

LTI System Characterized by Difference Equation

Prob. An LTI system characterized by following DE:

$$y[n-1] - \frac{5}{2} y[n] + y[n+1] = x[n]$$

Taking z-transform,

$$Y(z) \cdot z^{-1} - \frac{5}{2} Y(z) + Y(z) \cdot z = X(z)$$

$$Y(z) \left[z^{-1} - \frac{5}{2} + z \right] = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{z - \frac{5}{2} + z^{-1}} = \frac{1}{z \left(1 - \frac{5}{2} z^{-1} + z^{-2} \right)}$$

$$H(z) = \frac{z^{-1}}{1 - \frac{5}{2} z^{-1} + z^{-2}} \quad \text{Transfer Function}$$

To analyze different system responses;

$$H(z) = \frac{z^{-1}}{1 - 2z^{-1} - \frac{1}{2}z^{-1} + z^{-2}} = \frac{z^{-1}}{1(1 - 2z^{-1}) - \frac{1}{2}z^{-1}(1 - 2z^{-1})}$$

$$H(z) = \frac{z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)(1 - 2z^{-1})} = \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 - 2z^{-1}}$$

$$z^{-1} = A(1 - 2z^{-1}) + B\left(1 - \frac{1}{2}z^{-1}\right)$$

Comparing powers;

$$z^{-1} \Rightarrow 1 = -2A - \frac{1}{2}B \Rightarrow 2 = -4A - B \quad \text{--- (1)}$$

$$z^0 \Rightarrow 0 = A + B \quad \text{--- (2)}$$

$$\text{(1) + (2)} \Rightarrow$$

$$2 = -4A - B$$

$$0 = A + B$$

$$2 = -3A \Rightarrow$$

$$A = -\frac{2}{3}$$

put in (2)

$$-\frac{2}{3} + B = 0 \Rightarrow B = \frac{2}{3}$$

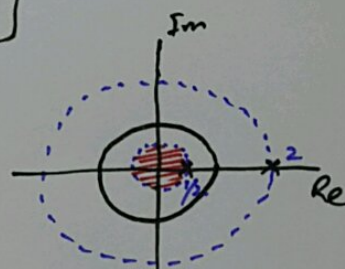
$$H(z) = \frac{-2/3}{1 - \frac{1}{2}z^{-1}} + \frac{2/3}{1 - 2z^{-1}}$$

Since no information provided about the system (like ~~causal~~ causality, stability), therefore based on pole locations, we can characterize the system types.

[1] $H(z) = \frac{-2/3}{1 - \frac{1}{2}z^{-1}} + \frac{2/3}{1 - 2z^{-1}}, \quad |z| < 1/2 \text{ (all LSS)}$

$$h[n] = \frac{2}{3} \left(\frac{1}{2}\right)^n u[-n-1] - \frac{2}{3} (2)^n u[-n-1]$$

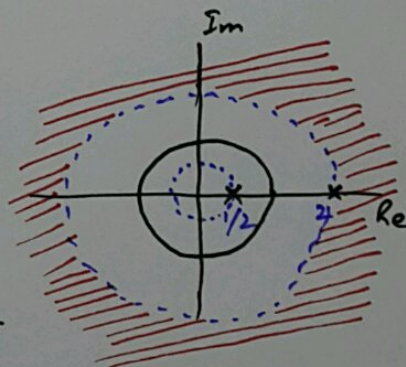
Stable = NO
Causal = NO
Anti-Causal = YES



[2] $H(z) = \frac{-2/3}{1 - \frac{1}{2}z^{-1}} + \frac{2/3}{1 - 2z^{-1}}, \quad |z| > 2 \text{ (all RSS)}$

$$h[n] = -\frac{2}{3} \left(\frac{1}{2}\right)^n u[n] + \frac{2}{3} (2)^n u[n]$$

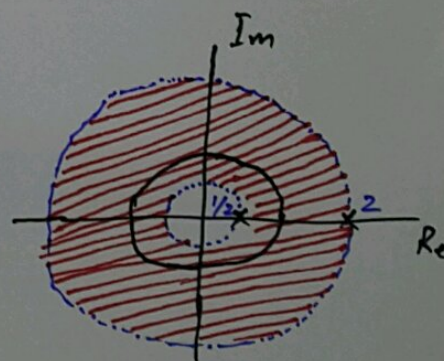
Stable = NO
Causal = YES
Anti-Causal = NO



[3] $H(z) = \frac{-2/3}{1 - \frac{1}{2}z^{-1}} + \frac{2/3}{1 - 2z^{-1}}, \quad \frac{1}{2} < |z| < 2 \text{ (Disc)}$
RSS LSS

$$h[n] = -\frac{2}{3} \left(\frac{1}{2}\right)^n u[n] - \frac{2}{3} (2)^n u[-n-1]$$

Stable = YES
Causal = NO
Anti-Causal = NO



If we are asked to seek for different combinations, like;

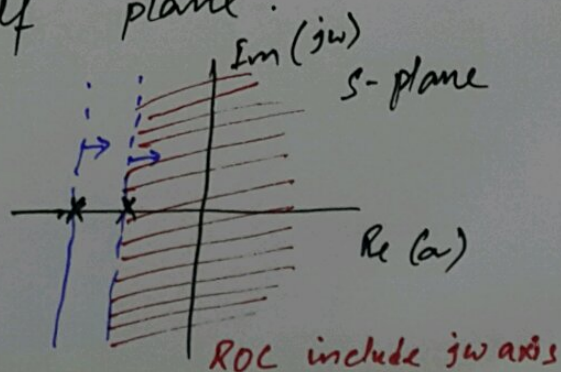
- 1 - Stable System = YES (Case 3)
- 2 - Causal System = YES (Case 2)
- 3 - Anti-Causal System = YES (Case 1)
- 4 - Causal and Stable = NO (Not possible because of pole at 2)
- 5 - Causal and Unstable = YES (Case 2)
- 6 - Anticausal and Stable = NO (NOT POSSIBLE)
because of pole at $\frac{1}{2}$
- 7 - Anticausal and Unstable = YES (Case 1)

General Requirement of LTI System:

"Causal and Stable"

Laplace Transform

poles should be on left half plane.



Z-transform

poles should be inside the unit circle.

