# Trigonometric Functions Cheatsheet - Exercise 11.2

## 1. Graphing Trigonometric Functions

### 1.1 Key Properties

| Function | Period | Range | Asymptotes |
|---|---|---|---|
| $y = \sin x$ | $2\pi$ | $[-1, 1]$ | None |
| $y = \cos x$ | $2\pi$ | $[-1, 1]$ | None |
| $y = \tan x$ | $\pi$ | $R$ | $x = (2n+1)\frac{\pi}{2}, n \in Z$ |

### 1.2 Transformations

- **Negation (e.g., $-\sin x$)**: Reflects graph across x-axis. - **Amplitude (e.g., $a \sin x$)**: Scales range to $[-|a|, |a|]$. - **Period Change (e.g., $\sin kx$)**: Period $= \frac{2\pi}{|k|}$ for sin, cos; $\frac{\pi}{|k|}$ for tan. - **Example**: $y = 2\cos x$ has amplitude 2, range $[-2, 2]$, period $2\pi$. - **Example**: $y = \tan 2x$ has period $\frac{\pi}{2}$, asymptotes at $2x = (2n+1)\frac{\pi}{2}$.



## 2. Plotting Graphs

**Steps:** 1. Identify function, period, and range. 2. Choose interval (e.g., $[0, 2\pi]$, $[-\pi, \pi]$) and sub-interval (e.g., $30^\circ$ or $\frac{\pi}{6}$). 3. Compute values using a table (e.g., $\sin x$, $\cos x$ at $0^\circ, 30^\circ, \ldots$). 4. Plot points and connect for sin, cos; note asymptotes for tan. 5. Use scale: 0.1 = small square, 1 = big square, $10^\circ$ = small square.

**Example:** $y = -\sin x$, $x \in [-2\pi, 2\pi]$ - Period: $2\pi$, Range: $[-1, 1]$. - Table (at $30^\circ$ intervals):

| $x$ | $0^\circ$ | $90^\circ$ | $180^\circ$ | $270^\circ$ |
|---|---|---|---|---|
| $y = -\sin x$ | 0 | $-1$ | 0 | 1 |

- Reflect $\sin x$ across x-axis.

## 3. Period and Range Adjustments

$$\sin kx, \cos kx : \text{Period} = \frac{2\pi}{|k|}, \text{Range} = [-1, 1]$$

$$a \sin kx, a \cos kx : \text{Period} = \frac{2\pi}{|k|}, \text{Range} = [-|a|, |a|]$$

$$\tan kx : \text{Period} = \frac{\pi}{|k|}, \text{Range} = R, \text{Asymptotes at } kx = (2n+1)\frac{\pi}{2}$$

**Examples:** - $y = \sin \frac{x}{2}$: Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$, Range $= [-1, 1]$. - $y = 2\cos x$: Period $= 2\pi$, Range $= [-2, 2]$. - $y = \tan 2x$: Period $= \frac{\pi}{2}$, Asymptotes at $x = \frac{(2n+1)\pi}{4}$.

# 4. Graphical Solutions

Solve equations like $\sin x = \cos x$ or $\sin x = x$ by finding intersections. - Plot both functions on the same axes. - Identify intersection points within the interval. - **Example**: Solve $\sin x = \cos x$, $x \in [0, \pi]$. - Table:

| $x$ | $0°$ | $45°$ | $90°$ | $180°$ |
|---|---|---|---|---|
| $\sin x$ | 0 | 0.707 | 1 | 0 |
| $\cos x$ | 1 | 0.707 | 0 | $-1$ |

- Intersection at $x = 45° = \frac{\pi}{4}$ (since $\sin \frac{\pi}{4} = \cos \frac{\pi}{4}$). - **Note**: For $\sin x = x$, convert $x$ to radians (e.g., $30° = \frac{\pi}{6} \approx 0.5236$).

# 5. Tips and Tricks

- Use $30°$ intervals for smooth curves; adjust for compressed periods (e.g., $\sin 2x$).
- For $\tan kx$, mark asymptotes at $kx = (2n + 1)\frac{\pi}{2}$.
- Constant coefficients (e.g., $2 \cos x$) affect amplitude, not period.
- Convert degrees to radians for equations involving linear functions (e.g., $y = x$).
- Plot multiple functions on the same axes to compare periods and shapes.



# 6. Applications

- **Physics**: Graphs model wave motion and oscillations.
- **Engineering**: Used in signal processing and circuit design.
- **Mathematics**: Graphical solutions approximate roots of complex equations.