Permutation and Factorial Cheatsheet - Exercises 7.1 and 7.2 (Class 11 Mathematics)

Prepared for Entry Test Preparation

1. Factorial Notation Basics (Ex. 7.1)

Factorial notation represents the product of all positive integers up to n. For a positive integer n:

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

Special case: $0! = 1$. Also, $n! = n \cdot (n-1)!$.

Key Formulas and Concepts

- **Factorial Evaluation**: Compute $n!$ directly, e.g., $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.
- **Factorial Ratios**: Simplify expressions like $\frac{n!}{(n-r)!} = n \cdot (n-1) \cdots (n-r+1)$.
- **Factorial Products**: For $\frac{n!}{k_1!k_2!\cdots k_m!}$, ensure $k_1 + k_2 + \cdots + k_m = n$.
- **Factorial Expressions**: Rewrite products like $n(n-1)(n-2)$ as $\frac{n!}{(n-3)!}$.
- **Binomial Coefficients**: $\frac{n!}{r!(n-r)!} = \binom{n}{r}$, used in permutations and combinations.

2. Examples (Ex. 7.1)

1. **Evaluate $\frac{10!}{7!}$**:
$$\frac{10)!}{(7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 10 \cdot 9 \cdot 8 = 720$$
2. **Evaluate $\frac{11!}{4!7!}$**:
$$\frac{11!}{4!7!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 7!} = \frac{11 \cdot 10 \cdot 9 \cdot 8}{24} = 330$$
3. **Write $12 \cdot 11 \cdot 10$ in factorial form**:
$$\frac{12 \cdot 11 \cdot 10 \cdot 9!}{9!} = \frac{12!}{9!}$$

3. Permutation Basics (Ex. 7.2)

A permutation is an arrangement of r objects from n distinct objects, denoted by ${}^nP_r$ or $P(n,r)$:

$${}^nP_r = \frac{n!}{(n-r)!} = n \cdot (n-1) \cdot (n-2) \cdots (n-r+1)$$

Fundamental Principle of Counting: If event A occurs in p ways and event B in q ways, total ways = $p \cdot q$.

Key Formulas and Concepts

- **Permutation Formula**: ${}^nP_r = \frac{n!}{(n-r)!}$.
- **Solving for n**: Solve equations like ${}^nP_r = k$ by expanding $n(n-1)\cdots(n-r+1) = k$.
- **Word Arrangements**: Arrange all letters of a word with n letters: ${}^nP_n = n!$.
- **Number Formation**: Form r-digit numbers from n digits without repetition: ${}^nP_r$.
- **Constraints**: Handle conditions like digits being together or not together using grouping or subtraction.
- **Alternate Arrangements**: For alternate seating (e.g., boys and girls), use ${}^mP_m \cdot {}^nP_n$.
- **Permutation Identities**: Prove ${}^nP_r = n \cdot {}^{n-1}P_{r-1}$ or ${}^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$.

Examples (Ex. 7.2)

1. **Evaluate ${}^{12}P_5$**:

$$ {}^{12}P_5 = \frac{12!}{(12-5)!} = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 = 95040 $$

2. **Solve ${}^nP_2 = 30$**:

$$ n(n-1) = 30 \implies n^2 - n - 30 = 0 \implies n = 6 $$

3. **Arrange letters of "PLANE"**:

$$ {}^5P_5 = 5! = 120 $$

4. **5-digit numbers with 2, 8 together**:

Treat (2,8) as one unit: ${}^4P_4 \cdot 2! = 24 \cdot 2 = 48$