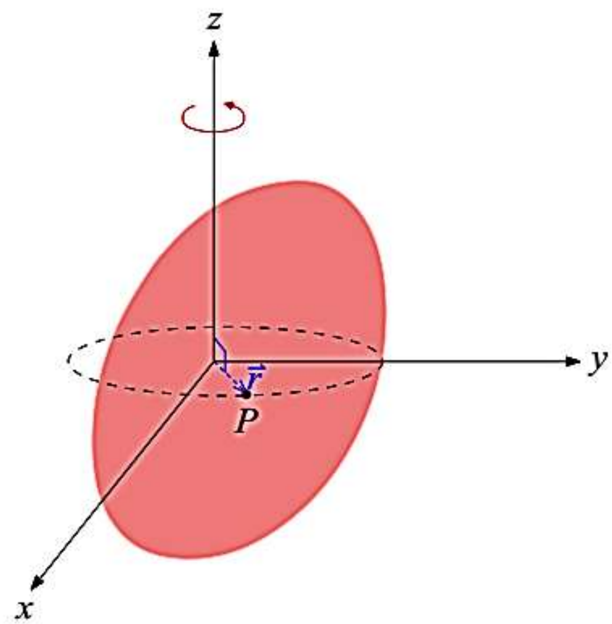
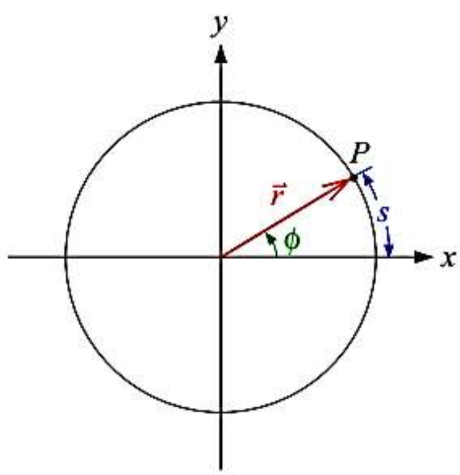
Chapter 8

Rotational Kinematics

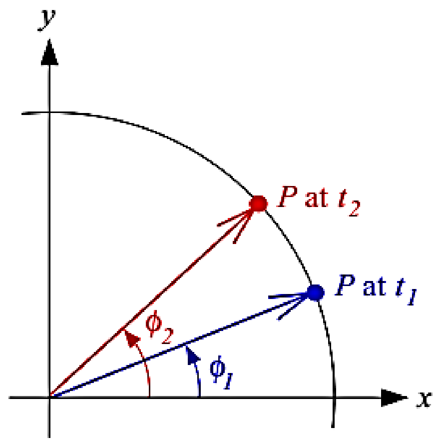


To describe the rotation of a rigid body about a fixed axis. We can observe the motion of a fixed point P in the rigid body. The motion of P is a circular motion about the axis of rotation.



arc length $s = r\phi$

angular displacement $\phi = \frac{s}{r}$ [unit: radian]



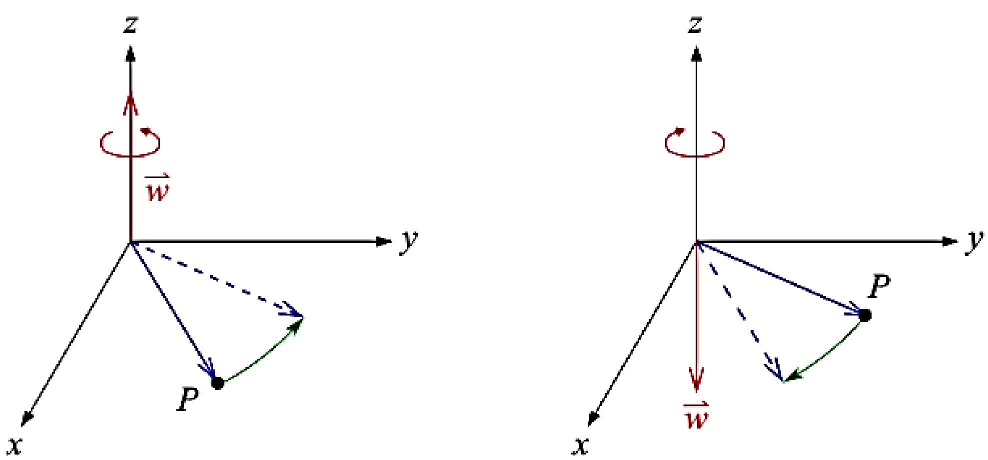
Like what is done in linear motion, average angular velocity ω_{av} and instantaneous angular velocity ω can be defined as:

$$\omega_{av} = \frac{\phi_2 - \phi_1}{t_2 - t_1} = \frac{\Delta\phi}{\Delta t} \qquad [\text{unit: rad s}^{-1}]$$
$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\phi}{\Delta t} = \frac{d\phi}{dt}$$

Similarly, average and angular instantaneous acceleration α_{av} and α are defined by:

$$\alpha_{av} = \frac{\omega(t_2) - \omega(t_1)}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \qquad [\text{unit: rad s}^{-2}]$$
$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Angular velocity as a vector



Right Hand Screw Rule

8.1 Rotation with constant angular acceleration

Suppose $\alpha = \dfrac{d\omega}{dt} = k$.

$$\therefore \quad \omega = kt + A, \qquad A = \text{constant}$$

At $t = 0$, $\omega = \omega_0 = A$ where ω_0 is the initial angular velocity.

$$\therefore \quad \boxed{\omega = \omega_0 + kt}$$

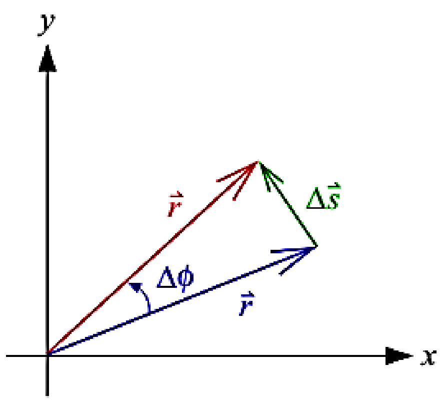
Therefore,

$$\omega = \frac{d\phi}{dt} = \omega_0 + kt \quad \Rightarrow \quad \phi = \omega_0 t + \frac{1}{2}kt^2 + B$$

At $t = 0$, $\phi = \phi_0 = B$ where ϕ_0 is the initial angular displacement.

$$\therefore \quad \boxed{\phi = \phi_0 + \omega_0 t + \tfrac{1}{2}kt^2}$$

8.2 Relation between linear and angular variables



- In a time interval Δt, the rotating vector $\vec{r}$ moves through an angle $\Delta\phi$.
- If $\Delta t \to 0$, $|\Delta\vec{s}| = \Delta s = r\Delta\phi$.

Thus the tangential velocity is:

$$v_T = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = r\frac{d\phi}{dt}$$

$$\therefore \quad \boxed{v_T = \omega r}$$

Moreover, tangential acceleration is given by:

$$a_T = \frac{dv_T}{dt} = r\frac{d\omega}{dt}$$

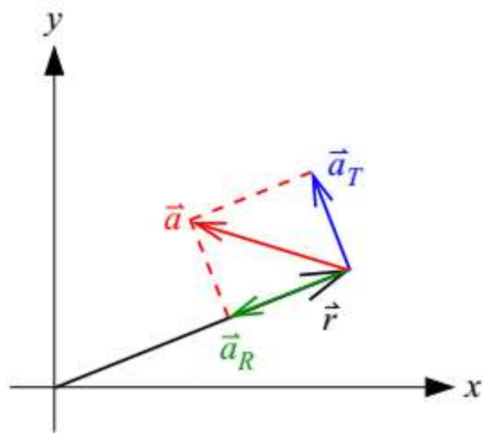
$$\therefore \quad \boxed{a_T = \alpha r}$$

From previous chapters, we also know for particle undergoing circular motion with constant speed, the particle indeed accelerates toward the center (centripetal acceleration). Thus the radial acceleration is equal to:

$$\boxed{a_R = \frac{v_T^2}{r} = \omega^2 r}$$

Hence, the resultant acceleration:

$$\vec{a} = \vec{a}_T + \vec{a}_R$$



N. B. For uniform circular motion, $\alpha = 0 \Rightarrow \vec{a} = \vec{a}_R$, pure radial force!!