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Question No 1

i) $1+i$

$z = 1+j = x+jy$

where $x=1$, $y=1$

olar Form

$z = r(\cos\theta + i\sin\theta)$

$r = \sqrt{x^2+y^2} = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{1}\right)$

$\theta = \tan^{-1}(1) = \frac{\pi}{4}$

$z = \sqrt{2}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right)$

$z = \sqrt{2}\,e^{j\pi/4}$

ii)

$z = 3+4j$

$r = \sqrt{9+16} = 5$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 52.4^\circ$

$z = 5(j\sin 52 + \cos 52)$

$\boxed{z = 5e^{j52^\circ}}$

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iii)

$z = -5 + 5j$

$r = \sqrt{5^2 + 5^2} = \sqrt{25 + 25} = 7$

$\theta = \tan^{-1}\left(\frac{5}{5}\right) = \pi/4$

$z = 7\left(\cos\frac{\pi}{4} + j\frac{\pi}{4}\right)$

$\boxed{z = 7e^{j\pi/4}}$

Question No 2

i) $\sqrt[3]{1+j}$

$z = (1+j)^{1/3}$

$= r(\cos\theta + jr\sin\theta)^{1/3} = r^{1/3}(\cos\theta + i\sin\theta)^{1/3}$

$= r^{1/3}\left[\cos(\theta + 2k\pi) + j\sin(\theta + 2\pi k)\right]^{1/3}$

$= r^{1/3}\left(\cos\frac{1}{3}(\theta + 2k\pi) + \sin\frac{1}{3}(\theta + 2\pi k)\right)$

$z_k = r^{1/3}\left[\mathrm{Cis}\,\frac{1}{3}(\theta + 2k\pi)\right]$

To find r & θ

P – T – O

To Find r and θ

$r\cos\theta = 1$

$r\sin\theta = 1$ Squaring on b/s and added

$$r^2(\cos^2\theta + \sin^2\theta) = 2$$

$$\sqrt{r^2} = \sqrt{2}$$

$$r = \sqrt{2}$$

$\sqrt{2}\cos\theta = 1 \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\sqrt{2}\sin\theta = 1 \Rightarrow \theta = \sin\frac{1}{\sqrt{2}}$ $\Bigg\}$ $\theta = \frac{\pi}{4}$

$$Z_k = 2^{1/6}\, Cis\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right)$$

$$Z_k = 2^{1/6}\left[\cos\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right)\right]$$

ii) $\sqrt[n]{-7+24i}$

Sol:- let

$$Z_k = (-7+24i)^{1/4}$$

$$Z_k = r^{1/4}(\cos\theta + \sin\theta)^{1/4}$$

$$Z_k = r^{1/4}\left\{\cos\frac{1}{3}(\theta + 2k\pi) + i\sin\frac{1}{4}(\theta + 2k\pi)\right.$$

$$Z_k = r^{1/4}\left[Cis\frac{1}{4}(\theta + 2k\pi)\right]$$

$\frac{\pi}{4} = 73^{\circ}$ $\quad 473^{\circ} = \pi$

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To find r & θ

$r\cos\theta = -7$

$r\sin\theta = 24$

squaring and added

$r^2(\cos^2\theta + \sin^2\theta) = 49 + 576$

$r^2 = 625$

$r = 25$

$25\cos\theta = -7 \Rightarrow \theta = \cos^{-1}\left(\frac{-7}{25}\right) = 73^{\circ}$

$25\sin\theta = 24 \Rightarrow \theta = \sin^{-1}\left(\frac{24}{25}\right) = 64^{\circ}$

$Z_k = 25^{1/4}\cos\left(\frac{\pi}{2\cdot 4} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{\pi}{2\cdot 8} + \frac{2k\pi}{3}\right)$

Question No 3

i) $z^2 - (5+j)z + (8+j) = 0$

Solution:-

$a = 1 \qquad b = -(5+j) \quad , \quad c = 8+j$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-(-(5+j)) \pm \sqrt{(5+j)^2 - 4(1)(8+j)}}{2(1)}$$

$$z = \frac{(5+j) \pm \sqrt{-8+6j}}{2} \longrightarrow ①$$

$$\boxed{\sqrt{z} = \pm\left[\frac{1}{2}\sqrt{|z|+x} + (\text{sin}y)j\,\frac{1}{2}\sqrt{|z|-x}\right]}$$

if $\text{sin}\, y = 1$ for $y \geq 0$ & $\text{sin}\, y = -1$ for $y < 0$

Now $z = -8+6j = x+jy$

$|z| = |8+6j| = \sqrt{(64+36)} = 10$

$$\sqrt{-8+6j} = \pm\left[\sqrt{\frac{1}{2}(10-8)} + (1)\,j\,\frac{1}{2}\sqrt{(10+8)}\right]$$

$= \pm(1+j3)$

So, eq ① $\Rightarrow$ $z = \dfrac{(5+j) \pm (1+3j)}{2} = \dfrac{6+4j}{2}$

$= 3+2j \quad , \quad 2j$

$$\boxed{z = 3+j \;,\; 2-j}$$

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Question No 4

i)

$$f = z^2 + 2z + 2 \quad \text{at} \quad z = 1 - j$$

$$f = (1-j)^2 + 2(1-j) + 2$$

$$f = 1 + j^2 - 2j + 2 - 2j + 2$$

$$f = \cancel{1} - \cancel{1} - 4j + 4$$

$$f = 4 - 4j$$

$$\text{Re} f = 4$$

$$\text{Im} f = -4$$

ii)

$$f = \frac{1}{1-z} \quad \text{at} \quad 7 + 2j$$

$$f = \frac{1}{1-(7+2j)} = \frac{1}{-(6+2j)}$$

$$f = \frac{1}{-6-2j} \times \frac{-6+2j}{-6+2j}$$

$$f = \frac{-6+2j}{36+4} \Rightarrow \frac{-6+2j}{40}$$

$$f = \frac{-6}{40} + \frac{2j}{40}$$

$$\text{Re} f = -6/40$$

$$\text{Im} f = \frac{1}{20}$$

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Question No 5

i) $(\text{Im } z)/|z|$

Given $f(z) = 0$

i.e $f(z)$ is define at "0"

Also $f(z) = \frac{\text{Im } z}{|z|}$ for $z \neq 0$

let $z = x + iy$

$$\lim_{z \to 0} f(z) = \lim_{z \to 0} \frac{\text{Im } z}{|z|}$$

$$= \lim_{z \to 0} \frac{y}{\sqrt{x^2+y^2}}$$

Two cases

Case 1 :-

1st $x \to 0$ then $y \to 0$

$$\lim_{z \to 0} f(z) = \lim_{y \to 0} \left(\lim_{x \to 0} \left(\frac{y}{\sqrt{x^2+y^2}} \right) \right) = \lim_{y \to 0} \left(\frac{y}{y} \right) = 1$$

Case 2 :- $y \to 0$ then $x \to 0$

$$= \lim_{x \to 0} \left(\lim_{y \to 0} \left(\frac{y}{\sqrt{x^2+y^2}} \right) \right) = \lim_{x \to 0} (0) = 0$$

So, CASE 1 $\neq$ CASE 2

Thus the function $f(z)$ is not continuous at origin.

ii) $\dfrac{\text{Im } z}{1+|z|}$

let $z = x + iy$, $|z| = \sqrt{x^2+y^2}$

$f(z) = \dfrac{\text{Im } z}{(1+|z|)} \Rightarrow \dfrac{y}{(1+\sqrt{x^2+y^2})}$

1) $f(z) = 0$ ($f(z)$ defined at "0")

2) $\lim\limits_{z\to 0} f(z) = \lim\limits_{x+iy\to 0}\left(\dfrac{y}{(1+\sqrt{x^2+y^2})}\right)$

CASE 1 :- $\lim\limits_{y\to 0}\left(\lim\limits_{x\to 0} \dfrac{y}{1+\sqrt{x^2+y^2}}\right)$

$\lim\limits_{x\to 0}\left(\dfrac{y}{1+y}\right) = 0$

CASE 2 :- $\lim\limits_{x\to 0}\left(\lim\limits_{y\to 0}\left(\dfrac{y}{1+\sqrt{x^2+y^2}}\right)\right)$

$\lim\limits_{x\to 0}\left(\dfrac{0}{1+x}\right) = 0$

Hence, CASE 1 = CASE 2

limit exist at origin

And, $\lim\limits_{z\to 0} f(z)$

Satisfied all three condition so the function is continous at origin.

Question No 6

a) Given $f(z) = \frac{z+j}{z-j}$

$$f'(z) = \frac{z-j-z-j}{(z-j)^2} = \frac{-2j}{(z-j)^2}$$

$$f'z\Big|_{z=-j} = \frac{-2j}{(-j-j)^2} = \frac{2j}{(-2j)^2} = \frac{1}{2}j$$

$$f'z\Big|_{z=-j} = \frac{1}{2j}$$

b)

$$f(z) = (z-4j)^8 \quad at \quad 5+4j$$

$$f'(z) = (8(z-4j)^7)$$

$$f'(z)\Big|_{z=5+4j} = 8(5+\cancel{4j}-\cancel{4j})^7$$

$$= 8(5)^7$$

$$f'z\Big|_{z=5+4j} = 625000$$

Qusetion No 7

$$u = x/x^2+y^2$$

$$U_x = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$U_{xx} = \frac{2x^5 - 8xy^4 - 4x^3y^2}{(x^2+y^2)^4} \longrightarrow (1)$$

Similarly,

$$U_y = (-1)(x)(2y) = \frac{-2xy}{(x^2+y^2)^2}$$

$$U_{yy} = \frac{-(2x^5 - 6xy^4 - 4x^3y^2)}{(x^2+y^2)^{4}} \longrightarrow (2)$$

Add eq (2) and (1)

$$U_{xx} + U_{yy} = 0$$

The founction is Hormonic

Now CRE's are

$$U_x = V_y$$

$$U_y = -V_x = -2xy/(x^2+y^2)^2 \longrightarrow (A)$$

$$V_y = \frac{2xy}{(x^2+y^2)^2} \longrightarrow (B)$$

Sing (A) and (B)

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$$\int v_x \, dx = y \int (2x)(x^2+y^2)^{-2} \, dx$$

$$= \frac{y(x^2+y^2)^{-2+1}}{-1} + k(y)$$

$$v_x = \frac{-y}{x^2+y^2} + (k(y)) \rightarrow (3)$$

Differentiate w.r.t "y"

$$\frac{\partial v}{\partial y} = y^2 - x^2/(x^2+y^2)^2 + k'y \longrightarrow (4)$$

Compare (3) in (4)

$$k'(y) = 0$$

Sing

$$f(z) = u + iv = \left(\frac{x}{x^2+y^2} + i \frac{-y}{x^2+y^2} \right) + ic'$$

$$f(z) = \frac{x - iy}{x^2 - i^2y^2} + c = \frac{\cancel{(x-iy)}}{(x+iy)\cancel{(x-iy)}} + c$$

$$f(z) = \frac{1}{x+iy} + c$$

$$f(z) = \frac{1}{z} + c$$

ii)

$U = \sin x \cosh x$

$U_x = \cos x \cosh y$

$U_{xx} = -\sin x \cosh y$

$U_y = \sin x \sinh y$

$U_{yy} = \cosh y \sin x$

$U_{xx} + U_{yy} = 0$

Hence, the function is harmonic

Now using CRE's

$V_y = \cos x \cosh y \longrightarrow (1)$

Ing on b/s

$V = \cos x \sinh y + k(x) \text{ --- } (A)$

P - diff w-r-t "x"

$V_x = -\sin x \sinh y + k'(x) \longrightarrow (B)$

Compare (B) in (1)

$k'(x) = 0$, Ing w-r-t "x"

$k(x) = c'$

$f(z) = U + iV = \sin x \cosh y i + \cos x \sinh y + ic'$

$f(z) = \sin(x + iy) + c$

$$\boxed{f(z) = \sin z + c}$$

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Qustion No 8

i)

$$z = 2 + 3\pi j$$

$$e^{z} = e^{2+3\pi j} = e^{2}\left[\cos(3\pi) + j\sin(3\pi)\right]$$

$$= e^{2}\left(0.98 + j\,0.16\right)$$

$$e^{z} = 7.24 + j\,1.18$$

$$u = 7.24 \quad , \quad v = 1.18$$

$$|e^{z}| = \sqrt{\left[e^{2}\left((\cos(3\pi))^{2} + (\sin(3\pi))^{2}\right]}$$

$$|e^{z}| = \sqrt{e^{4}(1)}$$

$$|e^{z}| \Rightarrow \boxed{e^{2} = 7.38}$$

ii)

$$z = 3\pi(1+j)$$

$$e^{z} = e^{3\pi}\left(\cos(3\pi) + j\sin(3\pi)\right)$$

$$= e^{3\pi}\left(0.98 + j\,0.16\right)$$

$$e^{z} = 12143.81 + j\,1982.6$$

$$\boxed{u = 12143.8} \cdot \boxed{v = 1982}$$

$$|e^{z}| = \sqrt{(e^{3\pi}\cos 3\pi)^{2} + (e^{3\pi}\sin 3\pi)^{2}}$$

$$\boxed{|e^{z}| = 12391.64}$$

iii) $-\pi j/2$

$$z = 0 - \frac{\pi j}{2}$$

$$e^{z} = e^{0}\left(\cos \pi/2 - j \sin \pi/2\right)$$

$$e^{z} = 0.99 - j\, 0.027$$

$\boxed{U = 0.99}$, $\boxed{V = 0.027}$

$$|e^{z}| = \sqrt{\left(\cos \frac{\pi}{2}\right)^2 + \left(\sin \frac{\pi}{2}\right)^2}$$

$$|e^{z}| = \sqrt{1}$$

$$\boxed{|e^{z}| = 1}$$

Question No 9

i) $\cosh z = \cosh x \cos y + j \sinh x \sin y$

Let $z = x + jy$

$$\cosh z = \cosh(x + jy)$$

$$= \left(\cosh x \cosh(jy) + \sinh x \sinh(jy)\right)$$

$$= \cosh x \cos y + \sinh x \, j \sin y$$

$\because$ $\cosh jy = \cos y$, $\sinh jy = j \sin y$

$$\boxed{\cosh z = \cosh x \cos y \, j + \sinh x \sin y}$$

ii) $\sinh z = \sinh x \cos y + j \cosh x \sin y$

let $z = x + jy$

$\sinh z = \sinh(x + yj)$

$= \sinh x \cosh yj + \cosh x \sinh yj$

$= \sinh x \cos y + \cosh x j \sin y$

$\boxed{\sinh z = \sinh \cos y + j \cosh x \sin y}$

Question No 10

a) $\cos(1+j)$

Sol: $\Rightarrow$

$\cos z = \cos x \cosh y - j \sin x \sinh y$

$= \cos(1) \cosh(1) - j \sin(1) \sinh(1)$

$\cos(1+j) = \boxed{0.8316 - j\, 0.98}$

$\boxed{u = 0.8316}$, $\boxed{v = j\, 0.98}$

b) $\cosh(-3-6j) = \cosh(-3)\cos(-6) + j \sinh(3) \sin(-6)$

$h(z)$ $= 9.65 - j\, 2.70$

$\boxed{u = 9.65}$, $\boxed{v = -2.70}$

c) $\cos\left(\frac{1}{2}\pi - \pi j\right) = \left(\cos\left(\frac{1}{2}\pi\right)\cosh(-\pi) - \sin\left(\frac{1}{2}\pi\right)\sinh(-\pi)\right)$

$= 9.65 - j\, 2.70$

$\boxed{u \cdot 9.65}$, $\boxed{v = -2.70}$