Solving Right Triangles Cheatsheet
Exercise 12.2

1 Right Triangle Fundamentals

1.1 Notation

In right triangle $\triangle ABC$, where $\angle C = 90^\circ$:

- Angles: $\angle A = \alpha$, $\angle B = \beta$, $\angle C = \gamma = 90^\circ$.
- Sides: $a = BC$ (opposite α), $b = AC$ (opposite β), $c = AB$ (hypotenuse).

1.2 Key Formulas

- Angle Sum: $\alpha + \beta = 90^\circ$.
- Trigonometric Ratios:

$$\sin\alpha = \frac{a}{c}, \quad \cos\alpha = \frac{b}{c}, \quad \tan\alpha = \frac{a}{b}$$
$$\sin\beta = \frac{b}{c}, \quad \cos\beta = \frac{a}{c}, \quad \tan\beta = \frac{b}{a}$$

- Pythagorean Theorem: $a^2 + b^2 = c^2$.
- Inverse Functions:

$$\alpha = \sin^{-1}\left(\frac{a}{c}\right), \quad \alpha = \cos^{-1}\left(\frac{b}{c}\right), \quad \alpha = \tan^{-1}\left(\frac{a}{b}\right)$$

Principal ranges: $\sin^{-1}, \cos^{-1} \in [0^\circ, 90^\circ]$, $\tan^{-1} \in (-90^\circ, 90^\circ)$.

2 Solving Right Triangles

2.1 Steps

1. Identify given data (angles: α, β; sides: a, b, c).
2. Find missing angle using $\alpha + \beta = 90^\circ$.
3. Choose appropriate trigonometric ratio ($\sin, \cos, \tan$) based on known values.
4. Solve for unknown sides by rearranging the ratio.
5. Verify using Pythagorean theorem or another ratio.
6. Report angles in degrees and minutes, sides exact or to two decimal places.

2.2 Example

Given: $\gamma = 90°, \alpha = 45°, a = 4$.

- Find β:

$$\alpha + \beta = 90° \implies 45° + \beta = 90° \implies \beta = 45°$$

- Find c (hypotenuse):

$$\sin 45° = \frac{a}{c} = \frac{4}{c} \implies \frac{\sqrt{2}}{2} = \frac{4}{c} \implies c = 4\sqrt{2}$$

- Find b:

$$\cos 45° = \frac{b}{c} = \frac{b}{4\sqrt{2}} \implies \frac{\sqrt{2}}{2} = \frac{b}{4\sqrt{2}} \implies b = 4$$

3 Common Trigonometric Values

Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

For non-standard angles (e.g., $37°20'$), use trigonometric tables or calculators.

4 Problem Types

- **One Angle + One Side**: Find other angle, then sides.

$$\text{E.g., } \gamma = 90°, \alpha = 37°20', a = 243 \implies \beta = 52°40', \text{ use } \sin, \cos \text{ for } c, b.$$

- **Two Sides**: Find one angle via $\tan\alpha = \frac{a}{b}$, then other angle and side.

$$\text{E.g., } a = 3.28, b = 5.74 \implies \alpha = \tan^{-1}\left(\frac{3.28}{5.74}\right) \approx 29°44'.$$

- **Side Ratios**: Find angle via inverse function.

$$\text{E.g., } \cos\beta = \frac{5}{10} \implies \beta = \cos^{-1}(0.5) = 60°.$$

- **Hypotenuse + One Side**: Find angles via $\sin\alpha = \frac{a}{c}$, then other side.

$$\text{E.g., } a = 5429, c = 6294 \implies \alpha = \sin^{-1}\left(\frac{5429}{6294}\right) \approx 59°36'.$$

5 Tips and Tricks

- Label sides correctly: opposite, adjacent, hypotenuse relative to the angle.
- Convert degrees and minutes: $\theta^\circ m' = \theta + \frac{m}{60}$ (e.g., $37^\circ 20' = 37.3333^\circ$).
- Verify with Pythagorean theorem: $a^2 + b^2 = c^2$.
- Use exact values for standard angles $(30^\circ, 45^\circ, 60^\circ)$.
- Round sides to two decimal places unless exact (e.g., $4\sqrt{2}$).

6 Applications

- Surveying: Measure heights or distances (e.g., pole and shadow).
- Engineering: Calculate structural angles or forces.
- Navigation: Determine bearings or distances via triangulation.