Exercise 1.3: Complex Numbers & Related Concepts Cheat Sheet

1. Argand (Complex) Plane Representation

• Any complex number $z = a + bi$ corresponds to the point (a, b) on the plane.
 – Horizontal axis (x) is the real part, a.
 – Vertical axis (y) is the imaginary part, b.
• Plotting steps:
 1. Move a units right (if $a>0$) or left (if $a<0$).
 2. Move b units up (if $b>0$) or down (if $b<0$).
 3. Mark the point.

2. Multiplicative Inverse (Reciprocal)

For $z = a + bi$, its inverse $z^{-1} = 1/z$ is:
 $z^{-1} = (a/(a^2 + b^2)) + (-b/(a^2 + b^2))\, i$
Examples:
 • $z = -3i \rightarrow a=0, b=-3 \rightarrow z^{-1} = (0/(0+9)) + (3/9)i = 1/3\, i$
 • $z = 1 - 2i \rightarrow a=1, b=-2 \rightarrow z^{-1} = (1/5) + (2/5)i$

3. Powers of i (Imaginary Unit)

$i^{-1} = -i$ (since $1/i = -i$)
$i^0 = 1$
$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$
...and repeats every 4:
 $i^n = i^{(n \bmod 4)}$

4. Conjugate and Reality Conditions

• Conjugate of $z = a + bi$ is $\bar{z} = a - bi$.
• $z = \bar{z} \Leftrightarrow b = 0 \Leftrightarrow z$ is real.
• $z + \bar{z} = 2a$ (always real).
• $(z - \bar{z})^2 = (2bi)^2 = -4b^2$ (always real).

5. Simplifying √(−k) and Binomial Products

- $\sqrt{(-k)} = i\sqrt{k}$. Example: $5 + 2\sqrt{(-4)} = 5 + 2(2i) = 5 + 4i$.
- Multiply conjugate pairs: $(2+\sqrt{-3})(3+\sqrt{-3})$

  $= 6 + 2\sqrt{-3} + 3\sqrt{-3} + (\sqrt{-3})^2$

  $= 6 + 5i\sqrt{3} - 3 = 3 + 5i\sqrt{3}$

6. Rationalizing Denominators

- To remove √ from denominator, multiply numerator and denominator by conjugate.

Examples:

$2/(\sqrt{5}+\sqrt{-8})$: multiply by $(\sqrt{5}-\sqrt{-8})/(\sqrt{5}-\sqrt{-8})$.

$3/(\sqrt{6}-\sqrt{12})$: multiply by $(\sqrt{6}+\sqrt{12})/(\sqrt{6}+\sqrt{12})$.

7. Real Combinations

- $z^2 + \bar{z}^2 = (a+bi)^2 + (a-bi)^2 = 2(a^2-b^2)$ (real).
- $(z - \bar{z})^2 = (2bi)^2 = -4b^2$ (real).

8. Cube of Complex Binomials

Use the expansion: $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.

Examples:

$(-\tfrac{1}{2} + (\sqrt{3}/2)i)^3 = 1$

$(-\tfrac{1}{2} - (\sqrt{3}/2)i)^3 = 1$