Multivariable Calculus (MT1008)

Date: 10th April 2025

Sessional-II

Course Instructor(s)

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Total Time (Hrs.):	**1**
Total Marks:	**55**
Total Questions:	**1**

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Attempt all the questions.

***CLO*#2. Evaluation of Multiple Integrals in Different Coordinate Systems and Their Applications to Work, Circulation, Flux, Green's Theorem, and Stokes' Theorem.**

Each part contains 10 marks.

1 (i). Change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral

$$\int_0^1 \int_{\sqrt{1-y^2}}^1 \frac{1}{(x^2+y^2)^2}\,dx\,dy$$

[10]

(ii). Find the volume of the solid enclosed by the cone $z = \sqrt{x^2+y^2}$ between the planes $z = 1$ and $z = 2$. **[10]**

(iii). Convert

$$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} 3\,dz\,r\,dr\,d\theta, \quad r \geq 0$$

to (a) Rectangular coordinates with order of integration $dz\,dx\,dy$ (b) spherical coordinates. Then (c) evaluate one of the integrals. **[15]**

(iv). Find the flux of $F = (x-y)\,i + x\,j$ across the circle $x^2+y^2 = 1$ in the xy-plane. **[10]**

(v). Find the centroid $(\bar{x}, \bar{y})$ of the region, with constant density $\delta(x,y)$, in the first quadrant bounded by the x-axis, the parabola $y^2 = 2x$, and the line $x + y = 4$. **[10]**

Hint: $\bar{x} = \frac{M_y}{M}$, $\bar{y} = \frac{M_x}{M}$, $M = \iint_R \delta\,dA$, $M_x = \iint_R y\,\delta\,dA$, $M_y = \iint_R x\,\delta\,dA$,