Physics

Kinematics

- Displacement
- Velocity
- Acceleration

- ***Constant*** **acceleration**
- **Vectors**

Position of a body at time t is denoted as $x(t)$

x is called a function of t

Displacement

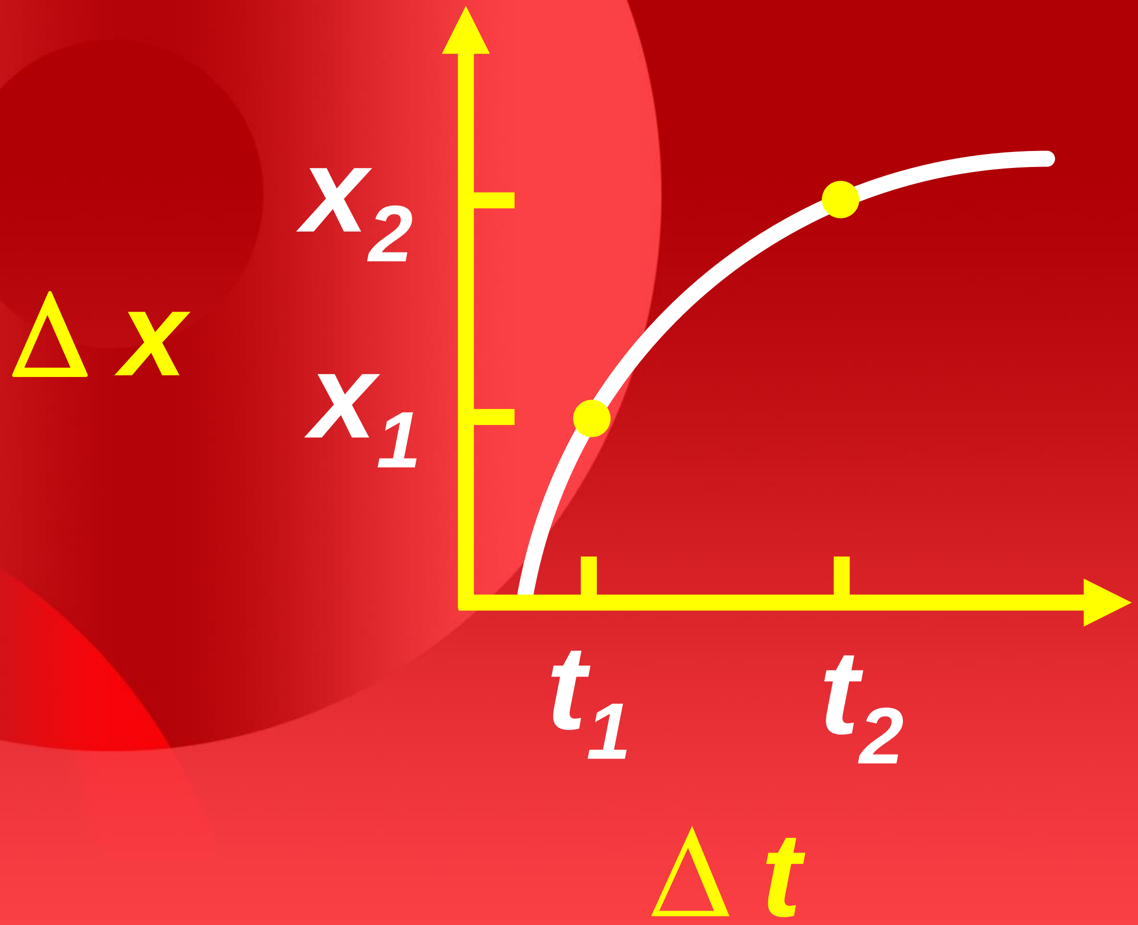
Position at time t_1 is denoted as $x(t_1)$

Position at time t_2 is denoted as $x(t_2)$

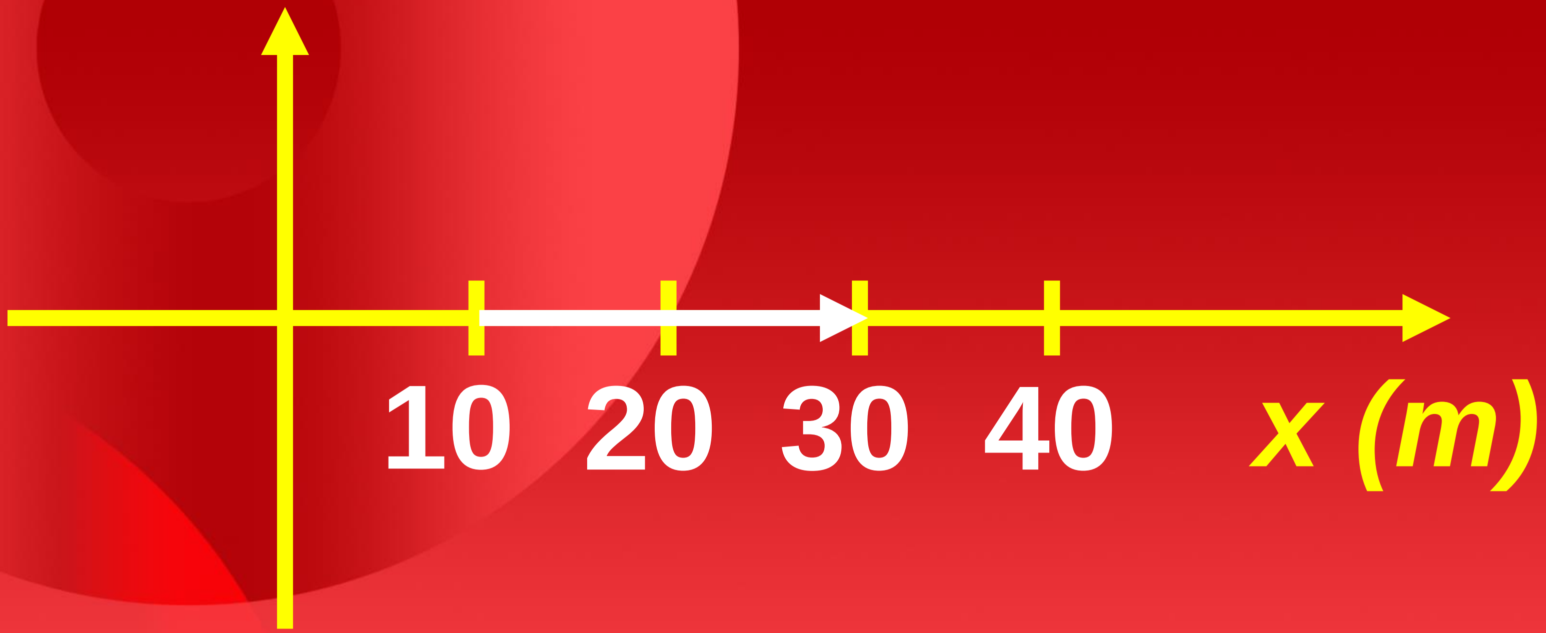
The displacement Δx in time interval $\Delta t = t_2 - t_1$ is:

$$\Delta x \equiv x(t_2) - x(t_1) = x_2 - x_1$$

Imagine a graph of a particle's position along the x-axis as a function of time...



x_2
x_1
Δx
t_1
t_2
Δt

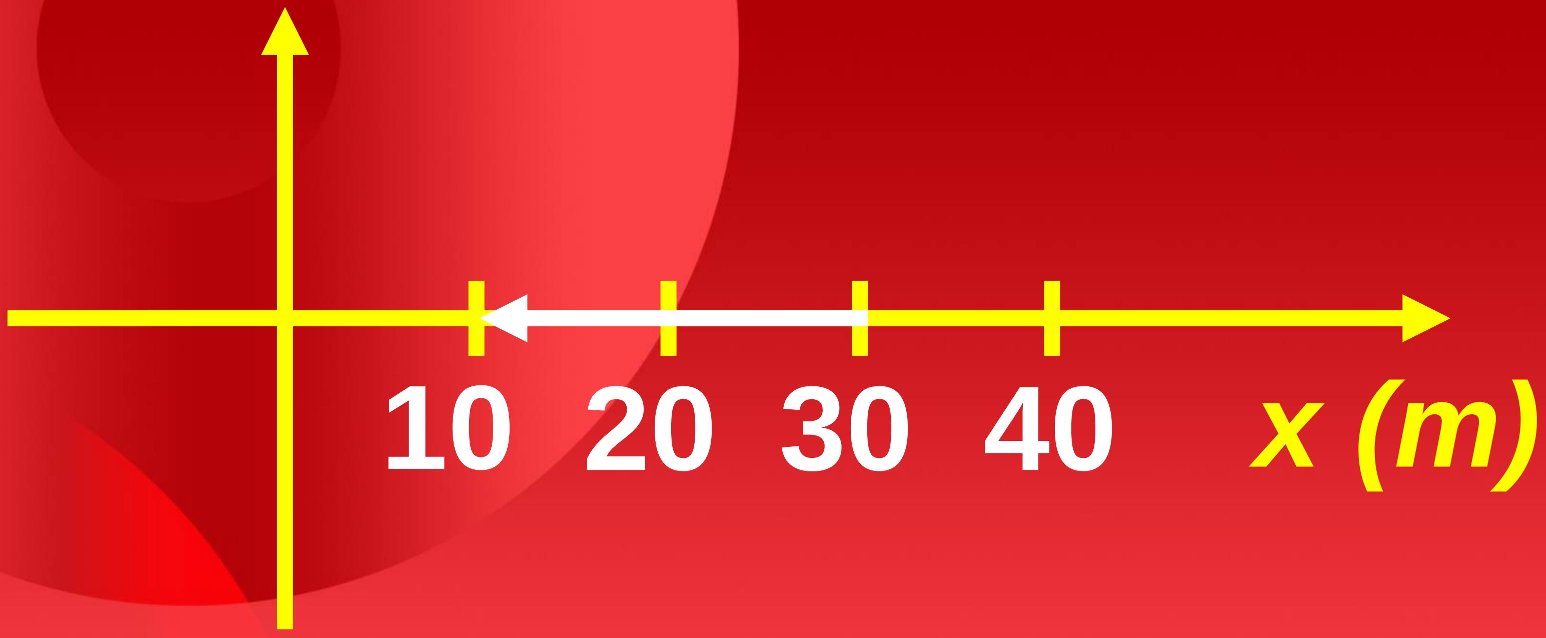


10
20
30
40
x (m)

$$\Delta x = x_2 - x_1$$

$$= 30\text{m} - 10\text{m}$$

$$= +20\text{ m}$$


10 20 30 40
x (m)

$$\Delta x = x_2 - x_1$$
$$= 10\text{ m} - 30\text{ m}$$
$$= -20\text{ m}$$

Notice that the sign indicates the direction!

Speed And Velocity

speed and velocity measure how position *changes* with time

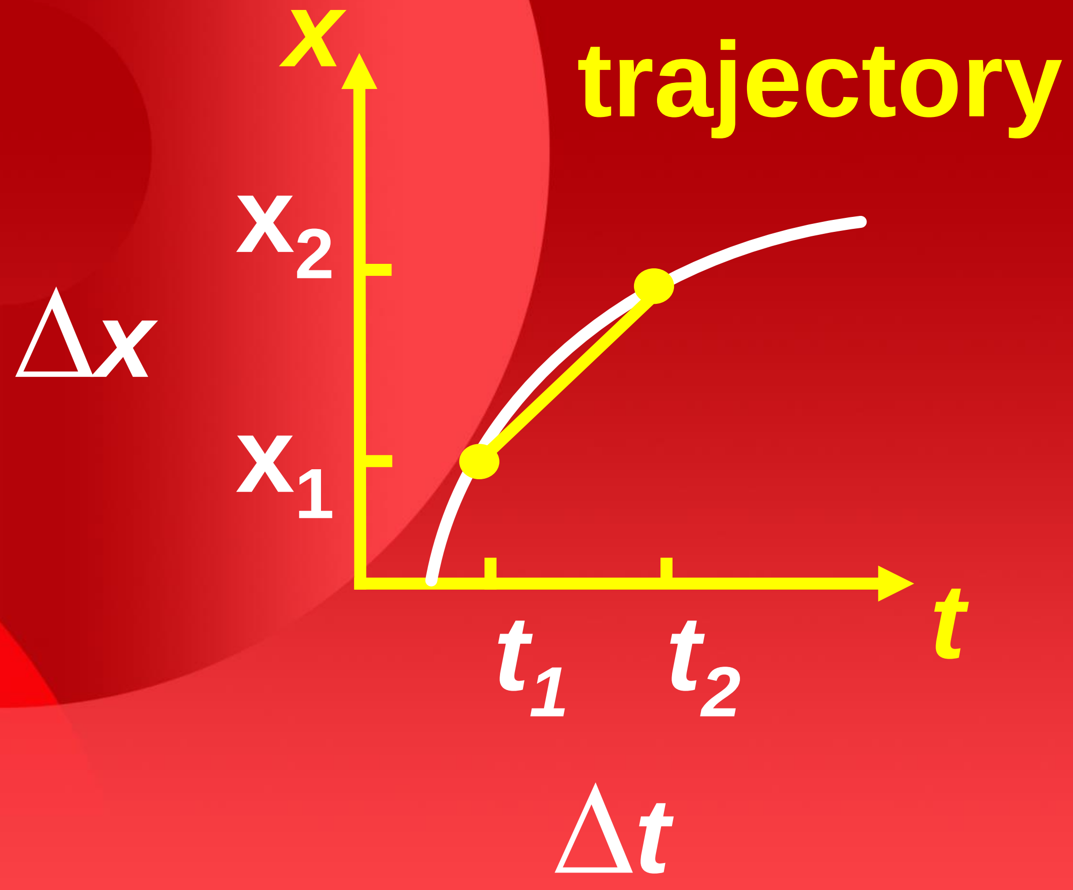
$$\text{average speed} = \frac{\text{total distance traveled}}{\text{total time}}$$

$$\text{average velocity} = \frac{\text{displacement}}{\text{total time}}$$

Average velocity

$$= \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

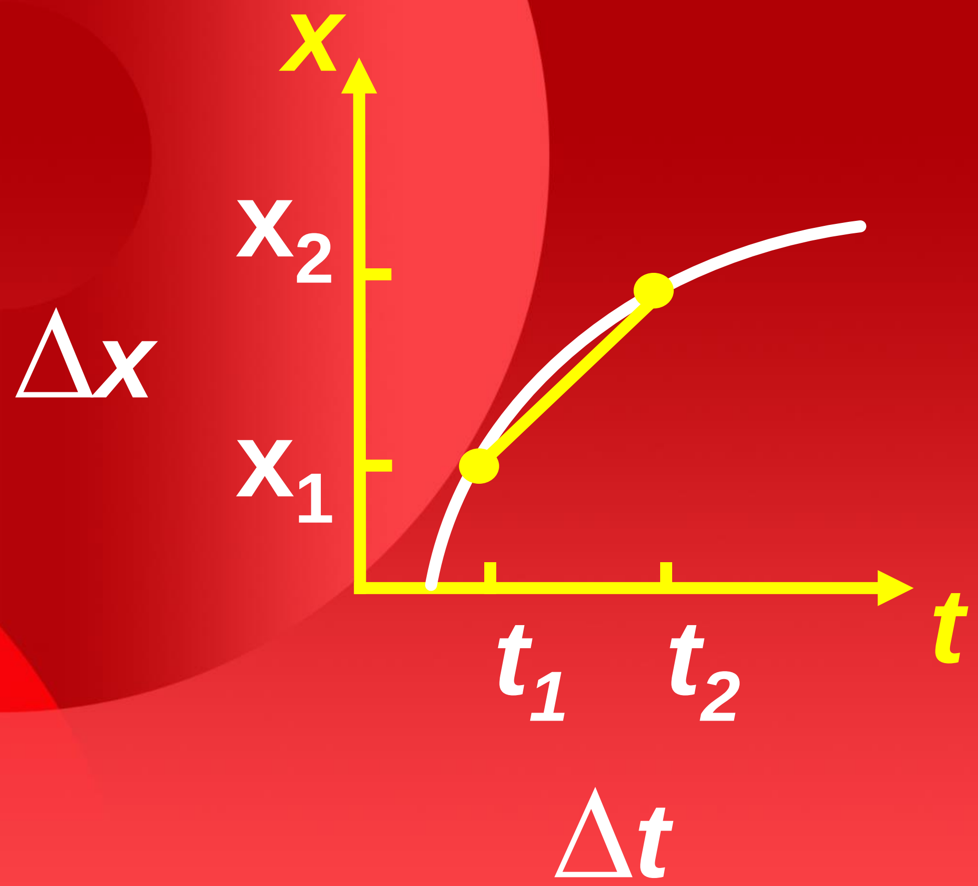
formula for a slope!



x
trajectory
x_2
Δx
x_1
t
t_1
t_2
Δt

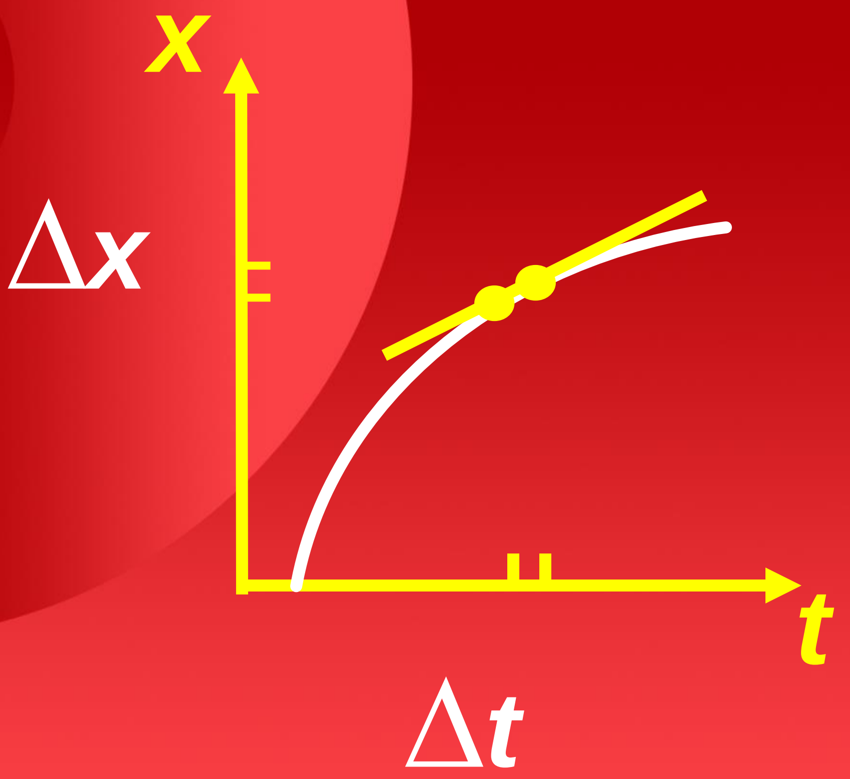
v_{av} = slope connecting line from t_1 to t_2

Instantaneous velocity is the velocity at a particular time


x
x_2
Δx
x_1
t
t_1
t_2
Δt

Instantaneous velocity

Take two times very close to each other so Δt is very small


x
Δx
t
Δt

Acceleration

Acceleration measures how the velocity *changes* with time

average acceleration = $\dfrac{\text{change in velocity}}{\text{time taken}}$

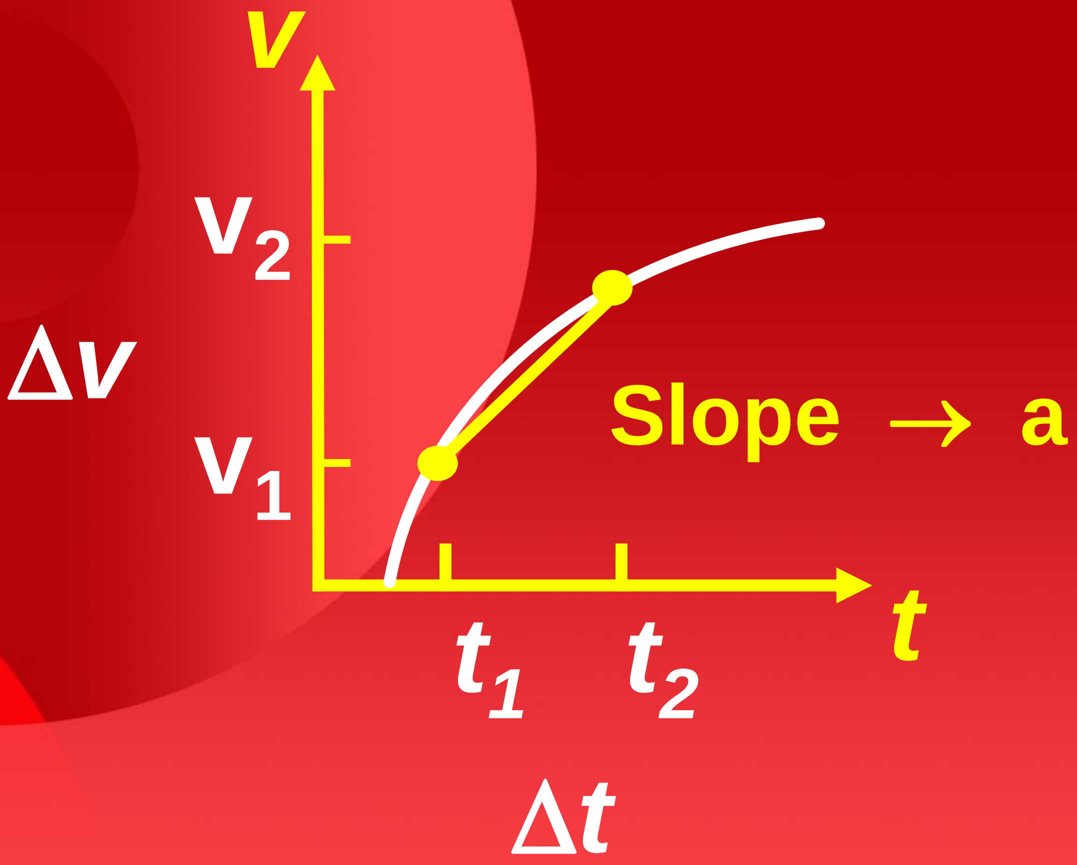
Average acceleration

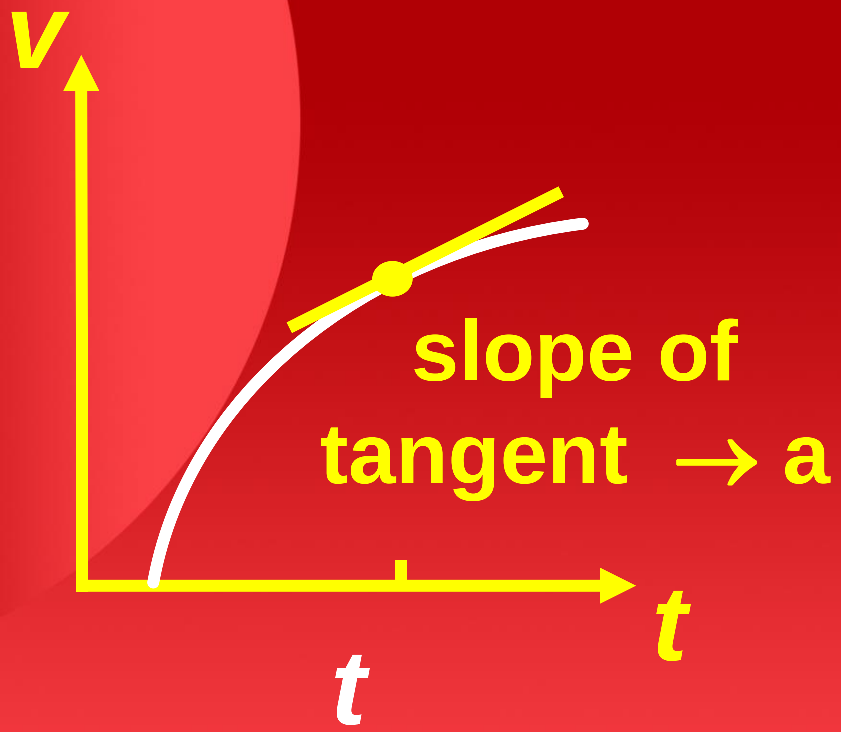
$$= \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

formula for a slope!

Instantaneous acceleration:
is the acceleration at a specific instant of time:

$$\boxed{a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}}$$


v
v2
Δv
v1
Slope → a
t
t1
t2
Δt



v
slope of
tangent → a
t
t

Note that acceleration *a* does *not* have to be in the same direction as velocity *v* !!

Acceleration = $-2 m/s^2$

at $t_1=0$, $v_1=15 m/s$

at t_2=5s,
v_2=5m/s

Constant Acceleration

Take for convenience

$t_1 = 0$ and $t_2 = t$

then:

$$x_1 = x_o \quad \text{and } x_2 = x$$

$$v_1 = v_o \quad \text{and } v_2 = v$$

If acceleration is constant, then $a = a_{av}$

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

$$\Downarrow$$

$$\boxed{v = v_o + at}$$

Definition of average

$$\Downarrow$$

$$v_{av} = \tfrac{1}{2}(v_o + v)$$

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\Downarrow$$

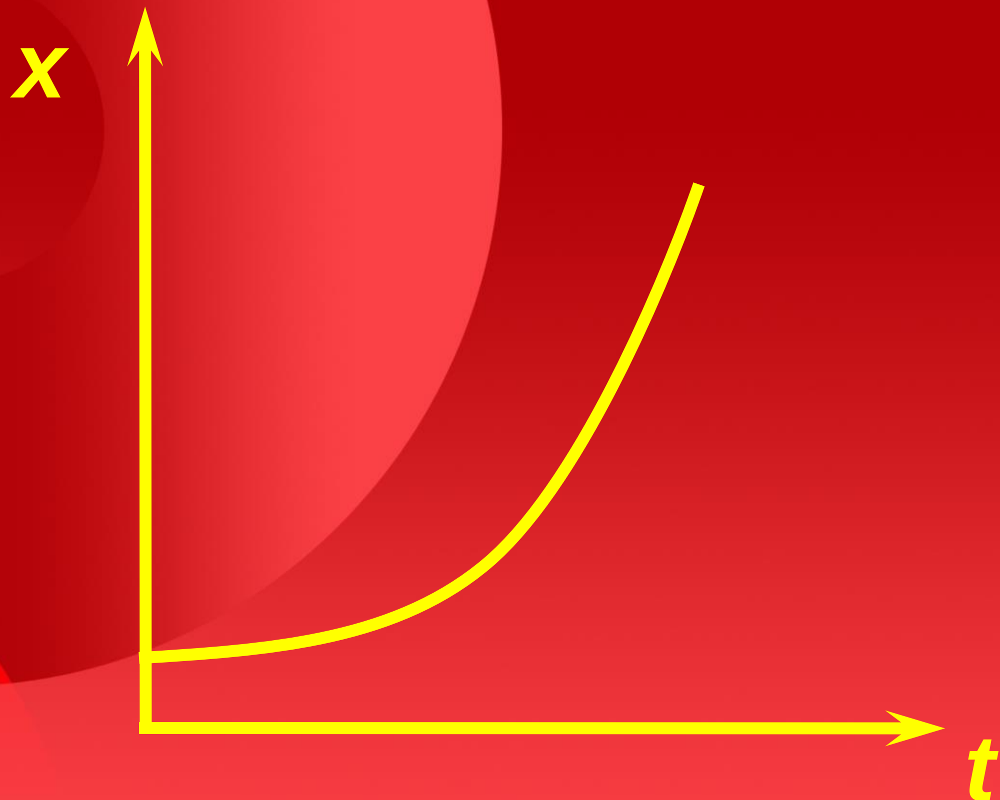
$$\boxed{x = x_o + v_{av}t}$$

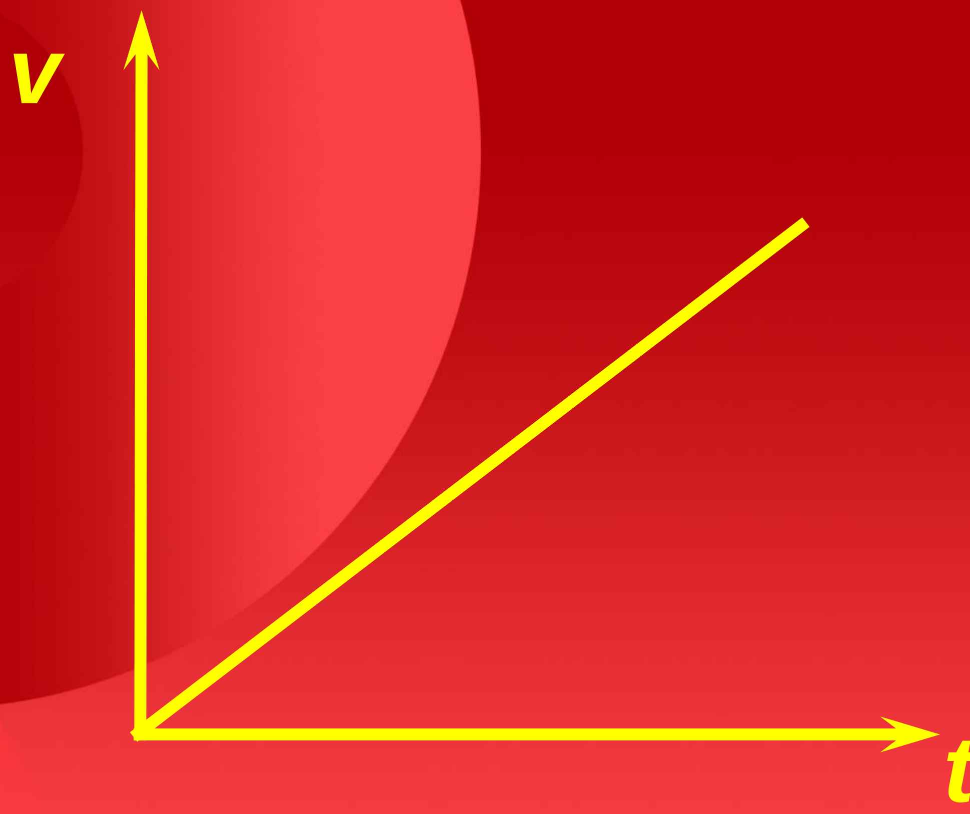
$$\boxed{x = x_o + v_o t + \tfrac{1}{2}at^2}$$

Two Main Equations

$$x = x_o + v_o t + \tfrac{1}{2}at^2$$

$$v = v_o + at$$


x
t


v
t

From these two:

$$x = x_o + v_o t + \tfrac{1}{2}at^2$$

$$v = v_o + at$$

$$v^2 = v_o^2 + 2a(x - x_o)$$

(v as a function of x)

Vectors

- **Vectors have magnitude and direction**
- **In 1-D, direction has a + or – sign**

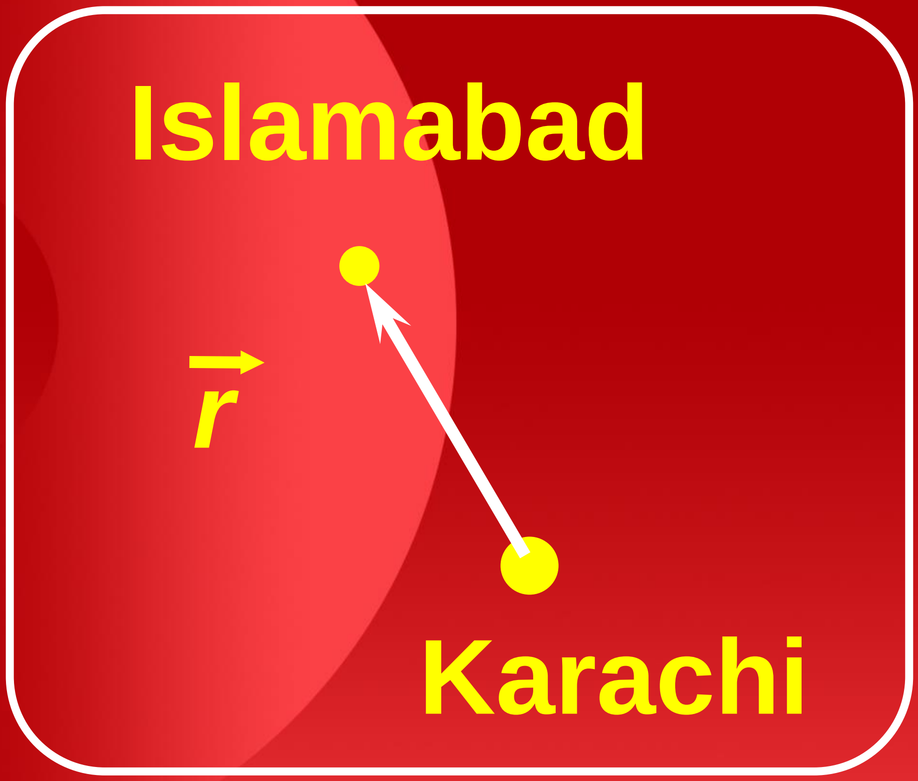
- Consider the ***position vector*** $\vec{r}$ in **2** dimensions

Example:

Where is Islamabad?

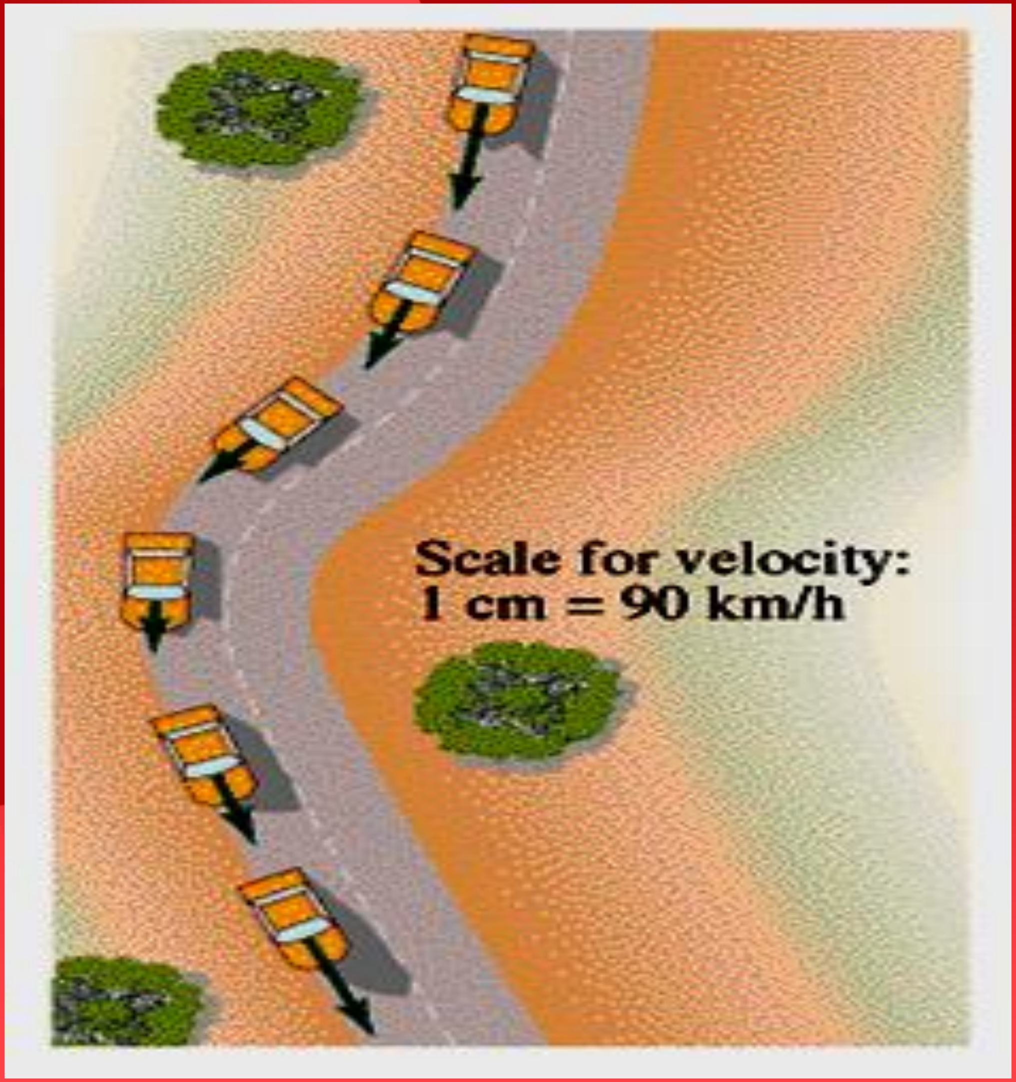
- Choose origin at **Karachi**

- Choose coordinates of distance **(km)** and direction **(N,S,E,W)**



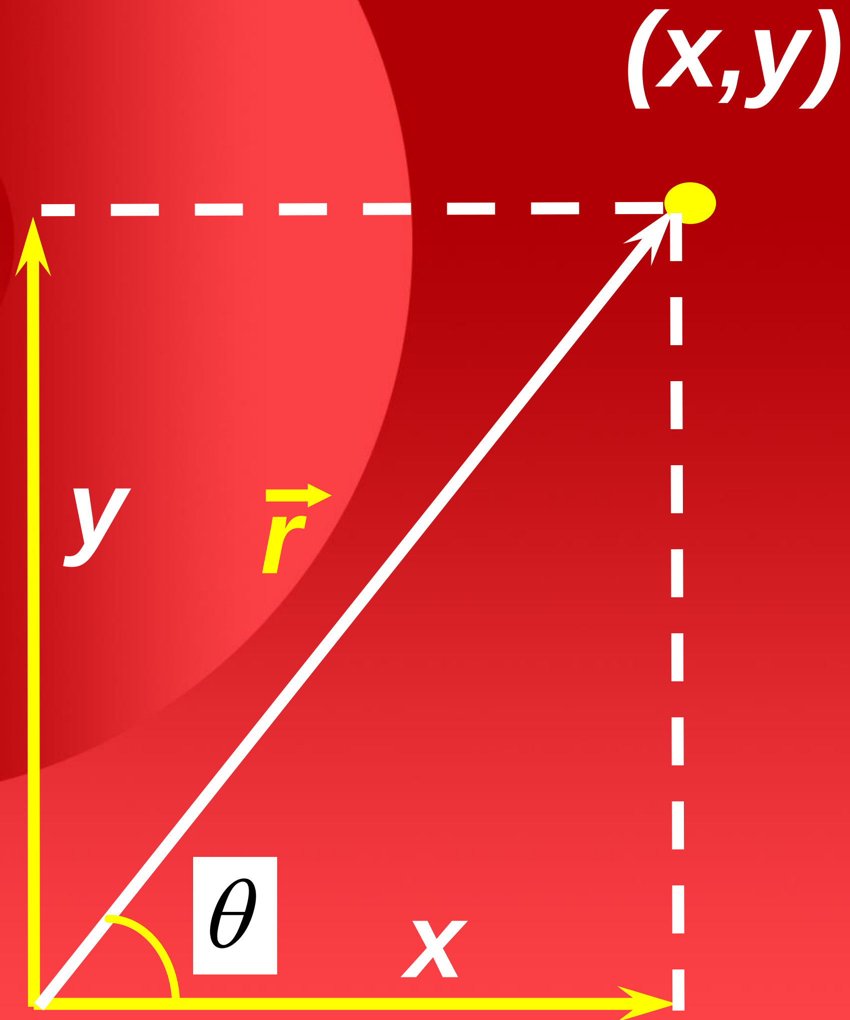
- **Vector $\vec{r}$ points 1200 km north west**

Velocity Vector


Scale for velocity:
1 cm = 90 km/h

- **Components of $\vec{r}$ are its (x,y) coordinates**

- $\vec{r} = (r_x, r_y) = (x,y)$



(x,y)
y
$\vec{r}$
θ
x

- Components can be expressed as:

$$r_x = x = r\cos\theta$$

$$r_y = y = r\sin\theta$$

$$\textit{where } r = |\vec{r}| \textit{ and}$$

$$\theta = \arctan(y / x)$$

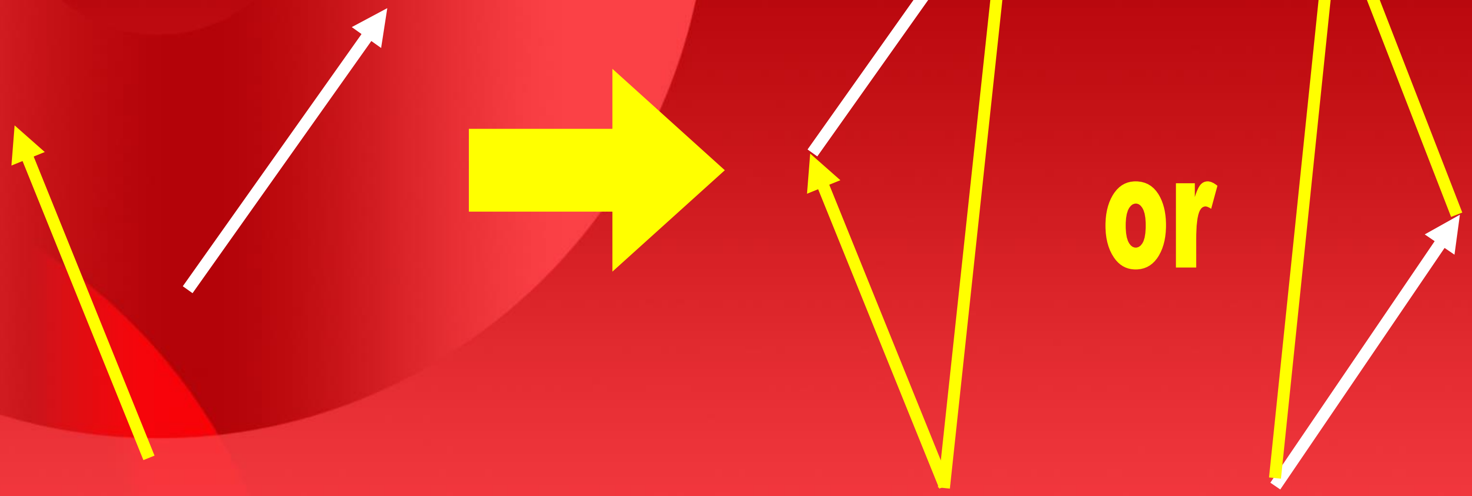
- **Magnitude (length) of $\vec{r}$ is found by Pythagorean theorem:**

$$\left|\vec{r}\right| = r = \sqrt{x^2 + y^2}$$

The length of a vector does not depend on its direction.

Vector Addition

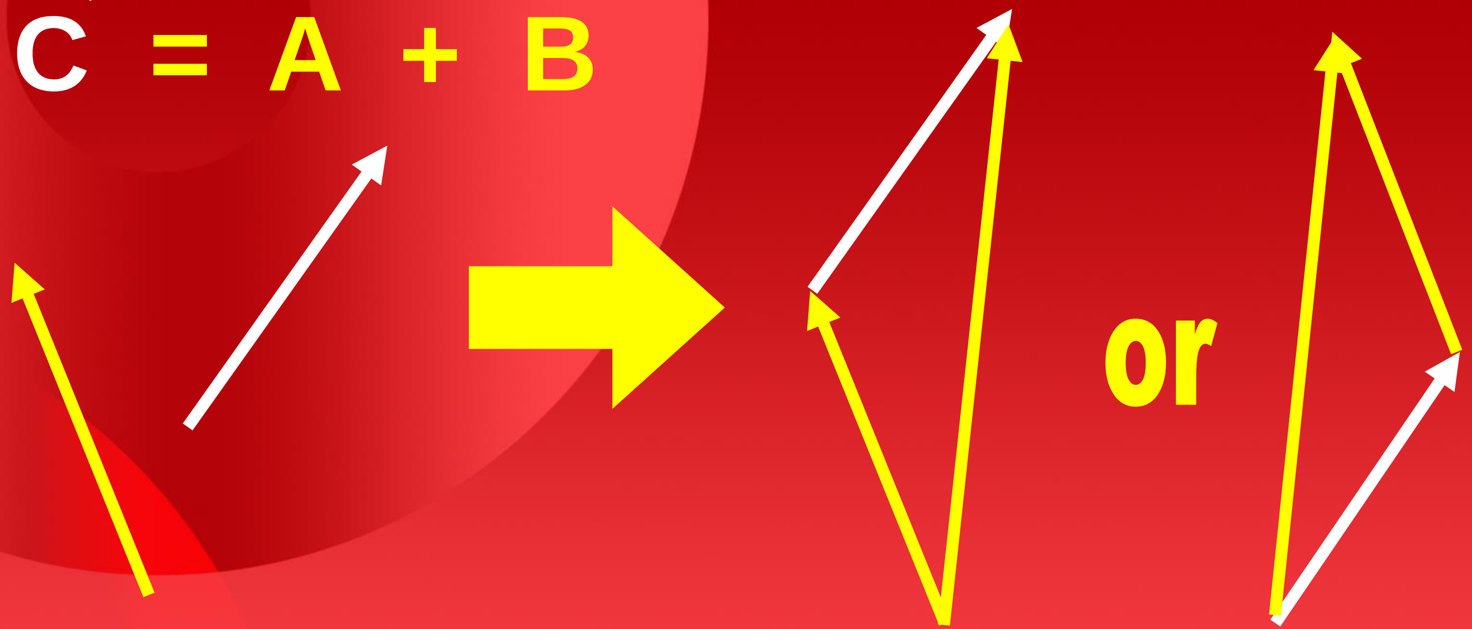
$$\vec{C} = \vec{A} + \vec{B}$$



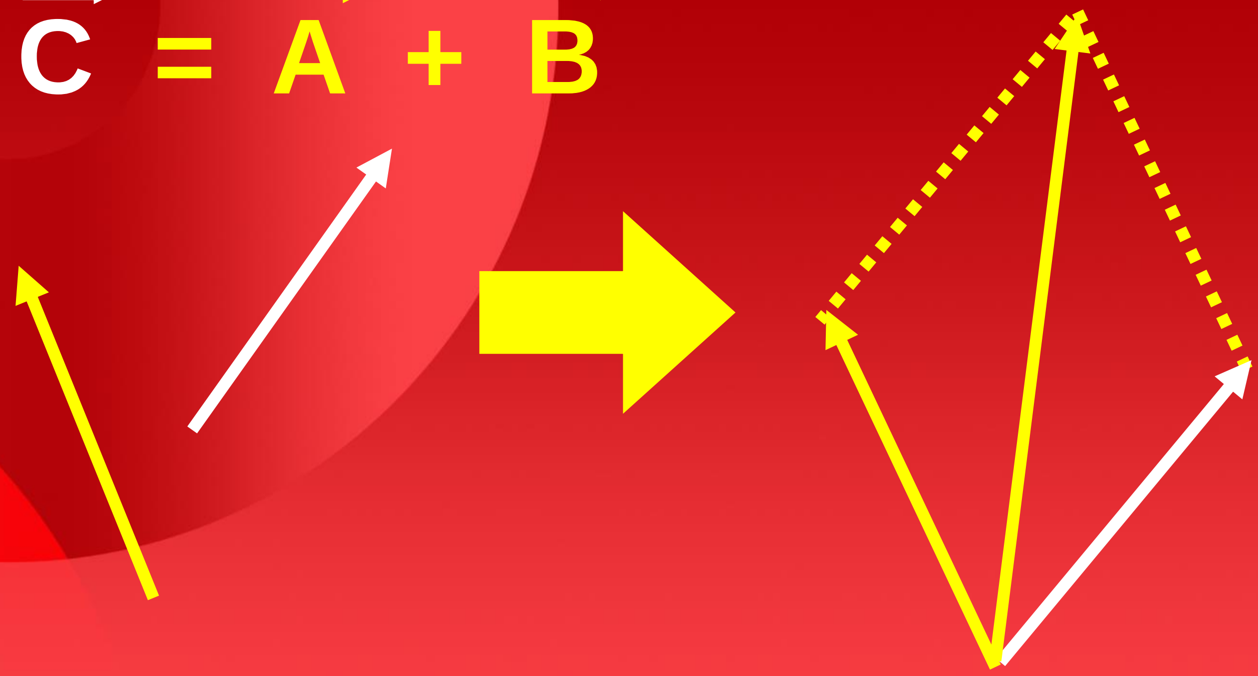
We can arrange the vectors any way we want, as long as we maintain their length and direction !

Parallelogram Method

$$\vec{C} = \vec{A} + \vec{B}$$



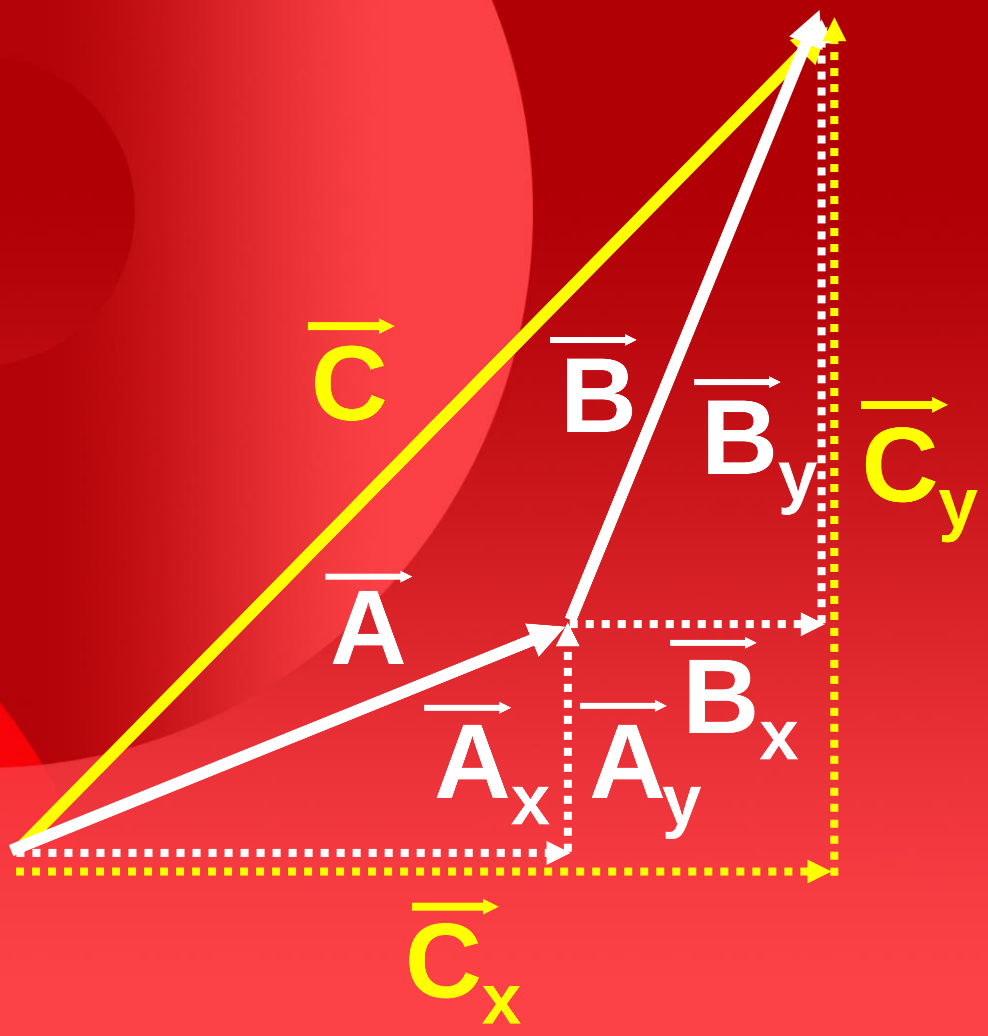
$$\vec{C} = \vec{A} + \vec{B}$$



Vector Addition

COMPONENT METHOD

If $\vec{C} = \vec{A} + \vec{B}$



$\vec{C}$
$\vec{B}$
$\vec{B}_y$
$\vec{C}_y$
$\vec{A}$
$\vec{B}_x$
$\vec{A}_x$
$\vec{A}_y$
$\vec{C}_x$

then $\vec{C}_x = \vec{A}_x + \vec{B}_x$

$$\vec{C}_y = \vec{A}_y + \vec{B}_y$$

Vector addition:

1. **add components**
2. **then use**

$$C = \sqrt{C_x^{\ 2} + C_y^{\ 2}}$$

3. **and the angle is**

$$\theta = \arctan(C_y / C_x)$$