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UNIVERSITY OF SOUTHAMPTON ELEC2222W1

SEMESTER 2 EXAMINATIONS 2016/17

CIRCUITS AND TRANSMISSION

Duration 120 mins (2 hours)

This paper contains 6 questions

Answer **ONE** question in **Section A, ONE** question in **Section B** and **ONE** question in **Section C.**

Section A carries 33% of the total marks for the exam paper.

Section B carries 33% of the total marks for the exam paper.

Section C carries 33% of the total marks for the exam paper.

Only University approved calculators may be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct 'Word to Word' translation dictionary AND it contains no notes, additions or annotations.

11 page examination paper (+ 2 page formula sheet, 1 page The Complete Smith Chart)

SECTION A

Answer ONE out of TWO questions in this section

1 (a) In the network shown in Figure 1, two voltage sources act on the load impedance connected between A, B. If the load is variable in both resistance and reactance, what load Z_L will receive the maximum power and what is the value of the maximum power? Use Millman's theorem.

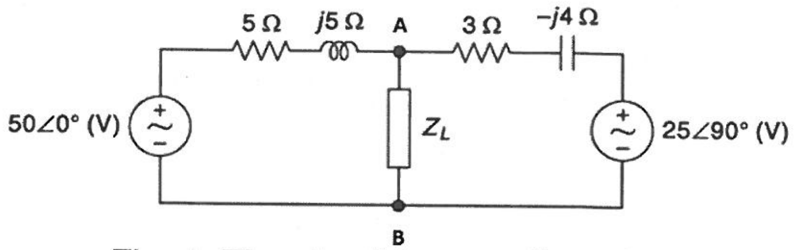


Fig. 1. The circuit for question 1(a)

[7 marks]

(b) Consider the circuit in Figure 2. Using node analysis with the ground reference node indicated, find the value of v_1, v_2, v_3 when $I_0 = 1\ A$ and $R_1 = R_2 = R_3 = R_4 = 1\ \Omega$.

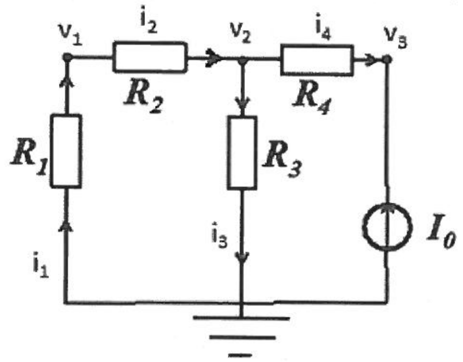


Fig. 2. The circuit for question 1(b)

[7 marks]

Question continues on following page

1 (a) In the network shown in Figure 1, two voltage sources act on the load impedance connected between A, B. If the load is variable in both resistance and reactance, what load Z_L ~~will receive the maximum~~ power and what is the value of the maximum power? Use Millman's theorem.

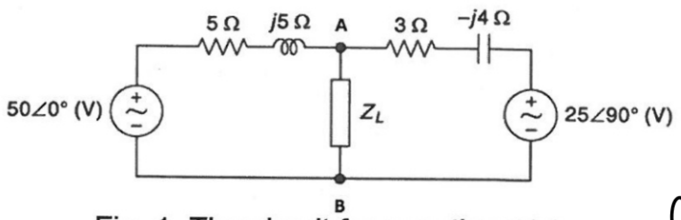


Fig. 1. The circuit for question 1(a)

[7 marks]

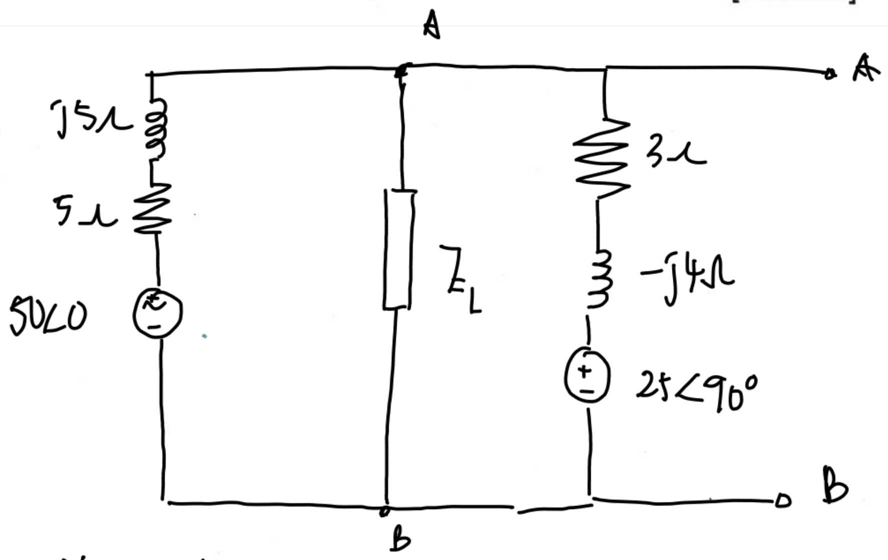
$$\frac{V^2}{Z} > 0$$

?

$$P = V I^*$$

α → x de soalan

AC



$$Y_1 = \frac{1}{5+j5} = 0.1 - j0.1 = \frac{\sqrt{2}}{10}\angle -45$$

$$Y_2 = \frac{1}{3-j4} = 0.12 + j\,0.16 = \frac{1}{5}\angle 53.1301^\circ$$

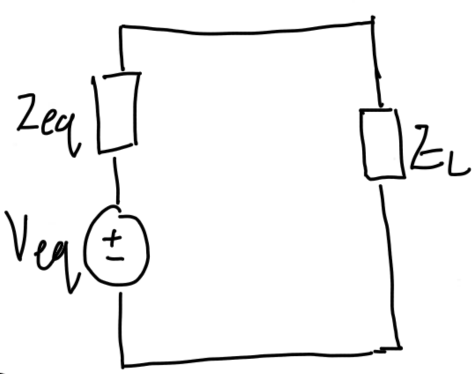
$$V_{AB} = \frac{50 \times \frac{\sqrt{2}}{10}\angle -45 + 25\angle 90 \times \frac{1}{5}\angle 53.1301}{\frac{\sqrt{2}}{10}\angle -45 + \frac{1}{5}\angle 53.1301 \; (Z_L)}$$

(need to include Z_L?)

$$= 9.806\angle -78.69^\circ$$

$$\frac{1}{Z_{eq}} = \frac{1}{5+j5} + \frac{1}{3-j4} = \frac{11}{50} + j\frac{3}{50}$$

$$Z_{eq} = \frac{55}{13} - j\frac{15}{13} = \frac{5\sqrt{130}}{13}\angle -15.255$$



$$I_L = \frac{V}{Z_{eq} + Z_L}$$

$$\frac{V^2}{R} > 0$$

$$9.806$$

$$= \frac{9.806\angle -78.69^\circ}{\frac{5\sqrt{130}}{13}\angle -15.255 + Z_L}$$

$$P = \frac{V^2}{R}$$

$$P = VI$$

$$\frac{9.806\angle -78.69}{\frac{5\sqrt{130}}{13}\angle -15.255 + Z_L} > 0$$

$$V_{th} = 1.9231 - 9.6155 j$$
$$Z_{eq} = 4.23077 - 1.1538 j$$

$$P = I^2 R_L$$

$$= \left(\frac{9.806\angle -78.69}{\frac{5\sqrt{130}}{13}\angle -15.255 + Z_L}\right)(Z_L)$$

$$P = \left[\frac{V_{TH}}{(Z_{TH} + Z_L)}\right]^2 Z_L$$

$$= \frac{Z_L V_{TH}^2}{(Z_{TH} + Z_L)^2}$$

$$u = V_{TH}^2 Z_L \qquad v = (Z_{TH} + Z_L)^2$$
$$u' = V_{TH}^2 \qquad v' = 2(Z_{TH} + Z_L)$$

$$\frac{dP}{dZ_L} = \frac{V_{TH}^2 (Z_{TH} + Z_L)^2 - 2(Z_{TH} + Z_L)(V_{TH}^2 Z_L)}{(Z_{TH} + Z_L)^2}$$

$$\frac{dP}{dZ_L} = 0$$

$$(Z_{TH} - Z_L)^2 = 0$$

$$\underline{Z_{TH}} = \underline{Z_L}$$

$$Z_L = 4.231 - j1.1538$$

$$P = \frac{(9.806\angle -78.69^\circ)^2}{\left(\frac{5\sqrt{130}}{13}\angle -5.255\right)}$$

$$=$$

[7 marks]

(b) Consider the circuit in Figure 2. Using node analysis with the ground reference node indicated, find the value of v_1, v_2, v_3 when $I_0 = 1\ A$ and $R_1 = R_2 = R_3 = R_4 = 1\ \Omega$.

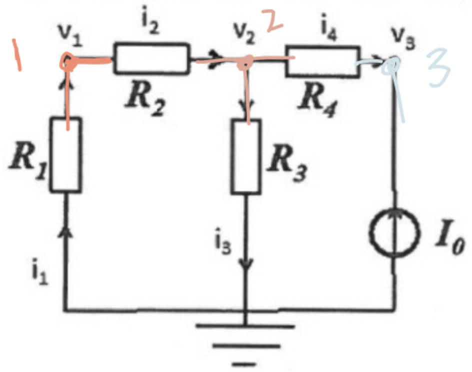


Fig. 2. The circuit for question 1(b)

[7 marks]

Nodal analysis

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}.$$

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} & -\frac{1}{R_4} \\ 0 & -\frac{1}{R_4} & \frac{1}{R_4} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} i_1 + i_2 \\ i_3 + i_4 \\ i_4 + I_0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 5 & -2 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 5 & -2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

$$3V_3 = 5$$
$$V_3 = \frac{5}{3}$$
$$= 1.667\,V$$

$$5V_2 - 2V_3 = 0$$
$$5V_2 = 2\left(\frac{5}{3}\right)$$
$$V_2 = \frac{2}{3}$$
$$= 0.667\,V$$

$$2V_1 - V_2 = 0$$
$$2V_1 = \frac{2}{3}$$
$$V_1 = \frac{1}{3}$$
$$= 0.333\,V$$

(c) Consider the two-port network in Figure 3, where the first impedance is $Z_1 = R$ (a resistance), the second one is $Z_2 = Ls$ (an inductor), and the admittance $Y = Cs$ (a capacitor). Derive the (A,B,C,D) representation of this network.

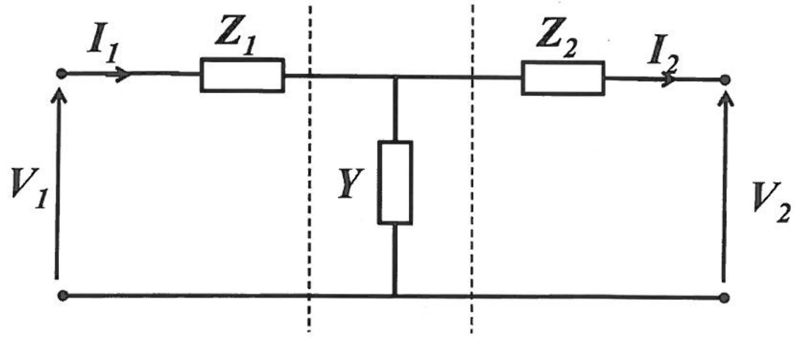


Fig. 3. The circuit for question 1(c), 1(d) and 1(e)

[5 marks]

(d) Consider the circuit in Figure 3. *Starting from the (A,B,C,D) representation* found answering Question 1 (c), derive its equivalent Z-representation. Show clearly how you arrived at your conclusions, i.e. how the Z-matrix is related to the (A,B,C,D) one.

[7 marks]

(e) Under which conditions is the two-port network in Figure 3 reciprocal? What are the consequences of reciprocity on the (A,B,C,D) representation of a two-port? Give a symbolic expression for the iterative impedance of a symmetric, reciprocal network as a function of its (A,B,C,D) parameters.

[7 marks]

TURN OVER

(c) Consider the two-port network in Figure 3, where the first impedance is $Z_1 = R$ (a resistance), the second one is $Z_2 = Ls$ (an inductor), and the admittance $Y = Cs$ (a capacitor). Derive the (A,B,C,D) representation of this network.

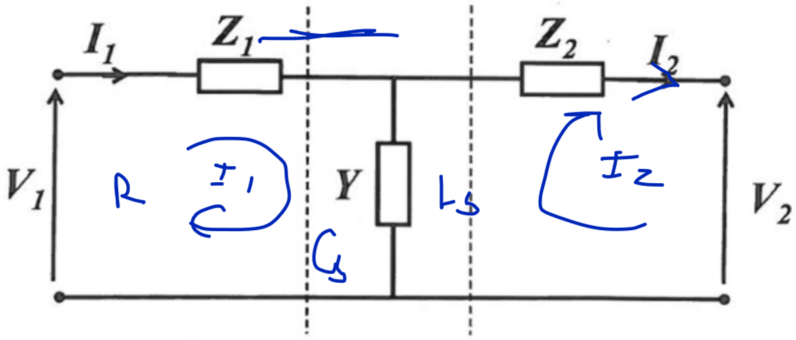


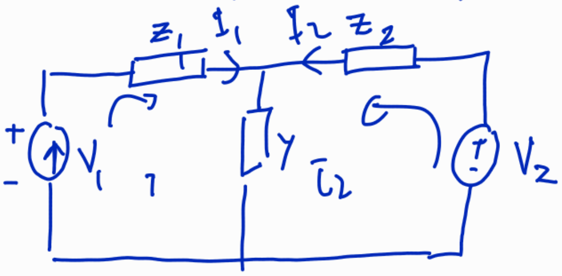
Fig. 3. The circuit for question 1(c), 1(d) and 1(e)

[5 marks]

$$\text{port 1} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{\text{transfer matrix}} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \text{port 2}$$

$Z = \frac{1}{Y}$ (?)

Close the terminals with two current source



$$V_1 = I_1 Z_1 + (I_1 + I_2)\frac{1}{Y}$$
$$= I_1\left(Z_1 + \frac{1}{Y}\right) + I_2\left(\frac{1}{Y}\right)$$
$$V_2 = I_2 Z_2 + (I_1 + I_2)\frac{1}{Y}$$
$$= I_2\left(Z_2 + \frac{1}{Y}\right) + I_1 \frac{1}{Y}$$

$$I_1 = Y\left(V_2 - I_2\left(Z_2 + \frac{1}{Y}\right)\right)$$
$$= YV_2 - I_2(YZ_2 + 1)$$
$$= YV_2 + I_2(-YZ_2 - 1)$$

$$V_1 = (YV_2 + I_2(-YZ_2 - 1))\left(Z_1 + \frac{1}{Y}\right) + I_2\left(\frac{1}{Y}\right)$$

$= V_2 Y Z_1 + V_2 + I_2(-YZ_1Z_2 - Z_1) + \cancel{I_2}(-Z_2 - \cancel{\tfrac{1}{Y}}) + \cancel{\tfrac{I_2}{Y}}$

$= V_2(YZ_1 + 1) + I_2(-YZ_1Z_2 - Z_1 - Z_2)$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} YZ_1 + 1 & -YZ_1Z_2 - Z_1 - Z_2 \\ Y & -YZ_2 - 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} YZ_1 + 1 & YZ_1Z_2 + Z_1 + Z_2 \\ Y & YZ_2 + 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Reciprocal ?

$AD - BC = 1$

$(YZ_1 + 1)(-YZ_2 - 1) - Y(-YZ_1Z_2 - Z_1 - Z_2)$

$= -Y^2\cancel{Z_1Z_2} - Y\cancel{Z_1} - Y\cancel{Z_2} - 1 + Y^2\cancel{Z_1Z_2} + Y\cancel{Z_1} + Y\cancel{Z_2}$

$= -1$

Not reciprocal

Symmetrical if $A = D$

$YZ_1 + 1 \neq -YZ_2 - 1$

(d) Consider the circuit in Figure 3. *Starting from the (A,B,C,D) representation* found answering Question 1 (c), derive its equivalent Z-representation. Show clearly how you arrived at your conclusions, i.e. how the Z-matrix is related to the (A,B,C,D) one.

[7 marks]

$V_1 = AV_2 - BI_2$ — (1) $V_1 = Z_{11}I_1 + Z_{12}I_2$ — (3)

$I_1 = CV_2 - DI_2$ — (2) $V_2 = Z_{21}I_1 + Z_{22}I_2$ — (4)

$I_1 + DI_2 = CV_2$ — (5)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} YZ_1+1 & YZ_1Z_2+Z_1+Z_2 \\ Y & YZ_2+1 \end{bmatrix}$$

$$V_2 = \frac{I_1 + DI_2}{C}$$

$V_2 \rightarrow$ (1)

$$V_1 = A\left[\frac{1}{C}I_1 + \frac{D}{C}I_2\right] - BI_2$$

$$= \frac{A}{C}I_1 + \frac{AD}{C}I_2 - BI_2$$

$$= \frac{A}{C}I_1 + I_2\left[\frac{AD}{C} - B\right]$$

$$\frac{AD-BC}{C} = \frac{|T|}{C}$$

$$V_1 = \frac{A}{C}I_1 + \frac{|T|}{C} \text{ — (6)}$$

$Z_{11} = \frac{A}{C}$ $Z_{12} = \frac{|T|}{C}$

$Z_{21} = \frac{1}{C}$ $Z_{22} = \frac{D}{C}$

$$Z_{11} = \frac{YZ_1+1}{Y} \qquad Z_{12} = \frac{(YZ_1+1)(YZ_2+1) - (YZ_1Z_2+Z_1+Z_2)(Y)}{Y}$$

$$= \frac{Y^2 Z_1 \cancel{Z_2} + \cancel{YZ_1} + \cancel{YZ_2} + 1 - Y^2 \cancel{Z_1 Z_2} - \cancel{YZ_1} - \cancel{YZ_2}}{Y}$$

$$= \frac{1}{Y}$$

$$Z_{21} = \frac{1}{Y} \qquad Z_{22} = \frac{YZ_2+1}{Y}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{YZ_1+1}{Y} & \frac{1}{Y} \\ \frac{1}{Y} & \frac{YZ_2+1}{Y} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

(e) Under which conditions is the two-port network in Figure 3 reciprocal? What are the consequences of reciprocity on the (A,B,C,D) representation of a two-port? Give a symbolic expression for the iterative impedance of a symmetric, reciprocal network as a function of its (A,B,C,D) parameters.

[7 marks]

Reciprocal when $AD - BC = 1$

$AD - BC$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} Yz_1+1 & Yz_1z_2+z_1+z_2 \\ Y & Yz_2+1 \end{bmatrix}$$

$$= (Yz_1+1)(Yz_2+1) - Y(Yz_1z_2+z_1+z_2)$$

$$= Y^2\cancel{z_1}z_2 + \cancel{Yz_1} + \cancel{Yz_2} - Y^2\cancel{z_1}z_2 - \cancel{Yz_1} - \cancel{z_2} + 1$$

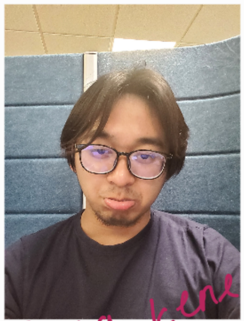
$$= 1$$

The circuit is reciprocal

But the circuit is not symmetrical

$A = D$

$Yz_1 + 1 \neq Yz_2 + 1$



Saya kene buli..

- Reciprocity ensures that power is conserved in a two-port network when the direction of signal flow is reversed, the power entering the network through one port is equal to the power leaving the network through the other part.
- In reciprocal network, the mutual impedance is equal to the mutual admittance.
- Identical forward & reverse transmission: Reciprocity implies that the forward and reverse transmission characteristics of two-port are equal.

$$Z = \frac{(A+D-2B)}{A+D+2B}$$

Assumes network both symmetric (B=C) and reciprocal A=D. The iterative impedance provides a measure of the characteristic impedance that would result of infinite chain of

Identical two-port networks were cascaded together

Question 2

(a) Using Rosen's theorem, find the equivalent "delta" circuit of the "star" circuit in Figure 4.

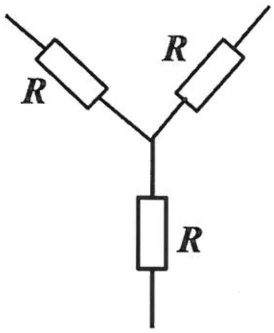


Fig. 4. The circuit for question 2(a)

[5 marks]

(b) Write down the A, B, C, D representation of the π-network in Figure 5. What is the A, B, C, D representation if two π-networks such as the one shown in Figure 5 are cascaded?

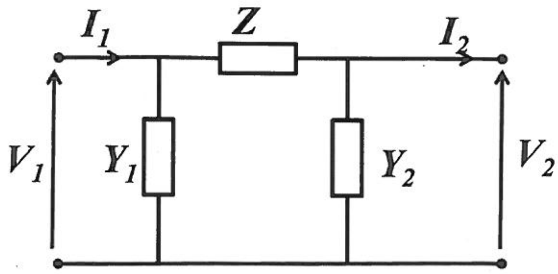
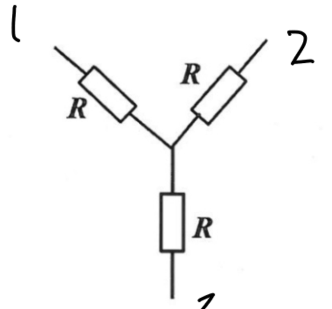


Fig. 5. The circuit for question 2(b)

[7 marks]

Question continues on following page

(a) Using Rosen's theorem, find the equivalent "delta" circuit of the "star" circuit in Figure 4.



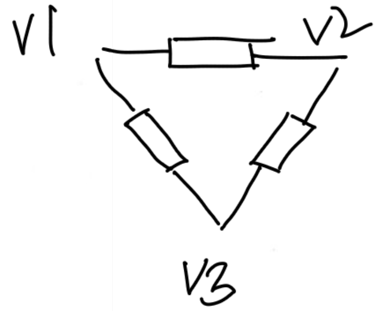


Fig. 4. The circuit for question 2(a)

[5 marks]

delta $Y_{12} = \dfrac{Y_1 Y_2}{Y_1 + Y_2 + Y_3}$ star

$$Y_{13} = \frac{Y_1 Y_3}{Y_1 + Y_2 + Y_3}$$

$$Y_{23} = \frac{Y_2 Y_3}{Y_1 + Y_2 + Y_3}$$

$$Y_{12} = \frac{\left(\frac{1}{R}\right)\left(\frac{1}{R}\right)}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}} = \frac{\frac{1}{R^2}}{\frac{3}{R}} = \frac{1}{3R}$$

$$Y_{13} = \frac{\left(\frac{1}{R}\right)\left(\frac{1}{R}\right)}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}} = \frac{\frac{1}{R^2}}{\frac{3}{R}} = \frac{1}{3R}$$

$$Y_{23} = \frac{\frac{1}{R}\frac{1}{R}}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}} = \frac{\frac{1}{R^2}}{\frac{3}{R}} = \frac{1}{3R}$$

(b) Write down the A, B, C, D representation of the π-network in Figure 5. What is the A, B, C, D representation if two π-networks such as the one shown in Figure 5 are cascaded?

Check back

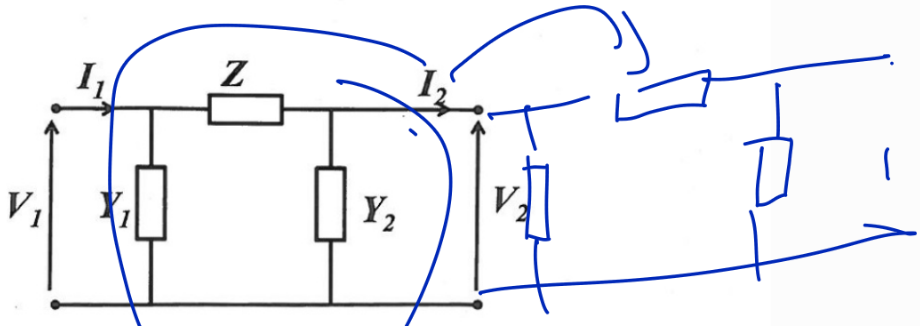


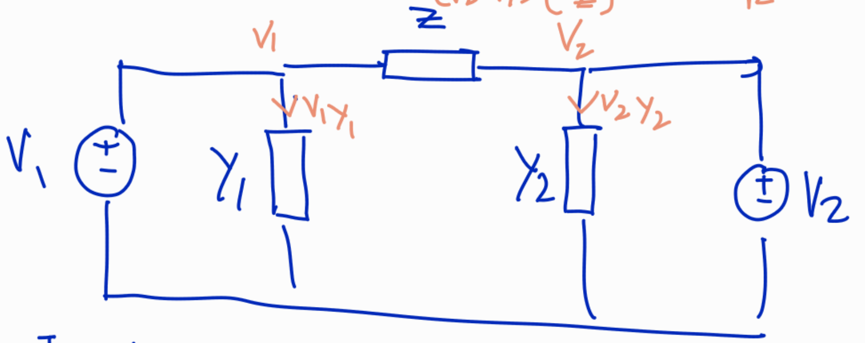
Fig. 5. The circuit for question 2(b)

[7 marks]

$V = IR$

$I = \frac{V}{R}$

Equivalent Circuit



$$I_1 = V_1 Y_1 + (V_1 - V_2)\left(\frac{1}{Z}\right)$$

$$= V_1\left(Y_1 + \frac{1}{Z}\right) - \frac{V_2}{Z}$$

$$I_2 = V_2 Y_2 + (V_2 - V_1)\left(\frac{1}{Z}\right)$$

$$I_2 = V_2 Y_2 + \frac{V_2}{Z} - \frac{V_1}{Z}$$

$$V_1 = Z\left(V_2 Y_2 + \frac{V_2}{Z} - I_2\right)$$

$$= V_2 Z Y_2 + V_2 - I_2 Z$$

$$= V_2 (Z Y_2 + 1) - I_2 Z$$

$$I_1 = (V_2 Z Y_2 + V_2 - I_2 Z)\left(Y_1 + \frac{1}{Z}\right) - \frac{V_2}{Z}$$

$$= V_2 Z Y_2 Y_1 + V_2 Y_1 - I_2 Z Y_1 + V_2 Y_2 + \cancel{\frac{V_2}{Z}} - I_2 - \cancel{\frac{V_2}{Z}}$$

$$= V_2(ZY_2Y_1 + Y_1 + Y_2) + I_2(-ZY_1 - 1)$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} ZY_2 + 1 & -Z \\ ZY_2Y_1 + Y_1 + Y_2 & -ZY_1 - 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} ZY_2 + 1 & Z \\ ZY_2Y_1 + Y_1 + Y_2 & ZY_1 + 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Cascaded π network

$$\begin{bmatrix} ZY_2+1 & Z \\ ZY_2Y_1 + Y_1 + Y_2 & ZY_1+1 \end{bmatrix} \begin{bmatrix} ZY_2+1 & Z \\ ZY_2Y_1 + Y_1 + Y_2 & ZY_1+1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} ZY_2+1 & Z \\ ZY_2Y_1 + Y_1 + Y_2 & ZY_1+1 \end{bmatrix} \begin{bmatrix} V_2(ZY_2+1) + I_2 Z \\ V_2(ZY_2Y_1 + Y_1 + Y_2) + I_2(ZY_1 + 1) \end{bmatrix}$$

$$= \begin{bmatrix} (ZY_2+1)(V_2ZY_2 + V_2 + I_2Z) + Z(V_2ZY_2Y_1 + V_2Y_1 + V_2Y_2 + I_2ZY_1 + I_2) \\ (ZY_2Y_1 + Y_1 + Y_2)(V_2ZY_2 + V_2 + I_2Z) + (ZY_1+1)(V_2ZY_2Y_1 + V_2Y_1 + V_2Y_2 + I_2ZY_1 + I_2) \end{bmatrix}$$

$$= \begin{bmatrix} V_2Z^2Y_2^2 + V_2ZY_2 + I_2Y_2Z^2 + V_2ZY_2 + V_2 + I_2Z + Z^2V_2Y_2Y_1 + ZV_2Y_1 + ZV_2Y_2 + Z^2I_2Y_1 + ZI_2 \\ \\ \end{bmatrix}$$

$$\begin{bmatrix}1&0\\Y_1&1\end{bmatrix}\begin{bmatrix}1&Z\\0&1\end{bmatrix}\begin{bmatrix}1&0\\Y_2&1\end{bmatrix}\begin{bmatrix}1&0\\Y_1&1\end{bmatrix}\begin{bmatrix}1&Z\\0&1\end{bmatrix}\begin{bmatrix}1&0\\Y_2&1\end{bmatrix}\begin{bmatrix}V_2\\I_2\end{bmatrix}$$

$$=\begin{bmatrix}1&0\\Y_1&1\end{bmatrix}\begin{bmatrix}1&Z\\0&1\end{bmatrix}\begin{bmatrix}1&0\\Y_2&1\end{bmatrix}\begin{bmatrix}1&0\\Y_1&1\end{bmatrix}\begin{bmatrix}1&Z\\0&1\end{bmatrix}\begin{bmatrix}V_2\\V_2Y_2+I_2\end{bmatrix}$$

$$=\begin{bmatrix}1&0\\Y_1&1\end{bmatrix}\begin{bmatrix}1&Z\\0&1\end{bmatrix}\begin{bmatrix}1&0\\Y_2&1\end{bmatrix}\begin{bmatrix}1&0\\Y_1&1\end{bmatrix}\begin{bmatrix}V_2+zV_2Y_2+zI_2\\V_2Y_2+I_2\end{bmatrix}$$

$$=\begin{bmatrix}1&0\\Y_1&1\end{bmatrix}\begin{bmatrix}1&Z\\0&1\end{bmatrix}\begin{bmatrix}1&0\\Y_2&1\end{bmatrix}\begin{bmatrix}V_2+zV_2Y_2+zI_2\\Y_1V_2+zV_2Y_1Y_2+Y_1zI_2+V_2Y_2+I_2\end{bmatrix}$$

$$=\begin{bmatrix}1&0\\Y_1&1\end{bmatrix}\begin{bmatrix}1&Z\\0&1\end{bmatrix}\begin{bmatrix}V_2+zV_2Y_2+zI_2\\V_2Y_2+V_2zY_2^2+I_2zY_2+V_2Y_1+V_2zY_1Y_2+I_2zY_1+V_2Y_2+I_2\end{bmatrix}$$

$$=\begin{bmatrix}1&0\\Y_1&1\end{bmatrix}\begin{bmatrix}V_2+zV_2Y_2+zI_2+V_2zY_2+V_2z^2Y_2^2+I_2z^2Y_2+V_2zY_1+V_2z^2Y_1Y_2+I_2z^2Y_1+V_2zY_2+I_2z\\V_2Y_2+V_2zY_2^2+I_2zY_2+V_2Y_1+V_2zY_1Y_2+I_2zY_1+V_2Y_2+I_2\end{bmatrix}$$

$$=\begin{bmatrix}V_2(1+zY_2+zY_2+z^2Y_2^2+zY_1+z^2Y_1Y_2+zY_2)+I_2(z+z^2Y_2+z^2Y_1+z)\\V_2(Y_1+zY_1Y_2+zY_2Y_1+z^2Y_2^2Y_1+zY_1^2+z^2Y_1^2Y_2+zY_2Y_1+Y_2+zY_2^2+Y_1+zY_1Y_2+Y_2)\\+I_2(zY_1+z^2Y_1Y_2+z^2Y_1^2+zY_1+zY_2+zY_1+1)\end{bmatrix}$$

$$=\begin{bmatrix}V_2(1+3zY_2+zY_1+z^2Y_2^2+z^2Y_1Y_2)+I_2(2z+z^2Y_2+z^2Y_1)\\V_2(2Y_1+4zY_1Y_2+z^2Y_2^2Y_1+z^2Y_2Y_1^2+zY_1^2+zY_2^2+2Y_2)\\+I_2(3zY_1+zY_2+z^2Y_1Y_2+z^2Y_1^2+1)\end{bmatrix}$$

(?)

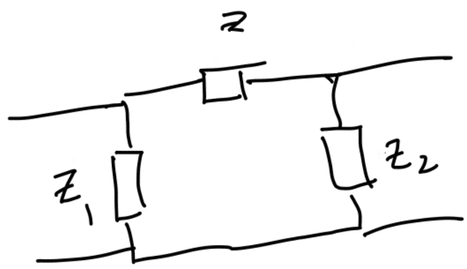
(c) A given resistive network has the (A,B,C,D) representation

$$\begin{bmatrix} 2 & 10 \\ \frac{3}{10} & 2 \end{bmatrix}.$$

Find an equivalent π-circuit containing only resistances. Make clear what the numerical values of the resistances in the equivalent network are.

[8 marks]

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ \frac{3}{10} & 2 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_1} & 1 \end{bmatrix}\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix}\begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & Z \\ \frac{1}{Z_1} & \frac{Z}{Z_1}+1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix}\begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+\frac{z}{Z_2} & Z \\ \frac{1}{Z_1}+\frac{z}{Z_1 Z_2}+\frac{1}{Z_2} & \frac{Z}{Z_1}+1 \end{bmatrix}\begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$z = 10$

$$1+\frac{z}{Z_2} = 2$$

$$Z_2 + z = 2z_2$$

$$Z_2 = z$$

$$\frac{Z}{Z_1}+1 = 2$$

$$\frac{Z}{Z_1} = 1$$

$z_2 = 10$

$$\frac{1}{Z_1}+\frac{z}{Z_1 Z_2}+\frac{1}{Z_2} = \frac{3}{10}$$

$$\frac{1}{Z_1}+\frac{10}{Z_1(10)}+\frac{1}{10} = \frac{3}{10}$$

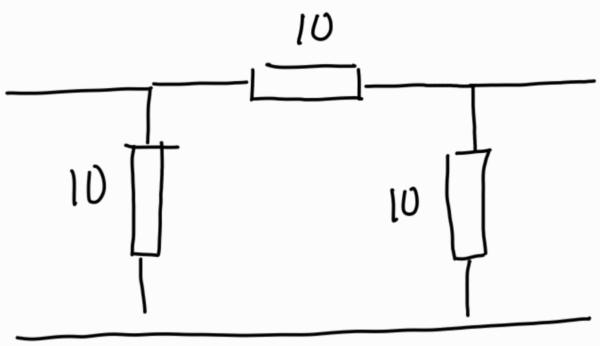
$$\frac{1}{Z_1} + \frac{1}{Z_1} = \frac{2}{10}$$

$$\frac{2}{Z_1} = \frac{2}{10}$$

$$\frac{Z_1}{2} = \frac{10}{2}$$

$$Z_1 = 0$$

$$Z = Z_1 = Z_2 = 10$$



Π-network

(d) Using Thevenin's theorem, find the equivalent circuit (with respect to the terminals A and B) to that in Figure 6.
Use such equivalent circuit to compute the numerical value of the current through R_3 when $R_1 = R_2 = R_3 = 1\ \Omega$ and $E_1 = 2\ V$.

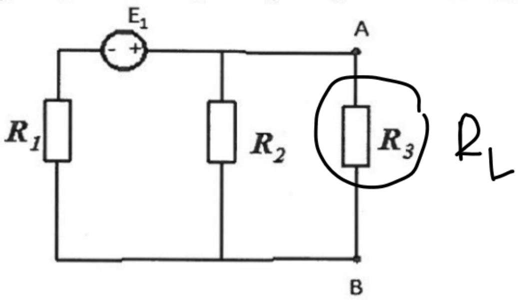
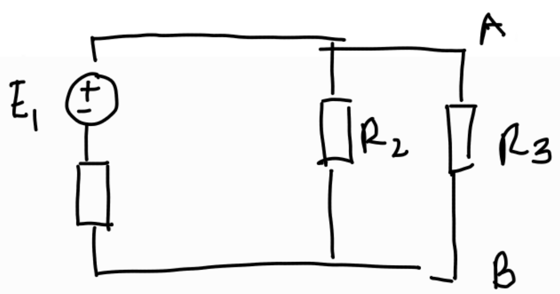


Fig. 6. The circuit for question 2(d)

[8 marks]



$$V_{AB} = \frac{\frac{E_1}{R_1} + \frac{0}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

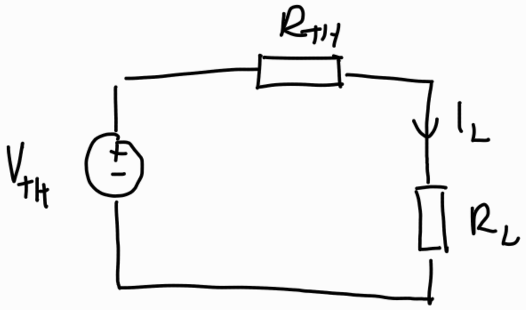
$$= \frac{2}{1+1}$$

$$= 1\,V$$

$$R_{TH} = \left(\frac{1}{1} + \frac{1}{1}\right)^{-1}$$

$$= \frac{1}{2}\,\Omega$$

$$= 0.5\,\Omega$$



$R_L = R_3 = 1$

$R_{TH} = 0.5\,\Omega$

$V_{TH} = V_{AB} = 1$

$$I_{R_3} = \frac{V_{TH}}{R_{TH} + R_L}$$

$$= \frac{1}{0.5+1}$$

$$= \frac{1}{1.5}$$

$$= 0.6667\,A$$

(e) Give expressions for the image impedances of the two-port in Figure 7.

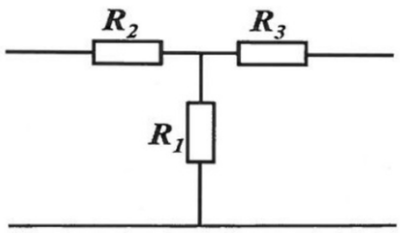


Fig. 7. The circuit for question 2(e)

[5 marks]

Image Impendances from ABCD parameters

$$Z_{I_1} = \frac{V_1}{I_1} = \qquad Z_{I_1} = \sqrt{\frac{AB}{CD}}$$

$$Z_{I_2} = \frac{V_2}{I'_2} \qquad Z_{I2} = \sqrt{\frac{DB}{CA}}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = A V_2 - B I_2$$

$$I_1 = C V_2 - D I_2$$

$$\frac{AV_2 - BI_2}{CV_2 - DI_2}$$

ABCD parameter

$$\begin{bmatrix} 1 & R_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & R_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{R_2}{R_1} & R_2 \\ \frac{1}{R_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & R_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{R_2}{R_1} & \frac{R_3 + R_2 R_3}{R_1} + R_2 \\ \frac{1}{R_1} & \frac{R_3}{R_1} + 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{R_2}{R_1} & \frac{R_3 + R_2 R_3}{R_1} + R_2 \\ \frac{1}{R_1} & \frac{R_3}{R_1} + 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$Z_{I_1} = \frac{V_1}{I_1} \cdot \sqrt{\frac{\left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_3 + R_2 R_3 + R_1 R_2}{R_1}\right)}{\left(\frac{1}{R_1}\right)\left(\frac{R_3 + R_1}{R_3}\right)}}$$

$$= \sqrt{\frac{\dfrac{R_3+R_2R_3+R_1R_2}{\cancel{R_1}} + \dfrac{R_2R_3+R_2^2R_3+R_1R_2^2}{R_1^2}}{\dfrac{R_3+R_1}{\cancel{R_1}R_3}}}$$

$$= \sqrt{\frac{R_3\left(R_1R_3+R_1R_2R_3+R_1^2R_2+R_2R_3+R_2^2R_3+R_1R_2^2\right)}{R_1(R_3+R_1)}}$$

$$= \sqrt{\frac{R_1R_3^2+R_1R_2R_3^2+R_1^2R_2R_3+R_2R_3^2+R_2^2R_3^2+R_1R_2^2R_3}{R_1(R_3+1)}}$$

$$Z_{12} = \sqrt{\frac{DB}{CA}} = \sqrt{\frac{\left(\dfrac{R_3+R_1}{R_1}\right)\left(\dfrac{R_3+R_2R_3+R_2R_1}{R_1}\right)}{\left(\dfrac{1}{R_1}\right)\left(\dfrac{R_1+R_2}{R_1}\right)}}$$

$$= \sqrt{\frac{(R_3+R_1)(R_3+R_2R_3+R_2R_1)}{R_1+R_2}}$$

SECTION B

Answer ONE out of TWO questions in this section

Question 3

(a) Describe what is meant by distortion of a lossy line, and state what relationship should hold among the line parameters per unit length (R, L, G and C) of a lossy line to obtain a distortion-free line, assuming the line operates at angular frequency ω.

[10 marks]

(b) A lossless transmission line has the following per unit parameters: $L = 0.4\ \mu H/m$ and $C = 130pF/m$. Calculate the propagation constant, characteristic impedance, wavelength and the phase velocity at 300 GHz. If the transmission line has a length of 10 cm, and is terminated with a load $Z_L = 30\Omega$, calculate the reflection coefficient at the load and the input impedance of the line.

[6 marks]

$L = 0.4\mu H/m$, $f = 300GHz$

$C = 130pF/m$, $l = 10cm$

$Z_L = 30\Omega$

$$\beta = \omega\sqrt{LC}$$
$$= 2\pi(300\times10^9)\sqrt{0.4\times10^{-6}\times130\times10^{-12}}$$
$$= 13592.61 \text{ rad/m}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.4\times10^{-6}}{130\times10^{-12}}}$$
$$= 55.47\Omega$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{13592.61}$$
$$= 4.62\times10^{-4} m$$

$$v = \frac{\omega}{\beta} = \frac{2\pi(300\times10^9)}{13592.61}$$
$$= 13.9\times10^7 m/s$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30-55}{30+55} = -0.294\Omega$$

Question continues on following page

$$Z_{in} = Z_0 \, \frac{Z_L + j Z_0 \tan(\beta l)}{Z_0 + j Z_z \tan(\beta l)}$$?

$$= 55 \times \frac{30 - j\,337.15}{55 - j\,183.9}$$

$$= 99.59 \angle -10$$

(c) Consider two lossless transmission lines operating at angular frequency ω carrying a signal with wavelength λ as shown in figure 8. Transmission line A is terminated with a load impedance Z_L while transmission line B is terminated with a load impedance (Z_0^2/Z_L). If the length of transmission lines A and B are $l+\lambda/4$ and l respectively and the characteristic impedance of both lines is Z_0, prove that the following equations are correct:

(i) $Z_{inA} = Z_{inB}$

(ii) $\Gamma_{LA} = -\Gamma_{LB}$

where Z_{inA}, Z_{inB}, Γ_{LA} and Γ_{LB} are the input impedances of line A and B and the reflection coefficients of transmission line A and B at the load respectively.

[17 marks]

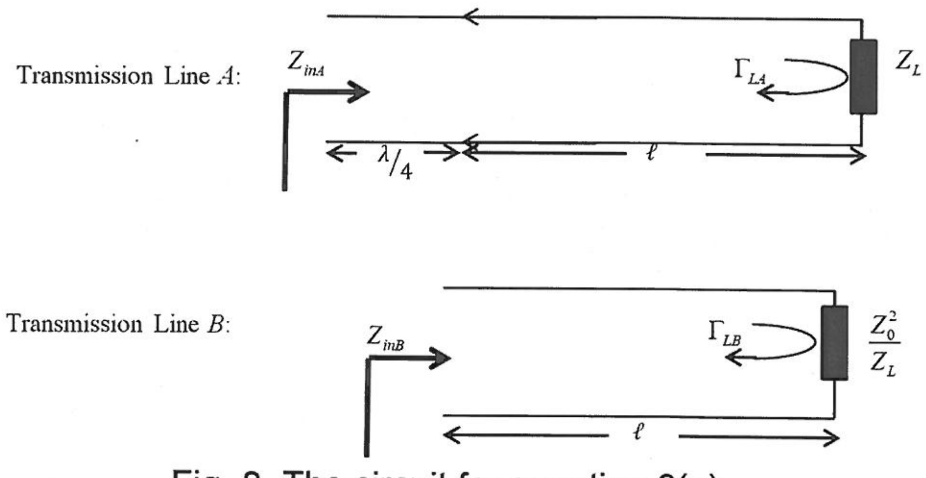


Fig. 8. The circuit for question 3(c)

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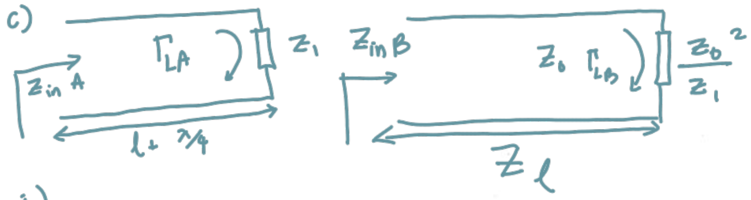
c)



i) $Z_{in\,A} = Z_{in\,B}$

ii) $\Gamma_{LA} = -\Gamma_{LB}$

$$Z_{in\,A} = Z_0\left(\frac{Z_L + jZ_0\tan\left(\beta\left(l+\frac{\lambda}{4}\right)\right)}{Z_0 + jZ_L\tan\left(\beta\left(l+\frac{\lambda}{4}\right)\right)}\right)$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\beta\left(l+\frac{\lambda}{4}\right) = \beta l + \frac{\beta\lambda}{4}$$

$$= \beta l + \frac{2\pi}{4}$$

$$= \beta l + \frac{\pi}{2}$$

$$Z_{in\,A} = Z_0\,\frac{Z_L + jZ_0\tan\left(\beta l + \frac{\pi}{2}\right)}{Z_0 + jZ_L\tan\left(\beta l + \frac{\pi}{2}\right)}$$

$$= Z_0\,\frac{Z_L - jZ_0\,0\tan\beta l}{Z_0 - jZ_L\,0\tan\beta l}$$

$$= Z_0\,\frac{Z_L\tan\beta l - jZ_0}{Z_0\tan\beta l - jZ_L}$$

$$= Z_0\,\frac{Z_0 + jZ_L\tan(\beta l)}{Z_L + jZ_0\tan(\beta l)} \quad -1$$

$$Z_{in\,B} = Z_0\,\frac{\frac{Z_0}{Z_L} + jZ_0\tan(\beta l)}{Z_0 + j\frac{Z_0^2}{Z_L}\tan(\beta l)}$$

$$= Z_0\,\frac{Z_0 + jZ_L\tan(\beta l)}{Z_L + jZ_0\tan(\beta l)} \quad -2$$

ii) $$\Gamma_{LA} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_{LB} = \frac{\frac{Z_0^2}{Z_L} - Z_0}{\frac{Z_0^2}{Z_L} + Z_0}$$

$$= \frac{Z_0 - Z_L}{Z_L + Z_0} = -\Gamma_{LA}$$

$$\Gamma_{LB} = -\Gamma_{LA}$$

Question 4

A strip line built on alumina substrate used at a frequency of 5 GHz has the following distributed circuit coefficients at that frequency:

$$R = 1.64\ \Omega/m;\ L = 5.2 \times 10^{-7}\ H/m;$$
$$G = 6.5 \times 10^{-3}\ S/m;\ C = 2.08 \times 10^{-10}\ F/m.$$

(a) Find the characteristic impedance of the line at the frequency of operation ($5\ GHz$) and comment on the result obtained in terms of losses of the line.

[5 marks]

(b) Assuming that the length of this transmission line is 10 mm, calculate the attenuation and phase difference that a sinusoidal voltage with amplitude of $5\ V$ and frequency of $5\ GHz$ will experience.

[3 marks]

(c) Assuming that the load impedance connected to this line is $100\ \Omega$ and transmission line is loss free, calculate the Voltage Standing Wave Ratio (VSWR) and Return Loss (RL).

[3 marks]

(d) Design a simple L network with the topology shown in figure (9) working at $5\ GHz$ to match a load impedance of $Z_L = 25 + j30\ \Omega$ to the $10\ mm$ strip line assuming that the strip line is loss free.

[12 marks]

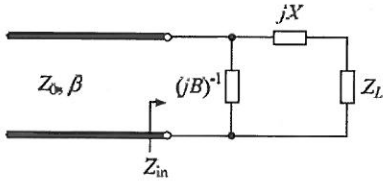


Fig. 9. The circuit for question 4(a) iv

Question continues on following page

A strip line built on alumina substrate used at a frequency of $5\ GHz$ has the following distributed circuit coefficients at that frequency:

$R = 1.64\ \Omega/m;\ L = 5.2 \times 10^{-7}\ H/m;$
$G = 6.5 \times 10^{-3}\ S/m;\ C = 2.08 \times 10^{-10}\ F/m.$

(a) Find the characteristic impedance of the line at the frequency ~~of operation ($5\ GHz$)~~ and comment on the result obtained in terms of losses of the line.

[5 marks]

$R = 1.64\ \Omega/m$ $\quad L = 5.2\times10^{-7}\ H/m$

$G = 6.5\times10^{-3}\ S/m$ $\quad C = 2.08\times10^{-10}\ F/m$ $\quad f = 5\times10^{9}\ Hz$

$\omega = 2\pi(5\times10^{9})$
$= 10\times10^{9}\pi$

$Z_0 = \sqrt{(R + j\omega L)(G + j\omega C)}$

$= \sqrt{(1.64 + j(10\times10^{9}\pi)(5.2\times10^{-7}))(6.5\times10^{-3} + j(10\times10^{9}\pi)(2.08\times10^{-10}))}$

$= \sqrt{(1.64 + j(5200\pi)) \times (6.5\times10^{-3} + j\frac{52}{25}\pi)}$

$=$

$= \sqrt{0.01066 - 106749.6412 + j116.9}$

$= \sqrt{-106749.63 + j116.9}$

$=$

(b) Assuming that the length of this transmission line is $10\ mm$, calculate the attenuation and phase difference that a sinusoidal voltage with amplitude of $5\ V$ and frequency of $5\ GHz$ will experience.

[3 marks]

(e) The $10\ mm$ strip line is connected between a power source with a source impedance of $Z_S = 50\ \Omega$ and frequency of $5\ GHz$ and a load with a load impedance $Z_L = 50 - j25\ \Omega$ as shown figure (10). Assuming that the losses on the strip line can be ignored calculate the complex reflection coefficient at the interface between the power source and the input of the loaded strip line using the provided Smith chart.

[10 marks]

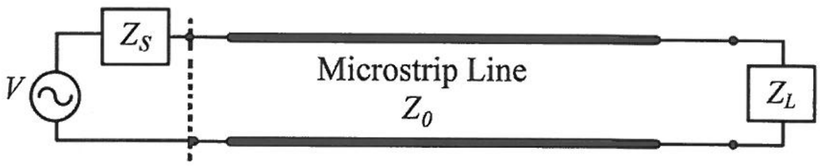


Fig. 10. The circuit for question 4(a) v

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SECTION C

Answer ONE out of TWO questions in this section

Question 5

A balanced three phase delta connected load has a per phase impedance $Z = 15\angle 30\ \Omega$.

The load is connected to a star connected voltage supply, where the neutrally is solidly grounded.

The lines which connect the load to the voltage supply have an entirely resistive impedance equal to 1 Ω.

The phase voltages of the supply are:

$$V_{AN} = 110\ \angle\ 0°\ V$$
$$V_{BN} = 105\ \angle\ \text{-}110°\ V$$
$$V_{CN} = 110\ \angle\ 105°\ V$$

(a) Obtain the sequence impedances, as seen from the terminals of the supply.

[6 marks]

(b) Use the symmetrical components method to obtain the line currents.

[18 marks]

(c) Find the complex power consumed by the load.

[9 marks]

Question 5

A balanced three phase delta connected load has a per phase impedance $Z = 15\angle 30\ \Omega$.

The load is connected to a star connected voltage supply, where the neutrally is solidly grounded.

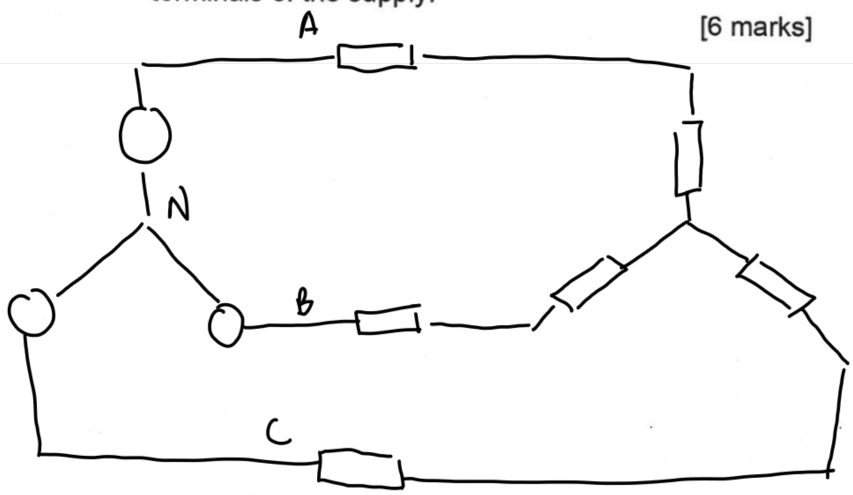
The lines which connect the load to the voltage supply have an entirely resistive impedance equal to 1 Ω.

The phase voltages of the supply are:

$$V_{AN} = 110 \angle 0° \ V$$
$$V_{BN} = 105 \angle -110° \ V$$
$$V_{CN} = 110 \angle 105° \ V$$

(a) Obtain the sequence impedances, as seen from the terminals of the supply.

[6 marks]



Transform the impedance into sequence form.

$Z_0 = \infty$

$$Z_2 = Z_1 = \frac{Z_\Delta}{3}$$

$$= 5\angle 30\ \Omega$$

(1)

$$Z_1 = \frac{Z_\Delta}{3} = \frac{15\angle 30}{3} = 5\angle 30\ \Omega$$ +1

(b) Use the symmetrical components method to obtain the line currents.

[18 marks]

$V_{an} = 110\angle 0°\ V$

$V_{bn} = 105\angle -110°\ V$

$V_{CN} = 110\angle 105°\ V$

$$\begin{pmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix}$$

$$V_{A0} = \frac{1}{3}(V_{AN} + V_{BN} + V_{CN})$$

$$= \frac{1}{3}(110\angle 0^\circ + 105\angle -110^\circ + 110\angle 105^\circ)$$

$$= 15.4146\angle 9.439^\circ$$

$$V_{A1} = \frac{1}{3}(V_{AN} + aV_{BN} + a^2 V_{CN})$$

$$= \frac{1}{3}(110\angle 0^\circ + (1\angle 120)(105\angle -110^\circ) + (1\angle -120)(110\angle 105))$$

$$= \frac{1}{3}(110\angle 0^\circ + 105\angle 10 + 110\angle -15)$$

$$= 106.607\angle -1.884^\circ$$

$$V_{A2} = \frac{1}{3}(V_{AN} + a^2 V_{BN} + aV_{CN})$$

$$= \frac{1}{3}(110\angle 0^\circ + (1\angle -120)(105\angle -110) + (1\angle 120)(110\angle 105))$$

$$= \frac{1}{3}(110\angle 0^\circ + 105\angle -230^\circ + 110\angle 225^\circ)$$

$$= 11.79135\angle 175.699^\circ$$

$$V_S = Z_S I_S$$

$$\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix} = \begin{bmatrix} \infty & 0 & 0 \\ 0 & 5\angle 30 & 0 \\ 0 & 0 & 5\angle 30 \end{bmatrix} \begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix}$$

$$I_{A0} = 0$$

$$V_{A1} = 5\angle 30 \; I_{A1}$$

$$I_{A1} = \frac{106.607\angle -1.834^\circ}{5\angle 30}$$

$$= 21.3214\angle -31.834^\circ$$

$$V_{A2} = 5\angle 30 \; I_{A2}$$

$$I_{A2} = \frac{11.79135\angle 175.699}{5\angle 30}$$

$$= 2.35827\angle 145.699$$

Now find the line currents using $I_p = A^{-1} I_s$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix}$$

$$I_A = I_{A0} + I_{A1} + I_{A2}$$

$$= \sqrt{2}\,(0 + 21.3214\angle -31.834^\circ + 2.35827\angle 145.699)$$

$$= 26.8214\angle -31.527$$

$$= 22.86 - j14.0249 \text{ A}$$

$$I_B = I_{A0} + a^2 I_{A1} + a I_{A2}$$

$$= \sqrt{2}\,(0 + (1\angle -120)(21.3214\angle -31.834) + (1\angle 120)(2.35827\angle 145.699))$$

$$= \sqrt{2}\,(21.3214\angle -151.834 + 2.35827\angle 265.699)$$

$$= 32.067\angle -146.8$$

$$= -26.8325 - j17.5587 \text{ A}$$

$$I_C = I_{A0} + a I_{A1} + a^2 I_{A2}$$

$$= \sqrt{2}\,(0 + (1\angle 120)(21.3214\angle -31.834^\circ) + (1\angle -120)(2.35827\angle 145.699))$$

$$= \sqrt{2}\,(21.3214\angle 88.166 + 2.35827\angle 25.699)$$

$$= 31.8324\angle 82.835^\circ$$

$$= 3.9704 + j31.5838 \text{ A}$$

(c) Find the complex power consumed by the load.

[9 marks]

$V_{an} = 110\angle 0^\circ V$ $\quad V_{CN} = 110\angle 105^\circ V$

$V_{bn} = 105\angle -110^\circ V$

$I_A = 22.86 - j14.0249 \ A \quad I_B = -26.8325 - j17.5587 \ A$

$I_C = 3.9704 + j31.5838 \ A$

$$S_A = \frac{1}{2}(110\angle 0^\circ)(3.9704 - j31.5838)$$

$$= 1750.78\angle -82.835$$

$$S_B = \frac{1}{2}(105\angle -110^\circ)(-26.8325 + j17.5587)$$

$$= 1683.516\angle 36.8$$

$$S_C = \frac{1}{2}(110\angle 105^\circ)(3.9704 - j31.5838)$$

$$= 1750.78\angle 22.1651^\circ$$

Total power

$$S_T = 1750.78\angle -82.835^\circ + 1683.516\angle 36.8^\circ + 1750.78\angle 22.165^\circ$$

$$= 3188.54\angle -1.2241^\circ$$

Question 6

A balanced, star-connected three-phase voltage source is connected to an unbalanced star-connected three-phase load as shown in the Figure 11 below:

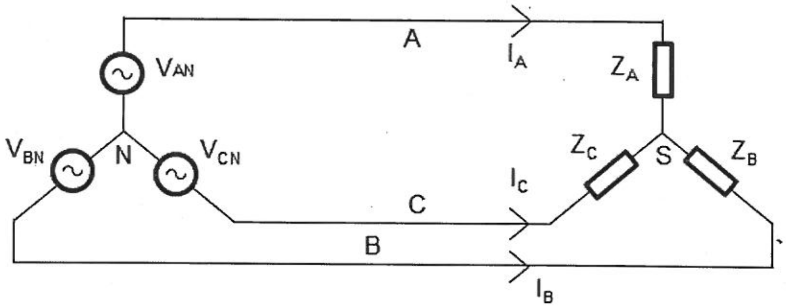


Fig. 11. The circuit for question 6

The phase voltage phasors and the loads are:

$V_{AN} = 230 \angle 0° \ V$ $\qquad$ $Z_A = 50 \angle 10° \ \Omega$

$V_{BN} = 230 \angle -120° \ V$ $\qquad$ $Z_B = 40 \angle -20° \ \Omega$

$V_{CN} = 230 \angle 120° \ V$ $\qquad$ $Z_C = 30 \angle 15° \ \Omega$

Note that all calculation of voltage and current phasors are to be referenced to V_A.

(a) Using Millman's theorem, calculate the phase voltages across each load as well as the line currents.

[13 marks]

(b) Determine the neutral current that flows from S to N if an impedance of $100 \angle 5° \ \Omega$ is connected between the two neutral terminals N and S.

[6 marks]

(c) Calculate the three-phase line currents as well as the active and reactive power supplied from the three phase voltage source when an impedance of $100 \angle 5° \ \Omega$ is connected between the two neutral terminals N and S.

[14 marks]

END OF PAPER

$V_{AN} = 230\angle 0^\circ$ V $\qquad Z_A = 50\angle 10 \ \Omega$

$V_{BN} = 230\angle -120^\circ$ V $\qquad Z_B = 40\angle -20 \ \Omega$

$V_{CN} = 230\angle 120^\circ$ V $\qquad Z_C = 30\angle 15 \ \Omega$

$$Y_A = \frac{1}{50\angle 10} = 0.02\angle -10^\circ \qquad Y_B = \frac{1}{40\angle -20} = 0.025\angle 20^\circ$$

$$Y_C = \frac{1}{30\angle 15}$$

$$= \frac{1}{30}\angle -15$$

$$= 0.0333\angle -15^\circ$$

$$V_{SN} = \frac{(230\angle 0^\circ)(0.02\angle -10^\circ) + (230\angle -120^\circ)(0.025\angle 20^\circ) + (230\angle 120^\circ)(\frac{1}{30}\angle -15)}{0.02\angle -10 + 0.025\angle 20 + \frac{1}{30}\angle -15}$$

$$= \frac{4.6\angle -10 + 5.75\angle 100 + \frac{23}{3}\angle 105}{0.07547\angle -2.6959}$$

$$= \frac{12.36648\angle 82.812}{0.07547\angle -2.6959}$$

$$= 163.8595\angle 85.51^\circ$$

$V_{AS} = V_{AN} - V_{SN}$

$= 230\angle 0 - 163.8595\angle 85.51^\circ$

$= 271.752\angle -36.95^\circ$

Peak voltage

$= \sqrt{2} \times 271.752\angle -36.95^\circ$

$= 384.315\angle -36.95^\circ$

$V_{BS} = V_{BN} - V_{SN}$

$= 230\angle -120^\circ - 163.8595\angle 85.51^\circ$

$= 384.418\ \angle -109.42^\circ$

Peak voltage

$= \sqrt{2} \times 384.418\angle -109.42^\circ$

$= 543.649\angle -109.42^\circ$

$V_{CS} = V_{CN} - V_{SN}$

$= 230\angle -120^\circ - 163.8595\angle 85.51^\circ$

$= 384.418\angle -109.42^\circ$

Peak voltage

$= \sqrt{2} \times 384.418\angle -109.42^\circ$

$= 543.649\ \angle -109.42^\circ$

$$Z_A = 50\angle 10 \ \Omega$$

$$Z_B = 40\angle -20 \ \Omega$$

$$Z_C = 30\angle 15 \ \Omega$$

$$I_A = \frac{V_{AS}}{Z_A}$$

$$= \frac{384.315\angle -36.95^\circ}{50\angle 10}$$

$$= 7.6863\angle -46.95^\circ$$

$$I_B = \frac{V_{BS}}{Z_B} = \frac{384.418\angle -109.42^\circ}{40\angle -20}$$

$$= 9.61045\angle -129.42^\circ$$

$$I_C = \frac{V_{CS}}{Z_C} = \frac{543.649\angle -109.42^\circ}{30\angle 15}$$

$$= 18.1216\angle -124.42^\circ$$

(b) Determine the neutral current that flows from S to N if an impedance of $100 \angle 5^\circ \, \Omega$ is connected between the two neutral terminals N and S.

[6 marks]

$$Z_N = 100\angle 5^\circ \ \Omega$$

$$V_{SN} = 163.8595\angle 85.51^\circ$$

$$I_N = I_A + I_B + I_C$$

$$= 7.6863\angle -46.95^\circ + 9.61045\angle -129.42^\circ + 18.1216\angle -124.42$$

$$= 30.11\angle -111.63^\circ$$

(c) Calculate the three-phase line currents as well as the active and reactive power supplied from the three phase voltage source when an impedance of $100 \angle 5^\circ\ \Omega$ is connected between the two neutral terminals N and S.

[14 marks]