Geometric Series Cheatsheet - Exercises 6.8 and 6.9 (Class 11 Mathematics)

Prepared for Entry Test Preparation

1. Geometric Series Basics (Ex. 6.8)

A geometric series is the sum of terms of a geometric progression (G.P.). The sum of n terms is given by:

- $S_n = \frac{a_1(1-r^n)}{1-r}$ if $|r| < 1$,
- $S_n = \frac{a_1(r^n-1)}{r-1}$ if $|r| > 1$.

For an infinite geometric series ($|r| < 1$):

$$S_\infty = \frac{a_1}{1-r}$$

2. Key Formulas and Concepts for Exercise 6.8

- **Sum of n Terms**: Use $S_n = \frac{a_1(1-r^n)}{1-r}$ for $|r| < 1$, or $S_n = \frac{a_1(r^n-1)}{r-1}$ for $|r| > 1$.
- **Sum of Special Series**: For series like $0.2 + 0.22 + 0.222 + \ldots$, rewrite as $2(0.1 + 0.11 + 0.111 + \ldots) = \frac{2}{9}(0.9 + 0.99 + \ldots)$, then use geometric series sum.
- **Algebraic Series**: For series like $1 + (a+b) + (a^2 + ab + b^2) + \ldots$, use $(a-b)S_n = \frac{a(a^n-1)}{a-1} - \frac{b(b^n-1)}{b-1}$.
- **Coefficient Series**: For series like $r + (1+k)r^2 + (1+k+k^2)r^3 + \ldots$, use $(1-k)S_n = \frac{r(r^n-1)}{r-1} - \frac{rk((rk)^n-1)}{rk-1}$.
- **Complex Series**: For series with complex terms, compute S_n using the geometric sum formula and simplify using $i^2 = -1$.
- **Infinite Series Sum**: Compute $S_\infty = \frac{a_1}{1-r}$ for $|r| < 1$.
- **Recurring Decimals**: Convert decimals like $0.\overline{abc}$ to fractions using $S_\infty = \frac{a_1}{1-r}$.

3. Key Concepts for Exercise 6.9

- **Applications of Geometric Series**: Compute total deposits, loan repayments, or population growth using S_n or $a_n = a_1 r^{n-1}$.
- **Population Growth**: For annual increase rate p%, use $P_n = P_0(1 + \frac{p}{100})^n$.
- **Doubling Periods**: For quantities doubling every k years, use $a_n = a_1 \cdot 2^{n-1}$.
- **Bacteria Growth**: For bacteria doubling every time period, use $a_n = a_1 \cdot 2^{2n}$ for n hours with doubling every half-hour.

- **Nested Triangles**: Sum perimeters of nested equilateral triangles using $S_\infty = \frac{a_1}{1-r}$.
- **Infinite Series with Variable Terms**: For series like $y = a_1x + a_1rx^2 + a_1r^2x^3 + \ldots$, solve $y = \frac{a_1x}{1-rx}$ to find x.
- **Convergence Interval**: Series converges if $|r| < 1$, e.g., for $r = 2x$, $|x| < \frac{1}{2}$.
- **Bouncing Ball**: Total distance = initial fall + $2 \cdot \frac{a_1}{1-r}$ for rebounds with ratio r.
- **Sum of Squares Series**: If sum of series is S and sum of squares is S^2, solve $\frac{a}{1-r} = S$, $\frac{a^2}{1-r^2} = S^2$.

4. Examples from Exercises 6.8 and 6.9

Sum of n Terms (Ex. 6.8, Q1)

Problem: Sum first 15 terms of $1, \frac{1}{3}, \frac{1}{9}, \ldots$.

- $a_1 = 1$, $r = \frac{1}{3}$, $n = 15$.
- $S_{15} = \frac{1\cdot(1-\left(\frac{1}{3}\right)^{15})}{1-\frac{1}{3}} = \frac{3}{2} \cdot \frac{14348906}{14348907} = \frac{7174453}{4782969}$.

Special Series (Ex. 6.8, Q2(i))

Problem: Sum $0.2 + 0.22 + 0.222 + \ldots$ to n terms.

- $S_n = \frac{2}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$.

Infinite Series (Ex. 6.8, Q5(iv))

Problem: Sum $2 + 1 + 0.5 + \ldots$.

- $a_1 = 2$, $r = \frac{1}{2}$. $S_\infty = \frac{2}{1-\frac{1}{2}} = 4$.

Recurring Decimal (Ex. 6.8, Q6(i))

Problem: Convert $1.\overline{34}$ to a fraction.

- $1.\overline{34} = 1 + \frac{0.34}{1-0.01} = \frac{133}{99}$.

Deposits (Ex. 6.9, Q1)

Problem: Deposits of Rs. 8, 24, 72, ... in 5 years.

- $a_1 = 8$, $r = 3$, $n = 5$. $S_5 = \frac{8(3^5-1)}{3-1} = 968$.

Bouncing Ball (Ex. 6.9, Q10)

Problem: Ball dropped from 27m, rebounds $\frac{2}{3}$. Total distance?

- Initial fall = 27, rebounds: $a_1 = 18$, $r = \frac{2}{3}$. Total = $27 + 2 \cdot \frac{18}{1-\frac{2}{3}} = 135$.

Infinite Series with Variable (Ex. 6.9, Q12)

Problem: Show $x = \frac{y-1}{2y}$ for $y = 1 + 2x + 4x^2 + \ldots$.

- $a_1 = 1$, $r = 2x$. $y = \frac{1}{1-2x}$. Solve: $x = \frac{y-1}{2y}$.