Oblique Triangles Cheatsheet
Exercise 12.4

1 Oblique Triangle Fundamentals

1.1 Definition and Notation

An oblique triangle has no right angle ($\alpha + \beta + \gamma \neq 90°$). In $\triangle ABC$:

- Angles: α (at A), β (at B), γ (at C).
- Sides: a (opposite α), b (opposite β), c (opposite γ).

1.2 Key Formulas

- **Angle Sum**: $\alpha + \beta + \gamma = 180°$.
- **Law of Sines**:

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

2 Solving Oblique Triangles

2.1 Steps (Given Two Angles and One Side)

1. Find the third angle: $\alpha = 180° - \beta - \gamma$.
2. Use the Law of Sines to find remaining sides:

$$\text{If given } b: \quad a = b \cdot \frac{\sin \alpha}{\sin \beta}, \quad c = b \cdot \frac{\sin \gamma}{\sin \beta}$$

$$\text{If given } a: \quad b = a \cdot \frac{\sin \beta}{\sin \alpha}, \quad c = a \cdot \frac{\sin \gamma}{\sin \alpha}$$

3. Verify using another ratio or sum of angles.
4. Report angles in degrees and minutes, sides exact or to two decimal places.

2.2 Example

Given: $\beta = 60°, \gamma = 15°, b = \sqrt{6}$.

- Find α:

$$\alpha = 180° - 60° - 15° = 105°$$

- Find a:

$$\frac{a}{\sin 105°} = \frac{\sqrt{6}}{\sin 60°} \implies a = \sqrt{6} \cdot \frac{\sin 105°}{\sin 60°} \approx \sqrt{6} \cdot \frac{0.9659}{0.8660} \approx 2.73$$

- Find c:

$$\frac{c}{\sin 15^\circ} = \frac{\sqrt{6}}{\sin 60^\circ} \implies c = \sqrt{6} \cdot \frac{\sin 15^\circ}{\sin 60^\circ} \approx \sqrt{6} \cdot \frac{0.2588}{0.8660} \approx 0.73$$

3 Common Trigonometric Values

Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

For non-standard angles (e.g., $89^\circ 35'$), use trigonometric tables or calculators.

4 Problem Types

- **Two Angles, One Side**: Find third angle, then other sides.

$$\text{E.g., } \beta = 60^\circ, \gamma = 15^\circ, b = \sqrt{6} \implies \alpha = 105^\circ, a \approx 2.73, c \approx 0.73.$$

- **Variations**: Side given can be a, b, or c; angles in degrees or degrees/minutes.

$$\text{E.g., } a = 89.35, \beta = 52^\circ, \gamma = 89^\circ 35' \implies \alpha = 38^\circ 25', b \approx 113.18, c \approx 143.79.$$

5 Tips and Tricks

- Ensure $\alpha + \beta + \gamma = 180^\circ$ before applying Law of Sines.
- Use exact values for standard angles $(30^\circ, 45^\circ, 60^\circ)$.
- Convert minutes to decimals for calculations: $\theta^\circ m' = \theta + \frac{m}{60}$.
- Round sides to two decimal places unless exact (e.g., $\sqrt{6}$).
- Verify results using alternative ratios (e.g., $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$).

6 Applications

- Surveying: Calculate distances/angles in irregular land shapes.
- Navigation: Determine bearings or distances in triangulation.
- Engineering: Analyze forces or structures with non-right angles.