Railroad Design

Your company has a contract to construct a track bed for a railroad line between towns A and B shown on the contour map in Figure 1. The bed can be created by cutting trenches through the surface or by using some combination of trenches and tunnels. As chief engineer, your assignment is to analyze the costs of trenches and tunnels and to propose a design strategy for minimizing the total construction cost.

Engineering Requirements

The Transportation Board submits the following engineering requirements to your company:

- The track bed is to be straight and 10 m wide. The grade is to increase at a constant rate from the existing elevation of 100 m at town A to an elevation of 110 m at point M and then decrease at a constant rate to the existing elevation of 88 m at town B.
- From town A to point M and from point N to town B the track bed is to be created by excavating a trench whose vertical cross sections are trapezoids with the dimensions shown in Figure 2.
- Between points M and N your company must decide whether to excavate a trench of the type in Figure 2 or to excavate a tunnel whose vertical cross sections have the dimensions shown in Figure 3.

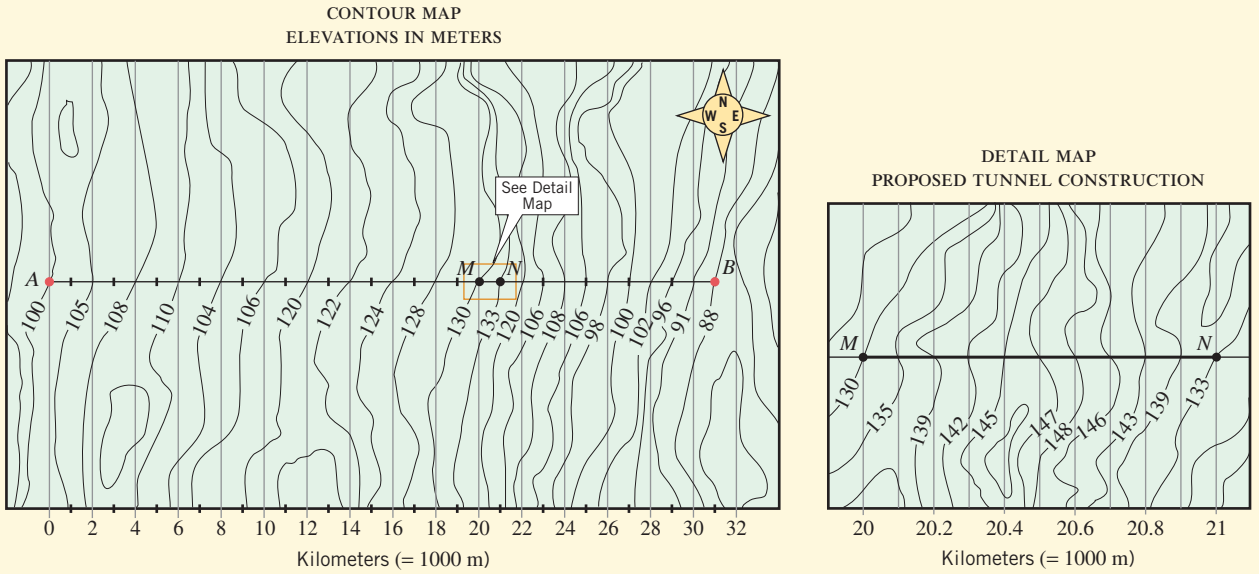

Figure 1

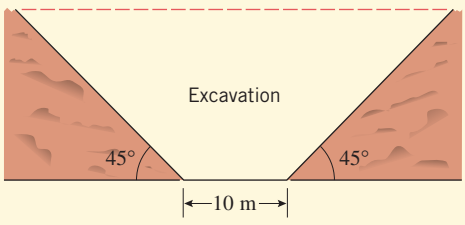

Figure 2

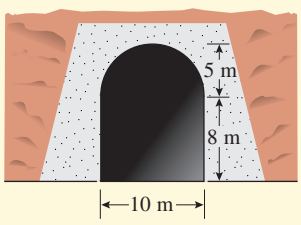

Figure 3

Cost Factors

Surface excavation of railbeds is performed using bulldozers, hydraulic excavators (backhoes), loading tractors, and other specialized equipment. Typically, the excavated dirt is piled at the side of the tracks to form sloped embankments, and the excavation cost is estimated from the volume of dirt to be removed and piled.

Tunnels in rock are often excavated by drilling shafts and inserting boring machines (called *moles*) to loosen and remove rock and dirt. Tunnels in soft ground are often excavated by starting at the tunnel face and using bucket or rotary excavators housed inside of shields. As the excavator progresses, tunnel liners are inserted behind it to support the earth and prevent cave-ins. Dirt removal is performed using conveyors or sometimes using railcars (called *muck cars*) that run on specially constructed tracks. Ventilation and air compression are other factors that add to the excavation cost of tunnels. In general, the excavation cost for a tunnel can be estimated from two components, the total volume of dirt to be removed and a cost that increases with the distance to the tunnel opening.

Make the following cost assumptions:

- The excavation and dirt-piling cost for a trench is \$4.00 per cubic meter.
- The drilling and dirt-piling cost for a tunnel is \$8.00 per cubic meter, and the costs involved in moving a load of dirt inside the tunnel a distance of 1 m toward the entrance along the track line is \$0.06 per cubic meter.

Cost Analysis of Trenches

Assume that variations in elevation are negligible for short distances at right angles to the track, so that the cross sections of the dirt to be excavated always have the trapezoidal shape shown in Figure 2 (straight horizontal edges at the surface).

Exercise 1 Complete Table 1, and then use the table and Simpson's rule with $n = 10$ to approximate the cost of a trench from town A to point M.

Table 1

DISTANCE x FROM TOWN A (m)	TERRAIN ELEVATION (m)	TRACK ELEVATION (m)	DEPTH OF CUT (m)	CROSS-SECTIONAL AREA $f(x)$ OF CUT (m^2)
0	100	100	0	0
2000	105	101	4	56
4000				
6000				
8000				
10,000				
12,000				
14,000				
16,000				
18,000				
20,000				

Exercise 2 As in Exercise 1, use Simpson's rule with $n = 10$ to approximate the cost of constructing a trench from (a) point M to point N, and (b) point N to town B.

Exercise 3 Find the total cost of the project if a trench is used along the entire line from town A to town B.

Cost Analysis of a Tunnel

Exercise 4

(a) Find the volume of dirt that must be removed from the tunnel, and calculate the drilling and dirt-piling cost.

(b) Find an integral for the cost of moving all of the dirt inside the tunnel to the tunnel entrance. [*Suggestion:* Use Riemann sums.]

(c) Find the total cost of excavating the tunnel.

Exercise 5 Find the total cost of the project using a trench from town A to point M, a tunnel from point M to point N, and a trench from point N to town B. Compare the cost to that obtained in Exercise 3 and state which method is cheaper.

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