

(d)

$$T(n) = 2T(n/2) + n/\lg n$$

$$T(n) -$$

$$\begin{array}{c} n \\ \swarrow \quad \searrow \\ n/2 \quad n/2 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ n/2^2 \quad n/2^2 \quad n/2^2 \quad n/2^2 \\ \vdots \end{array}$$

$$- n/\lg n$$

$$- 2[(n/2)/\lg(n/2)]$$

$$T(1) - n/2^k$$

$$- 2^k [(n/2^k)/\lg(n/2^k)]$$

$$* \text{ As, } T(1) = T(n/2^k)$$

$$1 = n/2^k$$

$$2^k = n$$

$$k = \log_2 n$$

$$* = n/\lg n + 2 \frac{n/2}{\lg(n/2)} + \dots + 2^k \frac{n/2^k}{\lg(n/2^k)}$$

$$= n/\lg n + n/\lg(n/2) + \dots + n/\lg(n/2^k)$$

$$= n \left(\frac{1}{\lg n} + \frac{1}{\lg(n/2)} + \dots + \frac{1}{\lg(n/2^k)} \right)$$

$$= n \left(\frac{1}{\lg n - 1} + \frac{1}{\lg(n/2) - 1} + \dots + \frac{1}{\lg(n) - 2^k} \right)$$

\therefore Harmonic Series $\therefore a_n = \frac{1}{a_1 + (n-1)d}$

$$= n \left(\frac{1}{\frac{1}{\lg n - 2} - \frac{1}{\lg n - 1}} \right) \Rightarrow n \left(\frac{1}{\frac{\lg n - 1 - \lg n + 2}{(\lg n - 2)(\lg n - 1)}} \right)$$

$$= n \left(\frac{1}{\frac{1}{\lg(\lg n)}} \right) \Rightarrow n(\lg(\lg n))$$

$$\# \text{ Big O} = O(n \lg(\lg n))$$

