EXCERCISE (13)

$\boxed{Q. no. \quad 9 — 10 — 11 — 13 — 14 — 16}$

Q.9. The pair of equations.

$$x_1 + 2x_2 = 3.0$$
$$3x_1 + x_2 = 4.0$$

Can be arranged

$$x_1 = 3 - 2x_2$$
$$x_2 = 4 - 3x_1$$

Apply Gauss seidel & Jacobi's method with $(1.01, 1.01)$

Solution.

Jacobi's Method.

First iteration.

$$\left.\begin{aligned} x_1 &= 3 - 2x_2 \\ x_2 &= 4 - 3x_1 \end{aligned}\right\} \longrightarrow A$$

Put $x_1 = 1.01$ & $x_2 = 1.01$ in R.H.S of A

$$x_1 = 3 - 2(1.01)$$
$$= 0.98$$
$$x_2 = 4 - 3(1.01)$$
$$= 0.97$$

Second Iteration. Put $x_1 = 0.98$ & $x_2 = 0.97$ in R.H.S of A

$$x_1 = 3 - 2(0.97)$$
$$= 1.06$$
$$x_2 = 4 - 3(0.98)$$
$$= 1.06$$

Third Iteration. Put x_1 & $x_2 = 1.06$ in R.H.S of A

$$x_1 = 3 - 2(1.06)$$
$$= 0.88$$
$$x_2 = 4 - 3(1.06)$$
$$= 0.82$$

Fourth Iteration. Put $x_1 = 0.88$ & $x_2 = 0.82$ in R.H.S of A

$$x_1 = 3 - \ldots (0.82)$$

$$x_2 = 4-3(0.88)$$
$$= 1.36$$

Fifth iteration. put x_1 & $x_2=1.36$ in R.H.S of A.

$$x_1 = 3-2(1.36)$$
$$= 0.28$$
$$x_2 = 4-3(1.36)$$
$$= -0.08$$

Sixth iteration put $x_1=0.28$ & $x_2=-0.08$ in R.H.S of A.

$$x_1 = 3-2(-0.08)$$
$$= 3.16$$
$$x_2 = 4-3(0.28)$$
$$= 3.16$$

After sixth iteration $\underline{x} = (3.16, 3.16)^T$
which is diverging.

By

Gauss. Seidel method.

Solution Again writing the equations

$$\left.\begin{aligned} x_1 &= 3-2x_2 \\ x_2 &= 4-3x_1 \end{aligned}\right\} \text{— A}$$

we set up the iteration

$$\left.\begin{aligned} x_1^{k+1} &= 3-2x_2^{k} \\ x_2^{k+1} &= 4-3x_1^{k+1} \end{aligned}\right\} \text{— B}$$

FIRST ITERATION put $x_1'=01.01$ & $x_2=1.01$ & $K=0$ in B

$$x_1' = 3-2(1.01)$$
$$= 0.98$$
$$x_2' = 4-3(0.98)$$
$$= 1.06$$

SECOND ITERATION Put $x_2'=1.06$ & $K=1$ in B

$$x_1^2 = 3-2(1.06)$$
$$= 0.88$$
$$x_2^2 = 4-3(0.88)$$
$$= 1.36$$

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<u>THIRD ITERATION</u>: Put $x_2^2 = 1.36$ & $K = 2$ in B

$$x_1^3 = 3 - 2(1.36)$$
$$= 0.28$$
$$x_2^3 = 4 - 3(0.28)$$
$$= 3.16$$

<u>FOURTH ITERATION</u>. Put $x_2^3 = 3.16$ & $K = 3$ in B

$$x_1^4 = 3 - 2(3.16)$$
$$= -3.32$$
$$x_2^4 = 4 - 3(-3.32)$$
$$= 13.96$$

<u>FIFTH ITERATION</u>. Put $x_2^4 = 13.96$ & $K = 4$ in B

$$x_1^5 = 3 - 2(13.96)$$
$$= -24.92$$
$$x_2^5 = 4 - 3(-24.92)$$
$$= 78.76$$

<u>SIXTH ITERATION</u> Put $x_2^5 = 78.76$ & $K = 5$ in B

$$x_1^6 = 3 - 2(78.76)$$
$$= -154.52$$
$$x_2^6 = 4 - 3(-154.52)$$
$$= 467.56$$

After six iteration solution vector for Gauss - Seidel method is $\underline{x} = (-154.52, 467.56)$ which is more rapidly divergent method than Jacobi's method.

Q. 10 — Solve the system of equations

$$6x_1 - 3x_2 + x_3 = 11$$
$$2x_1 + x_2 - 8x_3 = -15$$
$$x_1 - 7x_2 + x_3 = 10$$

by using gauss seidel method.

Solution. The system is not in diagonally dominant form for this purpose we interchange 2nd & 3rd rows

$$6x_1 - 3x_2 + x_3 = 11$$
$$x_1 - 7x_2 + x_3 = 10$$
$$2x_1 + x_2 - 8x_3 = -15$$

The system of equation is given by by writing the system in this form.

$$\left.\begin{aligned} x_1 &= \frac{1}{6}\left[11 + 3x_2 - x_3\right] \\ x_2 &= \frac{1}{7}\left[-10 + x_1 + x_3\right] \\ x_3 &= \frac{1}{8}\left[15 + 2x_1 + x_2\right] \end{aligned}\right\} \longrightarrow A$$

we set up the iteration.

$$\left.\begin{aligned} x_1^{k+1} &= 1/6\left[11 + 3x_2^{k} - x_3^{k}\right] \\ x_2^{k+1} &= 1/7\left[-10 + x_1^{k+1} + x_3^{k}\right] \\ x_3^{k+1} &= 1/8\left[15 + 2x_1^{k+1} + x_2^{k+1}\right] \end{aligned}\right\} \text{—— } B$$

FIRST ITERATION.

$k = 0$ Put initial solution $\underline{x} = (0,0,0)^T$ in B

$$x_1^1 = 1/6\left[11 + 3(0) - 0\right]$$
$$= 1.8333$$
$$x_2^1 = \frac{1}{7}\left[-10 + 1.8333 + 0\right]$$
$$= -1.1667$$
$$x_3^1 = \frac{1}{8}\left[15 + 2(1.8333) + (-1.1667)\right]$$
$$= 2.1875$$

SECOND ITERATION. Put $x_2^1 = -1.1667$, $x_3^1 = 2.1875$ & $k = 1$ in B and get

$$x_1^2 = 1/6\left[11 - (2.1875) + 3(-1.1667)\right]$$
$$= 0.8854$$
$$x_2^2 = \frac{1}{7}\left[-10 + 0.8854 + 2.1875\right]$$
$$= 0.9896$$
$$x_3^2 = \frac{1}{8}\left[15 + 2(0.8854) + (-0.9896)\right]$$

$$x_3^2 = 1/10\,[27 + 0.8869 + 1.9523 + 2(-0.1368)]$$
$$= 2.9566$$

$$x_4^2 = 1/10\,[-9 + 0.8869 + 1.9523 + 2(2.9566)]$$
$$= -0.0248$$

<u>THIRD ITERATION</u>: Put $x_2^2 = 1.9523$, $x_3^2 = 2.9566$, $x_4^2 = -0.0248$ & $K = 2$ in C and get

$$x_1^3 = 1/10\,[3 + 2(1.9523) + 2.9566 - 0.0248]$$
$$= 0.9836$$

$$x_2^3 = 1/10\,[15 + 2(0.9836) + 2.9566 + (-0.0248)]$$
$$= 1.9899$$

$$x_3^3 = 1/10\,[27 + 0.9836 + 1.9899 + 2(0.0248)]$$
$$= 2.9924$$

$$x_4^3 = 1/10\,[-9 + 0.9836 + 1.9899 + 2(2.9924)]$$
$$= -0.0042$$

<u>FOURTH ITERATION</u>: Put $x_2^3 = 1.9899$, $x_3^3 = 2.9924$, $x_4^3 = -0.0042$ & $K = 3$ in C and get

$$x_1^4 = 1/10\,[3 + 2(1.9899) + 2.9924 - 0.0042]$$
$$= 0.9968$$

$$x_2^4 = 1/10\,[15 + 2(0.9968) + 2.9924 - 0.0042]$$
$$= 1.9982$$

$$x_3^4 = 1/10\,[27 + 0.9968 + 1.9982 + 2(-0.0042)]$$
$$= 2.9987$$

$$x_4^4 = 1/10\,[-9 + 0.9968 + 1.9982 + 2(2.9987)]$$
$$= -0.0008$$

<u>FIFTH ITERATION</u>: Put $x_2^4 = 1.9982$, $x_3^4 = 2.9987$, $x_4^4 = -0.0008$ & $K = 4$ in C and get.

$$x_1^5 = 1/10\,[3 + 2(1.9982) + 2.9987 - 0.0008]$$
$$= 0.9994$$

$$x_2^5 = 1/10\,[15 + 2(0.9994) + 2.9987 + (-0.0008)]$$
$$= 1.9997$$

$$x_3^5 = 1/10\,[27 + 0.9994 + 1.9997 + 2(-0.0008)]$$
$$= 2.9998$$

$$x_4^5 = 1/10\,[-9 + 0.9994 + 1.9997 + 2(2.9998)]$$
$$= -0.0001$$

Sixth Iteration: Put $x_2^5 = 1.9997$, $x_3^5 = 2.9998$, $x_4^5 = -0.0001$ & K=5 in C and get.

$$x_1^6 = \frac{1}{10}\left[3+2(1.9997)+2.9998+(-0.0001)\right]$$
$$= 0.9999 \cong 1.00$$
$$x_2^6 = \frac{1}{10}\left[15+2(0.9999)+2.9998+(-0.0001)\right]$$
$$= 1.9999 \cong 2.00$$
$$x_3^6 = \frac{1}{10}\left[27+0.9999+1.9999+2(-0.0001)\right]$$
$$= 2.9999 \cong 3.00$$
$$x_4^6 = \frac{1}{10}\left[-9+0.9999+1.9999+2(2.9999)\right]$$
$$= -0.0000 \cong 0.00$$

Seventh Iteration: Put $x_2^6 = 2.00$, $x_3^6 = 3.00$ & $x_4^6 = 0.00$ & K=6 in C and get

$$x_1^7 = \frac{1}{10}\left[3+2(2)+3+0\right]$$
$$= 1.0000$$
$$x_2^7 = \frac{1}{10}\left[15+2(1.0000)+3+0\right]$$
$$= 2.0000$$
$$x_3^7 = \frac{1}{10}\left[27+1.0000+2.0000+2(0)\right]$$
$$= 3.0000$$
$$x_4^7 = \frac{1}{10}\left[-9+1.0000+2.0000+2(3.0000)\right]$$
$$= 0.0000$$

Hence solution vector upto four decimal place is $\underline{x} = (1, 2, 3, 0)^T$

Q14 Solve the system using Gauss-Seidel method

$$2x_1 + x_2 = -1$$
$$x_1 - 2x_2 + x_3 = 0$$
$$x_2 - 2x_3 + x_4 = 0$$
$$x_3 - 2x_4 = 0$$

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Third Iteration. Put $x_2^2 = -0.9896$, $x_3^2 = 1.9726$ & $K = 2$ in B.S get

$$x_1^3 = \frac{1}{6}\left[11 + 3(-0.9896) - 1.9726\right]$$
$$= 1.0097$$
$$x_2^3 = \frac{1}{7}\left[-10 + 1.0097 + 1.9726\right]$$
$$= -1.0025$$
$$x_3^3 = \frac{1}{8}\left[15 + 2(1.0097) - 1.0025\right]$$
$$= 2.0021$$

Fourth Iteration. Put $x_2^3 = -1.0025$, $x_3^3 = 2.0021$ & $K = 3$ in B.S get

$$x_1^4 = 1/6\left[11 + 3(-1.0025) - 2.0021\right]$$
$$= 0.9984$$
$$x_2^4 = 1/7\left[-10 + 0.9984 + 2.0021\right]$$
$$= -0.9999$$
$$x_3^4 = 1/8\left[15 + 2(0.9984) - 0.9999\right]$$
$$= 1.9996$$

Fifth Iteration Put $x_2^4 = -0.9999$, $x_3^4 = 1.9996$ & $K = 4$ in B and get

$$x_1^5 = 1/6\left[11 + 3(-0.9999) - 1.9996\right]$$
$$= 1.0001 \cong 1.0000$$
$$x_2^5 = \frac{1}{7}\left[-10 + 1.0001 + 1.9996\right]$$
$$= -1.0000$$
$$x_3^5 = \frac{1}{8}\left[15 + 2(1.0001) - 1.0000\right]$$
$$= 2.0000$$

Sixth Iteration Put $x_2^5 = -1.0000$, $x_3^5 = 2.0000$ & $K = 5$ in B and get

$$x_1^6 = \frac{1}{6}\left[11 + 3(-1.0000) - 2.0000\right]$$
$$= 1.0000$$
$$x_2^6 = \frac{1}{7}\left[-10 + 1.0000 + 2.0000\right]$$
$$= -1.0000$$
$$\therefore x_3^6 = \frac{1}{8}\left[15 + 2(1.0000) - 1.0000\right]$$
$$= 2.0000$$

Hence solution vector is $\underline{x}\ (1, -1, 2)^T$

i.e. $x_1 = 1$, $x_2 =$ & $x_3 = 2$.

Q.11 Calculate to three decimal places the solution of the system of equations using Gauss Seidel method. (20)

$$11x_1 + 2x_2 + x_3 = 15$$
$$x_1 + 10x_2 + 2x_3 = 16$$
$$2x_1 + 3x_2 - 8x_3 = 1$$

<u>Solution</u>: The system of equation is in diagonally dominant form. It can be written as:

$$\left.\begin{aligned} x_1 &= \tfrac{1}{11}\left[15 - 2x_2 - x_3\right] \\ x_2 &= \tfrac{1}{10}\left[16 - x_1 - 2x_3\right] \\ x_3 &= \tfrac{1}{8}\left[-1 + 2x_1 + 3x_2\right] \end{aligned}\right\} \longrightarrow (A)$$

we set up the iteration

$$\left.\begin{aligned} x_1^{k+1} &= \tfrac{1}{11}\left[15 - 2x_2^{k} - x_3^{k}\right] \\ x_2^{k+1} &= \tfrac{1}{10}\left[16 - x_1^{k+1} - 2x_3^{k}\right] \\ x_3^{k+1} &= \tfrac{1}{8}\left[-1 + 2x_1^{k+1} + 3x_2^{k+1}\right] \end{aligned}\right\} \Longrightarrow (B)$$

<u>FIRST ITERATION</u>: Put initial solution $\underline{x}(0,0,0)$ in (B) & $K=0$ and get

$$x_1^1 = \tfrac{1}{11}\left[15 - 2(0) - 0\right]$$
$$= 1.3636 \cong 1.364 \quad \text{(for three decimal)}$$
$$x_2^1 = \tfrac{1}{10}\left[16 - 1.3636 - 2(0)\right]$$
$$= 1.464$$
$$x_3^1 = \tfrac{1}{8}\left[-1 + 2(1.364) + 3(1.464)\right]$$
$$= 0.765$$

<u>Second Iteration</u>: Put $x_2^1 = 1.464$, $x_3^1 = 0.765$ & $K=1$ and get

$$x_1^2 = \tfrac{1}{11}\left[15 - 2(1.464) - 0.765\right]$$
$$= 1.028$$
$$x_2^2 = \tfrac{1}{10}\left[16 - 1.028 - 2(0.765)\right]$$

$= 1.344$

$x_3^2 = \frac{1}{8}[-1 + 2(1.028) + 3(1.344)]$
$= 0.636$

THIRD ITERATION: Put $x_2^2 = 1.344$, $x_3^2 = 0.636$ & $K = 2$ in Ⓑ & get.

$x_1^3 = \frac{1}{11}[15 - 2(1.344) - 0.636]$
$= 1.061$

$x_2^3 = \frac{1}{10}[16 - 1.061 - 2(0.636)]$
$= 1.367$

$x_3^3 = \frac{1}{8}[-1 + 2(1.061) + 3(1.367)]$
$= 0.653$

FOURTH ITERATION: Put $x_2^3 = 1.367$, $x_3^3 = 0.653$ & $K = 3$ in Ⓑ and get

$x_1^4 = \frac{1}{11}[15 - 2(1.367) - 0.653]$
$= \underline{1.056}$

$x_2^4 = \frac{1}{10}[16 - 1.056 - 2(0.653)]$
$= 1.364$

$x_3^4 = \frac{1}{8}[-1 + 2(1.056) + 3(1.364)]$
$= 0.651$

FIFTH ITERATION: Put $x_2^4 = 1.364$, $x_3^4 = 0.651$ & $K = 4$ in Ⓑ & get

$x_1^5 = \frac{1}{11}[15 - 2(1.364) - 0.651]$
$= 1.056$

$x_2^5 = \frac{1}{10}[16 - 1.056 - 2(0.651)]$
$= 1.364$

$x_3^5 = \frac{1}{8}[-1 + 2(1.056) + 3(1.364)]$
$= 0.651$

Hence solution vector is.

$\underline{X} = (1.056, 1.364, 0.651)^T$

i.e

$x_1 = 1.056$

$x_2 = 1.364$

$x_3 = 0.651$

Q 13(i)

Solve the following by Gauss Seidel method

$$10x_1 + 2x_2 + x_3 = 9$$
$$2x_1 + 20x_2 - 2x_3 = -44$$
$$-2x_1 + 3x_2 + 10x_3 = 22$$

<u>Solution</u>: The system of given equation is in diagonally dominant form. It can be written as;

$$\left.\begin{aligned} x_1 &= \tfrac{1}{10}[9 - 2x_2 - x_3] \\ x_2 &= \tfrac{1}{20}[-44 - 2x_1 + 2x_3] \\ x_3 &= \tfrac{1}{10}[22 + 2x_1 - 3x_2] \end{aligned}\right] \longrightarrow A$$

we set up the iteration.

$$\left.\begin{aligned} x_1^{k+1} &= \tfrac{1}{10}[9 - 2x_2^{k} - x_3^{k}] \\ x_2^{k+1} &= \tfrac{1}{20}[-44 - 2x_1^{k+1} + 2x_3^{k}] \\ x_3^{k+1} &= \tfrac{1}{10}[22 + 2x_1^{k+1} - 3x_2^{k+1}] \end{aligned}\right] \Longrightarrow B$$

<u>FIRST ITERATION</u>: Put initial solution vector $\underline{x} = (0,0,0)^T$ in B and $k=0$ & get.

$$x_1^1 = \tfrac{1}{10}[9 - 2(0) - 0]$$
$$= 0.9$$

$$x_2^1 = \tfrac{1}{20}[-44 - 2(0.9) + 2(0)]$$
$$= -2.29$$
$$x_3^1 = \tfrac{1}{10}[22 + 2(0.9) - 3(-2.29)]$$
$$= 3.067$$

<u>SECOND ITERATION</u>: Put $x_2^1 = -2.29$ $x_3^1 = 3.067$ & $k=1$ in B and get

$$x_1^2 = \tfrac{1}{10}[9 - 2(-2.29) - 3.067]$$
$$= 1.0513$$
$$x_2^2 = \tfrac{1}{20}[-44 - 2(1.0513) + 2(3.067)]$$
$$= -1.9984$$
$$x_3^2 = \tfrac{1}{10}[22 + 2(1.0513) - 3(-1.9984)]$$
$$= 3.0098$$

THIRD ITERATION: Put $x_2^{2} = -1.9984$, $x_3^{2} = 3.0098$ & $K=2$ in B and get.

$$x_1^{3} = \tfrac{1}{10}\left[9 - 2(-1.9984) - 3.0098\right]$$
$$= 0.9987$$
$$x_2^{3} = \frac{1}{20}\left[-44 - 2(0.9987) + 2(3.0098)\right]$$
$$= -1.9989$$
$$x_3^{3} = \frac{1}{10}\left[22 + 2(0.9987) - 3(-1.9989)\right]$$
$$= 2.9994$$

FOURTH ITERATION: Put $x_2^{3} = -1.9989$, $x_3^{3} = 2.9994$ & $K=3$ in B and get

$$x_1^{4} = \tfrac{1}{10}\left[9 - 2(-1.9989) - 2.9994\right)$$
$$= 0.9998 \cong 1.0$$
$$x_2^{4} = \frac{1}{20}\left[-44 - 2(0.9998) + 2(2.9994)\right]$$
$$= -2.0000$$
$$x_3^{4} = \tfrac{1}{10}\left[22 + 2(0.9998) - 3(-2)\right]$$
$$= 2.9999 \cong 3.0$$

FIFTH ITERATION: Put $x_2^{4} = -2$, $x_3^{4} = 3$ & $K=4$ in B and get

$$x_1^{5} = \tfrac{1}{10}\left[9 - 2(-2) - 3\right]$$
$$= 1.0$$
$$x_2^{5} = \frac{1}{20}\left[-44 - 2(1) + 2(3)\right]$$
$$= -2.0$$
$$x_3^{5} = \tfrac{1}{10}\left[22 + 2(1) - 3(-2)\right]$$
$$= 3.0$$

$\therefore$ Hence solution vector is $X = (1.0, -2.0, 3.0)^T$

Q. 13(ii)

$$\begin{bmatrix} 10 & -2 & -1 & -1 \\ -2 & 10 & -1 & -1 \\ -1 & -1 & 10 & -2 \\ -1 & -1 & -2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 15 \\ 27 \\ -9 \end{bmatrix}$$

Solve by Gauss Seidel method.

Solution: It can be written as

$$\left.\begin{aligned} 10x_1 - 2x_2 - x_3 - x_4 &= 3 \\ -2x_1 + 10x_2 - x_3 - x_4 &= 15 \\ -x_1 - x_2 + 10x_3 - 2x_4 &= 27 \\ -x_1 - x_2 - 2x_3 + 10x_4 &= -9 \end{aligned}\right] \text{—— A}$$

Since A is in diagnally dominant form

So It can be written as,

$$\left.\begin{aligned} x_1 &= \tfrac{1}{10}\left[3 + 2x_2 + x_3 + x_4\right] \\ x_2 &= \tfrac{1}{10}\left[15 + 2x_1 + x_3 + x_4\right] \\ x_3 &= \tfrac{1}{10}\left[27 + x_1 + x_2 + 2x_4\right] \\ x_4 &= \tfrac{1}{10}\left[-9 + x_1 + x_2 + 2x_3\right] \end{aligned}\right] \Longrightarrow B$$

We set up the iteration

$$\left.\begin{aligned} x_1^{k+1} &= \tfrac{1}{10}\left[3 + 2x_2^{k} + x_3^{k} + x_4^{k}\right] \\ x_2^{k+1} &= \tfrac{1}{10}\left[15 + 2x_1^{k+1} + x_3^{k} + x_4^{k}\right] \\ x_3^{k+1} &= \tfrac{1}{10}\left[27 + x_1^{k+1} + x_2^{k+1} + 2x_4^{k}\right] \\ x_4^{k+1} &= \tfrac{1}{10}\left[-9 + x_1^{k+1} + x_2^{k+1} + 2x_3^{k+1}\right] \end{aligned}\right] \Longrightarrow C$$

FIRST ITERATION: Put initial solution vector $\underline{x} = (0,0,0,0)^T$ and $k=0$ in C and get

$$x_1^1 = \tfrac{1}{10}\left[3 + 2(0) + 0 + 0\right]$$
$$= 0.3$$
$$x_2^1 = \tfrac{1}{10}\left[15 + 2(0.3) + 0 + 0\right]$$
$$= 1.56$$

$$x_3^1 = \tfrac{1}{10}\left[27 + 0.3 + 1.56 + 2(0)\right]$$
$$= 2.886$$
$$x_4^1 = \tfrac{1}{10}\left[-9 + 0.3 + 1.56 + 2(2.886)\right]$$
$$= -0.1368$$

SECOND ITERATION: Put $x_2^1 = 1.56$, $x_3^1 = 2.886$, $x_4^1 = -0.1368$ & $k=1$ in C and get

$$x_1^2 = \tfrac{1}{10}\left[3 + 2(1.56) + 2.886 + (-0.1368)\right]$$
$$= 0.8869$$
$$x_2^2 = \tfrac{1}{10}\left[15 + 2(0.8869) + 2.886 + (-0.1368)\right]$$
$$= \cancel{1.9532}\ 1.9523$$

Solution: It can be written as, (upto six decimal place)

$$\left.\begin{aligned} x_1 &= \tfrac{1}{2}[1 + x_2] \\ x_2 &= \tfrac{1}{2}[x_1 + x_3] \\ x_3 &= \tfrac{1}{2}[x_2 + x_4] \\ x_4 &= \tfrac{1}{2}[x_3] \end{aligned}\right\} \longrightarrow A$$

we set up the iteration

(for easeness write missing terms)

$$\left.\begin{aligned} x_1^{k+1} &= \tfrac{1}{2}\left[1 + x_2^k + 0x_3^k + 0x_4^k\right] \\ x_2^{k+1} &= \tfrac{1}{2}\left[x_1^{k+1} + x_3^k + 0x_4^k\right] \\ x_3^{k+1} &= \tfrac{1}{2}\left[0x_1^{k+1} + x_2^{k+1} + x_4^k\right] \\ x_4^{k+1} &= \tfrac{1}{2}\left[0x_1^{k+1} + 0x_2^{k+1} + x_3^{k+1}\right] \end{aligned}\right\} \longrightarrow B$$

It can also be written as;

$$\left.\begin{aligned} x_1^{k+1} &= \tfrac{1}{2}\left[1 + x_2^k\right] \\ x_2^{k+1} &= \tfrac{1}{2}\left[x_1^{k+1} + x_3^k\right] \\ x_3^{k+1} &= \tfrac{1}{2}\left[x_2^{k+1} + x_4^k\right] \\ x_4^{k+1} &= \tfrac{1}{2}\left[x_3^{k+1}\right] \end{aligned}\right\} \Longrightarrow C$$

FIRST ITERATION: put initial solution vector $\underline{x} = (0, 0, 0, 0)^T$ & $k = 0$ in C and get.

$$x_1^1 = \tfrac{1}{2}[1 + 0]$$
$$= 0.5$$
$$x_2^1 = \tfrac{1}{2}[0.5 + 0]$$
$$= 0.25$$
$$x_3^1 = \tfrac{1}{2}[0.25 + 0]$$
$$= 0.125$$
$$x_4^1 = \tfrac{1}{2}[0.125]$$
$$= 0.0625$$

SECOND ITERATION: Put $x_2^1 = 0.25$, $x_3^1 = 0.125$, $x_4^1 = 0.0625$ & $k = 1$ in C and get.

$$x_1^2 = \tfrac{1}{2}[1.25]$$

$$x_2^2 = \frac{1}{2}[0.625 + 0.125]$$
$$= 0.375$$
$$x_3^2 = \frac{1}{2}[0.375 + 0.0625]$$
$$= 0.21875$$
$$x_4^2 = \frac{1}{2}[0.21875]$$
$$= 0.109375$$

<u>Third Iteration</u> :. Put $x_2^2 = 0.375$, $x_3^2 = 0.21875$, $x_4^2 = 0.109375$ & $k = 2$ in c and get.

$$x_1^3 = \frac{1}{2}[1 + 0.375]$$
$$= 0.6875$$
$$x_2^3 = \frac{1}{2}[0.6875 + 0.21875]$$
$$= 0.453125$$
$$x_3^3 = \frac{1}{2}[0.453125 + 0.109375]$$
$$= 0.28125$$
$$x_4^3 = \frac{1}{2}[0.28125]$$
$$= 0.140625$$

<u>Fourth Iteration</u> :. Put $x_2^3 = 0.453125$, $x_3^3 = 0.28125$, $x_4^3 = 0.140625$ & $k = 3$ in c and get

$$x_1^4 = \frac{1}{2}[1 + 0.453125]$$
$$= 0.726563$$
$$x_2^4 = \frac{1}{2}[0.726563 + 0.28125]$$
$$= 0.503907$$

$$x_3^4 = \frac{1}{2}[0.503907 + 0.140625]$$
$$= 0.322266$$
$$x_4^4 = \frac{1}{2}[0.322266]$$
$$= 0.161133$$

<u>Fifth Iteration</u> :. Put $x_2^4 = 0.503907$, $x_3^4 = 0.322266$, $x_4^4 = 0.161133$ & $k = 4$ in c and get

$$x_1^5 = \frac{1}{2}[0.503907 + 1]$$
$$= 0.751954$$
$$x_2^5 = \frac{1}{2}[0.751954 + 0.322266]$$
$$= 0.53711$$
$$x_3^5 = \frac{1}{2}[0.53711 + 0.161133]$$
$$= 0.349122$$

$x_4^5 = \frac{1}{2}[0.349122]$
$= 0.174561$

Sixth Iteration. Put $x_2^5 = 0.53711$, $x_3^5 = 0.349122$, $x_4^5 = 0.174561$ & $k = 5$ in C and get

$x_1^6 = \frac{1}{2}[1 + 0.53711]$
$= 0.768555$

$x_2^6 = \frac{1}{2}[0.768555 + 0.349122]$
$= 0.558839$

$x_3^6 = \frac{1}{2}[0.558839 + 0.174561]$
$= 0.3667$

$x_4^6 = \frac{1}{2}[0.3667]$
$= 0.18335$

Seventh Iteration. Put $x_2^6 = 0.558839$, $x_3^6 = 0.3667$, $x_4^6 = 0.18335$ & $k = 6$ in C and get

$x_1^7 = \frac{1}{2}[1 + 0.558839]$
$= 0.779420$

$x_2^7 = \frac{1}{2}[0.779420 + 0.3667]$
$= 0.57306$

$x_3^7 = \frac{1}{2}[0.57306 + 0.18335]$
$= 0.378205$

$x_4^7 = \frac{1}{2}[0.378205]$
$= 0.189103$

After seven (7) iterations it seems that solution is converging to

$x_1 = 0.8$, $x_2 = 0.6$, $x_3 = 0.4$, $x_4 = 0.2$

If we make some few iterations we can get the desired solution.

Q.15 Solve the system of equations

$$x_1 + 0.1x_2 = 1.0$$
$$0.1x_1 + x_2 + 0.1x_3 = 2.0$$
$$0.1x_2 + x_3 = 3.0$$

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Using

(i) Jacobi's met d

(ii) Gauss – Se

Solution: It can be written as (28)

$$\left.\begin{aligned} x_1 &= [1 - 0.1x_2] \\ x_2 &= [2 - 0.1x_1 - 0.1x_3] \\ x_3 &= [3 - 0.1x_2] \end{aligned}\right\} \Rightarrow A$$

JACOBI'S METHOD

FIRST ITERATION: put initial solution $x_1=0, x_2=0$ & $x_3=0$ in R.H.S of A and get

$$x_1 = [1 - 0.1(0)] = 1$$

$$x_2 = [2 - 0.1(0) - 0.1(0)] = 2$$

$$x_3 = [3 - 0.1(0)] = 3$$

SECOND ITERATION: Put $x_2 = 2$ & $x_3 = 3, x_1 = 1$ in R.H.S of A and get

$$x_3 = [3 - 0.1(2)] = [3 - 0.2] = 2.8$$

THIRD ITERATION: Put $x_1 = 0.8$, $x_2 = 1.6$ & $x_3 = 2.8$ in R.H.S of A and get

$$x_1 = [1 - 0.1(1.6)] = [1 - 0.16] = 0.84$$

$$x_2 = [2 - 0.1(0.8) - 0.1(2.8)]$$

$= [2 - 0.08 - 0.28]$

$= 1.64$

$x_3 = [3 - 0.1(1.6)]$

$= [3 - 0.16]$

$= 2.836$

FOURTH ITERATION :- Put $x_1 = 0.84$, $x_2 = 1.64$ & $x_3 = 2.836$ in R.H.S of A and get

$x_1 = [1 - 0.1(1.64)]$

$= [1 - 0.164]$

$= [0.836]$

$x_2 = [2 - 0.1(0.84) - 0.1(2.836)]$

$= [2 - 0.084 - 0.2836]$

$= 1.6324$

$x_3 = [3 - 0.1(1.64)]$

$= [3 - 0.164]$

$= 2.836$

FIFTH ITERATION :- $x_1 = 0.836$, $x_2 = 1.6324$ & $x_3 = 2.836$ in R.H.S of A and get

$x_1 = [1 - 0.1(1.6324)]$

$= [1 - 0.16324]$

$= 0.8367$

$x_2 = [2 - 0.1(0.836) - 0.1(2.836)]$

$= [2 - 0.0836 - 0.2836]$

$= 1.6328$

$x_3 = [3 - 0.1(1.6324)]$

$= [3 - 0.16324]$

$= 2.8367$

Hence solution to three decimal places is

$x_1 = 0.836$

$x_2 = 1.632$

$x_3 = 2.836$

Gauss Seidel method (30)

$$\left.\begin{aligned} x_1 &= [1 - 0.1x_2] \\ x_2 &= [2 - 0.1x_1 - 0.1x_3] \\ x_3 &= [3 - 0.1x_2] \end{aligned}\right\} \Longrightarrow A$$

we set up the iterations

$$x_1^{k+1} = [1 - 0.1x_2^k + 0x_3^k]$$
$$x_2^{k+1} = [2 - 0.1x_1^{k+1} - 0.1x_3^k]$$
$$x_3^{k+1} = [3 + 0x_1^{k+1} - 0.1x_2^{k+1}]$$

It can be written as,

$$\left.\begin{aligned} x_1^{k+1} &= [1 - 0.1x_2^k] \\ x_2^{k+1} &= [2 - 0.1x_1^{k+1} - 0.1x_3^k] \\ x_3^{k+1} &= [3 - 0.1x_2^{k+1}] \end{aligned}\right] \Longrightarrow B$$

FIRST ITERATION: put initial solution vector $\underline{x} = (0,0,0)^T$ & $k=0$ in B and get.

$$x_1^1 = [1 - 0.1(0)]$$
$$= 1$$
$$x_2^1 = [2 - 0.1(1) - 0.1(0)]$$
$$= 1.99$$
$$x_3^1 = [3 - 0.1(1.99)]$$
$$= 2.801$$

SECOND ITERATION: Put $x_2^1 = 1.99$, $x_3^1 = 2.801$ & $k=1$ in B and get

$$x_1^2 = [1 - 0.1(1.99)]$$
$$= 0.801$$
$$x_2^2 = [2 - 0.1(0.801) - 0.1(2.801)]$$
$$= 1.6398$$
$$x_3^2 = [3 - 0.1(1.6398)]$$
$$= 2.83602$$

THIRD ITERATION: Put $x_2^2 = 1.6398$, $x_3^2 = 2.83602$ and $k=2$ in B and get

$$x_1^3 = [1 - 0.1(1.6398)]$$
$$= 0.83602$$
$$x_2^3 = [2 - 0.1(0.83602) - 0.1(2.83602)]$$
$$= 1.6328$$
$$x_3^3 = [3 - 0.1(1.6327)]$$
$$= 2.8367$$

<u>FOURTH ITERATION</u>: Put $x_2^3 = 1.6327$, $x_3^3 = 2.8367$ & $k=3$ in 8 and get

$$x_1^4 = [1 - 0.1(1.6327)]$$
$$= 0.8367$$
$$x_2^4 = [2 - 0.1(0.8367) - 0.1(2.8367)]$$
$$= 1.6326$$
$$x_3^4 = [3 - 0.1(1.6326)]$$
$$= 2.8367$$

<u>FIFTH ITERATION</u>: Put $x_2^4 = 1.6326$, $x_3^4 = 2.8367$ & $k = 4$ in 8 and get

$$x_1^5 = [1 - 0.1(1.6326)]$$
$$= 0.8367$$
$$x_2^5 = [2 - 0.1(0.8367) - 0.1(2.8367)]$$
$$= 1.6326$$
$$x_3^5 = [3 - 0.1(1.6326)]$$
$$= 2.8367$$

Hence Solution vector $\underline{x} = (0.8367, 1.6326, 2.8367)^T$

THE END

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