Equivalent Circuit Model Of Transmission Lines

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***Abstract*—This report aims to verify that transmission lines can be examined by building an equivalent model on Multisim. Lossless and lossy transmission lines are to be analyzed with different conditions. The equivalent circuit can be built using lumped parameters consisting of inductor, capacitor, resistor and Conductance.**

I. INTRODUCTION

Lossless transmission lines are ideal lines with no loss. It only consists of an inductor and a capacitor which are frequency independent. Lossy lines represent real transmission lines which consist of resistance and conductance in addition to inductor and capacitor. Various factors cause losses in a transmission line, some of which are frequency dependent.

II. LOSSLESS AND LOSSY MODELS

A. Extraction of Static Line Capacitance

To calculate the capacitance of the coaxial transmission line, the electric field of the structure was calculated initially[1]. The coaxial cable comprises of concentric conductors with a uniform charge density on the inner conductor which is analogous to the electric field of line charge in free space having uniform charge density. Hence, by Gauss's Law, the electric field of the structure having uniform charge density of ρ , radius *r* and relativity permittivity ε_r is given by (1):

$$E = \frac{\rho}{2\pi\varepsilon_0\,\varepsilon_r\,r} \tag{1}$$

Next, the voltage between the conductors was obtained by integrating the electric field using the limits as the radius of the conductors as shown below (2).

$$V = -\int_a^b E.dl \;=\; \frac{\rho}{2\pi\varepsilon_0\,\varepsilon_r}\; ln\,\frac{b}{a} \tag{2}$$

Where b and a are the radiuses of outer and inner conductors respectively. The capacitance is computed as the ratio of assumed charge to the potential difference. Thus, the capacitance of the structure is gives as (3):

$$C' = \frac{Q}{V} = \frac{2\pi\varepsilon_0\,\varepsilon_r\,l}{ln\frac{b}{a}} \tag{3}$$

Thus, the value of capacitance per unit length to be used in equivalent circuit model is given as (4):

$$C = \frac{2\pi\varepsilon_0\,\varepsilon_r}{ln\,\frac{b}{a}} \tag{4}$$

By using the values of radiuses of inner and outer conductors and the relative permittivity as given in the coursework, the value of capacitance per unit length obtained is 66.02 pF.

B. Extraction of Static Line Inductance

To calculate the inductance of the transmission line of inner and outer radiuses as a and b respectively, it was assumed that a current I flowing in +z direction gives rise to a magnetic field inside the coaxial structure. Hence, by Ampere's Law, the magnetic flux is obtained as given below (5):

$$B = \frac{\mu_0 I}{2\pi\rho}\;,\;\; a \le \rho \le b \tag{5}$$

Next the magnetic flux, $\emptyset$, is obtained by integrating the magnetic field using the limits as the radiuses of the conductors as shown below (6):

$$\emptyset = \int_a^b B.dS \;=\; \frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right) \tag{6}$$

The inductance of the coaxial cable can thus be calculated as the ratio of flux to source current. Hence, the inductance of the structure is given as (7):

$$L' = \frac{\Delta\emptyset}{I} = \frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right) \tag{7}$$

Thus, the value of capacitance per unit length to be used in equivalent circuit model is given as (9):

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) \tag{9}$$

By using the values of radiuses of inner and outer conductors and the relative permeability as given in the coursework, the value of inductance per unit length obtained is 0.371 uH.

C. Characteristic Impedance

The characteristic impedance of a lossless transmission line is given by (10):

$$Z_0 = \sqrt{\frac{L}{C}} \tag{10}$$

Where L and C are the inductance per unit length and capacitance per unit length respectively. By substituting L and C from sections *A* and *B*, a value of 74.94 ohm is obtained as the characteristic impedance.

D. Time Delay and Velocity of transmission line

The time delay and velocity of the transmission line is given by (11) and (12) respectively:

$$t_{pd} = \sqrt{LC} \tag{11}$$

$$v_{pd} = \frac{1}{\sqrt{(LC)}} \tag{12}$$

Where L and C are the inductance per unit length and capacitance per unit length respectively. By substituting L and C from I and II , a value of 4.95ns and 2.021 * 10^8 m/s obtained as the time delay and velocity of the electromagnetic wave. Since the transmission line is 32m long, the total propagation delay is 32 * 4.95 ns = 158.4ns.

E. Equivalent Circuit Model of a lossless Line

The model has to correctly simulate a 32 m long transmission line up to 700MHz. By using the velocity of the electromagnetic wave obtained in section IV, the wavelength corresponding to 700MHz was calculated. Thus, by using (13), a value of 0.289 m was obtained as the wavelength.

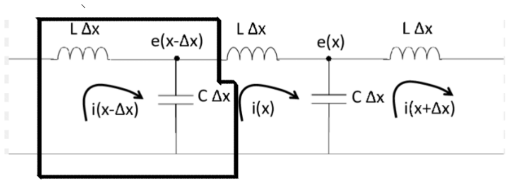
$$\lambda = \frac{c}{f} \tag{13}$$

A cable is classified as electrically long cable if the length of the cable is greater than one-tenth of the wavelength. As the length of the cable is 32m and the value of one-tenth of the wavelength is 0.0289m, it can be concluded that the cable is electrically long.

In order to simulate an electrically long cable, we need to divide the electrically long cable into a series of electrically short cables of length Δx such that (14):

$$\Delta x \le \frac{\lambda}{10} < l\ ,\ l = 32\text{m} \tag{14}$$

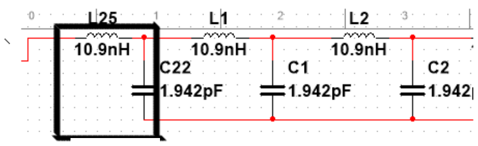
Hence, I have assumed Δx= 0.0289 m (one -tenth the value of wavelength). Δx corresponds to one segment of transmission line as shown in fig.1.



Δx

Fig.1. Example of Δx segment of a transmission line.

The value of capacitance and inductance per unit length are 66.02 pF and 0.371 uH as per sections *A* and *B.* respectively. I have hence divided the 32 m transmission line into 32 segments of 1 m each. The length of each segment comprising of a capacitor and inductor in 1m line is of Δx m. The number of Δx segments to be used in 1 m are 34 as $\frac{1m}{0.0289\ m}$=34. The value of capacitance of Δx network is 1.942pF and the inductance of Δx network is 10.9nH as per fig . Δx network with the calculated values of parameters is shown in fig.2.



Δx

Fig.2. Δx segment of transmission line with scaled values of Inductance and capacitance

The 1m transmission line is shown in fig.3.

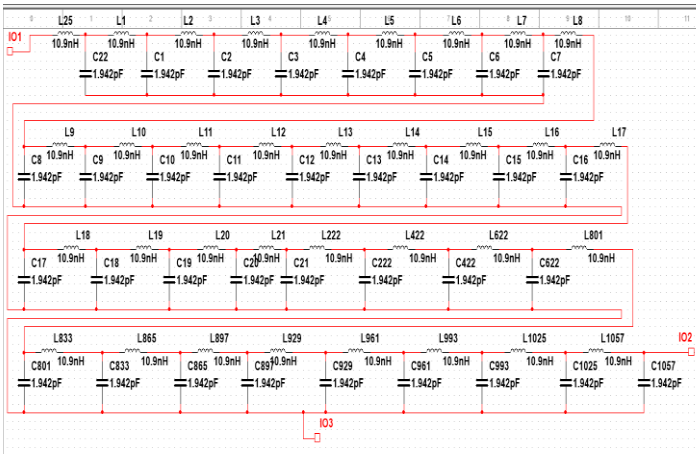


Fig.3. 1m transmission line.

32 blocks of 1m have been cascaded together to constitute for a 32m long coaxial transmission line. A resistor equivalent to characteristic impedance has been attached in series next to the source to account for internal resistance of the source. The equivalent circuit model has been shown in fig.4.

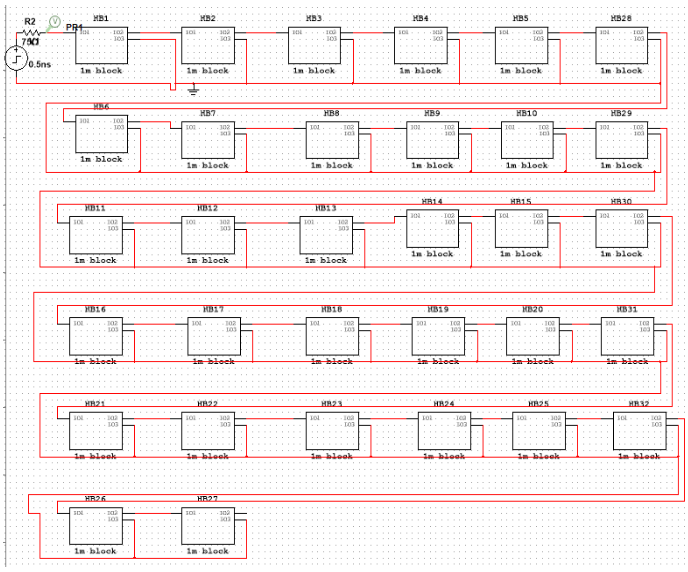


Fig.4. 32m transmission line made of 1m blocks

A similar approach was followed to setup the Multisim TL model. Internal resistance of the source and the characteristic impedance of the transmission line has been specified to construct the equivalent model. Since the transmission line is 32m long, the total propagation delay through the line is 32 * 4.95 ns = 158.4ns which has also been considered while constructing the TL model. The equivalent Multisim TL model is shown in fig.5.

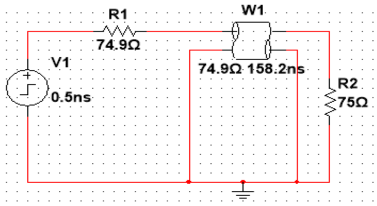


Fig.5. TL model of a lossless line with 75 Ω load

F. *Time Domain Analysis of lossless circuit under Step Voltage Excitation*

a) 75-ohm load

The initial input voltage, V_{in}, is 2.5V when $Z_L = Z_0$. The output/load voltage, V_{out} is zero initially due to the propagation delay of the signal through the transmission line. The signal takes 158.4ns to reach from the source to the load. Since the load is matched to Z_0, the reflection coefficient at the load, Γ_L is almost 0 and therefore the signal is not reflected, Hence, V_{out}=V_{in} for matched Transmission line. Since no reflection occurs, V_{in} remains the same. Steady state is achieved at 158.4 ns where V_{out}=$V_{in,}$ 2.5V. The transient response of matched transmission line is shown in fig.6.

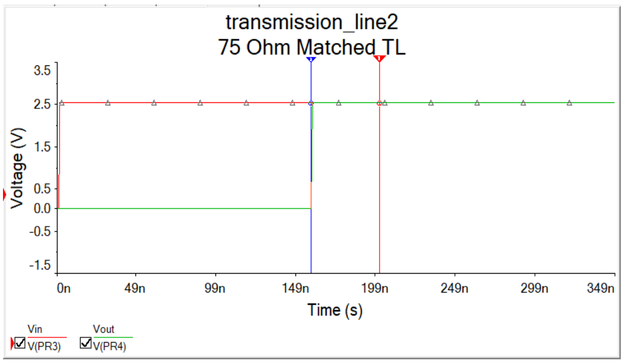


Fig.6. Transient response for Matched Transmission line, $Z_L = Z_0$. $V_{out} = V_{in}$ at 158.4ns.

b) Open Circuit Load

At t =0, $Z_{in} = Z_0$. Therefore, the incident wave, Vin, has an amplitude of 2.5V. For t < 158.4 ns, V_{out} = 0. For open circuit load $Z_L = \infty$. Thus, from (15), $\Gamma_L = 1$ at the load which accounts for 0 phase angle difference of reflected wave as compared to the incident wave. Thus, at t > 158.4 ns, the incident wave and reflected wave superimpose each other and $V_{out} = 2 * V_{in}$ = 5V. At t = $\frac{2l}{v_p}$ = 316.68 ns, the reflected wave reaches back to the source, superimposing the incident wave at the source making V_{in} = 5V. Current is very small for open circuit. The transient response of open circuit is show in fig.7.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{15}$$

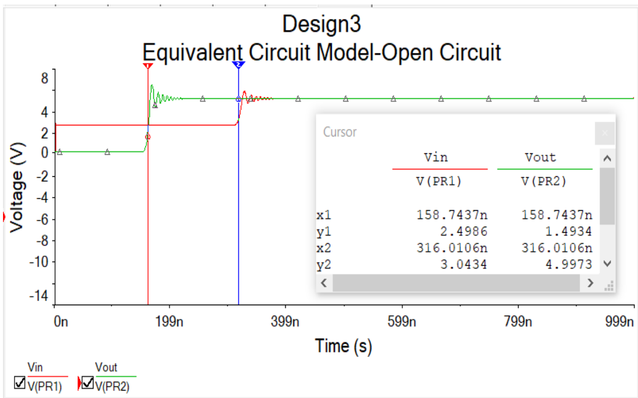


Fig.7. Transient response for Matched Transmission. V_{out} = 5V at 158.4ns, $V_{in} = V_{out}$ = 5V at 316.68ns

c) Short Circuit Load

For t < 316.68ns, $Z_{in} = Z_0$ and therefore V_{in} = 2.5V. Z_L=0 and Γ_L = -1 for short circuit indicating that the reflected wave has ∓ 180° phase difference as compared to the incident wave. Thus, at t > 158.4ns, the reflected and incident waves cancel each other out resulting in V_{out} becoming 0. For t >316.68ns, the reflected wave reaches the source and cancels out with the incident wave resulting in V_{in} becoming 0. Current is maximum for Short circuit. The transient response of short circuit is shown in fig.8.

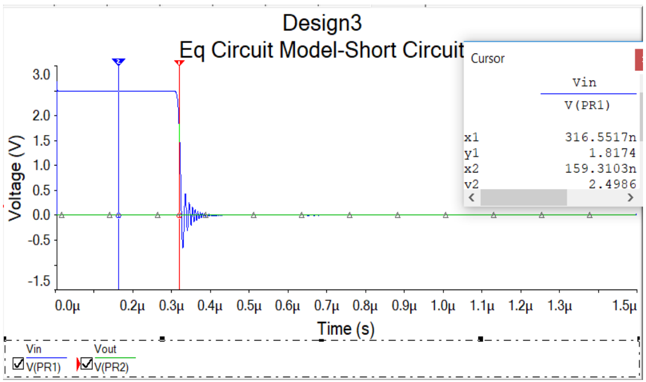


Fig.8. Transient response for Matched Transmission. V_{out} = 0V at 158.4ns, $V_{in} = V_{out}$ = 0V at 316.68ns.

d) 10Ω load

For t < 316.68 ns, $Z_{in} = Z_0$ and therefore V_{in} = 2.5V. for t < 158.4ns, V_{out} = 0. For Z_L = 10 ohm, Γ_L = -0.765 at the load as per equation. Thus, at t >158.4 ns, the reflected wave cancels out with 76.5% of the incident wave, resulting V_{out} = 0.588V. For t >316.68ns, the reflected wave reaches the source and cancels out with 76. 5% of the incident pulse resulting in V_{in} becoming 0.588V. The current for Z_L = 10 ohm is 58.8mA as per Ohm's Law. The transient response of 10-ohm load resistance is shown in fig.9.

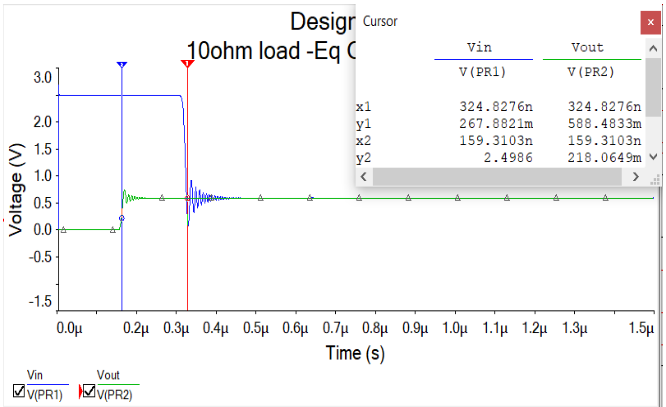


Fig.9. Transient response for Matched Transmission. V_{out} = 0.588V at 158.4ns, $V_{in} = V_{out}$ = 0.588V at 316.68ns.

Transient analysis using TL Model for different conditions is shown in figure. As seen from the figure, the output for all the cases is 0 for t < 158.4ns and V_{out} =V_{in} after t > 316.68ns. Furthermore, the steady state in both the models is achieved at the same time. But unlike the equivalent circuit model, The Multisim TL model (fig.10.) is exact and there is no ringing behaviour observed at the step edge. The ringing behaviour in equivalent LC circuit is caused due to Gibbs Phenomenon. The ringing behaviour depends on the rise time and bandwidth of the circuit. Thus, by increasing the number of LC segments and decreasing Δx, the bandwidth of the signal can be increased which in turn increases the accuracy of the LC circuit model.

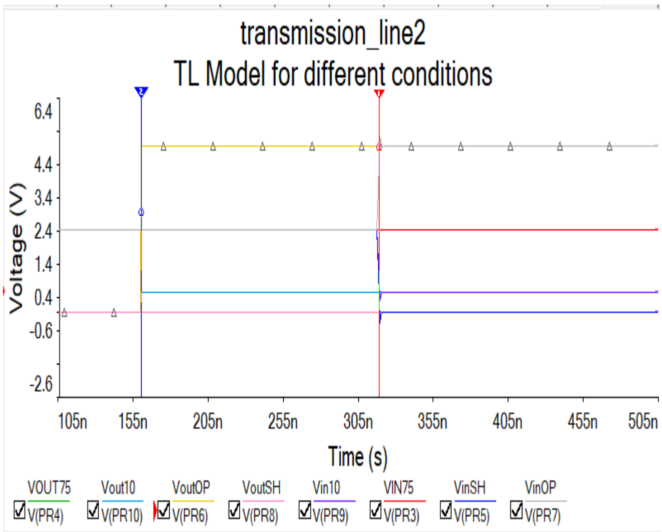


Fig.10. TL Model simulation for different conditions

G. AC Analysis of Lossless Lines

Like the Step Voltage input, $V_{out} = 0$ until the transit time (158.4 ns) but unlike the step voltage input, $V_{in} \neq V_{out}$ after t > (2 * transit time = 316.68ns) except for 75-ohm load. V_{out} does change after 316.68 ns but there is a phase shift observed as well. The phase shift and the magnitude of V_{in} can be explained by (16).

$$V(z) = V_0^+ (e^{-\beta l} + \Gamma\, e^{\beta l}) \tag{16}$$

Where $\beta = (2\pi/\lambda)$ where λ depends on the frequency of AC source and is calculated as per equation. Γ is the reflection coefficient at the load. l = -32m when calculating V(z) at the source and l = -0 When calculating reflection coefficient at the load. AC inputs have finite frequencies which causes a change in the value of β as per the chosen frequency leading to a change is V(z).Thus, a change in V(z) at the source depends on the frequency used and the reflection coefficient of the load.

a) Open Circuit Load

$\Gamma = 1$ for open circuit load. The behaviour of V_{out} is the same as observed with step voltage. However, there is a change in phase and magnitude of V_{in} as per equation. As $\Gamma = 1$, the phase difference between V_{in} and V_{out} will be either 0° or 180°. Simulations have been shown in fig.11 and fig.12.

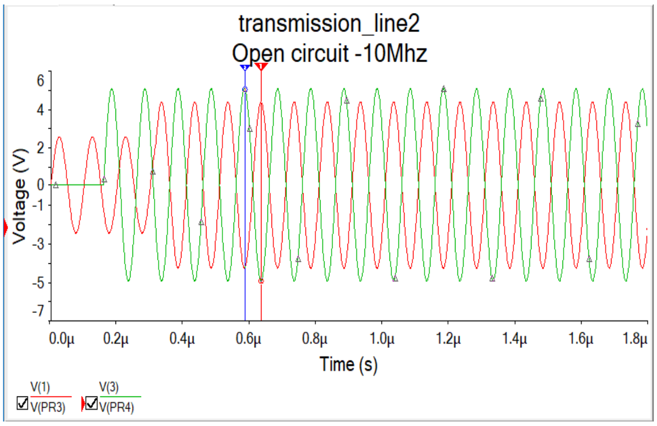


Fig.11. open circuit load at 1MHz with V_{out} = 5V after 158.4ns and V_{in} = 4.3V after 316.68ns. Phase difference between V_{in} and V_{out} is 180°.

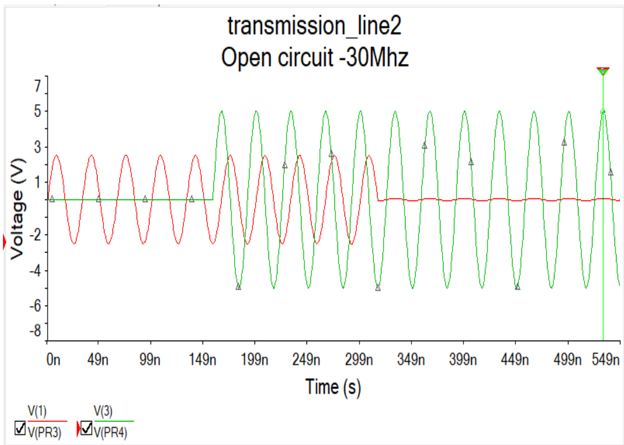


Fig.12. open circuit load at 1MHz with V_{out} = 5V after 158.4ns and V_{in} = 0.062V after 316.68ns. Phase difference between V_{in} and V_{out} is 0°.

b) Short Circuit Load

$\Gamma = -1$ for open circuit load. The behaviour of V_{out} is the same as observed with step voltage. However, there is a change in phase and magnitude of V_{in} as per equation. As $\Gamma = -1$, the phase difference between V_{in} and V_{out} will be either 90° or 270°. Simulations have been shown in fig.13 and fig.14.

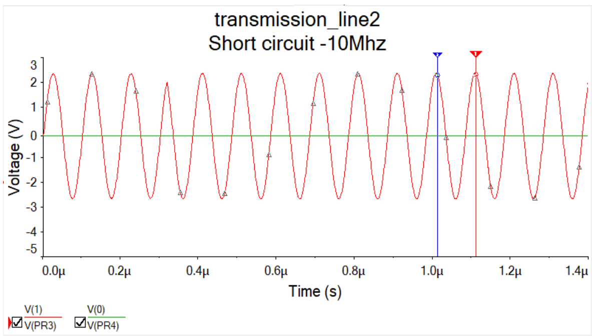


Fig.13. Short circuit load at 10MHz with V_{out} = 0V after 158.4ns and V_{in}= 2.5V after 316.68ns. Phase difference between V_{in} and V_{out} is 270°.

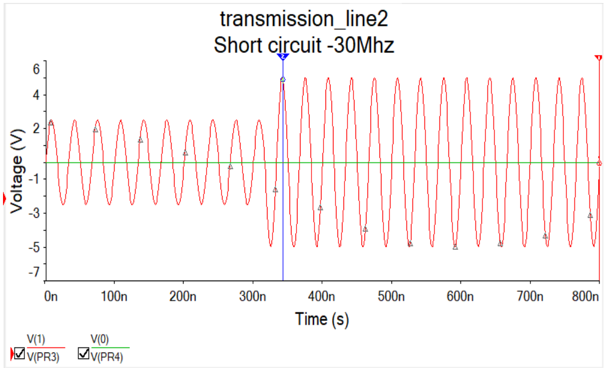


Fig.14. Short circuit load at 30MHz with V_{out} = 0V after 158.4ns and V_{in}= 4.98V after 316.68ns. Phase difference between V_{in} and V_{out} is 90°.

c) Matched Load – 75 Ohm

$\Gamma = 0$ for 75-ohm load. The behaviour of V_{out} is the same as observed with step voltage source. As $\Gamma = 0$, the magnitude of $V_{in} = V_{out}$. However, the phase difference between V_{in} and V_{out} depends on the frequency of the source. Simulations have been shown in fig.15 and fig. 16.

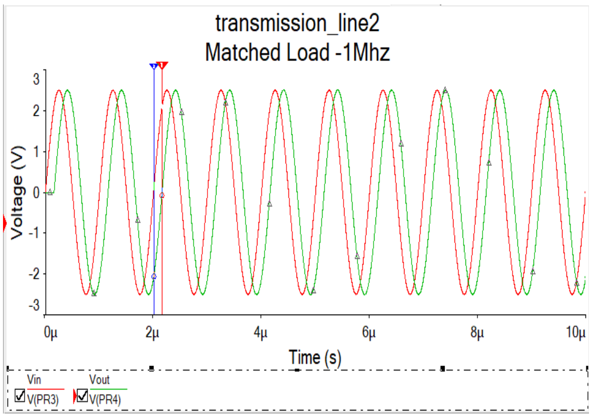


Fig.15. 75-ohm load at 1MHz with V_{out} = 2.5V after 158.4ns and V_{in}= 2.5V after 316.68ns. Phase difference between V_{in} and V_{out} is 57̊.

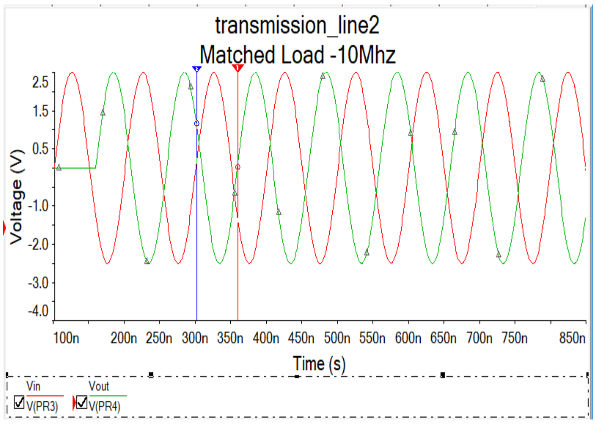


Fig.16. 75-ohm load at 10MHz with V_{out} = 2.5V after 158.4ns and V_{in}= 2.5V after 316.68ns. Phase difference between V_{in} and V_{out} is 115.56̊.

d) 10-ohm load

Γ = -0.764 for open circuit load. The behaviour of V_{out} is the same as observed with step voltage. However, there is a change in phase and magnitude of V_{in} as per equation. The phase difference between V_{in} and V_{out} will depend on the source frequency. Simulation at 1MHz is shown in fig17.

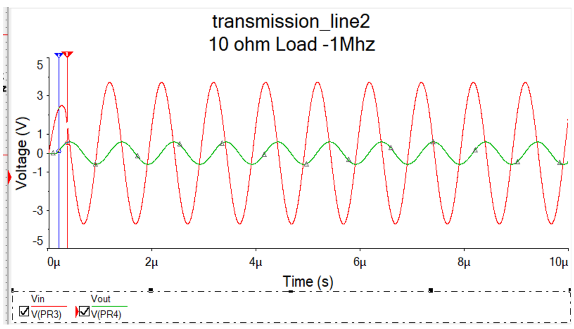


Fig.17. 10-ohm load at 1MHz with V_{out} =3.7V after 158.4ns and V_{in} = 0.588V after 316.68ns. Phase difference between V_{in} and V_{out} is 76.̊

As per fig.12, it can be observed that at 30 MHz open circuit load has V_{in} as 0.22 mV indicating Z_{in} = 0 at steady state. In contrast with open circuit load, short circuit load has V_{in} = 4.9V at 30 MHz indicating $Z_{in} = \infty$ at steady state as per fig. 14.

The simulations conducted using TL model and equivalent circuit yielded same results. No ringing behaviour was observed at transient time unlike step voltage source. This is because there is no drastic sudden change in voltage level of an AC source.

H. Equivalent Circuit of Lossy Model

The losses in transmission is characterized by attenuation constant α, which depends on various factors. The losses due to frequency, f, is shown in fig 18.

TABLE I. FREQUENCY DEPENDENT LOSSES

α_C	Metal loss	$\alpha_C \, \alpha \, \sqrt{f}$
α_D	Dielectric tangent loss	$\alpha_C \, \alpha \, f$

Fig18. Frequency Dependent Losses

As seen in table, α_C and α_G cause loss at microwave frequencies. Resistance, R depends on α_C and Conductance, G depends on α_D. Hence, the values of R and G would change with frequency. L and C remains the same as previous sections. The values of R and C extracted at different frequencies is shown in fig.19. As per section 2, the construction of segment of Δx m changes according to the values of R and L obtained at different frequencies which results in the change in the overall circuit model.Δx m segment at 1Mhz is shown in fig.20.

TABLE II. R AND G PARAMETERS OF LOSSY LINE AT VARIOUS FREQUENCIES

f (MHz)	R per unit length (Ω/m)	G per unit length (mho/m)	R per segment of Δx m (Ω)	G per segment of Δx m (mho)	Shunt Resistance of Δx m ($1/G_x$) (Ω)
1	0.20286	830n	5.46m	24.4n	40.96M
10	0.54924	8.3u	16.5m	244.1n	4.096M
30	0.92586	24.9u	27.23m	732.53n	1.365M
Step voltage	0.06774	0.00033p	1.99m	9.94 * 10^{-18}	10E

L = 0.371 uH/m , C = 66.02 pF/m

Fig19. Resistance and Conductance of lossy line at different frequencies

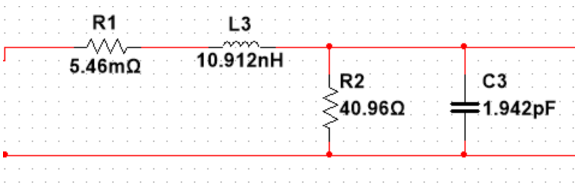


Fig.20 Δx m segment at 1MHz

I. AC ANALYSIS OF LOSSY MODEL

The simulation graphs of lossy line are shown in figure. On comparison with 75 Ω load of lossless lines with AC source, it is observed that unlike lossless lines, the Magnitude of V_{in} ≠ V_{out}, indicating that there is some loss in the line. This is because attenuation constant, α, increases with frequency which causes signal attenuation and increases the difference between V_{in} and V_{out}. The attenuation is not significantly big indicating a low loss line. However, the phase difference between V_{in} and V_{out} is the same as lossless lines. The simulations at various frequencies are shown in fig.21, fig.22 and fig.23. The attenuation Constant for a low loss lines is given by (17):

$$\alpha = 0.5\left(\frac{R}{Z_0} + GZ_0\right) \tag{17}$$

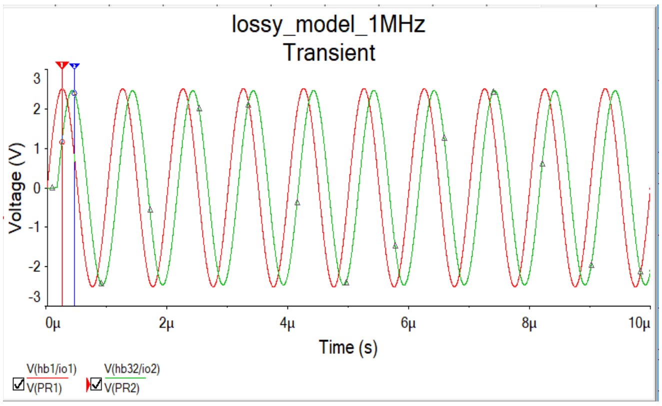


Fig.21. 75-ohm load at 1MHz with V_{out} = 2.4V after 158.4ns and V_{in}= 2.5V after 316.68ns. Phase difference between V_{in} and V_{out} is 57˚. α = 0.00138

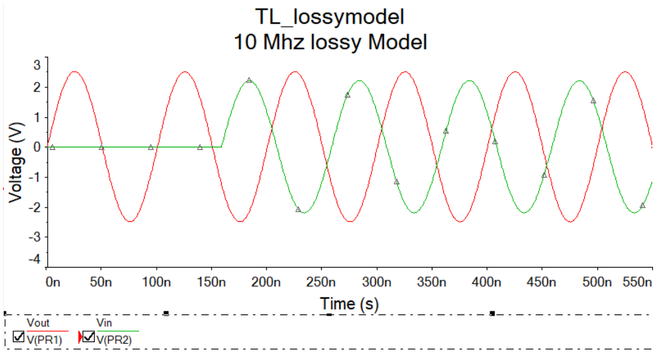


Fig.22. 75-ohm load at 10MHz with V_{out} = 2.2V after 158.4ns and V_{in}= 2.5V after 316.68ns. Phase difference between V_{in} and V_{out} is 115.56˚. α = 0.00397.

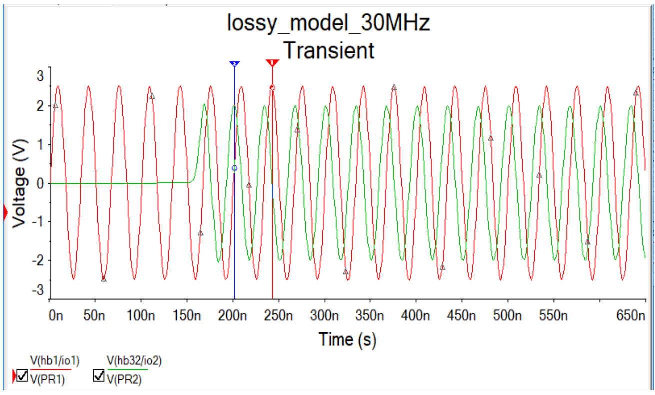


Fig.23. 75-ohm load at 30MHz with V_{out} = 1.984V after 158.4ns and V_{in}= 2.5V after 316.68ns. Phase difference between V_{in} and V_{out} is 270˚. α = 0.00712.

J. Step Voltage Analysis

With step voltage source, it was observed that the magnitude of V_{in} and V_{out} was not equal like the lossless model although the transit time is the same. This is also due to the attenuation constant α. Moreover, since (R/L) ≠ (G/C), there is distortion at transit time due to different frequency components reaching the end of the line at different frequencies. The simulation is shown in fig.24.

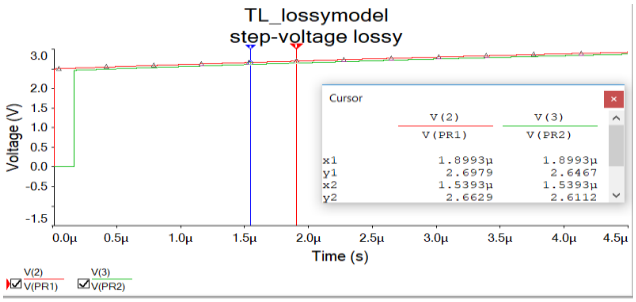


Fig.24 Lossy line with Step Voltage Source

III. Conclusion

As per section 1, the phase velocity is reduced by 32.6% which indicates that the speed of the photons will be lesser than the speed of light due permittivity of the line. For lossless lines, it is concluded that V_{load} depends on the load used as the it changes the reflection coefficient Γ. the magnitude of V_{out} is maximum for open load and minimum for short circuit load. For matched load, V_{out}=V_{in} as Γ= 0. For step voltage source, there was a ringing behaviour observed at transit time for equivalent circuit which can be reduced by increasing the number Δx segments. V_{in} changed after 316.68ns for AC source but not with step voltage source which indicates that β varies with frequency. The phase difference observed between input and output voltages also depends on Γ and β. Different factors cause losses in transmission line which depend on frequency used. R and G are frequency dependent whereas L and C are not. Losses in transmission line cause signal attenuation causing a difference in V_{in} and V_{out} for matched load.

References

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[2] Microwaves 101.com,Encyclopedia, Transmission Line Loss, https://www.microwaves101.com/encyclopedias/transmission-line-loss