CSE 221: Algorithms

Greedy algorithms

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References

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- Main challenge is to determine the criteria on which the next item will be selected
- Often leads to very efficient solutions to optimization problems.
- However, not all problems have greedy solutions.

Optimal Solution

What is an Optimal Solution?

- Given a problem, more than one solution exist
- One of the solution is the best based on some given constraints, that solution is called the optimal solution

What is Global Optimal Solution?

- Optimal Solution to the main problem

What is local Optimal Solution?

- Optimal Solution to the subproblems

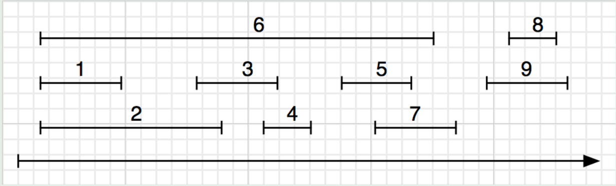
Contents

Designing a greedy algorithm

Definition (Interval scheduling problem)

Given a set of schedules $I = \{I_i\}$, find the largest set $A \subseteq I$ such that the members of A are non-conflicting.

Example



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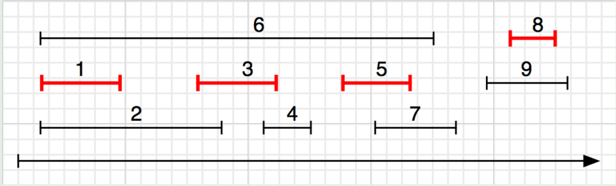

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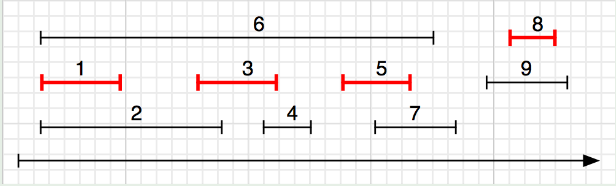

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Question

Is this the only "correct" answer?

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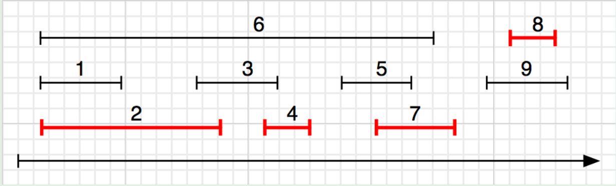

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How about $\{2, 4, 7, 8\}$?

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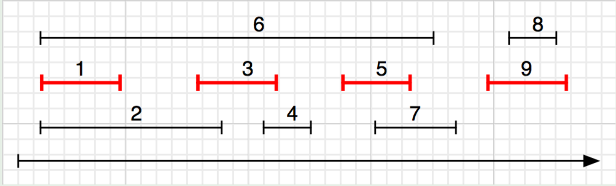

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Question

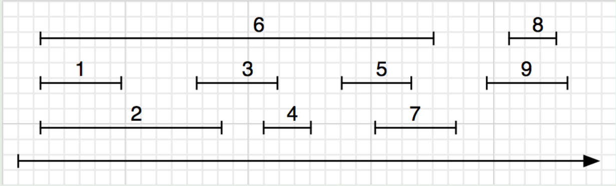
How about $\{2, 4, 7, 8\}$? $\{1, 3, 5, 9\}$?

Designing a greedy algorithm

Definition (Interval scheduling problem)

Given a set of schedules $I = \{I_i\}$, find $A \subseteq I$ such that the members of A are non-conflicting *and* $|A|$ is maximized.

Example



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Question

$\{1, 3, 5, 8\}$? $\{2, 4, 7, 8\}$? $\{1, 3, 5, 9\}$? ... **How many?**

Brute force solution

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1. Enumerate all possible *configurations* (i.e., all possible subsets of the intervals).
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3. Pick (any one of) the largest subset from the ones that survive.

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Complexity

- There are $2^n - 1$ non-empty subsets, one or more of which may be a feasible solution.

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- There are $2^n - 1$ non-empty subsets, one or more of which may be a feasible solution.
- Each feasible solution must be scanned for conflict, which takes $O(n)$ time.
- The algorithm runs in $\Theta(n2^n)$ time $\Rightarrow$ an exponential time algorithm!

Designing a greedy algorithm (continued)

Basic steps

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1. Use a “simple” rule (or strategy) to select the first interval i_1 to be accepted.
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3. Select the second interval i_2 to be accepted, and remove all the intervals that conflict with i_2.

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4. And so on until there are no more requests remain.

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4. And so on until there are no more requests remain.

Key challenge

How to choose the "simple" rule to select the next interval that leads to an optimal solution?

Designing a greedy algorithm (continued)

Strategy 1. *Earliest First*

The idea is to start using the resource as early as possible.

1. Sort the intervals by starting time, breaking ties arbitrarily.
2. Pick the first one, removing it from the list along with all the intervals that conflict with it.
3. Repeat Step 2, until the list is empty.

Example

$|A| = ???.$

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Example (using ***Earliest First*** **strategy)**

$$|A| = 1.$$

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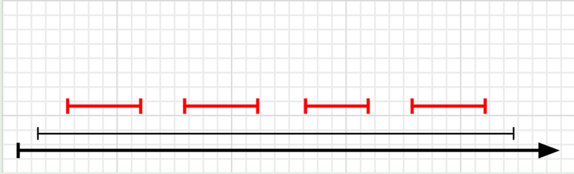
Example (using an optimal strategy)



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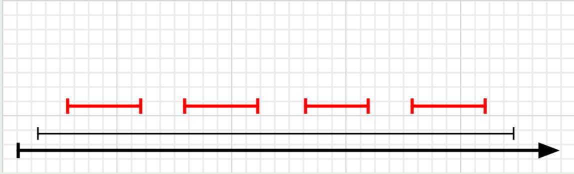
This strategy does not lead to an optimal solution.

Example (using an optimal strategy)



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Designing a greedy algorithm (continued)

Strategy 2. *Shortest First*

The *Earliest First* strategy failed perhaps because it missed the shorter intervals, which would accommodate more intervals.

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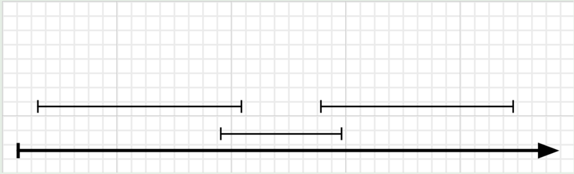

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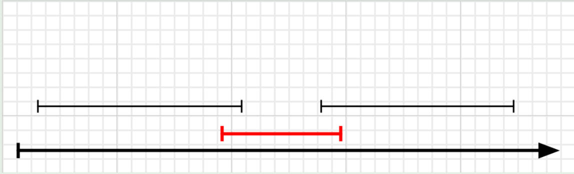

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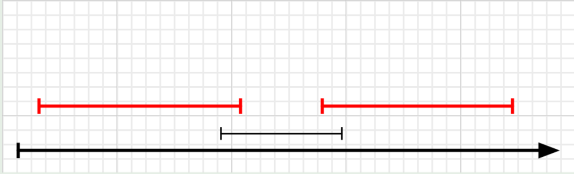

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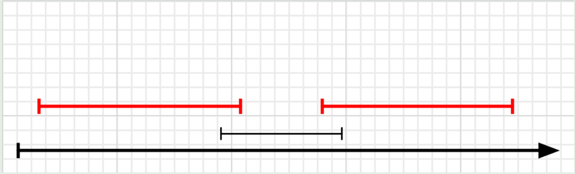

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Designing a greedy algorithm (continued)

Strategy 3. *Least-conflict First*

The *Shortest First* strategy failed perhaps because the shorter ones had more conflicts, and ruled out too many intervals in the process.

1. Sort the intervals by the number of other intervals which conflict with it.
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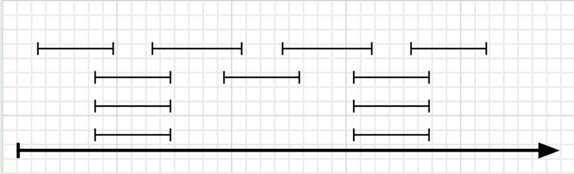

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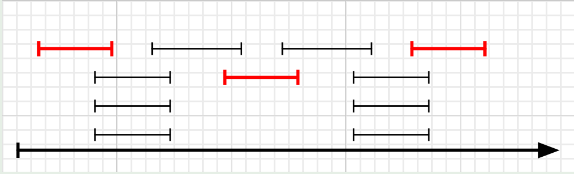

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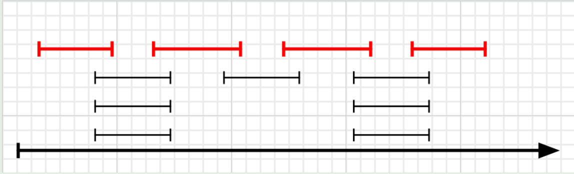

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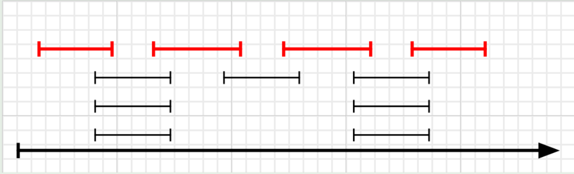

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Designing a greedy algorithm (continued)

Strategy 4. *Finish First*

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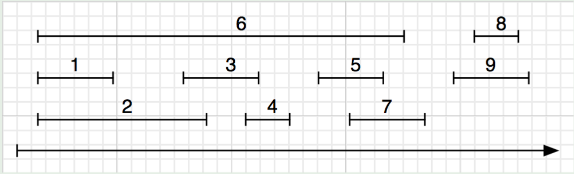

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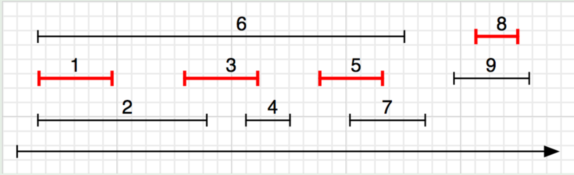

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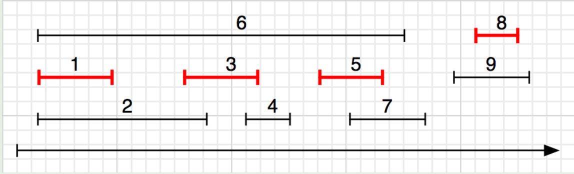

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This strategy is the one that works. But can you prove that it works?

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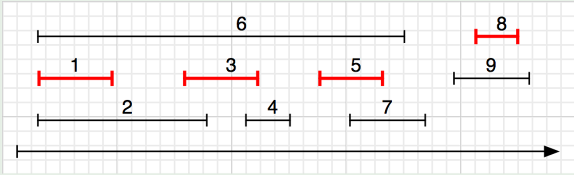

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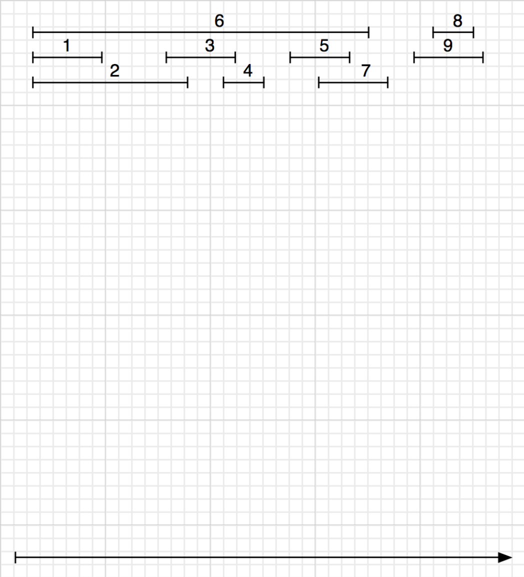
Interval scheduling in action



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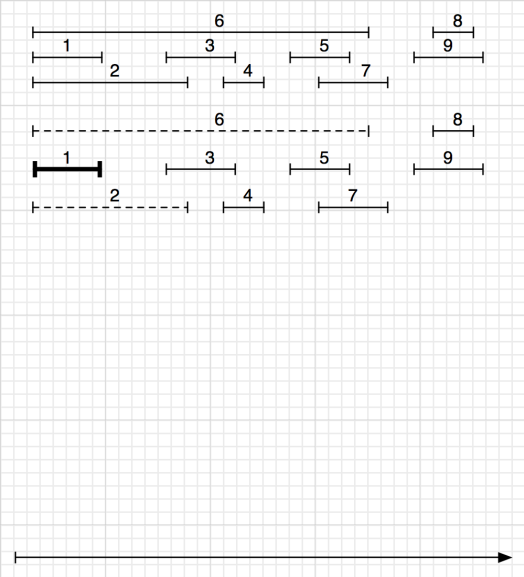

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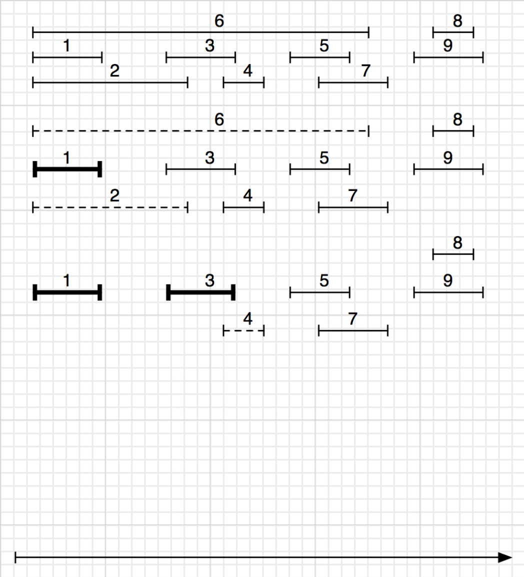

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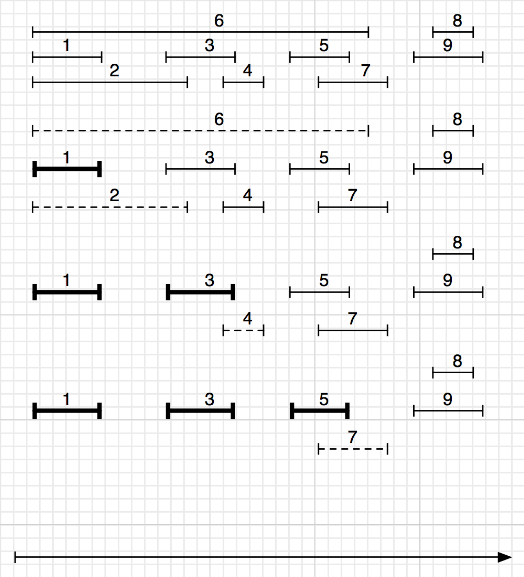

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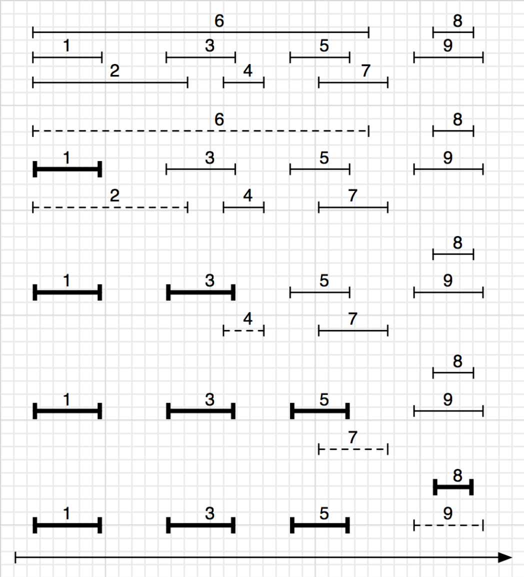

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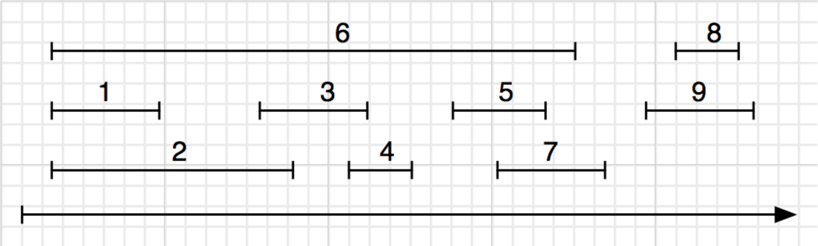

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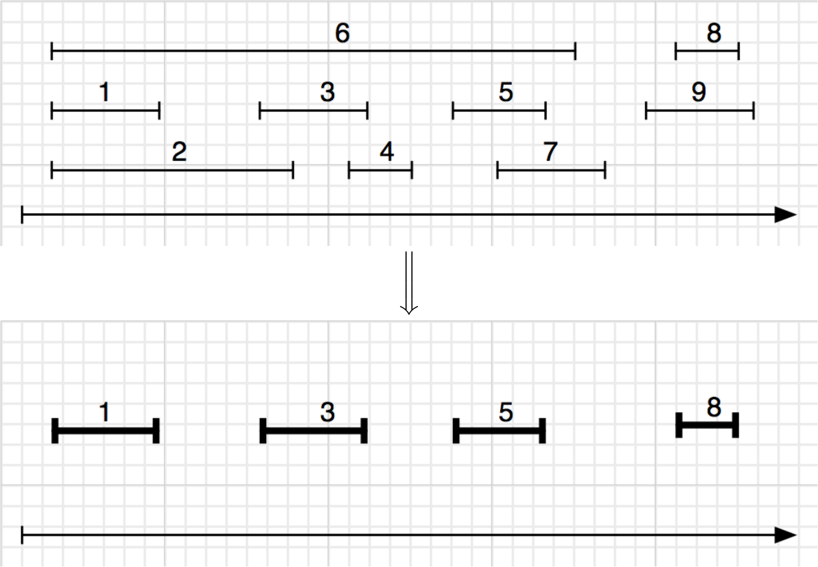

An $O(n \lg n)$ greedy algorithm for interval scheduling

SCHEDULE-INTERVALS(I) $\quad \triangleright\ I = \{I_i\}, I_i = (s_i, f_i)$

R = Sorted requests in order of finishing times such that $f_i \le f_j$ when $i < j$.
Create an array $S[1 \,..\, n]$ with starting times such that $S[i]$ contains s_i.
$A = \{R_1\}$ $\quad \triangleright$ select first interval from sorted list
$f = f_1$
while there are more intervals in S to look at
    do j = first interval for which $s_j \ge f$
        $A \leftarrow A \cup \{j\}$
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return A

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- Creating the starting time array $S[1\,..\,n]$ takes $O(n)$ time.

An $O(n \lg n)$ greedy algorithm for interval scheduling

```
SCHEDULE-INTERVALS(I)    ▷ I = {I_i}, I_i = (s_i, f_i)
R = Sorted requests in order of finishing times such that f_i ≤ f_j when i < j.
Create an array S[1..n] with starting times such that S[i] contains s_i.
A = {R_1}                              ▷ select first interval from sorted list
f = f_1
while there are more intervals in S to look at
      do j = first interval for which s_j ≥ f
         A ← A ∪ {j}
         f ← f_j
return A
```

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- The sorting step in takes $O(n \lg n)$ time.
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    $A \leftarrow A \cup \{j\}$
    $f \leftarrow f_j$
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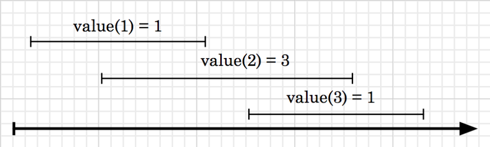
- An $\boxed{O(n \lg n)}$ time algorithm for a problem with a natural search space of $O(n2^n)$.

Extension: weighted interval scheduling problem

Definition (Weighted interval scheduling problem)

Given a set of schedules $I = \{I_i\}$, with associated weights $W = \{w_i\}$, find $A \subseteq I$ such that the members of A are non-conflicting *and* the total weight $\sum_{i \in A} w_i$ is maximized.

Example



$$|A| = ???,\ \sum_{i \in A} w_i = ???.$$

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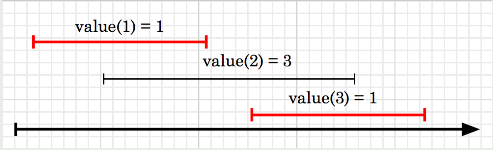
Example (using our greedy strategy)



$$|A| = 2,\ \sum_{i \in A} w_i = 2.$$

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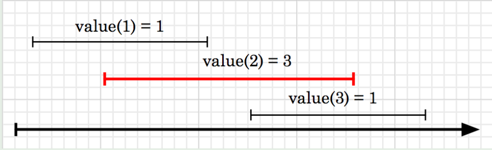
Example (using an optimal strategy)



$$|A| = 1, \sum_{i \in A} w_i = 3.$$

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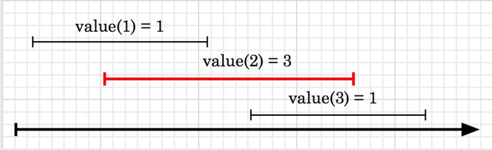

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Hmmm...

There is no greedy solution for the weighted interval scheduling problem!

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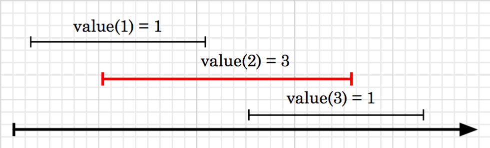

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There is no greedy solution for the weighted interval scheduling problem! Why? (see Greedy Choice property later)

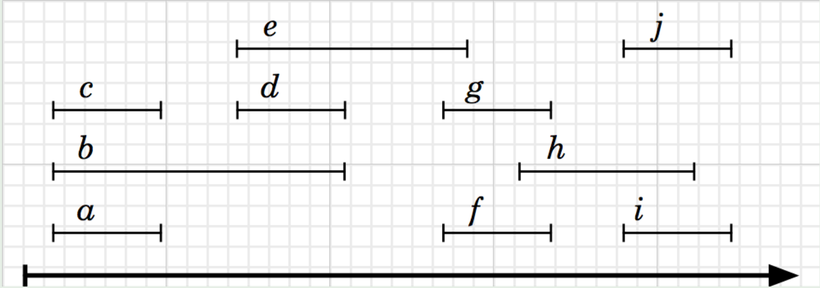
Contents

Scheduling all intervals greedy algorithm

Definition

Given a set of schedules $I = \{I_i\}$, find the minimum number of resources needed to schedule I such that the intervals on each resource are non-conflicting.

Example



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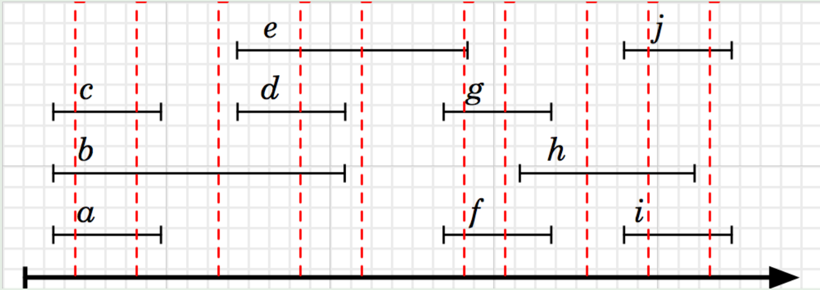

Depth = Maximum number of intervals at any point in time.

Scheduling all intervals greedy algorithm

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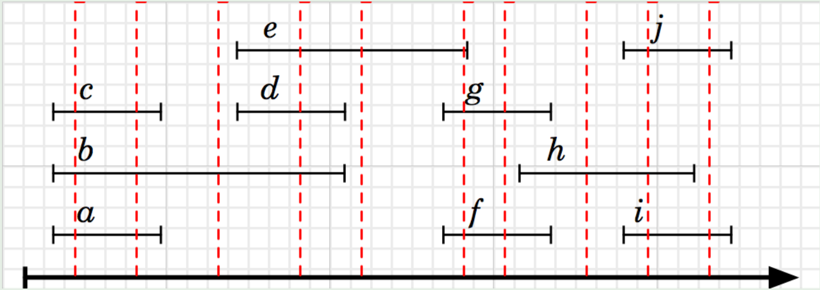

$\text{Depth} = 3$

Scheduling all intervals greedy algorithm

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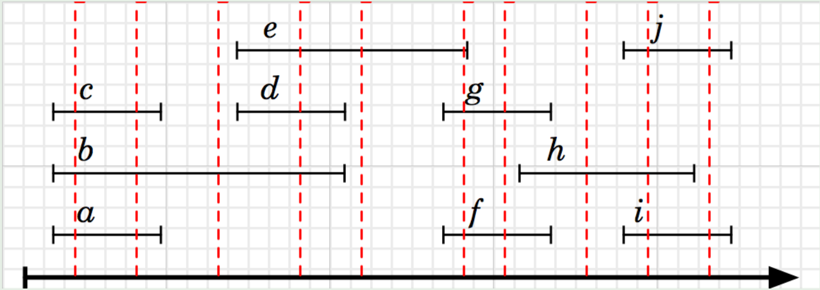

Depth $= 3$ $\implies$ Minimum # of resources needed $= 3$

A greedy algorithm for scheduling all intervals

SCHEDULE-INTERVALS(I) $\quad \triangleright\ I = \{I_i\}, I_i = (s_i, f_i)$

R = Sorted requests in order of starting times, breaking ties arbitrarily, such that $s_i \leq s_j$ when $i < j$.
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$T(n) = O(n \lg n)$.

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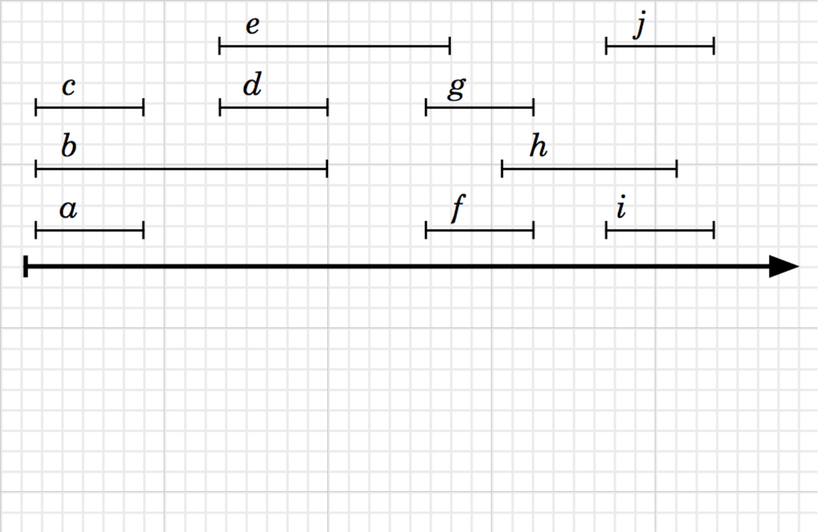
$T(n) = O(n \lg n)$. Prove it.

Scheduling all intervals in action



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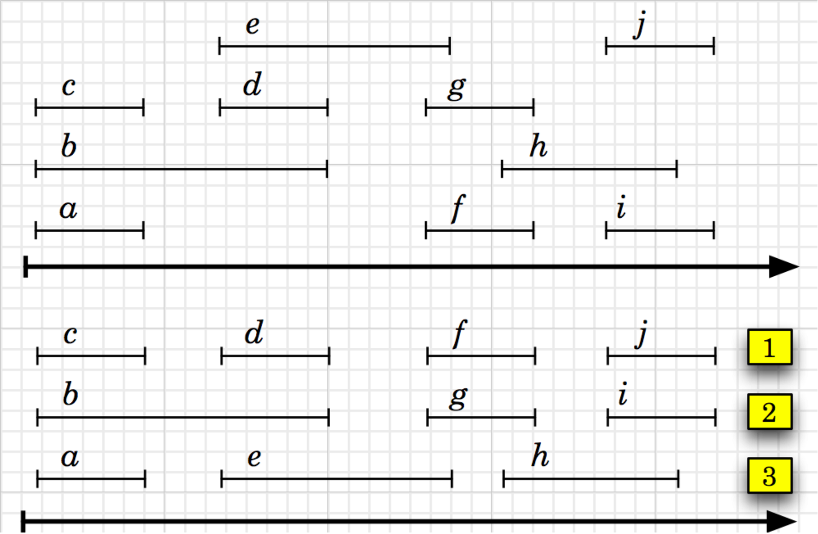

Contents

Fractional knapsack problem

Definition (fractional knapsack problem)

Given a set S of n items, such that each item i has a positive benefit b_i and a positive weight w_i, the goal is to find the maximum-benefit subset that does not exceed a given weight W, allowing for fractional items.

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- Maximum-benefit subset is then maximizing $\sum_{i \in S} b_i(x_i/w_i)$.

Fractional knapsack problem

Key question

- What strategy to use to select the next item (and the amount of it)?

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Let's see which measure to optimize...

Fractional knapsack problem

Item	Benefit	Weight
A	25	18 kg
B	15	10 kg
C	24	15 kg

Capacity W=20

Fractional knapsack problem

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First Strategy: Greedy method using **Profit** as it's measure. At each step it will choose an object that increases the profit the **most**.

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- So, finally, total benefit$= 25 + 3.2 = 28.2$

Fractional knapsack problem

- Note here, If we select C as first element and B as second element then,

Fractional knapsack problem

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- Total benefit$= 24 + 15(5/10) = 31.5$
- So, previous solution is not the optimal one.
- Thus, First Strategy fails.

Fractional knapsack problem

Second Strategy: Greedy method using **Capacity** as it's measure will, at each step choose an object that increases the capacity the **least**.

- B has the least weight 10. So select this. Remaining Capacity: $20 - 10 = 10$

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- So, total benefit$= 15 + 16 = 31$.

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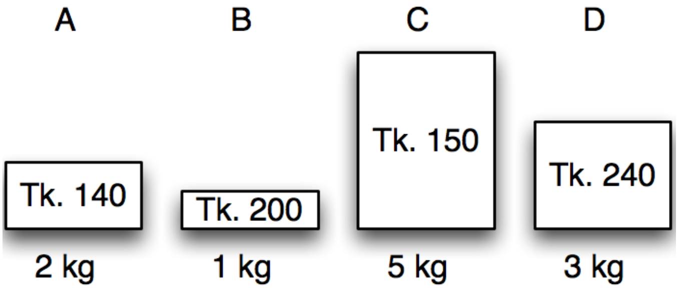
- Accumulated profit by adding $x_C = 10$ to the knapsack is: $24(10/15)=16$
- So, total benefit$= 15 + 16 = 31$. Again, it is not the optimal one.
- Thus, second strategy fails.

Fractional knapsack problem

Third Strategy: Strives to achieve a balance between the rate at which profit increases and the rate at which capacity is used.

- At each step, include the object which has the maximum profit per unit of capacity used.
- That means, objects are considered in order of the ratio b_i/w_i.

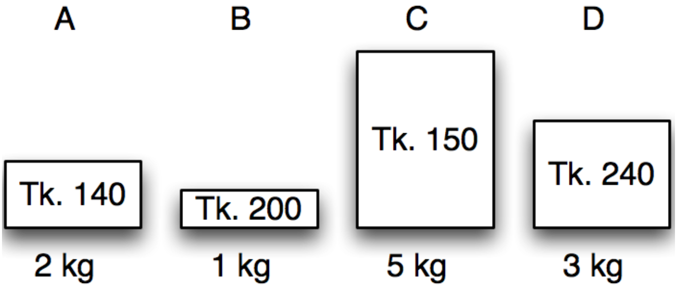
Fractional knapsack in action



Item	Benefit	Weight
A	140	2 kg
B	200	1 kg
C	150	5 kg
D	240	3 kg

Calculate benefit/kg – the value index.

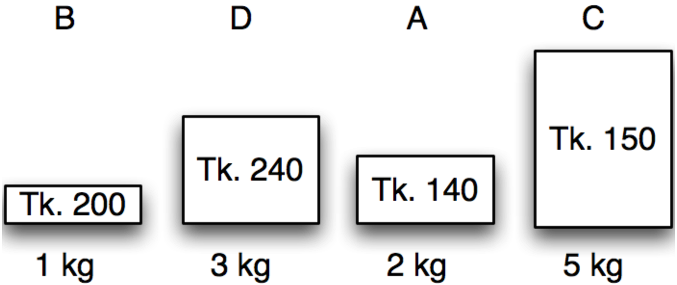
Fractional knapsack in action



Item	Benefit	Weight	Value index
A	140	2 kg	70
B	200	1 kg	200
C	150	5 kg	30
D	240	3 kg	80

Sort by non-increasing value index.

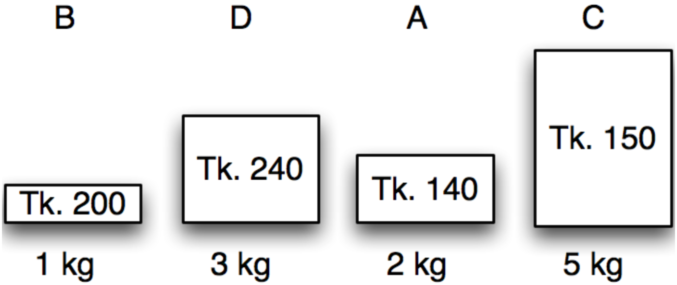
Fractional knapsack in action



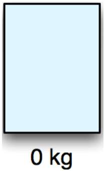
Item	Benefit	Weight	Value index
B	200	1 kg	200
D	240	3 kg	80
A	140	2 kg	70
C	150	5 kg	30

Maximum weight: 5 kg

Fractional knapsack in action

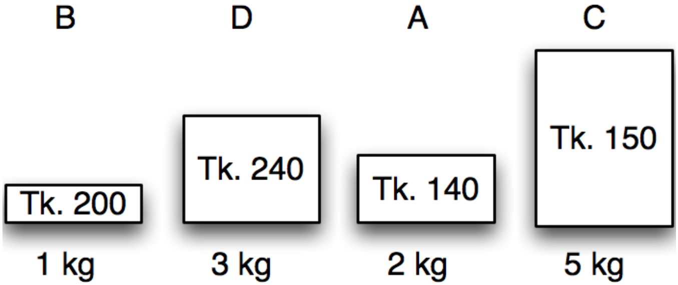


Item	Benefit	Weight	Value index	Chosen
B	200	1 kg	200	0 kg
D	240	3 kg	80	0 kg
A	140	2 kg	70	0 kg
C	150	5 kg	30	0 kg



Maximum weight: 5 kg   Remaining: 5 kg   Benefit: 0 kg

Fractional knapsack in action



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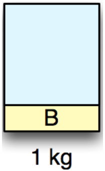
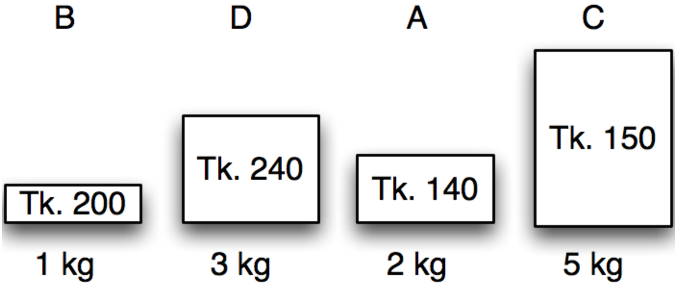


Maximum weight: 5 kg Remaining: 4 kg Benefit: 200 kg

Fractional knapsack in action



Item	Benefit	Weight	Value index	Chosen
B	200	1 kg	200	1 kg
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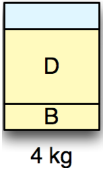
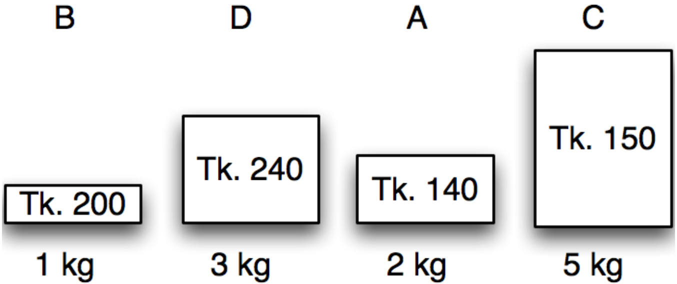


Maximum weight: 5 kg Remaining: 1 kg Benefit: 440 kg

Fractional knapsack in action



Item	Benefit	Weight	Value index	Chosen
B	200	1 kg	200	1 kg
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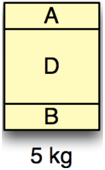
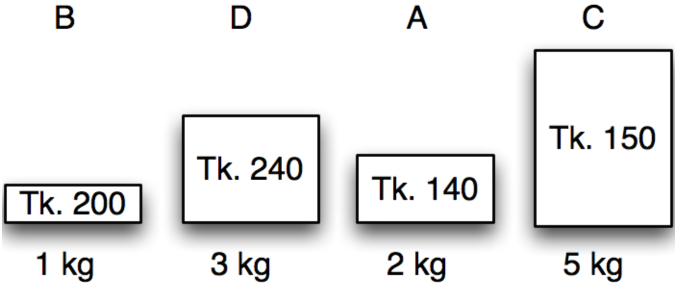
A	140	2 kg	70	1 kg
C	150	5 kg	30	0 kg

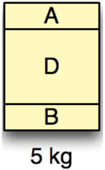


Maximum weight: 5 kg Remaining: 0 kg Benefit: 510 kg

Fractional knapsack in action



Item	Benefit	Weight	Value index	Chosen
B	200	1 kg	200	1 kg
D	240	3 kg	80	3 kg
A	140	2 kg	70	1 kg
C	150	5 kg	30	0 kg



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Fractional knapsack greedy algorithm

FRACTIONAL-KNAPSACK(S, W) $\quad \triangleright\ S = \{(w_i, b_i)\}$

for each item $i \in S$
    do $x_i \leftarrow 0$ $\quad \triangleright$ amount of item i chosen $(0 \leq x \leq w_i)$
        $v_i \leftarrow b_i / w_i$ $\quad \triangleright$ compute *value index*
Sort the items in non-increasing order of the ratio b_i / w_i
$w \leftarrow 0$
while $w < W$
    do i = extract from S the item with highest value index
        $\triangleright$ greedy choice
        if $w + w_i \leq W$
            then $x_i = w_i$
            else $x_i = W - w$ $\triangleright$ fill up the remaining with i
        $w \leftarrow w + x_i$
return x $\quad \triangleright\ x_i$ contains amount of item i chosen

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$T(n) = O(n \lg n)$.

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Extension: 0/1 knapsack problem

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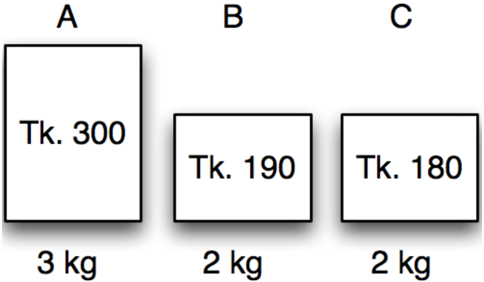

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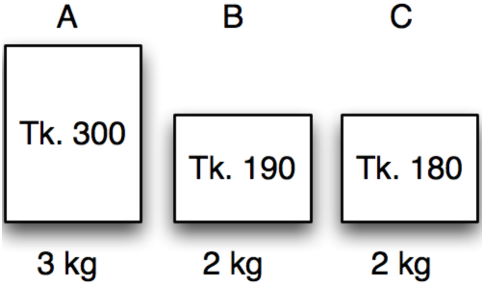

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Benefit: 300

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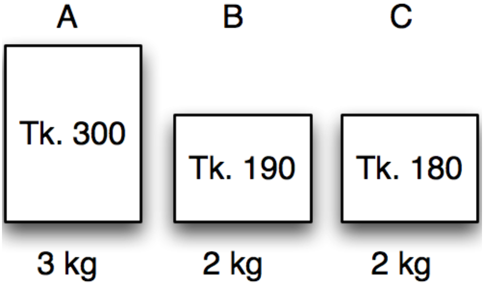

Maximum weight: $\boxed{4 \text{ kg}}$

Greedy solution: item A — Benefit: $\boxed{300}$

Optimal solution: items B and C — Benefit: $\boxed{370}$

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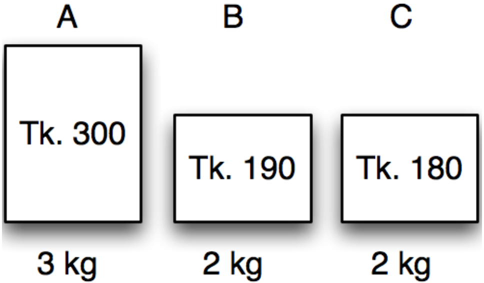

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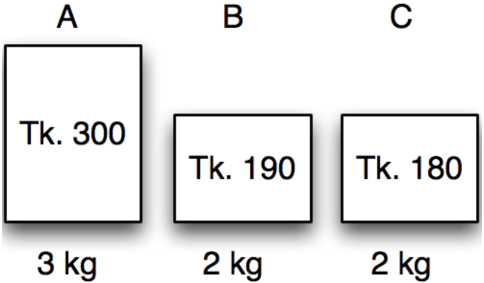

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Key question

Does a greedy approach always produce the optimal solution?

Coin changing problem (continued)

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Key observation

Correctness depends on the choice of coins, so greedy strategy does not provide a general solution to this problem!

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  Subproblem optimality: If the optimal solution to the entire problem contain optimal solution to the subproblems, then it has the subproblem optimality property.

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- Except for a few select problems, it is far better to use Dynamic Programming to solve such optimization problems.
- So why study greedy algorithms? Because there are very efficient provably correct greedy algorithms for many common problems (wait till we study graph algorithms).