# CassandrEAS: Highly Available and Storage-Efficient Distributed Key-Value Store with Erasure Coding

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Abstract—In this work, we propose an erasure coding-based protocol that implements a key-value store with atomicity and near-optimal storage cost. Our protocol supports concurrent read and write operations while tolerating asynchronous communication and crash failures of any client and some fraction of servers. One novel feature is a tunable knob between the number of supported concurrent operations, availability, and storage cost.

We implement our protocol into Cassandra, namely CassandrEAS (Cassandra + Erasure-coding Atomic Storage). Extensive evaluation using YCSB on Google Cloud Platform shows that CassandrEAS incurs moderate penalty on latency and throughput, yet saves significant amount of storage space.

Index Terms—atomicity, erasure-coding, KV store

# I. INTRODUCTION

Storage systems is the foundational platform for modern Internet services. Ever increasing reliance on massive data sets has forced developers to move towards a new class of scalable storage systems known as NoSQL key-value stores (KV-store). Most distributed NoSQL KV-stores (e.g., Cassandra [1], Riak [2], Dynamo [28]) support a flexible data schema and simple GET/PUT (or Read/Write) interface for reading and writing data items. The data items are replicated at multiple servers to provide high fault-tolerance and availability.

One drawback of the replication-based approach is the high storage cost. Erasure coding (EC) has been integrated with other types of KV-store to reduce storage cost (e.g., DepSky [8] for cloud-of-clouds, Cocytus [32] for in-memory KV-store and Giza [11] for cross-datacenter KV-store). This work is motivated by the following question: is it possible to use erasure coding in a NoSQL KV-store to reduce storage cost while maintaining strong consistency with moderate performance penalty? We provide an affirmative answer by introducing CassandrEAS (Cassandra + Erasure-coding

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**Atomic Storage**), a customized version of Cassandra [1] with a new EC-based storage protocol that achieves *atomicity* [21], a form of strong consistency.

In this paper, we first present a novel quorum and erasure-code-based and storage cost optimized protocol called Two-Round Erasure coded Atomic Storage-Optimized (TREAS-OPT). In addition to the improved storage cost, when compared to the existing erasure-code based algorithms, TREAS-OPT algorithm is simple to implement as the high-level structure of the algorithm is very similar to the classic replication-based algorithm by Attiya, Bar-Noy and Dolev [6] and practical quorum storage [7], [28]. TREAS-OPT is a two-round protocol that uses logical timestamps, which obviates the reliance on any physical (machine) clock to implement atomic KV-store. We also modify TREAS-OPT to use physical timestamp that results in a single-round distributed storage protocol One-Round Erasure coding Atomic Storage (OREAS). Both algorithms are suitable for quorum-based KV-store.

Cassandra: We chose to develop our system on top of Cassandra. Even though Cassandra was released in 2008, it is still one of the most used NoSQL KV-stores in industry. It is also an active open-source project (recent stable release in February 2020) [4]. Plus, its quorum-based replication [7] shares similarly with other popular systems like Riak [2] and Dynamo [28]. For many big data applications, Cassandra offers a good balance of properties, e.g., high availability, scalability, and tunable consistency. Our system CassandrEAS provides similar guarantees in scalability and availability.

Cassandra offers tunable consistency guarantees, ranging from eventual consistency to strong consistency. Popular storage systems in industry, such as Google Spanner [14] and CockroachDB [3], are focusing on providing strong consistency. Inspired by the observation, this paper focuses on *atomicity* [21] (or linearizability [18]), one of the more popular forms of strong consistency, because atomicity is composable [18] and more intuitive for developing applications.

Cassandra provides high availability and scalability using a quorum-based (or peer-to-peer) design. It replicates data on multiple servers, and each server can serve any read/write request from clients using the practical quorum approach (or Dynamo-style quorum) [7] for serving read and write operations. The approach does not use master node or consensus protocol to coordinate servers; hence, Cassandra has high performance without a single point of failure.\* Cassandra's quorum-based design make it difficult to use prior EC-based solutions [8], [11], [32]. Cocytus [32] uses a master-based design, whereas Giza [11] is a consensus-based protocol. DepSky [8] does not provide atomicity.

## Main Contributions

In the theoretical contribution, we propose two novel EC-based atomic storage protocols: TREAS-OPT and OREAS. From the engineering perspective, we demonstrate that our protocols can be seamlessly integrated with a real-world system, Cassandra. We made minimum modification to the original Cassandra codebase, and Cassandra users can call Cassandra's original read and write API directly *without* knowing the details of the algorithm. Our implementation can be found at https://github.com/yingjianwu199868/cassandra/

Using TREAS-OPT and OREAS, CassandrEAS also provides availability and scalability at a similar level of Cassandra, because both systems do not use master or consensus. One interesting feature of our protocols is a *tunable knob* that allows clients to choose the desired level of availability (i.e., the maximum number of tolerated server failures), storage cost, and the number of supported concurrent operations. If there are more server failures or more concurrent operations than the specified parameters, then CassandrEAS may return stale value or fail to serve client's request. A protocol for self-stabilization, reconfiguration, or recovery is left as an interesting future work.

For evaluation, we deploy CassandrEAS in Google Cloud Platform (GCP) and conduct extensive experiments using the Yahoo! Cloud Service Benchamarking (YCSB) workload generator [12]. YCSB is a practical tool used to benchmark many KV-stores. Our evaluation results in Section V-B indicate that CassandrEAS incurs moderate performance penalty, yet saves significant amount of storage space.

Cassandra vs. CassandrEAS: Table I summarizes the comparison between Cassandra and CassandrEAS when storing one unit of data (i.e., data size is normalized to 1). We do not count the size of meta-data such as timestamps. Erasure coding is more useful when dealing with larger data (say in terms of 100~Bs), so meta-data size is practically negligible. In the paper, n denotes the number of servers that store the data,  $\delta$  represents the maximum number of writes concurrent with any read on the same key-value pair, and f is the maximum number of tolerated crash servers. For compactness, let k = n - 2f. For Cassandra, we choose the majority quorum configuration.

In practical systems, the number of concurrent writes on the *same* KV-pair is small in most cases, since each operation completes in the order of  $100 \ ms$ . If CassandrEAS

	Cassandra	CassandrEAS	ARES
Storage cost/server	1	$\frac{1}{\left\lceil \frac{k}{\delta+1} \right\rceil}$	$(\delta+1)\frac{1}{k}$
f, max crashes	$\frac{n}{2} - 1$	$\frac{n-k}{2}$	$\frac{n-k}{2}$

TABLE I: Comparison of fault-tolerance level and storage-cost for Cassandra (Quorum), CassandrEAS and ARES

with n=9 is configured to tolerate f=2 crashed servers, and support  $\delta=3$  concurrent writes. In this case, k=5, so the data storage cost at each server becomes  $\frac{1}{\left\lceil \frac{k}{\delta+1} \right\rceil} = \frac{1}{\left\lceil \frac{5}{\delta+1} \right\rceil} = \frac{1}{2}$ . In comparison, Cassandra's storage cost is 1, as each server

In comparison, Cassandra's storage cost is  $\hat{1}$ , as each server stores the original data. Hence, CassandrEAS saves 50% of storage space. If we have the configuration  $n=7, f=1, \delta=1$ , then k=5 and the storage cost of CassandrEAS is  $\frac{1}{3}$ , a 67% reduction in storage space.

## II. PRELIMINARIES

## A. Erasure Coding Storage Systems

Erasure coding (EC) is a space-efficient solution for data storage. EC has been traditionally used with great success for storage cost reduction in write-once, read-many-times data stores (e.g., [8], [19], [25], [27]). Recently there is an increasing interest in using EC in update-many-times, read-many-times data stores. As observed in [11], [32], with the advancement of hardware, it is possible to perform encoding/decoding in a real-time fashion. EC also has the potential to significantly reduce network bandwidth, as well as for system maintenance (such as repairing failed servers). Therefore, both information theory and system research communities have investigated the usage of erasure coding to reduce various kind of costs, e.g., [26], [30]. Notably, [31] developed code constructions with constraints motivated by consistency requirements in storage, and the coding methodology of TREAS-OPT and OREAS are based on [31].

Three recent systems DepSky [8], Cocytus [32] and Giza [11] studied the applicability of erasure coding in other types of KV-stores. DepSky [8] uses erasure coding for efficient storage in cloud-of-clouds; however, it does not support atomicity. Giza [11], Microsoft's proprietary storage, is a Fast-Paxosbased multi-version cross-data center strongly consistent object store used in Microsoft's OneDrive storage system. Giza servers store erasure-coded elements instead of the original data to significantly reduce storage cost. Cocytus [32] is a master-based in-memory KV-store that guarantees strong consistency and reduces storage cost using erasure coding. For each key, value is erasure coded and the coded elements are stored among a subset of servers. In addition, the master server maintains a full copy of the value to provide high availability for read operations. As elaborated above, Cassandra's quorum-based design does not fit well with consensus protocol or masterbased design. Therefore, we have to design a new EC-based protocol for using erasure coding in Cassandra.

The distributed computing community also shows interest in using erasure coding. EC-based algorithms for strongly consistent storage are an active area of research in theory

<sup>\*</sup>Cassandra supports many operations beyond simple reads and writes, e.g., it uses Paxos [22] to support transactions. Here we do not support transactions; we restrict its use to support only reads and writes, for which Cassandra does *not* use Paxos (see Sec. V for more detail).

community, e.g., [9], [16], [17], [24]. SODA [17] described an algorithm to achieve optimal storage cost; however, it pays a higher write communication cost. It is not clear how to adapt or integrate them into real-world practical systems.

#### B. Erasure Codes

We adopt an [n,l] linear Maximum Distance Separable (MDS) code [20] over a finite field  $\mathbb{F}_q$  to encode and store the value v among the n servers. The value refers to a specific version of the data in our context. An [n,l] MDS code has the property that any l out of the n coded elements can be used to recover (or decode) the original value v. For encoding, v is divided  $\dagger$  into l elements  $v_1, v_2, \ldots, v_l$  with each element having size  $\frac{1}{l}$  (assuming size of v is 1). The encoder takes the l elements as input and produces n coded elements  $c_1, c_2, \ldots, c_n$  as output, i.e.,  $[c_1, \ldots, c_n] = \Phi^{(n,l)}([v_1, \ldots, v_l])$ , where  $\Phi^{(n,l)}$  denotes the encoder. For ease of notation, we simply write  $\Phi^{(n,l)}(v)$  to mean  $[c_1, \ldots, c_n]$ . The vector  $[c_1, \ldots, c_n]$  is referred to as the codeword corresponding to the value v. Each coded element  $c_i$  also has size  $\frac{1}{l}$ . In our scheme, we store one coded element per key-value pair at each server.

## C. Main Challenge

The key challenge in an EC-based storage protocol that is compatible with quorum-based system is to handle concurrent operations. To ensure high availability, the readers need to receive sufficient coded elements from the servers to be able to decode the version of the data that satisfies *atomicity*. This issue becomes complicated in practice due to the following reasons: (i) there might be concurrent write operations that write different versions of the data simultaneously; (ii) servers and clients might crash so that the servers do not have enough coded element for a particular version; and (iii) messages might arrive in an arbitrary order due to the asynchrony assumption of the underlying network.

## D. Model and Definitions

A shared atomic storage (or atomic KV-store) can be emulated by composing individual atomic objects. Therefore, we aim to implement a single atomic read/write memory object. A read/write object takes a value from a set  $\mathcal V$ . We assume a system consisting of three distinct sets of processes: a set  $\mathcal W$  of writers, a set  $\mathcal R$  of readers and  $\mathcal S$ , a set of servers. Servers host data elements (replicas or encoded data fragments). Each writer is allowed to modify the value of a shared object, and each reader is allowed to obtain the value of that object. Nodes communicate through *asynchronous*, but *reliable* channels.

**Executions.** An *execution* of an algorithm A is an alternating sequence of states and actions of A starting with the initial state. An execution  $\xi$  is *well-formed* if any process invokes one operation at a time and it is *fair* if enabled actions perform

a step infinitely often. In the rest of the paper we consider executions that are fair and well-formed. A node *crashes* in an execution if it stops taking steps; otherwise it is *non-faulty*.

Write and Read Operations. An implementation of a read or a write operation contains an *invocation* action and a *response* action (such as a return from the procedure). An operation  $\pi$  is *complete* in an execution, if it contains both the invocation and the *matching* response actions for  $\pi$ ; otherwise  $\pi$  is *incomplete*. We say that an operation  $\pi$  *precedes* an operation  $\pi'$  in an execution  $\xi$ , denoted by  $\pi \to \pi'$ , if the response step of  $\pi$  appears before the invocation step of  $\pi'$  in  $\xi$ . Two operations are *concurrent* if neither precedes the other. An implementation A of a read/write object satisfies the atomicity property if the conditions of Lemma 13.16 in [23] holds.

**Storage and Communication Costs.** We define the total storage cost as the size of the data stored *across all servers*, at any point during the execution of the algorithm. The communication cost associated with a read or write operation is the size of the total data transmitted in the messages sent as part of the operation. We assume that metadata, such as version number, process ID, etc. used by various operations is of negligible size. Further, we normalize both the costs w.r.t. to the size of the value v; in other words, we compute the costs assuming that v has size 1 unit.

**Quorum system**. We define our quorum system, Q, to be the set of all subsets of S that have at least  $\frac{n+k}{2}$  servers. We refer to the members of Q, as quorum sets and they satisfy the following property.

**Lemma.** For any k,  $1 \le k \le n-2f$ . (i) If  $Q_1$ ,  $Q_2 \in \mathcal{Q}$ , then  $|Q_1 \cap Q_2| \ge k$ . (ii) If the number of faulty servers is at most f, then Q contains at least one quorum set Q of non-faulty servers.

Liveness of operations. We require algorithms to satisfy certain liveness properties, specifically, in every fair execution that satisfies certain restrictions in terms of the number of failed nodes, we require every operation by a non-faulty client completes eventually, irrespective of the behavior of other clients. Most replication-based atomic memory algorithms guarantee liveness, as long as certain number of servers remain non-faulty; however, similar fault-tolerance levels can be achieved erasure-code based systems only in carefully designed algorithms. Moreover, there are some lower bound results on the storage-cost for erasure-coded atomic memory algorithms in the presence of faulty clients [10], [29]. As a result, to circumvent this restriction many authors assume some restricting assumptions. CASGC, ORCAS-A and ORCAS-B [13] assume a bound on the number of concurrent operations. In the same vein, our protocol assume a known bound on the number of concurrent writes with a read to achieve liveness.

# III. TREAS-OPT: STORAGE-OPTIMIZED TWO-ROUND ALGORITHM

The pseudo-code for TREAS-OPT is presented in Alg. 1. TREAS-OPT is parameterized by the number of servers n, the

 $<sup>^\</sup>dagger$ In practice, v can be viewed as a byte array, which is divided into many stripes based on the choice of the code, various stripes are individually encoded and stacked against each other. We omit details of represent-ability of v by a sequence of symbols of  $\mathbb{F}_q$ , and the mechanism of data striping, since these are fairly standard in the coding theory literature.

quorum intersection size k, and the degree of concurrency that can be tolerated  $\delta$ . Note that k does not represent the dimension of the code here. The algorithm uses an MDS code, with encoding function  $\Phi^{(n,\ell)}:\mathbb{F}^\ell\to\mathbb{F}^n$  where,  $\mathbb{F}$  represents the finite field over which encoding is performed, n represents the length of the code, and  $\ell=\lceil\frac{k}{\delta+1}\rceil$  represents the dimension of the code.

A tag  $\tau$  is defined as a pair (z,w), where  $z\in\mathbb{N}$  and  $w\in\mathcal{W}$ , an ID of a writer. Let  $\mathcal{T}$  be the set of all tags. Notice that tags could be defined in any totally ordered domain and given that this domain is countably infinite, then there can be a direct mapping to the domain we assume. For any  $\tau_1,\tau_2\in\mathcal{T}$ , where  $\tau_i=(z_i,w_i)$ , we define  $\tau_2>\tau_1$  if (i)  $\tau_2.z_2>\tau_1.z_1$  or (ii)  $\tau_2.z_2=\tau_1.z_1$  and  $\tau_2.w_2>\tau_1.w_1$ .

Each server  $s_i$  stores one state variable, List, which is a set of up to  $(\delta+1)$  (tag, coded-element) pairs. Initially the set at  $s_i$  contains a single element,  $List=\{(t_0,\Phi_i(v_0)\}$ . A read operation at a client is implemented based on the get-data and put-data steps; and a write operation is based on the get-tag and put-data steps.

get-tag(): A client, during the execution of a get-tag() primitive, queries all the servers in  $\mathcal S$  for the highest tags in their Lists, and awaits responses from  $\left\lceil \frac{n+k}{2} \right\rceil$  servers. A server upon receiving the GET-TAG request, responds to the client with the highest tag, as  $\tau_{max} \equiv \max_{(t,c) \in List} t$ . Once the client receives the tags from  $\left\lceil \frac{n+k}{2} \right\rceil$  servers, it selects the highest tag t and returns it .

put-data( $\langle t_w,v\rangle$ ): During put-data( $\langle t_w,v\rangle$ ), a client sends the pair  $(t_w,\Phi_i(v))$  to each server  $s_i\in\mathcal{S}$ . When a server  $s_i$  receives a message (PUT-DATA,  $t_w,c_i$ ), it adds the pair in its local List, trims the pairs with the smallest tags exceeding the length  $(\delta+1)$  of the List, and replies with an ack to the client. In particular,  $s_i$  replaces the coded-elements of the older tags with  $\bot$ , and maintains only the coded-elements associated with the  $(\delta+1)$  highest tags in the List (see Line Alg. 2:13-17). The client completes the primitive operation after getting acks from  $\left\lceil \frac{n+k}{2} \right\rceil$  servers.

get-data(): A client, during the execution of a get-data() primitive, queries all the servers in  $\mathcal S$  for their local variable List, and awaits responses from  $\lceil \frac{n+k}{2} \rceil$  servers. Once the client receives Lists from  $\lceil \frac{n+k}{2} \rceil$  servers, it selects the highest tag t, such that: (i) its corresponding value v is decodable from the coded elements in the lists; and (ii) t is the highest tag seen from the responses of at least k Lists (see lines Alg. 1:21-24) and returns the pair (t,v). Note that in the case where anyone of the above conditions is not satisfied the corresponding read operation does not complete.

The main technical aspect in which TREAS-OPT differs with ARES is the liveness proof. We argue that despite the changes, TREAS-OPT guarantees termination of every read operation whose concurrency is below  $\delta$ .

whose concurrency is below  $\delta$ . **Safety and Liveness properties** Now we state the safety and liveness properties of TREAS-OPT.

**Theorem** (Atomicity). Any well-formed and fair execution of TREAS-OPT is atomic.

*Proof.* Consider any well-formed execution  $\beta$  of TREAS-OPT, all of whose invoked read or write operations complete. Let  $\Pi$ denote the set of all completed read and write operations in  $\beta$ . We first define a partial order  $(\prec)$  on  $\Pi$ . For any completed write operation  $\pi$ , we define  $tag(\pi)$  as the variable  $t_w$ . For any completed read operation  $\pi$ , we define  $tag(\pi)$  as the value of  $t_r$ . Now, in  $\Pi$  the relation  $\prec$  is defined as follows: For any  $\pi, \phi \in \Pi$ , we say  $\pi \prec \phi$  if one of the following holds: (i)  $tag(\pi) < tag(\phi)$ , or (ii)  $tag(\pi) = tag(\phi)$ , and  $\pi$  and  $\phi$  are write and read operations, respectively. Atomicity is proved by using Lemma 13.16 in [23], which essentially requires any execution to hold four properties to guarantee atomicity. Let us denote them by P1, P2, P3 and P4. Property P1 is easily satisfied by our executions, so we show that any execution satisfies the remaining properties. Let  $\phi$  and  $\pi$  denote two operations in  $\Pi$  such that  $\phi$  completes before  $\pi$  starts in  $\beta$ . Let  $c_{\phi}$  and  $c_{\pi}$  denote the clients that invokes  $\phi$  and  $\pi$ , respectively.

*Property P2* We want to show that  $\pi \not\prec \phi$ . Below we consider the four possible cases of  $\phi$  and  $\pi$ .

 $\phi$ ,  $\pi$  are writes: It is enough to prove that  $tag(\pi) > tag(\phi)$ . Consider the put-data phase of  $\phi$ , where the writer  $c_{\phi}$  sends the pair  $(t_w, v)$  to all servers in S. Let us denote the set  $S_\phi$ of  $\lceil \frac{n+k}{2} \rceil$  servers that responds to  $c_\phi$  during the put-data phase. Now, observe that the maximum tag in any server's List is monotonically non-decreasing, because in algorithm B any server add tags to List only in lines Alg. 2:13-17. Once added, a tag is never removed from List. Therefore, at each server in  $S_{\phi}$  the maximum tag in List at the time of sending the responses to  $c_{\phi}$  in the put-data phase is at least  $tag(\phi)$ Now, suppose  $S_{\pi}$  be the set of  $\lceil \frac{n+k}{2} \rceil$  servers that responds to  $c_{\pi}$  during the get-tag phase of  $\pi$ . Therefore, at the point of the execution when  $\pi$  is invoked, the maximum tag in List of each server is at least  $tag(\phi)$ . Since  $|S_{\phi}| = |S_{\pi}| = \lceil \frac{n+k}{2} \rceil$ hence  $S_{\phi} \cap S_{\pi} \neq \emptyset$ . Therefore, there is at least one of the responses from the servers in the get-tag (see lines Alg. 2:15) has a tag at least  $tag(\phi)$ . So, the  $t_w$  is greater than  $tag(\phi)$ . So  $tag(\pi) \ge t_w > tag(\phi)$  hence,  $\pi \not\prec \phi$ .

 $\phi$  is a read,  $\pi$  is a write: By virtue of the definition of  $\prec$ , it is enough to prove that  $tag(\pi) \geq tag(\phi)$ . The rest of the argument is very similar to the previous case.

 $\phi$  is a write,  $\pi$  is a read: From the definition of  $\prec$ , it is enough to prove that  $taq(\pi) > taq(\phi)$ . Consider the put-data phase of  $\phi$ , where the reader  $c_{\phi}$  sends the pair  $(t_w, v)$  to all servers in  $\mathcal{S}$ . Let us denote the set  $\mathcal{S}_{\phi}$  of  $\lceil \frac{n+k}{2} \rceil$  servers that responds to  $c_{\phi}$ . Note that for each server in  $\bar{S}_{\phi}$  the maximum tag in List at the time of sending the responses to  $c_{\phi}$  in the put-data phase is at least  $tag(\phi)$  (see lines Alg. 2:13–17). Now, suppose  $S_{\pi}$  be the set of  $\lceil \frac{n+k}{2} \rceil$  servers that responds to  $c_{\pi}$ , with the values in their *Lists* during the get-data phase of  $\pi$ . Since the maximum tag in any server's List is monotonically non-decreasing therefore, at the point of the execution when  $\pi$  is invoked, the maximum tag in List of each server is at least  $tag(\phi)$  because for  $\phi$  to complete we must have  $t_{max}^{dec} =$  $t_{max}^*$  in line Alg. 1:25. Since  $|S_{\phi}| = |S_{\pi}| = \lceil \frac{n+k}{2} \rceil$  hence  $S_{\phi} \cap S_{\pi} \neq \emptyset$ . Therefore, there is at least one of the reponses from the servers in the get-data (see lines Alg. 1:25) has

# Algorithm 1 The reader/writer client-side steps for implementing TREAS-OPT.

```
/* at reader r */
                                                                                                                             end procedure
      operation read()
          \langle t_r, v \rangle \leftarrow \text{get-data}()
                                                                                                                      18: procedure get-data()
                                                                                                                                \mathbf{send} \,\, (\mathtt{QUERY-LIST}) \,\, \mathsf{to} \,\, \mathsf{each} \,\, s \in \mathcal{S}
          \mathsf{put-data}(\langle t_r, v \rangle)
          return v
                                                                                                                                until receives List_s from each server s \in S_g s.t. |S_g| = \left| \frac{n+k}{2} \right|
      end operation
                                                                                                                                 Tags_*^{\geq k} = set of tags that appears in k lists
                                                                                                                                Tags \frac{\geq \ell}{dec} = \text{tags appearing in } \ell \text{ lists with values}
                                                                                                                      22:
      /* at writer w */
                                                                                                                                 t_{max}^* \leftarrow \max Tags_{*,s}^{\geq k}
 6: operation write(v)
          t_{max} \leftarrow \text{get-tag}()
                                                                                                                                t_{max}^{dec} \leftarrow \max Tags_{dec}^{\geq \iota}
                                                                                                                      24:
          t_w \leftarrow (t_{max}.z+1,w)
                                                                                                                                if t_{max}^{dec} \geq t_{max}^* then
          \mathsf{put\text{-}data}(\langle t_w, v \rangle)
                                                                                                                                    v \leftarrow decode value for t_{max}^{dec}
                                                                                                                      26:
10: end operation
                                                                                                                                 return \langle t_{max}^{dec}, v \rangle
        at each process p_i \in \mathcal{I}
                                                                                                                      28: end procedure
12: procedure get-tag()
                                                                                                                             procedure put-data(\langle \tau, v \rangle))
          send (QUERY-TAG) to each s \in \mathcal{S}
                                                                                                                                code\text{-}elems = [(\tau, e_1), \dots, (\tau, e_n)], e_i = \Phi_i(v)
          until receives \langle t_s, e_s \rangle from \left| \frac{n+k}{2} \right| servers
14:
                                                                                                                                send (PUT-DATA, \langle \tau, e_i \rangle) to each s_i \in \mathcal{S}
          t_{max} \leftarrow \max(\{t_s : \text{ received } \langle t_s, v_s \rangle \text{ from } s\})
                                                                                                                                until receives ACK from \left| \frac{n+k}{2} \right|
                                                                                                                                                                                   servers
16:
          return t_{max}
                                                                                                                             end procedure
```

**Algorithm 2** The response protocols at any server  $s_i \in \mathcal{S}$  in TREAS-OPT for client requests.

```
/* at each server s_i \in \mathcal{S} */
                                                                                                                       end receive
2: State Variables:
                                                                                                                 10:
     List \subseteq \mathcal{T} \times \mathcal{C}_s, initially \{(t_0, \Phi_i(v_0))\}
                                                                                                                       Upon receive (PUT-DATA, \langle 	au, e_i 
angle) from q
                                                                                                                12:
                                                                                                                           \tau_{max} = \max_{(t,c) \in List} t
     Upon receive (QUERY-TAG) from q
                                                                                                                           if \tau \geq \tau_{max} then
4:
                                                                                                                 14.
                                                                                                                              List \leftarrow List \setminus \{\langle \tau_{max}, e_i \rangle : \langle \tau_{max}, e_i \rangle \in List \}
         \tau_{max} = \max_{(t,c) \in List} t
                                                                                                                               List \leftarrow List \cup \{\langle \tau, e_i \rangle, \langle \tau_{max}, \bot \rangle\}
         Send \tau_{max} to q
6: end receive
                                                                                                                 16:
                                                                                                                               List \leftarrow List \cup \{\langle \tau, \bot \rangle\}
     Upon receive (QUERY-LIST) from q
                                                                                                                 18:
                                                                                                                          Send ACK to q
8:
        Send List to a
                                                                                                                       end receive
```

a tag at least  $tag(\phi)$ . So, the  $t_r$  is as large as  $tag(\phi)$ . So  $tag(\pi) \ge t_r \ge tag(\phi)$  hence,  $\pi \not\prec \phi$ .

 $\phi$ ,  $\pi$  are reads: From the definition of  $\prec$ , it is enough to prove that  $tag(\pi) \geq tag(\phi)$ . Consider the put-data phase of  $\phi$ , where the reader  $c_{\phi}$  sends the pair  $(t_r, v)$  to all servers in  $\mathcal{S}$ . Let us denote the set  $S_{\phi}$  of  $\lceil \frac{n+k}{2} \rceil$  servers that responds to  $c_{\phi}$ . Note that for each server in  $S_{\phi}$  the maximum tag in List at the time of sending the responses to  $c_{\phi}$  in the put-data phase is at least  $tag(\phi)$  (see lines Alg. 2:13–17). Now, suppose  $S_{\pi}$  be the set of  $\lceil \frac{n+k}{2} \rceil$  servers that responds to  $c_{\pi}$ , with the data in their Lists during the get-data phase of  $\pi$ . Since the maximum tag in any server's List is monotonically non-decreasing therefore, at the point of the execution when  $\pi$  is invoked, the maximum tag in List of each server is at least  $tag(\phi)$  because for  $\phi$  to complete we must have  $t_{max}^{dec}=t_{max}^{*}$  in line Alg. 1:25. Since  $|S_{\phi}| = |S_{\pi}| = \lceil \frac{n+k}{2} \rceil$  hence  $S_{\phi} \cap S_{\pi} \neq \emptyset$ . Therefore, there is at least one of the reponses from the servers in the get-data (see lines Alg. 2:25) has a tag at least  $tag(\phi)$ . So, the  $t_r$  is as large as  $tag(\phi)$ . So  $tag(\pi) \ge t_r \ge tag(\phi)$  hence,  $\pi \not\prec \phi$ .

Property P3 This follows from the construction of tags, and the definition of the partial order ( $\prec$ ).

Property P4 This follows from the definition of partial order  $(\prec)$ , and by noting that value returned by a read operation  $\pi$  is simply the value associated with  $taq(\pi)$ .

**Theorem** (Liveness). Let  $\beta$  denote a well-formed and fair execution of TREAS-OPT with parameters  $[n,k,\delta]$  over a system of n servers. If the number of write operations that are with any valid read operation in  $\beta$  bounded by  $\delta$ , and the number of server failures is bounded by  $\lceil \frac{n+k}{2} \rceil - 1$ , then every operation in  $\beta$  terminates.

*Proof.* Note that in the read and write operation the get-tag and put-data operations initiated by any non-faulty client always complete. Therefore, the liveness property with respect to any write operation is clear because it uses only get-tag and put-data operations. So, we focus on proving the liveness property of any read operation  $\pi$ , specifically, the get-data operation completes. Let  $\alpha$  be an execution of TREAS-OPT and let  $c_{\sigma^*}$  and  $c_{\pi}$  be the clients that invoke the write operation  $\sigma^*$  and read operation  $c_{\pi}$ , respectively.

Let  $S_{\sigma^*}$  be the set of  $\left\lceil \frac{n+k}{2} \right\rceil$  servers that responds to  $c_{\sigma^*}$ , in the put-data operations, in  $\sigma^*$ . Let  $S_{\sigma^\pi}$  be the set of  $\left\lceil \frac{n+k}{2} \right\rceil$  servers that responds to  $c_\pi$  during the get-data step of  $\pi$ . Note that in  $\alpha$  at the point execution  $T_1$ , just before the execution of  $\pi$ , none of the write operations in  $\Lambda$  is complete. Observe that, by algorithm design, the coded-elements corresponding to  $t_{\sigma^*}$  are garbage-collected from the List variable of a server only if more than  $\delta$  higher tags are introduced by subsequent writes into the server. Since the number of concurrent writes  $|\Lambda|$ , s.t.  $\delta > |\Lambda|$  the corresponding value of tag  $t_{\sigma^*}$  is not garbage

collected in  $\alpha$ , at least until execution point  $T_2$  in any of the servers in  $S_{\sigma^*}$ .

Therefore, during the execution fragment between the execution points  $T_1$  and  $T_2$  of the execution  $\alpha$ , the tag and coded-element pair is present in the List variable of every in  $S_{\sigma^*}$  that is active. As a result, the tag and codedelement pairs,  $(t_{\sigma^*}, \Phi_s(v_{\sigma^*}))$  exists in the List received from any  $s \in S_{\sigma^*} \cap S_{\pi}$  during operation  $\pi$ . Note that since  $|S_{\sigma^*}| = |S_{\pi}| = \left\lceil \frac{n+k}{2} \right\rceil$  hence  $|S_{\sigma^*} \cap S_{\pi}| \geq k$  and hence  $t_{\sigma^*} \in Tags_{dec}^{\geq k}$ , the set of decodable tag, i.e., the value  $v_{\sigma^*}$  can be decoded by  $c_{\pi}$  in  $\pi$ , which demonstrates that  $Tags_{dec}^{\geq k} \neq \emptyset$ . Next we want to argue that  $t_{max}^* = t_{max}^{dec}$  via a contradiction: we assume  $\max Tags_*^{\geq k} > \max Tags_{dec}^{\geq k}$ . Now, consider any tag t, which exists due to our assumption, such that,  $t \in Tags^{\geq k}_*$ ,  $t \notin Tags^{\geq k}_{dec}$  and  $t > t^{dec}_{max}$ . Let  $S^k_\pi \subset S$  be any subset of k servers that responds with  $t^*_{max}$  in their List variables to  $c_\pi$ . Note that since k>n/3 hence  $|S_{\sigma^*}\cap S_\pi|\geq \left\lceil\frac{n+k}{2}\right\rceil+\left\lceil\frac{n+1}{3}\right\rceil\geq 1$ , i.e.,  $S_{\sigma^*}\cap S_\pi\neq\emptyset$ . Then t must be in some servers in  $S_{\sigma^*}$  at  $T_2$  and since  $t > t_{max}^{dec} \geq t_{\sigma^*}.$  Now since  $|\Lambda| < \delta$  hence  $(t, \perp)$  cannot be in any server at  $T_2$  because there are not enough concurrent write operations (i.e., writes in  $\Lambda$ ) to garbage-collect the codedelements corresponding to tag t, which also holds for tag  $t^*_{max}$ . In that case, t must be in  $Tag^{\geq k}_{\overline{dec}}$ , a contradiction.  $\square$ 

TREAS-OPT vs. ARES and ABD: TREAS-OPT shares some similarities with ABD [6] as well as ARES [24]. Specifically, in TREAS-OPT, like ARES, erasure coding is used, and each server stores a list of all the tags that it receives in the execution; even if the coded element corresponding to a tag t, logical timestamp, is garbage collected, the server stores a tuple of the form  $(t, \bot)$ . Unlike ARES, in TREAS-OPT each server stores only coded element, similar to ABD [6]. When a new tag-coded-element pair arrives at a server, if the tag is larger than the highest locally stored tag  $\tau_{max}$ , then the server simply replaces the stored coded element by the newly arriving coded element; however, the server will continue to store tuple  $(\tau_{max}, \bot)$ .

While in ARES, each server stores  $\delta+1$  coded elements of a code with dimension k, in TREAS-OPT, each server stores one coded element of dimension  $\lceil \frac{k}{\delta+1} \rceil$ . Since the size of each coded element is effectively a fraction  $\frac{1}{k}$  the size of the value, the server storage cost in ARES normalized by the size of the object is  $\frac{\delta+1}{k}$ , whereas the storage cost of TREAS-OPT is  $\frac{1}{\left\lceil \frac{k}{\delta+1} \right\rceil} \le \frac{\delta+1}{k}$  the storage cost of TREAS-OPT is no worse than the storage cost of ARES, and can be up to twice as efficient. For instance, if we have n=9 servers, and we use quorums of size 6 so that k=3, for a concurrency of  $\delta=1$ , the storage cost of ARES is 2/3, whereas the cost of TREAS-OPT is  $\frac{1}{2}$ . For the same system, if quorums of size 7 are used, and a concurrency can be bounded by  $\delta=3$ , then the storage cost of ARES is 4/5=0.8 whereas TREAS-OPT has a storage cost of 1/2=0.5.

IV. ONE-ROUND ERASURE CODING ATOMIC STORAGE Instead of logical timestamp (tags), OREAS relies on physical timestamps, i.e., machine time at the coordinator. The advantage

# Algorithm 3 The reader/writer steps for implementing OREAS.

```
/* at reader r */
operation read()
2: \langle t_r, v \rangle \leftarrow \text{get-data}()
put-data(\langle t_r, v \rangle)
end operation

/* at writer w */
6: operation write(v)

t_w \leftarrow (t, w) where t is machine time at w

8: put-data(\langle t_w, v \rangle)
end operation

/* at writer w */

t_w \leftarrow (t, w) where t is machine time at w

8: put-data(\langle t_w, v \rangle)
end operation
```

of using logical timestamp is that strong consistency is guaranteed even in the presence of clock drift and absence of synchronized clocks. The downside is that it requires, as in TREAS-OPT, an additional communication round in order to gather the largest tag. On the other hand, in OREAS, the strong consistency guarantee is reliant on correctly synchronized clocks; however, write operations complete in one round.

## V. OUR SYSTEM: CASSANDREAS

## A. Implementation

We share our implementation experience here in hope to lower the barrier of implementing other replication- or ECbased protocols in Cassandra in the future. We have two main goals in our implementation: (i) make minimum modification to the original Cassandra codebase; (ii) allow Cassandra users to use our implementation without knowing the details of OREAS. That is, users can use the same Cassandra API in our system. The implementation turns out to be a difficult task because we are constrained to using only Cassandra's internal tools and data structures for communication and coordination. We spent enormous amount of time to decompose and understand Cassandra's internal logic because the codebase is not well documented and often provide poor comments. The version of Cassandra (version 3.11) that we used to develop contains around 57,000 LoC in Java. We also implemented the algorithm ARES, which uses logical timestamp to deal with loosely synchronized clock. Our implementation of both algorithms consists of around 2,000 LoC. We did not implement our protocols as a middleware because the protocols require system information (e.g., timestamps), which is not revealed externally. **Data Model** Cassandra adopts the column-family data model, which provides a richer functionalities than a simple read-write object (KV-pair). At a high level, Cassandra's data structure can be viewed as a table, where each row corresponds to data sharing the same key. CassandrEAS uses a row to store one KV-pair. Recall that the maximum number of writes concurrent with any read is denoted by  $\delta$ . We use  $2 + 2\delta$  columns for each row: key, value (a place holder for original data), and additional  $2\delta$  columns for each version of the write. For each version, we use one column for timestamp (either physical or logical timestamp), and the other for the corresponding coded element. The reason that we need the value column is that it allows Cassandra to return the original data (decoded element) to the client. That column does *not* store any data in storage.

<sup>&</sup>lt;sup>‡</sup>We also has a separate paper that documents the implementation and benchmarking of other replication-based storage protocols in Cassandra [15].

Each server only stores one coded element. The rest would be  $\perp$  as discussed previously.

Technical Details: We mainly modify the following two files in the Cassandra codebase. In Cassandra's terminology, "mutation" is essentially a write operation that modifies the internal state of the servers. (i) StorageProxy.java is where the coordinator server handles the user's read or write operation. Specifically, MUTATE function handles Cassandra user's write operation whereas FETCHROWS function handles user's read operation. The size of read/write quorums is also specified in this file; and (ii) MutationVerbHandler.java where each server's database engine handles incoming write requests from the coordinator. Specifically, DOVERB function applies the mutation onto local storage. One major challenge we encountered is that Cassandra does not provide an easy way to fetch users' requests and modify their mutations. We have to figure out a way to construct a new mutation when we need to add new fields such as timestamp and coded elements. Recall that we do not want to introduce new data structure, so we choose to use Cassandra's column family data model (i.e., an ordered collection of rows) to store the List variable at each server. CassandrEAS uses BackBlaze Reed-Solomon Code.

## B. Evaluation

We evaluate the performance of CassandrEAS by comparing it to vanilla Cassandra (version 3.11). Cassandra-All requires the coordinator to hear from every server, whereas Cassandra-Quorum only requires to hear from a majority.

Cluster Configuration and Workload: We use the Google Cloud Platform (GCP) as our testing platform. Each virtual machine (VM) is equipped with 4 virtual CPUs, 16 GB memory, and hosting Ubuntu 14.04 LTS. All VMs are located in datacenter us-east1-c (South Carolina). VMs use internal IP's for communication. The average RTT between any two VMs is around 0.3 ms, and TCP bandwidth measured by Iperf is around 7.5 Gbits/sec. For most of evaluation, our cluster consists of 5 VMs, and 3 YCSB single-threaded clients. Thus,  $\delta=3$ . We use YCSB to generate realistic workload. We first insert a total of 30,000 KV-pairs, and each user performs 100,000 read or write operations. Recall that we have 3 YCSB clients, so we have 300,000 operations in total. We report the aggregated throughput, i.e., the sum of total operations per second across three YCSB clients.

Performance: Figure 1 presents our evaluation results in different configurations. Figures 1a to 1c show latency and throughput under different write ratio with data size equal to 128B. Read latency is comparable across all four algorithms. Ares has poor write latency due to the usage of logical timestamps, which requires an extra round-trip. Oreas has moderate penalty in write latency. Throughput is comparable in the case of write ratio 0.1, a common case in NoSQL KV-stores. Figures 1d to 1f show latency and throughput under different data size with write ratio 0.1. We only show average latency, as 95 percentile has the same pattern. Figure 1f demonstrates that CassandrEAS suffers moderate penalty on throughput.

Availability, Fault-tolerance, and Scalability: CassandrEAS is highly available, fault-tolerant, and scalable, because

its core is based on Cassandra. It can continue serving client's operations as long as the conditions specified by f and  $\delta$  are satisfied. We have tested our system with 7 servers and 1 crashed server, and observed minimal disruption on throughput (ranging from 0.1% to 2.5% decrease). We also observed that each YCSB client's throughput decreased a bit right after a server crashed, and later came back to normal throughput (compared with a cluster without any fault). Finally, we also tested clusters with 7, 9, and 11 servers. The throughput of OREAS is in the range of 77-80% of Cassandra's. This demonstrates that CassandrEAS' performance is also scalable, i.e., the throughput increases when n increases.

Correctness: We develop a consistency checker based on the approach specified in [23] to ensure that our implementation also provides this guarantee. Validating strong consistency requires precise clock synchronization across all servers. This is impossible to achieve in a distributed system where clock drift is inevitable. To circumvent it, we deploy our CassandrEAS servers on a single machine and use Mininet [5] to simulate the underlying network communication. Our checker then uses the machine time as the global clock. We collect multiple traces under different configurations with failures. The checker verifies that all traces we tested satisfy atomicity.

### VI. SUMMARY

Strong consistency, storage efficiency, and availability are three key features for NoSQL KV-stores. We demonstrated that through CassandrEAS – erasure coding can be used to reduce storage cost while incurring moderate performance penalty. One interesting future work is to investigate how to apply TREAS-OPT and OREAS in other NoSQL KV-stores, or to Cassandra to support transaction-related primitives.

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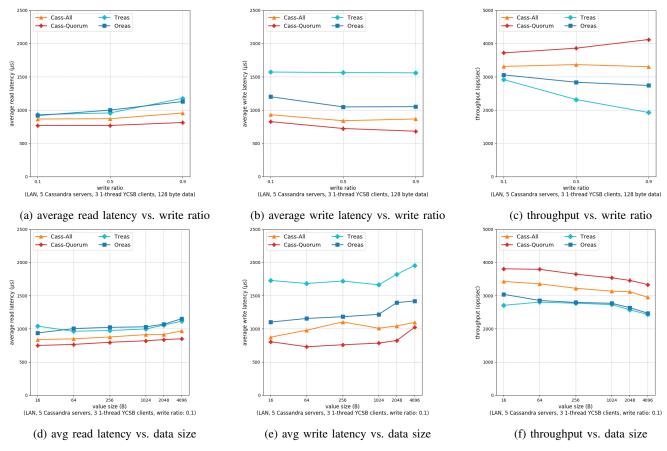


Fig. 1: Comparison of latency and throughput of different configurations

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