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Q 1:

$$y[n] = \frac{1}{6} y[n-1] - \frac{1}{6} y[n-2] = x[n-1]$$

→ Transfer function:

$$y[n] = \frac{1}{6} y[n-1] - \frac{1}{6} y[n-2] = x[n-1]$$

$$y[z] - \frac{1}{6} z^{-1} y[z] - \frac{1}{6} z^{-2} y[z] = z^{-1} X[z]$$

$$y[z]\left[1 - \frac{z^{-1}}{6} - \frac{z^{-2}}{6}\right] = z^{-1} X[z]$$

$$\frac{X[z]}{X[z]} = \frac{2}{1 - \frac{z^{-1}}{6} - \frac{z^{-2}}{6}}$$

$$H(z) = \frac{2}{z^2 - \frac{z}{6} - \frac{1}{6}}$$

(45) Find all posible impulse response h(n) of LTI system.

$$H(z) = \frac{z}{z^2 - \frac{2}{6} - \frac{1}{6}}$$

$$H(z) = \frac{z}{(z-1)(z+\frac{2}{3})}$$

* partial fraction

$$\frac{z}{(z-1)(z+\frac{2}{3})} = \frac{A}{(z-1)} + \frac{B}{(z+\frac{2}{3})}$$

Put $z = 1$

$$1 = A\left(\frac{5}{3}\right) \rightarrow A = \frac{3}{5}$$

Put $z = -\frac{2}{3}$

$$-\frac{2}{3} = B\left(\frac{-5}{2}\right) \rightarrow B = \frac{2}{5}$$

$$H(z) = \frac{3}{5\left(z - \frac{1}{2}\right)} + \frac{2}{5\left(z + \frac{1}{3}\right)}$$

(B) Impulse Response

ROC $|z| > \frac{1}{2}$: causal

Both are RSS so,

$$h(n) = \frac{3}{5}\left(\frac{1}{2}\right)^n u(n) + \frac{2}{5}\left(-\frac{1}{3}\right)^n u(n)$$

ROC : $\frac{1}{3} < |z| < \frac{1}{2}$ (Two sided)

Pole at $z = \frac{1}{2}$ to Right side

Pole at $z = -\frac{1}{3}$ to left side

So

$$h(n) = \frac{3}{5}\left(\frac{1}{2}\right)^n u(n) - \frac{2}{5}\left(-\frac{1}{3}\right)^n u(-n-1)$$

ROC $= |z| < \frac{1}{3}$ (Anti causal

Both are LSS:

$$h(n) = \frac{-3}{5}\left(\frac{1}{2}\right)^n u(-n-1) - \frac{2}{5}\left(\frac{-1}{3}\right)^n u(-n-1)$$

(c)

Impulse Response for causal:

$|z| > \frac{1}{2}$

$$h(n) = \frac{3}{5}\left(\frac{1}{2}\right)^n u(n) + \frac{2}{5}\left(\frac{1}{3}\right)^n u(n)$$

Impulse Response for stable:

Stability require ROC inside and circle. $R = 1$ so its not stable

Nither causal nor stable

ROC : $|z| < \frac{1}{3}$

$$h(n) = -\frac{3}{5}\left(\frac{1}{2}\right)^n u(-n-1) - \frac{2}{5}\left(\frac{-1}{3}\right)^n u(-n-1)$$

M T W T F S

Both stable & causal

→ causality require ROC $|z| > \frac{1}{2}$

→ Stability require ROC include unit circle $|z| = 1$

∴ So there is a conflict of condition thats why its not possible.