Oblique Triangles Cheatsheet
Exercise 12.5

1 Oblique Triangle Fundamentals

1.1 Definition and Notation

An oblique triangle has no right angle ($\alpha + \beta + \gamma \neq 90^\circ$). In $\triangle ABC$:

- Angles: α (at A), β (at B), γ (at C).
- Sides: a (opposite α), b (opposite β), c (opposite γ).

1.2 Key Formulas

- **Angle Sum**: $\alpha + \beta + \gamma = 180^\circ$.
- **Law of Cosines**:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha, \quad \cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

  (Similar for b^2, c^2, β, γ).

- **Law of Tangents**:

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$$

  (Similar for $\beta - \gamma, \gamma - \alpha$).

- **Law of Sines**:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

2 Solving Oblique Triangles

2.1 Steps: Two Sides and Included Angle

1. Use Law of Cosines to find third side:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

2. Find second angle using Law of Cosines:

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}, \quad \beta = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

3. Find third angle: $\gamma = 180^\circ - \alpha - \beta$.
4. Verify using Law of Sines or another angle.

2.2 Steps: Two Sides and Opposite Angle

1. Find sum of other angles: $\alpha + \beta = 180° - \gamma$.
2. Use Law of Tangents to find angle difference:

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$$

3. Solve for angles: Add/subtract equations for α, β.
4. Find third side using Law of Sines:

$$c = b \cdot \frac{\sin\gamma}{\sin\beta}$$

2.3 Example: Two Sides and Included Angle

Given: $b = 59, c = 34, \alpha = 52°$.

- Find a:

$$a^2 = 59^2 + 34^2 - 2 \cdot 59 \cdot 34 \cdot \cos 52° \approx 3481 + 1156 - 2466.6 \approx 6204, \quad a \approx 78.76$$

- Find β:

$$\cos\beta = \frac{78.76^2 + 34^2 - 59^2}{2 \cdot 78.76 \cdot 34} \approx 0.3146, \quad \beta \approx \cos^{-1}(0.3146) \approx 71°53'$$

- Find γ:

$$\gamma = 180° - 52° - 71°53' \approx 56°7'$$

3 Common Trigonometric Values

Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

For non-standard angles (e.g., $38°13'$), use trigonometric tables or calculators.

4 Problem Types

- **Two Sides, Included Angle**: Find third side and angles using Law of Cosines.

$$\text{E.g., } b = 59, c = 34, \alpha = 52° \implies a \approx 78.76, \beta \approx 71°53', \gamma \approx 56°7'.$$

- **Two Sides, Opposite Angle**: Find angles using Law of Tangents, third side using Law of Sines.

$$\text{E.g., } a = 36.21, b = 42.09, \gamma = 44°29' \implies \alpha \approx 57°22', \beta \approx 78°10', c \approx 30.13.$$

- **Side Ratio, Opposite Angle**: Use ratio to assign sides, solve as above.

$$\text{E.g., } a : b = 3 : 2, \gamma = 57° \implies \alpha \approx 81°44', \beta \approx 41°16'.$$

- **Resultant Force**: Use Law of Cosines for resultant magnitude.

$$\text{E.g., } c = 40, a = 30, \beta = 147°25' \implies b \approx 67.25.$$

5 Tips and Tricks

- Verify $\alpha + \beta + \gamma = 180°$.
- Use exact values for standard angles $(30°, 45°, 60°)$.
- Convert minutes to decimals for calculations: $\theta° m' = \theta + \dfrac{m}{60}$.
- Round sides to two decimal places unless exact (e.g., $\sqrt{6}$).
- For Law of Tangents, compute $\tan\left(\frac{\alpha+\beta}{2}\right)$ using $\alpha + \beta = 180° - \gamma$.
- In force problems, identify the included angle between vectors.

6 Applications

- Mechanics: Calculate resultant forces or vector magnitudes.
- Surveying: Measure distances/angles in irregular shapes.
- Navigation: Determine bearings or distances in triangulation.
- Engineering: Analyze structures with non-right angles.