UNIT-4: Graph theory

Graph theory: Basic concepts, Graph Theory and its Applications, Subgraphs, Graph Representations: Adeency and Incidence matrices, Isomorphic Graphs paths. and Circuits, Eulerian and Hamiltonian Graphs.

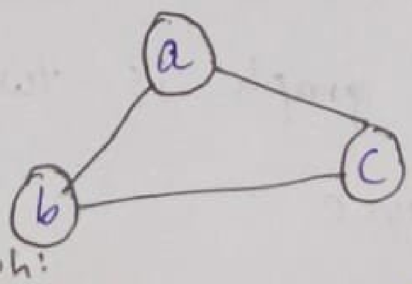
Graph:

The graph is a collection of vertices and edges usually denoted as G(V,E). where, V denoted no.of vertices and E denoted no.of edges.

Ex:

Let us consider G(V,E), where the set V defined as V = {a, b, c} and E is defined as E = {a-b, b-c, c-a} the corresponding graph G is as follow

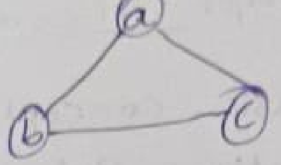
V = {a, b, c} , E = {a-b, b-c, c-a}



undirected graph:

The graph contains undirected edges is known as "Undirected Graph".

Ex:



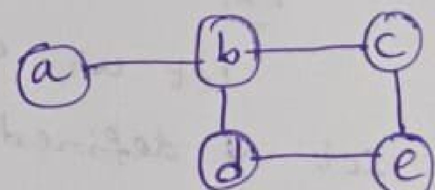
Direct graph:

If atleast one edge in the given graph G(V, E) is having direction then such graph is known as "Directed graph."

Ex:

Degree of Graph:

The graph G(V, E) vertex with the highest degree is called "the degree of graph."



Ex: deg(a) = 1
deg(b) = 3
deg(c) = 2
deg(d) = 2
deg(e) = 2

The direct graph is two types

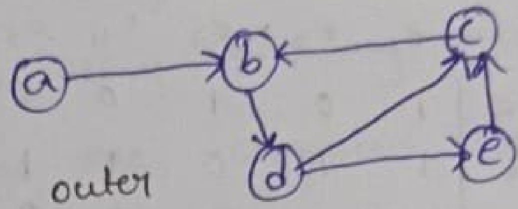
1. Inner degree
2. outer degree

Inner degree: The no.of incoming edges of a vertex in graph G(V, E) is called "In-degree".

Outer degree:

The no. of outgoing edges of a vertex in graph G is called "outer degree".

The following graph find inner & outer degree



	Inner	+	outer	
deg(a)	0	+	1	= 1
deg(b)	2	+	1	= 3
deg(c)	2	+	1	= 3
deg(d)	1	+	2	= 3
deg(e)	1	+	1	= 2

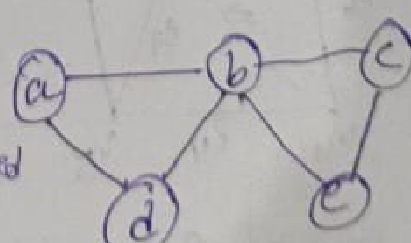
The degree of the above graph is 3.

Matrix Representation:

The graph G(V,E) can be represented in two ways.

1. Adjacency Matrix
2. Incidence Matrix

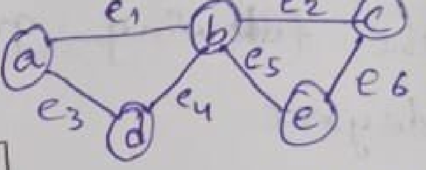
Adjacency Matrix:



A vertex is connected another vertex which matrix representation,

$$\begin{array}{c|c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

2) Incidence Matrix



$$\begin{array}{c|c} & \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

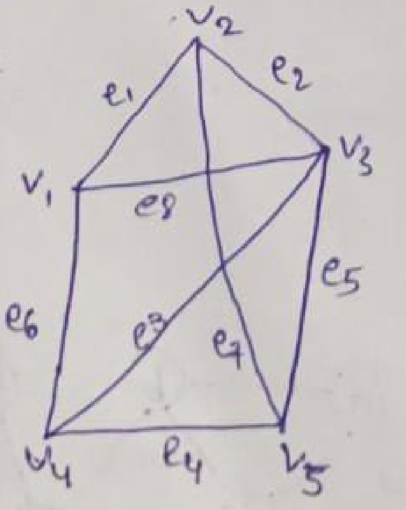
A vertex have which edge.

Subgraph:

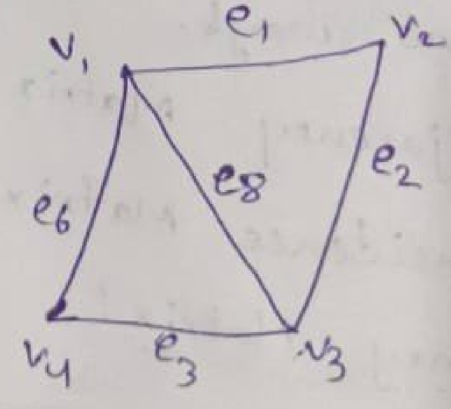
Given two graphs $G(V, E)$ and $G_1(V_1, E_1)$ we say that G_1 is a subgraph of G the following conditions are

1. All the vertex and all the edges of G_1 in G. $(V_1 \subseteq V, E_1 \subseteq E)$
2. Eeach edge of G_1 as the same and vertex in G as in G_1

Ex:

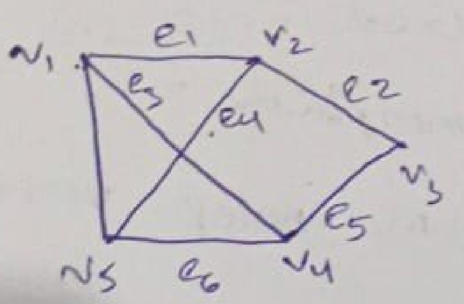


Graph $G(V, E)$

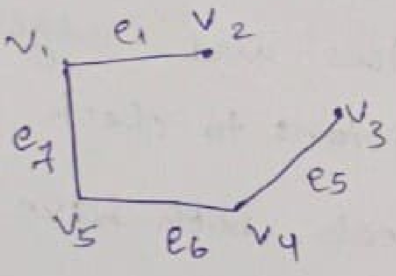


Subgraph $G_1(V_1, E_1)$

Ex:



Graph G (V, E)



Subgraph G_1 (V_1, E_1)

Applications of Graph theory

- The Graph theory using so many areas. the Specific areas are, computer Science technology.

Some of them are given below:

1. Graph are used to define the flow of communication.
2. Graphs are used to represent network of communication.
3. Graphs are used to data organization
4. Graph theory is used to find Slorest path in road and google

Isomorphism:-

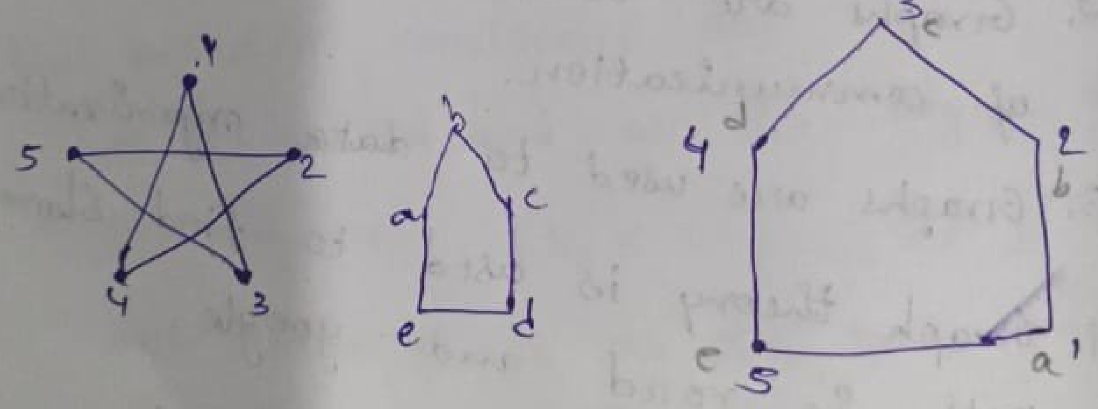
Two Simple graphs G_1 (V_1, E_1) and G_2 (V_2, E_2) are said to be isomorphism If there is one-to-one correspondence between there vertices & edges. but They will have same structure different only in the way. there vertices and edges label

conditions to check Isomorphism:

- check both have equal no.of vertices and edges.
- check both graphs have same degree of vertices
- Verify whether there is one-to-one correspondence between vertices.
- Verify Adjaceny matrix and equal w.k.t the vertices.

Ex:

Graph $G_1(V_1, E_1)$ Graph $G_2(V_2, E_2)$



1) $V(G_1) = 5$ $V(G_2) = 5$

$$V(G_1) = V(G_2)$$

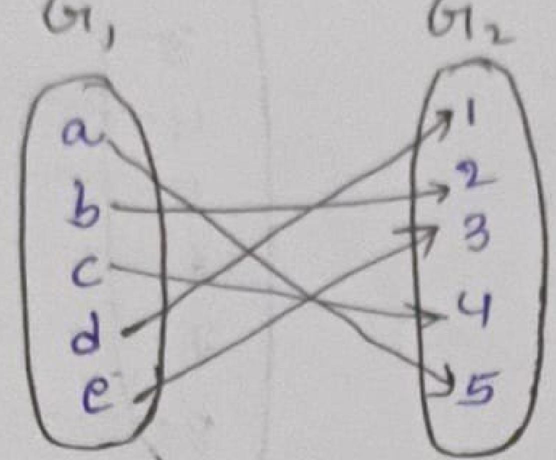
$E(G_1) = 5$ $E(G_2) = 5$

$$E(G_1) = E(G_2)$$

2) $G_1 : 1 \to 2,\ 2 \to 2,\ 3 \to 2,\ 4 \to 2,\ 5 \to 2$

$G_2 : a \to 2,\ b \to 2,\ c \to 2,\ d \to 2,\ e \to 2$

3)



4) $G_1(V_1, E_1)$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 0 & 0 & 1 \\ 4 & 1 & 1 & 0 & 0 & 0 \\ 5 & 0 & 1 & 1 & 0 & 0 \end{array}$$

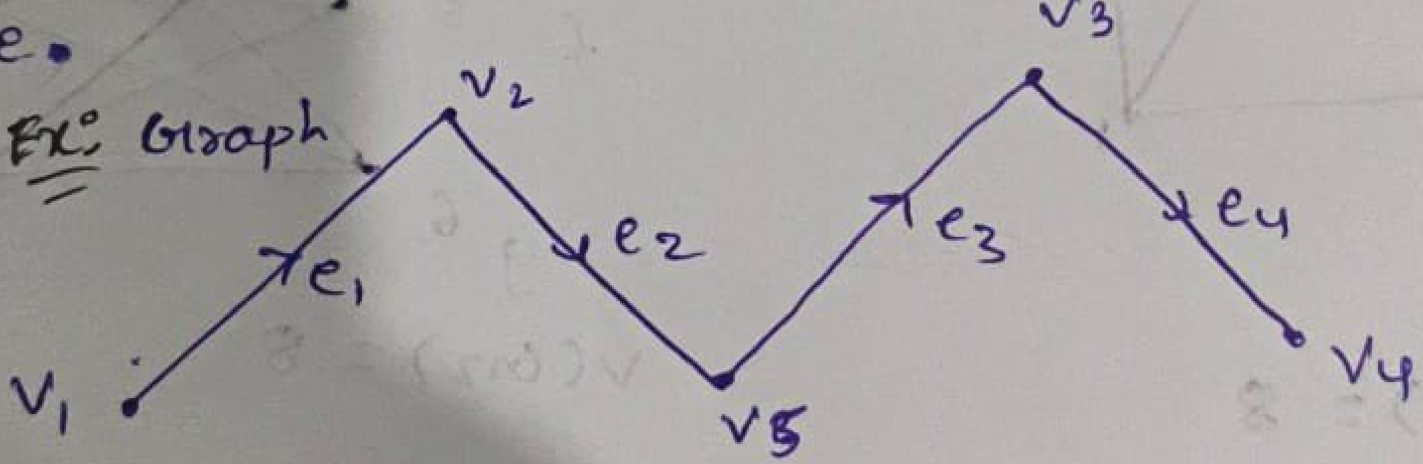
$G_2(V_2, E_2)$

$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 1 & 0 & 0 & 1 \\ b & 1 & 0 & 1 & 0 & 0 \\ c & 0 & 1 & 0 & 1 & 0 \\ d & 0 & 0 & 1 & 0 & 1 \\ e & 1 & 0 & 0 & 1 & 0 \end{array}$$

Path & Circuite

path: A path is a open walk it which no vertices appear more than once.

Ex: Graph

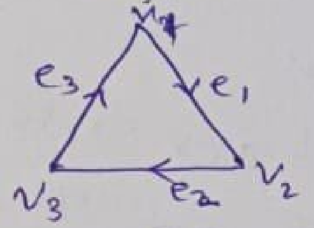


Sequence walk is

$v_1 e_1 v_2 e_2 v_5 e_3 v_3 e_4 v_4$

Circuit: circuit is a closed walk in which no edges appear more than once.

Ex: Graph



walk sequence is

$v_1 e_1 v_2 e_2 v_3 e_3 v_1$

Eular circuit:

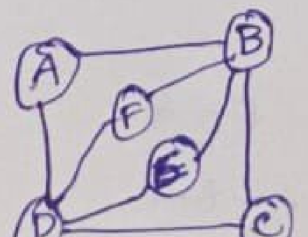
It is a closed walk which visit every edge of the graph exactly once. here no repeated edges.

Eular Graph:

A connected graph it which contain Eular circuit or cycle is called Eular circuit graph.

Note: A graph will contain an Eular circuit if and only if all vertices are even degree.

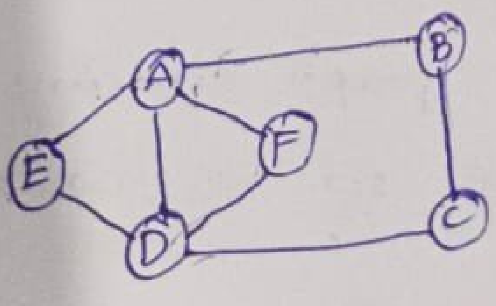
Ex:



walk sequence is

A B C D E B F D A

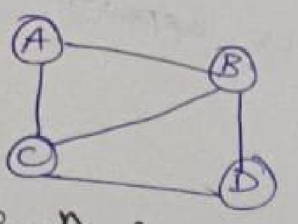
Ex:



walk seqence is

EA BC DFA DE

Example of non-regular Graph:



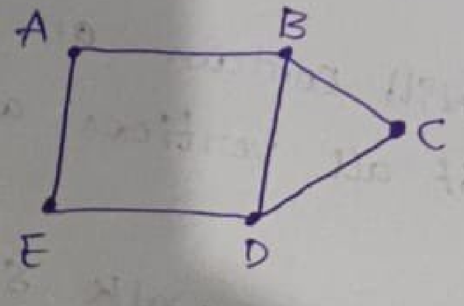
Hamiltonian Graph:

A graph $G(V, E)$ is said to be Hamiltoniam graph and it contains hamiltoniam cycle or circuit.

Hamiltonian cycle or circuit:

In a connected graph a closed wall that visits every vertex of the graph $G(V, E)$ exactly once except the starting and ending vertex/vertices it is called hamiltoniam cycle.

Ex-1:



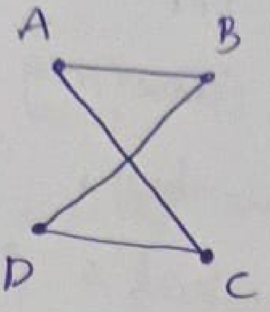
Sequence walk

A - B - C - D - E - A

C - D - E - A - B - C

So the above graph contains hamiltoniam cycle. so we called as hamiltoniam graph.

Ex-2:

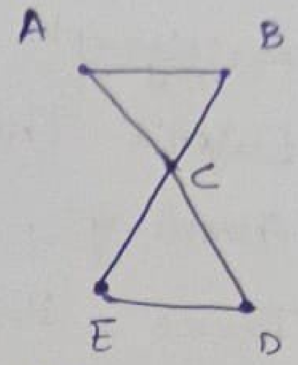


walk sequence

A - B - D - C - A

It is a hamitoniam graph

Ex-3:



walk sequence

A - B - C - E - D - C - A

It is not hamiltoniam graph.