CHAPTER: 4

Boolean Algebra and Logic Simplification

BOOLEAN OPERATIONS AND EXPRESSIONS :-

Example 4-1 :

Determine the values of A, B, C and D that make the sum term $A+\overline{B}+C+\overline{D}$ equal to 0.

Solution:

For the sum term to be 0, each of the literal in the term must be 0.

Therefore,

$A=0$, $B=1$ so $\overline{B}=0$

$C=0$ and $D=1$ so $\overline{D}=0$

$$\Rightarrow A+\overline{B}+C+\overline{D} = 0+\overline{1}+0+\overline{1}$$

$$= 0+0+0+0$$

$$= 0$$

Related Problem

Determine the values of A and B that make the sum term $\overline{A}+B$ equal to 0

Solution:

To make sum term 0 each of literal must be 0. Therefore,

$A = 1$ so $\overline{A} = 0$ and $B = 0$

$$\Rightarrow \overline{A}+B = \overline{1}+0$$
$$= 0+0$$
$$= 0$$

Example 4-2

Determine the values of A, B, C and D that make the product term $A\overline{B}C\overline{D}$ equal to 1.

Solution:

For the product term to be 1, each of the literals in the term must be 1. Therefore, $A = 1$, $B = 0$ so $\overline{B} = 1$, $C = 1$ and $D = 0$ so $\overline{D} = 1$

$$\Rightarrow A\overline{B}C\overline{D} = 1.\overline{0}.1.\overline{0}$$
$$= 1.1.1.1$$
$$= 1$$

Related Problem

Determine the values of A and B that make the product term $\overline{A}.\overline{B}$ equal to 1.

Solution:

To make product term to be 1, each of the literals must be 1.

Therefore, $A = 0$ so $\overline{A} = 1$

$B = 0$ so $\overline{B} = 1$

$$\Rightarrow \overline{A}\,\overline{B} = \overline{0}.\overline{0}$$
$$= 1.1$$
$$= 1$$

Section 4-1

1. Using Boolean notation, write an expression that is a 0 only when all of its variables (A, B, C and D) are 0s.

Solution:

In Boolean addition, the sum term is 0 when all of its variables are 0.

$$\Rightarrow X = A + B + C + D$$
$$0 = 0 + 0 + 0 + 0$$

2. Write an expression that is a 1 when one or more of its variable (A, B, C, D and E) are 0s.

Solution:

$\Rightarrow X = A + B + C + D + E$

Boolean addition expression.

3. Write an expression that is a 0 when one or more of its variables (A, B and C) are 0s.

Solution:

$\Rightarrow Y = A \cdot B \cdot C$

Boolean Multiplication expression.

4. Evaluate the following operations:

a) $0+0+0+0$

$\Rightarrow 0+0+0+0=0$

b) $0+0+0+1$

$\Rightarrow 0+0+0+1=1$

c) $1+1+1+1$

$\Rightarrow 1+1+1+1=1$

d) $1.1+0.0+1$

$\Rightarrow 1+0+1=1$

e) $1.0.1.0$

$\Rightarrow 1.0.1.0=0$

f) $1.0+1.0+0.1+0.1$

$\Rightarrow 0+0+0+0=0$

5. Find the values of the variables that make each product term 1 and each sum term 0.

a) ABC

To make product term 1 each of the variables must be 1. Therefore,

$\Rightarrow A = 1$, $B = 1$ and $C = 1$

b) $A + B + C$

To make sum term term 0 ~~sar~~

$\Rightarrow A + B + C = 0 + 0 + 0$

$= 0$

c) $\bar{A}\bar{B}C$

To make product term 1

$\Rightarrow A = 0$ so $\bar{A} = 1$

$\Rightarrow B = 0$ so $\bar{B} = 1$

$\Rightarrow C = 1$

$\Rightarrow \bar{A}\bar{B}C = \bar{0} . \bar{0} . 1$

$= 1.1.1$

$= 1$

d) $\bar{A} + \bar{B} + C$

To make sum term 0

$\Rightarrow A = 1$ so $\bar{A} = 0$

$B = 1$ so $\bar{B} = 0$

$C = 0$

$\Rightarrow \bar{A} + \bar{B} + C = \bar{1} + \bar{1} + 0$

$= 0 + 0 + 0 = 0$

e) $A + \bar{B} + \bar{C}$

To make sum term 0

$A = 0$, $B = 1$ so $\bar{B} = 0$

$C = 1$ so $\bar{C} = 0$

$\Rightarrow A + \bar{B} + \bar{C} = 0 + \bar{1} + \bar{1}$

$= 0 + 0 + 0 = 0$

f) $\bar{A} + \bar{B} + \bar{C}$

To make sum term 0

$A = 1$ so $\bar{A} = 0$

$B = 1$ so $\bar{B} = 0$

$C = 1$ so $\bar{C} = 0$

$\Rightarrow \bar{A} + \bar{B} + \bar{C} = \bar{1} + \bar{1} + \bar{1}$

$= 0 + 0 + 0 = 0$

6. Find the value of X for all possible values of the variables.

a) $X = A + B + C$

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

b) $X = (A + B)C$

A	B	C	A+B	X
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

c) $X = (A+B)\,(\overline{B+C})$

A	B	C	A+B	B+C	$\overline{B+C}$	X
0	0	0	0	0	1	0
0	0	1	0	1	0	0
0	1	0	1	1	0	0
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	0
1	1	1	1	1	0	0

d) $X = (A+B) + (\overline{AB+BC})$

A	B	C	A+B	AB	BC	AB+BC	$\overline{AB+BC}$	X
0	0	0	0	0	0	0	1	1
0	0	1	0	0	0	0	1	1
0	1	0	1	0	0	0	1	1
0	1	1	1	0	1	1	0	1
1	0	0	1	0	0	0	1	1
1	0	1	1	0	0	0	1	1
1	1	0	1	1	0	1	0	1
1	1	1	1	1	1	1	0	1

e) $X = (\bar{A}+\bar{B})(A+B)$

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}+\bar{B}$	A+B	X
0	0	1	1	1	0	0
0	1	1	0	1	1	1
1	0	0	1	1	1	1
1	1	0	0	0	1	0

Section 4-2

LAWS AND RULES OF BOOLEAN ALGEBRA:-

7. Identify the law of Boolean algebra upon which each of the following equalities is based:

a) $A + AB + ABC + \overline{ABCD} = \overline{ABCD} + ABC + AB + A$

Commutative law of Addition.

b) $A + \overline{AB} + ABC + \overline{ABCD} = \overline{DCBA} + CBA + \overline{BA} + A$

Commutative law of Multiplication and Addition.

c) $AB(CD + \overline{CD} + EF + \overline{EF}) = ABCD + AB\overline{CD} + ABEF + AB\overline{EF}$

Distributive law.

8. Identify the Boolean rule(s) on which each of the following equalities is based.

a) $\overline{\overline{AB+CD}} + EF = AB + CD + \overline{\overline{EF}}$

Rule : 9

$\Rightarrow \overline{\overline{A}} = A$

b) $A\overline{A}B + AB\overline{C} + AB\overline{B} = AB\overline{C}$

Rule 8 $\Rightarrow A \cdot \overline{A} = 0$

Applied on 1st and 3rd term.

c) $A(BC + BC) + AC = A(BC) + AC$

Rule 5

$\Rightarrow A + A = A$

d) $AB(C+\overline{C}) + AC = AB + AC$

Rule 6

$\Rightarrow A + \overline{A} = 1$

e) $\cancel{A(BC+BC)}$ $A\overline{B} + A\overline{B}C = A\overline{B}$

Rule 10

$\Rightarrow A + AB = A$

f) $ABC + \overline{AB} + \overline{ABC}D = ABC + \overline{AB} + D$

Rule 11 $A + \overline{A}B = A + B$

Applied to 1'st and 3rd values.

DE MORGAN'S THEOREMS :-

Example 4-3

Apply De Morgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X+Y+Z}$

Solution:

$\Rightarrow \overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$

$\Rightarrow \overline{X+Y+Z} = \overline{X}\,\overline{Y}\,\overline{Z}$

Example 4-4

Apply De Morgan Theorem to the expressions $\overline{WXYZ}$ and $\overline{W+X+Y+Z}$

Solution:

$\Rightarrow \overline{WXYZ} = \overline{W} + \overline{X} + \overline{Y} + \overline{Z}$

$\Rightarrow \overline{W+X+Y+Z} = \overline{WXYZ}$

Example 4-5

Apply De Morgan Theorem to each of the following expressions:

a) $\overline{(A+B+C)D}$

Solution

Let $A+B+C = X$ and $D = Y$. The expression $\overline{(A+B+C)D}$ is of the form $\overline{XY} = \overline{X} + \overline{Y}$ and can be rewritten as:

$$\Rightarrow \overline{(A+B+C)D} = \overline{A+B+C} + \overline{D}$$

Apply DeMorgan Law to the term $\overline{A+B+C} + \overline{D}$

$$\Rightarrow \overline{A+B+C} + \overline{D} = \overline{A}\,\overline{B}\,\overline{C} + \overline{D}$$

b) $\overline{ABC + DEF}$

Solution:

Let $ABC = X$ and $DEF = Y$. The expression $\overline{ABC + DEF}$ is one of the forms $\overline{X+Y} = \overline{X}\,\overline{Y}$ and can be rewritten as

$$\overline{ABC + DEF} = (\overline{ABC})(\overline{DEF})$$

Apply De Morgan's theorem to each of the terms $\overline{ABC}$ and $\overline{DEF}$

$$\Rightarrow (\overline{ABC})(\overline{DEF}) = (\bar{A}+\bar{B}+\bar{C})(\bar{D}+\bar{E}+\bar{F})$$

c) $\overline{A\bar{B}+\bar{C}D+EF}$

Solution:

Let $A\bar{B}=X$, $\bar{C}D=Y$ and $EF=Z$.

The expression $\overline{A\bar{B}+\bar{C}D+EF}$ is of the form $\overline{X+Y+Z}=\bar{X}\,\bar{Y}\,\bar{Z}$ and can be rewritten as

$$\Rightarrow \overline{A\bar{B}+\bar{C}D+EF} = (\overline{A\bar{B}})(\overline{\bar{C}D})(\overline{EF})$$

Apply De Morgan's theorem to each of the terms $\overline{A\bar{B}}$, $\overline{\bar{C}D}$ and $\overline{EF}$.

$$\Rightarrow (\overline{A\bar{B}})(\overline{\bar{C}D})(\overline{EF}) = (\bar{A}+B)(C+\bar{D})(\bar{E}+\bar{F})$$

Example 4-6

Apply DeMorgan's theorems to each expression:

a) $\overline{(\overline{A+B})+\bar{C}}$

Solution:

$$\Rightarrow (\overline{\overline{A+B}})\bar{\bar{C}} = (A+B)C$$

b) $\overline{(\bar{A}+B)+CD}$

Solution:

$\Rightarrow \overline{(\bar{A}+B)}\;\overline{CD} = (\bar{\bar{A}}\bar{B})(\bar{C}+\bar{D})$

$= A\bar{B}(\bar{C}+\bar{D})$

c) $\overline{(A+B)\bar{C}\bar{D}+E+\bar{F}}$

Solution:

$\Rightarrow \overline{((A+B)\bar{C}\bar{D})}\;(\overline{E+\bar{F}})$

$= (\bar{A}\bar{B}+C+D)\bar{E}F$

Example 4-7

The Boolean expression for an exclusive-OR gate is $A\bar{B}+\bar{A}B$. With this as a starting point, use DeMorgan's theorems and any other rules or laws that are applicable to develop an expressive for the exclusive-NOR gate.

Solution:

Start by complementing the exclusive-OR expression and then applying DeMorgan's theorem as follow:

$\Rightarrow \overline{A\bar{B}+\bar{A}B} = (\overline{A\bar{B}})(\overline{\bar{A}B})$

$= (\bar{A}+\bar{\bar{B}})(\bar{\bar{A}}+\bar{B})$

$= (\bar{A}+B)(A+\bar{B})$

Next, apply the distributive law and rule 8 ($A.\bar{A} = 0$)

$\Rightarrow (\bar{A}+B)(A+\bar{B}) =$

$\bar{A}A + \bar{A}\bar{B} + AB + B\bar{B} = \bar{A}\bar{B} + AB$

The final expression for the XNOR is $\bar{A}\bar{B} + AB$. Note that this expression equals 1 any time both variables are 0s or both variables are 1s.

Section 4-3

9. Apply De Morgan's theorems to each expression:

a) $\overline{A+\bar{B}}$

$\Rightarrow \overline{A+\bar{B}} = \bar{A}\bar{\bar{B}} = \bar{A}B$

b) $\overline{\bar{A}B}$

$\Rightarrow \overline{\bar{A}B} = \bar{\bar{A}} + \bar{B} = A+\bar{B}$

c) $\overline{A+B+C}$

$\Rightarrow \overline{A+B+C} = \bar{A}.\bar{B}.\bar{C}$

d) $\overline{ABC}$

$\Rightarrow \bar{A} + \bar{B} + \bar{C}$

e) $\overline{A(B+C)}$

$\Rightarrow \bar{A} + \overline{(B+C)} = \bar{A} + \bar{B}\bar{C}$

f) $\overline{AB} + \overline{CD}$

$\Rightarrow \bar{A} + \bar{B} + \bar{C} + \bar{D}$

g) $\overline{AB + CD}$

$\Rightarrow \overline{(AB)}\,\overline{(CD)} \Rightarrow (\bar{A}+\bar{B})(\bar{C}+\bar{D})$

h) $\overline{(A+\bar{B})(\bar{C}+D)}$

$\Rightarrow \overline{(A+\bar{B})\,(\bar{C}+D)} = \overline{A+\bar{B}} + \overline{\bar{C}+D}$

$= \bar{A}B + C\bar{D}$

10. Apply De Morgan's theorems to each expression.

a) $\overline{A\bar{B}(C+\bar{D})}$

$= \overline{A\bar{B}} + \overline{(C+\bar{D})} = \bar{A} + B + \bar{C}D$

b) $\overline{AB(CD+EF)}$

$= \overline{AB} + \overline{(CD+EF)}$

$= \bar{A} + \bar{B} + \overline{(CD)}\,\overline{(EF)}$

$= \bar{A} + \bar{B} + (\bar{C}+\bar{D})(\bar{E}+\bar{F})$

c) $\overline{(A+\overline{B}+C+\overline{D})} + \overline{ABC\overline{D}}$

$= \overline{A\overline{B}C\overline{D}} + \overline{A} + \overline{B} + \overline{C} + \overline{\overline{D}}$

$= \overline{A}B\overline{C}D + \overline{A} + \overline{B} + \overline{C} + D$

d) $\overline{\overline{(\overline{A}+B+C+D)}\,(A\overline{B}\overline{C}D)}$

$= \overline{(\overline{A}BCD)(A+\overline{B}+\overline{C}+D)}$

$\Rightarrow \overline{A\overline{B}CD} + \overline{A}+B+C+\overline{D}$

$= \overline{A}+B+C+D+A\overline{B}\overline{C}D$

e) $\overline{\overline{(A+B)}\,\overline{(C+D)}\,\overline{(E+F)}\,\overline{(G+H)}}$

$= \cancel{\overline{\overline{A+B}}}(AB)+(CD)+(EF)+(GH)$

11. Apply DeMorgan's theorems to the following

a) $\overline{\overline{(ABC)}\,\overline{(EFG)} + \overline{(HIJ)}\,\overline{(KLM)}}$

$= \overline{\overline{ABC} + \overline{EFG}} + \overline{\overline{HIJ} + \overline{KLM}}$

$= \overline{ABC + EFG + HIJ + KLM}$

$= (\overline{ABC})(\overline{EFG})(\overline{HIJ})(\overline{KLM})$

$= (\overline{A}+\overline{B}+\overline{C})(\overline{E}+\overline{F}+\overline{G})(\overline{H}+\overline{I}+J)$

$(\overline{K}+\overline{L}+\overline{M})$

b) $\overline{(A + \overline{B\overline{C}} + CD)} + \overline{\overline{BC}}$

$= \overline{A}(\overline{\overline{B\overline{C}}})(\overline{CD}) + BC$

$= \overline{A}(B\overline{C})(\overline{C}\,\overline{D}) + BC$

$= \overline{A}B\overline{C}(\overline{C}\,\overline{D}) + BC$

$= \overline{A}B\overline{C}(\overline{C} + \overline{D}) + BC$

$= \overline{A}B\overline{C} + \overline{A}B\overline{C}\,\overline{D} + BC$

$= \overline{A}B\overline{C}(1 + \overline{D}) + BC$

$= \overline{A}B\overline{C} + BC$ Ans.

c) $\overline{(\overline{A+B})(\overline{C+D})(\overline{E+F})(\overline{G+H})}$

$= (\overline{A+B})(\overline{C+D})(\overline{E+F})(\overline{G+H})$

$= \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\,\overline{E}\,\overline{F}\,\overline{G}\,\overline{H}$ Ans.

LOGIC SIMPLIFICATION USING BOOLEAN ALGEBRA:-

Example 4-9

Using Boolean algebra techniques simplify this expression:

$AB + A(B+C) + B(B+C)$

Solution:

1) Apply distributive law to second and third terms:

$\Rightarrow AB + AB + AC + BB + BC$

2) Apply rule 7 ($BB = B$) to fourth term

$\Rightarrow AB + AB + AC + B + BC$

3) Apply rule 5 ($AB + AB = AB$) to first and second term

$\Rightarrow AB + AC + B + BC$

4) Apply rule 10 ($B + BC = B$) to third and fourth term

$\Rightarrow AB + AC + B$

5) Apply rule 10 ($AB + B = B$) to first and third terms.

$\Rightarrow B + AC$

Example 4-10

Simplify the following Boolean expression:

$$[A\bar{B}(C+BD)+\bar{A}\bar{B}]C$$

Solution:

1) Apply Distributive law

$\Rightarrow (A\bar{B}C + A\bar{B}BD + \bar{A}\bar{B})C$

2) Apply rule 8 ($\bar{B}B = 0$)

$\Rightarrow (A\bar{B}C + A.0.D + \bar{A}\bar{B})C$

3) Apply rule 3 ($A \cdot 0 \cdot D = 0$)

$\Rightarrow (A\bar{B}C + 0 + \bar{A}\bar{B})\,C$

4) Apply rule 1 (drop 0)

$\Rightarrow (A\bar{B}C + \bar{A}\bar{B})\,C$

5) Apply Distributive law.

$\Rightarrow A\bar{B}CC + \bar{A}\bar{B}C$

6) Apply rule 7 ($CC = C$)

$\Rightarrow A\bar{B}C + \bar{A}\bar{B}C$

7) Factor out $\bar{B}C$

$\Rightarrow \bar{B}C\,(A + \bar{A})$

8) Apply rule 6 ($A + \bar{A} = 1$)

$\Rightarrow \bar{B}C\,(1)$

9) Apply rule 4 (drop 1)

$\Rightarrow \bar{B}C$

Example 4-11

Simplify the following expression

$\bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C + ABC$

Solution:

1) Factor out BC

$\Rightarrow BC(\bar{A}+A) + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C$

2) Apply rule 6 $(\bar{A}+A=1)$

$\Rightarrow BC(1) + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C$

3) Factor out $A\bar{B}$

$\Rightarrow BC(1) + \bar{A}\bar{B}\bar{C} + A\bar{B}(\bar{C}+c)$

4) Apply rule 6 $(\bar{C}+C=1)$

$\Rightarrow BC(1) + \bar{A}\bar{B}\bar{C} + A\bar{B}(1)$

5) Apply rule 4 (drop 1)

$\Rightarrow BC + \bar{A}\bar{B}\bar{C} + A\bar{B}$

6) Factor out $\bar{B}$

$\Rightarrow BC + \bar{B}(\bar{A}\bar{C}+A)$

7) Apply rule 11 $(\bar{A}\bar{C}+A=A+\bar{C})$

$\Rightarrow BC + \bar{B}(A+\bar{C})$

8) Apply Distributive law.

$\Rightarrow BC + A\bar{B} + \bar{B}\bar{C}$

Example 4-12

Simplify the following Boolean expression : $\overline{AB + AC} + \bar{A}\bar{B}C$

Solution:

1) Apply De Morgan's theorem

$\Rightarrow (\overline{AB})(\overline{AC}) + \bar{A}\bar{B}C$

$\Rightarrow (\bar{A} + \bar{B})(\bar{A} + \bar{C}) + \bar{A}\bar{B}C$

2) Apply Distributive law.

$\Rightarrow \bar{A}\bar{A} + \bar{A}\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{B}C$

3) Apply rule 7 ($\bar{A}\bar{A} = \bar{A}$)

$\cancel{\Rightarrow \bar{A} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{A}\bar{B}(1 + C)}$

$\Rightarrow \bar{A} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{A}\bar{B} + \bar{A}\bar{B}C$

4)

4) Apply rule 10 ($\bar{A}\bar{B} + \bar{A}\bar{B}C = \bar{A}\bar{B}$)

($\bar{A} + \bar{A}\bar{C} = \bar{A}$)

$\Rightarrow \bar{A} + \bar{B}\bar{C} + \bar{A}\bar{B}$

5) Apply rule 10 ($\bar{A} + \bar{A}\bar{B} = \bar{A}$)

$\Rightarrow \bar{A} + \bar{B}\bar{C}$

BOOLEAN ANALYSIS OF LOGIC CIRCUITS

Section 4-4

12. Write Boolean expression for each of the logic gates.

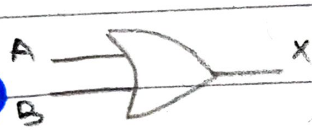
a)

$\Rightarrow AB = X$

b)

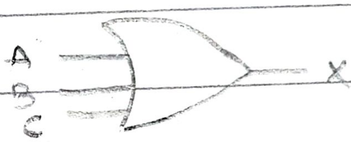
$\Rightarrow \overline{A} = X$

c)



$\Rightarrow A + B = X$

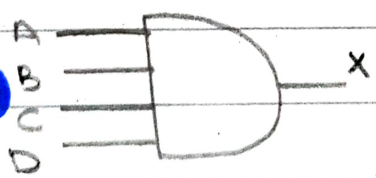
d)



$\Rightarrow A + B + C = X$

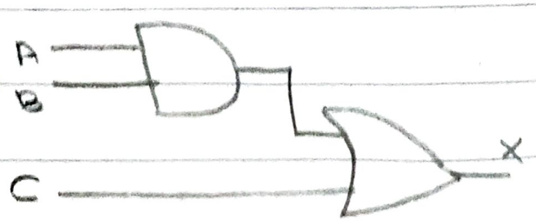
13. Write Boolean expression for each of the logic circuits.

a)



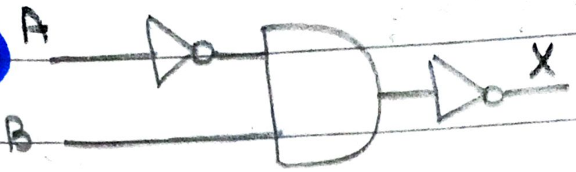
$\Rightarrow X = ABCD$

b)



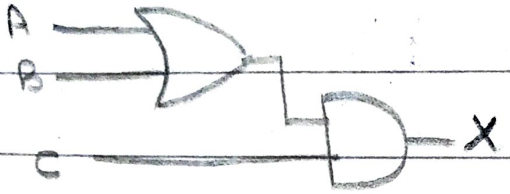
$\Rightarrow X = AB + C$

c)



$\Rightarrow \quad X = \overline{\bar{A}B}$

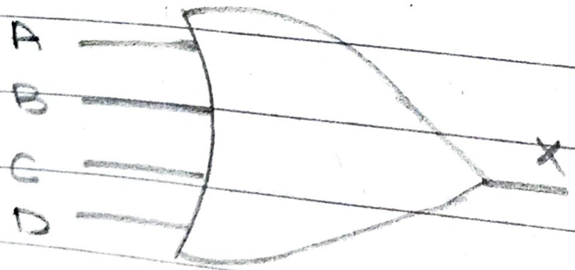
d)



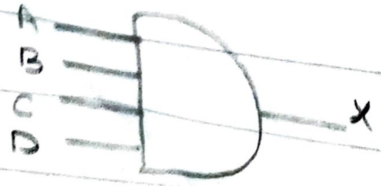
$\Rightarrow \quad X = (A+B)C$

14. Draw logic circuit represented by following expressions.

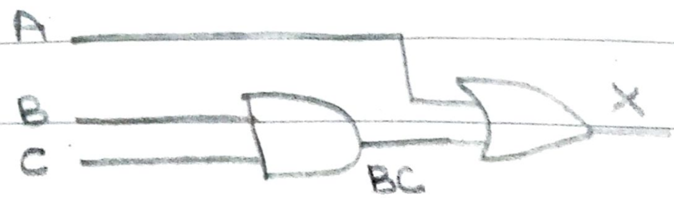
a) $A+B+C+D$



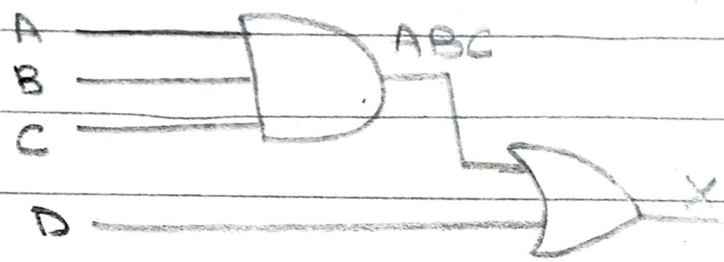
b) $ABCD$



c) $A + BC$

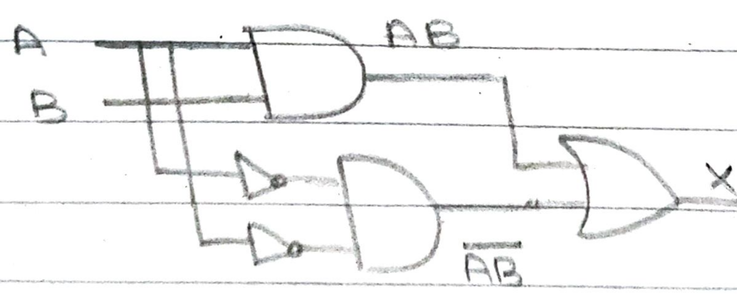


d) $ABC + D$

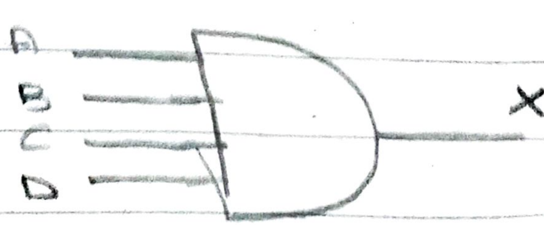


15. Draw logic circuit represented by each expression.

a) $AB + \overline{AB}$



b) $ABCD$



18. Construct a truth table for each of the following Boolean

a) $A + B + C$

$\Rightarrow X = A + B + C$

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

b) ABC

$\Rightarrow X = ABC$

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

c) $AB + BC + CA$

$\Rightarrow X = AB + BC + CA$

A	B	C	AB	BC	CA	X
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	1
1	0	0	0	0	0	0
1	0	1	0	0	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

d) $(A+B)(B+C)(C+A)$

$\Rightarrow X = (A+B)(B+C)(C+A)$

A	B	C	A+B	B+C	C+A	X
0	0	0	0	0	0	0
0	0	1	0	1	1	0
0	1	0	1	1	0	0
0	1	1	1	1	1	1
1	0	0	1	0	1	0
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

) $A\overline{B} + B\overline{C} + C\overline{A}$

A	B	C	$\overline{A}$	$\overline{B}$	$\overline{C}$	$A\overline{B}$	$B\overline{C}$	$C\overline{A}$	X
0	0	0	1	1	1	0	0	0	0
0	0	1	1	1	0	0	0	1	1
0	1	0	1	0	1	0	1	0	1
0	1	1	1	0	0	0	0	1	1
1	0	0	0	1	1	1	0	0	1
1	0	1	0	1	0	1	0	0	1
1	1	0	0	0	1	0	1	0	1
1	1	1	0	0	0	0	0	0	0

Section 4-5

19. Using Boolean algebra techniques, simplify the following expressions.

a) $A(A+B)$

1) Apply distributive law

$\Rightarrow AA + AB$

2) Apply rule 7 $(AA = A)$

$\Rightarrow A + AB$

3) Apply rule 10 $(A + AB = A)$

$\Rightarrow$ A Ans.

b) $A(\overline{A + AB})$

1) Using distributive law

$\Rightarrow$ $A\bar{A} + AAB$

2) Apply rule 7 $(AA = A)$

$\Rightarrow$ $A\bar{A} + AB$

3) Apply rule 8 $(A\bar{A} = 0)$

$\Rightarrow$ $0 + AB$

4) Apply rule 1 (drop 0)

$\Rightarrow$ AB Ans.

c) $BC + \bar{B}C$

1) Factor out C

$\Rightarrow$ $C(B + \bar{B})$

2) Apply rule 6 $(B + \bar{B} = 1)$

$\Rightarrow$ $C(1)$

3) Apply rule 4 (drop 1)

$\Rightarrow$ C Ans.

d) $A(A + \bar{A}B)$

1) Distributive law

$\Rightarrow$ $AA + A\bar{A}B$

2) Rule 7 $(AA = A)$

$\Rightarrow$ $A + A\bar{A}B$

3) Rule 8 ($A\bar{A} = 0$)

$\Rightarrow A + (0 . B)$

4) Rule 3 (~~drop 0~~) ($0 . B = 0$)

$\Rightarrow A + 0$

5) Rule 1 (drop 0)

$\Rightarrow A$ ans.

e) $A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C$

1) Factor out C

$\Rightarrow C(A\bar{B} + \bar{A}B + \bar{A}\bar{B})$

2) Factor out $\bar{A}$

$\Rightarrow C[A\bar{B} + \bar{A}(B + \bar{B})]$

3) Rule 6 ($B + \bar{B} = 1$)

$\Rightarrow C[A\bar{B} + \bar{A}(1)]$

4) Rule 4 (drop 1)

$\Rightarrow C(A\bar{B} + \bar{A})$

5) Rule 11 ($A\bar{B} + \bar{A} = \bar{A} + \bar{B}$)

$\Rightarrow C(\bar{A} + \bar{B})$

$\Rightarrow \bar{A}C + \bar{B}C$ Ans.

20. Using Boolean algebra, simplify the following expression:

a) $(\bar{A}+B)(A+C)$

$\Rightarrow \bar{A}A + AB + \bar{A}C + BC$

$\Rightarrow 0 + AB + \bar{A}C + BC$

$\Rightarrow AB + \bar{A}C + BC$ Ans

b) $A\bar{B} + A\bar{B}C + A\bar{B}CD + A\bar{B}CDE$

$\Rightarrow A\bar{B}(1 + C + CD + CDE)$

$\Rightarrow A\bar{B}\{1 + C(1 + D + DE)\}$

$\Rightarrow A\bar{B}[1 + C\{1 + D(1+E)\}]$

$\Rightarrow A\bar{B}\{1 + C\{1 + D(1)\}\}$

$\Rightarrow A\bar{B}(1)$ $\because A + 1 = 1$

$\Rightarrow A\bar{B}$ Ans.

c) $BC + \overline{BCD} + B$

$\Rightarrow B(1+C) + \overline{BCD}$

$\Rightarrow B(1) + \overline{BCD}$

$\Rightarrow B + \overline{BCD}$ ~~$\because A + \bar{A}B = A + B$~~

~~$\Rightarrow B + \overline{CD}$ Ans.~~

$\Rightarrow B + \bar{B} + \bar{C} + \bar{D}$

$\Rightarrow 1 + \bar{C} + \bar{D}$

$\Rightarrow \bar{C} + \bar{D}$ Ans.

d) $(B+\overline{B})(BC+BC\overline{D})$

$\Rightarrow (1)\{BC(1+\overline{D})\}$

$\Rightarrow BC(1)$

$\Rightarrow BC$ Ans.

e) $BC+(\overline{B}+\overline{C})D+BC$

$\Rightarrow \cancel{BC+BC+\overline{B}D+\overline{C}D}$

$\Rightarrow \cancel{BC+\overline{B}D+\overline{C}D}$

$\Rightarrow BC+BC+(\overline{BC})D$

$\Rightarrow BC+(\overline{BC})D$ Ans.

21. Using Boolean algebra, simplify the following expression

a) $CE+C(E+F)+\overline{E}(E+G)$

$\Rightarrow CE+CE+CF+\overline{E}E+\overline{E}G$

$\Rightarrow CE+CF+(0)+\overline{E}G$

$\Rightarrow CE+CF+\overline{E}G$ Ans.

b) $\overline{B}\,\overline{C}D+(\overline{B+C+D})+\overline{B}\,\overline{C}\,\overline{D}E$

$\Rightarrow \overline{B}\,\overline{C}D+(\overline{B}\,\overline{C}\,\overline{D})+\overline{B}\,\overline{C}\,\overline{D}E$

$\Rightarrow \overline{B}\,\overline{C}D+\overline{B}\,\overline{C}\,\overline{D}(1+E)$

$\Rightarrow \overline{B}\,\overline{C}(D+\overline{D})(1)$

$\Rightarrow \overline{B}\,\overline{C}(1)$

$\Rightarrow \overline{B}\,\overline{C}$ Ans.

c) $(C+CD)(C+\bar{C}D)(C+E)$

$\Rightarrow \{C(1+D)\}\{(C+\bar{C}D)(C+E)$

$\Rightarrow C(1)(C+D)(C+E)$

$\Rightarrow C(C+D)(C+E)$

$\Rightarrow (CC+CD)(C+E)$

$\Rightarrow (C+CD)(C+E)$ $\qquad A+AB=A$

$\Rightarrow (C)(C+E)$

$\Rightarrow CC+CE$

$\Rightarrow C+CE \Rightarrow C(1+E)$

$\Rightarrow C(1) \Rightarrow C$ Ans.

d) $BCDE+BC(\overline{DE})+(\overline{BC})DE$

$\Rightarrow BCDE+BC(\bar{D}+\bar{E})+(\bar{B}+\bar{C})DE$

$\Rightarrow BCDE+BC\bar{D}+BC\bar{E}+\bar{B}DE+\bar{C}DE$

$\Rightarrow DE(BC+\bar{B}+\bar{C})+BC\bar{D}+BC\bar{E}$

$\Rightarrow DE(BC+\overline{BC})+BC\bar{D}+BC\bar{E}$

$\Rightarrow DE(1)+BC\bar{D}+BC\bar{E}$

$\Rightarrow DE+BC(\bar{D}+\bar{E})$

e) $BCD[BC+\bar{D}(CD+BD)]$

$\Rightarrow$ ~~BCD (BC~~

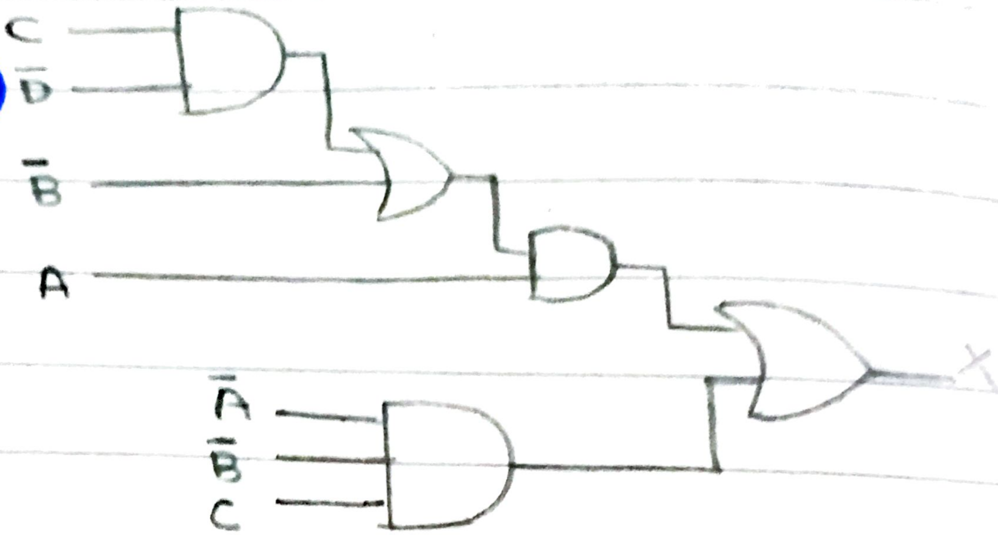
$\Rightarrow BCBCD+BCD\bar{D}(CD+BD)$

$\Rightarrow BCD+0(CD+BD)$ $\qquad A.A=A$

$\Rightarrow BCD$ Ans. $\qquad A\bar{A}=0$

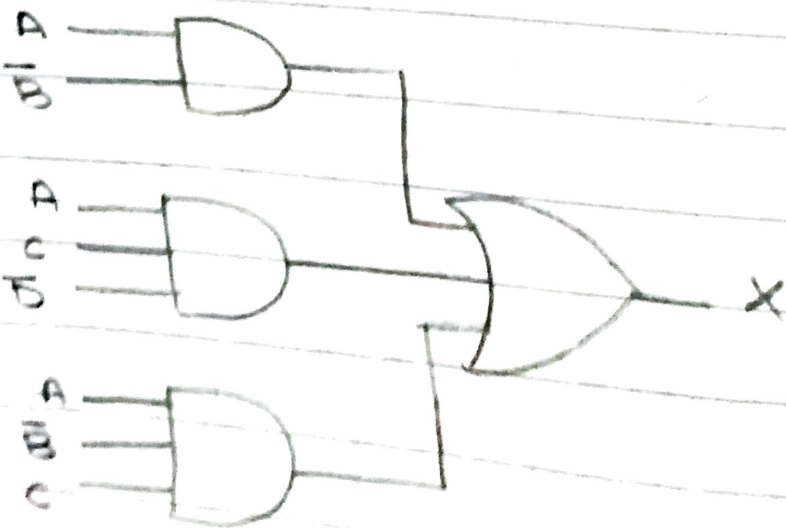
22. Determine which of the logic circuits are equivalent.

a)



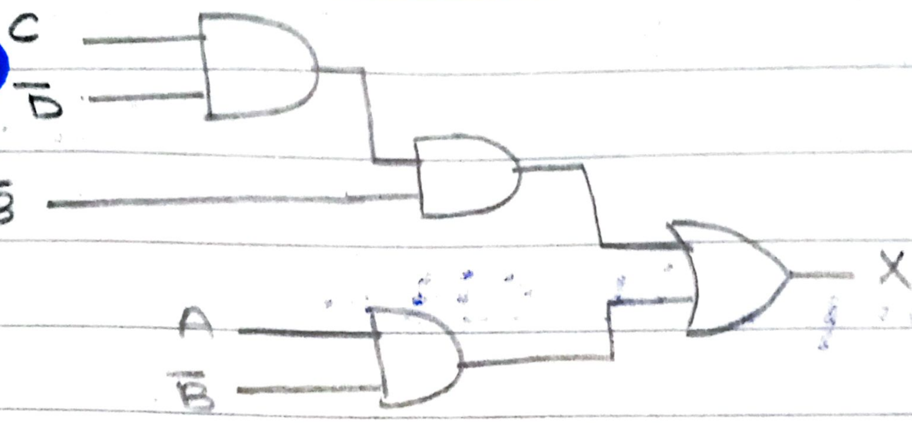
$\Rightarrow X = (C\bar{D} + B)A + \bar{A}\bar{B}C$

b)



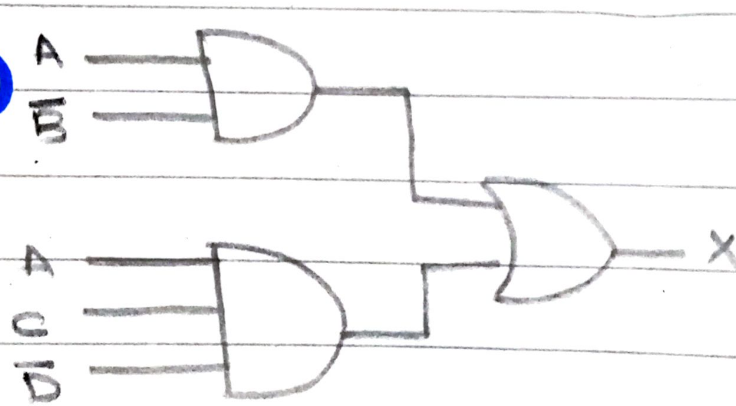
$\Rightarrow X = A\bar{B} + AC\bar{D} + A\bar{B}C$

c)



$$\Rightarrow \quad X = (C\overline{D})\overline{B} + A\overline{B}$$

d)



$$\Rightarrow \quad X = A\overline{B} + AC\overline{D}$$