Cheatsheet: Partial Fractions (Exercise 5.2)

Class 11 Mathematics (Chapter 5)

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Overview

Exercise 5.2 focuses on resolving rational functions into partial fractions where the denominator includes repeated linear factors, e.g., $(x-a)^n$, often combined with distinct linear factors. The goal is to decompose $\frac{P(x)}{Q(x)}$ into simpler fractions, handling both proper and improper fractions.

Note

For repeated linear factors $(x-a)^n$, include terms $\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \cdots + \frac{A_n}{(x-a)^n}$. Always check if the fraction is improper.

Key Concepts

1. Rational Function A rational function is $\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials, $Q(x) \neq 0$, with no common factors.

- **Proper**: Degree of $P(x) <$ degree of $Q(x)$. E.g., $\frac{1}{(x-1)^2(x+1)}$.
- **Improper**: Degree of $P(x) \geq$ degree of $Q(x)$. E.g., $\frac{2x^4}{(x-3)(x+2)^2}$.

2. Partial Fraction Resolution for Repeated Linear Factors For a denominator with a repeated linear factor $(x-a)^n$, the partial fraction form includes:

$$\frac{P(x)}{(x-a)^n} = \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \cdots + \frac{A_n}{(x-a)^n}$$

If combined with distinct factors, e.g., $(x-a)^n(x-b)$, include:

$$\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \cdots + \frac{A_n}{(x-a)^n} + \frac{B}{x-b}$$

3. Steps for Partial Fraction Decomposition

1. **Check Fraction Type**: If improper, divide $P(x)$ by $Q(x)$ to get a polynomial plus a proper fraction.
2. **Factor Denominator**: Identify repeated and distinct linear factors.
3. **Set Up Partial Fractions**: Include a term for each power of repeated factors and each distinct factor.
4. **Solve for Constants**: Use substitution (set x to roots of denominators) and/or equate coefficients.
5. **Combine**: Write the final sum of partial fractions.

Techniques for Exercise 5.2

1. Repeated Linear Factors Only For $\frac{P(x)}{(x-a)^n}$, assume:

$$\frac{P(x)}{(x-a)^n} = \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \cdots + \frac{A_n}{(x-a)^n}$$

Multiply through, equate numerators, and solve. E.g., $\frac{2x^2-3x+4}{(x-1)^3}$ (Q.1, page 358):

- Set $x = 1$ to find A_n.
- Equate coefficients of x^2, x to find A_1, A_2.

2. Repeated and Distinct Linear Factors For $\frac{P(x)}{(x-a)^n(x-b)}$, assume:

$$\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \cdots + \frac{A_n}{(x-a)^n} + \frac{B}{x-b}$$

E.g., $\frac{4x}{(x+1)^2(x-1)}$ (Q.3, page 360):

- Set $x = a, b$ to find A_n, B.
- Equate coefficients for remaining constants.

3. Improper Fractions Divide to obtain:

$$\frac{P(x)}{Q(x)} = \text{Polynomial} + \frac{\text{Remainder}}{Q(x)}$$

Resolve the proper remainder. E.g., $\frac{2x^4}{(x-3)(x+2)^2} = 2x - 2 + \frac{18x^2+8x-24}{(x-3)(x+2)^2}$ (Q.12, page 369).

4. Multiple Repeated Factors For $\frac{P(x)}{(x-a)^n(x-b)^m}$, include terms for each power up to n and m. E.g., $\frac{x-1}{(x-2)(x+1)^3}$ (Q.9, page 366).

Solving for Constants

- **Substitution Method**: Set x to roots of each denominator factor (e.g., $x = a$ for $(x-a)^n$) to find constants like A_n, B.
- **Coefficient Method**: Equate coefficients of like powers of x to solve for remaining constants.

Common Errors to Avoid

- Omitting terms for higher powers of repeated factors.
- Incorrect polynomial division for improper fractions.
- Arithmetic errors in coefficient equations.
- Failing to verify by recombining fractions.

Tip

Always include a term for each power of a repeated factor up to its multiplicity. Verify by substituting back or checking the numerator degree.