# Applied Physics

BS Software Engineering/Information Technology

1st Semester

Lecture # 21

# Magnetic Fields Due to Currents

Presented By

Dr. Arifa Mirza

Punjab University College of Information Technology

# Lecture # 21

- Magnetic field of a current
- The Biot- Savart law
- Two parallel conductors

# SECTION

# Electromagnetism

One basic observation of physics is that a moving charged particle produces a magnetic field around itself. Thus a current of moving charged particles produces a magnetic field around the current. This feature of *electromagnetism*, which is the combined study of electric and magnetic effects, came as a surprise to the people who discovered it. Surprise or not, this feature has become enormously important in everyday life because it is the basis of countless electromagnetic devices. For example, a magnetic field is produced in maglev trains and other devices used to lift heavy loads.

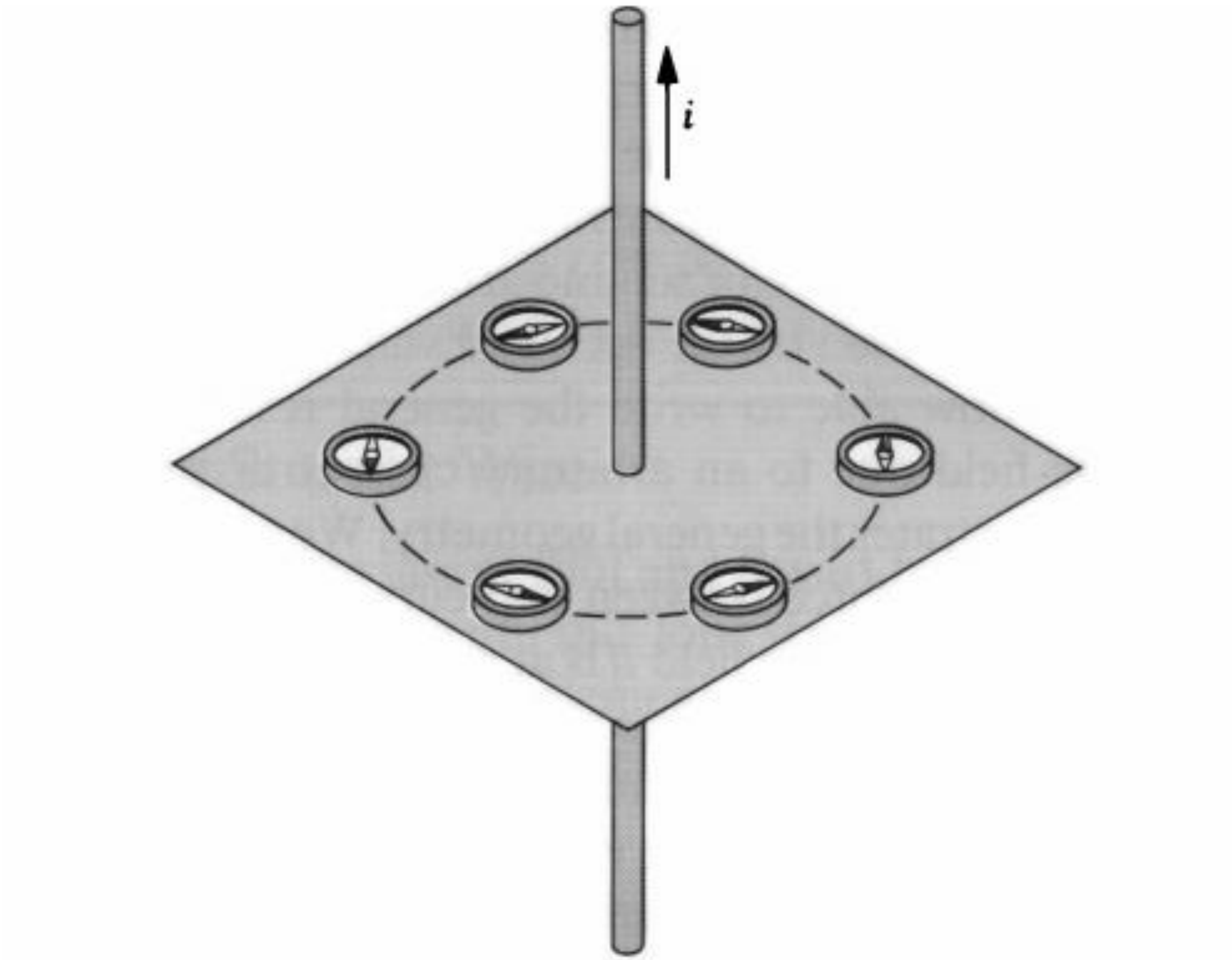
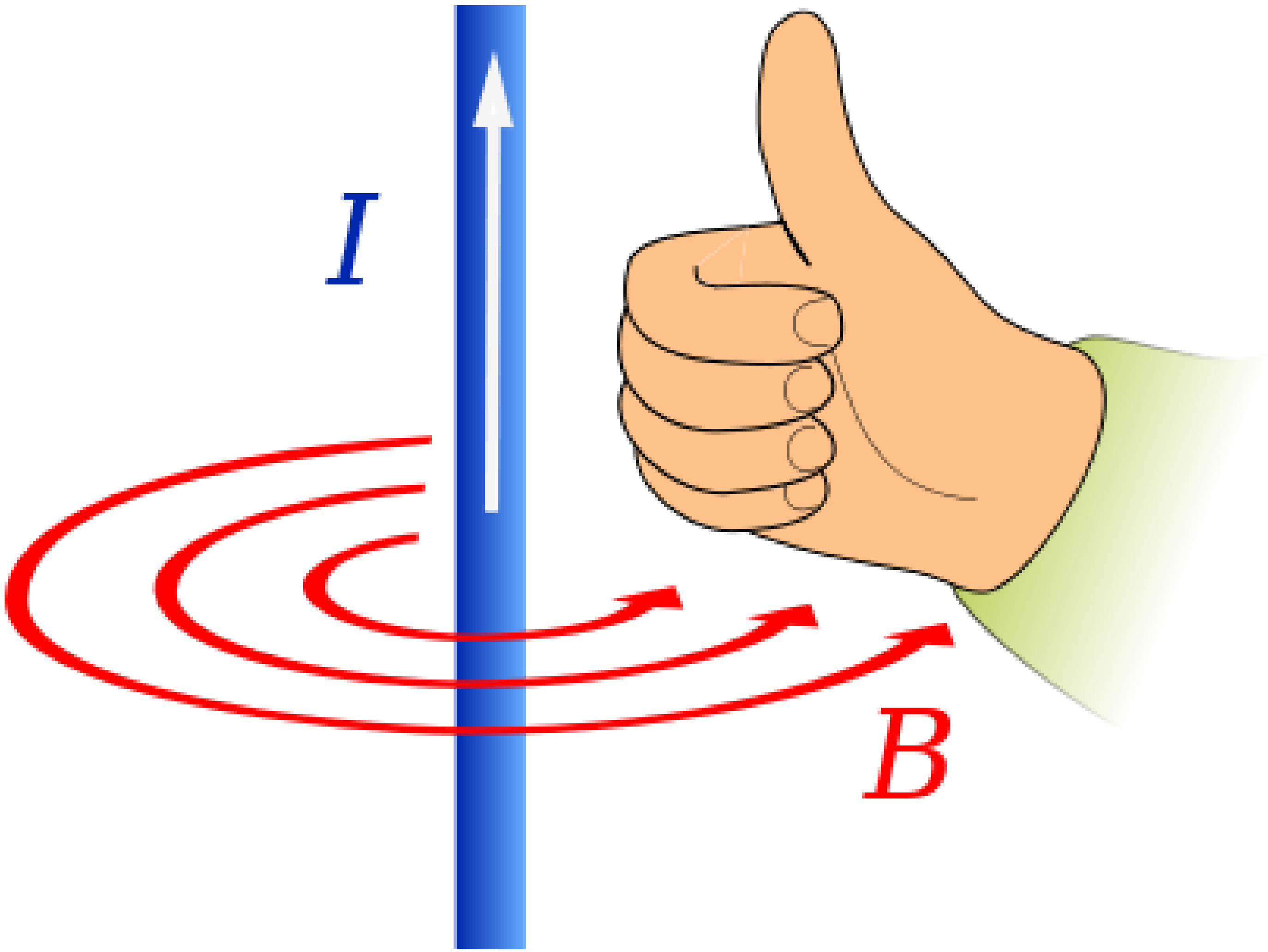
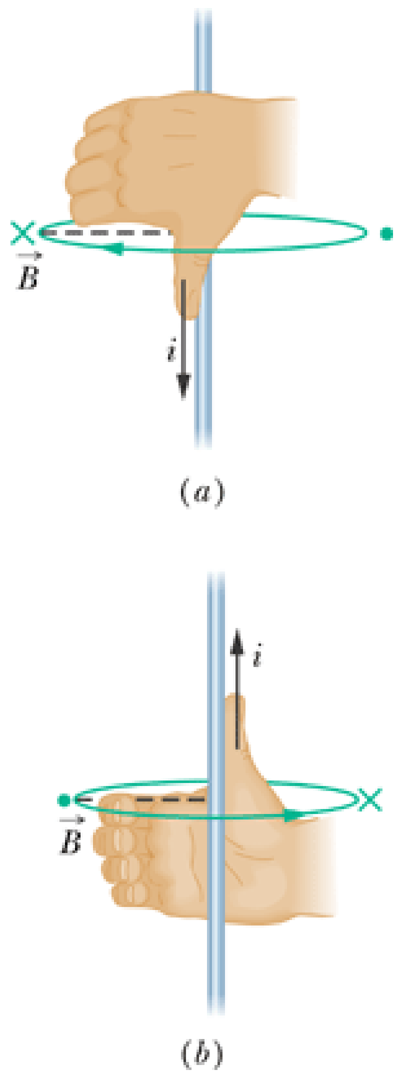



**Figure 1** Oersted's experiment. The direction of the compass needle is always perpendicular to the direction of the current

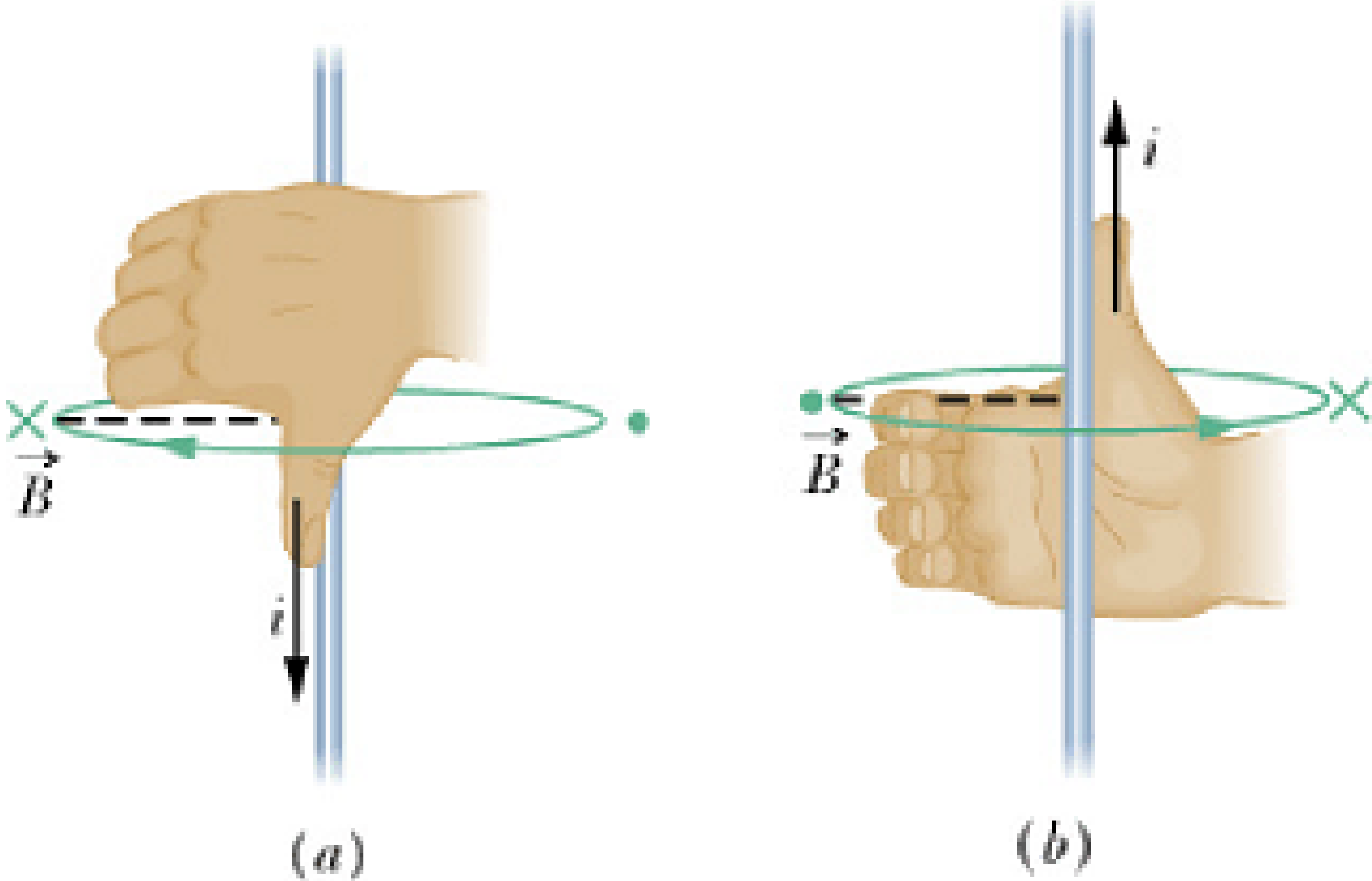
# Magnetic Field Due to a Current



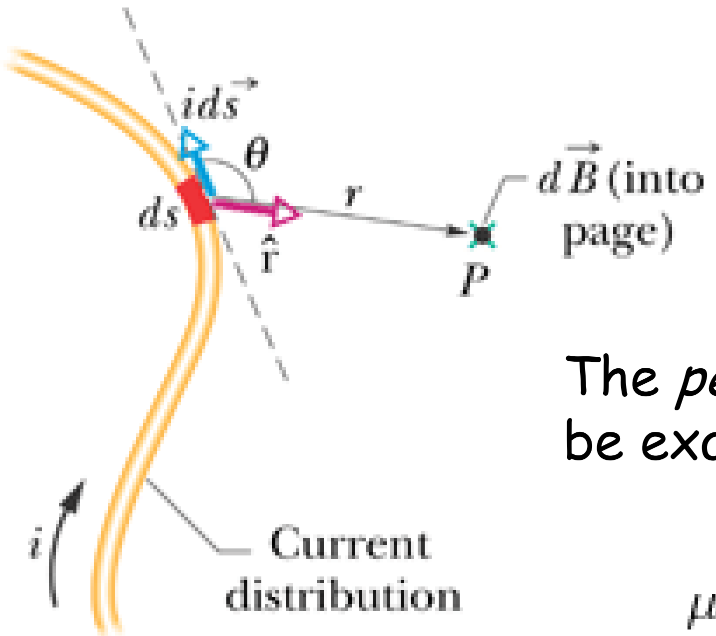
# Right-hand rule




Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.


$\vec{B}$
$i$
$i$
$\vec{B}$
(a)
(b)

# Magnetic Field Due to a Current

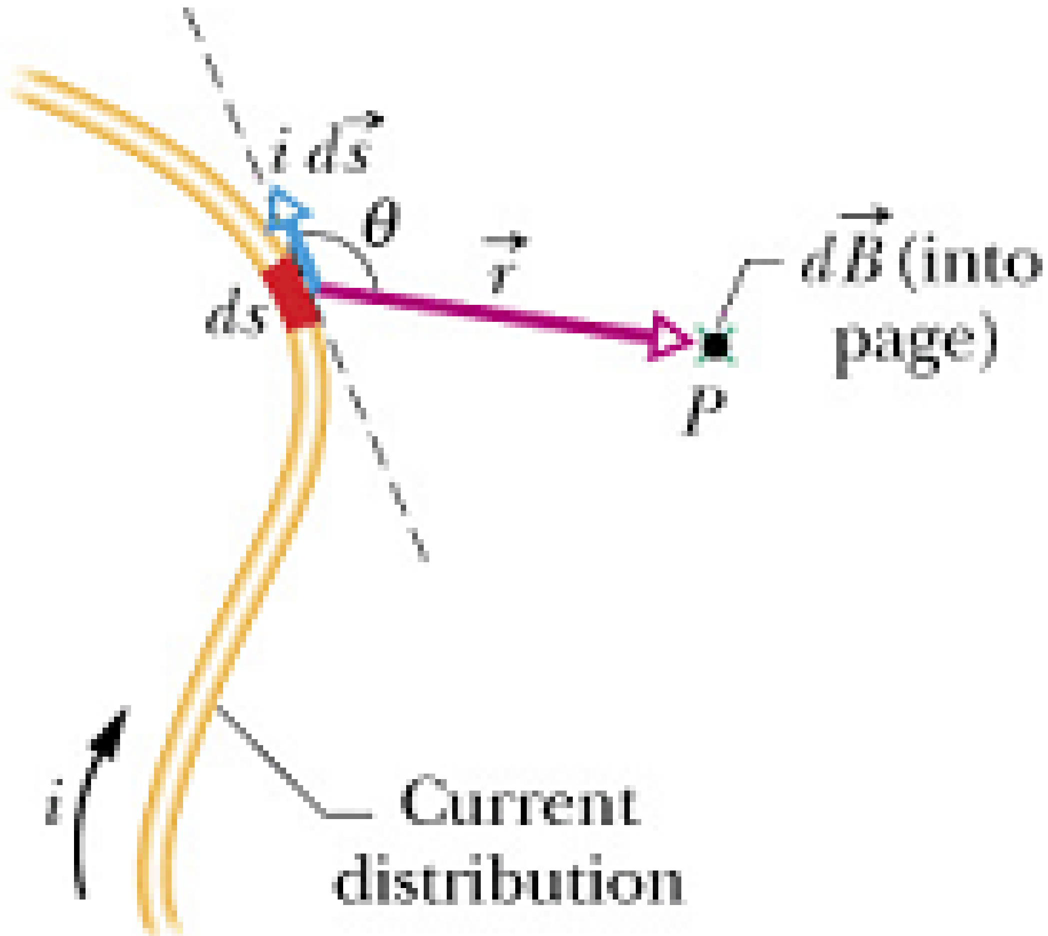



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{r}}{r^2} \quad \text{(Biot–Savart law)}.$$

The *permeability constant*, whose value is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A} \approx 1.26 \times 10^{-6}\,\mathrm{T \cdot m/A}.$$

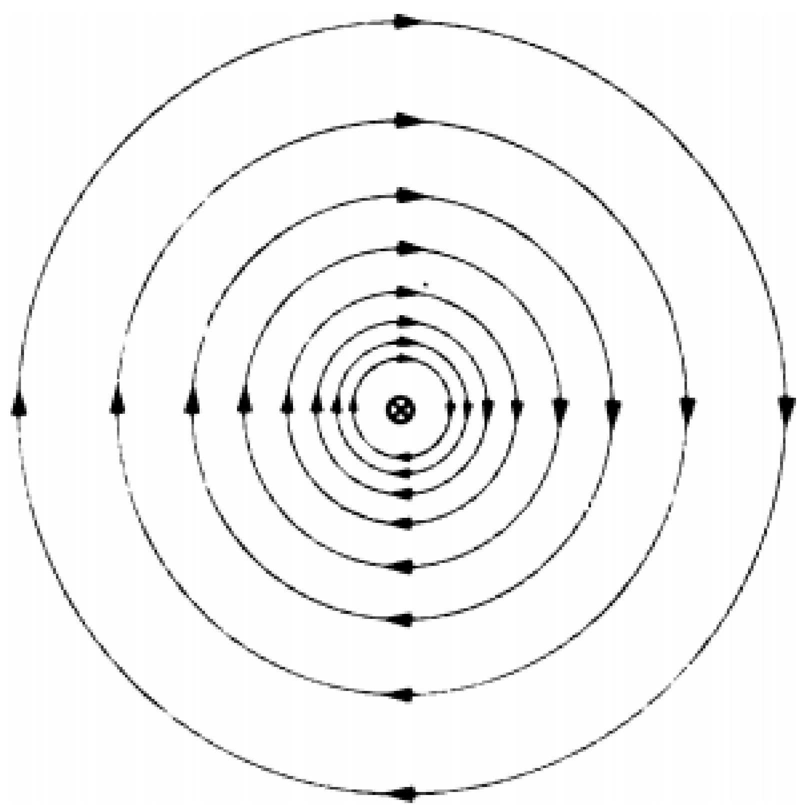
A length vector $d\vec{s}$ that has length *ds* and whose direction is the direction of the current in *ds*.



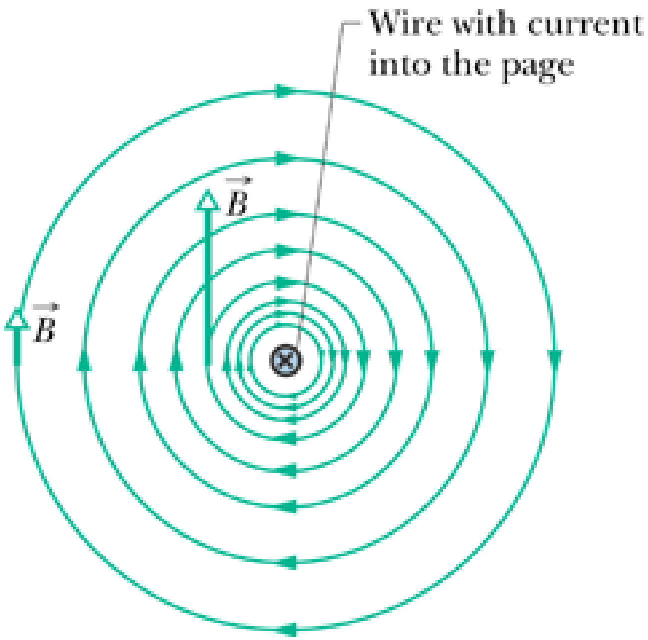
$i\,d\vec{s}$
$\theta$
$\vec{r}$
$d\vec{B}$ (into page)
$ds$
$P$
$i$
Current distribution

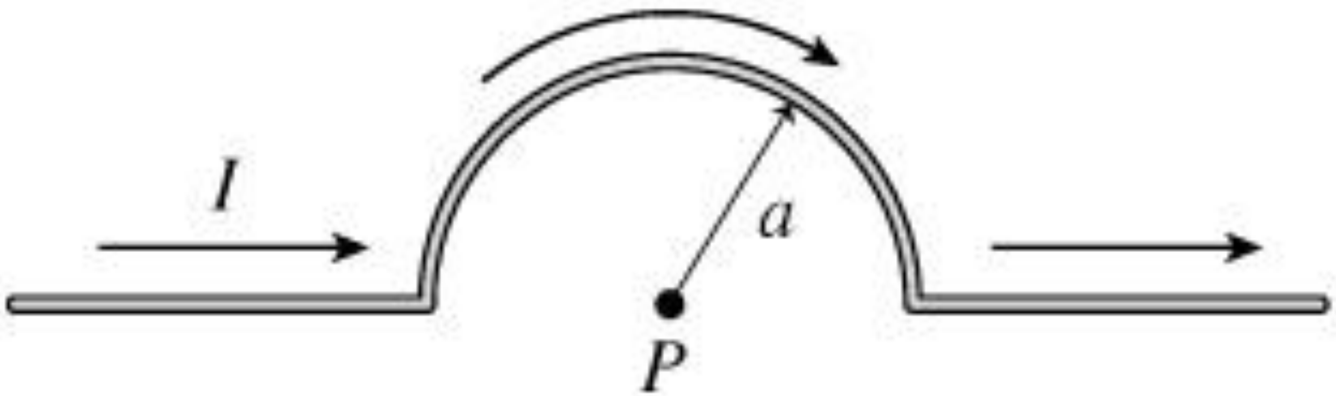
# 35-3 LINES OF B

Figure 9 shows lines representing the magnetic field **B** near a long straight wire. Note the increase in the spacing of the lines with increasing distance from the wire. This represents the $1/r$ decrease in $B$ predicted by Eq. 11.



# Magnetic field lines produced by a current in a long straight wire



$I$
$a$
$P$

## Sample Problem 29.01 Magnetic field at the center of a circular arc of current

The wire in Fig. 29-8*a* carries a current $i$ and consists of a circular arc of radius $R$ and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center $C$ of the arc. What magnetic field $\vec{B}$ (magnitude and direction) does the current produce at $C$?

### KEY IDEAS

We can find the magnetic field $\vec{B}$ at point $C$ by applying the Biot–Savart law of Eq. 29-3 to the wire, point by point along the full length of the wire. However, the application of Eq. 29-3 can be simplified by evaluating $\vec{B}$ separately for the three distinguishable sections of the wire—namely, (1) the straight section at the left, (2) the straight section at the right, and (3) the circular arc.

***Straight sections:*** For any current-length element in section 1, the angle $\theta$ between $d\vec{s}$ and $\hat{r}$ is zero (Fig. 29-8*b*); so Eq. 29-1 gives us

$$dB_1 = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 0}{r^2} = 0.$$

Thus, the current along the entire length of straight section 1 contributes no magnetic field at $C$:

$$B_1 = 0.$$

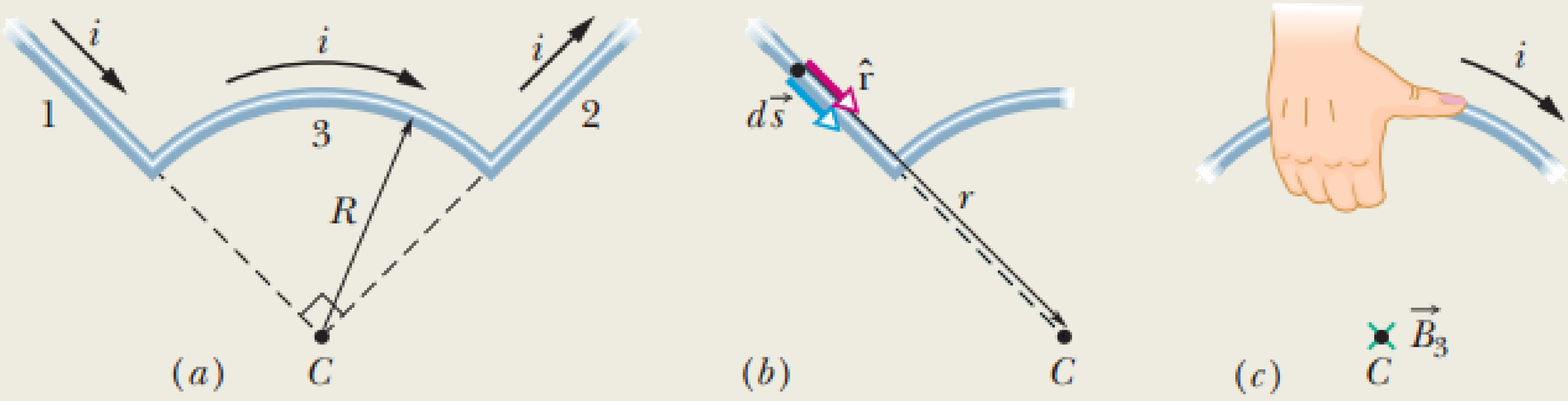
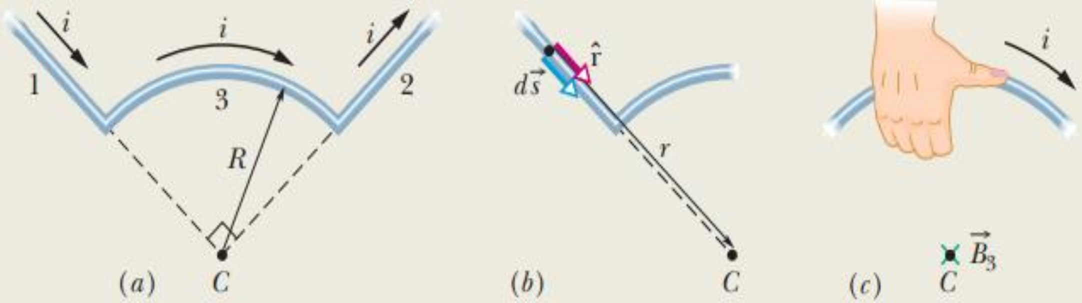


**Figure 29-8** (*a*) A wire consists of two straight sections (1 and 2) and a circular arc (3), and carries current *i*. (*b*) For a current-length element in section 1, the angle between $d\vec{s}$ and $\hat{r}$ is zero. (*c*) Determining the direction of magnetic field $\vec{B}_3$ at *C* due to the current in the circular arc; the field is into the page there.




The same situation prevails in straight section 2, where the angle $\theta$ between $d\vec{s}$ and $\hat{r}$ for any current-length element is 180°. Thus,

$$B_2 = 0.$$

***Circular arc:*** Application of the Biot–Savart law to evaluate the magnetic field at the center of a circular arc leads to Eq. 29-9 ($B = \mu_0 i\phi/4\pi R$). Here the central angle $\phi$ of the arc is $\pi/2$ rad. Thus from Eq. 29-9, the magnitude of the magnetic field $\vec{B}_3$ at the arc's center *C* is

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}.$$

To find the direction of $\vec{B}_3$, we apply the right-hand rule displayed in Fig. 29-5. Mentally grasp the circular arc with your right hand as in Fig. 29-8*c*, with your thumb in the direction of the current. The direction in which your fingers curl around the wire indicates the direction of the magnetic field lines around the wire. They form circles around the wire, coming out of the page above the arc and going into the page inside the arc. In the region of point *C* (inside the arc), your fingertips point *into the plane* of the page. Thus, $\vec{B}_3$ is directed into that plane.

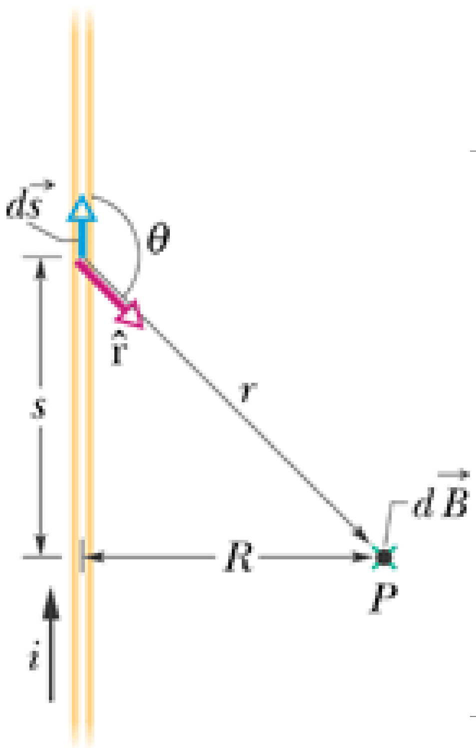
***Net field:*** Generally, we combine multiple magnetic fields as vectors. Here, however, only the circular arc produces a magnetic field at point *C*. Thus, we can write the magnitude of the net field $\vec{B}$ as

$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}. \qquad \text{(Answer)}$$

The direction of $\vec{B}$ is the direction of $\vec{B}_3$—namely, into the plane of Fig. 29-8.

# Magnetic Field Due to a Current in a Long Straight Wire

$$dB = \frac{\mu_0}{4\pi}\frac{i\sin\theta g ds}{r^2}$$




$$B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{\sin\theta\, ds}{r^2}.$$

$$r = \sqrt{s^2 + R^2} \qquad \sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2+R^2}}.$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{R\,ds}{\left(s^2+R^2\right)^{3/2}}$$

$$= \frac{\mu_0 i}{2\pi R}\left[\frac{s}{\left(s^2+R^2\right)^{1/2}}\right]_0^\infty = \frac{\mu_0 i}{2\pi R},$$

$$B = \frac{\mu_0 i}{2\pi R} \quad \text{(long straight wire)}.$$

$$B = \frac{\mu_0 i}{4\pi R} \quad \text{(semi–infinite straingt wire)}.$$

# TWO PARALLEL CONDUCTORS

Soon after Oersted's discovery that a current-carrying conductor would deflect the needle of a magnetic compass, Ampère concluded that two such conductors would attract each other with a force of magnetic origin.

We analyze the magnetic interaction of two currents in a manner similar to that of our analysis of the electric interaction between two charges:

$$\text{charge} \rightleftarrows \mathbf{E} \rightleftarrows \text{charge.}$$

That is, one charge sets up an electric field, and the other charge interacts with the field at its particular location. We use a similar procedure for the magnetic interaction:

$$\text{current} \rightleftarrows \mathbf{B} \rightleftarrows \text{current.}$$

Here a current sets up a magnetic field, and the other current then interacts with that field.

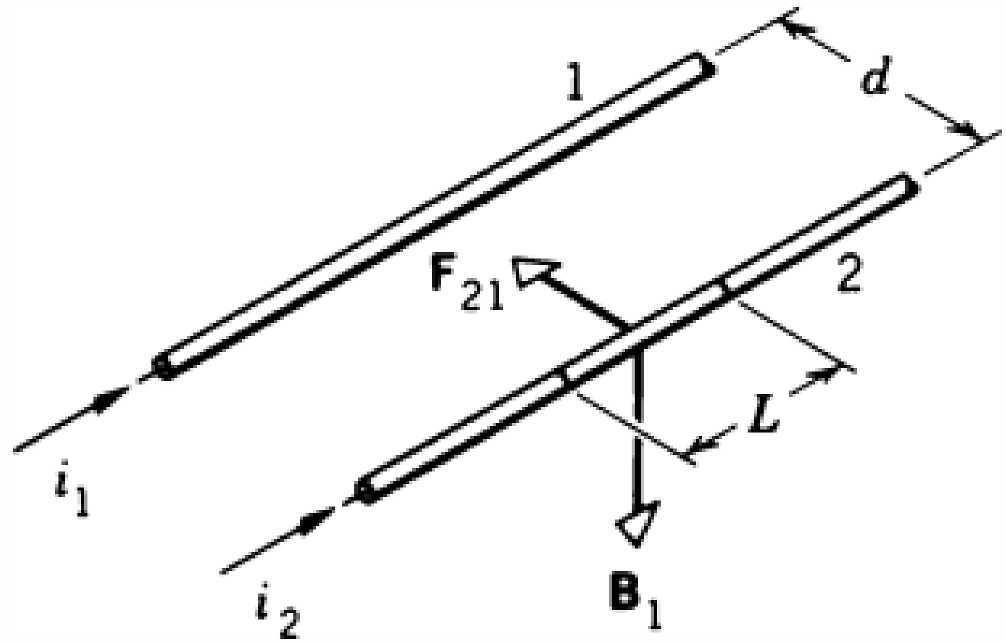



**Figure** Two parallel wires carrying currents in the same direction attract each other. The field $\mathbf{B}_1$ at wire 2 is that due to the current in wire 1.

# HOME WORK # 3

## Magnetic Field Due to a Current

•1 A surveyor is using a magnetic compass 6.1 m below a power line in which there is a steady current of 100 A. (a) What is the magnetic field at the site of the compass due to the power line? (b) Will this field interfere seriously with the compass reading? The horizontal component of Earth's magnetic field at the site is 20 $\mu$T.

**•4** A straight conductor carrying current $i = 5.0$ A splits into identical semicircular arcs as shown in Fig. 29-36. What is the magnetic field at the center $C$ of the resulting circular loop?

**•5** In Fig. 29-37, a current $i = 10$ A is set up in a long hairpin conductor formed by bending a wire into a semicircle of radius $R = 5.0$ mm. Point $b$ is midway between the straight sections and so distant from the semicircle that each straight section can be approximated as being an infinite wire. What are the (a) magnitude and (b) direction (into or out of the page) of $\vec{B}$ at $a$ and the (c) magnitude and (d) direction of $\vec{B}$ at $b$?

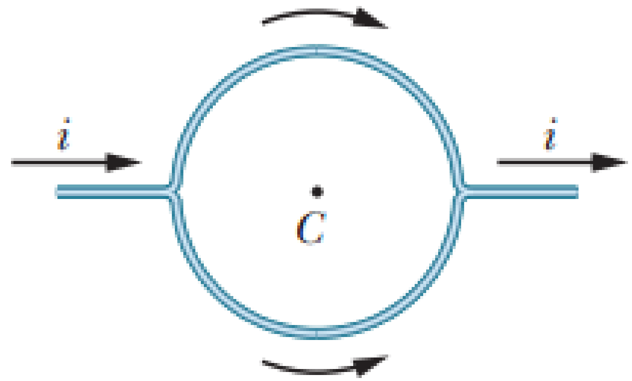



**Figure 29-36** Problem 4.


**Figure 29-37** Problem 5.

**•7** GO In Fig. 29-39, two circular arcs have radii $a = 13.5$ cm and $b = 10.7$ cm, subtend angle $\theta = 74.0°$, carry current $i = 0.411$ A, and share the same center of curvature $P$. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at $P$?

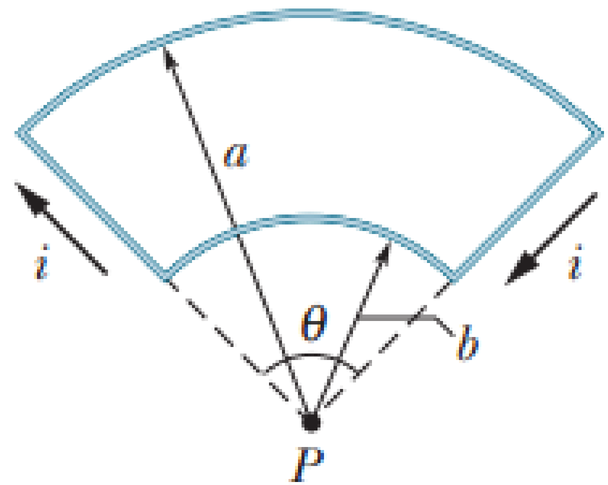



**Figure 29-39** Problem 7.