Advanced Probability Cheatsheet - Class 11 Mathematics

Prepared for Entry Test Preparation

1. Empirical Probability

Empirical probability is calculated from observed frequencies in experiments, using:

$$P(E) = \frac{\text{Frequency of event } E}{\text{Total number of trials}}$$

Key Concepts

- **Frequency Tables**: Summarize outcomes (e.g., heads/tails, die faces).
- **Applications**: Estimate probabilities from coin tosses, die rolls, or defective items.
- **Expected Outcomes**: Use probability to predict future occurrences (e.g., broken eggs).

Examples

1. **Probability of heads in 30 coin tosses (14 heads)**:

$$P(\text{head}) = \frac{14}{30} = \frac{7}{15}$$

2. **Probability of rolling a 3 in 100 die rolls (20 times)**:

$$P(3) = \frac{20}{100} = \frac{1}{5}$$

3. **Expected broken eggs from 7000 with daily breakage rate $\frac{9}{7}$%**:

$$7000 \times \frac{9}{7} \times \frac{1}{100} = 90$$

2. Mutually Exclusive and Equally Likely Events

- **Mutually Exclusive Events**: Events A and B are disjoint ($A \cap B = \emptyset$).
- **Equally Likely Events**: Each outcome has an equal chance of occurring.

3. Addition of Probabilities

For two events A and B:

- **Mutually Exclusive**: $P(A \cup B) = P(A) + P(B)$
- **Non-Mutually Exclusive**: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Key Concepts

- **Union**: Probability of A or B occurring.
- **Intersection**: Probability of both A and B (compute for non-mutually exclusive cases).
- **Applications**: Card draws, dice sums, or selections with overlapping conditions.

Examples

1. **Probability of red or white marble from 10 red, 30 white, 20 black (mutually exclusive)**:

$$P(\text{red}) = \frac{10}{60}, P(\text{white}) = \frac{30}{60}, P(\text{red} \cup \text{white}) = \frac{10}{60} + \frac{30}{60} = \frac{2}{3}$$

2. **Probability of diamond or ace from 52 cards (non-mutually exclusive)**:

$$P(\text{diamond}) = \frac{13}{52}, P(\text{ace}) = \frac{4}{52}, P(\text{diamond} \cap \text{ace}) = \frac{1}{52}, P(\text{diamond} \cup \text{ace}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} =$$

4. Independent Events

Events A and B are independent if the occurrence of one does not affect the other. The probability of both occurring is:

$$P(A \cap B) = P(A) \cdot P(B)$$

Example

1. **Probability of two heads in two independent coin tosses**:

$$P(\text{head}) = \frac{1}{2}, P(\text{both heads}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$