Cheatsheet: Systems of Quadratic Equations (Exercise 4.8)

Class 11 Mathematics (Chapter 4)

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Overview

Exercise 4.8 focuses on solving systems of simultaneous equations where at least one equation is quadratic. The primary methods are substitution (for one linear and one quadratic equation) and elimination (for two quadratic equations). Solutions typically yield two ordered pairs, but the discriminant determines the number of real solutions.

> Note
>
> Always check the discriminant of the resulting quadratic equation to confirm real solutions. Verify solutions in both original equations to avoid extraneous roots.

One Linear and One Quadratic Equation

Method

1. Solve the linear equation for one variable (e.g., y in terms of x).
2. Substitute into the quadratic equation to form a single quadratic equation in one variable.
3. Solve the quadratic equation using factoring, completing the square, or the quadratic formula.
4. Substitute back to find the corresponding values of the other variable.

Example For $2x - y = 4$, $2x^2 - 4xy - y^2 = 6$:

- From $2x - y = 4$, get $y = 2x - 4$.
- Substitute into $2x^2 - 4x(2x - 4) - (2x - 4)^2 = 6$.
- Simplify: $5x^2 - 16x + 11 = 0$.
- Solve: $x = 1, \frac{11}{5}$.
- Find y: When $x = 1$, $y = -2$; when $x = \frac{11}{5}$, $y = \frac{2}{5}$.
- Solution set: $\{(1, -2), \left(\frac{11}{5}, \frac{2}{5}\right)\}$.

Tip For equations like $\frac{a}{x} + \frac{b}{y} = c$, clear denominators after substitution to form a quadratic equation.

Two Quadratic Equations

Types and Methods

(i) **Both contain only x^2 and y^2 terms**: Eliminate one variable (e.g., y^2) by subtraction or multiplication to get a linear or quadratic equation in x^2.

(ii) **One equation is homogeneous**: Factorize the homogeneous equation (e.g., $x^2 - 3xy + 2y^2 = 0$) into linear factors (e.g., $(x-y)(x-2y) = 0$) and solve with the other equation.

(iii) **Both are non-homogeneous**: Multiply equations by constants to eliminate constant terms, yielding a homogeneous equation, then factorize.

Example For $x^2 + (y+1)^2 = 18$, $(x+2)^2 + y^2 = 21$:

- Expand: $x^2 + y^2 + 2y = 17$, $x^2 + y^2 + 4x = 17$.
- Subtract: $4x - 2y = 0 \implies y = 2x$.
- Substitute into first: $5x^2 + 4x - 17 = 0$.
- Solve: $x = \frac{-2 \pm \sqrt{89}}{5}$.
- Find y: $y = \frac{-4 \pm 2\sqrt{89}}{5}$.
- Solution set: $\left\{ \left(\frac{-2+\sqrt{89}}{5}, \frac{-4+2\sqrt{89}}{5} \right), \left(\frac{-2-\sqrt{89}}{5}, \frac{-4-2\sqrt{89}}{5} \right) \right\}$.

Key Reminders

- Check for restrictions (e.g., $x \neq 0, y \neq 0$ in reciprocal equations).
- Verify solutions in both equations to ensure correctness.
- For two quadratic equations, subtraction often yields a linear equation.
- Use the discriminant to determine the number of real solutions.

Tip

When factoring quadratics, test factors systematically. For complex discriminants, use the quadratic formula to ensure accuracy.