Department of Computer Science,
University of Karachi
BSCS: 306: Probability and Statistical Methods
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Assignment: 03: Exponential Distribution

Q 1. The mileage (in thousands of miles) which car owner get with certain kind of tire is a random variable having the exponential pdf.

$$f(x) = \begin{cases} \dfrac{1}{20} e^{-x/20} & , x > 0 \\ 0 & , x \le 0. \end{cases}$$

Find the probabilities that one of these tires will last.
(1).   At most 12,000 miles.
(2).   No less than 18,000 or no more than 24,000 miles.
(3).   At least 30,000 miles.

Q 2. Compute E(X), V(X), F(x) and $P(4.2 < X < 4.7)$. for the exponential pdf
$$f(x) = 4e^{-4x}, x \ge 0$$

Q 3. If $x \sim U(0,2)$ and $y \sim \exp(\lambda)$ with mean $\dfrac{1}{\lambda}$. Find the value of λ for which

$P\left(X < \dfrac{3}{4}\right) = P(Y < 1)$. (Hint: calculate the two sides separately and then solve for λ.)

Q 4. The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. (i). What is the probability that a person is served in less than 3 minutes. (ii). What is the probability that the person is served in less than 3 minutes on at least 4 of the next 6 days. (Hint: for (ii) use n=6, $y \ge 5$ and p= P(x<3) form part (i).

Q 5. Computer response time is an important application of the gamma and exponential distribution. Suppose that a study of a certain computer system reveals that the response time in seconds has an exponential distribution with mean 3 seconds.

(1).   What is the probability that response time exceeds 5 seconds?
(2).   What is the probability that response time exceeds 10 seconds?

Q 6. The mean number of automobiles that arrives at a certain intersection per minute has a Poisson distribution with a mean of 10. Interest centers around the time that elapses before 15 automobiles appear at the intersection. (Use $\beta = \frac{1}{10}$ for Exponential distribution). (i). What is the probability that more than 15 automobiles appear at the intersection during any given minute of time? (ii) What is the probability that more than 1 minute elapses between arrivals? (iii). What is the mean number of minutes that elapse between arrivals?

Q 7. The exponential distribution is frequently applied to the waiting times between successes in a Poisson process. If the number of calls received per hour by a telephone answering service is a Poisson random variable with $\lambda = 6$, we know that the time, in hours, between successive calls has an exponential distribution with parameter $\beta = \frac{1}{6}$. What is the probability of waiting more than 15 minutes between any two successive calls?