1) When σ is known:-

$$\Rightarrow \mu = \bar{x} \pm Z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right)$$

2) When σ is unknown:-

i) when $n > 30$

$$\Rightarrow \mu = \bar{x} \pm Z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

ii) when $n < 30$

$$\Rightarrow \mu = \bar{x} \pm t_{\left(\frac{\alpha}{2}, n-1\right)}\left(\frac{s}{\sqrt{n}}\right)$$

3) Error:

$$Z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) / \left(\frac{s}{\sqrt{n}}\right)$$

9.3 Many cardiac patients wear an implanted pacemaker to control their heartbeat. A plastic connector module mounts on the top of the pacemaker. Assuming a standard deviation of 0.0015 inch and an approximately normal distribution, find a 95% confidence interval for the mean of the depths of all connector modules made by a certain manufacturing company. A random sample of 75 modules has an average depth of 0.310 inch.

9.4 The heights of a random sample of 50 college students showed a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters.

(a) Construct a 98% confidence interval for the mean height of all college students.

(b) What can we assert with 98% confidence about the possible size of our error if we estimate the mean height of all college students to be 174.5 centimeters?

9.3 Data:

$\Rightarrow \sigma = 0.0015$

$\Rightarrow 1-\alpha = 95\%$ $\Rightarrow \alpha = 5\%$

$\Rightarrow n = 75$ $\Rightarrow \bar{X} = 0.310$

Solution :-

$\Rightarrow \alpha/2 = 0.005/2$ $\Rightarrow \alpha/2 = 0.0025$

Since σ is known and $n \geq 30$ so

$$\Rightarrow \mu = \bar{X} \pm Z_{\left(\frac{\alpha}{2}\right)} \frac{\sigma}{\sqrt{n}}$$

$$= 0.310 \pm Z_{(0.0025)} \frac{0.0015}{\sqrt{75}}$$

$$= 0.310 \pm 2.807\left(1.73 \times 10^{-4}\right)$$

$\Rightarrow \mu = 0.310 \pm 4.86 \times 10^{-4}$

~~$0.310 > \mu < 4.86 \times 10^{-4}$~~

$\Rightarrow \boxed{\mu = 0.309 < \mu < 0.310}$

9.5 Data :-

$\Rightarrow n = 100$ $\Rightarrow \bar{x} = 23,500$

$\Rightarrow S = 3900$ a) $1-\alpha = 99\%$ $\alpha = 1$

Solution :-

$\Rightarrow \alpha/2 = 0.01/2$ $\Rightarrow \alpha/2 = 0.005$

$\Rightarrow \mu = \bar{x} \pm Z_{\alpha/2} \dfrac{S}{\sqrt{n}}$ Since $n > 30$ and σ unkno

$= 23500 \pm Z_{(0.005)} \dfrac{3900}{\sqrt{100}}$

$= 23500 \pm 2.576(390)$

(a) $= 23500 \pm 1004.64$

$\Rightarrow$ ~~$\mu =$~~ $\boxed{22495.36 < \mu < 24504.64}$

(b) ~~$\bar{x} = 23500$~~ $\mu = \boxed{\text{Error} \leq 1004.64}$

9.9 D

9.9 Regular consumption of presweetened cereals contributes to tooth decay, heart disease, and other degenerative diseases, according to studies conducted by Dr. W. H. Bowen of the National Institute of Health and Dr. J. Yudben, Professor of Nutrition and Dietetics at the University of London. In a random sample consisting of 20 similar single servings of Alpha-Bits, the average sugar content was 11.3 grams with a standard deviation of 2.45 grams. Assuming that the sugar contents are normally distributed, construct a 95% confidence interval for the mean sugar content for single servings of Alpha-Bits.

9.9 Data:

$\Rightarrow n = 20$ $\Rightarrow \bar{X} = 11.3$ $\Rightarrow S = 2.45$

$\Rightarrow 1-\alpha = 95\% \Rightarrow \alpha = 0.05$

Solution:-

$\Rightarrow \frac{\alpha}{2} = \frac{0.05}{2} \Rightarrow \frac{\alpha}{2} = \frac{0.05}{2}$ $\frac{\alpha}{2} = 0.025$

Since σ is unknown & $n \leq 30$

$$\Rightarrow \mu = \bar{x} \pm t_{(\frac{\alpha}{2}, n-1)} \frac{s}{\sqrt{n}}$$

$$= 11.3 \pm t_{(0.025, 19)} \frac{2.45}{\sqrt{20}}$$

$$= 11.3 \pm 2.086 (0.547)$$

$$= 11.3 \pm 1.142$$

$$\Rightarrow \boxed{10.158 < \mu < 12.442}$$

9.10 A random sample of 12 graduates of a certain secretarial school typed an average of 79.3 words per minute with a standard deviation of 7.8 words per minute. Assuming a normal distribution for the number of words typed per minute, find a 95% confidence interval for the average number of words typed by all graduates of this school.

9.11 A machine produces metal pieces that are cylindrical in shape. A sample of pieces is taken, and the diameters are found to be 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, and 1.03 centimeters. Find a 99% confidence interval for the mean diameter of pieces from this machine, assuming an approximately normal distribution.

9.12 A random sample of 10 chocolate energy bars of a certain brand has, on average, 230 calories per bar, with a standard deviation of 15 calories. Construct a 99% confidence interval for the true mean calorie content of this brand of energy bar. Assume that the distribution of the calorie content is approximately normal.

9.13 A random sample of 12 shearing pins is taken in a study of the Rockwell hardness of the pin head. Measurements on the Rockwell hardness are made for each of the 12, yielding an average value of 48.50 with a sample standard deviation of 1.5. Assuming the measurements to be normally distributed, construct a 90% confidence interval for the mean Rockwell hardness.

9.14 The following

9.2 An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.