Hamburg University of Technology

Traffic Engineering

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Part 1 : Network Planning and Optimization
Exercise 3

Exercise 3

Planning of Optical Networks

3.0 Aims

This exercise introduces you to basic design approaches of optical networks. After finishing this exercise you should be able to use the optimization tool in solving the problems.

3.1 Introduction

In the following you will be introduced to the optical networks' planning problem, mainly based on the dissertation by Beckmann [1]. Several additional references are listed at the end of the exercise. As before, we consider two approaches: first by using LP solvers and then by using an appropriate heuristic.

Optical networks employing *wavelength division multiplexing* (WDM) offer the promise of meeting the high rate/bandwidth requirements of emerging communication applications, by dividing the huge transmission rate of an optical fiber into multiple communication channels. In these networks transmission and switching are both implemented in optical technology without the bottleneck of intermediate optical-to-electrical conversions. To be able to send data from one node to another, one needs (to establish) a connection in the optical layer similar to the one in a circuit-switched network. This can be realized by determining a path in the network between the two nodes and allocating a free wavelength (λ) on all of the links on the path. Such an all optical path is commonly referred to as an optical path or a lightpath. Generally, there exist two different kinds of optical paths: the first one, called *virtual wavelength path* (VWP), assumes that the signal is not necessarily carried by the same wavelength during its travel through the network. This concept requires the possibility of optical wavelength conversion in the nodes of the network. Unfortunately, providing such wavelength converters requires extra budget. For this reason, in the second scheme known as the concept of *wavelength paths* (WP), each connection is assigned just one wavelength for its transmission and thus no wavelength conversions are necessary. Thus, two arbitrarily paths can only be assigned the same wavelength if there are no common links used by these paths. This implies that in this scenario we must then assign wavelengths globally throughout the network. Because of the reason, in general, the WP scheme needs more wavelengths than the VWP scheme for the same network. The following figure illustrates both WP and VWP schemes.

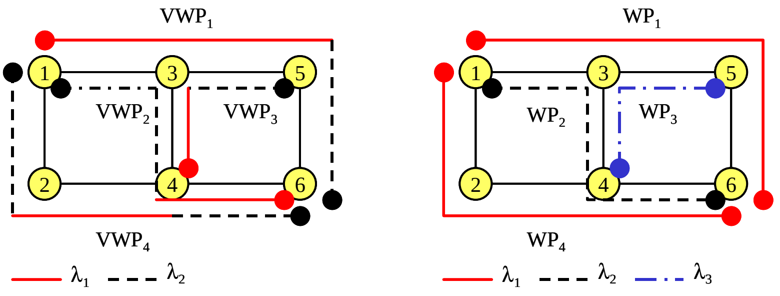


Figure 1 : The VWP and WP schemes

In both schemes, there are 4 predefined optical paths. For the VWP network (left) only two wavelengths are needed, while for the WP network (right) three are needed. Given a set of connections, the problem of setting up optical paths by routing and assigning a wavelength to each connection while optimizing a certain performance metric is called *Routing and Wavelength Assignment* (RWA) problem. Now we will introduce some notations and formulate our RWA problems.

Problem-1

Notations

$m=1, \dots, M$	is the index referring to node pairs
$r=1, \dots, R$	is the index referring to a possible route for every z (node pair)
$l=1, \dots, L$	is the index referring to a link of the network

Variables

N	is the maximum total number of wavelengths per fiber for each link of the network (homogeneous)
$x_{m,r}$	corresponds to the number of wavelengths belonging to the node pair m, which are established along route r

Parameters

d_m	denotes the wavelength demand for node pair m
$\delta^l_{m,r}$	are elements of a binary incidence matrix, which take on the value 1 if route r of node pair m contains link l, and the value 0 otherwise

VWP(1) : Formulation : minimizing the maximum number of required wavelengths

$\min \{N\}$	(1)	This is the objective function for minimizing the maximum number of required wavelengths.
$\sum_{r=1}^{R} x_{m,r} = d_m \;;\; \forall m$	(2)	The constraints assure that for every z we establish the number of wavelengths which are required by the corresponding value d_z in the traffic matrix.
$\sum_{m=1}^{M} \sum_{r=1}^{R} \delta^l_{m,r} \cdot x_{m,r} \leq N \;;\; \forall l$	(3)	The constraints are needed in order to guarantee that on each link l the number of assigned wavelengths is upper bounded by N.
$\text{int} \;\; x_{m,r} \;;\; \forall m, \forall r$	(4)	

Notations

$\lambda=1, \dots, N$	is the index referring to wavelengths

Variables:

$x^{\lambda}_{m,r}$	corresponds to binary variables, which take on the value 1 if the wavelength λ is used for routing one unit of d_m through the route r and the value 0 otherwise.
F	is the maximum link-load on the network (sum of all wavelength assignments per link)

Parameters

$\delta_{m,r}^{l}$	are elements of a binary incidence matrix, which take on the value 1 if route r of node pair m uses the link l, and the value 0 otherwise

WP(1) : Formulation (1) : minimizing the maximum number of required wavelengths

$\min\{N\}$	(5)	This is the objective function for minimizing the maximum number of required wavelengths.
$\sum_{\lambda=1}^{N}\sum_{r=1}^{R} x_{m,r}^{\lambda} = d_m \ ; \ \forall m$	(6)	The constraints assure that for every m we establish the number of wavelengths which are required by the corresponding value d_m in the traffic matrix.
$\sum_{m=1}^{M}\sum_{r=1}^{R} \delta_{m,r}^{l} \cdot x_{m,r}^{\lambda} \le 1 \ ; \ \forall l \ , \ \forall \lambda$	(7)	The constraints are needed in order to guarantee that every wavelength on each link is either not used or used only once.
$x_{m,r}^{\lambda} \in \{0,1\} \ ; \ \forall m, \forall r, \forall \lambda$	(8)	

WP(1) : Formulation (2) : minimizing the maximum link-load in the network

$\min\{F\}$	(9)	This is the objective function for minimizing the maximum link-load in the network.
$\sum_{\lambda=1}^{N}\sum_{m=1}^{M}\sum_{r=1}^{R} \delta_{m,r}^{l} \cdot x_{m,r}^{\lambda} \le F \ ; \ \forall l$	(10)	The constraints are needed in order to guarantee that the load on each link l is upper bounded by F.
(6)(7)(8)		The required constraints as in the first formulation.

Problem-2

Variables:

e_l	are the number of fibers on the link l

VWP(2) : Formulation : minimizing the number of fibers on the network

$\min\left\{\sum_{l}^{L} e_l\right\}$	(11)	This is the objective function for minimizing the number of fibers.
$\sum_{m=1}^{M}\sum_{r=1}^{R} \delta_{m,r}^{l} \cdot x_{m,r} \le N \cdot e_l \ ; \ \forall l$	(12)	The constraints are needed in order to guarantee that on each link l the number of assigned wavelengths is below the capacity of that link.
int $e_l \ ; \ \forall l$	(13)	
(2)(4)		The other required constraints

WP(2) : Formulation : minimizing the number of fibers on the network

$$\min \left\{ \sum_{l}^{L} e_l \right\}$$

This is the objective function minimizing the number of fibers.

$$\sum_{m=1}^{M} \sum_{r=1}^{R} \delta_{m,r}^{l} \cdot x_{m,r}^{\lambda} \leq e_l \;;\; \forall l \;,\; \forall \lambda \qquad (14)$$

The constraints are needed in order to guarantee that every wavelength on a certain fiber on each link is either not used or used only once.

$$\text{int } \; x_{m,r}^{\lambda} \;;\; \forall m, \forall r, \forall \lambda \qquad (15)$$

(6)(13)

The other required constraints

Notes:

1. For the first problem we restrict ourselves to just one fiber installed per link and the number of wavelengths available on each link should be the same in the whole network. The problem is then to assign routes and wavelengths in a way that the maximum number of λs needed to carry the demanded traffic is minimized.
   For the second problem there are no restrictions regarding the number of fibers installed per link. But as seen from the formulations, we are searching for solutions that minimize the total number of fibers needed in the network.
2. In the VWP case, wavelength assignment is trivial and can be done on per link basis. This means we start with the lowest wavelength index and increment this index if the corresponding wavelength is already assigned. It could be said, that in the VWP scheme we reduce the RWA problem to the plain routing problem.
3. If we take a look at WP1-Formulation 1 (WP1-F1), one could be somehow confused because we try to minimize N while the value of this N should be given. In this case the problem is only to find a feasible solution for a smallest value of N. To obtain this smallest value, one could guess first and then increase/decrease the value of N if necessary. For WP1-F2 and WP2 the value of N still has to be given but we have the more sensible objective functions rather than a constant function as in WP1-F1.

Greedy Heuristic

The problem of routing and wavelength assignment is known to be NP-complete. This means that the calculation time required for exact methods to solve the optimization problem increases exponentially with the size of the problem. In our cases, this depends on the number of connections in the traffic matrix and of course also on the size of the network itself. Thus in this part we will discuss two simple greedy-heuristics that can be applied to Problem-1 and Problem-2 respectively. It is called greedy, because after finding a free path from the source to the destination in the network, it immediately assigns the free capacity to the currently processed connection without testing further possibilities. In the following we consider only the WP scenario. However the heuristics are also applicable to VWP scenario by a slight modification.

```
for all connections m
  for all routes r
    for all possible wavelengthe λ
      if for all links l of route r, λ has not yet been used
             on one of the links belonging to route r
        assign route r and wavelength λ to the current connection;
        quit all loops;
      end if;
    end for;
  end for;
end for;
```

Figure 2 : A greedy-heuristic for problem-1 (WP scheme)

Consider the algorithm above. This heuristic assigns to each connection the first possible combination of route and wavelength, which does not interfere with one of the previously installed connections. Thereby the routes are ordered increasingly according to their length. This means that we always first try to assign the shortest possible route. We estimate the number of N of needed wavelengths in advance by guessing or using some techniques discussed in [7]. Afterwards we increase the number of wavelengths step by step and apply again the greedy algorithm to the remaining connections until all connections of the traffic matrix are installed in the network. Thus it is clear that the required number of wavelengths depends significantly on the order in which the connections are processed by the algorithm.

```
for all connections m
  for all routes r
    for all possible wavelengths λ
      if for all links l of route r, λ has not yet been used
             more than e_i times
        assign to the current connection route r and wavelength λ
                   on the first possible fiber on each link;
        quit all loops;
      end if;
    end for;
  end for;
end for;
```

Figure 3 : A greedy-heuristic for problem-2 (WP scheme)

The appropriate version of the greedy heuristic for problem-2 is given in Figure 3. In order to detect unnecessary fibers, we continuously number the fibers of each link and try to install as many connections as possible on the fibers with the lowest number. This means that the greedy algorithm on each link allocates the fiber with the lowest possible number where the proposed wavelength has not been occupied by another connection so far.

3.2 Tasks

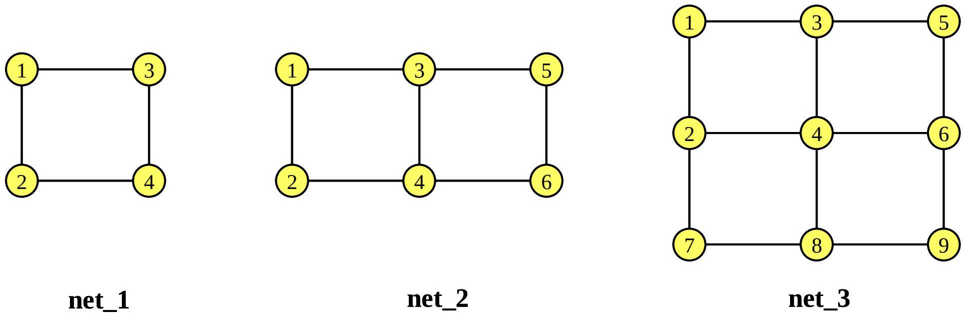


Figure 4 : Some test networks

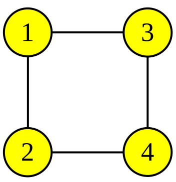
(4) **Complexity**. Consider the networks in Figure 4.

a. Derive the relationship for the number of variables and the number of constraints in terms of M, L, N, R for all formulations (problem-1 and problem-2) !

b. Assume that R=2 and N=4. How many variables and constraints are there? Fill out the table below !

			VWP1		WP1 (F1)		WP1 (F2)		VWP2		WP2	
	Z	L	Var.	Cons.	Var.	Cons.	Var.	Cons.	Var.	Cons.	Var.	Cons.
net_1												
net_2												
net_3												

(5) **Global Optimization**. Given is the following network topology and the corresponding traffic matrix.



node \ node	1	2	3	4
1		1	1	2
2	1		3	2
3	1	3		1
4	2	2	1	

node pair	(1,2) (2,1)	(1,3) (3,1)	(1,4) (4,1)	(2,3) (3,2)	(2,4) (4,2)	(3,4) (4,3)
r \ *m*	1	2	3	4	5	6
1	(1,2) (2,1)	(1,3) (3,1)	(1,2,4) (4,2,1)	(2,1,3) (3,1,2)	(2,4) (4,2)	(3,4) (4,3)
2	(1,3,4,2) (2,4,3,1)	(1,2,4,3) (3,4,2,1)	(1,3,4) (4,3,1)	(2,4,3) (3,4,2)	(2,1,3,4) (4,3,1,2)	(3,1,2,4) (4,2,1,3)

node pair	(1,2) (2,1)	(1,3) (3,1)	(1,4) (4,1)	(2,3) (3,2)	(2,4) (4,2)	(3,4) (4,3)
m	1	2	3	4	5	6
d(m)	1	1	2	3	2	1

between nodes	(1,2) (2,1)	(1,3) (3,1)	(1,4) (4,1)	(2,3) (3,2)	(2,4) (4,2)	(3,4) (4,3)
link *l*	1	2	-	-	3	4

$\delta^{l}_{m,r}$ for l = 1 (link between node 1 and node 2)

node pair	(1,2) (2,1)	(1,3) (3,1)	(1,4) (4,1)	(2,3) (3,2)	(2,4) (4,2)	(3,4) (4,3)
r \ *m*	1	2	3	4	5	6
1	1	0	1	1	0	0
2	0	1	0	0	1	1

Figure 5 : Obtaining the parameters for MIP formulation

Using plain solvers (without AMPL)

a. Find $\delta^l_{m,r}$ and $\delta^l_{m,r}$ for all l ! (Hint: use the tables in appendix A).

b. For each formulations, write the objective function, the constraints, integer declaration etc. and save them in a file ! (Notation: use **x_m_r** – e.g. **x_1_1** for m=1 and r=1 – and **x_m_r_λ** – e.g. **x_1_1_1** for m=1, r=1 and λ=1). For problem-2 use N=2!

c. Run the solver! e.g. `lp_solve < input.lp > output.out` !

Using AMPL

d. For each formulations :
   - Write a model file according to the formulation and save it as a file ! (e.g. *vwp1.mod*)
   - Write a corresponding data file and save it as a file ! (e.g. *vwp1a.dat*)
   - If necessary write the *run* file containing a certain sequence of AMPL commands !
   - Load the model and data file and run the AMPL as shown in Exercise 1!

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e. Open the results files and fill out the table below!

<table>
<tr><th>VWP1</th><th>VWP2</th><th>WP1 (F1)</th><th>WP1 (F2)</th><th>WP2</th></tr>
<tr><td rowspan="3">N =</td><td>N =</td><td rowspan="3">N =</td><td>N =</td><td>N =</td></tr>
<tr><td>$\sum e_l =$</td><td rowspan="2">F =</td><td>$\sum e_l =$</td></tr>
<tr><td>$\max\{e_l\} =$</td><td>$\max\{e_l\} =$</td></tr>
</table>

f. Repeat the tasks (2a – 2e) for the following traffic matrices! (use N=8 for the formulations problem-2). Fill out the corresponding results table!

(a) node × node

node	1	2	3	4
1		2	3	5
2	2		6	4
3	3	6		3
4	5	4	3	

(a)

(b) node × node

node	1	2	3	4
1		8	10	10
2	8		15	7
3	10	15		8
4	10	7	8	

(b)

Figure 6 : Traffic matrices

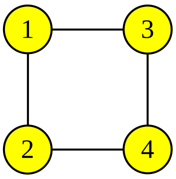
<table>
<tr><th>VWP1</th><th>VWP2</th><th>WP1 (F1)</th><th>WP1 (F2)</th><th>WP2</th></tr>
<tr><td rowspan="3">N =</td><td>N =</td><td rowspan="3">N =</td><td>N =</td><td>N =</td></tr>
<tr><td>$\sum e_l =$</td><td rowspan="2">F =</td><td>$\sum e_l =$</td></tr>
<tr><td>$\max\{e_l\} =$</td><td>$\max\{e_l\} =$</td></tr>
</table>

(a)

<table>
<tr><th>VWP1</th><th>VWP2</th><th>WP1 (F1)</th><th>WP1 (F2)</th><th>WP2</th></tr>
<tr><td rowspan="6">N =</td><td rowspan="2">N =</td><td rowspan="6">N =</td><td rowspan="3">N =</td><td rowspan="2">N =</td></tr>
<tr></tr>
<tr><td rowspan="2">$\sum e_l =$</td><td rowspan="2">$\sum e_l =$</td></tr>
<tr><td rowspan="3">F =</td></tr>
<tr><td rowspan="2">$\max\{e_l\} =$</td><td rowspan="2">$\max\{e_l\} =$</td></tr>
<tr></tr>
</table>

(b)

4.1 Exercise 3 : Optical Networks ($\delta^{l}_{z,r}$ and $\delta^{l,\lambda}_{z,r}$)



node \ node	1	2	3	4
1		1	1	2
2	1		3	2
3	1	3		1
4	2	2	1	

node pair	(1,2) (2,1)	(1,3) (3,1)	(1,4) (4,1)	(2,3) (3,2)	(2,4) (4,2)	(3,4) (4,3)
r \ *m*	1	2	3	4	5	6
1	(1,2) (2,1)	(1,3) (3,1)	(1,2,4) (4,2,1)	(2,1,3) (3,1,2)	(2,4) (4,2)	(3,4) (4,3)
2	(1,3,4,2) (2,4,3,1)	(1,2,4,3) (3,4,2,1)	(1,3,4) (4,3,1)	(2,4,3) (3,4,2)	(2,1,3,4) (4,3,1,2)	(3,1,2,4) (4,2,1,3)

node pair	(1,2) (2,1)	(1,3) (3,1)	(1,4) (4,1)	(2,3) (3,2)	(2,4) (4,2)	(3,4) (4,3)
m	1	2	3	4	5	6
d(*m*)	1	1	2	3	2	1

between nodes	(1,2) (2,1)	(1,3) (3,1)	(1,4) (4,1)	(2,3) (3,2)	(2,4) (4,2)	(3,4) (4,3)
link *l*	1	2	-	-	3	4

$\delta^{l}_{m,r}$ for l = 1 (link between node 1 and node 2)

node pair	(1,2) (2,1)	(1,3) (3,1)	(1,4) (4,1)	(2,3) (3,2)	(2,4) (4,2)	(3,4) (4,3)
r \ *m*	1	2	3	4	5	6
1	1	0	1	1	0	0
2	0	1	0	0	1	1

$\delta^{l}_{m,r}$ for l = 2 (link between node 1 and node 3)

node pair	(1,2) (2,1)	(1,3) (3,1)	(1,4) (4,1)	(2,3) (3,2)	(2,4) (4,2)	(3,4) (4,3)
r \ *m*	1	2	3	4	5	6
1						
2						

$\delta^{l}_{m,r}$ for l = 3 (link between node 2 and node 4)

node pair	(1,2) (2,1)	(1,3) (3,1)	(1,4) (4,1)	(2,3) (3,2)	(2,4) (4,2)	(3,4) (4,3)
r \ *m*	1	2	3	4	5	6
1						
2						

$\delta^{l}_{m,r}$ for l = 4 (link between node 3 and node 4)

node pair	(1,2) (2,1)	(1,3) (3,1)	(1,4) (4,1)	(2,3) (3,2)	(2,4) (4,2)	(3,4) (4,3)
r \ *m*	1	2	3	4	5	6
1						
2						