Problem Chosen	2024	Team Control Number
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Lampreys Have Feelings Too

Summary

It is observed that sea lamprey populations exhibit deviations from the expected 50:50 sex ratio, influenced by local environmental conditions. To understand this phenomenon, we develop and utilize three mathematical models, and subsequently explore their impact on the ecosystem.

Model 1: Modified Logistic Growth Model for Sea Lampreys
We adopt the **logistic growth model** to reflect the sea lamprey's life cycle, accounting for male and female populations and an intermediary larval stage. This stage's growth, influenced by the female population, birth rate, and resource availability, feeds into a **Bayesian hierarchical logistic regression model**, which determines the probability of larvae developing into male or female adults. We find that the ability to alter sex ratios allows lampreys to recover more effectively from disturbances, particularly in bad resource environments **(Recovery Period & Resource Utilization)**. Altering the sex ratio leads to different recovery times for males and females, unlike when the ratio is balanced. This implies that sex ratio manipulation is a strategy for population resilience in fluctuating environments **(Population Resilience)**.

Model 2: Parasite-Host Interaction Model
Building on Model 1, we explore the sea lamprey's role as a **parasite**. We estimate the mean number of lamprey attacks on host species, factoring in **feeding season length** and **host density** while omitting host search rates due to presumed host abundance. This model calculates **lamprey-induced host mortality**, considering higher mortality contributions from male parasites. We use the modified Lotka-Volterra model to conclude that Lampreys are more lethal in a bad resource environment as compared to a good resource environment **(Attack Rates)**. Moreover, changes in lamprey sex ratios impact competing and host species, underscoring their pivotal role in community dynamics **(Crucial Species Interaction)**.

Model 3: Competition Model
Also built upon Model 1, this model incorporates **inter-species competition** by constructing various logistic growth models with inter-species relationship coefficients. We show that some species such as plant eaters, and species that are in direct competition with lampreys benefit when their numbers fall **(Advantage to Other Species)**. We also show that some species are completely unaffected by Lampreys in both bad and good resource environments. Moreover, the ecosystem responds to disturbances with a return to initial rates of decrease in host populations within approximately 2 years **(Stability of Ecosystem)**.

In conclusion, our analysis of the model through disruptions to lamprey populations and a sensitivity analysis on growth rates and inter-species coefficients, confirms the model's robustness and clarifies the lamprey's ecosystem role.

Keywords: Adaptive sex ratio variation, Sea lamprey populations, Ecosystem dynamics

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1 Introduction

1.1 Background

Sex ratio in animal populations plays a crucial role in shaping the dynamics and stability of ecosystems. While some species exhibit a balanced sex ratio for male and female at birth, others deviate from this equilibrium due to various factors, including environmental conditions and resource availability [1]. Sea lampreys are one of the species that display adaptive sex ratio variation, in which they are intriguing organisms found in lake or sea habitats that migrate up rivers for spawning [2]. The sex ratio of sea lampreys is dependent on their growth rates during the larval stage, which are influenced by food availability. Analyzing the consequences of altering sex ratios within a population is crucial for comprehending the broader ecological system's dynamics.

1.2 Restatement of the Problem

- Develop a model to examine the interactions of various species in an ecological system under the variation of sex ratio of sea lampreys.
- Evaluate the factors that affect the population of sea lampreys due to their varying sex ratio based on resource availability.
- Analyze the impact of sex ratio changes in lampreys on the stability of the ecosystem.
- Investigate whether an ecosystem with variable sex ratios in the lamprey population can provide advantages to other organisms.

1.3 Our Approach

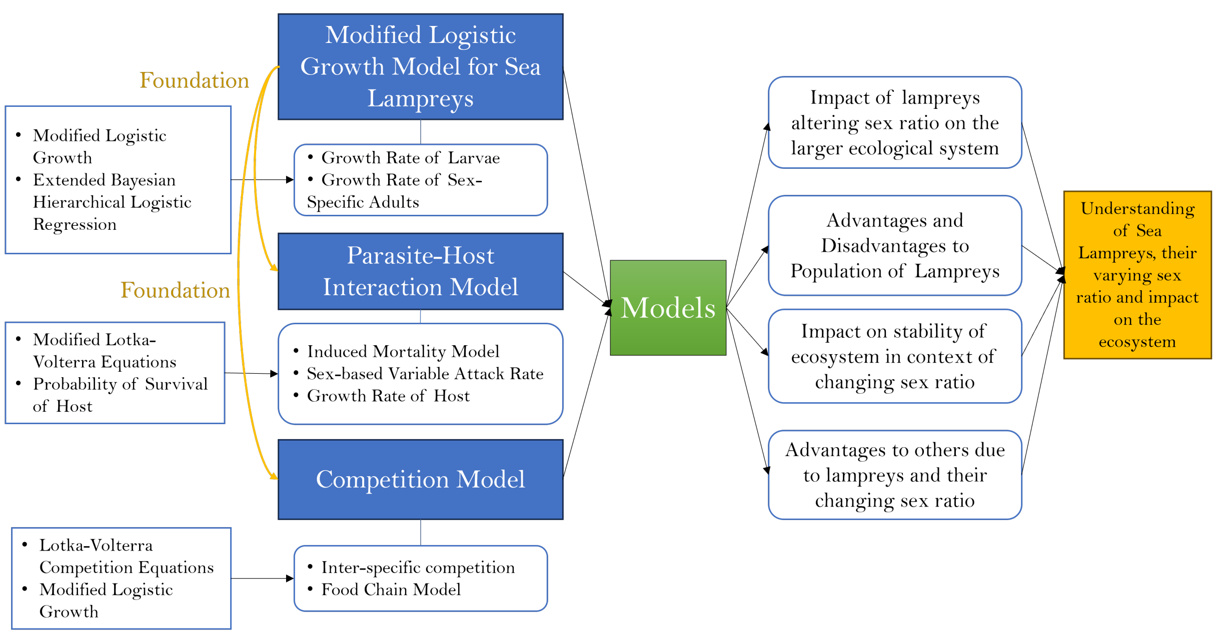


Figure 1: The flow chart of our approach

2 Notations and Assumptions

2.1 Notations

Symbol	Description
L	Number of larvae at time t
K	Carrying capacity
r	Intrinsic growth rate
P	Population size
b_L	Number of eggs hatched per pair
b	Birth rate
d	Death rate
ϕ	Rate of larvae survive to adulthood
β_{ij}	Inter-specific relationship coefficient between species i and j
C	Catch rate of adult lampreys
S_i	Population of species i
$R(t)$	Logistic regression function
a	Attack rate of sea lampreys
A	Mean total number of attacks by sea lampreys
f	Frequency
α	Predation rate coefficient of male lamprey on host population
ω	Predation rate coefficient of female lamprey on host population
F	Length of feeding season
N_i	Population density of host i
λ	Effective search rate
h	Attachment type for host i
Γ	Parasite population
M_i	Lamprey-induced mortality rate

2.2 Assumptions

To simplify the modeling, we establish the following main assumptions and provide justifications:

- Assumption: Sea lamprey populations and other species in the same environment follow a logistic growth pattern.
  - Justification: Due to the limitation of environmental resources, sea lamprey and any other species cannot grow indefinitely. Hence, we assume a logistic growth rate for simplicity.
- Assumption: The sex ratio of the sea lamprey population remains constant after the transition period from the larval stage.
  - Justification: When larvae change into the adult stage, their sex is fixed [3]. Thus, it is sufficient to consider a constant sex ratio of sea lampreys.

- Assumption: Sea lampreys are monogamous and mate only once.
  - Justification: Literature suggests that sea lampreys are generally monogamous, yet polyamory was reported [4]. Hence, to simplify the model, we consider the monogamy of sea lampreys.
- Assumption: Lampreys are apex predators as parasites within their ecological niche and the hosts for them are abundant.
  - Justification: We focus on how the ecosystem is being impacted by sea lampreys that act as parasites. Hence, we assume they as top predators with a sufficient host.

3 Models

In this paper, we model any species in consideration based on either a Logistic Growth model or a Modified Logistic Growth model. The rationale is as follows. Initially, when the said species' population is low, the growth rate is high due to the abundance of resources. As the population size of the species increases, the resources get scarce, leading to a reduction in its growth rate. Moreover, the more food available, the lower the mortality rate of the species. Furthermore, each environment can support a certain maximum number of species based on available resources.

Also, we solve all the differential equations mentioned below using MATLAB ode45() function which uses the fifth-order Runge-Kutte method to solve the equations numerically.

3.1 Modified Logistic Growth Model for Sea Lampreys

We begin with modeling the population dynamics of lampreys using a Modified Logistic Growth Model that takes into account their adaptive sex ratio. We model the male adult, female adult, and larva lampreys separately. Below is a rough overview of our model:

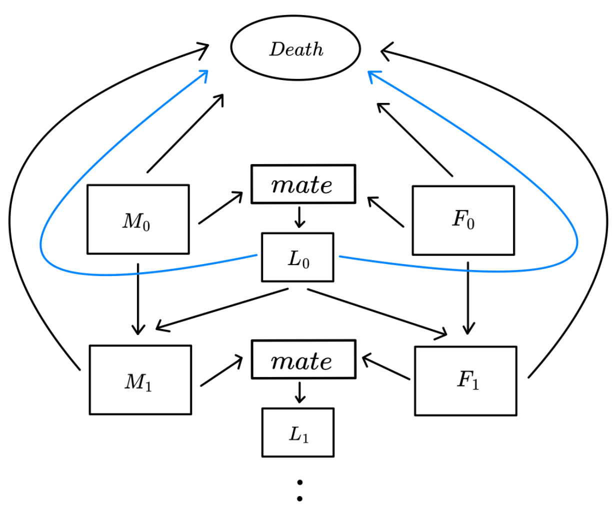


Figure 2: Overview of Modified Logistic Model for Sea Lampreys

3.1.1 Growth Rate of Larvae

We first derive an ODE representing the growth rate of lamprey larvae $\frac{dL}{dt}$:

$$\frac{dL}{dt} = L \times r_L \times (1 - \frac{L}{K_L}) \tag{1}$$

where K_L is the carrying capacity of larvae, which sets the maximum value that larva can attain in a system due to limitation in resources, and L is the number of larvae at time t. r_L is the Malthusian Parameter or the intrinsic growth rate which we determine using these parameters: b_L is the number of eggs hatched per each adult pair of sea lampreys, d_L is the natural mortality rate of larva, and ϕ is the rate at which larvae turn into adults.

Assuming that lampreys are monogamous and mate only once and all adults are reproductive[4], we can estimate the intrinsic growth rate of larvae. This is given by the total birth rate minus d_L and ϕ. The total birth of eggs is given by multiplying b_L by the minimum value between male and female lampreys, dividing this by L we get the total birth rate. Hence we obtain:

$$\frac{dL}{dt} = L \times \Big[\frac{b_L \times \min(P_m, P_f)}{L} - (d_L + \phi)\Big] \times (1 - \frac{L}{K_L}) \tag{2}$$

where $min(P_m, P_f)$ denotes the minimum value between males and females in the lampreys population. We will explain how to get b_L in section 3.1.2..

3.1.2 Growth Rate of Sex-Specific Adults Lamprey

Next, we derive an ODE to model the growth of the population of male adult lampreys $\frac{dP_m}{dt}$ and female adult lampreys $\frac{dP_f}{dt}$ based on the Modified Logistic Growth Model [5]. The equations are defined as follows:

$$\frac{dP_m}{dt} = r_m P_m \Big(1 - \frac{P_m}{K_m}\Big) - D_m P_m \tag{3}$$

$$\frac{dP_f}{dt} = r_f P_f \Big(1 - \frac{P_f}{K_f}\Big) - D_f P_f \tag{4}$$

where P_m and P_f represent the number of male and female adult lampreys at time t, r_m and r_f represent the intrinsic growth rate of the male and female lampreys, K_m and K_f represent the carrying capacity of the male and female lampreys, and D_m and D_f represent the rate of mortality of the male and female lampreys, induced by external events, such as fishing.

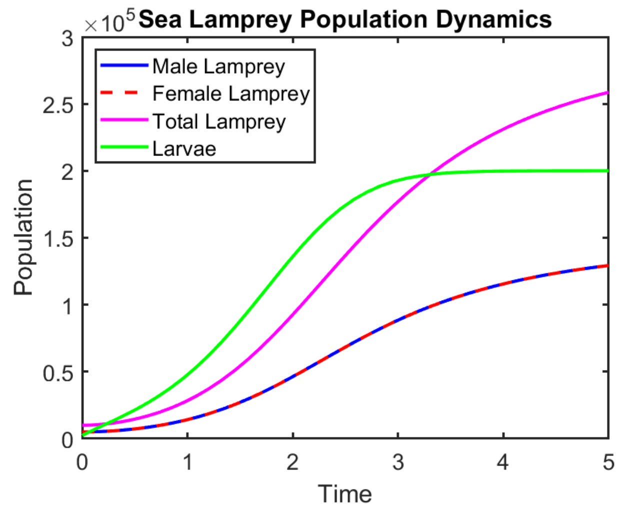


Figure 3: Logistic Growth Model with sex ratio at 1:1

Figure 3 shows the logistic model where the sex ratio for males and females is 1:1. So the P_m and P_f lines are exactly overlapping, so we sketch P_n (total population) line which is the summation of P_m and P_f.

We note that the intrinsic growth rate of the adult populations is dependent on the probability of a larva becoming a male or female when it matures into adulthood. This probability depends on the growth rate of larvae, and, as mentioned in the problem statement, the growth rate of larvae depends on the local conditions.

Hence, we define the intrinsic growth rates of male and female adult lampreys as follows:

$$r_m = \frac{L \times \Pr(\mathbf{M} = t)}{P_m} - d_m \tag{5}$$

$$r_f = \frac{L \times \Pr(\mathbf{F} = t)}{P_f} - d_f \tag{6}$$

Where L is the larvae population at time t, P_m and P_f are the male and female populations, and $\Pr(\mathbf{M} = t)$ and $\Pr(\mathbf{F} = t)$ are the probabilities of an individual larvae becoming male or female respectively. We subtract d_m and d_f which are the male and female natural mortality rate respectively, to account for the natural dying rate of adult lampreys in each sex.

To get the probability functions, we consider two cases for the sake of simplicity: an environment with 'good' resources (stream environment) and an environment with 'bad' resources (lentic environment). Based on a Bayesian hierarchical logistic regression model [2], the logistic regression equations for the two differing environments are as follows:

$$R_{bad}(t) = 0.876 + 0.122t, \quad 0 < t \leq 5, \quad \text{for bad resources} \tag{7}$$

$$R_{good}(t) = 0.930 - 0.236t, \quad 0 < t \leq 5, \quad \text{for good resources} \tag{8}$$

where R_{bad} is the probability function of being a male in the 'bad' resources scenario on a logit scale and R_{good} is the probability function of being a male in the 'good' resources scenario on a logit scale.

Since time t is restricted to five year, we attempt to extend the functions by deriving separate equations for the half-cycles of each 10-year cycle. Intuitively, the resources in an environment vary with time as they are being used by the population. Thus, we cautiously assumed that the regression model would reverse itself after every five years:

$$R(t) = \begin{cases} 0.879 + 0.122 \cdot \text{mod}\,(t, 5), & 0 < \text{mod}\,(t, 10) \leq 5 \\ 0.930 - 0.236 \cdot \text{mod}\,(t, 5), & 5 < \text{mod}\,(t, 10) \leq 10 \end{cases} \tag{9}$$

We visualize our extended resource function R in Figure 4:

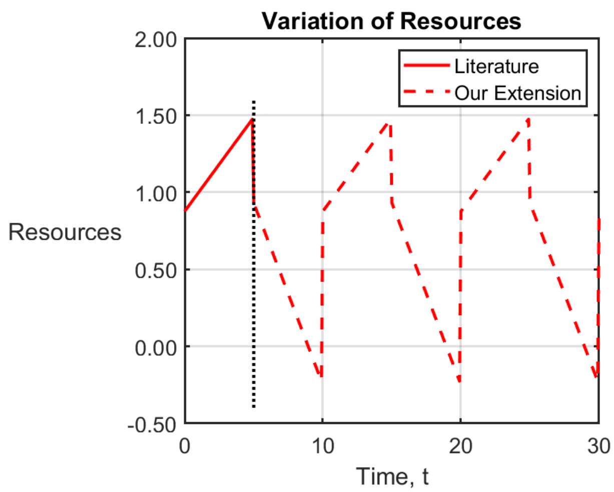


Figure 4: Extended Bayesian Hierarchical Logistic regression model

It can be observed in the figure that we reverse the resource function after each period of 5 years, depicting the change in resource availability. Hence, by combining our resource functions, we obtain the following function for the probability of a larvae becoming a male:

$$\Pr(\mathbf{M} = t) = \frac{1}{1 + e^{-R(t)}} \tag{10}$$

where $\Pr(\mathbf{M} = t)$ is the male probability function. Consequently, we define the female probability as follows:

$$\Pr(\mathbf{F} = t) = 1 - \Pr(\mathbf{M} = t) \tag{11}$$

Moreover, we define the numbers of eggs hatched per pair b_L (Section 3.1.1) based on the availability of resources. Referring again to the two different environments, we consider the egg-laying rate of each female adult and the hatching success of the eggs to obtain b_L, 8.34 for 'good' resources and 7.93 for bad resources [6]. Assuming these environments to be the two extreme environments, we propose a cosine curve to make the birth rate dynamic based on the time-varying resource availability with a frequency of 10 years.

$$b_L = b_{mean} + b_{amp} \cdot \cos(2\pi \cdot f \cdot t) \tag{12}$$

where b_L represents the eggs hatched per pair at time t, b_{mean} represents the mean eggs hatched per pair, b_{amp} represents the amplitude of the eggs hatched curve, and f is the frequency. Below is the visualization of the b_L function over 20 years.

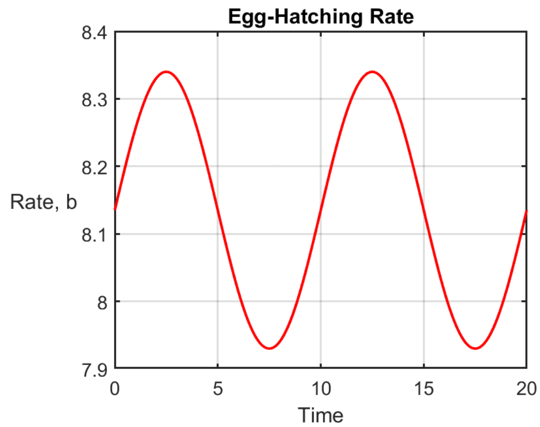


Figure 5: Larval eggs hatched over a 20-year period

3.2 Parasite-Host Interaction Model

We integrate a parasite-host interaction component to simulate the dynamic interplay between the sea lamprey and their host species within the ecosystem. This integration is crucial, as lampreys are parasitic during their adult stage and their population dynamics are inextricably linked to the abundance of host species.

We denote the host population at time t as P_h, which constitutes the various fish species that lampreys prey upon in their respective environments. To model the host population, we use modified Lotka-Volterra equations adjusted for predation pressure from male and female lampreys separately:

$$\frac{dP_h}{dt} = r_h P_h \left(1 - \frac{P_h}{K_h}\right) - (\alpha + \omega) P_h \tag{13}$$

where r_h represents the intrinsic growth rate of the host population, K_h is the carrying capacity of the host environment, α and ω are the induced mortality coefficients of male and female that capture the impact of male and female lamprey parasites on the host population, respectively.

3.2.1 Induced Mortality Model

To measure the lamprey-induced mortality of the host population, we start by modeling the attack rate of sea lamprey which is given by [6]:

$$a_i = \frac{F\lambda_i N_i}{1 + \sum_j \lambda_j N_j h_j} \tag{14}$$

where F is the length of the feeding season, N_i is the population density of host i. λ_i is the effective search rate. We sum over all the host species j in the denominator. Literature suggests that if the host population is abundant, the attack rate no longer depends on the search rate. Thus, for simplicity, we also assume that the host is abundant.

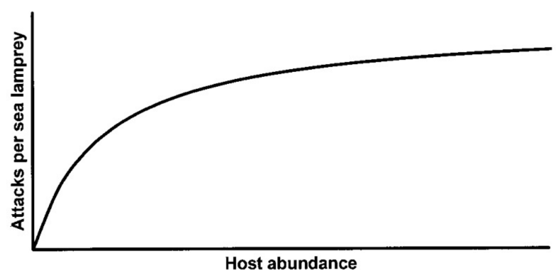


FIG. 1. Relationship between total attacks per sea lamprey in a season (a) and the abundance of hosts (N) assumed by equation 1. The graphed relationship is calculated with just one type of host present, but shows the general effect of increasing host density that is incorporated in the multi-species functional response equation.

Figure 6: Host Abundance[6]

Therefore, the new attack equation can be rewritten as:

$$a_i = \frac{FN_i}{1 + \sum_j N_j h_j} \tag{15}$$

Following this, we get the mean total number of attacks per individual of host type i which is given by this equation:

$$A_i = \frac{a_i \Gamma}{P_{h_i}} \tag{16}$$

where A_i is the mean total number of attacks, P_{h_i} is the population of host i, Γ is the parasite population that consists of male and female lampreys, and a_i is the attack rate given by (16).

After this we get the lamprey-induced mortality rate of the host with this formula:

$$M_i = (1 - \Pr(S_i))A_i \tag{17}$$

where $\Pr(S_i)$ means the probability of survival of species i, and A_i is the mean total number of attacks given by (16). The probability of survival $\Pr(S_i)$ is dependent on the ratio of lamprey length to the host weight; however, we assume it to be a constant value of 0.25 according to the literature [6].

We measure the impact of male and female lamprey predation by adjusting the number of attacks on host per individual parasite accordingly. We differentiate the number of attacks on host by males and females, a_m and a_f. Literature suggests that in scenarios of abundant parasitism, the male population of lampreys is prevalent [7]. Thus, we make a cautious assumption to increase the number of attacks by male lampreys by 20% and decrease the number of attacks by female lampreys by the same amount.

3.3 Competition Model

In every ecosystem, there exists competition and interaction between different species and lampreys. For simplicity, we assume that the competitions and interactions are only for resources among different species and no intra-species competition is occurring.

Competitive ranking is the measure of the advantage/disadvantage of two different species in competition under the same circumstances. We incorporate this ranking into the logistic model of species population growth. There exists an inter-species relationship index β_{ij} between species i and j, which denotes the effect of species j on the carrying capacity of species i in a system. Therefore, the general equation for growth rate of species i denoted by $\frac{dS_i}{dt}$ with only one competing species j can be defined as [8]:

$$\frac{dS_i}{dt} = r_i S_i \times (1 - \frac{S_i + \beta_{ij} S_j}{K_{S_i}}) \tag{18}$$

where r_i is the intrinsic growth rate of species i and S_i is the number of species i at time t. β_{ij} is the effect of species j on the carrying capacity of species i. K_{S_i} is the carrying capacity of S_i.

We define lampreys as species 1. Additionally, the growth rate of lampreys can be further decomposed as the growth rate of male and female lampreys:

$$\frac{dS_1}{dt} = \frac{dP_m}{dt} + \frac{dP_f}{dt} \tag{19}$$

Using (3), (4), (5) & (6), (19) can be rewritten as follows:

$$\frac{dS_1}{dt} = L\Big[1 - \frac{\Pr(\mathbf{M} = t)(P_m + \beta_{1j} S_j)}{K_m} - \frac{\Pr(\mathbf{F} = t)(P_f + \beta_{1j} S_j)}{K_f}\Big] - D_m P_m - D_f P_f \tag{20}$$

where $(P_m + \beta_{1j} S_j)$ and $(P_f + \beta_{1j} S_j)$ are the effects of competition between species. The remaining terms are gotten through basic substitution and algebra and then simplified.

Finally, we can generalize (20) to obtain the growth rate of lampreys that depends on n species:

$$\frac{dS_1}{dt} = L\left[1 - \frac{\Pr(\mathbf{M} = t)(P_m - \sum_{2 \le j \le n} \beta_{1j} S_j)}{K_m} - \frac{\Pr(\mathbf{F} = t)(P_f - \sum_{2 \le j \le n} \beta_{1j} S_j)}{K_f}\right] - D_m P_m - D_f P_f \tag{21}$$

The growth rate of other species in the same ecosystem can also be determined by the Logistic Growth Model with the inter-specific relationship index β_{ij} between species i and j. The equation building on (18) is as follows:

$$\frac{dS_i}{dt} = r_i S_i \left(1 - \frac{S_i - \sum_{\substack{1 \le j \le n \\ i \ne j}} \beta_{ij} S_j}{K_i}\right), \quad for \;\; i = 2, 3, ... \, n, \tag{22}$$

We also define the inter-specific relation matrix B_{ij} below. The values in the matrix are set through literature or are assumed carefully.

$$B_{ij} = \begin{bmatrix} \beta_{11} & \beta_{12} & \dots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \dots & \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{n1} & \beta_{n2} & \dots & \beta_{nn} \end{bmatrix} \tag{23}$$

4 Model Simulation Results

Here we discuss the simulation results with the parameters used. It is useful to note that all rates for parameters that were discrete were converted to annual rates and then were passed through this function to get the continuous rate:

$$f(x) = ln(1 + x)$$

where x is the discrete annual rate and $f(x)$ is the continuous rate.

4.1 Model 1: Logistic Growth

Table 1 provides a summary of parameters.

Table 1: Parameter Values

Parameter	**Value**	**Parameter**	**Value**
K_m	40000	K_f	40000
K_l	20000	D_m	0.2
D_f	0.2	b_{mean}	8.135
b_{amp}	0.2055	frequency	0.1
D_l	0.408	ϕ	0.25
P_{m_0}	5000	P_{f_0}	5000
L_0	2430		

Based on these parameters we obtained the following results for these ‘bad’ and ‘good’ resource scenarios:

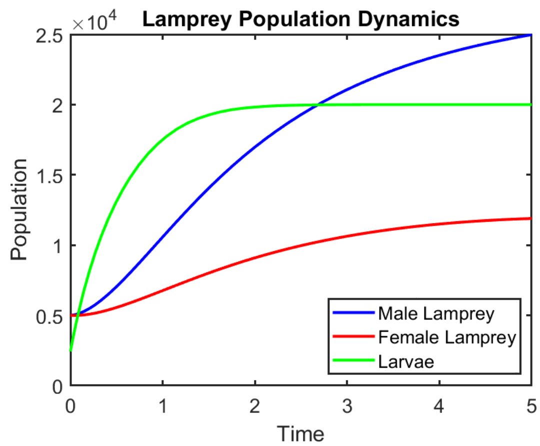


Figure 7: Bad Resources

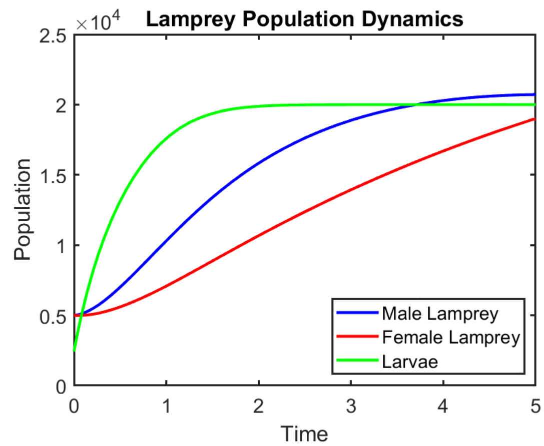


Figure 8: Good Resources

Figure 7 and 8 gives us a good insight into the changing sex ratio. As shown in the bad resource scenario in Figure 7, we can see that the male population keeps on increasing as a percentage of the total population, whereas in figure 8, in the good resource scenario the female population almost catches up to males. The sex ratios at different timestamps are summarized in the Table 2 below:

Table 2: Sex Ratio in Good and Bad resources

		Bad	**Good**
End of Year 2	**Male**	61.3%	59.39%
	Female	38.7%	40.61%
End of Year 5	**Male**	67.7%	52.25%
	Female	32.3%	47.75%

Our results are approximately similar to the situation described in the problem statement which validates our model.

4.2 Model 2: Parasite-Host Model

We simulate the Parasite-Host model based on these parameters:

Table 3: Parameter Values

Parameter	**Value**	**Parameter**	**Value**	**Parameter**	**Value**
K_{n_1}	400000	P_{h_0}	300000	r_{n_1}	0.5
h	0.054	$Area$	19000	N	15.789
a	0.034	a_m	0.051	a_f	0.017
$a_{m_{\text{mean}}}$	0.034	$a_{f_{\text{mean}}}$	0.011	M_m	0.0255

Based on these parameters we have visualized the results below on a logit scale:

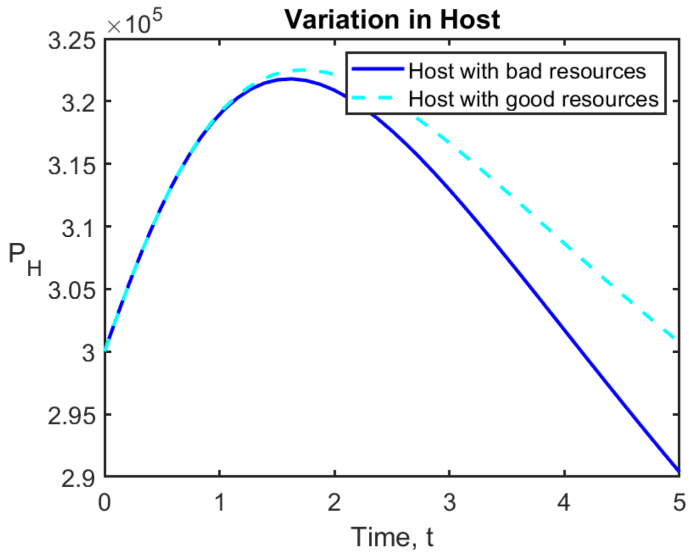


Figure 9: Difference in Host Population caused by differences in resources

Based on Figure 9, we can see that the host population decreases faster when resources are bad than in good. With bad resources the host population also reaches a slightly lower peak and achieves it earlier than in good resource environment. This is expected as we are assuming that males are more aggressive than female Lampreys, and in bad resource environment the population is male dominated.

4.3 Model 3: Competition Model

In order to simulate an ecological system, we fist make our own food chain. Here we assume lampreys are apex predators. Below is an overview of our hypothetical food chain:

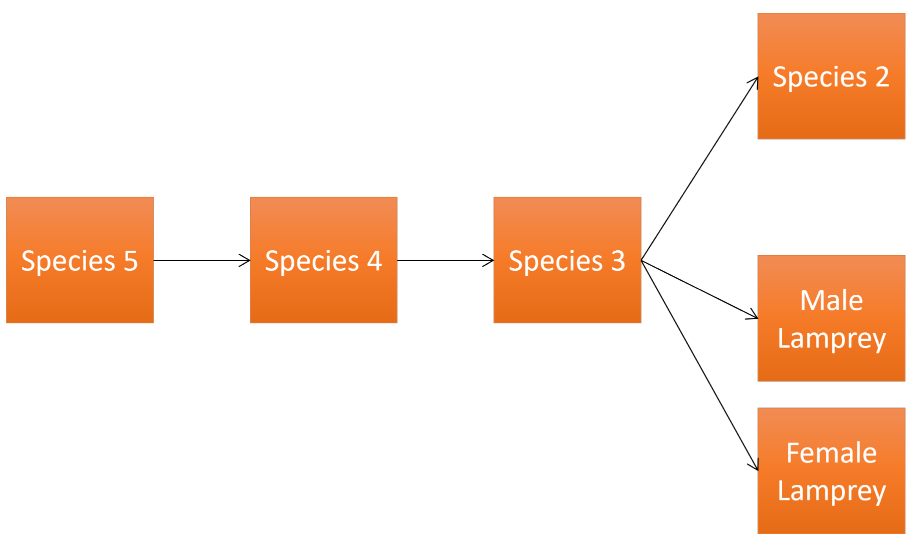


Figure 10: Food Chain

Here Species 5 plays the role of producer, Species 4 plays the role of plant eater, Species 3 plays the role of a carnivore, and Species 2 can play the role of either another apex predator or another parasite, and Male and Female lampreys are playing the role of parasites. Male and Female Lampreys are denoted by Species 1 and 6 respectively.

Then we define the B_{ij} matrix and the intrinsic growth rates R_i of the species as well. The specific values can be obtained either through literature or field surveys; however, the following data is to only show how our model simulates the ecosystem :

$$B_{ij} = \begin{bmatrix} 0 & 0.2 & 0.2 & -0.2 & -0.1 & 0 \\ 0.7 & 0 & -0.5 & -0.2 & -0.1 & 0.5 \\ 1.3 & 0.5 & 0 & -0.5 & -0.1 & 1.1 \\ -0.3 & -0.1 & 0.5 & 0 & -0.5 & -0.1 \\ 0.2 & 0.1 & -0.2 & 0.5 & 0 & -0.1 \\ 0 & 0.2 & 0.2 & -0.2 & -0.7 & 0 \end{bmatrix}$$

After this we also define the intrinsic growth rates of each species in the ecosystem.

$$R_i = [\frac{r_m + r_f}{2},\ 0.05,\ 0.5,\ 0.08,\ 0.15]$$

where r_m and r_f are the intrinsic growth rates of male and female lampreys respectively.

After this we set the initial population values P_i and carrying capacity K_i:

Parameter	Value	Parameter	Value
P_2	20000	K_2	50000
P_3	400000	K_3	1000000
P_4	10000000	K_4	50000000
P_5	50000000	K_5	100000000

It is important to note that we are assuming no intra-species competition and no competition between adult male and female lampreys in the ecosystem. Now based on the above parameters we get these results:

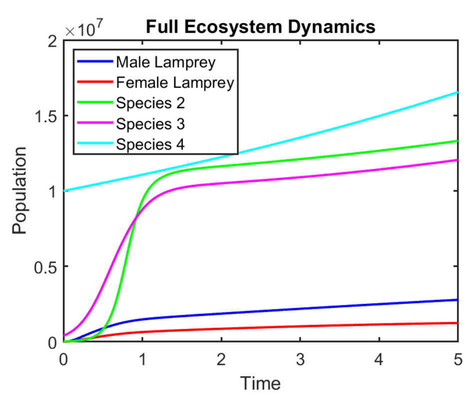


Figure 11: Difference in Host Population caused by differences in resources

This simulation is conducted in a bad resource environment as can be seen from the difference in the male and female Lamprey population. This shows us that it is possible to reach a steady stable solution in an ecosystem where lampreys exist. We will use this model in section 5.4 section to study the effect of altering sex ratio by altering some parameters and its overall effect on the ecosystem.

5 Application of Model

In this section, we use our model defined above to attempt to study the affect of the ability to alter sex ratio on the ecosystem, individual, and other species.

5.1 Simulating Disturbance

The disturbance pattern in the logistic growth model is defined as:

$$r_m^{new}(t) = r_m \cdot dist(t) \tag{24}$$

$$r_f^{new}(t) = r_f \cdot dist(t) \tag{25}$$

where $r_m^{new}(t)$ and $r_f^{new}(t)$ are the intrinsic growth rates for the disturbed male and female lampreys respectively, r_m and r_f are the intrinsic growth rate for the undisturbed male and female lampreys, and $dist(t)$ is the disturbance multiplier given by:

$$dist(t) = \begin{cases} 0.5 & \text{for } t = 10 \\ 0.5 + \left(\frac{1-d}{1+S(t)}\right) & \text{for } 10 < t < 15 \\ 1 & \text{for } t \geq 15 \end{cases} \tag{26}$$

where we set d = 0.30 which measures how much the disturbance affects the rate, and $S(t)$ is defined:

$$S(t) = e^{-\frac{6}{r}\cdot(t-\frac{s+g}{2})} \tag{27}$$

where g is the time it takes for the growth rate to recover and s is the year disturbance starts.

The disturbance pattern introduced into the logistic growth model is designed to simulate a temporary environmental shock or change in conditions that causes a significant decrease in the intrinsic growth rate of male and female sea lampreys. This shock is represented by a sudden reduction in the growth rate, which is then followed by a non-linear recovery back to normal levels over the subsequent years.

The recovery phase is modeled using a sigmoid function, ensuring a smooth and gradual return to the pre-disturbance growth rate. This non-linear recovery mirrors natural processes, where the adjustment to a disturbance is often initially slow, then accelerates as the system adapts or conditions improve, before finally slowing down as the system stabilizes at its original state.

Here we plot a sample growth rate with depth d = 0.5 to show the smooth recovery after disturbance as modeled by the function above. We assume, for this graph, that the growth stays constant except during the disturbance period.

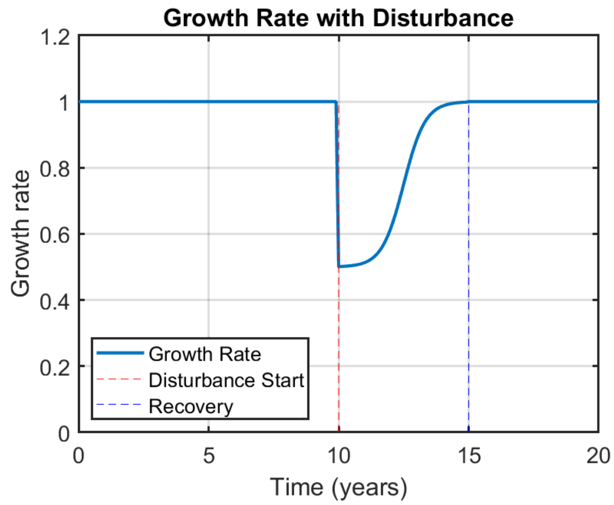


Figure 12: Disturbance and Recovery in Growth Rate with Disturbance

5.2 Application of Model 1 : Logistic Growth

We attempt to understand the individual advantages and disadvantages to the sea lampreys population using this model. Here we alternate between good and bad resources every 5 years. The first 5 years are bad resource environment, then good resource environment and so on. Next, we introduce a disturbance at the start of a good resource cycle and a bad resource cycle, and compare the results. We first start by modelling it on a control environment where the sex ratio of lampreys is 50:50. The results are as follows:

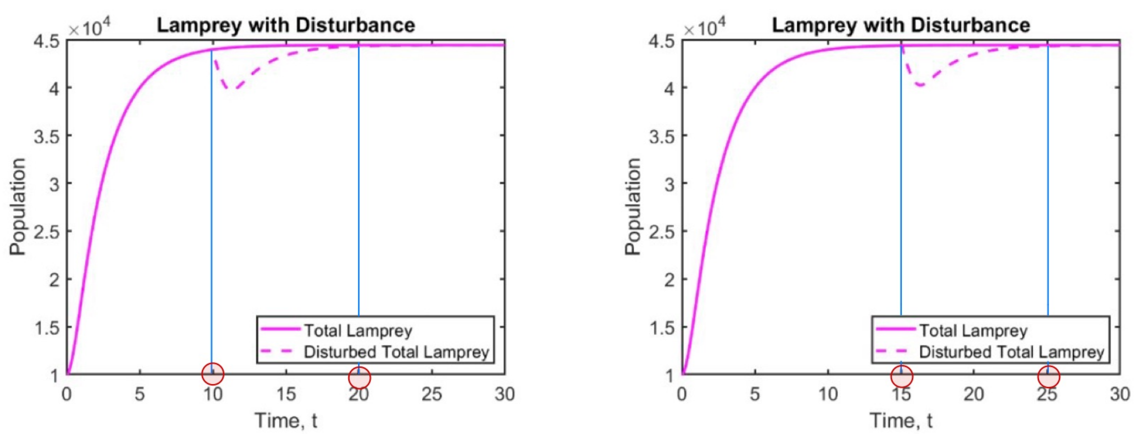


Figure 13: Effect of disturbance on bad resource environment (left) and good resource environment (right)

Interestingly, the recovery period in both environments when the sex ratio is 50:50 is 10 years. Now we model the disturbance in our Model 1:

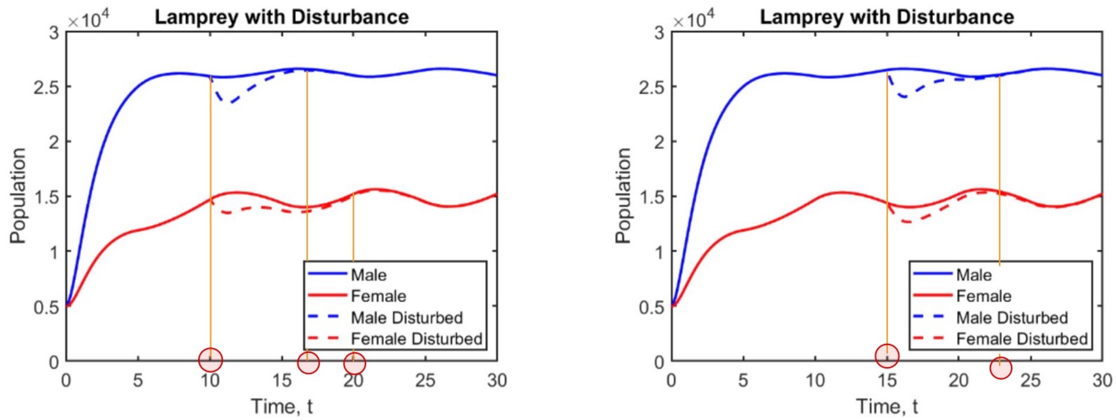


Figure 14: Effect of disturbance on bad resource environment (left) and good resource environment (right)

When compared to the control population, the differences can be observed clearly. In a bad resource environment, the male population recovers in around 6-7 years, whereas the female population recovers in 10 years. In a good resource environment, both male and female population recover in around 7-8 years. It is also noticeable that the drop in total population is much less than the one in control case with the male population dropping more on average than the female population. Hence, we can conclude that altering its sex ratio offers an evolutionary advantage to the lampreys, as they become more resilient due to this ability.

5.3 Application of Model 2: Parasite Model

5.3.1 Different Attack Rates

Literature suggests that males are more aggressive than females, so we attempt to model the effects of different attack rates. We keep the attack rates as depended on the mean total number of attacks per individual of host type i which we calculate from (15). The different attack rates we simulated for are:

Table 4: Attack Rates

	Male	**Female**
Rate 1	$1.2 \times a_i$	$0.8 \times a_i$
Rate 2	$1.4 \times a_i$	$0.6 \times a_i$
Rate 3	$1.6 \times a_i$	$0.4 \times a_i$

The results show that in a bad resource environment, the host population peaks earlier than in the good resource environment. Furthermore, it is also noted that all the host population were below the initial population level at t = 5. Furthermore the Host population peaked earlier and at a lower level than in We also found that Rate 1 in bad resource environment is similar to Rate 3 in a good resource environment. Meaning that a bad resource environment is much more dangerous to host population than a good resource environment is, which is unsurprising because males are modelled much more aggressively than females. However, an alternate explanation could be that Lampreys are more lethal when resources get scarce.

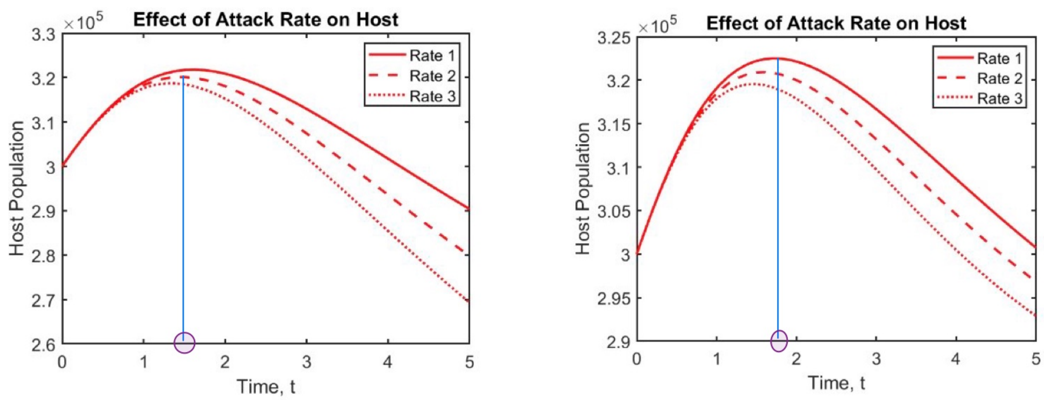


Figure 15: Effect of different attack rates in bad resource (left) and good resource (right) environment

5.3.2 Simulating disturbance

We introduce disturbance in the lamprey population at timestamp $t = 2$.

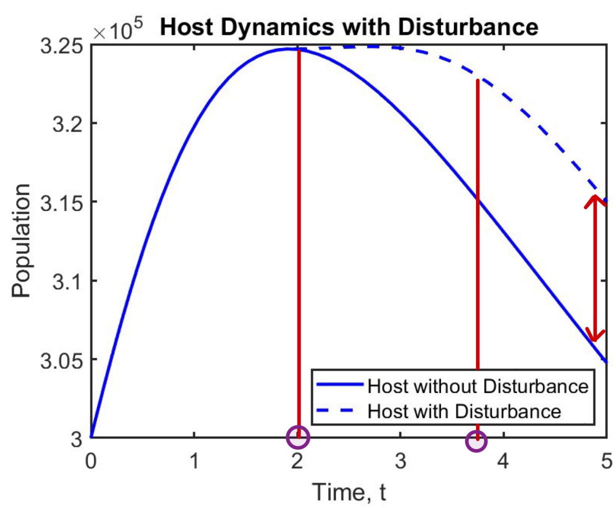
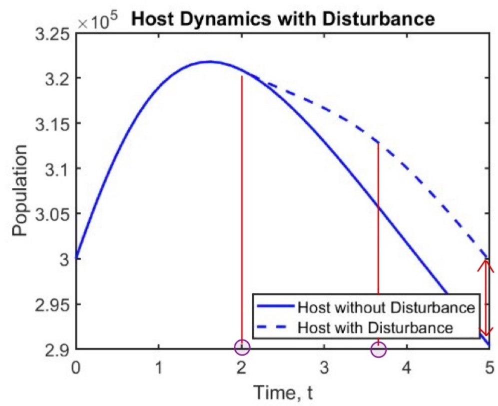


Figure 16: Control Host Dynamics with Disturbance

In Figure 16, after the disturbance is introduced in the lamprey population, the host population starts to increase; however it retrieves to its initial rate of decrease in about 2 years.

Then we model disturbance in our original model and the results are as follows:



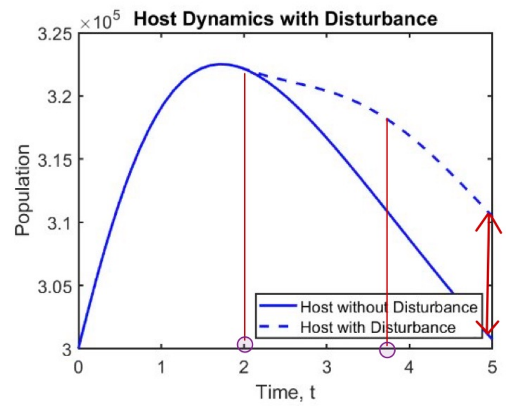


Figure 17: Control Host Dynamics with Disturbance on bad resources (left) and good resources (right)

Here we can see that in a bad resource environment even after disturbance the host does not recover relative to the good resource and control environment. However, in both situations the host population gets back its initial rate of decrease in approximately 2 years. Our model suggests that in a bad environment when disturbance is introduced lampreys utilize their resources more effectively; however regardless of whatever the sex ratio was in the population the time taken for hosts to come back to its original rate of decrease was approximately the same.

5.4 Application of Model 3 : Competition Model

Introducing disturbance in this model helps us understand the stability of the ecosystem and whether lampreys affect some species positively or negatively. In this section we do not include lampreys population in the graphs for good presentability purposes.

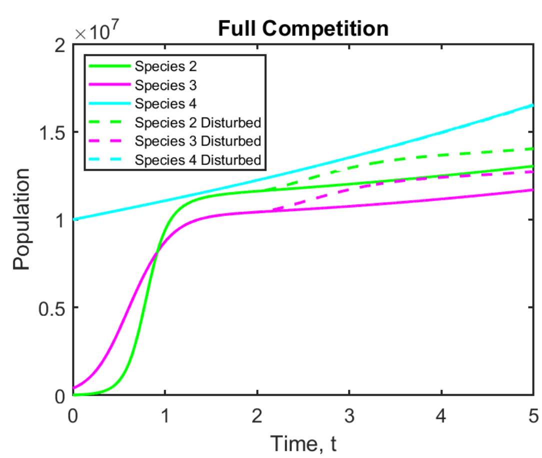


Figure 18: Disturbance introduce in control Lamprey population with 50:50 sex ratio

Here we can see that the effect of disturbance on species in the ecosystem as a result of the effect of disturbance on Lampreys. Species 2 plays the role of predator and is in direct competition with

Lampreys, and species 3 plays the role of host population and is being simulated independent of the parasite model hence species 3 is also competing for resources like oxygen, or space with Lampreys. Hence, a change in the population of Lampreys cause both Species 2 and 3 to increase. Species 4 is modelled as a plant eater and is source food for Species 3 and 4; however because of our values assumed in the B_{ij} matrix we do not see any significant changes in the population of this species.

Now we move to modelling the Lampreys in good and bad resources with their respective probability functions for the sex ratios:

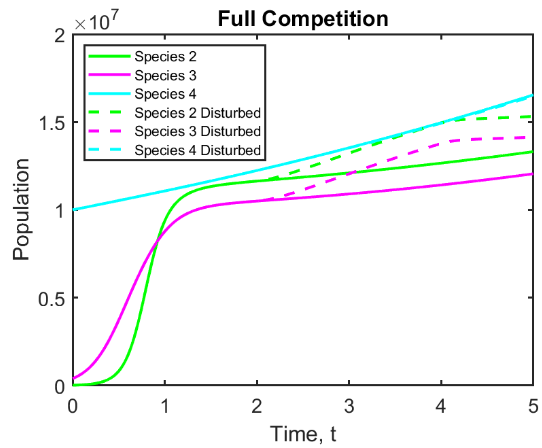


Figure 19: Disturbance in ecosystem in bad resource environment

In the bad resource environment, as shown by results in section 5.2, we find that total recovery is slower than in good. Hence as the male population decreases more in this disturbance we see an increased growth in species 3 and species 2. However, species 4 remains undisturbed and shows no change in growth rate.

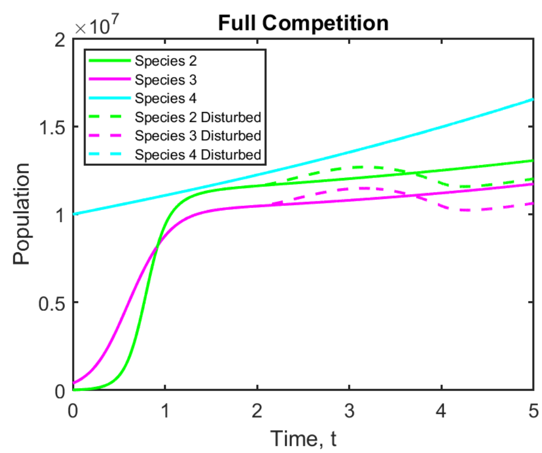


Figure 20: Disturbance in ecosystem in good resource environment

Again as seen in section 5.2, the recovery rate of Lampreys in a good resource environment is quicker than in bad. Hence the change in the growth rate of species 3 and 2 is not that significant. Moreover, because of the B_{ij} matrix values they even fall below its undisturbed population levels as the the disturbance effects Species 5 which is not shown in the graph because of a larger scale requirement to present it all in one picture.

However, based on these simulations we can conclude that disturbing the lampreys population does affect the stability of the ecosystem immensely and it may also in some cases harm the species that the disruptive measures are trying to protect from the parasitical behaviour of lampreys as demonstrated by figure 20.

6 Sensitivity Analysis

In this section we perform sensitivity analysis on two of our main parameters that we have determined in this model. The intrinsic growth rate of male and female adult lampreys, and the coefficients in the B_{ij} matrix.

First we start by varying our intrinsic growth rate parameters and the results are given below:

Table 5: Sensitivity Analysis of Intrinsic Growth Rate of Sea Lampreys

Change in r_m and r_f	**End of Year 1**	**End of Year 5**
-30%	-1.29%	+6.51%
-20%	-0.79%	+5.99%
-10%	-0.41%	+5.55%
0%	0%	0%
+10%	+1.04%	+4.49%
+20%	+1.29%	+3.99%
+30%	+1.50%	+3.48%

Here we can see that at the end of year 1 their is a approximate change of 1% even when our parameters are changed by around 30%. However, when we move to end of year 5 the fluctuations increase. This shows that our model is robust in a short term basis; however on a longer term it fluctuates more.

Table 6: Population changes for different multipliers of β_{ij}

	Percentage Change in Population (Year 1)				Percentage Change in Population (Year 5)			
M	Species 2	Species 3	Species 4	Species 5	Species 2	Species 3	Species 4	Species 5
1.1	34.18%	24.07%	1.37%	0.56%	26.42%	23.14%	6.67%	1.63%
1.2	64.13%	47.38%	2.79%	1.10%	65.52%	57.86%	13.56%	3.16%
1.3	96.38%	71.55%	4.25%	1.63%	128.32%	113.91%	20.11%	4.70%
0.9	-44.64%	-23.92%	-1.31%	-0.57%	-19.21%	-16.51%	-6.22%	-1.77%
0.8	-79.01%	-45.58%	-2.56%	-1.15%	-34.10%	-29.08%	-11.88%	-3.69%
0.7	-92.74%	-63.35%	-3.80%	-1.90%	-49.62%	-43.65%	-19.54%	-5.60%

However, for the B_{ij} matrix, we can see that the fluctuations are much more than compared to the intrinsic growth rate, hence our model is not that robust when it comes to modelling inter species competition.

7 Strengths and Weaknesses

7.1 Strengths

- **Universality**: Our model is based on a very flexible logistic growth equations and Lotka-Voltera competition models, hence to adapt it to any other species that exhibits this sex ratio behaviour we only need initial data values and this would be sufficient to present a reasonable and robust solution.
- **Excellent Cost-Benefit Ratio**: Our model has very modest data requirements and gives critical results for management and prediction of the Lamprey population and its ecosystem as well
- **Accuracy**: The results given by our model are in line with previous literature and common sense.
- **Stability**: Since we use the fifth order Runge-Kutte method our models results are robust and accurate.
- **Flexibility**: Our model is extremely dynamic and can easily be altered to simulate extreme scenarios and give a useful estimate of how that scenario would look like.

7.2 Weaknesses

- **Weak Estimated Parameters**: The parameters used for the models are estimated by considering different researches and literature, which may not be consistent with the actual situation in nature.

- **limited inter and intra species interactions**: Our models only assume that species interactions are based on competition for resources and parasitism by sea lampreys.
- **Lack of age structure**: Our model ignores the size and structure and dynamics of different parts of the population.
- **Lack of time Delays**: Our model does not incorporate time delays between reproduction and recruitment.

8 Conclusion

We started by modelling the population of lamprey's using modified logistic growth models for both its larval stage and adult stage. We then used that model as a foundation to develop two models. One of them is to model the Parasite - Host relationship, where Lampreys are the parasites. Then we proceed to model a competition model that simulates inter - species competition for resources. Both of these models help us examine the role of Lamprey in an ecosystem, and assess how their ability to alter their sex ratio affects the stability of the system, and offer insight into what species are at an advantage or disadvantage because of this ability of theirs. Than we end our report by doing a sensitivity analysis of the most important parameters that we assumed to check the robustness of our model.

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