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Lec #10

4.3 Properties of Continuous-Time Fourier Transform:

→ Linearity:-

$$x(t) \xrightarrow{F} X(j\omega)$$

$$y(t) \xrightarrow{F} Y(j\omega)$$

$$ax(t) + by(t) \xrightarrow{F} aX(j\omega) + bY(j\omega)$$

→ Time shifting:-

$$x(t) \xrightarrow{F} X(j\omega)$$

$$x(t-t_0) \xrightarrow{F} e^{-j\omega t_0} X(j\omega)$$

Lets first understand the implimentation of Linearity and time shifting through example 4.9.

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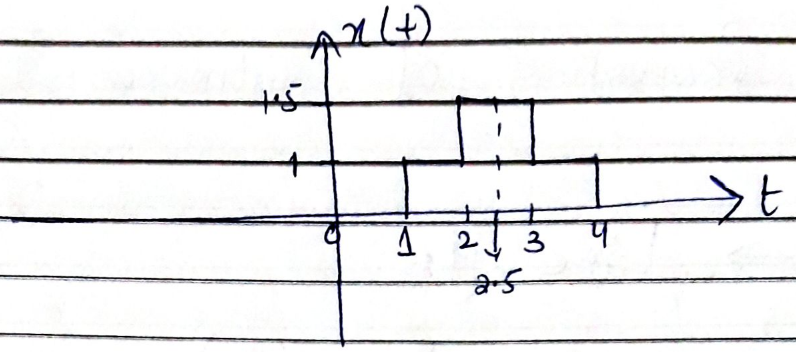
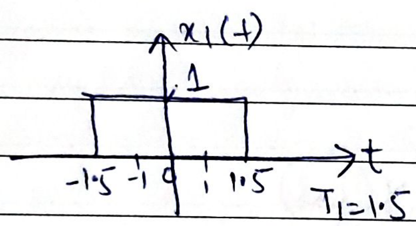
Example 4.9



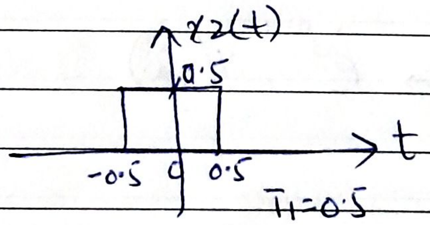
Some process can be adopted to find the Fourier Transform $X(j\omega)$ of time domain signal $x(t)$ just as we have adopted for F.S.C

generic form:-

$$X(j\omega) = \frac{2\sin\omega T_1}{\omega}$$

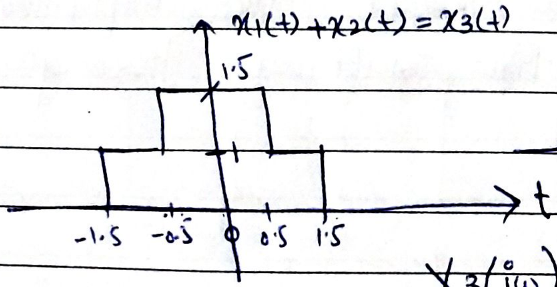


$$\longrightarrow X_1(j\omega) = \left(\frac{2\sin\omega(1.5)}{\omega}\right)\times 1$$



$$\longrightarrow X_2(j\omega) = \left(\frac{2\sin\omega(0.5)}{\omega}\right)\times 0.5$$

Property of Linearity



$$\longrightarrow X_1(j\omega) + X_2(j\omega) = X_3(j\omega)$$

$$X_3(j\omega) = \frac{2\sin 3\omega/2}{\omega} + \frac{\sin\omega/2}{\omega}.$$

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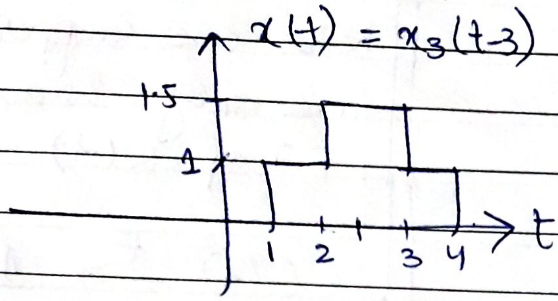
Now shifting $x_3(t)$ 2.5 units towards Right (Delay by 2.5 units) to get original $x(t)$ as:-

Applying the property of shifting



$$\longrightarrow X(j\omega) = e^{-j5/2\omega} X_3(j\omega)$$

So finally:-

$$X(j\omega) = e^{-j5/2\omega}\left[\frac{\sin \omega/2 + 2\sin 3\omega/2}{\omega}\right] \longrightarrow *$$

Next Property:

$\rightarrow$ Differentiation and Integration:

$$x(t) \xrightarrow{F} X(j\omega)$$

$$\frac{dx(t)}{x(t)} \xrightarrow{F} j\omega X(j\omega)$$

$$\int x(\tau)d\tau \rightarrow \frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$$

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Now lets solve an example for the property mentioned above:-

Example 4.11

$x(t) = u(t)$

$X(j\omega) = ?$

here we suppose that we only know the F.T of $\delta(t)$

$$\delta(t) \xrightarrow{F} 1$$

knowing this how we can find the F.T of $u(t)$, also knowing the relation b/w $\delta(t)$ and $u(t)$ i-e

$$\frac{du(t)}{dt} = \delta(t)$$

$$\int \delta(t) = u(t)$$

So As we already know that

$$g(t) = \delta(t) \xrightarrow{F} G(j\omega) = 1$$

So now making $\delta(t)$, such that it becomes equivalent of $u(t)$

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$x(t) = \int g(t)\,dt$
$u(t) = \int \delta(t)\,dt$ $\longrightarrow$ $X(j\omega) = \frac{G(j\omega)}{j\omega} + \pi G(0)\delta(\omega)$

where we already know
$G(j\omega) = 1$

so

$$\Rightarrow X(j\omega) = \frac{(1)}{j\omega} + \pi\delta(\omega)$$

so

$$x(t) = u(t) \xrightarrow{F} X(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega) \quad \text{---}* \; 1$$

Similarly if we know 1 and have to find the F.T of $\delta(t)$, ~~if~~ knowing

$$\frac{du(t)}{dt} = \delta(t)$$

so Applying the property of differentiation,

$$\delta(t) = \frac{du(t)}{dt} \xrightarrow{F} j\omega\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]$$

$$= \frac{\cancel{j\omega}}{\cancel{j\omega}} + \pi j\omega\delta(\omega)$$

$$= 1 + 0$$ how

$\delta(\omega)$ is 1 at $\omega = 0$, when 0 got multiplied all become 0

$$\boxed{\delta(t) \xrightarrow{F} 1} \quad \text{---}* \; 2$$

Same as calculated before.

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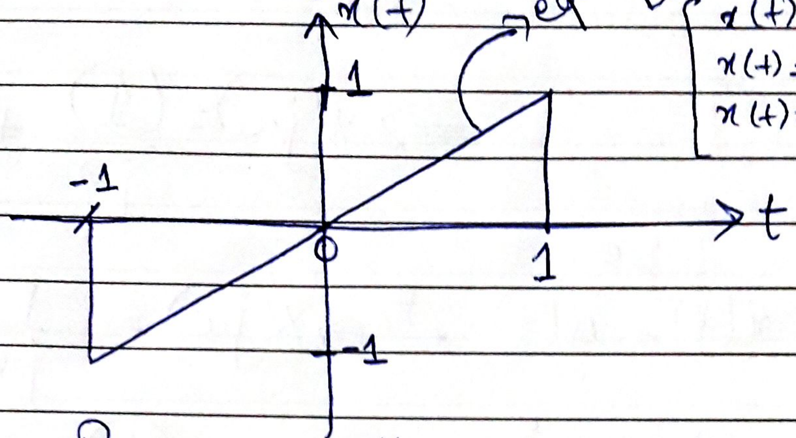
Example (4.12) (v.imp)

Find $X(j\omega)$



equation t because

$$\begin{bmatrix} x(t) = 1 & \rightarrow t = 1 \\ x(t) = -1 & \rightarrow t = -1 \\ x(t) = 0 & \rightarrow t = 0 \end{bmatrix}$$

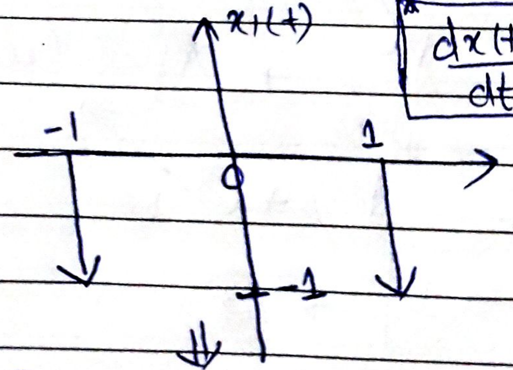
Always Remember, the only way to come out of slope figure is differentiation the resultant Ans. can be integrated back to get original $X(j\omega)$.

So first differentiate $x(t)$ to simplify the figure:-

slope $\xrightarrow{}$ when differentiated gives CONSTANT

point of discontinuity $\rightarrow$ " " " given IMPULSES

$$\boxed{\frac{dx(t)}{dt} = x_1(t) + x_2(t)}$$



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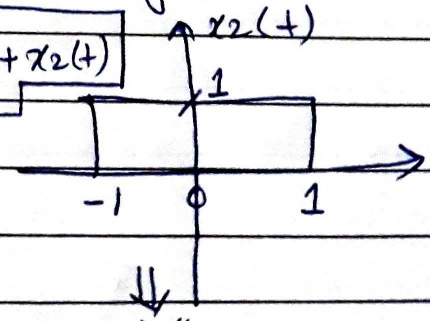


$\Downarrow$

Point of discontinuity at -1 and 1 both facing downward.

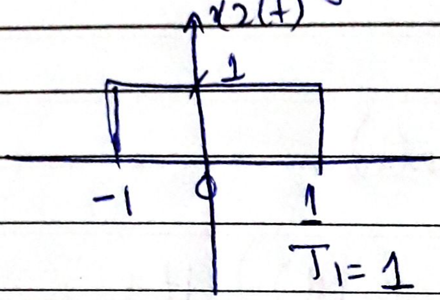
$\Downarrow$

Slope "t" when diff gives 1

first solving $x_2(t)$

$\left(\frac{2\sin\omega T_1}{\omega}\right)$

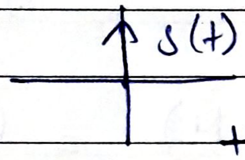
$\Downarrow$



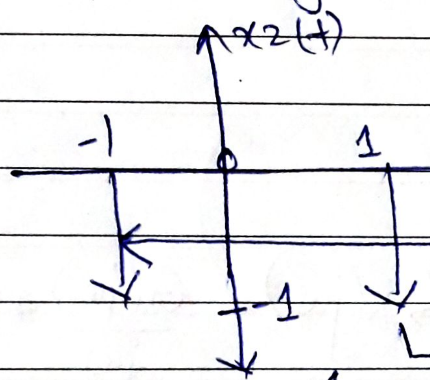
$\Rightarrow X_2(j\omega) = \frac{2\sin\omega(1)}{\omega}$

$X_2(j\omega) = \frac{2\sin\omega}{\omega}$ —— * (1)

Now solving $x_1(t)$:-



$\delta(t) \xrightarrow{F} 1$



$\Rightarrow X_2(j\omega) = -e^{+j\omega} - e^{-j\omega}$ —— * (2)

Advance   Delay

(Applying property of shifting)

$X_2(j\omega) = (-)e^{+j\omega}\ (-)e^{-j\omega}$ $\Rightarrow$ (the (-)ive sign is due to the downward position of impulses)

so

Now

$\frac{dx(t)}{dt} = x_1(t) + x_2(t) \xrightarrow{F.T} G(j\omega) = X_1(j\omega) + X_2(j\omega)$

$G(j\omega) = \boxed{\frac{2\sin\omega}{\omega} - e^{-j\omega} - e^{+j\omega}}$ *

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Now integrating $\frac{dx(t)}{dt}$ to get back $x(t)$ as:-

$$\int \frac{dx(t)}{dt} = \int x_1(t) + \int x_2(t)$$

$$= \int \Big(\underbrace{x_1(t) + x_2(t)}_{\text{suppose } g(t)}\Big)\, dt$$

$$\int \frac{dx(t)}{\cancel{dt}} = \int g(t)\, dt$$

$$x(t) = \int g(t)\, dt \xrightarrow{F.T} X(j\omega) = \frac{G(j\omega)}{j\omega} + \pi G(0)\delta(\omega)$$

where

$$G(j\omega) = \frac{2\sin\omega}{\omega} - e^{-j\omega} - e^{+j\omega}$$

$$G(0) = 0$$

plz calculate yourself

$$X(j\omega) = \left[\frac{2\sin\omega}{\omega} - 2\left[\frac{e^{-j\omega} + e^{+j\omega}}{2}\right]\right] \times \frac{1}{j\omega}$$

$$\boxed{X(j\omega) = \frac{2\sin\omega}{j\omega^2} - \frac{2\cos\omega}{j\omega}}$$

is the final Ans.