0.1 Section A: Linear, Fractional, Quadratic and Cubic Equations

Solve for the unknown variable in the following equations below: (Use factorization and / or the quadratic formula to arrive at the answer - in some cases only the formula may apply)

1. $x\left(x-1\right)=20$
2. $\left(x+1\right)\left(x-2\right)+\left(x+3\right)\left(x-1\right)=2\left(3x-1\right)$
3. $x\left(x-1\right)=4\left(3x-10\right)$
4. $\frac{x}{7}+2x=-\frac{3}{5}x+7$

5. $y^2 - 7y + 6 = 0$

6. $4(z-1)(z+1) - 4(z-2) = 12(z-1) + 1$

7. $(\theta - 2)(\theta + 1) = 7(\theta - 1) - 7$

8. $\frac{x+3}{x^2-4} + \frac{1}{x^2+x-2} = \frac{1}{2-x} - \frac{7}{4-x^2}$. **Hint:** For equations involving fractions, note any and all restrictions!

9. $\frac{x-3}{x^2+3x+2} - \frac{5}{x^2-4} = \frac{4}{-x-1}$

10. $\frac{3}{w} + w + 5 = \frac{w+3}{w}$

11. $\sqrt{y+6} = y$. **Hint:** Remember to check the value of y once you solve the equation!

12. $\sqrt{6+y} = \sqrt{7-y} + 1$

13. Using the technique of completing the square in the following problems:

13.1 $x^2 - 4x - 21 = 0$

13.2 $y^2 + 7y + 4 = 0$

13.3 $2z^2 - 7z + 3 = 0$

13.4 $3q^2 + 5q - 4 = 0$

14. $2x^2 + 5x - 4 = 0$

15. $x^2 - 4x + 1 = 0$

16. $x^2 + 2x + 3 + \frac{12}{x^2+2x+3} = 7$. **Hint:** Use an appropriate substitution to solve this equation!

17. $x^3 - 2x + 1 = 0$

18. $y^3 + y = 4y^2 - 2$

19. $x^2 = \frac{x^2-3x+2}{2-x}$

20. $2z^3 + 2z^2 - 5z - 2 = 0$

0.2 Section B: Linear, Quadratic and Fractional Inequalities

Solve the inequalities below:

1. $x^2 > 25$

2. $x^2 - 3x - 40 \leq 0$

3. $x^3 + 2x^2 - 15x < 0$

4. $\frac{2}{x-3} \geq \frac{1}{x+2}$

5. $-\frac{4}{5-2x} > 0$

6. $(y-3)(y+3) < 14(y-3)$

7. $\frac{2y}{2y-3} \leq \frac{4y}{y+3}$

8. $\frac{30}{y^2+9y-10} > -\frac{2}{y-1}$

9. $\frac{x}{x^2+x-2} + \frac{2x}{x+2} < \frac{x}{x-1}$

10. $\frac{28}{x-5} > 2x$

0.3 Section C: Word Problems With Equations

The purpose of these problems is to ascertain whether you can translate sentences into equations (mathematical formulation), solve the corresponding equations and then check whether the solution makes sense.

1. The product of two consecutive integers is 72. Find the integers.

2. A small rectangular vegetable garden is enlarged by increasing the length by 3 m and the width by 1 m. The area of the garden is three times larger than that of the original garden. Determine the original dimesnions of the garden if the area was 6 m^2.

3. The product of two consecutive odd integers is 143. Find the integers.

4. The difference between two natural numbers is 3 and their product is 40. Calculate the numbers.

5. The perimeter of a rectangle is 14 m and its area is 12 m^2. Find the dimensions of the rectangle.

6. A square plot is divided by decreasing the one side by 6 m and decreasing the other side by 8 m. The rectangular area thus formed is 24 m^2. Find the original dimensions of the plot.

7. A Jojo tank has two pipes entering it. When operating together, the tank is filled in 40 minutes. Operating independently, one of the pipes fills the tank 60 minutes faster than the other is able to on its own. Find the time taken by each

pipe to fill the tank on its own.

8. A man travels 180 km from his house in Stellenbosch to somewhere in Cape Town with a few passangers in order that he may drop them off. On his return journey he travels at 30 km/h faster to his home. He saves 1 hour on the return journey by travelling faster. How fast did he travel to Cape Town?

9. Two machines working together complete a job in 2 hours and 24 minutes. Working on its own, one of the machines takes 2 hours longer than the other to complete the job. How long does the slower machine take?

0.4 Section D: Simultaneous Equations

Solve simultaneously for the unknown variables in the systems of equations below:

1.

$$2x + y = 2$$

$$y^2 + 2x^2 = 3xy$$

2.

$$2x + 3y = 1$$

$$2x^2 + xy + x - 2y - y^2 - 1 = 0$$

3.

$$y - x = 2$$

$$x^2 + 2xy - 4 = 0$$

4.

$$a - b = 5$$

$$ab = 6$$

5.

$$2y + x = 1$$

$$6x^2 - 5xy + 10x + 11y - 6y^2 - 4 = 0$$

0.5 Mathematics of Finance

Simple Interest:

$$\boxed{F_v = P_v\,(1 + in) \text{ or } A = P\,(1 + rt)}$$

Compound Interest:

$$\boxed{F_v = P_v\,(1 + i)^n \text{ or } A = P\,(1 + r)^t}$$

Nominal and Effective Interest Rates:

$$\boxed{1 + i_{\text{eff}} = \left(1 + \frac{i_{\text{nom}}}{m}\right)^m \text{ or } 1 + r_{\text{eff}} = \left(1 + \frac{r_{\text{nom}}}{m}\right)^m}$$

Period of Compounding	m
Annually	$m = 1$
Semi-annually	$m = 2$
Quarterly	$m = 4$
Monthly	$m = 12$
Daily	$m = 365$

Straight-line Depreciation:

$$\boxed{F_v = P_v\,(1 - in) \text{ or } A = P\,(1 - rt)}$$

Reducing-balance Depreciation:

$$\boxed{F_v = P_v\,(1 - i)^n \text{ or } A = P\,(1 - r)^t}$$

1. A new car costs R52 000. If inflation is calculated at 18% per annum, compounded, how much would a similar new car cost in 10 years time?

2. You take out a loan from the bank to start a small business. The loan is repaid 3 years later in a lump sum payment of R48 000. If the interest rate is 14% per annum, compounded, calculate the amount of money borrowed from the bank.

3. R1 000 is invested at 15% per annum, coumpounded. The investment amount to R3 517.88. Calculate the period of the investment in years.

4. An investment of R1 000 increases to R3 000 over a period of 9 years. Calculate

the rate of interest, assuming that the interest is compounded annually.

5. The current value of a car is R80 000. Determine its value 5 years from now, if the rate of deprecation is 18% per annum on:

5.1. the straight-line basis,

5.2. the reducing-balance basis.

6. A 65-inch Samsung UHD TV is currently worth R10 000. In 5 years time, it is estimated to have a value of R5 000. Calculate the annual rate of depreciation if it is calculated on the reducing balance method.

7. Convert an effective (annual) interest rate of 15% p.a. to a:

7.1. semi-annual rate,

7.2. quarterly rate,

7.3. monthly rate,

7.4. daily rate.

8. Convert a rate of 18% p.a. compounded quarterly to a:

8.1. effective / annual rate,

8.2. semi-annual rate,

8.3. monthly rate,

8.4. daily rate.

9. A loan of R18 000 is repaid over 1 year in the following manner: R3 000 after 2 months, R7 500 at the end of 5 months and R4 200 3 months later. If the interest

rate is 12% p.a. componded monthly, what would the final payment be?

10. A father has three children aged 12, 15 and 17. He plans to give each of them R30 000 on their 21^{st} birthday. How much money should he invest now if the interest paid is 14% p.a. componded monthly?

0.6 Trignometric Identities

Prove the following identities listed below: **Hint:** Note the following useful identities

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

$$\boxed{\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta}$$

$$\boxed{\sin(A+B) = \sin A\cos B + \cos A\sin B}$$

$$\boxed{\cos(A+B) = \cos A\cos B - \sin A\sin B}$$

$$\boxed{(a-b)^2 = a^2 - 2ab + b^2}$$

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$$

$$\boxed{a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)}$$

1. $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

2. $\frac{1-2\cos^2\theta}{\sin\theta\cos\theta} = \tan\theta - \frac{\cos\theta}{\sin\theta}$

3. $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$

4. $\sin^3 A - \cos^3 A = (\sin A - \cos A)(1 + \sin A \cos A)$

5. $\frac{1}{\tan^2 \phi} - \cos^2 \phi = \frac{\cos^2 \phi}{\tan^2 \phi}$

6. $\left(\tan \alpha + \frac{1}{\tan \alpha}\right) \sin \alpha = \frac{1}{\cos \alpha}$

7. $\frac{\sin \alpha + \sin \beta}{\frac{1}{\sin \alpha} + \frac{1}{\sin \beta}} = \sin \alpha \sin \beta$

8. $\frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$

9. $\frac{1 - 2\sin^2 \theta}{\sin \theta \cos \theta} = \frac{\cos \theta}{\sin \theta} - \tan \theta$

0.7 Trignometric Reduction Formulae

Without the use of a calculator, simplify the following expressions:

1. $\frac{\tan(180° - x) \sin(360° - x) \cos(90° - x)}{\sin(180° + x) \cos(90° + x) \tan(180° + x)}$

2. $\frac{\tan(360° - \alpha) \sin(90° + \alpha) \tan(90° - \alpha) \sin \alpha}{\cos(270° + \alpha) \sin(180° + \alpha)}$

3. $\frac{2\sin(90° - \theta) + \cos(180° - \theta)}{\sin(90° - \theta) - \cos(180° - \theta)}$

4. $\frac{\sin(180° - \phi) \cos(90° - \phi)}{1 - \sin(90° + \phi) \cos \phi}$

5. $\frac{\sin 150° \cos 225° \sin 330° \sin 45° \sin 260°}{\cos 10°}$

6. $\sin 240° \left(\sqrt{3} - \frac{\sin 310°}{\tan 120° \cos 40°}\right)$

7. $\frac{\tan 150° \sin 120° \sin 70°}{\cos 20° \cos 135° \sin 135°}$

8. $\sin^2 150° \tan 110° \tan 200°$

9. $\frac{\cos(-\theta) \sin(180° + \theta)}{\tan(\theta - 180°) \sin(\theta + 90°)}$

10. $\frac{\sin(90° + \phi) \sin(-\phi)}{\tan(270° - \phi) \cos(360° - \phi) \tan(-180° - \phi)}$

11. $\frac{\sin(-20^\circ)}{\cos 430^\circ}$

12. $\frac{\sin(-239^\circ)}{\sin(-301^\circ)} + \tan 585^\circ$

13. $\frac{1}{\tan(-620^\circ)\tan(-10^\circ)} - \sin^2 217^\circ - \cos(-397^\circ)\cos 397^\circ$