Homework 2

Due date: Thu Sep 26th, 2019

Problem 1

For the following system of equations

$$\begin{bmatrix} \frac{10}{3} & \frac{5}{3} & \frac{10}{7} \\ \frac{10}{3} & \frac{30}{7} & \frac{50}{9} \\ \frac{20}{9} & 5 & \frac{50}{7} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} \frac{5}{3} \\ \frac{9}{7} \\ \frac{2}{3} \end{Bmatrix}$$

determine the values for x_i. Verify the solution.

Problem 2

Given

$$\delta = \left[\frac{1}{n} \sum_{i=1}^{n} x_i^{\beta} \right]^{1/\beta}$$

For $\beta = 2.65$ and $x = \{73, 81, 98, 102, 114, 116, 127, 125, 124, 140, 153, 160, 198, 208\}$, determine δ.

Problem 3

Evaluate the following series when $x_1 = 0.4$, $y_1 = 0.65$, $x_2 = 0.3$, and $y_2 = 0.45$

$$s = \sum_{k=1,3,\ldots}^{11} \sum_{m=2,4,\ldots}^{12} \frac{\sin\big((2m+k)x_1/3\big)\sin(ky_1)\sin\big((2m+k)x_2/3\big)\sin(ky_2)}{\big(k^2+km+m^2\big)^2}$$

Problem 4

Show that $\boldsymbol{B}^2 - 12\boldsymbol{B} - 45\boldsymbol{I} = 0$, where $\boldsymbol{I}$ is the identity matrix, if

$$\boldsymbol{B} = \begin{bmatrix} 3 & 6 & 6 \\ 6 & 3 & 6 \\ 6 & 6 & 3 \end{bmatrix}$$

Problem 5

If

$$k = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad T = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

then determine $\boldsymbol{T'kT}$, where the prime indicates the transpose.

Problem 6

The equation of a circle can be determined for three noncolinear points (x_i, y_i), $i = 1, 2, 3$ from

$$\det \begin{vmatrix} x^2 + y^2 & x & y & 1 \\ x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

Determine the equation for the circle that passes through the points $(-2,2)$, $(0,0)$, and $(1,1)$.

Problem 7

Determine the largest real eigenvalue of

$$\begin{pmatrix} 0 & 0 & 0 & 0.83 & 0.83 & 0.5 & 0.5 & 0.11 & 0.11 & 0 \\ 0.94 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.98 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.98 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.98 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.98 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.98 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.98 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.97 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.97 & 0 \end{pmatrix}$$