Harmonic Progression and Series Cheatsheet: Exercises 6.10 & 6.11

Class 11 Mathematics (Part-I)
Prepared for Entry Test Preparation

Exercise 6.10: Harmonic Progression (H.P.)

Basics

A sequence is in **Harmonic Progression (H.P.)** if the reciprocals of its terms form an **Arithmetic Progression (A.P.)**. If the A.P. has first term a and common difference d, the nth term of the H.P. is $\frac{1}{a+(n-1)d}$.

Key Formulas

- nth term of H.P.: $T_n = \frac{1}{a+(n-1)d}$
- Harmonic Mean (H.M.) of a and b: $H = \frac{2ab}{a+b}$
- k H.M.s between a and b: Reciprocals form A.P. with $k+2$ terms from $\frac{1}{a}$ to $\frac{1}{b}$.
- A.M., G.M., H.M. relation: $G^2 = AH$, where A, G, H are arithmetic, geometric, and harmonic means.
- A.P. formulas (for reciprocals):
  - nth term: $a_n = a + (n-1)d$
  - Sum: $S_n = \frac{n}{2}[2a + (n-1)d]$

Key Concepts

- Find nth term by computing the A.P. of reciprocals.
- Insert H.M.s by solving for the A.P. of reciprocals.
- Solve for numbers given H.M. and A.M.: Use $a + b = 2 \cdot \text{A.M.}$, $\frac{2ab}{a+b} = \text{H.M.}$.
- If a, b, c are in H.P., then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Example Problems

1. **9th term of H.P. $\frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \ldots$:**
   - Reciprocals: $5, 8, 11, \ldots$ (A.P., $a = 5$, $d = 3$).
   - 9th term of A.P.: $5 + (9-1) \cdot 3 = 29$.
   - 9th term of H.P.: $\frac{1}{29}$.
2. **Insert 3 H.M.s between 2 and $\frac{2}{9}$:**
   - Reciprocals: $\frac{1}{2}, \ldots, \frac{9}{2}$ (A.P., 5 terms).

- $d = \frac{\frac{9}{2} - \frac{1}{2}}{4} = 1$.
- H.M.s: $\frac{2}{3}, \frac{1}{2}, \frac{2}{5}$.

3. **Find numbers with H.M. = 6, A.M. = 5**:
   - $a + b = 10$, $\frac{2ab}{a+b} = 6 \implies ab = 30$.
   - Solve: $a, b = 3, 7$.

Exercise 6.11: Summation of Series

Basics

Find the nth term (T_n) of the series and compute the sum $S_n = \sum_{k=1}^{n} T_k$ using standard summation formulas.

Key Formulas

- $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
- $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$
- $\sum_{k=1}^{n} 1 = n$

Key Concepts

- Identify T_n as a polynomial or product (e.g., $k \cdot a_k$ where a_k is in A.P.).
- Split T_n into sums of powers of k or constants.
- Series may involve squares or products of A.P. terms.

Example Problems

1. **Sum $1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \ldots$ to n terms**:
   - $T_n = n \cdot (2n + 1) = 2n^2 + n$.
   - $S_n = 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)(4n+5)}{6}$.
2. **Sum $1^2 + 3^2 + 5^2 + \ldots$ to n terms**:
   - $T_n = (2n - 1)^2 = 4n^2 - 4n + 1$.
   - $S_n = \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n = \frac{n(4n^2-1)}{3}$.
3. **Sum with $T_n = 2n^2 + 3n + 1$ to n terms**:
   - $S_n = 2 \cdot \frac{n(n+1)(2n+1)}{6} + 3 \cdot \frac{n(n+1)}{2} + n = \frac{n(2n^2+7n+7)}{2}$.