ELEC 3224 Guidance Navigation and Control — Tutorial Questions Two

Q1. Consider the linear time invariant system described by the state-space model

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix} (t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 3 & -2 \end{bmatrix} x(t) \end{aligned}$$

a) Is this system stabilizable, i.e., there exists a state feedback law that will place all eigenvalues of the closed-loop system in the open left-half of the complex plane?

b) Is the state-space model in this question minimal?

c) Compute the transfer-function of the system in this question.

d) What are the stability properties of the system in this question?

Q2. Consider the linear time-invariant system with the state equation

$$\dot{x}(t) = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} (t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

a) Determine the Riccati equation with cost function matrices $Q = I_2$ (the 2×2 identity matrix) and $R = 1$.

b) Find (if possible) a stabilizing state feedback control law.

c) Find a new pair of weighting matrices such that the closed-loop system dynamics converge faster than the case of b) above.

Q3. In a linear quadratic optimal control problem, the solution matrix of the algebraic Riccati equation P is defined by

$$P = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} = \begin{bmatrix} p_1/4 + 1 & p_2 \\ p_2 & 4p_3 + 1 \end{bmatrix} - \frac{1}{p_3 + 1} \begin{bmatrix} p_2^2/4 & p_2 p_3 \\ p_2 p_3 & 4p_3^2 \end{bmatrix}$$

Determine the entries in P from these equations.

Q3. In nonlinear control, the equilibrium point is a critical issue and here the focus is on a system with state dynamics

$$\dot{x}(t) = f(x(t))$$

i.e., the state dynamics with any input and/or disturbance terms set to zero. The equilibrium points for this system are the solutions of

$$f(x) = 0$$

Consider the nonlinear system

$$ml\ddot{\Theta}(t) = mg \sin \Theta - kl\dot{\Theta}$$

(This is the model for an inverted pendulum of mass m, length l and Θ is the displacement angle measured anti-clockwise from the vertically down position.) Using the state variables $x_1 = \Theta$, $x_2 = \dot{\Theta}$, obtain a state-space model for this system, and calculate the equilibrium points.