since a singularity on one side of the equation must be balanced by a corresponding singularity on the other side.

In dealing with problems with impulsive forcing the use of the delta function usually simplifies the mathematical calculations, often quite significantly. However, if the actual excitation extends over a short, but nonzero, time interval, then an error will be introduced by modeling the excitation as taking place instantaneously. This error may be negligible, but in a practical problem it should not be dismissed without consideration. In Problem 16 you are asked to investigate this issue for a simple harmonic oscillator.

$\int_0^\infty g(t)\,\delta(t-a)\,dt = g(a)$

PROBLEMS

In each of Problems 1 through 12:

(a) Find the solution of the given initial value problem.

(b) Draw a graph of the solution.

1. $y'' + 2y' + 2y = \delta(t-\pi);\quad y(0) = 1,\quad y'(0) = 0$
2. $y'' + 4y = \delta(t-\pi) - \delta(t-2\pi);\quad y(0) = 0,\quad y'(0) = 0$
3. $y'' + 3y' + 2y = \delta(t-5) + u_{10}(t);\quad y(0) = 0,\quad y'(0) = 1/2$
4. $y'' - y = -20\delta(t-3);\quad y(0) = 1,\quad y'(0) = 0$
5. $y'' + 2y' + 3y = \sin t + \delta(t-3\pi);\quad y(0) = 0,\quad y'(0) = 0$
6. $y'' + 4y = \delta(t-4\pi);\quad y(0) = 1/2,\quad y'(0) = 0$
7. $y'' + y = \delta(t-2\pi)\cos t;\quad y(0) = 0,\quad y'(0) = 1$
8. $y'' + 4y = 2\delta(t-\pi/4);\quad y(0) = 0,\quad y'(0) = 0$
9. $y'' + y = u_{\pi/2}(t) + 3\delta(t-3\pi/2) - u_{2\pi}(t);\quad y(0) = 0,\quad y'(0) = 0$
10. $2y'' + y' + 4y = \delta(t-\pi/6)\sin t;\quad y(0) = 0,\quad y'(0) = 0$
11. $y'' + 2y' + 2y = \cos t + \delta(t-\pi/2);\quad y(0) = 0,\quad y'(0) = 0$
12. $y^{(4)} - y = \delta(t-1);\quad y(0) = 0,\quad y'(0) = 0,\quad y''(0) = 0,\quad y'''(0) = 0$
13. Consider again the system in Example 1 of this section, in which an oscillation is excited by a unit impulse at $t = 5$. Suppose that it is desired to bring the system to rest again after exactly one cycle—that is, when the response first returns to equilibrium moving in the positive direction.

    (a) Determine the impulse $k\delta(t-t_0)$ that should be applied to the system in order to accomplish this objective. Note that k is the magnitude of the impulse and t_0 is the time of its application.

19. (b) $y = 1 - \cos t + 2\sum_{k=1}(-1)^k u_{k\pi}(t)[1 - \cos(t - k\pi)]$

21. (b) $y = 1 - \cos t + \sum_{k=1}^{n}(-1)^k u_{k\pi}(t)[1 - \cos(t - k\pi)]$

23. (a) $y = 1 - \cos t + 2\sum_{k=1}^{n}(-1)^k u_{11k/4}(t)[1 - \cos(t - 11k/4)]$

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1. (a) $y = e^{-t}\cos t + e^{-t}\sin t + u_\pi(t)e^{-(t-\pi)}\sin(t-\pi)$
2. (a) $y = \frac{1}{2}u_\pi(t)\sin 2(t-\pi) - \frac{1}{2}u_{2\pi}(t)\sin 2(t-2\pi)$
3. (a) $y = -\frac{1}{2}e^{-2t} + \frac{1}{2}e^{-t} + u_5(t)[-e^{-2(t-5)} + e^{-(t-5)}] + u_{10}(t)[\frac{1}{2} + \frac{1}{2}e^{-2(t-10)} - e^{-(t-10)}]$
4. (a) $y = \cosh(t) - 20u_3(t)\sinh(t-3)$
5. (a) $y = \frac{1}{4}\sin t - \frac{1}{4}\cos t + \frac{1}{4}e^{-t}\cos\sqrt{2}t + (1/\sqrt{2})\,u_{3\pi}(t)e^{-(t-3\pi)}\sin\sqrt{2}(t-3\pi)$
6. (a) $y = \frac{1}{2}\cos 2t + \frac{1}{2}u_{4\pi}(t)\sin 2(t-4\pi)$
7. (a) $y = \sin t + u_{2\pi}(t)\sin(t-2\pi)$
8. (a) $y = u_{\pi/4}(t)\sin 2(t-\pi/4)$
9. (a) $y = u_{\pi/2}(t)[1-\cos(t-\pi/2)] + 3u_{3\pi/2}(t)\sin(t-3\pi/2) - u_{2\pi}(t)[1-\cos(t-2\pi)]$
10. (a) $y = (1/\sqrt{31})\,u_{\pi/6}(t)\exp[-\frac{1}{4}(t-\pi/6)]\sin(\sqrt{31}/4)(t-\pi/6)$
11. (a) $y = \frac{1}{5}\cos t + \frac{2}{5}\sin t - \frac{1}{5}e^{-t}\cos t - \frac{3}{5}e^{-t}\sin t + u_{\pi/2}(t)e^{-(t-\pi/2)}\sin(t-\pi/2)$
12. (a) $y = u_1(t)[\sinh(t-1) - \sin(t-1)]/2$
13. (a) $-e^{-T/4}\delta(t-5-T), \quad T = 8\pi/\sqrt{15}$
14. (a) $y = (4/\sqrt{15})\,u_1(t)e^{-(t-1)/4}\sin(\sqrt{15}/4)(t-1)$
    (b) $t_1 \cong 2.3613, \quad y_1 \cong 0.71153$
    (c) $y = (8\sqrt{7}/21)\,u_1(t)e^{-(t-1)/8}\sin(3\sqrt{7}/8)(t-1); \quad t_1 \cong 2.4569, \quad y_1 \cong 0.83351$
    (d) $t_1 = 1 + \pi/2 \cong 2.5708, \quad y_1 = 1$
15. (a) $k_1 \cong 2.8108$ (b) $k_1 \cong 2.3995$ (c) $k_1 = 2$
16. (a) $\phi(t,k) = [u_{4-k}(t)h(t-4+k) - u_{4+k}(t)h(t-4-k)]/2k, \quad h(t) = 1 - \cos t$
    (b) $\phi_0(t) = u_4(t)\sin(t-4)$ (c) Yes
17. (b) $y = \sum_{k=1}^{20} u_{k\pi}(t)\sin(t-k\pi)$
18. (b) $y = \sum_{k=1}^{20}(-1)^{k+1}u_{k\pi}(t)\sin(t-k\pi)$
19. (b) $y = \sum_{k=1}^{20} u_{k\pi/2}(t)\sin(t-k\pi/2)$
20. (b) $y = \sum_{k=1}^{20}(-1)^{k+1}u_{k\pi/2}(t)\sin(t-k\pi/2)$
21. (b) $y = \sum_{k=1}^{15} u_{(2k-1)\pi}(t)\sin[t-(2k-1)\pi]$
22. (b) $y = \sum_{k=1}^{40}(-1)^{k+1}u_{11k/4}(t)\sin(t-11k/4)$
23. (b) $y = \frac{20}{\sqrt{399}}\sum_{k=1}^{20}(-1)^{k+1}u_{k\pi}(t)e^{-(t-k\pi)/20}\sin[\sqrt{399}(t-k\pi)/20]$
24. (b) $y = \frac{20}{\sqrt{399}}\sum_{k=1}^{15}u_{(2k-1)\pi}(t)e^{-[t-(2k-1)\pi]/20}\sin\{\sqrt{399}[t-(2k-1)\pi]/20\}$