

Sorting Algorithms



Pool Game

Eight ball, also called stripes and solids, popular American pocket-billiards game in which 15 balls numbered consecutively and a white cue ball are used.



Pool Game: Problem

Our goal is to sort these balls in ascending order so that these can easily be placed on the table.



Pool Game: Problem

Let's consider the following balls for simplicity.













Pool Game: Problem

Since, we want to implement these algorithms on Computer and computers can only compare 2 elements in a unit time, therefore, we will only see/compare 2













elements at a time.

In-Place VS Out-Of-Place Sorting

A sorting algorithm is In-place if the algorithm does not use extra space for manipulating the input but may require a small though non-constant extra space for its operation.

Or we can say, a sorting algorithm sorts in-place if only a constant number of elements of the input array are ever stored outside the array.

Stable VS Unstable Sorting

A sorting algorithm is stable if it does not change the order of elements with the same value.



Stable VS Unstable Sorting

A sorting algorithm is unstable if it changes the order of elements with the same value.





Solution 1





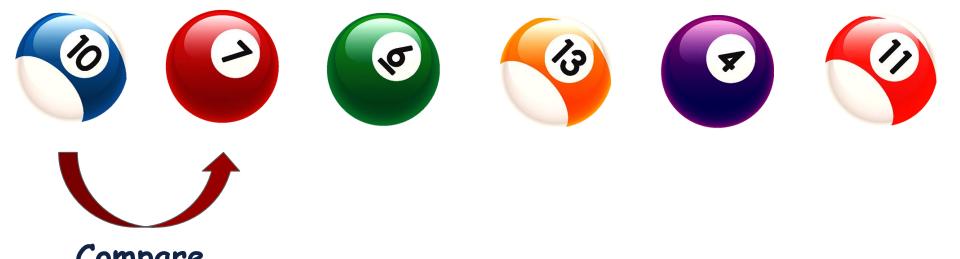


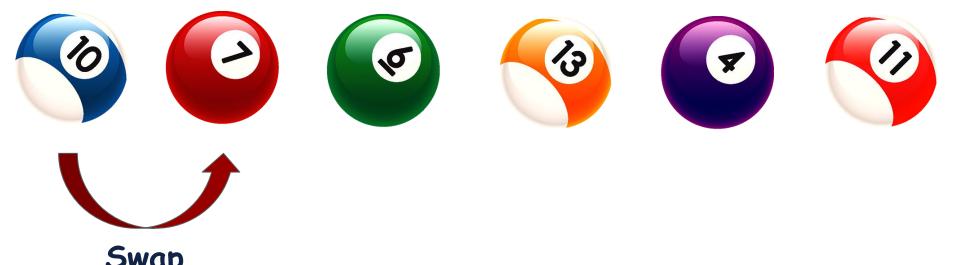














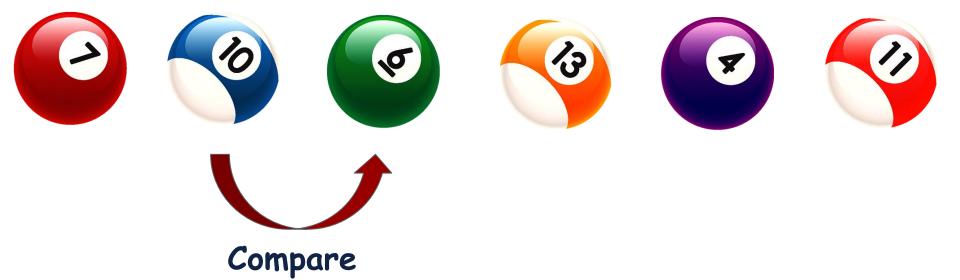


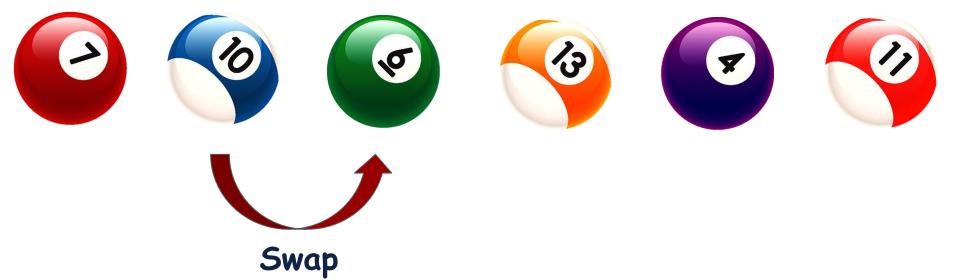


































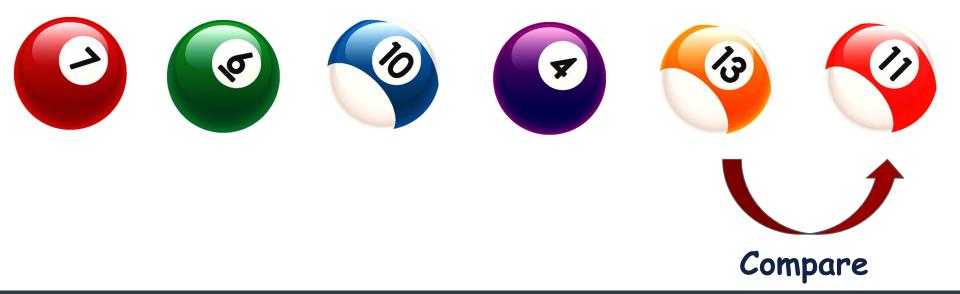


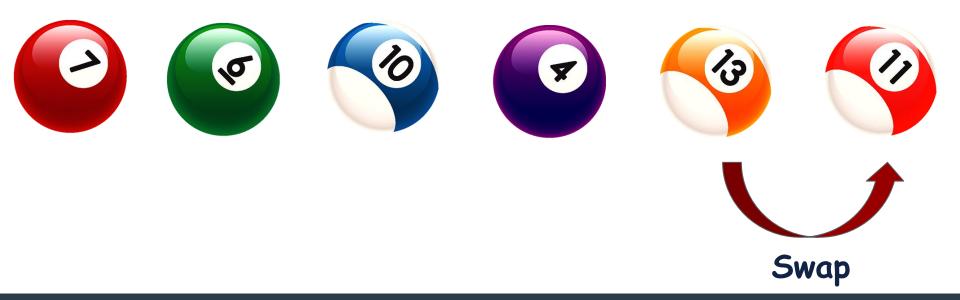
























Sorting: Bubble Sort

After 1 pass of the loop, the largest element is bubbled at the end of the array. Therefore, this Algorithm of Sorting is called Bubble Sort.

















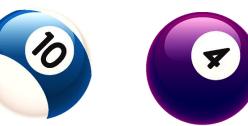






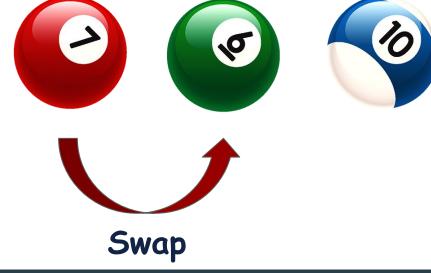




















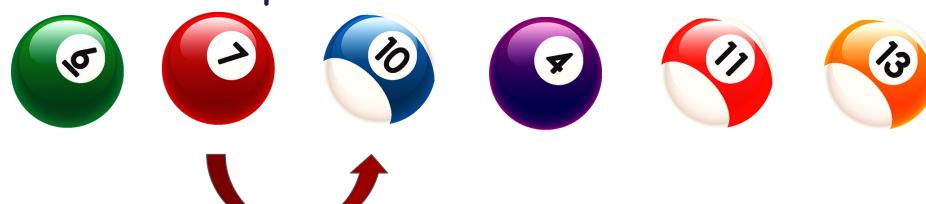




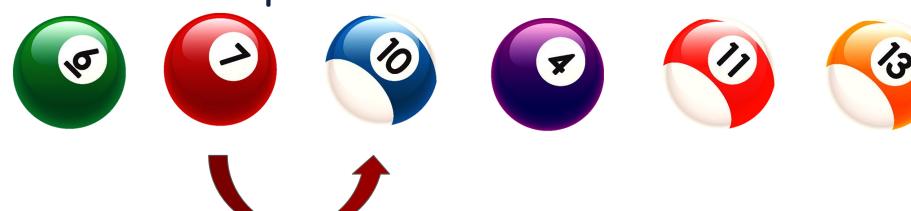




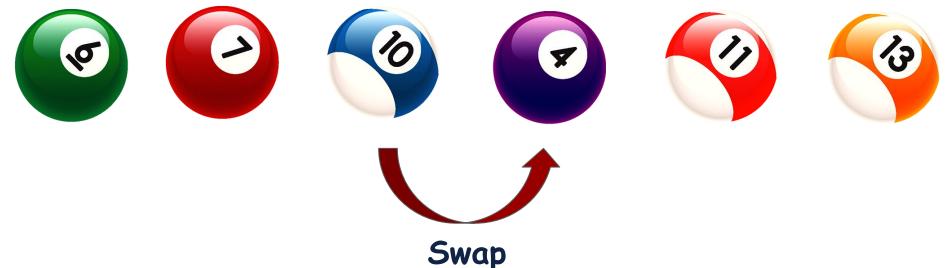




No Swap





































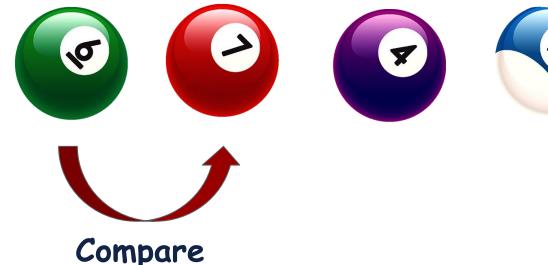
















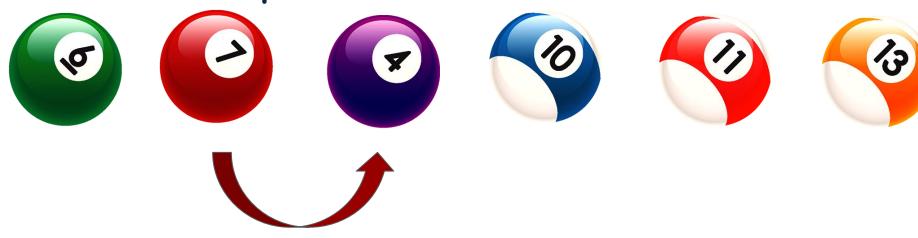


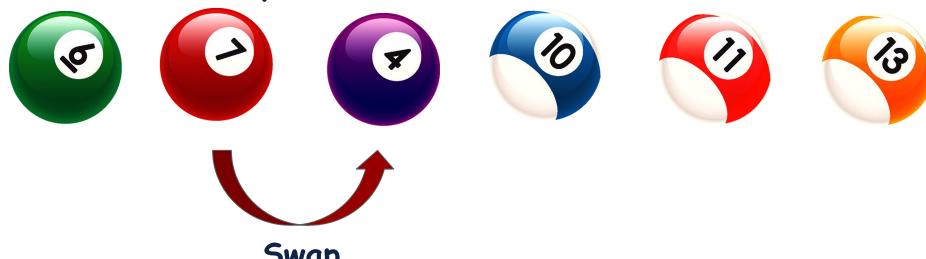








































































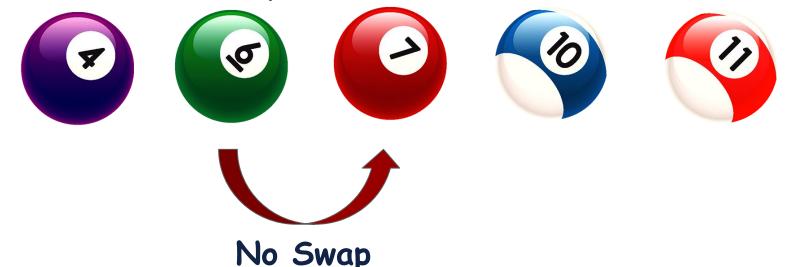
















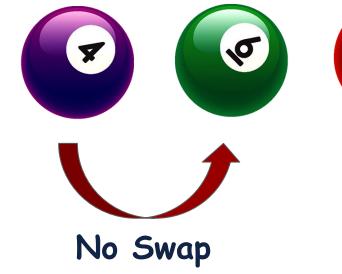


















After n-1 passes on the Array, the data is sorted.













Bubble Sort: Implementation

Let's implement the algorithm now.



Bubble Sort: Implementation

https://www.geeksforgeeks.org/bubble-sort/

```
main() {
    int arr[6] = {10, 7, 6, 13, 4, 11};
    bubbleSort(arr, 6);
    for (int x = 0; x < 6; x++)
    {
        cout << arr[x] << " ";
    }
}</pre>
```

```
void swap(int &a, int &b) {
    int c;
    c = a;
    a = b;
    b = c;
}
```

```
void bubbleSort(int arr[], int n)
    for (int x = 0; x < n - 1; x++)
        for (int y = 0; y < n - x - 1; y++)
            if (arr[y] > arr[y + 1])
                swap(arr[y], arr[y + 1]);
```

Bubble Sort: Time Complexity

What is the Time Complexity?

```
main() {
    int arr[6] = {10, 7, 6, 13, 4, 11};
    bubbleSort(arr, 6);
    for (int x = 0; x < 6; x++)
    {
        cout << arr[x] << " ";
    }
}</pre>
```

```
void swap(int &a, int &b) {
    int c;
    c = a;
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void bubbleSort(int arr[], int n)
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    }
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```

```
void swap(int &a, int &b) {
    int c;
    c = a;
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void bubbleSort(int arr[], int n)
    for (int x = 0; x < n - 1; x++)
        for (int y = 0; y < n - x - 1; y++)
            if (arr[y] > arr[y + 1])
                swap(arr[y], arr[y + 1]);
```



Bubble Sort: Point to Ponder

What if the data is already given in sorted order? How many iterations will this algorithm perform?













Bubble Sort: Point to Ponder

What if the data is already given in sorted order? How many iterations will this algorithm perform?



Both loops will run the same way. Just the swaps will not be made.

Best Case Time Complexity: O(n²)

Bubble Sort: Point to Ponder

Can we update the previous Algorithm so that our time complexity for the best case improves from $O(n^2)$ to O(n)?













Bubble Sort: Can we improve?

```
main() {
    int arr[6] = {10, 7, 6, 13, 4, 11};
    bubbleSort(arr, 6);
    for (int x = 0; x < 6; x++)
    {
        cout << arr[x] << " ";
    }
}</pre>
```

```
void swap(int &a, int &b) {
    int c;
    c = a;
    a = b;
    b = c;
}
```

```
void bubbleSort(int arr[], int n)
    for (int x = 0; x < n - 1; x++)
        for (int y = 0; y < n - x - 1; y++)
            if (arr[y] > arr[y + 1])
                swap(arr[y], arr[y + 1]);
```

Bubble Sort:

```
main() {
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    bubbleSort(arr, 6);
    for (int x = 0; x < 6; x++)
    {
        cout << arr[x] << " ";
    }
}</pre>
```

```
void swap(int &a, int &b) {
    int c;
    c = a;
    a = b;
    b = c;
}
```

```
void bubbleSort(int arr[], int n)
    for (int x = 0; x < n - 1; x++)
        bool isSwapped = false;
        for (int y = 0; y < n - x - 1; y++)
            if (arr[y] > arr[y + 1])
                swap(arr[y], arr[y + 1]);
                isSwapped = true;
        if (!isSwapped)
            break:
```



Solution 2





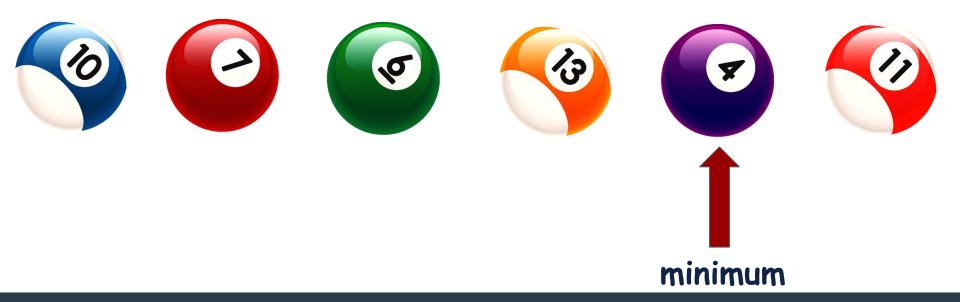


























Sorting: Selection Sort

In this Algorithm we are selecting the minimum element and then swapping it with the element on the specific index. Therefore, it is called as Selection Sort.













Selection Sort: 2nd Pass

Let's find the minimum element starting from the second element and swap it with the second element.













Selection Sort: 2nd Pass

Let's find the minimum element starting from the second element and swap it with the second element.



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Selection Sort: 3rd Pass













Selection Sort: 3rd Pass



Selection Sort: 3rd Pass





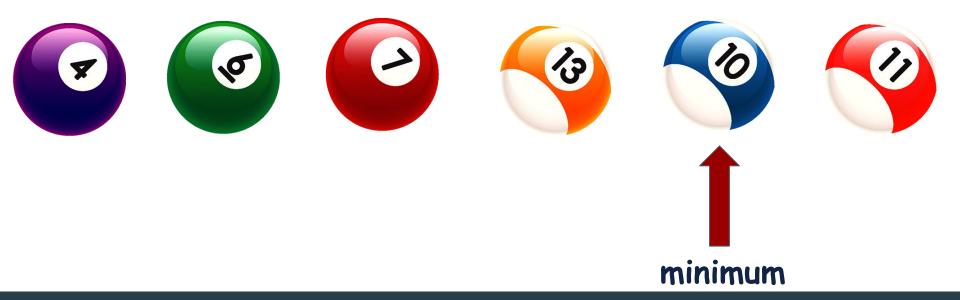




























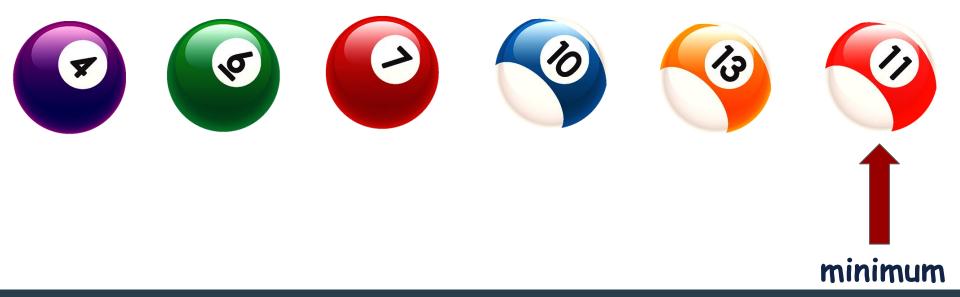


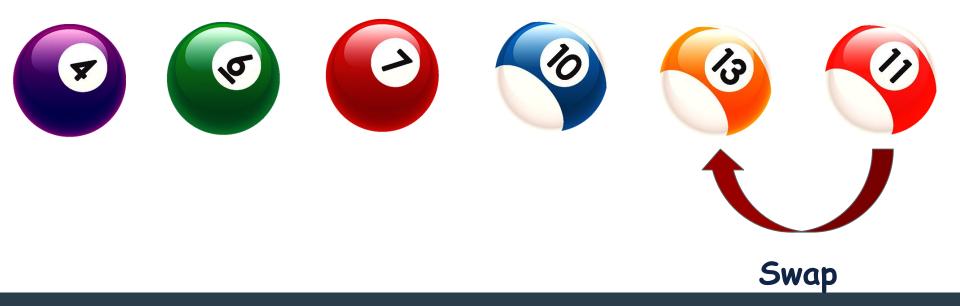
























Selection Sort: Implementation

Let's implement the algorithm now.



Selection Sort: Implementation

```
main()
{
    int arr[6] = {10, 7, 6, 13, 4, 11};
    selectionSort(arr, 6);
    for (int x = 0; x < 6; x++)
    {
       cout << arr[x] << " ";
    }
}</pre>
```

```
void swap(int &a, int &b)
{
    int c;
    c = a;
    a = b;
    b = c;
}
```

Selection Sort

```
int findMinimum(int arr[], int start, int end)
    int min = arr[start];
    int minIndex = start;
    for (int x = start; x < end; x++)
        if (min > arr[x])
            min = arr[x];
            minIndex = x;
    return minIndex;
```

```
void selectionSort(int arr[], int n) {
    for(int x = 0; x < n - 1; x++)
    {
        int minIndex = findMinimum(arr, x, n);
        swap(arr[x], arr[minIndex]);
    }
}</pre>
```

Selection Sort

What is the Time Complexity?

```
int findMinimum(int arr[], int start, int end)
    int min = arr[start];
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    for (int x = start; x < end; x++)
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    for(int x = 0; x < n - 1; x++)
    {
        int minIndex = findMinimum(arr, x, n);
        swap(arr[x], arr[minIndex]);
    }
}</pre>
```

O(n2)



Solution 3















The array is virtually split into a sorted and an unsorted part. Values from the unsorted part are picked and placed at the correct position in the sorted part.







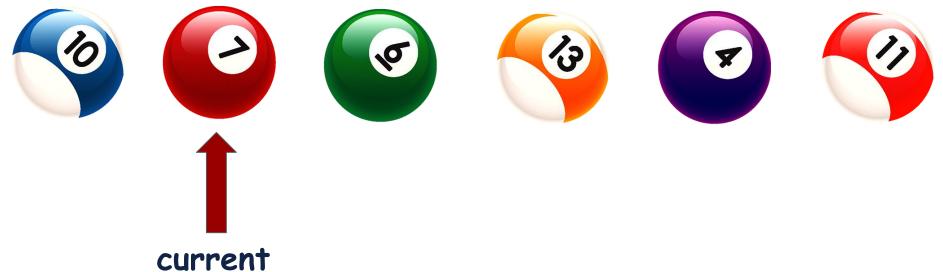






sorted

unsorted



The array is virtually split into a sorted and an unsorted part. Values from the unsorted part are picked and placed at the correct position in the sorted part.







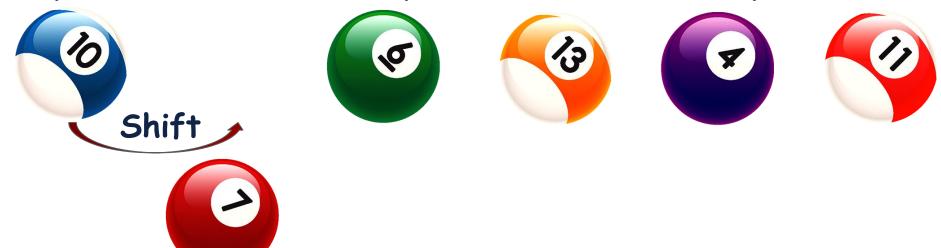






Store current element separately

















The array is virtually split into a sorted and an unsorted part. Values from the unsorted part are picked and placed at the correct position in the sorted part.













Place at the correct location













The array is virtually split into a sorted and an unsorted part. Values from the unsorted part are picked and placed at the correct position in the sorted part.



sorted

unsorted

Insertion Sort

In this Algorithm, we are inserting the current element in the already sorted Array. Therefore, this algorithm is called as Insertion Sort.



sorted

unsorted



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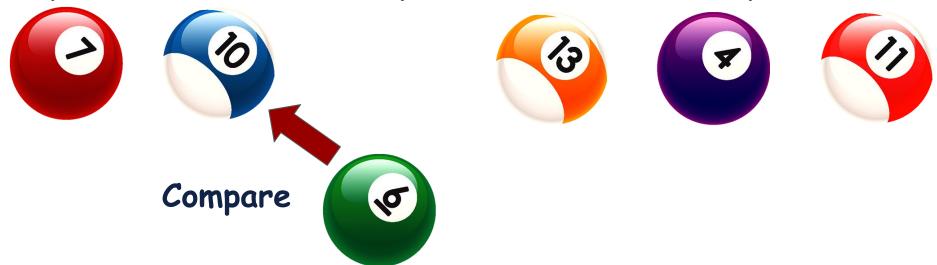






Store current element separately









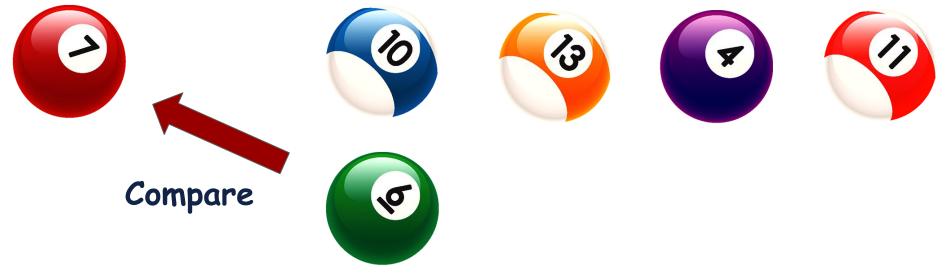




































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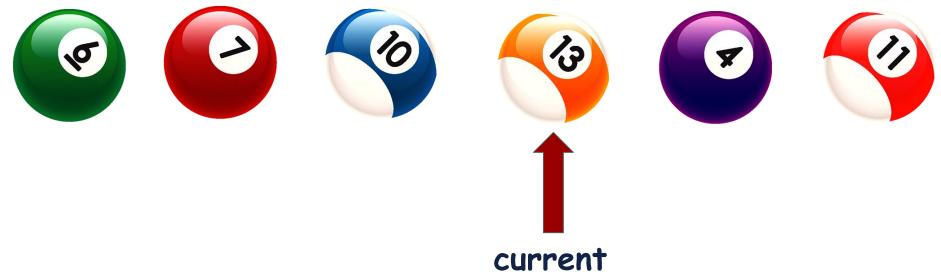


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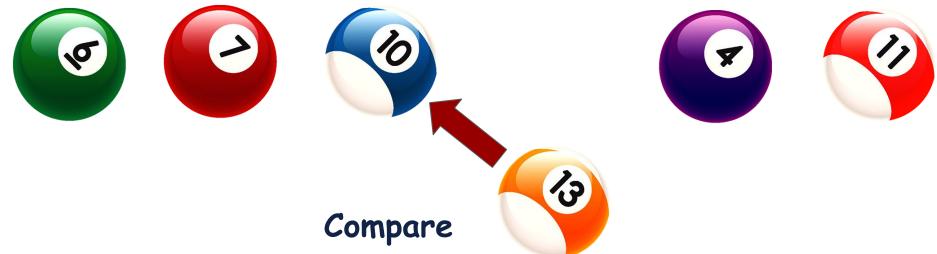








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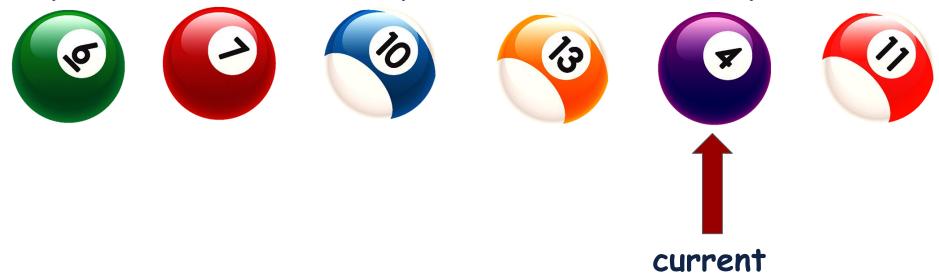






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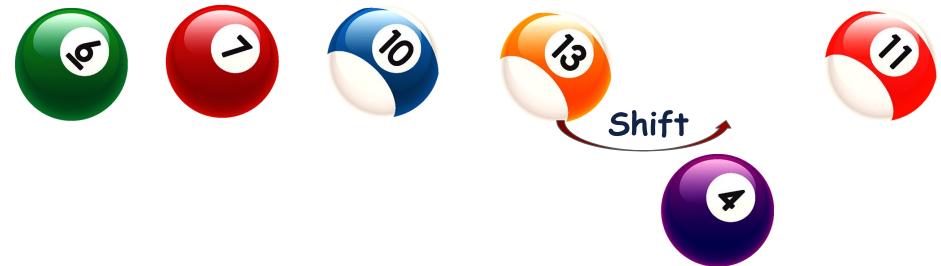




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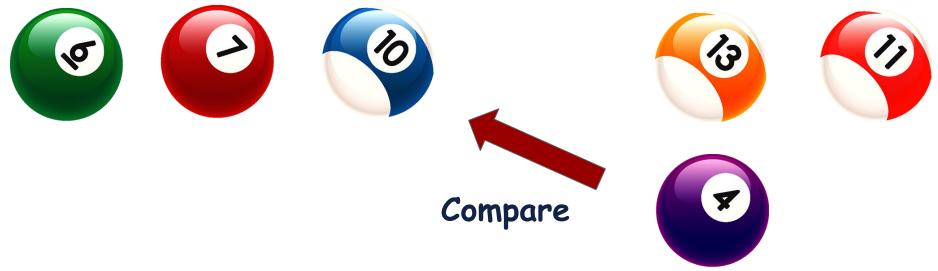


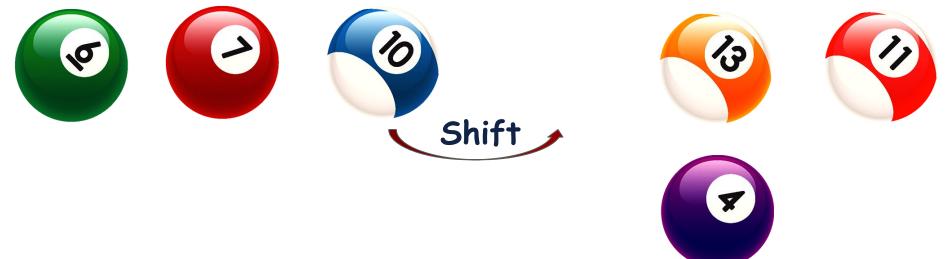














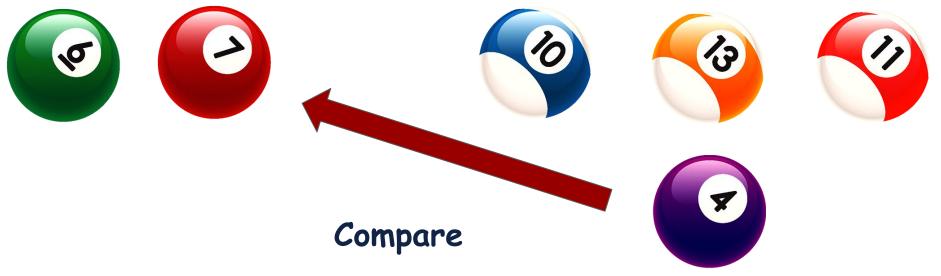
























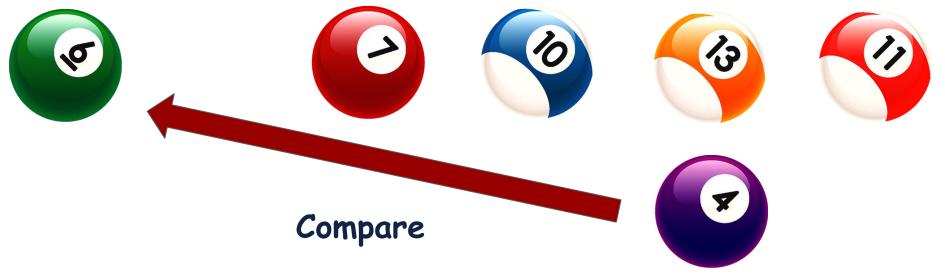




































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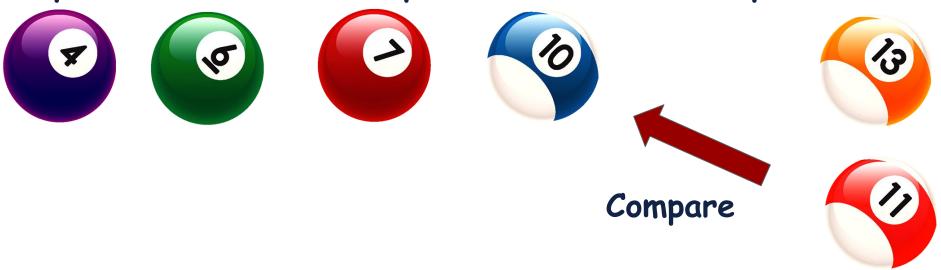












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Place at the correct location



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sorted

Let's implement the algorithm now.



```
main()
{
    int arr[6] = {10, 7, 6, 13, 4, 11};
    insertionSort(arr, 6);
    for (int x = 0; x < 6; x++)
    {
       cout << arr[x] << " ";
    }
}</pre>
```

```
void insertionSort(int arr[], int n)
    for (int x = 1; x < n; x++)
        int key = arr[x];
        int y = x - 1;
        while (y \ge 0 \&\& arr[y] > key)
            arr[y + 1] = arr[y];
            y--;
        arr[y + 1] = key;
```

What is the Time Complexity?

```
main()
{
    int arr[6] = {10, 7, 6, 13, 4, 11};
    insertionSort(arr, 6);
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```

Bubble VS Selection VS Insertion

Sorting Algorithm	Time Complexity			Space Complexity
	Best Case	Average Case	Worst Case	Worst Case
Bubble Sort	O(N)	O(N ²)	O(N ²)	O(1)
Selection Sort	O(N ²)	O(N ²)	O(N ²)	O(1)
Insertion Sort	0(N)	O(N ²)	O(N ²)	O(1)

Bubble VS Selection VS Insertion

https://www.geeksforgeeks.org/bubble-sort/

https://www.geeksforgeeks.org/selection-sort/

https://www.geeksforgeeks.org/insertion-sort/

https://visualgo.net/en/sorting

Sorting Algorithm	In-Place	Stable
Bubble Sort	Yes	Yes
Selection Sort	Yes	No
Insertion Sort	Yes	Yes

Learning Objective

Students should be able to apply sorting using Bubble, Selection and Insertion Sort.



Self Assessment

To visually see the Algorithms Running

https://visualgo.net/en/sorting

Self Assessment

We have studied the Bubble, Selection and Insertion Sort algorithm. The worst time complexity of all of these algorithms are same. So, Why and in which cases they are useful. Why the scientists thought to invent these algorithms?