SEMESTER 2 EXAMINATION 2022 - 2023

GUIDANCE NAVIGATION AND CONTROL

DURATION 120 MINS (2 Hours)

This paper contains 5 questions

Answer **three questions**

An outline marking scheme is shown in brackets to the right of each question.

This examination contributes 100% of the marks for the module

University approved calculators MAY be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct Word to Word translation dictionary AND it contains no notes, additions or annotations.

7 page examination paper.

Question 1.

(a) A differential linear system is described by the state-space model

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & -2 \\ 1 & -0.4 \end{bmatrix} x + \begin{bmatrix} 1 \\ -2 \end{bmatrix} u \\ y &= \begin{bmatrix} 3 & 4 \end{bmatrix} x \end{aligned}$$

Show that this system is controllable and observable and determine its transfer-function. [10 marks]

(b) Apply the state transformation $z = Tx$, $\det(T) \neq 0$, $T = \begin{bmatrix} B & AB \end{bmatrix}$, where A is the state matrix and B the input matrix, to the system in part (a) of this question and then design a state feedback control law to place the closed-loop system poles at $-3, -3$. Give the control law in terms of the state vector x. [10 marks]

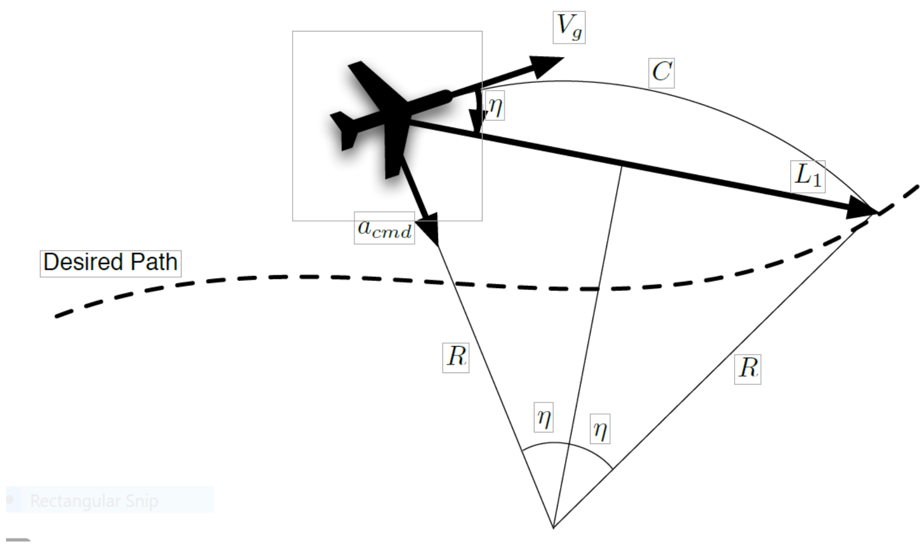
(c) Consider a rocket vehicle moving vertically above the earth, with vehicle thrust $T(t) = k\dot{m}(t)$ where k is a constant and $\dot{m}(t)$ is the rate of mass expulsion that can be controlled. Assume also that the drag force is a nonlinear function of velocity. Let $m(t)$ denote the instantaneous mass of the vehicle and, with $h(t)$ denoting the height above some datum, model the drag D as $D = f(\dot{h})$. Then Newton's second law gives the dynamic force balance, on assuming an inverse square gravity law, as

$$m\ddot{h}(t) = T(t) - f(\dot{h}) - \frac{\alpha k^2 m(t)}{(k+h)^2}$$

where α is a constant. Determine the state-space model for this system with state variables $x_1 = h,\ x_2 = \dot{h}$, and $x_3 = m$. Determine also the equations that define the equilibrium points. [13 marks]

Question 2.

(a) Figure Q2 shows a schematic of one form of pursuit guidance, where all variables have their normal meanings. Develop the equation for the bank angle command, show all working and explain all assumptions made.



[18 marks]

(b) By means of diagrams with all variables defined and marked, give the basic principles of i) LOS guidance, ii) PP guidance and iii) CB guidance. [10 marks]

(c) Let

$$p_t^n = \begin{bmatrix} N_t & E_t \end{bmatrix}^T$$

be the 2D position of a target in North-East coordinates. Formulate the control objective of a target-tracking scenario for this case.

[5 marks]

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Question 3.

(a) Consider the system described by the state-space model

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)\end{aligned}$$

and a state feedback control law is to be designed by minimising the cost function

$$J = \frac{1}{2}\int_0^\infty (x^T(t)C^TCx(t) + u^T(t)Ru(t))dt$$

The solution of this problem involves the following matrix equation where all symbols have their normal meanings

$$A^TP + PA + Q - PBR^{-1}B^TP = 0$$

What property must $Q = C^TC$ and R have in the solution of this problem? In the case of $R = 1$ and

$$P = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix}$$

find the set of equations governing the choice of the entries in P. Also, compute the resulting state feedback control law and show that the resulting controlled system is stable. [18 marks]

(b) The state feedback control law in the previous part of this question is to be implemented using an observer. Explain what is meant by the separation principle for this problem? Design a full state observer to implement this control law where the observer poles are to be those of the controlled dynamics multiplied by a constant selected in line with best practice. Construct **but do not solve** the equation that defines the observer gain matrix.

[15 marks]

Question 4.

(a) The Singer acceleration model for a target is described by

$$\frac{d}{dt}\ddot{x}(t) = -\gamma\ddot{x}(t) + w(t)$$

where $w(t)$ denotes the driving white noise. If position, velocity and acceleration are chosen as the state variables with resulting state vector $x(t),$ show that the corresponding state-space model for the dynamics is

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\gamma \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w(t)$$

Suppose now that the system dynamics are sampled with period $\alpha.$ Show that the corresponding discrete state-space model matrix, denoted by $G,$ is given by

$$G_{sm} = \begin{bmatrix} 1 & \alpha & h \\ 0 & 1 & \frac{1}{\gamma}(1 - e^{-\gamma\alpha}) \\ 0 & 0 & e^{-\gamma\alpha} \end{bmatrix}$$

where

$$h = \frac{1}{\gamma^2}(e^{-\gamma\alpha} + \gamma\alpha - 1)$$

[10 marks]

(b) In an application, it is decided to replace the Singer acceleration model of the previous part of this question by

$$\ddot{x}(t) = w(t)$$

In this case show that the discrete state-space model matrix, denoted

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by G, is given by

$$G = \begin{bmatrix} 1 & \alpha & \frac{\alpha^2}{2} \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{bmatrix}$$

Confirm that G is a particular case of G_{sm} of the previous part of this question by considering the case when $\gamma\alpha \to 0$. [16 marks]

(c) In application, filter banks grow with combinatorial complexity and hence quickly become unmanageable. Describe the basis of i) pruning and ii) merging in this context.

[7 marks]

Question 5.

(a) Let $P(s)$ be the transfer-function of a differential linear time-invariant system subject to uncertainty modeled as a) additive described by Δ and b) multiplicative described by δ. State the $\mathcal{H}_\infty$ condition for robust stability in each case. [8 marks]

(b) Consider the system described by

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + w_1(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + w_2(t)\end{aligned}$$

where $w_1(t)$ and $w_2(t)$ are uncorrelated white noise processes with intensity matrices

$$V_1 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \;\; V_2 = 1$$

Compute the minimum observer error covariance and the optimal Kalman filter gain.

You may make use of the following equation where all terms have their normal meanings.

$$AP + PA^T + V_1 - PC^T V_2^{-1} CP = 0$$

[25 marks]

END OF PAPER