Arithmetic Series Cheatsheet - Exercises 6.4 and 6.5 (Class 11 Mathematics)

Prepared for Entry Test Preparation

1. Arithmetic Series Basics (Ex. 6.4)

A series is the sum of terms in a sequence. For an arithmetic progression (A.P.) with first term a_1, common difference d, and n terms, the sum S_n is:

$$S_n = \frac{n}{2}[2a_1 + (n-1)d] \quad \text{or} \quad S_n = \frac{n}{2}(a_1 + a_n)$$

where $a_n = a_1 + (n-1)d$.

2. Key Formulas for Exercise 6.4

- **Sum of A.P.**: $S_n = \frac{n}{2}[2a_1 + (n-1)d]$.
- **n-th Term**: $a_n = a_1 + (n-1)d$.
- **Number of Terms**: If S_n is given, solve $S_n = \frac{n}{2}[2a_1 + (n-1)d]$ for n.
- **Ratio of Sums**: For two A.P.s, if $\frac{S_n}{S'_n} = \frac{pn+q}{rn+s}$, the ratio of their m-th terms is found by substituting $n = 2m-1$.
- **Sum of Special Series**: For series like $a_r = kr + c$, compute a_1, d, then use S_n.
- **Numbers in A.P.**: For k numbers in A.P. with sum S and product P, solve for a and d using sum and product equations.

3. Key Concepts for Exercise 6.5

Exercise 6.5 applies A.P. to real-world problems, such as financial deposits, physical arrangements, and motion. Key approaches:

- **Installments/Deposits**: Model as A.P. with increasing/decreasing terms, find S_n or a_1.
- **Physical Arrangements**: Triangular or layered patterns form A.P.s; compute a_n or S_n.
- **Motion**: Distances in equal time intervals may form A.P.; find a_n or S_n.
- **Sum of Angles**: Polygonal angles form A.P.; use $a_n = a_1 + (n-1)d$.

4. Examples from Exercises 6.4 and 6.5

Sum of Multiples (Ex. 6.4, Q1)

Problem: Sum multiples of 3 between 4 and 97.

- Sequence: $6, 9, \ldots, 96$. $a_1 = 6$, $d = 3$, $a_n = 96$.
- Find n: $96 = 6 + (n-1) \cdot 3 \implies n = 31$.
- Sum: $S_{31} = \frac{31}{2}[2 \cdot 6 + (31-1) \cdot 3] = 1581$.

Complex Series (Ex. 6.4, Q4(i))

Problem: Sum $3 + 5 - 7 + 9 + 11 - 13 + \ldots$ to $3n$ terms.

- Group: $(3 + 5 - 7) + (9 + 11 - 13) + \ldots = 1 + 7 + 13 + \ldots$.
- A.P.: $a_1 = 1$, $d = 6$. Sum: $S_n = n(3n - 2)$.

Numbers in A.P. (Ex. 6.4, Q14)

Problem: Three numbers in A.P. with sum 24, product 440.

- Numbers: $a - d, a, a + d$. Sum: $3a = 24 \implies a = 8$.
- Product: $a(a^2 - d^2) = 440 \implies d = \pm 3$. Numbers: $5, 8, 11$.

Deposits (Ex. 6.5, Q1)

Problem: Deposits $10, 15, 20, \ldots$ for 9 months.

- A.P.: $a_1 = 10$, $d = 5$, $n = 9$.
- Sum: $S_9 = \frac{9}{2}[2 \cdot 10 + (9-1) \cdot 5] = 270$.

Motion (Ex. 6.5, Q6)

Problem: Object falls 9m, 27m, 45m, ...

- A.P.: $a_1 = 9$, $d = 18$. Fifth term: $a_5 = 9 + 4 \cdot 18 = 81$.
- Sum to 5 seconds: $S_5 = \frac{5}{2}[2 \cdot 9 + 4 \cdot 18] = 225$.