Cheatsheet: Nature of Roots and Systems of Equations (Exercise 4.7)

Class 11 Mathematics (Chapter 4)

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Overview

Exercise 4.7 focuses on determining the nature of roots of quadratic equations using the discriminant and finding conditions for equal or rational roots. It also introduces solving systems of simultaneous equations where one is linear and one is quadratic.

Note

The discriminant $D = b^2 - 4ac$ determines the nature of roots. Always simplify expressions carefully to ensure accurate results.

Nature of Roots of a Quadratic Equation

Concept For a quadratic equation $ax^2 + bx + c = 0$, the roots are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The discriminant $D = b^2 - 4ac$ determines the nature of the roots:

- $D = 0$: Roots are real, equal, and rational (if coefficients are rational).
- $D > 0$: Roots are real and distinct. If D is a perfect square, roots are rational; otherwise, irrational.
- $D < 0$: Roots are complex and distinct.

Example For $4x^2 + 6x + 1 = 0$:

$$a = 4, b = 6, c = 1 \implies D = 6^2 - 4 \cdot 4 \cdot 1 = 36 - 16 = 20$$

Since $D > 0$ and not a perfect square, roots are real, distinct, and irrational.

Conditions for Equal Roots

Concept Roots are equal when $D = 0$. Solve for parameters by setting the discriminant to zero.

Example For $(m+1)x^2 + 2(m+3)x + m + 8 = 0$:

$$a = m + 1, b = 2(m + 3), c = m + 8$$

$$D = [2(m + 3)]^2 - 4(m + 1)(m + 8) = 4(m^2 + 6m + 9) - 4(m^2 + 9m + 8) = -12m + 4$$

Set $D = 0$:

$$-12m + 4 = 0 \implies m = \frac{1}{3}$$

Conditions for Rational Roots

Concept Roots are rational if $D \geq 0$ and D is a perfect square. This ensures the roots are expressible as fractions.
Example For $(p+q)x^2 - px - q = 0$:

$$a = p + q, b = -p, c = -q$$

$$D = (-p)^2 - 4(p+q)(-q) = p^2 + 4pq + 4q^2 = (p + 2q)^2$$

Since D is a perfect square, roots are rational.

Systems of Simultaneous Equations

Concept For a system with one linear and one quadratic equation:

1. Solve the linear equation for one variable.
2. Substitute into the quadratic equation to form a single quadratic equation.
3. Solve the resulting quadratic equation.

Example For:

$$x + y = 3, \quad x^2 + y^2 = 5$$

From the linear equation, $y = 3 - x$. Substitute into the quadratic:

$$x^2 + (3 - x)^2 = 5 \implies 2x^2 - 6x + 4 = 0 \implies x^2 - 3x + 2 = 0$$

Solve to find $x = 1, 2$, then $y = 2, 1$.

Tip

Check the discriminant for real roots before solving systems, as complex roots may indicate no real solutions.

Key Reminders

- Verify if D is a perfect square for rational roots.
- For equal roots, ensure the discriminant simplifies to zero.
- When solving systems, substitution from the linear equation simplifies the process.
- Always check for restrictions (e.g., $a \neq 0$, $m \neq 0$).