Multivariable Calculus (MT1008)

Date: 2nd June 2025

Final Examination

Course Instructor(s)

Total Time (Hrs.):

Dr. Mazhar Hussain, Dr. Akhlag Ahmad,

Total Marks:

120

Dr. Hina Firdous, Dr. Sidra Afzal, Tasaduque Hussain, Total Questions:

15

Muhammad Yaseen.

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There are fifteen (15) questions attempt all of them.

CLO#1. Defining Functions of Several Variables, Computing Partial Derivatives, Directional and Gradient Vectors. [Weightage: 8]

1. This nonlinear differential equation, which describes wave motion on shallow water surfaces, is given by

$$4u_t + u_{xxx} + 12uu_x = 0$$
Show that $u(x, t) = \operatorname{sech}^2(x - t)$ satisfies the above equation. [6]

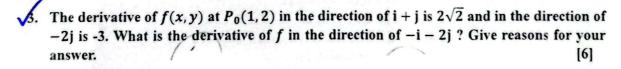
2. Let T = g(x, y) be the temperature at the point (x, y) on the ellipse

$$x = 2\sqrt{2}\cos t, y = \sqrt{2}\sin t, 0 \le t \le 2\pi,$$

and suppose that

$$\frac{\partial T}{\partial x} = y, \qquad \frac{\partial T}{\partial y} = x$$

- a) Locate the maximum and minimum temperatures on the ellipse by examining dT/dt and d^2T/dt^2 .
- b) Suppose that T = xy 2. Find the maximum and minimum values of T on the



CLO#2. Evaluation of Multiple Integrals in Different Coordinate Systems and Their Applications to Work, Circulation, Flux, Green's Theorem, and Stokes' Theorem. [Weightage: 23]

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Using polar integration, find the area of the region R in the xy-plane enclosed by the circle $x^2 + y^2 = 4$, above the line y = 1, and below the line $y = \sqrt{3}x$. [6]

- Find the volume of the tetrahedron D whose vertices are (0,0,0), (1,1,0), (0,1,0), and (0,1,1). Use the rectangular and cylindrical coordinates to set up the volume integral then solve both triple integrals.
- 6. Find the volume of the smaller region cut from the solid sphere $\rho \le 2$ by the plane z = 1.

7.

(a) Find a potential function for the gravitational field

$$\mathbf{F} = -GmM \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{3/2}}$$

(G, m, and M are constants).



Let P_1 and P_2 be points at distance s_1 and s_2 from the origin. Show that the work done by the gravitational field in part (a) in moving a particle from P_1 to P_2 is

$$GmM\left(\frac{1}{s_2} - \frac{1}{s_1}\right)$$

[10]

8. Use Green's Theorem to find the counterclockwise circulation and outward flux for the field F and curve C

$$\mathbf{F} = \left(\tan^{-1}\frac{y}{x}\right)\mathbf{i} + \ln\left(x^2 + y^2\right)\mathbf{j} \qquad \qquad C = 0$$

C: The boundary of the region defined by the polar coordinate inequalities $1 \le r \le 2, 0 \le \theta \le \pi$. [10]

Is the Stokes' Theorem to calculate the flux of the curl of the field F across the surface S in the direction of the outward unit normal n.

$$F = 2zi + 3xj + 5yk$$

$$S: r(r, \theta) = (r\cos \theta)i + (r\sin \theta)j + (4 - r^2)k,$$

$$0 \le r \le 2, 0 \le \theta \le 2\pi$$

Use the both sides of Divergence Theorem to find the outward flux of F across the boundary of the region D. [9]

$$F = x^2i + xzj + 3zk$$
D: The solid sphere $x^2 + y^2 + z^2 \le 4$

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[10]

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CLO 3: Some Integral Transforms, Orthogonal Functions, and Their Applications in Expanding Functions into Series, Including Laplace and Fourier Transforms, and Concepts of Sequences and Series. [Weightage: 19]

11. Use the integral test to determine if the given series converge or diverge? Be sure to check that the conditions of integral test are satisfied. [10]

$$\sum_{n=1}^{\infty} \frac{2}{1+e^n}$$

OR

Find the Fourier transform of the function

$$f(x) = e^{-ax^2}$$

12. Use the Laplace Transform to solve the given initial value problem

Use the Laplace Transform to solve the given initial value problem
$$y'' + 4y = f(t), \quad y(0) = 0, \quad y'(0) = -1$$
Where $f(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$

$$(1 - e^{-s}) \quad (1 - \cos t) \quad (1 - \cos t) \quad (2 - \cos t) \quad (3 - \cos t) \quad (3 - \cos t) \quad (4 - \cos t) \quad$$

[10] 13. Which of the following sequences $\{a_n\}$ in converge, and which diverge? Find the limit of

each convergent sequence.
 (i)
$$a_n = \frac{2n+1}{1-3\sqrt{n}}$$
,
 (ii) $a_n = \frac{1}{\sqrt{n^2-1}-\sqrt{n^2+n}}$

14. Which series give below converge, and which diverge? Give reasons for your answers. If a series converges, find its sum. [9]

$$(i) \sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n \qquad (ii) \sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n}$$

$$\cancel{15}. \text{ Expand } f(x) = \begin{cases} 0, & -\pi < x < 0 \\ & \text{in Fourier series.} \end{cases}$$

$$(6)$$

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