SEMESTER 2 EXAMINATION 2021 - 2022

GUIDANCE, NAVIGATION AND CONTROL

DURATION MINS (Hours)

This paper contains 5 questions

Answer **three** questions

An outline marking scheme is shown in brackets to the right of each question.

This examination contributes 100% of the marks for the module

University approved calculators MAY be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct Word to Word translation dictionary AND it contains no notes, additions or annotations.

8 page examination paper.

Question 1.

(a) The errors in one axis of an inertial navigation system are described by

$$\begin{aligned}\Delta\dot{P} &= \Delta V\\ \Delta\dot{V} &= -g\Delta\Theta + B\\ \Delta\dot{\Theta} &= \frac{1}{R}\Delta V + W\end{aligned}$$

where ΔP is the position error, ΔV is the velocity error, $\Delta\Theta$ is the tilt error, W is the gyro drift error, B is the accelerator bias error, R is the radius of the earth, and g is the acceleration due to gravity. Use the state variables $\Delta P = x_1$, $\Delta V = x_2$ and $\Delta\Theta = x_3$ to obtain a state-space model of the dynamics of this system. Determine the equation that governs the stability of this system. [13 marks]

(b) Consider the system described by

$$\dot{x}(t) = \begin{bmatrix} -3 & -3 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

which is to be controlled by minimising the cost function

$$J = \frac{1}{2}\int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))dt$$

where $R = 1$ and

$$Q = \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix}$$

Compute the optimal input in this case. You may make use of the following equation where all symbols have their normal meanings

$$A^TP + PA + Q - PBR^{-1}B^TP = 0$$

and assume that P is a diagonal matrix. [20 marks]

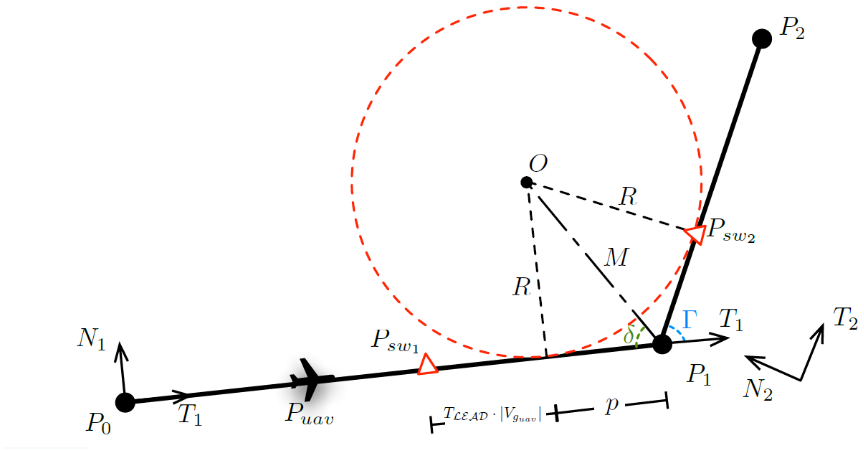

Figure Q2

Question 2.

(a) State the purpose of waypoint switching for UAVs. What is the distinctive feature of the L_2^+ implementation?

[6 marks]

(b) Figure Q2 shows the waypoint switching geometry for an UAV, where all terms have their usual meanings.

The radius of the circle in this figure is

$$R = \frac{(U_c + |\text{wind}|)^2}{a_{\max}}$$

Define all the variables in this formula. The actual radius of curvature of the vehicle can change. Explain why and give a possible solution.

[8 marks]

(c) What is the meaning of the point P_{sw1} in Figure Q2? Show also that

$$p = \frac{R}{\tan \delta}$$

[11 marks]

(d) Why in flight tests can commencing the turn just at the switch point cause a problem? Explain the use of a lead time as a counter to this

TURN OVER

problem and develop the formula for computing the resulting switch point. Also, what happens in this case if the vehicle is already beyond the next segment switching point? [8 marks]

Question 3.

(a) The equations of motion of a space vehicle in the earth's gravitational field are

$$\begin{aligned} \ddot{r} - r\dot{\theta}^2 &= -\frac{ga^2}{r^2} \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} &= 0 \end{aligned}$$

where all terms have their usual meanings. Show that the second equation of motion in this case can be written as

$$r^2\dot{\theta} = r_0 V \cos\gamma$$

and again define the terms in this equation. Give also the conditions under which this equation is valid. [13 marks]

(b) What is the reasoning for using complementary filtering in aerospace applications? Support your answer by explaining how this approach can be applied to **any three of** i) roll angle estimation, ii) pitch angle estimation, iii) altitude estimation, and iv) altitude rate estimation. [20 marks]

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Question 4.

(a) Figure Q4 shows a control scheme where all symbols have their normal meanings. Write down the equations that govern the dynamics of this system and give the formulas for i) the sensitivity function S, ii) the load sensitivity function PS, iii) the complementary sensitivity function T and iv) the noise sensitivity function CS. What is the significance of the relationship

$$S + T = 1$$

[10 marks]

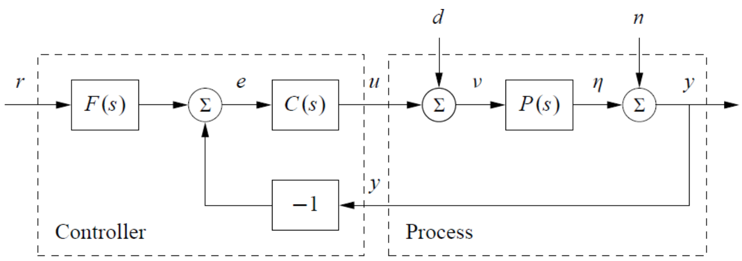

Figure Q4

(b) Consider the case of a system, for which

$$P(s) = \frac{1}{s-a}$$

where $a > 0$, is to be controlled by applying the scheme of the previous part of this question with $F(s) = 1$ and

$$C(s) = k\frac{s-a}{s}, \quad k > 0$$

Compute the functions S, PS, T and CS in this case and hence explain why this scheme cannot be applied. What are the implications when $a < \epsilon$ in the cases when i) ϵ is a very small negative number and ii) ϵ is a very large negative number? [10 marks]

(c) Consider a differential linear time-invariant system described by

$$P(s) = \frac{1}{s^2}$$

for which the performance specifications are less than 1% steady state error for step inputs and less than 10% tracking error up to 10 rad/sec. Give a sketch of the gain and phase responses of this system and explain why increasing the gain is not a feasible design. **Specify, but do not analyse,** the structure of a controller $C(s)$ that can be used to meet this specification. [13 marks]

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Question 5.

(a) Explain the following for target tracking categories with respect to size, sensor resolution, and target-sensor distance: i) point target, ii) extended target, iii) unresolved targets, and iv) dim targets.

[10 marks]

(b) Calculate the transition matrix e^{AT} for the following cases

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Use the resulting matrices to give the target tracking state-space model for i) the nearly constant velocity and ii) the nearly constant acceleration scenarios.

[14 marks]

(c) Explain the principles of the guidance laws for i) proportional navigation, ii) beam rider and iii) command-to-line-of-sight.

[9 marks]

END OF PAPER

