STANDARD FORMS OF BOOLEAN EXPRESSIONS:-

Example 4-14 :-

Convert each of the following Boolean expressions to SOP form.

a) $AB + B(CD + EF)$

Solution:

$\Rightarrow AB + BCD + BEF$

b) $(A+B)(B+C+D)$

Solution:

$\Rightarrow AB + AC + AD + BB + BC + BD$

c) $\overline{(\overline{A+B}) + C}$

Solution:

$\Rightarrow (\overline{\overline{A+B}})\,\overline{C} = (A+B)\overline{C}$

$\Rightarrow A\overline{C} + B\overline{C}$

Example 4-15

Convert the following Boolean expression into standard SOP form:

$$A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D$$

Solution:

Domain of this SOP expression is A, B, C, D.

The first term $A\bar{B}C$ is missing variable D or $\bar{D}$ so:

$$\Rightarrow A\bar{B}C = A\bar{B}C(D+\bar{D})$$
$$= A\bar{B}CD + A\bar{B}C\bar{D}$$

The ~~fir~~ second term $\bar{A}\bar{B}$ is missing variables C or $\bar{C}$ and D or $\bar{D}$.

$$\Rightarrow \bar{A}\bar{B} = \bar{A}\bar{B}(C+\bar{C})$$
$$= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$$
$$\Rightarrow \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} \{= \bar{A}\bar{B}C(D+\bar{D}) + \bar{A}\bar{B}\bar{C}(D+\bar{D})$$
$$= \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

The third term is already in standard form.

The complete standard SOP form of the original expression is as follows:

$$\Rightarrow A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D = A\bar{B}CD + A\bar{B}C\bar{D} +$$
$$\bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$$

Example 4-16

Determine the binary values for which the following standard SOP expression is equals to 1:

$$ABCD + A\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

Solution:

The term $ABCD$ equals to 1 when $A=1$, $B=1$, $C=1$, $D=1$

$\Rightarrow ABCD = 1.1.1.1 = 1$

The term $A\bar{B}\bar{C}D$ equals to 1 when $A=1$, $B=0$, $C=0$, $D=1$

$\Rightarrow A\bar{B}\bar{C}D = 1.\bar{0}.\bar{0}.1$

$= 1.1.1.1 = 1$

The term $\bar{A}\bar{B}\bar{C}\bar{D}$ equals to 1 when $A=0$, $B=0$, $C=0$, $D=0$

$\Rightarrow \bar{A}\bar{B}\bar{C}\bar{D} = \bar{0}.\bar{0}.\bar{0}.\bar{0}$

$= 1.1.1.1 = 1$

The SOP expression equals 1 when any or all of the three product terms is 1.

Example 4-17 :-

Convert the following Boolean expression into standard POS form:

$(A+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$

Solution:

The domain of this POS expression is A, B, C, D.

The first term $A+\overline{B}+C$ is missing D or $\overline{D}$, so add $D\overline{D}$ and apply rule 12:

$\Rightarrow A+\overline{B}+C = A+\overline{B}+C+D\overline{D}$

$= (A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})$

The second term is missing A or $\overline{A}$

$\Rightarrow \overline{B}+C+\overline{D} = A\overline{A}+\overline{B}+C+\overline{D}$

$= (A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})$

The third term is already in standard form.

The standard POS form of original expression is:

$\Rightarrow (\overline{A}+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D) =$

$(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+C+\overline{D})$

$(\overline{A}+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$

Example 4-18

Determine the binary values of variables for which the following standard POS expression is equal to 0.

$$(A+B+C+D)(A+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}$$

Solution:

The term $A+B+C+D$ is 0 when

$A=0$, $B=0$, $C=0$, $D=0$

$\Rightarrow A+B+C+D = 0+0+0+0 = 0$

The term $A+\overline{B}+\overline{C}+D$ is 0 when

$A=0$, $B=1$, $C=1$, $D=0$

$\Rightarrow A+\overline{B}+\overline{C}+D = 0+\overline{1}+\overline{1}+0$

$= 0+0+0+0 = 0$

The term $\overline{A}+\overline{B}+\overline{C}+\overline{D}$ is 0 when

$A=1$, $B=1$, $C=1$, $D=1$

$\Rightarrow \overline{A}+\overline{B}+\overline{C}+\overline{D} = \overline{1}+\overline{1}+\overline{1}+\overline{1}$

$= 0+0+0+0 = 0$

The POS expression equals 0 when any of the three sums equals 0.

Example 4-19

Convert the following SOP expression to an equivalent POS expression.

$$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

Solution:

The evaluation is as follows:

$$000 + 010 + 011 + 101 + 111$$

Since there are three variables in the domain of this expression, there are total of eight (2^3) possible combinations. The SOP expression contains five of these combinations so POS must contain the other three which are 001, 100 and 110.

These are binary values that make the sum term 0. The equivalent POS expression is

$$(A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$$

Section 4-6:

23. Convert following expression to sum of product (SOP) form.

a) $(C+D)(A+\bar{D})$

$\Rightarrow AC + AD + C\bar{D} + D\bar{D}$

$\Rightarrow AC + AD + C\bar{D}$ $\quad \because A\bar{A}=0$

b) $A(A\bar{D}+C)$

$\Rightarrow AA\bar{D} + AC$

$\Rightarrow A\bar{D} + AC$ $\quad \because AA = A$

c) $(A+C)(CD+AC)$

$\Rightarrow ACD + AAC + CCD + ACC$

$\Rightarrow ACD + AC + CD + AC$ $\quad \because AA=A$

$\Rightarrow ACD + AC + CD$

24. Convert the following expressions to sum of product (SOP) forms.

a) $BC + DE(B\bar{C} + DE)$

$\Rightarrow BC + DEB\bar{C} + DEDE$

$\Rightarrow BC + DEB\bar{C} + DE$

$\Rightarrow BC + DE(B\bar{C} + 1)$

$\Rightarrow BC + DE$

b) $BC(\overline{C}\overline{D} + CE)$

$\Rightarrow BC\overline{C}\overline{D} + BCCE$

$\Rightarrow 0 + BCE$

$\Rightarrow BCE$

$\because A\overline{A} = 0$
$CC = C$

c) $B + C[BD + (C+\overline{D})E]$

$\Rightarrow B + C[BD + CE + \overline{D}E)$

$\Rightarrow B + CBD + CCE + C\overline{D}E$

$\Rightarrow B + CBD + CE + C\overline{D}E$

$\Rightarrow B(1 + CD) + CE(1 + \overline{D})$

$\Rightarrow B + CE.$

25. Define the domain of each SOP expression in Problem 23 and convert the expression to standard SOP form.

a) $(C+D)(A+\overline{D})$

$\Rightarrow AC + AD + C\overline{D}$

Domain of this expression is A, C, D

First term is missing D or $\overline{D}$

$\Rightarrow AC = AC(D + \overline{D})$

$= ACD + AC\overline{D}$

Second term is missing C or $\overline{C}$

$\Rightarrow AD = AD(C + \overline{C})$

$= ACD + A\overline{C}D$

Third term is missing A or $\overline{A}$

$\Rightarrow C\overline{D} = C\overline{D}(A + \overline{A})$

$= AC\overline{D} + \overline{A}C\overline{D}$

The complete SOP form is :

$\Rightarrow AC + AD + C\overline{D} =$

$ACD + AC\overline{D} + A\overline{C}D + \overline{A}C\overline{D}$

b) $A(A\overline{D} + C)$

$\Rightarrow A\overline{D} + AC$

Domain of this expression is

A, C, D

The first expression has missing C or $\overline{C}$

$\Rightarrow A\overline{D} = A\overline{D}(C + \overline{C})$

$= AC\overline{D} + A\overline{C}\,\overline{D}$

The second term is missing D or $\overline{D}$

$$\Rightarrow AC = AC(D+\bar{D})$$
$$= ACD + AC\bar{D}$$

The complete SOP form is:

$$\Rightarrow A\bar{D} + AC = AC\bar{D} + \bar{A}\bar{C}\bar{D} + ACD.$$

c) $(A+C)(CD+AC)$

$$\Rightarrow ACD + AC + CD$$

Domain of this expression is

A, C, D.

The first term ACD is in standard form.

The second term AC is missing variable (D or $\bar{D}$)

$$\Rightarrow AC = AC(D+\bar{D})$$
$$= ACD + AC\bar{D}$$

The third term has missing variable A or $\bar{A}$

$$\Rightarrow CD = CD(A+\bar{A})$$
$$= ACD + \bar{A}CD$$

The complete SOP form is:

$$\Rightarrow ACD + AC\bar{D} + \bar{A}CD$$

27. Determine the binary value of each term in standard SOP expression from Problem 25.

a) $ACD + AC\overline{D} + A\overline{C}D + \overline{A}C\overline{D}$

The term ACD equals to 1 when

$\Rightarrow A = 1, C = 1, D = 1$

$\Rightarrow ACD = 1.1.1 = 1$

The term $AC\overline{D}$ equals to 1 when

$A = 1, C = 1$ and $D = 0$

$\Rightarrow AC\overline{D} = 1.1.\overline{0} = 1.1.1 = 1$

The term $A\overline{C}D$ equals to 1

when $A = 1, C = 0, D = 1$

$\Rightarrow A\overline{C}D = 1.\overline{0}.1$

$= 1.1.1 = 1$

The term $\overline{A}C\overline{D}$ is equals to 1

when $A = 0, C = 1, D = 0$

$\Rightarrow \overline{A}C\overline{D} = \overline{0}.1.\overline{0} = 1.1.1$

b) $AC\bar{D} + A\bar{C}\bar{D} + ACD$

The term $AC\bar{D}$ equals to 1 when
$A = 1, C = 1, D = 0$
$\Rightarrow AC\bar{D} = 1.1.\bar{0} = 1.1.1 = 1$

The term $A\bar{C}\bar{D}$ equals to 1
when $A = 1, C = 0, D = 0$
$\Rightarrow A\bar{C}\bar{D} = 1.\bar{0}.\bar{0} = 1.1.1 = 1$

The term ACD equals to 1
when $A = 1, C = 1, D = 1$
$\Rightarrow ACD = 1.1.1 = 1$

c) $ACD + AC\bar{D} + \bar{A}CD$

The term ACD equals 1 when
$A = 1, C = 1, D = 1$
$\Rightarrow ACD = 1.1.1 = 1$

The term $AC\bar{D}$ equals 1 when
$A = 1, C = 1, D = 0$
$\Rightarrow AC\bar{D} = 1.1.\bar{0} = 1.1.1 = 1$

The term $\bar{A}CD$ equals 1 when
$A = 0, C = 1, D = 1$
$\Rightarrow \bar{A}CD = \bar{0}.1.1 = 1.1.1 = 1$

29. Convert each standard SOP expression in Problem 25 to standard POS form.

a) $AC D + AC\overline{D} + A\overline{C}D + \overline{A}C\overline{D}$

The evaulation is as follows:

$\Rightarrow 111 + 110 + 101 + 010$

There are three variables in domain so total $(2^3) = 8$ possible combinations. Four are present so remaining four will be;

000, 011, 100, 001

The equivalent POS ~~eq~~ expression is:

$\Rightarrow$

$\Rightarrow (\overline{A} + \overline{C} + \overline{D})(\overline{A} + C + D)(A + \overline{C} + \overline{D})$
$(\overline{A} + \overline{C} + D)$

b) $AC\bar{D} + A\bar{C}\bar{D} + ACD$

The evaluation is as follows:

$\Rightarrow$ 110 + 100 + 111

There are three variables in domain so total $(2^3) = 8$ possible combinations. Three are present so remaining five will be:

000, 001, 010, 011, 101

The equivalent POS expression is

$\Rightarrow (\bar{A}+\bar{C}+\bar{D})(\bar{A}+\bar{C}+D)(\bar{A}+C+\bar{D})$
$(\bar{A}+C+D)(A+\bar{C}+D)$

c) $ACD + AC\bar{D} + \bar{A}CD$

The evaluation is as follows:

$\Rightarrow$ 111 + 110 + 011

There are three variable in domain so total $(2^3) = 8$ possible combinati

Three are present so ~~eg~~ five remaining will be:

000, 001, 010, 100, 101

The equivalent POS expression is:

$\Rightarrow (\bar{A}+\bar{C}+\bar{D})(\bar{A}+\bar{C}+D)(\bar{A}+C+\bar{D})$
$(A+\bar{C}+\bar{D})(A+\bar{C}+D)$