Conceptual Multiple Choice Questions: Quadratic Equations and Roots (Exercise 4.6)

Class 11 Mathematics (Chapter 4)

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MCQs

1. If α, β are roots of $3x^2 - 2x + 4 = 0$, the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ is:

(a) $-\frac{5}{4}$
(b) $\frac{5}{4}$
(c) $-\frac{20}{9}$
(d) $\frac{20}{9}$

2. For the same equation, the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is:

(a) $-\frac{5}{3}$
(b) $\frac{5}{3}$
(c) $-\frac{20}{9}$
(d) $\frac{20}{9}$

3. For the same equation, the value of $\alpha^4 + \beta^4$ is:

(a) $\frac{112}{81}$
(b) $\frac{400}{81}$
(c) $\frac{32}{9}$
(d) $\frac{288}{81}$

4. For the same equation, the value of $\alpha^3 + \beta^3$ is:

(a) $-\frac{64}{27}$
(b) $\frac{64}{27}$
(c) $-\frac{32}{9}$
(d) $\frac{32}{9}$

5. For the same equation, the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ is:

(a) -1
(b) 1
(c) $-\frac{64}{27}$
(d) $\frac{64}{27}$

6. If α, β are roots of $x^2 - px - p - c = 0$, the value of $(1 + \alpha)(1 + \beta)$ is:

(a) $1 - c$
(b) $1 + c$
(c) $p - c$
(d) $p + c$

7. The condition for one root of $x^2 + px + q = 0$ to be double the other is:

(a) $2p^2 = 9q$
(b) $9p^2 = 2q$
(c) $p^2 = 4q$
(d) $4p^2 = q$

8. The condition for one root to be the square of the other in $x^2 + px + q = 0$ is:

(a) $p^3 + q^2 + q = 3pq$
(b) $p^3 + q^2 - q = 3pq$
(c) $p^2 + q = 3pq$
(d) $p^3 - q^2 = 3pq$

9. The condition for one root to be the additive inverse of the other in $x^2 + px + q = 0$ is:

(a) $p = 0$
(b) $q = 0$
(c) $p = q$
(d) $p^2 = q$

10. The condition for one root to be the multiplicative inverse of the other in $x^2 + px + q = 0$ is:

(a) $q = 1$
(b) $p = 1$
(c) $q = -1$
(d) $p = -1$

11. If the roots of $x^2 - px + q = 0$ differ by 1, the condition is:

(a) $p^2 = 4q + 1$
(b) $p^2 = 4q - 1$
(c) $p^2 = q + 1$
(d) $p^2 = q - 1$

12. For $\frac{a}{x-a} + \frac{b}{x-b} = 5$ to have roots equal in magnitude but opposite in signs, the condition is:

(a) $a + b = 0$

(b) $a - b = 0$
(c) $ab = 0$
(d) $a + b = 5$

13. If α, β are roots of $px^2 + qx + q = 0$, the value of $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}}$ is:

(a) 0
(b) 1
(c) $\sqrt{\frac{q}{p}}$
(d) $-\sqrt{\frac{q}{p}}$

14. The equation with roots α^2, β^2 for $ax^2 + bx + c = 0$ is:

(a) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$
(b) $a^2x^2 - (b^2 + 2ac)x + c^2 = 0$
(c) $ax^2 - (b^2 - 2ac)x + c = 0$
(d) $ax^2 - (b^2 + 2ac)x + c = 0$

15. The equation with roots $\frac{1}{\alpha}, \frac{1}{\beta}$ for $ax^2 + bx + c = 0$ is:

(a) $cx^2 + bx + a = 0$
(b) $ax^2 + bx + c = 0$
(c) $cx^2 - bx + a = 0$
(d) $ax^2 - bx + c = 0$

16. The equation with roots $\frac{3}{\alpha}, \frac{3}{\beta}$ for $5x^2 - x - 2 = 0$ is:

(a) $2x^2 + 3x - 45 = 0$
(b) $2x^2 - 3x + 45 = 0$
(c) $x^2 + 3x - 45 = 0$
(d) $x^2 - 3x + 45 = 0$

17. The equation with roots $\frac{1-\alpha}{1+\alpha}, \frac{1-\beta}{1+\beta}$ for $x^2 - 3x + 5 = 0$ is:

(a) $9x^2 + 8x + 3 = 0$
(b) $9x^2 - 8x + 3 = 0$
(c) $x^2 + 8x + 3 = 0$
(d) $x^2 - 8x + 3 = 0$

18. The equation with roots α^3, β^3 for $ax^2 + bx + c = 0$ is:

(a) $a^3x^2 + b(b^2 - 3ac)x + c^3 = 0$
(b) $a^3x^2 - b(b^2 - 3ac)x + c^3 = 0$
(c) $ax^2 + b(b^2 - 3ac)x + c = 0$

(d) $ax^2 - b(b^2 - 3ac)x + c = 0$

19. The sum of roots $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ for $x^2 - 2x + 3 = 0$ is:

(a) $-\frac{2}{3}$
(b) $\frac{2}{3}$
(c) -2
(d) 2

20. The condition for roots of $x^2 + px + q = 0$ to satisfy $\alpha^2 = \beta$ is:

(a) $p^3 + q^2 + q = 3pq$
(b) $p^3 + q^2 - q = 3pq$
(c) $p^2 = q$
(d) $p^3 = q^2$

Answers and Explanations

1. **Answer: a**
Explanation: For $3x^2 - 2x + 4 = 0$, $\alpha + \beta = \frac{2}{3}$, $\alpha\beta = \frac{4}{3}$. Compute $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$. Since $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(\frac{2}{3}\right)^2 - 2 \cdot \frac{4}{3} = \frac{4}{9} - \frac{8}{3} = \frac{-20}{9}$, we get $\frac{\frac{-20}{9}}{\left(\frac{4}{3}\right)^2} = \frac{-20}{9} \cdot \frac{9}{16} = -\frac{5}{4}$. Option (a) is correct; others do not match.

2. **Answer: a**
Explanation: Using the same roots, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$. With $\alpha^2 + \beta^2 = -\frac{20}{9}$, compute $\frac{-\frac{20}{9}}{\frac{4}{3}} = -\frac{20}{9} \cdot \frac{3}{4} = -\frac{5}{3}$. Option (a) is correct; others yield incorrect values.

3. **Answer: a**
Explanation: Compute $\alpha^4 + \beta^4 = [(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$. Since $(\alpha+\beta)^2 - 2\alpha\beta = \frac{4}{9} - \frac{8}{3} = -\frac{20}{9}$, we get $\left(-\frac{20}{9}\right)^2 - 2 \cdot \left(\frac{4}{3}\right)^2 = \frac{400}{81} - \frac{32}{9} = \frac{112}{81}$. Option (a) is correct; others are incorrect.

4. **Answer: a**
Explanation: Compute $\alpha^3 + \beta^3 = (\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta] = \frac{2}{3} \cdot \left[\left(\frac{2}{3}\right)^2 - 3 \cdot \frac{4}{3}\right] = \frac{2}{3} \cdot \left[\frac{4}{9} - 4\right] = \frac{2}{3} \cdot \frac{-32}{9} = -\frac{64}{27}$. Option (a) is correct; others do not match.

5. **Answer: a**
Explanation: Compute $\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3+\beta^3}{(\alpha\beta)^3}$. Using $\alpha^3 + \beta^3 = -\frac{64}{27}$, $\frac{-\frac{64}{27}}{\left(\frac{4}{3}\right)^3} = -\frac{64}{27} \cdot \frac{27}{64} = -1$. Option (a) is correct; others are incorrect.

6. **Answer: a**
Explanation: For $x^2 - px - p - c = 0$, $\alpha + \beta = p$, $\alpha\beta = -p - c$. Compute $(1+\alpha)(1+\beta) = 1 + \alpha + \beta + \alpha\beta = 1 + p + (-p - c) = 1 - c$. Option (a) is correct; others do not simplify to $1 - c$.

7. **Answer: a**
Explanation: For roots $\alpha, 2\alpha$, $\alpha + 2\alpha = -p \implies 3\alpha = -p$, $\alpha \cdot 2\alpha = q \implies 2\alpha^2 = q$. Substitute $\alpha = -\frac{p}{3}$ into $q = 2 \cdot \left(-\frac{p}{3}\right)^2 = \frac{2p^2}{9}$, so $2p^2 = 9q$. Option (a) is correct; others yield incorrect relations.

8. **Answer: a**
Explanation: For roots α, α^2, $\alpha + \alpha^2 = -p$, $\alpha^3 = q$. Cube the sum: $(\alpha + \alpha^2)^3 = (-p)^3 \implies \alpha^3 + (\alpha^2)^3 + 3\alpha^3(\alpha + \alpha^2) = -p^3$. Substitute $\alpha^3 = q$, $\alpha + \alpha^2 = -p$ to get $q + q^2 - 3pq = -p^3 \implies p^3 + q^2 + q = 3pq$. Option (a) is correct.

9. **Answer: a**
Explanation: For roots $\alpha, -\alpha$, $\alpha + (-\alpha) = -p \implies p = 0$, $\alpha \cdot (-\alpha) = q \implies q = -\alpha^2$. The condition is $p = 0$. Option (a) is correct; others do not satisfy the sum.

10. **Answer: a**
Explanation: For roots $\alpha, \frac{1}{\alpha}$, $\alpha + \frac{1}{\alpha} = -p$, $\alpha \cdot \frac{1}{\alpha} = q \implies q = 1$. Option (a) is correct; others do not yield $q = 1$.

11. **Answer: a**
Explanation: For roots $\alpha, \alpha - 1$, $2\alpha - 1 = p$, $\alpha(\alpha - 1) = q$. Substitute $\alpha = \frac{p+1}{2}$ into $q = \left(\frac{p+1}{2}\right)^2 - \frac{p+1}{2} = \frac{p^2-1}{4}$, so $p^2 = 4q + 1$. Option (a) is correct; others are incorrect.

12. **Answer: a**
Explanation: Simplify $\frac{a}{x-a} + \frac{b}{x-b} = 5$ to $5x^2 - 6(a+b)x + 7ab = 0$. For roots $\alpha, -\alpha$, the sum is $\alpha + (-\alpha) = \frac{6(a+b)}{5} = 0 \implies a + b = 0$. Option (a) is correct; others do not satisfy the sum.

13. **Answer: a**
Explanation: For $px^2 + qx + q = 0$, $\alpha + \beta = -\frac{q}{p}$, $\alpha\beta = \frac{q}{p}$. Compute $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \sqrt{\frac{q}{p}} = \frac{\alpha+\beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{q}{p}} = \frac{-\frac{q}{p}}{\sqrt{\frac{q}{p}}} + \sqrt{\frac{q}{p}} = 0$. Option (a) is correct.

14. **Answer: a**
Explanation: For roots α^2, β^2, sum = $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{b^2-2ac}{a^2}$, product = $\alpha^2\beta^2 = (\alpha\beta)^2 = \frac{c^2}{a^2}$. The equation is $x^2 - \frac{b^2-2ac}{a^2}x + \frac{c^2}{a^2} = 0 \implies a^2x^2 - (b^2 - 2ac)x + c^2 = 0$. Option (a) is correct.

15. **Answer: a**
Explanation: For roots $\frac{1}{\alpha}, \frac{1}{\beta}$, sum = $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = -\frac{\frac{b}{a}}{\frac{c}{a}} = -\frac{b}{c}$, product = $\frac{1}{\alpha\beta} = \frac{a}{c}$. The equation is $x^2 + \frac{b}{c}x + \frac{a}{c} = 0 \implies cx^2 + bx + a = 0$. Option (a) is correct.

16. **Answer: a**
Explanation: For $5x^2 - x - 2 = 0$, $\alpha + \beta = \frac{1}{5}$, $\alpha\beta = -\frac{2}{5}$. Sum = $\frac{3}{\alpha} + \frac{3}{\beta} = \frac{3(\alpha+\beta)}{\alpha\beta} = \frac{3 \cdot \frac{1}{5}}{-\frac{2}{5}} = -\frac{3}{2}$, product = $\frac{9}{\alpha\beta} = \frac{9}{-\frac{2}{5}} = -\frac{45}{2}$. The equation is $x^2 + \frac{3}{2}x - \frac{45}{2} = 0 \implies 2x^2 + 3x - 45 = 0$. Option (a) is correct.

17. **Answer: a**
Explanation: For $x^2 - 3x + 5 = 0$, $\alpha + \beta = 3$, $\alpha\beta = 5$. Sum = $\frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta} = \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-10}{1+3+5} = -\frac{8}{9}$, product = $\frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)} = \frac{1-(\alpha+\beta)+\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{3}{9} = \frac{1}{3}$. The equation is $x^2 + \frac{8}{9}x + \frac{1}{3} = 0 \implies 9x^2 + 8x + 3 = 0$. Option (a) is correct.

18. **Answer: b**
Explanation: For roots α^3, β^3, sum = $\alpha^3 + \beta^3 = (\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta] = -\frac{b}{a} \cdot \frac{b^2-3ac}{a^2} = -\frac{b(b^2-3ac)}{a^3}$, product = $(\alpha\beta)^3 = \frac{c^3}{a^3}$. The equation is $x^2 + \frac{b(b^2-3ac)}{a^3}x + \frac{c^3}{a^3} = 0 \implies a^3x^2 - b(b^2 - 3ac)x + c^3 = 0$. Option (b) is correct.

19. **Answer: a**
Explanation: For $x^2 - 2x + 3 = 0$, $\alpha + \beta = 2$, $\alpha\beta = 3$. Compute $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$, where $\alpha^2 + \beta^2 = 2^2 - 2 \cdot 3 = 4 - 6 = -2$, so $\frac{-2}{3^2} = -\frac{2}{9}$. Option (a) is correct; others do not match.

20. **Answer: a**
Explanation: For $\alpha^2 = \beta$, the roots are β, β^2. Sum: $\beta + \beta^2 = -p$, product: $\beta^3 = q$. Cube the sum: $(\beta + \beta^2)^3 = (-p)^3 \implies \beta^3 + (\beta^2)^3 + 3\beta^3(\beta + \beta^2) = -p^3$. Substitute $\beta^3 = q$, $\beta + \beta^2 = -p$ to get $q + q^2 - 3pq = -p^3 \implies p^3 + q^2 + q = 3pq$. Option (a) is correct.