Physics

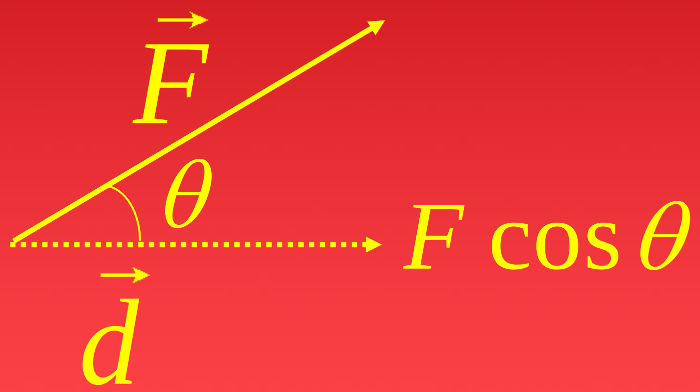
Work and Energy

Definition of Work

Work is: force applied in direction of displacement $\times$ displacement

$$W = \vec{F} \cdot \vec{d}$$

$$= Fd\cos\theta$$



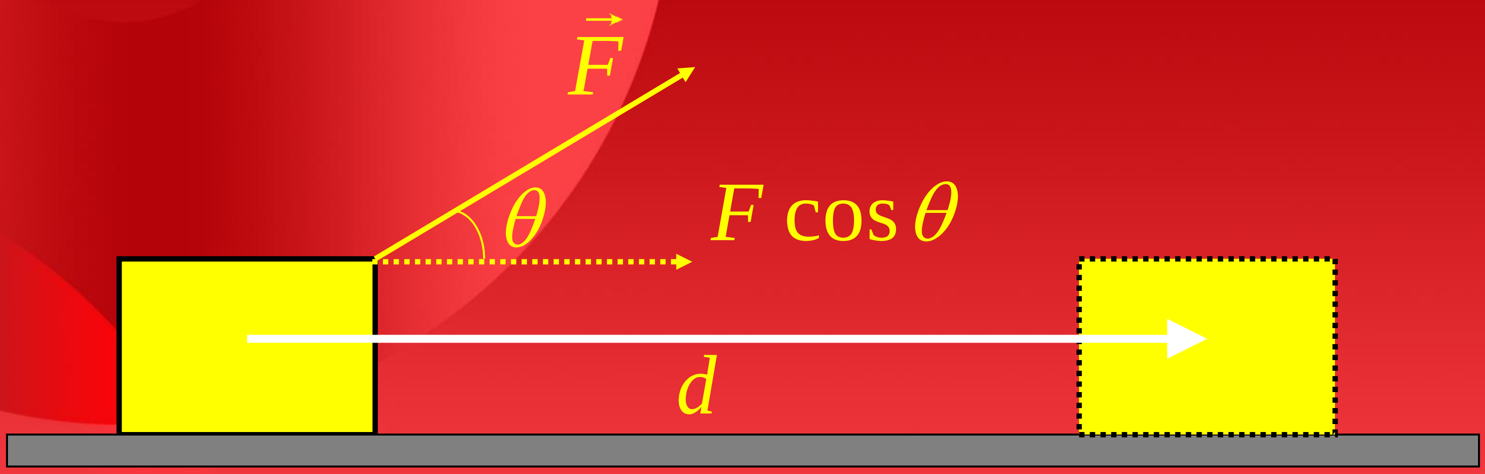
- Work is a *scalar*
- Work has dimensions:

$$M\,L\,T^{-2}\,L = M\,L^{2}\,T^{-2}$$

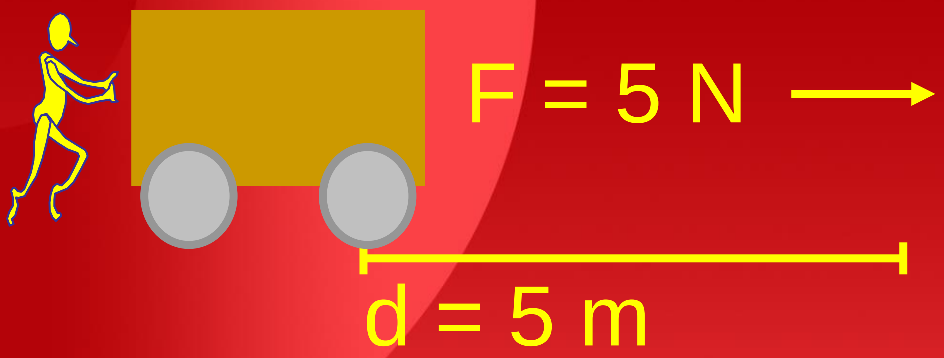
- Work has units:

$$\text{Newton} \cdot \text{Metre} \equiv \text{Joule (J)}$$

If a crate is pulled along the floor, only the force component *parallel* to the *displacement d* contributes to the work!



Forces do work on objects



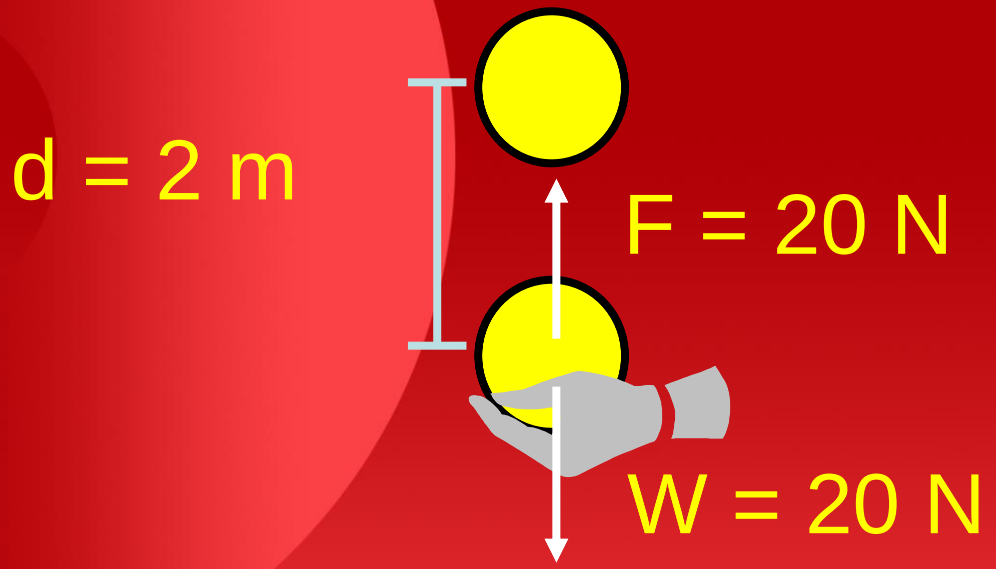
$$\text{Work} = Fd\cos\theta$$

$$= 25\ N.m = 25\ J$$

Forces do work on objects

Work on apple by gravity = 3 J

Forces do work on objects



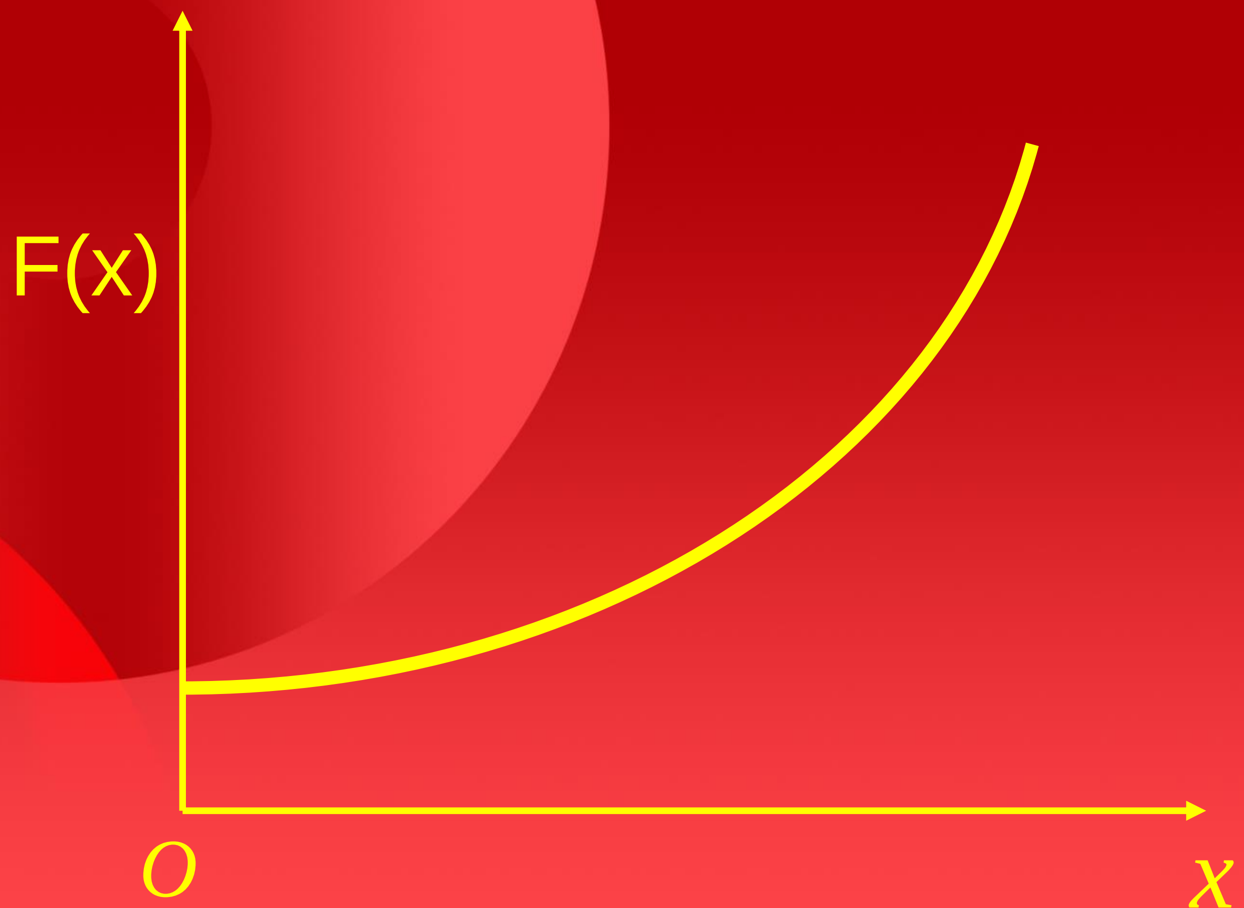
Work on ball by F_{hand} = 40 J

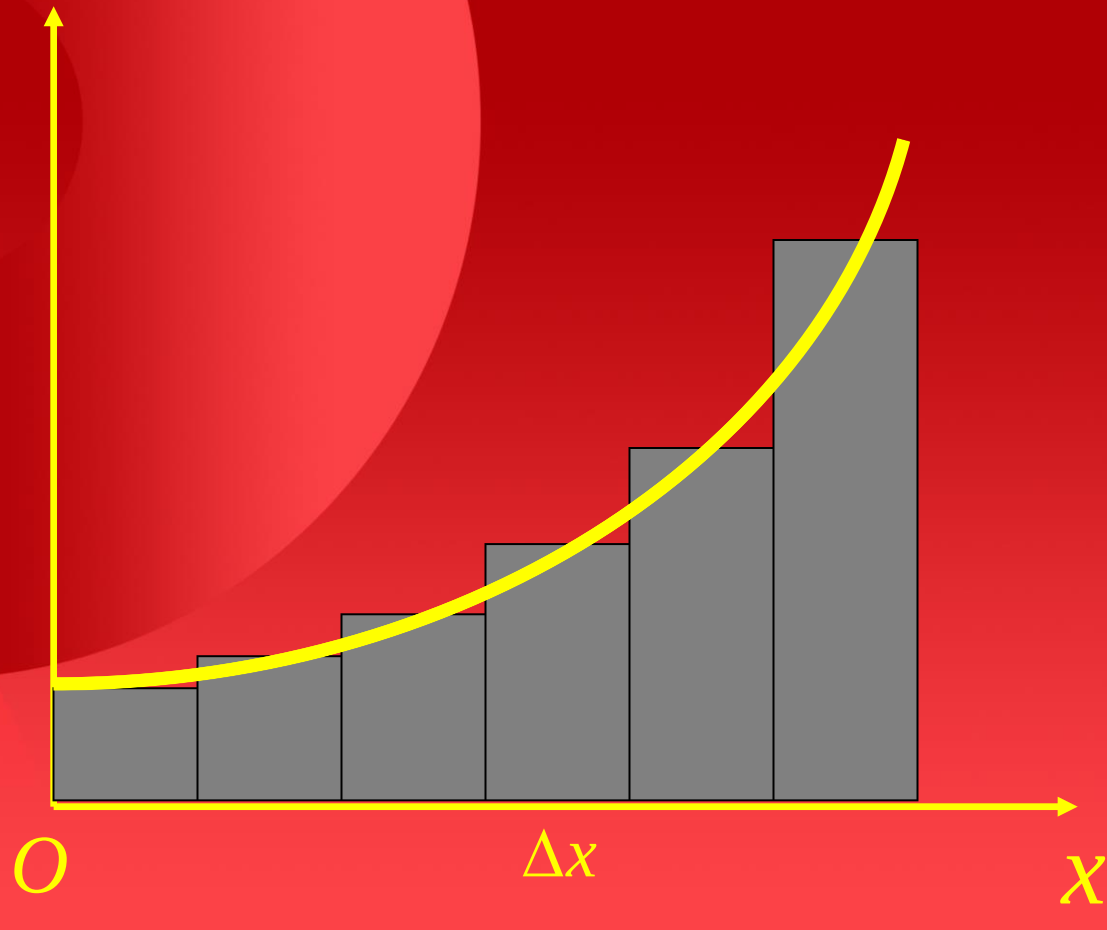
Work on ball by $F_{gravity}$ = – 40 J

Work done by a variable force

Let the force act in the x direction, and let it vary in magnitude with x according to the function F(x).

What is the work done when the body moves from some initial position to some final position ?


F(x)
O
x


O
Δx
x

$$\Delta W_1 = F_1 \Delta x$$

$$\Delta W_2 = F_2 \Delta x$$

$$\Delta W_3 = F_3 \Delta x$$

$$W = \Delta W_1 + \Delta W_2 + \cdots + \Delta W_N$$

$$= F_1 \Delta x + F_2 \Delta x + \cdots + F_N \Delta x$$

or

$$W = \sum_{n=1}^{N} F_n \Delta x$$

To get the exact result let $\Delta x \to 0$ and the number of intervals N $\to \infty$:

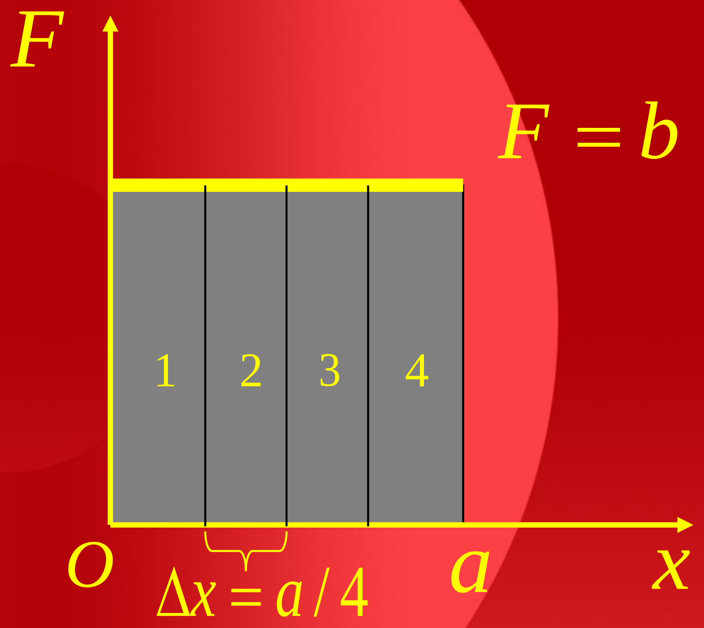
$$W = \lim_{\Delta x \to 0} \sum_{n=1}^{\infty} F_n \Delta x$$

Definition: $\lim_{\Delta x \to 0} \sum_{n=1}^{\infty} F_n \Delta x \equiv \int_{x_i}^{x_f} \mathrm{F(x)dx}$

is the integral of F with respect to x from x_i to x_f .

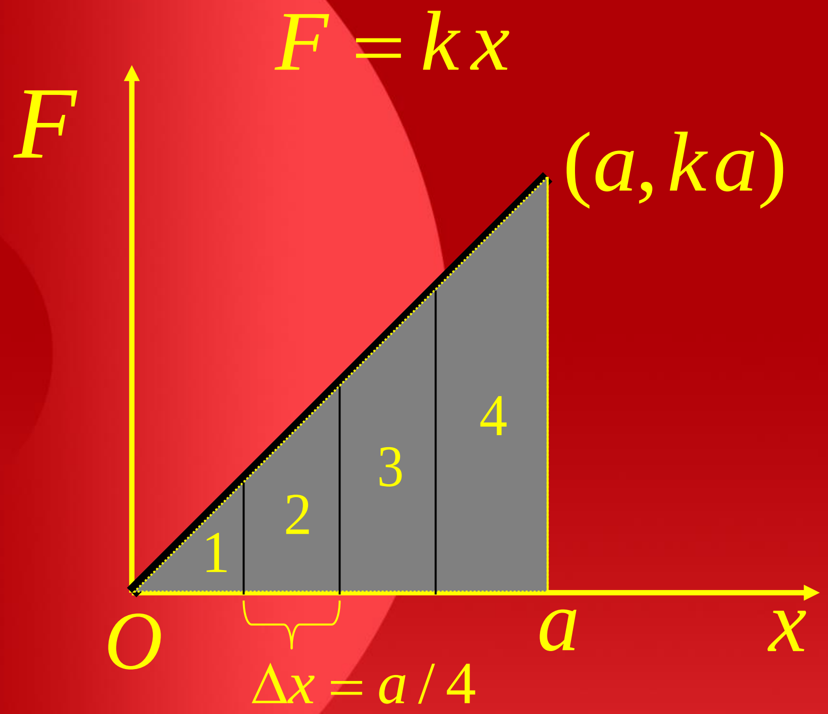
The total work done by F in moving a body from x_i and x_f is:

$$\boxed{W = \int_{x_i}^{x_f} \mathrm{F(x)dx}}$$



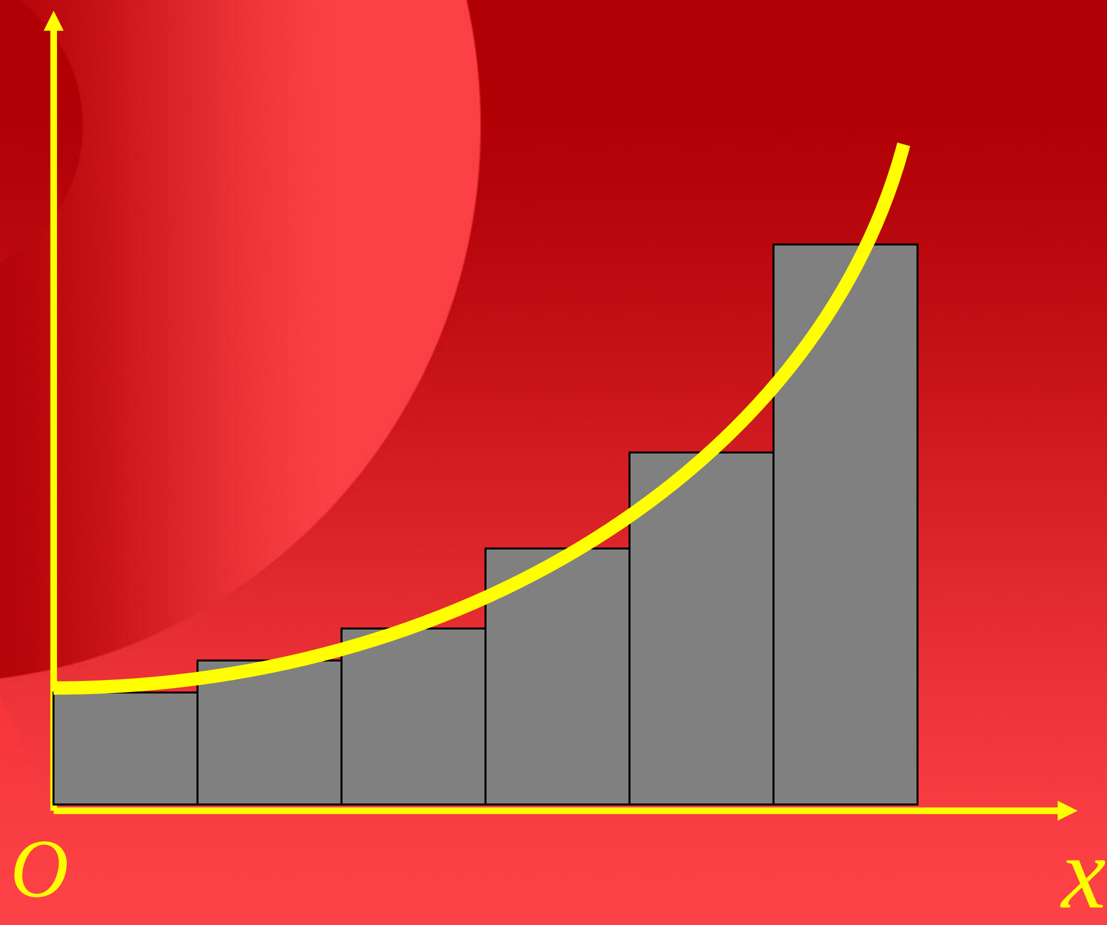
$$\frac{1}{4}a(b)+\frac{1}{4}a(b)+\frac{1}{4}a(b)+\frac{1}{4}a(b)=ab$$

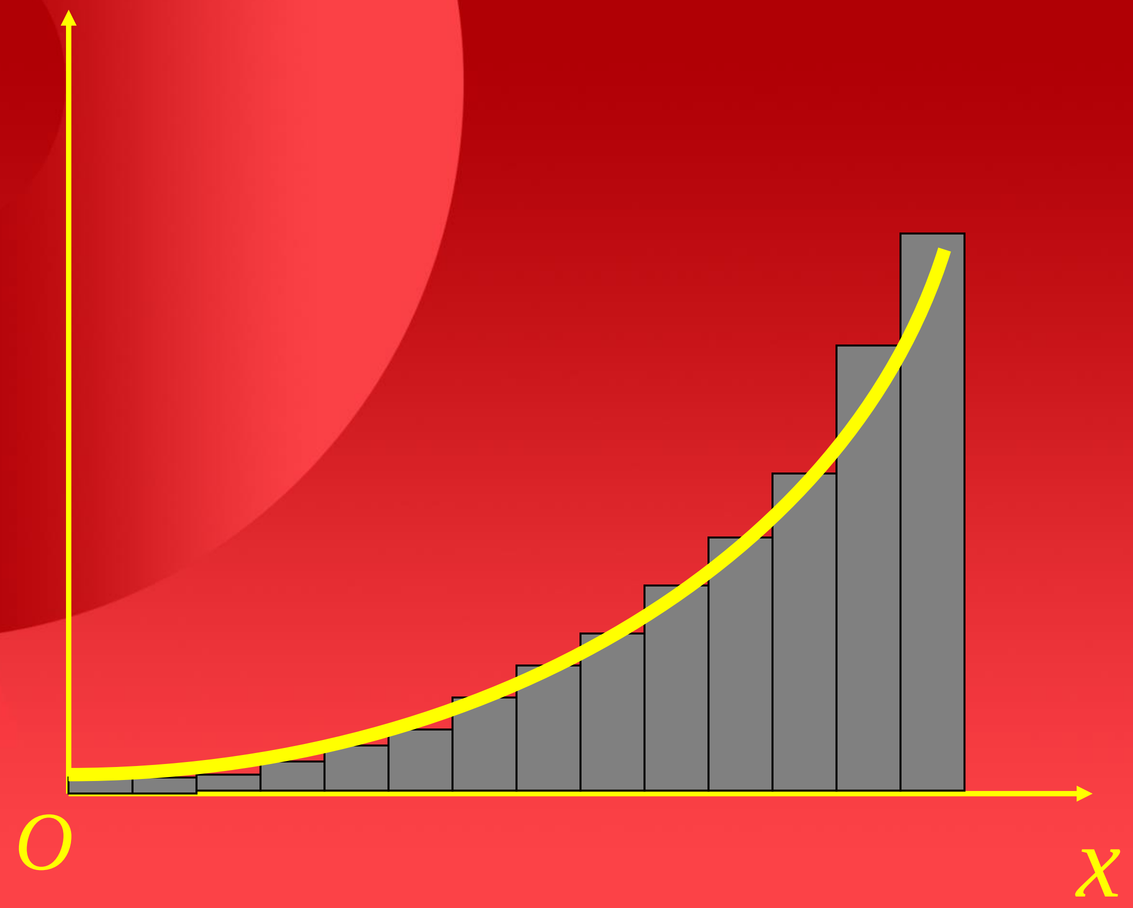
$$\int_0^a F\,dx = ab$$



$$\text{Area of shaded region} = \frac{1}{2}(a)(ka) = k\frac{a^2}{2}$$

$$\therefore \quad \int_0^a F\,dx = k\frac{a^2}{2}$$


O
x

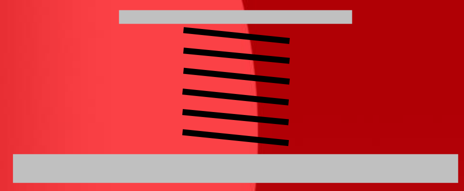


O
x

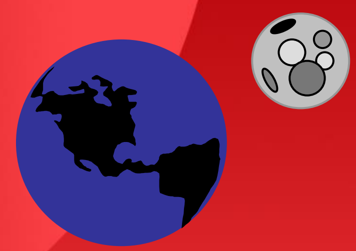
Energy is the capacity of a physical system to do work

- it comes in many forms
- it can be stored
- it can be converted into different forms
- it can never be *created* or *destroyed*

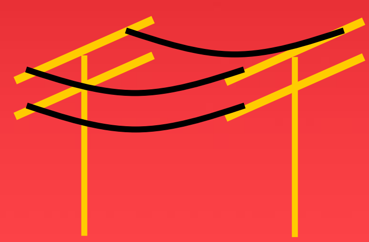
Some types of energy:



elastic

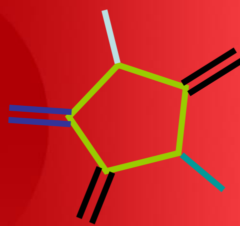


gravitational



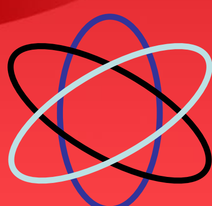
electrical

More types of energy:



chemical

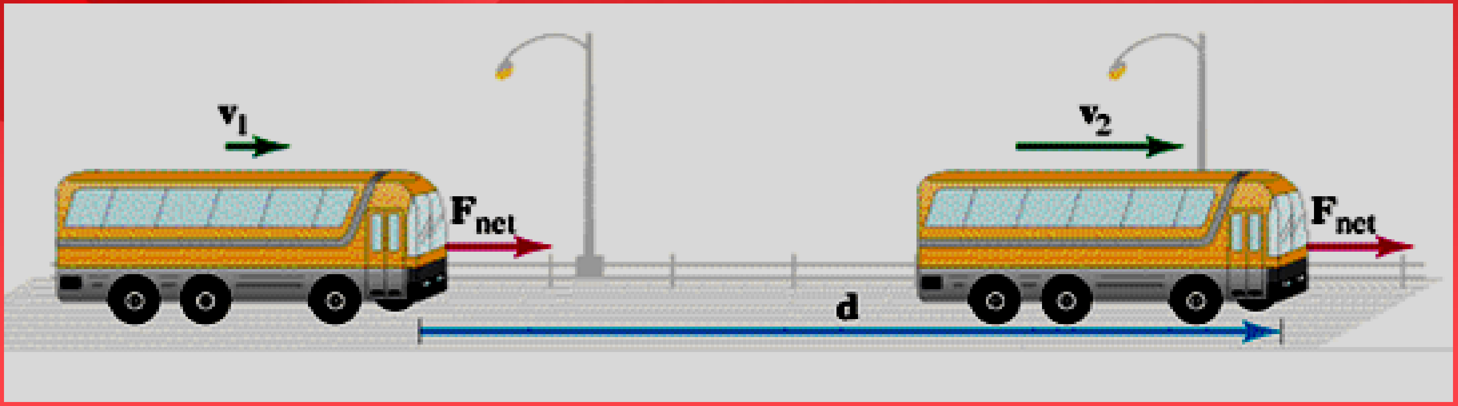
thermal



nuclear

ENERGY OF MOTION

A *constant force* accelerates a bus (mass *m*) from speed v_1 to speed v_2 over a *distance d*. What work is done by the engine?



Recall: $v_2^2 - v_1^2 = 2a\,(x_2 - x_1)$

where: v_2 = final velocity

x_2 = final position

v_1 = initial velocity

x_1 = initial position

$$\therefore \quad a = \frac{v_2^2 - v_1^2}{2d}$$

Calculate work:

$$W = F d$$

$$= m a d$$

$$= m \frac{v_2^2 - v_1^2}{2d} d$$

$$= \tfrac{1}{2} m {v_2}^2 - \tfrac{1}{2} m {v_1}^2$$

Define KINETIC ENERGY:

$$\boxed{KE = \tfrac{1}{2} m v^2}$$

A truck weighs 20 times more than a rickshaw but is moving 5 times slower. Which has more kinetic energy?

$$\text{KE (rickshaw)} = \tfrac{1}{2} m\text{v}^2$$

$$\text{KE (truck)} = \tfrac{1}{2}(20m)(\text{v}/5)^2$$

$$= \frac{20}{25} \cdot \frac{1}{2} m\text{v}^2$$

Work Kinetic-Energy Principle

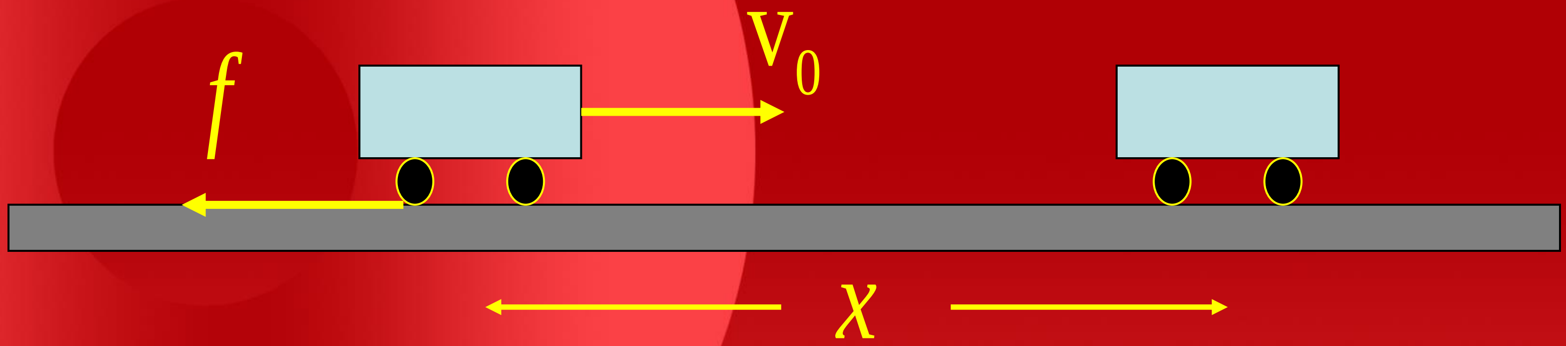
Net work done on object
= *Change* in *KE* of object

Work can be:

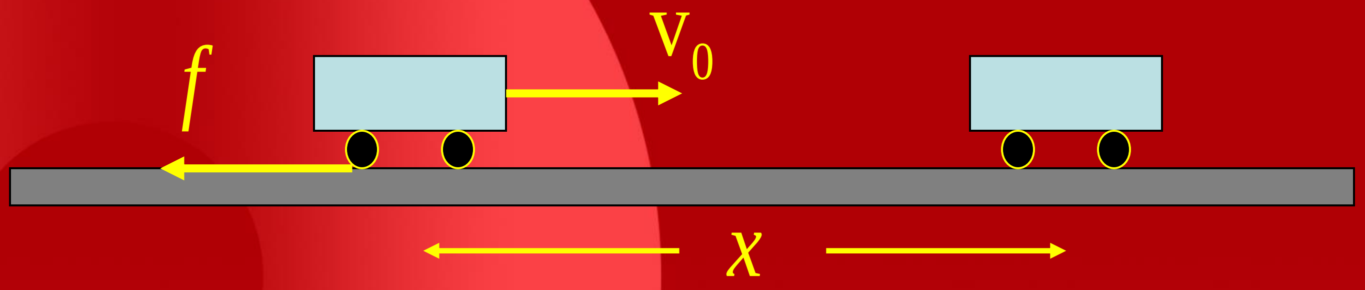
- Positive (KE increases)
- Negative (KE decreases)

Energy has the same units as work:

Joule = Newton Metre



Q: How far will the car travel before it comes to rest?



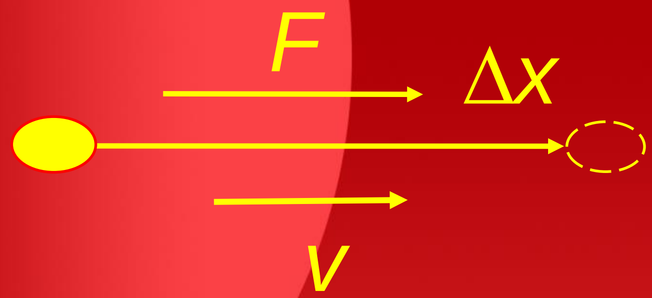
$$W_{net} = K_f - K_i = 0 - K_i$$

$$-f\,x = -\frac{1}{2}mv_0^{\,2} \quad \text{but } f = \mu mg$$

$$\mu mgx = \frac{1}{2}mv_0^{\,2} \quad \Rightarrow \quad x = \frac{v_0^{\,2}}{2\mu g}$$

$$W = F\,\Delta x$$

Work does not depend on time!



- Time does matter for <u>*power*</u> !
- Power is the “rate of doing work”

$$\boxed{\text{Power} = \frac{\text{Work done}}{\text{Time taken}}}$$

If the force does not depend on time:

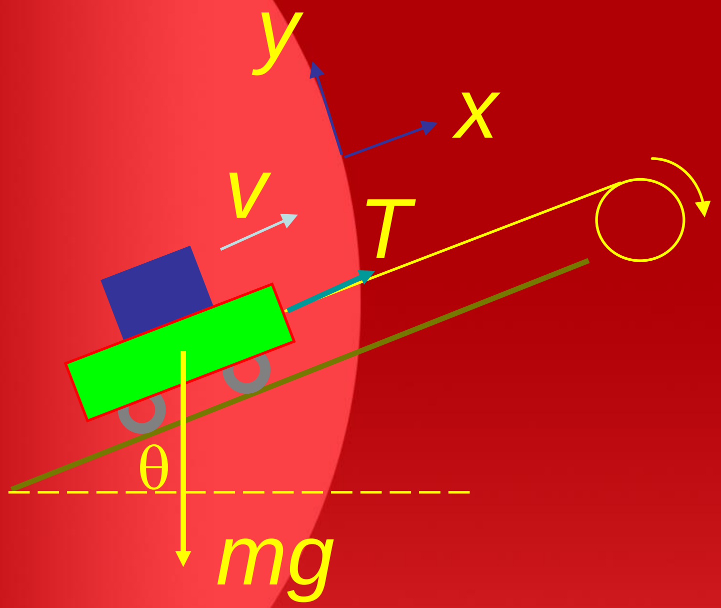
$$\frac{\text{Work}}{\text{Time}} = \frac{F\,\Delta x}{\Delta \text{t}} = F\,\text{v}$$

$$\therefore \text{Power} = F\,\text{v}$$

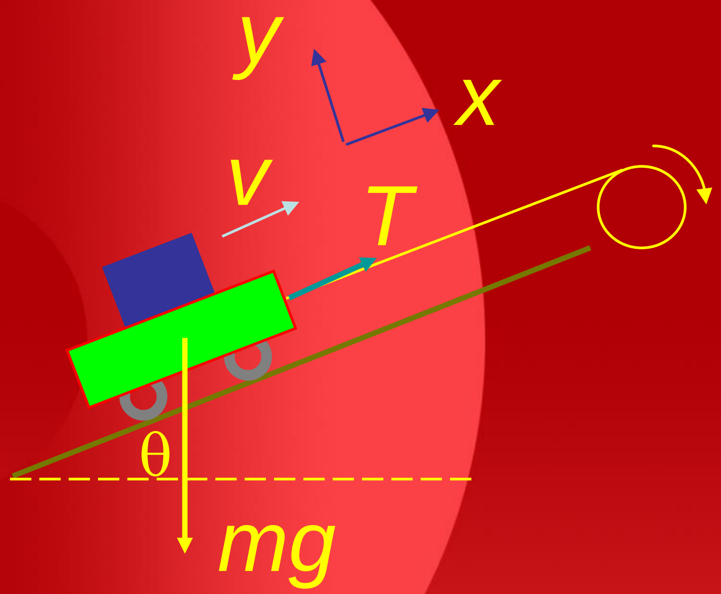
Units of power: *J/sec* = *Watts*

Old units: horsepower (hp)

1 hp = 746 W = 0.746 kW



Q: A *2000 kg* trolley is pulled up a *30°* hill at *20 mi/hr* by a rope. How much power is the machine providing ?



- The power is $P = Fv = Tv$
- No acceleration $\Rightarrow$ no net force
- Balance forces along <u>and</u> normal to plane

In the x direction: $T = mg\sin\theta$

$v = 20\ mi/hr = 8.93\ m/s$

$g = 9.8\ m/s^2$

$m = 2000\ kg$

$\sin\theta = \sin(30^o) = 0.5$

$P = (2000\ kg)(9.8\ m/s^2)(8.93\ m/s)(0.5)$

$= 88{,}000$ W (power of machine)

CAR POWER

Speed	*Friction*	*Air*	*P(kW)*	*P(hp)*
10	*180*	*40*	*2.2*	*2.9*
15	*180*	*90*	*4.1*	*5.5*
30	*180*	*360*	*16*	*22.0*