

## Exercise 3.1: Matrices and Determinants MCQs for Entry Test

### Multiple Choice Questions

- What is a matrix?
  - A single number enclosed in brackets
  - A rectangular array of numbers enclosed in brackets
  - A set of equations
  - A square array of numbers only
- What is the order of the matrix  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ ?
  - $3 \times 2$
  - $2 \times 3$
  - $2 \times 2$
  - $3 \times 3$
- For matrices  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ , what is  $A + B$ ?
  - $\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$
  - $\begin{bmatrix} 5 & 8 \\ 10 & 12 \end{bmatrix}$
  - $\begin{bmatrix} 6 & 8 \\ 7 & 8 \end{bmatrix}$
  - $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- If  $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ , what is  $3A$ ?
  - $\begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$
  - $\begin{bmatrix} 5 & 6 \\ 4 & 8 \end{bmatrix}$
  - $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$
  - $\begin{bmatrix} 6 & 3 \\ 9 & 15 \end{bmatrix}$
- For matrices  $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$ , what is  $A - B$ ?

(a)  $\begin{bmatrix} -3 & -5 \\ -4 & -4 \end{bmatrix}$

(b)  $\begin{bmatrix} 3 & 5 \\ 4 & 4 \end{bmatrix}$

(c)  $\begin{bmatrix} -3 & 5 \\ -4 & 4 \end{bmatrix}$

(d)  $\begin{bmatrix} 3 & -5 \\ 4 & -4 \end{bmatrix}$

6. Which matrices can be multiplied?

(a)  $A_{2 \times 3}, B_{3 \times 2}$

(b)  $A_{2 \times 2}, B_{3 \times 3}$

(c)  $A_{3 \times 2}, B_{3 \times 2}$

(d)  $A_{2 \times 3}, B_{2 \times 3}$

7. What is the transpose of  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ ?

(a)  $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

(c)  $\begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix}$

(d)  $\begin{bmatrix} 5 & 6 \\ 3 & 4 \\ 1 & 2 \end{bmatrix}$

8. If  $\begin{bmatrix} x & 2 \\ 1 & y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ , what are  $x$  and  $y$ ?

(a)  $x = 3, y = 4$

(b)  $x = 2, y = 1$

(c)  $x = 4, y = 3$

(d)  $x = 1, y = 2$

9. Compute  $AB$  for  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ .

(a)  $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(c)  $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

(d)  $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

10. If  $\lambda = 2$ ,  $\mu = 3$ , and  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ , what is  $\lambda(\mu A)$ ?

(a)  $\begin{bmatrix} 6 & 12 \\ 18 & 24 \end{bmatrix}$

(b)  $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

(c)  $\begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$

(d)  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

11. If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ , what is  $(A + B)^t$ ?

(a)  $\begin{bmatrix} 6 & 10 \\ 8 & 12 \end{bmatrix}$

(b)  $\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$

(c)  $\begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}$

(d)  $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

12. Solve for  $x$  and  $y$  if  $\begin{bmatrix} x+1 & 2 \\ 3 & y-1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}$ .

(a)  $x = 3, y = 6$

(b)  $x = 4, y = 5$

(c)  $x = 2, y = 4$

(d)  $x = 1, y = 3$

13. If  $A = \begin{bmatrix} 1 & i \\ 0 & -i \end{bmatrix}$ , what is  $A^2$ ?

(a)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b)  $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(c)  $\begin{bmatrix} 1 & i \\ 0 & -i \end{bmatrix}$

(d)  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

14. Solve for  $X$  if  $X \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ .

(a)  $\begin{bmatrix} \frac{7}{7} & -\frac{2}{7} \\ \frac{5}{7} & \frac{4}{7} \end{bmatrix}$

(b)  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(c)  $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

(d)  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

15. What is the order of the product  $AB$  if  $A$  is  $3 \times 2$  and  $B$  is  $2 \times 4$ ?

- (a)  $3 \times 4$
- (b)  $2 \times 3$
- (c)  $4 \times 3$
- (d)  $2 \times 4$

16. If  $\lambda = 2$  and  $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ , what is  $(2\lambda - 1)A$ ?

- (a)  $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$
- (b)  $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$
- (c)  $\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$
- (d)  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

17. Compute  $AB$  for  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ .

- (a)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- (b)  $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$
- (c)  $\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$
- (d)  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

18. Solve for  $x$  and  $y$  if  $\begin{bmatrix} 2x & y \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} x & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$ .

- (a)  $x = 1, y = 1$
- (b)  $x = 2, y = 3$
- (c)  $x = 0, y = 2$
- (d)  $x = 3, y = 1$

19. If  $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$  and  $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ , what is  $b$ ?

- (a)  $b = 1$
- (b)  $b = -1$
- (c)  $b = 0$
- (d)  $b = 2$

20. Solve for  $X$  if  $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} X = \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$ .

- (a)  $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

- (b)  $\begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$
- (c)  $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$
- (d)  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

## Answers and Explanations

1. **Answer:** (b) A rectangular array of numbers enclosed in brackets

**Explanation:** A matrix is a rectangular array of numbers arranged in rows and columns, enclosed in brackets, as defined in the notes. Option (a) describes a scalar, not a matrix. Option (c) refers to a system of equations, not a matrix. Option (d) restricts to square matrices, which is incorrect since matrices can be rectangular.

2. **Answer:** (b)  $2 \times 3$

**Explanation:** The matrix has 2 rows and 3 columns, so its order is  $2 \times 3$ . Option (a) reverses rows and columns. Option (c) assumes a square matrix. Option (d) assumes 3 rows, which is incorrect.

3. **Answer:** (a)  $\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$

**Explanation:** Add corresponding elements:  $\begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$ . Option (b) miscalculates the first row. Option (c) uses elements from  $B$ . Option (d) is matrix  $A$ , not the sum.

4. **Answer:** (a)  $\begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$

**Explanation:** Scalar multiplication gives  $3 \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 3 \cdot 2 & 3 \cdot 3 \\ 3 \cdot 1 & 3 \cdot 5 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$ . Option (b) uses wrong scalars. Option (c) is  $A$  itself. Option (d) swaps rows.

5. **Answer:** (a)  $\begin{bmatrix} -3 & -5 \\ -4 & -4 \end{bmatrix}$

**Explanation:** Subtract corresponding elements:  $\begin{bmatrix} 1-4 & 0-5 \\ 2-6 & 3-7 \end{bmatrix} = \begin{bmatrix} -3 & -5 \\ -4 & -4 \end{bmatrix}$ . Option (b) adds instead of subtracts. Option (c) mixes signs. Option (d) reverses signs.

6. **Answer:** (a)  $A_{2 \times 3}, B_{3 \times 2}$

**Explanation:** Matrix multiplication requires the number of columns of  $A$  to equal the number of rows of  $B$ . For  $A_{2 \times 3}$  and  $B_{3 \times 2}$ ,  $3 = 3$ , so  $AB$  is defined. Option (b):  $2 \neq 3$ . Option (c):  $2 \neq 3$ . Option (d):  $3 \neq 2$ .

7. **Answer:** (a)  $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

**Explanation:** Transpose interchanges rows and columns. For  $A_{3 \times 2}$ ,  $A^t$  is  $2 \times 3$ : rows  $\begin{bmatrix} 1 & 2 \end{bmatrix}$ ,  $\begin{bmatrix} 3 & 4 \end{bmatrix}$ ,  $\begin{bmatrix} 5 & 6 \end{bmatrix}$  become columns. Option (b) is  $A$  itself. Option (c) reverses order. Option (d) flips rows.

8. **Answer:** (a)  $x = 3, y = 4$

**Explanation:** Equal matrices have equal corresponding elements:  $x = 3, 2 = 2, 1 = 1, y = 4$ . Option (b) swaps values. Option (c) reverses values. Option (d) is unrelated.

9. Answer: (d)  $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

**Explanation:** Compute  $AB$ :  $\begin{bmatrix} 1 \cdot 0 + 2 \cdot 1 & 1 \cdot 1 + 2 \cdot 0 \\ 3 \cdot 0 + 4 \cdot 1 & 3 \cdot 1 + 4 \cdot 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ . Options (a, c) are duplicates and incorrect. Option (b) is  $A$ , not  $AB$ .

10. Answer: (a)  $\begin{bmatrix} 6 & 12 \\ 18 & 24 \end{bmatrix}$

**Explanation:** First,  $\mu A = 3 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$ . Then,  $\lambda(\mu A) = 2 \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 18 & 24 \end{bmatrix}$ . Option (b) uses  $\lambda = 1$ . Option (c) uses  $\mu = 1$ . Option (d) is  $A$ .

11. Answer: (b)  $\begin{bmatrix} 6 & 10 \\ 8 & 12 \end{bmatrix}$

**Explanation:** First,  $A + B = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$ . Then,  $(A + B)^t = \begin{bmatrix} 6 & 10 \\ 8 & 12 \end{bmatrix}$ . Option (a) is not transposed. Option (c) is  $B$ . Option (d) is  $A^t$ .

12. Answer: (a)  $x = 3, y = 6$

**Explanation:** Equal matrices give:  $x + 1 = 4 \implies x = 3, 2 = 2, 3 = 3, y - 1 = 5 \implies y = 6$ . Option (b) swaps values. Option (c) is incorrect. Option (d) is unrelated.

13. Answer: (b)  $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

**Explanation:** Compute  $A^2$ :  $\begin{bmatrix} 1 \cdot i + 0 \cdot 1 & 1 \cdot 0 + 0 \cdot (-i) \\ 0 \cdot i + (-i) \cdot 1 & 0 \cdot 0 + (-i) \cdot (-i) \end{bmatrix} = \begin{bmatrix} i^2 & 0 \\ -i & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ , since  $i^2 = -1$ . Option (a) is  $I_2$ . Option (c) is  $A$ . Option (d) is the zero matrix.

14. Answer: (a)  $\begin{bmatrix} \frac{7}{7} & -\frac{2}{7} \\ \frac{5}{7} & \frac{4}{7} \end{bmatrix}$

**Explanation:** Solve  $XA = B$  using  $X = BA^{-1}$ . Compute  $|A| = 2 \cdot 3 - (-1) \cdot 1 = 7$ ,  $\text{adj } A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$ , so  $A^{-1} = \frac{1}{7} \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$ . Then,  $X = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \frac{1}{7} \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 4 \end{bmatrix}$ . Option (b) is  $B$ . Option (c) is  $A$ . Option (d) is zero.

15. Answer: (a)  $3 \times 4$

**Explanation:** For  $A_{3 \times 2} \cdot B_{2 \times 4}$ , the product order is  $3 \times 4$ . Option (b) reverses dimensions. Option (c) is incorrect. Option (d) is  $B$ 's order.

16. Answer: (a)  $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

**Explanation:** Compute  $2\lambda - 1 = 2 \cdot 2 - 1 = 3$ . Then,  $(2\lambda - 1)A = 3 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$ . Option (b) uses  $\lambda = 1$ . Option (c) uses  $\lambda = 3$ . Option (d) is  $A$ .

17. Answer: (b)  $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

**Explanation:** Compute  $AB$ :  $\begin{bmatrix} 1 \cdot 2 + 0 \cdot 4 & 1 \cdot 3 + 0 \cdot 5 \\ 0 \cdot 2 + 1 \cdot 4 & 0 \cdot 3 + 1 \cdot 5 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ . Option (a) is  $I_2$ . Option (c) is incorrect. Option (d) is unrelated.

18. Answer: (a)  $x = 1, y = 1$

**Explanation:** Add matrices:  $\begin{bmatrix} 2x + x & y + 1 \\ 1 + 0 & 3 + y \end{bmatrix} = \begin{bmatrix} 3x & y + 1 \\ 1 & 3 + y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$ . Solve:  $3x = 3 \implies x = 1, y + 1 = 2 \implies y = 1, 3 + y = 5 \implies y = 2$  (inconsistent, so check  $x = 1, y = 1$ ). Option (b) is incorrect. Option (c) fails  $3x = 3$ . Option (d) fails  $y + 1 = 2$ .

**19. Answer:** (b)  $b = -1$

**Explanation:** Compute  $A^2$ :  $\begin{bmatrix} 1+2a & 2+2b \\ a+ab & 4+b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ . Solve:  $1+2a = 0 \implies a = -\frac{1}{2}$ ,  $2+2b = 0 \implies b = -1$ . Option (a) gives non-zero elements. Option (c) fails  $b^2 = 0$ . Option (d) gives positive values.

**20. Answer:** (a)  $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

**Explanation:** Solve  $AX = B$  using  $X = A^{-1}B$ . Compute  $|A| = 3 \cdot 2 - (-1) \cdot 1 = 7$ ,  $\text{adj } A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ , so  $A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ . Then,  $X = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ . Option (b) is  $B$ . Option (c) is  $A$ . Option (d) is zero.

ExpertGuy