SYSTEMS AND CLASSIFICATION OF SYSTEMS

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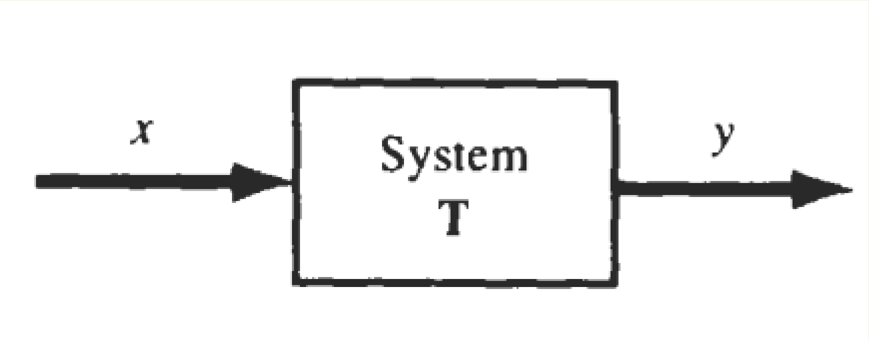
System

- A system may be defined as a set of elements or functional blocks which are connected together and produces an output in response to the input signal.

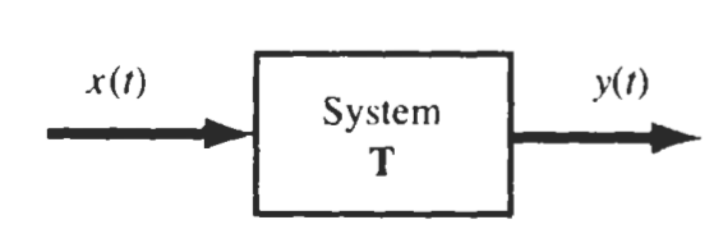
- Let x and y be the input and output signals, respectively of a system. Then system is viewed as transformation (or mapping) of x into y.

- It is represented as $y = Tx$ where T is defined as some well defined rule to transform the x into y.

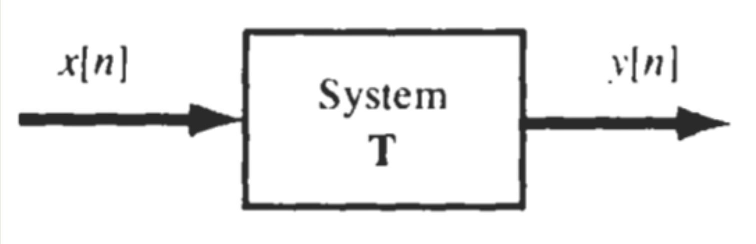


Types of Systems

- Continuous Time System
  - Systems in which input and output, both signals are continuous in nature.
  - $x(t) \rightarrow y(t)$
  - e.g. Audio and video amplifier, power supplies etc

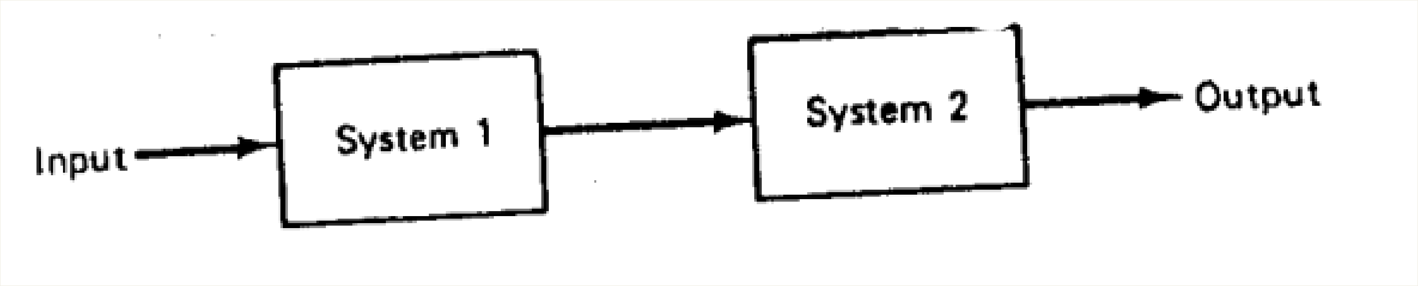


- Discrete Time System
  - Systems in which input and output, both signals are discrete in nature.
  - $x[n] \rightarrow y[n]$
  - e.g. microprocessors, shift registers

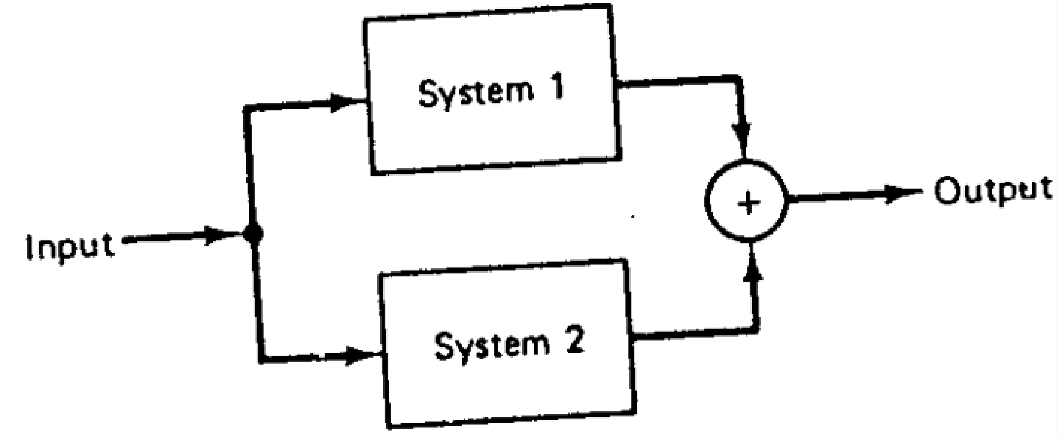


Interconnection of Systems

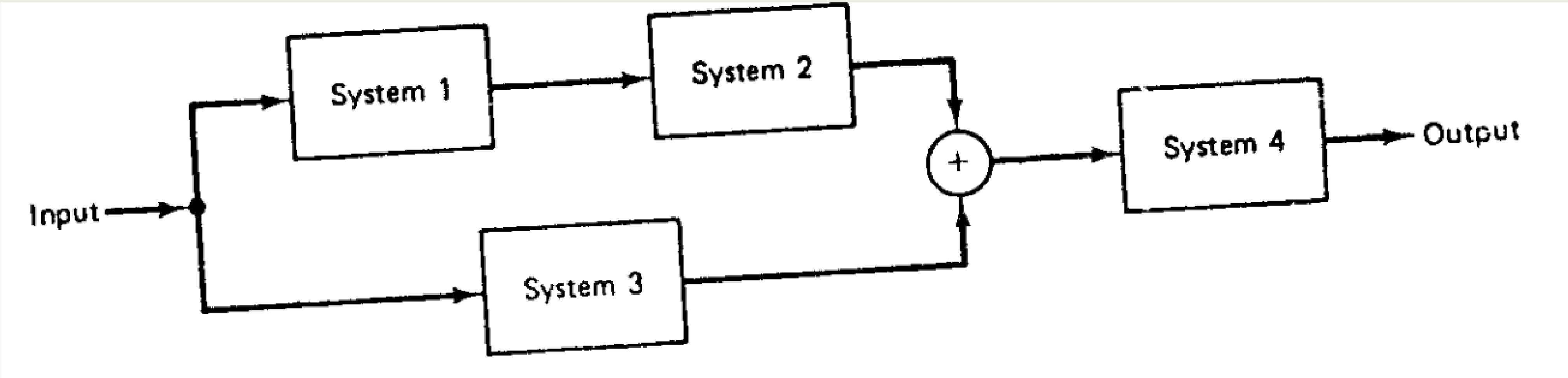
- Systems can be interconnected in multiple ways
  - 1) Series(Cascaded) Interconnection
    - Output of system 1 is input of other system
    - Overall impulse response is equal to convolution of individual impulse responses.



- 2) Parallel Interconnection
  - Same input is applied to all systems.
  - Overall impulse response is equal to addition of individual impulse responses

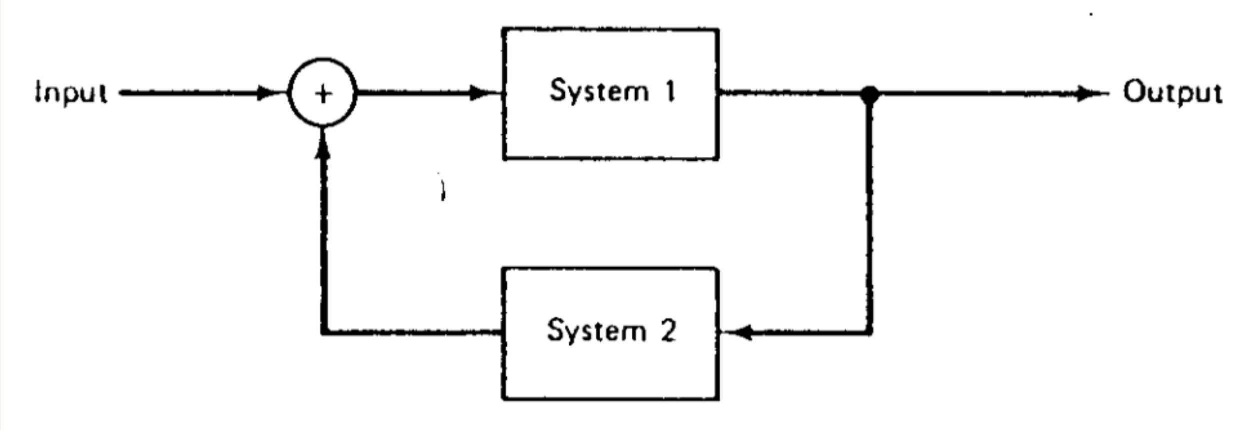


- 3) Series/Parallel Interconnection
  - Combination of series and parallel combination

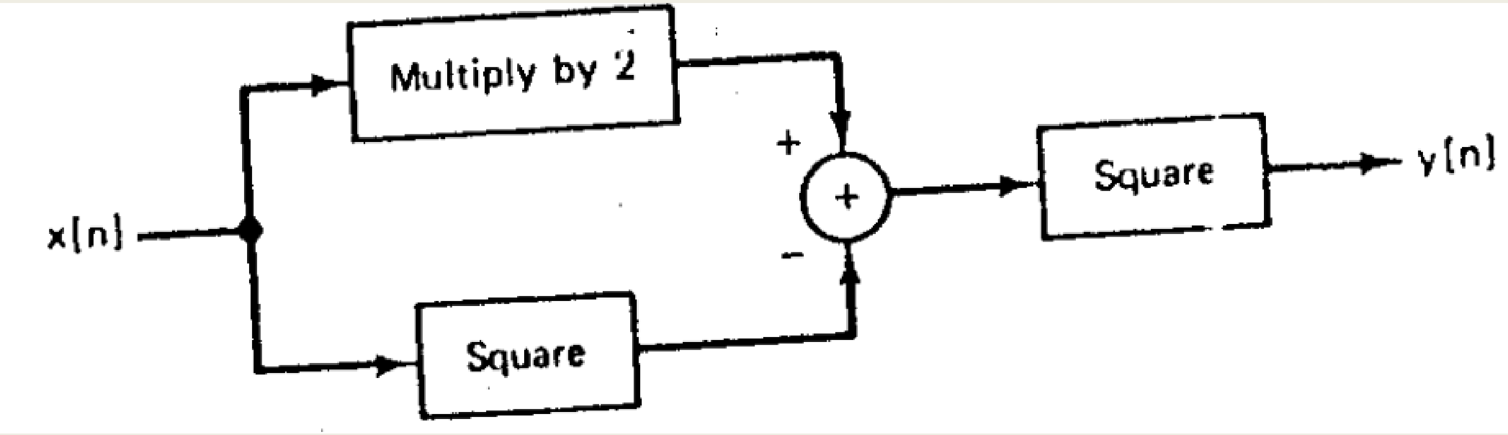


4) Feedback Interconnection

- Output of system 1 is the input to system 2, while the output of system 2 is fed back and added to the external input to produce the actual input to system 1.



Exp 7.1: Write relationship between input and output signals



- Multiple input $x[n]$ by 2 → $2x[n]$
- Square of input $x[n]$ → $x[n]^2$

- Relationship is: $y[n] = (2x[n] - x[n]^2)^2$

Classification of Systems

- i) Causal Systems and Non-causal Systems
- ii) Static Systems and Dynamic Systems
- iii) Time-invariant Systems and Time-variant Systems
- iv) Stable Systems and Unstable Systems
- v) Linear Systems and Non-linear Systems
- vi) Invertible Systems and Inverse Systems

1) Causal and Non-Causal Systems

- A system is said to be causal if the response or output does not begin before the input function is applied.

- In other words,
  - Response of the causal system to an input does not depend on future values of that input but depends only on the present and past values of the input.

Exp 7.2: Determine whether the systems are causal or non-causal

- (i) $\boldsymbol{y(t) = 0.2x(t) - x(t-1)}$
- **Solution:**
- Put different values of t in above equation to check whether it is causal or not
  - Put $t = 0$
    - $y(0) = 0.2x(0) - x(0-1) \Rightarrow y(0) = \underbrace{0.2x(0)}_{\text{Present value of input}} - \underbrace{x(-1)}_{\text{Past value of input}}$
  - Put $t = 1$
    - $y(1) = 0.2x(1) - x(1-1) \Rightarrow y(1) = 0.2x(1) - x(0)$
  - Put $t = -1$
    - $y(-1) = 0.2x(-1) - x(-1-1) \Rightarrow y(-1) = 0.2x(-1) - x(-2)$

- $y(t)$ is depending on previous and present values of $x(t)$ only so it is a causal system

- (ii) $\boldsymbol{y(t) = x(t+1) - x(t-1)}$
- **Solution:**
- Put different values of t in above equation to check whether it is causal or not
  - Put $t = 0$
    - Future value of input
    - $y(0) = x(0+1) - x(0-1) \Rightarrow y(0) = x(1) - x(-1)$
  - Put $t = 1$
    - Future value of input
    - $y(1) = x(1+1) - x(1-1) \Rightarrow y(1) = x(2) - x(0)$
  - Put $t = -1$
    - Future value of input
    - $y(-1) = x(-1+1) - x(-1-1) \Rightarrow y(-1) = x(0) - x(-2)$

- $y(t)$ is depending on future values of $x(t)$ only so it is a non-causal system

- (iii) $\boldsymbol{y(t) = x(t) cos\ (t+1)}$
- Solution:
- Put different values of t in above equation to check whether it is causal or not
    - Put $t=0$

      present value of input
        - $y(0) = x(0)\cos(0+1) \Rightarrow y(0) = x(0)\cos(1)$
    - Put $t=1$

      present value of input
        - $y(1) = x(1)\cos(1+1) \Rightarrow y(1) = x(1)\cos(2)$
    - Put $t=-1$

      present value of input
        - $y(-1) = x(-1)\cos(-1+1) \Rightarrow y(-1) = x(-1)\cos(0)$

- $y(t)$ is depending on present values of $x(t)$ only so it is a causal system

- (iv) $\boldsymbol{y(n) = x(n^2)}$
- **Solution:**
- Put different values of n in above equation to check whether it is causal or not
  - Put $n = 0$
    - $y(0) = x(0^2) \Rightarrow y(0) = x(0)$
  - Put $n = 1$
    - $y(1) = x(1^2) \Rightarrow y(1) = x(1)$
  - Put $n = -1$
    - $y(-1) = x(-1^2) \Rightarrow y(-1) = x(1)$

- $y(n)$ is depending on present and future values of $x(n)$ only so it is a non-causal system

- (v) $\boldsymbol{y(n) = 2x(n-1)}$
- Solution:
- Put different values of n in above equation to check whether it is causal or not
  - Put $n = 0$
    - $y(0) = 2x(0-1) \Rrightarrow y(0) = 2x(-1)$
  - Put $n = 1$
    - $y(1) = 2x(1-1) \Rrightarrow y(1) = 2x(0)$
  - Put $n = -1$
    - $y(-1) = 2x(-1-1) \Rrightarrow y(-1) = 2x(-2)$

- $y(n)$ is depending on previous values of $x(n)$ only so it is a causal system

2) Static (memoryless) and Dynamic (with memory) Systems

- Static systems are also known as memoryless systems
- Static systems contain no storage elements (thus, no integrals, derivatives or signal delays)
- A static or memoryless system is a system with an output signal whose values depends upon the present value of the input signal *only*. Otherwise the system is dynamic or with memory.

Exp 7.3: Determine whether the systems are static or dynamic

- (i) $\boldsymbol{y(t) = x(t)cos\ (t+1)}$
- **Solution:**
- Put different values of t in above equation to check whether it is static or not
  - Put $t = 0$
    - $y(0) = x(0)\cos(0+1) \Rightarrow y(0) = x(0)\cos(1)$
  - Put $t = 1$
    - $y(1) = x(1)\cos(1+1) \Rightarrow y(1) = x(1)\cos(2)$
  - Put $t = -1$
    - $y(-1) = x(-1)\cos(-1+1) \Rightarrow y(-1) = x(-1)\cos(0)$
- $y(t)$ is depending on present values of $x(t)$ only so it is a static system

- (ii) $\boldsymbol{y(t) = 0.2x(t) - x(t-1)}$
- **Solution:**
- Put different values of t in above equation to check whether it is static or not
  - Put $t = 0$
    - $y(0) = 0.2x(0) - x(0-1) \Rrightarrow y(0) = 0.2x(0) - x(-1)$
  - Put $t = 1$
    - $y(1) = 0.2x(1) - x(1-1) \Rrightarrow y(1) = 0.2x(1) - x(0)$
  - Put $t = -1$
    - $y(-1) = 0.2x(-1) - x(-1-1) \Rrightarrow y(-1) = 0.2x(-1) - x(-2)$

- $y(t)$ is depending on previous and present of $x(t)$ values only so it is a dynamic system

- (ii) $\boldsymbol{y(n) = x(n) - x(n-1)}$
- Solution:
- Put different values of n in above equation to check whether it is static or not
  - Put $n = 0$
    - $y(0) = x(0) - x(0-1) \Rrightarrow y(0) = x(0) - x(-1)$
  - Put $n = 1$
    - $y(1) = x(1) - x(1-1) \Rrightarrow y(1) = x(1) - x(0)$
  - Put $n = -1$
    - $y(-1) = x(-1) - x(-1-1) \Rrightarrow y(-1) = x(-1) - x(-2)$
- $y(n)$ is depending on previous and present of $x(n)$ values only so it is a dynamic system

3) Time Invariant and Time Variant Systems

- A system is **time invariant** if the time shift in the input signal results in corresponding time shift in the output.
- Let $y(t) = f[x(t)]$ i.e. $y(t)$ is response of $x(t)$.
- Then if $x(t)$ is delayed by time t_1 then output $y(t)$ will also be delayed by the same time. i.e.
- $f[x(t - t_1)] = y(t - t_1)$

Sr. #	Operation or system	Type
1	Bus and train arrival and departure times	Time Invariant
2	Rainfall per month	Time variant
3	Thermal noise in components	Time variant
4	Noise effects in radio communication	Time variant
5	File handling in C language	Time invariant
6	Printing documents by printer	Time invariant

- Time invariant systems are independent of time/day/year whereas time variant systems are effected at which time you are analyzing the system.

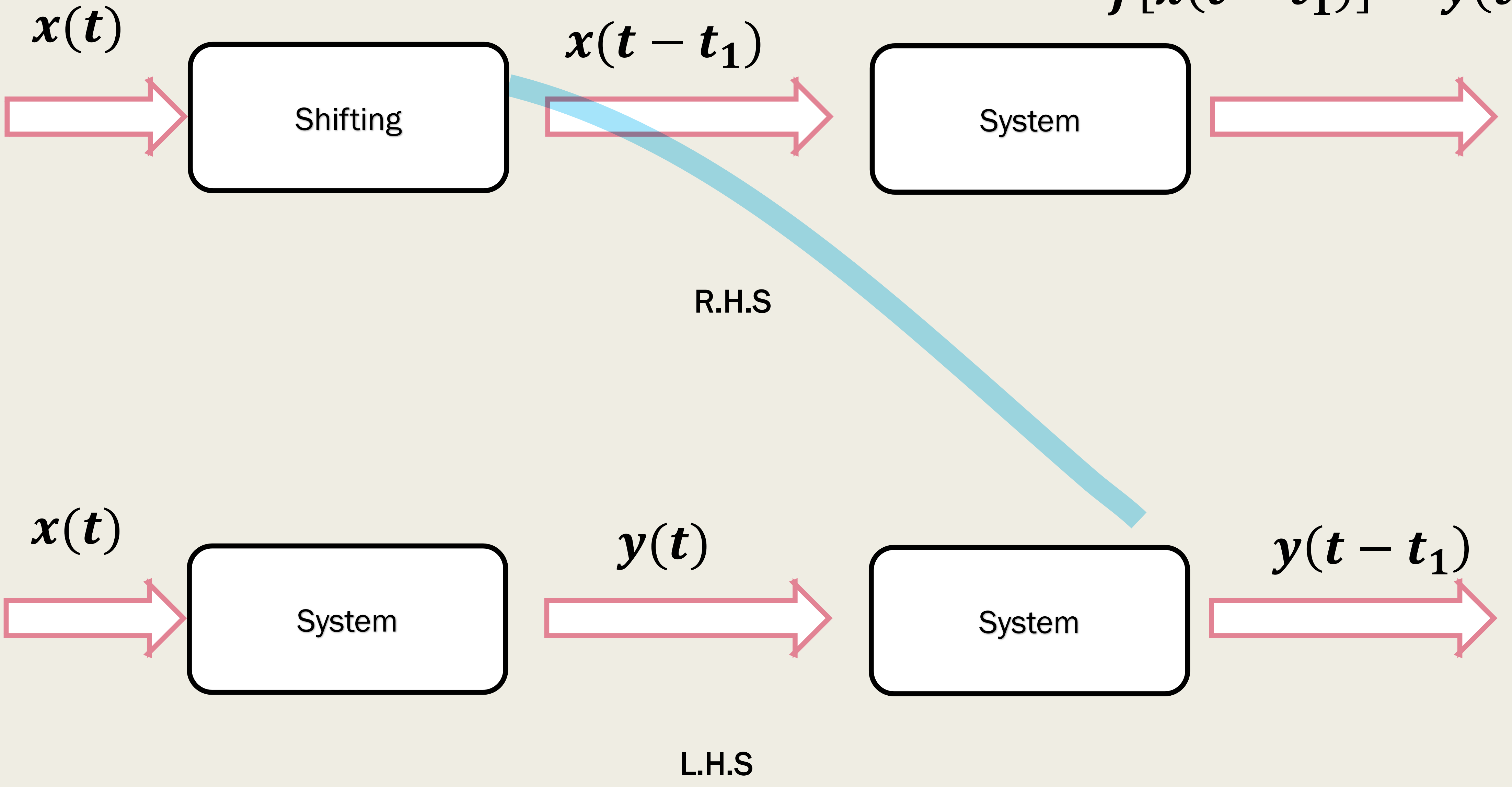
Steps to test for time invariance property

- Step 1: First delay the input $x(t)$ by t_1 i.e. $x(t - t_1)$ and then pass it through the system
  - $y(t, t_1) = f[x(t - t_1)]$

- Step 2: Then delay the output $y(t)$ itself by t_1 i.e. $y(t - t_1)$

- Step 3: Now compare both outputs. If
  - $y(t, t_1) \neq y(t - t_1)$ → Time variant
  - $y(t, t_1) = y(t - t_1)$ → Time invariant

$$f[x(t - t_1)] = y(t, t_1)$$



Exp 7.4: Determine whether the systems are time invariant or time variant

- $(i)\ y(t) = \sin x(t)$
- Step 1: First delay the input $x(t)$ by t_1 i.e. $x(t - t_1)$ and then pass it through the system
    - $y(t, t_1) = f[x(t - t_1)]$
    - $y(t, t_1) = \sin x(t - t_1)$
- Step 2: Now delay the output by t_1. Hence we have to replace t by $t - t_1$
    - $y(t - t_1) = \sin x(t - t_1)$
- Step 3: Compare $y(t, t_1)$ with $y(t - t_1)$
    - $y(t, t_1) = y(t - t_1)$
- Both are same so system is time invariant

- $(ii)\ y(t) = t\,x(t)$
- Step 1: First delay the input $x(t)$ by t_1 i.e. $x(t - t_1)$ and then pass it through the system
    - $y(t, t_1) = f[x(t - t_1)]$
    - $y(t, t_1) = tx(t - t_1)$
- Step 2: Now delay the output by t_1. Hence we have to replace t by $t - t_1$
    - $y(t - t_1) = (t - t_1)\,x(t - t_1)$
- Step 3: Compare $y(t, t_1)$ with $y(t - t_1)$
    - $y(t, t_1) \neq y(t - t_1)$
- Both are not same so system is *time variant*

- $(iii)\ y(t) = x(t)\cos 200\,\pi t$
- Step 1: Determine the output for delayed input i.e. . $x(t - t_1)$
  - $y(t, t_1) = f[x(t - t_1)]$
  - $y(t, t_1) = x(t - t_1)\cos 200\,\pi t$

- Step 2: Now delay the output by t_1. Hence we have to replace t by $t - t_1$
  - $y(t - t_1) = x(t - t_1)\cos 200\,\pi(t - t_1)$

- Step 3: Compare $y(t, t_1)$ with $y(t - t_1)$
  - $y(t, t_1) \neq y(t - t_1)$

- Both are not same so system is *time variant*

- $(iv)\ y(n) = x(n) - x(n-1)$
- Step 1: Determine the output for delayed input i.e. . $x(n-n_1)$
  - $y(n, n_1) = f[x(n-n_1)]$
  - $y(n, n_1) = x(n-n_1) - x(n-n_1-1)$
- Step 2: Now delay the output by n_1. Hence we have to replace n by $n-n_1$
  - $y(n-n_1) = x(n-n_1) - x(n-n_1-1)$
- Step 3: Compare $y(n, n_1)$ with $y(n-n_1)$
  - $y(n, n_1) = y(n-n_1)$
- Both are same so system is *time invariant*

- $(v)\ y(n) = nx(n)$
- Step 1: Determine the output for delayed input i.e. . $x(n - n_1)$
  - $y(n, n_1) = f[x(n - n_1)]$
  - $y(n, n_1) = nx(n - n_1)$
- Step 2: Now delay the output by n_1. Hence we have to replace n by $n - n_1$
  - $y(n - n_1) = (n - n_1)x(n - n_1)$
- Step 3: Compare $y(n, n_1)$ with $y(n - n_1)$
  - $y(n, n_1) \neq y(n - n_1)$
- Both are not same so system is *time variant*

Thank You !!!