

Homework 3

Due date: Thu Oct 3rd, 2019

Problem 1

Consider the following series

$$\sum_{n=2}^M \frac{1}{(n^2 - 1)^2} = S_M$$

where

$$S_{\infty} = \frac{\pi^2}{12} - \frac{11}{16}$$

Find the value of M for which

$$\left| \frac{S_M - S_{\infty}}{S_{\infty}} \right| < 0.0001$$

Problem 2

The elements of an $(N \times N)$ matrix are given by

$$h_{nm} = \begin{cases} 0 & n + m - 1 > N \\ n + m - 1 & \text{otherwise} \end{cases}$$

Use **Array** to generate these elements for $N = 8$ and display the results in matrix form.

Problem 3

Solve for x when

$$\eta = \sqrt{2} \left[\tanh^{-1} \left(\sqrt{\frac{2}{3}} \right) - \tanh^{-1} \left(\frac{\sqrt{x+2}}{\sqrt{3}} \right) \right]$$

Problem 4

Given the function

$$f(x) = e^{\sin(x+c)}$$

The value of $x = x_{\max}$ is that value at which the maximum value of $f(x)$ occurs. It can be determined from the solution to $df(x)/dx = 0$ provided that $d^2f(x_{\max})/dx^2 < 0$. Determine x_{\max} .

Problem 5

If

$$f_m(\alpha) = m + \frac{\alpha^2}{m} \quad m = 1, 2, \dots$$

then determine an expression for α , $\alpha > 0$, that satisfies $f_m(\alpha) = f_{m+1}(\alpha)$

Problem 6

Obtain the coefficients of a five-term expansion of the following expression about $\varepsilon = 0$.

$$f = \frac{\pi\varepsilon\sqrt{\pi^2(1-\varepsilon^2)+16\varepsilon^2}}{(1-\varepsilon^2)^2}$$

Problem 7

Evaluate the following integral when $b > 0$ and b is real.

$$f(b) = \int_0^b \frac{2x+5}{x^2+4x+5} dx$$

Problem 8

Determine the general solution to the following equation

$$\left(\frac{d^2}{dr^2} + \frac{d}{rdr} - \frac{m^2}{r^2}\right) \left(\frac{d^2w}{dr^2} + \frac{dw}{rdr} - \frac{m^2}{r^2}w\right) = q_o$$

where $w = w(r)$, q_o is a constant, and $m = 0, 1, \dots, 5$.

