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FA23-BCF-113

ASSIGNMENT - 02

Q 1:

We have two coins A & B, when we chose a coin random:

$P[A] = 1/2$, $P[B] = 1/2$

Coin A comes up head with probability $1/4$

$P[H|A] = \frac{1}{4}$, $P[T|A] = 1 - \frac{1}{4} = \frac{3}{4}$

Coin B comes up with head probability $3/4$

$P[H|B] = \frac{3}{4}$, $P[T|B] = 1 - \frac{3}{4} = \frac{1}{4}$

If the flip is head you guess The flip coin is B.

Case1 → we flips head and guess B (correct if B)

So

Case2 → we flip tail and guess A (correct if A).

Case 1:

$P[B|H] = P[H|B] \times \frac{P[B]}{P[H]}$ — ①

$P[H] = ?$

$P[H] = P[H \cap A] + P[H \cap B]$ ∴ $P[A \cap B] = P[A|B] \times P(B)$

$P[H] = P[H|A] \times P(A) + P[H|B] \times P[B]$

$= \frac{1}{4} \times \frac{1}{2} + \frac{3}{4} \times \frac{1}{2}$

$P[H] = \frac{1}{2}$ So, $P[T] = 1 - P[H] = 1 - \frac{1}{2} = \frac{1}{2}$

put values in eq 1

$P[B|H] = \frac{3}{4} \times \frac{1/2}{1/2} = 3/4$

$P[B|H] = 3/4$

Case 2:

~~$P[T|A] = P[A|T] \times$~~

$$P[A|T] = P[T|A] \times \frac{P[A]}{P[T]} \quad -② $$

$P[T] = ?$

$$P[T] = 1 - P[H]$$
$$= 1 - 1/2$$
$$P[T] = 1/2$$

putting values in eq 2

$$P[A|T] = \frac{3}{4} \times \frac{1/2}{1/2}$$

$$P[A|T] = \frac{3}{4}$$

Alternative Method:

We know:

$P[B|H] = 3/4$ So,

$$P[B|T] = 1 - \frac{3}{4} = 1/4$$

$$P[A|T] = 1 - P[B|T]$$
$$= 1 - 1/4$$
$$P[A|T] = 3/4$$

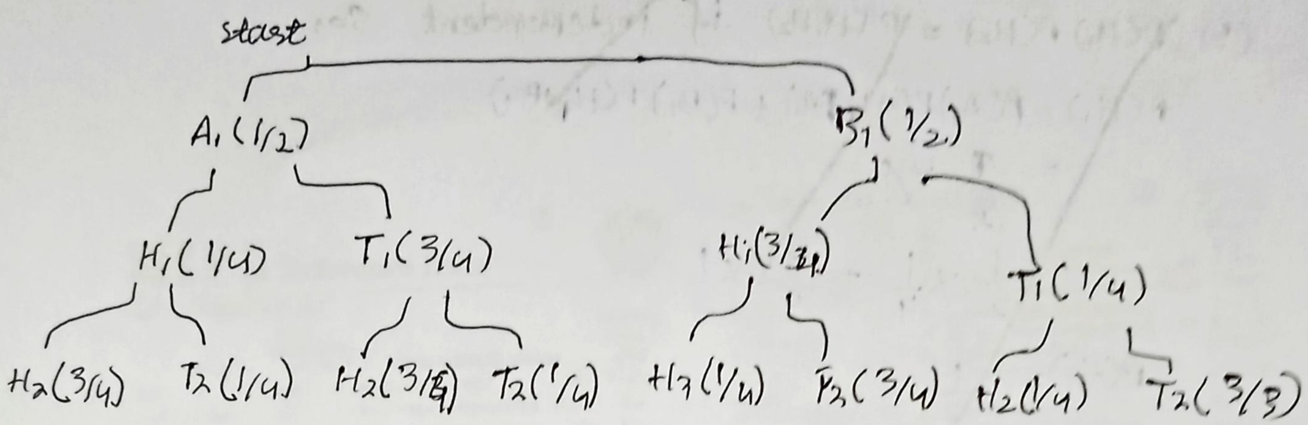
Now we compute $P[C]$ ∵ correct guess

$$P[C] = P(A \cap T) + P(B \cap H)$$
$$= (P(A|T) \times P(T)) + (P[B|H] \times P(H))$$
$$= \frac{3}{4} \times \frac{1}{2} + \frac{3}{4} \times 1/2$$
$$= \frac{3}{4}\left(\frac{1}{2} \times 1/2\right) = 3/4\,(1)$$

$$\boxed{P[C] = 3/4}$$

Q2:

From the given information in Question let first draw tree for probability distribution for better understanding of problem:



① Case 1: A_1 is chosen first

$P[H_1 | A_1] = 1/4$

$P[H_2 | A_1] = \frac{3}{4}$

now, B is second coin

$P[H | B] = 3/4$

$P[C_1] = 1/4 + 3/4 = \frac{3}{16}$

② case 2: B_1 is chosen first

$P[H_1 | B_1] = 3/4$

$P[H_2 | B_1] = 1/4$

now A is second coin

$P[H | A] = 1/4$

$P[C_2] = 3/4 + 1/4 = 3/16$

(a) $P(H_1 H_2) = ?$

Now:

$P(H_1 H_2) = P(A_1) \times P(C_1) + P(B_1) \times P(C_2)$

$= \frac{1}{2} \times \frac{3}{16} + \frac{1}{2} \times \frac{3}{16}$

$\boxed{P(H_1 H_2) = \frac{3}{16}}$

(b) $P(H_1)\,P(H_2) = P(H_1, H_2)$ if independent so,

$P(H_1) = P(A_1)\,P(H_2|A_1) + P(B_1)\,P(H_1|B_1)$

$= \frac{B}{B} \times A$

$= \frac{1}{2} \times \frac{1}{4} + \frac{3}{4} \times \frac{1}{2}$

$P(H_1) = \frac{1}{2}$

$P(H_2) = P(A_1)\,P(H$

(b) $P(H_1)$ $P(H_2)$, are event H_1, H_2 independent

$P(H_1) = P[A_1] \cdot P[H_1|A_1] + P[B_1]\,P[H_1|B_1]$

$= \frac{1}{2} \times \frac{1}{4} + \frac{3}{4} \times \frac{1}{2}$

$= \frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

$P[H_1] = \frac{1}{2}$

$P(H_2) = P[A_1] \cdot P[H_2|A_1] + P[B_1]\,P[H_2|B_1]$

$= \frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{4}$

$= \frac{3}{8} + \frac{1}{8} = \frac{4}{8}$

$P[H_2] = \frac{1}{2}$

Alternativley

$P[T_1] = 1 - P[H_1]$

$= 1 - \frac{1}{2}$

$P[T_1] = 1/2$

$P(T_1) = P[H_2]$

$P[H_2] = \frac{1}{2}$

Now

$P(H_1) \cdot P(H_2) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$\boxed{P[H_1] \cdot P[H_2] = \frac{1}{4}}$

CHECK for Independent:

$P(H_1 H_2) = P[H_1]\,P[H_2]$

$\frac{3}{16} \neq \frac{1}{4}$

So event H_1 and H_2 are not independent.