Lec # 17

Prob :-

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$$

Compute.

A. Transfer function $H(s)$

B. Find all possible impulse responses $h(t)$

C. Find impulse response $h(t)$ for each of the following cases

(i) Causal

(ii) Stable

(iii) Neither Causal nor Stable

(iv) Both causal and stable

Solution :- $\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$

A. Transfer function :-

Apply property on differential equation

$$s^2Y(s) + 4sY(s) + 3Y(s) = 2sX(s) + 4X(s)$$

$$(s^2 + 4s + 3)Y(s) = (2s + 4)X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{2s+4}{s^2+4s+3}$$

$$\therefore H(s) = \frac{2s+4}{s^2+4s+3}$$

Apply partial fraction

$$\frac{2s+4}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1}$$

Solve it, you will get

$A = 1$, $B = 1$

$$\therefore \quad H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$

B. All possible Combinations of impulse responses $h(t)$.

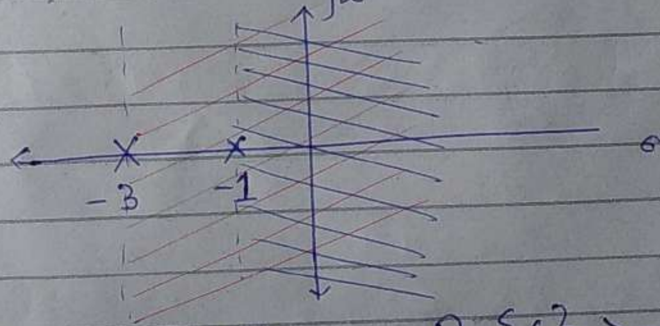
(i) When both sequences are right sided.

$$H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$

RSS RSS

pole : -3 -1

ROC : $Re\{s\} > -3$ $Re\{s\} > -1$



$\frac{1}{s+3} \xrightarrow{RSS} e^{-3t}u(t)$

$\frac{1}{s+1} \xrightarrow{RSS} e^{-t}u(t)$

$\therefore$ ROC of $H(s)$: $Re\{s\} > -1$

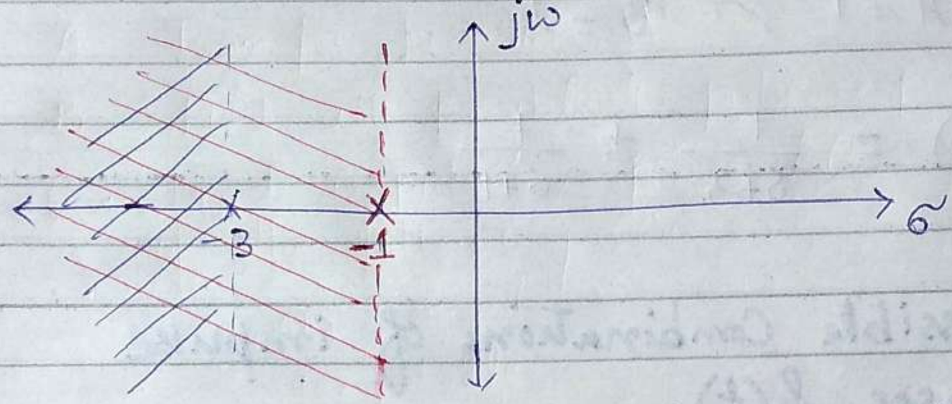
$$h(t) = e^{-3t}u(t) + e^{-t}u(t)$$

(ii) When both sequences are left sided (8)

$$H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$

pole : -3 $\quad\quad$ -1

ROC : $Re\{s\} < -3$ $\quad\quad$ $Re\{s\} < -1$



ROC of $H(s)$: $Re\{s\} < -3$

$$\frac{1}{s+3} \xrightarrow{LSS} -e^{-3t}u(-t)$$

$$\frac{1}{s+1} \xrightarrow{LSS} -e^{-t}u(-t)$$

$$h(t) = \left[-e^{-3t}u(-t)\right] + \left[-e^{-t}u(-t)\right]$$

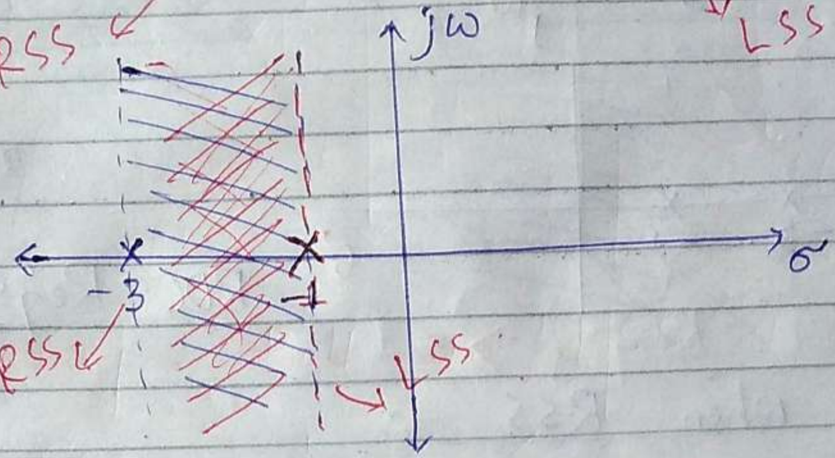
(iii) When ROC is in the form of strip (9)

$$H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$

pole : -3 $\quad -1$

ROC : $Re\{s\} > -3$ $\quad Re\{s\} < -1$

RSS $\quad$ LSS



ROC of H(s) : $-3 < Re\{s\} < -1$

$$\frac{1}{s+3} \xrightarrow{RSS} e^{-3t}u(t).$$

$$\frac{1}{s+1} \xrightarrow{LSS} -e^{-t}u(-t)$$

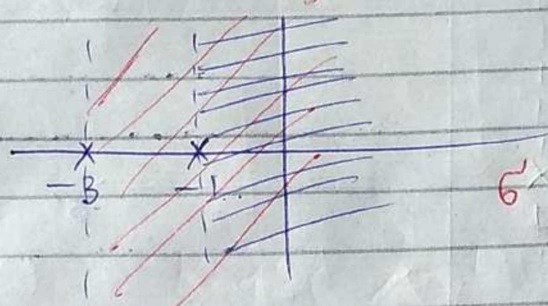
$$\therefore\ h(t) = \left[e^{-3t}u(t)\right] + \left[-e^{-t}u(-t)\right]$$

C. find $h(t)$ when system is ;

(i) Causal

$$H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$

$-3 \qquad -1$



right most pole

ROC: $Re\{s\} > -1$

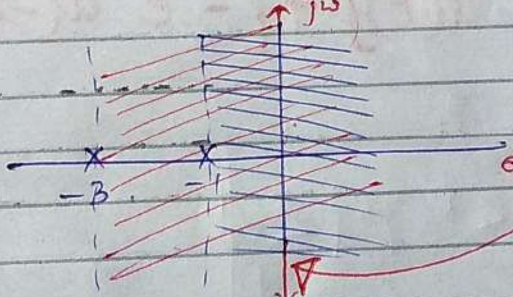
Causal when RSS ∴

$$\frac{1}{s+3} \xrightarrow{RSS} e^{-3t}u(t)$$

$$\frac{1}{s+1} \xrightarrow{RSS} e^{-t}u(t)$$

$$h(t) = e^{-3t}u(t) + e^{-t}u(t)$$

(ii) Stable :- includes jw - axis

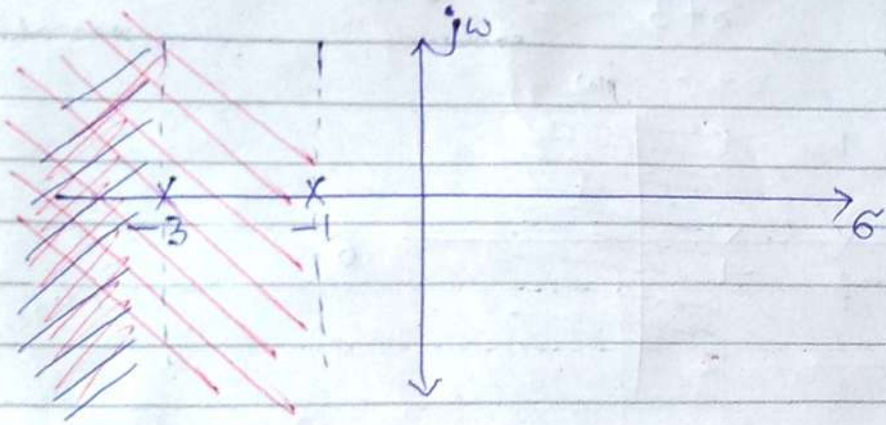


jw-axis is included in ROC

$$h(t) = \underset{\substack{\downarrow \\ RSS}}{e^{-3t}u(t)} + \underset{\substack{\downarrow \\ RSS}}{e^{-t}u(t)}$$

(iii) Neither Causal Nor Stable :- (4)

$$H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$

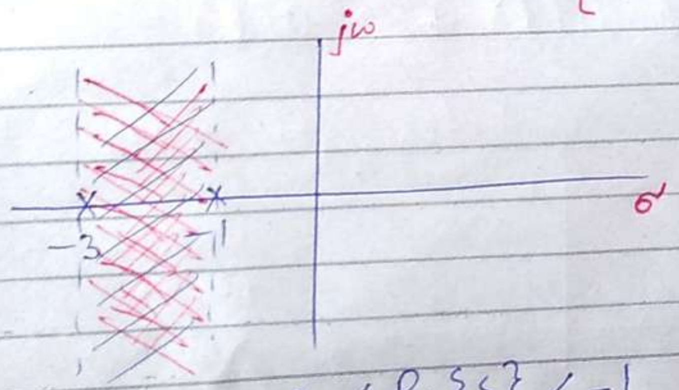


One way :- Both are LSS

ROC : $Re\{s\} < -3$

$$h(t) = \left[-e^{-3t}u(-t)\right] + \left[-e^{-t}u(-t)\right]$$

2nd way: ROC is strip $\begin{cases} \text{one is LSS} \\ \text{one is RSS} \end{cases}$

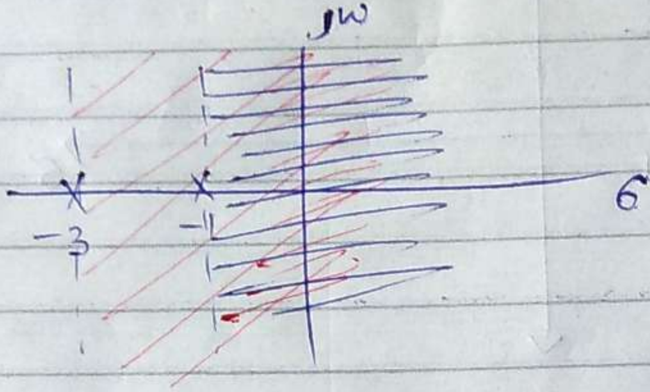


ROC : $\underbrace{-3 < Re\{s\}}_{RSS} \underbrace{< -1}_{LSS}$

$$h(t) = \left[e^{-3t}u(t)\right] + \left[-e^{-t}u(-t)\right]$$

(iv) Both Stable & Causal. 5

$$H(s) = \frac{1}{s+3} + \frac{1}{s+1}$$



ROC : $Re\{s\} > -1$

$$h(t) = \left[e^{-3t}u(t)\right] + \left[e^{-t}u(t)\right]$$