



INTRODUCTION TO SIGNALS AND SYSTEMS

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Course Contents

- Introduction
- Basic operations on signals
- Basic system properties
- Time domain analysis of continuous and discrete time systems
- Fourier series analysis of CTS and DTS
- Fourier transform analysis of CTS and DTS
- Laplace Transform
- Z Transform

Text Book:

Signals and Systems

By A.V. Oppenheim and A. S. Willsky

Second Edition Prentice Hall,
2012

Links for Video Lectures

- **Introduction to Signals and Systems:**

- https://www.youtube.com/watch?v=s8rsR_TStaA&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=1

- **Continuous and Discrete Time Signals**

- <https://www.youtube.com/watch?v=H4hk6N5vC1Q&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=2>

- **Time Shifting**

- <https://www.youtube.com/watch?v=9Cd5nVCFfc0>
- <https://www.youtube.com/watch?v=3Qzpj6UUxhE&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=273>

Signal

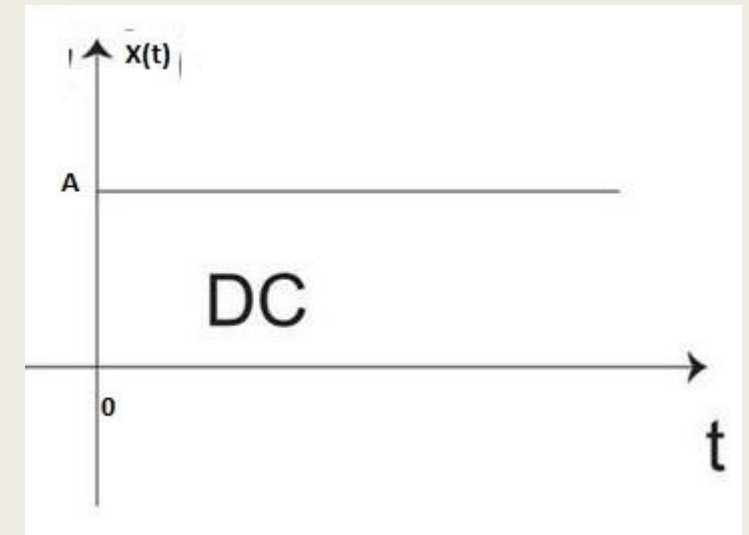
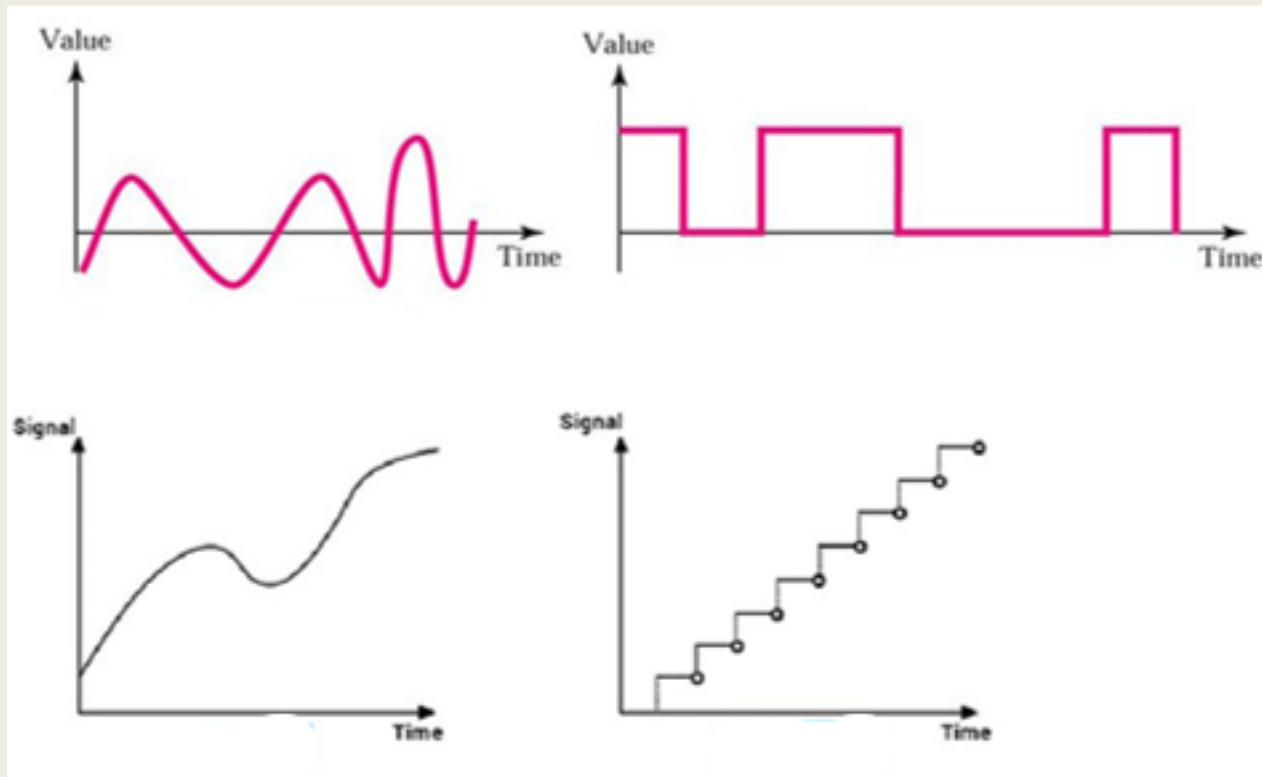
- Signal is defined as:
 - “A quantity used to convey information” e.g. human speech, temperature
 - “a dependent variable or function of one or more independent variables

$$\underbrace{f(x_1, x_2, \dots x_n)}_{\text{signal} \quad \text{Independent Variables}}$$

- Single Variable Signal → If signal is dependent on one variable only. $f(x), g(t)$
- Multi Variable Signal → If signal is depending on more than one variable.
 $f(x_1, x_2)$

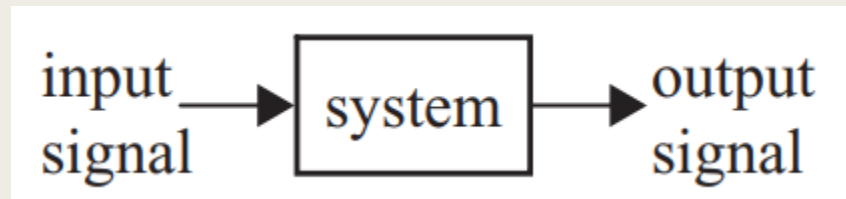
Difference btw signal and a dc value

- anything which is varying is a signal but a constant value is not a signal
- e.g. AC is a signal because current is changing with time. Whereas DC is not a signal because in DC, current is not changing with time.



System

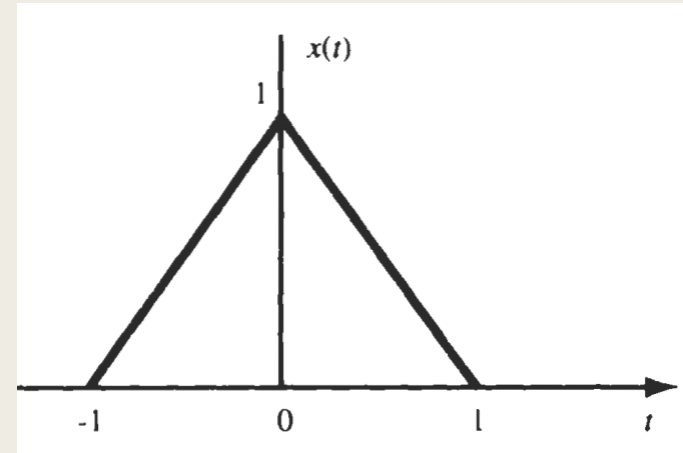
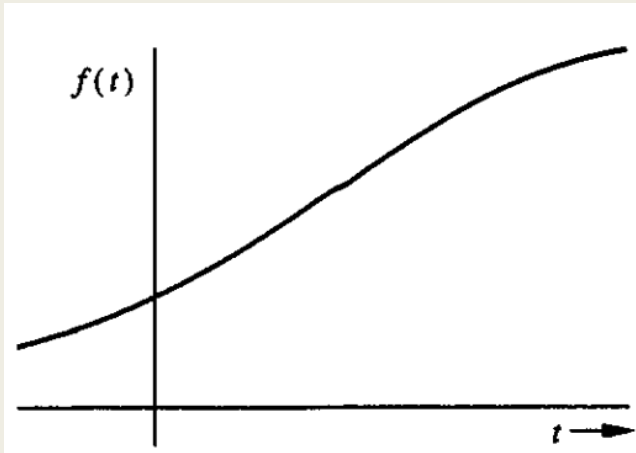
- It is defined as
 - The meaningful interconnection of physical devices and components is called a system
 - An entity that process a set of signals (input signal) and produces another set of signals (output signal).
- System alone can not achieve anything so it must be linked with a signal.



desirable signal

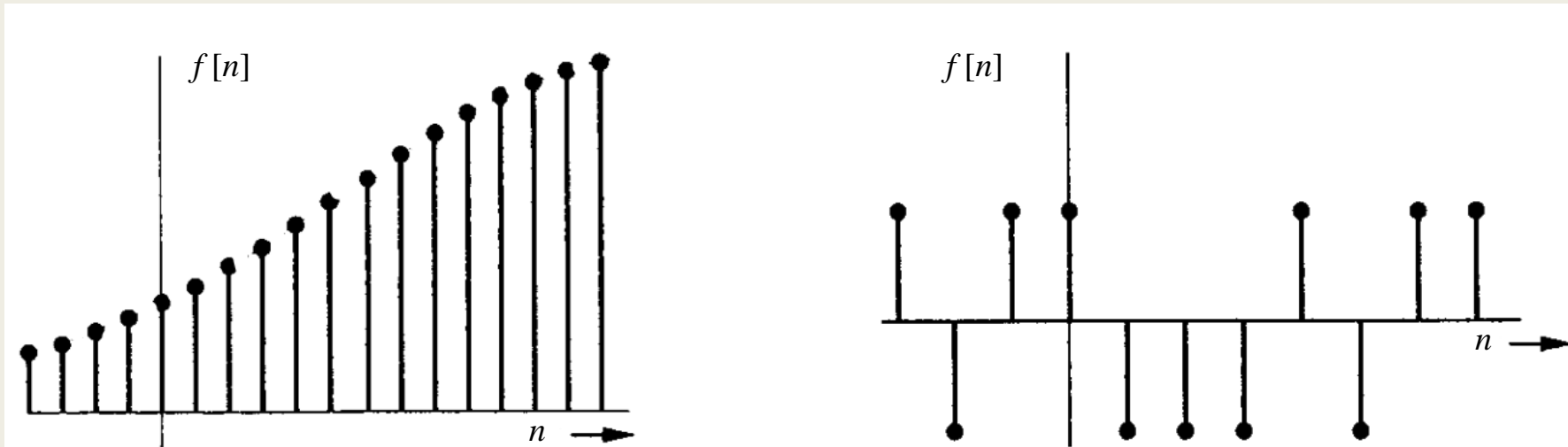
i) Continuous Time Signal (CTS)

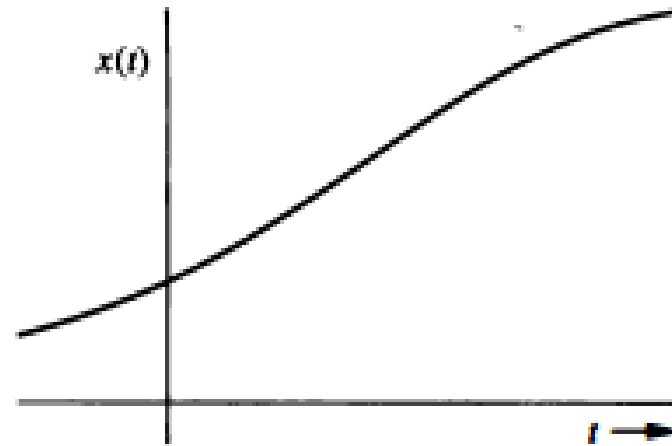
- Signals which are specified for every value of time (t)
- It is written as $f(t)$, $x(t)$, or $g(t)$



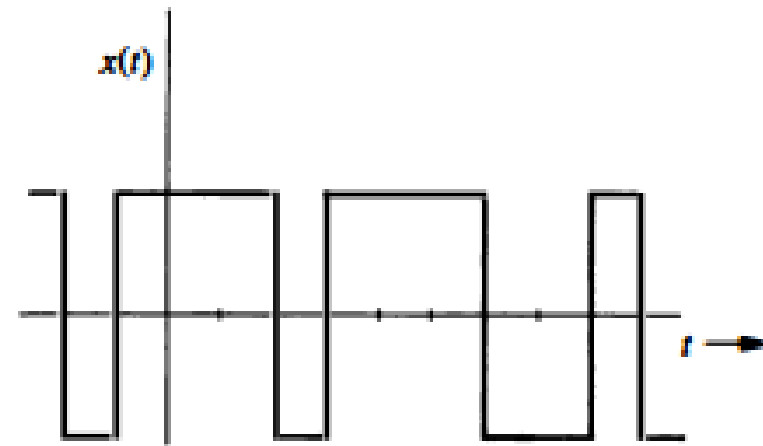
ii) Discrete Time Signals (DTS)

- Signals specified at discrete time intervals
- It is written as $f[n]$, $x[n]$, or $g[n]$ / $f(n)$, $x(n)$, $g(n)$

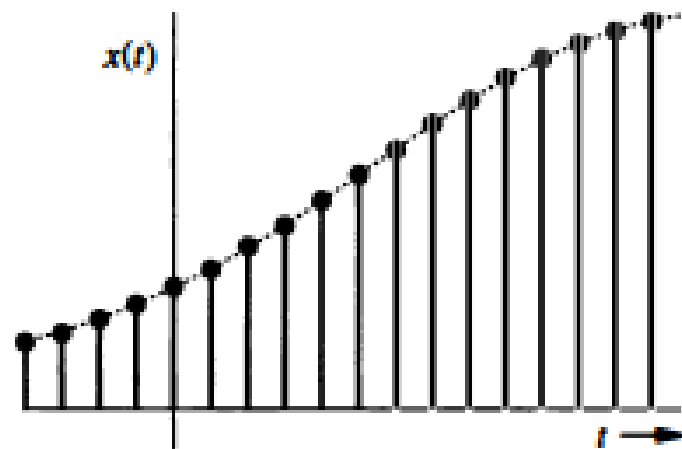




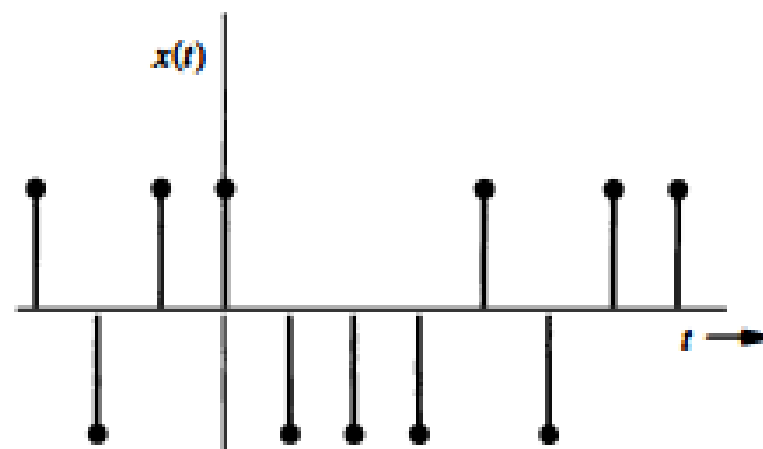
(a)



(b)



(c)



(d)

Figure 1.11 Examples of signals: (a) analog, continuous time, (b) digital, continuous time, (c) analog, discrete time, and (d) digital, discrete time.



OPERATIONS ON SIGNALS

(INDEPENDENT VARIABLE)

Links for Video Lectures

- **1) Time Shifting**

- <https://www.youtube.com/watch?v=9Cd5nVCFfc0>

- <https://www.youtube.com/watch?v=3Qzpj6UUxhE&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=273>

- **2) Time Scaling**

- <https://www.youtube.com/watch?v=jnB-U5KBvN4&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=5>

- **3) Time Reversal/Flipping/Folding**

- <https://www.youtube.com/watch?v=BzAbZfT6RxQ&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=9>

Operations on Signals

- **Operations with respect to x-axis (Time axis)** / Transformations on the independent variable
 - Time Shifting
 - Time Reversal/Folding
 - Time Scaling
- **Operations with respect to y-axis (Amplitude)** / Transformations on the dependent variable
 - Amplitude Multiplication
 - Amplitude Scaling
 - Addition
 - Subtraction

1) Time Shifting

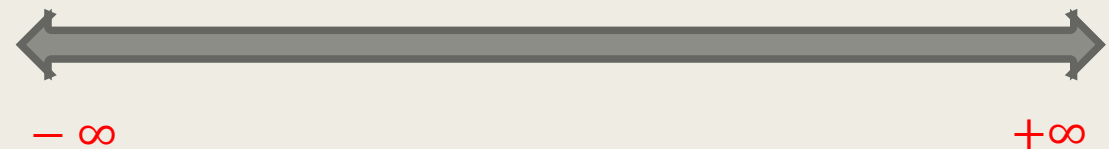
■ Time Delay

- When the signal is delayed, it is shifted right
i.e. $x(t - k)$ or $x[n - k]$ \therefore where k is positive
e.g. $x(t - 2)$, $x(t - 1.5)$, $x[n - 2]$, $x[n - 3]$

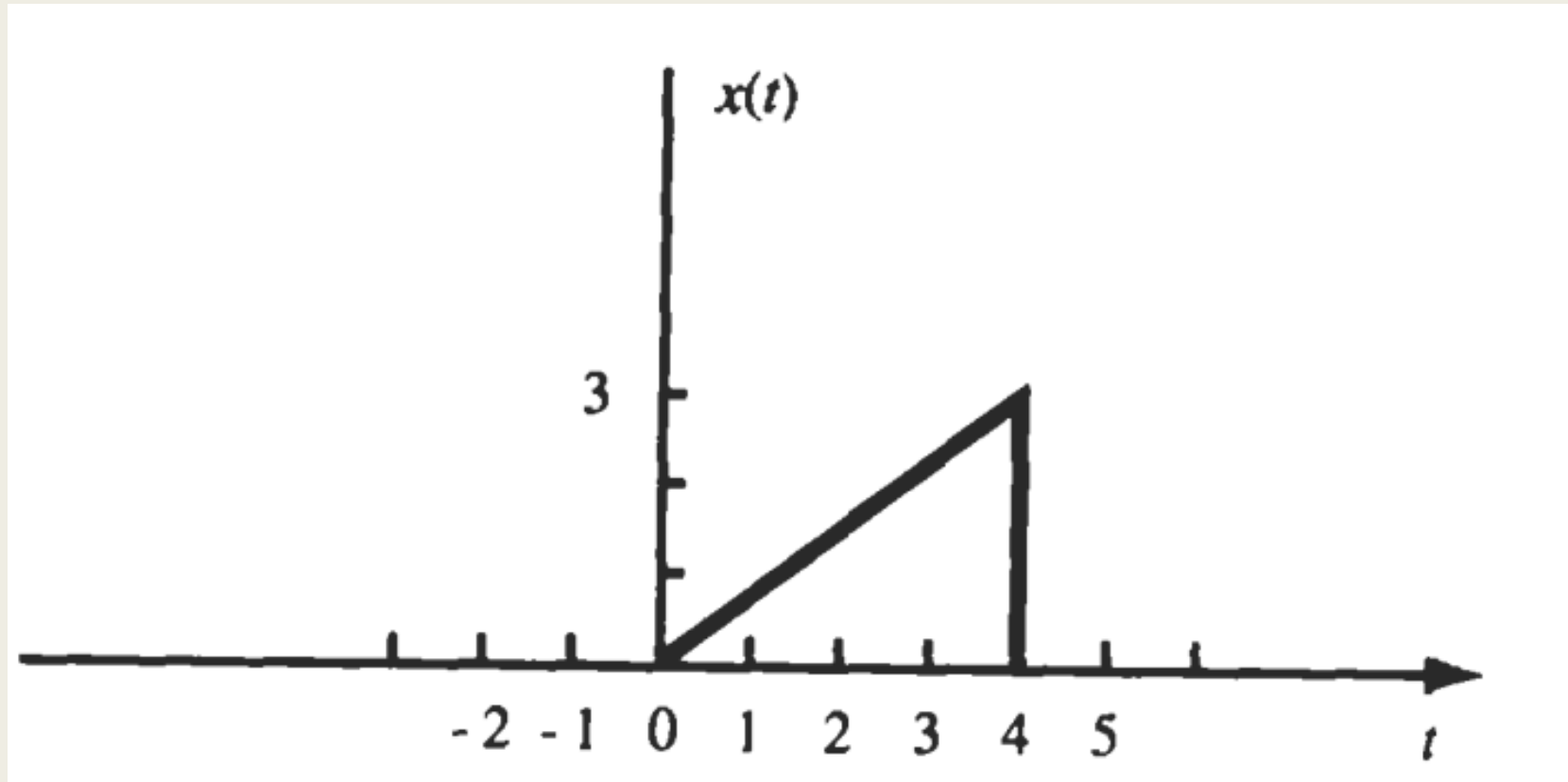


■ Time Advance


- When the signal is advanced, it is shifted left
i.e. $x(t + k)$ or $x[n + k]$ \therefore where k is positive
e.g. $x(t + 2)$, $x(t + 1.5)$, $x[n + 2]$, $x[n + 3]$



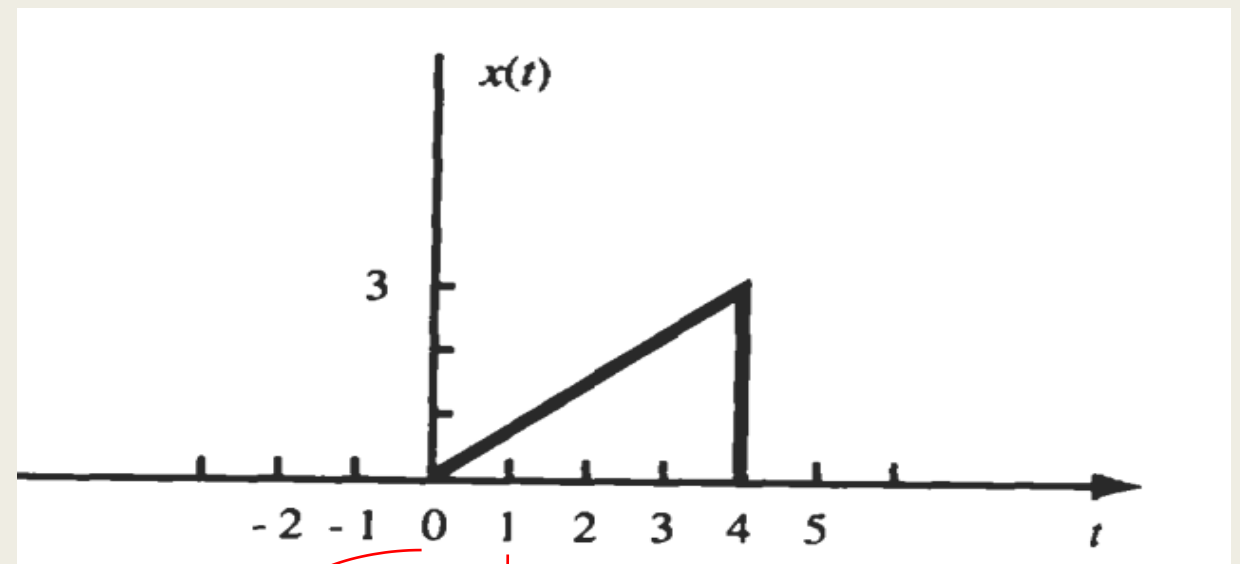
Exp 1.1: For $x(t)$, sketch $x(t - 2)$ and $x(t + 2)$



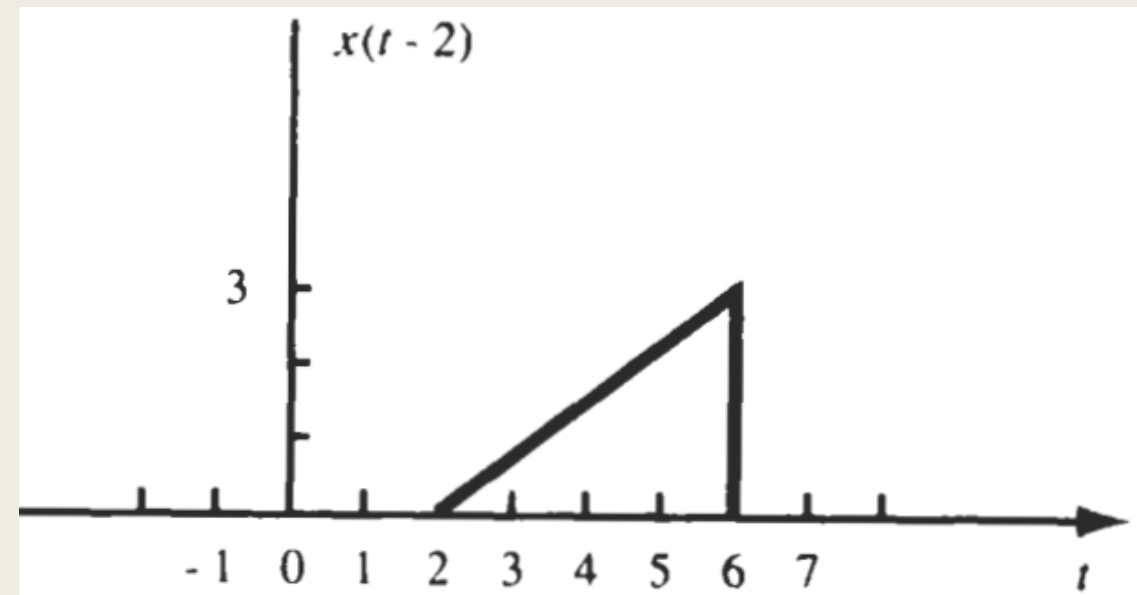
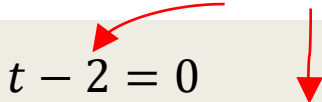
$x(t - 2)$:

- 
- $t - 2 = 0 \rightarrow t = 0 + 2 = 2$
 - $t - 2 = 1 \rightarrow t = 1 + 2 = 3$
 - $t - 2 = 2 \rightarrow t = 2 + 2 = 4$
 - $t - 2 = 3 \rightarrow t = 3 + 2 = 5$
 - $t - 2 = 4 \rightarrow t = 4 + 2 = 6$

Shifted towards right by 2 steps so it is a delayed signal



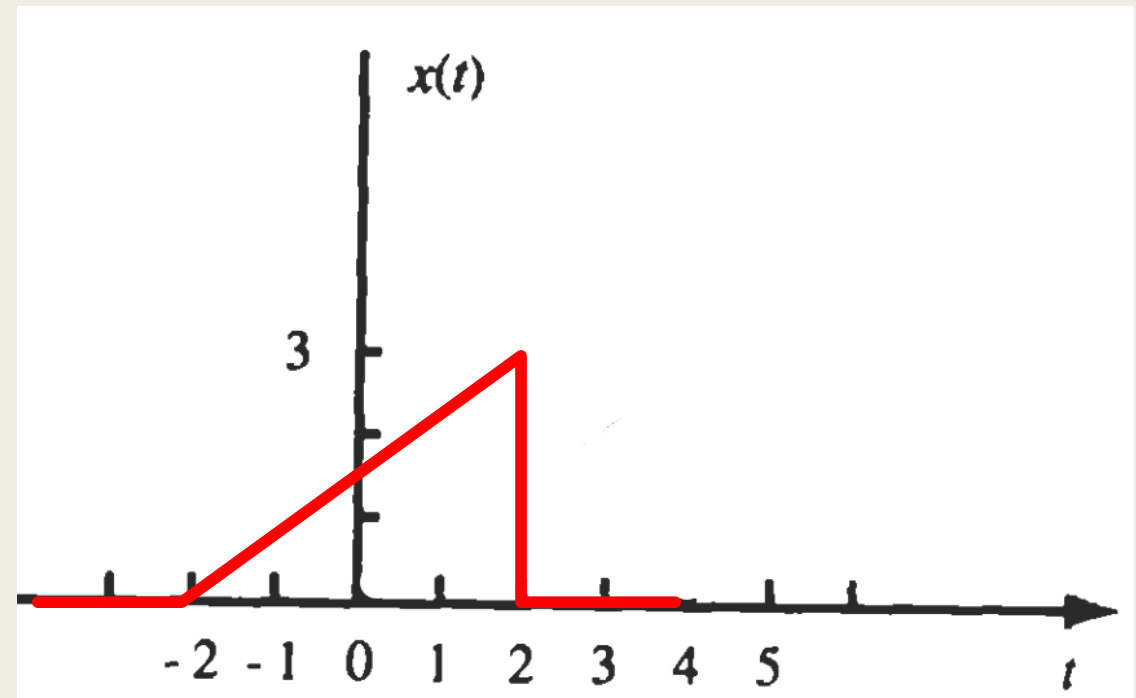
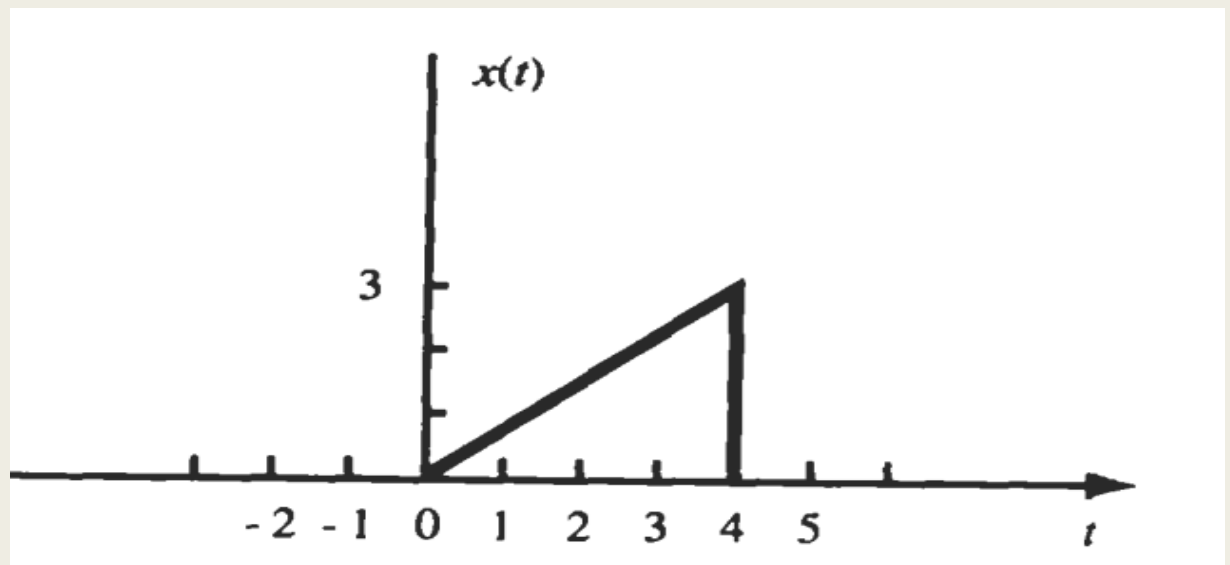
$t - 2 = 0$ $t - 2 = 1$



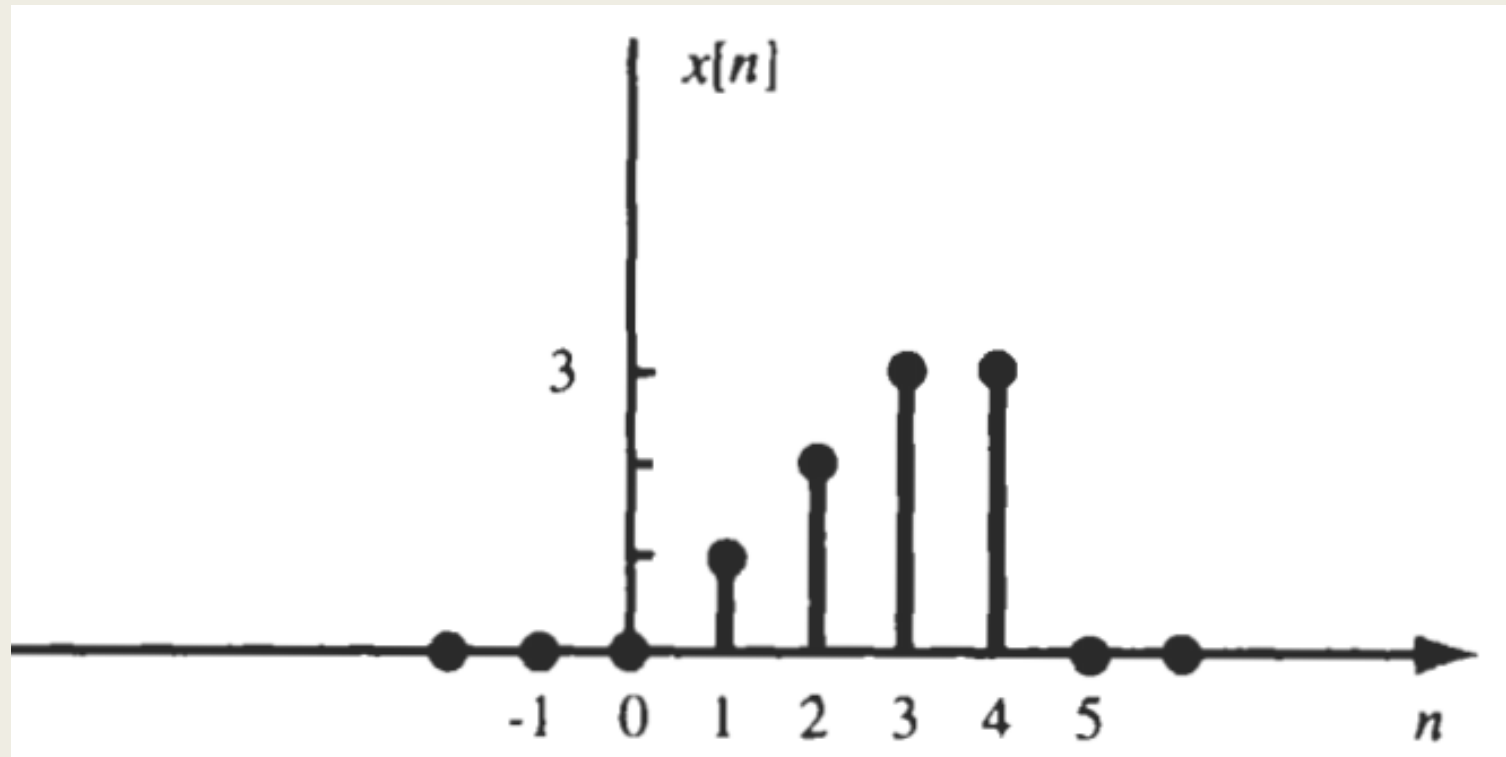
$x(t + 2)$:



- $t + 2 = 0 \rightarrow t = 0 + 2 = -2$
- $t + 2 = 1 \rightarrow t = 1 - 2 = -1$
- $t + 2 = 2 \rightarrow t = 2 - 2 = 0$
- $t + 2 = 3 \rightarrow t = 3 - 2 = 1$
- $t + 2 = 4 \rightarrow t = 4 - 2 = 2$

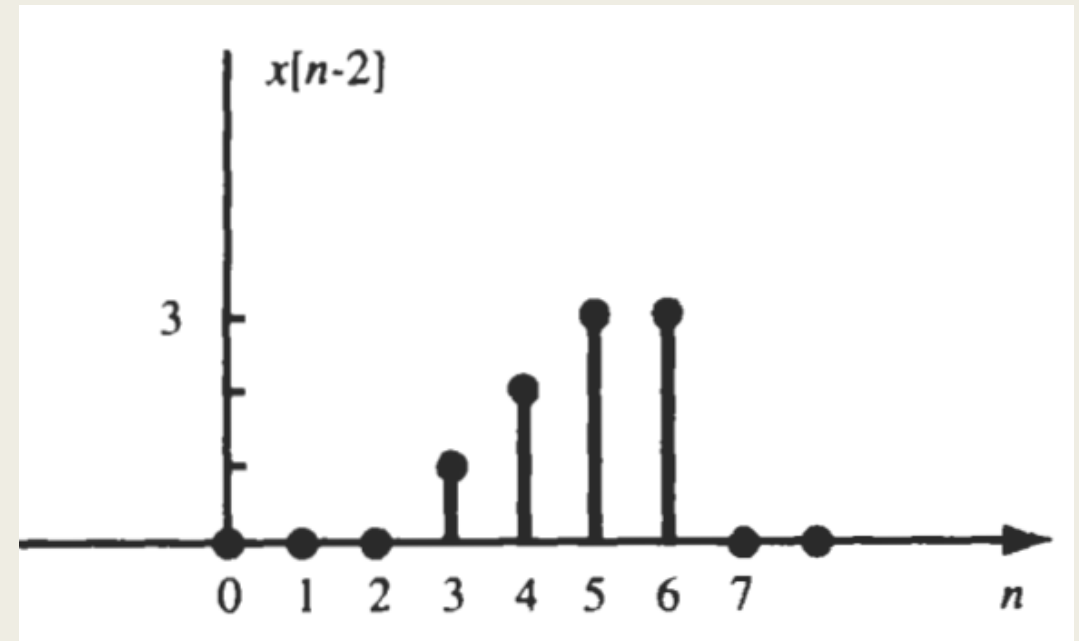
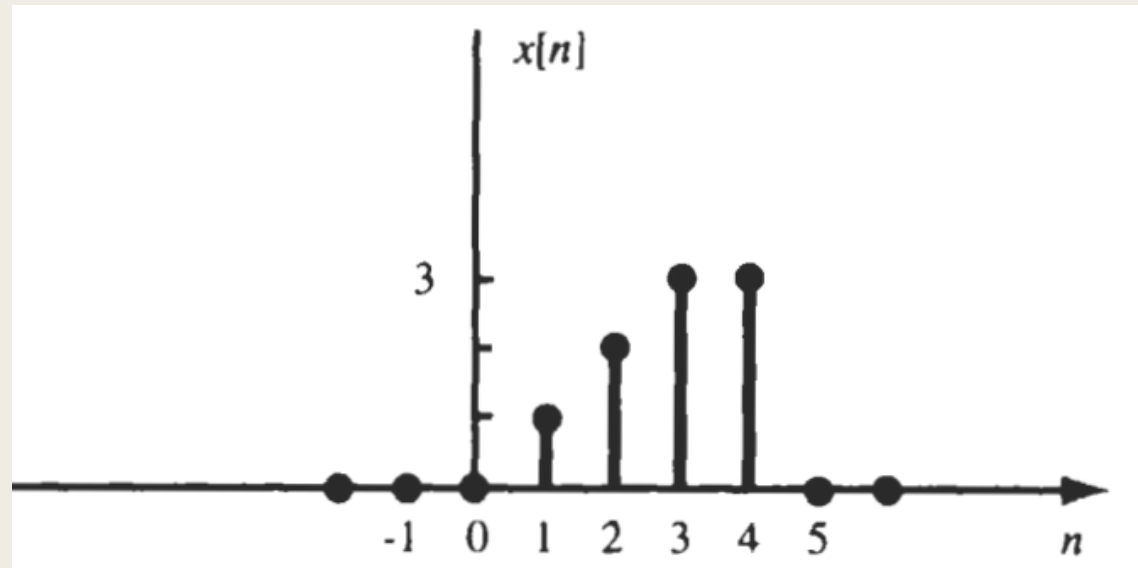


Exp 1.2: For $x[n]$, sketch $x[n - 2]$





- $n - 2 = 0 \rightarrow n = 2 - 0 = 2$
- $n - 2 = 1 \rightarrow n = 1 + 2 = 3$
- $n - 2 = 2 \rightarrow n = 2 + 2 = 4$
- $n - 2 = 3 \rightarrow n = 3 + 2 = 5$
- $n - 2 = 4 \rightarrow n = 4 + 2 = 6$

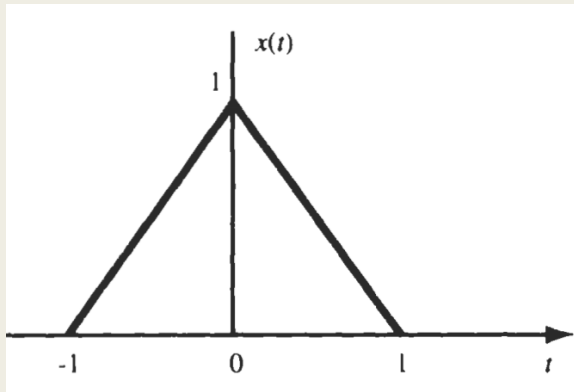


Shifted towards right by 2 steps so it is a delayed signal

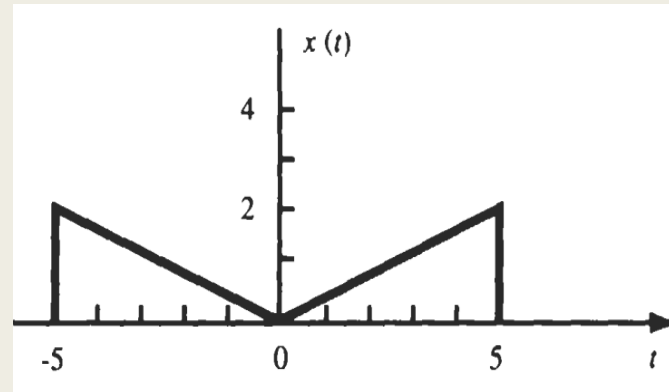
PP. 1.1) For signals given, sketch

i) $x(t - 2.5)$

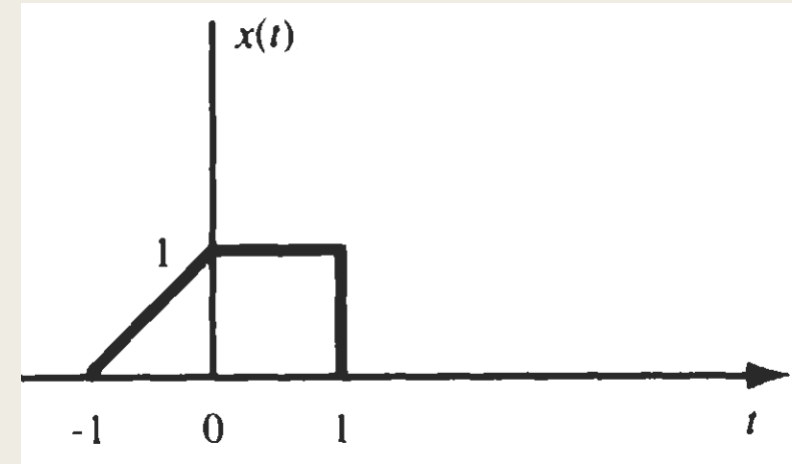
ii) $x(t + 1)$



(a)



(b)

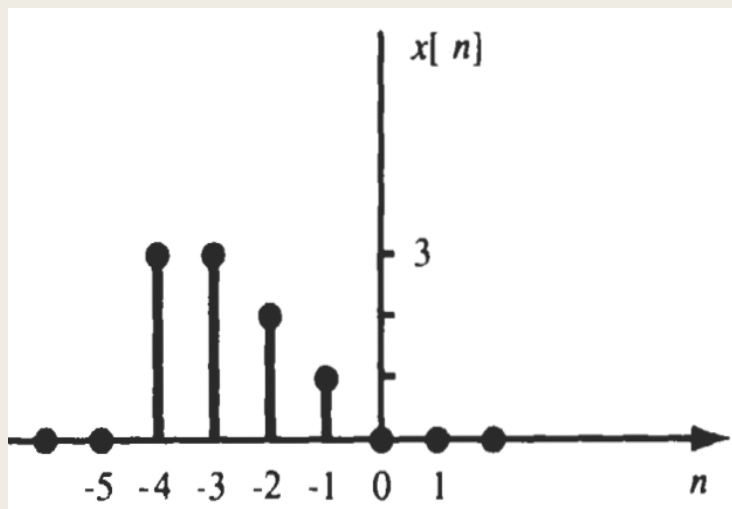


(c)

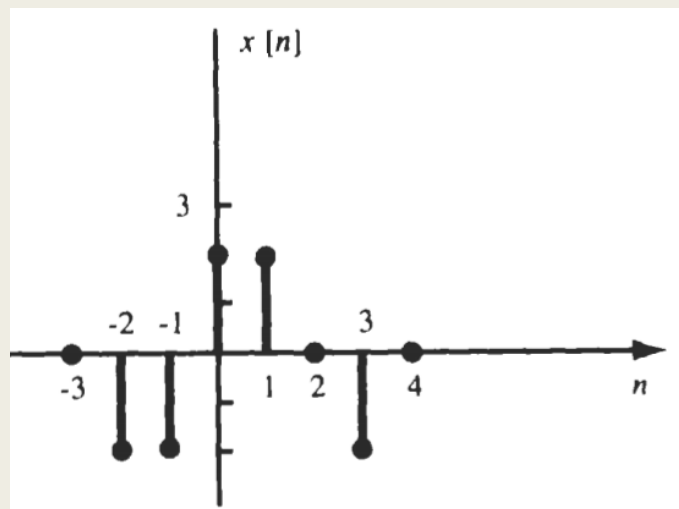
PP. 1.2) For signals given, sketch

i) $x[n - 1]$

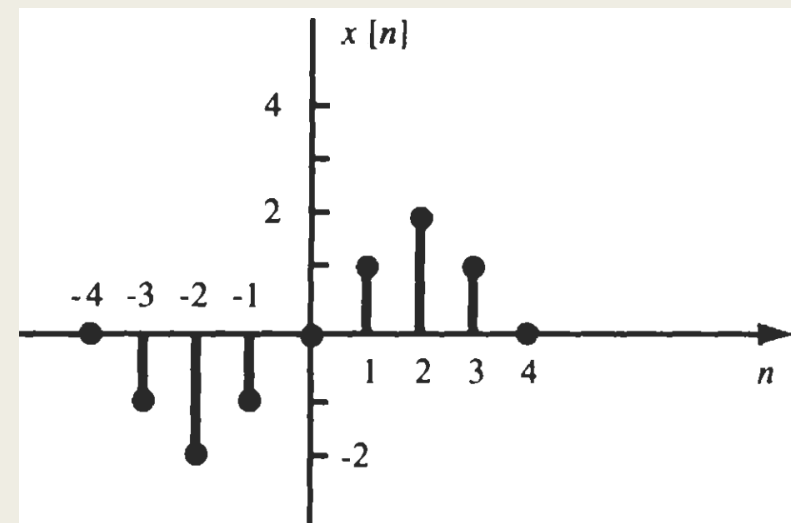
ii) $x[n + 3]$



(a)



(b)



(c)

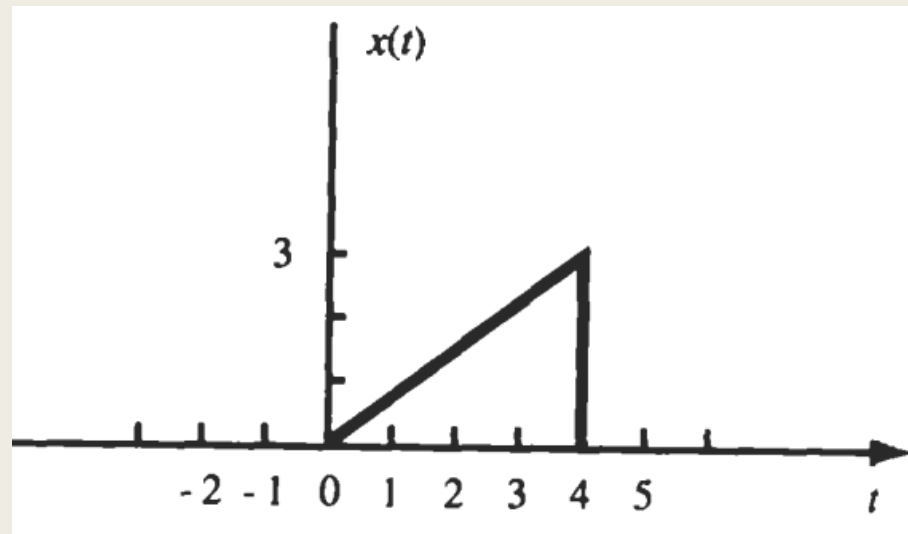
2) Time Reversal/Folding/Flipping

- Reversal of signal about the vertical axis (y-axis) is known as time reversal.
- It converts $x(t)$ into $x(-t)$
- Therefore, mirror image of the signal $x(t)$ about vertical axis is $x(-t)$

$$\blacksquare x(t) \rightarrow x(-t)$$

- Note: Mirror image of the signal $x(t)$ about horizontal axis is $-x(t)$

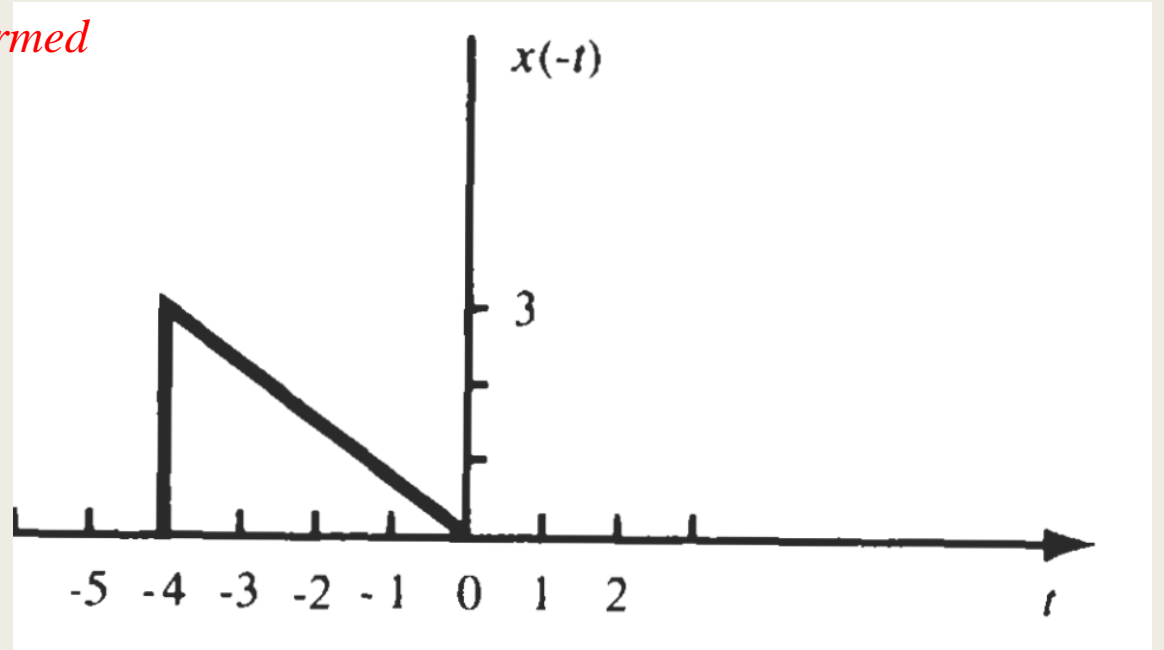
Exp 2.1: For $x(t)$, sketch $x(-t)$



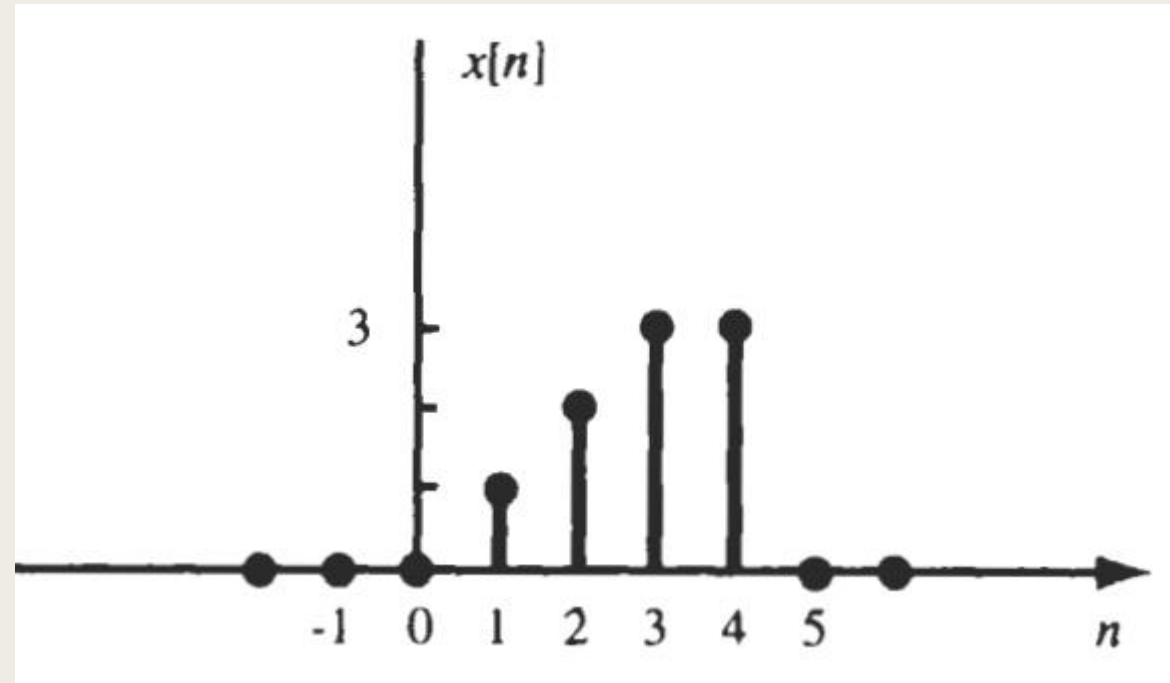
$$x(-t) = x(t)$$

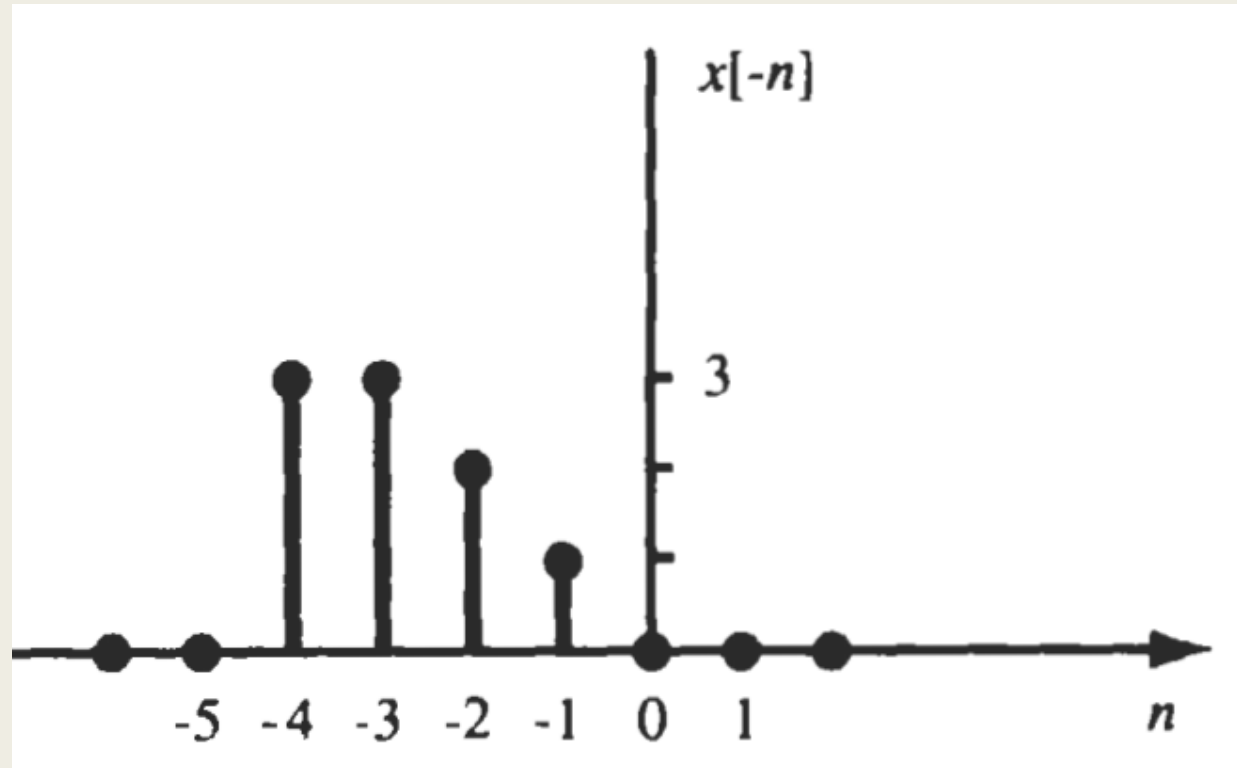
- $-t = 0 \rightarrow t = 0$
- $-t = 1 \rightarrow t = -1$
- $-t = 2 \rightarrow t = -2$
- $-t = 3 \rightarrow t = -3$
- $-t = 4 \rightarrow t = -4$

*Points for transformed
signal i.e. $x(-t)$*



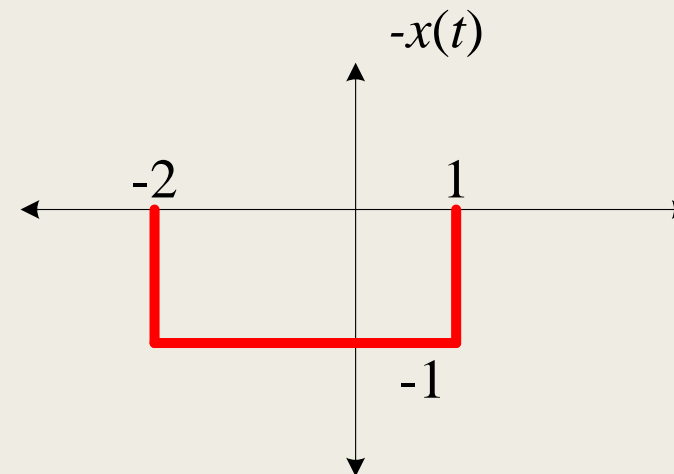
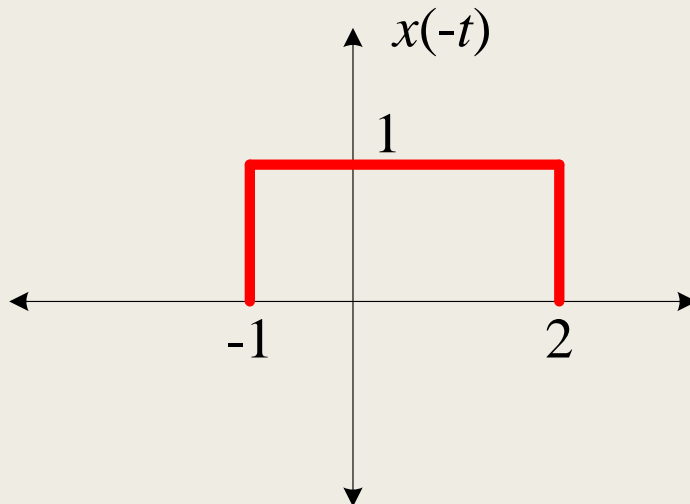
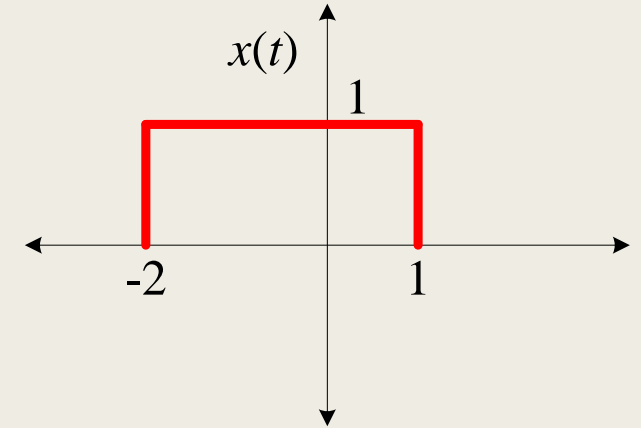
Exp 2.2: For $x[n]$, sketch $x[-n]$





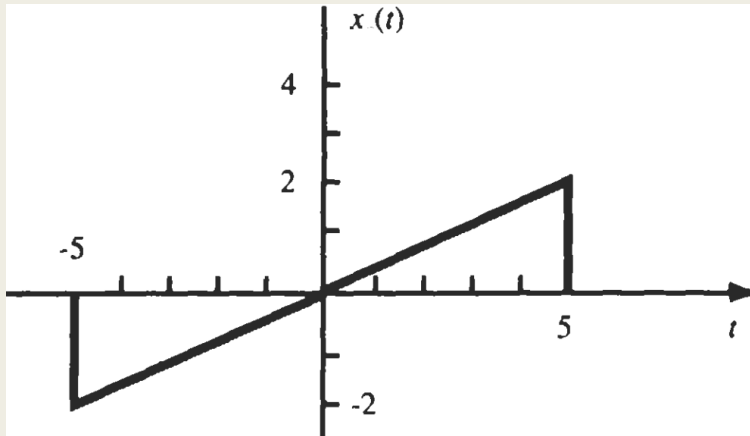
Difference between $x(-t)$ and $-x(t)$

- $x(t) \rightarrow x(-t)$: Flipping around vertical axis (y-axis)
- $x(t) \rightarrow -x(t)$: Flipping around horizontal axis (x-axis)

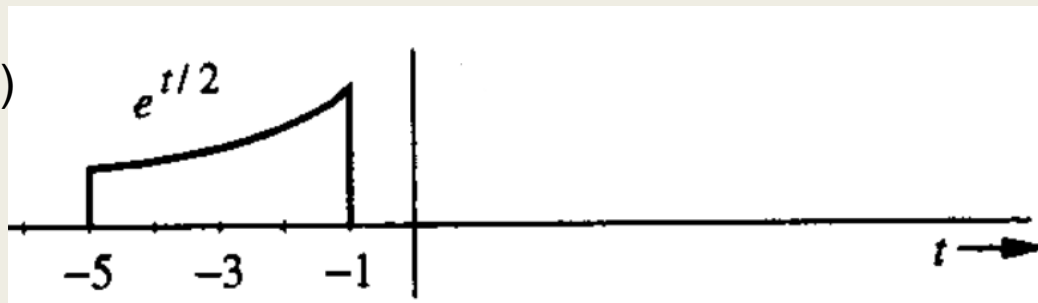


PP. 2.1) For signals given, sketch $x(-t)$ and $x[-n]$

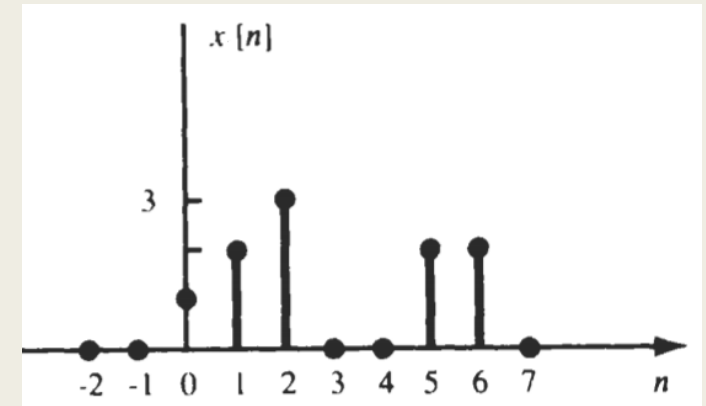
(a)



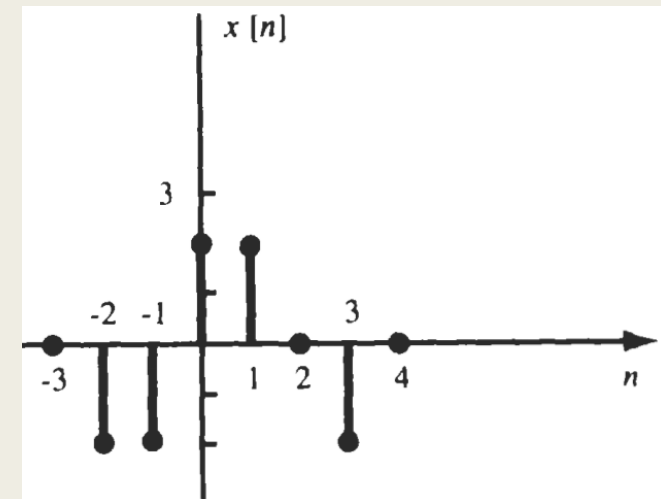
(b)



(c)



(d)



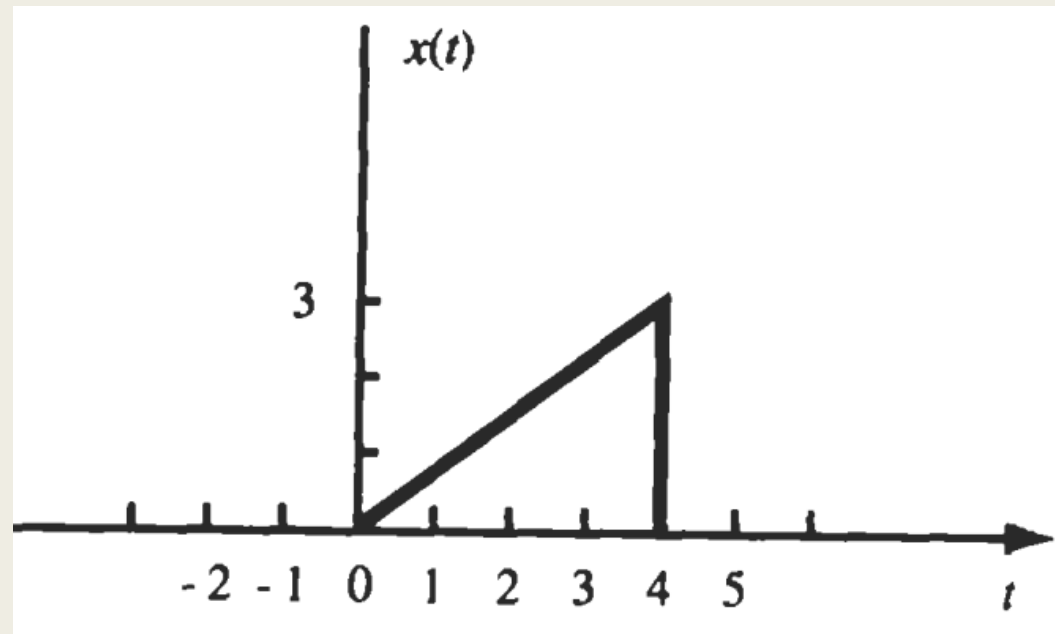
3) Time Scaling

- i) **Time Compression:** Time axis is compressed
- ii) **Time Expansion:** Time axis is expanded

$$\blacksquare x(t) \rightarrow x(\alpha t)$$

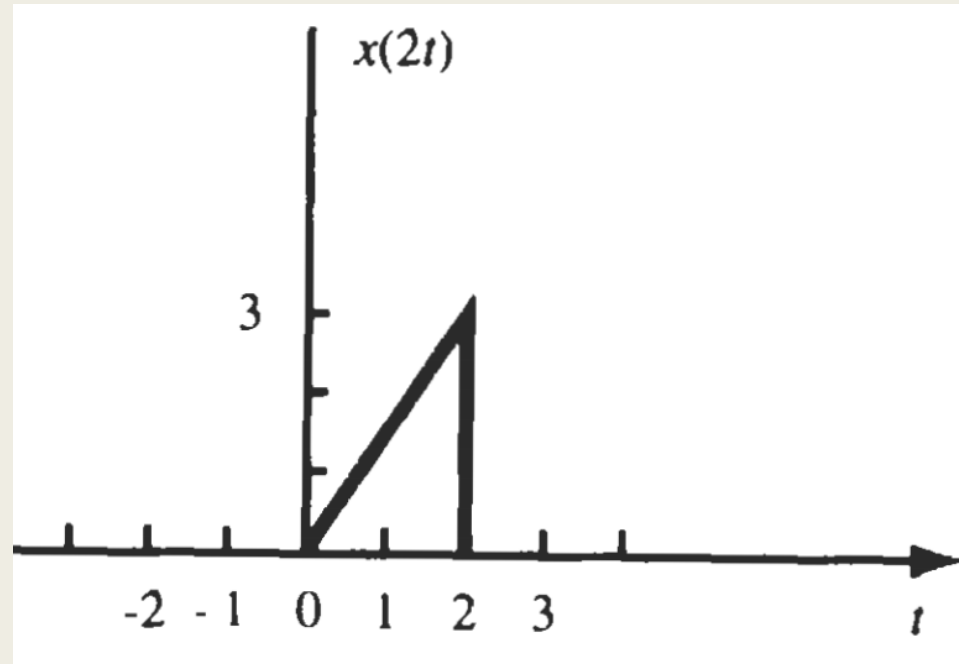
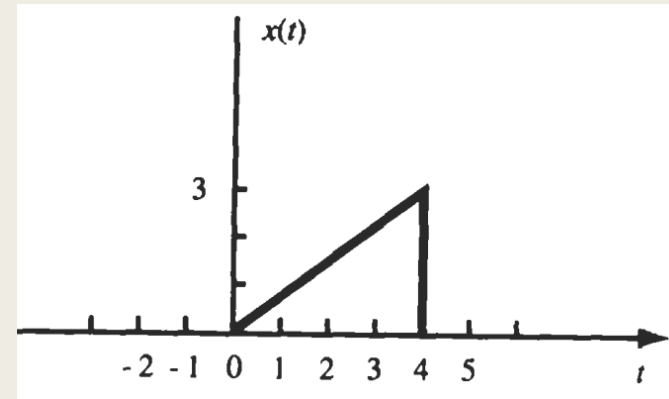
- If $\alpha > 1$ then scaling results in time compression
- If $\alpha < 1$ then scaling results in time expansion.

Exp 3.1: For the signal given, sketch $x(2t)$ and $x(t/2)$



$x(2t)$:

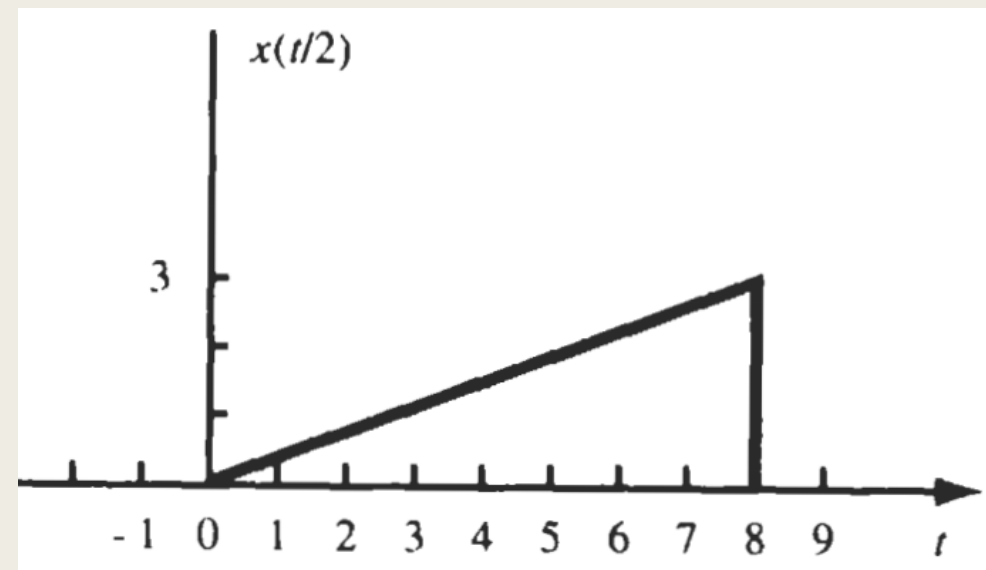
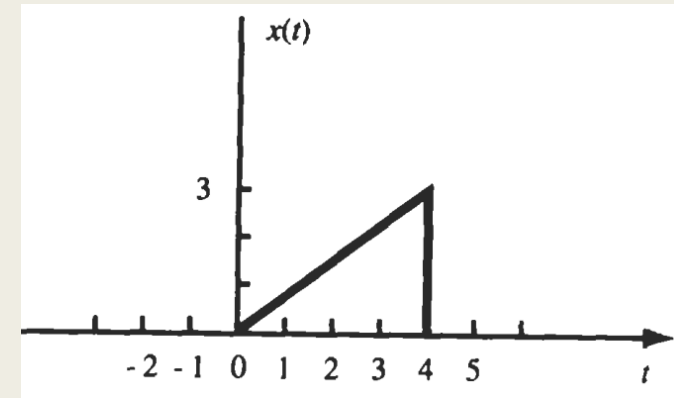
- $2t = 0 \rightarrow t=0$
- $2t = 1 \rightarrow t=1/2=0.5$
- $2t = 2 \rightarrow t=2/2=1$
- $2t = 3 \rightarrow t=3/2=1.5$
- $2t = 4 \rightarrow t=4/2 = 2$



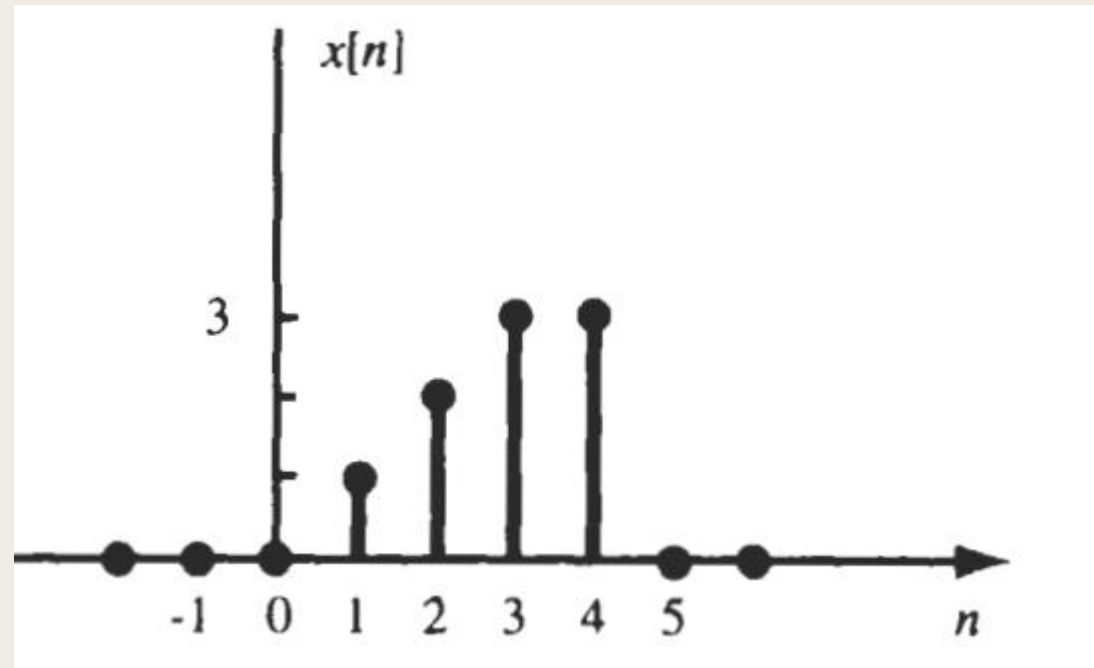
Signal is compressed by 2 times

$x(t/2)$:

- $t/2 = 0 \rightarrow t=0$
- $t/2 = 1 \rightarrow t=2 \times 1=2$
- $t/2 = 2 \rightarrow t=2 \times 2=4$
- $t/2 = 3 \rightarrow t=2 \times 3=6$
- $t/2 = 4 \rightarrow t=2 \times 4=8$

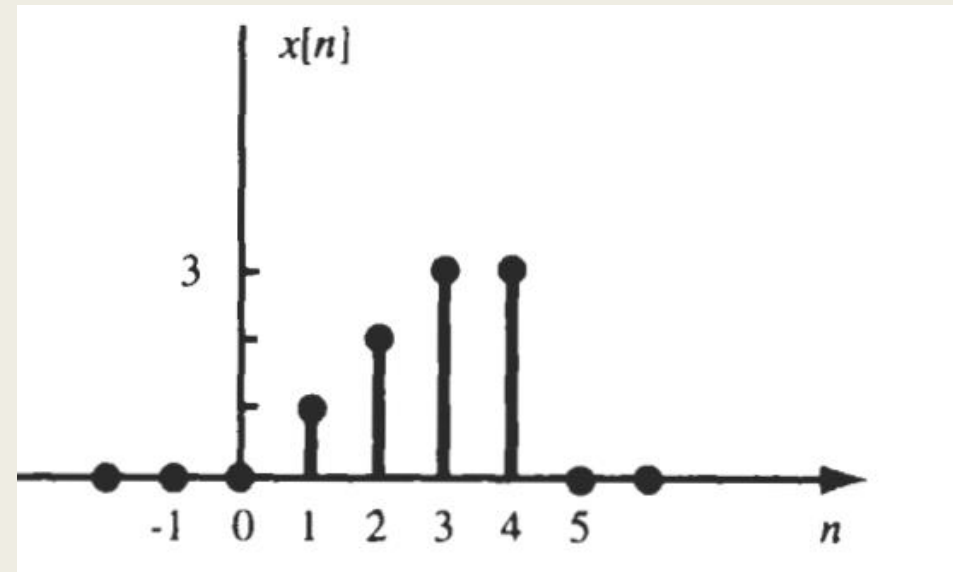


Exp 3.2: For the signal given, sketch $x[2n]$ and $x[n/2]$

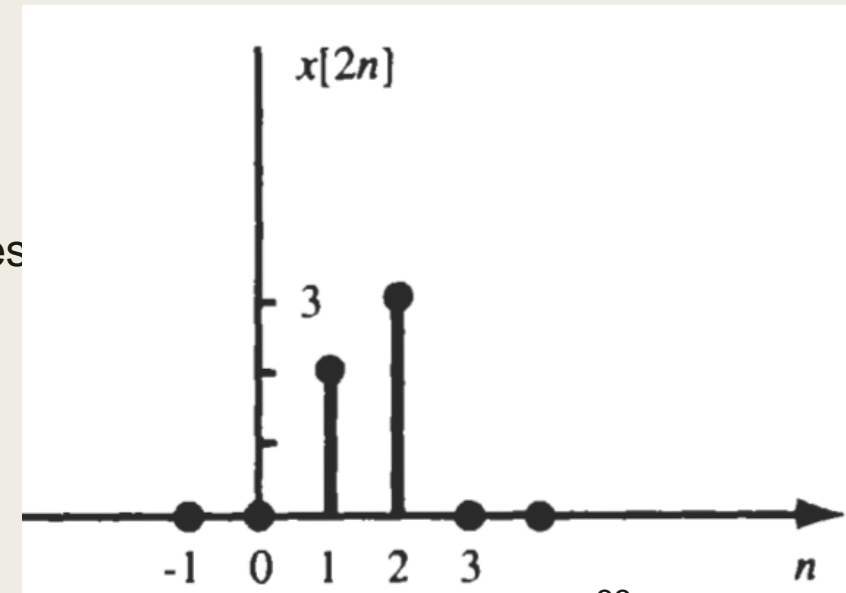


$x[2n] \rightarrow$ Down-sampling

- $2n = -2 \rightarrow n = -1$
 - $2n = -1 \rightarrow n = -1/2 = -0.5$
 - $2n = 0 \rightarrow n = 0$
 - $2n = 1 \rightarrow n = 1/2 = 0.5$
 - $2n = 2 \rightarrow n = 2/2 = 1$
 - $2n = 3 \rightarrow n = 3/2 = 1.5$
 - $2n = 4 \rightarrow n = 4/2 = 2$
-
- Thus, for $x[2n]$, pick every 2nd sample and discard other samples
 - Similarly for $x[3n]$, pick every 3rd sample and discard other samples

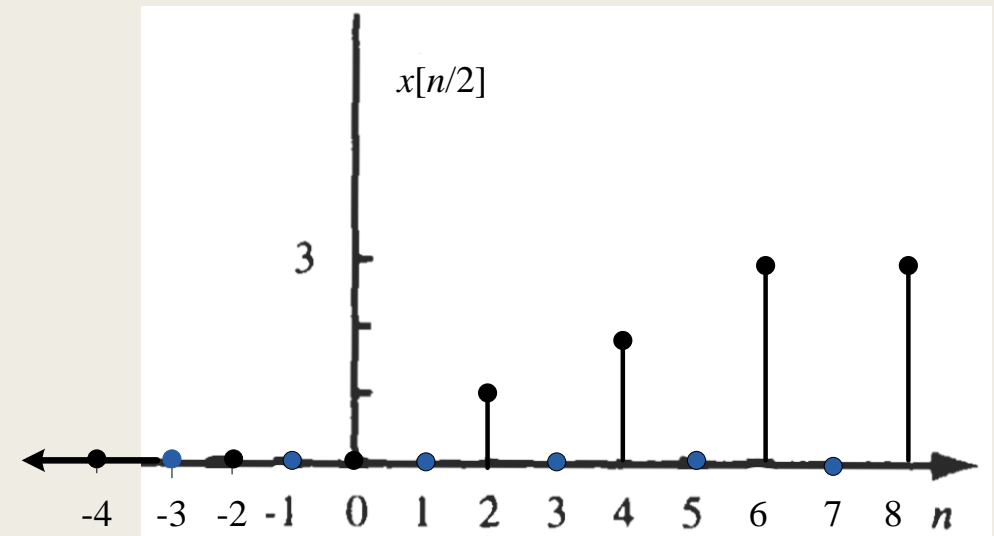
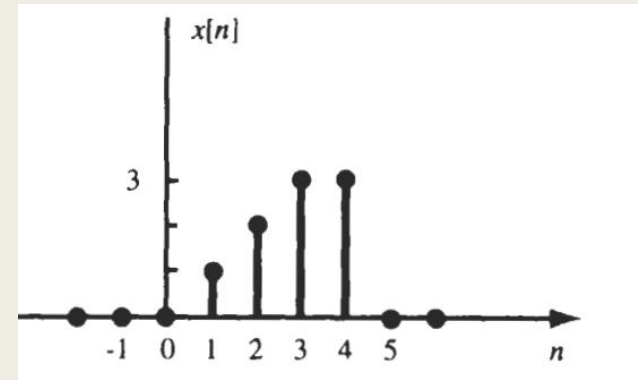


$$\frac{-1}{2} \quad \frac{0}{2} \quad \frac{1}{2} \quad \frac{2}{2} \quad \frac{3}{2} \quad \frac{4}{2}$$



$x[n/2] \rightarrow$ Up-sampling

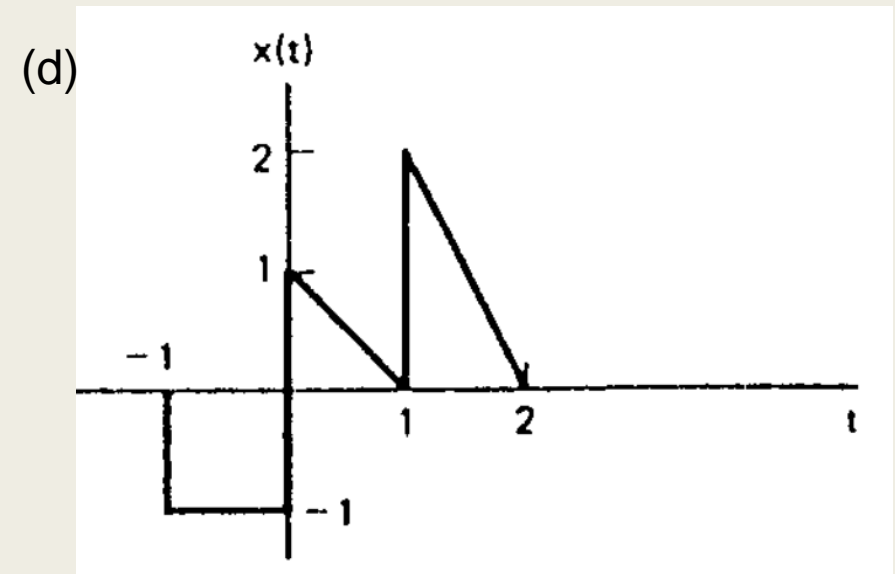
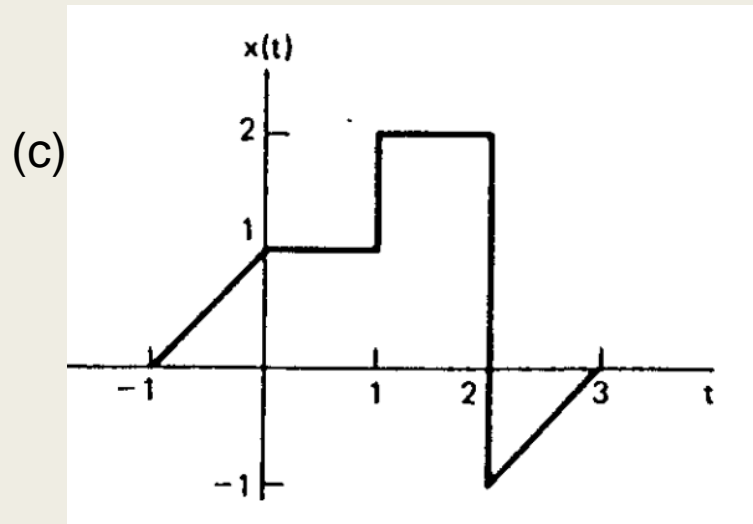
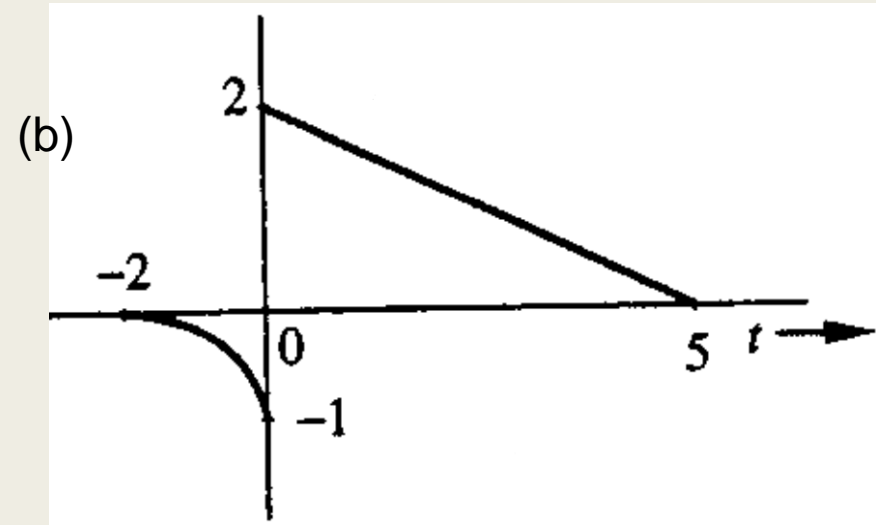
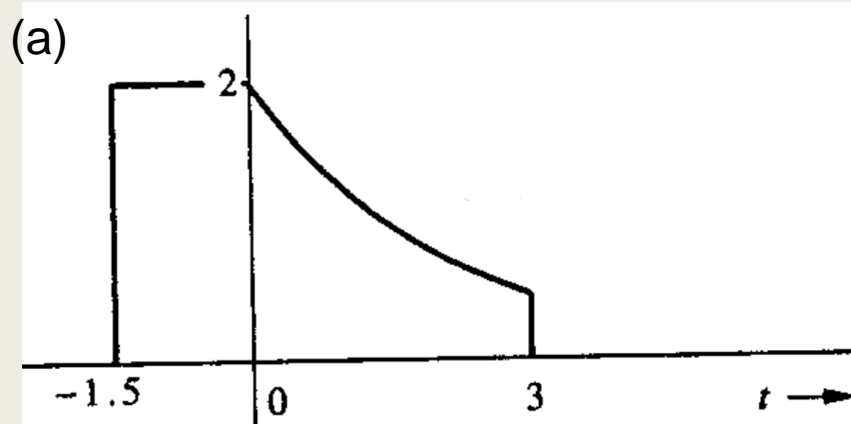
- $n/2 = -2 \rightarrow n = -4$
- $n/2 = -1 \rightarrow n = -1 \times 2 = -2$
- $n/2 = 0 \rightarrow n = 0$
- $n/2 = 1 \rightarrow n = 1 \times 2 = 2$
- $n/2 = 2 \rightarrow n = 2 \times 2 = 4$
- $n/2 = 3 \rightarrow n = 3 \times 2 = 6$
- $n/2 = 4 \rightarrow n = 4 \times 2 = 8$



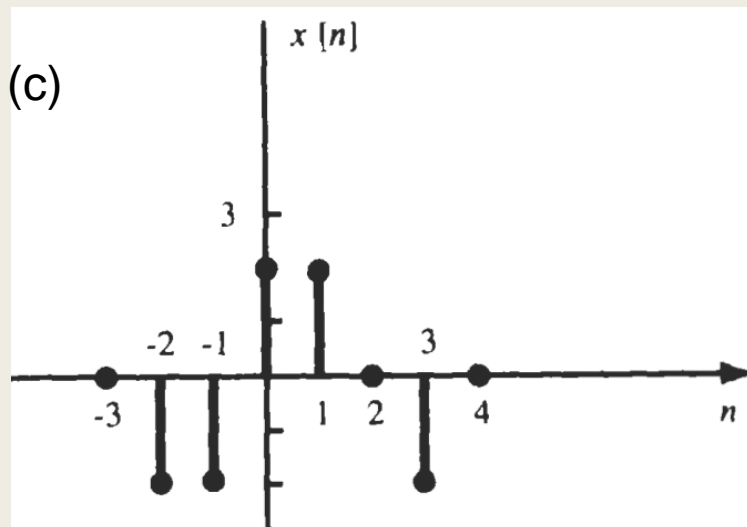
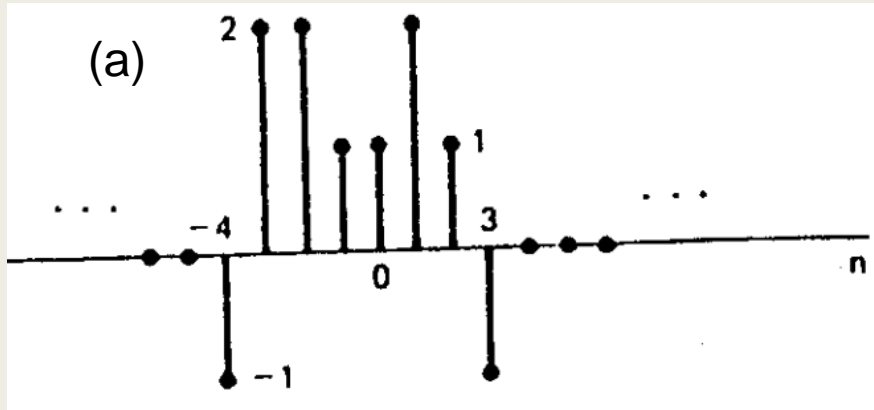
- Thus, $x[n/2] \rightarrow$ place samples at every 2nd place whereas zeros will be placed in between the samples
- Thus, $x[n/3] \rightarrow$ place samples at every 3rd place whereas zeros will be placed in between the samples

PP. 3.1) For signals given, sketch

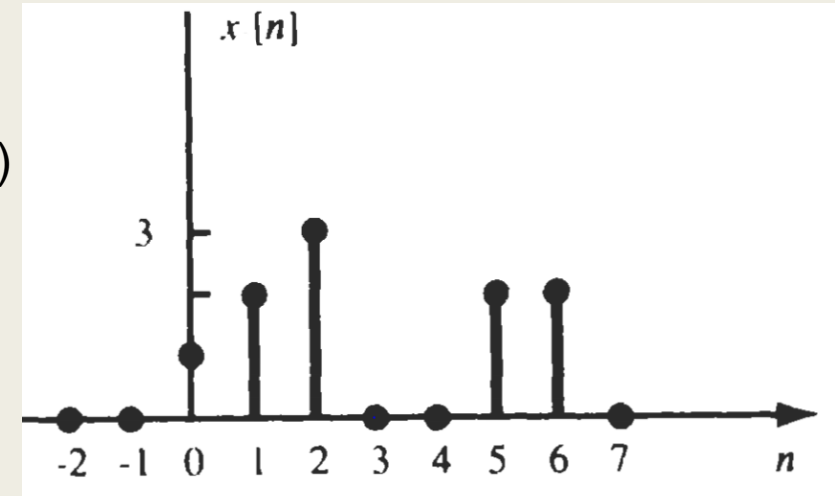
(i) $x(2t)$ (ii) $x(t/3)$ (iii) $x(1.5t)$



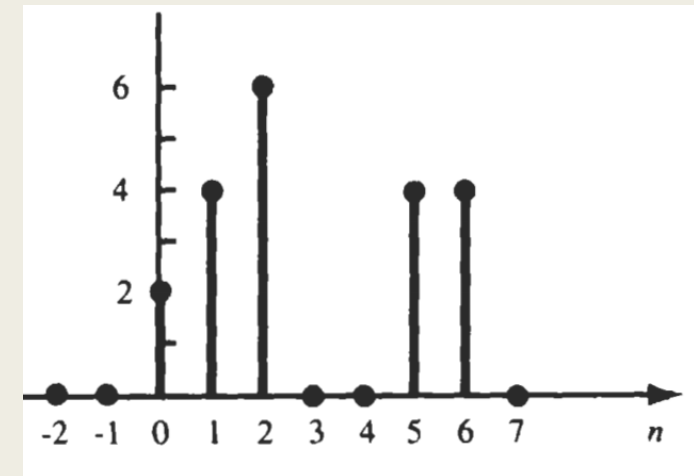
PP. 3.2) For signals given, sketch
(i) $x[2n]$ (ii) $x[3n]$ (iii) $x[n/3]$



(b)



(d)



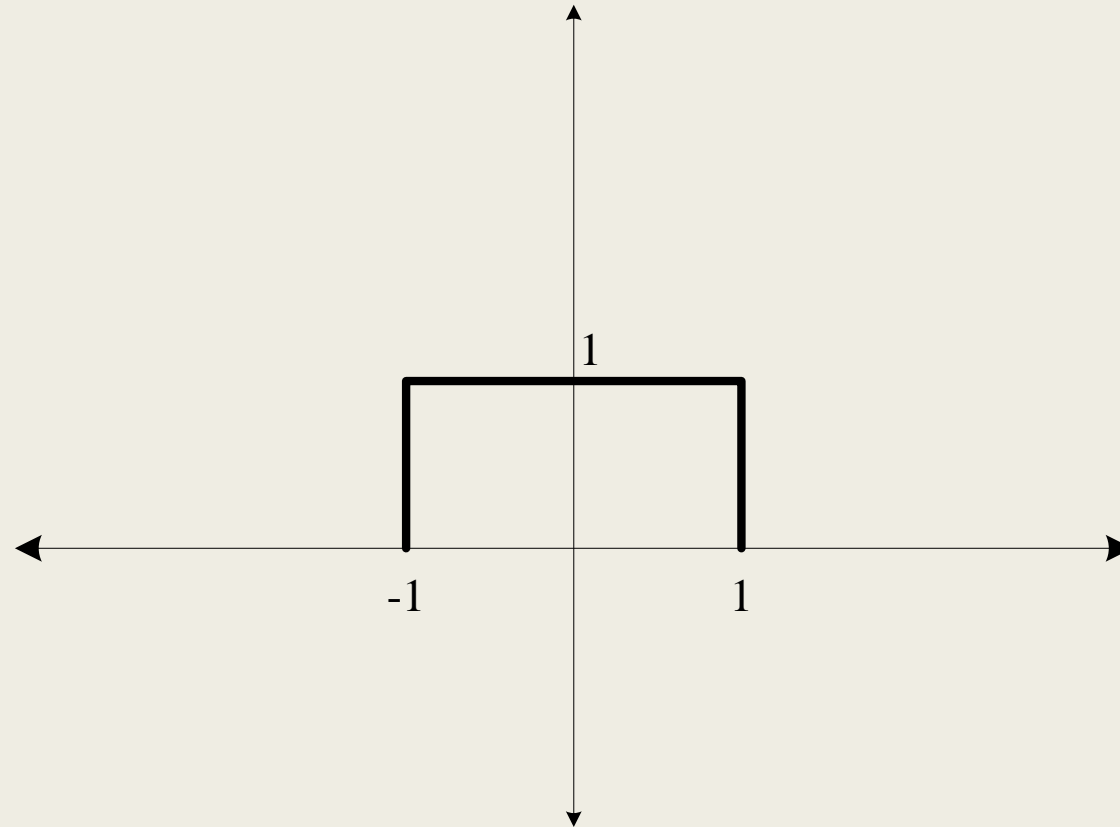
4. Precedence Rule for Combined Operations

- **e.g. Method 1:** $x(t)$ is the given signal
 - i) Time shifting operation $x(t - k), x(t + k)$
 - ii) Time scaling operation $x(\alpha t - k), x(\alpha t + k)$
 - iii) Time flipping $x(-\alpha t - k), x(-\alpha t + k)$

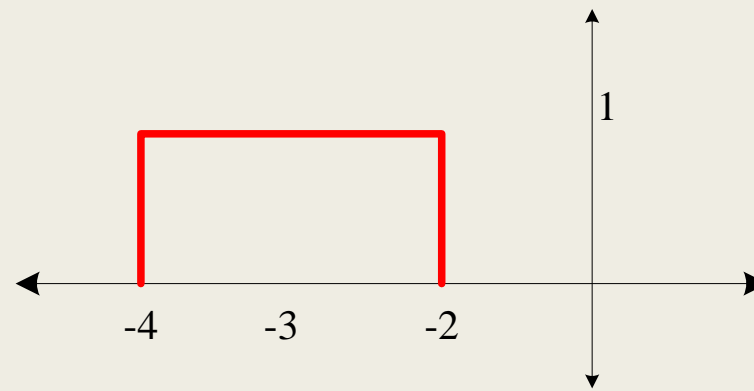
- **e.g. Method 2:** $x(t)$ is the given signal
 - i) Time scaling operation $x(\alpha t)$
 - ii) Time shifting operation $x(\alpha(t - k/\alpha)), x(\alpha(t + k/\alpha))$
 - iii) Time flipping $x(-\alpha(t - k/\alpha)), x(-\alpha(t + k/\alpha))$

**There is no precedence rule.
You can apply operations in
any order**

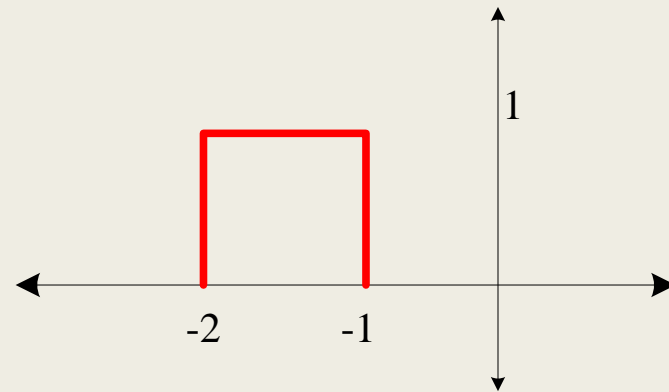
Exp 4.1: For the signal given, sketch $x(-2t + 3)$



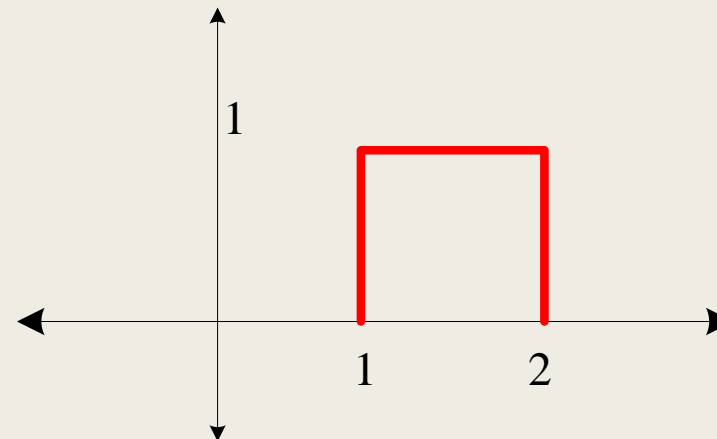
$$x(t+3)$$



$$x(2t+3)$$

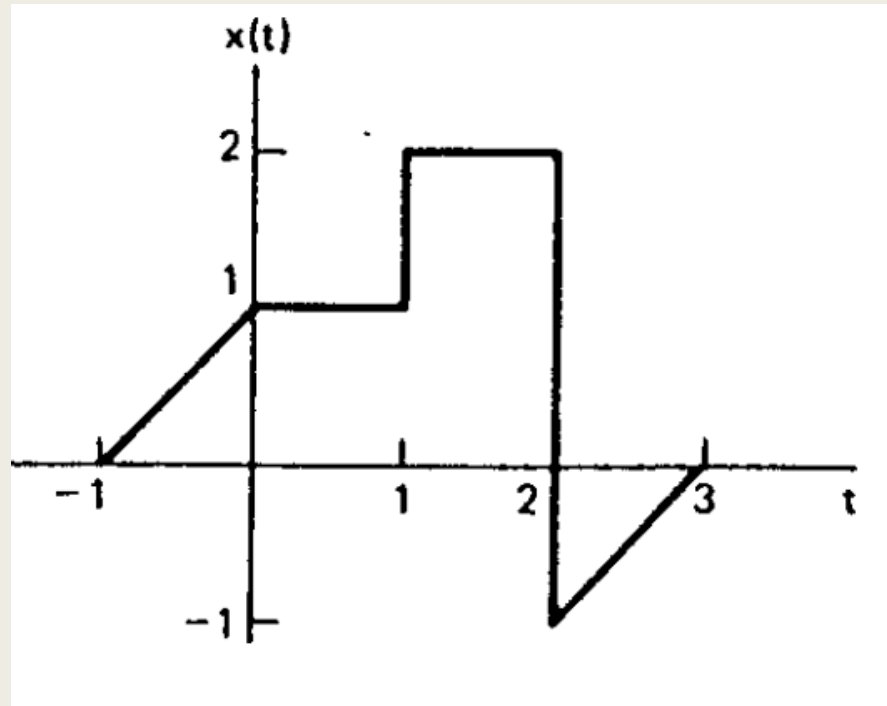


$$x(-2t+3)$$



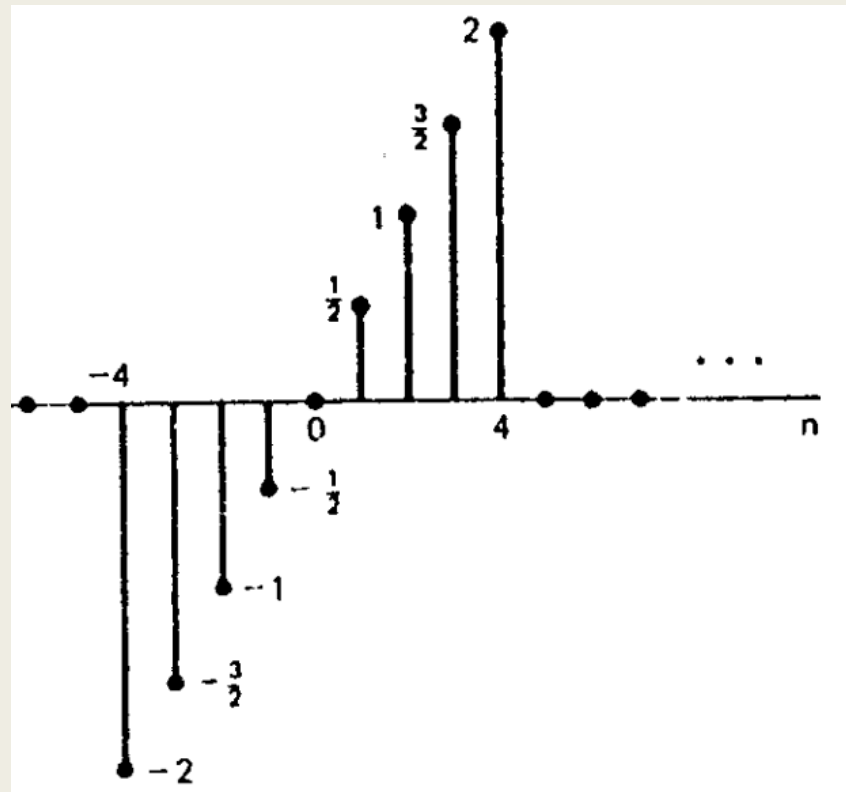
PP. 4.1) For signals given, sketch

(i) $x(t/2 - 2)$ (ii) $x(1 - 2t)$ (iii) $x(2 - t/3)$



PP. 4.2) For signals given, sketch

(i) $x[4 - n]$ (ii) $x[2n + 1]$ (iii) $x[-\frac{n}{3} + 2]$



Thank You !!!