Physics-PHY101-Lecture#05

APPLICATIONS OF NEWTON'S LAWS–I

In today's lecture, we will be focusing on Newton's laws of motion and how they can be applied to different situations and circumstances. We will be solving various examples to gain a better understanding of how important Newton Laws are in understanding the world around us. Living in the 21st century, we can sometimes forget the significant difference between the latest and old concepts/laws. Before Newton's laws, it was generally believed that every physical body or particle naturally wanted to remain at rest. However, Newton's first two laws of motion state that every physical body or particle tends to maintain its state of motion, whether it's at rest or in motion. If the state of motion of a body changes, such as a change in speed, it's because some force has acted upon it.

Equilibrium

If a body or a particle is in a state of rest, then we refer to it as an equilibrium state. Equilibrium means that in the state of a system, there is no force acting upon it, i.e., also called the state of rest (no motion). Newton's law means that if the sum of all the forces is equal to zero on that body or combination of bodies, then we get a state of rest. As per the definition, "State of a system when the sum of all forces acting upon it vanishes" is called 'Equilibrium'.

What is meant by the system here? The system here refers to many bodies which are connected to each other or isolated from each other. When you consider all these together then it is called a system.

Examples of equilibrium:

1) The gravity acting on the apple continuously pulls it downwards but when placed on the table as shown in Figure 5.1, the apple remains at rest. It doesn't move further down. Which forces are acting on this apple? Gravity is one as mentioned earlier but there must be at least one more force.

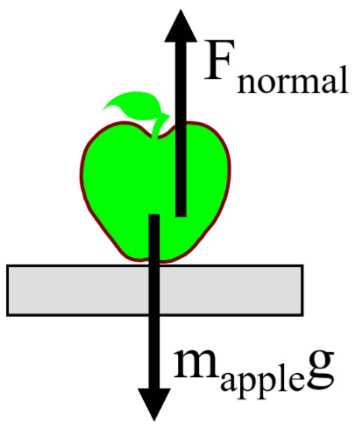


Figure 5.1: Apple placed on the horizontal top surface of the table. Normal force F_{normal} and weight of apple ($W_{apple} = m_{apple}g$).

Newton's law governs that whenever a force is applied to a body, the body starts accelerating. The table applies a force opposite to the gravitational force which stops this apple from falling. This force is called the Normal Force (F_{normal}). The word "normal" has two meanings in English, one is "as per usual" and the other is "perpendicular" as in geometry. In Figure 5.1, both these forces are indicated, the normal force (F_{normal}) and the $m_{apple}g$ which is the force due to gravity (also called weight). Mathematically Newton's second law (for y-axis or vertical direction) can be written as,

$$\sum F = m\,a_y = F_{Normal} - m_{apple}\,g$$

Normal force (F_{Normal}) is written as positive and gravitational force ($m_{apple}g$) is written with the negative sign as it is opposite to the direction of F_{Normal}. All forces along the x-axis (horizontal direction) are zero as gravity does not act along this direction and also table is not applying any force sideways. As the apple is at rest, acceleration (a_y) is also zero. The above equation can be simplified to

$$F_{Normal} - m_{apple}\,g = 0 \text{ (as } a_y = 0)$$
$$F_{Normal} = m_{apple}\,g$$

So normal force applied to the apple by the table is equal to its weight under the equilibrium (at rest/no motion). It is normally supposed that equilibrium means there is no force acting on the body which is a wrong presumption. Equilibrium or a state of rest arises when the total net force is zero (sum of all forces).

2) <u>Equilibrium Example: Rubber band as slingshot</u>

A rubber band can be considered a system and when pulled tension is created in it. This tension increases as it is pulled more and more. If a piece of paper in the form of a pellet is attached to a rubber band and stretched only and kept in place (at rest, equilibrium) like a slingshot, the tension in the rubber pulls the paper pellet forward. If released, the paper pellet will go forward like a projectile. So, it remains in equilibrium only under the force applied by us which is equal and opposite to tension in the rubber band. Now the question is whether this paper pellet was in the state of equilibrium when released or when it reached its maximum height considering that it is projectile motion. The answer is no, it was not in the equilibrium state. Although its velocity along the y-direction was zero momentarily at the highest point, its acceleration at this point was not zero as acceleration due to gravity was continuously acting on it. There was no force acting other than force due to gravity so it was not in the state of equilibrium. The total net force acting on it was force due to gravity (weight) which is nonzero.

3) Equilibrium Example: Aircraft flying

Consider an aircraft craft flying at a constant speed and same elevation. This is an example of equilibrium motion as there is no acceleration (constant speed) indicating net force (sum of all forces) acting on the aircraft is zero.

Now we will solve another problem considering another aircraft of mass m has a position vector given by $\vec{r} = (\text{at} + \text{bt}^3)\hat{i} + (\text{ct}^2 + d\,\text{t}^4)\hat{j}$. What force is acting upon it? Is it in equilibrium?

As velocity $\vec{v}$ is the time derivative of position $\vec{r}$ by

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left((\text{at} + \text{bt}^3)\hat{i} + (\text{ct}^2 + d\,\text{t}^4)\hat{j}\right)$$

$$\vec{v} = (\text{a} + 3\text{bt}^2)\hat{i} + (2\,\text{ct} + 4d\,\text{t}^3)\hat{j}$$

Now as acceleration $\vec{a}$ itself is time derivative of $\vec{v}$, so

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left((\text{a} + 3\text{bt}^2)\hat{i} + (2\,\text{ct} + 4d\,\text{t}^3)\hat{j}\right)$$

$$\vec{a} = (6\,\text{bt})\hat{i} + (2c + 12d\,\text{t}^2)\hat{j}$$

Now force $\vec{F}$ is a product of mass m and acceleration $\vec{a}$, so

$$\vec{F} = m\vec{a} = m((6\,\text{bt})\hat{i} + (2c + 12d\,\text{t}^2)\hat{j}) = 6m\,\text{bt}\,\hat{i} + m(2c + 12d\,\text{t}^2)\hat{j}$$

The above equation shows the force acting on the aircraft. Force along with acceleration is nonzero so the aircraft is not in equilibrium.

Tension

Now consider a string tied to a body as shown in Figure 5.2(a). The body has a certain weight (downward force) due to gravity which causes tension in the string. This tension force in the string is in an upward direction as it balances the weight. If we cut the string which is horizontally stretched, we will perceive tension towards the right as well as towards the left as shown in Figure 5.2 (b).

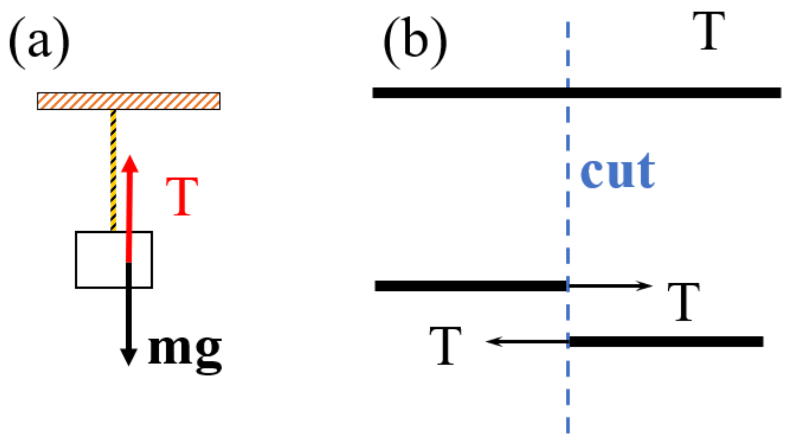


Figure 5.2: (a) A body having mass m tied to a string. The other end of the string is tied to a fixed support. Mass hanging vertically. (b) Tension T is considered in horizontal string. Blue dashed line indicating cut-line.

Now the big advantage of ropes is that we can pull a body from a distance and the direction of the force can be changed. Consider a horizontal string under tension as shown in figure 5.3. the yellow part represents a small part of the string. T_2 acts to the right-side direction and T_1 to the left-side direction of that small part of the string. According to Newton's law, total net force action on that small part of the rope is given by

$$\sum F = T_2 - T_1 = ma \quad \ldots\ldots\ldots\ldots 5.1$$

Where *m* is the mass and *a* is the acceleration of that small part of the string. Considering that string is very light in terms of mass, the mass of the small part of the string can be neglected and assumed to be zero.

$$m = 0 \text{ (small part of string whose mass is very less)}$$

Eq 5.1 becomes

$$T_2 - T_1 = 0 * a$$

$$T_2 - T_1 = 0$$

$$T_2 = T_1$$

As T_2 and T_1 are the same, we can equate both quantities to be T. This means that tension in a string has very little mass and is constant throughout the string.

An ideal (massless) string has constant tension

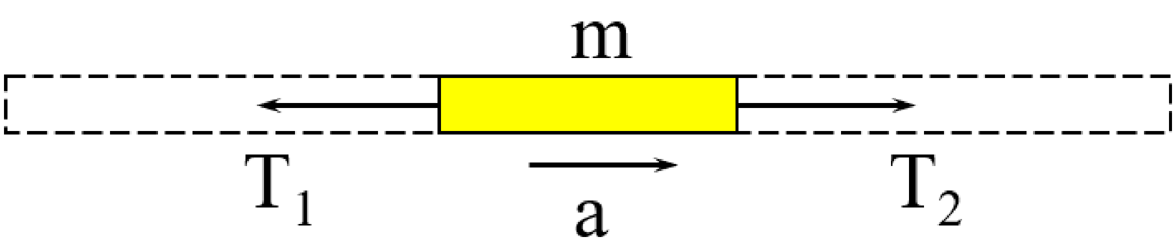


Figure 5.3: A string having horizontal tension. The central yellow part shows a small part of the string having mass m and accelerating by a being pulled to the left and right by tensions T_1 and T_2, respectively.

Newton's third law applies as forces act on all segments of a string, but these action and reaction forces are equal to each other. As shown in Figure 5.2(a), the body having mass is under equilibrium and is at rest. Newton's second law can be as applied as

$$\sum F = T - mg = ma_y$$

Where a_y is acceleration along the y-axis and is zero ($a_y = 0$).

$$T - mg = 0$$

$$T = mg$$

So, for this mass system, the tension is equal to the weight and the value of this tension will be equal everywhere inside the string. Now for string having as considerable mass such as rope, will the tension be equal in all parts of this rope if it is allowed to hang vertically without any additional body? The answer is NO, because the lower end of the rope has to support less mass as compared to the higher part of the rope. So, tension at the higher end is greater as compared to tension at the lower end.

A massive string (rope) does not have constant tension

Changing the direction of Tension

We can change the direction of tension as per requirement. As mentioned earlier, if the string is considered massless tension will be constant throughout the string even for the physical configurations shown in Figure 5.4. Figure 5.4(a) shows mass-II m_{II} hanging vertically from an ideal string and the direction of tension is changed using a pulley. Now tension is acting horizontally on mass-I m_I.

Figure 5.4 (b) shows another example of the change of tension direction. The weight ($W = mg$) is being lifted. A worker is pulling a rope and with it, the weight is rising upwards. So, F_T is shown on two sides and both of them are adding up. Under equilibrium, the net force upward is two times F_T and it is equal to weight ($W = mg$).

$$\sum F = F_T + F_T - mg = ma_y \text{ ....} \quad 5.2$$

Under equilibrium $a_y = 0$

$$F_T + F_T - mg = 0$$

$$2F_T = mg$$

If $2F_T$ is greater than weight ($W=mg$) such as $2F_T > mg$ then eq. 5.2 can be written as

$$2F_T - mg = ma_y$$

$$ma_y > 0$$

$$a_y > 0 \text{ (Upward direction or lifting up)}$$

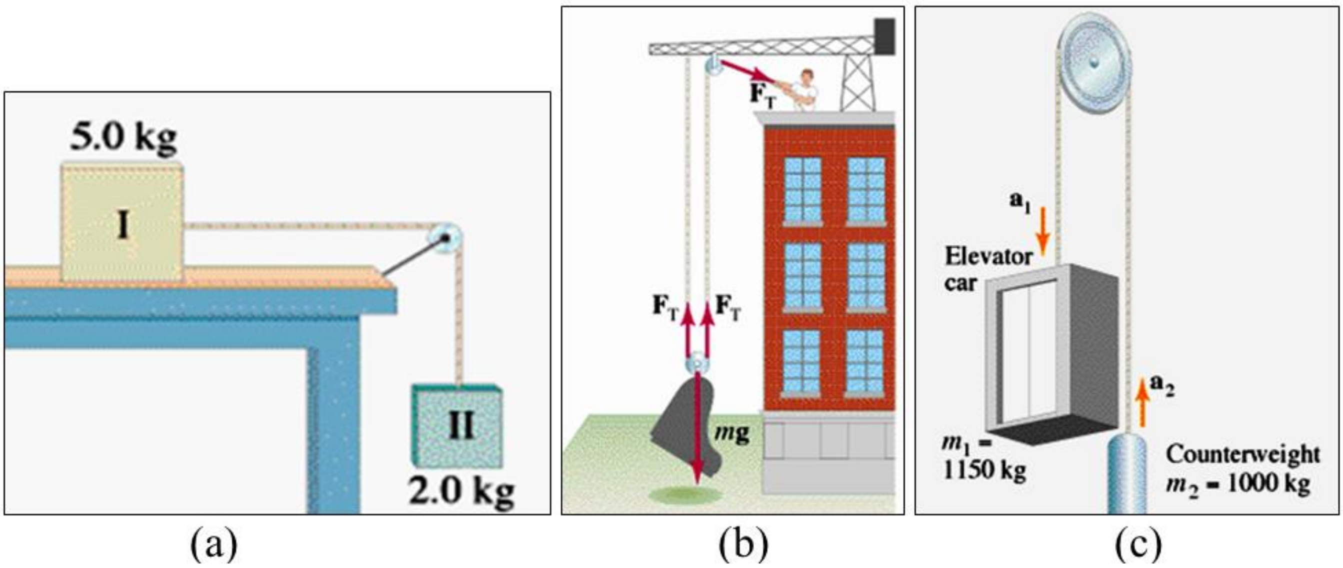
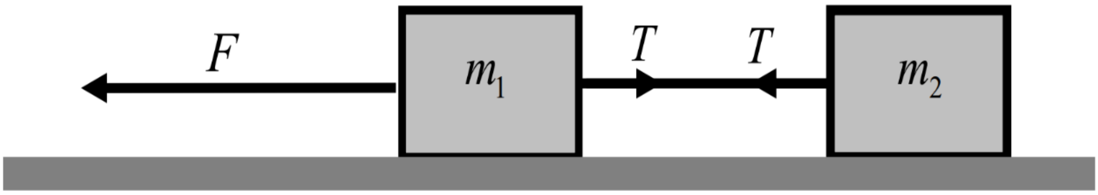


Figure 5.4: Examples of change of tension direction by pulley.

Figure 5.3(c) shows another example elevator car. The tension direction has been changed through the pulley. The mass of the elevator car is 1150 kg and on the other side, there is a counterweight which is equal to 1000 kg. Now the acceleration a_1 is downward as the elevator is more massive as compared to counterweight. The acceleration of the counterweight a_2 is downward.

Problem:

Question: Consider the two blocks shown below on a frictionless surface. Determine the tension and acceleration.



Solution:

$$F \text{ on } m_1:$$
$$\sum F = m_1 a$$
$$F - T = m_1 a \text{.........1)}$$
$$F \text{ on } m_2:$$
$$\sum F = T$$
$$T = m_2 a \text{...............2)}$$
To find "a"
Plugging eq. 2) in eq. 1)
$$F - m_2 a = m_1 a \Rightarrow F = m_1 a + m_2 a$$
$$a = \frac{F}{m_1 + m_2} \text{................3)}$$
To find "T"
Plugging eq. 3) in eq. 2)
$$T = m_2 a = \frac{m_2 F}{m_1 + m_2} \text{.......4)}$$

Friction

This is another important type of force that we encounter most frequently in our daily lives and it is called force of friction. The force of friction opposes every kind of motion. For example, if you place a body having a certain weight on the table and pull that body, then the force which is produced due to applied force and contact between the body and table is called frictional force as shown in Figure 5.5. There are countless examples of this in our everyday life. Suppose there is a plate lying on a book on this table, and it will not move on its own unless some force is applied to it. Now when we pull it (as per demonstration in video lecture), we will observe that it moves forward, because we have offset the frictional force. We pulled it with more force and it started moving. The normal force N is upwards and equal to its weight *mg*. Simultaneously, the greater the weight, the greater will force that opposes its motion. If we double the weight of this plate, then the force required to move the plate is also doubled. As depicted in Figure 5.5, friction results in a force parallel to the surface in the direction opposite to the direction of motion. So whichever direction we pull the body ($F_{Applied}$), the force of friction will be in the opposite direction. Additionally, the frictional force is perpendicular to the normal force N which is always upward.

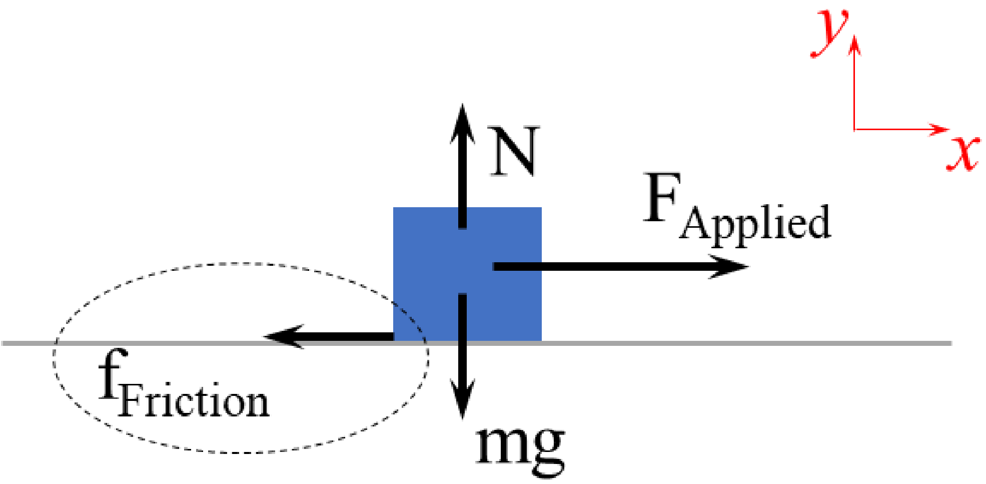


Figure 5.5: Mass m placed on a horizontal surface. Normal force N and weigh mg balanced along the y-axis. Frictional force $f_{Friction}$ produced along the negative x-axis as response to external force $F_{Applied}$ which is along the positive x-axis.

Now the pertinent question that why does the friction occur? We observe friction in various situations, for example, when we rub our hands, then we can feel opposing force due to contact of hands. To understand the underlying reason for friction, we need to observe the two surfaces in contact under a microscope. Any surface if seen under the microscope has very fine ups and downs in it and due to this unevenness, friction is created. The more the ups and downs, the more the roughness, then the friction will be higher accordingly. If we look at contact between surfaces of bodies at a microscopic level as shown in Figure 5.6, we will be able to observe the roughness caused by ups and downs which fundamentally causes the frictional force. If we can reduce this roughness then the frictional force or simply friction will reduce, and the fun will reduce.

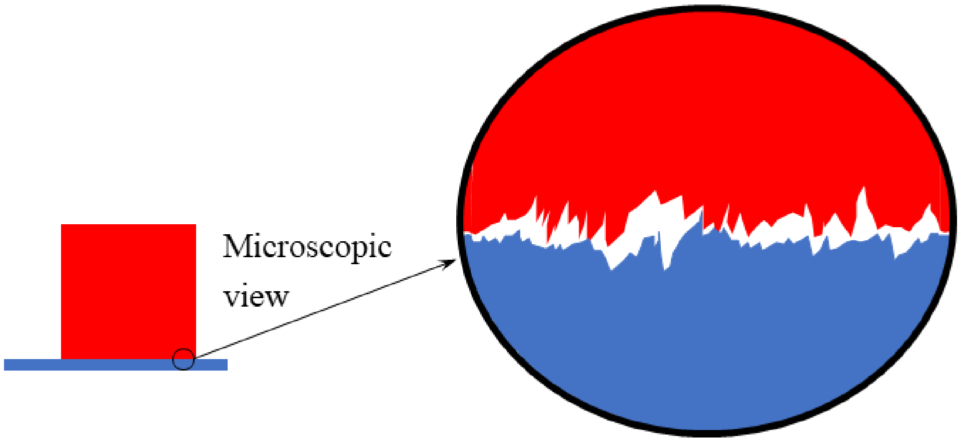


Figure 5.6: Microscopic view of two surfaces in contact which causes friction.

Characteristics of Frictional force f_F:

There are some special characteristics of frictional force. When the body is in motion, the friction that is produced is called kinetic friction. Friction force is proportional to the normal force N.

$$f_F \propto N$$

This sign of proportionality can be removed by the introduction of constant "μ_k" of proportionality and is called the "coefficient of kinetic friction".

$$f_F = \mu_k N$$

$$f_F = \mu_k mg \text{ (as } N = W = mg)$$

This equation shows that the "heavier" the object or a body, the greater the friction will be. This is an example of empirical law where $f_F = \mu_k N = \mu_k mg$. Can it be compared to fundamental laws such as Newton's second law? This is an example of an empirical law, it means that it is based on the observation that if the weight of the body/object is doubled then you have to apply double the force, if it is three times then, three times more force is required and so on. These statements are correct to some extent but it is obvious that if we keep increasing the weight of this body/object, then eventually a situation will arise when this law will not be applicable. If the body/object becomes too heavy then the surface (table) under the body/object might break and friction will not arise. So, this is an example of an Empirical Law which means that that law is based on observations and their scope is limited. Unlike empirical laws, the scope of Newton's law is very wide as we can apply it everywhere under every situation such as on the earth, on the moon, on small bodies and heavy bodies etc. Frictional law is not a such fundamental law. The frictional force is independent of the contact area. It does not matter whether this contact area is more or less. The force is just proportional to the Normal force, not to the area. We can say that it is correct to a great extent but we cannot claim that that is completely true. If the contact area is too large or too small, it can make a difference. Imagine that the entire weight of the plate is concentrated to something finer than a needle then it is quite possible that it will pierce the surface below and frictional force law would not be applicable. These things are very important to keep in mind that there is a difference between empirical laws and fundamental laws.

Frictional Dynamics

Now we will discuss the frictional dynamics, that is, how a body moves in the presence of friction. Now consider the forces along the x-axis (as shown in Figure 5.5).

$$\sum F_x = F_{Applied} - f_{Friction} = ma_x$$

$$\sum F_x = F_{Applied} - \mu_k N = ma_x \text{ ....5.3}$$

Where a_x is the acceleration body having mass m along the x direction. Similarly, for forces along the y-axis

$$\sum F_y = N - mg = ma_y = 0 \text{ (as } a_y = 0\text{)}$$

$$N = mg$$

So eq. 5.3 can be written as

$$F_{Applied} - \mu_k mg = ma_x$$

By this equation we can determine the acceleration for any given force and any given mass provided we are given the coefficient of Friction μ_k. Friction acts whenever the body is in motion but if a body is at rest then friction is also present. There is a difference between friction at rest and kinetic friction. Kinetic friction means the friction that exists when the body is in motion. Its value ($\mu_k N = \mu_k mg$) is a constant. But when the body is at rest, the value of friction force is not fixed. As we start pulling this plate it does not move (as per demonstration in the video lecture). We pulled it by applying a small force and its reaction is also a small frictional force accordingly. Only when we pull it with more than a certain limit force then it will move by balancing and overcoming frictional force. This means that static friction adjusts its value depending upon applied force, so static friction is different from kinetic friction in this regard. Now as the body is at rest, a_x will also be zero. Forces along the x-axis will be

$$\sum F_x = F_{Applied} - f_{Friction} = 0 \text{ (as } a_x = 0)$$

$$F_{Applied} = f_{Friction}$$

and for the y-axis, it will be similar to the previous (as in both kinetic and static cases, there is no motion along the y-axis)

$$\sum F_y = N - mg = ma_y = 0 \text{ (as } a_y = 0)$$

$$N = mg$$

The maximum possible force that can be produced by static frictions is f_{MAX} and is given by

$$f_{MAX} = \mu_s N$$

Here μ_s is the coefficient of static friction. The higher its value, the greater will be the static frictional force accordingly. There is a difference between kinetic friction and static friction. **Kinetic friction** is a constant force that means whether you go fast or slow, you have to counter that friction but static friction is different as it is not constant. For example, in the case of pulling a plate on the surface (as demonstrated in the video lecture), static friction adjusts depending on how hard we pull (apply the force) in that direction. The values μ_s may vary. If it is a frictionless surface, μ_s will be zero. But if there is a lot of friction, the value of μ_s would probably be one.

However, it is dependent on materials as different materials have different friction with each other. We can increase or decrease the friction between two materials, for example, if there are two materials and we smoothen them then friction gets reduced. If we lubricate any surface by using oil, then a layer of oil is formed which reduces the friction. When a body starts moving, the coefficient of friction is relatively less. If a body is at rest, the force required to initiate the motion is higher as compared to the force required to counter the friction when the body is in motion.

In simple words, the kinetic frictional force is less than the static frictional force. This is also depicted in Figure 5.7. We have to apply more force to make the body move and less force to keep it in motion. We talked about static and kinetic/sliding friction but there is a third one. There is also a type of friction which is called sliding friction. For example, when a wheel passes over a railway track, there is friction there too, but it is the rolling friction. The value of rolling friction is very low in comparison to static and kinetic friction.

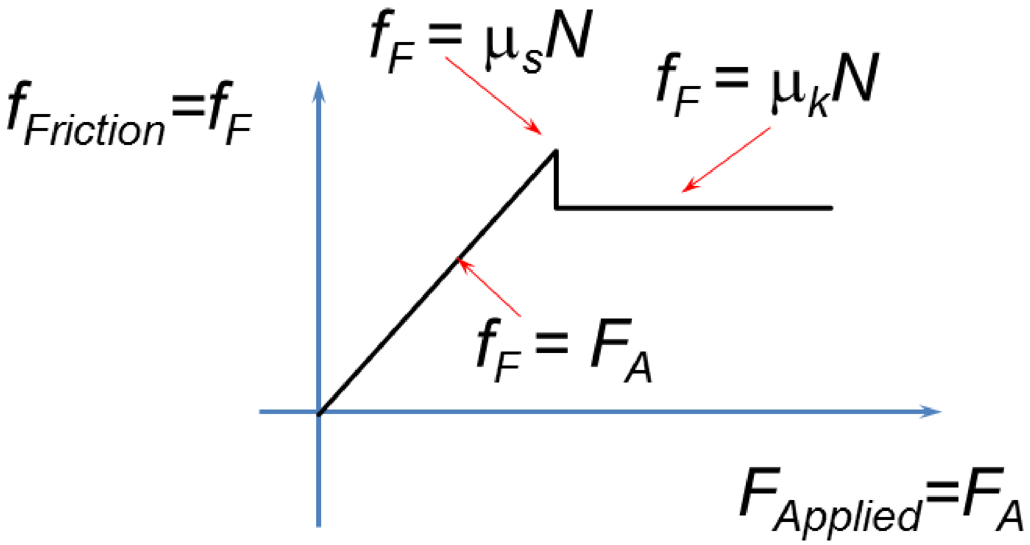
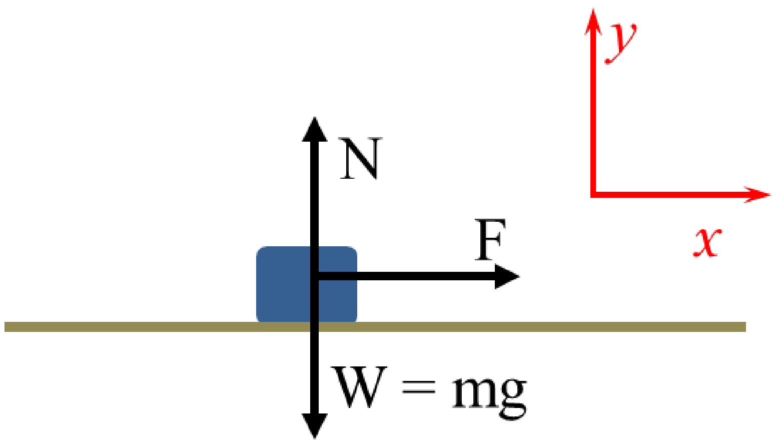


Figure 5.7: Frictional force ($f_{Friction} = f_F$) against applied force ($F_{Applied} = F_A$) showing that initially magnitude of frictional force is equal to the applied force but in the opposite direction thus body remains at rest. Once f_{MAX} is achieved the body starts moving and frictional force becomes constant i.e. $\mu_k N = \mu_k mg$ which is mostly less than f_{MAX}.

Problem:

Question: A box of mass m = 2 kg slides on a frictionless floor. A force F_x = 10N pushes on it in the x direction. What is the acceleration of the box? What forces acting on the box?

Solution:



We will solve it by applying Newton's second law along the x-axis and y-axis as shown in the figure below.

$$\sum F_x = F = ma_x$$

$$a_x = \frac{F}{m} = \frac{10N}{2kg} = 5\frac{m}{s^2}$$

$$\sum F_y = N - mg = ma_x$$

$a_y = 0$ (as no motion in y-direction) and $N = mg$

Body on an inclined plane and Angle of Repose:

Now we will consider a physical configuration where the body (plate shown in the video lecture) is on a surface/book which can be tilted. As we tilt the surface, the normal component acting on the body, because of the surface below will decrease. If we continue tilting the surface (raising one end of the surface while keeping the second end fixed), the body will start moving/sliding at a certain angle. This angle is called the Angle of Repose as shown by 'α' in Figure 5.8(a).

Now we will determine the value of the angle of repose for the physical configuration shown in Figure 5.8(a), where the body having mass M is placed on an inclined plane having angle α with horizontal which can be increased. The coefficient of static friction μ_s is also provided. The angle of repose can be calculated by determining the point till where the equilibrium condition persists. Figure 5.8(a) shows force Mg which is the weight of the body. Its components are also shown.

An unconventional xy-plane coordinate system (shown in by red inset of Figure 5.8(a)) is utilized for the sake of convenience, where weight Mg (which is in the vertical direction) is now considered making an angle α with the x-axis. Mg is resolved into its x and y components as per elaboration in Figure 5.8(b). Normal force N is perpendicular to the inclined surface and static frictional force f_s is parallel to the inclined surface. Under equilibrium, forces parallel to the plane (f_s and Mgsinα) are balanced. At an angle of repose (the angle at which the body starts sliding), acceleration due to weight (Mgsinα) will barely overcome static frictional force. Forces perpendicular to the plane (N and Mgcosα) will always be balanced.

$$f_s = Mg\sin\alpha \;\ldots\ldots.5.4$$

$$N = Mg\cos\alpha \;\ldots\ldots.5.5$$

Dividing eq. 5.4 by eq. 5.5 will give

$$\frac{f_s}{N} = \frac{Mg\sin\alpha}{Mg\cos\alpha} = \tan\alpha$$

As $f_s = \mu_s N$, above equation can be written as

$$\tan\alpha = \frac{\mu_s N}{N} = \mu_s$$

$$\alpha = \tan^{-1}(\mu_s)$$

Conclusion: The result we have derived is very reasonable and is also very easy to understand. If the friction is eliminated then the angle of repose will become zero.

$$\alpha = \tan^{-1}(\mu_s) \qquad \therefore \mu_s = 0$$
$$\alpha = \tan^{-1}(0) = 0$$

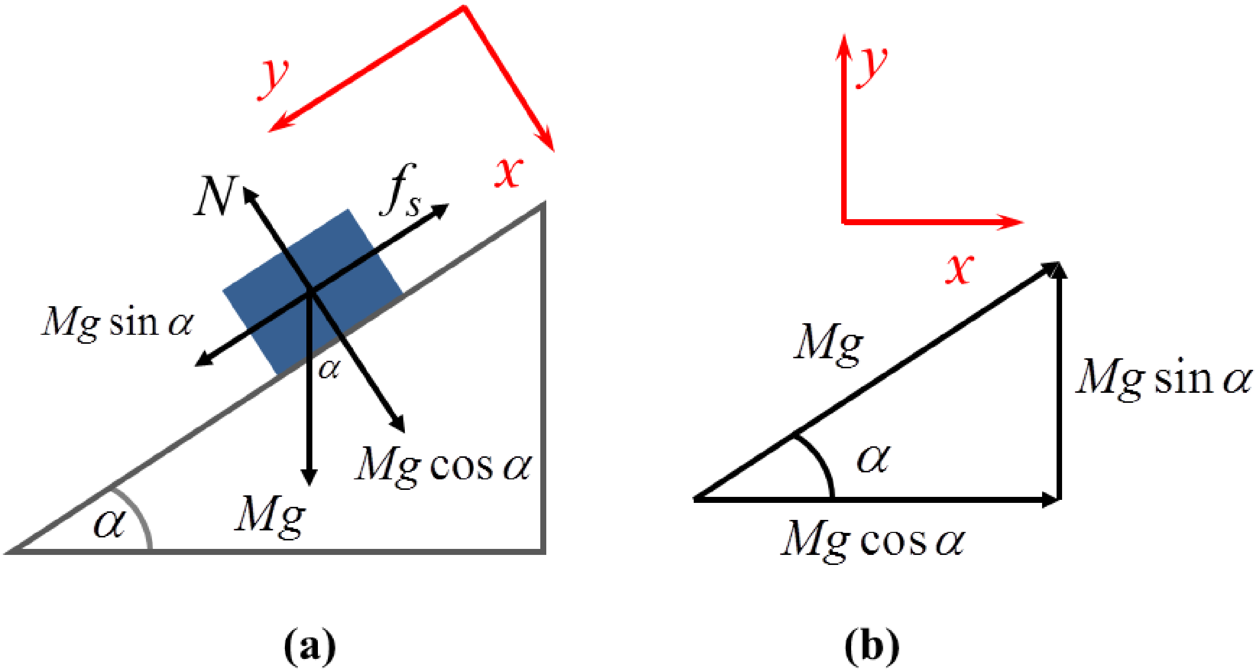


Figure 5.8: (a) Body having mass M on an inclined plane having an angle of inclination α. Note: xy-plane orientation differs from conventional representation where x and y correspond to

horizontal and vertical, respectively. (b) Components of weight Mg are shown as per normal xy-plane orientation.

Question: Why are the brakes of a car not so effective on an inclined road as compared to a levelled road?

The reason is that the normal force inclined road gets reduced. Remember that that component is $Mg\cos\alpha$. If α is zero then it is Mg and if α is 90° then it becomes zero. The brakes of a car do not work so well on an inclined road.

Question: Why a four-wheeler can use the brakes better and why does it grip better on the road?

The normal force is the same for that given coefficient of friction but instead of multiplying by two, if we multiply by four then obviously the frictional force increases. And still many concepts related to friction are still to discussed, especially that if we move through a fluid or liquid then why does friction occur? How does it occur? Can we include all effects in equations? We will discuss all these scenarios in our next lecture.