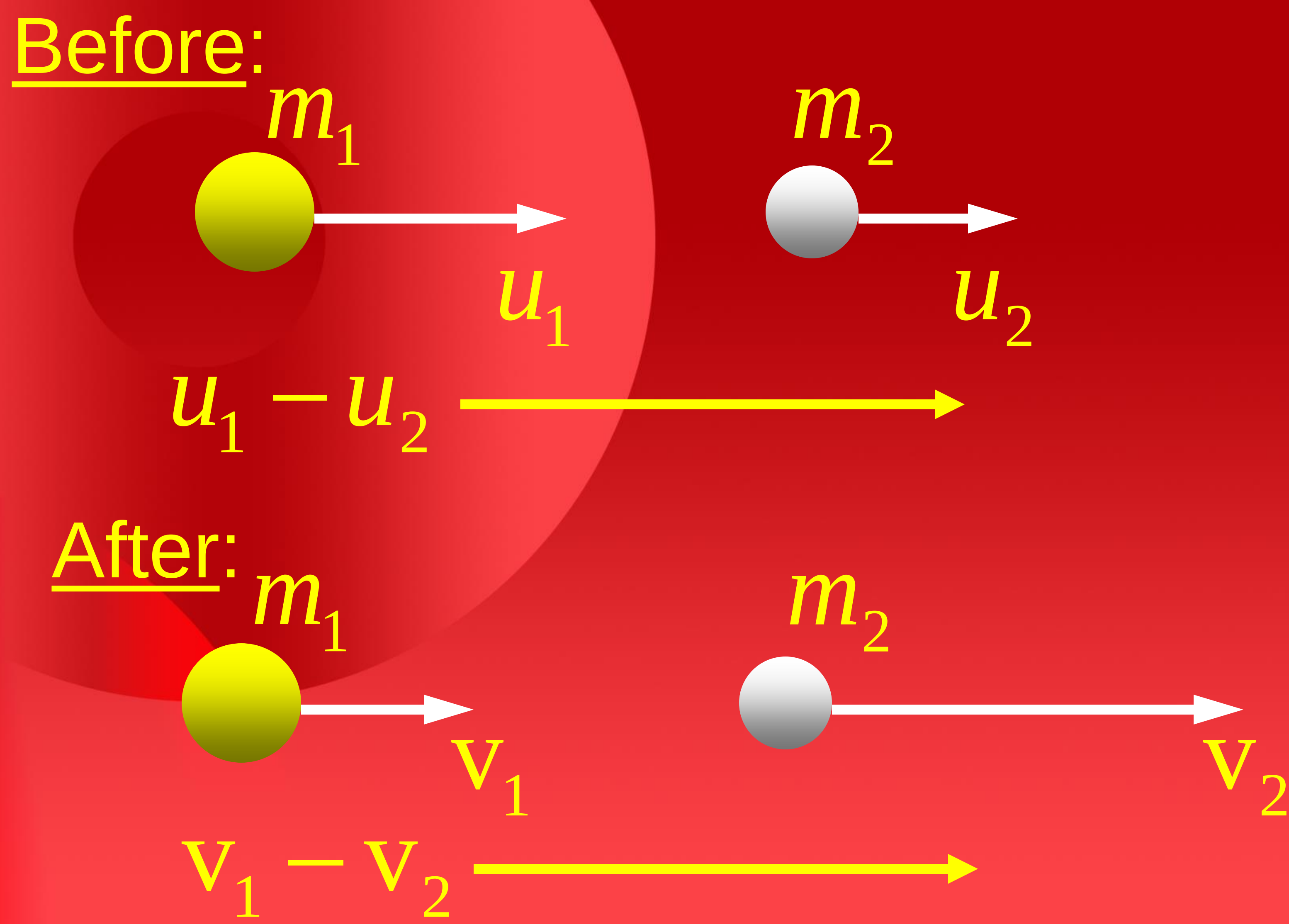
Physics

Collisions


Before:
m_1
m_2
u_1
u_2
$u_1 - u_2$
After:
m_1
m_2
v_1
v_2
$v_1 - v_2$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

$$\tfrac{1}{2} m_1(u_1^2 - v_1^2) = \tfrac{1}{2} m_2(v_2^2 - u_2^2)$$

$$\Downarrow$$

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1$$

Velocity of approach = velocity of receding

On solving,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

Holds in all inertial frames !!!

Equal masses: $m_1 = m_2$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

$$v_1 = u_2$$

$$v_2 = u_1$$

velocities exchanged!!

Newton’s cradle

animation of air track

Massive target at rest

$$m_2 >> m_1, \qquad u_2 = 0$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

$$\Rightarrow \; v_1 = -u_1 \quad \text{and} \quad v_2 = 0$$

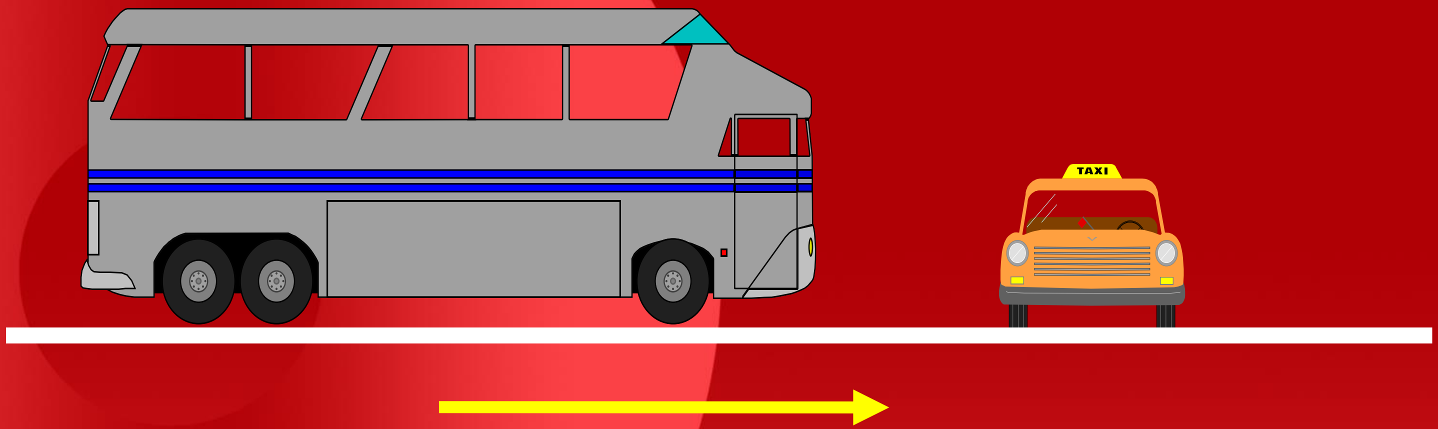
Lighter target at rest

$$m_2 << m_1 \qquad u_2 = 0$$

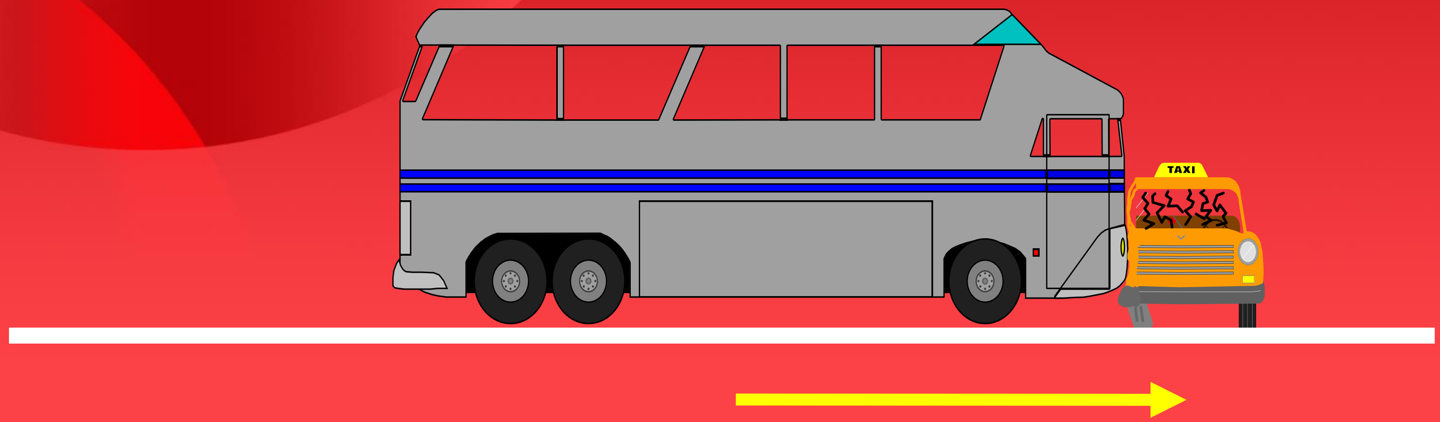
$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

$$\Rightarrow \quad v_1 = u_1 \text{ and } v_2 = 0$$



An accident



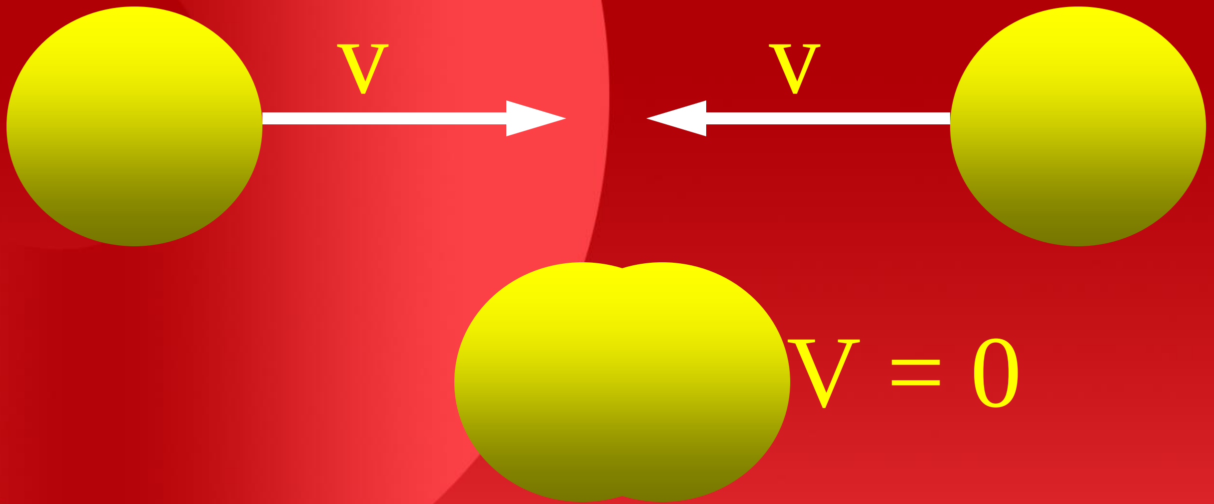
Elastic

- Momentum <u>*conserved*</u>
- Kinetic energy <u>*conserved*</u>

Inelastic

- Momentum <u>*conserved*</u>
- Kinetic energy <u>*not conserved*</u>

A COMPLETELY INELASTIC COLLISION



initial momentum = final momentum = 0

initial kinetic energy = $2\ \dfrac{1}{2} mv^2$

Q: By what fraction is the kinetic energy of a neutron (mass m_1) decreased in a head-on collision with an atomic nucleus (mass m_2) initially at rest?

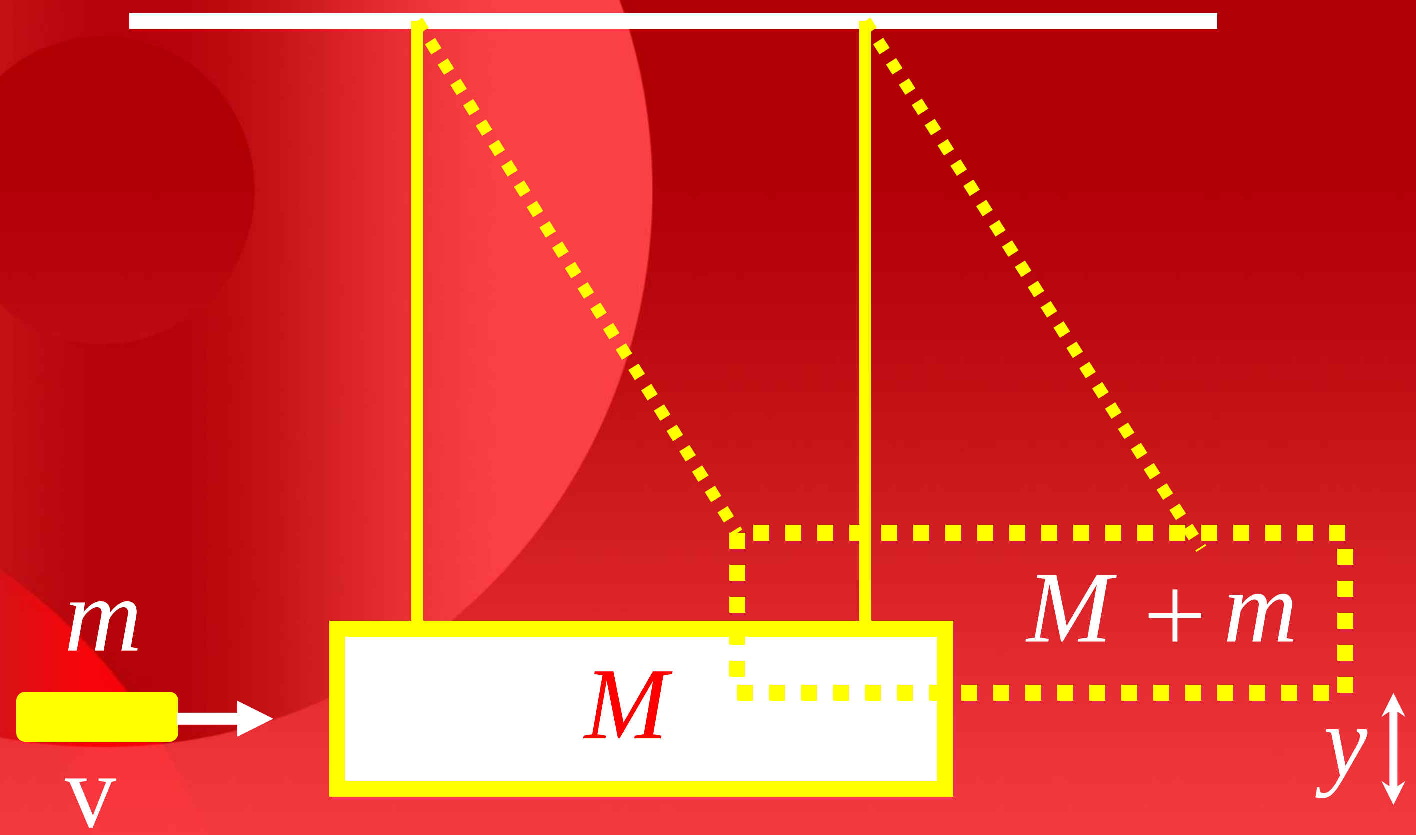
Fractional decrease in neutron K.E :

$$\frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{v_f^{\,2}}{v_i^{\,2}}$$

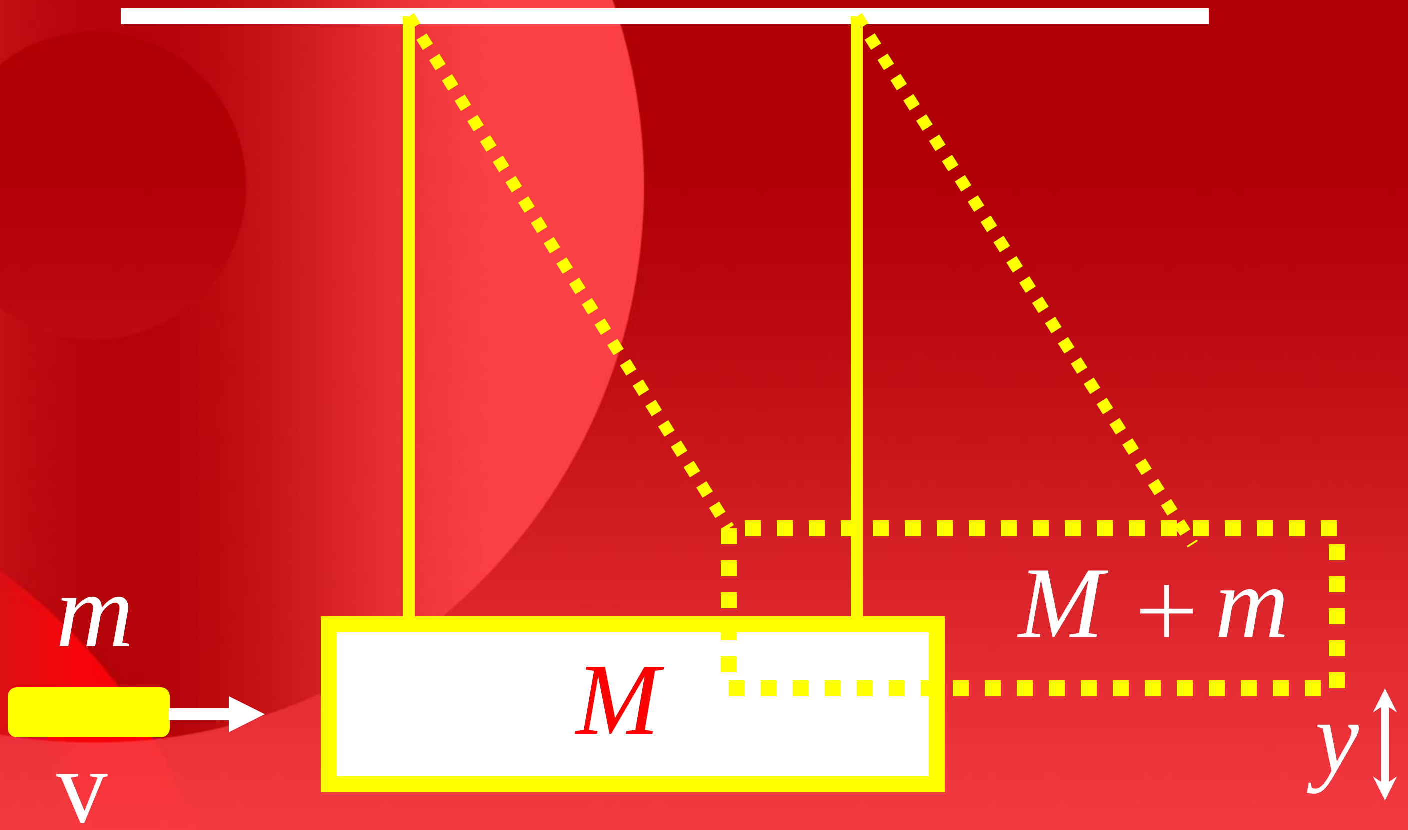
For a target at rest:

$$v_f = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_i$$

$$\frac{K_i - K_f}{K_i} = \frac{4 m_1 m_2}{(m_1 + m_2)^2}$$


m
v
M
M + m
y

Q: A bullet with mass m, is fired into a block of wood with mass M, suspended like a pendulum and makes a completely inelastic collision with it. After the impact, the block swings up to a maximum height y. What is the initial speed of the bullet?


m
v
M
M + m
y

$$m\text{v} = (m + M)V$$

$$\text{v} = \frac{(m + M)}{m}V$$

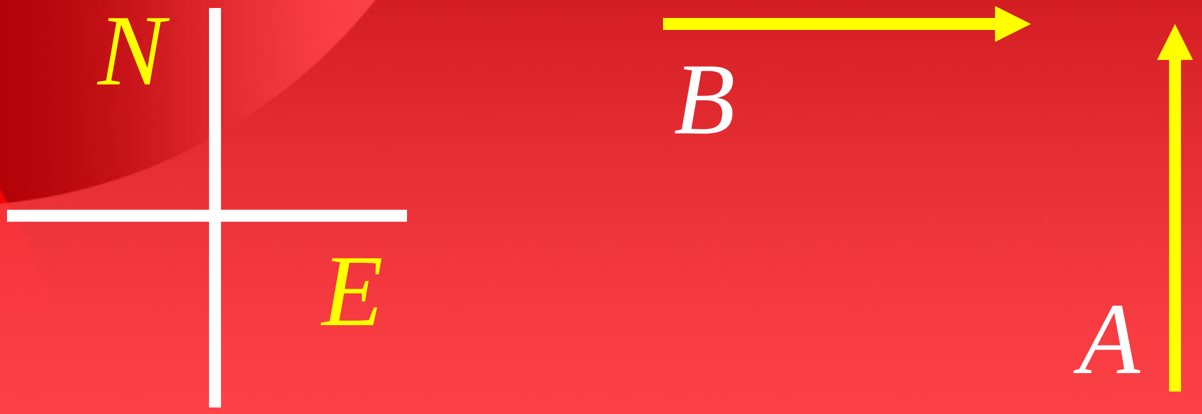
Conservation of energy gives,

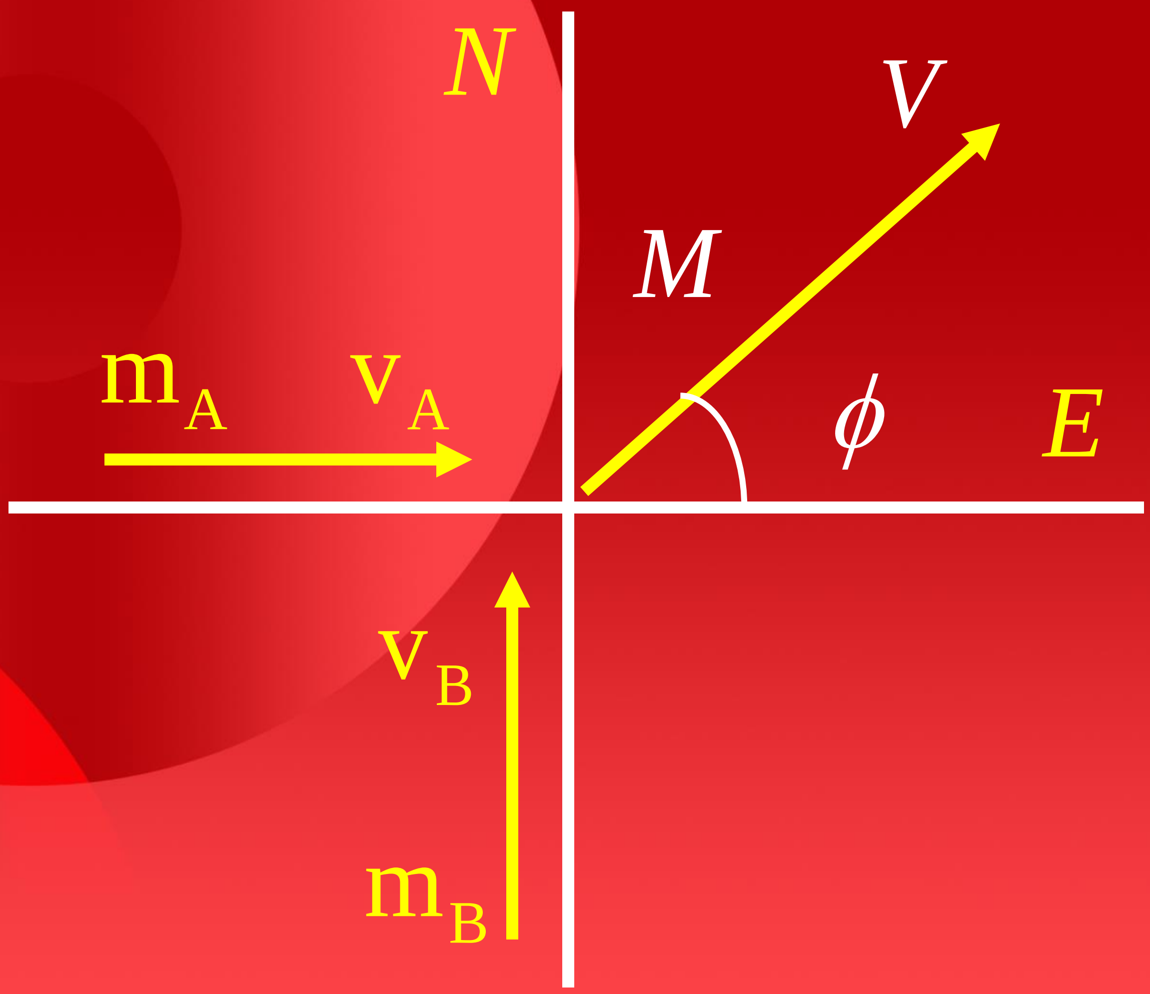
$$\frac{1}{2}(m + M)\text{V}^2 = (m + M)gy$$

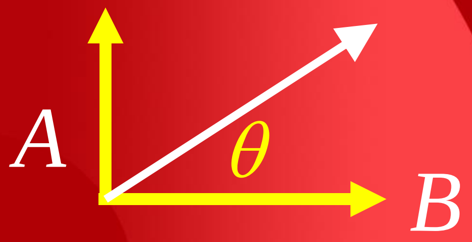
$$V = \sqrt{2gy}$$

$$\text{v} = \frac{(m + M)}{m}\sqrt{2gy}$$

A car A of mass 1000 kg is traveling north at 15 m/s collides with another car B of mass 2000 kg traveling east at 10 m/s. After collision they move as one mass. Find the total momentum just after the collision.




N
V
M
m_A
v_A
ϕ
E
v_B
m_B



$$P_x = p_{Ax} + p_{Bx} = \mathrm{m}_\mathrm{A}\mathrm{v}_{Ax} + \mathrm{m}_\mathrm{B}\mathrm{v}_{Bx}$$

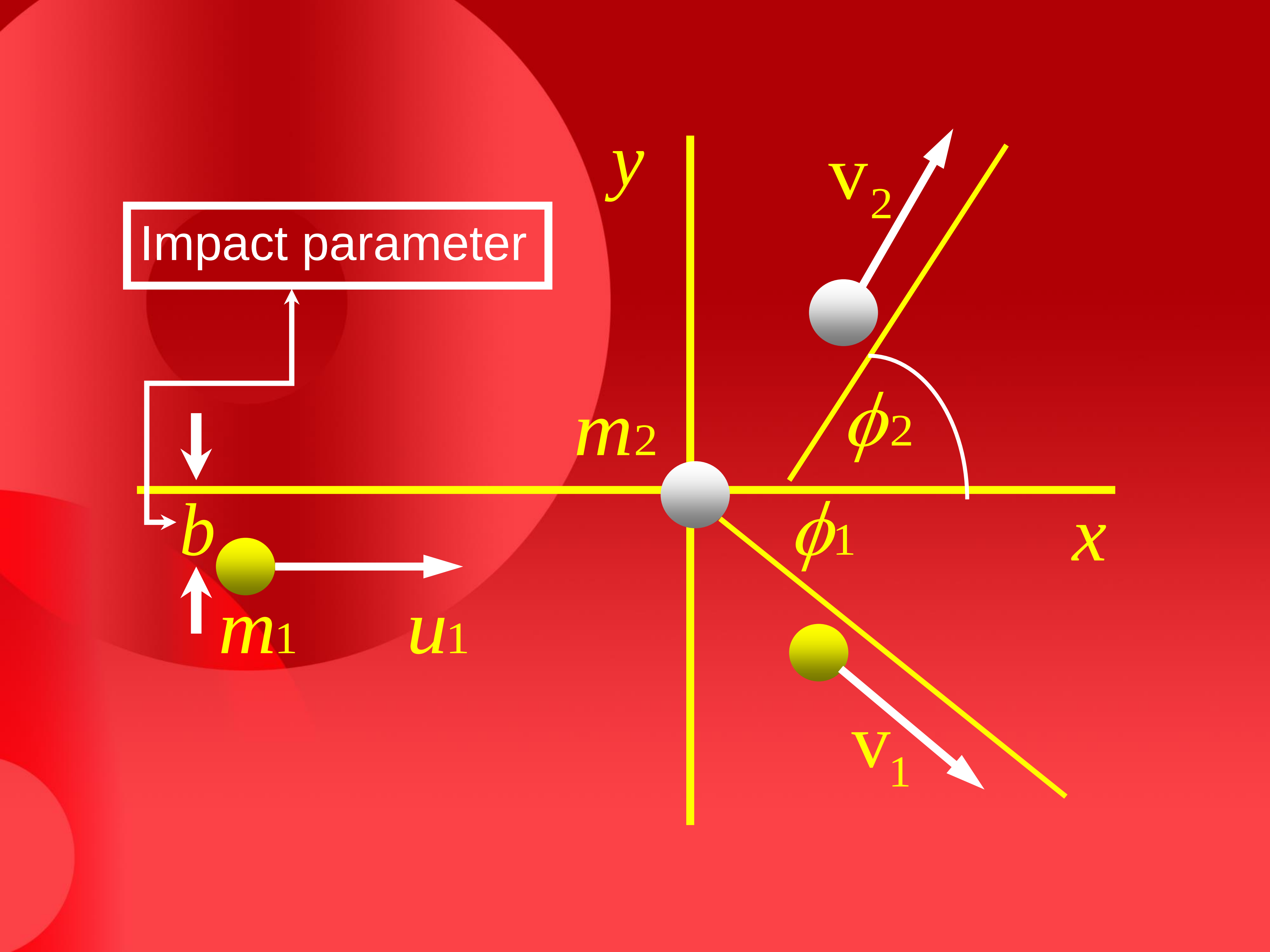
$$= 2\times10^4 kg\, m/s$$

$$P_y = p_{Ay} + p_{By} = \mathrm{m}_\mathrm{A}\mathrm{v}_{Ay} + \mathrm{m}_\mathrm{B}\mathrm{v}_{By}$$

$$= 1.5\times10^4 kg\, m/s$$

$$P = \sqrt{P_x{}^2 + P_y{}^2} = 2.5\times10^4 kg\, m/s$$

$$\tan\theta = \frac{P_y}{P_x} = 0.75 \quad \Rightarrow \quad \theta = 37^0$$



Impact parameter
y
x
$\mathbf{v}_2$
ϕ_2
m_2
ϕ_1
b
m_1
u_1
$\mathbf{v}_1$

1) $p_{ix} = p_{fx}$

$$\Rightarrow m_1 u_1 = m_1 v_1 \cos\phi_1 + m_2 v_2 \cos\phi_2$$

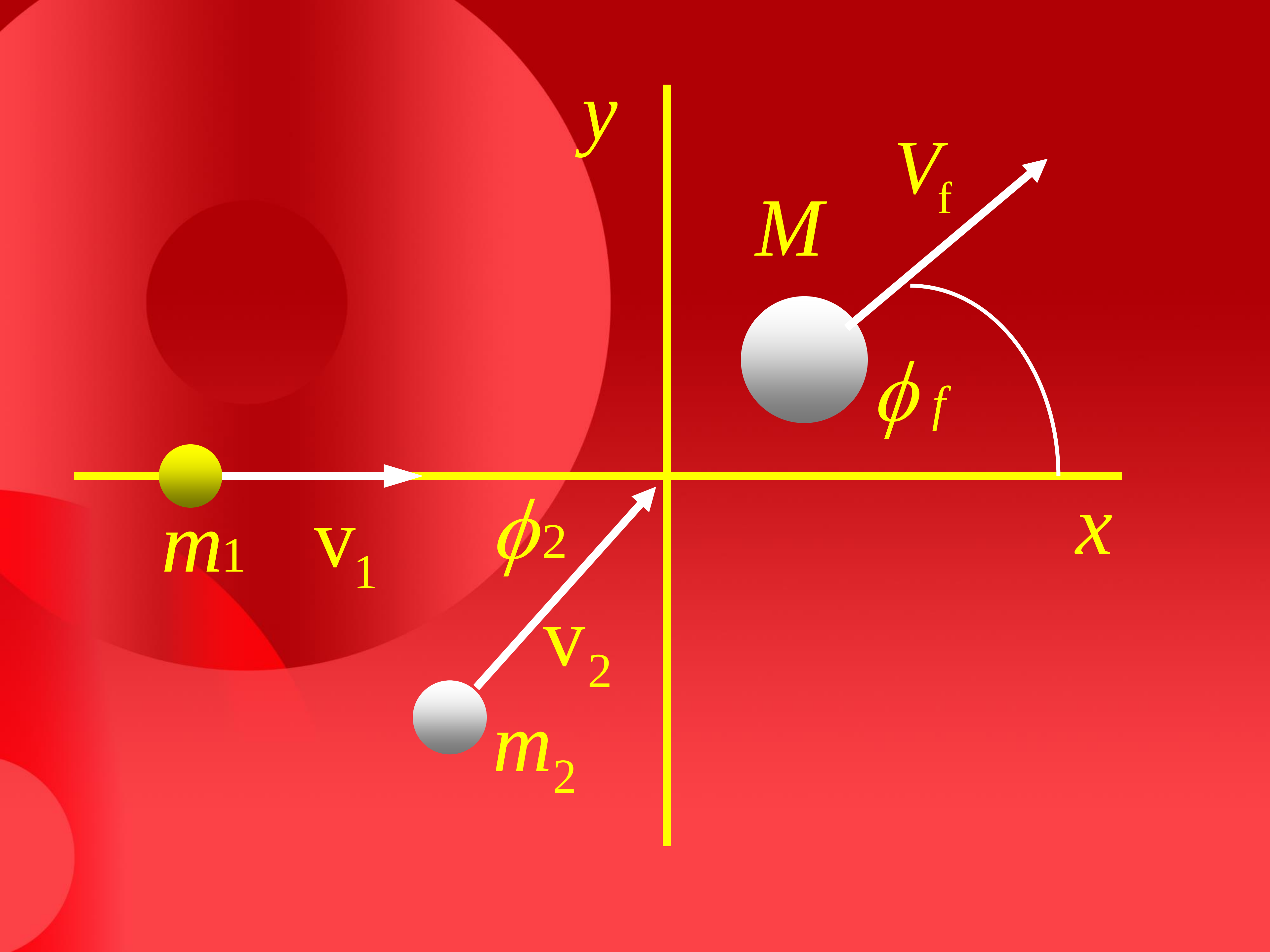
2) $p_{iy} = p_{fy}$

$$\Rightarrow 0 = m_1 v_1 \sin\phi_1 - m_2 v_2 \sin\phi_2$$

3) $KE_i = KE_f$

$$\Rightarrow \tfrac{1}{2} m_1 u_1^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

4-unknowns, 3-equations: cannot be solved !!


y
V_f
M
ϕ_f
x
m_1
V_1
ϕ_2
V_2
m_2

$$1)\ \mathrm{p}_{\mathrm{ix}} = \mathrm{p}_{\mathrm{fx}}$$

$$\Rightarrow \mathrm{m}_1 \mathrm{v}_1 + \mathrm{m}_2 \mathrm{v}_2 \cos \phi_2$$

$$= (m_1 + m_2) V_f \cos \phi_f$$

$$2)\ \mathrm{p}_{\mathrm{iy}} = \mathrm{p}_{\mathrm{fy}}$$

$$\Rightarrow \mathrm{m}_2 \mathrm{v}_2 \sin \phi_2 = M V_f \sin \phi$$

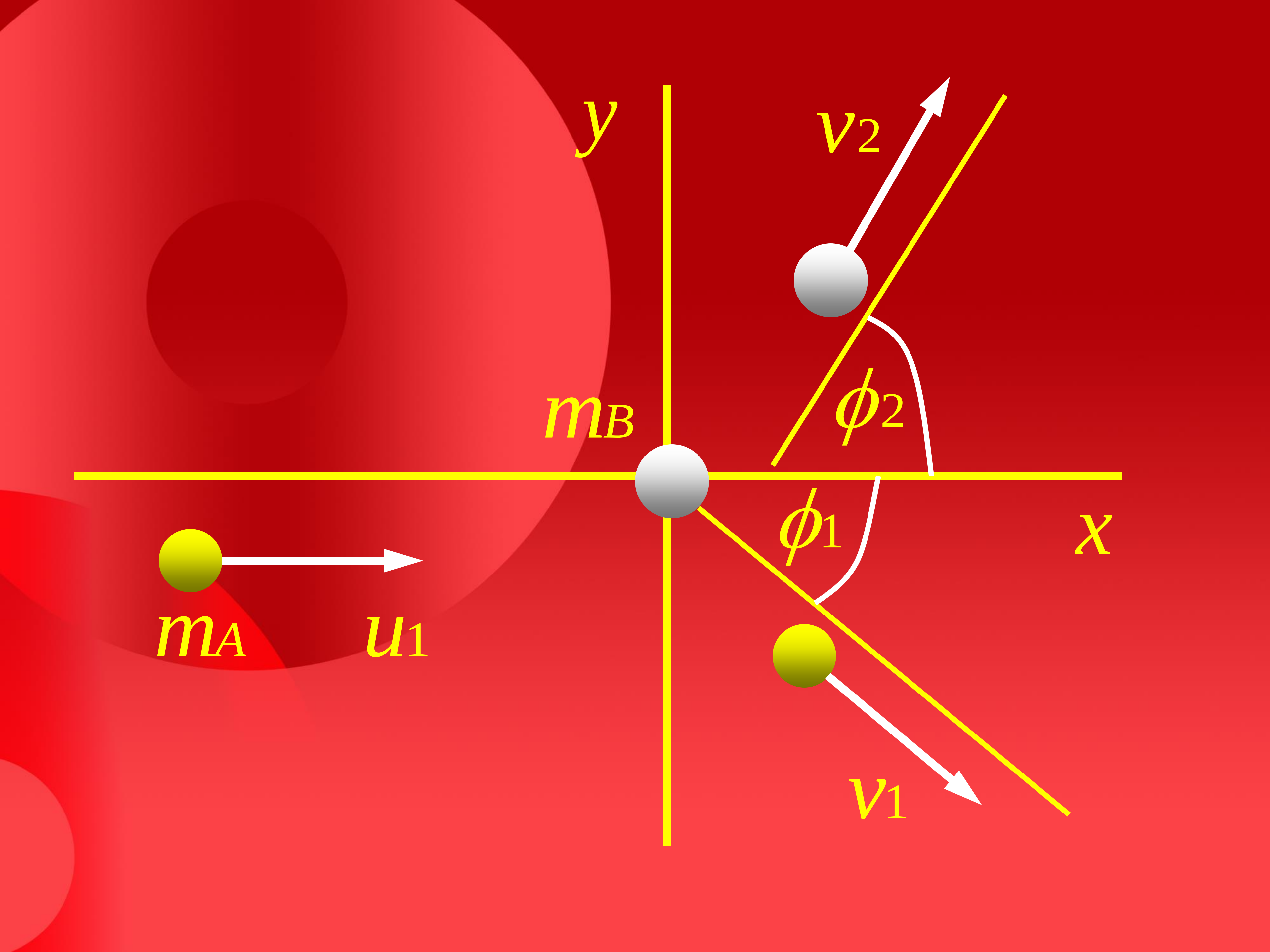
2-unknowns 2-equations: <u>can be solved!!</u>

- What happens when water is poured from one bottle into another ?
- Sand on a sheet of paper

Conclude:

- Momentum is always conserved in collisions, but energy may or may not be.
- We have come to trust momentum conservation very much: discovery of the neutrino, hints of black holes, discovery of dark matter.

Figure shows an elastic collision of two pucks on a frictionless table. Puck A has mass m_A=0.500 kg, and puck B has mass m_B=0.300 kg. Puck A has an initial velocity of 4 m/s in the positive x-direction and a final velocity of 2.00 m/s in an unknown direction. Puck B is initially at rest. Find the final velocity of puck B and the angles in the figure?



y
v_2
m_B
ϕ_2
ϕ_1
x
m_A
u_1
v_1

$$K.E_i = K.E_f$$

$$\frac{1}{2} m_A u_A^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$v_B^2 = \frac{m_A u_A^2 - m_A v_A^2}{m_B}$$

$$v_B = 4.47 m/s$$

$$p_{ix} = p_{fx}$$

$$m_A u_A = m_A v_A \cos\phi_1 + m_B v_B \cos\phi_2$$

$$p_{iy} = p_{fy}$$

$$0 = m_A v_A \sin\phi_1 - m_B v_B \sin\phi_2$$

$$\phi_1 = 26.6^0, \qquad \phi_2 = 36.9^0$$

Two skaters collide and embrace in a completely inelastic collision, A has mass m_A=83 kg, is originally moving east with speed v_A=6.4 km/h. B has mass m_B=55 kg, is originally moving north with a speed v_B=8.8 km/h. a): What is the combined velocity V after impact? b): What is the fractional change in the kinetic energy due to collision?

$$a)\ x-component: \qquad m_A v_A = MV\cos\phi$$

$$y-component: \qquad m_B v_B = MV\sin\phi$$

$$M = m_A + m_B$$

$$\tan\phi = \frac{m_B v_B}{m_A v_A} = 0.911$$

$$\phi = 42.3^0$$

$$V = \frac{m_B v_B}{M\sin\phi} = 5.21 km/h$$

$$b)\ K_i = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$= 3839 kg - km^2 / h^2$$

$$K_f = \frac{1}{2} M V^2$$

$$= 1870 kg - km^2 / h^2$$

$$fraction = \frac{K_f - K_i}{K_i} = -0.51$$

$\boxed{\text{51\% loss}}$