Electronics and
Computer Science

ELEC2208 Power Electronics and Drives

DC-DC Converter

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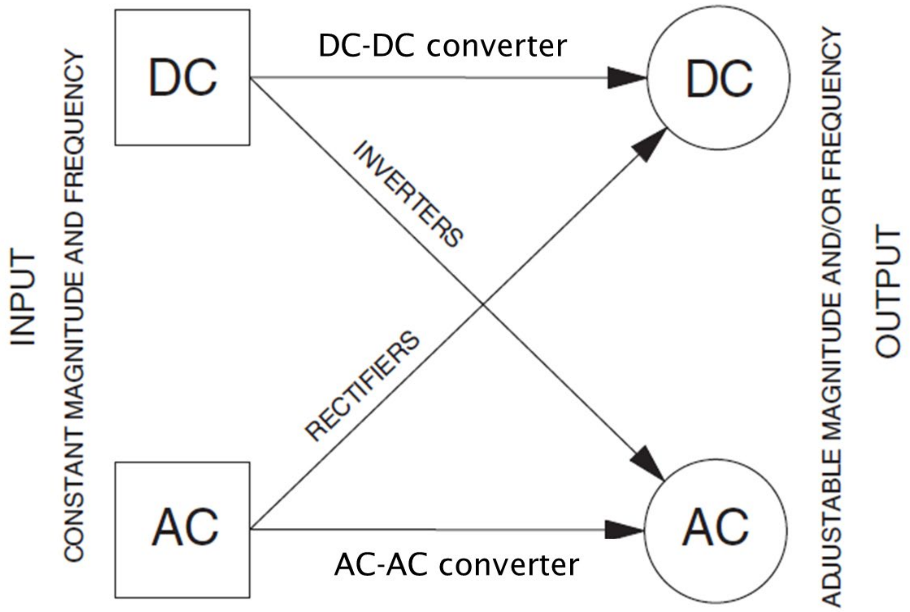
Classification

- Phase-Controlled Thyristor Converter **AC-AC, Voltage**
- Rectifier **AC-DC**
- Cycloconverter **AC-AC, Frequency**
- Inverter **DC-AC**
- DC-to-DC Converter **DC-DC**



Outline

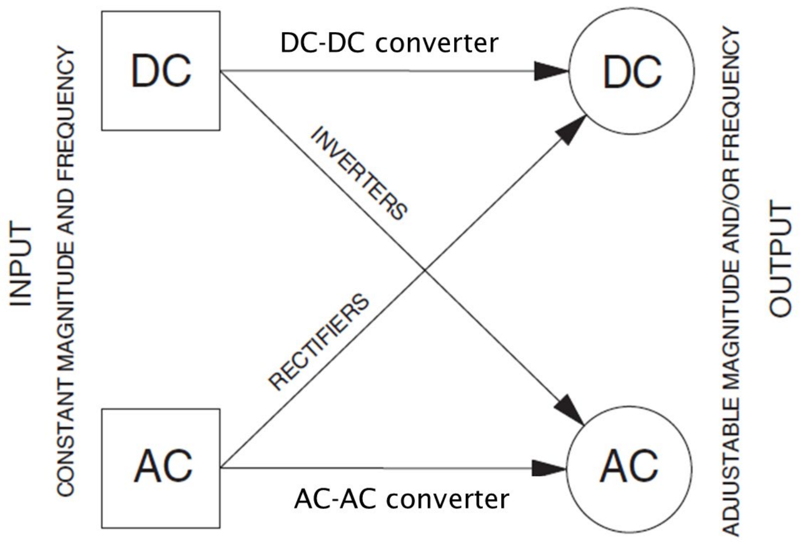
Switching converter

Step-down (Buck) converter

Step-up (Boost) converter

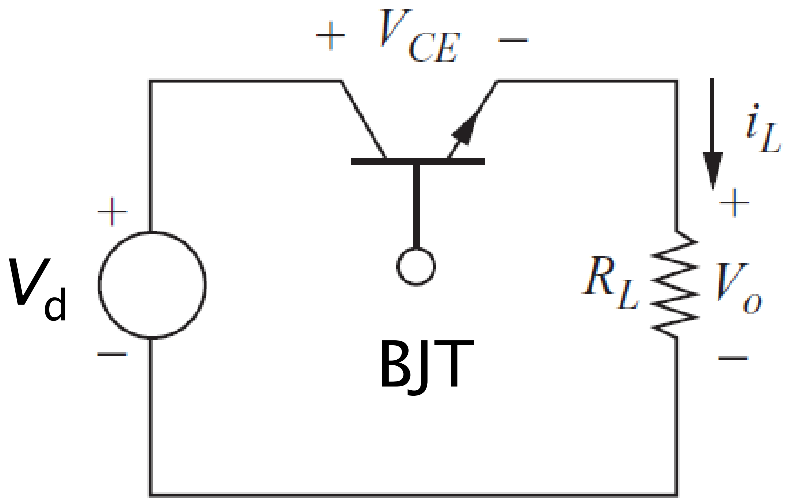
What is DC-DC converter?

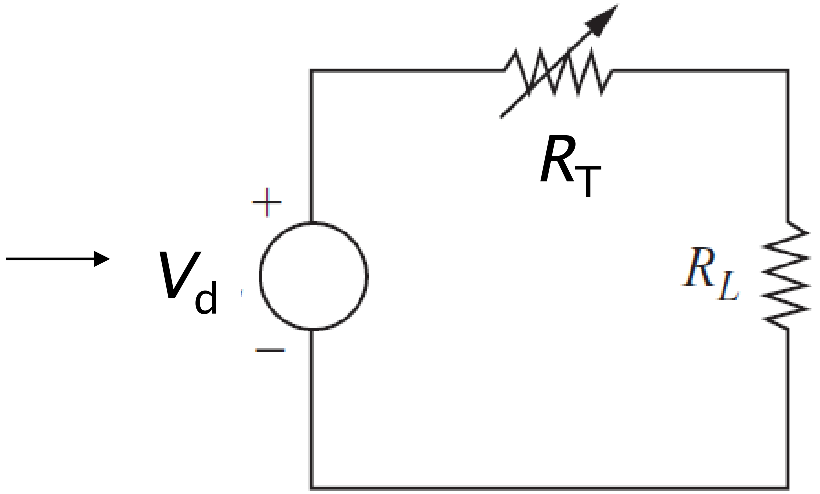
DC-DC converters are power electronic circuits that convert a dc voltage to a different DC voltage level, often providing a regulated output.



- Linear voltage regulator
- Switching converter (DC chopper)

Linear voltage regulator





$V_o = R_L i_L$

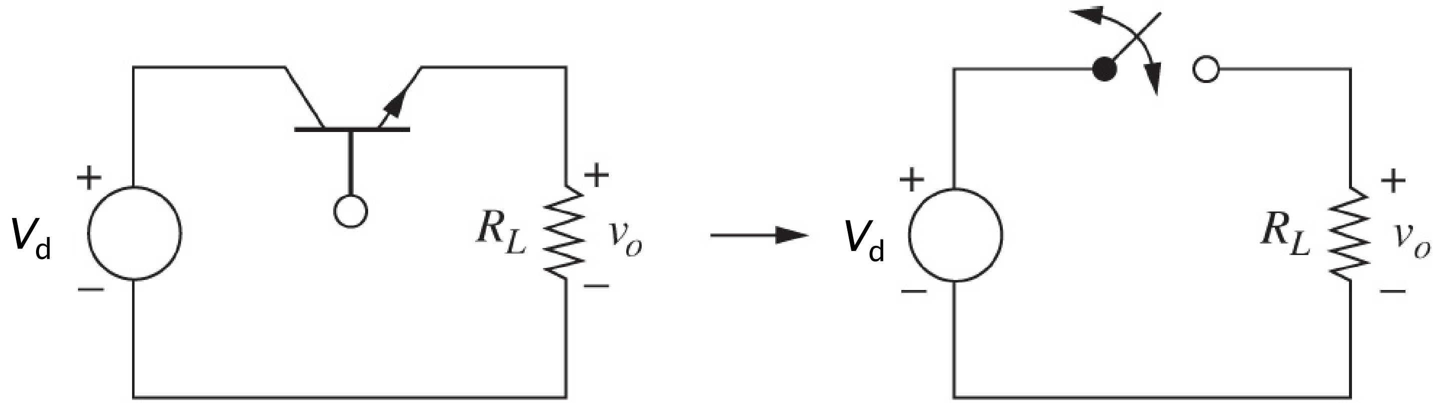
eg. $V_d = 9$ V, $R_L = 3\ \Omega$, $R_T = 6\ \Omega$

$i_L = 1$ A, $V_o = 3$ V

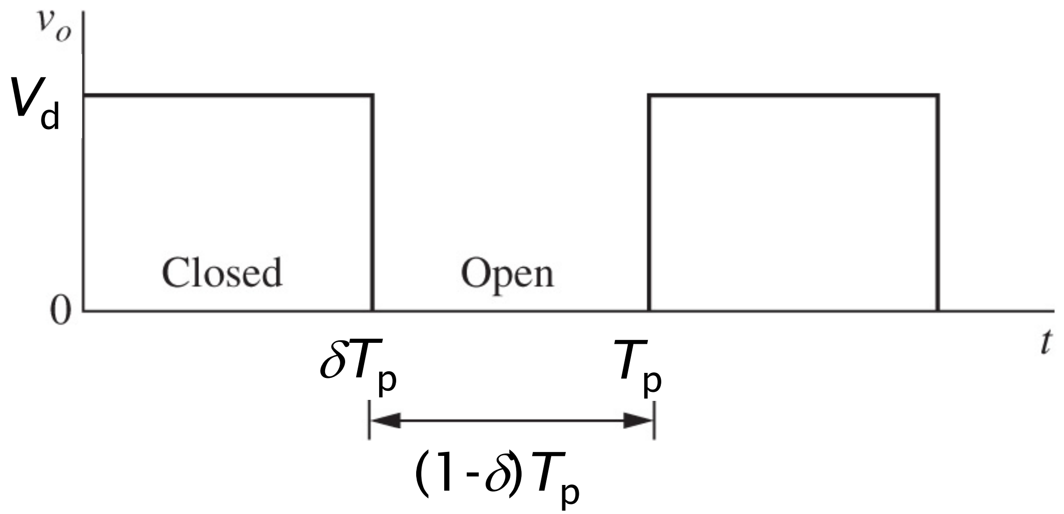
Voltage is converted from 9 V (Source) to 3 V (Output).

What is problem? → Power efficiency 33.333 %

Switching Converter (DC Chopper)

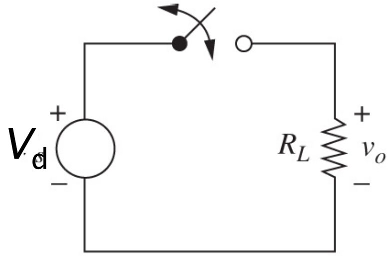


- When the switch is open, there is no current in it; when the switch is closed, there is no voltage across it.

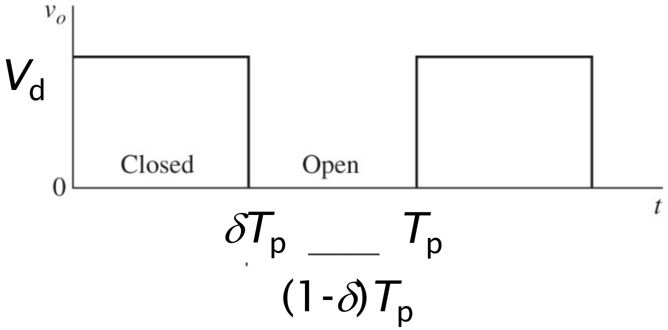


- Duty ratio δ is the fraction of the switching period that the switch is closed.
- The average or dc component of the output voltage is $V_o = \delta V_d$.

Switching Converter (DC Chopper)



- Transistor operates as an electronic switch by being completely **on** or completely **off** (saturation or cutoff for a BJT or the triode and cutoff regions of a MOSFET).



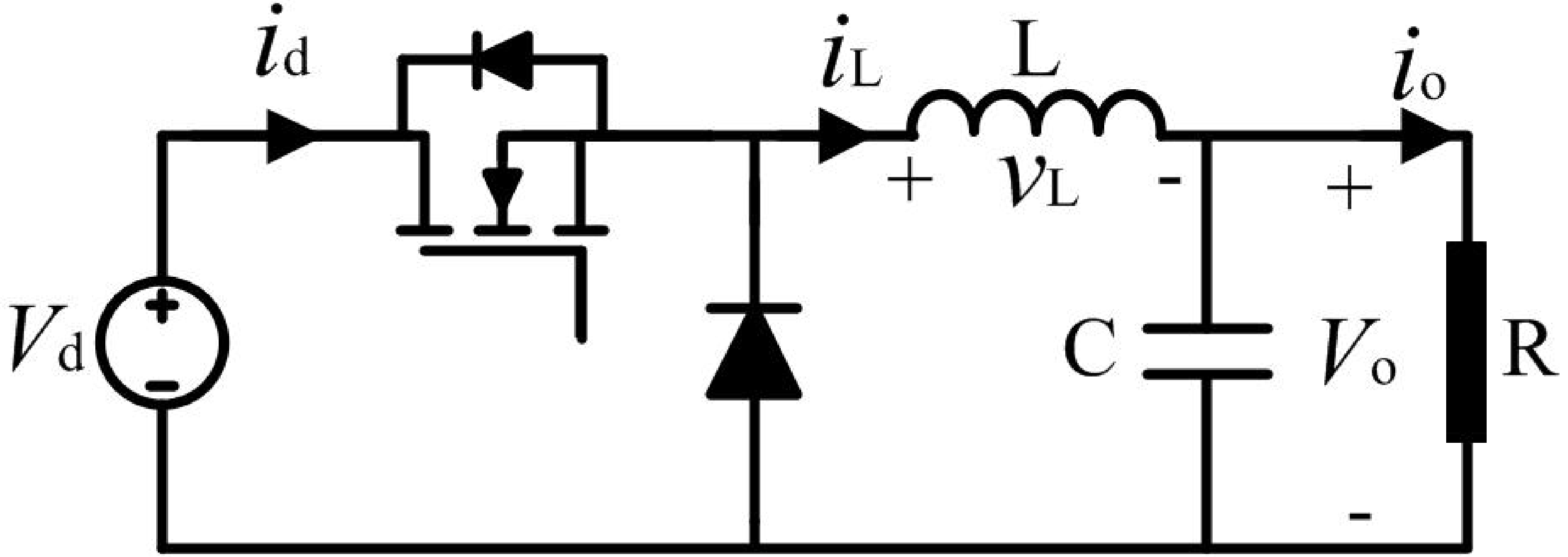
- The power absorbed by the ideal switch is zero.

- Therefore, all power is absorbed by the load, and the energy efficiency is 100%.

- Losses will occur in a real switch because the voltage across it will not be zero when it is on (conduction loss).

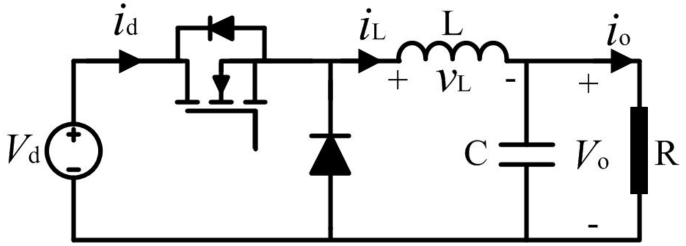
- The switch must pass through the linear region when making a transition from one state to the other (switching loss).

Step down (Buck) Converter

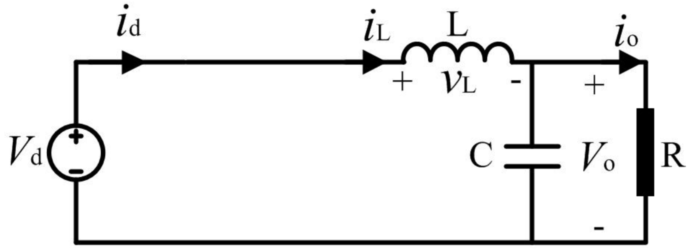


- Three states in operation
- Three modes of operation

Three states in operation



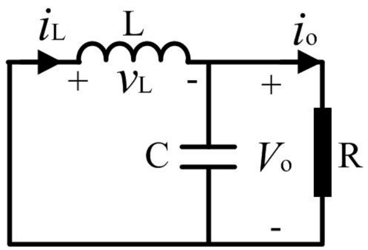
State 1



- MOSFET ON
- Diode reverse biased
- Inductor stores energy

$$v_L = V_d - V_o$$

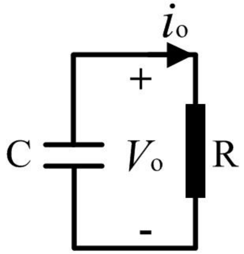
State 2



- MOSFET OFF
- Inductor current freewheels through diode

$$v_L = -V_o$$

State 3



- MOSFET OFF
- Stored energy (inductor) fully discharged

$$i_L = 0$$

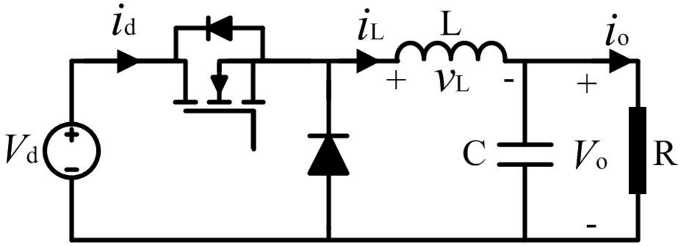
Three operation modes

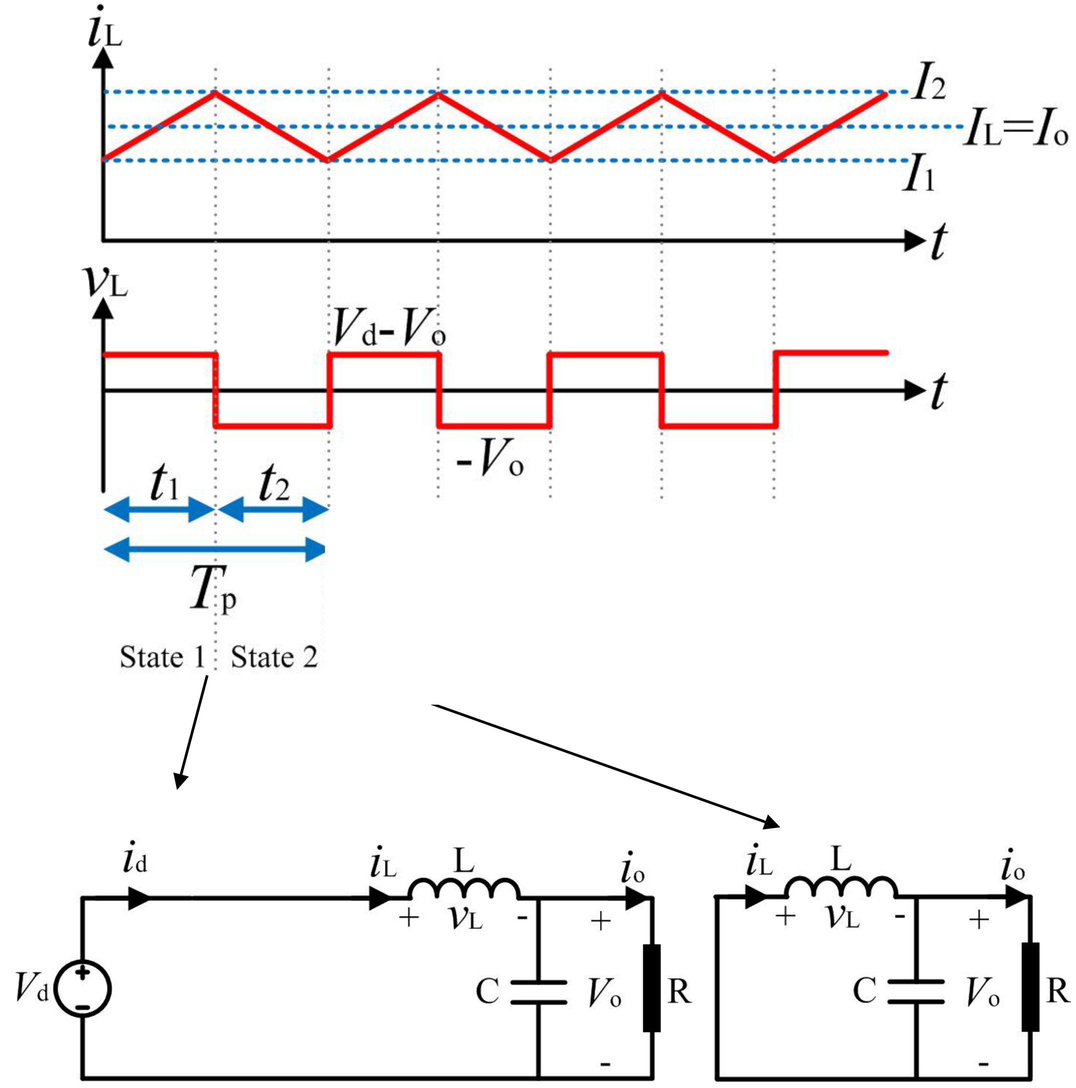
1. Continuous current operation — State 1 & 2

2. Boundary between continuous and discontinuous current operation — State 1 & 2

3. Discontinuous current operation — State 1, 2 & 3

Continuous current operation





δ: Duty ratio $\quad$ T_p: Period

$t_1 = \delta T_p$ $\quad$ On time

$t_2 = (1-\delta) T_p$ $\quad$ Off time

Inductor voltage v_L

V_d - V_o $\quad$ State 1 (On state)

-V_o $\quad$ State 2 (Off state)

Inductor current i_L

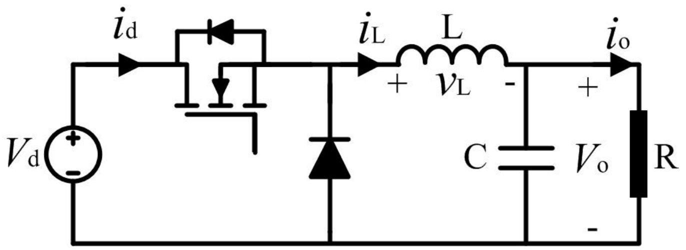
I_1: Maximum current

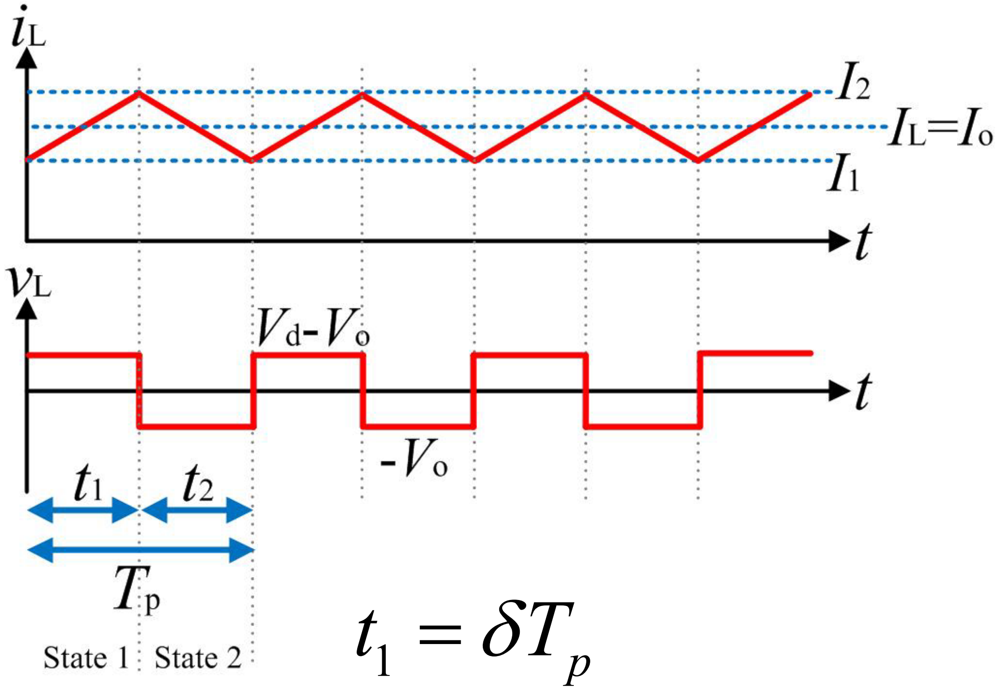
I_2: Minimum current

I_L: Average inductor current

$I_L = I_o$

Continuous current operation





$$t_1 = \delta T_p$$

$$t_2 = (1-\delta)T_p$$

$$I_L = I_o = \frac{I_1 + I_2}{2}$$

At periodic steady-state, the average inductor voltage is zero.

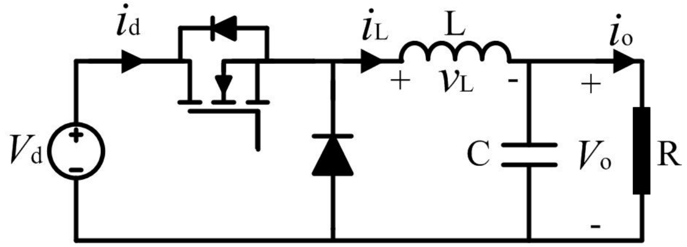
$$V_L = \frac{1}{T_p}\int_0^{T_p} v_L dt = 0$$

$$\frac{1}{T_p}\left[(V_d - V_o)t_1 + (-V_o)t_2\right] = 0$$

$$\frac{1}{T_p}\left[(V_d - V_o)\delta T_p + (-V_o)(1-\delta)T_p\right] = 0$$

$$V_o = \delta V_d$$

Continuous current operation



Example question

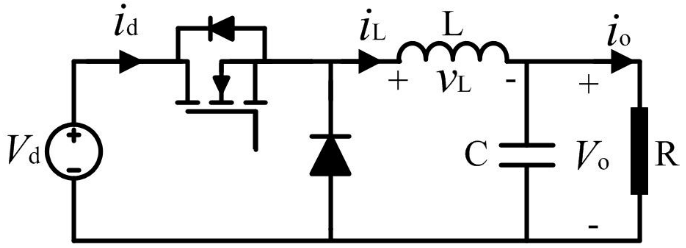
The buck dc-dc converter has the following parameters:

$$V_d = 50\ \text{V},\ \ \delta = 0.4,\ L = 400\ \mu\text{H}$$
$$C = 100\ \mu\text{F},\ f = 20\ \text{kHz},\ R = 20\ \Omega$$

Assuming ideal components, calculate

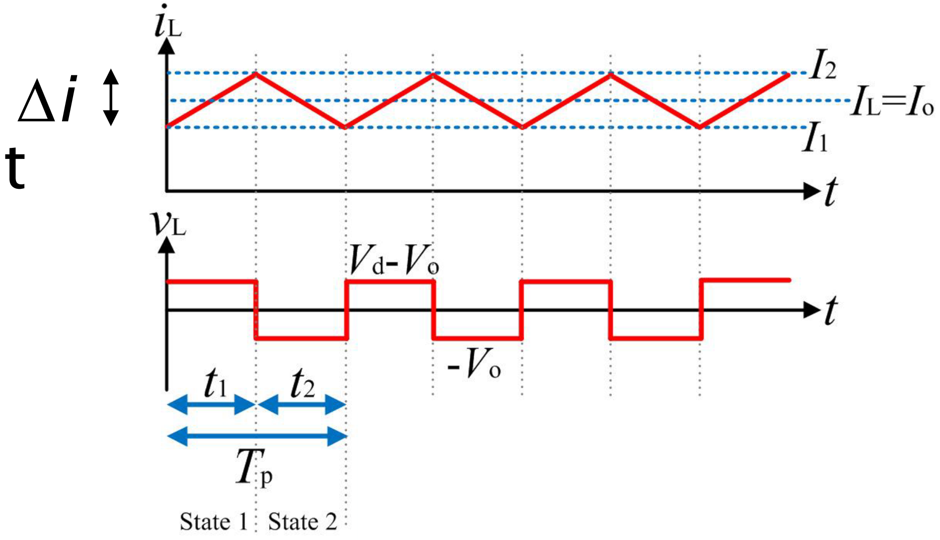
(a) the output voltage V_o
(b) the maximum and minimum inductor current.

Answer



(a) The output voltage V_o

$$V_o = \delta V_d = 50 \times 0.4 = 20 \text{ V}$$



(b) The maximum and minimum inductor current

$$I_o = V_o/R = 20/20 = 1 \text{ A}$$

$$v_L = L\frac{di_L}{dt}$$

$$\Delta i = (V_d\text{-}V_o)t_1/L = (V_d\text{-}V_o)\delta T_p/L$$

$$= (V_d\text{-}V_o)\delta/fL$$

$$= (30 \times 0.4)/(20 \times 10^3 \times 400 \times 10^{-6}) = 1.5 \text{ A}$$

$$\Delta i/2 = 0.75 \text{ A}$$

$$I_2 = I_o + \Delta i/2 = 1.75 \text{ A} \qquad I_1 = I_o - \Delta i/2\ 0.25 \text{ A}$$

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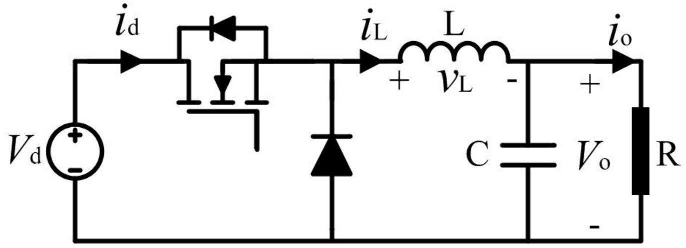
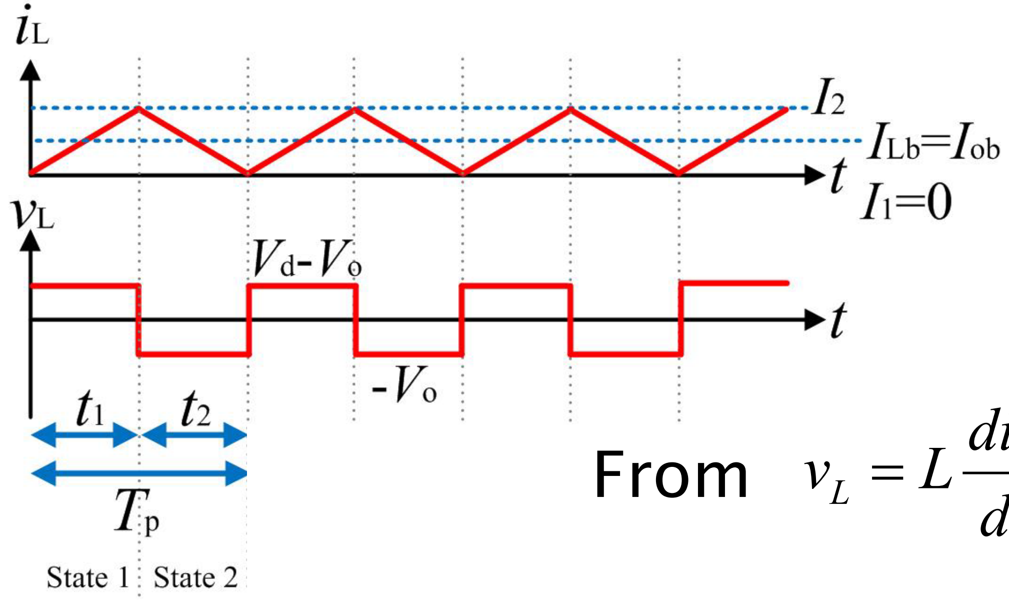
Boundary condition





The minimum inductor current $I_1 = 0$

The average inductor current at boundary $I_{Lb} = \frac{I_2}{2}$

From $v_L = L\frac{di_L}{dt}$ at State 1 (On state), $V_d - V_o = L\frac{I_2}{\delta T_p}$

$$t_1 = \delta T_p$$

$$t_2 = (1-\delta)T_p$$

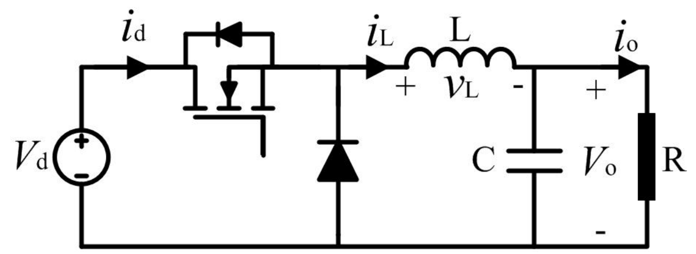
$$V_o = \delta V_d$$

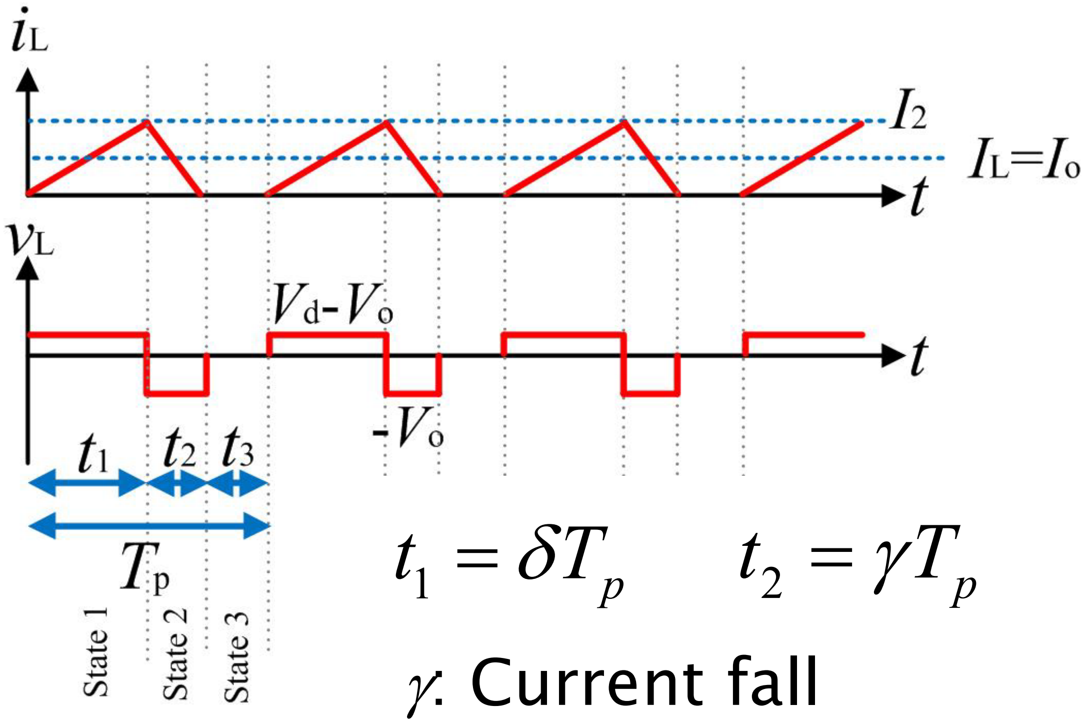
The maximum current $I_2 = \frac{(V_d - V_o)\delta T_p}{L}$

$$I_{Lb} = \frac{(V_d - V_o)\delta T_p}{2L} = \frac{(V_d - \delta V_d)\delta T_p}{2L} = \frac{(1-\delta)\delta T_p V_d}{2L}$$

$$I_{ob} = I_{Lb} = \frac{(1-\delta)\delta T_p V_d}{2L}$$

Discontinuous current operation





$$t_1 = \delta T_p \qquad t_2 = \gamma T_p$$

γ: Current fall time ratio

t_3: Current off time

$$i_L = 0$$

$$V_L = \frac{1}{T_p}\int_0^{T_p} v_L dt = 0$$

$$\frac{1}{T_p}\left[\left(V_d - V_o\right)t_1 + \left(-V_o\right)t_2\right] = 0$$

$$\frac{1}{T_p}\left[\left(V_d - V_o\right)\delta T_p + \left(-V_o\right)\gamma T_p\right] = 0$$

$$V_o = \frac{\delta}{\delta + \gamma} V_d$$

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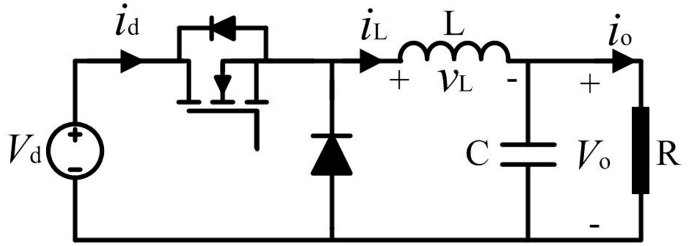
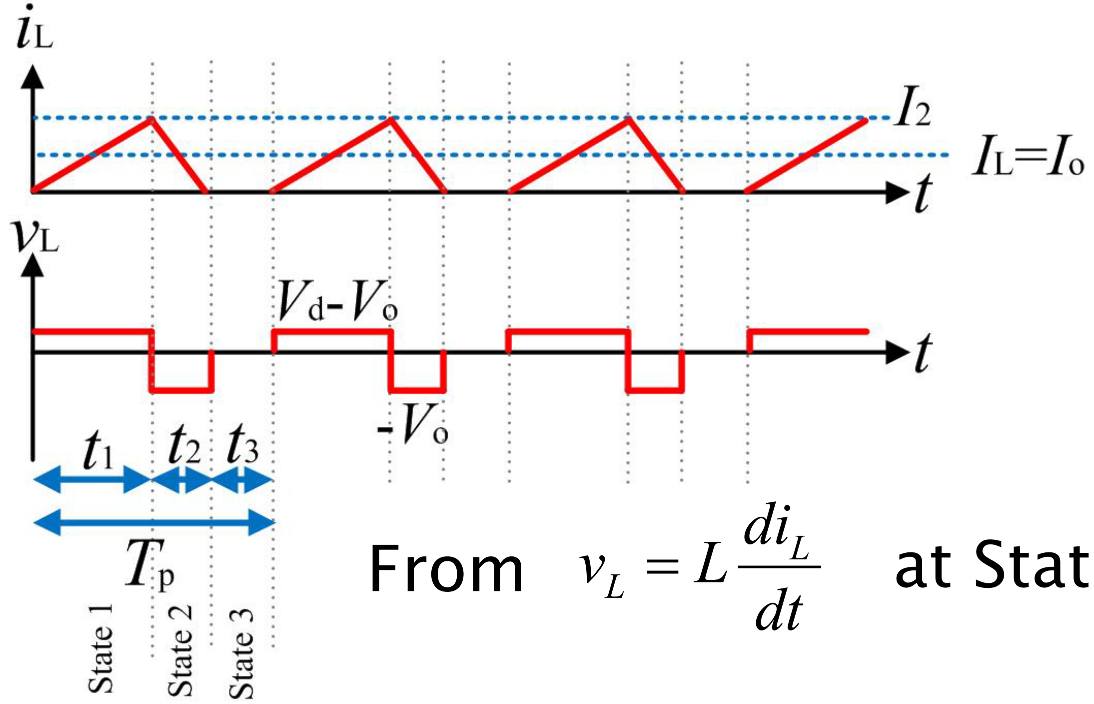
Discontinuous current operation





From $v_L = L\dfrac{di_L}{dt}$ at State 1 (On state),

$$V_d - V_o = L\frac{I_2}{\delta T_p} \qquad I_2 = \frac{(V_d - V_o)\delta T_p}{L}$$

From $v_L = L\dfrac{di_L}{dt}$ at State 2 (Off state), $-V_o = -L\dfrac{I_2}{\gamma T_p}$ $\quad I_2 = \dfrac{\gamma T_p V_o}{L}$

$$t_1 = \delta T_p$$

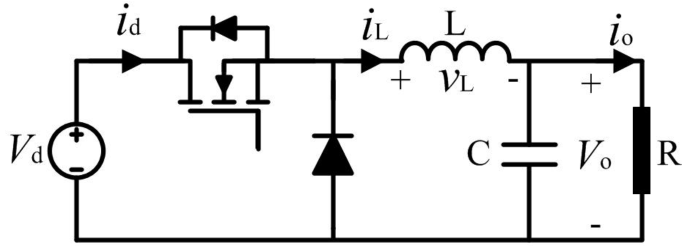
By equalising I_2, $\dfrac{(V_d - V_o)\delta T_p}{L} = \dfrac{\gamma T_p V_o}{L}$

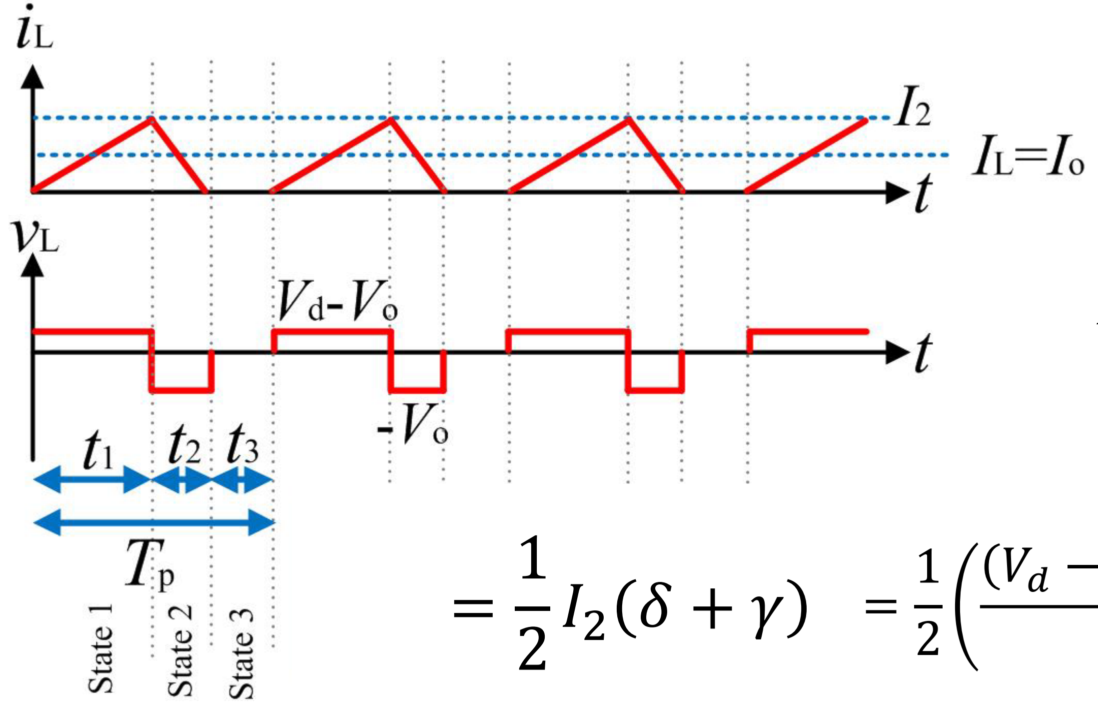
$$t_2 = \gamma T_p$$

Current fall time ratio $\quad \gamma = \delta\dfrac{(V_d - V_o)}{V_o}$

$$V_o = \frac{\delta}{\delta + \gamma}V_d$$

Discontinuous current operation





$$t_1 = \delta T_p$$

$$t_2 = \gamma T_p$$

$$V_o = \frac{\delta}{\delta + \gamma} V_d$$

Average inductor current

$$I_L = \frac{1}{T_p}\int_0^{T_p} i_L dt \quad = \frac{1}{T_p}\left[\frac{1}{2}\delta T_p I_2 + \frac{1}{2}\gamma T_p I_2\right]$$

$$= \frac{1}{2} I_2(\delta + \gamma) \quad = \frac{1}{2}\left(\frac{(V_d - V_o)\delta T_p}{L}\right)\left(\delta + \delta\frac{(V_d - V_o)}{V_o}\right) \quad = \frac{\delta^2 T_p V_d (V_d - V_o)}{2LV_o}$$

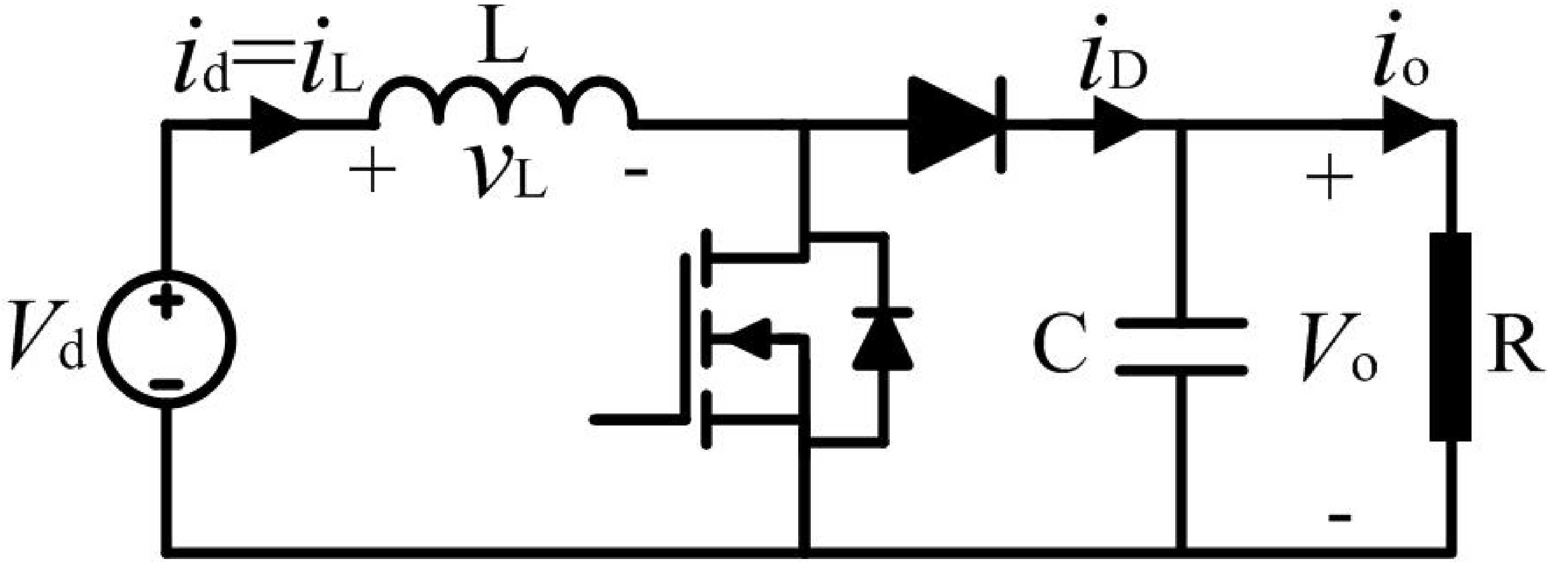
Output current

$$I_o = I_L = \frac{\delta^2 T_p V_d (V_d - V_o)}{2LV_o}$$

Step-down (Buck) converter

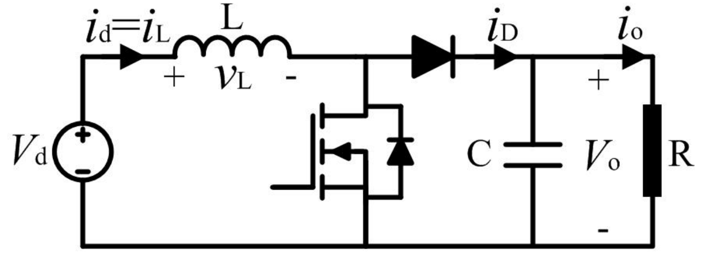
Continuous current operation	Boundary Condition	Discontinuous current operation
$V_o = \delta V_d$	$V_o = \delta V_d$	$V_o = \dfrac{\delta}{\delta + \gamma} V_d$
$I_L = I_o = \dfrac{I_1 + I_2}{2}$	$I_{Lb} = \dfrac{(1-\delta)\delta T_p V_d}{2L}$ $I_{Lb} = I_{Lo}$	$I_L = \dfrac{\delta^2 T_p V_d (V_d - V_o)}{2LV_o}$ $I_L = I_o$

Step-up (Boost) converter

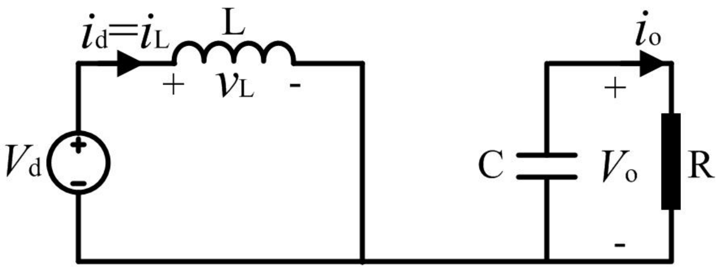


- Three states in operation
- Three modes of operation

Three states in operation

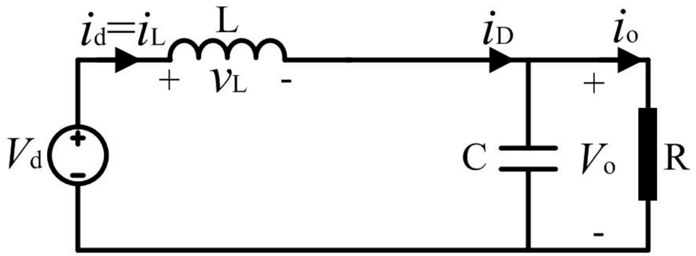


State 1



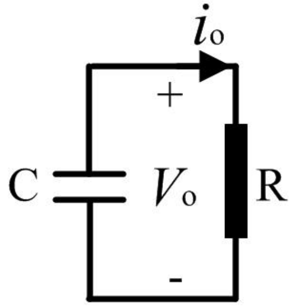
- MOSFET ON
- Diode reverse biased
- Inductor stores energy

State 2



- MOSFET OFF
- Inductor discharges energy through diode

State 3



$i_L = 0$

- MOSFET OFF
- Stored energy (inductor) fully discharged
- Diode reverse biased

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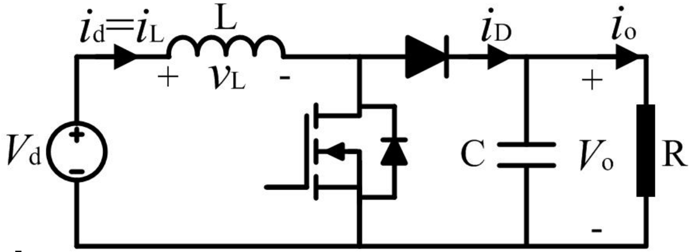
Three operation modes

1. Continuous current operation — State 1 & 2

2. Boundary between continuous and discontinuous current operation — State 1 & 2

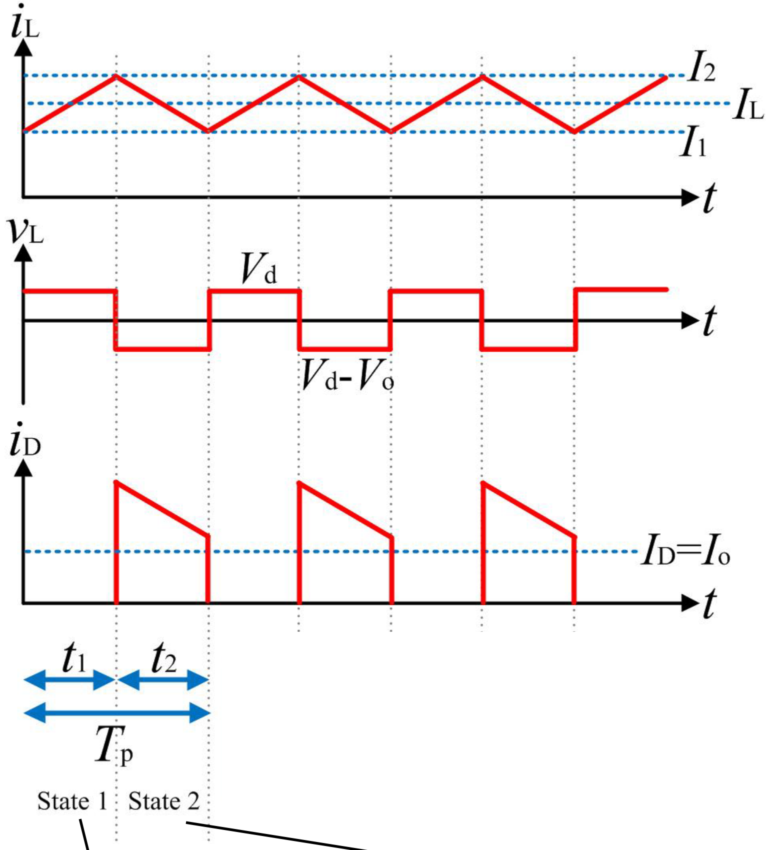
3. Discontinuous current operation — State 1, 2 & 3

Continuous current condition



δ: Duty ratio T_p: Period

$t_1 = \delta T_p$ On time $t_2 = (1-\delta)T_p$ Off time



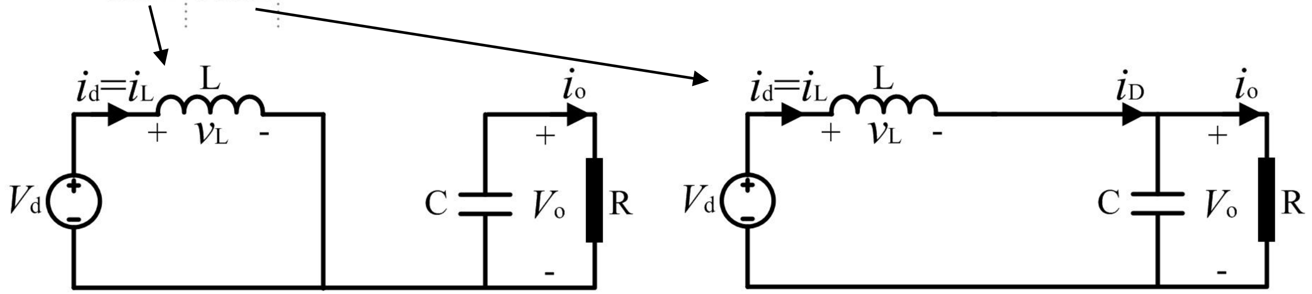
Inductor voltage v_L

V_d State 1 (On state)

V_d - V_o State 2 (Off state)

Inductor current i_L I_L: Average inductor current

I_1: Maximum current I_2: Minimum current

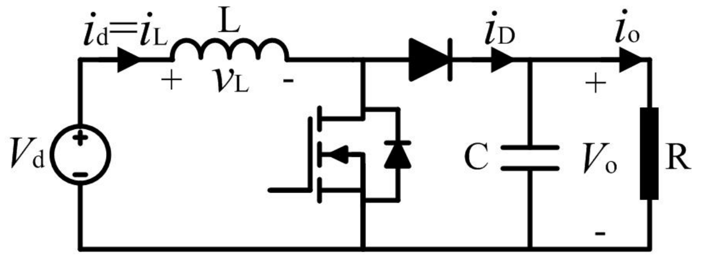


Diode current i_d

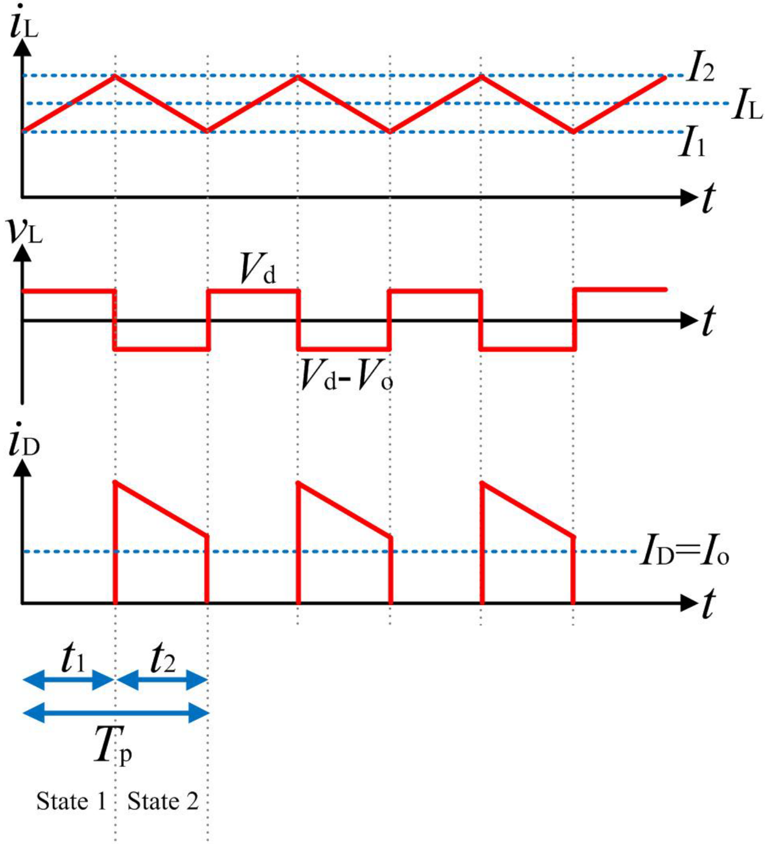
I_D: Average diode current

$I_o = I_D$

Continuous current condition



At periodic steady-state, the average inductor voltage is zero.



$$V_L = \frac{1}{T_p}\int_0^{T_p} v_L\,dt = 0$$

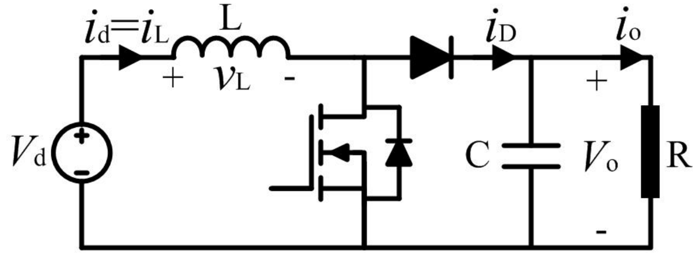
$$\frac{1}{T_p}\left[V_d t_1 + (V_d - V_o)t_2\right] = 0$$

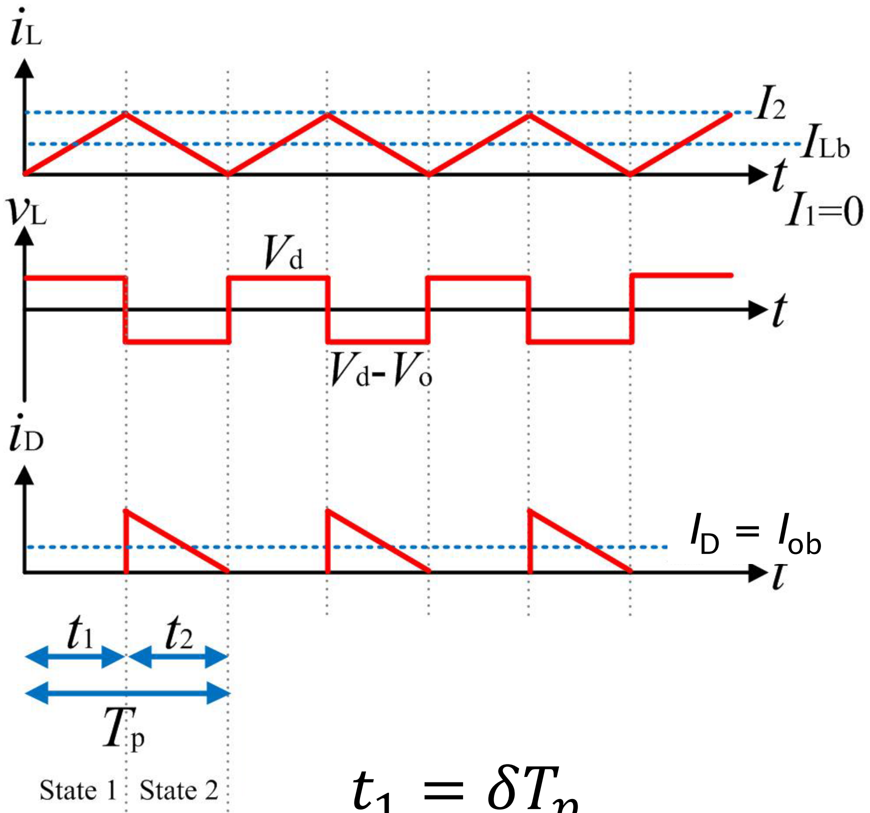
$$\frac{1}{T_p}\left[V_d \delta T_p + (V_d - V_o)(1-\delta)T_p\right] = 0$$

$$t_1 = \delta T_p$$
$$t_2 = (1-\delta)T_p$$

$$V_o = \frac{V_d}{1-\delta} \qquad I_L = \frac{I_1 + I_2}{2}$$

Boundary condition





From $v_L = L\dfrac{di_L}{dt}$ at State 1 (On state),

$$V_d = L\frac{I_2}{t_1} \qquad I_2 = \frac{V_d \delta T_p}{L}$$

$$I_{Lb} = \frac{I_1+I_2}{2} = \frac{I_2}{2} = \frac{V_d \delta T_p}{2L} = \frac{(1-\delta)V_o\, \delta T_p}{2L}$$

$$t_1 = \delta T_p$$
$$t_2 = (1-\delta)T_p$$
$$V_d = (1-\delta)V_o$$

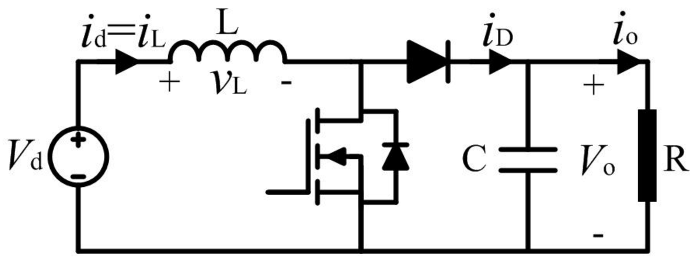
Assume efficiency = 100%

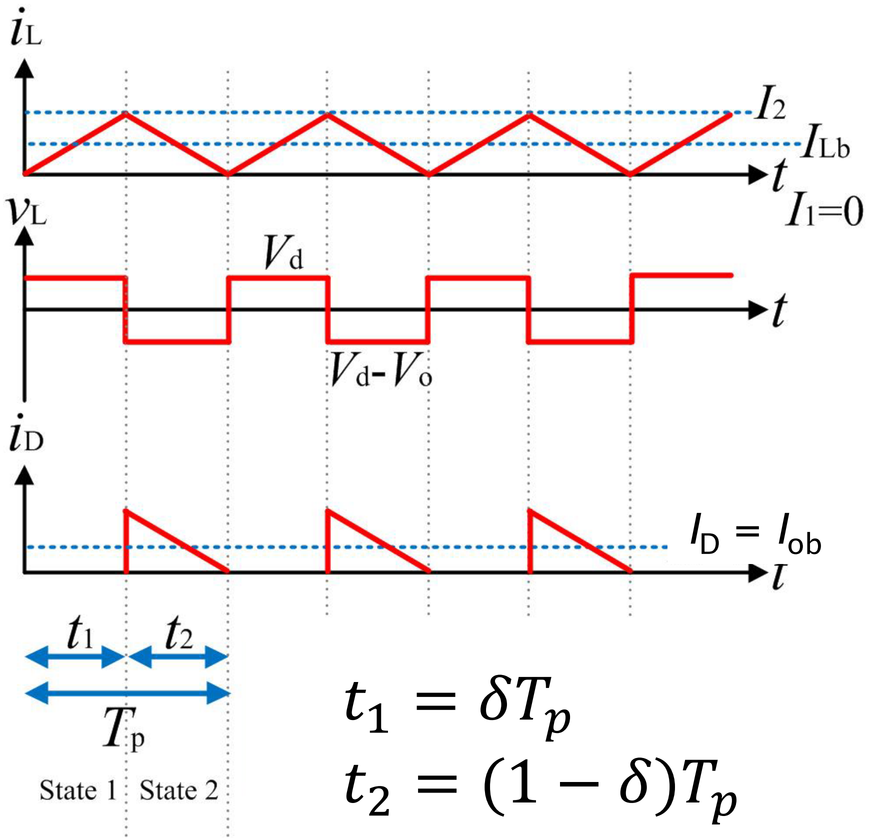
$$P_{in} = P_{out} \qquad V_d I_d = V_o I_{ob}, \quad I_d = I_{Lb}$$

$$I_{ob} = \frac{V_d I_{Lb}}{V_o} = (1-\delta)I_{Lb} = \frac{(1-\delta)^2 V_o\, \delta T_p}{2L}$$

Output current at the boundary condition

Boundary condition





$$t_1 = \delta T_p$$
$$t_2 = (1-\delta)T_p$$
$$V_d = (1-\delta)V_o$$

$$I_2 = \frac{V_d \delta T_p}{L},$$

Alternative derivation for I_{ob}

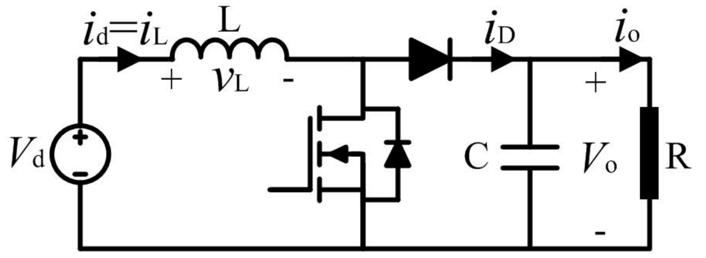
At periodic steady-state, the average capacitor current is zero. Hence, average diode current equal to average load current.

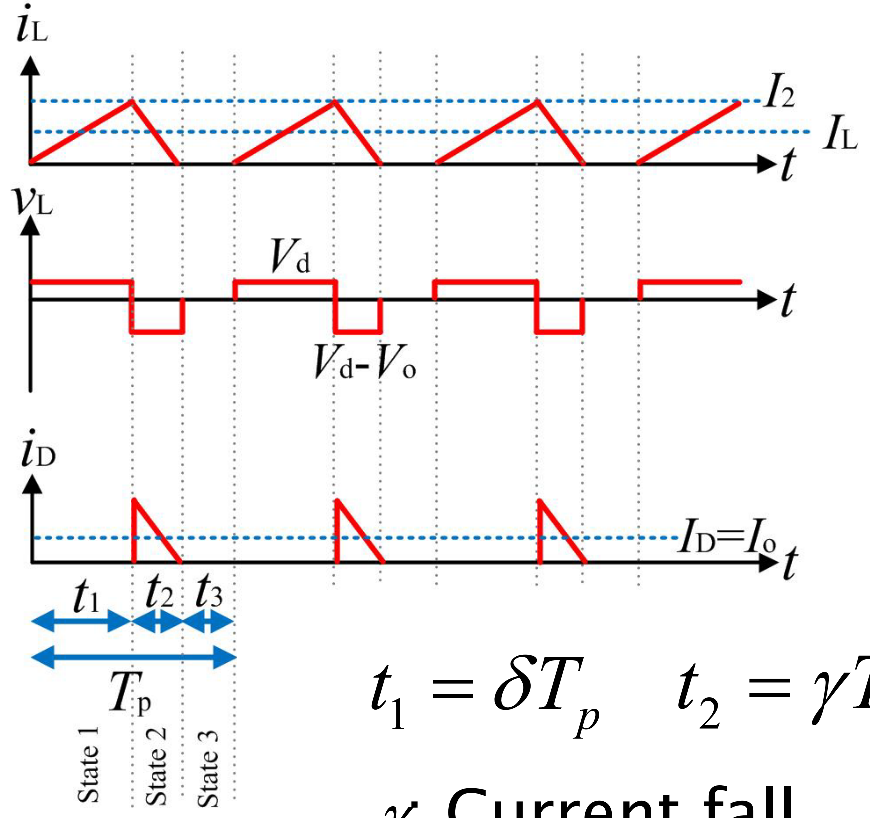
$$I_{ob} = I_D = \frac{1}{T_p}\int_0^{T_p} i_D dt = \frac{1}{T_p}\left[\frac{1}{2}(1-\delta)T_p I_2\right]$$

$$= \frac{I_2(1-\delta)}{2}$$

$$= \frac{(1-\delta)^2 V_o\, \delta T_p}{2L}$$

Discontinuous current condition





$$t_1 = \delta T_p \quad t_2 = \gamma T_p$$

γ: Current fall time ratio

$i_L = 0$

t_3: Current off time

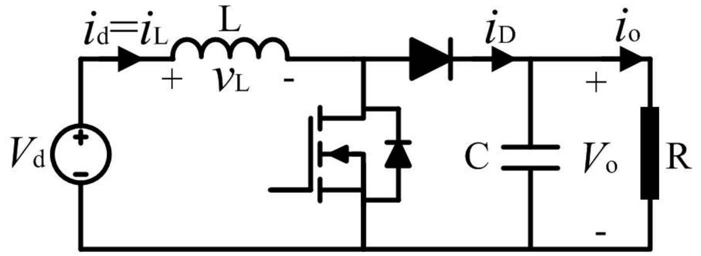
$$V_L = \frac{1}{T_p}\int_0^{T_p} v_L dt = 0$$

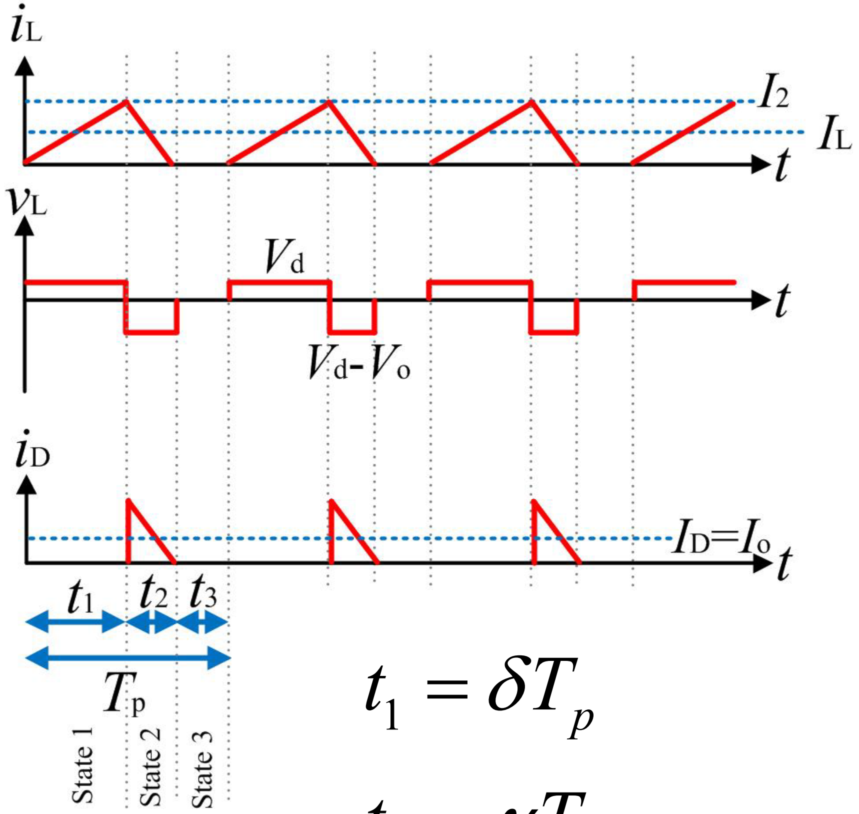
$$\frac{1}{T_p}\left[V_d \delta T_p + (V_d - V_o)\gamma T_p\right] = 0$$

$$V_d\delta + V_d\gamma - V_o\gamma = 0$$

$$\frac{V_o}{V_d} = \frac{\delta + \gamma}{\gamma}$$

Discontinuous current condition





$$i_L = 0$$

$$t_1 = \delta T_p$$

$$t_2 = \gamma T_p$$

$$\frac{V_o}{V_d} = \frac{\delta + \gamma}{\gamma}$$

From $v_L = L\dfrac{di_L}{dt}$ at State 1 (On state),

$$V_d = L\frac{I_2}{\delta T_p} \qquad I_2 = \frac{\delta T_p V_d}{L}$$

From $v_L = L\dfrac{di_L}{dt}$ at State 2 (Off state),

$$V_d - V_o = -L\frac{I_2}{\gamma T_p} \qquad I_2 = \frac{-\gamma T_p (V_d - V_o)}{L}$$

By equalising I_2, $\dfrac{\delta T_p V_d}{L} = \dfrac{-\gamma T_p (V_d - V_o)}{L}$

Current fall time ratio $\gamma = \dfrac{\delta V_d}{V_o - V_d}$

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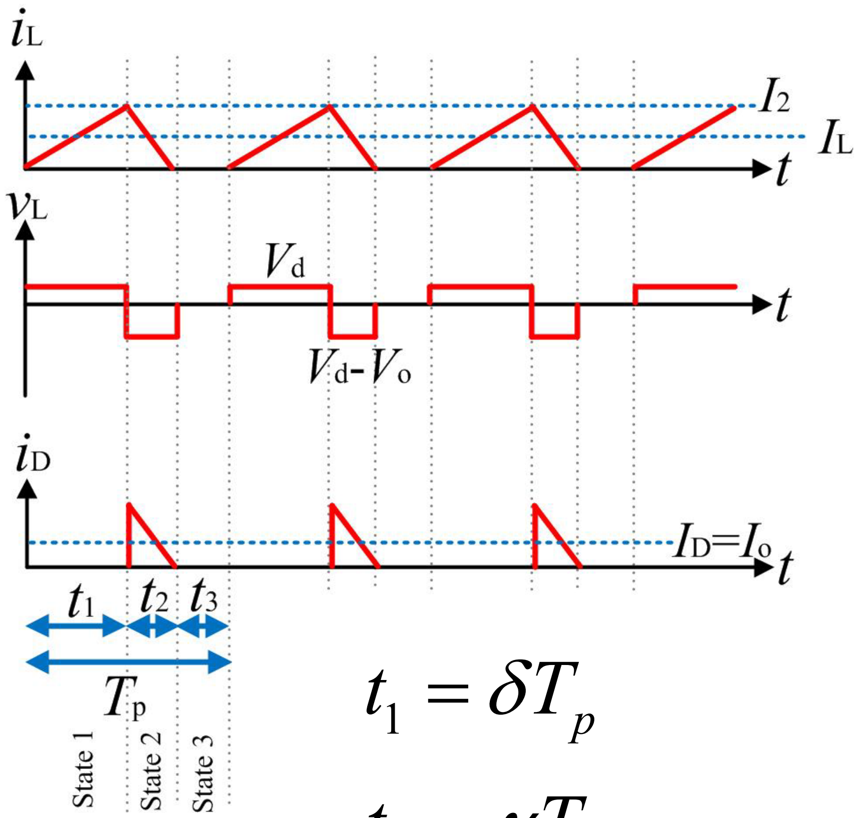
Discontinuous current condition



$$t_1 = \delta T_p$$

$$t_2 = \gamma T_p$$

$$I_2 = \frac{\delta T_p V_d}{L} \qquad \gamma = \frac{\delta V_d}{V_o - V_d}$$

Average inductor current

$$I_L = \frac{1}{T_p}\int_0^{T_p} i_L dt = \frac{1}{T_p}\left[\frac{1}{2}\delta T_p I_2 + \frac{1}{2}\gamma T_p I_2\right] = \frac{1}{2}(\delta + \gamma)\, I_2$$

$$= \frac{1}{2}(\delta + \gamma)\frac{\delta T_p V_d}{L} = \frac{1}{2}(\delta + \frac{\delta V_d}{V_o - V_d})\frac{\delta T_p V_d}{L}$$

$$= \frac{\delta^2 T_p V_d V_o}{2L(V_o - V_d)}$$

Duty ratio $\delta = \sqrt{\dfrac{2L(V_o - V_d)I_L}{T_p V_d V_o}}$

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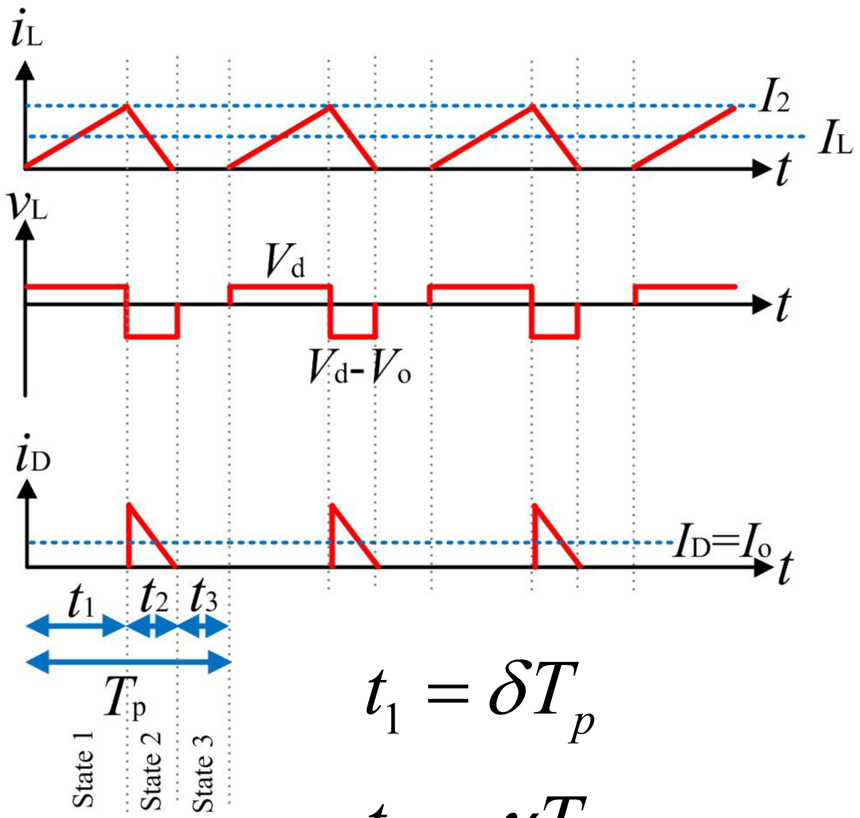
Discontinuous current condition



$$t_1 = \delta T_p$$

$$t_2 = \gamma T_p$$

$$I_2 = \frac{\delta T_p V_d}{L} \quad \gamma = \frac{\delta V_d}{V_o - V_d}$$

At steady-state, average capacitor current is zero. Hence, average diode current equal to average load current.

$$I_o = I_D = \frac{1}{T_p}\int_0^{T_p} i_D dt = \frac{1}{T_p}\left[\frac{1}{2}\gamma T_p I_2\right] = \frac{1}{2}\gamma I_2,$$

$$= \frac{1}{2} I_2 \frac{\delta V_d}{V_o - V_d} = \frac{1}{2}\left(\frac{\delta V_d}{V_o - V_d}\right)\left(\frac{\delta T_p V_d}{L}\right)$$

$$= \frac{\delta^2 T_p V_d^2}{2L(V_o - V_d)}$$

Output current

Summary – Step-up (Boost) converter

Continuous current operation	Boundary Condition	Discontinuous current operation
$V_o = \dfrac{V_d}{1-\delta}$	$V_o = \dfrac{V_d}{1-\delta}$	$\dfrac{V_o}{V_d} = \dfrac{\delta+\gamma}{\gamma}$
$I_L = \dfrac{I_1 + I_2}{2}$	$I_{Lb} = \dfrac{(1-\delta)V_o\ \delta\, T_p}{2L}$ $I_{ob} = \dfrac{(1-\delta)^2 V_o\ \delta\, T_p}{2L}$	$I_L = \dfrac{\delta^2 T_p V_d V_o}{2L(V_o - V_d)}$ $I_o = \dfrac{\delta^2 T_p V_d^2}{2L(V_o - V_d)}$

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Summary – DC-DC Converter

- DC-DC converters are power electronic circuits that convert a dc voltage to a different dc voltage level, often providing a regulated output.

- Step-down (Buck) converter and Step-up (Boost) converter have been analysed.