UNIVERSITY OF KARACHI

Probability and Statistical Methods

BSCS-306

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ASSIGNMENT : 03

EXPONENTIAL DISTRIBUTION:-

QUESTION : 01

$$f(x) = \begin{cases} \frac{1}{20} e^{-x/20} & , \quad x > 0 \\ 0 & , \quad x \leq 0 \end{cases}$$

(1) $P(x \leq 12) = \int_0^{12} \frac{e^{-x/20}}{20} \, dx$

$= \frac{1}{20} \left(-20 \, e^{-x/20}\right)_0^{12}$

$= -\left(e^{-12/20} - e^{0}\right)$

$= -0.54 - 1$

$P(x \leq 12) = 0.45$

(2) $P(18 < x < 24) = \int_{18}^{24} \frac{e^{-x/20}}{20} \, dx$

$= \frac{1}{20} \left(-20 e^{-x/20}\right)_{18}^{24}$

$= -\left[e^{-24/20} - e^{-18/20}\right]$

$= -(0.30 - 0.40)$

$= 0.1$

(3) $P(x \geq 30) = \int_{30}^{\infty} \frac{e^{-x/20}}{20} \, dx$

$= \frac{1}{\cancel{20}} \left(-e^{x/20} \frac{\cancel{20}}{\cancel{20}} \right)_{30}^{\infty}$

$= -\left[e^{\infty/20} - e^{-30/20} \right]$

$= -\left(0 - e^{-30/20}\right)$

$=$ ~~0.1053~~ 0.223

Question : 02

$f(x) = 4e^{-4x}, \quad x \geq 0$

$f(x) = \lambda e^{-\lambda x}$

$\Rightarrow \quad \lambda = 4$

In Exponential pdf

$E(x) = \frac{1}{\lambda} = \frac{1}{4}$

$V(x) = \frac{1}{\lambda^2} = \frac{1}{16}$

$F(x) = 1 - e^{-\lambda x} = 1 - e^{-4x}$

$P(4.2 < x < 4.7) = \int_{4.2}^{4.7} 4e^{-4x} \, dx$

$= 4 \left(-\frac{e^{-4x}}{4} \right)_{4.2}^{4.7}$

$$= -\left[e^{-4(4.7)} - e^{-4(4.2)}\right]$$

$$= -\left[6.8 \times 10^{-9} - 5.0 \times 10^{-8}\right].$$

$$= -\left(-4.3 \times 10^{-8}\right).$$

$$= 4.3 \times 10^{-8}$$

QUESTION : 03

$$x \sim U(\overset{a}{0}, \overset{b}{2})$$

for uniform distribution.

$$f(x) = \frac{1}{b-a} = \frac{1}{2-0} = \frac{1}{2}.$$

$$y \sim \exp(\lambda).$$

for exponential function.

$$f(x) = \lambda e^{-\lambda x}.$$

$$P(x < 3/4) = \int_0^{3/4} 1/2 \, dx.$$

$$= \frac{1}{2}(x)_0^{3/4}$$

$$= \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

$$P(Y<1) = \int_0^1 \lambda e^{-\lambda x}\, dx.$$

$$= \lambda \left[-\frac{e^{-\lambda x}}{\lambda} \right]_0^1.$$

$$= -(e^{-\lambda} - 1).$$

According to given condition.

$$P\left(X < \frac{3}{4}\right) = P(Y<1)$$

$$\frac{3}{8} = -(e^{-\lambda} - 1).$$

$$\cancel{3 = -8e^{-\lambda} - 8.}$$

$$\cancel{8e^{-\lambda} = -8 - 3.}$$

$$\cancel{e^{-\lambda} = \frac{11}{8}}$$

$$0.375 = -e^{-\lambda} + 1$$

$$e^{-\lambda} = 0.625$$

$$-\lambda = -0.47.$$

$$\boxed{\lambda = 0.47}$$

QUESTION : 04

$$\text{Mean} = 4 = \frac{1}{\lambda} = \frac{1}{4}$$

(i) $f(x) = \int \lambda e^{-\lambda x} \, dx$

$f(x) = \int_0^3 \lambda e^{-\lambda x} \, dx$

$= -|e^{-\lambda x}|_0^3 = \frac{-1}{4} - |e^{-\frac{1}{4}x}|_0^3$

$f(x) = -(e^{-\frac{3}{4}} - 1)$

$f(x) = 0.5276$

(2) By Binomial distribution.

$P(X = x) = {}^nC_x \, p^x \, q^{n-x}$

$n = 6 \; , \; x = 4 \quad , \; q = 1 - p$

$= {}^6C_4 \, (0.5276)^4 \, q(1 - 0.5276)^{6-4} +$

${}^6C_5 \, (0.5276)^5 (1 - 0.5276)^{6-5} + {}^6C_6 (0.5276)^6$
$(1 - 0.5276)^{6-6}$

$= 0.395$

Question : 05.

Mean = 3 =

$\frac{1}{\lambda} = 3$, $\lambda = \frac{1}{3}$.

$f(x) = \lambda e^{-\lambda x}$

(1) $P(x > 5) = 1 - \int_0^5 \lambda e^{-\lambda x} dx$.

$= 1 - \frac{1}{3}\int_0^5 e^{-\frac{x}{3}} dx = 1 + \left| e^{-\frac{x}{3}} \right|_0^5$

$= 1 + 0.811 - 1$

$P(x \geq 5) = 0.188$

(2) $P(X \geq 10) = 1 - P(X \leq 10)$

$= 1 - \frac{1}{3}\int_0^{10} e^{-x/3} dx$.

$= 1 + \left| e^{-x/3} \right|_0^{10}$

$= 1 + 0.0356 - 1$

$P(X \geq 10) = \cancel{1.96}\ 0.0356$.

QUESTION : 06

$\lambda = 10$

(i) $P(X > 15) = 1 - (X \leq 15)$

$P(X > 15) = 1 - \sum_{x=0}^{x=15} P(X \leq 15)$.

$\therefore P(X = x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$

$= 1 - e^{-\lambda}\left[\dfrac{\lambda^0}{0!} + \dfrac{\lambda^1}{1!} + \dfrac{\lambda^2}{2!} + \dfrac{\lambda^3}{3!} + \dfrac{\lambda^4}{4!} + \right.$

$\dfrac{\lambda^5}{5!} + \dfrac{\lambda^6}{6!} + \dfrac{\lambda^7}{7!} + \dfrac{\lambda^8}{8!} + \dfrac{\lambda^9}{9!} + \dfrac{\lambda^{10}}{10!} +$

$\left.\dfrac{\lambda^{11}}{11!} + \dfrac{\lambda^{12}}{12!} + \dfrac{\lambda^{13}}{13!} + \dfrac{\lambda^{14}}{14!} + \dfrac{\lambda^{15}}{15!}\right]$

$\therefore \lambda = 10$

$= 1 - e^{-10}[20952.88697]$.

$= 1 - 0.9512$.

$P(X > 15) = 0.048$

(2) $P(x \geq 1) = 1 - P(x \leq 1)$

$= 1 - \int_0^1 f(x)\,dx$

$\because$ $f(x) = \lambda e^{-\lambda x}$

$P(x \geq 1) = 1 - 10\int_0^1 e^{-10x}\,dx$

$= 1 + \left(e^{-10x}\right)_0^1$

$= 1 + 4.53 \times 10^{-5} - 1$

$P(x \geq 1) = 4.53 \times 10^{-5}$

(3) Mean $= \dfrac{1}{\lambda}$

Mean $=$ 0.01 minutes

Question .07 :

$\lambda = 6$. $\frac{15}{60} = 0.25$ hours.

$f(x) = \lambda e^{-\lambda x}$.

$P(x > 0.25) = 1 - P(x \leq 0.25)$

$= 1 - \int_0^{0.25} \lambda e^{-\lambda x} \, dx$.

$= 1 - 6 \int_0^{0.25} e^{-6x} \, dx$

$= 1 + \left(e^{-6x}\right)_0^{0.25}$

$= 1 + 0.223 - 1$

$P(x > 0.25) = 0.223$ hours.