9/NOV/21

Hypothesis Testing For Variance

1) H_0 $\sigma^2 = \sigma_0^2$

2) H_A $\sigma^2 < \sigma_0^2$

   or H_A $\sigma^2 > \sigma_0^2$

   or H_A $\sigma^2 \neq \sigma_0^2$

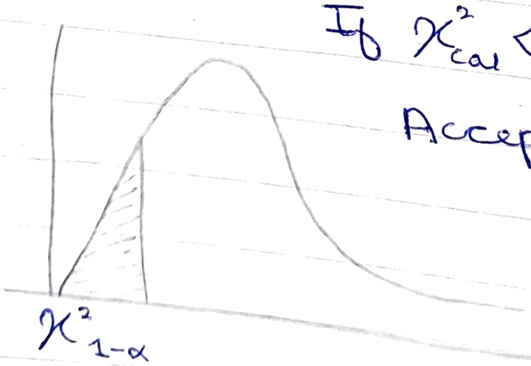
3) Level of significance (α)

4) Test Statistic

$$\chi^2_{cal} = \frac{(n-1)s^2}{\sigma_0^2}$$

5) Critical Region (The area where H_A is true)

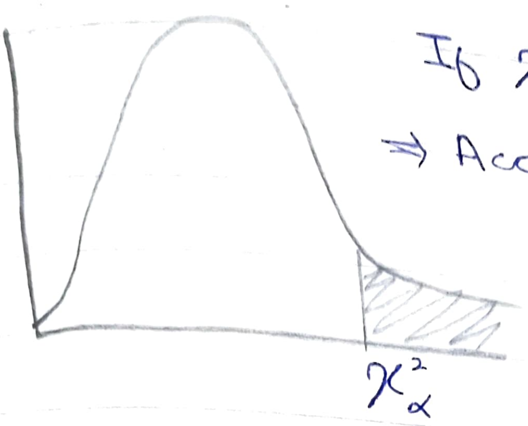
a) If H_A $\sigma^2 < \sigma_0^2$



If $\chi^2_{cal} < \chi^2_{~~cal~~\,tab}$

Accept H_A

b) If H_A $\sigma^2 > \sigma_0^2$



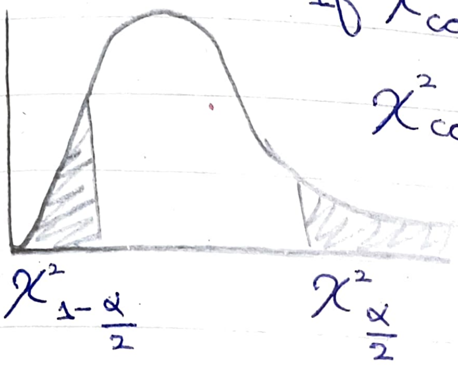
If $\chi^2_{cal} > \chi^2_{tab}$

$\Rightarrow$ Accept H_A

c) If H_A $\sigma^2 \neq \sigma_0^2$

If $\chi^2_{cal} < \chi^2_{1-\frac{\alpha}{2}}$ OR

$\chi^2_{cal} > \chi^2_{\frac{\alpha}{2}}$



$\Rightarrow$ Accept H_A

Example 10.12:

$\Rightarrow n = 10 \quad S = 1.2 \quad \sigma_0 = 0.9$

$$\chi^2_{cal} = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(10-1)(1.2)^2}{(0.9)^2}$$

$$= 16.0$$

we will check this calculated value from table. approximately where it is lying.

$P \approx 0.7$ (lying b/w. 10% and 5%)

Hypothesis Testing for σ_1^2/σ_2^2 OR σ_2^2/σ_1^2

1) $H_0 : \sigma_1^2 = \sigma_2^2 \Rightarrow H_0 : \dfrac{\sigma_1^2}{\sigma_2^2} = 1$

2) $H_A : \sigma_1^2 < \sigma_2^2$ OR

$H_A \quad \sigma_1^2 > \sigma_2^2$ OR

$H_A \quad \sigma_1^2 \neq \sigma_2^2$

3) Define α

4) $f_{cal} = \dfrac{S_1^2}{S_2^2}$ OR $f_{cal} = \dfrac{S_2^2}{S_1^2}$

(whichever is larger)

$\sigma_1^2 < \sigma_2^2$

5) $f_{cal} < f_{1-\alpha}(v_1, v_2) = $ Accept H_A

$\sigma_1^2 > \sigma_2^2$

$f_{cal} > f_{\alpha}(v_1, v_2) \Rightarrow$ Accept H_A

$\sigma_1^2 \neq \sigma_2^2$

$$\left.\begin{array}{l} f_{cal} < f_{1-\frac{\alpha}{2}}(v_1, v_2) \text{ OR} \\ f_{cal} > f_{\frac{\alpha}{2}}(v_1, v_2) \end{array}\right\} \text{Accept } H_A$$

Example 10.13

$n_1 = 12$ $\qquad$ $v_1 = 11$

$n_2 = 10$ $\qquad$ $v_2 = 9$

$\alpha = 10\%$ $\qquad$ $\frac{\alpha}{2} = 5\%$

$$f_{\frac{\alpha}{2}}(v_1, v_2) = f_{0.05}(11, 9) = 3.11$$

$$f_{1-\frac{\alpha}{2}}(v_1, v_2) = f_{0.95}(11, 9)$$

$$= \frac{1}{f_{0.05}(9, 11)} = \frac{1}{2.90}$$

$$= 0.34$$

Expected Frequency

$$e_f = \frac{R_i \times C_j}{G}$$

$O_{i,j}$	$e_{i,j}$	$(O_i - e_i)^2/e_i$
182	200.9	
213	209.9	
203	187.2	
154	135.1	
135	141.1	
110	125.8	

$$\chi^2_{cal} = \sum_{i=1}^{r \times c} \left(\frac{(O_i - e_i)^2}{e_i} \right)$$ here $r = 2$, $c = 3$

$= 7.85$

Critical Region

$$\chi^2_{tab} = \chi^2_{\alpha}((r-1)(c-1))$$

$$= \chi^2_{0.05} \underline{(2-1)(3-1)}$$

$= 2$

[If any value in table/matrix is less than 5 then we merge two values called Yaches correction].