

UNIVERSITY OF KARACHI



Probability and Statistical Methods

BSCS-306

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ASSIGNMENT

09

HYPOTHESIS TESTING : II

QUESTION : 01

DATA:-

$$n_1 = 12$$

$$n_2 = 12$$

$$\bar{x}_1 = 37900$$

$$\bar{x}_2 = 39800$$

$$s_1 = 5100$$

$$s_2 = 4900$$

$$\alpha = 0.05, \alpha/2 = 0.025$$

$$H_0 : \mu_1 = \mu_2 = 0$$

$$H_A : \mu_1 \neq \mu_2 \neq 0$$

SOLUTION:-

Since $n < 30$ and equal variances

$$s_p = \sqrt{\frac{(n_1 - 1)(s_1)^2 + (n_2 - 1)(s_2)^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$= \sqrt{\frac{(12 - 1)(5100)^2 + (12 - 1)(4900)^2}{(12 - 1) + (12 - 1)}}$$

$$s_p = 5000.1$$

$$t_{cal} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t_{cal} = \frac{(37900 - 39800) - (0)}{5000.1 \sqrt{\frac{1}{12} + \frac{1}{12}}} = -0.930$$

$$t_{\text{tab}} = t_{(\alpha, n)} = t_{(0.05, 22)} = 1.71$$

$$v = n_1 + n_2 - 2$$

$$= 12 + 12 - 2$$

$$v = 22$$

P-value approach

$$= 2P(t < -0.930)$$

$$= 2(0.15)$$

$$= 0.3$$

$$t_{\text{tab}} (0.05, 22)$$

$$t_{\text{tab}} = 1.71$$

$$P\text{-value} > \alpha$$

$$0.3 > 0.025$$

Now,

$$t_{\text{cal}} < t_{\text{tab}}$$

$$-0.930 < 1.71$$

Since the P-value is greater than the level of significance so we can not reject H_0 . Hence there is insufficient evidence to suggest difference in the average wear of 2 brands of tires.

QUESTION : 02

DATA :-

$$n_1 = 12$$

$$n_2 = 10$$

$$\bar{x}_1 = 85$$

$$\bar{x}_2 = 81$$

$$s_1 = 4$$

$$s_2 = 5$$

$$\alpha = 0.05, \alpha/2 = 0.025$$

$$H_0 : \mu_1 - \mu_2 = 2$$

$$H_A : \mu_1 - \mu_2 > 2$$

SOLUTION:-

Given that variances are equal and $n < 30$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$
$$= \sqrt{\frac{(12 - 1)(4)^2 + (10 - 1)(5)^2}{(12 - 1) + (10 - 1)}}$$

$$s_p = 4.477$$

$$t_{cal} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{1/n_1 + 1/n_2}}$$
$$= \frac{(85 - 81) - 2}{4.477 \sqrt{1/12 + 1/10}} = 1.043$$

P-value approach

$$P(t > 1.043)$$

$$0.16 > 0.025$$

Since the P-value is greater than $\alpha/2$ so we'll accept H_0 .

QUESTION : 03

DATA:-

$$\alpha = 5\% \quad , \quad \alpha/2 = 0.025$$

$$n = 15$$

$$\bar{x} = 9.84$$

$$s = 18.47$$

$$H_0 : \mu_1 = \mu_2$$

$$H_a : \mu_1 \neq \mu_2$$

Solution:-

$$\begin{aligned} t_{\text{cal}} &= \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \\ &= \frac{9.84 - 0}{18.47 / \sqrt{15}} \end{aligned}$$

$$t_{\text{cal}} = 2.06$$

P-value approach

$$2P(t > 2.06)$$

$$2(0.03)$$

$$0.06 > 0.025$$

Since P-value is greater than the level of significance. Therefore we can't reject the Null hypothesis.

QUESTION : 04

DATA :-

$$n = 15$$

Subject d_i $\bar{d} = 54.13$

1 -67 $SD = 83.002$

2 -33

$$\alpha = 0.05, \alpha/2 = 0.025$$

3 150

$$H_0: \mu_1 = \mu_2$$

4 128

$$H_a: \mu_1 \neq \mu_2$$

5 190

6 2

SOLUTION :-

7 -56

$$t_{cal} = \frac{\bar{d} - d_0}{SD / \sqrt{n}}$$

8 119

9 -8

$$= \frac{54.13 - 0}{83.002 / \sqrt{15}} = 2.52$$

10 -1

11 79

12 153

P-value approach :

13 34

$$2P(t > 2.52)$$

14 107

$$2(0.015)$$

15 15

$$0.03 > 0.025$$

Since the P-value is greater than the $\alpha/2$ hence we can not reject H_0 in favour of H_a .

QUESTION : 05

DATA:-

$$n_1 = 5$$

$$n_2 = 7$$

$$\bar{x}_1 = 97.4$$

$$\bar{x}_2 = 110$$

$$s_1 = 8.87$$

$$s_2 = 30.22$$

$$\alpha = 0.01$$

$$H_0 : \mu_2 - \mu_1 = 10$$

$$H_a : \mu_2 - \mu_1 < 10$$

SOLUTION:-

Since the variances are unequal

$$\begin{aligned} v &= \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}} \\ &= \frac{\left[(8.87)^2/5 + (30.22)^2/7 \right]^2}{\frac{[(8.87)^2/5]^2}{5-1} + \frac{[(30.22)^2/7]^2}{7-1}} \\ &= 7.37 \approx 7 \end{aligned}$$

P value approach:

$$P(t < 1.86)$$

$$0.05 < 0.1$$

Since the P value is less than α

so we reject the H_0 in favour of H_a .

QUESTION : Q6

DATA:-

$$\sigma^2 = 0.03$$

$$H_0 = \sigma^2 = 0.03$$

$$n = 10$$

$$H_A = \sigma^2 \neq 0.03$$

$$\bar{x} = 10.06$$

$$s = 0.2458$$

$$\alpha = 0.05, \quad \alpha/2 = 0.025$$

SOLUTION:-

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$
$$= \frac{(10-1)(0.24)^2}{0.03}$$

$$\chi^2_{\text{cal.}} = 18.12$$

P value approach,

$$2 P(\chi^2 > 18.12)$$

$$2 (0.025)$$

$$0.05 > 0.025$$

The P value is greater than the level of significance hence we can not reject H_0 in favour of H_A .

QUESTION 207

DATA:-

$$n = 12$$

$$\alpha = 0.01$$

$$s = 1.75$$

$$H_0 = \sigma^2 = 1.96$$

$$H_a = \sigma^2 > 1.96$$

SOLUTION:-

$$\chi^2 = \frac{(n-1) s^2}{\sigma^2}$$

$$= \frac{(12-1)(1.75)^2}{1.96}$$

$$\chi^2 = 18.75$$

P value approach

$$P(\chi^2 > 18.75)$$

$$0.05 > 0.01$$

Since the P value is greater than α
so we can't reject H_0 .

QUESTION • 08

DATA:-

$$n_1 = 20$$

$$n_2 = 20$$

$$v_1 = 19$$

$$v_2 = 19$$

$$S_1 = 281.06$$

$$S_2 = 119.394$$

$$\alpha = 0.02 \quad , \quad \alpha/2 = 0.01$$

$$H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_A : \sigma_1^2 \neq \sigma_2^2$$

SOLUTION:-

$$F_{cal} = \frac{S_1^2}{S_2^2}$$

$$= \frac{(281.06)^2}{(119.394)^2}$$

$$F_{cal} = \frac{S_1}{S_2} = 5.54$$

$$F_{tab} = f_{(19,19)} = 2.49$$

Since $F_{cal} > F_{tab}$, we reject H_0 in favour of H_A .