Combination and Probability Cheatsheet - Class 11 Mathematics

Prepared for Entry Test Preparation

1. Combinations

The number of ways to choose r objects from n distinct objects, where order does not matter, is:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}, \quad r \leq n$$

Key property: $\binom{n}{r} = \binom{n}{n-r}$.

Key Concepts

- **Selection**: Choose groups (e.g., committees) without regard to order.
- **Geometric Applications**: Use combinations to count diagonals ($\binom{n}{2} - n$) or triangles ($\binom{n}{3}$) in an n-sided polygon.
- **Constrained Selection**: Select specific numbers of objects from distinct groups (e.g., k men and m women).
- **At Least/At Most Constraints**: Sum combinations for cases like "at least 4 women" or "at most 4 women."
- **Identities**: Prove identities like $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$.
- **Solving for** n, r: Use equations involving $\binom{n}{r}$ or ratios to find unknowns.

Examples

1. **Evaluate** $\binom{12}{3}$:

$$\binom{12}{3} = \frac{12!}{3!9!} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220$$

2. **Solve** $\binom{n}{5} = \binom{n}{4}$:

$$\frac{n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} \implies \frac{1}{5} = \frac{1}{n-4} \implies n-4 = 5 \implies n = 9$$

3. **Number of diagonals in an 8-sided polygon**:

$$\binom{8}{2} - 8 = \frac{8 \cdot 7}{2} - 8 = 28 - 8 = 20$$

4. **Committee of 3 boys, 2 girls from 12 boys, 8 girls**:

$$\binom{12}{3} \cdot \binom{8}{2} = 220 \cdot 28 = 6160$$

2. Probability Basics

Probability is the numerical measure of the likelihood of an event occurring, defined as:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

where outcomes are equally likely.

Key Concepts

- **Sample Space**: The set of all possible outcomes.
- **Event**: A subset of the sample space.
- **Combinations in Probability**: Use $\binom{n}{r}$ to count favorable or total outcomes in selection problems.
- **Applications**: Compute probabilities for events like selecting defective items or forming specific committees.

Examples

1. **Probability of selecting 2 defective items from 5 items (2 defective, 3 non-defective)**:

$$P = \frac{\binom{2}{2} \cdot \binom{3}{0}}{\binom{5}{2}} = \frac{1 \cdot 1}{10} = \frac{1}{10}$$

2. **Probability of a committee of 5 including 2 specific persons from 8**:

$$P = \frac{\binom{6}{3}}{\binom{8}{5}} = \frac{20}{56} = \frac{5}{14}$$