
Below are a list of formulae applicable to financial mathematics:

1. **Simple Interest:** $F_v = P_v\left(1+in\right)$ or $A = P\left(1+rt\right)$
2. **Compound Interest:** $F_v = P_v\left(1+i\right)^n$ or $A = P\left(1+r\right)^t$
3. **Straight-line Depreciation:** $F_v = P_v\left(1-in\right)$ or $A = P\left(1-rt\right)$
4. **Reducing-balance depreciation:** $F_v = P_v\left(1-i\right)^n$ or $A = P\left(1-r\right)^t$
5. **Effective and nominal interest rate conversion:** $1+i_{\text{eff}} = \left(1+\frac{i_{\text{nom}}}{m}\right)^m$ or $1+r_{\text{eff}} = \left(1+\frac{r_{\text{nom}}}{m}\right)^m$
6. **Most General Compound Interest Formula:** $F_v = P_v\left(1+\frac{i}{m}\right)^{mn}$ or $A = P\left(1+\frac{r}{m}\right)^{mt}$

| **Period of Compounding** | **Value of $m$** |
|---|---|
| Annually / Per Annum | $m = 1$ |
| Semi-annually / Bi-Annually | $m = 2$ |
| Quarterly | $m = 4$ |
| Monthly | $m = 12$ |
| Weekly | $m = 52$ |
| Daily | $m = 365$ |

**Effective Interest Rate:** An annual interest rate, i.e. an interest rate given per annum.

**Nominal Interest Rate:** A non-annual interest rate, which includes semi-annual, quarterly, monthly, weekly and daily rates.

**Sinking Fund:** A *sinking fund* is a fund where money is invested at regular or irregular intervals in order to replace equipment and machinery at the end of its useful life.

**Amortised:** *Amortised* means that a loan, together with its interest, gets paid off.

## 0.1 Section A: Time Line Problems

1. A loan of R18 000 is repaid in the following manner: R3 000 after two months, R7 500 at th end of 5 months and R4 000 3 months later. If the interest rate is 12% compounded monthly, what would the final payment be?

2. R8 000 is invested for 10 years. For the first 2 years, the interest rate is 12% p.a. The rate then changes to 12% p.a. compounded monthly for the next 3 years and then to 14% p.a. compounded quarterly for the final 5 years. Calculate the final amount paid out at the end of 5 years.

3. A father has three children aged 12, 15 and 17. He plans to give each of them R30 000 on their $21^{\text{st}}$ birthdays. How much money should he invest now if the interest paid is 14% p.a. compounded monthly.

4. Anthony starts a small business and takes out a loan of R35 000. He repays R15 000 at the end of 2 years, and then follows up with another payment of R22 000 1 year later. After 4 years, he expands his business and takes out an additional loan of R56 000, and agrees to repay R45 000 1 year later. He follows this up with two equal payments of R$x$ of the next 2 consecutive years to pay off his loan. Calculate $x$ if the interest is 16% p.a. compounded annually.

5. Peter repays a loan of R10 000 in 3 equal payments of R$x$ at the end of the $2^{\text{nd}}, 3^{\text{rd}}$ and $4^{\text{th}}$ years. If interest is calculated at 22% p.a. compounded annually, calculate $x$.

6.1. Mrs. Roux starts her own printing business and buys machinery costing R850 000. Depreciation is calculated at 15% p.a. on the reducing balance method. Calculate the scrap value of the machinery after 8 years.

6.2. The replacment cost of the new machinery is expected to be R1.5 mil. How much money would be needed to purchase the new machinry?

6.3. A sinking fund is set up to replace the equipment, starting at the end of the $1^{st}$ year. How much money must be invested each year if the interest on the money invested in the sinking fund is expected to be 21%?

## 0.2 Section B: Future Value Annuities and Annuity Immediates

**Note:** Whenever an annuity problem involves the word "immediate" or "at the beginning of every period (typically month)" , it is referring to an annuity immediate problem!

1. Twenty equal payments of R8 000 are made into a savings account annually at the beginning of each year, starting immediately. Calculate the total accumulated amount at the end of 20 years if the interest rate is 9% p.a. compounded annually.

2. R200 is paid into a sinking fund every month for 5 years starting immediately.

Interest is 18% p.a. compounded monthly. Calculate the value of the fund at the end of 5 years.

3. Sam starts saving at the age of 20 for his pension. He pays R700 every month into a pension fund starting on his $20^{\text{th}}$ birthday. Calculate how much money he will have saved by the time he turns 60, assuming that his last payment is made 1 month before his $60^{\text{th}}$ birthday. The interest is calculated at 11.4% p.a. compounded monthly.

4. R500 is paid at the beginning of each month into a pension fund, starting immediately.

4.1. Calculate how much money will accumulate in the fund at the end of 15 years is the interest is calculated at 12% p.a. compounded monthly.

4.2. How much money will be in the fund if the payments continue to the end of 25 years?

## 0.3 Section D: Present Value Annuities

1. A loan of R90 000 is taken to renovate a kitchen. The loan is repaid by equal monthly installments over a period of 3 years. The interest is 7.5% p.a. compounded monthly. Calculate the monthly repayments.

2. Mr. Smith starts his own business and takes out a loan of R500 000. He repays the loan by means of equal quarterly payments, starting 3 months after the loan

was granted. The loan is repaid over 10 years and the rate of interest is 8% p.a. compounded quarterly. Calculate his quarterly payments.

3. Joanna buys a house and takes out a loan for R450 000. The interest is 9.5% p.a. compounded monthly.

3.1. Calculate the monthly repayments if the loan is to be repaid over 20 years.

3.2. How much money does she pay in total over the 20 year period?

3.3. Calculate the balance on the loan after 8 years.

3.4. Calculate the monthly repayments if she chose to repay the loan over 15 years.

## 0.4 Section E: Outstanding Balances on Loans

1. A loan of R180 000 is repaid over a period of 20 years by means of equal monthly payments starting one month after the loan granted. The interest rate is 16% p.a. compounded monthly.

1.1. Calculate the monthly repayment.

1.2. Calculate the balance on the loan after 10 years.

2. Mark takes out a loan of R64 000 to finance a car. He pays back the loan over a period of 5 years, starting one month after he bought the car. The interest charged is 19% p.a. compounded monthly.

2.1. Determine his monthly repayments in order to repay the loan over the required period.

2.2. If he decides to sell the car after 2 years, how much money does he still owe on the loan?

2.3. If he decides to repay the original loan over a period of 3 years, how much money would he have to pay each month?

3. You buy an engagement ring worth R3 600 and you pay for the ring by using extended credit on your card. You are required to repay the debt by means of equal monthly payments over a period of 3 years. The interest rate is 21% p.a. compounded.

3.1. How much will you have paid for the ring at the end of 3 years?

3.2. After 1 year, you restructure your loan and decide to pay off the balance in 6 equal installments. Calculate how much you will be required to pay for each of the final 6 installments.

## 0.5 Section F: Deferred Annuities

1. Leslie borrows R420 000 to start his own business. He agrees to repay the loan quarterly, over a period of 5 years, from the time the loan was granted. However, the first payment would only begin after 9 months. The interest rate is 18% p.a. compounded quarterly.

1.1. Calculate the quarterly installments.

1.2. Determine the balaance on the loan at the end of 3 years.

2. You take out a loan for R840 000 to start your own business. The loan is amortised after 10 years. The repayments begin 12 months after the loan was granted. The interest rate is 18% p.a. compounded monthly.

2.1. Calculate the monthly repayments.

2.2. Determine the balance on the loan at the end of 5 years.

## 0.6 Section G: Fixed Payment Annuities

1. Bob buys a car for R38 000. He pays 15% of the purchase price in cash. The balance is paid through a bank loan where the interest rate is 20% p.a. compounded monthly. His salary is R3 500 and he decides to spend 30% of his salary to repay the bank loan.

1.1. How long will it takw him to repay the loan?

1.2. Calculate the value of his final payment.

2. John buys a holiday apartment in Cape Town worth R160 000. He is able to obtain a home loan for the full amount and agrees to repay the loan over 20 years. The interest is 16% p.a. compounded monthly.

2.1. How much will he have to pay each month?

2.2. If he paid an extra R300 per month, how long will it take him to repay the loan?

3. Mr. de Blasio borrows R70 000 at 15% p.a. compounded monthly. He repays

the debt at R1 600 per month. After 5 years he increases the payment to R2 100 per month.

3.1. How long does it take to amortise the loan?

3.2. Determine his final payment.