UNIVERSITY OF SOUTHAMPTON ELEC3224W1

SEMESTER 1 EXAMINATION 2019 - 2020

GUIDANCE, NAVIGATION AND CONTROL

DURATION 120 MINS (2 Hours)

This paper contains 5 questions

Answer **three** questions

An outline marking scheme is shown in brackets to the right of each question.

This examination contributes 100% of the marks for the module

University approved calculators MAY be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct Word to Word translation dictionary AND it contains no notes, additions or annotations.

9 page examination paper.

Question 1.

(a) Give a block diagram representation of a general motion control system. Give the distinguishing features of i) setpoint regulation, ii) trajectory tracking control and iii) path-following control.

[6 marks]

(b) By means of diagrams with all variables defined and marked, give the basic principles of i) LOS guidance, ii) PP guidance and iii) CB guidance. [6 marks]

(c) Let

$$p_t^n = \begin{bmatrix} N_t & E_t \end{bmatrix}^T$$

be the 2D position of a target in North-East coordinates. Formulate the control objective of a target-tracking scenario for this case.

[4 marks]

(d) List the main considerations in forming a waypoint database.

[4 marks]

Question 2.

(a) State the purpose of waypoint switching for UAVs. What is the distinctive feature of the L_2^+ implementation?

[4 marks]

(b) Figure Q1 shows the waypoint switching geometry for an UAV, where all terms have their usual meanings.

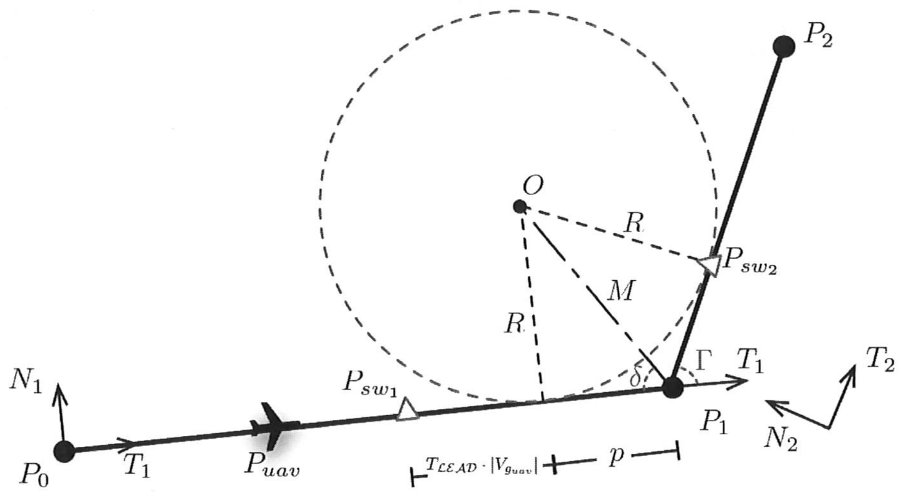

Figure Q1

The radius of the circle in this figure is

$$R = \frac{(U_c + |\text{wind}|)^2}{a_{\max}}$$

Define all the variables in this formula. The actual radius of curvature of the vehicle can change. Explain why and give a possible solution.

[4 marks]

(c) What is the meaning of the point P_{sw1} in Figure Q1? Show also that

$$p = \frac{R}{\tan \delta}$$

[6 marks]

TURN OVER

(d) Why in flight tests can commencing the turn just at the switch point cause a problem? Explain the use of a lead time as a counter to this problem and develop the formula for computing the resulting switch point. Also what happens in this case if the vehicle is already beyond the next segment switching point?

[6 marks]

Question 3.

(a) A unity negative feedback control scheme for a single-input single-output linear time-invariant system with input u and output y is described by the following equations

$$\begin{aligned} y &= \eta + n \\ \eta &= P(s)(u+d) \\ u &= C(s)e \\ e &= F(s)r - y \end{aligned}$$

where $P(s)$ is the transfer-function, $C(s)$ is the transfer-function of the controller, $F(s)$ is the feedforward controller, r is the reference signal, d is the load disturbance and n is the measurement noise. Give a block diagram of this system with all variables specified. In the special case of $F(s) = 1$, give the transfer-functions of i) the sensitivity function S, ii) the load sensitivity function PS, iii) the complementary sensitivity function T and the noise sensitivity function CS.

[6 marks]

(b) Consider the case of a system for which

$$P(s) = \frac{1}{s-a}$$

where $a > 0$ is to be controlled by applying the scheme of the previous part of this question with $F(s) = 1$ and

$$C(s) = k\frac{s-a}{s}, \quad k > 0$$

Compute the functions S, PS, T and CS in this case and hence explain why this scheme cannot be applied. What are the implications when $a < \epsilon$ in the cases when i) ϵ is a very small negative number and ii) ϵ is a very large negative number? [7 marks]

TURN OVER

(c) Consider a differential linear time-invariant system described by

$$P(s) = \frac{1}{s^2}$$

for which the performance specifications are less than 1% steady state error and less than 10% tracking error up to 10 rad/sec. Give a sketch of the gain and frequency responses of this system and explain why increasing the gain is not a feasible design. **Specify, but do not analyse,** the structure of a controller $C(s)$ that can be used to meet this specification. [7 marks]

Question 4.

(a) A loop transfer-function $L(s)$ of a feedback control system goes to zero faster than $\frac{1}{s}$ as $s \to \infty$ and has p_k poles in the right half-plane. Then

$$\int_0^\infty \log(|S(j\omega)|)d\omega = \pi \sum p_k$$

holds where S denotes the sensitivity function. Explain the implications of this result in terms of control design, where your answer must include discussion, including a diagram with all relevant terms marked, of the waterbed effect. Apply this result, if possible, to the following examples and in cases where this is not possible explain why.

$$L(s) = \frac{k}{s+5}, \quad L(s) = \frac{k}{(s+1)^2}, \quad L(s) = k\frac{(s-3)}{(s-1)(s-2)(s+5)}$$

[10 marks]

(b) Let $P(s)$ be the transfer-function of a differential linear time-invariant system subject to uncertainty modeled as a) additive described by Δ and b) multiplicative described by δ. State the $\mathcal{H}_\infty$ condition for robust stability in each case. [4 marks]

(c) Consider a unity negative feedback control scheme formed from system transfer-function P and controller C and transfer-function from load to disturbances to output given by

$$G_{yd} = \frac{P}{1+PC}$$

Express G_{yd} in terms of the sensitivity function S and show that

$$\frac{dG_{yd}}{G_{yd}} = S\frac{dP}{P}$$

and discuss the implications of this result. [6 marks]

TURN OVER

Question 5.

(a) A single-input single-output differential linear time-invariant system is described by the state-space model

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t)\end{aligned}$$

where the state vector $x(t) \in \mathbb{R}^n$. Give the matrix rank based conditions under which this system is controllable and observable. If these conditions hold what property does the system transfer-function have?

[4 marks]

(b) In an application, the uncontrolled system is described by the state-space model

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} u(t)\\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)\end{aligned}$$

The state feedback control law is to be implemented using an observer. What is meant by the separation principle? Design a state feedback control law in this case and a full state observer for implementation. The closed-loop poles are to be the roots of the polynomial

$$\rho(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

where $0 < \zeta < 1$ and the observer poles are to be those of the controlled dynamics times a constant selected in line with best practice. **In each case, construct, but do not solve, the equation that defines the solution.**

[10 marks]

(c) A differential linear time-invariant system has the state equation

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

and a state feedback control law is to be designed by minimising the cost function

$$J = \int_0^\infty (x^T(t)C^TCx(t) + u^T(t)Ru(t))dt$$

and $R = 1$. In this case the candidate solution of the associated algebraic Riccati equation is obtained from the following set of simultaneous equations

$$\begin{aligned} 2p_1 - p_1^2 + 4p_2 &= 0 \\ p_2 + 2p_3 - p_1p_2 &= 0 \\ 1 - p_2^2 &= 0 \end{aligned}$$

where p_1, p_2 and p_3 are real scalars. Compute the state feedback law in this case and show that the resulting controlled system stable.

[6 marks]

END OF PAPER