Equivalent Circuit Model of Transmission Lines

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Abstract—**The following paper will study and analyse the differences between lossy and lossless transmissions lines based on different load conditions with varying types of voltage sources.**

I. INTRODUCTION

A coaxial transmission line will be simulated in this paper. Analytical calculations will be first done in order to have values of components that will be extracted into equivalent circuit models. Comparisons of built circuits with lumped parameters and exact models will undergo simulations of DC voltages and AC voltages with varying frequencies to investigate the differences between each model. The differences between a close to real world example of a transmission line(lossy line) will be compared to the ideal line(lossless).

II. COAXIAL TRANSMISSION LINE

A. Characteristic Impedance

The model of the coaxial geometry of the wire is given in Figure 1. The diameter of the inner conductor is d = 0.58mm, while the inner diameter of the outside conductor is D =3.7mm, the relative permittivity of the insulating medium is ϵ_r =2.2 while the relative permeability of this medium is μ_r = 1. The length of the line is assumed to be 100mm.
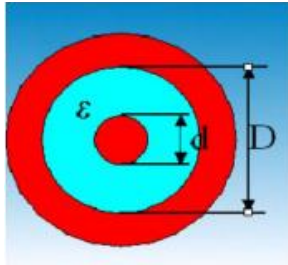

Figure 1:Coaxial transmission line

The characteristic impedance, Z_0 of the coaxial loss free line is calculated using values from inductance per unit length, L and capacitance, C per unit length. Using equations (1) and (2), L is 3.7061x10^-7 H/m while C is 6.60476x10^-11 F/m. Value of Z_0 can be calculated using equation (3) which is 74.909Ω.

$$\frac{L}{l} = \frac{\mu_r \mu_0}{2\pi} ln \frac{D}{d} \tag{1}$$

$$\frac{C}{l} = \frac{2\pi \epsilon_0 \epsilon_r}{ln\frac{D}{d}} \tag{2}$$

$$Z_0 = \sqrt{\frac{L}{C}} \tag{3}$$

B. Time Delay & Velocity of Transmission line

The propagation delay of the transmission wave in the line per meter, 4.9475x10^-9s which is obtained from equation (4). The phase velocity or the velocity of the electromagnetic wave from equation (5) is 2.02121x10^8 m/s^-1. From that, the velocity ratio which is the phase velocity divided by speed of light as in equation (6) is 0.674188.

$$T_D = \sqrt{LC} \tag{4}$$

$$V_{Ph} = \frac{1}{\sqrt{LC}} \tag{5}$$

$$V_r = \frac{V_{Ph}}{c} \tag{5}$$

III. EQUIVALENT CIRCUIT MODEL

A. Bandwidth of the LC equivalent circuit model

The equivalent circuit used to model the transmission line is a cascade of LC circuits. Before the circuit can be extracted to be a lossless transmission line and exact model in MultiSim, the cut-off frequency of a LC circuit must be calculated using equation (6) which means that frequencies below the targeted cut-off point will pass through, whereas frequency above this point will be heavily attenuated. In this case, a frequency of 700Mhz is used as the bandwidth. The value of capacitance and inductance is then calculated using the equation (7) and (8). The values acquired are used to divide the capacitance and inductance per length calculated in previous section.

$$f = \frac{1}{\pi\sqrt{LC}} \tag{6}$$

$$C = \frac{1}{2\pi f Z_0} \tag{7}$$

$$L = \frac{Z_0}{2\pi f} \tag{8}$$

The capacitance for each element is 3.0352pF and the inductance for each element is 17.0316nH. After dividing, the value acquired is 21.76 per meter then rounded off to 22 per meter. The 22 elements are built in MultiSim as seen in Fig. 2 before converting into a hierarchal block for the 32m transmission line.
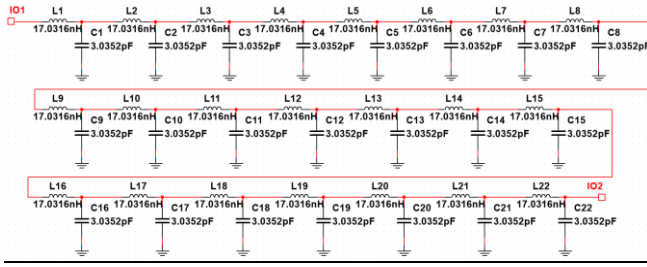
Figure 2:22 LC elements per meter

After acquiring the value of LC elements per meter which is 22 per meter. The equivalent circuit model of a transmission line can be built in MultiSim, so for this assignment, a 32 meters long transmission line will be built as seen in Fig. 3.
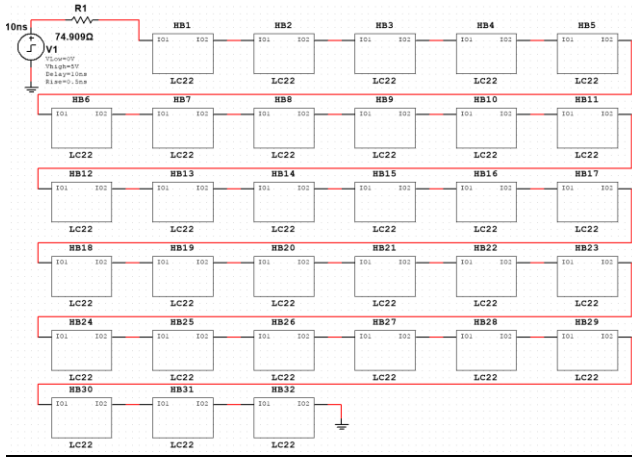

Figure 3:32m LC equivalent circuit model

After building the 704 LC element equivalent circuit model, the Multisim TL exact model is built as shown in Fig and the values of Z_0 and T_D are set to 74.909ohm and 158.32ns due to the product of T_D with the total length of the transmission line which is 32m in the exact model to simulate the same circuit.
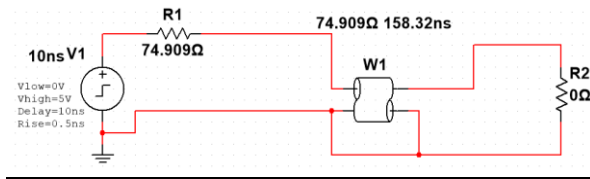

Figure 4:MultiSim TL Exact Model

In order to prevent the signal from the load to no reflect back to source, a source impedance that is equal to Z_0 is included in both models.

B. Time domain study and analysis under step voltage excitation

After building the two models of circuits, both models are simulated under a step voltage source that is set to a maximum of 5V and a rise time of 0.5ns. The input and output voltages of both models are observed under four load conditions which are 75ohm load, open circuit, closed circuit and 10ohm load from the four figures below.
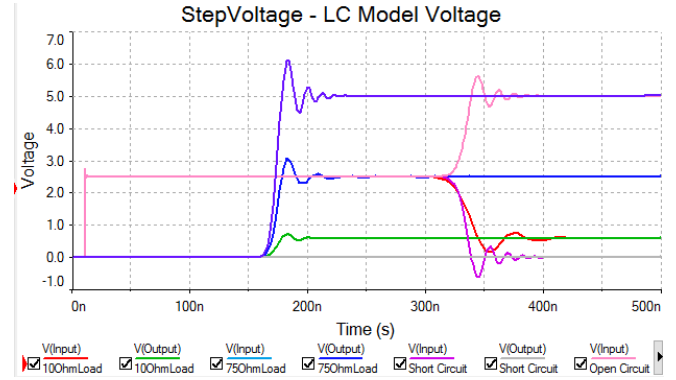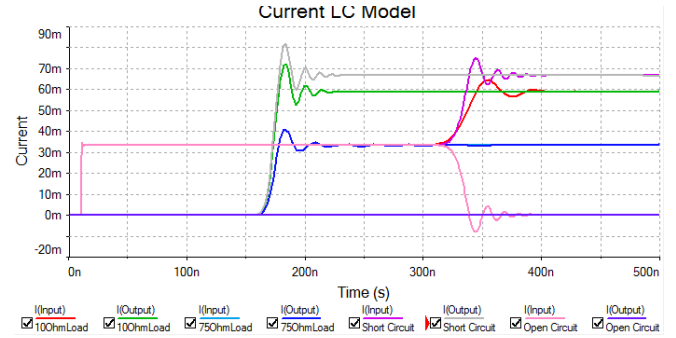

Figure 5:Step Voltage - LC Model Voltage

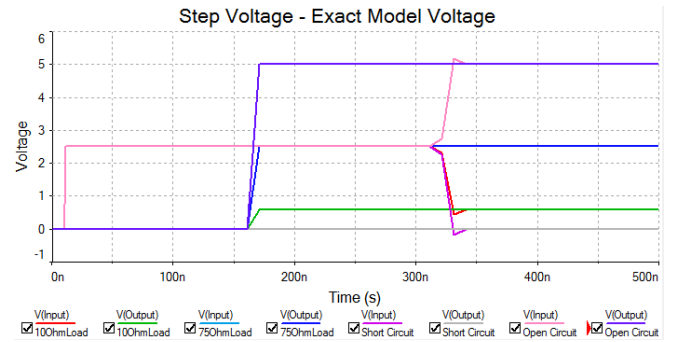

Figure 6:Step Voltage - LC Model Current
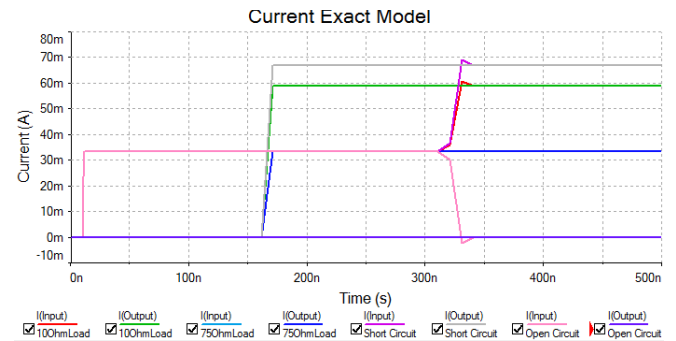

Figure 7:Step Voltage - Exact Model Voltage

Figure 8:Step Voltage - Exact Model Current

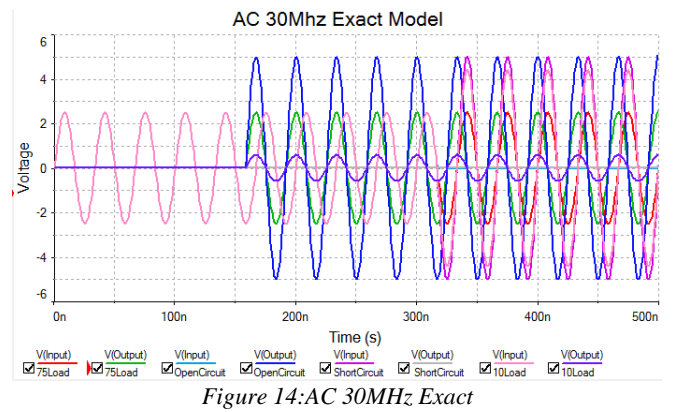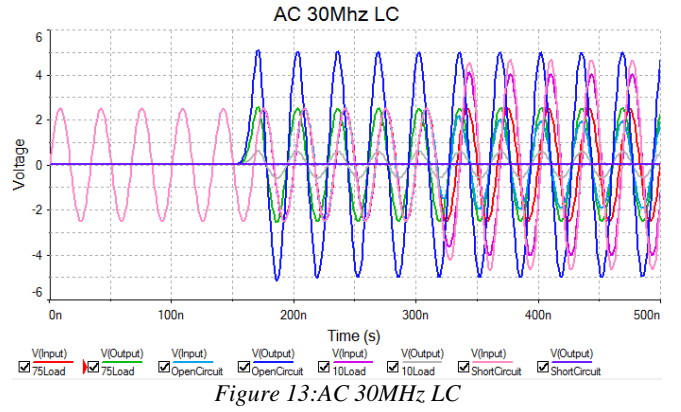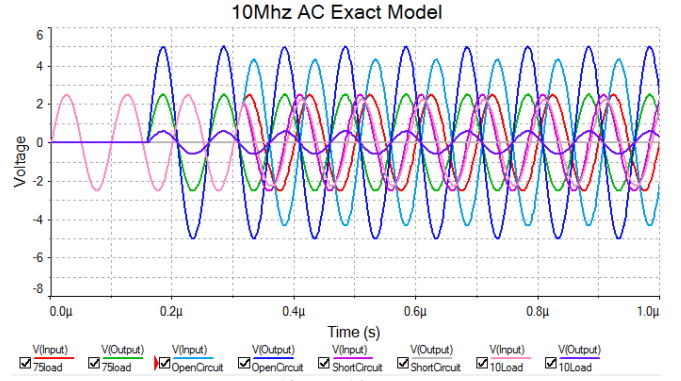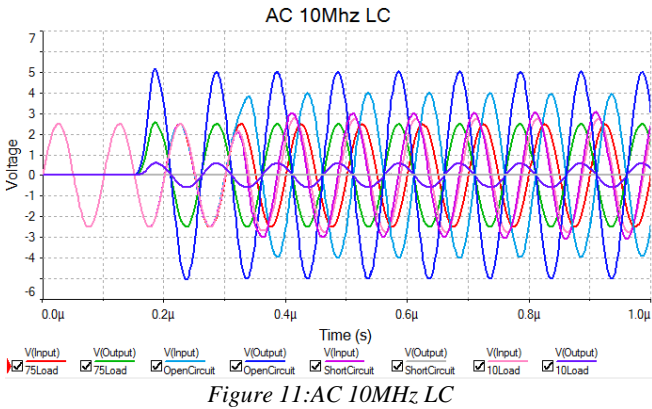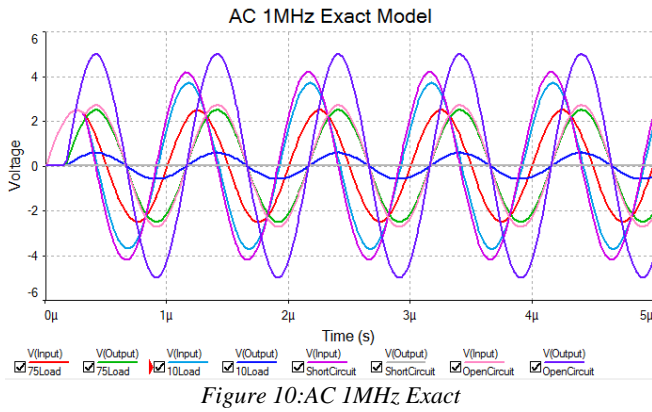From the above figures, there exists a time delay of ~158ns from the input to the output for both models which was calculated analytically previously. The input voltages are around half the DC source which is 5V. The load voltages have oscillations or attenuations in the beginning of the step input while the exact models have steep lines. Also, with higher impedance at the load, it is observed that the output voltages are higher, with short circuit being the

lowest(0V) while open circuit being the closest to the maximum voltage source, 5V. This applies to both models.

The current exhibits opposing behaviour to the voltage graphs. The higher the impedance at the load, the lower the output current observed, with short circuit having the highest current output while the

C. Time domain study and analysis under harmonic (AC) voltage excitation

The AC voltage excitation for both LC and exact is carried out. An AC voltage source with a 5V amplitude and a frequency of 1 MHz, 10MHz and 30MHz is observed with the same load cases as before. The following figures are the results acquired.

Figure 9: AC 1MHz LC

Figure 10:AC 1MHz Exact

Figure 11:AC 10MHz LC

Figure 12:AC 10MHz Exact

Figure 13:AC 30MHz LC

Figure 14:AC 30MHz Exact

From the above figures, it is observed that the output waveform voltages are lagging due to the T_D of the transmission waves though the line. The same case applies to the circuit under AC, the output voltages have differing magnitudes depending on the load impedance of the circuit. The higher the value of impedance, the higher the magnitude of the voltage outputs. This as seen from the figures above, is regardless of the frequency value. Furthermore, it is observed that both models have very little differences between each other except for certain dips in values on the LC model's simulations.

D. Study on Zero Input Impedance Effect

When the load is left open (open circuit), the circuit should have an zero input impedance when the line length is λ/4, 3λ/4, 5λ/4… . The frequency of the voltage source that is set-up such that the line has a zero input impedance can be calculated using equation (9) and equation (10). The list of wavelength and frequencies for each line length is listed in table 1.

$$f = \frac{v}{\lambda} \qquad (9)$$

$$l = \frac{\lambda}{4} + \frac{n\lambda}{2} \quad n = 0,1,2\ldots \qquad (10)$$

TABLE I. CALCULATION OF FREQUENCIES AND WAVELENGTHS THAT CAUSE INPUT IMPEDANCE = 0

n	Line length	Frequency(Hz)	Wavelength(m)
0	$\lambda/4$	1.58x10^6	128
1	$3\lambda/4$	4.75x10^6	42.6667
2	$5\lambda/4$	7.91x10^6	25.6
3	$7\lambda/4$	11.07x10^6	18.286

IV. LOSSY EQUIVALENT CIRCUIT MODEL.

A. Extraction of Circuit Model

The extraction circuit model of lossy transmission line requires parameter resistance per unit length, R and conductance per unit length, G to represent the losses of resistive and dielectric in the circuit. The G and R parameter is obtained from setting up the correct equivalent model on the excel file, Coax_rev3.4.xls that was provided [1]. After inputting the required frequency of for each AC frequency in the excel sheet, the value of R and G was obtained in table 2. The equivalent circuit can be built using the lossy equivalent circuit in figure 15. As seen below, the R is connected in series with the inductance while the G is connected in parallel with capacitance.
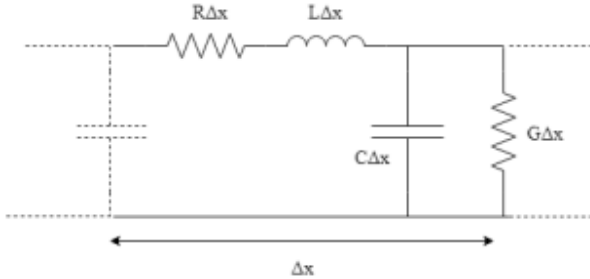

Figure 15:Equivalent Lossy Circuit Model

TABLE II. RESULTS OF CALCUALATION IN EXCEL SHEET

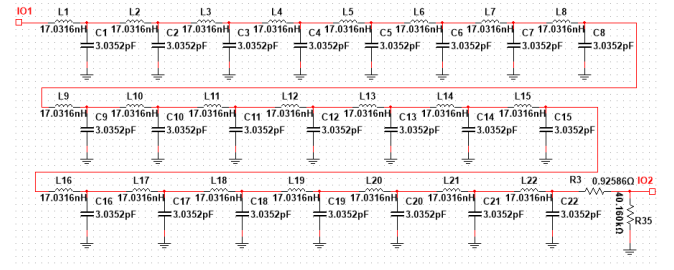| Frequency/ Hz	R(Ω /m)	G(S/m)	1/G(S/ m)
0	0.067759	3.93x10^-3	1.2048 T
1M	0.202865	8.30x10^-7	1.2048 M
10M	0.549242	8.30x10^-6	120k
30M	0.92586	2.49x10^-5	40.160k

Figure 16:LCGR Circuit per meter Model

The figure above shows the per meter circuit model that replaces the first per meter model in the lossless circuit previously.

B. Time domain analysis under harmonic (AC) voltage excitation

After the lossy equivalent circuit is built, the circuit is simulated under a 75ohm load with a step excitation voltage source like the lossless section. As seen in the figures below, there are differences between the lossy and lossless by the slight decrease in output voltages due to the addition of the G and R parameter. For all cases of different frequencies, this behaviour is constant with bigger differences at higher frequencies.
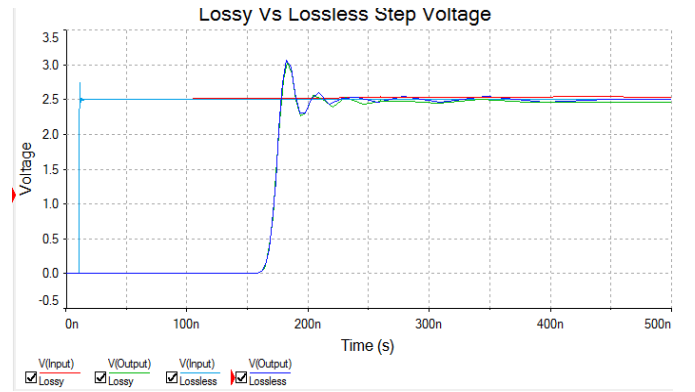

Figure 17:Lossy vs Lossless Step Voltage

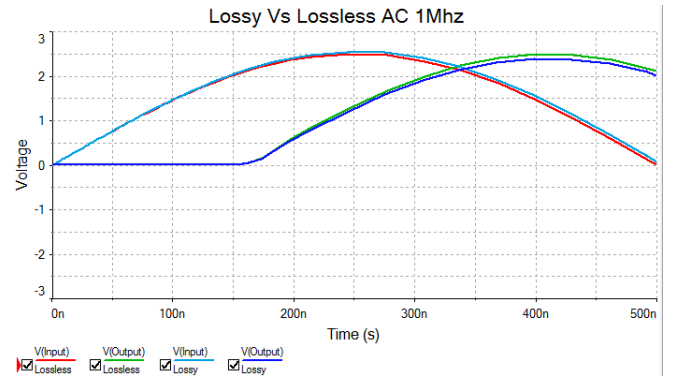### C. Time domain analysis under harmonic (AC) voltage excitation
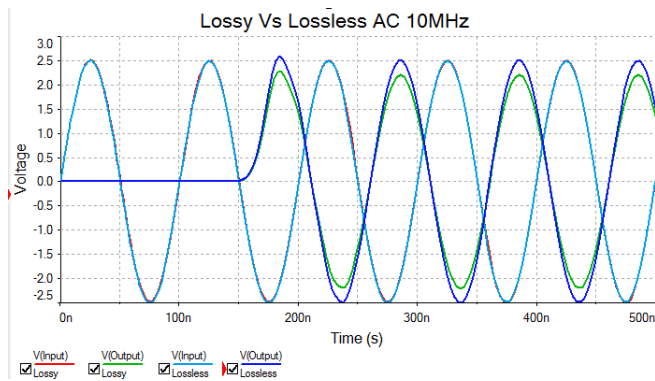

Figure 18:Lossy vs Lossless AC 1MHz
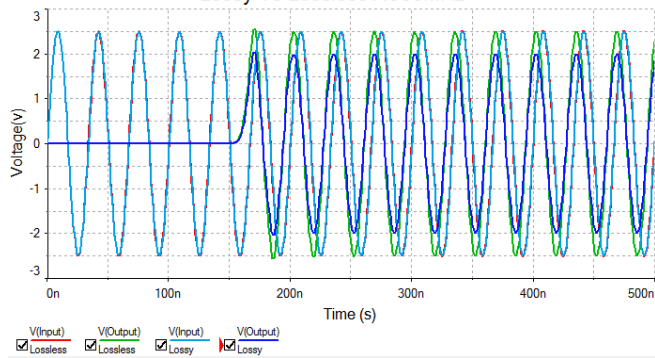
Figure 19:Lossy vs Lossless AC 10MHz

Figure 20:Lossy vs Lossless AC 30MHz

V. CONCLUSION

In conclusion, the simulation of both cases of lossless and lossy transmission lines can be done when the parameters for the transmission lines are acquired. Analytical calculations must be done in order to take into account various different factors that must influence the magnitudes of voltages that flow through the transmissions lines. The extraction of LC and exact model to simulate with various voltage sources can be done with minor differences between the two circuit models. However, the higher the frequency the higher the differences between the two models will occur. There also exists attenuation with LC models that differ from exact models. So, analytical simulations can be used however, 3D simulations for transmission lines might prove to produce better and more accurate to real world results.

REFERENCES

[1]   https://secure.ecs.soton.ac.uk/notes/elec2229/Assignment/Coax_re
       v3.4.xls