APPENDIX 2

Answers to Odd-Numbered Problems

Problem Set 1.1, page 8

1. $y = \dfrac{1}{\pi}\cos 2\pi x + c$
3. $y = ce^x$
5. $y = 2e^{-x}(\sin x - \cos x) + c$
7. $y = \dfrac{1}{5.13}\sinh 5.13x + c$
9. $y = 1.65e^{-4x} + 0.35$
11. $y = (x + \frac{1}{2})e^x$
13. $y = 1/(1 + 3e^{-x})$
15. $y = 0$ and $y = 1$ because $y' = 0$ for these y
17. $\exp(-1.4 \cdot 10^{-11}t) = \frac{1}{2}$, $t = 10^{11}(\ln 2)/1.4$ [sec]
19. Integrate $y'' = g$ twice, $y'(t) = gt + v_0$, $y'(0) = v_0 = 0$ (start from rest), then $y(t) = \frac{1}{2}gt^2 + y_0$, where $y(0) = y_0 = 0$

Problem Set 1.2, page 11

11. Straight lines parallel to the x-axis
13. $y = x$
15. $mv' = mg - bv^2$, $v' = 9.8 - v^2$, $v(0) = 10$, $v' = 0$ gives the limit/9.8 = 3.1 [meter/sec]
17. Errors of steps 1, 5, 10: 0.0052, 0.0382, 0.1245, approximately
19. $x_5 = 0.0286$ (error 0.0093), $x_{10} = 0.2196$ (error 0.0189)

Problem Set 1.3, page 18

1. If you add a constant later, you may not get a solution.
Example: $y' = y$, $\ln|y| = x + c$, $y = e^{x+c} = \tilde{c}e^x$ but not $e^x + c$ (with $c \neq 0$)
3. $\cos^2 y\, dy = dx$, $\frac{1}{2}y + \frac{1}{4}\sin 2y + c = x$
5. $y^2 + 36x^2 = c$, ellipses
7. $y = x\arctan(x^2 + c)$
9. $y = x/(c - x)$
11. $y = 24/x$, hyperbola
13. $dy/\sin^2 y = dx/\cosh^2 x$, $-\cot y = \tanh x + c$, $c = 0$, $y = -\mathrm{arccot}(\tanh x)$
15. $y^2 + 4x^2 = c = 25$
17. $y = x\arctan(x^3 - 1)$
19. $y_0e^{kt} = 2y_0$, $e^k = 2$ (1 week), $e^{2k} = 2^2$ (2 weeks), $e^{4k} = 2^4$
21. 69.6% of y_0
23. $PV = c = \text{const}$
25. $T = 22 - 17e^{-0.5306t} = 21.9$ [°C] when $t = 9.68$ min
27. $e^{-k\cdot 10} = \frac{1}{2}$, $k = \frac{1}{10}$, $\ln\frac{1}{2}$, $e^{-kt_0} = 0.01$, $t = (\ln 100)/k = 66$ [min]
29. No. Use Newton's law of cooling.
31. $y = ax$, $y' = g(y/x) = a = \text{const}$, independent of the point (x, y)
33. $\Delta S = 0.15S\Delta\phi$, $dS/d\phi = 0.15S$, $S = S_0e^{0.15\phi} = 1000S_0$, $\phi = (1/0.15)\ln 1000 = 7.3 \cdot 2\pi$. Eight times.

Problem Set 1.4, page 26

1. Exact, $2x = 2x$, $x^2y = c, y = c/x^2$

3. Exact, $y = \arccos(c/\cos x)$

5. Not exact, $y = \sqrt{x^2 + cx}$

7. $F = e^{x^2}$, $e^{x^2} \tan y = c$

9. Exact, $u = e^{2x} \cos y + k(y)$, $u_y = -e^{2x} \sin y + k'$, $k' = 0$. *Ans.* $e^{2x} \cos y = 1$

11. $F = \sinh x$, $\sinh^2 x \cos y = c$

13. $u = e^x + k(y)$, $u_y = k' = -1 + e^y$, $k = -y + e^y$. *Ans.* $e^x - y + e^y = c$

15. $b = k$, $ax^2 + 2kxy + ly^2 = c$

Problem Set 1.5, page 34

3. $y = ce^x - 5.2$

5. $y = (x + c)e^{-kx}$

7. $y = x^2(c + e^x)$

9. $y = (x - 2.5/e)e^{\cos x}$

11. $y = 2 + c \sin x$

13. Separate. $y - 2.5 = c \cosh^4 1.5x$

15. $(y_1 + y_2)' + p(y_1 + y_2) = (y_1' + py_1) + (y_2' + py_2) = 0 + 0 = 0$

17. $(y_1 + y_2)' + p(y_1 + y_2) = (y_1' + py_1) + (y_2' + py_2) = r + 0 = r$

19. Solution of $cy_1' + pcy_1 = c(y_1' + py_1) = cr$

21. $y = uy^*$, $y' + py = u'y^* + uy^{*\prime} + puy^* = u'y^* + u(y^{*\prime} + py^*) = u'y^* + u \cdot 0 = r$, $u' = r/y^* = re^{\int p\, dx}$, $u = \int e^{\int p\, dx} r\, dx + c$. Thus, $y = uy_h$ gives (4). We shall see that this method extends to higher-order ODEs (Secs. 2.10 and 3.3).

23. $y^2 = 1 + 8e^{-x^2}$

25. $y = 1/u$, $u = ce^{-3.2x} + 10/3.2$

27. $dx/dy = 6e^y - 2x$, $x = ce^{-2y} + 2e^y$

31. $T = 240e^{kt} + 60$, $T(10) = 200$, $k = -0.0539$, $t = 102$ min

33. $y' = A - ky$, $y(0) = 0$, $y = A(1 - e^{-kt})/k$

35. $y' = 175(0.0001 - y/450)$, $y(0) = 450 \cdot 0.0004 = 0.18$,
$y = 0.135e^{-0.3889t} + 0.045 = 0.18/2$,
$e^{-0.3889t} = (0.09 - 0.045)/0.135 = 1/3$,
$t = (\ln 3)/0.3889 = 2.82$. *Ans.* About 3 years

37. $y' = y - y^2 - 0.2y$, $y = 1/(1.25 - 0.75e^{-0.8t})$, limit 0.8, limit 1

39. $y' = By^2 - Ay = By(y - A/B)$, $A > 0$, $B > 0$. Constant solutions $y = 0$, $y = A/B$, $y' > 0$ if $y > A/B$ (unlimited growth), $y' < 0$ if $0 < y < A/B$ (extinction). $y = A/(ce^{At} + B)$, $y(0) > A/B$ if $c < 0$, $y(0) < A/B$ if $c > 0$.

Problem Set 1.6, page 38

1. $x^2/(c^2 + 9) + y^2/c^2 - 1 = 0$

3. $y - \cosh(x - c) - c = 0$

5. $y/x = c$, $y'/x = y/x^2$, $y' = y/x$, $\tilde{y}' = -x/\tilde{y}$, $\tilde{y}^2 + x^2 = \tilde{c}$, circles

7. $2\tilde{y}^2 - x^2 = \tilde{c}$

9. $y' = -2xy$, $\tilde{y}' = 1/(2x\tilde{y})$, $x = \tilde{c}e^{\tilde{y}^2}$

11. $\tilde{y} = \tilde{c}x$

13. $y' = -4x/9y$. Trajectories $\tilde{y}' = 9\tilde{y}/4x$, $\tilde{y} = \tilde{c}x^{9/4}$ ($\tilde{c} > 0$). Sketch or graph these curves.

15. $u = c$, $u_x\, dx + u_y\, dy = 0$, $y' = -u_x/u_y$. Trajectories $\tilde{y}' = u_{\tilde{y}}/u_x$. Now $v = \tilde{c}$, $v_x\, dx + v_y\, dy = 0$, $y' = -v_x/v_y$. This agrees with the trajectory ODE in u if $u_x = v_y$ (equal denominators) and $u_y = -v_x$ (equal numerators). But these are just the Cauchy–Riemann equations.

Problem Set 1.7, page 42

1. $y' = f(x, y) = r(x) - p(x)y$; hence $\partial f/\partial y = -p(x)$ is continuous and is thus bounded in the closed interval $|x - x_0| \leqq a$.

3. In $|x - x_0| < a$; just take b in $\alpha = b/K$ large, namely, $b = \alpha K$.

5. R has sides $2a$ and $2b$ and center $(1, 1)$ since $y(1) = 1$. In R, $f = 2y^2 \leqq 2(b + 1)^2 = K$, $\quad \alpha = b/K = b/(2(b + 1)^2)$, $\quad d\alpha/db = 0$ gives $b = 1$, and $\alpha_{\text{opt}} = b/K = \frac{1}{8}$. Solution by $dy/y^2 = 2\,dx$, etc., $y = 1/(3 - 2x)$.

7. $|1 + y^2| \leqq K = 1 + b^2$, $\quad \alpha = b/K$, $\quad d\alpha/db = 0$, $\quad b = 1$, $\quad \alpha = \frac{1}{2}$.

9. No. At a common point (x_1, y_1) they would both satisfy the "initial condition" $y(x_1) = y_1$, violating uniqueness.

Chapter 1 Review Questions and Problems, page 43

11. $y = ce^{-2x}$

13. $y = 1/(ce^{-4x} + 4)$

15. $y = ce^{-x} + 0.01 \cos 10x + 0.1 \sin 10x$

17. $y = ce^{-2.5x} + 0.640x - 0.256$

19. $25y^2 - 4x^2 = c$

21. $F = x$, $x^3e^y + x^2y = c$

23. $y = \sin\left(x + \frac{1}{4}\pi\right)$

25. $3 \sin x + \frac{1}{3} \sin y = 0$

27. $e^k = 1.25$, $\quad (\ln 2)/\ln 1.25 = 3.1$, $\quad (\ln 3)/\ln 1.25 = 4.9$ [days]

29. $e^k = 0.9$, 6.6 days. 43.7 days from $e^{kt} = 0.5$, $e^{kt} = 0.01$

Problem Set 2.1, page 53

1. $F(x, z, z') = 0$

3. $y = c_1e^{-x} + c_2$

5. $y = (c_1x + c_2)^{-1/2}$

7. $(dz/dy)z = -z^3 \sin y$, $-1/z = -dx/dy = \cos y + \tilde{c}_1$, $x = -\sin y + c_1y + c_2$

9. $y_2 = x^3 \ln x$

11. $y = c_1e^{2x} + c_2$

13. $y(t) = c_1e^{-t} + kt + c_2$

15. $y = 3 \cos 2.5x - \sin 2.5x$

17. $y = -0.75x^{3/2} - 2.25x^{-1/2}$

19. $y = 15e^{-x} - \sin x$

Problem Set 2.2, page 59

1. $y = c_1e^{-2.5x} + c_2e^{2.5x}$

3. $y = c_1e^{-2.8x} + c_2e^{-3.2x}$

5. $y = (c_1 + c_2x)e^{-\pi x}$

7. $y = c_1 + c_2e^{-4.5x}$

9. $y = c_1e^{-2.6x} + c_2e^{0.8x}$

11. $y = c_1e^{-x/2} + c_2e^{3x/2}$

13. $y = (c_1 + c_2x)e^{5x/3}$

15. $y = e^{-0.27x}(A \cos(\sqrt{\pi}\,x) + B \sin(\sqrt{\pi}\,x))$

17. $y'' + 2\sqrt{5}y' + 5y = 0$

19. $y'' + 4y' + 5y = 0$

21. $y = 4.6 \cos 5x - 0.24 \sin 5x$

23. $y = 6e^{2x} + 4e^{-3x}$

25. $y = 2e^{-x}$

27. $y = (4.5 - x)e^{-\pi x}$

29. $y = \dfrac{1}{\sqrt{\pi}} e^{-0.27x} \sin(\sqrt{\pi}\,x)$

31. Independent

33. $c_1x^2 + c_2x^2 \ln x = 0$ with $x = 1$ gives $c_1 = 0$; then $c_2 = 0$ for $x = 2$, say. Hence independent

35. Dependent since $\sin 2x = 2 \sin x \cos x$

37. $y_1 = e^{-x}$, $\quad y_2 = 0.001e^{x} + e^{-x}$

Problem Set 2.3, page 61

1. $4e^{2x}$, $-e^{-x} + 8e^{2x}$, $-\cos x - 2 \sin x$
3. 0, 0, $(D - 2I)(-4e^{-2x}) = 8e^{-2x} + 8e^{-2x}$
5. 0, $5e^{2x}$, 0
7. $(2D - I)(2D + I)$, $y = c_1 e^{0.5x} + c_2 e^{-0.5x}$
9. $(D - 2.1I)^2$, $y = (c_1 + c_2 x)e^{2.1x}$
11. $(D - 1.6I)(D - 2.4I)$, $y = c_1 e^{1.6x} + c_2 e^{2.4x}$
15. Combine the two conditions to get $L(cy + kw) = L(cy) + L(kw) = cLy + kLw$. The converse is simple.

Problem Set 2.4, page 69

1. $y' = y_0 \cos \omega_0 t + (v_0/\omega_0) \sin \omega_0 t$. At integer t (if $\omega_0 = \pi$), because of periodicity.
3. (i) Lower by a factor $\sqrt{2}$, (ii) higher by $\sqrt{2}$
5. 0.3183, 0.4775, $\sqrt{(k_1 + k_2)/m}/(2\pi) = 0.5738$
7. $mL\theta'' = -mg \sin \theta \approx -mg\theta$ (tangential component of $W = mg$), $\theta'' + \omega_0{}^2\theta = 0$, $\omega_0/(2\pi) = \sqrt{g/L}/(2\pi)$
9. $my'' = -\tilde{a}\gamma y$, where $m = 1$ kg, $ay = \pi \cdot 0.01^2 \cdot 2y$ meter^3 is the volume of the water that causes the restoring force $a\gamma y$ with $\gamma = 9800$ nt (= weight/meter^3). $y'' + \omega_0{}^2 y = 0$, $\omega_0{}^2 = a\gamma/m = a\gamma = 0.000628\gamma$. Frequency $\omega_0/2\pi = 0.4$ [sec^{-1}].
13. $y = [y_0 + (v_0 + \alpha y_0)t]e^{-\alpha t}$, $y = [1 + (v_0 + 1)t]e^{-t}$; (ii) $v_0 = -2, -\frac{3}{2}, -\frac{4}{3}, -\frac{5}{4}, -\frac{6}{5}$
15. $\omega^* = [\omega_0{}^2 - c^2/(4m^2)]^{1/2} = \omega_0[1 - c^2/(4mk)]^{1/2} \approx \omega_0(1 - c^2/8mk) = 2.9583$
17. The positive solutions of $\tan t = 1$, that is, $\pi/4$ (max), $5\pi/4$ (min). etc
19. $0.0231 = (\ln 2)/30$ [kg/sec] from $\exp(-10 \cdot 3c/2m) = \frac{1}{2}$.

Problem Set 2.5, page 73

3. $y = (c_1 + c_2 \ln x)x^{-1.8}$
5. $\sqrt{x}\,(c_1 \cos(\ln x) + c_2 \sin(\ln x))$
7. $y = c_1 x^2 + c_2 x^3$
9. $y = (c_1 + c_2 \ln x)x^{0.6}$
11. $y = x^2(c_1 \cos(\sqrt{6} \ln x) + c_2 \sin(\sqrt{6} \ln x))$
13. $y = x^{-3/2}$
15. $y = (3.6 + 4.0 \ln x)/x$
17. $y = \cos(\ln x) + \sin(\ln x)$
19. $y = -0.525x^5 + 0.625x^{-3}$

Problem Set 2.6, page 79

3. $W = -2.2e^{-3x}$
5. $W = -x^4$
7. $W = a$
9. $y'' + 25y = 0$, $W = 5$, $y = 3 \cos 5x - \sin 5x$
11. $y'' + 5y + 6.34 = 0$, $W = 0.3e^{-5x}$, $3e^{-2.5} \cos 0.3x$
13. $y'' + 2y' = 0$, $W = -2e^{-2x}$, $y = 0.5(1 + e^{-2x})$
15. $y'' - 3.24y = 0$, $W = 1.8$, $y = 14.2 \cosh 1.8x + 9.1 \sinh 1.8x$

Problem Set 2.7, page 84

1. $y = c_1 e^{-x} + c_2 e^{-4x} - 5e^{-3x}$
3. $y = c_1 e^{-2x} + c_2 e^{-x} + 6x^2 - 18x + 21$
5. $y = (c_1 + c_2 x)e^{-2x} + \frac{1}{2}e^{-x} \sin x$
7. $y = c_1 e^{-x/2} + c_2 e^{-3x/2} + \frac{4}{5}e^x + 6x - 16$
9. $y = c_1 e^{4x} + c_2 e^{-4x} + 1.2xe^{4x} - 2e^x$
11. $y = \cos(\sqrt{3}x) + 6x^2 - 4$

13. $y = e^{x/4} - 2e^{x/2} + \frac{1}{5}e^{-x} + e^x$ **15.** $y = \ln x$
17. $y = e^{-0.1x}(1.5 \cos 0.5x - \sin 0.5x) + 2e^{0.5x}$

Problem Set 2.8, page 91

3. $y_p = 1.0625 \cos 2t + 3.1875 \sin 2t$
5. $y_p = -1.28 \cos 4.5t + 0.36 \sin 4.5t$
7. $y_p = 25 + \frac{4}{3} \cos 3t + \sin 3t$
9. $y = e^{-1.5t}(A \cos t + B \sin t) + 0.8 \cos t + 0.4 \sin t$
11. $y = A \cos \sqrt{2}t + B \sin \sqrt{2}t + t(\sin \sqrt{2}t - \cos \sqrt{2}t)/(2\sqrt{2})$
13. $y = A \cos t + B \sin t - (\cos \omega t)/(\omega^2 - 1)$
15. $y = e^{-2t}(A \cos 2t + B \sin 2t) + \frac{1}{4} \sin 2t$
17. $y = \frac{1}{3} \sin t - \frac{1}{15} \sin 3t - \frac{1}{105} \sin 5t$
19. $y = e^{-t}(0.4 \cos t + 0.8 \sin t) + e^{-t/2}(-0.4 \cos \frac{1}{2}t + 0.8 \sin \frac{1}{2}t)$
25. CAS Experiment. The choice of ω needs experimentation, inspection of the curves obtained, and then changes on a trail-and-error basis. It is interesting to see how in the case of beats the period gets increasingly longer and the maximum amplitude gets increasingly larger as $\omega/(2\pi)$ approaches the resonance frequency.

Problem Set 2.9, page 98

1. $RI' + I/C = 0, \quad I = ce^{-t/(RC)}$
3. $LI' + RI = E, \quad I = (E/R) + ce^{-Rt/L} = 4.8 + ce^{-40t}$
5. $I = 2(\cos t - \cos 20t)/399$
7. I_0 is maximum when $S = 0$; thus, $C = 1/(\omega^2 L)$.
9. $I = 0$ **11.** $I = 5.5 \cos 10t + 16.5 \sin 10t$ A
13. $I = e^{-5t}(A \cos 10t + B \sin 10t) - 400 \cos 25t + 200 \sin 25t$ A
15. $R > R_{\text{crit}} = 2\sqrt{L/C}$ is Case I, etc.
17. $E(0) = 600, I'(0) = 600, I = e^{-3t}(-100 \cos 4t + 75 \sin 4t) + 100 \cos t$
19. $R = 2\ \Omega, \quad L = 1$ H, $\quad C = \frac{1}{12}$ F, $\quad E = 4.4 \sin 10t$ V

Problem Set 2.10, page 102

1. $y = A \cos 3x + B \sin 3x + \frac{1}{9}(\cos 3x) \ln|\cos 3x| + \frac{1}{3}x \sin 3x$
3. $y = c_1 x + c_2 x^2 - x \sin x$ **5.** $y = A \cos x + B \sin x + \frac{1}{2}x(\cos x + \sin x)$
7. $y = (c_1 + c_2 x)e^{2x} + x^{-2}e^{2x}$ **9.** $y = (c_1 + c_2 x)e^x + 4x^{7/2}e^x$
11. $y = c_1 x^2 + c_2 x^3 + 1/(2x^4)$ **13.** $y = c_1 x^{-3} + c_2 x^3 + 3x^5$

Chapter 2 Review Questions and Problems, page 102

7. $y = c_1 e^{-4.5x} + c_2 e^{-3.5x}$ **9.** $y = e^{-3x}(A \cos 5x + B \sin 5x)$
11. $y = (c_1 + c_2 x)e^{0.8x}$ **13.** $y = c_1 x^{-4} + c_2 x^3$
15. $y = c_1 e^{2x} + c_2 e^{-x/2} - 3x + x^2$ **17.** $y = (c_1 + c_2 x)e^{1.5x} + 0.25x^2 e^{1.5x}$
19. $y = 5 \cos 4x - \frac{3}{4}\sin 4x + e^x$ **21.** $y = -4x + 2x^3 + 1/x$
23. $I = -0.01093 \cos 415t + 0.05273 \sin 415t$ A

25. $I = \frac{1}{73}(50 \sin 4t - 110 \cos 4t)$ A
27. RLC-circuit with $R = 20\ \Omega$, $L = 4$ H, $C = 0.1$ F, $E = -25 \cos 4t$ V
29. $\omega = 3.1$ is close to $\omega_0 = \sqrt{k/m} = 3$, $y = 25(\cos 3t - \cos 3.1t)$.

Problem Set 3.1, page 111

9. Linearly independent
11. Linearly independent
13. Linearly independent
15. Linearly dependent

Problem Set 3.2, page 116

1. $y = c_1 + c_2 \cos 5x + c_3 \sin 5x$
3. $y = c_1 + c_2x + c_3 \cos 2x + c_4 \sin 2x$
5. $y = A_1 \cos x + B_1 \sin x + A_2 \cos 3x + B_2 \sin 3x$
7. $y = 2.398 + e^{-1.6x}(1.002 \cos 1.5x - 1.998 \sin 1.5x)$
9. $y = 4e^{-x} + 5e^{-x/2} \cos 3x$
11. $y = \cosh 5x - \cos 4x$
13. $y = e^{0.25x} + 4.3e^{-0.7x} + 12.1 \cos 0.1x - 0.6 \sin 0.1x$

Problem Set 3.3, page 122

1. $y = (c_1 + c_2x + c_3x^2)e^{-x} + \frac{1}{8}e^{x} - x + 2$
3. $y = c_1 \cos x + c_2 \sin x + c_3 \cos 3x + c_4 \sin 3x + 0.1 \sinh 2x$
5. $y = c_1x^2 + c_2x + c_3x^{-1} - \frac{1}{12}x^{-2}$
7. $y = (c_1 + c_2x + c_3x^2)e^{3x} - \frac{1}{4}(\cos 3x - \sin 3x)$
9. $y = \cos x + \frac{1}{2}\sin 4x$
11. $y = e^{-3x}(-1.4 \cos x - \sin x)$
13. $y = 2 - 2 \sin x + \cos x$

Chapter 3 Review Questions and Problems, page 122

7. $y = c_1 + e^{-2x}(A \cos 3x + B \sin 3x)$
9. $y = c_1 \cosh 2x + c_2 \sinh 2x + c_3 \cos 2x + c_4 \sin 2x + \cosh x$
11. $y = (c_1 + c_2x + c_3x^2)e^{-1.5x}$
13. $y = (c_1 + c_2x + c_3x^2)e^{-2x} + x^2 - 3x + 3$
15. $y = c_1x + c_2x^{1/2} + c_3x^{3/2} - \frac{10}{3}$
17. $y = 2e^{-2x} \cos 4x + 0.05\, x - 0.06$
19. $y = 4e^{-4x} + 5e^{-5x}$

Problem Set 4.1, page 136

1. Yes
5. $y_1' = 0.02(-y_1 + y_2)$, $y_2' = 0.02(y_1 - 2y_2 + y_3)$, $y_3' = 0.02(y_2 - y_3)$
7. $c_1 = 1$, $c_2 = -5$
9. $c_1 = 10$, $c_2 = 5$
11. $y_1' = y_2$, $y_2' = y_1 + \ + \frac{15}{4}y_2$, $\mathbf{y} = c_1[1 \quad 4]^{\mathsf{T}} e^{4t} + c_2 [1 \quad -\frac{1}{4}]^{\mathsf{T}} e^{-t/4}$
13. $y_1' = y_2$, $y_2' = 24y_1 - 2y_2$, $y_1 = c_1e^{4t} + c_2e^{-6t} = y$, $y_2 = y'$
15. **(a)** For example, $C = 1000$ gives -2.39993, -0.000167. **(b)** -2.4, 0.
(d) $a_{22} = -4 + 2\sqrt{6.4} = 1.05964$ gives the critical case. C about 0.18506.

Problem Set 4.3, page 147

1. $y_1 = c_1e^{-2t} + c_2e^{2t}, \quad y_2 = -3c_1e^{-2t} + c_2e^{2t}$

3. $y_1 = 2c_1e^{2t} + 2c_2, \quad y_2 = c_1e^{2t} - c_2$

5. $y_1 = 5c_1 + 2c_2e^{14.5t}$
$y_2 = -2c_1 + 5c_2e^{14.5t}$

7. $y_1 = -c_2 \cos \sqrt{2}t + c_3 \sin \sqrt{2}t + c_1$
$y_2 = c_2\sqrt{2} \sin \sqrt{2}t + c_3\sqrt{2} \cos \sqrt{2}t$
$y_3 = c_2 \cos \sqrt{2}t - c_3 \sin \sqrt{2}t + c_1$

9. $y_1 = \frac{1}{2}c_1e^{-18t} + 2c_2e^{9t} - c_3e^{18t}$
$y_2 = c_1e^{-18t} + c_2e^{9t} + c_3e^{18t}$
$y_3 = c_1e^{-18t} - 2c_2e^{9t} - \frac{1}{2}c_3e^{18t}$

11. $y_1 = -20e^t + 8e^{-t/2}$
$y_2 = 4e^t - 4e^{-t/2}$

13. $y_1 = 2 \sinh t, \quad y_2 = 2 \cosh t$

15. $y_1 = \frac{1}{2}e^t$
$y_2 = \frac{1}{2}e^t$

17. $y_2 = y_1' + y_1, \quad y_2' = y_1'' + y_1' = -y_1 - y_2 = -y_1 - (y_1' + y_1),$
$y_1'' + 2y_1' + 2y_1 = 0, \quad y_1 = e^{-t}(A \cos t + B \sin t),$
$y_2 = y_1' + y_1 = e^{-t}(B \cos t - A \sin t)$. Note that $r^2 = y_1^2 + y_2^2 = e^{-2t}(A^2 + B^2)$.

19. $I_1 = c_1e^{-t} + 3c_2e^{-3t}, I_2 = -3c_1e^{-t} - c_2e^{-3t}$

Problem Set 4.4, page 151

1. Unstable improper node, $y_1 = c_1e^t, \quad y_2 = c_2e^{2t}$

3. Center, always stable, $y_1 = A \cos 3t + B \sin 3t, \quad y_2 = 3B \cos 3t - 3A \sin 3t$

5. Stable spiral, $y_1 = e^{-2t}(A \cos 2t + B \sin 2t), \quad y_2 = e^{-2t}(B \cos 2t - A \sin 2t)$

7. Saddle point, always unstable, $y_1 = c_1e^{-t} + c_2e^{3t}, \quad y_2 = -c_1e^{-t} + c_2e^{3t}$

9. Unstable node, $y_1 = c_1e^{6t} + c_2e^{2t}, \quad y_2 = 2c_1e^{6t} - 2c_2e^{2t}$

11. $y = e^{-t}(A \cos t + B \sin t)$. Stable and attractive spirals

15. $p = 0.2 \neq 0$ (was 0), $\quad \Delta < 0$, spiral point, unstable.

17. For instance, **(a)** -2, **(b)** -1, **(c)** $= -\frac{1}{2}$, **(d)** $=1$, **(e)** 4.

Problem Set 4.5, page 159

5. Center at (0, 0). At (2, 0) set $y_1 = 2 + \tilde{y}_1$. Then $\tilde{y}_2' = \tilde{y}_1$. Saddle point at (2, 0).

7. (0, 0), $y_1' = -y_1 + y_2, \quad y_2' = -y_1 - y_2$, stable and attractive spiral point; $(-2, 2)$,
$y_1 = -2 + \tilde{y}_1, \quad y_2 = 2 + \tilde{y}_2, \quad \tilde{y}_1' = -\tilde{y}_1 - 3\tilde{y}_2, \quad \tilde{y}_2' = -\tilde{y}_1 - \tilde{y}_2$, saddle point

9. (0, 0) saddle point, $(-3, 0)$ and (3, 0) centers

11. $(\frac{1}{2}\pi \pm 2n\pi, 0)$ saddle points; $(-\frac{1}{2}\pi \pm 2n\pi, 0)$ centers.
Use $-\cos(\pm\frac{1}{2}\pi + \tilde{y}_1) = \sin(\pm\tilde{y}_1) \approx \pm\tilde{y}_1$.

13. $(\pm 2n\pi, 0)$ centers; $y_1 = (2n + 1)\pi + \tilde{y}_1', \quad (\pi \pm 2n\pi, 0)$ saddle points

15. By multiplication, $y_2y_2' = (4y_1 - y_1^3)y_1'$. By integration,
$y_2^2 = 4y_1^2 - \frac{1}{2}y_1^4 + c^* = \frac{1}{2}(c + 4 - y_1^2)(c - 4 + y_1^2)$, where $c^* = \frac{1}{2}c^2 - 8$.

Problem Set 4.6, page 163

3. $y_1 = c_1e^{-t} + c_2e^t, \quad y_2 = -c_1e^{-t} + c_2e^t - e^{3t}$

5. $y_1 = c_1e^{5t} + c_2e^{2t} - 0.43t - 0.24, \quad y_2 = c_1e^{5t} - 2c_2e^{2t} + 1.12t + 0.53$

7. $y_1 = c_1e^t + 4c_2e^{2t} - 3t - 4 - 2e^{-t}$, $y_2 = -c_1e^t - 5c_2e^{2t} + 5t + 7.5 + e^{-t}$

9. The formula for $\mathbf{v}$ shows that these various choices differ by multiples of the eigenvector for $\lambda = -2$, which can be absorbed into, or taken out of, c_1 in the general solution $y^{(h)}$.

11. $y_1 = -\frac{8}{3}\cosh t - \frac{4}{3}\sinh t + \frac{11}{3}e^{2t}$, $y_2 = -\frac{8}{3}\sinh t - \frac{4}{3}\cosh t + \frac{4}{3}e^{2t}$

13. $y_1 = \cos 2t + \sin 2t + 4\cos t$, $y_2 = 2\cos 2t - 2\sin 2t + \sin t$

15. $y_1 = 4e^{-t} - 4e^t + e^{2t}$, $y_2 = -4e^{-t} + t$

17. $I_1 = 2c_1e^{\lambda_1 t} + 2c_2e^{\lambda_2 t} + 100$,
$I_2 = (1.1 + \sqrt{0.41})c_1e^{\lambda_1 t} + (1.1 - \sqrt{0.41})c_2e^{\lambda_2 t}$,
$\lambda_1 = -0.9 + \sqrt{0.41}$, $\lambda_2 = -0.9 - \sqrt{0.41}$

19. $c_1 = 17.948$, $c_2 = -67.948$

Chapter 4 Review Questions and Problems, page 164

11. $y_1 = c_1e^{4t} + c_2e^{-4t}$, $y_2 = 2c_1e^{4t} - 2c_2e^{-4t}$. Saddle point

13. $y_1 = e^{-4t}(A\cos t + B\sin t)$, $y_2 = \frac{1}{5}e^{-4t}[(B - 2A)\cos t - (A + 2B)\sin t]$; asymptotically stable spiral point

15. $y_1 = c_1e^{-5t} + c_2e^{-t}$, $y_2 = c_1e^{-5t} - c_2e^{-t}$. Stable node

17. $y_1 = e^{-t}(A\cos 2t + B\sin 2t)$, $y_2 = e^{-t}(B\cos 2t - A\sin 2t)$. Stable and attractive spiral point

19. Unstable spiral point

21. $y_1 = c_1e^{-4t} + c_2e^{4t} - 1 - 8t^2$, $y_2 = -c_1e^{-4t} + c_2e^{4t} - 4t$

23. $y_1 = 2c_1e^{-t} + 2c_2e^{3t} + \cos t - \sin t$, $y_2 = -c_1e^{-t} + c_2e^{3t}$

25. $I_1' + 2.5(I_1 - I_2) = 169\sin t$, $2.5(I_2' - I_1') + 25I_2 = 0$,
$I_1 = (19 + 32.5t)e^{-5t} - 19\cos t + 62.5\sin t$,
$I_2 = (-6 - 32.5t)e^{-5t} + 6\cos t + 2.5\sin t$

27. $(0, 0)$ saddle point; $(-1, 0)$, $(1, 0)$ centers

29. $(n\pi, 0)$ center when n is even and saddle point when n is odd

Problem Set 5.1, page 174

3. $\sqrt{|k|}$

5. $\sqrt{3/2}$

7. $y = a_0(1 - x^2 + x^4/2! - x^6/3! + - \cdots) = a_0e^{-x^2}$

9. $y = a_0 + a_1x - \frac{1}{2}a_0x^2 - \frac{1}{6}a_1x^3 + \cdots = a_0\cos x + a_1\sin x$

11. $a_0(1 - \frac{1}{12}x^4 - \frac{1}{60}x^5 - \cdots) + a_1(x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{24}x^5 - \cdots)$

13. $a_0(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4 + \frac{13}{720}x^6 + \cdots) + a_1(x - \frac{1}{6}x^3 - \frac{1}{24}x^5 + \frac{5}{1008}x^7 + \cdots)$

15. $\displaystyle\sum_{m=1}^{\infty}\frac{(m+1)(m+2)}{(m+1)^2+1}x^m$, $\displaystyle\sum_{m=5}^{\infty}\frac{(m-4)^2}{(m-3)!}x^m$

17. $s = 1 + x - x^2 - \frac{5}{6}x^3 + \frac{2}{3}x^4 + \frac{11}{24}x^5$, $s(\frac{1}{2}) = \frac{923}{768}$

19. $s = 4 - x^2 - \frac{1}{3}x^3 + \frac{1}{30}x^5$, $s(2) = -\frac{8}{5}$; but $x = 2$ is too large to give good values. Exact: $y = (x - 2)^2e^x$

Problem Set 5.2, page 179

5. $P_6(x) = \frac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5)$,
$P_7(x) = \frac{1}{16}(429x^7 - 693x^5 + 315x^3 - 35x)$

11. Set $x = az$. $y = c_1 P_n(x/a) + c_2 Q_n(x/a)$

15. $P_1^1 = \sqrt{1 - x^2}$, $P_2^1 = 3x\sqrt{1 - x^2}$, $P_2^2 = 3(1 - x^2)$, $P_4^2 = (1 - x^2)(105x^2 - 15)/2$

Problem Set 5.3, page 186

3. $y_1 = 1 - \dfrac{x^2}{3!} + \dfrac{x^4}{5!} - + \cdots = \dfrac{\sin x}{x}$, $y_2 = \dfrac{1}{x} - \dfrac{x}{2!} + \dfrac{x^3}{4!} - + \cdots = \dfrac{\cos x}{x}$

5. $b_0 = 1$, $c_0 = 0$, $r^2 = 0$, $y_1 = e^{-x}$, $y_2 = e^{-x} \ln x$

7. $y_1 = 1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{30}x^5 + \frac{1}{144}x^6 - \cdots$,
$y_2 = x + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \frac{1}{120}x^5 - \frac{1}{120}x^6 + \cdots$

9. $y_1 \sqrt{x}$, $y_2 = 1 + x$

11. $y_1 = e^x$, $y_2 = e^x/x$

13. $y_1 = e^x$, $y_2 = e^x \ln x$

15. $y = AF(1, 1, -\frac{1}{2}; x) + Bx^{3/2}F(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}; x)$

17. $y = A(1 - 8x + \frac{32}{5}x^2) + Bx^{3/4}F(\frac{7}{4}, -\frac{5}{4}, \frac{7}{4}; x)$

19. $y = c_1 F(2, -2, -\frac{1}{2}; t - 2) + c_2(t - 2)^{3/2}F(\frac{7}{2}, -\frac{1}{2}, \frac{5}{2}; t - 2)$

Problem Set 5.4, page 195

3. $c_1 J_0(\sqrt{x})$

5. $c_1 J_\nu(\lambda x) + c_2 J_{-\nu}(\lambda x)$, $\nu \neq 0, \pm 1, \pm 2, \cdots$

7. $c_1 J_{1/2}(\frac{1}{2}x) + c_2 J_{-1/2}(\frac{1}{2}x) = x^{-1/2}(\tilde{c}_1 \sin \frac{1}{2}x + \tilde{c}_2 \cos \frac{1}{2}x)$

9. $x^{-\nu}(c_1 J_\nu(x) + c_2 J_{-\nu}(x))$, $\nu \neq 0, \pm 1, \pm 2, \cdots$

13. $J_n(x_1) = J_n(x_2) = 0$ implies $x_1^{-n} J_n(x_1) = x_2^{-n} J_n(x_2) = 0$ and $[x^{-n} J_n(x)]' = 0$ somewhere between x_1 and x_2 by Rolle's theorem. Now use (21b) to get $J_{n+1}(x) = 0$ there. Conversely, $J_{n+1}(x_3) = J_{n+1}(x_4) = 0$, thus $x_3^{n+1} J_{n+1}(x_3) = x_4^{n+1} J_{n+1}(x_4) = 0$ implies $J_n(x) = 0$ in between by Rolle's theorem and (21a) with $\nu = n + 1$.

15. By Rolle, $J_0' = 0$ at least once between two zeros of J_0. Use $J_0' = -J_1$ by (21b) with $\nu = 0$. Together $J_1 = 0$ at least once between two zeros of J_0. Also use $(xJ_1)' = xJ_0$ by (21a) with $\nu = 1$ and Rolle.

19. Use (21b) with $\nu = 0$, (21a) with $\nu = 1$, (21d) with $\nu = 2$, respectively.

21. Integrate (21a).

23. Use (21a) with $\nu = 1$, partial integration, (21b) with $\nu = 0$, partial integration.

25. Use (21d) to get

$$\int J_5(x)\,dx = -2J_4(x) + \int J_3(x)\,dx = -2J_4(x) - 2J_2(x) + \int J_1(x)\,dx$$
$$= -2J_4(x) - 2J_2(x) - J_0(x) + c.$$

Problem Set 5.5, page 200

1. $c_1 J_4(x) + c_2 Y_4(x)$

3. $c_1 J_{2/3}(x^2) + c_2 Y_{2/3}(x^2)$

5. $c_1 J_0(\sqrt{x}) + c_2 Y_0(\sqrt{x})$

7. $\sqrt{x}\,(c_1 J_{1/4}(\frac{1}{2}kx^2) + c_2 Y_{1/4}(\frac{1}{2}kx^2))$
9. $x^3(c_1 J_3(x) + c_2 Y_3(x))$
11. Set $H^{(1)} = kH^{(2)}$ and use (10).
13. Use (20) in Sec. 5.4.

Chapter 5 Review Questions and Problems, page 200

11. $\cos 2x$, $\sin 2x$
13. $(x-1)^{-5}$, $(x-1)^7$; Euler–Cauchy with $x-1$ instead of x
15. $J_{\sqrt{5}}(x)$, $J_{-\sqrt{5}}(x)$
17. e^x, $1 + x$
19. $\sqrt{x}\,J_1(\sqrt{x})$, $\sqrt{x}\,Y_1(\sqrt{x})$

Problem Set 6.1, page 210

1. $3/s^2 + 12/s$
3. $s/(s^2 + \pi^2)$
5. $1/((s-2)^2 - 1)$
7. $(\omega \cos\theta + s\sin\theta)/(s^2 + \omega^2)$
9. $\dfrac{1}{s} + \dfrac{e^{-s} - 1}{s^2}$
11. $\dfrac{1 - e^{-bs}}{s^2} - \dfrac{be^{-bs}}{s}$
13. $\dfrac{(1 - e^{-s})^2}{s}$
15. $\dfrac{e^{-s} - 1}{2s^2} - \dfrac{e^{-s}}{2s} + \dfrac{1}{s}$
19. Use $e^{at} = \cosh at + \sinh at$.
23. Set $ct = p$. Then $\mathcal{L}(f(ct)) = \displaystyle\int_0^\infty e^{-st} f(ct)\,dt = \int_0^\infty e^{-(s/c)p} f(p)\,dp/c = F(s/c)/c$.
25. $0.2\cos 1.8t + \sin 1.8t$
27. $\dfrac{1}{L^2}\cos\dfrac{n\pi t}{L}$
29. $2t^3 - 1.9t^5$
31. $\mathcal{L}^{-1}\left(\dfrac{4}{s-2} - \dfrac{3}{s+1}\right) = 4e^{2t} - 3e^{-t}$
33. $\dfrac{2}{(s+3)^3}$
35. $\dfrac{0.5 \cdot 2\pi}{(s+4.5)^2 + 4\pi^2}$
37. $\pi t e^{-\pi t}$
39. $\frac{7}{2}t^3 e^{-t\sqrt{2}}$
41. $e^{-5\pi t}\sinh \pi t$
43. $e^{3t}(2\cos 3t + \frac{5}{3}\sin 3t)$
45. $(k_0 + k_1 t)e^{-at}$

Problem Set 6.2, page 216

1. $y = 1.25e^{-5.2t} - 1.25\cos 2t + 3.25\sin 2t$
3. $(s-3)(s+2) = 11s + 28 - 11 = 11s + 17$, $\quad Y = 10/(s-3) + 1/(s+2)$,
$y = 10e^{3t} + e^{-2t}$
5. $(s^2 - \frac{1}{4})Y = 12s$, $\quad y = 12\cosh\frac{1}{2}t$
7. $y = \frac{1}{2}e^{3t} + \frac{5}{2}e^{-4t} + \frac{1}{2}e^{-3t}$
9. $y = e^t - e^{3t} + 2t$
11. $(s + 1.5)^2 Y = s + 31.5 + 3 + 54/s^4 + 64/s$,
$Y = 1/(s+1.5) + 1/(s+1.5)^2 + 24/s^4 - 32/s^3 + 32/s^2$,
$y = (1+t)e^{-1.5t} + 4t^3 - 16t^2 + 32t$
13. $t = \tilde{t} - 1$, $\quad \tilde{Y} = 4/(s-6)$, $\quad \tilde{y} = 4e^{6t}$, $\quad y = 4e^{6(t+1)}$

15. $t = \tilde{t} + 1.5$, $(s - 1)(s + 4)\tilde{Y} = 4s + 17 + 6/(s - 2)$, $y = 3e^{t-1.5} + e^{2(t-1.5)}$

17. $\dfrac{1}{(s + a)^2}$

19. $\dfrac{2\omega^2}{s(s^2 + 4\omega^2)}$

21. $\mathcal{L}(f') = \mathcal{L}(\sinh 2t) = s\mathcal{L}(f) - 1$. *Answer:* $(s^2 - 2)/(s^3 - 4s)$

23. $12(1 - e^{-t/4})$

25. $(1 - \cos \omega t)/\omega^2$

27. $\frac{1}{9}(1 + t - \cos 3t - \frac{1}{3}\sin 3t)$

29. $\dfrac{1}{a^2}(e^{-at} - 1) + \dfrac{t}{a}$

Problem Set 6.3, page 223

3. $\mathcal{L}((t - 2)u(t - 2)) = e^{-2s}/s^2$

5. $\left(e^t\left(1 - u\left(t - \dfrac{1}{2}\pi\right)\right)\right) = \dfrac{1}{s - 1}(1 - e^{-\pi s/2 + \pi/2})$

7. $\dfrac{1}{s + \pi}(e^{-2(s+\pi)} - e^{-4(s+\pi)})$

9. $e^{-3s/2}\left(\dfrac{2}{s^3} + \dfrac{3}{s^2} + \dfrac{\frac{9}{4}}{s}\right)$

11. $(se^{-\pi s/2} + e^{-\pi s})/(s^2 + 1)$

13. $2[1 + u(t - \pi)] \sin 3t$

15. $(t - 3)^3 u(t - 3)/6$

17. $e^{-t} \cos t \ (0 < t < 2\pi)$

19. $\frac{1}{3}(e^t - 1)^3 e^{-5t}$

21. $\sin 3t + \sin t \ (0 < t < \pi)$; $\frac{4}{3} \sin 3t \ (t > \pi)$

23. $e^t - \sin t \ (0 < t < 2\pi)$, $e^t - \frac{1}{2}\sin 2t \ (t > 2\pi)$

25. $t - \sin t \ (0 < t < 1)$, $\cos (t - 1) + \sin (t - 1) - \sin t \ (t > 1)$

27. $t = 1 + \tilde{t}$, $\tilde{y}'' + 4\tilde{y} = 8(1 + \tilde{t})^2(1 - u(\tilde{t} - 4))$, $\cos 2t + 2t^2 - 1$ if $t < 5$, $\cos 2t + 49 \cos (2t - 10) + 10 \sin (2t - 10)$ if $t > 5$

29. $0.1i' + 25i = 490e^{-5t}[1 - u(t - 1)]$,
$i = 20(e^{-5t} - e^{-250t}) + 20u(t - 1)[-e^{-5t} + e^{-250t+245}]$

31. $Rq' + q/C = 0$, $Q = \mathcal{L}(q)$, $q(0) = CV_0$, $i = q'(t)$,
$R(sQ - CV_0) + Q/C = 0$, $q = CV_0 e^{-t/(RC)}$

33. $10I + \dfrac{100}{s}I = \dfrac{100}{s^2}e^{-2s}$, $I = e^{-2s}\left(\dfrac{1}{s} - \dfrac{1}{s + 10}\right)$, $i = 0$ if $t < 2$ and
$1 - e^{-10(t-2)}$ if $t > 2$

35. $i = (10 \sin 10t + 100 \sin t)(u(t - \pi) - u(t - 3\pi))$

37. $(0.5s^2 + 20)I = 78s(1 + e^{-\pi s})/(s^2 + 1)$,
$i = 4 \cos t - 4 \cos \sqrt{40}t - 4u(t - \pi)[\cos t + \cos (\sqrt{40}(t - \pi))]$

39. $i' + 2i + 2\displaystyle\int_0^t i(\tau)\, d\tau = 1000(1 - u(t - 2))$, $I = 1000(1 - e^{-2s})/(s^2 + 2s + 2)$,

$i = 1000e^{-t} \sin t - 1000u(t - 2)e^{-t+2} \sin (t - 2)$

Problem Set 6.4, page 230

3. $y = 8 \cos 2t + \frac{1}{2}u(t - \pi) \sin 2t$

5. $\sin t \ (0 < t < \pi)$; $0 \ (\pi < t < 2\pi)$; $-\sin t \ (t > 2\pi)$

7. $y = e^{-t} + 4e^{-3t} \sin \frac{1}{2}t + \frac{1}{2}u(t - \frac{1}{2})e^{-3(t-1/2)} \sin (\frac{1}{2}t - \frac{1}{4})$

9. $y = 0.1[e^t + e^{-2t}(-\cos t + 7 \sin t)] + 0.1u(t - 10)[-e^{-t} +$
$e^{-2t+30}(\cos (t - 10) - 7 \sin (t - 10))]$

11. $y = -e^{-3t} + e^{-2t} + \frac{1}{6}u(t-1)(1 - 3e^{-2(t-1)} + 2e^{-3(t-1)}) + u(t-2)(e^{-2(t-2)} - e^{-3(t-2)})$

15. $ke^{-ps}/(s - se^{-ps})\ (s > 0)$

Problem Set 6.5, page 237

1. t

3. $(e^t - e^{-t})/2 = \sinh t$

5. $\frac{1}{2}t \sin \omega t$

7. $e^t - t - 1$

9. $y - 1 * y = 1, \quad y = e^t$

11. $y = \cos t$

13. $y(t) + 2\int_0^t e^{t-\tau} y(\tau)\, d\tau = te^t, \quad y = \sinh t$

17. $e^{4t} - e^{-1.5t}$

19. $t \sin \pi t$

21. $(\omega t - \sin \omega t)/\omega^2$

23. $4.5(\cosh 3t - 1)$

25. $1.5t \sin 6t$

Problem Set 6.6, page 241

3. $\dfrac{\frac{1}{2}}{(s+3)^2}$

5. $\dfrac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$

7. $\dfrac{2s^3 + 24s}{(s^2 - 4)^3}$

9. $\dfrac{\pi(3s^2 - \pi^2)}{(s^2 + \pi^2)^3}$

11. $\dfrac{4s^2 - \pi^2}{(s^2 + \frac{1}{4}\pi^2)^2}$

15. $F(s) = -\dfrac{1}{2}\left(\dfrac{1}{s^2 - 9}\right)', \quad f(t) = \frac{1}{6}t \sinh 3t$

17. $\ln s - \ln(s-1); \quad (-1 + e^t)/t$

19. $[\ln(s^2 + 1) - 2\ln(s-1)]' = 2s/(s^2+1) - 2/(s-1); \quad 2(-\cos t + e^t)/t$

Problem Set 6.7, page 246

3. $y_1 = -e^{-5t} + 4e^{2t}, \quad y_2 = e^{-5t} + 3e^{2t}$

5. $y_1 = -\cos t + \sin t + 1 + u(t-1)[-1 + \cos(t-1) - \sin(t-1)]$
$y_2 = \cos t + \sin t - 1 + u(t-1)[1 - \cos(t-1) - \sin(t-1)]$

7. $y_1 = -e^{-2t} + 4e^t + \frac{1}{3}u(t-1)(-e^{3-2t} + e^t)$,
$y_2 = -e^{-2t} + e^t + \frac{1}{3}u(t-1)(-e^{3-2t} + e^t)$

9. $y_1 = (3 + 4t)e^{3t}, \quad y_2 = (1 - 4t)e^{3t}$

11. $y_1 = e^t + e^{2t}, \quad y_2 = e^{2t}$

13. $y_1 = -4e^t + \sin 10t + 4\cos t, \quad y_2 = 4e^t - \sin 10t + 4\cos t$

15. $y_1 = e^t, \quad y_2 = e^{-t}, \quad y_3 = e^t - e^{-t}$

19. $4i_1 + 8(i_1 - i_2) + 2i_1' = 390\cos t, \quad 8i_2 + 8(i_2 - i_1) + 4i_2' = 0$,
$i_1 = -26e^{-2t} - 16e^{-8t} + 42\cos t + 15\sin t$,
$i_2 = -26e^{-2t} + 8e^{-8t} + 18\cos t + 12\sin t$

Chapter 6 Review Questions and Problems, page 251

11. $\dfrac{5s}{s^2 - 4} - \dfrac{3}{s^2 - 1}$

13. $\frac{1}{2}(1 - \cos \pi t), \quad \pi^2/(2s^3 + 2\pi^2 s)$

15. $e^{-3s+3/2}/(s - \frac{1}{2})$

17. Sec. 6.6; $2s^2/(s^2+1)^2$

19. $12/(s^2(s+3))$

21. $tu(t-1)$

23. $\sin(\omega t+\theta)$

25. $3t^2+t^3$

27. $e^{-2t}(3\cos t - 2\sin t)$

29. $y = e^{-2t}(13\cos t + 11\sin t) + 10t - 8$

31. $e^{-t} + u(t-\pi)[1.2\cos t - 3.6\sin t + 2e^{-t+\pi} - 0.8e^{2t-2\pi}]$

33. $0\ (0 \leqq t \leqq 2), \quad 1 - 2e^{-(t-2)} + e^{-2(t-2)} \quad (t > 2)$

35. $y_1 = 4e^t - e^{-2t}, \quad y_2 = e^t - e^{-2t}$

37. $y_1 = \cos t - u(t-\pi)\sin t + 2u(t-2\pi)\sin^2 \frac{1}{2}t,$
$y_2 = -\sin t - 2u(t-\pi)\cos^2\frac{1}{2}t + u(t-2\pi)\sin t$

39. $y_1 = (1/\sqrt{10})\sin\sqrt{10}t, \quad y_2 = -(1/\sqrt{10})\sin\sqrt{10}t$

41. $1 - e^{-t}\ (0 < t < 4), \quad (e^4 - 1)e^{-t} \quad (t > 4)$

43. $i(t) = e^{-4t}(\frac{3}{26}\cos 3t - \frac{10}{39}\sin 3t) - \frac{3}{26}\cos 10t + \frac{8}{65}\sin 10t$

45. $5i_1' + 20(i_1 - i_2) = 60, \quad 30i_2' + 20(i_2' - i_1') + 20i_2 = 0,$
$i_1 = -8e^{-2t} + 5e^{-0.8t} + 3, \quad i_2 = -4e^{-2t} + 4e^{-0.8t}$

Problem Set 7.1, page 261

3. $3\times 3, \quad 3\times 4, \quad 3\times 6, \quad 2\times 2, \quad 2\times 3, \quad 3\times 2$

5. $\mathbf{B} = \frac{1}{5}\mathbf{A}, \quad \frac{1}{10}\mathbf{A}$

7. No, no, yes, no, no

9. $\begin{bmatrix} 0 & 6 & 12 \\ 18 & 15 & 15 \\ 3 & 0 & -9 \end{bmatrix}, \quad \begin{bmatrix} 0 & 2.5 & 1 \\ 2.5 & 1.5 & 2 \\ -1 & 2 & -1 \end{bmatrix}, \quad \begin{bmatrix} 0 & 8.5 & 13 \\ 20.5 & 16.5 & 17 \\ 2 & 2 & -10 \end{bmatrix},$ undefined

11. $\begin{bmatrix} 0 & 26 \\ 34 & 32 \\ 28 & -10 \end{bmatrix},$ same, $\begin{bmatrix} 5.4 & 0.6 \\ -4.2 & 2.4 \\ -0.6 & 0.6 \end{bmatrix},$ same

13. $\begin{bmatrix} 70 & 28 \\ -28 & 56 \\ 14 & 0 \end{bmatrix},$ same, $-\mathbf{D}$, undefined

15. $\begin{bmatrix} 5.5 \\ 33.0 \\ -11.0 \end{bmatrix},$ same, undefined, undefined

17. $\begin{bmatrix} -4.5 \\ -27.0 \\ 9.0 \end{bmatrix}$

Problem Set 7.2, page 270

5. $10, \ n(n+1)/2$

7. $\mathbf{0}, \quad \mathbf{I}, \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

11. $\begin{bmatrix} 10 & -14 & -6 \\ -5 & 7 & -12 \\ -5 & -1 & -4 \end{bmatrix}$, same, $\begin{bmatrix} 10 & -5 & -15 \\ -14 & 7 & -33 \\ -2 & -4 & -4 \end{bmatrix}$, same

13. $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 13 & -6 \\ 0 & -6 & 4 \end{bmatrix}$, $\begin{bmatrix} -9 & -5 \\ 3 & -1 \\ 4 & 0 \end{bmatrix}$, undefined, $\begin{bmatrix} -9 & 3 & 4 \\ -5 & -1 & 0 \end{bmatrix}$

15. Undefined, $\begin{bmatrix} 8 \\ -4 \\ -3 \end{bmatrix}$, $[7 \quad -1 \quad 3]$, same

17. $\begin{bmatrix} -30 & -18 \\ 45 & 9 \\ 5 & -7 \end{bmatrix}$, undefined, $\begin{bmatrix} 22 \\ 4 \\ -12 \end{bmatrix}$, undefined

19. Undefined, $\begin{bmatrix} 10.5 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 7 \\ -3 \\ 1 \end{bmatrix}$, same

25. **(d)** $\mathbf{AB} = (\mathbf{AB})^\mathsf{T} = \mathbf{B}^\mathsf{T}\mathbf{A}^\mathsf{T} = \mathbf{BA}$; etc.
(e) *Answer.* If $\mathbf{AB} = -\mathbf{BA}$.
29. $\mathbf{p} = [85 \quad 62 \quad 30]^\mathsf{T}$, $\mathbf{v} = [44{,}920 \quad 30{,}940]^\mathsf{T}$

Problem Set 7.3, page 280

1. $x = -2$, $y = 0.5$
3. $x = 1$, $y = 3$, $z = -5$
5. $x = 6$, $y = -7$
7. $x = -3t$, $y = t$ arb., $z = 2t$
9. $x = 3t - 1$, $y = -t + 4$, $z = t$ arb.
11. $w = 1$, $x = t_1$ arb., $y = 2t_2 - t_1$, $z = t_2$ arb.
13. $w = 4$, $x = 0$, $y = 2$, $z = 6$
17. $I_1 = 2$, $I_2 = 6$, $I_3 = 8$
19. $I_1 = (R_1 + R_2)E_0/(R_1R_2)$ A, $I_2 = E_0/R_1$ A, $I_3 = E_0/R_2$ A
21. $x_2 = 1600 - x_1$, $x_3 = 600 + x_1$, $x_4 = 1000 - x_1$. No
23. C: $3x_1 - x_3 = 0$, H: $8x_1 - 2x_4 = 0$, O: $2x_2 - 2x_3 - x_4 = 0$, thus $C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$

Problem Set 7.4, page 287

1. 1; $[2 \quad -1 \quad 3]$; $[2 \quad -1]^\mathsf{T}$
3. 3; $\{[3 \quad 5 \quad 0], \ [0 \quad 3 \quad 5], \ [0 \quad 0 \quad 1]\}$
5. 3; $\{[2 \quad -1 \quad 4], \ [0 \quad 1 \quad -46], \ [0 \quad 0 \quad 1]\}$; $\{[2 \quad 0 \quad 1], \ [0 \quad 3 \quad 23], \ [0 \quad 0 \quad 1]\}$

7. 2; $[8 \quad 0 \quad 4 \quad 0]$, $[0 \quad 2 \quad 0 \quad 4]$; $[8 \quad 0 \quad 4]$, $[0 \quad 2 \quad 0]$
9. 3; $[9 \quad 0 \quad 1 \quad 0]$, $[0 \quad 9 \quad 8 \quad 9]$, $[0 \quad 0 \quad 1 \quad 0]$
11. **(c)** 1
17. No
19. Yes
21. No
23. Yes
25. Yes
27. 2, $[-2 \quad 0 \quad 1]$, $[0 \quad 2 \quad 1]$
29. No
31. No
33. 1, solution of the given system $c[1 \quad \frac{10}{3} \quad 3]$, basis $[1 \quad \frac{10}{3} \quad 3]$
35. 1, $[4 \quad 2 \quad \frac{4}{3} \quad 1]$

Problem Set 7.7, page 300

7. $\cos(\alpha + \beta)$
9. 1
11. 40
13. 289
15. -64
17. 2
19. 2
21. $x = 3.5, \quad y = -1.0$
23. $x = 0, \quad y = 4, \quad z = -1$
25. $w = 3, \quad x = 0, \quad y = 2, \quad z = -2$

Problem Set 7.8, page 308

1. $\begin{bmatrix} 1.20 & 4.64 \\ 0.50 & 3.60 \end{bmatrix}$

3. $\begin{bmatrix} 54 & 0.9 & -3.4 \\ 2 & 0.2 & -0.2 \\ -30 & -0.5 & 2 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}$

7. $\mathbf{A}^{-1} = \mathbf{A}$

9. $\begin{bmatrix} 0 & 0 & \frac{1}{2} \\ \frac{1}{8} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \end{bmatrix}$

11. $(\mathbf{A}^2)^{-1} = (\mathbf{A}^{-1})^2 = \begin{bmatrix} 3.760 & 22.272 \\ 2.400 & 15.280 \end{bmatrix}$

15. $\mathbf{AA}^{-1} = \mathbf{I}, \quad (\mathbf{AA}^{-1})^{-1} = (\mathbf{A}^{-1})^{-1}\mathbf{A}^{-1} = \mathbf{I}$. Multiply by $\mathbf{A}$ from the right.

Problem Set 7.9, page 318

1. $[1 \quad 0]^\mathsf{T}$, $[0 \quad 1]^\mathsf{T}$; $[1 \quad 0]^\mathsf{T}$, $[0 \quad -1]^\mathsf{T}$; $[1 \quad 1]^\mathsf{T}$, $[-1 \quad 1]^\mathsf{T}$
3. 1, $[1 \quad 11 \quad -7]^\mathsf{T}$
5. No
7. Dimension 2, basis xe^{-x}, e^{-x}
9. 3; basis $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$
11. $x_1 = 5y_1 - y_2, \quad x_2 = 3y_1 - y_2$
13. $x_1 = 2y_1 - 3y_2, \quad x_2 = -10y_1 + 16y_2 + y_3, \quad x_3 = -7y_1 + 11y_2 + y_3$

15. $\sqrt{26}$

17. $\sqrt{5}$

19. 1

21. $k = -20$

23. $\mathbf{a} = [3 \quad 1 \quad -4]^\mathsf{T}, \quad \mathbf{b} = [-4 \quad 8 \quad -1]^\mathsf{T}, \quad \|\mathbf{a} + \mathbf{b}\| = \sqrt{107} \leqq 5.099 + 9$

25. $\mathbf{a} = [5 \quad 3 \quad 2]^\mathsf{T}, \quad \mathbf{b} = [3 \quad 2 \quad -1]^\mathsf{T}, \quad 90 + 14 = 2(38 + 14)$

Chapter 7 Review Questions and Problems, page 318

11. $\begin{bmatrix} -1 & 6 & 1 \\ -18 & 8 & -7 \\ -13 & -2 & -7 \end{bmatrix}, \quad \begin{bmatrix} 1 & 18 & 13 \\ -6 & -8 & 2 \\ -1 & 7 & 7 \end{bmatrix}$

13. $[21 \quad -8 \quad -31]^\mathsf{T}, \quad [21 \quad -8 \quad 31]$

15. 197, 0

17. $-5, \quad \det \mathbf{A}^2 = (\det \mathbf{A})^2 = 25, \quad 0$

19. $\begin{bmatrix} -2 & -12 & -12 \\ -12 & 16 & -9 \\ -12 & -9 & -14 \end{bmatrix}$

21. $x = 4, \quad y = -2, \quad z = 8$

23. $x = 6, \quad y = 2t + 2, \quad z = t$ arb.

25. $x = 0.4, \quad y = -1.3, \quad z = 1.7$

27. $x = 10, \quad y = -2$

29. Ranks 2, 2, ∞

31. Ranks 2, 2, 1

33. $I_1 = 16.5$ A, $I_2 = 11$ A, $I_3 = 5.5$ A

35. $I_1 = 4$ A, $I_2 = 5$ A, $I_3 = 1$ A

Problem Set 8.1, page 329

1. $3, [1 \quad 0]^\mathsf{T}; \quad -0.6, [0 \quad 1]^\mathsf{T}$

3. $-4, [2 \quad 9]^\mathsf{T}; \quad 3, [1 \quad 1]^\mathsf{T}$

5. $-3i, [1 \quad -i]; \quad 3i, [1 \quad i], i = \sqrt{-1}$

7. $\lambda^2 = 0, \quad [1 \quad 0]^\mathsf{T}$

9. $0.8 + 0.6i, [1 \quad -i]^\mathsf{T}; \quad 0.8 - 0.6i, [1 \quad i]^\mathsf{T}$

11. $-(\lambda^3 - 18\lambda^2 + 99\lambda - 162)/(\lambda - 3) = -(\lambda^2 - 15\lambda + 54); \quad 3, [2 \quad -2 \quad 1]^\mathsf{T};$
$6, [1 \quad 2 \quad 2]^\mathsf{T}; \quad 9, [2 \quad 1 \quad -2]^\mathsf{T}$

13. $-(\lambda - 9)^3; \quad 9, [2 \quad -2 \quad 1]^\mathsf{T}$, defect 2

15. $(\lambda + 1)^2(\lambda^2 + 2\lambda - 15); \quad -1, [1 \quad 0 \quad 0 \quad 0]^\mathsf{T}, [0 \quad 1 \quad 0 \quad 0]^\mathsf{T};$
$-5, [-3 \quad -3 \quad 1 \quad 1]^\mathsf{T}, 3, [3 \quad -3 \quad 1 \quad -1]^\mathsf{T}$

17. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Eigenvalues $i, -i$. Corresponding eigenvectors are complex, indicating that no direction is preserved under a rotation.

19. $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}; \quad 1, \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad 0, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. A point onto the x_2-axis goes onto itself, a point on the x_1-axis onto the origin.

23. Use that real entries imply real coefficients of the characteristic polynomial.

Problem Set 8.2, page 333

1. 1.5, $[1 \quad -1]^T$, $-45°$; 4.5, $[1 \quad 1]^T$, $45°$
3. 1, $[-1/\sqrt{6} \quad 1]^T$, $112.2°$; 8, $[1 \quad 1/\sqrt{6}]^T$, $22.2°$
5. 0.5, $[1 \quad -1]^T$; 1.5, $[1 \quad 1]^T$; directions $-45°$ and $45°$
7. $[5 \quad 8]^T$
9. $[11 \quad 12 \quad 16]^T$
11. 1.8
13. $c[10 \quad 18 \quad 25]^T$
15. $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} = [0.6747 \quad 0.7128 \quad 0.7543]^T$
17. $\mathbf{A}\mathbf{x}_j = \lambda_j \mathbf{x}_j\ (\mathbf{x}_j \neq 0)$, $(\mathbf{A} - k\mathbf{I})\mathbf{x}_j = \lambda_j \mathbf{x}_j - k\mathbf{x}_j = (\lambda_j - k)\mathbf{x}_j$.
19. From $\mathbf{A}\mathbf{x}_j = \lambda_j \mathbf{x}_j\ (\mathbf{x}_j \neq \mathbf{0})$ and Prob. 18 follows $k_p \mathbf{A}^p \mathbf{x}_j = k_p \lambda_j^p \mathbf{x}_j$ and $k_q \mathbf{A}^q \mathbf{x}_j = k_q \lambda_j^q \mathbf{x}_j\ (p \geqq 0, q \geqq 0$, integer). Adding on both sides, we see that $k_p \mathbf{A}^p + k_q \mathbf{A}^q$ has the eigenvalue $k_p \lambda_j^p + k_q \lambda_j^q$. From this the statement follows.

Problem Set 8.3, page 338

1. $0.8 \pm 0.6i$, $[1 \quad \pm i]^T$; orthogonal
3. $2 \pm 0.8i$, $[1 \quad \pm i]$. Not skew–symmetric!
5. 1, $[0 \quad 2 \quad 1]^T$; 6, $[1 \quad 0 \quad 0]^T$, $[0 \quad 1 \quad -2]^T$; symmetric
7. 0, $\pm 25i$, skew–symmetric
9. 1, $[0 \quad 1 \quad 0]^T$; i, $[1 \quad 0 \quad i]^T$; $-i$, $[1 \quad 0 \quad -i]^T$, orthogonal
15. No
17. $\mathbf{A}^{-1} = (-\mathbf{A}^T)^{-1} = -(\mathbf{A}^{-1})^T$
19. No since $\det \mathbf{A} = \det(\mathbf{A}^T) = \det(-\mathbf{A}) = (-1)^3 \det(\mathbf{A}) = -\det(\mathbf{A}) = 0$.

Problem Set 8.4, page 345

1. $\begin{bmatrix} -25 & 12 \\ -50 & 25 \end{bmatrix}$, -5, $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$; 5, $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$; $\mathbf{x} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

3. $\begin{bmatrix} 3.008 & -0.544 \\ 5.456 & 6.992 \end{bmatrix}$, 4, $\begin{bmatrix} -17 \\ 31 \end{bmatrix}$; 6, $\begin{bmatrix} -2 \\ 11 \end{bmatrix}$; $\mathbf{x} = \begin{bmatrix} 25 \\ 25 \end{bmatrix}$, $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

5. $\begin{bmatrix} 4 & 3 & -9 \\ 0 & -5 & 15 \\ 0 & -5 & 15 \end{bmatrix}$, 0, $\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$; 4, $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; 10, $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$; $\mathbf{x} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

9. $\begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} \mathbf{A} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}$

11. $\begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix} \mathbf{A} \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \mathbf{A} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

15. $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \mathbf{A} \begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

17. $\mathbf{C} = \begin{bmatrix} 7 & 3 \\ 3 & 7 \end{bmatrix}$, $4y_1^2 + 10y_2^2 = 200$, $\mathbf{x} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$, ellipse

19. $\mathbf{C} = \begin{bmatrix} 3 & 11 \\ 11 & 3 \end{bmatrix}$, $14y_1^2 - 8y_2^2 = 0$, $\mathbf{x} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \mathbf{y}$; pair of straight lines

21. $\mathbf{C} = \begin{bmatrix} 1 & -6 \\ -6 & 1 \end{bmatrix}$, $7y_1^2 - 5y_2^2 = 70$, $\mathbf{x} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \mathbf{y}$, hyperbola

23. $\mathbf{C} = \begin{bmatrix} -11 & 42 \\ 42 & 24 \end{bmatrix}$, $52y_1^2 - 39y_2^2 = 156$, $\mathbf{x} = \frac{1}{\sqrt{13}} \begin{bmatrix} 2 & 3 \\ 3 & -2 \end{bmatrix} \mathbf{y}$, hyperbola

Problem Set 8.5, page 351

1. Hermitian, 5, $[-i \quad 1]^\mathsf{T}$, 7, $[i \quad 1]^\mathsf{T}$
3. Unitary, $(1 - i\sqrt{3})/2$, $[-1 \quad 1]^\mathsf{T}$; $(1 + i\sqrt{3})/2$, $[1 \quad 1]^\mathsf{T}$
5. Skew-Hermitian, unitary, $-i$, $[0 \quad -1 \quad 1]^\mathsf{T}$, i, $[1 \quad 0 \quad 0]^\mathsf{T}$, $[0 \quad 1 \quad 1]^\mathsf{T}$
7. Eigenvalues -1, 1; eigenvectors $[1 \quad -1]^\mathsf{T}$, $[1 \quad 1]^\mathsf{T}$; $[1 \quad -i]^\mathsf{T}$, $[1 \quad i]^\mathsf{T}$; $[0 \quad 1]^\mathsf{T}$, $[1 \quad 0]^\mathsf{T}$, resp.
9. Hermitian, 16
11. Skew-Hermitian, $-6i$
13. $\overline{(\mathbf{ABC})}^\mathsf{T} = \overline{\mathbf{C}}^\mathsf{T}\overline{\mathbf{B}}^\mathsf{T}\overline{\mathbf{A}}^\mathsf{T} = \mathbf{C}^{-1}(-\mathbf{B})\mathbf{A}$
15. $\mathbf{A} = \mathbf{H} + \mathbf{S}$, $\mathbf{H} = \frac{1}{2}(\mathbf{A} + \overline{\mathbf{A}}^\mathsf{T})$, $\mathbf{S} = \frac{1}{2}(\mathbf{A} - \overline{\mathbf{A}}^\mathsf{T})$ (**H** Hermitian, **S** skew-Hermitian)
19. $\mathbf{A}\overline{\mathbf{A}}^\mathsf{T} - \overline{\mathbf{A}}^\mathsf{T}\mathbf{A} = (\mathbf{H} + \mathbf{S})(\mathbf{H} - \mathbf{S}) - (\mathbf{H} - \mathbf{S})(\mathbf{H} + \mathbf{S}) = 2(-\mathbf{HS} + \mathbf{SH}) = \mathbf{0}$ if and only if $\mathbf{HS} = \mathbf{SH}$.

Chapter 8 Review Questions and Problems, page 352

11. 3, $[1 \quad 1]^\mathsf{T}$; 2, $[1 \quad -1]^\mathsf{T}$
13. 3, $[1 \quad 5]^\mathsf{T}$; 7, $[1 \quad 1]^\mathsf{T}$
15. 0, $[2 \quad -2 \quad 1]^\mathsf{T}$; $9i$, $[-1 + 3i \quad 1 + 3i \quad 4]^\mathsf{T}$; $-9i$, $[-1 - 3i \quad 1 - 3i \quad 4]^\mathsf{T}$
17. $-1, 1$; $\mathbf{A} = \frac{1}{16} \begin{bmatrix} 5 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 23 & 2 \\ 39 & 1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -1 & 1 \\ 63 & 1 \end{bmatrix}$

19. $\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \mathbf{A} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -0.9 & 0 \\ 0 & 0.6 \end{bmatrix}$

21. $\frac{1}{3}\begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \mathbf{A} \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -20 & 0 \\ 0 & 0 & 22 \end{bmatrix}$

23. $\mathbf{C} = \begin{bmatrix} 4 & 12 \\ 12 & -14 \end{bmatrix}$, $10y_1^2 - 20y_2^2 = 20$, $\mathbf{x} = \frac{1}{\sqrt{5}}\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}\mathbf{y}$, hyperbola

25. $\mathbf{C} = \begin{bmatrix} 3.7 & 1.6 \\ 1.6 & 1.3 \end{bmatrix}$, $4.5y_1^2 + 0.5y_2^2 = 4.5$, $\mathbf{x} = \frac{1}{\sqrt{5}}\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}\mathbf{y}$, ellipse

Problem Set 9.1, page 360

1. 5, 1, 0; $\sqrt{26}$; $[5/\sqrt{26}, 1/\sqrt{26}, 0]$
3. 8.5, −4.0, 1.7; $\sqrt{91.14}$, $[0.890, -0.419, 0.178]$
5. 2, 1, −2; $\mathbf{u} = [\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}]$, position vector of Q
7. Q: $(4, 0, \frac{1}{2})$, $|\mathbf{v}| = \sqrt{16.25}$
9. Q: $(0, 0, -8)$, $|\mathbf{v}| = 8$
11. $[6, 4, 0]$, $[\frac{3}{2}, 1, 0]$, $[-3, -2, 0]$
13. $[1, 5, 8]$
15. $7[9, -7, 8] = [63, -49, 56]$
17. $[12, 8, 0]$
21. $[4, 9, -3]$, $\sqrt{106}$
23. $[0, 0, 5]$, 5
25. $[6, 2, -14] = 2\mathbf{u}$, $\sqrt{236}$
27. $\mathbf{p} = [0, 0, -5]$
29. $\mathbf{v} = [v_1, v_2, 3]$, v_1, v_2 arbitrary
31. $k = 10$
33. $|\mathbf{p} + \mathbf{q} + \mathbf{u}| \leqq 18$. Nothing
35. $v_B - v_A = [-19, 0] - [22/\sqrt{2}, 22/\sqrt{2}] = [-19 - 22/\sqrt{2}, -22/\sqrt{2}]$
37. $\mathbf{u} + \mathbf{v} + \mathbf{p} = [-k, 0] + [l, l] + [0, -1000] = \mathbf{0}$, $-k + l + 0 = 0$,
$0 + l - 1000 = 0$, $l = 1000, k = 1000$

Problem Set 9.2, page 367

1. 44, 44, 0
3. $\sqrt{35}$, $\sqrt{320}$, $\sqrt{86}$
5. $|[2, 9, 9]| = \sqrt{166} = 12.88 < \sqrt{80} + \sqrt{86} = 18.22$
7. $|-24| = 24$, $|a||c| = \sqrt{35}\sqrt{86} = \sqrt{3010} = 54.86$; cf. (6)
9. 300; cf. (5a) and (5b)
13. Use (1) and $|\cos\gamma| \leqq 1$.
15. $|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = \mathbf{a} \cdot \mathbf{a} + 2\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{b} + (\mathbf{a} \cdot \mathbf{a} - 2\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{b})$
$= 2|\mathbf{a}|^2 + 2|\mathbf{b}|^2$
17. $[2, 5, 0] \cdot [2, 2, 2] = 14$
19. $[0, 4, 3] \cdot [-3, -2, 1] = -5$ is negative! Why?
21. Yes, because $W = (\mathbf{p} + \mathbf{q}) \cdot \mathbf{d} = \mathbf{p} \cdot \mathbf{d} + \mathbf{q} \cdot \mathbf{d}$.
23. arccos 0.5976 = 53.3°
27. $\beta - \alpha$ is the angle between the unit vectors **a** and **b**. Use (2).
29. $\gamma = \arccos(12/(6\sqrt{13})) = 0.9828 = 56.3°$ and 123.7°
31. $a_1 = -\frac{28}{3}$
33. $\pm[\frac{3}{5}, -\frac{4}{5}]$
35. $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = |\mathbf{a}|^2 - |\mathbf{b}|^2 = 0$, $|\mathbf{a}| = |\mathbf{b}|$. A square.
37. 0. Why?
39. If $|\mathbf{a}| = |\mathbf{b}|$ or if **a** and **b** are orthogonal.

Problem Set 9.3, page 374

5. $-\mathbf{m}$ instead of $\mathbf{m}$, tendency to rotate in the opposite sense.

7. $|\mathbf{v}| = |[0, 20, 0] \times [8, 6, 0]| = |[0, 0, -160]| = 160$

9. Zero volume in Fig. 191, which can happen in several ways.

11. $[0, 0, 7], \quad [0, 0, -7], \quad -4$

13. $[6, 2, 7], \quad [-6, -2, -7]$

15. $\mathbf{0}$

17. $[-32, -58, 34], \quad [-42, -63, 19]$

19. $1, \quad -1$

21. $[-48, -72, -168], \quad 12\sqrt{248} = 189.0, 189.0$

23. $0, \quad 0, \quad 13$

25. $\mathbf{m} = [-2, -2, 0] \times [2, 3, 0] = [0, 0, -10]$, $m = 10$ clockwise

27. $[6, 2, 0] \times [1, 2, 0] = [0, 0, 10]$

29. $\frac{1}{2}|[-12, 2, 6]| = \sqrt{46}$

31. $3x + 2y - z = 5$

33. $474/6 = 79$

Problem Set 9.4, page 380

1. Hyperbolas

3. Parallel straight lines (planes in space) $y = \frac{3}{4}x + c$

5. Circles, centers on the y-axis

7. Ellipses

9. Parallel planes

11. Elliptic cylinders

13. Paraboloids

Problem Set 9.5, page 390

1. Circle, center (3, 0), radius 2

3. Cubic parabola $x = 0, z = y^3$

5. Ellipse

7. Helix

9. A "Lissajous curve"

11. $\mathbf{r} = [3 + \sqrt{13}\cos t, 2 + \sqrt{13}\sin t, 1]$

13. $\mathbf{r} = [2 + t, 1 + 2t, 3]$

15. $\mathbf{r} = [t, 4t - 1, 5t]$

17. $\mathbf{r} = [\sqrt{2}\cos t, \sin t, \sin t]$

19. $\mathbf{r} = [\cosh t, (\sqrt{3}/2)\sinh t, -2]$

21. Use $\sin(-\alpha) = -\sin\alpha$.

25. $\mathbf{u} = [-\sin t, 0, \cos t]$. At P, $\mathbf{r}' = [-8, 0, 6]$. $\mathbf{q}(w) = [6 - 8w, i, 8 + 6w]$.

27. $\mathbf{q}(w) = [2 + w, \frac{1}{2} - \frac{1}{4}w, 0]$

29. $\sqrt{\mathbf{r}' \bullet \mathbf{r}'} = \cosh t$, $l = \sinh l = 1.175$

31. $\sqrt{\mathbf{r}' \bullet \mathbf{r}'} = a$, $l = a\pi/2$

33. Start from $\mathbf{r}(t) = [t, f(t)]$.

35. $\mathbf{v} = \mathbf{r}' = [1, 2t, 0], \quad |\mathbf{v}| = \sqrt{1 + 4t^2}, \quad \mathbf{a} = [0, 2, 0]$

37. $\mathbf{v}(0) = (\omega + 1)R\mathbf{i}$, $\mathbf{a}(0) = -\omega^2 R\mathbf{j}$

39. $\mathbf{v} = [-\sin t - 2\sin 2t, \cos t - 2\cos 2t], \quad |\mathbf{v}|^2 = 5 - 4\cos 3t,$

$$\mathbf{a} = [-\cos t - 4\cos 2t, -\sin t + 4\sin 2t], \text{ and } \mathbf{a}_{\tan} = \frac{6\sin 3t}{5 - 4\cos 3t}\mathbf{v}.$$

41. $\mathbf{v} = [-\sin t, 2\cos 2t, -2\sin 2t], \quad |\mathbf{v}|^2 = 4 + \sin^2 t,$

$$\mathbf{a} = [-\cos t, -4\sin 2t, -4\cos 2t], \text{ and } \mathbf{a}_{\tan} = \frac{\frac{1}{2}\sin 2t}{4 + \sin^2 t}\mathbf{v}.$$

43. 1 year $= 365 \cdot 86{,}400$ sec, $\quad R = 30 \cdot 365 \cdot 86{,}400/2\pi = 151 \cdot 10^6$ [km],
$|\mathbf{a}| = \omega^2 R = |\mathbf{v}|^2/R = 5.98 \cdot 10^{-6}$ [km/sec^2]

45. $R = 3960 + 80$ mi $= 2.133 \cdot 10^7$ ft, $\quad g = |\mathbf{a}| = \omega^2 R = |\mathbf{v}|^2/R, \quad |\mathbf{v}| = \sqrt{gR} =$
$\sqrt{6.61 \cdot 10^8} = 25{,}700$ [ft/sec] $= 17{,}500$ [mph]

49. $\mathbf{r}(t) = [t, y(t), 0], \quad \mathbf{r}' = [1, y', 0]$ $\mathbf{r} \bullet \mathbf{r}' = 1 + y'^2$, etc.

51. $\dfrac{d\mathbf{r}}{ds} = \dfrac{d\mathbf{r}}{dt}\Big/\dfrac{ds}{dt}, \qquad \dfrac{d^2\mathbf{r}}{ds^2} = \dfrac{d^2\mathbf{r}}{dt^2}\Big/\left(\dfrac{ds}{dt}\right)^2 + \cdots, \qquad \dfrac{d^3\mathbf{r}}{ds^3} = \dfrac{d^3\mathbf{r}}{dt^3}\Big/\left(\dfrac{ds}{dt}\right)^3 + \cdots$

53. $3/(1 + 9t^2 + 9t^4)$

Problem Set 9.7, page 402

1. $[2y - 1, 2x + 2]$
3. $[-y/x^2, 1/x]$
5. $[4x^3, 4y^3]$
7. Use the chain rule.
9. Apply the quotient rule to each component and collect terms.
11. $[y, x]$, $[5, -4]$
13. $[2x/(x^2 + y^2), 2y/(x^2 + y^2)]$, $[0.16, 0.12]$
15. $[8x, 18y, 2z]$, $[40, -18, -22]$
17. For P on the x- and y-axes.
19. $[-1.25, 0]$
21. $[0, -e]$
23. Points with $y = 0, \pm\pi, \pm 2\pi, \cdots$.
25. $-\nabla T(P) = [0, 4, -1]$
31. $\nabla f = [32x, -2y]$, $\nabla f(P) = [160, -2]$
33. $[12x, 4y, 2z]$, $[60, 20, 10]$
35. $[-2x, -2y, 1]$, $[-6, -8, 1]$
37. $[2, 1] \cdot [1, -1]/\sqrt{5} = 1/\sqrt{5}$
39. $[1, 1, 1] \cdot [-3/125, 0, -4/125]/\sqrt{3} = -7/(125\sqrt{3})$
41. $\sqrt{8/3}$
43. $f = xyz$
45. $f = \int v_1\,dx + \int v_2\,dy + \int v_3\,dz$

Problem Set 9.8, page 405

1. $2x + 8y + 18z$; 7
3. 0, after simplification; solenoidal
5. $9x^2y^2z^2$; 1296
7. $-2e^x(\cos y)z$
9. **(b)** $(fv_1)_x + (fv_2)_y + (fv_3)_z = f[(v_1)_x + (v_2)_y + (v_3)_z] + f_xv_1 + f_yv_2 + f_zv_3$, etc.
11. $[v_1, v_2, v_3] = \mathbf{r}' = [x', y', z'] = [y, 0, 0]$, $z' = 0, z = c_3$, $y' = 0, y = c_2$, and $x' = y = c_2, x = c_2t + c_1$. Hence as t increases from 0 to 1, this "shear flow" transforms the cube into a parallelepiped of volume 1.
13. $\operatorname{div}(\mathbf{w} \times \mathbf{r}) = 0$ because v_1, v_2, v_3 do not depend on x, y, z, respectively.
15. $-2\cos 2x + 2\cos 2y$
17. 0
19. $2/(x^2 + y^2 + z^2)^2$

Problem Set 9.9, page 408

3. Use the definitions and direct calculation.
5. $[x(z^2 - y^2), y(x^2 - z^2), z(y^2 - x^2)]$
7. $e^{-x}[\cos y, \sin y, 0]$
9. curl $\mathbf{v} = [-6z, 0, 0]$ incompressible, $\mathbf{v} = \mathbf{r}' = [x', y', z'] = [0, 3z^2, 0]$, $x = c_1$, $z = c_3$, $y' = 3z^2 = 3c_3^2$, $y = 3c_3^2t + c_2$
11. curl $\mathbf{v} = [0, 0, -3]$, incompressible, $x' = y$, $y' = -2x$, $2xx' + yy' = 0$, $x^2 + \frac{1}{2}y^2 = c$, $z = c_3$
13. curl $\mathbf{v} = 0$, irrotational, div $\mathbf{v} = 1$, compressible, $\mathbf{r} = [c_1e^t, c_2e^t, c_3e^{-t}]$. Sketch it.
15. $[-1, -1, -1]$, same (why?)
17. $-yz - zx - xy$, 0 (why?), $-y - z - x$
19. $[-2z - y, -2x - z, -2y - x]$, same (why?)

Chapter 9 Review Questions and Problems, page 409

11. $-10, \quad 1080, \quad 1080, \quad 65$
13. $[-10, -30, 0], \quad [10, 30, 0], \quad \mathbf{0}, \quad 40$
15. $[-1260, -1830, -300], \quad [-210, 120, -540]$, undefined
17. $-125, \quad 125, \quad -125$
19. $[70, -40, -50], \quad 0, \quad \sqrt{35^2 + 20^2 + 25^2} = \sqrt{2250}$
21. $[-2, -6, -13]$
23. $\gamma_1 = \arccos(-10/\sqrt{65 \cdot 40}) = 1.7682 = -101.3°, \gamma_2 = 23.7°$
25. $[5, 2, 0] \bullet [4 - 1, 3 - 1, 0] = 19$
27. $\mathbf{v} \bullet \mathbf{w}/|\mathbf{w}| = 22/\sqrt{8} = 7.78$
29. $[0, 0, -14]$, tendency of clockwise rotation
31. 4
33. $1, \quad -2y$
35. 0, same (why?), $2(y^2 + x^2 - xz)$
37. $[0, -2, 0]$
39. $9/\sqrt{225} = \frac{3}{5}$

Problem Set 10.1, page 418

3. 4
5. $\mathbf{r} = [2\cos t, \quad 2\sin t], \quad 0 \leqq t \leqq \pi/2; \quad \frac{8}{5}$
7. "Exponential helix," $(e^{6\pi} - 1)/3$
9. $23.5, \quad 0$
11. $2e^{-t} + 2te^{-t^2}, \quad -2e^{-2} - e^{-4} + 3$
15. $18\pi, \quad \frac{4}{3}(4\pi)^3, \quad 18\pi$
17. $[4\cos t, \quad + \sin t, \quad \sin t, \quad 4\cos t], \quad [2, 2, 0]$
19. $144t^4, \quad 1843.2$

Problem Set 10.2, page 425

3. $\sin \frac{1}{2}x \cos 2y, \quad 1 - 1/\sqrt{2} = 0.293$
5. $e^{xy} \sin z, \quad e - 0$
7. $\cosh 1 - 2 = -0.457$
9. $e^x \cosh y + e^z \sinh y, \quad e - (\cosh 1 + \sinh 1) = 0$
13. $e^{a^2} \cos 2b$
15. Dependent, $x^2 \neq -4y^2$, etc.
17. Dependent, $4 \neq 0$, etc.
19. $\sin(a^2 + 2b^2 + c^2)$

Problem Set 10.3, page 432

3. $8y^3/3, \quad 54$
5. $\int_0^1 [x - x^3 - (x^2 - x^5)]\, dx = \dfrac{1}{12}$
7. $\cosh 2x - \cosh x, \quad \frac{1}{2}\sinh 4 - \sinh 2$
9. $36 + 27y^2, \quad 144$
11. $z = 1 - r^2, \quad dx\, dy = r\, dr\, d\theta, \quad$ *Answer:* $\pi/2$
13. $\bar{x} = 2b/3, \quad \bar{y} = h/3$
15. $\bar{x} = 0, \quad \bar{y} = 4r/3\pi$
17. $I_x = bh^3/12, \quad I_y = b^3h/4$
19. $I_x = (a + b)h^3/24, \quad I_y = h(a^4 - b^4)/(48(a - b))$

Problem Set 10.4, page 438

1. $(-1 - 1) \cdot \pi/4 = -\pi/2$
3. $9(e^2 - 1) - \frac{8}{3}(e^3 - 1)$
5. $2x - 2y, \quad 2x(1 - x^2) - (2 - x^2)^2 + 1, \quad x = -1 \cdots 1, \quad -\frac{56}{15}$
7. 0. Why?
9. $\frac{16}{5}$
13. $\nabla^2 w = \cosh x, \quad y = x/2 \cdots 2, \quad \frac{1}{2}\cosh 4 - \frac{1}{2}$

15. $\nabla^2 w = 6xy, \quad 3x(10 - x^2)^2 - 3x, \quad 486$ **17.** $\nabla^2 w = 6x - 6y, \quad -38.4$
19. $|\text{grad } w|^2 = e^{2x}, \quad \frac{5}{2}(e^4 - 1)$

Problem Set 10.5, page 442

1. Straight lines, $\mathbf{k}$
3. $z = c\sqrt{x^2 + y^2}$, circles, straight lines, $[-cu \cos v, \quad -cu \sin v, \quad u]$
5. $z = x^2 + y^2$, circles, parabolas, $[-2u^2 \cos v, \quad -2u^2 \sin v, \quad u]$
7. $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1, \quad [bc \cos^2 v \cos u, \quad ac \cos^2 v \sin u, \quad ab \sin v \cos v]$, ellipses
11. $[\tilde{u}, \quad \tilde{v}, \quad \tilde{u}^2, \quad + \tilde{v}^2], \quad \tilde{\mathbf{N}} = [-2\tilde{u}, \quad -2\tilde{v}, \quad 1]$
13. Set $x = u$ and $y = v$.
15. $[2 + 5 \cos u, \quad -1 + 5 \sin u, \quad v], \quad [5 \cos u, \quad 5 \sin u, \quad 0]$
17. $[a \cos v \cos u, \quad -2.8 + a \cos v \sin u, \quad 3.2 + a \sin v], \quad a = 1.5;$
$[a^2 \cos^2 v \cos u, \quad a^2 \cos^2 v \sin u, \quad a^2 \cos v \sin v]$
19. $[\cosh u, \quad \sinh u, \quad v], \quad [\cosh u, \quad -\sinh u, \quad 0]$

Problem Set 10.6, page 450

1. $\mathbf{F}(\mathbf{r}) \bullet \mathbf{N} = [-u^2, \quad v^2, \quad 0] \bullet [-3, \quad 2, \quad 1] = 3u^2 + 2v^2, \quad 29.5$
3. $\mathbf{F}(\mathbf{r}) \bullet \mathbf{N} = \cos^3 v \cos u \sin u$ from (3), Sec. 10.5. *Answer*: $\frac{1}{3}$
5. $\mathbf{F}(\mathbf{r}) \bullet \mathbf{N} = -u^3, -128\pi$
7. $\mathbf{F} \bullet \mathbf{N} = [0, \quad \sin u, \quad \cos v] \bullet [1, -2u, 0], \quad 4 + (-2 + \pi^2/16 - \pi/2)\sqrt{2} = -0.1775$
9. $\mathbf{r} = [2 \cos u, \quad 2 \sin u, \quad v], \quad 0 \leqq u \leqq \pi/4, \quad 0 \leqq v \leqq 5$. Integrate $2 \sinh v \sin u$ to get $2(1 - 1/\sqrt{2})(\cosh 5 - 1) = 42.885$.
13. $7\pi^3/\sqrt{6} = 88.6$
15. $G(\mathbf{r}) = (1 + 9u^4)^{3/2}, \quad |\mathbf{N}| = (1 + 9u^4)^{1/2}$. *Answer*: 54.4
21. $I_{x=y} = \iint_S [\frac{1}{2}(x - y)^2 + z^2]\, \sigma\, dA$
23. $[u \cos v, \quad u \sin v, \quad u], \quad \int_0^{2\pi} \int_0^h u^2 \cdot u\sqrt{2}\, du\, dv = \frac{\pi}{\sqrt{2}} h^4$
25. $[\cos u \cos v, \quad \cos u \sin v, \quad \sin u], \quad dA = (\cos u)\, du\, dv$, B the z-axis, $\quad I_B = 8\pi/3,$
$I_K = I_B + 1^2 \cdot 4\pi = 20.9.$

Problem Set 10.7, page 457

1. 224
3. $-e^{-1-z} + e^{-y-z}, \quad -2e^{-1-z} + e^{-z}, \quad 2e^{-3} - e^{-2} - 2e^{-1} + 1$
5. $\frac{1}{2}(\sin 2x)\,(1 - \cos 2x), \quad \frac{1}{8}, \quad \frac{3}{4}$
7. $[r \cos u \cos v, \quad \cos u \sin v, \quad r \sin u], \quad dV = r^2 \cos u\, dr\, du\, dv, \quad \sigma = v, \quad 2\pi^2 a^3/3$
9. div $\mathbf{F} = 2x + 2z$, 48
11. $12(e - 1/e) = 24 \sinh 1$
13. div $\mathbf{F} = -\sin z$, 0
15. $1/\pi + \frac{5}{24} = 0.5266$
17. $h^4\pi/2$
19. $8abc(b^2 + c^2)/3$
21. $(a^4/4) \cdot 2\pi \cdot h = ha^4\pi/2$
23. $h^5\pi/10$
25. Do Prob. 20 as the last one.

Problem Set 10.8, page 462

1. $x = 0, y = 0, z = 0$, no contributions. $x = a$: $\partial f/\partial n = \partial f/\partial x = -2x = -2a$, etc. Integrals $x = a$: $(-2a)bc$, $y = b$: $(-2b)ac$, $z = c$: $(4c)\,ab$. Sum 0

3. The volume integral of $8y^2 + [0, 8y] \cdot [2x, 0] = 8y^2$ is $8y^3/3 = \frac{8}{3}$. The surface integral of $f\partial g/\partial n = f \cdot 2x = 2f = 8y^2$ over $x = 1$ is $8y^3/3 = \frac{8}{3}$. Others 0.

5. The volume integral of $6y^2 \cdot 4 - 2x^2 \cdot 12$ is 0; $8(x = 1)$, $-8(y = 1)$, others 0.

7. $\mathbf{F} = [x, \ 0, \ 0]$, $\operatorname{div} \mathbf{F} = 1$, use (2*), Sec. 10.7, etc.

9. $z = 0$ and $z = \sqrt{a^2 - x^2 - y^2} = \sqrt{a^2 - r^2}$, $dx\,dy = r\,dr\,d\theta$, $-2\pi \cdot \frac{1}{2}(a^2 - r^2)^{3/2} \cdot \frac{2}{3}\Big|_0^a = \frac{2}{3}\pi a^3$

11. $r = a$, $\phi = 0$, $\cos\phi = 1$, $v = \frac{1}{3}a \cdot (4\pi a^2)$

Problem Set 10.9, page 468

1. S: $z = y\ (0 \leqq x \leqq 1, 0 \leqq y \leqq 4)$, $[0, 2z, -2z] \cdot [0, -1, 1]$, ± 20

3. $[2e^{-z}\cos y, \ -e^{-z}, \ 0] \cdot [0, \ -y, \ 1] = ye^{-z}$, $\pm(2 - 2/\sqrt{e})$

5. $[0, 2z, \frac{3}{2}] \cdot [0, 0, 1] = \frac{3}{2}$, $\pm\frac{3}{2}a^2$

7. $[-e^z, \ -e^x, \ -e^y] \cdot [-2x, \ 0, \ 1]$, $\pm(e^4 - 2e + 1)$

9. The sides contribute a, $3a^2/2$, $-a$, 0.

11. -2π; curl $\mathbf{F} = \mathbf{0}$

13. $5\mathbf{k}$, 80π

15. $[0, -1, 2x - 2y] \cdot [0, 0, 1]$, $\frac{1}{3}$

17. $\mathbf{r} = [\cos u, \ \sin u, \ v]$, $[-3v^2, \ 0, \ 0] \cdot [\cos u, \ \sin u, \ 0]$, -1

19. $\mathbf{r} = [u\cos v, \ u\sin v, \ u]$, $0 \leqq u \leqq 1$, $0 \leqq v \leqq \pi/2$, $[-e^z, \ 1, \ 0] \cdot [-u\cos v, \ -u\sin v, \ u]$. *Answer*: 1/2

Chapter 10 Review Questions and Problems, page 469

11. $\mathbf{r} = [4 - 10t, \ 2 + 8t]$, $\mathbf{F}(\mathbf{r}) \cdot d\mathbf{r} = [2(4 - 10t)^2, \ -4(2t + 8t)^2] \cdot [-10, \ 8]\,dt$; $-4528/3$. Or using exactness.

13. Not exact, curl $\mathbf{F} = (5\cos x)\mathbf{k}$, ± 10

15. 0 since curl $\mathbf{F} = \mathbf{0}$

17. By Stokes, $\pm 18\pi$

19. $\mathbf{F} = \operatorname{grad}(y^2 + xz)$, 2π

21. $M = 8$, $\bar{x} = \frac{8}{5}$, $\bar{y} = \frac{16}{5}$

23. $M = \frac{63}{20}$, $\bar{x} = \frac{8}{7} = 1.14$, $\bar{y} = \frac{118}{49} = 2.41$

25. $M = 4k/15$, $\bar{x} = \frac{5}{16}$, $\bar{y} = \frac{4}{7}$

27. $288(a + b + c)\pi$

29. div $\mathbf{F} = 20 + 6z^2$. *Answer*: 21

31. $24 \sinh 1 = 28.205$

33. Direct integration, $\frac{224}{3}$

35. 72π

Problem Set 11.1, page 482

1. $2\pi, 2\pi, \pi, \pi, 1, 1, \frac{1}{2}, \frac{1}{2}$

5. There is no *smallest* $p > 0$.

13. $\dfrac{4}{\pi}\left(\cos x + \dfrac{1}{9}\cos 3x + \dfrac{1}{25}\cos 5x + \cdots\right) + 2\left(\sin x + \dfrac{1}{3}\sin 3x + \dfrac{1}{5}\sin 5x + \cdots\right)$

15. $\frac{4}{3}\pi^2 + 4\left(\cos x + \frac{1}{4}\cos 2x + \frac{1}{9}\cos 3x + \cdots\right) - 4\pi\left(\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right)$

17. $\dfrac{\pi}{2} + \dfrac{4}{\pi}\left(\cos x + \dfrac{1}{9}\cos 3x + \dfrac{1}{25}\cos 5x + \cdots\right)$

19. $\frac{\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right) + \sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - + \cdots$

21. $2(\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \frac{1}{4}\sin 4x + \frac{1}{5}\sin 5x + \cdots)$

Problem Set 11.2, page 490

1. Neither, even, odd, odd, neither **3.** Even **5.** Even

9. Odd, $L = 2$, $\frac{4}{\pi}\left(\sin\frac{\pi x}{2} + \frac{1}{3}\sin\frac{3\pi x}{2} + \frac{1}{5}\sin\frac{5\pi x}{2} + \cdots\right)$

11. Even, $L = 1$, $\frac{1}{3} - \frac{4}{\pi^2}\left(\cos \pi x - \frac{1}{4}\cos 2\pi x + \frac{1}{9}\cos 3\pi x - + \cdots\right)$

13. Rectifier, $L = \frac{1}{2}$, $\frac{1}{8} - \frac{1}{\pi^2}\left(\cos 2\pi x + \frac{1}{9}\cos 6\pi x + \frac{1}{25}\cos 10\pi x + \cdots\right) + \frac{1}{\pi}\left(\frac{1}{2}\sin 2\pi x - \frac{1}{4}\sin 4\pi x + \frac{1}{6}\sin 6\pi x - \frac{1}{8}\sin 8\pi x + - \cdots\right)$

15. Odd, $L = \pi$, $\frac{4}{\pi}\left(\sin x - \frac{1}{9}\sin 3x + \frac{1}{25}\sin 5x - + \cdots\right)$

17. Even, $L = 1$, $\frac{1}{2} + \frac{4}{\pi^2}\left(\cos \pi x + \frac{1}{9}\cos 3\pi x + \frac{1}{25}\cos 5\pi x + \cdots\right)$

19. $\frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$

23. $L = 4$, **(a)** 1, **(b)** $\frac{4}{\pi}\left(\sin\frac{\pi x}{4} + \frac{1}{3}\sin\frac{3\pi x}{4} + \frac{1}{5}\sin\frac{5\pi x}{4} + \cdots\right)$

25. $L = \pi$, **(a)** $\frac{\pi}{2} + \frac{4}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right)$,
(b) $2(\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \frac{1}{4}\sin 4x + \cdots)$

27. $L = \pi$, **(a)** $\frac{3\pi}{8} + \frac{2}{\pi}\left(\cos x - \frac{1}{2}\cos 2x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x - \frac{1}{18}\cos 6x + \frac{1}{49}\cos 7x + \frac{1}{81}\cos 9x - \frac{1}{50}\cos 10x + \frac{1}{121}\cos 11x + \cdots\right)$
(b) $\left(1 + \frac{2}{\pi}\right)\sin x + \frac{1}{2}\sin 2x + \left(\frac{1}{3} - \frac{2}{9\pi}\right)\sin 3x + \frac{1}{4}\sin 4x + \left(\frac{1}{5} + \frac{2}{25\pi}\right)\sin 5x + \frac{1}{6}\sin 6x + \cdots$

29. Rectifier, $L = \pi$,
(a) $\frac{2}{\pi} - \frac{4}{\pi}\left(\frac{1}{1\cdot 3}\cos x + \frac{1}{3\cdot 5}\cos 3x + \frac{1}{5\cdot 7}\cos 5x + \cdots\right)$, **(b)** $\sin x$

Problem Set 11.3, page 494

3. The output becomes a pure cosine series.

5. For A_n this is similar to Fig. 54 in Sec. 2.8, whereas for the phase shift B_n the sense is the same for all n.

7. $y = C_1 \cos \omega t + C_2 \sin \omega t + a(\omega) \sin t, \quad a(\omega) = 1/(\omega^2 - 1) = -1.33,$ $-5.26, 4.76, 0.8, 0.01$. Note the change of sign.

11. $y = C_1 \cos \omega t + C_2 \sin \omega t + \dfrac{4}{\pi}\left(\dfrac{1}{\omega^2 - 9} \sin t + \dfrac{1}{\omega^2 - 49} \sin 3t + \dfrac{1}{\omega^2 - 121} \sin 5t + \cdots\right)$

13. $y = \sum_{n=1}^{N} (A_n \cos nt + B_n \sin nt), \quad A_n = [(1 - n^2)a_n - nb_n c]/D_n,$ $B_n = [(1 - n^2)b_n + nca_n]/D_n, \quad D_n = (1 - n^2)^2 + n^2c^2$

15. $b_n = (-1)^{n+1} \cdot 12/n^3$ (n odd), $\quad y = \sum_{n=1}^{\infty} (A_n \cos nt + B_n \sin nt),$ $A_n = (-1)^n \cdot 12nc/n^3D_n, \quad B_n = (-1)^{n+1} \cdot 12(1 - n^2)/(n^3D_n)$ with D_n as in Prob. 13.

17. $I = 50 + A_1 \cos t + B_1 \sin t + A_3 \cos 3t + B_3 \sin 3t + \cdots, A_n = (10 - n^2)a_n/D_n,$ $B_n = 10na_n/D_n, \quad a_n = -400/(n^2\pi), D_n = (n^2 - 10)^2 + 100n^2$

19. $I(t) = \sum_{n=1}^{\infty} (A_n \cos nt + B_n \sin nt), \quad A_n = (-1)^{n+1} \dfrac{2400(10 - n^2)}{n^2D_n},$ $B_n = (-1)^{n+1} \dfrac{24{,}000}{nD_n}, \quad D_n = (10 - n^2)^2 + 100n^2$

Section 11.4, page 498

3. $F = \dfrac{\pi}{2} - \dfrac{4}{\pi}\left(\cos x + \dfrac{1}{9} \cos 3x + \dfrac{1}{25} \cos 5x + \cdots\right), E^* = 0.0748,$ $0.0748, 0.0119, 0.0119, 0.0037$

5. $F = \dfrac{4}{\pi}\left(\sin x + \dfrac{1}{3} \sin 3x + \dfrac{1}{5} \sin 5x + \cdots\right), E^* = 1.1902, 1.1902, 0.6243, 0.6243,$ 0.4206 $\quad$ (0.1272 when $N = 20$)

7. $F = 2[(\pi^2 - 6) \sin x - \frac{1}{8}(4\pi^2 - 6) \sin 2x + \frac{1}{27}(9\pi^2 - 6) \sin 3x - + \cdots];$ $E^* = 674.8, 454.7, 336.4, 265.6, 219.0$. Why is E^* so large?

Section 11.5, page 503

3. Set $x = ct + k$.

5. $x = \cos \theta, dx = -\sin \theta \, d\theta$, etc.

7. $\lambda_m = (m\pi/10)^2, m = 1, 2, \cdots; y_m = \sin(m\pi x/10)$

9. $\lambda = [(2m + 1)\pi/(2L)]^2, m = 0, 1, \cdots, y_m = \sin((2m + 1)\pi x/(2L))$

11. $\lambda_m = m^2, m = 1, 2, \cdots, y_m = x \sin(m \ln |x|)$

13. $p = e^{8x}, q = 0, r = e^{8x}, \lambda_m = m^2, y_m = e^{-4x} \sin mx, m = 1, 2, \cdots$

Section 11.6, page 509

1. $8(P_1(x) - P_3(x) + P_5(x))$

3. $\frac{4}{5}P_0(x) - \frac{4}{7}P_2(x) - \frac{8}{35}P_4(x)$

9. $-0.4775P_1(x) - 0.6908P_3(x) + 1.844P_5(x) - 0.8236P_7(x) + 0.1658P_9(x) + \cdots,$ $m_0 = 9$. ***Rounding*** seems to have considerable influence in Probs. 8–13.

11. $0.7854P_0(x) - 0.3540P_2(x) + 0.0830P_4(x) - \cdots, m_0 = 4$
13. $0.1212P_0(x) - 0.7955P_2(x) + 0.9600P_4(x) - 0.3360P_6(x) + \cdots, m_0 = 8$
15. (c) $a_m = (2/J_1^2(\alpha_{0,m}))(J_1(\alpha_{0,m})/\alpha_{0,m}) = 2/(\alpha_{0,m}J_1(\alpha_{0,m}))$

Section 11.7, page 517

1. $f(x) = \pi e^{-x}(x > 0)$ gives $A = \displaystyle\int_0^\infty e^{-v} \cos wv\, dv = \frac{1}{1 + w^2}$, $B = \dfrac{w}{1 + w^2}$ (see Example 3), etc.

3. Use (11); $B = \displaystyle\frac{2}{\pi}\int_0^\infty \frac{\pi}{2} \sin wv\, dv = \frac{1 - \cos \pi w}{w}$

5. $B(w) = \displaystyle\frac{2}{\pi}\int_0^1 \frac{1}{2}\pi v \sin wv\, dv = \frac{\sin w - w \cos w}{w^2}$

7. $\displaystyle\frac{2}{\pi}\int_0^\infty \frac{\sin w \cos xw}{w}\, dw$

9. $A(w) = \displaystyle\frac{2}{\pi}\int_0^\infty \frac{\cos wv}{1 + v^2}\, dv = e^{-w}\,(w > 0)$

11. $\displaystyle\frac{2}{\pi}\int_0^\infty \frac{\cos \pi w + 1}{1 - w^2} \cos xw\, dw$

15. For $n = 1, 2, 11, 12, 31, 32, 49, 50$ the value of $\mathrm{Si}(n\pi) - \pi/2$ equals $0.28, -0.15, 0.029, -0.026, 0.0103, -0.0099, 0.0065, -0.0064$ (rounded).

17. $\displaystyle\frac{2}{\pi}\int_0^\infty \frac{1 - \cos w}{w} \sin xw\, dw$

19. $\displaystyle\frac{2}{\pi}\int_0^\infty \frac{w - e(w \cos w - \sin w)}{1 + w^2} \sin xw\, dw$

Section 11.8, page 522

1. $\hat{f}_c(w) = \sqrt{(2/\pi)}\,(2 \sin w - \sin 2w)/w$
3. $\hat{f}_c(w) = \sqrt{(2/\pi)}\,(\cos 2w + 2w \sin 2w - 1)/w^2$
5. $\hat{f}_c(w) = \sqrt{\dfrac{2}{\pi}}\,\dfrac{(w^2 - 2) \sin w + 2w \cos w}{w^3}$
7. Yes. No
9. $\sqrt{2/\pi}\, w/(a^2 + w^2)$
11. $\sqrt{2/\pi}\,((2 - w^2) \cos w + 2w \sin w - 2)/w^3$
13. $\mathscr{F}_s(e^{-x}) = \dfrac{1}{w}\left(-\mathscr{F}_c(e^{-x}) + \sqrt{\dfrac{2}{\pi}} \cdot 1\right) = \dfrac{1}{w}\left(\sqrt{\dfrac{2}{\pi}} \cdot \dfrac{1}{w^2 + 1} + \sqrt{\dfrac{2}{\pi}}\right) = \sqrt{\dfrac{2}{\pi}}\,\dfrac{w}{w^2 + 1}$

Problem Set 11.9, page 533

3. $i(e^{-ibw} - e^{-iaw})/(w\sqrt{2\pi})$ if $a < b$; 0 otherwise
5. $[e^{(1-iw)a} - e^{-(1-iw)a}]/(\sqrt{2\pi}(1 - iw))$
7. $(e^{-iaw}(1 + iaw) - 1)/(\sqrt{2\pi}w^2)$
9. $\sqrt{2/\pi}(\cos w + w \sin w - 1)/w^2$
11. $i\sqrt{2/\pi}\,(\cos w - 1)/w$
13. $e^{-w^2/2}$ by formula 9

17. No, the assumptions in Theorem 3 are not satisfied.

19. $[f_1 + f_2 + f_3 + f_4,\quad f_1 - if_2 - f_3 + if_4,\quad f_1 - f_2 + f_3 - f_4,\quad f_1 + if_2 - f_3 - if_4]$

21. $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} f_1 + f_2 \\ f_1 - f_2 \end{bmatrix}$

Chapter 11 Review Questions and Problems, page 537

11. $1 + \dfrac{4}{\pi}\left(\sin\dfrac{\pi x}{2} + \dfrac{1}{3}\sin\dfrac{3\pi x}{2} + \dfrac{1}{5}\sin\dfrac{5\pi x}{2} + \cdots\right)$

13. $\dfrac{1}{4} - \dfrac{2}{\pi^2}\left(\cos \pi x + \dfrac{1}{9}\cos 3\pi x + \dfrac{1}{25}\cos 5\pi x + \cdots\right) +$

$\dfrac{1}{\pi}\left(\sin \pi x - \dfrac{1}{2}\sin 2\pi x + \dfrac{1}{3}\sin 3\pi x - + \cdots\right)$

15. $\cosh x$, $\sinh x$ $(-5 < x < 5)$, respectively

17. Cf. Sec. 11.1.

19. $\dfrac{1}{2} - \dfrac{4}{\pi^2}\left(\cos \pi x + \dfrac{1}{9}\cos 3\pi x + \cdots\right),\quad \dfrac{2}{\pi}\left(\sin \pi x - \dfrac{1}{2}\sin 2\pi x + - \cdots\right)$

21. $y = C_1 \cos \omega t + C_2 \sin \omega t + \dfrac{\pi^2}{\omega^2} - 12\left(\dfrac{\cos t}{\omega^2 - 1} - \dfrac{1}{4}\cdot\dfrac{\cos 2t}{\omega^2 - 4} + \dfrac{1}{9}\cdot\dfrac{\cos 3t}{\omega^2 - 9}\right.$

$\left.- \dfrac{1}{16}\cdot\dfrac{\cos 4t}{\omega^2 - 16} + - \cdots\right)$

23. 0.82, 0.50, 0.36, 0.28, 0.23

25. 0.0076, 0.0076, 0.0012, 0.0012, 0.0004

27. $\dfrac{1}{\pi}\displaystyle\int_0^\infty \dfrac{(\cos w + w \sin w - 1)\cos wx + (\sin w - w \cos w)\sin wx}{w^2}\, dw$

29. $\sqrt{2/\pi}\,(\cos aw - \cos w + aw \sin aw - w \sin w)/w^2$

Problem Set 12.1, page 542

1. $L(c_1u_1 + c_2u_2) = c_1L(u_1) + c_2L(u_2) = c_1 \cdot 0 + c_2 \cdot 0 = 0$

3. $c = 2$

5. $c = a/b$

7. Any c and ω

9. $c = \pi/25$

15. $u = 110 - (110/\ln 100)\ln(x^2 + y^2)$

17. $u = a(y)\cos 4\pi x + b(y)\sin 4\pi x$

19. $u = c(x)\, e^{-y^3/3}$

21. $u = e^{-3y}(a(x)\cos 2y + b(x)\sin 2y) + 0.1e^{3y}$

23. $u = c_1(y)x + c_2(y)/x^2$ (Euler–Cauchy)

25. $u(x, y) = axy + bx + cy + k$; a, b, c, k arbitrary constants

Problem Set 12.3, page 551

5. $k \cos 3\pi t \sin 3\pi x$

7. $\dfrac{8k}{\pi^3}\left(\cos \pi t \sin \pi x + \dfrac{1}{27}\cos 3\pi t \sin 3\pi x + \dfrac{1}{125}\cos 5\pi t \sin 5\pi x + \cdots\right)$

9. $\dfrac{0.8}{\pi^2}\left(\cos \pi t \sin \pi x - \dfrac{1}{9}\cos 3\pi t \sin 3\pi x + \dfrac{1}{25}\cos 5\pi t \sin 5\pi x - + \cdots\right)$

11. $\dfrac{2}{\pi^2}\Big((2 - \sqrt{2}) \cos \pi t \sin \pi x - \dfrac{1}{9}(2 + \sqrt{2}) \cos 3\pi t \sin 3\pi x$

$+ \dfrac{1}{25}(2 + \sqrt{2}) \cos 5\pi t \sin 5\pi x - + \cdots\Big)$

13. $\dfrac{4}{\pi^3}\Big((4 - \pi) \cos \pi t \sin \pi x + \cos 2\pi t \sin 2\pi x + \dfrac{4 + 3\pi}{27} \cos 3\pi t \sin 3\pi x$

$+ \dfrac{4 - 5\pi}{125} \cos 5\pi t \sin 5\pi x + \cdots\Big)$. No terms with $n = 4, 8, 12, \cdots$.

17. $u = \dfrac{8L^2}{\pi^3}\left(\cos\left[c\left(\dfrac{\pi}{L}\right)^2 t\right] \sin \dfrac{\pi x}{L} + \dfrac{1}{3^3} \cos\left[c\left(\dfrac{3\pi}{L}\right)^2 t\right] \sin \dfrac{3\pi x}{L} + \cdots\right)$

19. **(a)** $u(0, t) = 0$, **(b)** $u(L, t) = 0$, **(c)** $u_x(0, t) = 0$, **(d)** $u_x(L, t) = 0$. $C = -A$, $D = -B$ from (a), (c). Insert this. The coefficient determinant resulting from (b), (d) must be zero to have a nontrivial solution. This gives (22).

Problem Set 12.4, page 556

3. $c^2 = 300/[0.9/(2 \cdot 9.80)] = 80.83^2$ [m^2/sec^2]

9. Elliptic, $u = f_1(y + 2ix) + f_2(y - 2ix)$

11. Parabolic, $u = xf_1(x - y) + f_2(x - y)$

13. Hyperbolic, $u = f_1(y - 4x) + f_2(y - x)$

15. Hyperbolic, $xy'^2 + yy' = 0$, $y = v$, $xy = w$, $u_w = z$, $u = \dfrac{1}{y} f_1(xy) + f_2(y)$

17. Elliptic, $u = f_1(y - (2 - i)x) + f_2(y - (2 + i)x)$. Real or imaginary parts of any function u of this form are solutions. Why?

Problem Set 12.6, page 566

3. $u_1 = \sin x\, e^{-t}$, $u_2 = \sin 2x\, e^{-4t}$, $u_3 = \sin 3x\, e^{-9t}$ differ in rapidity of decay.

5. $u = \sin 0.1\pi x\, e^{-1.752\pi^2 t/100}$

7. $u = \dfrac{800}{\pi^3}\left(\sin 0.1\pi x\, e^{-0.01752\pi^2 t} + \dfrac{1}{3^3} \sin 0.3\pi x\, e^{-0.01752(3\pi)^2 t} + \cdots\right)$

9. $u = u_{\mathrm{I}} + u_{\mathrm{II}}$, where $u_{\mathrm{II}} = u - u_{\mathrm{I}}$ satisfies the boundary conditions of the text,

so that $u_{\mathrm{II}} = \displaystyle\sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{L}\, e^{-(cn\pi/L)^2 t}$, $B_n = \dfrac{2}{L}\displaystyle\int_0^L [f(x) - u_{\mathrm{I}}(x)] \sin \frac{n\pi x}{L}\, dx$.

11. $F = A \cos px + B \sin px$, $F'(0) = Bp = 0$, $B = 0$, $F'(L) = -Ap \sin pL = 0$, $p = n\pi/L$, etc.

13. $u = 1$

15. $\dfrac{1}{2} + \dfrac{4}{\pi^2}\left(\cos x\, e^{-t} + \dfrac{1}{9} \cos 3x\, e^{-9t} + \dfrac{1}{25} \cos 5x\, e^{-25t} + \cdots\right)$

17. $-\dfrac{K\pi}{L} \displaystyle\sum_{n=1}^{\infty} nB_n\, e^{-\lambda_n^2 t}$

19. $u = 1000 (\sin \frac{1}{2}\pi x \sinh \frac{1}{2}\pi y)/\sinh \pi$

21. $u = \dfrac{100}{\pi} \displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n - 1) \sinh (2n - 1)\pi} \sin \frac{(2n - 1)\pi x}{24} \sinh \frac{(2n - 1)\pi y}{24}$

23. $u = A_0 x + \sum_{n=1}^{\infty} A_n \frac{\sinh(n\pi x/24)}{\sinh n\pi} \cos\frac{n\pi y}{24},$

$$A_0 = \frac{1}{24^2}\int_0^{24} f(y)\,dy, \quad A_n = \frac{1}{12}\int_0^{24} f(y)\cos\frac{n\pi y}{24}\,dy$$

25. $\sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{a}\sinh\frac{n\pi(b-y)}{a}, A_n = \frac{2}{a\sinh(n\pi b/a)}\int_0^a f(x)\sin\frac{n\pi x}{a}\,dx$

Problem Set 12.7, page 574

3. $A = \frac{2}{\pi}\int_0^{\infty}\frac{\cos pv}{1+v^2}\,dv = \frac{2}{\pi}\cdot\frac{\pi}{2}e^{-p}, u = \int_0^{\infty} e^{-p-c^2p^2t}\cos px\,dp$

5. $A = \frac{2}{\pi}\int_0^1 v\cos pv\,dv = \frac{2}{\pi}\cdot\frac{\cos p + p\sin p - 1}{p^2}$, etc.

7. $A = \frac{2}{\pi}\int_0^{\infty}\frac{\sin v}{v}\cos pv\,dv = \frac{2}{\pi}\cdot\frac{\pi}{2} = 1$ if $0 < p < 1$ and 0 if $p > 1$,

$$u = \int_0^1 \cos px\, e^{-c^2p^2t}\,dp$$

9. Set $w = -v$ in (21) to get $\operatorname{erf}(-x) = -\operatorname{erf} x$.

13. In (12) the argument $x + 2cz\sqrt{t}$ is 0 (the point where f jumps) when $z = -x/(2c\sqrt{t})$. This gives the lower limit of integration.

15. Set $w = s/\sqrt{2}$ in (21).

Problem Set 12.9, page 584

1. **(a)**, **(b)** It is multiplied by $\sqrt{2}$. **(c)** Half

5. $B_{mn} = (-1)^{n+1}8/(mn\pi^2)$ if m odd, 0 if m even

7. $B_{mn} = (-1)^{m+n}4ab/(mn\pi^2)$

11. $u = 0.1\cos\sqrt{20}t\sin 2x\sin 4y$

13. $\frac{6.4}{\pi^2}\sum_{\substack{m=1\\ m,n}}^{\infty}\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\frac{1}{m^3n^3}\cos(t\sqrt{m^2+n^2})\sin mx\sin ny$

17. $c\pi\sqrt{260}$ (corresponding eigenfunctions $F_{4,16}$ and $F_{16,14}$), etc.

19. $\cos\left(\pi t\sqrt{\frac{36}{a^2}+\frac{4}{b^2}}\right)\sin\frac{6\pi x}{a}\sin\frac{4\pi y}{b}$

Problem Set 12.10, page 591

5. $110 + \frac{440}{\pi}\left(r\cos\theta - \frac{1}{3}r^3\cos 3\theta + \frac{1}{5}r^5\cos 5\theta - + \cdots\right)$

7. $55\pi - \frac{440}{\pi}\left(r\cos\theta + \frac{1}{9}r^3\cos 3\theta + \frac{1}{25}r^5\cos 5\theta + \cdots\right)$

11. Solve the problem in the disk $r < a$ subject to u_0 (given) on the upper semicircle and $-u_0$ on the lower semicircle.

$$u = \frac{4u_0}{\pi}\left(\frac{r}{a}\sin\theta + \frac{1}{3a^3}r^3\sin 3\theta + \frac{1}{5a^5}r^5\sin 5\theta + \cdots\right)$$

13. Increase by a factor $\sqrt{2}$

15. $T = 6.826\rho R^2 f_1^2$

17. No

25. $\alpha_{11}/(2\pi) = 0.6098$; See Table A1 in App. 5.

Problem Set 12.11, page 598

5. $A_4 = A_6 = A_8 = A_{10} = 0, \quad A_5 = 605/16, \quad A_7 = -4125/128, \quad A_9 = 7315/256$

9. $\nabla^2 u = u'' + 2u'/r = 0, \quad u''/u' = -2/r, \quad \ln|u'| = -2\ln|r| + c_1,$
$u' = \tilde{c}/r^2$, $u = c/r + k$

13. $u = 320/r + 60$ is smaller than the potential in Prob. 12 for $2 < r < 4$.

17. $u = 1$

19. $\cos 2\phi = 2\cos^2\phi - 1$, $2w^2 - 1 = \frac{4}{3}P_2(w) - \frac{1}{3}$, $u = \frac{4}{3}r^2 P_2(\cos\phi) - \frac{1}{3}$

25. Set $1/r = \rho$. Then $u(\rho, \theta, \phi) = rv(r, \theta, \phi)$, $u_\rho = (v + rv_r)(-1/\rho^2)$,
$u_{\rho\rho} = (2v_r + rv_{rr})(1/\rho^4) + (v + rv_r)(2/\rho^3)$, $u_{\rho\rho} + (2/\rho)u_\rho = r^5(v_{rr} + (2/r)v_r)$.
Substitute this and $u_{\phi\phi} = rv_{\phi\phi}$ etc. into (7) [written in terms of ρ] and divide by r^5.

Problem Set 12.12, page 602

5. $W = \dfrac{c(s)}{x^s} + \dfrac{x}{s^2(s+1)}$, $W(0, s) = 0$, $c(s) = 0$, $w(x, t) = x(t - 1 + e^{-t})$

7. $w = f(x)g(t)$, $xf'g + f\dot{g} = xt$, take $f(x) = x$ to get $g = ce^{-t} + t - 1$ and $c = 1$ from $w(x, 0) = x(c - 1) = 0$.

11. Set $x^2/(4c^2\tau) = z^2$. Use z as a new variable of integration. Use $\operatorname{erf}(\infty) = 1$.

Chapter 12 Review Questions and Problems, page 603

17. $u = c_1(x)e^{-3y} + c_2(x)e^{2y} - 3$

19. Hyperbolic, $f_1(x) + f_2(y + x)$

21. Hyperbolic, $f_1(y + 2x) + f_2(y - 2x)$

23. $\frac{3}{4}\cos 2t \sin x - \frac{1}{4}\cos 6t \sin 3x$

25. $\sin 0.01\pi x\, e^{-0.001143t}$

27. $\frac{3}{4}\sin 0.01\pi x\, e^{-0.001143t} - \frac{1}{4}\sin 0.03\pi x\, e^{-0.01029t}$

29. $100\cos 2x\, e^{-4t}$

39. $u = (u_1 - u_0)(\ln r)/\ln(r_1/r_0) + (u_0 \ln r_1 - u_1 \ln r_0)/\ln(r_1/r_0)$

Problem Set 13.1, page 612

1. $1/i = i/i^2 = -i, \quad 1/i^3 = i/i^4 = i$

3. $4.8 - 1.4i$

5. $x - iy = -(x + iy), \quad x = 0$

9. $-117, \quad 4$

11. $-8 - 6i$

13. $-120 - 40i$

15. $3 - i$

17. $-4x^2y^2$

19. $(x^2 - y^2)/(x^2 + y^2), \quad 2xy/(x^2 + y^2)$

Problem Set 13.2, page 618

1. $\sqrt{2}\,(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi)$

3. $2(\cos\frac{1}{2}\pi + i\sin\frac{1}{2}\pi), \quad 2(\cos\frac{1}{2}\pi - i\sin\frac{1}{2}\pi)$

5. $\frac{1}{2}(\cos \pi + i \sin \pi)$

7. $\sqrt{1 + \frac{1}{4}\pi^2}\,(\cos \arctan \frac{1}{2}\pi + i \sin \arctan \frac{1}{2}\pi)$

9. $3\pi/4$

11. $\pm\arctan(\frac{4}{3}) = \pm 0.9273$

13. -1024. *Answer:* π

15. $-3i$

17. $2 + 2i$

21. $\sqrt[6]{2}\,(\cos \frac{1}{12}k\pi + i \sin \frac{1}{12}\pi), \quad k = 1, 9, 17$

23. $6, \quad -3 \pm 3\sqrt{3}i$

25. $\cos(\frac{1}{8}\pi + \frac{1}{2}k\pi) + i \sin(\frac{1}{8}\pi + \frac{1}{2}k\pi), \quad k = 0, 1, 2, 3$

27. $\cos \frac{1}{5}\pi \pm i \sin \frac{1}{5}\pi, \quad \cos \frac{3}{5}\pi \pm i \sin \frac{3}{5}\pi, \quad -1$

29. $i, \quad -1 - i$

31. $\pm(1 - i), \quad \pm(2 + 2i)$

33. $|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)} = (z_1 + z_2)(\bar{z}_1 + \bar{z}_2)$. Multiply out and use $\operatorname{Re} z_1\bar{z}_2 \leqq |z_1\bar{z}_2|$ (Prob. 34).
$z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2 = |z_1|^2 + 2\operatorname{Re} z_1\bar{z}_2 + |z_2|^2 \leqq |z_1|^2 + 2|z_1||z_2| + |z_2|^2 = (|z_1| + |z_2|)^2$. Hence $|z_1 + z_1|^2 \leqq (|z_1| + |z_2|)^2$. Taking square roots gives (6).

35. $[(x_1 + x_2)^2 + (y_1 + y_2)^2] + [(x_1 - x_2)^2 + (y_1 - y_2)^2] = 2(x_1^2 + y_1^2 + x_2^2 + y_2^2)$

Problem Set 13.3, page 624

1. Closed disk, center $-1 + 5i$, radius $\frac{3}{2}$

3. Annulus (circular ring), center $4 - 2i$, radii π and 3π

5. Domain between the bisecting straight lines of the first quadrant and the fourth quadrant.

7. Half-plane extending from the vertical straight line $x = -1$ to the right.

11. $u(x, y) = (1 - x)/((1 - x)^2 + y^2), \quad u(1, -1) = 0,$
$v(x, y) = y((1 - x)^2 + y^2), \quad v(1, -1) = -1$

15. Yes, since $\operatorname{Im}(|z|^2/z) = \operatorname{Im}(|z|^2\bar{z}/(z\bar{z})) = \operatorname{Im}\bar{z} = -r\sin\theta \to 0$.

17. Yes, because $\operatorname{Re} z = r\cos\theta \to 0$ and $1 - |z| \to 1$ as $r \to 0$.

19. $f'(z) = 8(z - 4i)^7$. Now $z - 4i = 3$, hence $f'(3 + 4i) = 8 \cdot 3^7 = 17{,}496$.

21. $n(1 - z)^{-n-1}i, \quad ni$

23. $3iz^2/(z + i)^4, \quad -3i/16$

Problem Set 13.4, page 629

1. $r_x = x/r = \cos\theta, \quad r_y = \sin\theta, \quad \theta_x = -(\sin\theta)/r, \quad \theta_y = (\cos\theta)/r$
(a) $0 = u_x - v_y = u_r\cos\theta + u_\theta(-\sin\theta)/r - v_r\sin\theta - v_\theta(\cos\theta)/r$
(b) $0 = u_y + v_x = u_r\sin\theta + u_\theta(\cos\theta)/r + v_r\cos\theta + v_\theta(-\sin\theta)/r$
Multiply (a) by $\cos\theta$, (b) by $\sin\theta$, and add. Etc.

3. Yes

5. No, $f(z) = \overline{(z^2)}$

7. Yes, when $z \neq 0$. Use (7).

9. Yes, when $z \neq 0, \quad -2\pi i, \quad 2\pi i$

11. Yes

13. $f(z) = -\frac{1}{2}i(z^2 + c)$, c real

15. $f(z) = 1/z + c$ (c real)

17. $f(z) = z^2 + z + c$ (c real)

19. No

21. $a = \pi, \quad v = e^{\pi x}\sin\pi y$

23. $a = 0, \quad v = \frac{1}{2}b(y^2 - x^2) + c$

27. $f = u + iv$ implies $if = -v + iu$.

29. Use (4), (5), and (1).

Problem Set 13.5, page 632

3. $e^{2\pi i}e^{-2\pi} = e^{-2\pi} = 0.001867$

5. $e^2(-1) = -7.389$

7. $e^{\sqrt{2}}i = 4.113i$

9. $5e^{i\arctan(3/4)} = 5e^{0.644i}$

11. $6.3e^{\pi i}$

13. $\sqrt{2}e^{\pi i/4}$

15. $\exp(x^2 - y^2)\cos 2xy, \quad \exp(x^2 - y^2)\sin 2xy$
17. $\text{Re}(\exp(z^3)) = \exp(x^3 - 3xy^2)\cos(3x^2y - y^3)$
19. $z = 2n\pi i, \quad n = 0, 1, \cdots$

Problem Set 13.6, page 636

1. Use (11), then (5) for e^{iy}, and simplify. **7.** $\cosh 1 = 1.543$, $i \sinh 1 = 1.175i$
9. Both $-0.642 - 1.069i$. Why? **11.** $i \sinh \pi = 11.55i$, both
15. Insert the definitions on the left, multiply out, and simplify.
17. $z = \pm(2n + 1)i/2$ **19.** $z = \pm n\pi i$

Problem Set 13.7, page 640

5. $\ln 11 + \pi i$ **7.** $\frac{1}{2}\ln 32 - \pi i/4 = 1.733 - 0.785i$
9. $i \arctan(0.8/0.6) = 0.927i$ **11.** $\ln e + \pi i/2 = 1 + \pi i/2$
13. $\pm 2n\pi i, \quad n = 0, 1, \cdots$
15. $\ln |e^i| + i \arctan \dfrac{\sin 1}{\cos 1} \pm 2n\pi i = 0 + i + 2n\pi i, \quad n = 0, 1, \cdots$
17. $\ln(i^2) = \ln(-1) = (1 \pm 2n)\pi i, \quad 2 \ln i = (1 \pm 4n)\pi i, n = 0, 1, \cdots$
19. $e^{4-3i} = e^4(\cos 3 - i \sin 3) = -54.05 - 7.70i$
21. $e^{0.6}e^{0.4i} = e^{0.6}(\cos 0.4 + i \sin 0.4) = 1.678 + 0.710i$
23. $e^{(1-i)\,\text{Ln}\,(1+i)} = e^{\ln\sqrt{2} + \pi i/4 - i\ln\sqrt{2} + \pi/4} = 2.8079 + 1.3179i$
25. $e^{(3-i)(\ln 3 + \pi i)} = 27e^{\pi}(\cos(3\pi - \ln 3) + i \sin(3\pi - \ln 3)) = -284.2 + 556.4i$
27. $e^{(2-i)\,\text{Ln}(-1)} = e^{(2-i)\pi i} = e^{\pi} = 23.14$

Chapter 13 Review Questions and Problems, page 641

1. $2 - 3i$ **3.** $27.46e^{0.9929i}, \quad 7.616e^{1.976i}$
11. $-5 + 12i$ **13.** $0.16 - 0.12i$
15. i **17.** $4\sqrt{2}e^{-3\pi i/4}$
19. $15e^{-\pi i/2}$ **21.** $\pm 3, \quad \pm 3i$
23. $(\pm 1 \pm i)/\sqrt{2}$ **25.** $f(z) = -iz^2/2$
27. $f(z) = e^{-2z}$ **29.** $f(z) = e^{-z^2/2}$
31. $\cos 3 \cosh 1 + i \sin 3 \sinh 1 = -1.528 + 0.166i$
33. $i \tanh 1 = 0.7616i$
35. $\cosh \pi \cos \pi + i \sinh \pi \sin \pi = -11.592$

Problem Set 14.1, page 651

1. Straight segment from (2, 1) to (5, 2.5).
3. Parabola $y = x^2$ from (1, 2) to (2, 8).
5. Circle through (0, 0), center $(3, -1)$, radius $\sqrt{10}$, oriented clockwise.
7. Semicircle, center 2, radius 4.
9. Cubic parabola $y = x^3 \quad (-2 \leqq x \leqq 2)$
11. $z(t) = t + (2 + t)i \quad (-1 \leqq t \leqq 1)$
13. $z(t) = 2 - i + 2e^{it} \quad (0 \leqq t \leqq \pi)$

15. $z(t) = 2\cosh t + i\sinh t\ (-\infty < t < \infty)$
17. Circle $z(t) = -a - ib + re^{-it} \quad (0 \leqq t \leqq 2\pi)$
19. $z(t) = t + (1 - \frac{1}{4}t^2)i \quad (-2 \leqq t \leqq 2)$
21. $z(t) = (1 + i)t \quad (1 \leqq t \leqq 3),\quad \text{Re}\, z = t,\quad z'(t) = 1 + i$. *Answer:* $4 + 4i$
23. $e^{2\pi i} - e^{\pi i} = 1 - (-1) = 2$
25. $\frac{1}{2}\exp z^2\big|_1^i = \frac{1}{2}(e^{-1} - e^1) = -\sinh 1$
27. $\tan\frac{1}{4}\pi i - \tan\frac{1}{4} = i\tanh\frac{1}{4} - 1$
29. $\text{Im}\, z^2 = 2xy = 0$ on the axes. $z = 1 + (-1 + i)t \quad (0 \leqq t \leqq 1)$, $(\text{Im}\, z^2)\,\dot{z} = 2(1 - t)y(-1 + i)$ integrated: $(-1 + i)/3$.
35. $|\text{Re}\, z| = |x| \leqq 3 = M$ on C, $L = \sqrt{8}$

Problem Set 14.2, page 659

1. Use (12), Sec. 14.1, with $m = 2$. **3.** Yes **5.** 5
7. **(a)** Yes. **(b)** No, we would have to move the contour across $\pm 2i$.
9. 0, yes
11. πi, no
13. 0, yes
15. $-\pi$, no
17. 0, no
19. 0, yes
21. $2\pi i$
23. $1/z + 1/(z - 1)$, hence $2\pi i + 2\pi i = 4\pi i$.
25. 0 (Why?)
27. 0 (Why?)
29. 0

Problem Set 14.3, page 663

1. $2\pi i z^2/(z - 1)\big|_{z=-1} = -\pi i$
3. 0
5. $2\pi i(\cos 3z)/6\big|_{z=0} = \pi i/3$
7. $2\pi i(i/2)^3/2 = \pi/8$
11. $2\pi i \cdot \left.\dfrac{1}{z + 2i}\right|_{z=2i} = \dfrac{\pi}{2}$
13. $2\pi i(z + 2)\big|_{z=2} = 8\pi i$
15. $2\pi i\cosh(-\pi^2 - \pi i) = -2\pi i\cosh\pi^2 = -60{,}739i$ since $\cosh\pi i = \cos\pi = -1$ and $\sinh\pi i = i\sin\pi = 0$.
17. $2\pi i \left.\dfrac{\text{Ln}(z + 1)}{z + i}\right|_{z=i} = 2\pi i\dfrac{\text{Ln}(1 + i)}{2i} = \pi(\ln\sqrt{2} + i\pi/4) = 1.089 + 2.467i$
19. $2\pi i e^{2i}/(2i) = \pi e^{2i}$

Problem Set 14.4, page 667

1. $(2\pi i/3!)(-\cos 0) = -\pi i/3$
3. $(2\pi i/(n - 1)!)e^0$
5. $\dfrac{2\pi i}{3!}(\cosh 2z)''' = \dfrac{\pi i}{3} \cdot 8\sinh 1 = 9.845i$
7. $(2\pi i/(2n)!)\,(\cos z)^{(2n)}\big|_{z=0} = (2\pi i/(2n)!)(-1)^n\cos 0 = (-1)^n 2\pi i/(2n)!$
9. $-2\pi i(\tan\pi z)'\Big|_{z=0} = \left.\dfrac{-2\pi i \cdot \pi}{\cos^2\pi z}\right|_{z=0} = -2\pi^2 i$
11.
$$\frac{2\pi i}{4}((1 + z)\sin z)'\Big|_{z=1/2} = \tfrac{1}{2}\pi i(\sin z + (1 + z)\cos z)\big|_{z=1/2} = \tfrac{1}{2}\pi i(\sin\tfrac{1}{2} + \tfrac{3}{2}\cos\tfrac{1}{2}) = 2.821i$$

13. $2\pi i \cdot \left.\dfrac{1}{z}\right|_{z=2} = \pi i$

15. 0. Why?

17. 0 by Cauchy's integral theorem for a doubly connected domain; see (6) in Sec. 14.2.

19. $(2\pi i/2!)4^{-3}(e^{3z})''|_{z=\pi i/4} = -9\pi(1+i)/(64\sqrt{2})$

Chapter 14 Review Questions and Problems, page 668

21. $\frac{1}{2}\cosh(-\frac{1}{4}\pi^2) - \frac{1}{2} = 2.469$

23. $2\pi i(e^z)^{(4)}|_{z=0} = ie^z/12|_{z=0} = \pi i/12$ by Cauchy's integral formula.

25. $-2\pi i(\tan \pi z)'|_{z=1} = -2\pi^2 i/\cos^2 \pi z|_{z=1} = -2\pi^2 i$

27. 0 since $z^2 + \bar{z} - 2 = 2(x^2 - y^2)$ and $y = x$

29. $-4\pi i$

Problem Set 15.1, page 679

1. $z_n = (2i/2)^n$; bounded, divergent, ± 1, $\pm i$

3. $z_n = -\frac{1}{2}\pi i/(1 + 2/(ni))$ by algebra; convergent to $-\pi i/2$

5. Bounded, divergent, $\pm 1 + 10i$

7. Unbounded, hence divergent

9. Convergent to 0, hence bounded

17. Divergent; use $1/\ln n > 1/n$.

19. Convergent; use $\Sigma 1/n^2$.

21. Convergent

23. Convergent

25. Divergent

29. By absolute convergence and Cauchy's convergence principle, for given $\epsilon > 0$ we have for every $n > N(\epsilon)$ and $p = 1, 2, \cdots$

$$|z_{n+1}| + \cdots + |z_{n+p}| < \epsilon,$$

hence $|z_{n+1} + \cdots + z_{n+p}| < \epsilon$ by (6*), Sec. 13.2, hence convergence by Cauchy's principle.

Problem Set 15.2, page 684

1. No! Nonnegative integer powers of z (or $z - z_0$) only!

3. At the center, in a disk, in the whole plane

5. $\Sigma a_n z^{2n} = \Sigma a_n (z^2)^n$, $|z^2| < R = \lim |a_n/a_{n+1}|$; hence $|z| < \sqrt{R}$.

7. $\pi/2, \infty$

9. $i, \sqrt{3}$

11. $0, \sqrt{\frac{26}{5}}$

13. $-i, \frac{1}{2}$

15. $2i$, 1

17. $1/\sqrt{2}$

Problem Set 15.3, page 689

3. $f = \sqrt[n]{n}$. Apply l'Hôpital's rule to $\ln f = (\ln n)/n$.

5. 2

7. $\sqrt{3}$

9. $1/\sqrt{2}$

11. $\sqrt{\frac{7}{3}}$

13. 1

15. $\frac{3}{4}$

Problem Set 15.4, page 697

3. $2z^2 - \dfrac{(2z^2)^3}{3!} + \cdots = 2z^2 - \dfrac{4}{3}z^6 + \dfrac{4}{15}z^{10} - + \cdots, \quad R = \infty$

5. $\frac{1}{2} - \frac{1}{4}z^4 + \frac{1}{8}z^8 - \frac{1}{16}z^{12} + \frac{1}{32}z^{16} - + \cdots, \quad R = \sqrt[4]{2}$

7. $\dfrac{1}{2} + \dfrac{1}{2}\cos z = 1 - \dfrac{1}{2 \cdot 2!}z^2 + \dfrac{1}{2 \cdot 4!}z^4 - \dfrac{1}{2 \cdot 6!}z^6 + - \cdots, \quad R = \infty$

9. $\displaystyle\int_0^z \left(1 - \frac{1}{2}t^2 + \frac{1}{8}t^4 - + \cdots\right) dt = z - \frac{1}{6}z^3 + \frac{1}{40}z^5 - + \cdots, \quad R = \infty$

11. $z^3/(1!3) - z^7/(3!7) + z^{11}/(5!11) - + \cdots, \quad R = \infty$

13. $(2/\sqrt{\pi})(z - z^3/3 + z^5/(2!5) - z^7/(3!7) + \cdots), \quad R = \infty$

17. Team Project. (a) $(\text{Ln}(1 + z))' = 1 - z + z^2 - + \cdots = 1/(1 + z)$.

(c) Use that the terms of $(\sin iy)/(iy)$ are all positive, so that the sum cannot be zero.

19. $\frac{1}{2} + \frac{1}{2}i + \frac{1}{2}i(z - i) + (-\frac{1}{4} + \frac{1}{4}i)(z - i)^2 - \frac{1}{4}(z - i)^3 + \cdots, \quad R = \sqrt{2}$

21. $1 - \dfrac{1}{2!}\left(z - \dfrac{1}{2}\pi\right)^2 + \dfrac{1}{4!}\left(z - \dfrac{1}{2}\pi\right)^4 - \dfrac{1}{6!}\left(z - \dfrac{1}{2}\pi\right)^6 + - \cdots, \quad R = \infty$

23. $-\frac{1}{4} - \frac{2}{8}i(z - i) + \frac{3}{16}(z - i)^2 + \frac{4}{32}i(z - i)^3 - \frac{5}{64}(z - i)^4 + \cdots, \quad R = 2$

25. $2\left(z - \dfrac{1}{2}i\right) + \dfrac{2^3}{3!}\left(z - \dfrac{1}{2}i\right)^3 + \dfrac{2^5}{5!}\left(z - \dfrac{1}{2}i\right)^5 + \cdots, \quad R = \infty$

Problem Set 15.5, page 704

3. $|z + i| \leqq \sqrt{3} - \delta, \quad \delta > 0$

5. $|z + \frac{1}{2}i| \leqq \frac{1}{4} - \delta, \quad \delta > 0$

7. Nowhere

9. $|z - 2i| \leqq 2 - \delta, \quad \delta > 0$

11. $|z^n| \leqq 1$ and $\Sigma 1/n^2$ converges. Use Theorem 5.

13. $|\sin^n |z|| \leqq 1$ for all z, and $\Sigma 1/n^2$ converges. Use Theorem 5.

15. $R = 4$ by Theorem 2 in Sec. 15.2; use Theorem 1.

17. $R = 1/\sqrt{\pi} > 0.56$; use Theorem 1.

Chapter 15 Review Questions and Problems, page 706

11. 1

13. 3

15. $\frac{1}{2}$

17. $\infty, \quad e^{2z}$

19. $\infty, \quad \cosh \sqrt{z}$

21. $\displaystyle\sum_{n=0}^{\infty} \frac{z^{4n}}{(2n + 1)!}, \quad R = \infty$

23. $\displaystyle\frac{1}{2} + \frac{1}{2}\cos 2z = 1 + \frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}(2z)^{2n}, \quad R = \infty$

25. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} z^{2n-2}, \quad R = \infty$

27. $\cos[(z - \frac{1}{2}\pi) + \frac{1}{2}\pi] = -(z - \frac{1}{2}\pi) + \frac{1}{6}(z - \frac{1}{2}\pi)^3 - + \cdots = -\sin(z - \frac{1}{2}\pi)$

29. $\ln 3 + \dfrac{1}{3}(z - 3) - \dfrac{1}{2 \cdot 9}(z - 3)^2 + \dfrac{1}{3 \cdot 27}(z - 3)^3 - + \cdots, \quad R = 3$

Problem Set 16.1, page 714

1. $z^{-4} - \frac{1}{2}z^{-2} + \frac{1}{24} - \frac{1}{720}z^2 + - \cdots, \quad 0 < |z| < \infty$
3. $z^{-3} + z^{-1} + \frac{1}{2}z + \frac{1}{6}z^3 + \frac{1}{24}z^5 + \cdots, \quad 0 < |z| < \infty$
5. $z^{-2} + z^{-1} + 1 + z + z^2 + \cdots, \quad 0 < |z| < 1$
7. $z^3 + \frac{1}{2}z + \frac{1}{24}z^{-1} + \frac{1}{720}z^3 + \cdots, \quad 0 < |z| < \infty$
9. $\exp[1 + (z-1)](z-1)^{-2} = e \cdot [(z-1)^{-2} + (z-1)^{-1} + \frac{1}{2} + \frac{1}{6}(z-1) + \cdots],$
$0 < |z-1| < \infty$
11. $\dfrac{[\pi i + (z-\pi i)]^2}{(z-\pi i)^4} = \dfrac{(\pi i)^2}{(z-\pi i)^4} + \dfrac{2\pi i}{(z-\pi i)^3} + \dfrac{1}{(z-\pi i)^2}$
13. $i^{-3}\left(1 + \dfrac{z-i}{i}\right)^{-3}(z-i)^{-2} = \displaystyle\sum_{n=0}^{\infty}\binom{-3}{n} i^{-3-n}(z-i)^{n-2} = i(z-i)^{-2}$
$-3(z-i)^{-1} - 6i + 10(z-i) + \cdots, \quad 0 < |z-i| < 1$
15. $(-\cos(z-\pi))(z-\pi)^{-2} = -(z-\pi)^{-2} + \frac{1}{2} - \frac{1}{24}(z-\pi)^2 + - \cdots,$
$0 < |z-\pi| < \infty$
19. $\displaystyle\sum_{n=0}^{\infty} z^{2n}, \quad |z| < 1, \qquad -\sum_{n=0}^{\infty}\frac{1}{z^{2n+2}}, \quad |z| > 1$
21. $-(z + \frac{1}{2}\pi)^{-1}\cos(z + \frac{1}{2}\pi) = -(z + \frac{1}{2}\pi)^{-1} + \frac{1}{2}(z + \frac{1}{2}\pi) - \frac{1}{24}(z + \frac{1}{2}\pi)^3 + - \cdots,$
$|z + \frac{1}{2}\pi| > 0$
23. $z^8 + z^{12} + z^{16} + \cdots, \quad |z| < 1, \quad -z^4 - 1 - z^{-4} - z^{-8} - \cdots, \quad |z| > 1$
25. $\dfrac{i}{(z-i)^2} + \dfrac{1}{z-i} + i + (z-i)$

Section 16.2, page 719

1. $0 \pm 2\pi, \pm 4\pi, \cdots$, fourth order
3. $-81i$, fourth order
5. $\pm 1, \pm 2, \cdots$, second order
7. $\pm(2 + 2i), \pm i$, simple
9. $\frac{1}{2}\sin 4z$, $z = 0, \pm\pi/4, \pm\pi/2, \cdots$, simple
11. $f(z) = (z - z_0)^n g(z)$, $g(z_0) \neq 0$, hence $f^2(z) = (z - z_0)^{2n} g^2(z)$.
13. Second-order poles at i and $-2i$
15. Simple pole at ∞, essential singularity at $1 + i$
17. Fourth-order poles at $\pm n\pi i$, $n = 0, 1, \cdots$, essential singularity at ∞
19. $e^z(1 - e^z) = 0$, $e^z = 1$, $z = \pm 2n\pi i$ simple zeros. *Answer:* simple poles at $\pm 2n\pi i$, essential singularity at ∞
21. $1, \infty$ essential singularities, $\pm 2n\pi i$, $n = 0, 1, \cdots$, simple poles

Section 16.3, page 725

3. $\frac{4}{15}$ at 0
5. $\pm 4i$ at $\mp i$
7. $1/\pi$ at $0, \pm 1, \cdots$
9. -1 at $\pm 2n\pi i$
11. $(e^z)''/2!|_{z=\pi i} = -\frac{1}{2}$ at $z = \pi i$
15. Simple pole at $\frac{1}{4}$ inside C, residue $-1/(2\pi)$. *Answer:* $-i$
17. Simple poles at $\pi/2$, residue $e^{\pi/2}/(-\sin\pi/2)$, and at $-\pi/2$, residue $e^{-\pi/2}/\sin\pi/2 = e^{-\pi/2}$. *Answer:* $-4\pi i \sinh \pi/2$
19. $2\pi i(\sinh\frac{1}{2}i)/2 = -\pi \sin\frac{1}{2}$
21. $z^{-5}\cos\pi z = \cdots + \pi^4/(4!z) - + \cdots$. *Answer:* $2\pi^5 i/24$

23. Residues $\frac{1}{2}$ at $z = \frac{1}{2}$, 2 at $z = \frac{1}{3}$. *Answer:* $5\pi i$

25. Simple poles inside C at $2i, -2i, 3i, -3i$, residues $(2i \cosh 2i)/(4z^3 + 26z)|_{z=2i} = \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}$, respectively. *Answer:* $2\pi i \cdot \frac{4}{10}$

Problem Set 16.4, page 733

1. $2\pi/\sqrt{k^2 - 1}$

3. $\pi/\sqrt{2}$

5. $5\pi/12$

7. $2a\pi/\sqrt{a^2 - 1}$

9. 0. Why? (Make a sketch.)

11. $\pi/2$

13. 0. Why?

15. $\pi/3$

17. 0. Why?

19. Simple poles at ± 1, i (and $-i$); $2\pi i \cdot \frac{1}{4}i + \pi i(-\frac{1}{4} + \frac{1}{4}) = -\frac{1}{2}\pi$

21. Simple poles at 1 and $\pm 2\pi i$, residues i and $-i$. *Answer:* $\dfrac{\pi}{5}(\cos 1 - e^{-2})$

23. $-\pi/2$

25. 0

27. Let $q(z) = (z - a_1)(z - a_2) \cdots (z - a_k)$. Use (4) in Sec. 16.3 to form the sum of the residues $1/q'(a_1) + \cdots + 1/q'(a_k)$ and show that this sum is 0; here $k > 1$.

Chapter 16 Review Questions and Problems, page 733

11. $6\pi i$

13. $2\pi i(-10 - 10)$

15. $2\pi i(25z^2)'|_{z=5} = 500\pi i$

17. 0 (n even), $(-1)^{(n-1)/2} 2\pi i/(n - 1)!$ (n odd)

19. $\pi/6$

21. $\pi/60$

23. 0. Why?

25. $\operatorname{Res}_{z=i} e^{iz}/(z^2 + 1) = 1/(2ie)$. *Answer:* π/e.

Problem Set 17.1, page 741

5. Only in size

7. $x = c, w = -y + ic$; $\quad y = k, w = -k + ix$

9. Parallel displacement; each point is moved 2 to the right and 1 up.

11. $|w| \leqq \frac{1}{4}$, $\quad -\pi/4 < \operatorname{Arg} w < \pi/4$

13. $-5 \leqq \operatorname{Re} z \leqq -2$

15. $u \geqq 1$

17. Annulus $\frac{1}{2} \leqq |w| \leqq 4$

19. $0 < u < \ln 4$, $\quad \pi/4 < v \leqq 3\pi/4$

21. $z^3 + az^2 + bz + c$, $\quad z = -\frac{1}{3}(a \pm \sqrt{a^2 - 3b})$

23. $z = (-1 \pm \sqrt{3})/2$

25. $\sinh z = 0$ at $z = 0$, $\quad \pm\pi i, \pm 2\pi i, \cdots$

29. $M = |z| = 1$ on the unit circle, $J = |z|^2$

31. $|w'| = 1/|z|^2 = 1$ on the unit circle, $J = 1/|z|^4$

33. $M = e^x = 1$ for $x = 0$, the y-axis, $J = e^{2x}$

35. $M = 1/|z| = 1$ on the unit circle, $J = 1/|z|^2$

Problem Set 17.2, page 745

7. $z = \dfrac{w + i}{2w}$

9. $z = \dfrac{4w + i}{-3iw + 1}$

11. $z = 0$, $\quad 1/(a + ib)$

13. $z = 0$, $\quad \pm\frac{1}{2}, \pm = \pm i/2$

15. $z = i, 2i$ **17.** $w = \dfrac{az}{cz + a}$ **19.** $w = \dfrac{az + b}{-bz + a}$

Problem Set 17.3, page 750

3. Apply the inverse g of f on both sides of $z_1 = f(z_1)$ to get $g(z_1) = g(f(z_1)) = z_1$.
9. $w = iz$, a rotation. Sketch to see.
11. $w = (z + i)/(z - i)$
13. $w = 1/z$, almost by inspection
15. $w = 1/z - 1$
17. $w = (2z - i)/(-iz - 2)$
19. $w = (z^4 - i)(-iz^4 + 1)$

Problem Set 17.4, page 754

1. Circle $|w| = e^c$
3. Annulus $1/\sqrt{e} \leqq |w| \leqq \sqrt{e}$
5. w-plane without $w = 0$
7. $1 < |w| < e, v > 0$
9. $\pm(2n + 1)\pi/2, \quad n = 0, 1, \cdots$
11. $u^2/\cosh^2 2 + v^2/\sinh^2 2 < 1, \quad u > 0, v > 0$
13. Elliptic annulus bounded by $u^2/\cosh^2 1 + v^2/\sinh^2 1 = 1$ and $u^2/\cosh^2 3 + v^2/\sinh^2 3 = 1$
15. $\cosh z = \cos iz = \sin(iz + \frac{1}{2}\pi)$
17. $0 < \operatorname{Im} t < \pi$ is the image of R under $t = z^2/2$. *Answer:* $e^t = e^{z^2/2}$.
19. Hyperbolas $u^2/\cos^2 c - v^2/\sin^2 c = \cosh^2 c - \sinh^2 c = 1$ when $c \neq 0, \pi$, and $u = \pm\cosh y$ (thus $|u| \geqq 1$), $v = 0$ when $c = 0, \pi$.
21. Interior of $u^2/\cosh^2 2 + v^2/\sinh^2 2 = 1$ in the fourth quadrant, or map $\pi/2 < x < \pi, 0 < y < 2$ by $w = \sin z$ (why?).
23. $v < 0$
25. The images of the five points in the figure can be obtained directly from the function w.

Problem Set 17.5, page 756

1. w moves once around the circle $|w| = \frac{1}{2}$.
3. Four sheets, branch point at $z = -1$
5. $-i/4$, three sheets
7. z_0, n sheets
9. $\sqrt{z(z - i)(z + i)}$, $0, \pm i$, two sheets

Chapter 17 Review Questions and Problems, page 756

11. $1 < |w| < 4, |\arg w| < \pi/4$
13. Horizontal strip $-8 < v < 8$
15. $u = 1 - \frac{1}{4}v^2$, same (why?)
17. $|w| > 1$
19. $\frac{1}{3} < |w| < \frac{1}{2}, \quad v < 0$
21. $w = 1 + iv, \quad v < 0$
23. $w = \dfrac{10z + 5i}{z + 2i}$
25. Rotation $w = iz$
27. $w = 1/z$
29. $z = 0$
31. $z = 2 \pm \sqrt{6}$
33. $z = 0, \pm i, \pm 3i$
35. $w = e^{4z}$
37. $w = iz^2 + 1$
39. $w = z^2/(2c)$

Problem Set 18.1, page 762

1. 2.5 mm = 0.25 cm; $\Phi = \text{Re}\, 110(1 + (\text{Ln}\, z)/\ln 4)$

3. $\Phi = \text{Re}\left(30 - \dfrac{20}{\ln 10}\,\text{Ln}\, z\right)$

5. $\Phi(x) = \text{Re}\,(375 + 25z)$

7. $\Phi(r) = \text{Re}\,(32 - z)$

13. Use Fig. 391 in Sec. 17.4 with the z- and w-planes interchanged and $\cos z = \sin(z + \frac{1}{2}\pi)$.

15. $\Phi = 220(x^3 - 3xy^2) = \text{Re}\,(220z^3)$

Problem Set 18.2, page 766

3. $w = iz^2$ maps R onto the strip $-2 \leqq u \leqq 0$; and $\Phi^* = U_2 + (U_1 - U_2)(1 + \frac{1}{2}u) = U_2 + (U_1 - U_2)(1 - xy)$.

5. **(a)** $\dfrac{(x-2)(2x-1) + 2y^2}{(x-2)^2 + y^2} = c$, **(b)** $x^2 - y^2 = c$, $xy = c$, $e^x \cos y = c$

7. See Fig. 392 in Sec. 17.4. $\Phi = \text{Re}\,(\sin^2 z)$, $\sin^2 x\ (y = 0)$, $\sin^2 x \cosh^2 1 - \cos^2 x \sinh^2 1\ (y = 1)$, $-\sinh^2 y\ (x = 0, \pi)$.

9. $\Phi(x, y) = \cos^2 x \cosh^2 y - \sin^2 x \sinh^2 y$; $\cosh^2 y\ (x = 0)$, $-\sinh y\ (x = \frac{\pi}{2})$, $\cos^2 x\ (y = 0)$, $\cos^2 x \cosh^2 1 - \sin^2 x \sinh^2 1\ (y = 1)$

13. Corresponding rays in the w-plane make equal angles, and the mapping is conformal.

15. Apply $w = z^2$.

17. $z = (2Z - i)/(-iZ - 2)$ by (3) in Sec. 17.3.

19. $\Phi = \dfrac{5}{\pi}\,\text{Arg}\,(z - 2)$, $F = -\dfrac{5i}{\pi}\,\text{Ln}\,(z - 2)$

Problem Set 18.3, page 769

1. $(80/d)y + 20$. Rotate through $\pi/2$.

5. $\dfrac{80}{\pi}\arctan\dfrac{y}{x} = \text{Re}\left(-\dfrac{80i}{\pi}\,\text{Ln}\, z\right)$

7. $T_1 + \dfrac{2}{\pi}(T_2 - T_1)\arctan\dfrac{y}{x} = \text{Re}\left(T_1 - \dfrac{2i}{\pi}(T_2 - T_1)\,\text{Ln}\, z\right)$

9. $\dfrac{T_1}{\pi}\left(\arctan\dfrac{y}{x-b} - \arctan\dfrac{y}{x-a}\right) = \text{Re}\left(\dfrac{iT_1}{\pi}\,\text{Ln}\,\dfrac{z-a}{z-b}\right)$

11. $\dfrac{100}{\pi}(\text{Arg}\,(z-1) - \text{Arg}\,(z+1)) = \text{Re}\left(\dfrac{100i}{\pi}\,\text{Ln}\,\dfrac{z+1}{z-1}\right)$

13. $\dfrac{100}{\pi}[\text{Arg}\,(z^2 - 1) - \text{Arg}\,(z^2 + 1)]$ from $w = z^2$ and Prob. 11.

15. $-20 + (320/\pi)\,\text{Arg}\, z = \text{Re}\left(-20 - \dfrac{320i}{\pi}\,\text{Ln}\, z\right)$

17. $\text{Re}\, F(z) = 100 + (200/\pi)\,\text{Re}\,(\arcsin z)$

Problem Set 18.4, page 776

1. $V(z)$ continuously differentiable.

3. $|F'(iy)| = 1 + 1/y^2$, $|y| \geqq 1$, is maximum at $y = \pm 1$, namely, 2.

5. Calculate or note that $\nabla^2 = \text{div grad}$ and curl grad is the zero vector; see Sec. 9.8 and Problem Set 9.7.
7. Horizontal parallel flow to the right.
9. $F(z) = z^4$
11. Uniform parallel flow upward, $V = \overline{F'} = iK$, $V_1 = 0$, $V_2 = K$
13. $F(z) = z^3$
15. $F(z) = z/r_0 + r_0/z$
17. Use that $w = \arccos z$ gives $z = \cos w$ and interchanging the roles of the z- and w-planes.
19. $y/(x^2 + y^2) = c$ or $x^2 + (y - k)^2 = k^2$

Problem Set 18.5, page 781

5. $\Phi = \frac{3}{2} r^3 \sin 3\theta$
7. $\Phi = \frac{1}{2} a + \frac{1}{2} ar^8 \cos 8\theta$
9. $\Phi = 3 - 4r^2 \cos 2\theta + r^4 \cos 4\theta$
11. $\Phi = \dfrac{2}{\pi}\left(r \sin\theta - \dfrac{1}{2}r^2 \sin 2\theta + \dfrac{1}{3}r^3 \sin 3\theta - + \cdots\right)$
13. $\Phi = \dfrac{2}{\pi} r \sin\theta + \dfrac{1}{2} r^2 \sin 2\theta - \dfrac{2}{9\pi} r^3 \sin 3\theta - \dfrac{1}{4} r^4 \sin 4\theta + + - \cdots$
15. $\Phi = \dfrac{1}{2} + \dfrac{2}{\pi}\left(r \cos\theta - \dfrac{1}{3}r^3 \cos 3\theta + \dfrac{1}{5}r^5 \cos 5\theta - + \cdots\right)$
17. $\Phi = \dfrac{1}{3} - \dfrac{4}{\pi^2}\left(r \cos\theta - \dfrac{1}{4}r^2 \cos 2\theta + \dfrac{1}{9}r^3 \cos 3\theta - + \cdots\right)$

Problem Set 18.6, page 784

1. Use (2). $F(z_0 + e^{i\alpha}) = (\frac{7}{2} + e^{i\alpha})^3$, etc. $F(\frac{5}{2}) = \frac{343}{8}$
3. Use (2). $F(z_0 + e^{i\alpha}) = (2 + 3e^{i\alpha})^2$, etc. $F(4) = 100$
5. No, because $|z|$ is not analytic.
7.
$$\Phi(2, -2) = -3 = \frac{1}{\pi}\int_0^1\int_0^{2\pi} (1 + r\cos\alpha)(-3 + r\sin\alpha) r\, dr\, d\alpha$$
$$= \frac{1}{\pi}\int_0^1\int_0^{2\pi} (-3r + \cdots)\, dr\, d\alpha = \frac{1}{\pi}\left(-\frac{3}{2}\right)\cdot 2\pi$$
9.
$$\Phi(1, 1) = 3 = \frac{1}{\pi}\int_0^1\int_0^{2\pi} (3 + r\cos\alpha + r\sin\alpha + r^2\cos\alpha\sin\alpha) r\, dr\, d\alpha$$
$$= \frac{1}{\pi}\cdot\frac{3}{2}\cdot 2\pi$$
13. $|F(z)| = [\cos^2 x + \sinh^2 y]^{1/2}$, $z = \pm i$, Max $= [1 + \sinh^2 1]^{1/2} = 1.543$
15. $|F(z)|^2 = \sinh^2 2x \cos^2 2y + \cosh^2 2x \sin^2 2y = \sinh^2 2x + 1 \cdot \sin^2 2y$, $z = 1$, Max $= \sinh 2 = 3.627$
17. $|F(z)|^2 = 4(2 - 2\cos 2\theta)$, $z = \pi/2$, $3\pi/2$, Max $= 4$
19. No. Make up a counterexample.

Chapter 18 Review Questions and Problems, page 785

11. $\Phi = 10(1 - x + y), \quad F = 10 - 10(1 + i)z$

13. $\Phi = \text{Re}\,(220 - 95.54 \,\text{Ln}\, z) = 220 - \dfrac{220}{\ln 10} \ln r = 220 - 95.54 \ln r.$

17. $2(1 - (2/\pi) \text{ Arg } z)$

19. $30(1 - (2/\pi) \text{ Arg } (z - 1))$

21. $\Phi = x + y = \text{const}, \quad V = \overline{F'(z)} = 1 - i,$ parallel flow

23. $T(x, y) = x(2y + 1) = \text{const}$

25. $\overline{F'(z)} = \bar{z} + 1 = x + 1 - iy$

Problem Set 19.1, page 796

1. $0.84175 \cdot 10^2, \quad -0.52868 \cdot 10^3, \quad 0.92414 \cdot 10^{-3}, \quad -0.36201 \cdot 10^6$

3. 6.3698, 6.794, 8.15, impossible

5. Add first, then round.

7. 29.9667, 0.0335; 29.9667, 0.0333704 (6S-exact)

9. 29.97, 0.035; 29.97, 0.03337; 30, 0.0; 30, 0.033

11. $|\epsilon| = |x + y - (\tilde{x} + \tilde{y})| = |(x - \tilde{x}) + (y - \tilde{y})| = |\epsilon_x + \epsilon_y|$
$\leqq |\epsilon_x| + |\epsilon_y| = \beta_x + \beta_y$

13. $\dfrac{a_1}{a_2} = \dfrac{\tilde{a}_1 + \epsilon_1}{\tilde{a}_2 + \epsilon_2} = \dfrac{\tilde{a}_1 + \epsilon_1}{\tilde{a}_2}\left(1 - \dfrac{\epsilon_2}{\tilde{a}_2} + \dfrac{\epsilon_2^2}{\tilde{a}_2^2} - + \cdots\right) \approx \dfrac{\tilde{a}_1}{\tilde{a}_2} + \dfrac{\epsilon_1}{\tilde{a}_2} - \dfrac{\epsilon_2}{\tilde{a}_2} \cdot \dfrac{\tilde{a}_1}{\tilde{a}_2},$
hence $\left|\left(\dfrac{a_1}{a_2} - \dfrac{\tilde{a}_1}{\tilde{a}_2}\right) \bigg/ \dfrac{a_1}{a_2}\right| \approx \left|\dfrac{\epsilon_1}{a_1} - \dfrac{\epsilon_2}{a_2}\right| \leqq |\epsilon_{r1}| + |\epsilon_{r2}| \leqq \beta_{r1} + \beta_{r2}$

15. **(a)** $1.38629 - 1.38604 = 0.00025$, **(b)** $\ln 1.00025 = 0.000249969$ is 6S-exact.

19. In the present case, (b) is slightly more accurate than (a) (which may produce nonsensical results; cf. Prob. 20).

21. $c_4 \cdot 2^4 + \cdots + c_0 \cdot 2^0 = (1\,0\,1\,1\,1.)_2$, NOT $(1\,1\,1\,0\,1.)_2$

23. The algorithm in Prob. 22 repeats 0011 infinitely often.

25. $n = 26$. The beginning is 0.09375 ($n = 1$).

27. $I_{14} = 0.1812$ (0.1705 4S-exact), $\quad I_{13} = 0.1812$ (0.1820), $\quad I_{12} = 0.1951$ (0.1951),
$I_{11} = 0.2102$ (0.2103), etc.

29. $-0.126 \cdot 10^{-2}, -0.402 \cdot 10^{-3}; \quad -0.266 \cdot 10^{-6}, -0.847 \cdot 10^{-7}$

Problem Set 19.2, page 807

3. $g = 0.5 \cos x, \quad x = 0.450184$ ($= x_{10}$, exact to 6S)

5. Convergence to 4.7 for all these starting values.

7. $x = x/(e^x \sin x)$; 0.5, 0.63256, $\cdots$ converges to 0.58853 (5S-exact) in 14 steps.

9. $x = x^4 - 0.12; \quad x_0 = 0, x_3 = -0.119794$ (6S-exact)

11. $g = 4/x + x^3/16 - x^5/576; \quad x_0 = 2, x_n = 2.39165$ ($n \geqq 6$), 2.405 4S-exact

13. This follows from the intermediate value theorem of calculus.

15. $x_3 = 0.450184$

17. Convergence to $x = 4.7, 4.7, 0.8, -0.5$, respectively. Reason seen easily from the graph of f.

19. 0.5, 0.375, 0.377968, 0.377964; (b) $1/\sqrt{7}$

21. 1.834243 ($= x_4$), 0.656620 ($= x_4$), −2.49086 ($= x_4$)

23. $x_0 = 4.5$, $x_4 = 4.73004$ (6S-exact)

25. **(a) ALGORITHM BISECT (f, a_0, b_0, ϵ, N) Bisection Method**

This algorithm computes the solution c of $f(x) = 0$ (f continuous) within the tolerance ϵ, given an initial interval $[a_0, b_0]$ such that $f(a_0)f(b_0) < 0$.

INPUT: Continuous function f, initial interval $[a_0, b_0]$, tolerance ϵ, maximum number of iterations N.

OUTPUT: A solution c (within the tolerance ϵ), or a message of failure.

For $n = 0, 1, \cdots, N - 1$ do:

$c = \frac{1}{2}(a_n + b_n)$

If $f(c) = 0$ then OUTPUT c Stop. [*Procedure completed*]

Else if $f(a_n)f(b_n) < 0$ then set $a_{n+1} = a_n$ and $b_{n+1} = c$.

Else set $a_{n+1} = c$, and $b_{n+1} = b_n$.

If $|a_{n+1} - b_{n+1}| < \epsilon|c|$ then OUTPUT c. Stop. [*Procedure completed*]

End

OUTPUT $[a_N, b_N]$ and a message "Failure". Stop.

[*Unsuccessful completion; N iterations did not give an interval of length not exceeding the tolerance.*]

End BISECT

Note that $[a_N, b_N]$ gives $(a_N + b_N)/2$ as an approximation of the zero and $(b_N - a_N)/2$ as a corresponding error bound.

(b) 0.739085; **(c)** 1.30980, 0.429494

27. $x_2 = 1.5$, $x_3 = 1.76471, \cdots$, $x_7 = 1.83424$ (6S-exact)

29. 0.904557 (6S-exact)

Problem Set 19.3, page 819

1. $L_0(x) = -2x + 19$, $L_1(x) = 2x - 18$, $p_1(9.3) = L_0(9.3) \cdot f_0 + L_1(9.3) \cdot f_1$ $= 0.1086 \cdot 9.3 + 1.230 = 2.2297$

3. $$p_2(x) = \frac{(x-1.02)(x-1.04)}{(-0.02)(-0.04)} \cdot 1.0000 + \frac{(x-1)(x-1.04)}{0.02\,(-0.02)} \cdot 0.9888$$
$$+ \frac{(x-1)(x-1.02)}{0.04 \cdot 0.02} \cdot 0.9784 = x^2 - 2.580x + 2.580; \quad 0.9943, 0.9835$$

5. 0.8033 (error −0.0245), 0.4872 (error −0.0148); quadratic: 0.7839 (−0.0051), 0.4678 (0.0046)

7. $p_2(x) = 1.1640x - 0.3357x^2$; −0.5089 (error 0.1262), 0.4053 (−0.0226), 0.9053 (0.0186), 0.9911 (−0.0672)

9. $p_2(x) = -0.44304x^2 + 1.30896x - 0.023220$, $p_2(0.75) = 0.70929$ (5S-exact 0.71116)

11. $L_0 = -\frac{1}{6}(x-1)(x-2)(x-3)$, $L_1 = \frac{1}{2}x(x-2)(x-3)$, $L_2 = -\frac{1}{2}x(x-1)(x-3)$, $L_3 = \frac{1}{6}x(x-1)(x-2)$; $p_3(x) = 1 + 0.039740x - 0.335187x^2 + 0.060645x^3$; $p_2(0.5) = 0.943654$, $p_3(1.5) = 0.510116$, $p_3(2.5) = -0.047991$

13. $2x^2 - 4x + 2$

15. $p_3(x) = 2.1972 + (x-9) \cdot 0.1082 + (x-9)(x-9.5) \cdot 0.005235$

17. $r = -1.5$, $p_2(0.3) = 0.6039 + (-1.5) \cdot 0.1755 + \frac{1}{2}(-1.5)(-0.5) \cdot (-0.0302)$ $= 0.3293$

Problem Set 19.4, page 826

9. $[-1.39(x-5)^2 + 0.58(x-5)^3]'' = 0.004$ at $x = 5.8$ (due to roundoff; should be 0).

11. $1 - \frac{5}{4}x^2 + \frac{1}{4}x^4$

13. $1 - x^2$, $-2(x-1) - (x-1)^2 + 2(x-1)^3$, $-1 + 2(x-2) + 5(x-2)^2 - 6(x-2)^3$

15. $4 + x^2 - x^3$, $-8(x-2) - 5(x-2)^2 + 5(x-2)^3$, $4 + 32(x-4) + 25(x-4)^2 - 11(x-4)^3$

17. Use the fact that the third derivative of a cubic polynomial is constant, so that g''' is piecewise constant, hence constant throughout under the present assumption. Now integrate three times.

19. Curvature $f''/(1 + f'^2)^{3/2} \approx f''$ if $|f'|$ is small.

Problem Set 19.5, page 839

1. 0.747131, which is larger than 0.746824. Why?

3. 0.5, 0.375, 0.34375, 0.335 (exact)

5. $\epsilon_{0.5} \approx 0.03452$ ($\epsilon_{0.5} = 0.03307$), $\epsilon_{0.25} \approx 0.00829$ ($\epsilon_{0.25} = 0.00820$)

7. 0.693254 (6S-exact 0.693147)

9. 0.073930 (6S-exact 0.073928)

11. 0.785392 (6S-exact 0.785398)

13. $(0.785398126 - 0.785392156)/15 = 0.39792 \cdot 10^{-6}$

15. **(a)** $M_2 = 2$, $|KM_2| = 2/(12n^2) = 10^{-5}/2$, $n = 183$. **(b)** $f^{\text{iv}} = 24/x^5$, $M_4 = 24$, $|CM_4| = 24/(180 \cdot (2m)^4) = 10^{-5}/2$, $2m = 12.8$, hence 14.

17. 0.94614588, 0.94608693 (8S-exact 0.94608307)

19. 0.9460831 (7S-exact)

21. 0.9774586 (7S-exact 0.9774377)

23. Set $x = \frac{1}{2}(t + 1)$, 0.2642411177 (10S-exact), $1 - 2/e$

25. $x = \frac{1}{2}(t + 1)$, $dx = \frac{1}{2}dt$, 0.746824127 (9S-exact 0.746824133)

27. 0.08, 0.32, 0.176, 0.256 (exact)

29. $5(0.1040 - \frac{1}{2} \cdot 0.1760 + \frac{1}{3} \cdot 0.1344 - \frac{1}{4} \cdot 0.0384) = 0.256$

Chapter 19 Review Questions and Problems, page 841

17. 4.375, 4.50, 6.0, impossible

19. $44.885 \leqq s \leqq 44.995$

21. The same as that of $\tilde{a}$.

23. $x = 20 \pm \sqrt{398} = 20.00 \pm 19.95$, $x_1 = 39.95$, $x_2 = 0.05$, $x_2 = 2/39.95 = 0.05006$ (error less than 1 unit of the last digit)

25. $x = x^4 - 0.1$, -0.1, -0.999, -0.99900399

27. 0.824

29. $-x + x^3$, $2(x-1) + 3(x-1)^2 - (x-1)^3$

31. 0.26, $M_2 = 6$, $M_2^* = 0$, $-0.02 \leqq \epsilon \leqq 0$, 0.01

33. 0.90443, 0.90452 (5S-exact 0.90452)

35. (a) $(0.4^3 - 2 \cdot 0.2^3 + 0)/0.04 = 1.2$, (b) $(0.3^3 - 2 \cdot 0.2^3 + 0.1^3)/0.01 = 1.2$ (exact)

Problem Set 20.1, page 851

1. $x_1 = 7.3, \quad x_2 = -3.2$ **3.** No solution **5.** $x_1 = 2, \quad x_2 = 1$

7. $\begin{bmatrix} -3 & 6 & -9 & -46.725 \\ 0 & 9 & -13 & -51.223 \\ 0 & 0 & -2.88889 & -7.38689 \end{bmatrix}$
$x_1 = 3.908, \quad x_2 = -1.998, \quad x_3 = 2.557$

9. $\begin{bmatrix} 13 & -8 & 0 & 178.54 \\ 0 & 6 & 13 & 137.86 \\ 0 & 0 & -16 & -253.12 \end{bmatrix}$
$x_1 = 6.78, \quad x_2 = -11.3, \quad x_3 = 15.82$

11. $\begin{bmatrix} 3.4 & -6.12 & -2.72 & 0 \\ 0 & 0 & 4.32 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
$x_1 = t_1$ arbitrary, $\quad x_2 = (3.4/6.12)t_1, \quad x_3 = 0$

13. $\begin{bmatrix} 5 & 0 & 6 & -0.329193 \\ 0 & -4 & -3.6 & -2.143144 \\ 0 & 0 & 2.3 & -0.4 \end{bmatrix}$
$x_1 = 0.142856, \quad x_2 = 0.692307, \quad x_3 = -0.173912$

15. $\begin{bmatrix} -1 & -3.1 & 2.5 & 0 & -8.7 \\ 0 & 2.2 & 1.5 & -3.3 & -9.3 \\ 0 & 0 & -1.493182 & -0.825 & 1.03773 \\ 0 & 0 & 0 & 6.13826 & 12.2765 \end{bmatrix}$
$x_1 = 4.2, \quad x_2 = 0, \quad x_3 = -1.8, \quad x_4 = 2.0$

Problem Set 20.2, page 857

1. $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 0 & -1 \end{bmatrix}, \quad \begin{matrix} x_1 = -4 \\ x_2 = 6 \end{matrix}$

3. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix} \begin{bmatrix} 5 & 4 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}, \quad \begin{matrix} x_1 = 0.4 \\ x_2 = 0.8 \\ x_3 = 1.6 \end{matrix}$

5. $\begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 3 & 9 & 1 \end{bmatrix} \begin{bmatrix} 3 & 9 & 6 \\ 0 & -6 & 3 \\ 0 & 0 & -3 \end{bmatrix}, \quad \begin{matrix} x_1 = -\frac{1}{15} \\ x_2 = \frac{4}{15} \\ x_3 = \frac{2}{5} \end{matrix}$

7. $\begin{bmatrix} 3 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$, $\begin{matrix} x_1 = 0.6 \\ x_2 = 1.2 \\ x_3 = 0.4 \end{matrix}$

9. $\begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0.3 & 0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 0.1 & 0 & 0.3 \\ 0 & 0.4 & 0.2 \\ 0 & 0 & 0.1 \end{bmatrix}$, $\begin{matrix} x_1 = 2 \\ x_2 = -11 \\ x_3 = 4 \end{matrix}$

11. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 3 & -1 & 3 & 0 \\ 2 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, $\begin{matrix} x_1 = 2 \\ x_2 = -3 \\ x_3 = 4 \\ x_4 = -1 \end{matrix}$

13. No, since $\mathbf{x}^\mathsf{T}(-\mathbf{A})\mathbf{x} = -\mathbf{x}^\mathsf{T}\mathbf{A}\mathbf{x} < 0$; yes; yes; no

15. $\begin{bmatrix} -3.5 & 1.25 \\ 3.0 & -1.0 \end{bmatrix}$

17. $\frac{1}{36}\begin{bmatrix} 584 & 104 & -66 \\ 104 & 20 & -12 \\ -66 & -12 & 9 \end{bmatrix}$

19. $\frac{1}{16}\begin{bmatrix} 21 & -6 & -14 & 6 \\ -6 & 36 & -12 & -4 \\ -14 & -12 & 20 & -4 \\ 6 & -4 & -4 & 4 \end{bmatrix}$

Problem Set 20.3, page 863

5. Exact 0.5, 0.5, 0.5

7. $x_1 = 2$, $x_2 = -4$, $x_3 = 8$

9. Exact 2, 1, 4

11. (a) $\mathbf{x}^{(3)\mathsf{T}} = [0.49983 \quad 0.50001 \quad 0.500017]$,
(b) $\mathbf{x}^{(3)\mathsf{T}} = [0.50333 \quad 0.49985 \quad 0.49968]$

13. 8, −16, 43, 86 steps; spectral radius 0.09, 0.35, 0.72, 0.85, approximately

15. $[1.99934 \quad 1.00043 \quad 3.99684]^\mathsf{T}$ (Jacobi, Step 5); $[2.00004 \quad 0.998059 \quad 4.00072]^\mathsf{T}$ (Gauss–Seidel)

19. $\sqrt{306} = 17.49$, 12, 12

Problem Set 20.4, page 871

1. 18, $\sqrt{110} = 10.49$, 8, $[0.125 \quad -0.375 \quad 1 \quad 0 \quad -0.75 \quad 0]$

3. 5.9, $\sqrt{13.81} = 3.716$, 3, $\frac{1}{3}[0.2 \quad 0.6 \quad -2.1 \quad 3.0]$

5. 5, $\sqrt{5}$, 1, $[1 \quad 1 \quad 1 \quad 1 \quad 1]$

7. $ab + bc + ca = 0$

9. $\kappa = 5 \cdot \frac{1}{2} = 2.5$ **11.** $\kappa = (5 + \sqrt{5})(1 + 1/\sqrt{5}) = 6 + 2\sqrt{5}$
13. $\kappa = 19 \cdot 13 = 247$; ill-conditioned
15. $\kappa = 20 \cdot 20 = 400$; ill-conditioned
17. $167 \leqq 21 \cdot 15 = 315$
19. $[-2 \quad 4]^{\mathsf{T}}$, $[-144.0 \quad 184.0]^{\mathsf{T}}$, $\kappa = 25{,}921$, extremely ill-conditioned
21. Small residual $[0.145 \quad 0.120]$, but large deviation of $\tilde{\mathbf{x}}$.
23. 27, 748, 28,375, 943,656, 29,070,279

Problem Set 20.5, page 875

1. $1.846 - 1.038x$ **3.** $1.48 + 0.09x$
5. $s = 90t - 675$, $v_{\text{av}} = 90$ km/hr **9.** $-11.36 + 5.45x - 0.589x^2$
11. $1.89 - 0.739x + 0.207x^2$
13. $2.552 + 16.23x$, $-4.114 + 13.73x + 2.500x^2$, $2.730 + 1.466x - 1.778x^2 + 2.852x^3$

Problem Set 20.7, page 884

1. 5, 0, 7; radii 6, 4, 6. Spectrum $\{-1, 4, 9\}$
3. Centers 0; radii 0.5, 0.7, 0.4. Skew-symmetric, hence $\lambda = i\mu$, $-0.7 \leqq \mu \leqq 0.7$.
5. 2, 3, 8; radii $1 + \sqrt{2}$, 1, $\sqrt{2}$; actually (4S) 1.163, 3.511, 8.326
7. $t_{11} = 100$, $t_{22} = t_{33} = 1$
9. They lie in the intervals with endpoints $a_{jj} \pm (n - 1) \cdot 10^{-5}$. Why?
11. $\rho(\mathbf{A}) \leqq$ Row sum norm $\|\mathbf{A}\|_\infty = \max_j \sum_k |a_{jk}| = \max_j(|a_{jj}| + \text{Gerschgorin radius})$
13. $\sqrt{122} = 11.05$
15. $\sqrt{0.52} = 0.7211$
17. Show that $\mathbf{A}\overline{\mathbf{A}}^{\mathsf{T}} = \overline{\mathbf{A}}^{\mathsf{T}}\mathbf{A}$.
19. 0 lies in no Gerschgorin disk, by (3) with $>$; hence $\det \mathbf{A} = \lambda_1 \cdots \lambda_n \neq 0$.

Problem Set 20.8, page 887

1. $q = 10, 10.9908, 10.9999$; $|\epsilon| \leqq 3, 0.3028, 0.0275$
3. $q \pm \delta = 4 \pm 1.633$, 4.786 ± 0.619, 4.917 ± 0.398
5. Same answer as in Prob. 3, possibly except for small roundoff errors.
7. $q = 5.5, 5.5738, 5.6018$; $|\epsilon| \leqq 0.5, 0.3115, 0.1899$; eigenvalues (4S) 1.697, 3.382, 5.303, 5.618
9. $\mathbf{y} = \mathbf{A}\mathbf{x} = \lambda\mathbf{x}$, $\mathbf{y}^{\mathsf{T}}\mathbf{x} = \lambda\mathbf{x}^{\mathsf{T}}\mathbf{x}$, $\mathbf{y}^{\mathsf{T}}\mathbf{y} = \lambda^2\mathbf{x}^{\mathsf{T}}\mathbf{x}$,
$\epsilon^2 \leqq \mathbf{y}^{\mathsf{T}}\mathbf{y}/\mathbf{x}^{\mathsf{T}}\mathbf{x} - (\mathbf{y}^{\mathsf{T}}\mathbf{x}/\mathbf{x}^{\mathsf{T}}\mathbf{x})^2 = \lambda^2 - \lambda^2 = 0$
11. $q = 1, \cdots, -2.8993$ approximates -3 (0 of the given matrix), $|\epsilon| \leqq 1.633, \cdots, 0.7024$ (Step 8)

Problem Set 20.9, page 896

1. $$\begin{bmatrix} 0.98 & -0.4418 & 0 \\ -0.4418 & 0.8702 & 0.3718 \\ 0 & 0.3718 & 0.4898 \end{bmatrix}$$

3. $\begin{bmatrix} 7 & -3.6056 & 0 \\ -3.6056 & 13.462 & 3.6923 \\ 0 & 3.6923 & 3.5385 \end{bmatrix}$

5. $\begin{bmatrix} 3 & -67.59 & 0 & 0 \\ -67.59 & 143.5 & 45.35 & 0 \\ 0 & 45.35 & 23.34 & 3.126 \\ 0 & 0 & 3.126 & -33.87 \end{bmatrix}$

7. Eigenvalues 16, 6, 2

$\begin{bmatrix} 11.2903 & -5.0173 & 0 \\ -5.0173 & 10.6144 & 0.7499 \\ 0 & 0.7499 & 2.0952 \end{bmatrix}, \begin{bmatrix} 14.9028 & -3.1265 & 0 \\ -3.1265 & 7.0883 & 0.1966 \\ 0 & 0.1966 & 2.0089 \end{bmatrix}, \begin{bmatrix} 15.8299 & -1.2932 & 0 \\ -1.2932 & 6.1692 & 0.0625 \\ 0 & 0.0625 & 2.0010 \end{bmatrix}$

9. Eigenvalues (4S) 141.4, 68.64, -30.04

$\begin{bmatrix} 141.1 & 4.926 & 0 \\ 4.926 & 68.97 & 0.8691 \\ 0 & 0.8691 & -30.03 \end{bmatrix}, \begin{bmatrix} 141.3 & 2.400 & 0 \\ 2.400 & 68.72 & 0.3797 \\ 0 & 0.3797 & -30.04 \end{bmatrix}, \begin{bmatrix} 141.4 & 1.166 & 0 \\ 1.166 & 68.66 & 0.1661 \\ 0 & 0.1661 & -30.04 \end{bmatrix}$

Chapter 20 Review Questions and Problems, page 896

15. $[3.9 \quad 4.3 \quad 1.8]^T$

17. $[-2 \quad 0 \quad 5]^T$

19. $\begin{bmatrix} 0.28193 & -0.15904 & -0.00482 \\ -0.15904 & 0.12048 & -0.00241 \\ -0.00482 & -0.00241 & 0.01205 \end{bmatrix}$

21. $\begin{bmatrix} 5.750 \\ 3.600 \\ 0.838 \end{bmatrix}, \begin{bmatrix} 6.400 \\ 3.559 \\ 1.000 \end{bmatrix}, \begin{bmatrix} 6.390 \\ 3.600 \\ 0.997 \end{bmatrix}$

Exact: $[6.4 \quad 3.6 \quad 1.0]^T$

23. $\begin{bmatrix} 1.700 \\ 1.180 \\ 4.043 \end{bmatrix}, \begin{bmatrix} 1.986 \\ 0.999 \\ 4.002 \end{bmatrix}, \begin{bmatrix} 2.000 \\ 1.000 \\ 4.000 \end{bmatrix}$

Exact: $[2 \quad 1 \quad 4]^T$

25. 42, $\sqrt{674} = 25.96$, 21

27. 30

29. 5

31. $115 \cdot 0.4458 = 51.27$

33. $5 \cdot \frac{21}{63} = \frac{5}{3}$

35. $1.514 + 1.129x - 0.214x^2$

37. Centers 15, 35, 90; radii 30, 35, 25, respectively. Eigenvalues (3S) 2.63, 40.8, 96.6

39. Centers 0, -1, -4; radii 9, 6, 7, respectively; eigenvalues 0, 4.446, -9.446

Problem Set 21.1, page 910

1. $y = 5e^{-0.2x}$, 0.00458, 0.00830 (errors of y_5, y_{10})
3. $y = x - \tanh x$ (set $y - x = u$), 0.00929, 0.01885 (errors of y_5, y_{10})
5. $y = e^x$, 0.0013, 0.0042 (errors of y_5, y_{10})
7. $y = 1/(1 - x^2/2)$, 0.00029, 0.01187 (errors of y_5, y_{10})
9. Errors 0.03547 and 0.28715 of y_5 and y_{10} much larger
11. $y = 1/(1 - x^2/2)$; error -10^{-8}, $-4 \cdot 10^{-8}$, $\cdots$, $-6 \cdot 10^{-7}$, $+9 \cdot 10^{-6}$; $\epsilon = 0.0002/15 = 1.3 \cdot 10^{-5}$ (use RK with $h = 0.2$)
13. $y = \tan x$; error $0.83 \cdot 10^{-7}$, $0.16 \cdot 10^{-6}$, $\cdots$, $-0.56 \cdot 10^{-6}$, $+0.13 \cdot 10^{-5}$
15. $y = 3 \cos x - 2 \cos^2 x$; error $\cdot 10^7$: 0.18, 0.74, 1.73, 3.28, 5.59, 9.04, 14.3, 22.8, 36.8, 61.4
17. $y' = 1/(2 - x^4)$; error $\cdot 10^9$: 0.2, 3.1, 10.7, 23.2, 28.5, -32.3, -376, -1656, -3489, $+80444$
19. Errors for Euler–Cauchy 0.02002, 0.06286, 0.05074; for improved Euler–Cauchy -0.000455, 0.012086, 0.009601; for Runge–Kutta. 0.0000011, 0.000016, 0.000536

Problem Set 21.2, page 915

1. $y = e^x$, $y_5^* = 1.648717$, $y_5 = 1.648722$, $\epsilon_5 = -3.8 \cdot 10^{-8}$, $y_{10}^* = 2.718276$, $y_{10} = 2.718284$, $\epsilon_{10} = -1.8 \cdot 10^{-6}$
3. $y = \tan x$, $y_4, \cdots, y_{10}$ (error $\cdot 10^5$) 0.422798 (-0.49), 0.546315 (-1.2), 0.684161 (-2.4), 0.842332 (-4.4), 1.029714 (-7.5), 1.260288 (-13), 1.557626 (-22)
5. RK error smaller in absolute value, error $\cdot 10^5 = 0.4, 0.3, 0.2, 5.6$ (for $x = 0.4, 0.6, 0.8, 1.0$)
7. $y = 1/(4 + e^{-3x})$, $y_4, \cdots, y_{10}$ (error $\cdot 10^5$) 0.232490 (0.34), 0.236787 (0.44), 0.240075 (0.42), 0.242570 (0.35), 0.244453 (0.25), 0.245867 (0.16), 0.246926 (0.09)
9. $y = \exp(x^3) - 1$, $y_4, \cdots, y_{10}$ (error $\cdot 10^7$) 0.008032 (-4), 0.015749 (-10), 0.027370 (-17), 0.043810 (-26), 0.066096 (-39), 0.095411 (-54), 0.133156 (-74)
13. $y = \exp(x^2)$. Errors $\cdot 10^5$ from $x = 0.3$ to 0.7: -5, -11, -19, -31, -41
15. **(a)** 0, 0.02, 0.0884, 0.215848, $y_4 = 0.417818$, $y_5 = 0.708887$ (poor)
(b) By 30–50%

Problem Set 21.3, page 922

1. $y_1 = -e^{-2x} + 4e^x$, $y_2 = -e^{-2x} + e^x$; errors of y_1 (of y_2) from 0.002 to 0.5 (from -0.01 to 0.1), monotone
3. $y_1' = y_2$, $y_2' = -\frac{1}{4}y_1$, $y = y_1 = 1$, 0.99, 0.97, 0.94, 0.9005, error -0.005, -0.01, -0.015, -0.02, -0.0229; exact $y = \cos \frac{1}{2}x$
5. $y_1' = y_2$, $y_2' = y_1 + x$, $y_1(0) = 1$, $y_2(0) = -2$, $y = y_1 = e^{-x} - x$, $y = 0.8$ (error 0.005), 0.61 (0.01), 0.429 (0.012), 0.2561 (0.0142), 0.0905 (0.0160)
7. By about a factor 10^5. $\epsilon_n(y_1) \cdot 10^6 = -0.082, \cdots, -0.27$, $\epsilon_n(y_2) \cdot 10^6 = 0.08, \cdots, 0.27$
9. Errors of y_1 (of y_2) from $0.3 \cdot 10^{-5}$ to $1.3 \cdot 10^{-5}$ (from $0.3 \cdot 10^{-5}$ to $0.6 \cdot 10^{-5}$)
11. $(y_1, y_2) = (0, 1)$, (0.20, 0.98), (0.39, 0.92), $\cdots$, (-0.23, -0.97), (-0.42, -0.91), (-0.59), (-0.81); continuation will give an "ellipse."

Problem Set 21.4, page 930

3. $-3u_{11} + u_{12} = -200, \quad u_{11} - 3u_{12} = -100$

5. 105, 155, 105, 115; Step 5: 104.94, 154.97, 104.97, 114.98

7. 0, 0, 0, 0. All equipotential lines meet at the corners (why?).
Step 5: 0.29298, 0.14649, 0.14649, 0.073245

9. 0.108253, 0.108253, 0.324760, 0.324760; Step 10: 0.108538, 0.108396, 0.324902, 0.324831

11. **(a)** $u_{11} = -u_{12} = -66$. **(b)** Reduce to 4 equations by symmetry.
$u_{11} = u_{31} = -u_{15} = -u_{35} = -92.92$, $u_{21} = -u_{25} = -87.45$,
$u_{12} = u_{32} = -u_{14} = -u_{34} = -64.22$, $u_{22} = -u_{24} = -53.98$,
$u_{13} = u_{23} = u_{33} = 0$

13. $u_{12} = u_{32} = 31.25, \quad u_{21} = u_{23} = 18.75, \quad u_{jk} = 25$ at the others

15. $u_{21} = u_{23} = 0.25, \quad u_{12} = u_{32} = -0.25, \quad u_{jk} = 0$ otherwise

17. $\sqrt{3}$, $u_{11} = u_{21} = 0.0849$, $u_{12} = u_{22} = 0.3170$. (0.1083, 0.3248 are 4S-values of the solution of the linear system of the problem.)

Problem Set 21.5, page 935

5. $u_{11} = 0.766, \quad u_{21} = 1.109, \quad u_{12} = 1.957$, $u_{22} = 3.293$

7. $\mathbf{A}$, as in Example 1, right sides $-220, -220, -220, -220$.
Solution $u_{11} = u_{21} = 125.7$, $u_{21} = u_{22} = 157.1$

13. $-4u_{11} + u_{21} + u_{12} = -3$, $u_{11} - 4u_{21} + u_{22} = -12$, $u_{11} - 4u_{12} + u_{22} = 0$,
$2u_{21} + 2u_{12} - 12u_{22} = -14$, $u_{11} = u_{22} = 2$, $u_{21} = 4$, $u_{12} = 1$.
Here $-\frac{14}{3} = -\frac{4}{3}(1 + 2.5)$ with $\frac{4}{3}$ from the stencil.

15. $\mathbf{b} = [-200, -100, -100, 0]^{\mathsf{T}}; \quad u_{11} = 73.68$, $u_{21} = u_{12} = 47.37$, $u_{22} = 15.79$ (4S)

Problem Set 21.6, page 941

5. 0, 0.6625, 1.25, 1.7125, 2, 2.1, 2, 1.7125, 1.25, 0.6625, 0

7. Substantially less accurate, 0.15, 0.25 ($t = 0.04$), 0.100, 0.163 ($t = 0.08$)

9. Step 5 gives 0, 0.06279, 0.09336, 0.08364, 0.04707, 0.

11. Step 2: 0 (exact 0), 0.0453 (0.0422), 0.0672 (0.0658), 0.0671 (0.0628), 0.0394 (0.0373), 0 (0)

13. 0.3301, 0.5706, 0.4522, 0.2380 ($t = 0.04$), 0.06538, 0.10603, 0.10565, 0.6543 ($t = 0.20$)

15. 0.1018, 0.1673, 0.1673, 0.1018 ($t = 0.04$), 0.0219, 0.0355, $\cdots$ ($t = 0.20$)

Problem Set 21.7, page 944

1. $u(x, 1) = 0, -0.05, -0.10, -0.15, -0.20, 0$

3. For $x = 0.2, 0.4$ we obtain 0.24, 0.40 ($t = 0.2$), 0.08, 0.16 ($t = 0.4$), $-0.08, -0.16$ ($t = 0.6$), etc.

5. 0, 0.354, 0.766, 1.271, 1.679, 1.834, $\cdots$ ($t = 0.1$); 0, 0.575, 0.935, 1.135, 1.296, 1.357, $\cdots$ ($t = 0.2$)

7. 0.190, 0.308, 0.308, 0.190, (3S-exact: 0.178, 0.288, 0.288, 0.178)

Chapter 21 Review Questions and Problems, page 945

17. $y = e^x$, 0.038, 0.125 (errors of y_5 and y_{10})
19. $y = \tan x$; 0 (0), 0.10050 (−0.00017), 0.20304 (−0.00033), 0.30981 (−0.00048), 0.42341 (−0.00062), 0.54702 (−0.00072), 0.68490 (−0.00076), 0.84295 (−0.00066), 1.0299 (−0.0002), 1.2593 (0.0009), 1.5538 (0.0036)
21. 0.1003346 $(0.8 \cdot 10^{-7})$ 0.2027099 $(1.6 \cdot 10^{-7})$, 0.3093360 $(2.1 \cdot 10^{-7})$, 0.4227930 $(2.3 \cdot 10^{-7})$, 0.5463023 $(1.8 \cdot 10^{-7})$
23. $y = \sin x$, $y_{0.8} = 0.717366$, $y_{1.0} = 0.841496$ (errors $-1.0 \cdot 10^{-5}$, $-2.5 \cdot 10^{-5}$)
25. $y_1' = y_2$, $y_2' = x^2 y_1$, $y = y_1 = 1, 1, 1, 1.0001, 1.0006, 1.002$
27. $y_1' = y_2$, $y_2' = 2e^x - y_1$, $y = e^x - \cos x$, $y = y_1 = 0, 0.241, 0.571, \cdots$; errors between 10^{-6} and 10^{-5}
29. 3.93, 15.71, 58.93
31. 0, 0.04, 0.08, 0.12, 0.15, 0.16, 0.15, 0.12, 0.08, 0.04, 0 ($t = 0.3$. 3 time steps)
33. $u(P_{11}) = u(P_{31}) = 270$, $u(P_{21}) = u(P_{13}) = u(P_{23}) = u(P_{33}) = 30$, $u(P_{12}) = u(P_{32}) = 90$, $u(P_{22}) = 60$
35. 0.043330, 0.077321, 0.089952, 0.058488 ($t = 0.04$), 0.010956, 0.017720, 0.017747, 0.010964 ($t = 0.20$)

Problem Set 22.1, page 953

3. $f(\mathbf{x}) = 2(x_1 - 1)^2 + (x_2 + 2)^2 - 6$; Step 3: (1.037, −1.926), value −5.992
9. Step 5: (0.11247, −0.00012), value 0.000016

Problem Set 22.2, page 957

7. No
9. x_3, x_4 is the unused time on M_1, M_2, respectively.
11. $f(2.5, 2.5) = 100$
13. $f(-\frac{11}{3}, \frac{26}{3}) = 198\frac{1}{3}$
15. $f(9, 6) = 360$
17. $0.5x_1 + 0.75x_2 \leqq 45$ (copper), $0.5x_1 + 0.25x_2 \leqq 30$, $f = 120x_1 + 100x_2$, $f_{max} = f(45, 30) = 8400$
19. $f = x_1 + x_2$, $2x_1 + 3x_2 \leqq 1200$, $4x_1 + 2x_2 \leqq 1600$, $f_{max} = f(300, 200) = 500$
21. $x_1/3 + x_2/2 \leqq 100$, $x_1/3 + x_2/6 \leqq 80$, $f = 150x_1 + 100x_2$, $f_{max} = f(210, 60) =$ 37,500

Problem Set 22.3, page 961

3. $f(120/11, 60/11) = 480/11$
5. Eliminate in Column 3, so that 20 goes. $f_{min} = f(0, \frac{1}{2}) = -10$.
7. $f_{max} = f(\frac{60}{21}, 0, \frac{1500}{105}, 0) = \frac{2200}{7}$
9. $f_{max} = 6$ on the segment from (3, 0, 0) to (0, 0, 2)
11. We minimize! The augmented matrix is

$$\mathbf{T}_0 = \begin{bmatrix} 1 & 1.8 & 2.1 & 0 & 0 & 0 \\ 0 & 15 & 30 & 1 & 0 & 150 \\ 0 & 600 & 500 & 0 & 1 & 3900 \end{bmatrix}.$$

The pivot is 600. The calculation gives

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & \frac{6}{10} & 0 & -\frac{3}{1000} & -\frac{117}{10} \\ 0 & 0 & \frac{35}{2} & 1 & -\frac{1}{40} & \frac{105}{2} \\ 0 & 600 & 500 & 0 & 1 & 3900 \end{bmatrix} \quad \begin{matrix} \text{Row 1} - \frac{1.8}{600}\text{ Row 3} \\ \text{Row 2} - \frac{15}{600}\text{ Row 3} \\ \text{Row 3} \end{matrix}$$

The next pivot is $\frac{35}{2}$. The calculation gives

$$\mathbf{T}_2 = \begin{bmatrix} 1 & 0 & 0 & -\frac{6}{175} & -\frac{3}{1400} & -\frac{27}{2} \\ 0 & 0 & \frac{35}{2} & 1 & -\frac{1}{40} & \frac{105}{2} \\ 0 & 600 & 0 & -\frac{200}{7} & \frac{12}{7} & 2400 \end{bmatrix} \quad \begin{matrix} \text{Row 1} - \frac{1.2}{35}\text{ Row 2} \\ \text{Row 2} \\ \text{Row 3} - \frac{1000}{35}\text{ Row 2} \end{matrix}$$

Hence $-f$ has the maximum value -13.5, so that f has the minimum value 13.5, at the point

$$(x_1, x_2) = \left(\frac{2400}{600}, \frac{105/2}{35/2}\right) = (4, 3).$$

13. $f_{\max} = f(5, 4, 6) = 478$

Problem Set 22.4, page 968

1. $f(6, 3) = 84$
3. $f(20, 20) = 40$
5. $f(10, 5) = 5500$
7. $f(1, 1, 0) = 13$
9. $f(4, 0, \frac{1}{2}) = 9$

Chapter 22 Review Questions and Problems, page 968

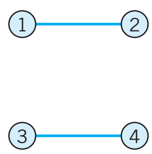
9. Step 5: $[0.353 \quad -0.028]^{\mathsf{T}}$. Slower. Why?
11. Of course! Step 5: $[-1.003 \quad 1.897]^{\mathsf{T}}$
17. $f(2, 4) = 100$
19. $f(3, 6) = -54$

Problem Set 23.1, page 974

9. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

11. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

13. $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

15.


17. If G is complete.

19.
$$\begin{array}{cc|c} & & \text{Edge} \\ & & \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \hline \text{Vertex} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} -1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Problem Set 23.2, page 979

1. 5

3. 4

5. The idea is to go backward. There is a v_{k-1} adjacent to v_k and labeled $k - 1$, etc. Now the only vertex labeled 0 is s. Hence $\lambda(v_0) = 0$ implies $v_0 = s$, so that $v_0 - v_1 - \cdots - v_{k-1} - v_k$ is a path $s \to v_k$ that has length k.

15. Delete the edge (2, 4).

17. No

Problem Set 23.3, page 983

1. (1, 2), (2, 4), (4, 3); $L_2 = 12, L_3 = 36, L_4 = 28$

5. (1, 2), (2, 4), (3, 4), (3, 5); $L_2 = 2, L_3 = 4, L_4 = 3, L_5 = 6$

7. (1, 2), (2, 4), (3, 4); $L_2 = 10, L_3 = 15, L_4 = 13$

9. (1, 5), (2, 3), (2, 6), (3, 4), (3, 5); $L_2 = 9, L_3 = 7, L_4 = 8, L_5 = 4, L_6 = 14$

Problem Set 23.4, page 987

1. $\begin{array}{ll} 2 \diagdown & \\ & 4 - 3 - 5 \\ 1 \diagup & \end{array} \qquad L = 10$

3. $5 - 3 - 6 \begin{array}{l} \diagup 1 \\ \diagdown 2 - 4 \end{array} \qquad L = 17$

5. $1 \begin{array}{l} \diagup 2 \\ \diagdown 4 \begin{array}{l} \diagup 3 \\ \diagdown 5 \end{array} \end{array} \qquad L = 12$

9. Yes

11. $1 - 3 - 4 \begin{array}{l} \diagup 2 \\ \diagdown 5 - 6 \end{array} \qquad L = 38$

13. New York–Washington–Chicago–Dalles–Denver–Los Angeles

15. G is connected. If G were not a tree, it would have a cycle, but this cycle would provide two paths between any pair of its vertices, contradicting the uniqueness.

19. If we add an edge (u, v) to T, then since T is connected, there is a path $u \longrightarrow v$ in T which, together with (u, v), forms a cycle.

Problem Set 23.5, page 990

1. If G is a tree.
3. A shortest spanning tree of the largest connected graph that contains vertex 1.
7. (1, 4), (1, 3), (1, 2), (2, 6), (3, 5); $L = 32$
9. (1, 4), (4, 3), (4, 2), (3, 5); $L = 20$
11. (1, 4), (4, 3), (4, 5), (1, 2); $L = 12$

Problem Set 23.6, page 997

1. $\{3, 6\}$, $11 + 3 = 14$
3. $\{4, 5, 6\}$, $10 + 5 + 13 = 28$
5. $\{3, 6, 7\}$, $8 + 4 + 4 = 16$
7. $S = \{1, 4\}$, $8 + 6 = 14$
9. One is interested in flows *from s to t,* not in the opposite direction.
13. $\Delta_{12} = 5, \Delta_{24} = 8, \Delta_{45} = 2$; $\Delta_{12} = 5, \Delta_{25} = 3$; $\Delta_{13} = 4, \Delta_{35} = 9$
P_1: $1 - 2 - 4 - 5, \Delta f = 2$; P_2: $1 - 2 - 5, \Delta f = 3$; P_3: $1 - 3 - 5, \Delta f = 4$
15. $1 - 2 - 5, \Delta f = 2$; $1 - 4 - 2 - 5, \Delta f = 2$, etc.
17. $f_{13} = f_{35} = 8$, $f_{14} = f_{45} = 5$, $f_{12} = f_{24} = f_{46} = 4$, $f_{56} = 13$, $f = 4 + 13 = 17$, $f = 17$ is unique.
19. For instance, $f_{12} = 10$, $f_{24} = f_{45} = 7$, $f_{13} = f_{25} = 5$, $f_{35} = 3$, $f_{32} = 2$, $f = 3 + 5 + 7 = 15$, $f = 15$ is unique.

Problem Set 23.7, page 1000

3. (2, 3) and (5, 6)
5. By considering only edges with one labeled end and one unlabeled end
7. $1 - 2 - 5, \Delta_t = 2$; $1 - 4 - 2 - 5, \Delta_t = 1$; $f = 6 + 2 + 1 = 9$, where 6 is the given flow
9. $1 - 2 - 4 - 6, \Delta_t = 2$; $1 - 3 - 5 - 6, \Delta_t = 1$; $f = 4 + 2 + 1 = 7$, where 4 is the given flow
15. $S = \{1, 2, 4, 5\}$, $T = \{3, 6\}$, $\text{cap}(S, T) = 14$

Problem Set 23.8, page 1005

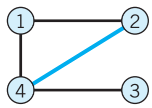
1. No
3. No
5. Yes, $S = \{1, 4, 5, 8\}$
7. Yes, $S = \{1, 3, 5\}$
11. $1 - 2 - 3 - 7 - 5 - 4$
13. $1 - 2 - 3 - 7 - 5 - 4$ is augmenting and gives $1 - 2 - 3 - 7 - 5 - 4$ and (1, 2), (3, 7), (5, 4) is of maximum cardinality.
15. $1 - 4 - 3 - 6 - 7 - 8$ is augmenting and gives $1 - 4 - 3 - 6 - 7 - 8$ and (1, 4), (3, 6), (7, 8) is of maximum cardinality.
19. 3
21. 2
23. 3
25. K_4

Chapter 23 Review Questions and Problems, page 1006

11. $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$

13. $\begin{array}{ll} & \text{To vertex} \quad 1 \quad\; 2 \quad\; 3 \quad\; 4 \\ \text{From vertex} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$

15.



17.

Vertex	Incident Edges
1	(1, 2), (1, 4)
2	(2, 1), (2, 4)
3	(3, 4)
4	(4, 1), (4, 2), (4, 3)

19. (1, 2), (1, 4), (2, 3); $L_2 = 2, L_3 = 5, L_4 = 5$

23. (1, 6), (4, 5), (2, 3), (7, 8)

Problem Set 24.1, page 1015

1. $q_L = 19, q_M = 20, q_U = 20.5$

3. $q_L = 138, q_M = 144, q_U = 154$

5. $q_L = 199, q_M = 201, q_U = 201$

7. $q_L = 1.3, q_M = 1.4, q_U = 1.45$

9. $q_L = 89.9, q_M = 91.0, q_U = 91.8$

11. $\bar{x} = 19.875, s = 0.835$, IQR $= 1.5$

13. $\bar{x} = 144.67, s = 8.9735$, IQR $= 16$

15. $\bar{x} = 1.355, s = 0.136$, IQR $= 0.15$

17. 3.54, 1.29

Problem Set 24.2, page 1017

1. 2^3 outcomes: *RRR, RRL, RLR, LRR, RLL, LRL, LLR, LLL*

3. $6^2 = 36$ outcomes (1, 1), (1, 2), $\cdots$, (6, 6), first number (second number) referring to the first die (second die)

5. Infinitely many outcomes *H TH TTH TTTH* $\cdots$ (*H* = *Head*, *T* = *Tail*)

7. The space of ordered pairs of numbers

9. 10 outcomes: *D ND NND* $\cdots$ *NNNNNNNNND*

11. Yes

17. $A \cup B = B$ implies $A \subseteq B$ by the definition of union. Conversely. $A \subseteq B$ implies that $A \cup B = B$ because always $B \subseteq A \cup B$, and if $A \subseteq B$, we must have equality in the previous relation.

Problem Set 24.3, page 1024

1. $1 - 4/216 = 98.15\%$, by Theorem 1
3. (a) $0.9^3 = 72.9\%$, **(b)** $\frac{90}{100} \cdot \frac{89}{99} \cdot \frac{88}{98} = 72.65\%$
5. $\frac{8}{9}$
7. Small sample from a large population containing *many* items in each class we are interested in (defectives and nondefectives, etc.)
9. $\frac{498}{500} \cdot \frac{497}{499} \cdot \frac{496}{498} \cdot \frac{495}{497} \cdot \frac{494}{496} \approx 0.98008$
11. (a) $\frac{100}{200} \cdot \frac{99}{199} = 24.874\%$, **(b)** $\frac{100}{200} \cdot \frac{100}{199} + \frac{100}{200} \cdot \frac{100}{199} = 50.25\%$, **(c)** same as (a). (a) + (b) + (c) = 1. Why?
13. $1 - 0.96^3 = 11.5\%$
15. $1 - 0.875^4 = 0.4138 < 1 - 0.75^2 = 0.4375 < 0.5 \quad (c < b < a)$
17. $A = B \cup (A \cap B^c)$, hence $P(A) = P(B) + P(A \cap B^c) \geqq P(B)$ by disjointedness of B and $A \cap B^c$

Problem Set 24.4, page 1028

1. In $10! = 3{,}628{,}800$ ways
3. $\frac{2}{6} \cdot \frac{1}{5} \cdot \frac{4}{4} \cdot \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{1}{1} = \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{2}{2} \cdot \frac{1}{1} = \frac{4!2!}{6!} = \frac{2}{6} \cdot \frac{1}{5} = \frac{1}{15}$
5. $\binom{10}{3}\binom{5}{2}\binom{6}{2} = 18{,}000$
7. 210, 70, 112, 28
9. In $6!/6 = 120$ ways
11. $9 \cdot 8 = 72$
13. (b) $1/(12n)$
15. P (*No two people have a birthday in common*) $= 365 \cdot 364 \cdots 346/365^{20} = 0.59$. *Answer:* 41%, which is surprisingly large.

Problem Set 24.5, page 1034

1. $k = \frac{1}{55}$ by (6)
3. $k = \frac{1}{4}$ by (10), $P(0 \leqq X \leqq 2) = \frac{1}{2}$
5. No, because of (6)
7. $k = \frac{1}{100}$ because of (6) and $1 + 8 + 27 + 64 = 100$
9. $k = 5$; 50%
11. $0.5^3 = 12.5\%$
13. $F(x) = 0$ if $x < -1$, $F(x) = \frac{1}{2}(x+1)^2$ if $-1 \leqq x < 0$
$F(x) = 1 - \frac{1}{2}(x-1)^2$ if $0 \leqq x < 1$, $F(x) = 1$ if $x \leqq 1$
Answer: 500 cans, $P = 0.125, 0$
15. $X > b, X \geqq b, X < c, X \leqq c$, etc.

Problem Set 24.6, page 1038

1. $k = \frac{1}{2}, \mu = \frac{4}{3}, \sigma^2 = \frac{2}{9}$
3. $\mu = \pi, \sigma^2 = \pi^2/3$; cf. Example 2
5. $\mu = \frac{1}{4}, \sigma^2 = \frac{1}{16}$
7. $C = \frac{1}{2}, \mu = 2, \sigma^2 = 4$
9. 750, 1, 0.002
11. $c = 0.073$
13. \$643.50
15. $\frac{1}{2}, \frac{1}{20}, (X - \frac{1}{2})\sqrt{20}$
17. X = *Product of the 2 numbers*. $E(X) = 12.25$, 12 cents
19. $(0 + 1 \cdot 3 + 3 \cdot 8 + 1 \cdot 27)/8 = 54/8 = 6 \cdot 75$

Problem Set 24.7, page 1044

3. 38%
5. $\binom{5}{x}\, 0.5^5$, 0.03125, 0.15625, $1 - f(0) = 0.96875$, 0.96875
7. 0.265
9. $f(x) = 0.5^x e^{-0.5}/x!$, $f(0) + f(1) = e^{-0.5}(1.0 + 0.5) = 0.91$. *Answer:* 9%
11. $13\frac{1}{4}$ %
13. 42%, 47.2%, 10.5%, 0.3%
15. $1 - e^{-0.2} = 18\%$

Problem Set 24.8, page 1050

1. 0.1587, 0.5, 0.6915, 0.6247
3. 45.065, 56.978, 2.022
5. 15.9%
7. 31.1%, 95.4%
9. About 58%
11. $t = 1084$ hours
13. About 683 (Fig. 521a)

Problem Set 24.9, page 1059

1. $\frac{1}{8}, \frac{3}{16}, \frac{3}{8}$
3. $\frac{2}{9}, \frac{1}{9}, \frac{1}{2}$
5. $f_2(y) = 1/(\beta_2 - \alpha_2)$ if $\alpha_2 < y < \beta_2$
7. 27.45 mm, 0.38 mm
11. 25.26 cm, 0.0078 cm
13. 50%
15. The distributions in Prob. 17 and Example 1
17. No

Chapter 24 Review Questions and Problems, page 1060

11. $Q_L = 110$, $Q_M = 112$, $Q_U = 115$
13. $\bar{x} = 111.9$, $s = 4.0125$, $s^2 = 16.1$
21. $x_{\min} \leqq x_j \leqq x_{\max}$. Sum over j from 1.
17. $\bar{x} = 6$, $s = 3.65$
19. $f(x) = \binom{50}{x} 0.03^x 0.97^{50-x} \approx 1.5^x e^{-1.5}/x!$
21. $f(x) = 2^{-x}$, $x = 1, 2, \cdots$
23. $1, \frac{1}{2}$
25. 0.1587, 0.6306, 0.5, 0.4950

Problem Set 25.2, page 1067

1. In Example 1, $\mu = 0$ so $\sum_{j=1}^{n} x_j = 0$. $\partial \ln \ell/\partial \ell = 0$ and $\tilde{\sigma}^2$ is as before.
3. $\ell = e^{-n\mu}\mu^{(x_1+\cdots+x_n)}/(x_1!\cdots x_n!)$, $\partial \ln \ell/\partial\mu = -n + (x_1 + \cdots + x_n)/\mu = 0$, $n\hat{\mu} = n\bar{x}$, $\hat{\mu} = \bar{x} = 15.3$
5. $l = p^k(1-p)^{n-k}$, $\hat{p} = k/n$, $k =$ number of successes in n trails
7. 7/12
9. $l = f = p(1-p)^{x-1}$, etc., $\hat{p} = 1/x$
11. $\hat{\theta} = n/\Sigma x_j = 1/\bar{x}$
13. $\hat{\theta} = 1$
15. Variability larger than perhaps expected

Problem Set 25.3, page 1077

3. Shorter by a factor $\sqrt{2}$ **5.** 4, 16

7. $c = 1.96, \bar{x} = 126, s^2 = 126 \cdot 674/800 = 106.155, k = cs/\sqrt{n} = 0.714$, $\text{CONF}_{0.95}\{125.3 \leqq \mu \leqq 126.7\}$, $\text{CONF}_{0.95}\{0.1566 \leqq p \leqq 0.1583\}$

9. $\text{CONF}_{0.99}\{63.72 \leqq \mu \leqq 66.28\}$

11. $n - 1 = 5, F(c) = 0.995, c = 4.03, \bar{x} = 9533.33, s^2 = 49{,}666.67$, $k = 366.66$ (Table 25.2), $\text{CONF}_{0.99}\{9166.7 \leqq \mu \leqq 9900\}$

13. $\text{CONF}_{0.95}\{0.023 \leqq \sigma^2 \leqq 0.085\}$

15. $n - 1 = 99$ degrees of freedom. $F(c_1) = 0.025$, $c_1 = 74.2$, $F(c_2) = 0.975$, $c_2 = 129.6$. Hence $k_1 = 12.41$, $k_2 = 7.10$. $\text{CONF}_{0.95}\{7.10 \leqq \sigma^2 \leqq 12.41\}$.

17. $\text{CONF}_{0.95}\{0.74 \leqq \sigma^2 \leqq 5.19\}$

19. $Z = X + Y$ is normal with mean 105 and variance 1.25. *Answer:* $P(104 \leqq Z \leqq 106) = 63\%$

Problem Set 25.4, page 1086

3. $t = (0.286 - 0)/(4.31/\sqrt{7}) = 0.18 < c = 1.94$; accept the hypothesis.

5. $c = 6090 > 6019$: do not reject the hypothesis.

7. $\sigma^2/n = 1.8$, $c = 57.8$, accept the hypothesis.

9. $\mu < 58.69$ or $\mu > 61.31$

11. Alternative $\mu \neq 5000$, $t = (4990 - 5000)/(20/\sqrt{50}) = -3.54 < c = -2.01$ (Table A9, Appendix 5). Reject the hypothesis $\mu = 5000$ g.

13. Two-sided. $t = (0.55 - 0)/\sqrt{0.546/8} = 2.11 < c = 2.37$ (Table A9, Appendix 5), no difference

15. $19 \cdot 1.0^2/0.8^2 = 29.69 < c = 30.14$ (Table A10. Appendix 5), accept the hypothesis

17. By (12), $t_0 = \sqrt{16}(20.2 - 19.6)/\sqrt{0.16 + 0.36} > c = 1.70$. Assert that B is better.

Problem Set 25.5, page 1091

1. $\text{LCL} = 1 - 2.58 \cdot 0.02/2 = 0.974$, $\text{UCL} = 1.026$

3. 27

5. Choose 4 times the original sample size

9. $2.58\sqrt{0.0004}/\sqrt{2} = 0.036$, $\text{LCL} = 3.464$, $\text{UCL} = 3.536$

11. $\text{LCL} = np - 3\sqrt{np(1-p)}$, $\text{CL} = np$, $\text{UCL} = np + 3\sqrt{np(1-p)}$

13. In about 30% (5%) of the cases

15. $\text{LCL} = \mu - 3\sqrt{\mu}$ is negative in (b) and we set $\text{LCL} = 0$, $\text{CL} = \mu = 3.6$, $\text{UCL} = \mu + 3\sqrt{\mu} = 9.3$.

Problem Set 25.6, page 1095

1. 0.9825, 0.9384, 0.4060 **3.** 0.8187, 0.6703, 0.1353

5. $e^{-25\theta}(1 + 25\theta)$, $P(A; 1.5) = 94.5$, $\alpha = 5.5\%$ **7.** 19.5%, 14.7%

9. $(1 - \theta)^n + n\theta(1 - \theta)^{n-1}$ **11.** $(1 - \frac{1}{2})^3 + 3 \cdot \frac{1}{2}(1 - \frac{1}{2})^2 = \frac{1}{2}$

13. $\displaystyle\sum_{x=0}^{9} \binom{100}{x} 0.12^x\, 0.88^{100-x} = 22\%$ (by the normal approximation)

15. $(1 - \theta)^5$, $[\theta(1 - \theta)^{5-1}]' = 0$, $\theta = \frac{1}{6}$, $\text{AOQL} = 6.7\%$

Problem Set 25.7, page 1099

3. $\chi_0^2 = (40 - 50)^2/50 + (60 - 50)^2/50 = 4 > c = 3.84$; no
5. $\chi_0^2 = \frac{16}{10} > 11.07$; yes
7. $\chi_0^2 = 10.264 < 11.07$; yes
9. 42 even digits, accept.
13. $\chi_0^2 = \dfrac{(355 - 358.5)^2}{358.5} + \dfrac{(123 - 119.5)^2}{119.5} = 0.137 < c = 3.84$ (1 degree of freedom, 95%)
15. Combining the last three nonzero values, we have $K - r - 1 = 9$ ($r = 1$ since we estimated the mean, $\frac{10{,}094}{2608} \approx 3.87$). $\chi_0^2 = 12.8 < c = 16.92$. Accept the hypothesis.

Problem Set 25.8, page 1102

3. $(\frac{1}{2})^8 + 8 \cdot (\frac{1}{2})^8 = 3.5\%$ is the probability that 7 cases in 8 trials favor A under the hypothesis that A and B are equally good. Reject.
5. $(\frac{1}{2})^{18}(1 + 18 + 153 + 816) = 0.0038$
7. $\bar{x} = 9.67$, $s = 11.87$. $t_0 = 9.67/(11.87/\sqrt{15}) = 3.16 > c = 1.76$ ($\alpha = 5\%$). Hypothesis rejected.
9. Hypothesis $\tilde{\mu} = 0$. Alternative $\tilde{\mu} > 0$, $\bar{x} = 1.58$,
$t = \sqrt{10} \cdot 1.58/1.23 = 4.06 > c = 1.83$ ($\alpha = 5\%$). Hypothesis rejected.
11. Consider $y_j = x_j - \tilde{\mu}_0$.
13. $n = 8$; 4 transpositions, $P(T \leqq 4) = 0.007$. Assert that fertilizing increases yield.
15. $P(T \leqq 2) = 2.8\%$. Assert that there is an increase.

Problem Set 25.9, page 1111

1. $y = 0.98 + 0.495x$
3. $y = -11{,}457.9 + 43.2x$
5. $y = -10 + 0.55x$
7. $y = 0.5932 + 0.1138x$, $R = 1/0.1138$
9. $y = 0.32923 + 0.00032x$, $y(66) = 0.35035$
13. $c = 3.18$ (Table A9), $k_1 = 43.2$, $q_0 = 54{,}878$, $K = 1.502$,
$\text{CONF}_{0.95}\{41.7 \leqq \kappa_1 \leqq 44.7\}$.
15. $y - 1.875 = 0.067(x - 25)$, $3s_x^2 = 500$, $q_0 = 0.023$, $K = 0.021$,
$\text{CONF}_{0.95}\{0.046 \leqq \kappa_1 \leqq 0.088\}$

Chapter 25 Review Questions and Problems, page 1111

15. $\hat{\mu} = 20.325$, $\hat{\sigma}^2 = (\frac{7}{8})s^2 = 3.982$
17. $\text{CONF}_{0.99}\{27.94 \leqq \mu \leqq 34.81\}$
19. $c = 14.74 > 14.5$, reject μ_0; $\Phi((14.74 - 14.50)/\sqrt{0.025}) = 0.9353$
21. $2.58 \cdot \sqrt{0.00024}/\sqrt{2} = 0.028$, LCL $= 2.722$, UCL $= 2.778$
23. $\alpha = 1 - (1 - \theta)^6 = 5.85\%$, when $\theta = 0.01$. For $\theta = 15\%$ we obtain $\beta = (1 - \theta)^6 = 37.7\%$. If n increases, so does α, whereas β decreases.
25. $y = 3.4 - 1.85x$