Conic Sections Cheatsheet: Class 12, Chapter 6, Exercise 6.4

This cheatsheet covers parabola properties and equation derivation based on Exercise 6.4, including focus, vertex, directrix, and sketching.

1. Definition and Standard Forms

A parabola is the locus of points equidistant from a focus and directrix.

Standard Forms:

- $y^2 = 4ax$ (opens right, vertex at $(0,0)$)
- $y^2 = -4ax$ (opens left)
- $x^2 = 4ay$ (opens up)
- $x^2 = -4ay$ (opens down)
- Shifted form: $(x-h)^2 = 4a(y-k)$ or $(y-k)^2 = 4a(x-h)$

Key Elements:

- Focus: For $y^2 = 4ax$, $(a,0)$; for $x^2 = 4ay$, $(0,a)$
- Vertex: (h,k) (or $(0,0)$ for standard forms)
- Directrix: For $y^2 = 4ax$, $x = -a$; for $x^2 = 4ay$, $y = -a$
- Latus Rectum: Length = $4a$

2. Finding Focus, Vertex, and Directrix

Use the standard form to identify elements: - $4a$ is the coefficient of x or y in the equation. - Focus and directrix are offset by a from the vertex.

Example 1: For $y^2 = 8x$:
$$4a = 8 \implies a = 2$$
$$\text{Focus} = (2,0), \quad \text{Vertex} = (0,0), \quad \text{Directrix} = x = -2$$

Example 2: For $x^2 = -16y$:
$$4a = -16 \implies a = 4$$
$$\text{Focus} = (0,-4), \quad \text{Vertex} = (0,0), \quad \text{Directrix} = y = 4$$

3. Deriving Equations from Given Elements

Use the definition: distance from point to focus equals distance to directrix.

Example 3: Focus $(-3,1)$, Directrix $x = 3$:

$$\text{Distance to focus} = \sqrt{(x+3)^2 + (y-1)^2}$$
$$\text{Distance to directrix} = |x-3|$$
$$(x+3)^2 + (y-1)^2 = (x-3)^2$$

$$x^2 + 6x + 9 + y^2 - 2y + 1 = x^2 - 6x + 9$$
$$y^2 - 2y + 1 = -12x \implies (y-1)^2 = -12x$$

Example 4: Focus $(2,5)$, Directrix $y = 1$:

$$\text{Distance to focus} = \sqrt{(x-2)^2 + (y-5)^2}$$

$$\text{Distance to directrix} = |y-1|$$
$$(x-2)^2 + (y-5)^2 = (y-1)^2$$
$$x^2 - 4x + 4 + y^2 - 10y + 25 = y^2 - 2y + 1$$
$$x^2 - 4x - 8y + 28 = 0$$

4. Shifted Parabolas

For equations like $(x-h)^2 = 4a(y-k)$, transform variables: - Let $X = x - h$, $Y = y - k$.

Example 5: For $(x-1)^2 = 8(y+2)$:

$$4a = 8 \implies a = 2$$

$$\text{Focus} = (1,0), \quad \text{Vertex} = (1,-2), \quad \text{Directrix} = y = -4$$

5. Graphing Tips

- Plot vertex, focus, and directrix. - Draw the axis of symmetry (through vertex and focus). - Sketch the curve opening toward the focus, using latus rectum points.