CLASSIFICATION OF SYSTEMS

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Classification of Systems

- i) Causal Systems and Non-causal Systems
- ii) Static Systems and Dynamic Systems
- iii) Time-invariant Systems and Time-variant Systems
- iv) Stable Systems and Unstable Systems
- v) Linear Systems and Non-linear Systems
- vi) Invertible Systems and Inverse Systems

1) Causal and Non-Causal Systems

- Causal systems are described as:
    - Response of the causal system to an input does not depend on future values of that input but depends only on the present and past values of the input.

2) Static (memoryless) and Dynamic (with memory) Systems

- Static systems are also known as memoryless systems
- Static systems contain no storage elements (thus, no integrals, derivatives or signal delays)
- A static or memoryless system is a system with an output signal whose values depends upon the present value of the input signal *only*. Otherwise the system is dynamic or with memory.

3) Time Invariant and Time Variant Systems

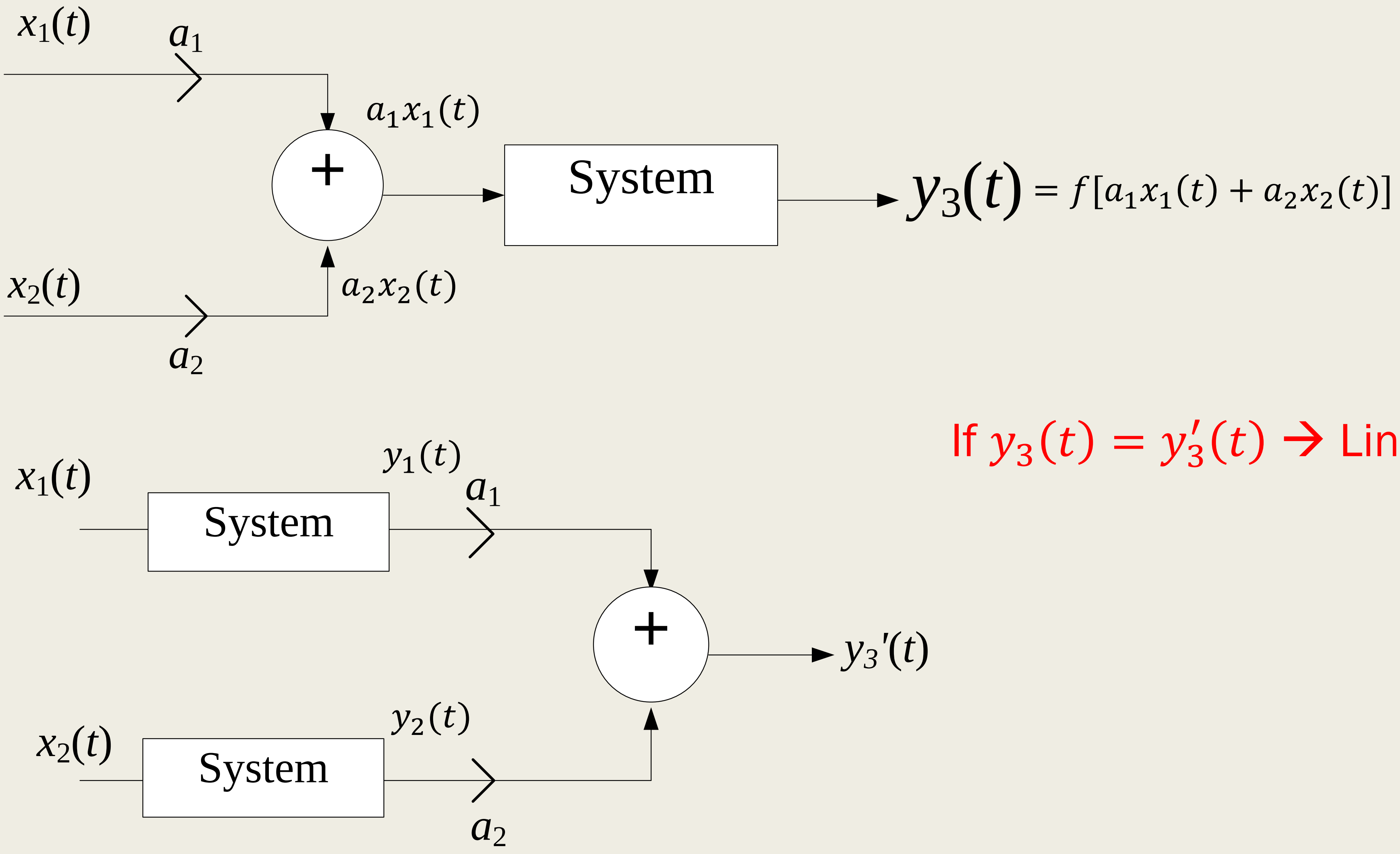
- A system is time invariant if the time shift in the input signal results in corresponding time shift in the output.
- Let $y(t) = f[x(t)]$ i.e. $y(t)$ is response of $x(t)$. Then if $x(t)$ is delayed by time t_1 then output $y(t)$ will also be delayed by the same time. i.e.
- $f[x(t - t_1)] = y(t - t_1)$

Steps to test for time invariance property

- **Step 1:** Determine the output of system for delayed input i.e. $x(t-t_1)$
  - $y(t, t_1) = f[x(t-t_1)]$
- **Step 2:** Then delay the output itself by t_1 i.e. $y(t-t_1)$
- **Step 3:** If
  - $y(t, t_1) \neq y(t-t_1)$ → Time variant
  - $y(t, t_1) = y(t-t_1)$ → Time invariant

4) Linear and Non-Linear Systems

- A system is said to be linear if it follows the superposition principle.
- Consider two systems defined as
  - $y_1(t) = f[x_1(t)]$ i.e. $x_1(t)$ is input and $y_1(t)$ is output
  - $y_2(t) = f[x_2(t)]$ i.e. $x_2(t)$ is input and $y_2(t)$ is output
- Then the system is linear if,
  - $f[a_1\, x_1(t) + a_2\, x_2(t)] = a_1\, y_1(t) + a_2\, y_2(t)$
- Similarly, the discrete time system is said to be linear if
  - $f[a_1\, x_1(n) + a_2\, x_2(n)] = a_1\, y_1(n) + a_2\, y_2(n)$



If $y_3(t) = y_3'(t)$ → Linear

Exp 8.1: Check whether the system is linear or not

$$y(t) = tx(t)$$

- For RHS
- Consider two systems
  - $y_1(t) = tx_1(t)$
  - $y_2(t) = tx_2(t)$
- $y_3'(t) = a_1y_1(t) + a_2y_2(t)$
- $y_3'(t) = a_1tx_1(t) + a_2tx_2(t)$
- For LHS
- $y_3(t) = f[a_1x_1(t) + a_2x_2(t)]$
- $= t\,(a_1x_1(t) + a_2x_2(t))$
- $= t\,a_1x_1(t) + ta_2x_2(t)$

Since $y_3(t) = y_3'(t)$ → Linear

ii) $y(t) = x(t)\cos\omega_c(t)$

- For RHS
- Consider two systems
  - $y_1(t) = x_1(t)\cos\omega_c(t)$
  - $y_2(t) = x_2(t)\cos\omega_c(t)$
- $y_3'(t) = a_1y_1(t) + a_2y_2(t)$
- $y_3'(t) = a_1x_1(t)\cos\omega_c(t) + a_2x_2(t)\cos\omega_c(t)$
- For LHS
- $y_3(t) = f[a_1x_1(t) + a_2x_2(t)]$
- $= (a_1x_1(t) + a_2x_2(t))\cos\omega_c(t)$
- $= a_1x_1(t)\cos\omega_c(t) + ta_2x_2(t)\cos\omega_c(t)$

Since $y_3(t) = y_3'(t) \rightarrow$ Linear

iii) $y(t) = x^2(t)$

- For RHS
- Consider two systems
  - $y_1(t) = x_1^2(t)$
  - $y_2(t) = x_2^2(t)$
- $y_3'(t) = a_1 y_1(t) + a_2 y_2(t)$
- $y_3'(t) = a_1 x_1^2(t) + a_2 x_1^2(t)$

- For LHS
- $y_3(t) = f[a_1 x_1(t) + a_2 x_2(t)]$
- $\qquad = [a_1 x_1(t) + a_2 x_2(t)]^2$
- $\qquad = a_1^2 x_1^2(t) + a_2^2 x_2^2(t) + 2a_1 a_2 x_1(t)\, x_2(\mathrm{t})$

Since $y_3(t) \neq y_3'(t)$ → Non-Linear

Exp 8.2: Check whether the system is linear or not

(a) $y(n) = x(n^2)$

(b) $y(n) = x^2(n) - x(n-1) + x(n+1)$

5) Stable and Unstable Systems

- When every bounded input produces bounded output. It is called Stable System.
- Follows BIBO

- If $|x(t)| \leq M_x < \infty$
- $|x(n)| \leq M_x < \infty$

Bounded Input

- Then O/P is
- $|y(t)| \leq M_y < \infty$
- $|y(n)| \leq M_y < \infty$

Bounded Output

Bounded Signals: dc, $\sin t$, $\cos t$, $u(t)$

$y(t) = 6$ -1 to 1 -1 to 1 0 or 1

ex 1: $y(t) = t \cdot x(t)$

$x(t) \longrightarrow \text{Sys.} \longrightarrow y(t) = t \cdot x(t)$

$u(t) \longrightarrow \text{sys.} \longrightarrow y'(t) = t \cdot u(t)$

$= r(t)$

unstable

Exp 8.3: Check whether the system is stable or not

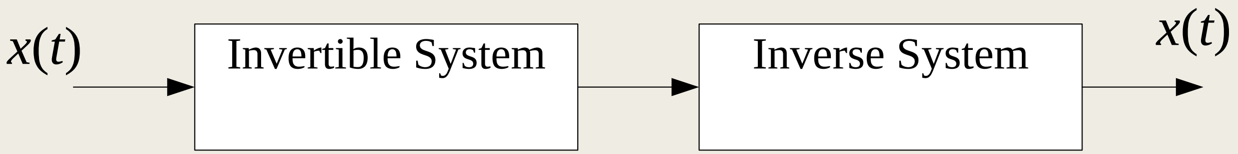
- (a) $y(t) = tx(t)$
- unStable

- (b) $y(t) = x(t)sin100\pi t$
- Stable

- (c) $y(n) = r^n x(n) \qquad r > 1$
- Unstable because the value of $y(n)$ is not only depending upon $\mathrm{x}(n)$ but also upon r which can be unbounded.

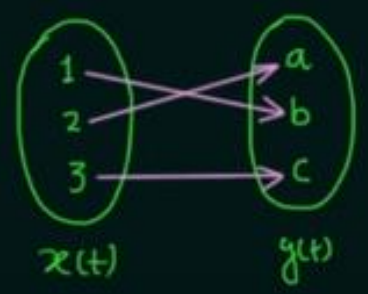
6) Invertible and Inverse Systems

- A system is said to be invertible if there is unique output for every unique input.
- For each invertible system, there is always an inverse system
- If invertible and inverse systems are connected in cascaded form, then the output remains same as input.
- $HH^{-1} = 1$

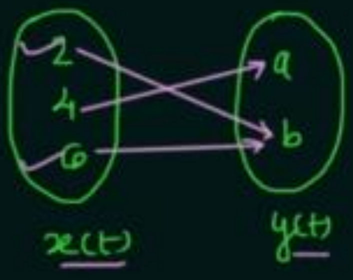


» For an invertible system, there should be *one to one mapping* between i/p and o/p at each and every instant of time

One to One mapping:



many to one mapping:



ex 1: $y(t) = x^2(t)$

$x(t)$	$y(t)$
-2 →	4
2 ↗	

Non-Invertible

many to one mapping

ex 2: $y(t) = x(t) + 2$

$x(t)$	$y(t)$
0 →	2
1 →	3
-1 →	1

Exp 8.4: Check whether the system is invertible or not

- (a) $y(t) = 10x(t)$
  - Invertible

- (b) $y(t) = x^2(t)$
  - It is non-invertible

PP 8.1: Determine whether the following systems are
i) Static or dynamic
ii) Linear or non-Linear
iii) Time Invariant or Time Variant
iv) Causal or non-Causal
v) Stable or unstable

(a) $y(t) = 10x(t) + 5$
(b) $y(t) = x(t + 10) + x^2(t)$
(c) $\frac{dy(t)}{dt} + ty(t) = x(t)$
(d) $y(t) = x(t)\cos(100\pi t)$
(e) $y(n) = x(n) + nx(n + 1)$
(f) $y(n) = x(n)u(n)$

Thank You !!!