Set Theory Exercise 2.2 Cheatsheet

Set Theory Concepts (Exercise 2.2)

This cheatsheet summarizes key set theory concepts from Exercise 2.2 with definitions and examples for quick reference.

1. **Equivalent Sets**
   - *Definition*: Sets with the same number of elements (cardinality).
   - *Example*: A = {a, b, c}, B = {1, 2, 3}. Both have 3 elements, so they are equivalent.
2. **Equal Sets**
   - *Definition*: Sets with exactly the same elements, regardless of order.
   - *Example*: A = {0, 1, 2, 3}, B = {3, 2, 1, 0}. A = B since they have the same elements.
3. **Venn Diagrams**
   - *Definition*: Diagrams with a rectangle (universal set) and circles (subsets) to show set operations.
   - *Example*: A = {1, 2}, B = {2, 3}, U = {1, 2, 3, 4}. A ∪ B shades both circles; A ∩ B shades the overlap.
4. **Union of Sets (A ∪ B)**
   - *Definition*: All elements in A, B, or both (no duplicates).
   - *Example*: A = {1, 2}, B = {2, 3}. A ∪ B = {1, 2, 3}.
5. **Intersection of Sets (A ∩ B)**
   - *Definition*: Elements common to both A and B.
   - *Example*: A = {1, 2}, B = {2, 3}. A ∩ B = {2}.
6. **Set Difference (A - B)**
   - *Definition*: Elements in A that are not in B.
   - *Example*: A = {1, 2, 3}, B = {2, 4}. A - B = {1, 3}.
7. **Complement of a Set (A')**
   - *Definition*: All elements in universal set U not in A.

- *Example*: U = {1, 2, 3, 4}, A = {1, 2}. A’ = {3, 4}.

8. **Disjoint Sets**
   - *Definition*: Sets with no common elements ($A \cap B = \emptyset$).
   - *Example*: A = {1, 2}, B = {3, 4}. $A \cap B = \emptyset$.
9. **Overlapping Sets**
   - *Definition*: Sets with at least one common element ($A \cap B \neq \emptyset$).
   - *Example*: A = {1, 2}, B = {2, 3}. $A \cap B = \{2\}$.
10. **Subset ($A \subseteq B$)**
    - *Definition*: Every element of A is in B.
    - *Example*: A = {1, 2}, B = {1, 2, 3}. $A \subseteq B$.
11. **Empty Set ($\emptyset$)**
    - *Definition*: A set with no elements, denoted $\emptyset$ or {}.
    - *Example*: $\{x \mid x \in \mathbb{N} \wedge x + 5 = 3\} = \emptyset$ (no solution in $\mathbb{N}$).
12. **Universal Set (U)**
    - *Definition*: The set containing all elements under consideration.
    - *Example*: For numbers 1 to 10, U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
13. **Commutative Property of Union ($A \cup B = B \cup A$)**
    - *Definition*: The order of sets in a union does not affect the result.
    - *Example*: A = {1, 2}, B = {2, 3}. $A \cup B = \{1, 2, 3\} = B \cup A$.
14. **Commutative Property of Intersection ($A \cap B = B \cap A$)**
    - *Definition*: The order of sets in an intersection does not affect the result.
    - *Example*: A = {1, 2}, B = {2, 3}. $A \cap B = \{2\} = B \cap A$.
15. **Associative Property of Union ($A \cup (B \cup C) = (A \cup B) \cup C$)**
    - *Definition*: The grouping of sets in a union does not affect the result.
    - *Example*: A = {1}, B = {2}, C = {3}. $A \cup (B \cup C) = \{1, 2, 3\} = (A \cup B) \cup C$.
16. **Associative Property of Intersection ($A \cap (B \cap C) = (A \cap B) \cap C$)**
    - *Definition*: The grouping of sets in an intersection does not affect the result.
    - *Example*: A = {1, 2}, B = {2, 3}, C = {2, 4}. $A \cap (B \cap C) = \{2\} = (A \cap B) \cap C$.
17. **Distributive Property of Union over Intersection**
    - *Definition*: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

- *Example*: A = $\{1\}$, B = $\{2, 3\}$, C = $\{3, 4\}$. A $\cup$ (B $\cap$ C) = $\{1, 3\}$ = (A $\cup$ B) $\cap$ (A $\cup$ C).

18. Distributive Property of Intersection over Union

- *Definition*: A $\cap$ (B $\cup$ C) = (A $\cap$ B) $\cup$ (A $\cap$ C).
- *Example*: A = $\{1, 2\}$, B = $\{2, 3\}$, C = $\{2, 4\}$. A $\cap$ (B $\cup$ C) = $\{2\}$ = (A $\cap$ B) $\cup$ (A $\cap$ C).

19. De Morgan's Laws

- *Definition*: (A $\cup$ B)' = A' $\cap$ B'; (A $\cap$ B)' = A' $\cup$ B'.
- *Example*: U = $\{1, 2, 3, 4\}$, A = $\{1, 2\}$, B = $\{2, 3\}$. (A $\cup$ B)' = $\{4\}$ = A' $\cap$ B'; (A $\cap$ B)' = $\{1, 3, 4\}$ = A' $\cup$ B'.

20. Cardinality of Union for Disjoint Sets

- *Definition*: If A $\cap$ B = $\emptyset$, then n(A $\cup$ B) = n(A) + n(B).
- *Example*: A = $\{1, 2\}$, B = $\{3, 4\}$, A $\cap$ B = $\emptyset$. n(A $\cup$ B) = 4 = 2 + 2.

21. Cardinality of Intersection (n(A $\cap$ B) = n(A) if A $\subseteq$ B)

- *Definition*: If A $\subseteq$ B, then n(A $\cap$ B) = n(A).
- *Example*: A = $\{1, 2\}$, B = $\{1, 2, 3\}$, A $\subseteq$ B. n(A $\cap$ B) = 2 = n(A).

22. Set Difference Property (A - B = A if A $\cap$ B = $\emptyset$ or B = $\emptyset$)

- *Definition*: If A and B are disjoint or B is empty, A - B = A.
- *Example*: A = $\{1, 2\}$, B = $\{3, 4\}$, A $\cap$ B = $\emptyset$. A - B = $\{1, 2\}$ = A.

23. Complement of Universal Set (U' = $\emptyset$)

- *Definition*: The complement of the universal set is the empty set.
- *Example*: U = $\{1, 2, 3\}$. U' = $\emptyset$.

24. Intersection Property (A $\cap$ B = B if B $\subseteq$ A)

- *Definition*: If B $\subseteq$ A, then A $\cap$ B = B.
- *Example*: A = $\{1, 2, 3\}$, B = $\{1, 2\}$, B $\subseteq$ A. A $\cap$ B = $\{1, 2\}$ = B.

25. Union with Complement (A $\cup$ B = U if A = B')

- *Definition*: If A is the complement of B, their union is the universal set.
- *Example*: U = $\{1, 2, 3\}$, B = $\{1, 2\}$, A = B' = $\{3\}$. A $\cup$ B = $\{1, 2, 3\}$ = U.