ELEC 3224 — Complementary Filters

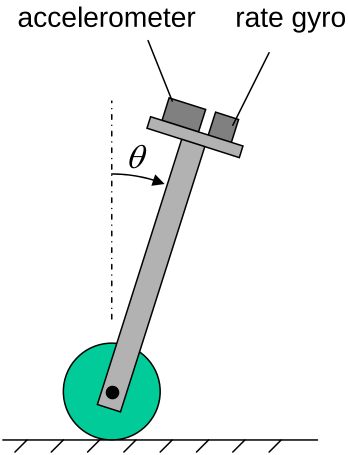
Professor Eric Rogers

School of Electronics and Computer Science
University of Southampton
etar@ecs.soton.ac.uk
Office: Building 1, Room 2037

Introduction

- Cases exist where there are two different measurement sources to estimate one variable.
- Moreover, the noise properties of these measurements are such that one source gives good information only in a low frequency region and the other is only good in a high frequency region.
- in such cases, a **complementary filter (CF)** can be used.
- An example: **Tilt angle estimation using an accelerometer and a rate giro.**

Problem variables



CF Filter

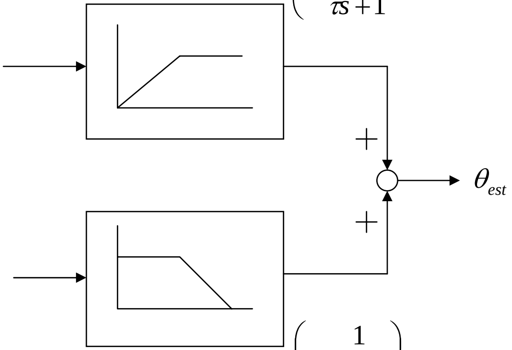
$$\theta \approx \int (\text{angular rate})\, dt$$

- not good in long term due to integration

$$\theta \approx \sin^{-1}\left(\frac{\text{accel. output}}{g}\right)$$

- only good in long term
- not proper during fast motion

High Pass Filter $\left(= \dfrac{\tau s}{\tau s + 1}, \text{ for example}\right)$

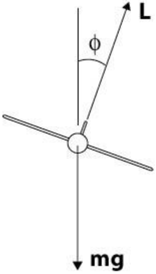


Low Pass Filter $\left(= \dfrac{1}{\tau s + 1}\right)$

Roll Angle Estimation

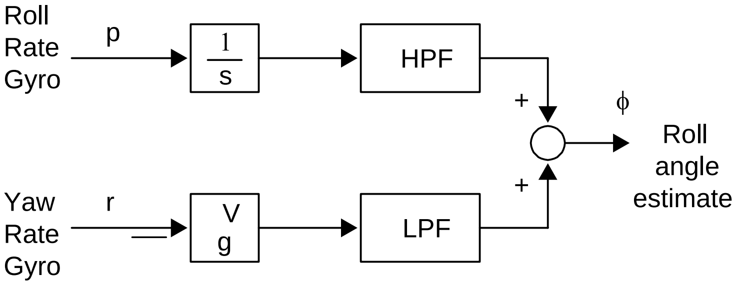
- **High frequency:** integrating roll rate (p) gyro output.
- **Low frequency:** uses aircraft kinematics.
- **Assuming steady state turn dynamics,** roll angle is related with turning rate, which is close to the yaw rate (r).

Problem variables



$$
\begin{aligned}
L\sin\phi &= mV\Omega \\
L &\approx mg \\
\Omega &\approx r \\
\sin\phi &\approx \phi
\end{aligned}
\quad \Rightarrow \quad \phi \approx \frac{V}{g} r
$$

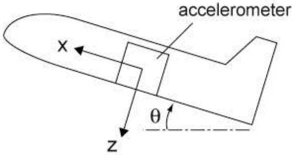
CF Filter


Roll
Rate
Gyro
p
1
s
HPF
+
ϕ
Roll
angle
estimate
+
Yaw
Rate
Gyro
r
V
g
LPF

Pitch Angle Estimation

- **High frequency:** integrating pitch rate (q) gyro output.
- **Low frequency:** using the sensitivity of accelerators to gravity direction – known as ‘gravity aiding’.

Problem variables



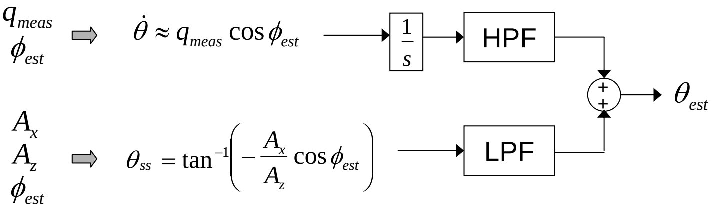
In steady state

$$A_X = g \sin\theta$$
$$A_Z = -g \cos\theta$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(-\frac{A_x}{A_z}\right)$$

A_X , A_Z – accelerometer outputs

CF Filter

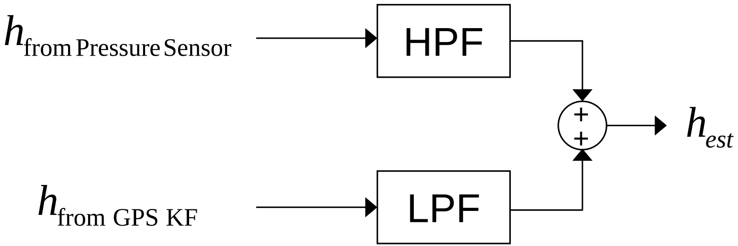


q_{meas}
ϕ_{est}
$\dot{\theta} \approx q_{meas} \cos\phi_{est}$
$\frac{1}{s}$
HPF
A_x
A_z
ϕ_{est}
$\theta_{ss} = \tan^{-1}\left(-\frac{A_x}{A_z}\cos\phi_{est}\right)$
LPF
+
+
θ_{est}

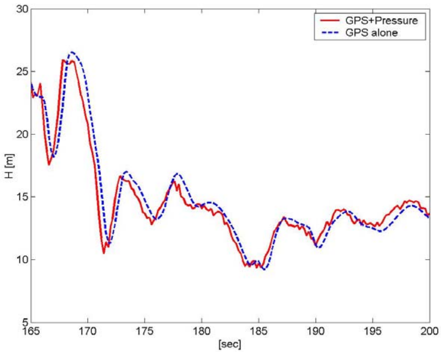
Altitude Estimation

- **Motivation:** A GPS receiver gives an altitude output but has 0.4 seconds delay.
- To overcome this delay, a pressure sensor is added.
- **Low frequency:** from the GPS receiver.
- **High frequency:** from the pressure sensor.

CF Filter



Flight Data

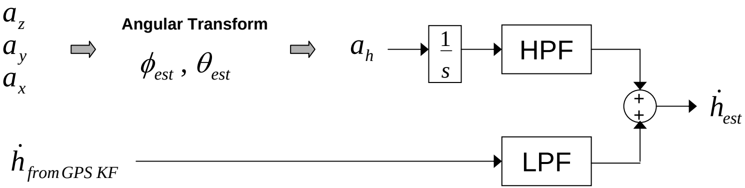


GPS+Pressure
GPS alone
H [m]
[sec]
30
25
20
15
10
5
165
170
175
180
185
190
195
200

Altitude Rate Estimation

- **Motivation:** A GPS receiver gives an altitude output but has 4 seconds delay.
- To overcome this delay, inertial sensor outputs were added.
- **Low frequency:** from the GPS receiver.
- **High frequency:** integrating acceleration estimate in altitude direction from inertial sensors.

Flight Data



$$\text{note}: \begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix} = \begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix} - [\phi_{est}][\theta_{est}] \begin{Bmatrix} 0 \\ 0 \\ -g \end{Bmatrix}$$

A_x , A_z – accelerometer outputs

$[\phi_{est}], [\theta_{est}]$: angular transformation matrices