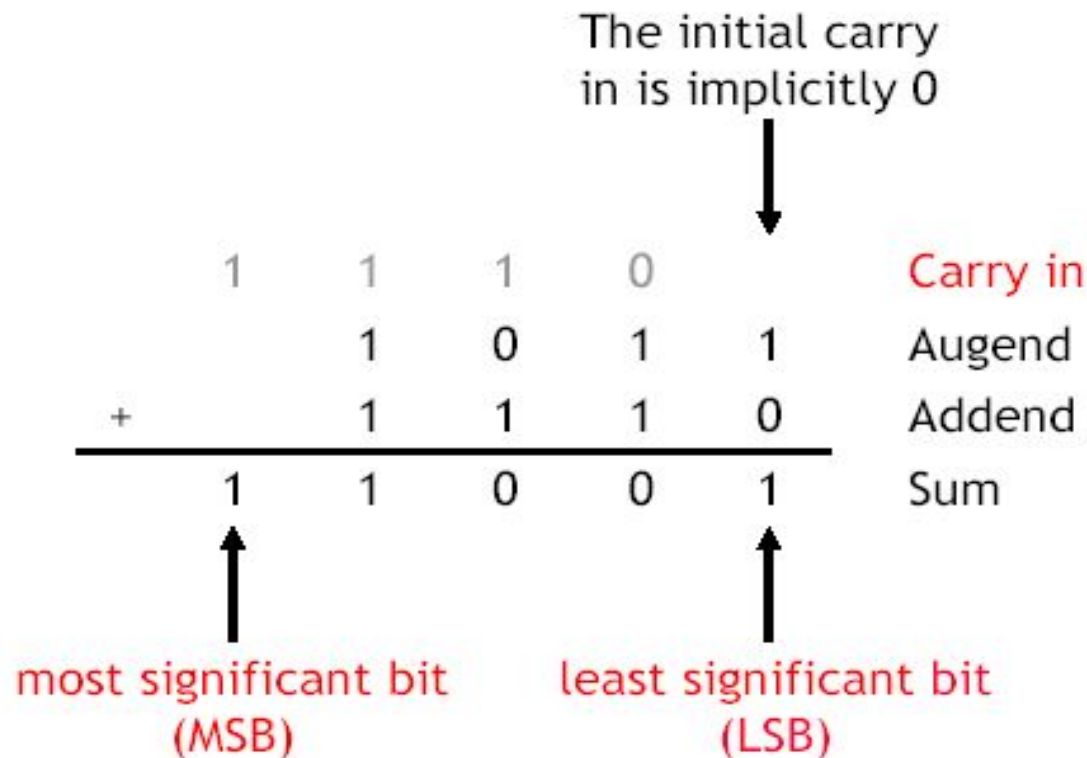# Overview

- Adder
- Ripple Carry Adder
- Subtraction
- Adder/Subtractor

Binary addition by hand

- You can add two binary numbers one column at a time starting from the right, just like you add two decimal numbers.
- But remember it's binary. For example, $1 + 1 = 10$ and you have to carry!

The initial carry in is implicitly 0

		1	1	1	0		Carry in
			1	0	1	1	Augend
+			1	1	1	0	Addend
		1	1	0	0	1	Sum

most significant bit (MSB)

least significant bit (LSB)

Adder

- Design an Adder for 1-bit numbers?

- **1. Specification:**
  2 inputs (X, Y)
  2 outputs (C,S)

Adder

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- **1. Specification:**
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  2 outputs (C,S)

- **2. Formulation:**

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0
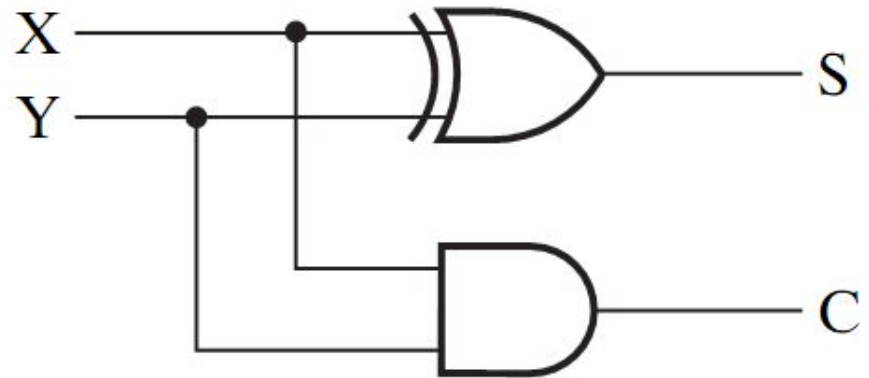
Adder

- Design an Adder for 1-bit numbers?

- **1. Specification:**

  2 inputs (X,Y)

  2 outputs (C,S)

- **2. Formulation:**

3. Circuit

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Full Adder

- A combinational circuit that adds 3 input bits to generate a Sum bit and a Carry bit

- A truth table and sum of minterm equations for C and S are shown below.

X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$0 + 1 + 1 = 10 \longrightarrow$

$1 + 1 + 1 = 11 \longrightarrow$

$C(X,Y,Z) = \Sigma m(3,5,6,7)$
$S(X,Y,Z) = \Sigma m(1,2,4,7)$

Full Adder

- A combinational circuit that adds 3 input bits to generate a Sum bit and a Carry bit

X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$S = X'Y'Z + X'YZ' + XY'Z' + XYZ$

$= X \oplus Y \oplus Z$

$C = X'YZ + XY'Z + XYZ' + XYZ$

$= (X'Y + XY') \cdot Z + XY (Z' + Z)$

$= (X \oplus Y)Z + XY$

Full Adder = 2 Half Adders

Manipulating the Equations:
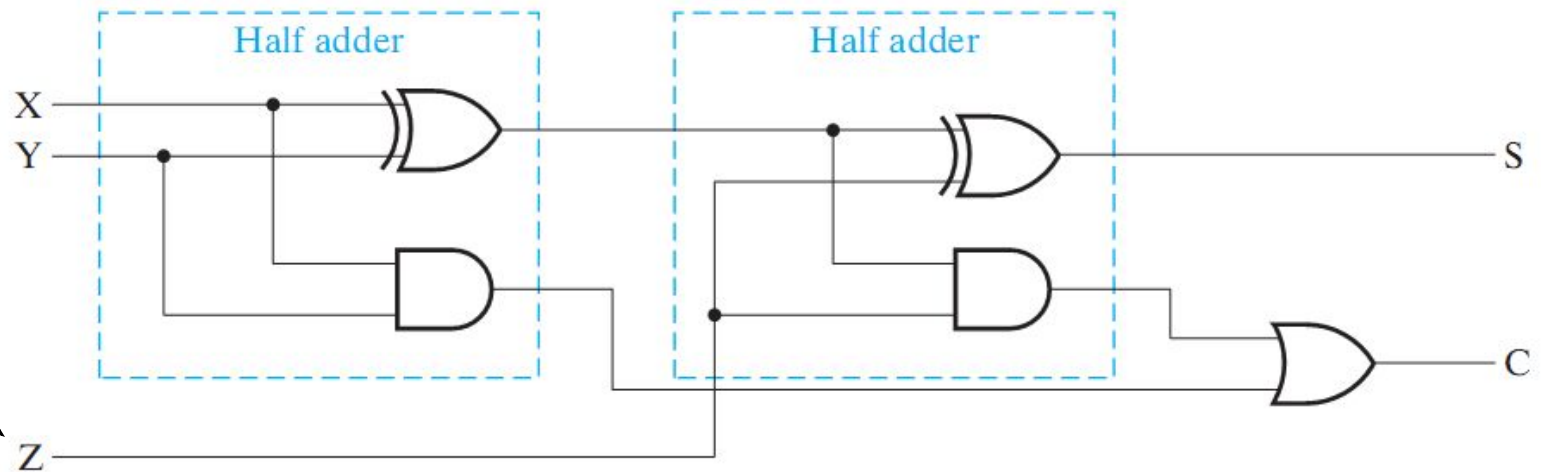
$$S = (X \oplus Y) \oplus Z$$

$$C = (X \oplus Y)Z + XY$$

Full Adder = 2 Half Adders

Manipulating the Equations:

$$S = (X \oplus Y) \oplus Z$$

$$C = (X \oplus Y)Z + XY$$

Think of Z as a carry in

Src: Mano's Book

Bigger Adders

- How to build an adder for n-bit numbers?
  - Example: 4-Bit Adder
    - Inputs ?
    - Outputs ?
    - What is the size of the truth table?
    - How many functions to optimize?

Bigger Adders

- How to build an adder for n-bit numbers?
  - Example: 4-Bit Adder
    - Inputs ? 9 inputs
    - Outputs ? 5 outputs
    - What is the size of the truth table? 512 rows!
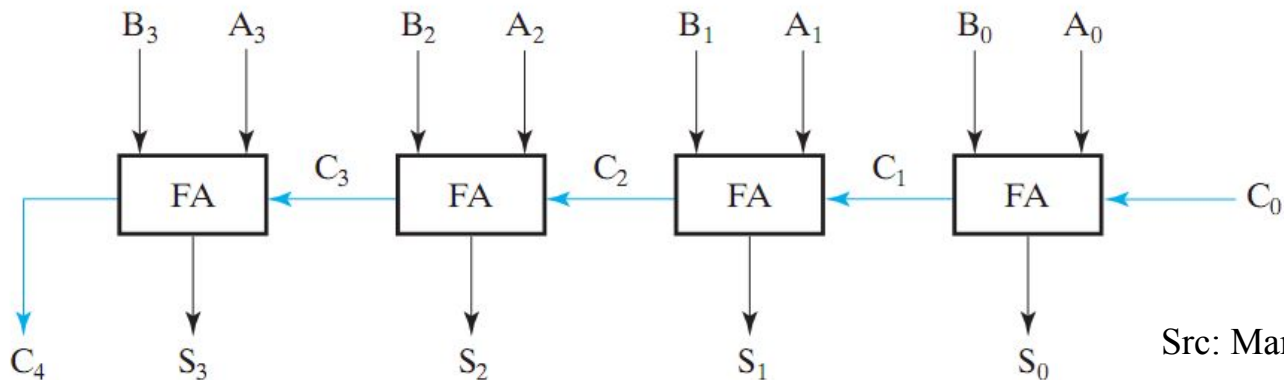   - How many functions to optimize? 5 functions

Ripple Carry Adder

- To add n-bit numbers:
- Use n Full-Adders
- The carries propagates as in addition by hand
- Use Z in the circuit as a C_{in}

- $\quad\quad$ 1 0 0 0
- $\quad\quad$ 0 1 0 1
- $\quad\quad$ 0 1 1 0
- $\quad\quad$ 1 0 1 1

Ripple Carry Adder

- To add n-bit numbers:
- Use n Full-Adders
- The carries propagates as in addition by hand
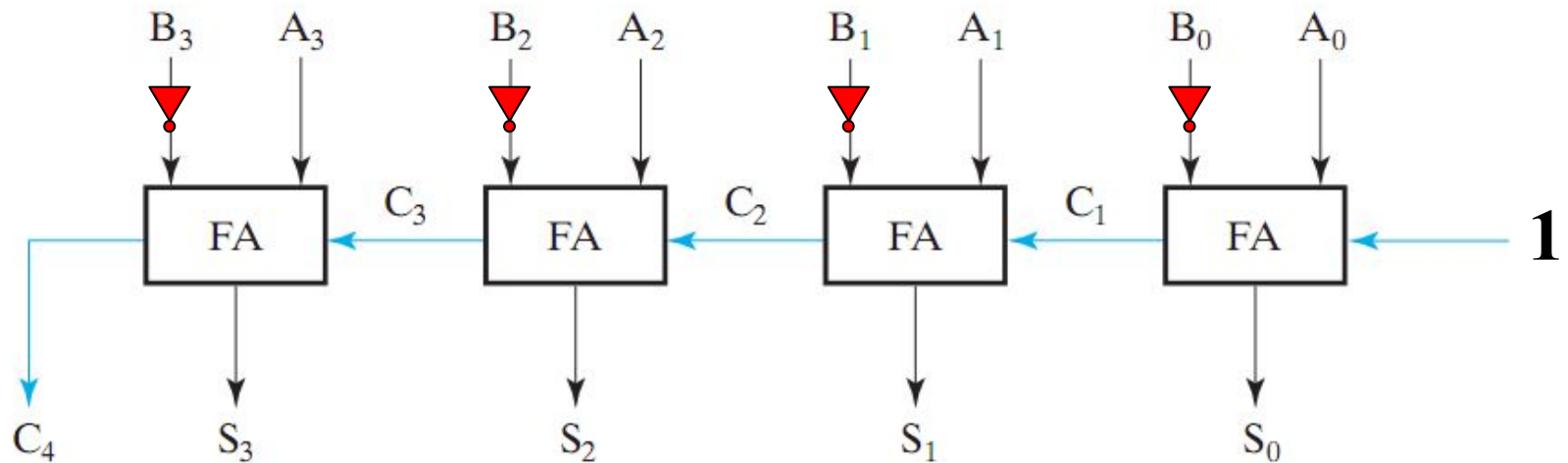

Src: Mano's Book

This adder is called *a 4-bit ripple carry adder*

Subtraction (2's Complement)

- How to build a subtractor using 2's complement?

Subtraction (2's Complement)
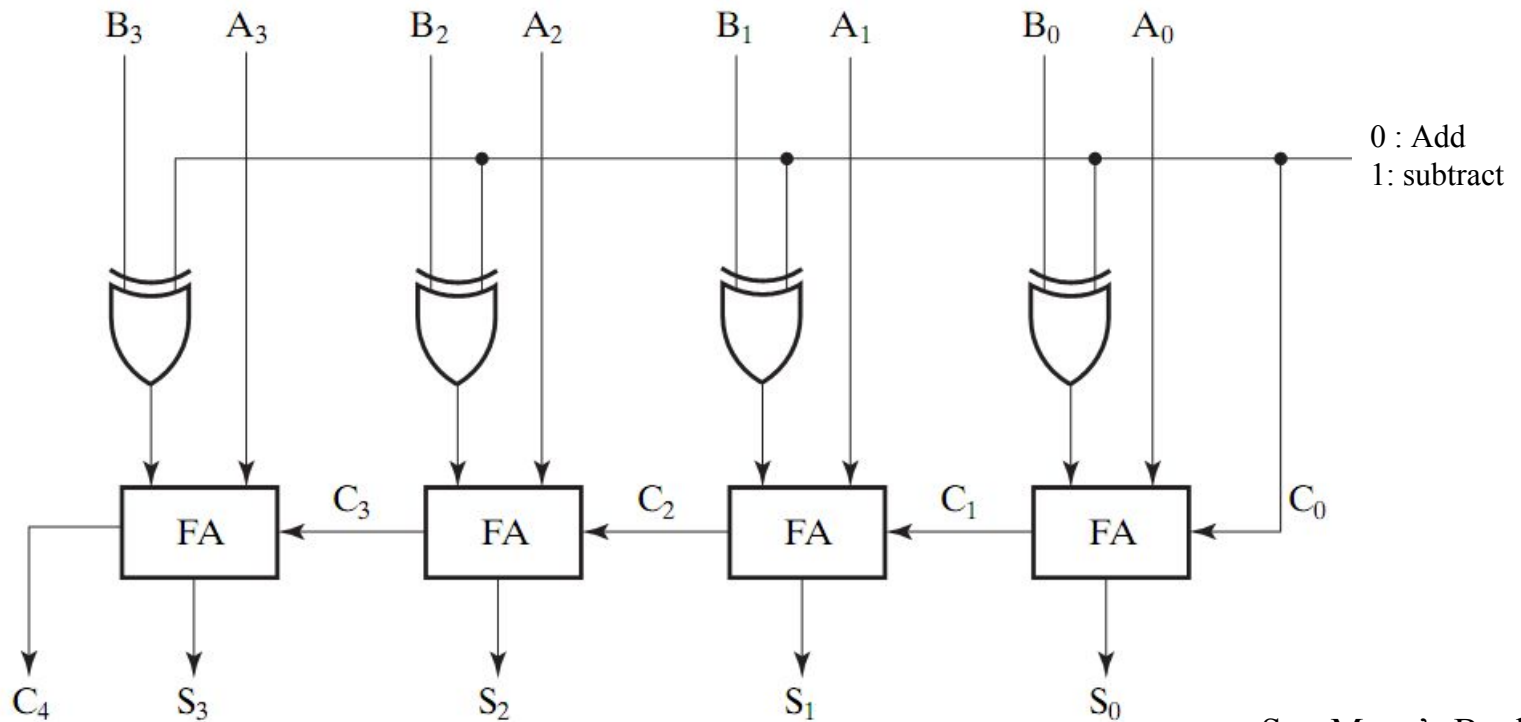
- How to build a subtractor using 2's complement?

$$S = A + (-B)$$

Adder/Subtractor

- How to build a circuit that performs both addition and subtraction?

Adder/Subtractor

Src: Mano's Book

Using full adders and XOR we can build an Adder/Subtractor!