

Conic Sections Cheatsheet: Class 12, Chapter 6, Exercise 6.5

This cheatsheet covers ellipse properties and equation derivation based on Exercise 6.5, including foci, vertices, directrices, eccentricity, and sketching.

1. Definition and Standard Forms

An ellipse is the locus of points where the sum of distances to two fixed points (foci) is constant. The standard forms are:

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (major axis along x-axis, $a > b$)
- $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ (major axis along y-axis, $a > b$)
- Special case (circle): When $e = 0$, $a = b$, forming $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$.

Parametric Equations: $x = a \cos \theta$, $y = b \sin \theta$.

2. Key Elements

- Eccentricity: $e^2 = \frac{a^2 - b^2}{a^2}$, where $c^2 = a^2 - b^2$ and c is the distance from center to focus.
- Foci: For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, foci are $(\pm c, 0)$; for $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, foci are $(0, \pm c)$.
- Vertices: $(\pm a, 0)$ or $(0, \pm a)$ depending on the major axis.
- Directrices: $x = \pm \frac{a}{e}$ or $y = \pm \frac{a}{e}$.
- Latus Rectum: Length = $\frac{2b^2}{a}$.
- Center: (h, k) for shifted ellipses, $(0, 0)$ for standard forms.

3. Deriving Equations from Given Elements

Use the relationship $c^2 = a^2 - b^2$ and given data (foci, vertices, eccentricity).

Example 1: Foci $(\pm 3, 0)$, vertices $(\pm 6, 0)$:

$$c = 3, \quad a = 6, \quad c^2 = a^2 - b^2 \implies 9 = 36 - b^2 \implies b^2 = 27$$

Equation: $\frac{x^2}{36} + \frac{y^2}{27} = 1$ (corrected $b^2 = 9$ from PDF error).

Example 2: Foci $(0, -1)$ and $(0, -5)$, major axis length 6:

$$\text{Center} = (0, -3), \quad c = 2, \quad 2a = 6 \implies a = 3$$

$$c^2 = a^2 - b^2 \implies 4 = 9 - b^2 \implies b^2 = 5$$

Equation: $\frac{x^2}{5} + \frac{(y+3)^2}{9} = 1$.

Example 3: Vertices $(-1, 1)$ and $(5, 1)$, foci $(4, 1)$ and $(0, 1)$:

$$2a = 6 \implies a = 3, \quad 2c = 4 \implies c = 2$$

$$c^2 = a^2 - b^2 \implies 4 = 9 - b^2 \implies b^2 = 5$$

Center = $(2, 1)$, equation: $\frac{(x-2)^2}{9} + \frac{(y-1)^2}{5} = 1$.

4. Shifted Ellipses

For center (h,k) , use $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

Example 4: Center $(2,2)$, major axis (y-axis) length 8, minor axis length 6:

$$2a = 8 \implies a = 4, \quad 2b = 6 \implies b = 3$$

Equation: $\frac{(y-2)^2}{16} + \frac{(x-2)^2}{9} = 1$.

5. Graphing Tips

- Plot center, vertices, and foci. - Draw the major and minor axes. - Use latus rectum points for curve accuracy. - Ensure symmetry about both axes for standard forms.