Physics-PHY101-Lecture 10

COLLISIONS

Collisions are extremely important to understand because they happen all the time - electrons collide with atoms, a bat with a ball, cars with trucks, star galaxies with other galaxies. In every case, the sum of the initial momenta equals the sum of the final momenta. This follows directly from Newton's Second Law, as we have already seen.

Elastic collision in one dimension:

Let us take the simplest collision. Consider two bodies of mass m_1 and m_2 moving with velocities u_1 and u_2. After the collision they are moving with velocities v_1 and v_2 as shown in figure 10.1. For elastic collision, the total linear momentum, and kinetic energies of the two bodies before and after collision must remain the same.

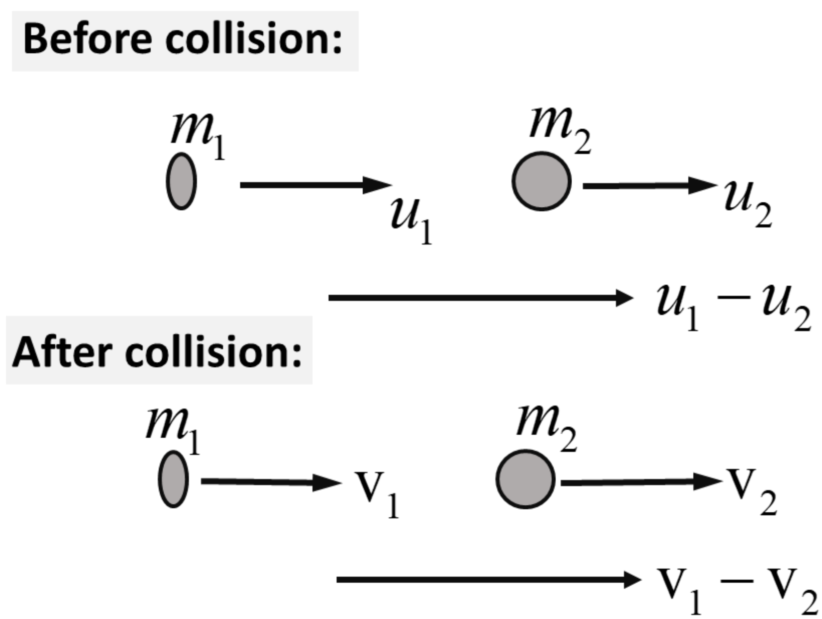


Figure 10.1. Illustrating one-dimensional elastic collision dynamics, where masses m_1 and m_2 exchange velocities u_1, u_2 to v_1, and v_2 after collision while preserving total linear momentum and kinetic energies.

From the law of conservation of linear momentum,

Total momentum before collision (p_i) = Total momentum after collision (p_f)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
$m_1(u_1 - v_1) = m_2(v_2 - u_2) \rightarrow 1$
For elastic collision,
Total kinetic energy before collision $(K.E_i) =$ Total kinetic energy after collision $(K.E_f)$
$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
$\frac{1}{2}m_1(u_1^2 - v_1^2) = \frac{1}{2}m_2(v_2^2 - u_2^2)$
$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \rightarrow 2$
Using the formula, $a^2 - b^2 = (a+b)(a-b)$, we can rewrite the above equation as
$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \rightarrow 3$
Dividing equation (3) by (1) gives,

$$\frac{m_1(u_1 + v_1)(u_1 - v_1)}{m_1(u_1 - v_1)} = \frac{m_2(v_2 + u_2)(v_2 - u_2)}{m_2(v_2 - u_2)}$$

$u_1 + v_1 = v_2 + u_2$
$u_1 - u_2 = v_2 - v_1$
$u_1 - u_2 = -(v_1 - v_2) \rightarrow 4$

This means that for any elastic head on collision, the relative speed of the two elastic bodies after the collision has the same magnitude as before collision but in opposite direction. Further note that this result is independent of mass.

Rewriting the equation 4 for v_1 and v_2,
$v_1 = v_2 + u_2 - u_1 \rightarrow 5$
$v_2 = u_1 + v_1 - u_2 \rightarrow 6$

To find the final velocities v_1 and v_2:

Substituting equation (5) in equation (1) gives the velocity of m1 as,

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$
$m_1(u_1 - v_1) = m_2(u_1 + v_1 - 2u_2)$
$m_1u_1 - m_1v_1) = m_2u_1 + m_2v_1 - 2m_2u_2$
$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$

$$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \frac{2m_2}{(m_1 + m_2)}u_2 \rightarrow 7$$

Similarly, by substituting equation (7) in equation (6), we get the final velocity of m_2 as,

$$v_2 = u_1 + \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1 + \frac{2m_2}{(m_1 + m_2)} u_2 - u_2$$

$$v_2 = \frac{2m_1}{(m_1 + m_2)} u_1 + \frac{(m_2 - m_1)}{(m_1 + m_2)} u_2 \rightarrow 8$$

These equations holds true in all inertial frames.

Case-I:

When bodies have same mass i.e., $m_1 = m_2$,

$$v_1 = (\frac{m_1 - m_2}{m_1 + m_2}) u_1 + (\frac{2m_2}{m_1 + m_2}) u_2$$

$$v_1 = (0) u_1 + \frac{2m_2}{2m_2} u_2$$

$$v_1 = u_2$$

$$v_2 = (\frac{2m_1}{m_1 + m_2}) u_1 + (\frac{m_2 - m_1}{m_1 + m_2}) u_2$$

$$v_2 = \frac{2m_1}{2m_1} u_1 + (0) u_2$$

$$v_2 = u_1$$

The equations show that in one dimensional elastic collision, when two bodies of equal mass collide after the collision their velocities are exchanged.

Case-II:

When bodies have the same mass i.e., $m_1 = m_2$ and second body (usually called target) is at rest ($u_2 = 0$), By substituting $m_1 = m_2$ and $u_2 = 0$ in equations we get,

$$v_1 = 0$$

$$v_2 = u_1$$

Equations show that when the first body comes to rest the second body moves with the initial velocity of the first body.

Case-III:

$m_1 \ll m_2$ and $u_2 = 0$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_1 = -u_1$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

$$v_2 = 0$$

The equations implies that the first body which is lighter, returns back in the opposite direction with the same initial velocity as it has a negative sign. The second body which is heavier in mass continues to remain at rest even after collision.

For example, if a ball is thrown at a fixed wall, the ball will bounce back from the wall with the same velocity with which it was thrown but in opposite direction.

Case-IV:

$m_2 << m_1 \quad u_2 = 0$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_1 = u_1$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

$$v_2 = 2u_1$$

The equations implies that the first body which is heavier continues to move with the same initial velocity. The second body, which is lighter, will move with twice the initial velocity of the first body. It means that the lighter body is thrown away from the point of collision.

Elastic and inelastic collision:

Elastic Collision	**Inelastic Collision**

The total kinetic energy is conserved	The total kinetic energy of the bodies at the beginning and the end of the collision is different.
Momentum is conserved	Momentum is conserved
No conversion of energy takes place	Kinetic energy is changed into other energy such as sound or heat energy.
Highly unlikely in the real world as there is almost always a change in energy.	This is the normal form of collision in the real world.
An example of this can be swinging balls or a spacecraft flying near a planet but not getting affected by its gravity in the end.	An example of an inelastic collision can be the collision of two cars.

Sometimes we wish to slow down particles by making them collide with other particles. In a nuclear reactor, neutrons can be slowed down in this way.

A completely inelastic collision:

A collision that is completely inelastic in physics is one in which the two colliding objects stick together to form a single mass as shown in figure 10.2. The total momentum is conserved in such collisions but kinetic energy is not conserved; instead, part or all of it is converted into deformation energy or internal kinetic energy.

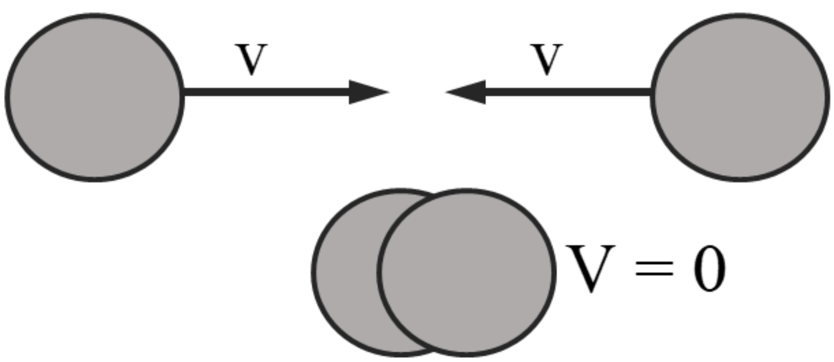


Figure 10.2. Illustrating one-dimensional in-elastic collision dynamics, where two masses moves with the same velocity V, conserving only the total linear momentum but the kinetic energies are not conserved.

Mathematically,

$$\text{Initial momentum} = \text{Final momentum} = 0$$

$$\text{Initial kinetic energy} = 2\ \frac{1}{2}\text{mv}^2 \left(\text{for two massess}\right)$$

$$\text{Final kinetic energy} = 0\ \left(\text{as V} = 0\right)$$

A common example of a totally inelastic collision is the result of two balls colliding and sticking to one another. After the collision, the combined mass that results travels at the same speed.

<u>Problem 1:</u> By what fraction is the kinetic energy of a neutron (mass m_1) decreased in a head-on collision with an atomic nucleus (mass m_2) initially at rest?

<u>Solution:</u>

$$m_1 \ll m_2, \quad \text{v}_{2i} = 0$$

$$\text{v}_{1\text{f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)u_{2i}$$

$$\text{v}_{1\text{f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_{1i} \Rightarrow \frac{\text{v}_\text{f}}{u_i} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$\left(\frac{\text{v}_\text{f}}{u_i}\right)^2 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

Fractional decrease in neutron K.E :

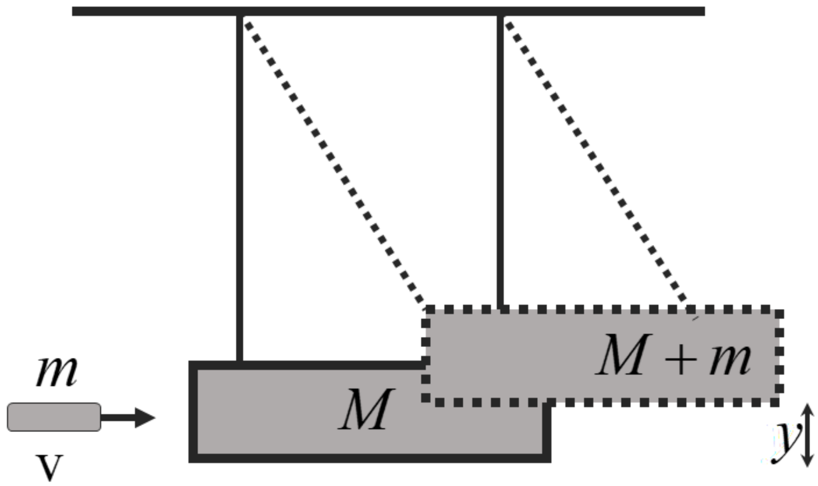
$$\frac{K_i - K_f}{K_i} = 1 - \frac{K_\text{f}}{K_\text{i}} = 1 - \frac{\text{v}_\text{f}^{\,2}}{\text{v}_\text{i}^{\,2}}$$

$$\frac{K_i - K_f}{K_i} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

$$\frac{K_i - K_f}{K_i} = \frac{\left(m_1 + m_2\right)^2 - \left(m_1 - m_2\right)^2}{\left(m_1 + m_2\right)^2}$$

$$\frac{K_i - K_f}{K_i} = \frac{4m_1 m_2}{(m_1 + m_2)^2}$$

Problem 2: A bullet with mass m is fired into a block of wood with mass M, suspended like a pendulum and makes a completely inelastic collision with it. After the impact, the block swings up to a maximum height y. What is the initial speed of the bullet?



Solution:

$$mv + 0 = (m+M)V$$

$$m\mathrm{v} = (m+M)V$$

$$\mathrm{v} = \frac{(m+M)}{m}V \rightarrow 1$$

Conservation of energy gives,

$$\frac{1}{2}(m+M)V^2 = (m+M)gy$$

$$y = \frac{(m+M)V^2}{2(m+M)g}$$

$$y = \frac{V^2}{2g}$$

$$V = \sqrt{2gy} \rightarrow 2$$

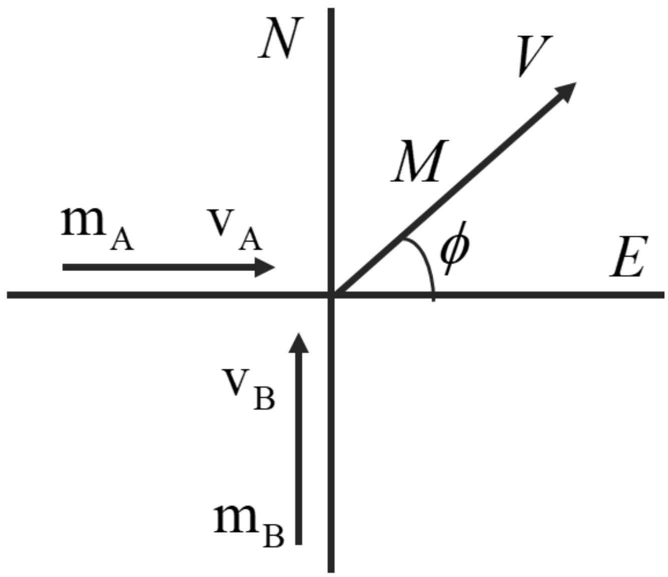
On putting equation (2) in equation (1),

$$\mathrm{v} = \frac{(m+M)}{m}\sqrt{2gy}$$

Conclusion: Equation shows the direct relation between the initial speed of bullet and the wooden block's height 'y'. The wooden block gains height as much as speedily the bullet enters the wooden block.

Problem 3: A car 'A' of mass 1000 kg is traveling north at 15 m/s collides with another car B of mass 2000 kg traveling east at 10 m/s. After a collision they move as one mass. Find the total momentum just after the collision.

Solution:

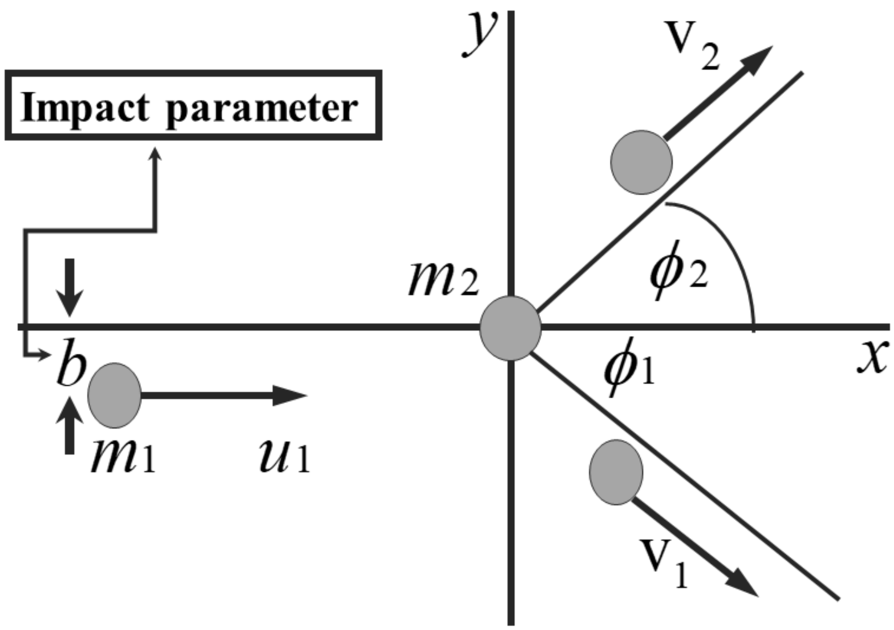


$$P_x = p_{Ax} + p_{Bx} = \mathrm{m_A} \mathrm{v}_{Ax} + \mathrm{m_B} \mathrm{v}_{Bx}$$
$$P_x = 0 + 2000 \times 10 = 20,000 \ kg \ m/s$$
$$P_y = p_{Ay} + p_{By} = \mathrm{m_A} \mathrm{v}_{Ay} + \mathrm{m_B} \mathrm{v}_{By}$$
$$P_y = 1000 \times 15 + 0 = 15000 \ kg \ m/s$$
$$P = \sqrt{P_x^2 + P_y^2} = \sqrt{20,000^2 + 15,000^2} = 25000 \ kg \ m/s$$
$$\tan\theta = \frac{P_y}{P_x} = \frac{15,000}{20,000} = 0.75 \quad \Rightarrow \quad \theta = 37^0$$

Problem 4:

Consider two masses m_1 moving with velocity u_1 and m_2 at rest (u_2=0) as shown in figure. This is not a central collision (i.e., m_1 directly collide to m_2), instead m_1 slightly touches m_2 by one of its sides. Apply the law of conservation of momentum to this case.

Solution:



$$
\begin{aligned}
&1)\ p_{ix} = p_{fx} \\
&\quad \Rightarrow m_1 u_1 = m_1 v_1 \cos\phi_1 + m_2 v_2 \cos\phi_2 \\
&2)\ p_{iy} = p_{fy} \\
&\quad \Rightarrow 0 = m_1 v_1 \sin\phi_1 - m_2 v_2 \sin\phi_2 \\
&3)\ KE_i = KE_f \\
&\quad \Rightarrow \tfrac{1}{2} m_1 u_1^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2
\end{aligned}
$$

As there are 4-unknowns and 3-equations, they cannot be solved.

Problem 5: Consider two masses m_1 moving with velocity v_1 and m_2 moving with velocity v_2. Both masses collied with each other. After collision, they become a one body with mass M moving with velocity v_f. Apply the law of conservation of momentum to this case.

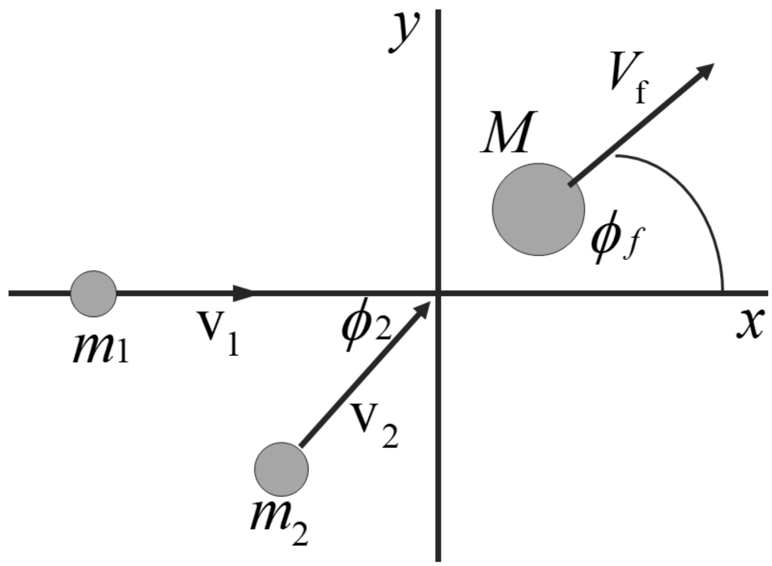
Solution:

1) $p_{ix} = p_{fx}$

$$\Rightarrow m_1 v_1 + m_2 v_2 \cos\phi_2$$
$$= (m_1 + m_2) V_f \cos\phi_f$$

2) $p_{iy} = p_{fy}$

$$\Rightarrow m_2 v_2 \sin\phi_2 = M V_f \sin\phi_f$$



Now there are 2-unknowns and 2-equations, hence, can be solved.

Conclusion:

- Momentum is always conserved in collisions, but energy may or may not be.
- We have come to trust momentum conservation very much: discovery of the neutrino, hints of black holes, discovery of dark matter.