Oblique Triangles MCQs for Entry Test - Exercise 12.6

1 Introduction

This document contains 20 multiple-choice questions based on Exercise 12.6 of the Application of Trigonometry chapter, designed for entry test preparation. Questions test solving oblique triangles using half-angle formulas and the Law of Cosines when all three sides are given, covering all angles, smallest/greatest angles, and specific angle proofs. Solutions provide detailed explanations. Notation: In $\triangle ABC$, angles α, β, γ are opposite sides a, b, c. Angles are in degrees and minutes, sides exact or to two decimal places.

2 Multiple-Choice Questions

- 1. In $\triangle ABC$, $a=7,\,b=7,\,c=9$. What is α ?
 - A) 50°
 - B) 80°
 - $C) 90^{\circ}$
 - D) 120°
- 2. In $\triangle ABC$, a = 7, b = 7, c = 9. What is γ ?
 - $A) 50^{\circ}$
 - $B) 80^{\circ}$
 - C) 90°
 - D) 120°
- 3. In $\triangle ABC$, a = 32, b = 40, c = 66. What is α ?
 - A) 20°56′
 - B) $26^{\circ}30'$
 - C) 132°34′
 - D) 150°
- 4. In $\triangle ABC$, a = 32, b = 40, c = 66. What is β ?
 - A) 20°56′
 - B) $26^{\circ}30'$
 - C) 132°34′

| D) 150° |
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| 5. In $\triangle ABC$, $a = 28.3$, $b = 31.7$, $c = 42.8$. What is α ? |
| A) 41°23′ |
| B) 47°46′ |
| C) 90°51′ |
| D) 100° |
| 6. In $\triangle ABC$, $a = 28.3$, $b = 31.7$, $c = 42.8$. What is γ ? |
| A) 41°23′ |
| B) 47°46′ C) 90°51′ |
| D) 100° |
| 7. In $\triangle ABC$, $a = 31.9$, $b = 56.31$, $c = 40.27$. What is β ? |
| A) 30°43′ |
| B) 33°39′ |
| C) 115°38′ |
| D) 150° |
| 8. In $\triangle ABC$, $a = 31.9$, $b = 56.31$, $c = 40.27$. What is γ ? |
| A) 30°43′ |
| B) 33°39′ |
| C) 115°38′ |
| D) 150° |
| 9. In $\triangle ABC$, $a = 4584$, $b = 5140$, $c = 3624$. What is α ? |
| A) 43°17′ |
| B) 60°9′ |
| C) 76°34′ D) 90° |
| 10. In $\triangle ABC$, $a = 4584$, $b = 5140$, $c = 3624$. What is γ ? |
| A) 43°17′ |
| B) 60°9′ |
| C) 76°34′ |
| D) 90° |
| 11. In $\triangle ABC$, $a = 37.34$, $b = 3.24$, $c = 35.06$. What is the smallest angle? |
| A) 3°39′ |

| B) 35°6′ |
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| C) 37°34′ |
| D) 90° |
| 12. In $\triangle ABC$, $a=16$, $b=20$, $c=23$. What is the greatest angle? |
| A) 30° |
| B) 60° |
| C) 84°18′ |
| D) 132°35′ |
| 13. In $\triangle ABC$, sides are $x^2 + x + 1$, $2x + 1$, $x^2 - 1$. What is α ? |
| A) 60° |
| B) 90° |
| C) 120° |
| D) 150° |
| 14. In $\triangle ABC$, $a = 413$, $b = 214$, $c = 375$. What is α ? |
| A) 31°2′ |
| B) 64°38′ |
| C) 84°20′ |
| D) 90° |
| 15. In $\triangle ABC$, $a = 413$, $b = 214$, $c = 375$. What is β ? |
| A) 31°2′ |
| B) 64°38′ |
| C) 84°20′ |
| D) 90° |
| 16. In $\triangle ABC$, $a = 6$, $b = 9$, $c = 13$. What is α ? |
| A) 23°51′ |
| B) 37°21′ |
| C) 118°46′ |
| D) 150° |
| 17. In $\triangle ABC$, $a = 6$, $b = 9$, $c = 13$. What is γ ? |
| A) 23°51′ |
| B) 37°21′ |
| C) 118°46′ |
| D) 150° |

18. In $\triangle ABC$, a = 10, b = 12, c = 15. What is the smallest angle?

- A) 36°52′
- B) 45°
- C) 80°36′
- D) 90°

19. In $\triangle ABC$, a = 8, b = 15, c = 10. What is the greatest angle?

- A) 30°
- B) 48°36′
- C) 97°36′
- D) 120°

20. In $\triangle ABC$, a = 25, b = 30, c = 40. What is β ?

- A) 36°15′
- B) 50°28′
- C) 93°17′
- D) 120°

3 Solutions and Explanations

1. Question 1:

$$S = \frac{7+7+9}{2} = 11.5, \quad S-a = 4.5, \quad \cos\frac{\alpha}{2} = \sqrt{\frac{11.5 \cdot 4.5}{7 \cdot 9}} \approx 0.9063, \quad \alpha \approx 2\cos^{-1}(0.9063) \approx 50$$

Answer: A) 50°

2. Question 2:

$$S = 11.5, \quad S - c = 2.5, \quad \cos \frac{\gamma}{2} = \sqrt{\frac{11.5 \cdot 2.5}{7 \cdot 7}} \approx 0.7659, \quad \gamma \approx 2 \cos^{-1}(0.7659) \approx 80^{\circ}.$$

Answer: B) 80°

3. Question 3:

$$S = \frac{32 + 40 + 66}{2} = 69, \quad S - a = 37, \quad \cos\frac{\alpha}{2} = \sqrt{\frac{69 \cdot 37}{40 \cdot 66}} \approx 0.9836, \quad \alpha \approx 2\cos^{-1}(0.9836) \approx 20^{\circ} 80^{\circ} = 0.9836$$

Answer: A) 20°56′

4. Question 4:

$$S = 69$$
, $S - b = 29$, $\cos \frac{\beta}{2} = \sqrt{\frac{69 \cdot 29}{32 \cdot 66}} \approx 0.9733$, $\beta \approx 2 \cos^{-1}(0.9733) \approx 26^{\circ}30'$.

Answer: B) 26°30′

5. Question 5:

$$S = \frac{28.3 + 31.7 + 42.8}{2} = 51.4, \quad S - a = 23.1, \quad \cos\frac{\alpha}{2} = \sqrt{\frac{51.4 \cdot 23.1}{31.7 \cdot 42.8}} \approx 0.9352, \quad \alpha \approx 2\cos^{-1}(0.5)$$

Answer: A) 41°23′

6. Question 6:

$$S = 51.4, \quad S - c = 8.6, \quad \cos\frac{\gamma}{2} = \sqrt{\frac{51.4 \cdot 8.6}{28.3 \cdot 31.7}} \approx 0.7020, \quad \gamma \approx 2\cos^{-1}(0.7020) \approx 90^{\circ}51'.$$

Answer: C) 90°51′

7. Question 7:

$$S = \frac{31.9 + 56.31 + 40.27}{2} = 64.24, \quad S - b = 7.93, \quad \cos\frac{\beta}{2} = \sqrt{\frac{64.24 \cdot 7.93}{31.9 \cdot 40.27}} \approx 0.6298, \quad \beta \approx 2\cos^{-2}(1.5) = 0.6298$$

Answer: C) 115°38′

8. Question 8:

$$S = 64.24, \quad S - c = 23.97, \quad \cos\frac{\gamma}{2} = \sqrt{\frac{64.24 \cdot 23.97}{31.9 \cdot 56.31}} \approx 0.9256, \quad \gamma \approx 2\cos^{-1}(0.9256) \approx 30^{\circ}43'.$$

Answer: A) $30^{\circ}43'$

9. Question 9:

$$S = \frac{4584 + 5140 + 3624}{2} = 6674, \quad S - a = 2090, \quad \cos\frac{\alpha}{2} = \sqrt{\frac{6674 \cdot 2090}{5140 \cdot 3624}} \approx 0.8660, \quad \alpha \approx 2\cos^{-1}$$

Answer: B) $60^{\circ}9'$

10. **Question 10**:

$$S = 6674$$
, $S - c = 3050$, $\cos \frac{\gamma}{2} = \sqrt{\frac{6674 \cdot 3050}{4584 \cdot 5140}} \approx 0.9295$, $\gamma \approx 2 \cos^{-1}(0.9295) \approx 43^{\circ}17'$.

Answer: A) 43°17′

11. **Question 11:**

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac} = \frac{37.34^2 + 35.06^2 - 3.24^2}{2 \cdot 37.34 \cdot 35.06} \approx 0.9982, \quad \beta \approx \cos^{-1}(0.9982) \approx 3^{\circ}39'.$$

Answer: A) 3°39′

12. **Question 12:**

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab} = \frac{16^2 + 20^2 - 23^2}{2 \cdot 16 \cdot 20} \approx 0.1975, \quad \gamma \approx \cos^{-1}(0.1975) \approx 84^{\circ}18'.$$

Answer: C) 84°18′

13. **Question 13:**

$$S = \frac{x^2 + x + 1 + 2x + 1 + x^2 - 1}{2} = \frac{2x^2 + 3x + 1}{2}, \quad S - a = \frac{x - 1}{2}, \quad \cos\frac{\alpha}{2} = \sqrt{\frac{\frac{2x^2 + 3x + 1}{2} \cdot \frac{x - 1}{2}}{(2x + 1)(x^2 - 1)^2}}$$

Answer: C) 120°

14. **Question 14:**

$$S = \frac{413 + 214 + 375}{2} = 501, \quad S - a = 88, \quad \cos\frac{\alpha}{2} = \sqrt{\frac{501 \cdot 88}{214 \cdot 375}} \approx 0.7412, \quad \alpha \approx 2\cos^{-1}(0.7412)$$

Answer: C) 84°20′

15. **Question 15:**

$$S = 501$$
, $S - b = 287$, $\cos \frac{\beta}{2} = \sqrt{\frac{501 \cdot 287}{413 \cdot 375}} \approx 0.9635$, $\beta \approx 2 \cos^{-1}(0.9635) \approx 31^{\circ}2'$.

Answer: A) 31°2′

16. **Question 16:**

$$S = \frac{6+9+13}{2} = 14, \quad S-a = 8, \quad \cos\frac{\alpha}{2} = \sqrt{\frac{14\cdot 8}{9\cdot 13}} \approx 0.9796, \quad \alpha \approx 2\cos^{-1}(0.9796) \approx 23^{\circ}51'.$$

Answer: A) 23°51′

17. **Question 17:**

$$S = 14$$
, $S - c = 1$, $\cos \frac{\gamma}{2} = \sqrt{\frac{14 \cdot 1}{6 \cdot 9}} \approx 0.5092$, $\gamma \approx 2 \cos^{-1}(0.5092) \approx 118^{\circ}46'$.

Answer: C) 118°46′

18. **Question 18:**

$$S = \frac{10 + 12 + 15}{2} = 18.5, \quad \cos \beta = \frac{10^2 + 15^2 - 12^2}{2 \cdot 10 \cdot 15} \approx 0.8033, \quad \beta \approx \cos^{-1}(0.8033) \approx 36^{\circ}52'.$$

Answer: A) 36°52′

19. **Question 19:**

$$S = \frac{8 + 15 + 10}{2} = 16.5, \quad \cos \beta = \frac{8^2 + 10^2 - 15^2}{2 \cdot 8 \cdot 10} \approx -0.1312, \quad \beta \approx \cos^{-1}(-0.1312) \approx 97^{\circ}36'.$$

Answer: C) 97°36′

20. **Question 20:**

$$S = \frac{25 + 30 + 40}{2} = 47.5, \quad S - b = 17.5, \quad \cos\frac{\beta}{2} = \sqrt{\frac{47.5 \cdot 17.5}{25 \cdot 40}} \approx 0.9115, \quad \beta \approx 2\cos^{-1}(0.9115)$$

Answer: B) 50°28′