Sir Syed University of Engineering & Technology

ANSWER SCRIPT

Date:	18 Jan 2021
Roll Number:	2016-CE-026
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Course Name:	CVIT
Degree Program:	BSCE
Total number of pages being submitted:	9

Start writing from here …

Q#5 (b)

i) $f(z) = \frac{e^z}{(z-R)^3}$

Residue at

$z = \cancel{0}$

$\Rightarrow \frac{e^0}{(0-2b)(0-2b)(0-2b)}$

$= \infty$

ii)

$f(z) = \frac{z - \sin 2bz}{z^3}$

z^3 : mean $z = 0$

Resedu
at $z = 0$

$\Rightarrow \frac{0 - \sin(0)}{0^3}$

$= 0.$

Ans.

$-2^{5/8}\left(\cos\left(19\pi/16\right) + i\sin\left(19\pi/16\right)\right)$

$= -2^{7/8}\cos\left(3\pi/16\right) - 2^{7/8}\cdot\sin\left(3\pi/16\right)$

$k = 3$

$\sqrt[4]{8\sqrt{2}}\left(\cos\left(\frac{(3\pi/4)+2\pi\cdot 3}{4}\right) + i\sin\left(\frac{(3\pi/4)+2\pi\cdot 3}{4}\right)\right)$

$= \sqrt[4]{2}\cdot 2^{5/8}\left(\cos\left(27\pi/16\right) + i\sin\left(\frac{27\pi}{16}\right)\right)$

$= 2^{7/8}\cos\left(5\pi/16\right) - 2^{7/8} i\sin\left(5\pi/16\right).$

Answer:

$\sqrt[4]{-8+8i} = 2^{7/8}\cos\left(\frac{3\pi}{16}\right) + 2^{7/8}\sin\left(3\pi/16\right) \approx$

$\approx 1.52492199256651 + 1.01892029992627i$

$\sqrt[4]{-8+8i} = -2^{7/8}\cos\left(5\pi/16\right) + 2^{7/8} i\sin\left(5\pi/16\right) \approx$

$-1.01892029992627 + 1.52492199256651i$

$\sqrt[4]{-8+8i} = -2^{7/8}\cos\left(3\pi/16\right) - 2^{7/8} i\sin\left(3\pi/16\right) =$

$-1.52492199256651 - 1.01892029992627i$

$\sqrt[4]{-8+8i} = 2^{7/8}\cos\left(5\pi/16\right) - 2^{7/8} i\sin\left(5\pi/16\right)$

$\approx 1.01892029992627 - 1.52492199256651i.$

Thus

$K = 0$

$$\sqrt[4]{8\sqrt{2}}\left(\cos\left(\frac{(3\pi/4)+2\pi\cdot 0}{4}\right)+i\sin\left(\frac{(3\pi/4)+2\pi\cdot 0}{4}\right)\right) = \cancel{i\sin}\ \sqrt[4]{2}$$

$$\cdot\ 2^{5/8}\left(\cos\left(3\pi/16\right)+i\sin\left(3\pi/16\right)\right)$$

$$= 2^{7/8}\cos\left(3\pi/16\right) + 2^{7/8}\, i\sin\left(3\pi/16\right)$$

(b) $K = 1$

$$\sqrt[4]{8\sqrt{2}}\left(\cos\left(\frac{(3\pi/4)+2\pi\cdot 1}{4}\right)+i\sin\left(\frac{(3\pi/4)+2\pi\cdot 1}{4}\right)\right) = \sqrt[4]{2}\cdot 2^{5/8}$$

$$\left(\cos\left(11\pi/16\right)+i\sin\left(11\pi/16\right)\right) =$$

$$\Rightarrow -2^{7/8}\cos\left(5\pi/16\right) + 2^{7/8}\, i\sin\left(5\pi/16\right)$$

$K = 2$

$$\sqrt[4]{8\sqrt{2}}\left(\cos\left(\frac{(3\pi/4+2\pi\cdot 2}{4}\right)+i\sin\left(\frac{(3\pi/4)+2\pi\cdot 2}{4}\right)\right)\cdot\sqrt[4]{2}$$

Q 5 (a)

Sol:

fourth root.

$$\sqrt[4]{-8+8i}$$

$-8+8i$

$$= 8\sqrt{2}\left(\cos\left(3\pi/4\right) + i\sin\left(3\pi/4\right)\right)$$

According to De Moivre formula of all n-th root of complex number.

$r\left(\cos(\theta) + i\sin(\theta)\right)$ are given by

$$= \sqrt[n]{r}\left(\cos\left(\theta + 2\pi k/n\right) + i\sin\left(\theta + \frac{2\pi k}{n}\right)\right) \cancel{+ i\sin}$$

$K = 0 \ldots n-1$

we have that $r = 8\sqrt{2}$

$\theta = 3\pi/4$, $n = 4$

Q#04

Ans.

$$I = \int_{-\infty}^{\infty} T/(1+x^2)^2$$

Sol:

$$\Rightarrow 8 \int_{-\infty}^{\infty} 1/(1+x^2)^2$$

let

$$f(z) = \frac{1}{(1+z^2)^2}$$

If is clear that for z large

$$f(z) \simeq \frac{1}{z^4}$$

Hypothesis of theorm of 9.1 satisfie

$$\int_{C_1+C_R} f(z)\, dz = 2\pi i \sum \text{residu of } f \text{ inside the combo}$$

Residue

An

$T = 8$

Q#3 Sol.

$$f_c(\omega) = \sqrt{\frac{2}{\pi}} \int_0^{T} f(x) \cos \omega x \, dx.$$

$$= \sqrt{2/\pi} \int_0^{T} x \cos \omega x \, dx.$$

Using tabular form

derivative		Integral
		$\cos \omega x$
x	$+$	$\sin \omega x/\omega$
1	$-$	$-\cos \omega x/\omega^2$
0		

$$= \sqrt{2/\pi} \left[T/\omega \sin \omega x \Big|_0^T + 1/\omega^2 \cos \omega x \Big|_0^T \right]$$

$$= \sqrt{2/\pi} \left[\frac{T \sin \omega T}{\omega} + \frac{1}{\omega^2} (\cos \omega T - 1) \right]$$

$$= \sqrt{2/\pi} \left[\frac{T \sin \omega T}{\omega} + \frac{\cos \omega T}{\omega^2} - 1/\omega^2 \right]$$

As $\because T = 8$

$$= \sqrt{2/\pi} \left[\frac{8\omega(\sin 8\omega) + \cos 8\omega - 1}{\omega^2} \right]$$

Ans.

Q#2 (a)

Sol

$$f(t) = \cos(26t - 7)$$

$R = 26$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\cos 26t - 7\}$$

Formula

$$\cos(at-b) = s\cos(b) - \frac{a\cos(b)}{s^2+a^2}$$

$$F(s) = s\cos(7)\ \frac{-26\cos(7)}{s^2+(26)^2}$$

(b)

$$F(s) = \frac{s}{R^2 s^2 + 16\pi^2}$$

$$= \frac{s}{26^2 s^2 + 16\pi}$$

$$\mathcal{L}^{-1}(F(s)) = f(t) = \frac{1}{676} \cdot \frac{s}{(s)^2 + (4\pi)^2}$$

$$= \frac{1}{676} \cos 4\pi t$$

Sol. of Q#1

$\Rightarrow$ $k = 26$

so) $y'' - 26y = 0$ $y(0) = 1$

$y'' - 676 = 0$ $y'(0) = 1$

$s^2 \mathcal{L}\{y\} - s\, y'(0) - y''(0) + 1225\, \mathcal{L}\{y\} = 0$

$s^2 \mathcal{L}\{y\} - s\, \mathcal{L}\{y\} + 676\, \mathcal{L}\{y\} = 1$

$\mathcal{L}\{y\}[s^2 - s + 676) = 1$

$\mathcal{L}\{y\} = \dfrac{1}{s^2 - s + 676}$

$= 1225\, s(t) - s'(t) + t$

Ans.