Cheatsheet: Systems of Quadratic Equations (Exercise 4.9)

Class 11 Mathematics (Chapter 4)

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Overview

Exercise 4.9 focuses on solving systems of two quadratic equations in two variables. Solutions typically yield up to four ordered pairs, depending on the discriminant. Key methods include elimination (for equations with x^2 and y^2), factorization (for homogeneous equations), and substitution (for equations like $xy = c$).

> **Note**
>
> Verify solutions in both original equations to avoid extraneous roots. Check the discriminant to confirm the number of real solutions.

Both Equations Contain Only x^2 and y^2

Method

1. Multiply equations by appropriate constants to align coefficients of x^2 or y^2.
2. Add or subtract to eliminate one variable, yielding an equation in the other.
3. Solve the resulting equation (linear or quadratic).
4. Substitute back to find corresponding values.

Example For $2x^2 - 3y^2 = 6$, $3x^2 - 5y^2 = 7$:

- Multiply first by 3, second by 2: $6x^2 - 9y^2 = 18$, $6x^2 - 10y^2 = 14$.
- Subtract: $y^2 = 4 \implies y = \pm 2$.
- Substitute into first: $x^2 = 9 \implies x = \pm 3$.
- Solution set: $\{(3,2), (-3,2), (3,-2), (-3,-2)\}$.

Tip Check if subtraction yields a positive value for x^2 or y^2, as negative values indicate no real solutions.

One Equation is Homogeneous

Method

1. Factorize the homogeneous equation (e.g., $ax^2+bxy+cy^2 = 0$) into two linear factors: $(px - qy)(rx - sy) = 0$.
2. Solve each linear equation with the second equation (usually by substitution).

3. Solve the resulting quadratic equations and find corresponding values.

Example For $x^2 - 5xy + 6y^2 = 0$, $x^2 + y^2 = 45$:

- Factorize: $(x - 2y)(x - 3y) = 0 \implies x = 2y, x = 3y$.
- For $x = 2y$, substitute into $x^2 + y^2 = 45$: $5y^2 = 45 \implies y = \pm 3 \implies (6, 3), (-6, -3)$.
- For $x = 3y$, substitute: $10y^2 = 45 \implies y = \pm\frac{3}{\sqrt{2}} \implies \left(\frac{9}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right), \left(-\frac{9}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right)$.
- Solution set: $\left\{\left(\frac{9}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right), \left(-\frac{9}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right), (6, 3), (-6, -3)\right\}$.

Tip Factorize by finding two numbers whose sum is the coefficient of xy and product is ac.

Both Equations are Non-Homogeneous

Method

1. Multiply equations by constants to eliminate constant terms, yielding a homogeneous equation.
2. Factorize the homogeneous equation into linear factors.
3. Solve each linear equation with one of the original equations.

Example For $y^2 - 2xy = 7$, $2x^2 + 3 = xy$:

- Rewrite: $y^2 - 2xy = 7$, $2x^2 - xy + 3 = 0$.
- Multiply first by 3, second by 7, add: $14x^2 - 13xy + 3y^2 = 0 \implies (2x - y)(7x - 3y) = 0$.
- For $2x - y = 0$, no solution. For $7x - 3y = 0$, solve: $y = \pm 7, x = \pm 3$.
- Solution set: $\{(3, 7), (-3, -7)\}$.

Equations Involving $xy = c$

Method

1. Solve $xy = c$ for one variable (e.g., $y = \frac{c}{x}$).
2. Substitute into the other equation to form a quadratic or biquadratic equation.
3. Solve using the quadratic formula or factoring.

Example For $x^2 + y^2 = 5$, $xy = 2$:

- Substitute $y = \frac{2}{x}$ into $x^2 + y^2 = 5$: $x^4 - 5x^2 + 4 = 0$.

- Solve: $x^2 = 4, 1 \implies x = \pm 2, \pm 1$.
- Find y: $(2, 1), (-2, -1), (1, 2), (-1, -2)$.

Key Reminders

- Verify all solutions in both equations.
- Check for complex solutions if required (e.g., $x^2 < 0$).
- For biquadratic equations, let $u = x^2$ to simplify.
- Ensure no division by zero in substitutions.

Tip

Simplify radical expressions in solutions for clarity, and rationalize denominators if possible.