Compiler Construction

Syntax Definitions - Grammers

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What is Grammer?

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- It is basically defined as a set of 4-tuple (V,T,P,S), where,
  1. **V** is set of nonterminals (variables)
  2. **T** is set of terminals (primitive symbols)
  3. **P** is set of productions (rules), which govern the relationship between non-terminal and terminals
  4. **S** is start symbol with which strings in grammar are derived

Grammar example in English Language

- Consider the following English grammar rules,

  $\langle$ `sentence` $\rangle \rightarrow \langle$ `noun` $\rangle \langle$ `verb` $\rangle$

  $\langle$ `noun` $\rangle \rightarrow \langle$ `com-noun` $\rangle \mid \langle$ `prop-noun` $\rangle$

  $\langle$ `verb` $\rangle \rightarrow$ **ate** | **sat** | **ran**

  $\langle$ `prop-noun` $\rangle \rightarrow$ **Ali** | **Mahad**

  $\langle$ `com-noun` $\rangle \rightarrow$ **She** | **He**

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  $\langle$ `prop-noun` $\rangle \rightarrow$ **Ali** | **Mahad**

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- Using these rules, many sentences can be derived

  1. Ali ate
  2. Mahad ran
  3. He sat
  4. He ran

Grammar ***notations***

Some of the ***notations*** used to represent grammar are,

1. **Terminal Symbols:** these can be represented by,
   - Lower-case letters of alphabet like a, c, z, etc
   - Operator symbols like +, -, etc
   - Punctuation symbols like (, {, ; etc
   - Digits 0-9
   - Bold face strings **int, main, if, else** etc

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4. Lower case letters u, v, w, x, y, z are generally used to represent a string of terminals

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d. Type 3 Grammers—Regular Grammers (RG)

Chomsky Hierarchy continued ...

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- It can be represented by the following productions in URG,

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Ca$\rightarrow$aaC
CB$\rightarrow$DB
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aD$\rightarrow$Da
AD$\rightarrow$AD
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AE$\rightarrow \phi$

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  S $\Longrightarrow$ ACaB $\Longrightarrow$ AaaCB $\Longrightarrow$ AaaE $\Longrightarrow$ AaEa $\Longrightarrow$ AEaa $\Longrightarrow$ aa

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  $S \Longrightarrow SBC \Longrightarrow SBCBC \Longrightarrow aCBCBC \Longrightarrow aBCCBC \Longrightarrow aaCCBC \Longrightarrow aaCBCC \Longrightarrow aaBCC \Longrightarrow aaaCCC \Longrightarrow aaabbb$

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  S $\Longrightarrow$ aSb $\Longrightarrow$ aaSbb $\Longrightarrow$ aaϕbb $\Longrightarrow$ aabb

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  A $\rightarrow \phi$
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- It describes the same language as by the regular expression a^*bc^*

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A Demonstrative Example:

- Give a CSG but not CFG for ($a^n \mid n \geq 1$),

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