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Lab Report:	10
Subject:	Signals & Systems
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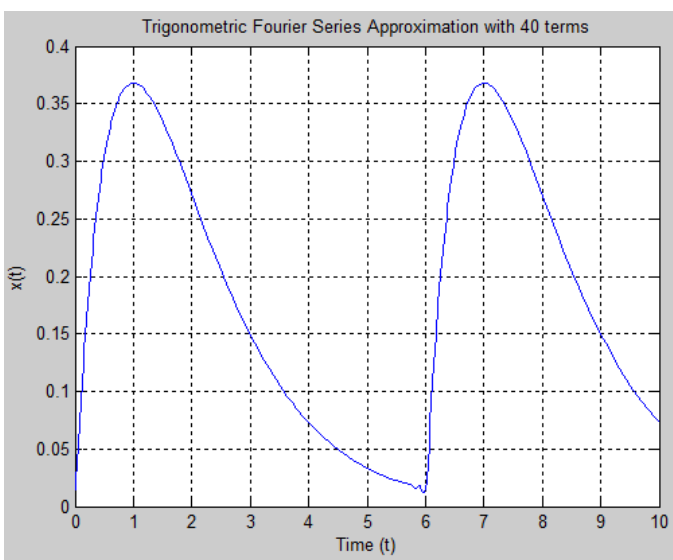
LAB 10

Trigonometric (Real) Fourier Series Representation and its Properties

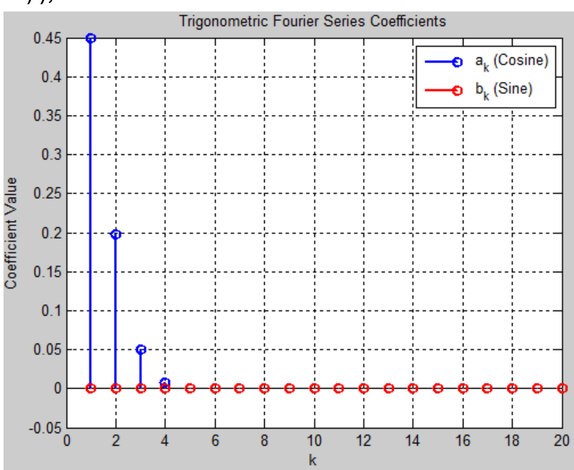
Task 01: The periodic signal $x(t)$ is defined in one period as $x(t) = te^{-t}, 0 \le t \le 6$. Plot approximate signal using 81 terms of trigonometric form of Fourier series.

```
clc; clear; close all;
syms t k
T = 6;
t0 = 0;
w = 2 * pi / T;
x = t * exp(-t);
a0 = (1/T) * int(x, t, t0, t0 + T);
k_vals = 1:40;
ak = (2/T) * int(x * cos(k_vals * w * t), t, t0, t0 + T);
bk = (2/T) * int(x * sin(k_vals * w * t), t, t0, t0 + T);
ak_num = double(ak);
bk_num = double(bk);
a0_num = double(a0);
x_approx = @(t) a0_num + sum(ak_num .* cos(k_vals * w * t) + bk_num .* sin(k_vals * w * t));
figure;
fplot(x_approx, [0, 10]);
grid on;
title('Trigonometric Fourier Series Approximation with 40 terms');
xlabel('Time (t)');
ylabel('x(t)');
```



Task 02: Plot the coefficients of the trigonometric Fourier series for the periodic signal that in one period is defined by $x(t) = e^{-t^2}, -3 \le t \le 3$.

```
clc; clear; close all;
syms t k
T = 6;
t0 = -3;
w = 2 * pi / T;
x = exp(-t^2);
a0 = (1/T) * int(x, t, t0, t0 + T);
k = 1:20;
ak = (2/T) * int(x * cos(k * w * t), t, t0, t0 + T);
bk = (2/T) * int(x * sin(k * w * t), t, t0, t0 + T);
figure;
stem(k, double(ak), 'b', 'LineWidth', 1.5); hold on;
stem(k, double(bk), 'r', 'LineWidth', 1.5);
grid on;
xlabel('k');
ylabel('Coefficient Value');
title('Trigonometric Fourier Series Coefficients');
legend('a_k (Cosine)', 'b_k (Sine)');
```



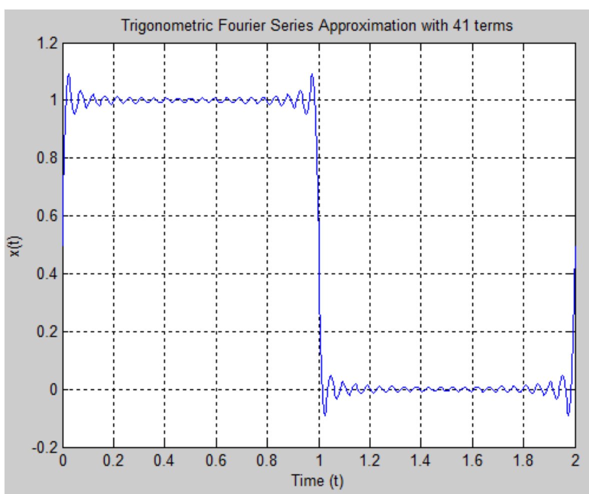
Task 03: The periodic signal $x(t)$ **in a period is given by**

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & 1 \le t \le 2 \end{cases}$$

Plot in one period the approximate signals using 41 and 201 term of the trigonometric Fourier series. Furthermore, each time plot the complex exponential coefficients.

```
clc; clear; close all;
syms t k
T = 2;
t0 = 0;
w = 2 * pi / T;
x = heaviside(t) - heaviside(t - 1);
```

```
a0 = (1/T) * int(x, t, t0, t0 + T);
terms = [41, 201];
for N = terms
   k_vals = 1:N;
   ak = (2/T) * int(x * cos(k_vals * w * t), t, t0, t0 + T);
   bk = (2/T) * int(x * sin(k_vals * w * t), t, t0, t0 + T);
   ak_num = double(ak);
   bk_num = double(bk);
   a0_num = double(a0);
   x_approx = @(t) a0_num + sum(ak_num .* cos(k_vals * w * t) + bk_num .* sin(k_vals * w * t));
   figure;
   fplot(x_approx, [0, T]);
   grid on;
   title(['Trigonometric Fourier Series Approximation with ', num2str(N), ' terms']);
   xlabel('Time (t)');
   ylabel('x(t)');
   figure;
   stem(k_vals, abs(ak_num + 1j * bk_num), 'b', 'LineWidth', 1.5);
   grid on;
   xlabel('k');
   ylabel('Magnitude');
   title(['Complex Exponential Coefficients for ', num2str(N), ' terms']);
end
```



Task 04: The periodic signal $x(t)$ in a period is given by

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 2 - t, & 1 \le t \le 2 \end{cases}$$

Calculate the approximation percentage when the signal $x(t)$ is approximated by 3, 5, 7, and 17 terms of the trigonometric Fourier series. Furthermore, plot the signal in each case.

```
clear;close all;clc;
T=2;w0=2*pi/T;t=linspace(0,2,1000);
x=(t>=0&t<1)+(2-t).*(t>=1&t<2);
```

```
Nvals=[3,5,7,17];
figure('Position',[100,100,800,600]);
for idx=1:length(Nvals)
N=Nvals(idx);
a0=(1/T)*(integral(@(t)1,0,1)+integral(@(t)(2-t),1,2));
x_fs=a0/2*ones(size(t));
for n=1:N
an=(2/T)*(integral(@(t)cos(n*w0*t),0,1)+integral(@(t)(2-t).*cos(n*w0*t),1,2));
bn=(2/T)*(integral(@(t)sin(n*w0*t),0,1)+integral(@(t)(2-t).*sin(n*w0*t),1,2));
x_fs=x_fs+an*cos(n*w0*t)+bn*sin(n*w0*t);
end
approx_pct=100*(1-sqrt(trapz(t,(x-x_fs).^2)/sqrt(trapz(t,x.^2))));
subplot(2,2,idx);
plot(t,x,'k','LineWidth',1.5);hold on;
plot(t,x_fs,'r--','LineWidth',1.2);
title(sprintf('%d terms: %.2f%%',N,approx_pct));
legend('Original','Approximation');
xlabel('Time (t)');ylabel('Amplitude');
grid on;hold off;
end
```

Critical Analysis

Using MATLAB for Trigonometric Fourier Series simplifies computation and visualization of the series components. It automates the calculation of Fourier coefficients and allows for easy plotting of the original and reconstructed signals. This helps in analyzing signal behavior, convergence, and frequency content efficiently

Lab Assessment		
Lab Task Evaluation	/6	/10
Lab Report	/4	
Instructor Signature and Comments		