# Cheatsheet: Quadratic Equations (Exercise 4.1)

**Class 11 Mathematics (Chapter 4)**

Prepared by ExpertGuy

## Quadratic Equation Overview

**Definition** A quadratic equation in $x$ is:

$$ax^2 + bx + c = 0, \quad a \neq 0, \quad a, b, c \in \mathbb{R}$$

Example: $x^2 - 7x + 10 = 0$ ($a = 1, b = -7, c = 10$).

**Solution Methods**

- Factorization
- Completing the Square
- Quadratic Formula

Note

Quadratic equations are second-degree polynomials. Always ensure the equation is in standard form before solving.

## Factorization Method

**Concept** Rewrite $ax^2 + bx + c = 0$ as:

$$(x - m)(x - n) = 0 \implies x = m \text{ or } x = n$$

Use splitting the middle term or difference of squares.

**Steps**

1. Write in standard form: $ax^2 + bx + c = 0$.
2. Find two numbers whose product is $ac$ and sum is $b$.
3. Split the middle term $bx$ using these numbers.
4. Factor by grouping or directly.
5. Set each factor to zero and solve for $x$.

**Example** Solve $9x^2 - 12x - 5 = 0$:

$$\begin{aligned} 9x^2 - 15x + 3x - 5 &= 0 \\ 3x(3x - 5) + 1(3x - 5) &= 0 \\ (3x - 5)(3x + 1) &= 0 \\ 3x - 5 = 0 \text{ or } 3x + 1 &= 0 \\ x = \frac{5}{3}, \quad x &= -\frac{1}{3} \end{aligned}$$

Solution set: $\{-\frac{1}{3}, \frac{5}{3}\}$.

**Fractional Equations** For equations like $\frac{x}{x+1} + \frac{x+1}{x} = \frac{5}{2}$, $x \neq -1, 0$:

1. Find a common denominator.
2. Simplify to a quadratic equation.
3. Solve using factorization.

Example:

$$\begin{aligned}
\frac{x^2+(x+1)^2}{x(x+1)} &= \frac{5}{2}\\
\frac{2x^2+2x+1}{x^2+x} &= \frac{5}{2}\\
5(x^2+x) &= 2(2x^2+2x+1)\\
x^2+x-2 &= 0\\
(x+2)(x-1) &= 0\\
x=-2, \quad x &= 1
\end{aligned}$$

Solution set: $\{-2, 1\}$.

**Tip**

Check for restrictions (e.g., denominators $\neq 0$) and verify solutions in fractional equations.

## Completing the Square

**Concept** Transform $ax^2+bx+c=0$ into:

$$\left(x+\frac{b}{2a}\right)^2 = k$$

Solve by taking the square root.
**Steps**

1. Move the constant: $x^2+\frac{b}{a}x = -\frac{c}{a}$.
2. Add $\left(\frac{b}{2a}\right)^2$ to both sides.
3. Factor the left side as a perfect square.
4. Take the square root and solve for $x$.

**Example** Solve $x^2+6x-567=0$:

$$\begin{aligned}
x^2+6x &= 567\\
x^2+6x+\left(\frac{6}{2}\right)^2 &= 567+9\\
(x+3)^2 &= 576 = 24^2\\
x+3 &= \pm 24\\
x=21, \quad x &= -27
\end{aligned}$$

Solution set: $\{-27, 21\}$.

**Tip**

For equations like $2x^2 + 12x - 110 = 0$, divide by the leading coefficient first to simplify.

## Quadratic Formula

**Formula** For $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant: $\Delta = b^2 - 4ac$

- $\Delta > 0$: Two distinct real roots.
- $\Delta = 0$: One real root (repeated).
- $\Delta < 0$: No real roots.

**Steps**

1. Identify $a, b, c$.
2. Compute $\Delta = b^2 - 4ac$.
3. Substitute into the quadratic formula.
4. Simplify the roots.

**Example** Solve $16x^2 + 8x + 1 = 0$:

$$a = 16, b = 8, c = 1$$

$$\Delta = 8^2 - 4 \cdot 16 \cdot 1 = 64 - 64 = 0$$

$$x = \frac{-8 \pm \sqrt{0}}{2 \cdot 16} = \frac{-8}{32} = -\frac{1}{4}$$

Solution set: $\{-\frac{1}{4}\}$.

**Parameter-Based Equations** For equations like $15x^2 + 2ax - a^2 = 0$:

$$x = \frac{-2a \pm \sqrt{(2a)^2 - 4 \cdot 15 \cdot (-a^2)}}{2 \cdot 15} = \frac{-2a \pm 8a}{30}$$

Solution set: $\{-\frac{a}{3}, \frac{a}{5}\}$.

**Note**

For complex expressions (e.g., $(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$), simplify carefully to standard form before applying the formula.

## Key Reminders

- Always verify solutions by substituting back into the original equation.
- For fractional equations, ensure denominators are non-zero.
- Repeated roots ($\Delta = 0$) yield a single solution.
- Parameter-based equations require careful algebraic simplification.