Set Theory Cheatsheet

Set Theory Concepts

This cheatsheet summarizes key set theory concepts with definitions and examples for quick reference.

1. **Set**
   - *Definition*: A well-defined collection of distinct objects (elements or members).
   - *Example*: Set of vowels: {a, e, i, o, u}.
2. **Methods to Describe a Set**
   - *Definition*: Sets can be described using:
     - Descriptive: Words describing the set.
     - Tabular (Roster): List elements in { }.
     - Set Builder: {x | condition}.
   - *Example*: First five natural numbers:
     - Descriptive: The set of the first five natural numbers.
     - Tabular: {1, 2, 3, 4, 5}.
     - Set Builder: $\{x \mid x \in \mathbb{N} \land x \leq 5\}$.
3. **Set Builder Notation**
   - *Definition*: Describes a set using a rule: {x | condition}.
   - *Example*: Even numbers from 2 to 10: $\{x \mid x \in \mathbb{Z} \land 2 \leq x \leq 10 \land x \text{ is even}\}$ = {2, 4, 6, 8, 10}.
4. **Finite and Infinite Sets**
   - *Definition*: Finite sets have a limited number of elements; infinite sets have unlimited elements.
   - *Example*:
     - Finite: Days of the week = {Monday, Tuesday, …, Sunday}.
     - Infinite: Natural numbers = {1, 2, 3, …}.
5. **Subset**

- *Definition*: Set A is a subset of B ($A \subseteq B$) if every element of A is in B.
- *Example*: $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$. $A \subseteq B$ since 1, 2 are in B.

6. Proper Subset

- *Definition*: $A \subset B$ if $A \subseteq B$ and $A \neq B$ (B has extra elements).
- *Example*: $A = \{1, 2\}$, $B = \{1, 2, 3\}$. $A \subset B$ since B has 3.

7. Improper Subset

- *Definition*: A is an improper subset of B if $A = B$.
- *Example*: $A = \{1, 2\}$, $B = \{1, 2\}$. $A \subseteq B$ and $A = B$.

8. Empty Set

- *Definition*: A set with no elements, denoted $\emptyset$ or $\{\}$.
- *Example*: Set of people 10 feet tall = $\emptyset$.

9. Power Set

- *Definition*: The set of all subsets of a set S, denoted P(S). If S has n elements, P(S) has 2^n elements.
- *Example*: $S = \{1, 2\}$. $P(S) = \{ \emptyset, \{1\}, \{2\}, \{1, 2\} \}$.

10. Equivalent Sets

- *Definition*: Sets with the same number of elements (same cardinality).
- *Example*: $A = \{a, b, c\}$, $B = \{1, 2, 3\}$. Both have 3 elements, so they are equivalent.

11. Equal Sets

- *Definition*: Sets with exactly the same elements, regardless of order.
- *Example*: $A = \{1, 2\}$, $B = \{2, 1\}$. $A = B$ since they have the same elements.

12. Set Membership

- *Definition*: $\in$ means an element is in a set; $\notin$ means it is not.
- *Example*: $A = \{1, 2, 3\}$. $2 \in A$, $4 \notin A$.

13. Union of Sets ($A \cup B$)

- *Definition*: All elements in A, B, or both (no duplicates).
- *Example*: $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$. $A \cup B = \{1, 2, 3, 4, 5\}$.

14. Intersection of Sets ($A \cap B$)

- *Definition*: Elements common to both A and B.
- *Example*: $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$. $A \cap B = \{3\}$.

15. Complement of a Set (A')

- *Definition*: All elements in the universal set U not in A.

- *Example*: U = {1, 2, 3, 4, 5}, A = {1, 2}. A' = {3, 4, 5}.

16. Disjoint Sets

- *Definition*: Sets with no common elements ($A \cap B = \emptyset$).
- *Example*: A = {1, 2}, B = {3, 4}. $A \cap B = \emptyset$.

17. Overlapping Sets

- *Definition*: Sets with at least one common element ($A \cap B \neq \emptyset$).
- *Example*: A = {1, 2, 3}, B = {2, 3, 4}. $A \cap B = \{2, 3\}$.

18. Venn Diagrams

- *Definition*: Diagrams using circles in a rectangle (universal set) to show set relationships.
- *Example*: For A = {1, 2, 3}, B = {3, 4, 5}, $A \cup B$ shades both circles; $A \cap B$ shades the overlap.

19. Common Set Notations

- *Definition*: Standard symbols for sets:
  - $\mathbb{N}$: Natural numbers {1, 2, 3, …}.
  - $\mathbb{Z}$: Integers {…, -2, -1, 0, 1, 2, …}.
  - $\mathbb{Q}$: Rational numbers (fractions).
  - $\mathbb{R}$: Real numbers.
  - E: Even integers {…, -2, 0, 2, …}.
  - O: Odd integers {…, -1, 1, 3, …}.
  - P: Prime numbers {2, 3, 5, 7, …}.
  - W: Whole numbers {0, 1, 2, …}.
- *Example*: Odd integers from 1 to 5: $\{x \mid x \in O \land 1 \leq x \leq 5\} = \{1, 3, 5\}$.

20. Empty Set as a Solution

- *Definition*: A set is empty if no elements satisfy the condition.
- *Example*: $\{x \mid x \in \mathbb{N} \land x + 4 = 0\} = \emptyset$ (no natural number satisfies x = -4).

21. Sets Defined by Equations

- *Definition*: Sets defined by elements satisfying an equation.
- *Example*: $\{x \mid x \in \mathbb{Q} \land x^2 = 2\} = \emptyset$ (no rational number satisfies $x^2 = 2$).

22. Difference Between {a, b} and {{a, b}}

- *Definition*: {a, b} has two elements; {{a, b}} has one element (the set {a, b}).
- *Example*: {1, 2} has elements 1, 2; {{1, 2}} has one element: {1, 2}.

23. True/False Statements About Sets

- *Definition*: Statements about sets (e.g., equality, membership) are evaluated as true or false.
- *Example*: $\{1, 2\} = \{2, 1\}$ is true (same elements, order doesn't matter).