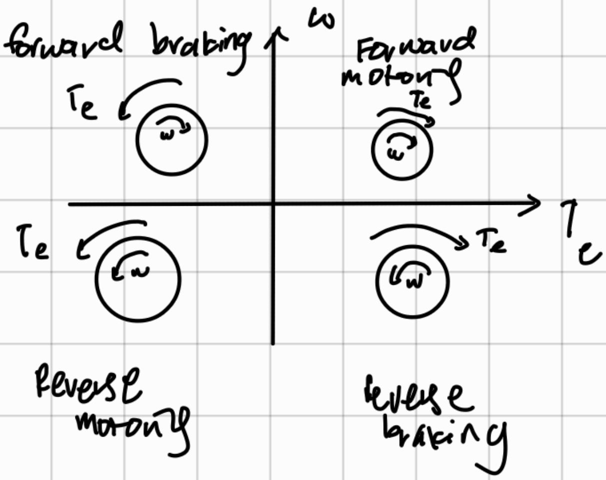
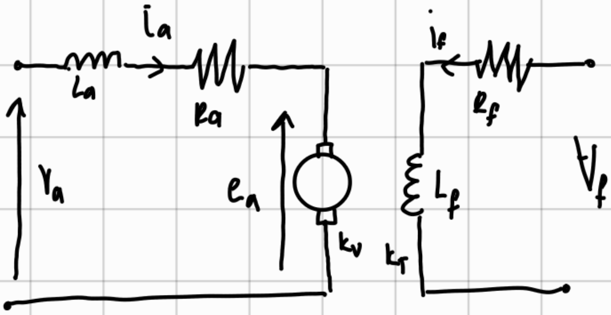
DC

Four Quadrant





$V_f = R_f I_f + L_f \frac{di_f}{dt}$ $\quad V_a = R_a i_a + L_a \frac{di_a}{dt} + e_a$

$e_a = K_v \omega i_f$

$T_e = K_T i_f i_a$ → electromagnetic torque

$= J\frac{d\omega}{dt} + B\omega + T_L$ → DC motor under electrical steady state

$= B\omega + T_L$

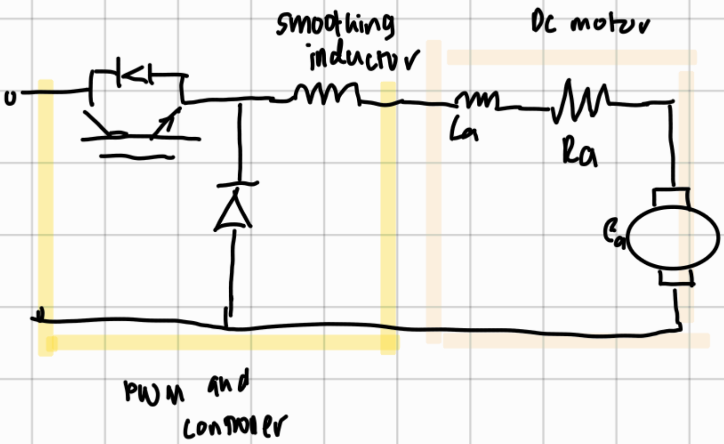
$P_{developed} = T_e \omega$ $\quad \omega = \frac{V_a - R_a I_a}{K_v V_f / R_f}$

Speed regulation $= \frac{\omega_{no\text{-}load} - \omega_{full\text{-}load}}{\omega_{full\text{-}load}} \times 100\%$

$V_p = \frac{2V_{pk}}{\pi}\cos\alpha_f$ → single phase converter

Chopper control of DC machines

- refer to power electronic converter with PWM-operated (MOSFET) IGBT



$V_{dc} = L\frac{di}{dt} + Ri + E$

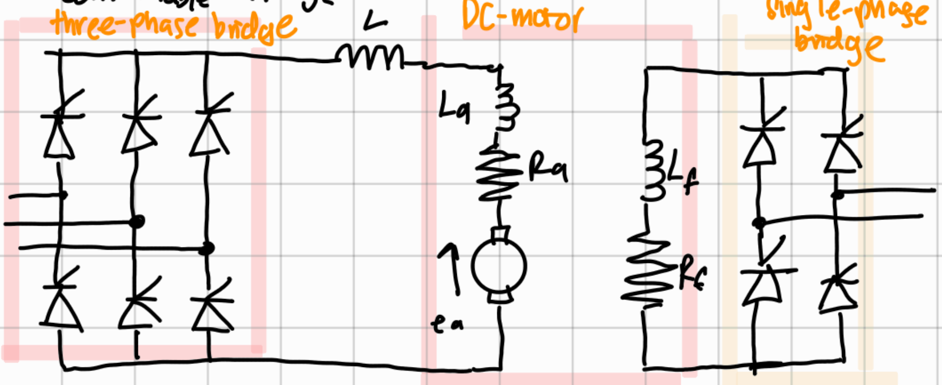
$i_1(t) = I_1 e^{-\frac{R}{L}t} - \frac{(E - V_{dc})}{R}\left[1 - e^{-\frac{R}{L}t}\right]$

$I_2 = I_1 e^{-\frac{R}{L}kT} - \frac{(E - V_{dc})}{R}\left[1 - e^{-\frac{R}{L}kT}\right]$

T_2 & T_3: V_{dc}

$-V_{dc} = L\frac{di}{dt} + Ri + E$

DC motor drive based on 3-phase fully controllable bridge



$V_{an} = V_{pk}\sin(\omega t)$

$V_{bn} = V_{pk}\sin(\omega t - \frac{2\pi}{3})$

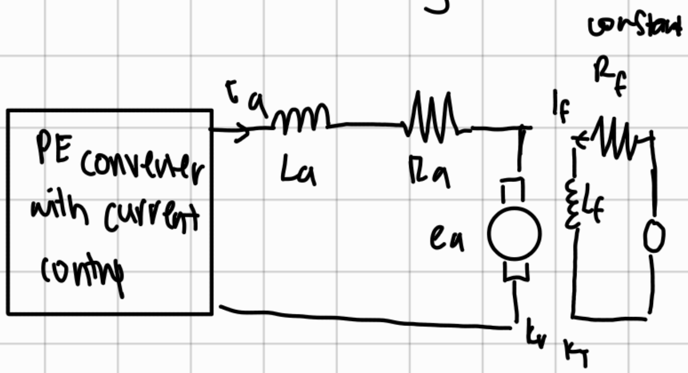
$V_{cn} = V_{pk}\sin(\omega t + 2\pi/3)$

$V_{ab} = V_{an} - V_{bn} = \sqrt{3}\, V_{pk}\sin(\omega t + \pi/6)$

$V_{bc} = \sqrt{3}\, V_{pk}\sin(\omega t - \pi/2)$

$V_{ca} = \sqrt{3}\, V_{pk}\sin(\omega t + \pi/2)$

Regenerative Braking



$$v_m = K_e\omega_m + i_a R_a + L_a \frac{di_a}{dt}$$

$$T = K_t i_a$$ speed rotation

$$= J\frac{d\omega_m}{dt} + B\omega_m$$

Copper loss at the armature

$$P_d = i_a^2 R_a$$

The braking current is controlled by the DC drive to remain constant I_R (negative)

$$V_m(t) = K_e\omega_m(t) + I_a R_a + 0$$

windage friction = 0, no load connected with constant braking armature current

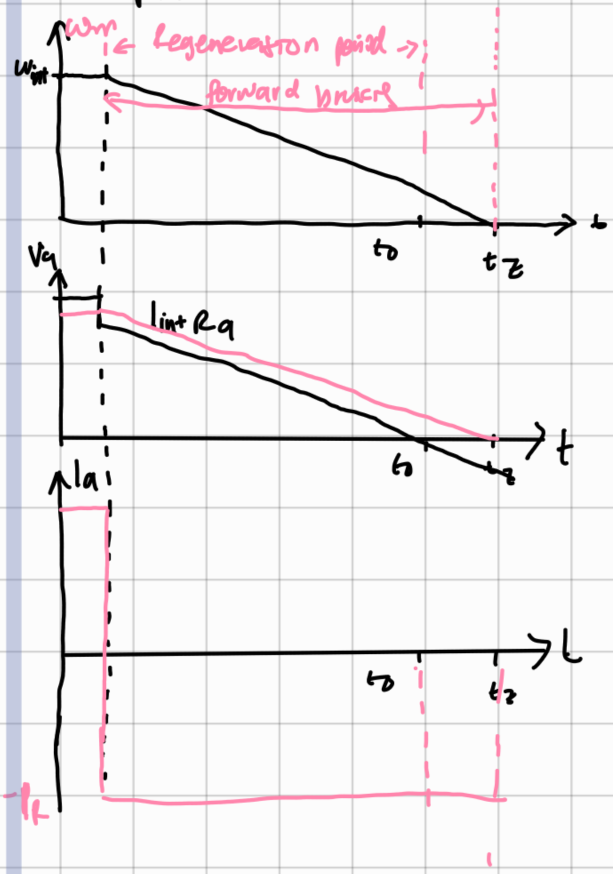
$T_{braking}$ constant during braking period

deceleration constant

$$\omega_m(t) = \left[1 - \frac{t}{t_z}\right]\omega_{int}$$

initial $t=0$ $\omega_{int} = \omega_m(0)$

Graph



Prior to deceleration from ω_{int},

$$V_m(t) = K_e\left[1-\frac{t}{t_z}\right]\omega_{int} + I_R R_a$$

Immediately following the application of the negative braking current the terminal voltage drop to ($K_e\omega_{int} + I_R R_a$, I_R negative)

$K_e\omega_{int}$ = initial back EMF

$V_m \downarrow$, $\omega_m \downarrow$ due to EMF $\downarrow$ until zero

$$t_0 = t_z\left[1 + \frac{I_R R_a}{K_e\omega_{int}}\right]$$

$$P_{regen} = V_m(t) I_R$$

$$E_{regen} = \int_0^{t_0} V_m(t) I_R \, dt$$

$$= \int_0^{t_0}\left[-\frac{K_e\omega_{int} t}{t_z} + K_e\omega_{int} + I_R R_a\right] I_R \, dt$$

$$= t_0\left[\frac{K_e\omega_{int} I_R}{2} + \frac{I_R^2 R_a}{2}\right]$$

Braking Torque, $T_{braking} = K_T I_R = J\alpha$

$$\alpha = \frac{K_T I_R}{J}$$

Deceleration: $\alpha = \frac{0-\omega_{int}}{t_z}$

$$\frac{K_T I_R}{J} = \frac{-\omega_{int}}{t_z}$$

$$t_z = -\frac{J\omega_{int}}{K_T I_R}$$

Calculated regenerated energy

$t=0$ to $t=t_0$

$$E_{regen} = t_0\left[\frac{K_e\omega_{int} I_R}{2} + \frac{I_R^2 R_a}{2}\right]$$

$$= -\frac{J\omega_{int}}{K_t I_R}\left[1+\frac{I_R R_a}{K_e\omega_{int}}\right]\left[\frac{K_e\omega_{int} I_R}{2}+\frac{I_R^2 R_a}{2}\right]$$

$$= -\frac{J\omega_{int}}{K_t}\left[\frac{K_e\omega_{int}}{2} + \frac{I_R^2 R_a^2}{2K_e\omega_{int}} + I_R R_a\right]$$

Relationship between t_0 and t_z

$$t_0 = -\frac{J\omega_{int}}{K_t I_R}\left[1+\frac{I_R R_a}{K_e\omega_{int}}\right]$$

Based on assumptions,
* constant braking current
* no load torque and no friction.

AC induction motor drives

2 categories: wound-rotor type & squirrel cage type

Synchronous speed

$$\omega_s = \frac{\omega}{P}$$

P = pole pairs $= \frac{p}{2}$

p = num. of poles

f = supply frequency

$$= \frac{\pi f}{P}$$

slip of the motor, $s = \frac{\omega_s - \omega_m}{\omega_s}$

motor speed, $\omega_m = \omega_s(1-s)$

stator phase voltage $v_s = V_{pk} \sin \omega_s t$

produces steady-state magnetic flux linkage $\phi(t)$ in the rotor at a given ω_m:

$$\phi(t) = \phi_{pk} \cos(\omega_m t + \delta - \omega_s t)$$

Induced back EMF at one phase of the rotor windings with equivalent N_r turns (ω_s & ω_m constant)

$$e_r(t) = \frac{d(N_r \phi(t))}{dt} = N_r \frac{d}{dt}\left(\phi_{pk} \cos(\omega_m t + \delta - \omega_s t)\right)$$

$$= -N_r \phi_{pk}(\omega_m - \omega_s) \sin((\omega_m - \omega_s)t + \delta)$$

$$= -N_r \phi_{pk}(\omega_s - \omega_m) \sin[(\omega_s - \omega_m)t - \delta]$$

$$= -N_r \phi_{pk}(\cancel{\omega_s - \omega_s} + s\omega_s) \sin[(\omega_s - \omega_s + s\omega_s)t + \delta]$$

$$= -sN_r \phi_{pk} \omega_s \sin[(s\omega_s)t + \delta]$$

δ = relative position of the rotor

N_r = num. of turns on each rotor phase

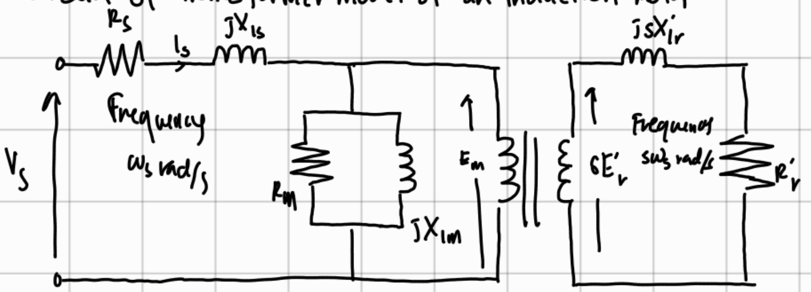
$$= -sE_{pk} \sin(s\omega_s t + \delta)$$

$E_{pk} = N_r \phi_{pk} \omega_s$ = peak value of the rotor induced back EMF

$$= -s\sqrt{2}\, E_r \sin(s\omega_s t - \delta)$$

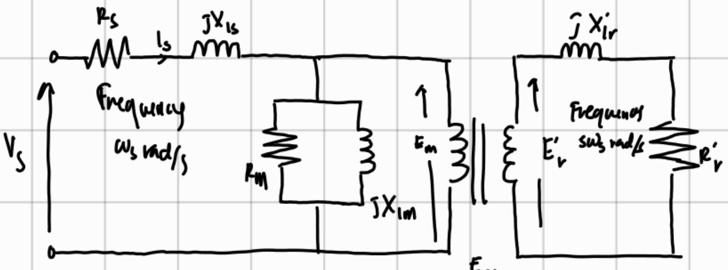
E_r = RMS value of the rotor induced back EMF

Circuit of transformer model of an induction motor



rotor current in rms phasor:

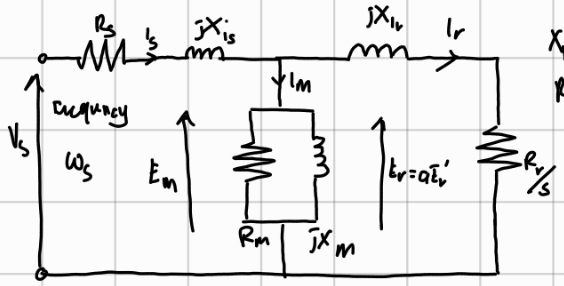
$$I_r' = \frac{sE_r'}{R_r' + jsX_{lr}'} = \frac{E_r'}{\frac{R_r'}{s} + jX_{lr}'}$$



$$\frac{E_m}{E_r} = a = \frac{\text{Stator}}{\text{winding}} \text{ ratio}$$

Lastly, by adjusting E_r', using a.

Equivalent circuit.



$X_{lr} = a^2 X_{lr}'$

$R_r = a^2 R_r'$

$I_r = \frac{I_r'}{a}$

V_s is rated phase to neutral rms voltage of the stator.

All of the reactance are at rated frequency.

(power transferred from stator to rotor)

Air gap power, $P_g = 3I_r^2 \frac{R_r}{s}$

Stator copper loss, $P_{scloss} = 3I_s^2 R_s$

Rotor copper loss, $P_{rcloss} = 3I_r^2 R_r$

Core loss, $P_{core\text{-}loss} = \frac{3E_m^2}{R_m} \approx \frac{3V_s^2}{R_m}$

Total input active power

θ_m = phase angle

$$P_{in} = 3V_s^2 I_s \cos\theta_m$$

$$= P_{coreloss} + P_{scloss} + P_{ag}$$

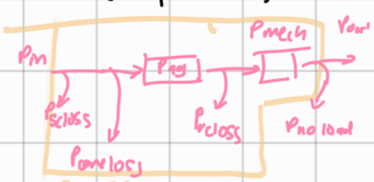
$$P_{mech} = P_{ag} - P_{rcloss}$$

$$= 3I_r^2 \frac{R_r}{s} - 3I_r^2 R_r$$

$$= 3I_r^2 R_r \frac{(1-s)}{s}$$

$$= P_{ag}(1-s)$$

components inside an induction motor



$P_{out} = P_{mech} - P_{noload}$

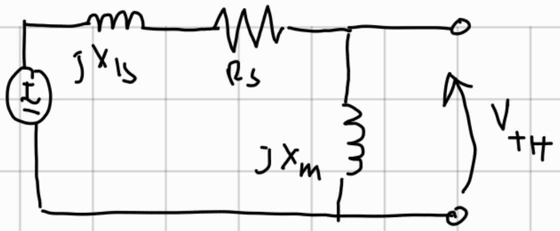
efficiency

$$\eta = \frac{P_{mech}}{P_{in}} = \frac{P_{mech}}{P_{core\text{-}loss} + P_{scloss} + P_{ag}}$$

$P_{ag} \gg P_{core\text{-}loss}$ $P_{ag} \gg P_{scloss}$

$$\eta \approx \frac{P_{mech}}{P_{ag}} = \frac{P_{ag}(1-s)}{P_{ag}}$$

$$= 1-s$$

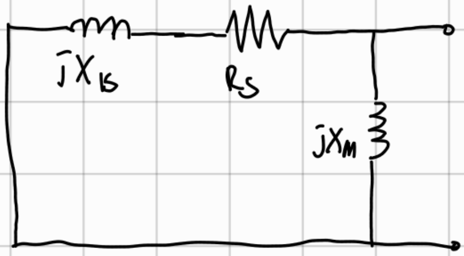


$$V_{TH} = \frac{jX_M}{R_s + jX_{ls} + jX_M} V_s$$

$$= \frac{X_M}{\sqrt{R_s^2 + (X_m + X_{ls})^2}} V_s$$

$X_M >> X_{Ls}$

$(X_M + X_{ls}) >> R_s$



$$Z_{TH} = \frac{jX_m(R_s + jX_{ls})}{(R_s + j(X_m + X_{ls}))}$$

assume

$X_m >> X_{ls}$

$X_m + X_{ls} >> R_s$

$$= \frac{jX_m R_s - X_m X_{ls}}{R_s + j(X_m + X_{ls})} \cdot \frac{(R_s - j(X_m + X_{ls}))}{(R_s - j(X_m + X_{ls}))}$$

$$= \frac{jX_m R_s^2 + X_m R_s(X_m + X_{ls}) - R_s X_m X_{ls} + jX_m X_{ls}(X_m + X_{ls})}{R_s^2 + (X_m + X_{ls})^2}$$

$$= \frac{j(X_m R_s^2 + X_m X_{ls}(X_m + X_{ls})) + X_m R_s(X_m + X_{ls}) - R_s X_m X_{ls}}{R_s^2 + (X_m + X_{ls})^2}$$

$$= \frac{j(X_m(R_s^2 + X_{ls}(X_m + X_{ls}))) + X_m R_s(X_m + \cancel{X_{ls}} - \cancel{X_{ls}})}{R_s^2 + (X_m + X_{ls})^2}$$

$$= \frac{jX_m(R_s^2 + X_{ls}(X_m + X_{ls})) + X_m^2 R_s}{R_s^2 + (X_m + X_{ls})^2}$$

$$= \frac{X_m^2 R_s}{R_s^2 + (X_m + X_{ls})} + \frac{jX_m(R_s^2 + X_{ls}(X_m + X_{ls}))}{R_s^2 + (X_m + X_{ls})^2}$$

$$\approx \frac{X_m^2 R_s}{(X_m + X_{ls})^2} + j\frac{R_s^2 X_m + X_m X_{ls}(X_m + X_{ls})}{(X_m + X_{ls})^2}$$

$$= \frac{X_m^2 R_s}{(X_m + X_{ls})^2} + j\left(\frac{R_s^2 X_m}{(X_m + X_{ls})^2} + \frac{X_m X_{ls}\cancel{(X_m + X_{ls})}}{(X_m + X_{ls})^{\cancel{2}}}\right)$$

$$= \frac{X_m^2 R_s}{(X_m + X_{ls})^2} + j\left(R_s \overbrace{\frac{R_s X_m}{(X_m + X_{ls})^2}}^{(\text{zero})} + \frac{\cancel{X_m} X_{ls}}{\cancel{X_m}}\right)$$

$$= \frac{X_m^2 R_s}{(X_m + X_{ls})^2} + jX_{ls}$$

$$= R_s\left[\frac{X_m}{X_m + X_{ls}}\right]^2 + jX_{ls}$$

$$= R_{TH} + jX_{TH} \quad //$$

$$E = \frac{R_r/s + jX_{lr}}{R_{TH} + \frac{R_r}{s} + j(X_{TH} + X_{lr})} V_s$$

$$I_r = \frac{V_{TH}}{R_{TH} + \frac{R_r}{s} + j(X_{TH} + X_{lr})}$$

$$= \frac{V_{TH}}{\sqrt{(R_{TH} + R_r/s)^2 + (X_{TH} + X_{lr})^2}}$$

Torque electromagnetic

$$T_e = \frac{P_{mech}}{\omega_m} = \frac{P_{ag}(1-s)}{(1-s)\omega_s} = \frac{P_{ag}}{\omega_s}$$

$$= \frac{3I_r^2 R_r}{s\omega_s} = \frac{3E^2\left(\frac{R_r}{s}\right)}{\omega_s\left[\left(\frac{R_r}{s}\right)^2 + X_{lr}^2\right]}$$

or in Thevenin

$$= \frac{3V_{TH}^2\left(\frac{R_r}{s}\right)}{\omega_s\left[\left(R_{TH} + \frac{R_r}{s}\right)^2 + (X_{TH} + X_{lr})^2\right]}$$

$$P_{mech} = P_{ag}(1-s)$$

$$= \frac{3V_{TH}^2 R_r (1-s)}{s\left[\left(R_{TH} + \frac{R_r}{s}\right)^2 + (X_{TH} + X_{lr})^2\right]}$$

$$I_s = \frac{V_s - E}{R_s + jX_{ls}}$$

$$E = \left[\frac{R_r/s + jX_{lr}}{R_{TH} + \frac{R_r}{s} + j(X_{TH} + X_{lr})}\right] V_{TH} \qquad V_{TH} = \frac{jX_m V_s}{R_s + j(X_m + X_{ls})}$$

If stator frequency is changed from ω_s to $\beta\omega_s$ where $0 < \beta \leq 1$.

$$T_e @ \beta\omega_s = \frac{3V_{TH}^2 (R_r/s)}{\beta\omega_s\left[\left(R_{TH} + \frac{R_r}{s}\right)^2 + (\beta X_{TH} + \beta X_{lr})^2\right]}$$

$$T_{max} @ \beta\omega_s = \frac{3V_{TH}^2}{2\beta\omega_s\left[R_{TH} + \sqrt{R_{TH}^2 + (\beta X_{TH} + \beta X_{lr})^2}\right]}$$

Assume R_{TH} is negligible

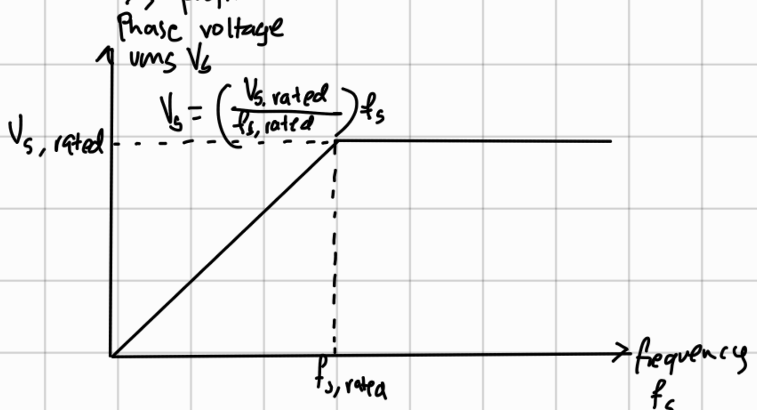
$$T_{max} @ \beta\omega_s = \frac{3V_{TH}^2}{2\beta\omega_s(\beta X_{TH} + \beta X_{lr})}$$

$$= \frac{3}{2\omega_s(X_{TH} + X_{lr})}\left[\frac{V_{TH}}{\beta}\right]^2$$

$$E_m = V_s - (R_s + jX_{ls}) I_s$$
$$= jX_m I_m$$

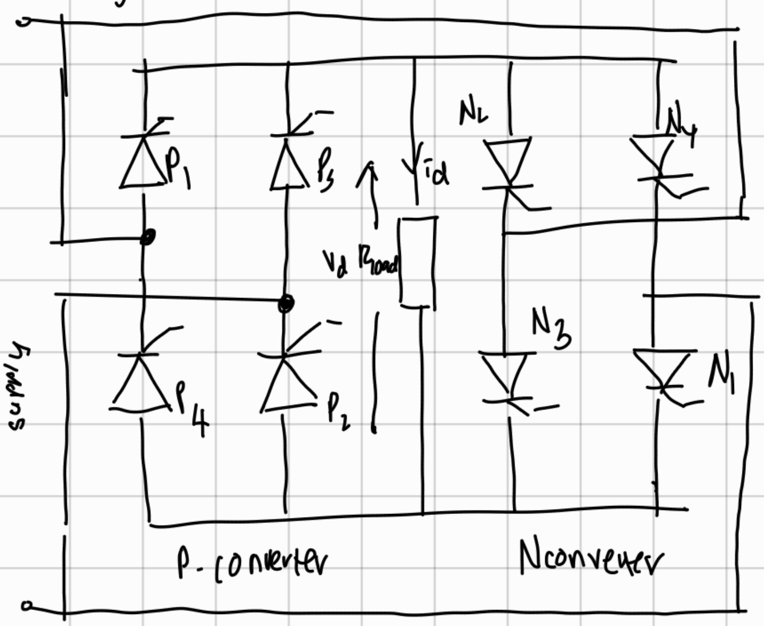
$$V_s = jX_m I_m + (R_s + jX_{ls}) I_s$$?

Ideal V/f profile



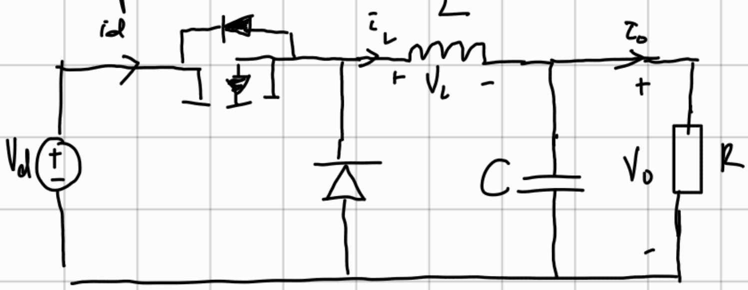
Yoshi

Cycloconverter

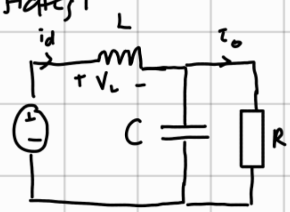


P-converter operates in rectification mode and produces positive voltage for first half cycle

Step-down (Buck) converter

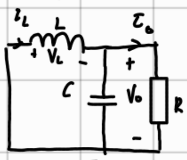


States 1



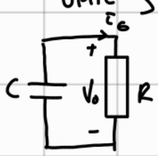
MOSFET ON
Diode · reverse biased
Inductor stores energy
$V_L = V_d - V_o$

State 2



. MOSFET OFF
. Inductor current freewheels through diode
$V_L = -V_o$

State 3



. MOSFET OFF
. Stored energy (inductor) fully discharged
$i_L = 0$

State

a) 1 & 2 Continuous current operation

b) 1 & 2 Boundary between continuous and discontinuous current operation

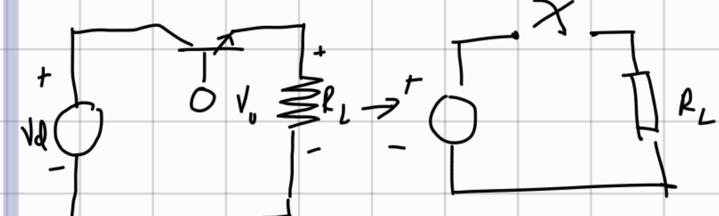
c) 1, 2, 3 Discontinuous current operation

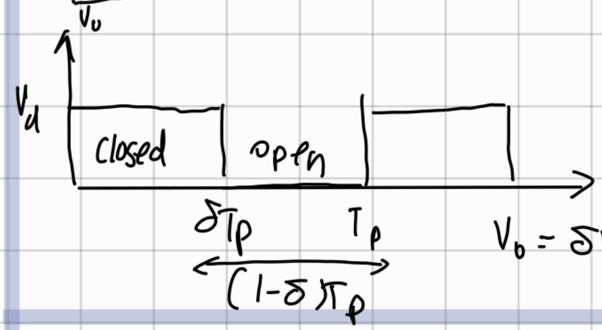
DC-DC converter

linear voltage regulator

$V_O = R_L i_L$

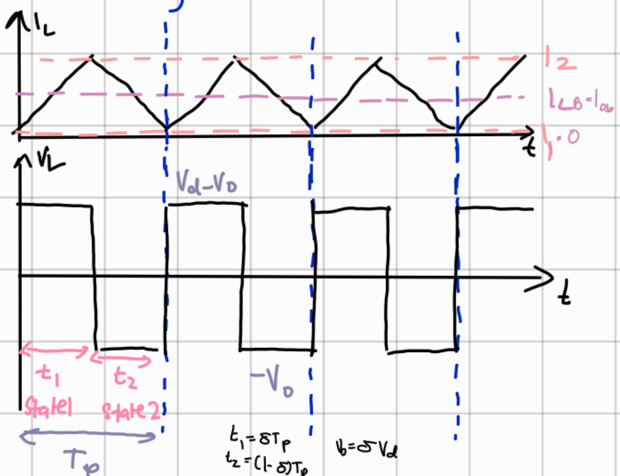
Switching Converter (DC chopper)





$V_o = \delta V_d$

b) Boundary condition



$t_1 = \delta T_p$
$t_2 = (1-\delta)T_p$

$V_o = \delta V_d$

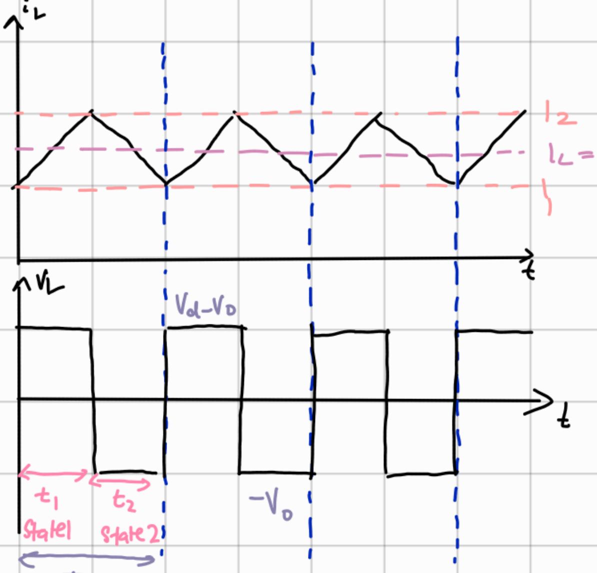
The min inductor current $I_1 = 0$
The average inductor current at boundary $I_{LB} = \frac{I_2}{2}$

From $V_L = L\frac{di_L}{dt}$ at State 1 (on state)
$V_d - V_o = L\frac{I_2}{\delta T_p}$

max current
$I_2 = \frac{(V_d - V_o)\delta T_p}{L}$

$$I_{LB} = \frac{(V_d - V_o)\delta T_p}{2L} = \frac{(V_d - \delta V_d)\delta T_p}{2L} = \frac{(1-\delta)\delta V_d T_p}{2L} = I_{ob}$$

a) Continuous Current Operation (State 1 & 2)



$\boxed{t_1 = \delta T}$ on time

$\boxed{t_2 = (1-\delta)T_p}$ off time

Inductor voltage v_L

(ON) $V_d - V_o$

(OFF) $-V_o$

Inductor current i_L

I_1 = Max current

I_2 = Min current

I_L = Average inductor current

$$\boxed{I_L = I_o = \frac{I_1 + I_2}{2}}$$

At periodic steady-state, the average inductor voltage is 0.

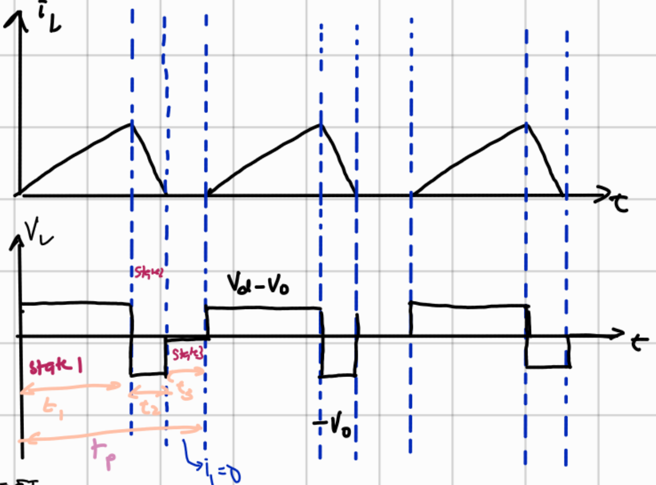
$$V_L = \frac{1}{T_p}\int_0^{t_p} v_L \, dt = 0$$

$$\frac{1}{T_p}\left[(V_d - V_o)t_1 + (-V_o)t_2\right] = 0$$

$$\frac{1}{T_p}\left[(V_d - V_o)\delta T_p + (-V_o)(1-\delta)T_p\right] = 0$$

$$\boxed{V_o = \delta V_d}$$

c) Discontinuous current operation



$t_1 = \delta T_p$

$t_2 = \gamma T_p$

γ = current fall time ratio

t_2 = Current off-time

$$V_L = \frac{1}{T_p}\int_0^{T_p} v_L \, dt = 0$$

$$\frac{1}{T_p}\left[(V_d - V_o)t_1 + (-V_o)t_2\right] = 0$$

$$\frac{1}{T_p}\left[(V_d - V_o)\delta T_p + (-V_o)\gamma T_p\right] = 0$$

$$V_o = \frac{\delta}{\delta + \gamma} V_d$$

From $v_L = L\frac{di_L}{dt}$ State 1 (ON state)

$$V_d - V_o = L\frac{I_2}{\delta T_p} \qquad I_2 = \frac{(V_d - V_o)\delta T_p}{L}$$

State 2 (off state) $-V_o = -L\frac{I_2}{\gamma T_p}$ $\qquad \boxed{I_2 = \frac{\gamma T_p V_o}{L}}$

By equality I_2

$$\frac{(V_d - V_o)\delta T_p}{L} = \frac{\gamma T_p V_o}{L}$$

Current fall time ratio $\qquad \boxed{\gamma = \frac{\delta(V_d - V_o)}{V_o}}$

Average inductor current

$$I_L = \frac{1}{T_p}\int_0^{T_p} i_L \, dt$$

$$= \frac{1}{T_p}\left[\frac{1}{2}\delta T_p I_2 + \frac{1}{2}\gamma T_p I_2\right]$$

$$= \frac{I_2}{2}[\delta + \gamma] = \frac{1}{2}\left(\frac{(V_d - V_o)\delta T_p}{L}\right)\left(\delta + \delta\frac{V_d - V_o}{V_o}\right)$$

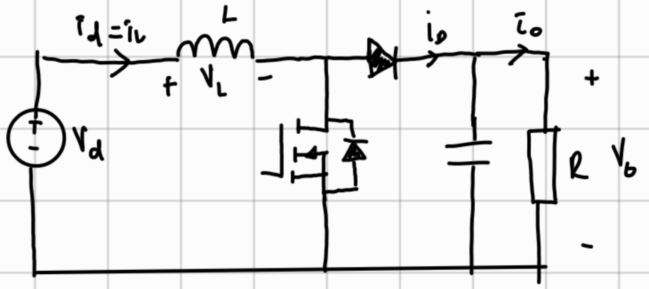
$$= \boxed{\frac{\delta^2 T_p V_d (V_d - V_o)}{2LV_o}} = I_o$$

output current

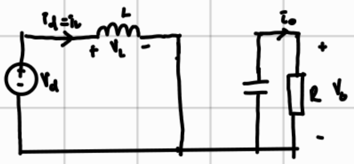
Summary

Continuous current operation	Boundary condition	Discontinuous current operation
$V_o = \delta V_d$	$V_o = \delta V_d$	$V_o = \frac{\delta}{\delta + \gamma} V_d$
$I_L = I_o = \frac{I_1 + I_2}{2}$	$I_{Lb} = \frac{(1-\delta)\delta T_p V_d}{2L}$ $I_{Lb} = I_o$	$I_L = \frac{\delta^2 T_p V_d (V_d - V_o)}{2LV_o}$ $I_L = I_o$

Step-up (BOOST) converter

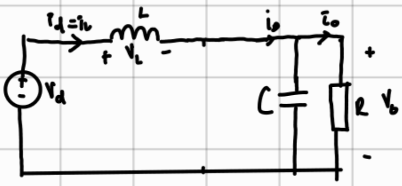


State 1



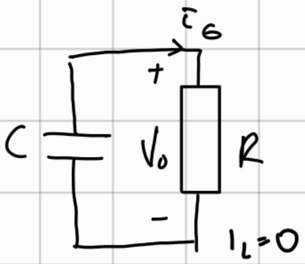
- MOSFET ON
- Diode reverse biased
- Inductor stores energy

State 2



MOSFET OFF
- Inductor discharges energy through diode

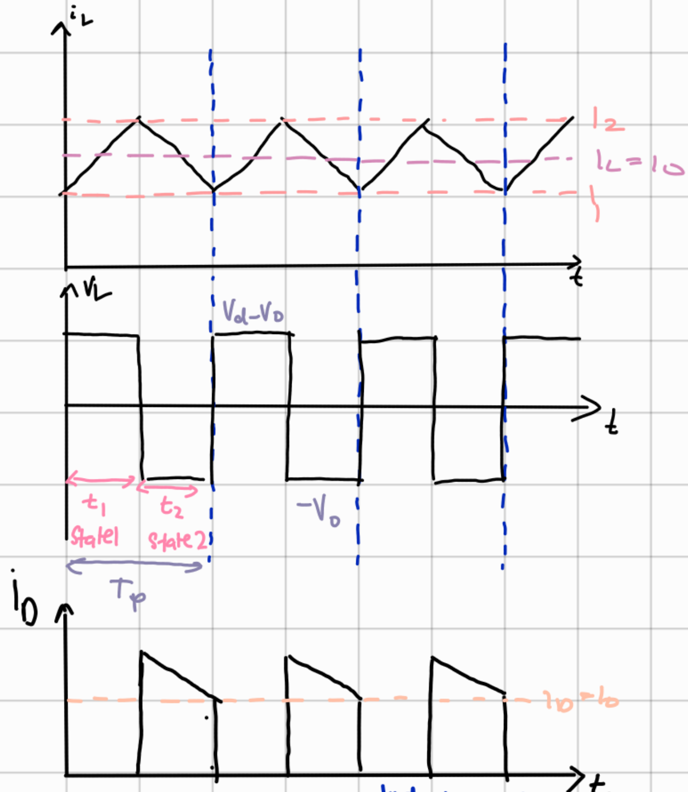
State 3



MOSFET OFF
- Stored energy discharged
- Diode reverse biased

State
a) 1 & 2 Continuous current operation
b) 1 & 2 Boundary between continuous and discontinuous current operation
c) 1, 2, 3 Discontinuous current operation

a) Continuous current condition



$\boxed{t_1 = \delta T_p}$ ON

$\boxed{t_2 = (1-\delta) T_p}$ OFF

Inductor voltage, V_L

V_d State 1 (ON state)

$V_d - V_o$ State 2 (OFF state)

Inductor current i_L I_L : Average inductor current

I_1 = Max. current I_2 = Min. current

Diode current, i_d

I_D = Average diode current

$I_o = I_D$

At periodic steady-state, the average inductor voltage = 0

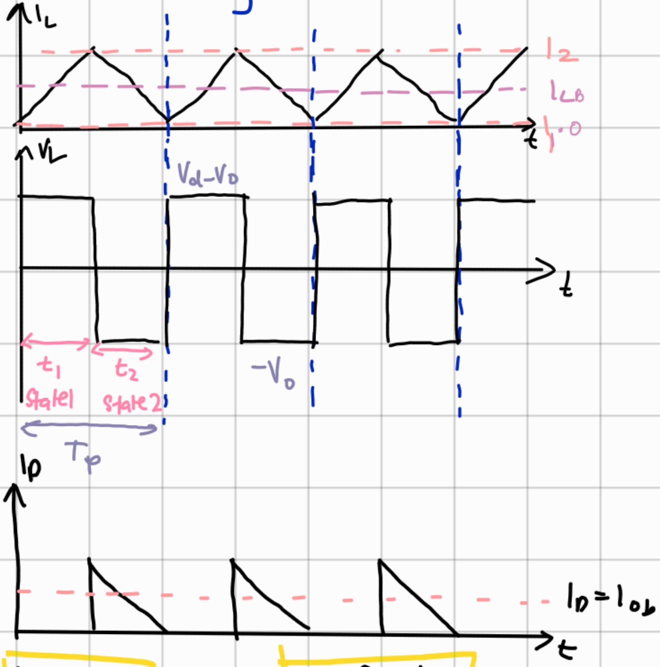
$$V_L = \frac{1}{T_p}\int_0^{T_p} V_L \, dt = 0$$

$$\frac{1}{T_p}\left[V_d t_1 + (V_d - V_o) t_2\right] = 0$$

$$\frac{1}{T_p}\left[V_d \delta T_p + (V_d - V_o)(1-\delta) T_p\right] = 0$$

$$\boxed{V_o = \frac{V_d}{1-\delta}} \quad \boxed{I_L = \frac{I_1 + I_2}{2}}$$

b) Boundary condition



$t_1 = \delta T_p$ $\quad V_d = (1-\delta)V_o$

$t_2 = (1-\delta)T_p$

From $v_L = L\frac{di_L}{dt}$ state 1 (ON state)

$V_d = L\frac{I_2}{t_1}$ $\quad \boxed{I_2 = \frac{V_d \delta T_p}{L}}$

$$I_{Lb} = \frac{I_1 + I_2}{2} = \frac{I_2}{2} = \frac{V_d \delta T_p}{L} = \frac{(1-\delta)V_o \delta T_p}{2L}$$

Assume efficiency = 100%

$P_{in} = P_{out}$, $V_d I_d = V_o I_{ob}$, $I_d = I_{Lb}$

$$I_{ob} = \frac{V_d I_{Lb}}{V_o} = (1-\delta)I_{Lb} = \frac{(1-\delta)^2 V_o \delta T_p}{2L}$$

Alternative derivation for I_{ob}

At periodic steady-state, the average capacitor current = 0

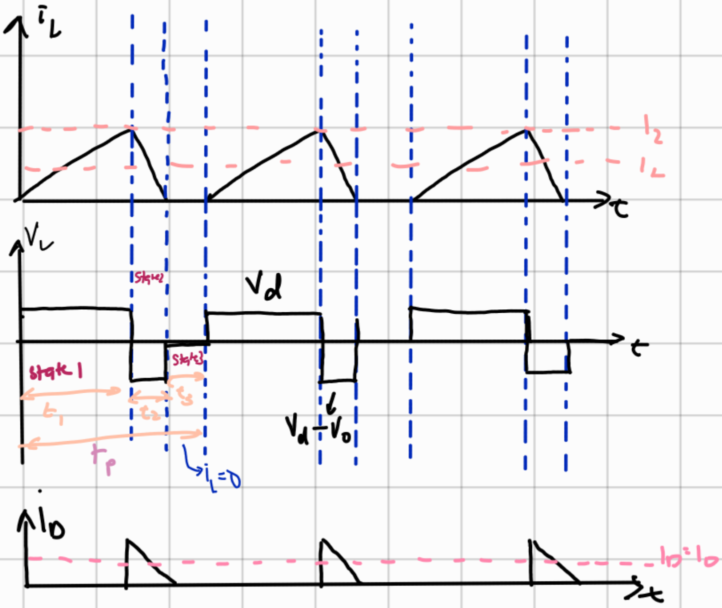
Hence, average $I_D = I_L$

$$I_{ob} = I_D = \frac{1}{T_p}\int_0^{T_p} i_D\, dt = \frac{1}{T_p}\left[\frac{1}{2}(1-\delta)T_p I_2\right]$$

$$= \frac{I_2(1-\delta)}{2}$$

$$I_{ob} = \boxed{\frac{(1-\delta)^2 V_o \delta T_p}{2L}}$$

c) Discontinuous condition



$t_1 = \delta T_p$ $\quad t_2 = \gamma T_p$ $\quad$ From $v_L = L\frac{di_L}{dt}$ state 1 (ON)

$\frac{V_o}{V_d} = \frac{\delta + \gamma}{\gamma}$

$V_d = L\frac{I_2}{\delta T_p}$ $\quad I_2 = \frac{\delta T_p V_d}{L}$

From $v_L = L\frac{di_L}{dt}$ state 2 (OFF)

$V_d - V_o = -L\frac{I_2}{\gamma T_p}$

$$\boxed{I_2 = -\frac{\gamma T_p (V_d - V_o)}{L}}$$

$$\frac{\delta T_p V_d}{L} = \frac{-\gamma T_p (V_d - V_o)}{L}$$

current fall time ratio $\quad \boxed{\gamma = \frac{\delta V_d}{V_o - V_d}}$

Average inductor current

$$I_L = \frac{1}{T_p}\int_0^{T_p} i_L\, dt = \frac{1}{2}\left[\frac{1}{2}\delta T_p I_2 + \frac{1}{2}\gamma T_p I_2\right] = \frac{1}{2}(\delta + \gamma)I_2$$

$$= \frac{1}{2}(\delta + \gamma)\frac{\delta T_p V_d}{L} = \frac{1}{2}\left(\delta + \frac{\delta V_d}{V_o - V_d}\right)\frac{\delta T_p V_d}{L} = \frac{\delta^2 T_p V_d V_o}{2L(V_o - V_d)} = I_L$$

Duty ratio $\delta = \sqrt{\frac{2L(V_o - V_d)I_L}{T_p V_d V_o}}$

At steady state, average $I_{capacitor} = 0$. Hence, average diode current = $I_L = I_D$

$$I_o = I_D = \frac{1}{T_p}\int_0^{T_p} i_D\, dt = \frac{1}{T_p}\left[\frac{1}{2}\gamma T_p I_2\right] = \frac{1}{2}\gamma I_2,$$

$$= \frac{1}{2} I_2 \frac{\delta V_d}{V_o - V_d} = \frac{1}{2}\left(\frac{\delta V_d}{V_o - V_d}\right)\left(\frac{\delta T_p V_d}{L}\right)$$

$$= \frac{\delta^2 T_p V_d^2}{2L(V_o - V_d)}$$

Step-Up Boost

Continuous	Boundary	Discontinuous Current operation
$V_o = \frac{V_d}{1-\delta}$	$V_o = \frac{V_d}{1-\delta}$	$\frac{V_o}{V_d} = \frac{\delta+\gamma}{\gamma}$
$I_L = \frac{I_1+I_2}{2}$	$I_{Lb} = \frac{(1-\delta)V_o \delta T_p}{2L}$	$I_L = \frac{\delta^2 T_p V_d V_o}{2L(V_o - V_d)}$
	$I_{ob} = \frac{(1-\delta)^2 V_o \delta T_p}{2L}$	$I_o = \frac{\delta^2 T_p V_d^2}{2L(V_o - V_d)}$

Step-down Buck

Continuous current operation	Boundary condition	Discontinuous current operation
$V_o = \delta V_d$	$V_o = \delta V_d$	$V_o = \frac{\delta}{\delta+\gamma} V_d$
$I_L = I_o = \frac{I_1+I_2}{2}$	$I_{Lb} = \frac{(1-\delta)\delta T_p V_d}{2L}$	$I_L = \frac{\delta^2 T_p V_d (V_d - V_o)}{2L V_o}$
	$I_{Lb} = I_o$	$I_L = I_o$