

SEMESTER 2 EXAMINATIONS 2015/16

CIRCUITS AND TRANSMISSION

Duration 120 mins (2 hours)

This paper contains 6 questions

Answer **ONE** question in **Section A**, **ONE** question in **Section B** and **ONE** question in **Section C**.

Section A carries 33% of the total marks for the exam paper.

Section B carries 33% of the total marks for the exam paper.

Section C carries 33% of the total marks for the exam paper.

Only University approved calculators may be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct 'Word to Word' translation dictionary AND it contains no notes, additions or annotations.

11 page examination paper (+ 2 page formula sheet, 1 page The Complete Smith Chart)

SECTION A**Answer ONE out of TWO questions in this section****Question A1**

- (a) Using Millman's theorem (pay attention to the orientation of the voltages and currents!), compute the numerical values of the currents I_1 , I_2 , I_3 in the circuit in Figure 1 when $v_1 = 10 \text{ V}$, $v_2 = v_3 = 7 \text{ V}$, $R_1 = 2 \Omega$, $R_2 = R_3 = 1 \Omega$.

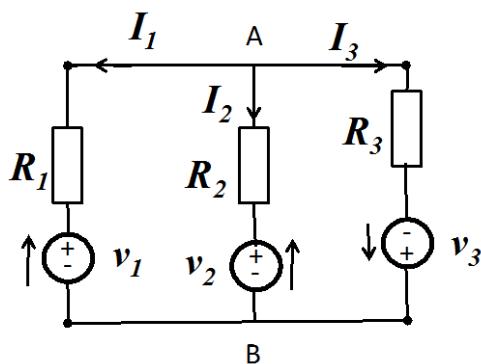


Fig. 1. The circuit for question 1(a)

[5 marks]

- (b) Consider the circuit in Figure 2. Using node analysis with the ground reference node indicated, find the value of v_1 , v_2 , v_3 when $I_0 = 1 \text{ A}$ and $R_1 = R_2 = R_3 = R_4 = 1 \Omega$.

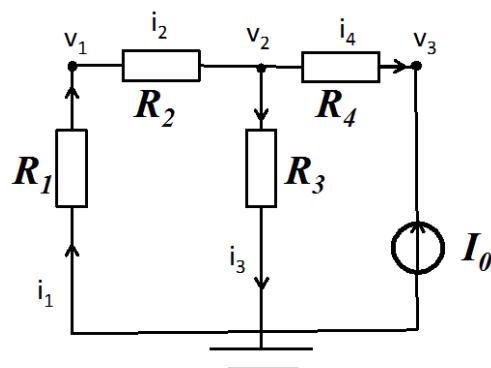


Fig. 2. The circuit for question 1(b)

[7 marks]

- (c) Consider the two-port network in Figure 3, where the first impedance is $Z_1 = R$ (a resistance), the second one is $Z_2 = L_s$ (an inductor), and the admittance $Y = C_s$ (a capacitor). Derive the (A,B,C,D) representation of this network.

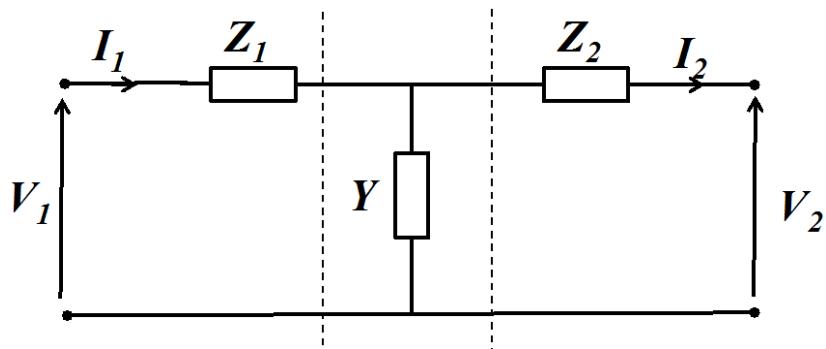


Fig. 3. The circuit for question 1(c), 1(d) and 1(e)

[6 marks]

- (d) Consider the two-port network in Figure 3. *Starting from the (A,B,C,D) representation* found answering Question 1 (c), derive its equivalent Z -representation. Show clearly how you arrived at your conclusions, i.e. how the Z -matrix is related to the (A,B,C,D) one.

[8 marks]

- (e) Under which conditions is the two-port network in Figure 3 reciprocal? What are the consequences of reciprocity on the (A,B,C,D) representation of a two-port? Give a symbolic expression for the iterative impedance of a symmetric, reciprocal network as a function of its (A,B,C,D) parameters.

[7 marks]

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Question A2

- (a) Using Rosen's theorem, find the equivalent "delta" circuit of the "star" circuit in Figure 4.

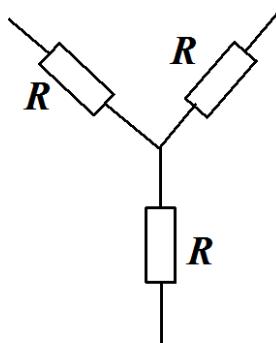


Fig. 4. The circuit for question 2(a)

[5 marks]

- (b) Using mesh analysis and the mesh currents indicated, find the values of i_1 , i_2 , i_3 , when $V_s = 1$ V and $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 1 \Omega$.

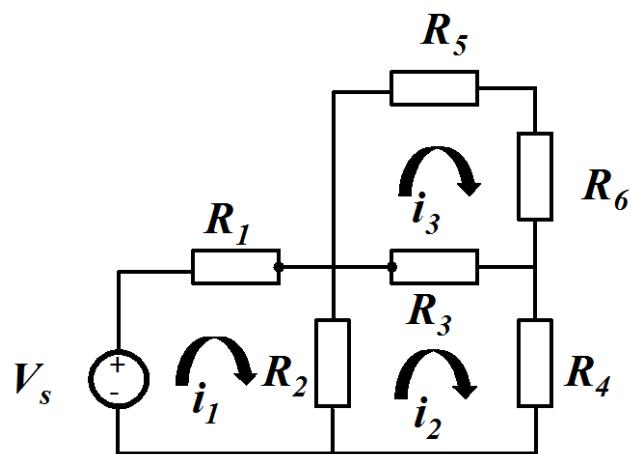


Fig. 5. The circuit for question 2(b)

[7 marks]

Question continues on following page

- (c) A given resistive network has the (A, B, C, D) representation

$$\begin{bmatrix} 2 & 10 \\ 3 & \frac{1}{10} \\ \hline 10 & 2 \end{bmatrix}.$$

Find an equivalent π -circuit containing only resistances. Make clear what the numerical values of the resistances in the equivalent network are.

[8 marks]

- (d) Using Thevenin's theorem, find the equivalent circuit (with respect to the terminals A and B) to that in Figure 6. Use such equivalent circuit to compute the numerical value of the current through R_3 when $R_1 = R_2 = R_3 = 1 \Omega$ and $E_1 = 2 V$.

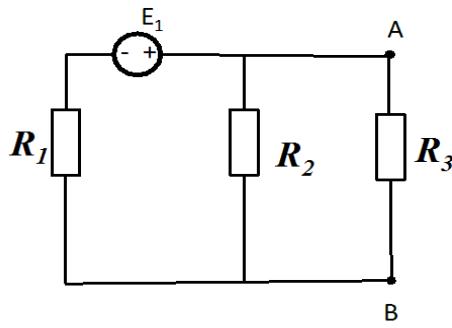


Fig. 6. The circuit for question 2(d)

[8 marks]

- (e) Give expressions for the image impedances of the two-port in Figure 7.

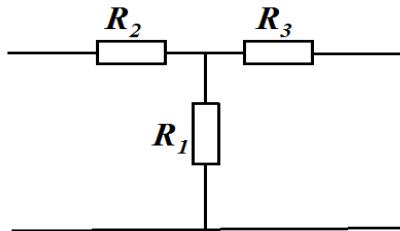


Fig. 7. The circuit for question 2(e)

[5 marks]

TURN OVER

SECTION B**Answer ONE out of TWO questions in this section****Question B1**

(a) A transmission line has the following per unit length parameters: $L = 0.2 \mu H/m$, $C = 250 pF/m$, $R = 3\Omega/m$, $G = 0.04 S/m$

- (i) Calculate the propagation constant, characteristic impedance, wavelength and the phase velocity for this line at $600 MHz$. Also calculate the length of the line at which the attenuation is $0.2 dB$. [5 marks]
- (ii) Assuming that at coordinate z on this transmission line, the instantaneous current is given by $i(t) = 200 \times 10^{-3} \cos(2\pi \times 600 \times 10^6 t) A$, find an expression for $\frac{\partial^2 i}{\partial z^2}$ along the line at the point z in A/m^2 . [8 marks]
- (iii) Assuming that the transmission line is terminated with a load $Z_L = 40\Omega$, find the length of the transmission line for which the reflection coefficient at its input is $-0.171j$ ($j = \sqrt{-1}$). Then calculate the input impedance of this transmission line. [8 marks]

Question continues on following page

$$\textcircled{a}) \quad L = 0.2 \mu\text{H m}^{-1} \quad f = 600 \text{ MHz}$$

$$C = 250 \text{ pF m}^{-1}$$

calc length of line
at which attenuation = 0.2 dB.

$$R = 3 \Omega \text{ m}^{-1}$$

$$G = 0.04 \text{ S m}^{-1}$$

find R, Z_0, λ, V_p

$$r = a + j\beta = \sqrt{(R+jWL)(G+jWC)}$$

$$= \sqrt{(3 + j2\pi \times 600 \times 10^6 \times 0.2 \times 10^{-6})(0.04 + j2\pi \times 600 \times 10^6 \times 250 \times 10^{-12})}$$

$$r = \sqrt{-710.5 + j32.98}$$

$$= \underbrace{0.618}_{\alpha} + j\underbrace{26.66}_{\beta}$$

$$\alpha = 0.618 \text{ Npm}^{-1}$$

$$\beta = 26.66 \text{ radm}^{-1}$$

$$Z_0 = \sqrt{\frac{R+jWL}{G+jWC}} = \sqrt{\frac{3+j \times 753.6}{0.04+j0.94}} = 28.26 + j0.54$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{26.66} = 0.236 \text{ m}$$

$$V_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = 1.41 \times 10^8 \text{ ms}^{-1}$$

$$\text{Attenuation} = 0.2 \text{ dB}$$

$$\alpha L = 0.2 \text{ dB}$$

$$\alpha = 0.618 \text{ Npm}^{-1}$$

$$0.618 \times 8.686 \times L = 0.2 \text{ dB}$$

$$L = 0.037 \text{ m}$$

$\downarrow \text{Nepitton?}$
 $\boxed{1 \text{ Np} = 8.686 \text{ dB}}$

$\downarrow \text{base e}$
 $A = e^{-\alpha L}$
 $\downarrow \text{base 10 note e}$

$$\log A = \alpha L$$

Sqrt of complex no.

$$C = A + jB = \sqrt{A^2 + B^2} e^{j\theta}$$

$$D = \sqrt{C} \quad \theta = \tan^{-1}\left(\frac{B}{A}\right)$$

$$D = \sqrt{\sqrt{A^2 + B^2} e^{j\theta}} = \left(\sqrt{A^2 + B^2} e^{j\frac{\theta}{2}}\right)^{\frac{1}{2}}$$

$$= (\sqrt{A^2 + B^2})^{\frac{1}{4}} e^{j\frac{\theta}{2}}$$

$$= (A^2 + B^2)^{\frac{1}{4}} e^{j\frac{\theta}{2}}$$

$$= (A^2 + B^2)^{\frac{1}{4}} \left(\cos\left(\frac{\theta}{2}\right) + j\sin\left(\frac{\theta}{2}\right)\right)$$

- (b) Consider the transmission line with characteristic impedance Z_0 connected to the lossless transmission line with characteristic impedance of Z_1 as shown in the following figure. The Z_1 section of the transmission line is open circuit at the end

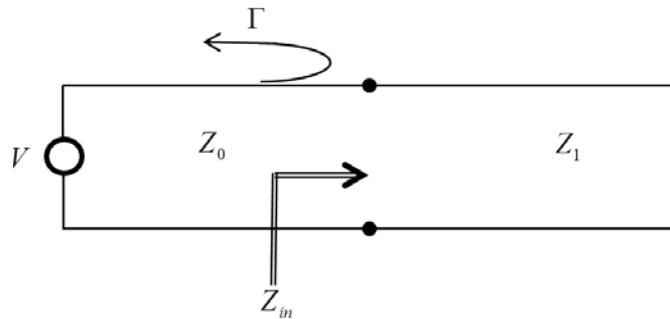


Fig. 8. The circuit for question 3(b)

- (i) Calculate the Z_{in} looking into the Z_1 section of the transmission line.

[6 marks]

- (ii) Find an expression relating Z_0 and Z_1 for the case when the reflection coefficient Γ looking into the Z_1 section of the transmission line is zero. Calculate also the length of the transmission line (in terms of the wavelength λ), if the relationship between Z_0 and Z_1 is $Z_0 = -jZ_1$.

[6 marks]

TURN OVER

Question B2

A microstrip line built on a dielectric FR4 substrate used at a frequency of 1 GHz has the following distributed circuit coefficients at that frequency:

$$R = 16.6 \Omega/m; L = 464.7 \times 10^{-6} \text{ H/m}; \\ G = 5.6 \times 10^{-3} \text{ S/m}; C = 74.7 \times 10^{-12} \text{ F/m}.$$

- (a) Find the characteristic impedance of the line at the frequency of operation (1 GHz) and comment on the result obtained in terms of losses of the line. [6 marks]
- (b) Assuming that the length of this transmission line is 10 cm , calculate the attenuation and phase difference that a phasor voltage with an amplitude of 5 V and frequency of 1 GHz will experience. What would be the amplitude of the voltage at the end of this line, assuming that the source impedance and load impedance connected to this line are equal and have only real parts. [8 marks]
- (c) Calculate the wavelength and phase velocity of a phasor voltage of 1 GHz frequency traveling along this microstrip line using the results obtained from (a) and (b) and compare them with the phase velocity and wavelength calculated knowing that the wave traveling along the line is a TEM wave and the effective permittivity of the medium is 3.13 , describe clearly all your assumptions. [7 marks]

Question continues on following page

- (d) The 10 cm microstrip line is connected between a power source with a source impedance of $Z_s = 50 \Omega$ and frequency of 1 GHz and a load with a load impedance $Z_L = 50 - j25 \Omega$ as shown figure (9). Assuming that the losses on the microstrip line can be ignored calculate the complex reflection coefficient at the interface between the power source and the input of the loaded microstrip line using the provided Smith chart.

[12 marks]

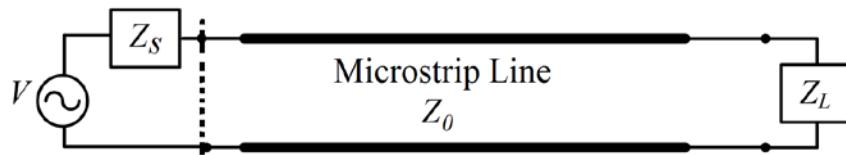


Fig. 9. The circuit for question 4(a) iv

TURN OVER

SECTION C

Answer ONE out of TWO questions in this section

Question C1

The impedances of an unbalanced, star connected three phase load can be represented as:

$$Z_{as} = (10 + j10) \Omega$$

$$Z_{bs} = (5 + j10) \Omega$$

$$Z_{cs} = (3 - j9) \Omega$$

The load is connected to a balanced, star connected, positive sequence rotation supply which has an rms line voltage of 400V. The neutral of the supply is grounded, but there is no connection between the supply neutral and the star point of the load. The three connecting leads (between the supply and the load) have a purely resistive impedance of 1Ω .

Determine the real power lost in the connecting leads as a percentage of the total real power supplied.

[33 marks]

Question C2

A star connected three phase supply is used to power two balanced loads, one star connected and one delta connected, in parallel.

The delta connected load has an impedance $Z_d = (6 + j9) \Omega$ per phase. The star connected load has a per phase impedance of $Z_s = (10 - j5) \Omega$, with the star point being connected to the supply neutral via an impedance $Z_n = j2 \Omega$.

The supply voltages are:

$$V_{AN} = 110 \angle 0^\circ V$$

$$V_{BN} = 100 \angle -120^\circ V$$

$$V_{CN} = 110 \angle 110^\circ V$$

- (a) Determine the sequence impedance representation of the load, as seen from the supply terminals.
[8 marks]
- (b) Use symmetrical components to find the phase currents drawn from the supply.
[20 marks]
- (c) Explain whether or not symmetrical components methods can be applied to cases of unbalanced loads.
[5 marks]

END OF PAPER