Conic Sections Cheatsheet: Class 12, Chapter 6, Exercise 6.7

This cheatsheet summarizes tangent and normal equations for conic sections, with examples from Exercise 6.7.

1. Tangent Equations

(i) Circle ($x^2+y^2=a^2$): - Point form at (x_1,y_1): $xx_1+yy_1=a^2$ - Slope form with slope m: $y=mx\pm a\sqrt{1+m^2}$ (where $c^2=a^2(1+m^2)$)

(ii) Parabola ($y^2=4ax$): - Point form at $(at^2,2at)$: $y\cdot 2at=2a(x+at^2)$ or $yt=x+at^2$ - Slope form: $y=mx+\frac{a}{m}$ (condition of tangency)

(iii) Ellipse ($\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$): - Point form at $(a\cos\theta,b\sin\theta)$: $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

(iv) Hyperbola ($\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$): - Point form at $(a\sec\theta,b\tan\theta)$: $\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$

2. Normal Equations

(i) Parabola ($y^2=4ax$): - At (x_1,y_1): $y-y_1=\frac{-y_1}{2a}(x-x_1)$

(ii) Ellipse ($\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$): - At (x_1,y_1): $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

(iii) Hyperbola ($\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$): - At (x_1,y_1): $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$

3. Tangent Through a Point

- **Circle**: Solve $c^2=a^2(1+m^2)$ with the point (x_0,y_0) in $y=mx+c$. - **Parabola**: Use $c=\frac{a}{m}$ and solve with the point. - **Hyperbola**: Use $c^2=a^2m^2-b^2$ and solve with the point.

4. Parallel Tangents

- Match slope m of the given line, then use the conic's tangency condition (e.g., $c^2=a^2m^2-b^2$ for hyperbola).

5. Common Tangents

- Solve for m and c using the discriminant condition (disc $=0$) for one conic and tangency for the other.

6. Examples from Exercise 6.7

Q.1(i) $y^2=4ax$ at $(at^2,2at)$: - Tangent: $yt=x+at^2$ - Normal: $tx+y-at-at^3=0$

Q.2(i) $3x^2=-16y$ at $y=-3$ (points $(4,-3)$, $(-4,-3)$): - Tangent at $(4,-3)$: $3x+2y-6=0$ - Tangent at $(-4,-3)$: $3x-2y+6=0$

Q.3(i) $x^2+y^2=25$ through $(7,-1)$: - Tangents: $4x+3y-25=0$, $4x+3y+25=0$

Q.5 $x^2/4+y^2=1$ parallel to $2x-4y+5=0$ (slope $m=1/2$): - Tangent: $x-2y\pm 2\sqrt{2}=0$

Q.6 $9x^2 - 4y^2 = 36$ parallel to $5x - 2y + 7 = 0$ (slope $m = 5/2$): - Tangent: $5x - 2y \pm 8 = 0$

Q.7(i) $x^2 = 80y$ and $x^2 + y^2 = 81$: - Common tangent: $\pm 3x - 4y - 45 = 0$

7. Tips

- Differentiate implicitly to find slopes for general conics. - Check tangency conditions to ensure the line touches the conic at one point. - Simplify equations by completing the square when shifting points.