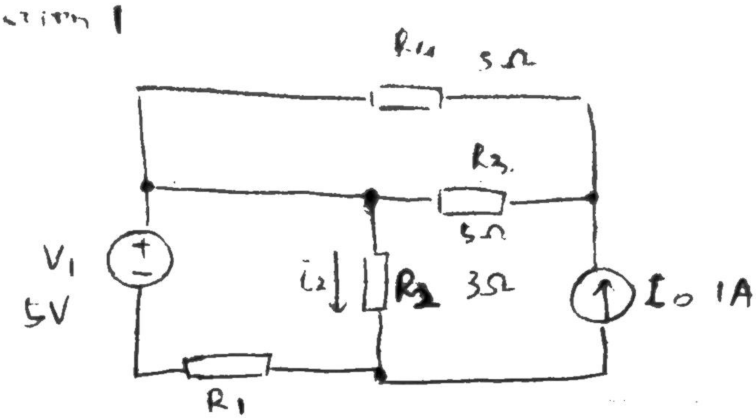
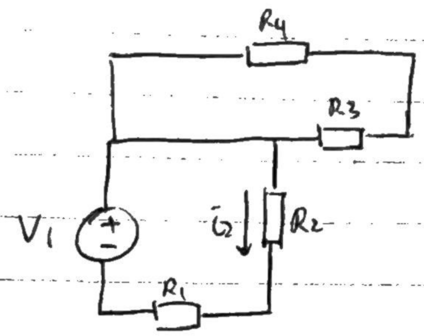
2021 2022 Transmission

Question 1

(a)

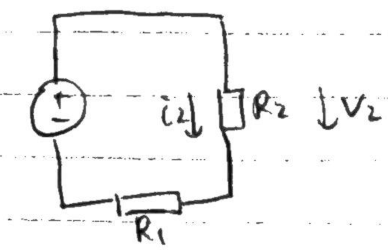


(i) only voltage source, current source ~~shorted~~. ⇒ OPEN.

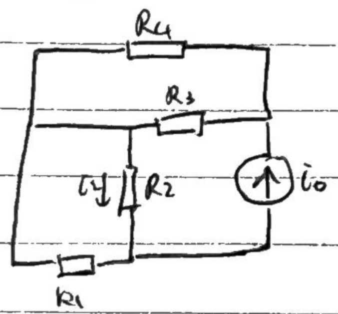


⇒

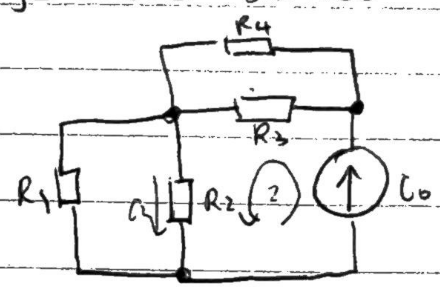
R4, R3 shorted.



(ii) only current source, voltage source shorted



equivalent to ⇒



(iii) from (i)

$$V_{2,1} = V_1 \frac{R_2}{R_2+R_1} \quad \text{(voltage divider)}$$

$$V_{2,1} = \frac{3}{5}\times V_1 = \frac{3}{5}\times 5 = 3\,V$$

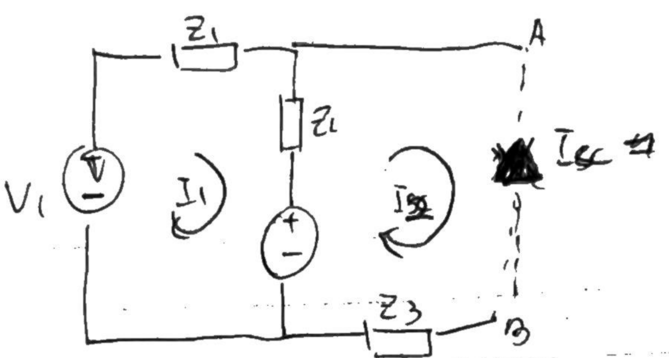
$$I_{2,1} = \frac{3V}{3\Omega} = 1\,A$$

from (ii) current across = 1 A

current $i_{2,2} = \frac{R_1}{R_2+R_1} i_o = \frac{2}{5}\times 1 = 0.4\,A$

$$i_{total} = i_{2,1} + i_{2,2} = 1.4\,A$$ //

(b)



Norton theorem

→ AB shorted.

$I_{sc} = -I_2$

Mesh 1

$$\cancel{V_1} + I_1 Z_1 + Z_2(I_1 - I_2) - \cancel{3 V_2} = 0$$

$$10 I_1 - 5 I_2 = 0$$

$$2 I_1 = I_2.$$

Mesh 2 $\quad I_2 Z_3 + 3 + (I_2 - I_1) Z_2 = 0$

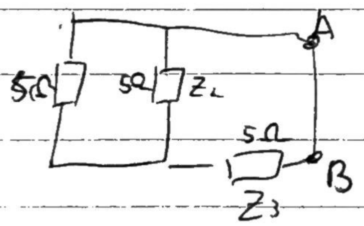
$$5 I_2 + 5 I_2 - 2.5 I_2 = -3$$

$$I_2 = -0.4 \text{ A}$$

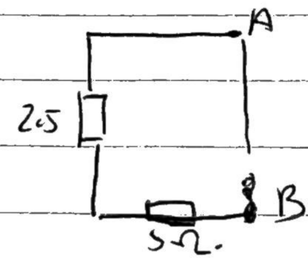
$$I_{sc} = 0.4 \text{ A.}$$

$I_{sc} \Rightarrow I_N$

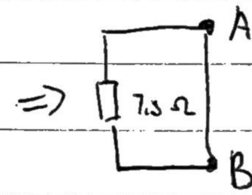
$R_N \Rightarrow$



$\Rightarrow$

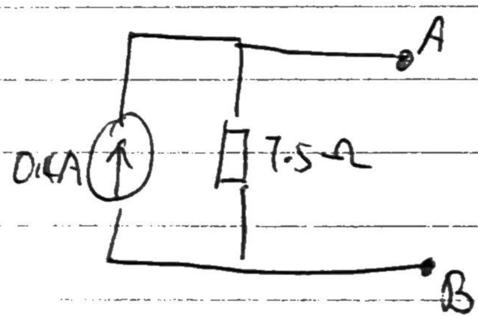


$\Rightarrow$



$R_N = 7.5\ \Omega$

→ Equivalent circuit



(c) $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ $V_1 = AV_2 + BI_2$

$I_1 = CV_2 + DI_2$

(i) $\begin{vmatrix} 1 & 0 \\ Y & 1 \end{vmatrix} \begin{vmatrix} 1 & Z \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ Y & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & Z \\ Y_1 & Y_1 Z + 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ Y_2 & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 + ZY_2 & Z \\ Y_1 + Y_1 Y_2 Z & Y_1 Z + 1 \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} = \begin{bmatrix} 2.5 & 3 \\ 1 & 1.6 \end{bmatrix}$

$1 + ZY_2 = 2.5$ $Z = 3$

$1 + 3Y_2 = 2.5$

$Y_2 = 0.5$

$Y_1 Z + 1 = 1.6$ $Y_1 = 0.2$

(ii) to be symmetrical, A has to $= D$

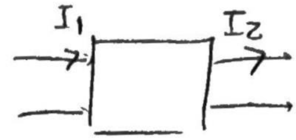
here $A \neq D \Rightarrow$ not symmetrical.

(iii) $AD - BC = 2.5 \times 1.6 - 3 = 1 \Rightarrow$ reciprocal

(iv) $Z_{I1} = \sqrt{\frac{AB}{CD}}$ $Z_{I2} = \sqrt{\frac{DB}{CA}}$

$= 2.16$ $= 3.19$

(d) $Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$



$V_1 = Z_{11} I_1 + Z_{12} I_2$ —① $\quad$ $V_1 = A V_2 + B I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2.$ —② $\quad$ $I_1 = C V_2 + D I_2.$

[Z] to [ABCD]

① ⇒ $V_1 - Z_{12} I_2 = Z_{11} I_1$

$$I_1 = \frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}} I_2. \quad \text{—③}$$

② $V_2 = Z_{21} I_1 + Z_{22} I_2$

$$Z_{21} I_1 = V_2 - Z_{22} I_2$$

$$I_1 = \frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}} I_2. \quad \text{—④}$$

④ = ③

$$\frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}} I_2 = \frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}} I_2.$$

$$\frac{V_1}{Z_{11}} = \frac{V_2}{Z_{21}} + \left(\frac{Z_{12}}{Z_{11}} - \frac{Z_{22}}{Z_{21}}\right) I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 + \frac{(\cancel{Z_{11}})(Z_{12}Z_{21} - Z_{22} * Z_{11})}{\cancel{Z_{11}} Z_{21}} I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 + \frac{Z_{12}Z_{21} - Z_{22}Z_{11}}{Z_{21}} I_2.$$

$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} \xrightarrow{Z} \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{Z_{12}Z_{21} - Z_{22}Z_{11}}{Z_{21}} \\ \frac{1}{Z_{21}} & -\frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

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Question 2.

(a) (i) $Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{4\times10^{-6}}{10^{-8}}} = 20\,\Omega$

$\Gamma_{L1} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{3+4j-20}{3+4j+20} = -0.688 + 0.294j$

$= 0.748\angle 2.74,$

$\Gamma_{L2} = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.366 + 0.293j$

$= 0.469\angle 2.47$

(ii) $Z_{in,L_1} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$

$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{6\times10^6} = 1.047\times10^{-6}$

$l_1 = 50$

$\beta l = 5.23\times1$

$\tan(\beta l) = 5.23\times1$

$= Z_0 \frac{3+4j + j(20)\tan(5.23\times10^{-5})}{20 + j(3+4j)\tan(5.23\times10^{-5})}$

$= 20\times(0.150003 + 0.20005j)$

$= 3+4j = 5\angle 0.927$

$Z_{in}L_2 = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$

$\beta l = \frac{2\pi}{6\times10^6}\times100$

$= 1.047\times10^{-4}$

$\tan\beta l = 1.047\times10^{-4}$

$= 20 \cdot \frac{8+6j + j20(1.047\times10^{-4})}{20 + j(8+6j)(1.04\times10^{-4})}$

$= 20\times 0.4 + 0.3j = 8+6j = 10\angle 0.64$

$Z_{AB} = \frac{1}{\frac{1}{Z_{in}L_1} + \frac{1}{Z_{in}l_2}}$ (parallel)

$= 2.26 + 2.49j = 3.36\angle 0.83$ //

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{3.36\angle 0.83 - 20}{3.36\angle 0.83 + 20} = -0.775 + 0.198$

$= 0.799\angle 2.89$ //

(b)(i) $\quad V^+ \qquad V^-$

$$V(l) = V_0^+ e^{-j\beta^+ l} + V_0^- e^{j\beta^- l}$$

$$I(l) = \frac{V_0^+ e^{-j\beta^+ l}}{Z_0^+} - \frac{V_0^- e^{-j\beta^- l}}{Z_0^-}$$

$$Z_L = \frac{V(l)}{I l} = \frac{V_0^+ e^{-j\beta^+ l} + V_0^- e^{j\beta^- l}}{\frac{V_0^+ e^{-j\beta^+ l}}{Z_0^+} - \frac{V_0^- e^{-j\beta^- l}}{Z_0^-}}$$

Γ at load

$$= \Gamma(0) = \frac{V^-(0)}{V^+(0)}$$

$\cancel{\Gamma = Z_L}$

$\cancel{\Gamma = \frac{V^-(l)}{V^+(l)}}$

$$Z_L = \frac{V_0^+ e^{-j\beta^+ l} + V_0^- e^{j\beta^- l}}{\frac{V_0^+ e^{-j\beta^+ l}}{Z_0^+} - \frac{V_0^- e^{j\beta^- l}}{Z_0^-}}$$

$$Z_L = \frac{V_0^+ \cancel{e^{-j\beta^+ l}} + V_0^- \cancel{e^{j\beta^- l}}}{\frac{Z_0^- V_0^+ \cancel{e^{-j\beta l}} - V_0^- \cancel{e^{j\beta l}}(Z_0^+)}{Z_0^+ Z_0^-}}$$

$$(l=0) \quad Z_L = \frac{V_0^+ + V_0^-}{Z_0^- V_0^+ - V_0^-(Z_0^+)}$$

$$Z_L(Z_0^- V_0^+ - V_0^- Z_0^+) = (V_0^+ + V_0^-)(Z_0^+ Z_0^-)$$

$$Z_L Z_0^- V_0^+ - Z_L Z_0^+ V_0^- = Z_0^+ Z_0^- V_0^+ + Z_0^+ Z_0^- V_0^-$$

$$Z_L Z_0^- V_0^+ - Z_0^+ Z_0^- V_0^+ = V_0^-(Z_L Z_0^+ + Z_0^+ Z_0^-)$$

$$\frac{V_0^-}{V_0^+} = \frac{Z_L Z_0^- - Z_0^+ Z_0^-}{Z_L Z_0^+ + Z_0^+ Z_0^-} \quad //$$

(b) find phase voltage (V_{AN}, V_{BN}, V_{CN})

$W_1 = 127W$ $\quad$ $W_2 = 460W$ $\quad$ $Z = 30 + 40j$

$W_{total} = W_1 + W_2 = 127 + 460 = 587$

Power per phase $= \frac{587}{3} = 195.7$

$|I_{rms}|^2 \times |R| = P$ $\quad$ $Z = 30 + 40j$

$I_{rms} = 2.55$ $\quad$ $= 50\angle 53.1$

$I_a = 2.55 \angle -53.1$ $\quad$ (~~line~~

line voltage. $\quad$ $+30°$

$P = |I_{rms}||V_{rms}| \cos(\phi_{Vab, Ia})$ $\quad$ $V_{an} \rightarrow V_{ab}$

W1 measure ab

$127 = (2.55)(V_{ab}) \cos(30 - 53.1)$

V_{ab} ~~#~~, rms $=$ ~~B~~ 54.1 V

phase voltage ~~#~~, peak $= \sqrt{2} \times$ ~~#~~ $\frac{54.1}{\sqrt{3}} = \sqrt{2} \times 31.26$

$= 44.2$

$V_{AN} = 44.2 \angle 0$

$V_{BN} = 44.2 \angle -120$

$V_{CN} = 44.2 \angle 120$

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