Physics-PHY101-Lecture 7

WORK AND ENERGY

Definition of Work:

"Force applied in direction of displacement x displacement".

This means that the force F acts at an angle θ with respect to the direction of motion as shown in figure 7.1.

$$W = \vec{F}.\vec{d} = Fd\cos\theta$$

a) Work is a scalar - it has magnitude but no direction.

b) Work has dimensions: ($MLT^{-2}. L = ML^{2}T^{-2}$)

c) Work has units: Newton · Meter ≡ Joule (J)

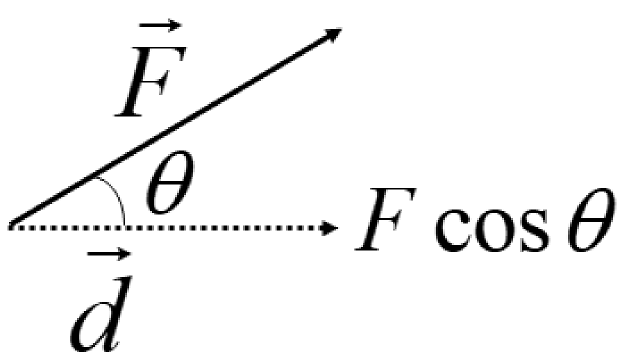


Figure 7.1: Work done by a constant force F.

Nature of work:

- **Positive work:** if the applied force displaces the object in its direction, then the work done is known as positive work. θ = 0°
- **Negative work:** if the force and displacement work in the opposite direction, then the work done is known as negative work. θ = 180°
- **Zero work;** If the force and displacement act perpendicular to each other, then the work done is known as zero work. θ = 90°

Forces do work on objects:

If a crate is pulled along the floor, only the force component parallel to the displacement d contributes to the work as shown in figure 7.2.

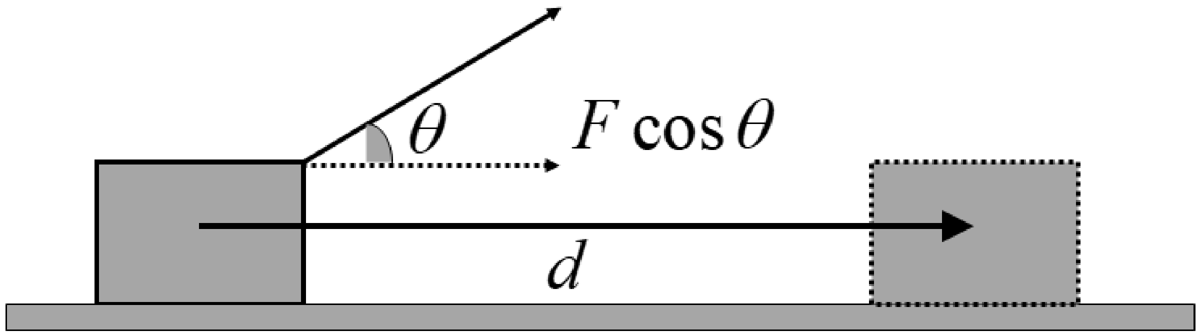
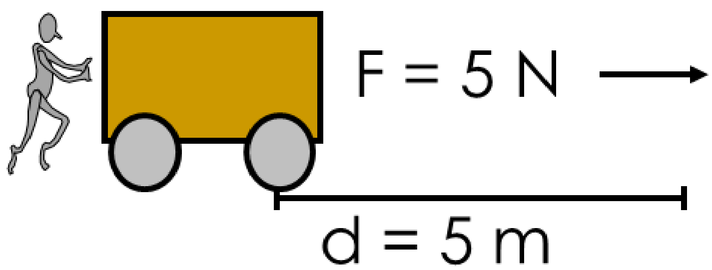


Figure 7.2: Work done by a force F in displacing object through displacement d.

We will justify this by following problem.

Problem 1: Suppose a man pushing a cart. A 5N force acts on it and displaces the cart through a displacement of 5m. As the force and displacement both are in same direction (parallel) to each other, calculate the work done on cart.



Solution: As we know,

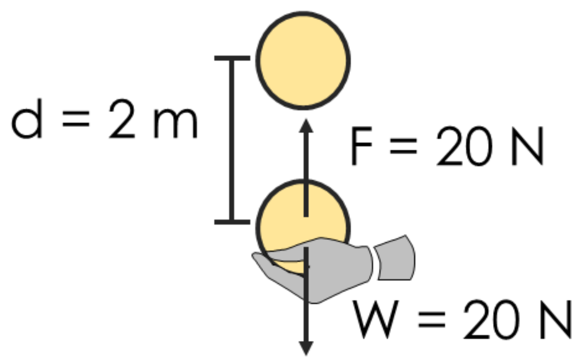
$$W = F.d\cos\theta = 5.5\cos 0 = 25\,(1)\ Nm = 25\ J$$

Problem 2: Suppose an apple falling from a tree. Due to its weight (1N), a force of gravity acts on it. Let suppose if it covers a displacement of 3m, then calculate the work done on apple due to gravity.

Solution: As we know,

$$W = F.d \cos\theta = 1N \times 3m = 3\ Nm = 3J$$

Problem 3: A heavy weight of 20 N is lifted from a height of 2m. calculate the work done on the heavy weight.



Solution: As we know,

$$W = F.d \cos\theta = 20N \times 2m \cos\theta$$

Here, we have to deal with two cases:

Work on ball by F_{hand} = 20 N x 2 m cos0 = 40 J (+ve work done)

Work on ball by $F_{gravity}$ = 20 N x 2 m cos180 = – 40 J (because force and displacement are anti-parallel to each other i.e., -ve work done)

Conclusion:

- If a mass is attached to a string and is moving in a circular motion, tension T is produced in a string. But this tension is not responsible for work as, T is perpendicular to the d (displacement).
- If the angle between two quantities is 90°, then it contributes towards no work.

Work done by a variable force:

Let the force F (x) act in the x direction, and let it vary in magnitude with x according to the function F(x) as shown in figure 7.3. What is the work done when the body moves from some initial position to some final position?

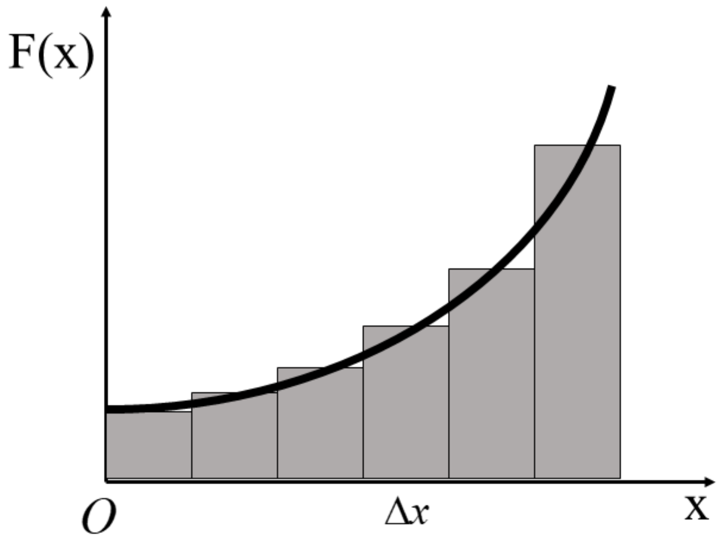


Figure 7.3. Variation of Force with Respect to x. The graph illustrates the dynamic nature of the force applied, showing fluctuations in response to changes in x.

What if the force varies with distance (say, a spring pulls harder as it becomes longer). In that case, we should break up the distance over which the force acts into small pieces so that the force is approximately constant over each bit. As we make the pieces smaller and smaller, we will approach the exact result.

$$\Delta W_1 = F_1 \Delta x$$
$$\Delta W_2 = F_2 \Delta x$$
$$\Delta W_3 = F_3 \Delta x$$

Now add up all the little pieces of work:

$$\begin{aligned} W &= \Delta W_1 + \Delta W_2 + \cdots + \Delta W_N \\ &= F_1 \Delta x + F_2 \Delta x + \cdots + F_N \Delta x \end{aligned}$$

To get the exact result let $\Delta x \to 0$ and the number of intervals N $\to \infty$:

$$W = \sum_{n=1}^{N} F_n \Delta x \qquad\qquad W = \lim_{\Delta x \to 0} \sum_{n=1}^{\infty} F_n \Delta x$$

Definition: $\lim_{\Delta x \to 0} \sum_{n=1}^{\infty} F_n \,\Delta x \equiv \int_{x_i}^{x_f} \mathrm{F(x)dx}$

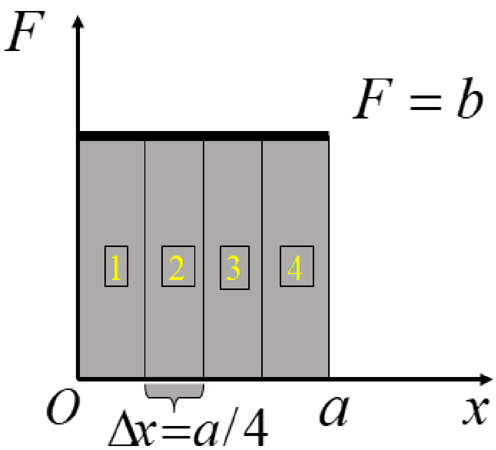
is the integral of F with respect to x from x_i to x_f.

This quantity is the work done by a force, constant or non-constant. So, if the force is known as a function of position, we can always find the work done by calculating the definite integral.

Example (for constant force):

Let suppose we have a rectangle, which is divided into 4 parts.

Area =



$$\frac{1}{4}a(b)+\frac{1}{4}a(b)+\frac{1}{4}a(b)+\frac{1}{4}a(b)=ab$$

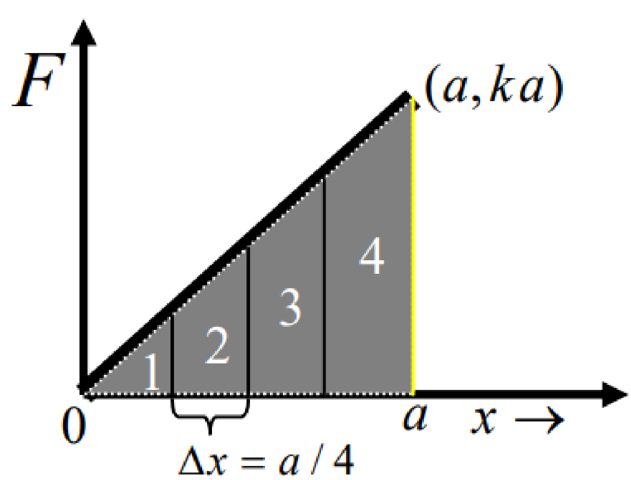
$$W=\int_0^a F dx$$

For a constant force, $W = F\int_0^a dx = F\left|x\right|_0^a$

$$\therefore F = b$$

$$W = b(a-0) = ab$$

Example (for variable force):



Let suppose we have a triangle, which is divided into 4 parts. Here, F = kx, force is proportional to x i.e., the force increases linearly with x.

$$\text{Area} = \frac{1}{2} * base * height$$

From the fig. base is "a" and height is "F", $\therefore$ F = ka

$$\text{Area of shaded region} = \frac{1}{2}(a)(ka) = k\frac{a^2}{2}$$

Mathematically, work done is given as, W=$\int Fdx$

$$W = \int_0^a Fdx = \int_0^a kxdx = \frac{k}{2}\left|x^2\right|_0^a = \frac{k}{2}\left(a^2 - 0\right) = k\frac{a^2}{2}$$

Example: Calculation of area under the curve:

Suppose we have a parabolic function y = x^2 as shown in figure 7.4. The parabola is divided into 4 parts so, Δx = 1/n = 1/4 (as maximum value on x and y-axis is “1”) and evaluated at four different points as shown in figure.

Area under the curve between 0 and 1 is approximately, $\sum_{i=1}^{n} y_i \Delta x$

As, number of rectangles, n=4, hence, $\Delta x = \frac{1.00}{n} \rightarrow 1$

$x_i = \left(i - \frac{1}{2}\right)\Delta x \Rightarrow$ putting the value of $\Delta x = \frac{1.00}{n}$

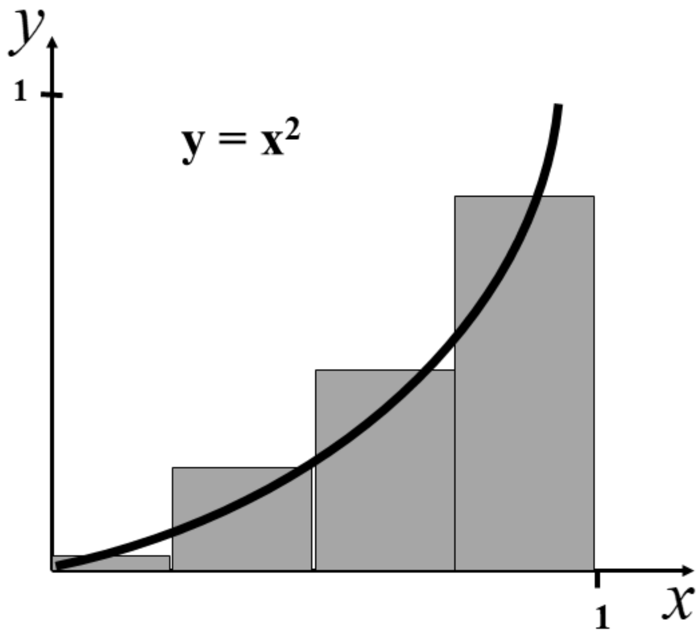


Figure 7.4: Graph of the Parabolic Function y = x^2. The curve represents the quadratic relationship between the independent variable x and the dependent variable y, forming a symmetric parabola centered at the origin.

$$x_i = \left(i-\frac{1}{2}\right)\frac{1.00}{n} = \frac{\left(i-\frac{1}{2}\right)}{n}$$

As, the function is, $y_i = x_i^2$, hence, $y_i = \frac{\left(i-\frac{1}{2}\right)^2}{n^2} \to 2$

Since, area under the curve is, $\sum_{i=1}^{n} y_i \Delta x, so$

$$\sum_{i=1}^{n} y_i \Delta x = \sum_{i=1}^{n} \frac{\left(i-\frac{1}{2}\right)^2}{n^2}.\frac{1.00}{n} = \sum_{i=1}^{n} \frac{\left(i-\frac{1}{2}\right)^2}{n^3}$$

Now lets solve R.H.S of the equation,

$$\sum_{i=1}^{n} \frac{\left(i-\frac{1}{2}\right)^2}{n^3} = \sum_{i=1}^{n} \frac{1}{n^3}\left(i^2+\frac{1}{4}-i\right) \Rightarrow = \frac{1}{n^3}\left[\sum_{i=1}^{n} i^2 + \frac{1}{4}\sum_{i=1}^{n} 1 - \sum_{i=1}^{n} i\right] \to 3$$

Using properties of summation,

$$\because \sum_{i=1}^{n} 1 = n, \because \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \because \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Now 3 equation becomes,

$$= \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6} + \frac{1}{4}n - \frac{n(n+1)}{2}\right] \Rightarrow = \frac{1}{n^3}\left[\frac{(n^2+n)(2n+1)}{6} + \frac{n}{4} - \frac{(n^2+n)}{2}\right]$$

$$= \frac{1}{n^3}\left[\frac{2n^3+n^2+2n^2+n}{6} + \frac{n}{4} - \frac{(n^2+n)}{2}\right] \Rightarrow = \frac{1}{n^3}\left[\frac{4n^3+2n^2+4n^2+2n+3n-6n^2-6n}{12}\right]$$

$$= \frac{1}{n^3}\left[\frac{4n^3-n}{12}\right] = \frac{n(4n^2-1)}{12n^3} = \frac{(4n^2-1)}{12n^2} = \frac{4n^2}{12n^2} - \frac{1}{12n^2} = \frac{1}{3} - \frac{1}{12n^2}$$

$$\sum_{i=1}^{n} \frac{\left(i-\frac{1}{2}\right)^2}{n^3} = \frac{1}{3} - \frac{1}{12n^2}$$

putting $n = 4$, $\sum_{i=1}^{n} \frac{\left(i-\frac{1}{2}\right)^2}{n^3} = 0.32812$

Now if we divide the parabola into 8 parts (n = 8) then the result will be: $\sum_{i=1}^{n} \frac{\left(i-\frac{1}{2}\right)^2}{n^3} = 0.33203$

And if n = 16, then, $\sum_{i=1}^{n} \frac{\left(i - \frac{1}{2}\right)^2}{n^3} = 0.33301$

And if n = 32, then, $\sum_{i=1}^{n} \frac{\left(i - \frac{1}{2}\right)^2}{n^3} = 0.33325$

Now if we divide the parabola into infinite parts (n = ∞) then the result will be:

$$\sum_{i=1}^{n} y_i \Delta x = \lim_{\Delta x \to 0} \sum_{i=1}^{n} y_i \Delta x = \int_0^1 x^2 dx = \lim_{n \to \infty} \left[\frac{1}{3} - \frac{1}{12n^2} \right] = \frac{1}{3} = 0.3333$$

In general, if we integrate a function $\int_0^1 x^n dx$ then the result would be $= \frac{1}{n+1}$. In this way we can calculate the total work done by a variable force.

Introduction to Energy:

In physics work and energy is of great importance. Energy is the capacity of a physical system to do work.

- It comes in many forms – mechanical, electrical, chemical, nuclear, etc.
- It can be stored
- It can be converted into different forms
- It can never be created or destroyed

Types of Energy:

1) **Elastic energy:**

   Elastic energy is stored in an elastic object - such as a coiled spring or a stretched elastic band. Elastic objects store elastic energy when a force causes them to be stretched.

2) **Gravitational energy:**

   Gravitational energy associated with gravity or gravitational force. In other words, the energy held by an object when it is in a high position compared to a lower position.

3) **Electrical energy:**

   Electrical energy is the movement of electrons. Lightning is an example of electrical energy.

4) **Chemical energy:**

Chemical energy is stored in the bonds of atoms and molecules. It is the energy that holds these particles together. Stored chemical energy is found in food, biomass, petroleum, and natural gas.

5) **Thermal energy:**

Thermal energy is created from the vibration of atoms and molecules within substances. The faster they move, the more energy they possess and the hotter they become. Thermal energy is also called heat energy.

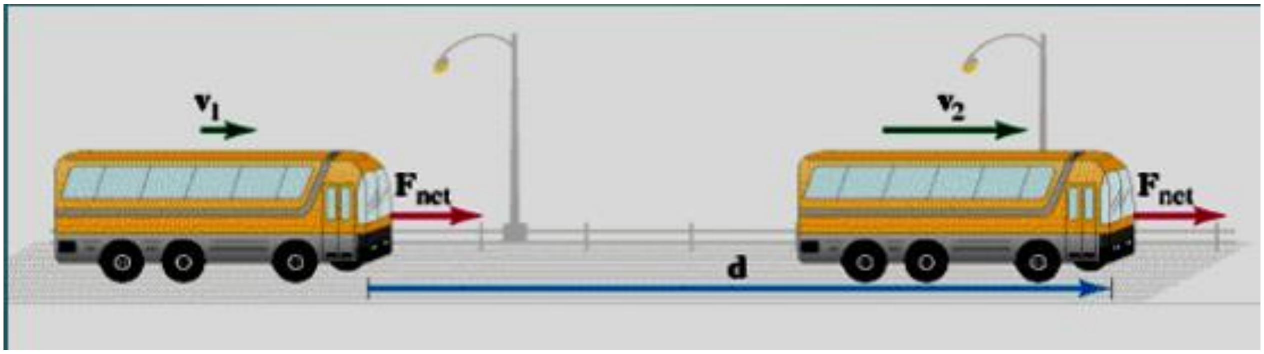
6) **Nuclear energy:**

Nuclear energy is stored in the nucleus of atoms. This energy is released when the nuclei are combined (fusion) or split apart (fission). Nuclear power plants split the nuclei of uranium atoms to produce electricity.

7) **Sound energy:**

Sound energy is the energy produced due to vibrations. There is usually much less energy in sound than in other forms of energy.

Problem 4: A constant force accelerates a bus (mass m) from speed v_1 to speed v_2 over a distance d. What work is done by the engine?



Solution:

Recall: $v_2^2 - v_1^2 = 2a(x_2 - x_1)$

where: v_2 = final velocity, x_2 = final position

v_1 = initial velocity, x_1 = initial position

$$\therefore \quad a = \frac{v_2^2 - v_1^2}{2d}$$

To calculate work:

$W = F\, d$

$= ma\, d$

$= m\, \dfrac{v_2^2 - v_1^2}{2d}\, d$

$= \frac{1}{2} m{v_2}^2 - \frac{1}{2} m{v_1}^2$

Define Kinetic Energy:

$KE = \frac{1}{2} mv^2$

so, work done is equal to the change in kinetic energy.

$W = \Delta K.E$

Problem 5: A truck weighs 20 times more than a rickshaw but moves 5 times slower. Which has more kinetic energy?

Solution:

KE (rickshaw) = $\frac{1}{2} mv^2$

KE (truck) = $\frac{1}{2}(20m)(v/5)^2$

$= \frac{20}{25} \cdot \frac{1}{2} mv^2$

As, the speed of truck is 5 times less than the rickshaw, hence, the kinetic energy of truck is less than the rickshaw.

Work Energy Principle:

Net work done on object = Change in *K.E* of object

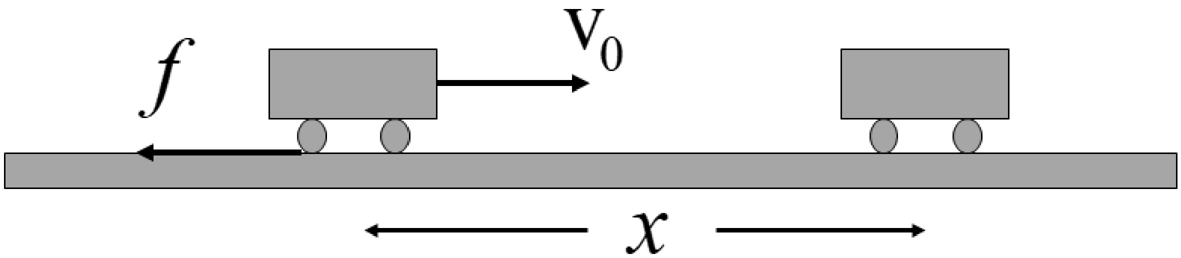
Work can be:

- Positive (K.E increases)
- Negative (K.E decreases)

Units of energy: Energy has the same units as work:

Joule = Newton. Meter = Nm

Problem 6: A car is moving on a road, and a frictional force acts on it. We want to know how far the car travel before it comes to rest.



Solution: According to Work Energy Principle,

$$W_{net} = K_f - K_i = 0 - K_i$$

$$-f\,x = -\frac{1}{2}m\mathrm{v}_0^{\ 2} \quad \text{but } f = \mu N = \mu mg$$

k.E consumed by the frictional work is, $f\,x = \frac{1}{2}m\mathrm{v}_0^{\ 2}$

$$\mu mgx = \frac{1}{2}m\mathrm{v}_0^{\ 2} \quad \Rightarrow \quad x = \frac{\mathrm{v}_0^{\ 2}}{2\mu \mathrm{g}}$$

Checking:

- If $\mu = 0$, then x = ∞, which means, in the absence of friction, car does not stop and move through an infinite distance.
- If $\mu = \infty$, then x = 0, which means, car would not move or cover a distance because of infinite friction.

Introduction to power:

The work done by a force is just the force multiplied by the distance – it does not depend upon time. But suppose that the same amount of work is done in half the time. We then say that the power is twice as much.

Power is the “rate of doing work”

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

If the force does not depend on time:

$$\frac{\text{Work}}{\text{Time}} = \frac{F\,\Delta x}{\Delta \text{t}} = F\text{ v}$$

$\therefore$ Power = F v

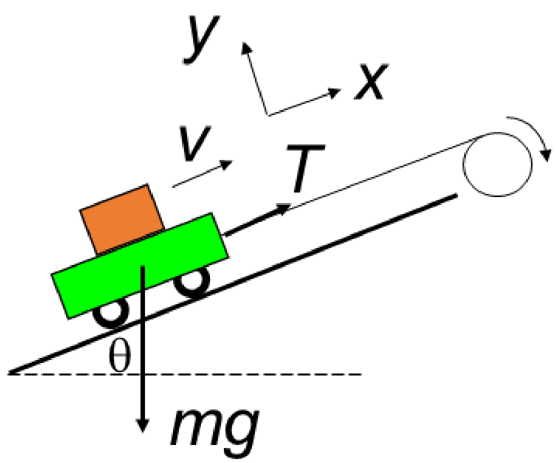
Units of power: J/sec = Watts

Old units: horsepower (hp)

1 hp = 746 W = 0.746 kW

Problem 7: A 2000 kg trolley is pulled up a 30° hill at 20 mi/hr by a rope. How much power is the machine providing?

Solution:



- The power is $P = F\,v = T\,v$
- No acceleration $\Rightarrow$ no net force
- Balance forces along and normal to plane

  In the x direction: $T = mg\sin\theta$

  v = 20 mi/hr = 8.93 m/s

  g = 9.8 m/s^2

  m = 2000 kg

  Sinθ = sin(30^o) = 0.5

  P = (2000 kg) . (9.8 m/s^2) (8.93 m/s) (0.5)

  P = 88,000 W (power of machine) = 88 kw