Name: Muhammad Ali, Registration No: 19pwcse1801, Section: A

$2x - y = 5$
$4x - 2y = t$

Sol:-

a) Determine values of t, so that system has a solution

$$2x - y = 5 \quad -(i)$$
$$4x - 2y = t \quad -(ii)$$

$$\cancel{4x} - \cancel{2y} = 10$$
$$\pm\cancel{4x} \mp \cancel{2y} = \pm t$$
$$0 = 10 - t \quad -(iii)$$
$$\boxed{t = 10}$$

b) Determine values of t so system has no solution

$$0 = 10 - 1 \Rightarrow 0 \neq 9$$

for $t = 0$ the system has no solution

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c) How many different values of t can be selected in part b

for all values of t except 10 The system has no solution.

$$(-\infty, 10) \cup (10, \infty)$$

Question NO 2

Solution :-

x_1 = Low suffer
x_2 = High suffer

Blending Plant $5x_1 + 4x_2 = 3h$
Refining plant $4x_1 + 2x_2 = 2h$
By converting hours into minutes

$$5x_1 + 4x_2 = 3 \times 60 = 180$$
$$4x_1 + 2x_2 = 2 \times 60 = 120$$

$$5x_1 + 4x_2 = 180 \quad \text{(i)}$$
$$4x_1 + 2x_2 = 120 \quad \text{(ii)}$$

multiplying equation (2) by (2)

$$2(4x_1 + 2x_2) = 2 \times 120$$
$$8x_1 + 4x_2 = 240$$

subtracting equation (1) from (3)

$$8x_1 + 4x_2 = 240$$
$$\pm 5x_1 \pm 4x_2 = \pm 180$$
$$3x_1 = 60$$
$$x_1 = 60/3$$
$$\boxed{x_1 = 20}$$

by putting $x_1 = 20$ in eq (1)

$$5(20) + 4x_2 = 180$$
$$100 + 4x_2 = 180$$
$$4x_2 = 180 - 100$$
$$4x_2 = \cancel{1000}\ 80$$

Each type of fuel should be manufactured by amounts of 20 tons

$x_1 = x_2 = 20$ tons

Solution is unique

Question 3 :-

Solution :-

Plant A : $2x + 2y = 8$

Plant B : $5x + 3y = 15$

xing eq (1) by 5 and equation 2 with 2 and Subract both equation

$$\cancel{10x} + 10y = 40$$

$$\pm \cancel{10x} \pm 6y = \pm 30$$

$$4y = 10$$

$$y = 10/4$$

$$y = 5/2 \qquad \boxed{y = 2.5}$$

(5)

put $y = 2.5$ in equation (1)

$2x + 2(2.5) = 8$

$2x + 5 = 8$

$2x = 8 - 5$

$2x = 3 \quad x = 3/2$

$\boxed{x = 1.5}$

$x = 1.5$ tons

$y = 2.5$ tons

Let 0 represent off and 1 represent ON

ON	ON	OFF
OFF	ON	OFF
OFF	ON	ON

⑥

<u>Solution:</u>

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

and

$$A+B = \begin{bmatrix} ON & ON & ON \\ ON & ON & ON \\ ON & ON & ON \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - A$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1-1 & 1-1 & 1-0 \\ 1-0 & 1-1 & 1-0 \\ 1-0 & 1-1 & 1-1 \end{bmatrix}$$

(7)

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

OFF form, the matrix is

$$= \begin{bmatrix} OFF & OFF & ON \\ ON & OFF & ON \\ ON & OFF & OFF \end{bmatrix}$$

Question No 4

Solution

$S_1 = [18.95 \quad 14.75 \quad 8.98]$

$S_2 = [17.80 \quad 13.50 \quad 10.79]$

Combined 2×3 Matrix

$$P = \begin{bmatrix} 18.95 & 14.75 & 8.98 \\ 17.80 & 13.50 & 10.79 \end{bmatrix}$$

a matrix that contain combined information about prices of item at stores

⑧

ii) Item prices reduced by 20

As item prices is reduced to 20% than actual prices, then a combined 2x3 matrix is

$$up = \begin{bmatrix} 18.95 \times 80\% & 14.75 \times 80\% & 8.98 \times 80 \\ 17.80 \times 80\% & 13.50 \times 80\% & 10.79 \times 80 \end{bmatrix}$$

$$= \begin{bmatrix} 18.95 \times 80/100 & 14.75 \times 80/100 & 8.98 \times 80/100 \\ 17.80 \times 80/100 & 13.50 \times 80/100 & 10.79 \times 80/100 \end{bmatrix}$$

$$up = \begin{bmatrix} 15.16 & 11.8 & 7.184 \\ 14.24 & 10.8 & 8.632 \end{bmatrix}$$

Question # 5

ⓐ

Solution

$$A = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$$

$k = 1/2$

R is a unit square (0,0) (0,1) (1,0) (1,1)

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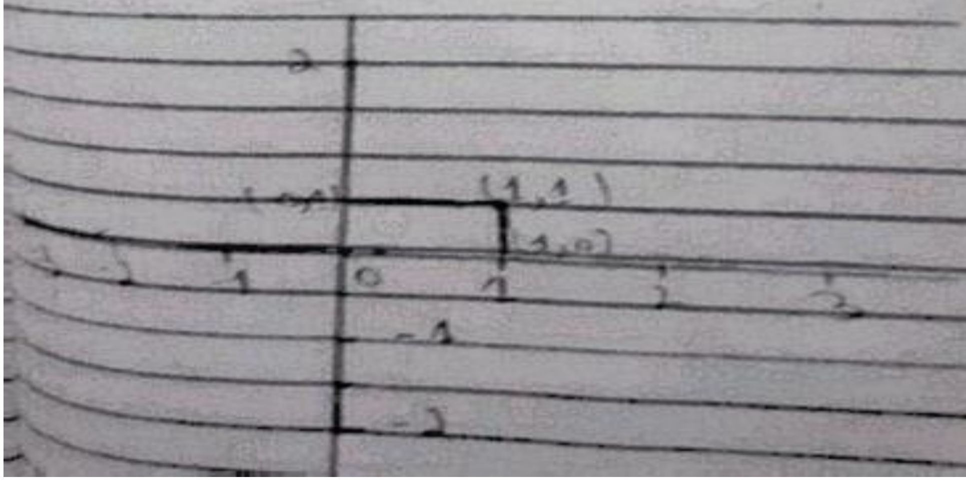
$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

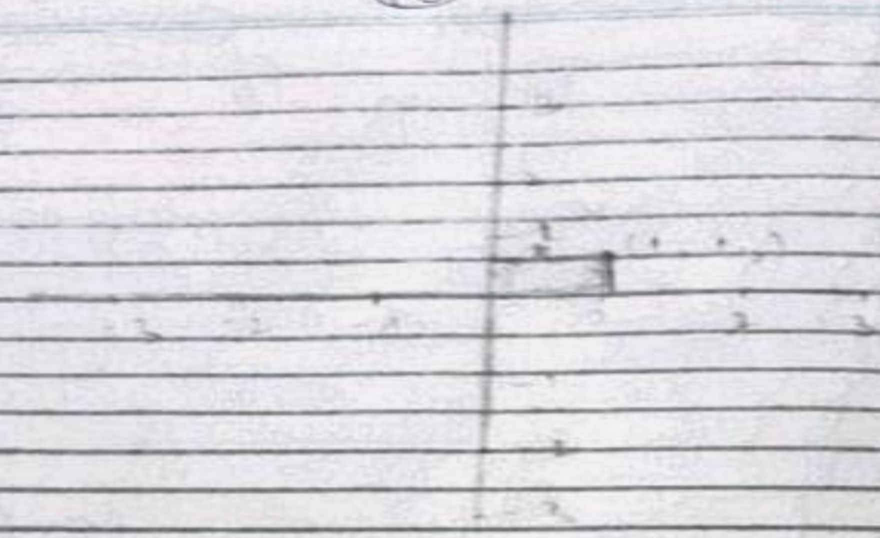
$v) = Av$

$$(v) = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1/2 & 0 & 1/2 \end{bmatrix}$$





b) $T = \begin{bmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{bmatrix}$

i) Regularity of T

$T^2 = T \cdot T = \begin{bmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 0 & 0.5 & 0.7 \end{bmatrix} \begin{bmatrix} 0 & 0.2 & \ldots \\ 0 & 0.3 & \ldots \\ 1 & .5 & \ldots \end{bmatrix}$

$T^2 = \begin{bmatrix} 0 & 0.06 & 0.06 \\ 0.3 & 0.24 & 0.2 \\ 0.7 & 0.7 & 0.64 \end{bmatrix}$

(11)

...there is a zero entry in T^2 so that T is regular

...e T^3

$$T \cdot T^2 = \begin{bmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{bmatrix} \begin{bmatrix} 0 & 0.06 & 0.0 \\ 0.3 & 0.24 & 0.3 \\ 0.7 & 0.7 & 0.9 \end{bmatrix}$$

$$T^3 = \begin{bmatrix} 0.06 & 0.048 & 0.06 \\ 0.3 & 0.282 & 0.282 \\ 0.64 & 0.67 & 0.658 \end{bmatrix}$$

$$T^3 = \begin{bmatrix} 0.06 & 0.048 & 0.06 \\ 0.3 & 0.282 & 0.282 \\ 0.64 & 0.67 & 0.658 \end{bmatrix}$$

Now T^3 have all position ent

let $x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ be a vector is steady state

$$TX = X$$

$$(T - I)X = 0$$

(11)

$$\left(\begin{bmatrix} 0 & 0.2 & 0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) x =$$

$$\begin{bmatrix} -1 & 0.2 & 0 \\ 0 & -0.7 & 0.3 \\ 1 & 0.5 & -0.3 \end{bmatrix} x = 0$$

Augmented matrix is

$$\sim \left[\begin{array}{ccc|c} -1 & 0.2 & 0 & 0 \\ 0 & -0.7 & 0.3 & 0 \\ 0 & 0.7 & -0.3 & 0 \end{array}\right] R_2 + R_1$$

$$\sim \left[\begin{array}{ccc|c} -1 & 0.2 & 0 & 0 \\ 0 & -0.7 & 0.3 & 0 \\ 0 & 0.7 & 0 & 0 \end{array}\right] \sim R_3 + R_2$$

from 2^{nd} Row

$$-0.7b + 0.3c = 0$$

$$0.3c = 0.7b$$

$$c = \left(0.7/0.3\right) b$$

from First Row

$-a + 0.2b + 0c = 0$

$-a + 0.2b = 0$

$0.2b = a$

$b = \frac{1}{0.2} a$

$\boxed{b = 5}$

Sum of probability = is 1

$a + b + c = 1$

$6a + 5a + 7/3\ b = 1$

$6a + 7/3\ b = 1$

$6a + 7/3\ (5a) = 1$

$6a + \frac{35}{3} a = 1$

$\frac{18 + 35a}{3} = 1$

$\frac{53\ a}{3} = 1$

$a = 3/53$

$$b = 5\left(\frac{3}{53}\right) \Rightarrow \boxed{b = 15/53}$$

$$\Rightarrow c = \frac{7}{3}\left(\frac{15}{53}\right)$$

$$\boxed{c = 35/53}$$

So the steady vector is

$$x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5/53 \\ 15/53 \\ 35/53 \end{bmatrix}$$

Question #6

Solution:-

$$A = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 1 & 0 & 0.25 & -1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix} \quad b = \begin{bmatrix} -2 \\ -0.5 \\ [illegible] \\ [illegible] \end{bmatrix}$$

Row operation on matrix to obtain upper triangular

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix}$$

$\to R_2$ [illegible]

R_2 [illegible]

$$\begin{bmatrix} 0 & -0.5 & 0.25 & 1 \\ 0 & -0.1 & 0.25 & 2.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix} R_3 - (1R_1)$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & -0.1 & 0.25 & 2.2 \\ 0 & 0.2 & 0.3 & 5.6 \end{bmatrix} \begin{array}{l} \sim R_4 \\ R_4 - 2R_1 \end{array}$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.2 & 2 \\ 0 & 0.2 & 0.3 & 5.6 \end{bmatrix} \begin{array}{l} \sim R_3 \\ R_3 - (0.2) R_2 \end{array}$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.2 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{array}{l} \sim R_4 \\ R_4 - 2R_3 \end{array}$$

Therefore the upper triangular matrix is

$$U = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.2 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

And the lower triangular

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ -1 & -0.2 & 1 & 0 \\ 2 & -0.1 & 2 & 1 \end{bmatrix}$$

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At
$Lz = b$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ -1 & 0.2 & 1 & 0 \\ 2 & -0.4 & 2 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -3 \\ -1.5 \\ 5.6 \\ 2.2 \end{bmatrix}$$

$z_1 = 3$

$0.5z_1 + z_2 = 1.5$

$-1z_1 + 0.2z_2 + z_3 = 5.6$

$2z_1 - 0.4z_2 + 2z_3 + z_4 = 2.2$

By Solving Equation (1)

$z_1 = -3$

$0.5(-3) + z_2 = -1.5$

$-1.5 + z_2 = -1.5$

$z_2 = -1.5 + 1.5$

Question no 7

Solution:

Let $A = \begin{bmatrix} \lambda-1 & -1 & -2 \\ 0 & \lambda-2 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}$

det A is

$$|A| = \begin{vmatrix} \lambda-1 & -1 & -2 \\ 0 & \lambda-2 & 2 \\ 0 & 0 & \lambda-3 \end{vmatrix}$$

Row operation Method

Expanding R_3

$$|A| = (\lambda-1)\begin{vmatrix} \lambda-2 & 2 \\ 0 & \lambda-3 \end{vmatrix} - (-1)\begin{vmatrix} 0 & 2 \\ 0 & \lambda-3 \end{vmatrix}$$

$$-2\begin{vmatrix} 0 & \lambda-3 \\ 0 & 0 \end{vmatrix}$$

det $|A|$ or $|A| = (\lambda-1)\left[(\lambda-2)(\lambda-3) - 0\times2\right.$

$+1\left[0\times(\lambda-3) - 0\times2\right] - 2\left[0\times0 - 0\times(\lambda-\right.$

[redacted]

(18)

$|A| = (\lambda-1)[(\lambda-2)(\lambda-3) - 1(0) - 2(0)]$

$|A| = (\lambda-1)[\lambda^2 - 3\lambda - 2\lambda + 6) + 0 - 0$

$|A| = (\lambda-1)[\lambda^2 - 5\lambda + 6)$

$|A| = \lambda^3 - 5\lambda^2 + 6\lambda - \lambda^2 + 5\lambda - 6$

$|A| = \lambda^3 - 6\lambda^2 + 11\lambda - 6$

Question part b

Solution:-

$p(x) = ax^2 + bx + c$

and $f(x) = xe^{x-1}$

put $x = 1$ in $p(x)$

$p(1) = a(1)^2 + b(1) + c$

$p(1) = a + b + c$

put $x = 1$ in $f(x)$

$f(1) = 1 - e^{1-1}$

$f(1) = 1$

But given $F(1) = f(1)$
$a+b+c = 1$
Differentiating $F(x)$ and $f(x)$ w.r.t. x
$F'(x) = 2ax + b$
$F'(1) = 2a(1) + b$
$F'(1) = 2a + b$
and $f'(x) = 1e^{x-1} + x(e^{x-1})\frac{d}{dx}(x-1)$
$f'(x) = e^{x-1} + xe^{x-1}(1)$
$f'(x) = e^{x-1} + xe^{x-1}$
$f'(1) = e^{1-1} + 1e^{1-1}$
$= e^{0} + e^{0} = 1 + 1$
$f'(1) = 2$
But given $F'(1) = f'(1)$
$\boxed{2a + b = 2}$ — (ii)
Now again differentiating $F'(x)$

(20)

$p''(1) = 2a$

and

$f''(x) = e^{x-1} + 1e^{x-1} + xe^{x-1}\frac{d}{dx}(x-1)$

$= e^{x-1} + e^{x-1} + xe^{x-1}(1)$

$= e^{x-1}(1+1+x)$

$f''(x) = e^{1-1}(2+1)$

$f''(1) = e^{0}(3)$

$f''(1) = 3$

put given $p''(1) = f''(1)$

$2a = 3$

from (iii) $a = 3/2$

put value of 'a' in eq (ii)

$2(3/2) + b = 2$

$3 + b = 2$

$\boxed{b = -1}$

Put values of 'a' and 'b' in eq (i)

$$\frac{3}{2} - 1 + c = 1$$

$$c = 1 + 1 - \frac{3}{2}$$

$$\boxed{c = \frac{1}{2}}$$

Therefore the quadratic polynomial that satisfies given eq n

$$\boxed{P(x) = \frac{3}{2}x^2 - x + \frac{1}{2}}$$

Question No 8

Solution:-

$$= \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix}$$

$$x = \begin{bmatrix} 0.8 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$Tx = x'$

$$x' = T \cdot x = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} 0.8 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$x' = \begin{bmatrix} 0.8 \times 0.8 + 0.3 \times 0.1 + 0.2 \times 0.1 \\ 0.1 \times 0.8 + 0.5 \times 0.1 + 0.2 \times 0.1 \\ 0.1 \times 0.8 + 0.2 \times 0.1 + 0.6 \times 0.1 \end{bmatrix}$$

$$x' = \begin{bmatrix} 0.64 + 0.03 + 0.02 \\ 0.08 + 0.05 + 0.02 \\ 0.08 + 0.02 + 0.06 \end{bmatrix}$$

$$x' = \begin{bmatrix} 0.69 \\ 0.15 \\ 0.16 \end{bmatrix}$$

This is the probality that grandchild of professionals will also be professionals

(2.4)

1st Row

$\Rightarrow 0.3a + 0.8b = 0$

$0.3a = 0.8b$

$a = \frac{0.8}{0.3} b$

$a = \frac{8}{3} b$

$\Rightarrow 3^{rd}$ Row

$0.7b - 0.6c = 0$

$0.7b = 0.6c$

$b = \frac{6}{7} c$

Probality vector is

$a + b + c = 1$

$\frac{8}{3} b + b + \frac{7}{6} b = 1$

$\frac{29}{6} b = 1$

$b = \frac{6}{29} = \boxed{0.207}$

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$a = 8/3\,(6/29)$

$a = 16/29 = 0.551$

and

$c = 7/_0\,(6/29)$

$\boxed{c = 7/29 = 0.241}$

$$x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 16/29 \\ 6/29 \\ 7/29 \end{bmatrix}$$

Solution b

Tug boat along negative x-axis = OA = -400

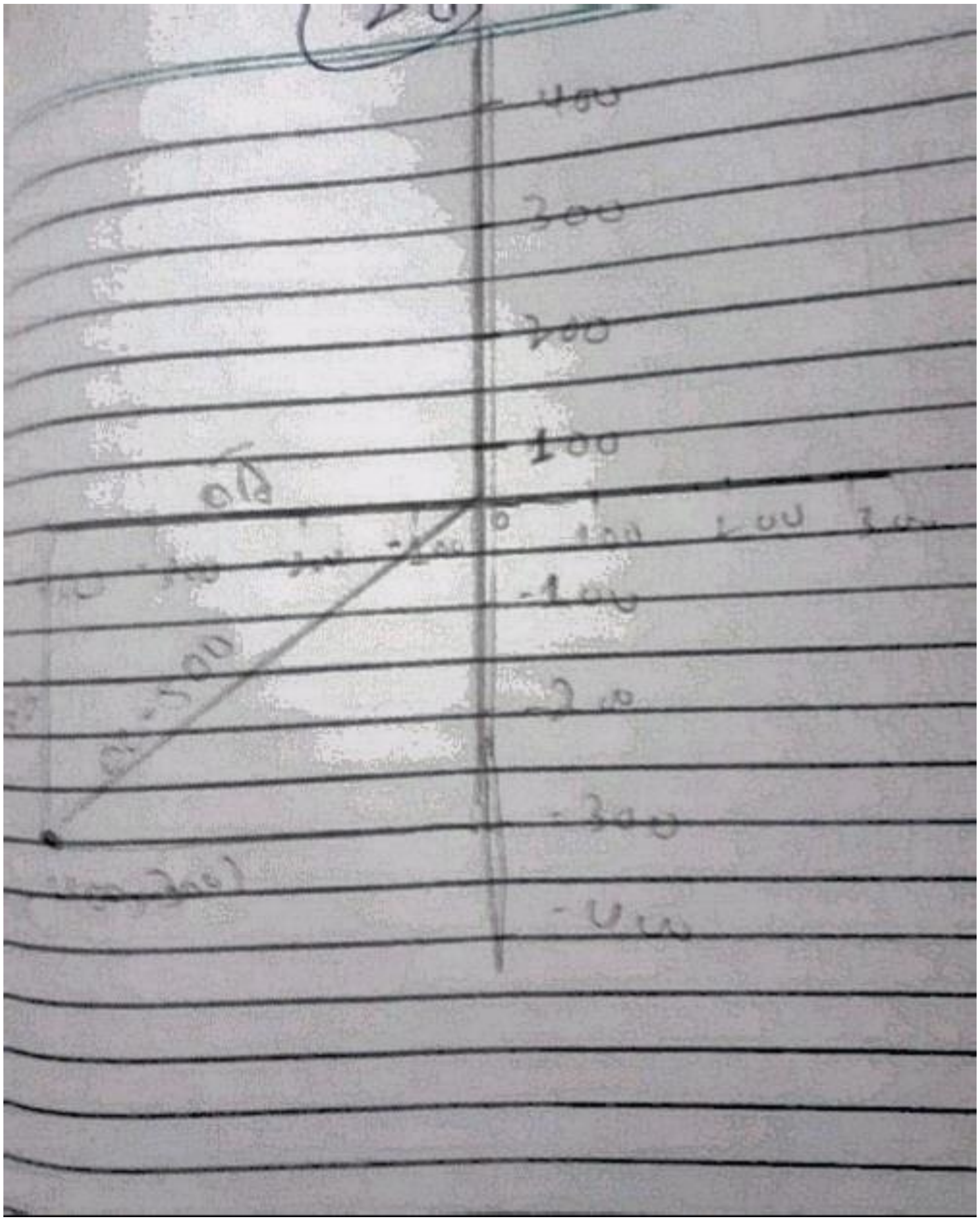
Tugboat along negative y-axis = OB = -300

Using pythogaras theorm

$$\bar{OC} = \bar{OA}^2 + \bar{OB}^2$$

$$OC = \sqrt{\bar{OA}^2 + \bar{OB}^2}$$

$$\bar{OC} = \sqrt{(-400)^2 + (-300)^2}$$



400
300
200
100
0
-100
-300