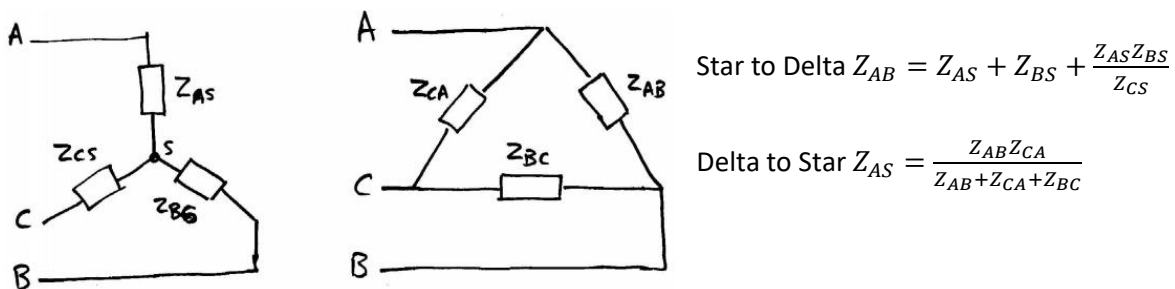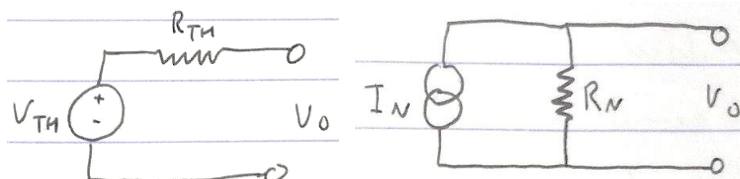## Milos 1 – Linear circuit analysis

- Nodal analysis works by choosing a reference node, applying KCL to all nodes but the reference, writing the currents as functions of the node voltages and the resistances (I = V/R), collecting the equations into matrices, finding the inverse of the matrix containing the multiples of voltages, and multiplying this by the matrix on the other side to get the unknown voltages.

- The inverse of matrix **A** is the adjoint of **A** divided by the determinant of **A**. The adjoint matrix is the transpose of the matrix of cofactors. The matrix of cofactors is the matrix of minors with each element in the matrix multiplied by 1 or -1 depending on the position in the matrix. The matrix of minors is a matrix where each element is replaced with the value of the matrix determinant with the row and column that contains the element removed from the matrix.

- Millman's theorem is provided in the formula sheet and is useful in the later three phase content $V_{SN} = \frac{V_{1N}Y_1 + V_{2N}Y_2 + V_{3N}Y_3 \cdots}{Y_1 + Y_2 + Y_3 + \cdots}$, the notation V_{SN} means going to S from N. It is used when you have parallel branches of voltage sources in series with resistors. In the three phase application it is used to find the voltage between the star point and the neutral point.

- Rosen's theorem links the star and delta configurations, and is very useful in the three phase part.
- It isn't provided in the formula sheet, so this is one you must *learn*. In this section it is given in terms on admittances, however in the three phase section impedances are used. It is easy to switch from impedance to admittance by applying the reciprocal. Find a way to remember them.

Star to Delta $Z_{AB} = Z_{AS} + Z_{BS} + \frac{Z_{AS}Z_{BS}}{Z_{CS}}$

Delta to Star $Z_{AS} = \frac{Z_{AB}Z_{CA}}{Z_{AB} + Z_{CA} + Z_{BC}}$

- Mesh analysis works by making an equation for each mesh in the circuit. Each equation contains the voltage equalling the sum of impedances multiplied by currents. This can be put into a matrix, where **V = Z I**. Find the inverse of **Z** and multiply it by **A** to get the currents **I**.

- The superposition theorem states you can analyse the circuit by looking at the currents due to one voltage source, the currents caused by the second voltage source etc etc… and adding up all the currents to find the overall circuit operation. This is also used in deriving op amp equations.

- Thévenin's theorem and Norton's theorem are two different ways to simplify a circuit into a source and an impedance. It applies to circuits with two terminals, to which you usually connect a load.
- Thévenin's theorem creates a voltage source in series with an impedance.
- Norton's theorem creates a current source in parallel with an impedance.

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = Z^{-1} \begin{bmatrix} V_1 & 0 & \cdots & 0 \\ 0 & V_2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & V_N \end{bmatrix}$$
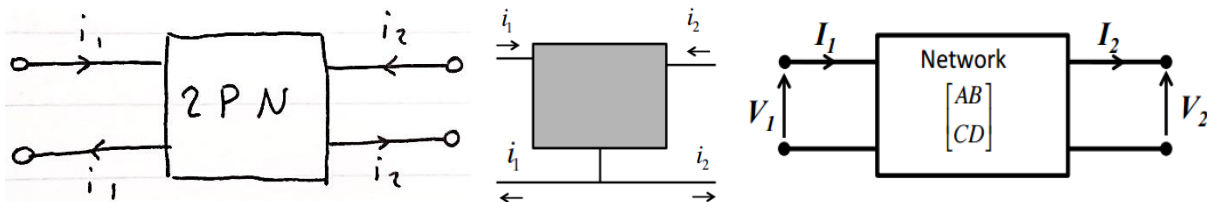
- V_{TH} is the voltage across the open terminals. R_{TH} is the equivalent resistance with the voltage sources and current sources in the original circuit turned off, short and open circuits respectively.
- I_N is the short circuit current (short the two output terminals). R_N is the total resistance with voltage sources turned off, so they become short circuits.

Equation	Meaning
$Z = R + jX$	Impedance with real resistance R, and imaginary reactance X. Measured in Ohms.
$Y = G + jB$	Admittance with real conductance G, and imaginary susceptance B. Measured in Siemens.
$A^{-1} = \dfrac{adj\,A}{\det A} = \dfrac{adj\,A}{|A|}$	The equation for the inverse of the matrix **A**.
$V_{SN} = \dfrac{V_{1N}Y_1 + V_{2N}Y_2 + V_{3N}Y_3 \ldots}{Y_1 + Y_2 + Y_3 + \cdots}$	Millman's parallel generator theorem. Finds the overall voltage between two terminals powered by multiple voltage sources in parallel.
$Z_{AB} = Z_{AS} + Z_{BS} + \dfrac{Z_{AS}Z_{BS}}{Z_{CS}}$	Rosen's theorem, star to delta transform. (sum of the two related impedances) + (product of two related impedances)/(unrelated impedance)
$Z_{AS} = \dfrac{Z_{AB}Z_{CA}}{Z_{AB} + Z_{CA} + Z_{BC}}$	Rosen's theorem, delta to star transform. (product of two related impedances)/(sum of three impedances)

Milos 2 – Two-port networks
- This is a way of breaking down a transmission line into simpler circuits. You consider the network as a 'black box' and are only concerned with the voltage and currents at the ports.
- A port is a pair of terminals where the current entering one equals the current exiting the other.
- A two-port network (2PN) is a system with four terminals, hence two ports. A transformer is a good example of a 2PN, the current i_1 entering and leaving the primary coil is constant, as is the current i_2 entering and leaving the secondary coil. 2PNs can also share a common terminal.

- Before we can characterise 2PNs, the impedances/admittances must be converted into the *s* domain by using Laplace transforms. Adding in series and parallel rules should be known, seen below.
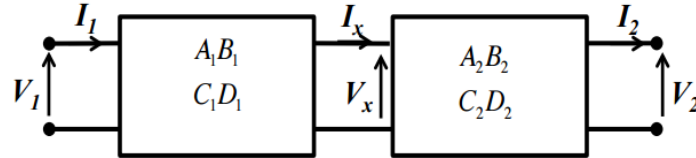
$$Series: Z_T(s) = Z_1(s) + Z_2(s) \qquad Parallel: \frac{1}{Z_T(s)} = \frac{1}{Z_1(s)} + \frac{1}{Z_2(s)}$$

$$Series: \frac{1}{Y_T(s)} = \frac{1}{Y_1(s)} + \frac{1}{Y_2(s)} \qquad Parallel: Y_T(s) = Y_1(s) + Y_2(s)$$

Quantity	Impedance	Admittance
Resistance R	R	$\dfrac{1}{R}$
Inductance L	Ls	$\dfrac{1}{Ls}$
Capacitance C	$\dfrac{1}{Cs}$	Cs

- 2PNs are represented using *transfer matrices*. A and D are constants, B is impedance, and C is admittance, in what is called the *ABCD* notation. There are other notations such as the admittance matrix (right), but *ABCD* is suited for transmission lines.
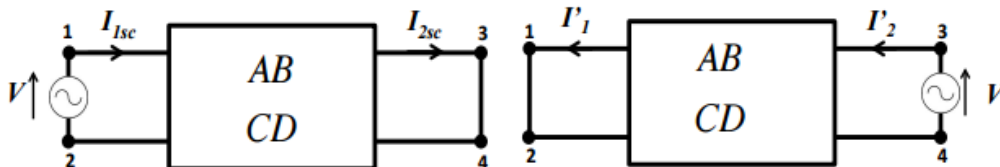
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}, \qquad V_1 = AV_2 + BI_2, \qquad I_1 = CV_2 + DI_2, \qquad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

- Finding the *ABCD* matrix using circuit analysis takes a long time. Instead there are simple networks that you can cascade to get the network you want. Cascading works by multiplying *ABCD* matrices.
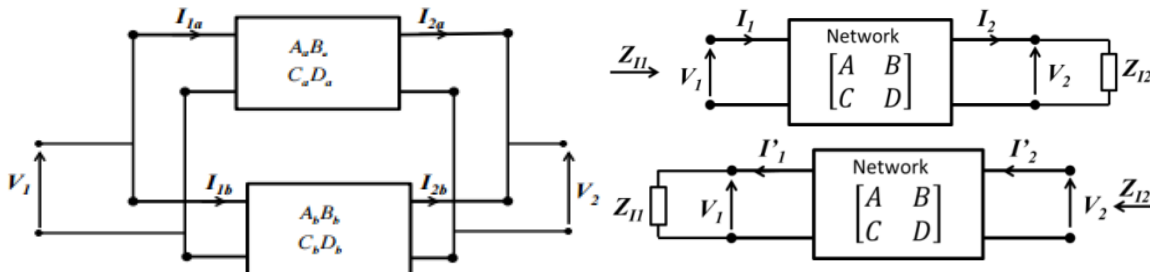

$$\text{Since } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_X \\ I_X \end{bmatrix} \text{ and } \begin{bmatrix} V_X \\ I_X \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \text{ then } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$
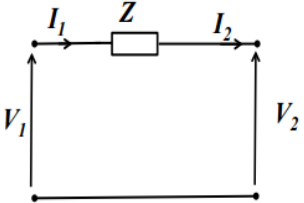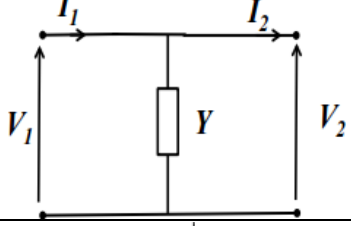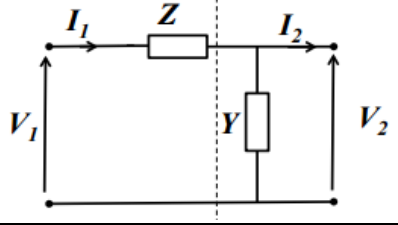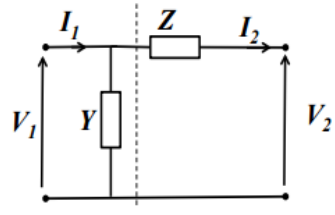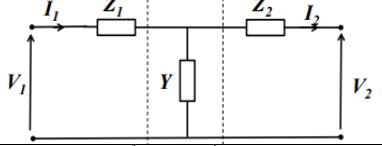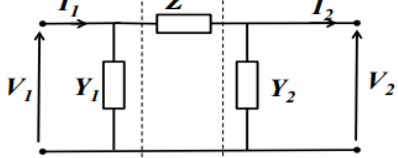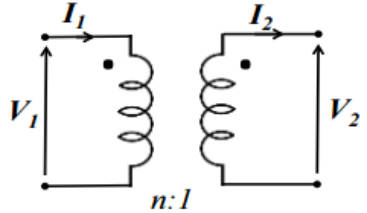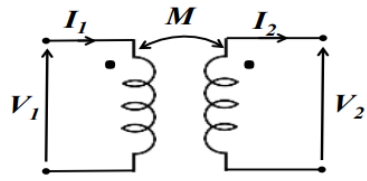
- All of the networks are shown in the table. Realistically by remembering just the series impedance, shunt admittance, transformer, and mutual inductance matrices you can make any network.
- You can experimentally find the *ABCD* parameters. First, by having an open circuit on the output terminals, $I_2 = 0$, which lets you find A and C. Second, by shorting the output terminals, $V_2 = 0$, which lets you find B and D.
- Reciprocity is the property found in two port networks, where the impedance one way is the same as the impedance the other way. This is tested by applying a voltage on one side, measuring the current on the other side, and then swapping the voltage source and current monitor around to see if the current is still the same. Passive circuits (R, L, C, transformers, most circuits) are reciprocal.
- Reciprocal networks have the property $AD - BC = 1$. This is the *determinant* of the *ABCD* matrix.
- Symmetrical networks have the property $A = D$, where the input impedance equals the output impedance. For symmetrical reciprocal networks, we can combine the two equations $A^2 - BC = 1$.
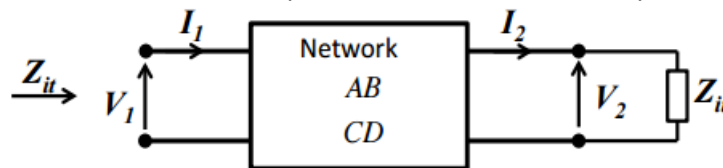

- Networks in parallel can be solved for their *ABCD* matrix by using the impedance matrix. You create the impedance matrix equation for each branch by rearranging the equations. Then the impedance matrices are added to get the full equation. You then rearrange the equation to get it in *ABCD* form.

- Image impedances ensure that no power is reflected back to the source. The maximum power transfer theorem states that, for a generator with resistive internal impedance, maximum power is transferred when a resistive load at the generator terminal has a resistance equal to the internal generator resistance. At the receiving end (load side) maximum power is received by the load through a transmission network from a source when the resistance seen at the load terminals is equal with the load resistance.

Network name	Image	Matrix
Series impedance		$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$
Shunt admittance		$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$
Half-T		$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$ $= \begin{bmatrix} 1+ZY & Z \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$
Half-π		$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$ $= \begin{bmatrix} 1 & Z \\ Y & 1+YZ \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$
T		$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$
π		$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$
Transformer		$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$
Pure mutual inductance		$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & -j\omega M \\ \frac{1}{j\omega M} & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$

- Remember the two equations for image impedance, $Z_{I1} = \sqrt{\frac{AB}{CD}}$ and $Z_{I2} = \sqrt{\frac{DB}{CA}}$, which are from the input side and output side respectively. For symmetrical networks where the output and input impedances are the same, $A = D$ applies and the image impedance on both sides simplifies to $\sqrt{\frac{B}{C}}$. The units are Ohms, think about the units of B and C and this will make sense.
- The second way to find the image impedance is to apply short and open circuits to both sides and obtain equations. Z_{oc1} is the impedance across the input terminals when there is an open circuit on the output terminals, found by $\frac{V_1}{I_1} = \frac{AV_2}{CV_2} = \frac{A}{C} = Z_{oc1}$. The input impedance Z_{sc1} when there is a short circuit on the output terminals, is found by $\frac{V_1}{I_1} = \frac{BI_2}{DI_2} = \frac{B}{D} = Z_{sc1}$. You apply the same logic going the other way, to find Z_{oc2} and Z_{sc2}, which equal $\frac{D}{C}$ and $\frac{B}{A}$ respectively. From this, the input and output image impedances are $Z_{I1} = \sqrt{Z_{oc1}Z_{sc1}}$ and $Z_{I2} = \sqrt{Z_{oc2}Z_{sc2}}$ respectively.
- The neat thing here is that for a symmetrical network, $Z_{oc1} = Z_{oc2}$ and $Z_{sc1} = Z_{sc2}$, as the impedance looking either way is the same. As $A = D$, the image impedance either way simplifies to $Z_0 = \sqrt{\frac{B}{C}}$, which is known as the *characteristic impedance*.
- Iterative impedance follows this, it's possible to find a load impedance that when connected to the output, it creates an input impedance equal to the load. This means that looking from the input side, you can "see right through" to the load connected on the other side of the 2PN, the 2PN acts as a direct connection to the load in terms of impedance, as it doesn't add any extra.
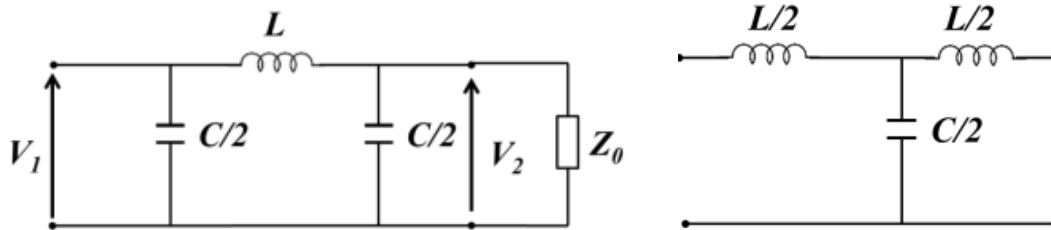

- To calculate this using *ABCD* parameters, you set $I_2 = \frac{V_2}{Z_{it}}$, $Z_{in} = \frac{V_1}{I_1}$, then using the equations for V_1 and I_1, subbing in the above form of I_2, you can rearrange to get Z_{it} as you know that $Z_{in} = Z_{it}$.
- The quick way for symmetrical networks is $Z_{it} = \sqrt{\frac{B}{C}}$, note this is the same as the image impedance and subsequently, the characteristic impedance for symmetrical networks.

- The *insertion loss ratio* is the ratio of the voltage across the load that's connected to a voltage source with and without a 2PN in between the voltage source and the load. This can be useful when creating filters out of 2PNs, you want a large insertion loss for attenuation band frequencies and ideally zero insertion loss for the transmission band frequencies. To turn this ratio into dB use the equation $loss\ (dB) = 20\log_{10}\left(\frac{V'_1}{V_2}\right)$. There also could be a phase change as a result of introducing the 2PN. This is found using $phase = \angle(V'_1) - \angle(V_2)$.

- The *propagation coefficient* γ applies to symmetrical networks, it is the natural log of the insertion loss. $\gamma = \ln\frac{I_1}{I_2} = \ln\frac{V_1}{V_2} = ke^{j\beta}$. This can be a complex number if there is a phase change.
$\gamma = \ln k + j\beta$, where the real $\ln k$ is the *attenuation coefficient* and the imaginary β is the *phase-change coefficient*.

- The derivation to get γ is complex, instead remember that for a symmetrical network $A = D$, and

$$\gamma = \cosh^{-1}(A) = \ln\left(A + \sqrt{A^2 - 1}\right)$$

- As mentioned, we can make low pass filters using 2PNs. Properties required include the attenuation being large/zero depending on the frequency (real part of γ), the phase delay to be constant (imaginary part of γ), and the characteristic impedance to be close in value to the terminal impedance to avoid unwanted reflections.
- We look at π and T section filters. They are ideally made up of just inductors and capacitors, so that there is no power dissipation.
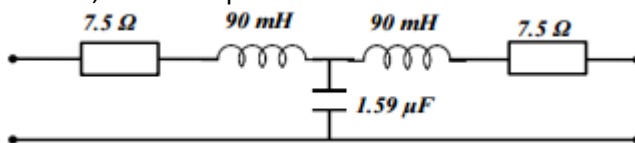

- For the π filter, the attenuation coefficient k is L/C, as it is independent of frequency, the circuit is called a constant k filter. Taking the assumption that $A^2 < 1$, the following can be derived.

$$\gamma = \ln\left(A + \sqrt{(-1) - A^2 + 1}\right) = \ln\left(A + \sqrt{(-1)}\sqrt{-A^2 + 1}\right) = \ln(A + j\sqrt{1 - A^2}) = \ln(ke^{j\beta})$$

You are then able to find the magnitude and phase of the expression. The magnitude is 1.
- By cascading the three sections to find $A = D = 1 - \frac{\omega^2 LC}{2}$, and applying the assumption that $A^2 < 1$, thus $-1 < A < 1$, we can rearrange the equation to find the limits of ω where there is no attenuation.
- After this you can easily find the cut off frequency.

- For the T-filter, the same principles apply.

- The delay line is a network of cascaded T or π sections, with no attenuation and a constant time delay. This is like how a filter works within the passband frequency. To find the total delay, you add together the phase-change coefficients β of each section.
- Transmission lines can be modelled by using the *lumped model*, where a section that models a metre (or mile) of transmission line is repeated to get the desired length. This is correct for when the line is short in comparison to the voltage wavelength; 50 Hz corresponds to a wavelength of 6000 km.
- As usual, the *ABCD* parameters can be found.

- The final topic is *per-unit systems*. In some scenarios it is more useful to state what percentage of the maximum that the component is working at, instead of the absolute value.

Per-unit voltage & current: **A, B, C, D parameters in per unit system:**

$$V_{1pu} = \frac{\text{actual voltage}}{\text{rated voltage}} = \frac{V_1}{V_r} \qquad I_{1pu} = \frac{\text{actual current}}{\text{rated current}} = \frac{I_1}{I_r} \qquad \begin{bmatrix} V_{1pu} \\ I_{1pu} \end{bmatrix} = \begin{bmatrix} A & \frac{B}{V_r}I_r \\ \frac{C}{I_r}V_r & D \end{bmatrix} \begin{bmatrix} V_{2pu} \\ I_{2pu} \end{bmatrix} =: \begin{bmatrix} A_{pu} & B_{pu} \\ C_{pu} & D_{pu} \end{bmatrix} \begin{bmatrix} V_{2pu} \\ I_{2pu} \end{bmatrix}$$