

Mid term paper



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Class Section: A

Date:(8, 1, 2021)

Complex variable

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Question No 1

a)

$$\begin{aligned} 5x_1 + 4x_2 &= 3 \times 60 = 180 \\ 4x_1 + 2x_2 &= 2 \times 60 = 120 \end{aligned}$$

$$\begin{aligned} 5x_1 + 4x_2 &= 180 & \text{--- A} \\ 4x_1 + 2x_2 &= 120 & \text{--- B} \end{aligned}$$

Xing 2 by \textcircled{B} the $(-)$ from \textcircled{A}

$$\begin{aligned} 5x_1 + 4x_2 &= 180 \\ \pm 8x_1 + 4x_2 &= -240 \end{aligned}$$

$$-3x_1 = -60 \Rightarrow \boxed{x_1 = 20}$$

Put in eq \textcircled{A}

$$5 \times 20 + 4x_2 = 180$$

$$4x_2 = 180 - 100$$

$$4x_2 = 80$$

$$\boxed{x_2 = 20}$$

$$\boxed{x_1 = x_2} \rightarrow \text{same solution}$$

2)

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Sol

$$A = \begin{bmatrix} \text{ON} & \text{ON} & \text{OFF} \\ \text{OFF} & \text{ON} & \text{OFF} \\ \text{OFF} & \text{ON} & \text{ON} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\text{Let } A+B = \begin{bmatrix} \text{ON} & \text{ON} & \text{ON} \\ \text{ON} & \text{ON} & \text{ON} \\ \text{ON} & \text{ON} & \text{ON} \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$B = (A+B) - A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1-1 & 1-1 & 1-0 \\ 1-0 & 1-1 & 1-0 \\ 1-0 & 1-1 & 1-1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Question No 2

9)

$$S_1 = \begin{bmatrix} 18.95 & 14.35 & 8.98 \\ 17.8 & 13.30 & 10.70 \end{bmatrix}$$

let combine S_1 and S_2 in
2x3 order as S_3

$$S_3 = \begin{bmatrix} 18.9 & 14.7 & 8.98 \\ 17.80 & 13.5 & 10.70 \end{bmatrix}$$

S_3 is information about price
of term

Prince reduce by 20%

$$S_3 = \begin{bmatrix} 18.95 \times 80\% & 14.7 \times 80\% & 8.9 \times 80\% \\ 17.5 \times 80\% & 13.5 \times 80\% & 10.7 \times 80\% \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 15.16 & 11.8 & 7.18 \\ 14.24 & 10.8 & 8.63 \end{bmatrix}$$

b) Solution :-

$$p(x) = ax^2 + bx + c$$

$$f(x) = xe^{x-1}$$

$$p(1) = a(1)^2 + b(1) + c$$

$$p(1) = a + b + c$$

$$f(1) = 1e^{1-1}$$

$$f(1) = 1$$

it is given that $f(1) = p(1)$, so
 $a + b + c = 1 \rightarrow \textcircled{A}$

$$p'(x) = 2ax + b$$

$$p'(1) = 2a + b$$

$$f'(x) = e^{x-1} + xe^{x-1}$$

$$f'(1) = e^{1-1} + 1e^{1-1}$$

$$f'(1) = 1 + 1$$

$$f'(1) = 2$$

$$p''(x) = 2a$$

$$p''(1) = 2$$

$$f''(x) = e^{x-1}(2+x)$$

$$f''(1) = e^{1-1}(2+1)$$

$$f''(1) = (1)(3)$$

$$f''(1) = 3$$

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it is given that

$$f'(1) = p'(1)$$
$$2a + b = 2 \longrightarrow \textcircled{B}$$

And,

$$p''(1) = f''(1)$$
$$2a = 3 \longrightarrow \textcircled{C}$$

put $a = 3/2$ value of "a" in eq B

$$2\left(\frac{3}{2}\right) + b = 2$$

$$3 + b = 2$$

$b = -1$

put a & b in eq A

$$a + b + c = 1$$

$$\frac{3}{2} - 1 + c = 1$$

$$c = 2 - \frac{3}{2}$$

$c = 1/2$

Quadratic Polynomial which
satisfies the eq

$$x(x) = \frac{3}{2}x^2 - x + \frac{1}{2}$$

Question No 3

a) Solution:-

$$A = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 7 & 0 & 0.25 & -1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 11 & 2.2 & 0.3 & -2.4 \end{bmatrix} \quad B = \begin{bmatrix} -3 \\ -1.5 \\ 5.6 \\ 2.2 \end{bmatrix}$$

By Rows Operation

$$R_2 = 0.5 R_1$$

$$\underline{R_3 + R_1}$$

$$\begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & -0.1 & 0.25 & 2.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix}$$

$$R_4 - 2R_1$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & -0.1 & 0.25 & 2.2 \\ 0 & 0.2 & 0.3 & 5.6 \end{bmatrix}$$

$$R_3 + (0.1) R_1$$

$P - I$

$$\sim \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.25 & 1.8 \\ 0 & 0.2 & 0.3 & 5.6 \end{bmatrix}$$

$$R_4 - 0.2 R_1$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.25 & 1.8 \\ 0 & 0 & 0.3 & 6.4 \end{bmatrix}$$

$$R_4 - 1.2 R_3$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.25 & 1.8 \\ 0 & 0 & 0 & 7.24 \end{bmatrix}$$

Upper triangular matrix

$$A = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.25 & 1.8 \\ 0 & 0 & 0 & 7.24 \end{bmatrix}$$

And

Lower Triangular matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ -1 & 0.2 & 1 & 0 \\ 3 & -0.4 & 2 & -1 \end{bmatrix}$$

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Using lower triangular matrix
 $AX = b$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ -1 & 0.2 & 1 & 0 \\ 2 & -0.4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -3 \\ -1.5 \\ 5.6 \\ 2.2 \end{bmatrix}$$

$$a = -3$$

$$0.5a + b = -1.5$$

$$0.5(-3) + b = -1.5$$

$$b = -1.5 + 1.5$$

$$b = 0$$

$$-a + 0.2b + c = 5.6$$

$$(-1)(-3) + 0.2(0) + c = +5.6$$

$$c = 2.6$$

$$2a - 0.4b + 2c + d = 2.2$$

$$2(-3) + 0.4(0) + 2(2.6) + d = 2.2$$

$$d = 3$$

b)

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Question No 3

part b)

Solution :-

$$A = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$$

R is a unit square :

$$(0,0) \quad (0,1) \quad (1,0)$$

$$V = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

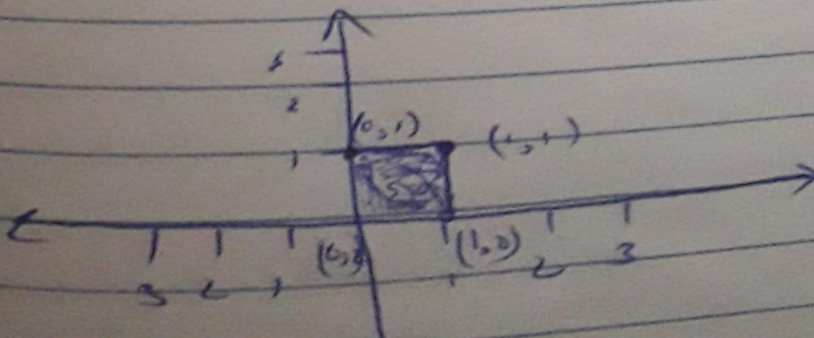
$$f(V) = AV$$

Now For $k = 1/2$

$$f(V) = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$f(V) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1/2 & 0 & 1/2 \end{bmatrix}$$

Plot :-



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Question No 4

a) Solution :-

$$T = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix}$$

$$x = \begin{bmatrix} 0.8 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$Tx = x'$$

$$x' = T \cdot x = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.8 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$x' = \begin{bmatrix} 0.69 \\ 0.15 \\ 0.16 \end{bmatrix}$$

This is probability for which
grandchild of a professional is
also professional

8)

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part b)

Solution :-

By Markov's chain

$$Tn = N$$

$$(T - I)N = 0$$

$$N = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$(T - I)N = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} N$$

$$\Rightarrow \begin{bmatrix} -0.2 & 0.3 & 0.2 \\ 0.1 & -0.5 & 0.2 \\ 0.1 & 0.2 & -0.4 \end{bmatrix} N = 0$$

$$\Rightarrow \left[\begin{array}{ccc|c} -0.2 & 0.3 & 0.2 & 0 \\ 0.1 & -0.5 & 0.2 & 0 \\ 0.1 & 0.2 & -0.4 & 0 \end{array} \right]$$

$$R_3 - R_2$$

$$\sim \left[\begin{array}{ccc|c} -0.2 & 0.3 & 0.2 & 0 \\ 0.1 & -0.5 & 0.2 & 0 \\ 0 & 0.7 & -0.6 & 0 \end{array} \right]$$

$$R_1 - R_2$$

$$\sim \left[\begin{array}{ccc|c} -0.2 & 0.8 & 0 & 0 \\ 0.1 & -0.5 & 0.2 & 0 \\ 0 & 0.7 & -0.6 & 0 \end{array} \right]$$

9)

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$$\Rightarrow -0.3a + 0.8b = 0$$

$$a = \frac{0.8b}{0.3}$$

$$\Rightarrow 0.7b - 0.6c = 0$$

$$b = \frac{6}{7}c$$

$$\Rightarrow c = \frac{7}{6}b$$

Probability vector:-

$$a + b + c = 1$$

$$\frac{8}{3}b + b + \frac{7}{6}b = 1$$

$$\frac{29}{6}b = 1$$

$$b = \frac{6}{29}$$

$$b = 0.208$$

$$a = \frac{8}{3} \left(\frac{6}{29} \right) \Rightarrow \frac{16}{29}$$

$$a = 0.56$$

And,

$$c = \frac{7}{6} \left(\frac{6}{29} \right) = \frac{7}{29}$$

$$c = 0.24$$

$$X = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.56 \\ 0.208 \\ 0.24 \end{bmatrix}$$