Physics

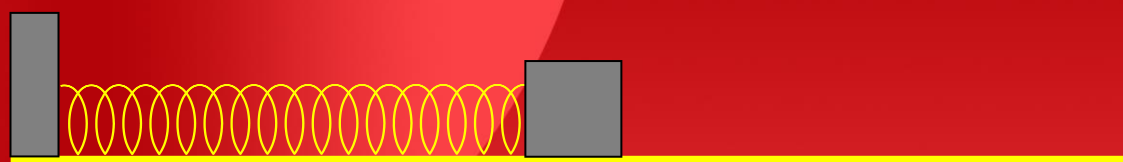
Conservation of Energy

POTENTIAL ENERGY

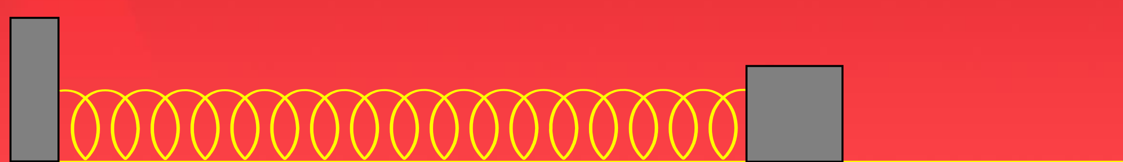
- Elastic
- Gravitational
- Electric

Elastic potential energy

Kinetic energy



Elastic potential energy



The spring pulls/pushes with a *restoring force proportional to the extension x* :

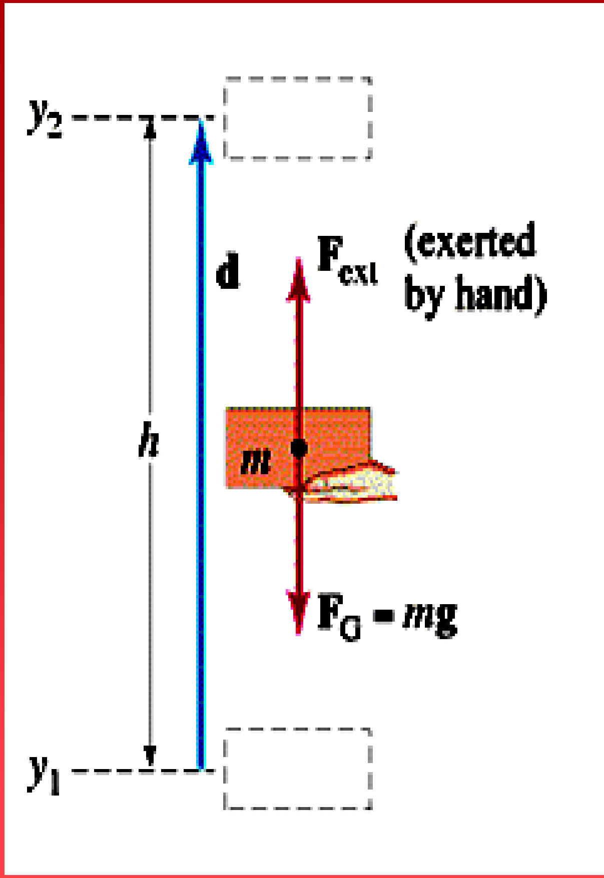
$$F_{spring} = -k\,x$$

Work due to external force gives the *elastic potential energy*:

$$W = \int_0^x F dx = \int_0^x k\,x\,dx = \frac{1}{2} k\,x^2$$

How much work does it take to lift mass *m* to height *h* ?

$$W_{ext} = F_{ext}\, d$$
$$= mg\, h$$
$$= m\, g\, (y_2 - y_1)$$



You did work on the object. Therefore its energy increased.

POTENTIAL ENERGY !!

- PE is measured with respect to some reference level.
- Only *changes* in PE actually have physical meaning.
- Changes in PE do not depend on path.
- Energy is a shared property !

- Work is force x distance
- Energy is the capacity to do work
- Power is the "rate of doing work"

$$\boxed{\text{Power} = \frac{\text{Work done}}{\text{Time taken}}}$$

YOUR HEART AS A PUMP

Volume of blood lifted daily = 8000 litres

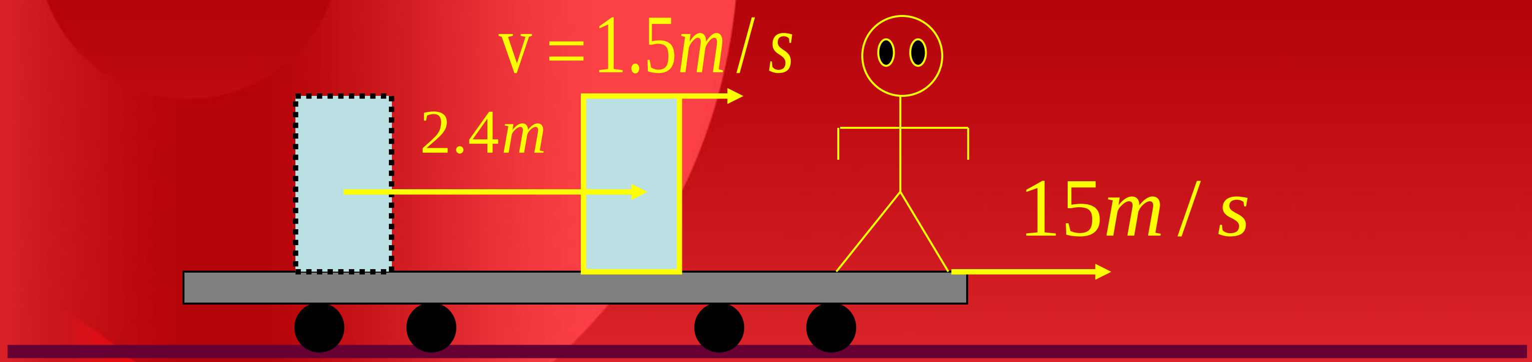
Average height lifted = 1.5 m

Density of blood $\approx$ 1 kg/litre

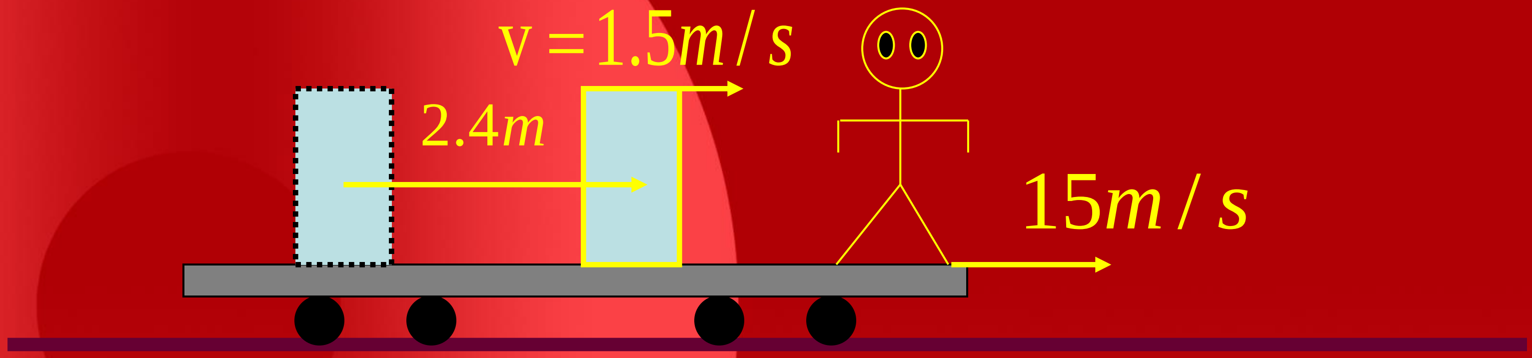
Work done = 8000 x 1 x 9.8 x 1.5

$\approx$ 120,000 J in 24 hours

$$\therefore \quad \text{Power} = \frac{120000}{24 \times 60 \times 60} \approx 1.4 \text{ W}$$



v = 1.5m / s
2.4m
15m / s



$$\text{mass of box} = 12\ kg$$

$$\Delta K = K_f - K_i = \frac{1}{2}(12kg)(1.5m/s)^2 - 0 = 13.5J$$

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(1.5m/s)^2 - 0}{2(2\cdot 4m)} = 0.469m/s^2$$

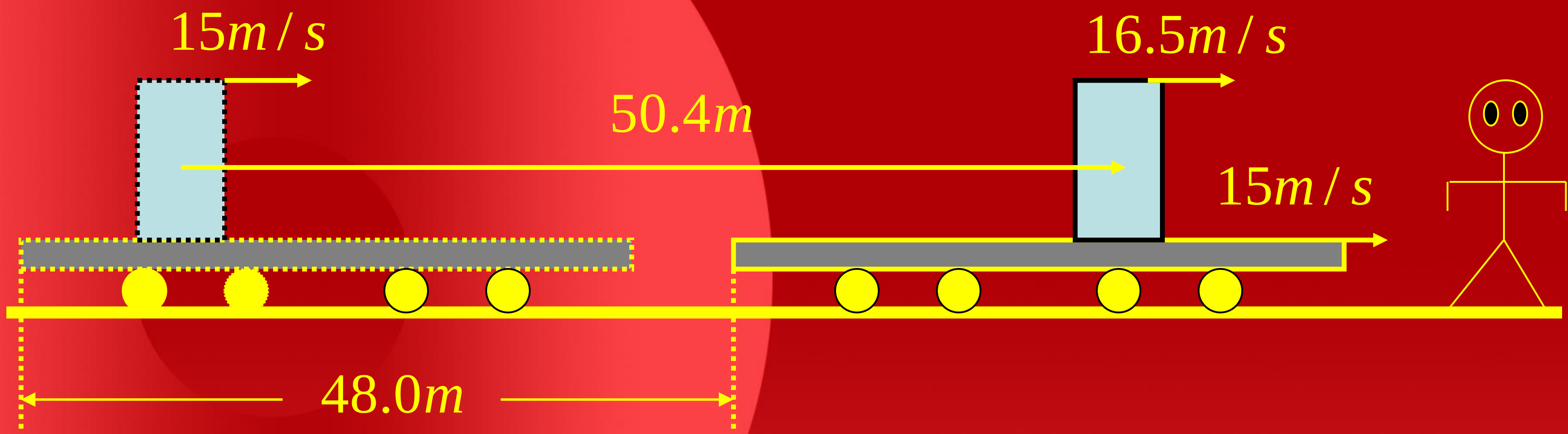
This acceleration results from a constant net force given by:

$$F = ma = (12kg)(0.469\,m/s^2) = 5.63N$$

Work done on the crate is:

$$W = F\Delta x = (5.63N)(2.4m) = 13.5J$$

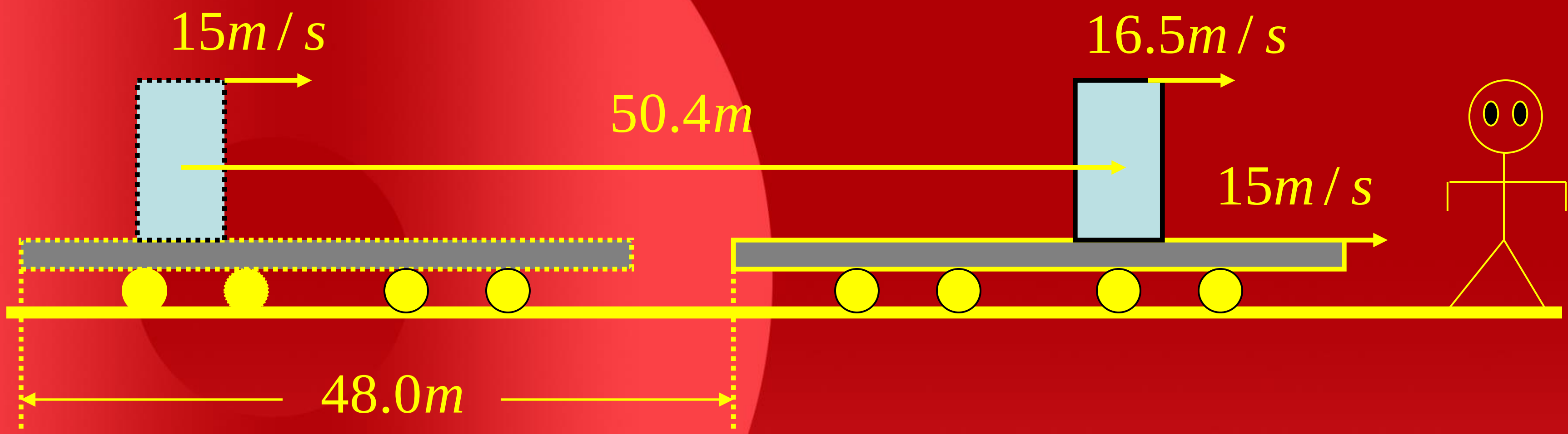
(same as $\Delta K = 13.5J$!)



How does an observer on the ground interpret a similar measurement?

$$v_i = 15.0 m/s$$

$$v_f = 15.0 m/s + 1.5 m/s = 16.5 m/s$$



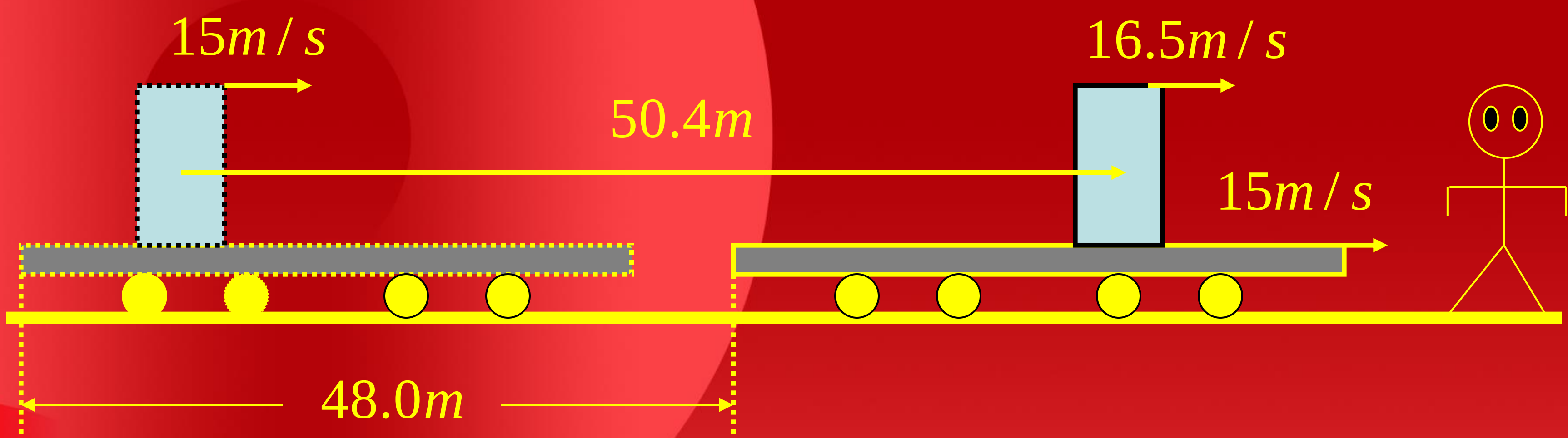
$$\Delta K' = K'_f - K'_i = \frac{1}{2} m \mathrm{v_f}^2 - \frac{1}{2} m \mathrm{v_i}^2$$

$$= \frac{1}{2}(12kg)(16.5m/s)^2 - \frac{1}{2}(12kg)(15.0m/s)^2$$

$$= 284J$$

(This is not equal to $\Delta K = 13.5J$)

$$\because \ a' = a \quad \therefore \ F' = F = 5.63N$$



$$t = \frac{v_f - v_i}{a} = \frac{1.5m/s}{0.469m/s^2} = 3.2s$$

and train moves (15m/s)(3.2s) $= 48m$

$$\text{Total displacement} = \Delta x'$$

$$= 48m + 2.4m = 50.4m$$

The ground based observer also concludes that the work is:

$$W' = F'\Delta x' = (5.63N)(50.4m) = 284J$$

Total mechanical energy is:

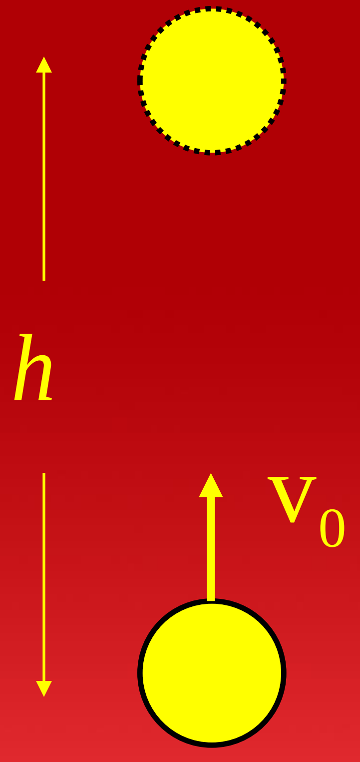
E_{mech} = $KE + PE$

IF no friction then E_{mech} is conserved :

$\Delta(\mathrm{E_{mech}}) = \Delta(KE) + \Delta(PE) = 0$

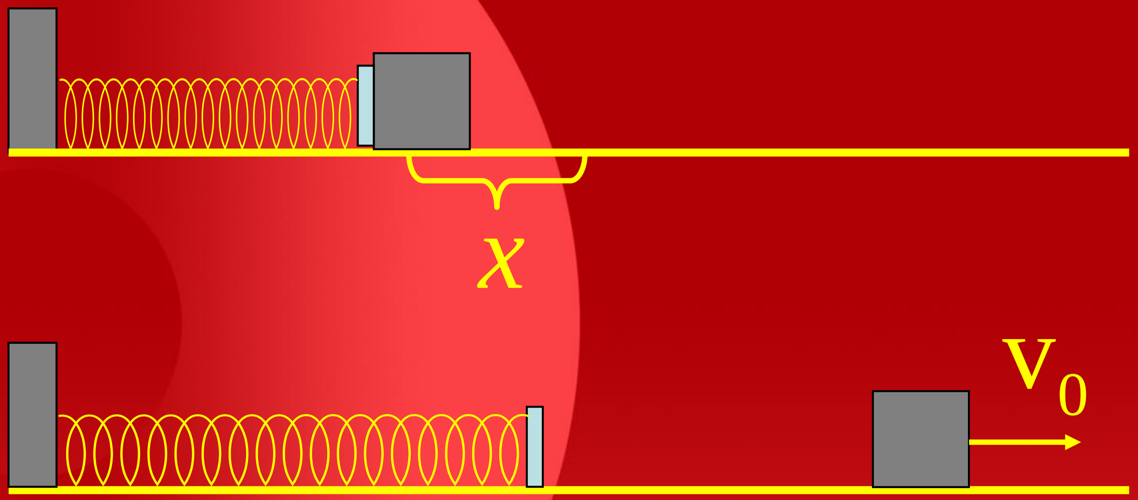
E_{mech} = $KE + PE$ is *constant !!!*

How high will the ball rise?

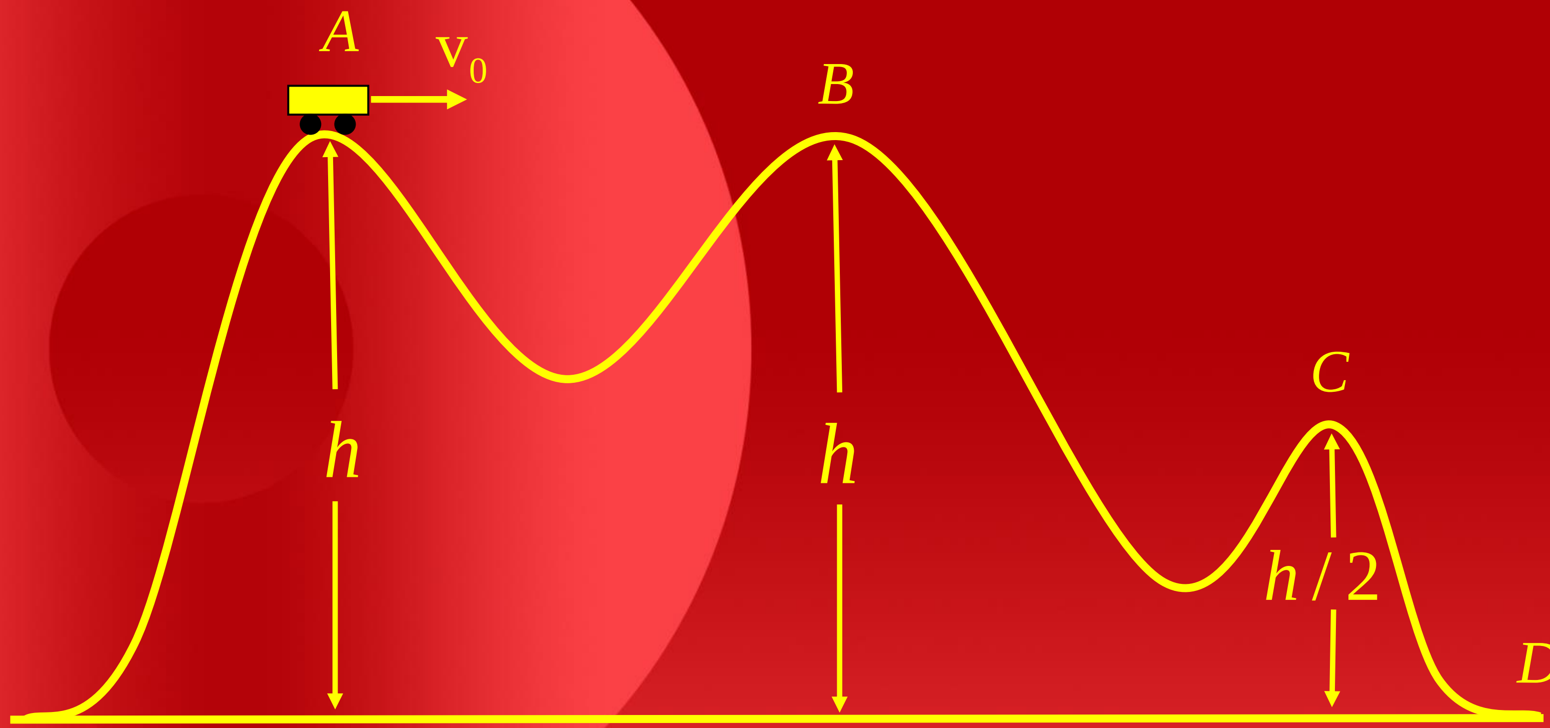


$$\frac{1}{2} m v_0^2 = mgh$$

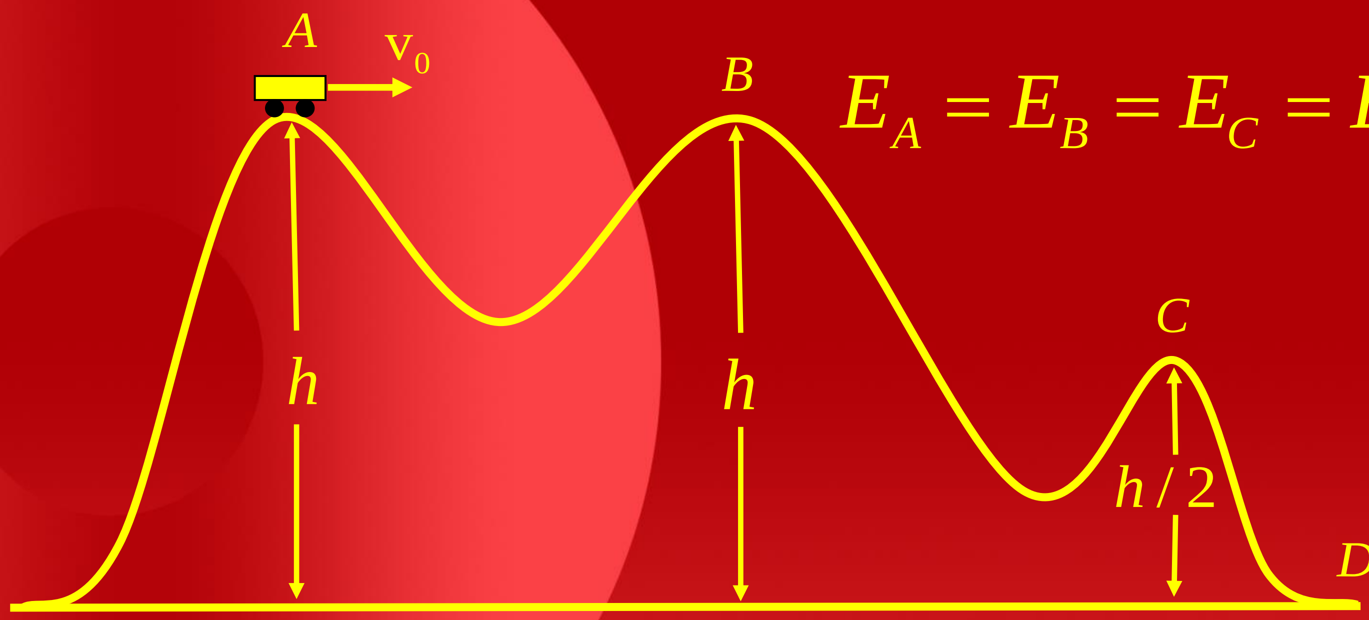
$$\Rightarrow \quad h = \frac{v_0^2}{2g}$$



$$\frac{1}{2}m v_0^2 = \frac{1}{2}kx^2 \quad \Rightarrow \quad v_0 = \sqrt{\frac{k}{m}}\,x$$



What will be the speed of the car a) at point B, b) at point C, and c) at point D?

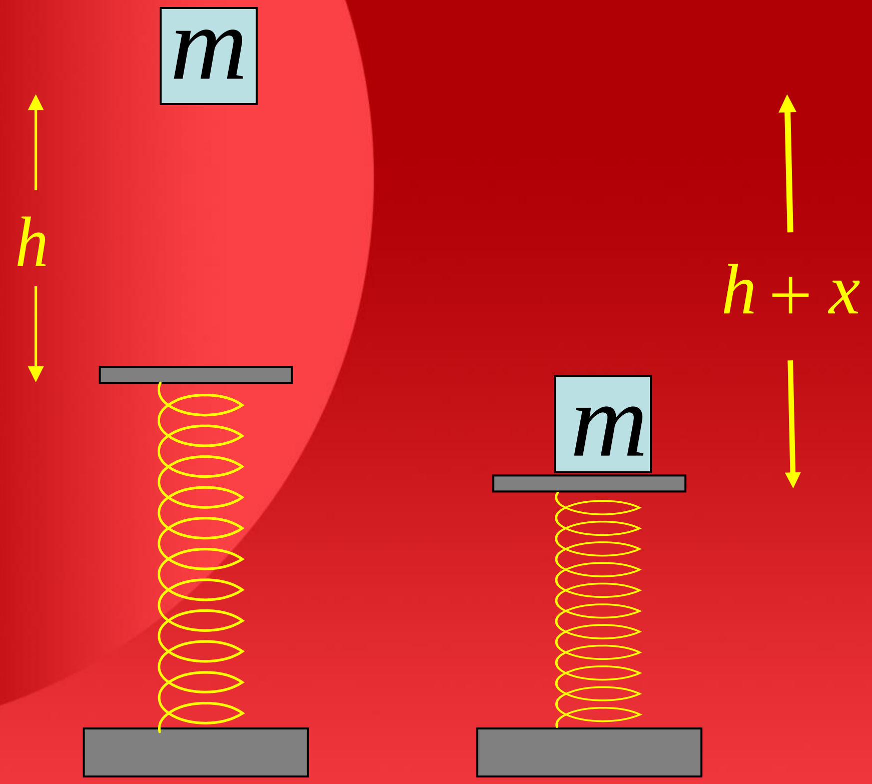


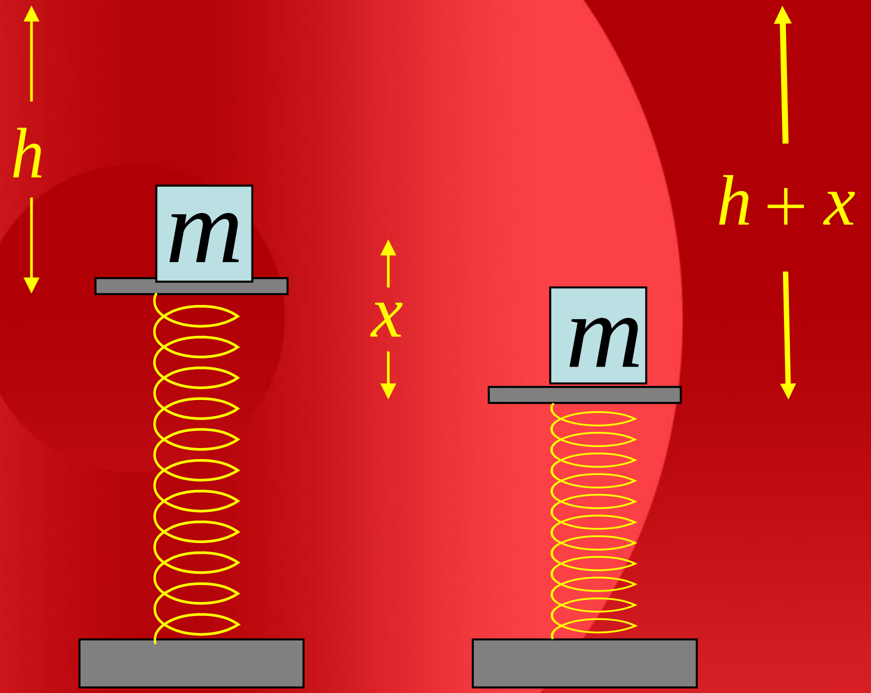
$$E_A = E_B = E_C = E_D$$

$$\frac{1}{2}mv_A{}^2 + mgh = \frac{1}{2}mv_B{}^2 + mgh$$

$$= \frac{1}{2}mv_C{}^2 + mg\frac{h}{2} = \frac{1}{2}mv_D{}^2$$

$$v_A = v_B \ , \ v_C = \sqrt{v_0{}^2 + gh} \ , v_D = \sqrt{v_0{}^2 + 2gh}$$

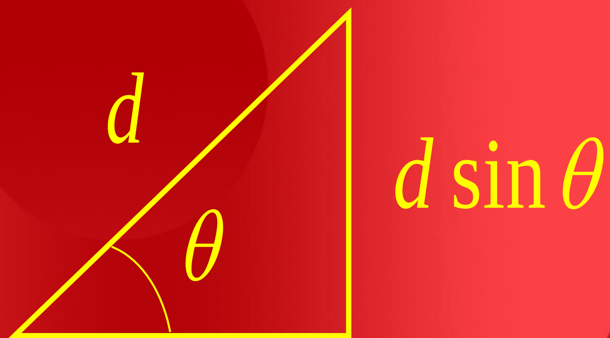

m
h
m
h + x

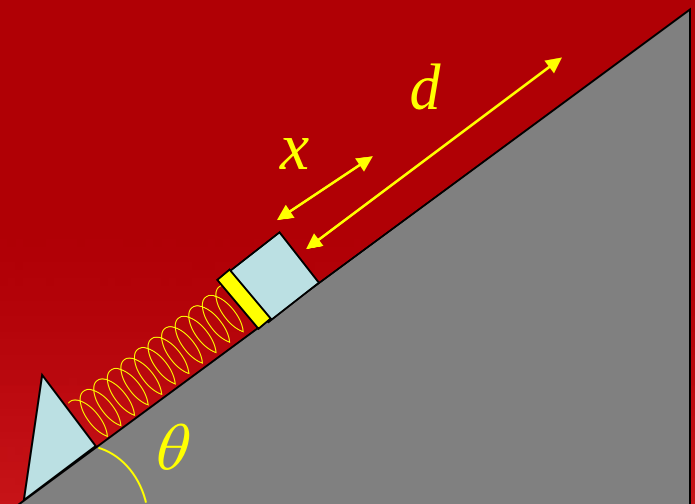


$$\frac{1}{2}kx^2 = mg(h+x)$$

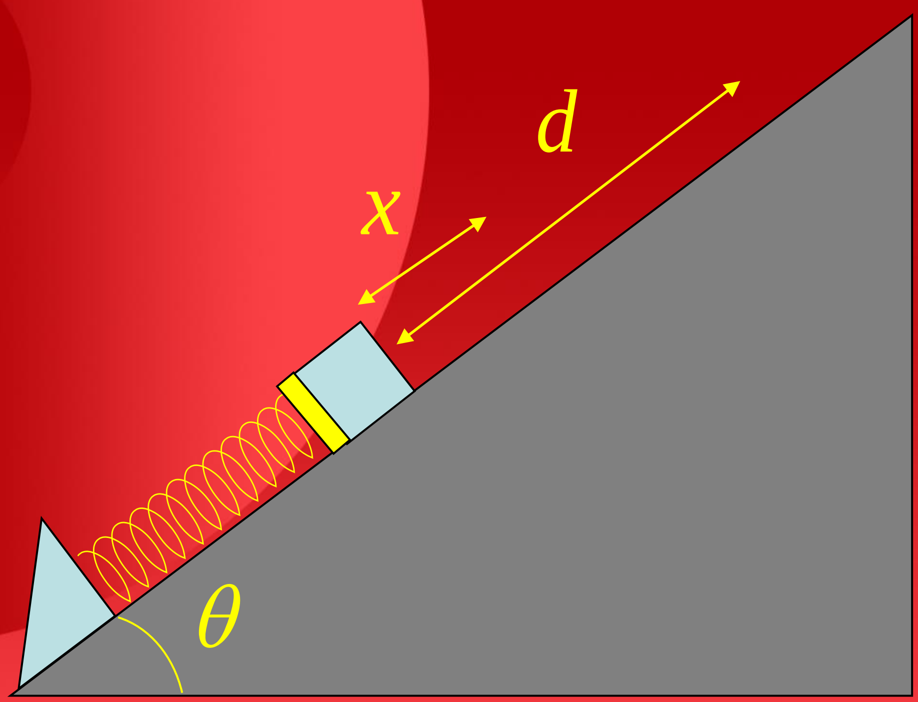
$$x^2 - \frac{2mg}{k}x - \frac{2mg}{k}h = 0$$

$$\Rightarrow x = \frac{mg}{k}\left(1 \pm \sqrt{1 + 2hk/mg}\right)$$

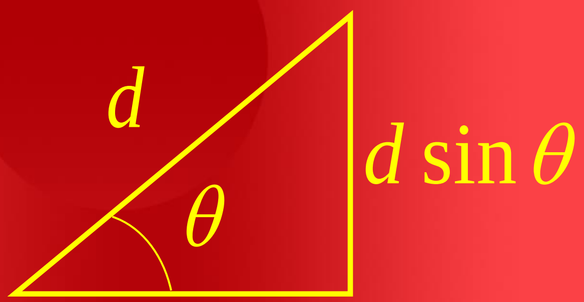


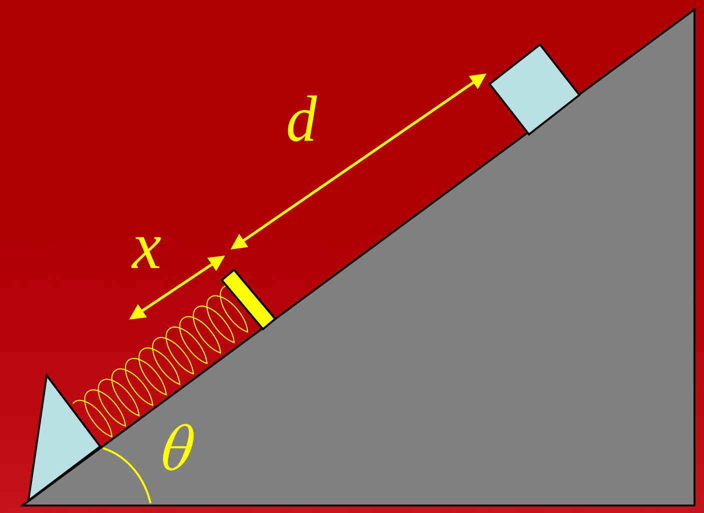


$$\frac{1}{2}kx^2 = mgd\sin\theta \quad \Rightarrow \quad d = \frac{kx^2}{2mg\sin\theta}$$

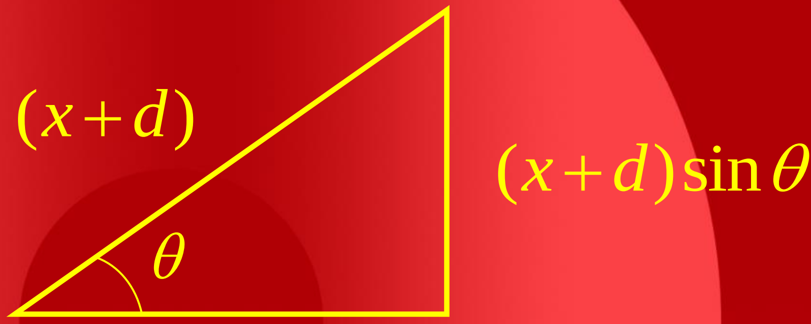


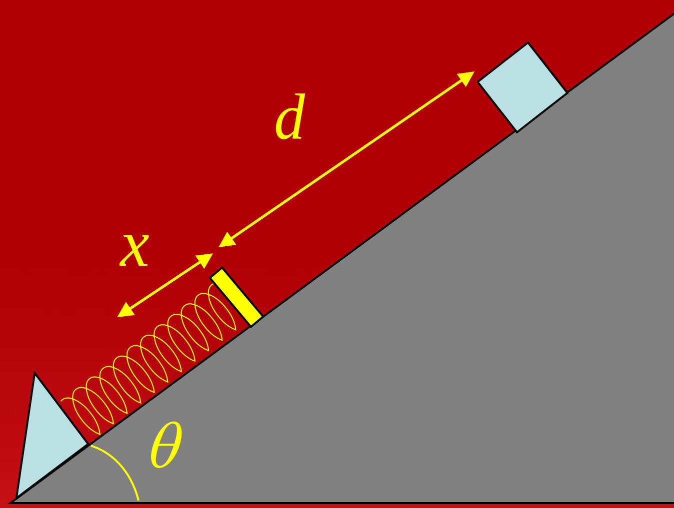
d
x
θ





$$mgd\sin\theta = \frac{1}{2}m\mathrm{v}^2 \quad \Rightarrow \quad \mathrm{v} = \sqrt{2gd\sin\theta}$$

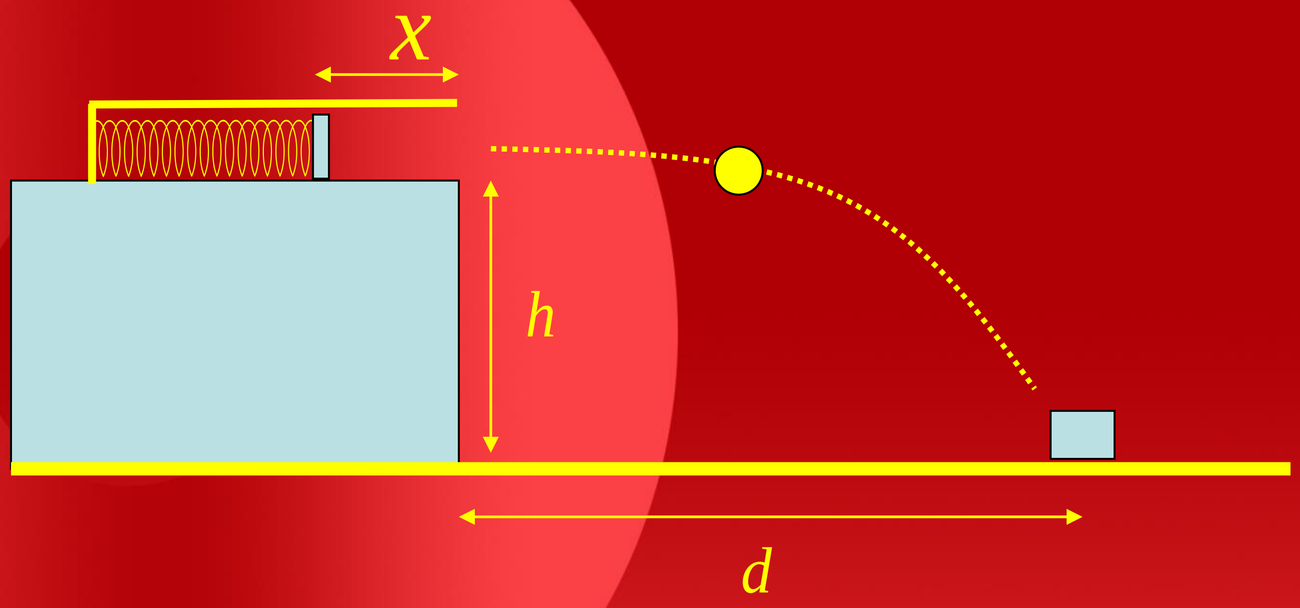




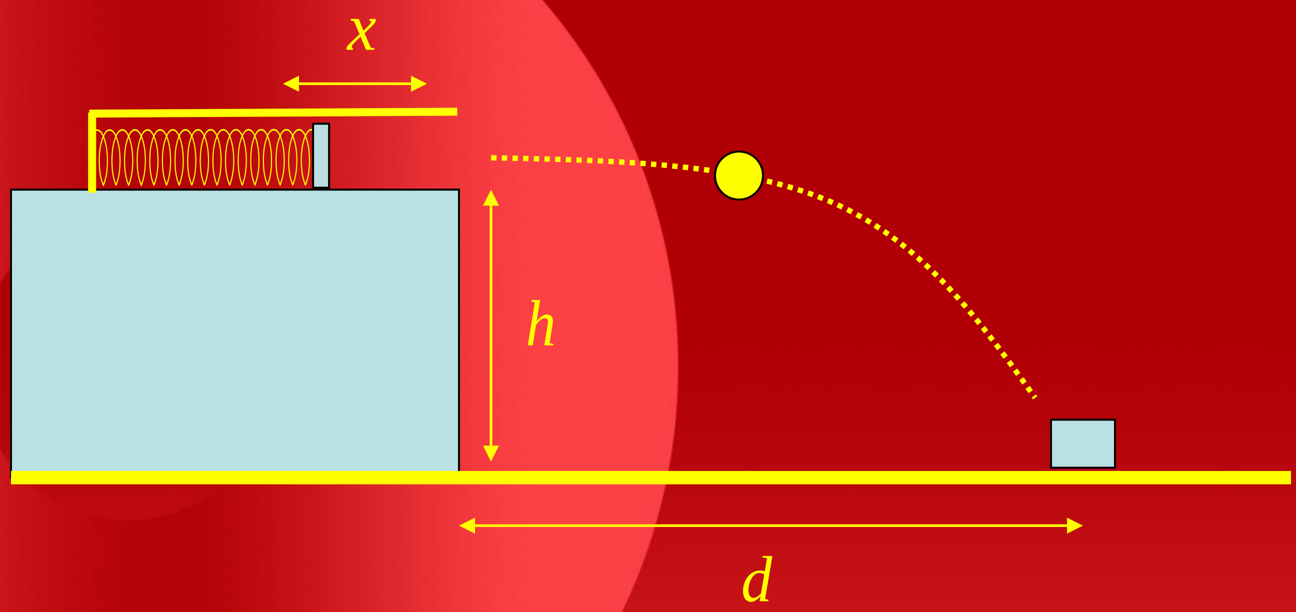
$$mg(x+d)\sin\theta = \frac{1}{2}kx^2$$

$$x^2 - 2\left(\frac{mg\sin\theta}{k}\right)x - 2\left(\frac{mg\sin\theta}{k}\right)d = 0$$

$$x = \frac{mg\sin\theta}{k}\left(1 \pm \sqrt{1 + \frac{2kd}{mg\sin\theta}}\right)$$



$$\frac{1}{2}m\mathrm{v}_0^2 = \frac{1}{2}kx^2 \quad \Rightarrow \quad x = \sqrt{\frac{m}{k}}\,\mathrm{v}_0$$



$$d = v_0 t \quad \text{and} \quad h = \frac{1}{2} g t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2h}{g}}$$

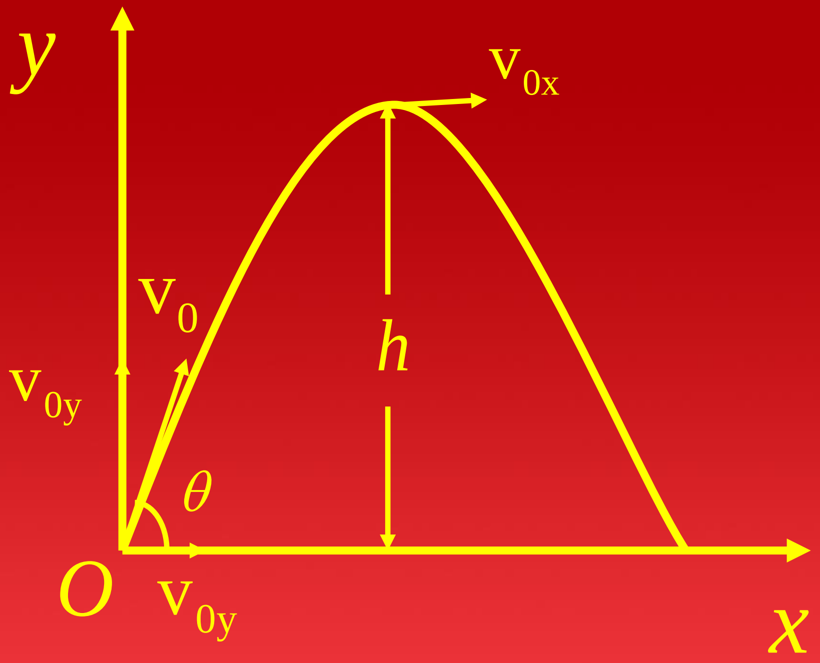
$$\therefore \; v_0 = \frac{d}{t} = d\sqrt{\frac{g}{2h}} \quad \Rightarrow \quad x = d\sqrt{\frac{mg}{2hk}}$$

Maximum height of a projectile, using energy methods

$$K_0 + U_0 = K + U$$

$$\frac{1}{2}m(\mathrm{v}_{0x}^2 + \mathrm{v}_{0y}^2) + 0$$

$$= \frac{1}{2}m\mathrm{v}_{0x}^2 + mgh$$



$$\frac{1}{2}m\mathrm{v}_{0y}^2 = mgh \quad \Rightarrow \quad h = \frac{\mathrm{v}_0^2 \sin^2\theta}{2g}$$

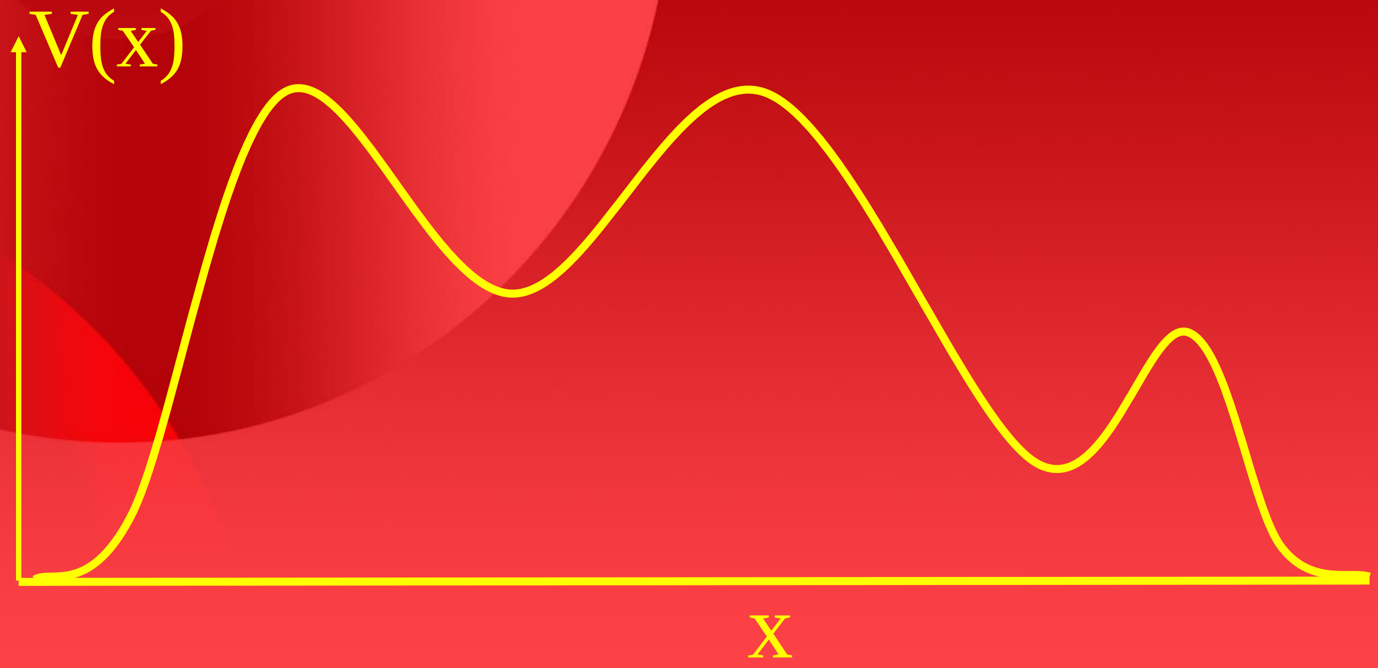
CONSERVATIVE FORCE

Work does not depend on path take

- gravity
- electric force
- springs

Potential energy can be defined!

Suppose a particle has potential energy V(x)



If the particle moves Δx

Then change in PE is $\Delta \mathrm{V}$

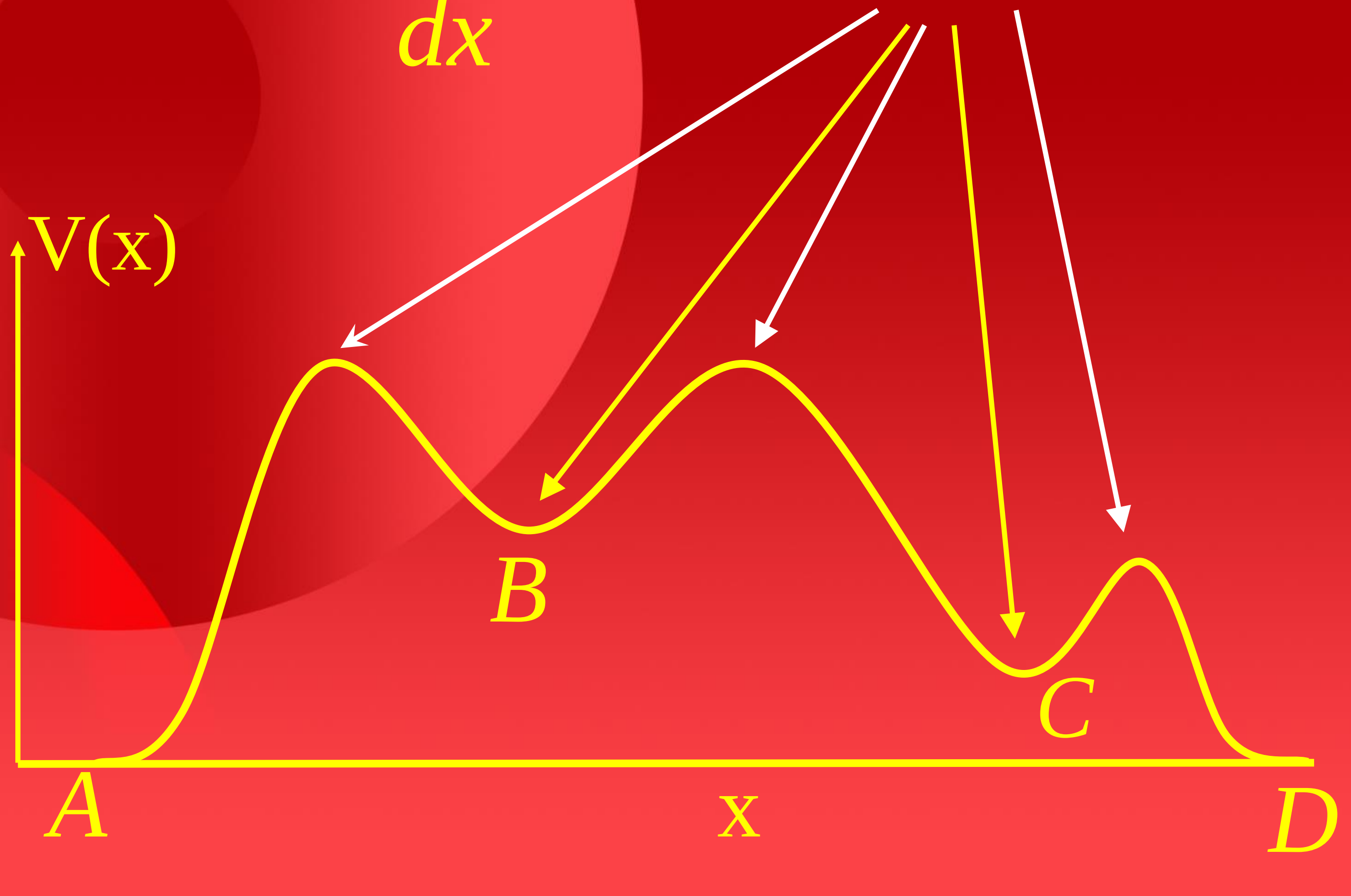
Where: $\Delta \mathrm{V} = - \mathrm{F}\, \Delta x$

$$\Rightarrow \qquad \mathrm{F} = - \frac{\Delta \mathrm{V}}{\Delta x}$$

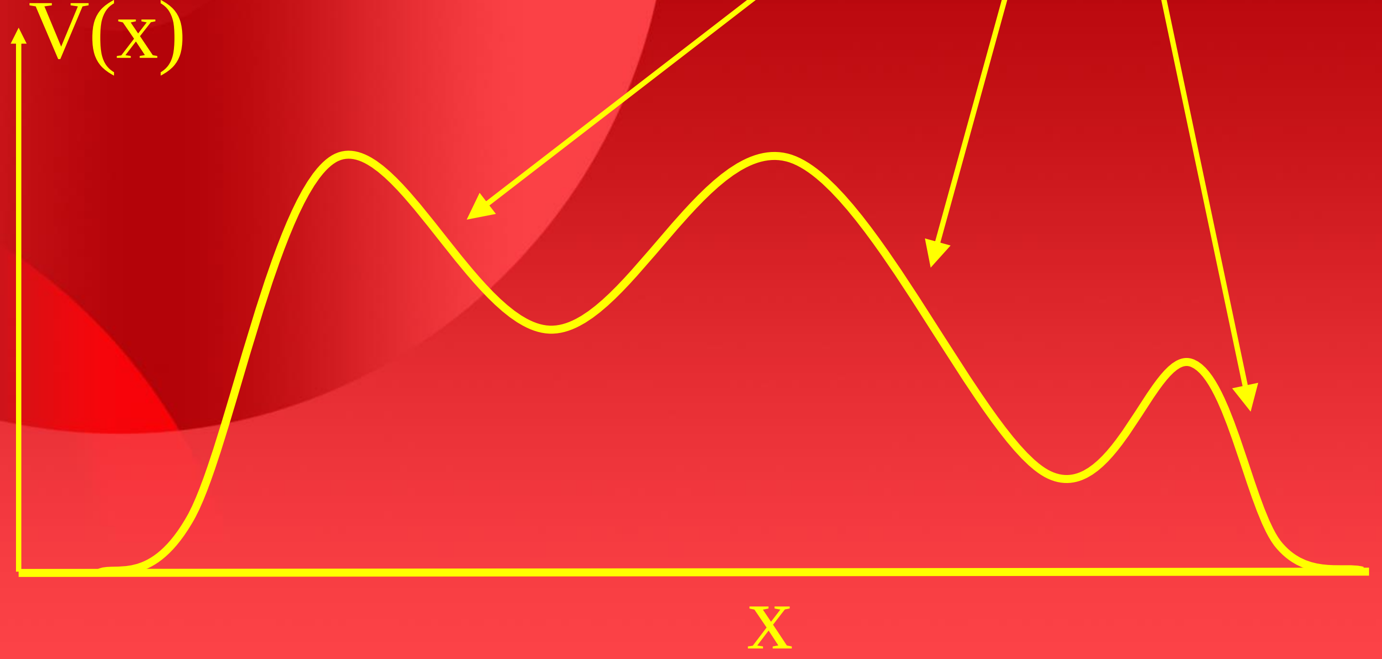
Now let $\Delta x \rightarrow 0$

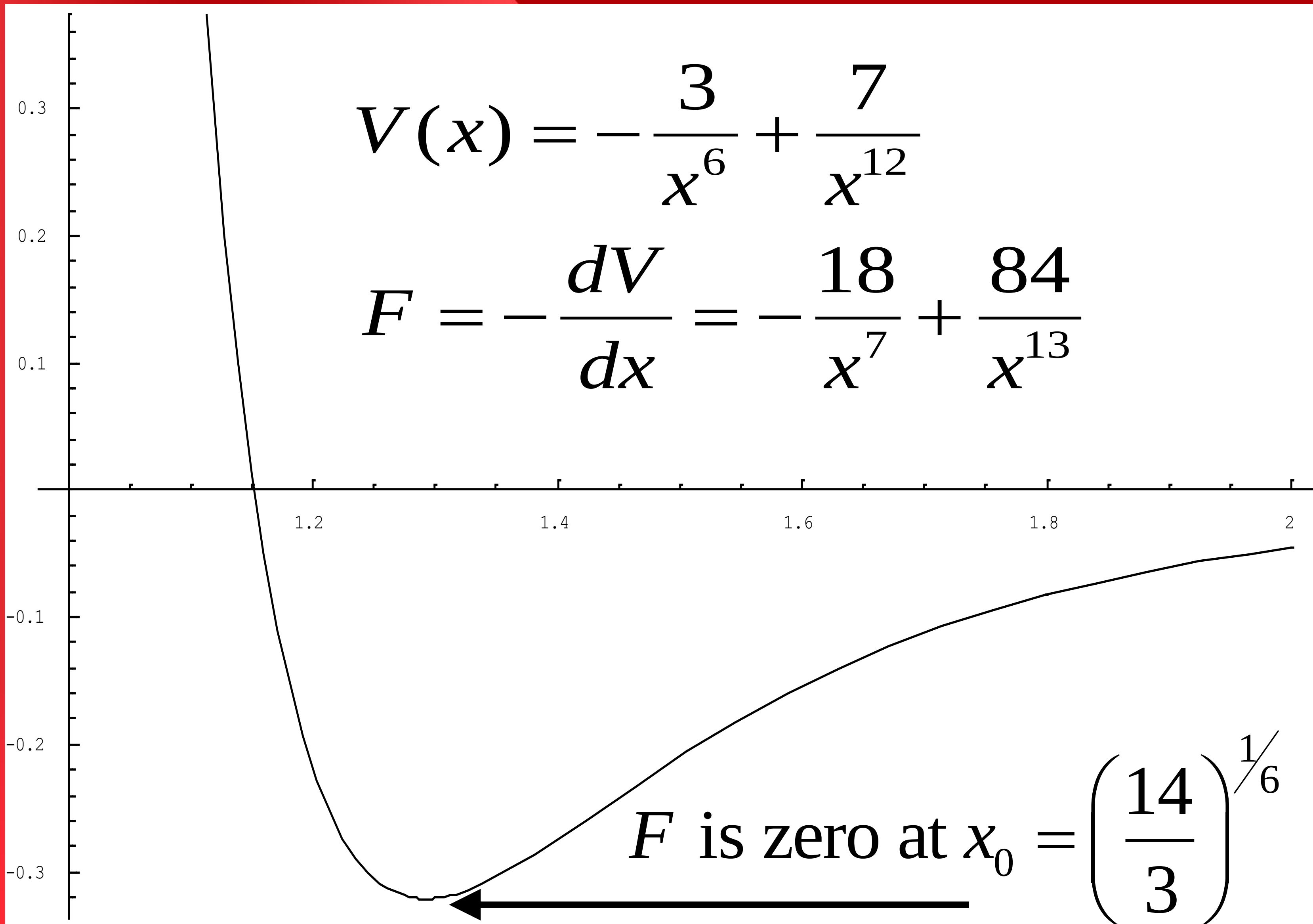
$$\boxed{\mathrm{F} = - \lim_{\Delta x \rightarrow 0} \frac{\Delta \mathrm{V}}{\Delta x} = - \frac{dV}{dx}}$$

$$F = -\frac{dV}{dx} \text{ is zero here}$$



$$F = -\frac{dV}{dx}$$ is positive here





$$V(x) = -\frac{3}{x^6} + \frac{7}{x^{12}}$$

$$F = -\frac{dV}{dx} = -\frac{18}{x^7} + \frac{84}{x^{13}}$$

F is zero at $x_0 = \left(\frac{14}{3}\right)^{1/6}$

NON-CONSERVATIVE FORCE

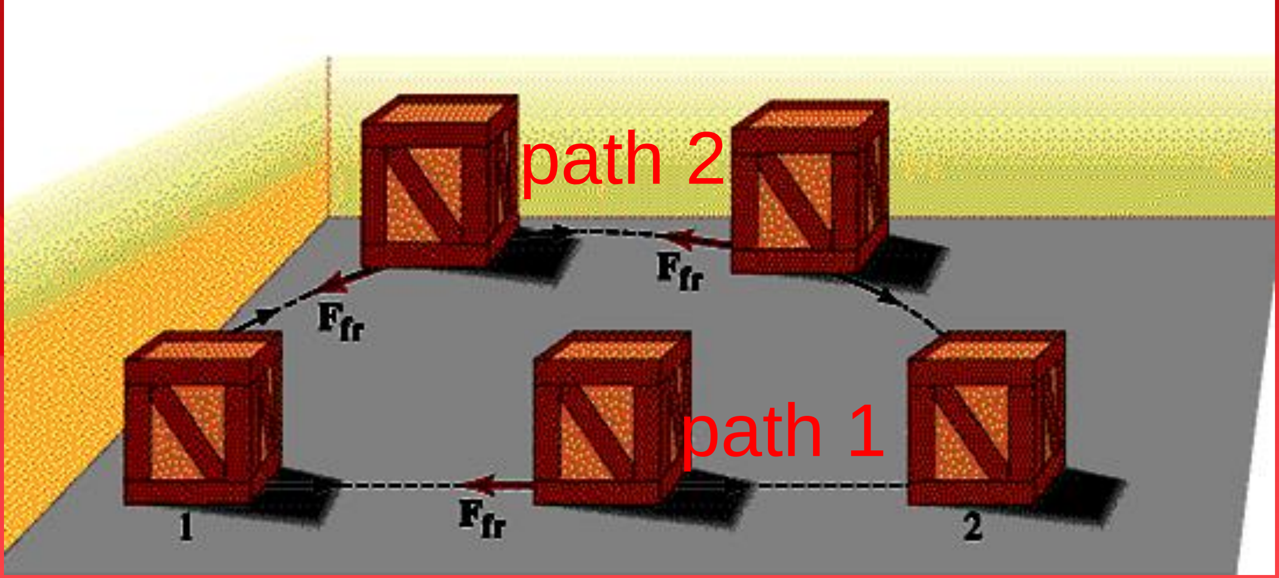
Work <u>does</u> depend on path taken.

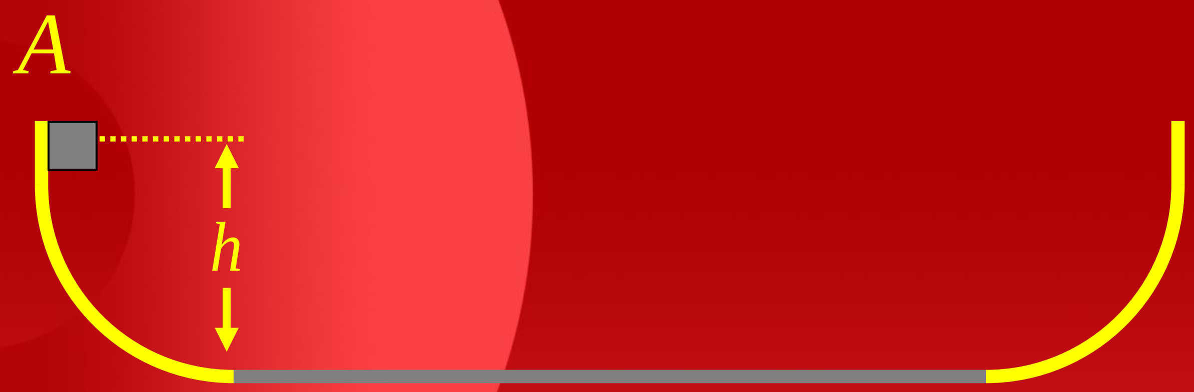
- friction
- air resistance
- velocity dependent forces

Potential energy cannot be defined!

Work due to friction

The work done to push the box along a path of length L is: $W_f = F_f L = \mu N L = \mu mg L$

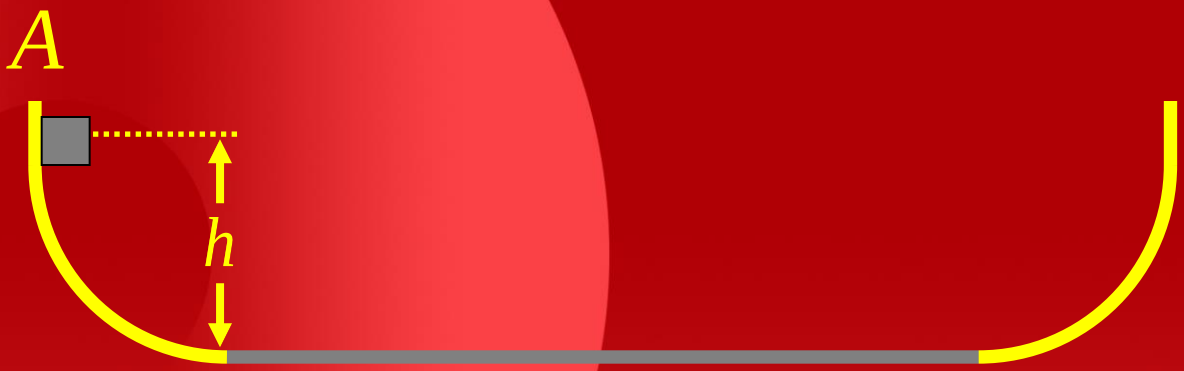




Where does the particle finally come to rest?

What happens to energy in a non-conservative system?

Is the energy lost when a driver puts the car brakes on?



Suppose the total distance moved on the flat part before it comes to rest is x.

$$mgh = fx = \mu mgx \quad \Rightarrow \quad x = \frac{h}{\mu g}$$

Conservation of energy is a truth

Mechanical energy conservation is Newton’s law.

There is something mysterious about it

Energy takes different forms

Yet each must be capable of doing work