DISCRETE CONVOLUTION

Dr. Arsla Khan

Convolution

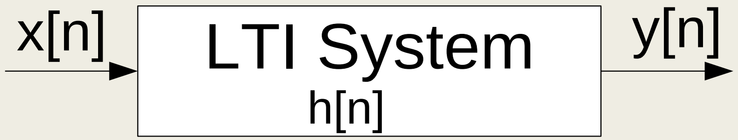
- Convolution operation is used to relate the input output relationship for LTI systems.

- Importance of convolution stems from the fact that knowledge of response of an LTI system to the unit impulse input allows us to find its output to any input signals.

- Convolution can be termed as
  - Convolution Integral → CT LTI Systems
  - Convolution Sum → DT LTI Systems

Impulse Response

- The impulse response (or unit sample response) $h[n]$ of a discrete-time LTI system is defined to be the output response of the system when the input $x[n]$ is $\delta[n]$



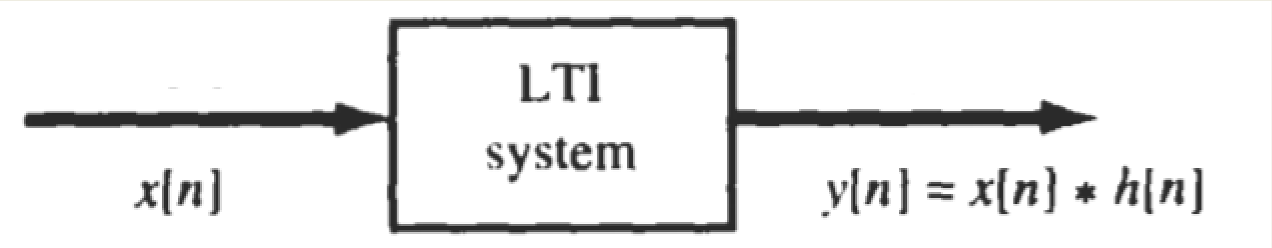
Methods to find Convolution Sum

- Analytical approach
- Multiplication approach
- Graphical / Formula approach

Convolution Sum

- Convolution Sum of two sequences $x[n]$ and $h[n]$ is denoted by

- $y[n] = x[n] * h[n]$
- $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$
- $y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$



h[n]

Properties of Convolution Sum

1. ***Commutative:***

$$x[n] * h[n] = h[n] * x[n]$$

2. ***Associative:***

$$\{x[n] * h_1[n]\} * h_2[n] = x[n] * \{h_1[n] * h_2[n]\}$$

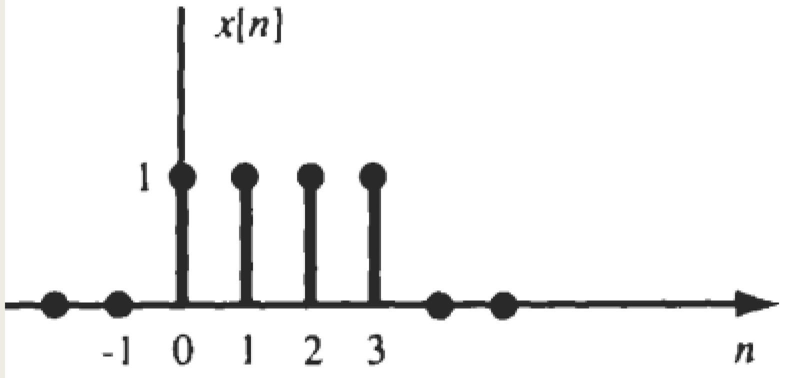
3. ***Distributive:***

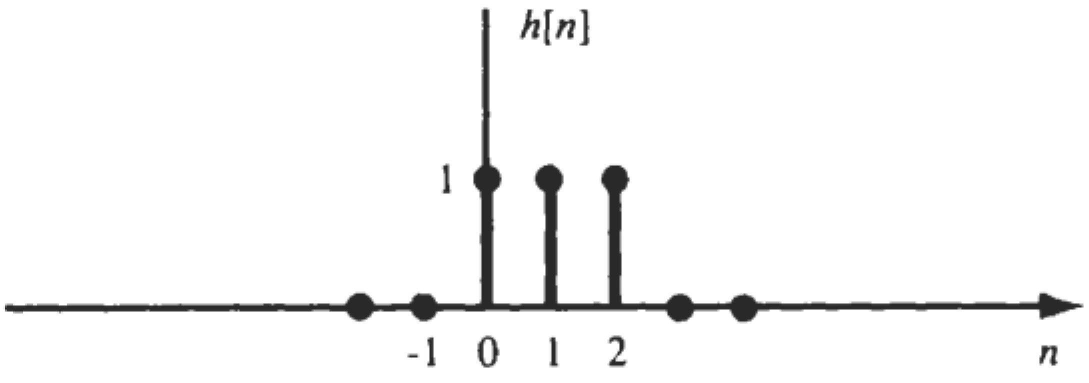
$$x[n] * \{h_1[n]\} + h_2[n]\} = x[n] * h_1[n] + x[n] * h_2[n]$$

Points to Remember

- $x[n]\delta[n-n_o] = x[n_o]$
- $x[n] * \delta[n-n_o] = x[n-n_o]$

Exp 1: Evaluate $x[n]$ and $h[n]$ by an analytical technique





Note that $x[n]$ and $h[n]$ can be expressed as

$$x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$

$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$$

Now, using Eqs.

$$\begin{aligned} x[n] * h[n] &= x[n] * \{\delta[n] + \delta[n-1] + \delta[n-2]\} \\ &= x[n] * \delta[n] + x[n] * \delta[n-1] + x[n] * \delta[n-2]\} \\ &= x[n] + x[n-1] + x[n-2] \end{aligned}$$

Thus,

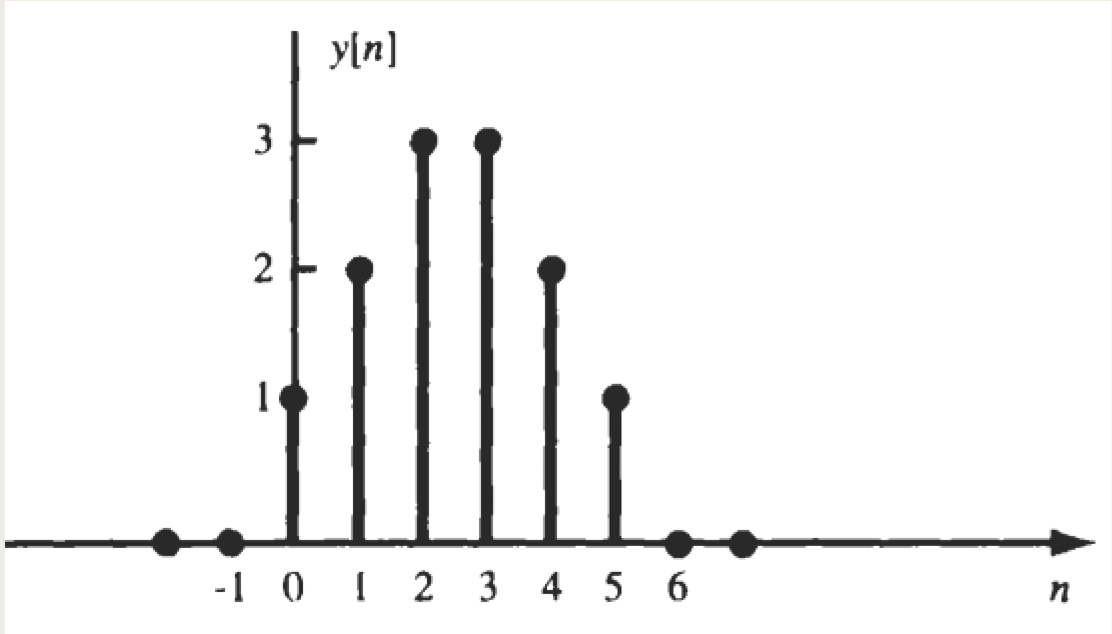
$$\begin{aligned} y[n] = \delta[n] &+ \delta[n-1] + \delta[n-2] + \delta[n-3] \\ &+ \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] \\ &+ \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5] \end{aligned}$$

or

$$y[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 3\delta[n-3] + 2\delta[n-4] + \delta[n-5]$$

or

$$y[n] = \{1, 2, 3, 3, 2, 1\}$$


y[n]
3
2
1
-1
0
1
2
3
4
5
6
n

Multiplication Method

EXAMPLE 4.25 Find the linear convolution between $x(n) = [1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1]$ and $h(n) = [1, 1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1, 1]$ by using the multiplication method.

Solution By using the multiplication method, we have

$$
\begin{array}{l|ccccccccccccccc}
h(n) \Rightarrow & & & & & & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
x(n) \Rightarrow & & & & & & & & \times & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\hline
 & & & & & & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
 & & & & & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \\
 & & & & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & \\
 & & & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & & \\
 & & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & & & \\
 & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & & & & \\
 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & & & & & \\
\hline
y(n) \Rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 7 & 7 & 6 & 5 & 4 & 3 & 2 & 1
\end{array}
$$

Since for both the multiplicands together, the number of sample points on left hand side from the origin is 2 + 3 = 5 and that on right hand side is 4 + 5 = 9, the convoluted sum is

$$y(n) = [1, 2, 3, 4, 5, \underset{\uparrow}{6}, 7, 7, 7, 6, 5, 4, 3, 2, 1]$$

EXAMPLE 4.27 Find the linear convolution between $x(n) = [\underset{\uparrow}{0}, 1, 2]$ and $h(n) = [\underset{\uparrow}{0}, 0, 1, 1]$ by using the multiplication method.

Solution By using the multiplication method, we have

$$
\begin{array}{rrrrrrr}
h(n) \Rightarrow & & & 0 & 0 & 1 & 1 \\
x(n) \Rightarrow & & & \times & 0 & 1 & 2 \\
\hline
 & & & 0 & 0 & 2 & 2 \\
 & & 0 & 0 & 1 & 1 & \\
 & 0 & 0 & 0 & 0 & & \\
\hline
y(n) \Rightarrow & 0 & 0 & 0 & 1 & 3 & 2
\end{array}
$$

There is no sample point on the left hand side from the origin. However, on the right hand side 2 + 3 = 5 samples are there. Hence, the convoluted sum is $y(n) = [\underset{\uparrow}{0}, 0, 0, 1, 3, 2]$.

Thank You !!!