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Exercise 11.2

$f(-x) = x$ Even

$f(-x) = -x$ odd

(1) e^x

$f(x) = e^x$, $f(-x) = e^{-x}$

Neither even nor odd.

$f(x) = e^{-|x|} = e^{-|-x|} = e^{-|x|} \Rightarrow$ Even function

$x^3 \cos nx = (-x)^3 \cos n(-x) = -(x^3 \cos nx)$ odd function

$x^2 \tan \pi x = (-x)^2 \tan \pi(-x) = -x \tan \pi x$ odd function

$\sinh x - \cosh x = \sinh(-x) - \cosh(-x)$

$= -\sinh(x) - \cosh x$

Neither Even NOR odd.

2 $\sin^2 x = [\sin(-x)]^2 = \sin^2 x$ Even function

$\sin(x^2) = \sin((-x))^2 = \sin(x^2)$ Even fun

$\ln x = \ln(-x) =$ ~~odd function~~

$\ln(-x)$ is not defined.

So, neither even nor odd

$\frac{x}{x^2+1}$, $\frac{-x}{(-x)^2+1} = -\frac{x}{x^2+1} \Rightarrow$ odd funct

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0 $x\cot x = (-x)\cot(-x) = +x\cot x$ Even fun

3 Sum & product of even functions is even.

4. Sum of odd functions is odd
Product of odd functions is even.

5 Absolute values of odd functions is even.

6 Product of an odd times an even function is even.

7 $f(-x) = -f(x)$ — ①

$f(-x) = f(x)$ — ②

From ① & ②

$-f(x) = f(x)$

$2f(x) = 0$

$f(x) = 0$

zero function is both even & odd function.

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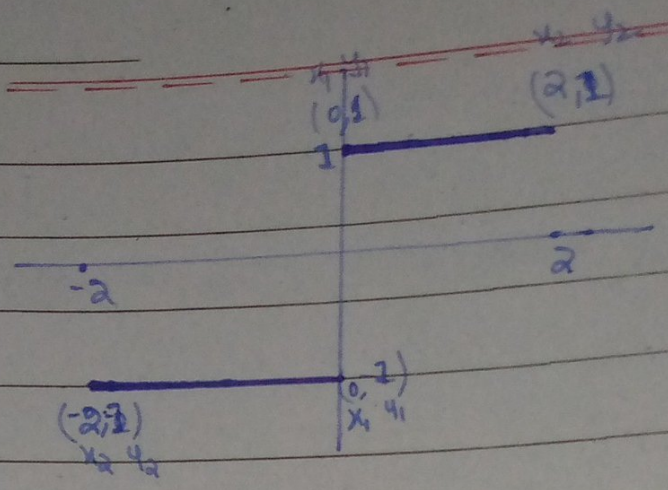

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y+1 = \frac{-1+1}{-2-0}(x-0)$$

$$y-1 = \frac{1-1}{2-0}(x-0)$$

$$y+1 = 0$$

$$y-1 = 0$$

$$y = 0$$

$$y = 1$$

$$y = -1$$

$$y = -1$$

$$f(x) = \begin{cases} -1 & -2<x<0 \\ 1 & 0<x<2 \end{cases}$$

$$2L = 4$$

$$L = 2$$

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$a_0 = 0$

$a_n = 0$

because the function is symmetric about origin so, these term will become zero.

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x) \sin\frac{n\pi x}{L}\, dx$$

Since, above function is even so,

$$b_n = \frac{2}{L}\int_{0}^{L} f(x) \sin\frac{n\pi x}{L}\, dx$$

By putting the value of L.

$$b_n = \frac{2}{2}\int_{0}^{2} f(x) \sin\frac{n\pi x}{2}\, dx$$

$$b_n = \int_{0}^{2} f(x) \sin\frac{n\pi x}{2}\, dx$$

$$b_n = \int_{0}^{2} (-1) \sin\frac{n\pi x}{2}\, dx + \int_{0}^{2} (1) \sin\frac{n\pi x}{2}\, dx$$

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$$b_n = 0 + \int_0^2 \sin \frac{n\pi x}{2}\, dx$$

$$b_n = \left| -\cos \frac{n\pi x}{2} \cdot \frac{2}{n\pi} \right|_0^2$$

$$b_n = \frac{-2}{n\pi} \left| \cos \frac{n\pi x}{2} \right|_0^2$$

$$b_n = \frac{-2}{n\pi} \left[\cos n\pi - 1 \right]$$

$$b_n = -\frac{2}{n\pi} \left[(-1)^n - 1 \right] = \begin{cases} 0 & n \text{ is even} \\ \frac{4}{n\pi} & n \text{ is odd} \end{cases}$$

<u>Fourier Series is:</u>

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L} = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin \frac{n\pi x}{2}$$

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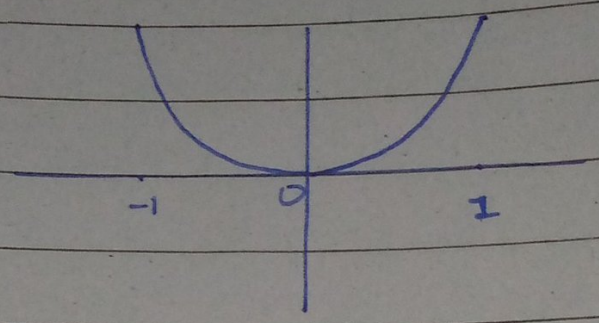
11 $f(x) = x^2 \quad (-1 < x < 1), \; p = 2$



The graph of a function is symmetric about y-axis. So, it is an even function. We will have fourier cosine series.

Also,

$$2L = 2$$
$$L = 1$$

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\,dx$$

Since, $f(x)$ is even so formula becomes

$$a_0 = \frac{2}{2L} \int_{0}^{L} f(x)\,dx = \frac{1}{L} \int_{0}^{L} f(x)\,dx$$

Putting the value of L.

$$a_0 = \int_{0}^{1} f(x)\,dx$$

$$f(x) = \begin{cases} x^2 & -1 < x < 0 \\ x^2 & 0 < x < 1 \end{cases}$$

Because $f(x)$ is a continuous function. i.e. (L.H.L = R.H.L)

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$$a_0 = \int_0^1 x^2 \, dx$$

$$a_0 = \left| \frac{x^3}{3} \right|_0^1 = \frac{1}{3}\left[1 - 0\right] = \frac{1}{3}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x) \cos\frac{n\pi x}{L} \, dx$$

Since, $f(x)$ is an even function.

$$a_n = \frac{2}{L}\int_0^L f(x) \cos\frac{n\pi x}{L} \, dx$$

putting the value of L

$$a_n = 2\int_0^1 x^2 \cos n\pi x \, dx$$

x^2		$\cos n\pi x$
$2x$	$+$	$\frac{1}{n\pi}\sin n\pi x$
2	$-$	$-\frac{1}{n^2\pi^2}\cos n\pi x$
0	$+$	$-\frac{1}{n^3\pi^3}\sin n\pi x$

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$$a_n = 2\left[\cancel{\frac{2}{n\pi}\sin n\pi} + 2x \cdot \frac{1}{n^2\pi^2}\cos n\pi x \cancel{- \frac{1}{n^3\pi^3}\sin n\pi x} \right]_0^1$$

$$a_n = 2\left[\left(\cancel{\frac{1}{n\pi}\sin n\pi} + \frac{2}{n^2\pi^2}\cos n\pi \cancel{- \frac{1}{n^3\pi^3}}\right) - \left(0 + 0 \cancel{- 0}\right)\right]$$

$$a_n = \cancel{\frac{2}{n\pi}\sin n\pi} + \frac{4}{n^2\pi^2}\cos n\pi \cancel{- \frac{4}{n^3\pi^3}}$$

$$a_n = \cancel{0} + \frac{4}{n^2\pi^2}(-1)^n \cancel{- \frac{4}{n^3\pi^3}}$$

$$a_n = \frac{4}{n^2\pi^2}(-1)^n$$

Fourier Series is:

$$\cancel{f(x) = a_0 + \sum_{n=1}^{\infty} a_n}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$

$$f(x) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2}\cos(n\pi x)$$

$$f(x) = \frac{1}{3} + \frac{4}{1^2\pi^2}\cos\pi x + \frac{4}{2^2\pi^2}\cos(2\pi x) + \frac{4}{3^2\pi^2}\cos(3\pi x) \ldots$$

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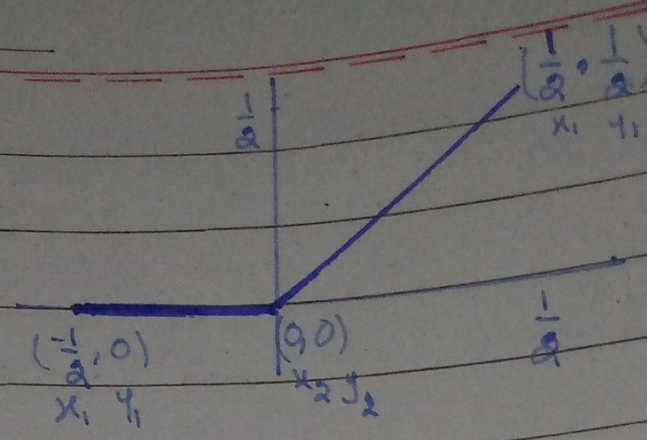

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{0 - 0}{0 + \frac{1}{2}}\left(x + \frac{1}{2}\right)$$

$$y = 0$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - \frac{1}{2} = \frac{0 - \frac{1}{2}}{0 - \frac{1}{2}}\left(x - \frac{1}{2}\right)$$

$$y - \frac{1}{2} = 1\left(x - \frac{1}{2}\right)$$

$$y = x - \frac{1}{2} + \frac{1}{2}$$

$$y = x$$

$$f(x) = \begin{cases} 0 & -\frac{1}{2} < x < 0 \\ x & 0 < x < \frac{1}{2} \end{cases}$$

$$2L = 1$$

$$L = \frac{1}{2}$$

The graph of the function is neither ~~odd~~ symmetric about y-axis nor origin. So,

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it is ar neither odd nor even function.

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

putting the value of L.

$$a_0 = \frac{1}{2\cdot\frac{1}{2}}\int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx$$

$$a_0 = \int_{-\frac{1}{2}}^{0} 0\,dx + \int_{0}^{\frac{1}{2}} x\,dx$$

$$a_0 = \left|\frac{x^2}{2}\right|_0^{\frac{1}{2}} = \frac{1}{2}\left[\left(\frac{1}{2}\right)^2 - (0)^2\right]$$

$$a_0 = \frac{1}{2}\left[\frac{1}{4}\right] = \frac{1}{8}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$a_n = \frac{1}{\frac{1}{2}}\int_{-1/2}^{1/2} f(x)\cos\frac{n\pi x}{1/2}\,dx$$

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$$a_n = 2\int_{-1/2}^{1/2} f(x)\cos 2n\pi x\,dx$$

$$\cancel{a_n = 2\int_{-1/2}^{0} 0\,du + 2\int_{0}^{1/2} x\,dx}$$

$$a_n = 2\int_{-\frac{1}{2}}^{0} 0.\cos 2n\pi x\,du + 2\int_{0}^{\frac{1}{2}} x.\cos 2n\pi x\,du$$

$$a_n = 2\int_{0}^{\frac{1}{2}} x\cos(2n\pi u)\,du$$

x	$\cos(2n\pi u)$
1	$\frac{1}{2n\pi}\sin(2n\pi u)$
0	$-\frac{1}{4n^2\pi^2}\cos(2n\pi u)$

$$a_n = 2\left|\frac{1}{4\pi^2 n^2}\cos(2n\pi u)\right|_0^{1/2}$$

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$$a_n = \frac{1}{2\pi^2 n^2} \left| \cos 2n\pi x \right|_0^{1/2}$$

$$a_n = \frac{1}{2\pi^2 n^2} \left[(-1)^n - 1 \right]$$

$$a_n = \frac{(-1)^n - 1}{2\pi^2 n^2}$$

★ $$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L} \, dx$$

$$b_n = \frac{1}{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} f(x) \sin 2n\pi x \, dx$$

$$b_n = 2 \int_{-\frac{1}{2}}^{0} 0 \sin 2n\pi x \, dx + 2 \int_{0}^{\frac{1}{2}} x \sin 2n\pi x \, d$$

$$b_n = 2 \int_{0}^{1/2} x \sin 2n\pi x \, dx$$

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x	$\sin 2n\pi x$	
1	$-\frac{1}{2n\pi}\cos 2n\pi x$	$\times$
0	$\frac{-1}{4n^2\pi^2}\sin 2n\pi x$	$-$

$$b_n = 2\left[-x\frac{1}{2n\pi}\cos 2n\pi x\right]_0^{1/2}$$

$$b_n = -\frac{1}{n\pi}\left[x\cos 2n\pi x\right]_0^{1/2}$$

$$\cancel{-\frac{1}{n\pi}\left[(-1)^n - 1\right] - (-1)^n}$$

$$b_n = -\frac{1}{n\pi}\left[\frac{1}{2}(-1)^n - 0\right]$$

$$b_n = \frac{-1}{2n\pi}\left[(-1)^n - 0\right]$$

$$b_n = \frac{(-1)^n}{2n\pi}$$

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$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}$$

$$f(x) = \frac{1}{8} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{2\pi^2 n^2} \cos\frac{n\pi x}{L} + \frac{(-1)^n}{2n\pi} \sin\frac{n\pi x}{L}$$

$$f(x) = \frac{1}{8} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{2\pi^2 n^2} \cos 2n\pi x + \frac{(-1)^n}{2n\pi} \sin 2n\pi x$$

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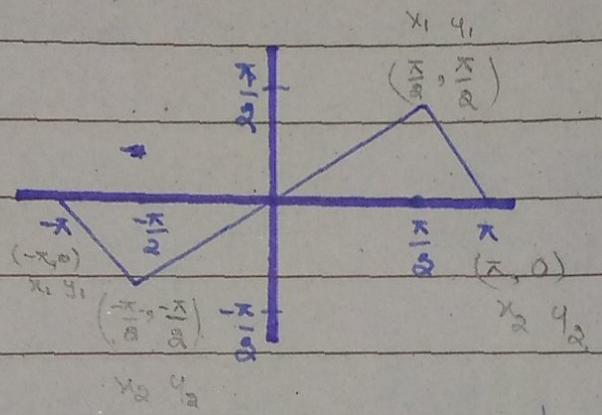

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{-\frac{\pi}{2} - 0}{-\frac{\pi}{2} + \pi}(x + \pi)$$

$$y = \frac{-\frac{\pi}{2}}{-\pi + \frac{2\pi}{2}}(x + \pi)$$

$$y = \frac{-\pi}{2} \cdot \frac{2}{\pi}(x + \pi)$$

$$y = -(x + \pi)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - \frac{\pi}{2} = \frac{-\frac{\pi}{2} - \frac{\pi}{2}}{-\frac{\pi}{2} - \frac{\pi}{2}}\left(x - \frac{\pi}{2}\right)$$

$$y - \frac{\pi}{2} = \frac{-\frac{2\pi}{2}}{-\frac{2\pi}{2}}\left(x - \frac{\pi}{2}\right)$$

$$y - \frac{\pi}{2} = x - \frac{\pi}{2}$$

$$y = x$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - \frac{\pi}{2} = \frac{0 - \frac{\pi}{2}}{\pi - \frac{\pi}{2}}\left(x - \frac{\pi}{2}\right)$$

$$y - \frac{\pi}{2} = \frac{-\frac{\pi}{2}}{\frac{\pi}{2}}\left(x - \frac{\pi}{2}\right)$$

$$y = -x + \frac{\pi}{2} + \frac{\pi}{2}$$

$$y = -x + \pi$$

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$$f(x) = \begin{cases} -x-\pi & -\pi < x < -\frac{\pi}{2} \\ x & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ -x+\pi & \frac{\pi}{2} < x < \pi \end{cases}$$

$2L = 2\pi$

$L = \pi$

Since, $f(x)$ is symmetric about origin, so, it is an odd function. We will have fourier sine series.

$a_0 = 0$

$a_n = 0$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x) \sin\frac{n\pi x}{L}\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\, dx$$

Since, above function is an even fn,

$$b_n = \frac{2}{\pi}\int_{0}^{\pi} f(x) \sin nx\, dx$$

$$b_n = \frac{2}{\pi}\int_{0}^{\frac{\pi}{2}} x \sin nx\, dx + \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} (-x+\pi) \sin nx\, dx$$

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~~Ex~~

x	$\sin nx$		$-x+\pi$	$\sin nx$
1	$-\frac{1}{n}\cos nx$		-1	$-\frac{1}{n}\cos nx$
0	$-\frac{1}{n^2}\sin nx$		0	$-\frac{1}{n^2}\sin nx$

$$\cancel{b_n = \frac{2}{\pi}\left[x\left(-\frac{1}{n}\cos nx\right)\right]_0^{\pi}}$$

$$b_n = \frac{2}{\pi}\left[x\left(-\frac{1}{n}\cos nx\right) + \frac{1}{n^2}\sin nx\right]_0^{\frac{\pi}{2}} + \frac{2}{\pi}\left[(-x+\pi)\left(\frac{-1}{n}\cos nx\right) - \frac{1}{n^2}\sin nx\right]_{\frac{\pi}{2}}^{\pi}$$

$$b_n = \frac{2}{\pi}\left[\left(0 + \frac{1}{n^2}\sin\frac{n\pi}{2}\right) - (0+0)\right] + \frac{2}{\pi}\left[(0-0) - \left(\cancel{\frac{-\pi}{2}\cdot\frac{0}{n}(-1)^n} - \frac{1}{n^2}\sin\frac{n\pi}{2}\right)\right]$$

$$b_n = \frac{2}{\pi}\left[\frac{1}{n^2}\sin\frac{n\pi}{2}\right] + \frac{2}{\pi}\left[\cancel{\frac{0}{n}(-1)^n} + \frac{1}{n^2}\sin\frac{n\pi}{2}\right]$$

$$b_n = \frac{2}{\pi n^2}\sin\frac{n\pi}{2} + \frac{2}{\pi n^2}\sin\frac{n\pi}{2}$$

$$b_n = \frac{4}{\pi n^2}\sin\frac{n\pi}{2}$$

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$$f(x) = \sum_{n=1}^{\infty} \cancel{f(x)}\, b_n \sin\frac{n\pi x}{L}$$

$$f(x) = \sum_{n=1}^{\infty} \left(\frac{4}{\pi n^2} \sin\frac{n\pi}{2} \right) \sin\frac{n\pi x}{\pi}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{4}{\pi n^2} \sin\left(\frac{n\pi}{2}\right) \cdot \sin(nx)$$

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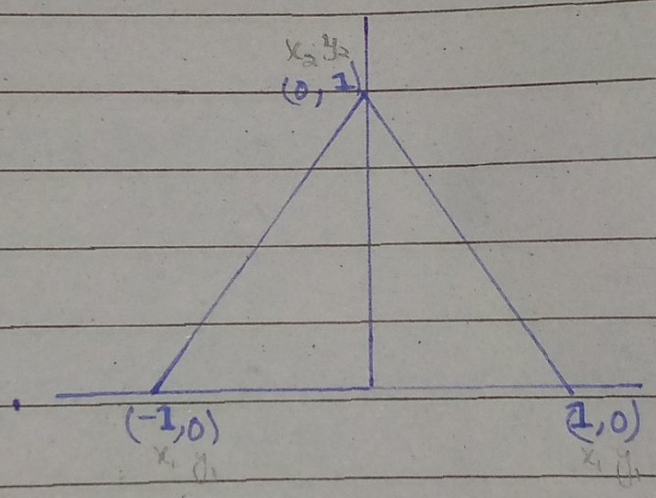

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{1-0}{0+1}(x+1)$$

$$y = x+1$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{1-0}{0-1}(x-1)$$

$$y = -1(x-1)$$

$$y = -x+1$$

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$$f(x) = \begin{cases} x+1 & -1 < x < 0 \\ -x+1 & 0 < x < 1 \end{cases}$$

$2L = 2$

$L = 1$

The graph of function is symmetric about y-axis. So, it is an even function. So we will have fourier cosine series.

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\,dx$$

$f(x)$ is an even. So,

$$a_0 = \frac{2}{2L} \int_{0}^{L} f(x)\,dx$$

$$a_0 = \frac{1}{L} \int_{0}^{L} f(x)\,dx$$

$\because L = 1$

$$a_0 = \int_{0}^{1} f(x)\,dx$$

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$$a_0 = \int_0^1 (-x+1)\,dx$$

$$a_0 = \left| \frac{-x^2}{2} + x \right|_0^1$$

$$a_0 = \left[\left(\frac{-1}{2} + 1 \right) - (0) \right] = \frac{1}{2}$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L}\,dx$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L}\,dx$$

$$a_n = \frac{2}{1} \int_0^1 f(x) \cos n\pi x\,dx$$

$$a_n = 2 \int_0^1 (-x+1) \cos n\pi x\,dx$$

$-x+1$	$+$	$\cos n\pi x$
-1	$-$	$\frac{1}{n\pi} \sin n\pi x$
0		$\frac{-1}{n^2\pi^2} \cos n\pi x$

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$$a_n = 2\left[-\frac{1}{n^2\pi^2}\cos n\pi x\right]_0$$

$$a_n = \frac{-2}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$a_n = \frac{2}{n^2\pi^2}\left[1 - (-1)^n\right]$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}\,dx$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2}\left(1-(-1)^n\right)\cos(n\pi x)\,d$$

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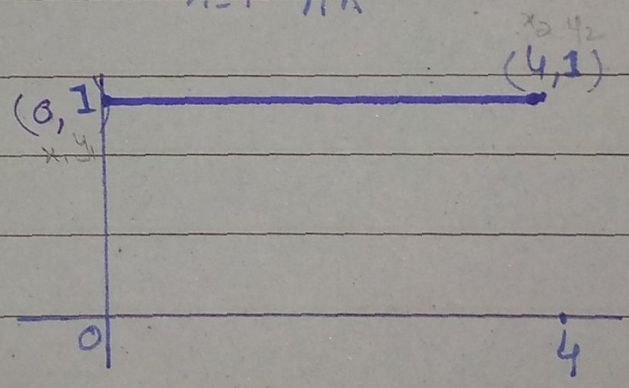

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{1-1}{4-0}(x-0)$$

$$y - 1 = 0$$

$$y = 1$$

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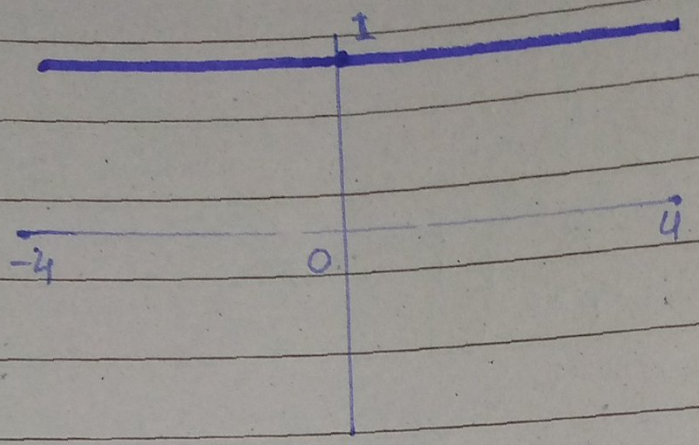
$f(x) = 1 \qquad 0 < x < 4$

(a) Fourier Cosine Series:-



$2L = 8$

$L = 4$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

$$a_0 = \frac{2}{2L}\int_{0}^{L} f(x)\,dx$$

$$a_0 = \frac{1}{L}\int_{0}^{L} f(x)\,dx$$

$$a_0 = \frac{1}{4}\int_{0}^{4} 1\,dx = \frac{1}{4}\left[\frac{x}{1}\right]_0^4 = \frac{1}{4}(4-0)$$

$a_0 = 1$

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$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$a_n = \frac{2}{4}\int_{0}^{4} f(x)\cos\left(\frac{n\pi x}{4}\right)dx$$

$$a_n = \frac{1}{2}\int_{0}^{4} f(x)\cos\left(\frac{n\pi x}{4}\right)dx$$

$$a_n = \frac{1}{2}\int_{0}^{4}\cos\frac{n\pi x}{4}\,dx = \frac{1}{2}\left|\frac{\sin\frac{n\pi x}{4}}{\frac{n\pi}{4}}\right|_{0}^{4}$$

$$\frac{1}{2}\cdot\frac{4}{n\pi}\left|\sin\left(\frac{n\pi x}{4}\right)\right|_{0}^{4}$$

$$\frac{2}{n\pi}\left[0-0\right] = 0$$

Fourier Series:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L}\,dx$$

$$f(x) = 1 + 0$$

$$f(x) = 1$$

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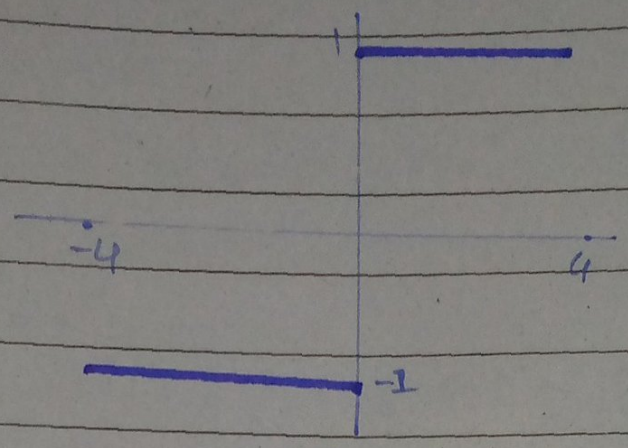
(b) Fourier Sine Series:



$2L = 8$

$L = 4$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{2}{4}\int_{0}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$b_n = \frac{2}{4}\int_{0}^{4} \sin\left(\frac{n\pi x}{4}\right)dx$$

$$b_n = \frac{1}{2}\left|\frac{-\cos\frac{n\pi x}{4}}{\frac{n\pi}{4}}\right|_0^4 = -\frac{1}{2}\cdot\frac{4}{n\pi}\left|\cos\left(\frac{n\pi x}{4}\right)\right|_0^4$$

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$$b_n = \frac{-2}{n\pi}\left[(-1)^n - 1\right]$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}\, dx$$

$$f(x) = \sum_{n=1}^{\infty} \frac{-2}{n\pi}\left[(-1)^n - 1\right] \sin\left(\frac{n\pi x}{4}\right) d$$

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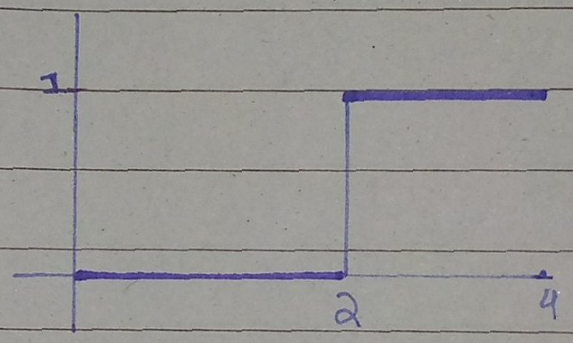
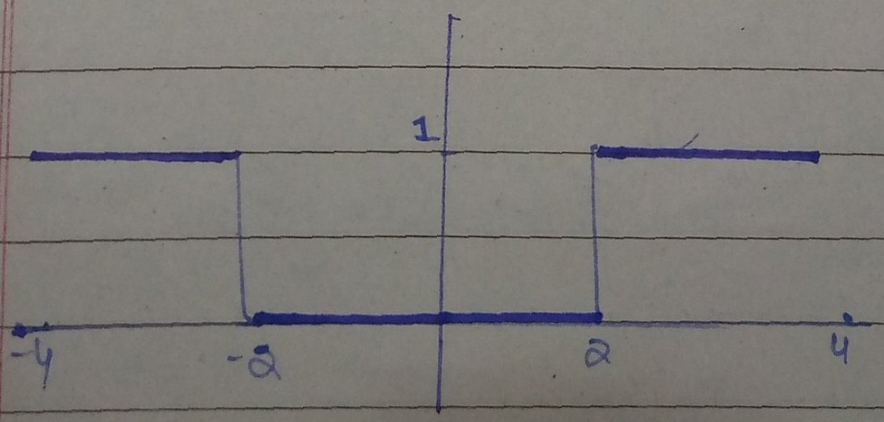


$$f(x) = \begin{cases} 0 & 0 < x < 2 \\ 1 & 2 < x < 4 \end{cases}$$

(a) Fourier Cosine Series:-



$2L = 8$, $\boxed{L = 4}$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

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$$a_0 = \frac{1}{2L}\cdot 2\int_{0}^{L} f(x)\,dx$$

$$a_0 = \frac{1}{L}\int_{0}^{L} f(x)\,dx$$

$$a_0 = \frac{1}{4}\int_{0}^{4} f(x)\,dx$$

$$a_0 = \frac{1}{4}\int_{0}^{2} 0\,dx + \frac{1}{4}\int_{2}^{4} 1\,dx$$

$$a_0 = \frac{1}{4}(0) + \frac{1}{4}\Big|x\Big|_{2}^{4}$$

$$a_0 = 0 + \frac{1}{4}(4-2)$$

$$a_0 = 0 + \frac{1}{4}(2) = \frac{1}{2}$$

B

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

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$$a_n = \frac{2}{4}\int_0^4 f(x)\cos\left(\frac{n\pi x}{4}\right)dx$$

$$a_n = \frac{1}{2}\int_0^2 0.\cos\left(\frac{n\pi x}{4}\right)dx + \frac{1}{2}\int_2^4 \cos\left(\frac{n\pi x}{4}\right)dx$$

$$a_n = \frac{1}{2}\int_2^4 \cos\left(\frac{n\pi x}{4}\right)dx$$

$$a_n = \frac{1}{2}\left|\sin\frac{n\pi x}{4}\right|_2^4 \cdot \frac{4}{n\pi}$$

$$a_n = \frac{1}{2}\cdot\frac{4}{n\pi}\left[0 - \sin\frac{n\pi 2}{4}\right]$$

$$a_n = \frac{2}{n\pi}\left[-\sin\frac{n\pi}{2}\right]$$

$$a_n = -\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a(n)\cos\frac{n\pi x}{4}$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-2}{n\pi}\sin\left(\frac{n\pi}{2}\right).\cos\frac{n\pi x}{4}$$

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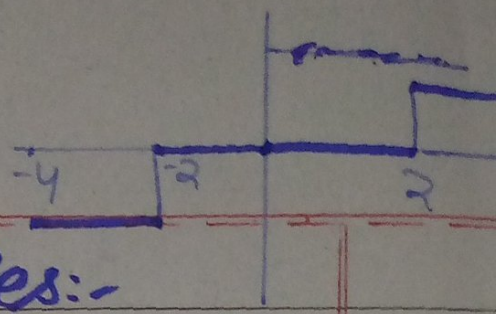

(b) Fourier Sine Series:-

$L = 4$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{2}{4}\int_{0}^{4} f(x)\sin\left(\frac{n\pi x}{4}\right)dx$$

$$b_n = \frac{1}{2}\int_{0}^{2} 0\sin\left(\frac{n\pi x}{4}\right)dx + \frac{1}{2}\int_{2}^{4}\sin\left(\frac{n\pi x}{4}\right)dx$$

$$b_n = \frac{1}{2}\cdot\frac{4}{n\pi}\left[-\cos\frac{n\pi x}{4}\right]_{2}^{4}$$

$$b_n = -\frac{2}{n\pi}\left[(-1)^n - 0\right]$$

$$b_n = \frac{-2}{n\pi}(-1)^n$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{4}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{-2}{n\pi}(-1)^n \sin\left(\frac{n\pi x}{4}\right)$$

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25 $f(x) = -x + \pi \quad 0 < x <$

$$a_0 = \frac{\pi}{2}$$

$$a_n = \frac{2}{n\pi^2}\left[1 - (-1)^n\right]$$

$$b_n = \frac{2}{n}$$

Do self

26 $$f(x) = \begin{cases} x & 0 < x < \frac{\pi}{2} \\ \frac{x}{2} & \frac{\pi}{2} < x < \pi \end{cases}$$

$$a_0 = \frac{3\pi}{8}$$

$$a_n = \frac{2\cos\left(\frac{n\pi}{2}\right) - 2}{n^2\pi}$$

$$b_n = \frac{(-1)^{n+1}}{n} + \frac{2\sin\left(\frac{n\pi}{2}\right)}{n^2\pi}$$

Do self.

27 $$f(x) = \begin{cases} \frac{\pi}{2} & 0 < x < \frac{\pi}{2} \\ -x + \pi & \frac{\pi}{2} < x < \pi \end{cases}$$

$$a_0 = \frac{3\pi}{8}$$

$$a_n = \frac{2\left[(-1)^{n+1} + \cos\left(\frac{\pi n}{2}\right)\right]}{n^2\pi}$$

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$$b_n = \frac{1}{n} + \frac{2\sin\left(\frac{n\pi}{2}\right)}{n^2\pi}$$

28 $f(x) = x$

$$a_0 = \frac{L}{2}$$

$$a_n = 2L\left[\frac{(-1)^n - 1}{n^2\pi^2}\right]$$

$$b_n = -\frac{2(-1)^n L}{n\pi}$$

29 $$a_0 = \frac{2}{\pi}$$

$$a_n = 2\,\frac{1-(-1)^{n+1}}{(1-n^2)\pi}$$