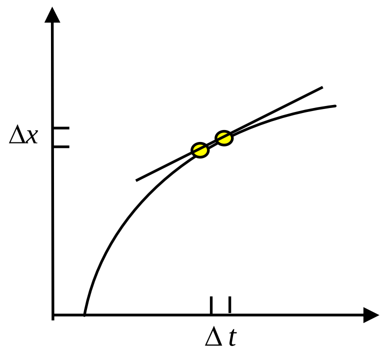
Summary of Lecture 3 – KINEMATICS II

1. The concept of the derivative of a function is exceedingly important. The derivative shows how fast a function changes when its argument is changed. (Remember that for $f(x)$ we say that f is a function that depends upon the argument x. You should think of f as a machine that gives you the value f when you input x.)

2. Functions do not always have to be written as $f(x)$. $x(t)$ is also a function. It tells us where a body is at different times t.

3. The derivative of $x(t)$ at time t is defined as:

$$\frac{dx}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}.$$



4. Let's see how to calculate the derivative of a simple function like $x(t) = t^2$. We must first calculate the difference in x at two slightly different values, t and $t + \Delta t$, while remembering that we choose Δt to be extremely small:

$$\Delta x = \left(t + \Delta t\right)^2 - t^2$$
$$= t^2 + \left(\Delta t\right)^2 + 2t\Delta t - t^2$$
$$\frac{\Delta x}{\Delta t} = \Delta t + 2t \Rightarrow \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = 2$$

5. In exactly the same way you can show that if $x(t) = t^n$ then:

$$\frac{dx}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = nt^{n-1}$$

This is an extremely useful result.

6. Let us apply the above to the function $x(t)$ which represents the distance moved by a body with constant acceleration (see lecture 2):

$$x(t) = x_0 + \mathrm{v}_0 t + \frac{1}{2}at^2$$

$$\frac{dx}{dt} = 0 + \mathrm{v}_0 + \frac{1}{2}a(2t) = \mathrm{v}_0 + a\,t$$

This clearly shows that $\frac{d\mathrm{v}}{dt} = 0 + a = a$ (acceleration is constant)

7. A stone dropped from rest increases its speed in the downward direction according to $\frac{d\mathrm{v}}{dt} = g \approx 9.8$ m/sec. This is true provided we are fairly close to the earth, otherwise the value of g decreases as we go further away from the earth. Also, note that if we measured distances from the ground up, then the acceleration would be negative.

8. A useful notation: write $\frac{d\mathrm{v}}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$. We call $\frac{d^2x}{dt^2}$ the second derivative of x with respect to t, or the rate of rate of change of x with respect to t.

9. It is easy to extend these ideas to a body moving in both the x and y directions. The position and velocity in 2 dimensions are:

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j}$$

$$\vec{\mathrm{v}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$= \mathrm{v}_x\hat{i} + \mathrm{v}_y\hat{j}$$

Here the unit vectors $\hat{i}$ and $\hat{j}$ are fixed, meaning that they do not depend upon time.

10. The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as:

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

You can think of:

$$\vec{A}\cdot\vec{B} = (A)(B\cos\theta)$$

$$= (\text{length of } \vec{A}) \times (\text{projection of } \vec{B} \text{ on } \vec{A})$$

OR,

$$\vec{A}\cdot\vec{B} = (B)(A\cos\theta)$$

$$= (\text{length of } \vec{B}) \times (\text{projection of } \vec{A} \text{ on } \vec{B}).$$

Remember that for unit vectors $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = 1$ and $\hat{i}\cdot\hat{j} = 0$.