
SEMESTER TWO EXAMINATIONS 2019/20

POWER CIRCUITS AND TRANSMISSION

Duration 120 mins (2 hours)

This paper contains 3 questions

Answer **all questions.**

Section A carries 33% of the total marks for the exam paper.

Section B carries 33% of the total marks for the exam paper.

Section C carries 33% of the total marks for the exam paper.

Only University approved calculators may be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct 'Word to Word' translation dictionary AND it contains no notes, additions or annotations.

7 page examination paper (+ 1 page formula sheet)

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SECTION A

Answer all questions in this section

Question 1:

(a) Consider the circuit shown in Figure 1. Using mesh analysis, find the values of i_1, i_2, and i_3 when $V_1 = 10\ V$, $I_0 = 1\ A$, $R_1 = R_2 = 5\ \Omega$, $R_3 = 7\ \Omega$, and $R_4 = 3\ \Omega$. [7 marks]

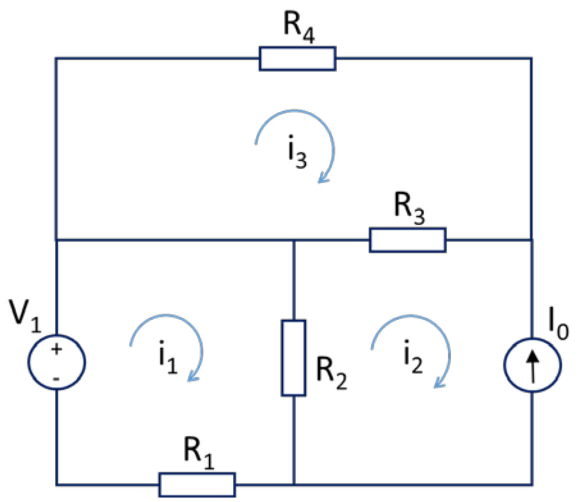

Fig. 1. The circuit for question (a)

(b) Using the Millman's (parallel generator) theorem, find the voltage V_{AB} across the node A and B, in the circuit shown in Figure 2. $V_1 = 5$ V, $V_2 = 10$ V, $R_1 = R_2 = 5\ \Omega$, $R_3 = 20\ \Omega$. [5 marks]

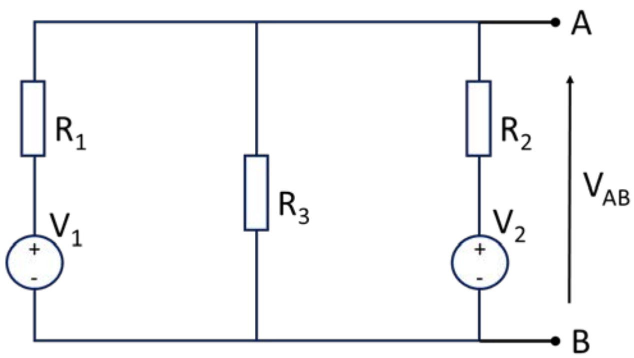

Fig. 2. The circuit for question (b)

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(a) Consider the circuit shown in Figure 1. Using mesh analysis, find the values of i_1, i_2, and i_3 when $V_1 = 10\ V$, $I_0 = 1\ A$, $R_1 = R_2 = 5\ \Omega$, $R_3 = 7\ \Omega$, and $R_4 = 3\ \Omega$. [7 marks]

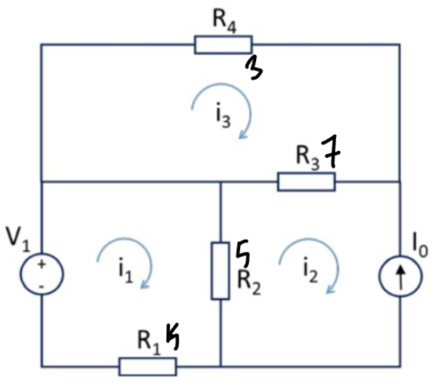
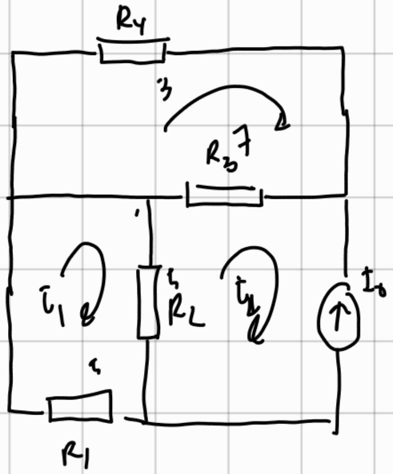


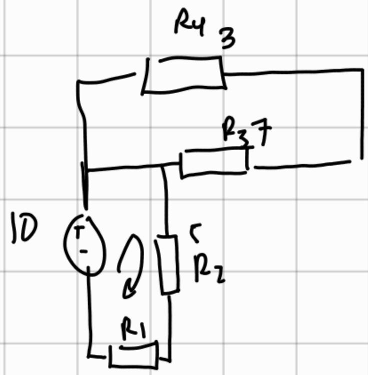
Fig. 1. The circuit for question (a)

Short circuit



$i_2 = -I_0$
$= -1$

$i_1 R_1 + (i_1 - i_2) R_2 = 0$
$5 i_1 + 5 i_1 + 5 = 0$
$10 i_1 = -5$
$i_1 = -0.5$



$i_3 R_4 + (i_3 - i_2) R_3 = 0$
$i_3 (3) + (i_3 + 1) 7 = 0$
$3 i_3 + 7 i_3 + 7 = 0$
$10 i_3 = -7$
$i_3 = \frac{-7}{10} A$

$i_1 R_2 + i_1 R_1 = V$
$i_1 (5) + i_1 (5) = 10$
$10 i_1 = 10$
$i_1 = 1$

$i_1 = 1 + -0.5 A$
$= 0.5 A$

(b) Using the Millman's (parallel generator) theorem, find the voltage V_{AB} across the node A and B, in the circuit shown in Figure 2. $V_1 = 5$ V, $V_2 = 10$ V, $R_1 = R_2 = 5\ \Omega$, $R_3 = 20\ \Omega$.

[5 marks]

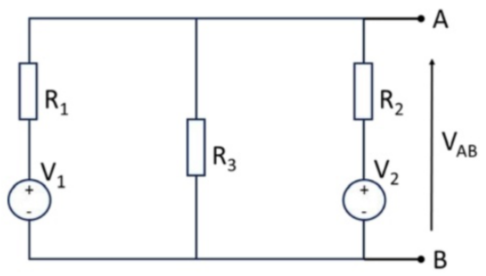


Fig. 2. The circuit for question (b)

$$Y_1 = \frac{1}{R_1} = \frac{1}{5} = 0.2\ \Omega^{-1}$$

$$Y_2 = \frac{1}{R_2} = \frac{1}{5} = 0.2\ \Omega^{-1}$$

$$Y_3 = \frac{1}{R_3} = \frac{1}{20} = 0.05\ \Omega^{-1}$$

$$V_{AB} = \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3}{Y_1 + Y_2 + Y_3}$$

$$= \frac{5(0.2) + 10(0.2) + 0(0.05)}{0.2 + 0.2 + 0.05}$$

$$= \frac{20}{3}$$

$$= 6.667\ V$$

(c) Consider the two-port network in Figure 3. There are two impedances Z, and two admittances *Y connected as shown in the network.* Derive the *(A,B,C,D)* representation of this two-port network. Please also write the image impedances Z_{I1} and Z_{I2} in terms of *A,B,C,D.*

[8 marks]

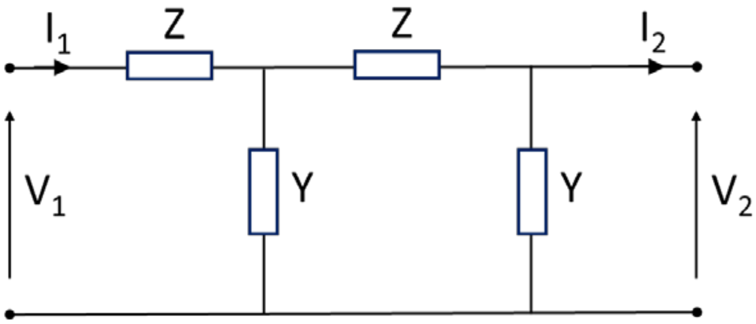


Fig. 3. The circuit for question (c)

(d) Starting from the *(A,B,C,D)* representation of a two-port network, derive its equivalent Y-representation. Show clearly how you arrived at your conclusions.

[6 marks]

(e) Describe what is symmetry of a two-port network? Write down the equation, in terms of the (*A,B,C,D*) representation, derived from a reciprocal and symmetrical two-port network. Write down the iterative impedance Z_{it} of a symmetrical two-port network.

[7 marks].

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SECTION B

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(c) Consider the two-port network in Figure 3. There are two impedances Z, and two admittances Y *connected as shown in the network.* Derive the *(A,B,C,D)* representation of this two-port network. Please also write the image impedances Z_{I1} and Z_{I2} in terms of *A,B,C,D*.

[8 marks]

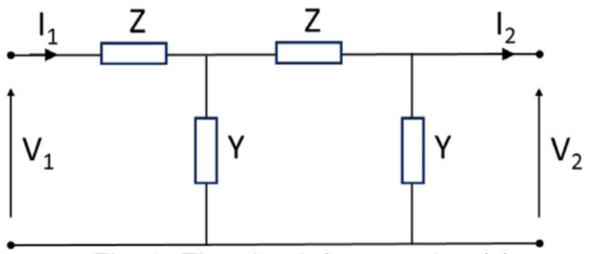
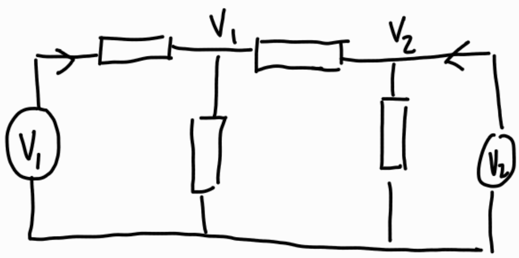


Fig. 3. The circuit for question (c)



$$I_1 = \frac{V_1}{Z} + V_1 Y + \frac{V_1 - V_2}{Z}$$

$$I_2 = V_2 Y + \frac{V_2 - V_1}{Z}$$

$$I_2 Z = V_2 Y Z + V_2 - V_1$$

$$V_1 = V_2 Y Z + V_2 - I_2 Z$$

$$= V_2(YZ + 1) - I_2 Z$$

$$I_1 = V_1\left(\frac{2}{Z} + Y\right) - \frac{V_2}{Z}$$

$$= \left(V_2(YZ+1) - I_2 Z\right)\left(\frac{2}{Z} + Y\right) - \frac{V_2}{Z}$$

$$= \frac{2V_2}{Z}(YZ+1) - 2I_2 + V_2 Y(YZ+1) - I_2 YZ - \frac{V_2}{Z}$$

$$= 2V_2 Y + \frac{2V_2}{Z} - 2I_2 + V_2 Y^2 Z + V_2 Y - I_2 YZ - \frac{V_2}{Z}$$

$$= 3V_2 Y + \frac{V_2}{Z} + V_2 Y^2 Z - 2I_2 - I_2 YZ$$

$$= V_2\left(3Y + \frac{1}{Z} + Y^2 Z\right) + I_2(-2 - YZ)$$

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} Yz+1 & -z \\ 3Y+\frac{1}{z}+Y^2 z & -2-Yz \end{bmatrix} \begin{bmatrix} V_2 \\ i_2 \end{bmatrix}$$

ABCD parameter

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} Yz+1 & z \\ 3Y+\frac{1}{z}+Y^2 z & 2+Yz \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

$$V_2 = \frac{I_1 - DI_2}{C}$$

$$V_1 = A\left(\frac{I_1 - DI_2}{C}\right) + BI_2$$

$$= \frac{A}{C} I_1 - \frac{AD}{C} I_2 + BI_2$$

$$= \frac{A}{C} I_1 + \frac{BC-AD}{C} I_2$$

$$Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{BC-AD}{C}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$I_1 = \frac{V_2 - Z_{22} I_2}{Z_{21}}$$

(d) Starting from the *(A,B,C,D)* representation of a two-port network, derive its equivalent Y-representation. Show clearly how you arrived at your conclusions.

[6 marks]

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$I_2 = \frac{AV_2 - V_1}{B}$$

$$I_1 = CV_2 - D\left(\frac{AV_2 - V_1}{B}\right)$$

$$= CV_2 - \frac{AD}{B}V_2 + \frac{D}{B}V_1$$

$$= \frac{D}{B} + \frac{CB - AD}{B}V_2$$

Y-parameters

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$I_2 = \frac{V_1}{B} - \frac{A}{B}V_2$$

$$Y_{11} = \frac{D}{B} \qquad Y_{12} = -\frac{(AD - CB)}{D} \qquad Y_{21} = -\frac{1}{B} \qquad Y_{22} = \frac{A}{B}$$

$$Y = \begin{bmatrix} \frac{D}{B} & \frac{BC - AD}{D} \\ -1/B & \frac{A}{B} \end{bmatrix}$$

(e) Describe what is symmetry of a two-port network? Write down the equation, in terms of the (A,B,C,D) representation, derived from a reciprocal and symmetrical two-port network. Write down the iterative impedance Z_{it} of a symmetrical two-port network.

[7 marks].

A symmetry of a two-port network is when $A=D$.

Reciprocal is when $AD-BC=1$ — ①

Symmetrical is when $A=D$ — ②

So $D^2-BC=1$

Iterative impedance

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} D & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = DV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

$$V_2 = \frac{I_1 - DI_2}{C} = \frac{I_1}{C} - \frac{D}{C} I_2$$

$$V_1 = D\left(\frac{I_1 - DI_2}{C}\right) + BI_2$$
$$= \frac{D}{C} \cdot I_1 - \frac{D^2}{C} I_2 + BI_2$$
$$= \frac{D}{C} I_1 + \frac{BC - D^2}{C} I_2$$

Z-parameter

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$Z_{11} = \frac{D}{C}$ $\qquad Z_{12} = \frac{BC - D^2}{C} = -\frac{D^2 - BC}{C} = -\frac{1}{C}$

$Z_{21} = \frac{1}{C}$ $\qquad Z_{22} = -\frac{D}{C}$

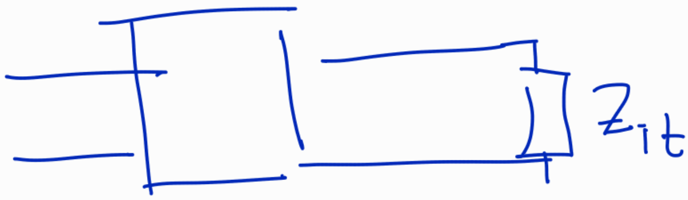
$Z_{It} = \frac{A + D - 2B}{A + D + 2B})$

$Z_{11} = \frac{V_1}{I_1} = \frac{A Z_{12} + B}{C Z_{12} + D}$

$\frac{V_2}{I'_2} = \frac{D Z_{11} + B}{C Z_{11} + A}$

Iterative impedance Z_{It} of a symmetrical TPN:

$Z_{I_1} = Z_{I_2} = \sqrt{\frac{B}{C}}$



$Z_{It} = Z_{in} = \frac{V_1}{I_1}$

Answer all questions in this section

Question 2:

a) **Explain** why there are *distortions* in signal transmission in lossy transmission lines. **Explain** what relation among the line parameters should hold to correct such distortions and why such a relation corrects the distortions.
[6 marks]

b) A lossless transmission line has the following per-unit-length parameters: $L = 0.5\,\mu H/m$, $C = 200\,pF/m$. **Calculate** the *propagation constant* and *characteristic impedance, wavelength* and the *phase velocity* for this lossless line at 800 MHz.
[5 marks]

c) **Calculate** the *propagation constant* and *characteristic impedance* of the line in section (b) at frequency 800 MHz for the case when the line is lossy with, $R = 4.0\,\Omega/m$ and $G = 0.02\,S/m$. If the line is 30 cm long, **Calculate** the *attenuation* in dB.
[6 marks]

d) A coaxial transmission line with the characteristic impedance of 75Ω has a length of 2.0 cm and is terminated with a load impedance of $37.5 + j75\Omega$. If the line has permeability of free space and the relative permittivity of 2.56 and the frequency is 3.0 GHz, **find** the *input impedance* of the line, the *reflection coefficient* at the load, the *reflection coefficient* at the input and the *SWR* on the line.
[9 marks]

e) Let Z_{sc} be the input impedance of a length of coaxial line when one end is short-circuited, and let Z_{oc} be the input impedance

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of the line when one end is open-circuited. **Derive** an expression for the *characteristic impedance* of the cable in terms of Z_{sc} and Z_{oc}.

[7 marks]

TURN OVER

SECTION C

Answer all questions in this section

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Question 3:

A balanced, star-connected three-phase voltage source is connected to an unbalanced star-connected three-phase load as shown in the Figure 4 below:

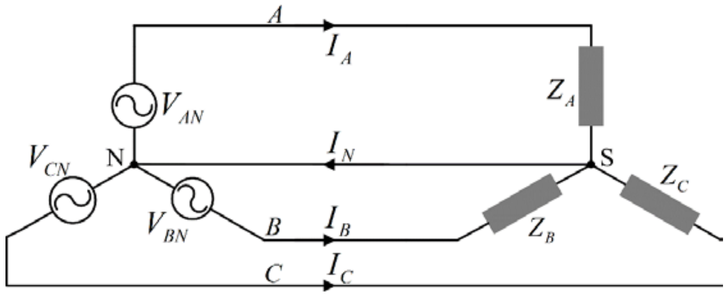


Fig. 4. The circuit for question 3

The phase voltage phasors and the loads are:

$V_{AN} = 230 \angle 0° \; V \, (rms)$ $\qquad$ $Z_A = 10 \angle 0° \; \Omega$

$V_{BN} = 230 \angle -120° \; V \, (rms)$ $\qquad$ $Z_B = 15 \angle 45° \; \Omega$

$V_{CN} = 230 \angle 120° \; V \, (rms)$ $\qquad$ $Z_C = 20.02 \angle 29.81 \; \Omega$

(a) Given that **total complex power** (S_{tot}) consumed by the three-phase load is $\boldsymbol{10772 \angle 20.7° \; VA}$, **calculate** phase currents I_A, I_B, I_C and neutral return current $\boldsymbol{I_N}$. **Calculate** complex powers S_A, S_B, S_C for each load. **Prove** that the phase impedance $Z_C = 17.37 + 9.95i \; (20.02 \angle 29.81°) \; \Omega$. *Show full workings.*

Hint: complex conjugate is required for power calculations.

[16 marks]

(b) Neutral current, I_N can be reduced by connecting an impedance, Z_N between the two neutral terminals N and S. **determine** Z_N such that neutral current reduces to $1 \angle 0° \; A$. **Calculate** the neutral line voltage V_{SN}. *Show full workings*

[6 marks]

(c) Use Millmans theorem to **calculate** the voltage between the two neutral terminals N and S for the case where the neutral

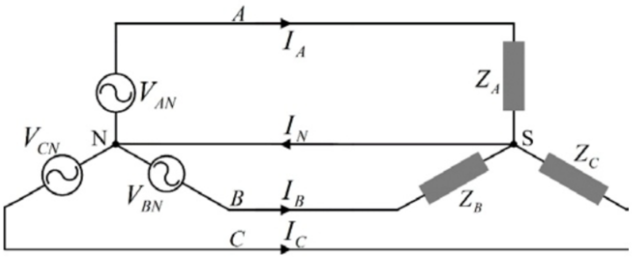


Fig. 4. The circuit for question 3

The phase voltage phasors and the loads are:

$V_{AN} = 230 \angle 0° \ V\ (rms)$	$Z_A = 10 \angle 0° \ \Omega$
$V_{BN} = 230 \angle -120° \ V\ (rms)$	$Z_B = 15 \angle 45° \ \Omega$
$V_{CN} = 230 \angle 120° \ V\ (rms)$	$Z_C = 20.02 \angle 29.81 \ \Omega$

(a) Given that **total complex power** (S_{tot}) consumed by the three-phase load is ***10772*** $\angle$ *20.7° VA*, **calculate** phase currents I_A, I_B, I_C and neutral return current $\boldsymbol{I_N}$. **Calculate** complex powers S_A, S_B, S_C for each load. **Prove** that the phase impedance Z_C *= 17.37 + 9.95i (20.02* $\angle$ *29.81°)* Ω*. Show full workings.*

Hint: complex conjugate is required for power calculations.

[16 marks]

$$I_A = \frac{V_{AN}}{Z_A} = \frac{\sqrt{2}\,(230\angle 0)}{10} = 32.527 \text{ A}$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{\sqrt{2}\,(230\angle -120°)}{15\angle 45°} = 21.6846\angle -165° \text{ A}$$

$$= -20.946 - j5.6124$$

$$I_C = \frac{V_{CN}}{Z_C} = \frac{\sqrt{2}\,(230\angle 120)}{20.02\angle 29.81} = 16.247\angle 90.19° \text{ A}$$

$$= -0.0539 + j16.247 \text{ A}$$

$$I_N = I_A + I_B + I_C$$

$$= 32.527 + 21.684\angle -165 + 16.247\angle 90.19$$

$$= 15.68\angle 42.7°$$

$$S = V I^*$$

$$S_A = \sqrt{2}\,(230\angle 0)(32.527)$$

$$= 10580.02864$$

$$S_A = \frac{(230\angle 0)(32.527)}{\sqrt{2}}$$

$$= 5290.014$$

$S_B = \sqrt{2}(230\angle -120)(-20.946+j5.6124)$

$= 7053.421\angle 45^\circ \div 2$

$= 3526.7105\angle 45^\circ$

$S_C = \sqrt{2}(230\angle 120)(-0.0539-j16.247)$

$= 5284.676\angle 29.81^\circ \div 2$

$= 2642.338\angle 29.81$

Total $S_A + S_B + S_C$

$= 5290.014 + 3526.7105\angle 45^\circ + 2642.338\angle 29.81$

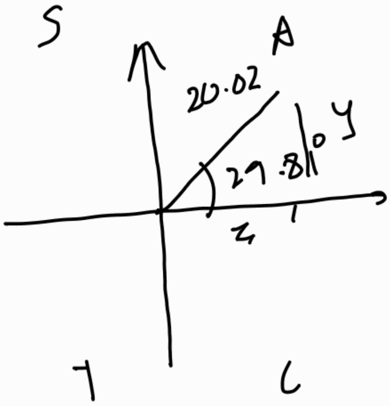
$= 10771.8\angle 20.7$

V_{SN}:

$54.463\angle 63.39$

$Z_C = 20.02\angle 29.81^\circ$

$S_N = 853.98\angle 20.69^\circ$



$\cos(29.81) = \dfrac{x}{20.02}$

$x = 17.371$

$\sin(29.81) = \dfrac{y}{20.02}$

$y = 9.9525$

$Z = 17.371 + j9.9525\,\Omega$

$= 17.37 + j9.95\,\Omega$

(c) Use Millmans theorem to **calculate** the voltage between the two neutral terminals N and S for the case where the neutral wire is disconnected (open circuit). **Calculate** voltages V_{AS}, V_{BS}, V_{CS} and currents I_A, I_B, I_C across each load. Calculate **total complex power** consumed by the three-phase load. *Show full workings.*

Hint: complex conjugate is required for power calculations.

[11 marks]

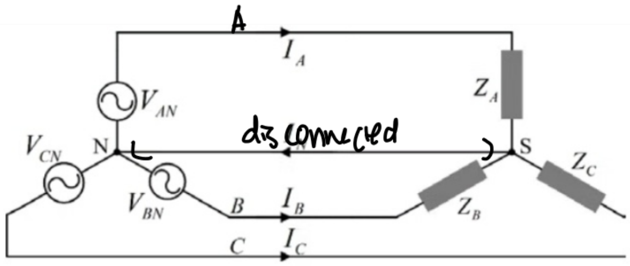


Fig. 4. The circuit for question 3

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$$V_{SN} = \frac{\frac{230\angle 0}{10\angle 0} + \frac{230\angle -120}{15\angle 45} + \frac{230\angle 120}{20.02\angle 29.81}}{\frac{1}{10\angle 0} + \frac{1}{15\angle 45} + \frac{1}{20.02\angle 29.81}}$$

$$= \frac{11.09\angle 42.694}{0.20362\angle -20.699}$$

$$= 54.463\angle 63.39$$

$V_{AS} = V_{AN} - V_{SN}$

$= 230 - 54.463\angle 63.39$

$= 211.293\angle -13.324$

Peak voltage V_{AS}

$= \sqrt{2} \times 211.293\angle -13.324$

$= 298.81\angle -13.324^\circ$

$V_{BS} = V_{BN} - V_{SN}$

$= 230\angle -120 - 54.463\angle 63.39$

$= 284.386\angle -119.35$

Peak voltage V_{BS}

$= \sqrt{2} \times 284.386\angle -119.35^\circ$

$= 402.1824\angle -119.35^\circ$

$V_{CS} = V_{CN} - V_{SN}$

$= 230\angle 120 - 54.463\angle 63.39$

$= 205.13\angle 132.808$

Peak voltage V_{CS}

$= \sqrt{2} \times 205.13\angle 132.808$

$= 290.099\angle 132.81^\circ$

$$I_A = \frac{V_{AS}}{Z_A}$$

$$= \frac{298.81\angle -13.324}{10}$$

$$= 29.881\angle -13.324 \text{ A}$$

$$= 29.08 - 6.886j \text{ A}$$

$$I_B = \frac{V_{BS}}{Z_B}$$

$$= \frac{402.1824\angle -119.35^\circ}{15\angle 45}$$

$$= 26.81\angle 164.35^\circ \text{ A}$$

$$= -25.816 + j7.232 \text{ A}$$

$$I_C = \frac{V_{CS}}{Z_C}$$

$$= \frac{205.13\angle 132.808}{20.02\angle 29.81}$$

$$= 10.246\angle 102.998^\circ \text{A}$$

$$= -2.3045 + J\,9.983 \text{ A}$$

Compex power

$$S = \frac{1}{2} V I^*$$

$$S_A = (298.81\angle -13.324)(29.08 + J\,6.886)$$

$$= \frac{8928.74}{2}$$

$$= 4464.37 \text{ VA}$$

$$S_B = (402.1824\angle -119.35^\circ)(-25.816 - j7.232) \div 2$$

$$= (2553.71 + j\,10475.74) \div 2$$

$$= (10782.51\angle 76.3) \div 2$$

$$= 5391.255\angle 76.3$$

$$S = V_{RMS}\, I_{RMS}^*$$

$$S_c = \frac{(290.099\angle 132.81)(-2.3045 - j9.783)}{2}$$

$$= (2578.997 + j1477.72) \div 2$$

$$= (2972.35 \angle 29.812) \div 2$$

$$= 1486.175$$

$$S_A + S_B + S_C = \underline{\underline{18455.60\angle 40.37^\circ}}$$

$$S_N = \sqrt{2}(54.463\angle 63.39)(0.956 - j10.329)$$

$$= 798.964 \angle -21.322$$

$$S_A + S_B + S_C + S_N = 18847.61\angle 38.23 \ (?)$$

(b) Neutral current, I_N can be reduced by connecting an impedance, Z_N between the two neutral terminals N and S. **determine** Z_N such that neutral current reduces to $1 \angle 0° A$. **Calculate** the neutral line voltage V_{SN}. *Show full workings*

[6 marks]

$$V_{SN} = \frac{\frac{230\angle 0}{10\angle 0} + \frac{230\angle -120}{15\angle 45} + \frac{230\angle 120}{20.02\angle 29.81}}{\frac{1}{10\angle 0} + \frac{1}{15\angle 45} + \frac{1}{20.02\angle 29.81}}$$

$$= \frac{11.09\angle 42.694}{0.20362\angle -20.699}$$

$$= 54.463\angle 63.39$$

$I_N = 15.68\angle 42.7°$

we want
$I_N = 1\angle 0°$

$V_{SN} = I_N Z_N$

$$Z_N = \frac{V_{SN}}{I_N}$$

$$= \frac{54.463\angle 63.39}{1\angle 0}$$

$$= 54.463\angle 63.39°$$

?

$$\frac{15.68\angle 42.7}{1\angle 0}$$

$V = IR$

$$= 15.68\angle 42.7$$