AI4M_C3_M3_lecture_notebook_permutation_method

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1 Course 3 Week 3 lecture notebook 01

2 Permutation Method

In this lecture notebook, we'll demonstrate the permutation method on a dataset you worked on in Week 1 of Course 2. By going through this short exercise, you'll hopefully develop some intuition on how to do the related task in this week's programming assignment.

2.1 Setup

First, let's import the packages we'll be using:

```
In [1]: import pandas as pd
    import pickle
    from util import cindex
```

Using TensorFlow backend.

Next, we load the model we trained during the said assignment in Course 2. This accepts a patient's age, systolic blood pressure, diastolic blood pressure, and cholesterol to output a risk score (i.e. probability of an event).

```
In [2]: model_X = pickle.load(open('C2M1_model.model', 'rb'))
```

We will also use the same test dataset from that programming assignment:

```
In [3]: # patient data (test set in the previous assignment)
    X_baseline = pd.read_csv('./lecture_nb_permutation_data/X_data_normalized.csv',index_cd
    # corresponding patient outcome
    y = pd.read_csv('./lecture_nb_permutation_data/y_data_test.csv',index_col=0)
```

Let's print the first 10 cells to peek into the dataset. Take note that these are already standardized and normalized as you did previously.

```
In [4]: X_baseline.head(10)
```

| Out[4]: | | Age | Systolic_BP | Diastolic_BP | Cholesterol |
|---------|------|-----------|-------------|--------------|-------------|
| | 4320 | 0.747640 | -1.211321 | -1.003548 | -1.072461 |
| | 2006 | -0.758669 | -1.476605 | -1.541427 | -0.434487 |
| | 5689 | 0.067833 | 0.399011 | 0.904060 | 0.694137 |
| | 472 | -0.257881 | 0.655984 | 1.157183 | 0.700589 |
| | 1370 | 0.800621 | -0.664058 | 0.228552 | 1.366445 |
| | 1457 | -0.988683 | 0.935349 | 0.752583 | -0.709739 |
| | 5783 | 0.145403 | 0.759190 | 0.155382 | 1.790223 |
| | 3484 | 0.149429 | -0.852439 | 0.671971 | 0.477288 |
| | 1145 | -0.250499 | -0.173969 | 0.006586 | -0.360930 |
| | 130 | -0.757791 | -0.781030 | 0.431331 | -0.617740 |

2.2 C-index and Baseline Performance

Now let's get the c-index of the trained model using our test dataset. Just to refresh:

- The c-index measures the discriminatory power of a risk score.
- Intuitively, a higher c-index indicates that the model's prediction is in agreement with the actual outcomes of a pair of patients.
- The formula for the c-index is

$$cindex = \frac{concordant + 0.5 \times ties}{permissible}$$

- A permissible pair is a pair of patients who have different outcomes.
- A concordant pair is a permissible pair in which the patient with the higher risk score also has the worse outcome.
- A tie is a permissible pair where the patients have the same risk score.

Computing this for our test dataset will give us our baseline performance. For efficiency, we will be using the concordance_index() method from the lifelines package instead of the homemade c-index function we developed in the Course 2 assignment. This is wrapped in a cindex() function in the util library we imported earlier.

This should look familiar as this is also the result you got in the Course 2 assignment.

2.3 Feature Importance and the Permutation Method

As discussed in the lecture videos, one way we can compute the importance of a feature in a given model is by shuffling the values of a particular column in our test set. Evaluating the performance of the model after this shuffling will allow us to quantify the feature's importance. For this exercise, we'll choose to shuffle the Age column. You will do this programmatically in this week's assignment but for now, you'll use a dataset that is shuffled beforehand.

```
In [6]: # patient data permuted at the Age column
        X_permuted_1 = pd.read_csv('./lecture_nb_permutation_data/X_data_normalized_perm1.csv'
        # Let's print the first 10 cells and compare with the baseline table above.
        # All are in the same place except for the values in the Age column
        X_permuted_1.head(10)
Out [6]:
                   Age Systolic_BP
                                      Diastolic_BP
                                                    Cholesterol
        4320 -0.494544
                          -1.211321
                                         -1.003548
                                                      -1.072461
        2006 -0.289638
                          -1.476605
                                         -1.541427
                                                      -0.434487
        5689 0.425693
                           0.399011
                                          0.904060
                                                       0.694137
        472 -2.244324
                           0.655984
                                          1.157183
                                                       0.700589
        1370 -0.078010
                                          0.228552
                          -0.664058
                                                       1.366445
        1457 -1.475322
                           0.935349
                                          0.752583
                                                      -0.709739
        5783 0.694167
                           0.759190
                                          0.155382
                                                       1.790223
        3484 -0.695555
                                          0.671971
                          -0.852439
                                                       0.477288
        1145 -0.761429
                          -0.173969
                                          0.006586
                                                      -0.360930
        130
              0.021294
                          -0.781030
                                          0.431331
                                                      -0.617740
```

Now, let's see how our model performs with this shuffled input:

We see a big drop in the performance. To quantify the importance of this feature, we simply subtract this drop from the baseline performance. This results in 0.8182 - 0.6749 = 0.1433

2.4 Additional Permutations

The result above (i.e. 0.1433) describes the feature importance of Age in our model given the shuffled dataset. However, you might deduce that the predicted risk scores are greatly affected by how the Age column is rearranged. If we arrange it differently, then the model will predict different values and this might lead to a different c-index, and consequently a different value of the feature importance. To illustrate, let's import two more datasets with different permutations

of the Age column. Since there is a random element when doing the permutation, the resulting order of the elements in the Age column will be different for each shuffle. You can see that in the two datasets below.

In [8]: # 2nd permutation

```
X_permuted_2 = pd.read_csv('./lecture_nb_permutation_data/X_data_normalized_perm2.csv'
        X_permuted_2.head(10)
Out [8]:
                                     Diastolic_BP
                   Age Systolic_BP
                                                    Cholesterol
                          -1.211321
        4320
             0.728099
                                         -1.003548
                                                      -1.072461
        2006 0.008176
                          -1.476605
                                         -1.541427
                                                      -0.434487
        5689 0.604035
                                                       0.694137
                           0.399011
                                          0.904060
        472 -1.179030
                           0.655984
                                          1.157183
                                                       0.700589
        1370 0.835747
                          -0.664058
                                          0.228552
                                                       1.366445
        1457 0.872547
                           0.935349
                                          0.752583
                                                      -0.709739
        5783 0.498609
                                          0.155382
                                                       1.790223
                           0.759190
        3484 0.324675
                          -0.852439
                                          0.671971
                                                       0.477288
        1145 0.671958
                                          0.006586
                          -0.173969
                                                      -0.360930
        130 -0.925788
                          -0.781030
                                          0.431331
                                                      -0.617740
In [9]: # 3rd permutation
        X_permuted_3 = pd.read_csv('./lecture_nb_permutation_data/X_data_normalized_perm3.csv'
        X_permuted_3.head(10)
Out [9]:
                   Age Systolic_BP Diastolic_BP Cholesterol
                          -1.211321
                                         -1.003548
        4320 -0.593886
                                                      -1.072461
        2006 0.303244
                          -1.476605
                                         -1.541427
                                                      -0.434487
        5689 -1.755198
                           0.399011
                                          0.904060
                                                       0.694137
        472 -0.500154
                           0.655984
                                          1.157183
                                                       0.700589
        1370 -0.109112
                                          0.228552
                          -0.664058
                                                       1.366445
        1457 0.439787
                           0.935349
                                          0.752583
                                                      -0.709739
        5783 1.335356
                                          0.155382
                                                       1.790223
                           0.759190
        3484 -0.428345
                          -0.852439
                                          0.671971
                                                       0.477288
        1145 0.199869
                          -0.173969
                                          0.006586
                                                      -0.360930
        130
              0.146330
                          -0.781030
                                          0.431331
                                                      -0.617740
  Now let's evaluate the performance of the model with these new datasets:
In [10]: # get the patient's risk scores by feeding a dataset with a different permutation of
         scores = model_X.predict_proba(X_permuted_2)[:, 1]
         # measure c-index
         c_index = cindex(y.values, scores)
```

print(f"c-index for 2nd permutation: {c_index:.4f}")

c-index for 2nd permutation: 0.6554

As you can see, there are slight differences in the resulting c-index depending on how the values are shuffled. This will then result in different values of the feature importance as well. Let's summarize our results in a table for clarity:

| Dataset | C-index | Importance |
|-----------------|---------|------------|
| Baseline | 0.8182 | n/a |
| 1st permutation | 0.6749 | 0.1433 |
| 2nd permutation | 0.6554 | 0.1628 |
| 3rd permutation | 0.6428 | 0.1754 |
| | | |

To take these variations into account, we can take the mean of these different results to have a single value of the feature importance. More formally:

$$I_x = |perf - perf_x| \tag{1}$$

where I_x is the importance of feature x and

$$perf_x = \frac{1}{n} \cdot \sum_{i=1}^{n} perf_i^{sx}$$
 (2)

where $perf_i^{sx}$ is the performance with the feature x shuffled in the ith permutation.

Applying this to our results, that would be abs(0.8182 - mean([0.6749, 0.6554, 0.6428]) = 0.1605. We would want to get more permutations to get a more accurate value of the feature importance. You will get the chance to work this out in the week's programming assignment.

That's it for this lecture on the permutation method! We can now measure the global importance of a feature in the predictions of a given model. In the next section of the course, you will learn how to determine the importance of a feature for an individual patient in the dataset.