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Simple Queue Model

Simple Queue Model

- **Characteristics of Queueing Systems**
- The key elements, of a queueing system are the customers and servers. The term "customer" can refer to people, machines, trucks, mechanics, patients—anything that arrives at a facility and requires service.
- The term "server" might refer to receptionists, repairpersons, CPUs in a computer, or washing machines….any resource (person, machine, etc. which provides the requested service.

Examples

System	*Customers*	*Server(s)*
Reception desk	People	Receptionist
Repair facility	Machines	Repairperson
Garage	Trucks	Mechanic
Tool crib	Mechanics	Tool-crib clerk
Hospital	Patients	Nurses
Warehouse	Pallets	Crane
Airport	Airplanes	Runway
Production line	Cases	Case packer
Warehouse	Orders	Order picker
Road network	Cars	Traffic light
Grocery	Shoppers	Checkout station
Laundry	Dirty linen	Washing machines/dryers
Job shop	Jobs	Machines/workers
Lumberyard	Trucks	Overhead crane
Saw mill	Logs	Saws
Computer	Jobs	CPU, disk, tapes
Telephone	Calls	Exchange
Ticket office	Football fans	Clerk
Mass transit	Riders	Buses, trains

Discount warehouse with three service centers

- Consider a discount warehouse where customers may either serve themselves; or Wait of three clerks, and finally leave after paying a single cashier
- The subsystem, consisting of queue 2 and service center 2, is shown in more detail in figure 2 below. Other variations of service mechanisms include batch service (a server serving several customers simultaneously) or a customer requiring several servers simultaneously.

Discount warehouse with three service centers

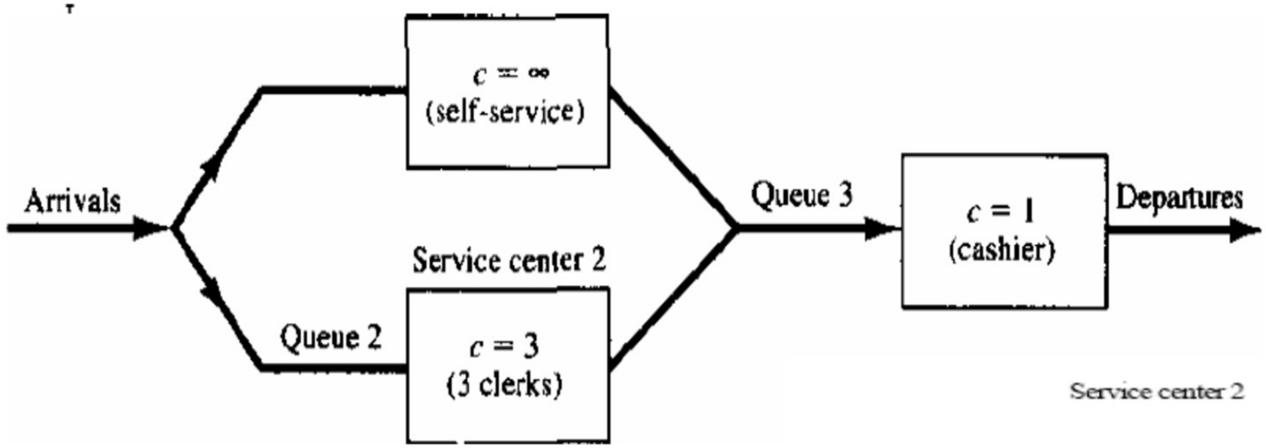


Figure 1: Discount warehouse with three service centers

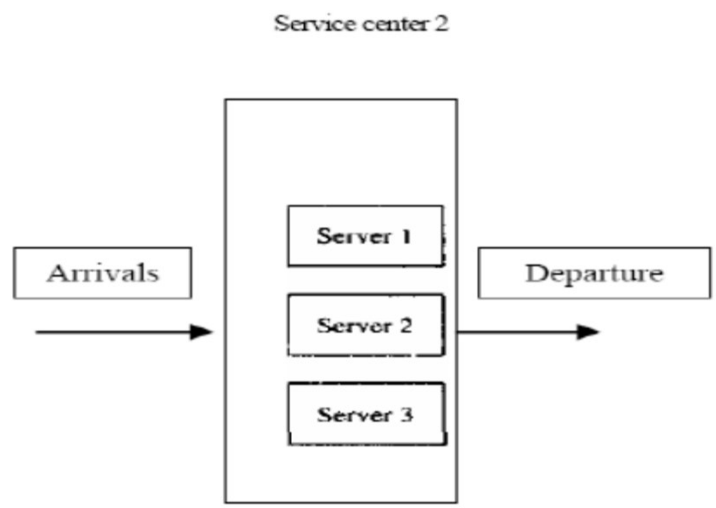
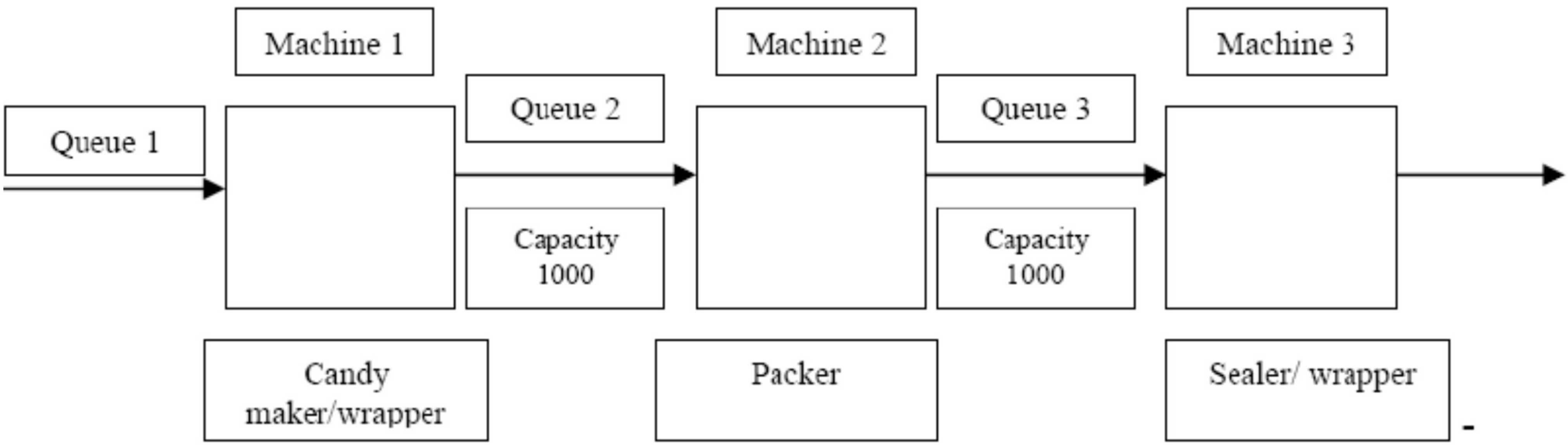


Figure 2: Service center 2, with c = 3 parallel servers.

Candy Production line

A candy manufacturer has a production line which consists of three machines separated by inventory-in-process buffers. The first machine makes and wraps the individual pieces of candy, the second packs 50 pieces in a box, and the third seals and wraps the box. The two inventory buffers have capacities of 1000 boxes each. As illustrated by Figure 3, the system is modeled as having three service centers, each center having c = 1 server (a machine), with queue-capacity constraints between machines. It is assumed that a sufficient supply of raw material is always available at the first queue. Because of the queue-capacity constraints, machine 1 shuts down whenever the inventory buffer fills to capacity, while machine 2 shuts down whenever the buffer empties. In brief, the system consists of three single-server queues in series with queue-capacity constraints and a continuous arrival stream at the first queue.

Candy Production line



Single Channel queue

- A small grocery store has only one checkout counter. Customers arrive at this checkout counter at random from 1 to 8 minutes apart. Each possible value of interarrival time has the same probability of occurrence. The service times vary from 1 to 6 minutes with the probabilities shown in "Service time distribution table". The problem is to analyze the system by simulating the arrival and service of 20 customers.
- A set of uniformly distributed random numbers is needed to generate the arrivals at the checkout counter. Random numbers have the following properties:

1. The set of random numbers is uniformly distributed between 0 and 1.
2. Successive random numbers are independent.

Service time distribution

Service Time (Min)	Probability	Cumulative Frequency	Random Digit Assignment
1	0.10	0.10	01-10
2	0.20	0.30	11-12
3	0.30	0.60	31-60
4	0.25	0.85	61-85
5	0.10	0.95	86-95
6	0.05	1.00	96-00

Time Between Arrivals Determination

Customers	Random Digits	Time Between Arrivals (Min)	Customers	Random Digits	Time Between Arrivals (Min)
1	--	--	11	109	1
2	913	8	12	093	1
3	727	6	13	607	5
4	015	1	14	738	6
5	948	8	15	359	3
6	309	3	16	888	8
7	922	8	17	106	1
8	753	.7	18	212	2
9	235	2	19	493	4
10	302	3	20	535	5

Service Time Generation

Customer	Random Digits	Service Time (Min)	Customer	Random Digits	Service Time (Min)
1	84	4	11	32	3
2	10	1	12	94	5
3	74	4	13	79	4
4	53	3	14	05	1
5	17	2	15	79	5
6	79	4	16	84	4
7	91	5	17	52	3
8	67	4	18	55	3
9	89	5	19	30	2
10	38	3	20	50	3

Simulation Table

A Customers	B Time since last Arrival (Min)	C Arrival Time	D Service Time	E Time Service Begins	F Time customer waits in queue	G Time Service Ends	H Time customer spends in system	I Idle Time of Server
1	--	0	4	0	0	4	4	0
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
11	1	47	3	53	6	56	9	0
12	1	48	5	56	8	61	13	0
13	5	53	4	61	8	65	12	0
14	6	59	1	65	6	66	7	0
15	3	62	5	66	4	71	9	0
16	8	70	4	71	1	75	5	0
17	1	71	3	75	4	78	7	0
18	2	73	3	78	5	81	8	0
19	4	77	2	81	4	83	6	0
20	5	82	3	83	1	86	4	0
			68		56		124	18

Simulation Parameters

1. Average delay in queue= Average waiting time of total time customers wait in queue/Total number of customers who wait

- = 56/13= 4.3 minutes

2. The average waiting time for a customer is 2.8minutes.

Average Waiting Time = Total time customers wait in queue (min)/ Total no. of customers

- = 56 /20 = 2.8 minutes

3. The probability that a customer has to wait in the queue is 0.65. Probability (wait) = number of customers who wait/ Total number of customers

13/20= = 0.65

Simulation Parameters

4. The fraction of idle time of the server is 0.21.

- Probability of idle server = Total idle time of server (minutes)/ Total run time of simulation (minutes)

= 18/86 = 0.21

- The probability of the server being busy is the complement of 0.21, or 0.79.

Simulation Parameters

5. The average service time is 3.4 minutes, determined as follows:

- Average service time (minutes) =Total service time/Total number of customers

= 68/20 = 3.4 minutes

- The average time a customer spends in the system is 6.2 minutes.
- Average time customer = Total time customers spend in the system/
- Total number of customers
- 124/20 = 6.2 Minutes

Simulation Parameters

- Average time customer Spends in the system = Average time customer spends waiting in the queue + Average time customer spends in service

- Average time customer spends in the system = 2.8+ 3.4 = 6.2 minutes.

The Ali Baber Carhop Problem

- This example illustrates the simulation procedure when there is more than one service channel. Consider a drive-in restaurant where carhops take orders and bring food to the car. Cars arrive in the manner shown in table 1. There are two carhops-Ali and Baber.
- Ali is better able to do the job and works a bit faster than Baber. The distribution of their service times are shown in tables 2 and 3.

Table 1: Interarrival distribution of Cars

Time Between arrivals (Min)	Probability	Cumulative Probability	Random Digit Assignment
1	0.25	0.25	01-25
2	0.40	0.65	26-65
3	0.20	0.85	66-85
4	0.15	1.00	86-00

The Able Baker Carhop Problem

Table 2: Service Distribution of Ali

Service Time	Probability	Cumulative Probability	Random-Digit Assignment
2	0.30	0.30	01-30
3	0.28	0.58	31-58
4	0.25	0.83	59-83
5	0.17	1.00	84-00

Table 3: Service Distribution of Baber

Service Time	Probability	Cumulative Probability	Random-Digit Assignment
3	0.35	0.35	01-35
4	0.25	0.60	36-60
5	0.20	0.80	61-80
6	1.00	1.00	81-00

The Ali Baber Carhop Problem

- The logic to compute who gets a given customer, and when that service begins, is more complex. The logic goes as follows when a customer arrives: if the customer finds Ali idle, the customer begins service immediately with Ali. If Ali is not idle but Baber is, then the customer begins service immediately with Baber. If both are busy, the customer begins service with the first server to become free.

The Ali Baber Carhop Problem

Ali

Baber

A Customer No.	B Random Digits for Arrival	C Time between Arrivals	D Clock time Arrival	E Random Digits for Service	F Time Service Begins	G Service Time	H Time Service Ends	I Time Service Begins	J Service Time	K Time Service Ends	L Time In Queue
1	--	--	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2	12	89				12	6	18	0
6	42	2	14	38	15	3	18				1
7	74	3	17	13	18	2	20				1
8	80	3	20	61	20	4	24				0
9	68	3	23	50				23	4	27	0
10	22	1	24	49	24	3	27				0
11	48	2	26	39	27	3	30				1
12	34	2	28	53				28	4	32	0
13	45	2	30	88	30	5	35				0
14	24	1	31	01				32	3	35	1
15	34	2	33	81	35	4	39				2
16	63	2	35	53				35	4	39	0
17	38	2	37	81	39	4	43				2
18	80	3	40	64				40	5	45	0
19	42	2	42	01	43	2	45				1
20	56	2	44	67	45	4	49				1
21	89	4	48	01				48	3	51	0
22	18	1	49	47	49	3	52				0
23	51	2	51	75				51	5	56	0
24	71	3	54	57	54	3	57				0
25	16	1	55	87				56	6	62	1
26	92	4	59	47	59	3	62				0
						56			43		11

The Ali Baber Carhop Problem

- Compute the following

1. Average delay in queue
2. The average waiting time for a customer
3. The probability that a customer has to wait in the queue
4. The average service time