# Exercise 2.8: Groups and Algebraic Structures MCQs for Entry Test

## Multiple Choice Questions

**1.** Which properties must a set with an operation satisfy to be a group?

(a) Closure, commutativity, identity
(b) Closure, associativity, identity, inverses
(c) Associativity, identity, commutativity
(d) Closure, associativity, commutativity

**2.** Consider the set $\{0, 1\}$ with operation $\oplus$ defined by the table below. What is the identity element?

| $\oplus$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

(a) 0
(b) 1
(c) Both 0 and 1
(d) None



**3.** Which set with addition forms an Abelian group?

(a) Natural numbers $\mathbb{N}$
(b) Whole numbers $\mathbb{W}$
(c) Rational numbers $\mathbb{Q}$
(d) Positive rationals $\mathbb{Q}^+$

**4.** Is the operation $\oplus$ on $\{0, 1, 2, 3\}$ (addition modulo 4) commutative?

| $\oplus$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 2 | 3 | 0 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 0 | 1 | 2 |

(a) Yes
(b) No
(c) Only for 0 and 1
(d) Only for 2 and 3

**5.** A semigroup is defined by which properties?

(a) Closure and identity
(b) Closure and associativity
(c) Associativity and inverses
(d) Closure, associativity, identity

**6.** For the set $\{a, b, c\}$ with operation $*$ given below, what value of $c * a$ ensures associativity?

| $*$ | $a$ | $b$ | $c$ |
|---|---|---|---|
| $a$ | $c$ | $a$ | $b$ |
| $b$ | $a$ | $b$ | $c$ |
| $c$ | ? | $c$ | $a$ |

(a) a
(b) b
(c) c
(d) None

**7.** The power set $P(S)$ with intersection ($\cap$) forms a:

(a) Group
(b) Semigroup
(c) Monoid
(d) Non-Abelian group

**8.** What is the identity element for $P(S)$ with intersection ($\cap$)?

(a) Empty set $\emptyset$
(b) Set $S$
(c) Any subset of $S$
(d) None

**9.** Which set with multiplication forms a group?

(a) Integers $\mathbb{Z}$
(b) Rational numbers $\mathbb{Q}$
(c) Positive rationals $\mathbb{Q}^+$
(d) Whole numbers $\mathbb{W}$

**10.** Why does the set of integers $\mathbb{Z}$ with multiplication not form a group?

(a) Lacks closure
(b) Lacks associativity
(c) Lacks identity
(d) Lacks inverses

**11.** In a group $G$ with operation $*$, the solution to $a * x = b$ is:

(a) $x = a * b$
(b) $x = a^{-1} * b$
(c) $x = b * a$
(d) $x = b * a^{-1}$

**12.** In a group $G$, solve $x * a = b$. The solution is:

(a) $x = a^{-1} * b$
(b) $x = b * a^{-1}$
(c) $x = a * b$
(d) $x = b * a$

**13.** The set of $2 \times 2$ non-singular matrices over reals with multiplication forms a:

(a) Abelian group
(b) Non-Abelian group
(c) Semigroup only
(d) Monoid only

**14.** Why is the group of $2 \times 2$ non-singular matrices non-Abelian?

(a) Lacks closure
(b) Lacks associativity
(c) Lacks commutativity
(d) Lacks inverses

**15.** In $\{1, \omega, \omega^2\}$, where $\omega^3 = 1$, what is the inverse of $\omega$?

| $\cdot$ | $1$ | $\omega$ | $\omega^2$ |
|---|---|---|---|
| $1$ | $1$ | $\omega$ | $\omega^2$ |
| $\omega$ | $\omega$ | $\omega^2$ | $1$ |
| $\omega^2$ | $\omega^2$ | $1$ | $\omega$ |



(a) 1
(b) $\omega$
(c) $\omega^2$
(d) None

**16.** Is $\{1, \omega, \omega^2\}$, where $\omega^3 = 1$, with multiplication an Abelian group?

(a) Yes
(b) No, lacks closure
(c) No, lacks associativity
(d) No, lacks commutativity

**17.** The set $\{a + \sqrt{3}b \mid a, b \in \mathbb{Q}\}$ with addition is a:

(a) Non-Abelian group
(b) Abelian group
(c) Semigroup only
(d) Monoid only

**18.** What is the identity element in $\{a + \sqrt{3}b \mid a, b \in \mathbb{Q}\}$ with addition?

(a) $1 + \sqrt{3}1$
(b) $0 + \sqrt{3}0$
(c) $0 + \sqrt{3}1$

(d) None

**19.** In $\{E, O\}$ with operation $\oplus$, what is the inverse of $E$?

| $\oplus$ | $E$ | $O$ |
|---|---|---|
| $E$ | $E$ | $O$ |
| $O$ | $O$ | $E$ |

(a) $E$

(b) $O$

(c) Both $E$ and $O$

(d) None

**20.** Does $\{E, O\}$ with $\oplus$ form an Abelian group?

(a) Yes

(b) No, lacks inverses

(c) No, lacks commutativity

(d) No, lacks identity

## Answers and Explanations



1. **Answer**: (b) Closure, associativity, identity, inverses
**Explanation**: A group requires closure ($a * b \in G$), associativity ($(a * b) * c = a * (b * c)$), an identity ($a * e = a$), and inverses ($a * a^{-1} = e$). Option (b) lists all four. Option (a) includes commutativity, which is not required for a general group. Option (c) omits closure, and option (d) omits identity and inverses, making them incomplete.

2. **Answer**: (a) 0
**Explanation**: The identity element satisfies $e \oplus a = a$. From the table, $0 \oplus 0 = 0$, $0 \oplus 1 = 1$, so 0 is the identity. Option (b): $1 \oplus 1 = 0 \neq 1$, so 1 is not the identity. Option (c): Only one identity exists in a group. Option (d): 0 exists as the identity.

3. **Answer**: (c) Rational numbers $\mathbb{Q}$
**Explanation**: An Abelian group requires closure, associativity, identity, inverses, and commutativity under addition. For $\mathbb{Q}$, $a + b \in \mathbb{Q}$, addition is associative, 0 is the identity, inverses are $-a \in \mathbb{Q}$, and $a + b = b + a$. Option (a): $\mathbb{N}$ lacks inverses (e.g., no $-1$). Option (b): $\mathbb{W}$ lacks inverses. Option (d): $\mathbb{Q}^+$ lacks 0 and inverses for negative numbers.

4. **Answer**: (a) Yes
**Explanation**: Commutativity requires $a \oplus b = b \oplus a$. The table is symmetric across the diagonal (e.g., $1 \oplus 2 = 3$, $2 \oplus 1 = 3$), confirming commutativity. Option (b): The operation is commutative. Options (c, d): Commutativity holds for all elements, not just specific pairs.

5. **Answer**: (b) Closure and associativity
**Explanation**: A semigroup requires closure ($a * b \in G$) and associativity ($(a * b) * c = a * (b * c)$). Option (b) is correct. Option (a): Identity is for monoids. Option (c): Inverses are for groups. Option (d): Identity makes it a monoid, not just a semigroup.

6. **Answer**: (b) b
**Explanation**: Associativity requires $(a * a) * a = a * (a * a)$. Compute: $a * a = c$, so left side is $c * a$, right side is $a * c = b$. Thus, $c * a = b$. Option (b) is correct. Option (a): $c * a = a$ fails (e.g., $c * a \neq b$). Option (c): $c * a = c \neq b$. Option (d): $b$ satisfies associativity.

7. **Answer**: (c) Monoid
**Explanation**: $P(S)$ with $\cap$ is closed ($A \cap B \in P(S)$), associative ($A \cap (B \cap C) = (A \cap B) \cap C$), and has an identity ($A \cap S = A$). It's a monoid but not a group, as inverses don't exist (e.g., no $B$ such that $A \cap B = S$ for all $A$). Option (a): No inverses. Option (b): It's more than a semigroup due to identity. Option (d): Intersection is commutative.

8. **Answer**: (b) Set $S$
**Explanation**: The identity for intersection satisfies $A \cap e = A$. Since $A \cap S = A$, $S$ is the identity. Option (a): $A \cap \emptyset = \emptyset \neq A$. Option (c): Subsets like $\{x\} \subset S$ give $A \cap \{x\} \neq A$. Option (d): $S$ exists as the identity.

9. **Answer**: (c) Positive rationals $\mathbb{Q}^+$
**Explanation**: A group under multiplication requires closure, associativity, identity, and inverses. For $\mathbb{Q}^+$, $a \cdot b \in \mathbb{Q}^+$, multiplication is associative, 1 is the identity, and inverses are $\frac{1}{a} \in \mathbb{Q}^+$. Option (a): $\mathbb{Z}$ lacks inverses for non-$\pm 1$. Option (b): $\mathbb{Q}$ includes 0, which has no inverse. Option (d): $\mathbb{W}$ lacks inverses for non-1.

10. **Answer**: (d) Lacks inverses
**Explanation**: $\mathbb{Z}$ with multiplication is closed ($a \cdot b \in \mathbb{Z}$), associative, and has identity 1, but lacks inverses (e.g., 2 has no inverse in $\mathbb{Z}$, as $\frac{1}{2} \notin \mathbb{Z}$). Option (a): Closure holds. Option (b): Associativity holds. Option (c): Identity exists.

11. **Answer**: (b) $x = a^{-1} * b$
**Explanation**: To solve $a * x = b$, multiply both sides by $a^{-1}$ on the left: $a^{-1} * (a * x) = a^{-1} * b$, so $(a^{-1} * a) * x = a^{-1} * b$, hence $e * x = a^{-1} * b$, or $x = a^{-1} * b$. Option (a): $a * b$ doesn't isolate $x$. Option (c): $b * a$ assumes commutativity. Option (d): $b * a^{-1}$ is for $x * a = b$.

12. **Answer**: (b) $x = b * a^{-1}$
**Explanation**: To solve $x * a = b$, multiply both sides by $a^{-1}$ on the right: $(x * a) * a^{-1} = b * a^{-1}$, so $x * (a * a^{-1}) = b * a^{-1}$, hence $x * e = b * a^{-1}$, or $x = b * a^{-1}$. Option (a): Incorrect for this equation. Option (c): Doesn't isolate $x$. Option (d): Assumes wrong operation order.

13. **Answer**: (b) Non-Abelian group
**Explanation**: $2 \times 2$ non-singular matrices are closed (product determinant non-zero), associative (matrix multiplication), have identity ($I_2$), and inverses (matrix inverses). They are non-Abelian since $A \cdot B \neq B \cdot A$. Option (a): Not commutative. Option (c): More than a semigroup. Option (d): More than a monoid.

14. **Answer**: (c) Lacks commutativity
**Explanation**: The matrix group is non-Abelian because matrix multiplication is not commutative (e.g., try $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$). Option (a): Closure holds. Option (b): Associativity holds. Option (d): Inverses exist.

15. **Answer**: (c) $\omega^2$
**Explanation**: The inverse of $\omega$ satisfies $\omega \cdot x = 1$. From the table, $\omega \cdot \omega^2 = 1$, so $\omega^{-1} = \omega^2$. Option (a): $\omega \cdot 1 = \omega$. Option (b): $\omega \cdot \omega = \omega^2$. Option (d): An inverse exists.

16. **Answer**: (a) Yes
**Explanation**: The set is closed (all products in set), associative (multiplication), has identity (1), inverses ($\omega^{-1} = \omega^2$), and is commutative (table symmetric, e.g., $\omega \cdot \omega^2 = \omega^2 \cdot \omega$). Option (b): Closure holds. Option (c): Associativity holds. Option (d): Commutativity holds.

17. **Answer**: (b) Abelian group
**Explanation**: The set is closed ($(a + \sqrt{3}b) + (c + \sqrt{3}d) \in \mathbb{Q}$), associative, has identity ($0 + \sqrt{3}0$), inverses ($-(a + \sqrt{3}b)$), and is commutative ($(a + \sqrt{3}b) + (c + \sqrt{3}d) = (c + \sqrt{3}d) + (a + \sqrt{3}b)$). Option (a): It's commutative. Option (c): More than a semigroup. Option (d): More than a monoid.

18. **Answer**: (b) $0+\sqrt{3}0$
**Explanation**: The identity satisfies $(a+\sqrt{3}b)+e=a+\sqrt{3}b$. Since $(a+\sqrt{3}b)+(0+\sqrt{3}0)=a+\sqrt{3}b$, $0+\sqrt{3}0$ is the identity. Option (a): Adds non-zero terms. Option (c): Adds $\sqrt{3}$. Option (d): Identity exists.

19. **Answer**: (a) $E$
**Explanation**: The inverse of $E$ satisfies $E\oplus x=E$. From the table, $E\oplus E=E$, so $E^{-1}=E$. Option (b): $E\oplus O=O\neq E$. Option (c): Only one inverse exists. Option (d): Inverse exists.

20. **Answer**: (a) Yes
**Explanation**: The set is closed (all entries in $\{E,O\}$), associative (check $(E\oplus O)\oplus O=E\oplus(O\oplus O)$), has identity ($E$), inverses ($E^{-1}=E$, $O^{-1}=O$), and is commutative ($E\oplus O=O\oplus E$). Option (b): Inverses exist. Option (c): Commutative. Option (d): Identity exists.