0405

National University of Computer and Emerging Sciences

Lahore Campus

Multivariable Calculus (MT1008)

Date: 2nd June 2025

Course Instructor(s)

Dr. Mazhar Hussain, Dr. Akhlaq Ahmad,

Dr. Hina Firdous, Dr. Sidra Afzal, Tasaduque Hussain,

Muhammad Yaseen.

Final Examination

Total Time (Hrs.): 3

Total Marks: 120

Total Questions: 15

Do not write below this line

There are fifteen (15) questions attempt all of them.

CLO#1. Defining Functions of Several Variables, Computing Partial Derivatives, Directional and Gradient Vectors. [Weightage: 8]

1. This nonlinear differential equation, which describes wave motion on shallow water surfaces, is given by

$$4u_t + u_{xxx} + 12uu_x = 0$$

Show that $u(x,t) = \text{sech}^2(x-t)$ satisfies the above equation. [6]

2. Let $T = g(x,y)$ be the temperature at the point (x,y) on the ellipse

$$x = 2\sqrt{2}\cos t, y = \sqrt{2}\sin t, 0 \le t \le 2\pi,$$

and suppose that

$$\frac{\partial T}{\partial x} = y, \quad \frac{\partial T}{\partial y} = x$$

a) Locate the maximum and minimum temperatures on the ellipse by examining dT/dt and d^2T/dt^2.

b) Suppose that $T = xy - 2$. Find the maximum and minimum values of T on the ellipse. [8]

3. The derivative of $f(x,y)$ at $P_0(1,2)$ in the direction of $\mathbf{i}+\mathbf{j}$ is $2\sqrt{2}$ and in the direction of $-2\mathbf{j}$ is -3. What is the derivative of f in the direction of $-\mathbf{i}-2\mathbf{j}$? Give reasons for your answer. [6]

CLO#2. Evaluation of Multiple Integrals in Different Coordinate Systems and Their Applications to Work, Circulation, Flux, Green's Theorem, and Stokes' Theorem. [Weightage: 23]

$\frac{\pi}{3} - \frac{1}{\sqrt{3}}$

Using polar integration, find the area of the region R in the xy −plane enclosed by the circle $x^2 + y^2 = 4$, above the line $y = 1$, and below the line $y = \sqrt{3}\,x$. [6]

5. Find the volume of the tetrahedron D whose vertices are $(0,0,0), (1,1,0), (0,1,0)$, and $(0,1,1)$. Use the rectangular and cylindrical coordinates to set up the volume integral then solve both triple integrals. $1/6$ [9]

6. Find the volume of the smaller region cut from the solid sphere $\rho \leq 2$ by the plane $z = 1$. [5]

$5\pi/3$

7.

a) Find a potential function for the gravitational field

$$\mathrm{F} = -GmM\frac{x\mathrm{i} + y\mathrm{j} + z\mathrm{k}}{(x^2+y^2+z^2)^{3/2}}$$

(G, m, and M are constants).

b) Let P_1 and P_2 be points at distance s_1 and s_2 from the origin. Show that the work done by the gravitational field in part (a) in moving a particle from P_1 to P_2 is

$$GmM\left(\frac{1}{s_2} - \frac{1}{s_1}\right)$$

[10]

8. Use Green's Theorem to find the counterclockwise circulation and outward flux for the field F and curve C

$$\mathrm{F} = \left(\tan^{-1}\frac{y}{x}\right)\mathrm{i} + \ln(x^2+y^2)\mathrm{j}$$

$F = 2$
$C = 0$

C : The boundary of the region defined by the polar coordinate inequalities $1 \leq r \leq 2, 0 \leq \theta \leq \pi$. [10]

9. Use the Stokes' Theorem to calculate the flux of the curl of the field F across the surface S in the direction of the outward unit normal n. [6]

32π

$$F = 2z\mathrm{i} + 3x\mathrm{j} + 5y\mathrm{k}$$
$$S: \mathrm{r}(r,\theta) = (r\cos\theta)\mathrm{i} + (r\sin\theta)\mathrm{j} + (4-r^2)\mathrm{k},$$
$$0 \leq r \leq 2, 0 \leq \theta \leq 2\pi$$

10. Use the both sides of Divergence Theorem to find the outward flux of F across the boundary of the region D. [9]

$$F = x^2\mathrm{i} + xz\mathrm{j} + 3z\mathrm{k}$$
$$D: \text{The solid sphere } x^2 + y^2 + z^2 \leq 4$$

CamScanner

National University of Computer and Emerging Sciences

Lahore Campus

CLO 3: Some Integral Transforms, Orthogonal Functions, and Their Applications in Expanding Functions into Series, Including Laplace and Fourier Transforms, and Concepts of Sequences and Series. [Weightage: 19]

11. Use the integral test to determine if the given series converge or diverge? Be sure to check that the conditions of integral test are satisfied. **[10]**

$$\sum_{n=1}^{\infty} \frac{2}{1+e^n}$$

OR

Find the Fourier transform of the function

$$f(x) = e^{-ax^2}$$

12. Use the Laplace Transform to solve the given initial value problem

$$y'' + 4y = f(t), \quad y(0) = 0, \quad y'(0) = -1$$

Where $f(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$ $(1-e^{-s})$ $\frac{(1-\cos 2t)}{4}$ $-\frac{1}{2}\sin 2t$

[10]

13. Which of the following sequences $\{a_n\}$ in converge, and which diverge? Find the limit of each convergent sequence. **[10]**

$$(i) \quad a_n = \frac{2n+1}{1-3\sqrt{n}}, \qquad (ii) \quad a_n = \frac{1}{\sqrt{n^2-1}-\sqrt{n^2+n}}$$

14. Which series give below converge, and which diverge? Give reasons for your answers. If a series converges, find its sum. **[9]**

$$(i) \sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^n \qquad (ii) \sum_{n=1}^{\infty}\frac{2^n+3^n}{4^n}$$

15. Expand $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 \le x < \pi \end{cases}$ **in Fourier series.** **[6]**

CS CamScanner