Triangle Radii MCQs for Entry Test - Exercise 12.8

1 Introduction

This document contains 20 multiple-choice questions based on Exercise 12.8 of the Application of Trigonometry chapter, designed for entry test preparation. Questions test proving identities involving inradius (r), circumradius (R), and exradii (r_1, r_2, r_3), numerical calculations, and equilateral triangle properties. Solutions provide detailed explanations. Notation: In $\triangle ABC$, angles α, β, γ are opposite sides a, b, c. Semi-perimeter $S = \frac{a+b+c}{2}$, area $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$. Numerical results are exact or to three decimal places.

2 Multiple-Choice Questions

1. Which identity correctly expresses the inradius r in terms of circumradius R?

   A) $r = 4R \sin\frac{\alpha}{2} \sin\frac{\beta}{2} \sin\frac{\gamma}{2}$

   B) $r = 4R \cos\frac{\alpha}{2} \cos\frac{\beta}{2} \cos\frac{\gamma}{2}$

   C) $r = 2R \sin\alpha \sin\beta \sin\gamma$

   D) $r = R \tan\frac{\alpha}{2} \tan\frac{\beta}{2} \tan\frac{\gamma}{2}$

2. Which identity relates the semi-perimeter S to circumradius R?

   A) $S = 4R \sin\frac{\alpha}{2} \sin\frac{\beta}{2} \sin\frac{\gamma}{2}$

   B) $S = 4R \cos\frac{\alpha}{2} \cos\frac{\beta}{2} \cos\frac{\gamma}{2}$

   C) $S = 2R \cos\alpha \cos\beta \cos\gamma$

   D) $S = R \cot\frac{\alpha}{2} \cot\frac{\beta}{2} \cot\frac{\gamma}{2}$

3. Which expression is equivalent to inradius r?

   A) $r = a \sin\frac{\beta}{2} \sin\frac{\gamma}{2} \sec\frac{\alpha}{2}$

   B) $r = a \cos\frac{\beta}{2} \cos\frac{\gamma}{2} \csc\frac{\alpha}{2}$

   C) $r = a \tan\frac{\beta}{2} \tan\frac{\gamma}{2}$

   D) $r = a \cot\frac{\beta}{2} \cot\frac{\gamma}{2}$

4. What is the correct expression for exradius r_1?

   A) $r_1 = 4R \sin\frac{\alpha}{2} \cos\frac{\beta}{2} \cos\frac{\gamma}{2}$

   B) $r_1 = 4R \cos\frac{\alpha}{2} \sin\frac{\beta}{2} \sin\frac{\gamma}{2}$

C) $r_1 = 2R \sin\alpha \cos\beta \cos\gamma$

D) $r_1 = R \tan\alpha$

5. Which identity holds for exradius r_1?

A) $r_1 = S \tan\frac{\alpha}{2}$

B) $r_1 = S \cot\frac{\alpha}{2}$

C) $r_1 = S \sin\frac{\alpha}{2}$

D) $r_1 = S \cos\frac{\alpha}{2}$

6. Which relation is true for the radii in $\triangle ABC$?

A) $r_1 r_2 + r_2 r_3 + r_3 r_1 = S^2$

B) $r_1 r_2 + r_2 r_3 + r_3 r_1 = \Delta^2$

C) $r_1 r_2 + r_2 r_3 + r_3 r_1 = 4R^2$

D) $r_1 r_2 + r_2 r_3 + r_3 r_1 = r^2$

7. What is the product $r_1 r_2 r_3$?

A) Δ^2

B) S^2

C) $4R^2$

D) r^2

8. Which identity relates the sum of exradii and inradius?

A) $r_1 + r_2 + r_3 - r = 4R$

B) $r_1 + r_2 + r_3 - r = 2R$

C) $r_1 + r_2 + r_3 - r = S$

D) $r_1 + r_2 + r_3 - r = \Delta$

9. In $\triangle ABC$ with $a = 13, b = 14, c = 15$, what is the circumradius R?

A) 4.000

B) 8.125

C) 10.500

D) 12.000

10. In $\triangle ABC$ with $a = 13, b = 14, c = 15$, what is the inradius r?

A) 4

B) 7

C) 8

D) 21

11. In $\triangle ABC$ with $a = 13, b = 14, c = 15$, what is exradius r_1?

A) 6

B) 10.5

C) 12

D) 14

12. In $\triangle ABC$ with $a = 34, b = 20, c = 42$, what is the area Δ?

A) 96

B) 336

C) 672

D) 1344

13. In $\triangle ABC$ with $a = 34, b = 20, c = 42$, what is the circumradius R?

A) 7.000

B) 12.000

C) 21.250

D) 24.000

14. In $\triangle ABC$ with $a = 34, b = 20, c = 42$, what is exradius r_3?

A) 12

B) 24

C) 48

D) 56

15. In an equilateral triangle, what is the ratio $r : R : r_1$?

A) $1 : 2 : 3$

B) $1 : 1 : 1$

C) $2 : 3 : 4$

D) $3 : 2 : 1$

16. In an equilateral triangle, what is the ratio $r : R : r_1 : r_2 : r_3$?

A) $1 : 2 : 3 : 3 : 3$

B) $1 : 2 : 3 : 2 : 3$

C) $1 : 1 : 1 : 1 : 1$

D) $2 : 3 : 4 : 4 : 4$

17. Which identity expresses the area Δ in terms of r and cotangents?

A) $\Delta = r^2 \cot \frac{\alpha}{2} \cot \frac{\beta}{2} \cot \frac{\gamma}{2}$

B) $\Delta = r^2 \tan \frac{\alpha}{2} \tan \frac{\beta}{2} \tan \frac{\gamma}{2}$

C) $\Delta = r^2 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2}$

D) $\Delta = r^2 \cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

18. Which identity relates r, R, and reciprocal products?

A) $\frac{1}{2rR} = \frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$
B) $\frac{1}{2rR} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$
C) $\frac{1}{2rR} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$
D) $\frac{1}{2rR} = \frac{1}{\Delta} + \frac{1}{S}$

19. Which identity involves the inradius and half-angles?

A) $r = \frac{a\sin\frac{\beta}{2}\sin\frac{\gamma}{2}}{\cos\frac{\alpha}{2}}$
B) $r = \frac{a\cos\frac{\beta}{2}\cos\frac{\gamma}{2}}{\sin\frac{\alpha}{2}}$
C) $r = \frac{a\tan\frac{\beta}{2}\tan\frac{\gamma}{2}}{\cot\frac{\alpha}{2}}$
D) $r = \frac{a\cot\frac{\beta}{2}\cot\frac{\gamma}{2}}{\tan\frac{\alpha}{2}}$

20. Which identity relates exradii and side c?

A) $(r_1 + r_2)\tan\frac{\gamma}{2} = c$
B) $(r_1 + r_2)\cot\frac{\gamma}{2} = c$
C) $(r_1 + r_2)\sin\frac{\gamma}{2} = c$
D) $(r_1 + r_2)\cos\frac{\gamma}{2} = c$

3 Solutions and Explanations

1. **Question 1:**

$$r = 4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2} = 4\cdot\frac{abc}{4\Delta}\cdot\sqrt{\frac{(S-b)(S-c)}{bc}}\cdot\sqrt{\frac{(S-a)(S-c)}{ac}}\cdot\sqrt{\frac{(S-a)(S-b)}{ab}} = \frac{\Delta}{S} = r.$$

Answer: A)

2. **Question 2:**

$$S = 4R\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2} = 4\cdot\frac{abc}{4\Delta}\cdot\sqrt{\frac{S(S-a)}{bc}}\cdot\sqrt{\frac{S(S-b)}{ac}}\cdot\sqrt{\frac{S(S-c)}{ab}} = S.$$

Answer: B)

3. **Question 3:**

$$r = a\sin\frac{\beta}{2}\sin\frac{\gamma}{2}\sec\frac{\alpha}{2} = a\cdot\sqrt{\frac{(S-a)(S-c)}{ac}}\cdot\sqrt{\frac{(S-a)(S-b)}{ab}}\cdot\sqrt{\frac{bc}{S(S-a)}} = \frac{\Delta}{S} = r.$$

Answer: A)

4. **Question 4:**

$$r_1 = 4R\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2} = 4\cdot\frac{abc}{4\Delta}\cdot\sqrt{\frac{(S-b)(S-c)}{bc}}\cdot\sqrt{\frac{S(S-b)}{ac}}\cdot\sqrt{\frac{S(S-c)}{ab}} = \frac{\Delta}{S-a} = r_1.$$

Answer: A)

5. **Question 5:**

$$r_1 = S\tan\frac{\alpha}{2} = S\cdot\sqrt{\frac{(S-b)(S-c)}{S(S-a)}} = \frac{\Delta}{S-a} = r_1.$$

Answer: A)

6. **Question 6:**

$$r_1r_2+r_2r_3+r_3r_1 = \frac{\Delta}{S-a}\cdot\frac{\Delta}{S-b}+\frac{\Delta}{S-b}\cdot\frac{\Delta}{S-c}+\frac{\Delta}{S-c}\cdot\frac{\Delta}{S-a} = \Delta^2\cdot\frac{3S-(a+b+c)}{(S-a)(S-b)(S-c)} = S^2.$$

Answer: A)

7. **Question 7:**

$$r_1r_2r_3 = \frac{\Delta}{S-a}\cdot\frac{\Delta}{S-b}\cdot\frac{\Delta}{S-c} = \frac{\Delta^3}{(S-a)(S-b)(S-c)} = \Delta^2.$$

Answer: A)

8. **Question 8:**

$$r_1+r_2+r_3-r = \frac{\Delta}{S-a}+\frac{\Delta}{S-b}+\frac{\Delta}{S-c}-\frac{\Delta}{S} = \Delta\cdot\frac{S(2S-(a+b+c))}{S(S-a)(S-b)(S-c)} = \frac{abc}{\Delta} = 4R.$$

Answer: A)

9. **Question 9:**

$$S = \frac{13+14+15}{2} = 21, \quad \Delta = \sqrt{21\cdot 8\cdot 7\cdot 6} = 84, \quad R = \frac{13\cdot 14\cdot 15}{4\cdot 84} = \frac{2730}{336} = 8.125.$$

Answer: B)

10. **Question 10:**

$$r = \frac{\Delta}{S} = \frac{84}{21} = 4.$$

Answer: A)

11. **Question 11:**

$$r_1 = \frac{\Delta}{S-a} = \frac{84}{21-13} = \frac{84}{8} = 10.5.$$

Answer: B)

12. **Question 12:**

$$S = \frac{34+20+42}{2} = 48, \quad \Delta = \sqrt{48 \cdot (48-34) \cdot (48-20) \cdot (48-42)} = \sqrt{48 \cdot 14 \cdot 28 \cdot 6} = 336.$$

Answer: B)

13. **Question 13:**

$$R = \frac{34 \cdot 20 \cdot 42}{4 \cdot 336} = \frac{28560}{1344} = 21.250.$$

Answer: C)

14. **Question 14:**

$$r_3 = \frac{\Delta}{S-c} = \frac{336}{48-42} = \frac{336}{6} = 56.$$

Answer: D)

15. **Question 15:** In an equilateral triangle, $r = \frac{\sqrt{3}a}{6}$, $R = \frac{a}{\sqrt{3}}$, $r_1 = \frac{\sqrt{3}a}{2}$, so $r : R : r_1 = \frac{\sqrt{3}a}{6} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}a}{2} = 1 : 2 : 3$. Answer: A)

16. **Question 16:** Since $r_1 = r_2 = r_3$, the ratio is $r : R : r_1 : r_2 : r_3 = 1 : 2 : 3 : 3 : 3$. Answer: A)

17. **Question 17:**

$$\Delta = r^2 \cot\frac{\alpha}{2} \cot\frac{\beta}{2} \cot\frac{\gamma}{2} = \left(\frac{\Delta}{S}\right)^2 \cdot \sqrt{\frac{S(S-a)}{(S-b)(S-c)}} \cdot \sqrt{\frac{S(S-b)}{(S-a)(S-c)}} \cdot \sqrt{\frac{S(S-c)}{(S-a)(S-b)}} = \Delta.$$

Answer: A)

18. **Question 18:**

$$\frac{1}{2rR} = \frac{1}{2 \cdot \frac{\Delta}{S} \cdot \frac{abc}{4\Delta}} = \frac{2S}{abc} = \frac{a+b+c}{abc} = \frac{1}{bc} + \frac{1}{ac} + \frac{1}{ab}.$$

Answer: A)

19. **Question 19:**

$$r = \frac{a \sin\frac{\beta}{2} \sin\frac{\gamma}{2}}{\cos\frac{\alpha}{2}} = a \cdot \sqrt{\frac{(S-a)(S-c)}{ac}} \cdot \sqrt{\frac{(S-a)(S-b)}{ab}} \cdot \sqrt{\frac{bc}{S(S-a)}} = \frac{\Delta}{S} = r.$$

Answer: A)

20. **Question 20:**

$$(r_1 + r_2) \tan\frac{\gamma}{2} = \left(\frac{\Delta}{S-a} + \frac{\Delta}{S-b}\right) \cdot \sqrt{\frac{(S-a)(S-b)}{S(S-c)}} = \frac{\Delta^2(2S-a-b)}{\Delta^2} = c.$$

Answer: A)