



4NF – Multivalued Dependency

Fourth Normal Form – 4NF

- Relation should be in Boyce-Codd Normal Form
- It should have no multivalued dependency

<i>name</i>	<i>street</i>	<i>city</i>	<i>title</i>	<i>year</i>
C. Fisher	123 Maple St.	Hollywood	Star Wars	1977
C. Fisher	5 Locust Ln.	Malibu	Star Wars	1977
C. Fisher	123 Maple St.	Hollywood	Empire Strikes Back	1980
C. Fisher	5 Locust Ln.	Malibu	Empire Strikes Back	1980
C. Fisher	123 Maple St.	Hollywood	Return of the Jedi	1983
C. Fisher	5 Locust Ln.	Malibu	Return of the Jedi	1983

- BCNF?
- 4NF?

4NF – Multivalued Dependency

- Based on knowledge of real world
- All instances of relation must adhere

$$R \quad \underline{\bar{A} \twoheadrightarrow \bar{B}} \quad A_1, \dots, A_n \quad B_1, \dots, B_n$$


$\forall t, u \in R : t[\bar{A}] = u[\bar{A}] \text{ then}$

$\exists v \in R : v[\bar{A}] = t[\bar{A}] \text{ and}$

$v[\bar{B}] = t[\bar{B}] \text{ and}$

$v[\text{rest}] = u[\text{rest}]$

	\bar{A}	\bar{B}	rest
t	a	b ₁	r ₁
u	a	b ₂	r ₂
v	a	b ₁	r ₂
w	a	b ₂	r ₁



Example

<i>name</i>	<i>street</i>	<i>city</i>	<i>title</i>	<i>year</i>
C. Fisher	123 Maple St.	Hollywood	Star Wars	1977
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name \rightarrow street city

Apply(SSN, cName, hobby)

SSN \rightarrow cName SSN \rightarrow hobby

	SSN	cName	hobby
t	123	Stanford.	trumpet
u	123	Berkeley	tennis.
v	123	Stanford	tennis
w	123	Berkeley	trumpet
	:	:	:

Example

<i>name</i>	<i>street</i>	<i>city</i>	<i>title</i>	<i>year</i>
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name →→ street city

Example

Modified example

Apply(SSN, cName, hobby) ★

Reveal hobbies to colleges selectively ★

MVDs? *None*

Good design? *Yes.*

Properties

Trivial Multivalued Dependency

$$\overline{A} \twoheadrightarrow \overline{B} \quad \overline{B} \subseteq \overline{A} \text{ or } \overline{A} \cup \overline{B} = \text{all attributes}$$

Nontrivial MVD

otherwise.

Transitive Rule:

- $A \twoheadrightarrow B$ and $B \twoheadrightarrow C$ then $A \twoheadrightarrow C$

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name \twoheadrightarrow street city

name \twoheadrightarrow street

Properties

Complementation Rule:

- If $A \twoheadrightarrow B$ then $A \twoheadrightarrow \text{rest}$

The complementation rule says that

$\text{name} \twoheadrightarrow \text{street city}$

$\text{name} \twoheadrightarrow \text{title year}$

Rules for Multivalued Dependencies

FD-is-an-MVD rule

$\bar{A} \rightarrow \bar{B}$ then $\bar{A} \twoheadrightarrow \bar{B}$

$\bar{b}_1 = \bar{b}_2$

	\bar{A}	\bar{B}	rest
t	$\bar{a} \cdot$	\bar{b}_1	\bar{r}_1
u	$\bar{a} \cdot$	\bar{b}_2	\bar{r}_2
$\rightarrow v$	\bar{a}	$\bar{b}_1 = \bar{b}_2$	\bar{r}_2
		\vdots	

Rules for Multivalued Dependencies

Intersection rule

$$\overline{A} \twoheadrightarrow \overline{B} \quad \overline{A} \twoheadrightarrow \overline{C} \quad \text{then} \quad \overline{A} \twoheadrightarrow \overline{B} \cap \overline{C}$$

Transitive rule

$$\overline{A} \twoheadrightarrow \overline{B} \quad \overline{B} \twoheadrightarrow \overline{C} \quad \text{then} \quad \underline{\overline{A} \twoheadrightarrow \underline{\overline{C} - \overline{B}}}$$

4NF decomposition algorithm

Input: relation R + FDs for R + MVDs for R

Output: decomposition of R into 4NF relations with "lossless join"

Compute keys for R ✓

Repeat until all relations are in 4NF: ✓

Pick any R' with nontrivial $A \twoheadrightarrow B$ that violates 4NF

Decompose R' into $R_1(A, B)$ and $R_2(A, \text{rest})$

Compute FDs and MVDs for R_1 and R_2

Compute keys for R_1 and R_2

4NF Decomposition Example #1

Apply(SSN, cName, hobby)

SSN \Rightarrow cName No keys.

A1(SSN, cName)

No FDs
No MVDs

A2(SSN, hobby)



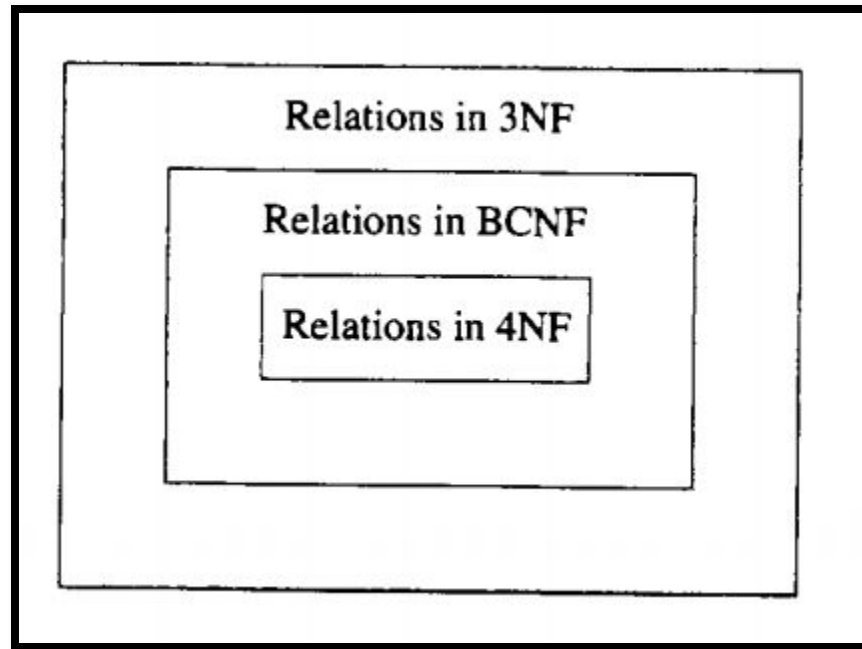
Example

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name \rightarrow street city

{name, street, city}
{name, title, year}

Relation among Normal Forms



Property	3NF	BCNF	4NF
Eliminates redundancy due to FD's	No	Yes	Yes
Eliminates redundancy due to MVD's	No	No	Yes
Preserves FD's	Yes	No	No
Preserves MVD's	No	No	No