Proportional Navigation and Optimal Evasion

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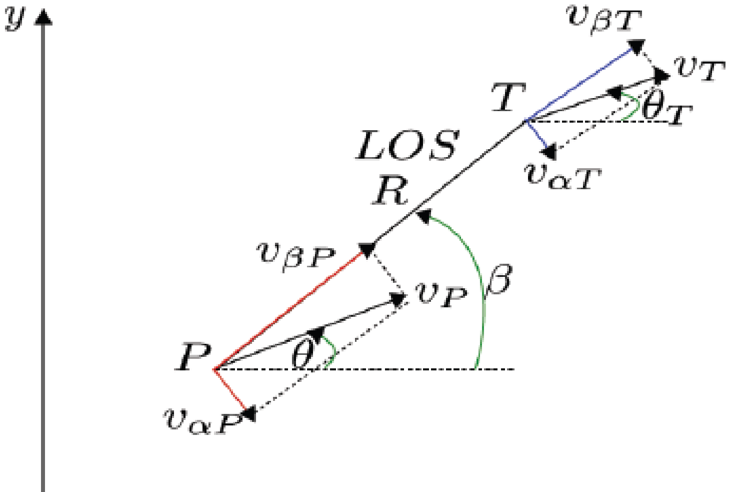
Background

- Proportional navigation is a prevalent method for homing guidance
- The problem of homing guidance is to cause a moving pursuer to hit or come close to a moving target, despite unpredictable maneuvers of the latter.
- Proportional navigation solves the homing problem by letting the turn rate of the pursuer be proportional to the turn rate of the line-of-sight, i.e., the line from pursuer to target. The evasion problem is for the target to avoid collision.

Assumptions for Homing Guidance

- The pursuer is actively controlled during the entirety of the engagement.
- The pursuer has a velocity with constant norm, i.e., has constant speed.
- The pursuer is able to sense the direction of the line-of-sight.

Geometry of Planar Homing



Variables

P represents the pursuer, T represents the target, LOS is the line-of-sight, β is the line-of-sight angle, i.e., the angle between a given, fixed direction and the line-of-sight, v_P is the pursuer's velocity, with heading angle Θ with components $v_{\alpha P}$, $v_{\beta P}$ orthogonal to and along the line-of-sight, respectively, v_T is the target velocity, with heading angle Θ_T and components $v_{\alpha T}$, $v_{\beta T}$ and R is the range, i.e., the distance from pursuer to the target.

Homing and Evasion Problems

Homing Problem: For the pursuer and target as in the figure above, find a guidance law for Θ as a function of β and $\dot{\beta}$ to cause a collision or near collision.
Evasion Problem: Find Θ_t to avoid a collision.

Fundamental Equations of Homing

$$\dot{R} = v_{\beta T} - v_{\beta P} = v_T \cos\left(\beta - \Theta_T\right) - v_P \cos\left(\beta - \Theta\right)$$

$$\dot{\beta} = -\frac{v_{\alpha T} - v_{\alpha P}}{R} = -\frac{v_T \sin\left(\beta - \Theta_T\right) - v_P \sin\left(\beta - \Theta\right)}{R}$$

Fundamental Equations of Homing

- Ina **plane, two vehicles moving with constant velocities are on a collision course if and only if their line-of-sight does not rotate.**
- Constant bearing — the LOS does not rotate.
- One way of accounting for the non-instantaneous responses of the pursuer and target is to use two integrators

$$\ddot{y}_P = u, \;\; \ddot{y}_T = v$$

where u and v are the lateral accelerations of the pursuer and target, respectively.

Fundamental Equations of Homing

- Introduce the state variables

$$x_1 = y_p,\ x_2 = y_T,\ \dot{x}_3\dot{y}_P,\ x_4 = \dot{y}_P$$

- Then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u \\ v \end{bmatrix}$$

Fundamental Equations of Homing

- The zero-sum cost function for this system is

$$J(u(\cdot), v(\cdot)) = \frac{1}{2}\int_{t_0}^{t_f}(t_f - t)(u^2(t)r_P - v^2(t)r_T)dt$$

- In this cost function t_0 is a given initial time and $r_P > 0$ and $r_T > 0$ are constants. The signs of the coefficients of u^2 and v^2 are opposite, to make thisa **game rather than a standard optimal control problem with linear dynamics and quadratic cost.**
- In linear quadratic control, the ratio of magnitudes of regulation and control reflects the relative importance or urgency.

Fundamental Equations of Homing

- If the so-called discount factor $(t_f - t)$ is close to t_f this factor is small – known as **cheap control**, where the weighting on the control energy tends to zero.
- This discount factor gives more urgency to times close to the nominal intercept.