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SECTION A

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Question No 1

Statement :-

Let n be an integer then $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Proof :-

CASE 1 :- (For n = positive)

Suppose n is a +ve integer So for solving, we use mathematical induction method.

Let, $n = 1$

$$(\cos\theta + i\sin\theta)^1 = \cos 1\theta + i\sin\theta$$

which is true for $n = 1$

Now, let suppose it is true for $n = k$

$$(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin\theta \longrightarrow \text{(A)}$$

For $n = k+1$

$$(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin(k+1)\theta \rightarrow$$

Xing eq (A) by $\cos\theta + i\sin\theta$

$$(\cos\theta + i\sin\theta)^k(\cos\theta + i\sin\theta) = (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta$$

P-T-O

$$(\cos\theta + i\sin\theta)^{k}(\cos\theta + i\sin\theta) = \cos k\theta\cos\theta - \sin k\theta\sin\theta + $$
$$+ (i\sin k\theta + \cos k\theta\sin\theta)$$
$$= \cos(k\theta + \theta) + i\sin(k\theta + \theta)$$

$$(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin\overset{(1+k)}{\theta}$$

Hence by mathematical induction it is true for i.e $n = k+1$

CASE 2 :- (For $n = -ve$)

Suppose is negative integer then we can write $n = -m$ where m is positive integer.

$$(\cos\theta + i\sin\theta)^{n} = (\cos\theta + i\sin\theta)^{-m}$$
$$= \frac{1}{(\cos\theta + i\sin\theta)^{m}}$$
$$= \frac{1}{(\cos m\theta + i\sin m\theta)} \times \frac{\cos m\theta - i\sin m\theta}{\cos m\theta - \sin m\theta}$$
$$= \frac{\cos m\theta - i\sin m\theta}{\cos^2 m\theta + \sin^2 m\theta}$$
$$= \cos m\theta - i\sin\theta m$$

$$= \cos(-m)\theta + i\sin(-m)\theta$$

$$\because n = -m$$

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

$\therefore$ it is true for $-ve$ integer.

~~Question No 2~~

CASE 3 :- (For $n = 0$)

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

$$(\cos\theta + i\sin\theta)^0 = \cos 0^\circ + i\sin 0^\circ$$

$$1 = 1$$

Hence als proved for $n = 0$

Question No 2

$$f(z) = xy^2 + jx^2y$$

Sol:-

$$f(z) = xy^2 + ix^2y$$

Let, $u = xy^2$, $v = x^2y$

$$U_x = \frac{\partial}{\partial x}(xy^2) = y^2$$

$$V_x = \frac{\partial}{\partial x}(x^2y) = 2xy$$

$$U_y = \frac{\partial}{\partial y}(xy^2) = 2xy$$

$$V_y = \frac{\partial}{\partial y}(x^2y) = x^2$$

Now from CRE's theorem:

$$U_x = V_y$$

$$xy^2 = x^2$$

$$x = 0 \quad \text{or} \quad x = y^2$$

And,

$$U_y = -V_x$$

$$2xy = -2xy$$

$$xy = 0$$

$$x = 0, \quad y = 0$$

$$D - T - O$$

The Values :-

$(x = 0$ or $y = x^4)$ in $(x = 0$ or $y = 0)$

$f'(z)$ exist at every point on the x-axis , $x = 0$

$f'(z) = u_x + i\, v_x$

$= y^2 + i\, 2xy$

f is not analytic at any point

→ But every neighbourhood points on $x = 0$, there is exist other point Which is not differentiatible at that point

ii) $\sqrt[n]{-7+24j}$ PART b)

Sol:- let

$$z_k = (-7+24j)^{1/4}$$

$$z_k = r^{1/4}(\cos\theta + \sin\theta)^{1/4}$$

$$z_k = r^{1/4}\left[\cos\frac{1}{3}(\theta+2k\pi) + j\sin\frac{1}{4}(\theta+2k\pi)\right]^{1/4}$$

$$z_k = r^{1/4}\left[\text{Cis}\frac{1}{4}(\theta+2k\pi)\right]$$

$\frac{\pi}{2.4} = 73°$ $473° = \pi$

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To find r in θ

$r\cos\theta = -7$

$r\sin\theta = 24$

squaring and added

$r^2(\cos^2\theta + \sin^2\theta) = 49 + 576$

$r^2 = 625$

$r = 25$

$25\cos\theta = -7 \Rightarrow \theta = \cos^{-1}\left(\frac{-7}{25}\right) = 73°$

$25\sin\theta = 24 \Rightarrow \theta = \sin^{-1}\left(\frac{24}{25}\right) = 64°$

$$\boxed{Z_K = 25^{1/4} \cos\left(\frac{\pi}{2.4} + \frac{2K\pi}{3}\right) + i\sin\left(\frac{\pi}{2.8} + \frac{2K\pi}{3}\right)}$$

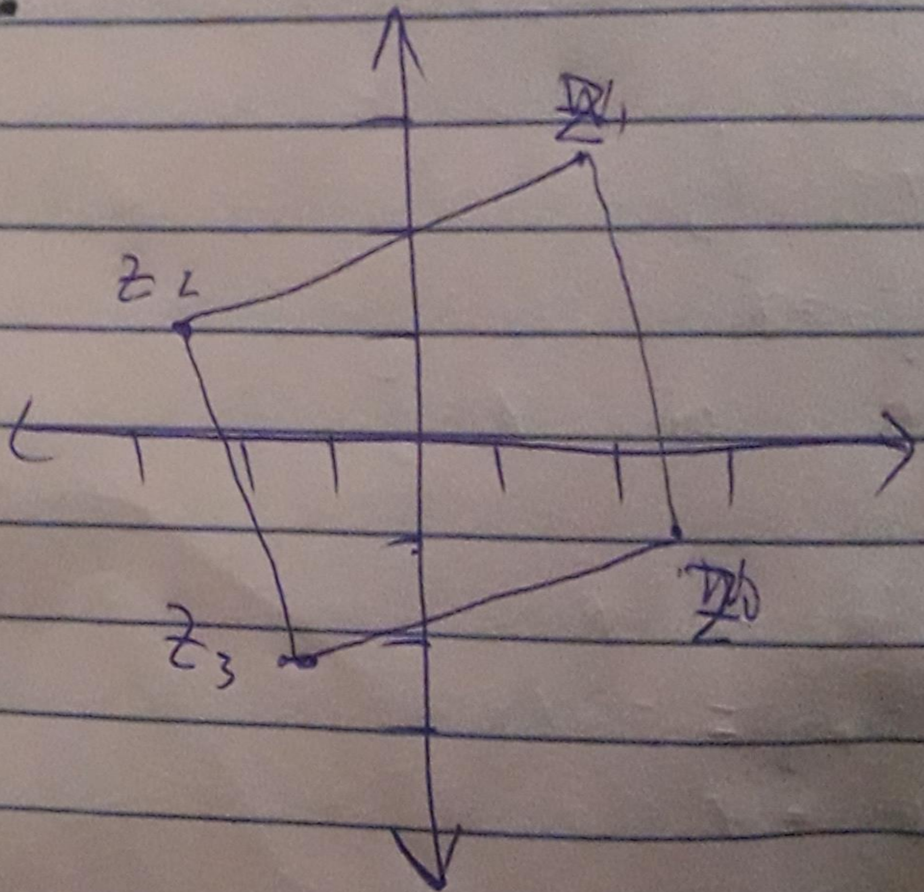
$Z_0 = (25)^{1/4} \cos\left(\frac{\pi}{2.4} + 0\right) + \sin\left(\frac{\pi}{2.8} + 0\right)$

$Z_0 = 2.1 = 0.7i$

$Z_1 = (25)^{1/4} \cos\left(\frac{\pi}{2.4} + \frac{2\pi}{3}\right) + \sin\left(\frac{\pi}{2.8} + \frac{2\pi}{3}\right)$

$Z_1 = 0.7 \times 21i$

Plot:-



Question No 3

Part a)

Analytic function:-

A function $f(z)$ is said to be analytic in a domain D if $f(z)$ is define and differentiable at all point of D. The $f(z)$ is analytic at a point $z = z_0$ in D if $f(z)$ is analytic in a neighborhood of z_0

Hence analyticity of function at z_0 means that $f(z)$ has a derivative at every point in some neighborhood of z_0 (may be z_0). This concept is explain that a function is differentiable merely at a single point z_0 but not throughout some neighborhood of z_0.

Example :-

The given function in part (b) in this Question,

$$f(z) = z + \frac{1}{z}$$

is an example of analytic function which is solve in part (b)

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part b)

$$f(z) = z + \frac{1}{z}$$

Sol:-

Let $z = x + jy$

$$f(z) = z + \frac{1}{z}$$

$$= x + jy + \frac{1}{x+jy}$$

$$= \frac{(x+jy)^2 + 1}{x+jy}$$

$$= \frac{x^2 - y^2 + 1 + j2xy}{x+jy}$$

$$= \frac{(x^2 - y^2 + 1) + j(2xy)}{x+jy} \times \frac{x-jy}{x-jy}$$

$$= \frac{(x^3 - xy^2 + x + 2xy^2) + j(2x^2y - x^2y + y^3 - y)}{x^2 + y^2}$$

$$u = \frac{x^3 + x + xy^2}{x^2+y^2} \quad , \quad v = \frac{x^2y + y^3 - y}{x^2+y^2}$$

Using CRE

$$U_x = \frac{(x^2+y^2)(3x^2+y^2+1) - (x^3+x+y^2x)(2x)}{(x^2+y^2)^2}$$

$$U_x = \frac{x^4 + 2x^2y^2 - x^2 + y^4 + y^2}{(x^2+y^2)^2} \longrightarrow A$$

$$V_y = \frac{(x^2+y^2)(x^2+3y^2-1)-(x^2y+y^3-y)(2y)}{(x^2+y^2)^2}$$

$$V_y = \frac{x^4+2x^2y^2-x^2+y^4+y^2}{(x^2+y^2)^2} \longrightarrow (B)$$

From eq (A) in (B)

$$\boxed{U_x = V_y}$$

Now, find

$$U_y = \frac{(x^2+y^2)(2xy)-(x^3+x+xy^2)(2y)}{(x^2+y^2)^2}$$

$$U_y = -2xy/(x^2+y^2)^2 \longrightarrow (C)$$

And,

$$V_x = \frac{(x^2+y^2)(2xy)-(x^2y+y^3-y)(2x)}{(x^2+y^2)^2}$$

$$V_x = 2xy/(x^2+y^2) \longrightarrow (D)$$

From eq (C) in (D)

$$\boxed{U_y = -V_x}$$

Hence satisfied by CRE that the given function is analytic.

Question No 4

Part a)

Continuity :-

A function $f(z)$ is said to be contineous at $z = z_0$ if it satisfied the following conditions

1) $f(z)$ is defined at z_0
2) $\lim_{z \to 0} f(z)$ exists
3) $\lim_{z \to} f(z) = f(z_0)$

Cauchy-Riemann's Equation :-

Consider a complex number given by $w = f(z)$

$$f(z) = u(x,y) + i\,v(x,y)$$

$f(z)$ is analytic iff it satisfied the relation

$$V_x = V_y \quad \text{and} \quad -V_x = U_y$$

Known as Cauchy-Riemann's equation

Cauch-Riemann's in polar form :-

If $f(z) = u(r \cdot \theta) + i(r \cdot \theta)$

then $V_r = \frac{1}{r} V_\theta$ and $V_r = -\frac{1}{r} V_\theta$

is called CRE in polar form.

Question No 4

$$u = x/x^2+y^2$$

$$U_x = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$U_{xx} = \frac{2x^5 - 8xy^4 - 4x^3y^2}{(x^2+y^2)^4} \longrightarrow (1)$$

Similarly,

$$U_y = (-1)(x)(2y) = \frac{-2xy}{(x^2+y^2)^2}$$

$$U_{yy} = \frac{-(2x^5 - 6xy^4 - 4x^3y^2)}{(x^2+y^2)^4} \longrightarrow (2)$$

Add eq (2) and (1)

$$U_{xx} + U_{yy} = 0$$

The function is Harmonic

Now CRE's are

$$U_x = V_y$$

$$U_y = -V_x = -2xy/(x^2+y^2)^2 \longrightarrow (A)$$

$$V_y = \frac{2xy}{(x^2+y^2)^2} \longrightarrow (B)$$

Sing (A) and (B)

$$\int V_x \, dx = y \int (2x)(x^2+y^2)^{-2} \, dx$$

$$= \frac{y\,(x^2+y^2)^{-2+1}}{-1} + k(y)$$

$$V_x = \frac{-y}{x^2+y^2} + (k(y)) \rightarrow ③$$

Differentiate w.r.t "y"

$$\frac{\partial v}{\partial y} = y^2 - x^2/(x^2+y^2)^2 + k'y \longrightarrow ④$$

Compare ③ in ④

$$k'(y) = 0$$

$$f(z) = u + iv = \left(\frac{x}{x^2+y^2} + i\,\frac{-y}{x^2+y^2}\right) + ic'$$

$$f(z) = \frac{x - iy}{x^2 - i^2y^2} + c = \frac{\cancel{(x-iy)}}{(x+iy)\cancel{(x-iy)}} + c$$

$$f(z) = \frac{1}{x+iy} + c$$

$$f(z) = \frac{1}{z} + c$$