

Exercise 6.8: Conic Sections Cheatsheet

Axis Shifting (Parallel Axes)

Transform equations by shifting origin to (h, k) : $x = X + h$, $y = Y + k$ Substitute into original equation and simplify to remove first-degree terms.

Steps: 1. Identify new origin (h, k) . 2. Substitute $x = X + h$, $y = Y + k$. 3. Expand and collect terms to eliminate X and Y linear terms. 4. Solve for transformed equation.

Examples: - $x^2 + 16y - 16 = 0$, $O'(0, 1)$: $X^2 + 16(Y + 1) - 16 = 0 \rightarrow X^2 + 16Y = 0$ - $4x^2 + y^2 + 16x - 10y + 37 = 0$, $O'(-2, 5)$: $4(X - 2)^2 + (Y + 5)^2 + 16(X - 2) - 10(Y + 5) + 37 = 0 \rightarrow 4X^2 + Y^2 - 4 = 0$

Find New Origin (Remove First-Degree Terms)

For $ax^2 + by^2 + cx + dy + e = 0$: Set coefficients of X and Y to zero: $c + 2ah + ah^2 = 0$, $d + 2bk + bk^2 = 0$ Solve for (h, k) .

Steps: 1. Expand $(X + h)^2$, $(Y + k)^2$ terms. 2. Equate coefficients of X and Y to zero. 3. Substitute (h, k) back to get transformed equation.

Examples: - $3x^2 - 2y^2 + 24x + 12y + 24 = 0$: $6h + 24 = 0 \rightarrow h = -4$, $4k - 12 = 0 \rightarrow k = 3$ $O'(-4, 3)$, Transformed: $3X^2 - 2Y^2 - 6 = 0$ - $25x^2 + 9y^2 + 50x - 36y - 164 = 0$: $50h + 50 = 0 \rightarrow h = -1$, $18k - 36 = 0 \rightarrow k = 2$ $O'(-1, 2)$, Transformed: $25X^2 + 9Y^2 - 225 = 0$

Axis Rotation (Remove xy-Term)

Transform using rotation angle θ : $x = X \cos \theta - Y \sin \theta$, $y = X \sin \theta + Y \cos \theta$ Angle θ where: $\tan 2\theta = \frac{2h}{a-b}$ (for $ax^2 + 2hxy + by^2 + \dots = 0$)

Steps: 1. Compute $\tan 2\theta$ using a , h , b . 2. Solve for θ and find $\cos \theta$, $\sin \theta$. 3. Substitute into transformation equations. 4. Simplify to remove XY -term.

Examples: - $xy = 1$, $\theta = 45^\circ$: $\cos 45^\circ = \sin 45^\circ = \frac{1}{\sqrt{2}}$ $\left(\frac{X-Y}{\sqrt{2}}\right) \left(\frac{X+Y}{\sqrt{2}}\right) = 1 \rightarrow X^2 - Y^2 = 2$ - $5x^2 - 6xy + 5y^2 - 8 = 0$, $\tan 2\theta = \frac{-6}{5-5} = \infty \rightarrow \theta = 45^\circ$: $X^2 + 4Y^2 - 4 = 0$

Key Formulas

- $\cos 45^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$ - $\tan 2\theta = \frac{2h}{a-b}$ - Transformed equation: Substitute and simplify.