Set Theory and Logic Exercise 2.3 New Concepts Cheatsheet

New Concepts in Exercise 2.3

This cheatsheet summarizes new set theory and logic concepts introduced in Exercise 2.3, not covered in Exercises 2.1 and 2.2, with definitions and examples.

1. **Associative Property of Intersection**
   - *Definition*: $(A \cap B) \cap C = A \cap (B \cap C)$. The grouping of sets in an intersection does not affect the result.
   - *Example*: $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{3, 4, 5\}$. $(A \cap B) \cap C = \{2, 3\} \cap \{3, 4, 5\} = \{3\}$. $A \cap (B \cap C) = \{1, 2, 3\} \cap \{3, 4\} = \{3\}$.
2. **Idempotent Laws**
   - *Definition*: $A \cup A = A$; $A \cap A = A$. The union or intersection of a set with itself is the set itself.
   - *Example*: $A = \{1, 2\}$. $A \cup A = \{1, 2\} \cup \{1, 2\} = \{1, 2\} = A$. $A \cap A = \{1, 2\} \cap \{1, 2\} = \{1, 2\} = A$.
3. **Identity Laws**
   - *Definition*: $A \cup \emptyset = A$; $A \cap U = A$. The union with the empty set or intersection with the universal set leaves the set unchanged.
   - *Example*: $A = \{1, 2\}$, $U = \{1, 2, 3, 4\}$. $A \cup \emptyset = \{1, 2\} \cup \{\} = \{1, 2\} = A$. $A \cap U = \{1, 2\} \cap \{1, 2, 3, 4\} = \{1, 2\} = A$.
4. **Complement Laws**
   - *Definition*: $A \cup A' = U$; $A \cap A' = \emptyset$. The union of a set and its complement is the universal set; their intersection is empty.
   - *Example*: $U = \{1, 2, 3\}$, $A = \{1, 2\}$, $A' = \{3\}$. $A \cup A' = \{1, 2\} \cup \{3\} = \{1, 2, 3\} = U$. $A \cap A' = \{1, 2\} \cap \{3\} = \emptyset$.
5. **Absorption Laws**
   - *Definition*: $A \cap (A \cup B) = A$; $A \cup (A \cap B) = A$. A set absorbs the result of its union or intersection with another set.
   - *Example*: $A = \{1, 2\}$, $B = \{2, 3\}$. $A \cap (A \cup B) = \{1, 2\} \cap \{1, 2, 3\} = \{1, 2\} = A$. $A \cup (A \cap B) = \{1, 2\} \cup \{2\} = \{1, 2\} = A$.

6. Set Difference Identities

- *Definition*:
  - $A \cap B' = A$ if and only if $A \cap B = \emptyset$.
  - $(A - B) \cup B = A \cup B$.
  - $(A - B) \cap B = \emptyset$.
- *Example*: $A = \{1, 2\}$, $B = \{3, 4\}$, $U = \{1, 2, 3, 4\}$.
  - $A \cap B = \emptyset$, $B' = \{1, 2\}$, $A \cap B' = \{1, 2\} = A$.
  - $(A - B) = \{1, 2\}$, $(A - B) \cup B = \{1, 2\} \cup \{3, 4\} = \{1, 2, 3, 4\} = A \cup B$.
  - $(A - B) \cap B = \{1, 2\} \cap \{3, 4\} = \emptyset$.

7. Union with Complement Intersection

- *Definition*: $A \cup B = A \cup (A' \cap B)$. The union of two sets equals the union of the first set with the intersection of its complement and the second set.
- *Example*: $A = \{1, 2\}$, $B = \{2, 3\}$, $U = \{1, 2, 3, 4\}$. $A \cup B = \{1, 2, 3\}$. $A' = \{3, 4\}$, $A' \cap B = \{3\}$, $A \cup (A' \cap B) = \{1, 2\} \cup \{3\} = \{1, 2, 3\}$.

8. Logic Concepts

- *Definitions*:
  - *Inductive Logic*: Generalizing from specific observations.
  - *Deductive Logic*: Concluding specifics from general facts.
  - *Proposition*: A true or false statement.
  - *Aristotelian Logic*: Statements are strictly true or false.
  - *Non-Aristotelian Logic*: Allows additional possibilities.
  - *Symbolic Logic*:
    - Negation ($\sim$p): Not p.
    - Conjunction ($p \wedge q$): p and q.
    - Disjunction ($p \vee q$): p or q.
    - Conditional ($p \rightarrow q$): If p, then q.
    - Biconditional ($p \leftrightarrow q$): p if and only if q.
- *Examples*:
  - *Inductive*: Sun rises daily, so it will rise tomorrow.
  - *Deductive*: All men are mortal; Socrates is a man; Socrates is mortal.
  - *Proposition*: "2 + 2 = 4" (true).
  - *Aristotelian*: "It is raining" is true or false.
  - *Non-Aristotelian*: "This statement is false" may be undefined.

– *Symbolic Logic*: p = "It is sunny," q = "It is warm." $\sim$p: "It is not sunny." p $\wedge$ q: "It is sunny and warm." p $\vee$ q: "It is sunny or warm." p $\rightarrow$ q: "If it is sunny, then it is warm." p $\leftrightarrow$ q: "It is sunny if and only if it is warm."