Chapter 13

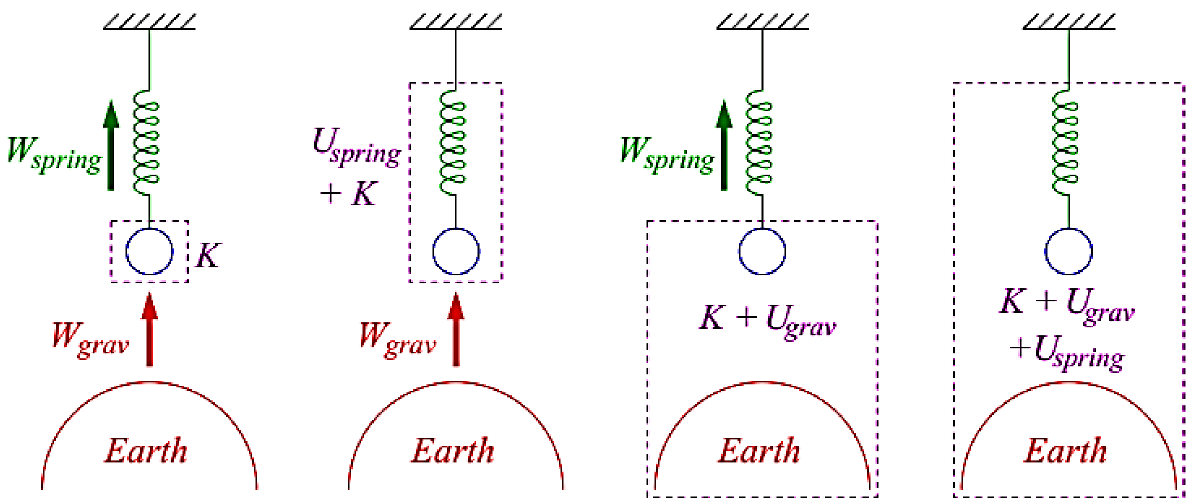
Conservation of Energy

If external force acting on the system is not zero, the conservation of energy becomes:

$$\Delta K + \Delta U = W_{\text{ext}}$$

where W_{ext} is the work done on the system by the external force.

Example



System = Mass $\Delta K =$ $W_{\text{spring}} + W_{\text{grav}}$	System = Mass + Spring $\Delta K + \Delta U_{\text{spring}} =$ W_{grav}	System = Mass + Earth $\Delta K + \Delta U_{\text{grav}} =$ W_{spring}	System = Mass + Earth + Spring $\Delta K + \Delta U_{\text{grav}} +$ $\Delta U_{\text{spring}} = 0$

13.1 Internal energy in a system of particle

$$\Delta K + \Delta U + \Delta E_{\text{int}} = W_{\text{ext}}$$

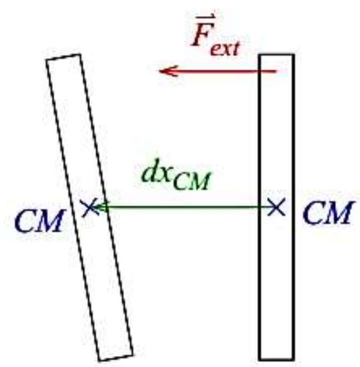
where E_{int} is the change in internal energy of the system.
Internal enery is the K.E. associated with the random motion of atoms and molecules (usually related to the object temperature), or the P.E. associated with forces between atoms.

$$E_{\text{int}} = K_{\text{int}} + U_{\text{int}}$$

Looking deeper into the cases of rigid body

$$F_{\text{ext}} = Ma_{\text{CM}} \qquad \text{for rigid body}$$

Notice that F_{ext} may not acting on the C.M.
Consider the force acting for a short period and in the period, C.M. displaces by dx_{CM}.



$$F_{\text{ext}} dx_{\text{CM}} = Ma_{\text{CM}} dx_{\text{CM}}$$

(Only multiply the previous equation by dx_{CM}. No special physical meaning. Notice that this is not the work done as in its definition $\vec{F} \cdot d\vec{x}$ — force is applied at a point and the point is displaced by $d\vec{x}$.)

$$F_{\text{ext}} dx_{\text{CM}} = Ma_{\text{CM}} dx_{\text{CM}} = M \frac{dv_{\text{CM}}}{dt} v_{\text{CM}} dt$$
$$\Rightarrow \qquad F_{\text{ext}} dx_{\text{CM}} = Mv_{\text{CM}} dv_{\text{CM}}$$

Consider the C.M. displaces from x_i to x_f and its velocity change from $v_{\text{CM},i}$ to $v_{\text{CM},f}$.

$$\therefore \int_{x_i}^{x_f} F_{\text{ext}} dx_{\text{CM}} = \frac{1}{2} M v_{\text{CM},f}^2 - \frac{1}{2} M v_{\text{CM},i}^2 = K_{\text{CM},f} - K_{\text{CM},i}$$
$$(\because K_{\text{CM}} \overset{\text{def}}{=} \tfrac{1}{2} M v_{\text{CM}}^2)$$

$$\text{or} \quad \boxed{F_{\text{ext}} s_{\text{CM}} = \Delta K_{\text{CM}}} \quad \text{if } F_{\text{ext}} \text{ is constant,}$$

— Center of mass (COM) energy equation

where s_{CM} is the displacement of the center of mass.

$$\Delta K + \Delta U + E_{\text{int}} = W_{\text{ext}}$$

— Conservation of energy (COE) equation

***The COM equation is not the work-energy theorem for a particle. s_{CM} is the center of mass displacment but not the displacement of the point that the force acts on.

13.2 Some examples of conservation of energy

1) A sliding block is stopped on a horizontal table with friction.

   Center of mass (COM) energy equation: $-f s_{\rm CM} = -\frac{1}{2} M v_{\rm CM}^2$

   Conservation of energy (COE) equation: $W_f = -\frac{1}{2} M v_{\rm CM}^2 + \Delta E_{\rm int,block}$

2) Pushing a stick on a horioznted frictionless table.

   Center of mass (COM) energy equation:

   $$F_{\rm ext} s_{\rm CM} = \frac{1}{2} M v_{\rm CM}^2$$

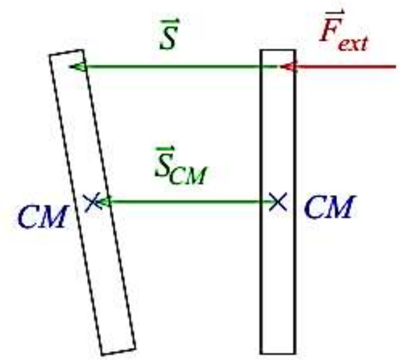
   Conservation of energy (COE) equation:

   $$F_{\rm ext} s = \frac{1}{2} M v_{\rm CM}^2 + \frac{1}{2} I \omega^2$$

   If $F_{\rm ext}$ is acted on center of mass,

   $$s = s_{\rm CM}$$

   $$F_{\rm ext} s = F_{\rm ext} s_{\rm CM} = \frac{1}{2} M v_{\rm CM}^2$$



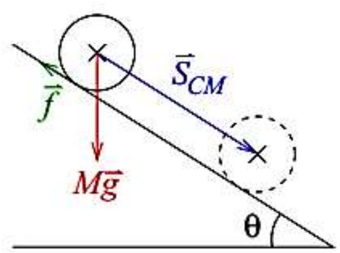
3) Ball rolling down an inclined plane without slipping

   Center of mass (COM) energy equation:

   $$(Mg\sin\theta - f) s_{\rm CM} = \frac{1}{2} M v_{\rm CM}^2$$

   Conservation of energy (COE) equation:

   $$\underbrace{Mg\, s_{\rm CM} \sin\theta}_{M\vec{g}\text{ acts on CM}} = \frac{1}{2} M v_{\rm CM}^2 + \frac{1}{2} I \omega^2$$



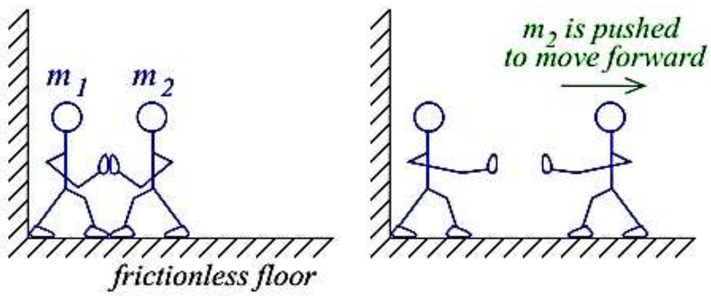
   Notice that the frictional force does no work in the COE eq. as the instantaneous point of contact between the ball and the plane does not move.

Example

Two men are pushing each other. m_2 is pushed away from m_1 by straightening their arms and the force between them is F.

(a) What is the speed of m_2 just after losing contact?

(b) What is the change in internal energies for m_1 and m_2?



Answer:

(a) Consider m_2 as one system, COM eq. is:

$$F s_{\mathrm{CM}} = \Delta K_{\mathrm{CM}} = \frac{1}{2} m_2 v_{\mathrm{CM},m_2}^2$$

where s_{CM} is the displacement of the center of mass of m_2.

$$\therefore \quad v_{\mathrm{CM},m_2} = \sqrt{\frac{2F s_{\mathrm{CM}}}{m_2}}$$

(b) For m_2, COE equation is

$$\Delta K + \Delta E_{\mathrm{int},m_2} = W_{\mathrm{ext}}$$

where $\begin{cases} \Delta K = \Delta K_{\mathrm{CM}} = |F s_{\mathrm{CM}}| \\ W_{\mathrm{ext}} = |F s| \end{cases}$.

Note that s is the total extension of m_1's hand (i.e. the displacement of m_2's hand when a force F is acting on it, where $s \neq s_{\mathrm{CM}}$).

$$\therefore \Delta E_{\mathrm{int},m_2} = |F s| - |F s_{\mathrm{cm}}|$$

For m_1, COE equation is

$$\Delta E_{\mathrm{int},m_1} = W_{\mathrm{ext}} = -|F s| \;\; (\vec{F} \text{ opposite to } \vec{s})$$