day / date

Lec #12

4.5 **Multiplication property:**

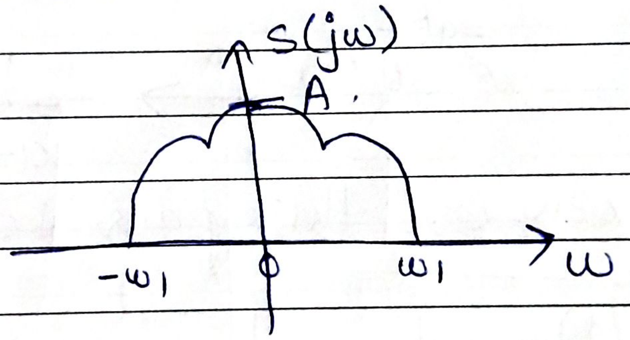
$$r(t) = s(t)\,p(t) \xrightarrow{F.T} R(j\omega) = \frac{1}{2\pi}\left[S(j\omega) * P(j\omega)\right]$$

Lets understand this property through example.

Example 4.21

$$P(t) = \cos \omega_0 t$$

and



and

$$r(t) = s(t)\,p(t)$$

find $r(t) = ?$

In order to solve this Question, we have two options:-

option #1: Solve in time domain:

+ → Simple multiplication is required.

- → But before multiplication, we have to find S(t) because we are given p(t) but with S(jw).

Finding S(t) is very difficult, because we need an equation for this.

option #2 Solve in frequency Domain.

+ → S(jw) is already in Frequency Domain.

+ → we know P(jw) can be calculated easily, because its in form of equation and can be calculated easily.

- → will have to perform convolution in frequency domain.

(Multiplication) x (time Domain) $\xrightarrow{F.T}$ * (Convolution) (Frequency Domain)

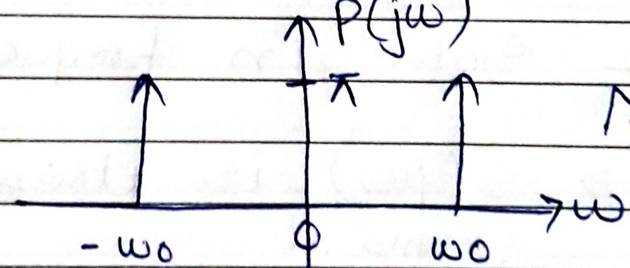
← and vice versa.

Therefore adopting option #2.

$p(t) = \cos\omega_0 t$ is periodic in nature. It's fourier Transform calculation can be consulted from Lec#9.

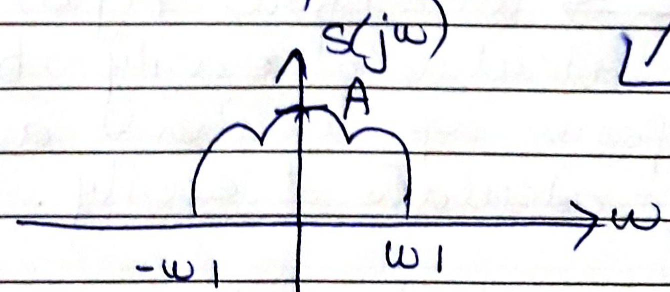
Therefore

$$P(j\omega) = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$$

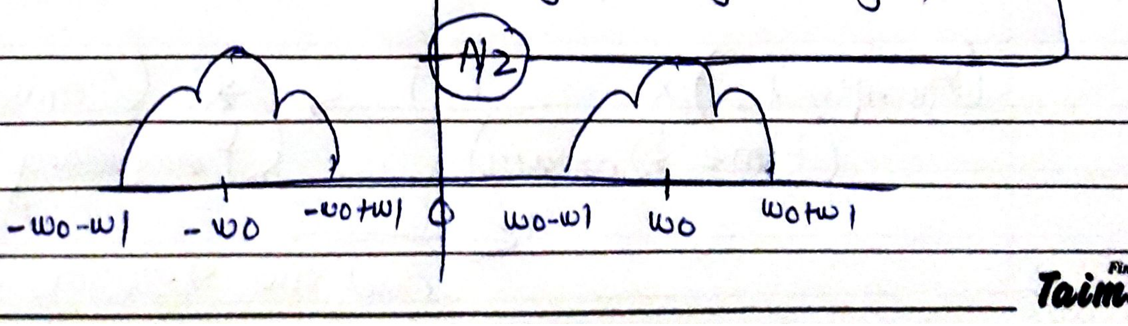


* Convolve toget R(jw)

and



$R(j\omega) = P(j\omega) * S(j\omega)$



Convolution with impulses is very easy.
As it shifts the original signal to the location of the impulses.

A/2 $\longrightarrow$ how?

$P(j\omega)$ ~~An~~ $\longrightarrow \pi$.

$S(j\omega) \longrightarrow A$

and from formula

$$r(t) = s(t)\,p(t) \xrightarrow{F.T} \cancel{\tfrac{1}{2}}\ R(j\omega) = \frac{1}{2\pi}\left[S(j\omega) * P(j\omega)\right]$$

$$= \left(\frac{1}{2\cancel{\pi}}\right)\left(\cancel{\pi} \times A\right) = A/2.$$

$R(j\omega)$ can be written in form of equation as:-

$$R(j\omega) = \frac{A}{2}\left[S(j(\omega - \omega_0)) + S(j(\omega + \omega_0))\right]$$

Apply shifting property.

and in time domain:-

$$r(t) = A/2\left[s(t)\,e^{-j\omega_0 t} + s(t)\,e^{+j\omega_0 t}\right]$$ ——— *

v.v. imp Plz Attempt Example 4.22 yourself.