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11.1

1 $\cos x = 2\pi$; $\sin x = 2\pi$

$f(ax) = f/a$

$f\left(\frac{x}{a}\right) = af$

$\cos 2x = \pi$; $\sin 2x = \pi$

$\cos \pi x = 2$; $\sin \pi x = 2$

$\cos 2\pi x = 1$; $\sin 2\pi x = 1$

2 $\cos nx = \frac{2\pi}{n}$; $\sin nx = \frac{2\pi}{n}$

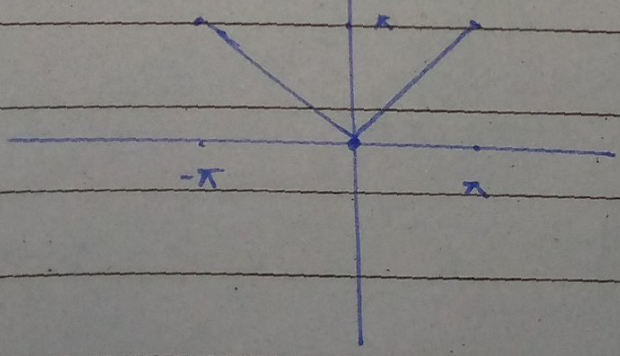
$\cos \frac{2\pi x}{k} = \frac{2\pi}{\left(\frac{2\pi}{k}\right)} = k$; $\sin \frac{2\pi x}{k} = \frac{2\pi}{\left(\frac{2\pi}{k}\right)} = k$

$\cos \frac{2\pi n x}{k} = \frac{2\pi}{\left(\frac{2\pi n}{k}\right)} = \frac{k}{n}$; $\sin \frac{2\pi n x}{k} = \frac{2\pi}{\left(\frac{2\pi n}{k}\right)} = \frac{k}{n}$

6 $f(x) = |x|$

$$f(x) = \begin{cases} -x & -\pi < x < 0 \\ x & 0 \le x < \pi \end{cases}$$

$$|x| = \begin{cases} -x & x < 0 \\ x & x \ge 0 \end{cases}$$



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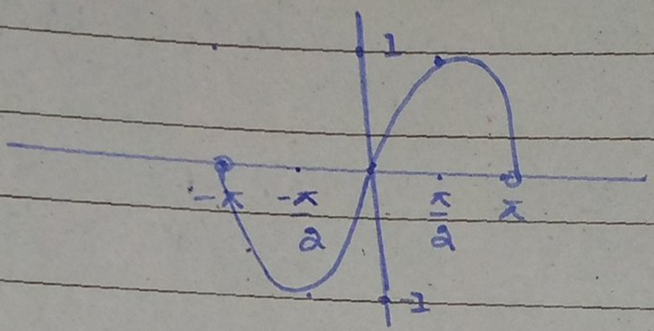
(7) $f(x) = |\sin x|$, $f(x) = \sin|x|$

$$f(x) = \begin{cases} -\sin x & -\pi < x < 0 \\ \sin x & 0 \le x < \pi \end{cases}$$

$$f(x) = \sin|x| = \begin{cases} -\sin x & -\pi < x < 0 \\ \sin x & 0 \le x < \pi \end{cases}$$

Graph will be same.



(8) $f(x) = e^{-|x|}$, $f(x) = |e^{-x}|$

$$f(x) = \begin{cases} e^{x} & -\pi < x < 0 \\ e^{-x} & 0 \le x < \pi \end{cases}$$

$$f(x) = \begin{cases} \cancel{-e^{-x}} & -\pi < x < 0 \\ e^{-x} & 0 \le x < \pi \end{cases}$$

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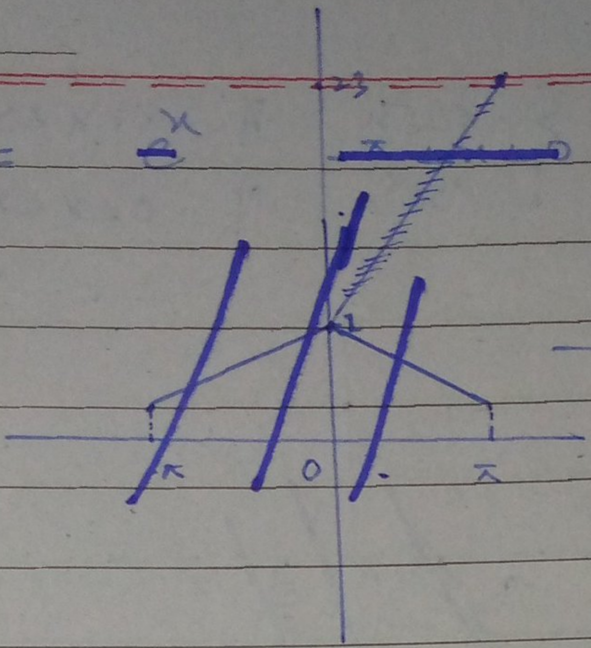
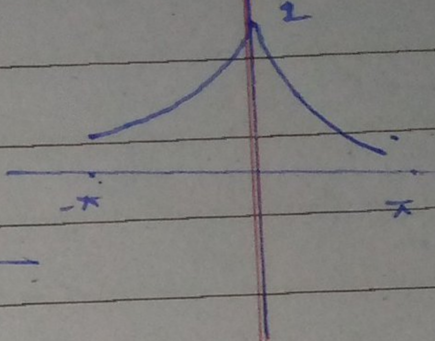
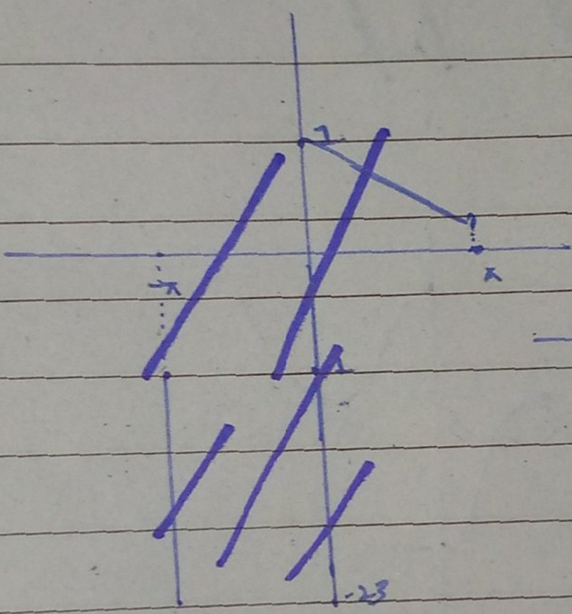
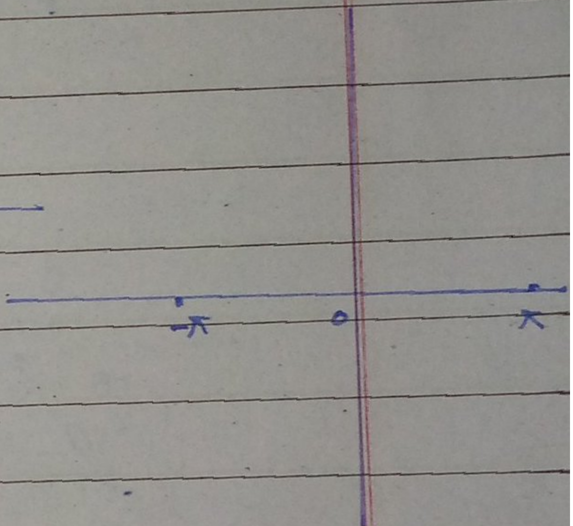
$f(x) = e^{x}$

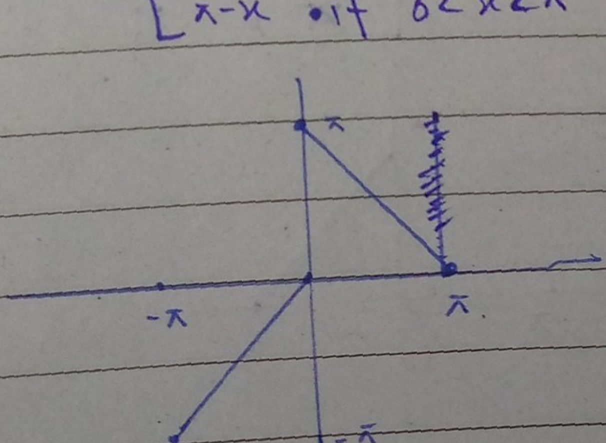








9 $f(x) = \begin{cases} x & \text{if } -\pi \le x < 0 \\ \pi - x & \text{if } 0 < x < \pi \end{cases}$



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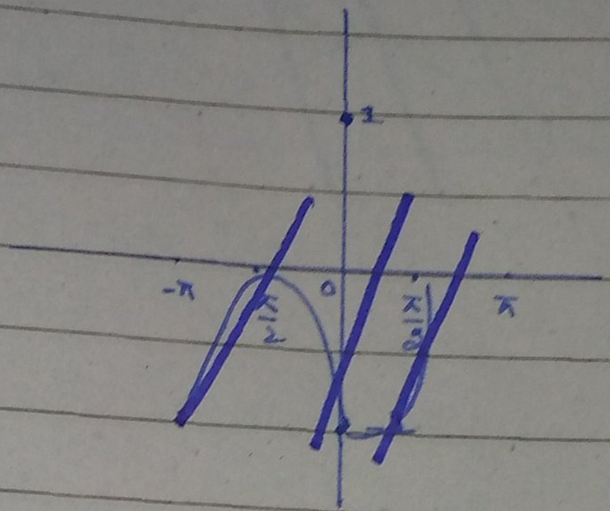
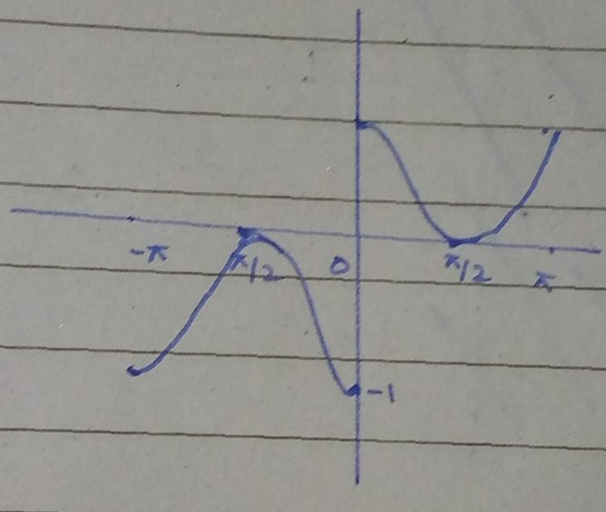
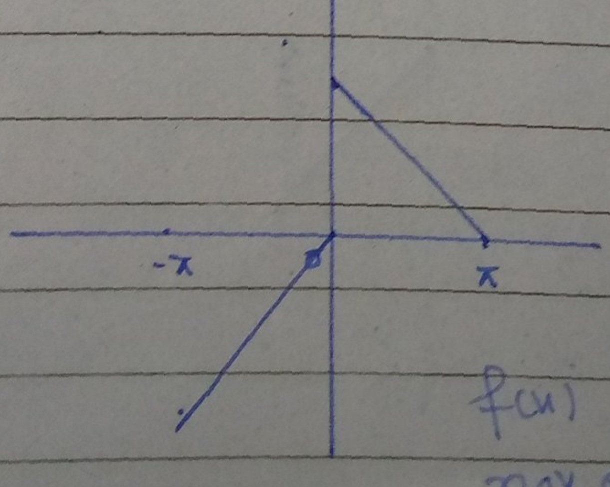
10 $f(x) = \begin{cases} -\cos^2 x & \text{if } -\pi < x < 0 \\ \cos^2 x & \text{if } 0 < x < \pi \end{cases}$





13. g $f(x) = \begin{cases} x & \text{if } -\pi < x < 0 \\ -x & \text{if } 0 < x < \pi \end{cases}$



f(x) is neither odd nor even.

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- $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} x\,dx + \int_{0}^{\pi}(\pi - x)\,dx\right]$

$a_0 = \frac{1}{2\pi}\left[\left|\frac{x^2}{2}\right|_{-\pi}^{0} + \left|\pi x - \frac{x^2}{2}\right|_{0}^{\pi}\right]$

$a_0 = \frac{1}{2\pi}\left[-\frac{\pi^2}{2} + \left(\pi^2 - \frac{\pi^2}{2}\right) - 0\right]$

$a_0 = \frac{1}{2\pi}\left[\frac{-\pi^2}{2} + \frac{2\pi^2 - \pi^2}{2}\right]$

$a_0 = \frac{1}{2\pi}\left[\frac{-\pi^2}{2} + \frac{\pi^2}{2}\right]$

$a_0 = 0$

- $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$

$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} x\,dx + \int_{0}^{\pi}(\pi - x)\,dx\right]\cos nx\,dx$

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$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} x\cos nx\,dx + \int_{0}^{\pi}(\pi - x)\cos nx\,dx\right]$$

$$\begin{array}{ccc|ccc}
x & & \cos nx & \pi - x & & \cos nx \\
 & + & & & + & \\
1 & & \frac{1}{n}\sin nx & -1 & & \frac{1}{n}\sin nx \\
 & - & & & - & \\
0 & & -\frac{1}{n^2}\cos nx & 0 & & -\frac{1}{n^2}\cos nx
\end{array}$$

$$a_n = \frac{1}{\pi}\left[\left|+\frac{1}{n^2}\cos nx\right|_{-\pi}^{0} + \left|-\frac{1}{n^2}\cos nx\right|_{0}^{\pi}\right]$$

$$a_n = \frac{1}{\pi}\left[\left(\frac{1}{n^2} - \frac{(-1)^n}{n^2}\right) - \frac{1}{n^2}\left[(-1)^n - 1\right]\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{1}{n^2}\left(1 - (-1)^n\right) - \frac{1}{n^2}\left((-1)^n - 1\right)\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{1}{n^2} - \frac{(-1)^n}{n^2} - \frac{(-1)^n}{n^2} + \frac{1}{n^2}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{2}{n^2} - \frac{2(-1)^n}{n^2}\right]$$

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$$= \frac{2}{\pi n^2}\left[1-(-1)^n\right]$$

$$a_n = \frac{2}{\pi n^2}\left(1-(-1)^n\right)$$

$$a_1 = \frac{2}{\pi}\cdot 2 = \frac{4}{\pi}$$

$$a_2 = 0$$

$$a_3 = \frac{2}{\pi 9}\left(1+1\right) = \frac{4}{9\pi}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} x\sin nx\,dx + \int_{0}^{\pi}(\pi-x)\sin nx\,dx\right]$$

x	$\sin nx$	:	$\pi-x$	$\sin nx$
1	$-\frac{1}{n}\cos nx$	:	-1	$-\frac{1}{n}\cos nx$
0	$-\frac{1}{n^2}\sin nx$	:	0	$-\frac{1}{n^2}\sin nx$

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$$b_n = \frac{1}{\pi}\left[\left| -\frac{1}{n} x\cos nx\right|_{-\pi}^{0} + \left| -(\pi - x)\left(+\frac{1}{n}\cos nx\right)\right|_{0}^{\pi}\right]$$

$$b_n = \frac{1}{\pi}\left[0 - \frac{\pi}{n}(-1)^n + \pi\left(+\frac{1}{n}(+1)\right)\right]$$

$$b_n = \frac{1}{\pi}\left[-\frac{\pi}{n}(-1)^n \cancel{- \frac{\pi}{n}(-1)^n} \quad \frac{\pi}{n}(1)\right]$$

$$\cancel{b_n = \frac{1}{\pi}\left[\frac{-2\pi(-1)^n}{n}\right]} = \left[-\frac{\pi}{n}(-1)^n + \frac{\pi}{n}\right]\frac{1}{\pi}$$

$$b_n = \frac{1}{n}\left(1 - (-1)^n\right)$$

$$\cancel{b_n = -\frac{2}{n}(-1)^n}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$$

$$= \sum_{n=1}^{\infty} \frac{2}{\pi n^2}\left(1 - (-1)^n\right)\cos nx + \frac{1}{n}\left(1 - (-1)^n\right)\sin nx$$

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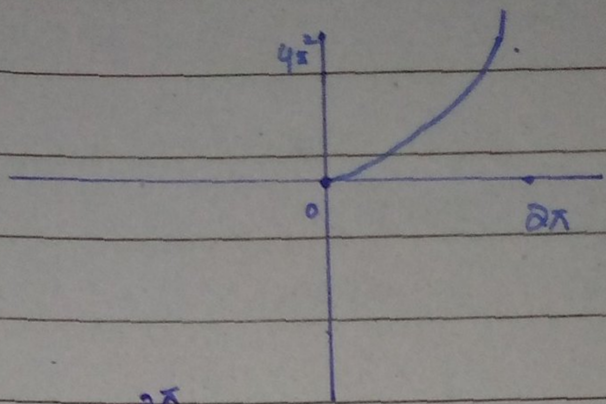
15 $f(x) = x^2 \quad 0 < x < 2\pi$



$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x^2\,dx = \frac{1}{2\pi}\left|\frac{x^3}{3}\right|_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[8\pi^3\right] = \frac{4}{3}\pi^2$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \cos nx$$

x^2	$\cos nx$
$2x$	$\frac{1}{n}\sin nx$
2	$-\frac{1}{n^2}\cos nx$
0	$\frac{-1}{n^3}\sin x$

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$$a_n = \frac{1}{\pi}\left| +2x\left(\frac{1}{n^2}\right)\cos nx \right|_0^{2\pi}$$

$$a_n = \frac{2}{n^2\pi}\left| x\cos nx \right|_0^{2\pi}$$

$$a_n = \frac{2}{n^2\pi}\left[2\pi(1)^n - 0\right]$$

$$a_n = \frac{2}{n^2\pi}\left(2\pi(1)^n\right) = \frac{4}{n^2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \sin nx\, dx$$

x^2	$\sin nx$
$2x$	$-\frac{1}{n}\cos nx$
2	$-\frac{1}{n^2}\sin nx$
0	$\frac{1}{n^3}\cos nx$

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$$b_n = \frac{1}{\pi}\left[-\frac{x^2}{n}\cos nx - \frac{2}{n^3}\cos nx\right]_0^{2\pi}$$

$$b_n = \frac{1}{\pi}\left[-\frac{4\pi^2}{n} - \frac{2}{n^3} + \frac{2}{n^3}\right]$$

$$b_n = \frac{1}{\pi}\left[-\frac{4\pi^2}{n} - \cancel{\frac{2}{n^3}} + \cancel{\frac{2}{n^3}}\right]$$

$$b_n = \frac{1}{\pi}\left(-\frac{4\pi^2}{n}\right) = -\frac{4\pi}{n}$$

$a_n = \frac{4}{n^2}$	$b_n = -\frac{4\pi}{n}$
$a_1 = 4$	$b_1 = -4\pi$
$a_2 = \frac{4}{4} = 1$	$b_2 = -2\pi$
$a_3 = \frac{4}{9}$	$b_3 = -\frac{4\pi}{3}$

Fourier Series is:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$f(x) = \frac{4}{3}\pi^2 + \sum_{n=1}^{\infty} \frac{4}{n^2}\cos nx + \sum_{n=1}^{\infty} \frac{-4\pi}{n}\sin nx$$

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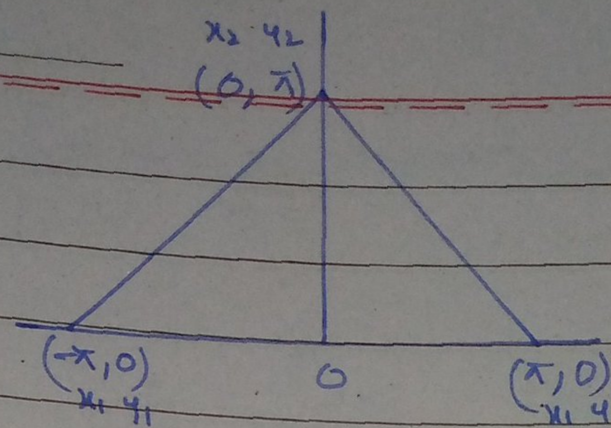

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{\pi - 0}{0 + \pi}(x + \pi)$$

$$y = \frac{\pi}{\pi}(x + \pi)$$

$$y = (x + \pi)$$

from $-\pi$ to 0

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{\pi - 0}{0 - \pi}(x - \pi)$$

$$y = \frac{\pi}{-\pi}(x - \pi)$$

$$y = \pi - x$$

from 0 to π

$$f(x) = \begin{cases} x + \pi & -\pi < x < 0 \\ \pi - x & 0 < x < \pi \end{cases}$$

Since, $f(x)$ is symmetric about y-axis, so it is an even function.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\cdot 2\int_{0}^{\pi} f(x)\,dx$$

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$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{\pi}\left[\int_0^{0} (x+\pi)\,dx + \int_0^{\pi} (\pi-x)\,dx\right]$$

$$a_0 = \frac{1}{\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{\pi}$$

$$a_0 = \frac{1}{\pi}\left[\pi^2 - \frac{\pi^2}{2}\right] = \frac{1}{\pi}\left[\frac{2\pi^2-\pi^2}{2}\right]$$

$$a_0 = \frac{1}{\pi}\left[\frac{\pi^2}{2}\right] = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx = \frac{1}{\pi}\cdot 2\int_0^{\pi} f(x)\cos nx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx$$

$$\cancel{a_n} = \cancel{\frac{2}{\pi}}\left[\cancel{(x+\pi)}\cos nx + \cancel{(\pi-x)\cos nx}\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi}\left[\int_0^{0} (x+\pi)\cos nx + \int_0^{\pi} (\pi-x)\cos nx\,dx\right]$$

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$$\begin{array}{ccc|ccc} x+\pi & & \cos nx & x-\pi & & \cos nx \\ 1 & & \frac{1}{n}\sin nx & 1 & & \frac{1}{n}\sin nx \\ 0 & & -\frac{1}{n^2}\cos nx & 0 & & -\frac{1}{n^2}\cos nx \end{array}$$

$$\frac{2}{\pi}\left[\left|\frac{1}{n^2}\cos nx\right|_{0}^{0} + \left| -\frac{1}{n^2}\cos nx\right|_{0}^{\pi}\right]$$

$$\frac{2}{\pi}\left[\frac{1}{n^2} - \frac{(-1)^n}{n^2}\right]$$

$$a_n = \frac{2}{n^2\pi}\left(1-(-1)^n\right)$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx$$

$$f(x) = \frac{\pi}{2} - \sum_{n=1}^{\infty} \frac{2}{n^2\pi}\left(1-(-1)^n\right)\cos nx$$

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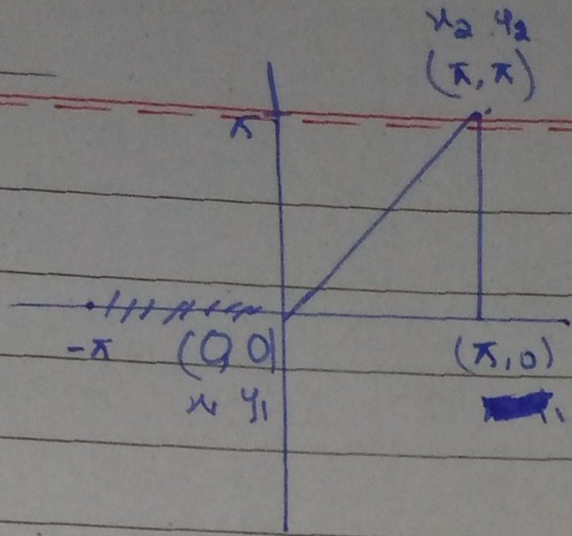

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\cancel{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$

$$y - 0 = \frac{\pi - 0}{\pi - 0}(x - 0)$$

$$\cancel{y - 0 = \frac{\pi - 0}{\pi - 0}(x - 0)}$$

$$\cancel{y = 0}$$

$$y = x$$

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} x\,dx\right]$$

$$a_0 = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{\pi} = \frac{1}{2\pi}\cdot\frac{1}{2}\pi^2 = \frac{\pi^2}{4}$$

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$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} x\cos nx\,dx\right]$$

$$a_n = \frac{1}{\pi}\int_{0}^{\pi} x\cos nx\,dx$$

$x \quad \cos nx$

$1 \quad \frac{1}{n}\sin nx$

$0 \quad -\frac{1}{n^2}\cos nx$

$$a_n = \frac{1}{\pi}\left|\frac{1}{n^2}\cos nx\right|_0^{\pi}$$

$$a_n = \frac{1}{n^2\pi}(-1)^n$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} x\sin nx\,dx\right]$$

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$$\frac{1}{\pi}\int_0^{\pi} x \sin nx \, dx$$

$$\begin{array}{ccc} x & & \sin nx \\ 1 & & -\frac{1}{n}\cos nx \\ 0 & & -\frac{1}{n^2}\sin nx \end{array}$$

$$= \frac{1}{\pi}\left| -\frac{1}{n} x \cos nx \right|_0^{\pi}$$

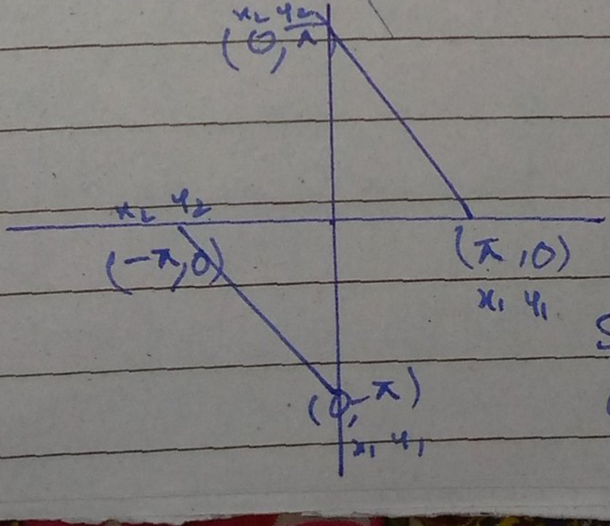
$$= \frac{1}{\pi}\left(-\frac{1}{n}\pi(-1)^n\right) = \frac{-1}{n\pi}\pi(-1)^n$$

$$b_n = \frac{-1}{n}(-1)^n$$

F.S

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left(\frac{1}{n^2\pi}(-1)^n \cos nx - \frac{1}{n}(-1)^n \sin nx\right)$$

Q1



Symmetric about origin. So, it is an odd function.

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$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y+\pi = \frac{0+\pi}{-\pi-0}(x-0)$$

$$y+\pi = \frac{\pi}{-\pi}(x)$$

$$y = -(x)-(\pi)$$

$$y = -x-\pi$$

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0 = \frac{\pi-0}{0-\pi}(x-\pi)$$

$$y = \frac{\pi}{-\pi}(x-\pi)$$

$$y = \pi-x$$

$$f(x) = \begin{cases} -x-\pi & -\pi < x < 0 \\ \pi-x & 0 < x < \pi \end{cases}$$

$$a_0 = 0$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-x-\pi)\sin nx\,dx + \int_{0}^{\pi}(\pi-x)\sin nx\,dx\right]$$

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$b_n = \frac{1}{\pi}$

$-x-\pi$	$\sin nx$	$\pi - x$	$\sin nx$
-1	$-\frac{1}{n}\cos nx$	-1	$-\frac{1}{n}\cos nx$
0	$-\frac{1}{n^2}\sin nx$	0	$-\frac{1}{n^2}\sin nx$

$$b_n = \frac{1}{\pi}\left[\left| (-x-\pi)\left(-\frac{1}{n}\right)\cos nx \right|_{-\pi}^{0} + \left| (\pi - x)\left(-\frac{1}{n}\right)\cos nx \right|_{0}^{\pi} \right]$$

$$b_n = \frac{1}{\pi}\left[\left(\frac{-\pi}{n}(-1) - 0\right) + 0 - (\pi)\left(-\frac{1}{n}\right)(1) \right]$$

$$b_n = \frac{1}{\pi}\left[+\frac{\pi}{n} + \frac{\pi}{n} \right]$$

$$b_n = \frac{1}{\pi}\left[\frac{2\pi}{n} \right] = \frac{2}{n}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$f(x) = \sum_{n=1}^{\infty} \left(\frac{2}{n} \sin nx \right)$$