Two populations :-

1) When σ_1^2 and σ_2^2 given :- $(\mu_1 - \mu_2)$

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

2) When σ^2 unknown, $n_1, n_2 > 30$

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Exercises

9.35 A random sample of size $n_1 = 25$, taken from a normal population with a standard deviation $\sigma_1 = 5$, has a mean $\bar{x}_1 = 80$. A second random sample of size $n_2 = 36$, taken from a different normal population with a standard deviation $\sigma_2 = 3$, has a mean $\bar{x}_2 = 75$. Find a 94% confidence interval for $\mu_1 - \mu_2$.

9.36 Two kinds of thread are being compared for strength. Fifty pieces of each type of thread are tested under similar conditions. Brand A has an average tensile strength of 78.3 kilograms with a standard deviation of 5.6 kilograms, while brand B has an average tensile strength of 87.2 kilograms with a standard deviation of 6.3 kilograms. Construct a 95% confidence interval for the difference of the population means.

Two population

9.35 Data :-

$\Rightarrow n_1 = 25$ $\Rightarrow \sigma_1 = 5$ $\Rightarrow \bar{X}_1 = 80$

$\Rightarrow n_2 = 36$ $\Rightarrow \sigma_2 = 3$ $\Rightarrow \bar{X}_2 = 75$

$\Rightarrow 1-\alpha = 94\%$ $\Rightarrow \alpha = 0.06$ $\frac{\alpha}{2} = 0.03$

Solution :-

$$\Rightarrow \mu_1 - \mu_2 = (\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$= (80 - 75) \pm Z_{0.03} \sqrt{\frac{(5)^2}{(25)} + \frac{(3)^2}{36}}$$

$$= 5 \pm 0.512\,(1.118)$$

$$= 5 \pm 0.572$$

$$\boxed{4.428 < \mu_1 - \mu_2 < 5.572}$$

9.36 Data :

$\Rightarrow n_1 = n_2 = 50$

$\Rightarrow \mu_1 - \mu_2 = ?$

$\Rightarrow \bar{X}_1 = 78.3 \quad \Rightarrow \sigma_1 = 5.6$

$\Rightarrow \bar{X}_2 = 87.2 \quad \Rightarrow \sigma_2 = 6.3$

$\Rightarrow 1 - \alpha = 95 \quad \Rightarrow \alpha = 0.05$

Solution:-

$\Rightarrow \alpha/2 = 0.05/2 \quad \Rightarrow \alpha/2 = 0.025$

Since $n > 30$ and σ_1, σ_2 known so,

$$\Rightarrow \mu_1 - \mu_2 = (\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$= (78.3 - 87.2) \pm Z_{0.025} \sqrt{\frac{(5.6)^2}{50} + \frac{(6.3)^2}{50}}$$

$$= -8.9 \pm 1.960 (1.192)$$

$$= -8.9 \pm 2.33$$

$$\Rightarrow \boxed{6.57 < \mu_1 - \mu_2 < -11.23}$$

9.37 A study was conducted to determine if a certain treatment has any effect on the amount of metal removed in a pickling operation. A random sample of 100 pieces was immersed in a bath for 24 hours without the treatment, yielding an average of 12.2 millimeters of metal removed and a sample standard deviation of 1.1 millimeters. A second sample of 200 pieces was exposed to the treatment, followed by the 24-hour immersion in the bath, resulting in an average removal of 9.1 millimeters of metal with a sample standard deviation of 0.9 millimeter. Compute a 98% confidence interval estimate for the difference between the population means. Does the treatment appear to reduce the mean amount of metal removed?

9.37 Data :

$\Rightarrow n_1 = 100 \quad \Rightarrow \bar{X}_1 = 12.2 \quad \Rightarrow s_1 = 1.1$

$\Rightarrow n_2 = 200 \quad \Rightarrow \bar{X}_2 = 9.1 \quad \Rightarrow s_2 = 0.9$

$\Rightarrow \mu_1 - \mu_2 = ? \quad \Rightarrow 1 - \alpha = 98\% \quad \Rightarrow \alpha = 0.02$

Solution :

$\Rightarrow \alpha/2 = 0.02/2 \quad \Rightarrow \alpha/2 = 0.01$

Since $n > 30$ and σ, σ_2 unknown

$$\Rightarrow \mu_1 - \mu_2 = (\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= \left(\overset{12.2-9.1}{\cancel{100-200}}\right) \pm Z_{0.01}\sqrt{\frac{(1.1)^2}{100} + \frac{(0.9)^2}{200}}$$

$$= 3.1 \pm 2.326(0.127)$$

$$= 3.1 \pm 0.295$$

$$\Rightarrow \quad 2.805 < \mu_1 - \mu_2 < 3.395$$

3) When σ is given equal but unknown

n_1 and $n_2 < 30$

First we find Sp (Pooled sample variance)

$$\Rightarrow Sp = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1)+(n_2-1)}}$$

$$\Rightarrow v = n_1 + n_2 - 2$$

Then

$$(\bar{X}_1 - \bar{X}_2) \pm t_{\left(\frac{\alpha}{2}, v\right)} \, Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

9.38 Two catalysts in a batch chemical process, are being compared for their effect on the output of the process reaction. A sample of 12 batches was prepared using catalyst 1, and a sample of 10 batches was prepared using catalyst 2. The 12 batches for which catalyst 1 was used in the reaction gave an average yield of 85 with a sample standard deviation of 4, and the 10 batches for which catalyst 2 was used gave an average yield of 81 and a sample standard deviation of 5. Find a 90% confidence interval for the difference between the population means, assuming that the populations are approximately normally distributed with equal variances.

9.39 Students may choose between a 3-semester-hour physics course without labs and a 4-semester-hour course with labs. The final written examination is the same for each section. If 12 students in the section with labs made an average grade of 84 with a standard deviation of 4, and 18 students in the section without labs made an average grade of 77 with a standard deviation of 6, find a 99% confidence interval for the difference between the average grades for the two courses. Assume the populations to be approximately normally distributed with equal variances.

9.38 Data:

$\Rightarrow n_1 = 12$ $\Rightarrow \bar{x}_1 = 85$ $\Rightarrow S_1 = 4$

$\Rightarrow n_2 = 10$ $\Rightarrow \bar{x}_2 = 81$ $\Rightarrow S_2 = 5$

$\Rightarrow 1-\alpha = 90\%$ $\Rightarrow \alpha = 0.1$ $\Rightarrow \mu_1 - \mu_2 = ?$

Solution:-

$\Rightarrow \alpha/2 = 0.1/2$ $\Rightarrow \alpha/2 = 0.05$

Since σ is equal but unknown and $n_1, n_2 < 30$ so,

$$\Rightarrow S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1)+(n_2-1)}}$$

$$= \sqrt{\frac{(12-1)(4)^2 + (10-1)(5)^2}{(12-1)+(10-1)}}$$

$$\sqrt{\frac{176+225}{11+9}}$$

$\Rightarrow S_p = 4.47$

$\Rightarrow v = n_1 + n_2 - 2$

$= 12 + 10 - 2$

$\Rightarrow v = 20$

Now,

$$\Rightarrow \mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \pm t_{(\frac{\alpha}{2}, v)} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= (85 - 81) \pm t_{(0.05, 20)} (4.47) \sqrt{\frac{1}{12} + \frac{1}{10}}$$

$$= (4) \pm 1.725 (4.47)(0.428)$$

$$= 4 \pm 3.30$$

$$\Rightarrow \boxed{0.7 < \mu_1 - \mu_2 < 7.3}$$

Stats

Two Population Cases:-

9.39 Data:

$\Rightarrow n_1 = 12$ $\Rightarrow \bar{X}_1 = 84$ $\Rightarrow S_1 = 4$

$\Rightarrow n_2 = 18$ $\Rightarrow \bar{X}_2 = 77$ $\Rightarrow S_2 = 6$

$\Rightarrow 1 - \alpha = 99\%$

$\alpha = 0.01$ $\Rightarrow \mu_1 - \mu_2 = ?$

Solutions:-

Since σ_1, σ_2 are equal but unknown and $n_1, n_2 < 30$ so,

$$\Rightarrow S_p = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$= \sqrt{\frac{(12-1)(4)^2 + (18-1)(6)^2}{(12-1) + (18-1)}}$$

$$= \sqrt{\frac{176 + 612}{11 + 17}}$$

$$\Rightarrow S_p = 5.304$$

$$\Rightarrow v = n_1 + n_2 - 2 = 12 + 18 - 2$$

$$\Rightarrow v = 28$$

$$\Rightarrow \frac{\alpha}{2} = \frac{0.01}{2} \Rightarrow \frac{\alpha}{2} = 0.005$$

Now,

$$\Rightarrow \mu_1 - \mu_2 = (\bar{X}_1 - \bar{X}_2) \pm t_{(\frac{\alpha}{2}, v)} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= (84-77) \pm t_{(0.005, 28)} (5.304) \sqrt{\frac{1}{12} + \frac{1}{18}}$$

$$= (7) \pm 2.763 (5.304) (0.372)$$

$$= 7 \pm 5.46$$

$$\Rightarrow \boxed{1.54 < \mu_1 - \mu_2 < 12.46}$$

9.41 The following data represent the length of time, in days, to recovery for patients randomly treated with one of two medications to clear up severe bladder infections:

Medication 1	Medication 2
$n_1 = 14$	$n_2 = 16$
$\bar{x}_1 = 17$	$\bar{x}_2 = 19$
$s_1^2 = 1.5$	$s_2^2 = 1.8$

Find a 99% confidence interval for the difference $\mu_2 - \mu_1$

in the mean recovery times for the two medications, assuming normal populations with equal variances.

9.42 An experiment reported in *Popular Science* compared fuel economies for two types of similarly equipped diesel mini-trucks. Let us suppose that 12 Volkswagen and 10 Toyota trucks were tested in 90-kilometer-per-hour steady-paced trials. If the 12 Volkswagen trucks averaged 16 kilometers per liter with a standard deviation of 1.0 kilometer per liter and the 10 Toyota trucks averaged 11 kilometers per liter with a standard deviation of 0.8 kilometer per liter, construct a 90% confidence interval for the difference between the average kilometers per liter for these two mini-trucks. Assume that the distances per liter for the truck models are approximately normally distributed with equal variances.

9.43 A taxi company is trying to decide whether to purchase brand *A* or brand *B* tires for its fleet of taxis. To estimate the difference in the two brands, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are

$$\begin{aligned} \text{Brand } A: \quad & \bar{x}_1 = 36,300 \text{ kilometers}, \\ & s_1 = 5000 \text{ kilometers}. \\ \text{Brand } B: \quad & \bar{x}_2 = 38,100 \text{ kilometers}, \\ & s_2 = 6100 \text{ kilometers}. \end{aligned}$$

Compute a 95% confidence interval for $\mu_A - \mu_B$ assuming the populations to be approximately normally distributed. You may not assume that the variances are equal.

9.50 Two levels (low and high) of insulin doses are given to two groups of diabetic rats to check the insulin-binding capacity, yielding the following data:

Low dose:	$n_1 = 8$	$\bar{x}_1 = 1.98$	$s_1 = 0.51$
High dose:	$n_2 = 13$	$\bar{x}_2 = 1.30$	$s_2 = 0.35$

Assume that the variances are equal. Give a 95% confidence interval for the difference in the true average insulin-binding capacity between the two samples.