

## Multivariable Calculus (MT1008)

Date: 25<sup>th</sup> Feb 2025

Sessional-I

Course Instructor(s)

Total Time (Hrs.): 1

Dr. Mazhar Hussain, Dr. Akhlaq Ahmad,

Total Marks: 55

Dr. Hina Firdous, Dr. Sidra Afzal, Tasaduque Hussain,

Total Questions: 2

Muhammad Yaseen.

Do not write below this line

Attempt all the questions.

**CLO#1. Defining Functions of Several Variables, Computing Partial Derivatives, Directional and Gradient Vectors.**

1(a). Find and sketch the domain of  $f$ . Then find an equation for the level curves of the function passing through the given point *identify and sketch LC*

$$f(x, y) = \sqrt{x^2 - y}$$

Also, determine if the domain is bounded or unbounded, and decide if open region or closed region.

(b). Check whether the given function has a limit or not as  $(x, y)$  approaches  $(0, 0)$ ,

$$f(x, y) = \frac{2x^2y}{x^4 + y^2}$$

(c). Show that for a function  $f(x, y)$  and a positive number  $\epsilon$  given below there exist a  $\delta > 0$  such that for all  $(x, y)$ ,  $|f(x, y) - f(0, 0)| < \epsilon$  whenever  $0 < \sqrt{x^2 + y^2} < \delta$

$$f(x, y) = \frac{x + y}{2 + \cos x}, \quad \epsilon = 0.02$$

(d). Does there exist a direction  $u$  in which the rate of change of the temperature function  $T(x, y, z) = x^4y - 3xy^2z + e^{xyz}$  (temperature in degree Celsius, distance in meter) at  $P(1, -1, 0)$  equals  $-5.74^\circ \text{C/m}$ ? If not, provide reasons for your answers. [10+10+10+10]

**CLO#2. Evaluation of Multiple Integrals in Different Coordinate Systems and Their Applications to Work, Circulation, Flux, Green's Theorem, and Stokes' Theorem.**

2. Sketch the region of integration and evaluate the integral

$$\iint_R xy \, dA$$

Where  $R$  is the region bounded by the lines  $y = x$ ,  $y = 2x$ , and  $x + y = 2$ .

[15]