UNIVERSITY OF KARACHI

MATHEMATICS III

BSCS-405

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UNIVERSITY OF KARACHI

EXERCISE 10.1

Date:

Q5 $x = 2\cos t$, $y = 5\sin t$ $(0 \leq t \leq 2\pi)$

SOLUTION:

$\frac{x}{2} = \cos^2 t$, $\frac{y}{5} = \sin^2 t$.

$\Rightarrow \frac{x}{4} + \frac{y}{25} = 1$ $\Rightarrow \frac{x^2}{4} = 4 - \frac{4y^2}{25}$

$x = \sqrt{4 - \frac{4y^2}{25}}$

t	0	$\pi/2$	π	$3\pi/2$	2π
x	2	0	-2	0	2
y	0	5	0	-5	0

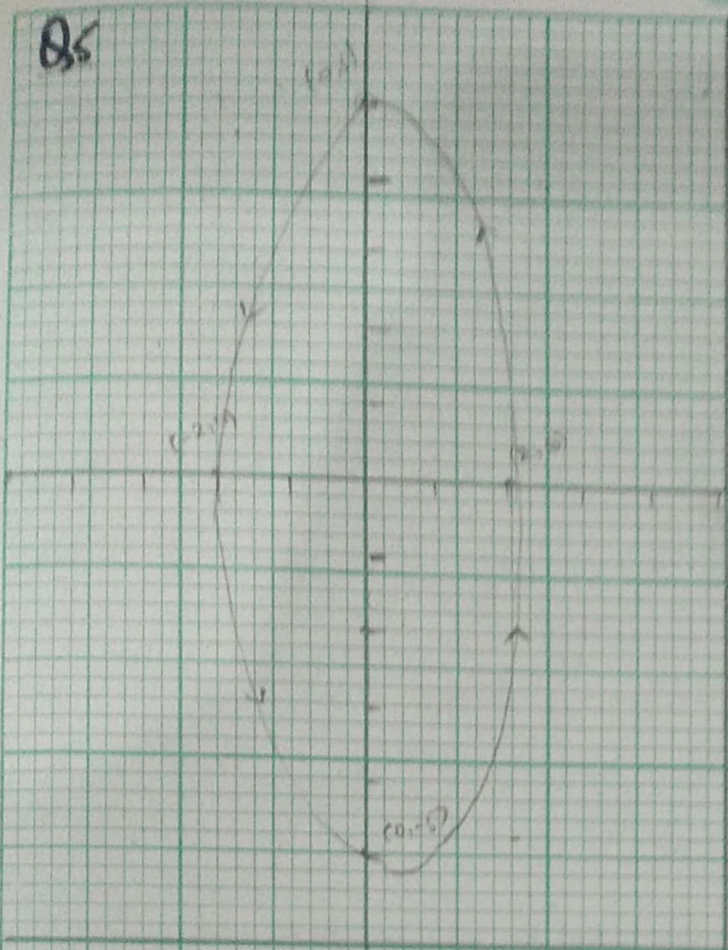
Q7 $x = 3 + 2\cos t$, $y = 2 + 4\sin t$ $(0 \leq t \leq 2\pi)$

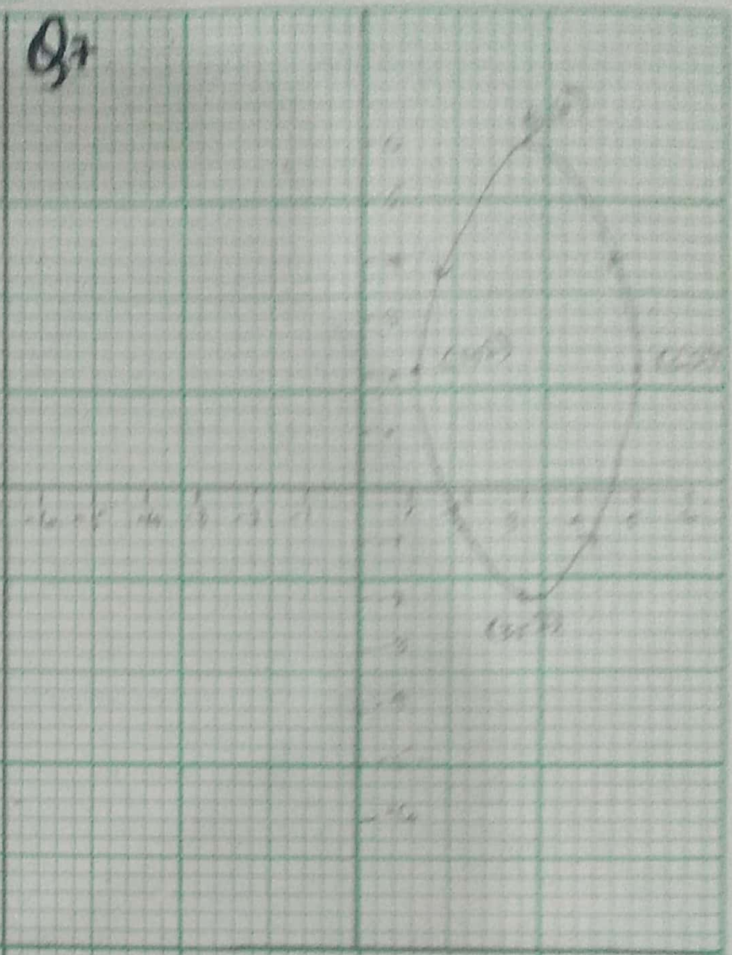
SOLUTION:-

$\frac{(x-3)^2}{4} = \cos^2 t$, $\frac{(y-2)^2}{4} = \sin^2 t$

$\Rightarrow \frac{(x-3)^2}{4} + \frac{(y-2)^2}{4} = 1$.

t	0	$\pi/2$	π	$3\pi/2$	2π
x	5	3	1	3	5
y	2	6	2	-2	2


Q5


Q7

Date:

Q11 $x = 2\sin^2 t$, $y = 3\cos^2 t$ $(0 \le t \le \pi/2)$

t	0	$\pi/5$	$\pi/4$	$\pi/3$	$\pi/2$
x	0	0.69	1	1.5	2
y	3	1.96	1.5	0.75	0

$$\frac{x}{2} + \frac{y}{3} = 1$$

$$3x + 2y - 6 = 0$$

Q13 A circle of radius 5, centered at origin, oriented clock-wise.

Eq^n of circle.

$x = a\cos t$, $y = a\sin t$

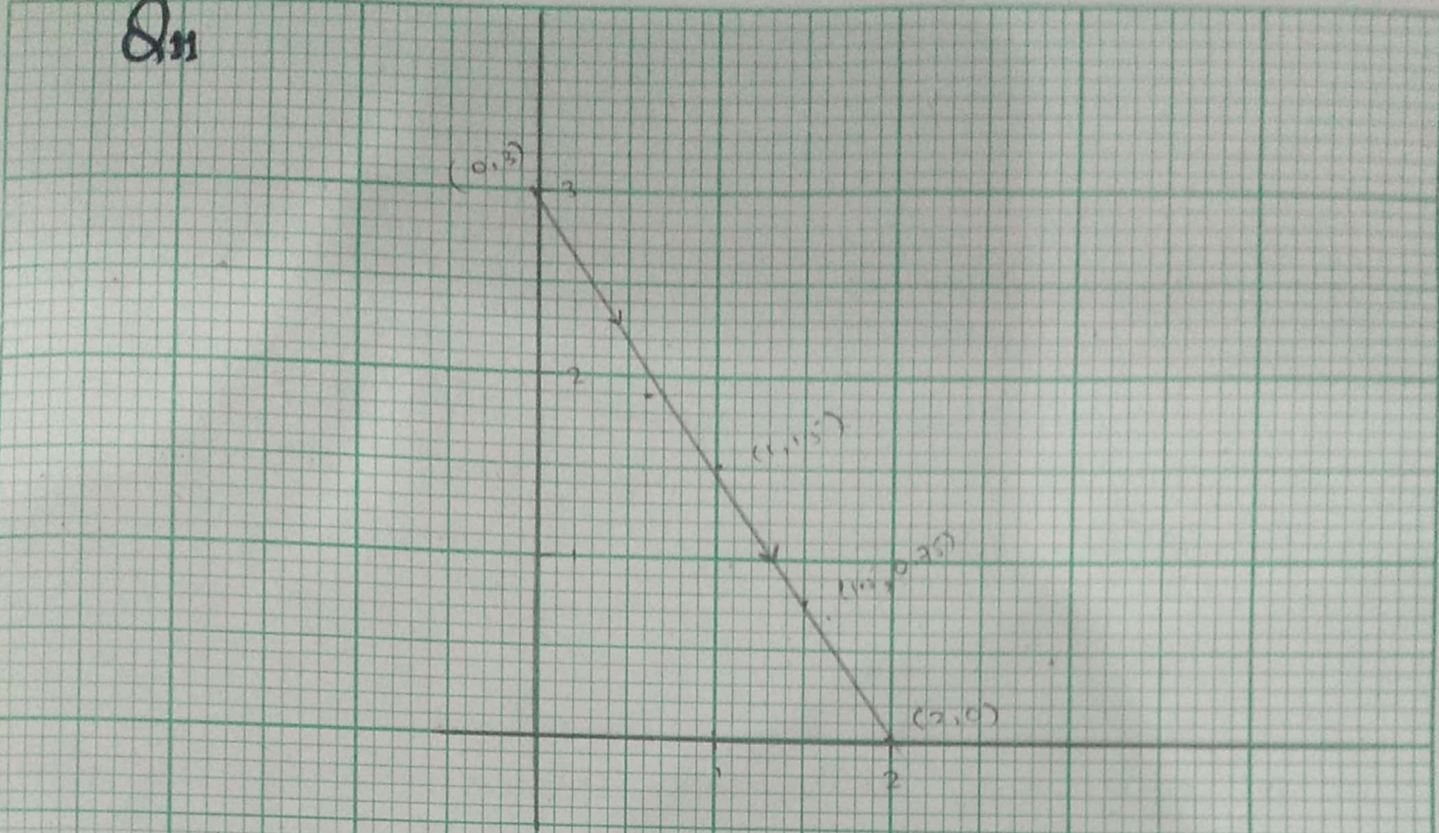
Since circle is oriented clockwise and radius is 5 i.e $t = -t$ & $a = 5$

$x = 5\cos t$, $y = -5\sin t$

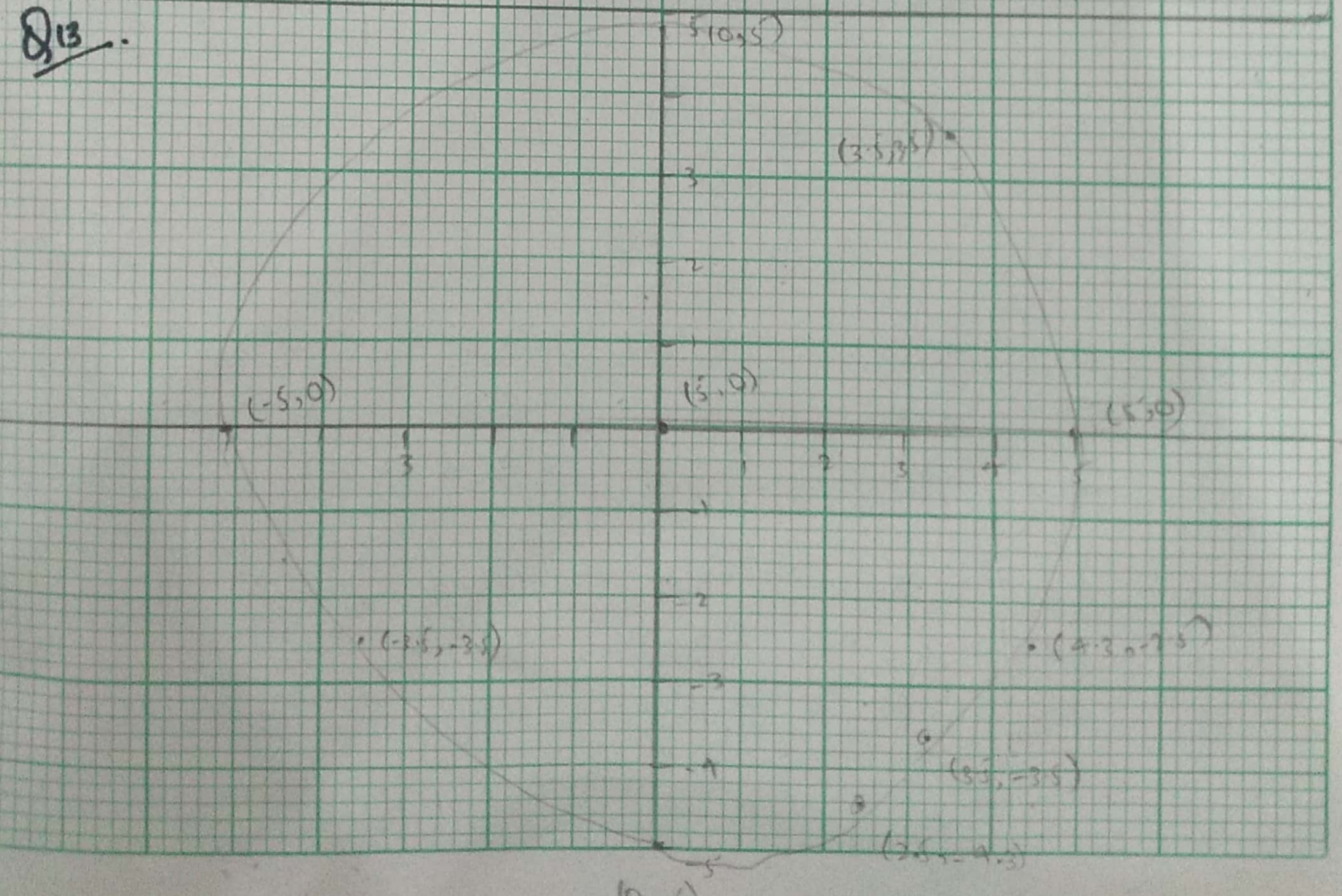
t	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
x	5	4.3	3.5	2.5	0
y	0	-2.5	-3.5	-4.3	-5

t	$3\pi/4$	π	$3\pi/2$	$7\pi/4$	2π
x	-3.5	-5	0	3.5	5
y	-3.5	0	5	3.5	0

Q11



Q13.



EXERCISE 10.1

~~Q14 $x = \frac{1}{2}t^2 + 1$, $y = \frac{1}{3}t^3 - t$; t~~

Q15 $x = \sqrt{t}$, $y = 2t + 4$; $t = 1$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{2}{1/2\sqrt{t}}$$

$$\frac{dy}{dx} = 4\sqrt{t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \Big/ \frac{dx}{dt}$$

$$= \frac{d}{dt}4\sqrt{t} \Big/ \frac{d\sqrt{t}}{dt}$$

$$= \frac{2/\sqrt{t}}{1/2\sqrt{t}}$$

$$\frac{d^2y}{dx^2} = 4 \Big|_{t=1}$$

$$\frac{d^2y}{dx^2} = 4$$ Ans

Q49 $x = \theta + \cos\theta$, $y = 1 + \sin\theta$; $\theta = \pi/6$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} =$$

$$\frac{dy}{dx} = \frac{\cos\theta}{1-\sin\theta} = \cancel{\cot\theta}$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right) \Big/ \frac{dx}{d\theta}$$

$$= \frac{(1-\sin\theta)(-\sin\theta) - \cos\theta(-\cos\theta)}{(1-\sin\theta)^2} \cdot \frac{1}{(1-\sin\theta)}$$

$$= \frac{(1-\sin\theta)(-\sin\theta) + \cos^2\theta}{(1-\sin\theta)^2(1-\sin\theta)}$$

$$= \frac{-\sin\theta + \sin^2\theta + \cos^2\theta}{(1-\sin\theta)^3}$$

$$= \frac{-\sin\theta + 1}{(1-\sin\theta)^3} = \frac{(1-\sin\theta)}{(1-\sin\theta)^3}$$

$$\frac{d^2y}{dx^2} = \frac{1}{(1-\sin\theta)^2}$$ Ans!

Q51. (a) $x = e^t$, $y = e^{-t}$

$$\frac{dx}{dt} = e^t \qquad \frac{dy}{dt} = -e^{-t}$$

$$\frac{dy}{dx} = \frac{-e^{-t}}{e^t} \qquad (\text{At } t=1)$$

$$\frac{dy}{dx} = -e^{-2t}$$

$y - y_1 = m(x - x_1)$

$y - e^{-t} = -e^{-2}(x - e^{t})$

$y = e^{-1} - e^{-2}x + \cancel{e^{-t}}$

$\boxed{y = 2e^{-1} - e^{-2}x}$

(b) $x = e^t \qquad y = e^{-t}$

$1/x = e^{-t} \qquad y = e^{-t}$

$$\frac{1}{x} = y \Rightarrow \frac{dy}{dx} = \frac{-1}{x^2} = \frac{1}{e^2}$$

$$y - e^{-1} = \frac{1}{e^2}(x - e)$$

$$\boxed{y = \frac{-x}{e^2} + \frac{2}{e}}$$

Ans

Q53 $x = 2\sin t$, $y = 4\cos t$ $(0 \le t \le 2\pi)$

$\frac{dx}{dt} = 2\cos t$, $\frac{dy}{dt} = -4\sin t$.

$\frac{dy}{dx} = \frac{-4\sin t}{2\cos t} = -2\tan t$.

(a)

for horizontal tangent line,

$\frac{dy}{dx} = 0$, If $\tan t = 0$.

$\tan t = 0$ when $t = 0, \pi, 2\pi$

(b) for vertical tangent line,

$\frac{dx}{dt}$ must be equals to 0.

$2\cos t = 0$

when $t = \frac{\pi}{2}, \frac{3\pi}{2}$.

Q65 $x = t^2$, $y = 1/3t^3$ $(0 \le t \le 1)$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^1 \sqrt{\left(\frac{d(t^2)}{dt}\right)^2 + \left(\frac{d}{dt}\frac{1}{3}t^3\right)^2}\, dt$$

$$= \int_0^1 \sqrt{(2t)^2 + (t^2)^2}\, dt$$

$$= \int_0^1 \sqrt{(4t^2 + t^4)}\, dt$$

$$L = \int_0^1 t\sqrt{4 + t^2}\, dt$$

let $U = 4 + t^2$ $\quad t = 0$, $t = 1$

$\frac{du}{dt} = 2t$ $\quad U = 4$ $\quad U = 5$

$\frac{du}{2} = t\, dt$

$$\Rightarrow L = \int_4^5 \sqrt{U}\, \frac{du}{2}$$

$$= \frac{1}{\cancel{2}} \frac{\cancel{2}}{3} U^{3/2} \Big|_4^5$$

$$= \frac{1}{3}\left[(5)^{3/2} - (4)^{3/2}\right]$$

$$= \frac{1}{3}\left[5\sqrt{5} - 8\right]$$ Ans

Q67 $x = \cos 3t$, $y = \sin 3t$ $(0 \le t \le \pi)$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$$

$$= \int_0^{\pi} \sqrt{(-3\sin 3t)^2 + (3\cos 3t)^2} \cdot dt$$

$$= \int_0^{\pi} 3\sqrt{\sin^2 3t + \cos^2 3t}\ dt$$

$$= 3 \int_0^{\pi} dt$$

$$= 3\ t \Big|_0^{\pi}$$

$$= 3\pi \quad \text{Ans!}$$

Q26. $P(2,-1)$, $Q(3,1)$

first finding slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1+1}{3-2}$$

$$m = \frac{2}{1} \quad \begin{matrix}(b)\\(a)\end{matrix}$$

$\Rightarrow$ $b = 2$, $a = 1$

point $P(2,-1) = P(x_0, y_0)$

$x = x_0 + at$, $y = y_0 + bt$

$x = 2 + (1)t$, $y = -1 + (2)t$

$x = 2 + t$, $y = -1 + 2t$

(a) The mid point between P and Q.

To find the mid point between P and Q, put $t = 1/2$

$x = 2 + 1/2$, $y = -1 + 2(1/2)$

$x = \frac{5}{2}$, $y = 0$

The mid point is $(5/2, 0)$

(b) The point that is one fourth of the way from P to Q

Put $t = 1/4$

$x = 2 + \frac{1}{4}$ $y = -1 + 2(1/4)$

$x = \frac{9}{4}$ $y = -\frac{1}{2}$.

$S\left(\frac{9}{4}, -\frac{1}{2}\right)$

(c) The point that is 3/4 the way from P to Q.

put $t = 3/4$

$x = 2 + \frac{3}{4}$ $\quad$ $y = -1 + 2(3/4)$

$x = \frac{11}{4}$ $\quad$ $y = \frac{1}{2}$

$T\left(\frac{11}{2}, \frac{1}{2}\right)$ A1

Q33 False

Q34 False

Q35 True.

Q36 False

EXERCISE 10.2

Q3 (a) $(6, \pi/6)$

$(r, \theta) = (6, \pi/6)$

$x = r\cos\theta$, $y = r\sin\theta$

$x = 6\cos(\pi/6)$ $y = 6\sin(\pi/6)$

$x = 3\sqrt{3}$ $y = 3$

rectangular cordinates are $(3\sqrt{3}, 3)$

Q5 (b) $(2\sqrt{3}, -2)$

$r^2 = x^2 + y^2$

$r^2 = (2\sqrt{3})^2 + (-2)^2$

$r^2 = 16$

$r = 4$

$\theta = \tan^{-1}(y/x) = \tan^{-1}(-2/2\sqrt{3})$

$\theta = -\frac{\pi}{6}$ on $\frac{11\pi}{6}$

$\left(4, \frac{-\pi}{6}\right) \left(4, \frac{11\pi}{6}\right)$ Ans!

Q9 (a) $r = 2$

$r^2 = x^2 + y^2$
$4 = x^2 + y^2$

The curve is circle.

(b) $r\sin\theta = 4$

$\therefore y = r\sin\theta$
$y = 4$

line.

(c) $r = 3\cos\theta$.

$r^2 = 3r\cos\theta$
$r^2 = 3x$.

$x^2 + y^2 = 3x$.
$x^2 - 3x + y^2 = 0$.
$x^2 - 3x + \left(\frac{3}{2}\right)^2 + y^2 = \left(\frac{3}{2}\right)^2$

$\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$
circle

(d) $r = \dfrac{6}{3\cos\theta + 2\sin\theta}$

$3r\cos\theta + 2r\sin\theta = 6$

$3x + 2y = 6$

line Ans!

Q11. (a) $x = 3$

$x = r\cos\theta$

$r\cos\theta = 3$

$x^2 + y^2 = 7$

$r^2 = x^2 + y^2$

$\sqrt{7} = x^2 + y^2.$

Q17 (a)

Give $r = 5$

$x^2 + y^2 = (5)^2$

$x^2 + y^2 = 25$

$r = 5$

(b) A circle is not at origin

$(x-h)^2 + (y-k)^2 = r^2$

$(x-3)^2 + (y-0)^2 = r^2$

$x^2 - 6x + 9 + y^2 = r^2$

$r^2\cos^2\theta - 6r\cos\theta + 9 + r^2\sin^2\theta = r^2$

$r^2(\cos^2\theta + \sin^2\theta) - 6r\cos\theta + 9 = r^2$

$r^2 - 6r\cos\theta + 9 = \cancel{r^2}\,(3)^2$

$\cancel{\dfrac{9^{3}}{{}_{2}6\cos\theta} = r}$

$\cancel{\dfrac{3}{2\cos\theta} = r}$

$r^2 - 6r\cos\theta + \cancel{9} = \cancel{9}$

$r^2 - 6r\cos\theta = 0$

$\boxed{r = 6\cos\theta}$ A-1

(c) for a cardiod

$r = a(1-\cos\theta)$ ∵ $a=1$

$r = 1-\cos\theta$ Ans.

Q19 (a) For a 4-petal rose.

$r = a\sin n\theta$.

∴ $a=3$, $n=2$.

$r = 3\sin 2\theta$.

(b) For limicon.

$r = a + b\sin\theta$.

∵ $a = 3$, $b = 5-a = 2$

$r = 3 + 2\sin\theta$.

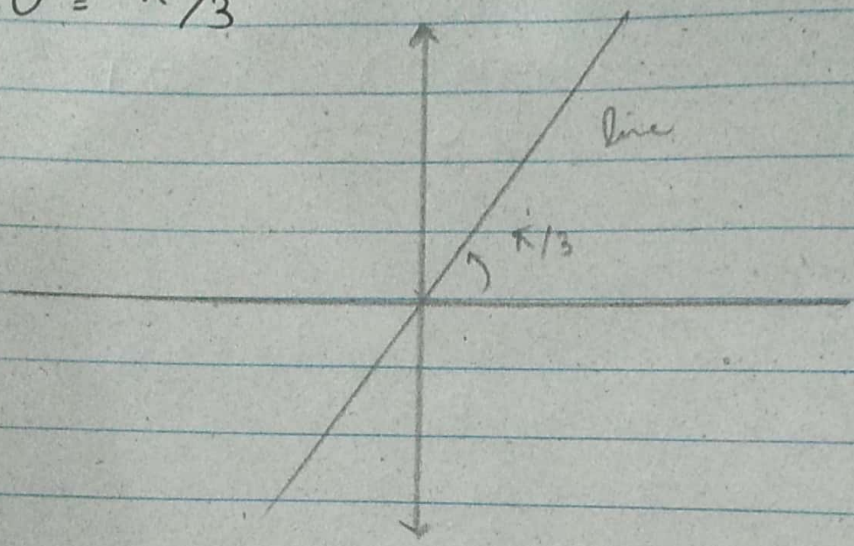
(c) for a lemniscate.

$r = a\sqrt{\cos 2\theta}$.

$r = 3\sqrt{\cos 2\theta}$.

$r^2 = 9\cos 2\theta$

Q21 $\theta = \pi/3$



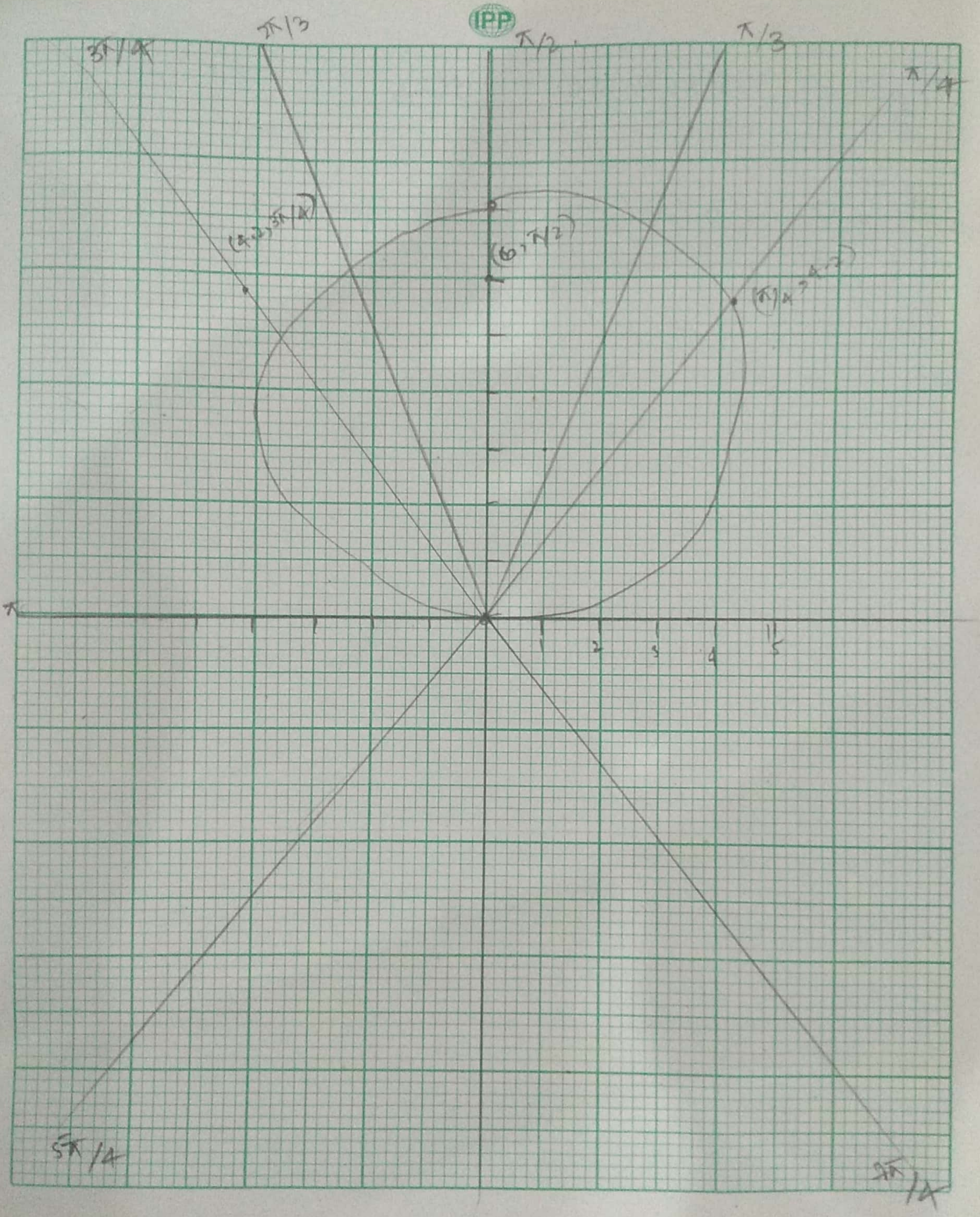
Date:

EXERCISE 10.2

Q. $r = 6\sin\theta$

		45	90	135	180	225
θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$
r	0	4.2	6	4.2	0	-4.24

	135	315	360	60	120
θ	$3\pi/4$	$7\pi/4$	2π	$\pi/3$	$2\pi/3$
r	4.2	-4.2	0	5.1	5.1



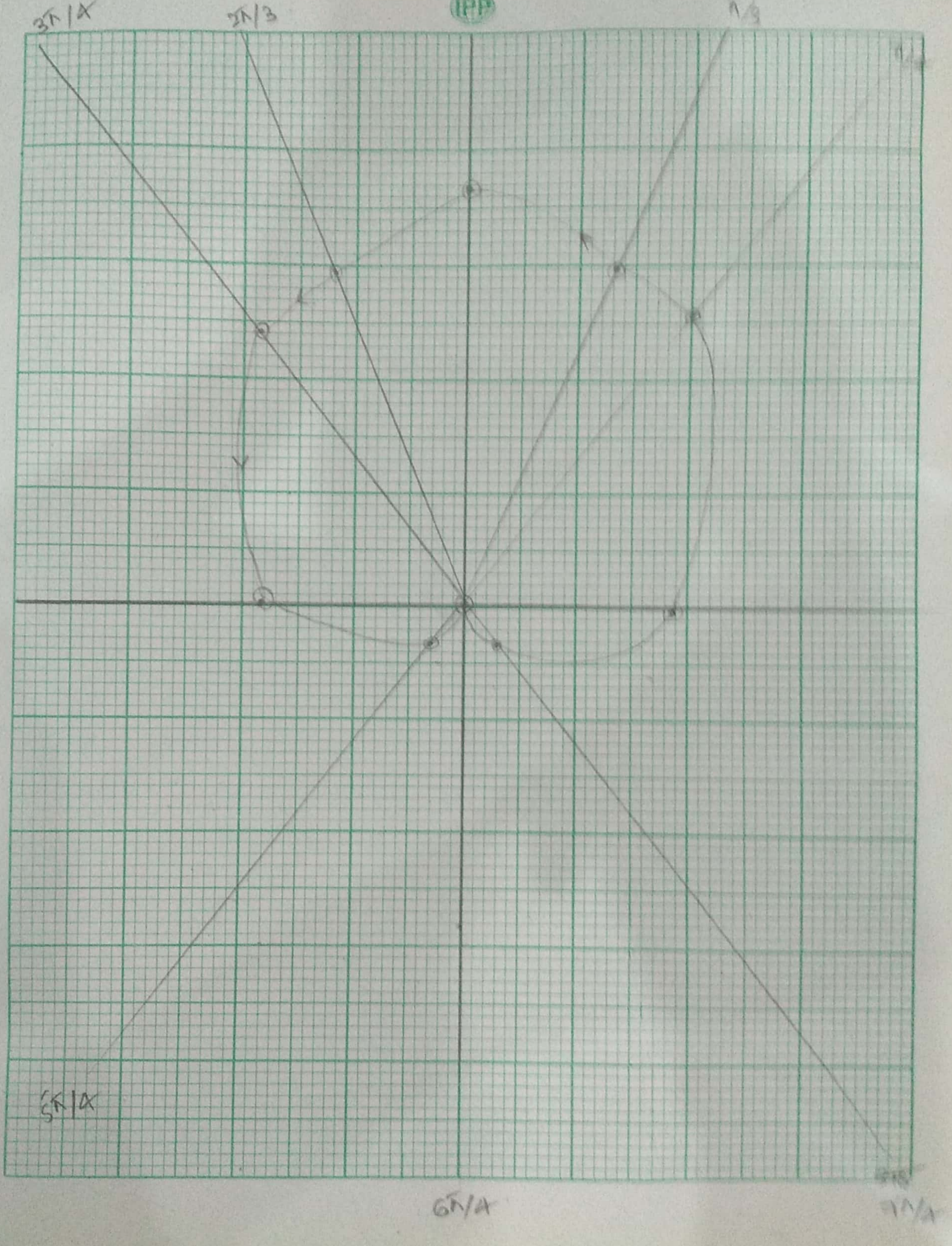
$3\pi/4$
$2\pi/3$
$\pi/2$
$\pi/3$
$\pi/4$
$(4.2, 3\pi/4)$
$(6, \pi/2)$
π
1
2
3
4
5
$5\pi/4$
$7\pi/4$

Date:

Q27 $r = 3(1 + \sin\theta)$

θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$6\pi/4$
r	3	5.1	5.6	6	5.1	3	0.8	0

$7\pi/4$	2π	$2\pi/3$.
0.8	3	5.5

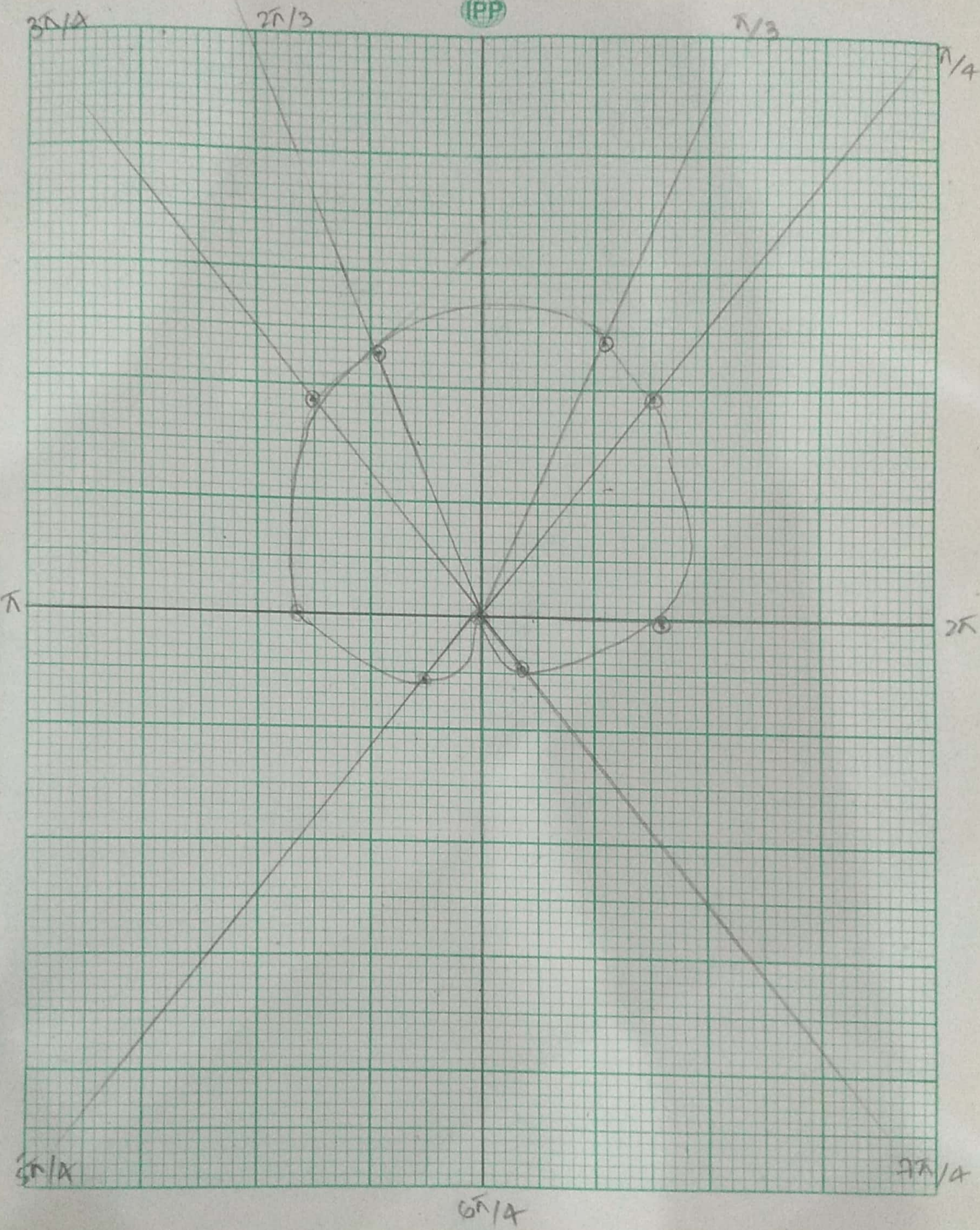

3π/4
2π/3
π/3
π/4
5π/4
6π/4
7π/4

Date:

Q $r = 1 + \sin\theta$

θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π	$5\pi/4$
r	1	1.7	1.8	2	1.8	1.7	1	0.29

$6\pi/4$	$7\pi/4$	2π
0	0.2	1


3π/4
2π/3
IPP
π/3
π/4
π
2π
5π/4
7π/4
6π/4

Q25

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
r	0	4.2	6	4.2	0	-4.2	-6	-4.2	0

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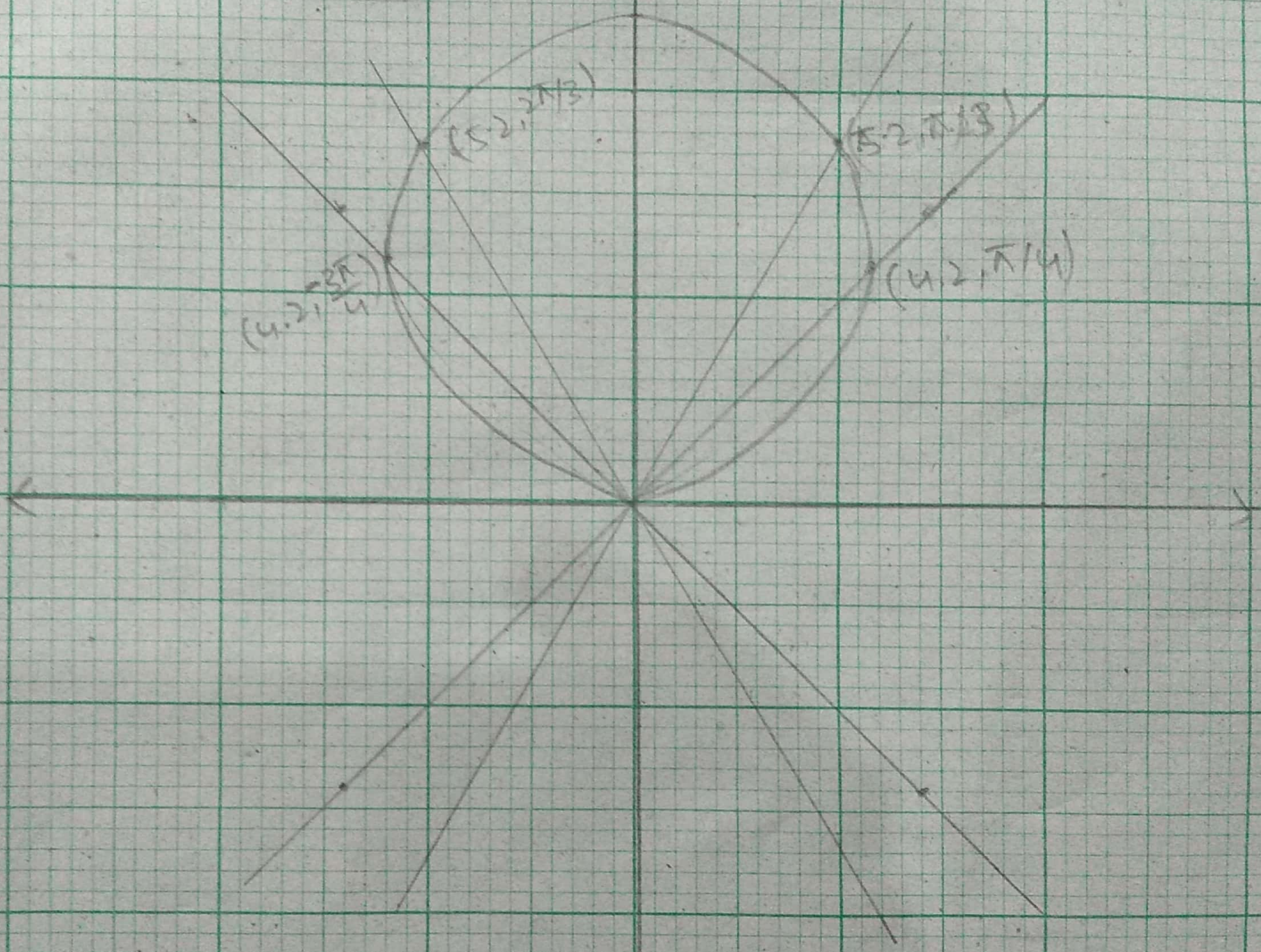
QUESTION: 25



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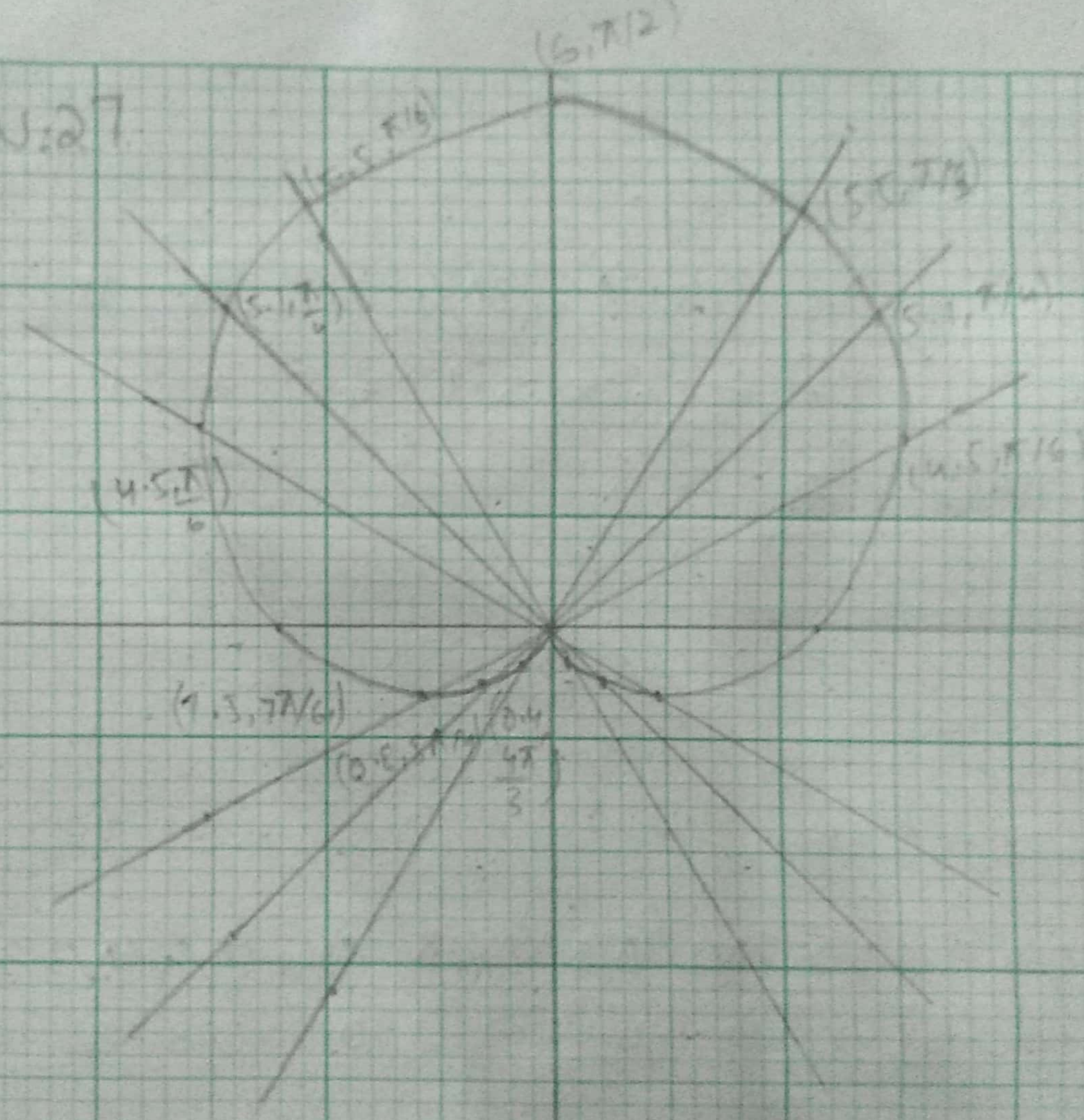
Q27

$$r = 3(1 + \sin\theta)$$

θ	0	$\pi/3$	$\pi/4$	$\pi/6$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π	$7\pi/6$	$3\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$
r	3	5.6	5.1	4.5	6	5.6	5.1	4.5	3	1.5	0.8	0.4	0	0.4

θ	$7\pi/4$	$11\pi/6$	2π
r	0.87	1.5	3

QUESTION:27



31.

Q31

$r = -1 - \cos\theta$.

θ	0	$\pi/3$	$\pi/4$	$\pi/6$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π	$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	$11\pi/6$
r	-2	-15	-1.7	-1.5	-1	-0.5	-0.3	-0.1	0	-0.1	-0.3	-0.5	-1	-1.5	-1.7	-1.86

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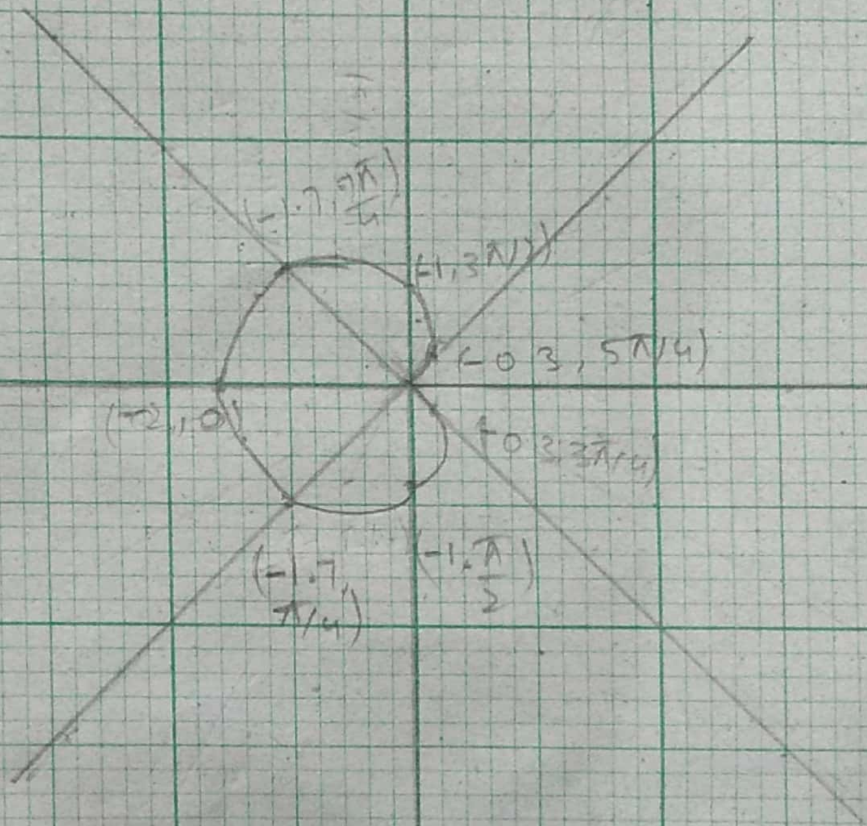
31.



Q33

$r = 3 - \sin\theta$.

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
r	3	2.3	2	2.3	3	3.7	4	3.7	3

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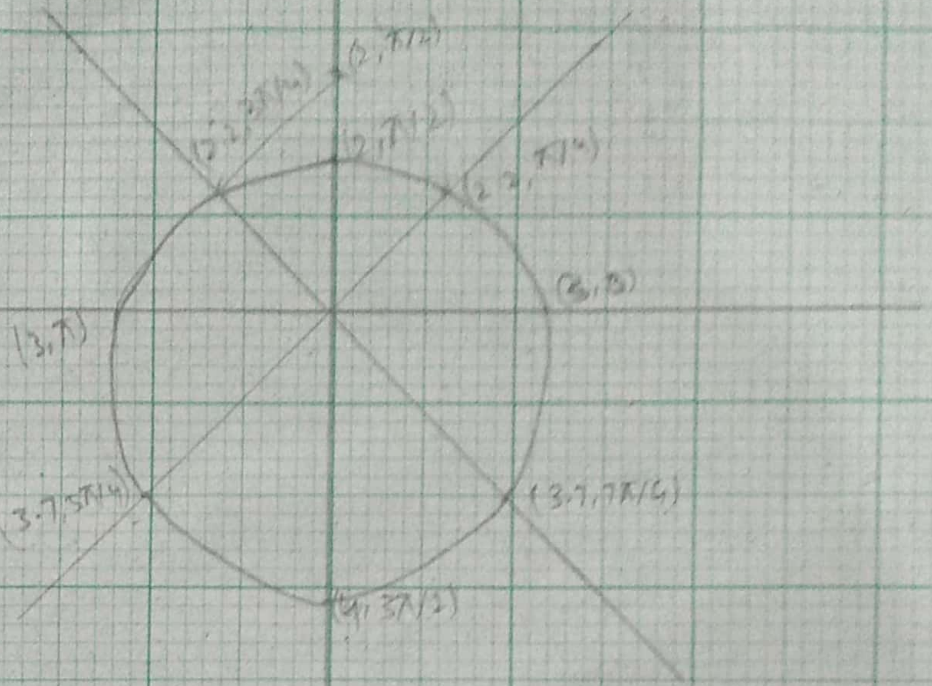
QUESTION : 33



Fine

39. $r^2 = 16\sin 2\theta$

$r = \pm 4\sqrt{\sin 2\theta}$

r	θ
0	0
3.8	$\pi/3$
0	$\pi/2$
4	$3\pi/4$
0	π
3.8	$4\pi/3$
0	$3\pi/2$
3.8	$2\pi/3$
0	2π

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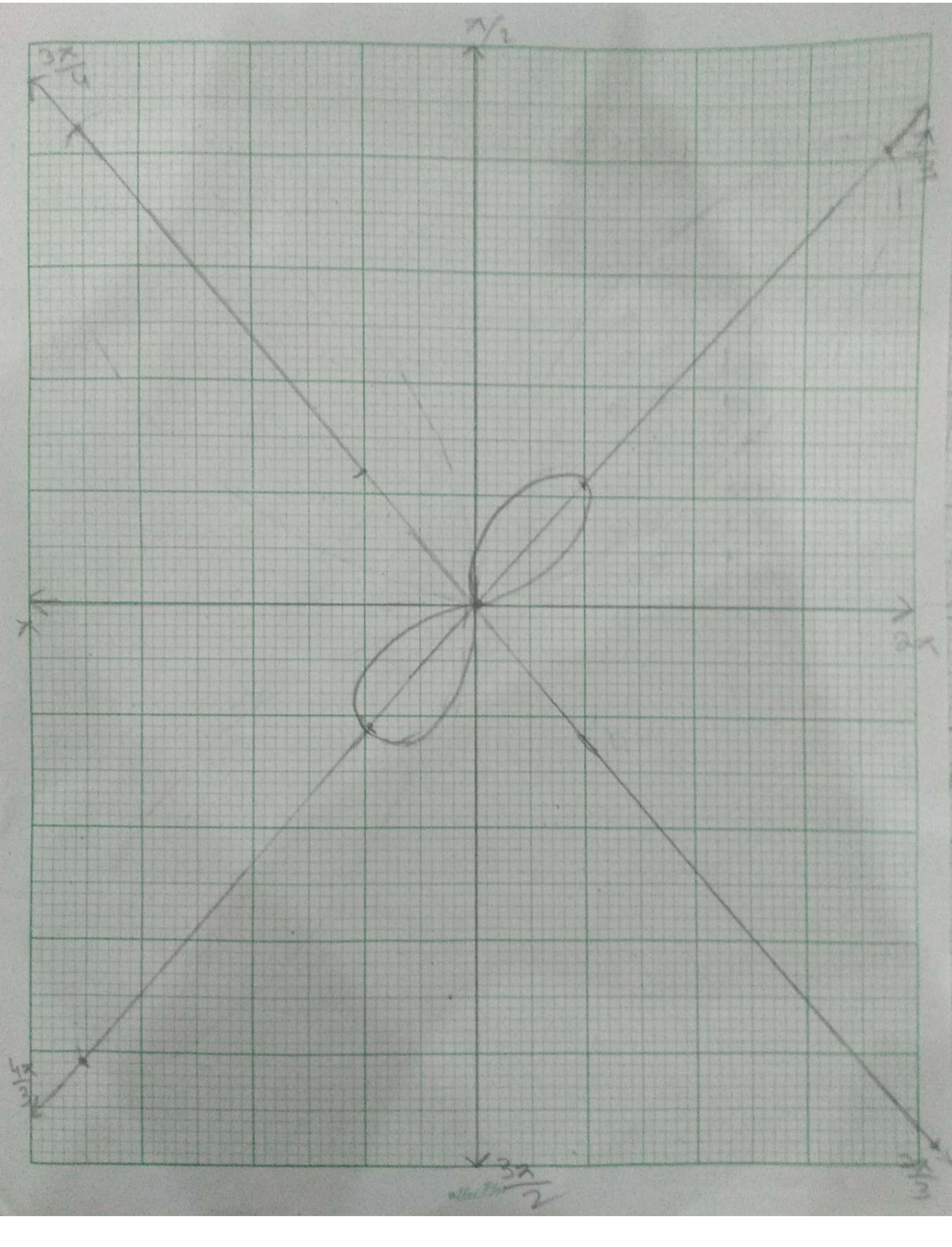
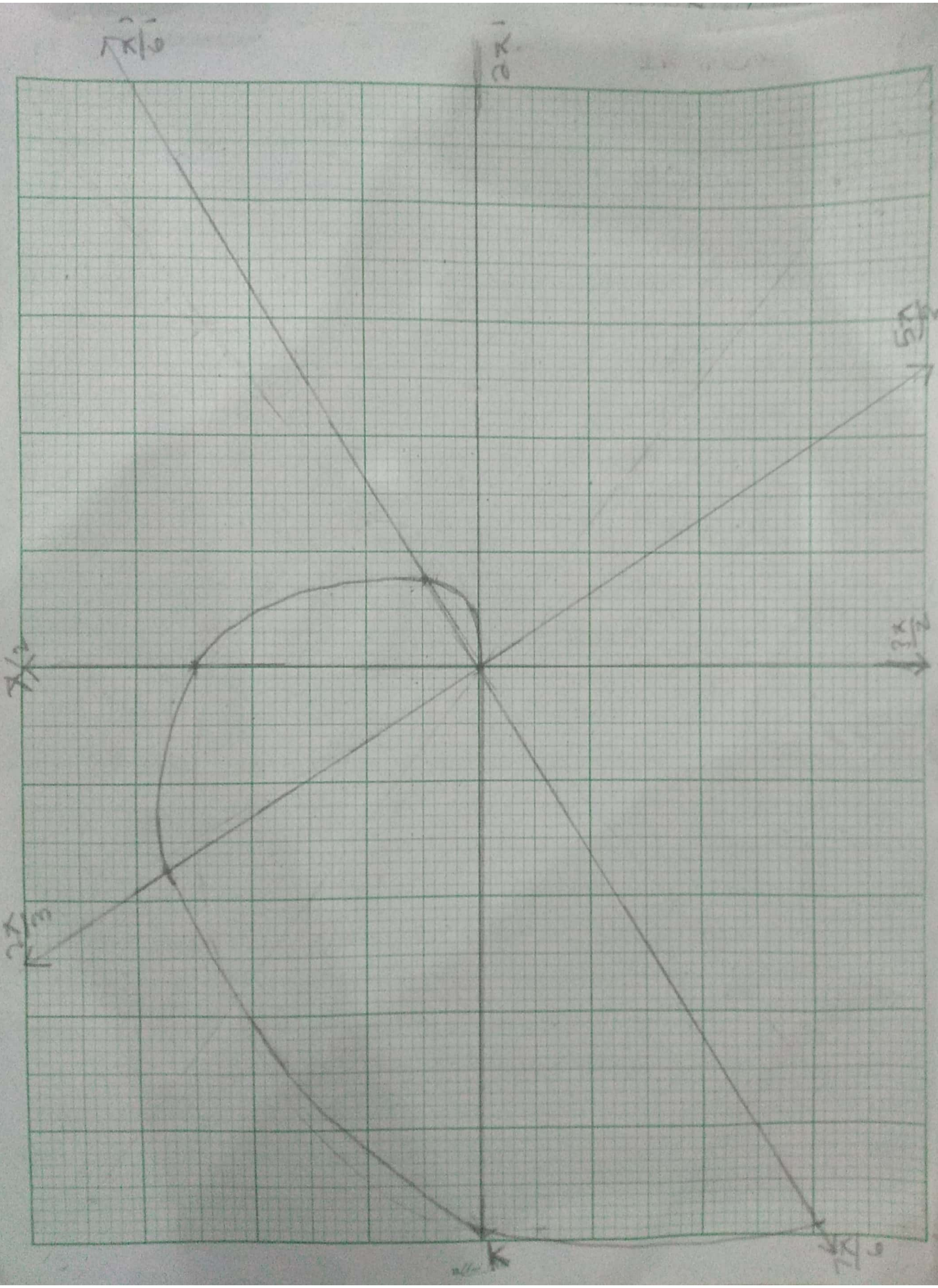

π/2
3π/4
3π/2

41. $r = 4\theta \quad (\theta \leq 0)$

$r=$	θ
0	0
2	$\pi/6$
6.2	$\pi/2$
8.3	$2\pi/3$
12.5	π
14.6	$7\pi/6$
18.8	$3\pi/2$
20.9	$5\pi/3$
25	2π



EXERCISE 10.3

Q7 $r = 2 + 2\sin\theta$

$x = r\cos\theta \Rightarrow x = 2\cos\theta + 2\sin\theta\cos\theta$

$y = r\sin\theta \Rightarrow y = 2\sin\theta + 2\sin^2\theta$

$$\frac{dx}{d\theta} = -2\sin\theta + 2\,[\sin\theta(-\sin\theta) + \cos\theta\cos\theta]$$

$$= -2\sin\theta - 2\sin^2\theta + 2\cos^2\theta$$

$$\frac{dx}{d\theta} = -2\,(\sin^2\theta + \cos^2\theta + \sin\theta)$$

$$\frac{dy}{d\theta} = 2\cos\theta + 2\sin\theta\cos\theta$$

$$m = \frac{dy/d\theta}{dx/d\theta} = \frac{2\cos\theta + 2\sin\theta\cos\theta}{-2\,(\sin^2\theta - \cos^2\theta + \sin\theta)}$$

$$= \frac{\cos\theta + 2\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta + \sin\theta}$$

Slope at $\theta = 0, \pi/2, \pi$

$m = +, 0, -1$

8a $r = a(1+\cos\theta)$

$x = r\cos\theta \Rightarrow a(1+\cos\theta)\cos\theta$

$y = r\sin\theta \Rightarrow a(1+\cos\theta)\sin\theta$.

$x = a\cos\theta + a\cos^2\theta$.

$y = a\sin\theta + a\sin\theta\cos\theta$

$\frac{dx}{d\theta} = -a\sin\theta - 2a\sin\theta\cos\theta$

$\frac{dy}{d\theta} = a\cos\theta - 2a(\sin\theta(-\sin\theta) + \cos\theta\cos\theta)$

$= a\cos\theta - 2a(-\sin^2\theta + \cos^2\theta)$.

$= a\cos\theta + 2a\sin^2\theta - 2a\cos^2\theta$.

$= a(\cos\theta + \sin^2\theta - \cos^2\theta)$

$= a(\cos\theta + 1 - \cos^2\theta - \cos^2\theta)$

$\frac{dy}{d\theta} = a(\cos\theta - 2\cos^2\theta + 1)$

Tangent line is horizontal when $\frac{dy}{d\theta} = 0$ &

$\frac{dx}{d\theta} \neq 0$

$\theta = \pi$

Q19 The entire circle $r = a$

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$$

$$\frac{dr}{d\theta} = \frac{da}{d\theta} = 0$$

Because it is a complete circle so the limits are $(0, 2\pi)$

$$L = \int_0^{2\pi} \sqrt{a^2 + 0} \; d\theta$$

$$= \int_0^{2\pi} a \, d\theta = \cancel{\frac{a^2}{2}\Big|_0^{2\pi}}$$

$$= \cancel{\frac{1}{2}(2\pi)^2} - a\,|\theta|_0^{2\pi}$$

$$= a(2\pi - 0)$$

$$\boxed{L = 2\pi a} \text{ Ans!}$$

Q19 The entire circle $r = a$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

$$\frac{dr}{d\theta} = \frac{da}{d\theta} = 0$$

Because it's the complete circle so the limits will be $(0, 2\pi)$

$$L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$$

$$= \int_0^{2\pi} \sqrt{a^2 + 0} \, d\theta$$

$$= \int_0^{2\pi} a \, d\theta$$

$$= a(2\pi - 0)$$

$$\boxed{L = 2\pi a}$$ Ans!

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Q21 The entire cardiod $r = a(1-\cos\theta)$

Range of entire cardiod $(0, 2\pi)$

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$\frac{dr}{d\theta} = \frac{d\,a(1-\cos\theta)}{d\theta} = 0 + a\sin\theta$$

$$\frac{dr}{d\theta} = a\sin\theta$$

$$L = \int_0^{2\pi} \sqrt{[a(1-\cos\theta)]^2 + (a\sin\theta)^2}\, d\theta$$

$$L = \int_0^{2\pi} a\sqrt{(1-2\cos\theta+\cos^2\theta+\sin^2\theta)}\, d\theta$$

$$= \int_0^{2\pi} a\sqrt{2-2\cos\theta}\, d\theta$$

$$= \sqrt{2}\,a \int_0^{2\pi} \sqrt{1-\cos\theta}\, d\theta$$

$$\because \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\sqrt{2}\sin\frac{\theta}{2} = \sqrt{1-\cos\theta}$$

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$$L = \sqrt{2}a\int_0^{2\pi} \sqrt{2}\,\sin\frac{\theta}{2}\,d\theta.$$

$$= 2a\int_0^{2\pi} \sin\frac{\theta}{2}\,d\theta.$$

$$= 2a \times 2 \left| -\cos\frac{\theta}{2} \right|_0^{2\pi}$$

$$= 4a(+1+1)$$

$$L = 8a \quad \text{Ans!}$$

Q25

$$A = \frac{1}{2}\int_\alpha^\beta r^2\,d\theta.$$

(a) $r = 1-\cos\theta$.

$$A = \frac{1}{2}\int_{\pi/2}^{\pi} (1-\cos\theta)^2\,d\theta$$

(b) $r = 2\cos\theta$

$$A = \frac{1}{2}\int_0^{\pi/2} (2\cos\theta)^2\,d\theta.$$

c) = $r = \sin 2\theta$.

$$A = \frac{1}{2}\int_0^{\pi/2} (\sin 2\theta)^2 \, d\theta$$

d) $r = \theta$

$$A = \frac{1}{2}\int_0^{2\pi} (\theta)^2 \, d\theta$$

e) $r = 1 - \sin\theta$

$$A = \frac{1}{2}\int_{-\pi/2}^{\pi/2} (1 - \sin\theta)^2 \, d\theta$$

f) $r = \cos 2\theta$.

$$A = \frac{1}{2}\int_{-\pi/4}^{\pi/4} (\cos 2\theta)^2 \, d\theta$$

Q27 (a) $r = 2a\sin\theta$

SOLUTION:-

for limits

$$2a\sin\theta = 0$$

$$\theta = \sin^{-1}(0)$$

θ is 0 at $0, \pi$

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2\, d\theta$$

$$A = \frac{1}{2}\int_{0}^{\pi} (2a\sin\theta)^2\, d\theta$$

$$= 2a^2 \int_0^{\pi} \sin^2\theta\, d\theta$$

$$= \cancel{2}a^2 \int_0^{\pi} \left(\frac{1-\cos 2\theta}{\cancel{2}}\right) d\theta$$

$$= a^2 \left| \theta - \frac{\sin 2\theta}{2} \right|_0^{\pi}$$

$$= a^2 \left[(\pi - 0) - \left(\frac{\sin 2\pi}{2} - \frac{\sin 0}{2} \right) \right]$$

$$= a^2 (\pi - 0 - 0)$$

$A = a^2\pi$ Ans!

Q27 (b) $r = 2a\cos\theta$

SOLUTION:

$2a\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = \frac{1}{2}\pi , \frac{3\pi}{2}$.

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$$

$$= \frac{1}{2}\int_{\pi/2}^{3\pi/2} (2a\cos^2\theta)\, d\theta$$

$$= 2a^2 \int_{\pi/2}^{3\pi/2} \cos^2\theta \, d\theta$$

$$= \cancel{2}a^2 \int_{\pi/2}^{3\pi/2} \left(\frac{1+\cos 2\theta}{\cancel{2}}\right) d\theta$$

$$= a^2 \left| \theta + \frac{\sin 2\theta}{2} \right|_{\pi/2}^{3\pi/2}$$

$$= a^2 \left[\frac{3\pi}{2} - \frac{\pi}{2} + \frac{\sin 3\pi}{2} - \frac{\sin \pi}{2} \right]$$

$$= a^2 (\pi)$$

$= a^2\pi$ Ans!

Q29 The region that is enclosed by the cardiod $r = 2 + 2\sin\theta$.

SOLUTION:-

for cardiod $\alpha = 0$, $\beta = 2\pi$.

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$$

$$= \frac{1}{2}\int_{0}^{2\pi} (2 + 2\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_{0}^{2\pi} (4 + 8\sin\theta + 4\sin^2\theta) \, d\theta$$

$$= 2\int_{0}^{2\pi} (1 + 2\sin\theta + \sin^2\theta) \, d\theta$$

$$= 2\int_{0}^{2\pi} \left(1 + 2\sin\theta + \frac{1 - \cos 2\theta}{2}\right) d\theta$$

$$= 2\left| \theta + 2\cos\theta + \frac{\theta}{2} - \frac{1\sin 2\theta}{4} \right|_{0}^{2\pi}$$

$$= 2(2\pi + 2 - 2 + \pi)$$

$$= 6\pi$$ Ans.

Q31 The region enclosed by the rose $r = 4\cos 3\theta$

SOLUTION:-

$4\cos 3\theta = 0$

$3\theta = \cos^{-1}(0)$

$\theta = \frac{\cos^{-1}(0)}{3}$

$\theta = \frac{\pi}{6}, \frac{3\pi}{2}$ one limit should be zero

$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$

$= \frac{1}{2}\int_{\pi/6}^{\pi/2} (4\cos 3\theta)^2 \, d\theta$

$= \frac{1\times16}{2}\int_{\pi/6}^{\pi/2} \cos^2 3\theta \, d\theta$

$= \frac{1\times16}{2}\int_{\pi/6}^{\pi/2} \frac{1+\cos 6\theta}{2} \, d\theta$

$= \frac{1}{\cancel{2}}\times\frac{\overset{8\ 4}{16}}{\cancel{2}}\left| \theta + \frac{\sin 6\theta}{6} \right|_{\pi/6}^{\pi/2}$

$= 4\left(\frac{\pi}{2} - \frac{\pi}{6} + \frac{\sin 3\pi}{6} - \frac{\sin \pi}{6}\right)$

$= 4\left(\frac{\pi}{3}\right)$

Q#1 The region inside the cardiod $r = 2 + 2\cos\theta$ and outside the circle $r = 3$

SOLUTION :-

cardiod $r = 2 + 2\cos\theta$.

circle $r = 3$.

$$2 + 2\cos\theta = 3$$

$$\theta = \cos^{-1}(1/2)$$

$$\theta = \pi/3 ,$$

$\beta = \pi/3$. $\alpha = 0$.

Subtract area of cardiod from area of circle.

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta - \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$$

$$= \frac{1}{2}\int_{0}^{\pi/3} (2 + 2\cos\theta)^2 \, d\theta - \frac{1}{2}\int_{0}^{\pi/3} 9 \, d\theta$$

$$= \frac{1}{2}\left[\int_{0}^{\pi/3} (4 + 8\cos\theta + 4\cos^2\theta)\, d\theta - 9|\theta|_{0}^{\pi/3}\right]$$

$$= \frac{1}{2}\left[|\theta|_{0}^{\pi/3} + 8|\sin\theta|_{0}^{\pi/3} + \frac{4}{2}\left(\theta + \frac{\sin 2\theta}{2}\right)_{0}^{\pi/3} - 3\pi\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{3} + 4\sqrt{3} + \frac{4\pi}{6} + \frac{\sqrt{3}}{2} - 3\pi\right]$$

$$-\pi + 9\sqrt{3} .$$

A !

Q45 The region inside the circle $r=2$ and to the right of the line $r=\sqrt{2}\sec\theta$

SOLUTION :-

circle $r=2$

line $r=\sqrt{2}\sec\theta$

$\sqrt{2}\sec\theta = 2$

$\theta = \sec^{-1}\left(\frac{2}{\sqrt{2}}\right)$

$\theta = \sec^{-1}(\sqrt{2})$

$\theta = 0, \pi/4$

line circle

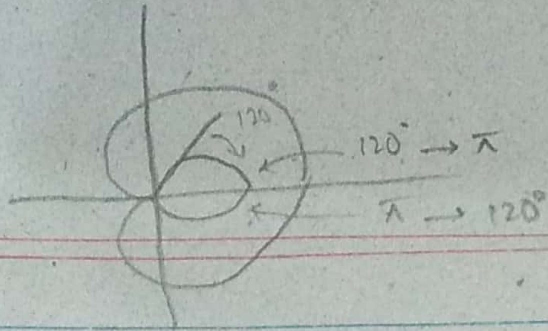
$A = \frac{2\times 1}{2}\int_{\alpha}^{\beta} r^2\,d\theta - \frac{1}{2}\int_{\alpha}^{\beta} r^2\,d\theta$

$= \frac{2\times 1}{2}\left[\int_0^{\pi/4}(4 - 2\sec^2\theta)\,d\theta\right]$

$= \frac{2\times 1}{2}\left[4\theta\Big|_0^{\pi/4} - 2\tan\theta\Big|_0^{\pi/4}\right]$

$= \frac{2\times 1}{2}(\pi - 2)$

$= \pi - 2$ Ans!



Q33 $\quad r = 1 + 2\cos\theta$

SOLUTION: $2\cos\theta = 0$

$\cos\theta = -1/2$

$\theta = \cos^{-1}(-1/2)$

$\theta = 2\pi/3, \ \pi$

$A = \frac{1}{2}\int_\alpha^\beta r^2\, d\theta$

$A = 2 \times \frac{1}{2}\int_\alpha^\beta r^2\, d\theta$

$A = \frac{\cancel{2}}{2}\int_\alpha^\beta r^2\, d\theta$. But,

$= \frac{\cancel{2}}{\cancel{2}}\int_{2\pi/3}^{\pi} (1 + 2\cos\theta)^2\, d\theta$

$= \frac{\cancel{2}}{\cancel{2}}\int_{2\pi/3}^{\pi} (1 + 4\cos\theta + 4\cos^2\theta)\, d\theta$

$= \frac{\cancel{2}}{\cancel{2}}\left[\theta + 4\sin\theta + \cancel{4}^{2}\int_{2\pi/3}^{\pi} \frac{1 + \cos 2\theta}{\cancel{2}}\, d\theta\right.$

$= \frac{\theta}{\cancel{2}} + 4\sin\theta + 2\theta + \sin 2\theta$

$= 3\theta + 4\sin\theta + \sin 2\theta \Big|_{2\pi/3}^{\pi}$

$= \Big[3(\pi - 2\pi/3) + 4(\sin(\pi) - \sin(2\pi/3)) + \{\sin 2(\pi) - \sin 2(2\pi/3)\}\Big]$

$= \pi \quad 0 - \frac{4\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right)$

$= \pi - \frac{3\sqrt{3}}{2}$ Ans!

EXERCISE 11.5

Q3 (a) $P_1(3, -2)$, $P_2(5, 1)$

SOLUTION:-

$\overrightarrow{P_1P_2} = \langle x - x_0 , y - y_0 \rangle$

$V = \langle 2 , 3 \rangle$

$x = 3 + 2t$

$y = -2 + 3t$

(b) $P_1(5, -2, 1)$, $P_2(2, 4, 2)$

SOLUTION:-

$\overline{P_1P_2} = V = \langle -3 \quad 6 , 1 \rangle$

$x = 5 - 3t$

$y = -2 + 6t$

$z = 1 + t$

Q9 (a) $x = -3 + t$, $y = 4 + 5t$

SOLUTION :-

$(x, y) = (x_0, y_0) + (a_1, a_2)t$

$= \langle -3, 4 \rangle + \langle 1, 5 \rangle t$

$v = -3i + 4j + (i + 5j)t$ Ans!

(b) $x = 2 - t$, $y = -3 + 5t$, $z = t$

SOLUTION :-

$(x, y) = \langle x_0, y_0, z_0 \rangle + (a_1, a_2, a_3)t$

$= \langle 2, -3, 0 \rangle + \langle -1, 5, 1 \rangle t$

$v = 2i - 3j + (-i + 5j + k)t$ Ans!

Q11 SOLUTION.-

False

Q13 SOLUTION.-

False

Q15 The line through $(-5, 2)$ that is parallel to $2i - 3j$.

SOLUTION.-

$P(-5, 2)$, $v = \langle 2, -3 \rangle$

$x = -5 + 2t$

$y = 2 - 3t$

Q17 The line that is tangent to the circle $x^2 + y^2 = 25$ at the point $(3, -4)$

SOLUTION:-

$$x^2 + y^2 = 25$$

$$2x + 2y\frac{dy}{dx} = 0.$$

$$\frac{dy}{dx} = \frac{-x}{y}.$$

$$\frac{dy}{dx} = \frac{-3}{-4}$$

$\Rightarrow$ ~~3i + 4j~~ $4i + 3j$

$x = 3 + 4t$

$y = -4 + 3t$ Ans!

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Q19 The line through $(-1, 2, 4)$ that is parallel to $3i - 4j + K$

SOLUTION:-

$P(-1, 2, 4)$ $V = \langle 3, -4, 1 \rangle$

$x = -1 + 3t$

$y = 2 - 4t$

$z = 4 + t$

Q25 $x = -2$, $y = 4 + 2t$, $z = -3 + t$

SOLUTION:-

for xy plane $z = 0$.

$0 = -3 + t$

$t = 3$.

$y = 4 + 6$. , $z = -3 + 3$

$y = 10$ $z = 0$

$(-2, 10, 0)$

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for xz plane. $y=0$.

$0 = 4 + 2t$.

$-2 = t$.

$x = -2$.

$y = 4 + 2(-2)$.

$y = 0$

$z = -3 + (-2)$.

$z = -5$

$(-2, 0, -5)$

for yz plane $x = 0$.

but x is alway -2. from the question.

Q29 SOLUTION -

L_1 : $x = 2+t_1$, $y = 2+3t_1$, $z = 3+t_1$
L_2 : $x = 2+t_2$, $y = 3+4t_2$, $z = 4+2t_2$

$2 + t_1 = 2 + t_2$ ——— (i)
$2 + 3t_1 = 3 + 4t_2$ ——— (ii)
$3 + t_1 = 4 + 2t_2$ ——— (iii)

Subtracting (i) & (iii)
$-1 = -2 - t_2$
$t_2 = -1$

Putting $t_2 = -1$ in eqn (ii).
$2 + 3t_1 = 3 + 4(-1)$
$3t_1 = -1 - 2$
$t_1 = -1$

Put t_1 & t_2 in eqn (i).
$2 + (-1) = 2 + (-1)$
Satisfied

Putting t_1 and t_2 in ~~t_1 and t_2~~ for eqn (i), (ii) & (iii) for point of intersection.
$(1, -1, 2)$

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Q33 **SOLUTION:-**

L_1 : $x = 3-2t$, $y = 4+t$, $z = 6-t$

L_2 : $x = 5+4t$, $y = -2+2t$, $z = 7-2t$.

first we find vectors of both line

$V_1 \langle -2 , 1 , -1 \rangle$

$V_2 \langle 4 , 2 , -2 \rangle$

Since both vectors are scalar multiple of each other

Hence lines are parallel.

Q37 **SOLUTION:-**

L_1 : $x = 3-t$, $y = 1+2t$

L_2 : $x = -1+3t$, $y = 9-6t$

Put $t = 0$ in L_1 we get

$(x, y) = (3, 1)$

Put $t = 4/3$ in L_2 we get the same point

$(x, y) = (3, 1)$

Now put $t = 1$ in L_1 & L_2 we get the same points.

$$(x, y) = (2, 3)$$

Since L_1 & L_2 are lying on same points $(3, -1)$ & $(2, 3)$

L_1 & L_2 are same lines.

Q43 SOLUTION:-

$$\langle x, y\rangle = \langle 1, 0\rangle + t\langle -2, 3\rangle \quad (0 \le t \le 2)$$

Put $t = 0$

$$(x, y) = (1, 0)$$

Put $t = 2$

$$(x, y) = (1, 0) + (-4, 6)$$

$$(x, y) = (-3, 6)$$

The line segment joining the points $(1, 0)$ and $(-3, 6)$

Q45 SOLUTION:-

Let $P(x_0, y_0)$, $P_1(3, 6)$, $P_2(8, -4)$

According to given condition.

$$\overrightarrow{P_1P} = \frac{2}{5}\,\overrightarrow{P_1P_2}$$

$$\langle x_0 - 3, y_0 - 6\rangle = \frac{2}{5}\langle 5, -10\rangle$$

$$\langle x_0 - 3, y_0 - 6\rangle = \langle 2, -4\rangle$$

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$x_0 - 3 = 2 \Rightarrow x_0 = 5$

$y_0 - 6 = -4 \qquad y_0 = 2$

$(5, 2)$ Ans!

Q47 SOLUTION:-

$P(-2, 1, 1)$

$L: x = 3 - t, \quad y = t, \quad z = 1 + 2t.$

$Q(3, 0, 1)$

$V \langle -1, 1, 2 \rangle$

$\overline{PQ} = \langle -5, 1, 0 \rangle$

$$D = \frac{|\overrightarrow{PQ} \times \vec{v}|}{|\vec{v}|}$$

$$\overrightarrow{PQ} \times \vec{v} = \begin{vmatrix} i & j & k \\ -5 & 1 & 0 \\ -1 & 1 & 2 \end{vmatrix}$$

$= \hat{i}(2-0) - \hat{j}(-10-0) + \hat{k}(-5+1)$

$= 2\hat{i} + 10\hat{j} - 5\hat{k}$

$|\overrightarrow{PQ} \times \vec{v}| = \sqrt{4 + 100 + 25}$

$= \sqrt{129}$

$|\vec{v}| = \sqrt{(-1)^2 + 1^2 + 2^2}$

$= \sqrt{6}$

$$O = \frac{\sqrt{129}}{\sqrt{6}}$$ اسم.

EXERCISE 11.6

Q3 SOLUTION:-

$P(2, 6, 1)$; $n = \langle 1, 4, 2 \rangle$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$1(x - 2) + (4)(y - 6) + (2)(z - 1) = 0$

$x - 2 + 4y - 24 + 2z - 2 = 0$

$x + 4y + 2z - 28 = 0$

Q15 (a) SOLUTION:-

$L: x = 4 + 2t, \; y = -t, \; z = -1 - 4t$

$3x + 2y + z - 7 = 0$

$n = \langle 3, 2, 1 \rangle$

$\vec{v} = \langle 2, -1, -4 \rangle$

for parallel: $v \cdot n = 0$

$(2i, -1j, -4k)(3i, 2j, 1k) = 0$

$6 - 2 - 4 = 0$

$0 = 0$

line and plane are parallel.

Q15 (b) **SOLUTION:-**

$x = t, \quad y = 2t, \quad z = 3t$

$x - y + 2z = 5$

$v\ \langle 1, 2, 3 \rangle \qquad n = \langle 1, -1, 2 \rangle$

for parallel $\quad v \cdot n = 0$

$(1i + 2j + 3k)(i - j + 2k) = 0$

$1 - 2 + 6 = 0$

$5 \neq 0$

Plane and line isn't parallel.

for perpendicular. $\quad v \times n = 0$.

$(i + 2j + 3k) \times (i - j + 2k) = 0$.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

$$\hat{i}\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = 0$$

$\hat{i}(4 + 3) - \hat{j}(2 - 3) + \hat{k}(-1 - 2) = 0$

$7\hat{i} + \hat{j} - 3\hat{k} = 0$

line and plane aren't perpendicular.

Q15 (e) **SOLUTION:-**

$x = -1 + 2t$, $y = 4 + t$, $z = 1 - t$

$4x + 2y - 2z = 7$.

$v \langle 2, 1, -1 \rangle$, $n = \langle 4, 2, -2 \rangle$

for parallel. $v \cdot n = 0$.

$(2i + j - k) \cdot (4i + 2j - 2k) = 0$.

$8 + 2 - 2 = 0$.

$8 \neq 0$.

for perpendicular $v \times n = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 4 & 2 & -2 \end{vmatrix} = 0$$

$$\hat{i} \begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} = 0$$

$\hat{i}(-2 + 2) - \hat{j}(-4 + 4) + \hat{k}(4 - 4) = 0$

$0 = 0$.

lines and plane are perpendicular.

Q17 (a) SOLUTION:-

L: $x = t$, $y = t$, $z = t$

$3x - 2y + z - 5 = 0$

Put L in plane eqn.

$3(t) - 2(t) + t - 5 = 0$.

$2t - 5 = 0$.

$t = 5/2$

Put t in L.

$(x, y, z) = \left(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}\right)$

(b) SOLUTION:-

L: $x = 2 - t$, $y = 3 + t$, $z = t$.

$2x + y + z = 1$

Putting L in plane.

$2(2 - t) + 3 + t + t = 1$

$4 - \cancel{2t} + 3 + \cancel{t} + \cancel{t} = 1$

$7 = 1$

Line has no solution therefore they aren't intersecting.

Q21 True

Q23 True.

Q25 SOLUTION:-

$4x - 2y + 7z + 12 = 0$

The plane eq^n will become.

$4x - 2y + 7z + d = 0$ —— (i)

The plane is passing through origin. which is (0, 0, 0)!

(i) => $0 - 0 + 0 + d = 0$.

$d = 0$.

Now eq^n (i) becomes

$4x - 2y + 7z = 0$.

Q29 **SOLUTION:-**

$P(1, 2, -1)$

$2x + y + z = 2$ $\quad n_1 \langle 2, 1, 1 \rangle$

$x + 2y + z = 3$ $\quad n_2 \langle 1, 2, 1 \rangle$

$$n_1 \times n_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1-2) - \hat{j}(2-1) + \hat{k}(4-1)$$

$$= -\hat{i} - \hat{j} + 3\hat{k}$$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$

$-1(x-1) + (-1)(y-2) + 3(z+1) = 0.$

$-x + 1 - y + 2 + 3z + 3 = 0$

$-x - y + 3z + 6 = 0$

$x + y - 3z = 6$ Ans

Q35 **SOLUTION:-**

$P(5, 0, -2)$

$x - 4y + 2z = 0$ $\quad n_1 \langle 1, -4, +2 \rangle$

$2x + 3y - z + 1 = 0$ $\quad n_2 \langle 2, 3, -1 \rangle$

$$n_1 \times n_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & 2 \\ 2 & 3 & -1 \end{vmatrix}$$

$= \cancel{\hat{i}(-4-6) - \hat{j}(1-4) + \hat{k}(-8-3)}$

$= \cancel{-10\hat{i} + 3\hat{j} - 11\hat{k}}$

$= \hat{i}(4-6) - \hat{j}(-1-4) + \hat{k}(3+8)$

$= -2\hat{i} + 5\hat{j} + 11\hat{k}$

$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct.$

$x = 5 + (-2)t, \quad y = 5t, \quad z = -2 + (11)t.$

$x = 5 - 2t, \quad y = 5t, \quad z = -2 + 11t.$

Ans.

Q27. P(-1, 4, 2)

$4x_0 - y_0 + z_0 - 2 = 0$ ———— (i)

$2x_0 + y_0 - 2z_0 - 3 = 0$ ———— (ii)

Adding eq^n (i) and (ii)

$6x_0 - z_0 - 5 = 0$.

Put $x_0 = 0$ $\boxed{z_0 = -5}$ Put $x_0 = 1$ $\boxed{z_0 = 1}$

Now putting x_0 and z_0 in eq^n (i).

$2(0) - y_0 + (-5) - 2 = 0$, $2(1) + y_0 - 2(1) - 3 = 0$

$y_0 = -7$ $y_0 = 3$.

$P_1(0, -7, -5)$ $P_2(1, 3, 1)$

$\cancel{\overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = (1, 11, 7) \times (0, 1, -5)}$

$\cancel{\overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 11 & 7 \\ 0 & 1 & -5 \end{vmatrix}}$

$\cancel{= \hat{i}\begin{vmatrix} 11 & 7 \\ 1 & -5 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 7 \\ 0 & -5 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 11 \\ 0 & 1 \end{vmatrix}}$

$\cancel{= i(-55 - 7) - j(-5) + k(1)}$

$\cancel{= -62\hat{i} + 5j + k.}$

$\overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = (1,-11,-7),(0,-1,-1)$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & -11 & -7 \\ 0 & -1 & -1 \end{vmatrix}$$

$$= \hat{x}(11-7) - \hat{y}(-1+14) + \hat{z}(-1+22)$$

$$= 4\hat{x} - 13\hat{y} + 21\hat{z}$$

$\cancel{(x_0, y_0, z_0) = (4, -13, 21)}$

$(x_0, y_0, z_0) = (-1, 4, 2)$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$4(x+1) - 13(y-4) + 21(z-2) = 0$

$4x + 4 - 13y + 52 + 21z - 42 = 0$

$4x - 13y + 21z = -14$ Ans

EXERCISE 1.1

Q17 (a) $$\begin{bmatrix} -3 & -1 & 2 & 4 \\ 2 & -3 & 3 & 2 \\ 0 & 2 & -3 & 1 \end{bmatrix}$$

SOLUTION:-

$$\begin{bmatrix} 1 & -7 & 8 & 8 \\ 2 & -3 & 3 & 2 \\ 0 & 2 & -3 & 1 \end{bmatrix} \quad 2R_2 + R_1$$

(b) $$\begin{bmatrix} 0 & 1 & -5 & 0 \\ 2 & -9 & 3 & 2 \\ 1 & 4 & -3 & 3 \end{bmatrix}$$

SOLUTION:-

$$\begin{bmatrix} 1 & 5 & -8 & 3 \\ 2 & -9 & 3 & 2 \\ 1 & 4 & -3 & 3 \end{bmatrix} \quad R_1 + R_3$$

Q.19 (a)

$$\left[\begin{array}{cc|c} 1 & K & -4 \\ 4 & 8 & 2 \end{array}\right]$$

$R_2 - 4R_1$

$$\left[\begin{array}{cc|c} 1 & K & -4 \\ 0 & 8-4K & 18 \end{array}\right]$$

If the value of K is not equal to 2 then

Rank (A) = 2 & Rank(Ab) = 2

∴ RANK (A) = Rank (Ab)

∴ System is consistent for all values of K except 2.

(b)

$$\left[\begin{array}{cc|c} 1 & k & -1 \\ 4 & 8 & -4 \end{array}\right]$$

$R_2 - 4R_1$

$$\left[\begin{array}{cc|c} 1 & K & -1 \\ 0 & 8-4K & 0 \end{array}\right]$$

System is consistent for all values of K.

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Rank (A) = 2 , Rank (AB) = 2

Rank (A) = Rank (Ab)

Q28

(a) True.

A homogenous system is always consistence since.

1- The zero solar the trival solution is always a solution to that system.

2- A homogenous eq^n with at least 1 free variable has infinitely many solutions.

(b) False

Multiply any row with a non-zero scalar isn't zero operation.

(c) True.

(d) A single linear eq^n with two or more unknowns must always have infinitely many solutions.

(e) False.

(f) False.

(g) True

(h) False.

EXERCISE 1.2

Q1 (a) Both

(b) Both

(c)

(d) Both

(e)

(f) Both

(g) Row Echleon form.

Q5 $x_1 + x_2 + 2x_3 = 8$

$-x_1 - 2x_2 + 3x_3 = 1$

$3x_1 - 7x_2 + 4x_3 = 10$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & +3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

$$\begin{array}{l} \\ \sim R_2 + R_1 \\ \sim 3R_1 - R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 10 & 2 & 14 \end{array}\right]$$

$$\begin{array}{l} \sim R_1 + R_2 \\ \sim (-1) \times R_2 \\ \sim R_3 + 10(R_2) \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 52 & 104 \end{array}\right]$$

$$\begin{array}{l} \\ \\ \sim \frac{1}{52}(R_3) \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\begin{array}{l} R_1 - 7R_3 \\ \sim 5R_3 + R_2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & +3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$x_1 = 3$, $x_2 = 1$, $x_3 = 2$.

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Q7 $x - y + 2z - w = -1$

$2x + y - 2z - 2w = -2$

$-x + 2y - 4z + w = 1$

$3x - 3w = -3$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array}\right]$$

$$\begin{array}{l} \sim R_2 - 2R_1 \\ R_3 + R_1 \\ R_4 + 3R_1 \end{array} \left[\begin{array}{cccc|c} 1 & -1 & -2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & 3 & 6 & 0 & 0 \end{array}\right]$$

$$R_2 \to R_2/3 \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & 6 & 0 & 0 \end{array}\right]$$

$$\begin{array}{l} \sim R_3 - R_2 \\ R_4 \to R_4 - 3R_2 \\ R_1 + R_2 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$x - w = -1$$
$$x = -1 + w$$
$$w = x + 1$$

- $y - 2z = 0 \Rightarrow z = 4/2$
  $y = 2z$

- $z = y$
  $w = x$.

Q27

$$x + 3y - z = a$$
$$x + y + 2z = b$$
$$2y - 3z = c$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 1 & 1 & 2 & b \\ 0 & 2 & -3 & c \end{array}\right]$$

$$R_2 - R_1 \quad \left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 0 & -2 & 3 & b-a \\ 0 & 2 & -3 & c \end{array}\right]$$

$c - (-2)\left(\frac{a-b}{2}\right)$ $\quad c - \frac{(-a+b)}{2}$

$c + a - b.$ $\quad c + a - b.$

$$(1/-2)R_2 \left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 0 & 1 & -3/-2 & -\left(\frac{b-a}{2}\right) \\ 0 & 2 & -3 & c \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 0 & 1 & -3/2 & \frac{-(b-a)}{2} \\ 0 & 2 & -3 & c \end{array}\right]$$

$$\begin{array}{l} -3(R_2)+R_1 \\ \\ R_3+(-2)R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 7/2 & \frac{3(a-b)}{2}+a \\ 0 & 1 & -3/2 & \frac{a-b}{2} \\ 0 & 0 & 0 & a-b+c \end{array}\right]$$

$a - b + c$ impose a condition on abc for any such abc the system will be consistent.

Q31 $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

$R_2 - 2R_1 \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ 1^{st} form

$R_1 - 3R_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 2^{nd} form.

Q35 $x^2 + y^2 + z^2 = 6$

$x^2 - y^2 + 2z^2 = 2$

$2x^2 + y^2 - z^2 = 3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 2 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_2 - R_1$, $R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -4 \\ 0 & -1 & -3 & -9 \end{array}\right]$$

$R_2(-1/2)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1/2 & 2 \\ 0 & -1 & -3 & -9 \end{array}\right]$$

$R_1 - R_2$, $R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3/2 & 4 \\ 0 & 1 & -1/2 & 2 \\ 0 & 0 & -7/2 & -7 \end{array}\right]$$

$R_3(-2/7)$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3/2 & 4 \\ 0 & 1 & -1/2 & 2 \\ 0 & 0 & 1 & +2 \end{array}\right]$$

$4 + (-3)$

$2 + \frac{1}{2}^{2}$.

$$R_1 + \left(\frac{-3}{2}\right)R_3 \quad R_2 + (-1/2)R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$x^2 = 1$, $y^2 = 3$, $z^2 = 2$.

$x^2 = \pm 1$, $y^2 = \pm\sqrt{3}$, $c = \pm\sqrt{2}$ Ans

EXERCISE 1.3

(a) True

(b) False m = row, n = column

(c) False

(d) False

(e) True.

(f) False.

(g) False. coz $A^T = B^T A^T$

(h) True.

(i) True

(j) True

(k) True

(l) False : matrices don't follow commutative law for product

(m) True

(n) True

(o) False.

EXERCISE 1.4

Q15 $(7A^{-1}) = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$

$$Adj.A = \begin{bmatrix} -2 & -7 \\ -1 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6-7} \begin{bmatrix} -2 & -7 \\ -1 & -3 \end{bmatrix}$$

$$7A^{-1} = -1 \begin{bmatrix} -2 & -7 \\ -1 & -3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2/7 & 1 \\ 1/7 & 3/7 \end{bmatrix}$$ Ans.

Q23 $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$$AC = CA$$

$$AC = \begin{bmatrix} b & 0 \\ d & 0 \end{bmatrix}$$

$$CA = \begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix}$$

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$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \end{bmatrix}$$

$0 = b$

$0 = 0$

$a = d$

$b = 0$

$$\boxed{a = d}$$

True False:-

(a) False

(b) False

(c) false

(d) false

(e) false

(f) True

(g) True

(h) True

(i) False

(j) True

(k) false

EXERCISE 1.5

Q7 (a) $EA = B$

We find that B is the product of interchanging first and third row of matrix A so, therefore; A

$$E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(b) $EB = A$

So we find the A is the product of interchanging first and 3rd row of matrix B.

So E, is the product of interchanging first and 3rd row identity matrix.

$$E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(C) $EA = C$

We found that it is the product of A when we apply row operation $R_3 - 2R_1$ to matrix A.

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

we found that it is the product of applying row operation $2R_1 + R_3$ to matrix C.

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Q (a) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ R_2 - 2R_1 \\ R_3 - R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} R_1 - 2R_2 \\ \\ R_3 + 2R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ \\ -(R_3) \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & -9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -40 & 16 & -9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} A^{-1}$$

(b). $\begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} -1 & 3 & -4 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} -R_1 \\ R_2 + 2R_1 \\ R_3 - 4R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & -3 & +4 & -1 & 0 & 0 \\ 0 & 10 & -7 & 2 & 1 & 0 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} \\ R_2/10 \\ R_3 + R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7/10 & 2/10 & 1/10 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right]$$

Since the row is 0 which means we can not find the inverse.

Q19 (a) $\begin{bmatrix} K_1 & 0 & 0 & 0 \\ 0 & K_2 & 0 & 0 \\ 0 & 0 & K_3 & 0 \\ 0 & 0 & 0 & K_4 \end{bmatrix}$

$$\left[\begin{array}{cccc|cccc} K_1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & K_2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & K_3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & K_4 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} R_1/K_1 \\ R_2/K_2 \\ R_3/K_3 \\ R_4/K_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/K_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1/K_2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/K_3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1/K_4 \end{array}\right]$$

$$\begin{bmatrix} 1/K_1 & 0 & 0 & 0 \\ 0 & 1/K_2 & 0 & 0 \\ 0 & 0 & 1/K_3 & 0 \\ 0 & 0 & 0 & 1/K_4 \end{bmatrix}$$

Q21 $\begin{bmatrix} c & c & c \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} c & c & c & 1 & 0 & 0 \\ 1 & c & c & 0 & 1 & 0 \\ 1 & 1 & c & 0 & 0 & 1 \end{array}\right]$$

$R_1(1/c)$ $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1/c & 0 & 0 \\ 1 & c & c & 0 & 1 & 0 \\ 1 & 1 & c & 0 & 0 & 1 \end{array}\right]$

$$\begin{array}{c} \\ R_2 - R_1 \\ R_3 - R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1/c & 0 & 0 \\ 0 & c-1 & c-1 & -1/c & 1 & 0 \\ 0 & 0 & c-1 & -1/c & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ R_2(1/c-1) \\ R_3(1/c-1) \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1/c & 0 & 0 \\ 0 & 1 & 1 & -(c-1)/c & 1/c-1 & 0 \\ 0 & 0 & 1 & -(c-1)/c & 0 & 1/c-1 \end{array}\right]$$

$$\begin{array}{c} R_1 - R_2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1/(c-1) & 0 \\ 0 & 1 & 1 & -c+1/c & 1/c-1 & 0 \\ 0 & 0 & 1 & -c+1/c & 0 & 1/c-1 \end{array}\right]$$

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$$R_2 - R_3 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1/(c-1) & 0 \\ 0 & 1 & 0 & 0 & 1/c-1 & -1/c-1 \\ 0 & 0 & 1 & -c+1/c & 0 & 1/c-1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1/c-1 & 0 \\ 0 & 1 & 0 & 0 & 1/c-1 & -1/c-1 \\ 0 & 0 & 1 & -\frac{(c^2+1)}{c} & 0 & 1/c-1 \end{array}\right]$$

as the denominator can't be equal to 0 so,

$c \neq 0$. $\quad c-1 \neq 0$.

$c \neq 1$

So the matrix is invertable when

$c \neq 0, 1$. A.1

EXERCISE 1.6

Q13 $x + 3x_2 = b_1$
$-2x_1 + x_2 = b_2$

$$\left[\begin{array}{cc|c} 1 & 3 & b_1 \\ -2 & 1 & b_2 \end{array}\right]$$

$R_2 + 2R_1$ $$\left[\begin{array}{cc|c} 1 & 3 & b_1 \\ 0 & 7 & 2b_1 + b_2 \end{array}\right]$$

$R_2(1/7)$ $$\left[\begin{array}{cc|c} 1 & 3 & b_1 \\ 0 & 1 & \frac{(2b_1 + b_2)}{7} \end{array}\right]$$

$R_1 - 3R_2$ $$\left[\begin{array}{cc|c} 1 & 0 & (b_1 - 3b_2)/7 \\ 0 & 1 & (2b_1 + b_2)/7 \end{array}\right]$$

No restriction on b_1 and b_2 system is consistent for all values of b_1 & b_2

Unique solution is

$X_1 = \frac{b_1 - 3b_2}{7}$, $X_2 = \frac{2b_1 + b_2}{7}$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} (b_1 - 3b_2)/7 \\ (2b_1 + b_2)/7 \end{bmatrix}$$

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Q A. $x_1 - x_2 + 3x_3 + 2x_4 = b_1$

$-2x_1 + x_2 + 5x_3 + x_4 = b_2$

$-3x_1 + 2x_2 + 2x_3 - x_4 = b_3$

$4x_1 - 3x_2 + x_3 + 3x_4 = b_4$

$$\left[\begin{array}{cccc|c} 1 & -1 & 3 & 2 & b_1 \\ -2 & 1 & 5 & 1 & b_2 \\ -3 & 2 & 2 & -1 & b_3 \\ 4 & 3 & 1 & 3 & b_4 \end{array}\right]$$

$$\begin{array}{c} \\ R_2 + 2R_1 \\ R_3 + 3R_1 \\ R_4 - 4R_1 \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 3 & 2 & b_1 \\ 0 & -1 & 11 & 5 & b_2 + 2b_1 \\ 0 & -1 & 11 & 5 & b_3 + 3b_1 \\ 0 & +1 & -11 & -5 & b_4 - 4b_1 \end{array}\right]$$

$$\begin{array}{c} \\ -R_2 \\ \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 3 & 2 & b_1 \\ 0 & 1 & -11 & -5 & -2b_1 - b_2 \\ 0 & -1 & 11 & 5 & b_3 + 3b_1 \\ 0 & 1 & -11 & -5 & b_4 - 4b_1 \end{array}\right]$$

$$\begin{array}{c} \\ \cancel{R_2 + R_3} \\ R_3 + R_2 \\ R_4 - R_2 \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 3 & 2 & b_1 \\ 0 & 1 & -11 & -5 & -b_2 - 2b_1 \\ 0 & 0 & 0 & 0 & b_3 + b_1 - b_2 \\ 0 & 0 & 0 & 0 & b_4 + b_2 - 2b_1 \end{array}\right]$$

we can see that we have 2 rows in bottom of the matrix in order for the system to be consistent it is necessary that

$$b_1 - b_2 + b_3 = 0$$
$$2b_1 + b_2 + b_4 = 0$$

It has 2 free variable the corresponding augmented matrix is

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 0 \\ -2 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$2R_1 + R_2 \left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 1 & 0 \end{array}\right]$$

$$\begin{array}{c} R_1 - R_2 \\ -R_2 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & -2 & -1 & 0 \end{array}\right]$$

$$b_1 - b_3 - b_4 = 0$$
$$b_2 - 2b_3 - b_4 = 0.$$

$$b_1 = b_4 + b_3.$$
$$b_2 = 2b_3 + b_4$$

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$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} b_1 + b_3 \\ b_1 + 2b_3 \\ b_3 \\ b_4 \end{bmatrix}$$

EXERCISE 1.7

Q25 $A = \begin{bmatrix} 4 & -3 \\ a+5 & -1 \end{bmatrix}$

In order for A to be symetric the eqn should be.

$$A = A^T$$

$$A^T = \begin{bmatrix} 4 & a+5 \\ -3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ a+5 & -1 \end{bmatrix} = \begin{bmatrix} 4 & a+5 \\ -3 & -1 \end{bmatrix}$$

$$a+5 = -3.$$

$$\boxed{a = -8} \text{ Ans}$$

Q27 $A = \begin{bmatrix} x-1 & x^2 & x^4 \\ 0 & x+2 & x^3 \\ 0 & 0 & x-4 \end{bmatrix}$

Since it is an upper triangular matrix.
Thus A will be invertible if

$$x-1 \neq 0;$$
$$x+2 \neq 0;$$
$$x-4 \neq 0;$$

Ans.

EXERCISE 1.8

Q13 $T(x_1, x_2) = (x_2, -x_1, x_1 + 3x_2, x_1 - x_2)$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -x_1 + 0 \\ 0 + x_2 \\ x_1 + 3x_2 \\ x_1 - x_2 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ -1 \end{bmatrix}$$

$$A = [T(e_1) \mid T(e_2)] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 1 & 3 \\ 1 & -1 \end{bmatrix} \text{ Ans!}$$

Standard Matrix.

(b) $T(x_1, x_2, x_3, x_4) = (7x_1 + 2x_2 - x_3 + x_4, x_2 + x_3, - x_1)$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 7x_1 + 2x_2 - x_3 + x_4 \\ 0 + x_2 + x_3 + 0 \\ -x_1 + 0 + 0 + 0 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 7 \\ 0 \\ -1 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

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$$A = \left[T(e_1) \mid T(e_2) \mid T(e_3) \mid T(e_4) \right]$$

$$= \begin{bmatrix} 7 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

(c) $T(x_1, x_2, x_3) = (0, 0, 0, 0, 0)$

$$A = \left[T(e_1) \mid T(e_2) \mid T(e_3) \mid T(e_4) \right]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Standard Matrix.

Q15 $w_1 = 3x_1 + 5x_2 - x_3$

$w_2 = 4x_1 - x_2 + x_3$

$w_3 = 3x_1 + 2x_2 - x_3$

$$T\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 3x_1 & 5x_2 & -x_3 \\ 4x_1 & -x_2 & x_3 \\ 3x_1 & 2x_2 & -x_3 \end{bmatrix}$$

$$T\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$A = \left[T(e_1) \mid T(e_2) \mid T(e_3) \right] = \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$$

$$T(-1,2,4) = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -3+10-4 \\ -\cancel{4}-2+\cancel{4} \\ -3+\cancel{4}-\cancel{4} \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}$$

Q19 (a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; $x \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$$T_A(x) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ Ans!}$$

(b) $A = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix}$; $x = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

$$T_A(x) = \begin{bmatrix} 1 + 2 \\ -3 + 1 + 15 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 13 \end{bmatrix} \text{ Ans!}$$

Q21 (a) $T(x, y) = (2x + y, -y + x)$

$$= T(2x_1 + 2x_2 + y_1 + y_2, \; x_1 + x_2 - y_1 - y_2)$$

$$= T(2x_1 + y_1 + 2x_2 + y_2, \; x_1 - y_1 + x_2 - y_2)$$

$$= T(2x_1 + y_1 + 2x_2 + y_2) + (x_1 - y_1 + x_2 - y_2)$$

$$= T(u) + T(v) = T(u + v)$$

Proved!

Q23 (a) $T(x,y) = (x^2, y)$

$T(u) = x_1^2 + y_1$

$T(v) = x_2^2 + y_2$.

~~$T(u) + T(v) = (x_1^2 + y_1 + x_2^2 + y_2$~~

$T(u) + T(v) = (x_1^2 + x_2^2, \; y_1 + y_2)$

proved!

(b) ~~$T(x_1, x_2)$~~ $T(x,y,z) = (x, y, xz)$

$T(u) = (x_1, y_1, x_1 z_1)$

$T(v) = (x_2, y_2, x_2 z_2)$

$T(u) + T(v) = (x_1 + x_2, \; y_1 + y_2, \; x_1 z_1 + x_2 z_2)$

Proved

EXERCISE 4.9

Q1 (a) x - axis

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

(b) y - axis

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(c) line $y = x$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Q3 (a) x-y plane.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix}$$

(b) x-z. plane.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

(c) y-z plane

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -5 \\ 3 \end{bmatrix}$$

Q7 (a) xy - plane

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

(b) xz - plane

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$$

(c) yz - plane.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

Q9. (a) $\theta = 30^\circ$

$$\because \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4.60 \\ 1.96 \end{bmatrix}$$

$\theta = -60^\circ$

$$\begin{bmatrix} 1/2 & +\sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}.$$

$$\begin{bmatrix} -1.96 \\ -4.60 \end{bmatrix}$$

$\theta = 45^\circ$

$$\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4.95 \\ -0.71 \end{bmatrix}$$

Q13.(a)

$$= \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 6 \end{bmatrix} \quad \text{Ans!}$$

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EXERCISE 2.2

Q6 $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Determinant $= (1)(1)(-5)(1)$

$= -5$ Ans!

Q7 $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$- \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim R_1 R_2$

Determinant $= -(1)(1)(1)(1)$

$= -1$ Ans!

Q4 $\begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$

$= - \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 2 & 0 & 1 & 0 \\ 1 & 0 & 2 & 3 \end{bmatrix}$ → $R_1 R_2$

$= - \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$

$= - \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ $R_2 - 2R_1$

$= - \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ $R_3 - 2R_2$, $R_4 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix} \quad R_4 + R_3.$$

$$\text{Determinant }(A) = -\{(1)(1)(-1)(6)\}$$
$$= 6 \quad \text{Ans!}$$

Combination of row and cofactor expansion.

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix} \quad R_1 - 2R_2$$

cofactor expansion along 1^{st} column.

$$= - \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

$R_3 \to R_3 - R_1$ $\quad R_2 \to R_2 - 2R_1$

$$= - \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

Cofactor expansion along 1^{st} column.

$$= - \begin{bmatrix} -1 & 2 \\ 1 & 4 \end{bmatrix}$$

$$= -(-4-2)$$

$$= 6$$

Q13 :

$$\begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$$\begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ 0 & -1 & 2 & 6 & 8 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_4 \longrightarrow R_4 - 2R_3$

$$\begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ 0 & -1 & 2 & 6 & 8 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_5 \longrightarrow R_5 - R_4$

$$\begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ 0 & -1 & 2 & 6 & 8 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Determinant

$$\det(A) = (1)(-1)(1)(1)(2)$$
$$= -2$$

Q17
$$\begin{bmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 4g & 4h & 4i \end{bmatrix}$$

$$\text{Determinant} = 3(-1)(4)\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\therefore \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -6$$

Determinant $= (3)(-1)(4)(-6)$

$= 72$

Q19 $\begin{bmatrix} a+g & b+h & c+i \\ d & e & f \\ g & h & i \end{bmatrix}$

$R_1 \rightarrow R_1 - R_3$

$$= \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$\det(A) = -6$

Q23 $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$

Taking L.H.S.

$= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

$C_2 \to C_2 - C_1 \qquad C_3 \to C_3 - C_1$

$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$

$= 1 \begin{vmatrix} b-a & c-a \\ b^2-a^2 & c^2-a^2 \end{vmatrix}$

$= (b-a)(c^2-a^2) - (c-a)(b^2-a^2)$

$= (b-a)(c-a)(c+a) - (c-a)(b-a)(b+a)$

$= (b-a)(c-a)(c+\cancel{a}-b-\cancel{a})$

$= (b-a)(c-a)(c-b)$ Ans.

Q_{2a}. $A = \begin{bmatrix} -2 & 8 & 1 & 4 \\ 3 & 2 & 5 & 1 \\ 1 & 10 & 6 & 5 \\ 4 & -6 & 4 & -3 \end{bmatrix}$.

since C_2 and C_4 are scalar multiple of each other since

det (A) = 0 .

EXERCISE 2.3

Q3 $A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & 4 & 5 \end{bmatrix}$; $K = -2$

det

$\det(KA) = K^n \det(A)$

$R_1 \to R_1/2$

$= \begin{bmatrix} 1 & -1/2 & 3/2 \\ 3 & 2 & 1 \\ 1 & 4 & 5 \end{bmatrix}$

$R_2 \to R_2 - 3R_1$, $R_3 = R_3 - R_1$

$= \begin{bmatrix} 1 & -1/2 & 3/2 \\ 0 & 7/2 & -7/2 \\ 0 & 9/2 & 7/2 \end{bmatrix}$

$=$

$R_2 \to 2R_2/7$

$= \begin{bmatrix} 1 & -1/2 & 3/2 \\ 0 & 1 & -1 \\ 0 & 9/2 & 7/2 \end{bmatrix}$

$R_3 \to R_3 - 9/2 R_2$

$$= \begin{bmatrix} 1 & -1/2 & 3/2 \\ 0 & 1 & -1 \\ 0 & 0 & 8 \end{bmatrix}$$

$$\text{Det}(A) = (1)(1)(8)$$
$$= 8$$

$$K^n \det(A) = (-2)^3 (8)$$
$$= -64$$

$$\det(KA) = -2 \begin{vmatrix} 1 & -1/2 & 3/2 \\ 0 & 1 & -1 \\ 0 & 0 & 8 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 1 & -3 \\ 0 & -2 & 2 \\ 0 & 0 & -16 \end{vmatrix}$$

$$= (-2)(-2)(-16)$$

$$\det(KA) = -64$$

Q17 $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 6 \\ K & 3 & 2 \end{bmatrix}$

$\det(A) = 1 \begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix} - 2 \begin{vmatrix} 3 & 6 \\ K & 2 \end{vmatrix} + 4 \begin{vmatrix} 3 & 1 \\ K & 3 \end{vmatrix}$

$= 1(2-18) - 2(6-6K) + 4(9-K)$

$= -16 - 12 + 12K + 36 - 4K$

$= 8 + 8K.$

$8 + 8K = 0$

$\boxed{K = -1}$

A matrix will be invertible when $\det(A) \neq 0$, if $K \neq 1$ then the matrix will be invertible for all values of k.

Q33. $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\det(A) = -7$

a. $\det(3A)$

$K = 3$; $n = 3$

$\det(KA) = K^n \det(A)$

$= (3)^3 (-7)$

$= -189$

b. $\det(A^{-1})$

$\det(A^{-1}) = \dfrac{1}{\det(A)}$

$= -\dfrac{1}{7}$

c. $\det(2A^{-1})$

$\det(2A^{-1}) = \dfrac{-2^3}{7}$

$= -\dfrac{8}{7}$

d. $\det((2A)^{-1})$

$2A = (2^3)(-7) = -56.$

$\det(2A^{-1}) = -\frac{1}{56}.$

Q35 (a)

$\det(3A)$

$k = 3$; $n = 3$

$= k^n \det(A)$

$= 3^3 (7)$

$= 189$ Ans!

(b) $\det(A^{-1})$

$\det(A^{-1}) = \frac{1}{\det(A)}$

$= \frac{1}{7}$. Ans!

(c) $\det(2A^{-1})$

$K = 2$, $n = 3$

$\det(2A^{-1}) = 2^3 \times \frac{1}{\det(A)}$

$= \frac{8}{7}$ Ans!

d) $\det((2A)^{-1})$

$2A = 2^3(A) = 8(7) = 56$

$\det((2A)^{-1}) = \frac{1}{56}$ Ans

True / False:

a) false

b) false

c) True

d) False

e) True

f) True

g) True

h) True

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i) True

j) True

k) True

l) False.

EXERCISE 5.1

Q5 (a) $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

SOLUTION.-

let $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$(\lambda I - A) = \begin{bmatrix} \lambda-1 & -4 \\ -2 & \lambda-3 \end{bmatrix}$

$= (\lambda-1)(\lambda-3) - (8)$

$= \lambda^2 - 3\lambda - \lambda + 3 - 8$

$= \lambda^2 - 4\lambda - 5$

$= \lambda^2 - 5\lambda + \lambda - 5$

$= \lambda(\lambda-5) + (\lambda-5)$

$= (\lambda-5)(\lambda+1)$

$\lambda = 5, -1$ — Eigen values

Put $\lambda = -1$ in $(\lambda I - A)$

$= \left[\begin{array}{cc|c} -2 & -4 & 0 \\ -2 & -4 & 0 \end{array}\right]$

$= \left[\begin{array}{cc|c} -2 & -4 & 0 \\ 0 & 0 & 0 \end{array}\right] \sim R_2 - R_1$

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$$-2x_1 - 4x_2 = 0$$

$$x_1 = -2x_2$$

x_2 is free variable

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2x_2 \\ \cancel{4}x_2 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Put $\lambda = 5$ in $(\lambda I - A)$

$$= \left[\begin{array}{cc|c} 4 & -4 & 0 \\ -2 & 2 & 0 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 4 & -4 & 0 \\ 0 & 0 & 0 \end{array}\right] \sim 2R_2 + R_1$$

$$4x_1 - 4x_2 = 0$$

$$x_1 = x_2$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

x_2 is free variable

(D) $\begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$

SOLUTION:-

~~$\lambda I - A$~~ $\quad \lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$$(\lambda I - A) = \begin{bmatrix} \lambda+2 & 7 \\ -1 & \lambda-2 \end{bmatrix}$$

$$|(\lambda I - A)| = 0$$

$$(\lambda-2)(\lambda+2) + 7 = 0$$

$$\lambda^2 + 2\lambda - 2\lambda - 4 + 7 = 0$$

$$\lambda^2 + 3 = 0 \text{ — characteristic eq}^n$$

There is no eigen values because of unreal eq^n.

Q5 (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

SOLUTION:-

$$\det(\lambda I - A) = 0$$

$$\begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda-1 \end{vmatrix} = 0$$

$$(\lambda-1)(\lambda-1) = 0$$

$$(\lambda-1)^2 = 0$$

$$\lambda = 1 \text{ Eigen value}$$

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Put $\lambda = 1$ in $(\lambda I - A)$

$$= \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

x_1 and x_2 are free variables.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(d) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

SOLUTION –

$\det(\lambda I - A) = 0$

$$(\lambda I - A) = \begin{vmatrix} \lambda - 1 & 2 \\ 0 & \lambda - 1 \end{vmatrix}$$

$$= (\lambda - 1)^2$$

$$\lambda = 1$$

Putting $\lambda = 1$

$$= \left[\begin{array}{cc|c} 0 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$2x_2 = 0$

$x_2 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Q7 $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$\lambda I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$\det(\lambda I - A) = 0$

$$\begin{vmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{vmatrix} \quad \text{——} \quad (i)$$

$$\lambda-4 \begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda-1 \end{vmatrix} - 1 \begin{vmatrix} 2 & \lambda-1 \\ 2 & 0 \end{vmatrix} = 0$$

$(\lambda-4)(\lambda-1)^2 + (2\lambda-2) = 0$

$\cancel{\lambda^2 - 4\lambda - \lambda + 4}$

$(\lambda-4)(\lambda^2-2\lambda+1) + 2\lambda - 2 = 0$

$\cancel{\lambda^3 - 2\lambda^3 + \lambda - 4\lambda^2 + 8\lambda - 4 + 2\lambda - 2 = 0}$

$\cancel{\lambda^3 - 4\lambda^2 + 11\lambda - 6 = 0}$

$\lambda^3 - 2\lambda^2 + \lambda - 4\lambda^2 + 8\lambda - 4 + 2\lambda - 2 = 0$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\lambda = 1, 2, 3$

Putting $\lambda = 1$ in (i)

$$\left[\begin{array}{ccc|c} -3 & 0 & -1 & 0 \\ 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{array}\right]$$

$-3x_1 + x_3 = 0$; $2x_1 = 0$

$x_3 = 3x_1$ $\quad$ $x_1 = 0$

x_2 is a free variable

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Putting $\lambda = 2$

$$\left[\begin{array}{ccc|c} -2 & 0 & -1 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \end{array}\right]$$

$-2x_1 - x_3 = 0$

$x_1 = -x_3/2$

Putting $\lambda = 1$ in (i)

$$\left[\begin{array}{ccc|c} -3 & 0 & -1 & 0 \\ 2\cancel{0} & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} -3 & 0 & -1 & 0 \\ 0 & 0 & -2\cancel{0} & 0 \\ 0 & 0 & -2 & 0 \end{array}\right]$$

$-3x_1 \quad - x_3 \quad = 0$

$x_1 = -x_3/3$

x_2 is free variable

$-2x_3 = 0$

$x_3 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Putting $\lambda = 2$

$$\left[\begin{array}{ccc|c} -2 & 0 & -1 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} -2 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ R_1 + R_2 \\ R_1 + R_3 \end{array}$$

$-2x_1 - x_3 = 0$.

$x_1 = -x_3/2$

$x_2 - x_3 = 0$.

$x_2 = x_3$.

x_3 is free variable.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3/2 \\ x_3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$$

Putting $\lambda = 3$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_2 + 2R_1 \\ R_3 + 2R_1 \end{array}$$

$-x_1 - x_3 = 0$, $2x_2 - 2x_3 = 0$

$x_1 = -x_3$ $\quad$ $x_2 = x_3$

x_3 is free variable

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ 3x_3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$$

Q13 $\begin{bmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{bmatrix}$

SOLUTION:-

$\det(\lambda I - A) = 0$

$$\begin{vmatrix} \lambda-3 & 0 & 0 \\ 2 & \lambda-7 & 0 \\ -4 & -8 & \lambda-1 \end{vmatrix} = 0$$

$$\lambda-3 \begin{vmatrix} \lambda-7 & 0 \\ -8 & \lambda-1 \end{vmatrix} = 0$$

$(\lambda-3)(\lambda-7)(\lambda-1) = 0$

EXERCISE 5.2

Q. $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$

$\lambda I = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$

$\lambda I - A = \begin{bmatrix} \lambda+1 & -4 & 2 \\ 3 & \lambda-4 & 0 \\ 3 & -1 & \lambda-3 \end{bmatrix}$ —— (A)

$\det(\lambda I - A) = (\lambda+1)(\lambda-3)(\lambda-4) + 3(\lambda-3) + (-3)(3)$

$(\lambda - 4).$

$= (\lambda^2 - 3\lambda + \lambda - 3)(\lambda-4) + 3\lambda - 3 - 9\lambda + 27$

$= \lambda^3 - 4\lambda^2 - 2\lambda^2 + 8\lambda + \cancel{\lambda} - \cancel{4\lambda} - \cancel{3\lambda}$

$+ 12 + \cancel{3\lambda} - 3 - 9\lambda + 27.$

$= \lambda^3 - 6\lambda^2 - 5\lambda + 36.$

$\lambda = 1, 2, 3$

Put $\lambda = 1$ in (A)

$$= \left[\begin{array}{ccc|c} 2 & -4 & 2 & 0 \\ 3 & -3 & 0 & 0 \\ 2 & -1 & -2 & 0 \end{array}\right]$$

$-3 - 6 -$

$-3 + 6$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 3 & -3 & 0 & 0 \\ 2 & -1 & -2 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & -3 & -4 & 0 \end{array}\right] \begin{array}{l} \\ \sim R_2 - 3R_1 \\ \sim R_2 - 2R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -3 & -4 & 0 \end{array}\right] \sim \tfrac{1}{3} R_2$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -7 & 0 \end{array}\right] \sim R_3 + (+3R_2)$$

$x_1 - 2x_2 + x_3 = 0 \qquad x_1 = 0$

$x_2 - x_3 = 0 \; ; \; \Rightarrow \quad x_2 = 0$

$-7x_3 = 0$

$\boxed{x_3 = 0}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \Big|$$

Put $\lambda = 2$ in (A)

$$= \left[\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ 3 & -\cancel{1}2 & 0 & 0 \\ 3 & -1 & -1 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \sim R_2 - R_1 \\ \sim R_3 - R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ R_3 - \frac{3}{2}R_2 \end{array}$$

$3x_1 - 4x_2 + 2x_3 = 0$

$2x_2 - 2x_3 = 0$.

$\boxed{x_2 = x_3.}$

$\boxed{x_3 = x_3.}$

$3x_1 - 4x_3 + 2x_3 = 0$.

$3x_1 = 2x_3 \Rightarrow x_1 = \frac{2}{3}x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2/3\, x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 2/3 \\ 1 \\ 1 \end{bmatrix}$$

Put $\lambda = 3$ in (A).

$$= \left[\begin{array}{ccc|c} 4 & -4 & 2 & 0 \\ +3 & -1 & 0 & 0 \\ +3 & -1 & 0 & 0 \end{array}\right]$$

$$\cancel{= \left[\begin{array}{ccc|c} 4 & -4 & 2 & 0 \\ 0 & -4 & 3/2 & 0 \\ 0 & & & 0 \end{array}\right] \begin{array}{l} R_2 \\ \sim R_2 - \frac{3}{4}R_1 \\ \sim R_3 - \frac{3}{4}R_1 \end{array}}$$

$$= \left[\begin{array}{ccc|c} 2 & -4 & 4 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & -1 & 3 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 2 & -4 & 4 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim R_3 - R_2$$

$$2x_3 - 4x_2 + 4x_1 = 0$$

$$-x_2 + 3x_1 = 0$$

$$\boxed{x_1 = \frac{x_2}{3}} \quad \text{—— (i)}$$

$$x_3 = x_3.$$

$$2x_3 - 4x_2 + \frac{4}{3}x_2 = 0.$$

$$2x_3 - \frac{12x_2 + 4x_2}{3} = 0$$

$$2x_3 - \frac{\cancel{16}8x_2}{3} = 0$$

$$x_3 = \frac{\cancel{16}8x_2}{6} = \frac{8}{3}x_2.$$

$$x_2 = \frac{{}^3\cancel{6}x_3}{\cancel{8}4} = \frac{3}{4}x_3.$$

Now (i) becomes.

$$x_1 = \frac{\cancel{3}x_3}{4 \times \cancel{3}} = \frac{x_3}{4}.$$

Now (i) can also becomes.

$$\frac{x_3}{4} = \frac{x_2}{3}.$$

Scanned with CamScanner

Q. $\begin{pmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{pmatrix}$

SOLUTION :-

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{bmatrix} \sim R_2 + R_1$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix} \sim 3R_3 - 3R_1$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 10 & 0 & -52 & -104 \end{bmatrix} \begin{matrix} \text{Echelon form} \\ \\ \sim R_3 - 10R_2 \end{matrix}$$

-2 -50

Rank (A) = 3.

Pivot row Numbers = R_1, R_2, R_3

Pivot column Numbers = C_1, C_2, C_3.

Pivot positions = a_{11}, a_{22}, a_{33}.

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{bmatrix} \begin{matrix} R_1 + R_2 \\ \sim (-1) R_2 \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix} \sim \left(-\tfrac{1}{52}\right) R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{matrix} R_1 - 7R_3 \\ R_2 + 5R_3 \\ \end{matrix}$$

Q3 $\begin{pmatrix} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{pmatrix}$

SOLUTION:-

$$\begin{bmatrix} 0 & -2 & 3 & 1 \\ 3 & 0 & 6 & 1 \\ 6 & 0 & 12\ \cancel{6} & 8 \end{bmatrix} \quad \begin{matrix} \\ 3R_1 + R_2. \\ 3R_1 + R_3. \end{matrix}$$

$$\begin{bmatrix} 0 & -2 & 3 & 1 \\ 3 & 0 & 6 & 1 \\ 0 & 0 & 6 & -6 \end{bmatrix} \quad \begin{matrix} \\ \\ 2R_2 - R_3. \end{matrix}$$

Rank (A) = 3

Pivot

$$\begin{bmatrix} 3 & 0 & 6 & 1 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 6 & -6 \end{bmatrix} \quad \begin{matrix} R_1R_2. \\ \\ \text{Echelon form} \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & 1/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & 0 & 6\,0 & \cancel{1/6}\ -6 \end{bmatrix} \quad \begin{matrix} (1/3)R_1 \\ (-1/2)R_2. \\ \cancel{(1/6)R_3} \end{matrix}$$

$$\left(\begin{array}{cccc} 1 & 0 & 0 & -7/3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & -6 \end{array}\right) \begin{array}{l} 2R_3 - R_1 \\ (3/2)R_3 + R_2 \\ \cancel{\left(\frac{3}{2}\right)R_3 + R_2} \end{array}$$

Reduced Echelon form

Q4 $\begin{pmatrix} 2 & -3 & -2 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$

SOLUTION :-

$$\begin{bmatrix} 2 & -3 & -2 \\ 0 & +4 & 3 \\ 0 & -1 & 1 \end{bmatrix} \quad \begin{matrix} 1-(-2),\ 1-(-3) \\ R_2 - R_1 \\ 3R_2 - 2R_3 . \end{matrix}$$

$$\begin{bmatrix} 2 & -3 & -2 \\ 0 & 4 & 3 \\ 0 & 0 & 7 \end{bmatrix} \quad 4R_3 + R_2 .$$

Rank (A) = 3 .

$$\begin{bmatrix} 1 & -3/2 & -1 \\ 0 & 4 & 3 \\ 0 & 0 & 7 \end{bmatrix} \quad 1/2\, R_1 .$$

$$\begin{bmatrix} 1 & -3/2 & -1 \\ 0 & 1 & 3/4 \\ 0 & 0 & 7 \end{bmatrix} \quad 1/4\, R_2 .$$

$$\begin{bmatrix} 1 & 0 & 1/8 \\ 0 & 1 & 3/4 \\ 0 & 0 & 7 \end{bmatrix} \quad \left(\tfrac{3}{2}\right)R_2 + R_1$$

$$\begin{bmatrix} 1 & 0 & 1/8 \\ 0 & 1 & 3/4 \\ 0 & 0 & 1 \end{bmatrix} \quad -1/7 R_3$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{matrix} 1/8 R_3 - R_1 \\ 3/4 R_3 - R_2. \\ \\ \end{matrix}$$

Rank (A) = 3.

Q5 $\begin{pmatrix} 2 & 1 & 3 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$

SOLUTION :-

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad 2R_2 - R_1$$

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & 0 & 6 & 0 \end{bmatrix} \quad 3R_3 - R_2.$$

Rank (A) = 0.

$$\begin{bmatrix} 1 & 1/2 & 3/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{matrix} 1/2\,R_1 \\ 1/3\,R_2 \\ 1/6\,R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad 1/2\,R_2 - R_1.$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{matrix} 2R_3 + R_1. \\ R_3 + R_2 \end{matrix}$$

Q1 $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 3 & 4 & 7 \end{pmatrix}$

SOLUTION:-

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 4 \end{bmatrix}$ $\begin{matrix} \\ -2R_1 + R_2 \\ -3R_1 + R_3 \end{matrix}$

R~ $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ $R_3 - R_2$

Rank (A) = 3.

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ $R_1 - R_2$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{matrix} 2R_3 + R_1 \\ -3R_3 + R_2 \end{matrix}$

Rank (A) = 3

Q8

$$\begin{matrix} 2 & 4 & 5 \\ 1 & 1 & 1 \\ 3 & 5 & 7 \\ 6 & 10 & 13 \end{matrix}$$

SOLUTION:-

$$\begin{bmatrix} 2 & 4 & 5 \\ 0 & -2 & -3 \\ 0 & 2 & 1 \\ 0 & 2 & 2 \end{bmatrix} \begin{matrix} \\ 2R_2 - R_1 \\ 3R_1 - 2R_3 \\ 3R_1 - R_4 \end{matrix}$$

$$\begin{bmatrix} 2 & 4 & 5 \\ 0 & 0 & -2 \\ 0 & 2 & 1 \\ 0 & 2 & 2 \end{bmatrix} \begin{matrix} \\ R_2 + R_3 \\ \\ \\ \end{matrix}$$

$$\begin{bmatrix} 2 & 4 & 5 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \\ 0 & 2 & 2 \end{bmatrix} \begin{matrix} \\ R_2R_3 \\ \\ \\ \end{matrix}$$

$$\begin{bmatrix} 2 & 4 & 5 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} \\ \\ \\ R_4 - R_2 \end{matrix}$$

$$\begin{bmatrix} 2 & 4 & 5 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \quad 2R_4 + R_3.$$

Rank (A) = 3.

$$\begin{bmatrix} 1 & 2 & 5/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 1/2\, R_1 \\ 1/2\, R_2 \\ -1/2\, R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -3/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad 2R_2 - R_1.$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 3/2\, R_3 + R_1 \\ -1/2\, R_3 + R_2 \end{matrix}$$

Rank (A) = 3.