Application of Trigonometry Cheatsheet - Chapter 12

1. Evaluating Trigonometric Functions

1.1 Key Functions

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

1.2 Method

- Use trigonometric tables or calculators for angles in degrees and minutes (e.g., $53°40'$). - For $\cot\theta$, compute $\tan\theta$ first, then take reciprocal. - Round to four decimal places for consistency.

Examples:

$$\sin 53°40' = 0.8056$$
$$\cot 33°50' = \frac{1}{\tan 33°50'} = 1.4919$$
$$\tan 25°34' = 0.4784$$

2. Inverse Trigonometric Functions

2.1 Definition

Find θ such that:

$$\sin\theta = a \implies \theta = \sin^{-1}(a), \quad \cos\theta = a \implies \theta = \cos^{-1}(a), \quad \tan\theta = a \implies \theta = \tan^{-1}(a)$$

- Principal ranges: $\sin^{-1}, \cos^{-1} : [0°, 90°]$, $\tan^{-1} : (-90°, 90°)$.

2.2 Method

- Use tables or calculators to find θ in degrees and minutes. - Ensure θ is in the appropriate quadrant (usually first for positive values).

Examples:

$$\sin\theta = 0.5791 \implies \theta = \sin^{-1}(0.5791) = 35°23'$$
$$\cos\theta = 0.9316 \implies \theta = \cos^{-1}(0.9316) = 21°18'$$
$$\tan\theta = 21.943 \implies \theta = \tan^{-1}(21.943) = 87°23'$$

3. Solving Right Triangles

3.1 Key Formulas

- **Angle Sum**: $\alpha+\beta+\gamma = 180^\circ$ (where one angle, usually $\beta = 90^\circ$). - **Trigonometric Ratios**:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

- **Pythagorean Theorem**: $a^2 + b^2 = c^2$ (where c is the hypotenuse). - **Notation**: For $\triangle ABC$ with $\angle C = 90^\circ$, sides opposite $\angle A$, $\angle B$, $\angle C$ are $a = BC$, $b = AC$, $c = AB$.

3.2 Steps

1. Find missing angle: $\gamma = 180^\circ - \alpha - \beta$. 2. Use given side and angle to find others via sin, cos, or tan. 3. Apply Pythagorean theorem if two sides are known.

Example:

$$\alpha = 45^\circ, \beta = 90^\circ, BC = 4$$

- Find γ:

$$\alpha + \beta + \gamma = 180^\circ \implies 45^\circ + 90^\circ + \gamma = 180^\circ \implies \gamma = 45^\circ$$

- Find hypotenuse AB:

$$\sin 45^\circ = \frac{BC}{AB} = \frac{4}{AB} \implies \frac{\sqrt{2}}{2} = \frac{4}{AB} \implies AB = 4\sqrt{2}$$

4. Common Trigonometric Values

Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

- For non-standard angles (e.g., $53^\circ 40'$), use tables or calculators.

5. Tips and Tricks

- Convert degrees and minutes to decimal degrees if needed: $\theta^\circ m' = \theta + \frac{m}{60}$.
- For $\cot\theta$, always compute $\tan\theta$ first.
- In right triangles, label sides relative to the angle (opp, adj, hyp).
- Use Pythagorean theorem to verify side lengths.
- Ensure inverse function results are in the principal range.

6. Applications

- **Physics**: Calculate angles in projectile motion or force components.
- **Engineering**: Determine heights, distances, or structural angles.
- **Navigation**: Find bearings or distances using triangulation.