day / date

Lec#9.

4.2 Fourier Transform for Periodic Signals:

Basic formula To remember:

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0) \quad \text{--- } (1)$$

The Fourier Transform of Periodic Signals is calculated using Fourier Series Coefficients (a_k) and is drawn in form of ~~Sam~~ Impulses $\delta(\omega)$.

Example 4.7)

$$x(t) = \sin\omega_0 t$$

as we have already calculated previously the F.S.C (a_k) for $\sin\omega_0 t$, i-e

$$a_1 = \frac{1}{2j} \quad , \quad a_{-1} = \frac{-1}{2j}$$

$$a_k = 0 \text{ when } k \neq 1, -1$$

Now how To calculate it's Fourier Transform.

using formula # (1)

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

As for $\sin\omega_0 t$, k exists only for $k = -1, 1$, therefore:-

$$X(j\omega) = \sum_{k=-1}^{1} 2\pi a_k \delta(\omega - k\omega_0)$$

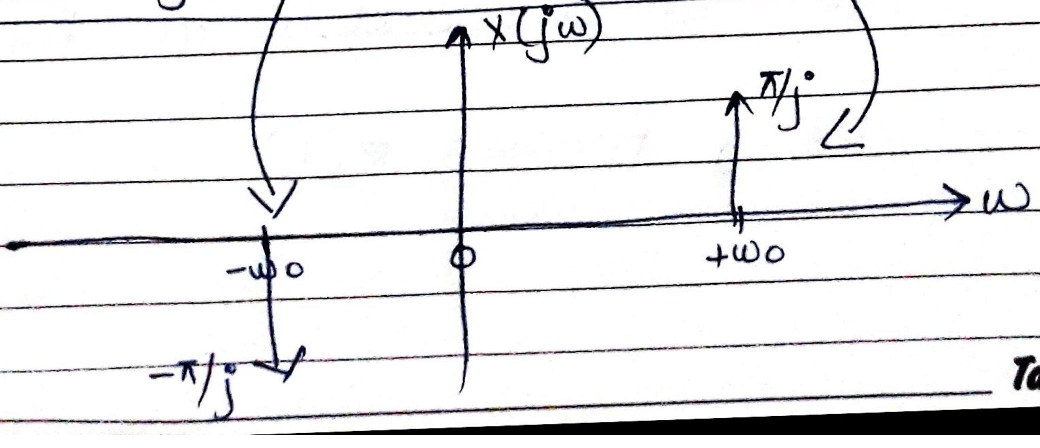
$$X(j\omega) = 2\pi a_{-1} \delta(\omega - (-1)\omega_0) + 0 + 2\pi a_{+1} \delta(\omega - (+1)\omega_0)$$

$$X(j\omega) = 2\pi a_{-1} \delta(\omega + \omega_0) + 2\pi a_1 \delta(\omega - \omega_0)$$

here $a_{-1} = -\frac{1}{2j}$ and $a_1 = \frac{1}{2j}$

$$X(j\omega) = \cancel{2}\pi\left(-\frac{1}{\cancel{2}j}\right)\delta(\omega + \omega_0) + \cancel{2}\pi\left(\frac{1}{\cancel{2}j}\right)\delta(\omega - \omega_0)$$

$$\boxed{X(j\omega) = -\frac{\pi}{j}\delta(\omega + \omega_0) + \frac{\pi}{j}\delta(\omega - \omega_0)} \quad \text{---}*$$



Please apply the Same procedure to calculate the Fourier Transform $X(j\omega)$ of a signal $x(t) = \cos\omega_0 t$ (a periodic Signal).

Example 4.8

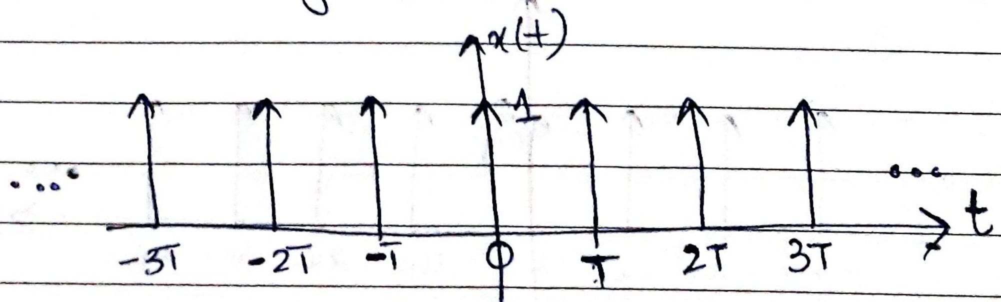
Consider another periodic Signal

$$x(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT)$$

The Signal can be expanded like:-

$$x(t) = \dots + \underset{k=-2}{\delta(t+2T)} + \underset{k=-1}{\delta(t+T)} + \underset{k=0}{\delta(t)} + \underset{k=1}{\delta(t-T)} + \underset{k=2}{\delta(t-2T)} + \dots$$

The Signal $x(t)$, can be drawn like:-



so it is a periodic Signal with $\boxed{F.P = T}$

where, we already know that

a_k for $\delta(t)$ is

$$x(t) = \delta(t) \xrightarrow{F.S.} a_k = \frac{1}{T}$$

(for all values of k)

Applying formula:-

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} 2\pi\left(\frac{1}{T}\right)\delta(\omega - k\omega_0)$$

$$X(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta(\omega - k\omega_0) \longrightarrow *$$