UNIVERSITY OF KARACHI

Probability and Statistical Methods

BSCS-306

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Semester No: **2nd**

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ASSIGNMENT : 01

Date:

Compute mean and variance for the following Pmf.

1) $f(y) = \left(\frac{1}{2}\right)^y y \; ; \; y = 1, 2, 3$

SOLUTION :-

y	1	2	3	Sum
f(y)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{7}{8}$

$\sum_{y=1}^{3} f(y) \neq 1$

Let K be the required question to make f(x) a Pmf.

$$K \times \sum_{x=1}^{3} f(y) = 1$$

$$K \times \frac{7}{8} = 1 \quad , \quad \boxed{K = \frac{8}{7}}$$

$$f(y) = \frac{8}{7}\left(\frac{1}{2}\right)^y \; ; \; y = 1, 2, 3.$$

$$f(y) = \frac{8}{7}\left(\frac{1}{2}\right)^1 , \frac{8}{7}\left(\frac{1}{2}\right)^2 , \frac{8}{7}\left(\frac{1}{2}\right)^3$$

$$E(Y) = 1\left(\frac{8}{7}\right)\left(\frac{1}{2}\right)^1 , 2\left(\frac{8}{7}\right)\left(\frac{1}{2}\right)^2 , 3\left(\frac{8}{7}\right)\left(\frac{1}{2}\right)^3 .$$

$$E(Y^2) = (1)^2\left(\frac{8}{7}\right)\left(\frac{1}{2}\right)^1 , (2)^2\left(\frac{8}{7}\right)\left(\frac{1}{2}\right)^2 , (3)^2\left(\frac{8}{7}\right)\left(\frac{1}{2}\right)^3$$

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y	1	2	3	Sum
f(y)	4/7	2/7	1/7	1
E(y)	4/7	4/7	3/7	11/7
$E(y^2)$	4/7	8/7	9/7	3

Mean $= E(Y) = \frac{11}{7} = 1.57$

Variance $= E(Y^2) - [E(Y)]^2$

$= 3 - (1.57)^2$

Variance $= 0.53$

2) $f(x) = \frac{6 - |x-7|}{36}$ for $x = 2, 3, 4, \ldots, 12$

SOLUTION:-

x	2	3	4	5	6	7	8	9	10	11	12	Sum
f(x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36	1

$\sum_{x=2}^{12} f(x) = 1$

It is a Pmf.

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x	2	3	4	5	6	7	8	9	10	11	12	sum
$E(x)$	$\frac{1}{18}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{5}{9}$	$\frac{5}{6}$	$\frac{7}{6}$	$\frac{10}{9}$	1	$\frac{5}{6}$	$\frac{11}{18}$	$\frac{1}{3}$	7
$E(x^2)$	$\frac{1}{18}$	$\frac{1}{2}$	$\frac{4}{3}$	$\frac{25}{9}$	5	$\frac{49}{6}$	$\frac{80}{9}$	9	$\frac{25}{3}$	$\frac{121}{18}$	4	$\frac{329}{6}$

$E(x) = x \cdot f(x)$

$E(x^2) = x^2 f(x)$.

Mean $= E(x)$

Mean $= 7$

Variance $= E(x^2) - [E(x)]^2$

Variance $= \frac{329}{6} - (7)^2$

Variance $= 5.833$

3) $f(x) = \frac{x+2}{25}$; for $x = 1, 2, 3, 4, 5$

SOLUTION:-

x	1	2	3	4	5	Sum
$f(x)$	$\frac{3}{25}$	$\frac{4}{25}$	$\frac{1}{5}$	$\frac{6}{25}$	$\frac{7}{25}$	1

$\sum_{x=1}^{5} f(x) = 1$ so it's Pmf.

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x	1	2	3	4	5	Sum
E(x)	$\frac{3}{25}$	$\frac{8}{25}$	$\frac{3}{5}$	$\frac{24}{25}$	$\frac{7}{5}$	3.4
$E(x^2)$	$\frac{3}{25}$	$\frac{16}{25}$	$\frac{9}{5}$	$\frac{96}{25}$	7	13.4

Mean = E(x)

Mean = 3.4

Variance = $E(x^2) - [E(x)]^2$

$= 13.4 - (3.4)^2$

Variance = 1.84

4) $f(x) = \frac{\binom{2}{x}\binom{4}{3-x}}{\binom{6}{3}}$; for $x = 0, 1, 2$

SOLUTION:-

x	0	1	2	Sum
f(x)	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	1

$\sum_{x=0}^{2} f(x) = 1$ ∴ it is pmf

Mean = E(x)

Mean = 1

Variance = $E(x^2) - [E(x)]^2 = 1.4 - (1)^2$

Variance = 0.4

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Compute the mean and variance of following P.d.f

1) $g(x) = \begin{cases} 6x(1-x) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$

SOLUTION :-

$$\int_0^1 g(x)\,dx = \int_0^1 6x(1-x)\,dx.$$

$$= \int_0^1 (6x - 6x^2)\,dx = \frac{6x^2}{2}\Big|_0^1 - \frac{6x^3}{3}\Big|_0^1$$

$$= [3(1)^2 - 3(0)^2] - [2(1)^3 - 2(0)^3] = 3 - 2 = 1$$

$\int_0^1 g(x)\,dx = 1$ it's P.d.f.

$$E(x) = \int_0^1 x \cdot (6x - 6x^2)\,dx.$$

$$= \int_0^1 (6x^2 - 6x^3)\,dx = \frac{6x^3}{3}\Big|_0^1 - \frac{6x^4}{4}\Big|_0^1$$

$$= \{2(0)^3 - 2(1)^3\} - \left\{\frac{3(1)^4}{2} - \frac{3(0)^4}{2}\right\}$$

$$= 2 - \frac{3}{2} \qquad \boxed{E(x) = \frac{1}{2}}$$

$\boxed{\text{Mean} = 0.5}$

$$E(x^2) = \int_0^1 x^2 (6x - 6x^2)\,dx$$

$$= \int_0^1 (6x^3 - 6x^4)\,dx$$

$$= \left|\frac{6x^4}{4}\right|_0^1 - \left|\frac{6x^5}{5}\right|_0^1$$

$$= \left\{\frac{3(1)^4}{2} - \frac{3(0)^4}{2}\right\} - \left\{\frac{6(1)^5}{5} - \frac{6(0)^5}{5}\right\}$$

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$= \frac{3}{2} - \frac{6}{5}$, $\boxed{E(x^2) = \frac{3}{10}}$

Variance $= E(x^2) - [E(x)]^2$

$= \frac{3}{10} - \left(\frac{1}{2}\right)^2 = 0.05$

$\boxed{\text{Variance} = 0.05}$.

2) $f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \leq x \leq 2 \\ 0 & \text{elsewhere.} \end{cases}$

SOLUTION:-

$f(x) = \int_0^1 x\,dx + \int_1^2 (2-x)\,dx.$

$= \left|\frac{x^2}{2}\right|_0^1 + \left|2x - \frac{x^2}{2}\right|_1^2$

$= \left\{\frac{(1)^2}{2} - \frac{(0)^2}{2}\right\} + \left\{\frac{2-(2)^2}{2} - \frac{2-(1)^2}{2}\right\}$

$= \frac{1}{2} + \frac{1}{2} = 1$

It's a Pdf.

$E(x) = \int_0^1 x \cdot x\,dx + \int_1^2 x \cdot (2-x)\,dx$

$= \int_0^1 x^2\,dx + \int_1^2 (2x - x^2)\,dx$

$= \left|\frac{x^3}{3}\right|_0^1 + \left|\frac{2x^2}{2} - \frac{x^3}{3}\right|_1^2$

$= \left\{\frac{(1)^3}{3} - \frac{(0)^3}{3}\right\} + \left\{\frac{2(2)^2}{2} - \frac{2(2)^3}{3}\right\}$

$= \frac{1}{3} + \left(\frac{4}{3} - \frac{2}{3}\right) = \frac{1}{3} + \frac{2}{3} = \frac{3}{3}$

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$E(x) = 1$

Mean = 1

$$E(x^2) = \int_0^1 x^2 \cdot x\,dx + \int_1^2 x^2 \cdot (2-x)\,dx$$

$$= \int_0^1 x^3\,dx + \int_1^2 (2x^2 - x^3)\,dx$$

$$= \frac{x^4}{4}\Big|_0^1 + \frac{2x^3}{3} - \frac{x^4}{4}\Big|_1^2$$

$$= \left\{\frac{(1)^4}{4} - \frac{(0)^4}{4}\right\} + \left\{\left(\frac{2(2)^3}{3} - \frac{(2)^4}{4}\right) - \left(\frac{2(1)^3}{3} - \frac{(1)^4}{4}\right)\right\}$$

$$= \frac{1}{4} + \left(\frac{4}{3} - \frac{5}{12}\right) = \frac{1}{4} + \frac{11}{12}$$

$$E(x^2) = \frac{7}{6}.$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= \frac{7}{6} - (1)^2$$

Variance = 0.1667

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