Prob :-

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$$

Compute.

A. Transfer function $H(s)$

B. Find all possible impulse responses $h(t)$

C. Find impulse response $h(t)$ for each of the following cases
   (i) Causal
   (ii) Stable
   (iii) Neither Causal nor Stable
   (iv) Both causal and stable

Solution :- $\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$

A. Transfer function :-

Apply property on differential equation

$$s^2 Y(s) + 4sY(s) + 3Y(s) = 2sX(s) + 4X(s)$$

$$\left(s^2 + 4s + 3\right)Y(s) = (2s + 4)X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{2s+4}{s^2+4s+3}$$

$$\therefore H(s) = \frac{2s+4}{s^2+4s+3}$$

Apply partial fraction

$$\frac{2S+4}{(S+3)(S+1)} = \frac{A}{S+3} + \frac{B}{S+1}$$

Solve it , you will get

$$A = 1 , \qquad B = 1$$

∴ $$H(S) = \frac{1}{S+3} + \frac{1}{S+1}$$

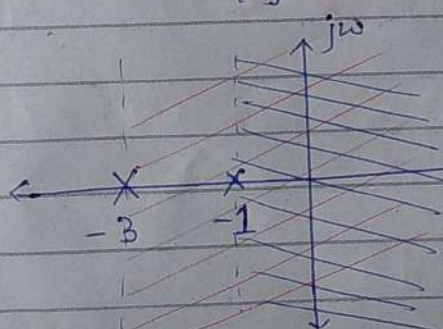**B.** All possible Combinations of impulse responses $h(t)$.

(i) When both sequences are right sided.

$$H(S) = \frac{1}{S+3} + \frac{1}{S+1}$$

$$\qquad\qquad RSS \qquad\qquad RSS$$

pole : $\qquad -3 \qquad\qquad -1$

ROC : $Re\{s\} > -3 \qquad Re\{s\} > -1$

$$\frac{1}{S+3} \xrightarrow{RSS} e^{-3t} u(t)$$
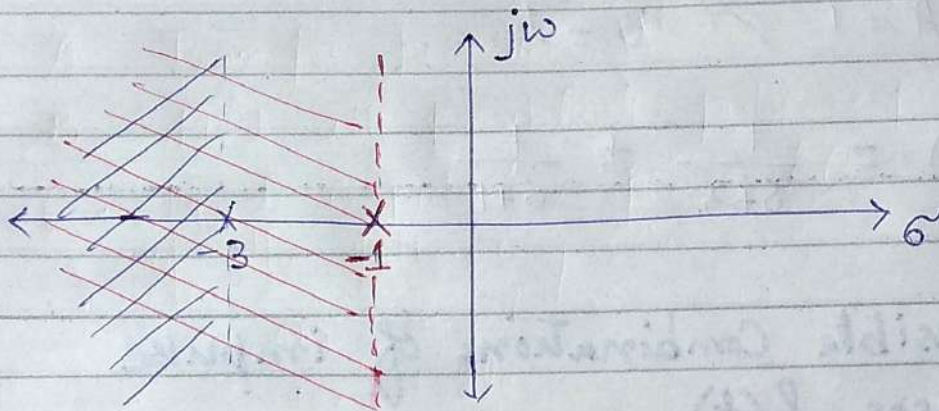
$$\frac{1}{S+1} \xrightarrow{RSS} e^{-t} u(t)$$

∴ ROC of $H(S)$: $Re\{s\} > -1$

$$h(t) = e^{-3t} u(t) + e^{-t} u(t)$$

(ii) When both sequences are left sided. ⑧

$$H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$

pole : -3 -1

ROC : $Re\{s\} < -3$ $Re\{s\} < -1$

ROC of $H(s)$: $Re\{s\} < -3$

$$\frac{1}{s+3} \xrightarrow{\ LSS\ } -e^{-3t} u(-t)$$

$$\frac{1}{s+1} \xrightarrow{\ LSS\ } -e^{-t} u(-t)$$

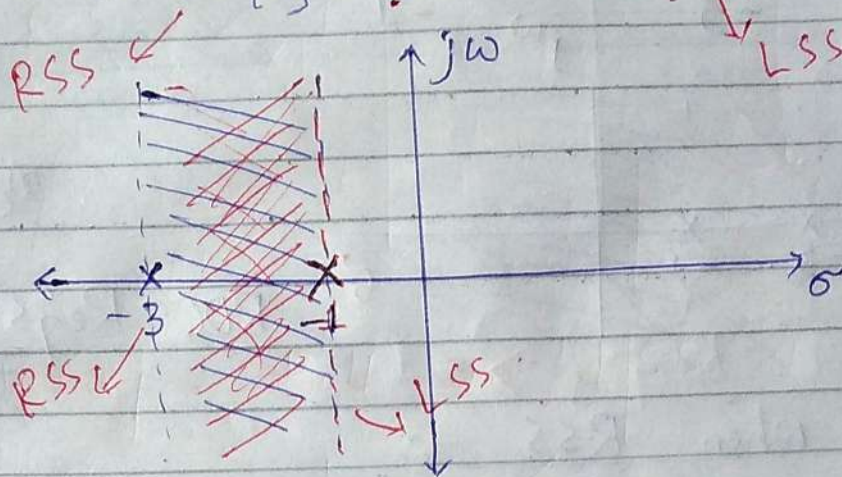$$h(t) = \left[-e^{-3t} u(-t)\right] + \left[-e^{-t} u(-t)\right]$$

(iii)   When ROC is in the form of strip ... ④

$$H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$

pole :   -3   -1
ROC : $Re\{s\} > -3$ $Re\{s\} < -1$
                                        $\downarrow$ LSS

ROC of $H(s)$: $-3 < Re\{s\} < -1$

$\frac{1}{s+3} \xrightarrow{\text{RSS}} e^{-3t} u(t)$

$\frac{1}{s+1} \xrightarrow{\text{LSS}} -e^{-t} u(-t)$

$\therefore h(t) = \left[e^{-3t} u(t) \right] + \left[-e^{-t} u(-t) \right]$
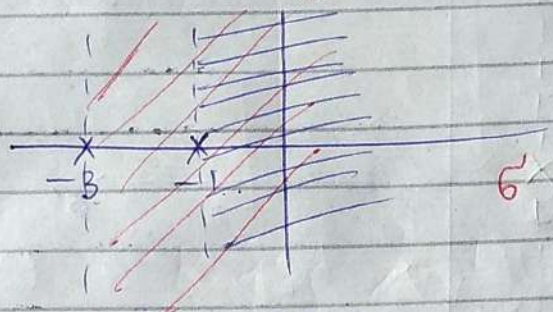
C. find $h(t)$ when system is ; ㉟

(i) Causal

$$H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$

right most
pole.

ROC: $Re\{s\} > -1$
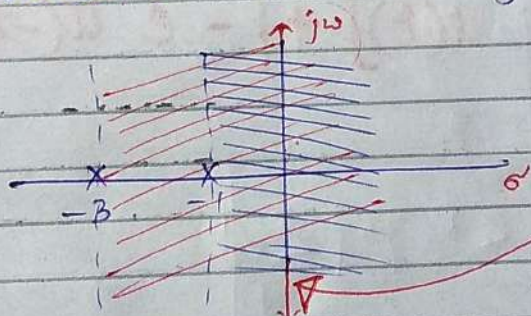
Causal when RSS :-

$$\frac{1}{s+3} \xrightarrow{RSS} e^{-3t} u(t)$$

$$\frac{1}{s+1} \xrightarrow{RSS} e^{-t} u(t)$$

$$h(t) = e^{-3t} u(t) + e^{-t} u(t)$$

(ii) Stable :- includes jw-axis
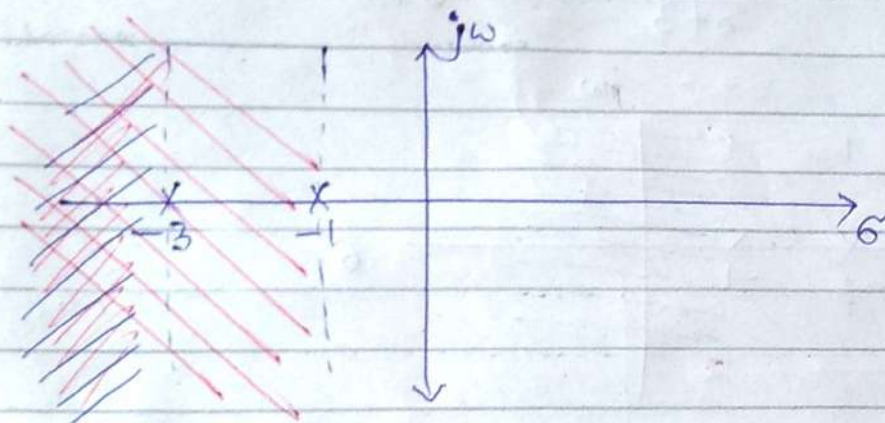

jw-axis is included
in ROC

$$h(t) = e^{-3t} u(t) + e^{-t} u(t)$$

$\downarrow$ $\downarrow$

RSS   RSS

(iii) Neither Causal Nor Stable :- (4)
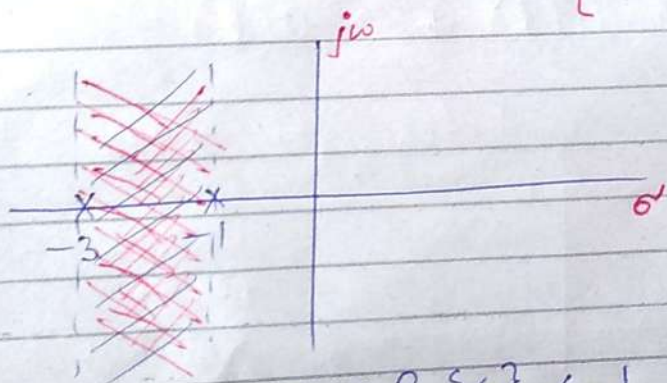
$$H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$

One way :- Both are LSS

ROC : $Re\{s\} < -3$

$$h(t) = \left[-e^{-3t} u(-t)\right] + \left[-e^{-t} u(-t)\right]$$

2nd way : ROC is strip $\begin{cases} \text{one is LSS} \\ \text{one is RSS} \end{cases}$

ROC : $\underbrace{-3}_{RSS} < Re\{s\} < \underset{LSS}{-1}$

$$h(t) = \left[e^{-3t} u(t)\right] + \left[-e^{-t} u(-t)\right]$$

(iv)   Both   Stable   & Causal.   (5)

$$H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$

ROC : $Re\{s\} > -1$

$$h(t) = \left[e^{-3t} u(t) \right] + \left[e^{-t} u(t) \right]$$