mhby1g21

2^{nd} Coursework ELEC2220:

Question 1: Root Locus plots, design of forward path controllers and closed loop system response

A highly oscillatory plant, *P(s)* is to be controlled using a forward path controller *K(s)* and a unity feedback loop. An approximate model of the plant is given by

$$P(s) = \frac{1}{s^2+4}$$

It is proposed to use an ideal PD controller to ensure that the overall system does not have an oscillatory impulse response. The controller is defined as

$$K(s) = 4k + ks$$

where k is a tuneable scalar gain.

(a) Sketch the root locus and determine the minimum allowable value of k to ensure that the closed loop impulse response is not oscillatory.

(12 Marks)

(b) It is decided not to use the ideal PD controller and instead the implemented controller is defined as

$$K(s) = \frac{120k+30ks}{50+s}$$

Sketch the new root locus and comment on system response as k is increased from zero to plus infinity.

(8 Marks)

(c) In practice it is found that there is a maximum value of k for which the closed loop system is stable. Further tests reveal that the plant has an additional open loop pole at s = -100. Sketch the root locus diagram and explain why there is a finite maximum value of k.

(5 Marks)

Question 2: Nyquist stability criterion, gain and phase margin, stability and the effect of time delays

An industrial process has the open loop transfer function

$$GH(s) = \frac{k}{s(s+2)(s+8)}$$

(a) Using the Nyquist stability criterion, plot the locus of the closed-loop system and clearly indicate the gain and phase margins.

(8 Marks)

(b) Determine the maximum allowable gain, k, and frequency, ω (rad s^{-1}), for the system to remain stable.

(7 Marks)

(c) Sketch the locus of the closed loop system if the open loop transfer function also includes a pure time delay of 10 ms.

(5 Marks)

(d) Show that the phase of the time delayed closed loop system is 180° when

$$\frac{\pi}{2} - 0.01\omega = \tan^{-1}\left(\frac{10\omega}{16-\omega^2}\right)$$

Given

$$\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right)$$

(5 Marks)

Question 1: Root Locus plots, design of forward path controllers and closed loop system response

long derivatives are calculated on different pages to reduce clutter.

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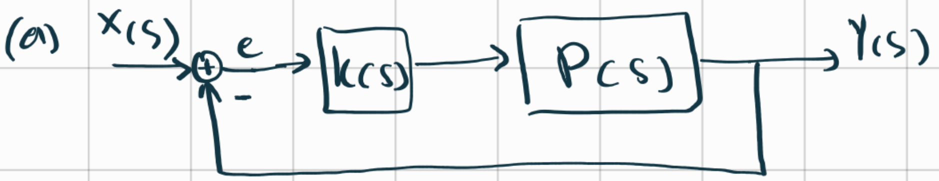
(a)



highest number of poles or zeros are 2 (from poles) so number of line = 2.

$$OLTF = \frac{Y(s)}{X(s)} = \frac{k(s)\,P(s)}{1} = \frac{k(4+s)}{(s^2+4)}$$

$\therefore$ OL zeros : $s = -4$

OL poles : $s = 2j, -2j$

Rank 1 as numerator highest order is 1 and denominator highest order is 2, thus $2-1=1$.

Angle of asymptote $= \phi_A = \frac{(2q+1)}{n-m}.180^\circ \therefore \theta_A = 180^\circ$

$q: 0, 1, 2, \ldots, (n-m-1)$

where n = number of pole/zero (whichever higher) = 2

m = number of pole or zero (whichever lower) = 1

$$\text{centroid}, \alpha = \frac{\Sigma \text{ real part of OL poles} - \Sigma \text{ real part of OL zeros}}{n-m} = \frac{0 - -4}{1} = 4$$

angle of departure from a complex pole, $\theta_{P_j} = 180^\circ + \Sigma$ angles from all zeros to P_j $- \Sigma$ angles from all other poles to P_j.

$\therefore$ @ $2j$, $\theta_{P_{j1}} = 180^\circ + \tan^{-1}\left(\frac{2}{4}\right) - 90 = 116.565^\circ$

$\therefore$ @ $-2j$, $\theta_{P_{j2}} = 180^\circ + \tan\left(\frac{-2}{4}\right) - 270^\circ = 243.434$

Finding break in point: ① Rewrite in form CLTF: $1 + G(s)H(s) = 0$

$\therefore \; 1 + \dfrac{k(s+4)}{(s^2+4)} = 0$ ② let k be subject.

$\therefore \; k = \dfrac{-(s^2+4)}{s+4}$ ③ differentiate, then equate to 0.

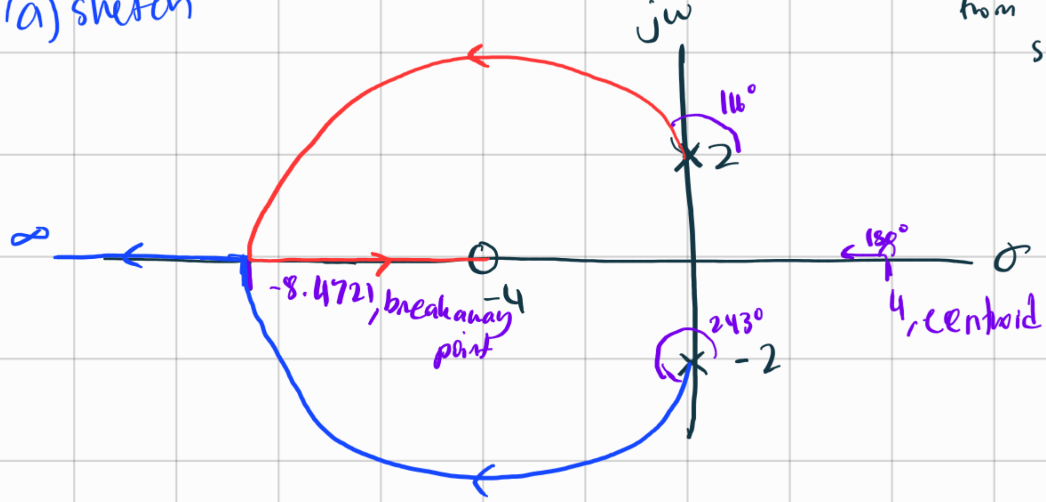
$\dfrac{dk}{ds} = -\dfrac{(s^2+8s-4)}{(s+4)^2} = 0$ ④ solve and pick suitable s as solution

$\therefore \; s \approx -8.4721$ & $s \approx 0.47214$

as poles loci move to the left from $(0,2)$ & $(0,-2)$, the solution is $s \approx -8.4721$

$\therefore$ break in point $= -8.4721$

(a) sketch



To find minimum allowable value of k so that closed loop impulse response is not oscillatory, we can find the gain at breakaway point. as the loci before that had damping ratio, $\zeta < 1$ (underdamped oscillatory in this case)

$\therefore \; s = -8.4721$

from CLTF char eqn:

$$k = \frac{-(s^2+4)}{s+4} = \frac{-[(-8.4721)^2+4]}{(-8.4721+4)} = 16.944$$

$\therefore \; k > 16.944$

(b) It is decided not to use the ideal PD controller and instead the implemented controller is defined as

$$K(s) = \frac{120k+30ks}{50+s}$$

Sketch the new root locus and comment on system response as k is increased from zero to plus infinity.

(8 Marks)

OLTF: $K(s)P(s) = \left(\frac{120k+30ks}{50+s}\right)\left(\frac{1}{s^2+4}\right) = k\,\frac{30s+120}{(s^2+4)(s+50)}$

∴ OL zeros: $30s = -120$, $s = -4$

∴ OL poles: $s = -50, -2j, 2j$.

rank 2 as $3 - 1 = 2$.

$\phi_A = \frac{(2q+1)\cdot 180^\circ}{n-m=2}$, $q = 0, 1, \ldots, (n-m-1)$

$\phi_{A0} = 90^\circ$

$\phi_{A1} = 270^\circ$

$\alpha = \frac{(-50)-(-4)}{2} = -23$

Number of lines = 3

@$2j$, $\theta_{pj1} = 114.3^\circ$ @$-2j$, $\theta_{p1} = -114.3^\circ = 245.7^\circ$

break-in point: ① $1 + \frac{k(30s+120)}{(s^2+4)(s+50)} = 0$

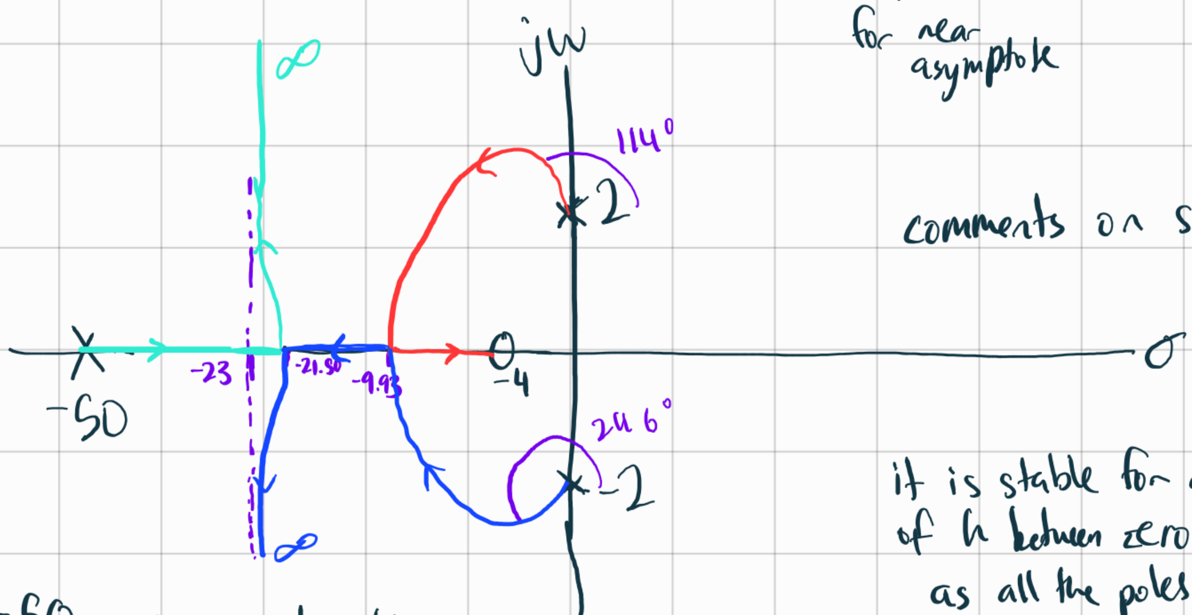
∴ $k = \frac{-(s^2+4)(s+50)}{(30s+120)}$ ② $\frac{dk}{ds} = \frac{-(s^3+31s^2+200s-92)}{15(s+4)^2} = 0$

∴ $s^3+31s^2+200s-92 = 0$, $s \approx -21.50, -9.93, 0.43$.

most suitable $s = -21.50$ and $s = -9.93$.

for near asymptote

near zero $= -4$.



comments on stability:

it is stable for all values of k between zero to infinity as all the poles loci are still in the left hand plane.

-50 poles go to right as it is even numbered poles.

(c) In practice it is found that there is a maximum value of *k* for which the closed loop system is stable. Further tests reveal that the plant has an additional open loop pole at s = -100. Sketch the root locus diagram and explain why there is a finite maximum value of k.

(5 Marks)

OLTF: $k C(s) P(s) = k \dfrac{(30s + 120)}{(s^2+4)(s+50)(s+100)}$

OL zeros: $s = -4$

OL poles: $s = -100, -50, -2j, 2j$

rank 3 as $4-1=3$

$\alpha = \dfrac{(-100-50)-(-4)}{3} = -48.67$

$\phi_A = \dfrac{(2q+1)\cdot 180^\circ}{\cancel{n-m}\ 3}$, $q = 0, 1, 2 \dots, (n-m-1)$

$\phi_{A0} = 60^\circ$

$\phi_{A1} = 180^\circ$

$\phi_{A2} = 300^\circ$

number of lines = 4.

@ 2j, $\theta_{pj1} = 113.1^\circ$

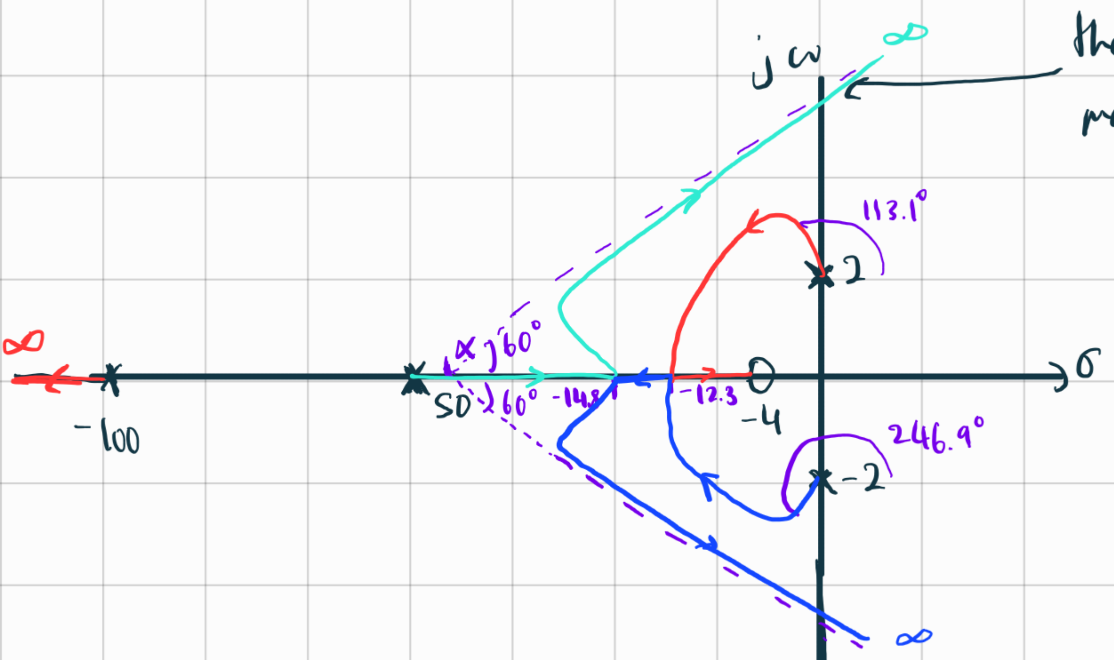
@ -2j, $\theta_{pj} = -113.1^\circ = 246.9^\circ$

breakaway point: ① CLTF: $1 + k \dfrac{(30s+120)}{(s^2+4)(s+50)(s+100)}$

② $k = \dfrac{-(s^2+4)(s+50)(s+100)}{(30s+120)}$ ③ $\dfrac{dk}{ds} = \dfrac{3s^4 + 316s^3 + 6804s^2 + 40032s - 17600}{30(s+4)^2} = 0$

④ solve $s \approx -78.7, -14.8, -12.3, 0.41$.

most suitable s as break-away and break-in point respectively: $s = -14.8$ & $s = -12.3$.



there is now a finite maximum value of k as some of the loci of the pole gets to right hand plane which causes the system to be unstable.

Question 2: Nyquist stability criterion, gain and phase margin, stability and the effect of time delays

An industrial process has the open loop transfer function

$$GH(s) = \frac{k}{s(s+2)(s+8)}$$

(a) Using the Nyquist stability criterion, plot the locus of the closed-loop system and clearly indicate the gain and phase margins.

(8 Marks)

OLTF : $GH(s) = \dfrac{k}{s(s+2)(s+8)}$

let $G(j\omega) = GH(j\omega)$

$\therefore$ let $s = j\omega$

$$G(j\omega) = \frac{k}{j\omega(j\omega+2)(j\omega+8)}$$

$$|G(j\omega)| = \frac{k}{\sqrt{\omega^2(\omega^2+2^2)(\omega^2+8^2)}} = \frac{k}{\omega\sqrt{(\omega^2+4)(\omega^2+64)}}$$

$$\angle G(j\omega) = \underline{\left| -\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{8}\right)\right.}$$

$$\lim_{\omega \to 0^+} = |\infty| \angle -\frac{\pi}{2} - 0 - 0 = |\infty| \angle -\frac{\pi}{2} = -90^\circ = 270^\circ$$

$$\lim_{\omega \to \infty} = |0| \angle -\frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} = |0| \angle -\frac{3\pi}{2} = -270^\circ = 90^\circ$$

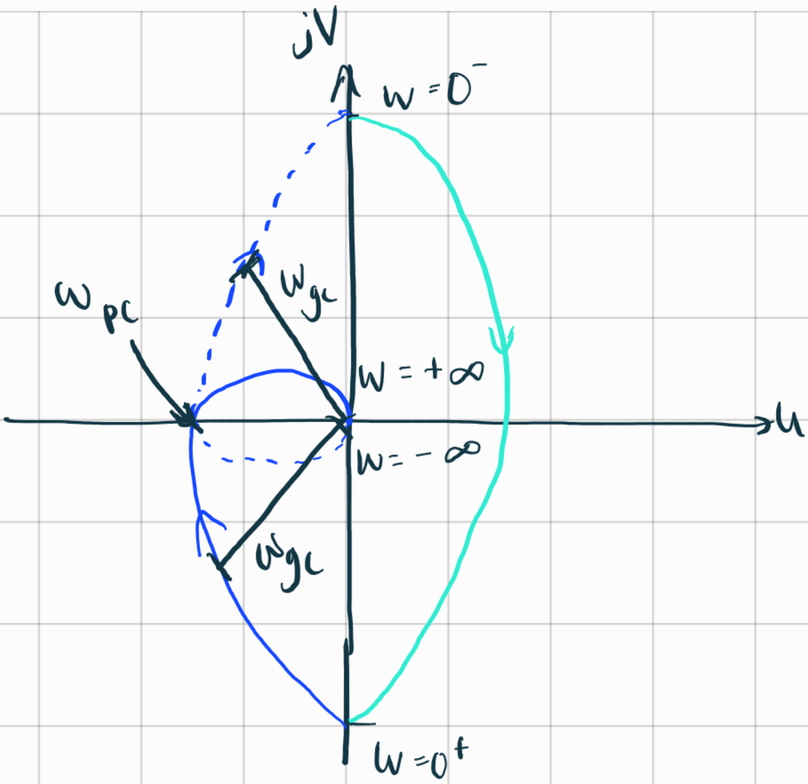
Phase Cross over Frequency

The frequency at which the Nyquist plot intersects the negative real axis (phase angle is 180^0) is known as the **phase cross over frequency**. It is denoted by ω_{pc} .

Gain Cross over Frequency

The frequency at which the Nyquist plot is having the magnitude of one is known as the **gain cross over frequency**. It is denoted by ω_{gc} .

$M = 1 = 0$ dB



Gain Margin

The gain margin GM is equal to the reciprocal of the magnitude of the Nyquist plot at the phase cross over frequency.

$$GM = \frac{1}{M_{pc}}$$

find w_{pc} by finding axis intersection:

@ w_{pc} $\angle G(jw) = -\frac{\pi}{2} - \tan^{-1}\left(\frac{w}{2}\right) - \tan^{-1}\left(\frac{w}{8}\right) = -\pi$

try 2 and 4 for w, $w = 4$.

$$\frac{1}{GM} = \frac{1}{4j\,(4j+2)\,(4j+8)} = \frac{-1}{160}, \quad GM = -160$$

GM in dB $= -20\log_{10}(-160) = -44.08$ dB

Phase Margin

The phase margin PM is equal to the sum of 180^0 and the phase angle at the gain cross over frequency.

$$PM = 180^0 + \phi_{gc}$$

Where, ϕ_{gc} is the phase angle at the gain cross over frequency.

$$|G(j\omega)| = \cancel{k}^{1} \frac{1}{\omega^2(\omega^2+4)(\omega^2+64)} = 1$$

$$\therefore \omega^6 + 68\omega^4 + 256\omega^2 - 1 = 0$$

let $x = \omega^2$.

$$x^3 + 68x^2 + 286x - 1 = 0$$

$$x = -64, \quad x = 0.0039, \quad x = -4.004$$

$$\omega^2 = -64, \quad \omega^2 = 0.0039, \quad \omega^2 = -4.004$$

$$\omega = \pm 8j, \quad \underline{\underline{\omega = \pm 0.0062}}, \quad \omega = \pm 2j$$

$$\therefore @ \omega = 0.0062, \ \angle G(j\omega) = -\frac{\pi}{2} - \tan^{-1}\left(\frac{0.0062}{2}\right) - \tan^{-1}\left(\frac{0.062}{8}\right)$$

$$\phi_{gc} = -92.2^\circ$$

$$PM = -180^\circ - (-92.2^\circ) = -87.78^\circ$$

(b) Determine the maximum allowable gain, k, and frequency, ω (rad s^{-1}), for the system to remain stable.

(7 Marks)

$$GH(s) = \frac{k}{s(s+2)(s+8)}$$

from gain margin:

max f, $w = 4$.

OL zero : N/A

OL poles : $s = -8, -2, 0$

rank $= 3 - 0 = 3$

$$GH(j\omega) = \frac{k}{(j\omega)(j\omega+2)(j\omega+8)} = \frac{k}{-j\omega^3 - 10\omega^2 + 16j\omega}$$

∴ system is stable as long as the contour didn't encircle point (-1, 0) according to nyquist stability criterion.

real part

$$\therefore \frac{k}{\cancel{-j(4)^3}^{0} - 10(4)^2 + \cancel{16j(4)}^{0}} = -1$$

$$\therefore k = -1 \times -160$$

$$k = 160.$$

imaginary part is removed as k is real only.

(c) Sketch the locus of the closed loop system if the open loop transfer function also includes a pure time delay of 10 ms.

(5 Marks)

Time delay in terms of $j\omega$: $e^{-0.01s}$

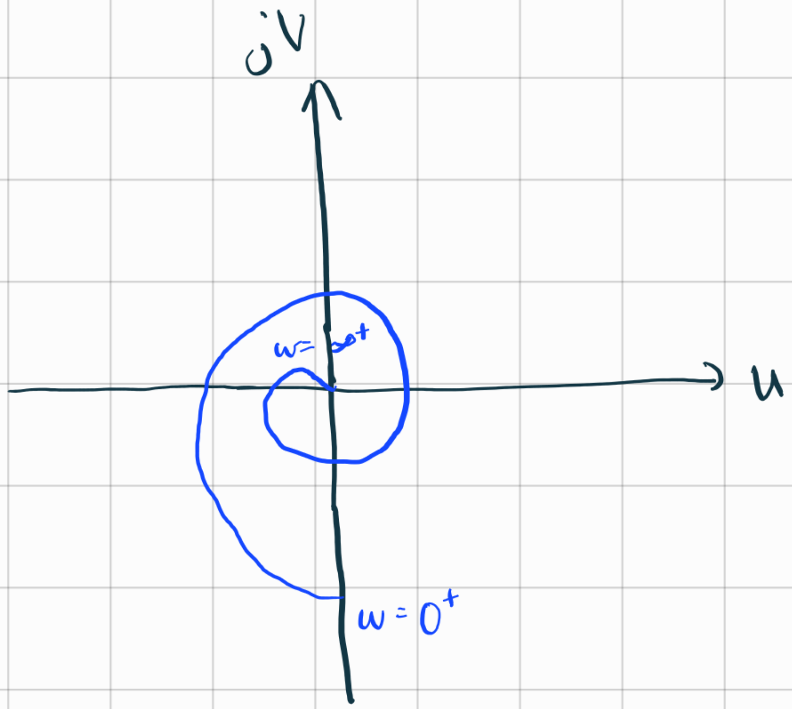
$$\therefore GH(s) = \frac{k e^{-0.01s}}{s(s+2)(s+8)}$$

$$GH(j\omega) = \frac{k e^{-0.01j\omega}}{\omega\sqrt{(\omega^2+4)(\omega^2+64)}} \angle -\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{8}\right) - 0.01\omega$$

$$\therefore \lim_{\omega \to 0} = |\infty| \angle -\frac{\pi}{2} - 0 - 0 - 0 = |\infty| \angle -\frac{\pi}{2}$$

$$\lim_{\omega \to \infty} = |0| \angle -\frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} - \infty = |0| \angle -\frac{3\pi}{2} - \infty$$

causes spiral



(d) Show that the phase of the time delayed closed loop system is 180° when

$$\frac{\pi}{2} - 0.01\omega = \tan^{-1}\left(\frac{10\omega}{16-\omega^2}\right) \quad (1)$$

Given

$$\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right) \quad (2)$$

(5 Marks)

phase of $G(H(j\omega))$, $\angle = \underline{\left| -\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{8}\right) - 0.01\omega \right.}$

using (2):

$$\therefore \angle = \underline{\left| -\frac{\pi}{2} - 0.01\omega - \left(\tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{8}\right)\right) \right.}$$

$$= \underline{\left| -\frac{\pi}{2} - 0.01\omega - \tan^{-1}\left(\frac{\frac{\omega}{2} + \frac{\omega}{8}}{1 - \frac{\omega^2}{16}}\right) \right.}$$

$$= \underline{\left| -\frac{\pi}{2} - 0.01\omega - \tan^{-1}\left(\frac{\frac{5\omega}{8}}{\frac{16-\omega^2}{16}}\right) \right.}$$

$$= \underline{\left| -\frac{\pi}{2} - 0.01\omega - \tan^{-1}\left(\frac{10\omega}{16-\omega^2}\right) \right.}$$

using (1),

$$\therefore \angle = \underline{\left| -\frac{\pi}{2} - 0.01\omega - \left(\frac{\pi}{2} - 0.01\omega\right) \right.}$$

$$= \underline{\left| -\frac{\pi}{2} - \frac{\pi}{2} - 0.01\omega + 0.01\omega \right.}$$

$$= \underline{\left| -\pi \right.} = \underline{\left| \pi \right.} = \underline{\left| 180^\circ \right.}$$

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