Exercise 3.2: Matrices and Determinants MCQs for Entry Test

Multiple Choice Questions

1. What is the result of multiplying a 2×3 matrix A by the 2×2 identity matrix I_2?

(a) I_2
(b) A
(c) I_3
(d) Zero matrix

2. For $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, what is the determinant $|A|$?

(a) 5
(b) 8
(c) 2
(d) 6

3. Solve the system $x + y = 3$, $2x - y = 0$ using matrix inverses. What is $\begin{bmatrix} x \\ y \end{bmatrix}$?

(a) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
(b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$
(c) $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$
(d) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

4. For $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$, what is $A - B$?

(a) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$
(d) $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$

5. Which property holds for matrices A, B, C of compatible orders?

(a) $(AB)C = A(BC)$

(b) $AB = BA$

(c) $(A+B)^2 = A^2 + 2AB + B^2$

(d) $(A-B)^2 = A^2 - 2AB + B^2$

6. For $A = \begin{bmatrix} 1 & i \\ 0 & -i \end{bmatrix}$, what is A^{-1} if it exists?

(a) $\begin{bmatrix} 1 & i \\ 0 & i \end{bmatrix}$

(b) $\begin{bmatrix} -1 & -i \\ 0 & -i \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -i \\ 0 & i \end{bmatrix}$

(d) Does not exist

7. If $(A+B)C = AC + BC$ holds for matrices A, B, C, which property is this?

(a) Associative

(b) Commutative

(c) Distributive

(d) Identity

8. For $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, what is A^tA?

(a) $\begin{bmatrix} 5 & 11 \\ 11 & 25 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 10 & 14 \\ 14 & 20 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

9. Solve $2X - A = B$ for X, where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$.

(a) $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

10. Why does $(A+B)^2 \neq A^2 + 2AB + B^2$ in general for square matrices A, B?

(a) Matrix addition is not commutative

(b) Matrix multiplication is not commutative

(c) Matrices cannot be squared

(d) Distributive property does not hold

11. For $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, what is $(A - B)$?

(a) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$

12. Solve the system $3x - y = 4$, $x + 2y = 5$ using matrix inverses. What is $\begin{bmatrix} x \\ y \end{bmatrix}$?

(a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

13. For $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, what is $(AB)C$?

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

14. For $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, what is the adjoint of A?

(a) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$

15. Why does $(A - B)^2 \neq A^2 - 2AB + B^2$ in general for square matrices A, B?

(a) Matrix subtraction is not associative

(b) Matrix multiplication is not commutative
(c) Matrix subtraction is not commutative
(d) Distributive property fails

16. For $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, what is AA^t?

(a) $\begin{bmatrix} 5 & 6 \\ 6 & 9 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 5 & 2 \\ 2 & 9 \end{bmatrix}$

17. Solve $3X - 2A = B$ for X, where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

18. Solve $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for X.

(a) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

19. Solve the system $2x + y = 3$, $x - y = 1$ using matrix inverses. What is $\begin{bmatrix} x \\ y \end{bmatrix}$?

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

20. For $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$, what is A^{-1}?

(a) $\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$

(b) Does not exist

(c) $\begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Answers and Explanations

1. **Answer**: (b) A
Explanation: For a 2×3 matrix A, multiplying by I_2 (left) gives $I_2 A = A$, as the identity matrix preserves A. Option (a) is I_2, not A. Option (c) is incorrect as I_3 is 3×3. Option (d) implies a zero result, which is false.

2. **Answer**: (a) 5
Explanation: For $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, $|A| = 2 \cdot 4 - 1 \cdot 3 = 8 - 3 = 5$. Option (b) is 8, ignoring subtraction. Option (c) is unrelated. Option (d) miscalculates.

3. **Answer**: (b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$
Explanation: Write as $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$. Compute $|A| = 1 \cdot (-1) - 1 \cdot 2 = -3$, adj $A = \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$, $A^{-1} = \frac{1}{-3} \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$. Then, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3} \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Option (a) reverses values. Option (c) fails equations. Option (d) is incorrect.

4. **Answer**: (a) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$
Explanation: Compute $A - B = \begin{bmatrix} 1-0 & 0-1 \\ 2-1 & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$. Option (b) misplaces signs. Option (c) is incorrect. Option (d) is A.

5. **Answer**: (a) $(AB)C = A(BC)$
Explanation: Matrix multiplication is associative, so $(AB)C = A(BC)$ holds for compatible matrices. Option (b) is false as $AB \neq BA$ generally. Options (c, d) fail due to non-commutativity ($BA \neq AB$).

6. **Answer**: (c) $\begin{bmatrix} 1 & -i \\ 0 & i \end{bmatrix}$
Explanation: For $A = \begin{bmatrix} 1 & i \\ 0 & -i \end{bmatrix}$, $|A| = 1 \cdot (-i) - i \cdot 0 = -i$, adj $A = \begin{bmatrix} -i & -i \\ 0 & 1 \end{bmatrix}$, so $A^{-1} = \frac{1}{-i} \begin{bmatrix} -i & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -i \\ 0 & i \end{bmatrix}$ (since $\frac{1}{-i} = i$). Option (a) is unrelated. Option (b) negates incorrectly. Option (d) is false as $|A| \neq 0$.

7. **Answer**: (c) Distributive
Explanation: $(A + B)C = AC + BC$ is the distributive property for matrices. Option (a) is $(AB)C = A(BC)$. Option (b) is $AB = BA$, which is false generally. Option (d) involves identity matrices.

8. **Answer**: (a) $\begin{bmatrix} 5 & 11 \\ 11 & 25 \end{bmatrix}$

Explanation: Compute $A^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, then $A^t A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1+9 & 2+12 \\ 2+12 & 4+16 \end{bmatrix} = \begin{bmatrix} 5 & 11 \\ 11 & 25 \end{bmatrix}$. Option (b) is A^t. Option (c) is unrelated. Option (d) is A.

9. **Answer**: (a) $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

Explanation: Solve $2X = A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$. Then, $X = \frac{1}{2} \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$. Option (b) is incorrect. Option (c) is A. Option (d) is B.

10. **Answer**: (b) Matrix multiplication is not commutative
Explanation: $(A+B)^2 = A^2 + AB + BA + B^2$, but $AB \neq BA$ generally, so it does not equal $A^2 + 2AB + B^2$. Option (a) is false as addition is commutative. Option (c) is false as matrices can be squared. Option (d) is false as distributive property holds.

11. **Answer**: (c) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Explanation: Compute $A - B = \begin{bmatrix} 2-1 & -1-0 \\ 0-(-1) & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. Option (a) misplaces signs. Option (b) is unrelated. Option (d) is A.

12. **Answer**: (c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Explanation: Write as $\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. Compute $|A| = 3 \cdot 2 - (-1) \cdot 1 = 7$, adj $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$, $A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$. Then, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Options (a, b, d) fail equations.

13. **Answer**: (c) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

Explanation: Compute $AB = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then $(AB)C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$. Option (a) is C. Option (b) is AB. Option (d) is I_2.

14. **Answer**: (a) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Explanation: For $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, adj $A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$. Option (b) is A^t. Option (c) is unrelated. Option (d) swaps signs.

15. **Answer**: (b) Matrix multiplication is not commutative
Explanation: $(A-B)^2 = A^2 - AB - BA + B^2$, but $AB \neq BA$ generally, so it does not equal $A^2 - 2AB + B^2$. Option (a) is false as subtraction is associative. Option (c) is false as subtraction is commutative. Option (d) is false as distributive property holds.

16. **Answer**: (a) $\begin{bmatrix} 5 & 6 \\ 6 & 9 \end{bmatrix}$

Explanation: Compute $A^t = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$, then $AA^t = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 6 & 9 \end{bmatrix}$. Option (b) is A. Option (c) is A. Option (d) is incorrect.

17. Answer: (a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Explanation: Solve $3X = 2A + B = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$. Then, $X = \frac{1}{3}\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Option (b) is unrelated. Option (c) is B. Option (d) is zero.

18. Answer: (a) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

Explanation: For $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$, $|A| = 2 \cdot 1 - 1 \cdot 1 = 1$, adj $A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$, so $A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$. Then, $X = A^{-1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$. Option (b) is A. Option (c) is I_2. Option (d) is zero.

19. Answer: (a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Explanation: Write as $\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Compute $|A| = 2 \cdot (-1) - 1 \cdot 1 = -3$, adj $A = \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$, $A^{-1} = \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$. Then, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Options (b, c, d) fail equations.

20. Answer: (b) Does not exist

Explanation: For $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$, $|A| = 2 \cdot 2 - 1 \cdot 4 = 0$, so A^{-1} does not exist. Option (a) assumes non-zero determinant. Option (c) is unrelated. Option (d) is I_2.