University of Computer and Emerging Sciences
(FAST)

Final Paper **Fall-2013**

Course: Probability and Statistics
Time Allowed: 3 hours Maximum marks: 50

Q.1: Suppose that the 4 inspectors at a film factory are supposed to stamp the expiration date on each package of film at the end of the assembly line. John who stamps 20% of the packages, fails to stamp the expiration date twice in every 100 packages; Tom who stamps 60% of the packages, fails to the stamp once in every 100 packages; Jeff, who stamps 15% of the packages, fails to stamp once in every 90 packages; and Pat, who stamps 5% of the packages, fails to stamp the expiration date once in every 150 packages. If a customer complains that his package of film does not show the expiration date, what is the probability that it was inspected by Tom? (5)

Q.2: Consider the density function: (8)

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

(a) Evaluate k (b) Find Mean and Variance of X.
(c) Find $F(x)$ and use it to evaluate $P(0.2 < x < 0.6)$.

Q.3: Suppose that X and Y are independent random variables having the joint probability distribution: (6)

f(x,y)		x = 2	x = 4
y	1	0.10	0.15
	3	0.20	0.30
	5	0.10	0.15

Find (a) $E(2X - 3Y)$; (b) $E(XY)$.

Q.4: A manufacturing company uses an acceptance scheme on items from a production line before they are shipped. The plan is a two-stage one. Boxes of 25 items are readied for shipment, and a sample of 3 items is tested for defectives. If any defectives are found, the entire box is sent back for 100% screening. If no defectives are found, the box is shipped.
(a) What is the probability that a box containing 3 defectives will be shipped?
(b) What is the probability that a box containing only 1 defective will be sent back for screening? (5)

Q.5: Suppose the probability that any given person will believe a tale about the transgressions of a famous actress is 0.7. What is the probability that:
(a) The fifth person to hear this tale is the third one to believe it.
(b) The fourth person to hear this tale is the first one to believe it. (4)

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Q.6: The average life of a certain type of small motor is 10 years with a S.D. of 2 years. The manufacturer replaces free all motors that fail while under guarantee. If he is willing to replace only 4% of the motors that fail, how long a guarantee should be offered? Assume that the lifetime of a motor follows a normal distribution. (5)

Q.7: The random variable X, representing the number of cherries in a cherry puff, has the following probability distribution:

x	4	5	6	7
P(X=x)	0.2	0.4	0.3	0.1

(a) Find the mean μ and the variance σ^2.

(b) Find the mean $\mu_{\bar{x}}$ and the variance $\sigma^2_{\bar{x}}$ of the mean $\bar{x}$ for random samples of 36 cherry puffs.

(c) Find the probability that the average number of cherries in 36 cherry puffs will be less than 5.1 or greater than 5.6. (6)

Q.8: A random sample of 12 shearing pins is taken in a study of the Rockwell hardness of the pin head. Measurements on the Rockwell hardness are made for each of the 12, yielding an average value of 48.50 with a sample S.D. of 1.5. Assuming the measurements to be normally distributed:

(a) Construct 95% confidence interval for the mean Rockwell hardness.

(b) What will be the 95% confidence interval if population variance is 4? (6)

Q.9: A manufacturer claims that the average tensile strength of thread A exceeds the tensile strength of thread B by at least 10 kg. To test this claim, 50 pieces of each type of thread were tested under similar conditions. Type A thread had an average tensile strength of 87 kg with a S.D. of 6 kg, while thread B had an average tensile strength of 78 with a S.D. of 5 kg. Test the manufacturer's claim using a 0.05 level of significance. (5)

FORMULAS of STATISTICS

$\times$ = multiplication

Rule of addition of probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Rule of multiplication of probability: $P(A \cap B) = P(A)P(B|A)$

Rule of subtraction of probability: $P(A') = 1 - P(A)$

Random Variables: X and Y are random variables, a and b are constants

Expected value of X=$E(X) = \mu_x = \sum(x_i * P(x_i))$, Variance of X = $Var\,(X) = \sigma^2 = \sum(x_i - E(x))^2 * P(x_i) = \sum(x_i - \mu_x)^2 * P(x_i)$

Normal random variable = $z = (x - \mu)/\sigma$

Chi-square statistic = $\chi^2 = [(n-1)S^2]/\sigma^2$, f statistic = $(s_1^2\,\sigma_2^2)/(s_2^2\,\sigma_1^2)$.

$Z = (\bar{x} - \mu)/[\sigma/\sqrt{n}]$

Expected value of sum of random variables = $E(X + Y) = E(X) + E(Y)$

Expected value of difference between random variables = $E(X - Y) = E(X) - E(Y)$

Variance of the sum of independent random variables = $Var(X + Y) = Var(X) + Var(Y)$

Variance of difference between independent random variables = $Var(X - Y) = \cancel{E(X) - E(X)}$ $Var(X) + Var(Y)$

Binomial formula: $P(X = x) = b(x; n, p) = {}^{n}C_{x} * p^x * (1-p)^{n-x} = {}^{n}C_{x} * p^x * (q)^{n-x}$

Mean of binomial distribution= $\mu_x = n * p$, Variance of binomial distribution=$\sigma_x^2 = np(1-p)$

Negative binomial formula: $P(X = x) = b^*(x; r, p) = {}^{x-1}C_{r-1} * p^r * (1-p)^{x-r}$

Mean of negative binomial distribution= $\mu_x = rq/p$, Variance of negative binomial distribution = $\sigma_x^2 = rq/p^2$

Geometric formula: $P(X = x) = g(x; p) = p * q^{x-1}$

Mean of geometric distribution = $\mu_x = q/p$, Variance of geometric distribution = $\sigma_x^2 = q/p^2$

Hypergeometric formula: $P(X = x) = h(x; N, n, k) = ({}^{k}C_{x})({}^{N-k}C_{n-x})/({}^{N}C_{n})$

Mean of hypergeometric distribution= $\mu_x = [n * k]/N$

Variance of hypergeometric distribution=$\sigma_x^2 = n * k * (N - k) * (N - n)/[N^2 * (N - 1)]$

Poisson formula: $P(x; \mu) = [e^{-\mu} \mu^x]/x!$

Mean of Poisson distribution $\mu_x = \mu$, Variance of Poisson distribution=$\sigma_x^2 = \mu$

Multinomial formula: $P = [n!/n_1! n_2! \ldots n_k!] * (p_1^{n_1} * p_2^{n_2} * \ldots * p_k^{n_k})$

Hypothesis Testing:

Standerdized test statistic= (Statistic-Parameter) / (Standard deviation of statistic)

One sample z-test for proportions: z-score= $z = (p - p_0)/\sqrt{p * q/n}$

Two sample z-test for proportions: z-score= $z = (\bar{p}_1 - \bar{p}_2)/\sqrt{\bar{p}\,\bar{q}(\frac{1}{n_1} + \frac{1}{n_2})}$

One sample t-test for means: t-score = $t = (\bar{x} - \mu)/[s/\sqrt{n}]$

Two-sample t-test for means, unknown but equal variances: t-score= $t = [(\bar{x}_1 - \bar{x}_2) - d_0] / S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

Confidence interval for μ, σ known : $\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

Confidence interval for $\mu_1 - \mu_2$; $\sigma_1^2 = \sigma_2^2$ but unknown, $n_1, n_2 \le 30$, is $(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}}(v) S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

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