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SEAT NO : B20102077

CLASS : BSCS I

SECTION : A

M. AMAS WASEEM

COURSE 303

APPENDIX E

Q3: Solution.

(a) Solution:-

$0.123123123\ldots$

let $x = 0.123123123\ldots$

$1000x = 123.123123123\ldots$

$999x = 122.99$

$x = \dfrac{122.99}{999}$

$x = \dfrac{41}{333}$ A.

(b) Solution:-

12.777

let $x = 12.777$

$10x = 127.777\ldots$

$9x = 115$.

$x = \dfrac{115}{9}$ A

(c) Solution:-

$38.0781818\ldots$

$x = 38.07818181\ldots$

$100x = 3807.818181\ldots$

$99x = 3769.74$.

$x = 3769.74/99$

$x = \dfrac{20943}{550}$ A.

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(d) Solution:-

$0.4296000\ldots$

let $x = 0.4296000\ldots$

$1000x = 429.6000\ldots$

$999x = 429.1704$

$$x = \frac{429.1704}{999}$$

$$x = \frac{537}{1250} \text{ Ans.}$$

Q9

(a) Solution:-

$a - 3 \leq b - 3$.

$a \leq b - 3 + 3$

$a \leq b$ correct.

(b) $-a \leq -b$

wrong, because if we multiply to cancel -ve sign so the signs will change and the equality will be wrong.

(c) Solution:-

$3 - a \leq 3 - b$.

$-a \leq -b$

wrong.

(d) Solution:-

$6a \le 6b$

multiply $1/6$ on B.S.

$a \le b$ correct.

(e) $a^2 \le ab$.

wrong, because we don't know the value of a whether it is positive or negative.

(f) Solution:-

$a^3 \le a^2 b$.

$a \le b$ proved!

Q" (a) $a \le a$.

Right, This inequality is valid for all the values of a.

(b) $a < a$

wrong, This inequality isn't valid for any values of a.

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Q27. Solution:-

$$3 \leq 4 - 2x < 7$$

$3 \leq 4 - 2x$

$-1 \leq -2x$.

$\frac{-1}{-2} \geq \frac{-2x}{-2}$.

$\frac{1}{2} \geq x$.

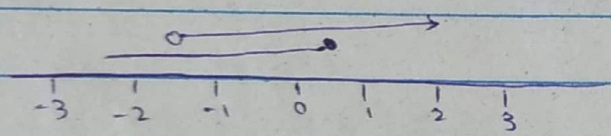
$4 - 2x < 7$.

$-2x < 3$

$\frac{-2x}{-2} > \frac{3}{-2}$.

$x > -\frac{3}{2}$

coordinate line



A = 1 = +ve

B = -1 = +ve

C = -2 = -ve.

$\left(-\frac{3}{2}, \frac{1}{2}\right]$

A

Q31. solution:

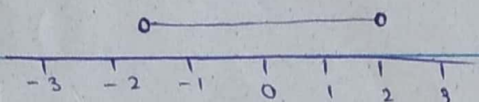
$$\frac{3x+1}{x-2} < 1$$

$$\frac{3x+1-x+2}{x-2} < 0$$

$$\frac{2x+3}{x-2} < 0.$$

$2x + 3 = 0$ or $x - 2 = 0.$

$x = -3/2$ $\quad$ $x = 2$



A = -2 = $-$ve.

B = 1 = $+$ve.

C = 3 = $-$ve.

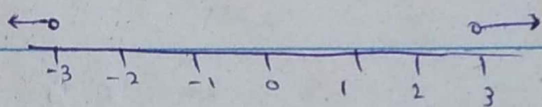
Q35. Solution:

$$x^2 > 9$$

$$x^2 - 9 > 0.$$

$$(x)^2 - (3)^2 > 0.$$

$$(x+3)(x-3) > 0.$$



$x + 3 = 0$ $\quad$ $x - 3 = 0.$

$x = -3$ $\quad$ $x = +3$

A = -3 = -4 = $+$ve.

B = 4 = $+$ve.

C = 3 = $-$ve.

Q39 Solution:-

$x^2 - 9x + 20 \le 0$

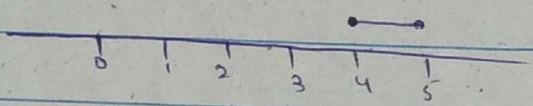
$x^2 - 4x - 5x + 20 \le 0$

$x(x-4) - 5(x-4) \le 0$

$(x-4)(x-5) \le 0$

$x - 4 = 0 \qquad x - 5 = 0$

$x = 4 \qquad x = 5$

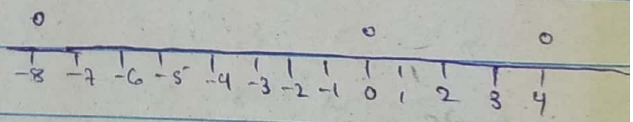


Q41. Solution:-

$\frac{2}{x} < \frac{3}{x-4}$

co-ordinate line

$\frac{2}{x} - \frac{3}{x-4} < 0$



$\frac{2x - 8 - 3x}{x(x-4)} < 0$

$\frac{-x - 8}{x(x-4)} < 0$

S.S $= (-8, 0) \cup (4, \infty)$

A

$-x - 8 = 0 \qquad x(x-4) = 0$

$x = -8 \qquad x = 0 \qquad x = 4$

Test points,

A = -9 = $-ve$

B = -4 = $+ve$

C = 5 = $+ve$

D = 2 = $-ve$

Q43. Solution:

$x^3 - x^2 - x - 2 > 0$.

$x^3 - 2x^2 + x^2 - 2x + x - 2 > 0$

$x^2(x-2) + x(x-2) + (x-2) > 0$.

$(x-2)(x^2 + x + 1) > 0$

Either $x - 2 > 0$ $\qquad$ $x^2 + x + 1 > 0$

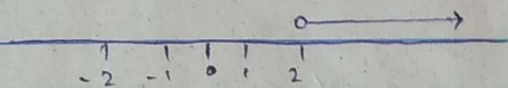
$x > 2$.

Test points

$A = 3 = +ve$.

$B = 5 = +ve$.

$C \quad -1 = -ve$



$(2, +\infty)$

Q45 solution:-

$\sqrt{x^2 + x - 6}$

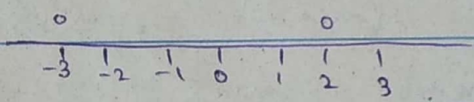
$\sqrt{x^2 + x - 6} \geq 0$

$\sqrt{x^2 + 3x - 2x - 6} \geq 0$

$\sqrt{x(x+3) - 2(x+3)} \geq 0$

$\sqrt{(x+3)(x-2)} \geq 0$

coordinate line



suppose $x + 3 = 0$ $\qquad$ $x - 2 = 0$

$x = -3$ $\qquad$ $x = 2$

Test points

$A = -4 = +ve$..

$B = -1 = -ve$.

$C = 3 = +ve$.

Solution Set = $(-\infty, -3] \cup [2, \infty)$.

APPENDIX : F

Q8 Solution:-

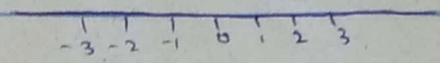
$$|x-3| = 3-x$$

$$\pm(x-3) = 3-x$$

$x - 3 = 3 - x$ $\qquad\qquad$ $-\cancel{x} + \cancel{3} = \cancel{3} - \cancel{x}$

$2x = 6$ $\qquad\qquad$ $0 = 0$

$x = 3$



Test points.

$A = -2 = +ve$ $\qquad\qquad$ $S.S = x \leq 3$

$B = 2 = +ve$

Q9 Solution:-

$$\sqrt{(x+5)^2} = x+5$$

$$|x+5| = x+5$$

$x+5 = x+5$ $\qquad\qquad$ $-(x+5) = x+5$

$0 = 0$ $\qquad\qquad$ $-x-5 = x+5$

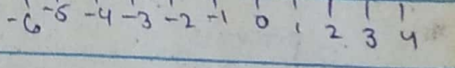
$-2x = 10$

$x = -5$

Test points.

$A = -6 = -ve$

$B = -2 = +ve$



$C = 2 = +ve$

$$x \geq -5$$

Q5 Solution.

$|x^2+9| = x^2+9$

x here $|x| \geq 0$

x = all real values.

'coz the square on x will positive the negative numbers ~~too~~ so x can be any number.

Q7 Solution :-

$|3x^2+2x| = x|3x+2|$

$x|3x+2| = x|3x+2|$

$x|3x+2| \geq 0$ $\quad\because |x| = x$

$x \geq 0$ $\qquad |3x+2| \geq 0$ $\qquad x \geq 0$.

$3x+2 \geq 0$ $\qquad -3x-2 \geq 0$

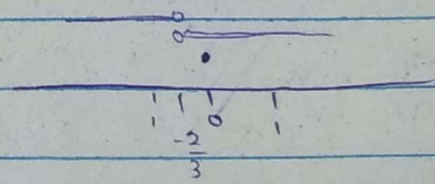
$x \geq \frac{-2}{3}$. $\qquad -3x \geq 2$.

$x \leq \frac{2}{-3}$

Test points

$A = 1 = +ve$

$B = -2 = -ve$



$[0, \infty) \cup \frac{-2}{3}$ A

Q17 Solution,-

$$|6x-2|=7.$$

$6x-2=7$

$6x=9.$

$x=\frac{9}{6}$

$x=3/2.$

$-6x+2=7.$

$-6x=\cancel{9}5$

$x=\frac{\cancel{9}5}{-6}$

$x=-\cancel{3}5/2.$

$$\frac{-5}{6},\ \frac{3}{2}\quad A$$

Q23 Solution,-

$$\left|\frac{x+5}{2-x}\right|=6.$$

$$\frac{\pm(x+5)}{2-x}=6.$$

$\frac{x+5}{2-x}=6$

$x+5=12-6x.$

$5-12=-6x-x.$

$-\cancel{17}=-7x.$

$\frac{-\cancel{17}}{\cancel{7}}=x$

$1=x$

$-\frac{(x+5)}{2-x}=6$

$-x-5=12-6x.$

$5x=17.$

$x=\frac{17}{5}.$

$$1,\ \frac{17}{5}\quad A$$

Q25 Solution :-

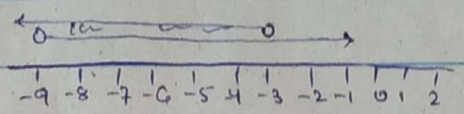
$|x + 6| < 3$.

$\pm(x + 6) < 3$.

$x + 6 < 3$. $\qquad$ $-x - 6 < 3$.

$x < -3$. $\qquad$ $-x < 9$.

$x > -9$.



Test points.

$A = -6 = +ve$.

$B = -10 = -ve$

$C = -2 = -ve$

$(-9, -3)$ A

Q33 Solution :-

$\frac{1}{|x-1|} < 2$.

$\frac{1}{\pm(x-1)} < 2$.

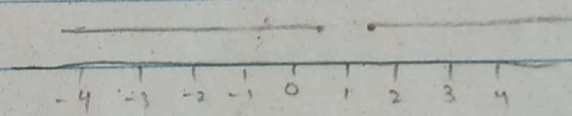
$\frac{1}{-(x-1)} < 2$

$\frac{1}{-x+1} < 2 = 1 > -2x + 2$

$-1 > -2x$.

$\frac{-1}{-2} < x = 1/2 > x$

co-ordinate line.



$(-\infty, \frac{1}{2}) \cup (\frac{3}{2}, +\infty)$

$\frac{1}{+(x-1)} < 2$.

$1 < 2x - 2$.

$3 < 2x$.

$\frac{3}{2} < x$

Test Points : $A = 2 = +ve$.

$B = 0 = +ve$

$C = 1 = -ve$.

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Q39 Solution :-

$|x-3|^2 - 4|x-3| = 12$ — (i)

let $u = |x-3|$

(i) $\Rightarrow u^2 - 4u = 12$

$u^2 - 4u - 12 = 0$.

$u^2 - 6u + 2u - 12 = 0$

$u(u-6) + 2(u-6) = 0$

$(u-6)(u+2) = 0$

$u - 6 = 0$ $\quad\quad$ $u + 2 = 0$

$u = 6$ $\quad\quad$ $u = -2$.

$|x-3| = 6$ $\quad\quad$ $|x-3| = -2$

$\pm(x-3) = 6$ $\quad\quad$ $\pm(x-3) = -2$.

$x - 3 = 6$ $\quad$ $-x + 3 = 6$. $\quad\quad$ $x - 3 = -2$. $\quad$ $-x + 3 = -2$.

$x = 9$ $\quad$ $-x = 3$ $\quad\quad$ $x = 1$ $\quad$ $-x = -5$

$x = -3$ $\quad\quad$ $x = 5$.

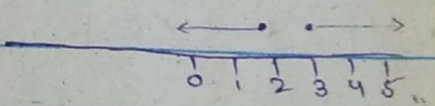
$-3, 9$ A

Q37 Solution :-

$\sqrt{(x^2 - 5x + 6)^2} = x^2 - 5x + 6$

$|x^2 - 5x + 6| = x^2 - 5x + 6$.

$x^2 - 5x + 6 \geq 0$



$x^2 - 3x - 2x + 6 \geq 0$

$x(x-3) - 2(x-3) \geq 0$

Test Points

$(x-3)(x-2) \geq 0$

$A = 1$ =

$x - 3 \geq 0$ $\quad\quad$ $x - 2 \geq 0$

$B = 2$ =

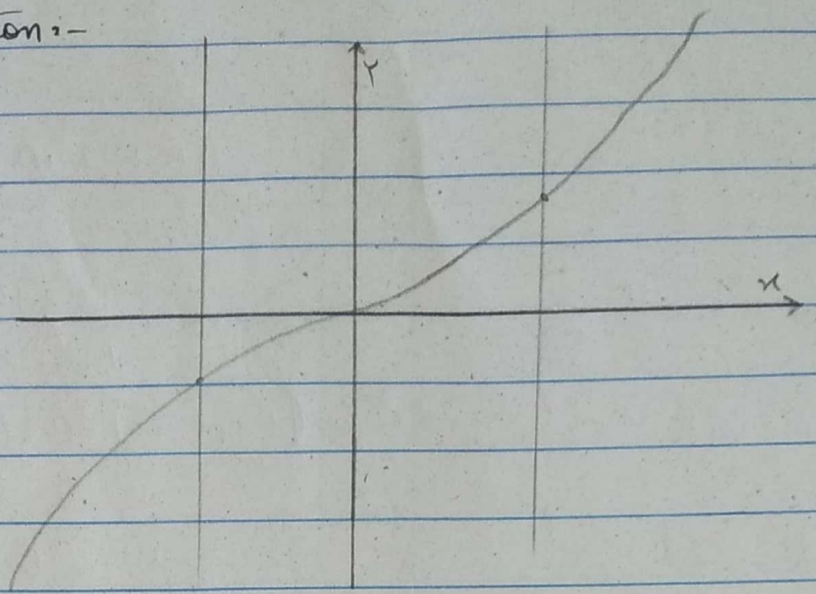
$x \geq 3$ $\quad\quad$ $x \geq 2$.

$C = 4$ =

EXERCISE 0.1

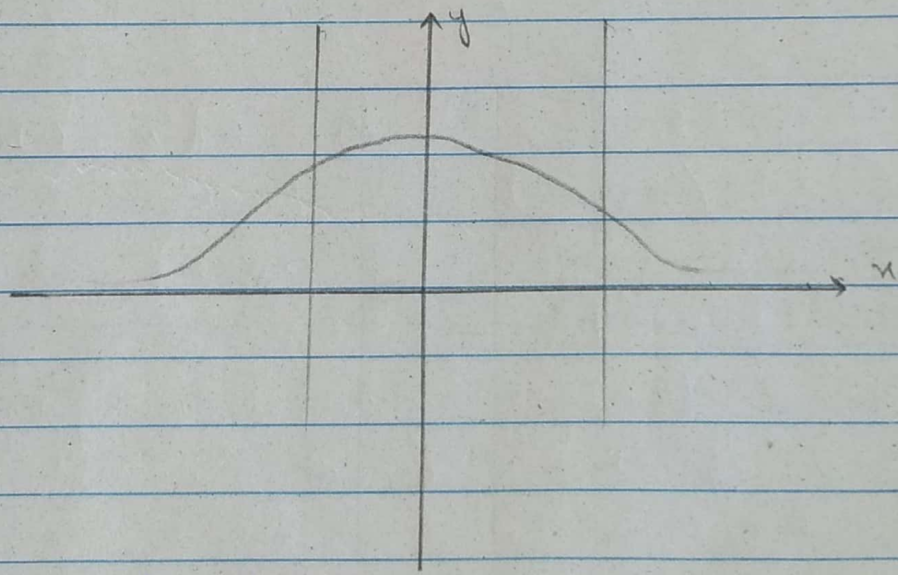
Q3 Solution:-

(a)



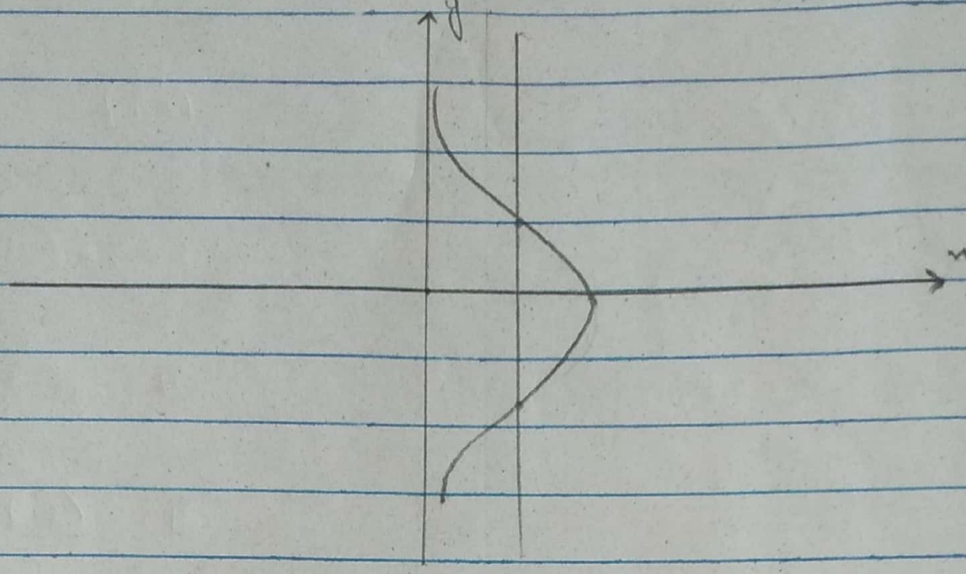
Yes, vertical lines intersect at only one points

(b)



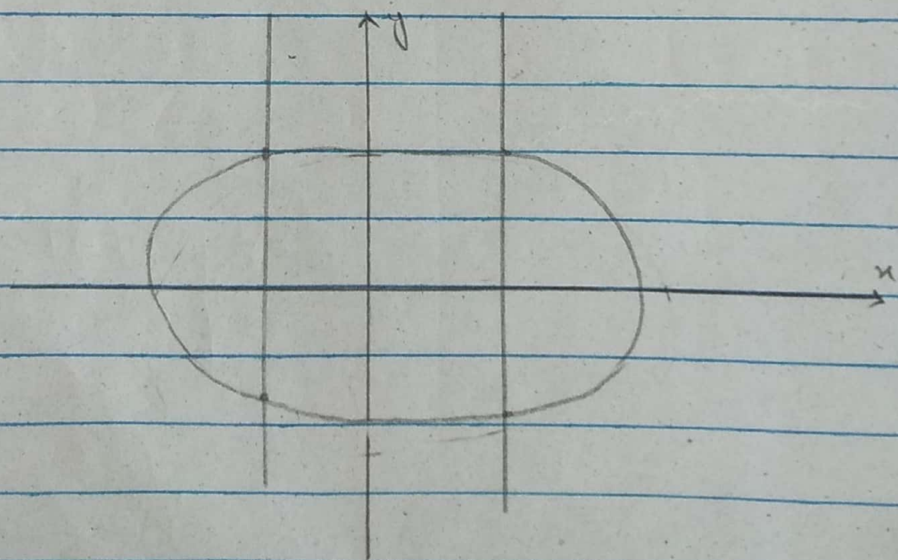
Yes, vertical lines intersect at only 1 points

(c)



No, vertical line intersecting at two points.

(d)



No, vertical lines intersecting at two points.

Q4

(a) Solution:-

$$f(x) = \frac{x^2 + x}{x + 1}.$$

for domain: $x + 1 = 0$.

$x = -1$.

Domain $f(x) = (-\infty, -1) \cup (-1, \infty)$

$g(x) = x$.

Domain $g(x) = (-\infty, \infty)$

Domain of f and g = $(-\infty, -1), (-1, \infty)$

(b) Solution:-

$$f(x) = \frac{x^2\sqrt{x} + \sqrt{x}}{x + 1}$$

$g(x) = \sqrt{x}$

Domain: $x \geq 0$

Domain: $x \geq 0$

Domain of f and g = $x \geq 0$

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Q7

(a) Solution :-

$$f(x) = 3x^2 - 2.$$

$f(0) = -2$

$f(2) = 10$

$f(-2) = 10$

$f(3) = 25$

$f(\sqrt{2}) = 4$

$f(3t) = 27t^2 - 2.$

(b) Solution :-

$$f(x) = \begin{cases} 1/x, & x > 3 \\ 2x, & x \leq 3. \end{cases}$$

$f(0) = 0$

$f(2) = 4$

$f(-2) = -4$

$f(3) = 6$

$f(\sqrt{2}) = 2\sqrt{2}$

$f(3t) = 1/3t$ when $t > 1$

$f(3t) = 6t$ when $t \leq 1$

Q9

(a) Solution :-

$$f(x) = \frac{1}{x-3}$$

Domain : $x - 3 \neq 0$

$x \neq 3$.

Range : let $y = \frac{1}{x-3}$.

$$x = \frac{1-3y}{y}$$

$$f(y) = \frac{1-3y}{y} \quad \therefore f(y) = f^{-1}(x).$$

$$f^{-1}(x) = \frac{1-3x}{x}.$$

Domain of $f^{-1}(x) \neq 0$.

So the Domain of $f(x) = \frac{1}{x-3}$; $x \neq 3$

and range $x \neq 0$

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Q9 (b) Solution:-

$$f(x) = \frac{x}{|x|}$$

Domain: $x \neq 0$

Range: $\{-1, 1\}$

Q9 (c) Solution:

$$g(x) = \sqrt{x^2 - 3}.$$

Domain: $x^2 - 3 \geq 0$

$x^2 \geq 3$. $\quad \Rightarrow |x| \geq 3$.

$x \geq \pm\sqrt{3}$ $\quad$ or $\quad x \leq -\sqrt{3}$.

Range:

Let $y = \sqrt{x^2 - 3}$.

$y^2 = x^2 - 3$

$y^2 + 3 = x^2$.

$f(y) = \sqrt{y^2 + 3}$.

$f^{-1}(x) = \sqrt{x^2 + 3}$.

Domain $f^{-1}(x) = [0, \infty)$

$g(x) = \sqrt{x^2 - 3}$

Domain: $(-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$

Range: $[0, \infty)$

Q9 (d) Solution:

$G(x) = \sqrt{x^2 - 2x + 5}$

Domain: x = all real numbers.

let $y = \sqrt{x^2 - 2x + 5}$

$y^2 = x^2 - 2x + 5$

$y^2 + 2x + 5 = x^2$.

$\cancel{f(y) = \sqrt{y^2 + 2x + 5}}$

$-x^2 + 2x \mp 5 + y^2 = 0$;

$D \geq 0 \Rightarrow b^2 - 4ac \Rightarrow 4 + 4(1)(-5+y^2) \geq 0$

$4 - 20 + 4y^2 \geq 0$

$+4y^2 - 16 \geq 0$

$4y^2 \geq 16$.

$y^2 \geq 4$

$y \geq 2$? $D \geq 0$.

$G(x) = \sqrt{x^2 - 2x + 5}$

Domain : $(-\infty, \infty)$

Range : $[2, \infty)$

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Q9 (e) Solution :-

$$h(x) = \frac{1}{1 - \sin x}$$

$$1 - \sin x \neq 0$$

$$x \neq \sin^{-1}(1)$$

$$x \neq \left(\frac{\pi}{2} + 2n\pi\right)$$

where $n = 0, \pm 1, \pm 2 \ldots$

$$\cancel{h(x)}\ y = \frac{1}{1 - \sin x}.$$

$$-1 \leq \sin x \leq 1$$

for maximum value $= \frac{1}{1-1} = \infty$

for minimum value $= \frac{1}{1-(-2)} = \frac{1}{2}.$

$$\cancel{h(x)}\ y \geq \frac{1}{2}$$

$$h(x) = \frac{1}{1 - \sin x}$$

Domain = $x \neq \left(2n + \frac{1}{2}\right)\pi$ where $n = 0, \pm 1, \ldots$

Range = $y \geq \frac{1}{2}$

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Q9 (f) solution:-

$$H(x) = \sqrt{\frac{x^2-4}{x-2}}$$

$$y = \sqrt{\frac{(x-2)(x+2)}{(x-2)}}$$

$$y = \sqrt{x+2}$$

$$y = [-2, 2) \cup (2, +\infty)$$

$$\Rightarrow \quad y^2 = x+2.$$

$$y^2 - 2 = x.$$

$$R = [0, 2) \cup (2, \infty)$$

$$H(x) = \sqrt{\frac{x^2-4}{x-2}}.$$

Domain = $x \geq 2$, $x \neq 2$.

Range = ~~*~~ $y \geq 0$, $y \neq 2$

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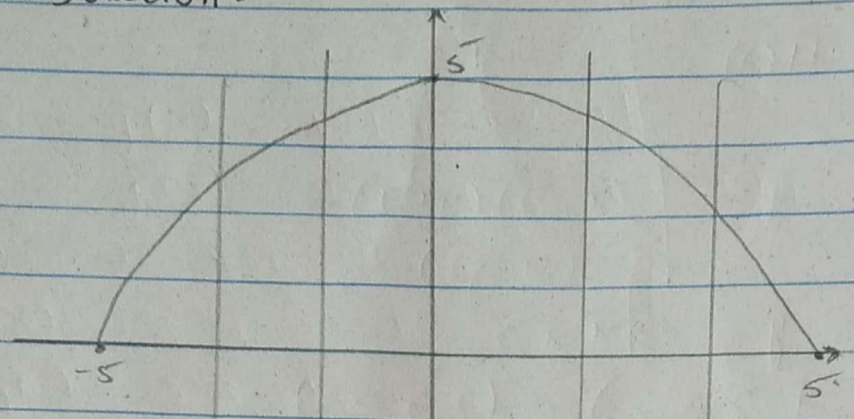
Q15 Solution :-

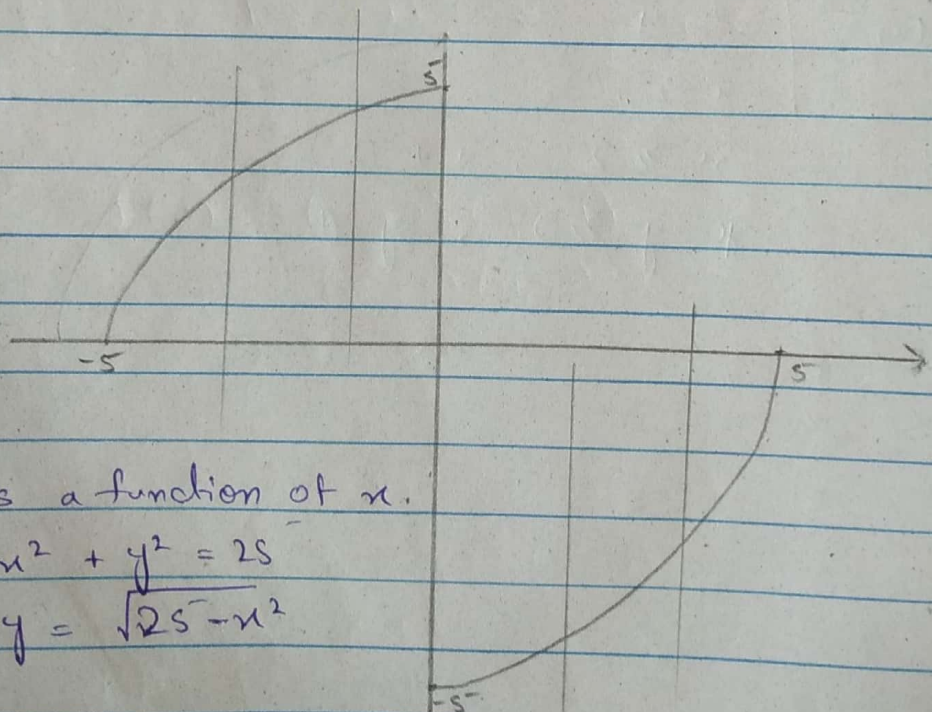


y is a function of x.

$$x^2 + y^2 = 25$$

$$y = \sqrt{25 - x^2}$$

Q17 Solution :-



y is a function of x.

$$x^2 + y^2 = 25$$

$$y = \sqrt{25 - x^2}$$

$$y = \begin{cases} \sqrt{25 - x^2}, & -5 \geq x \leq 0 \\ -\sqrt{25 - x^2} & 0 \geq x \leq 5 \end{cases}$$

Q19 Solution:-

False; The function which is plotted on graph can cross the x - axis on two or many points ~~but~~ ~~on~~ ~~diffe~~ like as $\sin(x)$.

Q27 (a)

Solution:-

$f(x) = |x| + 3x + 1$

$f(x) = \pm x + 3x + 1$

$f(x) = x + 3x + 1$

$f(x) = 4x + 1$ when $x > 0$

$f(x) = -x + 3x + 1$

$f(x) = 2x + 1$ when $x < 0$

$$f(x) = \begin{cases} 4x + 1 , & x \geq 0 \\ 2x + 1 , & x < 0 \end{cases}$$

Q27 (b)

Solution :-

$$g(x) = |x| + |x-1|$$
$$g(x) = \pm x + \{\pm (x-1)\}$$

when $x > 0$.

$$g(x) = x + x - 1$$
$$g(x) = 2x - 1$$

when $x < 0$.

$$g(x) = -x - (x-1) \Rightarrow -x - x + 1$$
$$g(x) = -2x + 1$$

$$g(x) = \begin{cases} 2x-1, & x \\ -2x+1, & x < 0 \end{cases}$$

Q35 Solution:-

$$f(x) = \frac{(x+2)(x^2-1)}{(x+2)(x-1)}$$

(i) Holes occur at $x = -2$ ~~or~~ $x = 1$

(ii)
$$f(x) = \frac{\cancel{(x+2)}\cancel{(x-1)}(x+1)}{\cancel{(x+2)}\cancel{(x-1)}}$$

$$g(x) = x + 1$$

$g(x)$ has all real values of x.

Q37 Solution:-

$$WCT = \begin{cases} T, & 0 \le v \le 3 \\ 35.74 + 0.6215T - 35.75v^{0.16} + 0.4275Tv^{0.16} & 3 < v \end{cases}$$

(a) $v = 3 mi/h$

$T = 3°F$

(b) $v = 15 mi/h$

$T = 33.74 + 0.6215(25) - 35.75(15)^{0.16} +$
$0.4275(25)(15)^{0.16}$.
$12.62°F$

(c) $v = 46 mi/h$

$T = 33.74 + 0.6215(25) - 35.75(48)^{0.16} +$
$0.4275(25)(46)^{0.16}$
$= 5°F$

Q29 (a) Solution:-

Volume = $l \times b \times h$

length = $15 - 2x$

breath = $8 - 2x$.

height = x.

Volume = $(15-2x)(8-2x)x$

(b)

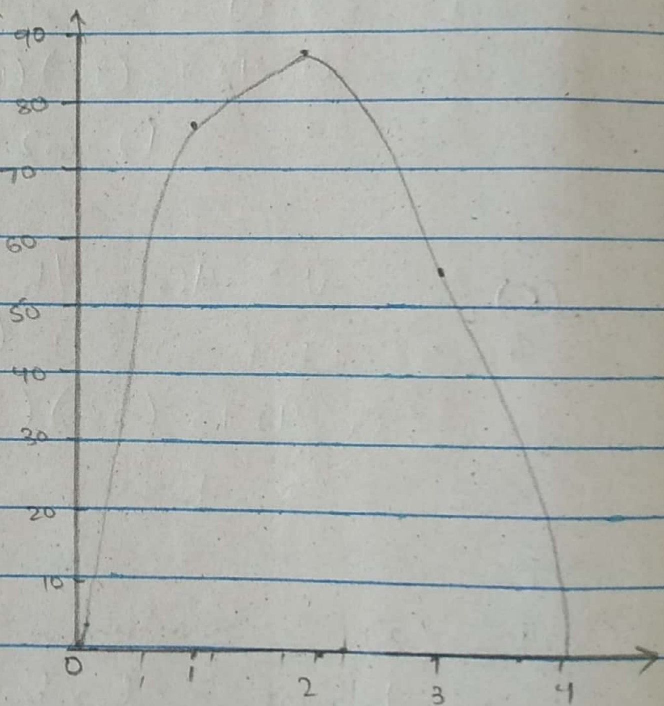
$$8 - 2x \geq 0$$
$$8 < 2x$$
$$4 < x.$$

Domain $(0, 4)$

(c)

$$V = (15 - 2x)(8 - 2x)\,x$$

- For $x = 0$.
  $V = 0$



- For $x = 1$
  $V = 78$

- For $x = 2$
  $V = 88$

- For $x = 3$
  $V = 54$

- For $x = 4$
  $V = 0$

Range:
$[0, 88]$

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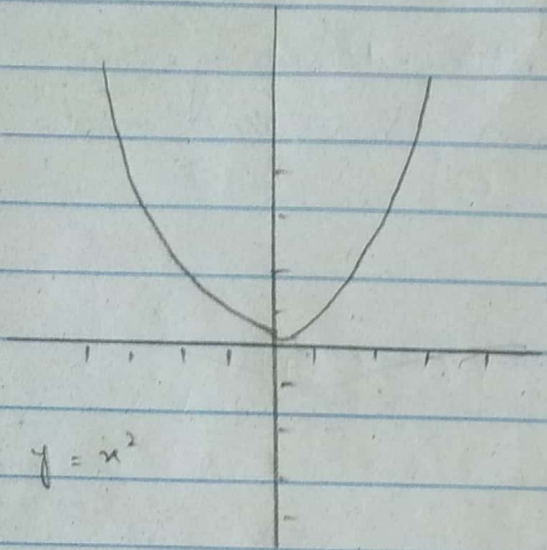
(d)

The v increases initially with x and when it reaches it's maximum value then it starts decreasing. One might construct boxes of max volumes at $x = 2$.

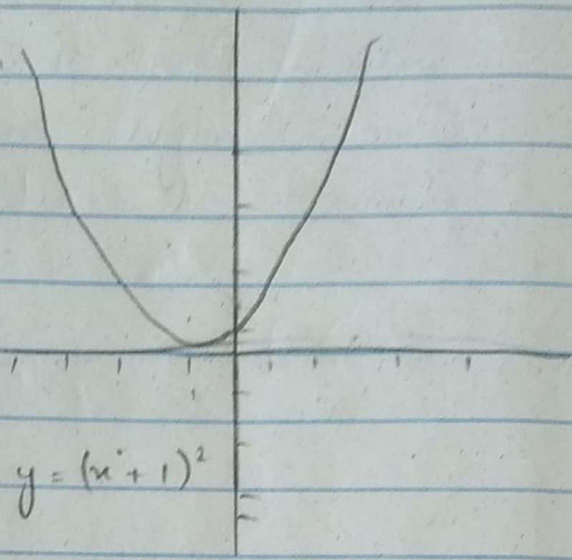
EXERCISE 0.2

Q5 Solution:-

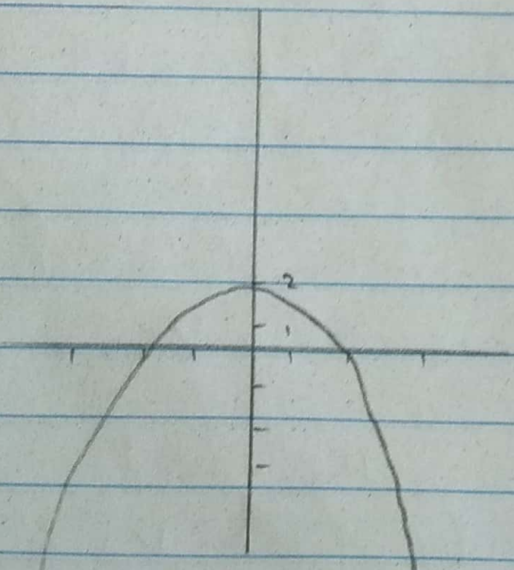
$$y = -2(x+1)^2 - 3$$



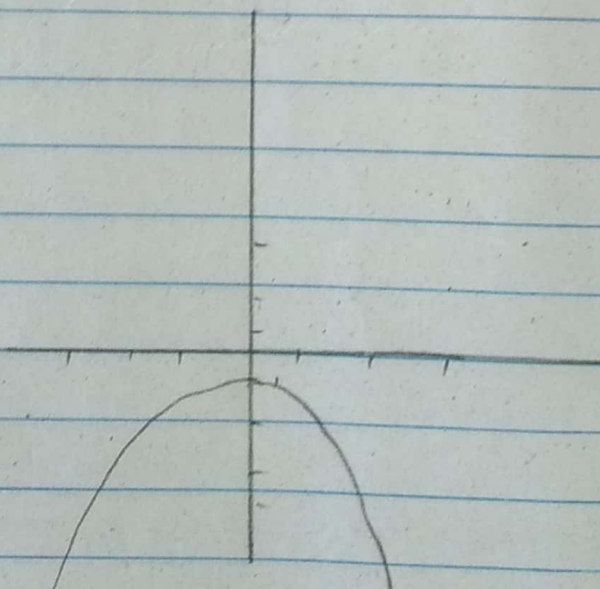
$y = x^2$



$y = (x+1)^2$



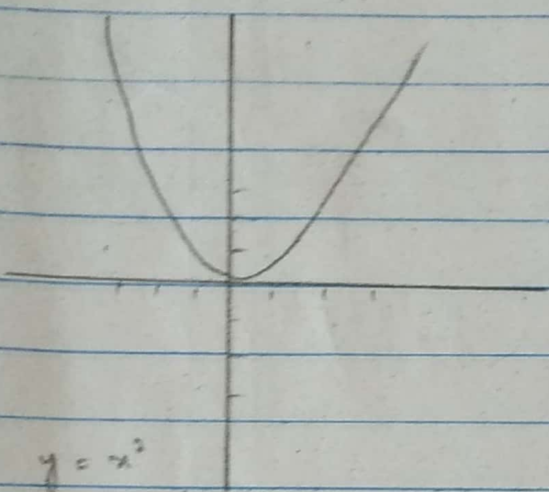
$y = -2(x+1)^2$



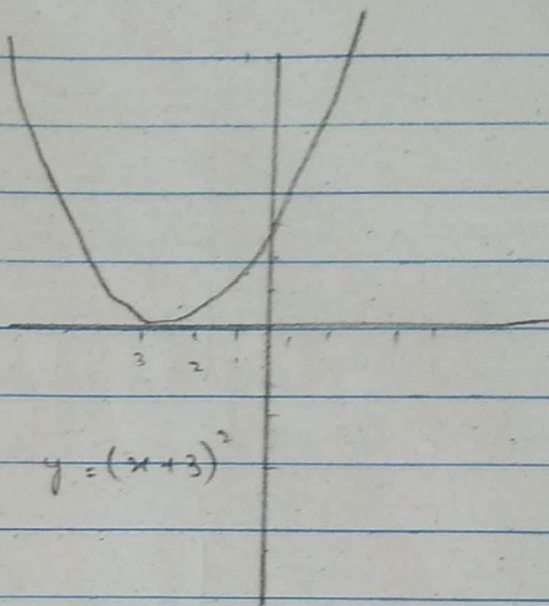
$y = -2(x+1)^2 - 3$

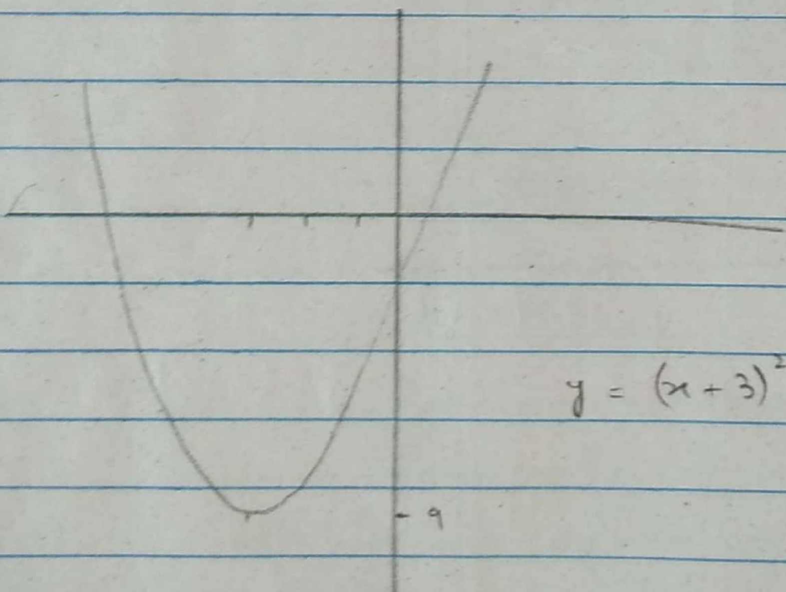
Q7 Solution:-

$$y = x^2 + 6x$$
$$= (x)^2 + 2(x)(3) + (3)^2 - (3)^2$$
$$= (x+3)^2 - 9.$$



$y = x^2$

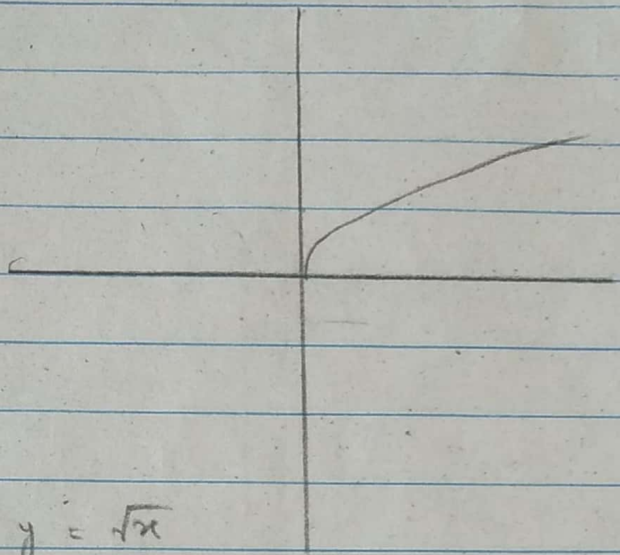




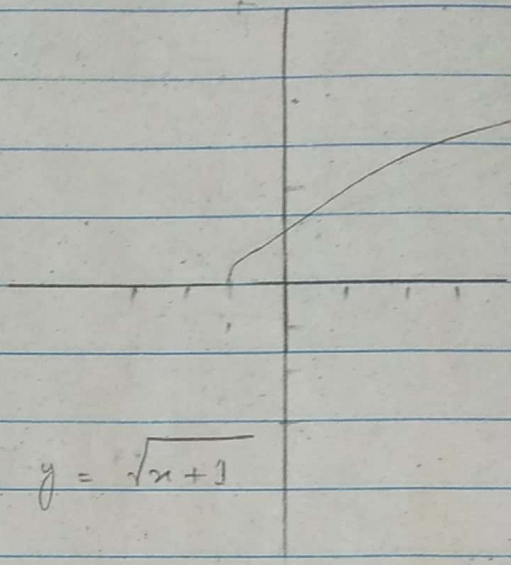
$$y = (x+3)^2 - 9 = x^2 + 6x$$

Q9 Solution:-

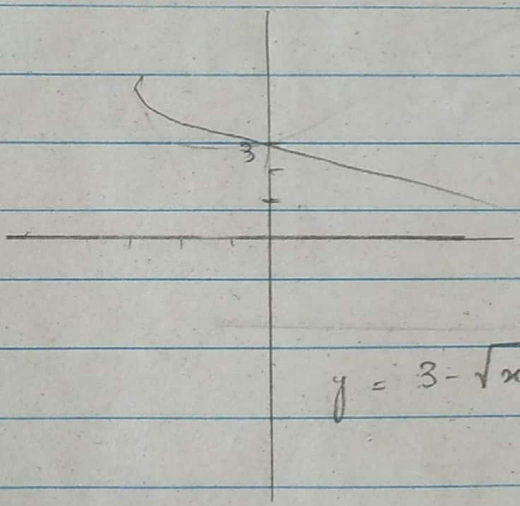
$$y = 3 - \sqrt{x+1}$$



$y = \sqrt{x}$



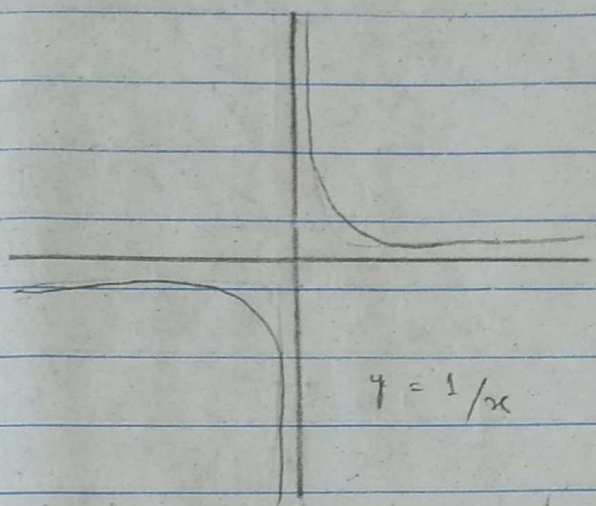
$y = \sqrt{x+1}$



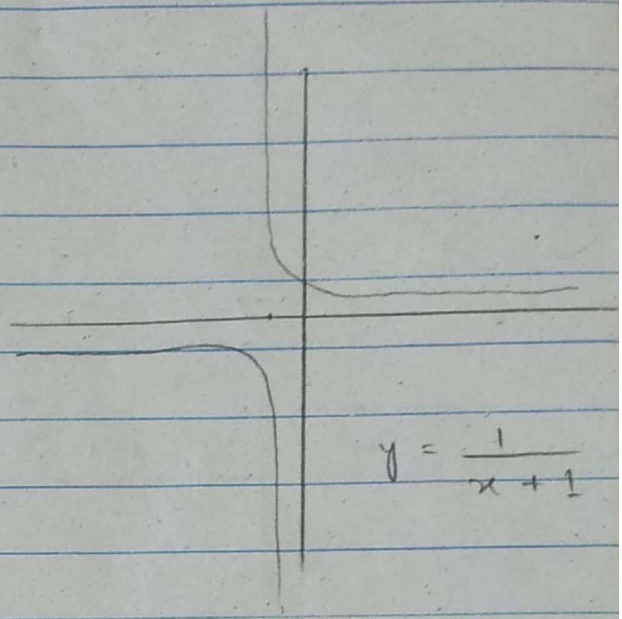
$y = 3 - \sqrt{x+1}$

Q 15 Solution :-

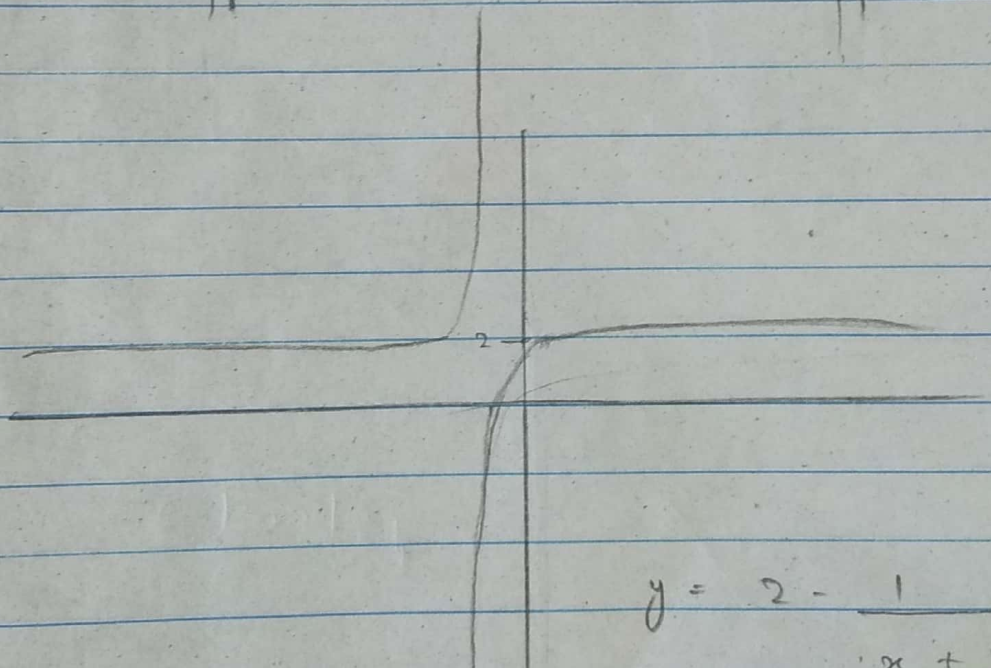
$$y = 2 - \frac{1}{x+1}$$



$y = 1/x$



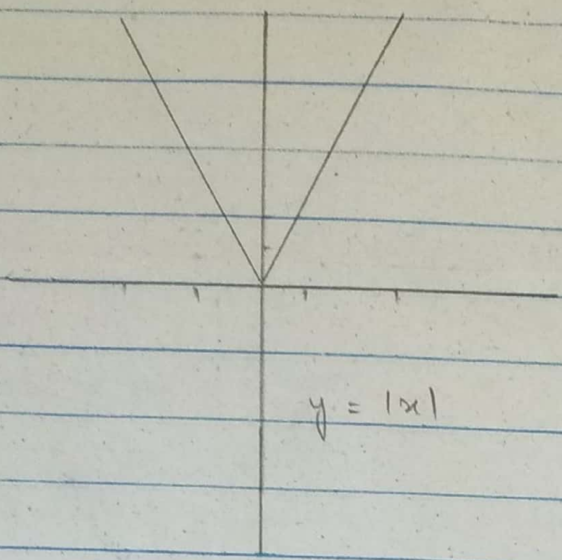
$$y = \frac{1}{x+1}$$



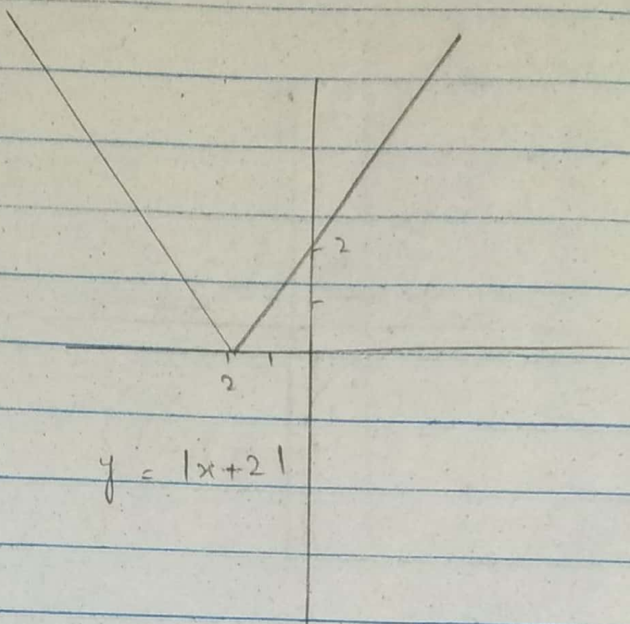
$$y = 2 - \frac{1}{x+1}$$

Q17 Solution:-

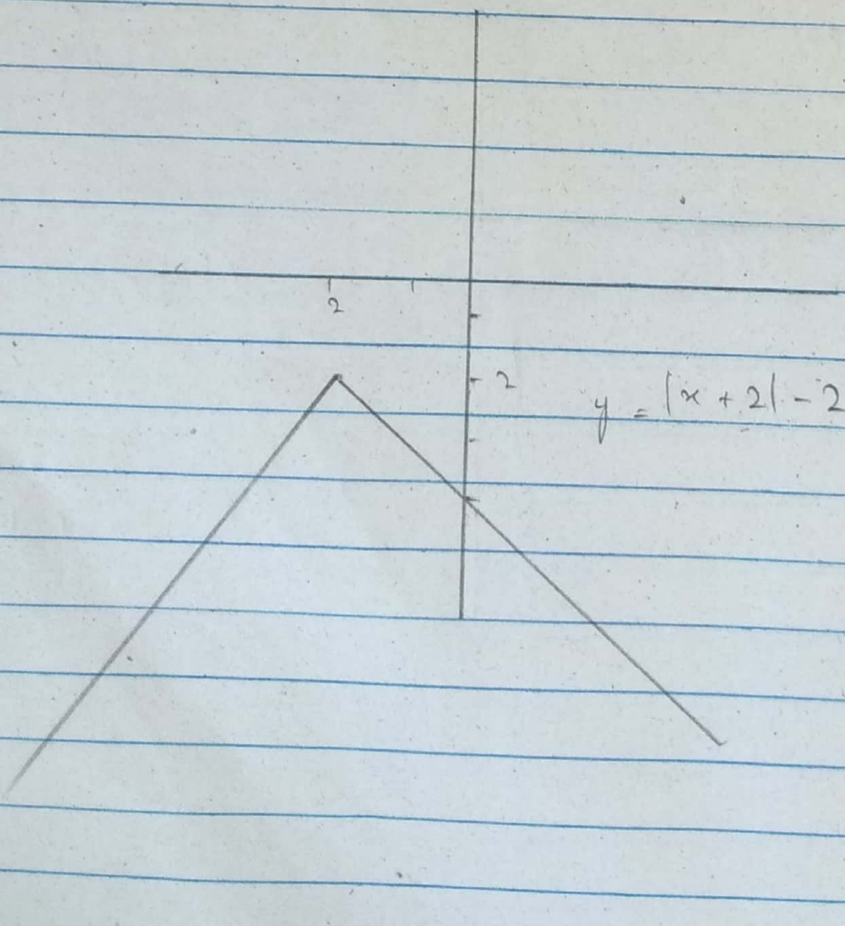
$y = |x+2| - 2$



$y = |x|$



$y = |x+2|$

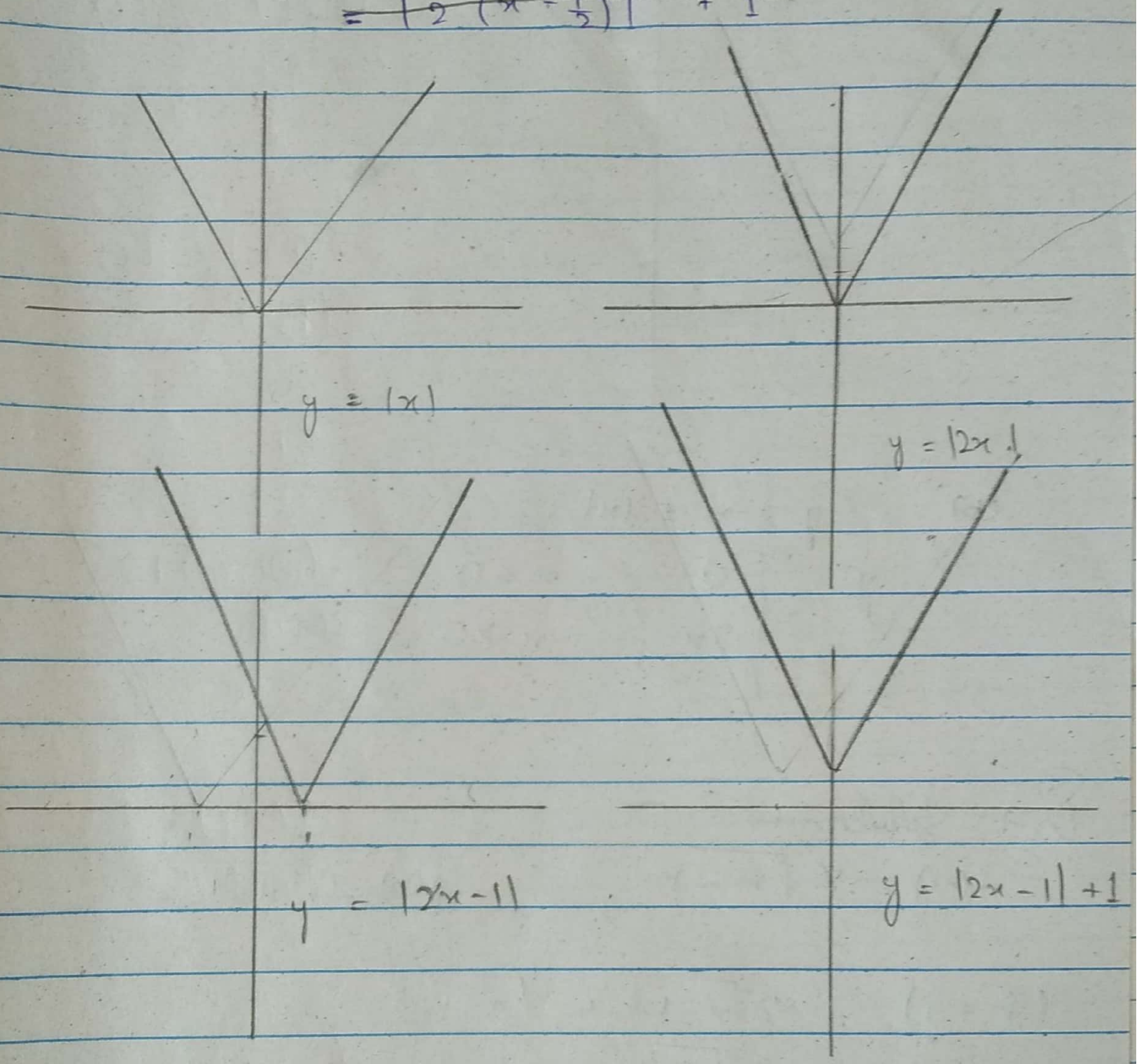


$y = |x+2| - 2$

Q19 Solution :-

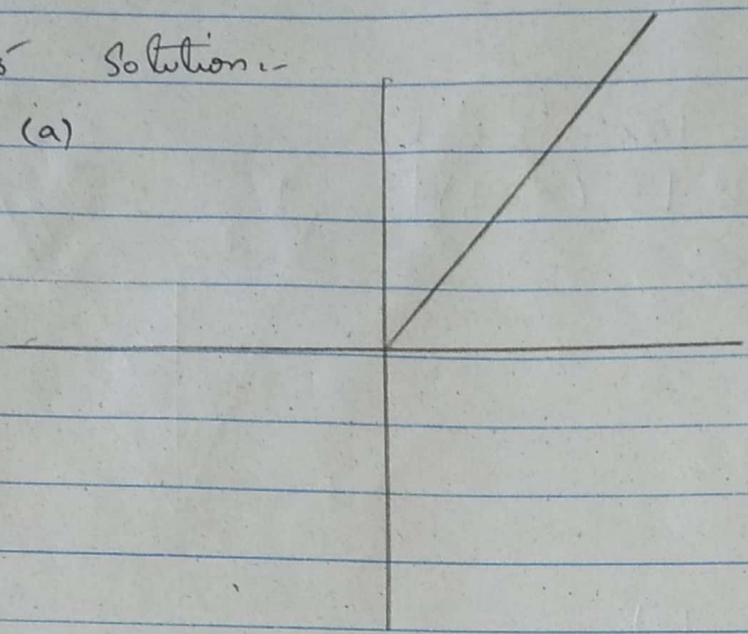
$$y = |2x - 1| + 1$$

$$\cancel{= |2(x - \tfrac{1}{2})| + 1}$$



Q25 Solution:-

(a)



(b)
$$y = x + |x|$$
$$y = \begin{cases} 0 & x \le 0 \\ 2x & x > 0 \end{cases}$$

Q27 Solution:-

$f(x) = 2\sqrt{x-1}$ $\qquad g(x) = \sqrt{x-1}$

$(f+g) = 2\sqrt{x-1} + \sqrt{x-1}$
$\qquad = 3\sqrt{x-1}$; $x \geq 1$

$(f-g) = \sqrt{x-1}$; $x \geq 1$

$(fg) = 2x - 2$; $x \geq 1$

$(f/g) = 2$ $\qquad x > 1$

Q29 Solution :-

(a) $f(g(2))$

$\sqrt{x^3+1}$

$= 3$ Ans.

(b) $g(f(4))$

$= (\sqrt{4})^3 + 1$

$= 8 + 1$

$= 9$ Ans.

(c) $f(f(16))$

$= \sqrt{(\sqrt{16})}$

$= 2$ Ans.

(d) $g(g(0))$

$= ((0)^3 + 1)^3 + 1$

$= 2$ Ans.

(e) $f(2+h)$.

$= \sqrt{2+h}$ Ans

(f) $g(3+h)$

$(3+h)^3 + 1$ Ans

Q31 Solution :-

$$fog = \left(\sqrt{1-x}\right)^2$$
$$= 1 - x. \quad , x \leq 1$$

$$gof = \sqrt{1 - x^2} \quad , |x| \leq 1$$

Q33 Solution :-

$$fog = f(g(x))$$
$$\frac{1 + (x/1-x)}{1 - (x/1-x)}$$
$$\frac{1 - \cancel{x} + \cancel{x}}{\cancel{1-x}} \div \frac{1 - x - x}{\cancel{1-x}}.$$
$$fog = \frac{1}{1-2x} \quad ; \quad x \neq \frac{1}{2}.$$

$$gof = g(f(x))$$
$$= \frac{1+x}{\cancel{1-x}} \div 1 - \frac{1+x}{\cancel{1-x}}.$$
$$= \frac{1+x}{\cancel{1-x}} \div \frac{1-x-(1+x)}{\cancel{1-x}}.$$
$$= \frac{1+x.}{\cancel{1}-x-\cancel{1}-x.}$$
$$= -\left(\frac{1+x}{2x}\right) \quad ; \quad x \neq 0$$

Q35 Solution.

$$fogoh = f(g(h))$$

$$= \left(\frac{1}{x^3}\right)^2 + 1$$

$$= \frac{1}{x^6} + 1$$

$$fogoh = x^{-6} + 1$$ Ans

Q39 Solution..

$h(x) = \sin x.$

$g(x) = x^2.$ Ans

Q61 Solution..

(a) $f(x) = x^2$ Even function.

(b) $f(x) = x^3$

$f(-x) = -x^3$ Odd function.

$-f(x) = -x^3$

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Q63 Solution :-

(a) $f(x) = x^2.$

$f(-x) = x$

$-f(x) = -x$ Even function.

(b) $f(x) = x^3.$

$f(-x) = -x^3$

$-f(x) = -x^3$ Odd function.

(c) $f(x) = |x|$

$f(-x) = x$

$-f(x) = -x$ Even function.

(d) $f(x) = x+1$

$f(-x) = -x+1$

$-f(x) = -x-1$ Nor Even neither odd

(e) $f(x) = \dfrac{x^5 - x}{1+x^2}$

$f(-x) = \dfrac{-x^5 + x}{1+x^2}.$

$-f(x) = \dfrac{-x^5 + x}{1+x^2}.$ odd function.

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(f) $f(x) = 2$. Even function.

EXERCISE 1.1

Q1 Solution:-

(a) $\lim_{x \to 0^-} g(x) = 3$

(b) $\lim_{x \to 0^+} g(x) = 3$

(c) $\lim_{x \to 0} g(x) = 3$

(d) $g(0) = 3$

Q3 Solution:-

(a) $\lim_{x \to 3^-} f(x) = -1$

(b) $\lim_{x \to 3^+} f(x) = 3$

(c) $\lim_{x \to 3} f(x)$ = limits doesn't exist.

(d) $f(3) = 1$

Q5 Solution:-

(a) $\lim_{x \to -2^-} f(x) = 0$

(b) $\lim_{x \to -2^+} f(x) = 0$

(c) $\lim_{x \to 2} f(x) = 0$

(d) $F(-2) = 3$

Q7 Solution:-

(a) $\lim_{x \to 3^-} f(x) = -\infty$

(b) $\lim_{x \to 3^+} f(x) = -\infty$

(c) $\lim_{x \to 3} f(x) = -\infty$

(d) $f(3) = 1$

Q9 Solution :-

$$\lim_{x \to -2} f(x) = +\infty$$

$$\lim_{x \to 0^-} f(x) = +\infty$$

$$\lim_{x \to 0^+} f(x) = 2$$

$$\lim_{x \to 2^-} f(x) = 2$$

$$\lim_{x \to 2^+} f(x) = -\infty$$

the vertical asymptotes of the graph of f :

$\Rightarrow$ $x = -2$, $x = 0$, $x = 2$

Q17: False:

It is possible that $f(a) = L$ but $\lim_{x \to a} f(x) \neq L$ 'coz it is also possible that

$\lim_{x \to a^+} \neq L$ and $\lim_{x \to a^-} \neq L$

Q18: True:

$\lim_{x \to a} f(x)$ exist when both right and left hand limits are equal and has real no.

Q19 False:

It is possible that $\lim_{x \to a^-}$ and $\lim_{x \to a^+}$ exist and limit $x \to a$ not exist. 'coz $\lim_{x \to a}$ exist when both right hand and left hand limits are equal.

Q20 false:

It is not understood that the limits of a function and actual ~~points~~ value of a point is same. Right hand limit and actual value can be unequal and anything.

EXERCISE 1.2

Q2 Solution:-

(a) $\lim\limits_{x \to 2} [f(x) + g(x)]$

$= \cancel{\lim} \; [0 + 0]$

$= 0$ Ans.

(b) $\lim\limits_{x \to 0} [f(x) + g(x)]$

$f(x)$ does not exist so limit doesn't exist.

(c) $\lim\limits_{x \to 0^+} [f(x) + g(x)]$

$= -2 + 2$

$= 0$ Ans

(d) $\lim\limits_{x \to 0^-} [f(x) + g(x)]$

$= 1 + 2$

$= 3$ Ans

(e) $\lim_{x \to 2} \dfrac{f(x)}{1 + g(x)}$

$= \dfrac{0}{(1 + 0)}$

$= 0$ Ans

(f) $\lim_{x \to 2} \dfrac{1 + g(x)}{f(x)}$

Limit doesn't exist 'coz function $f(x)$ is which is denominator tends to zero.

(g) $\lim_{x \to 0^+} \sqrt{f(x)}$.

Limit doesnt exist cous $f(x)$ is undefined at $x = 0$.

(h) $\lim_{x \to 0^-} \sqrt{f(x)}$

$= \sqrt{1}$

$= 1$ Ans.

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Q5 Solution:

$$\lim_{x \to 3} \frac{x^2 - 2x}{x + 1}$$

$$\frac{(3)^2 - 2(3)}{3 + 1}$$

$$\frac{3}{4} \quad \text{Ans}$$

Q13 Solution:-

$$\lim_{t \to 2} \frac{t^3 + 3t^2 - 12t + 4}{t^3 - 4t}$$

$\because$ $t^3 + 3t^2 - 12t + 4$

	1	+3	-12	+4
2	1	2	10	-4
	1	5	-2	0

$(t - 2)(t^2 + 5t - 2)$

$$\Rightarrow \lim_{t \to 2} \frac{(t-2)(t^2 + 5t - 2)}{t^3 - 4t}$$

$$= \lim_{t \to 2} \frac{\cancel{(t-2)}(t^2 + 5t - 2)}{t\,\cancel{(t-2)}(t+2)}$$

$$= \lim_{t \to 2} \frac{t^2 + 5t - 2}{t^2 + 2t}$$

$$= \frac{12}{8} = \frac{3}{2} \quad \text{Ans}$$

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Q15 Solution:-

$$\lim_{x \to 3^+} \frac{x}{x-3}.$$

$$= \frac{3}{0}$$

$$= +\infty$$

Q19 Solution:-

$$\lim_{x \to 2^-} \frac{x}{x^2-4}$$

$$= \frac{2}{4-4} = \frac{2}{0}$$

$$= -\infty$$

Q23 Solution:-

$$\lim_{y \to 6} \frac{y+6}{y^2-36}.$$

$$\lim_{y \to 6} \frac{\cancel{y+6}}{\cancel{(y+6)}(y-6)}$$

$$= \frac{1}{0} = \infty$$

Limit doesn't exist

Q 27 Solution:

$$\lim_{x \to 2^+} \frac{1}{|2-x|}$$

$$= \frac{1}{0} = +\infty$$ Ans

Q 29 Solution:-

$$\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3}$$

$$= \lim_{x \to 9} \frac{\cancel{(\sqrt{x}-3)}(\sqrt{x}+3)}{\cancel{(\sqrt{x}-3)}}$$

$$= \lim_{x \to 9} \sqrt{x}+3$$

$$= 3+3 = 6$$ Ans.

Q 31 Solution:-

(a) 2

(b) 2

(c) 2

Q33 Solution:-

Yes, ~~if~~ both ~~funtion~~ has its existed limits so the summed function obviously has it's limits.

Q34

False, At a time 2 functions have the ~~same~~ limit but the functions can be different.

Q35 False, At a time two functions may have the same limits but the functions can be different.

Q36 True,

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Q37 Solution:-

$$\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x}$$

$$= \lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x} \times \frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2}$$

$$= \lim_{x \to 0} \frac{\cancel{x} + \cancel{4} - \cancel{4}}{\cancel{x}\left(\sqrt{x+4} + 2\right)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{x+4} + 2}$$

$$= \frac{1}{4} \quad \text{Ans}$$

Q39 Solution:-

$$f(x) = \frac{x^3 - 1}{x - 1}$$

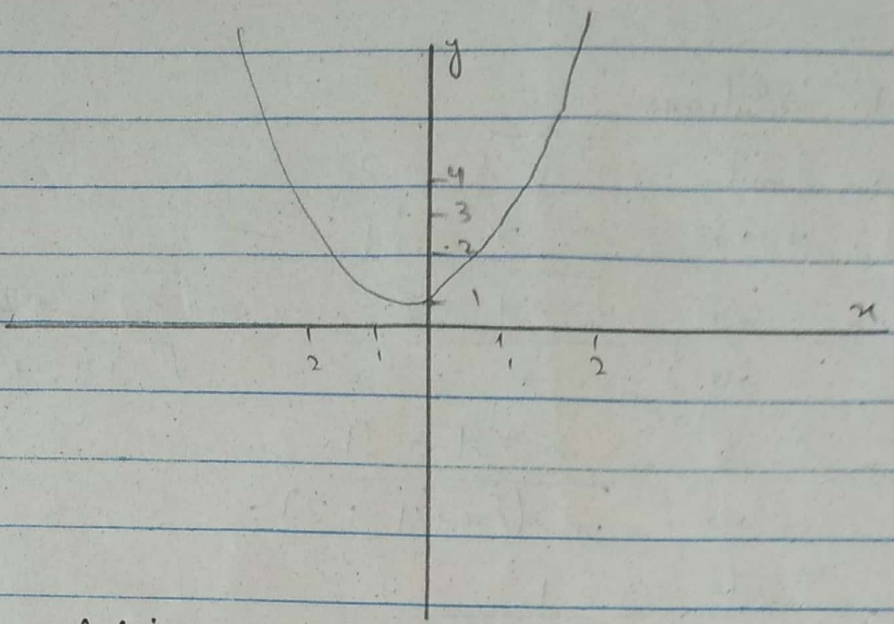
$$= \frac{\cancel{(x-1)}\left(x^2 + x + 1\right)}{\cancel{x-1}}$$

$$f(x) = x^2 + x + 1$$

(a) $$\lim_{x \to 1} f(x) = \lim_{x \to 1} x^2 + x + 1$$

$$= 3.$$

(b)



Q 41 Solution.

(a)

Infinities can't be summed.

(b) $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{x^2} \right) = \lim_{x \to 0^+} \left(\frac{x-1}{x^2} \right)$

$= \frac{0-1}{0}$

$= +\infty$

EXERCISE 1.3

Q3 Solution:-

(a)

$$\lim_{x \to -\infty} \phi(x) = 0$$

(b)

$$\lim_{x \to +\infty} \phi(x) = -1$$

Q13 Solution:-

$$= \lim_{x \to +\infty} \frac{3x + 1}{2x - 5}$$

$$= \lim_{x \to +\infty} \frac{3x + 1 / x}{2x - 5 / x}$$

$$= \lim_{x \to +\infty} \frac{3 + 1/x}{2 - 5/x}$$

$$= \frac{3 + 0}{2 - 0} = \frac{3}{2} \quad \text{Ans}$$

Q21 Solution:-

$$\lim_{t \to +\infty} \frac{6 - t^3}{7t^3 + 3} = \frac{6 - t^3 / t^3}{7t^3 + 3 / t^3}$$

$$= \lim_{t \to +\infty} \frac{6/t^3 - 1}{7 + 3/t^3} = \frac{(1/\infty) - 1}{7 + (1/\infty)}$$

$$= \frac{-1}{7} \quad \text{Ans}$$

Q_{23} Solution:-

$$\lim_{x \to +\infty} \sqrt[3]{\frac{2 + 3x - 5x^2}{1 + 8x^2}}$$

$$= \lim_{x \to +\infty} \sqrt[3]{\frac{\cancel{x^2}\left(2/x^2 + 3/x - 5\right)}{\cancel{x^2}\left(1/x^2 + 8\right)}}$$

$$= \sqrt[3]{\frac{0 - 0 - 5}{0 + 8}}$$

$$= \sqrt[3]{\frac{-5}{8}} = \frac{\sqrt[3]{-5}}{8} \quad A$$

Q_{27} Solution :-

$$\lim_{y \to -\infty} \frac{2 - y}{\sqrt{7 + 6y^2}}$$

$$= \lim_{y \to -\infty} \frac{y\left(2/y - 1\right)}{y\sqrt{7/y^2 + 6}}$$

$$= \lim_{y \to -\infty} \frac{2/y - 1}{\sqrt{7/y^2 + 6}}$$

$$= \frac{0 - 1}{\sqrt{0 + 6}} = \frac{-1}{\sqrt{6}} \quad \text{Ans}$$

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Q32 Solution :-

$$= \lim_{x \to +\infty} \left(\sqrt{x^2 - 3x} - x\right)$$

$$= \lim_{x \to +\infty} \left(\sqrt{x^2 - 3x} - x\right) \cdot \frac{\sqrt{x^2 - 3x} + x}{\sqrt{x^2 - 3x} + x}$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^2} - 3x - \cancel{x^2}}{\sqrt{x^2 - 3x} + x}$$

$$= \lim_{x \to +\infty} \frac{-3x}{x\sqrt{1 - 3/x} + x}$$

$$= \lim_{x \to +\infty} \frac{-3}{\sqrt{1 - 3/x} + 1}$$

$$= \frac{-3}{\sqrt{1 - 0} + 1}$$

$$= \frac{-3}{2} \quad \text{Ans}$$

Q35 Solution :-

$$\lim_{x \to +\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$= \lim_{x \to +\infty} \frac{(e^x + e^{-x})/e^x}{(e^x - e^{-x})/e^x}$$

$$= \lim_{x \to +\infty} \frac{1 + e^{-2x}}{1 - e^{-2x}} = \lim_{x \to +\infty} \frac{1 + 1/e^{2x}}{1 - 1/e^{2x}}$$

$$= \frac{1 + (1/\infty)}{1 - 1/\infty} = 1 \quad \text{Ans}$$

Q37 Solution:-

$$\lim_{x \to +\infty} \ln\left(\frac{2}{x^2}\right)$$

$$= \lim_{x \to +\infty} \ln\left(\frac{2}{x^2}\right) \Rightarrow \ln(0)$$

$$= -\infty \qquad \because \ln(0) = -\infty$$

Ans

Q39 Solution:-

$$\lim_{x \to +\infty} \frac{(x+1)^x}{x^x}$$

$$= \lim_{x \to +\infty} \frac{\{x(1 + 1/x)\}^x}{x^x}$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^x}(1 + 1/x)^x}{\cancel{x^x}} \qquad \because e = (1 + 1/x)$$

$$= \lim_{x \to +\infty} (1 + 1/x)^x = e$$

Ans

Q47 Solution :-

(a) $\lim_{x \to -\infty} f(x)$

$= \lim_{x \to -\infty} 2x^2 + 5$

$= 2(-\infty)^2 + 5$

$= +\infty$ Ans.

(b) $\lim_{x \to +\infty} f(x)$

$= \lim_{x \to +\infty} \dfrac{3 - 5x^3}{1 + 4x + x^3}$

$= \lim_{x \to +\infty} \dfrac{\cancel{x^3}(3/x^3 - 5)}{\cancel{x^3}(1/x^3 + 4/x^2 + 1)}$

$= \dfrac{-5}{1} = -5$ Ans.

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EXERCISE 0.4

Q 3 (a) Solution:-

$f(x) = 3x+2$

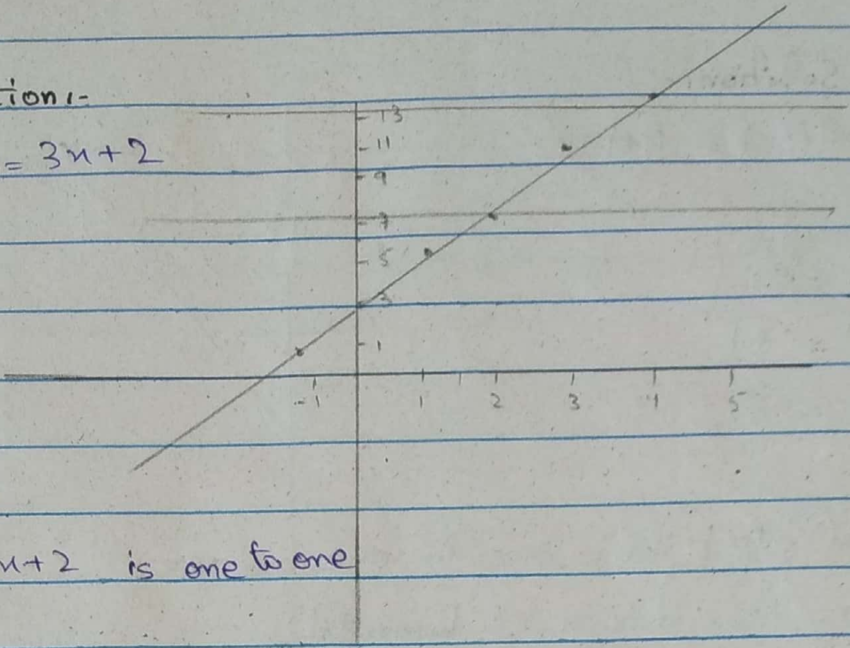
$f(1) = 5$

$f(2) = 7$

$f(3) = 11$

$f(4) = 14$

$f(-1) = -1$



$f(x) = 3x+2$ is one to one

(b) Solution:-

$f(x) = \sqrt{x-1}$

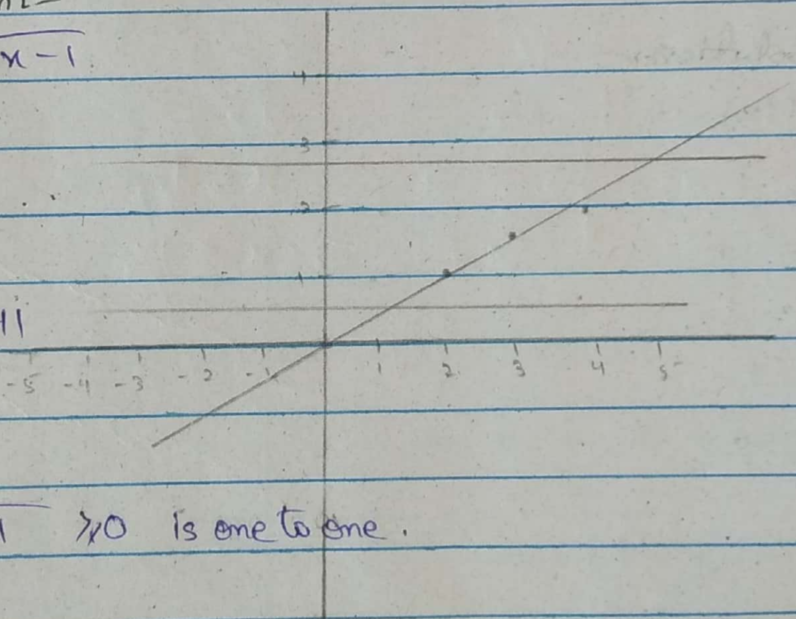
$x-1 \geq 0$

$f(1) = 0$

$f(2) = 1$

$f(3) = 1.41$

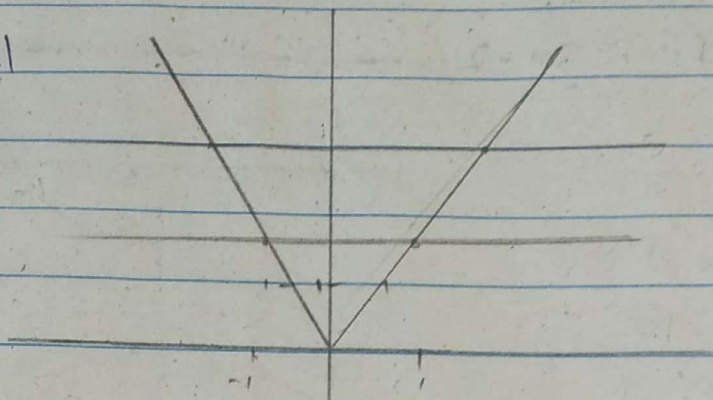
$f(4)$ 1.73



$f(x) = \sqrt{x-1} \geq 0$ is one to one.

(c) Solution:-

$f(x) = |x|$



$f(1) = \pm 1$

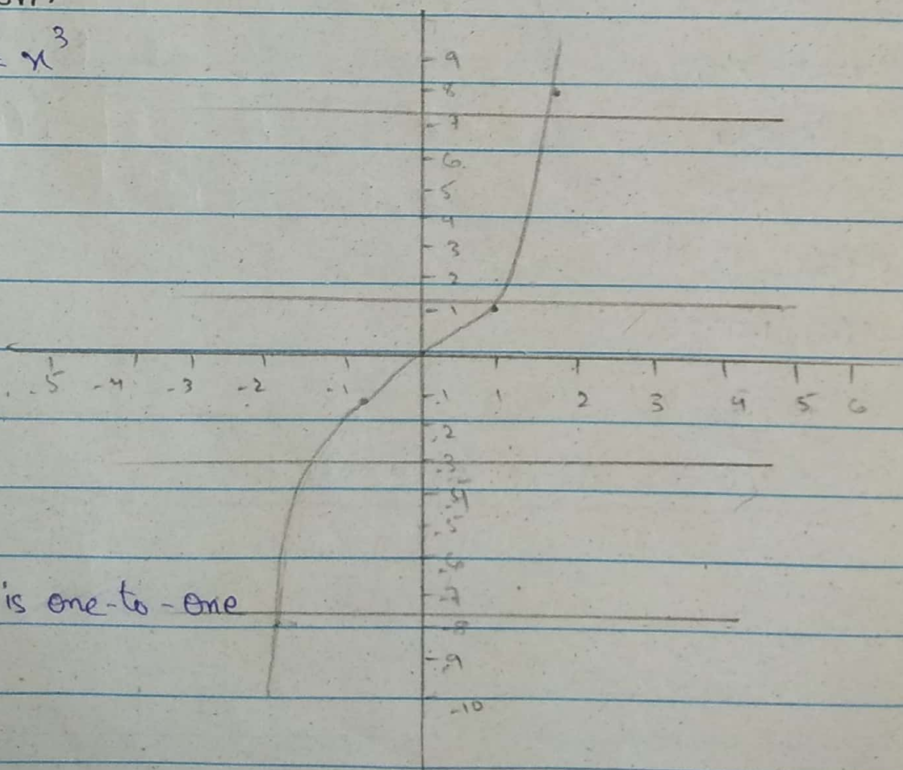
$f(-1) = \pm 1$

$f(0) = 0$

$f(x) = |x|$ isn't one to one 'coz it intersects horizontal line at two points.

(d) Solution:-

$f(x) = x^3$



$f(1) = 1$

$f(0) = 0$

$f(-1) = -1$

$f(2) = 8$

$f(-2) = -8$

$f(x) = x^3$ is one-to-one

(e) Solution:-

$f(x) = x^2 - 2x + 2$

$f(1) = 1$

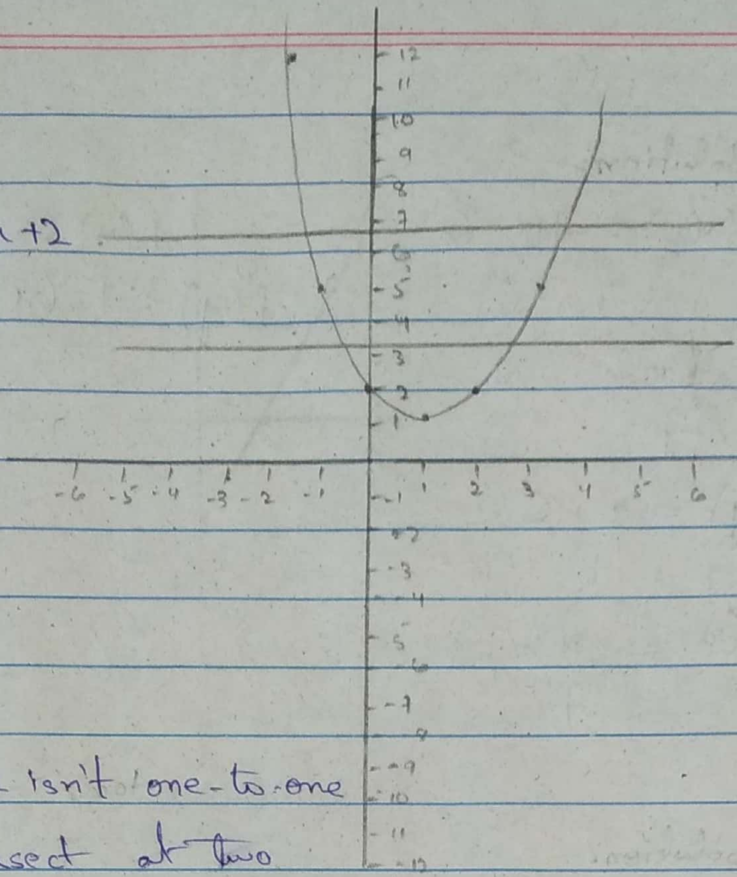
$f(-1) = 5$

$f(0) = 2$

$f(2) = 2$

$f(-2) = 12$

$f(3) = 5$



$f(x) = x^2 - 2x + 2$ isn't one-to-one 'coz it intersect at two points.

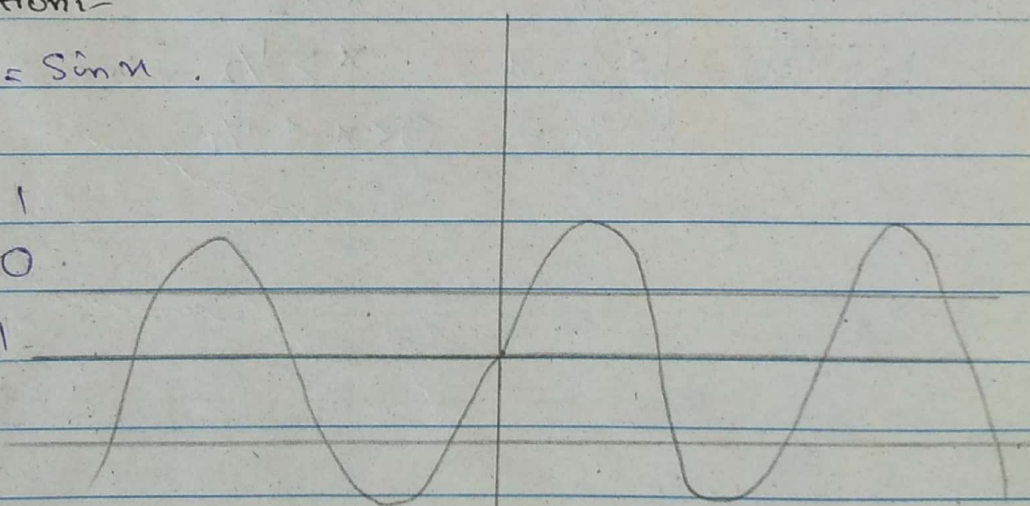
(f) Solution:-

$f(x) = \sin x$.

$f(1) = 1$

$f(0) = 0$

$f(-1) = 1$



$f(x) = \sin x$ isn't one-to-one 'coz it's intersecting horizontal line at two points.

Q.9 Solution:-

$f(x) = 7x-6$; $y = f(x)$

let $y = 7x-6$ $\qquad f(y) = f^{-1}(x)$

$\frac{y+6}{7} = x$,

$f(y) = \frac{y+6}{7}$

$f^{-1}(x) = \frac{x+6}{7}$

Q.15 Solution:-

$$f(x) = \begin{cases} 5/2 - x, & x < 2 \\ 1/x, & x \geq 2. \end{cases}$$

$$f^{-1}(x) = \begin{cases} 5/2 - x, & x > 1/2 \\ 1/x, & 0 < x \leq 1/2. \end{cases}$$

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Q17 Solution:-

$f(x) = (x+2)^4$

$f(x) = y$

$f(y) = f^{-1}(x)$

$y = (x+2)^4$

$y^{1/4} - 2 = x$

$f(y) = y^{1/4} - 2$

$f^{-1}(x) = x^{1/4} - 2$

Domain of $f^{-1}(x)$:

$x^{1/4} - 2 \geq 0$

$x^{1/4} \geq 2$

$x \geq 16$

Q27 Solution:-

$f(x) = 2x^3 + 5x + 3$

if $f^{-1}(x) = 1$ then $f(1) = x$

$f(1) = 2(1)^3 + 5(1) + 3$

$f(1) = 10$

$\Rightarrow x = 10$

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EXERCISE 1.5

Q1

(a) Solution:- $[1, 3]$

Function is not continous.

$\lim_{x \to 2} f(x)$ does not exist.

(b) Solution:- $(1, 3)$

Function is not continous.

$\lim_{x \to 2} f(x)$ doesn't exist.

(c) Solution:- $[1, 2]$

No, 'coz $f(x) = 2$ not defined

(d) Solution:-

Yes, it's continous

(e) Solution:-

Yes, it's continous

(f) Solution:-

Yes its continous

Q11 Solution:-

$$f(x) = 5x^4 - 3x + 7$$

Since it is polynomial, so it's continous at all real numbers.

Q15 Solution:-

$$f(x) = \frac{x}{2x^2 + x}$$

let $2x^2 + x = 0$

$x(2x + 1) = 0$

$x = 0 \qquad 2x + 1 = 0$

$x = -1/2$.

function is not contionous at $x = 0$ & $x = -1/2$ because function is not defined.

Q19 Solution:-

$$f(x) = \frac{x^2 + 6x + 9}{|x| + 3}$$

This function is continous at all real numbers of x.

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Q21 Solution:-

$$f(x) = \begin{cases} 2x + 3, & x \leq 4 \\ 7 + \frac{16}{x} & x > 4 \end{cases}$$

$$f(4) = 2x + 3 = 2(4) + 3 = 11$$

$$\lim_{x \to 4^-} f(x) = 2(x) + 3 = 2(4) + 3 = 11$$

$$\lim_{x \to 4^+} (x) = 7 + \frac{16}{x} = 7 + \frac{16}{4} = 11.$$

NO, since $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$ so the function is continous at all real numbers.

Q29 Solution (a)

$$f(x) = \begin{cases} 7x - 2, & x \leq 1 \\ kx^2 & x > 1 \end{cases}$$

$$f(1) = 7x - 2 = 7(1) - 2 = 5$$

$$\lim_{x \to 1^+} f(x) = kx^2 = k$$

$$\lim_{x \to 1^-} f(x) = 7x - 2 = 7(1) - \cancel{2} = 5$$

for continous function.

$$\lim_{x \to 1^+} f(x) = \lim_{x \to ^-} f(x)$$

$$\boxed{K = 5}$$

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Q29 (b) Solution:-

$$f(x) = \begin{cases} Kx^2, & x \le 2 \\ 2x+K, & x > 2 \end{cases}$$

$f(2) = Kx^2 = 4K.$

$\lim\limits_{x \to 2^+} f(x) = 2x+K = 4+K$

$\lim\limits_{x \to 2^-} f(x) = Kx^2 = 4K$

for continous function

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^-} f(x)$

$4+K = 4K.$

$\boxed{K = 4/3}$

Q31 Solution:-

$$f(x) = \begin{cases} x^2+5 & x > 2 \\ m(x+1)+K & -1 < x \le 2 \\ 2x^3+x+7 & x \le -1 \end{cases}$$

$f(2) = x^2+5 = 9.$

$\lim\limits_{x \to 2^-} f(x) = m(x+1)+K = 2m+m+K = 3m+K$

$\lim\limits_{x \to 2^+} f(x) = x^2+5 = 9$

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^-} f(x)$

$9 = 3m+K$ ——— (i)

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$\lim_{x \to -1^+} f(x) \quad m(x+1) + K = K$

$\lim_{x \to -1^-} f(x) = 2x^3 + x + 7 = 4$

$\lim_{x \to -1^+} f(x) = \lim_{x \to 1^-} f(x)$

$K = 4$ ——— (ii)

Putting the value of (ii) in (i).

$\Rightarrow \quad 9 = 3m + 4$

$5 = 3m$

OR $\quad 5/3 = m$

f(x) is continous everywhere if $K = 4$ &

$m = 5/3$.

Q 35 Solution :- (a)

$$f(x) = \frac{|x|}{x}$$

$\lim_{x \to 0^-} f(x) = -1 \quad , \quad \lim_{x \to 0^+} = 1$

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

so, the $\lim_{x \to 0} f(x)$ doesn't exist.

the continuity is not removable.

(b) Solution:-

$$f(x) = \frac{x^2 + 3x}{x + 3}$$

$f(x)$ is discontinous when

$x + 3 = 0 \Rightarrow x = -3$

$$\lim_{x \to 3} f(x) = \lim_{x \to -3} \frac{x^2 + 3x}{x + 3} \Rightarrow \lim_{x \to -3} \frac{x \cancel{(x+3)}}{\cancel{x+3}}$$

$$\lim_{x \to 3} f(x) = -3$$

$\lim_{x \to -3} f(x)$ exist but $f(-3)$ is undefined

So, The discontinuty is removable

(C) Solution:-

$$f(x) = \frac{x - 2}{|x| - 2}$$

$f(x)$ is discontinous when $|x| - 2 = 0$

$x = \pm 2$

$$\lim_{x \to 2} f(x) = 1$$

$$\lim_{x \to -2} f(x) = \infty$$

$\lim_{x \to 2} f(x)$ exist but $f(2)$ is undefined, so

the discontinuty is removable at $f(2)$

$\lim_{x \to -2} f(x)$ does not exist

so the discontinuty is non removable at $f(-2)$

EXERCISE 1.6

Q5 Solution:-

$$f(x) = \csc x$$

$$f(x) = \frac{1}{\sin x}.$$

$\sin x \neq 0$

$x = \sin^{-1}(0).$

$x = 0, 180, 360$

$x = 0, \pi, 2\pi$

where $n = 0, 1, 2, 3, \ldots$

$f(x)$ is continous at $x = n\pi$.

Q17 Solution:-

$$\lim_{x \to \infty} \cos\left(\frac{1}{x}\right)$$

$$\cos\left(\frac{1}{\infty}\right)$$

$$\cos(0)$$

$$1 \quad \text{Ans}$$

Q21 Solution:-

$$\lim_{x \to 0} e^{\sin x}$$

$$e^{\sin(0)}$$

$$e^{0}$$

$$1$$

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Q27 Solution:-

$$\lim_{x \to 0} \frac{\tan 7x}{\sin 3x}$$

$$= \lim_{x \to 0} \frac{7x \frac{\tan 7x}{7x}}{3x \frac{\sin 3x}{3x}}$$

$$= \lim_{x \to 0} \frac{7\cancel{x} \cdot 1}{3\cancel{x} \cdot 1}$$

$$= \frac{7}{3} \text{ Ans}$$

Q35 Solution:-

$$\lim_{\theta \to 0} \frac{\theta^2}{1 - \cos\theta}$$

$$= \lim_{\theta \to 0} \frac{\theta^2 \cdot (1 + \cos\theta)}{(1 - \cos\theta)(1 + \cos\theta)}$$

$$= \lim_{\theta \to 0} \frac{\theta^2 (1 + \cos\theta)}{1 - \cos^2\theta}$$

$$= \lim_{\theta \to 0} \frac{\theta^2}{\sin^2\theta} (1 + \cos\theta)$$

$$= \lim_{\theta \to 0} \frac{1}{\sin^2\theta / \theta^2} (1 + \cos\theta)$$

$$= \lim_{\theta \to 0} \frac{1}{1} (1 + \cos\theta)$$

$$= 1 + 1$$

$$= 2 \text{ Ans}$$

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Q49 Solution:-

$$f(x) = \begin{cases} \frac{\tan kx}{x} & x < 0 \\ 3x + 2k^2 & x \geq 0 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = \frac{\tan kx}{x} = \frac{\sin kx}{x \cos kx}$$

$$= \lim_{x \to 0^-} k \frac{\sin kx}{kx} \cdot \frac{1}{\cos kx}$$

$$= \lim_{x \to 0^-} k\,(1) \cdot (1/\cos kx)$$

$$= k$$

$$f(x) = \lim_{x \to 0^+} 3x + 2k^2$$

$$= 3(0) + 2k^2$$

$$= 2k^2.$$

for non zero value.

$$\lim_{x \to 0^-} = \lim_{x \to 0^+} \Rightarrow k = 2k^2 = \frac{1}{2}$$

$$\boxed{k = 1/2}$$

Q51 Solution:-

(a)

$$\lim_{x \to +\infty} x \sin \frac{1}{x}$$

$$= \lim_{t \to +\infty} \frac{1}{t} \sin t \quad ; \quad t = \frac{1}{x}$$

$$= \lim_{t \to +\infty} \frac{\sin t}{t}$$

$$= \lim_{t \to +\infty} 1$$

$$= 1 \quad \text{Ans}$$

(b) Solution:-

$$\lim_{x \to -\infty} x\left(1 - \cos\frac{1}{x}\right) \quad ; \quad t = \frac{1}{x}.$$

$$= \lim_{t \to -\infty} \frac{1}{t}(1 - \cos t)$$

$$= \lim_{t \to -\infty} \frac{1 - \cos t}{t}$$

$$= \lim_{t \to -\infty} 0 \qquad \because \frac{1 - \cos t}{t} = 0$$

$$= 0 \quad \text{Ans.}$$

(c) Solution:-

$$\lim_{x \to \pi} \frac{\pi - x}{\sin x} \quad ; \quad t = \pi - x.$$

$$= \lim_{t \to \pi} \frac{t}{\sin(\pi - t)} \qquad \because \sin(\pi - t) = \sin(t)$$

$$= \lim_{t \to \pi} \frac{t}{\sin t}$$

$$= \lim_{t \to \pi} \frac{1}{\frac{\sin t}{t}}$$

$$= \lim_{t \to \pi} 1/1$$

$$= \lim_{t \to \pi} 1$$

Q27 Solution:-

$$\lim_{x \to 0} \frac{\tan 7x}{\sin 3x}$$

$$= \lim_{x \to 0} \frac{\sin 7x}{\cos 7x} \div \sin 3x.$$

$$= \lim_{x \to 0} \frac{\sin 7x}{7x} \cdot \frac{7x}{\cos 7x} \cdot \frac{3x}{\sin 3x} \cdot \frac{1}{3x}.$$

$$= \lim_{x \to 0} 1 \cdot \frac{7\cancel{x}}{3\cancel{x} \cos 7x} \cdot (1)$$

$$= \lim_{x \to 0} \frac{7}{3 \ (\cos 7x)}$$

$$= \frac{7}{3(1)}$$

$$= \frac{7}{3} \text{ Ans}$$

EXERCISE 2.1

Q3 solution:

(a)

Given that, $f(x_0+h) = 10$, $f(x_0) = 10$.

$x_0 = 0$, $x_0 + h = 3$

$$\text{Velocity} = \frac{f(x_0+h) - f(x_0)}{(x_0+h) - x_0}.$$

$$= \frac{10 - 10}{3 - 0}.$$

$= 0$ cm/s.

(b)

Instantaneous velocity is zero at horizontal tangents. which are:

$x = 0$, $x = 2$, $x = 4$ & $x = 8$.

(c)

(b) $m_{\cancel{tan}} = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}.$

$$m_{tan} = \lim_{x \to 0} \frac{2x_1^2 - 0}{x_1 - 0}$$

$$= \lim_{x \to 0} 2x_1.$$

$$m_{tan} = 0 \quad \underline{A}$$

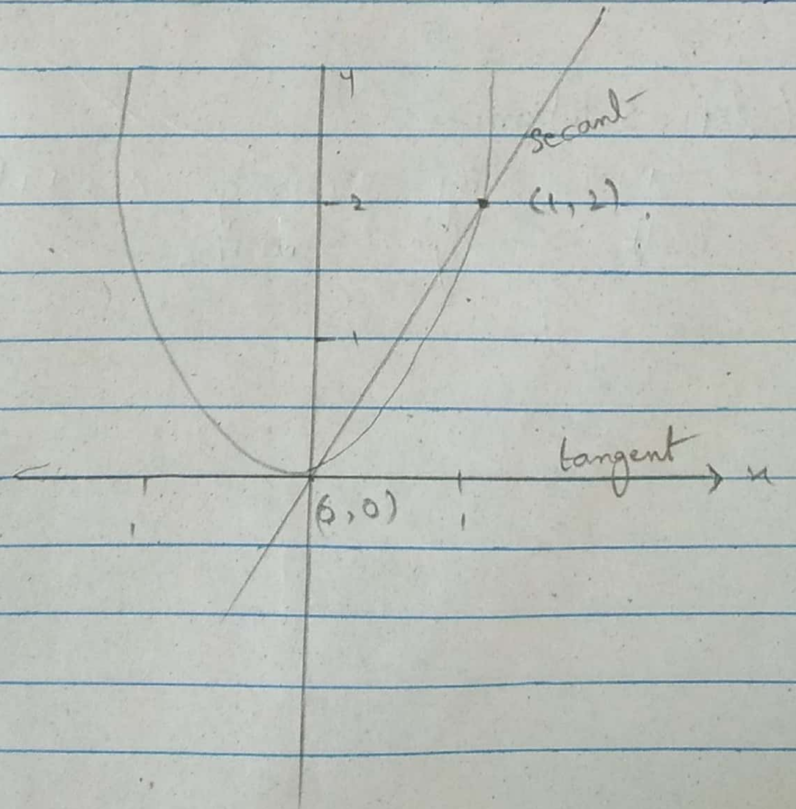
(c) $$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{2x_1^2 - 2x_0^2}{x_1 - x_0}.$$

$$= \lim_{x \to x_0} (2x_1 + 2x_0) = 4x_0.$$

$\underline{A}$

(d)



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Q15 Solution:-

(a)

$$= \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{x^2 - 1 - (x_0^2 - 1)}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{x^2 - x_0^2}{x - x_0}$$

$$= \lim_{x \to x_0} x^2 + x_0^2$$

$$= x_0^2 + x_0^2$$

$$m_{tan} = 2x_0^2 \quad \text{Ans}$$

(b) $m_{tan} = 2x_0^2$

$$= 2(-1) = -2 \quad \text{Ans}$$

Q19 Solution:-

True, By supposing $x = 1+h$

Both eq^n satisfies.

Q20 Solution:-

(a)

$$S = 0.3t^3 \text{ ft.}$$
$$= 0.3(40)^3 \text{ ft.}$$
$$= 19200 \text{ ft}$$

(b)

$$V_{av} = \frac{\text{Distance}}{\text{Time taken}}$$
$$= \frac{19200}{40}$$
$$= 480 \text{ ft/s}$$

(c)

$$S = 0.3t^3 \text{ ft}$$
$$1000 = 0.3t^3$$
$$14.938 \text{ sec} = t$$

$$V_{av} = \frac{1000}{14.938}$$
$$V_{av} = 66.94 \text{ ft/sec}$$

(d)

$$V_{inst} = \lim_{h\to 0} \frac{0.3(40+h)^3 - (0.3)(40)^3}{h}$$
$$= \lim_{h\to 0} \frac{0.3(4800h + 120h^2 + h^3)}{h}$$
$$= \lim_{h\to 0} 0.3(4800 + 120h + h^2)$$
$$V_{inst} = 1440 \text{ ft/s} \quad \text{Ans}$$

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EXERCISE 2.2

Q. Solution :-

$f'(1)$

By Taking points on the graph.

$(1, 3)$ & $(2.1, 6)$

$x \quad y \qquad x_1 \quad y_1$

$$m = \frac{y - y_1}{x - x_1}$$

$$= \frac{3 - 6}{1 - 2.1}$$

$m = 2.7$ A

$f'(3)$

By taking points on the graph.

i.e; $(3, 6)$ & $(1, 6)$

$x, y \qquad x_1, y_1$

$$m = \frac{6 - 6}{3 - 1}$$

$m = 0$ A

$f'(5)$

By taking points on the graph

i.e. $(5, 3)$ & $(6, 1)$

x, y $\quad x_1, y_1$

$$m = \frac{3 - 1}{5 - 6}$$

$$m = -2 \quad \text{Ans}$$

$f'(6)$

By taking points on the graph

i.e. $(6, 1)$ & $(4, 3)$

x, y $\quad x_1, y_1$

$$m = \frac{1 - 3}{6 - 4} = \frac{-2}{2}$$

$$m = -1 \quad \text{Ans}$$

Q7 Solution:-

Given that $x_1 = 3$ & $y_1 = -1$, $f'(3) = 5$

$$(y - y_1) = m(x - x_1)$$

$$y - (-1) = 5(x - 3)$$

$$y + 1 = 5x - 15$$

$$5x - y - 16 = 0 \quad \text{Ans}$$

Q" Solution:-

$f(x) = x^3$; $a = 0$.

$f(x+h) = (x+h)^3$

$f(x) = x^3$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}.$$

$$= \lim_{h \to 0} 3x^2 + 3hx + h^2.$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2.$$

$f'(x) = 3x^2$.

At $x = 0$ which is $a = 0$,

$f'(0) = 0$.

Eqn of tangent line.

$(y - y_1) = m(x - x_1)$

$(y - 0) = 0(x - 0)$

$\boxed{y = 0}$

Q13 Solution:-

$f(x) = \sqrt{x+1}$; $a = 8$.

$f(x) = \sqrt{x+1}$; $f(8) = \sqrt{8+1}$

$f(x+h) = \sqrt{x+h+1}$; $f(8) = 3$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}.$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \times \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \lim_{h\to 0} \frac{\cancel{x} + \cancel{h} + \cancel{1} - \cancel{x} - \cancel{1}}{\cancel{h}\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \frac{1}{\sqrt{x+1} + \sqrt{x+1}}$$

$$f'(x) = \frac{1}{2\sqrt{x+1}}$$

$$f'(8) = \frac{1}{2\sqrt{8+1}} = \frac{1}{6}.$$

Eqn of tangent line.

$y - 3 = 1/6\,(x - 8)$.

$6y - 18 = x - 8$.

$6y = x + 10 \implies y = \frac{x}{6} + \frac{10}{6}$

$y = \frac{1}{6}x + \frac{5}{3}$ A

Q15 Solution:-

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

$\because$ $f(x) = 1/x.$

$f(x+h) = 1/(x+h)$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{1/x+h - 1/x}{h}.$$

$$= \lim_{h \to 0} \frac{\frac{x - h - x}{x(x+h)}}{h}.$$

$$= \lim_{h \to 0} \frac{-h}{x(x+h)} \div h$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)}.$$

$$f'(x) = -\frac{1}{x^2} \quad \text{Ans}$$

Q23 Solution:-

Q 47 Solution:-

$$\lim_{x \to 1^+} f(x) = 2(1)$$

$$\lim_{x \to 1^+} f(x) = 2$$

$$\lim_{x \to 1^-} f(x) = (1)^2 + 1$$

$$\lim_{x \to 1^-} f(x) = 2.$$

for continity of function.

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x).$$

$$2 = 2.$$

$$\lim_{x \to 1} f(x) = 2.$$

so the function is continous at $x=1$

$$= \lim_{h \to 0^-} \cancel{f(x)} = \frac{(x+h)^2 + 1 - (x^2+1)}{h}$$

$$= \lim_{h \to 0} \cancel{f(x)} = \frac{\cancel{x^2} + 2xh + h^2 + \cancel{1} - \cancel{x^2} - \cancel{1}}{h}$$

$$= \lim_{h \to 0} = 2x + h$$

$$= 2x$$

$= x = 1$

$1 = 2.$

$$= \lim_{h \to 0^+} \frac{2(x+h) - 2(x)}{h}$$

$$= \lim_{h \to 0^+} \frac{2(\cancel{x} + \cancel{h} - \cancel{x})}{\cancel{h}}.$$

$$= \lim_{h \to 0^+} 2$$

$$= 2.$$

$$\lim_{h \to 0^-} f(x) = \lim_{h \to 0^+} f(x).$$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = 2.$$

Hence proved function is both continous and differentiable.

Q27 False;
If a tangent line is horizontal.
~~sat~~ $x = a$ so $f'(a)$ its slope is 0

Q28: True;
Slope at $(x = -2) = f'(-2)$

Q29. False;

For any function to be ~~continous~~ differentiable it's necessary that it should be continous at that point and it's limit $\lim_{x \to 0} f'(x)$ must also exist.

Q30 True;

If a function is differentiable so it first must be continous at that point.

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EXERCISE 2.3

Q7 Solution:-

$$y = \frac{-1}{3}(x^7 + 2x - 9)$$

$$\frac{dy}{dx} = -\frac{1}{3}(7x^6 + 2).$$

$$\frac{dy}{dx} = -\left[\frac{7}{3}x^6 + \frac{2}{3}\right]$$

A

Q9 Solution:-

let $y = f(x) = x^{-3} + \frac{1}{x^7}$

$$\frac{dy}{dx} = -3x^{-4} + \cancel{x}(-7x^{-8})$$

$$f'(x) = \frac{dy}{dx} = -3x^{-4} - 7x^{-8}$$

A

Q13 Solution:-

let $y = f(x) = x^e + \frac{1}{x^{\sqrt{10}}}$

$$\frac{dy}{dx} = ex^{e-1} + (-\sqrt{10}\, x^{-\sqrt{10}-1})$$

$$f'(x) = ex^{e-1} - \sqrt{10}\, x^{-\sqrt{10}-1}$$

A

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Q15 Solution:-

Let $y = f(x) = (3x^2+1)^2$

$\frac{dy}{dx} = 9x^4 + (2)3x^2 + 1$

$= 36x^3 + 6x(2)$

$f'(x) = 36x^3 + 12x$ Ans.

Q17 Solution:-

$y = 5x^2 - 3x + 1$

$f'(x)\ \frac{dy}{dx} = 10x - 3.$

$f'(1) = 10(1) - 3.$

$= 7$ Ans.

Q19 Solution:-

$x = t^2 - t$

$\frac{dx}{dt} = 2t - 1$ Ans.

Q23 Solution. -

$$y = (1-x)(1+x)(1+x^2)(1+x^4)$$
$$= (1-x^2)(1+x^2)(1+x^4).$$
$$= (1-x^4)(1+x^4).$$
$$y = (1-x^8)$$
$$\frac{dy}{dx} = \cancel{0 - 16x^8}$$

$$\frac{dy}{dx} = 0 - 8x^7$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -8$$ A.

Q33 True;

By putting $x = 2$ you got the same answers 'coz the function is the same which was being derivative.

Q35: False;

$$\left.\frac{d}{dx}\left[4f(x) + x^3\right]\right|_{x=2} = \left.\frac{d}{dx}\left[4f(x) + 8\right]\right|_{x=2} =$$

$$4f'(2) + 3x^2 = 4(5) + 3(2)^2 = 32$$

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Q 37. Solution:-

(a) $V = \frac{4}{3} \pi r^3$.

$$\frac{dV}{dr} = \frac{4}{\cancel{3}} \pi (\cancel{3}r^2).$$

$$\frac{dV}{dr} = 4 \pi r^2$$

(b) $$\left.\frac{dV}{dr}\right|_{x=5} = 4 \pi (5)^2.$$

$$= 100 \pi$$

Q 39 Solution:-

* $y - y_1 = m (x - x_1)$.

$x_1 = -3$, $y_1 = 2$, $m = f'(-3) = 5$.

$\Rightarrow$ $y - 2 = 5 (x + 3)$

$y = 5x + 17$ Ans

Q41 Solution :-

(a)

$$y = 7x^3 - 5x^2 + x.$$

$$\frac{dy}{dx} = 21x^2 - 10x + 1.$$

$$\frac{d^2y}{dx^2} = 42x - 10.$$

A

(b)

$$y = 12x^2 - 2x + 3.$$

$$\frac{dy}{dx} = 24x - 2$$

$$\frac{d^2y}{dx^2} = 24.$$

A

(c)

$$y = \frac{x+1}{x}. = 1 + x^{-1}$$

$$\frac{dy}{dx} = 0 - 1(x^{-2}).$$

$$\frac{d^2y}{dx^2} = 2x^{-3}$$

A

~~(d)~~

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(d) $y = (5x^2 - 3)(7x^3 + x)$

$$\frac{dy}{dx} = (5x^2 - 3)\frac{dy}{dx}(7x^3 + x) + (7x^3 + x)\frac{dy}{dx}(5x^2 - 3)$$

$$\frac{dy}{dx} = (5x^2 - 3)(21x^2 + 1) + (7x^3 + x)(10x).$$

$$\frac{d^2y}{dx^2} = (5x^2 - 3)\frac{dy}{dx}(21x^2 + 1) + (21x^2 + 1)$$

$$\frac{dy}{dx}(5x^2 - 3) + (7x^3 + x)\frac{dy}{dx}(10x).$$

$$+ (10x) \frac{dy}{dx}(7x^3 + x)$$

$$= (5x^2 - 3)(42x) + (21x^2 + 1)(10x)$$
$$+ (7x^3 + x)(10) + (10x)(21x + 1)$$

A

Q43 Solution.

(a) $y = x^{-5} + x^5$.

$$\frac{dy}{dx} = -5x^{-6} + 5x^4.$$

$$\frac{d^2y}{dx^2} = 30x^{-7} + 20x^3.$$

$$y''' = \frac{d^3y}{dx^3} = -210x^{-8} + 60x^2$$ A

(b) $y = \frac{1}{x}$.

$$\frac{dy}{dx} = -\frac{1}{x^2}.$$

$$\frac{d^2y}{dx^2} = 2\frac{1}{x^3}.$$

$$y''' = \frac{d^3y}{dx^3} = -6\frac{1}{x^4}$$

A

(c) $y = ax^3 + bx + c$.

$$\frac{dy}{dx} = 3ax^2 + b$$

$$\frac{d^2y}{dx^2} = 6ax$$

$$y''' = \frac{d^3y}{dx^3} = 6a$$

A

Q47 Solution:-

$$y = x^3 + 3x + 1$$

$$y' = \frac{dy}{dx} = 3x^2 + 3.$$

$$y'' = \frac{d^2y}{dx^2} = 6x$$

$$y''' = \frac{d^3y}{dx^3} = 6$$

Given That

$$y''' + xy'' - 2y' = 0$$

$$6 + x(6x) - 2(3x^2 + 3) = 0$$

$$\cancel{6} + \cancel{6x^2} - 6x^2 - 6 = 0$$

$$0 = 0$$

Proved!

Q67 Solution:-

$$f(x) = \begin{cases} x^2, & x \le 1 \\ \sqrt{x}, & x > 1 \end{cases}$$

$$\cancel{f'(x) = \begin{cases} 2x & x \le 1/2 \\ 1/2\sqrt{x} & x > 1 \end{cases}}$$

$$f'(x) = \frac{dx^2}{dx} = 2x.$$

$$f'(x) = \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\lim_{x \to 1^-} f'(x) = 2(1) = 2.$$

$$\lim_{x \to 1^+} f'(x) = \frac{1}{2\sqrt{1}} = \frac{1}{2}.$$

Hence; $\lim_{x \to 1^-} \neq \lim_{x \to 1^+}$

So the function is not differentiable.

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Q9.

(a) Solution:-

$f(x) = |3x - 2|$

let $|3x - 2| = 0$

$x = \pm \frac{2}{3}$

$3x - 2 = 0$, $-3x + 2 = 0$

$$f(x) = \begin{cases} 3x - 2 & x \geq 2/3 \\ 2 - 3x & x < 2/3 \end{cases}$$

$$f'(x) = \begin{cases} 3 & x \geq 2/3 \\ -3 & x < 2/3 \end{cases}$$

$f(2/3)^{+} = 3$

$f'(-2/3)^{-} = -3$

Hence $f'(2/3)^{+} \neq f'(2/3)^{-}$

$f(x)$ is not differentiable at $x = 2/3$

(b) Solution:-

$$f(x) = |x^2 - 4|$$

let $|x^2 - 4| = 0$.

$(-\infty, -2) \cup (2, \infty)$

$$f(x) = \begin{cases} x^2 - 4 & x \geq 2 \\ 4 - x^2 & x \leq -2 \end{cases}$$

$$f'(x) = \begin{cases} 2x & x \geq 2 \\ -2x & x \leq -2 \end{cases}$$

$$\lim_{x \to 2^+} f(x) = 4$$

$$f(-2) = -4$$

Since $f(2) \neq f(-2)$

so $f(x)$ is not differentiable at $x = \pm 2$.

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EXERCISE 2.4

Q5. Solution:-

$$f(x) = (3x^2+6)(2x - 1/4)$$

$$f'(x) = (3x^2+6)(2) + (2x - 1/4)(6x).$$

$$= 6x^2 + 12 + 12x^2 - 6/4\, x.$$

$$= 18x^2 - \frac{3}{2}x + 12 \quad \text{A}$$

Q9 Solution:-

$$f(x) = (x-2)(x^2+2x+4)$$

$$f'(x) = (x-2)(2x+2) + (x^2+2x+4)(1).$$

$$= 2x^2 + \cancel{2x} - \cancel{4x} - \cancel{4} + x^2 + \cancel{2x} + \cancel{4}.$$

$$= 3x^2 + 0$$

$$= 3x^2 \quad \text{A}$$

Q11. Solution:-

$$f(x) = \frac{3x+4}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)(3) - (3x+4)(2x)}{(x^2+1)^2}.$$

$$= \frac{3x^2+3-6x^2-8x}{x^4+2x^2+1}.$$

$$= \frac{-3x^2-8x+3}{x^4+2x^2+1} \quad \text{A}$$

Q15 Solution:-

$$f(x) = \frac{(2\sqrt{x} + 1)(x-1)}{x+3}.$$

$$f(x) = \frac{2x^{3/2} - 2\sqrt{x} + x - 1}{x+3}.$$

$$f'(x) = \frac{(x+3)(3x^{1/2} - x^{-1/2} + 1)}{(x+3)^2} - \frac{(2x^{3/2} - 2\sqrt{x} + x - 1)(1)}{(x+3)^2}.$$

$$= \frac{(x+3)(3\sqrt{x} - 1/\sqrt{x} + 1) - [2(\sqrt[3]{x})^2 - 2\sqrt{x} + x - 1}{(x+3)^2}$$

$$\cancel{= 3(\sqrt[3]{x})^2 - x/\sqrt{x} + x + 9\sqrt{x} - 9/\sqrt{x} + 9 -}$$

$$\cancel{\frac{2(\sqrt[3]{x})^2 + 2\sqrt{x} + x + 1}{(x+3)^2}}$$

$$\cancel{= \frac{(\sqrt[3]{x})^2 - 10/\sqrt{x} + 11\sqrt{x} + 10}{(x+3)^2}.}$$

$$= \frac{3(\sqrt[3]{x})^2 - \sqrt{x} + \cancel{x} + 9\sqrt{x} - 3/\sqrt{x} + 3 -}{(x+3)^2}$$

$$\frac{2(\sqrt[3]{x})^2 + 2\sqrt{x} - \cancel{x} + 1}{(x+3)^2}$$

$$= \frac{x^{3/2} + 10x^{1/2} - 3/\sqrt{x} + 4}{(x+3)^2}$$

A

Q19 Solution :-

$$f(x) = (x^7 + 2x - 3)^3$$

$$f'(x) = 3(x^7 + 2x - 3)^2 (7x^6 + 2).$$

$$= (x^7 + 2x - 3)^2 (21x^6 + 6)$$ Ans

Q23 Solution :-

$$y = \left(\frac{3x + 2}{x}\right)(x^{-5} + 1)$$

$$y = (3 + 2x^{-1})(x^{-5} + 1).$$

$$= 3x^{-5} + 3 + 2x^{-6} + 2x^{-1}$$

$$\frac{dy}{dx} = -15x^{-6} + 0 + (-12x^{-7}) + (-2).$$

$$\frac{dy}{dx} = -15(x^{-6}) - 12(x^{-7}) - 2$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -15 - 12 - 2.$$

$$= -29$$ Ans

Q31 Solution :-

$$y = \frac{x^2 - 1}{x + 2}.$$

$$\frac{dy}{dx} = \frac{(x+2)(2x) - (x^2-1)(1)}{(x+2)^2}$$

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$$= \frac{2x^2 + 4x - x^2 + 1}{(x+2)^2}$$

$$= \frac{x^2 + 4x + 1}{(x+2)^2}.$$

Given that the line is horizontal.

i.e $\frac{dy}{dx} = 0.$

$$\frac{x^2 + 4x + 1}{(x+2)^2} = 0.$$

$$x^2 + 4x + 1 = 0.$$

$a = 1, \quad b = 4, \quad c = 1$

$$x = \frac{-4 \pm \sqrt{16 - 4}}{2}.$$

$$= \frac{-4 \pm \sqrt{12}}{2}.$$

$$= \frac{-4 \pm 2\sqrt{3}}{2}.$$

$$x = -2 \pm \sqrt{3}.$$

The tangent line is horizontal at $x = -2 \pm \sqrt{3}$.

EXERCISE 2.5

Q5 Solution.-

$$f(x) = \frac{5 - \cos x}{5 + \sin x}.$$

$$f'(x) = \frac{(5+\sin x)(\sin x) - (5-\cos x)(\cos x)}{(5+\sin x)^2}$$

$$= \frac{5\sin x + \sin^2 x - 5\cos x + \cos^2 x}{(5+\sin x)^2}$$

$$= \frac{5\sin x + 1 - 5\cos x}{(5+\sin x)^2}$$

$$= \frac{1 + 5(\sin x - \cos x)}{(5+\sin x)^2} \quad \text{A.}$$

Q11 Solution.-

$$f(x) = \sec x \tan x.$$

$$f'(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$$

$$= \sec^3 x + \tan^2 x \sec x \quad \text{A.}$$

Q17 Solution.-

$$f(x) = \frac{\sin x \sec x}{1 + x\tan x}.$$

$$= \frac{\sin x\,(1/\cos x)}{1 + x\tan x}$$

$$f(x) = \frac{\tan x}{1 + x\tan x}.$$

$$f'(x) = \frac{(1+x\tan x)(\sec^2 x) - \tan x\left\{0 + \frac{d}{dx} x\tan x\right\}}{(1+x\tan x)^2}$$

$$= (1+x\tan x)(\sec^2 x) - \tan x(\sec^2 x + \tan x) / (1+x\tan x)^2$$

$$f'(x) = \frac{\sec^2 x + x\sec^2 x\tan x - x\sec^2 x\tan x - \tan^2 x}{(1+x\tan x)^2}$$

$$= \frac{\sec^2 x - \tan^2 x}{(1+x\tan x)^2}$$

$$f'(x) = \frac{1}{(1+x\tan x)^2}$$

Q21 Solution:-

$$y = x\sin x - 3\cos x.$$

$$\frac{dy}{dx} = x\cos x + \sin x - 3(-\sin x)$$

$$= x\cos x + \sin x + 3\sin x.$$

$$\frac{dy}{dx} = x\cos x + 4\sin x.$$

$$\frac{d^2y}{dx^2} = x(-\sin x) + \cos x + 4(\cos x).$$

$$= -x\sin x + \cos x + 4\cos x.$$

$$= -x\sin x + 5\cos x.$$

Q25 Solution:-

$f(x) = \tan x.$

$f'(x) = \sec^2 x.$

(a) $f'(0) = 1.$; $f(0) = 0.$

So the tangent line becomes.

$m(x - x_1) = (y - y_1).$

$1(x - 0) = (y - 0).$

$$\boxed{x = y}.$$

(b) $x = \pi/4$

$f(\pi/4) = 1$; $f'(\pi/4) = 2$

So the tangent line becomes.

$(y - y_1) = f'(x)(x - x_1).$

$(y - 1) = 2(x - \pi/4)$

$y - 1 = 2x - \pi/2.$

$$\boxed{y = 2x + 1 - \frac{\pi}{2}}$$

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(c) Solution.-

$x = -\pi/4$

$f(-\pi/4) = -1$; $f'(-\pi/4) = 2$

So the tangent line becomes.

$(y - y_1) = f'(x)(x - x_1)$

$y - (-1) = 2\{x - (-\pi/4)\}$

$y + 1 = 2x + \pi/2$.

$\boxed{y = 2x - 1 + \pi/2}$

Q27 Solution:-

(a) $y = x \sin x$.

$y' = \frac{dy}{dx} = x \cos x + \sin x$.

$y'' = \frac{d^2y}{dx^2} = x(-\sin x) + \cos x + \cos x$.

$y'' = -x\sin x + \cos x + \cos x$.

Given that

$y'' + y = 2\cos x$.

$-\cancel{x\sin x} + \cos x + \cos x + \cancel{x\sin x} = 2\cos x$.

$2\cos x = 2\cos x$. Proved.

(b) $y''' = \frac{d^3y}{dx^3} = -\left(x\cos x + \sin x\right) + 2(-\sin x)$

$= -x\cos x - \sin x - 2\sin x.$

$= -x\cos x - 3\sin x.$

$y^{(4)} = \frac{d^4y}{dy^4} = -\left[\{x \cdot (-\sin x) + \cos x\} - 3\cos x\right]$

$y^{(4)} = x\sin x - 4\cos x.$

Given that

$y^{(4)} + y'' = -2\cos x$

$\cancel{x\sin x} - 4\cos x + (-\cancel{x\sin x} + 2\cos x) = -2\cos x$

$-2\cos x = -2\cos x$

Proved!

Q29 Solution:-

(a) $f(x) = \sin x$

$f'(x) = \cos x.$

for horizontal tangent line

$f'(x) = 0.$

$\cos x = 0.$

$x = \cos^{-1}(0).$

$x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}.$

(b) $f(x) = x + \cos x$.

$f'(x) = 1 - \sin x$.

for tangent line of slope 0 i.e horizontal

$f'(x) = 0$.

$1 - \sin x = 0$.

$x = \sin^{-1}(1)$.

$=$

(c) $f(x) = \tan x$.

$f'(x) = \sec^2 x = 1/\cos^2 x$.

which is alway greater than or equals to 1, i.e $x \geq 1$,

So no horizontal tangent line,

(d) $f(x) = \sec x$.

$f'(x) = \sec x \tan x$.

for horizontal tangent line

$\sec x \tan x = 0$.

$\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = 0$.

$\sin x = 0$.

$x = \sin^{-1}(0)$

$x = 0, \pm \pi, \pm 2\pi$

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EXERCISE 2.6

Q7 Solution :-

$f(x) = (x^3 + 2x)^{37}$

$f'(x) = 37(x^3 + 2x)^{36}(3x^2 + 2)$ A

Q11 Solution :-

$f(x) = \frac{4}{(3x^2 - 2x + 1)^3}$

$f'(x) = \frac{(3x^2 - 2x + 1)^3(0) - 4(3)(3x^2 - 2x + 1)^2(6x - 2)}{(3x^2 - 2x + 1)^6}$

$= \frac{12(3x^2 - 2x + 1)^2(6x - 2)}{(3x^2 - 2x + 1)^6}$

$= \frac{12(6x - 2)}{(3x^2 - 2x + 1)^4}$ A

Q15 Solution :-

$f(x) = \sin\left(\frac{1}{x^2}\right)$

$f'(x) = \cos\frac{1}{x^2}(-2)(x^{-3})$

$= \frac{-2}{x^3}\cos\left(\frac{1}{x^2}\right)$ A

Q21 Solution:-

$f(x) = 2\sec^2(x^7)$

$f'(x) = 2(2)\sec(x^7) \cdot \sec(x^7)\tan(x^7) \cdot (7x^6)$

$= 28x^6 \sec^2(x^7)\tan(x^7)$

A

Q25 Solution:-

$f(x) = [x + \csc(x^3+3)]^{-3}$

$f'(x) = -3[x + \csc(x^3+3)]^{-4}(1 - \csc(x^3+3)\cot(x^3+3)]$

$(3x^2)$

$= -3[x + \csc(x^3+3)]^{-4}[1 - 3x^2\csc(x^3+3)$

$\cot(x^3+3)]$

A

Q35 Solution:-

$y = (5x+8)^7(1-\sqrt{x})^6$

$\frac{dy}{dx} = (5x+8)^7 \frac{dy}{dx}(1-\sqrt{x})^6 + (1-\sqrt{x})^6 \frac{dy}{dx}(5x+8)^7$

$= (5x+8)^7(6)(1-\sqrt{x})^5(1/2 \cdot 1/\sqrt{x}) +$

$(1-\sqrt{x})^6(7)(5x+8)^6(5).$

$= 6(5x+8)^7(1-\sqrt{x})^5(1/2\sqrt{x}) +$

$35(1-\sqrt{x})^6(5x+8)^6.$

X

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Q39 Solution :-

$$y = \frac{(2x+3)^3}{(4x^2-1)^8}$$

$$\frac{dy}{dx} = \frac{(4x^2-1)^8 (3)(2x+3)^2(2) - (2x+3)^3 (8)(4x^2-1)^7(8x)}{[(4x^2-1)^8]^2}$$

$$= \frac{6(4x^2-1)^8(2x+3)^2 - (2x+3)^3(64x)(4x^2-1)^7}{(4x^2-1)^{16}}$$

$$= \frac{\cancel{(4x^2-1)^7}\left[6(2x+3)^2 - (2x+3)^3(64x)\right]}{(4x^2-1)^{\cancel{16}\,9}}$$

$$= \frac{6(2x+3)^2 - (2x+3)^3(64x)}{(4x^2-1)^9}$$

Ans.

Q45 Solution :-

~~$y = x\cos(3x)$~~

$$y = \sec^3\left[\frac{\pi}{2} - x\right], \quad x = -\frac{\pi}{2}.$$

$$y' = -3\sec^2\left[\frac{\pi}{2} - x\right]\sec\left[\frac{\pi}{2} - x\right]\tan\left[\frac{\pi}{2} - x\right]$$

$$y' = -3\sec^3\left[\frac{\pi}{2} - x\right]\tan\left[\frac{\pi}{2} - x\right]$$

$$y'\left(-\frac{\pi}{2}\right) = 0$$

$$y\left(-\frac{\pi}{2}\right) = -1$$

Eq^n of tangent line.

$y + 1 = 0$ or $\boxed{y = -1}$

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Q:41 Solution:-

$$y = x\cos(5x) - \sin^2 x$$

$$\frac{dy}{dx} = x\{-\sin(5x)\}(5) - 2\sin x\cos x + \cos(5x)$$

$$\frac{d^2y}{dx^2} = -5\{x\cos(5x)(5) + \sin(5x)\} -$$

$$2\{\sin x(-\sin x) + \cos(x)\cdot\cos x\} + \cos 5x.$$

$$= -25x\cos(5x) + 5\sin 5x + 2\sin^2 x - 2\cos^2 x$$

$$+ \cos 5x.$$

Ans

$=$

Q:55 Solution:-

$$y = \cot^3(\pi - \theta)$$

$$\frac{dy}{d\theta} = 3\cot^2(\pi - \theta)\left[-\csc^2(\pi - \theta)\right](-1).$$

$$= +3\cot^2(\pi - \theta)\csc^2(\pi - \theta)$$

$$= +3\left[\frac{\cos(\pi - \theta)}{\sin(\pi - \theta)} \times \frac{1}{\sin(\pi - \theta)}\right]^2$$

$$= 3\left[\frac{\cos\pi\cos\theta + \sin\pi\sin\theta}{((\sin\pi\cos\theta - \cos\pi\sin\theta)^2}\right]^2.$$

$$= 3\left[\frac{-\cos\theta + 0}{(0 + \sin\theta)^2}\right]^2$$

$$= 3\frac{\cos^2\theta}{\sin^4\theta}.$$

$$= 3\frac{\cos^2\theta}{\sin^2\theta} \times \frac{1}{\sin^2\theta}.$$

EXERCISE 3.1

Q5 Solution:-

$$xy^2 + 3xy^3 - x = 3$$

$$x^2\frac{dy}{dx} + y\,2x + 3\left[x\,3y^2\frac{dy}{dx} + y^3\right] - 1 = 0$$

$$x^2\frac{dy}{dx} + 2xy + 9xy^2\frac{dy}{dx} + 3y^3 - 1 = 0$$

$$\left(x^2 + 9xy^2\right)\frac{dy}{dx} = 1 - 2xy - 3y^3$$

$$\frac{dy}{dx} = \frac{1 - 2xy - 3y^3}{x^2 + 9xy^2} \quad \text{Ans}$$

Q7 Solution:-

$$\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = 1$$

$$\sqrt{y} + \sqrt{x} = \sqrt{xy}$$

$$\frac{1}{2}y^{-1/2} + \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{xy}}\left(x\frac{dy}{dx} + y\right)$$

$$\frac{1}{2\sqrt{y}} + \frac{1}{2\sqrt{x}} = \frac{\sqrt{x}}{2\sqrt{y}}\frac{dy}{dx} + \frac{1}{2\sqrt{x}}$$

$$\frac{2\sqrt{x} + 2\sqrt{y}}{4\sqrt{xy}} = dy$$

Q7 Solution:-

$$\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = 1$$

$$\frac{1}{x^{3/2}} + \frac{1}{y^{3/2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{y^{3/2}}{x^{3/2}} \quad A$$

Q9 Solution:-

$$\sin(x^2y^2) = x$$

$$\cos(x^2y^2)\left[x^2\, 2y\frac{dy}{dx} + y^2\, 2x\right] = 1$$

$$\cos(x^2y^2)\left[2x^2y\frac{dy}{dx} + 2xy^2\right] = 1$$

$$2x^2y\frac{dy}{dx} + 2xy^2 = \frac{1}{\cos(x^2y^2)}$$

$$2x^2y\frac{dy}{dx} = \frac{1 - 2xy^2\cos(x^2y^2)}{\cos(x^2y^2)}$$

$$\frac{dy}{dx} = \frac{1 - 2xy^2\cos(x^2y^2)}{2x^2y\cos(x^2y^2)} \quad A$$

Q11 Solution:-

$\tan^3(xy^2+y) = x.$

$3\tan^2(xy^2+y)\sec^2(xy^2+y)\left[x2y\frac{dy}{dx}+y^2+\frac{dy}{dx}\right]=1$

$3\tan^2(xy^2+y)\sec^2(xy^2+y)\left[y^2+(2xy+1)\frac{dy}{dx}\right]=1.$

$(2x+1)\frac{dy}{dx} = \frac{1 - y^2(3)\tan^2(xy^2+y)\sec^2(xy^2+y)}{3\tan^2(xy^2+y)\sec^2(xy^2+y)}$

$\frac{dy}{dx} = \frac{1-3y^2\tan^2(xy^2+y)\sec^2(xy^2+y)}{3\cdot(2x+1)\tan^2(xy^2+y)\sec^2(xy^2+y)}$ *

Q13 Solution:-

$2x^2 - 3y^2 = 4$

$4x - 6y = 0$

$\Rightarrow \cancel{4 - 6 = 0}$

$\cancel{\frac{d^2y}{dx^2} = -2 = 0}$

$4x - 6y\frac{dy}{dx} = 0$ —— (i)

$4x = 6y\frac{dy}{dx}.$

$\frac{4x}{6y} = \frac{dy}{dx}.$

$\frac{2x}{3y} = \frac{dy}{dx}.$

$$\cancel{\frac{d^2y}{dx^2} = 6y\,(4) - 6\frac{dy}{dx}\,(4x)}$$

(i) $\Rightarrow$ $4 - 6y\frac{d^2y}{dx^2} - 6\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) = 0$

$$-6y\frac{d^2y}{dx^2} - 6\frac{d^2y}{dx^2} = -4.$$

$$(6y+6)\frac{dy^2}{dx^2} = 4,$$

$$\frac{d^2y}{dx^2} = \frac{4}{6y+6}.$$

$$\frac{d^2y}{dx^2} = \frac{2}{3y+3}$$

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$$\frac{d^2y}{dx^2} = \frac{3y(2) - 2x\, 3\frac{dy}{dx}}{9y^2}$$

$$= 6y - 6x\frac{dy}{dx} \Big/ 9y^2$$

By Putting $dy/dx = 2x/3y$

$$\Rightarrow \frac{d^2y}{dx^2} = 6y - 6x\left(\frac{2x}{3y}\right) \Big/ 9y^2$$

$$= \frac{18y^2 - 12x^2}{3y\,(9y^2)}$$

$$= \frac{18y^2 - 12x^2}{27y^3}$$

$$\frac{d^2y}{dx^2} = \frac{6y^2 - 4x^2}{9y^3}$$

Q17 Solution.-

$$y + \sin y = x$$

$$\frac{dy}{dx} + \cos y\frac{dy}{dx} = 1$$

$$\frac{d^2y}{dx^2}\left(1 + \cos y\right) = 1$$

$$\frac{dy}{dx} = \frac{1}{1+\cos y}$$

$$\frac{dy}{dx} = (1+\cos y)^{-1}$$

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$$\frac{d^2y}{dx^2} = -(1+\cos y)^{-2}\left(0+(-\sin y)\right)\frac{dy}{dx}.$$

$$\frac{d^2y}{dx^2} = \frac{+\sin y}{(1+\cos y)^2}\cdot\frac{dy}{dx}.$$

$$\frac{d^2y}{dx^2} = \frac{\sin y}{(1+\cos y)^2\,(1+\cos y)}.$$

$$\frac{d^2y}{dx^2} = \frac{\sin y}{(1+\cos y)^3} \quad A$$

Q19 Solution:-

$$x^2 + y^2 = 1.$$

$$y^2 = 1 - x^2.$$

$$2y\frac{dy}{dx} = 0 - 2x.$$

$$\frac{dy}{dx} = -\frac{\cancel{2}x}{\cancel{2}y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\left.\frac{dy}{dx}\right|_{(1/2,\ \sqrt{3}/2)} = \frac{-1/2}{\sqrt{3}/2}.$$

$$= -\frac{1}{\sqrt{3}}.$$

$$\left.\frac{dy}{dx}\right|_{(1/2,\ -\sqrt{3}/2)} = \frac{1}{\sqrt{3}}$$

By implicit differentiation.

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = 0 - 2x.$$

$$\frac{dy}{dx} = \frac{0 - \cancel{2}x}{\cancel{2}y}.$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\left.\frac{dy}{dx}\right|_{(1/2,\ \sqrt{3}/2)} = -\frac{1}{\sqrt{3}}.$$

$$\left.\frac{dy}{dx}\right|_{(1/2,\ -\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$ A

EXERCISE 3.2

Q5 Solution:-

$$y = \ln|x^2 - 1|$$

$$\frac{dy}{dx} = \frac{1}{x^2-1}(2x - 0)$$

$$\frac{dy}{dx} = \frac{2x}{x^2-1}$$

Q7 Solution:-

$$y = \ln\left(\frac{x}{1+x^2}\right)$$

$$y = \ln x(1+x^2)^{-1}$$

$$y = \ln x - \ln(1+x^2)$$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{1+x^2}(2x)$$

$$= \frac{1}{x} - \frac{2x}{1+x^2}$$

$$= \frac{1+x^2-2x^2}{x(1+x^2)}$$

$$\frac{dy}{dx} = \frac{1-x^2}{x(1+x^2)} \quad A$$

Q_{15} Solution:-

$$y = x^2 \log_2 (3-2x).$$

$$\frac{dy}{dx} = x^2 \frac{d}{dx} \log_2 (3-2x) + \log_2 (3-2x) \frac{d}{dx} x^2$$

$$= x^2 \left[\frac{-2}{(3-2x)\ln 2}\right] + \log_2 (3-2x)(2x)$$

$$\frac{dy}{dx} = 2x \log_2 (3-2x) - \frac{2x^2}{(3-2x)\ln 2}.$$

Q_{17} Solution:-

$$y = \frac{x^2}{1+\log x}$$

$$\frac{dy}{dx} = \frac{(1+\log x)(2x) - x^2 \frac{1}{x \ln 10}}{(1+\log x)^2}$$

$$= \frac{2x(1+\log x) - \frac{1}{\ln 10}}{(1+\log x)^2}$$

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Q25 Solution:-

$$y = \log(\sin^2 x)$$

$$\frac{dy}{dx} = \frac{1}{\sin^{\cancel{2}} x \ln 10} \; 2 \sin \cancel{x} \cos x$$

$$= 2 \frac{\cos x}{\sin x \ln 10}$$

$$\frac{dy}{dx} = \frac{2 \cot x}{\ln 10} \quad A$$

Q27 Solution:-

$$\frac{d}{dx}\left[\ln\left((x-1)^3 (x^2+1)^4\right)\right]$$

$$\frac{d}{dx}\left(\ln (x-1)^3 (x^2+1)^4\right)$$

$$= \frac{d}{dx} \ln (x-1)^3 + \frac{d}{dx} \ln (x^2+1)^4$$

$$= \frac{d}{dx}\left[3 \ln (x-1)\right] + \frac{d}{dx}\left[4 \ln (x^2+1)\right]$$

$$= \frac{3}{x-1} + \frac{8x}{x^2+1}$$

$$= \frac{3x^2 + 3 + 8x^2 - 8x}{(x-1)(x^2+1)}$$

$$= \frac{11x^2 - 5x + 3}{(x-1)(x^2+1)}$$ A.

Q29 Solution:-

$$\frac{d}{dx}\left[\ln \frac{\cos x}{\sqrt{4-3x^2}}\right]$$

$$= \frac{d}{dx}\ln\cos x - \frac{d}{dx}\ln\sqrt{4-3x^2}$$

$$= \frac{1}{\cos x}(-\sin x) - \frac{1}{\sqrt{4-3x^2}\cdot\sqrt{4-3x^2}}\cdot(-6x)$$

$$= \frac{-\sin x}{\cos x} + \frac{6x}{4-3x^2}$$

$$= \cancel{\cot x} - \tan x + \frac{6x}{4-3x^2}$$ A

Q35 Solution:-

$$y = x\sqrt[3]{1+x^2}$$

$$\ln y = \ln x + \ln\sqrt[3]{1+x^2}$$

$$\ln y = \ln x + \frac{\cancel{3}}{3}\ln\sqrt{1+x^2}$$

$$\ln y = \ln x + \frac{1}{3}\ln(1+x^2)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{3} \cdot \frac{2x}{1+x^2}$$

$$\frac{dy}{dx} = y\left[\frac{1}{x} + \frac{2x}{3(1+x^2)}\right]$$

$$= x\sqrt[3]{1+x^2}\left[\frac{1}{x} + \frac{2x}{3(1+x^2)}\right] \text{ A.}$$

Q37 Solution –

$$y = \frac{(x^2-8)^{1/3}\sqrt{x^3+1}}{x^6-7x+5}$$

$$\ln y = \ln \frac{(x^2-8)^{1/3}\sqrt{x^3+1}}{x^6-7x+8}$$

$$\ln y = \ln (x^2-8)^{1/3}\sqrt{x^3+1} - \ln(x^6-7x+5)$$

$$\cancel{\ln y} = \ln(x^2-8)^{1/3} + \ln\sqrt{x^3+1} - \ln(x^6-7x+5)$$

$$\cancel{\frac{1}{y}\frac{dy}{dx} = \frac{1}{(x^2-8)}}$$

$$\ln y = \frac{1}{3}\ln(x^2-8) + \ln\sqrt{x^3+1} - \ln(x^6-7x+5)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x}{3(x^2-8)} + \frac{3x^2}{2(x^3+1)} - \frac{(6x^5-7)}{x^6-7x+5}$$

$$\frac{dy}{dx} = \frac{(x^2-8)^{1/3}\sqrt{x^3+1}}{x^6-7x+5}\left[\frac{2x}{3(x^2-8)} + \frac{3x^2}{2(x^3+1)} - \frac{6x^5-7}{x^6-7x+5}\right] \text{ Ans}$$

EXERCISE 3-3

Q27 Solution:-

$f(x) = 2^x$

$\because \frac{d}{dx} b^u = b^u \ln b \cdot \frac{du}{dx}$

$f'(x) = 2^x \ln 2 \frac{dx}{dx}$

$f'(x) = 2^x \ln 2$ A

By Using Logarithmic function.

$f(x) = 2^x$.

let $y = 2^x$.

$\ln y = x \ln 2$

$\frac{1}{y} \frac{dy}{dx} = \ln 2$.

$\frac{dy}{dx} = 2^x \ln 2$ A.

Q43 Solution..

$y = \sin^{-1}(3x)$.

$\frac{dy}{dx} = \frac{1}{\sqrt{1-9x^2}}$ A

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Q4 Solution:-

$$y\ (\tan^{-1} x)^{-1}$$

$$y = \frac{1}{\tan x} = \cot x$$

$$\frac{dy}{dx} = -\csc^2 x$$

Q5 Solution:-

$$y = e^x \sec^{-1} x$$

$$\frac{dy}{dx} = e^x \frac{1}{x\sqrt{x^2-1}} + \sec^{-1} x\ e^x$$

EXERCISE 3.6

Q7 Solution:-

$$\lim_{x \to 0} \frac{e^x - 1}{\sin x}.$$

$$\lim_{x \to 0} \frac{e^x}{\cos x}.$$

$$= 1 \quad A.$$

Q11 Solution:-

$$\lim_{x \to \pi^+} \frac{\sin x}{x - \pi}$$

$$= \lim_{x \to \pi^+} \frac{\cos x}{1}$$

$$= -1 \quad A.$$

Q17 Solution:-

$$\lim_{x \to +\infty} \frac{x^{100}}{e^x}$$

$$= \lim_{x \to +\infty} \frac{100\, x^{99}}{e^x},$$

$$= \lim_{x \to +\infty} \frac{100\,(99)\, x^{98} \cdots\cdots}{e^x}$$

$$= \lim_{x \to +\infty} \frac{100\,!}{e^x} = \frac{100!}{\infty}$$

$$= 0 \quad A$$

Q18 Solution:

$$= \lim_{x \to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)}$$

$$= \lim_{x \to 0^+} \frac{1/\sin x \cdot \cos x}{1/\tan x \cdot \sec^2 x}$$

$$= \lim_{x \to 0^+} \frac{\cos x}{\sin x} \cdot \frac{\tan x}{\sec^2 x}$$

$$= \lim_{x \to 0^+} \frac{\cancel{\cos x}}{\cancel{\sin x}} \cdot \cos^2 x \cdot \frac{\cancel{\sin x}}{\cancel{\cos x}}$$

$$= \lim_{x \to 0^+} \cos^2 x$$

$$= 1 \quad \text{A}$$

Q25 Solution:

$$\lim_{x \to \pi/2} \sec 3x \cos 5x$$

$$= \lim_{x \to \pi/2} \frac{\cos 5x}{\cos 3x}$$

$$= \lim_{x \to \pi/2} \frac{-5 \sin 5x}{-3 \sin 3x}$$

$$= \frac{-5 (1)}{-3(-1)}$$

$$= -\frac{5}{3} \quad \text{A}$$

Q29 Solution:-

$$\lim_{x \to 0} (e^x + x)^{1/x}$$

$$\text{let } \lim_{x \to 0} y = \lim_{x \to 0} (e^x + x)^{1/x}$$

$$\lim_{x \to 0} \ln y = \lim_{x \to 0} \ln (e^x + x)^{1/x}$$

$$\lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{\ln e^x + x}{x}$$

$$\lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{e^x + 1}{e^x + x}$$

$$\lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{e^x + 1}{e^x + x}$$

$$\ln y = 1 + 1/1$$

$$\ln y = 2$$

$$y = e^2 \quad A$$

Q3. Solution:-

$$\lim_{x\to 1} y = \lim_{x\to 1} (2-x)^{[\tan(\pi/2)x]}$$

$$\lim_{x\to 1} \ln y = \lim_{x\to 1} \frac{\ln(2-x)}{\cot\left(\frac{\pi x}{2}\right)}$$

$$\lim_{x\to 1} \ln y = \lim_{x\to 1} \frac{-1/(2-x)}{-\operatorname{cosec}^2\left(\frac{\pi x}{2}\right) \times \pi/2}$$

$$\lim_{x\to 1} \ln y = \lim_{x\to 1} \frac{1}{(2-x)\operatorname{cosec}^2\left(\frac{\pi x}{2}\right) \pi/2}$$

$$\ln y = \frac{1}{(2-1)(1)(\pi/2)}$$

$$\ln y = \frac{2}{\pi}$$

$$y = e^{2/\pi} \quad A$$

Q4. Solution:-

$$\lim_{x\to 0^+} \left[-\frac{1}{\ln x}\right]^x$$

$$\lim_{x\to 0} \ln y = \lim_{x\to 0^+} x \ln\left[-\frac{1}{\ln x}\right]$$

$$= \lim_{x\to 0^+} \frac{\ln\left[-\frac{1}{\ln x}\right]}{1/x}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{(-1/\ln x)} \cdot \frac{1}{x}}{-\frac{1}{x^2}}$$

$$= \lim_{x \to 0^+} -\frac{\ln x}{x} \div -\frac{1}{x^2}$$

$$= \lim_{x \to 0^+} x \ln x$$

$$= \lim_{x \to 0^+} \frac{\ln x}{1/x}$$

$$= \lim_{x \to 0^+} \frac{1}{x} \div -\frac{1}{x^2}$$

$$= \lim_{x \to 0^+} -x$$

$$\ln y = 0$$

$$y = e^0$$

$$y = 1 \quad A$$

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EXERCISE 5.3

Q19 Solution:-

$\int \sec 4x \tan 4x \, dx.$

$= \frac{1}{4} \sec 4x + C$

Q23 Solution:-

$\int \frac{dx}{\sqrt{1-4x^2}}$

$\int \frac{dx}{\sqrt{1-u^2}}.$

Let $u = (2x)$.

$\frac{du}{} = 2dx$

$\frac{du}{2} = dx.$

$\Rightarrow \frac{1}{2} \int \frac{du}{\sqrt{1-u^2}}$

$= \frac{1}{2} \sin^{-1}(u)$

$= \frac{1}{2} \sin^{-1}(2x) + C$

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Q31 Solution:-

$$\int e^{\sin x} \cos x \, dx$$

let $u = \sin x$.

$du = \cos x \, dx$.

$\Rightarrow \int e^{u} \, du$.

$= e^{u} + C$

$= e^{\sin x} + C$ — A.

Q35 Solution:-

$$\int \frac{e^{x}}{1 + e^{2x}} \, dx$$

let $u = e^{x}$ $\Rightarrow u^2 = e^{2x}$.

$du = e^{x} \, dx$.

$$\Rightarrow \int \frac{du}{1 + u^2}$$

$= \tan^{-1} u + C$

$= \tan^{-1} e^{x} + C$ — A.

Q45. Solution,-

$$\int \frac{\sec^2 x \, dx}{\sqrt{1-\tan^2 x}}$$

let $u = \tan x$.

$du = \sec^2 x \, dx$.

$\Rightarrow$ $$\int \frac{du}{\sqrt{1-u^2}}$$

$= \sin^{-1}(u) + C$

$= \sin^{-1}(\tan x) + C$ A.

EXCERCISE 7.2

Q9 Solution:-

$= \int x \ln x \, dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$\int dv = \int x \, dx$

$v = \frac{x^2}{2}$

$\int u \, dv = uv - \int v \, du$

$\int x \ln x \, dx = \ln x \frac{x^2}{2} - \int \frac{x^{\cancel{2}}}{2} \frac{1}{\cancel{x}} dx$

$= \frac{x^2}{2} \ln x - \frac{1}{2} \frac{x^2}{2}$

$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$

Q13 Solution:-

$\int \ln (3x-2) \, dx$

$u = \ln (3x-2)$

$\int dv = \int dx$

$du = \frac{3}{3x-2} dx$

$v = x$

$\int u \, dv = uv - \int v \, du$

$= \frac{3}{3x-2} (x) - \int x \frac{3}{(3x-2)} dx$

$= \frac{3x}{3x-2} - \int \left(1 + \frac{2}{3x-2}\right) dx$

$= x \ln|3x-2| - \cancel{3}x - \frac{2}{3} \ln|3x-2| + C$

A.

Q.15 Solution-

$\int \sin^{-1} x \, dx$

$u = \sin^{-1} x.$ $\int dv = \int dx$

$du = \frac{1}{\sqrt{1-x^2}} dx.$ $v = x.$

$\int u \, dv = uv - \int v \, du.$

$= x \sin^{-1} x - \int x \frac{1}{\sqrt{1-x^2}} dx.$

$= x \cdot \sin^{-1} x - \frac{1}{2} \int 2x (1-x^2)^{-1/2} dx.$

$= x \sin^{-1} x - \frac{1}{2} \frac{(1-x^2)^{-\cancel{3}/2}}{-\cancel{3}/\cancel{2}}.$

$= x \sin^{-1} x + \cancel{3} \frac{1}{(1-x^2)^{-1}}$

$= x \sin^{-1} x + \sqrt{1-x^2} + C$ A.

Q19 Solution:-

$\int e^x \sin x \, dx$

$u = \sin x$ $\int dv = \int e^x dx$

$du = \cos x \, dx$ $v = e^x$

$\int u \, dv = uv - \int v \, du$

$\int e^x \sin x \, dx = \sin x \, e^x - \int e^x \cos x \, dx$ ——— (i)

$I = \int e^x \cos x \, dx$

$u = \cos x$ $\int dv = \int e^x dx$

$du = -\sin x \, dx$ $v = e^x$

$\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx$

(i) $\Rightarrow$

$\int e^x \sin x \, dx = \sin x e^x - e^x \cos x - \int e^x \sin x \, dx$

$2 \int e^x \sin x \, dx = e^x (\sin x - \cos x)$

$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) + C$

Q29 Solution:-

$$\int_1^e x^2 \ln x \, dx.$$

$u = \ln x.$ $\qquad \int dv = \int x^2 dx.$

$du = \frac{dx}{x}$ $\qquad v = \frac{x^3}{3}$

$$\int u\,dv = uv - \int v\,du$$

$$= \left[\ln x \frac{x^3}{3} - \int \frac{x^{\cancel{3}\,2}}{3} \frac{dx}{\cancel{x}}\right]_1^e$$

$$= \left[\frac{x^3 \ln x}{3} - \frac{1}{3}\int x^2 dx\right]_1^e$$

$$= \left[\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right]_1^e$$

$$= \frac{1}{3}\left(e^3 \ln e - (1)\ln(1)\right) - \frac{1}{9}(e^3 - 1)$$

$$= \frac{1}{3}(e^3 - 0) - \frac{1}{9}(e^3 - 1).$$

$$= \frac{1e^3}{3} - \frac{1e^3}{9} + \frac{1}{9}.$$

$$= \frac{2e^3 + 1}{9} + \cancel{C}$$

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Q35 Solution:-

$$\int_0^{\pi} x \sin 2x \, dx.$$

$u = x$, $\qquad \int dv = \int \sin 2x \, dx$

$du = dx$ $\qquad v = \dfrac{\cos 2x}{2}$

$$\int u\,dv = uv - \int v\,du.$$

$$= \left[x \frac{\cos 2x}{2} - \int \frac{\cos 2x}{2} dx \right]_0^{\pi}$$

$$= \left[x \frac{\cos 2x}{2} + \frac{1}{2} \frac{\sin 2x}{2} \right]_0^{\pi}$$

$$\cancel{= \frac{1}{2}\left[(\pi)(\sin 2\pi) - 0\right] + \frac{1}{4}\left[(\sin 2\pi) - 1\right]}$$

$$= \left[\frac{x \cos 2x}{2} + \frac{1}{4} \frac{\sin 2x}{2} \right]_0^{\pi}$$

$$= \frac{1}{2}\left[(\pi)(\cos 2\pi) - (0)\right] + \frac{1}{4}\left[\sin 2\pi - 0\right]$$

$$= -\frac{\pi}{2} + C$$

EXCERCISE 7.3

Q3. Solution:-

$\int \sin^2 5\theta \, d\theta$

$\int \frac{1 - \cos 10\theta}{2} \, d\theta$

$= \frac{\theta}{2} - \frac{\sin 10\theta}{20} + C$ A

Q4 Solution:-

$= \int \sin^2 x \cos^2 x \, dx.$

$= \int \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right) dx.$

$= \frac{1}{4} \int \left(1 - \frac{\cos^2 2x}{2}\right) dx$

$= \frac{1}{4} \left[\int dx - \int \left(\frac{1 + \cos 4x}{2}\right) dx\right]$

$= \frac{1}{4} \int dx - \frac{1}{8} \int dx - \frac{1}{8} \int \cos 4x$

$= \frac{x}{4} - \frac{x}{8} - \frac{1}{8} \frac{\sin 4x}{4} + C.$

$= \frac{1}{8} \left[x - \frac{\sin 4x}{4}\right] + C$ A.

Q_{15} Solution:-

$$\int \sin x \cdot \cos(x/2)\, dx$$

$$\int \frac{1}{2} \sin\left(x + \frac{x}{2}\right) + \sin\left(x - \frac{x}{2}\right) dx$$

$$= \frac{1}{2} \int \left[\sin(3x/2) + \sin\left(\frac{x}{2}\right)\right] dx$$

$$= \frac{1}{2} \cdot \frac{\cancel{2} \cos(3x/2)}{3\cancel{x}} + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2} \cos(x/2)}{\cancel{x}}$$

$$= \frac{\cos(3x/2)}{3\cancel{x}} + \cos(x/2) + C \quad A.$$

Q_{21} Solution:-

$$= \int_0^{\pi/6} \sin 4x \cos 2x \, dx.$$

$$= \frac{1}{2} \int_0^{\pi/6} (\sin 6x + \sin 2x)\, dx$$

$$= \frac{1}{2} \left[\frac{-\cos 6x}{6} \Big|_0^{\pi/6} + \frac{-\cos 2x}{2} \Big|_0^{\pi/6} \right]$$

$$= \frac{1}{2} \left[\frac{-1}{6} (\cos \pi - \cos 0) - \frac{1}{2} \cos(\pi/3) - \cos(0) \right]$$

$$= \frac{1}{2} \left[\frac{-1}{6} (-1 - 1) - \frac{1}{2} \left(\frac{1}{2} - 1\right) \right]$$

$$= \frac{1}{2} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) + \left(-\frac{1}{12}\right)(-2)$$

$$= +\frac{1}{8} + \frac{1}{6} + C \qquad = \frac{7}{24} + C \quad A$$

Q27 Solution:-

$$\int \sec 4x \, dx.$$

$$\frac{1}{4} \ln |\sec 4x + \tan 4x| + C$$

A.

Q31 Solution:-

$$\int \tan 4x \sec^4 4x \, dx$$

$$= \int \tan 4x \,(1+\tan^2 4x) \sec^2 4x \, dx.$$

$$= \int \tan 4x \sec^2 4x \, dx + \int \tan^3 4x \sec^2 4x \, dx$$

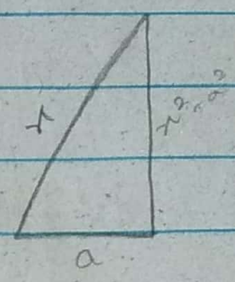
$$= \frac{\tan^2 4x}{8} + \frac{\tan^4 4x}{16} + C$$

A.

EXERCISE 7.4

Q7 Solution:-

$$= \int \frac{\sqrt{x^2-9}}{x}\, dx$$



let $x = a\sec\theta$.

$x = 3\sec\theta$.

$dx = 3\sec\theta\tan\theta\, d\theta$.

$$\int \frac{\sqrt{9\sec^2\theta - 9}}{\cancel{3\sec\theta}}\, \cancel{3\sec\theta}\, \tan\theta\, d\theta$$

$$= 3\int \sqrt{\sec^2\theta - 1}\, \tan\theta\, d\theta$$

$$= 3\int \sqrt{\tan^2\theta}\, \tan\theta\, d\theta.$$

$$= 3\int \tan^2\theta\, d\theta.$$

$$= 3\int (\sec^2\theta - 1)\, d\theta.$$

$$= 3\tan\theta - 3\theta.$$

$$= \cancel{3\tan\frac{x^2-a^2}{3} - 3\sec^{-1}\frac{x}{3}}.$$

$$= \cancel{3}\frac{\sqrt{x^2-9}}{\cancel{3}} - 3\sec^{-1}\frac{x}{3} + C$$

$$= \sqrt{x^2-9} - 3\sec^{-1}\frac{x}{3} + C$$

Q11 Solution:-

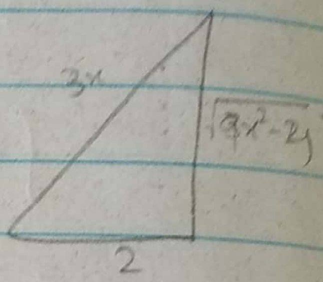
$$\int \frac{dx}{x^2\sqrt{9x^2-4}}$$

$1/\cos$

$\sec\theta = \frac{3x}{2}$

~~$x = a\sec\theta$~~

$$\int \frac{dx}{x^2\sqrt{x^2-4/9}}$$



Let

$$x = a\sec\theta$$

$$x = \frac{2}{3}\sec\theta$$

$$dx = \frac{2}{3}\sec\theta\tan\theta\, d\theta$$

$$\Rightarrow \int \frac{2/3 \sec\theta\tan\theta\, d\theta}{4/9 \sec^2\theta\sqrt{4/9\sec^2\theta - 4/9}}$$

$$= \int \frac{2/3 \cancel{\sec\theta}\tan\theta\, d\theta}{(4/9)(2/3)\sec^{\cancel{2}}\theta\sqrt{\tan^2\theta}}$$

$$= \frac{9}{4}\int \frac{\cancel{\tan\theta}\, d\theta}{\sec\theta\,\cancel{\tan\theta}}$$

$$= \frac{9}{4}\int \cos\theta\, d\theta$$

$$= \frac{9}{4}\sin\theta + C$$

$$= \frac{9}{4}\frac{\sqrt{9x^2-4}}{2} + C$$

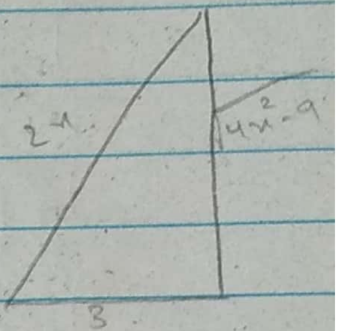
Q4 Solution:-

$$= \int \frac{dx}{(4x^2-9)^{3/2}} = \int \frac{dx}{8\left(x^2-9/4\right)^{3/2}}$$

Let $x = a\sec\theta$

$$x = \frac{3}{2}\sec\theta \quad , \quad dx = \frac{3}{2}\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{(3/2)\sec\theta\tan\theta\,d\theta}{8\left(9/4\sec^2\theta - 9/4\right)^{3/2}}$$



$$= \int \frac{(3/2)\sec\theta\tan\theta\,d\theta}{\cancel{8}\,(27/8)\left(\tan^2\theta\right)^{3/2}}$$

$$= \frac{1}{18}\int \frac{\sec\theta\tan\theta\,d\theta}{\tan^3\theta}$$

$$= \frac{1}{18}\int \frac{\cos\theta\,d\theta}{\sin^2\theta}$$

let $t = \sin\theta$

$dt = \cos\theta\,d\theta$

$$\Rightarrow \frac{1}{18}\int \frac{dt}{t^2} = \frac{1}{18}\int t^{-2}\,dt$$

$$= \frac{1}{18}\,\frac{t^{-1}}{-1} + C = \frac{1}{18}\left[-\frac{1}{t}\right] + C$$

$$= \frac{1}{18}\left[\frac{-1}{\sin\theta}\right] + C$$

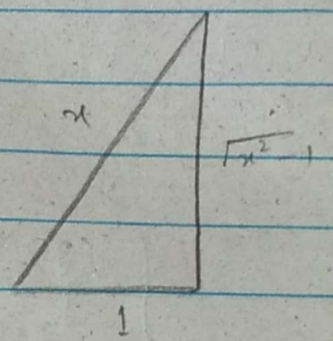
$$= \frac{1}{18}\left[\frac{-2x}{\sqrt{4x^2-9}}\right] + C \quad A$$

Q23 Solution

$$\int_{\sqrt{2}}^{2} \frac{dx}{x^2\sqrt{x^2-1}}$$

let $x = a\sec\theta$

$dx = \sec\theta\tan\theta \, d\theta$



$$\Rightarrow \int_{\sqrt{2}}^{2} \frac{\cancel{\sec\theta}\tan\theta \, d\theta}{\sec^{\cancel{2}}\theta\sqrt{\sec^2\theta-1}}$$

$$= \int_{\sqrt{2}}^{2} \frac{\cancel{\tan\theta}\, d\theta}{\sec\theta\,\cancel{\tan\theta}}$$

$$= \int_{\sqrt{2}}^{2} \cos\theta \, d\theta$$

$$= \left|\sin\theta \cancel{d\theta}\right|_{\sqrt{2}}^{2} + C$$

$$= \left|\frac{\sqrt{x^2-1}}{x}\right|_{\sqrt{2}}^{2} + C$$

$$= \frac{\sqrt{4-1}}{2} - \frac{\sqrt{(\sqrt{2})^2-1}}{\sqrt{2}} + C$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} + C$$

A

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EXERCISE 7.5

Q11 Solution.

$$\int \frac{11x + 17}{2x^2 + 7x - 4}\, dx$$

$$\int \frac{11x + 17}{2x^2 + 8x - x - 4}$$

$$\int \frac{11x + 17}{2x(x+4) - 1(x+4)}$$

$$= \int \frac{11x + 17}{(x+4)(2x-1)}.$$

$$\frac{11x + 17}{(x+4)(2x-1)} = \frac{A}{(x+4)} + \frac{B}{(2x-1)}$$

$$11x + 17 = A(2x-1) + (x+4)B.$$

when $x = 1/2.$

$$\frac{11}{2} + 17 = 0 + \left(\frac{1}{2} + 4\right) B.$$

$$\frac{45}{2} = \frac{9}{2} B.$$

$$\boxed{5 = B}$$

when $x = -4$

$$-44 + 17 = A(-8-1) + 0.$$

$$-27 = -9A.$$

$$\boxed{3 = A}$$

$$\int \frac{11x+17}{(x+4)(2x-1)} = \int \frac{3}{x+4} + \int \frac{5}{2x-1}$$

$$= 3 \ln|x+4| + \frac{5}{2} \ln|2x-1| + C$$

A

Q15 Solution:-

$$\int \frac{x^2-8}{x+3}\, dx.$$

$$\cancel{\int \frac{(x+4)(x-4)}{}}$$

$$= \int \frac{x^2-9+1}{x+3}\, dx$$

$$= \int \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}\, dx + \int \frac{1}{x+3}\, dx$$

$$= \int x-3\, dx + \int \frac{1}{x+3}\, dx.$$

$$= \frac{x^2}{2} - 8x + \ln|x+3| + C$$

A.

Q19 Solution:-

$$\int \frac{2x - 3}{x^2 - 3x - 10} dx.$$

$$\int \frac{2x - 3}{x^2 - 5x + 2x - 10}$$

$$\int \frac{2x - 3}{x(x-5) + 2(x-5)}$$

$$\int \frac{2x-3}{(x-5)(x+2)}.$$

$$\frac{2x-3}{(x-5)(x+2)} = \frac{A}{x-5} + \frac{B}{x+2}$$

$$2x - 3 = A(x+2) + B(x-5)$$

when $x = -2$.

$$-4 - 3 = 0 + B(-7).$$

$$-7 = -7B.$$

$$\boxed{1 = B}$$

when $x = 5$.

$$10 - 3 = A(7) + 0.$$

$$7 = 7A$$

$$\boxed{1 = A}$$

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$$\Rightarrow \int \left[\frac{1}{x-5} + \frac{1}{x+2} \right] dx.$$

$$= \ln|x-5| + \ln|x+2| + C.$$

$$= \ln|(x-5)(x+2)|$$

$$= \ln|x^2 - 3x - 10| + C$$

Q27 Solution:-

$$\int \frac{x^2}{(x+1)^3} dx.$$

$$\frac{x^2}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$$\rightarrow (a)\quad x^2 = A(x+1)^2 + B(x+1) + C.$$

when $x = -1$

$$(-1)^2 = 0 + 0 + C.$$

$$\boxed{1 = C}$$

$$(a) \Rightarrow x^2 = Ax^2 + 2Ax + A + B(x+1) + C.$$

~~when $x = -1$~~

$$\cancel{(-1)^2 = A - 2A + A + B(x+1) + C.}$$

$$x^2 = Ax^2 + (2A+B)x + A + B + C.$$

Equating corresponding coefficients.

$Ax^2 = x^2$ ——— (i) $\quad A = 1$

$(2A+B)x = 0$ ——— (ii)

$A + B + C = 0$ ——— (iii)

Put $A = 1$ in (ii)

$2 + 2B = 0$

$\boxed{B = -2}$

$$\Rightarrow \int \frac{x^2}{(x+1)^3} = \int \frac{1}{x+1}\,dx + \frac{(-2)}{(-1)}\int \frac{1}{(x+1)^2}\,dx + \int \frac{dx}{(x+1)^3}$$

$$= \ln|x+1| + (-2)\frac{1}{(x+1)} + \frac{1}{(-2)(x+1)^2}$$

$$= \ln|x+1| + \frac{2}{x+1} - \frac{1}{2(x+1)^2} + C$$

Q33 Solution:-

$$\int \frac{x^3 - 2x^2 + 2x - 2}{x^2 + 1}\, dx.$$

$$
\begin{array}{l}
x^2+1 \;)\; x^3 - 2x^2 + 2x - 2 \;(\; x - 2 \\
\qquad\quad \;\; x^3 \qquad\qquad\;\; + x \\
\hline
\qquad\qquad -2x^2 + x - 2 \\
\qquad\qquad -2x^2 \qquad\; - 2 \\
\hline
\qquad\qquad\qquad\quad x
\end{array}
$$

$$= \int (x-2)\, dx + \int \frac{x}{x^2+1}\, dx.$$

$$= \frac{x^2}{2} - 2x + \frac{1}{2} \ln |x^2 + 1| + C$$

EXERCISE 4.1

Q15 Solution:-

$$f(x) = x^2 - 3x + 8$$
$$f'(x) = 2x - 3$$
$$f''(x) = 2$$

$2x - 3 > 0$ $\qquad$ $2x - 3 < 0$.

$x < 3/2$ $\qquad$ $x > 3/2$.

Test points

A = less Than $3/2$ = 1 = $-$ve.

B = Greater than $3/2$ = 2 = $+$ve

$f'(x) < 0$ when $x < 3/2$

$f'(x) > 0$ when $x > 3/2$.

(a) function is increasing at $(3/2, +\infty)$

(b) function is decreasing at $(-\infty, 3/2)$

(c) f is concave up everywhere.

(d) f is never concave down.

(e) There is no inflation points.

Q21 Solution:-

$$f(x) = \frac{x-2}{(x^2-x+1)^2}$$

$$f'(x) = \frac{(x^2-x+1)^2 - (x-2)\,2(x^2-x+1)(2x-1)}{(x^2-x+1)^4}$$

$$= \frac{(x^2-x+1) - 2(x-2)(2x-1)\cancel{(x^2-x+1)}}{(x^2-x+1)^3}$$

$$= \frac{(x^2-x+1) - 2(2x^2-x-4x+2)}{(x^2-x+1)^3}$$

$$= \frac{(x^2-x+1) - (4x^2-10x+4)}{(x^2-x+1)^3}$$

$$= \frac{x^2-x+1-4x^2+10x-4}{(x^2-x+1)^3}$$

$$= \frac{-3x^2+9x-3}{(x^2-x+1)^3}$$

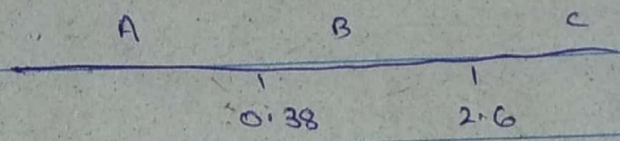
$$= \frac{-3(x^2-3x+1)}{(x^2-x+1)^3}$$

$(x^2 - 3x + 1$

$$x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

$$\Rightarrow f'(x) = \quad x > \frac{3+\sqrt{5}}{2}$$

$$x < \frac{3-\sqrt{5}}{2}$$



A = 0 $f'(x)$ -ve.

B = 1 $f'(x)$ +ve.

C = 3. $f'(x)$. -ve.

(a) increasing on $\left[\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right]$

(b) decreasing on $\left(-\infty, \frac{3-\sqrt{5}}{2}\right] \cup \left[\frac{3+\sqrt{5}}{2}, \infty\right)$

$$f''(x) = \frac{6x(2x^2 - 8x + 5)}{(x^2 - x + 1)^4}$$

$$2x^2 - 8x + 5 = 0$$

$$x = \frac{8 \pm \sqrt{24}}{4} = \frac{2 \pm \sqrt{6}}{2}.$$

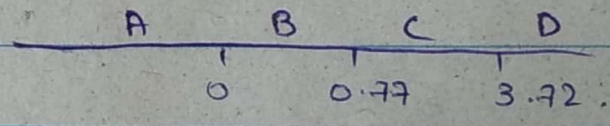
$$= \frac{2 \pm \sqrt{6}}{2}. \quad \text{or} \quad x = 0.$$

A = -1 $f''(x)$ -ve

B = 0.6 $f''(x)$ +ve

C = 1 $f''(x)$ -ve

D = 4 $f''(x)$ +ve



(c) concave up $\left(0, 2-\frac{\sqrt{6}}{2}\right) \cup \left(2+\frac{\sqrt{6}}{2}, +\infty\right)$

(d) concave down $(\infty, 0) \cup \left(2-\frac{\sqrt{6}}{2}, 2+\frac{\sqrt{6}}{2}\right)$

(e) inflation points are $0, 2 \pm \frac{\sqrt{6}}{2}$

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Q25 Solution

$$f(x) = (x^{2/3} - 1)^2$$

$$f'(x) = 2(x^{2/3} - 1)(2/3)x^{-1/3}$$

$$= \frac{4}{3}\left[(x^{2/3 - 1/3}) - x^{-1/3}\right]$$

$$= \frac{4}{3}(x^{1/3} - x^{-1/3})$$

$$f''(x) = \frac{4}{3}\left[\frac{1}{3}x^{-2/3} + \frac{1}{3}x^{-4/3}\right]$$

$$= \frac{4}{9}(x^{-2/3} + x^{-4/3})$$

$$= \frac{4x^{-4/3}}{9}(x^{2/3} + 1)$$

Q29 Solution:-

$$f(x) = \ln \sqrt{x^2+4}$$

$$f'(x) = \frac{2x}{\sqrt{x^2+4}} \left(\frac{1}{2}\right)(x^2-4)^{-1/2}.$$

$$f'(x) = \frac{x}{x^2+4}$$

(a) Increasing up $(0, +\infty)$

(b) Increasing down $(-\infty, 0)$

0

$A = -1$ -ve

$B = +1$ +ve.

$$f''(x) = \frac{(x^2+4)1 - (x)(2x)}{(x^2+4)^2}$$

$$= \frac{x^2+4-2x^2}{(x^2+4)^2}.$$

$$= \frac{-x^2+4}{(x^2+4)^2}$$

(c) Concave up on $[-2, 2]$

A B C

-2 2

(d) concave down on $(-\infty, -2] \cup [2, +\infty)$

$A = -3$ -ve

(e) Inflation points $2, -2$.

$B = 0$ +ve

$C = 3$ -ve.

EXERCISE 4.2

Q 25. Solution:-

$f'(x) = x^2 (x^3 - 5)$

for critical points.

$f'(x) = 0$.

$x^2 (x^3 - 5) = 0$.

$x^2 = 0$ $\qquad\qquad$ $x^3 - 5 = 0$.

$x = 0$ $\qquad\qquad$ $x^3 = 5$

$\qquad\qquad\qquad x = \sqrt[3]{5}$

A = -1 $\quad$ $-ve$

B = 1 $\quad$ $-ve$

C = 2 $\quad$ $+ve$

A		B		C
	0		$\sqrt[3]{5}$	

$x = 0$, $\sqrt[3]{5}$; $x = 0$: neither ;

$x = \sqrt[3]{5}$: relative minimum.

Q29 Solution:-

$f'(x) = x e^{1-x^2}$.

critical point $x = 0$.

$f''(x) = x(e^{1-x^2})(-2x) + e^{1-x^2}$

$= -2x^2 e^{1-x^2} + e^{1-x^2}$.

$f''(0) = 2.71$ +ve.

relative minimum at $x = 0$.

Q33 Solution:-

$f(x) = 1 + 8x - 3x^2$

from first derivative test.

$f'(x) = 8 - 6x$.

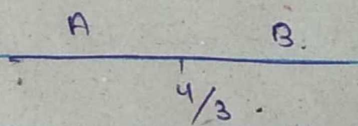
critical points

$8 - 6x = 0$.

$x = 8/6 = 4/3$.

$A = 0$ +ve.

$B = 2$. -ve.



Relative maximum at $4/3 = x$.

from second derivative test.

$$f''(x) = -6.$$
$$f''(4/3) = -6.$$

Relative maximum at $x = 4/3$.

$$f(4/3) = 1 + 8(4/3) - 3(4/3)^2$$
$$f(4/3) = 19/3$$

$f(x)$ has relative maximum of $19/3$.

Q35 Solution

$$f(x) = \sin 2x \qquad 0 < x < \pi.$$
$$f'(x) = 2\cos 2x.$$

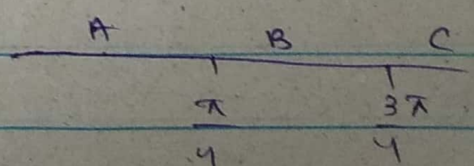
critical points

$$\cancel{2}\cos 2x = 0.$$
$$\cos 2x = 0.$$
$$2x = \cos^{-1}(0).$$
$$2x = \frac{\pi}{2}, \frac{3\pi}{2}$$
$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$

$A = \pi/6$ +ve

$B = \pi/3$. -ve

$C = 6\pi/5$ +ve.



relative maximum at $\pi/4 = x$
relative minimum at $3\pi/4 = x$.

from 2nd derivative test.

$$f''(x) = -4 \sin 2x.$$

$f''(\pi/4) = -4$ relative maxima

$f''(3\pi/4) = 4$ relative minima

$f(\pi/4) = \sin 2(\pi/4) = 1$
$f(3\pi/4) = \sin 2(3\pi/4) = -1.$

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