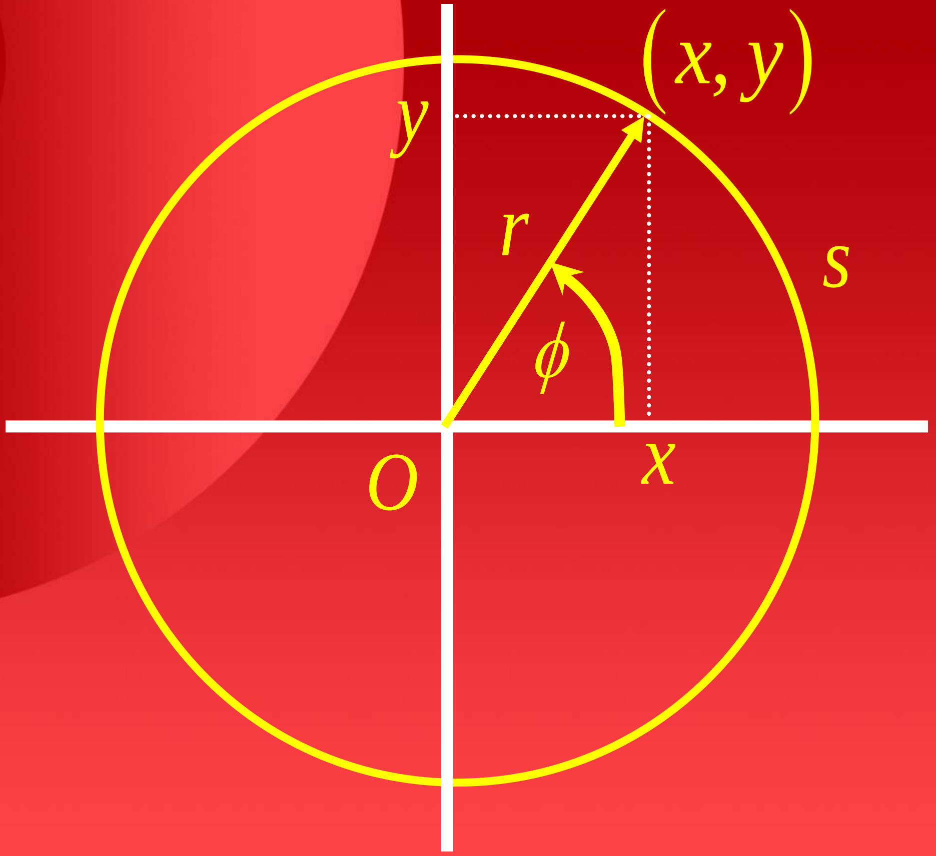
Physics

Rotational Kinematics

- Rotation involves angles
- Polar coordinates are more natural



arc length = radius × angular displacement

$$\boxed{s = r\phi}$$

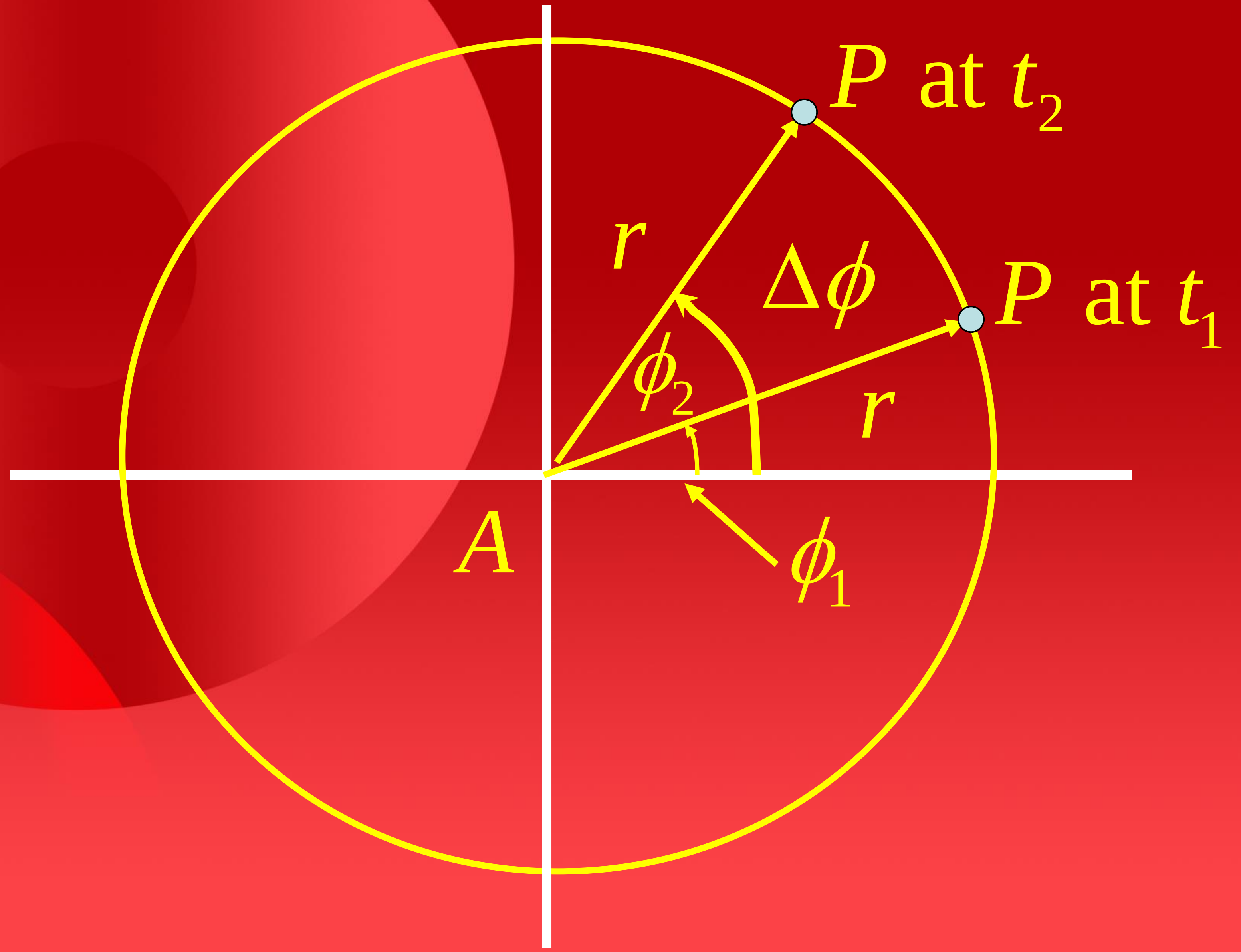
$$\text{one revolution} = 2\pi \text{ radians}$$

$$= 360 \text{ degrees}$$

$$1 \text{ radian} = 57.3^0$$

$$1 \text{ radian} = 0.159 \text{ revolution}$$

$$s = 2\pi r = \text{total circumference}$$

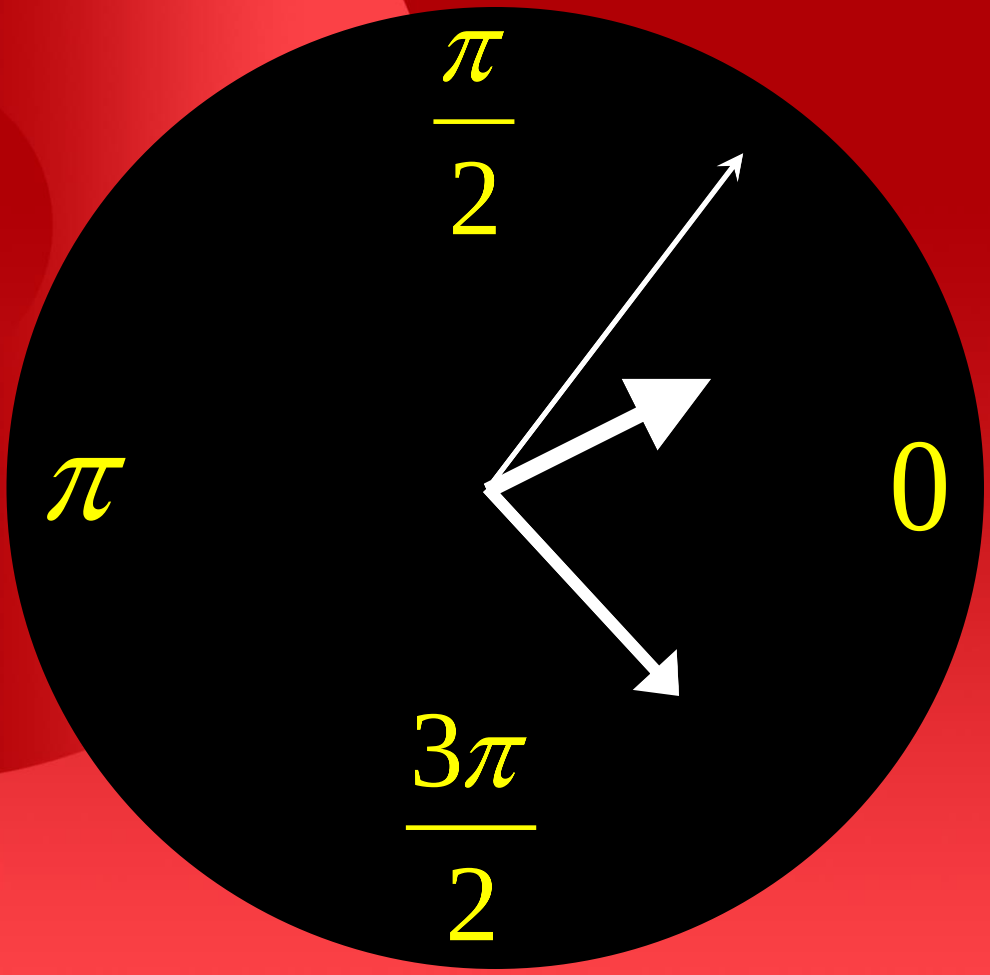


P at t_2
P at t_1
r
r
$\Delta\phi$
ϕ_2
ϕ_1
A

$$\bar{\omega} = \frac{\phi_2 - \phi_1}{t_2 - t_1} = \frac{\Delta\phi}{\Delta t}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\phi}{\Delta t}$$

$$\omega = \frac{d\phi}{dt}$$ Angular speed !



$\frac{\pi}{2}$
π
0
$\frac{3\pi}{2}$

$$\omega = \frac{2\pi}{T}$$

$$\omega_{\text{second}} = \frac{2\pi}{60} = 0.105\, rad / s$$

$$\omega_{\text{minute}} = \frac{2\pi}{60 \times 60} = 1.75 \times 10^{-3}\, rad / s$$

$$\omega_{\text{hour}} = \frac{2\pi}{60 \times 60 \times 12} = 1.45 \times 10^{-4}\, rad / s$$

Our sun is 2.3×10^4 light years away from the centre of our Milky Way galaxy. It moves in a circle around this centre at 250 km/s.

(a) How long does it take the sun to make one revolution about the galactic center?

(b) How many revolutions has the sun completed since it was formed about 4.5×10^9 years ago?

$$\text{a) 1 Light Year} = 9.46 \times 10^{15}\, m$$

$$\text{v} = R\omega = R\frac{\theta}{t} = R\frac{2\pi}{T}$$

$$\therefore \text{ for one revolution } T = \frac{2\pi R}{\text{v}}$$

$$T = 5.5 \times 10^{15}\, s = 1.74 \times 10^{8}\text{ years}$$

$$b)\ \frac{4.5 \times 10^{9}}{1.74 \times 10^{8}} = 26\text{ revolutions}$$

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\frac{d\phi}{dt} = \frac{d^2\phi}{dt^2}$$

Angular acceleration !

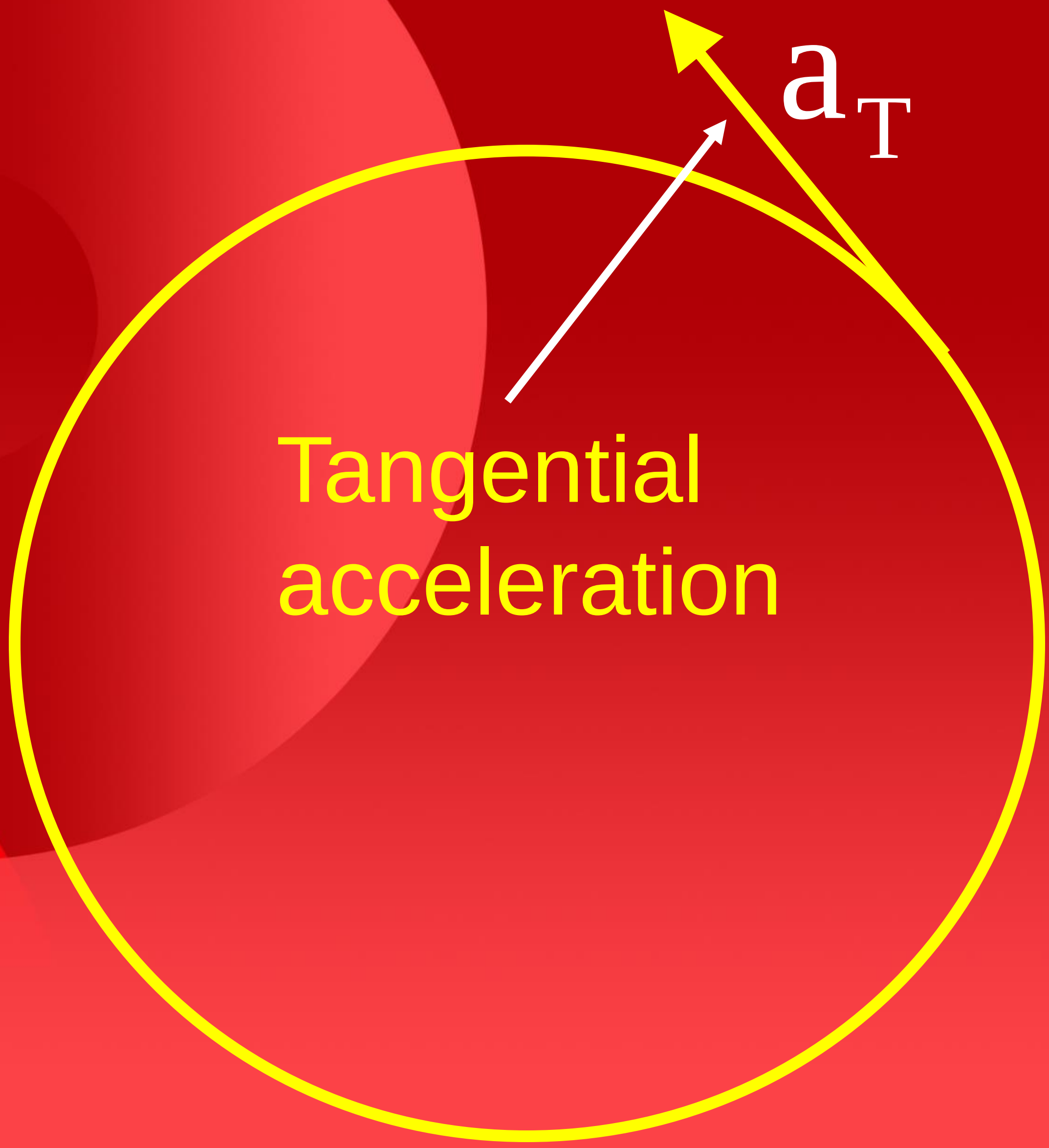
$$s = r\phi$$

$$\frac{ds}{dt} = r\frac{d\phi}{dt}$$

$$\boxed{\mathrm{v} = r\omega}$$

$$\frac{d\mathrm{v}}{dt} = r\frac{d\omega}{dt}$$

$$\boxed{\mathrm{a}_{\mathrm{T}} = r\alpha}$$



a T
Tangential acceleration

Relationship between linear and angular variables

Translational Motion	Rotational Motion
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2} at^2$	$\phi = \phi_0 + \omega_0 t + \frac{1}{2} \alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\phi - \phi_0)$

A point on the rim of a 0.75-m diameter grinding wheel changes speed from 12 m/s to 25 m/s in 6.2 s. What is the angular acceleration during this interval?

$$\mathrm{a} = \frac{\mathrm{v}_f - \mathrm{v}_i}{t} = 2.1\,\mathrm{m/s^2}$$

$$\mathrm{a} = r\alpha$$

$$\alpha = \frac{\mathrm{a}}{r} = 5.6\,\mathrm{rad/s^2}$$

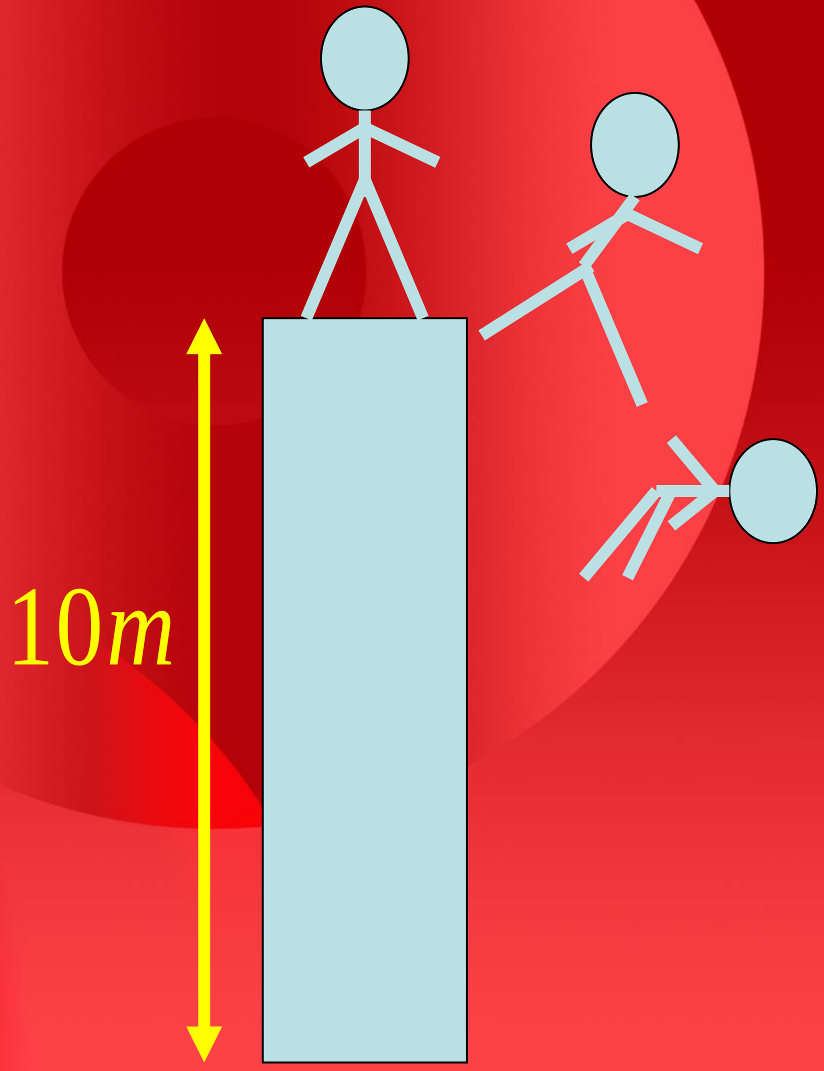
The angular speed of a car engine is increased from 1170 rev/min to 2880 rev/min in 12.6 s.

(a) Find the average angular acceleration in rev/ min^2.

(b) How many revolutions does the engine make during this time?

$$\alpha = \frac{\omega_f - \omega_i}{t} = 8140\,\mathrm{rev/min}^2$$

$$\phi = \omega_i t + \frac{1}{2}\alpha t^2 = 425\,\mathrm{rev}$$



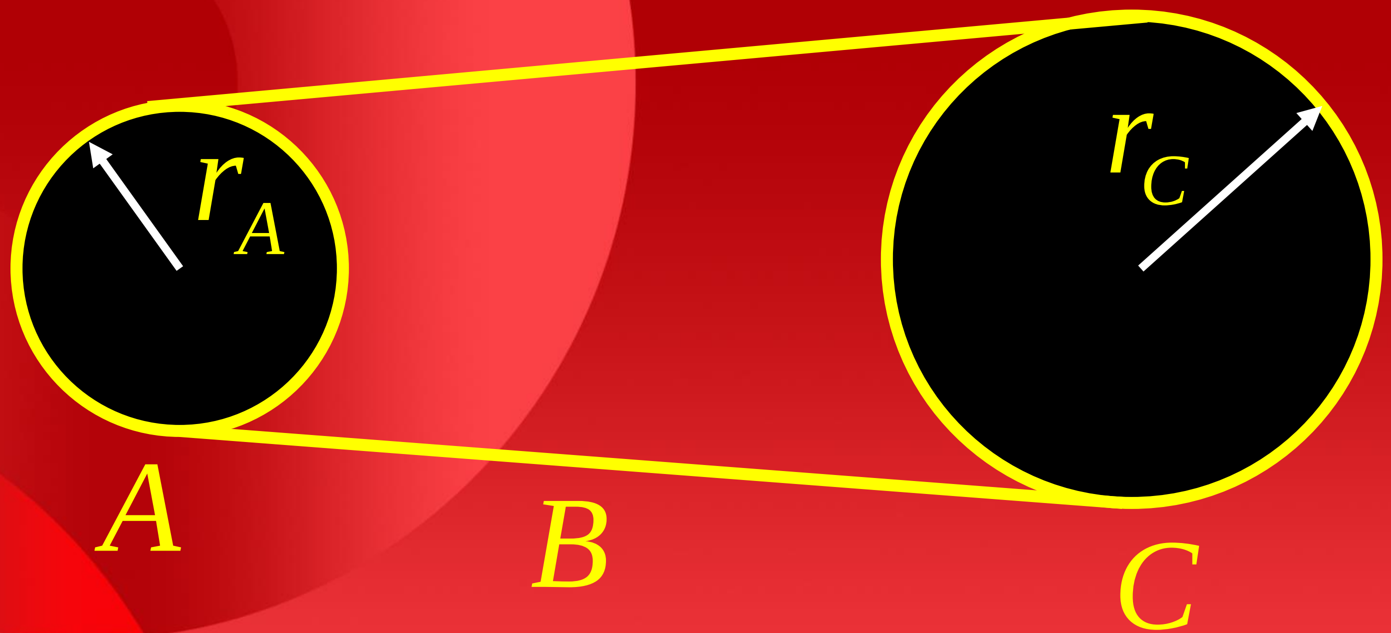
A diver makes 2.5 complete revolutions on the way from a 10-m platform to the water below. Assuming zero initial vertical velocity, calculate the average angular velocity.

$$h = \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2h}{g}} = 1.43\ s$$

$$\omega = \frac{\phi}{t}$$

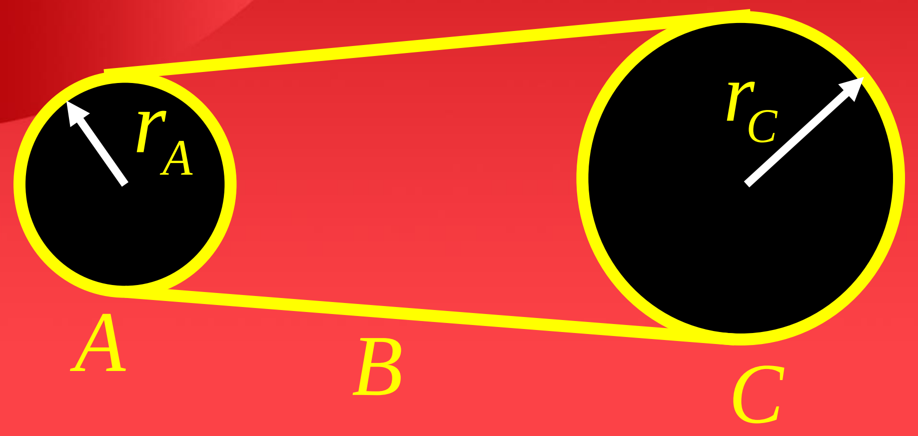
$$\omega = \frac{2\pi \times 2.5}{1.43} = 11\,\text{rad/s}$$



r_A
r_C
A
B
C

Wheel A of radius r_A = 10.0 cm is coupled by a chain B to wheel C of radius r_C = 25.0 cm. Wheel A increases its angular speed from rest at a uniform rate of 1.60 rad/s^2.

Determine the time for wheel C to reach a rotational speed of 100 rev/min.



$$\mathrm{v}_A = \mathrm{v}_C \Rightarrow \; r_A \omega_A = r_C \omega_C$$

$$\omega_A = \frac{r_C \omega_C}{r_A}$$

$$\alpha = \frac{\omega_A - 0}{t}$$

$$t = \frac{\omega_A}{\alpha} = \frac{r_C \omega_C}{r_A \alpha} = 16.4\ s$$

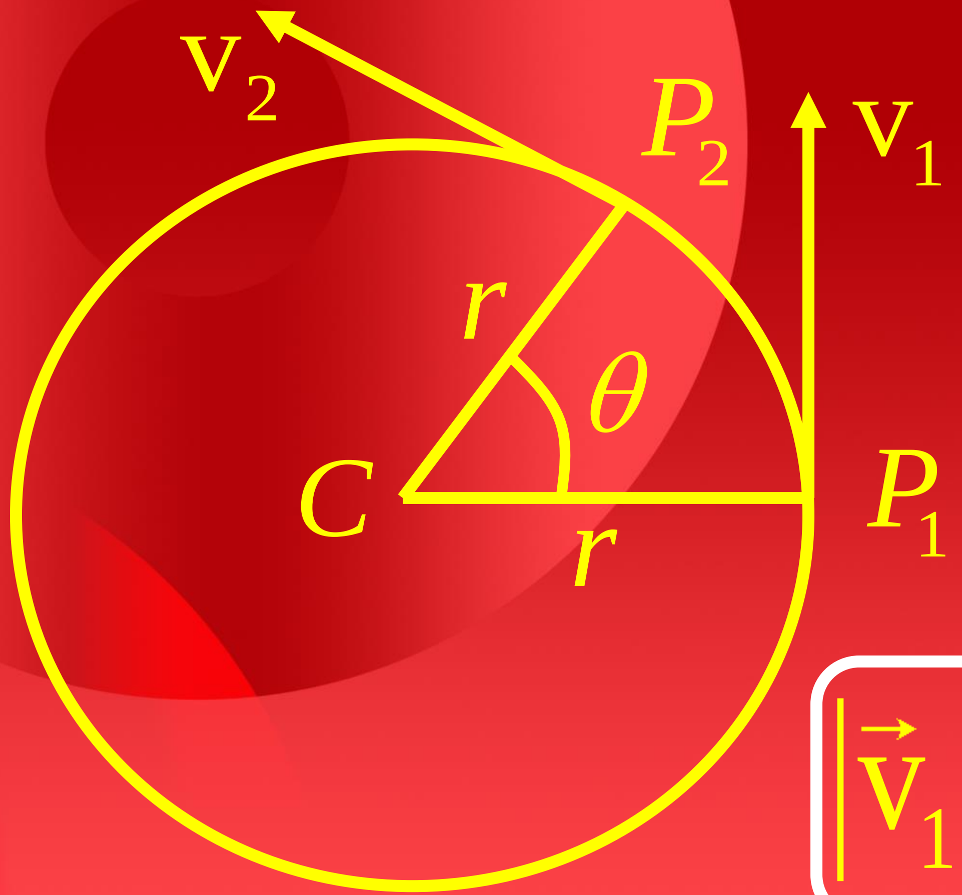
Rotation with constant angular acceleration

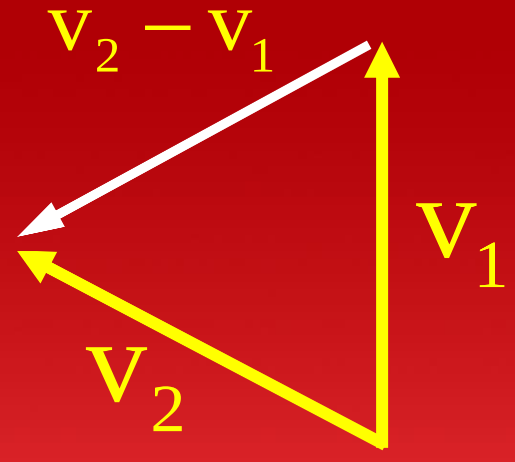
All particles will have same 'ω' and 'α' but different 'v' and 'a'

'ω' and 'α' are simpler choices !!

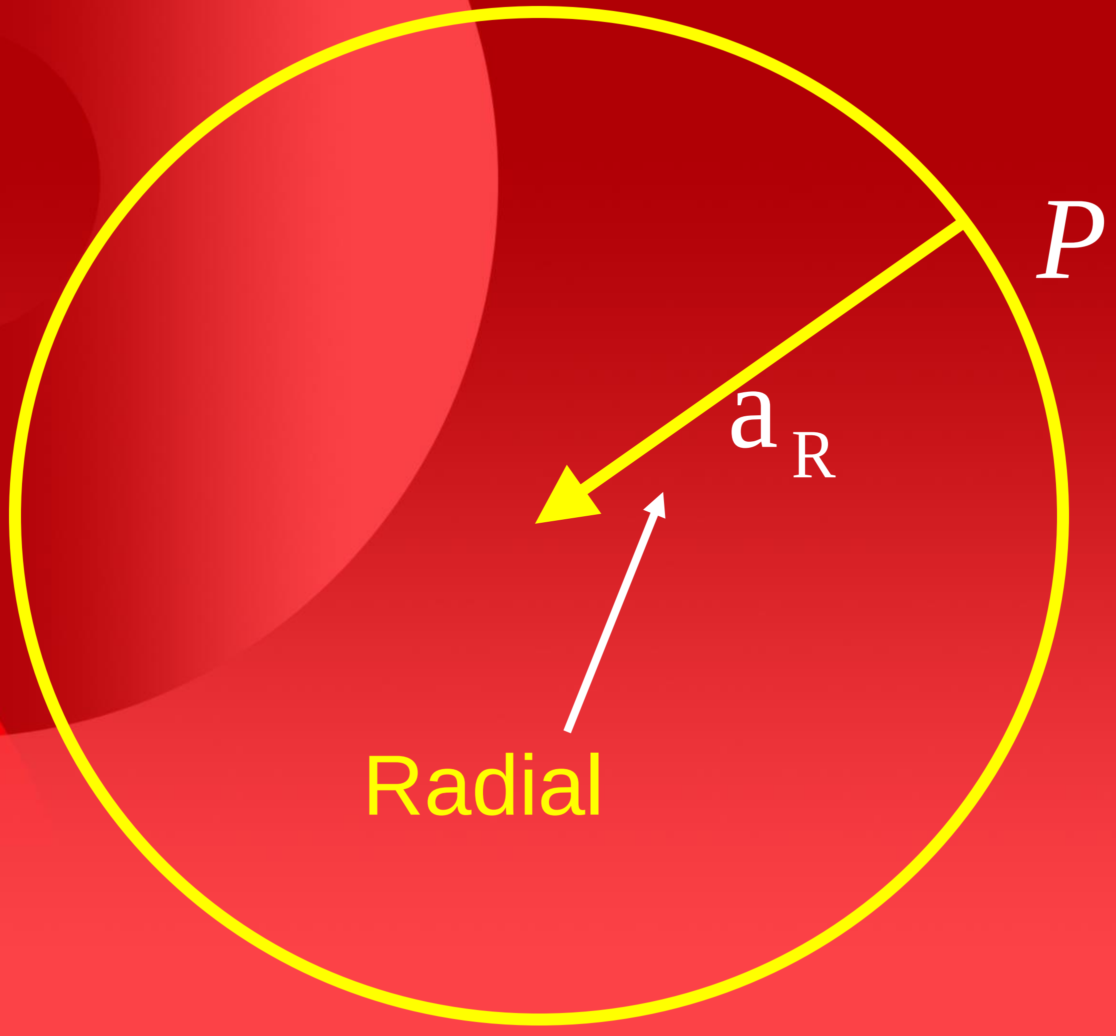
Centripetal acceleration

Uniform circular motion

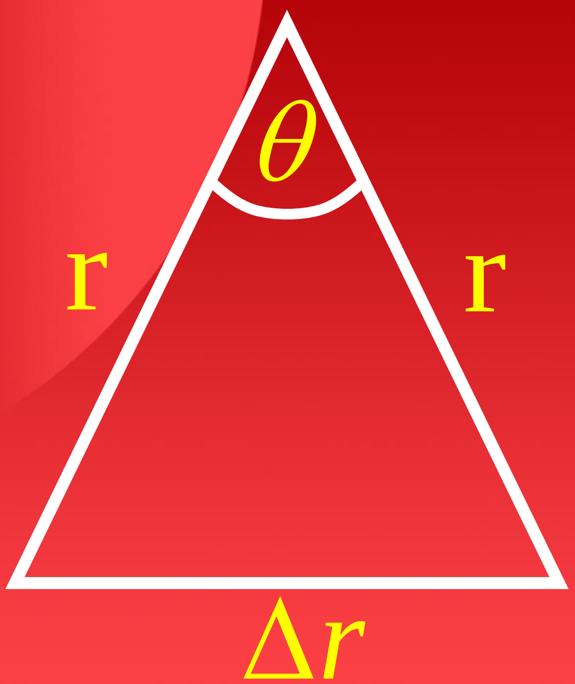




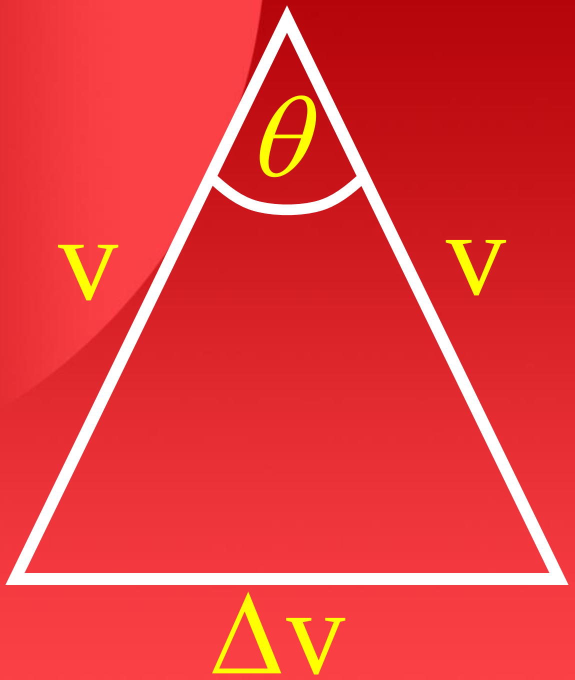
$$\boxed{\left|\vec{v}_1\right| = \left|\vec{v}_2\right| = v}$$


P
a_R
Radial

$$\Delta r = v\Delta t \approx r\theta$$



$$\Delta v \approx v\theta$$



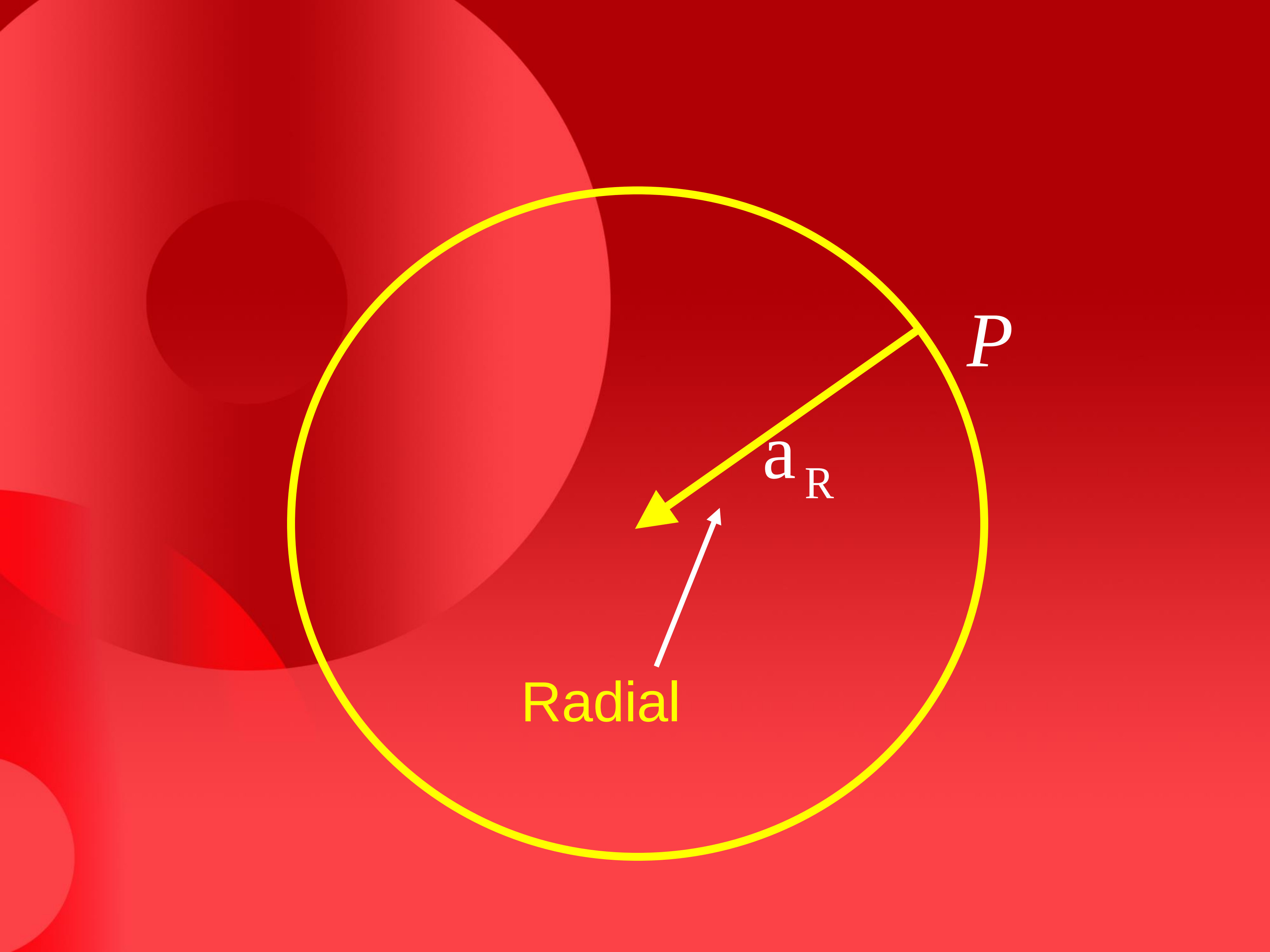
$$\Delta \mathrm{v} \approx \mathrm{v}\theta$$

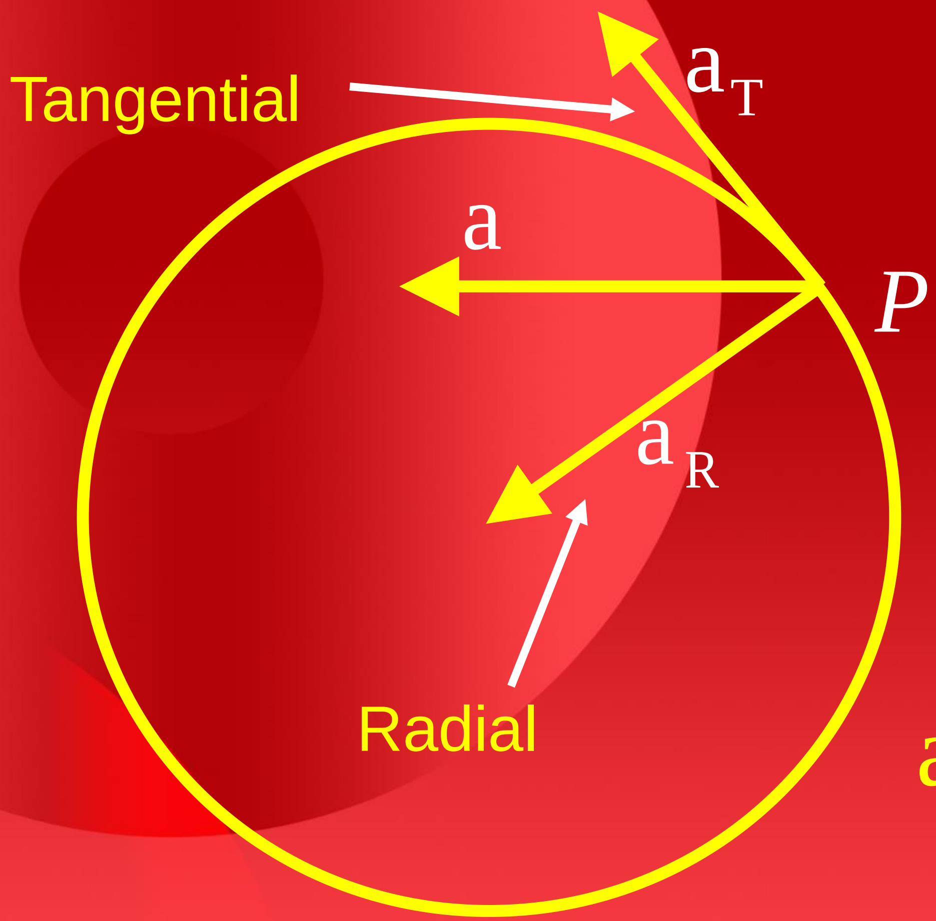
$$\bar{\mathrm{a}} = \frac{\Delta \mathrm{v}}{\Delta t} \approx \frac{\mathrm{v}\theta}{r\theta / \mathrm{v}} = \frac{\mathrm{v}^2}{r}$$

$$\mathrm{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathrm{v}}{\Delta t} = \frac{\mathrm{v}^2}{r}$$

$$\vec{\mathrm{a}}_R = -\frac{\mathrm{v}^2}{r}\hat{r}$$

direction is radially inward !!

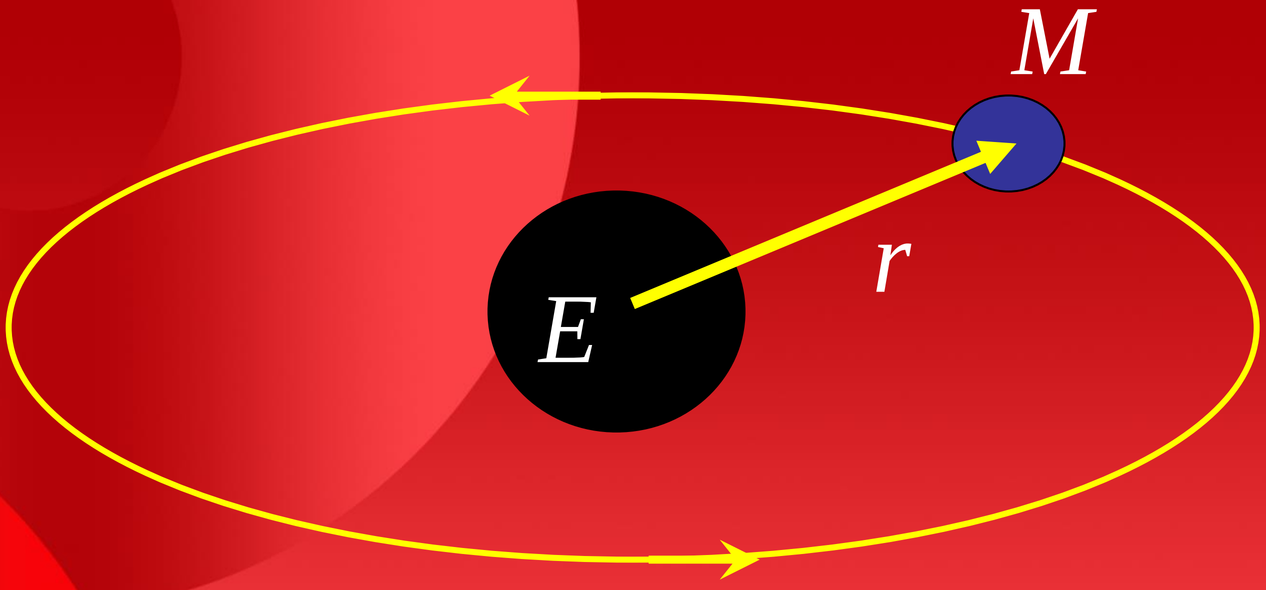

P
a_R
Radial



$$a_T = r\alpha$$

$$a_R = \frac{v^2}{r} = r\omega^2$$

The Moon revolves about the Earth, making a complete revolution in 27.3 days. Assume that the orbit is circular and has a radius of 238,000 miles. What is the magnitude of the acceleration of the Moon towards the Earth?


M
E
r

$$r = 238{,}000\,\mathrm{mi} = 3.28\times10^{8}\,m$$

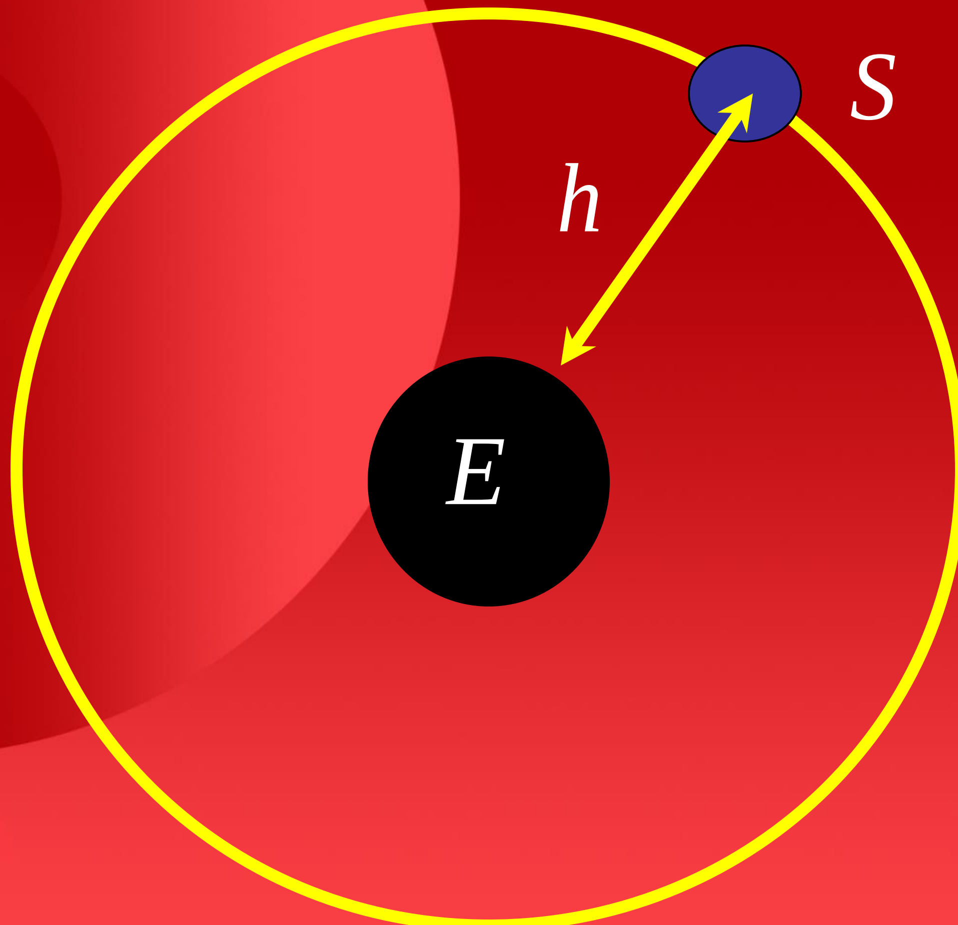
$$\mathrm{v} = \frac{2\pi r}{T} = 1018\,m/s$$

$$\mathrm{a} = \frac{\mathrm{v}^2}{\mathrm{r}} = 0.00271\,m/s^2$$

$$= 2.76\times10^{-4}\,g$$

(where g=9.81 m/sec^2)

Calculate the speed of an Earth satellite that it is traveling at an altitude h of 210 km where g = 9.2 m/s^2. The radius R of the Earth is 6370 km.


S
h
E

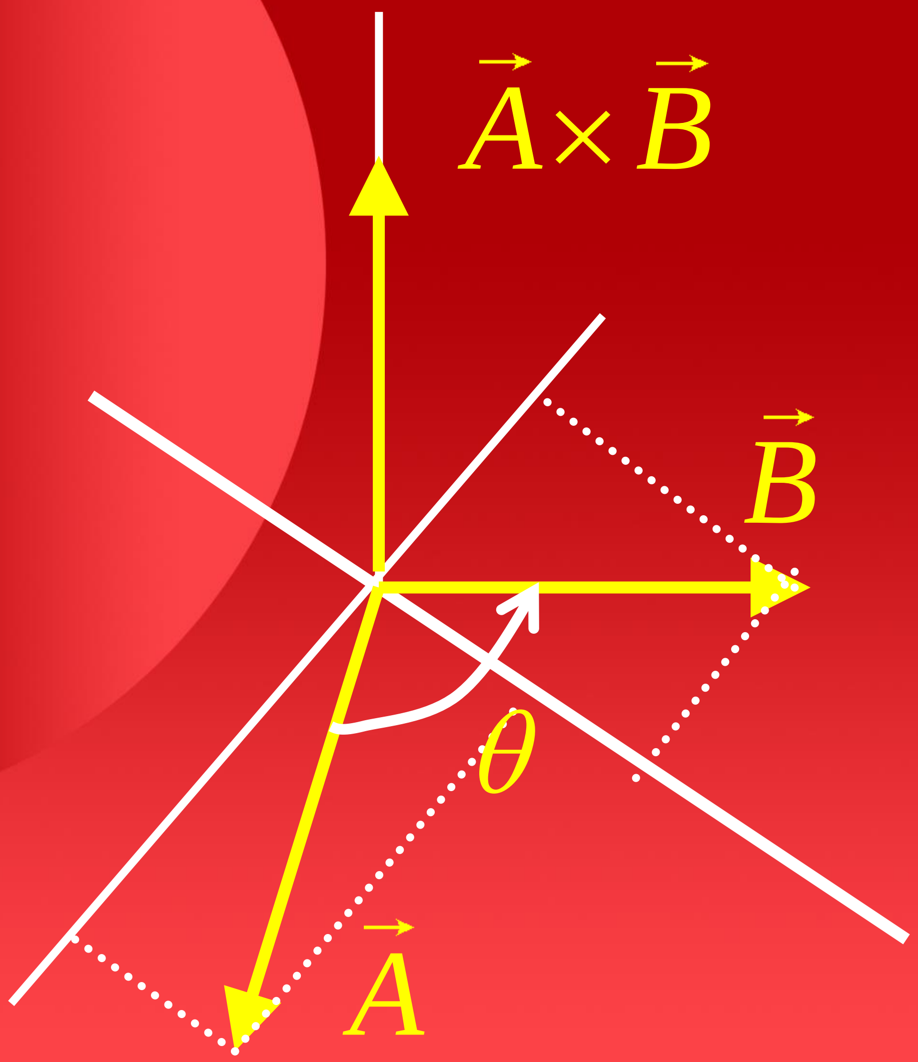
$$a = \frac{v^2}{r}$$

$$a = g \text{ and } r = R + h$$

$$g = \frac{v^2}{R + h}$$

$$v = \sqrt{(R + h)g} = 7780\, m/s$$

Vector Cross Products



$\vec{A} \times \vec{B}$
$\vec{B}$
θ
$\vec{A}$

Cross product (vector product)

is defined as,

$$\boxed{\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}}$$

$\hat{n}$ is perpendicular to AB-plane

i

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

Important properties

$$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$

$$\vec{A}\times\vec{A}=0$$

$$\left(\vec{A}+\vec{B}\right)\times\vec{C}=\left(\vec{A}\times\vec{B}\right)+\left(\vec{A}\times\vec{C}\right)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j}$$

$$+ (A_x B_y - A_y B_x)\hat{k}$$