Permutation and Combination Cheatsheet - Class 11 Mathematics

Prepared for Entry Test Preparation

1. Permutations of Objects Not All Distinct

For n objects with n_1 alike of one kind, n_2 alike of another, etc., the number of permutations taken all at a time is:

$$\frac{n!}{n_1!n_2!\cdots n_k!} = \binom{n}{n_1, n_2, \ldots, n_k}$$

where $n = n_1 + n_2 + \cdots + n_k$.

Key Concepts

- **Word Arrangements**: Arrange letters of a word with repeated letters, e.g., ”PAKPATTAN” ($n = 9, n_1 = 3(\text{A}), n_2 = 2(\text{P}), n_3 = 2(\text{T})$).
- **Constrained Arrangements**: Fix specific letters at the start or end, reducing n for remaining letters.
- **Grouped Objects**: Treat objects of the same type (e.g., books by subject) as a single unit if they must stay together.

Examples

1. **Arrange letters of ”PAKISTAN”**:

$$n = 8, n_1 = 2(\text{A}) \implies \frac{8!}{2!} = \frac{40320}{2} = 20160$$

2. **Arrange ”ATTACKED” with C at start, K at end**:

$$n = 6, n_1 = 2(\text{A}), n_2 = 2(\text{T}) \implies \frac{6!}{2!2!} = \frac{720}{4} = 180$$

3. **3 English, 5 Urdu books, same subjects together**:

$$\text{Forms: EEEEEUUU or UUUEEEEE} \implies 5! \cdot 3! + 3! \cdot 5! = 720 + 720 = 1440$$

2. Circular Permutations

The number of ways to arrange n distinct objects in a circle is:

$$(n-1)!$$

For necklaces (where rotations and reflections are identical):

$$\frac{1}{2} \cdot (n-1)!$$

If k objects are treated as one unit (e.g., sitting together), reduce n to $n - k + 1$.

Key Concepts

- **Circular Arrangements**: Fix one object to account for rotational symmetry.
- **Grouped Objects**: Treat objects that must stay together as a single unit.
- **Necklaces/Bracelets**: Divide by 2 for reflectional symmetry.
- **Alternate Seating**: Arrange two groups (e.g., men and women) alternately in a circle.

Examples

1. **12 officers at a round table**:

$$(12-1)! = 11! = 39916800$$

2. **6 beads in a necklace**:

$$\frac{1}{2} \cdot 5! = \frac{120}{2} = 60$$

3. **5 men, 5 women, no same-sex neighbors**:

Fix one man, alternate: $5! \cdot 4! = 120 \cdot 24 = 2880$

3. Combinations

The number of ways to choose r objects from n distinct objects, where order does not matter, is:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}, \quad r \leq n$$

Property: $\binom{n}{r} = \binom{n}{n-r}$.

Key Concepts

- **Selection**: Choose groups without regard to order, e.g., forming committees.
- **Partitioning**: Divide n objects into groups with $n_1, n_2, \ldots$ members:

$$\binom{n}{n_1, n_2, \ldots, n_k} = \frac{n!}{n_1! n_2! \cdots n_k!}$$

- **Applications**: Use in committee formation, probability, and partitioning problems.

Examples

1. **Form 4 committees (3, 4, 2, 2 members) from 11 people**:

$$\binom{11}{4,3,2,2} = \frac{11!}{4!3!2!2!} = 69300$$

2. **Choose 3 books from 5**:

$$\binom{5}{3} = \frac{5!}{3!2!} = 10$$