Mathematical Induction Cheatsheet - Class 11 Mathematics

Prepared for Entry Test Preparation

1. Principle of Mathematical Induction

The principle states: If $S(1)$ is true (base case), and if $S(k+1)$ is true whenever $S(k)$ is true for any positive integer k (inductive step), then $S(n)$ is true for all positive integers n.

Procedure

- **Condition 1 (Base Case)**: Verify the statement $S(n)$ is true for $n = 1$ (or the smallest applicable integer).
- **Condition 2 (Inductive Step)**: Assume $S(k)$ is true for some positive integer k. Prove that $S(k + 1)$ is true.

2. Key Applications

Mathematical induction is used to prove formulas involving sums, products, divisibility, inequalities, and sequences (e.g., arithmetic/geometric progressions, factorials).

Examples

1. **Sum of odd numbers (Q.2)**: Prove $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.

   Base Case (n=1) : $1 = 1^2$ (True).

   Inductive Step : Assume $1+3+\cdots+(2k-1) = k^2$. Prove $1+3+\cdots+(2k-1)+(2k+1) = (k+1)^2$.

   $$k^2 + (2k + 1) = k^2 + 2k + 1 = (k + 1)^2.$$

2. **Geometric series (Q.4)**: Prove $1 + 2 + 4 + \cdots + 2^{n-1} = 2^n - 1$.

   Base Case (n=1) : $2^0 = 2^1 - 1 \implies 1 = 1$ (True).

   Inductive Step : Assume $1+2+\cdots+2^{k-1} = 2^k-1$. Prove $1+2+\cdots+2^k = 2^{k+1}-1$.

   $$(2^k - 1) + 2^k = 2 \cdot 2^k - 1 = 2^{k+1} - 1.$$

3. **Divisibility (Q.21(i))**: Prove $n^2 + n$ is divisible by 2.

   Base Case (n=1) : $1^2 + 1 = 2$ (Divisible by 2).

   Inductive Step : Assume k^2+k is divisible by 2. Prove $(k+1)^2+(k+1)$ is divisible by 2.

   $$(k + 1)^2 + (k + 1) = k^2 + 2k + 1 + k + 1 = (k^2 + k) + 2(k + 1).$$

   Both terms are divisible by 2.

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4. **Inequality (Q.33)**: Prove $n^2 > n + 3$ for $n \geq 3$.

Base Case (n=3) : $3^2 = 9 > 3 + 3 = 6$ (True).

Inductive Step : Assume $k^2 > k + 3$. Prove $(k + 1)^2 > (k + 1) + 3$.

$$(k + 1)^2 = k^2 + 2k + 1 > k + 3 + 2k + 1 = 3k + 4 > k + 4 \quad (\text{since } 2k > 0).$$

3. Tips for Proofs

- Simplify algebraic manipulations in the inductive step.
- For divisibility, express $S(k + 1)$ in terms of $S(k)$.
- For inequalities, ensure the base case and inductive step align with the given constraints.

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