

THE KARNAUGH MAP

Section 4-8

37. Draw a 3-variable Karnaugh map and label each cell according to its binary values.

AB \ C 0 1

AB \ C	0	1
00	000	001
01	010	011
10	100	101
11	110	111

38. Draw a 4-variable Karnaugh map and label each cell according to its binary values.

AB \ CD 00 01 10 11

AB \ CD	00	01	10	11
00	0000	0001	0010	0011
01	0100	0101	0110	0111
10	1000	1001	1010	1011
11	1100	1101	1110	1111

39. Write the standard product term for each cell in a 3-variable Karnaugh map.

A \ B	C	0	1	...
0	0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	
1	0	$\bar{A}B\bar{C}$	$\bar{A}BC$	
1	1	$AB\bar{C}$	ABC	

KARNAUGH MAP SOP MINIMIZATION:-

Example 4-23:-

Map the following standard SOP expression on a Karnaugh map:

$$\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + AB\bar{C} + ABC$$

Solution:

Place a 1 on the 3-variable Karnaugh map for each standard product term in the expression:

$$\begin{array}{cccc} \bar{A}\bar{B}\bar{C} & \bar{A}\bar{B}\bar{C} & AB\bar{C} & ABC \\ 001 & 010 & 110 & 111 \end{array}$$

A	B	C	D	1	
0	0	0	0	1	$\rightarrow \bar{A}BC$
0	1	0	1	1	$\rightarrow \bar{A}\bar{B}\bar{C}$
1	0	1	0	1	$\rightarrow A\bar{B}\bar{C}$
1	1	1	1	1	$\rightarrow ABC$
				\downarrow	ABC

Example 4-24

Map the following standard SOP expression on a Karnaugh map.

$$\overline{AB}CD + \overline{A}\overline{B}\overline{C}\overline{D} + AB\overline{C}D + ABCD$$

$$+ AB\overline{C}\overline{D} + \overline{ABC}D + A\overline{B}C\overline{D}$$

Solution:

$$\overline{AB}CD + \overline{A}\overline{B}\overline{C}\overline{D} + AB\overline{C}D$$

$$0011 \quad 0100 \quad 1101$$

$$ABCD + ABC\overline{D} + \overline{ABC}D + A\overline{B}C\overline{D}$$

$$1111 \quad 1100 \quad 0001 \quad 1010$$

$$AB \backslash CD \quad 00 \quad 01 \quad 10 \quad 11$$

A	B	00	01	10	11
0	0		1		1
0	1	1			
1	0			1	
1	1	1	1		1

Example 4-25

Map the following SOP expression

on a Karnaugh map

$$\bar{A} + A\bar{B} + AB\bar{C}$$

Solution:

\bar{A}	$A\bar{B}$	$AB\bar{C}$
000	100	110
001	101	
010		
011		

A	B	C	O	1
00	1		1	
01	1		1	
10	1		1	
11	1			

Example 4-26

Map the following SOP expression
on a Karnaugh map.

$$\begin{aligned} & \bar{B}\bar{C} + A\bar{B} + ABC\bar{C} + A\bar{B}C\bar{D} + \bar{A}BCD \\ & + A\bar{B}CD \end{aligned}$$

Solution:

$\bar{B}\bar{C}$	$A\bar{B} + \bar{A}B\bar{C}$	$A\bar{B}\bar{C} + A\bar{B}C\bar{D}$	
0000	1000	1100	1010
0001	1001	1101	
1000	1010		
1001	1011		

$$\bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}CD$$

⋮

0001	1011
------	------

AB	CD	00	01	10	11
00	1	1			
01					
10	1	1	1	1	1
11	1	1			

Section 4-9

40. Use a Karnaugh map to find minimum SOP form for each expression

a) $A\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C}$

$$\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C}$$

000	001	101
-----	-----	-----

\bar{A}	B	C	0	1
0	0		1	1
0	1			
1	0			1
1	1			

b) $AC(\bar{B} + C)$

$$\Rightarrow A\bar{B}C + AC$$

101 101

111

\bar{A}	B	C	0	1
0	0			
0	1			
1	0			1
1	1			1

c) $\bar{A}(BC + B\bar{C}) + A(BC + B\bar{C})$

$$\Rightarrow \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C}$$

011 010 111 110

\bar{A}	B	C	0	1
0	0			
0	1		1	1
1	0			
1	1		1	1

$$d) \quad \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + AB\bar{C}$$

$$\Rightarrow \begin{array}{cccc} \bar{A}\bar{B}\bar{C} & + & A\bar{B}\bar{C} & + \bar{A}\bar{B}\bar{C} & + AB\bar{C} \\ 000 & & 100 & & 010 \\ & & & & 110 \end{array}$$

~~A B C~~ 0 1

00	1	
01	1	
10	1	
11	1	

41. Use a Karnaugh map to simplify each expression to a minimum SOP form:

$$a) \quad \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + AB\bar{C}$$

$$\bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + AB\bar{C}$$

$$000 \quad 101 \quad 011 \quad 110$$

~~A B C~~ 0 1

00	(1)	
01		(1)
11	(1)	
10		(1)

NO Simplification

b) $AC[\bar{B} + B(B + \bar{C})]$

$$\Rightarrow AC(\bar{B} + B + B\bar{C})$$

$$\Rightarrow A\bar{B}C + ABC + AB\bar{C}\bar{C}$$

$$\Rightarrow A\bar{B}C + ABC$$

101 111

A B C O 1

00

01

11

10

(1)

(1)

$$x = AC$$

c) $DEF + \bar{D}EF + \bar{D}\bar{E}F$

$$\Rightarrow DEF + \bar{D}EF + \bar{D}\bar{E}F$$

110 010 000

D E F O 1

00

01

11

10

(1)

(1)

(1)

$$x = \bar{D}\bar{E} + EF$$

42. Expand each expression to a standard SOP form:

a) $AB + A\bar{B}C + ABC$

$$\Rightarrow AB(C + \bar{C}) + A\bar{B}C + ABC$$

$$\Rightarrow ABC + AB\bar{C} + A\bar{B}C + A\cancel{BC}$$

$$\Rightarrow ABC + AB\bar{C} + A\bar{B}C$$

b) $A + BC$

$$\Rightarrow A(B + \bar{B}) + BC(A + \bar{A})$$

$$\Rightarrow AB(C + \bar{C}) + A\bar{B}(C + \bar{C}) + ABC + \bar{A}BC$$

$$\Rightarrow ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \cancel{ABC} + \bar{A}BC$$

$$\Rightarrow ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$$

c) $A\bar{B}\bar{C}D + AC\bar{D} + B\bar{C}D + \bar{A}BC\bar{D}$

$$\Rightarrow A\bar{B}\bar{C}D + AC\bar{D}(B + \bar{B}) + B\bar{C}D(A + \bar{A}) + * \bar{A}BC\bar{D}$$

$$\Rightarrow A\bar{B}\bar{C}D + ABC\bar{D} + A\bar{B}C\bar{D} + AB\bar{C}D + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D}$$

d) $A\bar{B} + A\bar{B}\bar{C}D + CD + B\bar{C}D$
+ $ABCD$

$$\rightarrow A\bar{B} (C + \bar{C}) + A\bar{B}\bar{C}D + CD (A + \bar{A})$$
$$+ B\bar{C}D (A + \bar{A}) + ABCD$$

$$\rightarrow A\bar{B}C (D + \bar{D}) + A\bar{B}\bar{C}(D + \bar{D}) + A\bar{B}\bar{C}D$$
$$+ ACD (B + \bar{B}) + \bar{A}CD (B + \bar{B})$$
$$+ AB\bar{C}D + \bar{A}B\bar{C}D + ABCD$$

$$\Rightarrow A\bar{B}CD + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D}$$
$$+ A\bar{B}\bar{C}D + ABCD + A\bar{B}\bar{C}D +$$
$$\bar{A}BCD + \bar{A}B\bar{C}D + AB\bar{C}D + \bar{A}B\bar{C}D$$
$$+ ABCD$$

$$\Rightarrow A\bar{B}CD + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D +$$
$$\bar{A}\bar{B}\bar{C}D + ABCD + \bar{A}BCD +$$
$$\bar{A}\bar{B}CD + AB\bar{C}D + \bar{A}B\bar{C}D$$

43. Minimize each expression in Problem 42 with a Karnaugh map.

$$ABC + AB\bar{C} + A\bar{B}C$$

$$ABC + AB\bar{C} + A\bar{B}C$$

111

110

101

AB \ C	0	1
00		
01		
11	1	1
10		1

$$X = AB + AC$$

$$ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$$

$$ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$$

111

110

101

100

011

AB \ C	0	1
00		
01		1
11	1	1
10	1	1

$$X = A + BC$$

$$\bar{A}\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D$$

$$+ \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD$$

$$\bar{A}\bar{B}CD + A\bar{B}CD + A\bar{B}\bar{C}\bar{D} +$$

$$1001 \quad 1110 \quad 1010$$

$$AB\bar{C}D + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD$$

$$1101 \quad 0101 \quad 0110$$

AB	CD	00	01	11	10
00					
01					
11			(1) (1)		
10		(1)			(1) (1)

$$X = B\bar{C}D + A\bar{C}D + BC\bar{D} + ACD$$

$$\bar{A}\bar{B}CD + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + A\bar{B}CD +$$

$$ABC\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}CD + AB\bar{C}D$$

$$+ \bar{A}\bar{B}\bar{C}D$$

$$\bar{A}\bar{B}CD + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D +$$

$$1011 \quad 1010 \quad 1001$$

$$\bar{A}\bar{B}CD + ABCD + \bar{A}\bar{B}CD +$$

$$1000 \quad 1111 \quad 0111$$

$$\bar{A}\bar{B}CD + AB\bar{C}D + \bar{A}\bar{B}\bar{C}D$$

$$0011 \quad 1101 \quad 0101$$

AB	CD	00	01	11	10
00				1	
01			1		1
11		(1)		1	
10		1	1	1	1

$$X = A\bar{B} + \bar{C}D + B\bar{C} + \bar{C}\bar{D}$$

KARNAUGH MAP POS MINIMIZATION:-

Example 4-33

Map the following standard POS expression on a Karnaugh map:

$$(\bar{A} + \bar{B} + C + D)(\bar{A} + B + \bar{C} + \bar{D})$$

$$(A + B + \bar{C} + D)(\bar{A} + \bar{B} + \bar{C} + \bar{D})(A + B + \bar{C} + \bar{D})$$

Solution:

$$(\bar{A} + \bar{B} + C + D) \quad (\bar{A} + B + \bar{C} + \bar{D})$$

1100 1011

$$(A + B + \bar{C} + D) \quad (\bar{A} + \bar{B} + \bar{C} + \bar{D})$$

0010 1111

$$(A + B + \bar{C} + \bar{D})$$

0011

AB	CD	00	01	11	10
00				0	1
01				0	
11		0		0	
10					

Example 4-34

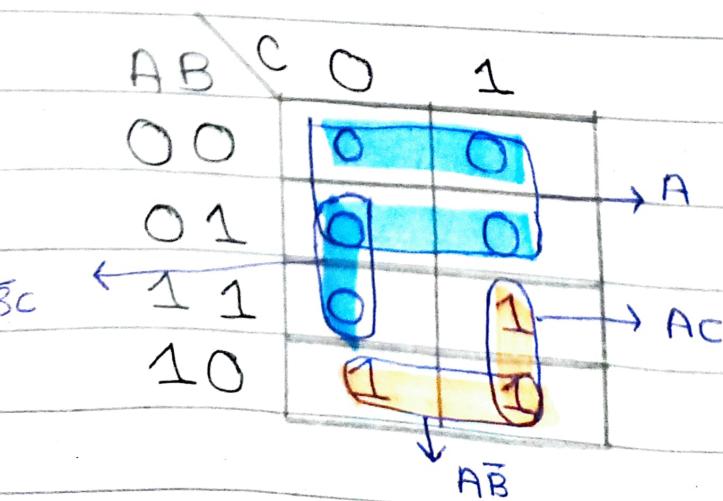
Use a Karnaugh map to minimize the following standard POS expression:

$$(A+B+C)(A+B+\bar{C})(A+\bar{B}+C) \\ (A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$$

Also, derive equivalent SOP expression,

Solution:

$$(0+0+0)(0+0+1)(0+1+0) \\ (0+1+1)(1+1+0)$$



Resulting minimum POS expression is
 $\Rightarrow A(\bar{B}C)$

Grouping the 1's yields an SOP expression

$$\Rightarrow AC + A\bar{B}$$

$$\Rightarrow A(\bar{B} + C)$$

Example 4-35

Use a Karnaugh map to minimize following POS expression:

$$(B+C+D)(A+B+\bar{C}+D)(\bar{A}+B+C+\bar{D})$$

$$(A+\bar{B}+C+D)(\bar{A}+\bar{B}+C+D)$$

Solution:

The first term must be expanded into $\bar{A}+B+C+D$ and $A+B+C+D$ to get a standard POS expression

$$\Rightarrow (\bar{A}+B+C+D)(A+B+C+D)(A+B+\bar{C}+D)$$

1000 0000 0010

$$(\bar{A}+B+C+\bar{D})(A+\bar{B}+C+D)(\bar{A}+\bar{B}+C+D)$$

1001 0100 1100

AB	CD	00	01	11	10	
00	0					\bar{C} $\rightarrow A+B+D$
01	0					
11	0					
10	0	0	0			

$C+D$ $\rightarrow \bar{A}+B+C$

The resulting sum term for each group is minimum POS expression
 $\Rightarrow (C+D)(A+B+D)(\bar{A}+B+C)$.

Example 4-36

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression and a minimum SOP expression

$$(\bar{A}+\bar{B}+C+D)(A+\bar{B}+C+D)(A+B+C+\bar{D}) \\ (A+B+\bar{C}+\bar{D})(\bar{A}+B+C+\bar{D})(A+B+\bar{C}+D)$$

Solution:

AB	CD	00	01	11	10
00	0	0	0	0	
01	0				
11	0				
10		0			

Minimum POS:

$$(A + B + \bar{C})(\bar{B} + \bar{C} + D)(B + C + \bar{D})$$

AB	000	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1

Standard SOP:

$$\overline{ABCD} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \\ ABC\bar{D} + A\bar{B}C\bar{D} + \overline{ABC\bar{D}} + A\bar{B}\bar{C}D + \\ A\bar{B}CD + ABCD$$

Minimum SOP:

$$AC + BC + BD + \bar{B}\bar{C}\bar{D}$$

Section 4-10

48. Use a Karnaugh map to find the minimum POS for each expression:

a) $(A+B+C)(\bar{A}+\bar{B}+\bar{C})(A+\bar{B}+C)$

(000) (111) (010)

~~AB~~ C 0 1

00	0	
01	0	
11		0
10		

$$X = A+C$$

b) $(X+\bar{Y})(\bar{X}+Z)(X+\bar{Y}+\bar{Z})(\bar{X}+\bar{Y}+Z)$

$$\Rightarrow (X+\bar{Y}+Z)(X+\bar{Y}+\bar{Z})(\bar{X}+Y+Z)$$

$$(\bar{X}+\bar{Y}+Z)(X+\bar{Y}+\bar{Z})(\bar{X}+\bar{Y}+Z)$$

$$\Rightarrow (X+\bar{Y}+Z)(X+Y+\bar{Z})(\bar{X}+Y+Z)$$

010

001

100

$$(\bar{X}+\bar{Y}+Z)$$

110

XY	Z	0	1
00		0	
01		0	
11		0	
10		0	

$$X = (\bar{X} + Z)(\bar{X} + \bar{Z})$$

49. Use a Karnaugh map to simplify each expression to minimum POS form.

a) $(A + \bar{B} + C + \bar{D})(\bar{A} + B + \bar{C} + D)$
 $(\bar{A} + \bar{B} + \bar{C} + \bar{D})$

$$(0101) (1010) (1111)$$

AB	CD	00	01	11	10
00					
01			0		
11				0	
10					0

No ~~exp~~ minimization.

b) $(x + \bar{y})(w + \bar{z})(\bar{x} + \bar{y} + \bar{z})(w + x + y + z)$

 $\Rightarrow (w + x + \bar{y})(\bar{w} + x + \bar{y})(w + x + \bar{z})$
 $(w + \bar{x} + \bar{z})(w + \bar{x} + \bar{y} + \bar{z})(\bar{w} + \bar{x} + \bar{y} + \bar{z})$
 $(w + x + y + z)$
 $\Rightarrow (w + x + \bar{y} + z)(\bar{w} + x + \bar{y} + \bar{z})(w + x + y + \bar{z})$
 $(w + x + y + z)(\bar{w} + x + \bar{y} + z)(\bar{w} + x + \bar{y} + \bar{z})$
 $(w + \bar{x} + y + \bar{z})(w + \bar{x} + \bar{y} + \bar{z})$
 $(w + \bar{x} + \bar{y} + \bar{z})(\bar{w} + \bar{x} + \bar{y} + \bar{z})(w + x + y + z)$

$(0010)(1011)(0001)(0000)$

$(1010)(\cancel{1011})(0101)(0111)$

$(\cancel{01})(1111)$

wx	vz	00	01	11	10
00	0	0	0	0	0
01		0	0	0	
11			0	0	
10			0	0	0

$X = (w + x + y)(w + y + \bar{z})(w + \bar{x} + \bar{z})$
 $(\bar{x} + \bar{y} + \bar{z})(\cancel{w} + \bar{y} + \bar{z})$
 $(\bar{w} + x + \bar{y})$

52.

Convert following POS expressions to minimum SOP expression using Karnaugh map.

a) $(A + \bar{B})(A + \bar{C})(\bar{A} + \bar{B} + C)$
 $\Rightarrow (\bar{A} + \bar{B} + C)(A + \bar{B} + \bar{C})(A + B + \bar{C})$
 ~~$(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + C)$~~
 $(010)(011)(001)(110)$

A	B	C	0	1
00		1	0	
01		0	0	
11		0	1	
10		1	1	

Standard SOP expression is:
 $\Rightarrow \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC$

Minimum SOP expression is:
 $\Rightarrow X = AC + \bar{B}\bar{C}$

b) $(\bar{A}+B)(\bar{A}+\bar{B}+\bar{C})(B+\bar{C}+D)$
 $(A+\bar{B}+C+\bar{D})$

$\Rightarrow (\bar{A}+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C+D)$
 $(\bar{A}+\bar{B}+\bar{C}+D)(A+B+\bar{C}+D)$
 $(\bar{A}+B+\bar{C}+D)(A+\bar{B}+C+D)$

$\Rightarrow (\bar{A}+B+C+D)(\bar{A}+B+C+\bar{D})$
 $(\bar{A}+B+\bar{C}+D)(\bar{A}+B+\bar{C}+\bar{D})$
 $(\bar{A}+\bar{B}+\bar{C}+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})$
 $(A+B+\bar{C}+D)(\bar{A}+B+\bar{C}+D)$
 $(A+\bar{B}+C+\bar{D})$

$$(1000)(1001)(1010)(1011)$$

$$(1110)(1111)(0010)(0101)$$

AB	CD	00	01	11	10
00	1	1	1		0
01	1	0	1		1
11	1	1	0		0
10	0	0	0		0

$$X = \bar{A}\bar{C}D + B\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}D$$

$$+ A\bar{B}\bar{C} + \bar{A}CD + \bar{A}BC$$