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FA23-BCE-113

ASSIGNMENT 2:

Q 1:

$x(n) = \{1, 2, 1\}$ for $n = 0, 1, 2$

$h[n] = \{1, -1, 2\}$ for $n = 0, 1, 2$

Find $y[n] = x[n] * h[n]$

$x(n)$			$\underset{\uparrow}{1}$	2	1
$h(n)$	x		$\underset{\uparrow}{1}$	-1	2
			2	4	2
		-1	-2	-1	x
	1	2	1	x	x
$y[n] =$	1	1	1	3	2

$y[n] = \underset{\uparrow}{1} \quad 1 \quad 1 \quad 3 \quad 2$

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Q No 2:

$x(t) = u(t)$

$h(t) = e^{-t} u(t)$

Find $y(t) = x(t) * h(t)$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(\tau) = 1$ for $\tau \geq 0$

$h(\tau) = e^{-t}$ for $\tau \geq 0$

$h(t-\tau) = e^{-(t-\tau)}$

→ No overlap $(t < 0)$:

$y(t) = 0$

→ partial overlap $(0 \leq t < \infty)$

$y(t) = \int_0^t e^{-(t-\tau)}\, d\tau$

$y(t) = e^{-t} \int_0^t e^{\tau}\, d\tau$

$= e^{-t}(e^{+t} - 1)$

$y(t) = 1 - e^{-t}$

→ complete overlap

$y(t) = \int_0^t e^{-(t-\tau)}\, d\tau$

$$y(t) = 1 - e^{-t}$$

so

$$y(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-t} & t \geq 0 \end{cases}$$

Q NO 3:

$$x(t) = \begin{cases} 1 & 0 < t < T/2 \\ -1 & T/2 < t < T \end{cases}$$

Fourier series coefficient a_n:

$$a_n = \frac{1}{T} \int_0^T x(t)\, e^{-jnw_o t}\, dt$$

where $w_o = \frac{2\pi}{T}$

$$a_n = \frac{1}{T} \left[\int_0^{T/2} e^{-jnw_o t}\, dt + \int_{T/2}^{T} -e^{-jnw_o t}\, dt \right]$$

$$a_n = \frac{1}{T} \left[\left[\frac{e^{-jnw_o t}}{-jnw_o} \right]_0^{T/2} + \left[\frac{e^{-jnw_o t}}{-jnw_o t} \right]_{T/2}^{T} \right]$$

$$a_n = \frac{1}{T} \left\{ \frac{1}{-jnw_o} \left(e^{-jnw_o T/2} - 1 \right) \right\} + \left[\frac{1}{-jnw_o} \left(e^{-jnw_o T} - e^{-jnw_o T/2} \right) \right.$$

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$$a_n = \frac{1}{T} \cdot \frac{1}{jn\omega_0} \left[e^{-jn\omega_0 T/2} - 1 \cdot e^{-jn\omega_0 T} - e^{-jn\omega_0 T/2} \right]$$

$$a_n = \frac{1}{T} \cdot \frac{1}{jn\omega_0} \left[2e^{-jn\omega_0 T/2} - 1 - e^{-jn\omega_0 T} \right.$$

now:

$$\omega_0 = \frac{2\pi}{T} \quad \text{so} \quad e^{-jn\omega_0 T} = e^{-j2\pi n} = 1$$

$$a_n = \frac{1}{T} \frac{1}{-jn\omega_0} \left[2e^{-jn\pi} - 2 \right]$$

$$a_n = \frac{2}{T} \frac{e^{-jn\pi} - 1}{-jn\omega_0}$$

$$\because e^{-jn\pi} = \cos(n\pi) - j\sin(n\pi) = (-1)^n$$

$$a_n = \frac{2}{T} \frac{(-1)^n - 1}{-jn\omega_0}$$

if n is even $(-1)^n - 1 = 0$

so $a_n = 0$

else $a_n = 1$ for odd n.

So:

$$a_n = \begin{cases} 0 & \text{for } n \% 2 = 0 \text{ (even)} \\ 2/jn\pi & \text{for } n \% 2 \,!\, 0 \text{ (odd)} \end{cases}$$

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Q No 4:

$a_0 = 0$

$a_1 = j$

$a_{-1} = -j$

Find time domain signal

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{-jn\omega_0 t}$$

$$x(t) = j e^{j\omega_0 t} - j e^{-j\omega_0 t}$$

$$x(t) = j (e^{j\omega_0 t} - e^{-j\omega_0 t})$$

$$x(t) = 2\sin(\omega_0 t)$$