UNIVERSITY OF KARACHI

Probability and Statistical Methods

BSCS-306

Name of Student: MUHAMMAD AMAS WASEEM

Seat No: B20102077

Class Roll No: 36

Section: A

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Submitted to: Dr. Tahseen A. Jilani

DEPARTMENT OF COMPUTER SCIENCE

UNIVERSITY OF KARACHI

MUHAMMAD AMAS WASEEM
B20102077

ASSIGNMENT 04

NORMAL DISTRIBUTION

QUESTION : 01

$\mu = 120$, $\sigma = 18$

(a) $Z = \frac{x - \mu}{\sigma} = \frac{96 - 120}{18} = -1.333$

$P(x < 96) = P(Z < -1.333)$

$= 0.09176$

(b) $Z = \frac{x - \mu}{\sigma} = \frac{81 - 120}{18} = -2.16$

$P(x > 81) = 1 - P(x < 81) = 1 - P(Z < -2.16)$

$= 0.984$

(c) for $x = 126$

$Z = \frac{126 - 120}{18} = 0.33$

for $x = 153$

$Z = \frac{153 - 120}{18} = 1.83$

$$P(126 < x < 153) = P(x < 153) - P(x > 126)$$
$$= P(z < 1.83) - P(z < 0.33)$$
$$= 0.9664 - 0.6293$$
$$= 0.3371$$

(d) $x = ?$ $\quad z = 45\% = 0.45$

$$\cancel{z = \frac{x - \mu}{\sigma}} \quad 0.45 = \frac{x - 120}{18}$$

$$\cancel{x = 118.02}$$

(e) $P(x < x) = 0.45$

$$P(z < -0.11) = 0.45$$

$$z = -0.11$$

$$z = \frac{x - \mu}{\sigma}$$

$$-0.11 = \frac{x - 120}{18}$$

$$\boxed{x = 118.02}$$

(e) $P(X > x) = 0.13$

$1 - P(X < x) = 0.13$

$P(X < x) = 1 - 0.13 = 0.87$

$P(z < 1.13) = 0.87$

$z = 1.13$

$\frac{x - \mu}{\sigma} = z$

$\frac{x - 120}{18} = 1.13$

$\boxed{x = 140.34}$

QUESTION : 02

$\mu = 100$, $\sigma = 5$

(a) $x = 107$

$z = \frac{x - \mu}{\sigma} = \frac{107 - 100}{5} = 1.4$

$P(z < 1.4) = 0.9192$

(b) $x = 89.5$

$$Z = \frac{89.5 - 100}{5} = -2.1$$

$$P(x > 89.5) = 1 - P(x < 89.5)$$
$$= 1 - P(Z < -2.1)$$
$$= 0.982.$$

(c) for $x = 94$

$$Z = \frac{94 - 100}{5} = -1.2$$

for $x = 103$

$$Z = \frac{103 - 100}{5} = 0.6$$

$$P(94 < x < 103) = P(103) - P(x < 94)$$
$$= P(Z < 0.6) - P(Z < -1.2)$$
$$= 0.7257 - 0.1150$$
$$= 0.6107$$

(d) $Z = 0.8$

$$\frac{x - \mu}{\sigma} = z$$

$$\frac{x - 100}{5} = 0.8$$

$$\boxed{x = 104}$$

QUESTION : 03

$\mu = 18$, $\sigma = 2.5$

(a) $x = 15$

$$z = \frac{15 - 18}{2.5} = -1.2$$

$$P(x < 15) = P(z < -1.2)$$

$$= 0.11507$$

(b) for $x = 16$

$$z = \frac{16 - 18}{2.5} = -0.8$$

for $x = 20$

$$z = \frac{20 - 18}{2.5} = 0.8$$

$$P(16 < x \leq 20) = P(x \leq 20) - P(x \leq 16)$$

$$= P(z < 0.8) - P(z < -0.8)$$

$$= 0.7881 - 0.21186$$

$$= 0.576$$

(c) $P(X > K) = 0.1539$

$1 - P(x < K) = 0.1539$

$1 - 0.1539 = P(x < K)$

$0.8461 = P(x < K)$

$P(z < 1.02) = 0.8461$

$z = 1.02$

$$z = \frac{x - \mu}{\sigma}$$

$$1.02 = \frac{K - 18}{2.5}$$

$$\boxed{K = 20.55}$$

QUESTION : 04

$P(x \leq c) = 0.1151$

$P(z \leq -1.2) = 0.1151$

$$\boxed{c = -1.2}$$

QUESTION : 05

$P(x > 7) = 0.05$

$1 - P(x < 7) = 0.05$

$P(x < 7) = 0.95$

$P(z < 1.65) = 0.95$

$z = 1.65$

$1.65 = \frac{7 - \mu}{\sigma}$

$\sigma = \frac{7 - \mu}{1.65}$ ——— (i)

$P(x < 5) = 0.5$

$P(z < 0) = 0.5$

$z = 0$

$\frac{5 - \mu}{\sigma} = 0$

$5 = \mu$ ——— (ii)

(i) => $\sigma = \frac{7 - 5}{1.65} = 1.21$

$P(4 < x < 6)$

for $x = 4$

$$Z = \frac{4-5}{1.21} = -0.82$$

for $x = 6$

$$Z = \frac{6-5}{1.21} = 0.82$$

$$P(4 < x < 6) = P(x < 6) - P(x > 4)$$

$$= P(Z < 0.82) - P(Z < -0.82)$$

$$= 0.7939 - 0.2061$$

$$= 0.5878$$

QUESTION : 06

$\mu = 3$, $\sigma^2 = 9$, $\sigma = 3$

(a) $P(2 < x < 5)$

for $x = 5$

$$Z = \frac{5-3}{3} = 0.66$$

for $x = 2$

$$Z = \frac{2-3}{3} = -0.33$$

$P(2 < x < 5) = P(x < 5) - P(x < 2)$

$= P(z < 0.66) - P(z < -0.33)$

$= 0.7454 - 0.3707$

$= 0.3747$

(b) for $x = 0$

$z = \frac{0-3}{3} = -1$

$P(x > 0) = 1 - P(x < 0)$

$= 1 - P(z < -1)$

$= 1 - 0.158$

$= 0.842$

(c) $P(|x-3| > 6)$

$|x-3| = 6$

$x - 3 = \pm 6$

$x < 9$, $x > -3$

for $x = 9$

$z = \frac{9-3}{3} = 2$

for $x = -3$

$z = \frac{-3-3}{3} \Rightarrow z = -2$

$$P(|x-3| > 6) = 1 \quad P(-3 < x < 9)$$
$$= P(x < 9) - P(x < -3)$$
$$= P(z < 2) - P(z < -2)$$
$$= (0.9773 - 0.0227)$$
$$= 0.9546$$

QUESTION : 07

$$\mu = 36 \quad , \quad \sigma = 5$$

(a) $$z = \frac{30 - 36}{5} = -1.2$$

$$P(x < 30) = P(z < -1.2)$$
$$= 0.115$$

(b) $$P(x < k) = 0.2578$$
$$P(z < -0.65) = 0.2578$$
$$z = -0.65$$

$$\frac{k - \mu}{\sigma} = -0.65 = \frac{k - 36}{5}$$

$$\boxed{k = 32.75}$$

(c) for $x = 34$

$$z = \frac{34 - 36}{5} = -0.4$$

for $x = 42$.

$$z = \frac{42 - 36}{5} = 1.2$$

$$P(34 < x < 42) = P(x < 42) - P(x < 34)$$
$$= P(z < 1.2) - P(z < -0.4)$$
$$= 0.8849 - 0.3445$$
$$= 0.5404.$$

(d) $P(x > k) = 0.1539$.

$$1 - P(x < k) = 0.1539.$$
$$P(x < k) = 1 - 0.1539$$
$$P(x < k) = 0.8461$$
$$P(z < 1.02) = 0.8461$$
$$\Rightarrow z = 1.02.$$
$$1.02 = \frac{k - 36}{5}$$

$$\boxed{k = 41.1}$$

Question : 08

$\mu = 24$ min $\quad \sigma = 3.8$ min.

(a) $P(x > 30) = 1 - (x < 30)$

for $x = 30$

$z = \frac{30 - 24}{3.8} = 1.58$

$\Rightarrow \quad = 1 - P(z < 1.58)$

$= 1 - 0.9429$

$= 0.0571$

(b) $P(x > 15) = 1 - P(x < 15)$

for $x = 15$

$z = \frac{15 - 24}{3.8} = -2.37$

$\Rightarrow \quad = 1 - P(z < -2.37)$

$= 1 - 0.00889$

$= 0.9911$

$$P(2 < x < 5) = P(x < 5) - P(x \leq 2)$$
$$= P(z < 0.66) - P(z < -0.33)$$
$$= 0.7454 - 0.3707$$
$$= 0.3747.$$

(b) for $x = 0$

$$z = \frac{0-3}{3} = -1.$$

$$P(x > 0) = 1 - P(x < 0)$$
$$= 1 - P(z < -1)$$
$$= 1 - 0.158$$
$$= 0.842.$$

(c) $P(|x-3| \geq 6)$

$$|x-3| = 6.$$
$$x - 3 = \pm 6$$
$$x \leq 9 \quad , \quad x \geq -3.$$

for $x = 9$

$$z = \frac{9-3}{3} = 2$$

for $x = -3$

$$z = \frac{-3-3}{3} \Rightarrow z = -2.$$

QUESTION : 09

$\mu = 500$, $\sigma = 100$

$x = 585$

$$z = \frac{585 - 500}{100} = 0.85$$

$P(x < 585) = P(z < 0.85)$

$= 0.8023$ or 80.23%

Tom scored is higher than required mark, so, he'll get admitted.

QUESTION : 10

$\mu = 5$ cm $\sigma = 0.02$ cm

(a) $P(4.98 < x < 5.02)$

for $x = 4.98$

$$z = \frac{4.98 - 5}{0.02} = -1$$

for $x = 5.02$

$$z = \frac{5.02 - 5}{0.02} = 1$$

$P(4.98 < x < 5.02) = P(x < 5.02) - P(x < 4.98)$

$= P(z < 1) - P(z < -1)$

$= 0.8413 - 0.1586$

$= 0.6827$

(b) $(4.96 < x < 5.04)$

for $z = \frac{4.96 - 5}{0.02} = -2$

for $x = 5.04$

$z = \frac{5.04 - 5}{0.02} = 2$

$P(4.96 < x < 5.04) = P(x < 5.04) - P(x < 4.96)$

$= P(z < 2) - P(z < -2)$

$= 0.9773 - 0.0227$

$= 0.9546$

QUESTION : 11

$\mu = 50$, $\sigma = 15$

for $x = 50$

$z = \frac{50 - 50}{15} = 0$

for $x = 70$

$z = \frac{70 - 50}{15} = 1.33$

$$P(50 < x < 70) = P(x < 70) - P(x < 50)$$
$$= P(z < 1.33) - P(z < 0)$$
$$= 0.9082 - 0.500$$
$$= 0.4082 \text{ or } 40\%$$

QUESTION : 12

$\mu = 12$, $\sigma = 2$

(a) $P(Y \leq 48) = P(4x + 2 \leq 48)$
$= P(x \leq 11.5)$.

for $x = 11.5$ =

$$z = \frac{11.5 - 12}{2} = -0.25$$

$$P(Y \leq 48) = P(z < -0.25)$$
$$= 0.4013$$

(b) $P(Y \geq 60) = 1 - P(Y < 60)$
$= 1 - P(4x + 2 < 60)$
$= 1 - P(x < 14.5)$.

$$z = \frac{14.5 - 12}{2} = 1.25$$

$$= 1 - P(z < 1.25).$$
$$= 1 - 0.8944$$
$$= 0.1056$$

(c) $P(46 \le Y \le 58) = P(Y \le 58) - P(Y \le 46)$

$= P(4x+2 \le 58) - P(4x+2 \le 46)$

$= P(x \le 14) - P(x \le 11)$

for $x = 14$.

$$z = \frac{14-12}{2} = 1$$

for $x = 11$

$$z = \frac{11-12}{2} = -0.5$$

$\Rightarrow$ $= P(z < 1) - P(z < -0.5)$

$= 0.8413 - 0.30854$

$P(46 \le Y \le 58) = 0.53276$.

QUESTION : 13

$\mu = 1200$, $\sigma = 300$

(a) $P(x < 1200) =$

$x = 1200$

$$z = \frac{1200-1200}{300} = 0$$

$P(z < 0) = 0.5$.

$P(x < 1200) = 0.5$.

(b) $P(1200 < x < 1500) = P(x < 1500) - P(x < 1200)$

for $x = 1500$

$$Z = \frac{1500 - 1200}{300} = 1$$

for $x = 1200$

$Z = 1$

$$= P(Z < 1) - P(Z < 0)$$
$$= 0.8413 - 0.5$$
$$= 0.3413.$$

(c) $P(x > 1500) = 1 - P(x \leq 1500)$

$$= 1 - P(Z < 1)$$
$$= 1 - 0.8413.$$
$$= 0.1587$$

(d) $P(1000 < x < 1500) = P(x < 1500) - P(x < 1000)$

for $x = 1000$

$$Z = \frac{1000 - 1200}{300} = -0.67.$$

$$= P(x < 1) - P(x < -0.67)$$
$$= 0.8413 - 0.2514$$
$$= 0.5899.$$