ELEC 3225 Guidance Navigation and Control — Tutorial Questions

Q1. Consider the differential linear system with state equation

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -10 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

Find the characteristic polynomial of this system. Also design a stae feedback control law that places the poles of the controlled system at

$$s = -1 \pm j1, -5$$

Repeat this design for the case where the controlled system poles are required to be at

$$s = -1, -1, -1$$

Q2. Consider the differential linear system with transfer-function

$$G(s) = \frac{46s + 13}{s^3 + 13s^2 + 33s + 13}$$

Confirm that

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -13 & -33 & -13 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 13 & 46 & 0 \end{bmatrix} x \end{aligned}$$

Also design a state feedback control law that places the closed-loop poles at $s = -1, -2, -3$.

Q3. Consider a differential linear time-invariant system described by the state-space model

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 2 & 0 \\ 1 & -3 & 4 \\ -1 & 1 & -9 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} x \\ y &= \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x \end{aligned}$$

Apply the state transformation

$$x = Tz$$

where

$$T = \begin{bmatrix} 51 & 16 & 1 \\ 19 & 17 & 2 \\ -5 & -3 & -1 \end{bmatrix}$$

and

$$T^{-1} = \begin{bmatrix} 0.0283 & -0.0334 & -0.0386 \\ -0.0231 & 0.1183 & 0.2134 \\ -0.0720 & -0.1877 & -1.4473 \end{bmatrix}$$

What is the special feature of the resulting state-space model?

Q4. Consider the differential linear time-invariant system described by

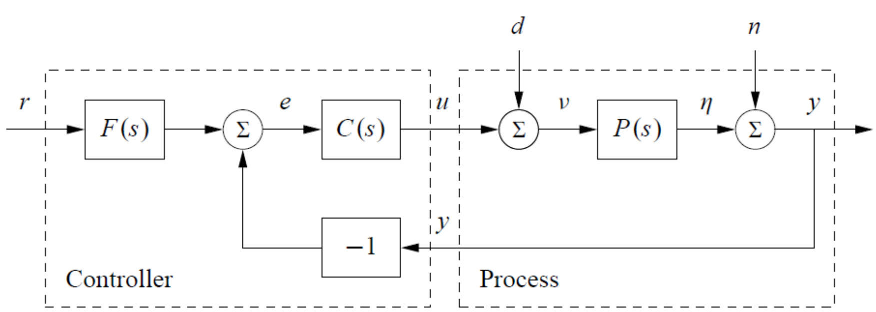
$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & \frac{1}{2} \end{bmatrix} x \end{aligned}$$

Design a state observer that places the poles of the observer at $s = -4, -4$.

Q5. Design a state feedback control law for the system of the previous example to place the closed-loop poles at $s = -1 \pm j2$. If this control law is to be implemented using a full order state observer, where should the observer poles be located? Illustrate the possibilities by one numerical example.

Q6.

Give the formulas for y, η, u and e in the case of the system represented by the block diagram below.



If $F(s) = 1$, give the transfer-functions of i) the sensitivity function, ii) the load sensitivity function, iii) the complementary sensitivity function and the iv) the noise sensitivity function.