Digital Image Processing (CPE-415)

Assignment #1 – Detailed Solutions

Topics: Image Sensors, Sampling & Quantization, Pixel Connectivity

Reference: Digital Image Processing, 3rd Edition
by Gonzalez & Woods

Problems: 2.1, 2.2, 2.5, 2.10, 2.11, 2.12, 2.13, 2.15, 2.16, 2.17

Name: _____

Roll No: _____

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Problem 2.1 – Smallest Dot the Eye Can Discern

Problem Statement

Using the background information provided in Section 2.1, and thinking purely in geometric terms, estimate the diameter of the smallest printed dot that the eye can discern if the page on which the dot is printed is 0.2 m away from the eyes. Assume for simplicity that the visual system ceases to detect the dot when the image of the dot on the fovea becomes smaller than the diameter of one receptor (cone) in that area of the retina. Assume further that the fovea can be modeled as a square array of dimensions 1.5 mm × 1.5 mm, and that the cones and spaces between the cones are distributed uniformly throughout this array.

Step 1: Understand What We Know (Given Information)

From the textbook (Section 2.1), we are given the following facts about the human eye:

- The **fovea** is the central part of the retina where cones are most dense. This is where sharp, detailed vision happens.
- The fovea is modeled as a **square array** of size 1.5 mm × 1.5 mm.
- The **density of cones** in the fovea is approximately 150,000 elements per mm^2.
- The distance from the center of the lens to the retina (along the visual axis) is approximately **17** mm.
- The page is at a distance of $d = 0.2$ m $= 200$ mm from the eye.

Step 2: Find the Total Number of Cones in the Fovea

The fovea has area:

$$A_{\text{fovea}} = 1.5 \times 1.5 = 2.25 \text{ mm}^2$$

Total number of cones:

$$N = 150{,}000 \times 2.25 = 337{,}500 \text{ cones}$$

Step 3: Find the Size of One Cone (One Receptor)

Since the fovea is modeled as a square array and the cones are uniformly distributed, the number of cones along one side of the fovea is:

$$n = \sqrt{337{,}500} \approx 581 \text{ cones per side}$$

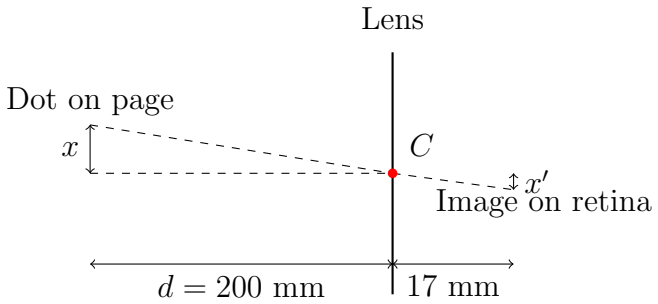
So the **diameter (size) of one cone** is:

$$d_{\text{cone}} = \frac{1.5 \text{ mm}}{581} \approx 0.00258 \text{ mm} = 2.58 \ \mu\text{m}$$

What this means: Each cone receptor on the fovea occupies roughly a 2.58 μm × 2.58 μm square.

Step 4: Use Geometry to Find the Smallest Dot

Now we use **similar triangles**. The eye works like a pinhole camera—an object in the real world creates an image on the retina. The geometry looks like this:



By similar triangles:

$$\frac{x}{d} = \frac{x'}{17}$$

where:

- x = diameter of the dot on the page (what we want to find)
- $d = 200$ mm (distance from page to eye)
- x' = size of the image on the retina = size of one cone = d_{cone}
- 17 mm = distance from lens to retina

We want the image of the dot to be **exactly the size of one cone** (any smaller and the eye can't see it). So:

$$\frac{x}{200} = \frac{0.00258}{17}$$

$$x = \frac{200 \times 0.00258}{17}$$

$$\boxed{x \approx 0.0304 \text{ mm} \approx 30.4\ \mu\text{m}}$$

Answer

The smallest dot the eye can discern at a distance of 0.2 m is approximately **0.0304 mm** (or about **30.4 μm**). This is roughly 1/3 of the thickness of a human hair.
In plain English: We figured out how big one "pixel" on our retina is (one cone receptor), then used simple geometry to calculate what real-world size that corresponds to at a distance of 20 cm. If the dot is smaller than this, its image on our retina would be smaller than one cone, and we simply cannot detect it.

Problem 2.2 – Dark Theater Adaptation

Problem Statement

When you enter a dark theater on a bright day, it takes an appreciable interval of time before you can see well enough to find an empty seat. Which of the visual processes explained in Section 2.1 is at play in this situation?

Step 1: Understand the Scenario

- You are outside in bright sunlight. Your eyes are adapted to **high light intensity** (photopic vision—cones are active).
- You walk into a dark movie theater. Suddenly, the light level drops **dramatically**.
- For several minutes, you can barely see anything. Then gradually, your vision improves and you can start to see seats, people, etc.

Step 2: Identify the Visual Process

The process at play is called **brightness adaptation** (also called **dark adaptation**).
From Section 2.1.3 of the textbook:

- The human visual system can handle an **enormous range of light intensities**—about 10^{10} from the dimmest to the brightest.
- However, the eye **cannot** operate over this entire range **simultaneously**. It works within a smaller window of intensity at any given time.
- The eye adjusts this window by changing its **overall sensitivity**—this is brightness adaptation.
- When you go from bright to dark, the eye needs time to shift its adaptation level from a high-intensity setting to a low-intensity setting.

Step 3: What Happens Physically

1. **Outside (bright):** Your **cones** are active (photopic vision). Cones need lots of light to work well. The **rods** are essentially "bleached out" (overwhelmed by the bright light and inactive).
2. **Entering the theater (dark):** The light level drops below what cones can handle effectively. Your vision must switch from **cone-based (photopic)** to **rod-based (scotopic)** vision.
3. **The delay:** The rods need time (typically **20–30 minutes** for full adaptation) to regenerate the visual pigment (rhodopsin) that was bleached by the bright light. During this regeneration period, your sensitivity to low light gradually increases.
4. **After adaptation:** The rods become fully functional, and you can see in the dim light of the theater. However, you won't see colors well because **rods are not sensitive to color**—everything looks grayish (scotopic vision).

Answer

The visual process at play is **brightness adaptation** (specifically, **dark adaptation**). When you move from a brightly lit environment to a dark one, the eye must shift its sensitivity from photopic (cone-based, bright-light) vision to scotopic (rod-based, dim-light) vision. This transition takes time because the rods need to regenerate their visual pigments (rhodopsin) that were bleached by the bright light. This is why you are essentially "blind" for several minutes until your eyes adjust.

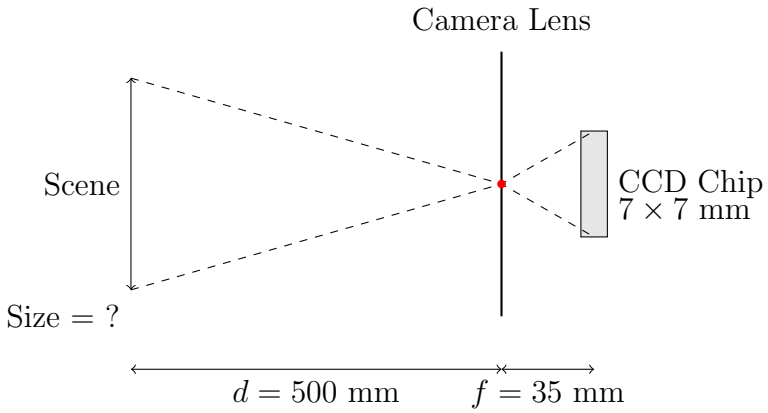
Problem 2.5 – CCD Camera Resolution (Line Pairs per mm)

Problem Statement

A CCD camera chip of dimensions 7×7 mm, and having 1024×1024 elements, is focused on a square, flat area, located 0.5 m away. How many line pairs per mm will this camera be able to resolve? The camera is equipped with a 35-mm lens. (Hint: Model the imaging process as in Fig. 2.3, with the focal length of the camera lens substituting for the focal length of the eye.)

Step 1: Understand the Setup

Think of this like the eye model from Section 2.1.2 (Fig. 2.3), but instead of an eye, we have a CCD camera:



Given:

- CCD chip size: 7×7 mm
- CCD resolution: 1024×1024 elements (pixels)
- Distance to scene: $d = 0.5$ m $= 500$ mm
- Focal length of lens: $f = 35$ mm

Step 2: Find the Size of the Scene Area Being Imaged

Using similar triangles (same geometry as the eye in Fig. 2.3):

$$\frac{\text{Scene size}}{d} = \frac{\text{Chip size}}{f}$$

$$\frac{\text{Scene size}}{500} = \frac{7}{35}$$

$$\text{Scene size} = 500 \times \frac{7}{35} = 500 \times 0.2 = 100 \text{ mm}$$

So the camera sees a 100×100 mm area of the scene.

Step 3: Find the Resolution on the Scene

The CCD has 1024 pixels spread across 100 mm of the scene. So:

$$\text{Pixels per mm (on the scene)} = \frac{1024}{100} = 10.24 \text{ pixels/mm}$$

Step 4: Convert to Line Pairs per mm

What is a "line pair"? A line pair consists of one **dark line** and one **bright line** (or one white + one black stripe). To resolve one line pair, you need **at least 2 pixels**—one pixel for the dark line and one for the bright line.

This is essentially the **Nyquist criterion**: to resolve a spatial frequency, you need at least 2 samples per cycle.

$$\text{Line pairs per mm} = \frac{\text{Pixels per mm}}{2} = \frac{10.24}{2} = 5.12 \text{ lp/mm}$$

Answer

The camera can resolve approximately **5.12 line pairs per mm** on the scene.
In plain English: The camera sees a 100×100 mm area with 1024 pixels across. That gives us about 10 pixels per mm. Since you need 2 pixels to see one "stripe" (one dark + one light = one line pair), we can resolve about 5 line pairs per mm.

Problem 2.10 – HDTV Movie Storage

Problem Statement

High-definition television (HDTV) generates images with 1125 horizontal TV lines interlaced (where every other line is painted on the tube face in each of two fields, each field being 1/60th of a second in duration). The width-to-height aspect ratio of the images is 16:9. A company has designed an image capture system that generates digital images from HDTV images. The resolution of each TV (horizontal) line in their system is in proportion to vertical resolution, with the proportion being the width-to-height ratio of the images. Each pixel in the color image has 24 bits of intensity resolution, 8 bits each for a red, a green, and a blue image. How many bits would it take to store a 2-hour HDTV movie?

Step 1: Find the Image Dimensions (in Pixels)

Vertical resolution:

- There are **1125 horizontal lines**, so the vertical resolution is **1125 pixels** (each line = one row of pixels).

Horizontal resolution:

- The problem says: horizontal resolution is proportional to vertical resolution, with the proportion being the aspect ratio (16:9).
- Aspect ratio = width/height = 16/9
- So: horizontal pixels = $1125 \times \frac{16}{9} = 1125 \times 1.7778 =$ **2000 pixels**

So each frame is **2000** $\times$ **1125** pixels.

Step 2: Find the Number of Bits per Frame

Each pixel has 24 bits (8 bits for Red + 8 bits for Green + 8 bits for Blue).

$$\text{Bits per frame} = 2000 \times 1125 \times 24 = 54{,}000{,}000 \text{ bits/frame}$$

$$= 5.4 \times 10^7 \text{ bits/frame}$$

Step 3: Find the Frame Rate

The system uses **interlaced scanning**:

- Each "field" paints every other line and takes 1/60th second.
- It takes **2 fields** to make one complete frame.
- So: frame rate = 60/2 = **30 frames per second**.

Step 4: Find the Total Duration in Seconds

$$\text{Duration} = 2 \text{ hours} = 2 \times 60 \times 60 = 7{,}200 \text{ seconds}$$

Step 5: Find Total Number of Frames

$$\text{Total frames} = 30 \times 7{,}200 = 216{,}000 \text{ frames}$$

Step 6: Calculate Total Bits

$$\text{Total bits} = \text{bits/frame} \times \text{total frames}$$

$$= 54{,}000{,}000 \times 216{,}000$$

$$= 11{,}664{,}000{,}000{,}000 \text{ bits}$$

$$\boxed{\approx 1.1664 \times 10^{13} \text{ bits}}$$

To put this in perspective:

$$\frac{1.1664 \times 10^{13}}{8} = 1.458 \times 10^{12} \text{ bytes} \approx 1.458 \text{ terabytes (TB)}$$

Answer

It would take approximately $\mathbf{1.1664 \times 10^{13}}$ **bits** (about **1.458 TB**) to store a 2-hour HDTV movie.

Breakdown:

- Frame size: 2000×1125 pixels
- Bits per pixel: 24 (color)
- Frame rate: 30 fps (interlaced, 2 fields per frame)
- Duration: 7200 seconds
- Total: $2000 \times 1125 \times 24 \times 30 \times 7200 = 1.1664 \times 10^{13}$ bits

This is why video compression (like H.264 or H.265) is absolutely necessary—without it, even a modern hard drive would struggle to store a single movie!

Problem 2.11 – Adjacency of Two Image Subsets

Problem Statement

Consider the two image subsets, S_1 and S_2, shown in the following figure. For $V = \{1\}$, determine whether these two subsets are (a) 4-adjacent, (b) 8-adjacent, or (c) m-adjacent.

0	0	0	0	0	0	0	1	1
0	1	0	0	1	0	0	1	0
0	1	1	0	0	1	0	1	1
0	0	0	0	0	1	1	1	0
0	0	0	0	0	0	1	1	1

S_1 is on the left and S_2 is on the right.

Step 1: Identify the Two Subsets

Let me label the pixel positions using (row, column), starting from (0,0) at top-left.

S_1 (the 1-valued pixels on the left side):

- $(2,1)$, $(3,1)$, $(3,2)$, $(2,4)$, $(3,5)$, $(4,5)$

S_2 (the 1-valued pixels on the right side):

- $(1,7)$, $(1,8)$, $(2,7)$, $(3,7)$, $(3,8)$, $(4,6)$, $(4,7)$, $(5,6)$, $(5,7)$, $(5,8)$

Since $V = \{1\}$, adjacency only matters between pixels that have value 1.

Step 2: Find the "Bridge" Pixels

The key question is: **Are any pixels from S_1 adjacent to any pixels from S_2?**

Let's look at the pixels in S_1 that are closest to S_2:

- Pixel $p = (4,5)$ from S_1 (value = 1)
- Pixel $q = (4,6)$ from S_2 (value = 1)
- Also: Pixel $(3,5)$ from S_1 and $(3,7)$ from S_2 — but these are too far apart.

The closest pair is $p = (4,5)$ and $q = (4,6)$.

Step 3: Check 4-adjacency

Two pixels are **4-adjacent** if one is in the N_4 (4-neighborhood) of the other. The 4-neighbors of a pixel are the pixels directly above, below, left, and right.

$N_4(p) = N_4(4,5) = \{(3,5), (5,5), (4,4), (4,6)\}$

Is $q = (4,6)$ in $N_4(p)$? **YES!** $(4,6)$ is directly to the right of $(4,5)$.

Answer

(a) 4-adjacent: YES. The pixel $(4,5) \in S_1$ and the pixel $(4,6) \in S_2$ are direct horizontal neighbors (4-adjacent). Since both have value 1 and $1 \in V$, S_1 and S_2 are 4-adjacent.

Step 4: Check 8-adjacency

Two pixels are **8-adjacent** if one is in the N_8 (8-neighborhood) of the other. The 8-neighbors include all surrounding pixels (horizontal, vertical, AND diagonal).

Since S_1 and S_2 are already 4-adjacent, they are **automatically 8-adjacent** too. (4-adjacency is a subset of 8-adjacency—if you're a 4-neighbor, you're definitely an 8-neighbor.)

Answer

(b) 8-adjacent: YES. Since they are already 4-adjacent, they are trivially 8-adjacent as well.

Step 5: Check m-adjacency

Two pixels p and q with values from V are **m-adjacent** if:

1. q is in $N_4(p)$, **OR**
2. q is in $N_D(p)$ (diagonal neighbor) AND $N_4(p) \cap N_4(q)$ has **no pixels with values from** V.

We already showed that $q = (4,6)$ is in $N_4(p)$ where $p = (4,5)$. Condition (i) is satisfied.

Answer

(c) m-adjacent: YES. Since $(4,6) \in N_4(4,5)$, condition (i) of m-adjacency is satisfied. S_1 and S_2 are m-adjacent.
Summary: S_1 and S_2 are **4-adjacent, 8-adjacent, and m-adjacent**—all three types—because there exist two pixels (one from each set) that are direct horizontal neighbors.

Problem 2.12 – Converting 8-Path to 4-Path

Problem Statement

Develop an algorithm for converting a one-pixel-thick 8-path to a 4-path.

Step 1: Understand the Problem

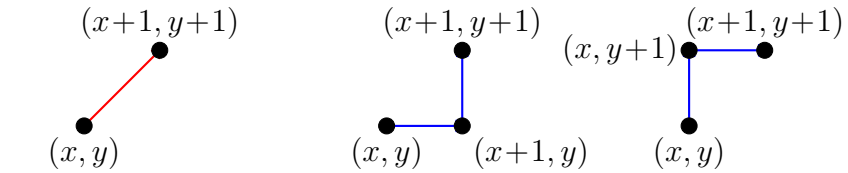
What is an 8-path? A sequence of pixels where each consecutive pair is 8-adjacent (they can be connected horizontally, vertically, or diagonally).

What is a 4-path? A sequence of pixels where each consecutive pair is 4-adjacent (they can ONLY be connected horizontally or vertically—NO diagonals allowed).

The Goal: Take any 8-path and convert it to a 4-path that goes between the same start and end points.

Step 2: The Key Idea

Whenever we encounter a **diagonal step** in the 8-path, we need to replace it with **two steps**: one horizontal and one vertical. A diagonal move goes from (x, y) to $(x+1, y+1)$—we can break this into: first go to $(x+1, y)$ then to $(x+1, y+1)$, OR first go to $(x, y+1)$ then to $(x+1, y+1)$.



Step 3: The Algorithm

Algorithm: Convert 8-Path to 4-Path

Input: An 8-path $P_8 = \{p_0, p_1, p_2, \ldots, p_n\}$ where $p_i = (x_i, y_i)$
Output: A 4-path P_4

1. Initialize $P_4 = \{p_0\}$ (start with the first pixel).
2. For each consecutive pair (p_i, p_{i+1}) in the 8-path:
   (a) Compute: $\Delta x = x_{i+1} - x_i$ and $\Delta y = y_{i+1} - y_i$
   (b) **If** $|\Delta x| + |\Delta y| = 1$ (already a 4-connected step, i.e., purely horizontal or purely vertical):
   - Add p_{i+1} to P_4.

   (c) **Else if** $|\Delta x| = 1$ AND $|\Delta y| = 1$ (diagonal step):
   - Insert an intermediate pixel: add $(x_i + \Delta x,\ y_i)$ to P_4.
   - Then add $p_{i+1} = (x_{i+1}, y_{i+1})$ to P_4.
3. Return P_4.

Step 4: Example

Consider the 8-path: $(0,0) \to (1,1) \to (2,1) \to (3,2)$

- $(0,0) \to (1,1)$: Diagonal! Insert $(1,0)$. Path becomes: $(0,0) \to (1,0) \to (1,1)$
- $(1,1) \to (2,1)$: Horizontal move. Already 4-connected. Just add. Path: $\ldots \to (2,1)$
- $(2,1) \to (3,2)$: Diagonal! Insert $(3,1)$. Path: $\ldots \to (3,1) \to (3,2)$

Final 4-path: $(0,0) \to (1,0) \to (1,1) \to (2,1) \to (3,1) \to (3,2)$

Answer

The algorithm scans the 8-path step by step. For every diagonal step (where both x and y change by 1), it inserts an additional intermediate pixel that decomposes the diagonal into two 4-connected moves (one horizontal + one vertical). Non-diagonal steps are kept as-is. The choice of whether to go horizontal-first or vertical-first is arbitrary (either produces a valid 4-path).

Problem 2.13 – Converting m-Path to 4-Path

Problem Statement

Develop an algorithm for converting a one-pixel-thick m-path to a 4-path.

Step 1: Understand m-adjacency vs 4-adjacency

Recall the three types of adjacency (for pixels with values in V):

- **4-adjacency:** Only horizontal/vertical neighbors.
- **8-adjacency:** All 8 surrounding neighbors (includes diagonals).
- **m-adjacency (mixed):** Two pixels p and q are m-adjacent if:
  - (i) $q \in N_4(p)$, **OR**
  - (ii) $q \in N_D(p)$ (diagonal) AND $N_4(p) \cap N_4(q)$ contains **no pixels from** V

Key difference between 8-path and m-path: An m-path only uses diagonal connections when there is NO common 4-neighbor with a value in V. This means m-paths are **more restrictive** than 8-paths and have fewer ambiguities.

Step 2: Observation

In an m-path, a diagonal step from $p = (x, y)$ to $q = (x + \Delta x, y + \Delta y)$ (where $|\Delta x| = |\Delta y| = 1$) only occurs when the two "corner" pixels $(x + \Delta x, y)$ and $(x, y + \Delta y)$ do NOT have values in V. However, for our 4-path conversion, we just need **any valid geometric 4-path**, regardless of pixel values.

Step 3: The Algorithm

The algorithm is **essentially the same as Problem 2.12**, since the conversion is purely geometric.

Algorithm: Convert m-Path to 4-Path

Input: An m-path $P_m = \{p_0, p_1, p_2, \ldots, p_n\}$ where $p_i = (x_i, y_i)$
Output: A 4-path P_4

1. Initialize $P_4 = \{p_0\}$.
2. For each consecutive pair (p_i, p_{i+1}) in the m-path:
   (a) Compute: $\Delta x = x_{i+1} - x_i$ and $\Delta y = y_{i+1} - y_i$
   (b) **If** $|\Delta x| + |\Delta y| = 1$ (4-connected step):
   - Add p_{i+1} to P_4 directly.

   (c) **Else if** $|\Delta x| = 1$ AND $|\Delta y| = 1$ (diagonal step):
   - Since this is an m-path, we know that the two "corner" pixels (the possible intermediate 4-connected pixels) do NOT have values in V. However, geometrically, they still exist as positions.
   - Insert the intermediate pixel $(x_i + \Delta x,\ y_i)$ to P_4.
   - Then add p_{i+1} to P_4.
3. Return P_4.

Step 4: Important Note

There is a subtle but important difference from Problem 2.12:

In the m-path, when we have a diagonal step from p to q, we know by the m-adjacency definition that $N_4(p) \cap N_4(q)$ has no pixels with values from V. This means the intermediate pixels we insert to form the 4-path will NOT have values from V—they will be background/zero pixels.

Therefore, the resulting 4-path may pass through pixels that are **not in** V. This is acceptable if we only care about the geometric path. If we need the 4-path to consist only of V-valued pixels, then **such a conversion may not always be possible** for m-paths with diagonal steps.

Answer

The algorithm is the same geometrical decomposition as Problem 2.12: scan each step in the m-path, and replace every diagonal step with two 4-connected steps (one horizontal + one vertical) by inserting an intermediate pixel. The key nuance is that in an m-path, diagonal steps only occur where the corner pixels are NOT in V, so the intermediate pixels inserted will not have values from V. The 4-path produced is a valid geometric path but may include pixels not in V.

Problem 2.15 – Shortest Paths Between p and q

Problem Statement

Consider the image segment shown.

3	1	2	1 (q)
2	2	0	2
1	2	1	1
(p) 1	0	1	2

(a) Let $V = \{0, 1\}$ and compute the lengths of the shortest 4-, 8-, and m-path between p and q. If a particular path does not exist between these two points, explain why.
(b) Repeat for $V = \{1, 2\}$.

Step 0: Label the Grid

Let me assign coordinates (row, col) starting from $(0, 0)$ at top-left:

	col 0	col 1	col 2	col 3
row 0	3	1	2	**1 (q)**
row 1	2	2	0	2
row 2	1	2	1	1
row 3	**1 (p)**	0	1	2

$p = (3, 0)$ with value 1, $q = (0, 3)$ with value 1.

Part (a): $V = \{0, 1\}$

Only pixels with values 0 or 1 can be used in the path.

Let me mark which pixels are "usable" (have values in $V = \{0, 1\}$):

	col 0	col 1	col 2	col 3
row 0	3	**1**	2	**1 (q)**
row 1	2	2	**0**	2
row 2	**1**	2	**1**	**1**
row 3	**1 (p)**	**0**	**1**	2

Green = usable (value in V), Red = NOT usable.

8.2.1 Shortest 4-path (V = {0,1})

In a 4-path, we can only move up, down, left, or right to adjacent pixels with values in V.

Starting from $p = (3, 0)$, the usable pixels and their 4-connections are:

- $(3, 0) \to (3, 1)$: values $1 \to 0$ ✓
- $(3, 1) \to (3, 2)$: values $0 \to 1$ ✓

- $(3,2) \to (2,2)$: values $1 \to 1$ ✓
- $(2,2) \to (1,2)$: values $1 \to 0$ ✓
- $(1,2) \to (0,2)$: value at $(0,2) = 2$, NOT in V. **Blocked!**
- $(2,2) \to (2,3)$: values $1 \to 1$ ✓
- $(2,3) \to (1,3)$: value at $(1,3) = 2$, NOT in V. **Blocked!**

Let me trace all possible routes systematically. We need to reach $(0,3)$.
Looking at the grid, to reach row 0 via 4-connectivity:

- The only usable pixels in row 0 near q are $(0,1)$ with value 1 and $(0,3)$ with value 1. But $(0,0) = 3$ and $(0,2) = 2$ are blocked.
- From $(1,2)$ value 0, can go up to $(0,2)$ value 2—blocked.
- From $(2,0)$ value 1, can go up to $(1,0)$ value 2—blocked.

Can we reach $(0,1)$? From $(1,2)$, go left to $(1,1) = 2$—blocked. From $(2,0)$ up to $(1,0) = 2$—blocked.

There is **no 4-path** from p to q because the usable pixels in row 1 and row 0 form a barrier. The only pixel in row 1 that's usable is $(1,2) = 0$, and from there we can't reach row 0 via 4-connectivity through V-valued pixels ($(0,2) = 2$ is blocked, and going left to $(1,1) = 2$ is blocked).

Wait—let me recheck more carefully. From $(1,2)$, going up reaches $(0,2) = 2$ (blocked). There's no way to reach $(0,1)$ or $(0,3)$ via 4-path.

Result: No 4-path exists between p and q for $V = \{0,1\}$.

8.2.2 Shortest 8-path (V = {0,1})

In an 8-path, we can also move diagonally. Let's try:

$(3,0) \to (3,1) \to (3,2) \to (2,2) \to (1,2) \to (0,3)$

Check: $(1,2)$ to $(0,3)$: diagonal move. Value at $(1,2) = 0 \in V$, value at $(0,3) = 1 \in V$. Are they 8-adjacent? $(1,2)$ to $(0,3)$: $|\Delta r| = 1$, $|\Delta c| = 1$. Yes, they are 8-adjacent! ✓

Path: $p(3,0) \to (3,1) \to (3,2) \to (2,2) \to (1,2) \to q(0,3)$

Length = 5 steps.

Can we do better? Let's try using diagonals more:

$(3,0) \to (2,0) \to (1,2)$? No—$(2,0)$ to $(1,2)$ is not 8-adjacent (distance too far).

$(3,0) \to (3,1) \to (2,2) \to (1,2) \to (0,3)$? – Check $(3,1)$ to $(2,2)$: diagonal, values $0 \to 1$, both in V. ✓

Path: $p(3,0) \to (3,1) \to (2,2) \to (1,2) \to q(0,3)$

Length = 4 steps. This is shorter!

Can we do even better?

$(3,0) \to (3,1) \to (2,2) \to (0,3)$? – $(2,2)$ to $(0,3)$: $|\Delta r| = 2$, not adjacent—impossible.

Result: Shortest 8-path has length 4.

Path: $(3,0) \to (3,1) \to (2,2) \to (1,2) \to (0,3)$

8.2.3 Shortest m-path (V = {0,1})

For m-adjacency, diagonal connections are only used when the two "corner" pixels (shared 4-neighbors) don't have values in V.

Let's check each diagonal step in the 8-path found above:
Step $(3,1) \to (2,2)$: diagonal. Shared 4-neighbors: $(3,2)$ and $(2,1)$.

- $(3,2) = 1 \in V$ — **shared neighbor has value in V!**
- So this diagonal connection is **NOT allowed** in m-adjacency!

Step $(1,2) \to (0,3)$: diagonal. Shared 4-neighbors: $(0,2)$ and $(1,3)$.

- $(0,2) = 2 \notin V$, $(1,3) = 2 \notin V$.
- Neither has value in V, so this diagonal IS allowed in m-adjacency. ✓

So we need an alternative for the $(3,1) \to (2,2)$ step. We must go through $(3,2)$:
$p(3,0) \to (3,1) \to (3,2) \to (2,2) \to (1,2) \to q(0,3)$
Check all steps for m-adjacency:

- $(3,0) \to (3,1)$: 4-adjacent. ✓
- $(3,1) \to (3,2)$: 4-adjacent. ✓
- $(3,2) \to (2,2)$: 4-adjacent. ✓
- $(2,2) \to (1,2)$: 4-adjacent. ✓
- $(1,2) \to (0,3)$: m-adjacent (diagonal, and shared 4-neighbors $(0,2) = 2$, $(1,3) = 2$, neither in V). ✓

Result: Shortest m-path has length 5.
Path: $(3,0) \to (3,1) \to (3,2) \to (2,2) \to (1,2) \to (0,3)$

Answer

Part (a): $V = \{0,1\}$

- **4-path:** Does NOT exist. The V-valued pixels don't form a 4-connected path from p to q because the barrier of 2-valued and 3-valued pixels blocks all vertical 4-connections between rows 1 and 0.
- **8-path:** Shortest length = **4**. Path: $(3,0) \to (3,1) \to (2,2) \to (1,2) \to (0,3)$.
- **m-path:** Shortest length = **5**. Path: $(3,0) \to (3,1) \to (3,2) \to (2,2) \to (1,2) \to (0,3)$.

Part (b): $V = \{1, 2\}$

Now usable pixels have values 1 or 2:

	col 0	col 1	col 2	col 3
row 0	3	**1**	**2**	**1 (q)**
row 1	**2**	**2**	0	**2**
row 2	**1**	**2**	**1**	**1**
row 3	**1 (p)**	0	**1**	**2**

Many more pixels are now usable!

8.3.1 Shortest 4-path (V = {1,2})

We can try: $p(3,0) \to (2,0) \to (1,0) \to (0,1) \to$?

Wait—$(1,0)$ to $(0,1)$ is diagonal, not 4-adjacent. Let me be careful.

$(3,0) \to (2,0) \to (1,0) \to (1,1) \to (0,1) \to (0,2) \to (0,3)$

Length = 6.

Can we do better?

$(3,0) \to (2,0) \to (2,1) \to (1,1) \to (0,1) \to (0,2) \to (0,3)$

Length = 6. Same.

$(3,0) \to (2,0) \to (2,1) \to (2,2) \to (2,3) \to (1,3) \to (0,3)$

Length = 6. Same.

The minimum Manhattan distance from $(3,0)$ to $(0,3)$ is $|3-0| + |0-3| = 6$, so the shortest possible 4-path has length 6.

Result: Shortest 4-path has length 6.

8.3.2 Shortest 8-path (V = {1,2})

$(3,0) \to (2,1) \to (1,1) \to (0,2) \to (0,3)$?

Check: $(3,0)$ value = 1, $(2,1)$ value = 2, 8-adjacent? Yes (diagonal). Both $\in V$. ✓

$(2,1) \to (1,1)$: values $2 \to 2$, 4-adjacent. ✓

$(1,1) \to (0,2)$: values $2 \to 2$, diagonal, 8-adjacent. ✓

$(0,2) \to (0,3)$: values $2 \to 1$, 4-adjacent. ✓

Length = 4.

Can we do better with length 3?

$(3,0) \to (2,1) \to (1,2) \to (0,3)$: values $1 \to 2 \to 1 \to 1$. But $(1,2) = 0$, NOT in V! **Blocked.**

$(3,0) \to (2,1) \to (1,1) \to (0,2) \to \ldots$ already length 3 and not at q yet.

So length 4 is the shortest 8-path.

Result: Shortest 8-path has length 4.

Path: $(3,0) \to (2,1) \to (1,1) \to (0,2) \to (0,3)$.

8.3.3 Shortest m-path (V = {1,2})

Check each diagonal step in the 8-path:

Step $(3,0) \to (2,1)$: diagonal. Shared 4-neighbors of $(3,0)$ and $(2,1)$:

- $(3,1) = 0 \notin V$, $(2,0) = 1 \in V$.

- Shared 4-neighbor $(2, 0)$ has value in V! So this diagonal is **NOT m-adjacent**.

Step $(1, 1) \to (0, 2)$: diagonal. Shared 4-neighbors:

- $(0, 1) = 1 \in V$, $(1, 2) = 0 \notin V$.
- Shared 4-neighbor $(0, 1)$ has value in V! So this diagonal is **NOT m-adjacent**.

Both diagonals fail m-adjacency! We need a purely 4-connected route (or m-adjacent diagonals that pass the test).

Let's look for m-adjacent diagonals:

$(2, 2) \to (1, 3)$: Shared 4-neighbors: $(2, 3) = 1 \in V$ and $(1, 2) = 0 \notin V$. Has one in V, so NOT m-adjacent.

$(2, 2) \to (1, 1)$: Shared 4-neighbors: $(2, 1) = 2 \in V$ and $(1, 2) = 0 \notin V$. Has one in V, so NOT m-adjacent.

$(2, 3) \to (1, 3)$?: 4-adjacent (same column). ✓

It seems hard to find valid m-diagonals. The shortest m-path may just be the 4-path.

Shortest m-path: same as 4-path with length 6.

Result: Shortest m-path has length 6.

Path: e.g., $(3, 0) \to (2, 0) \to (2, 1) \to (1, 1) \to (0, 1) \to (0, 2) \to (0, 3)$.

Answer

Part (b): $V = \{1, 2\}$

- **4-path:** Shortest length = **6**. E.g., $(3, 0) \to (2, 0) \to (2, 1) \to (1, 1) \to (0, 1) \to (0, 2) \to (0, 3)$.
- **8-path:** Shortest length = **4**. Path: $(3, 0) \to (2, 1) \to (1, 1) \to (0, 2) \to (0, 3)$.
- **m-path:** Shortest length = **6**. Same as 4-path since all useful diagonals are blocked by shared 4-neighbors with values in V.

Problem 2.16 – D4 Distance and 4-Path

Problem Statement

(a) Give the condition(s) under which the D_4 distance between two points p and q is equal to the shortest 4-path between these points.
(b) Is this path unique?

Step 1: Review D_4 Distance

The D_4 distance (city-block distance or Manhattan distance) between two pixels $p = (x, y)$ and $q = (s, t)$ is:

$$D_4(p, q) = |x - s| + |y - t|$$

This is literally: how far apart are they horizontally + how far apart are they vertically.
Example: If $p = (1, 3)$ and $q = (5, 6)$, then $D_4 = |1 - 5| + |3 - 6| = 4 + 3 = 7$.

Step 2: What is the Shortest 4-Path?

A 4-path is a sequence of pixels where consecutive pixels are 4-adjacent (share an edge—horizontal or vertical neighbors only). The "shortest 4-path" is the one with the fewest steps (fewest intermediate pixels).

Key observation: The shortest 4-path between p and q depends on **which pixels are allowed** to be in the path (i.e., which pixels have values in V). The D_4 distance, on the other hand, is a **pure geometric/mathematical formula** that ignores pixel values entirely.

Part (a): When Does D_4 = Shortest 4-Path?

Answer

(a) The D_4 distance between p and q equals the length of the shortest 4-path between them when the **path is unconstrained**—meaning:

1. **All pixels** along the path (between p and q) have values in V (no obstacles), OR equivalently, the V set is unrestricted or all pixel values belong to V.
2. The path can proceed **freely** without being blocked by pixels with values outside V.

In other words, when every pixel is a valid step (no barriers), the shortest 4-path length is exactly $|x - s| + |y - t| = D_4(p, q)$.
Formally: $D_4(p, q)$ = shortest 4-path if and only if there exists a 4-path from p to q of length $D_4(p, q)$ consisting entirely of pixels with values in V. Since the minimum possible length of any 4-path between p and q is $D_4(p, q)$ (you need at least $|x - s|$ horizontal steps and $|y - t|$ vertical steps, and in 4-connectivity you can't combine them), this happens when no obstacles force a detour.
Simple version: The D_4 distance equals the shortest 4-path when the set V includes all intensity values in the image (or when no pixel along a Manhattan-distance-optimal route has a value outside V).

Part (b): Is This Path Unique?

Answer

(b) No, the path is generally NOT unique.
When the shortest 4-path length equals $D_4(p, q)$, the path has $|x - s|$ horizontal moves and $|y - t|$ vertical moves. These moves can be **interleaved in any order**, producing multiple distinct paths.
The number of distinct shortest 4-paths is given by the binomial coefficient:

$$\text{Number of paths} = \binom{|x - s| + |y - t|}{|x - s|} = \frac{(|x - s| + |y - t|)!}{|x - s|! \cdot |y - t|!}$$

Example: If $p = (0, 0)$ and $q = (2, 1)$, then $D_4 = 3$. The number of shortest paths is $\binom{3}{2} = 3$:

1. $(0, 0) \to (1, 0) \to (2, 0) \to (2, 1)$ (right, right, down)
2. $(0, 0) \to (1, 0) \to (1, 1) \to (2, 1)$ (right, down, right)
3. $(0, 0) \to (0, 1) \to (1, 1) \to (2, 1)$ (down, right, right)

The path is unique **only** when p and q are on the same row ($|y - t| = 0$) or same column ($|x - s| = 0$), in which case there is only one straight-line path.

Problem 2.17 – D8 Distance and 8-Path

Problem Statement

Repeat Problem 2.16 for the D_8 distance.
(a) Give the condition(s) under which the D_8 distance between two points p and q is equal to the shortest 8-path between these points.
(b) Is this path unique?

Step 1: Review D_8 Distance

The D_8 distance (chessboard distance) between two pixels $p = (x, y)$ and $q = (s, t)$ is:

$$D_8(p, q) = \max(|x - s|, |y - t|)$$

This is the number of moves a **king in chess** would need to go from p to q.

Why? Because in each step along an 8-path, you can move in any of 8 directions (including diagonals). A diagonal move reduces both the horizontal and vertical distance by 1 simultaneously. So the total number of steps is the larger of the two distances—you make diagonal moves to reduce both distances, and when one distance hits 0, you continue straight in the remaining direction.

Example: If $p = (1, 3)$ and $q = (5, 6)$, then $D_8 = \max(4, 3) = 4$.

Part (a): When Does D_8 = Shortest 8-Path?

Answer

(a) The D_8 distance between p and q equals the length of the shortest 8-path between them when the **path is unconstrained**—meaning:

1. **All pixels** along the path have values in V (no obstacles).
2. The path can proceed freely without detours.

This is the same condition as in Problem 2.16, but now for 8-paths. When every pixel value is in V, the shortest 8-path has length $\max(|x - s|, |y - t|) = D_8(p, q)$.
The optimal strategy (when unconstrained):

1. First, make $\min(|x - s|, |y - t|)$ diagonal moves. After this, one of the two distances reaches 0.
2. Then, make the remaining $| \, |x-s|-|y-t| \, |$ moves in a straight line (horizontal or vertical) to reach q.
3. Total steps: $\min(|x - s|, |y - t|) + | \, |x - s| - |y - t| \, | = \max(|x - s|, |y - t|) = D_8(p, q)$.

If pixels outside V block this direct route, the shortest 8-path may be longer than D_8 or may not exist at all.

Part (b): Is This Path Unique?

Answer

(b) No, the path is generally NOT unique.
Consider the case where both $|x - s| > 0$ and $|y - t| > 0$. During the diagonal phase, you must make $\min(|x - s|, |y - t|)$ diagonal moves. But if $|x - s| \neq |y - t|$, you also need straight-line moves, and these can be **interleaved** with the diagonal moves in different orders, producing multiple distinct paths.
Even when $|x-s| = |y-t|$ (all moves are diagonal), if the number of diagonal steps > 1, there may be multiple paths with the same length that take slightly different routes.
The path is unique only when p and q are on the same row, same column, or same diagonal AND the distance is 0 or 1. In general, there are multiple shortest 8-paths.
Example: $p = (0, 0)$, $q = (2, 1)$. $D_8 = \max(2, 1) = 2$.
Two possible shortest 8-paths:

1. $(0, 0) \to (1, 1) \to (2, 1)$: one diagonal + one vertical. Length 2.
2. $(0, 0) \to (1, 0) \to (2, 1)$: one horizontal + one diagonal. Length 2.

Both have length $2 = D_8$, demonstrating non-uniqueness.

Summary of All Answers

Problem	**Key Answer**
2.1	Smallest dot ≈ 0.0304 mm (30.4 μm) at 0.2 m distance
2.2	Brightness adaptation (dark adaptation): transition from photopic (cone) to scotopic (rod) vision
2.5	Camera resolves ≈ 5.12 line pairs/mm on the scene
2.10	2-hour HDTV movie needs $\approx 1.1664 \times 10^{13}$ bits (≈ 1.458 TB)
2.11	S_1 and S_2 are 4-adjacent, 8-adjacent, AND m-adjacent (all three)
2.12	Replace each diagonal step with two 4-connected steps (horizontal + vertical)
2.13	Same as 2.12, but note intermediate pixels won't have values in V
2.15(a)	$V = \{0, 1\}$: no 4-path; 8-path length=4; m-path length=5
2.15(b)	$V = \{1, 2\}$: 4-path length=6; 8-path length=4; m-path length=6
2.16	D_4 = shortest 4-path when all pixels in V (no obstacles); path not unique
2.17	D_8 = shortest 8-path when all pixels in V (no obstacles); path not unique