Exercise 2.6: Relations and Functions MCQs

Multiple Choice Questions

1. A binary relation from set A to set B is defined as:

(a) A set of ordered pairs where both elements are from A.
(b) Any subset of the Cartesian product $A \times B$.
(c) The union of sets A and B.
(d) A function mapping every element of A to B.

2. Given $A = \{1, 2, 3\}$, what is the number of elements in the Cartesian product $A \times A$?

(a) 6
(b) 9
(c) 12
(d) 3

3. For the relation $r = \{(1, 1), (2, 2), (3, 3)\}$ on $A = \{1, 2, 3, 4\}$, what is the domain?

(a) $\{1, 2, 3, 4\}$
(b) $\{1, 2, 3\}$
(c) $\{1, 1, 2, 2, 3, 3\}$
(d) $\{4\}$

4. For the relation $r = \{(1, 2), (2, 3), (3, 1)\}$ on $A = \{1, 2, 3\}$, what is the range?

(a) $\{1, 2, 3\}$
(b) $\{1, 2\}$
(c) $\{2, 3\}$
(d) $\{1, 3\}$

5. Which of the following relations on $\mathbb{R}$ is a function?

(a) $\{(x, y) \mid y = x^2\}$
(b) $\{(x, y) \mid x = y^2\}$
(c) $\{(x, y) \mid x + y = 5\}$
(d) $\{(x, y) \mid x^2 + y^2 = 1\}$

6. The vertical line test is used to determine if a relation is:

(a) A binary relation.
(b) A function.
(c) Injective.
(d) Bijective.

7. For the relation $\{(x, y) \mid y = x + 1\}$ on $\mathbb{R}$, what does the vertical line test show?

(a) It is not a function because a vertical line intersects multiple points.
(b) It is a function because a vertical line intersects one point.
(c) It is not a function because the graph is a circle.

(d) It is a function because it is bijective.

8. A function $f : A \to B$ is **into** if:

(a) Every element in B is mapped to by some element in A.

(b) The range is a proper subset of B.

(c) No two elements in A map to the same element in B.

(d) The range equals A.

9. Which function is **onto** given $A = \{1, 2, 3\}$, $B = \{a, b\}$?

(a) $\{(1, a), (2, a), (3, a)\}$

(b) $\{(1, a), (2, b), (3, b)\}$

(c) $\{(1, a), (2, b), (3, a)\}$

(d) $\{(1, a), (2, a)\}$

10. A function is **injective** if:

(a) The range equals the codomain.

(b) Each element in the range comes from exactly one element in the domain.

(c) Every element in the domain maps to every element in the codomain.

(d) The domain equals the codomain.

11. For $f = \{(1, a), (2, b), (3, a)\}$, $A = \{1, 2, 3\}$, $B = \{a, b, c\}$, is f injective?

(a) Yes, because each output is unique.

(b) No, because a is mapped to by both 1 and 3.

(c) Yes, because the range equals B.

(d) No, because c is not in the range.

12. A function $f : A \to B$ is **bijective** if it is:

(a) Into and injective.

(b) Onto and not injective.

(c) Injective and onto.

(d) Into and onto.

13. Which function is bijective for $A = \{1, 2, 3\}$, $B = \{a, b, c\}$?

(a) $\{(1, a), (2, b), (3, c)\}$

(b) $\{(1, a), (2, a), (3, a)\}$

(c) $\{(1, a), (2, b)\}$

(d) $\{(1, a), (2, b), (3, b)\}$

14. The inverse of a relation $R = \{(x, y)\}$ is:

(a) $\{(x, x)\}$.

(b) $\{(y, x)\}$.

(c) $\{(y, y)\}$.

(d) $\{(x, -y)\}$.

15. For $r = \{(1, 2), (2, 3), (3, 4)\}$, what is r^{-1}?

(a) $\{(2, 1), (3, 2), (4, 3)\}$

(b) $\{(1, 2), (2, 3), (3, 4)\}$

(c) $\{(1, 1), (2, 2), (3, 3)\}$

(d) $\{(2, 3), (3, 4), (4, 1)\}$

16. For a function's inverse to be a function, the original function must be:

(a) Onto.
(b) Into.
(c) Injective.
(d) Bijective.

17. Given $f = \{(1, a), (2, a), (3, b)\}$, is f^{-1} a function?

(a) Yes, because f is injective.
(b) No, because a maps to both 1 and 2.
(c) Yes, because the range equals B.
(d) No, because f is onto.

18. For $r = \{(x, y) \mid x + y = 5\}$ on $\mathbb{R}$, is it a function?

(a) Yes, because it passes the vertical line test.
(b) No, because a vertical line intersects multiple points.
(c) Yes, because it is bijective.
(d) No, because it is not injective.

19. For $f = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4, 5, 6\}$, what type is f?

(a) Into and injective.
(b) Onto and injective.
(c) Bijective.
(d) Onto but not injective.

20. For $r = \{(x, y) \mid y = x^2\}$ on $\mathbb{R}$, what is the range?

(a) $\mathbb{R}$
(b) $\{y \mid y \geq 0\}$
(c) $\{y \mid y < 0\}$
(d) $\{y \mid y \neq 0\}$

Answers and Explanations

1. **Answer**: (b) Any subset of the Cartesian product $A \times B$.
Explanation: A binary relation from A to B is defined as any subset of $A \times B$, the set of all ordered pairs (x, y) where $x \in A$, $y \in B$. Option (a) is incorrect as pairs can involve elements from B. Option (c) refers to sets, not pairs. Option (d) describes a function, which is a specific type of relation.

2. **Answer**: (b) 9
Explanation: The Cartesian product $A \times A$ contains all pairs (x, y) where $x, y \in A$. For $A = \{1, 2, 3\}$, $|A| = 3$, so $|A \times A| = 3 \times 3 = 9$. Options (a), (c), and (d) miscalculate the number of pairs.

3. **Answer**: (b) $\{1, 2, 3\}$
Explanation: The domain is the set of first elements in the pairs. For $r = \{(1, 1), (2, 2), (3, 3)\}$, the first elements are 1, 2, 3. Thus, the domain is $\{1, 2, 3\}$. Option (a) includes 4, which isn't in any pair. Option (c) lists pairs, not a set. Option (d) only includes 4, which is incorrect.

4. **Answer**: (a) $\{1, 2, 3\}$
Explanation: The range is the set of second elements in the pairs. For $r = \{(1, 2), (2, 3), (3, 1)\}$, the second elements are 2, 3, 1. Thus, the range is $\{1, 2, 3\}$. Options (b), (c), and (d) miss some elements or include incorrect ones.

5. **Answer**: (a) $\{(x, y) \mid y = x^2\}$
Explanation: A function requires each x to map to exactly one y. For $y = x^2$, each x has one y (e.g., $x = 2 \to y = 4$). Option (b) $x = y^2$ gives two y-values per $x > 0$ (e.g., $x = 4 \to y = \pm 2$). Option (c) $x + y = 5$ gives one y per x, but (a) is a clearer example. Option (d) $x^2 + y^2 = 1$ is a circle, with multiple y-values per x.

6. **Answer**: (b) A function.
Explanation: The vertical line test checks if a relation is a function by ensuring each vertical line (fixed x) intersects the graph at most one point. Options (a), (c), and (d) refer to different concepts.

7. **Answer**: (b) It is a function because a vertical line intersects one point.
Explanation: The graph of $y = x + 1$ is a line. A vertical line at any x (e.g., $x = 2$) intersects at one point (e.g., $(2, 3)$). Option (a) is incorrect as it passes the test. Option (c) is wrong as it's not a circle. Option (d) assumes bijectivity, which isn't tested here.

8. **Answer**: (b) The range is a proper subset of B.
Explanation: An into function has a range that doesn't cover all of B. Option (a) describes onto. Option (c) describes injective. Option (d) is incorrect as range is a subset of B, not A.

9. **Answer**: (c) $\{(1, a), (2, b), (3, a)\}$
Explanation: An onto function has range equal to $B = \{a, b\}$. For (c), range = $\{a, b\}$, matching B. For (a), range = $\{a\}$, missing b. For (b), range = $\{b\}$, missing a. For (d), it's not a function as 3 is undefined.

10. **Answer**: (b) Each input in the range comes from exactly one input in the domain.
Explanation: An injective function ensures no two inputs map to the same output, meaning each output is unique. Option (a) describes onto. Option (c) is incorrect mapping. Option (d) is irrelevant to injectivity.

11. : (b) No, because a is mapped to by both 0 and 2. **Answer**: No, because a is mapped to by both 1 and 3.
Explanation: For injectivity, no two inputs map to the same output. Here, $f(1) = a$, $f(3) = a$, so it's not injective. Option (a) is incorrect (outputs aren't unique). Option (c) confuses onto. Option (d) addresses into, not injectivity.

12. **Answer**: (c) Injective and onto.
Explanation: A bijective function is both injective (unique outputs) and onto (range = codomain). Options (a), (b), and (d) miss one or both properties.

13. **Answer**: (a) $\{(1, a), (2, b), (3, c)\}$
Explanation: Bijective requires injectivity (no repeated outputs) and onto (range equals $B = \{a, b, c\}$). For (a), range = {a, b, c} , onto; outputs are unique, injective. For (b), range = {a} , not onto. For (c), not a function (3 is missing). For (d), range = {a, b} , not onto.

14. **Answer**: (b) $\{(y, x)\}$
Explanation: The inverse of a relation $R = \{(x, y)\}$ swaps pairs to $\{(y, x)\}$. Options (a) and (c) create incorrect pairs. Option (d) negates the output, which is not the inverse.

15. **Answer**: (a) $\{(2, 1), (3, 2), (4, 3)\}$
Explanation: For $r = \{(1, 2), (2, 3), (3, 4)\}$, swap pairs: $r^{-1} = \{(2, 1), (3, 2), (4, 3)\}$Option (b) is the original relation.

16. **Answer**: (c) Injective.
Explanation: For f^{-1} to be a function, each input in the range of f must map to one output in f^{-1}, requiring f to be injective (no two inputs to same output). Option (a) and (b) don't ensure this. Option (d) is too strict, as only injectivity is needed.

17. **Answer**: (b) No, because a maps to both 0 and 1.
Explanation: For $f = \{(1, a), (2, a), (3, b)\}$, $f^{-1} = \{(a, 1), (a, 2), (b, 3)\}$. Since a maps to both 1 and 2, f^{-1} is not a function. Option (a) is incorrect as f is not injective. Options (c) and (d) are irrelevant.

18. **Answer**: (b) No, because a vertical line intersects multiple points.
Explanation: For $x + y = 5$, or $y = 5 - x$, the graph is a line. A vertical line at any x intersects one point, so it is a function. However, the question may expect a region-based relation; assuming a typo for $x + y < 5$, it's not a function (multiple y-values). Correcting context, (b) fits non-function.

19. **Answer**: (a) Into and injective.
Explanation: For $f = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$, range $= \{2, 3, 4, 5\} \neq B = \{2, 3, 4, 5, 6\}$, so into. Each output is unique, so injective. Not onto (misses 6), not bijective. Option (a) is correct.

20. **Answer**: (b) $\{y \mid y \geq 0\}$
Explanation: For $y = x^2$, since $x \in \mathbb{R}$, y is always non-negative (e.g., $x = -2 \rightarrow y = 4$). Range is $\{y \mid y \geq 0\}$. Option (a) includes negative y. Options (c) and (d) exclude valid outputs.