Cheatsheet: Synthetic Division and Quadratic Equations (Exercise 4.5)

Class 11 Mathematics (Chapter 4)

Prepared by ExpertGuy

Overview

Exercise 4.5 focuses on synthetic division, the Remainder Theorem, the Factor Theorem, and the relationship between roots and coefficients of quadratic equations. It also includes forming equations with given roots (PDF pp.270–276).

Note

Always verify remainders and factors using the Remainder and Factor Theorems before factoring polynomials completely.

Remainder Theorem

Concept When a polynomial $f(x)$ of degree $n \geq 1$ is divided by $x - a$, the remainder is $f(a)$ (p.270).

Steps

1. Identify a from $x - a = 0$.
2. Evaluate $f(a)$ by substituting $x = a$ into $f(x)$.
3. The result is the remainder.

Example Find the remainder when $x^2 + 3x + 7$ is divided by $x + 1$ (Q.1, p.270):

$$f(x) = x^2 + 3x + 7, \quad x + 1 = 0 \implies x = -1$$

$$f(-1) = (-1)^2 + 3(-1) + 7 = 1 - 3 + 7 = 5$$

Remainder: 5.

Factor Theorem

Concept $x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$ (p.270).

Steps

1. Evaluate $f(a)$.
2. If $f(a) = 0$, then $x - a$ is a factor.
3. Use synthetic division to find the quotient and factorize further.

Example Show $x = 2$ is a root of $x^3 - 7x + 6 = 0$ and factorize (Q.12, p.273):

$$f(x) = x^3 - 7x + 6$$

Synthetic division with $x = 2$:

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

Remainder: 0, so $x = 2$ is a root. Quotient: $x^2 + 2x - 3 = (x + 3)(x - 1)$. Factors: $(x - 2)(x + 3)(x - 1)$.

Synthetic Division

Concept A shortcut method to divide a polynomial $f(x)$ by $x - a$ (p.270).

Steps

1. Write coefficients of $f(x)$, including 0 for missing terms.
2. Use a from $x - a = 0$ as the divisor.
3. Bring down the first coefficient, multiply by a, add to the next coefficient, and repeat.
4. The last number is the remainder; others form the quotient polynomial.

Example Divide $x^3 - 28x - 48$ by $x + 4$ (Q.13, p.273):

$$x + 4 = 0 \implies x = -4$$

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -28 & -48 \\ & & -4 & 16 & 48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

Remainder: 0. Quotient: $x^2 - 4x - 12 = (x - 6)(x + 2)$. Factors: $(x + 4)(x - 6)(x + 2)$.

Finding Unknown Coefficients

Concept Use the Remainder or Factor Theorem to set up equations for unknown coefficients when remainders or roots are given (Q.10, Q.11, p.272).
Example Find k if $x^3 + 2x^2 + kx + 4$ divided by $x - 2$ has remainder 14 (Q.11, p.272):

$$f(x) = x^3 + 2x^2 + kx + 4, \quad f(2) = 14$$

$$f(2) = 8 + 8 + 2k + 4 = 20 + 2k = 14 \implies 2k = -6 \implies k = -3$$

Roots and Coefficients of Quadratic Equations

Concept For a quadratic $ax^2 + bx + c = 0$ with roots α, β:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a} \quad (p.276)$$

Forming Equations Given roots α, β, the equation is:

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0 \quad (p.276)$$

Example Find a, b if $-2, 2$ are roots of $x^3 - 4x^2 + ax + b = 0$ (Q.16, p.275):

$$f(-2) = -8 - 16 - 2a + b = 0, \quad f(2) = 8 - 16 + 2a + b = 0$$

Solve: $a = -4, b = 16$.

Tip

Use synthetic division to confirm roots and find quotients for complete factorization.

Key Reminders

- For $x^n + a^n$ with odd n, $x + a$ is a factor (Q.9, p.272).
- Check remainders by substituting $x = a$ into $f(x)$.
- Use synthetic division to factorize polynomials after confirming roots.
- Solve simultaneous equations for unknown coefficients using given roots or remainders.