

If we consider the form of the move to be:

$$\theta_t = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \quad (1)$$

The speed and acceleration can be defined as:

$$\dot{\theta}_t = a_1 + 2a_2 t + 3a_3 t^2 \quad (2)$$

$$\ddot{\theta}_t = 2a_2 + 6a_3 t \quad (3)$$

Given the following initial conditions, for a move t_f :

$$\theta_0 = \theta_1, \dot{\theta}_0 = \dot{\theta}_1, \text{ and } \ddot{\theta}_0 = \ddot{\theta}_{t_f} = 0$$

From Eqn. , $t = 0$ and $t_0 = \theta_1$

$$\theta_1 = a_0$$

From Eqn. , $t = 0$ and $\dot{\theta}_0 = 0$

$$0 = a_1$$

To solve for a_3 , Eqn can be written as

$$\theta_2 = \theta_1 + a_2 t_f^2 + a_3 t_f^3$$

$$\theta_2 - \theta_1 = a_2 t_f^2 + a_3 t_f^3$$

From Eqn , when $t = t_f$

$$0 = 2a_2 t_f + 3a_3 t_f^2 \quad (4)$$

Gives

$$a_2 = \frac{-3a_3 t_f}{2}$$

Substituting into Eqn.

$$\theta_2 - \theta_1 = \frac{-3a_3 t_f^3}{2} + a_3 t_f^3 = \frac{-a_3 t_f^3}{2}$$

Hence

$$a_3 = -\frac{2(\theta_2 - \theta_1)}{t_f^3} \quad (5)$$

Substituting Eqn. 5 into Eqn , when $t = t_f$, gives

$$a_2 = \frac{3(\theta_2 - \theta_1)}{t_f^2}$$

On substitution:

$$\begin{aligned}\theta_t &= \theta_1 + \frac{3(\theta_2 - \theta_1)}{t_f^2}t^2 - \frac{2(\theta_2 - \theta_1)}{t_f^3}t^3 \\ \dot{\theta}_t &= \frac{6(\theta_2 - \theta_1)}{t_f^2}t - \frac{6(\theta_2 - \theta_1)}{t_f^3}t^2 \\ \ddot{\theta}_t &= \frac{6(\theta_2 - \theta_1)}{t_f^2} - \frac{12(\theta_2 - \theta_1)}{t_f^3}t\end{aligned}$$