Quiz #02

Question #01

A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives. Also find Mean and Variance.

Solution:

Total

$n = 20$

defective

$p = \frac{3}{20}$

$q = \frac{17}{20}$

x = no. of defective

x	$P(x)$	$xP(x)$	$x^2P(x)$
0	68/95	0	0
1	51/190	51/190	51/190
2	3/190	3/95	6/95

$$P(0) = \frac{({}^3C_0)({}^{17}C_2)}{{}^{20}C_2} = \frac{68}{95}$$

$$P(1) = \frac{({}^3C_1)({}^{17}C_1)}{{}^{20}C_2} = \frac{51}{190}$$

$$P(2) = \frac{({}^3C_2)({}^{17}C_0)}{{}^{20}C_2} = \frac{3}{190}$$

$$E(X) = \Sigma xP(x)$$

$$= 0 + \frac{51}{190} + \frac{3}{95}$$

$$= \frac{3}{10} = 0.3$$

$$\text{var}(X) = \Sigma x^2P(x) - (\Sigma xP(x))^2$$

$$= \left(\frac{51}{190} + \frac{6}{95}\right) - \left(\frac{3}{10}\right)^2$$

$$= 0.2416$$

Question #02

An investment firm offers its customers municipal bonds that mature after varying numbers of years. Given that the cumulative distribution function of T, the number of years to maturity for a randomly selected bond, is

$$F(t) = \begin{cases} 0, & t < 1, \\ \frac{1}{4}, & 1 \le t < 3, \\ \frac{1}{2}, & 3 \le t < 5, \\ \frac{3}{4}, & 5 \le t < 7, \\ 1, & t \ge 7, \end{cases}$$

Find

(a) P(T = 5)

(b) P(T > 3)

(c) P(1.4 < 6)

(d) P(T ≤ 5 | T ≥ 2).

Solution:

(a) $P(T=5) = F(5) - F(4)$

$= \frac{3}{4} - \frac{1}{2}$

$= 1/4$

(b) $P(T>3) = 1 - F(T<3)$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

(c) $P(1.4<T<6) = F(6) - F(1)$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{1}{2}$

(d) $P(T \le 5 \mid T \ge 2) = \frac{P(2 \le T \le 5)}{P(T \ge 2)}$

$= \frac{F(5) - F(2)}{1 - F(2)} = \frac{3/4 - 1/4}{1 - 1/4} = \frac{1/2}{3/4} = \frac{2}{3}$