To prove whether f(n) is $O(g(n))/\theta(g(n))/\Omega(g(n))$ using the definition you need to find the constants C_1/C_2 and n_o. But sometimes it might seem difficult to search for such constants. Instead we can apply limit method. Note following rules:

1. If $0 < \lim_{n\to\infty} f(x)/g(x) < \infty$, then $f(x)$ is $\theta(g(x))$.
2. If $\lim_{n\to\infty} f(x)/g(x) < \infty$, then $f(x)$ is $O(g(x))$.
3. If $\lim_{n\to\infty} f(x)/g(x) > 0$, then $f(x)$ is $\Omega(g(x))$.

Examples:

1. Prove that $9n^2 - 14n + 25 = \Omega(n^2)$.

   We can prove it one of two ways - using the limit of $f(n)/g(n)$ as $n \to \inf$, or using the definition of $\Omega(\cdot)$.

$$\begin{aligned}\lim_{n\to\inf}\tfrac{f(n)}{g(n)} &= \lim_{n\to\inf}\tfrac{9n^2-14n+25}{n^2}\\ &= \lim_{n\to\inf} 9 - \tfrac{14}{n} + \tfrac{25}{n^2}\\ &= 9\\ &> 0\end{aligned}$$

   Since $\lim_{n\to\inf} f(n)/g(n)$ exists, and is > 0, $9n^2 - 14n + 25 = \Omega(n^2)$.

2. $n^2 + 4$ is *not* $\Omega(n^3)$

$$\lim_{n\to\infty}\frac{n^2+4}{n^3} = \frac{1}{n} + \frac{4}{n^3} = 0$$

   $\lim_{n\to\infty}\frac{n^2+4}{n^3} = 0$ proves that $n^2 + 4 \neq \Omega(n^3)$.

3. Prove that $13n^3 - n^2 + 5n - 3 = \Theta(n^3)$.

   At least two ways of doing it, the simplest being the one using limits as $n \to \infty$.

$$\lim_{n\to\infty}\frac{13n^3 - n^2 + 5n - 3}{n^3} = \lim_{n\to\infty}\left(13 - \frac{1}{n} + \frac{5}{n^2} - \frac{3}{n^3}\right) = 13 = \text{constant}$$

   This proves that $13n^3 - n^2 + 5n - 3 = \Theta(n^3)$.

4. Prove that $3n^3 - 17n^2 + 3n + 5 = O(n^3)$.

   We can prove it one of two ways - using the limit of $\frac{f(n)}{g(n)}$ as $n \to \infty$, or using the definition of $O(\cdot)$.

$$\begin{aligned} \lim_{n\to\infty} \tfrac{f(n)}{g(n)} &= \lim_{n\to\infty} \tfrac{3n^3-17n^2+3n+5}{n^3} \\ &= \lim_{n\to\infty} 3 - \tfrac{17}{n} + \tfrac{3}{n^2} + \tfrac{5}{n^3} \\ &= 3 \\ &\neq \infty \end{aligned}$$

   Since $\lim_{n\to\infty} f(n)/g(n)$ exists, and is $\neq \infty$, it proves that $3n^3 - 17n^2 + 3n + 5 = O(n^3)$.