Chapter 1

Dimensional Analysis

All the physical quantities used in mechanics are originated from the three basis dimensions:

	SI units
Mass	kg
Length	m
Time	s

For example:	acceleration	$[LT^{-2}]$
	force	$[MLT^{-2}]$
	Energy	$[ML^2T^{-2}]$
	momentum	$[MLT^{-1}]$

For any equation to be held, dimension of right hand side must be equal to that of left hand side.

Example

We know a force is required to keep an object in circular motion. The force is dependent on the object mass, speed of the object and the radius of the motion.

Solution

$$F \sim m^a v^b r^c$$

$$\begin{aligned}[F] &= [m^a][v^b][r^c] \\ [\mathrm{MLT}^{-2}] &= [\mathrm{M}]^a[\mathrm{LT}^{-1}]^b[\mathrm{L}]^c \\ &= [\mathrm{M}]^a[\mathrm{L}]^{b+c}[\mathrm{T}]^{-b}\end{aligned}$$

$$\therefore\ a = 1,\ b = 2,\ c = -1,\quad F \sim \frac{mv^2}{r}.$$

Example

The frequency of vibration f of a mass m at the end of a spring that has a stiff constant k is related to m and k by a relation of the form $f = (\text{constant})m^a k^b$. Use dimensional analysis to find a and b. It is known that $[f] = [\mathrm{T}^{-1}]$ and $[k] = [\mathrm{MT}^{-2}]$.

Solution

Using dimensional analysis,

$$\begin{gathered}f \propto m^a k^b \ \Rightarrow\ [\mathrm{T}^{-1}] = [\mathrm{M}]^a[\mathrm{MT}^{-2}]^b = [\mathrm{M}^{a+b}\mathrm{T}^{-2b}] \\ \therefore\ a + b = 0 \ \text{ and } \ 2b = 1 \\ \Rightarrow\ a = -b = -\frac{1}{2}\end{gathered}$$