(iv) Required area $= [0.5 - \text{Area from 0 to } 2.52] + [0.5 - \text{Area from 0 to } 1.83]$

$= (0.5 - 0.4941) + (0.5 - 0.4664) = 0.0059 + 0.0336 = 0.0395$

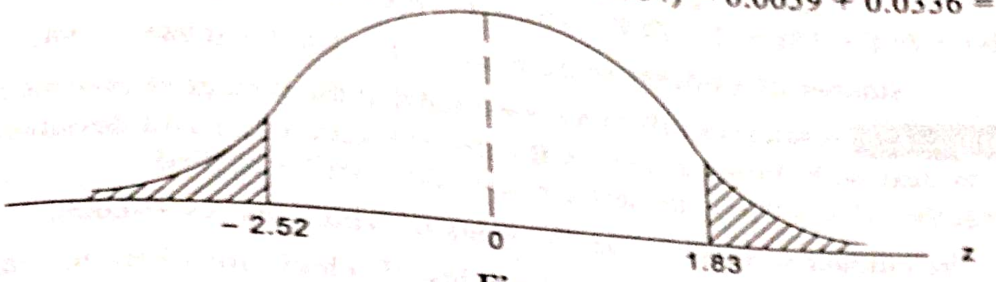


Fig.

Example 11 : In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find

(i) how many students score between 12 and 15 ?

(ii) how many score above 18 ?

(iii) how many score below 18 ?

[JNTU 2004 S, 2007S (Set No. 2,3)]

Solution : Let μ be the mean and σ the sandard deviation of the normal distribution. Then we are given

$\mu = 14$ and $\sigma = 2.5$

Let the variable X denote the score in a test.

(i) When $X = 12$, $z = \dfrac{X-\mu}{\sigma} = \dfrac{12-14}{2.5} = -0.8 = z_1$ (say)

When $X = 15$, $z = \dfrac{X-\mu}{\sigma} = \dfrac{15-14}{2.5} = 0.4 = z_2$ (say)

$\therefore \quad P(12 < X < 15) = P(-0.8 < z < 0.4)$

$= A(z_2) + A(z_1) = A(0.4) + A(-0.8)$

$= A(0.4) + A(0.8)$ (due to symmetry)

$= 0.1554 + 0.2881$

$= 0.4435$

Hence number of students score between 12 and 15

$= 1000 \times 0.4435 = 443$ (approximately)

(ii) When $X = 18$, $z = \dfrac{X-\mu}{\sigma} = \dfrac{18-14}{2.5} = 1.6$

$\therefore \quad P(X > 18) = P(z > 1.6)$

$= 0.5 - A(1.6) = 0.5 - 0.4452$

$= 0.0548$

Hence number of students score above 18 $= 1000 \times 0.0548$

$= 54.8 = 55$ (approximately)

(iii) $P(X < 18) = P(z < 1.6)$

$= 0.5 + A(1.6) = 0.5 + 0.4452 = 0.9452$

Aliter : $P(X < 18) = 1 - P(X > 18) = 1 - 0.0548 = 0.9452$

$\therefore$ Number of students score below 18 = $1000 \times 0.9452 = 945$

Example 12 : A sales tax officer has reported that the average sales of the 500 business that he has to deal with during a year is Rs. 36,000 with a standard deviation of 10,000. Assuming that the sales in these business are normally distributed, find

(i) the number of business as the sales of which are Rs. 40,000.

(ii) the percentage of business the sales of which are likely to range between Rs. 30,000 and Rs. 40,000.

[JNTU 2005 (Set No. 1,2,...

Solution : Let μ be the mean and σ the standard deviation of the sales. Then we are given that $\mu = 36000$ and $\sigma = 10000$

Let the variable X denote the sales in the business

When $X = 40000$, $z = \dfrac{X-\mu}{\sigma} = \dfrac{40000-36000}{10000} = 0.4$

When $X = 30000$, $z = \dfrac{X-\mu}{\sigma} = \dfrac{30000-36000}{10000} = -0.6$

(i) $P(X > 40000) = P(z > 0.4)$

$= 0.5 - A(0.4) = 0.5 - 0.1554 = 0.3446$

Number of business as the sales of which are Rs. 40,000

$= 500 \times 0.3446 = 172$ (approximately)

(ii) $P(30000 < X < 40000) = P(-0.6 < z < 0.4)$

$= A(0.4) + A(-0.6) = A(0.4) + A(0.6)$

$= 0.1554 + 0.2257 = 0.3811$

$\therefore$ The required percentage of business = 38.11%

Example 13 : If the masses of 300 students are normally distributed with mean ... kgs and standard deviation 3 kgs, how many students have masses

(i) Greater than 72 kgs (ii) Less than or equal to 64 kgs

(iii) Between 65 and 71 kgs inclusive.

[JNTU 2005, 2008, (A) Nov. 2010, (H) Dec. 2011, (K) May 2013, Dec. 2015 (Set No...

Solution : Let μ be the mean and σ the standard deviation of the distribution. Th...

$\mu = 68$ kgs and $\sigma = 3$ kgs

Let the variable X denote the masses of students

(i) When X = 72, $z = \dfrac{X-\mu}{\sigma} = \dfrac{72-68}{3} = 1.33$

$\therefore \quad P(X > 72) = P(z > 1.33)$

$= 0.5 - A(1.33) = 0.5 - 0.4082 = 0.0918$

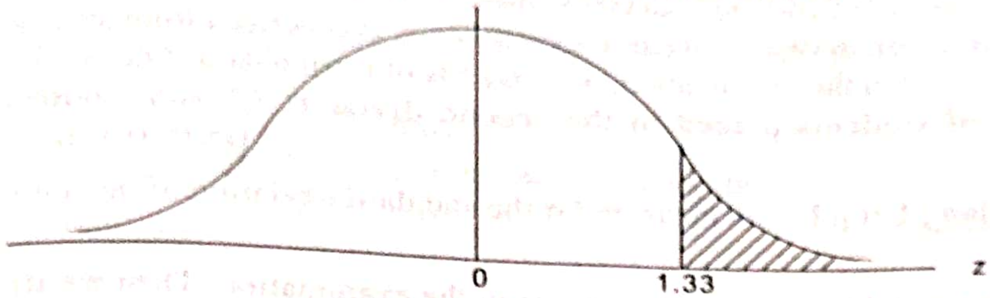


Fig.

Number of students with more than 72 kgs = 300 × 0.0918 = 28 (approximately)

(ii) When $X = 64$, $z = \dfrac{X-\mu}{\sigma} = \dfrac{64-68}{3} = -1.33$

$\therefore \quad P(X \le 64) = P(z \le -1.33)$

$= 0.5 - A(1.33) = 0.5 - 0.4082 = 0.0918$

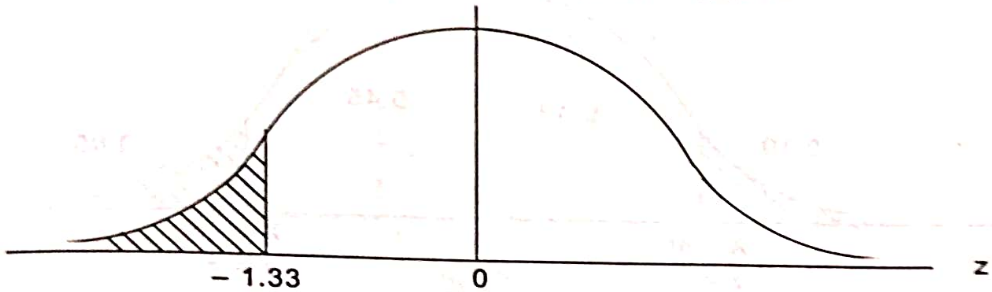


Fig.

Number of students have masses less than or equal to 64 Kg

= 300 × 0.0918 = 28 (approximately)

(iii) When $X = 65$, $z = \dfrac{X-\mu}{\sigma} = \dfrac{65-68}{3} = -1 = z_1$ (say)

When $x = 71$, $z = \dfrac{X-\mu}{\sigma} = \dfrac{71-68}{3} = 1 = z_2$ (say)

$\therefore \quad P(65 \le X \le 71) = P(-1 \le z \le 1)$

$= A(z_2) + A(z_1) = A(1) + A(-1) = A(1) + A(1)$

$= 2.A(1) = 2(0.3413) = 0.6826$

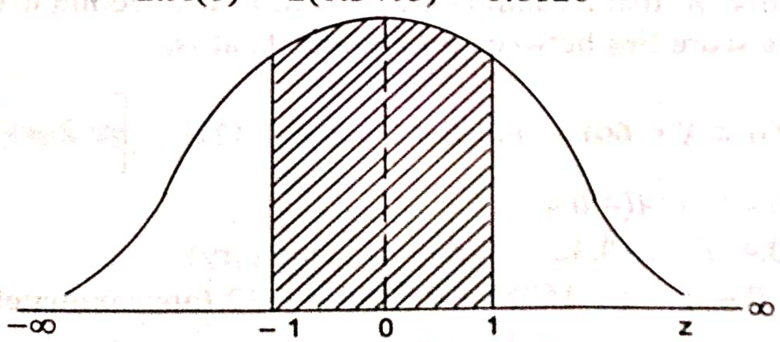


Fig.

∴ Required number of students = 300 × 0.6826 = 205 (approximately)

Example 14 : In an examination it is laid down that a student passes if he secures percent or more. He is placed in the first, second and third division according as he secures 60% or more marks, between 50% and 60% marks and marks between 40% and 50% respectively. He gets a distinction in case he secures 75% or more. It is noticed from the results that 10% the students failed in the examination, whereas 5% of them obtained distinction. Calculate percentage of students placed in the second division. (Assume normal distribution of marks).

[JNTU (K) May 2013 (Set No.

Solution : Let μ be the mean and σ the standard deviation of the normal distribution of marks.

Let the variable X denote the marks in the examination. Then we are given

$P(X < 40) = 0.10$ and $P(X \geq 75) = 0.05$

When $X = 40$, $z = \dfrac{X-\mu}{\sigma} = \dfrac{40-\mu}{\sigma} = -z_1$ (say)

When $X = 75$, $z = \dfrac{X-\mu}{\sigma} = \dfrac{75-\mu}{\sigma} = z_2$ (say)

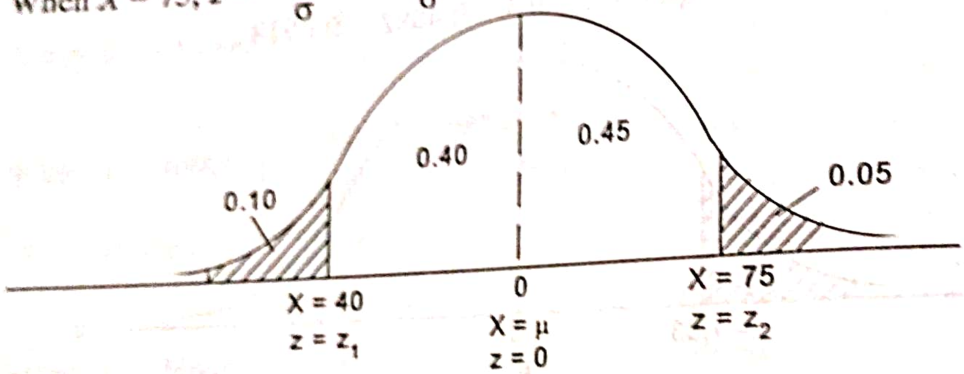


Fig.

$\therefore \quad P(0 < z < z_2) = 0.5 - 0.05 = 0.45$

and $\quad P(0 < z < z_1) = P(-z_1 < z < 0) = 0.5 - 0.10 = 0.40$

$\therefore \quad$ From normal tables, we have $z_1 = 1.28$ and $z_2 = 1.64$

Hence $\dfrac{40-\mu}{\sigma} = -1.28 \Rightarrow \dfrac{\mu-40}{\sigma} = 1.28$ .... (1) and $\dfrac{75-\mu}{\sigma} = 1.64$ ....

(1) + (2) gives $\dfrac{35}{\sigma} = 2.92 \Rightarrow \sigma = \dfrac{35}{2.92} = 11.99 \simeq 12$

From (1), $\mu = 40 + \sigma\,(1.28) = 40 + 12\,(1.28) \simeq 55$

The probability 'p' that a candidate is placed in the second division is equal to probability that his score lies between 50 and 60. That is,

$$p = P(50 < X < 60) = P(-0.42 < Z < 0.42) \qquad \left[\because Z = \frac{X-55}{12}\right]$$

$$= A(0.42) + A(-0.42)$$

$$= A(0.42) + A(0.42) \quad \text{(due to symmetry)}$$

$$= 2A(0.42) = 2(0.1628) = 0.3256 = 0.32 \text{ (approximately)}$$

Hence 32% candidates get second division in the examination.

Example 15 : A manufacturer knows from experience that the resistance of resistors he produces is normal with mean 100 ohms and standard deviation 2 ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms ?

Solution : Let μ be the mean and σ the standard deviation of the normal distribution. Then we are given that

μ = 100 ohms and σ = 2 ohms

Let the variable X represents the resistance of resistors

When $X = 98$ ohms, $z = \dfrac{X-\mu}{\sigma} = \dfrac{98-100}{2} = -1$

and when $X = 102$ ohms, $z = \dfrac{X-\mu}{\sigma} = \dfrac{102-100}{2} = 1$

$$\therefore P(98 < X < 102) = P(-1 < z < 1)$$
$$= A(1) + A(-1) = A(1) + A(1) = 2A(1)$$
$$= 2(0.3413) = 0.6826$$

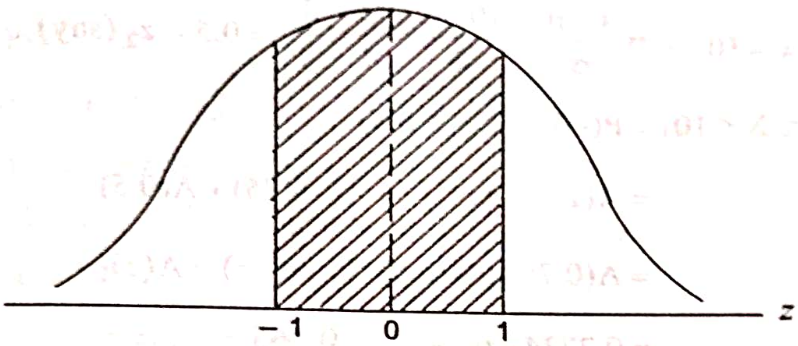


Fig.

$\therefore$ Percentage of resistors having resistance between 98 ohms and 102 ohms = 68.26 = 68 (approximately).

Example 16 : Given that the mean height of students in a class is 158 cms with standard deviation of 20 cms. Find how many students heights lie between 150 cms and 170 cms, if there are 100 students in the class. **[JNTU 2007, 2007S, (H) Dec. 2011 (Set No. 4)]**

Solution : We have

Mean, μ = 158 cm and standard deviation, σ = 20 cms

$$\therefore \quad z = \frac{X-\mu}{\sigma} = \frac{X-158}{20}$$

When $X = 150$, $z = \dfrac{150-158}{20} = \dfrac{-2}{5} = -0.4$

When $X = 170$, $z = \dfrac{170-158}{20} = \dfrac{3}{5} = 0.6$

$$\therefore \quad P(150 \le X \le 170) = P(-0.4 \le z \le 0.6)$$
$$= P(-0.4 \le z \le 0) + P(0 \le z \le 0.6)$$
$$= P(0 \le z \le 0.4) + P(0 \le z \le 0.6), \text{ due to symmetry}$$
$$= 0.1554 + 0.2257 = 0.3811$$

Number of students whose height lie between 150 cms and 170 cms

$= \text{Probability} \times \text{total no. of students}$

$= 0.3811 \times 100 = 38$ ($\because$ Number of students should be integer.)

Example 17 : Suppose 2% of the people on the average are left handed. Find

(*i*) The probability of finding 3 or more left handed

(*ii*) The probability of finding ≤ 1 left handed

(b) The mean and standard deviation of a normal variable are 8 and 4 respectively.

Find (*i*) $p(5 \leq X \leq 10)$ (*ii*) $p(x \geq 5)$ [JNTU (H) 2009 (Set No. 1)]

Solution : Given mean, $\mu = 8$ and standard deviation, $\sigma = 4$.

(*i*) When $x = 5$, $z = \dfrac{x-\mu}{\sigma} = \dfrac{5-8}{4} = \dfrac{-3}{4} = -0.75 = z_1$ (say)

When $x = 10$, $z = \dfrac{x-\mu}{\sigma} = \dfrac{10-8}{4} = \dfrac{2}{4} = \dfrac{1}{2} = 0.5 = z_2$ (say)

$\therefore P(5 \leq X \leq 10) = P(-0.75 \leq z \leq 0.5)$

$= A(z_2) + A(z_1) = A(-0.75) + A(0.5)$

$= A(0.75) + A(0.5)$ [$\because A(-z) = A(z)$]

$= 0.2734 + 0.1916 = 0.465$

(*ii*) When $x = 5$, $z_1 = -0.75$, from above

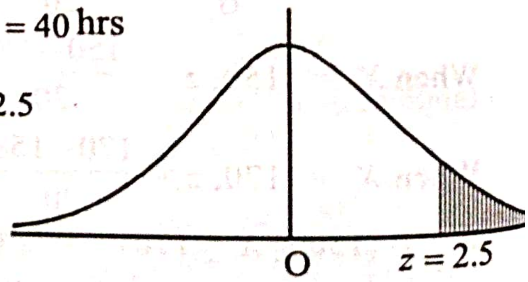
$\therefore P(X \geq 5) = P(z_1 \geq -0.75) = 0.5 - A(z_1)$

$= 0.5 - A(-0.75) = 0.5 - 0.2734 = 0.2266$

Example 18 : In a test on 2000 electrical bulbs, it was found that the life of a particul make, was normally distributed with an average life of 2040 hours and S.D. of 40 hrs. Estima the number of bulbs likely to burn for (*i*) more than 2140 hrs. (*ii*) between 1920 and 2080 h (*iii*) less than 1960 hrs. [JNTU (K) June 2015 (Set No.1

Solution : Given mean, $\mu = 2040$ hrs and S.D., $\sigma = 40$ hrs

(*i*) When $x = 2140$, $z = \dfrac{x-\mu}{\sigma} = \dfrac{2140-2040}{40} = \dfrac{100}{40} = 2.5$



$\therefore P(x > 2140) = P(z > 2.5) = 0.5 - P(0 \leq z \leq 2.5)$

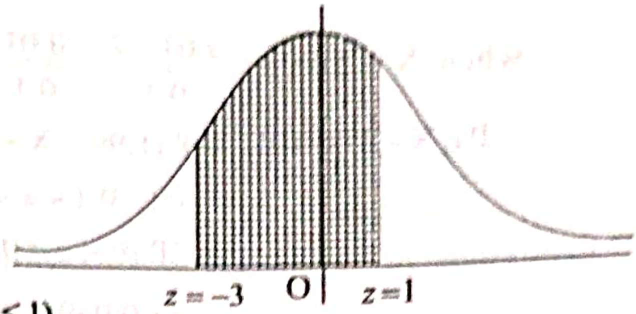
$= 0.5 - 0.4938 = 0.0062$

Hence the number of bulbs likely to burn for more than 2140 hours $= 0.0062 \times 2000 = 12.4 \simeq 12$.

(*ii*) When $x_1 = 1920, z = \dfrac{x_1-\mu}{\sigma} = \dfrac{1920-2040}{40} = \dfrac{-120}{40} = -3$

When $x_2 = 2080, z = \frac{x_2 - \mu}{\sigma} = \frac{2080 - 2040}{40}$

$$= \frac{40}{40} = 1$$



$\therefore P(1920 \le x \le 2080) = P(-3 \le z \le 1)$

$= P(-3 \le z \le 0) + P(0 \le z \le 1)$

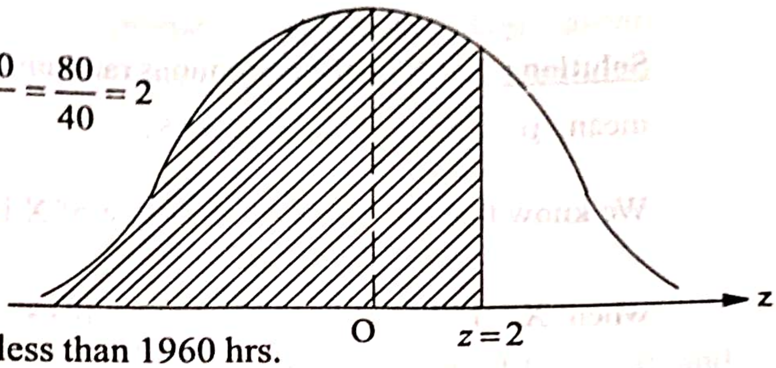
$= P(0 \le z \le 3) + P(0 \le z \le 1)$ (by symmetry)

$= 0.4986 + 0.3413 = 0.8399$

Hence number of bulbs which are likely to burn between 1920 hrs. and 2080 hrs.

$= 0.8399 \times 2000 = 1679.8 \approx 1680$

(iii) When $x = 1960, z = \frac{x - \mu}{\sigma} = \frac{1960 - 2040}{40} = \frac{80}{40} = 2$



$\therefore P(x \le 1960) = P(z \le 2)$

$= 0.5 + P(0 \le z \le 2)$

$= 0.5 + 0.4772 = 0.9772$

Hence number of bulbs likely to burn less than 1960 hrs.

$= 0.9772 \times 2000 = 1954$

Example 19 : If the marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If three students are selected at random from this group what is the probability that at least one of them would have scored above 75? **[JNTU (K) May 2010 (Set No. 2)]**

Solution : Here μ = mean = 65 and σ = S.D. = 5

We have $z = \frac{x - \mu}{\sigma}$ when $x = 75, z = \frac{75 - 65}{5} = \frac{10}{5} = 2$

$\therefore p = P(x > 75) = P(z > 2) = 0.5 - P(0 \le z \le 2) = 0.5 - 0.4772 = 0.0228$

Hence the required probability that out of 3 students, at least one of them will have marks over 75 is given by

$${}^3C_1 pq^2 + {}^3C_2 p^2 q^{3-2} + {}^3C_3 p^3 q^{3-3} = 3p(1-p)^2 + 3p^2(1-p) + p^3 \quad [\because q = 1 - p]$$

$$= 3(0.0228)(1 - 0.0228)^2 + 3(0.0228)^2(1 - 0.0228) + (0.0228)^3 = 0.0668.$$

Example 20 : If X is normally distributed with mean 2 and variance 0.1, then find $P(|X - 2| \ge 0.01)$? **[JNTU (K) May 2010 (Set No. 3)]**

Solution : Here $\mu = 2, \sigma = 0.1$

when $X = 1.99, z = \frac{x - \mu}{\sigma} = \frac{1.99 - 2}{0.1} = -0.1$

when $X = 2.01, z = \frac{2.01-2}{0.1} = \frac{0.01}{0.1} = 0.1$

$\therefore P(|X-2|<0.01) = P(1.99 < X < 2.01)$

$= P(-0.1 < z < 0.1)$

$= 2P(0 < z < 0.1)$ [by symmetry]

$= 2 \times 0.0398 = 0.0796$

$\therefore P(|X-2| \geq 0.01) = 1 - P(|X-2| < 0.01) = 1 - 0.0796 = 0.9204$

Example 21 : The mean height of students in a college is 155 cms and standard dev is 15. What is the probability that the mean height of 36 students is less than 157 cms. **[JNTU (H) Dec. 2011 (Set**

Solution : Let X be the continuous random variable denoting the height of students.

mean, $\mu = 155$ and S. D., $\sigma = 15$.

We know that the standardized value of X is $z = \frac{X-\mu}{\sigma}$

when $X = 157, z = \frac{157-155}{15} = \frac{2}{15} = 0.13$

Probability that the height of student is less than 157 cm.

$= P(X < 157) = P(z < 0.13)$

= area under the normal curve to the right of $z = 0.13$

$= 0.0517$

$\therefore$ Required probability $= 0.0517 \times 36 = 1.86$

Example 22 : The mean inside diameter of a sample of 200 washers produced Machine is 500 cms with standard deviation 0.005 cms. The purpose of which these w are intended a maximum tolerance in the diameter 0.495 to 0.505 cms. otherwise the w are considered defective. Determine the precentage of defective washers produced machine, assuming the diameters are normally distributed. **[JNTU (H) Apr. 2012 (Set**

Solution : We are given

$\mu =$ Mean of the inside diameters $= 0.500$ cm

$\sigma =$ Standard deviation $= 0.005$ cm

The tolerance limits of non - defective tubes are 0.495 cm and 0.505 cm.

When $x = 0.495$ (standardized value of 0.495),

$$z = \frac{x-\mu}{\sigma} = \frac{0.495-0.500}{0.005} = -1$$

When $x = 0.505$ (*i.e.* standardized value of 0.505),

$$z = \frac{x-\mu}{\sigma} = \frac{0.505 - 0.500}{0.005} = 1$$

$\therefore$ Proportion of non-defective washers

$= $ Area under the standard normal curve between $z = -1$ and $z = +1$

$= 2$ (Area between $z = 0$ and $z = 1$)

$= 2 \times 0.3413 = 0.6826 = 68.26\,\%$

Hence the required percentage of defective washers

$= 100\% - 68.26\% = 31.74\%$

[(or) Probability of defective washers $= 1 -$ probability of non-defective washers

$$= 1 - p\,(0.495 \le x \le 0.505)$$
$$= 1 - p\,(-1 \le z \le 1)$$
$$= 1 - 2 \times p\,(0 \le z \le 1)$$
$$= 1 - 2\,(0.3413) = 0.3174$$

$\therefore$ Percentage of defective washers $= 31.74 \approx 32$]

Example 23 : 1000 students have written an examination the mean of test is 35 and standard deviation is 5. Assuming the distribution to be normal, find:

(*i*) How many students marks lie between 25 and 40?

(*ii*) How many students get more than 40?

(*iii*) How many students get below 20?

(*iv*) How many students get more than 50? **[JNTU(K) Dec. 2013 (Set No. 1)]**

Solution : Here $\mu = 35$ and $\sigma = 5$

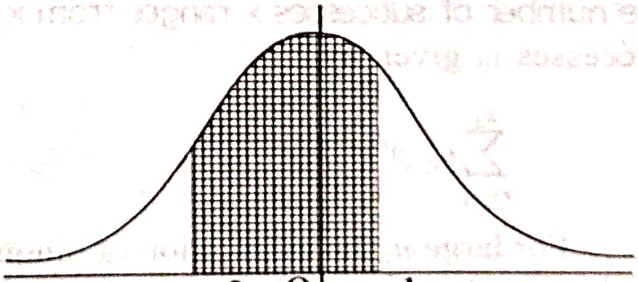
We know, standarised variate, $z = \dfrac{x-\mu}{\sigma} = \dfrac{x-35}{5}$

(*i*) When $x_1 = 25$, then

$$z = \frac{25-35}{5} = \frac{-10}{5} = -2$$

When $x_2 = 40$, then

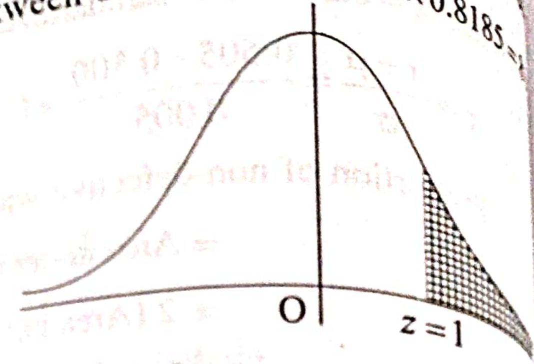
$$z = \frac{40-35}{5} = \frac{5}{5} = 1$$



$\therefore$ Probability of students whose marks lie between 25 and 40 $= P(25 \le X \le 40)$

$$= P(2 \le z \le 1) \quad = P(-2 \le z \le 0) + P(0 \le z \le 1)$$
$$= P(0 \le z \le 2) + P(0 \le z \le 1)\text{, by symmetry}$$
$$= 0.4772 + 0.3415 \text{ [From Tables]}$$
$$= 0.8185$$

Number of students whose marks lie between 25 and 40 $=1000\times0.8185 \approx$
(approximately)



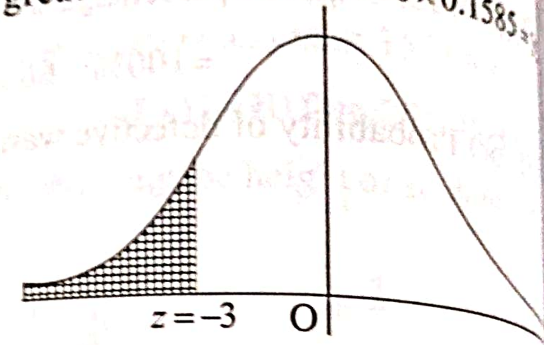
(ii) When $x=40$, then $z=\frac{40-35}{5}=\frac{5}{5}=1$

$P(X>40)=P(z>1)$

$=0.5-P(z\le 1)$

$=0.5-0.3415=0.1585$ [From Tables]

$\therefore$ Number of students whose marks are greater than 40 $=1000\times0.1585 \approx$
(approximately)



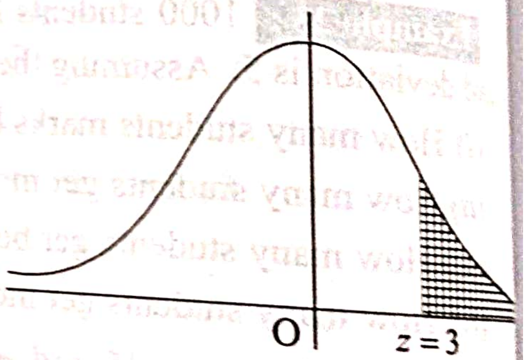
(iii) When $x=20$, then $z=\frac{20-35}{5}=\frac{-15}{5}=-3$

$P(X<20)=P(z\le -3)$ $=0.5-P(-3\le z\le 0)$

$=0.5-P(0\le z\le 3)$, by symmetry

$=0.5-0.499=0.001$ [From Tables]

$\therefore$ Number of students whose marks are less than 20 $=1000\times0.001=1$



(iv) When $x=50$, $z=\frac{50-35}{5}=\frac{15}{5}=3$

$\therefore$ $P(X>50)=P(z>3)=0.5-P(0\le z\le 3)$

$=0.5-0.499=0.001$ [From Tables]

Hence number of students whose marks are greater than 50 $=1000\times0.001=1$.

3.16 NORMAL APPROXIMATION TO THE BINOMIAL DISTRIBUTION

The Normal distribution can be used to approximate the Binomial distribution. Supp… the number of successes x ranges from x_1 to x_2. Then the probability of getting x_1 … successes is given by

$$\sum_{r=x_1}^{x_2} {}^{n}c_r\, p^r q^{n-r}$$

For large n, the calculation of binomial probabilities is very difficult. In such ca… the binomial curve can be replaced by the normal curve and the required probabili… computed. We consider two cases.

Case 1 : When $p=q=\frac{1}{2}$

Even when n is not large, Binomial distribution (B.D) can be approximated by No… distribution (N.D).