SEMESTER 2 EXAMINATIONS 2016/17

CIRCUITS AND TRANSMISSION

Duration 120 mins (2 hours)

This paper contains 6 questions

Answer **ONE** question in **Section A, ONE** question in **Section B** and **ONE** question in **Section C.**

Section A carries 33% of the total marks for the exam paper.

Section B carries 33% of the total marks for the exam paper.

Section C carries 33% of the total marks for the exam paper.

Only University approved calculators may be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct 'Word to Word' translation dictionary AND it contains no notes, additions or annotations.

11 page examination paper (+ 2 page formula sheet, 1 page The Complete Smith Chart)

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SECTION A

Answer ONE out of TWO questions in this section

1 (a) In the network shown in Figure 1, two voltage sources act on the load impedance connected between A, B. If the load is variable in both resistance and reactance, what load Z_L will receive the maximum power and what is the value of the maximum power? Use Millman's theorem.

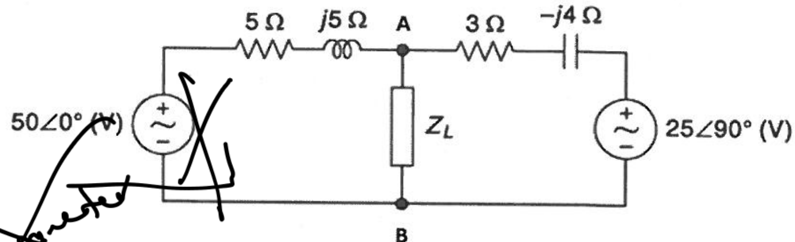
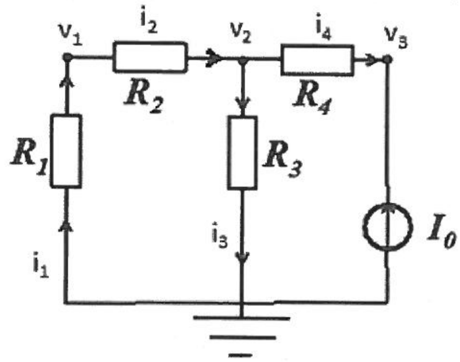


Fig. 1. The circuit for question 1(a)

[7 marks]

(b) Consider the circuit in Figure 2. Using node analysis with the ground reference node indicated, find the value of v_1, v_2, v_3 when $I_0 = 1\ A$ and $R_1 = R_2 = R_3 = R_4 = 1\ \Omega$.



$V_1 = 0.334$ V

$V_2 = 0.6667$ V

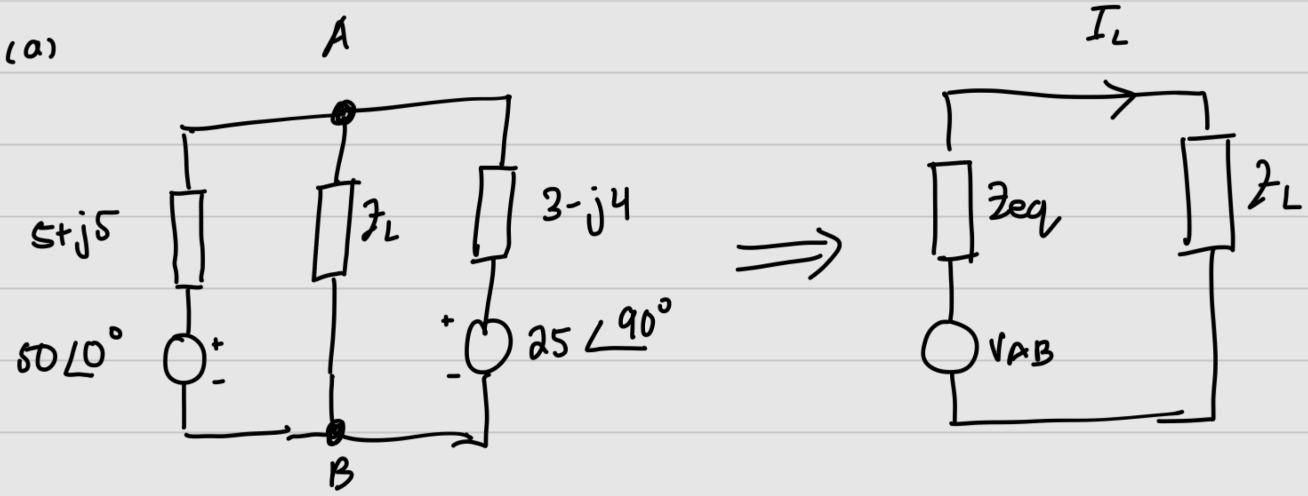
$V_3 = 1.6667$ V

Fig. 2. The circuit for question 1(b)

[7 marks]

Question continues on following page

1. (a)



$$50\angle 0^\circ \Rightarrow 50 + j0$$

$$25\angle 90^\circ \Rightarrow 0 + j25$$

$$V_{AB} = \frac{\frac{50}{5+j5} + \frac{j25}{3-j4}}{\frac{1}{5+j5} + \frac{1}{3-j4}}$$

$$= \frac{(5-j5) + (-4+j3)}{(0.1 - j0.1) + (0.12 + j0.16)}$$

$$= \frac{1 - j2}{0.22 + j0.06}$$

$$= \frac{25}{13} - j\frac{125}{13}$$

$$\frac{1}{Z_{eq}} = \frac{1}{5+j5} + \frac{1}{3-j4}$$

$$\frac{1}{Z_{eq}} = \frac{11}{50} + \frac{3}{50}j$$

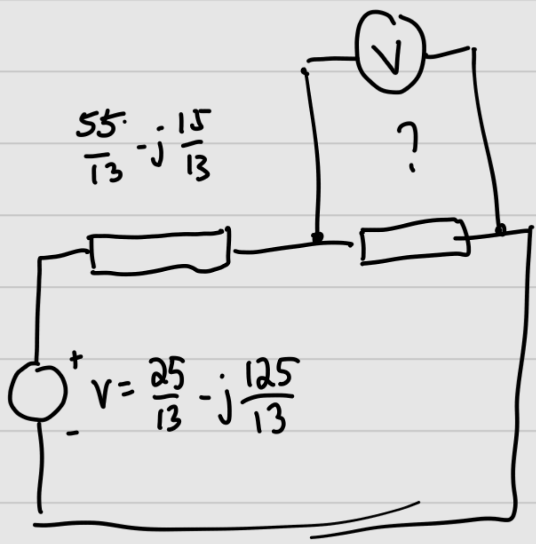
$$Z_{eq} = \frac{55}{13} - j\frac{15}{13}$$

$$Z_{total} = Z_{eq} + Z_L$$

$$P = VI$$

$$V = IR$$

$$I = \frac{V}{R}, \quad P = \frac{V^2}{R}$$



$$V_L = \frac{Z_L}{Z_{eq} + Z_L} \times V_{AB}$$

$$Z_L = a + jb$$

$$= \frac{Z_L\left(\frac{25 - j125}{13}\right)}{\frac{55 - j15}{13} + Z_L}$$

$$= \frac{Z_L(25 - j125)}{55 - j15 + \frac{Z_L}{13}}$$

$$P = \frac{V^2}{R}$$

$$= \frac{Z_L\,(25 - j125)^2}{\left(55 - j15 + \frac{Z_L}{13}\right)^2}$$

$$= \frac{Z_L\,(-15000 - j6250)}{2800 - j1650 + \frac{55 Z_L}{13} - \frac{j15 Z_L}{13} + \frac{55 Z_L}{13} - \frac{j15 Z_L}{13} + \frac{Z_L^2}{169}}$$

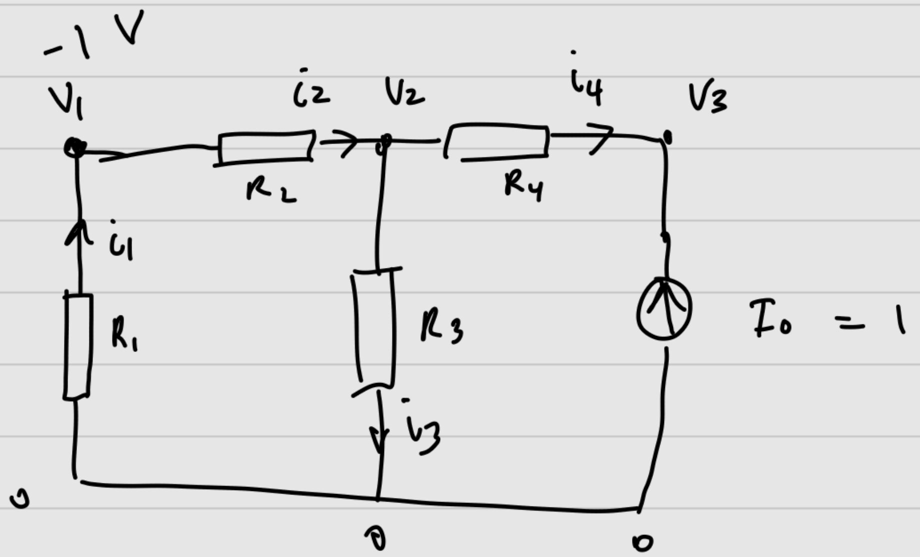
$$= \frac{Z_L\,(-15000 - j6250)}{2800 - j1650 + \frac{Z_L(110 - j30)}{13} + \frac{Z_L^2}{169}}$$

$$\frac{dP}{dZ_L} = 0 = vu' - uv'$$

$$= \left(2800 - j1650 + \frac{Z_L(110 - j30)}{13} + \frac{Z_L^2}{169}\right)(-1500 - j6250)$$

$$- Z_L(-15000 - j6250)\left(\frac{110 - j30}{13} + \frac{2Z_L}{169}\right)$$

(b) $R = 1\ \Omega$



①

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ $R_2 + \frac{R_1}{2}$

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2.5 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ $\frac{R_1}{2}$, $\frac{R_2}{2.5}$

$$\begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 1 & -0.4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ $R_3 + R_2$

$$\begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 1 & -0.4 \\ 0 & 0 & 0.6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$0.6V_3 = 1$$

$$V_3 = 1.667 \text{ V}$$

$$V_2 - 0.4V_3 = 0$$

$$0.4(1.667) = V_2$$

$$V_2 = 0.667 \text{ V}$$

$$V_1 - 0.5V_2 = 0$$

$$V_1 = 0.5(0.667)$$

$$V_1 = 0.333 \text{ V}$$

(c) Consider the two-port network in Figure 3, where the first impedance is $Z_1 = R$ (a resistance), the second one is $Z_2 = Ls$ (an inductor), and the admittance $Y = Cs$ (a capacitor). Derive the (A,B,C,D) representation of this network.

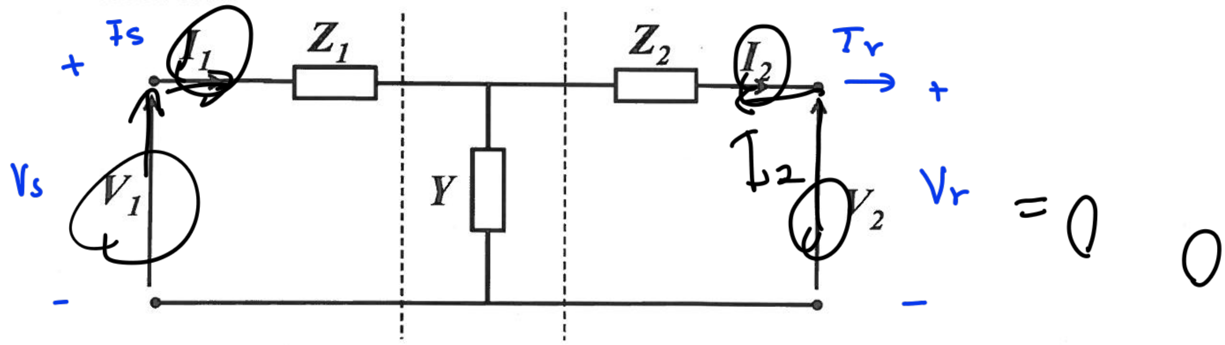


Fig. 3. The circuit for question 1(c), 1(d) and 1(e)

[5 marks]

(d) Consider the circuit in Figure 3. *Starting from the (A,B,C,D) representation* found answering Question 1 (c), derive its equivalent Z-representation. Show clearly how you arrived at your conclusions, i.e. how the Z-matrix is related to the (A,B,C,D) one.

[7 marks]

(e) Under which conditions is the two-port network in Figure 3 reciprocal? What are the consequences of reciprocity on the (A,B,C,D) representation of a two-port? Give a symbolic expression for the iterative impedance of a symmetric, reciprocal network as a function of its (A,B,C,D) parameters.

[7 marks]

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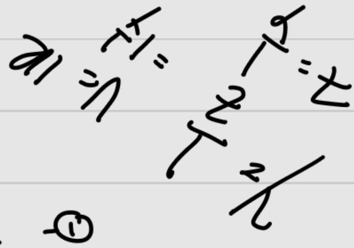
(c) $V_s = AV_r + BI_r$ 

$I_s = CV_r + DI_r$

$V_1 = AV_2 + BI_2$ — ①

$I_1 = CV_2 + DI_2$ — ②

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ oc $\qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$ sc

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ oc $\qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$ sc

KVL :

$V_1 = Z_1 I_1 + \frac{1}{Y}(I_1 + I_2)$ — ①

$V_2 = Z_2 I_2 + \frac{1}{Y}(I_1 + I_2)$ — ②

$V_2 = Z_2 I_2 + \frac{1}{Y} I_1 + \frac{1}{Y} I_2$

$\frac{1}{Y} I_1 = V_2 - Z_2 I_2 - \frac{1}{Y} I_2$

$I_1 = Y V_2 + \left(\frac{-Z_2 - 1/Y}{1/Y}\right) I_2$ — ③

$V_1 = Z_1 I_1 + \frac{1}{Y} I_1 + \frac{1}{Y} I_2$

$V_1 = Z_1\left(Y V_2 + \left(\frac{-Z_2 - 1/Y}{1/Y}\right) I_2\right) + \frac{1}{Y}\left(Y V_2 + \left(\frac{-Z_2 - 1/Y}{1/Y}\right) I_2\right)$

$+ \frac{1}{Y} I_2$

$V_1 = YZ_1 V_2 + \left(\frac{-Z_1 Z_2 - Z_1 1/Y}{1/Y}\right) I_2 + V_2 + \cancel{(-Z_2 - \frac{1}{Y}) I_2}$

$+ \cancel{\frac{1}{Y} I_2}$

$V_1 = (Z_1 Y + 1) V_2 + \left(\frac{-Z_1 Z_2 - Z_1 1/Y}{1/Y} - Z_2\right) I_2$

$$V_1 = \left(\frac{Z_1 + 1/Y}{1/Y}\right) V_2 + \left(\frac{-Z_1 Z_2}{1/Y} - Z_1 - Z_2\right) I_2$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \dfrac{Z_1 + 1/Y}{1/Y} & -Z_1 Z_2 Y - (Z_1 + Z_2) \\ Y & -\dfrac{Z_2 - 1/Y}{1/Y} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$[T] = \begin{bmatrix} Z_1 Y + 1 & Z_1 Z_2 Y + (Z_1 + Z_2) \\ Y & 1 + Y Z_2 \end{bmatrix}$$

(d) $V_1 = Z_{11} I_1 - Z_{12} I_2$ — ①

$V_2 = Z_{21} I_1 - Z_{22} I_2$ — ②

$V_1 = A V_2 + B I_2$

$I_1 = C V_2 + D I_2$

$C V_2 = I_1 - D I_2$

$V_2 = \frac{1}{C} \cdot I_1 - \frac{D}{C} I_2$ — ③

$V_1 = A V_2 + B I_2$

$V_1 = A\left(\frac{1}{C} \cdot I_1 - \frac{D}{C} I_2\right) + B I_2$

$V_1 = \frac{A}{C} \cdot I_1 - \frac{AD}{C} I_2 + B I_2$

$V_1 = \frac{A}{C} \cdot I_1 + \left(-\frac{AD}{C} + B\right) I_2$ — ④

compare ① & ④

$Z_{11} = \frac{A}{C}$

$= \frac{Z_1 Y + 1}{Y}$

$$-Z_{12} = -\frac{AD}{C} + B$$

$$Z_{12} = \frac{(Z_1Y+1)(1+YZ_2)}{Y} - \left(Z_1Z_2Y + (Z_1+Z_2)\right)$$

$$= \frac{Z_1Y + Z_1Z_2Y^2 + 1 + YZ_2}{Y} - Z_1Z_2Y - Z_1 - Z_2$$

$$= \cancel{Z_1} + Z_1\cancel{Z_2}Y + \frac{1}{Y} + \cancel{Z_2} - Z_1\cancel{Z_2}Y - \cancel{Z_1} - \cancel{Z_2}$$

$$= \frac{1}{Y}$$

Compare (2) and (3)

$$Z_{21} = \frac{1}{C}$$

$$= \frac{1}{Y}$$

$$-Z_{22} = -\frac{D}{C}$$

$$Z_{22} = \frac{1+YZ_2}{Y}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{Z_1Y+1}{Y} & \frac{1}{Y} \\ \frac{1}{Y} & \frac{1+YZ_2}{Y} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

(e)

- Reciprocity ensures that power is conserved in a two-port network when the direction of signal flow is reversed, the power entering the network through one port is equal to the power leaving the network through the other part.
- In reciprocal network, the mutual impedance is equal to the mutual admittance.
- Identical forward & reverse transmission: Reciprocity implies that the forward and reverse transmission characteristics of two-port are equal.

$$Z = \frac{(A+D-2B)}{A+D+2B}$$

Assumes network both symmetric (B=C) and reciprocal A=D. The iterative impedance provides a measure of the characteristic impedance that would result if infinite chain of

13/32

Identical two-port networks were cascaded together


Question 2

(a) Using Rosen's theorem, find the equivalent "delta" circuit of the "star" circuit in Figure 4.

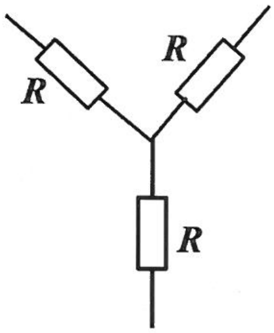


Fig. 4. The circuit for question 2(a)

[5 marks]

(b) Write down the A, B, C, D representation of the π-network in Figure 5. What is the A, B, C, D representation if two π-networks such as the one shown in Figure 5 are cascaded?

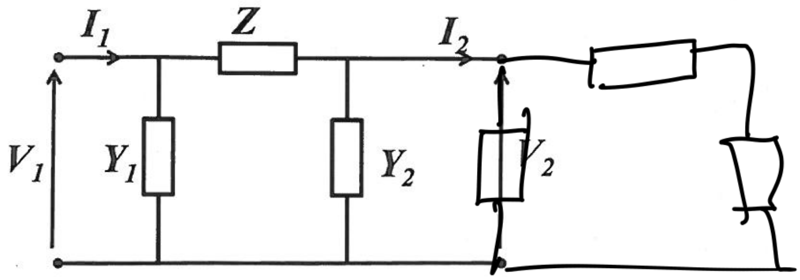
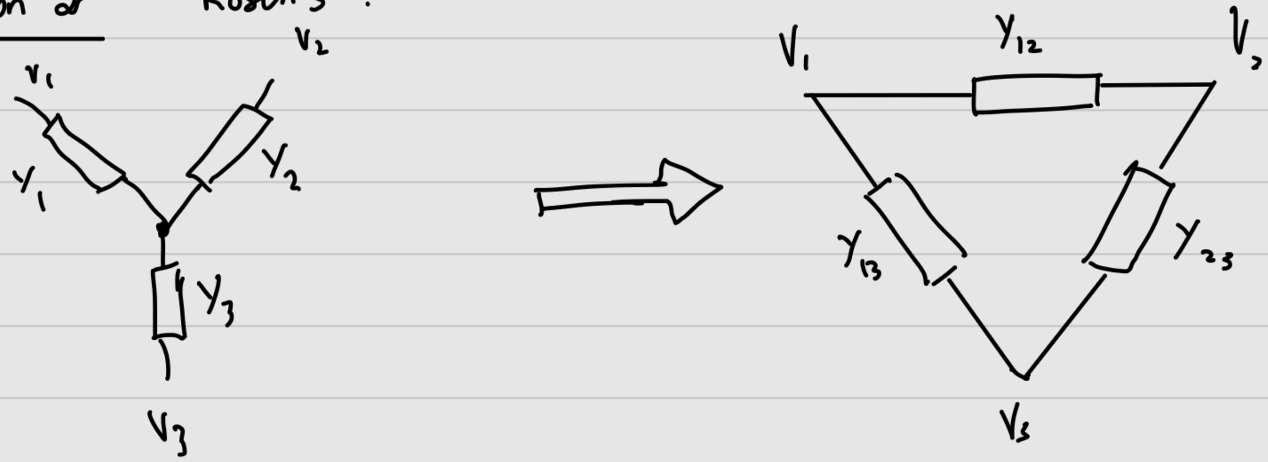


Fig. 5. The circuit for question 2(b)

[7 marks]

Question continues on following page

Question 2 Rosen's :



$$Y_{12} = \frac{Y_1 Y_2}{Y_1 + Y_2 + Y_3}$$

$$Y_{23} = \frac{Y_2 Y_3}{Y_1 + Y_2 + Y_3}$$

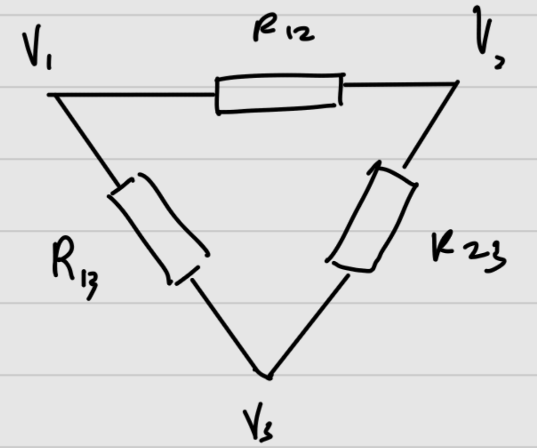
$$Y_{31} = \frac{Y_3 Y_1}{Y_1 + Y_2 + Y_3}$$

$$Y_1 = Y_{31} + Y_{12} + \frac{Y_{31} Y_{12}}{Y_{23}}$$

$$Y_2 = Y_{12} + Y_{23} + \frac{Y_{12} Y_{23}}{Y_{13}}$$

$$Y_3 = Y_{23} + Y_{31} + \frac{Y_{23} Y_{31}}{Y_{12}}$$

(a)



$$R_{12} = R_{23} = R_{31} = \frac{R^2}{3R}$$

$$= \frac{R}{3}$$

(b)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ Y_1 & ZY_1+1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1+ZY_2 & Z \\ Y_1+ZY_1Y_2+Y_2 & ZY_1+1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Cascaded :

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_3 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_4 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ Y_1 & Z_1Y_1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_2+Y_3 & 1 \end{bmatrix} \begin{bmatrix} 1+Z_2Y_4 & Z_2 \\ Y_4 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1+Z_1Y_2+Z_1Y_3 & Z_1 \\ Y_1+Z_1Y_1Y_2+Z_1Y_1Y_3 & Z_1Y_1 \end{bmatrix} \begin{bmatrix} 1+Z_2Y_4 & Z_2 \\ Y_4 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} (1+Z_1Y_2+Z_1Y_3)(1+Z_2Y_4)+Z_1Y_4 & Z_2+Z_1Z_2Y_2+Z_1Z_2Y_3+Z_1 \\ (Y_1+Z_1Y_1Y_2+Z_1Y_1Y_3)(1+Z_2Y_4)+Z_1Y_1Y_4 & Z_2Y_1+Z_1Z_2Y_1Y_2+Z_1Z_2Y_1Y_3+Z_1Y_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

if $Z_1 = Z_2 = Z$, $Y_1 = Y_3$, $Y_2 = Y_4$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} (1+ZY_2+ZY_1)(1+ZY_2)+ZY_2 & Z+Z^2Y_2+Z^2Y_1+Z \\ (Y_1+ZY_1Y_2+ZY_1^2)(1+ZY_2)+ZY_1Y_2 & ZY_1+Z^2Y_1Y_2+Z^2Y_1^2+ZY_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

(c) A given resistive network has the (A,B,C,D) representation

$$\begin{bmatrix} 2 & 10 \\ \frac{3}{10} & 2 \end{bmatrix}.$$

Find an equivalent π-circuit containing only resistances. Make clear what the numerical values of the resistances in the equivalent network are.

[8 marks]

(d) Using Thevenin's theorem, find the equivalent circuit (with respect to the terminals A and B) to that in Figure 6.
Use such equivalent circuit to compute the numerical value of the current through R_3 when $R_1 = R_2 = R_3 = 1\ \Omega$ and $E_1 = 2\ V$.

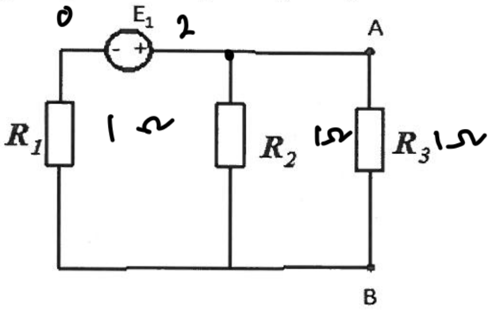


Fig. 6. The circuit for question 2(d)

[8 marks]

(e) Give expressions for the image impedances of the two-port in Figure 7.

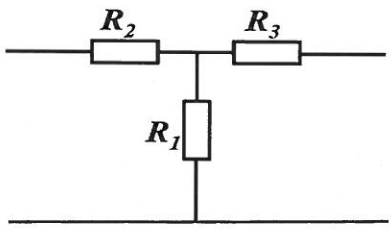


Fig. 7. The circuit for question 2(e)

[5 marks]

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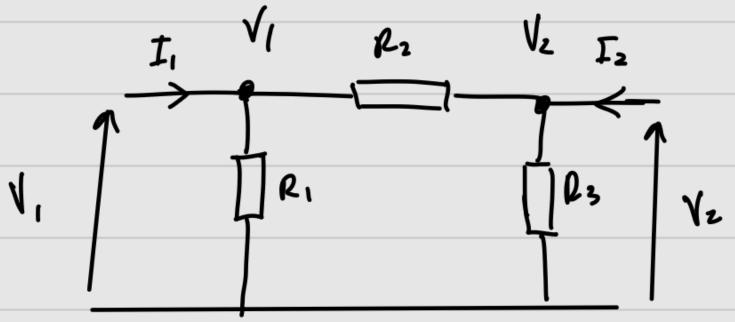
(c) $\begin{bmatrix} 2 & 10 \\ \frac{3}{10} & 2 \end{bmatrix}$ $A = D$, symetry

$$AD - BC = 2(2) - 10\left(\frac{3}{10}\right)$$

$$= 4 - 3$$

$$= 1 \implies \text{Reciprocal}$$



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ \frac{3}{10} & 2 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \implies \begin{aligned} V_1 &= 2V_2 - 10 I_2 \quad (1) \\ I_1 &= \frac{3}{10} V_2 - 2 I_2 \quad (2) \end{aligned}$$

KCL :

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} \quad (3)$$

$$I_2 = \frac{V_2}{R_3} - \frac{V_1 - V_2}{R_2} \quad (4)$$

$$\frac{V_1 - V_2}{R_2} = \frac{V_2}{R_3} - I_2$$

$$V_1 - V_2 = \frac{R_2}{R_3} V_2 - R_2 I_2$$

$$V_1 = \frac{R_2}{R_3} V_2 + V_2 - R_2 I_2$$

$$V_1 = \left(\frac{R_2}{R_3} + 1\right) V_2 - R_2 I_2$$

$\therefore$ $R_2 = 10$

$$\frac{10}{R_3} + 1 = 2$$

$$R_3 = 10$$

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

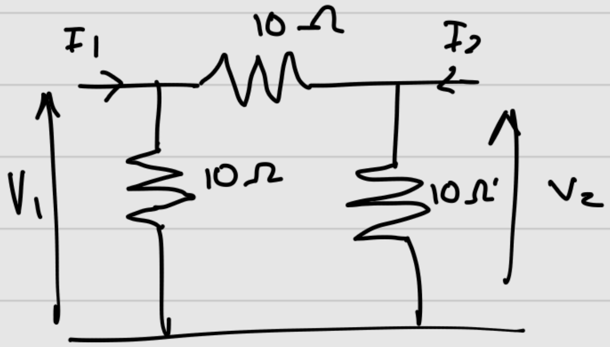
$$I_1 = \frac{2V_2 - 10 I_2}{R_1} + \frac{2V_2 - 10 I_2 - V_2}{10}$$

$$I_1 = \frac{2}{R_1} V_2 - \frac{10}{R_1} I_2 + \frac{1}{5} V_2 - I_2 - \frac{1}{10} V_2$$

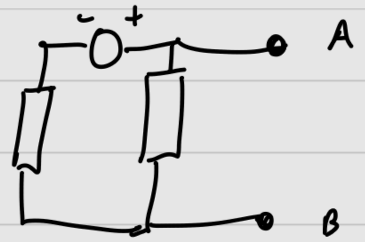
$$I_1 = \left(\frac{2}{R_1} + \frac{1}{5} - \frac{1}{10}\right) V_2 - \left(\frac{10}{R_1} + 1\right) I_2$$

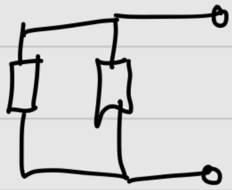
$$\frac{10}{R_1} + 1 = 2$$

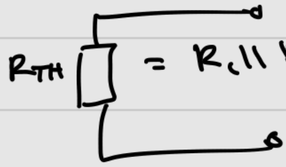
$$R_1 = 10$$



(d)



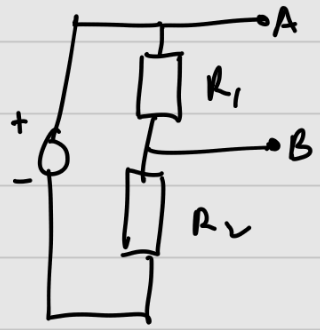




$R_{TH} = R_1 \parallel R_2$

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2}$$

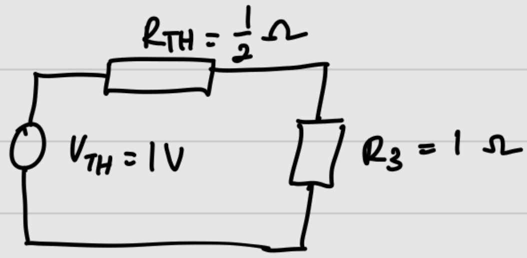
$$= \frac{1}{2}$$



$$V_{TH} = \frac{R_1}{R_1 + R_2} E_1$$

$$= \frac{1}{2} E_1$$

$$= 1 \ V$$

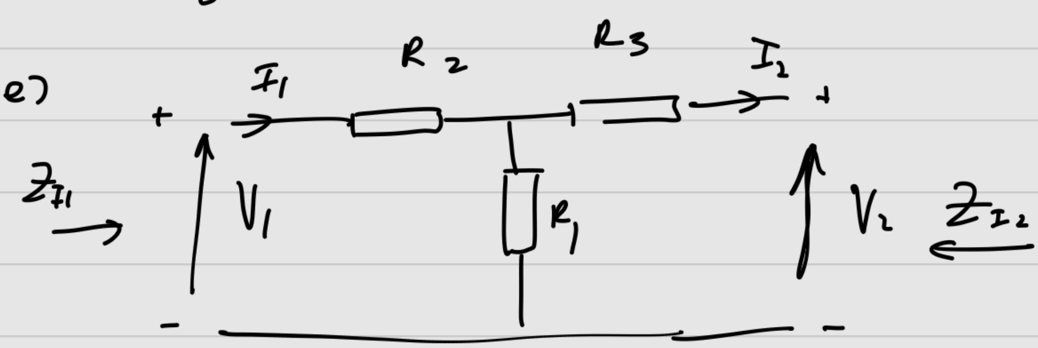


$$I_{R3} = \frac{V_{TH}}{R_{TH} + R_3}$$

$$= \frac{1}{\frac{1}{2} + 1}$$

$$= \frac{2}{3} \ A$$

(e)



$$Z_{I1} = \sqrt{\frac{AB}{CD}} \qquad Z_{I2} = \sqrt{\frac{DB}{CA}}$$

KVL

$$V_1 = I_1R_2 + (I_1+I_2)R_1$$
$$V_2 = I_2R_3 + (I_1+I_2)R_1$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_2 = I_2R_3 + I_1R_1 + I_2R_1$$
$$I_1R_1 = V_2 - I_2R_3 - I_2R_1$$
$$I_1 = \frac{1}{R_1}V_2 - \left(\frac{R_3+R_1}{R_1}\right)I_2 \quad (1)$$

$$V_1 = \frac{R_2}{R_1}V_2 - \left(\frac{R_2R_3+R_2R_1}{R_1}\right)I_2 + V_2 - (R_3+R_1)I_2 + I_2R_1$$

$$= \left(\frac{R_2}{R_1}+1\right)V_2 - \left(\frac{R_2R_3}{R_1} + R_2 + R_3 + R_1 - R_1\right)I_2$$

$$= \left(\frac{R_2+R_1}{R_1}\right)V_2 - \left(\frac{R_2R_3}{R_1} + R_2 + R_3\right)I_2 \quad (2)$$

$$Z_{I1} = \frac{R_2^2 + R_1R_2R_3}{R_1} + \frac{R_2^2+R_1R_2}{R_1} + \frac{R_2R_3+R_1R_3}{R_1}$$

SECTION B

Answer ONE out of TWO questions in this section

Question 3

(a) Describe what is meant by distortion of a lossy line, and state what relationship should hold among the line parameters per unit length (R, L, G and C) of a lossy line to obtain a distortion-free line, assuming the line operates at angular frequency ω.

[10 marks]

(b) A lossless transmission line has the following per unit parameters: $L = 0.4\ \mu H/m$ and $C = 130 pF/m$. Calculate the propagation constant, characteristic impedance, wavelength and the phase velocity at 300 GHz. If the transmission line has a length of 10 cm, and is terminated with a load $Z_L = 30\Omega$, calculate the reflection coefficient at the load and the input impedance of the line.

[6 marks]

$$\gamma = \alpha + j\beta$$
$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= \sqrt{(j\omega L)(j\omega C)}$$
$$= \sqrt{j^2 (\omega)^2 LC}$$
$$= j\omega\sqrt{LC}$$
$$\alpha = 0$$

Question continues on following page

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Q3 (b)

Lossless line

$L = 0.4\,\mu H/m$, $f = 300\,GHz$

$C = 130\,pF/m$, $l = 10\,cm$

$\beta = ?$ $Z_L = 30\,\Omega$

$Z_0 = ?$ $\Gamma_L = ?$

$\lambda = ?$ $Z_{in} = ?$

$v_p = ?$

$R = G = \alpha = 0$

$$\beta = \omega\sqrt{LC} = 2\pi(300\times10^9)\sqrt{(0.4\times10^{-6})(130\times10^{-12})}$$

$=$

$\beta = 13592.61$ rad/m

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.4\times10^{-6}}{130\times10^{-12}}}$$

$= 55\,\Omega$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{13592.61} = 4.62\times10^{-4}\ m$$

$$V = \frac{\omega}{\beta} = \frac{2\pi\times300\times10^9}{13592.61} = 13.9\times10^7\ m/s$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30-55}{30+55} = -0.294$$

$$Z_{in} = Z_0\,\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$$

$$Z_{in} = 50\times\frac{30 - j337.15}{55 - j183.9}$$

$= 99.59\ \angle -10^\circ$

(c) Consider two lossless transmission lines operating at angular frequency ω carrying a signal with wavelength λ as shown in figure 8. Transmission line A is terminated with a load impedance Z_L while transmission line B is terminated with a load impedance (Z_0^2/Z_L). If the length of transmission lines A and B are $l+\lambda/4$ and l respectively and the characteristic impedance of both lines is Z_0, prove that the following equations are correct:

(i) $Z_{inA} = Z_{inB}$

(ii) $\Gamma_{LA} = -\Gamma_{LB}$

where Z_{inA}, Z_{inB}, Γ_{LA} and Γ_{LB} are the input impedances of line A and B and the reflection coefficients of transmission line A and B at the load respectively.

[17 marks]

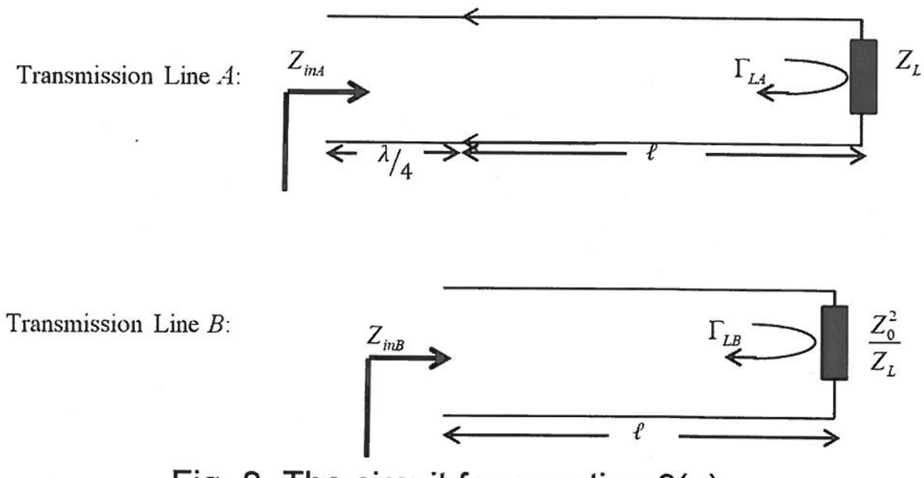

Fig. 8. The circuit for question 3(c)

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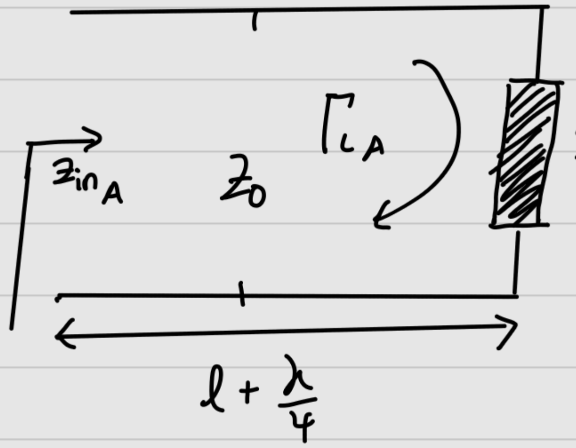
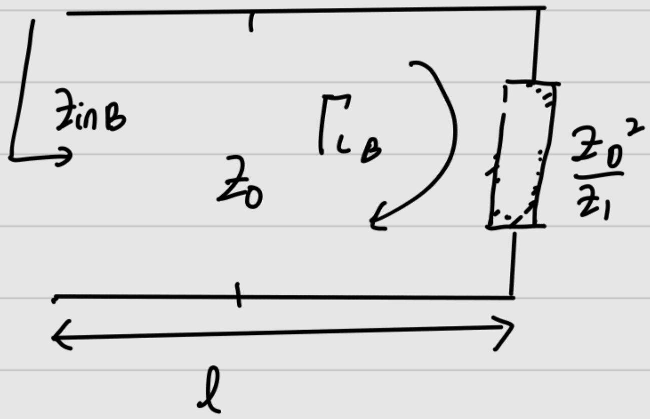
Q3 (c)



$$Z_{inA} = Z_0 \frac{Z_L + jZ_0 \tan\left(\beta\left(l + \frac{\lambda}{4}\right)\right)}{Z_0 + jZ_L \tan\left(\beta\left(l + \frac{\lambda}{4}\right)\right)}$$

$$Z_{inA} = Z_0 \frac{Z_L + jZ_0 \tan\left(\beta l + \frac{\lambda}{2}\right)}{Z_0 + jZ_L \tan\left(\beta l + \frac{\lambda}{2}\right)}$$

$$Z_{inA} = Z_0 \frac{Z_L - jZ_0 \omega \tan(\beta l)}{Z_0 - jZ_L \omega \tan(\beta l)}$$



$Z_{inA} = Z_{inB}$

$\Gamma_{LA} = \Gamma_{LB}$

$\beta = \frac{2\pi}{\lambda}$

$\beta = \left(l + \frac{\lambda}{4}\right) = \beta l + \frac{\beta\lambda}{4}$

$= \beta l + \frac{2\pi}{\lambda} \times \frac{\lambda}{4}$

$= \beta l + \frac{\pi}{2}$

$$Z_{inA} = Z_0 \frac{Z_L \tan\beta l - jZ_0}{Z_0 \tan\beta l - jZ_L} = Z_0 \frac{Z_0 + jZ_L \tan\beta l}{Z_L + jZ_0 \tan\beta l}$$

$$Z_{inB} = Z_0 \frac{\left(\frac{Z_0^2}{Z_L}\right) + jZ_0 \tan\beta l}{Z_0 + j\frac{Z_0^2}{Z_L}\tan\beta l}$$

$$Z_{inB} = Z_0 \frac{Z_0 + jZ_L \tan\beta l}{Z_L + jZ_0 \tan\beta l}$$

$$\Gamma_{LA} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_{LB} = \frac{\frac{Z_0^2}{Z_L} - Z_0}{\frac{Z_0^2}{Z_L} + Z_0} = \frac{Z_0 - Z_L}{Z_L + Z_0} = -\Gamma_{LA}$$

$$\Gamma_{LA} = -\Gamma_{LB}$$

Question 4

A strip line built on alumina substrate used at a frequency of *5 GHz* has the following distributed circuit coefficients at that frequency:

$R = 1.64\ \Omega/m;\ L = 5.2 \times 10^{-7}\ H/m;$
$G = 6.5 \times 10^{-3}\ S/m;\ C = 2.08 \times 10^{-10}\ F/m.$

(a) Find the characteristic impedance of the line at the frequency of operation (*5 GHz*) and comment on the result obtained in terms of losses of the line.

[5 marks]

(b) Assuming that the length of this transmission line is *10 mm*, calculate the attenuation and phase difference that a sinusoidal voltage with amplitude of *5 V* and frequency of *5 GHz* will experience.

[3 marks]

(c) Assuming that the load impedance connected to this line is $100\ \Omega$ and transmission line is loss free, calculate the Voltage Standing Wave Ratio (VSWR) and Return Loss (RL).

[3 marks]

(d) Design a simple L network with the topology shown in figure (9) working at *5 GHz* to match a load impedance of $Z_L = 25 + j30\ \Omega$ to the *10 mm* strip line assuming that the strip line is loss free.

[12 marks]

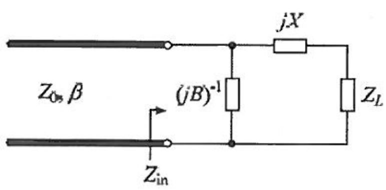


Fig. 9. The circuit for question 4(a) iv

Question continues on following page

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(e) The $10\ mm$ strip line is connected between a power source with a source impedance of $Z_S = 50\ \Omega$ and frequency of $5\ GHz$ and a load with a load impedance $Z_L = 50 - j25\ \Omega$ as shown figure (10). Assuming that the losses on the strip line can be ignored calculate the complex reflection coefficient at the interface between the power source and the input of the loaded strip line using the provided Smith chart.

[10 marks]

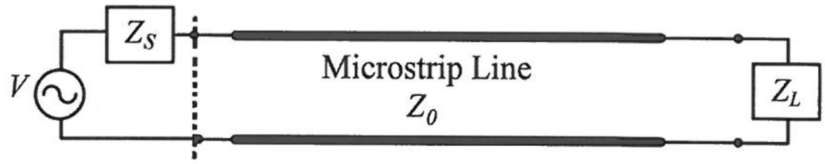

Fig. 10. The circuit for question 4(a) v

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SECTION C

Answer ONE out of TWO questions in this section

Question 5

A balanced (three phase delta) connected load has a per phase impedance $Z = 15\angle 30\ \Omega$.

The load is connected to a star connected voltage supply, where the neutrally is solidly grounded.

The lines which connect the load to the voltage supply have an entirely resistive impedance equal to 1 Ω.

The phase voltages of the supply are:

$$V_{AN} = 110\ \angle\ 0°\ V$$
$$V_{BN} = 105\ \angle\ \text{-}110°\ V$$
$$V_{CN} = 110\ \angle\ 105°\ V$$

(a) Obtain the sequence impedances, as seen from the terminals of the supply.

[6 marks]

(b) Use the symmetrical components method to obtain the line currents.

[18 marks]

(c) Find the complex power consumed by the load.

[9 marks]

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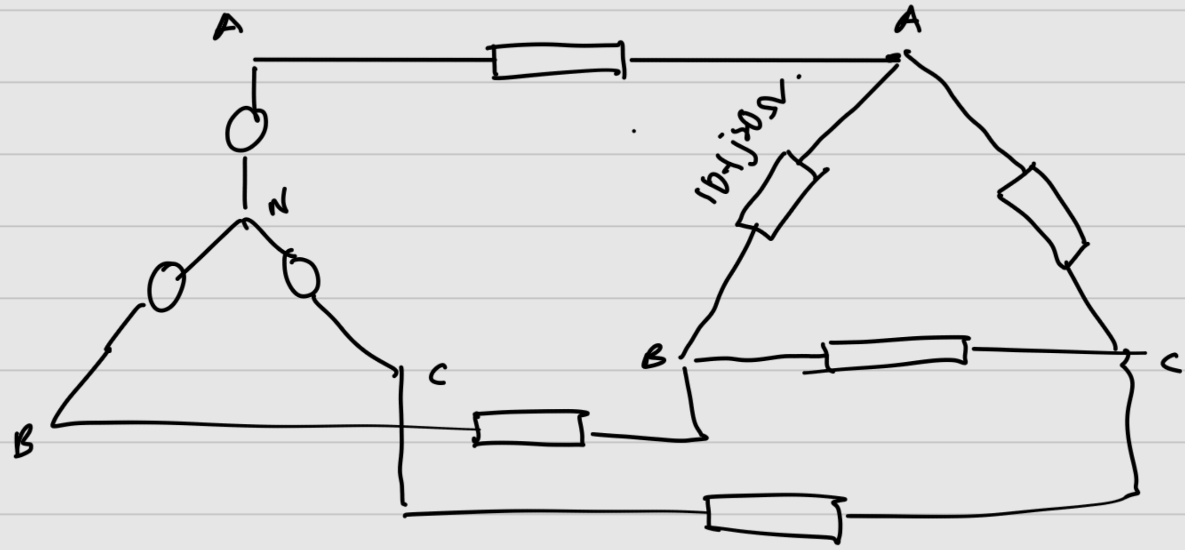
(a)

$Y_1 = Y_{12} + Y_{13} + \frac{Y_{12} Y_{13}}{Y_{23}}$



$V_{AN} = 110\angle 0^\circ \text{ V} \Rightarrow 110 + j0$

$V_{BN} = 105\angle -110^\circ \text{ V} \rightarrow -35.91 - j98.67$

$V_{CN} = 110\angle 105^\circ \text{ V} \Rightarrow -28.47 + j106.25$

(a) $\Delta \rightarrow$ star :

$$\frac{1}{Z_{AN}} = \frac{1}{Z_{AB}} + \frac{1}{Z_{AC}} + \frac{Z_{BC}}{Z_{AB}Z_{AC}}$$

$$= \frac{Z_{AC} + Z_{AB} + Z_{BC}}{Z_{AB}Z_{AC}}$$

$$Z_{AN} = \frac{Z_{AB}Z_{AC}}{Z_{AC} + Z_{AB} + Z_{BC}}$$

$$= \frac{(15\angle 30^\circ)^2}{3(15\angle 30^\circ)}$$

$$= 5\angle 30^\circ$$

$$Z_{BN} = \frac{Z_{AB}Z_{BC}}{Z_{AC} + Z_{AB} + Z_{BC}}$$

$$= 5\angle 30^\circ$$

$$Z_{CN} = 5\angle 30^\circ$$

$Z_0 = \infty$

$Z_1 = Z_2 = 5\angle 30^\circ + 1$

$= 5.89\angle 25.13^\circ$

$\Rightarrow 5.33 + j2.5$.

$$Z_0 = \left(\frac{Z_0}{3} + 3Z_N\right)$$

$Z_\Delta = \infty$

$\therefore Z_0 = \infty$

$$Z_1 = Z_2 = \frac{Z_\Delta}{3} + 1$$

$= 5\angle 30^\circ + 1$

$= 5.89\angle 25.13^\circ$

$\Rightarrow 5.33 + j2.5$

$$Z_s = \begin{bmatrix} \infty & 0 & 0 \\ 0 & 5.33+j2.5 & 0 \\ 0 & 0 & 5.33+j2.5 \end{bmatrix}$$

(b) $V_s = AV_p$

$a = 1\angle 120^\circ$

$a^2 = 1\angle -120^\circ$

$$\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

Zero Sequence : $V_{A0} = \frac{1}{3}(V_A + V_B + V_C)$

$= \frac{1}{3}(110\angle 0^\circ + 105\angle -110^\circ + 110\angle 105^\circ)$

$= \frac{1}{3}(46.24\angle 9.44)$

$= 15.41\angle 9.44^\circ$

+ve Sequence : $V_{A1} = \frac{1}{3}(V_a + aV_b + a^2V_c)$

$= \frac{1}{3}(110\angle 0^\circ + (1\angle 120^\circ)(105\angle -110^\circ) + (1\angle -120^\circ)(110\angle 105^\circ))$

$= \frac{1}{3}(319.82\angle -1.83^\circ)$

$= 106.61\angle -1.83^\circ$

-ve sequence : $V_{A2} = \frac{1}{3}(V_A + a^2V_B + aV_C)$

$= \frac{1}{3}((110\angle 0^\circ) + (1\angle -120^\circ)(105\angle -110^\circ) + (1\angle 120^\circ)(110\angle 105^\circ))$

$= \frac{1}{3}(35.374\angle 175.70^\circ)$

$= 11.79\angle 175.70^\circ$

$I_{A0} = \frac{V_{A0}}{Z_{A0}} = 0$

$I_{A1} = \frac{V_{A1}}{Z_{A1}} = \frac{106.61\angle -1.83^\circ}{5.89\angle 25.13^\circ} = 18.1\angle -26.96^\circ$

$I_{A2} = \frac{V_{A2}}{Z_{A2}} = \frac{11.79\angle 175.70^\circ}{5.89\angle 25.13^\circ} = 2.00\angle 150.57^\circ$

$I_p = A^{-1} I_s$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix}$$

$I_A = I_{A1} + I_{A2} = 18.1\angle -26.96^\circ + 2.00\angle 150.57^\circ$

$= 16.1\angle -26.65$

$$I_B = a^2 I_{A1} + a I_{A2}$$

$$= (1\angle -120^\circ)(18.1\angle -26.96^\circ) + (1\angle 120^\circ)(2.00\angle 150.57^\circ)$$

$$= 19.25\angle -141.93$$

$$I_C = a I_{A1} + a^2 I_{A2}$$

$$= (1\angle 120)(18.1\angle -26.96^\circ) + (1\angle -120^\circ)(2.00\angle 150.57^\circ)$$

$$= 19.11\angle 87.71^\circ$$

Question 6

A balanced, star-connected three-phase voltage source is connected to an unbalanced star-connected three-phase load as shown in the Figure 11 below:

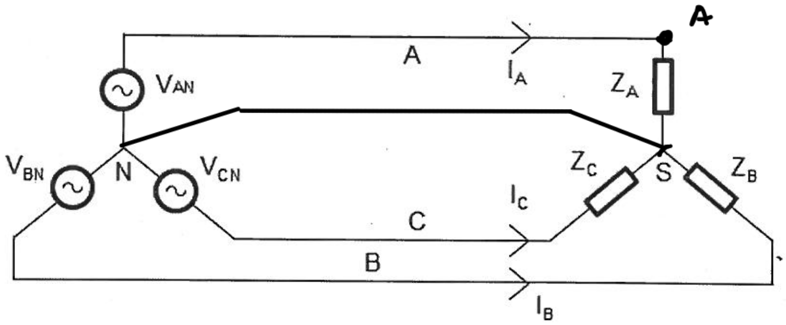


Fig. 11. The circuit for question 6

The phase voltage phasors and the loads are:

$V_{AN} = 230 \angle 0° \ V$ $\qquad$ $Z_A = 50 \angle 10° \ \Omega$

$V_{BN} = 230 \angle -120° \ V$ $\qquad$ $Z_B = 40 \angle -20° \ \Omega$

$V_{CN} = 230 \angle 120° \ V$ $\qquad$ $Z_C = 30 \angle 15° \ \Omega$

Note that all calculation of voltage and current phasors are to be referenced to V_A.

(a) Using Millman's theorem, calculate the phase voltages across each load as well as the line currents.

[13 marks]

(b) Determine the neutral current that flows from S to N if an impedance of $100 \angle 5° \ \Omega$ is connected between the two neutral terminals N and S.

[6 marks]

(c) Calculate the three-phase line currents as well as the active and reactive power supplied from the three phase voltage source when an impedance of $100 \angle 5° \ \Omega$ is connected between the two neutral terminals N and S.

[14 marks]

END OF PAPER

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Question 6

(a) $Z_A = 50\angle 10^\circ$, $Y_A = 0.02\angle -10^\circ$

$Z_B = 40\angle -20^\circ$, $Y_B : 0.025\angle 20^\circ$

$Z_C = 30\angle 15^\circ$, $Y_C = \frac{1}{30}\angle -15^\circ$

$$V_{SN} = \frac{V_{AN}Y_A + V_{BN}Y_B + V_{CN}Y_C}{Y_A + Y_B + Y_C}$$

$$= \frac{(230\angle 0^\circ)(0.02\angle -10^\circ) + (230\angle -120)(0.025\angle 20^\circ) + (230\angle 120^\circ)(\frac{1}{30}\angle -15^\circ)}{(0.02\angle -10^\circ) + (0.025\angle 20^\circ) + (\frac{1}{30}\angle -15^\circ)}$$

$$= 24.02\angle 34.08^\circ \Rightarrow 19.89 + j13.46$$

$$V_{AS} = V_{AN} - V_{SN}$$

$$= 230\angle 0^\circ - 24.02\angle 34.08^\circ$$

$$= 210.54\angle -3.67^\circ$$

$$I_A = \frac{V_{AS}}{Z_A}$$

$$= \frac{210.54\angle -3.67^\circ}{50\angle 10^\circ}$$

$$= 4.21\angle -13.67^\circ$$

$$V_{BS} = V_{BN} - V_{SN}$$

$$= 230\angle -120 - 24.02\angle 34.08^\circ$$

$$= 251.82\angle -122.39^\circ$$

$$I_B = \frac{V_{BS}}{Z_B}$$

$$= \frac{257.82\angle -122.39^\circ}{40\angle -20^\circ}$$

$$= 6.30\angle -102.39$$

$$V_{cs} = V_{CN} - V_{SN}$$
$$= 230 \angle 120^\circ - 24.02 \angle 34.08^\circ$$
$$= 229.54 \angle 125.99^\circ$$

$$I_c = \frac{V_{cs}}{Z_c}$$
$$= \frac{229.54 \angle 125.99^\circ}{30 \angle 15^\circ}$$
$$= 7.65 \angle 110.99^\circ$$

(b) $Z_N = 100 \angle 5^\circ$, $\Rightarrow Y_N = 0.01 \angle -5^\circ$

$$V_{SN} = \frac{V_{NN} Y_N + V_{AN} Y_A + V_{BN} Y_B + V_{CN} Y_C}{Y_N + Y_A + Y_B + Y_C}$$
$$= \frac{0 + (230\angle 0^\circ)(0.02\angle -10^\circ) + (230\angle -120^\circ)(0.025\angle 20^\circ) + (230\angle 120^\circ)(\frac{1}{30}\angle -15^\circ)}{(0.01\angle 5^\circ) + (0.02\angle -10^\circ) + (0.025\angle 20^\circ) + (\frac{1}{30}\angle -15^\circ)}$$
$$= 21.23 \angle 33.18^\circ$$

$$I_N = \frac{V_{SN}}{Z_N}$$
$$= \frac{21.23 \angle 33.18^\circ}{100 \angle 5^\circ}$$
$$= 0.2123 \angle 28.18^\circ \text{ A}$$