Oblique Triangles Cheatsheet
Exercise 12.6

1 Oblique Triangle Fundamentals

1.1 Definition and Notation

An oblique triangle has no right angle ($\alpha + \beta + \gamma \neq 90°$). In $\triangle ABC$:

- Angles: α (at A), β (at B), γ (at C).
- Sides: a (opposite α), b (opposite β), c (opposite γ).

1.2 Key Formulas

- **Angle Sum**: $\alpha + \beta + \gamma = 180°$.
- **Semi-perimeter**:

$$S = \frac{a+b+c}{2}$$

- **Half-Angle Formulas**:

$$\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}}, \quad \cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}}, \quad \cos\frac{\gamma}{2} = \sqrt{\frac{S(S-c)}{ab}}$$

$$\alpha = 2\cos^{-1}\sqrt{\frac{S(S-a)}{bc}}, \quad \text{similar for } \beta, \gamma$$

- **Law of Cosines** (for specific angles):

$$\cos\beta = \frac{a^2+c^2-b^2}{2ac}, \quad \beta = \cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right)$$

2 Solving Oblique Triangles

2.1 Steps: All Three Sides Given

1. Calculate semi-perimeter: $S = \frac{a+b+c}{2}$.
2. Compute differences: $S-a$, $S-b$, $S-c$.
3. Use half-angle formulas to find angles:

$$\alpha = 2\cos^{-1}\sqrt{\frac{S(S-a)}{bc}}, \quad \beta = 2\cos^{-1}\sqrt{\frac{S(S-b)}{ac}}, \quad \gamma = 2\cos^{-1}\sqrt{\frac{S(S-c)}{ab}}$$

4. Verify: Ensure $\alpha + \beta + \gamma = 180°$.
5. Report angles in degrees and minutes.

2.2 Steps: Find Smallest/Greatest Angle

1. Identify smallest/greatest angle by comparing sides (smallest angle opposite shortest side, greatest opposite longest).
2. Use Law of Cosines:
$$\cos\beta = \frac{a^2+c^2-b^2}{2ac}, \quad \beta = \cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right)$$
3. Compute remaining angles if needed using half-angle formulas or angle sum.

2.3 Example: All Three Sides

Given: $a = 7, b = 7, c = 9$.

- Semi-perimeter:
$$S = \frac{7+7+9}{2} = 11.5$$
- Differences: $S - a = 11.5 - 7 = 4.5$, $S - b = 4.5$, $S - c = 11.5 - 9 = 2.5$.
- Angles:

$$\cos\frac{\alpha}{2} = \sqrt{\frac{11.5\cdot 4.5}{7\cdot 9}} \approx 0.9063, \quad \alpha \approx 2\cos^{-1}(0.9063) \approx 50^\circ$$
$$\cos\frac{\beta}{2} = \sqrt{\frac{11.5\cdot 4.5}{7\cdot 9}} \approx 0.9063, \quad \beta \approx 50^\circ$$
$$\cos\frac{\gamma}{2} = \sqrt{\frac{11.5\cdot 2.5}{7\cdot 7}} \approx 0.7659, \quad \gamma \approx 2\cos^{-1}(0.7659) \approx 80^\circ$$

3 Common Trigonometric Values

Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

For non-standard angles (e.g., $84^\circ 20'$), use trigonometric tables or calculators.

4 Problem Types

- **All Three Sides**: Find all angles using half-angle formulas.
$$\text{E.g., } a = 7, b = 7, c = 9 \implies \alpha \approx 50^\circ, \beta \approx 50^\circ, \gamma \approx 80^\circ.$$

- **Smallest Angle**: Use Law of Cosines for angle opposite shortest side.

$$\text{E.g., } a = 37.34, b = 3.24, c = 35.06 \implies \beta \approx 3^\circ 39' \text{ (opposite } b\text{)}.$$

- **Greatest Angle**: Use Law of Cosines for angle opposite longest side.

$$\text{E.g., } a = 16, b = 20, c = 23 \implies \gamma \approx 84^\circ 18' \text{ (opposite } c\text{)}.$$

- **Algebraic Sides**: Prove specific angle (e.g., 120°) using half-angle formulas.

$$\text{E.g., } a = x^2 + x + 1, b = 2x + 1, c = x^2 - 1 \implies \alpha = 120^\circ.$$

5 Tips and Tricks

- Verify $\alpha + \beta + \gamma = 180^\circ$.
- For smallest/greatest angle, use Law of Cosines for efficiency.
- Simplify algebraic expressions in denominators before computing half-angles.
- Round angles to degrees and minutes; sides to two decimal places unless exact.
- Use exact values for standard angles $(30^\circ, 45^\circ, 60^\circ)$.
- In applications, interpret sides as distances (e.g., roads, plots).

6 Applications

- Surveying: Calculate corner angles of triangular plots (e.g., Q.9).
- Navigation: Determine angles between roads connecting locations (e.g., Q.10).
- Engineering: Analyze triangular structures or force distributions.