Applications of Trigonometry Cheatsheet
Exercise 12.3

1 Angles of Elevation and Depression

1.1 Definitions

- **Angle of Elevation**: Angle above the horizontal when looking up (e.g., to a tree top).
- **Angle of Depression**: Angle below the horizontal when looking down (e.g., from a cliff to a boat).
- Note: Angle of depression equals the alternate interior angle in the right triangle formed.

1.2 Key Formulas

For a right triangle with angle θ, perpendicular (height), and base (distance):

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}, \quad \theta = \tan^{-1}\left(\frac{\text{perpendicular}}{\text{base}}\right)$$
$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}, \quad \cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

Pythagorean Theorem: $\text{perpendicular}^2 + \text{base}^2 = \text{hypotenuse}^2$.

2 Solving Right Triangle Problems

2.1 Steps

1. Draw a diagram, labeling the right triangle with known heights, distances, and angles.
2. Identify the angle of elevation or depression and the corresponding right triangle.
3. Select tan, sin, or cos based on given and required values.
4. Solve for the unknown (height, distance, or hypotenuse).
5. For multiple triangles, set up equations and solve simultaneously.
6. Report angles in degrees and minutes, lengths to two decimal places or exact.

2.2 Example

A pole is 8 m high, its shadow is 6 m. Find the sun's angle of elevation.

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{8}{6} \approx 1.3333 \implies \theta = \tan^{-1}(1.3333) \approx 53°7'.$$

3 Oblique Triangle Laws

3.1 Key Formulas

For $\triangle ABC$ with angles α, β, γ opposite sides a, b, c, and semi-perimeter $S = \dfrac{a+b+c}{2}$:

- **Law of Sines**:
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$
- **Law of Cosines**:
$$a^2 = b^2 + c^2 - 2bc\cos\alpha, \quad b^2 = a^2 + c^2 - 2ac\cos\beta, \quad c^2 = a^2 + b^2 - 2ab\cos\gamma$$
- **Law of Tangents**:
$$\frac{a-b}{a+b} = \frac{\tan\frac{\alpha-\beta}{2}}{\tan\frac{\alpha+\beta}{2}}$$
- **Half-Angle Formulas** (e.g., for α):
$$\sin\frac{\alpha}{2} = \sqrt{\frac{(S-b)(S-c)}{bc}}, \quad \cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}}, \quad \tan\frac{\alpha}{2} = \sqrt{\frac{(S-b)(S-c)}{S(S-a)}}$$

4 Common Trigonometric Values

Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

For non-standard angles (e.g., $32°, 55°$), use trigonometric tables or calculators.

5 Problem Types

- **Single Right Triangle**: Find height, distance, or angle.

$$\text{E.g., Pole (8 m), shadow (6 m): } \tan\theta = \frac{8}{6} \implies \theta \approx 53°7'.$$

- **Multiple Right Triangles**: Solve for distances or heights using two triangles.

  E.g., Lighthouse (100 m), ships with depression angles $17^\circ, 19^\circ$: Distance = 36.58 m.

- **Changing Positions**: Find new angles or distances after moving.

  E.g., Tower (60 m), move 20 m closer: New angle = $28^\circ 54'$.E.g., Tower (60 m), move 20 m close

- **Oblique Triangles**: Use laws of sines/cosines (not detailed in problems but listed).

6 Tips and Tricks

- Draw clear diagrams to identify right triangles and angles of elevation/depression.
- Use $\tan\theta$ for height/base problems, $\sin\theta$ or $\cos\theta$ for hypotenuse-related problems.
- Convert units (e.g., 18 dm = 1.8 m) before calculations.
- For multiple triangles, set up equations equating shared variables (e.g., height).
- Round lengths to two decimal places, angles to degrees and minutes.

7 Applications

- Surveying: Measure heights (trees, buildings) or distances (boats, ships).
- Aviation: Calculate plane heights or angles.
- Architecture: Determine structural angles or ladder lengths.