INTRODUCTION TO SIGNALS AND SYSTEMS

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Course Contents

- Introduction
- Basic operations on signals
- Basic system properties
- Time domain analysis of continuous and discrete time systems
- Fourier series analysis of CTS and DTS
- Fourier transform analysis of CTS and DTS
- Laplace Transform
- Z Transform

Text Book:
Signals and Systems
By A.V. Oppenheim and A. S. Willsky
Second Edition Prentice Hall, 2012

Links for Video Lectures

- **Introduction to Signals and Systems:**
- https://www.youtube.com/watch?v=s8rsR_TStaA&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=1
- **Continuous and Discrete Time Signals**
- https://www.youtube.com/watch?v=H4hk6N5vC1Q&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=2
- **Time Shifting**
- https://www.youtube.com/watch?v=9Cd5nVCFfc0
- https://www.youtube.com/watch?v=3Qzpj6UUxhE&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=273

Signal

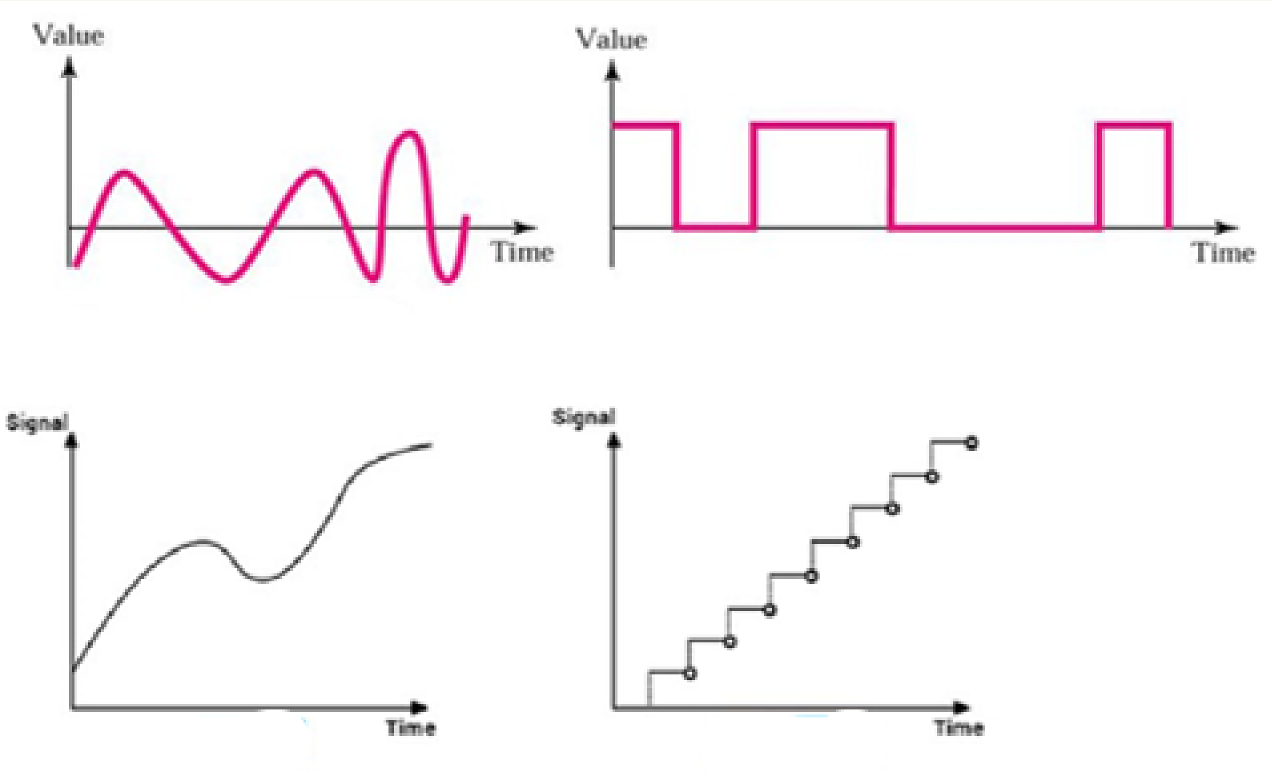
- Signal is defined as:
  - "A quantity used to convey information" e.g. human speech, temperature
  - "a dependent variable or function of one or more independent variables

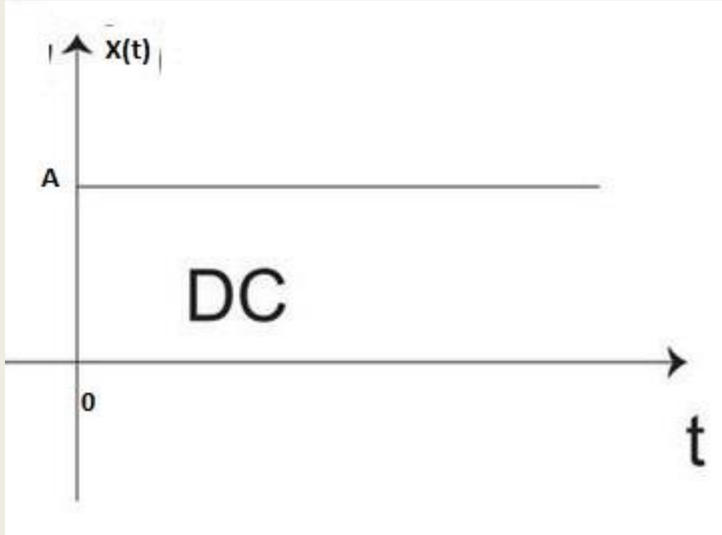
$$\underbrace{f}_{\text{signal}}\underbrace{(x_1, x_2, \dots x_n)}_{\text{Independent Variables}}$$

- Single Variable Signal → If signal is dependent on one variable only. $f(x), g(t)$
- Multi Variable Signal → If signal is depending on more than one variable. $f(x_1, x_2)$

Difference btw signal and a dc value

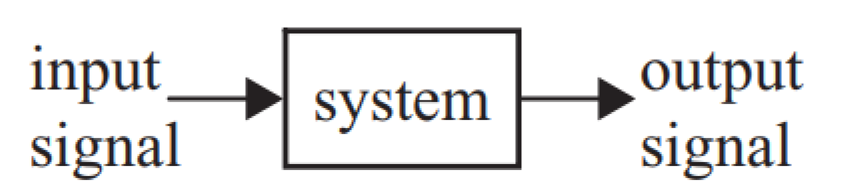
- anything which is varying is a signal but a constant value is not a signal
- e.g. AC is a signal because current is changing with time. Whereas DC is not a signal because in DC, current is not changing with time.





System

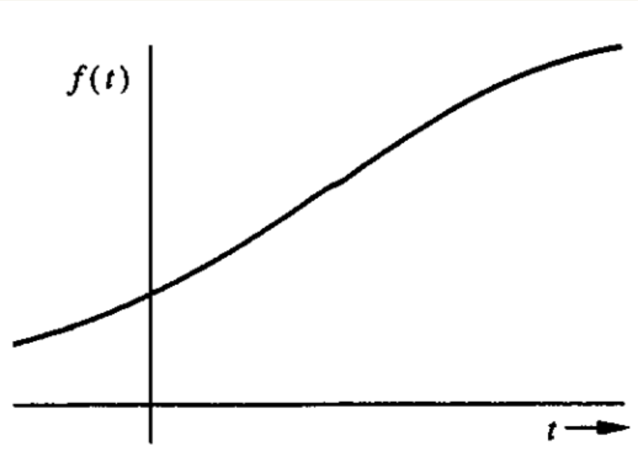
- It is defined as
  - The meaningful interconnection of physical devices and components is called a system
  - An entity that process a set of signals (input signal) and produces another set of signals (output signal).
- System alone can not achieve anything so it must be linked with a signal.

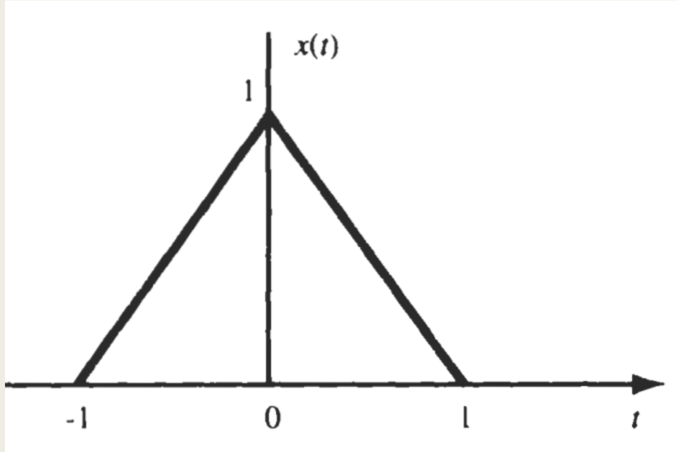


desirable signal

i) Continuous Time Signal (CTS)

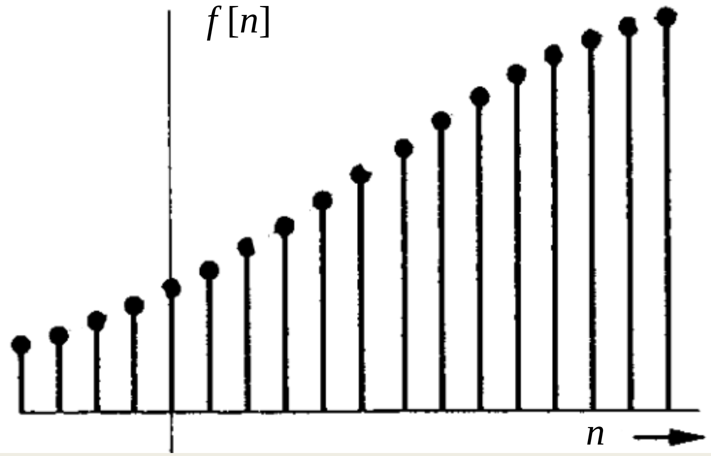
- Signals which are specified for every value of time (*t*)
- It is written as $f(t), x(t), or\ g(t)$

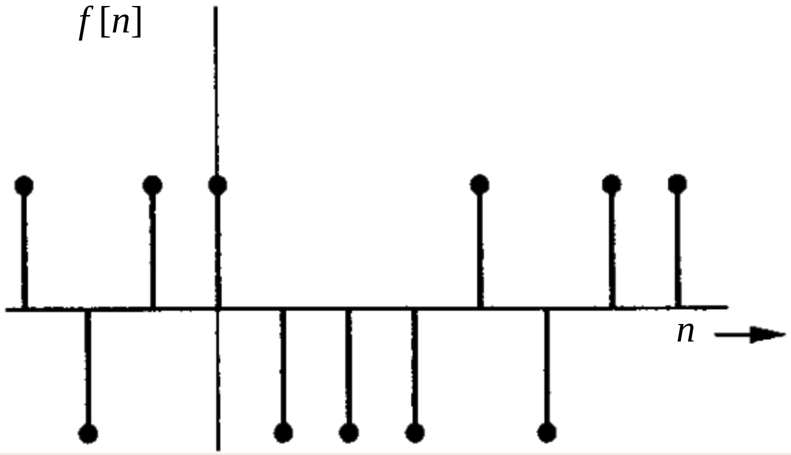




ii) Discrete Time Signals (DTS)

- Signals specified at discrete time intervals
- It is written as $f[n],\ x[n], or\ g[n]\ /\ f(n), x(n), g(n)$





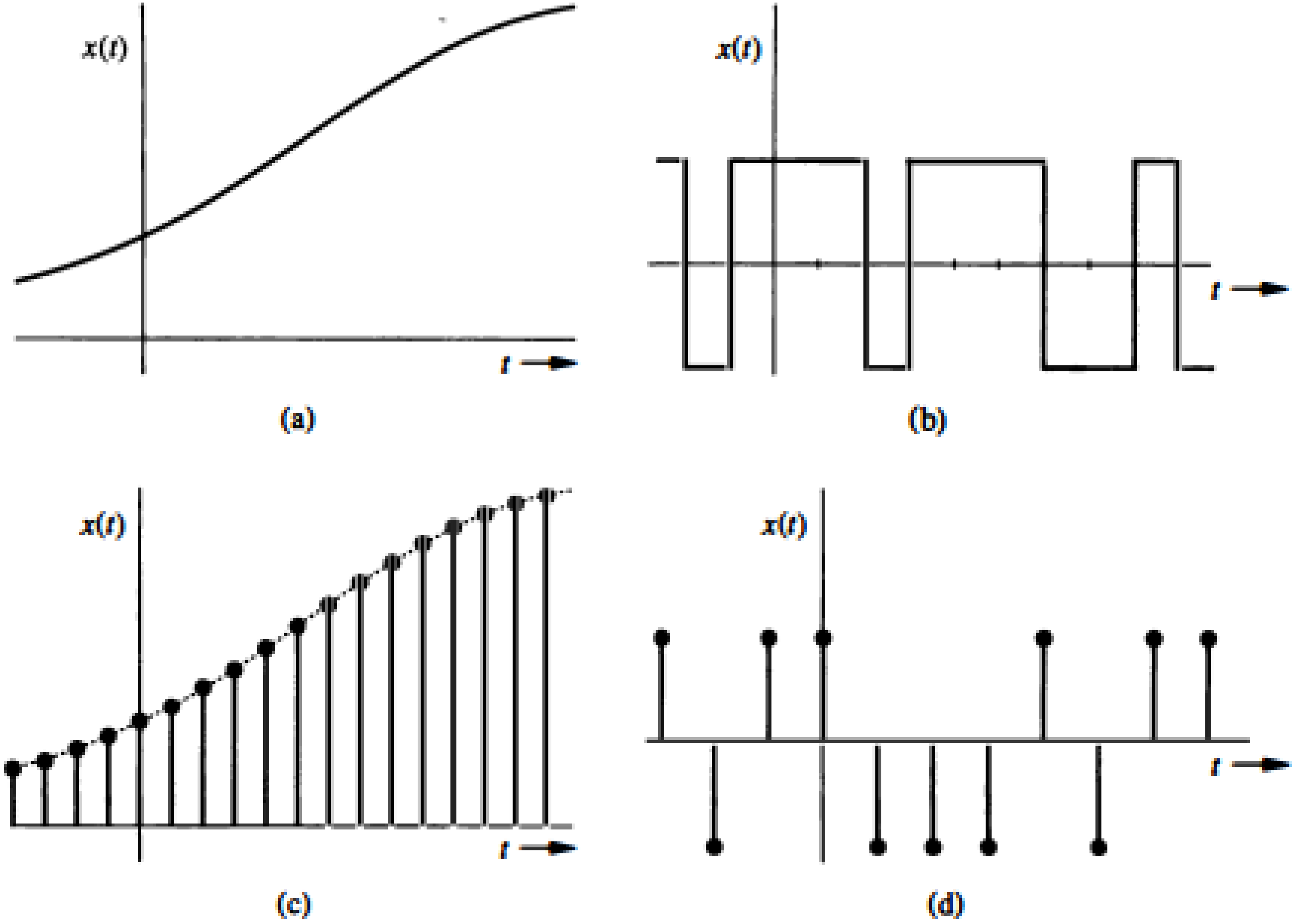


Figure 1.11 Examples of signals: (**a**) analog, continuous time, (**b**) digital, continuous time, (**c**) analog, discrete time, and (**d**) digital, discrete time.

OPERATIONS ON SIGNALS

(INDEPENDENT VARIABLE)

Links for Video Lectures

- **1) Time Shifting**
- https://www.youtube.com/watch?v=9Cd5nVCFfc0
- https://www.youtube.com/watch?v=3Qzpj6UUxhE&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=273
- **2) Time Scaling**
- https://www.youtube.com/watch?v=jnB-U5KBvN4&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=5
- **3) Time Reversal/Flipping/Folding**
- https://www.youtube.com/watch?v=BzAbZfT6RxQ&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=9

Operations on Signals

- **Operations with respect to x-axis (Time axis)** / Transformations on the independent variable
  - Time Shifting
  - Time Reversal/Folding
  - Time Scaling
- **Operations with respect to y-axis (Amplitude)** / Transformations on the dependent variable
  - Amplitude Multiplication
  - Amplitude Scaling
  - Addition
  - Subtraction

1) Time Shifting

- **Time Delay**
  - *When the signal is delayed, it is shifted right*

    i.e. $x(t - k)$ or $x[n - k]$ ∴ where k is positive

    e.g. $x(t - 2)$, $x(t - 1.5)$, $x[n - 2]$, $x[n - 3]$

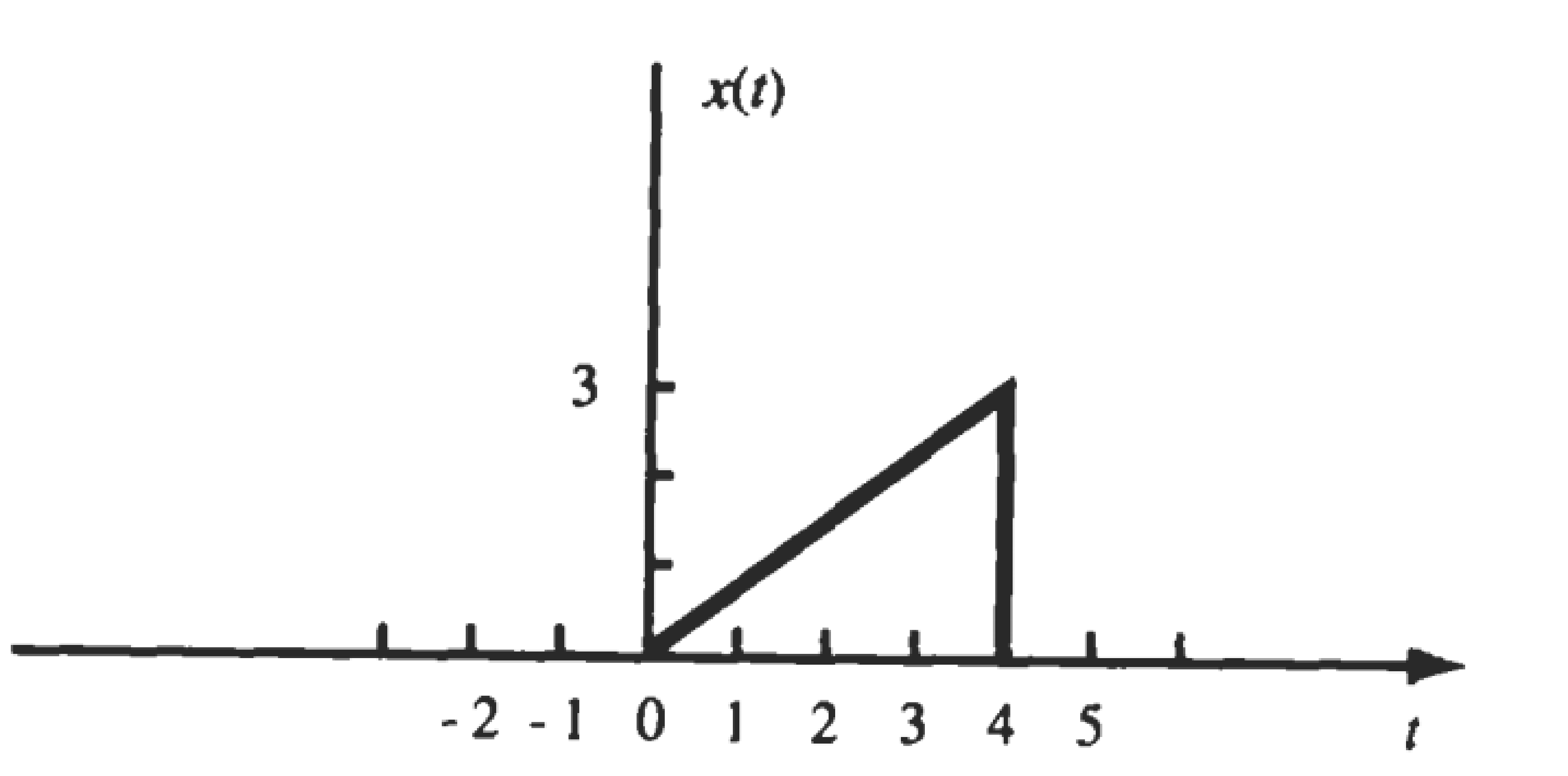
- **Time Advance**
  - *When the signal is advanced, it is shifted left*

    i.e. $x(t + k)$ or $x[n + k]$ ∴ where k is positive

    e.g. $x(t + 2)$, $x(t + 1.5)$, $x[n + 2]$, $x[n + 3]$

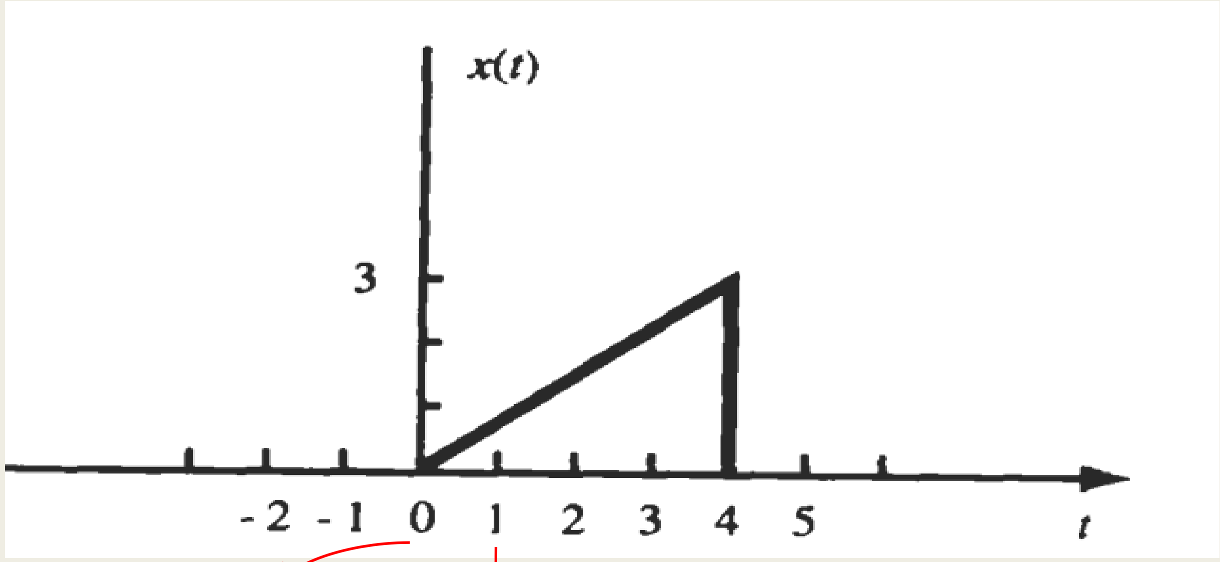
$-\infty$ $+\infty$

Exp 1.1: For $x(t)$, sketch $x(t-2)$ and $x(t+2)$



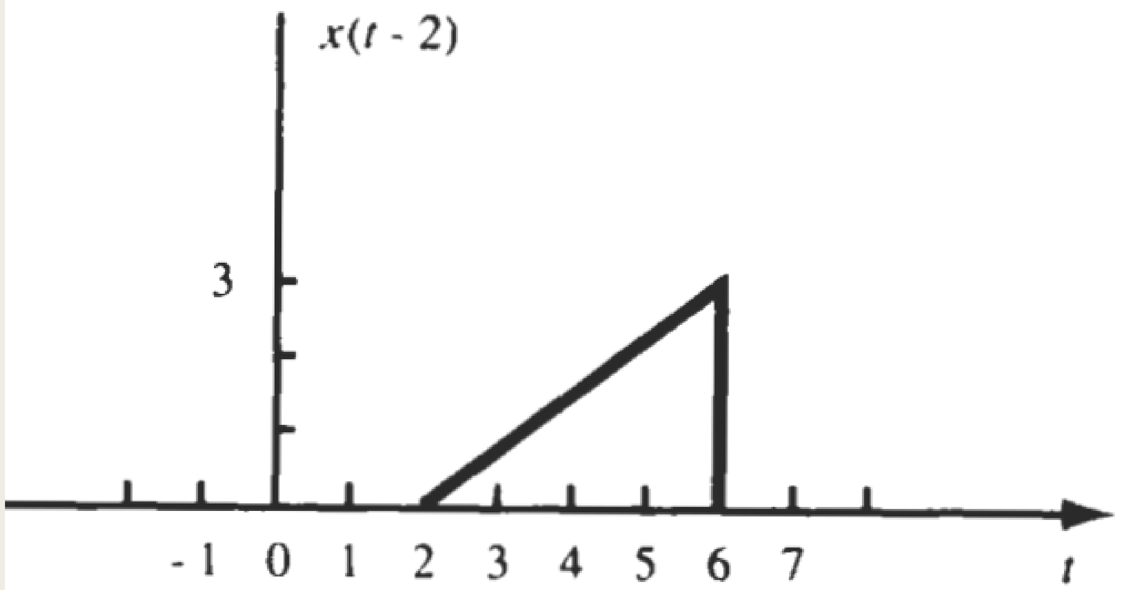
$x(t - 2)$:

- $t - 2 = 0 \rightarrow t = 0 + 2 = 2$
- $t - 2 = 1 \rightarrow t = 1 + 2 = 3$
- $t - 2 = 2 \rightarrow t = 2 + 2 = 4$
- $t - 2 = 3 \rightarrow t = 3 + 2 = 5$
- $t - 2 = 4 \rightarrow t = 4 + 2 = 6$



$t - 2 = 0$

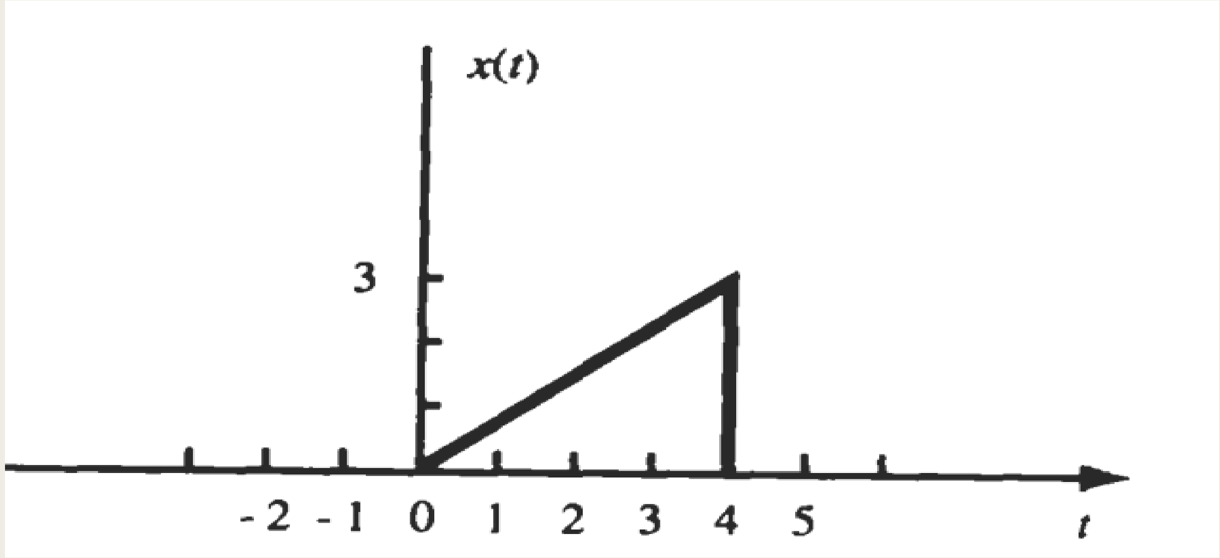
$t - 2 = 1$

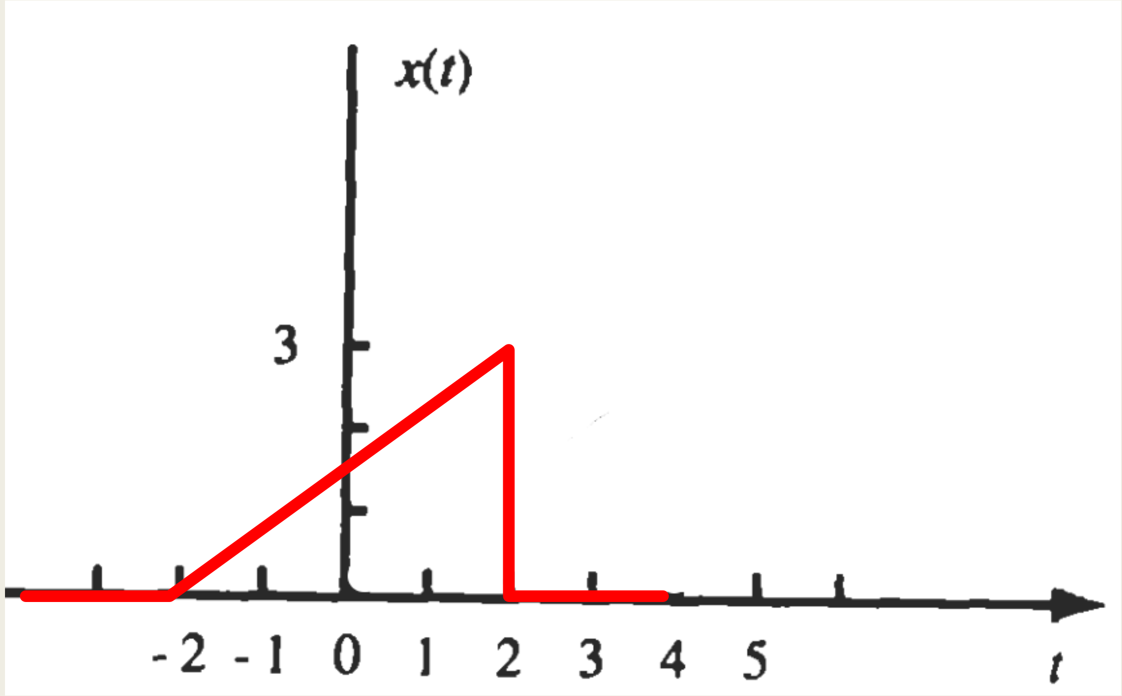


Shifted towards right by 2 steps so it is a delayed signal

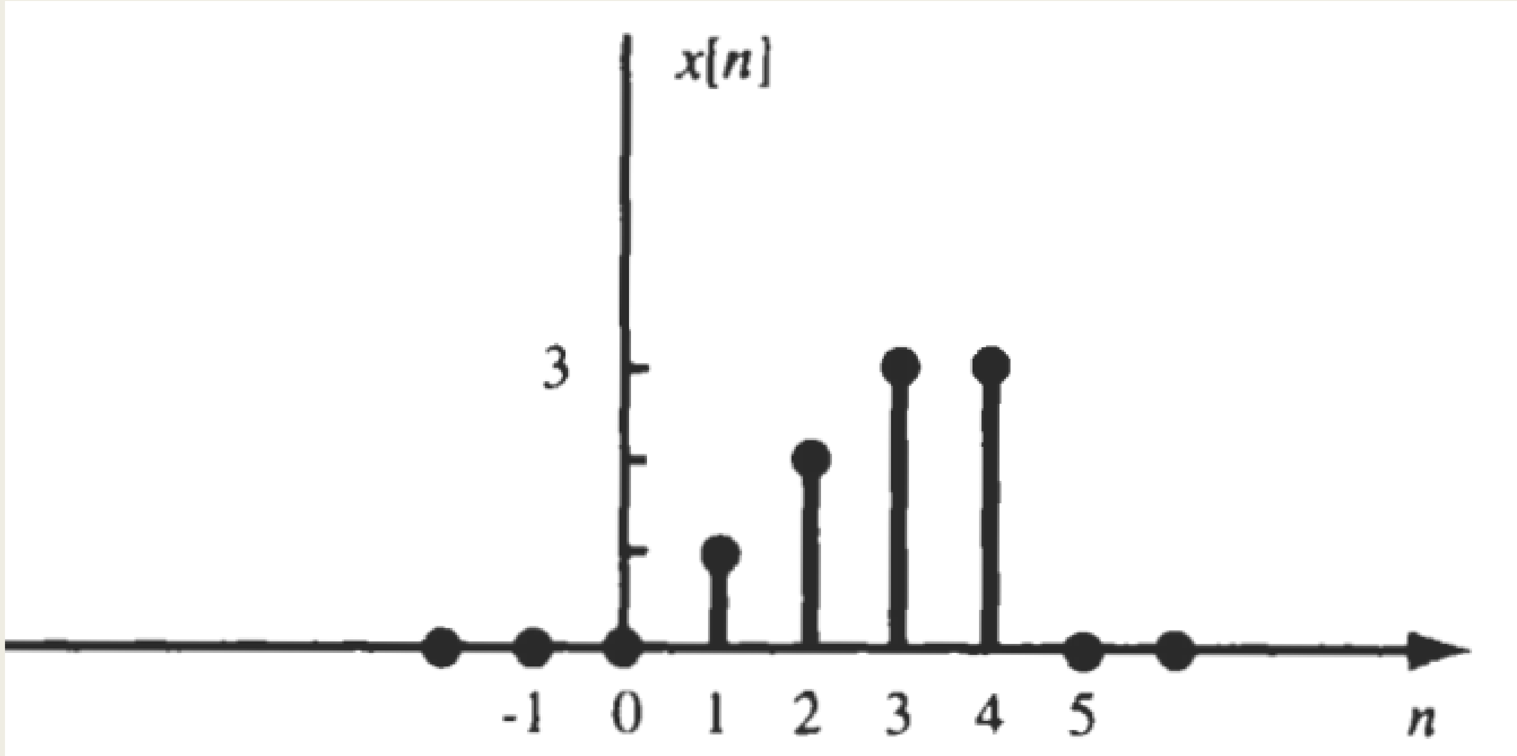
$x(t + 2)$:

- t + 2 = 0 → t = 0 + 2 = - 2
- t + 2 = 1 → t = 1 - 2 = -1
- t + 2 = 2 → t = 2 - 2 = 0
- t + 2 = 3 → t = 3 - 2 = 1
- t + 2 = 4 → t = 4 - 2 = 2

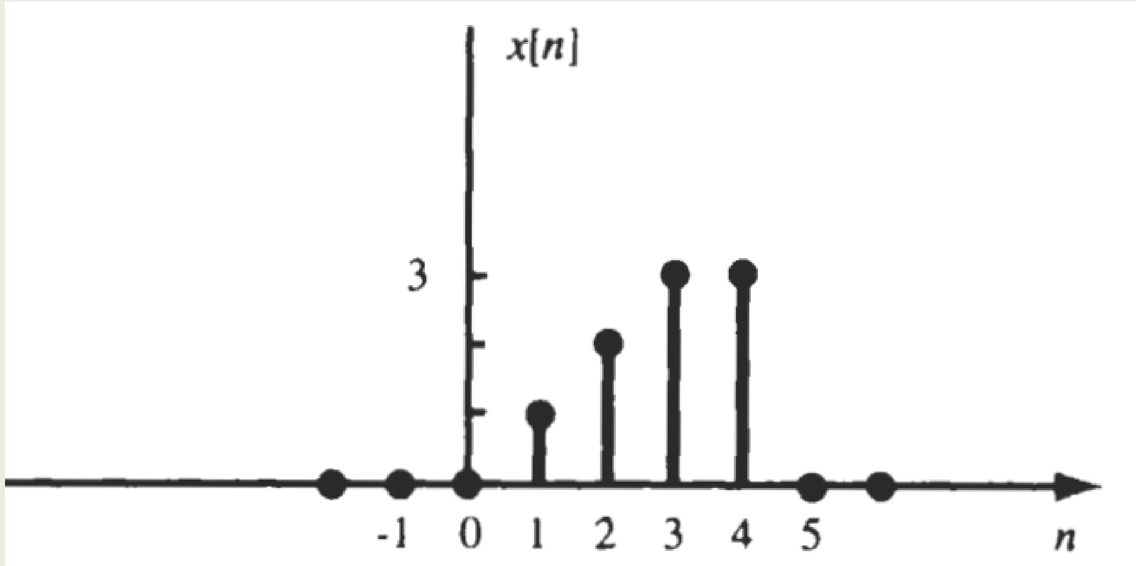


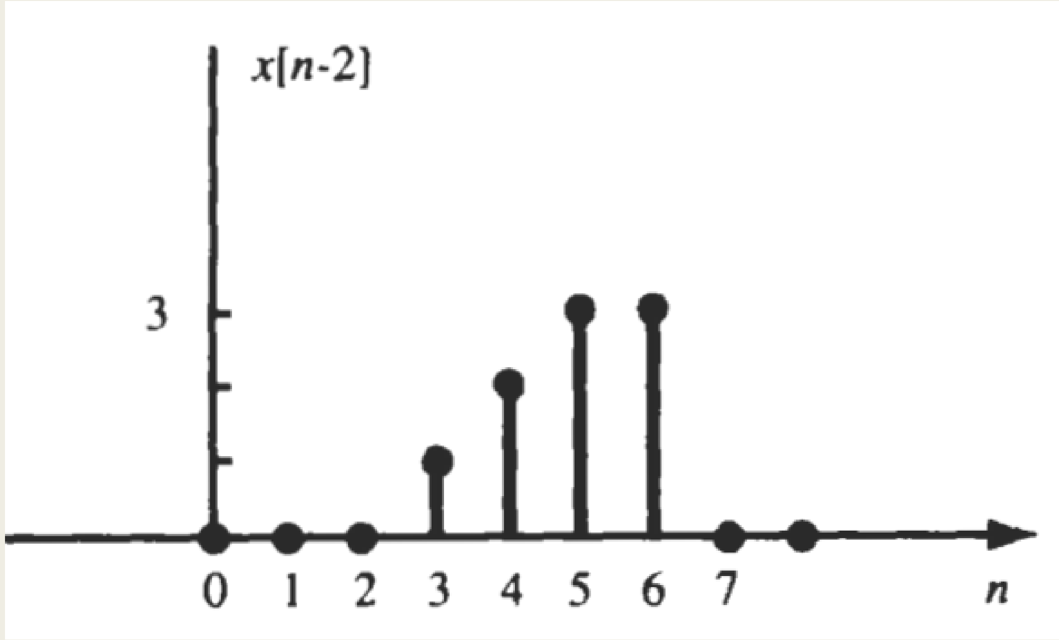


Exp 1.2: For $x[n]$, sketch $x[n - 2]$



- $n - 2 = 0 \rightarrow n = 2 - 0 = 2$
- $n - 2 = 1 \rightarrow n = 1 + 2 = 3$
- $n - 2 = 2 \rightarrow n = 2 + 2 = 4$
- $n - 2 = 3 \rightarrow n = 3 + 2 = 5$
- $n - 2 = 4 \rightarrow n = 4 + 2 = 6$

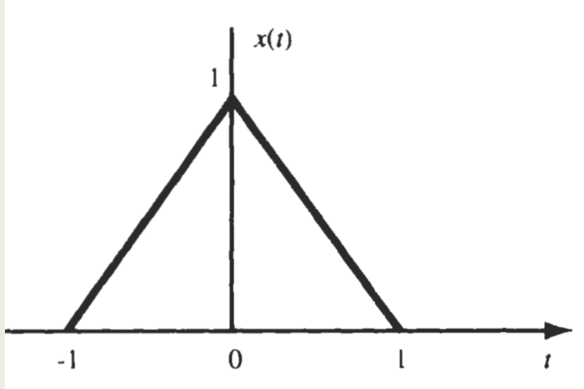




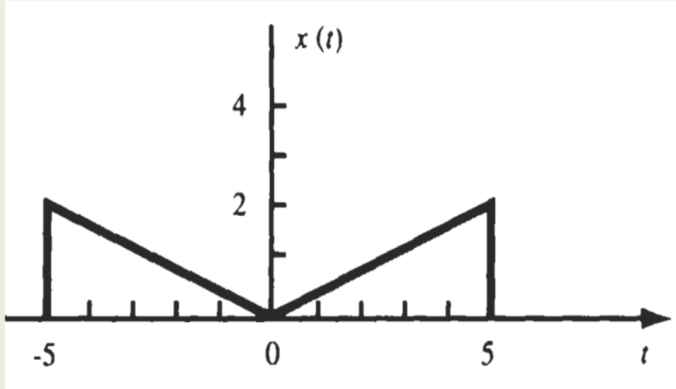
Shifted towards right by 2 steps so it is a delayed signal

PP. 1.1) For signals given, sketch

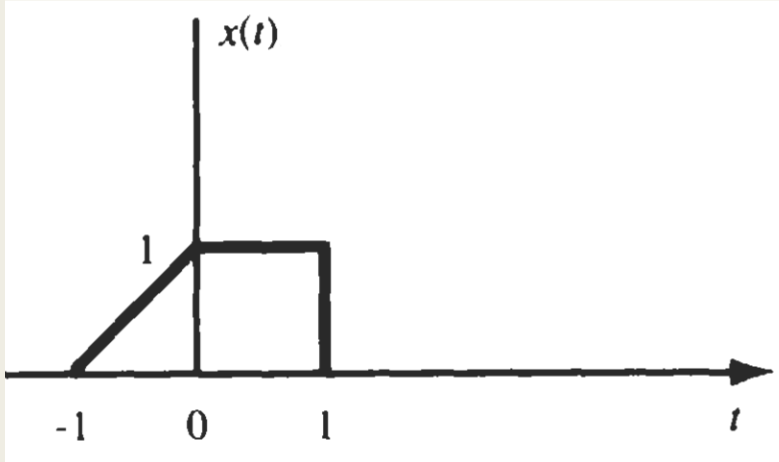
i) $x(t - 2.5)$
ii) $x(t + 1)$



(a)



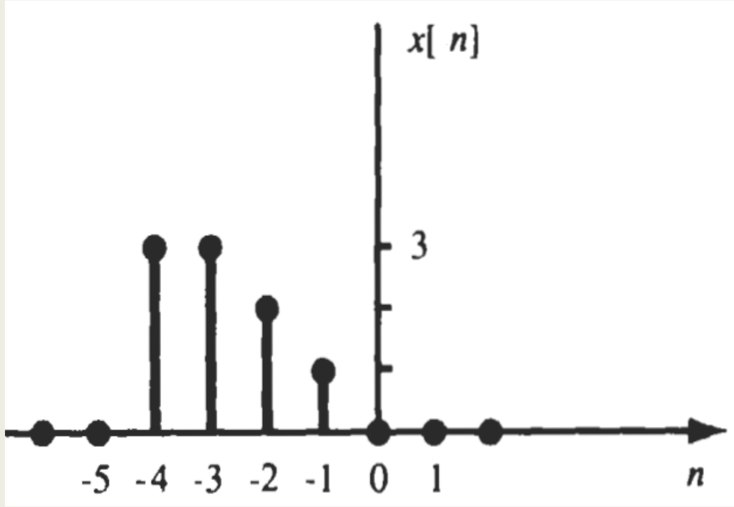
(b)



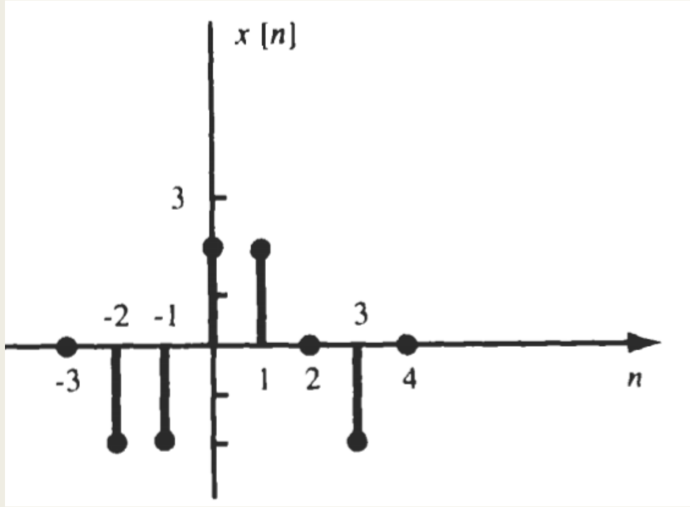
(c)

PP. 1.2) For signals given, sketch

i) $x[n-1]$
ii) $x[n+3]$



(a)



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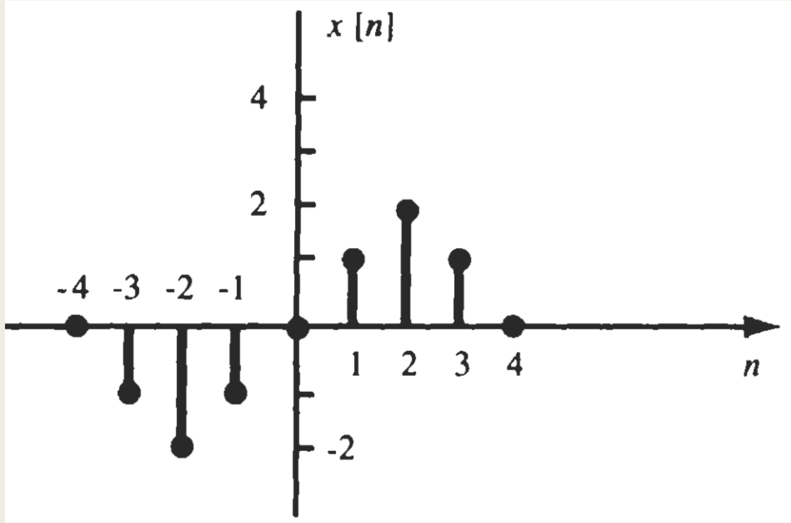


(c)

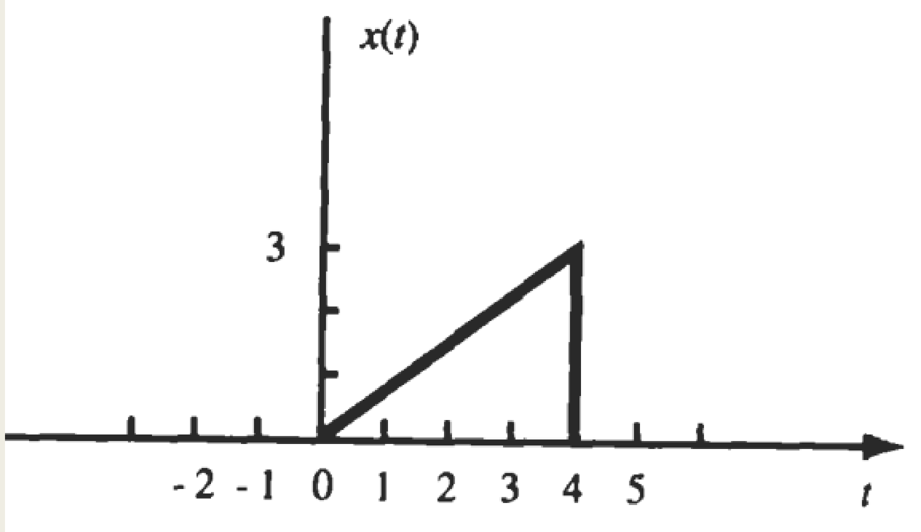
2) Time Reversal/Folding/Flipping

- Reversal of signal about the vertical axis (y-axis) is known as time reversal.
- It converts $x(t)$ into $x(-t)$
- Therefore, mirror image of the signal $x(t)$ about vertical axis is $x(-t)$

  - $x(t) \rightarrow x(-t)$

- Note: Mirror image of the signal $x(t)$ about horizontal axis is $-x(t)$

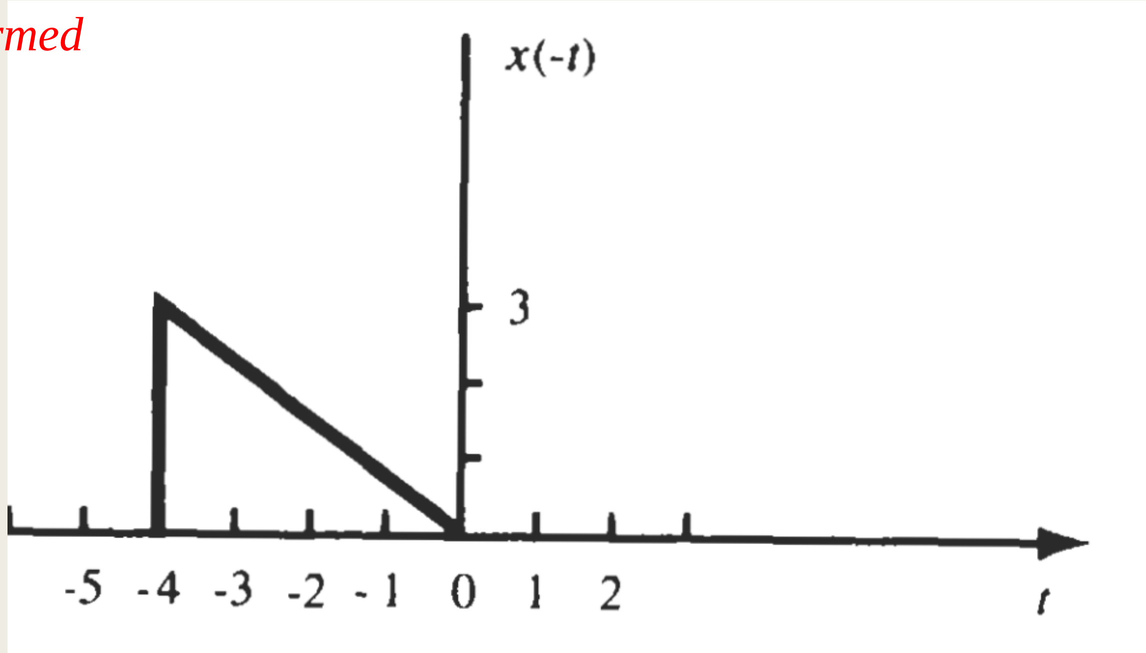
Exp 2.1: For $x(t)$, sketch $x(-t)$



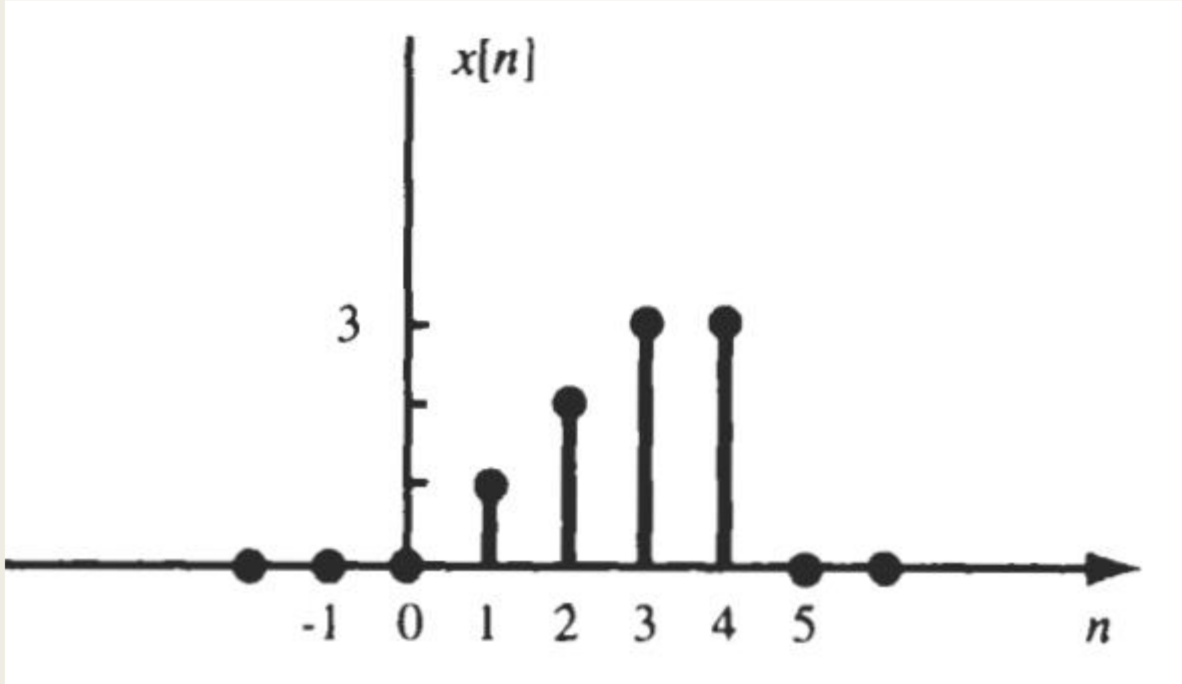
Points for transformed signal i.e. $x(-t)$

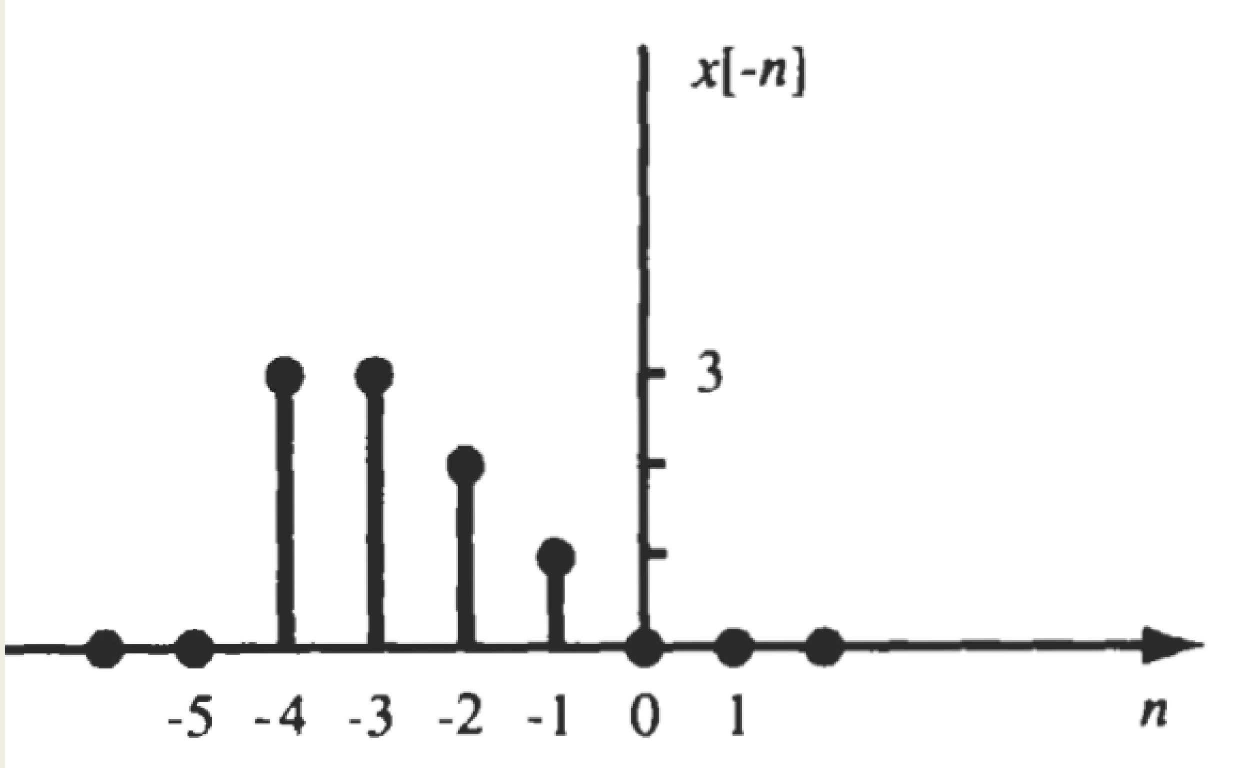
$x(-t) = x(t)$

- $-t = 0 \rightarrow t = 0$
- $-t = 1 \rightarrow t = -1$
- $-t = 2 \rightarrow t = -2$
- $-t = 3 \rightarrow t = -3$
- $-t = 4 \rightarrow t = -4$



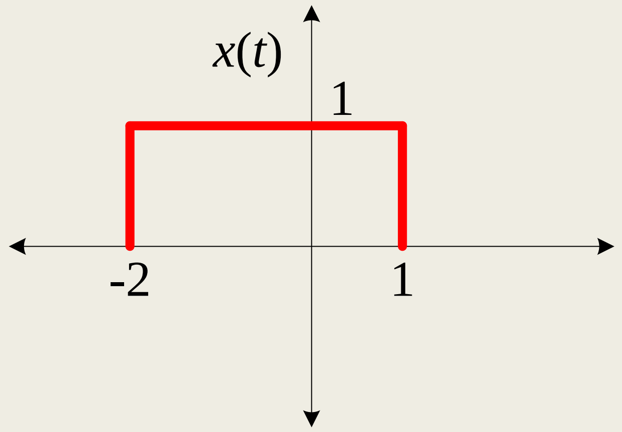
Exp 2.2: For $x[n]$, sketch $x[-n]$

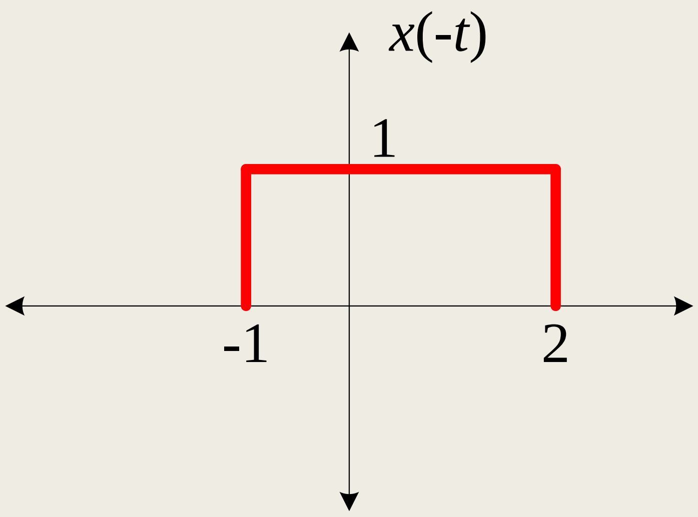


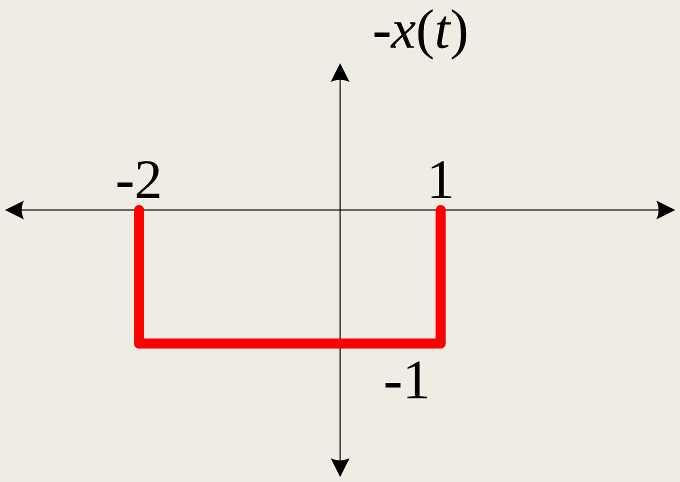

x[-n]
3
-5
-4
-3
-2
-1
0
1
n

Difference between $x(-t)$ and $-x(t)$

- $x(t) \rightarrow x(-t)$: Flipping around vertical axis (y-axis)
- $x(t) \rightarrow -x(t)$: Flipping around horizontal axis (x-axis)

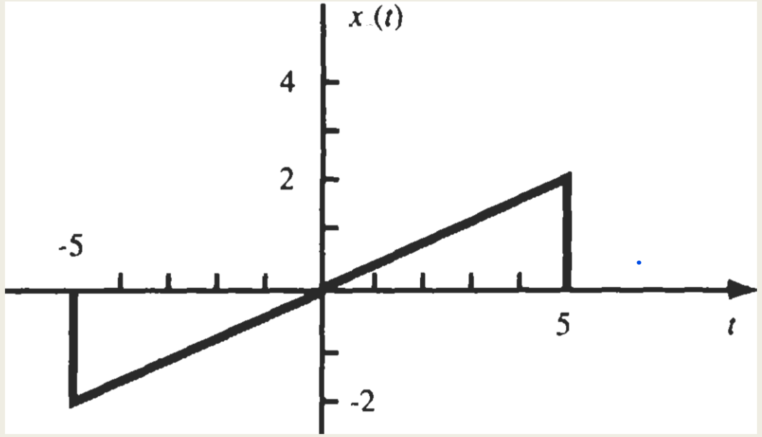


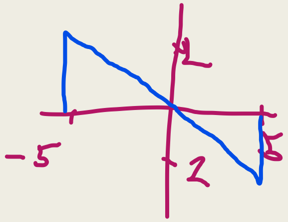




PP. 2.1) For signals given, sketch $x(-t)$ and $x[-n]$

(a)





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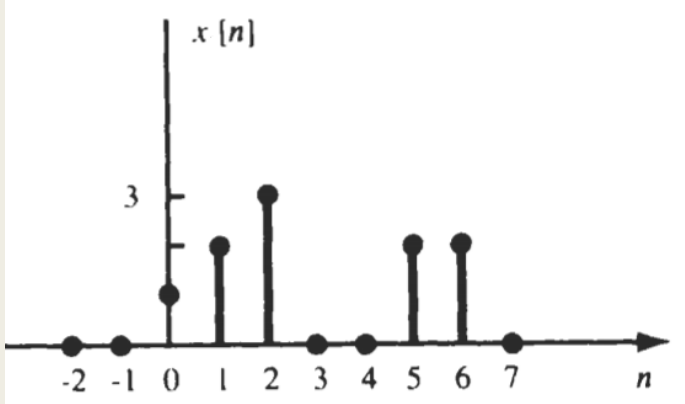

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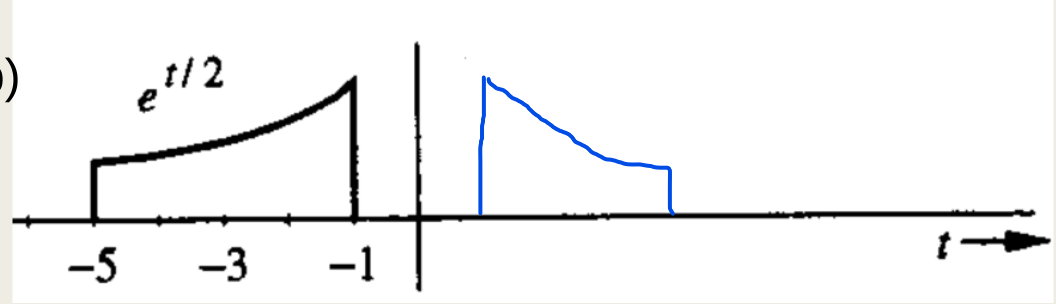
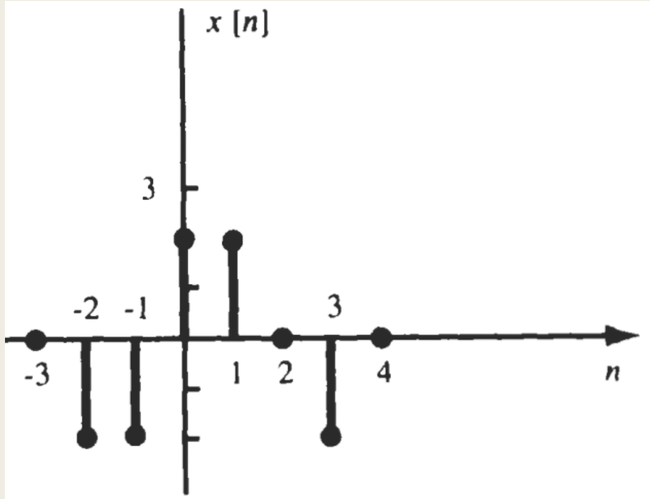


(d)



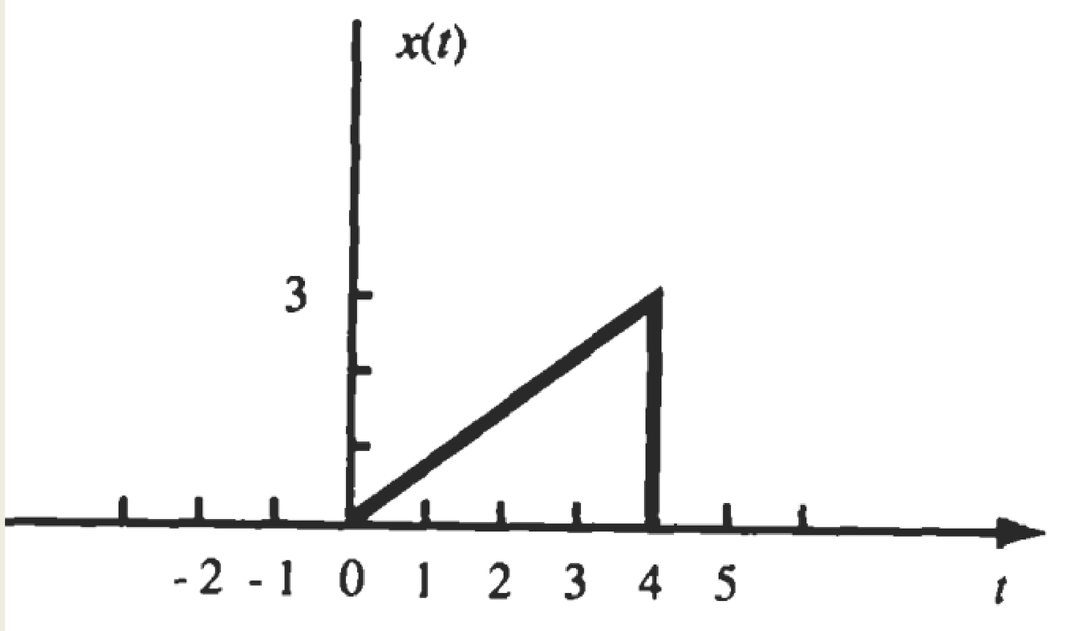
3) Time Scaling

- **i) Time Compression:** Time axis is compressed
- **ii) Time Expansion:** Time axis is expanded

- $x(t) \rightarrow x(\alpha t)$

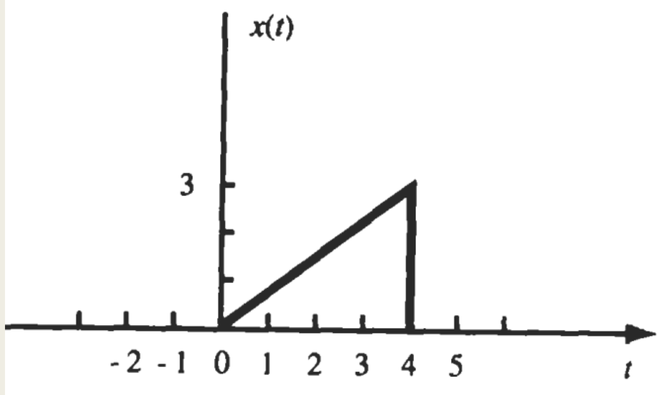
- If $\alpha > 1$ then scaling results in time compression
- If $\alpha < 1$ then scaling results in time expansion.

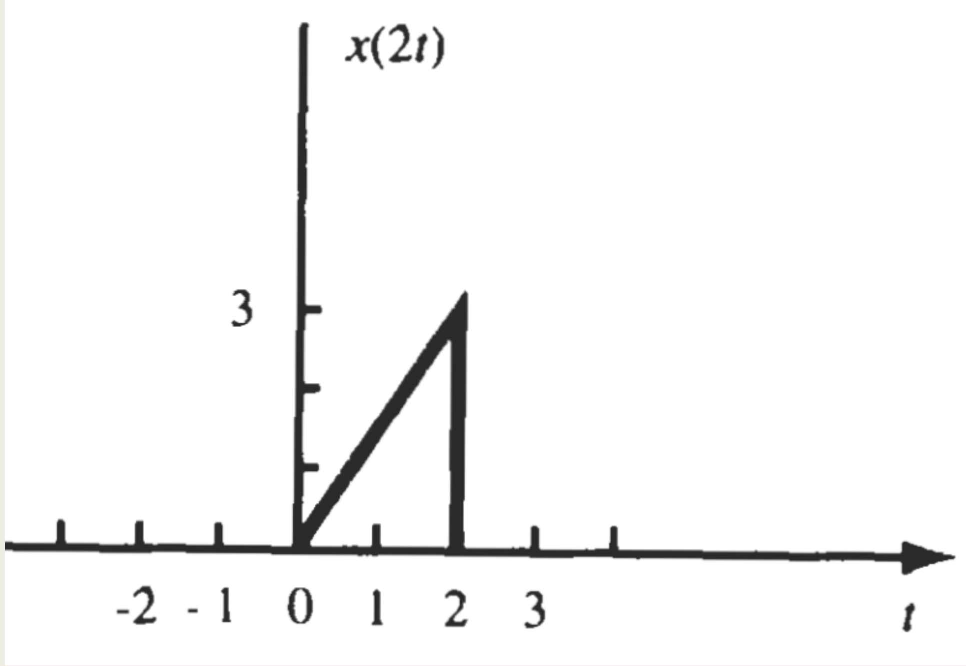
Exp 3.1: For the signal given, sketch $x(2t)$ and $x(t/2)$



$x(2t)$:

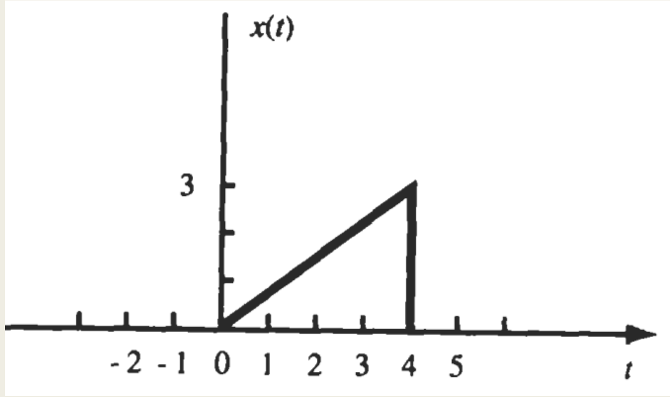
- 2t = 0 → t=0
- 2t = 1 → t=1/2=0.5
- 2t = 2 → t=2/2=1
- 2t = 3 → t=3/2=1.5
- 2t = 4 → t=4/2 = 2



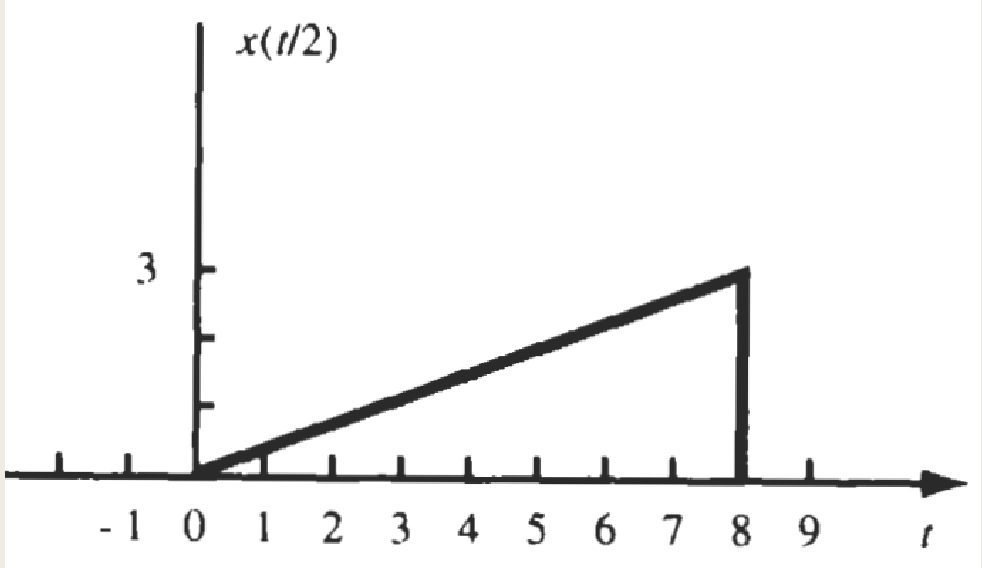


Signal is compressed by 2 times

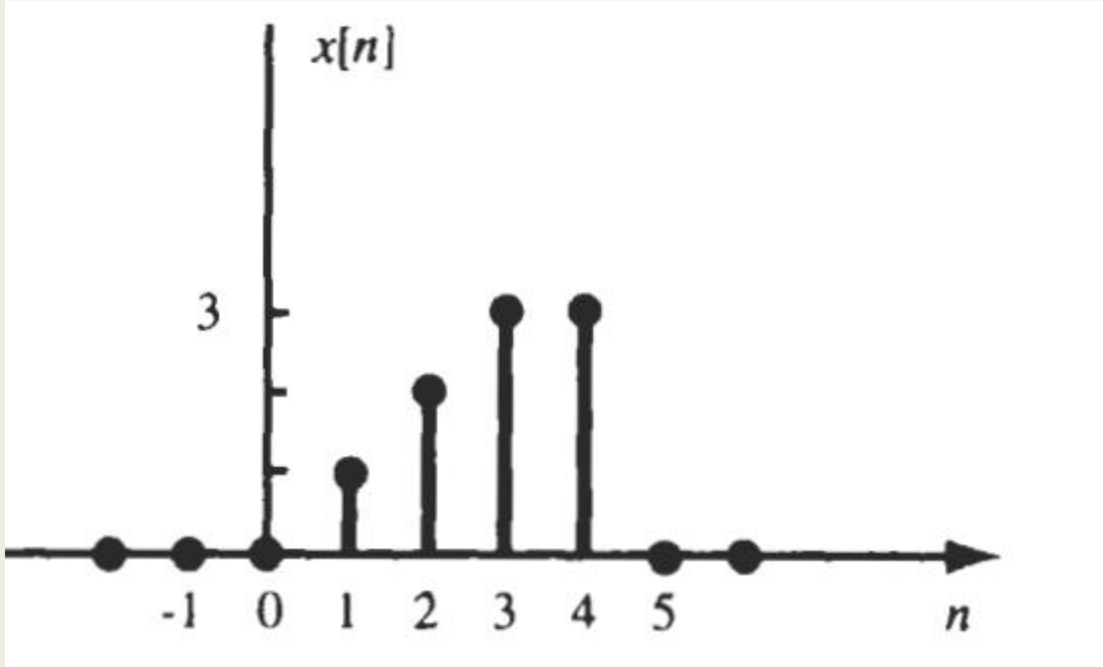
$x(t/2)$:



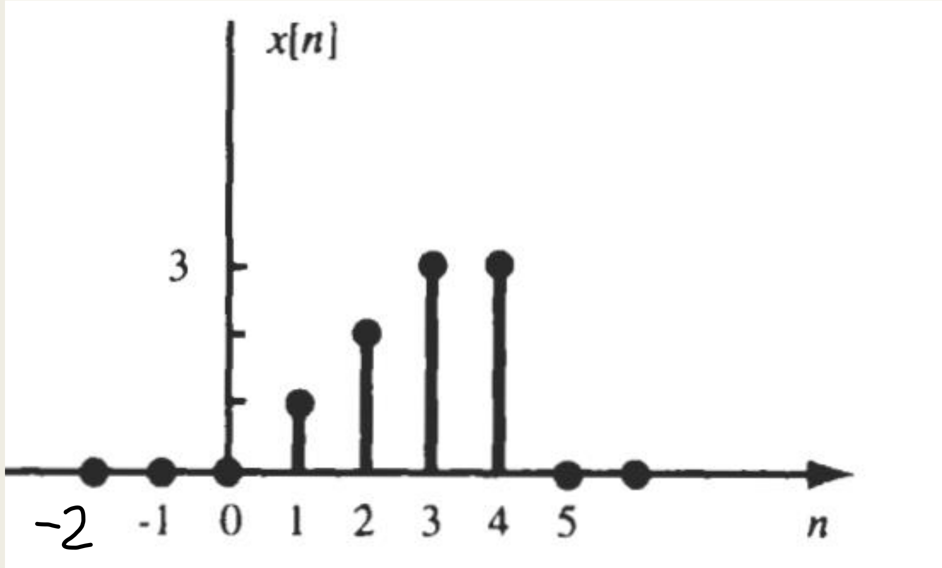
- t/2 = 0 → t=0
- t/2 = 1 → t=2x1=2
- t/2 = 2 → t=2x2=4
- t/2 = 3 → t=2x3=6
- t/2 = 4 → t=2x4=8



Exp 3.2: For the signal given, sketch $x[2n]$ and $x[n/2]$



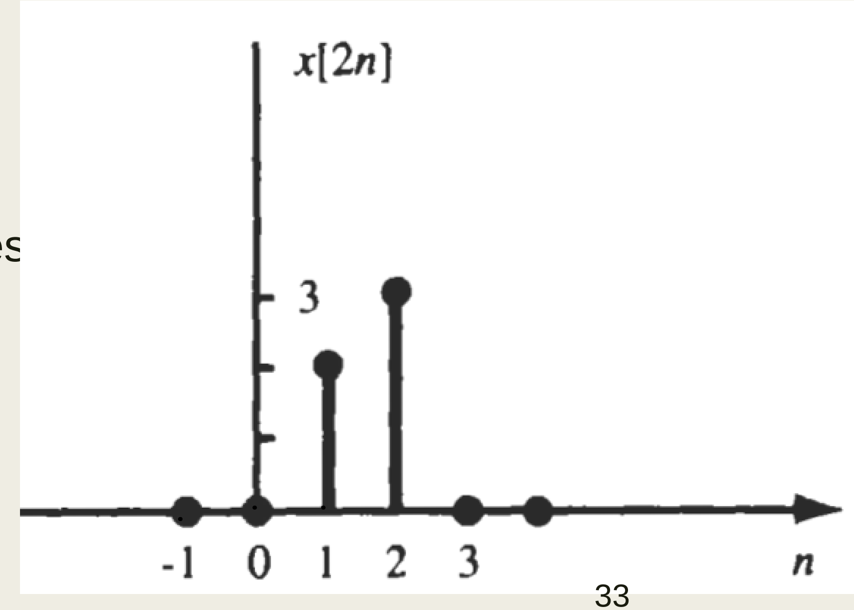
$x[2n]$ → Down-sampling



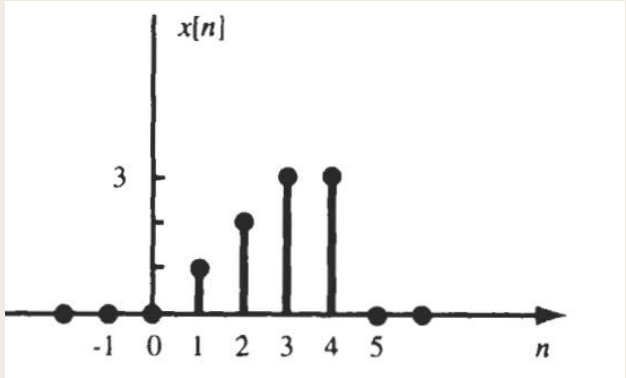
$$\frac{-2}{2} \quad \frac{-1}{2} \quad \frac{0}{2} \quad \frac{1}{2} \quad \frac{2}{2} \quad \frac{3}{2} \quad \frac{4}{2}$$

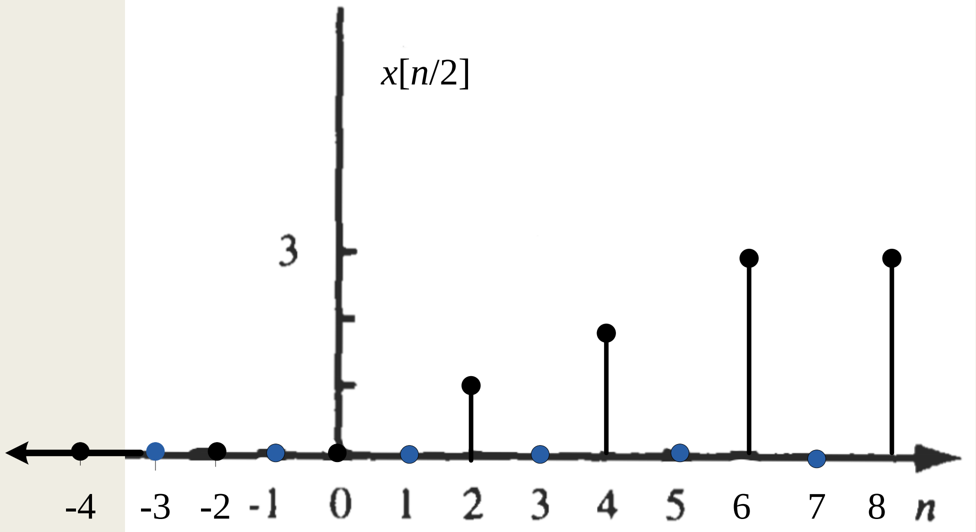
- 2n = -2 → n=-1
- 2n = -1 → n=-1/5=-0.5
- 2n = 0 → n=0
- 2n = 1 → n=1/2=0.5
- 2n = 2 → n=2/2=1
- 2n = 3 → n=3/2=1.5
- 2n = 4 → n=4/2 = 2

- Thus, for $x[2n]$, pick every 2nd sample and discard other samples
- Similarly for $x[3n]$, pick every 3rd sample and discard other samples



$x[n/2]$ → Up-sampling





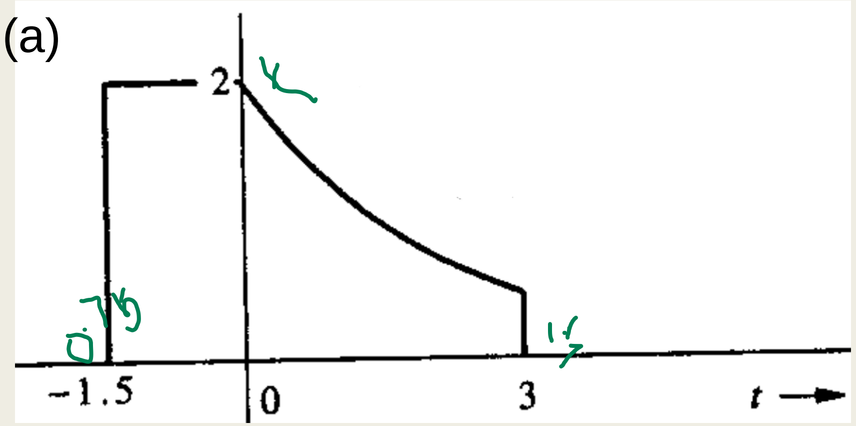
- n/2 = -2 → n=-4
- n/2 = -1 → n= -1 x 2= - 2
- n/2 = 0 → n=0
- n/2 = 1 → n=1 x 2= 2
- n/2 = 2 → n=2 x2=4
- n/2 = 3 → n=3 x 2=6
- n/2 = 4 → n=4 x 2 = 8

- Thus, $x[n/2]$→place samples at every 2nd place whereas zeros will be placed in between the samples
- Thus, $x[n/3]$→place samples at every 3rd place whereas zeros will be placed in between the samples

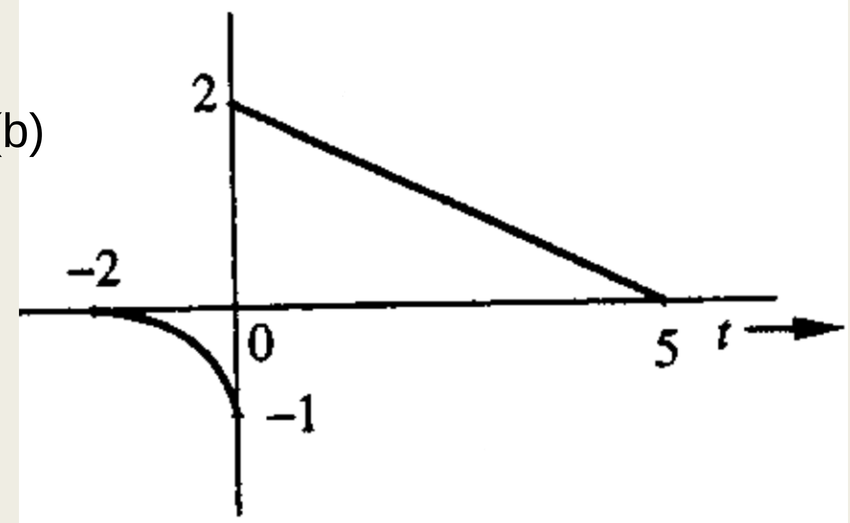
PP. 3.1) For signals given, sketch

(i) $x(2t)$ (ii) $x(t/3)$ (iii) $x(1.5t)$

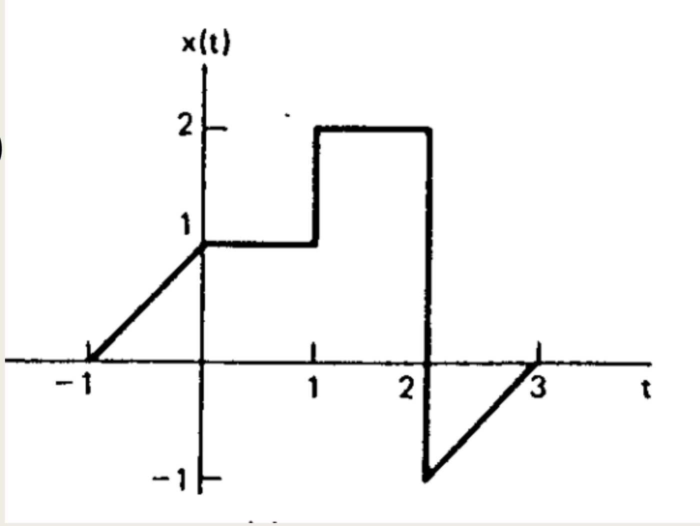
(a)



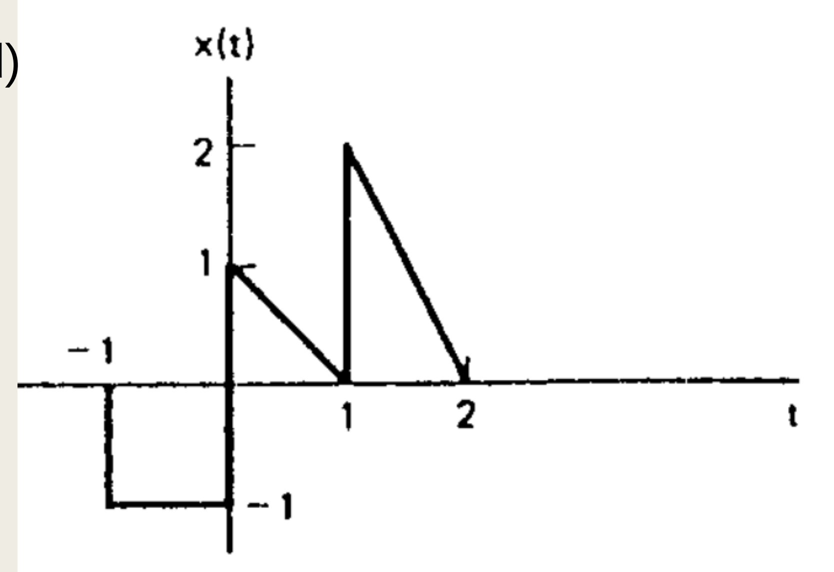
(b)



(c)

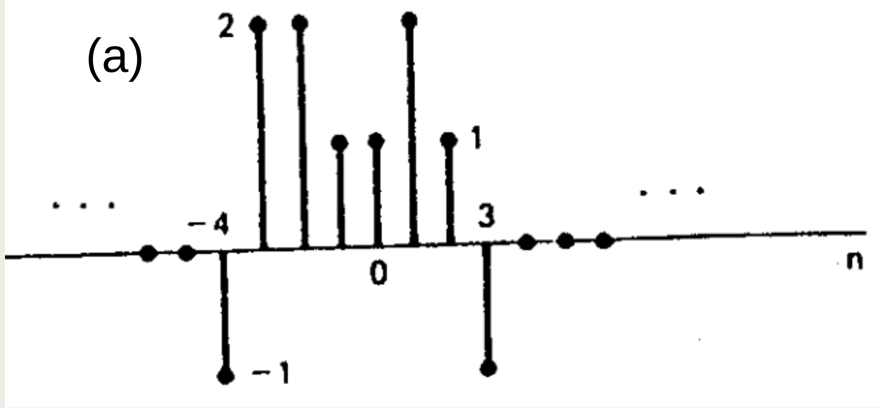


(d)

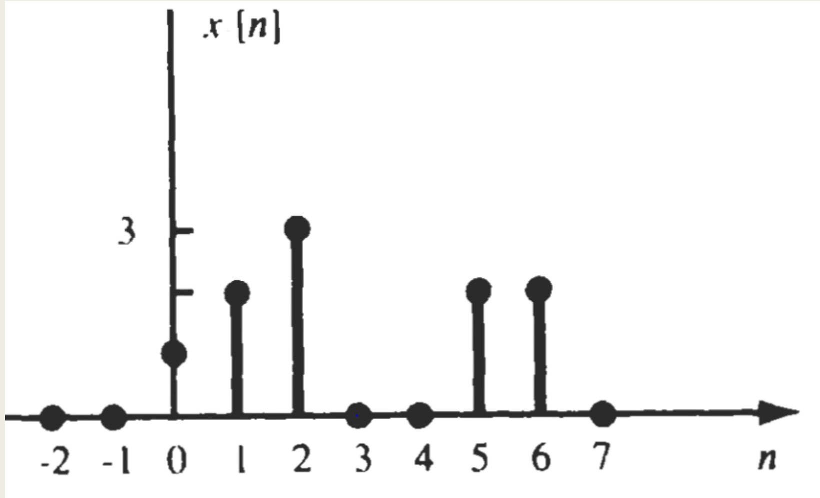


PP. 3.2) For signals given, sketch
(i) $x[2n]$ (ii) $x[3n]$ (iii) $x[n/3]$

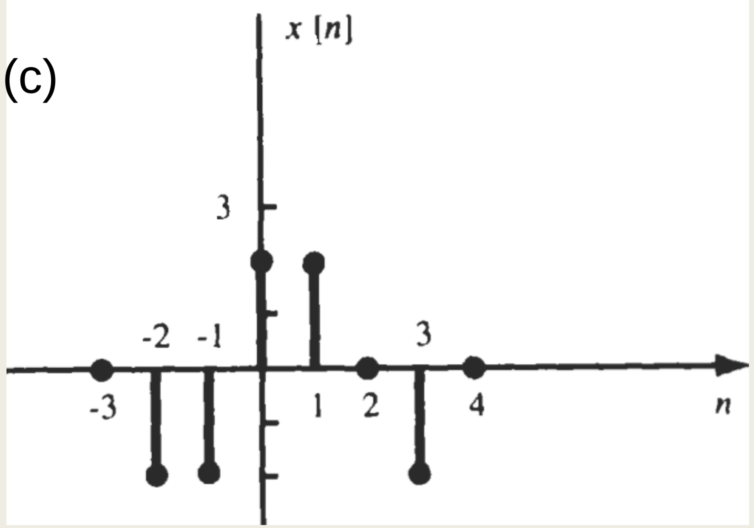
(a)



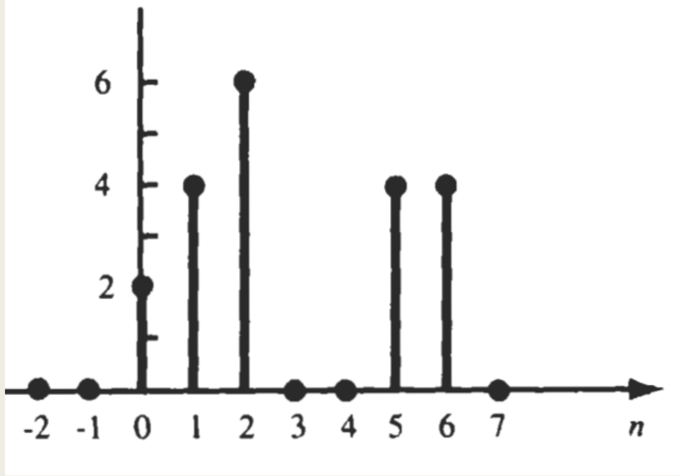
(b)



(c)



(d)



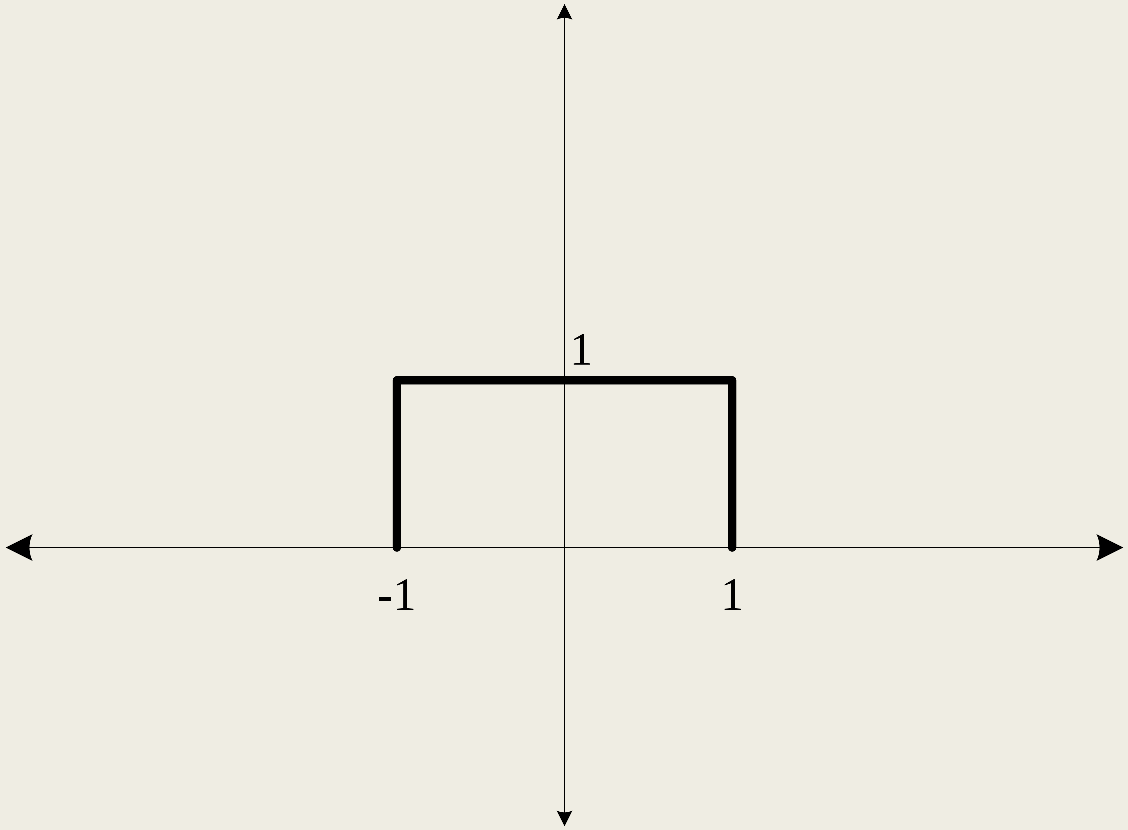
4. Precedence Rule for Combined Operations

- **e.g. Method 1:** $x(t)$ is the given signal
- i) Time shifting operation $x(t-k)$, $x(t+k)$
- ii) Time scaling operation $x(\alpha t-k)$, $x(\alpha t+k)$
- iii) Time flipping $x(-\alpha t-k)$, $x(-\alpha t+k)$

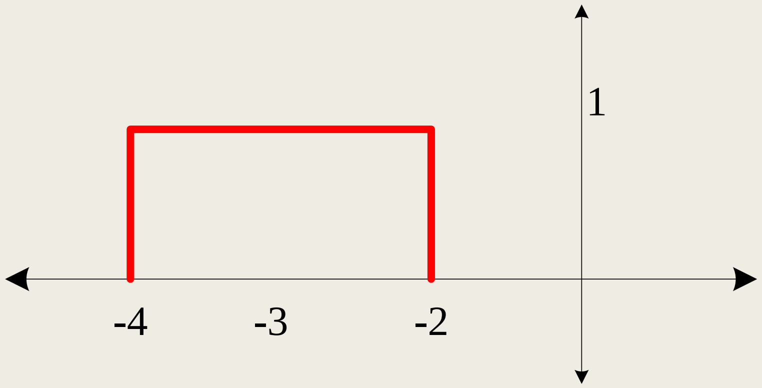
- **e.g. Method 2:** $x(t)$ is the given signal
- i) Time scaling operation $x(\alpha t)$
- ii) Time shifting operation $x(\alpha(t-k/\alpha)$, $x(\alpha(t+k/\alpha)$
- iii) Time flipping $x(-\alpha(t-k/\alpha)$, $x(-\alpha(t+k/\alpha)$

There is no precedence rule. You can apply operations in any order

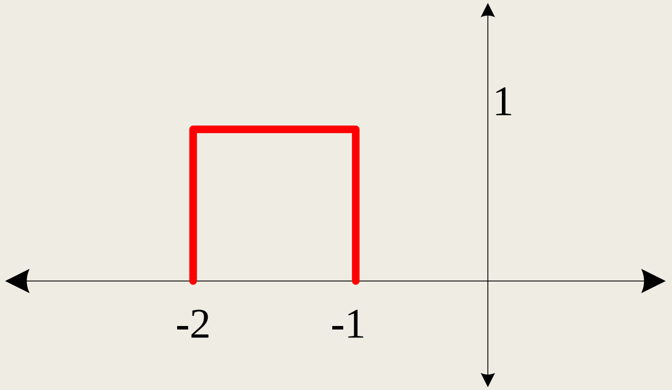
Exp 4.1: For the signal given, sketch $x(-2t + 3)$



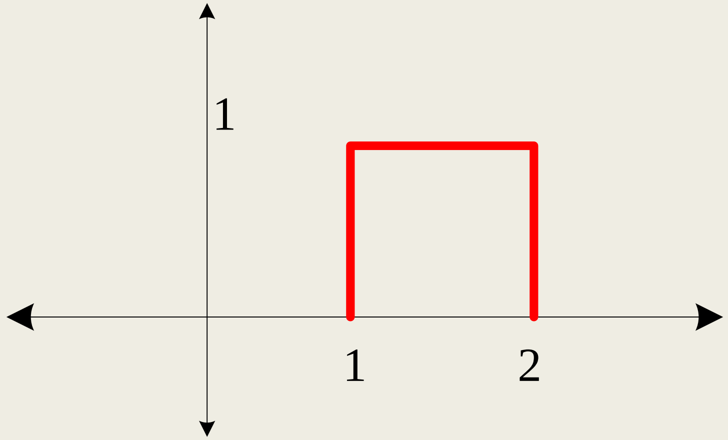
$x(t+3)$



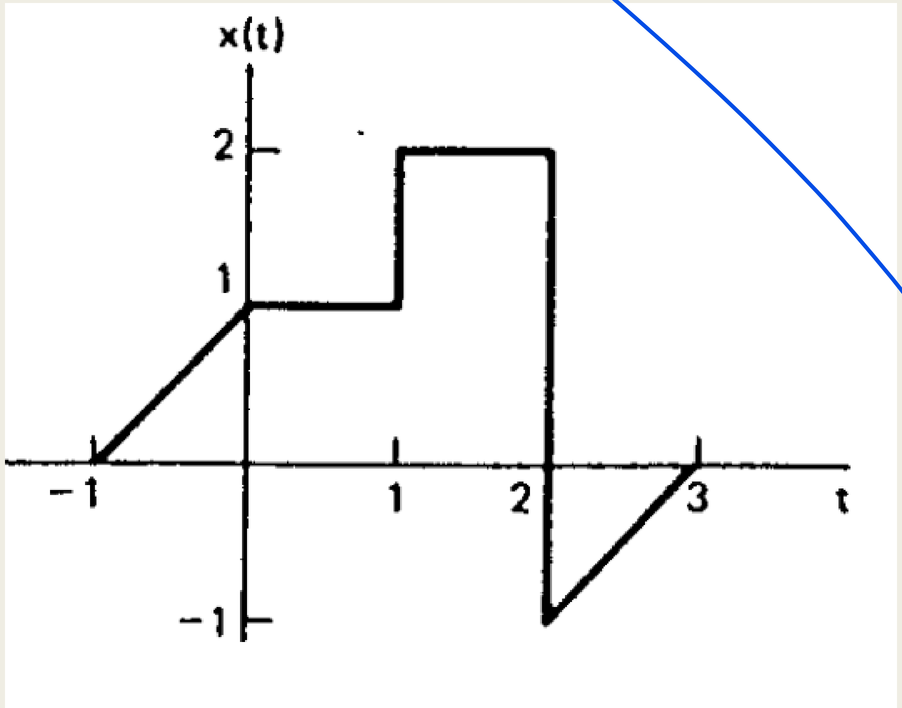
$x(2t+3)$



$x(-2t+3)$

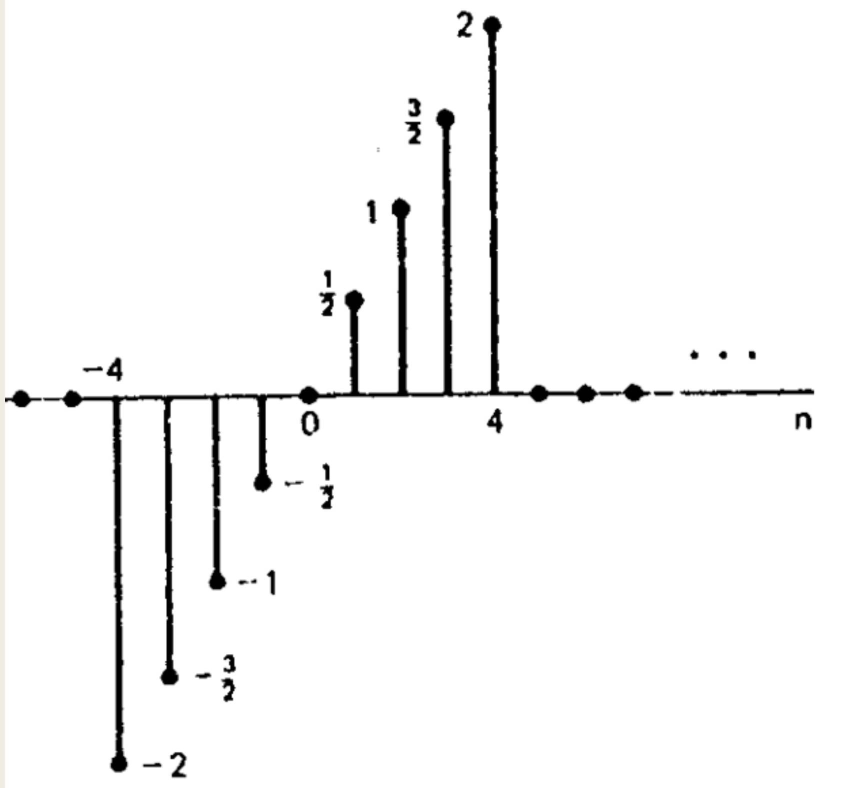


PP. 4.1) For signals given, sketch
(i) $x(t/2\ -2)$ (ii) $x(1-2t)$ (iii) $x(2-t/3)$



PP. 4.2) For signals given, sketch

(i) $x[4-n]$ (ii) $x[2n+1]$ (iii) $x[-\frac{n}{3}+2]$



Thank You !!!