SEMESTER 1 EXAMINATION 2019 - 2020

GUIDANCE, NAVIGATION AND CONTROL

DURATION 120 MINS (2 Hours)

This paper contains 5 questions

Answer **three** questions

An outline marking scheme is shown in brackets to the right of each question.

This examination contributes 100% of the marks for the module

University approved calculators MAY be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct Word to Word translation dictionary AND it contains no notes, additions or annotations.

18 page examination paper.

Question 1.

(a) Give a block diagram representation of a general motion control system. Give the distinguishing features of i) setpoint regulation, ii) trajectory tracking control and iii) path-following control.

[6 marks]

(b) By means of diagrams with all variables defined and marked, give the basic principles of i) LOS guidance, ii) PP guidance and iii) CB guidance. [6 marks]

(c) Let

$$p_t^n = \begin{bmatrix} N_t & E_t \end{bmatrix}^T$$

be the 2D position of a target in North-East coordinates. Formulate the control objective of a target-tracking scenario for this case.

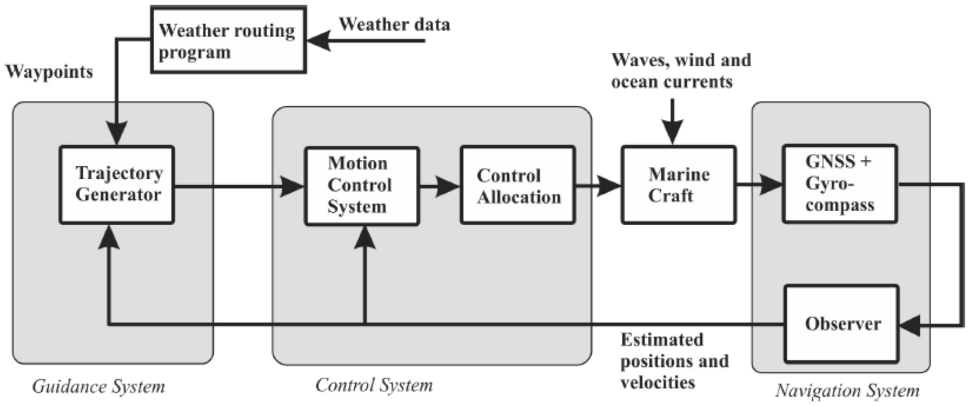
[4 marks]

(d) List the main considerations in forming a waypoint database.

[4 marks]

Indicative Solution for Question 1.

(a) [6 marks]



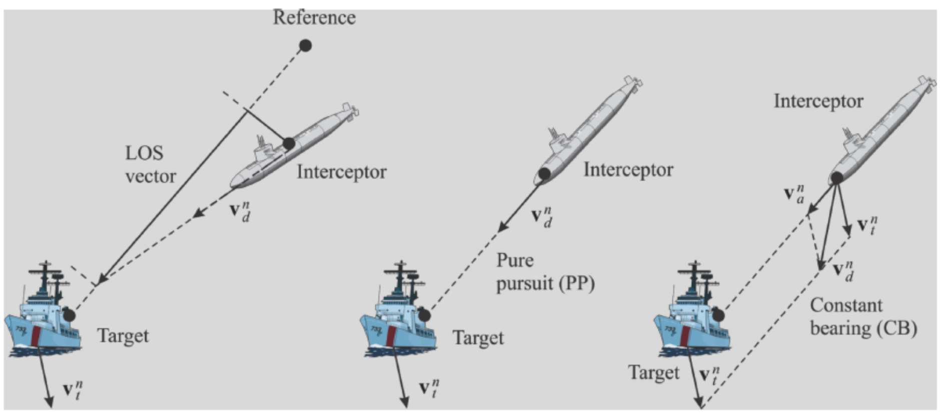
(For an ASV — for air vehicles the disturbances and information for waypoints change).

Setpoint regulation: The most basic guidance system is a constant input or setpoint provided by a human operator. The corresponding controller will then be a regulator.

Trajectory-tracking Control: The position and velocity of the vehicle should track desired time-varying position and velocity reference signals. The corresponding feedback is a trajectory-tracking controller.

Path-following Control: This is to follow a predefined path independent of time, i.e., no temporal constraints. Also no restrictions are placed on the temporal propagation along the path.

(b) [6 marks]



(This is in marine terms but applies equally to aerospace.)

(c) [4 marks]

TURN OVER

$$\lim_{t\to\infty}[p^n(t) - p_t^n(t)] = 0$$

where $p_t^n(t)$ **is either a stationary point or an object moving with non-zero and bounded North-East direction velocity**

$$v_t^n(t) \in \mathbf{R}^2$$

item [4 marks]

Mission: the vessel should move from some starting point (x_o, y_o, z_o) **to the terminal point** (x_n, y_n, z_n) **via the waypoints** (x_i, y_i, z_i).

Environmental data: information about wind etc can be used for energy optimal routing (or avoidance of bad weather for safety reasons).

Geographical data: information specific to the domain, e.g., shallow waters, islands etc in the marine area, should be included

Obstacles: in e.g., the marine case, floating constructions and other obstacles must be avoided.

Collision avoidance: avoid by introducing safety margins.

Feasibility: each waypoint must be feasible, in that it must be possible to maneuver to the next waypoint without exceeding maximum speed etc.

Question 2.

(a) State the purpose of waypoint switching for UAVs. What is the distinctive feature of the L_2^+ implementation?

[4 marks]

(b) Figure Q1 shows the waypoint switching geometry for an UAV, where all terms have their usual meanings.

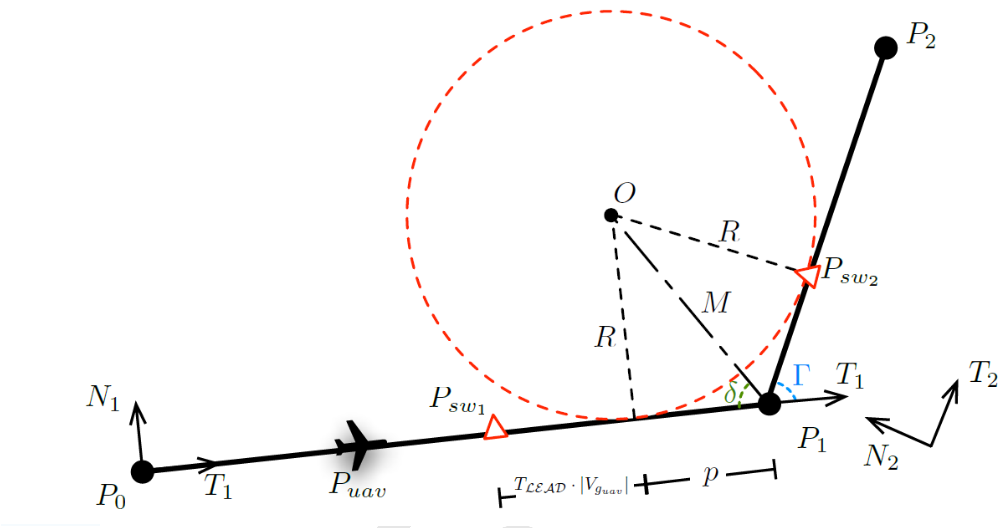


Figure Q1

The radius of the circle in this figure is

$$R = \frac{(U_c + |\text{wind}|)^2}{a_{\max}}$$

Define all the variables in this formula. The actual radius of curvature of the vehicle can change. Explain why and give a possible solution.

[4 marks]

(c) What is the meaning of the point P_{sw1} in Figure Q1? Show also that

$$p = \frac{R}{\tan \delta}$$

[6 marks]

TURN OVER

(d) Why in flight tests can commencing the turn just at the switch point cause a problem? Explain the use of a lead time as a counter to this problem and develop the formula for computing the resulting switch point. Also what happens in this case if the vehicle is already beyond the next segment switching point?

[6 marks]

Indicative Solution for Question 2.

(a) [4 marks] The guidance strategy is to move from one segment to the next of the flight mission by connecting the two segments with a circular arc. An L_2^+ implementation uses early waypoint switching to give priority to path-following over precision in the waypoint.

(b) [4 marks]

U_c **— commanded airspeed**
wind **— the wind speed**
$a_{\mathbf{max}}$ **— maximum acceleration of the UAV**

In the presence of wind the actual radius of curvature of the vehicle over the ground changes as the track angle changes. To avoid this problem in defining switching points, the maximum possible ground speed should be used in the numerator, which creates circle with radius larger than any curve over the ground.

(c) [6 marks]

The first tangent point, $P_{\mathbf{sw1}}$, determines the location where the UAV will switch to start tracking the next waypoint segment.

$$\Gamma = \arccos T_1^T T_2$$
$$\delta = \frac{\Gamma - \pi}{2}$$

Also

$$p = M \cos\delta, \,: M = \frac{R}{\sin\delta}$$

Hence

$$p = \frac{R}{\tan\delta}$$

(d) [6 marks]

Initiating the turn just at the switch point given by the last equation is not adequate due to the lag time of the roll dynamics of the UAV. To counter this a lead time, say $\tau_{\mathbf{lead}}$ can be introduced to initiate the turn. Multiplying $\tau_{\mathbf{lead}}$ by the groundspeed, gives the extra distance from the waypoint to the switch point. The new switch point distance is

$$p = \tau_{\mathbf{lead}}|V_g| + \frac{R}{\tan\delta}$$

and

$$P_{\mathbf{sw1}} = P_1 - pT_1$$

During the transition from missions, the L_2 vector intercepts the circular arc, not the straight line segments. Also, depending on the exact geometry of the waypoints, and the aircraft speed, it may be that just after switching legs, the aircraft is already beyond the next segment switching point. In this case, the logic immediately switches again to the next leg.

Question 3.

(a) A unity negative feedback control scheme for a single-input single-output linear time-invariant system with input u and output y is described by the following equations

$$\begin{aligned} y &= \eta + n \\ \eta &= P(s)(u+d) \\ u &= C(s)e \\ e &= F(s)r - y \end{aligned}$$

where $P(s)$ is the transfer-function, $C(s)$ is the transfer-function of the controller, $F(s)$ is the feedforward controller, r is the reference signal, d is the load disturbance and n is the measurement noise. Give a block diagram of this system with all variables specified. In the special case of $F(s) = 1$, give the transfer-functions of i) the sensitivity function S, ii) the load sensitivity function PS, iii) the complementary sensitivity function T and iv) the noise sensitivity function CS.

[6 marks]

(b) Consider the case of a system for which

$$P(s) = \frac{1}{s-a}$$

where $a > 0$ is to be controlled by applying the scheme of the previous part of this question with $F(s) = 1$ and

$$C(s) = k\frac{s-a}{s}, \;\; k > 0$$

Compute the functions S, PS, T and CS in this case and hence explain why this scheme cannot be applied. What are the implications when $a < \epsilon$ in the cases when i) ϵ is a very small negative number and ii) ϵ is a very large negative number? [7 marks]

(c) Consider a differential linear time-invariant system described by

$$P(s) = \frac{1}{s^2}$$

for which the performance specifications are less than 1% steady state error and less than 10% tracking error up to 10 rad/sec. Give a sketch of the gain and frequency responses of this system and explain why increasing the gain is not a feasible design. **Specify, but do not analyse,** the structure of a controller $C(s)$ that can be used to meet this specification. [7 marks]

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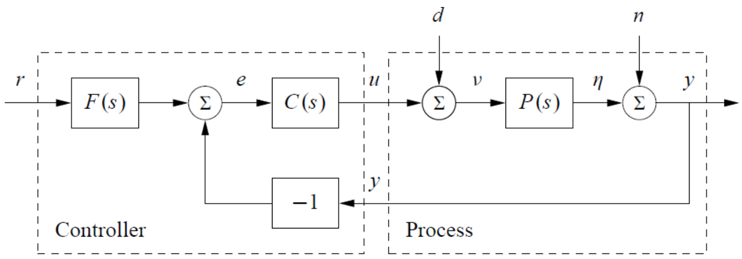
Indicative Solution for Question 3.

(a) [6 marks]



Sensitivity Function

$$S = \frac{1}{1+PC}$$

Load Sensitivity Function

$$PS = \frac{P}{1+PC}$$

Complementary Sensitivity Function

$$T = \frac{PC}{1+PC}$$

Noise Sensitivity Function

$$CS = \frac{C}{1+PC}$$

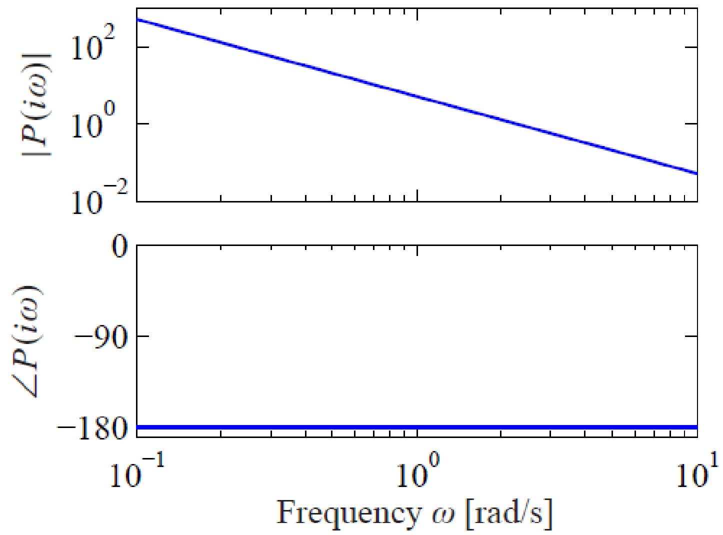
(b) [7 marks] Routine algebra gives

$$\begin{aligned} T &= \frac{k}{s+k}, \quad PS = \frac{s}{(s-b)(s+k)} \\ CS &= k\frac{s-a}{s+k}, \quad S = \frac{s}{s+k} \end{aligned}$$

Unstable pole-zero cancellation when $b > 0$ and PS is unstable. Hence the design is not feasible. In case i) this is a stable pole-zero cancellation but very close to the stability boundary — impinges on the transient dynamics and poor relative stability unless the cancellation is exact. Case ii) is feasible as the pole-zero cancellation is far away from the stability boundary.

[7 marks] To achieve the specification, wa gain of at least 10 at 10 rad/s is needed, requiring the gain crossover frequency to be at a higher frequency. From the figure below, increasing the gain would give a very low phase margin. Hence the phase at the desired crossover frequency must be increased. This can be achieved by a phase lead controller

$$C(s) = \frac{s+a}{s+b}, \quad a < b$$


10^2
10^0
10^{-2}
$|P(i\omega)|$
0
−90
−180
$\angle P(i\omega)$
10^{-1}
10^0
10^1
Frequency ω [rad/s]

Question 4.

(a) A loop transfer-function $L(s)$ of a feedback control system goes to zero faster than $\frac{1}{s}$ as $s \to \infty$ and has p_k poles in the right half-plane. Then

$$\int_0^\infty \log(|S(j\omega)|)d\omega = \pi \sum p_k$$

holds where S denotes the sensitivity function. Explain the implications of this result in terms of control design, where your answer must include discussion, including a diagram with all relevant terms marked, of the waterbed effect. Apply this result, if possible, to the following examples and in cases where this is not possible explain why.

$$L(s) = \frac{k}{s+5}, \;\; L(s) = \frac{k}{(s+1)^2}, \;\; L(s) = k\frac{(s-3)}{(s-1)(s-2)(s+5)}$$

[10 marks]

(b) Let $P(s)$ be the transfer-function of a differential linear time-invariant system subject to uncertainty modeled as a) additive described by Δ and b) multiplicative described by δ. State the $\mathcal{H}_\infty$ condition for robust stability in each case. [4 marks]

(c) Consider a unity negative feedback control scheme formed from system transfer-function P and controller C and transfer-function from load to disturbances to output given by

$$G_{yd} = \frac{P}{1+PC}$$

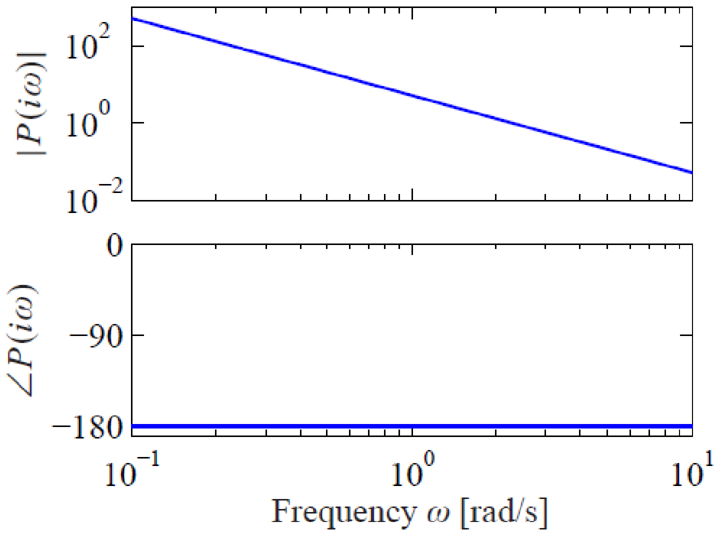
Express G_{yd} in terms of the sensitivity function S and show that

$$\frac{dG_{yd}}{G_{yd}} = S\frac{dP}{P}$$

and discuss the implications of this result. [6 marks]

Indicative Solution for Question 4.

(a) [10 marks]



Bode's integral formula can be regarded as a conservation law. The figure above shows the waterbed effect — if the sensitivity is to be made smaller up to some frequency and hence reduce the effect of disturbances this must be balanced by increased sensitivity at other frequencies, for which disturbance amplification occurs.

$L_1(s)$ **— result is not applicable as the faster than $\frac{1}{s}$ requirement is not true.**
$L_2(s)$ **– result is applicable as the faster than $\frac{1}{s}$ requirement is true. Both poles are at $s = -1$ and hence the integral is zero.**
$_3L(s)$ **— result is applicable as the faster than $\frac{1}{s}$ requirement is true. Poles at $s = 1, 2, -5$. The integral hence has value 3π.**

(b) [4 marks]

Let $||\cdot||_\infty$ denote the norm. Then for additive uncertainty, i.e. $P + \Delta$ the condition is

$$||CS\Delta||_\infty < 1$$

and for multiplicative uncertainty, i.e., $P(1 + \delta)$

$$||T\delta||_\infty < 1$$

where T is the complementary sensitivity function.

(c) [6 marks] Form the fact that $S = \frac{1}{1+PC}$ it follows immediately that

$$G_{yd} = \frac{P}{1 + PC} = PS$$

TURN OVER

Also

$$\frac{dG_{yd}}{dP} = \frac{1}{(1+PC)^2} = \frac{SP}{P(1+PC)} = S\frac{G_{yd}}{P}$$

and the result follows.

The response to load disturbances is thus insensitive to process variations for frequencies where $|S(j\omega)|$ is small, i.e., for frequencies where load disturbances are important.

Question 5.

(a) A single-input single-output differential linear time-invariant system is described by the state-space model

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t)\end{aligned}$$

where the state vector $x(t) \in \mathbb{R}^n$. Give the matrix rank based conditions under which this system is controllable and observable. If these conditions hold what property does the system transfer-function have?

[4 marks]

(b) In an application, the uncontrolled system is described by the state-space model

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} u(t)\\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)\end{aligned}$$

The state feedback control law is to be implemented using an observer. What is meant by the separation principle? Design a state feedback control law in this case and a full state observer for implementation. The closed-loop poles are to be the roots of the polynomial

$$\rho(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

where $0 < \zeta < 1$ and the observer poles are to be those of the controlled dynamics times a constant selected in line with best practice. **In each case, construct, but do not solve, the equation that defines the solution.**

[10 marks]

(c) A differential linear time-invariant system has the state equation

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)\\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)\end{aligned}$$

TURN OVER

and a state feedback control law is to be designed by minimising the cost function

$$J = \int_0^\infty (x^T(t)C^TCx(t) + u^T(t)Ru(t))dt$$

and $R = 1$. In this case the candidate solution of the associated algebraic Riccati equation is obtained from the following set of simultaneous equations

$$\begin{aligned} 2p_1 - p_1^2 + 4p_2 &= 0 \\ p_2 + 2p_3 - p_1p_2 &= 0 \\ 1 - p_2^2 &= 0 \end{aligned}$$

where p_1, p_2 and p_3 are real scalars. Compute the state feedback law in this case and show that the resulting controlled system stable.

[6 marks]

Indicative Solution for Question 5.

(a) [4 marks] Controllability

$$\mathbf{rank}\left[\begin{array}{ccccc} B & AB & A^2B & \dots & A^{n-1}B \end{array}\right] = n$$

Alternatively, the following matrix is nonsingular

$$\left[\begin{array}{ccccc} B & AB & A^2B & \dots & A^{n-1}B \end{array}\right]$$

Observability

$$\mathbf{rank}\left[\begin{array}{c} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{array}\right] = n$$

Alternatively, the following matrix is nonsingular

$$\left[\begin{array}{c} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{array}\right]$$

(b) [10 marks]

State feedback control law

$$u = -Kx = -\left[\begin{array}{cc} k_1 & k_2 \end{array}\right] x(t)$$

The pole placement problem has a solution if and only if the open-loop system is controllable. This is the case since

$$\mathbf{det}\left[\begin{array}{cc} \frac{1}{2} & -\frac{1}{2} \\ 1 & \frac{1}{2} \end{array}\right] \neq 0$$

The design problem now is

$$\mathbf{det}\left[sI_2 - A + BK\right] = s^2 + 2\zeta\omega_n s + \omega_n^2$$

By the separation principle the observer poles but implementation experience has concluded that 5 to 10 times faster than the systems poles gives good performance. The observer design problem has a solution if and only if the open-loop system is observable. This is the case since

$$\mathbf{det}\left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right] \neq 0$$

Let

$$o_p(s) = s^2 + 2a^{'}s + b^{'}$$

TURN OVER

denote the polynomial describing the observer poles selected in line with the criteria above. The design problem is

$$\mathbf{det}\left[sI_2 - A + LC\right] = o_p(s)$$

(c) [6 marks]

The solution matrix of the algebraic Riccati equation must be symmetric positive definite and assume it has the form

$$P = \left[\begin{array}{cc} p_1 & p_2 \\ P_2 & p_3 \end{array}\right]$$

where p_1, P_2 **and** p_3 **satisfy the equations given in the question. The** 3**rd equation gives** $p_2 = \pm 1.$ **The choice of** $p_2 = -1$ **means that the first equation would have complex roots and this is not appropriate. Hence** $p_2 = 1.$ **Also** $p_1 = 1 \pm \sqrt{5}$ **and for the matrix to be positive-definite** $p_1 > 0$**. Consequently** $p_1 = 1 + \sqrt{5} = 3.236$ **and** $p_3 = \sqrt{5}/2 = 1.118.$

Stabilising control law

$$U(t) = -R^{-1}B^T Px(t) = -\left[\begin{array}{cc} 3.236 & 1 \end{array}\right] x(t)$$

In this case, the closed-loop poles are $-\sqrt{5}/2 \pm j\sqrt{3}/2$**, i.e., the closed-loop system is stable.**

END OF PAPER