CONTENT

Reference

[1] M. H. Rashid. *Power Electronics – Circuits, Devices, and Applications*, 3rd ed. Pearson. 2003.

[2] Emil Levi. *Power Electronics, Drives and Systems*. Course material. 2014.

[3] Richard Crowder. *Electric Drives and Electromechanical Systems.* Elsevier. 2006.

1. ELECTROMAGNETIC AND MECHANICAL FUNDAMENTALS

1.1. Introduction

Modern electric motor drives can control starting currents, maintain precise set speeds, quickly change or reverse speeds, and enable quick stop. This makes machinery of all types more productive and flexible (e.g. by allowing quick change over to run different materials with minimum of downtime), and of higher quality. Multiple machines can be coordinated through networked motor-drive systems. Another advantage of motor drives that is gaining attention is its ability to reduce energy usage by a proper matching of the motor torque and the load. Variable torque loads like fans and pumps that vary their output by mechanical means can now adjust the motor's speed with the simple addition of a motor drive and subsequently leads to reduction of energy input by 25% to 50%. In short, motor drives can be found in machinery of all kinds: metal-machining, woodworking, chemical processing, water treatment, conveyors, heating, cooling, refrigeration, hoists, and just about every industrial or commercial process. The penetration will continue to grow.

Electric motor drive is indeed a unique area of electrical and electronic engineering study as this topic encompasses a wide range of domain knowledge. It requires the understanding of the law of electromagnetism that governs the electro-mechanical energy conversion process inside electric machines; the understanding of control law that governs the dynamics of electric machines; the understanding of power electronic converters as different types of converter topologies can fulfill different design requirements; the understanding of the auxiliary devices such as rotary-to-linear power transmission tools for application sizing. To ensure an effective learning of this introductory subject, this first chapter reviews several basic mechanical and electromagnetism concepts.

1.2. Moment of inertia

Moment of inertia, or rotational inertia of a point mass *m* about an axis of rotation is defined as the mass *m* times the perpendicular distance (from the axis) squared, $I = mr^2$. Moment of inertia of a non-point mass object is obtained by integration. In a three-dimensional Cartesian coordinate system, inertia of the cube about the three perpendicular axes are the respective integration of the inertia of a point mass *dm* over the entire cube body about the respective axes.

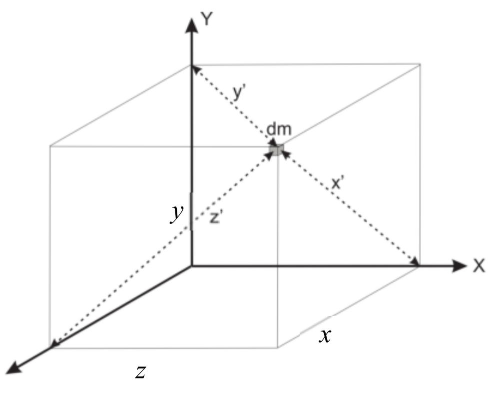


Fig. 1.1: A point mass of a cube in the three-dimensional Cartesian coordinate.

Based on Fig. 1.1, the moment of inertia of the cube about the *x*-, *y*- and *z*-axis can be found by:

$$I_{xx} = \int x'^2 \cdot dm = \int (y^2 + z^2) \cdot dm$$
$$I_{yy} = \int y'^2 \cdot dm = \int (x^2 + z^2) \cdot dm$$
$$I_{zz} = \int z'^2 \cdot dm = \int (x^2 + y^2) \cdot dm$$

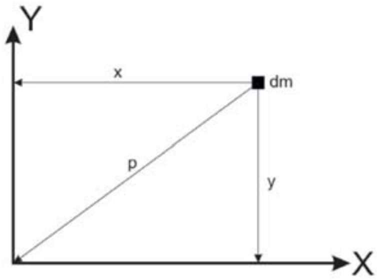


Fig. 1.2: A point mass of a thin plate on the *xy*-plane.

On the other hand, for a thin plate on the *xy*-plane with a small but non-zero thickness, as shown in Fig. 1.2, its moment of inertia along the *z*-axis is the sum of the inertia about the *x*- and *y*-axis, as shown below (note that $I_{xx} \neq I_x$, $I_{yy} \neq I_y$, $I_{zz} \neq I_z$):

$$I_x = \int y^2 \cdot dm$$
$$I_y = \int x^2 \cdot dm$$
$$I_{zz} = \int p^2 \cdot dm$$
$$= \int (x^2 + y^2) \cdot dm = I_y + I_x$$

Thin disc calculation example - the moment inertia about the *x*-, *y*-, and *z*-axis of a thin disc as shown in Fig. 1.3 are derived as follows:

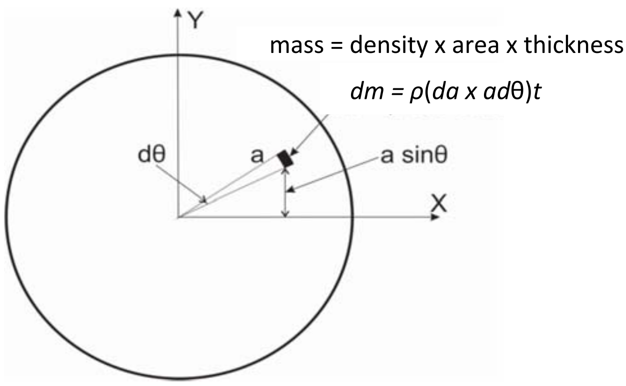


Fig. 1.3: A thin disc on the *xy*-plane.

Let ρ be the density and t be the disc thickness (*z*-axis not shown):

$$\text{Total volume} = \pi R^2 t$$
$$\text{Total mass } M = \rho \pi R^2 t$$

$$\begin{aligned}
I_x &= \int a^2 \sin^2\theta \cdot dm \\
&= \int_0^{2\pi}\int_0^R a^2 \sin^2\theta \cdot \rho t \times da \times a d\theta \\
&= \rho t \int_0^{2\pi}\int_0^R a^3 \sin^2\theta \cdot da \cdot d\theta \\
&= \rho t\left(\frac{1}{4}\pi R^4\right) \\
&= \frac{1}{4}\left(\rho\pi R^2 t\right)R^2 \\
&= \frac{1}{4}MR^2 = I_y
\end{aligned}$$

Hence, the inertia of the thin disc about the *z*-axis is

$$I_z = I_x + I_y = \frac{1}{2}MR^2$$

Cylinder calculation example - the moment of inertia about the *z*-axis of the solid cylinder as shown in Fig. 1.4 is derived as follows:

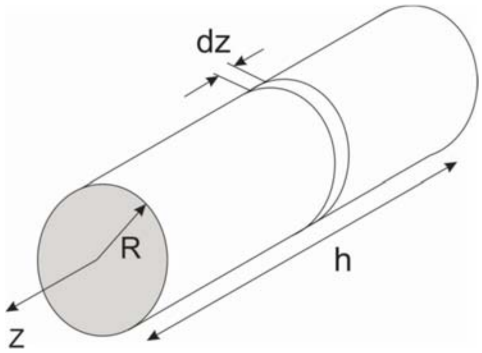


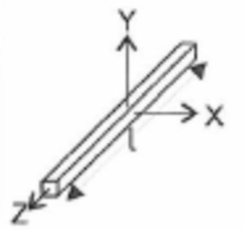
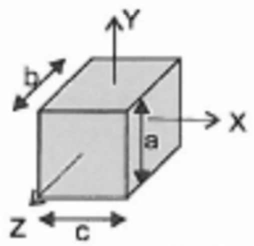
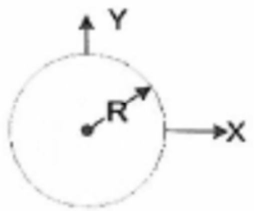
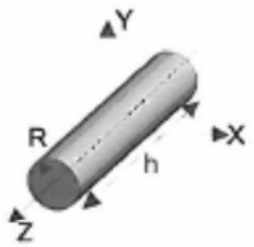
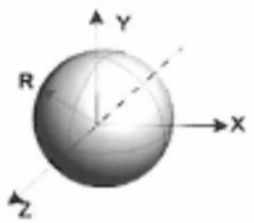
Fig. 1.4: A cylinder of radius *R* and length *h*.

Consider the cylinder of length *h* and mass *M* to be constructed of an array of thin discs with thickness *dz* and mass *M*(*dz*/*h*), we can make use of the previous derivation:

$$\begin{aligned}
I_z &= \int_0^h \frac{1}{2}MR^2\left(\frac{dz}{h}\right) \\
&= \frac{1}{2}M\frac{R^2}{h}\left[z\right]_0^h \\
&= \frac{1}{2}MR^2
\end{aligned}$$

The same method can be extended to other common bodies with uniform density and their moment of inertia can be obtained. The results for several common bodies are shown in Table 1.

Table 1: Moment of inertia for some common bodies with uniform density.

Body		I_{xx}	I_{yy}	I_{zz}
Slender bar		$\frac{ml^2}{12}$	$\frac{ml^2}{12}$	–
Cuboid		$\frac{m}{12}(a^2 + b^2)$	$\frac{m}{12}(b^2 + c^2)$	$\frac{m}{12}(a^2 + c^2)$
Thin disc*		$\frac{mR^2}{4}$	$\frac{mR^2}{4}$	$\frac{mR^2}{2}$
Cylinder		$\frac{m}{12}(3R^2 + h^2)$	$\frac{m}{12}(3R^2 + h^2)$	$\frac{mR^2}{2}$
Sphere		$\frac{2}{5}mR^2$	$\frac{2}{5}mR^2$	$\frac{2}{5}mR^2$

* A thin disc is considered a special case of a cylinder where h = 0.

Moment of inertia of a body about a particular axis of rotation is at its lowest value if the axis passes through its centre of mass – as the cases shown above. Parallel axis theorem states that the moment of inertia about any other axis parallel to the centre of mass' axis is given by the moment of inertia about the centre of mass' axis plus the moment of inertia of the entire object treated as a point mass about the new parallel axis at a distance *d* from the centre of mass' axis.

$$I_{new} = I_{com} + Md^2$$

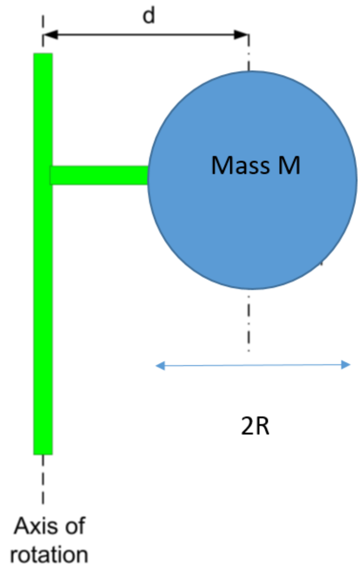


Fig. 1.5: A thin disc with a new parallel axis.

Taking the example shown in Fig. 1.5, the thin disc's new moment of inertia about the new axis of rotation (green, vertical) is

$$J_{thin\ disc,new} = \frac{MR^2}{4} + Md^2$$

1.3. Friction

Fig. 1.6 shows a block of mass *m* placed on an inclined surface. The friction imposes on the block by the inclined surface is:

$$F_{friction} = \mu \cdot F_{normal}$$

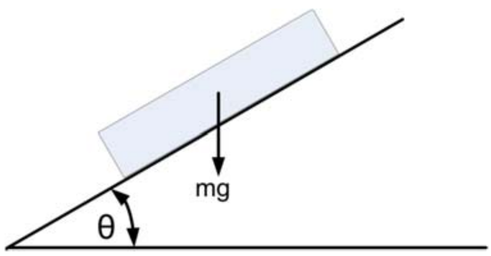


Fig. 1.6: A block of mass *m* on a surface of inclination angle *θ*.

If the mass *m* is stationary, we have the static friction coefficient:

$$F_{static} = \mu_s \cdot mg\cos\theta$$

If the mass *m* is moving, we have the kinetic friction coefficient:

$$F_{kinetic} = \mu_k \cdot mg\cos\theta$$

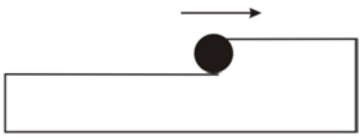
The coefficient of static friction μ_s is typically larger than the coefficient of kinetic friction μ_k.

1.4. Path control

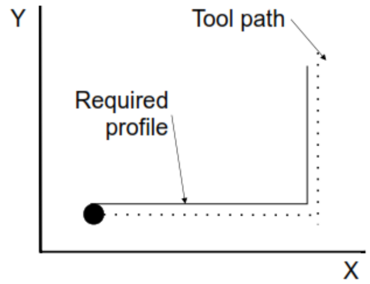
Path/position control is usually needed in process automation (note that this is not the machine angular internal position closed-loop control but rather a simpler, open-loop position control to meet the purpose of the application). Three basic strategies for path control can be employed:

1.4.1. Straight cut

An example of straight cut is in the milling process. Fig. 1.7 shows two simple profiles of straight cut.



(a)



(b)

Fig. 1.7: (a) Straight line; (b) linear movement with junctions.

1.4.2. Point-to-point (Trapezoidal)

An application example that uses trapezoidal point-to-point movement is CNC machining. The power requirement of the electric drives can be known if the peak acceleration/deceleration and peak speed of the trapezoidal profile are known. With the known load inertia, then the highest possible torque assuming that the peak acceleration and peak speed occurring at the same time ($T_{e,peak} = J\alpha_{peak} + B\omega_{peak}$), and its power requirement, can be estimated.

What follows is the calculation to get the peak speed, peak acceleration, and acceleration/deceleration duty cycle of the trapezoidal movement. The following derivation takes the angular motion as an example, but the same trapezoidal profile can be extended to linear motion. Fig. 1.8 shows one period of the rotational speed profile of a repetitive point-to-point movement.

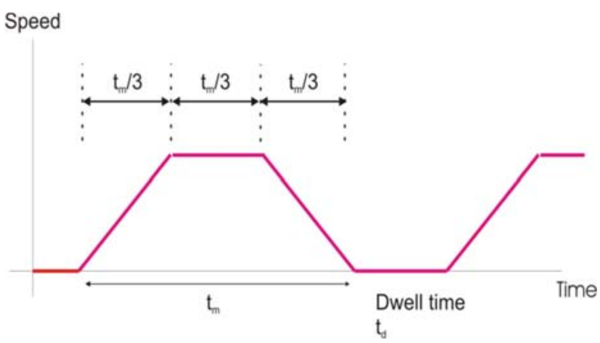


Fig. 1.8: Trapezoidal speed profile with equal $t_m/3$ duration for acceleration, constant-speed, and deceleration, and a dwell time t_d.

Note that the duration for acceleration, constant-speed (non-zero), and deceleration duration are the same and equal to $t_m/3$, respectively. An inactive period of dwell time t_d is a part of the profile. For a rotary move of angle θ in the whole period of ($t_m + t_d$), the peak speed of the movement can be determined by equating the area under the curve to θ, and with a simple manipulation, we get:

$$\omega_{peak} = \frac{3\theta}{2t_m}$$

The acceleration can then be determined as:

$$\alpha_{slope} = \frac{3\omega_{peak}}{t_m}$$

Lastly, the acceleration/deceleration duty cycle, which is defined as the ratio of the duration with acceleration and deceleration to the full period of the repetitive movements:

$$d = \frac{2t_m}{3(t_m + t_d)}$$

It should be noted that the motion is not limited to rotation but can be extended to linear movement with a proper change of variables.

1.4.3. Point-to-point (Triangular)

Similarly, Fig. 1.9 shows the triangular counterpart of the point-to-point movement for the rotational motion similar to the last subsection.

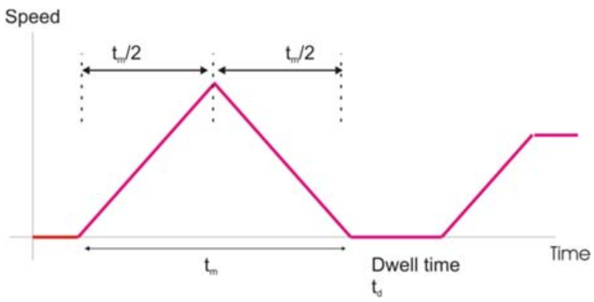


Fig. 1.9: Triangular speed profile with equal $t_m/2$ duration for acceleration and deceleration, and a dwell time t_d.

For a rotary move of angle θ in the whole period of (t_m + t_d), the peak speed of the movement can be determined as:

$$\omega_{peak} = \frac{2\theta}{t_m}$$

The acceleration can then be determined as:

$$\alpha_{slope} = \frac{2\omega_{peak}}{t_m}$$

And lastly, the acceleration/deceleration duty cycle is:

$$d = \frac{2t_m}{2t_m + t_d}$$

1.4.4. Contouring (e.g. in rotational motion)

An example of contouring path control is in CNC machine.

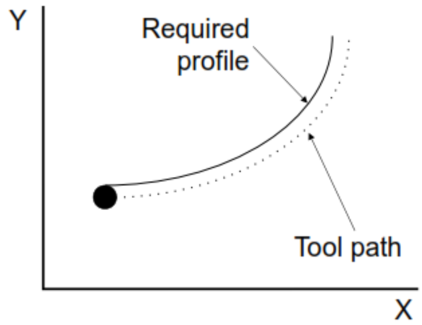


Fig. 1.10: Contouring

A smooth contour can be generated by a polynomial path. Cubic polynomial is chosen here to illustrate the angular position path (again, the same can be extended to linear path):

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

It can be noticed from the cubic function that there are four unknown parameters. Therefore, if there are four known points (of the angular position, speed, or acceleration), then these parameters can be determined. Taking the following four points of the position and speed profiles with zero starting and ending velocity as an example (note that the velocity can be respectively of any value, but they should be the same if the motion is meant to be continuous without dwell time; the position shall be the travelled position instead of displacement):

$$\begin{matrix} \theta(0) = \theta_1 \\ \theta(t_f) = \theta_2 \end{matrix} \quad \text{and} \quad \begin{matrix} \dot{\theta}(0) = 0 \\ \dot{\theta}(t_f) = 0 \end{matrix}$$

where t_f is the time required to complete the path from θ_1 to θ_2. The corresponding speed and acceleration profiles can be derived through differentiation:

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$

The four known points on the position and speed profiles can form four simultaneous equations. Solving them gives a_1, a_2, a_3, and a_4 values in terms of θ_1, θ_2, and t_f.

$$a_0 = \theta_1$$
$$a_1 = 0$$
$$a_2 = \frac{3}{t_f^2}(\theta_2 - \theta_1)$$
$$a_3 = -\frac{2}{t_f^3}(\theta_2 - \theta_1)$$

Then, the cubic angular position profile can be expressed in terms of the knowns variables as:

$$\theta(t) = \theta_1 + \frac{3}{t_f^2}(\theta_2 - \theta_1)t^2 - \frac{2}{t_f^3}(\theta_2 - \theta_1)t^3$$

The corresponding speed and acceleration profiles are:

$$\dot{\theta}(t) = \frac{6}{t_f^2}(\theta_2 - \theta_1)t - \frac{6}{t_f^3}(\theta_2 - \theta_1)t^2$$

$$\ddot{\theta}(t) = \frac{6}{t_f^2}(\theta_2 - \theta_1) - \frac{12}{t_f^3}(\theta_2 - \theta_1)t$$

These angular, speed, and acceleration profiles with the four given points (θ_1 = 15; θ_2 = 75; initial speed = 0; final speed = 0; t_f = 3s) are plotted in the Fig. 1.11 below.

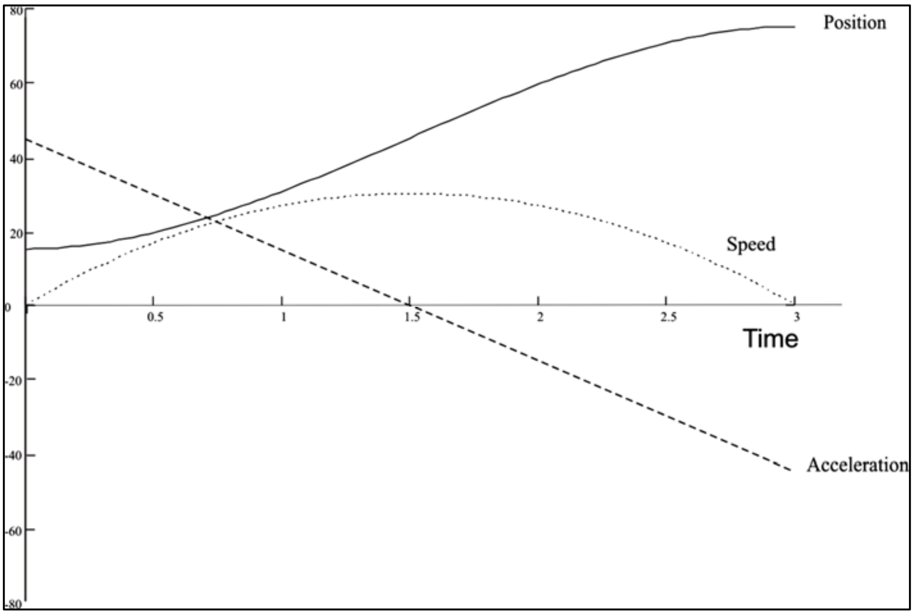
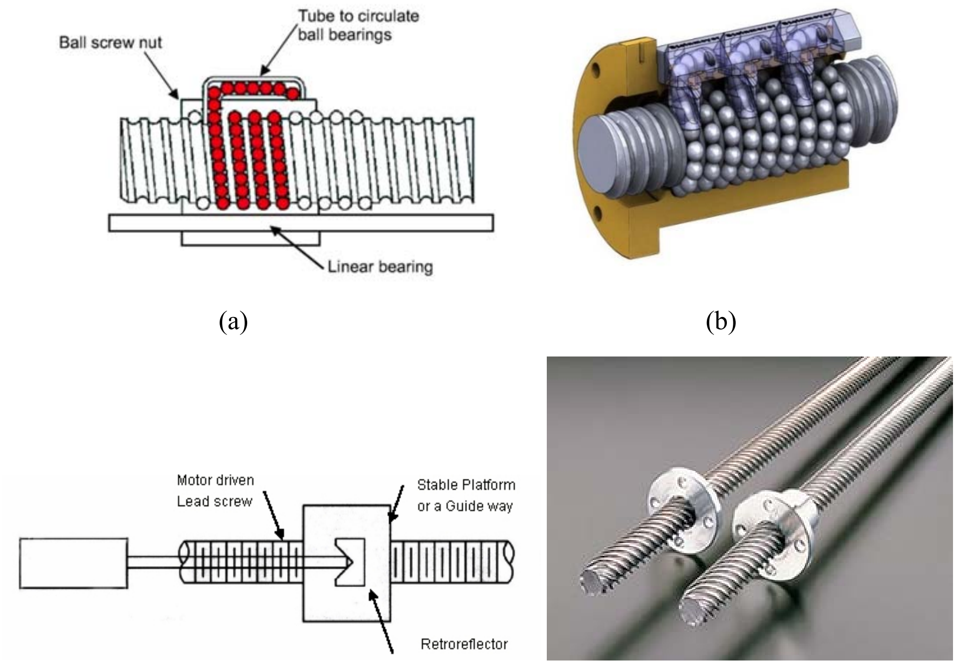


Fig. 1.11: A cubic position profile and the corresponding speed and acceleration profiles.

1.5. Power transmission (linkage)

Rotating electrical machines is the main form of electrical-to-mechanical energy "converter" that produces rotational motion. In actual application, there is sometimes a need to have linear motion in applications such as belt-conveyor and hydraulic press. A variety of power transmission mechanism (linkage) such as pulley, hydraulic, and screw mechanism are available to meet different design requirement. As an example, screw power transmission mechanism of the types of ballscrew and leadscrew are the preferred choices over belt mechanism in low-power position automation with different load levels as this type can maintain high accuracy in position control. They are illustrated further in this section.

Both ballscrew and leadscrew convert rotary motion into linear motion. Ballscrews have low friction (if non-ideal ballscrew is considered) and hence good dynamic response. Leadscrews have direct contact between the screw and the nut and this leads to a relatively higher friction than ballscrew and therefore less efficient. They are shown respectively in Fig. 1.12.

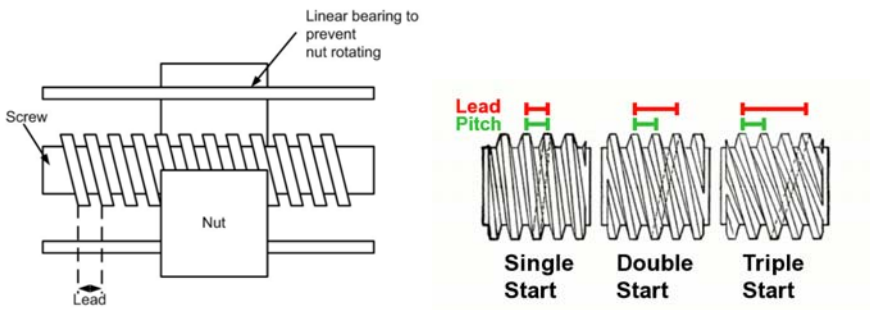


(a) (b)

(c) (d)

Fig. 1.12: (a) Ballscrew cross-sectional view; (b) ballscrew 3D view; (c) leadscrew's cross-sectional view; (d) leadscrew.

The distance moved by one turn of the leadscrew is termed as "lead". Lead should not be confused with the pitch of the screw, which is the distance between the threads, illustrated graphically in Fig. 1.13.



(a) (b)

Fig. 1.13: (a) Lead; (b) lead and pitch.

The torque-force transmission through the screw mechanism is first discussed based on a free-body diagram, then it is followed by its kinematic aspect that includes the relationship between the linear and angular displacement/velocity.

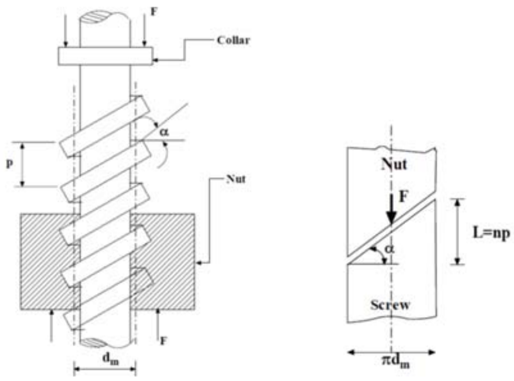


Fig. 1.14: Cross-sectional view of the leadscrew.

The screw-raising torque expression is derived in the following section.

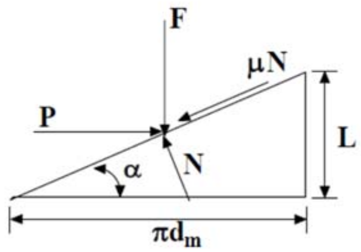


Fig. 1.15: Force diagram for the screw-raising case.

At equilibrium:

$$P - \mu N\cos\alpha - N\sin\alpha = 0$$
$$F + \mu N\sin\alpha - N\cos\alpha = 0$$

This gives:

$$N = \frac{F}{\cos\alpha - \mu\sin\alpha}$$
$$P = \frac{F(\mu\cos\alpha + \sin\alpha)}{\cos\alpha - \mu\sin\alpha}$$

Torque transmitted during raising of the load is given by:

$$\begin{aligned} T_{raise} &= P\frac{d_m}{2} \\ &= \frac{d_m}{2}\frac{F(\mu\cos\alpha + \sin\alpha)}{\cos\alpha - \mu\sin\alpha} \\ &= \frac{d_m}{2}\frac{F(\mu + \tan\alpha)}{(1-\mu\tan\alpha)} \end{aligned}$$

Since $\tan\alpha = \dfrac{L}{\pi d_m}$, we have the screw-raising torque as

$$T_{raise} = \frac{d_m}{2}\frac{F(\mu\pi d_m + L)}{(\pi d_m - \mu L)}$$

Friction coefficient μ can also be expressed as tan λ, where λ is the specific inclination angle with which a block on the inclined surface is about to slide down (basic definition). We then get:

$$\begin{aligned} T_{raise} &= \frac{d_m}{2}\frac{F(\mu\pi d_m + L)}{(\pi d_m - \mu L)} \\ &= \frac{Fd_m}{2}\frac{(\mu + \frac{L}{\pi d_m})}{(1-\mu\frac{L}{\pi d_m})} \\ &= \frac{Fd_m}{2}\frac{(\tan\lambda + \tan\alpha)}{(1-\tan\lambda\tan\alpha)} \\ &= \frac{Fd_m}{2}\tan(\lambda+\alpha) \end{aligned}$$

The screw-lowering torque expression is derived next.

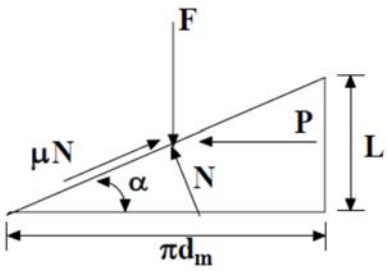


Fig. 1.16: Force diagram for the screw-lowering case.

At equilibrium:

$$\begin{aligned} P - \mu N\cos\alpha + N\sin\alpha = 0 \\ F - \mu N\sin\alpha - N\cos\alpha = 0 \end{aligned}$$

This gives:

$$\begin{aligned} N &= \frac{F}{\cos\alpha + \mu\sin\alpha} \\ P &= \frac{F(\mu\cos\alpha - \sin\alpha)}{\cos\alpha + \mu\sin\alpha} \end{aligned}$$

Torque transmitted during lowering of the load is given by:

$$
\begin{aligned}
T_{lower} &= P\frac{d_m}{2} \\
&= \frac{d_m}{2}\frac{F(\mu\cos\alpha - \sin\alpha)}{\cos\alpha + \mu\sin\alpha} \\
&= \frac{d_m}{2}\frac{F(\mu - \tan\alpha)}{(1+\mu\tan\alpha)}
\end{aligned}
$$

Alternatively,

$$
\begin{aligned}
T_{lower} &= \frac{d_m}{2}\frac{F(\mu\pi d_m - L)}{(\pi d_m + \mu L)} \\
&= \frac{Fd_m}{2}\frac{(\mu - \frac{L}{\pi d_m})}{(1+\mu\frac{L}{\pi d_m})} \\
&= \frac{Fd_m}{2}\frac{(\tan\lambda - \tan\alpha)}{(1+\tan\lambda\tan\alpha)} \\
&= \frac{Fd_m}{2}\tan(\lambda - \alpha)
\end{aligned}
$$

However, if the friction were to be ignored, the two expressions can be simplified to:

$$T_{raise} = \frac{d_m}{2}\frac{FL}{\pi d_m} = \frac{FL}{2\pi}$$

$$T_{lower} = \frac{d_m}{2}\frac{F(-L)}{(\pi d_m)} = -\frac{FL}{2\pi}$$

What follows is the kinematic aspects of a screw during the power transmission (linkage). We want to derive the relation between their rotational motion and the linear motion for displacement and speed. From Fig. 1.14, we know that the lead is

$$L = \pi d_m \tan\alpha = 2\pi r \tan\alpha$$

where d_m and r are the screw diameter and radius, and α is the lead angle. Angular speed is obtained from:

$$N = \frac{V_{linear}}{L}$$

where N has the unit revolution per second or rps, V_{linear} has the unit ms^{-1}, and L is the lead in m. Angular revolution per minute RPM is simply equal to $60N$.

In order to obtain an accurate linear/rotational displacement control, optical encoder is usually installed at the screw shaft. A common parameter of an optical encoder is the number of pulses generated per revolution. A single pulse is the smallest angular step that the encoder can detect, which essentially becomes the resolution of the angular displacement θ. Then, the relationship between the linear displacement S, in m, to angular displacement θ, in rad, can be obtained using

$$S = \frac{\theta L}{2\pi}$$

This relation can be easily derived by equating the ratio of S/θ to the ratio of $L/2\pi$ as one full 2π revolution of the screw will produce a linear displacement of L. As an example, an angular displacement of 10π rad for ballscrew with lead L = 0.01 m has a linear displacement of $(10\pi/2\pi)$*0.01 = 0.05m. Then, for a given linear displacement step/resolution $S_{resolution}$ and a screw lead L, the number of pulses per revolution (a specification for digital encoder) required to meet the displacement resolution can be determined from:

$$\text{Pulses per revolution} = \frac{L}{S_{resolution}}$$

Ballscrew/leadscrew mechanism converts a rotational motion to a linear motion, and since the rotational motional dynamics depends on the rotating body's moment of inertia, the equivalent moment of inertia imposed by a linear moving body attached to the rotating screw shall be determined. Fig. 1.17 shows a general representation of a linear body attached to a leadscrew.

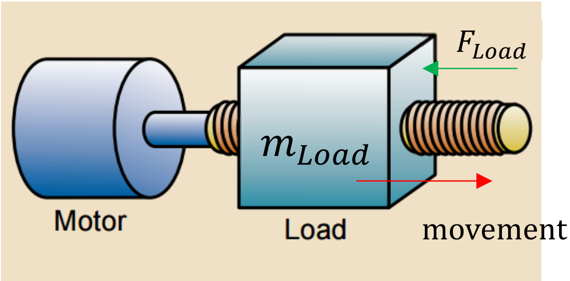


Fig. 1.17: A typical leadscrew application.

The total inertia of the screw-cube body can be calculated bby

$$J_{total} = J_{screw} + J_{load}$$

where $J_{load} = m_{load}\left(\frac{L}{2\pi}\right)^2$ and $J_{screw} = \frac{m_{screw} r^2}{2}$.

Lastly, from previous derivation, the linear force that opposes the movement of the linear body is seen by the rotating motor as a torque with magnitude:

$$T_{load} = \frac{LF_{load}}{2\pi}$$

Accounting for the losses due to friction, instead of using the exact formulae derived earlier, the opposing/load torque is "amplified" by the non-ideal condition and is approximated using:

$$T_{eff} = \frac{T_{load}}{e}$$

The screw efficiency e usually has a value in between 0.3 to 0.9. Some common values are shown in

Table 2.

Table 2: Typical values for screw efficiency.

System type	Efficiency
Ball screw	0.95
Lead screw	0.90
Rolled-ball lead screw	0.80
ACME threaded lead screw	0.40

1.6. Rotational motion dynamics and kinetic energy

Electrical machines exist in two major categories based on the nature of the mechanical movement: linear and rotary. A vast majority of the existing machinery belong to the rotary type. This includes for example the mega-watt motors or generators used in heavy industries or power plants, the medium-power electric motors used in compressors, pumps, and also the low-power motors used in automation industry. Linear motors are mainly used in very specialized applications. Whenever a linear work is required, e.g. in conveyor system that move the loads in linear manner, power transmission mechanism such as ballscrew, leadscrew, pulley, and etc. are commonly used.

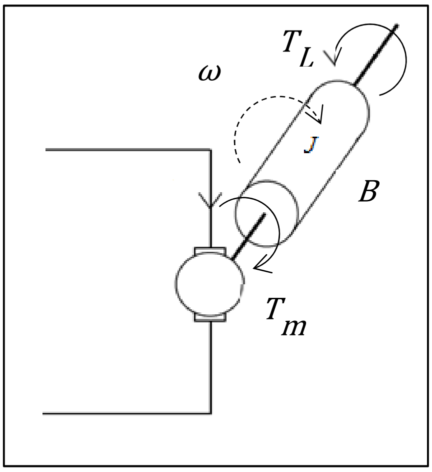


Fig. 1.18: A standard rotating motor with a coupled load of inertia *J*.

Fig. 1.18 shows a typical motor-load system. Apply the Newtow's second law of motion to the rotating body, we form the ideal torque balanced equation (which is equivalent to $F = ma$ but now in the rotational domain):

$$T_m - T_L = J\frac{d\omega}{dt}$$

where T_m is the torque developed by the motor, in Nm; T_L is the torque required to drive the load (physical load is not shown in Fig. 1.6), in Nm; J is the combined moment of inertia of the motor-load system, in kgm^2; and ω is the angular velocity of the motor shaft, in rads^{-2}. In practice, the windage friction or damping constant B, which is typically assumed to be linearly proportional to the speed of rotation, opposes the driving torque T_m and this is reflected in the following torque balanced equation.

$$T_m - T_L - B\omega = J\frac{d\omega}{dt}$$

$$T_m = T_L + J\frac{d\omega}{dt} + B\omega$$

where B is in Nm/(rad/s).

An acceleration or deceleration of a rotating mass with its moment of inertia J leads to kinetic energy content changes. The change in kinetic energy for changing of speeds from ω_1 from ω_2 is given by:

$$\Delta E_k = \frac{1}{2} J(\omega_2^2 - \omega_1^2)$$

1.7. Magnetic field and magnetomotive force

Operating principle of electric machines are governed by two sets of basic laws (of classical electromagnetism): the Faraday's law of electromagnetic induction and the Bio-Savart's law of force creation in an electromagnetic field. Both of these laws basically explain the following phenomena that occur during the electrical-mechanical energy conversion in electrical machines (for a detailed revision, readers shall be referred to other source of materials):

(a) Current following in a wire produces a magnetic field around the wire.
(b) Voltage is induced in a coil of wire if the field passes through the closed contour formed by the wire is varying with time.
(c) Force is induced upon a current-carrying wire in the presence of a magnetic field
(d) Voltage is induced in a moving wire in the presence of a magnetic field.

The basic law that governs the production of magnetic field due to electric current is the Ampere's law:

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{net}$$

where **H** [A-turn/m] is the magnetic field intensity produced by the current I_{net} [A-turn] that passes through the closed contour form by the path with differential length $d\mathbf{l}$ [m].

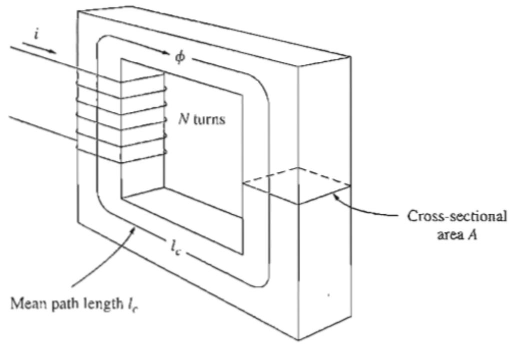


Fig. 1.19: Diagram of rectangular iron core

Considering a closed, rectangular-shaped iron core (or core with other ferromagnetic material) with I_{net} current-carrying wire wrapped around the core, as shown in Fig. 1.20, the magnetic field intensity **H** can be interpreted as a measure of "effort" that a current is putting into the establishment of magnetic field in the iron core. The strength of the magnetic flux produced in the core depends on the property of the ferromagnetic material, which is known as magnetic permeability:

$$\mathbf{B} = \mu\mathbf{H}$$

where **B** [Wb/m^2] is the magnetic flux density and μ [H/m] is the magnetic permeability of the material. μ can be expressed in terms of the relative permeability (μ_r) and the permeability of free space (μ_o) as:

$$\mu = \mu_r \mu_o$$

Then, in a given area **A** in the magnetic field, the total magnetic flux Φ [Wb] is given by the surface integral of **B** over the area **A**:

$$\Phi = \int_A \mathbf{B} \cdot d\mathbf{A}$$

In the electric circuit, the presence of a voltage or electromagnetic force drives the flow of electric current, $v = iR$. By analogy, in the magnetic circuit, the magnetomotive force MMF [A-turn] produces magnetic flux ϕ in a "circuit" with reluctance $\Re$ [A-turn/Wb], i.e.:

$$MMF = \phi \Re$$

For current i that flows through a conductor that makes N turns around a ferromagnetic core, the magnetomotive force MMF "created" inside the core is:

$$MMF = Ni$$

1.8. Transformer

Transformers convert AC electrical energy at one voltage level to another voltage level. Despite being an "old" technology, they are indeed one of the most critical component in today's power system. Transformer is used also for low-power applications such as, to name a few, voltage instrumentation transformer, current instrumentation transformer, and impedance transformer.

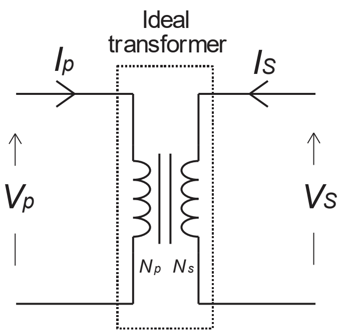


Fig. 1.20: A circuit representation of an ideal transformer.

An ideal transformer's schematic representation is shown in Fig. 1.20. It has N_p turns of wire on its primary side and N_s turns of wire on its secondary side. The following primary-secondary voltage and current relationships are:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = a$$

$$\frac{I_p}{I_s} = \frac{1}{a}$$

where a is the transformer turn ratio, N_p/N_s.

The equivalent circuit of a realistic transformer is reviewed next as it forms the fundamentals for the AC machines/drives study. Additional circuit elements are added appropriately into the ideal transformer circuit to account for the major imperfections. The equivalent circuit of the form shown in Fig. 1.21 has been proposed.

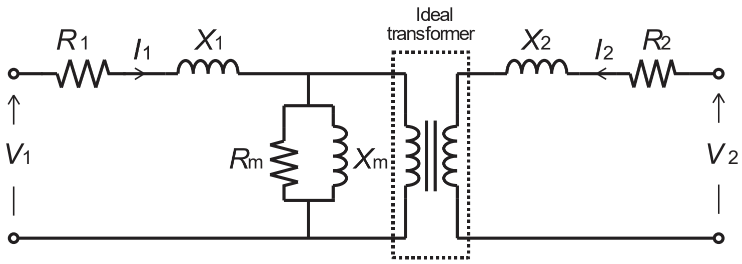


Fig. 1.21: A realistic representation of a transformer with consideration on the stator copper loss, rotor copper loss, magnetizing core loss, and the presence of magnetizing current I_m.

The equivalent circuit in Fig. 1.21 takes into account the major imperfection in a real transformer, which include the copper loss (by including R_1 and R_2), leakage flux (by including X_1 and X_2), the magnetizing current (by including X_m) and the core loss (by including R_m). N_1 and N_2 determine the primary and secondary voltage relation in the way similar to that of N_p and N_s in the ideal transformer schematic in Fig. 1.20.

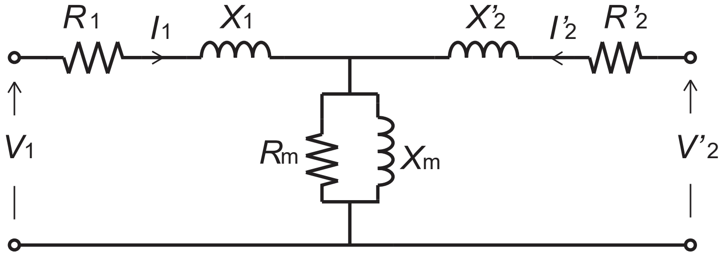


Fig. 1.22: A transformer model with the secondary circuitry's parameters and variables being referred to the primary circuit.

Although the model in Fig. 1.21 is accurate, but it is not useful when come to circuit analysis. The main reason is that the primary and secondary circuits have different voltage levels (and, in the case of AC induction machines, also their frequencies). Fig. 1.22 shows a transformer model with the secondary circuit being scaled appropriately to match the primary and secondary back EMFs. This secondary circuit is said to have been "referred" to the primary circuit. The change of variables include:

$$R'_2 = a^2 R_2$$
$$X'_2 = a^2 X_2$$
$$V'_2 = aV_2$$
$$I'_2 = \frac{I_2}{a}$$

where I_2 is the secondary circuit current in Fig. 1.21 and I_2' is the secondary circuit current in Fig. 1.22. The secondary back EMF is scaled by a ($=N_1/N_2$) times to match with the primary back EMF. For power conservation, the current is scaled by $1/a$ times. Similarly, recall that the power across the resistor is the current squared times the resistance, the resistance value is scaled by a^2 times to preserve the power content. The same applies to the reactance.