

UNIVERSITY OF KARACHI



Probability and Statistical Methods

BSCS-306

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Semester No: 2nd

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ASSIGNMENT : 02 (A)

→ BINOMIAL DISTRIBUTIONS :-

QUESTION : 01

$$P = 0.10, \quad q = 1 - P, \quad n = 20$$

$$P(X=x) = {}^nC_x P^x q^{n-x}$$

$$P(X=x) = {}^{20}C_x (0.10)^x (0.9)^{20-x}$$

$$P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$$

$$= {}^{20}C_0 (0.10)^0 (0.9)^{20-0} + {}^{20}C_1 (0.10)^1 (0.9)^{20-1} + {}^{20}C_2 (0.10)^2 (0.9)^{20-2} + {}^{20}C_3 (0.10)^3 (0.9)^{20-3}$$

$$P(X \leq 3) = 0.86 \quad \text{Ans}$$

QUESTION : 02

$$P = 70\% = 0.7, \quad q = 0.3$$

$$(a) \quad n = 10$$

$$P(X=x) = {}^nC_x P^x q^{n-x}$$

$$P(X < 5) = P(0) + P(1) + P(2) + P(3) + P(4)$$

$$= {}^{10}C_0 P^0 q^{10-0} + {}^{10}C_1 P^1 q^{10-1} + {}^{10}C_2 P^2 q^{10-2} + {}^{10}C_3 P^3 q^{10-3} + {}^{10}C_4 P^4 q^{10-4}$$

$$P(X < 5) = 0.047$$

$$(b) \quad n = 20$$

$$P(X < 10) = \sum_{x=0}^9 {}^{20}C_x (0.7)^x (0.3)^{20-x}$$

$$P(X \leq 9) = \sum_{x=0}^9 b(x; 20; 0.7)$$

$$= 0.001 + 0.004 + 0.012$$

$$P(X \leq 9) = 0.017 \quad \text{---}$$

QUESTION : 03

$$P = 75\% = 0.75 \quad n = 5$$

$$(a) \quad P(X = 2) = b(2; 5; 0.75)$$

$$P(X = 2) = 0.088$$

$$(b) \quad P(X \leq 3) = \sum_{x=0}^3 b(x; 5; 0.75)$$

$$= 0.001 + 0.015 + 0.088 + 0.264$$

$$P(X \leq 3) = 0.368$$

QUESTION : 04

$$P = 30\% = 0.3, \quad n = 20$$

(a)

$$\begin{aligned} P(X \geq 10) &= 1 - P(X \leq 9) \\ &= 1 - \sum_{x=0}^9 b(x; 20, 0.3) \\ &= 1 - 0.001 + 0.007 + 0.028 + 0.092 + \\ &\quad 0.130 + 0.149 + 0.192 + 0.164 + \\ &\quad 0.114 + 0.065 \\ &= 1 - 0.952 \end{aligned}$$

$$P(X \geq 10) = 0.048$$

(b)

$$P(X \leq 4) = \sum_{x=0}^4 b(x; 20; 0.3)$$

$$= 0.001 + 0.007 + 0.028 + 0.092 + 0.130$$

$$P(X \leq 4) = 0.238$$

$$(c) \quad n = 20, \quad x = 5, \quad p = 30\% = 0.3$$

$$P(X=5) = \text{Bin}(20, 0.3) \\ = 0.179 \approx 18\%$$

NO, 30% is an incorrect value / Percentage of errors.

QUESTION 2 05

$$p = 60\% = 0.6, \quad n = 8$$

$$(a) \quad P(X=3) = b(3; 8; 0.6)$$

$$P(X=3) = 0.124$$

$$(b) \quad p = 40\% = 0.4$$

$$P(X \geq 5) = 1 - (X \leq 4)$$

$$= 1 - \sum_{x=0}^4 b(x; 8; 0.4)$$

$$= 1 - 0.07 + 0.090 + 0.209 +$$

$$0.279 + 0.232 +$$

$$= 1 - 0.827$$

$$P(X \geq 5) = 0.173$$

QUESTION 2 06

$$P = 25\% = 0.25, \quad n = 15$$

$$(a) P(3 \leq x \leq 6)$$

$$= P(x \leq 6) - P(x \leq 2)$$

$$= \sum_{x=0}^6 b(x; 15, 0.25) - \sum_{x=0}^2 b(x; 15, 0.25)$$

$$= 0.225 + 0.225 + 0.165 + 0.092$$

$$= 0.707$$

$$(b) P(x < 4) = P(x \leq 3)$$

$$= \sum_{x=0}^3 b(x; 15, 0.25)$$

$$= 0.03 + 0.067 + 0.156 + 0.225$$

$$= 0.461$$

$$(c) P(x > 5) = 1 - P(x \leq 5)$$

$$= \sum_{x=0}^5 b(x; 15, 0.25)$$

$$= 1 - (0.461 + 0.225 + 0.165)$$

$$= 1 - 0.851$$

$$P(x > 5) = 0.149$$

QUESTION : 07

$$q = 0.4$$

$$p = 1 - q = 0.6$$

for 4 engines :

$$x = \frac{4}{2} = 2, \quad n = 4$$

$$\begin{aligned} P(x \geq 2) &= 1 - P(x \leq 1) \\ &= 1 - \sum_{x=0}^1 \binom{n}{x} p^x q^{n-x} \\ &= 1 - 0.18 \\ &= 0.82 \end{aligned}$$

for 2-Engine Plane:

$$n = 2, \quad x = \frac{2}{2} \times \frac{1}{2} = 1$$

$$\begin{aligned} P(x \geq 1) &= 1 - P(x \leq 0) \\ &= 1 - \sum_{x=0}^0 \binom{n}{x} p^x q^{n-x} \\ &= 1 - 0.026 \\ &= 0.974 \end{aligned}$$

2 engine plane has a higher probability of success i.e.

$$0.974 > 0.82$$

QUESTION 208

$$n = 20$$

$$P = 0.9$$

$$\begin{aligned} (a) \quad P(X=18) &= \sum_{x=0}^{18} (x; 20; 0.9) - \sum_{x=0}^{17} (x; 20; 0.9) \\ &= 0.608 - 0.323 \\ &= 0.285 \end{aligned}$$

$$\begin{aligned} (b) \quad P(X=15) &= \sum_{x=0}^{15} (x; 20; 0.9) - \sum_{x=0}^{14} (x; 20; 0.9) \\ &= 0.043 - 0.011 \\ &= 0.032 \end{aligned}$$

$$P = 0.1$$

$$\begin{aligned} (c) \quad P(X \geq 2) &= 1 - P(X \leq 1) \\ &= 1 - \sum_{x=0}^1 (x; 20; 0.1) \\ &= 1 - 0.392 \\ &= 0.608 \end{aligned}$$

QUESTION 209

$$\begin{aligned} (a) \quad \text{Mean} &= np = (10)(0.5) \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{Variance} &= npq = (10)(0.5)(0.5) \\ &= 2.5 \end{aligned}$$

$$\begin{aligned}
 (b) \quad P(X \geq 7) &= 1 - P(X \leq 6) \\
 &= 1 - \sum_{n=0}^6 (n; 10; 0.5) \\
 &= 1 - 0.828 \\
 &= 0.172
 \end{aligned}$$

QUESTION 11

$$P = 50\% = 0.5 \quad n = 18$$

(a)

$$\begin{aligned}
 P(X=10) &= C_x^n p^x q^{n-x} \\
 &= C_{10}^{18} (0.5)^{10} (0.5)^{18-10} \\
 P(X=10) &= 0.1669
 \end{aligned}$$

(b)

$$\begin{aligned}
 P(X \geq 10) &= 1 - P(X \leq 9) \\
 &= 1 - \{0.1854 + 0.1669 + 0.1213 + \\
 &\quad 0.0708 + 0.0326 + 0.0116 + 0.0003 \\
 &\quad + 5.83 \times 10^{-4} + 6.8 \times 10^{-5} + 3.81 \times 10^{-6}\} \\
 &= 1 - 0.5895 \\
 P(X \geq 10) &= 0.4105
 \end{aligned}$$

$$(c) \quad P(X \leq 8) = \sum_{n=0}^8 C_n^n p^n q^{n-n}$$

$$= 3.81 \times 10^{-6} + 6.8 \times 10^{-5} + 5.8 \times 10^{-4} + 3.11 \times 10^{-3} + 0.011 + 0.032 + 0.070 + 0.121 + 0.166$$

$$P(X \leq 8) = 0.403$$

QUESTION 212

$$n = 12, \quad p = 0.5 \quad \text{'coz guessing}$$

$$P(X=3) = b(3; 12; 0.5)$$

$$P(X=3) = 0.054$$