Department of Computer Science, University of Karachi
BSCS: 306: Probability and Statistical Methods
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Assignment: 05: Sampling and Sampling Distribution for Sample Mean

Q.1. Consider a hypothetical population of N=5 (consisting of multiples of last digit of your seat#). e.g. if the last digit is 3 then use 3,6,9,12,15. Simulate samples of n=2 with and without repetition. Construct sampling distributions for sample mean for both cases.

Q.2.   An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed, with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a random sample of 16 bulbs will have an average life of less than 775 hours.

Q.3. A certain type of thread is manufactured with a mean tensile strength of 78.3 kilograms and a standard deviation of 5.6 kilograms. How is the variance of the sample mean changed when the sample size is (a) increased from 64 to 196? (b) Decreased from 784 to 49?

Q.4. A soft-drink machine is being regulated so that the amount of drink dispensed averages 240 ml with a SD of 15 ml. Periodically, the machine is checked by taking a sample of 40 drinks and computing the average content. If the mean of the 40 drinks is a value within the interval $\mu_{\bar{x}} \pm \sigma_{\bar{x}}$, the machine is thought to be operating satisfactorily; otherwise, adjustments are made. The company official found the mean of 40 drinks to be 236 ml and concluded that the machine needed no adjustment. Was this a reasonable decision?

Q.5. The heights of 1000 students are approximately normally distributed with a mean of 174.5 cm and a SD of 6.9 cm. If 200 random samples of size 25 are drawn from this population and the means recorded to the nearest tenth of a centimeter, determine (a) the mean and SD of the sampling distribution of $\bar{X}$; (b) the number of sample means that fall between 172.5 and 175.8 cm inclusive; (c) the number of sample means falling below 172.0 cm.

Q.6. The random variable X, representing the number of cherries in a cherry puff, has the following probability distribution:

x	4	5	6	7
$P(X = x)$	0.2	0.4	0.3	0.1

(a) Find the mean and the variance of X. (b) Find the mean, and the variance of the sample mean X for random samples of 36 cherry puffs. (c) Find the probability that the average number of cherries in 36 cherry puffs will be less than 5.5.

Q.7. The average life of a bread-making machine is 7 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution, find (a) the probability that the mean life of a random sample of 9 such machines fall between 6.4 and 7.2 years; (b) *the value of a; to the right of which 15% of the means computed from random samples of size 9 would fall.*

Q.8. The amount of time that a drive-through bank teller spends on a customer is a random variable with a mean $p = 3.2$ min. and a SD = 1.6 min. If a random sample of 64 customers is observed, find the probability that their mean time at the teller's counter is (a) at most 2.7 min: (b) more than 3.5 min; (c) at least 3.2 min but less than 3.4 min.

Q.9. In a chemical process the amount of a certain type of impurity in the output is difficult to control and is thus a random variable. Speculation is that the populations mean amount of the impurity is 0.20 grams per gram of output. It is known that the standard deviation is 0.1 grams per gram. An experiment is conducted to gain more insight regarding the speculation that $p = 0.2$. The process was run on a lab scale 50 times and the sample average x turned out to be 0.23 grams per gram. Comment on the speculation that the mean amount of impurity is 0.20 grams per gram. Make use of the central limit theorem in your work.

Q.10. The mean score for freshmen on an aptitude test at a certain college is 540, with a standard deviation of 50. What is the probability that two groups of students selected at random, consisting of 32 and 50 students, respectively, will differ in their mean scores by, (a) more than 20 points? (b) an amount between 5 and 10 points? Assume the means to be measured to any degree of accuracy.