...nstant coefficients.

Supplementary Problems

In Problems 23.26 through 23.34, determine whether the given values of x are ordinary points or singular poi... of the given differential equations.

23.26. $x = 1;\ y'' + 3y' + 2xy = 0$

23.27. $x = 2;\ (x - 2)y'' + 3(x^2 - 3x + 2)y' + (x - 2)^2 y = 0$

23.28. $x = 0;\ (x + 1)y'' + \dfrac{1}{x}y' + xy = 0$

23.29. $x = -1;\ (x + 1)y'' + \dfrac{1}{x}y' + xy = 0$

23.30. $x = 0;\ x^3y'' + y = 0$

23.31. $x = 0$; $x^3y'' + xy = 0$

23.32. $x = 0$; $e^xy'' + (\sin x)y' + xy = 0$

23.33. $x = -1$; $(x + 1)^3y'' + (x^2 - 1)(x + 1)y' + (x - 1)y = 0$

23.34. $x = 2$; $x^4(x^2 - 4)y'' + (x + 1)y' + (x^2 - 3x + 2)y = 0$

23.35. Find the general solution near $x = 0$ of $y'' - y' = 0$. Check your answer by solving the equation by the method of Chapter 8 and then expanding the result in a power series about $x = 0$.

In Problems 23.36 through 23.47, find (*a*) the recurrence formula and (*b*) the general solution of the given differential equation by the power series method around the given value of x.

23.36. $x = 0$; $y'' + xy = 0$

23.37. $x = 0$; $y'' - 2xy' - 2y = 0$

23.38. $x = 0$; $y'' + x^2y' + 2xy = 0$

23.39. $x = 0$; $y'' - x^2y' - y = 0$

23.40. $x = 0$; $y'' + 2x^2y = 0$

23.41. $x = 0$; $(x^2 - 1)y'' + xy' - y = 0$

23.42. $x = 0$; $y'' - xy = 0$

23.43. $x = 1$; $y'' - xy = 0$

23.44. $x = -2$; $y'' - x^2y' + (x + 2)y = 0$

23.45. $x = 0$; $(x^2 + 4)y'' + y = x$

23.46. $x = 1$; $y'' - (x - 1)y' = x^2 - 2x$

23.47. $x = 0$; $y'' - xy' = e^{-x}$

23.48. Use the Taylor series method described in Problem 23.23 to solve $y'' - 2xy' + x^2y = 0$; $y(0) = 1$, $y'(0) = -1$.

23.49. Use the Taylor series method described in Problem 23.23 to solve $y'' - 2xy = x^2$; $y(1) = 0$, $y'(1) = 2$.

23.26. Ordinary point

23.27. Ordinary point

23.28. Singular point

23.29. Singular point

23.30. Singular point

23.31. Singular point

23.32. Ordinary point

23.33. Singular point

23.34. Singular point

23.35. $y = a_0 + a_1\left(x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots\right) = c_1 + c_2e^x$, where $c_1 = a_0 - a_1$ and $c_2 = a_1$

23.36. RF (recurrence formula): $a_{n+2} = \frac{-1}{(n+2)(n+1)}a_{n-1}$

$$y = a_0\left(1 - \frac{1}{6}x^3 + \frac{1}{180}x^6 + \cdots\right) + a_1\left(x - \frac{1}{12}x^4 + \frac{1}{504}x^7 + \cdots\right)$$

23.37. RF: $a_{n+2} = \frac{2}{n+2}a_n$

$$y = a_0\left(1 + x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6 + \cdots\right) + a_1\left(x + \frac{2}{3}x^3 + \frac{4}{15}x^5 + \frac{8}{105}x^7 + \cdots\right)$$

23.38. RF: $a_{n+2} = \frac{-1}{n+2}a_{n-1}$

$$y = a_0\left(1 - \frac{1}{3}x^3 + \frac{1}{18}x^6 + \cdots\right) + a_1\left(x - \frac{1}{4}x^4 + \frac{1}{28}x^7 + \cdots\right)$$

23.39. RF: $a_{n+2} = \frac{n-1}{(n+2)(n+1)}a_{n-1} + \frac{1}{(n+2)(n+1)}a_n$

$$y = a_0\left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{20}x^5 + \cdots\right) + a_1\left(x + \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{120}x^5 + \cdots\right)$$

23.40. RF: $a_{n+2} = \frac{-2}{(n+2)(n+1)}a_{n-2}$

$$y = a_0\left(1 - \frac{1}{6}x^4 + \frac{1}{168}x^8 + \cdots\right) + a_1\left(x - \frac{1}{10}x^5 + \frac{1}{360}x^9 + \cdots\right)$$

23.41. RF: $a_{n+2} = \frac{n-1}{n+2}a_n$

$$y = a_0\left(1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6 - \cdots\right) + a_1x$$

23.42. RF: $a_{n+2} = \frac{1}{(n+2)(n+1)}a_{n-1}$

$$y = a_0\left(1 + \frac{1}{6}x^3 + \frac{1}{180}x^6 + \cdots\right) + a_1\left(x + \frac{1}{12}x^4 + \frac{1}{504}x^7 + \cdots\right)$$

23.43. RF: $a_{n+2} = \dfrac{1}{(n+2)(n+1)}(a_n + a_{n-1})$

$$y = a_0\left[1 + \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 + \frac{1}{24}(x-1)^4 + \cdots\right]$$
$$+ a_1\left[(x-1) + \frac{1}{6}(x-1)^3 + \frac{1}{12}(x-1)^4 + \cdots\right]$$

23.44. RF: $a_{n+2} = \dfrac{n-2}{(n+2)(n+1)}a_{n-1} - \dfrac{4n}{(n+2)(n+1)}a_n + \dfrac{4}{n+2}a_{n+1}$

$$y = a_0\left[1 - \frac{1}{6}(x+2)^3 - \frac{1}{6}(x+2)^4 + \cdots\right]$$
$$+ a_1\left[(x+2) + 2(x+2)^2 + 2(x+2)^3 + \frac{2}{3}(x+2)^4 + \cdots\right]$$

23.45. RF: $a_{n+2} = -\dfrac{n^2 - n + 1}{4(n+2)(n+1)}a_n, \qquad n > 1$

$$y = \left(\frac{1}{24}x^3 - \frac{7}{1920}x^5 + \cdots\right) + a_0\left(1 - \frac{1}{8}x^2 + \frac{1}{128}x^4 + \cdots\right) + a_1\left(x - \frac{1}{24}x^3 + \frac{7}{1920}x^5 + \cdots\right)$$

23.46. RF: $a_{n+2} = \dfrac{n}{(n+2)(n+1)}a_n, \qquad n > 2$

$$y = -\frac{1}{2}(x-1)^2 + a_0 + a_1\left[(x-1) + \frac{1}{6}(x-1)^3 + \frac{1}{40}(x-1)^5 + \cdots\right]$$

23.47. RF: $a_{n+2} = \dfrac{n}{(n+2)(n+1)}a_n + \dfrac{(-1)^n}{n!\,(n+2)(n+1)}$

$$y = \left(\frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{8}x^4 - \frac{1}{30}x^5 + \cdots\right) + a_0 + a_1\left(x + \frac{1}{6}x^3 + \frac{1}{40}x^5 + \cdots\right)$$

23.48. $y = 1 - x - \dfrac{1}{3}x^3 - \dfrac{1}{12}x^4 - \cdots$

23.49. $y = 2(x-1) + \dfrac{1}{2}(x-1)^2 + (x-1)^3 + \cdots$

CHAPTER 24

$a_n = \dfrac{1}{[2(\lambda+n)-1][(\lambda+n)-1]}a_{n-1}$