$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = \frac{dE}{dt}(t). \tag{33}$$

Suitable initial conditions on u, Q, or I must also be prescribed.

We have noted previously, in Section 3.7, that Eq. (31) for the spring–mass system and Eq. (32) or (33) for the electric circuit are identical mathematically, differing only in the interpretation of the constants and variables appearing in them. There are other physical problems that also lead to the same differential equation. Thus, once the mathematical problem is solved, its solution can be interpreted in terms of whichever corresponding physical problem is of immediate interest.

In the problem lists following this and other sections in this chapter are numerous initial value problems for second order linear differential equations with constant coefficients. Many can be interpreted as models of particular physical systems, but usually we do not point this out explicitly.

OBLEMS

In each of Problems 1 through 10 find the inverse Laplace transform of the given function.

1. $F(s) = \dfrac{3}{s^2+4}$
2. $F(s) = \dfrac{4}{(s-1)^3}$
3. $F(s) = \dfrac{2}{s^2+3s-4}$
4. $F(s) = \dfrac{3s}{s^2-s-6}$
5. $F(s) = \dfrac{2s+2}{s^2+2s+5}$
6. $F(s) = \dfrac{2s-3}{s^2-4}$
7. $F(s) = \dfrac{2s+1}{s^2-2s+2}$
8. $F(s) = \dfrac{8s^2-4s+12}{s(s^2+4)}$
9. $F(s) = \dfrac{1-2s}{s^2+4s+5}$
10. $F(s) = \dfrac{2s-3}{s^2+2s+10}$

In each of Problems 11 through 23 use the Laplace transform to solve the given initial value problem.

11. $y'' - y' - 6y = 0;\quad y(0) = 1,\quad y'(0) = -1$
12. $y'' + 3y' + 2y = 0;\quad y(0) = 1,\quad y'(0) = 0$
13. $y'' - 2y' + 2y = 0;\quad y(0) = 0,\quad y'(0) = 1$
14. $y'' - 4y' + 4y = 0;\quad y(0) = 1,\quad y'(0) = 1$
15. $y'' - 2y' + 4y = 0;\quad y(0) = 2,\quad y'(0) = 0$
16. $y'' + 2y' + 5y = 0;\quad y(0) = 2,\quad y'(0) = -1$
17. $y^{(4)} - 4y''' + 6y'' - 4y' + y = 0;\quad y(0) = 0,\quad y'(0) = 1,\quad y''(0) = 0,\quad y'''(0) = 1$
18. $y^{(4)} - y = 0;\quad y(0) = 1,\quad y'(0) = 0,\quad y''(0) = 1,\quad y'''(0) = 0$
19. $y^{(4)} - 4y = 0;\quad y(0) = 1,\quad y'(0) = 0,\quad y''(0) = -2,\quad y'''(0) = 0$
20. $y'' + \omega^2 y = \cos 2t,\quad \omega^2 \neq 4;\quad y(0) = 1,\quad y'(0) = 0$

14. $F(s) = \dfrac{\quad}{(s-a)^2+b^2}$, ... $(s-a)^2$

16. $F(s) = \dfrac{2as}{(s^2+a^2)^2}, \quad s > 0$

17. $F(s) = \dfrac{s^2+a^2}{(s-a)^2(s+a)^2}, \quad s > |a|$

18. $F(s) = \dfrac{n!}{(s-a)^{n+1}}, \quad s > a$

19. $F(s) = \dfrac{2a(3s^2-a^2)}{(s^2+a^2)^3}, \quad s > 0$

20. $F(s) = \dfrac{2a(3s^2+a^2)}{(s^2-a^2)^3}, \quad s > |a|$

21. Converges

22. Converges

23. Diverges

24. Converges

26. (d) $\Gamma(3/2) = \sqrt{\pi}/2; \quad \Gamma(11/2) = 945\sqrt{\pi}/32$

Section 6.2, page 320

1. $f(t) = \frac{3}{2}\sin 2t$
2. $f(t) = 2t^2 e^t$
3. $f(t) = \frac{2}{5}e^t - \frac{2}{5}e^{-4t}$
4. $f(t) = \frac{9}{5}e^{3t} + \frac{6}{5}e^{-2t}$
5. $f(t) = 2e^{-t}\cos 2t$
6. $f(t) = 2\cosh 2t - \frac{3}{2}\sinh 2t$
7. $f(t) = 2e^t\cos t + 3e^t\sin t$
8. $f(t) = 3 - 2\sin 2t + 5\cos 2t$
9. $f(t) = -2e^{-2t}\cos t + 5e^{-2t}\sin t$
10. $f(t) = 2e^{-t}\cos 3t - \frac{5}{3}e^{-t}\sin 3t$
11. $y = \frac{1}{5}(e^{3t} + 4e^{-2t})$
12. $y = 2e^{-t} - e^{-2t}$
13. $y = e^t\sin t$
14. $y = e^{2t} - te^{2t}$
15. $y = 2e^t\cos\sqrt{3}t - (2/\sqrt{3})e^t\sin\sqrt{3}t$
16. $y = 2e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t$
17. $y = te^t - t^2e^t + \frac{2}{3}t^3e^t$
18. $y = \cosh t$
19. $y = \cos\sqrt{2}t$
20. $y = (\omega^2-4)^{-1}[(\omega^2-5)\cos\omega t + \cos 2t]$
21. $y = \frac{1}{5}(\cos t - 2\sin t + 4e^t\cos t - 2e^t\sin t)$
22. $y = \frac{1}{5}(e^{-t} - e^t\cos t + 7e^t\sin t)$
23. $y = 2e^{-t} + te^{-t} + 2t^2e^{-t}$
24. $Y(s) = \dfrac{s}{s^2+4} + \dfrac{1-e^{-\pi s}}{s(s^2+4)}$
25. $Y(s) = \dfrac{1}{s^2(s^2+1)} - \dfrac{e^{-s}(s+1)}{s^2(s^2+1)}$
26. $Y(s) = (1-e^{-s})/s^2(s^2+4)$

29. $F(s) = 1/(s-a)^2$

30. $F(s) = 2b(3s^2-b^2)/(s^2+b^2)^3$