Cheatsheet: Matrices and Determinants (Exercise 3.4)

Class 11 Mathematics (Chapter 3)

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1. Matrix Types

Symmetric Matrix

- **Definition**: $A = A^t$ (square matrix, $a_{ij} = a_{ji}$).
- **Example**: $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.
- **Property**: $(A+B)^t = A + B$ if A, B symmetric (Q1).

Skew-Symmetric Matrix

- **Definition**: $A^t = -A$ (diagonal elements are 0).
- **Example**: $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.
- **Property**: $A - A^t$ is skew-symmetric (Q2, Q3).

Hermitian Matrix

- **Definition**: $(\overline{A})^t = A$ (complex entries).
- **Example**: $\begin{bmatrix} 0 & 2+i \\ 2-i & -2 \end{bmatrix}$ (Q6).
- **Property**: $A + (\overline{A})^t$ is Hermitian.

Skew-Hermitian Matrix

- **Definition**: $(\overline{A})^t = -A$.
- **Example**: $\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ (Q6).
- **Property**: $A - (\overline{A})^t$ is skew-Hermitian.

2. Echelon and Reduced Echelon Forms

Echelon Form

- **Conditions**:
  1. Leading entry in each non-zero row is 1.
  2. Zeros before leading 1 increase in successive rows.
- **Example**: $\begin{bmatrix} 0 & 1 & -2 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Reduced Echelon Form

- **Additional Condition**: Column of leading 1 has zeros elsewhere.
- **Example**: $\begin{bmatrix} 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

3. Rank of a Matrix

- **Definition**: Number of non-zero rows in reduced echelon form (Q10).
- **Steps**:
  1. Apply row operations to reach echelon form.
  2. Count non-zero rows.
- **Example**: $\begin{bmatrix} 1 & -4 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ has rank 2.

4. Matrix Properties

- $A + A^t$: Always symmetric (Q2, Q3).
- $A - A^t$: Always skew-symmetric.
- AB: Symmetric if A, B symmetric and $AB = BA$ (Q4).
- AA^t, A^tA: Symmetric for any matrix (Q5).
- A^2: Symmetric if A is symmetric or skew-symmetric (Q7).

5. Matrix Inverse (Q9)

Adjoint Method

- **Formula**: $A^{-1} = \frac{\text{Adj } A}{|A|}$, where Adj $A = (\text{cofactor matrix})^t$.
- **Steps**:
  1. Compute $|A| \neq 0$.
  2. Find cofactors $A_{ij} = (-1)^{i+j} \cdot$ minor.
  3. Transpose cofactor matrix.
  4. Divide by $|A|$.

Row/Column Operations

- **Steps**:
  1. Form $[A|I]$.
  2. Use row/column operations to make $A \to I$.
  3. Right side becomes A^{-1}.
- **Example**: For $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} -1 & -1 & \frac{3}{2} \\ 0 & -\frac{1}{2} & 0 \\ -1 & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$.

6. Complex Matrix Operations (Q8)

- **Conjugate**: $\overline{A}$ replaces each entry $a + bi$ with $a - bi$.
- **Example**: For $A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$, $A(\overline{A})^t = \begin{bmatrix} 3 \\ 3-2i \\ 2+i \end{bmatrix}$.