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CHAPTER 1

MAP WORK CALCULATIONS - FOR GRADES 10, 11 AND 12

The **golden rule** of map work calculations is to **show all calculations**. This is because even if you get the final answer wrong, you will get part marks!

1.1 Scale of a Topographical Map

$$\boxed{1 : 50\,000} \tag{1.1}$$

This means that 1 cm on the map represents 50 000 cm on the actual ground!

1.2 Scale of an Orthophoto Map

$$\boxed{1 : 10\ 000} \tag{1.2}$$

This means that 1 cm on the map represents 10 000 cm on the actual ground!

1.3 Conversion of Units

These are the most commonly occuring conversions, of course more other conversions are naturally possible.

1.3.1 cm to m

$$\boxed{\text{Divide by 100}} \tag{1.3}$$

1.3.2 cm to km

$$\boxed{\text{Divide by 100 000}} \tag{1.4}$$

Note that this is equivalnet to taking the measurement in cm and multiplying by 10^{-5}.

1.3.3 m to cm

$$\boxed{\text{Multiply by 100}} \tag{1.5}$$

1.3.4 km to cm

$$\boxed{\text{Multiply by 100 000}} \tag{1.6}$$

Note that this is equivalnet to taking the measurement in cm and multiplying by 10^5.

1.4 Which Map Scale is Bigger?

Answer: The orthophoto map scale is bigger. The reason is that the orthophoto map is reduced 10 000 times whereas the topographic map is reduced 50 000 times.

1.5 Distance Calculations

1. Distance on a map is calculated between two points.

2. Using your ruler, simply draw a line between the two points.

3. Thereafter, measure the length of the line.

4. Map distance then needs to be converted to actual distance.

The formula for calculating the actual distance is

$$\boxed{\text{Actual Distance} = \text{Map Distance} \times \text{Scale}} \tag{1.7}$$

Symbolically, this is written as

$$\boxed{AD = MD \times S} \tag{1.8}$$

Example: The distance on a topographic map between two points is 6.2 cm. What is the actual distance in

(1) m?

(2) km?

Solution: Recall that the sclae of a topographic map is 1 : 50 000.

(1)

$$\begin{aligned} AD &= MD \times S \\ &= 6.2 \text{ cm} \times 50\ 000 \text{ cm} \\ &= 310\ 000 \text{ cm} \\ &= \frac{310\ 000 \text{ cm}}{100} \\ &= 3\ 100 \text{ m.} \end{aligned}$$

(2)

$$
\begin{aligned}
AD &= MD \times S \\
&= 6.2 \text{ cm} \times 50\ 000 \text{ cm} \\
&= 310\ 000 \text{ cm} \\
&= \frac{310\ 000 \text{ cm}}{100\ 000} \\
&= 3.1 \text{ km.}
\end{aligned}
$$

1.6 How to Calculate the Curve / Winding Distance

1. There are several methods, the most common being the **string method** and the **paper method**.

2. The most accurate is the paper method.

3. Take an A4 sheet of paper, or even your examination sheet, and curve the piece of paper while marking off distances as you traverse the curve.

4. The string method is not preferable.

5. Examiners are very strict and only give a 0.1 cm margin of error, so make sure you do this as accurately as possible.

Example: Using the paper method, you calculate the winding distance between two points on on a topographic map to be 7 cm. What is the actual distance?

Solution:

$$
\begin{aligned}
AD &= MD \times S \\
&= 7 \text{ cm } \times \text{ } 50\ 000 \text{ cm} \\
&= 350\ 000 \text{ cm} \\
&= \frac{350\ 000 \text{ cm}}{100\ 000} \\
&= 3.5 \text{ km.}
\end{aligned}
$$

1.7 Calculating Area

1. Area is calculated to determine the actual (size on the ground) size of a feature / region / demarcated area.

The formulae to calculate the area of a demarcated region is

$$\boxed{\text{Area} = (\text{Length } \times \text{ Scale}) \times (\text{Breadth } \times \text{ Scale})} \tag{1.9}$$

Symbolically, this is written as

$$\boxed{A = (L \times S) \times (B \times S)} \tag{1.10}$$

Example: A demarcated block on a topographic map has the following dimensions:

Length: 3.7 cm

Breadth: 3.3 cm.

Calculate the area of this region.

Solution:

$$
\begin{aligned}
A &= (L \times S) \times (B \times S) \\
&= (3.7 \text{ cm } \times 50\ 000 \text{ cm}) \times (3.3 \text{ cm } \times 50\ 000 \text{ cm}) \\
&= (185\ 000 \text{ cm}) \times (165\ 000 \text{ cm}) \\
&= \frac{185\ 000 \text{ cm}}{100\ 000} \times \frac{165\ 000 \text{ cm}}{100\ 000} \\
&= (1.85 \text{ km}) \times (1.65 \text{ km}) \\
&= 3.05 \text{ km}^2 \text{ (Correct to two decimal places).} \\
&= 3.1 \text{ km}^2 \text{ (Correct to one decimal place).}
\end{aligned}
$$

> Note: If you do not have km^2, even if you have the correct value, the answer is **wrong**!

1.8 How to Convert Degrees, Minutes and Seconds (D°M′S″) to Decimal Degrees

Note: $1^\circ = 60' = 3\ 600''$

$$
\boxed{\mathcal{D}^\circ = D^\circ + \frac{M'}{60} + \frac{S''}{3\ 600}} \tag{1.11}
$$

An easy way to remember the conversion factors:

1. Degrees remain the same.
2. Since there are 60 minutes in an hour, we simply divide M' by 60.
3. Since there are 3 600 seconds in an hour, we simply divide S'' by 3 600.

Example: Convert $48°28'32''$ to decimal degrees.

Solution:

$$\begin{aligned} 48°28'32'' &= 48° + \frac{28}{60} + \frac{32}{3\ 600} \\ &= 48.48°. \end{aligned}$$

Example: Convert $38°41'27''$ to decimal degrees.

Solution:

$$\begin{aligned} 38°41'27'' &= 38° + \frac{41}{60} + \frac{27}{3\ 600} \\ &= 38.69°. \end{aligned}$$

1.9 How to Convert Decimal Degrees to Degrees, Minutes

This is best illustrated with an example.

Example: Convert $38.4256°$ to $D°M'S''$.

Solution: The degree part is $38°$. Now consider the decimal part, 0.4256.

$$0.4256 \times 60 = \boxed{25}.536 \implies \text{ The minutes part is 25.}$$

Now take the remainder part, 0.536.

$$0.536 \times 60 = \boxed{32}.16 \implies \text{ The seconds part is 32.}$$

Thus, $38.4256° = 38°25'32''$.

Example: Convert $52.3175°$ to $D°M'S''$.

Solution: The degree part is $52°$. Now consider the decimal part, 0.3175.

$$0.3175 \times 60 = \boxed{19}.05 \implies \text{ The minutes part is 19.}$$

Now take the remainder part, 0.05.

$$0.05 \times 60 = \boxed{3}.00 \implies \text{ The seconds part is 3.}$$

Thus, $52.3175° = 52°19'3''$

1.10 Finding Places by Means of Bearings

1.11 Magnetic Declination and Magnetic Bearing

You need to know the following:

- Magnetic declination is the horizontal angle between the true meridian and the magnetic meridian through the point under consideration.
- Magnetic North **is not** True North!
- The magnetic field changes over time.
- In Southern Africa, Magnetic North is **west** of True North! This is important when working with maps of regions in Southern Africa.
- Magnetic Declination = Magnetic North.

In order to perform a calculation, you need to know the following:

1. Mean magnetic declination and direction of declination.
2. In which year was it given.
3. What is the mean annual change and in which direction
4. For which year must the magnetic delination be calculated?

The steps to performing the calculation is as follows:

1. Calculate the difference between the year you are asked to calculate the magnetic declination and the referece year.

2. Calculate the magnetic change. This is done by multiplying the average annual change by the difference between years found in step 1.

3. Add / subtract the mean magnetic declination to the magnetic change. If the direction is **west**, you **add**, and if the direction is **east**, you **subtract**.

We also note the formula

$$\boxed{\text{Magnetic Bearing} = \text{Normal Bearing} + \text{Magnetic Declination}} \tag{1.12}$$

Example: Suppose your current bearing is $195°$. The mean magnetic declination is $13°45'$ west of True North in 2008. The average annual change is $5'$ west. Calculate the magnetic declination for 2018.

Solution:

Step 1

$$\begin{aligned} \text{Difference in years} &= 2018 - 2008 \\ &= 10 \text{ years.} \end{aligned}$$

Step 2

$$\begin{aligned}\text{Magnetic change} &= 5' \times 10 \text{ years} \\ &= 50'.\end{aligned}$$

Step 3

$$\begin{aligned}\text{Magnetic declination} &= 13°45' + 50' \\ &= 13°95' \\ &= 13° + (95' - 60') + \left(60' \times \frac{1°}{60'}\right) \\ &= 13° + 35' + 1° \\ &= 14°35'.\end{aligned}$$

Step 4

$$\begin{aligned}\text{Magnetic Bearing} &= \text{Normal Bearing} + \text{Magnetic Declination} \\ &= 195° + 14°35' \\ &= 209°35'.\end{aligned}$$

- The two were added because they both were directed west.
- In line 3, since in $1°$ there is $60'$, we have to effectively convert $95'$ to degrees. Effectively this is accomplished by firstly subtracting $60'$ and thereafter adding $60'$ by converting it to its equivalent degree form.

Example: Find the true bearing if the magnetic bearing of a line is $128°30'$ and the declination is $5°$W.

Solution: This is the inverse problem, given the magnetic bearing, we are asked to calculate the true bearing.

$$\begin{aligned}
\text{Magnetic Bearing} &= \text{True Bearing} + \text{Magnetic Declination} \\
\implies \text{True Bearing} &= \text{Magnetic Bearing} - \text{Magnetic Declination} \\
&= 128°30' - (+5°) \\
&= 123°30'.
\end{aligned}$$

Example: Find the true bearing if the magnetic bearing of a line is $85°30'$ and the declination is $6°$E.

Solution:

$$\begin{aligned}
\text{Magnetic Bearing} &= \text{True Bearing} + \text{Magnetic Declination} \\
\implies \text{True Bearing} &= \text{Magnetic Bearing} - \text{Magnetic Declination} \\
&= 85°30' - (-6°) \\
&= 91°30'.
\end{aligned}$$

Remember in the above two examples,

- **west** is **positive**.
- **east** is **negative**.

Example: In 1981, the mean magnetic declination is $18°7'$ W of true north. The mean annual change is $2'$ W. Calculate the magnetic declination for the year 2012.

Solution:

Step 1

$$\begin{aligned}\text{Difference in years} &= 2012 - 1981 \\ &= 31 \text{ years.}\end{aligned}$$

Step 2

$$\begin{aligned}\text{Magnetic change} &= 2' \times 31 \text{ years} \\ &= 62'.\end{aligned}$$

Step 3

$$\begin{aligned}\text{Magnetic declination} &= 18°7' + 62' \\ &= 18°69' \\ &= 18° + (69' - 60') + \left(60' \times \frac{1°}{60'}\right) \\ &= 18° + 9' + 1° \\ &= 19°9'.\end{aligned}$$

This implies that the magnetic declination is $19°9'$ W of true north.

1.12 Gradient Calculations

1. *Gradient* is essentially how steep a slope is between two places.

2. The *vertical interval* is the difference in height between two locations on a map.

3. The *horizontal equivalent* is simply the distance between the two locations on a map.

4. A *contour line* is a line that connects all points with the same altitude.

The formula to calculate gradient is

$$\boxed{\text{Gradient} = \frac{\text{Vertical Interval}}{\text{Horizontal Equivalent}}} \tag{1.13}$$

Symbolically, this is written as

$$\boxed{G = \frac{VI}{HE}} \tag{1.14}$$

Gradient is unitless and is always expressed as a ratio, i.e. 1:something.

Example: On a topographic map, the distance between two locations is measured to be 25 cm. point A is at an inclination of 1 020 m above sea level and point B is 1 100 m above sea level. Calculate the gradient between points A and B.

Solution:

$$
\begin{aligned}
G &= \frac{VI}{HE} \\
&= \frac{(1\ 100 - 1\ 020)\ \text{m}}{25 \times 50\ 000\ \text{cm}} \\
&= \frac{80\ \text{m}}{1\ 250\ 000\ \text{cm}} \\
&= \frac{80\ \cancel{\text{m}}}{1\ 250\ 000\ \cancel{\text{cm}}} \times \frac{100\ \cancel{\text{cm}}}{1\ \cancel{\text{m}}} \\
&= \frac{8\ 000}{1\ 250\ 000} \\
&= \frac{\frac{8\ 000}{8\ 000}}{\frac{1\ 250\ 000}{8\ 000}} \\
&= \frac{1}{156.25}.
\end{aligned}
$$

This means that for every 156.25 m that one travels from point A to point B, there will be an increase in height of 1 m. This indicate that this is a very gentle slope because you would have to travel quite a long distance before the road increases in height.

- In line 2 of the above calculation, where does the 50 000 come from? Remember that we are working with a topographic map and therefore the scale is 1 : 50 000.

- In line 4 of the above calculation, why do we multiply by $\frac{100 \text{ cm}}{1 \text{ m}}$? This is to ensure that the gradient is unitless.

- In line 6 of the above calculation, why did we divide the numerator and denominator by 8 000? This is so we can express the gradient as a ratio.

1.13 Cross Sections

Refer to your class notes!

1.14 How to Calculate Vertical Exaggeration

- *Vertical exaggeration* is a scale used to emphasize vertical features like mountains, valleys, etc. which might be too small to identify relative to the horizontal scale.

The calculation essentially involves two steps:

1. Converting the vertical scale to a ratio in the form 1:something.

2. Applying the formula.

$$\boxed{\text{Vertical Exaggeration} = \frac{\text{Vertical Scale}}{\text{Horizontal Scale}}} \tag{1.15}$$

Symbolically, this is written as

$$\boxed{VE = \frac{VS}{HS}} \tag{1.16}$$

Example: Given a a map with a vertical scale of 4 mm = 20 m and horizontal scale of 1 : 50 000, calculate the vertical exaggeration.

Solution: Firstly, we have to re-write 4 mm = 20 m as a ratio.

$$\begin{aligned}
&\frac{4\ \cancel{\text{mm}}}{20\ \cancel{\text{m}}} \times \frac{1\ \cancel{\text{m}}}{1\ 000\ \cancel{\text{mm}}} \\
&= \frac{4}{20\ 000} \\
&= \frac{\frac{4}{4}}{\frac{20\ 000}{4}} \\
&= \frac{1}{5\ 000}.
\end{aligned}$$

- In line 1, we note that 1 m = 1000 mm.

- In line 3, we want to make the ratio 1:something. In order to do this, we divide by the numerator.

Lastly, we apply the formula

$$
\begin{aligned}
VE &= \frac{VS}{HS} \\
&= \frac{\frac{1}{5\,000}}{\frac{1}{50\,000}} \\
&= 10 \text{ times.}
\end{aligned}
$$

Thus, the vertical exaggeration is 10 times. This means that the vertical scale has been exaggerated 10 times.

Example: Calculate the vertical exaggeration between two points having a vertical scale of 1 cm : 20 m.

Solution:

Firstly, we will convert the vertical scale to its cm equivalent

$$
\begin{aligned}
VS &= 1 \text{ cm} : 20 \ \cancel{\text{m}} \times \frac{100 \text{ cm}}{1 \ \cancel{\text{m}}} \\
&= 1 \text{ cm} : 2\,000 \text{ cm.}
\end{aligned}
$$

Now, we calculate the vertical exaggeration

$$
\begin{aligned}
VE &= \frac{\frac{1}{2\,000}}{\frac{1}{50\,000}} \\
&= 25 \text{ times.}
\end{aligned}
$$

Thus, the vertical exaggeration is 25 times.

1.15 Speed, Distance and Time Problems

Unlike physics and mathematics, we will adopt the following symbols:

$$\text{Speed} = S$$

$$\text{Distance} = D$$

$$\text{Time} = T$$

1.15.1 How to Calculate Speed

$$\boxed{\text{Speed} = \frac{\text{Distance}}{\text{Time}}} \tag{1.17}$$

Symbolically, this is written as

$$\boxed{S = \frac{D}{T}} \tag{1.18}$$

1.15.2 How to Calculate Distance

$$\boxed{\text{Distance} = \text{Speed} \times \text{Time}} \tag{1.19}$$

Symbolically, this is written as

$$\boxed{D = S \times T} \tag{1.20}$$

- Don't forget to use the map's scale to convert the map distance to an actual distance!
- If you are not told the type of map, or the scale of the map is not explicitly stated, you can do one of two things:
  1. Pick your hand up and ask your teacher what is the scale of the map.
  2. Assume it is a topographic map and the scale is 1 : 50 000.

1.15.3 How to Calculate Time

$$\boxed{\text{Time} = \frac{\text{Distance}}{\text{Speed}}} \tag{1.21}$$

Symbolically, this is written as

$$\boxed{T = \frac{D}{S}} \tag{1.22}$$

1.15.4 Words to Look Out For...

"How far" $\Longrightarrow$ calculate distance

"How fast" $\Longrightarrow$ calculate speed

"How long" $\Longrightarrow$ calculate time

Example: How long will it take a car travelling at 60 km/h to drive from point R to point P along the N4? Using you brilliant measurement skills you calculated the distance between points R and P to be 13.8 cm. Assume it is a topographic map.

Solution:

Firstly, we will have to use the map's scale to calculate the actual distance and thereafter convert cm to m.

$$
\begin{aligned}
D &= 13.8 \times 50\,000 \text{ cm} \\
&= 690\,000 \text{ cm} \\
&= 690\,000 \cancel{\text{cm}} \times \frac{1 \cancel{\text{m}}}{100 \cancel{\text{cm}}} \times \frac{1 \text{ km}}{1\,000 \cancel{\text{m}}} \\
&= 6.9 \text{ km.}
\end{aligned}
$$

Now, we simply apply the formula

$$
\begin{aligned}
T &= \frac{D}{S} \\
&= \frac{6.9 \text{ km}}{60 \text{ km/h}} \\
&= 0.115 \text{ h} \\
&= 0.115 \cancel{\text{h}} \times \frac{60 \text{ min}}{1 \cancel{\text{h}}} \\
&= 6.9 \text{ min.}
\end{aligned}
$$

Example: At what speed were you travelling at if it took you 12 min to travel from point R to point S along the N4 with the map distance being 13.8 cm?

$$
\begin{aligned}
D &= 13.8 \times 50\ 000 \text{ cm} \\
&= 690\ 000 \text{ cm} \\
&= 690\ 000\ \cancel{\text{cm}} \times \frac{1\ \cancel{\text{m}}}{100\ \cancel{\text{cm}}} \times \frac{1 \text{ km}}{1\ 000\ \cancel{\text{m}}} \\
&= 6.9 \text{ km}.
\end{aligned}
$$

Now, we convert the time in minutes to a time in hours

$$
\begin{aligned}
T &= 12\ \cancel{\text{min}} \times \frac{1 \text{ h}}{60\ \cancel{\text{min}}} \\
&= 0.2 \text{ h}.
\end{aligned}
$$

Now, we simply apply the formula

$$
\begin{aligned}
S &= \frac{D}{T} \\
&= \frac{6.9 \text{ km}}{0.2 \text{ h}} \\
&= 34.5 \text{ km/h}.
\end{aligned}
$$

1.16 Intervisibility

- *Intervisibility* essentially refers to what can and what cannot be seen on a map.
- The item that cannot be seen is referred to as *dead ground*.
- The item, area, region, or object that obstructs your view, called the *line of visibility*, from the area to be observed is called the *blocking feature*.
- Given two points on a map, you are asked if point A is visible from point B or is point B visible from point A or some other point say X, Y, etc. while giving reasons for your answer.

1.17 How to Calculate Drainage Density

- *Drainage density* is a measure of how well or how poorly a watershed is drained by stream channels.

$$\boxed{\text{Drainage Density} = \frac{\text{Total Stream Length}}{\text{Total Basin Area}}} \tag{1.23}$$

1.18 How to Calculate the Scale of Vertical Photographs

- Scale is defined as the ratio of the map distance to the actual ground distance.