UNIT-II

1b) $Z = 10x_1 + 8x_2$ $\quad x_1 \geq 0, x_2 \geq 0$

s.t.c

$x_1 + 2x_2 \leq 1000$

$x_1 \leq 300$

$x_2 \leq 500$

$x_1 + 2x_2 + S_1 = 1000$

$x_1 + S_2 = 300$

$x_2 + S_3 = 500$

$Z = 10x_1 + 8x_2 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3$

C_B	BV	C_j / X_B	10 / x_1	8 / x_2	0 / S_1	0 / S_2	0 / S_3	min ratio
0	S_1	1000	1	2	1	0	0	1000
0	S_2	300	1	0	0	1	0	300 ←
0	S_3	500	0	1	0	0	1	—
		$Z_j = C_B X_i$	0	0	0	0	0	
		$Z_j - C_j$	−10	−8	0	0	0	
			↑					

$R_1^N \to R_1^O - R_2^N$

C_B	BV	C_j / X_B	10 / x_1	8 / x_2	0 / S_1	0 / S_2	0 / S_3	min ratio
0	S_1	700	0	2	1	−1	0	350 ←
10	x_1	300	1	0	0	1	0	—
0	S_3	500	0	1	0	0	1	500
		$Z_j = C_B X_i$	10	0	0	10	0	
		$Z_j - C_j$	0	−8	0	10	0	
				↑				

$R_1 \to R_1/2$

$R_3^N \to R_3^O - R_1^N$

		C_j	10	8	0	0	0
C_B	BV	X_B	x_1	x_2	S_1	S_2	S_3
8	x_2	350	0	1	$1/2$	$-1/2$	0
10	x_1	300	1	0	0	1	0
0	S_3	150	0	0	$-1/2$	$1/2$	1
		Z_j =	10	8	4	6	0
		$Z_j - C_j$	0	0	4	6	0

$Z_j - C_j$ all are positive.

$x_1 = 300$

$x_2 = 350$

$x_3 = 0$

$Z_{max} = 10(300) + 8(350)$

$= 3000 + 2800$

$\boxed{Z_{max} = 5800}$

2. maximize $Z = 6x_1 + 9x_2$

s.t.c

$2x_1 + 2x_2 \le 24$

$x_1 + 5x_2 \le 44$

$6x_1 + 2x_2 \le 60$

$x_1 \ge 0 \quad x_2 \ge 0$

$2x_1 + 2x_2 + S_1 = 24$

$x_1 + 5x_2 + S_2 = 44$

$6x_1 + 2x_2 + S_3 = 60$

$Z = 6x_1 + 9x_2 + 0 S_1 + 0 \cdot S_2 + 0 S_3$

		C_j	6	9	0	0	0	
C_B	BV	X_B	x_1	x_2	S_1	S_2	S_3	min ratio
0	S_1	24	2	2	1	0	0	12
0	S_2	44	1	5	0	1	0	44/5 ←
0	S_3	60	6	2	0	0	1	30
		Z_j	0	0	0	0	0	
		$Z_j - C_j$	-6	-9	0	0	0	

↑

$12 - \frac{44}{5}$

$\frac{120-44}{5}$

$= 24 - \frac{44}{5}$

$\frac{120-88}{10}$

$R_1^N \to R_1^0 - 2R_2^N$

$R_3^N \to R_3^0 - 2R_2^N$

		C_j	6	9	0	0	0	min ratio
C_B	BV	X_B	x_1	x_2	S_1	S_2	S_3	
0	S_1	32/5	8/5	0	1	-2/5	0	4 ←
9	x_2	44/5	1/5	1	0	1/5	0	44
0	S_3	212/5	$\frac{28}{5}$	0	0	-2/5	1	7.5
		z_j	9/5	9	0	9/5	0	
		$z_j - c_j$	-21/5	0	0	9/5	0	

↑

$R_1 \to R_1 / 8/5$

$R_2^N \to R_2^0 - 1/5\, R_1^N$

$R_3^N \to R_3^0 - \frac{28}{5} R_1^N$

		C_j	6	9	0	0	0
C_B	BV	X_B	x_1	x_2	S_1	S_2	S_3
6	x_1	4	1	0	5/8	-1/4	0
9	x_2	8	0	1	-1/8	1/4	0
0	S_3	20	0	0	-7/2	1	1
		z_j	6	9	$\frac{21}{8}$	$\frac{3}{4}$	0
		$z_j - c_j$	0	0	$\frac{21}{8}$	$\frac{3}{4}$	0

all are +ve

$x_1 = 4$

$x_2 = 8$

$x_3 = 0$

$Z = 6(4) + 9(8)$

$= 24 + 72$

$= 96$

$\boxed{Z_{max} = 96}$

3. minimize $z = x_1 - 3x_2 + 2x_3$

s.t.

$3x_1 - x_2 + 2x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$4x_1 + x_2 + x_3 \le 6$

max $z = -(\min z)$

$z = -x_1 + 3x_2 - 2x_3$

Sub to $3x_1 - x_2 + 2x_3 + S_1 = 7$

$-2x_1 + 4x_2 + S_2 = 12$

$4x_1 + x_2 + x_3 = 6$

		C_j	-1	3	-2	0	0	0	
C_B	B_V	X_B	x_1	x_2	x_3	S_1	S_2	S_3	min ratio
0	S_1	7	3	-1	2	1	0	0	$\rightarrow$
0	S_2	12	-2	4	0	0	1	0	$3 \leftarrow$
0	S_3	6	4	1	1	0	0	1	6
	z_j		0	0	0	0	0	0	
	$z_j - c_j$		1	-3	2	0	0	0	
				$\uparrow$					

$R_1^N \rightarrow R_1^0 + R_2^N$

$R_3^N \rightarrow R_3^0 - R_2^N$

		C_j	-1	3	-2	0	0	0	
C_B	B_V	X_B	x_1	x_2	x_3	S_1	S_2	S_3	min ratio
0	S_1	10	$5/2$	0	2	1	$1/4$	0	4
3	x_2	3	$-1/2$	1	0	0	$1/4$	0	-6
0	S_3	3	$9/2$	0	1	0	$-1/4$	1	$2/3$ (min) $\leftarrow$
	z_j		$-3/2$	3	0	0	$3/4$	0	
	$z_j - c_j$		$-1/2$	0	2	0	$3/4$	0	
			$\uparrow$						

$R_1^N \rightarrow R_1^0 - 5/2 R_3^N$

$R_2^N \rightarrow R_2^0 + 1/2 R_3^N$

		C_j	-1	3	-2	0	0	0
C_B	BV	X_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	$25/3$	0	0	$13/9$	1	$7/18$	$-5/9$
3	x_2	$10/3$	0	1	$1/9$	0	$2/9$	$1/9$
-1	x_1	$2/3$	1	0	$2/9$	0	$-1/18$	$2/9$
		Z_j	-1	3	$1/9$	0	$13/18$	$1/9$
		$Z_j - C_j$	0	0	$19/9$	0	$13/18$	$1/9$

$Z_j - C_j$ All are +ve

$$-2/3 + 3(10/3) = \frac{-2}{3} + 10$$

$$= \frac{-2+30}{3}$$

$$x_1 = 2/3$$

$$x_2 = 10/3$$

$$x_3 = 0$$

$$\text{Max } Z = \frac{28}{3}$$

$$\boxed{Z_{min} = \frac{-28}{3}}$$

3 b) $Z = 20x_1 + 10x_2$

s.t.c

$x_1 + 2x_2 \leq 40$

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \geq 60$

$x_1 + 2x_2 = 40$ — 1

$3x_1 + x_2 = 30$ — 2

$4x_1 + 3x_2 = 60$ — 3

Put $x_1 = 0$ in eq 1, $\cancel{x_2} = 0$

$2x_2 = 40$	$x_1 = 40$
$x_2 = 20$	$\cancel{Q_1}(40,0)$
$P_1(0,20)$	

Put $x_1 = 0$ in eq 2, $x_2 = 0$

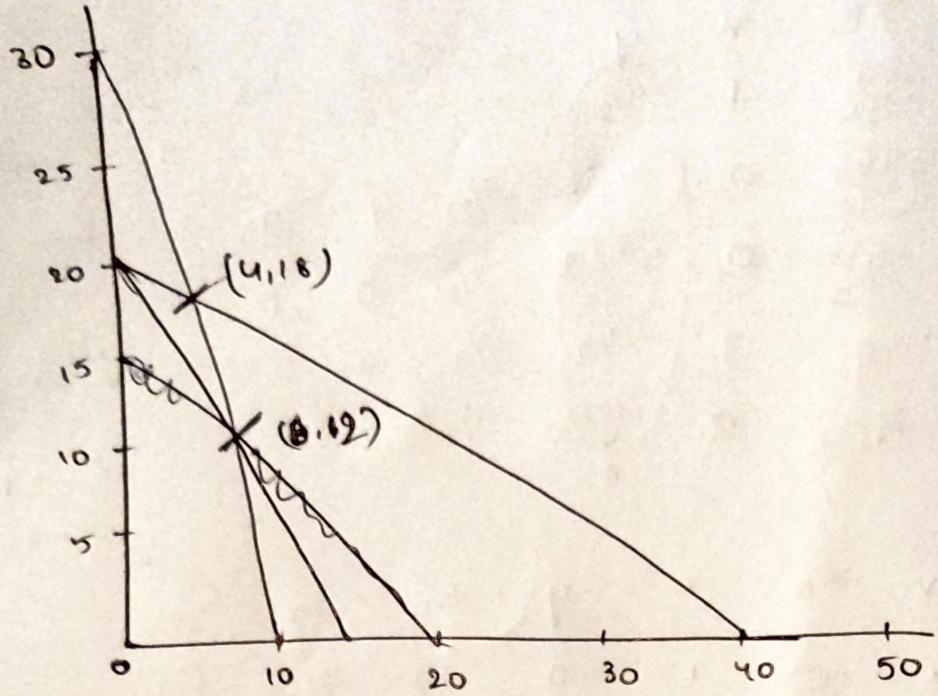
$x_2 = 30$	$3x_1 = 30$
	$x_1 = 10$

$P_2(0,30)$ $Q_2(10,0)$

Put $x_1 = 0$ in eq 3 $x_2 = 0$

$3x_2 = 60$	$\cancel{4}x_1 = \cancel{60}$
	$2 \quad 30$
$x_2 = 20$	$x_1 = 15$

$P_3(0,20)$ $Q_3(15,0)$



$x_1+2x_2=40$ — ①

$3x_1+x_2=30$ — ②

Solving ① & ②

$3x_1+6x_2=120$

$3x_1+x_2=30$

$5x_2=90$

$x_2=18$

$x_1+36=40$

$x_1=4$

(4,18)

$3x_1+x_2=30$ — ②

$4x_1+3x_2=60$ — ③

Solving ② & ③

$9x_1+3x_2=90$

$4x_1+3x_2=60$

$5x_1=30$

$x_1=6$

$18+x_2=30$

$x_2=12$

	Point	$Z=20x_1+10x_2$
G	(4,18)	$20(4)+10(18)=260$
P1	(0,20)	$20(0)+10(20)=200$
G2	(6,12)	$20(6)+10(12)=240$

optimal solution is

$x_1=0, x_2=20$ with

$\boxed{Z_{min}=200}$

$\max z = -x_1 + 3x_2 - 2x_3$

4a) maximize $z = x_1 + 2x_2 + x_3$

s.t c

$2x_1 + x_2 - x_3 \leq 2$

$-2x_1 + x_2 - 5x_3 \geq -6$

$4x_1 + x_2 + x_3 \leq 6$ and

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

$z = x_1 + 2x_2 + x_3$

$2x_1 + x_2 - x_3 + S_1 = 2$

$2x_1 - x_2 + 5x_3 + S_2 = 6$

$4x_1 + x_2 + x_3 + S_3 = 6$

$z = x_1 + 2x_2 + x_3 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3$

		C_j	1	2	3	0	0	0	
C_B	BV	X_B	x_1	x_2	x_3	S_1	S_2	S_3	Min ratio
0	S_1	2	2	1	-1	1	0	0	2 ←
0	S_2	6	2	-1	5	0	1	0	-6
0	S_3	6	4	1	1	0	0	1	6
		$Z_j = C_B \times X_i$	0	0	0	0	0	0	
		$Z_j - C_j$	-1	-2	-1	0	0	0	

↑

$R_2^N \rightarrow R_2^O + R_1^N$

$R_3^N \rightarrow R_3^O - R_1^N$

		C_j	1	2	1	0	0	0	
C_B	BV	X_B	x_1	x_2	x_3	S_1	S_2	S_3	Min ratio
2	x_2	2	2	1	-1	1	0	0	-2
0	S_2	8	4	0	4	1	1	0	2 ←
0	S_3	4	2	0	2	-1	0	1	2
		$Z_j = C_B \times X_i$	4	2	-2	2	0	0	
		$Z_j - C_j$	3	0	-3	2	0	0	

↑

$R_2 \rightarrow R_2/4$

$-1 - 1/2$

$R_1^N \rightarrow R_1^O + R_2^N$

$R_3^N \rightarrow R_3^O - 2R_2^N$

		C_j	1	2	1	0	0	0
C_B	BV	X_B	x_1	x_2	x_3	S_1	S_2	S_3
2	x_2	4	3	1	0	5/4	1/4	0
1	x_3	2	1	0	1	1/4	1/4	0
0	S_3	0	0	0	0	-3/2	-1/2	0
		Z_j	7	2	1	11/4	3/4	0
		$Z_j - C_j$	6	0	0	11/4	3/4	0

$Z_j - C_j$ are all +ve values

$x_1 = 0$

$x_2 = 4$

$x_3 = 2$

4 b) $Z = 4x_1 + 3x_2$

Subject to

$x_1 + 2x_2 \le 6$

$x_1, x_2 \ge 0$ $\quad 2x_1 + x_2 \le 8$

$x_1 \ge 7$

$x_1 + x_2 = 6$ —①

$2x_1 + x_2 = 8$ —②

$x_1 = 7$ —③

Put $x_1 = 0$ in eq① | $x_2 = 0$

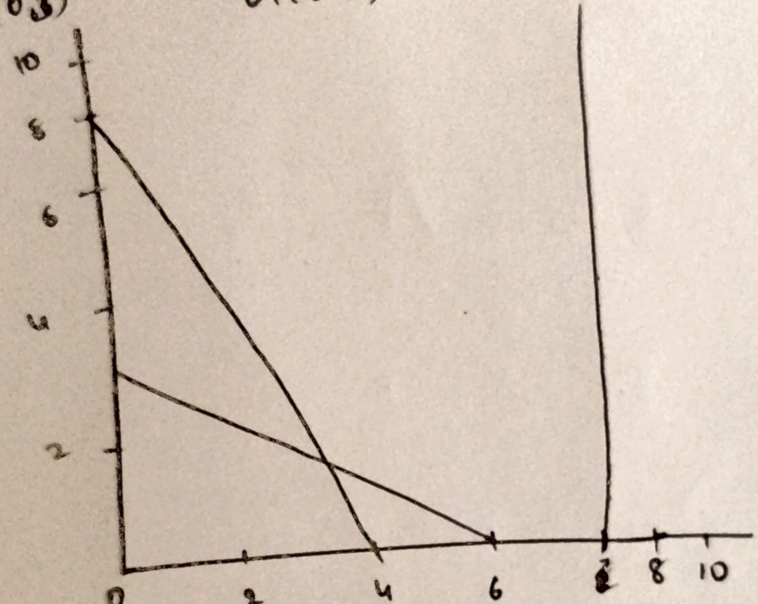
$2x_2 = 6$ | $x_1 = 6$

P_1 (0,3) | $Q_1(6,0)$

Put $x_1 = 0$ in eq② | $x_2 = 0$

$x_2 = 8$ | $x_1 = 4$

P_2 (0,8) | Q_2 (4,0)



The feasible region does not exist, $x_1 \ge 7$ is outside the valid intersection of first two constraints.

$\therefore$ Thus, LPP has no feasible solution.

5 a)