

Cheatsheet: Word Problems on Quadratic Equations (Exercise 4.10)

Class 11 Mathematics (Chapter 4)

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Overview

Exercise 4.10 involves solving word problems that lead to quadratic equations in one or two variables. Problems include number relations, geometric figures (rectangles, triangles), work rates, and financial scenarios. Solutions typically involve defining variables, forming equations based on conditions, and solving using factoring or the quadratic formula.

Note

Always define variables clearly, translate conditions into equations, and verify solutions against the problem's constraints (e.g., positive values for physical quantities).

General Strategy

Steps to Solve

1. **Define Variables:** Assign unknowns (e.g., x for a number, x, y for dimensions).
2. **Translate Conditions:** Form equations based on given relationships (e.g., area, product, sum).
3. **Simplify:** Combine terms to form a quadratic equation in standard form ($ax^2 + bx + c = 0$).
4. **Solve:** Use factoring, quadratic formula, or substitution for systems of equations.
5. **Verify:** Check solutions for physical sense (e.g., reject negative values for lengths).

Tip Read the problem carefully to identify all conditions. For two-variable problems, look for opportunities to express one variable in terms of the other.

Common Problem Types

Number Problems

- **Example:** The sum of a number and its square is 380. Let number = x . Equation: $x + x^2 = 380 \Rightarrow x^2 + x - 380 = 0$. Solve: $x = 19, -20$. Positive number: 19.
- **Tip:** For digit problems, let tens digit = x , units digit = y . Number = $10x + y$, reversed = $10y + x$.

Geometric Problems

- **Rectangle/Triangle:** Use area ($xy = A$) or Pythagorean theorem ($x^2 + y^2 = c^2$) for right triangles/diagonals.
- **Example:** Rectangle area = 1680 m², diagonal = 58 m. Equations: $xy = 1680$, $x^2 + y^2 = 3364$. Substitute: $y^4 - 3364y^2 + 2822400 = 0$. Solutions: $y = 40, 42$.
- **Tip:** Substitute one variable to form a biquadratic equation (y^4).

Work-Rate Problems

- **Method:** Use work rate = $\frac{1}{\text{time}}$. Combined rate: $\frac{1}{x} + \frac{1}{y} = \frac{1}{\text{total time}}$.
- **Example:** A and B finish in 4 days, A takes twice as long as B. Let B take x days, A take $2x$. Equation: $\frac{1}{x} + \frac{1}{2x} = \frac{1}{4} \Rightarrow x = 6$.

Financial Problems

- **Method:** Define investment amounts and rates. Form equations based on total investment and profit.
- **Example:** Rs. 100,000 invested, total profit Rs. 3080, one at $y\%$, other at $(y + 1)\%$. Equations: $xy = 198000$, $(y + 1)(100000 - x) = 308000$.

Solving Quadratic Equations

Factoring Find two numbers whose product is ac and sum is b . Example: $x^2 - 7x - 494 = 0$. Numbers: $-26, 19$. Factors: $(x - 26)(x + 19) = 0$.

Quadratic Formula For $ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Check discriminant ($b^2 - 4ac$) for real solutions.

Biquadratic Equations For equations like $y^4 - 3364y^2 + 2822400 = 0$, let $u = y^2$. Solve for u , then $y = \pm\sqrt{u}$.

Key Reminders

- Reject negative solutions for physical quantities (e.g., lengths, time).
- Verify solutions in all given conditions.
- Simplify fractions before cross-multiplying to avoid errors.
- For biquadratic equations, ensure positive roots for squared variables.

Tip

Always check the discriminant to confirm real solutions. For digit problems, ensure digits are between 0 and 9.