MCQs: Solutions of Trigonometric Equations (Chapter 14, High Difficulty)

This document contains 20 high-difficulty multiple-choice questions (MCQs) for Chapter 14: Solutions of Trigonometric Equations, based on Exercise 14.1, Mathematics (Part-I). Each question has four options, with only one correct answer. All angles are in radians, and solutions are exact. These questions test advanced concepts, including quadratic equations, multiple-angle identities, sum-to-product transformations, and general solutions.

1. What is the general solution of $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$? (Q.3)

   (a) $\theta = \frac{5\pi}{6} + n\pi, \frac{11\pi}{6} + n\pi$

   (b) $\theta = \frac{\pi}{3} + n\pi, \frac{4\pi}{3} + n\pi$

   (c) $\theta = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$

   (d) $\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

   Answer: (a) $\theta = \frac{5\pi}{6} + n\pi, \frac{11\pi}{6} + n\pi$

2. What is the general solution of $\tan^2\theta - \sec\theta - 1 = 0$? (Q.4)

   (a) $\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \pi + 2n\pi$

   (b) $\theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

   (c) $\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

   (d) $\theta = \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

   Answer: (a) $\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \pi + 2n\pi$

3. For $2\sin\theta + \cos^2\theta - 1 = 0$, what is the general solution? (Q.5)

   (a) $\theta = n\pi, \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

   (b) $\theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

   (c) $\theta = \frac{\pi}{2} + n\pi, \frac{3\pi}{2} + n\pi$

   (d) $\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

   Answer: (a) $\theta = n\pi, \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

4. What is the general solution of $4\sin^2\theta - 8\cos\theta + 1 = 0$? (Q.8)

   (a) $\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

   (b) $\theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

   (c) $\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

   (d) $\theta = \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

Answer: (a) $\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

5. What is the general solution of $\sqrt{3}\tan x - \sec x - 1 = 0$? (Q.9)

   (a) $\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \pi + 2n\pi$

   (b) $\theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

   (c) $\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

   (d) $\theta = \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

   Answer: (a) $\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \pi + 2n\pi$

6. For $\cos 2x = \sin 3x$, what is the general solution? (Q.10)

   (a) $x = \frac{\pi}{2} + 2n\pi, \frac{\pi}{10} + 2n\pi, \frac{9\pi}{10} + 2n\pi, \frac{13\pi}{10} + 2n\pi, \frac{17\pi}{10} + 2n\pi$

   (b) $x = \frac{\pi}{3} + n\pi, \frac{4\pi}{3} + n\pi$

   (c) $x = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$

   (d) $x = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

   Answer: (a) $x = \frac{\pi}{2} + 2n\pi, \frac{\pi}{10} + 2n\pi, \frac{9\pi}{10} + 2n\pi, \frac{13\pi}{10} + 2n\pi, \frac{17\pi}{10} + 2n\pi$

7. What is the general solution of $\sec 3\theta = \sec \theta$? (Q.11)

   (a) $\theta = \frac{n\pi}{2}$

   (b) $\theta = \frac{\pi}{3} + n\pi, \frac{4\pi}{3} + n\pi$

   (c) $\theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

   (d) $\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

   Answer: (a) $\theta = \frac{n\pi}{2}$

8. What is the general solution of $\tan 2\theta + \cot \theta = 0$? (Q.12)

   (a) $\theta = (2n+1)\frac{\pi}{2}$

   (b) $\theta = \frac{\pi}{3} + n\pi, \frac{4\pi}{3} + n\pi$

   (c) $\theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

   (d) $\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

   Answer: (a) $\theta = (2n+1)\frac{\pi}{2}$

9. What is the general solution of $\sin 2x + \sin x = 0$? (Q.13)

   (a) $x = n\pi, \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

   (b) $x = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

   (c) $x = \frac{\pi}{2} + n\pi, \frac{3\pi}{2} + n\pi$

   (d) $x = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

   Answer: (a) $x = n\pi, \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

10. What is the general solution of $\sin 4x - \sin 2x = \cos 3x$? (Q.14)

    (a) $x = (2n+1)\frac{\pi}{6}, \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$

    (b) $x = \frac{n\pi}{3}, \frac{\pi}{12} + \frac{n\pi}{2}, \frac{5\pi}{12} + \frac{n\pi}{2}$

(c) $x = \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$

(d) $x = \frac{7\pi}{24} + \frac{n\pi}{2}, \frac{11\pi}{24} + \frac{n\pi}{2}$

Answer: (a) $x = (2n+1)\frac{\pi}{6}, \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$

11. What is the general solution of $\sin x + \cos 3x = \cos 5x$? (Q.15)

(a) $x = n\pi, \frac{7\pi}{24} + \frac{n\pi}{2}, \frac{11\pi}{24} + \frac{n\pi}{2}$

(b) $x = \frac{\pi}{3} + n\pi, \frac{4\pi}{3} + n\pi$

(c) $x = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

(d) $x = \frac{\pi}{2} + n\pi, \frac{3\pi}{2} + n\pi$

Answer: (a) $x = n\pi, \frac{7\pi}{24} + \frac{n\pi}{2}, \frac{11\pi}{24} + \frac{n\pi}{2}$

12. What is the general solution of $\sin 3x + \sin 2x + \sin x = 0$? (Q.16)

(a) $x = \frac{n\pi}{2}, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

(b) $x = \frac{\pi}{3} + n\pi, \frac{4\pi}{3} + n\pi$

(c) $x = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

(d) $x = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

Answer: (a) $x = \frac{n\pi}{2}, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

13. What is the general solution of $\sin 7x - \sin x = \sin 3x$? (Q.17)

(a) $x = \frac{n\pi}{3}, \frac{\pi}{12} + \frac{n\pi}{2}, \frac{5\pi}{12} + \frac{n\pi}{2}$

(b) $x = \frac{\pi}{3} + n\pi, \frac{4\pi}{3} + n\pi$

(c) $x = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

(d) $x = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

Answer: (a) $x = \frac{n\pi}{3}, \frac{\pi}{12} + \frac{n\pi}{2}, \frac{5\pi}{12} + \frac{n\pi}{2}$

14. What is the general solution of $\sin x + \sin 3x + \sin 5x = 0$? (Q.18)

(a) $x = \frac{n\pi}{3}, \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$

(b) $x = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

(c) $x = \frac{\pi}{2} + n\pi, \frac{3\pi}{2} + n\pi$

(d) $x = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

Answer: (a) $x = \frac{n\pi}{3}, \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$

15. What is the general solution of $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$? (Q.19)

(a) $\theta = \frac{n\pi}{4}, (2n+1)\frac{\pi}{4}, (2n+1)\frac{\pi}{2}$

(b) $\theta = \frac{\pi}{3} + n\pi, \frac{4\pi}{3} + n\pi$

(c) $\theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

(d) $\theta = \frac{\pi}{2} + n\pi, \frac{3\pi}{2} + n\pi$

Answer: (a) $\theta = \frac{n\pi}{4}, (2n+1)\frac{\pi}{4}, (2n+1)\frac{\pi}{2}$

16. What is the general solution of $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$? (Q.20)

(a) $\theta = (2n+1)\frac{\pi}{4}, (2n+1)\frac{\pi}{2}$

(b) $\theta = \frac{\pi}{3} + n\pi, \frac{4\pi}{3} + n\pi$

(c) $\theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

(d) $\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

Answer: (a) $\theta = (2n+1)\frac{\pi}{4}, (2n+1)\frac{\pi}{2}$

17. Which identity is key to solving $\sin 7x - \sin x = \sin 3x$? (Q.17)

(a) $\sin a - \sin b = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$

(b) $\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$

(c) $\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$

(d) $\sin 2x = 2\sin x \cos x$

Answer: (a) $\sin a - \sin b = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$

18. In solving $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$, what is the first step? (Q.19)

(a) Group as $(\sin 7\theta + \sin\theta) + (\sin 5\theta + \sin 3\theta)$

(b) Apply $\sin 2x = 2\sin x \cos x$

(c) Factor out $\sin\theta$

(d) Use $\tan^2\theta = \sec^2\theta - 1$

Answer: (a) Group as $(\sin 7\theta + \sin\theta) + (\sin 5\theta + \sin 3\theta)$

19. Why does $4\sin^2\theta - 8\cos\theta + 1 = 0$ yield $\cos\theta = -\frac{5}{2}$ as an invalid solution? (Q.8)

(a) $\cos\theta \leq 1$

(b) $\sin\theta \leq 1$

(c) $\tan\theta$ is undefined

(d) The equation has no real roots

Answer: (a) $\cos\theta \leq 1$

20. What is a common mistake when solving $\cos 2x = \sin 3x$? (Q.10)

(a) Forgetting to account for all solutions from the cubic equation

(b) Using the wrong period (π instead of 2π)

(c) Applying $\sin^2 x + \cos^2 x = 1$ incorrectly

(d) Ignoring quadrant rules for $\sin x$

Answer: (a) Forgetting to account for all solutions from the cubic equation