

2.1

# CHAPTER : 02

→ Express decimal numbers as a sum of values of each digit

## Example 2.1

$$\Rightarrow (47) = (4 \times 10^1) + (7 \times 10^0)$$

$$= 40 + 7$$

$$= 47$$

Related Problem,

$$\Rightarrow (939) = (9 \times 10^2) + (3 \times 10^1) + (9 \times 10^0)$$

$$= 900 + 30 + 9$$

$$= 939$$

## Example 2.2

$$\Rightarrow (568.23) = (5 \times 10^2) + (6 \times 10^1) + (8 \times 10^0)$$

$$+ (2 \times 10^{-1}) + (3 \times 10^{-2})$$

$$= (5 \times 100) + (6 \times 10) + (8 \times 1) +$$

$$(2 \times 0.1) + (3 \times 0.01)$$

$$= 500 + 60 + 8 + 0.2 + 0.03 = 568.23$$

Related Problem

$$\Rightarrow (67.924) = (6 \times 10^1) + (7 \times 10^0) + (9 \times 10^{-1})$$

$$+ (2 \times 10^{-2}) + (4 \times 10^{-3})$$

$$= (6 \times 10) + (7 \times 1) + (9 \times 0.1) +$$

$$(2 \times 0.01) + (4 \times 0.001)$$

$$= 60 + 7 + 0.9 + 0.02 + 0.004$$

$$= 67.924,$$

## Section 2.1

### DECIMAL NUMBERS.

1. Weight of 7 in each numbers.

(a) 1947

$$\Rightarrow (7 \times 10^0) = (7 \times 1) = 7$$

(b) 1799

$$\Rightarrow (7 \times 10^2) = (7 \times 100) = 700$$

(c) 1979

$$\Rightarrow (7 \times 10^1) = (7 \times 10) = 70$$

2. Express each as power of ten.

(a) 1000

$$\Rightarrow 10^3$$

(b) 10000000

$$\Rightarrow 10^7$$

(c) 1000000000

$$\Rightarrow 10^9$$

3. Give value of each digit.

(a) 263

$$\begin{aligned}\Rightarrow & (2 \times 10^2) + (6 \times 10^1) + (3 \times 10^0) \\ \Rightarrow & 2 \times 100 + 6 \times 10 + 3 \times 1 \\ \Rightarrow & 200 + 60 + 3\end{aligned}$$

(b) 5436

$$\begin{aligned}\Rightarrow & (5 \times 10^3) + (4 \times 10^2) + (3 \times 10^1) + (6 \times 10^0) \\ \Rightarrow & 5 \times 1000 + (4 \times 100) + (3 \times 10) + (6 \times 1) \\ \Rightarrow & 5000 + 400 + 30 + 6\end{aligned}$$

(c) 234543

$$\begin{aligned}\Rightarrow & (2 \times 10^5) + (3 \times 10^4) + (4 \times 10^3) + (5 \times 10^2) \\ & + (4 \times 10^1) + (3 \times 10^0) \\ \Rightarrow & (2 \times 100000) + (3 \times 10000) + (4 \times 1000) + \\ & (5 \times 100) + (4 \times 10) + (3 \times 1) \\ \Rightarrow & 200000 + 30000 + 4000 + 500 + \\ & 40 + 3.\end{aligned}$$

4. 999999

## BINARY NUMBERS :-

### Example 2.3

$$\begin{aligned} & \text{1101101 to decimal.} \\ (1101101)^2 &= (1 \times 10^6) + (1 \times 10^5) + \\ & (0 \times 10^4) + (1 \times 10^3) + (1 \times 10^2) + \\ & (0 \times 10^1) + (1 \times 10^0) \\ &= 64 + 32 + 0 + 8 + \end{aligned}$$

$$\begin{aligned} \Rightarrow (1101101)^2 &= (1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) \\ &+ (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\ &= 64 + 32 + 0 + 8 + 4 + 1 \\ &= (109)_{10} \text{ Ans.} \end{aligned}$$

### Example 2.4

0.1011 to decimal.

$$\begin{aligned} \Rightarrow (0.1011)_2 &= (1 \times 2^{-1}) + (0 \times 2^{-2}) \\ &+ (1 \times 2^{-3}) + (1 \times 2^{-4}) \\ &= 0.5 + 0 + 0.125 + 0.0625 \\ &= (0.6875)_{10} \text{ Ans.} \end{aligned}$$

## Section 2.2

5. Convert following binary numbers

a) 001

$$\begin{aligned} \Rightarrow (001)_2 &= (0 \times 10^2) + (0 \times 10^1) \\ &= (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\ &= 0 + 0 + 1 \\ &= (1)_{10} \text{ Ans.} \end{aligned}$$

**b) 010**

$$\Rightarrow (010)_2 = (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$$
$$= 0 + 2 + 0$$
$$= (2)_{10} \text{ Ans.}$$

**(c) 101**

$$\Rightarrow (101)_2 = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$
$$= 4 + 0 + 1$$
$$= (5)_{10} \text{ Ans.}$$

**(d) 110**

$$\Rightarrow (110)_2 = (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$$
$$= 4 + 2 + 0$$
$$= (6)_{10} \text{ Ans.}$$

**(e) 1010**

$$\Rightarrow (1010)_2 = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$$
$$= 8 + 0 + 2 + 0$$
$$= (10)_{10} \text{ Ans.}$$

**(f) 1011**

$$\Rightarrow (1011)_2 = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$
$$= 8 + 0 + 2 + 1$$
$$= (11)_{10} \text{ Ans.}$$

**(g) 1110**

$$\Rightarrow (1110)_2 = (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$$
$$= 8 + 4 + 2 + 0$$
$$= (14)_{10} \text{ Ans.}$$

(h) 1111

$$\begin{aligned}\Rightarrow (1111)_2 &= (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\ &= 8 + 4 + 2 + 1 \\ &= (15)_{10} \quad \text{Ans.}\end{aligned}$$

6. Convert binary numbers into decimal.

a) 100001

$$\begin{aligned}\Rightarrow (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) \\ &\quad + (0 \times 2^1) + (1 \times 2^0) \\ \Rightarrow 32 + 0 + 0 + 0 + 0 + 1 \\ \Rightarrow 33 \quad \text{Ans.}\end{aligned}$$

b) 100111

$$\begin{aligned}\Rightarrow (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) \\ &\quad + (1 \times 2^1) + (1 \times 2^0) \\ \Rightarrow 32 + 0 + 0 + 4 + 2 + 1 \\ \Rightarrow 39 \quad \text{Ans.}\end{aligned}$$

c) 101010

$$\begin{aligned}\Rightarrow (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) \\ &\quad + (1 \times 2^1) + (0 \times 2^0) \\ \Rightarrow 32 + 0 + 8 + 0 + 2 + 0 \\ \Rightarrow 42 \quad \text{Ans.}\end{aligned}$$

d) 111001

$$\begin{aligned}\Rightarrow & (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + \\ & (0 \times 2^1) + (1 \times 2^0) \\ \Rightarrow & 32 + 16 + 8 + 0 + 0 + 1 \\ \Rightarrow & 57 \text{ Ans.}\end{aligned}$$

e) 1100000

$$\begin{aligned}\Rightarrow & (1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + \\ & (0 \times 2^2) + (0 \times 2^1) + (0 \times 2^0) \\ \Rightarrow & 64 + 32 + 0 + 0 + 0 + 0 + 0 \\ \Rightarrow & 96 \text{ Ans.}\end{aligned}$$

f) 11111101

$$\begin{aligned}\Rightarrow & (\cancel{2} \times 1 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + \\ & + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\ \Rightarrow & 128 + 64 + 32 + 16 + 8 + 4 + 0 + 1 \\ \Rightarrow & 253 \text{ Ans.}\end{aligned}$$

g) 11110010

$$\begin{aligned}\Rightarrow & (1 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + \\ & (0 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) \\ \Rightarrow & 128 + 64 + 32 + 16 + 0 + 0 + 2 + 1 \\ \Rightarrow & 243 \text{ Ans.}\end{aligned}$$

**h)** 11111111

$$\Rightarrow \cancel{(1 \times 2^7)} + (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) \\ + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\ \Rightarrow 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 \\ \Rightarrow 255 \text{ Ans.}$$

**7.** Convert binary numbers to decimal.

**a)** 110011.11

$$\Rightarrow (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + \\ (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2}) \\ \Rightarrow 32 + 16 + 0 + 0 + 2 + 1 + 0.5 + 0.25 \\ \Rightarrow 51.75 \text{ Ans.}$$

**b)** 101010.01

$$\Rightarrow (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + \\ (1 \times 2^1) + (0 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) \\ \Rightarrow 32 + 0 + 8 + 0 + \cancel{+ 0} + 1 2 + 0 + 0.25 \\ \Rightarrow 42.25 \text{ Ans.}$$

**c)** 1000001.111

$$\Rightarrow (1 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + \\ (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + \\ (1 \times 2^{-2}) + (1 \times 2^{-3}) \\ \Rightarrow 64 + 0 + 0 + 0 + 0 + 1 + 0.5 + 0.25 + 0.125 \\ \Rightarrow 65.875 \text{ Ans.}$$

d) 1111000.101

$$\Rightarrow (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + \\ (0 \times 2^2) + (0 \times 2^1) + (0 \times 2^0) + (1 \times 2^{-1}) + \\ (0 \times 2^{-2}) + (1 \times 2^{-3})$$

$$\Rightarrow 64 + 32 + 16 + 8 + 0.5 + 0.125$$

$$\Rightarrow 120.625 \text{ Ans.}$$

e) 1011100.10101

$$\Rightarrow (1 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + \\ (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0) + (1 \times 2^{-1}) + \\ (0 \times 2^{-2}) + (1 \times 2^{-3}) + (0 \times 2^{-4}) + (1 \times 2^{-5})$$

$$\Rightarrow 64 + 16 + 8 + 4 + 0.5 + 0.125 + 0.03125$$

$$\Rightarrow 92.65625 \text{ Ans.}$$

f) 1110001.0001

$$\Rightarrow (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + \\ (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + \\ (0 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4})$$

$$\Rightarrow 64 + 32 + 16 + 1 + 0.0625$$

$$\Rightarrow 113.0625 \text{ Ans.}$$

g) 1011010.1010

$$\Rightarrow (1 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) + (0 \times 2^{-4})$$

$$\Rightarrow 64 + 16 + 8 + 2 + 0.5 + 0.125$$

$$\Rightarrow 90.625 \text{ Ans.}$$

h) 1111111.11111

$$\Rightarrow (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3}) + (1 \times 2^{-4}) + (1 \times 2^{-5})$$

$$\Rightarrow 64 + 32 + 16 + 8 + 4 + 2 + 1 + 0.5 + 0.25 + 0.125 + 0.0625 + 0.03125$$

$$\Rightarrow 127.96875 \text{ Ans.}$$

$2^{-3}$

## DECIMAL TO BINARY CONVERSION:-

### Example 2.5

a) 12

$$= 8 + 4 = 2^3 + 2^2 = 1100$$

b) 25

$$= 16 + 8 + 1 = 2^4 + 2^3 + 2^0 = 11001$$

c) 58

$$= 32 + 16 + 8 + 2 = 2^5 + 2^4 + 2^3 + 2^1 = 111010$$

d) 82

$$= 64 + 16 + 2 = 2^6 + 2^4 + 2^1 = 1010010$$

## Example 2.6

a) 19

$$\begin{array}{r} 19 \\ 2 \Big| \\ 9 - 1 \\ 2 \Big| \\ 4 - 1 \\ 2 \Big| \\ 2 - 0 \\ 1 - 0 \end{array} \quad (19)_{10} = (10011)_2$$

Ans.

b) 45

$$\begin{array}{r} 45 \\ 2 \Big| \\ 22 - 1 \\ 2 \Big| \\ 11 - 0 \\ 2 \Big| \\ 5 - 1 \\ 2 \Big| \\ 2 - 1 \\ 1 - 0 \end{array} \quad (45)_{10} = (101101)_2$$

Ans.

## Section 2.3

13. Convert each decimal number to binary.

a) 13

$$\begin{array}{r} 13 \\ 2 \Big| \\ 6 - 1 \\ 2 \Big| \\ 3 - 0 \\ 1 - 1 \end{array} \quad (13)_{10} = (1101)_2$$

Ans.

b) 17

$$\begin{array}{r} 17 \\ 2 \Big| 8 - 1 \\ 2 \Big| 4 - 0 \\ 2 \Big| 2 - 0 \\ 1 - 0 \end{array}$$

$(17)_{10} = (10001)_2$

Ans.

c) 23

$$\begin{array}{r} 23 \\ 2 \Big| 11 - 1 \\ 2 \Big| 5 - 1 \\ 2 \Big| 2 - 1 \\ 1 - 0 \end{array}$$

$(23)_{10} = (10111)_2$

Ans.

d) 30

$$\begin{array}{r} 30 \\ 2 \Big| 15 - 0 \\ 2 \Big| 7 - 1 \\ 2 \Big| 3 - 1 \\ 1 - 1 \end{array}$$

$(30)_{10} = (1110)_2$

Ans.

(e) 35

$$\begin{array}{r} 35 \\ 2 \bigg| \\ 17 - 1 \\ 2 \bigg| \\ 8 - 1 \\ 2 \bigg| \\ 4 - 0 \\ 2 \bigg| \\ 2 - 0 \\ 2 \bigg| \\ 1 - 0 \end{array}$$

$$(35)_{10} = (100011)_2$$

Ans.

f) 40

$$\begin{array}{r} 40 \\ 2 \bigg| \\ 20 - 0 \\ 2 \bigg| \\ 10 - 0 \\ 2 \bigg| \\ 5 - 0 \\ 2 \bigg| \\ 2 - 1 \\ 2 \bigg| \\ 1 - 0 \end{array}$$

$$(40)_{10} = (101000)_2$$

Ans.

g) 49

$$\begin{array}{r} 49 \\ 2 \bigg| \\ 24 - 1 \\ 2 \bigg| \\ 12 - 0 \\ 2 \bigg| \\ 6 - 0 \\ 2 \bigg| \\ 3 - 0 \\ 2 \bigg| \\ 1 - 1 \end{array}$$

$$(49)_{10} = (110001)_2$$

Ans.

h) 60

$$\begin{array}{r} 60 \\ 2 \Big| \\ 30 - 0 \\ 2 \Big| \\ 15 - 0 \\ 2 \Big| \\ 7 - 1 \\ 2 \Big| \\ 3 - 1 \\ 1 - 1 \end{array}$$

$$(60)_{10} = (111100)_2$$

Ans.

## BINARY ARITHMETIC

### Example 2.7

Add following binary numbers.

a)  $11 + 11$

$$\begin{array}{r} 11 \\ + 11 \\ \hline 110 \end{array} \quad \begin{array}{r} 3 \\ + 3 \\ \hline 6 \end{array}$$

b)  $100 + 10$

$$\begin{array}{r} 100 \\ + 10 \\ \hline 110 \end{array} \quad \begin{array}{r} 4 \\ + 2 \\ \hline 6 \end{array}$$

c)  $111 + 11$

$$\begin{array}{r}
 111 \\
 + 11 \\
 \hline
 1010
 \end{array}$$

~~111 + 3~~

$$\begin{array}{r}
 111 \\
 + 3 \\
 \hline
 10
 \end{array}$$

d)  $110 + 100$

$$\begin{array}{r}
 110 \\
 + 100 \\
 \hline
 1010
 \end{array}
 \quad
 \begin{array}{r}
 6 \\
 + 4 \\
 \hline
 10
 \end{array}$$

### Example 2.8

Perform following subtraction:-

a)  $11 - 01$

$$\begin{array}{r}
 11 \\
 - 01 \\
 \hline
 10
 \end{array}$$

b)  $11 - 10$

$$\begin{array}{r}
 11 \\
 - 10 \\
 \hline
 01
 \end{array}$$

### Example 2.9

Subtract 011 from 101

$$\begin{array}{r}
 101 \\
 - 011 \\
 \hline
 010
 \end{array}
 \quad
 \begin{array}{r}
 5 \\
 - 3 \\
 \hline
 2
 \end{array}$$

## Example 2.10

Binary Multiplication.

a)  $11 \times 11$

$$\begin{array}{r} 11 \\ \times 11 \\ \hline 11 \\ 11 \\ \hline 1001 \end{array}$$

b)  $111 \times 101$

$$\begin{array}{r} 111 \\ \times 101 \\ \hline 111 \\ 000 \\ \hline 111 \\ \hline 10011 \end{array}$$

## Example 2.11

Binary Division.

a)  $110 \div 101$

$$\begin{array}{r} 10 \\ \hline 11 \sqrt{110} \\ 11 \\ \hline 000 \\ 0 \\ \hline x \end{array}$$

b)  $110 \div 10$

$$\begin{array}{r} 11 \\ \hline 10 \sqrt{110} \\ 10 \\ \hline 010 \\ 10 \\ \hline x \end{array}$$

## Section 2.4 :-

15. Add the binary numbers.

a)  $10 + 10$

$$\begin{array}{r} 10 \\ + 10 \\ \hline 100 \end{array}$$

b)  $10 + 11$

$$\begin{array}{r} 10 \\ + 11 \\ \hline 101 \end{array}$$

c)  $100 + 11$

$$\begin{array}{r} 100 \\ + 11 \\ \hline 111 \end{array}$$

d)  $111 + 101$

$$\begin{array}{r} 111 \\ + 101 \\ \hline 1100 \end{array}$$

16. Subtract the following numbers.

a)  $10 - 1$

$$\begin{array}{r} 10 \\ - 1 \\ \hline 01 \end{array}$$

b)  $100 - 11$

$$\begin{array}{r} 100 \\ - 11 \\ \hline 001 \end{array}$$

c)  $110 - 100$

$$\begin{array}{r} 110 \\ - 100 \\ \hline 010 \end{array}$$

d)  $1111 - 11$

$$\begin{array}{r} 1111 \\ - 11 \\ \hline 1100 \end{array}$$

**17.** Perform binary multiplication.

a)  $11 \times 10$

$$\begin{array}{r} 11 \\ \times 10 \\ \hline 00 \end{array}$$

$$\begin{array}{r} 11 \\ + 11 \\ \hline 110 \end{array}$$

b)  $101 \times 11$

$$\begin{array}{r} 101 \\ \times 11 \\ \hline 101 \end{array}$$

$$\begin{array}{r} + 101 \\ \hline 1111 \end{array}$$

c)  $111 \times 110$

$$\begin{array}{r} 111 \\ \times 110 \\ \hline 000 \end{array}$$

$$\begin{array}{r} 111 \\ + 111 \\ \hline 101010 \end{array}$$

d)  $1100 \times 101$

$$\begin{array}{r} 1100 \\ \times 101 \\ \hline 1100 \end{array}$$

$$\begin{array}{r} 0000 \\ + 1100 \\ \hline 111100 \end{array}$$

**18.** Perform binary division.

a)  $110 \div 11$

$$\begin{array}{r} 10 \\ \boxed{11} \boxed{110} \\ 11 \\ \hline 0 \end{array}$$

b)  $1010 \div 10$

$$\begin{array}{r} 11 \\ \boxed{10} \boxed{1010} \\ 10 \\ \hline 10 \end{array}$$

C)  $1111 \div 101$

$$\begin{array}{r}
 & 11 \\
 101 & \boxed{1111} \\
 101 & \hline
 & 101 \\
 101 & \hline
 & 0
 \end{array}$$

## COMPLEMENTS OF BINARY NUMBERS :-

### Example 2.12

Find 2's complement of 10110010

$$\begin{array}{r}
 10110010 \\
 01001101 \\
 + \quad \quad \quad 1 \\
 \hline
 100000000
 \end{array}$$

1's complement

$$\begin{array}{r}
 10110010 \\
 01001101 \\
 \hline
 ; \quad +1 \\
 \hline
 01001110
 \end{array}$$

1's complement

## Section 2.5

21. Determine 1's complement in each number.

a) 100  
011

b) 111  
000

c) 1100  
0011

d) 10111011  
01000100

22. Determine 2's complement

a) 11  
11  
00      1's comp.  
+ 1  

---

01

b) 110  
110  
0'01      1's comp.  
+ 1  

---

010

c) 1010  
1010  
01'01  
+ 1  

---

0110

d) 1001  
1001  
0110  
+ 1  

---

0111

# SIGNED NUMBERS.

## Example 2.14

Express -39 as 8 bit number in sign magnitude, 1's complement 2's complement

8 bit number for -39

|   |        |          |
|---|--------|----------|
| 2 | 39     |          |
| 2 | 19 - 1 |          |
| 2 | 9 - 1  | 00100111 |
| 2 | 4 - 1  |          |
| 2 | 2 - 0  |          |
|   | 1 - 0  |          |

Sign magnitude form

10100111 Ans.

1's complement form.

11011000 Ans.

2's complement form.

11011000

$$\begin{array}{r} + \\ \hline 11011000 \\ \hline 11011001 \end{array}$$

Ans.

Signed magnitude

### Example 2.15

10010101

$$\Rightarrow (0 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + \\ (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$

$$\Rightarrow 16 + 4 + 1 \Rightarrow 21$$

Sign-bit is 1 so decimal number is -21. Ans.

1's complement.

### Example 2.16

a) 00010111

$$\Rightarrow -(0 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + \\ (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

$$\Rightarrow 16 + 4 + 2 + 1$$

$$\Rightarrow +23 \text{ Ans.}$$

b) 11101000

$$\Rightarrow -(1 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + \\ (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$$

$$\Rightarrow -128 + 64 + 32 + 8$$

$$\Rightarrow -24$$

Adding 1 to result.

$$\Rightarrow -24 + 1 \Rightarrow -23 \text{ Ans.}$$

## Example 2.17

a) 01010110

$$\begin{aligned} & \Rightarrow -(0 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) \\ & \quad + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) \\ & \Rightarrow 64 + 16 + 4 + 2 \\ & \Rightarrow +86 \text{ Ans.} \end{aligned}$$

b) 10101010

$$\begin{aligned} & \Rightarrow -(1 \times 2^7) + (0 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) \\ & \quad + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) \\ & \Rightarrow -128 + 32 + 8 + 2 \\ & \Rightarrow -86 \text{ Ans.} \end{aligned}$$

## Example 2.18

Convert decimal to binary

$$\Rightarrow 3.248 \times 10^4 = 32480$$

$$= 111111011100000_2$$

$$= 1.11111011100000 \times 2^{14}$$

$$\Rightarrow 14 + 127 = 141$$

$$= 10001101_2$$

The complete floating-point number is,

|   |          |                      |
|---|----------|----------------------|
| 0 | 10001101 | 11111101110000000000 |
|---|----------|----------------------|

|   |           |
|---|-----------|
| 2 | 32480     |
| 2 | 16240 - 0 |
| 2 | 8120 - 0  |
| 2 | 4060 - 0  |
| 2 | 2030 - 0  |
| 2 | 1015 - 0  |
| 2 | 507 - 1   |
| 2 | 253 - 1   |
| 2 | 126 - 1   |
| 2 | 63 - 0    |
| 2 | 32 - 1    |
| 2 | 16 - 0    |
| 2 | 8 - 0     |
| 2 | 4 - 0     |
| 2 | 2 - 0     |

## Section 2.6

23. Express each decimal number in binary as an 8 bit sign magnitude number.

a) +29

8-bit number for +29

$$\Rightarrow 0011101 \text{ Ans.}$$

|   |       |
|---|-------|
| 2 | 29    |
| 2 | 14 -1 |
| 2 | 7 -0  |
| 2 | 3 -1  |
| 2 | 1 -1  |
| 2 | 0 -1  |

b) -85

8-bit number for -85

$$\Rightarrow 1010101 \text{ Ans.}$$

$$\Rightarrow -85 = 11010101$$

|   |       |
|---|-------|
| 2 | 85    |
| 2 | 42 -1 |
| 2 | 21 -0 |
| 2 | 10 -1 |
| 2 | 5 -0  |
| 2 | 2 -1  |
| 2 | 1 -0  |

c) +100

8-bit number for +100

~~$$\Rightarrow 11001000$$~~

$$\Rightarrow 1100100$$

$$\Rightarrow +100 = 01100100 \text{ Ans.}$$

|   |       |
|---|-------|
| 2 | 100   |
| 2 | 50 -0 |
| 2 | 25 -0 |
| 2 | 12 -1 |
| 2 | 6 -0  |
| 2 | 3 -0  |
| 2 | 1 -1  |

d) -123

binary number for 123

$$\Rightarrow 1111011$$

8-bit signed number

for -123

$$\Rightarrow 11111011 \text{ Ans.}$$

|   |       |
|---|-------|
| 2 | 123   |
| 2 | 61 -1 |
| 2 | 30 -1 |
| 2 | 15 -1 |
| 2 | 7 -1  |
| 2 | 3 -1  |
| 2 | 1 -1  |

4. Express each decimal number as an 8-bit number in 1's complement.

a) -34

Binary representation for 34

$$\Rightarrow 0100010$$

$$\Rightarrow -34 = \cancel{1100010}10100010$$

$$1\text{'s complement} = 11011101 \text{ Ans.}$$

|   |        |
|---|--------|
| 2 | 34     |
| 2 | 17 - 0 |
| 2 | 8 - 1  |
| 2 | 4 - 0  |
| 2 | 2 - 0  |
| 2 | 1 - 0  |

b) +57

$$\Rightarrow 57 = 0111001_2$$

$$\Rightarrow +57 = 00111001 \text{ Ans.}$$

|   |        |
|---|--------|
| 2 | 57     |
| 2 | 28 - 1 |
| 2 | 14 - 0 |
| 2 | 7 - 0  |
| 2 | 3 - 1  |
| 1 | - 1    |

~~Ans~~

c) -99

$$\Rightarrow 99 = 1100011_2$$

$$\Rightarrow -99 = 11100011$$

$$1\text{'s complement} = 10011100 \text{ Ans.}$$

|   |        |
|---|--------|
| 2 | 99     |
| 2 | 49 - 1 |
| 2 | 24 - 1 |
| 2 | 12 - 0 |
| 2 | 6 - 0  |
| 2 | 3 - 0  |
| 1 | - 1    |

d) +115

$$\Rightarrow 115 = 1110011$$

$$\Rightarrow +115 = 01110011 \text{ Ans.}$$

|   |        |
|---|--------|
| 2 | 115    |
| 2 | 57 - 1 |
| 2 | 28 - 1 |
| 2 | 14 - 0 |
| 2 | 7 - 0  |
| 2 | 3 - 1  |
| 2 | 1 - 1  |

**25.** Express each decimal number as an 8-bit number in 2's complement

a) +12

$$\Rightarrow 12 = 0001101_2$$

$$\Rightarrow +12 = 00001101 \text{ Ans.}$$

|   |       |
|---|-------|
| 2 | 12    |
| 2 | 6 - 0 |
| 2 | 3 - 0 |
| 2 | 1 - 1 |

b) -68

$$\Rightarrow 68 = 1000100_2$$

$$\Rightarrow -68 = 11000100$$

|   |        |
|---|--------|
| 2 | 68     |
| 2 | 34 - 0 |
| 2 | 17 - 0 |
| 2 | 8 - 1  |
| 2 | 4 - 0  |
| 2 | 2 - 0  |
| 2 | 1 - 0  |

Negative No so taking 1's and 2's complement

$$\Rightarrow 1\text{'s complement} = 10111011$$

$$\Rightarrow 2\text{'s complement} = 10111011 \\ + 1$$

$$\hline 10111100 \text{ Ans.}$$

c) +101

$$\Rightarrow 101 = 1100101$$

$$\Rightarrow +101 = 01100101 \text{ Ans.}$$

|   |        |
|---|--------|
| 2 | 101    |
| 2 | 50 - 1 |
| 2 | 25 - 0 |
| 2 | 12 - 1 |
| 2 | 6 - 0  |
| 2 | 3 - 0  |
| 2 | 1 - 1  |

the number so no need of 1's and 2's complement.

d) - 125

$$\Rightarrow 125 = 1111101_2$$

$$\Rightarrow -125 = 1111101$$

|   |        |
|---|--------|
| 2 | 125    |
| 2 | 62 - 1 |
| 2 | 31 - 0 |
| 2 | 15 - 1 |
| 2 | 7 - 1  |
| 2 | 3 - 1  |
| 2 | 1 - 1  |

$$\Rightarrow 1\text{'s complement} = 1000010$$

$$\Rightarrow 2\text{'s complement} =$$

$$1000010$$

$$+ 1$$

$$\hline 1000011 \text{ Ans.}$$

26. Determine decimal value of each signed binary number in sign-magnitude form.

a) 10011001

$$\Rightarrow \cancel{(1 \times 2^7)} + (0 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + \\ (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$

$$\Rightarrow \cancel{128} + 16 + 8 + 1$$

$$\Rightarrow 103 \text{ 25}$$

Sign bit is 1 so decimal number is ~~-103~~ Ans ~~-25~~ - 25 Ans.

b) 01110100

$$\Rightarrow (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) \\ + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$$

$$\Rightarrow 64 + 32 + 16 + 4$$

$$\Rightarrow 116$$

Sign bit is 0 so decimal number  
is +116 Ans.

c) 10111111

$$\Rightarrow (0 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + \\ (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

$$\Rightarrow 32 + 16 + 8 + 4 + 2 + 1$$

$$\Rightarrow 64$$

Sign bit is 1 so decimal number  
is -64 Ans.

27. Determine decimal number of  
each signed binary number in  
1's complement.

a) 10011001

$$\Rightarrow -(1 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) \\ + (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$

$$\Rightarrow -128 + 16 + 8 + 1 \Rightarrow -103$$

Adding 1 to -103

$$\Rightarrow -102 \text{ Ans.}$$

b) 01110100

$$\Rightarrow -(0 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + \\ (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$$
$$\Rightarrow 64 + 32 + 16 + 4$$
$$\Rightarrow +116 \text{ Ans.}$$

c) 10111111

$$\Rightarrow -(1 \times 2^7) + (0 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + \\ (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$
$$\Rightarrow -128 + 32 + 16 + 8 + 4 + 2 + 1$$
$$\Rightarrow -65$$

Adding 1 to -65

$$\Rightarrow -64 \text{ Ans.}$$

28. Determine the decimal value of each signed binary in 2's complement.

a) 10011001

$$\Rightarrow (-1 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + \\ (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$
$$\Rightarrow -128 + 16 + 8 + 1$$
$$\Rightarrow -103 \text{ Ans.}$$

b) 01110100

$$\Rightarrow -(0 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) \\ + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$$
$$\Rightarrow 64 + 32 + 16 + 4 =$$
$$\Rightarrow + 116 \text{ Ans.}$$

c) 10111111

$$\Rightarrow -(1 \times 2^7) + (0 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) \\ + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$
$$\Rightarrow -128 + 32 + 16 + 8 + 4 + 2 + 1$$
$$\Rightarrow -65 \text{ Ans.}$$

29. Express each of the following sign-magnitude binary numbers in single-precision floating point.

a) 0111110000101011

$$\Rightarrow 1.11110000101011 \times 2^{14}$$

$$\Rightarrow \text{Exponent} = 14 + 127$$

$$= 141 = 10001101_2$$

Mantissa = 1111000010101100000000

Complete floating point number is:

$\rightarrow 0100011011110000101011$   
0000000000 Ans.

b) 100110000011000

Sign = 1

$$\Rightarrow 1.10000011000 \times 2^{11}$$

$$\text{Exponent} = 11 + 127$$

$$= 138 = 10001010_2$$

$$\begin{array}{r}
 138 \\
 -69 \\
 \hline
 69 \\
 -34 \\
 \hline
 34 \\
 -17 \\
 \hline
 17 \\
 -8 \\
 \hline
 8 \\
 -4 \\
 \hline
 4 \\
 -2 \\
 \hline
 2 \\
 -1 \\
 \hline
 1
 \end{array}$$

Mantissa = 1100000110000000000000

Complete floating point number can be expressed as

110001010110000011000000

○○○○○○○○○○ Ans I didn't stop it

**30.** Determine the values of following single-precision floating point numbers,

a) 1100000101001001110001000000000

$$\Rightarrow \text{sign} = 1$$

$$\frac{(1 \times 2^7) + (1 \times 2^0)}{= 128 + 1}$$

$\Rightarrow$  Exponent = 10000001

$$= 129 - 127 = 2$$

$$\Rightarrow \text{Mantissa} = 1.01001001110001 \times 2^2$$

$$= 101.001001110001$$

$$\Rightarrow -101.001001110001 = -5.15258789$$

Arno.

b) 01100110010000111101001  
000000000

⇒ Sign = 0

⇒ Exponent = 11001100

$$= 204 - 127 = 77$$

⇒ Mantissa = 1.10000111101001

$$1.10000111101001 \times 10^{77}$$

## ARITHMETIC OPERATIONS WITH SIGNED NUMBERS:-

### Example 2.19

Add signed numbers. 01000100,  
00011011, 00001110, 00010010

$$\begin{array}{r} 01000100 & 1st \\ + 00011011 & 2nd \\ \hline 01'0'1'1'111 \\ + 00001110 & 3rd \\ \hline 01101101 \\ + 00010010 & Hth. \\ \hline 01111111 & Ans. \end{array}$$

## Example 2.20

Perform Subtraction.

a)  $00001000 - 00000011$

2's complement of 00000011

$$\begin{array}{r} \cancel{00000011} & 11111100 \\ \underline{-11111100} & +1 \\ \hline 11111101 \end{array}$$

Now,

$$\begin{array}{r} '0b'001000 \\ +11111101 \\ \hline 100000101 \end{array}$$

b)  $00001100 - 11110111$

2's complement 11110111

$$\begin{array}{r} 00001000 \\ +1 \\ \hline 00001001 \end{array}$$

$$\begin{array}{r} 00001100 \\ +00001001 \\ \hline 00010101 \end{array}$$

**Example 2.21 -**

Multiply the signed binary numbers  
01001101 and 00000100  
Decimal value of multiplier is 4  
so multiplicand is added to itself  
4 times.

$$\begin{array}{r} 01001101 & \text{1'st time} \\ + 01001101 & \text{2nd time} \\ \hline 10011010 \\ + 01001101 & \text{3rd time} \\ \hline 11100111 \\ + 01001101 & \text{4th time} \\ \hline 100110100 \end{array}$$

Ans.

**Example 2.22**