Physics-PHY101-Lecture#06

APPLICATIONS OF NEWTON'S LAWS – II

Introduction:

In previous lecture, we have applied Newton's law on frictional problems for solid objects. As we know, frictional force is produced because of the rubbing of two objects together. But this is not necessary for only solid objects, fluid friction is also possible.

Example:

1. If you wave your hand in the air, apparently you will not feel any frictional force. But if you take an umbrella in your hand and then wave your hand you will feel a frictional force because of air resistance with umbrella.
2. If you pass your hand from water, then you will feel a frictional force easily.

Importance of fluid friction:

Newton's law is very important in fluid friction also because when an object is passing through a fluid, object has to make some space to move in fluid, where the use of pair of forces (action-reaction force) is important. For Example, as a body moves through a body it displaces the fluid. It has to exert a force on the fluid to push it out of the way. By Newton's third law, the fluid pushes back on the body with an equal and opposite force.

The two main conclusions of fluid friction are.

- The direction of the fluid resistance force on a body is always opposite to the direction of the body's velocity relative to the fluid.
- The magnitude of the fluid resistance force usually increases with the speed of the body through the fluid

Q. Think, what will happen without fluid friction in sea water?

Ans: Off course fishes would die there. Fishes would not swim in water and eventually leads to death without fluid friction.

Dependence of fluid power:

The power of fluid to resist the object is depends on,

- The speed of object moving in fluid
- The higher the speed of object usually, maximizes to resist the motion of object.

Fluid frictional force and velocity of object:

According to a linear law, we can say that "The fluid resistance force is proportional to the velocity but if the velocity of the object has increased then fluid resistance force becomes proportional to the square or cube of the velocity".

Limitations of this law:

This law would not be applicable to the case where the speed of the object becomes faster. For solid objects, we use empirical laws (which can be calculated experimentally) but with some limitations.

Example of equilibrium under two forces:

To see how friction/resistance affects different bodies, scientist Galileo performed an experiment at the leaning tower of Pisa. He dropped two balls of different masses (M and m, where M >m) from the top of Pisa tower. Both balls hit the ground at the same time regardless of their masses. These results shocked him for a while because before his experiment, it was thought that the heavier ball hit the ground before the lighter ball, but the conclusion was different. Then Galileo again performed the experiment with a ball and a feather. This time he finds that the ball hit the ground before feather even though both have same gravitational force act on them. This experiment leads to the conclusion that there is some other resistive force (frictional force) which restricts their motion. Now in order to see, how resistance affects acceleration and velocity of feather, we have the following,

Let's, the magnitude of fluid resistance 'f' is almost proportional to the speed of the body through the fluid given as,

$$f \propto v$$
$$f = kv$$

Where, “k” is called the proportionality constant. As, the upward resistive force on feather when it’s falling downward is always in opposite direction to the motion of object. Hence,

Upward resistive force= $F_r = -\ kv$

Downward weight force = $F_w = mg$

The net force acting on the object is,

$$F_w + F_r = F$$

$$mg + (-kv) = ma$$

After certain interval, acceleration ‘a’ of the feather becomes constant (a=0) and attains a velocity called as terminal velocity (V_T) which is given as,

$$mg - kv_T = 0$$

$$v_T = \frac{mg}{k}$$

This was a simple example of equilibrium between two forces.

Problem solving steps:

In general, while solving actual problems one should do the following steps,

- Draw a free body diagram.
- Define an origin for a system of coordinates.
- Define x, y coordinate system.
- Identify all forces (tension, normal, friction, weight, etc.) and their x & y-components.
- Apply Newton's 2nd law separately on x-axis and y-axis.

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

- Find the unknown quantities i.e.,
- Acceleration, velocity, and displacement:

$$F = ma_x$$

$$V = V_o x + a_x t$$

$$X = X_o + V_o x + \frac{1}{2} a_x t^2$$

- Forces:

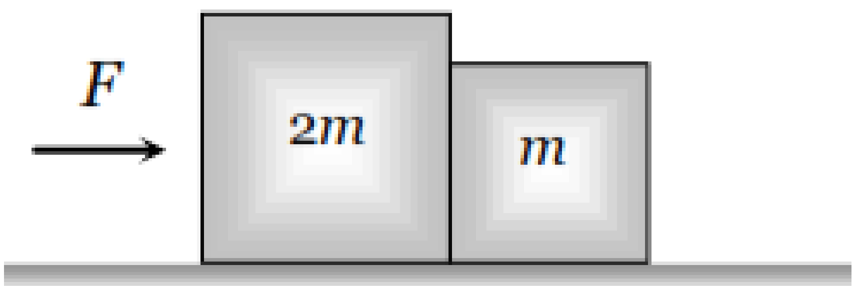
$$F_w = mg \text{ (gravitational force)}$$

$$f = \mu N \text{ (frictional force)}$$

Applications to solve frictional problems:

Problem 1:

Consider two blocks of mass 2m and m on a frictionless surface and a force F is applied on 2m block from left side. As the two blocks acts as one body, hence, begins to move towards right side. We have to calculate force on mass ‘m’ due to mass ‘2m’. As shown in figure.



Solution:

Acceleration of the blocks can be calculated as,

$F = ma$

$Total\ \ mass = 2m + m$

$F = (2m + m)a \Rightarrow F = 3ma \Rightarrow\Rightarrow a = \frac{F}{3m}$

Now isolate block 'm' and then apply newtons 2nd law,

$\sum F = ma \qquad \therefore a = \frac{F}{3m}$

$F_{2on1} = m\left(\frac{F}{3m}\right) \Rightarrow F_{2on1} = \frac{F}{3m}$

This is the force exerted on mass m from mass 2m. Similarly, the force on mass 2m from mass m is,

$F_{1on2} = 2m\left(\frac{F}{3m}\right) = \frac{2F}{3}$

Conclusion: Results show that the force acts on m block because the force of 2m block is just one third (1/3rd) of the total force.

Limitation of newtons law:

Newtons laws are only applicable to inertial frame of reference.

Difference between inertial and non-inertial frame of reference:

Inertial frame	▪ The frame of reference which is moving with uniform velocity and does not accelerate (a=0). ▪ Obeys Newton's law of motion. ▪ **For example**, a train moving with uniform velocity is an inertial frame of reference.
Non-inertial frame	▪ The frame of reference which is accelerating ($a \neq 0$) is called non-inertial frame of reference. ▪ Does not obey Newton's law of motion. ▪ **For example**, a freely falling elevator is taken as non-inertial frame of reference.

Problems in non-inertial frame of reference:

Problem 2:

Suppose you are in a lift. Here we have to deal with different cases:

- **Case-1:** where, you are at rest or moving at constant velocity (a=0) then normal force N (i.e., the force with which the floor of the lift is pushing on you) and the force due to gravity are exactly equal,

$$N - Mg = 0$$
$$N = Mg$$

The relation shows the apparent weight is equal to the true weight just for the condition that the acceleration must be equal to zero.

- **Case-2:** if the lift is accelerating upwards then,

$$N - Mg = Ma$$
$$N = M\left(g + a\right)$$

These relations show that the apparent weight is more than the true weight (you feel heavier) as there is an added gravitational (g) force of 9.8 m/s^2. Acceleration would add an extra gravitational force, making you feel twice as heavy.

- **Case-3:** if the lift is accelerating downwards then,

$$N - Mg = Ma$$
$$N = M\left(g - a\right)$$

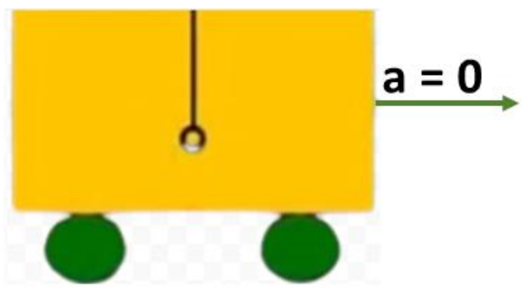
This relation shows that the apparent weight is less than the true weight (you feel lighter).

- **Case-4:** if the cable, supporting the lift is breaks, the lift falls downward with a= g. then

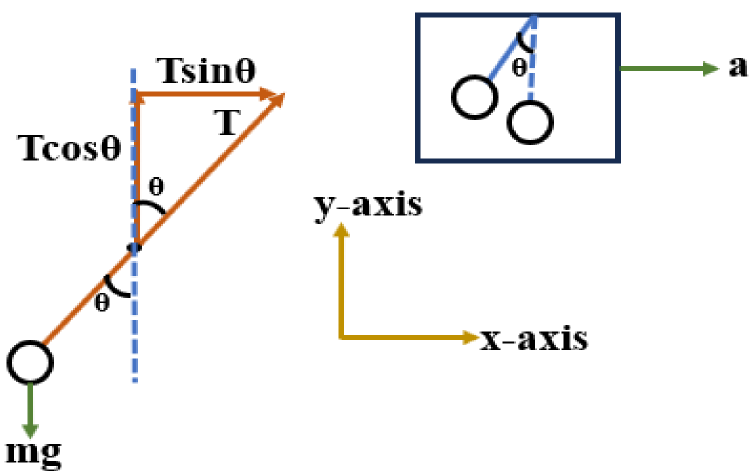
$$N = M\left(g - g\right) = 0$$

This shows the apparent weight under the free fall is zero and you will experience weightlessness just like astronauts in space do.

<u>Problem 3:</u> Imagine that you are in a railway wagon and want to know how much you are accelerating. You are not able to look out of the windows. A mass is hung from the roof. Find the acceleration of the car from the angle made by the mass.



Solution:



According to the balancing of forces,

Forces acting horizontally,

$$\sum F_x = T\sin\theta$$

$$ma = T\sin\theta \rightarrow 1)$$

Forces acting vertically,

$$\sum F_y = T\cos\theta - mg$$

$$0 = T\cos\theta - mg$$

$$T\cos\theta = mg \rightarrow 2)$$

On dividing equation 1) and 2), we get

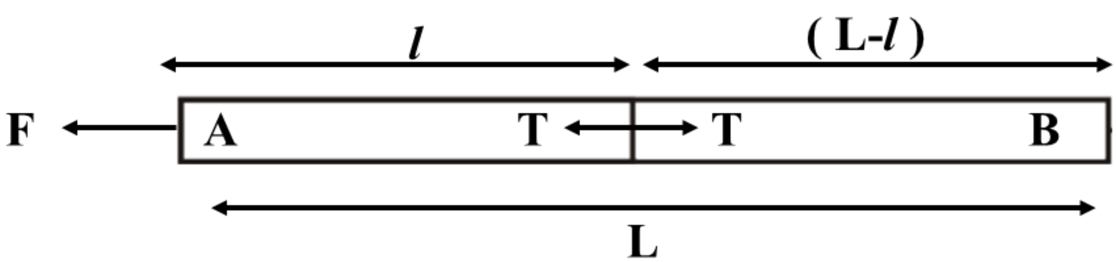
$$\frac{T\sin\theta}{T\cos\theta} = \frac{ma}{mg}$$

$$\tan\theta = \frac{a}{g}$$

Conclusion: Result shows that the mass doesn't matter during the motion.

Problems in inertial frame of reference:

Problem 4: A uniform rope of length L, lying on a horizontal smooth floor, is pulled by a horizontal force F. What is the tension in the rope at a distance l from the end where the force is applied?



Solution:

Massive rope have different value of tention at different position, $\sum F_x = F - T$

$(\text{mass of rope upto length } 'l')\, a = F - T$

$(ml)\, a = F - T \Rightarrow\Rightarrow T = F - mla \rightarrow 1)$

$\because m = \dfrac{M}{L} = \dfrac{mass}{length} = \text{linear mass density}$

$\because F = Ma, \because F = mLa \rightarrow 2)$

plug in equation 2) in 1), we get,

$T = mLa - mla = ma(L - l)$

$T = maL\left(1 - \dfrac{l}{L}\right) = F\left(1 - \dfrac{l}{L}\right) \rightarrow 3$

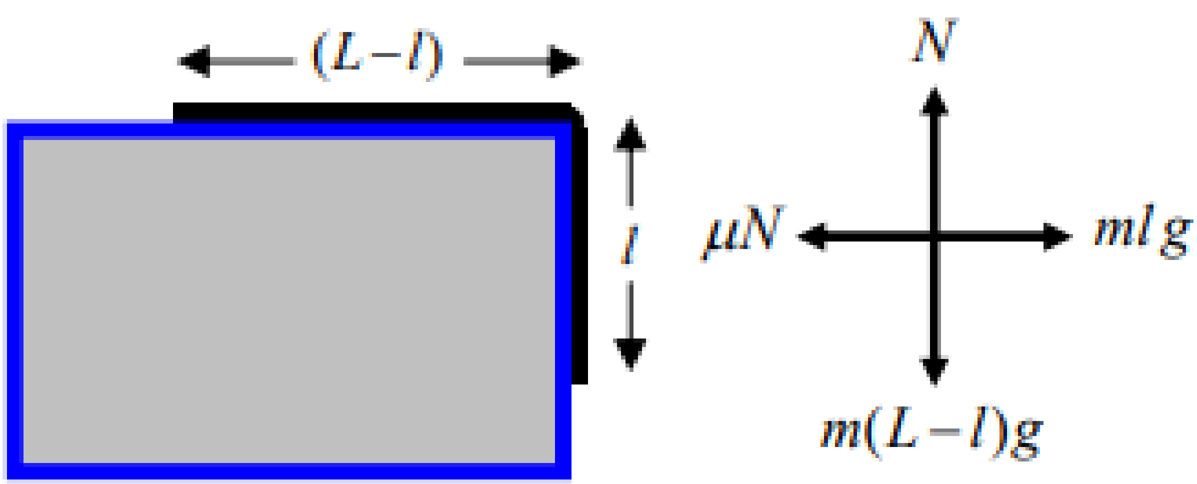
Checking:

a. If $l = 0$ then, T=F (as, this is the point where we have applied force)

b. If $l = L$ then, T=0 (as, at the end of the rope, there is nothing (no force) to act upon)

Problem 5: A rope of total length L and mass per unit length m is put on a table with a length l hanging from one edge. What should be length l such that the rope just begins to slip?

Solution: To solve this, look at the balance of forces in the diagram.



Total length=L,   Total mass=M

mass per unit length=m= $\frac{M}{L}$

$\because$ Force of gravity on hanging rope=$\left(\text{mass of hanging rope}\right)g$

Force of gravity on hanging rope $= m\,\text{lg}$

Force of gravity on rope lies on the table=$\left(\text{mass of rope on table}\right)g$

Force of gravity on rope lies on the table=$m\left(L-l\right)g$

Forces acting on the rope lies on the table;

$$\sum F_x = \mu N - mgl$$

$$0 = \mu N - mgl \Rightarrow \mu N = mgl \rightarrow 1$$

$$\sum F_y = N - mg\left(L-l\right)$$

$$0 = N - mg\left(L-l\right) \Rightarrow N = mg\left(L-l\right) \rightarrow 2$$

Putting eq.2 in eq.1,we get

$$\mu N = mgl$$

$$\mu\left(mg\left(L-l\right)\right) = mgl$$

$$\mu mgL - \mu mgl = mgl$$

$$\mu mgL = \mu mgl + mgl$$

$$\mu mgL = mg\left(l + \mu l\right) \Rightarrow \mu L = \left(\mu + 1\right)l$$

$$l = \frac{\mu L}{\left(\mu + 1\right)}$$

Conclusion:

- Note that if μ is very small (μ=0) then, even a small piece of string that hangs over the edge will cause the entire string to slip down.
- And if you take μ to be a very large number then, a very small portion of the length of rope is enough to stay on table.

Problem 6: A massive rope of total length l is passing through a pulley as shown in figure. The larger length of rope is moving downward with some acceleration which we have to calculate here.

Solution:

Total length $= l$

mass per unit length= m

on the right side of fig.

$\sum F_y = m(l-x)g - mgx$

$(\text{total mass of the rope})a = m\lg - mxg - mxg$

$mla = ml\,g - 2mxg$

$mla = m(\lg - 2xg) \Rightarrow la = (\lg - 2xg)$

$a = \frac{\lg}{l} - \frac{2xg}{l} \Rightarrow g - \frac{2xg}{l}$

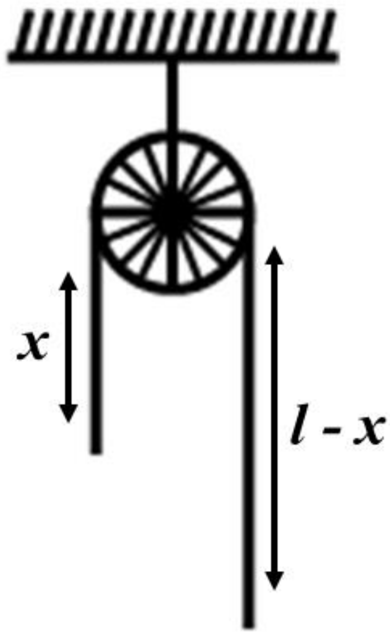
$a = g\left(1 - \frac{2x}{l}\right)$

Checking; if,

$x = \frac{l}{2} \quad \Rightarrow a = g\left(1 - \frac{2\frac{l}{2}}{l}\right)$

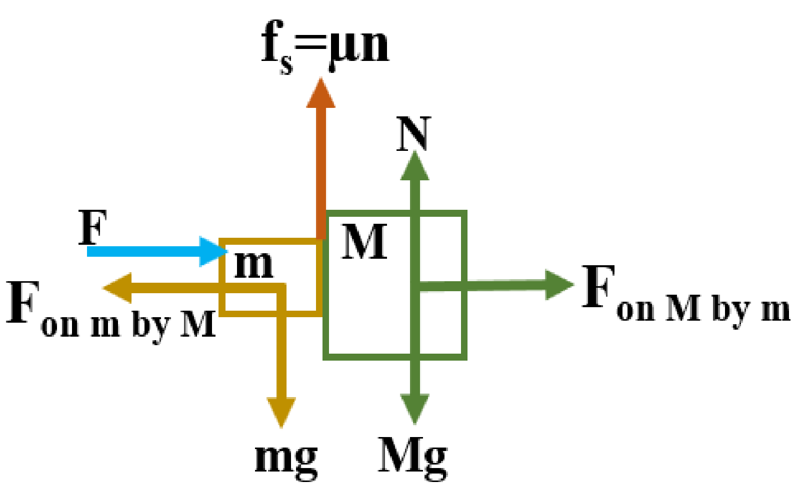
$a = g$

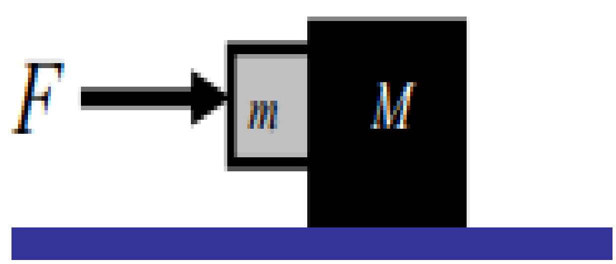
Both sides of the rope will have equal acceleration.



Problem 7: In this problem, two masses M and m attached together. We would like to calculate the minimum force such that the small block does not slip down.

Solution:





Since masses moves together so, $\sum F_x = F$

$(m+M)a = F \Rightarrow a = \dfrac{F}{m+M} \to 1$

At contact surface, $\sum F_y = f_s - mg$

$0 = f_s - mg \Rightarrow f_s = mg \quad \Rightarrow\Rightarrow mg \le f_s \to 2$

Where, $f_s = \mu n \Rightarrow mg \le \mu n \to 3$

$n =$ force by M on m $\Rightarrow n = ma$

$n = (\text{mass of box on which force act})(\text{acceleration of the system})$

$n = m\left(\dfrac{F}{m+M}\right) \to 4 \qquad \because a = \dfrac{F}{m+M}$

Using eq. 4 in eq.3, we get,

$mg \le \mu m\left(\dfrac{F}{m+M}\right) \Rightarrow g \le \mu\left(\dfrac{F}{m+M}\right)$

$\mu \ge \dfrac{g(m+M)}{F} \to 5$

Where, μ is a dimensionless quantity as it's the ratio of two forces.

Conclusion:

- We want the friction 'μn' to be at least as large as the downwards force 'mg' so that, it minimizes the horizontal force to prevent it from slipping.

Problem 8: Suppose two bodies m_1 and m_2 attached to a string and pass over the pulley. Mass m_1 is on the frictionless table and the forces acted on the masses are shown in figure. We have to calculate the tension and acceleration produced in the string.

Solution: If the string is massless then the tension in rope would be same at all points. So according to the balancing of forces, one can calculate as following:

Lets calculate the forces on m_1 & m_2 first,

Forces on m_1;

$$\sum F_x = T \Rightarrow m_1 a = T \rightarrow 1$$

$$\sum F_y = N - m_1 g$$

$$0 = N - m_1 g \Rightarrow N = m_1 g$$

Forces on m_2;

$$\sum F_y = m_2 g - T \Rightarrow m_2 a = m_2 g - T \rightarrow 2$$

To find "a", putting eq. 1 in eq. 2,

$$m_2 a = m_2 g - m_1 a$$

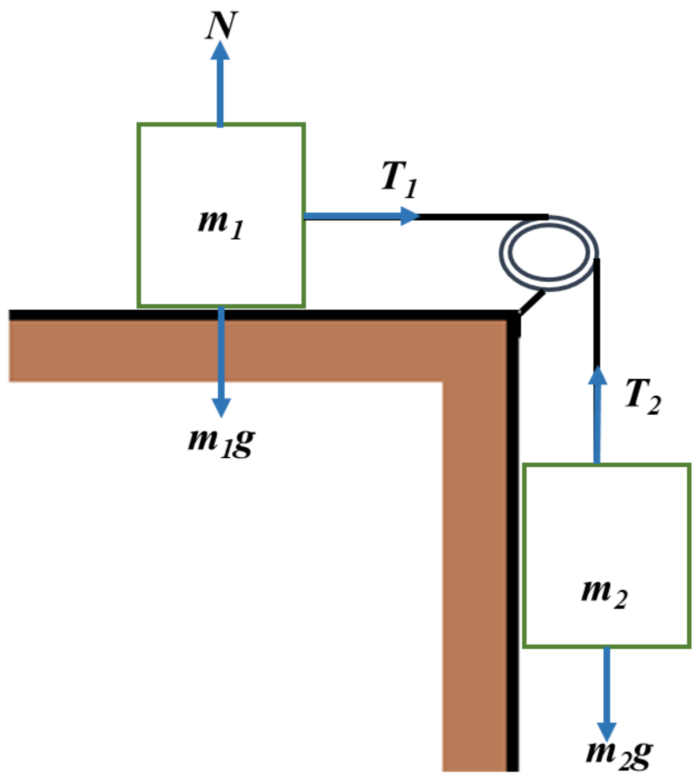
$$(m_1 + m_2) a = m_2 g \Rightarrow a = \left(\frac{m_2}{m_1 + m_2}\right) g \rightarrow 3$$

Now, to calculate "T", using eq.1 in eq.2,

$$m_2 \frac{T}{m_1} = m_2 g - T \Rightarrow m_2 T = m_1 m_2 g - m_1 T$$

$$(m_1 + m_2) T = m_1 m_2 g$$

$$T = \left(\frac{m_1 m_2}{(m_1 + m_2)}\right) g \rightarrow 4$$



Checking:

- If we put $m_1=m_2$ in eq.3 and consider both masses as one mass, then it concludes that half of the mass is accelerated downward, and rest of the half is not. We can say that half of the mass is responsible for this acceleration.
- If m_1 or $m_2 = 0$ in eq.4, then T (tension in string) becomes zero.

Problem 9: Consider two bodies of unequal masses m_1 and m_2 connected by the ends of a string, which passes over a frictionless pulley as shown in the diagram. We have to calculate the tension in the string and acceleration of the masses.

Solution:

Forces on m_1;

$$\sum F_y = T - m_1 g$$

$$m_1 a = T - m_1 g \rightarrow 1$$

Forces on m_2;

$$\sum F_y = m_2 g - T$$

$$m_2 a = m_2 g - T \rightarrow 2$$

On subtracting, eq.1 & eq.2, we will get acceleration as,

$$m_1 a = T - m_1 g$$

$$\underline{m_2 a = m_2 g - T}$$

$$(m_1 + m_2)a = (m_2 - m_1)g$$

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g \rightarrow 3$$

To calculate 'T', multiply eq.1 to m_2

and multiply eq.2 to m_1 and then on subtracting, we get

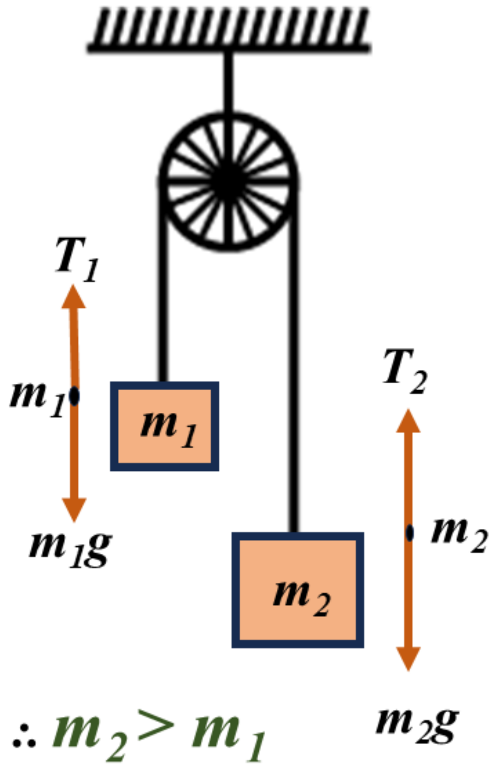
$$m_1 m_2 a = m_2 T - m_1 m_2 g$$

$$\underline{m_1 m_2 a = m_1 m_2 g - m_1 T}$$

$$0 = (m_1 + m_2)T - 2m_1 m_2 g$$

$$(m_1 + m_2)T = 2m_1 m_2 g$$

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g \rightarrow 4$$



Conclusion: If $m_1 = m_2$ then, the body would not be able to accelerate because at both sides, masses are equal so, $a = 0$.