

Trigonometric Identities MCQs for Entry Test - Exercise 10.3

Introduction

This document contains 20 multiple-choice questions based on Exercise 10.3 of the trigonometry chapter, designed for entry test preparation. Each question tests concepts such as double-angle identities, half-angle identities, triple-angle identities, cotangent and secant identities, specific angle evaluations, power reduction, and advanced identities. Solutions with detailed explanations are provided at the end.

Multiple-Choice Questions

1. If $\sin \alpha = \frac{12}{13}$, $0 < \alpha < \frac{\pi}{2}$, what is $\sin 2\alpha$?

- A) $\frac{120}{169}$
- B) $\frac{119}{169}$
- C) $\frac{144}{169}$
- D) $\frac{25}{169}$

2. If $\cos \alpha = \frac{3}{5}$, $0 < \alpha < \frac{\pi}{2}$, what is $\cos 2\alpha$?

- A) $\frac{7}{25}$
- B) $\frac{-7}{25}$
- C) $\frac{9}{25}$
- D) $\frac{-9}{25}$

3. What is $\tan 2\alpha$ if $\tan \alpha = \frac{12}{5}$?

- A) $\frac{60}{11}$
- B) $\frac{-60}{11}$
- C) $\frac{120}{119}$
- D) $\frac{-120}{119}$

4. Which identity holds for $\frac{1-\cos \alpha}{\sin \alpha}$?

- A) $\tan \alpha$
- B) $\tan \frac{\alpha}{2}$

C) $\cot \alpha$

D) $\cot \frac{\alpha}{2}$

5. **What is $\cot \alpha - \tan \alpha$?**

A) $\cot 2\alpha$

B) $2 \cot 2\alpha$

C) $\tan 2\alpha$

D) $2 \tan 2\alpha$

6. **What is the value of $\sin 18^\circ$?**

A) $\frac{\sqrt{5}+1}{4}$

B) $\frac{\sqrt{5}-1}{4}$

C) $\sqrt{\frac{10+2\sqrt{5}}{16}}$

D) $\sqrt{\frac{10-2\sqrt{5}}{16}}$

7. **What is $\cos 36^\circ$?**

A) $\frac{\sqrt{5}+1}{4}$

B) $\frac{\sqrt{5}-1}{4}$

C) $\sqrt{\frac{10+2\sqrt{5}}{16}}$

D) $\sqrt{\frac{10-2\sqrt{5}}{16}}$

8. **What is $\sin 54^\circ$?**

A) $\frac{\sqrt{5}+1}{4}$

B) $\frac{\sqrt{5}-1}{4}$

C) $\sqrt{\frac{10+2\sqrt{5}}{16}}$

D) $\sqrt{\frac{10-2\sqrt{5}}{16}}$

9. **What is $\cos 144^\circ$?**

A) $\frac{\sqrt{5}+1}{4}$

B) $-\frac{\sqrt{5}+1}{4}$

C) $\frac{\sqrt{5}-1}{4}$

D) $\sqrt{\frac{10-2\sqrt{5}}{16}}$

10. **Simplify $\sin^4 \theta$.**

A) $\frac{3-4 \cos 2\theta+\cos 4\theta}{8}$

B) $\frac{3-4 \cos 2\theta+\cos 4\theta}{8}$

C) $\frac{1-4\cos 2\theta+\cos 4\theta}{8}$

D) $\frac{3-4\cos 2\theta+\cos 4\theta}{8}$

11. What is $\frac{\sin 2\alpha}{1+\cos 2\alpha}$?

A) $\tan \alpha$

B) $\cot \alpha$

C) $\tan \frac{\alpha}{2}$

D) $\cot \frac{\alpha}{2}$

12. Which identity holds for $\frac{\cos \alpha - \sin \alpha}{\cos \alpha + \sin \alpha}$?

A) $\sec 2\alpha - \tan 2\alpha$

B) $\sec \alpha - \tan \alpha$

C) $\csc 2\alpha - \cot 2\alpha$

D) $\csc \alpha - \cot \alpha$

13. What is $\sqrt{\frac{1+\sin \alpha}{1-\sin \alpha}}$?

A) $\frac{\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2}}$

B) $\frac{\sin \alpha + \cos \alpha}{\sin \alpha - \cos \alpha}$

C) $\frac{\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}}$

D) $\tan \frac{\alpha}{2}$

14. What is $\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta}$?

A) 1

B) 2

C) 3

D) 4

15. What is $\frac{\cos 3\theta}{\cos \theta} + \frac{\sin 3\theta}{\sin \theta}$?

A) $2 \cos 2\theta$

B) $4 \cos 2\theta$

C) $2 \cot 2\theta$

D) $4 \cot 2\theta$

16. What is $\frac{2 \sin \theta \sin 2\theta}{\cos \theta + \cos 3\theta}$?

A) $\tan \theta \tan 2\theta$

B) $\cot \theta \cot 2\theta$

C) $\tan \theta \cot 2\theta$

D) $\cot \theta \tan 2\theta$

17. What is $\frac{\sin 3\theta}{\cos \theta} + \frac{\cos 3\theta}{\sin \theta}$?

- A) $2 \cot 2\theta$
- B) $4 \cot 2\theta$
- C) $2 \cos 2\theta$
- D) $4 \cos 2\theta$

18. If $\cos 36^\circ \cos 72^\circ \cos 108^\circ \cos 144^\circ = k$, what is k ?

- A) $\frac{1}{8}$
- B) $\frac{1}{16}$
- C) $\frac{1}{32}$
- D) $\frac{1}{4}$

19. What is $\frac{\csc \theta + 2 \csc 2\theta}{\sec \theta}$?

- A) $\cot \theta$
- B) $\cot \frac{\theta}{2}$
- C) $\tan \theta$
- D) $\tan \frac{\theta}{2}$

20. What is $1 + \tan \alpha \tan 2\alpha$?

- A) $\sec 2\alpha$
- B) $\csc 2\alpha$
- C) $\sec \alpha$
- D) $\csc \alpha$

Solutions and Explanations

1. Solution to Question 1:

$$\sin \alpha = \frac{12}{13}, \quad \cos \alpha = \sqrt{1 - \frac{144}{169}} = \frac{5}{13}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = 2 \cdot \frac{12}{13} \cdot \frac{5}{13} = \frac{120}{169}$$

Answer: A) $\frac{120}{169}$

2. Solution to Question 2:

$$\cos \alpha = \frac{3}{5}, \quad \sin \alpha = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \frac{9}{25} - \frac{16}{25} = \frac{-7}{25}$$

Answer: B) $\frac{-7}{25}$

3. Solution to Question 3:

$$\tan \alpha = \frac{12}{5}, \quad \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{2 \cdot \frac{12}{5}}{1 - \left(\frac{12}{5}\right)^2} = \frac{\frac{24}{5}}{1 - \frac{144}{25}} = \frac{\frac{24}{5}}{\frac{-119}{25}} = \frac{24 \cdot 25}{-119 \cdot 5} = \frac{-120}{119}$$

Answer: D) $\frac{-120}{119}$

4. Solution to Question 4:

$$\frac{1 - \cos \alpha}{\sin \alpha} = \frac{2 \sin^2 \frac{\alpha}{2}}{2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}} = \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} = \tan \frac{\alpha}{2}$$

Answer: B) $\tan \frac{\alpha}{2}$

5. Solution to Question 5:

$$\cot \alpha - \tan \alpha = 2 \cot 2\alpha = 2 \frac{\cos 2\alpha}{\sin 2\alpha} = \frac{2(\cos^2 \alpha - \sin^2 \alpha)}{2 \sin \alpha \cos \alpha} = \frac{\cos^2 \alpha}{\sin \alpha \cos \alpha} - \frac{\sin^2 \alpha}{\sin \alpha \cos \alpha} = \cot \alpha - \tan \alpha$$

Answer: B) $2 \cot 2\alpha$

6. Solution to Question 6:

$$\sin 18^\circ = \frac{\sqrt{5} - 1}{4} \quad (\text{derived using } \cos 3\theta = \sin 2\theta \text{ for } \theta = 18^\circ)$$

Answer: B) $\frac{\sqrt{5}-1}{4}$

7. Solution to Question 7:

$$\cos 36^\circ = \frac{\sqrt{5} + 1}{4} \quad (\text{derived using } \sin 3\theta = \sin 2\theta \text{ for } \theta = 36^\circ)$$

Answer: A) $\frac{\sqrt{5}+1}{4}$

8. Solution to Question 8:

$$\sin 54^\circ = \sin(90^\circ - 36^\circ) = \cos 36^\circ = \frac{\sqrt{5} + 1}{4}$$

Answer: A) $\frac{\sqrt{5}+1}{4}$

9. Solution to Question 9:

$$\cos 144^\circ = \cos(180^\circ - 36^\circ) = -\cos 36^\circ = -\frac{\sqrt{5} + 1}{4}$$

Answer: B) $-\frac{\sqrt{5}+1}{4}$

10. Solution to Question 10:

$$\sin^4 \theta = \left(\frac{1 - \cos 2\theta}{2} \right)^2 = \frac{1 - 2 \cos 2\theta + \cos^2 2\theta}{4} = \frac{1 - 2 \cos 2\theta + \frac{1 + \cos 4\theta}{2}}{4} = \frac{3 - 4 \cos 2\theta + \cos 4\theta}{8}$$

Answer: A) $\frac{3-4 \cos 2\theta + \cos 4\theta}{8}$

11. **Solution to Question 11:**

$$\frac{\sin 2\alpha}{1 + \cos 2\alpha} = \frac{2 \sin \alpha \cos \alpha}{2 \cos^2 \alpha} = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha$$

Answer: A) $\tan \alpha$

12. **Solution to Question 12:**

$$\frac{\cos \alpha - \sin \alpha}{\cos \alpha + \sin \alpha} = \frac{1 - \sin 2\alpha}{\cos 2\alpha} = \frac{1}{\cos 2\alpha} - \frac{\sin 2\alpha}{\cos 2\alpha} = \sec 2\alpha - \tan 2\alpha$$

Answer: A) $\sec 2\alpha - \tan 2\alpha$

13. **Solution to Question 13:**

$$\sqrt{\frac{1 + \sin \alpha}{1 - \sin \alpha}} = \sqrt{\frac{\sin^2 \frac{\alpha}{2} + \cos^2 \frac{\alpha}{2} + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}{\sin^2 \frac{\alpha}{2} + \cos^2 \frac{\alpha}{2} - 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}} = \frac{\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2}}$$

Answer: A) $\frac{\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2}}$

14. **Solution to Question 14:**

$$\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} = \frac{\sin 3\theta \cos \theta - \cos 3\theta \sin \theta}{\sin \theta \cos \theta} = \frac{\sin 2\theta}{\sin \theta \cos \theta} = \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2$$

Answer: B) 2

15. **Solution to Question 15:**

$$\frac{\cos 3\theta}{\cos \theta} + \frac{\sin 3\theta}{\sin \theta} = \frac{\cos 3\theta \sin \theta + \sin 3\theta \cos \theta}{\cos \theta \sin \theta} = \frac{\sin 4\theta}{\cos \theta \sin \theta} = \frac{2 \sin 2\theta \cos 2\theta}{\sin \theta \cos \theta} = 4 \cos 2\theta$$

Answer: B) $4 \cos 2\theta$

16. **Solution to Question 16:**

$$\frac{2 \sin \theta \sin 2\theta}{\cos \theta + \cos 3\theta} = \frac{2 \sin \theta \sin 2\theta}{4 \cos^3 \theta - 2 \cos \theta} = \frac{2 \sin \theta \sin 2\theta}{2 \cos \theta (2 \cos^2 \theta - 1)} = \frac{\sin 2\theta}{\cos 2\theta} \cdot \frac{\sin \theta}{\cos \theta} = \tan 2\theta \tan \theta$$

Answer: A) $\tan \theta \tan 2\theta$

17. **Solution to Question 17:**

$$\frac{\sin 3\theta}{\cos \theta} + \frac{\cos 3\theta}{\sin \theta} = \frac{\sin 3\theta \sin \theta + \cos 3\theta \cos \theta}{\cos \theta \sin \theta} = \frac{\cos 2\theta}{\cos \theta \sin \theta} = 2 \cdot \frac{\cos 2\theta}{2 \sin \theta \cos \theta} = 2 \cot 2\theta$$

Answer: A) $2 \cot 2\theta$

18. **Solution to Question 18:**

$$\cos 108^\circ = \cos(180^\circ - 72^\circ) = -\cos 72^\circ$$

$$\cos 36^\circ \cos 72^\circ \cos 108^\circ \cos 144^\circ = \left(\frac{\sqrt{5}+1}{4}\right) \left(\frac{\sqrt{5}-1}{4}\right) \left(-\frac{\sqrt{5}-1}{4}\right) \left(-\frac{\sqrt{5}+1}{4}\right) = \left(\frac{4}{16}\right)^2 =$$

Answer: B) $\frac{1}{16}$

19. **Solution to Question 19:**

$$\frac{\csc \theta + 2 \csc 2\theta}{\sec \theta} = \left(\frac{1}{\sin \theta} + \frac{2}{2 \sin \theta \cos \theta} \right) \cos \theta = \frac{\cos \theta + 1}{\sin \theta \cos \theta} \cdot \cos \theta = \frac{2 \cos^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} = \cot \frac{\theta}{2}$$

Answer: B) $\cot \frac{\theta}{2}$

20. **Solution to Question 20:**

$$1 + \tan \alpha \tan 2\alpha = 1 + \frac{\sin \alpha}{\cos \alpha} \cdot \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{\cos \alpha \cos 2\alpha + \sin \alpha \sin 2\alpha}{\cos \alpha \cos 2\alpha} = \frac{\cos \alpha}{\cos \alpha \cos 2\alpha} = \frac{1}{\cos 2\alpha} = \sec 2\alpha$$

Answer: A) $\sec 2\alpha$

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