Geometric Progression Cheatsheet - Exercises 6.6 and 6.7 (Class 11 Mathematics)

Prepared for Entry Test Preparation

1. Geometric Progression Basics (Ex. 6.6)

A sequence is a geometric progression (G.P.) if the ratio $\frac{a_n}{a_{n-1}} = r$ (common ratio, non-zero) for all $n > 1$. General term: $a_n = a_1 r^{n-1}$.

2. Key Formulas and Concepts for Exercise 6.6

- **n-th Term**: $a_n = a_1 r^{n-1}$.
- **Complex Numbers in G.P.**: For sequences with complex terms, compute $r = \frac{a_2}{a_1}$, then simplify $a_n = a_1 r^{n-1}$ using $i^2 = -1$.
- **Depreciation**: Value after n years with depreciation rate p%: $V_n = V_0(1 - \frac{p}{100})^n$.
- **Finding n**: If a_n is given, solve $a_n = a_1 r^{n-1}$ for n.
- **Properties of G.P.**: If a, b, c, d are in G.P., then:
  - $a - b, b - c, c - d$ are in G.P.
  - $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are in G.P.
  - $a^2 + b^2, b^2 + c^2, c^2 + d^2$ are in G.P.
- **Reciprocals**: Reciprocals of a G.P. form a G.P. with ratio $\frac{1}{r}$.
- **Three Terms in G.P.**: Sum S, product P, terms: $\frac{a}{r}, a, ar$. Solve $a^3 = P$, $\frac{a}{r} + a + ar = S$.
- **Four Terms in G.P.**: Sum S, A.M. of second and fourth terms M, terms: a, ar, ar^2, ar^3. Solve $a(1 + r + r^2 + r^3) = S$, $ar(1 + r^2) = 2M$.
- **Reciprocals' Common Ratio**: If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P., common ratio is $\pm\sqrt{\frac{a}{c}}$.
- **A.P. to G.P.**: If A.P. terms modified by constants form a G.P., solve for a, d using G.P. ratio condition.

3. Key Concepts for Exercise 6.7

- **Geometric Mean (G.M.)**: For numbers a, b, G.M. = $\pm\sqrt{ab}$.
- **Inserting G.M.s**: To insert k G.M.s between a and b, form G.P. $a, G_1, \ldots, G_k, b$. Solve $ar^{k+1} = b$, compute $G_i = ar^i$.
- **G.M. vs. A.M.**: For positive distinct x, y, G.M. $<$ A.M., i.e., $\sqrt{xy} < \frac{x+y}{2}$.

- **G.M. in Expressions**: Solve for n if $\frac{a^n+b^n}{a^{n-1}+b^{n-1}} = \sqrt{ab}$.
- **A.M. and G.M. Relations**: Given A.M. and G.M., solve $a + b = 2 \cdot \text{A.M.}, ab = (\text{G.M.})^2$.

4. Examples from Exercises 6.6 and 6.7

Finding n-th Term (Ex. 6.6, Q1)

Problem: Find 5th term of G.P. $3, 6, 12, \ldots$.

- $a_1 = 3$, $r = \frac{6}{3} = 2$, $n = 5$.
- $a_5 = 3 \cdot 2^{5-1} = 3 \cdot 16 = 48$.

Complex G.P. (Ex. 6.6, Q2)

Problem: Find 11th term of $1 + i, 2, \frac{4}{1+i}, \ldots$.

- $a_1 = 1 + i$, $r = \frac{2}{1+i}$, $n = 11$.
- $a_{11} = (1 + i) \left(\frac{2}{1+i}\right)^{10} = 2^{10} \cdot \frac{1+i}{(1+i)^{10}} = 32(1 - i)$.

Depreciation (Ex. 6.6, Q5)

Problem: Automobile worth Rs. 12,000 depreciates 5% annually. Value after 4 years?

- $V_4 = 12000 \cdot (1 - 0.05)^4 = 12000 \cdot (0.95)^4 = 9774$.

Three Terms in G.P. (Ex. 6.6, Q10)

Problem: Find three G.P. terms with sum 26, product 216.

- Terms: $\frac{a}{r}, a, ar$. $a^3 = 216 \implies a = 6$. $\frac{6}{r} + 6 + 6r = 26 \implies r = \frac{1}{3}, 3$.
- Numbers: $18, 6, 2$ or $2, 6, 18$.

Inserting G.M.s (Ex. 6.7, Q2(i))

Problem: Insert two G.M.s between 1 and 8.

- G.P.: $1, G_1, G_2, 8$. $1 \cdot r^3 = 8 \implies r = 2$.
- $G_1 = 1 \cdot 2 = 2$, $G_2 = 1 \cdot 2^2 = 4$. G.M.s: $2, 4$.

G.M. vs. A.M. (Ex. 6.7, Q5)

Problem: Show G.M. $<$ A.M. for positive distinct x, y.

- G.M. = $\sqrt{xy}$, A.M. = $\frac{x+y}{2}$. Prove: $(\sqrt{x} - \sqrt{y})^2 > 0$.