Arithmetic Means Cheatsheet - Exercise 6.3 (Class 11 Mathematics)

Prepared for Entry Test Preparation

1. Concept of Arithmetic Mean (A.M.)

A number A is the arithmetic mean between two numbers a and b if a, A, b form an arithmetic progression (A.P.), i.e., $A - a = b - A$. This implies:

$$A = \frac{a+b}{2}$$

In general, for a sequence $a_1, a_2, \ldots, a_n$, the k-th term is the A.M. between a_{k-1} and a_{k+1}:

$$a_k = \frac{a_{k-1} + a_{k+1}}{2}$$

2. Inserting Multiple A.M.s

To insert n A.M.s $A_1, A_2, \ldots, A_n$ between a and b, the sequence $a, A_1, A_2, \ldots, A_n, b$ forms an A.P. with: - First term: $a_1 = a$ - $(n+2)$-th term: $a_{n+2} = b$ - Number of terms: $n+2$ The common difference d is:

$$d = \frac{b-a}{n+1}$$

The A.M.s are:

$$A_1 = a + d, \quad A_2 = a + 2d, \quad \ldots, \quad A_n = a + nd$$

3. Key Formulas for Exercise 6.3

- **Single A.M.**: $A = \frac{a+b}{2}$.
- **Common Difference for n A.M.s**: $d = \frac{b-a}{n+1}$.
- **k-th A.M.**: $A_k = a + k \cdot \frac{b-a}{n+1}$.
- **Sum of n A.M.s**: The sum of n A.M.s between a and b is:

$$n \cdot \frac{a+b}{2}$$

- **A.M. of Powers**: If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A.M. between a and b, then $n = 1$.

4. Examples from Exercise 6.3

Single A.M.

Problem: Find the A.M. between $3\sqrt{5}$ and $5\sqrt{5}$.

- Compute: $A = \frac{3\sqrt{5}+5\sqrt{5}}{2} = \frac{8\sqrt{5}}{2} = 4\sqrt{5}$.

Multiple A.M.s

Problem: Insert 6 A.M.s between 2 and 5.

- Sequence: $2, A_1, A_2, \ldots, A_6, 5$. Here, $a_1 = 2$, $a_8 = 5$, $n = 6$.
- Common difference: $d = \frac{5-2}{6+1} = \frac{3}{7}$.
- A.M.s: $A_1 = 2 + \frac{3}{7} = \frac{17}{7}$, $A_2 = \frac{20}{7}$, $A_3 = \frac{23}{7}$, $A_4 = \frac{26}{7}$, $A_5 = \frac{29}{7}$, $A_6 = \frac{32}{7}$.

Sum of A.M.s

Problem: Show the sum of n A.M.s between a and b equals n times their A.M.

- Proof: Sum = $A_1 + A_2 + \cdots + A_n = na + \frac{b-a}{n+1} \cdot \frac{n(n+1)}{2} = n \cdot \frac{a+b}{2}$.

A.M. of Powers

Problem: Find n such that $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A.M. between a and b.

- Solve: $\frac{a^n+b^n}{a^{n-1}+b^{n-1}} = \frac{a+b}{2} \implies n = 1$.