ISE 754 Logistic Engineering

Final Project Report

5/5/2019

Military Aircraft Maintenance Scheduling

Submitted by:

Muhammad Ali Haider (200224608)

Manjunath Jois (200286077)

Introduction

Due to the increasing traffic demand and the limited availability of airport resources, aviation authorities are seeking methods to better use the existing infrastructure during operations and to better manage aircraft movements in the proximity of airports, trying to improve punctuality while maintaining the required safety level. From a logical point of view, it is possible to divide Air Traffic Control (ATC) decisions in a Terminal Control Area (TCA) into: (i) Routing decisions, where an origin-destination route for each aircraft has to be chosen regarding air segments and runways; (ii) Scheduling decisions, where feasible aircraft sequencing and timing have to be determined in each air segment, runway and (eventually) holding circle, satisfying scheduling regulation and giving optimized solutions. Aircraft scheduling problem (ASP) that is one of the challenging problems in air traffic control during disturbed traffic situations (Sama, 2013). The aircraft maintenance scheduling is one among the major decisions an airline has to make during its operation (Sriram, 2003). Though maintenance scheduling comes as an end stage in an airline operation, it has potential for cost savings. Maintenance scheduling is an easily understood but difficult to solve problem. Given a flight schedule with aircraft assigned to it, the aircraft maintenance-scheduling problem is to determine which aircraft should fly which segment and when and where each aircraft should undergo different levels of maintenance check required by the Federal Aviation Administration. In an effort to control costs, airlines have begun to concentrate on their maintenance operations as a potential source for savings (Feo, 1989). The objective is to minimize the maintenance cost and any costs incurred during the re-assignment of aircraft to the flight segments.

Military use of aviation dates to the 18th century when the French observation balloon was used to monitor Austrian troop movements. Since then aviation has grown to be a vital part of every Military organization be it Airforce, Army, Navy or the Coast Guard. While aviation plays a crucial role in these organizations, it is imperative to maintain these aircraft continuously to ensure availability of aircraft to carry missions as required.

Aircraft maintenance is the performance of tasks required to ensure the continuing airworthiness of an aircraft, including overhaul, inspection, replacement, defect rectification, and the embodiment of modifications, compliance with airworthiness directives and repair.

The maintenance of aircraft is highly regulated, to ensure safe and correct functioning during flight. National regulations are coordinated under international standards, maintained by bodies such as the International Civil Aviation Organization (ICAO). The maintenance tasks, personnel and inspections are all tightly regulated, and staff must be licensed for the tasks they carry out. This in turn results in lengthy maintenance schedules that needs to be scheduled to avoid conflict with availability to carry out required missions.

Modelling using MILP

Here we explore a typical problem of developing schedule for aircraft maintenance while being mindful of constraints associated with flight operations. While the constraints considered here are not exhaustive they form a basis for military aircraft maintenance scheduling problems.

To define the problem, let us consider an air station with n aircraft which need to be assigned to m missions. Let us assume for the sake of tractability that each aircraft flies a fixed number of hours each time-period when it's on a mission and different fixed number of flight hours when its flying at the home station. This can later be relaxed to allow a variable number of hours each time period. Also, there's usually a target number of hours an aircraft must be flown each year. Each aircraft must be maintained after flying a certain number of hours and this cannot occur when an aircraft is undertaking a mission. The main objective in this problem would be to find a feasible schedule while minimizing the number aircraft that need to be maintained on the same time, as this would increase strain on maintenance capacity and resources.

Problem can be stated in mathematical terms as below:

Let

$n - number\ of\ aircraft\ available\ at\ airstation$

$i - index\ for\ aircraft$

$m - number\ of\ missions\ that\ need\ to\ be\ undertaken$

$j - index\ for\ missions$

$t - number\ of\ time\ periods\ for\ which\ a\ schedule\ needs\ to\ be\ generated$

$k - index\ for\ time\ period$

$p - number\ of\ time\ periods\ t\ take\ to\ complete\ a\ maintenance\ once\ started$

$y - number\ of\ hours\ an\ aircraft\ can\ be\ flown\ in\ an\ year$

$z - number\ of\ hours\ after\ which\ an\ aircraft\ must\ be\ maintained$

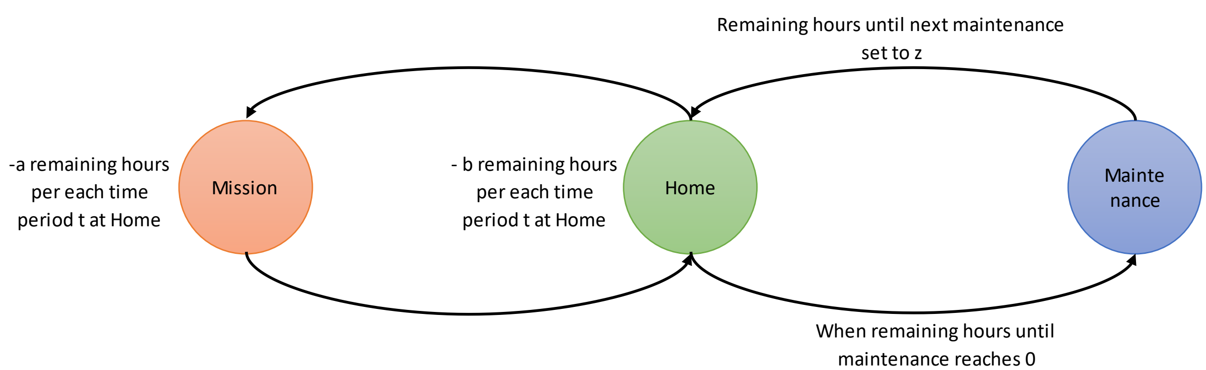
$a - number\ of\ hours\ flown\ by\ an\ aircraft\ when\ its\ on\ a\ mission$

$b - number\ of\ hours\ flown\ by\ an\ aircraft\ when\ its\ at\ home\ airstation$

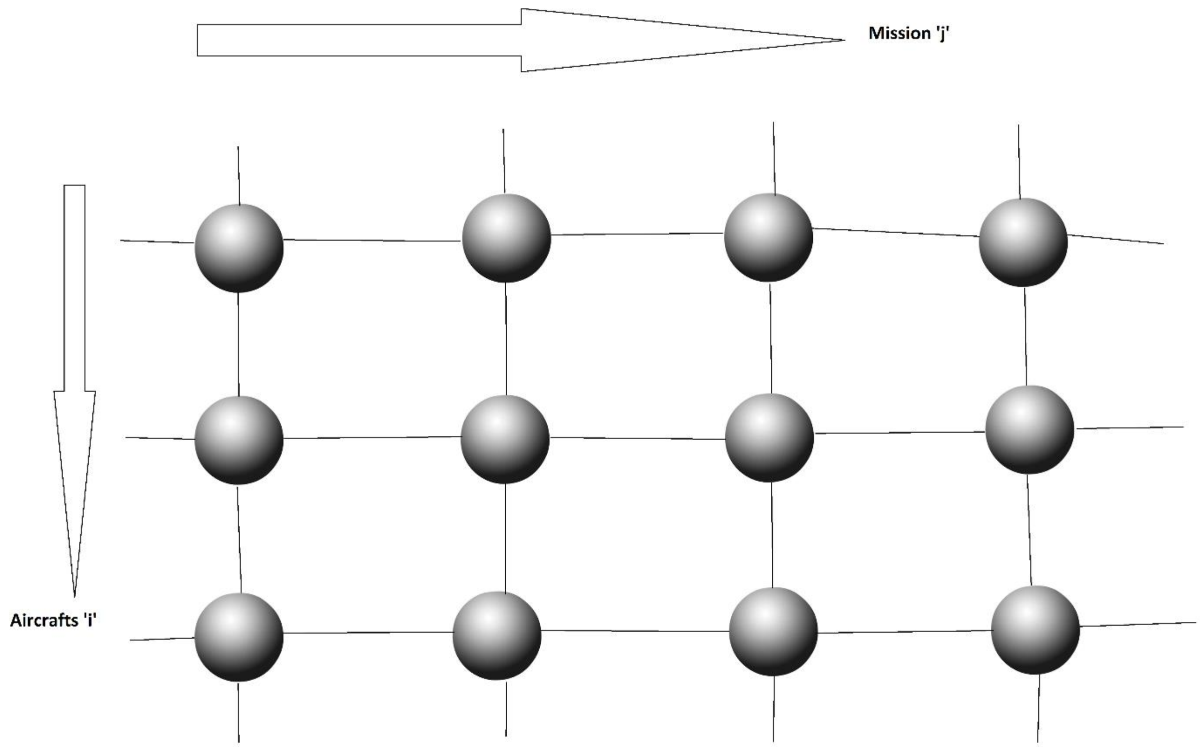
$s_i - \ starting\ remaining\ flight\ hours\ until\ maintenance\ for\ aircraft\ i$

$d_{jk} - be\ a\ binary\ variable\ indicating\ if\ mission\ j\ occurs\ in\ time\ period\ k$

$f_{ik} - be\ number\ of\ flight\ hours\ remaining\ before\ next\ maintennace\ is\ due$



$x_{ij} - be\ a\ binary\ variable\ indicating\ assignment\ of\ aircraft\ to\ mission\ j$



With the variables and input parameters defined we can construct constraints for flight operations as given below:

1. Only 1 aircraft assigned to each mission

$$\sum_{i}^{n} x_{ij} = 1 \qquad \forall\, mission\ j$$

2. Considering that each aircraft can only undergo maintenance once a year

$$\sum_{j}^{m} x_{ij} = 1 \qquad \forall\, aircraft\ i$$

3. To ensure an aircraft is not assigned to 2 separate mission that overlap during any particular time period k, we can multiply the assignment variable with the deployment schedule and ensure all their elements as less than or equal to 1, indicating no overlap of missions:

$$x_{ij} \times \ d_{jk} \ \leq 1 \qquad \forall \ aircraft \ i \ during \ time \ period \ k$$

4. We can have a constraint to incorporate the starting remaining flight hours as shown below:

   $$f_{i1} \ = s_i \qquad \forall \ aircraft \ i$$

5. To determine the remaining flight hours until maintenance is due, we can have the following flow balance constraint that reduces the remaining flight hours by b when it's not assigned to any mission and is not being maintained. Otherwise the remaining flight hours by a or 0 when it's on a mission or being maintained respectively.

   $$f_{ik} \ = f_{ik-1} - b + ((b-a) * \ x_{ij} \ \times \ \ d_{jk} \ ) \qquad \forall \ aircraft \ i$$

6. Once f_{ik} reaches, it undergoes maintenance and after p time periods from the start of maintenance, the remaining flight hours until next maintenance is reset to z

   To accomplish this without reducing model tractability, we consider the fact that there are finite time periods t during which a maintenance can start. Hence, we can uniquely identify every possible maintenance event from their starting time period. Let's call this variable mt_{lk} .We can also preprocess this variable to be 1 for the next (p-1) time periods indicating continuing maintenance operation.

   Lastly, to represent the fact that the remaining flight hours on each aircraft is reset to z after maintenance completion, we can create another binary variable mc_{lk} that is 1 for the time period k that is p time periods after the start of maintenance event l

   Lastly, because there cannot be a maintenance during mission progress, we have an opportunity to combine d_{jk} and mt_{lk} by appending mt_{lk} to d_{jk}. This would also require us to insert a j rows of 0s into mc_{lk} and we can call the resulting variable mc_{jk} .

7. Now we can modify equation 4, to include reset of remaining flight hours using variable mc_{lk} as shown below:

$$f_{ik} = f_{ik-1} - b + \left((b-a) * x_{ij} \times d_{jk}\right) + (x_{ij} \times mc_{jk} * (z + b)) \qquad \forall\, aircraft\ i$$

8. We define cost constraints and binary variables CZ_{ik} and CY_{ik} to capture the number of times we have a maintenance occurring too soon (maintenance starts with more than 10% of z remaining) or too late (maintenance starts after remaining hours goes beyond 10% of z below 0) as shown below:

$$f_{ik} + CZ_{ik} * (-1.1 * z) \leq 1.1 * z \qquad \forall\, aircraft\ i\ durng\ time\ period\ k$$

$$f_{ik} + CY_{ik} * (z) \geq -0.1 * z \qquad \forall\, aircraft\ i\ durng\ time\ period\ k$$

9. We also define a cost function CX_{ik} to capture the number of times multiple maintenances are taking place during same time period k,

$$CX_k = \sum_{i}^{n} x_{ij} \times mc_{jk} \qquad \forall\, time\ period\ k$$

10. Lastly since we combined maintenances with missions, and we only care about assigning an aircraft to each mission, and since it's not feasible to assign an aircraft to every possible maintenance start, we split the two as below and only restrict mandatory assignments to indexes below that of total number of missions as shown below:

$$\sum_{i}^{n} x_{ij} = 1 \qquad \forall\, mission\ j \ni j \leq m$$

$$\sum_{i}^{n} x_{ij} <= 1 \qquad \forall\, mission\ j \ni j > m$$

Combining all of these we have the following formulation of MILP:

Objective Function: Minimize overlapping, early and late maintenances:

Minimize:

$$\sum_{k}^{t}\sum_{i}^{n} CX_{ik} + CY_{ik} + CZ_{ik}$$

Subject to constraints:

$$\sum_{i}^{n} x_{ij} = 1 \qquad \forall\, mission\ j\ \ni j \leq m$$

$$\sum_{j}^{m} x_{ij} = 1 \qquad \forall\, aircraft\ i$$

$$x_{ij} \times d_{jk} \leq 1 \qquad \forall\, aircraft\ i\ during\ time\ period\ k$$

$$f_{i1} = s_i \qquad \forall\, aircraft\ i$$

$$f_{ik} = f_{ik-1} - b + \Big((b-a) * x_{ij} \times d_{jk}\Big) + (x_{ij} \times mc_{jk} * (z + b)) \qquad \forall\, aircraft\ i$$

$$CX_k = \sum_{i}^{n} x_{ij} \times mc_{jk} \qquad \forall\, time\ period\ k$$

$$f_{ik} + CY_{ik} * (z) \geq -0.1 * z \qquad \forall\, aircraft\ i\ durng\ time\ period\ k$$

$$f_{ik} + CZ_{ik} * (-1.1 * z) \leq 1.1 * z \qquad \forall\, aircraft\ i\ durng\ time\ period\ k$$

$$x_{ij}, CX_{ik}, CY_{ik}, CZ_{ik} -\ binary\ variables$$

$$f_{ik} \geq -0.1 * z$$

$$f_{ik} \leq 1.1 * z$$

Model built on AMPL and solved using CPLEX

We solve one instance of this problem using data provided:

```
n = 12              %number of aircraft
m = 15              %number of missions
t = 53              %number of time periods
p = 4;              %time taken for one maintenance cycle
z = 600;            %hours flown before maintenance
a = 11;             %hours flown per time period when on mission
b = 18;             %hours flown by an aircraft at home air station
```

s_i	
Acft Num	**Starting Flight Hours**
1	440
2	270
3	0
4	100
5	520
6	35
7	280
8	240
9	290
10	320
11	450
12	270

MISSION ID	START TIME PRD	END TIME PRD
1	1	16
2	3	13
3	8	16
4	9	22
5	14	23
6	14	27
7	18	28
8	21	32
9	24	37
10	25	32
11	26	37
12	31	42
13	34	44
14	37	46
15	41	50

AMPL model file:

```
set ACFT;
set MSSN;
set TPRD;
param d{MSSN,TPRD};
param mc{MSSN,TPRD};
param s{ACFT};

var x{i in ACFT, j in MSSN} binary;
var f{i in ACFT, k in TPRD};
var Cx{k in TPRD};
```

```
var Cy{i in ACFT,k in TPRD} binary;
var Cz{i in ACFT,k in TPRD} binary;


minimize Totalsum :  sum{i in ACFT,k in TPRD}(Cx[k]+Cy[i,k]+Cz[i,k]);

   subject to ACFTperMSSN {j in MSSN: j<16}      : sum{i in ACFT} x[i,j]              = 1;
   subject to MNTCperACFT {i in ACFT}            : sum{j in MSSN: j>15} x[i,j]        = 1;
   subject to ACFTonMSSN {i in ACFT,  k in TPRD} : sum{j in MSSN}(x[i,j] * d[j,k])  <= 1;

   subject to initREMFLHR {i in ACFT}            : f[i,1]    = s[i];

   subject to calcREMFLHR {i in ACFT, k in TPRD : k>1}:
                  f[i,k]      = f[i,k-1] - 18 + (sum{j in MSSN}(x[i,j] * mc[j,k])*618)
                                                + (7*(sum{j in MSSN}(x[i,j] * d[j,k]))) ;

   subject to COSTX      {k in TPRD} : Cx[k] = sum{i in ACFT}(sum{j in MSSN}(x[i,j] * mc[j,k]));
   subject to COSTZ      {i in ACFT,  k in TPRD}    : f[i,k]  + Cz[i,k]*(-660)    <= 660;
   subject to COSTY      {i in ACFT,  k in TPRD}    : f[i,k]  + Cy[i,k]*(660)     >= -60;
```

Results and discussions

The mixed integer linear program solved using CPLEX results in an objective function of 135.

We obtain an assignment of aircraft to missions and maintenances as shown below:

x[i,k]	Missions															Maintenances																
Acft	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
3	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
5	0	1	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
8	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
11	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

x[i,k]	Maintenances																															
Acft	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65
1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

A heatmap of the resulting remaining flight hours each time period for each aircraft is as shown below:

f[i,k]	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	440	422	404	386	368	350	332	314	296	278	260	242	224	206	188	170	152	134
2	270	252	234	216	198	180	162	144	126	108	90	72	54	36	18	0	-11	-22
3	0	-11	-22	-33	-44	556	538	527	516	505	494	483	472	461	450	439	421	403
4	100	82	64	46	28	10	-1	-12	-23	-34	566	548	530	512	494	476	458	447
5	520	502	491	480	469	458	447	436	425	414	403	392	381	370	359	348	337	326
6	35	17	-1	-12	-23	-34	-45	555	544	533	522	511	500	489	478	467	456	445
7	280	262	244	226	208	190	172	154	136	118	100	82	64	53	42	31	20	620
8	240	222	204	186	168	150	132	114	96	85	74	63	52	659	648	637	626	615
9	290	272	254	236	218	200	182	164	146	128	110	92	81	70	59	48	648	630
10	320	302	284	266	248	230	212	194	176	158	140	122	104	93	82	71	60	660
11	450	439	428	417	406	395	384	373	362	351	340	329	318	307	296	285	267	249
12	270	252	234	216	198	180	162	144	126	108	90	79	68	57	46	646	628	610

f[i,k]	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	116	98	87	76	65	54	654	636	618	600	582	564	546	528	510	492	474	456
2	-33	-44	556	538	520	502	484	466	448	430	412	394	376	358	340	322	304	286
3	385	367	349	331	313	302	291	280	269	258	247	236	225	214	203	192	181	170
4	436	425	414	403	392	381	370	359	348	337	319	301	290	279	268	257	246	235
5	315	304	293	282	271	253	242	231	220	209	198	187	176	165	147	129	111	93
6	434	423	412	401	383	365	347	336	325	314	303	292	281	270	259	248	237	226
7	602	584	566	548	530	512	494	476	458	440	422	404	386	368	350	332	314	296
8	604	593	582	571	560	549	538	527	516	498	480	462	444	426	408	390	372	354
9	612	594	576	558	540	522	504	486	468	450	432	414	396	378	360	342	324	306
10	642	624	606	588	570	552	534	516	498	480	462	444	426	408	390	372	354	336
11	231	213	202	191	180	169	158	147	136	125	114	103	92	81	63	52	41	30
12	592	574	556	538	520	502	484	466	448	430	412	394	376	358	340	322	304	286

f[i,k]	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53
1	438	420	402	384	366	348	330	312	294	276	258	240	222	204	186	168	150
2	268	250	232	214	196	178	160	142	124	106	88	70	52	34	16	-2	-20
3	159	141	123	105	94	83	72	61	50	39	28	17	6	-5	-23	-41	-59
4	224	213	202	191	180	169	151	133	115	97	79	61	43	25	7	-11	-29
5	82	71	60	49	38	27	16	5	-6	-17	-35	-53	-71	-82	-93	-104	-115
6	215	197	179	161	143	125	107	89	71	53	35	17	-1	-19	-37	-55	-73
7	278	260	242	224	206	188	170	152	134	116	98	80	62	44	26	8	-10
8	336	318	300	282	264	246	228	210	192	174	156	138	120	102	84	66	48
9	288	270	252	234	216	198	180	162	144	126	108	90	72	54	36	18	0
10	318	300	282	264	246	228	210	192	174	156	138	120	102	84	66	48	30
11	19	8	-3	-14	-25	-36	-47	-58	-76	-94	-112	-130	-148	-159	-170	-181	-192
12	268	250	232	214	196	178	160	142	124	106	88	70	52	34	16	-2	-20

We can identify points where maintenances are being completed resulting in “reset” of remaining flight hours. We can also see how many of these maintenances are being completed around the same time which could be resulting in instances of overlap as captured in our objective function value.

In this project we have considered a frequently encountered problem in aircraft maintenance scheduling and provide a Mixed integer linear programming formulation to solve the same. An objective function of 135, although optimal suggests that it’s violating the requirements of schedule at multiple places. This could very well be due to the problem instance and reassessing some of the parameters such as hours an aircraft is flown at home air station and during mission can be adjusted until an appropriate schedule is obtained.

Alternatively, we could also formulate this problem to allow variable flight hours during a mission and at home station. This level of granularity could yield better optimal solutions than obtained through the formulation shown here.

References

Samà, Marcella, Andrea D’Ariano, and Dario Pacciarelli. "Rolling horizon approach for aircraft scheduling in the terminal control area of busy airports." Procedia-Social and Behavioral Sciences 80 (2013): 531-552.

Sriram, Chellappan, and Ali Haghani. "An optimization model for aircraft maintenance scheduling and re-assignment." Transportation Research Part A: Policy and Practice 37.1 (2003): 29-48.

Feo, Thomas A., and Jonathan F. Bard. "Flight scheduling and maintenance base planning." Management Science 35.12 (1989): 1415-1432.