Physics

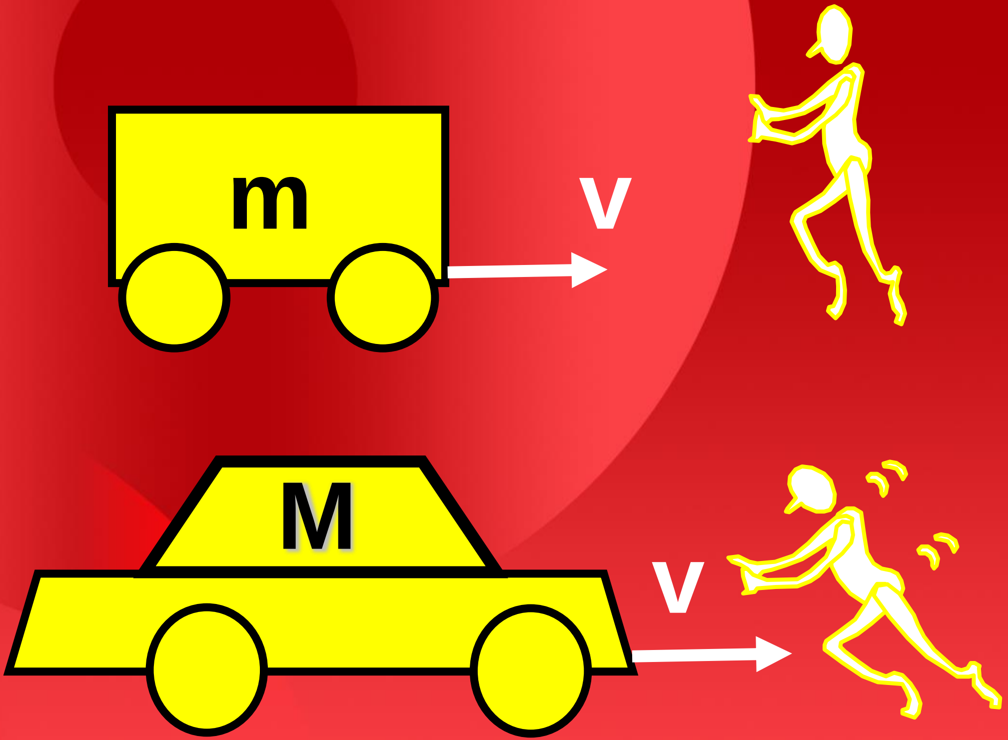
Momentum

Linear Momentum

Momentum is "quantity of motion"

Easy to start Hard to start

Momentum depends on *mass* and *velocity*



Easy to stop

Hard to stop

momentum = mass × velocity

Q: A truck is 20 times more massive than a motorcycle but is moving at 1/25 the speed. Which has more momentum?

$$p\ (\text{motorcycle}) = mv$$

$$p\ (\text{truck}) = (20m)(v/25)$$

$$= \frac{20}{25}\ mv$$

Linear Momentum

$$\boxed{\vec{P} = m\vec{\mathbf{v}}}$$

Dimensions of momentum: MLT^{-1}

Units of momentum: kg-m/s

Force $\Longleftrightarrow$ **Momentum**

NEW FORM OF SECOND LAW

The rate of change of momentum of a body is equal to the resultant force acting on the body and is in the direction of that force.

$$m\vec{\mathbf{a}} = \vec{\mathbf{F}} \text{ (old form)}$$

$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}} \quad \text{(new form of Newton's Law)}$$

They are the same:

$$\frac{d\vec{\mathbf{p}}}{dt} = \frac{d(m\vec{\mathbf{v}})}{dt} = m\frac{d\vec{\mathbf{v}}}{dt} = m\vec{\mathbf{a}} = \vec{\mathbf{F}}$$

Newton's 2nd law for several particles

$$\vec{\mathbf{P}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \cdots \vec{\mathbf{p}}_N$$

$$\frac{d}{dt}\vec{\mathbf{P}} = \frac{d}{dt}\vec{\mathbf{p}}_1 + \frac{d}{dt}\vec{\mathbf{p}}_2 + \cdots \frac{d}{dt}\vec{\mathbf{p}}_N$$

$$= \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots \vec{\mathbf{F}}_N$$

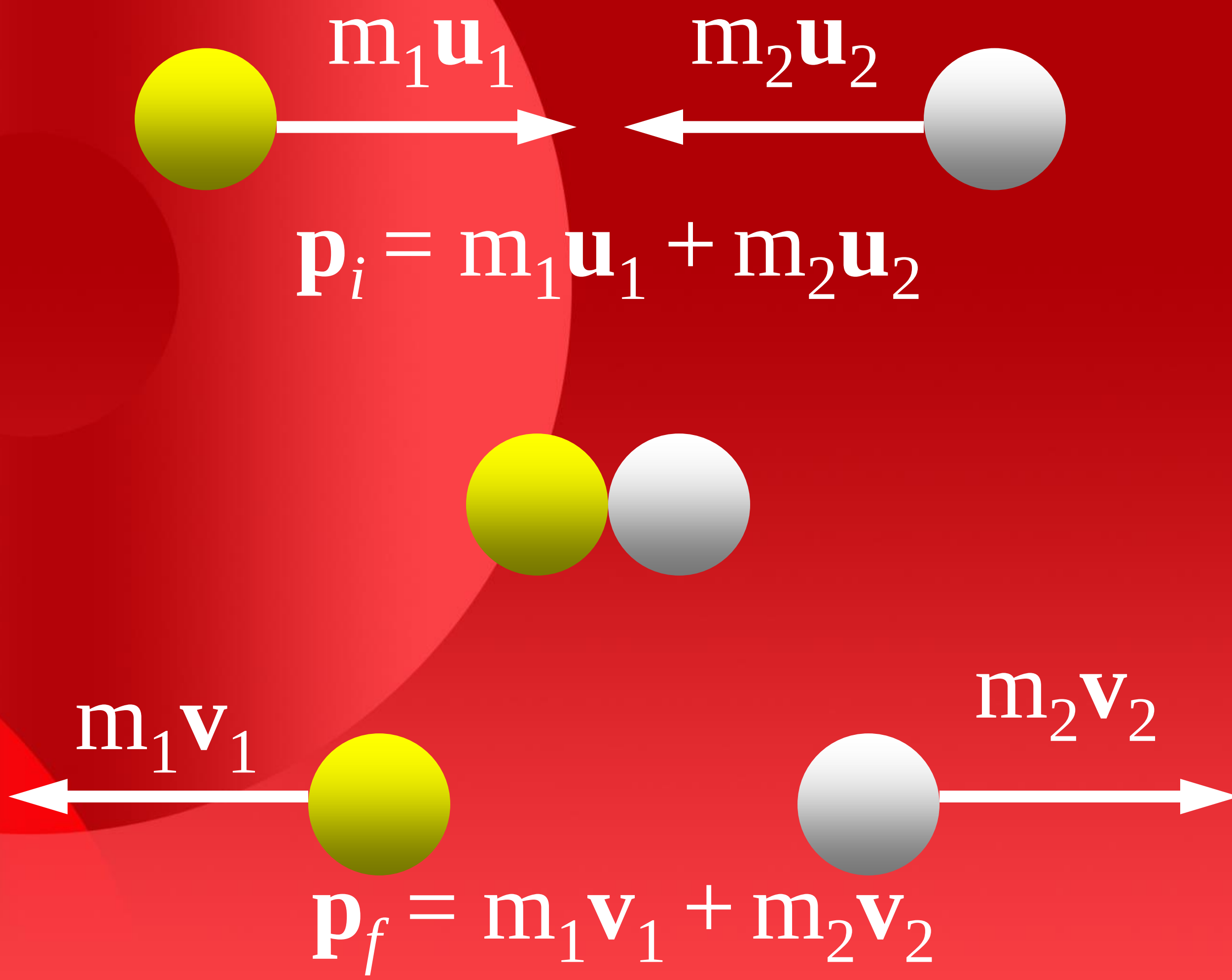
$$= \sum_{i=1}^{i=N} \vec{\mathbf{F}}_i = \text{total external force}$$

Conservation of Linear Momentum

If $\sum \vec{\mathbf{F}}_{ext} = 0 \Rightarrow \dfrac{d\vec{\mathbf{P}}}{dt} = 0$

$\Rightarrow \vec{\mathbf{P}} = \text{constant}$

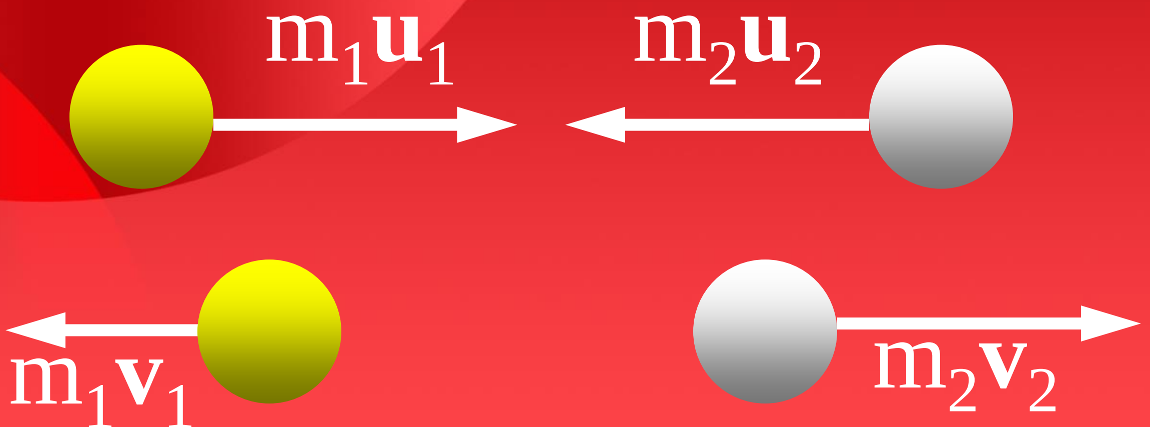
Momentum is conserved for an isolated system


$m_1\mathbf{u}_1$
$m_2\mathbf{u}_2$
$\mathbf{p}_i = m_1\mathbf{u}_1 + m_2\mathbf{u}_2$
$m_1\mathbf{v}_1$
$m_2\mathbf{v}_2$
$\mathbf{p}_f = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$

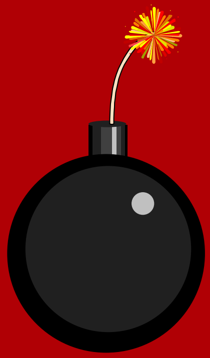
Momentum is conserved:

initial momentum = final momentum

$$\boxed{m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2}$$



Before explosion:



$$\vec{\mathbf{P}}_i = \mathbf{0}$$

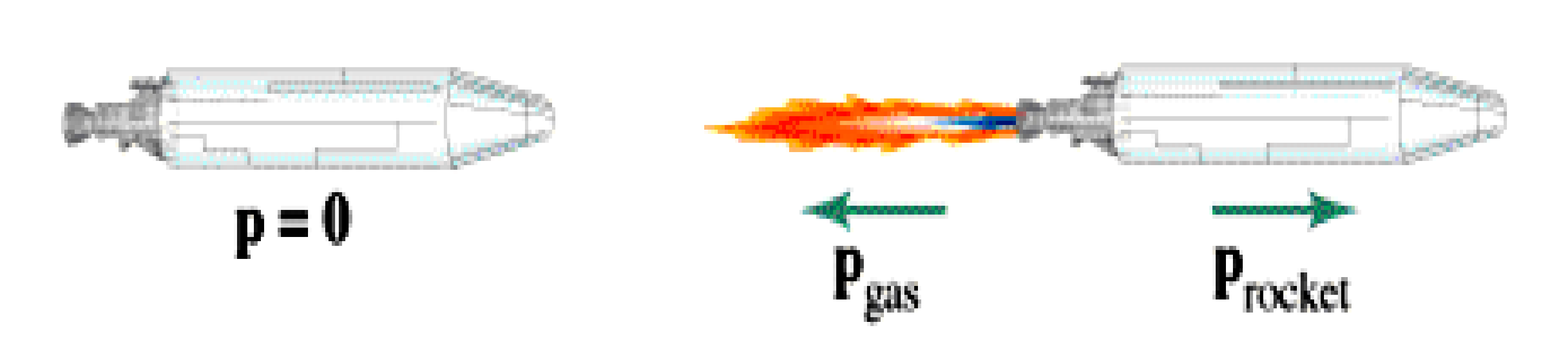
After explosion:

$$\mathbf{P}_f = \mathbf{0} \quad \text{but } \mathbf{P}_f = m_1 \mathrm{v}_1 + m_2 \mathrm{v}_2$$

$$\therefore \; m_1 \mathrm{v}_1 = -m_2 \mathrm{v}_2$$

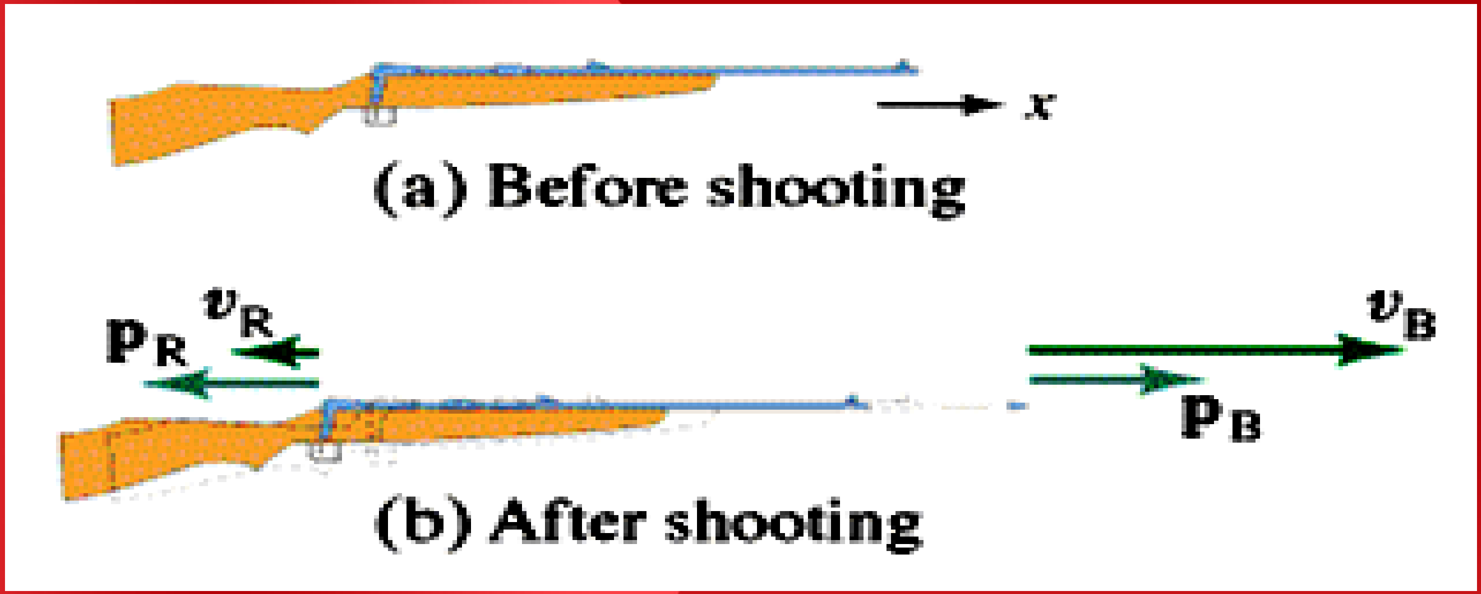
Q: A bomb of mass 10 kg, initially at rest, explodes into two pieces of masses 4 kg and 6 kg. If the speed of the 4 kg piece is 12 m/s, find the speed of the 6 kg piece.

$$ANSWER: \quad 6\mathrm{v} = 4 \times 12 \;\Rightarrow\; \mathrm{v} = 8\,m/s$$



$$0 = p_i = p_f = p_{\text{gas}} + p_{\text{rocket}}$$

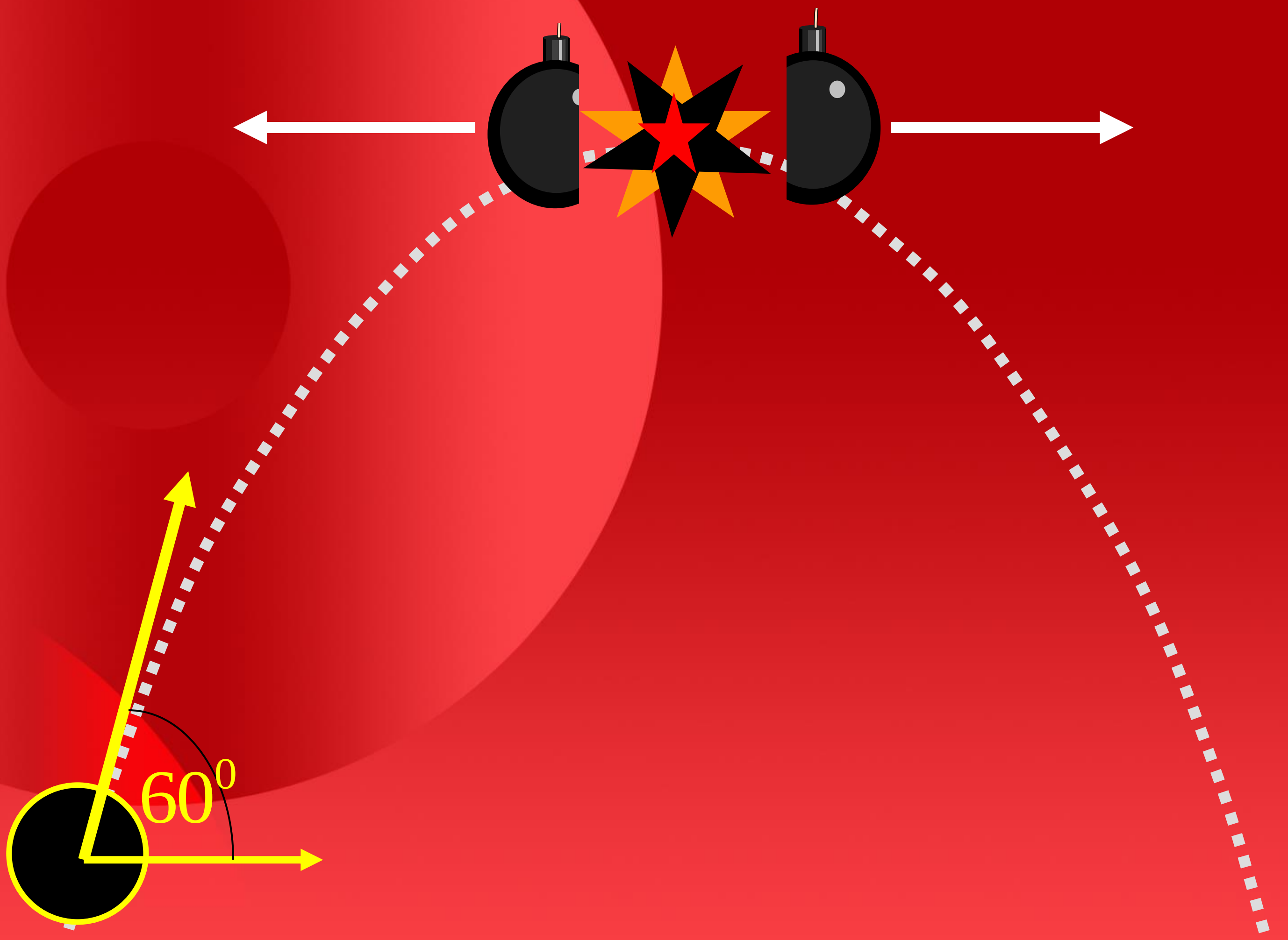
$$\Rightarrow \quad p_{\text{gas}} = -p_{\text{rocket}}$$

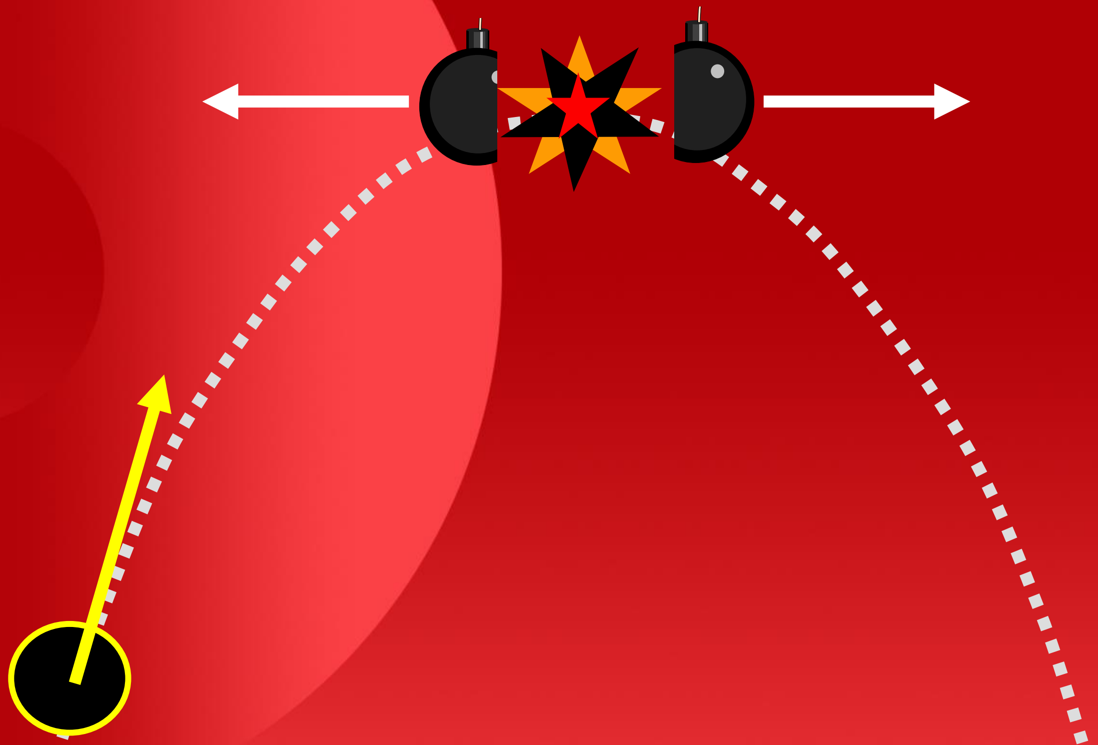


$$0 = p_i = p_f = p_{\text{rifle}} + p_{\text{bullet}}$$

$$\Rightarrow \quad p_{\text{rifle}} = -p_{\text{bullet}}$$

Q: A shell is fired from a cannon with a speed 10 m/s at an angle 60^0 with the horizontal. At the highest point in its path it explodes into two pieces of equal masses. One of the pieces retraces its path to the cannon. Find the velocity of the other piece immediately after the explosion.


60⁰



BEFORE EXPLOSION:

$$V_x = 10\cos 60 = 5\, m/s$$

$$P_x = 5\,M \quad \text{kg}\, m/s$$

AFTER EXPLOSION:

$$P_{1x} = -5\frac{M}{2} \quad \text{(why?)}$$

But $\;P_{1x} + P_{2x} = P_x \quad \Rightarrow \quad P_{2x} = 5M + 5\frac{M}{2}$

Use $\;P_{2x} = \frac{M}{2} v_{2x} \quad \Rightarrow \quad v_{2x} = 15\, m/s$

Q: A stream of bullets, each of mass m, is fired horizontally with a speed v into a large wooden block of mass M that is initially at rest on a horizontal table. If the block is free to slide without friction, what speed will it acquire after it has absorbed N bullets?

Linear momentum is conserved:

$$P_f = P_i$$

$$(M + Nm)V = N(m\text{v})$$

$$V = \frac{Nm}{(M + Nm)}\text{v}$$

A cannon with mass M equal to 1300 kg fires a 72 kg ball in horizontal direction with a muzzle speed v of 55 m/s. The cannon is mounted so that it can recoil freely.

a): What is the velocity V of the recoiling cannon with respect to the earth?

b): What is the initial velocity v_E of the ball with respect to the earth?

a) Linear momentum is conserved:

$$P_f = P_i$$

$$MV + m(\mathrm{v} + V) = 0$$

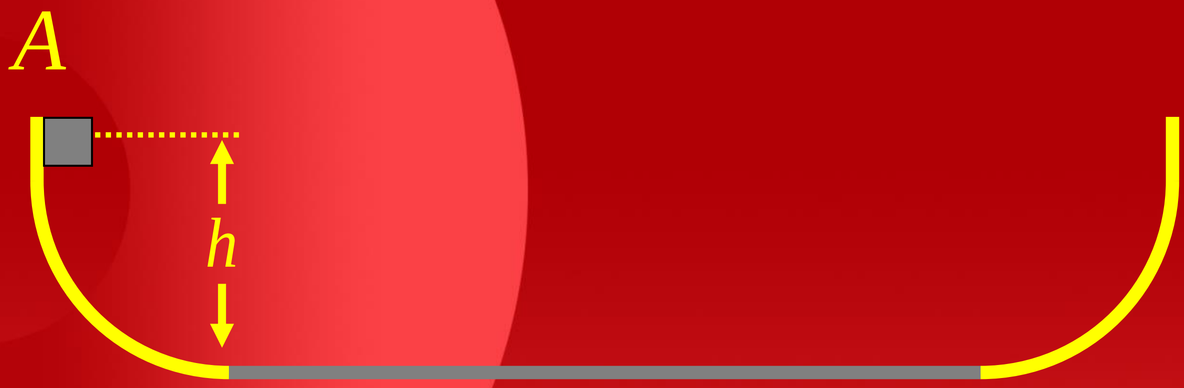
$$V = -\frac{m\mathrm{v}}{m+M} = -\frac{(72kg)(55m/s)}{1300kg + 72kg}$$

$$V = -2.9\ m/s$$

b) $\mathrm{v}_{\mathrm{E}} = \mathrm{v} + V$

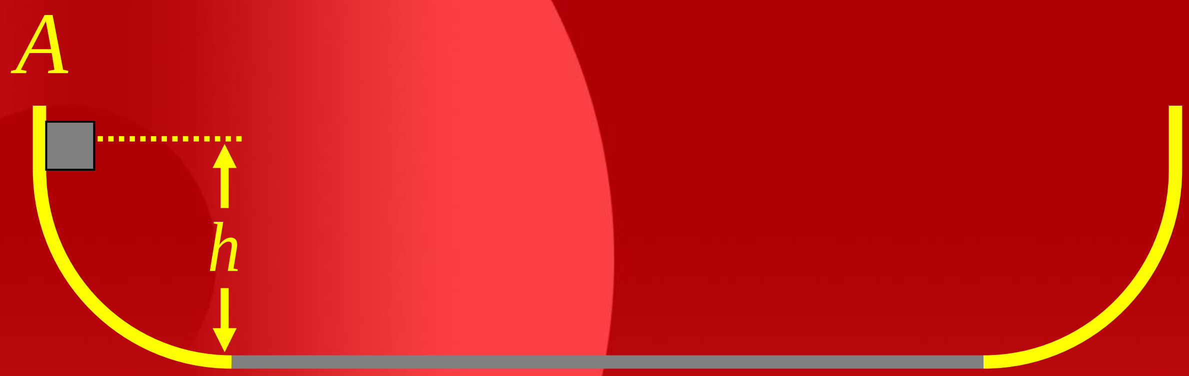
$$= 55m/s + (-2.9\ m/s)$$

$$\mathrm{v}_{\mathrm{E}} = 52m/s$$



Is the momentum conserved ?

Where does the particle finally come to rest ?



Suppose the total distance moved on the flat part before it comes to rest is x.

$$mgh = fx = \mu mgx \quad \Rightarrow \quad x = \frac{h}{\mu g}$$

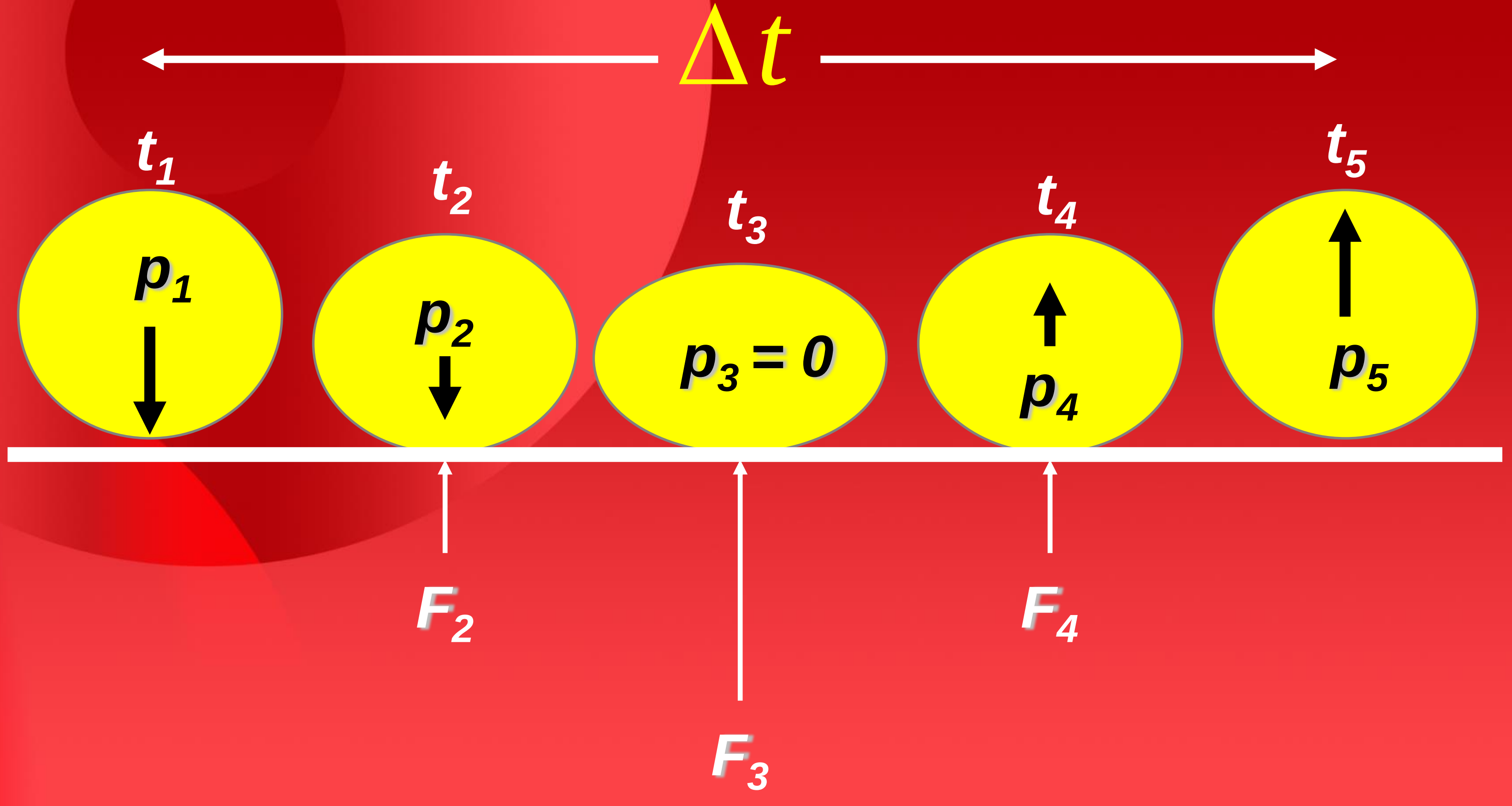
Impulse and Momentum

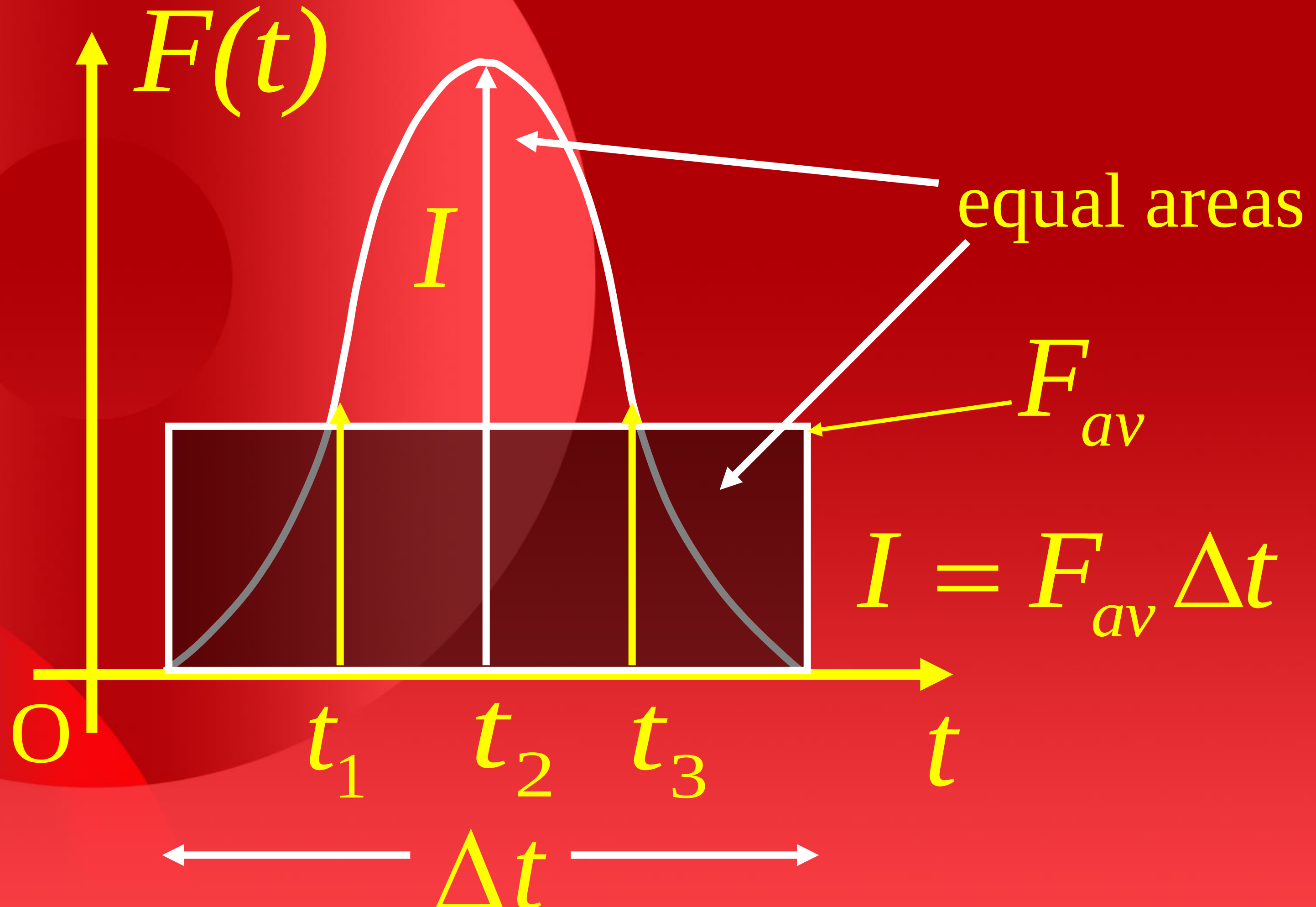
$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \quad \Rightarrow \quad d\mathbf{p} = \mathbf{F}dt$$

Define: $$\mathrm{I} \equiv \int_{t_1}^{t_2} \mathrm{F}dt$$

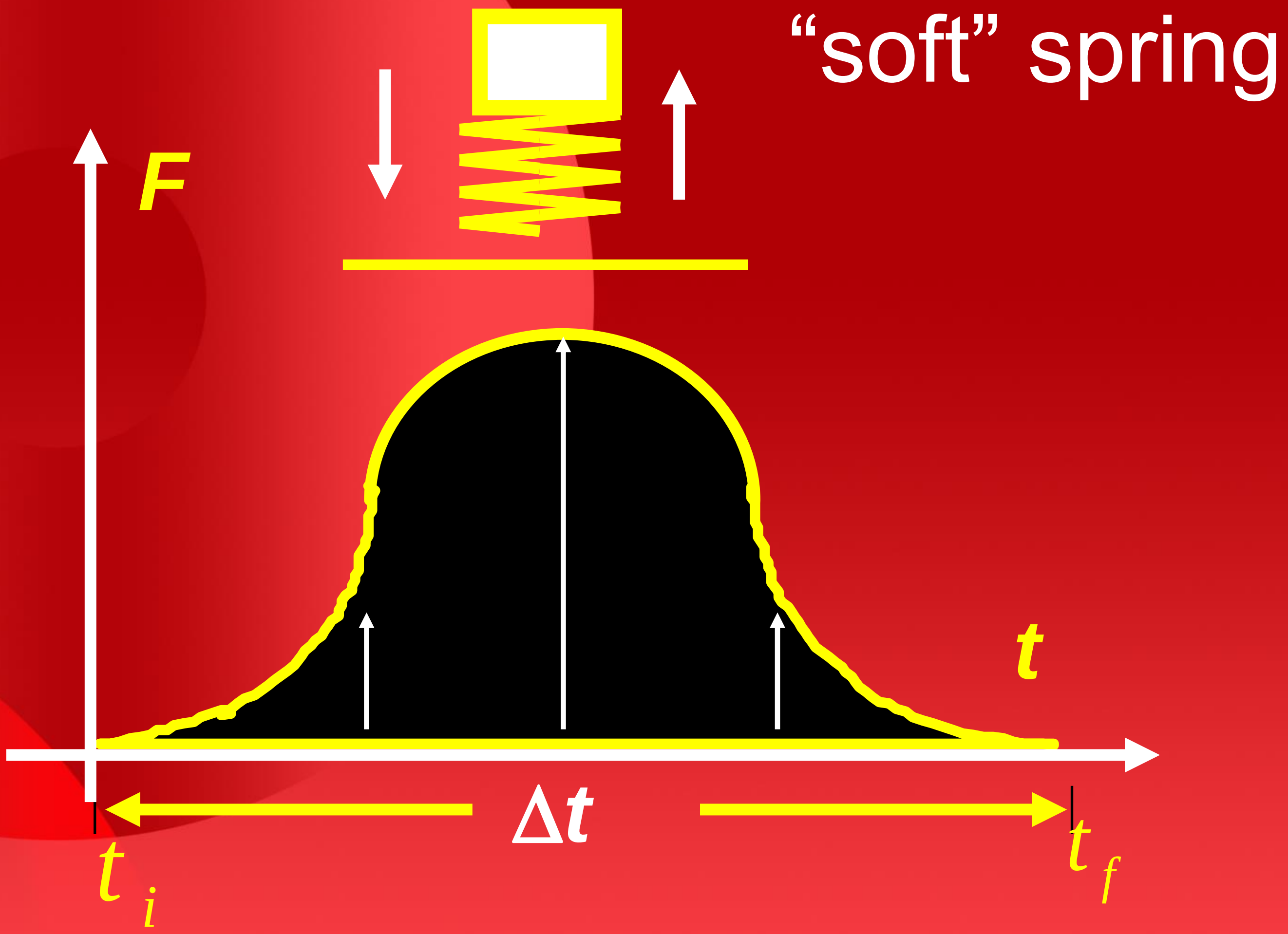
Since $$\int_{t_1}^{t_2} \mathrm{F}dt = \int_{p_i}^{p_f} dp \quad \therefore \quad \boxed{\mathrm{I} = \mathbf{p}_f - \mathbf{p}_i}$$

A ball bounces off the floor :

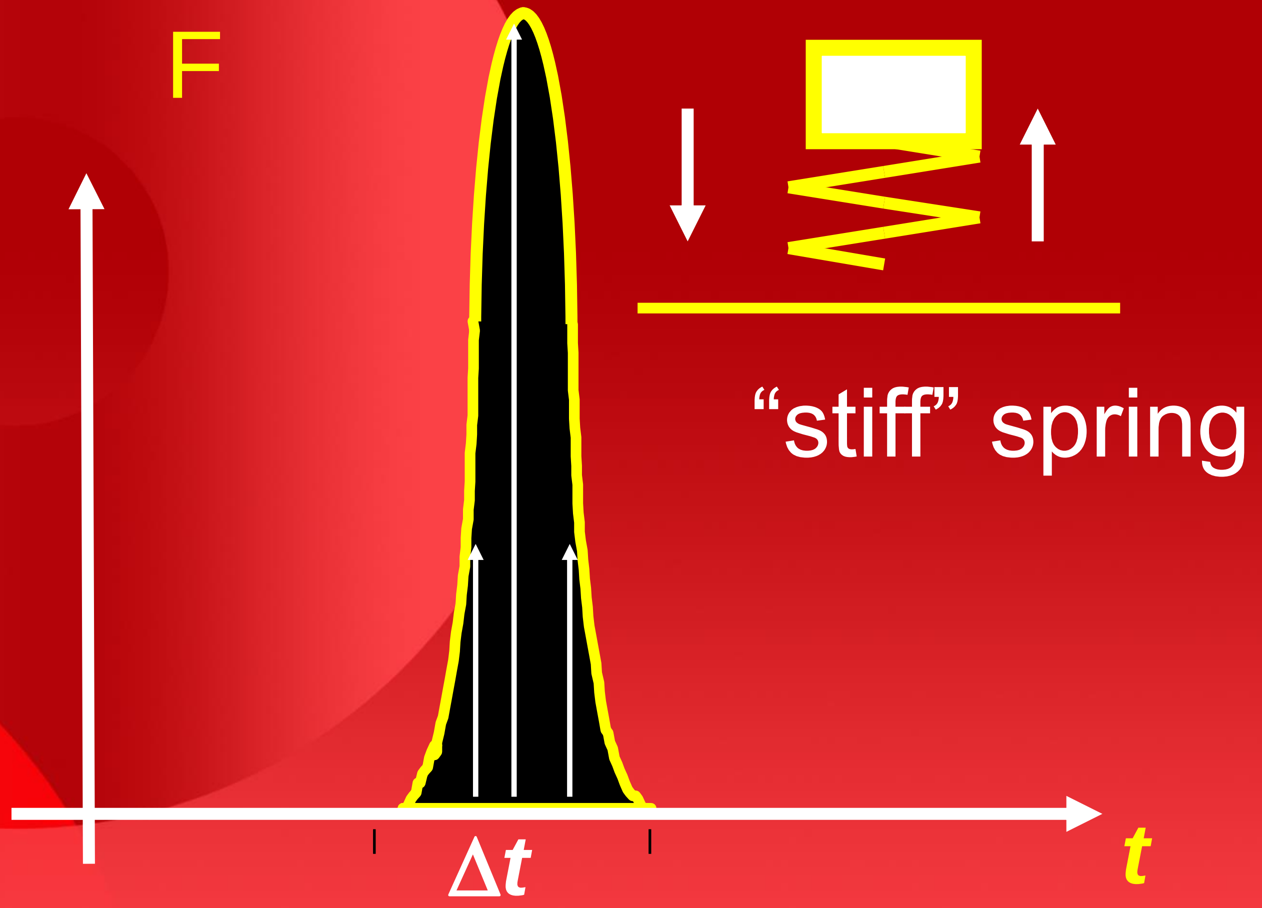




Area under the curve = *impulse*



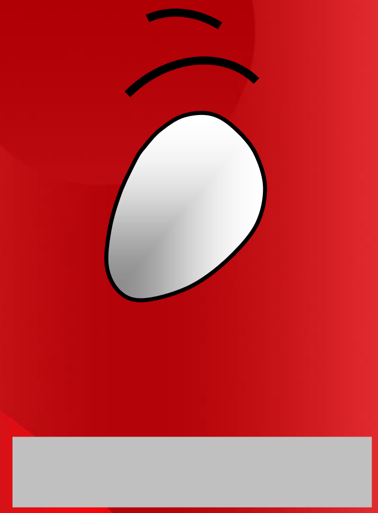
Δt is BIG, F is SMALL



Δt is SMALL, F is BIG

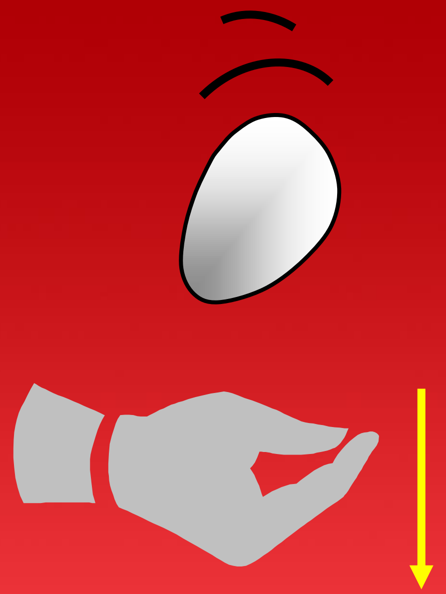
Drop egg on foam

$F\Delta t \to \Delta p$



(breaks)

$F\Delta t \to \Delta p$



(doesn't break)

Q: Would you rather land with your legs bending or stiff?

Q: Why do cricket fielders move their hands backwards when catching a fast ball?

Q: Why do railway carriages have dampers at the front and back?

Conclude:

Momentum is a concept that is useful because of Newton's 2nd law. Physics is a quantitative field.

It turns out that momentum conservation still holds even when we go beyond Newton's laws. But momentum is not just mv.