Question:-

Strain displacement relation from cylinderical coordinates to spherical coordinates.

Answer:-

The relation b/w cylinderical and spherical coordinate is

$$r = \rho \sin\phi \quad , \quad z = \rho\cos\phi \quad , \quad \theta = \theta$$

where $\rho = \sqrt{r^2 + z^2}$, $\theta = \tan^{-1}(y/x)$, $\phi = \arccos\left(\frac{z}{\rho}\right)$

The partial derivatives for the above equations are

$$\frac{\partial}{\partial r} = \frac{\partial \rho}{\partial r} \cdot \frac{\partial}{\partial \rho} + \frac{\partial \phi}{\partial r} \cdot \frac{\partial}{\partial \phi}$$

$$= \sin\phi \frac{\partial}{\partial \rho} + \frac{r^2}{\sqrt{r^2+z^2} \cdot \rho^{3/2}} \cdot \frac{\partial}{\partial \phi}$$

$$\frac{\partial}{\partial z} = \cos\phi \frac{\partial}{\partial \rho} + \frac{\partial z}{\sqrt{r^2 - z^2} \cdot \rho^{3/2}} \cdot \frac{\partial}{\partial \phi}$$

Now

$$U_r = U_\rho \sin\phi + U_\phi \frac{r^2}{\sqrt{r^2 - z^2}\, \rho^{3/2}} , \quad U_z = U_\rho \cos\phi + U_\phi \frac{rz}{\sqrt{r^2 - z^2} \cdot \rho^{3/2}}$$

$$U_\theta = U_\theta$$

calculating $e_\rho = \frac{\partial U_r}{\partial r}$

$$\hat{e}_\rho = \sin\phi \left[\frac{\partial}{\partial \rho} \left(U_\rho \sin\phi + U_\phi \frac{r^2}{\sqrt{r^2 - z^2}\, \rho^{3/2}} \right) \right] + \frac{r^2}{\sqrt{r^2 - z^2} \cdot \rho^{3/2}} \frac{\partial}{\partial \phi} \left[U_\rho \sin\phi + U_\phi \frac{r^2}{\sqrt{r^2 - z^2} \cdot \rho^{3/2}} \right.$$

$$= \left[\frac{\partial U_\rho}{\partial \rho} \sin\phi + \frac{\partial U_\phi}{\partial \rho} \cdot \frac{r^2 \sin\phi}{\rho^{3/2} \sqrt{r^2 - z^2}} + \frac{U_\phi r^2}{\sqrt{r^2 - z^2}} \cdot \frac{\sin\phi}{\rho^{-5/2}} + \frac{\partial U_\rho}{\partial \phi} \cdot \frac{\sin\phi\, r^2}{\sqrt{r^2 - z^2}\, \rho^{3/2}} \right.$$

$$\left. + \frac{r^2 U_\rho \cos\phi}{\sqrt{r^2 - z^2}} + \frac{\partial U_\phi}{\partial \phi} \cdot \frac{r^4}{\rho^3 (r^3 - z^3)} \right]$$

$$\hat{e}_\rho = \frac{\partial U_\rho}{\partial \rho} \sin^2\phi + \left(\frac{\partial U_\phi}{\partial \rho} \frac{1}{\rho^{3/2}} + \frac{U_\rho}{\rho^{-5/2}} + \frac{\partial U_\rho}{\partial \phi} \frac{1}{\rho^{3/2}} \right) \frac{r^3 \sin\phi}{\sqrt{r^2 - z^2}} +$$

$$\left(U_\rho \cos\phi + \frac{\partial U_\phi}{\partial \phi} \cdot \frac{1}{\rho^3} \right) \cdot \frac{r^3}{\sqrt{r^2 - z^2}} \Bigg) \frac{r^2}{\sqrt{r^2 - z^2}}$$

$$\hat{e}_\phi = \frac{\partial U_z}{\partial z}$$

$$\hat{e}_\phi = \cos\phi \frac{\partial}{\partial \rho}\left[U_\rho \cos\phi + U_\phi \cdot \frac{\gamma z}{\rho^{3/2}\sqrt{\gamma^2 - z^2}}\right] +$$

$$\frac{\gamma}{\sqrt{\gamma^2 - z^2}\,\rho^{3/2}} \frac{\partial}{\partial z}\left[U_\rho \cos\phi + U_\phi \frac{\gamma z}{\sqrt{\gamma^2 - z^2}\,\rho^{3/2}}\right]$$

$$= \frac{\partial U_\rho}{\partial \rho}\cos^2\phi + \frac{\partial U_\phi}{\partial \rho} \cdot \frac{\partial z}{\rho^{3/2}} \frac{\cos\phi}{\sqrt{\gamma^2 - z^2}} + \frac{U_\phi \gamma z}{\sqrt{\gamma^2 - z^2}} \cdot \frac{\cos\phi}{\rho^{-5/2}} +$$

$$\frac{\partial U_\rho}{\partial \phi} \frac{\cos\phi}{\rho^{3/2}} \frac{\gamma z}{\sqrt{\gamma^2 - z^2}} - \frac{\sin\phi}{\sqrt{\gamma^2 - z^2}} \frac{\gamma z}{\rho^{3/2}} U_\rho + \frac{\partial U_\phi}{\partial \phi} \cdot \frac{\gamma^2 z^2}{(\gamma^2 - z^2)\rho^3}$$

$$\hat{e}_\phi = \frac{\partial U_\rho}{\partial \rho}\cos^2\phi + \left[\frac{\partial U_\rho}{\partial \rho} \cdot \frac{1}{\rho^{3/2}} + \frac{U_\phi}{\rho^{-5/2}} + \frac{\partial U_\rho}{\partial \phi} \frac{1}{\rho^{3/2}}\right] \frac{\cos\phi\, \gamma z}{\sqrt{\gamma^2 - z^2}}$$

$$+ \left[\frac{\partial U_\phi}{\partial \phi} \frac{\gamma z}{\sqrt{\gamma^2 - z^2}\,\rho^{3/2}} - \frac{U_\rho}{\rho^{3/2}} \sin\phi\right] \frac{\gamma z}{\sqrt{\gamma^2 - z^2}}$$

Therefore the strain displacement relation becomes

$$e_\rho = \frac{\partial U_\gamma}{\partial \gamma}, \qquad e_\phi = \frac{1}{\rho}\left(U_\gamma + \frac{\partial U_z}{\partial \phi}\right)$$

$$e_\theta = \frac{1}{\rho \sin\phi}\left(\frac{\partial U_\theta}{\partial \theta} + \sin\phi\, U_\gamma + \cos\phi\, U_z\right)$$

$$e_{\rho\phi} = \frac{1}{\gamma}\left[\frac{1}{\rho}\frac{\partial U_\gamma}{\partial \phi} + \frac{\partial U_z}{\partial \rho} \cdot \frac{U_z}{\rho}\right]$$

$$e_{\phi\theta} = \frac{1}{\partial \rho}\left[\frac{1}{\sin\theta} \cdot \frac{\partial U_z}{\partial \theta} + \frac{\partial U_\theta}{\partial \phi} - \cot\phi\, U_\theta\right]$$

$$e_{\theta\rho} = \frac{1}{\gamma}\left[\frac{1}{\rho \sin\phi} \cdot \frac{\partial U_\gamma}{\partial \theta} + \frac{\partial U_\theta}{\partial \rho} - \frac{U_\theta}{\rho}\right]$$

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