

# BOOLEAN EXPRESSIONS AND TRUTH TABLE -

## Example 4-20

Develop a truth table for the standard SOP expression

$$\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$$

Solution:

There are three values in domain so there are eight possible combinations. The binary values that make the product term equal to 1 are.

Inputs			Output	Product Terms.
A	B	C	X	
0	0	0	0	
0	0	1	1	$\bar{A}\bar{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	$ABC$

$\bar{A}\bar{B}C : 001$ ,  $A\bar{B}C = 100$ , and  
 $ABC : 111$

## Example 4-21

Determine truth table for the following standard POS expression:

$$(A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C}) \\ (\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$$

Solution:

There are three variables in domain so eight possible binary values. The binary values that make sum term equals to 0 are

$$A+B+C : 000, A+\bar{B}+C = 010,$$

$$A\bar{B}\bar{C} = 011, \bar{A}\bar{B}\bar{C} = 101, \bar{A}+\bar{B}+C = 110$$

Inputs	Output			Sum Term
A	B	C	X	
0	0	0	0	$(A+B+C)$
0	0	1	1	
0	1	0	0	$(A+\bar{B}+C)$
0	1	1	0	$(A+\bar{B}+\bar{C})$
1	0	0	1	
1	0	1	0	$(\bar{A}+B+\bar{C})$
1	1	0	0	$(\bar{A}+\bar{B}+C)$
1	1	1	1	

## Example 4-22

From the truth table, determine the standard SOP expression and the equivalent standard POS expression.

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

solution:

There are four 1's in the output column and the corresponding binary values are 011, 100, 110 and 111

Convert these binary values to product terms.

$$011 \rightarrow \bar{A}BC$$

$$100 \rightarrow A\bar{B}\bar{C}$$

$$110 \rightarrow AB\bar{C}$$

$$111 \rightarrow ABC$$

The resulting standard SOP expression is:

$$\Rightarrow X = \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + A\bar{B}C$$

For POS expression, the output 0 for binary values 000, 001, 010 and 101.

Convert these binary values to sum terms:

$$000 \rightarrow A + B + C$$

$$001 \rightarrow A + B + \bar{C}$$

$$010 \rightarrow A + \bar{B} + C$$

$$101 \rightarrow \bar{A} + B + \bar{C}$$

The resulting standard POS expression for the output X is

$$\Rightarrow X = (A + B + C)(A + B + \bar{C})$$

$$(A + \bar{B} + C)(\bar{A} + B + \bar{C})$$

# SECTION 4-7

**31.** Develop a truth table for each of the following standard SOP expressions:

a)  $ABC + \bar{A}\bar{B}C + A\bar{B}\bar{C}$

Inputs			Output	Product Term
A	B	C	X	
0	0	0	0	
0	0	1	1	$\bar{A}\bar{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	1	$ABC$
1	1	1	1	$ABC$

$$ABC \rightarrow 111$$

$$\bar{A}\bar{B}C \rightarrow 001$$

$$A\bar{B}\bar{C} \rightarrow 110$$

b)  $\bar{X}YZ + \bar{X}Y\bar{Z} + X\bar{Y}Z + \bar{X}YZ + XY\bar{Z}$

Inputs			Output	Product Term
A	B	C	X	
0	0	0	1	$A\bar{X}\bar{Y}\bar{Z}$
0	0	1	0	
0	1	0	1	$\bar{X}X\bar{Z}$
0	1	1	1	$\bar{X}YZ$
1	0	0	0	
1	0	1	1	$X\bar{Y}Z$
1	1	0	1	$XY\bar{Z}$
1	1	1	0	

$$\bar{X}YZ \rightarrow 000$$

$$\bar{X}Y\bar{Z} \rightarrow 010$$

$$X\bar{Y}Z \rightarrow 101$$

$$\bar{X}YZ \rightarrow 011$$

$$XY\bar{Z} \rightarrow 110$$

**33.** Develop a truth table for each of the SOP expression:

a)  $\bar{A}B + AB\bar{C} + \bar{A}\bar{C} + A\bar{B}\bar{C}$

Inputs			Output	Product Term
A	B	C	X	
0	0	0	1	$\bar{A}\bar{C}$
0	0	1		
0	1	0		
0	1	1	1	$\bar{A}B$
1	0	0		
1	0	1	1	$A\bar{B}\bar{C}$
1	1	0	1	$AB\bar{C}$
1	1	1	1	

$$\Rightarrow \bar{A}B = \bar{A}B(C + \bar{C})$$

$$= \bar{A}BC + \bar{A}B\bar{C}$$

$$\Rightarrow \bar{A}\bar{C} = \bar{A}\bar{C}(B + \bar{B})$$

$$= \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}$$

The standard SOP expression is:  
 $\bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} + AB\bar{C} + A\bar{B}\bar{C}$

Inputs			Output	Product:
A	B	C	X	Term.
0	0	0	1	$\bar{A}\bar{B}\bar{C}$
0	0	1	0	
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	1	$\bar{A}BC$
1	0	0	0	
1	0	1	1	$A\bar{B}C$
1	1	0	1	$ABC$
1	1	1	0	

$$\bar{A}BC \rightarrow 011$$

$$\bar{A}B\bar{C} \rightarrow 010$$

$$\bar{A}\bar{B}\bar{C} \rightarrow 000$$

$$ABC \rightarrow 110$$

$$AB\bar{C} \rightarrow 101$$

$$\begin{aligned}
 b) \quad & \bar{x} + y\bar{z} + wz + x\bar{y}z \\
 \Rightarrow \bar{x} &= \bar{x}(w + \bar{w}) \\
 &= w\bar{x} + \bar{w}\bar{x} \\
 \Rightarrow \star &= w\bar{x}(y + \bar{y}) + \bar{w}\bar{x}(y + \bar{y}) \\
 &= w\bar{x}y + w\bar{x}\bar{y} + \bar{w}\bar{x}y + \bar{w}\bar{x}\bar{y} \\
 &= w\bar{x}y(z + \bar{z}) + w\bar{x}\bar{y}(z + \bar{z}) + \\
 &\quad \bar{w}\bar{x}y(z + \bar{z}) + \bar{w}\bar{x}\bar{y}(z + \bar{z}) \\
 &= w\bar{x}yz + w\bar{x}y\bar{z} + w\bar{x}\bar{y}z + w\bar{x}\bar{y}\bar{z} + \\
 &\quad \bar{w}\bar{x}yz + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}\bar{y}\bar{z}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow y\bar{z} &= y\bar{z}(w + \bar{w}) \\
 &= w y\bar{z} + \bar{w} y\bar{z} \\
 &= w y\bar{z}(x + \bar{x}) + \bar{w} y\bar{z}(x + \bar{x}) \\
 &= wxyz + w\bar{x}\bar{y}z + \bar{w}xyz + \bar{w}\bar{x}\bar{y}z
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow wz &= wz(x + \bar{x}) \\
 &= wxz + w\bar{x}z \\
 &= wxz(y + \bar{y}) + w\bar{x}z(y + \bar{y}) \\
 &= wxyz + wx\bar{y}z + w\bar{x}yz + w\bar{x}\bar{y}z
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow xy\bar{z} &= xy\bar{z}(w + \bar{w}) \\
 &= wxy\bar{z} + w\bar{x}y\bar{z} + \bar{w}xy\bar{z} + \bar{w}\bar{x}y\bar{z}
 \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow \overline{WXYZ} + \overline{WXYZ} + \overline{WXY\bar{Z}} + \overline{W\bar{X}YZ} \\
 & + \overline{W}\bar{X}\bar{Y}Z + \overline{W}\bar{X}Y\bar{Z} + W\overline{XYZ} + \\
 & W\overline{X}\bar{Y}Z + W\bar{X}\bar{Y}\bar{Z} + W\bar{X}YZ + W\bar{X}\bar{Y}Z \\
 & + WXY\bar{Z} + WXYZ
 \end{aligned}$$

Inputs				Output	Product Term
W	X	Y	Z	A	
0	0	0	0	1	$\overline{WXYZ}$
0	0	0	1	1	$\overline{WXYZ}$
0	0	1	0	1	$\overline{WXY\bar{Z}}$
0	0	1	1	1	$\overline{WXY\bar{Z}}$
0	1	0	0	0	
0	1	0	1	1	$\overline{W}\bar{X}\bar{Y}Z$
0	1	1	0	1	$\overline{WXY\bar{Z}}$
0	1	1	1	0	
1	0	0	0	1	$W\overline{XYZ}$
1	0	0	1	1	$W\overline{XYZ}$
1	0	1	0	1	$W\bar{X}Y\bar{Z}$
1	0	1	1	1	$W\bar{X}YZ$
1	1	0	0	0	
1	1	0	1	1	$W\bar{X}\bar{Y}Z$
1	1	1	0	1	$WXY\bar{Z}$
1	1	1	1	1	$WXYZ$

**34.** Develop a truth table for each of the standard POS expression.

a)  $(\bar{A} + \bar{B} + \bar{C})(A + B + C)(A + B + \bar{C})$

Inputs			Output	Sum Term
A	B	C	X	
0	0	0	0	$A + B + C$
0	0	1	0	$A + B + \bar{C}$
0	1	0	1	
0	1	1	1	
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	0	$\bar{A} + \bar{B} + \bar{C}$

b)  $(A + \bar{B} + C + \bar{D})(\bar{A} + B + \bar{C} + D)$

 $(A + B + \bar{C} + \bar{D})(\bar{A} + \bar{B} + C + D)$

Inputs				X	Output
A	B	C	D		Sum Term
0	0	0	0		
0	0	0	1		
0	0	1	0		
0	0	1	1	0	$A + B + \bar{C} + \bar{D}$
0	1	0	0		
0	1	0	1	0	$A + \bar{B} + C + \bar{D}$
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0	0	$\bar{A} + B + \bar{C} + D$
1	0	1	1		
1	1	0	0	0	$\bar{A} + \bar{B} + C + D$
1	1	0	1		
1	1	1	0		
1	1	1	1		

**35.** Develop a truth table for each of the standard POS expressions:

a)  $(A+B)(A+C)(A+B+C)$

Domain of this expression is A, B, C.

$$\Rightarrow A+B = A+B+C\bar{C}$$

$$= (A+B+C)(A+B+\bar{C})$$

$$\Rightarrow A+C = A+B\bar{B} + C$$

$$= \cancel{(A+B+C)}(A + \bar{B} + C)$$

$$\Rightarrow (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)$$

Inputs			Outputs	Sum Term
A	B	C	X	
0	0	0	0	$A+B+C$
0	0	1	0	$A+B+\bar{C}$
0	1	0	0	$A+\bar{B}+C$
0	1	1	1	
1	0	0	1	
1	1	0	1	
1	1	1	1	

$$\text{b) } \frac{(A+\bar{B})(A+\bar{B}+\bar{C})(B+C+\bar{D})}{(\bar{A}+B+\bar{C}+D)}$$

$$\Rightarrow A + \bar{B} = A + \bar{B} + C\bar{C}$$

$$= (A + \bar{B} + C + D\bar{D})(A + \bar{B} + \bar{C} + D\bar{D})$$

$$= (A + \bar{B} + C + D)(A + \bar{B} + C + \bar{D})$$

$$(A + \bar{B} + \bar{C} + D)(A + \bar{B} + \bar{C} + \bar{D})$$

$$\Rightarrow (A + \bar{B} + \bar{C}) = (A + \bar{B} + \bar{C} + D\bar{D})$$

$$= (A + \bar{B} + \cancel{\bar{C}} + D)(A + \bar{B} + \bar{C} + \bar{D})$$

$$\Rightarrow (B + C + \bar{D}) = (A\bar{A} + B + C + \bar{D})$$

$$= (A + B + C + \bar{D})(\bar{A} + B + C + \bar{D})$$

$$\Rightarrow (A + \bar{B} + C + D)(A + \bar{B} + C + \bar{D})$$

$$(A + \bar{B} + \bar{C} + D)(A + \bar{B} + \bar{C} + \bar{D})$$

$$(A + B + C + \bar{D})(\bar{A} + B + C + \bar{D})$$

$$(\bar{A} + B + \bar{C} + D)$$

Inputs				X	Sum Term
A	B	C	D	X	
0	0	0	0	1	
0	0	0	1	0	$A + B + C + \bar{D}$
0	0	1	0	1	
0	0	1	1	1	
0	1	0	0	0	$A + \bar{B} + C + D$
0	1	0	1	0	$A + \bar{B} + C + \bar{D}$
0	1	1	0	0	$A + \bar{B} + \bar{C} + D$
0	1	1	1	0	$A + \bar{B} + \bar{C} + \bar{D}$
1	0	0	0	1	
1	0	0	1	0	$\bar{A} + B + C + \bar{D}$
1	0	1	0	0	$\bar{A} + B + \bar{C} + D$
1	0	1	1	1	
1	1	0	0	1	
1	1	0	1	1	
1	1	1	0	1	
1	1	1	1	1	