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Lab Report:	9
Subject:	Signals & Systems
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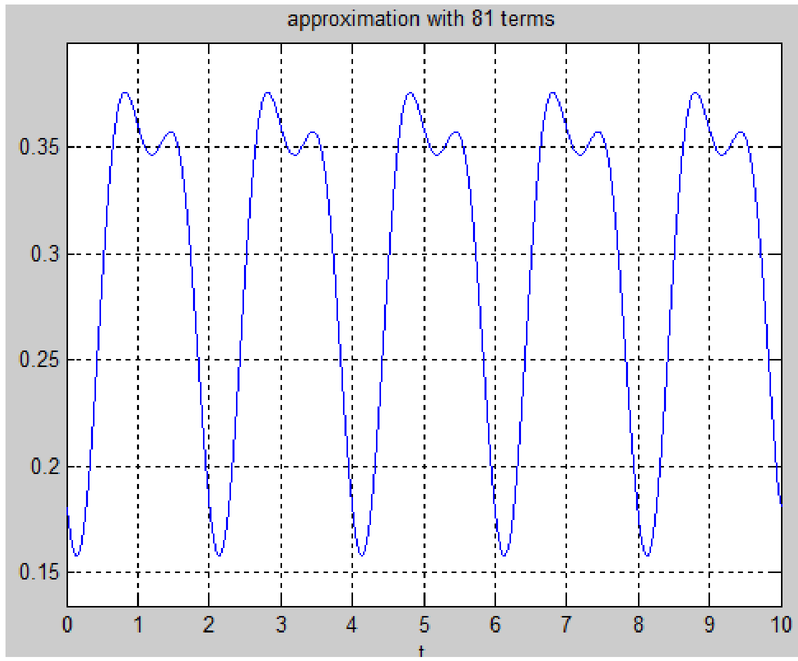
LAB 9

Complex Fourier Series Representation of Signals

Task 01: The periodic signal $x(t)$ is defined in one period as $x(t) = te^{-t}, 0 \le t \le 6$. Plot in time of four periods the approximate signals using 81 terms of complex exponential form of Fourier series.

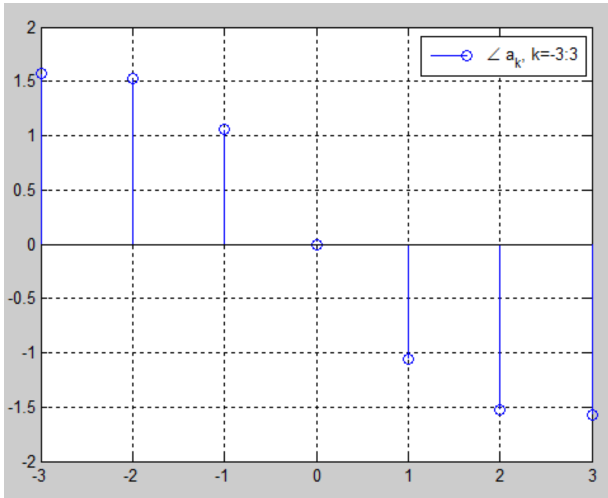
```
syms t
x=t.*exp(-t);
ezplot(x,[0 2]);
k=-2:2;
t0 =0;
T=2;
w=2*pi/T;
a=(1/T)*int(x*exp(-j*k*w*t),t,t0,t0+T)
x=sum(a.*exp(j*k*w*t))
ezplot(x,[0 10]); grid on;
title('approximation with 5 terms')
k=-5:5;
a=(1/T)*int(x*exp(-j*k*w*t),t,t0,t0+T);
x=sum(a.*exp(j*k*w*t));
ezplot(x,[0 10]); grid on;
title('approximation with 11 terms')
k=-10:10;
a=(1/T)*int(x*exp(-j*k*w*t),t,t0,t0+T);
x=sum(a.*exp(j*k*w*t));
ezplot(x,[0 10]); grid on;
title('approximation with 21 terms')
k=-40:40;
a=(1/T)*int(x*exp(-j*k*w*t),t,t0,t0+T);
x=sum(a.*exp(j*k*w*t));
ezplot(x,[0 10]); grid on;
title('approximation with 81 terms')
```



Task 02: Plot the coefficients of the complex exponential Fourier series for the periodic signal that in one period is defined by $x(t) = e^{-t^2}, -3 \le t \le 3$.

```
syms t k n;
x = exp(-t.^(2));
t0=0;
T=3;
w=2*pi/T;
a=(1/T)*int(x*exp(-j*k*w*t),t,t0,t0+T);
k1=-3:3;
ak=subs(a,k,k1);
```

```
stem(k1,abs(ak)); grid on;
legend('|a_k|, k=-3:3');
stem(k1,angle(ak));
legend('\angle a_k, k=-3:3');
grid on;
```

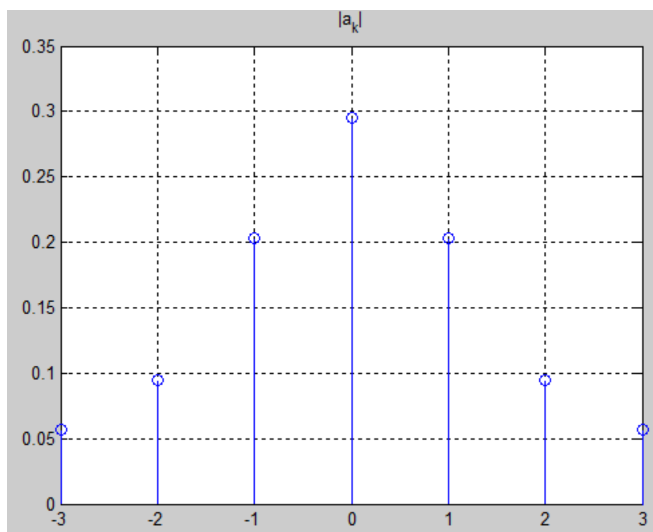


Task 03: The periodic signal $x(t)$ in a period is given by

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & 1 \le t \le 2 \end{cases}$$

Plot in one period the approximate signals using 41 and 201 term of the complex exponential Fourier series. Furthermore, each time plot the complex exponential coefficients.

```
syms t k real
assume(k, 'integer');
x = t * exp(-t);
T = 2; w = 2*pi/T; t0 = 0;
k = -40:40;
a = (1/T) * int(x * exp(-1i*k*w*t), t, t0, t0+T);
x_fs = sum(a .* exp(1i*k*w*t));
figure; ezplot(real(x_fs), [0 10]); grid on;
title('Approximation with 81 terms');
x = exp(-t^2); T = 3; w = 2*pi/T;
a = (1/T) * int(x * exp(-1i*k*w*t), t, 0, T);
k1 = -3:3; ak = subs(a, k, k1);
figure; stem(k1, abs(ak)); title('|a_k|'); grid on;
figure; stem(k1, angle(ak)); title('angle(a_k)'); grid on;
clc; clear;
syms t k real
assume(k, 'integer');
x = exp(-t^2); T = 3; w = 2*pi/T;
a = (1/T) * int(x * exp(-1i*k*w*t), t, 0, T);
k1 = -3:3; ak = subs(a, k, k1);
figure; stem(k1, abs(ak)); title('|a_k|'); grid on;
```



```
figure; stem(k1, angle(ak)); title('angle(a_k)'); grid on;
```

Task 04: The periodic signal $x(t)$ in a period is given by

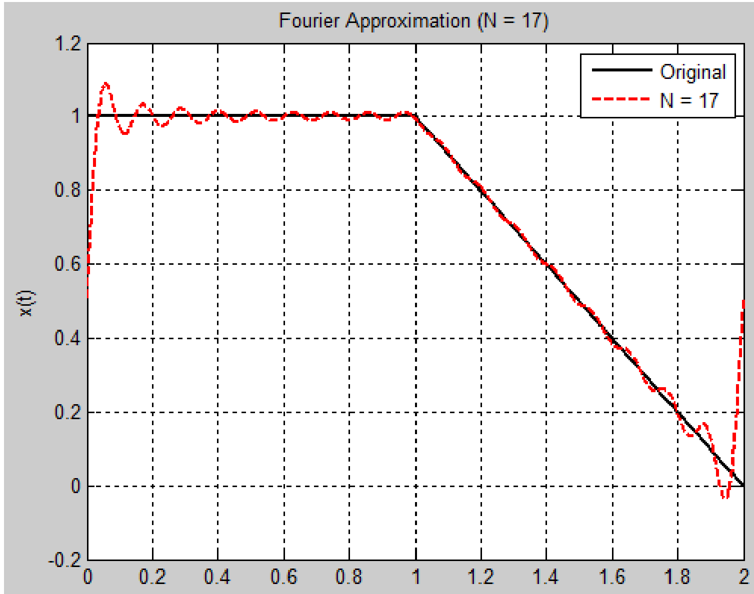
$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 2-t, & 1 \le t \le 2 \end{cases}$$

Calculate the approximation percentage when the signal $x(t)$ is approximated by 3, 5, 7, and 17 terms of the complex exponential Fourier series. Furthermore, plot the signal in each case.

```
clc; clear;
syms t k real; assume(k, 'integer');
T = 2; w0 = pi;
x = 1 * (heaviside(t) - heaviside(t - 1)) + (2 - t) * (heaviside(t - 1) - heaviside(t - 2));
ck_expr = (1/T) * int(x * exp(-1i * k * w0 * t), t, 0, T);
tt = linspace(0, 2, 1000);
x_true = double(subs(x, t, tt));
N_vals = [3, 5, 7, 17];



for N = N_vals
  k_vals = -N:N;
  x_approx = zeros(size(tt));
  for i = 1:length(k_vals)
    kk = k_vals(i);
    if kk == 0
      ck_val = double((1/T) * int(x, t, 0, T));  % c0 separately
    else
      ck_val = double(subs(ck_expr, k, kk));
    end
    x_approx = x_approx + ck_val * exp(1i * kk * w0 * tt);
  end
  err = norm(real(x_approx) - x_true) / norm(x_true) * 100;
  fprintf('N = %d ? Error = %.2f%%\n', N, err);
  figure;
  plot(tt, x_true, 'k', tt, real(x_approx), 'r--', 'LineWidth', 1.5);
  legend('Original', ['N = ', num2str(N)]);
  title(['Fourier Approximation (N = ', num2str(N), ')']);
  xlabel('t'); ylabel('x(t)'); grid on;
end
```



Critical Analysis

In this lab I learnt that, MATLAB efficiently computes and visualizes complex Fourier series but requires precise handling of coefficients and phase to avoid errors. Its built-in functions speed up analysis, but manual validation ensures accuracy.

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