0.1 Section A: Counting Methods - Permutations and Combinations

Permutation: Order is important. People, job roles are specifically referred to or are specifically named.

Combination: Order does not matter.

Factorial: A special notation that makes use of an exclamation mark (!) to denotes the product of a natural number and the sequence of numbers that precede it all the way till 1.

"With replacment" : When objects are removed from their container, they are

put back into their container.

"Without replacment" : When objects are removed from their container, they are not put back into their container.

Fair Coin: A *fair coin* has one side which is heads and the other side which is tails.

Unfair / biased coin: An *unfair coin* has both sides that are heads or both sides that are tails.

Standard Deck of Cards: A standard deck of cards consists of the following:

(A) Two (2) colours - Typically red and black.

(B) Four (4) suits - Hearts, diamonds, clubs and spades. Traditionally, hearts and diamonds are red while spades and clubs are black. Each suit has 13 cards, given a total of 52 cards in the deck.

(C) The above assumptions ignore jokers in the deck! When you are not told whether the deck has jokers or not, you may assume it does not have jokers.

1. **Factorial notation:** $n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$
2. **Permutation formula:** $N = {}^nP_k = \frac{n!}{(n-k)!}$
3. **Combination formula:** $N = {}^nC_k = \frac{n!}{(n-k)! \times k!} = \binom{n}{k}$

Solve the following counting problems:

1. In a club there are 20 members, in how many ways can I choose:

1.1. a president, vice-president, secretary and treasurer?

1.2. a committee of four?

2. A basket contains 3 red balls, 4 blue balls and 2 yellow balls. If 3 balls are taken out of the basket, without replacement, in how many ways can the balls be selected so that: 2.1. 1 is red and 2 are blue?

2.2. 1 is red, 1 is blue and 1 is yellow?

2.3. all are blue?

2.4. at least 1 is red?

3. A shipping consignment of 40 machines contains 4 that are faulty. If a sample of 10 machines are chosen at random, without replacment, in how many ways can the following be chosen:

3.1. 2 of the defective machines?

3.2. 3 of the defective machines?

3.3. all the defective machines?

4. A student must answer 8 out of 10 questions in an examination. How many choices does she have if:

4.1. there are no compulsory questions?

4.2. the first 3 questions are compulsory?

4.3. she must choose 4 from the first 5?

5. In how many different ways can the letters of the name **ANNAMARIE** be

arranged?

0.2 Section B: Probability

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Independent Event: $P(A \cap B) = P(A \text{ and } B) = P(A) \times P(B)$

Mutually Exclusive / Disjoint Events: $P(A \cap B) = \emptyset$

Conditional Probability: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Not / Complement of an Event: $P(A') = P\left(\bar{A}\right) = 1 - P(A)$

Solve the following probability problems:

1. Suppose you take the letters of the word **PICTURE** and arrange them in any order, without repitition. What is the probability that the new word will start with **P** and end with **E**?

2. In Gauteng, number plates for vehicles are constructed in the following manner: Three letters, followed by three digits from $\{0, 1, 2, \cdots, 9\}$. The letters must not contain vowels or the letter **Q**. What is the probability that a number plate, chosen at random, starts with the letter **B** and end with the number 0?

3. If the letters of the word **STATISTICS** are arranged randomly, what is the probability that the word would start and end with the same letter?

4. In rolling a die, consider the following events:

$$S = \{1, 2, 3, 4, 5, 6\},$$

$$A = \{2, 4, 6\},$$

$$B = \{3, 4, 5, 6\}.$$

Find the following probabilities:

4.1. $P\left(\bar{B}\right)$

4.2. $P\left(A \cap B\right)$

4.3. $P\left(A \cup B\right)$

4.4. $P\left(\overline{A \cup B}\right)$

4.5. $P\left(A \cap \bar{B}\right)$

4.6. $P\left(B \cup \bar{A}\right)$

4.7. $P\left(\bar{A} \cap B\right)$

5. Janice writes down the numbers from 1 to 10 on some cards and puts it into a bag. Find the following probabilities:

5.1. A prime number or a number less than 6

5.2. A prime number which is less than 6

5.3. Neither a prime number nor a number less than 6

5.4. A number which is less than 6 but not pirime

6. If for two events A and B, it is given that $P(A) = 0.25, P(B) = 0.5$ and $P(A \cap B) = 0.15$, calculate:

6.1. $P(A \cup B)$

6.2. $P\left(A \cap \bar{B}\right)$

6.3. $P\left(A \cup \bar{B}\right)$

7. There are 300 grade 10 pupils at Durban Girls' High. Of these pupils, 267 take Setswana, 175 take English and 150 take Afrikaans. Also, 37 pupils take all three languages, 167 take English and Setswana, 127 take Setswana and Afrikaans and no pupil takes only English. Calculate the probability that a learner, chosen at random, will

7.1. take all three languages

7.2. takes Setswana and English, but not Afrikaans

7.3. Takes none of the languages under consideration

7.4. takes only Afrikaans

8. At Westville Girls' High, there are 126 pupils in grade 11. Of these pupils, 44 take Additional Mathematics, 112 take Mathematics and 90 take Biology. All the pupils that take additional Mathematics also take Mathematics, 30 pupils take Additional Mathematics and Biology. Also, 80 pupils take Additional Mathematics

and Biology. Calculate the probability that a learner, chosen at random

8.1. takes none of the three subjects,

8.2. takes Mathematics but neither Additional Mathematics nor Biology

8.3. takes Mathematics but not Biology

8.4. takes Mathematics and Biology but not Additional Mathematics

8.5. takes Mathematics or Biology

9. At a supermarket, 320 shoppers bought food. Of these, 162 bought meat, 128 bought milk and 150 bought bread. Also, 52 bought meat and milk, 46 bought milk and bread, 62 bought meat and bread and 22 bought all three items. What is the probability that a shopper, chosen at random, bought

9.1. bread, but not meat nor milk?

9.2. none of the specified items?

9.3. meat or milk?

9.4. bread and milk, but not meat?

9.5. meat or bread, but not milk?

10. Andrea, Bonny and Cassy decide to enter the New York Marathon. The probabilities that they will complete the marathon are 0.7, 0.9 and 0.6 respectively. What is the probability that at least two will complete the marathon, assuming that the performance of each is independent of the performance of the others?

11. A bag contains 8 toffees, 6 jelly babies and 7 Lindt chocolate balls. If 9 sweets

are selected at random, what is the probability that there will be

11.1. 3 of each type?

11.2. 5 toffees?

11.3. at least 1 Lindt chocolate ball?