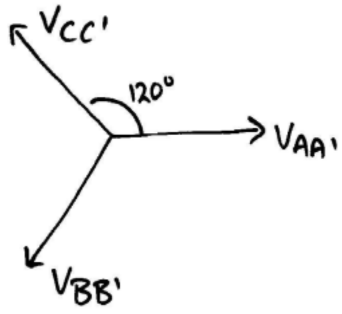
ELEC2222 – Circuits and Transmission topic overview and formulae - Ricki Tura rst1g15

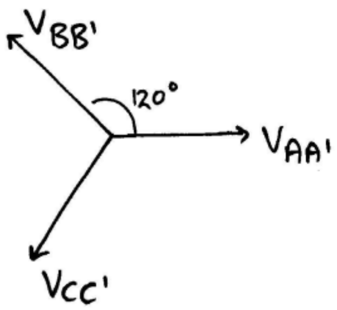
James 1 – Three Phase Connections

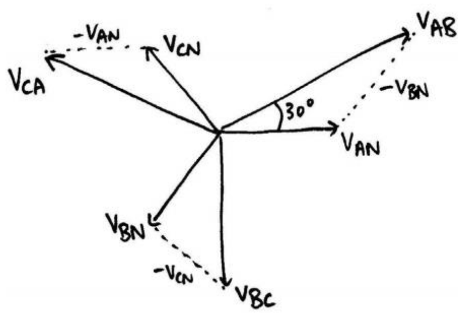
- Phasors are covered extensively in this section.

Equation	**Info**
$\mathrm{V(t)} = \mathrm{V}_{max}\cos(\omega t + \phi)$	Standard description of an AC wave.
$\mathrm{V(j\omega)} = \mathrm{V}_{max}\, e^{j\phi}$	The corresponding phasor (a complex number).
$\mathrm{V(j\omega)} = \mathrm{V}_{max}\, e^{j(\omega t+\phi)} = \mathrm{V}_{max}\, e^{j\phi}e^{j\omega t}$	The exponential version.
$\mathrm{V(j\omega)} = \mathrm{V}_{max}\angle\phi$	The polar version.
$\mathrm{V(j\omega)} = \mathrm{V}_{max}\cos(\phi) + j\mathrm{V}_{max}\sin(\phi)$ $\mathrm{V(j\omega)} = a + jb$	The Cartesian version.
$\mathrm{V} = \sqrt{a^2+b^2}, \phi = \tan^{-1}\left(\frac{b}{a}\right)$	Finding the magnitude and angle.
$A\angle B \times C\angle D = (A\times C)\angle(B+D)$	Multiplying phasors.
$A\angle B\ \div\ C\angle D = (A \div C)\angle(B-D)$	Dividing phasors.

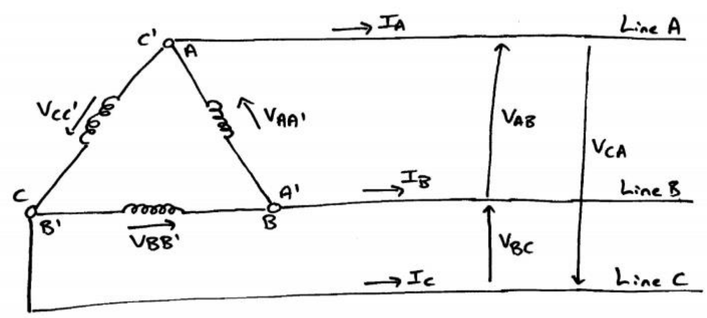
- Inductors cause the current to *lag* w.r.t the voltage. A purely inductive load has a 90° lag, and +jΩ.
- Capacitors cause the current to *lead* w.r.t to the voltage. A capacitive load has a 90° lead, and -jΩ.
- Two-phase systems are inferior to three-phase systems. The latter has a uniform power dissipation, causing less vibrations in machines, and yields better transmission efficiency.
- There are two three-phase configurations, known as *phase sequences*. They are the positive and negative phase sequences, shown below left and right respectively.

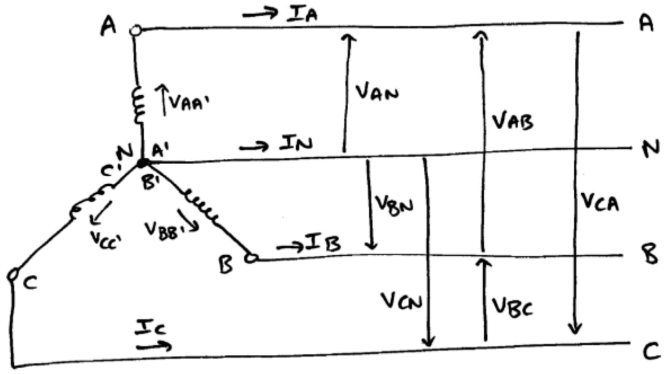






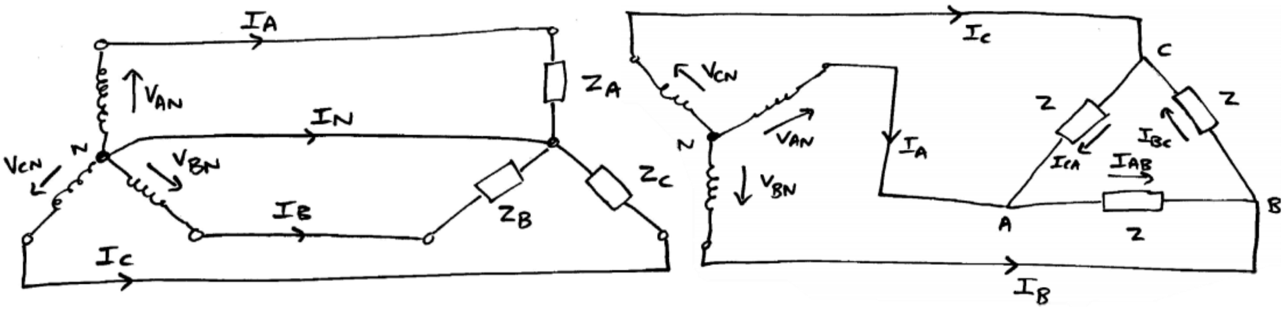
- There are two ways to connect the coils in the generator – the delta, and the star.





- The *phase currents* and *phase voltages* are the values across and in the source and load components.
- The *line currents* and *line voltages* are the values across and in the connecting lines.
- In the delta configuration, the line voltages *equal* the phase voltages, and the line currents *don't equal* the phase currents.
- In the star configuration, the line voltages *don't equal* the phase voltages, and the line currents *do equal* the phase currents.

- The star line to line voltages are $\sqrt{3}$ times greater than the line to neutral voltages, and lead by 30°.
- Load systems can also be in either the delta or star configuration.

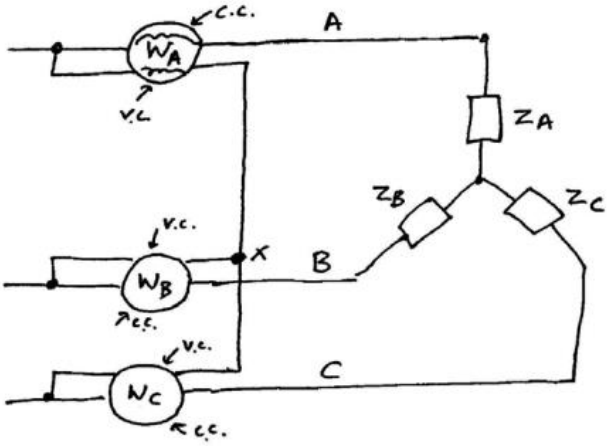


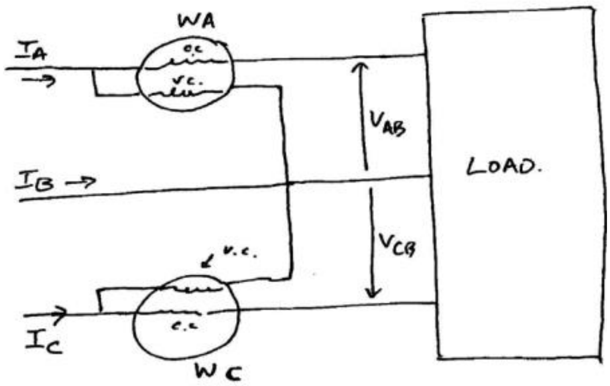
- In a star-star configuration, if the load impedances are all equal then the system is balanced. There will be no current in the neutral line which means it can be removed.
- In a star-delta configuration, the load phase voltage equals the line voltage, from which you can calculate the line currents. The line currents are $\sqrt{3}$ times greater than the phase currents.
- You can convert stars into deltas and vice versa using Rosen's theorem, outlined in Milos' section.

James 2 – Three Phase Power and Measurement

Equation	Info
$\mathrm{V(t)} = \mathrm{V}_{max}\cos(\omega\mathrm{t}+\phi)$ $i_r(t) = I_{Rmax}\cos(\omega\mathrm{t}+\phi)$ $p_r(t) = \mathrm{V}_{max}\mathrm{I}_{Rmax}\cos^2(\omega\mathrm{t}+\phi)$	Power in a purely resistive load.
$\mathrm{V(t)} = \mathrm{V}_{max}\cos(\omega\mathrm{t}+\phi)$ $i_r(t) = I_{Lmax}\cos(\omega\mathrm{t}+\phi-90)$ $p_r(t) = \frac{1}{2}\mathrm{V}_{max}\mathrm{I}_{Lmax}\,cos(2[\omega\mathrm{t}+\phi)]-90)$	Power in a purely inductive load, using a trig identity for the final line. $2cosAcosB = \cos(A-B)+\cos(A-B)$
$P_{av} = \frac{1}{T}\int_0^T p(t)dt$ $= \frac{\mathrm{V}_{max}\mathrm{I}_{max}}{T}\int_0^T cos(\omega\mathrm{t}+\phi_1)\,cos(\omega\mathrm{t}+\phi_2)\,dt$ $P_{av} = \frac{\mathrm{V}_{max}\mathrm{I}_{max}}{2}cos(\phi_1-\phi_2)$	Average power, where $T = \frac{2\pi}{\omega}$, a period of the waveform. Working through the integration gives the final line. The phase shift is caused by the presence of either capacitance or inductance.
$P_{real} = \Re P(j\omega) = \Re\left(\frac{1}{2}V(j\omega)\times I^*(j\omega)\right)$ $P = V_{RMS}I_{RMS}\cos(\phi)$	**P,** real part of the complex power is the average power, energy dissipated through heat. Has the unit watt.
$P_{reactive} = \Im P(j\omega) = \frac{\mathrm{V}_{max}\mathrm{I}_{max}}{2}\sin(\phi_1-\phi_2)$ $Q = V_{RMS}I_{RMS}\sin(\phi)$	**Q**, imaginary part of the complex power is the reactive power, energy which isn't dissipated. Has the unit var (volt-amp reactive).
$S = V_{RMS}I_{RMS} = \frac{1}{2}V_p\times I_p^*$	Apparent power, magnitude of complex power, units VA.
$Power\ factor = \cos(\phi) = \frac{P}{S}$	The ratio of the real power that is used to do work and the apparent power that is supplied.
$S^2 = P^2 + Q^2$	
$P = \sqrt{3}V_L I_L\cos(\phi)$ $Q = \sqrt{3}V_L I_L\sin(\phi)$	The power in a balanced 3-phase system. This considers the 3 generators, and applies to both star-star and star-delta systems.

- Power measurement can be done by either the 2 or 3 watt meter method. Watt meters output the average (real) power and consist of a voltage measuring coil and a current measuring coil.
- In the 3 watt meter method, a watt meter is put on each line with a common reference X. The power readings are simply added to get the total average power.

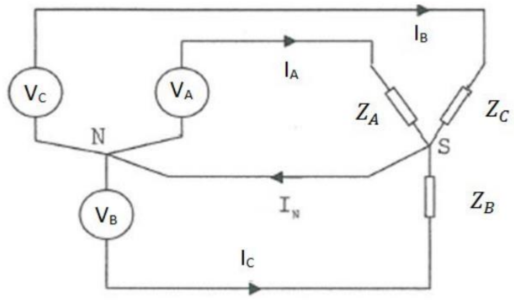


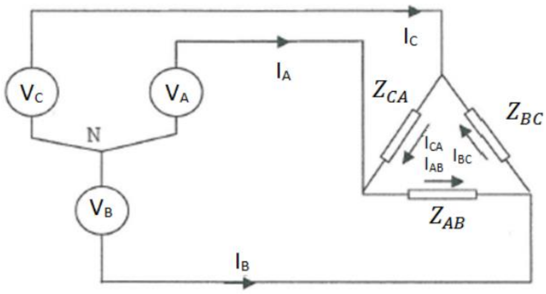


- In the 2 watt meter method, the common reference is one of the lines. The watt meter on that line will give a reading of zero, which means you can remove it, leaving only 2 watt meters.

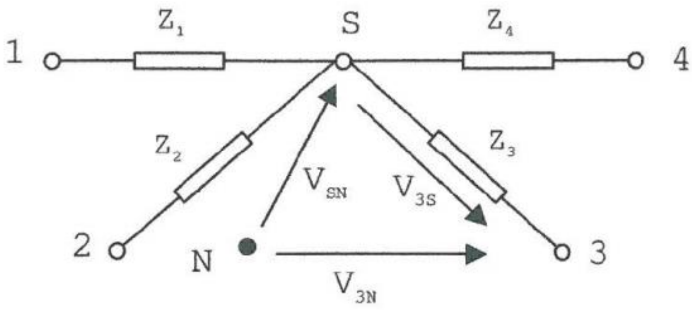
<u>James 3 – Unbalanced Three Phase Circuits</u>

- In some cases, for unbalanced loads we can consider each phase and corresponding load as a separate circuit, which makes calculations much easier. Always check if this applies.





- The unbalanced load 4 wire star-star circuit is an example of where this can be applied. Provided there is no impedance in the neutral line, you can treat each phase separately and find each line current. After this you can find the neutral current by applying KCL at the star point.
- The imbalance of one of the loads doesn't affect the power dissipation in the other loads, as the neutral line acts to 'isolate' that particular phase.
- For unbalanced loads in star-delta circuits, using the line voltages the phase currents can be found, and then the line currents using KCL.
- In three wire star-star system, there is no neutral line to 'isolate' each phase, so an unbalanced load will affect the other phases. How can we find line currents? We use *Millman's theorem*, which calculates the voltage between the star point and the neutral point.



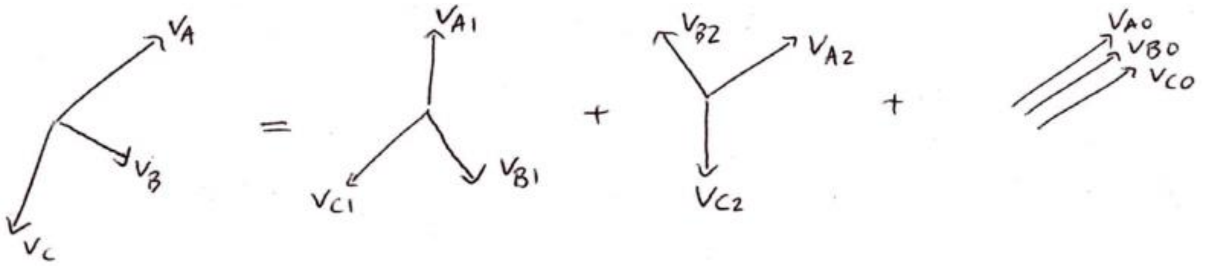
$$V_{SN} = \frac{V_{1N}Y_1 + V_{2N}Y_2 + V_{3N}Y_3 + V_{4N}Y_4}{Y_1 + Y_2 + Y_3 + Y_4}$$

Y_4 is 0 unless you have a neutral wire, at which point it needs to be accounted for.

For $A\angle B$, Y is found by inverting the phasor magnitude A, and flipping the sign of the angle B.

James 4 – Symmetrical Components

- You can represent a set of unbalanced voltages or currents as a sum of three balanced sets of phasors, one set rotating counter clockwise (positive phase sequence), one set rotating clockwise (negative phase sequence), and the final set being identical (same magnitude, direction) and rotating clockwise (zero phase sequence).



- The 'a' operator is used here, and is defined below:

Equation	Info
$a = \cos(120°) + j\sin(120°) = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$	The 'a' operator. Multiplying a vector by a corresponds to rotating the vector 120° counter clockwise.
$a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$	Multiplying a vector by a^2 corresponds to rotating the vector 240° counter clockwise.
$a^3 = 1$	Multiplying a vector by a^3 corresponds to a full 360° counter clockwise rotation.

- By a derivation on the slides, you can make up the symmetrical components out of equations in V_A, V_B, and V_C. The equations are nicely summarised in matrix form, $V_s = \boldsymbol{A}\, V_P$, where V_s is the symmetrical component vector, $\boldsymbol{A}$ is the transformation matrix, and V_P the phase vector.

$$\begin{pmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix} \quad A = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \quad \begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}\begin{pmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{pmatrix}$$

- ***A*** is provided in the exam.
- You can also apply matrices to other equations, for example $V_p = \boldsymbol{Z}_p I_P$ relates the phase voltages to the phase currents and impedances.
- The zero phase sequence currents I_{A0}, I_{B0}, and I_{C0} are all $(I_A + I_B + I_C)/3$, as the neutral current I_N is $(I_A + I_B + I_C)$, you can find it by multiplying the zero phase sequence current by 3.
- A typical question gives you an unbalanced supply, from which you have to find the sequence voltages and currents. By applying the matrix equation above you can find the values of V_{A0}, V_{A1}, and V_{A2}, after which you can find the components for V_B and V_C. The currents for each phase sequence component is found by dividing by the impedance, a quick way is to find $\boldsymbol{Z}_p$ and use $V_p = \boldsymbol{Z}_p I_P$.
- You apply this method in cases where the system is unbalanced and there is impedance in the neutral. We can't use the simpler method as you need the line currents to find V_{SN}.
- When there is no neutral connected, the zero sequence component of the current is 0.
- Likewise, the zero sequence component of the impedance is ∞.

- You can also apply symmetrical components to power.