Department of Computer Science, University of Karachi

BSCS: 306: Probability and Statistical Methods

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Assignment: 02(A): Binomial Distributions

Note: Binomial probability Table attached (Page-03 to Page -06)

- Q 1. Suppose that for a very large shipment of integrated-circuit chips, the probability of failure for any one chip is 0.10. Assuming that the assumptions underlying the binomial distribution are met; find the probability that at most 3 chips fail in a random sample of 20.
- **Q 2.** A prominent physician claims that 70% of those with lung cancer are chain smokers. If his assertion is correct
 - (a) Find the probability that out of 10 such patients recently admitted to a hospital, fewer than half are chain smokers.
 - (b) Find the probability that of 20 such patients recently admitted to a hospital, fewer than half are chain smokers.
- **Q 3.** In a certain city district the need for money to buy drugs is stated as the reason for 75% of all thefts. Find the probability that among the next 5 theft cases reported in this district.
 - (a). Exactly 2 resulted from the need for money to buy drugs.
 - (b). At most 3 resulted from the need for money to buy drugs.
- **Q 4.** According to Chemical Engineering progress approximately 30% of all pipe work failures in the chemical plant are caused by operator error.
 - (a). What is the probability that out of next 20 pipe work failures at least 10 are due to operator error?
 - (b). What is the probability that no more than 4 out of 20 such failures are due to operator error?
 - (c). Think of a particular plant, that out of the random sample of 20 such failures are exactly 5 are operational errors. Do you feel that the 30% figure stated above applies to this plant? Comment.
- **Q 5.** According to study published by a group of University of Massachusetts sociology, approximately 60% of the valium users in the state of Massachusetts first took valium for psychological problems. Find the probability that among next 8 users interviewed from the state.
 - (a). Exactly 3 begin taking valium for psychological problems:
 - (b). At least 5 began taking valium for problems were not psychological.
- **Q 6.** In testing a certain kind of truck tire over rugged terrain, it is found that 25% of the trucks fail to complete the test run without a blowout. Of the next 15 trucks tested, find the probability that
 - (a). from 3 to 6 have blowouts;
 - (b). fewer than 4 have blowouts;
 - (c). more than 5 have blowouts.

- **Q 7.** Suppose that airplane engines operate independently and fail with probability equal to 0.4. Assuming that a plane makes a safe flight if **at least one-half** of its engine runs, determine whether a 4-engine plane or a 2-engine plane has the higher probability for a successful flight.
- **Q 8.** If the probability that a florescent light has a useful life of at least 800 hours is 0.9, find the probabilities that among 20 such lights
 - (a). exactly 18 will have a useful life of at least 800 hours;
 - (b). exactly 15 will have a useful life of at least 800 hours;
 - (c). at least 2 will not have a useful life of at least 800 hours;
- **Q 9.** The random variable X, the number of radar signals properly identified in a 30-minutes period, in which 10 signals are received. Assume that X is a binomial random variable with parameters n = 10 and p = 1/2.
 - (a). Find mean and variance
 - (b). Find the probability that at least 7 signals are received.
- **Q 10.** It is known that the probability of being log on to a computer from a remote terminal at any given time is 0.7. Let X denote the number of attempts that must be made to gain access to the computer.
 - (a). Find the first four terms of the density function.
 - (b). Find a close form expression for f(x).
 - (c). Find P[X = 6].
 - (d). Find a close form for F(x).
 - (e). Use F to find the probability that at most four attempts must be made to gain access to the computer.
- **Q 11.** A social scientist claims that only 50 percent of all high school seniors capable of doing college work actually go to college. Assuming that his claim is true, Find the probability that among 18 high school seniors capable of doing college work
 - (a). exactly 10 will go to college.
 - (b). at least 10 will go to college.
 - (c). at most eight will go to college.
- **Q 12.** Twelve people are given two identical speakers to listen for different. If any. Suppose that these people answered by guessing only. Find the probability that three people claim to have a difference between the two speakers.

Binomial probabilities:

$$\binom{n}{x} p^x (1-p)^{n-x}$$

| | | | | | | | P | | | | | |
|---|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| п | x | 0.1 | 0.2 | 0.25 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.75 | 0.8 | 0.9 |
| 1 | 0 | 0.900 | 0.800 | 0.750 | 0.700 | 0.600 | 0.500 | 0.400 | 0.300 | 0.250 | 0.200 | 0.100 |
| | 1 | 0.100 | 0.200 | 0.250 | 0.300 | 0.400 | 0.500 | 0.600 | 0.700 | 0.750 | 0.800 | 0.900 |
| 2 | 0 | 0.810 | 0.640 | 0.563 | 0.490 | 0.360 | 0.250 | 0.160 | 0.090 | 0.063 | 0.040 | 0.010 |
| | 1 | 0.180 | 0.320 | 0.375 | 0.420 | 0.480 | 0.500 | 0.480 | 0.420 | 0.375 | 0.320 | 0.180 |
| | 2 | 0.010 | 0.040 | 0.063 | 0.090 | 0.160 | 0.250 | 0.360 | 0.490 | 0.563 | 0.640 | 0.810 |
| 3 | 0 | 0.729 | 0.512 | 0.422 | 0.343 | 0.216 | 0.125 | 0.064 | 0.027 | 0.016 | 0.008 | 0.001 |
| | 1 | 0.243 | 0.384 | 0.422 | 0.441 | 0.432 | 0.375 | 0.288 | 0.189 | 0.141 | 0.096 | 0.027 |
| | 2 | 0.027 | 0.096 | 0.141 | 0.189 | 0.288 | 0.375 | 0.432 | 0.441 | 0.422 | 0.384 | 0.243 |
| | 3 | 0.001 | 0.008 | 0.016 | 0.027 | 0.064 | 0.125 | 0.216 | 0.343 | 0.422 | 0.512 | 0.729 |
| 4 | 0 | 0.656 | 0.410 | 0.316 | 0.240 | 0.130 | 0.063 | 0.026 | 0.008 | 0.004 | 0.002 | 0.000 |
| | 1 | 0.292 | 0.410 | 0.422 | 0.412 | 0.346 | 0.250 | 0.154 | 0.076 | 0.047 | 0.026 | 0.004 |
| | 2 | 0.049 | 0.154 | 0.211 | 0.265 | 0.346 | 0.375 | 0.346 | 0.265 | 0.211 | 0.154 | 0.049 |
| | 3 | 0.004 | 0.026 | 0.047 | 0.076 | 0.154 | 0.250 | 0.346 | 0.412 | 0.422 | 0.410 | 0.292 |
| | 4 | 0.000 | 0.002 | 0.004 | 0.008 | 0.026 | 0.063 | 0.130 | 0.240 | 0.316 | 0.410 | 0.656 |
| 5 | 0 | 0.590 | 0.328 | 0.237 | 0.168 | 0.078 | 0.031 | 0.010 | 0.002 | 0.001 | 0.000 | 0.000 |
| | 1 | 0.328 | 0.410 | 0.396 | 0.360 | 0.259 | 0.156 | 0.077 | 0.028 | 0.015 | 0.006 | 0.000 |
| | 2 | 0.073 | 0.205 | 0.264 | 0.309 | 0.346 | 0.313 | 0.230 | 0.132 | 0.088 | 0.051 | 0.008 |
| | 3 | 0.008 | 0.051 | 0.088 | 0.132 | 0.230 | 0.313 | 0.346 | 0.309 | 0.264 | 0.205 | 0.073 |
| | 4 | 0.000 | 0.006 | 0.015 | 0.028 | 0.077 | 0.156 | 0.259 | 0.360 | 0.396 | 0.410 | 0.328 |
| | 5 | 0.000 | 0.000 | 0.001 | 0.002 | 0.010 | 0.031 | 0.078 | 0.168 | 0.237 | 0.328 | 0.590 |
| 6 | 0 | 0.531 | 0.262 | 0.178 | 0.118 | 0.047 | 0.016 | 0.004 | 0.001 | 0.000 | 0.000 | 0.000 |
| | 1 | 0.354 | 0.393 | 0.356 | 0.303 | 0.187 | 0.094 | 0.037 | 0.010 | 0.004 | 0.002 | 0.000 |
| | 2 | 0.098 | 0.246 | 0.297 | 0.324 | 0.311 | 0.234 | 0.138 | 0.060 | 0.033 | 0.015 | 0.001 |
| | 3 | 0.015 | 0.082 | 0.132 | 0.185 | 0.276 | 0.313 | 0.276 | 0.185 | 0.132 | 0.082 | 0.015 |
| | 4 | 0.001 | 0.015 | 0.033 | 0.060 | 0.138 | 0.234 | 0.311 | 0.324 | 0.297 | 0.246 | 0.098 |
| | 5 | 0.000 | 0.002 | 0.004 | 0.010 | 0.037 | 0.094 | 0.187 | 0.303 | 0.356 | 0.393 | 0.354 |
| | 6 | 0.000 | 0.000 | 0.000 | 0.001 | 0.004 | 0.016 | 0.047 | 0.118 | 0.178 | 0.262 | 0.531 |
| 7 | 0 | 0.478 | 0.210 | 0.133 | 0.082 | 0.028 | 0.008 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 1 | 0.372 | 0.367 | 0.311 | 0.247 | 0.131 | 0.055 | 0.017 | 0.004 | 0.001 | 0.000 | 0.000 |
| | 2 | 0.124 | 0.275 | 0.311 | 0.318 | 0.261 | 0.164 | 0.077 | 0.025 | 0.012 | 0.004 | 0.000 |
| | 3 | 0.023 | 0.115 | 0.173 | 0.227 | 0.290 | 0.273 | 0.194 | 0.097 | 0.058 | 0.029 | 0.003 |
| | 4 | 0.003 | 0.029 | 0.058 | 0.097 | 0.194 | 0.273 | 0.290 | 0.227 | 0.173 | 0.115 | 0.023 |
| | 5 | 0.000 | 0.004 | 0.012 | 0.025 | 0.077 | 0.164 | 0.261 | 0.318 | 0.311 | 0.275 | 0.124 |
| | 6 | 0.000 | 0.000 | 0.001 | 0.004 | 0.017 | 0.055 | 0.131 | 0.247 | 0.311 | 0.367 | 0.372 |
| | 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.002 | 0.008 | 0.028 | 0.082 | 0.133 | 0.210 | 0.478 |

(continued)

Binomial probabilities:

 $\binom{n}{x} p^x (1-p)^{n-x}$

| п | x | 0.1 | 0.2 | 0.25 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.75 | 0.8 | 0.9 |
|----|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 8 | 0 | 0.430 | 0.168 | 0.100 | 0.058 | 0.017 | 0.004 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 1 | 0.383 | 0.336 | 0.267 | 0.198 | 0.090 | 0.031 | 0.008 | 0.001 | 0.000 | 0.000 | 0.000 |
| | 2 | 0.149 | 0.294 | 0.311 | 0.296 | 0.209 | 0.109 | 0.041 | 0.010 | 0.004 | 0.001 | 0.000 |
| | 3 | 0.033 | 0.147 | 0.208 | 0.254 | 0.279 | 0.219 | 0.124 | 0.047 | 0.023 | 0.009 | 0.000 |
| | 4 | 0.005 | 0.046 | 0.087 | 0.136 | 0.232 | 0.273 | 0.232 | 0.136 | 0.087 | 0.046 | 0.005 |
| | 5 | 0.000 | 0.009 | 0.023 | 0.047 | 0.124 | 0.219 | 0.279 | 0.254 | 0.208 | 0.147 | 0.033 |
| | 6 | 0.000 | 0.001 | 0.004 | 0.010 | 0.041 | 0.109 | 0.209 | 0.296 | 0.311 | 0.294 | 0.149 |
| | 7 | 0.000 | 0.000 | 0.000 | 0.001 | 0.008 | 0.031 | 0.090 | 0.198 | 0.267 | 0.336 | 0.383 |
| | 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.004 | 0.017 | 0.058 | 0.100 | 0.168 | 0.430 |
| 9 | 0 | 0.387 | 0.134 | 0.075 | 0.040 | 0.010 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 1 | 0.387 | 0.302 | 0.225 | 0.156 | 0.060 | 0.018 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 2 | 0.172 | 0.302 | 0.300 | 0.267 | 0.161 | 0.070 | 0.021 | 0.004 | 0.001 | 0.000 | 0.000 |
| | 3 | 0.045 | 0.176 | 0.234 | 0.267 | 0.251 | 0.164 | 0.074 | 0.021 | 0.009 | 0.003 | 0.000 |
| | 4 | 0.007 | 0.066 | 0.117 | 0.172 | 0.251 | 0.246 | 0.167 | 0.074 | 0.039 | 0.017 | 0.001 |
| | 5 | 0.001 | 0.017 | 0.039 | 0.074 | 0.167 | 0.246 | 0.251 | 0.172 | 0.117 | 0.066 | 0.007 |
| | 6 | 0.000 | 0.003 | 0.009 | 0.021 | 0.074 | 0.164 | 0.251 | 0.267 | 0.234 | 0.176 | 0.045 |
| | 7 | 0.000 | 0.000 | 0.001 | 0.004 | 0.021 | 0.070 | 0.161 | 0.267 | 0.300 | 0.302 | 0.172 |
| | 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.004 | 0.018 | 0.060 | 0.156 | 0.225 | 0.302 | 0.387 |
| | 9 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.002 | 0.010 | 0,040 | 0.075 | 0.134 | 0.387 |
| 10 | 0 | 0.349 | 0.107 | 0.056 | 0.028 | 0.006 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 1 | 0.387 | 0.268 | 0.188 | 0.121 | 0.040 | 0.010 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 2 | 0.194 | 0.302 | 0.282 | 0.233 | 0.121 | 0.044 | 0.011 | 0.001 | 0.000 | 0.000 | 0.000 |
| | 3 | 0.057 | 0.201 | 0.250 | 0.267 | 0.215 | 0.117 | 0.042 | 0.009 | 0.003 | 0.001 | 0.000 |
| | 4 | 0.011 | 0.088 | 0.146 | 0.200 | 0.251 | 0.205 | 0.111 | 0.037 | 0.016 | 0.006 | 0.000 |
| | 5 | 0.001 | 0.026 | 0.058 | 0.103 | 0.201 | 0.246 | 0.201 | 0.103 | 0.058 | 0.026 | 0.001 |
| | 6 | 0.000 | 0.006 | 0.016 | 0.037 | 0.111 | 0.205 | 0.251 | 0.200 | 0.146 | 0.088 | 0.011 |
| | 7 | 0.000 | 0.001 | 0.003 | 0.009 | 0.042 | 0.117 | 0.215 | 0.267 | 0.250 | 0.201 | 0.057 |
| | 8 | 0.000 | 0.000 | 0.000 | 0.001 | 0.011 | 0.044 | 0.121 | 0.233 | 0.282 | 0.302 | 0.194 |
| | 9 | 0.000 | 0.000 | 0.000 | 0.000 | 0.002 | 0.010 | 0.040 | 0.121 | 0.188 | 0.268 | 0.387 |
| | 10 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.006 | 0.028 | 0.056 | 0.107 | 0.349 |
| 11 | 0 | 0.314 | 0.086 | 0.042 | 0.020 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 1 | 0.384 | 0.236 | 0.155 | 0.093 | 0.027 | 0.005 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 2 | 0.213 | 0.295 | 0.258 | 0.200 | 0.089 | 0.027 | 0.005 | 0.001 | 0.000 | 0.000 | 0.000 |
| | 3 | 0.071 | 0.221 | 0.258 | 0.257 | 0.177 | 0.081 | 0.023 | 0.004 | 0.001 | 0.000 | 0.000 |
| | 4 | 0.016 | 0.111 | 0.172 | 0.220 | 0.236 | 0.161 | 0.070 | 0.017 | 0.006 | 0.002 | 0.000 |
| | 5 | 0.002 | 0.039 | 0.080 | 0.132 | 0.221 | 0.226 | 0.147 | 0.057 | 0.027 | 0.010 | 0.000 |
| | 6 | 0.000 | 0.010 | 0.027 | 0.057 | 0.147 | 0.226 | 0.221 | 0.132 | 0.080 | 0.039 | 0.002 |
| | 7 | 0.000 | 0.002 | 0.006 | 0.017 | 0.070 | 0.161 | 0.236 | 0.220 | 0.172 | 0.111 | 0.016 |
| | 8 | 0.000 | 0.000 | 0.001 | 0.004 | 0.023 | 0.081 | 0.177 | 0.257 | 0.258 | 0.221 | 0.071 |
| | 9 | 0.000 | 0.000 | 0.000 | 0.001 | 0.005 | 0.027 | 0.089 | 0.200 | 0.258 | 0.295 | 0.213 |
| | 10 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.005 | 0.027 | 0.093 | 0.155 | 0.236 | 0.384 |
| | 11 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.004 | 0.020 | 0.042 | 0.086 | 0.314 |

Binomial probabilities:

$$\binom{n}{x} p^x (1-p)^{n-x}$$

| | <i>p</i> | | | | | | | | | | | | | |
|----|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--|--|
| n | x | 0.1 | 0.2 | 0.25 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.75 | 0.8 | 0.9 | | |
| 12 | 0 | 0.282 | 0.069 | 0.032 | 0.014 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | | |
| | 1 | 0.377 | 0.206 | 0.127 | 0.071 | 0.017 | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | | |
| | 2 | 0.230 | 0.283 | 0.232 | 0.168 | 0.064 | 0.016 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 | | |
| | 3 | 0.085 | 0.236 | 0.258 | 0.240 | 0.142 | 0.054 | 0.012 | 0.001 | 0.000 | 0.000 | 0.000 | | |
| | 4 | 0.021 | 0.133 | 0.194 | 0.231 | 0.213 | 0.121 | 0.042 | 0.008 | 0.002 | 0.001 | 0.000 | | |
| | 5 | 0.004 | 0.053 | 0.103 | 0.158 | 0.227 | 0.193 | 0.101 | 0.029 | 0.011 | 0.003 | 0.000 | | |
| | 6 | 0.000 | 0.016 | 0.040 | 0.079 | 0.177 | 0.226 | 0.177 | 0.079 | 0.040 | 0.016 | 0.000 | | |
| | 7 | 0.000 | 0.003 | 0.011 | 0.029 | 0.101 | 0.193 | 0.227 | 0.158 | 0.103 | 0.053 | 0.004 | | |
| | 8 | 0.000 | 0.001 | 0.002 | 0.008 | 0.042 | 0.121 | 0.213 | 0.231 | 0.194 | 0.133 | 0.021 | | |
| | 9 | 0.000 | 0.000 | 0.000 | 0.001 | 0.012 | 0.054 | 0.142 | 0.240 | 0.258 | 0.236 | 0.085 | | |
| | 10 | 0.000 | 0.000 | 0.000 | 0.000 | 0.002 | 0.016 | 0.064 | 0.168 | 0.232 | 0.283 | 0.230 | | |
| | 11 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.003 | 0.017 | 0.071 | 0.127 | 0.206 | 0.377 | | |
| | 12 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.002 | 0.014 | 0.032 | 0.069 | 0.282 | | |
| 13 | 0 | 0.254 | 0.055 | 0.024 | 0.010 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | | |
| | 1 | 0.367 | 0.179 | 0.103 | 0.054 | 0.011 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | | |
| | 2 | 0.245 | 0.268 | 0.206 | 0.139 | 0.045 | 0.010 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | | |
| | 3 | 0.100 | 0.246 | 0.252 | 0.218 | 0.111 | 0.035 | 0.006 | 0.001 | 0.000 | 0.000 | 0.000 | | |
| | 4 | 0.028 | 0.154 | 0.210 | 0.234 | 0.184 | 0.087 | 0.024 | 0.003 | 0.001 | 0.000 | 0.000 | | |
| | 5 | 0.006 | 0.069 | 0.126 | 0.180 | 0.221 | 0.157 | 0.066 | 0.014 | 0.005 | 0.001 | 0.000 | | |
| | 6 | 0.001 | 0.023 | 0.056 | 0.103 | 0.197 | 0.209 | 0.131 | 0.044 | 0.019 | 0.006 | 0.000 | | |
| | 7 | 0.000 | 0.006 | 0.019 | 0.044 | 0.131 | 0.209 | 0.197 | 0.103 | 0.056 | 0.023 | 0.001 | | |
| | 8 | 0.000 | 0.001 | 0.005 | 0.014 | 0.066 | 0.157 | 0.221 | 0.180 | 0.126 | 0.069 | 0.006 | | |
| | 9 | 0.000 | 0.000 | 0.001 | 0.003 | 0.024 | 0.087 | 0.184 | 0.234 | 0.210 | 0.154 | 0.028 | | |
| | 10 | 0.000 | 0.000 | 0.000 | 0.001 | 0.006 | 0.035 | 0.111 | 0.218 | 0.252 | 0.246 | 0.100 | | |
| | 11 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.010 | 0.045 | 0.139 | 0.206 | 0.268 | 0.245 | | |
| | 12 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.002 | 0.011 | 0.054 | 0.103 | 0.179 | 0.367 | | |
| | 13 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.010 | 0.024 | 0.055 | 0.254 | | |
| 4 | 0 | 0.229 | 0.044 | 0.018 | 0.007 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | | |
| | 1 | 0.356 | 0.154 | 0.083 | 0.041 | 0.007 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | | |
| | 2 | 0.257 | 0.250 | 0.180 | 0.113 | 0.032 | 0.006 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | | |
| | 3 | 0.114 | 0.250 | 0.240 | 0.194 | 0.085 | 0.022 | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 | | |
| | 4 | 0.035 | 0.172 | 0.220 | 0.229 | 0.155 | 0.061 | 0.014 | 0.001 | 0.000 | 0.000 | 0.000 | | |
| | 5 | 0.008 | 0.086 | 0.147 | 0.196 | 0.207 | 0.122 | 0.041 | 0.007 | 0.002 | 0.000 | 0.000 | | |
| | 6 | 0.001 | 0.032 | 0.073 | 0.126 | 0.207 | 0.183 | 0.092 | 0.023 | 0.008 | 0.002 | 0.000 | | |
| | 7 | 0.000 | 0.009 | 0.028 | 0.062 | 0.157 | 0.209 | 0.157 | 0.062 | 0.028 | 0.009 | 0.000 | | |
| | 8 | 0.000 | 0.002 | 0.008 | 0.023 | 0.092 | 0.183 | 0.207 | 0.126 | 0.073 | 0.032 | 0.001 | | |
| | 9 | 0.000 | 0.000 | 0.002 | 0.007 | 0.041 | 0.122 | 0.207 | 0.196 | 0.147 | 0.086 | 0.008 | | |
| | 10 | 0.000 | 0.000 | 0.000 | 0.001 | 0.014 | 0.061 | 0.155 | 0.229 | 0.220 | 0.172 | 0.035 | | |
| | 11 | 0.000 | 0.000 | 0.000 | 0.000 | 0.003 | 0.022 | 0.085 | 0.194 | 0.240 | 0.250 | 0.114 | | |
| | 12 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.006 | 0.032 | 0.113 | 0.180 | 0.250 | 0.257 | | |
| | 13 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.007 | 0.041 | 0.083 | 0.154 | 0.356 | | |
| | 14 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.007 | 0.018 | 0.044 | 0.229 | | |

(continued)

Binomial probabilities:

 $\binom{n}{x} p^x (1-p)^{n-x}$

| n | x | 0.1 | 0.2 | 0.25 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.75 | 0.8 | 0.9 |
|----|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 15 | 0 | 0.206 | 0.035 | 0.013 | 0.005 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 1 | 0.343 | 0.132 | 0.067 | 0.031 | 0.005 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 2 | 0.267 | 0.231 | 0.156 | 0.092 | 0.022 | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 3 | 0.129 | 0.250 | 0.225 | 0.170 | 0.063 | 0.014 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 4 | 0.043 | 0.188 | 0.225 | 0.219 | 0.127 | 0.042 | 0.007 | 0.001 | 0.000 | 0.000 | 0.000 |
| | 5 | 0.010 | 0.103 | 0.165 | 0.206 | 0.186 | 0.092 | 0.024 | 0.003 | 0.001 | 0.000 | 0.000 |
| | 6 | 0.002 | 0.043 | 0.092 | 0.147 | 0.207 | 0.153 | 0.061 | 0.012 | 0.003 | 0.001 | 0.000 |
| | 7 | 0.000 | 0.014 | 0.039 | 0.081 | 0.177 | 0.196 | 0.118 | 0.035 | 0.013 | 0.003 | 0.000 |
| | 8 | 0.000 | 0.003 | 0.013 | 0.035 | 0.118 | 0.196 | 0.177 | 0.081 | 0.039 | 0.014 | 0.000 |
| | 9 | 0.000 | 0.001 | 0.003 | 0.012 | 0.061 | 0.153 | 0.207 | 0.147 | 0.092 | 0.043 | 0.002 |
| | 10 | 0.000 | 0.000 | 0.001 | 0.003 | 0.024 | 0.092 | 0.186 | 0.206 | 0.165 | 0.103 | 0.010 |
| | 11 | 0.000 | 0.000 | 0.000 | 0.001 | 0.007 | 0.042 | 0.127 | 0.219 | 0.225 | 0.188 | 0.043 |
| | 12 | 0.000 | 0.000 | 0.000 | 0.000 | 0.002 | 0.014 | 0.063 | 0.170 | 0.225 | 0.250 | 0.129 |
| | 13 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.003 | 0.022 | 0.092 | 0.156 | 0.231 | 0.267 |
| | 14 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.005 | 0.031 | 0.067 | 0.132 | 0.343 |
| | 15 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.005 | 0.013 | 0.035 | 0.206 |
| 20 | 0 | 0.122 | 0.012 | 0.003 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 1 | 0.270 | 0.058 | 0.021 | 0.007 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 2 | 0.285 | 0.137 | 0.067 | 0.028 | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 3 | 0.190 | 0.205 | 0.134 | 0.072 | 0.012 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 4 | 0.090 | 0.218 | 0.190 | 0.130 | 0.035 | 0.005 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 5 | 0.032 | 0.175 | 0.202 | 0.179 | 0.075 | 0.015 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 6 | 0.009 | 0.109 | 0.169 | 0.192 | 0.124 | 0.037 | 0.005 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 7 | 0.002 | 0.055 | 0.112 | 0.164 | 0.166 | 0.074 | 0.015 | 0.001 | 0.000 | 0.000 | 0.000 |
| | 8 | 0.000 | 0.022 | 0.061 | 0.114 | 0.180 | 0.120 | 0.035 | 0.004 | 0.001 | 0.000 | 0.000 |
| | 9 | 0.000 | 0.007 | 0.027 | 0.065 | 0.160 | 0.160 | 0.071 | 0.012 | 0.003 | 0.000 | 0.000 |
| | 10 | 0.000 | 0.002 | 0.010 | 0.031 | 0.117 | 0.176 | 0.117 | 0.031 | 0.010 | 0.002 | 0.000 |
| | 11 | 0.000 | 0.000 | 0.003 | 0.012 | 0.071 | 0.160 | 0.160 | 0.065 | 0.027 | 0.007 | 0.000 |
| | 12 | 0.000 | 0.000 | 0.001 | 0.004 | 0.035 | 0.120 | 0.180 | 0.114 | 0.061 | 0.022 | 0.000 |
| | 13 | 0.000 | 0.000 | 0.000 | 0.001 | 0.015 | 0.074 | 0.166 | 0.164 | 0.112 | 0.055 | 0.002 |
| | 14 | 0.000 | 0.000 | 0.000 | 0.000 | 0.005 | 0.037 | 0.124 | 0.192 | 0.169 | 0.109 | 0.009 |
| | 15 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.015 | 0.075 | 0.179 | 0.202 | 0.175 | 0.032 |
| | 16 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.005 | 0.035 | 0.130 | 0.190 | 0.218 | 0.090 |
| | 17 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.012 | 0.072 | 0.134 | 0.205 | 0.190 |
| | 18 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.003 | 0.028 | 0.067 | 0.137 | 0.285 |
| | 19 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.007 | 0.021 | 0.058 | 0.270 |
| | 20 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.003 | 0.012 | 0.122 |