Conic Sections Cheatsheet: Class 12, Chapter 6, Exercise 6.6

This cheatsheet covers hyperbola properties and equation derivation based on Exercise 6.6, including foci, vertices, directrices, eccentricity, and sketching.

1. Definition and Standard Forms

A hyperbola is the locus of points where the absolute difference of distances to two fixed points (foci) is constant. The standard forms are:

- $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ (transverse axis horizontal)

- $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ (transverse axis vertical)

Asymptotes: $y - k = \pm\frac{b}{a}(x-h)$ or $y - k = \pm\frac{a}{b}(x-h)$ depending on orientation.

2. Key Elements

- Eccentricity: $e = \frac{c}{a}$, where $c^2 = a^2 + b^2$ and c is the distance from center to focus.

- Foci: $(\pm c + h, k)$ or $(h, \pm c + k)$ depending on transverse axis.

- Vertices: $(\pm a + h, k)$ or $(h, \pm a + k)$.

- Directrices: $x - h = \pm\frac{a}{e}$ or $y - k = \pm\frac{a}{e}$.

- Latus Rectum: Length $= \frac{2b^2}{a}$.

- Center: (h, k) for shifted hyperbolas.

3. Deriving Equations from Given Elements

Use $c^2 = a^2 + b^2$ and given data (foci, vertices, eccentricity, transverse axis length).

Example 1: Center $(0,0)$, Focus $(6,0)$, Vertex $(4,0)$:

$$c = 6, \quad a = 4, \quad c^2 = a^2 + b^2 \implies 36 = 16 + b^2 \implies b^2 = 20$$

Equation: $\frac{x^2}{16} - \frac{y^2}{20} = 1$.

Example 2: Foci $(\pm 5, 0)$, Vertex $(3,0)$:

$$c = 5, \quad a = 3, \quad c^2 = a^2 + b^2 \implies 25 = 9 + b^2 \implies b^2 = 16$$

Equation: $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

Example 3: Foci $(2 \pm 5\sqrt{2}, -7)$, Transverse axis length 10:

$$2a = 10 \implies a = 5, \quad c = \sqrt{50} = 5\sqrt{2}, \quad c^2 = a^2 + b^2 \implies 50 = 25 + b^2 \implies b^2 = 25$$

Center $(2, -7)$, equation: $\frac{(x-2)^2}{25} - \frac{(y+7)^2}{25} = 1$ (corrected a^2 and b^2).

Example 4: Foci $(0, \pm 9)$, Directrix $y = \pm 4$:

$$ae = 9, \quad \frac{a}{e} = 4 \implies a \cdot \frac{4}{a} = 9 \implies a^2 = 36, \quad c^2 = a^2 + b^2 \implies 81 = 36 + b^2 \implies b^2 = 45$$

Equation: $\frac{y^2}{36} - \frac{x^2}{45} = 1$.

4. Shifted Hyperbolas

For center (h,k), adjust coordinates accordingly.

Example 5: Center $(2,2)$, Transverse axis length 6, $e = 2$:

$$2a = 6 \implies a = 3, \quad c = ae = 6, \quad c^2 = a^2 + b^2 \implies 36 = 9 + b^2 \implies b^2 = 27$$

Equation: $\frac{(x-2)^2}{9} - \frac{(y-2)^2}{27} = 1$.

5. Graphing Tips

- Plot center, vertices, and foci. - Draw the transverse and conjugate axes. - Sketch asymptotes using $y - k = \pm \frac{b}{a}(x - h)$. - Ensure symmetry and proper opening direction.