Physics-PHY101-Lecture #02

Kinematics

"Kinematic" is derived from the Greek word "kinesis," which means to move or motion. Our world is full of motion. For example, cars move, birds can fly, and fish can move underwater. It is necessary to understand motion and its causes.

The main topics of this lecture are:

- Displacement
- Velocity
- Acceleration

Whenever we discuss the motion of a body, it is essential to locate its position. To achieve this, we need an origin point. For example, a point x has a value of x = 0 when stationary, but as it starts moving, its values begin to increase.

If the position of the vector x depends on "t," then we can express its position as x(t), where x is referred to as a function of "t." (Think of a function as a factory where you input raw material and receive products, like inputting a number into a function and obtaining a result as output.)

In kinematics, we require a specific function denoted as x (t). This function allows us to input a value of t and obtain the position of a moving particle.

Now, let's delve into the concept of "Displacement."

Displacement:

As we know, the position at time t is denoted as x(t). Here, "t" can be any number, such as 9, 7, 6, etc. Similarly, when a particle moves at different times "t", it may be denoted with different notations, such as $x(t_1)$ and $x(t_2)$, and its function may be defined as x(t). Now, when we subtract these two quantities, we will obtain a value $\triangle x$. The displacement $\triangle x$ in time interval $\triangle t$ = t_2-t_1 is:

$$\triangle x = x(t_2) - x(t_1)$$

Sometimes we don't want to write equation as $x(t_1)$ and $x(t_2)$ so we do write the above equation as:

$$\triangle x = x_2 - x_1$$

This is what we call displacement. Now consider that a particle is moving in one dimension. Imagine a graph of particle's position along the x-axis as a function of time.

From the Figure. 2.1 we can see that when the particle is at position x_1 the time is t_1 and when at x_2 the time is t_2.

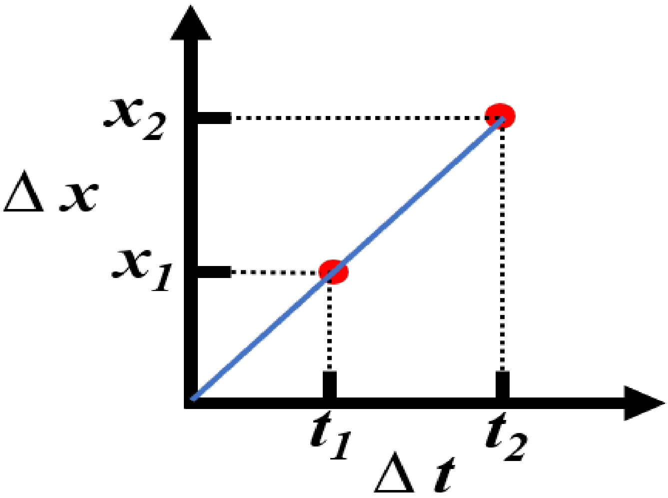


Figure 2.1. Displacement vs. time graph of an object.

If an object as shown in figure 1. is at 10 m from origin at t_1 and reach at 30 m at time t_2, then, we can define displacement $\triangle x$ as:

$$\triangle x = x_2 - x_1$$

$$=30 - 10$$

$$\triangle x = 20 \text{ m}$$

But if the object at $t_1$is at 30 m and reach at 10 m at t_2, then the magnitude will remain same but negative sign appears due to net displacement in negative direction.

$$\triangle x = x_2 - x_1$$

$$=10 - 30$$

$$\triangle x = - 20 \text{ m}$$

This is what we call displacement, which can be both positive and negative.

Speed and Velocity:

Speed and velocity measure how position changes with time. There are two major concepts to consider. First, let's look at average speed:

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

If we divide the particle's total distance travelled by the total time taken, we can determine the average speed of that particle. Average speed can vary, being either maximum or minimum. Meanwhile, average velocity can be defined as:

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{x_2 - x_1}{t_2 - t_1}$$

$$v = \frac{\Delta x}{\Delta t}$$

Note that distance is always positive, while displacement can be positive or negative. It's important to clarify that this is average velocity because we are dividing displacement by time, not distance by time. This is also known as slope or gradient. For example, consider a car moving on a slope – the steeper the slope, the greater the gradient.

Now, consider the Figure. 2.2 (a) where the distance covered by two points creates a slope from which average velocity is calculated.

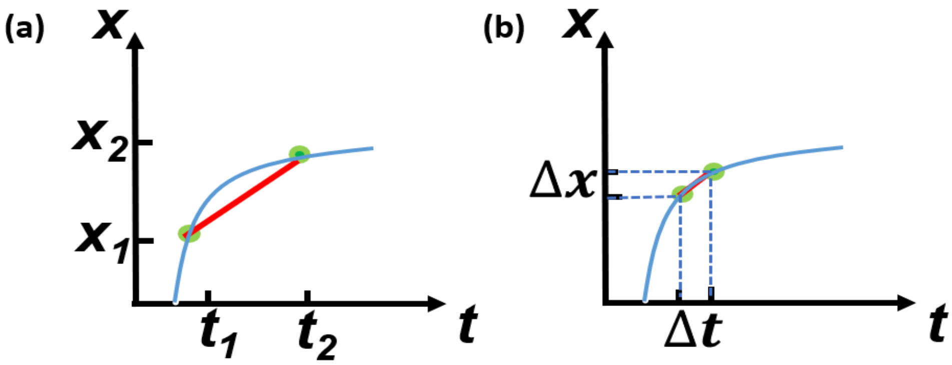


Figure 2.2. Displacement vs. time graph of an object presenting (a) constant velocity, (b) Instantaneous velocity.

In contrast**, instantaneous velocity** is determined by taking two times very close to each other, with the interval between them approaching zero. In Figure 2.2 (b). if we take two values so close to each other that they approach zero, we can call it instantaneous velocity. Now, let's discuss what is meant by "being too close." Being too close implies approaching a distance of zero. (It's important to note that approaching zero doesn't mean you are making the distance zero.)

Acceleration

Acceleration measures how the velocity changes with time. As we have defined average velocity, we can also define average acceleration as:

$$\text{Average acceleration} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\triangle v}{\triangle t}$$

We can say that average acceleration is change in velocity divided by time taken to undergo that change. It is also known as slope of graph of velocity against time.

$$a = \lim_{\triangle t \to 0} \frac{\triangle v}{\triangle t}$$

Now to understand definition better we will again construct a graph as shown in Figure. 3:

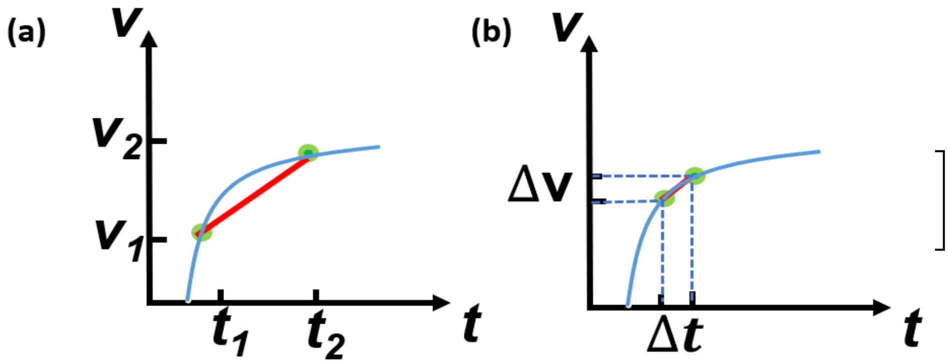


Figure 2.3. Velocity vs. time graph of an object presenting (a) constant acceleration, (b) Instantaneous acceleration.

When t_1 and t_2 come closer, v_1 and v_2 also come closer and we draw a tangent line at that point which is known as slope of tangent acceleration as shown in Figure 2.3 (b). Remember that we are approaching t to zero, not making it zero.

The acceleration can be positive and negative. Moreover, it is not necessary for acceleration to be in the direction of velocity. They can have different direction as shown in Figure 2.4, in which velocity of train is decreasing as time passes which presents deacceleration. By convention we take positive x-axis in right direction and negative x-axis in left direction. If the velocity of an object is increasing in positive x-axis direction than its **acceleration is positive.** If the velocity of the object is increasing in negative x-axis, then its **acceleration will be negative**, but it never be deacceleration or retardation. The SI unit of acceleration is m s^{-2}.

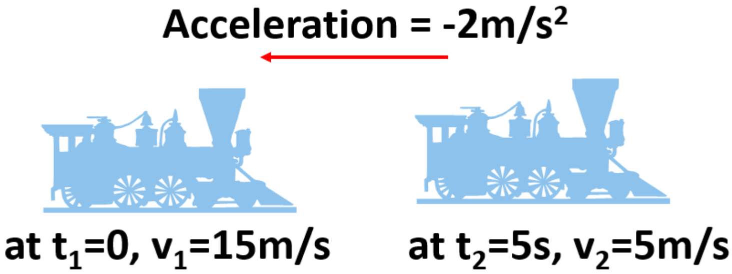


Figure 2.4. The negative acceleration of a train.

Constant Acceleration

When an object's acceleration stays constant over time, it's referred to as having constant acceleration. Stated differently, the object's velocity changes at a constant rate.

Mathematically:

Let us talk about constant acceleration in more detail. For this purpose, let's do simple calculations.

For convenience, take:

$$t_1 = 0,\ t_2 = \mathrm{t}$$

Then: $x_1 = x_o$ and $x_2 = x$

$$v_1 = v_o \text{ and } v_2 = v$$

At time t_1, the velocity of particle is v_1 which is v_o, and at time t_2 its velocity becomes $v_2 = v$. If acceleration is constant, then we can write the average acceleration as:

$$a_{vg} = \frac{v_2 - v_1}{t_2 - t_1}$$

And as acceleration is constant, so, v is equals to,

$$v = v_o + at \text{ ………. (2)}$$

$$\therefore \mathrm{a = v/t,\ so\ v = at}$$

The average velocity as "it is the average of v and v_o divided by 2".

$$v_{av} = \frac{1}{2}(v_o + v)$$

If we want to write in previous notation, then:

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1}$$

If we substitute values from equation given above, then:

$$\frac{x - x_o}{t - 0} = \frac{v + v_o}{2}$$

$$\frac{x - x_o}{t} = \frac{v_o + at + v_0}{2}$$

$$\frac{x - x_o}{t} = v_o + \frac{1}{2}at$$

$$x = x_0 + v_o t + \frac{1}{2}at^2$$

As we have discussed earlier that x is function of t. So, by putting t = 0 in above equation, we will have value of x as x_o, which is the initial point from, we started.

From all that procedure we have got two main equations:

$$x = x_0 + v_o t + \frac{1}{2}at^2 \ldots\ldots. \text{(A)}$$

$$v = v_o + at \ \ldots\ldots\ldots. \text{(B)}$$

Now let's plot again them on a graph for better understanding:

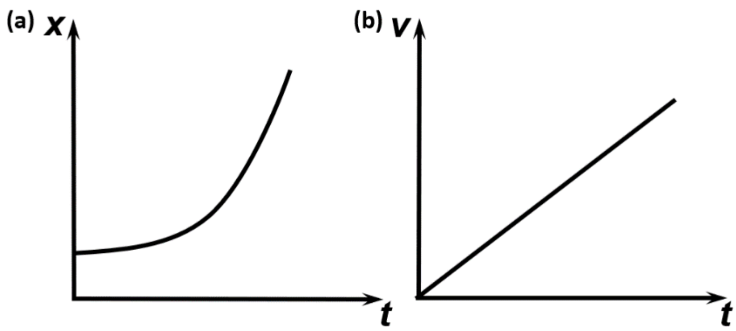


Figure 2.5. (a) Displacement vs. time, (b) velocity vs. time graph of an object with constant acceleration.

From the graphs in Figure. 2.5 it can be observed that as the time increases particle start moving away from the origin and its velocity increases linearly, while its displacement increases quadratically.

From equation (A) and (B),

$$t = \left[\frac{v - v_o}{a}\right]$$

$$x = x_o + v_o\left[\frac{v - v_o}{a}\right] + \frac{1}{2}a\left[\frac{v - v_o}{a}\right]^2$$

$$x - x_o = \frac{v_o v - v_o^2}{a} + \frac{v^2 + v_o^2 - 2vv_o}{2a}$$

$$x - x_o = \frac{2v_o v - 2v_o^2 + v^2 + v_o^2 - 2vv_o}{2a}$$

$$2a(x - x_o) = v^2 - v_o^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Where v is function of x. This equation tells us when the value of x changes, the value of v also changes.

Example:

For better understanding, let's consider an example with cars. In the context of cars equipped with a speedometer ranging from 0 to 160 km/h, the initial state of the car is at rest, denoted as v = 0. When the car is started, its speed, let's consider it as = 70 km/h, increases to this value and moves at this speed for some time. Afterward, it changes its velocity to 60 km/h, indicating negative acceleration. If we bring the car to a stop, its velocity will gradually decrease and eventually become zero, signifying that the car is at rest.

Introduction to Vectors

Up until now, the discussion has focused on motion in one dimension. Now, the exploration will extend to motion in two and three dimensions. Vectors exhibit two primary properties:

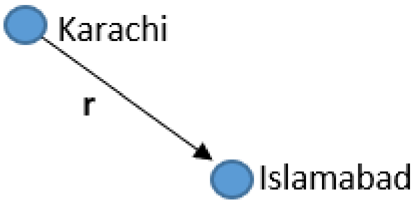
- Magnitude
- Direction

Consider the position vector r in two dimensions. For example, envision drawing a vector on a map of Pakistan, where one end (origin) is in Karachi, and the other end is in Islamabad. To achieve this, two essential considerations are needed:

- Set the origin at Karachi.

- Choose coordinates for distance (in kilometres) and direction (North, West, East, South)

Two sets of numbers or coordinates are necessary. Subsequently, two arrows will be drawn, representing vectors originating from Islamabad and Karachi.



Consider a car moving on a non-uniform surface. We will notice that's its velocity is changing its direction. Although it is moving with constant speed, but its direction keep changing. The components of vector in two dimensions can be expressed as,

$$r_X = x = r cos\theta$$

$$r_y = y = r sin\theta$$

The position vector is written as,

$$r = (r_X, r_y) = (x, y)$$

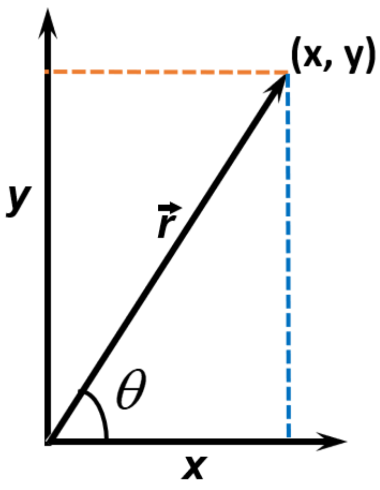


Figure 2.6. The point (x, y) is in two-dimensional plane with origin (0,0). So, we write this vector as r or in two coordinates (x, y). We call these coordinates as components.

For x = rcos(θ):

From the first trigonometric identity, we have:

$$\cos(\theta) = x/r$$

Multiplying both sides by r, we get:

$$r\cos(\theta) = x$$

Therefore, x = rcos(θ)

For y = rsin(θ):

Similarly, from the second trigonometric identity, we have:

$$\sin(\theta) = y/r$$

Multiplying both sides by r, we get:

$$r\sin(\theta) = y$$

Therefore, y = rsin(θ).

Mathematically we write them as x and y as:

$$r_X = x = rcos\theta$$
$$r_y = y = rsin\theta$$

We are already familiar from these basic trigonometric formulas.

$$x^2 + y^2 = r^2(\sin^2\theta + \cos^2\theta)$$
$$x^2 + y^2 = r^2(1) = r^2$$

Which proves that if we square two rectangular components and then add them, we get a constant number.

Although the length of the vector doesn't depend on the direction of vectors. We can write it mathematically as:

Where **r** = |**r**|

$$\theta = \tan^{-1}\frac{y}{x}$$

Magnitude of r can be found by Pythagorean theorem.

$$|\mathbf{r}| = r = \sqrt{x^2 + y^2}$$

Where r is independent of value of angle.

Vectors can be of different types. We have discussed position vector till now. For instance, we have also discussed velocity vector with example of moving car. Acceleration is a vector itself although it made up from velocity but still it is independent of it.

Vector addition:

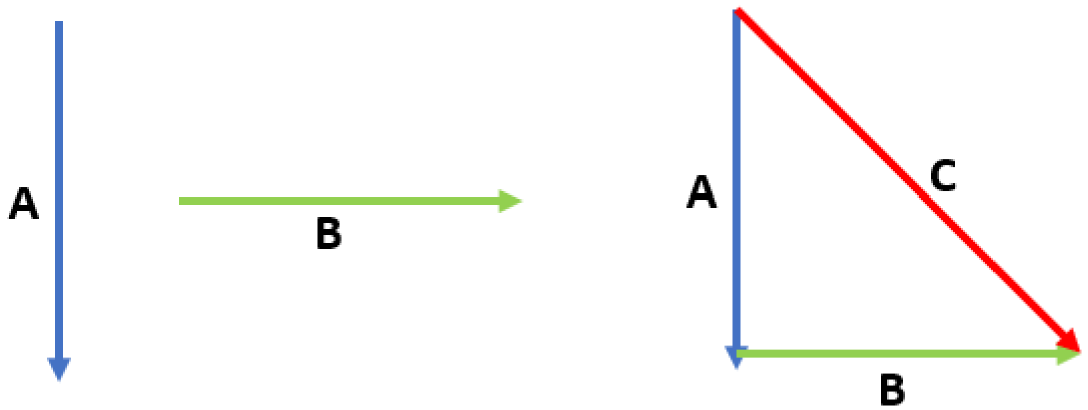
We can add vectors in one dimension. For example, consider a vector in one direction with a length of 10 m and another in the opposite direction with a length of 20 m.

20 m −10 m = 10 m

So, adding vectors in one dimension is an easy task. Now, let's discuss two dimensions. We want to add two vectors **A and B and** resultant as **C.** As,

$$\mathbf{C} = \mathbf{A}+\mathbf{B}$$

Graphically if we add these two vectors, they form a triangle with a resultant vector **C.**



We can arrange the vectors any way we want if we maintain their length and direction.

Parallelogram method for vector addition:

We can also add two vectors by parallelogram method, Continuing from previous example,

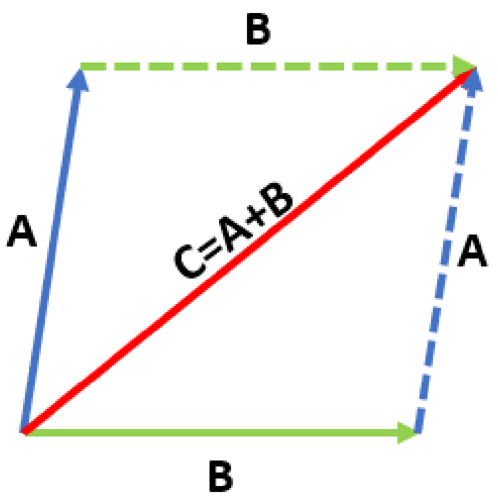


Figure 2.7. We translate the vectors A and B and form the sides of a parallelogram. The diagonal between them represents their resultant vector, denoted as C. This method is known as the parallelogram method.

In the parallelogram method for vector addition, the vectors are translated, (i.e., moved) to a common origin and the parallelogram constructed as follows:

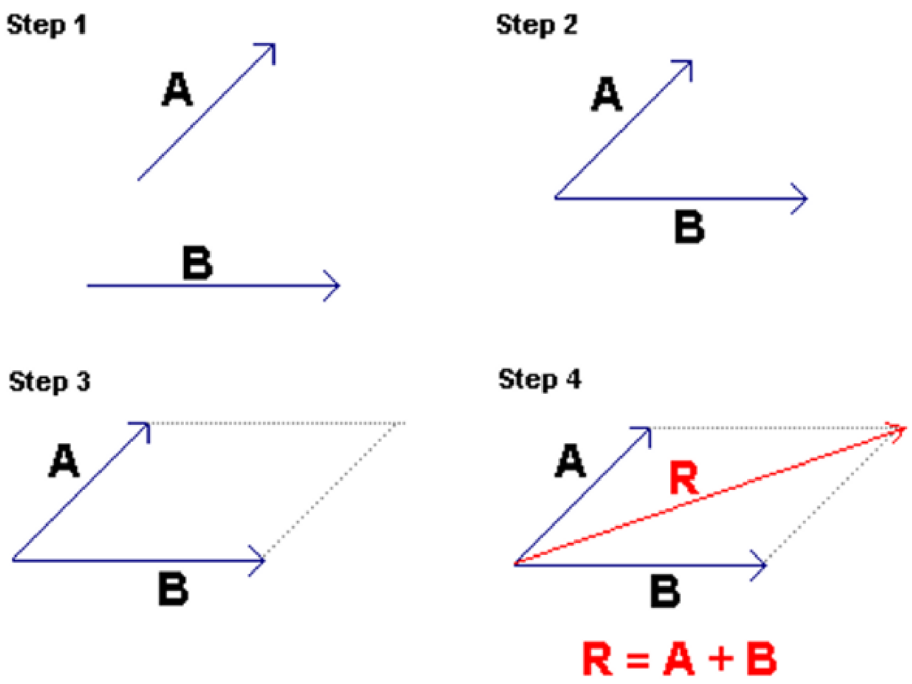


Figure 2.8. The resultant R is the diagonal of the parallelogram drawn from the common origin.

Component Method:

The components of a vector are those vectors which, when added together, give the original vector. The sum of the components of two vectors is equal to the sum of these two vectors. When we add two vectors their components do get added.

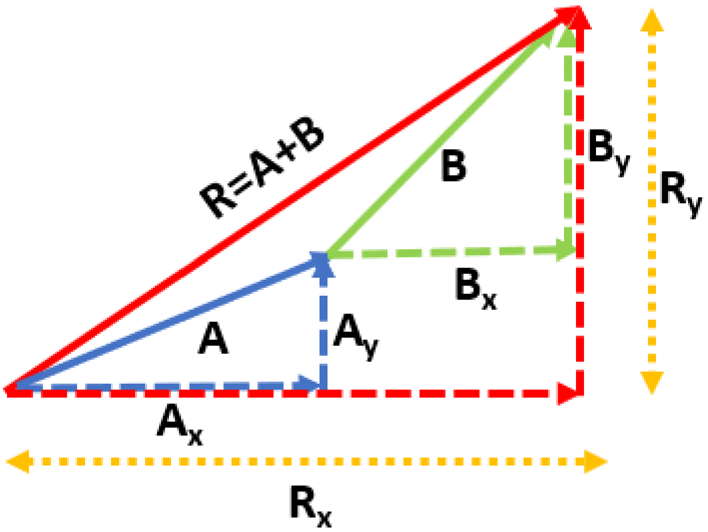


Figure 2.9. Figure presents the concept of vector addition by component method.

Here vector **A** and **B** have two components along x-axis and along y-axis we get a resultant.

vector **R** by adding these vectors components in x and y direction.

Summarizing it as,

- Add components.

$$\boldsymbol{R_x} = \boldsymbol{A_x} + \boldsymbol{B_x}$$

$$\boldsymbol{R_y} = \boldsymbol{A_y} + \boldsymbol{B_y}$$

  Bold letters present the vector nature.

- Then calculate the magnitude from following formula,

$$R = \sqrt{{R_X}^2 + {R_y}^2}$$

- Calculate the angle by using formula.

$$\theta = tan^{-1}\frac{R_y}{R_X}$$