Inverse Trigonometric Functions MCQs for Entry Test - Exercise 13.1

1 Introduction

This document contains 20 multiple-choice questions based on Exercise 13.1 of the Inverse Trigonometric Functions chapter, designed for entry test preparation. Questions test evaluating inverse trigonometric functions, proving equalities, and computing composite expressions. Solutions provide detailed explanations. Notation: $y = \sin^{-1} x \iff x = \sin y$, with restricted domains. All evaluations are exact, without calculators.

2 Multiple-Choice Questions

1. What is the value of $\sin^{-1}(1)$?

    A) $\frac{\pi}{2}$

    B) $\frac{\pi}{3}$

    C) $\frac{\pi}{4}$

    D) 0

2. What is the value of $\sin^{-1}(-1)$?

    A) $-\frac{\pi}{2}$

    B) $-\frac{\pi}{3}$

    C) $-\frac{\pi}{4}$

    D) $-\pi$

3. What is the value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$?

    A) $\frac{\pi}{6}$

    B) $\frac{\pi}{3}$

    C) $\frac{\pi}{4}$

    D) $\frac{\pi}{2}$

4. What is the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$?

    A) $-\frac{\pi}{6}$

    B) $-\frac{\pi}{4}$

C) $-\frac{\pi}{3}$

D) $\frac{\pi}{6}$

5. What is the value of $\cos^{-1}\left(\frac{1}{2}\right)$?

    A) $\frac{\pi}{3}$

    B) $\frac{\pi}{6}$

    C) $\frac{\pi}{4}$

    D) $\frac{\pi}{2}$

6. What is the value of $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$?

    A) $\frac{\pi}{6}$

    B) $\frac{\pi}{4}$

    C) $\frac{\pi}{3}$

    D) 0

7. What is the value of $\cot^{-1}(-1)$?

    A) $\frac{3\pi}{4}$

    B) $\frac{\pi}{4}$

    C) $\frac{\pi}{2}$

    D) $-\frac{\pi}{4}$

8. What is the value of $\csc^{-1}\left(-\frac{2}{\sqrt{3}}\right)$?

    A) $-\frac{\pi}{3}$

    B) $-\frac{\pi}{4}$

    C) $-\frac{\pi}{6}$

    D) $\frac{\pi}{3}$

9. What is the value of $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$?

    A) $-\frac{\pi}{4}$

    B) $-\frac{\pi}{3}$

    C) $-\frac{\pi}{6}$

    D) $\frac{\pi}{4}$

10. Which equality is true?

    A) $\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$

    B) $\tan^{-1}\frac{5}{12} = \cos^{-1}\frac{5}{13}$

    C) $\tan^{-1}\frac{5}{12} = \cot^{-1}\frac{5}{13}$

    D) $\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{12}{13}$

11. Which equality is true?

    A) $2\cos^{-1}\frac{4}{5} = \sin^{-1}\frac{24}{25}$

    B) $2\cos^{-1}\frac{4}{5} = \cos^{-1}\frac{24}{25}$

    C) $2\cos^{-1}\frac{4}{5} = \tan^{-1}\frac{24}{25}$

    D) $2\cos^{-1}\frac{4}{5} = \sin^{-1}\frac{7}{25}$

12. Which equality is true?

    A) $\cos^{-1}\frac{4}{5} = \cot^{-1}\frac{4}{3}$

    B) $\cos^{-1}\frac{4}{5} = \tan^{-1}\frac{4}{3}$

    C) $\cos^{-1}\frac{4}{5} = \sin^{-1}\frac{4}{3}$

    D) $\cos^{-1}\frac{4}{5} = \cot^{-1}\frac{3}{4}$

13. What is the value of $\cos\left(\sin^{-1}\frac{1}{\sqrt{2}}\right)$?

    A) $\frac{1}{\sqrt{2}}$

    B) $\frac{1}{2}$

    C) $\frac{\sqrt{3}}{2}$

    D) 0

14. What is the value of $\sec\left(\cos^{-1}\frac{1}{2}\right)$?

    A) 2

    B) $\sqrt{2}$

    C) $\frac{2}{\sqrt{3}}$

    D) 1

15. What is the value of $\tan\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$?

    A) $\frac{1}{\sqrt{3}}$

    B) $\sqrt{3}$

    C) 1

    D) $\frac{\sqrt{3}}{2}$

16. What is the value of $\csc\left(\tan^{-1}(-1)\right)$?

    A) $-\sqrt{2}$

    B) $\sqrt{2}$

    C) -1

    D) 1

17. What is the value of $\sec\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$?

    A) $\frac{2}{\sqrt{3}}$

B) 2

C) $\sqrt{2}$

D) $\frac{\sqrt{3}}{2}$

18. What is the value of $\tan\left(\tan^{-1}(-1)\right)$?

    A) -1

    B) 1

    C) $-\sqrt{3}$

    D) $\sqrt{3}$

19. What is the value of $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$?

    A) $\frac{1}{2}$

    B) $\frac{\sqrt{3}}{2}$

    C) $\frac{1}{\sqrt{2}}$

    D) 1

20. What is the value of $\sin\left(\tan^{-1}(-1)\right)$?

    A) $-\frac{1}{\sqrt{2}}$

    B) $\frac{1}{\sqrt{2}}$

    C) $-\frac{1}{2}$

    D) $\frac{1}{2}$

3 Solutions and Explanations

1. **Question 1:**

$$\sin^{-1}(1) \implies \sin y = 1,\ y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \implies y = \frac{\pi}{2}.$$

Answer: A)

2. **Question 2:**

$$\sin^{-1}(-1) \implies \sin y = -1,\ y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \implies y = -\frac{\pi}{2}.$$

Answer: A)

3. **Question 3:**

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \implies \cos y = \frac{\sqrt{3}}{2},\ y \in [0, \pi] \implies y = \frac{\pi}{6}.$$

Answer: A)

4. **Question 4:**

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) \implies \tan y = -\frac{1}{\sqrt{3}}, \; y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \implies y = -\frac{\pi}{6}.$$

Answer: A)

5. **Question 5:**

$$\cos^{-1}\left(\frac{1}{2}\right) \implies \cos y = \frac{1}{2}, \; y \in [0, \pi] \implies y = \frac{\pi}{3}.$$

Answer: A)

6. **Question 6:**

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \implies \tan y = \frac{1}{\sqrt{3}}, \; y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \implies y = \frac{\pi}{6}.$$

Answer: A)

7. **Question 7:**

$$\cot^{-1}(-1) \implies \cot y = -1, \; y \in (0, \pi) \implies \tan y = -1 \implies y = \frac{3\pi}{4}.$$

Answer: A)

8. **Question 8:**

$$\csc^{-1}\left(-\frac{2}{\sqrt{3}}\right) \implies \csc y = -\frac{2}{\sqrt{3}}, \; \sin y = -\frac{\sqrt{3}}{2}, \; y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\} \implies y = -\frac{\pi}{3}.$$

Answer: A)

9. **Question 9:**

$$\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) \implies \sin y = -\frac{1}{\sqrt{2}}, \; y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \implies y = -\frac{\pi}{4}.$$

Answer: A)

10. **Question 10:**

Let $\theta = \tan^{-1}\frac{5}{12}$, $\tan \theta = \frac{5}{12}$. Right triangle: opposite 5, adjacent 12, hypotenuse $\sqrt{5^2 + 12^2} = $

Answer: A)

11. **Question 11:**

Let $\theta = \cos^{-1}\frac{4}{5}$, $\cos \theta = \frac{4}{5}$, $\sin \theta = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$. $\sin 2\theta = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25} \implies 2\theta = \sin^{-1}\frac{24}{25}.$

Answer: A)

12. **Question 12:**

Let $\theta = \cos^{-1} \dfrac{4}{5}$, $\cos\theta = \dfrac{4}{5}$, $\sin\theta = \dfrac{3}{5}$. $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{4/5}{3/5} = \dfrac{4}{3} \implies \theta = \cot^{-1} \dfrac{4}{3}$.

Answer: A)

13. **Question 13:**

$$\text{Let } y = \sin^{-1} \dfrac{1}{\sqrt{2}}, \ \sin y = \dfrac{1}{\sqrt{2}}, \ y = \dfrac{\pi}{4}. \ \cos y = \cos \dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}.$$

Answer: A)

14. **Question 14:**

$$\text{Let } y = \cos^{-1} \dfrac{1}{2}, \ \cos y = \dfrac{1}{2}, \ y = \dfrac{\pi}{3}. \ \sec y = \sec \dfrac{\pi}{3} = \dfrac{1}{\cos \frac{\pi}{3}} = 2.$$

Answer: A)

15. **Question 15:**

$$\text{Let } y = \cos^{-1} \dfrac{\sqrt{3}}{2}, \ \cos y = \dfrac{\sqrt{3}}{2}, \ y = \dfrac{\pi}{6}. \ \tan y = \tan \dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}}.$$

Answer: A)

16. **Question 16:**

Let $y = \tan^{-1}(-1)$, $\tan y = -1$, $y = -\dfrac{\pi}{4}$. $\csc y = \csc\left(-\dfrac{\pi}{4}\right) = \dfrac{1}{\sin\left(-\frac{\pi}{4}\right)} = \dfrac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$.

Answer: A)

17. **Question 17:**

Let $y = \sin^{-1}\left(-\dfrac{1}{2}\right)$, $\sin y = -\dfrac{1}{2}$, $y = -\dfrac{\pi}{6}$. $\sec y = \sec\left(-\dfrac{\pi}{6}\right) = \dfrac{1}{\cos \frac{\pi}{6}} = \dfrac{1}{\frac{\sqrt{3}}{2}} = \dfrac{2}{\sqrt{3}}$.

Answer: A)

18. **Question 18:**

$$\text{Let } y = \tan^{-1}(-1), \ \tan y = -1, \ y = -\dfrac{\pi}{4}. \ \tan y = \tan\left(-\dfrac{\pi}{4}\right) = -1.$$

Answer: A)

19. **Question 19:**

$$\text{Let } y = \sin^{-1}\left(\dfrac{1}{2}\right), \ \sin y = \dfrac{1}{2}, \ y = \dfrac{\pi}{6}. \ \sin y = \sin \dfrac{\pi}{6} = \dfrac{1}{2}.$$

Answer: A)

20. **Question 20:**

$$\text{Let } y = \tan^{-1}(-1), \ \tan y = -1, \ y = -\dfrac{\pi}{4}. \ \sin y = \sin\left(-\dfrac{\pi}{4}\right) = -\dfrac{1}{\sqrt{2}}.$$

Answer: A)