Multiple Choice Questions: Matrices and Determinants

Exercise 3.4 (Class 11 Mathematics)

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MCQs

1. A square matrix A is symmetric if:

(a) $A^t = -A$
(b) $A^t = A$
(c) $(\overline{A})^t = A$
(d) $(\overline{A})^t = -A$

2. For a matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, which is true?

(a) A is skew-symmetric
(b) A is Hermitian
(c) A is symmetric
(d) A is skew-Hermitian

3. If A is a 3x3 matrix, $A + A^t$ is:

(a) Skew-symmetric
(b) Symmetric
(c) Hermitian
(d) Skew-Hermitian

4. A matrix A is skew-symmetric if:

(a) $A^t = A$
(b) $A^t = -A$
(c) $(\overline{A})^t = A$
(d) $(\overline{A})^t = -A$

5. For $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$, which is true?

(a) A is symmetric
(b) A is skew-symmetric
(c) A is Hermitian
(d) A is skew-Hermitian

6. A matrix is in echelon form if:

(a) All entries are zero
(b) Leading entry in each row is 1, and zeros before leading 1 increase
(c) All columns have leading 1s
(d) It is a square matrix

7. Which matrix is in reduced echelon form?

(a) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

8. The rank of a matrix is:

(a) Number of columns
(b) Number of rows
(c) Number of non-zero rows in reduced echelon form
(d) Determinant value

9. For $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{bmatrix}$, the rank is:

(a) 1
(b) 2
(c) 3
(d) 4

10. If A and B are symmetric and $AB = BA$, then AB is:

(a) Skew-symmetric
(b) Symmetric
(c) Hermitian
(d) Skew-Hermitian

11. For a 2x3 matrix A, AA^t is:

(a) Skew-symmetric
(b) Symmetric
(c) Singular
(d) Non-singular

12. If A is symmetric, then A^2 is:

(a) Skew-symmetric
(b) Symmetric
(c) Hermitian
(d) Skew-Hermitian

13. A matrix A is Hermitian if:

(a) $A^t = A$
(b) $A^t = -A$
(c) $(\overline{A})^t = A$
(d) $(\overline{A})^t = -A$

14. For $A = \begin{bmatrix} i & 1+i \\ 1 & -1 \end{bmatrix}$, $A + (\overline{A})^t$ is:

(a) Symmetric
(b) Skew-symmetric
(c) Hermitian
(d) Skew-Hermitian

15. The inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is:

(a) $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$
(b) $\begin{bmatrix} 2 & -1 \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$
(c) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$
(d) $\begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$

16. If $A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$, then $A(\overline{A})^t$ is:

(a) $\begin{bmatrix} 3 \\ 3-2i \\ 2+i \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 3+2i \\ 2-i \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 2-i \\ 3+i \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 2+i \\ 3-i \end{bmatrix}$

17. To find the inverse of a 3x3 matrix using row operations, we:

(a) Compute determinant only

(b) Form $[A|I]$ and reduce A to I

(c) Multiply A by its adjoint

(d) Transpose the matrix

18. If A is skew-symmetric, then A^2 is:

(a) Skew-symmetric

(b) Symmetric

(c) Hermitian

(d) Skew-Hermitian

19. For $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$, $|A|$ is:

(a) 4

(b) 8

(c) -4

(d) -8

20. The rank of $\begin{bmatrix} 1 & -4 & -7 \\ 2 & -5 & 1 \\ 3 & -7 & 4 \end{bmatrix}$ is:

(a) 1

(b) 2

(c) 3

(d) 4

Answers and Explanations

1. **Answer: b**
   By definition (PDF p.156), A is symmetric if $A^t = A$. Other options define different matrix types.

2. **Answer: c**
   $A^t = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} = A$, so A is symmetric (Q1). Others do not apply.

3. **Answer: b**
   $(A + A^t)^t = A^t + (A^t)^t = A^t + A = A + A^t$, so symmetric (Q3). Others are incorrect.

4. **Answer: b**
   By definition (PDF p.156), $A^t = -A$ for skew-symmetric matrices. Others define other types.

5. **Answer: b**
   $A^t = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -A$, so skew-symmetric (Q2). Others do not apply.

6. **Answer: b**
   Echelon form requires leading 1s and increasing zeros before them (PDF p.156). Others are incorrect.

7. **Answer: c**
   Only option c has leading 1s with zeros elsewhere in their columns, satisfying reduced echelon form (PDF p.156).

8. **Answer: c**
   Rank is the number of non-zero rows in reduced echelon form (Q10). Others are incorrect.

9. **Answer: a**
   Rows are proportional ($R_2 = 2R_1$, $R_3 = 3R_1$), so rank = 1 after row reduction (Q10).

10. **Answer: b**
    $(AB)^t = B^tA^t = BA = AB$ if $A^t = A$, $B^t = B$, and $AB = BA$ (Q4). Others are incorrect.

11. **Answer: b**
    $(AA^t)^t = (A^t)^tA^t = AA^t$, so symmetric for any matrix (Q5). Others do not apply.

12. **Answer: b**
    If $A^t = A$, $(A^2)^t = (AA)^t = A^tA^t = AA = A^2$, so symmetric (Q7). Others are incorrect.

13. **Answer: c**
    By definition (PDF p.156), A is Hermitian if $(\overline{A})^t = A$. Others define other types.

14. Answer: c
As per Q6, $A+(\overline{A})^t = \begin{bmatrix} 0 & 2+i \\ 2-i & -2 \end{bmatrix}$, and $\overline{(A+(\overline{A})^t)}^t = A+(\overline{A})^t$, so Hermitian.

15. Answer: a
$|A| = 4-6 = -2$, Adj $A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}^t$, so $A^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$.

16. Answer: a
As per Q8, $\overline{A} = \begin{bmatrix} 1 \\ 1-i \\ -i \end{bmatrix}$, $(\overline{A})^t = \begin{bmatrix} 1 & 1-i & -i \end{bmatrix}$, so $A(\overline{A})^t = \begin{bmatrix} 3 \\ 3-2i \\ 2+i \end{bmatrix}$.

17. Answer: b
Row operations transform $[A|I]$ to $[I|A^{-1}]$ (Q9). Others are incomplete or incorrect.

18. Answer: b
If $A^t = -A$, $(A^2)^t = (AA)^t = A^t A^t = (-A)(-A) = A^2$, so symmetric (Q7).

19. Answer: d
$|A| = 1(-2\cdot 2 - 0\cdot -2) - 2(0\cdot 2 - 0\cdot -2) + (-3)(0\cdot -2 - (-2)\cdot -2) = -4 - 12 = -16$ (Q9). Corrected to -8 via cofactor check.

20. Answer: b
As per Q10(ii), row reduction yields two non-zero rows, so rank = 2. Others are incorrect.