \sin 150° = \frac{1}{2} \cdot 200 \cdot 120 \cdot 0.5 = 6000 \text{ sq. units}$$

3 Problem Types

- **Two Sides, Included Angle**: Use $\Delta = \frac{1}{2}ab\sin\gamma$.

  $$\text{E.g., } a = 200, b = 120, \gamma = 150° \implies \Delta = 6000 \text{ sq. units.}$$

- **One Side, Two Angles**: Find third angle, use $\Delta = \frac{b^2 \sin\alpha \sin\gamma}{2\sin\beta}$.

  $$\text{E.g., } b = 25.4, \gamma = 36°41', \alpha = 45°17' \implies \beta = 98°2', \Delta \approx 138.29 \text{ sq. units.}$$

- **Three Sides**: Use Heron's formula.

  $$\text{E.g., } a = 18, b = 24, c = 30 \implies S = 36, \Delta = 216 \text{ sq. units.}$$

- **Find Angle Given Area**: Use $\Delta = \frac{1}{2}ac\sin\beta$ to solve for β.

  $$\text{E.g., } \Delta = 2437, a = 79, c = 97 \implies \beta \approx 39°30'.$$

- **Find Side Given Area**: Use $\Delta = \frac{1}{2}c^2\frac{\sin\alpha\sin\beta}{\sin\gamma}$ to solve for c.

  $$\text{E.g., } \Delta = 121.34, \alpha = 32°15', \beta = 65°37' \implies c \approx 22.24.$$

4 Tips and Tricks

- Verify $\alpha + \beta + \gamma = 180°$.
- Use exact sine values for standard angles (e.g., $\sin 150° = 0.5$).
- Convert minutes to decimals for calculations: $\theta° m' = \theta + \frac{m}{60}$.
- Round areas to two decimal places unless exact.
- For real-world problems, interpret sides as distances and compute costs as area $\times$ rate.
- Check triangle inequality ($a + b > c$) for three-side problems.

5 Applications

- Surveying: Calculate land area for triangular plots.
- Landscaping: Estimate costs for grass planting or tiling.
- Engineering: Determine surface areas in structural designs.