

# Compiler Construction

## Trees

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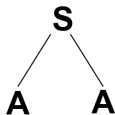
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- $S \rightarrow AA \rightarrow bAA \rightarrow bAAb \rightarrow baab$

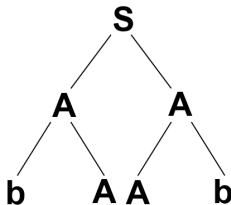
# Syntactic Tree continued ...

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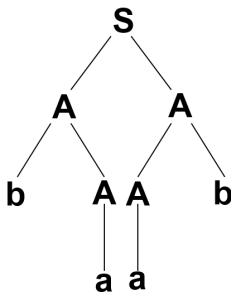
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# Total Language Tree

- For a given CFG, a tree with the start Symbol  $S$  as its root
- whose nodes are working strings of terminals and non-terminals
- The descendants of each node are all possible results of applying every production to the working string
- This tree is called **total language tree**

# Total Language Tree example 1

- Consider the following CFG,

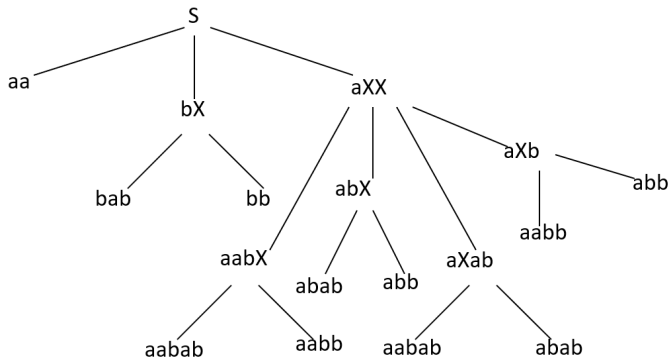
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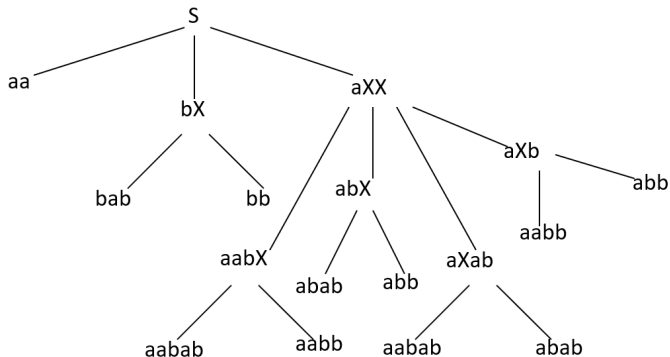
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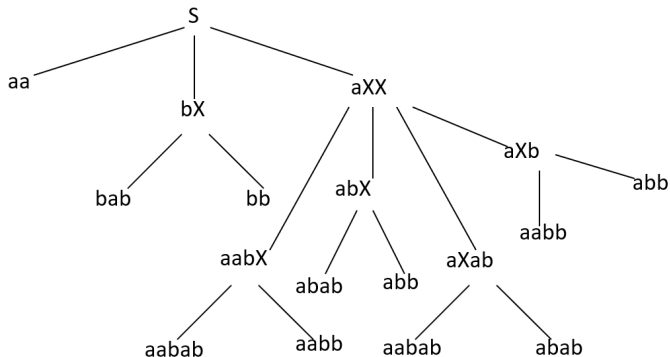
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- the the total language tree for the give CFG may be,
- Ignoring the repetitive words, the total words generated by the above CFG is,
- $\{aa, bab, bb, aabab, aabb, abab, abb\}$



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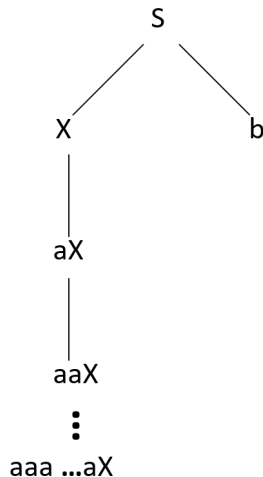


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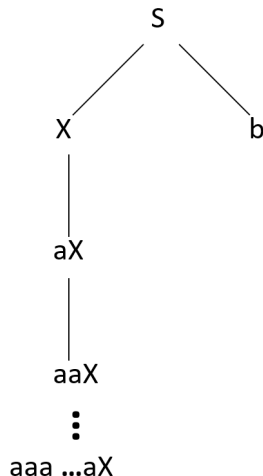
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- Consider the following CFG,  
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- then the total language tree for the give CFG is,
- It is to be noted that the only word generated by this language is,  $\{b\}$



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- It is rather difficult to write grammar directly
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  - Each state 'i' of the FA creates a non-terminal symbol 'A'
  - If a state 'i' has a transition to the state 'j' on a symbol 'a', i.e.,  $\delta(i,a) = j$ , then introduce a production rule of the following,  
 $A_i \rightarrow aA_j$



- If state 'i' goes to 'j' on input ' $\epsilon$ ', then introduce a production rule of the form  $A_i \rightarrow A_j$

# FA to CFG conversion

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- If state 'i' is an accepting state, then introduce a production rule of the form  $A_i \rightarrow \epsilon$
- If state 'i' is the start state, then  $A_i$  is the start symbol (non-terminal) of the grammar.

# RE to CFG: Example 1

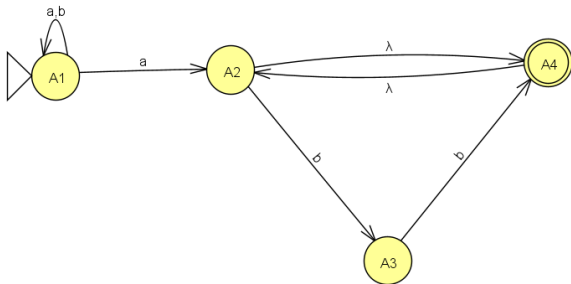
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 $\Sigma = \{a, b\}$   
 $N = \{A_1, A_2, A_3, A_4\}$   
 $S = \{A_1\}$

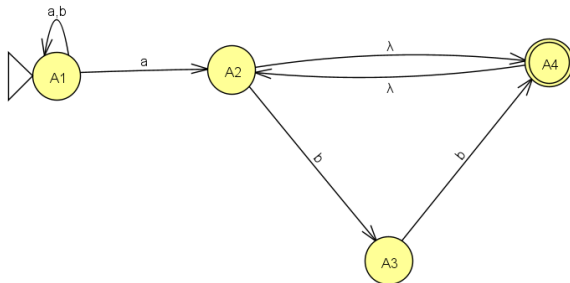
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$$A_1 \rightarrow aA_1$$

$$A_1 \rightarrow bA_1$$

$$A_1 \rightarrow aA_2$$

$$A_2 \rightarrow bA_3$$

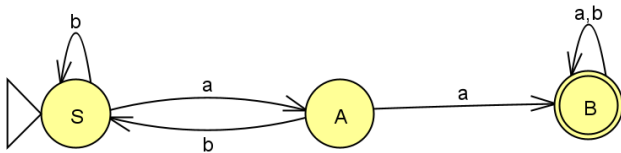
$$A_2 \rightarrow A_4$$

$$A_3 \rightarrow bA_4$$

$$A_4 \rightarrow A_2$$

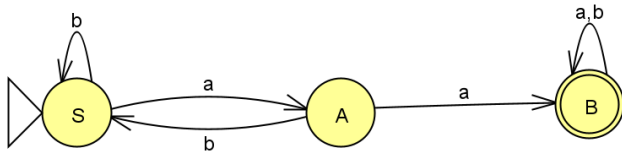
$$A_4 \rightarrow \epsilon$$

# RE to CFG: Example 1





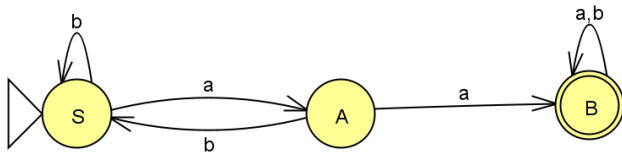
# RE to CFG: Example 1



- The corresponding CFG may be,

$$S \rightarrow bS \mid aA$$
$$A \rightarrow aB \mid bS$$
$$B \rightarrow aB \mid bB \mid \epsilon$$

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- The corresponding CFG may be,

$$S \rightarrow bS \mid aA$$

$$A \rightarrow aB \mid bS$$

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- It may be noted that the number of terminals in the above CFG is equal to the number of states of corresponding FA
- where S corresponds to the initial state
- each transition defines a production

## RE to CFG: Example 3

- Construct FA and grammar for the following RE,

$$(a|b)^* bbb(a|b)^*$$