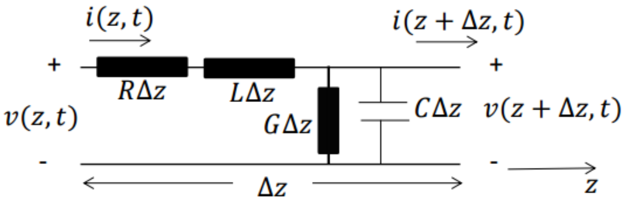
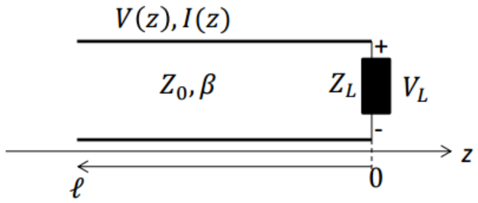
ELEC2222 – Circuits and Transmission topic overview and formulae - Ricki Tura rst1g15

Sasan 1 – Transmission Line Theory

Equations highlighted are provided in the formula sheet.

- A transmission line can be modelled by linked together short identical sections of line, known as the *lumped parameters* model. A small length of line, say Δx or Δz, will have the following quantities per meter: series resistance, series inductance, shunt conductance, and shunt capacitance.





- *Telegrapher's equations* (1)* and (2)* are derived from applying KVL and KCL to the above system.

$$KVL:\ v(z,t) = R\Delta z i(z,t) + L\Delta z\frac{\partial i(z,t)}{\partial t} + v(z+\Delta z,t)$$

$$\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t} \tag{1}$$

$$KCL:\ i(z,t) = G\Delta z v(z+\Delta z,t) + C\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t} + i(z+\Delta z,t)$$

$$\frac{i(z+\Delta z,t) - i(z,t)}{\Delta z} = -Gv(z+\Delta z,t) - C\frac{\partial v(z+\Delta z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t} \tag{2}$$

- By making the following substitutions, $v(z,t) = V(z)e^{j\omega t}$ and $i(z,t) = I(z)e^{j\omega t}$, and substituting them into the telegrapher's equations, we can get the following equations (3) and (4).

$$\frac{dV(z)}{dz} = -(R+j\omega L)I(z) \tag{3}$$

$$\frac{dI(z)}{dz} = -(G+j\omega C)V(z) \tag{4}$$

- By finding the second derivative, and substituting in the first derivative where needed, you end up with the eigenvalue problem for both the voltage and for the current. Solutions are (5) and (6).

$$V(z) = V_0^+e^{-\gamma z} + V_0^-e^{+\gamma z} \tag{5}$$

$$I(z) = I_0^+e^{-\gamma z} + I_0^-e^{+\gamma z} \tag{6}$$

- $\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$, and is known as the *propagation* constant, the same one mentioned in Milos' section.
- The terms are the same, α is attenuation and β is phase change.
- The $e^{-\gamma z}$ term is propagation in the +*z* direction, likewise the $e^{+\gamma z}$ term is propagation in the -*z* direction.
- From this you can find the *characteristic impedance*, calculated as $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$, compare this with $Z_0 = \sqrt{\frac{B}{C}}$ from the *ABCD* notation from Milos' part.

Sasan 2 – Lossless Lines

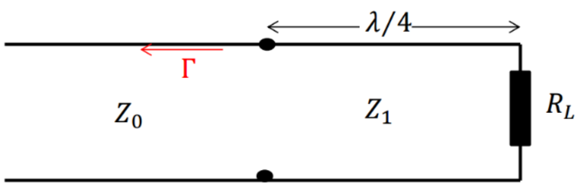
- For a lossless line, there is no resistance or conductance, so it simplifies to $Z_0 = \sqrt{\frac{L}{C}}$. You can also find the wavelength and phase velocity, $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}$, $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$.
- Lines are usually assumed to be lossless in practical examples, as the loss is negligible.
- Applying a load Z_L to the end of the line *terminates* the line. Voltage reflections form and bounce back.
- This is where the $V_0^- e^{+\gamma z}$ part of (5) comes in, it takes into account these reflections.
- The ratio of V_0^- to V_0^+ is known as the *voltage reflection coefficient*, the symbol is a capital γ, Γ.
- $\Gamma = \frac{(Z_L - Z_0)}{(Z_L + Z_0)} = \frac{V_0^-}{V_0^+}$, note that if $Z_L = Z_0$, $\Gamma = 0$ and there are no reflections, the impedance is *matched*.
- Through a derivation, we can incorporate Γ into (5) like so:

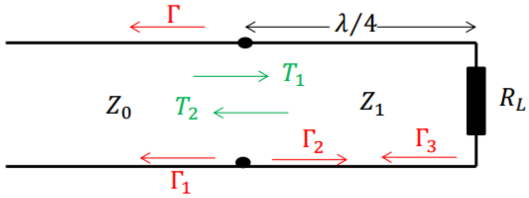
$$V(z) = V_0^+\left(e^{-j\beta z} + \Gamma e^{+j\beta z}\right) \tag{7}$$

- The current can be found by another derivation, which gives (8).

$$I(z) = \frac{V_0^+\left(e^{-j\beta z} - \Gamma e^{+j\beta z}\right)}{Z_0} \tag{8}$$

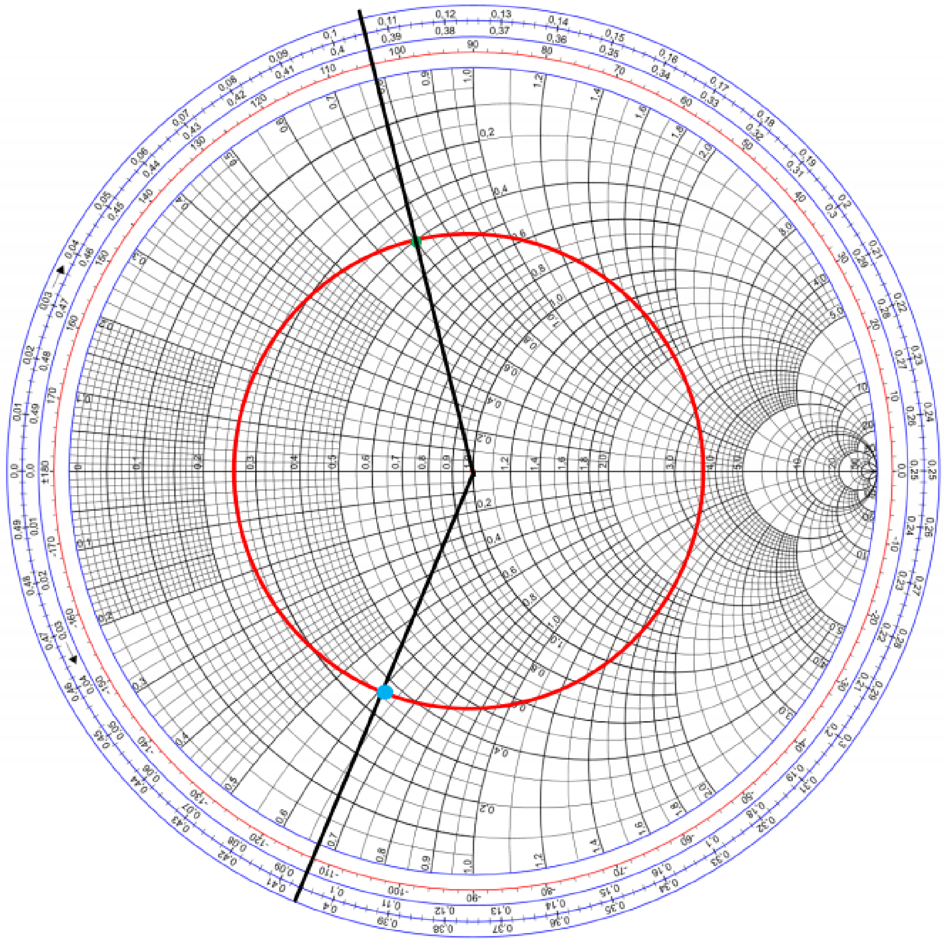
- Learn (7) and (8), they are useful in examples where you have to find the input impedance.
- Maximum power is delivered when $\Gamma = 0$, and no power is delivered when $\Gamma = 1$.
- The *return loss* in power due to reflections is measured as $RL = -20\log|\Gamma|\,dB$.
- When the load isn't matched to the characteristic impedance of the line, there will be voltage reflections and $\Gamma \neq 0$, a *standing wave* will be formed from the incoming and reflected voltages.
- The *standing wave ratio (SWR)* ranges from 1 to ∞, and is the ratio between the voltage maximum and minimum along the line. $SWR = \frac{|1+|\Gamma||}{|1-|\Gamma||}$.
- Γ is defined at the load, we can also find Γ at any point l along the line, so it can become $\Gamma(l)$ where l = 0 is the load point and we go backwards (z = - l), so $\Gamma(0) = \Gamma$. $\Gamma(l) = \Gamma(0)e^{-2j\beta l}$
- In cases where the load is a short circuit or open circuit, the SWR is ∞ in both cases.
- The input/transmission line impedance is defined as $Z_{in} = Z_0 \frac{Z_L + jZ_0\tan(\beta x)}{Z_0 + jZ_L\tan(\beta x)}$, x can be l if preferred.
- There are questions where you have to find the input impedance of shorted or open loads. The equation above isn't the most suitable in this case, instead divide (7) by (8), put in the value of Γ, and simplify the exponentials using $e^{j(\beta z)} = \cos(\beta z) + j\sin(\beta z)$.
- *Quarter-wave transformers* are lines of a particular length that change the input impedance without changing the load impedance. In other words, it changes what impedances the input end "sees", even though the load impedance is unchanged.
- When the length is defined $l = \frac{\lambda}{4} + \frac{n\lambda}{2}$ for any integer *n*, multiplying this by β to find the tan value in the equation for Z_{in} will give infinity, as the point is on one of the asymptotes of the tan graph. This allows us to assume that $jZ_0\tan(\beta x) \gg Z_L$ for the numerator and $jZ_L\tan(\beta x) \gg Z_0$ for the denominator. We can remove the small terms to get $Z_{in} = Z_0 \frac{jZ_0\tan(\beta x)}{jZ_L\tan(\beta x)} = Z_0 \frac{Z_0}{Z_L} = \frac{{Z_0}^2}{Z_L}$.
- This is incredibly useful in cases where you have a transmission line with a characteristic impedance that isn't matched to the load impedance. You can use quarter-wave transformers in between the load and the end of the line to match the impedances and make $\Gamma = 0$, so no reflections!
- In questions, you may be asked to find the characteristic impedance Z_1 of the quarter-wave line, this must be a certain value for it all to work. Find Z_{in} at this point in terms of Z_1 and R_L, then equate this with Z_0 as we're matching it, then you can rearrange to find Z_1.
- You may also be asked about the multiple reflection coefficients. Use the provided formula.





<u>Sasan 3 – Smith Charts</u>

- *Smith charts* are an easy way to find the magnitude of Γ, the SWR, and the input impedance. A copy is provided in the exam. The load impedance is divided by the characteristic impedance to *normalise* it. The centre of the chart is the normalised load impedance of 1 + j0. The x axis is the real part, above the x axis is the positive imaginary part, and below the x axis is the negative imaginary part. For any normalised load impedance, you can find its point on the Smith Chart.
- Drawing a circle with its centre at the centre of the chart and the circumference crossing through the point of the normalised load impedance will produce two points of intersection at the x axis. The intersection point that is greater than 1 is the SWR. The other point will be less than 1, and can't be the SWR.
- The straight distance from this normalised load impedance point to the centre of the chart is the magnitude of the reflection coefficient Γ.
- Extending this line to the edge of the circle will make it intersect several scales, the wavelength-towards-generator (WTG), the wavelength-towards-source (WTS), and the angle (phase) of Γ.
  WTG is used in the slide examples, and tells you the phase of Γ in terms of λ. This is useful when finding the input impedance, if you know the length of the line in terms of multiples of λ, you can rotate clockwise along the scale this amount of λ, draw a line from this point to the centre of the chart, and the intersection between the line and the circle is the point corresponding to the input impedance.

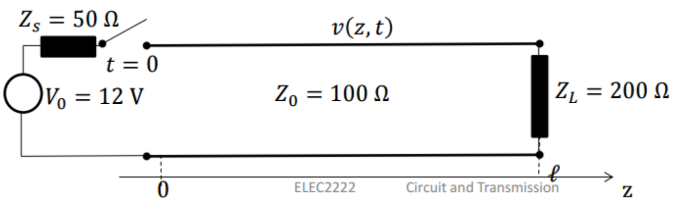


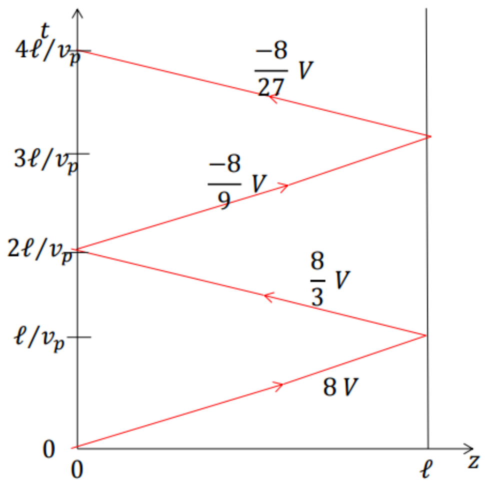
Sasan 4 – Lossy Transmission Lines

- In lossy cases, (5) and (6) have to be edited, as β becomes γ to include the real part of the propagation constant ($\gamma = a + j\beta$). Key differences are outlined in the table below.

Symbol	Lossless	Lossy	Info
Z_0	$\sqrt{\frac{L}{C}}$	$\sqrt{\frac{R + j\omega L}{G + j\omega C}}$	Characteristic impedance
γ	$j\beta$	$\alpha + j\beta$	Propagation coefficient
$V(z)$	$V_0^+\left(e^{-j\beta z} + \Gamma e^{+j\beta z}\right)$	$V_0^+\left(e^{-j\gamma z} + \Gamma e^{+\gamma z}\right)$	Voltage along the line
$I(z)$	$\frac{V_0^+\left(e^{-j\beta z} - \Gamma e^{+j\beta z}\right)}{Z_0}$	$\frac{V_0^+\left(e^{-j\gamma z} - \Gamma e^{+\gamma z}\right)}{Z_0}$	Current along the line
Z_{in}	$Z_0 \frac{Z_L + jZ_0 \tan(\beta x)}{Z_0 + jZ_L \tan(\beta x)}$	$Z_0 \frac{Z_L + Z_0 \tanh(\gamma x)}{Z_0 + Z_L \tanh(\gamma x)}$	Input impedance
	$\frac{Z_0 V_0^+\left(e^{-j\beta z} + \Gamma e^{+j\beta z}\right)}{V_0^+\left(e^{-j\beta z} - \Gamma e^{+j\beta z}\right)}$	$\frac{Z_0 V_0^+\left(e^{-j\gamma z} + \Gamma e^{+\gamma z}\right)}{V_0^+\left(e^{-j\gamma z} - \Gamma e^{+\gamma z}\right)}$	
$\Gamma(l)$	$\Gamma(0)e^{-2j\beta l}$	$\Gamma(0)e^{-2\gamma l}$	Reflection coefficient along the line
$\lvert\Gamma(l)\rvert$	$\lvert\Gamma(0)\rvert$	$\lvert\Gamma(0)\rvert e^{-2\alpha l}$	Magnitude of reflection coefficient

- The calculations in lossy cases are mostly the same, you just substitute the relevant terms for the ones that apply to the lossy case as outlined in the table above.
- Bounce diagrams are a way of visualising the propagation of a wave as it reflects back and forth along the transmission line. The x axis represents the length of the line, and the y axis represents time. The amplitude of the wave is simply labelled on the plot.





- The magnitude of the reflected wave is determined by the reflection coefficients at the source (Γ_s) and at the load (Γ_l), which are found using the provided formula and the given source and load impedances, and the characteristic impedance of the line.