Inverse Trigonometric Functions MCQs for Entry Test - Exercise 13.2

1 Introduction

This document contains 20 multiple-choice questions based on Exercise 13.2 of the Inverse Trigonometric Functions chapter, designed for entry test preparation. Questions test proving sum/difference identities, double-angle identities, general trigonometric identities, and evaluating trigonometric functions. Solutions provide detailed explanations. Notation: $y = \sin^{-1} x \iff x = \sin y$, with restricted domains. All evaluations are exact, without calculators.

2 Multiple-Choice Questions

1. Which equality is true?

   A) $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}$
   B) $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \sin^{-1}\frac{253}{325}$
   C) $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \tan^{-1}\frac{253}{325}$
   D) $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{12}{13}$

2. What is the value of $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5}$?

   A) $\tan^{-1}\frac{9}{19}$
   B) $\tan^{-1}\frac{1}{9}$
   C) $\tan^{-1}\frac{2}{9}$
   D) $\tan^{-1}\frac{5}{19}$

3. Which equality is true?

   A) $2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{12}{13}$
   B) $2\tan^{-1}\frac{2}{3} = \cos^{-1}\frac{12}{13}$
   C) $2\tan^{-1}\frac{2}{3} = \tan^{-1}\frac{12}{13}$
   D) $2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{3}{13}$

4. What is the value of $\tan\left(2\cos^{-1}\frac{12}{13}\right)$?

   A) $\frac{120}{119}$
   B) $\frac{119}{120}$

C) $\frac{5}{12}$

D) $\frac{12}{5}$

5. Which equality is true?

A) $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{4}$

B) $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{3}$

C) $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{6}$

D) $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{2}$

6. What is the value of $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17}$?

A) $\sin^{-1}\frac{77}{85}$

B) $\sin^{-1}\frac{48}{85}$

C) $\sin^{-1}\frac{45}{85}$

D) $\sin^{-1}\frac{24}{85}$

7. Which equality is true?

A) $\sin^{-1}\frac{77}{85} - \sin^{-1}\frac{3}{5} = \cos^{-1}\frac{15}{17}$

B) $\sin^{-1}\frac{77}{85} - \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{15}{17}$

C) $\sin^{-1}\frac{77}{85} - \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{15}{17}$

D) $\sin^{-1}\frac{77}{85} - \sin^{-1}\frac{3}{5} = \cos^{-1}\frac{36}{85}$

8. What is the value of $\sin\left(\cos^{-1}\frac{63}{65} + \tan^{-1}\frac{5}{12}\right)$?

A) $\frac{3}{5}$

B) $\frac{16}{65}$

C) $\frac{5}{13}$

D) $\frac{12}{13}$

9. Which equality is true?

A) $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$

B) $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{3}$

C) $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{6}$

D) $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{2}$

10. What is the value of $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}$?

A) $\frac{\pi}{2}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{3}$

D) $\frac{\pi}{6}$

11. Which equality is true?

A) $\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6}=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$

B) $\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6}=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{4}$

C) $\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6}=\tan^{-1}\frac{2}{3}+\tan^{-1}\frac{1}{2}$

D) $\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6}=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{2}{3}$

12. What is the value of $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}$?

A) $\frac{\pi}{4}$

B) $\frac{\pi}{3}$

C) $\frac{\pi}{6}$

D) $\frac{\pi}{2}$

13. What is the value of $\cos\left(\sin^{-1}\frac{1}{2}\right)$?

A) $\sqrt{1-\left(\frac{1}{2}\right)^2}=\frac{\sqrt{3}}{2}$

B) $\frac{1}{2}$

C) $\frac{1}{\sqrt{2}}$

D) 1

14. What is the value of $\sin\left(2\cos^{-1}\frac{3}{5}\right)$?

A) $2\cdot\frac{3}{5}\cdot\sqrt{1-\left(\frac{3}{5}\right)^2}=\frac{24}{25}$

B) $\frac{3}{5}$

C) $\frac{4}{5}$

D) $\frac{6}{5}$

15. What is the value of $\cos\left(2\sin^{-1}\frac{1}{\sqrt{5}}\right)$?

A) $1-2\left(\frac{1}{\sqrt{5}}\right)^2=\frac{3}{5}$

B) $\frac{1}{\sqrt{5}}$

C) $\frac{2}{\sqrt{5}}$

D) $\frac{4}{5}$

16. Which equality is true?

A) $\tan^{-1}\left(-\frac{1}{2}\right)=-\tan^{-1}\frac{1}{2}$

B) $\tan^{-1}\left(-\frac{1}{2}\right)=\tan^{-1}\frac{1}{2}$

C) $\tan^{-1}\left(-\frac{1}{2}\right)=\pi-\tan^{-1}\frac{1}{2}$

D) $\tan^{-1}\left(-\frac{1}{2}\right)=\frac{\pi}{2}-\tan^{-1}\frac{1}{2}$

17. Which equality is true?

A) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)=-\sin^{-1}\frac{1}{\sqrt{2}}$

B) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \sin^{-1}\frac{1}{\sqrt{2}}$

C) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \pi - \sin^{-1}\frac{1}{\sqrt{2}}$

D) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{\pi}{2} - \sin^{-1}\frac{1}{\sqrt{2}}$

18. Which equality is true?

A) $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\frac{1}{2}$

B) $\cos^{-1}\left(-\frac{1}{2}\right) = \cos^{-1}\frac{1}{2}$

C) $\cos^{-1}\left(-\frac{1}{2}\right) = -\cos^{-1}\frac{1}{2}$

D) $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{2} - \cos^{-1}\frac{1}{2}$

19. What is the value of $\tan\left(\sin^{-1}\frac{3}{5}\right)$?

A) $\frac{3}{\sqrt{1-\left(\frac{3}{5}\right)^2}} = \frac{3}{4}$

B) $\frac{4}{3}$

C) $\frac{3}{5}$

D) $\frac{5}{3}$

20. If $x = \sin^{-1}\frac{1}{2}$, what is the value of $\sec x$?

A) $\frac{2}{\sqrt{3}}$

B) 2

C) $\sqrt{2}$

D) $\frac{\sqrt{3}}{2}$

3 Solutions and Explanations

1. **Question 1:**

Let $\alpha = \sin^{-1}\frac{5}{13}$, $\beta = \sin^{-1}\frac{7}{25}$. $\cos\alpha = \sqrt{1-\frac{25}{169}} = \frac{12}{13}$, $\cos\beta = \sqrt{1-\frac{49}{625}} = \frac{24}{25}$.

$$\cos(\alpha+\beta) = \frac{12}{13}\cdot\frac{24}{25} - \frac{5}{13}\cdot\frac{7}{25} = \frac{288-35}{325} = \frac{253}{325} \implies \alpha+\beta = \cos^{-1}\frac{253}{325}.$$

Answer: A)

2. **Question 2:**

$$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} = \tan^{-1}\left(\frac{\frac{1}{4}+\frac{1}{5}}{1-\frac{1}{4}\cdot\frac{1}{5}}\right) = \tan^{-1}\left(\frac{\frac{5+4}{20}}{\frac{20-1}{20}}\right) = \tan^{-1}\frac{9}{19}.$$

Answer: A)

3. **Question 3:**

Let $\theta = \tan^{-1}\frac{2}{3}$. $\sec\theta = \sqrt{1+\frac{4}{9}} = \frac{\sqrt{13}}{3}$, $\cos\theta = \frac{3}{\sqrt{13}}$, $\sin\theta = \sqrt{1-\frac{9}{13}} = \frac{2}{\sqrt{13}}$.

$$\sin 2\theta = 2\cdot\frac{2}{\sqrt{13}}\cdot\frac{3}{\sqrt{13}} = \frac{12}{13} \implies 2\theta = \sin^{-1}\frac{12}{13}.$$

Answer: A)

4. **Question 4:**

$$\text{Let } \theta = \cos^{-1}\frac{12}{13},\ \cos\theta = \frac{12}{13},\ \sin\theta = \sqrt{1-\frac{144}{169}} = \frac{5}{13}.$$

$$\tan 2\theta = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{2\cdot\frac{5}{13}\cdot\frac{12}{13}}{\frac{144}{169}-\frac{25}{169}} = \frac{\frac{120}{169}}{\frac{119}{169}} = \frac{120}{119}.$$

Answer: A)

5. **Question 5:**

Let $\alpha = \sin^{-1}\frac{1}{\sqrt{5}}$, $\beta = \cot^{-1}3$. $\cos\alpha = \sqrt{1-\frac{1}{5}} = \frac{2}{\sqrt{5}}$, $\tan\beta = \frac{1}{3}$, $\cos\beta = \frac{3}{\sqrt{10}}$, $\sin\beta = \frac{1}{\sqrt{10}}$.

$$\sin(\alpha+\beta) = \frac{1}{\sqrt{5}}\cdot\frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}}\cdot\frac{1}{\sqrt{10}} = \frac{3+2}{\sqrt{50}} = \frac{1}{\sqrt{2}} \implies \alpha+\beta = \frac{\pi}{4}.$$

Answer: A)

6. **Question 6:**

$$\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17} = \sin^{-1}\left(\frac{3}{5}\cdot\sqrt{1-\frac{64}{289}} + \frac{8}{17}\cdot\sqrt{1-\frac{9}{25}}\right) = \sin^{-1}\left(\frac{3}{5}\cdot\frac{15}{17} + \frac{8}{17}\cdot\frac{4}{5}\right) = \sin^{-1}\frac{77}{85}$$

Answer: A)

7. **Question 7:**

$$\text{Let } \alpha = \sin^{-1}\frac{77}{85},\ \beta = \sin^{-1}\frac{3}{5}.\ \cos\alpha = \sqrt{1-\frac{5929}{7225}} = \frac{36}{85},\ \cos\beta = \sqrt{1-\frac{9}{25}} = \frac{4}{5}.$$

$$\cos(\alpha-\beta) = \frac{36}{85}\cdot\frac{4}{5} + \frac{77}{85}\cdot\frac{3}{5} = \frac{144+231}{425} = \frac{15}{17} \implies \alpha-\beta = \cos^{-1}\frac{15}{17}.$$

Answer: A)

8. **Question 8:**

$$\text{Let } \alpha = \cos^{-1}\frac{63}{65},\ \beta = \tan^{-1}\frac{5}{12}.\ \sin\alpha = \sqrt{1-\frac{3969}{4225}} = \frac{16}{65},\ \cos\beta = \frac{12}{13},\ \sin\beta = \frac{5}{13}.$$

$$\sin(\alpha+\beta) = \frac{16}{65}\cdot\frac{12}{13} + \frac{63}{65}\cdot\frac{5}{13} = \frac{192+315}{845} = \frac{3}{5}.$$

Answer: A)

9. **Question 9:**

$$\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\cdot\frac{3}{5}}\right)=\tan^{-1}\frac{27}{11}.$$

$$\tan^{-1}\frac{27}{11}-\tan^{-1}\frac{8}{19}=\tan^{-1}\left(\frac{\frac{27}{11}-\frac{8}{19}}{1+\frac{27}{11}\cdot\frac{8}{19}}\right)=\tan^{-1}\frac{425}{425}=\tan^{-1}1=\frac{\pi}{4}.$$

Answer: A)

10. **Question 10:**

$$\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}=\sin^{-1}\left(\frac{4}{5}\cdot\frac{12}{13}+\frac{5}{13}\cdot\frac{3}{5}\right)=\sin^{-1}\frac{63}{65}.$$

$$\sin^{-1}\frac{63}{65}+\sin^{-1}\frac{16}{65}=\sin^{-1}\left(\frac{16}{65}\cdot\frac{63}{65}+\frac{63}{65}\cdot\frac{16}{65}\right)=\sin^{-1}1=\frac{\pi}{2}.$$

Answer: A)

11. **Question 11:**

$$\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6}=\tan^{-1}\left(\frac{\frac{1}{11}+\frac{5}{6}}{1-\frac{1}{11}\cdot\frac{5}{6}}\right)=\tan^{-1}1=\frac{\pi}{4}.$$

$$\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}=\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\cdot\frac{1}{2}}\right)=\tan^{-1}1=\frac{\pi}{4}.$$

Answer: A)

12. **Question 12:**

$$2\tan^{-1}\frac{1}{3}=\tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}}=\tan^{-1}\frac{3}{4}.$$

$$\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7}=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}}\right)=\tan^{-1}\frac{25}{25}=\tan^{-1}1=\frac{\pi}{4}.$$

Answer: A)

13. **Question 13:**

$$\text{Let } \alpha=\sin^{-1}\frac{1}{2},\ \sin\alpha=\frac{1}{2}.\ \cos\alpha=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}.$$

Answer: A)

14. **Question 14:**

$$\text{Let } \theta=\cos^{-1}\frac{3}{5},\ \cos\theta=\frac{3}{5},\ \sin\theta=\sqrt{1-\frac{9}{25}}=\frac{4}{5}.$$

$$\sin 2\theta=2\cdot\frac{4}{5}\cdot\frac{3}{5}=\frac{24}{25}.$$

Answer: A)

15. **Question 15:**

$$\text{Let } \alpha = \sin^{-1}\frac{1}{\sqrt{5}},\ \sin\alpha = \frac{1}{\sqrt{5}},\ \cos\alpha = \sqrt{1-\frac{1}{5}} = \frac{2}{\sqrt{5}}.$$

$$\cos 2\alpha = 1 - 2\left(\frac{1}{\sqrt{5}}\right)^2 = 1 - \frac{2}{5} = \frac{3}{5}.$$

Answer: A)

16. **Question 16:**

$$\tan^{-1}\left(-\frac{1}{2}\right) + \tan^{-1}\frac{1}{2} = \tan^{-1}\left(\frac{-\frac{1}{2}+\frac{1}{2}}{1-\left(-\frac{1}{2}\right)\cdot\frac{1}{2}}\right) = \tan^{-1}\frac{0}{1+\frac{1}{4}} = \tan^{-1} 0 = 0.$$

$$\implies \tan^{-1}\left(-\frac{1}{2}\right) = -\tan^{-1}\frac{1}{2}.$$

Answer: A)

17. **Question 17:**

$$\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) + \sin^{-1}\frac{1}{\sqrt{2}} = \sin^{-1}\left(\left(-\frac{1}{\sqrt{2}}\right)\cdot\sqrt{1-\frac{1}{2}} + \frac{1}{\sqrt{2}}\cdot\sqrt{1-\frac{1}{2}}\right) = \sin^{-1} 0 = 0.$$

$$\implies \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\sin^{-1}\frac{1}{\sqrt{2}}.$$

Answer: A)

18. **Question 18:**

$$\text{Let } \theta = \cos^{-1}\frac{1}{2},\ \cos\theta = \frac{1}{2},\ \theta = \frac{\pi}{3}.\ \cos^{-1}\left(-\frac{1}{2}\right) = \cos^{-1}(-\cos\theta) = \pi - \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}.$$

Answer: A)

19. **Question 19:**

$$\text{Let } \alpha = \sin^{-1}\frac{3}{5},\ \sin\alpha = \frac{3}{5},\ \cos\alpha = \sqrt{1-\frac{9}{25}} = \frac{4}{5}.$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}.$$

Answer: A)

20. **Question 20:**

$$x = \sin^{-1}\frac{1}{2} \implies \sin x = \frac{1}{2},\ x = \frac{\pi}{6}.\ \cos x = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}.$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}.$$

Answer: A)