Cheatsheet: Partial Fractions (Exercise 5.1)

Class 11 Mathematics (Chapter 5)

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Overview

Partial fractions involve decomposing a rational function $\frac{p(x)}{q(x)}$ into a sum of simpler fractions. Exercise 5.1 focuses on resolving proper and improper rational fractions with linear and quadratic denominators into partial fractions, using techniques like substitution and coefficient comparison.

Note

Ensure the denominator is fully factored, and check if the fraction is proper. If improper, perform polynomial division first.

Key Concepts

1. Rational Function A rational function is $\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials, $Q(x) \neq 0$, with no common factors.

- **Proper**: Degree of $P(x) <$ degree of $Q(x)$. E.g., $\frac{1}{x^2-1}$.
- **Improper**: Degree of $P(x) \geq$ degree of $Q(x)$. E.g., $\frac{x^4}{x^2-1}$.

2. Partial Fraction Resolution Express $\frac{P(x)}{Q(x)}$ as a sum of simpler fractions based on the factors of $Q(x)$. Types of denominators:

- Linear: $\frac{A}{x-a}$.
- Repeated linear: $\frac{A}{x-a} + \frac{B}{(x-a)^2} + \cdots$.
- Quadratic: $\frac{Ax+B}{x^2+bx+c}$ (if irreducible).

3. Steps for Partial Fraction Decomposition

1. **Check Fraction Type**: If improper, divide $P(x)$ by $Q(x)$ to get a polynomial plus a proper fraction.
2. **Factor Denominator**: Fully factor $Q(x)$ into linear or quadratic factors.
3. **Set Up Partial Fractions**: Assign terms based on denominator factors.
4. **Solve for Constants**: Use substitution (set x to roots of denominators) or equate coefficients.
5. **Combine**: Write the final sum of partial fractions.

Techniques by Denominator Type

1. Distinct Linear Factors For $\frac{P(x)}{(x-a)(x-b)(x-c)}$, assume:

$$\frac{P(x)}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

Multiply through, equate numerators, and solve for A, B, C by substituting $x = a, b, c$.

- **Example**: $\frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$. Set $x = 1$: $1 = 2B \implies B = \frac{1}{2}$. Set $x = -1$: $1 = -2A \implies A = -\frac{1}{2}$.

2. Improper Fractions Divide numerator by denominator to obtain:

$$\frac{P(x)}{Q(x)} = \text{Polynomial} + \frac{\text{Remainder}}{Q(x)}$$

Resolve the proper remainder. E.g., $\frac{x^4+x^2}{x^2-1} = x^2 + 2 + \frac{2}{x^2-1}$.
3. Quadratic Denominators For denominators like (x^2+b^2), use substitution $y = x^2$. E.g., $\frac{x^2+a^2}{(x^2+b^2)(x^2+c^2)(x^2+d^2)}$ becomes $\frac{y+a^2}{(y+b^2)(y+c^2)(y+d^2)}$.
4. Non-Standard Linear Factors For denominators like $(1-ax)(1-bx)$, set $x = \frac{1}{a}, \frac{1}{b}$. E.g., $\frac{1}{(1-ax)(1-bx)(1-cx)}$.

Solving for Constants

- **Substitution Method**: Set x to roots of each denominator factor to isolate constants.
- **Coefficient Method**: Equate coefficients of like powers of x in the numerator.

Common Errors to Avoid

- Forgetting to divide for improper fractions.
- Incorrectly factoring the denominator.
- Omitting terms for repeated factors.
- Arithmetic errors in solving for constants.

Tip

Verify the solution by combining partial fractions back to the original fraction or checking with a common denominator.