Sequences and Series Cheatsheet - Exercises 6.1 and 6.2 (Class 11 Mathematics)

Prepared for Entry Test Preparation

1. Concept of Sequences (Exercise 6.1)

A sequence is a function with a domain as a subset of natural numbers ($\mathbb{N}$). The n-th term is denoted a_n, with $a_1, a_2, a_3, \ldots$ as the first, second, third terms, etc. Sequences can be defined explicitly (by a formula) or recursively (using previous terms).

2. Types of Sequences in Exercise 6.1

- **Explicit Sequences**: Formula-based, e.g., $a_n = 2n - 3$.
- **Alternating Sequences**: Use $(-1)^n$ for sign alternation, e.g., $a_n = (-1)^n n^2$.
- **Recursive Sequences**: Defined using previous terms, e.g., $a_n = na_{n-1}$, $a_1 = 1$.
- **Arithmetic-Like Sequences**: Differences follow a pattern, e.g., $a_n - a_{n-1} = n + 2$.
- **Fractional Sequences**: Involve fractions, e.g., $a_n = \frac{n}{2n+1}$.
- **Reciprocal Arithmetic Sequences**: Denominators form A.P., e.g., $a_n = \frac{1}{a+(n-1)d}$.

3. Key Formulas for Exercise 6.1

- **Explicit**: $a_n = f(n)$.
- **Recursive**: $a_n = g(a_{n-1}, n)$, with initial condition.
- **Difference-Based**: If $a_n - a_{n-1} = k(n)$, compute iteratively.
- **Reciprocal A.P.**: $a_n = \frac{1}{a+(n-1)d}$.

4. Examples from Exercise 6.1

Explicit Sequence

Problem: $a_n = 3n - 5$

- Compute: $a_1 = 3 \cdot 1 - 5 = -2$, $a_2 = 1$, $a_3 = 4$, $a_4 = 7$.
- Result: $-2, 1, 4, 7$.

Alternating Sequence

Problem: $a_n = (-1)^n(2n-3)$

- Compute: $a_1 = (-1)^1(2-3) = 1$, $a_2 = 1$, $a_3 = -3$, $a_4 = 5$.
- Result: $1, 1, -3, 5$.

Recursive Sequence

Problem: $a_n = (n+1)a_{n-1}$, $a_1 = 1$

- Compute: $a_2 = 3 \cdot 1 = 3$, $a_3 = 4 \cdot 3 = 12$, $a_4 = 5 \cdot 12 = 60$.
- Result: $1, 3, 12, 60$.

Reciprocal A.P.

Problem: $a_n = \frac{1}{a+(n-1)d}$

- Compute: $a_1 = \frac{1}{a}$, $a_2 = \frac{1}{a+d}$, $a_3 = \frac{1}{a+2d}$, $a_4 = \frac{1}{a+3d}$.

5. Arithmetic Progression (A.P.) - Exercise 6.2

An A.P. is a sequence where $a_n - a_{n-1} = d$ (constant common difference). The n-th term is:

$$a_n = a_1 + (n-1)d$$

General form: $a_1, a_1 + d, a_1 + 2d, \ldots$.

6. Key Formulas for Exercise 6.2

- **n-th Term**: $a_n = a_1 + (n-1)d$.
- **Common Difference**: $d = \frac{a_n - a_m}{n-m}$.
- **First Term**: If $a_m = p$, $a_n = q$, then $a_1 = p - (m-1)d$.
- **Term Number**: If $a_n = k$, then $n = \frac{k-a_1}{d} + 1$.
- **Arithmetic Mean**: For a, A, b in A.P., $A = \frac{a+b}{2}$.
- **Reciprocal A.P.**: If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., then $b = \frac{2ac}{a+c}$, and $d = \frac{a-c}{2ac}$.
- **Proof for Terms**: For p-th, q-th, r-th terms l, m, n:

$$l(q-r) + m(r-p) + n(p-q) = 0, \quad p(m-n) + q(n-l) + r(l-m) = 0$$

7. Examples from Exercise 6.2

Given Two Terms

Problem: $a_5 = 17$, $a_9 = 37$

- Solve: $a_1 + 4d = 17$, $a_1 + 8d = 37$. Subtract: $4d = 20 \implies d = 5$. Then, $a_1 = -3$.
- Compute: $a_2 = 2$, $a_3 = 7$, $a_4 = 12$.
- Result: $-3, 2, 7, 12$.

Find n-th Term

Problem: $a_{n-3} = 2n - 5$

- Compute: Set $k = n - 3$, so $a_k = 2(k + 3) - 5 = 2k + 1$. Thus, $a_n = 2n + 1$.

Reciprocal A.P.

Problem: Show if $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., then $b = \frac{2ac}{a+c}$.

- Proof: $\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b} \implies \frac{2}{b} = \frac{a+c}{ac} \implies b = \frac{2ac}{a+c}$.