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Lec#11

4.4 The Convolution Property:-

$$x(t) \xrightarrow{F} X(j\omega)$$
$$h(t) \xrightarrow{F} H(j\omega)$$

$$y(t) = \underline{x(t) * h(t)} \xrightarrow{F} Y(j\omega) = \underline{H(j\omega)\, X(j\omega)}$$

Convolution   multiplication

Example 4.19

$$h(t) = e^{-at} u(t) \quad a>0$$

$$x(t) = e^{-bt} u(t) \quad b>0$$

find $y(t) = x(t) * h(t)$

Now instead of solving $x(t)$ and $h(t)$ in time domain, where, we will have to convolve, why not take them to frequency domain where, we just have to multiply their Fourier Transforms and final ans can be converted back to time domain to get $y(t)$ back.

See Solution:

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As we already know:-

$$h(t) = e^{-at} u(t) \xrightarrow{F.T} H(j\omega) = \frac{1}{a+j\omega}$$

$$x(t) = e^{-bt} u(t) \xrightarrow{F.T} X(j\omega) = \frac{1}{b+j\omega}$$

$$Y(j\omega) = H(j\omega)\, X(j\omega)$$

$$Y(j\omega) = \left(\frac{1}{a+j\omega}\right)\left(\frac{1}{b+j\omega}\right) \quad \text{—— } A$$

Now in order to Apply the property of Linearity, the above equation A needs to be converted in added form that can be done by partial fraction

$$\left(\frac{1}{a+j\omega}\right)\left(\frac{1}{b+j\omega}\right) = \frac{A}{a+j\omega} + \frac{B}{b+j\omega}$$

Solve for A and B

$$A = \frac{1}{b-a} \qquad \boxed{A = -B}$$

$$B = \frac{1}{a-b}$$

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Now Substituting back the values, we get:-

$$Y(j\omega) = \frac{(1/b-a)}{a+j\omega} + \frac{(1/a-b)}{b+j\omega}$$

$$Y(j\omega) = \frac{1}{b-a}\left[\frac{1}{a+j\omega} - \frac{1}{b+j\omega}\right] \text{ — B}$$

Now converting B back to Time domain ~~to get~~ back $y(t)$

As we know the pairs

$$e^{-at}u(t) \longrightarrow \frac{1}{a+j\omega}$$

I can use this pair to get $y(t)$ as:-

$$\boxed{y(t) = \frac{1}{b-a}\left[e^{-at}u(t) - e^{-b(t)}u(t)\right]} \text{ — *}$$

is the final Answer.