

<b>Department of Computer Science, UET Lahore</b> Course Preparation Report			
Course Code		Semester	Spring 2024
Course Title	Graph Theory		
Subject Teachers (names of all teachers)	Tazeem Haider		
Course Type	Theory		

## Course Description

Graph theory is a branch of mathematics that studies the properties of graphs, which are mathematical structures used to model a wide variety of relationships between objects. Complex data can be represented as a graph of relationships between objects. Such networks are a fundamental tool for modeling social, technological, and biological systems. Graph theory provides a mathematical framework for representing and analyzing networks and systems, which is a useful tool in many fields, from computer science to physics. It provides a powerful toolset for solving problems in many fields, such as finding the shortest path between two points, scheduling tasks, and analyzing social networks. Graph theory is used in a wide variety of real-world applications, including transportation systems, electrical power grids, social networks, the World Wide Web, and many others. Learning graph theory also provides a solid theoretical background for understanding more advanced topics in related areas such as Machine Learning, Artificial Intelligence and Data Science. This course focuses on the computational, algorithmic, and modeling challenges specific to the analysis of massive graphs. At the end of this course, by means of studying the underlying graph structure and its features, students are introduced to machine learning techniques and data mining tools to reveal insights on a variety of networks.

## Course Learning Outcomes

CLO1	Understand and apply the fundamental concepts in graph theory
CLO2	Model problems using graphs and to solve these problems algorithmically.
CLO3	Describe the concepts of colorings, matching, flows, connectivity and planarity
CLO4	Apply graph theory concepts to solve real world applications

## Text Books

Please add the latest text books

T1	A Textbook of Graph Theory, 5 <sup>th</sup> Edition by R. Balakrishnan, K. Ranganathan
T2	Graph Theory & Applications, 3 <sup>rd</sup> Edition by Jean-Clause Fournier
T3	Introduction to Graph Theory, 2 <sup>nd</sup> Edition, by Douglas B. West.
T4	Graph Theory, 5 <sup>th</sup> edition by Reinhard Diestel

## Course Plan

Week wise content with mapping of text book

Week	Content	CLO	Text Book
W1	<b>Basic Concepts</b>  The origin of the graph concept, Definition of graphs, Notation, Representation, Isomorphism and unlabeled graphs, Complete graphs, Subgraphs  Customary notation, Paths and cycles, Paths, Cycles, Paths and cycles as graphs, Degrees, Regular graphs, Connectedness, Bipartite graphs, Characterization,  Algorithmic aspects, Representations of graphs inside a machine, Weighted	CLO1  CLO2	T2 Ch#1

	graphs		
W2	<b>Trees</b> Definitions and properties, First properties of trees, Forests, Bridges, Tree characterizations, Spanning trees, , Spanning trees in a weighted graph, Application: minimum spanning tree problem, Kruskal's algorithm, Connectivity, Block decomposition, k-connectivity, k-connected graphs, Menger's theorem, Edge connectivity, k-edge-connected graphs	CLO1 CLO2	T2 Ch#2
W3	<b>Connectivity</b> Introduction, Vertex Cuts and Edges Cuts, Connectivity and Edge Connectivity, Blocks, Cyclical Edge Connectivity of a Graph, Menger's Theorem	CLO1, CLO2, CLO3	T1 CH#3
W4	<b>Search Algorithms</b> Depth-first search of an arborescence, Iterative form, Visits to the vertices, Optimization of a sequence of decisions, Application to game theory: finding a winning strategy, The minimax algorithm, Pruning, Depth-first search of a digraph	CLO2, CLO4	T2 CH#5
W5	<b>Optimal Paths</b> Distances and shortest paths	CLO1, CLO2, CLO3	T2 CH#6

	<p>problems, Types of problems, Case of non-weighted digraphs: breadth-first search, Application to calculation of distances, Determining the shortest paths, Digraphs without circuits, Shortest paths, Longest paths, Formulas, Application to scheduling, Potential task graph, Earliest starting times, Latest starting times, Total slacks and critical tasks, Free slacks, More general constraints, Positive lengths, Associated shortest paths, Undirected graphs</p>		
W6	<p><b>Matchings</b></p> <p>Matchings and alternating paths, Concept of alternating paths and Berge's theorem, Matchings in bipartite graphs, Matchings and transversals, The Hungarian method, Concept of alternating trees, Maximum matching algorithm, Kuhn-Munkres algorithm</p>	CLO3	T2 CH#7
W7	<p><b>Graph Colorings</b></p> <p>Introduction, Vertex Colorings, Applications of Graph Coloring, Critical Graphs, Brooks' Theorem, Other Coloring Parameters, b-Colorings, Homomorphisms and Colorings, Quotient Graphs</p>	CLO3	T1 CH#7
W9	<p><b>Flows</b></p> <p>Flows in transportation networks,</p>	CLO3, CLO4	T2 CH#8

	<p>Interpretation, Single-source single-sink networks, The max-flow min-cut theorem, Concept of unsaturated paths, Maximum flow algorithm, Flow with stocks and demands, Revisiting theorems, Menger's theorem, Hall's theorem, Kőnig's theorem</p>		
W10	<p><b>Planarity</b></p> <p>Introduction, Planar and Nonplanar Graphs, Euler Formula and Its Consequences, <math>K_5</math> and <math>K_{3,3}</math> are Nonplanar Graphs, Dual of a Plane Graph, The Four-Color Theorem and the Heawood Five-Color Theorem, Kuratowski's Theorem, Hamiltonian Plane Graphs, Tait Coloring</p>	CLO3, CLO4	T1 CH#8
W11	<p><b>Euler Tours</b></p> <p>Euler trails and tours, Principal result, Algorithms</p> <p>Elimination of recursion, The Rosenstiehl algorithm, The Chinese postman problem, The Edmonds-Johnson algorithm</p> <p><b>Hamilton Cycles</b></p> <p>A few simple properties, The traveling salesman problem, Approximation of a difficult problem, Concept of approximate algorithms, Approximation of the metric TSP, An</p>	CLO3, CLO4	T2 CH#9, T2 CH#10

	approximate algorithm, Amelioration Christofides' algorithm, Upper and lower bounds for the optimal value		
W12	<b>Graph Databases:</b> Neo4J <b>Graph Query Languages:</b> Cypher	CLO4	NEO4J  Documentation + Cypher Query Language developer Guide + Graph Databases book by Ian Robinson
W13	<b>Graph – Knowledge Representation Learning:</b> <b>Basics of network and graph analysis (Introductory Concepts)</b> Link prediction Graph and node classification Spectral clustering Node embeddings Knowledge graph embeddings Graph signal processing Deep generative models of graphs	CLO4	
W14	<b>Graph Neural Networks (Introduction level)</b> Graph Convolutional Networks Graph Attention Networks Graph Transformers	CLO2, CLO4	
W15	<b>Identify using Graph Algorithms:</b> What's Important What's Unusual?	CLO2, CLO4	

	What's Next?		
W16	Smarter Predictive Analytics with Graph Graph Level Prediction	CLO2, CLO4	
W17	Graph Generation Graph Evolution	CLO2, CLO4	
W18	Graph visualization	CLO2, CLO4	
W19	Graph Centrality algorithms Graph Similarity algorithms Graph Pathfinding algorithms Node embedding algorithms Community detection Algorithms	CLO2, CLO4	