Lecture 1

Lec 1

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"FOURIER SERIES"

Complex exponential signals $\rightarrow e^{st}$

s = complex number

$$s = \underbrace{\sigma}_{\text{Real}} + j\underbrace{\omega}_{\text{Imaginary}}$$

"The response of an LTI system to a complex exponential input is the same complex exponential with only a change in amplitude."

$$\underset{e^{st}}{x(t)} \longrightarrow \boxed{\text{System}} \longrightarrow y(t) = H(s)e^{st}$$

Here $H(s)$ = Complex Amplitude factor
↓
Eign value

e^{st} = Eign function.

One of the methods to find output response is Convolution.

$$\text{Convolution} \to \text{CTS} \quad \text{Continuous Integral}$$

$$\text{Convolution} \to \text{DTS} \quad \text{Convolution sum}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \quad \text{--- (1)}$$

or $$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Now suppose $x(t) = e^{st}$

Use eq (1)

$$x(t-\tau) = e^{s(t-\tau)} = e^{st} \cdot e^{-s\tau}$$

Put in eq (1)

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{st} \cdot e^{-s\tau}\, d\tau$$

$$= e^{st} \underbrace{\int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau}_{H(s)}$$

$$\therefore \boxed{y(t) = e^{st} H(s)}$$

Input is Combination of multiple Inputs.

$$x(t) = a_1 e^{s_1 t} + a_2 e^{s_2 t} + a_3 e^{s_3 t}$$

$$x(t) \longrightarrow \boxed{SYS} \longrightarrow y(t) = ???$$

Use property

$$y(t) = H(s_1) a_1 e^{s_1 t} + H(s_2) e^{s_2(t)} a_2 + H(s_3) e^{s_3 t} a_3 + H(s_4) e^{s_4 t} a_4 .$$

$$H(s_1) = \int_{-\infty}^{\infty} h(\tau) e^{-s_1 \tau} d\tau$$

$$H(s_2) = \int_{-\infty}^{\infty} h(\tau) e^{-s_2 \tau} d\tau$$

$$H(s_3) = \int_{-\infty}^{\infty} h(\tau) e^{-s_3 \tau} d\tau$$

$$H(s_4) = \int_{-\infty}^{\infty} h(\tau) e^{-s_4 \tau} d\tau .$$

$$\therefore \boxed{x(t) = \sum_k a_k e^{s_k t} \longrightarrow y(t) = \sum_k a_k H(s_k) e^{s_k t}}$$

Exp 3.1 :- $h(t) = \delta(t-3)$

$x(t) = e^{j2t}$

$y(t) = ???$

Method 1 :- Use Convolution Integral.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Method 2 :-

$$y(t) = x(t) H(s)$$

$\because$ $h(t) = \delta(t-3)$

$x(t) = e^{j2t}$

$$e^{j2t} \leftrightarrow e^{st}$$

$$s = 2j$$

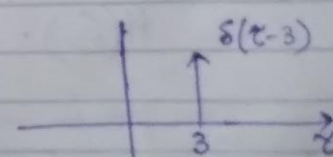
We have to find $H(s)$.

$$H(s) = \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

$$H(s) = \int_{-\infty}^{\infty} \delta(\tau-3)\, e^{-s\tau}\, d\tau$$

$\downarrow$

$\tau = 3$



$$H(s) = e^{-s\tau}\Big|_{\tau=3}$$

$$\boxed{H(s) = e^{-3s}}$$

$\therefore \quad y(t) = x(t)\,H(s)$

$$\underbrace{y(t)}_{\text{Time Domain}} = \underset{\text{TD}}{e^{j2t}} \cdot \underbrace{e^{-3s}}_{\text{SD}}$$

$s = 2j$

$$y(t) = e^{j2t} \cdot e^{-3(2j)}$$

$$= e^{j2t} \cdot e^{-6j}$$

$$\boxed{y(t) = e^{2j(t-3)}}$$

Prob :-

$$x(t) = \cos(4t) + \cos(7t).$$

$$y(t) = x(t-3) \qquad h(t) = \delta(t-3).$$

Use Euler Identity

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\cos(4t) = \frac{e^{j(4t)} + e^{-j(4t)}}{2}$$

$$\cos(7t) = \frac{e^{j(7t)} + e^{-j(7t)}}{2}$$

So, $x(t)$ can be written as;

$$x(t) = \frac{1}{2} e^{j(4t)} + \frac{1}{2} e^{-j(4t)} +$$

$$\frac{1}{2} e^{j(7t)} + \frac{1}{2} e^{-j(7t)}$$

e^{st}

$\frac{1}{2} e^{j4t} \Rightarrow s_1 = 4j$

$\frac{1}{2} e^{-j4t} \Rightarrow s_2 = -4j$

$\frac{1}{2} e^{j7t} \Rightarrow s_3 = 7j$

$\frac{1}{2} e^{-j7t} \Rightarrow s_4 = -7j$

$$y(t) = x(t) H(s)$$

First find $H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$

$$H(s) = \int_{-\infty}^{\infty} \delta(\tau - 3) e^{-s\tau} d\tau$$

$H(s) = e^{-3s}$ → previously calculated.

$H(s) = e^{-3s}$

$$y(t) = H(s_1)\,x_1(t) + H_2(s)\,x_2(t) + H(s_3)\,x_3(t) + H(s_4)\,x_4(t)$$

$$= e^{-3s_1}\left(\frac{1}{2}e^{4jt}\right) + e^{-3s_2}\left(\frac{1}{2}e^{-4jt}\right) + e^{-3s_3}\left(\frac{1}{2}e^{7jt}\right) + e^{-3s_4}\left(\frac{1}{2}e^{-7jt}\right)$$

$s_1 = 4j$, $s_2 = -4j$

$s_3 = 7j$, $s_4 = -7j$

$$y(t) = e^{-3(4j)}\,\frac{1}{2}e^{4jt} + e^{-3(-4j)}\,\frac{1}{2}e^{-4jt} + e^{-3(7j)}\,\frac{1}{2}e^{7jt} + e^{-3(7j)}\,\frac{1}{2}e^{-7jt}$$

$$y(t) = e^{-12j}\cdot\frac{1}{2}e^{4jt} + e^{12j}\cdot\frac{1}{2}e^{-4jt} + e^{-21j}\cdot\frac{1}{2}e^{7jt} + e^{-21j}\,\frac{1}{2}e^{-7jt}$$

$$y(t) = \underbrace{\frac{1}{2}e^{4j(t-3)} + \frac{1}{2}e^{-4j(t-3)}}_{\cos 4(t-3)} + \underbrace{\frac{1}{2}e^{+7j(t-3)} + \frac{1}{2}e^{-7j(t-3)}}_{\cos 7(t-3)}$$

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y :- $y(t) = x(t-3)$

$x(t) = \cos 4t + \cos 7t$

$y(t) = \cos 4(t-3) + \cos 7(t-3)$