

Calculus & Analytical Geometry

Final Exam

(MT1003)

Date: 26th December, 2024

Course Instructor(s)

Dr. Mazhar Hussain (Moderator)

Dr. Sonia Hanif

Dr. Komal Hassan

Dr. Ayesha Razzaq

Dr. Atta Ullah

Mr. Abdul Hafeez

Mr. Muhammad Adeel

Ms. Eesha Meer

Total Time (Hrs.): 3

Total Marks: 100

Total Questions: 13

Roll No.

Section

Student/Signature

Do not write below this line

- Attempt all the questions in the given order
- Write question number on your answer with bold faced marker.

CLO#2: (Analyze the function and sketching the curve by using properties horizontal/ vertical and compressing / stretching)

- ✓ 1. Explain how $g(x)$ and $h(x)$ are related to $f(x)$ where $f(x) = x^2$, $g(x) = 3x^2$, and $h(x) = (x - 2)^2 - 4$.

[3]

CLO#3: (Investigation of continuity through limits analytically / graphically)

2. Prove that $\lim_{x \rightarrow \sqrt{3}} \frac{1}{x^2} = \frac{1}{3}$ by using precise definition of limit.

[7]

CLO#3: (Investigation of continuity through limits analytically / graphically)

3. Identify the vertical, horizontal, or oblique asymptotes of the given function. Also sketch the graph of the function and its asymptotes.

$$f(x) = \frac{x^3 + x^2 - 6x}{x^2 - 9}$$

[6]

CLO#4: (Apply the concept of differentiation in real life problem.)

4. A person is standing 350 feet away from a model rocket that is fired straight up into the air at a rate of 15 ft/sec. At what rate is the distance between the person and the rocket increasing 1 minute after liftoff?

[6]

CLO#4: (Apply the concept of differentiation in real life problem)

5. Find the equations of the tangent and normal to the curve of $x^3 + x^2y + y^3 - 7 = 0$ at (2, 3).

[7]

CLO#5: (Curve sketching using extrema theory)

6. A printer needs to make a poster that will have a total area of 125 in.^2 and will have 1/2 inch margin on the bottom, 1 inch margin on the right, 2 inch margin on the left and 4 inch margin on the top. What dimensions of the poster will give the largest printed area?

[8]

CLO#5: (Curve sketching using extrema theory)

7. Discuss the curve $f(x) = x^2 - x - \ln x$ by analyzing its critical points, intervals of increase and decrease, local extreme values, concavity, and points of inflection. Use this information to sketch the curve.

[10]

CLO#6: (Riemann sum, evaluation of definite & indefinite integral and their applications to compute lengths of curves / area of regions / volume of solids.)

8. Let $f(x) = x^2 + 2x - 5$ is defined over the interval $[1, 4]$.
a) Find a formula for the Riemann sum obtained by dividing the interval into n equal subintervals.
b) Take a limit of the sum to calculate the area under the curve over the interval.

[10]

CLO#6: (Riemann sum, evaluation of definite & indefinite integral and their applications to compute lengths of curves / area of regions / volume of solids.)

9. Use L' Hopital's Rule to find the following limit.

$$\lim_{x \rightarrow 0^+} (\sin x)^{\frac{1}{\ln x}}$$

[6]

CLO#6: (Riemann sum, evaluation of definite & indefinite integral and their applications to compute lengths of curves / area of regions / volume of solids.)

10. Find the length of the curve $x = 4(3 + y)^2$, $1 \leq y \leq 4$.

[10]

CLO#6: (Riemann sum, evaluation of definite & indefinite integral and their applications to compute lengths of curves / area of regions / volume of solids.)

11. Determine the area of the region enclosed by $y = 2x^2 + 10$, $y = 4x + 16$, $x = -2$, and $x = 5$.

[7] 2

CLO#6: (Riemann sum, evaluation of definite & indefinite integral and their applications to compute lengths of curves / area of regions / volume of solids.)

12. Determine if the following integral is convergent or divergent.

$$\int_0^5 \frac{x}{x-2} dx$$

[10]

CLO#6: (Riemann sum, evaluation of definite & indefinite integral and their applications to compute lengths of curves / area of regions / volume of solids.)

13. Find the volume of the solid obtained by rotating the region bounded by $y = 2\sqrt{x} - 1$ and $y = x - 1$ about the line $x = 6$.

[10] 2

GOOD LUCK