

Independent Probability Cheatsheet - Class 11 Mathematics

Prepared for Entry Test Preparation

1. Independent Events

Events A and B are independent if the occurrence of one does not affect the probability of the other. For independent events, the probability of both occurring is:

$$P(A \cap B) = P(A) \cdot P(B)$$

For n independent events A_1, A_2, \dots, A_n :

$$P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1) \cdot P(A_2) \cdot \dots \cdot P(A_n)$$

Key Concepts

- **Independence:** Common in experiments with replacement (e.g., card draws with replacement, multiple dice rolls) or unrelated events (e.g., survival probabilities).
- **Applications:** Coin tosses, dice rolls, card draws with replacement, and sequential selections with replacement.
- **Verification:** Check if $P(A \cap B) = P(A) \cdot P(B)$ to confirm independence.

Examples

1. **Probability both persons A and B are alive after 15 years** ($P(A) = \frac{5}{7}, P(B) = \frac{7}{9}$):

$$P(A \cap B) = \frac{5}{7} \cdot \frac{7}{9} = \frac{5}{9}$$

2. **Probability of two heads in two coin tosses:**

$$P(\text{head}) = \frac{1}{2}, P(\text{two heads}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

3. **Probability of drawing two aces with replacement from 52 cards:**

$$P(\text{ace}) = \frac{4}{52} = \frac{1}{13}, P(\text{both aces}) = \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169}$$

4. **Probability of red, white, black balls drawn with replacement (8 red, 5 white, 7 black):**

$$P(\text{red}) = \frac{8}{20}, P(\text{white}) = \frac{5}{20}, P(\text{black}) = \frac{7}{20}, P(\text{all}) = \frac{8}{20} \cdot \frac{5}{20} \cdot \frac{7}{20} = \frac{7}{200}$$

2. Verifying Independence

To prove events A and B are independent, show:

$$P(A \cap B) = P(A) \cdot P(B)$$

Compute $P(A \cap B)$ directly from the sample space and compare with $P(A) \cdot P(B)$.

Example

1. **Verify independence for even number and >4 dots in two die rolls:**

$$P(\text{even}) = \frac{9}{36}, P(> 4) = \frac{4}{36}, P(\text{even} \cap > 4) = \frac{1}{36}, \frac{9}{36} \cdot \frac{4}{36} = \frac{1}{36}$$

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