Logic and Sets Cheatsheet

1. Sets and Set Operations

Definition: A set is a collection of distinct objects. Key operations include union ($\cup$), intersection ($\cap$), and complement ($'$).

Key Identity: $A \cup B = A \cup (A' \cap B)$

Example: Let $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$.

- Union: $A \cup B = \{1, 2, 3, 4\}$
- Intersection: $A \cap B = \{2, 3\}$
- Complement: $A' = \{4, 5\}$
- Verify: $A' \cap B = \{4, 5\} \cap \{2, 3, 4\} = \{4\}$, so $A \cup (A' \cap B) = \{1, 2, 3\} \cup \{4\} = \{1, 2, 3, 4\} = A \cup B$.

2. Inductive and Deductive Logic

Induction: Generalizing from specific observations.

- **Example**: Sun rises in the east daily, so it always rises in the east.

Deduction: Specific conclusions from general facts.

- **Example**: All humans are mortal. Socrates is human. Thus, Socrates is mortal.

3. Proposition

Definition: A statement that is either true or false, but not both.

- **Example**: "$2 + 2 = 4$" (true). "The moon is cheese" (false).

4. Aristotelian vs. Non-Aristotelian Logic

Aristotelian: Statements are true or false (binary).

- **Example**: "It is raining" is true or false.

Non-Aristotelian: Allows other possibilities (e.g., "maybe").

- **Example**: "The room is warm" may have a truth value of 0.7 in fuzzy logic.

5. Symbolic Logic

Definition: Uses symbols for logical operations.

Symbol	Meaning	Expression	Read As
$\sim$	Not	$\sim p$	Not p
$\wedge$	And	$p \wedge q$	p and q
$\vee$	Or	$p \vee q$	p or q
$\rightarrow$	If...then	$p \rightarrow q$	If p, then q
$\leftrightarrow$	If and only if	$p \leftrightarrow q$	p if and only if q

Example: Let p: "It is sunny," q: "I go hiking."

- $\sim p$: "It is not sunny."
- $p \wedge q$: "It is sunny and I go hiking."
- $p \to q$: "If it is sunny, then I go hiking."
- $p \leftrightarrow q$: "I go hiking if and only if it is sunny."

6. Truth Tables

Definition: Lists all possible truth values for logical expressions.
Example: Truth table for $p \to q$ and $p \leftrightarrow q$.

p	q	$p \to q$	$p \leftrightarrow q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

7. Converse, Inverse, Contrapositive

Definition: For $p \to q$:

- Converse: $q \to p$
- Inverse: $\sim p \to \sim q$
- Contrapositive: $\sim q \to \sim p$

Example (from Exercise 2.4, Q.1): Given $\sim p \to q$ ("If it is not raining, I go hiking").

- Converse: $q \to \sim p$ ("If I go hiking, it is not raining").
- Inverse: $p \to \sim q$ ("If it is raining, I do not go hiking").
- Contrapositive: $\sim q \to p$ ("If I do not go hiking, it is raining").

p	q	$\sim p$	$\sim q$	$\sim p \to q$	$q \to \sim p$	$p \to \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	T	T
F	T	T	F	T	T	F
F	F	T	T	F	T	T

8. Tautology, Absurdity, Contingency

Tautology: Always true. **Absurdity**: Always false. **Contingency**: True or false depending on values.
Examples (from Exercise 2.4, Q.4):

- $p \wedge \sim p$: Absurdity (always false).

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

- $p \to (q \to p)$: Tautology (always true).

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

9. Logical Equivalence

Definition: Two statements are equivalent if their truth values are identical.

Example (from Exercise 2.4, Q.5): Prove $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$.

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q)$	$p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	F
F	F	T	T	T	T	T

10. De Morgan's Theorem

Definition: $(A \cap B)' = A' \cup B'$, in logical form: $\sim (p \wedge q) = \sim p \vee \sim q$.

Example (from Exercise 2.5, Q.1):

p	q	$p \wedge q$	$\sim (p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Set Example: $U = \{1, 2, 3, 4\}$, $A = \{1, 2\}$, $B = \{2, 3\}$. Then $(A \cap B)' = \{2\}' = \{1, 3, 4\}$, and $A' \cup B' = \{3, 4\} \cup \{1, 4\} = \{1, 3, 4\}$.