

### Input Sentence:

[0.3,0.4,0.7] This  
[0.1,0.2,0.7] is  
[0.2,0.2,0.8] a  
[0.3,0.7,0.8] book

$$W^Q$$

1	1	0
1	1	0
1	1	0

$$W^K$$

1	0	1
1	0	1
1	0	1

$$W^V$$

0	1	1
0	1	1
0	1	1

Compute self-attention for the 3<sup>rd</sup> word.

$$\text{Attn} = \text{Softmax} \left( \frac{Q \times K^T}{\sqrt{d_{\text{model}}}} \right) \cdot V$$

Dimensions

$$\text{Attn}(Q, K, V) = \text{Softmax} \left( \frac{(1 \times 3) \times (3 \times 4)^T}{\sqrt{3}} \right) \cdot (4 \times 3)$$

$(1 \times 3)$   $(1 \times 4)$

$$Q = \underset{1 \times 3}{X} \underset{1 \times 3}{\times} \underset{3 \times 3}{W^Q} \rightarrow [1.2, 1.2, 0]$$

$$K = X \times W^K \rightarrow$$

$$\begin{bmatrix} 0.3 & 0.4 & 0.7 \\ 0.1 & 0.2 & 0.7 \\ 0.2 & 0.2 & 0.8 \\ 0.3 & 0.7 & 0.8 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(3x3)

(4x3)

$$K = \begin{bmatrix} 1.4 & 0 & 1.4 \\ 1 & 0 & 1 \\ 1.2 & 0 & 1.2 \\ 1.8 & 0 & 1.8 \end{bmatrix}$$

(4x3)

$$V = \begin{bmatrix} 0 & 1.4 & 1.4 \\ 0 & 1 & 1 \\ 0 & 1.2 & 1.2 \\ 0 & 1.8 & 1.8 \end{bmatrix}$$

(4x3)

$$Q \times K^T \Rightarrow \underset{1 \times 3}{[1.2, 1.2, 0]} \underset{3 \times 4}{\begin{bmatrix} 1.4 & 1 & 1.2 & 1.8 \\ 0 & 0 & 0 & 0 \\ 1.4 & 1 & 1.2 & 1.8 \end{bmatrix}}$$

$$= [1.68, 1.2, 1.44, 2.16]$$

$$\frac{Q \times K^T}{\sqrt{3}} = [0.97, 0.69, 0.83, 1.25]$$

$$\text{softmax}\left(\frac{Q \times K^T}{\sqrt{3}}\right) = [0.25, 0.19, 0.22, 0.33]$$

$$\text{Softmax} \left( \frac{Q \times K^T}{\sqrt{3}} \right) \cdot V$$

(1x4) (4x3)

$$= \begin{bmatrix} 0.25 & 0.19 & 0.22 & 0.33 \end{bmatrix} \begin{bmatrix} 0 & 1.4 & 1.4 \\ 0 & 1 & 1 \\ 0 & 1.2 & 1.2 \\ 0 & 1.8 & 1.8 \end{bmatrix}$$

(1x4) (4x3)

$$= \begin{bmatrix} 0 & 1.398 & 1.398 \end{bmatrix}$$