DYNAMICS ANALYSIS OF A TWO-BODY SYSTEM USING NUMERICAL METHODS

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Muhaimenur Rahman

Abstract

In this study, we have dealt with a simple two-body classical problem where an object rotates on a frictionless surface and another object moves up and down under the surface and these two objects are connected with a massless string that passes through a hole on the surface. The Euler method and the 4^{th} Order Runge Kutta method have been applied to analyze the two-body problem. The result shows that the 4^{th} Order Runge Kutta method gives more accurate results in comparison to the Euler method. The analysis also shows if the angular momentum is zero then for any values of equal mass condition and the same initial conditions, the time required to destroy the system is the same. The time period is high when the mass of the object over the surface is much greater than the object under the surface and the time period is less when the mass of the object over the surface is much lesser than the object under the surface.

Keywords: Euler Lagrange equation, Euler method, 4^{th} Order Runge Kutta method.

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1. Introduction

There are two types of objects in our universe; classical objects and quantum objects. The objects which we can see with the naked eye are called macroscopic or classical objects and which need a microscope for studying are called microscopic or quantum objects. As there are two types of objects, two different mechanics are developed to describe them; classical mechanics [1, 2, 3] and quantum mechanics [6]. Classical mechanics describes classical objects. The Euler Lagrange equation [1, 2, 3] is one of the classical mechanics mathematical tool that describes the dynamics of a classical object. It is a function of generalized coordinate, generalized velocity and time [1]. A classical system can have one or more than one generalized coordinates. An example of one generalized coordinate is simple harmonic motion and more than one generalized coordinates is double pendulum. By knowing the dynamics of a classical problem, we can know a lot about the system and can use them in technology and also can discover many unknown things about nature (the moon revolving around the earth).

To know the dynamics of a classical system, we need to solve the Euler-Lagrange equation. But sometimes it is hard to find an analytical solution. In that case, it is easier to solve them using numerical methods [4, 5]. Such a case we take here, where an object can moves above a frictionless plane surface and another object can moves under the surface and the two objects are connected with a massless string through a hole in the surface. It's a simple two-body classical problem. The Euler-Lagrange equations of this system don't have any specific analytical solutions. If we try to find out the analytical solution, it will give different solutions for different conditions and also a complicated process. So, if we want to know about its dynamics, it is easier and more convenient to solve it numerically. There are many numerical methods. For different conditions, different methods are used. As an example, the Taylor series, Picard method, Euler method, Runge Kutta method are used to solve ordinary differential equations (ODE) for initial value problems, and for partial differential equations (PDE) Finite Difference method is used [4, 5]. In our system, there were two ordinary differential equations; one was a first-order ordinary differential equation and the other one was a second-order ordinary differential equation. To solve these two differential equations, we used two numerical methods; one was Euler Method and another one was the 4^{th} Order Runge Kutta Method [4, 5]. For solving second-order differential equation, first it turned into two first-order differential equations and then applied Euler and 4^{th} Order Runge Kutta methods. Using these two methods, we tried to find the trajectory of the system. We also tried to find out how the values of the solution differ from these two methods and tried to examine how the results vary from the values of h (step size).

2. Theory

2.1. Euler Lagrange Equation

Lagrangian (L) is the difference between kinetic(T) and potential energy(V). In the mathematical form we can write,

$$L = T - V \tag{1}$$

Lagrangian is a function of generalized coordinate, the velocity of this coordinate which is called generalized velocity and time ($L(q, \dot{q}, t)$). The configuration of a physical system can be specified completely by generalized coordinates which are a set of parameters at each point in time.

Euler Lagrange equation or Lagrange equation of motion is a mathematical tool to determine a classical system's dynamics. The Euler Lagrange equation is,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0 \tag{2}$$

2.2. Formulation of the Two-body System

2.2.1. Description of the system

Figure 1 is our system, where an object is over a frictionless plane surface and the other one is hanging under the surface and the two objects are connected with a massless string through a hole in the surface. The object under the surface doesn't allow any swinging.

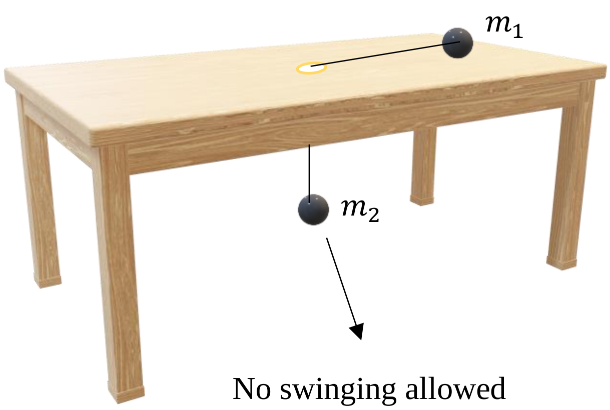


Figure 1: System diagram.

In this system, the length of the string is constant. Let us consider the length is l. We consider the hole as the origin of the coordinate system. The mass of the object on the surface is m_1 and under the surface is m_2. The distance between the hole and m_1 is r and the distance between the hole and m_2 is s. So,

$$l = r + s \tag{3}$$

2.2.2. Generalized coordinates

If we work with cartesian coordinates then we have x, y and z coordinates. And if we work with polar coordinates then we have r, Θ and s. The conversion of cartesian to polar coordinates is,

$$\left.\begin{aligned} x &= r\cos\Theta \\ y &= r\sin\Theta \\ z &= s \end{aligned}\right\} \tag{4}$$

s is related to r. So if we take polar coordinates then r or s and Θ are our generalized coordinates. That means our generalized coordinates are two. If we take cartesian coordinate then it also shows two generalized coordinates but we take polar coordinates for our convenience. We take r and Θ as generalized coordinates.

2.2.3. Energy

The kinetic energy of the m_1,

$$\begin{aligned} T_{m_1} &= \frac{1}{2}m_1 v_1^2 \\ &= \frac{1}{2}m_1(v_x^2 + v_y^2) \\ &= \frac{1}{2}m_1\{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2\} \\ &= \frac{1}{2}m_1\{(\frac{d}{dt}r\cos\Theta)^2 + (\frac{d}{dt}r\sin\Theta)^2\} \\ &= \frac{1}{2}m_1\{(-r\sin\Theta\frac{d\Theta}{dt} + \cos\Theta\frac{dr}{dt})^2 + (r\cos\Theta\frac{d\Theta}{dt} + \sin\Theta\frac{dr}{dt})^2\} \\ &= \frac{1}{2}m_1\{\cos^2\Theta\left(\frac{dr}{dt}\right)^2 - 2r\sin\Theta\cos\Theta\frac{dr}{dt}\frac{d\Theta}{dt} + r^2\sin^2\Theta\left(\frac{d\Theta}{dt}\right)^2 + \\ &\qquad r^2\cos^2\Theta\left(\frac{d\Theta}{dt}\right)^2 + 2r\sin\Theta\cos\Theta\frac{dr}{dt}\frac{d\Theta}{dt} + \sin^2\Theta\,(\frac{dr}{dt})^2\} \\ &= \frac{1}{2}m_1\{\left(\frac{dr}{dt}\right)^2(\sin^2\Theta + \cos^2\Theta) + r^2\left(\frac{d\Theta}{dt}\right)^2(\sin^2\Theta + \cos^2\Theta)\} \\ &= \frac{1}{2}m_1\left(\dot{r}^2 + r^2\dot{\Theta}^2\right) \qquad [\sin^2\Theta + \cos^2\Theta = 1] \end{aligned} \tag{5}$$

Kinetic energy of m_2,

$$\begin{aligned} T_{m_2} &= \frac{1}{2}m_2 v_2^2 \\ &= \frac{1}{2}m_2 v_z^2 \end{aligned}$$

$$= \frac{1}{2} m_2 v_s^2$$

$$= \frac{1}{2} m_2 (\frac{ds}{dt})^2$$

$$= \frac{1}{2} m_2 (\frac{d}{dt} (l - r))^2$$

$$= \frac{1}{2} m_2 (\frac{dl}{dt} - \frac{dr}{dt})^2$$

$$= \frac{1}{2} m_2 (-\frac{dr}{dt})^2 \quad [since\ l\ is\ constant, its\ derivative\ is\ zero]$$

$$= \frac{1}{2} m_2 \dot{r}^2 \tag{6}$$

Total kinetic energy of the system,

$$T = T_{m_1} + T_{m_2}$$

$$= \frac{1}{2} m_1 \left(\dot{r}^2 + r^2 \dot{\theta}^2\right) + \frac{1}{2} m_2 \dot{r}^2 \tag{7}$$

We consider that, the potential is zero at the surface, and negative when it goes down. As m_1 moves only over the surface so its potential energy is always zero. On the other hand, m_2 moves under the surface on a straight path so it has either a negative potential or zero potential. So the total potential energy of the system,

$$V = V_{m_1} + V_{m_2}$$

$$= 0 - m_2 g s$$

$$= -m_2 g (l - r)$$

$$= -m_2 g l + m_2 g r \tag{8}$$

Therefore the total energy of the system,

$$E = T + V$$

$$= \frac{1}{2} m_1 \left(\dot{r}^2 + r^2 \dot{\theta}^2\right) + \frac{1}{2} m_2 \dot{r}^2 - m_2 g l + m_2 g r \tag{9}$$

2.2.4. Lagrangian of the system

The Lagrangian of the system is,

$$L = T - V$$

$$= \frac{1}{2} m_1 \left(\dot{r}^2 + r^2 \dot{\theta}^2\right) + \frac{1}{2} m_2 \dot{r}^2 + m_2 g l - m_2 g r \tag{10}$$

2.2.5. Euler Lagrange equation (equation of motion)

Euler Lagrange equation or equation of motion for Θ coordinate is,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\Theta}}\right) - \frac{\partial L}{\partial \Theta} = 0$$

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{\Theta}}\left(\frac{1}{2}m_1\left(\dot{r}^2 + r^2\dot{\Theta}^2\right) + \frac{1}{2}m_2\dot{r}^2 + m_2gl - m_2gr\right)\right) - \frac{\partial}{\partial \Theta}\left(\frac{1}{2}m_1\left(\dot{r}^2 + r^2\dot{\Theta}^2\right) + \frac{1}{2}m_2\dot{r}^2 + m_2gl - m_2gr\right) = 0$$

$$\frac{d}{dt}\left(m_1r^2\dot{\Theta}\right) - 0 = 0$$

$$m_1r^2\dot{\Theta} = constant \tag{11}$$

As we see, $\frac{dL}{d\dot{\Theta}} = 0$ that means Θ is an ignorable coordinate [1]. Ignorable coordinates are those coordinates which don't appear in the system's Lagrangian function of this mechanical system. Ignorable coordinates conjugate momentums are always conserved. As Θ represents angular coordinate so the constant is angular momentum. Here we are denoting the angular momentum as p_Θ. So, the equation of motion for Θ coordinate becomes,

$$m_1r^2\dot{\Theta} = p_\Theta \tag{12}$$

$$\dot{\Theta} = \frac{p_\Theta}{m_1r^2} \tag{13}$$

Euler Lagrange equation or equation of motion for r coordinate,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$$

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{\Theta}}\left(\frac{1}{2}m_1\left(\dot{r}^2 + r^2\dot{\Theta}^2\right) + \frac{1}{2}m_2\dot{r}^2 + m_2gl - m_2gr\right)\right) - \frac{\partial}{\partial \Theta}\left(\frac{1}{2}m_1\left(\dot{r}^2 + r^2\dot{\Theta}^2\right) + \frac{1}{2}m_2\dot{r}^2 + m_2gl - m_2gr\right) = 0$$

$$\frac{d}{dt}(m_1\dot{r} + m_2\dot{r}) - \left(m_1r\dot{\Theta}^2 - m_2g\right) = 0$$

$$m_1\ddot{r} + m_2\ddot{r} - m_1r\dot{\Theta}^2 + m_2g = 0$$

$$\ddot{r}(m_1 + m_2) = m_1r\dot{\Theta}^2 - m_2g$$

$$\ddot{r} = \frac{m_1r\dot{\Theta}^2}{m_1+m_2} - \frac{m_2g}{m_1+m_2}$$

$$\ddot{r} = \frac{m_1r}{m_1+m_2}\left(\frac{p_\Theta}{m_1r^2}\right)^2 - \frac{m_2g}{m_1+m_2}$$

$$\ddot{r} = \frac{{p_\Theta}^2}{m_1(m_1+m_2)r^3} - \frac{m_2g}{m_1+m_2} \tag{14}$$

When r becomes negative or greater than l that means when m_1 and m_2 try to pass through the hole then equation (13) and equation (14) get violated because the system will get destroyed.

2.2.6. Angular momentum at equilibrium

Here equilibrium means m_2 is at rest. It will happen only if the weight of m_2 will be equal to centrifugal force of m_1. That means,

$$m_2 g = m_1 r \dot{\Theta}^2$$

$$\dot{\Theta}^2 = \frac{m_2 g}{m_1 r}$$

$$\dot{\Theta} = \sqrt{\frac{m_2 g}{m_1 r}} \tag{15}$$

From equation (12),

$$m_1 r^2 \dot{\Theta} = p_\Theta$$

$$m_1 r^2 \sqrt{\frac{m_2 g}{m_1 r}} = p_\Theta$$

$$p_\Theta = \sqrt{\frac{m_1^2 m_2 r^4 g}{m_1 r}}$$

$$p_\Theta = \sqrt{m_1 m_2 g r^3} \tag{16}$$

3. Numerical Methods

3.1. Euler Method

Let,

$$\frac{dy}{dx} = f(x, y) \tag{17}$$

is the differential equation.

The solution of this differential equation by Euler Method is,

$$y_{n+1} = y_n + hf(x_n, y_n) \qquad where,\ n = 0,1,2,3 \ldots \ldots . \tag{18}$$

Here, h is the difference between two consecutive values of x which is called step size and it has to take a constant value. y_0 stands for the initial value of the function and x_0 stands for the initial value of x. This method gives us the solution in tabular form. If the differential equation is second order differential equation then just make it two first order differential equations and the rest of the solving process is in the same manner.

3.1.1. Solutions of the equation of motion of the system by Euler Method

In our system, there are two differential equations. One is first order and another one is second order. At first, we will solve equation (13) which is for the Θ coordinate.

Now,

$$\dot{\Theta} = \frac{d\Theta}{dt} = \frac{p_\Theta}{m_1 r^2} = f(t, \Theta) \tag{19}$$

So the solution is,

$$\Theta_{n+1} = \Theta_n + hf(t_n, \Theta_n) = \Theta_n + h\frac{p_\Theta}{m_1 r^2} \tag{20}$$

For equation (14), first we need to make it two first order differential equations. For this purpose, let,

$$\dot{r} = \frac{dr}{dt} = v_r = q(t, r, v_r) \tag{21}$$

and,

$$\ddot{r} = \frac{d^2 r}{dt^2} = \frac{dv_r}{dt} = \frac{{p_\Theta}^2}{m_1(m_1+m_2)r^3} - \frac{m_2 g}{m_1+m_2} = s(t, r, v_r) \tag{22}$$

So the solutions are,

$$r_{n+1} = r_n + hq\left(t_n, r_n, v_{r_n}\right) = r_n + hv_{r_n} \tag{23}$$

and,

$$v_{r_{n+1}} = v_{r_n} + hs(t_n, r_n, v_{r_n}) = v_{r_n} + h(\frac{{p_\Theta}^2}{m_1(m_1+m_2)r_n^3} - \frac{m_2 g}{m_1+m_2}) \tag{24}$$

3.2. 4th Order Runge Kutta Method

Let,

$$\frac{dy}{dx} = f(x, y) \tag{25}$$

is the differential equation.

The solve of this differential equation by 4th Order Runge Kutta Method is,

$$y_n = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \qquad where,\ n = 0,1,2,3 \ldots \ldots \tag{26}$$

Where,

$$\left.\begin{aligned} k_1 &= hf(x_n, y_n) \\ k_2 &= hf(x_n + \tfrac{h}{2}, y_n + \tfrac{k_1}{2}) \\ k_3 &= hf(x_n + \tfrac{h}{2}, y_n + \tfrac{k_2}{2}) \\ k_4 &= hf(x_n + h, y_n + k_3) \end{aligned}\right\} \tag{27}$$

$$where, n = 0,1,2,3 \ldots \ldots$$

Here, h (step size) is the difference between two consecutive values of x and it has to take a constant value. y_0 stands for the initial value of the function and x_0 stands for the initial value of x. This method also gives us the solution in tabular form. If the differential equation is second order differential equation then just make them two first order differential equations and the rest of the solve process is in same manner.

3.2.1. Solution of the equation of motion of the system by 4th Order Runge Kutta Method

As we know, in our system, there are two differential equations. One is first order and another one is second order. At first, we solve equation (13).

Equation (13) is,

$$\dot{\Theta} = \frac{d\Theta}{dt} = \frac{p_\Theta}{m_1 r^2} = f(t, \Theta) \tag{28}$$

So the solution is,

$$\Theta_{n+1} = \Theta_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{29}$$

Where,

$$\left.\begin{aligned}
k_1 &= hf(t_n, \Theta_n) = h\frac{p_\Theta}{m_1 r^2} \\
k_2 &= hf\left(t_n + \frac{h}{2}, \Theta_n + \frac{k_1}{2}\right) = h\frac{p_\Theta}{m_1 r^2} \\
k_3 &= hf\left(t_n + \frac{h}{2}, \Theta_n + \frac{k_2}{2}\right) = h\frac{p_\Theta}{m_1 r^2} \\
k_4 &= hf(t_n + h, \Theta_n + k_3) = h\frac{p_\Theta}{m_1 r^2}
\end{aligned}\right\} \tag{30}$$

$$where, n = 0,1,2,3 \ldots \ldots ..$$

For equation (14), first we need to make two first order differential equations of this second order differential equation. For this purpose, let,

$$\dot{r} = \frac{dr}{dt} = v_r = q(t, r, v_r) \tag{31}$$

and,

$$\ddot{r} = \frac{d^2 r}{dt^2} = \frac{dv_r}{dt} = \frac{{p_\Theta}^2}{m_1(m_1+m_2)r^3} - \frac{m_2 g}{m_1+m_2} = s(t, r, v_r) \tag{32}$$

So the solutions are,

$$r_{n+1} = r_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{33}$$

$$v_{r_{n+1}} = v_{r_n} + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) \tag{34}$$

where,

$$\left.\begin{aligned}
k_1 &= hq(t_n, r_n, v_{r_n}) = hv_{r_n} \\
l_1 &= hs(t_n, r_n, v_{r_n}) = h\left(\frac{{p_\Theta}^2}{m_1(m_1+m_2)r_n^3} - \frac{m_2 g}{m_1+m_2}\right) \\
k_2 &= hq\left(t_n + \frac{h}{2}, r_n + \frac{k_1}{2}, v_{r_n} + \frac{l_1}{2}\right) = h\left(v_{r_n} + \frac{l_1}{2}\right) \\
l_2 &= hs\left(t_n + \frac{h}{2}, r_n + \frac{k_1}{2}, v_{r_n} + \frac{l_1}{2}\right) = h\left(\frac{{p_\Theta}^2}{m_1(m_1+m_2){(r_n+\frac{k_1}{2})}^3} - \frac{m_2 g}{m_1+m_2}\right) \\
k_3 &= hq\left(t_n + \frac{h}{2}, r_n + \frac{k_2}{2}, v_{r_n} + \frac{l_2}{2}\right) = h\left(v_{r_n} + \frac{l_2}{2}\right) \\
l_3 &= hs\left(t_n + \frac{h}{2}, r_n + \frac{k_1}{2}, v_{r_n} + \frac{l_2}{2}\right) = h\left(\frac{{p_\Theta}^2}{m_1(m_1+m_2){(r_n+\frac{k_2}{2})}^3} - \frac{m_2 g}{m_1+m_2}\right) \\
k_4 &= hq(t_n + h, r_n + k_3, v_{r_n} + l_3) = h(v_{r_n} + l_3) \\
l_4 &= hs(t_n + h, r_n + k_3, v_{r_n} + l_3) = h\left(\frac{{p_\Theta}^2}{m_1(m_1+m_2){(r_n+k_3)}^3} - \frac{m_2 g}{m_1+m_2}\right)
\end{aligned}\right\} \tag{35}$$

$$where, n = 0,1,2,3 \ldots \ldots ..$$

4. Results and Analysis

4.1. Trajectory for Different Angular Momenta (p_Θ)

For,

$$m_1 = 6, \quad m_2 = 4, \quad g = 9.8, \quad l = 3, \quad r = 2.5, \quad v_{r_0} = 0, \quad \Theta_0 = 15^0$$

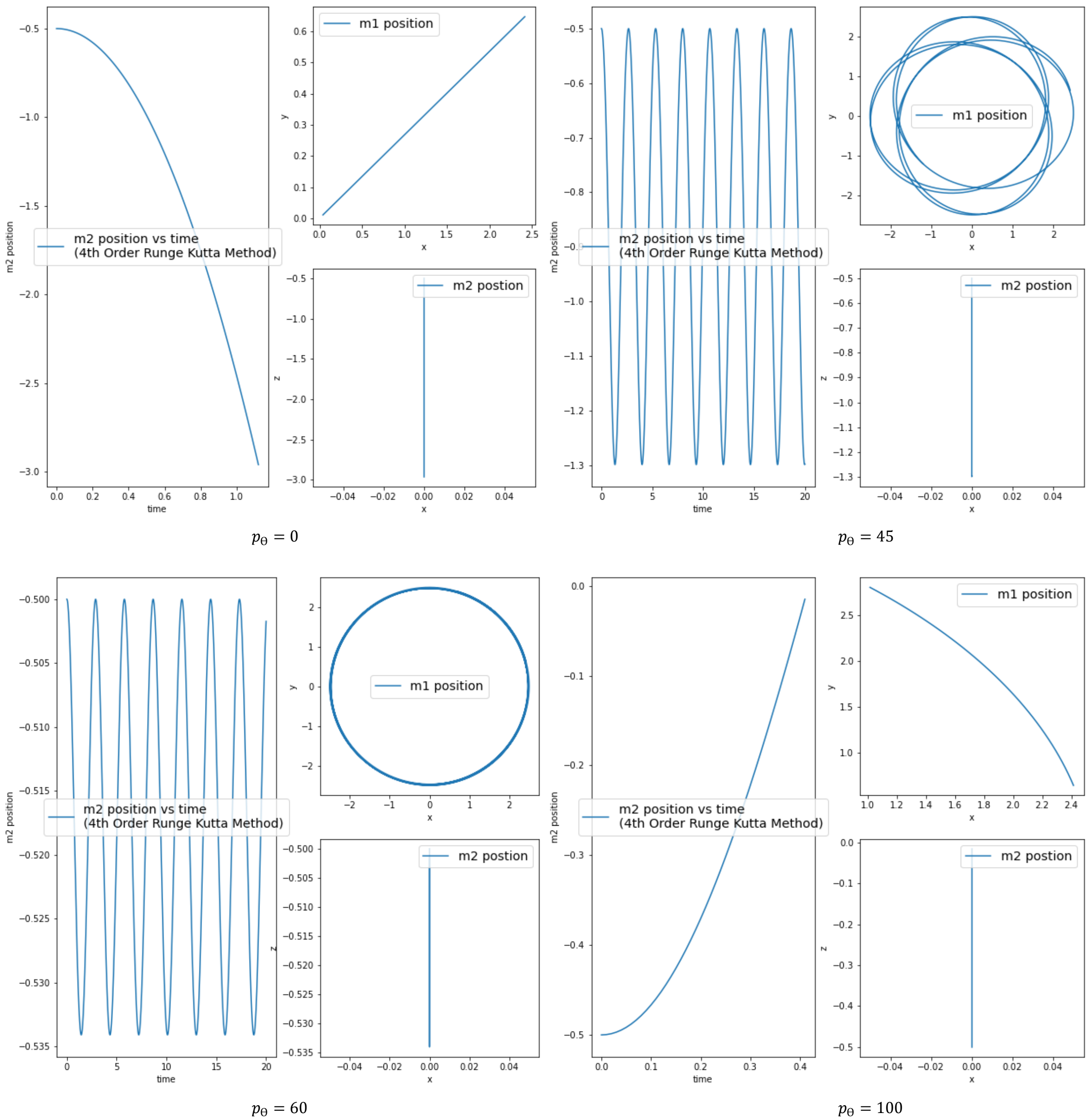


Figure 2: Trajectory for different p_Θ.

We take here four different p_Θ. They are 0, 45, 60 and 100. We can see that, for different angular momentum (p_Θ) m_1 object travels completely in different manner. It starts moving in a straight line and as p_Θ starts increasing, it turns into circular shape and at a particular value of p_Θ, it moves completely in a circular path. And as p_Θ increases more, after a certain time, it destroys the system.

4.2. Error Finding for the Two Methods Comparing with Equilibrium Condition

As we know, at equilibrium, angular moment, $p_\Theta = \sqrt{m_1 m_2 g r^3}$

We take, $m_1 = 10,\ m_2 = 5,\ g = 9.8,\ r = 2.5$

which gives, $p_\Theta = 87.5$

Now, we are going to see the solution by Euler and 4th Order Runge methods.

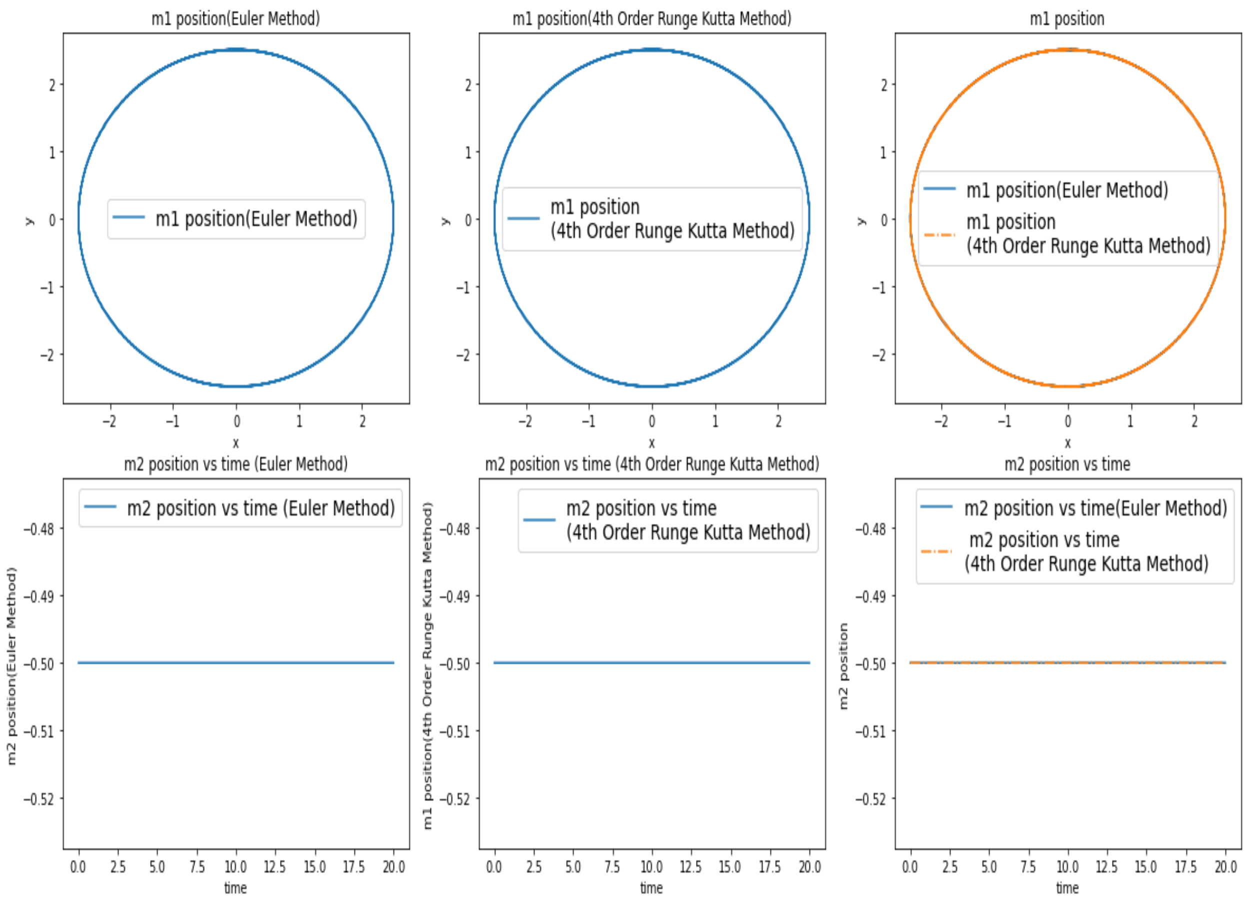
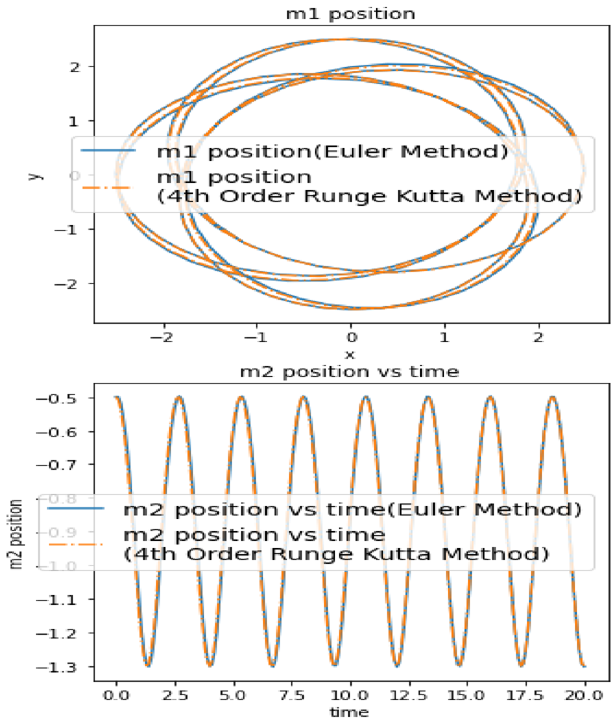


Figure 3: Trajectory at equilibrium.

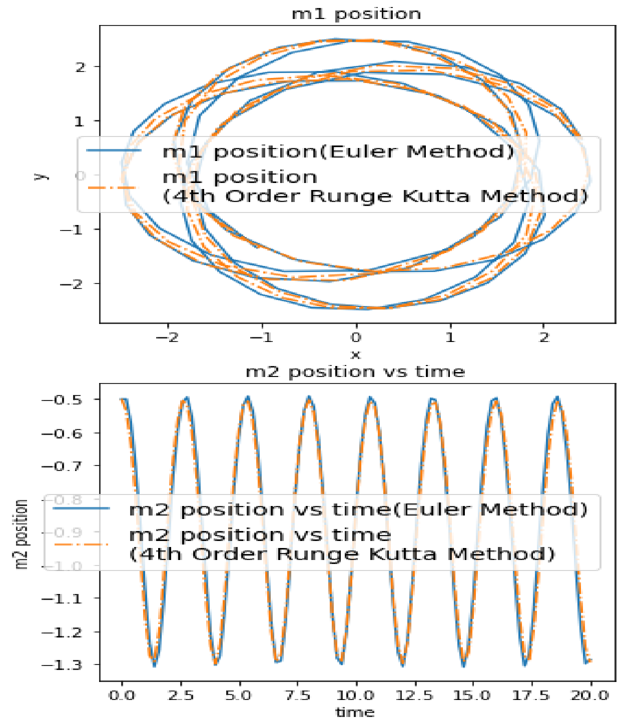
The upper row of the graph shows the trajectory of m_1 and the lower row shows the position of m_2 with time. We can see that the distance from the origin to m_1 is always the same and m_2 is at rest. Which exactly match with theory. Because, in theory portion we describe that equilibrium means, m_2 is in rest position and m_1 moves completely in a circular path. Graph also shows that, the solution is same in both methods.

4.3. Difference Between the Methods for Different Values of h

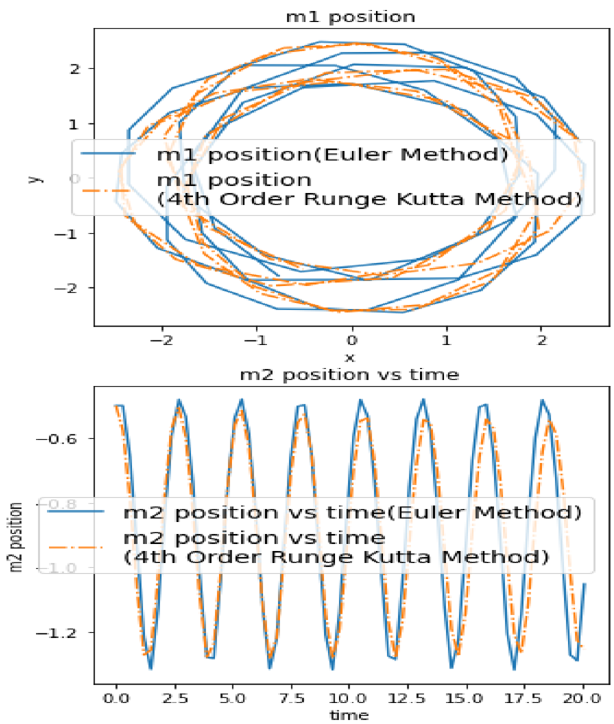
We take, $m_1 = 6,\ m_2 = 4,\ g = 9.8,\ p_\Theta = 45,\ l = 3,\ r = 2.5,\ v_{r_0} = 0,\ \Theta_0 = 15^0$



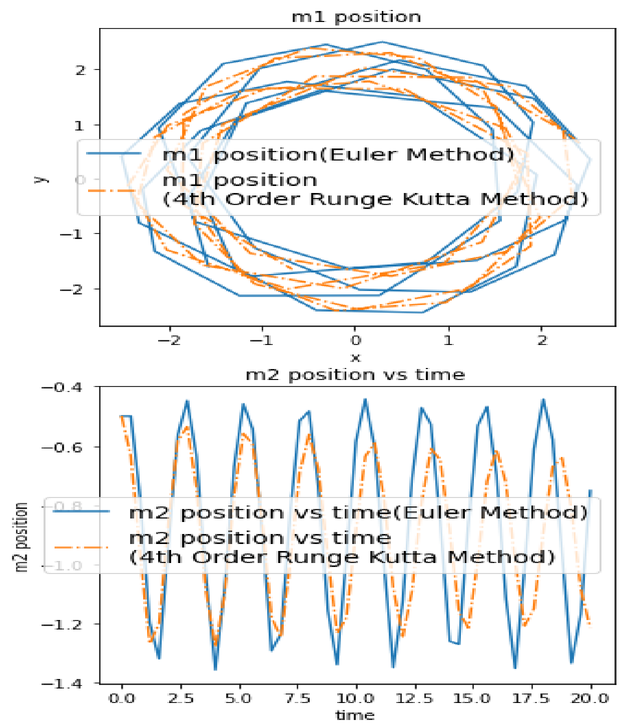
h=0.1



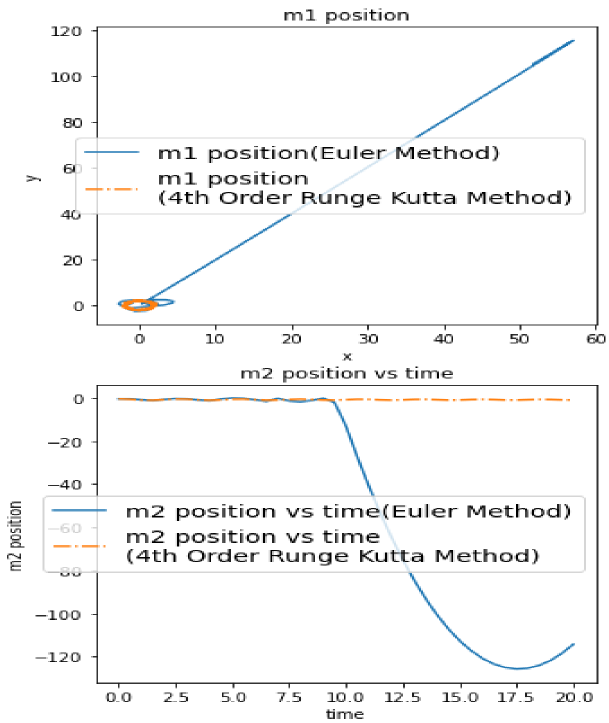
h=0.2



h=0.3



h=0.4



h=0.5

Figure 4: Solution for different h.

Above graph is drawn for five different values of step size (h). They are 0.1, 0.2, 0.3, 0.4, 0.5. Figure 4 shows thats, for h=0.1, results between the two methods are almost same. For h=0.2, it shows a small difference. But as h starts increasing more the difference also get increasing more. Which we can see for h=0.3 and h=0.4. But after a particular value it starts show a huge difference (h=0.5). If we compare the five graphs, we can say that 4th Order Runge Kutta gives more accurate result, because as step size increase(h) it affects Euler method more with compared to 4th Order Runge Kutta method.

4.4. Energy vs Time Graph

For different values of $m_1, m_2, p_\Theta, l, r, v_{r_0}, \Theta_0$ we plot energy vs time graph. The graph (figure 5) is drown for different initial conditions and masses. In all the graphs we can see that, the total energy is conserved which indicates the methods give us accurate result.

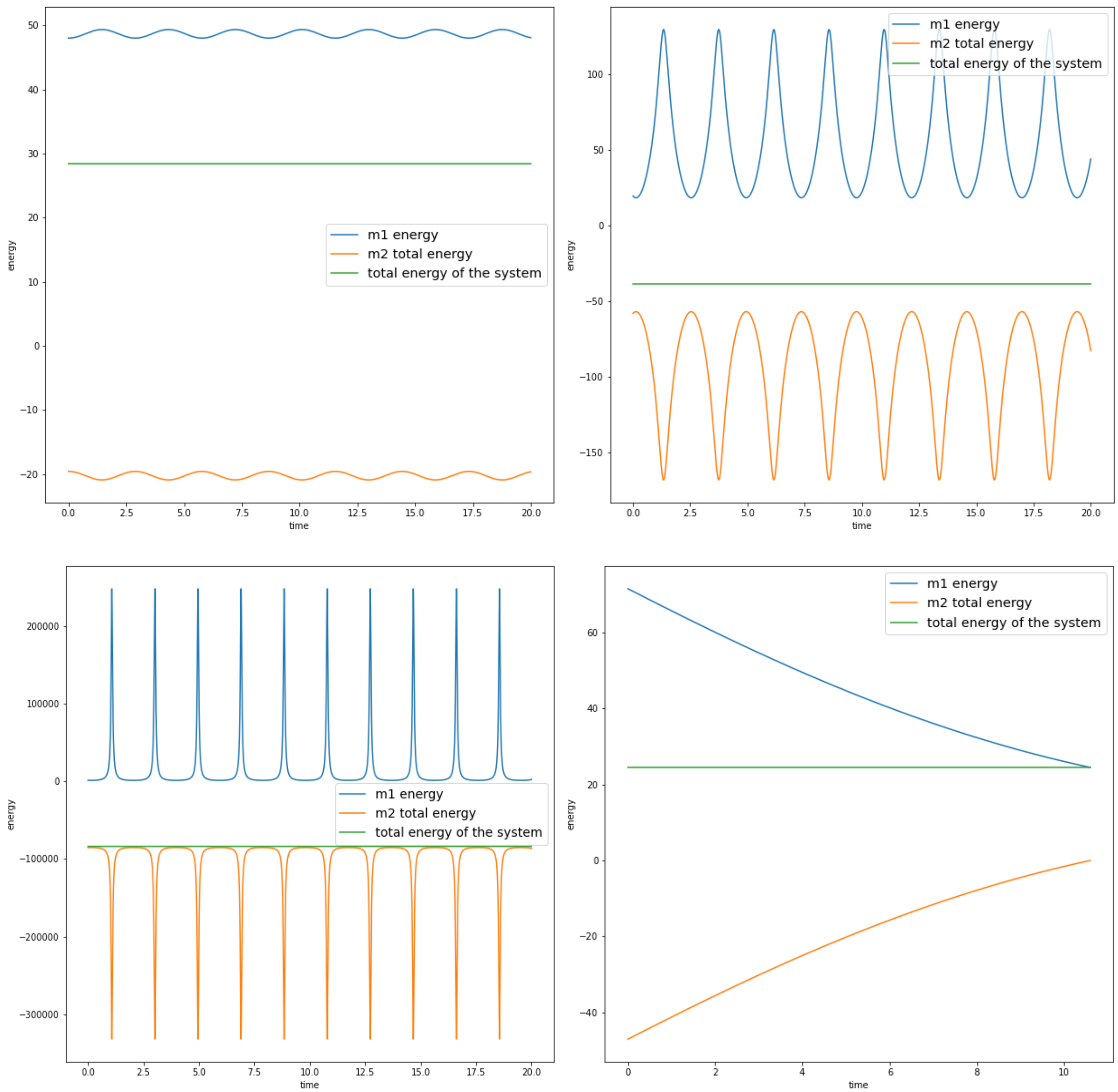


Figure 5: Energy vs Time.

4.5. Different Conditions of m_1 and m_2

In this section, we see how the result differs from the extreme conditions of m_1 and m_2 and normal values of m_1 and m_2.

4.5.1. $m_1 \ll m_2$

First two columns (Figure 6) is for $m_1 \ll m_2$ and last two columns (Figure6) is for $m_1 < m_2$. For $m_1 \ll m_2$ condition we take, , $m_1 = 1,\ m_2 = 1000,\ g = 9.8,\ l = 3,\ r = 2.5,\ v_{r_0} = 0,\ \Theta_0 = 15^0$ and for $m_1 < m_2$, we take $m_1 = 4,\ m_2 = 6,\ g = 9.8,\ l = 3,\ r = 2.5,\ v_{r_0} = 0,\ \Theta_0 = 15^0$. For first raw $p_\Theta = 0$ and for second raw $p_\Theta = 60$.

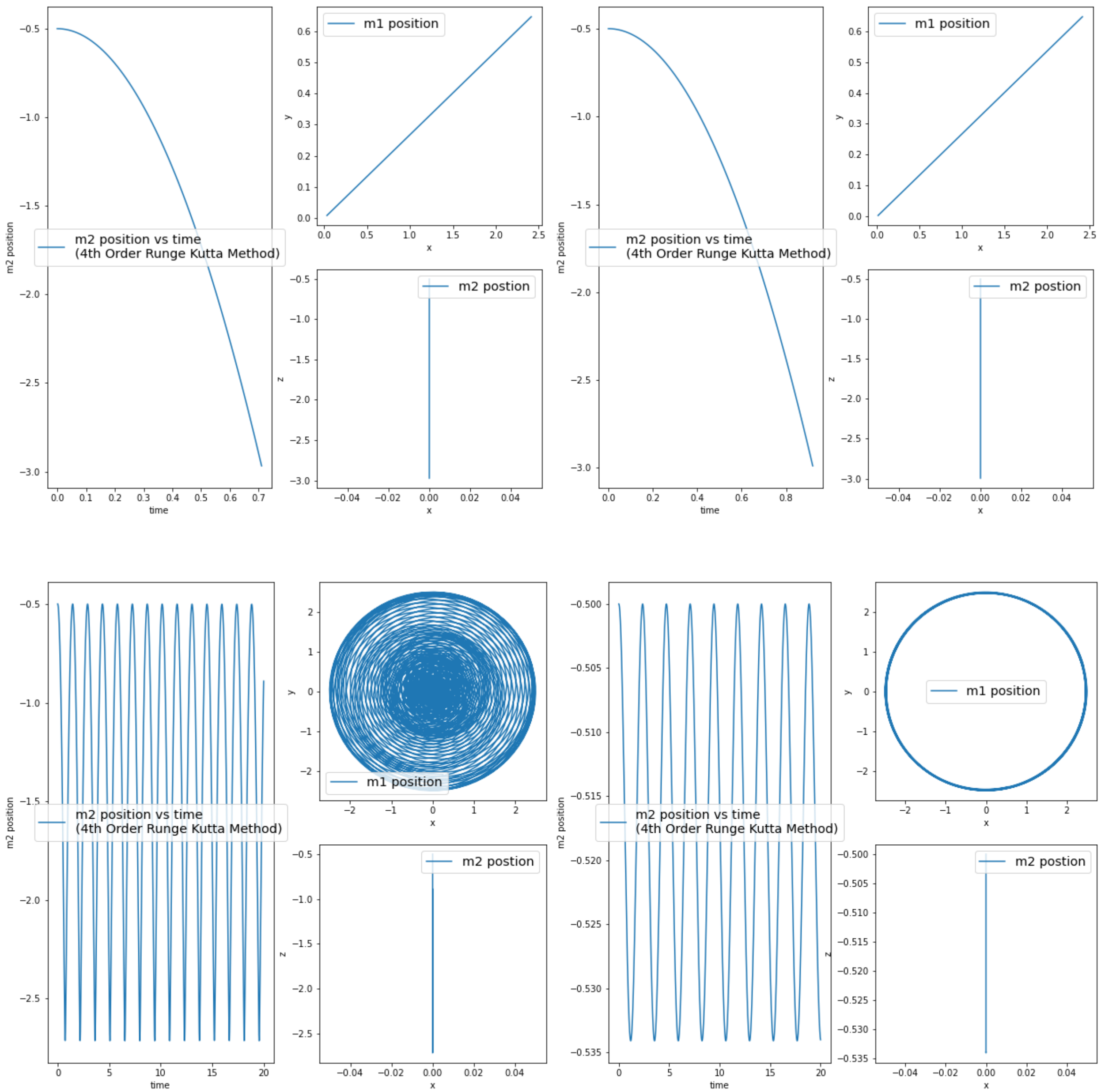


Figure 6: Comparing between $m_1 \ll m_2$ and $m_1 < m_2$.

In first row (figure 6), we can see, when $m_1 \ll m_2$, it takes less time to destroy the system than $m_1 < m_2$. In second row (figure 7), we also see that, for $m_1 \ll m_2$, time period is shorter than

$m_1 < m_2$. So, it is clear that, when $m_1 \ll m_2$ then its time period is shorter than the condition $m_1 < m_2$.

4.5.2. $\boldsymbol{m_1 \gg m_2}$

First two columns (Figure 7) is for $m_1 \gg m_2$ and last two columns (Figure 7) is for $m_1 > m_2$. For $m_1 \gg m_2$ condition we take, , $m_1 = 1000,\ m_2 = 1,\ g = 9.8,\ l = 3,\ r = 2.5,\ v_{r_0} = 0,\ \Theta_0 = 15^0$ and for $m_1 > m_2$, we take $m_1 = 6,\ m_2 = 4,\ g = 9.8,\ l = 3,\ r = 2.5,\ v_{r_0} = 0,\ \Theta_0 = 15^0$. For first raw $p_\Theta = 0$ and for second raw $p_\Theta = 60$.

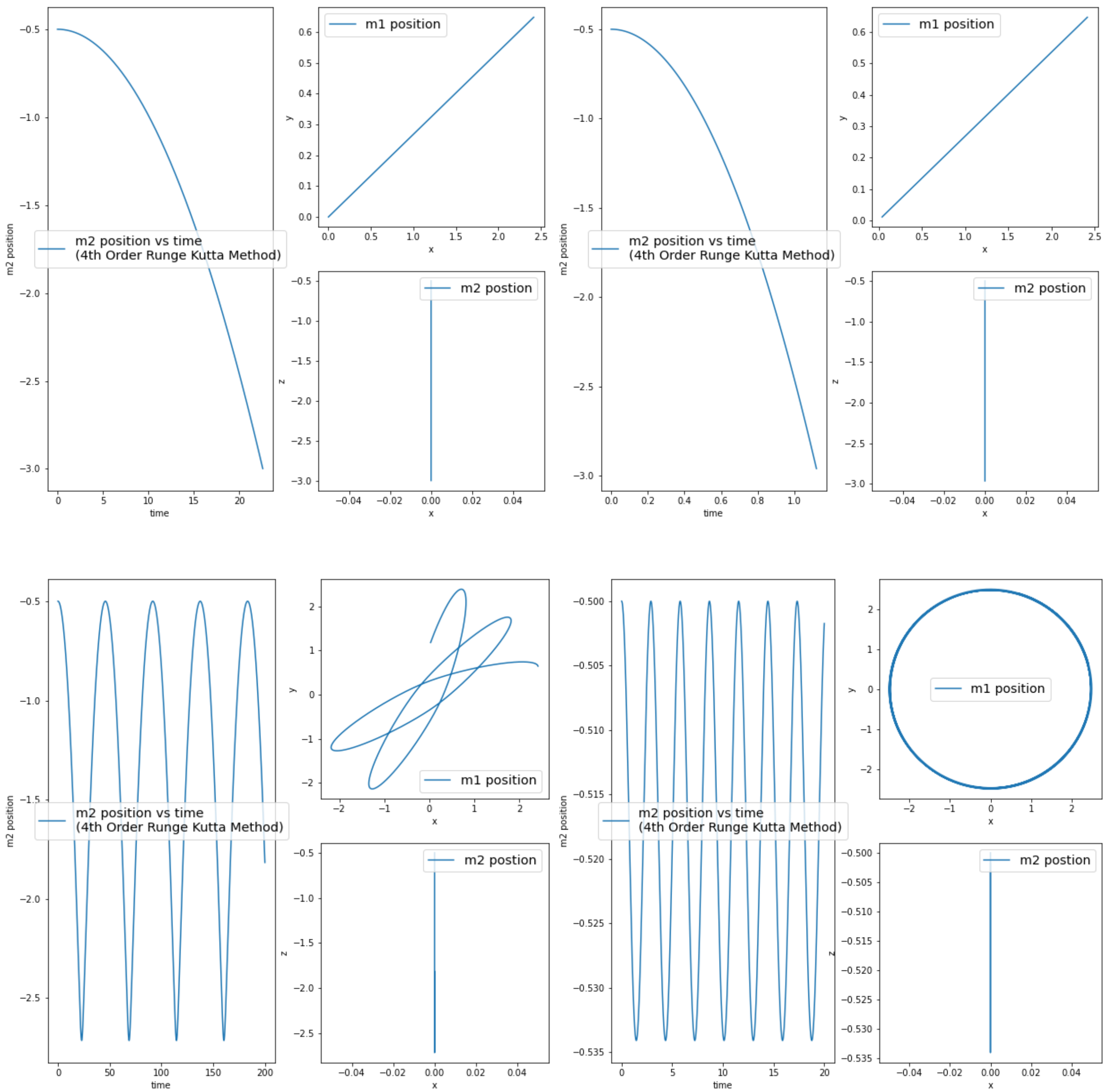


Figure 7: Comparing between $m_1 \gg m_2$ and $m_1 > m_2$.

In first row (figure 7), we can see, when $m_1 \gg m_2$, it takes huge time to destroy the system than $m_1 > m_2$. In second row (figure 7), we also see that, for $m_1 \gg m_2$, time period is longer than $m_1 > m_2$. So, it is clear that, when $m_1 \gg m_2$ then its time period is longer than the condition $m_1 > m_2$.

4.5.3. $\boldsymbol{m_1 = m_2}$

We plot for two different values of m; 0.6 and 60. For first row, we take initial velocity, $v_{r_0} = 0$ and initial angular position, $\Theta_0 = 15^0$. For second row, initial velocity is $v_{r_0} = 2$ and initial angular positon is$\Theta_0 = 25^0$. The rest initial for first rows $s = 3, r = 2.5, g = 9.8$ and for second row $s = 6, r = 5, g = 9.8$. And we plot all the graphs for $p_\Theta = 0$. In first two columns, $m_1 = m_2 = 0.6$ and in last two columns $m_1 = m_2 = 60$.

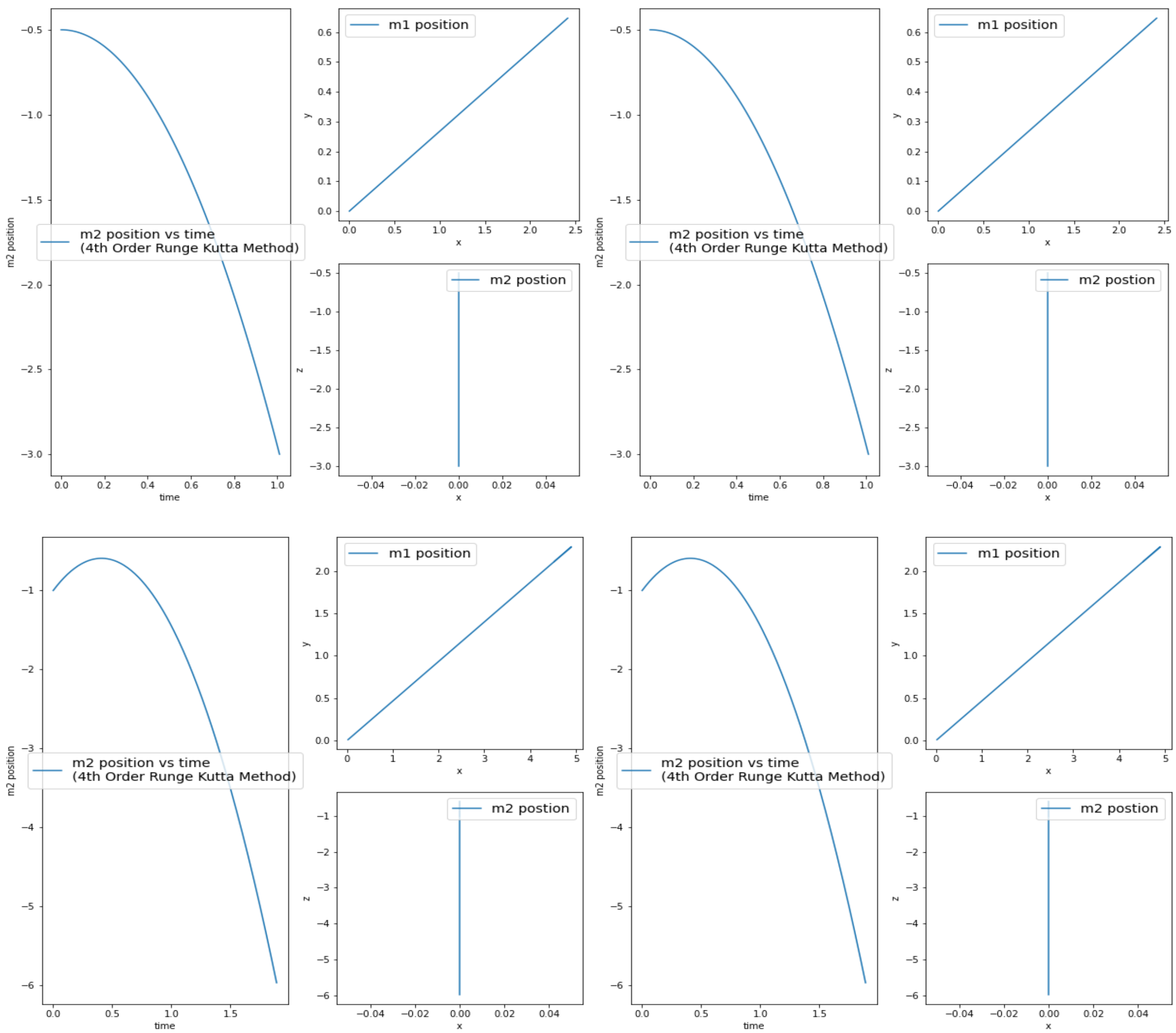
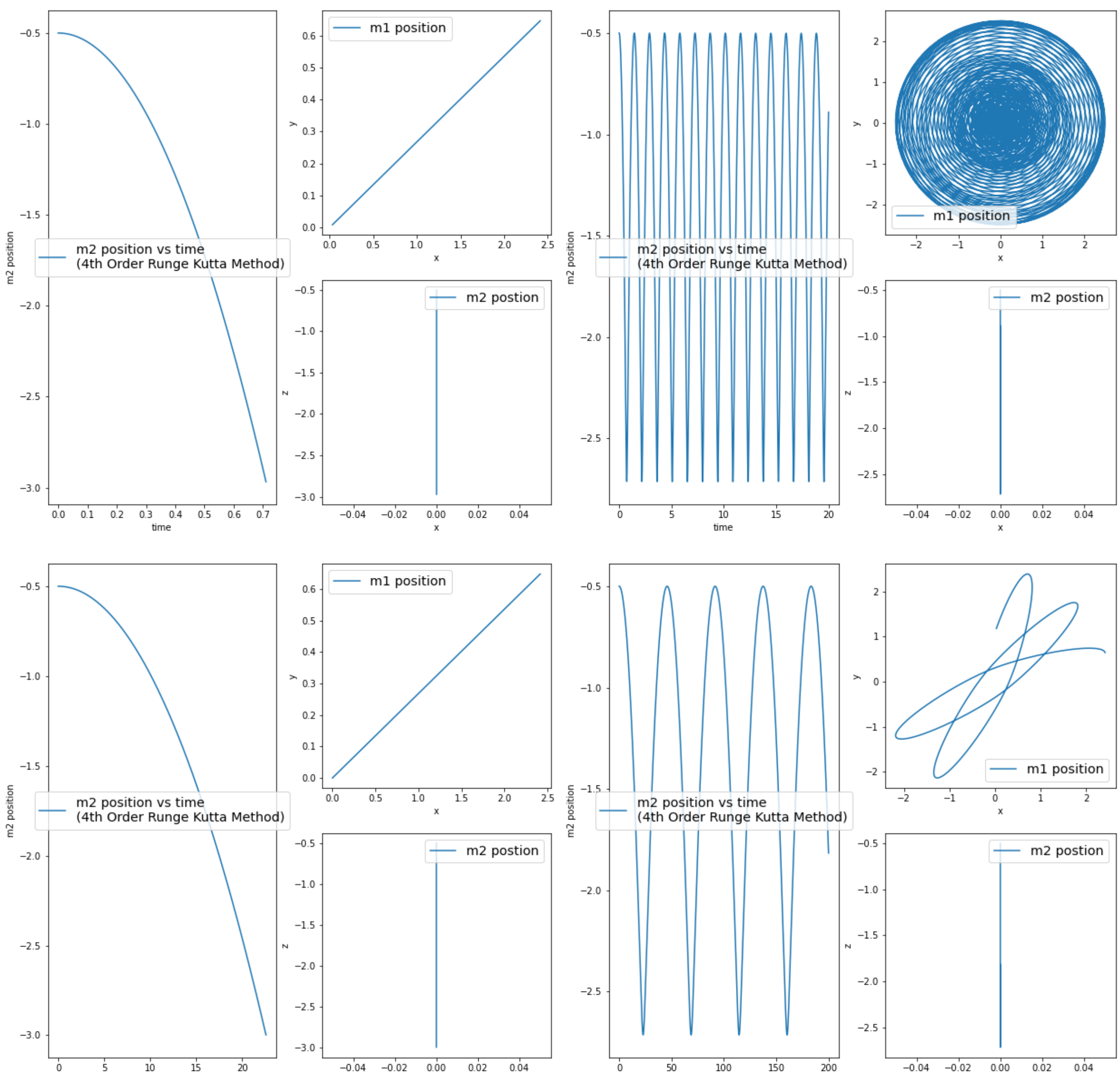


Figure 8: Comparison between various $m_1 = m_2$ values.

In first row we can see that, for different $m_1 = m_2$ values , the time required for destroying the system is same and that is 1. In second row we can see the same pattern, for different $m_1 = m_2$ values, the time is same for destroying the system and that is 1.89. So we can say that, for all $m_1 = m_2$ values, if initial conditions are same then time period is also same.

4.5.4. Comparison between $m_1 \ll m_2$, $m_1 \gg m_2$ and $m_1 = m_2$

First row (Figure 9) is for $m_1 \ll m_2$, second row (Figure 9) is for $m_1 \gg m_2$ and third row (Figure 9) is for $m_1 = m_2$. For the condition $m_1 \ll m_2$ we take $m_1 = 1,\ m_2 = 1000,\ g = 9.8,\ l = 3,\ r = 2.5,\ v_{r_0} = 0,\ \Theta_0 = 15^0$, for $m_1 \gg m_2$ we take $m_1 = 1000,\ m_2 = 1,\ g = 9.8,\ l = 3,\ r = 2.5,\ v_{r_0} = 0,\ \Theta_0 = 15^0$and for $m_1 = m_2$ we take $m_1 = 6,\ m_2 = 6, g = 9.8,\ l = 3,\ r = 2.5,\ v_{r_0} = 0,\ \Theta_0 = 15^0$. For first column $p_\Theta = 0$ and for second raw $p_\Theta = 60$.



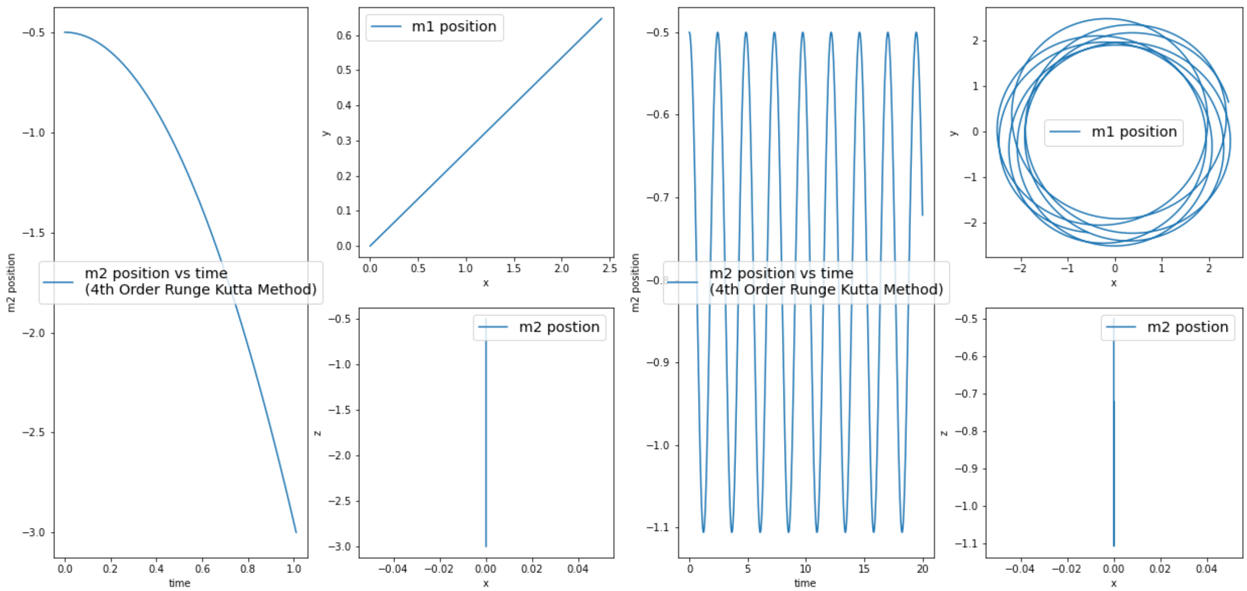


Figure 9: Comparison between $m_1 \ll m_2$ and $m_1 \gg m_2$ and $m_1 = m_2$.

In figure 9 we can see that, when $m_1 \ll m_2$, time period or the time required to destroy the system is less than $m_1 \gg m_2$ and $m_1 = m_2$. We also can see that, time period or the time required to destroy the system is higher than $m_1 \ll m_2$ and $m_1 = m_2$.

So the information we got from this graph are, for$m_1 \gg m_2$ time period is too long, for $m_1 \ll m_2$ is shorter in comparison to $m_1 = m_2$.

5. Conclusion

The Euler Lagrange equation or equation of motion of our system doesn't have any specific analytical solution, it gives different solutions for different conditions and also a complicated process. So it is better to use numerical method to get the result. Between Euler and 4th Order Runge Kutta method, 4th Order Runge Kutta method gives the more accurate result. If step size (h) is low then these two methods give almost same result but as h increases, the difference gets higher. In comparison between two methods, 4th Order Runge Kutta method gives better result than Euler method when the value of h is high. At equilibrium both the methods give the same result. In comparison between values of mass of the objects, we can see that if the mass of the object above the surface is much greater than the mass of the object under the surface then the time period of the system has a large value and when the mass of the object above the surface is much lesser than the mass of the object under the surface then the time period of the system has a lower value in comparison to other mass condition. We also see, when angular momentum is zero and the objects have same mass values then the time period is same in all values of equal mass condition for same initial conditions.

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