6.4 ① $y'' + y = f(t)$ $\quad y(t) = \begin{cases} 1 & 0 \le t < 3\pi \\ 0 & 3\pi \le t < \infty \end{cases}$

$y(0) = 0;\ y'(0) = 1$

$\cancel{Y(s^2+s) = \frac{1}{s^2} - \frac{1}{s^2} e^{-3\pi s}}$

$y'' + y = 1 - 1\,U(t-3\pi)$

$s^2 Y - 1 + Y = \frac{1}{s} - \frac{1}{s} e^{-3\pi s}$

$Y(s^2+1) = \frac{1}{s} - \frac{1}{s} e^{-3\pi s} + 1$

$Y = \frac{1}{s(s^2+1)} - \frac{1}{s(s^2+1)} e^{-3\pi s} + \frac{1}{s^2+1}$

$\frac{1}{s(s^2+1)} = \frac{1}{s} \cdot \frac{1}{s^2+1}$

$f(t) = 1$

$g(t) = \cancel{\frac{1}{s^2}}$ Sint

$f(t-\tau) = 1$

$g(\tau) = \sin\tau$

$h(t) = \int_0^t \sin\tau = -\cos\tau \Big|_0^t = -\cos t + 1$

Now,

$Y = 1 - \cos t - (1 - \cos t) e^{-3\pi s} + \sin t$

$$Y = 1 - \cos t - [1 - \cos(t - 3\pi)]\, U(t-3\pi) + \sin t$$

$$Y = 1 - \cos t - [1 - (-\cos t)]\, U(t-3\pi) + \sin t$$

$$Y = 1 - \cos t - (1 + \cos t)\, U(t-3\pi) + \sin t$$

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$y(0)=0,\ y'(0)=0$

3. $y'' + 4y = \sin t - U_{2\pi}(t)\sin(t-2\pi)$

$s^2\mathcal{L}\{y\} - sf(0) - f'(0) + 4\mathcal{L}\{y\} = \mathcal{L}\{\sin t\} - \mathcal{L}\{\sin(t-2\pi)U(t-2\pi)\}$

$$s^2Y - 0 - 0 + 4Y = \frac{1}{s^2+1} - \frac{1}{s^2+1}e^{-2\pi s}$$

$$Y(s^2+4) = \frac{1}{s^2+1} - \frac{1}{s^2+1}e^{-2\pi s}$$

$$Y = \frac{1}{(s^2+1)(s^2+4)} - \frac{1}{(s^2+1)(s^2+4)}e^{-2\pi s} \quad \text{--- AA}$$

$$\frac{1}{(s^2+1)(s^2+4)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4} \quad \text{--- A}$$

$$1 = (As+B)(s^2+4) + (Cs+D)(s^2+1)$$

$$1 = As^3 + 4As + Bs^2 + 4B + Cs^3 + Cs + Ds^2 + D$$

$$1 = As^3 + Cs^3 + Bs^2 + Ds^2 + 4As + Cs + 4B + D$$

$A + C = 0$ --- (1)

$B + D = 0$ --- (2)

$4A + C = 0$ --- (3)

$4B + D = 1$ --- (4)

put $A = -C$ in eq (3)

$4(-C) + C = 0$

$-4C + C = 0$

$-3C = 0 \qquad C = 0$

Put $c = 0$ in eq. (1)

$A = 0$

Subtractin eq (2) & eq (4)

$$B + D = 0$$
$$\underline{4B + D = 1}$$
$$-3B = -1$$
$$B = \frac{1}{3}$$

Put $B = \frac{1}{3}$ in eq. (2)

$$D = -\frac{1}{3}$$

Now eq. (A) becomes.

$$\frac{1}{(s^2+1)(s^2+4)} = \frac{\frac{1}{3}}{s^2+1} + \frac{-\frac{1}{3}}{s^2+4}$$

Now eq. (AA) becomes

$$Y = \left(\frac{\frac{1}{3}}{s^2+1} + \frac{-\frac{1}{3}}{s^2+4}\right) - \left(\frac{\frac{1}{3}}{s^2+1} + \frac{-\frac{1}{3}}{s^2+4}\right)e^{-2\pi s}$$

Taking I.L.T

$$f(t) = \frac{1}{3}\mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right] - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}e^{-2\pi s}\right\} + \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}e^{-2\pi s}\right\}$$

$$f(t)=\frac{1}{3}\sin t-\frac{1}{6}\sin 2t-\frac{1}{3}\sin(t-2\pi)\,U(t-2\pi)$$
$$+\frac{1}{6}\sin 2(t-2\pi)\,U(t-2\pi)$$

$$\therefore Y=\frac{1}{3}\sin t-\frac{1}{6}\sin 2t-\frac{1}{6}\sin(t-2\pi)\,U(t-2\pi)$$

$$\therefore \sin(t-2\pi)=\sin t\cos 2\pi-\cos t\sin 2\pi=\sin t$$

$$Y=\frac{1}{3}\sin t-\frac{1}{6}\sin 2t-\frac{1}{6}\sin t\,U(t-2\pi)$$

$$\therefore \sin(t-2\pi)=\sin t\cos 2\pi-\cos t\sin 2\pi=\sin t$$
$$\therefore \sin 2(t-2\pi)=\sin 2t$$

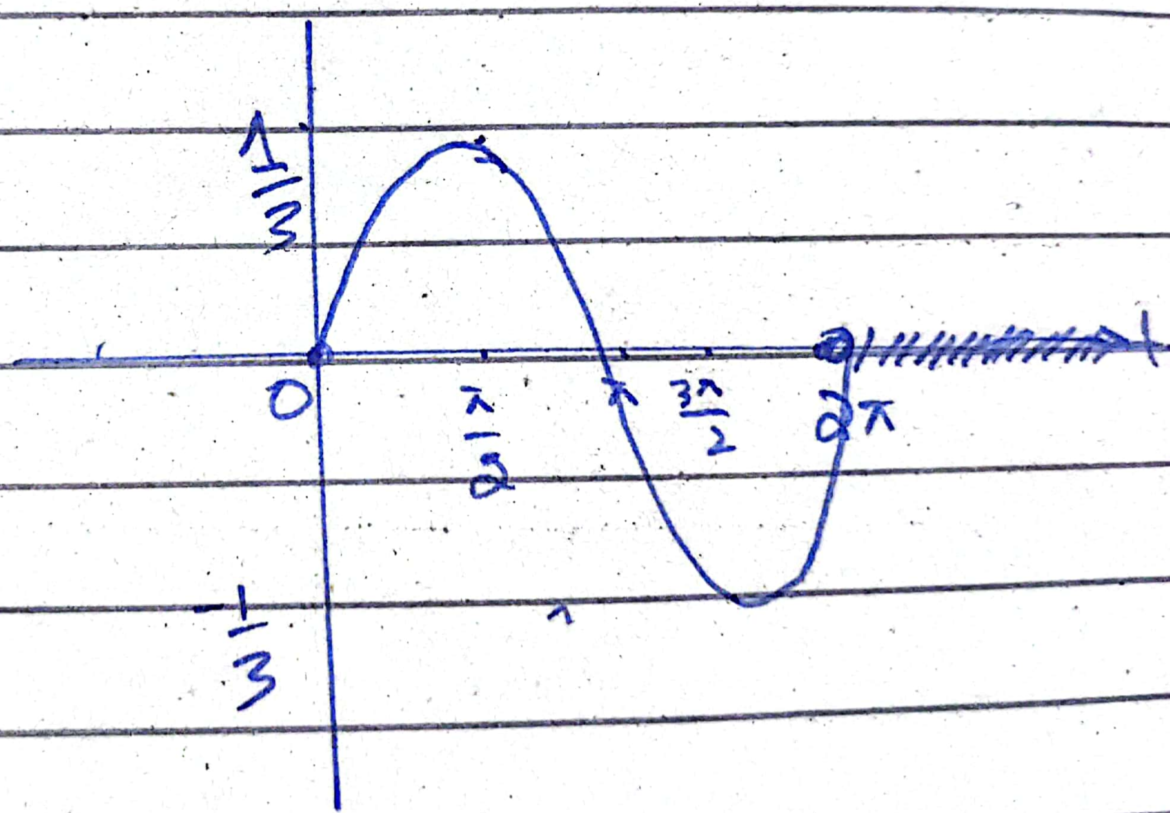
$$f(t)=\frac{1}{3}\sin t-\frac{1}{6}\sin 2t-\frac{1}{3}\sin t\,U(t-2\pi)+\frac{1}{6}\sin 2t\,U(t-2\pi)$$

$$f(t)=\frac{1}{3}\sin t-\frac{1}{6}\sin 2t-\frac{1}{3}\sin t\,U(t-2\pi)+\frac{1}{6}2\sin t\cos t\,U(t-2\pi)$$

$$f(t)=\frac{1}{3}\sin t-\frac{1}{6}2\sin t\cos t-\frac{1}{3}\sin t\,U(t-2\pi)+\frac{1}{3}\sin t\cos t\,U(t-2\pi)$$

$$f(t)=\frac{1}{3}\sin t(1-\cos t)-\frac{1}{3}\sin t(1-\cos t)\,U(t-2\pi)$$

$$f(t) = \begin{cases} \frac{1}{3}\sin t\,(1-\cos t) & 0 \le t < 2\pi \\ 0 & t \geq 2\pi \end{cases}$$



5. $y'' + 3y' + 2y = f(t)$; $y(0) = 0$, $y'(0) = 0$;

$$f(t) = \begin{cases} 1 & 0 < t < 10 \\ 0 & t \geq 10 \end{cases}$$

7/

$$y'' + 3y' + 2y = 1 - 1u(t-10)$$

{ taking LT }

$$s^2Y + 3sY + 2Y = \frac{1}{s} - \frac{1}{s}e^{-10s}$$

$$Y = \frac{1}{s(s^2+3s+2)} - \frac{1}{s(s^2+3s+2)}e^{-10s}$$

$$\frac{1}{s(s+1)(s+2)} = \frac{\frac{1}{2}}{s} + \frac{-1}{s+1} + \frac{\frac{1}{2}}{s+2}$$

Now,

$$Y = \frac{\frac{1}{2}}{s} + \frac{-1}{s+1} + \frac{\frac{1}{2}}{s+2} - \left[\frac{\frac{1}{2}}{s} + \frac{-1}{s+1} + \frac{\frac{1}{2}}{s+2}\right]e^{-10s}$$

taking I.L.T

$$y(t) = \frac{1}{2}(1) - e^{-t} + \frac{1}{2}e^{-2t} - \left[\frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}\right]u(t-10)$$

$$y(t) = \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t} - \left[\frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}\right]u(t-10)$$

$y'' + y = 1U(t-3\pi)$; $y(0)=1, y'(0)=0$

7/ $s^2\mathcal{L}\{y\} - sy(0) - y'(0) + \mathcal{L}\{y\} = \frac{1}{s}e^{-3\pi s}$

$$s^2Y - s + Y = \frac{1}{s}e^{-3\pi s}$$

$$Y = \frac{1}{s(s^2+1)}e^{-3\pi s} + \frac{s}{s^2+1} \quad \text{--- (1)}$$

$$\frac{1}{s(s^2+1)} = \frac{1}{s} \cdot \frac{1}{s^2+1} \quad \text{--- (2)}$$

$f(t) = 1$; $f(t-T) = 1$

$g(t) = \sin t$; $g(T) = \sin T$

$$h(t) = \int_0^t \sin T \, dT = \left| -\cos T \right|_0^t$$

$$h(t) = -\cos t + 1$$

Now eq. (1) becomes.

$$Y = (1 - \cos t)e^{-3\pi s} + \frac{s}{s^2+1}$$

taking I.L.T

$$y(t) = [1 - \cos(t-3\pi)]U(t-3\pi) + \cos t$$

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$y'' + y = g(t)$, $y(0) = 0$, $y'(0) = 1$

$$g(t) = \begin{cases} \frac{t}{2} & 0 \le t < 6 \\ 3 & t \geq 6 \end{cases}$$

9 $\cancel{Y = \frac{1}{2}\sin t + \frac{1}{2}t - \frac{1}{2}u(t-6)\left[t-6-\sin(t-6)\right]}$

$\cancel{Y = \frac{1}{2}\sin t + \frac{1}{2}t - \frac{1}{2}}$

$$y'' + y = \frac{t}{2} - \frac{t-6}{2}u(t-6)$$

$$y'' + y = \frac{1}{2}\cdot\frac{1}{s^2} - \frac{1}{2}\cdot\frac{1}{s^2}e^{-6s}$$

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) + \mathcal{L}\{y\} = \frac{1}{2s^2} - \frac{1}{2s^2}e^{-6s}$$

$$s^2Y - 0 - 1 + Y = \frac{1}{2s^2} - \frac{1}{2s^2}e^{-6s}$$

$$Y(s^2+1) = \frac{1}{2s^2} - \frac{1}{2s^2}e^{-6s} + 1$$

$$Y = \frac{1}{2s^2(s^2+1)} - \frac{1}{2s^2(s^2+1)}e^{-6s} + \frac{1}{s^2+1}$$

$$\frac{1}{s^2(s^2+1)} = \frac{1}{s^2}\cdot\frac{1}{s^2+1}$$

$f(t) = t \quad ; \quad f(t-\tau) = t - \tau$

$g(t) = \sin t \quad ; \quad g(\tau) = \sin \tau$

$$h(t) = \int_0^t (t-T)\sin T$$

$t-T$		$\sin T$
	$+$	
-1		$-\cos T$
	$-$	
0		$-\sin T$

$$= \Big| (t-T)(-)(\cos T) + \sin T \Big|_0^t$$

$$h(t) = -\sin t + t$$

Now,

$$Y = \frac{1}{2}(-\sin t + t) - \frac{1}{2}(-\sin t + t)e^{-6s} + \sin t$$

$$Y = \frac{1}{2}(t - \sin t) - \frac{1}{2}\left[t-6 - \sin(t-6)\right]u(t-6) + \sin t$$

$$Y = \frac{1}{2}(t-\sin t) - \frac{1}{2}\left[t-6 - \sin(t-6)\right]u(t-6) + \sin t$$