Trigonometry Cheatsheet - Exercise 9.3

1. Verifying Trigonometric Identities

1.1 Angle Sum and Difference Identities

Use identities to verify expressions involving multiple angles:

- $\sin(A - B) = \sin A \cos B - \cos A \sin B$

Example: Verify $\sin 60° \cos 30° - \cos 60° \sin 30° = \sin 30°$.

$$\begin{aligned}
\text{LHS} &= \sin 60° \cos 30° - \cos 60° \sin 30° \\
&= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \\
\text{RHS} &= \sin 30° = \frac{1}{2} \\
\text{LHS} &= \text{RHS}
\end{aligned}$$

1.2 Pythagorean Identities

Use to simplify or verify expressions:

- $\sin^2 \theta + \cos^2 \theta = 1$

Example: Verify $\sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4} = 2$.

$$\begin{aligned}
\text{LHS} &= \left(\sin \frac{\pi}{6}\right)^2 + \left(\sin \frac{\pi}{3}\right)^2 + \left(\tan \frac{\pi}{4}\right)^2 \\
&= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2 = \frac{1}{4} + \frac{3}{4} + 1 = 2 \\
\text{RHS} &= 2
\end{aligned}$$

2. Evaluating Trigonometric Expressions

2.1 Tangent Difference Formula

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

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Example: Evaluate $\frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}}$.

$$\begin{aligned}
&= \frac{\tan 60^\circ - \tan 30^\circ}{1+\tan 60^\circ \tan 30^\circ} = \frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\cdot\frac{1}{\sqrt{3}}} \\
&= \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{2}{\sqrt{3}}\cdot\frac{1}{2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}
\end{aligned}$$

2.2 Double Angle Identities

Use to verify or evaluate expressions:

- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
- $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Example: Verify $\sin 2\theta = 2\sin\theta\cos\theta$ for $\theta = 30^\circ$.

$$\begin{aligned}
&\text{LHS} = \sin 60^\circ = \frac{\sqrt{3}}{2} \\
&\text{RHS} = 2\sin 30^\circ\cos 30^\circ = 2\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \\
&\text{LHS} = \text{RHS}
\end{aligned}$$

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3. Solving Trigonometric Equations

Solve for variables in equations involving trigonometric functions. **Example:** Find x if $\tan^2 45^\circ - \cos^2 60^\circ = x\sin 45^\circ\cos 45^\circ\tan 60^\circ$.

$$\begin{aligned}
\text{LHS} &= (1)^2 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4} \\
\text{RHS} &= x\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\cdot\sqrt{3} = x\cdot\frac{\sqrt{3}}{2} \\
\frac{3}{4} &= x\cdot\frac{\sqrt{3}}{2} \Rightarrow x = \frac{3}{4}\cdot\frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{2}
\end{aligned}$$

4. Quadrantal Angles

Evaluate trigonometric functions at angles that are multiples of $\frac{\pi}{2}$ or 90°.

- Use coterminal angles: $\theta = 2k\pi + \alpha$ (radians) or $\theta = k\cdot 360^\circ + \alpha$ (degrees).
- Common values:

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	1	0	-1	0
$\cos\theta$	1	0	-1	0	1
$\tan\theta$	0	undef.	0	undef.	0

Example: Find trigonometric functions for $-\pi$.

$$-\pi = -2\pi + \pi = \pi$$
$$\sin(-\pi) = \sin\pi = 0, \quad \cos(-\pi) = \cos\pi = -1, \quad \tan(-\pi) = \tan\pi = 0$$

5. Non-Quadrantal Angles

Reduce large or negative angles to coterminal angles in $[0, 2\pi)$ or $[0^\circ, 360^\circ)$.

- For radians: $\theta = 2k\pi + \alpha$
- For degrees: $\theta = k \cdot 360^\circ + \alpha$

Example: Find trigonometric functions for 390°.

$$390^\circ = 360^\circ + 30^\circ$$
$$\sin 390^\circ = \sin 30^\circ = \frac{1}{2}, \quad \cos 390^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \tan 390^\circ = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

6. Applications

- **Physics:** Use double-angle identities in harmonic motion.
- **Engineering:** Apply angle difference formulas in signal processing.

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