1. Suppose $C = C_1 \cup C_2$, where $C_1 \cap C_2 = \phi$. Then prove/disprove that: C is convex, C is not convex, or C can be, but not necessarily, convex.

2. Provide a conversion to/from the general affine combination $x = \sum_{i=1}^{n} \theta_i x_i,\ \sum_i \theta_i = 1$ and the α-combination studied in lectures:

$$x = \alpha_n x_n + (1-\alpha_n)\Big(\alpha_{n-1}x_{n-1} + (1-\alpha_{n-1})\big(\alpha_{n-2}x_{n-2} + (1-\alpha_{n-2})(\cdots)\big)\Big) \quad \text{(recursively)}$$

$$= \Big(\big((1-\alpha_2)x_1 + \alpha_2 x_2\big)(1-\alpha_3) + \alpha_3 x_3\Big)(1-\alpha_4) + \alpha_4 x_4 + \cdots \quad \text{(iteratively)}$$

Hint: use the iterative expression; the recursive one is written just for clarification. The solution should be in the form:

$$\theta_i = \theta_i(\alpha_j,\ j = 1,\cdots,n),\ i = 1,\cdots,n$$
$$\alpha_i = \alpha_i(\theta_j,\ j = 1,\cdots,n),\ i = 1,\cdots,n$$