DATE: _____

Laplace transform of Derivatives.

(II) $y'' - y' - 6y = 0; \quad y(0) = 1 \;\&\; y'(0) = -1$

<u>Solution:</u>

$$\mathcal{L}\{y''\} - \mathcal{L}\{y'\} - 6\mathcal{L}\{y\} = 0$$

$$\left(s^2\mathcal{L}\{f\} - sf(0) - f'(0)\right) - \left(s\mathcal{L}\{f\} - f(0)\right) - 6\mathcal{L}\{y\} = 0$$

$$\left(s^2 Y - s(1) - (-1)\right) - \left(sY - 1\right) - 6Y = 0$$

$$s^2Y - s + 1 - sY + 1 - 6Y = 0$$

$$s^2Y + Y(-s-6) - s + 2 = 0$$

$$Y(s^2 - s - 6) = s - 2$$

$$Y = \frac{s-2}{s^2 - s - 6}$$

$$Y = \frac{s-2-2+2}{(s-3)(s+2)} = \frac{(s+2)-4}{(s-3)(s+2)}$$

$$Y = \frac{\cancel{(s+2)}}{(s-3)\cancel{(s+2)}} - \frac{4}{(s-3)(s+2)}$$

$$Y = \frac{1}{(s-3)} - \frac{4}{(s-3)(s+2)}$$

$$Y = \frac{1}{(s-3)} - \left[\frac{4}{s-3} - \frac{4}{s-2}\right]$$

<u>By Partial Fraction.</u>

DATE: _____

$$Y = \frac{1}{s-3} - \frac{4}{s-3} + \frac{4}{s-2}$$

$$Y = \frac{-3}{s-3} + \frac{4}{s-2}$$

Taking inverse laplace transform.

$$\mathcal{L}^{-}\{Y\} = -3\,\mathcal{L}\left\{\frac{1}{s-3}\right\} + 4\,\mathcal{L}\left\{\frac{1}{s-2}\right\}$$

$$y(t) = -3 \cdot e^{3t} + 4e^{2t}$$

13 $y'' - 2y' + 2y = 0$; $y(0) = 0$; $y'(0) = 1$

Solution

$$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = 0$$

$$\left(s^2\mathcal{L}\{f\} - sf(0) - f'(0)\right) - 2\left(s\mathcal{L}\{f\} - f(0)\right) + 2\mathcal{L}\{y\} = 0$$

$$s^2Y - sf(0) - f'(0) - 2sY + 2f(0) + 2Y = 0$$

$$s^2Y - 0 - 1 - 2sY + 2(0) + 2Y = 0$$

$$s^2Y - 1 - 2sY + 2Y = 0$$

$$s^2Y - 2sY + 2Y = 1$$

$$Y[s^2 - 2s + 2] = 1$$

$$Y = \frac{1}{s^2 - 2s + 2} = \cancel{\frac{1}{s^2 - 1s - 1s + 2}}$$

$$= \cancel{\frac{1}{s(s-1) - 1(s-2}} = \frac{1}{s^2 - 2s + 2 + 1 - 1} = \frac{1}{(s-1)^2 + 1}$$

DATE: _____

$$Y = \frac{1}{(s-1)^2 + (1)^2}$$

Taking inverse laplace transform

$$y(t) = \sin t \cdot e^{1t} = e^{t} \sin t$$

15 $y'' - 2y' + 4y = 0$; $y(0) = 2$, $y'(0) = 0$

Taking laplace transform.

$$\left[s^2 \mathcal{L}\{y\} - s y(0) - y'(0)\right] - 2\left[s\mathcal{L}\{y\} - y(0)\right] + 4\mathcal{L}\{y\} = 0$$

$$s^2 Y - s(2) - 0 - 2(sY - 2) + 4Y = 0$$

$$s^2 Y - 2s - 2sY + 4 + 4Y = 0$$

$$s^2 Y - 2sY + 4Y = 2s - 4$$

$$Y = \frac{2s-4}{s^2 - 2s + 4} = \cancel{\frac{2(s-2)}{(s-2)^2}}$$

$$Y = \frac{2s-4}{s^2 - 2s + 4 + 1 - 1} = \frac{2s-4}{(s-1)^2 + 3}$$

$$Y = \frac{2s}{(s-1)^2 + (\sqrt{3})^2} - \frac{4}{(s-1)^2 + (\sqrt{3})^2}$$

$$Y = 2\,\frac{s}{(s-1)^2 + (\sqrt{3})^2} - 2.31 \cdot \frac{\sqrt{3}}{(s-1)^2 + (\sqrt{3})^2}$$

DATE: _____

19 $y^{(4)} - 4y = 0$ $\quad y(0)=1,\ y'(0)=0,\ y''(0)=-2,\ y'''(0)=0$

$$s^4\mathcal{L}\{y\} - s^3y(0) - s^2y'(0) - sy''(0) - y'''(0) - 4\mathcal{L}\{y\} = 0$$

$$s^4Y - s^3(1) - 0 - s(-2) - 0 - 4Y = 0$$

$$s^4Y - s^3 + 2s - 4Y = 0$$

$$Y(s^4-4) = s^3 - 2s$$

$$Y = \frac{s^3-2s}{s^4-4} = \frac{s\cancel{(s^2-2)}}{\cancel{(s^2-2)}(s^2+2)}$$

$$Y = \frac{s}{s^2+2} = \frac{s}{s^2+(\sqrt{2})^2} = \cos\sqrt{2}\,t$$