University of Engineering and Technology Lahore

End Term Examination of BSc Civil Engineering

Subject: Applied Math II

Semester: 2nd

Time allowed: 90 Minutes

Session: 2021

Max. Marks:40

Q#1. Sketch the function state whether even, odd or neither even nor odd, hence find its Fourier series

$$f(x) = \begin{cases} x & -\pi < x < 0 \\ -x & 0 \le x < \pi \end{cases}$$ (5)

Q#2.Sketch the function and its two periodic extensions, hence find Fourier sine series

f(x)=x 0< x< L (5)

Q#3. Using convolution theorem find inverse Laplace transform of

$H(s)=\frac{1}{(s+1)(s^2+4)}$ (5)

Q#4. Find the solution of initial value problem using Laplace transform. Draw the graph of forcing function

$y''+y=g(t),\ y(0)=0,\ y'(0)=1 \quad g(t)=\begin{cases} \frac{t}{2} & 0 \le t < 6 \\ 3 & t \ge 6 \end{cases}$

$\sin \frac{n\pi x}{L}$

Q#5.*Solve the system of initial value problem using Laplace transform*

$y'_1=y_1+y_2,\quad y'_2=-y_1+3y_2 \quad y_1(0)=1,\ y_2(0)=0$ (5)

$s(s-3)-1(s-3)$
$s^2-3s-s+3$
s^2-4s+3

Q#6. Find the deflection u(x,t) for the string of length L=1 ,c^2=1 initial velocity is zero and initial deflection with small K is as follows

f(x)=Kx(1-x) (5)

Q#7.Solve the following partial differential equation by method of separating variables

$\frac{1}{\frac{n\pi}{L}}$

$U_{xy}=3x^2y^2u$ (5)

ILATE

$\frac{d}{dy}(y^3$
$3y^2(y)^2$
$9y^4 \times 3y^2$
$27y^6$

Q#8.Evaluate the double integral

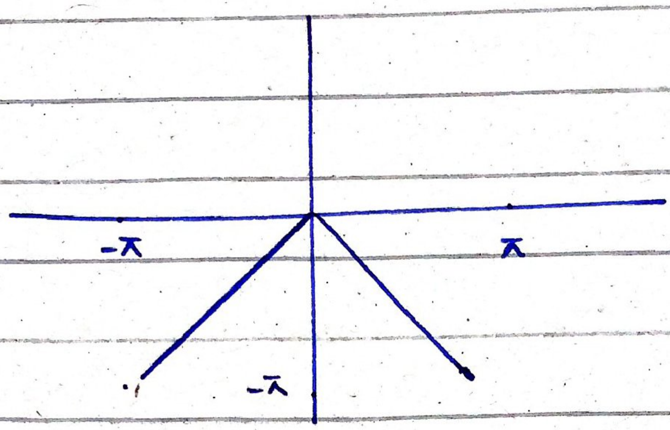
$\int_0^1 \int_0^{y^2} 3y^3 e^{xy} dx\, dy$ (5)

$(-\pi, -\pi)$ $(\pi, -\pi)$

$\frac{5}{9}e-2$

$\frac{y-0}{x-0} = \frac{-\pi-0}{-\pi-0}$ $\frac{y-0}{x-0} = \frac{-\pi-0}{\pi-0}$

Q:1 $f(x) = \begin{cases} x & -\pi < x < 0 \\ -x & 0 \leq x < \pi \end{cases}$



Since, the function is symmetric about y-axis. So, it is an even function.

We know that

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx \quad \text{(1)}$$

$$b_n = 0$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

Since function is even.

$$a_0 = \frac{1}{\pi} \int_0^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{\pi} \int_0^{\pi} -x\,dx$$

$$a_0 = -\frac{1}{\pi}\left|\frac{x^2}{2}\right|_0^{\pi}$$

$$a_0 = -\frac{1}{2\pi}\left[\pi^2 - 0\right] = -\frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \, dx$$

$$a_n = \frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx \, dx$$

$$a_n = \frac{2}{\pi}\int_{0}^{\pi} (-x)\cos nx \, dx$$

$$a_n = -\frac{2}{\pi}\int_{0}^{\pi} x\cos nx \, dx$$

x		$\cos nx$
	$+$	
1		$\frac{\sin nx}{n}$
	$-$	
0		$\frac{-\cos nx}{n^2}$

$$= \left(\cancel{\pi}\right) \frac{-2}{\pi} \left| \frac{\cos nx}{n^2} \right|_0^{\pi} = \frac{-2}{n^2\pi} \left| \cos n\pi \right|_0^{\pi}$$

$$a_n = \frac{-2}{n^2\pi}\left((-1)^n - 1\right)$$

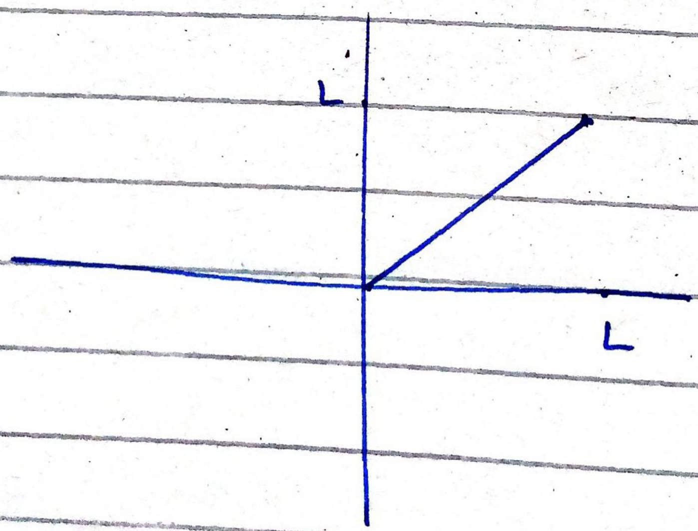
Equation (I) becomes.

$$f(x) = -\frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{-2}{n^2\pi}\left[(-1)^n - 1\right]\cos nx + 0$$

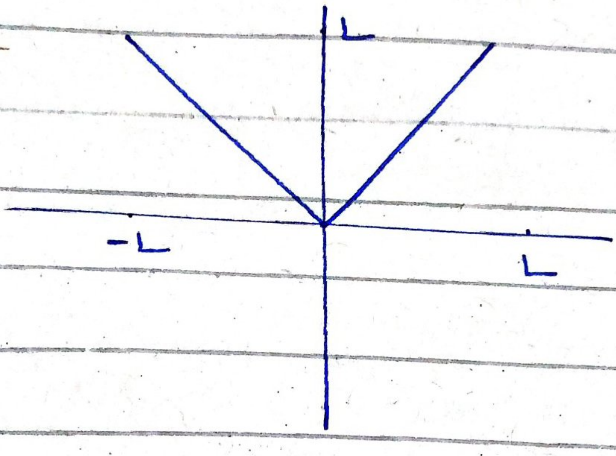
$$f(x) = \frac{-\pi}{2} + \sum_{n=1}^{\infty} \frac{-2}{n^2\pi}\left((-1)^n - 1\right)\cos nx$$

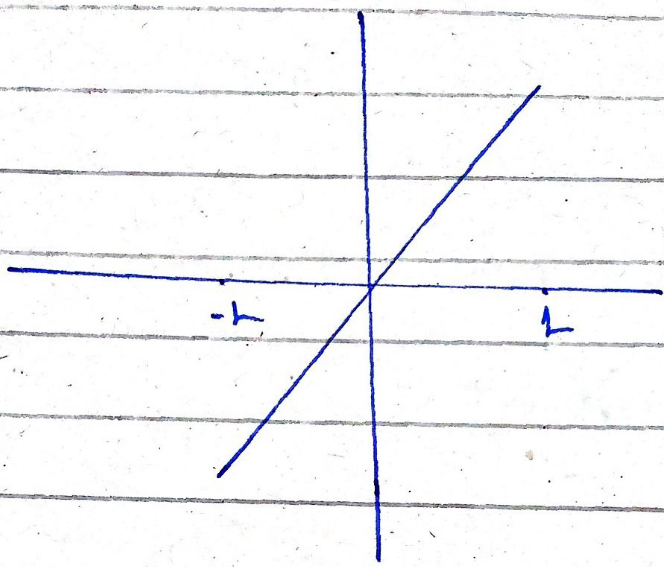
Q No. 2

$$f(x) = x \qquad 0 < x < L$$



Two periodic extensions:





Fourier Sine Series:-

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L} \, dx$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L} \, dx$$

$$b_n = \frac{2}{L} \int_{0}^{L} f(x) \sin \frac{n\pi x}{L} \, dx$$

$$= \frac{2}{L} \int_{0}^{L} x \sin \frac{n\pi x}{L} \, dx$$

$$x \qquad\qquad \sin\frac{n\pi x}{L}$$

$$+$$

$$1 \qquad\qquad -\frac{L}{n\pi}\cos\frac{n\pi x}{L}$$

$$-$$

$$0 \qquad\qquad -\frac{L^2}{n^2\pi^2}\sin\frac{n\pi x}{L}$$

$$b_n = \frac{2}{L}\left|(x)\left(-\frac{L}{n\pi}\cos\frac{n\pi x}{L}\right)\right|_0^L$$

$$b_n = -\frac{2}{\cancel{L}}\cdot\frac{\cancel{L}}{n\pi}\left[x\cos\frac{n\pi x}{L}\right]_0^L$$

$$b_n = -\frac{2}{n\pi}\left[L(-1)^n - 0\right]$$

$$b_n = \frac{-2}{n\pi}L(-1)^n$$

Now,

$$f(x) = \sum_{n=}^{\infty}\frac{-2}{n\pi}L(-1)^n \sin\frac{n\pi x}{L}\,dx$$

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$y'' + y = g(t)$ $\qquad g(t) = \begin{cases} \frac{t}{2} & 0 \le t < 6 \\ 3 & t \geq 6 \end{cases}$

$y(0) = 0, \; y'(0) = 1$

$$y'' + y = \frac{t}{6} - \frac{t-6}{2} u(t-6)$$

$$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = \frac{1}{6}\mathcal{L}\{t\} - \frac{1}{2}\mathcal{L}\{(t-6)u(t-6)\}$$

$$\left[s^2\mathcal{L}\{y\} - sy(0) - y'(0)\right] + \mathcal{L}\{y\} = \quad //$$

$$s^2 Y - 0 - 1 + Y = \frac{1}{6}\frac{1}{s^2} - \frac{1}{2}\frac{1}{s^2}e^{-6s}$$

$$s^2 Y + Y - 1 = \frac{1}{6s^2} - \frac{1}{2s^2}e^{-6s}$$

$$Y(s^2+1) = \frac{1}{6s^2} - \frac{1}{2s^2}e^{-6s} + 1$$

$$Y = \frac{1}{6s^2(s^2+1)} - \frac{1}{2s^2(s^2+1)}e^{-6s} + \frac{1}{s^2+1} \quad (1)$$

$$\frac{1}{s^2(s^2+1)} = \frac{1}{s^2} - \frac{1}{s^2+1}$$ By partial fraction

$$Y = \frac{1}{6}\left(\frac{1}{s^2} - \frac{1}{s^2+1}\right) - \frac{1}{2}\left(\frac{1}{s^2} - \frac{1}{s^2+1}\right)e^{-6s} + \left(\frac{1}{s^2} - \frac{1}{s^2+1}\right)$$

$\cancel{Y =}$ Taking g. L T

$$Y = \frac{1}{6}(t - \sin t) - \frac{1}{2}\left((t-6)U(t-6) - \sin(t-6)U(t-6)\right)$$

$$+ t - \sin t$$

$$Y = \frac{7t}{6} - \frac{7\sin t}{6} - \frac{1}{2}\left[(t-6)U(t-6) - \sin(t-6)U(t-6)\right]$$

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$$y_1' = y_1 + y_2$$

$$y_2' = -y_1 + 3y_2 \qquad y_1(0) = 1, \; y_2(0) = 0$$

$$\mathcal{L}\{y_1'\} = \mathcal{L}\{y_1\} + \mathcal{L}\{y_2\} \; ; \; \mathcal{L}\{y_2'\} = -\mathcal{L}\{y_1\} + 3\mathcal{L}\{y_2\}$$

$$s\mathcal{L}\{y_1\} - y_1(0) = \;\; '' \;\; '' \; ; \; s\mathcal{L}\{y_2\} - y_2(0) = \;\; '' \;\; ''$$

$$sY_1 - 1 = Y_1 + Y_2 \; ; \; sY_2 - 0 = -Y_1 + 3Y_2$$

$$sY_1 - Y_1 - Y_2 = 1 \; ; \; Y_1 + sY_2 - 3Y_2 = 0$$

$$Y_1(s-1) - Y_2 = 1 \qquad Y_1 + Y_2(s-3) = 0$$

$Y_1(s-1) - Y_2 = 1$

$Y_1 + Y_2(s-3) = 0$

$\because a_1Y_1 + b_1Y_2 = c_1$

$a_2Y_1 + b_2Y_2 = c_2$

$$\because Y_1 = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

&

$$\because Y_2 = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Now,

$$Y_1 = \frac{\begin{vmatrix} 1 & -1 \\ 0 & s-3 \end{vmatrix}}{\begin{vmatrix} s-1 & -1 \\ 1 & s-3 \end{vmatrix}} = \frac{(s-3)+0}{(s-3)(s-1)+1}$$

$$Y_1 = \frac{s-3}{s^2-s-3s+3+1} = \frac{s-3}{s^2-4s+4} = \frac{s-3}{(s-2)^2}$$

$$Y_i = \frac{s-2-1}{(s-2)^2} = \frac{s-2}{(s-2)^2} - \frac{1}{(s-2)^2}$$

$$Y_1 = \frac{1}{s-2} - \frac{1}{(s-2)^2}$$

taking I.L.T

$$y_1 = e^{2t} - t e^{2t}$$

Now,

$$Y_2 = \frac{\begin{vmatrix} 1 & s-1 \\ 0 & 1 \end{vmatrix}}{(s-2)^2} = \frac{1-0}{(s-2)^2}$$

$$Y_2 = \frac{1}{(s-2)^2}$$

taking I.L.T

$$y_2 = t e^{2t}$$