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Exercise 6.1

1. $3t+12$

$f(t) = 3t+12$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{3t+12\}$

$= 3\mathcal{L}\{t\} + 12\mathcal{L}\{1\}$

$\boxed{\because \mathcal{L}\{1\} = \frac{1}{s} \quad \therefore \mathcal{L}\{t\} = \frac{1}{s^2}}$

$= 3 \cdot \frac{1}{s^2} + 12 \cdot \frac{1}{s} = \frac{3}{s^2} + \frac{12}{s^2}$

3 $\cos \pi t$

$f(t) = \cos \pi t$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{\cos \pi t\}$

$\because \cos \omega t = \frac{s}{s^2+\omega^2}$

$= \frac{s}{s^2+\pi^2}$

5 $e^{2t} \sinh t$

$f(t) = e^{2t} \sinh t$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{2t} \sinh t\}$

$= \frac{1}{(s-1)^2 - 1}$

According to Ist shifting theorem:

$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$

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7 $\sin(\omega t + \theta)$

$\because \sin(A+B) = \sin A \cos B + \cos A \sin B$

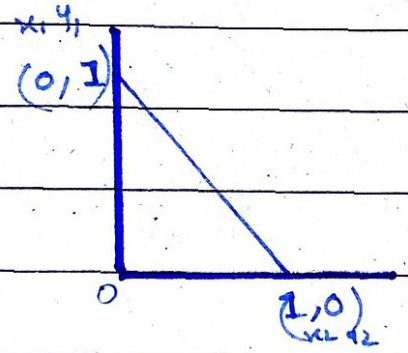
$= \sin(\omega t)\cos\theta + \cos(\omega t)\sin\theta$

$= \cos\theta \, \mathcal{L}\{\sin(\omega t)\} + \sin\theta \, \mathcal{L}\{\cos(\omega t)\}$

$= \cos\theta \cdot \dfrac{\omega}{s^2+\omega^2} + \sin\theta \cdot \dfrac{s}{s^2+\omega^2}$

$= \left[\cos\theta \cdot (\omega) + \sin\theta \cdot (s)\right] \dfrac{1}{s^2+\omega^2}$

9



$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 1 = \dfrac{0-1}{1-0}(x - 0)$

$y - 1 = -1(x - 0)$

$y = -x + 1$

$y = -t + 1 \quad 0 < t < 1$

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$f(t) = -t + 1$

$\mathcal{L}\{f(t)\} = -\mathcal{L}\{t\} + \mathcal{L}\{1\}$

$= -1 \frac{1}{s^2} + \frac{1}{s}$

$\because \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)$

$=$

$\mathcal{L}\{f(t)\} = \int_0^1 e^{-st}(-t+1)\,dt$

$= \int_0^1 -t e^{-st}\,dt + \int_0^1 e^{-st}\,dt$

$= \frac{-1}{s^2} + \frac{1}{s}$

$f(t) = 1 - t$

$\because \mathcal{L}f(t) = \int_0^\infty e^{-st} \cdot f(t)\,dt$

$\mathcal{L}f(t) = \int_0^1 e^{-st} \cdot (1-t) + \int_1^\infty 0$

$= \int_0^1 e^{-st}(1-t)$

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$1-t$		e^{-st}
-1	$+$	$\frac{e^{-st}}{-s}$
0	$-$	$\frac{e^{-st}}{s^2}$

$$= \left| (1-t)\frac{e^{-st}}{-s} + \frac{e^{-st}}{s^2} \right|_0^1$$

$$= \left(0 + \frac{e^{-s}}{s^2}\right) - \left(\frac{e^{0}}{-s} + \frac{1}{s^2}\right)$$

$$= \frac{e^{-s}}{s^2} + \frac{1}{s} - \frac{1}{s^2} = \frac{e^{-s}}{s^2} - \frac{1}{s^2} + \frac{1}{s}$$

$$= \frac{e^{-s}-1}{s^2} + \frac{1}{s}$$

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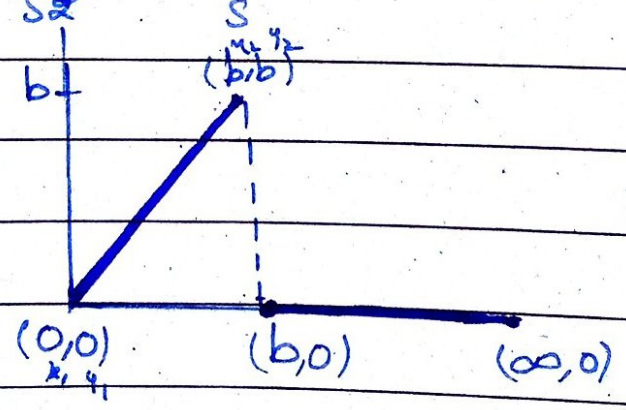

$\cancel{y_2 - y_1 =}$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y = \frac{b}{b}(x)$

$y = x$

$y = t$

$$f(t) = \begin{cases} t & 0 < t < b \\ 0 & b < t < \infty \end{cases}$$

$$\therefore \mathcal{L}\{f(t)\} = \int_0^\infty f(t) \cdot e^{-st}\, dt.$$

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$$\mathcal{L}\{f(t)\} = \int_0^b (t) \cdot e^{-st}\, dt + \int_b^\infty 0$$

t		e^{-st}
1	$+$	$\dfrac{e^{-st}}{-s}$
0	$-$	$\dfrac{e^{-st}}{s^2}$

$$\mathcal{L}\{f(t)\} = \left| t \cdot \frac{e^{-st}}{-s} - 1 \cdot \frac{e^{-st}}{s^2} \right|_0^b$$

$$= \left(\frac{b\, e^{-sb}}{-s} - \frac{e^{-sb}}{s^2} \right) - \left(- \frac{1}{s^2} \right)$$

$$= \frac{b e^{-sb}}{-s} - \frac{e^{-sb}}{s^2} + \frac{1}{s^2}$$

$$= \cancel{\frac{b e^{-sb}}{-s} - \frac{1}{s^2} \left(e^{-sb} + 1 \right)}$$

$$= \frac{b e^{-sb}}{-s} + \frac{-e^{-sb} + 1}{s^2}$$

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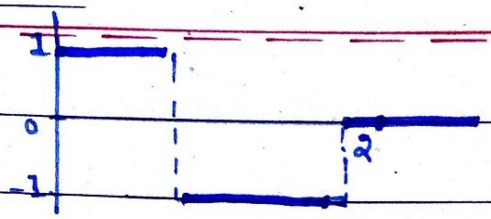

$$f(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \\ 0 & 2 < t < \infty \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^1 1\, e^{-st}\, dt + \int_1^2 -1\, e^{-st}\, dt + \int_2^\infty 0$$

$$= \left| \frac{e^{-st}}{-s} \right|_0^1 + \left| \frac{e^{-st}}{s} \right|_1^2 + 0$$

$$= \left[\frac{e^{-s}}{-s} - \frac{1}{-s} \right] + \left[\frac{e^{-2s}}{s} - \frac{e^{-s}}{s} \right]$$

$$= -\frac{e^{-s}}{+s} + \frac{1}{s} + \frac{e^{-2s}}{s} - \frac{e^{-s}}{s}$$

$$= -\frac{2e^{-s}}{s} + \frac{1 + e^{-2s}}{s} = \frac{-2e^{-s} + 1 + e^{-2s}}{s}$$

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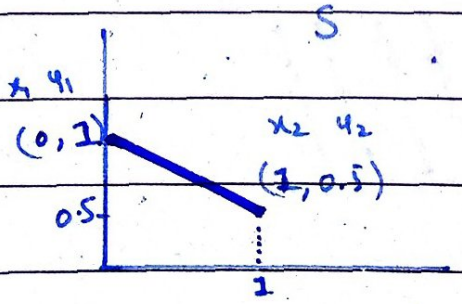

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{0.5 - 1}{1 - 0}(x - 0)$$

$$y - 1 = \frac{-0.5}{1}(x)$$

$$y - 1 = -0.5(x)$$

$$y - 1 = -\frac{1}{2}(x)$$

$$y = -\frac{1}{2}x + 1$$

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$$f(t) = \begin{cases} -\frac{1}{2}t + 1 & 0 < t < 1 \\ 0 & 1 < t < \infty \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^1 \left(-\frac{1}{2}t + 1\right) e^{-st}\, dt + \int_0^\infty 0 = \int_0^1 \left(-\frac{t}{2} + 1\right) e^{-st}$$

$$\begin{array}{ll} -\frac{t}{2} + 1 & e^{-st} \\ -\frac{1}{2} & \frac{e^{-st}}{-s} \\ 0 & \frac{e^{-st}}{s^2} \end{array}$$

$$= \left| \left(-\frac{t}{2} + 1\right)\left(\frac{e^{-st}}{s}\right) + \left(\frac{-1}{2}\right)\left(-\frac{e^{-st}}{s^2}\right) \right|_0^1$$

$$= \left| \left(-\frac{t}{2} + 1\right)\left(\frac{-e^{-st}}{s}\right) + \frac{e^{-st}}{2s^2} \right|_0^1$$

$$= \left[\left(\frac{-1}{2} + 1\right)\left(-\frac{e^{-s}}{s}\right) + \frac{e^{-s}}{2s^2}\right] - \left[\frac{-1}{s} + \frac{1}{2s^2}\right]$$

$$= \frac{-e^{-s}}{2s} + \frac{e^{-s}}{2s^2} + \frac{1}{s} - \frac{1}{2s^2} = \frac{e^{-s} - 1}{2s^2} + \frac{-e^{-s} + 2}{2s}$$

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25 $\dfrac{0.2s + 1.8}{s^2 + 3.24}$

$$F(s) = \frac{0.2s}{s^2+3.24} + \frac{1.8}{s^2+3.24}$$

$$\mathcal{L}\{F(s)\} = \mathcal{L}\left\{\frac{0.2s}{s^2+3.24} + \frac{1.8}{s^2+3.24}\right\}$$

$$\boxed{\begin{aligned} \frac{s}{s^2+w^2} &= \cos wt \\ \frac{w}{s^2+w^2} &= \sin wt \end{aligned}}$$

$$= 0.2\,\mathcal{L}\left\{\frac{s}{s^2+3.24}\right\} + \mathcal{L}\left\{\frac{1.8}{s^2+3.24}\right\}$$

$$= 0.2 \cdot \cos(1.8t) + \sin(1.8t)$$

27 $\dfrac{s}{L^2 s^2 + n^2\pi^2}$

$$= \frac{s}{\frac{L^2}{L^2}(L^2 s^2 + n^2\pi^2)}$$

$$= \frac{s}{L^2\left[\frac{L^2 s^2}{L^2} + \frac{n^2\pi^2}{L^2}\right]}$$

$$= \frac{s}{L^2\left(s^2 + \frac{n^2\pi^2}{L^2}\right)}$$

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$$\mathcal{L}\{F(s)\} = \frac{1}{L^2}\mathcal{L}\left\{\frac{s}{s^2+\left(\frac{n\pi}{L}\right)^2}\right\}$$

$$\frac{1}{L^2}\cos\frac{n\pi}{L}t$$

$$\boxed{\because \mathcal{L}\cos\omega t = \frac{s}{s^2+\omega^2}}$$

29 $\frac{12}{s^4} - \frac{228}{s^6}$

$$\therefore \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$= \frac{2 \cdot 3!}{s^4} - 1.9\,\frac{5!}{s^6}$$

$$= 2\,\mathcal{L}\left\{\frac{3!}{s^4}\right\} - 1.9\,\mathcal{L}\left\{\frac{5!}{s^6}\right\}$$

$$= 2t^3 - 1.9t^5$$

31 $\frac{s+10}{s^2-s-2} = \frac{s+10}{(s+1)(s-2)}$

By partial fraction.

$$\frac{s+10}{(s+1)(s-2)} = \frac{A}{s+1} + \frac{B}{s-2} \quad \text{——} \; (1)$$

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Multiply by L.C.M.

$$s+10 = A(s-2) + B(s+1) \quad \text{--- (2)}$$

$s-2=0$

$s=2$

put in eq. (2)

$12 = B(3)$

$\boxed{B=4}$

$s+1=0$

$s=-1$

Put in (2)

$9 = A(-1-2)$

$9 = -3B$

$\boxed{B=-3}$

Now, eq. (1) becomes.

$$\frac{s+10}{(s+1)(s-2)} = \frac{-3}{s+1} + \frac{4}{s-2}$$

$$\mathcal{L}\left\{\frac{-3}{s+1} + \frac{4}{s-2}\right\} = -3\mathcal{L}\frac{1}{s+1} + 4\mathcal{L}\frac{1}{s-2}$$

$$= -3e^{-t} + 4e^{2t}$$

$$\boxed{\because \mathcal{L}\{e^{at}\} = \frac{1}{s-a}}$$

33 $\mathcal{L}\{e^{2-3t}\} = \dfrac{2!}{(s+3)^3}$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

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35 $0.5e^{-4.5t}\sin 2\pi t$

$$0.5 \cdot \frac{2\pi}{(s+4.5)^2 + 4\pi^2}$$

According to s-shifting them.
$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$

37 $\dfrac{\pi}{(s+\pi)^2}$

$$= \pi \mathcal{L}^{-1} \frac{1}{(s+\pi)^2}$$

$$\pi \cdot t \cdot e^{-\pi t} = \pi t e^{-\pi t}$$

s-shifting theorem.
$\boxed{\therefore \mathcal{L}^{-1}\{F(s-a)\} = e^{at} f(t)}$

39 $\dfrac{21}{(s+\sqrt{2})^4} = (3.5)\dfrac{6}{(s+\sqrt{2})^4} = 3.5\dfrac{3!}{(s+\sqrt{2})^4}$

$= 3.5\, t^3 e^{-\sqrt{2}t}$

$\boxed{\therefore \text{Apply s-shifting theorm}}$

41 $\dfrac{\pi}{s^2+10\pi s+24\pi^2} = \dfrac{\pi}{s^2+10\pi s+24\pi^2+\left(\frac{5\pi}{2}\right)^2-\left(\frac{5\pi}{2}\right)^2}$

$\therefore$ By completing square.

$$= \frac{\pi}{\left(s+\frac{5\pi}{2}\right)^2+24\pi^2-25\pi^2} = \frac{\pi}{(s+5\pi)^2-\pi^2}$$

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$$= \sinh \pi t \cdot e^{-5\pi t}$$

$$= e^{-5\pi t} \sinh \pi t$$

43, $$\frac{2s-1}{s^2-6s+18} = \frac{2s-1}{s^2-6s+18} = \frac{2s-1}{s^2-6s+18+(3)^2-(3)^2}$$

$$\frac{2s-1}{s^2-6s+(3)^2+18-9} = \frac{2s-1}{(s-3)^2+(3)^2}$$

$$= \frac{2s}{(s-3)^2+(3)^2} - \frac{1}{(s-3)^2+(3)^2}$$

$$= \frac{2s}{(s-3)^2+(3)^2} - \frac{\frac{1}{3}\cdot 3}{(s-3)^2+(3)^2}$$

$$= 2\mathcal{L}^{-1}\left\{\frac{s}{(s-3)^2+(3)^2}\right\} - \frac{1}{3}\mathcal{L}\left\{\frac{3}{(s-3)^2+(3)^2}\right\}$$

$$= 2 \cdot \cos 3t \cdot e^{3t} - \frac{1}{3}\sin 3t \cdot e^{3t}$$

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45 $\dfrac{k_0(s+a)+k_1}{(s+a)^2}$

$= \dfrac{k_0(s+a)}{(s+a)^2} + \dfrac{k_1}{(s+a)^2}$

$= k_0 \dfrac{1}{s+a} + k_1 \cdot \dfrac{1}{(s+a)^2}$

$= k_0 \cdot 1 \cdot e^{-at} + k_1 \cdot t \cdot e^{-at}$

$= k_0 e^{-at} + k_1 t e^{-at}$