Question: Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Explain your reasoning.

Solution:

Let,

$1)$ Either Kevin or Heather, or both, are chatting : $K \vee H$

$2)$ Either Randy or Vijay, but not both, are chatting : $R \oplus V$

$3)$ If Abby is chatting, so is Randy : $A \rightarrow R$

$4)$ Vijay and Kevin are either both chatting or neither is :
$(V \wedge K) \vee (\sim V \wedge \sim K)$ i.e. $V \Leftrightarrow K$

$5)$ If Heather is chatting, then so are Abby and Kevin : $H \rightarrow A \wedge K$

Now, we have to find the assignment for K, H, R, V and A so that whole system will be consistent means some assignment of truth values for these variables so that all the statements must be true.

Now, Let's start with $5)$ $H \rightarrow A \wedge K$ and $1)$ $K \vee H$,

For this, Possible assignments of truth values(T = True and F = False) are :-

$1) A = F, K = T, H = F$

$2) A = T, K = T, H = F$

$3) A = T, K = T, H = T$

Now, statement $3) \ A \rightarrow R$ will be true when :-

$1) A = F, K = T, H = F, R = T$

$2) A = F, K = T, H = F, R = F$

$3) A = T, K = T, H = F, R = T$

$4) A = T, K = T, H = T, R = T$

Now, statement $2) \ R \oplus V$ will be true when either $R = T, V = F$ (or) $V = T, R = F$, So, possible assignments will be :-

$1) A = F, K = T, H = F, R = T, V = F$

$2) A = F, K = T, H = F, R = F, V = T$

$3) A = T, K = T, H = F, R = T, V = F$

$4) A = T, K = T, H = T, R = T, V = F$

Now, last statement $4) \ (V \wedge K) \vee (\sim V \wedge \sim K)$ i.e. $V \Leftrightarrow K$ will be true when both K and V are True or False.

So, in above assignments $1), 3)$ and $4)$ are not possible. Only possibility is assignment $2)$ i.e. $A = F, K = T, H = F, R = F, V = T$. It means only Kevin and Vijay are chatting.

So, Answer is :- **Only Kevin and Vijay are chatting.**