CLASSIFICATION OF SIGNALS

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Classification of Signals

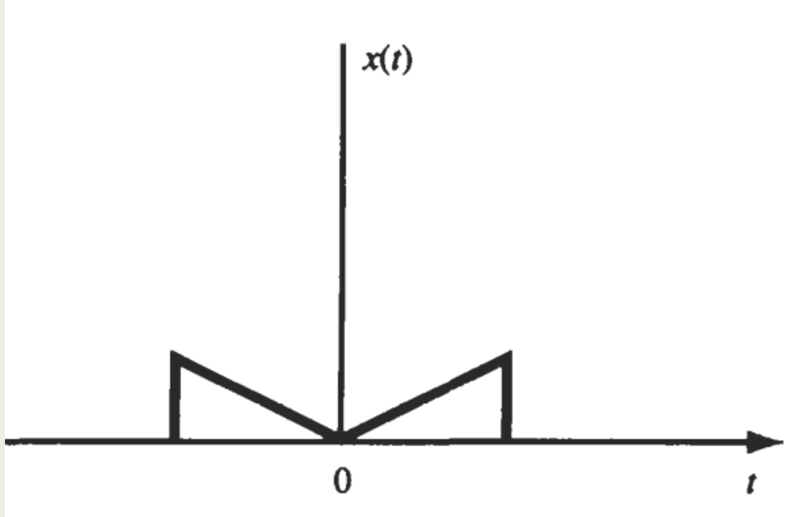
- Based upon their characteristics and nature of availability, the signals can be classified
  - *Continuous Time Signal*
  - *Discrete Time Signal*
  - *Even Signal*
  - *Odd Signal*
  - *Periodic Signal*
  - *Aperiodic Signal*
  - *Energy Signal*
  - *Power Signal*
  - *Deterministic Signal*
  - *Random Signal*

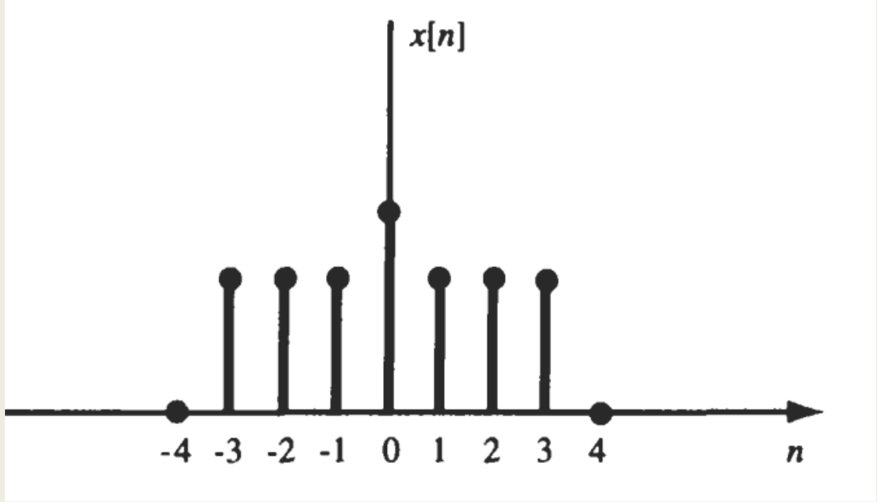
1. Even and Odd Signals

- **Even Signal:** A signal is said to be *even signal* if inversion of time-axis does not change the amplitude.
- Condition for signal to be even: $\begin{cases} x(t) = x(-t) \\ x[n] = x[-n] \end{cases}$
- Even signals are known as symmetric signals.

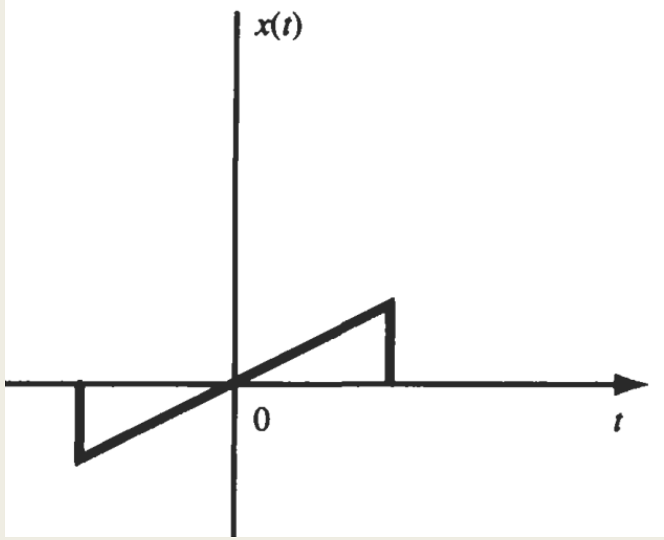
- **Odd Signal:** A signal is said to be *odd signal* if inversion of time-axis also inverts the amplitude.
- Condition for signal to be odd: $\begin{cases} x(t) = -x(-t) \\ x[n] = -x[-n] \end{cases}$
- Odd signals are known as anti-symmetric signals.

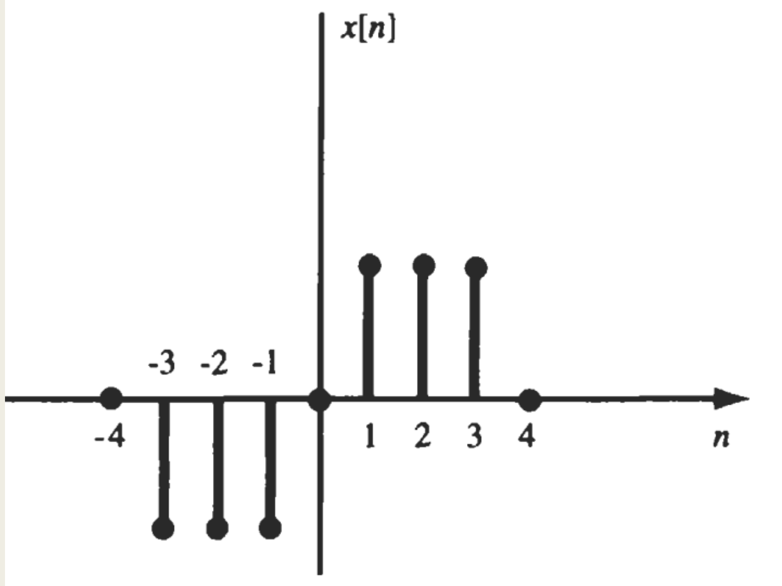
Examples of Even Signals





Examples of Odd Signals





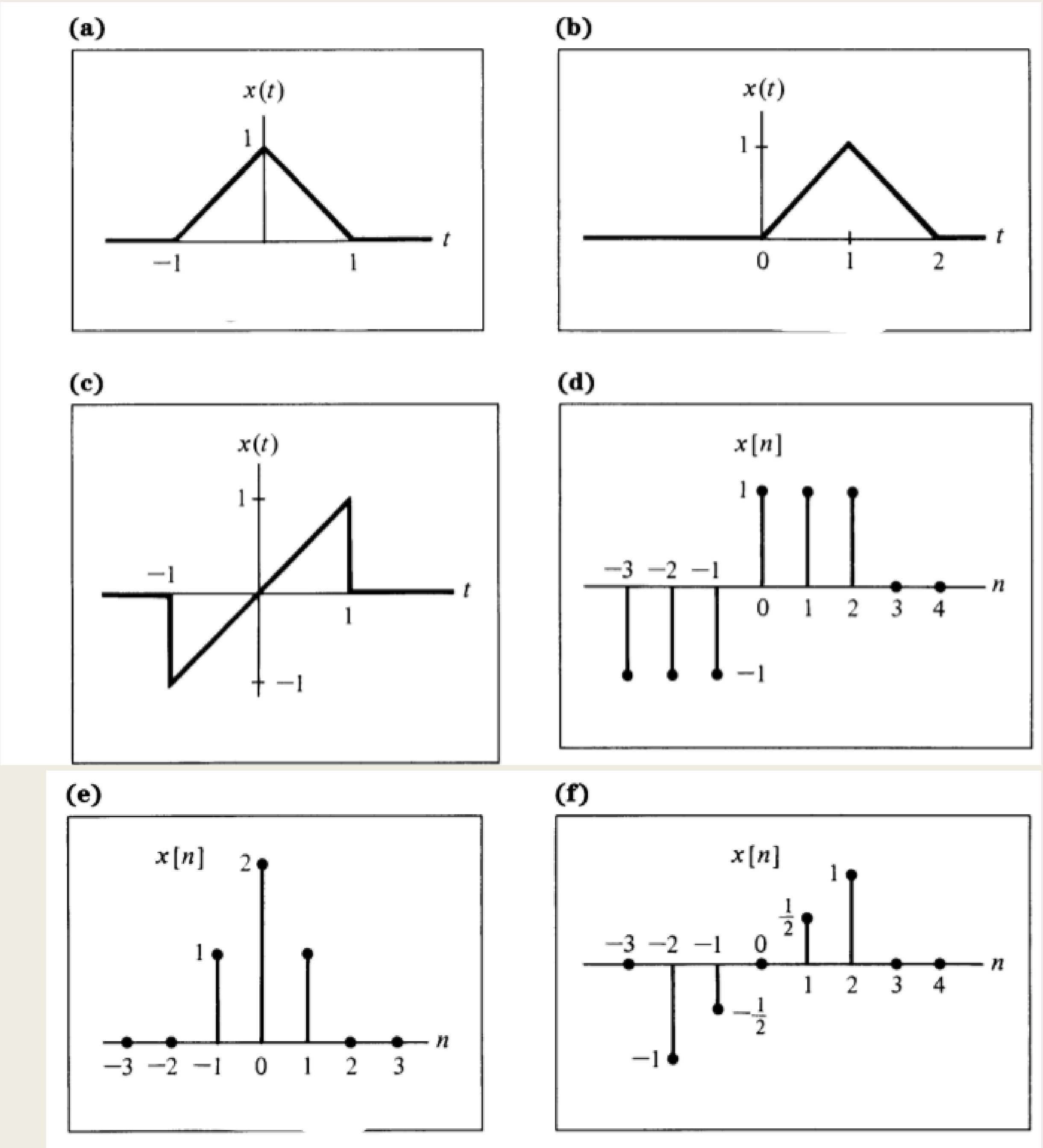
- Any signal can be expressed as a sum of two parts, one of which is even part and the other is odd part.
- $x(t) = x_e(t) + x_o(t)$

- How can we calculate even and odd parts of any signal???
- For CT Signal:
  - Even part: $x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$
  - Odd part: $x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$
- For DT Signal:
  - Even part: $x_e(n) = \frac{1}{2}\{x(n) + x(-n)\}$
  - Odd part: $x_o(n) = \frac{1}{2}\{x(n) - x(-n)\}$

Exp 5.1:

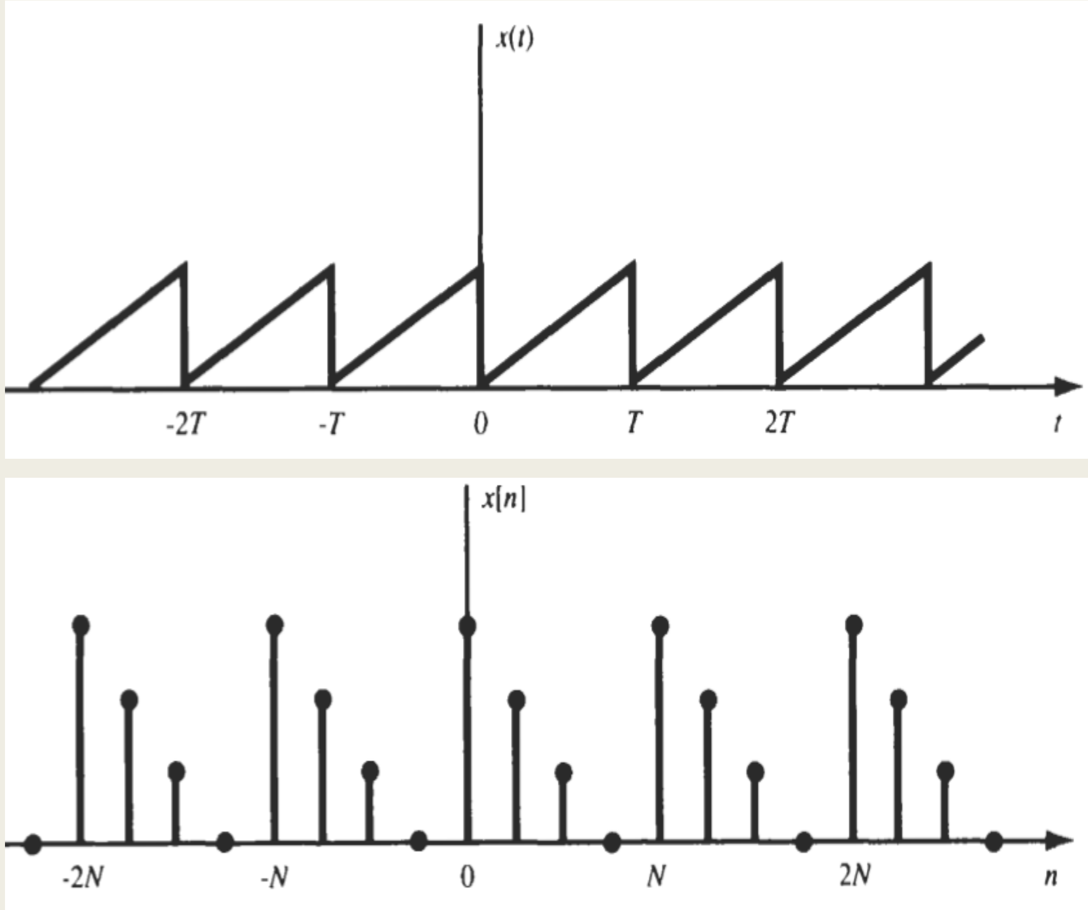
Determine whether signals shown are even, odd or neither. Also show that $x(t) = x_e(t) + x_o(t)$

- (a) Even
- (b) Neither even nor odd
- (c) Odd
- (d) Neither even nor odd
- (e) Even
- (f) Odd



2. Periodic and Aperiodic Signals

- A signal is said to be periodic if it repeats itself after a regular interval of time
- Periodic Signal: A signal is said to be periodic with time T if there is a positive nonzero value of T for which
  - $x(t+T) = x(t) \quad for\ all\ t$
  - $x[n+N] = x[n] \quad for\ all\ n$
  - T and N are known as fundamental time period (smallest time period for which above equations hold)
- Aperiodic Signal: Signals which are not periodic are known as aperiodic signals.



Examples of periodic signals.

Periodicity for CT signals

- $x(t)$ is periodic if it repeats itself for the **smallest positive period of time $\boldsymbol{T}$.**

- If $x(t)$ is a summation of two signals $x_1(t)$ and $x_2(t)$ then the overall system is periodic if and only if;

$$\boxed{\frac{T_1}{T_2} = \frac{q}{p} = rational\ number}$$

Rational Number: Numbers which can be written in the form of q/p where both q and p are integer numbers.

- The overall/ fundamental time period is obtained by

$$\boxed{T_o = pT_1 = qT_2}$$

Exp 5.3: Find periodicity

- i) $x(t) = \cos(5\pi t)$
- $\omega = 2\pi f$
- $\frac{\omega}{2\pi} = f$
- $\frac{5\pi}{2\pi} = f$
- $f = \frac{5}{2}$
- $T = \frac{1}{f}$
- $T = \frac{2}{5}$
- $T = 0.4$ Smallest positive number so it is a periodic signal with FTP (fundamental time period) equal to 0.4

- ii) $x(t) = \sin(5t)$
- $\omega = 2\pi f$
- $\frac{\omega}{2\pi} = f$
- $\frac{5}{2\pi} = f$
- $f = \frac{5}{2\pi}$
- $T = \frac{1}{f}$
- $T = \frac{2\pi}{5}$
- $T = 1.25663706144$ Smallest positive number so it is a periodic signal with FTP (fundamental time period) equal to 1.25663706144

- iii) $x(t) = \cos(5\pi t) - \sin(6\pi t)$
- **Step 1: Determine individual TP of all components**
- $\omega_1 = 2\pi f_1$
- $\frac{\omega_1}{2\pi} = f_1$
- $\frac{5\pi}{2\pi} = f_1$
- $f_1 = \frac{5\pi}{2\pi}$
- $T_1 = \frac{1}{f_1}$
- $T_1 = \frac{2\pi}{5\pi}$
- $T_1 = \frac{2}{5}$
- $T_1 = 0.4$ Smallest positive number so it is a periodic signal with TP (time period) equal to 0.4

- $\omega_2 = 2\pi f_2$
- $\frac{\omega_2}{2\pi} = f_2$
- $\frac{6\pi}{2\pi} = f_2$
- $f_2 = \frac{6\pi}{2\pi}$
- $T_2 = \frac{1}{f_2}$
- $T_2 = \frac{2\pi}{6\pi}$
- $T_2 = \frac{1}{3}$
- $T_2 = 0.33$ Smallest positive number so it is a periodic signal with TP (time period) equal to 0.33

- **Step 2: Determine whether $x(t)$ is periodic or not**
- The signal $x(t)$ is periodic only when the ratio is a rational number
- $$\frac{T_1}{T_2} = \frac{2/5}{1/3} = \frac{6}{5}$$
- Ratio of $\frac{T_1}{T_2}$ is a rational number so $x(t)$ is a periodic signal
- **Step 3: Determine overall FTP**
- The overall FTP is given as
- $$T = pT_1 = qT_2$$
- $T = 5T_1 = 6T_2$
- $T = 5\left(\frac{2}{5}\right) = 6\left(\frac{1}{3}\right)$
- $T = 2$

P.P 5.1

- Check whether the given signals are periodic or not. If periodic, then determine their FTP.
- $(i)\ x(t) = \cos\left(\frac{\pi t}{4}\right) + \sin(t)$
- $(ii)\ x(t) = \cos\left(\sqrt{2}t\right) + \cos(t)$

- Note: π and square root ($\sqrt{\ }$) are not rational numbers.

Periodicity for DT signals

- A discrete-time signal $x(n)$ is said to be periodic for all values of n only if its frequency is rational.

  - $f = \frac{k}{N}$

- where N is the fundamental time period and k is some integer.

- If $x(n)$ is a summation of two signals $x_1(n)$ and $x_2(n)$ then the overall system is periodic if N_1 and N_2 are integer numbers

- The overall/ fundamental time period is obtained by

  - $N_o = LCM(N_1, N_2)$

Exp 5.4: Find periodicity

- (i) $\cos(0.1\pi n)$
- $\omega = 0.1\pi$
- $\frac{\omega}{2\pi} = f$
- $\frac{0.1\pi}{2\pi} = f$
- $f = \frac{1}{20}$
- $k = 1$
- $N = 20$
- Periodic sequence with FTP = 20

- (ii) $\cos(\frac{n}{10})\cos(\frac{n\pi}{10})$
- $\omega_1 = {}^{1}/_{10}$
- $\frac{\omega_1}{2\pi} = f_1$
- $\frac{{}^{1}/_{10}}{2\pi} = f_1$
- $f_1 = \frac{1}{20\pi}$
- $N_1 = 20\pi$
- N_1 is not an integer value so it is not a periodic signal.
- $\omega_2 = {}^{\pi}/_{10}$
- $\frac{\omega_2}{2\pi} = f_2$
- $\frac{1}{20} = f_2$
- $f_2 = \frac{1}{20}$
- $N_2 = 20$
- N_2 is an integer value so it is a periodic signal.

Therefore, the given signal is non=periodic

EXAMPLE 1.7 Determine whether the following discrete-time signals are periodic or not. If periodic, determine the fundamental period.

(a) $\sin(0.02\pi n)$ (b) $\sin(5\pi n)$

(c) $\cos 4n$ (d) $\sin \frac{2\pi n}{3} + \cos \frac{2\pi n}{5}$

(e) $\cos\left(\frac{n}{6}\right)\cos\left(\frac{n\pi}{6}\right)$ (f) $\cos\left(\frac{\pi}{2} + 0.3n\right)$

(g) $e^{j(\pi/2)n}$ (h) $1 + e^{j2\pi n/3} - e^{j4\pi n/7}$

Solution:

(a) Given $x(n) = \sin(0.02\pi n)$

Comparing it with $x(n) = \sin(2\pi fn)$

we have $0.02\pi = 2\pi f$ or $f = \frac{0.02\pi}{2\pi} = 0.01 = \frac{1}{100} = \frac{k}{N}$

Here f is expressed as a ratio of two integers with $k = 1$ and $N = 100$. So it is rational. Hence the given signal is periodic with fundamental period $N = 100$.

(b) Given $x(n) = \sin(5\pi n)$

Comparing it with $x(n) = \sin(2\pi fn)$

we have $2\pi f = 5\pi$ or $f = \frac{5}{2} = \frac{k}{N}$

Here f is a ratio of two integers with $k = 5$ and $N = 2$. Hence it is rational. Hence the given signal is periodic with fundamental period $N = 2$.

(c) Given $x(n) = \cos 4n$

Comparing it with $x(n) = \cos 2\pi fn$

we have $2\pi f = 4$ or $f = \dfrac{2}{\pi}$

Since $f = (2/\pi)$ is not a rational number, $x(n)$ is not periodic.

(d) Given $x(n) = \sin \dfrac{2\pi n}{3} + \cos \dfrac{2\pi n}{5}$

Comparing it with $x(n) = \sin 2\pi f_1 n + \cos 2\pi f_2 n$

we have $2\pi f_1 = \dfrac{2\pi}{3}$ or $f_1 = \dfrac{1}{3} = \dfrac{k_1}{N_1}$

$\therefore \; N_1 = 3$

and $2\pi f_2 n = \dfrac{2\pi}{5}$ or $f_2 = \dfrac{1}{5}$

$\therefore \; N_2 = 5$

Since $\dfrac{N_1}{N_2} = \dfrac{3}{5}$ is a ratio of two integers, the sequence $x(n)$ is periodic. The period of $x(n)$ is the LCM of N_1 and N_2. Here LCM of $N_1 = 3$ and $N_2 = 5$ is 15. Therefore, the given sequence is periodic with fundamental period $N = 15$.

(e) Given $x(n) = \cos\left(\dfrac{n}{6}\right) \cos\left(\dfrac{n\pi}{6}\right)$

Comparing it with $x(n) = \cos(2\pi f_1 n) \cos(2\pi f_2 n)$

we have $2\pi f_1 n = \dfrac{n}{6}$ or $f_1 = \dfrac{1}{12\pi}$

which is not rational.

And $2\pi f_2 n = \dfrac{n\pi}{6}$ or $f_2 = \dfrac{1}{12}$

which is rational.

Thus, $\cos(n/6)$ is non-periodic and $\cos(n\pi/6)$ is periodic. $x(n)$ is non-periodic because it is the product of periodic and non-periodic signals.

(f) Given $x(n) = \cos\left(\dfrac{\pi}{2} + 0.3n\right)$

Comparing it with $x(n) = \cos(2\pi fn + \theta)$

we have $2\pi fn = 0.3n$ and phase shift $\theta = \dfrac{\pi}{2}$

$$\therefore \qquad f = \frac{0.3}{2\pi} = \frac{3}{20\pi}$$

which is not rational.

Hence, the signal $x(n)$ is non-periodic.

(g) Given $$x(n) = e^{j(\pi/2)n}$$

Comparing it with $$x(n) = e^{j2\pi fn}$$

we have $$2\pi f = \frac{\pi}{2} \quad \text{or} \quad f = \frac{1}{4} = \frac{k}{N}$$

which is rational.

Hence, the given signal $x(n)$ is periodic with fundamental period $N = 4$.

(h) Given $$x(n) = 1 + e^{j2\pi n/3} - e^{j4\pi n/7}$$

Let $$x(n) = 1 + e^{j2\pi n/3} - e^{j4\pi n/7} = x_1(n) + x_2(n) + x_3(n)$$

where $$x_1(n) = 1,\ x_2(n) = e^{j2\pi n/3} \quad \text{and} \quad x_3(n) = e^{j4\pi n/7}$$

$x_1(n) = 1$ is a d.c. signal with an arbitrary period $N_1 = 1$

$$x_2(n) = e^{j2\pi n/3} = e^{j2\pi f_2 n}$$

$$\therefore \qquad \frac{2\pi n}{3} = 2\pi f_2 n \quad \text{or} \quad f_2 = \frac{1}{3} = \frac{k_2}{N_2} \quad \text{where } N_2 = 3$$

Hence $x_2(n)$ is periodic with period $N_2 = 3$.

$$x_3(n) = e^{j4\pi n/7} = e^{j2\pi f_3 n}$$

$$\therefore \qquad \frac{4\pi n}{7} = 2\pi f_3 n \quad \text{or} \quad f_3 = \frac{2}{7} = \frac{k_3}{N_3} \quad \text{where } N_3 = \frac{7}{2}$$

Now, $$\frac{N_1}{N_2} = \frac{1}{3} = \text{Rational number}$$

$$\frac{N_1}{N_3} = \frac{1}{7/2} = \frac{2}{7} = \text{Rational number}$$

The LCM of $$N_1, N_2, N_3 = \frac{7}{2} \times 3 = \frac{21}{2}$$

$\therefore$ The given signal $x(n)$ is periodic with fundamental period $N = 10.5$.

3. Energy and Power Signals

Energy of a signal is defined as the area under the square of the magnitude of the signal .

The energy of a signal x(*t*) is :

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

The units of signal energy depends on the unit of the signal .
If the signal unit is volt (V) , the energy of that signal is expressed in V^2.s .

Some signals have infinite signal energy . In that case it is more convenient to deal with **average signal power** .

The average power of a signal x(*t*) is :

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

For a periodic signal x(*t*) , the average signal power is :

$$P_x = \frac{1}{T} \int_{T} |x(t)|^2 \, dt$$

where *T* is any period of the signal .

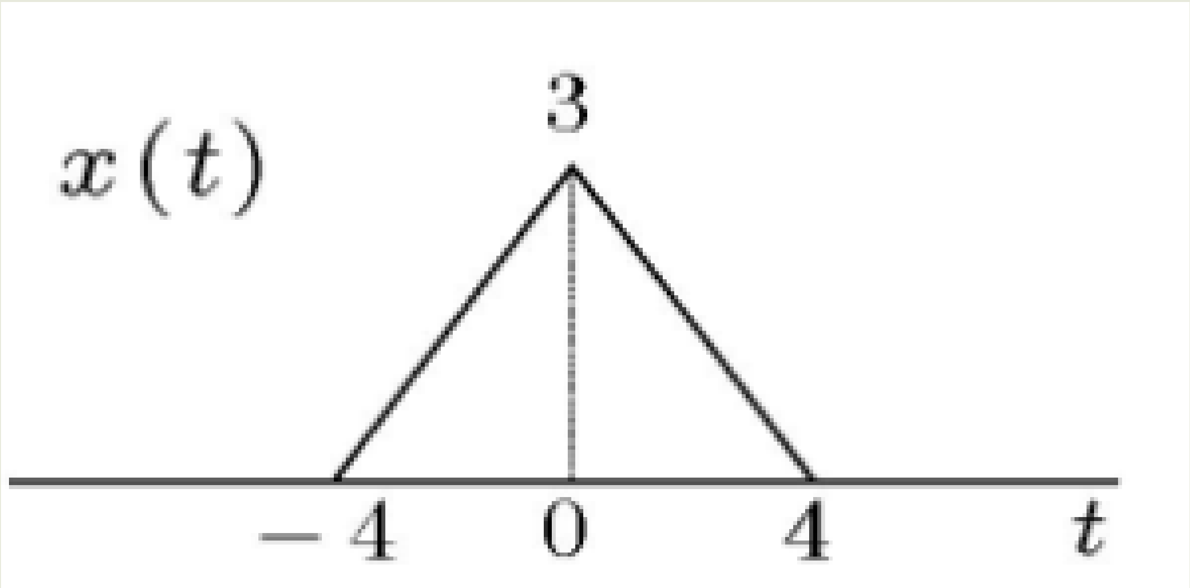
A signal with finite energy is called an **energy signal** .

$$0 < E < \infty \quad (P = 0)$$

A signal with infinite energy and finite average signal power is called a **power signal** .

$$0 < P < \infty \quad (E = \infty)$$

Example 1: Consider the signal given below. Is this power or energy type signal?



$$x(t) = \begin{cases} 3\left(1 - \frac{t}{4}\right) & \text{if } 0 \le t \le 4 \\ 3\left(1 + \frac{t}{4}\right) & \text{if } -4 \le t \le 0 \\ 0 & \text{otherwise} \end{cases}$$

Therefore,

$$\begin{aligned} E_x &= \int_{-4}^{4} |x(t)|^2 dt \\ &= \int_{-4}^{4} x^2(t) dt \\ &= 9 \int_{-4}^{0} (1 + \frac{t}{4}))^2 dt + 9 \int_{0}^{4} (1 - \frac{t}{4}))^2 dt \\ &= 9 \frac{(1 + \frac{t}{4})^3}{\frac{3}{4}} \Big|_{-4}^{0} + 9 \frac{(1 - \frac{t}{4})^3}{\frac{-3}{4}} \Big|_{0}^{4} \\ &= 9 \frac{4}{3} + 9 \frac{4}{3} \\ &= 24 \end{aligned}$$

So, $x(t)$ is an **Energy Signal**.

Example 2: Determine whether the signal x (t) described by :

x (t) = e^{-at} u (t) , a > 0 is a power signal or energy signal or neither .

Ans.

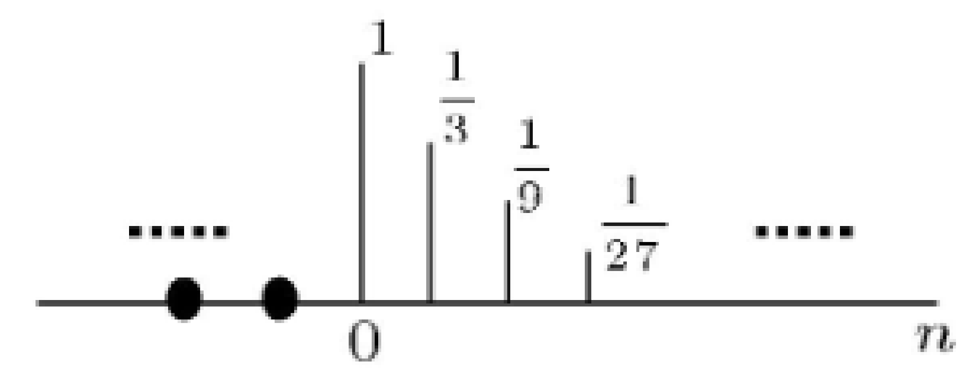
x(t) is a non-periodic signal .

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{0}^{\infty} e^{-2at}\, dt = \left.\frac{e^{-2at}}{-2a}\right|_{0}^{\infty} = \frac{1}{2a} \,(\mathit{finite}\ ,\ \mathit{positive})$$

The energy is finite and deterministic .
Hence , x(t) is an energy signal .

Example 4: Compute the energy of the signal $x[n]$ given by

$$x[n] = \begin{cases} \left(\frac{1}{3}\right)^n & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



$$E_x = \sum_{-\infty}^{\infty} x^2[n] = \sum_{0}^{\infty} x^2[n] = \sum_{0}^{\infty} \left(\left(\frac{1}{3}\right)^n\right)^2$$

$$= \sum_{0}^{\infty} \left(\frac{1}{3}\right)^{2n} = \sum_{0}^{\infty} \left(\frac{1}{9}\right)^n = \frac{1}{1-\frac{1}{9}}$$

$$= \frac{9}{8}$$

Thank You !!!