Trigonometric Identities MCQs for Entry Test - Exercise 10.2

Introduction

This document contains 20 multiple-choice questions based on Exercise 10.2 of the trigonometry chapter, designed for entry test preparation. Each question tests concepts such as angle sum and difference identities, specific angle transformations, evaluating trigonometric functions, cotangent identities, triangle angle identities, linear combination expressions, and quadrant-based calculations. Solutions with detailed explanations are provided at the end.

Multiple-Choice Questions

1. **What is** $\sin(180° + \theta)$**?**

   A) $\sin\theta$

   B) $-\sin\theta$

   C) $\cos\theta$

   D) $-\cos\theta$

2. **Evaluate** $\cos(180° + \theta)$**.**

   A) $\cos\theta$

   B) $-\cos\theta$

   C) $\sin\theta$

   D) $-\sin\theta$

3. **What is** $\tan(270° - \theta)$**?**

   A) $\tan\theta$

   B) $-\tan\theta$

   C) $\cot\theta$

   D) $-\cot\theta$

4. **Find** $\cos(\theta - 180°)$**.**

   A) $\cos\theta$

   B) $-\cos\theta$

C) $\sin\theta$

D) $-\sin\theta$

5. **Evaluate** $\cos(270° + \theta)$.

   A) $\sin\theta$

   B) $-\sin\theta$

   C) $\cos\theta$

   D) $-\cos\theta$

6. **What is** $\sin(270° + \theta)$**?**

   A) $\cos\theta$

   B) $-\cos\theta$

   C) $\sin\theta$

   D) $-\sin\theta$

7. **Find** $\tan(180° + \theta)$.

   A) $\tan\theta$

   B) $-\tan\theta$

   C) $\cot\theta$

   D) $-\cot\theta$

8. **Evaluate** $\cos(360° - \theta)$.

   A) $\cos\theta$

   B) $-\cos\theta$

   C) $\sin\theta$

   D) $-\sin\theta$

9. **What is the value of** $\sin 15°$**?**

   A) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

   B) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

   C) $\frac{1+\sqrt{3}}{2\sqrt{2}}$

   D) $\frac{1-\sqrt{3}}{2\sqrt{2}}$

10. **Find** $\cos 105°$.

    A) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

    B) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

    C) $\frac{1-\sqrt{3}}{2\sqrt{2}}$

    D) $\frac{1+\sqrt{3}}{2\sqrt{2}}$

11. **Which identity holds for** $\sin(45° + \alpha)$**?**

    A) $\frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$

    B) $\frac{1}{\sqrt{2}}(\sin\alpha - \cos\alpha)$

    C) $\frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$

    D) $\frac{1}{\sqrt{2}}(\cos\alpha + \sin\alpha)$

12. **Evaluate** $\tan(45° + A)\tan(45° - A)$**.**

    A) 0

    B) 1

    C) $\tan A$

    D) $-\tan A$

13. **What is** $\sin\left(\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{\pi}{3}\right)$**?**

    A) $\sin\theta$

    B) $\cos\theta$

    C) $\tan\theta$

    D) $\cot\theta$

14. **If** $\sin\alpha = \frac{4}{5}$, $\cos\beta = \frac{40}{41}$, $0 < \alpha, \beta < \frac{\pi}{2}$, **what is** $\sin(\alpha - \beta)$**?**

    A) $\frac{133}{205}$

    B) $\frac{27}{205}$

    C) $\frac{160}{205}$

    D) $\frac{133}{41}$

15. **If** $\sin\alpha = \frac{4}{5}$, $\sin\beta = \frac{12}{13}$, $\frac{\pi}{2} < \alpha, \beta < \pi$, **what is** $\sin(\alpha + \beta)$**?**

    A) $\frac{56}{65}$

    B) $-\frac{56}{65}$

    C) $\frac{33}{65}$

    D) $-\frac{33}{65}$

16. **If** $\tan\alpha = \frac{3}{4}$, $\cos\beta = \frac{5}{13}$, **both in 3rd quadrant, what is** $\cos(\alpha + \beta)$**?**

    A) $\frac{33}{65}$

    B) $\frac{56}{65}$

    C) $-\frac{56}{65}$

    D) $-\frac{33}{65}$

17. **If** $\alpha + \beta + \gamma = 180°$, **what is** $\cot\alpha\cot\beta + \cot\beta\cot\gamma + \cot\gamma\cot\alpha$**?**

    A) 0

    B) 1

C) 2

D) -1

18. **Express** $12\sin\theta + 5\cos\theta$ **as** $r\sin(\theta + \phi)$**. What is** r**?**

   A) 13

   B) $\sqrt{169}$

   C) 17

   D) 7

19. **What is** $\frac{\cos 8° - \sin 8°}{\cos 8° + \sin 8°}$**?**

   A) $\tan 37°$

   B) $\tan 45°$

   C) $\cot 37°$

   D) $\cot 45°$

20. **If** α, β, γ **are angles of a triangle, what is** $\cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2}$**?**

   A) $\cot\frac{\alpha}{2} \cot\frac{\beta}{2} \cot\frac{\gamma}{2}$

   B) $\tan\frac{\alpha}{2} \tan\frac{\beta}{2} \tan\frac{\gamma}{2}$

   C) 1

   D) 0

Solutions and Explanations

1. **Solution to Question 1:**

   $$\sin(180° + \theta) = \sin 180° \cos\theta + \cos 180° \sin\theta = 0 \cdot \cos\theta + (-1) \cdot \sin\theta = -\sin\theta$$

   Answer: B) $-\sin\theta$

2. **Solution to Question 2:**

   $$\cos(180° + \theta) = \cos 180° \cos\theta - \sin 180° \sin\theta = (-1) \cdot \cos\theta - 0 \cdot \sin\theta = -\cos\theta$$

   Answer: B) $-\cos\theta$

3. **Solution to Question 3:**

   $$\tan(270° - \theta) = \frac{\sin(270° - \theta)}{\cos(270° - \theta)} = \frac{-\cos\theta}{-\sin\theta} = \cot\theta$$

   Answer: C) $\cot\theta$

4. **Solution to Question 4:**

   $$\cos(\theta - 180°) = \cos\theta \cos 180° + \sin\theta \sin 180° = \cos\theta \cdot (-1) + \sin\theta \cdot 0 = -\cos\theta$$

   Answer: B) $-\cos\theta$

5. **Solution to Question 5:**

$$\cos(270° + \theta) = \cos 270° \cos \theta - \sin 270° \sin \theta = 0 \cdot \cos \theta - (-1) \cdot \sin \theta = \sin \theta$$

Answer: A) $\sin \theta$

6. **Solution to Question 6:**

$$\sin(270° + \theta) = \sin \theta \cos 270° + \cos \theta \sin 270° = \sin \theta \cdot 0 + \cos \theta \cdot (-1) = -\cos \theta$$

Answer: B) $-\cos \theta$

7. **Solution to Question 7:**

$$\tan(180° + \theta) = \frac{\tan 180° + \tan \theta}{1 - \tan 180° \tan \theta} = \frac{0 + \tan \theta}{1 - 0 \cdot \tan \theta} = \tan \theta$$

Answer: A) $\tan \theta$

8. **Solution to Question 8:**

$$\cos(360° - \theta) = \cos 360° \cos \theta + \sin 360° \sin \theta = 1 \cdot \cos \theta + 0 \cdot \sin \theta = \cos \theta$$

Answer: A) $\cos \theta$

9. **Solution to Question 9:**

$$\sin 15° = \sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45° = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

Answer: B) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

10. **Solution to Question 10:**

$$\cos 105° = \cos(60° + 45°) = \cos 60° \cos 45° - \sin 60° \sin 45° = \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1 - \sqrt{3}}{2\sqrt{2}}$$

Answer: C) $\frac{1-\sqrt{3}}{2\sqrt{2}}$

11. **Solution to Question 11:**

$$\sin(45° + \alpha) = \sin 45° \cos \alpha + \cos 45° \sin \alpha = \frac{1}{\sqrt{2}} \cos \alpha + \frac{1}{\sqrt{2}} \sin \alpha = \frac{1}{\sqrt{2}}(\cos \alpha + \sin \alpha)$$

Answer: A) $\frac{1}{\sqrt{2}}(\sin \alpha + \cos \alpha)$

12. **Solution to Question 12:**

$$\tan(45° + A)\tan(45° - A) = \frac{\tan 45° + \tan A}{1 - \tan 45° \tan A} \cdot \frac{\tan 45° - \tan A}{1 + \tan 45° \tan A} = \frac{1 + \tan A}{1 - \tan A} \cdot \frac{1 - \tan A}{1 + \tan A} = 1$$

Answer: B) 1

13. **Solution to Question 13:**

$$\sin\left(\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{\pi}{3}\right) = \left(\sin\theta\frac{\sqrt{3}}{2} + \cos\theta\frac{1}{2}\right) + \left(\cos\theta\frac{1}{2} - \sin\theta\frac{\sqrt{3}}{2}\right) = \cos\theta$$

Answer: B) $\cos\theta$

14. **Solution to Question 14:**

$$\sin\alpha = \frac{4}{5}, \cos\alpha = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}, \cos\beta = \frac{40}{41}, \sin\beta = \sqrt{1 - \frac{1600}{1681}} = \frac{9}{41}$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta = \frac{4}{5} \cdot \frac{40}{41} - \frac{3}{5} \cdot \frac{9}{41} = \frac{160}{205} - \frac{27}{205} = \frac{133}{205}$$

Answer: A) $\frac{133}{205}$

15. **Solution to Question 15:**

$$\sin\alpha = \frac{4}{5}, \cos\alpha = -\sqrt{1 - \frac{16}{25}} = -\frac{3}{5}, \sin\beta = \frac{12}{13}, \cos\beta = -\sqrt{1 - \frac{144}{169}} = -\frac{5}{13}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{4}{5} \cdot \left(-\frac{5}{13}\right) + \left(-\frac{3}{5}\right) \cdot \frac{12}{13} = \frac{-20}{65} - \frac{36}{65} = -\frac{56}{65}$$

Answer: B) $-\frac{56}{65}$

16. **Solution to Question 16:**

$$\tan\alpha = \frac{3}{4}, \sec\alpha = -\sqrt{1 + \frac{9}{16}} = -\frac{5}{4}, \cos\alpha = -\frac{4}{5}, \sin\alpha = -\frac{3}{5}, \cos\beta = \frac{5}{13}, \sin\beta = -\sqrt{1 - \frac{25}{169}} =$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \left(-\frac{4}{5}\right) \cdot \frac{5}{13} - \left(-\frac{3}{5}\right) \cdot \left(-\frac{12}{13}\right) = \frac{-20}{65} - \frac{36}{65} = -\frac{56}{65}$$

Answer: C) $-\frac{56}{65}$

17. **Solution to Question 17:**

$$\alpha + \beta + \gamma = 180° \implies \alpha + \beta = 180° - \gamma, \tan(\alpha + \beta) = -\tan\gamma$$

$$\frac{\cot\alpha + \cot\beta}{\cot\alpha\cot\beta - 1} = \frac{-1}{\cot\gamma} \implies \cot\alpha\cot\gamma + \cot\beta\cot\gamma + \cot\alpha\cot\beta = 1$$

Answer: B) 1

18. **Solution to Question 18:**

$$12\sin\theta + 5\cos\theta = r\sin(\theta + \phi), \quad r = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

Answer: A) 13

19. **Solution to Question 19:**

$$\frac{\cos 8° - \sin 8°}{\cos 8° + \sin 8°} = \frac{1 - \tan 8°}{1 + \tan 8°} = \tan(45° - 8°) = \tan 37°$$

Answer: A) $\tan 37°$

20. **Solution to Question 20:**

$$\alpha + \beta + \gamma = 180° \implies \frac{\alpha}{2} + \frac{\beta}{2} = 90° - \frac{\gamma}{2}, \tan\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \cot\frac{\gamma}{2}$$

$$\frac{\cot\frac{\alpha}{2} + \cot\frac{\beta}{2}}{\cot\frac{\alpha}{2}\cot\frac{\beta}{2} - 1} = \cot\frac{\gamma}{2} \implies \cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

Answer: A) $\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$