UNIVERSITY OF KARACHI

MATHEMATICS-II
(Differential Calculus)

BSCS-304

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EXERCISE 2.2

Q1 $\frac{dy}{dx} = \sin 5x.$

SOLUTION:-

$dy = \sin 5x \, dx.$

$\int dy = \int \sin 5x \, dx.$

$y = -\frac{1}{5} \cos 5x + C$ Ans!

Q3 $dx + e^{3x} dy = 0.$

SOLUTION

$\frac{dx}{-e^{3x}} = dy.$

$-e^{-3x} dx = dy$

$-\int e^{-3x} dx = \int dy.$

$+\frac{1}{3} e^{-3x} = y + C$ Ans!

Q5 $x \frac{dy}{dx} = 4y.$

SOLUTION:-

$\frac{dy}{4y} = \frac{dx}{x}.$

$\frac{1}{4} \int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln|y| = 4\ln|x|$ Ans!

Q7 $\frac{dy}{dx} = e^{3x+2y}$

SOLUTION

$\frac{dy}{dx} = e^{3x} \cdot e^{2y}$

$\frac{dy}{e^{2y}} = e^{3x}\,dx$

$\int e^{-2y}\,dy = \int e^{3x}\,dx$

$-\frac{1}{2}e^{2y} = \frac{1}{3}e^{3x} + C$ Ans!

Q9 $y\ln x \frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2$

SOLUTION:-

$\frac{y}{(y+1)^2} \cdot \frac{1}{dy} \ln x\,dx = \frac{1}{x^2}$

$x^2 \ln x\,dx = \frac{(y+1)^2}{y}\,dy$

$\int x^2 \ln x\,dx = \int\left(y + 2 + \frac{1}{y}\right)dy$

$\int x^2 \ln x\,dx = \frac{y^2}{2} + 2y + \ln y$

Solving L.H.S

$\int x^2 \ln x \, dx$.

$u = \ln x$. $\quad \int dv = \int x^2 \, dv$.

$x \, du = dx$ $\quad V = \frac{x^3}{3}$

$$\int u \, dv = uv - \int v \, du.$$

$$= \ln x \frac{x^3}{3} - \int \frac{x^{\cancel{3}\,2}}{3} \frac{dx}{\cancel{x}}.$$

$$= \frac{x^3 \ln x}{3} - \frac{1}{3} \frac{x^3}{3}.$$

$$= \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + C.$$

ⓐ $\frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) = \frac{y^2}{2} + 2y + \ln y + C$

A

Q11 $\csc y \, dx + \sec^2 x \, dy = 0$

SOLUTION:-

$$\frac{dx}{\sec^2 x} = -\frac{dy}{\csc y}.$$

$$\cos^2 x \, dx = -\sin y \, dy.$$

$$\int \cos^2 x \, dx = -\int \sin y \, dy.$$

$$\cancel{\sin} \int \left(\frac{1 + \cos 2x}{2} \right) dx = -\int \sin y \, dy.$$

$$\int \frac{1}{2} dx + \int \frac{\cos 2x}{2} dx = -\int \sin y \, dy.$$

$$\frac{1}{2} x + \frac{1}{4} \sin 2x = \cos y + C$$

Ans

Q13 $(e^y + 1)^2 e^{-y} dx + (e^x + 1)^3 e^{-x} dy = 0$

SOLUTION:-

$$\int \frac{dx}{(e^x + 1)^3 e^{-x}} \cancel{=} \int \frac{dy}{(e^y + 1)^2 e^{-y}}.$$

$$-\frac{1}{2} (e^x + 1)^{-2} \cancel{e^x} = + (e^y + 1)^{-1} + C$$

Q15 $\frac{ds}{dr} = KS$

SOLUTION:-

$$\frac{ds}{s} = K\,r$$

$$\int \frac{ds}{s} = K \int dr$$

$$\ln|s| = Kr + C$$

$$s = e^{Kr} + C \quad \underline{Ans}$$

$$\frac{1}{P - P^2}$$

$$\frac{1}{P} + \frac{P}{P - P^2}$$

$$\frac{1}{P} + \frac{1}{P - P}$$

Q17 $\frac{dP}{dt} = P - P^2$

SOLUTION:-

$$\frac{dP}{dt} = P - P^2$$

$$\cancel{\int \frac{dP}{P} - \int \frac{dP}{P^2} = \int dt}$$

$$\cancel{\ln|P| + P^{-1} = t}$$

$$\cancel{P + e^{1/P} = e^t}$$

$$\int \frac{dP}{P - P^2} = \int dt$$

$$\int \left(\frac{1}{P} + \frac{1}{1-P} \right) dP = \int dt$$

$$\ln|P| + \left[-\ln(1-P) \right] = t + C$$

$$\ln\left(\frac{P}{1-P}\right) = t + C$$

$$\frac{P}{1-P} = ce^{t}$$

$$P = ce^{t} - Pce^{t}$$

$$P(1 + ce^{t}) = ce^{t}$$

$$P = \frac{ce^{t}}{1 + ce^{t}}$$ Ans!

Q19. $$\frac{dy}{dx} = \frac{xy + 3x - y - 3}{xy - 2x + 4y - 8}$$

SOLUTION:-

$$\frac{dy}{dx} = \frac{x(y+3) - (y+3)}{x(y-2) + 4(y-2)}$$

$$\frac{dy}{dx} = \frac{(y+3)(x-1)}{(y-2)(x+4)}$$

$y+3 \,\overline{)\, y - 2}$; $y + 3$; -5

$$\frac{y-2}{y+3}\,dy = \frac{x-1}{x+4}\,dx$$

$$\int\left(1 - \frac{5}{y+3}\right)dy = \int\left(1 - \frac{5}{x+4}\right)dx$$

$x+4 \,\overline{)\, x - 1}$; $x + 4$; -5

$$y - 5\ln|y+3| = x - 5\ln|x+4| + C$$

$$y - x = 5\left[\ln|y+3| - \ln|x+4|\right] + C$$

$$y - x = 5\ln\left(\frac{y+3}{y+4}\right) + C$$

$$y - x = e^{5}\left(\frac{y+3}{y+4}\right) + C$$ Ans!

Q21. $\frac{dy}{dx} = x\sqrt{1-y^2}$

SOLUTION:-

$$\int \frac{dy}{\sqrt{1-y^2}} = \int x\,dx.$$

$$\sin^{-1} y = \frac{x^2}{2} + C.$$

$$y = \sin\left(\frac{x^2}{2} + C\right) \quad Ans$$

Q23 $\frac{dx}{dt} = 4(x^2+1)$, $x(\pi/4) = 1$

SOLUTION:-

$$\int \frac{dx}{x^2+1} = 4\int dt$$

$$\tan^{-1} x = 4t + C. \quad \text{--- (a)}$$

$$\tan^{-1}(1) = \cancel{4}\left(\frac{\pi}{\cancel{4}}\right) + C.$$

$$\frac{\pi}{4} - \pi = C.$$

$$-\frac{3\pi}{4} = C.$$

(a) $\rightarrow$ $\tan^{-1} x = 4t - 3\pi/4$

$$x = \tan\left(4t - \frac{3\pi}{4}\right) \quad A.$$

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Q25 $x^2 \frac{dy}{dx} = y - xy \quad , \quad y(-1) = -1$

SOLUTION :-

$$\frac{x^2}{dx} = \frac{y(1-x)}{dy}$$

$$\frac{x^2}{(1-x)\,dx} = \frac{y}{dy}$$

$$\frac{(1-x)}{x^2}\,dx = \frac{dy}{y}$$

$$\frac{1}{x^2}\,dx - \frac{1}{x}\,dx = \frac{dy}{y}$$

$$\int \frac{1}{x^2}\,dx - \int \frac{1}{x}\,dx = \int \frac{dy}{y}$$

$$\frac{1}{x} - \ln|x| = \cancel{\frac{y^2}{2}}\ \ln|y| + C$$

$$\frac{1}{x} = \ln|y| + \ln|x| + C$$

$$\frac{1}{x} = \ln xy + C \quad \text{———} \; (a)$$

$$ce^{1/x} = \cancel{\ln}\ xy \quad \text{———} \; (b)$$

$$c(e^{-1}) = (-1)(-1) \qquad \because y(-1) = -1$$

$$c = e^{-1}$$

$$(a) \Rightarrow \frac{1}{x} = \ln xy + e^{-1}$$

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(b) => $\frac{e^{-1}\, e^{-1/x}}{}$ (÷) xy :

$$\frac{e^{-(1+1/x)}}{x} = y \quad \text{Ans!}$$

Q27 $\sqrt{1-y^2}\,dx - \sqrt{1-x^2}\,dy = 0 \; ; \; y(0) = \frac{\sqrt{3}}{2}$.

SOLUTION :-

$$\frac{dx}{\sqrt{1-x^2}} = \frac{dy}{\sqrt{1-y^2}}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \int \frac{dy}{\sqrt{1-y^2}}$$

$$\sin^{-1} x = \sin^{-1} y + C$$

$$\sin^{-1} x - \sin^{-1} y = C \quad \text{———— (a)}$$

$$\sin^{-1}(0) - \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = C \quad \text{———— (b)}$$

$$0 - \frac{\pi}{3} = C$$

$$C = -\pi/3$$

(a) => $\sin^{-1} x - \sin^{-1} y = \frac{-\pi}{3}$

$$\sin^{-1} x + \frac{\pi}{3} = \sin^{-1} y$$

$$\sin\left(\sin^{-1} x + \frac{\pi}{3}\right) = y \quad \text{A.}$$

EXERCISE 2.3

Q1 $\frac{dy}{dx} = 5y$.

SOLUTION:-

$$\frac{dy}{dx} - 5y = 0 \quad \text{------ (i)}$$

compare $\frac{dy}{dx} + P(x)y = Q(x)$.

$$P(x) = -5$$
$$\int P(x) = -5\int dx.$$
$$\int P(x) = -5x$$

we know that

$$I.F = e^{\int P(x)dx}.$$
$$= e^{-5x}$$

$$\left(\frac{dy}{dx} - 5y\right)e^{-5x} = 0 \cdot e^{-5x}$$

$$\frac{d}{dx}ye^{-5x} = 0.$$

$$\int \frac{d}{dx} ye^{-5x} = 0$$

$$ye^{-5x} = c.$$
$$y = ce^{5x}$$

Q3 $\frac{dy}{dx} + y = e^{3x}$

comp with: $\frac{dy}{dx} + P(x)y = Q(x)$

$$P(x) = 1.$$

$$\int P(x)dx = x.$$

$$I.F = e^{x}.$$

Question will become.

$$\left(\frac{dy}{dx} + y\right)e^{x} = e^{3x} \cdot e^{x}$$

$$\frac{d}{dx} ye^{x} = e^{4x}$$

$$ye^{x} = \frac{1}{4} e^{4x} + c.$$

$$y = \frac{1}{4} e^{3x} + ce^{-x}$$

Q5 $y' + 3x^2y = x^2$

$P(x) = 3x^2$

$\int P(x) = x^3$

I.F $= e^{x^3}$

Question will become.

$$\left(\frac{dy}{dx} + 3x^2y\right)e^{x^3} = x^2e^{x^3}$$

$$\int \frac{d}{dx}\, ye^{x^3} = \int x^2e^{x^3} = I$$

—— (i)

$I = \int x^2e^{x^3}$

$u = x^2$

$\frac{du}{dx} = 2x$

$\int v\,du = e^{x^3}$

$= \frac{e^{x^3}}{3x^2}$

$= uv - \int v\,du$

$= x^2\frac{e^{x^3}}{3x^2} - \int \frac{e^{x^3}}{3x^2}\, 2x\, dx$

$= e^{x^3} - \frac{2}{3}\int \frac{e^{x^3}}{x}\, dx$

Q5 $y' + 3x^2y = x^2$

$$\frac{dy}{dx} + P(x)y = x^2$$

$P(x) = 3x^2$

$\int P(x) = x^3$

I.F $= e^{\int P(x)} = e^{x^3}$

Question will become:

$$\left(\frac{dy}{dx} + 3x^2y\right)e^{x^3} = x^2e^{x^3}$$

$$\int \frac{d}{dx} e^{x^3}y = \int x^2 e^{x^3} dx$$

$$e^{x^3}y = \int x^2 e^{x^3} dx \quad \text{------} \; (i)$$

let $u = x^3$

$du = 3x^2 dx$

(i) $\Rightarrow$ $$e^{x^3}y = \frac{1}{3}\int e^{u} du$$

$$e^{x^3}y = \frac{1}{3}e^{x^3} + C$$

$$y = \frac{1}{3}(0) + Ce^{x^{-3}}$$ Ans!

Q. $x^2y' + xy = 1$

$$y' + \frac{y}{x} = \frac{1}{x^2} \quad \text{—— (i)}$$

$$\frac{dy}{dx} + P(x)y = Q(x).$$

$P(x) = 1/x$.

$\int P(x) = \ln|x|$

$$I.F = e^{\ln x}$$
$$= x$$

Question will become.

$$\text{(i)} \Rightarrow \cancel{x^2y'}\left(y' + \frac{y}{x}\right)x = \frac{1}{x^2} \cdot x.$$

$$\int \frac{d}{dx} xy = \int \frac{1}{x} \, dx.$$

$$xy = \ln|x| + c.$$

$$y = \frac{1}{x}\ln|x| + \frac{c}{x}. \quad \text{A}$$

Q9 $x\frac{dy}{dx} - y = x^2 \sin x$

$$\frac{dy}{dx} - \frac{y}{x} = x\sin x \longleftarrow (i)$$

$$P(x) = -\frac{1}{x}$$

$$\int P(x) = -\ln|x|$$

$$I.F = e^{-\ln(x)}$$

$$= \frac{1}{x}$$

$$(i) \Rightarrow \left(\frac{dy}{dx} - \frac{y}{x}\right)\frac{1}{x} = \cancel{x^2} \sin x \frac{1}{\cancel{x}}$$

$$\int \frac{dy}{dx} \frac{1}{x} y = \int \sin x \, dx$$

$$y \cdot \frac{1}{x} = -\cos x + C$$

$$y = -x\cos x + C$$

$$y = C - x\cos x \quad \text{Ans!}$$

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Q4 $x\dfrac{dy}{dx} + 4y = x^3 - x$.

$$\frac{dy}{dx} + \frac{4}{x}y = x^2 - 1 \quad \text{—— (i)}$$

$P(x) = 4/x$.

$\int P(x) = 4\ln x. = \ln x^4$

$$I.F = e^{\ln x^4}$$
$$= x^4$$

$$(i) \Rightarrow \left(\frac{dy}{dx} + \frac{4}{x}y\right)x^4 = (x^2-1)\,x^4$$

$$\int \frac{d}{dx}x^4 y = \int (x^2-1)\,x^4\,dx.$$

$$x^4 y = \int x^6\,dx - \int x^4\,dx$$

$$x^4 y = \frac{x^7}{7} - \frac{x^5}{5} + c$$

$$y = \frac{x^3}{7} - \frac{x}{5} + cx^{-4}$$

Ans.

Q13: $x^2y' + x(x+2)y = e^x$

$$y' + \frac{(x+2)}{x}y = \frac{e^x}{x^2} \quad \text{———} \quad (i)$$

$$P(x) = \frac{(x+2)}{x} = 1 + \frac{2}{x}$$

$$\int P(x) = x + 2\ln|x|$$

$$I.F = e^{x + 2\ln x} = e^{x + \ln x^2}$$
$$= e^x x^2$$

$$(i) \Rightarrow \left(y' + \frac{(x+2)}{x} y \right) e^x x^2 = \frac{e^x}{\cancel{x^2}} \cdot e^x \cancel{x^2}$$

$$\int \frac{d}{dx} e^x x^2 y = \int e^{2x} \, dx$$

$$e^x x^2 y = e^{2x} \cdot \frac{1}{2} + C$$

$$y = \frac{1 e^x}{2x^2} + \frac{C e^{-x}}{x^2}$$ Ans!

Q15 $y\,dx - 4(x + y^6)\,dy = 0$

$$\frac{dx}{dy} = \frac{4(x+y^6)}{y}.$$

$$= \frac{4x}{y} + 4y^5$$

$$\frac{dx}{dy} - \frac{4}{y}x = 4y^5 \quad \text{—— (i)}$$

$$P(y) = -\frac{4}{y}.$$

$$\int P(y) = -4\ln y. = \ln y^{-4}$$

$$I.f = e^{\ln y^{-4}} = y^{-4}$$

$$= \frac{1}{y^4}$$

(i) $\Rightarrow$ $$\left(\frac{dx}{dy} - \frac{4x}{y}\right)\frac{1}{y^4} = 4y^5 \cdot \frac{1}{y^4}$$

$$\int \frac{d}{dy}\, y^{-4}x = \int 4y\, dy.$$

$$\frac{x}{y^{-4}} = 2y^2 + c.$$

$$x = 2y^6 + cy^4 \quad \text{Ans!}$$

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EXERCISE 2.4

Q1 $(2x-1)\,dx + (3y+7)\,dy = 0.$

SOLUTION:-

let $M = 2x-1$, $N = 3y+7$.

$\frac{\partial M}{\partial y} = 0$ $\frac{\partial N}{\partial x} = 0$

The Given eqn is exact D.E.

$\frac{\partial f}{\partial x} = M(x,y).$

$\frac{\partial f}{\partial x} = 2x-1.$

$\partial f = \cancel{\partial 2x}\ (2x-1)\,\partial x.$

$\int \partial f = \int (2x-1)\,\partial x$

$f = x^2 - x + g(y)$ ——— (i)

$\frac{\partial f}{\partial y} = N(x,y)$

$\frac{\partial (x^2 - x + g(y))}{\partial y} = 3y+7$

$g'(y) = 3y+7$ ——— (ii)

integrating on B.S.

$y = \frac{3}{2}y^2 + 7y$

(i) => $x^2 - x + \frac{3}{2}y^2 + 7y = c$.

~~Q3 $(2x-1)dx + (3y+7)dy = 0$.~~

Q3 $(5x+4y)dx + (4x-8y^3)dy = 0$

SOLUTION:-

let $M = 5x + 4y$, $N = 4x - 8y^3$.

$\frac{\partial M}{\partial y} = 4$, $\frac{\partial N}{\partial x} = 4$

Since eq^n is exact.

$\frac{\partial f}{\partial x} = M(x,y)$.

$\frac{\partial f}{\partial x} = 5x + 4y$.

$\int \partial f = \int (5x + 4y)\, \partial x$.

$f = \frac{5}{2}x^2 + 4xy + h(y)$. ——— (i)

$\frac{\partial f}{\partial y} = N(x,y)$.

$\partial\left(\frac{5}{2}x^2 + 4xy + h(y)\right) = (4x - 8y^3)\, \partial y$

$4x + h'y = 4x - 8y^3$

$h'(y) = \cancel{4x} - \cancel{4x} - 8y^3 = 0x - 8y^3$

Integrating Both sides.

$y = -2y^4$

(i) $\Rightarrow \frac{5}{2}x^2 + 4x + (-2y^4) = c$ Ans!

Q5 $(2xy^2 - 3)\,dx + (2x^2y + 4)\,dy = 0$

SOLUTION :-

Let $M = (2xy^2 - 3)$, $N = (2x^2y + 4)$

$\frac{\partial M}{\partial y} = 4xy$, $\frac{\partial N}{\partial x} = 4xy$.

Eqn is Exact.

$\frac{\partial f}{\partial x} = 2xy^2 - 3$.

$f = \int (2xy^2 - 3)\,\partial x$

$f = x^2y^2 - 3x + h(y)$ ——— (i)

$\frac{\partial f}{\partial y} = 2x^2y + 4$

$\frac{\partial}{\partial y}(x^2y^2 - 3x + h(y)) = 2x^2y + 4$.

$$2x^2y + h'(y) = 2x^2y + 4$$

$$\cancel{2x^2y} + h'(y) = \cancel{2x^2y} + 4 \Rightarrow h'(y) = 4$$

integrating B.S.

$$\cancel{y = \frac{x^3y}{3} + 4x}$$

$$\cancel{(i) \Rightarrow \quad xy^{22} - 3x + \frac{x^3y + 4x}{3}}$$

$$\cancel{xy^{22} + \frac{x^3y}{3} + x = c} \quad \text{~~Ans!~~}$$

$$y = 4y$$

$$(i) \Rightarrow \quad xy^{22} - 3x + 4y = c \quad \text{Ans!}$$

Q7 $(x^2 - y^2)dx + (x^2 - 2xy)dy = 0$

SOLUTION:-

let $M = x^2 - y^2$.

$$\frac{\partial M}{\partial y} = -2y \quad , \quad N = x^2 - 2xy.$$

$$\frac{\partial N}{\partial x} = 2x$$

The eq^n is non exact.

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Q9 $(x - y^3 + y^2 \sin x)\,dx = (3xy^2 + 2y\cos x)\,dy.$

SOLUTION :-

$(x - y^3 + y^2 \sin x)dx - (3xy^2 + 2y\cos x)dy = 0$

$-\{(-x + y^3 - y^2 \sin x)dx + (3xy^2 + 2y\cos x)dy\} = 0.$

$(-x + y^3 - y^2 \sin x)dx + (3xy^2 + 2y\cos x)dy = 0$

Let $M = (-x + y^3 - y^2 \sin x).$

$\frac{\partial M}{\partial y} = 3y^2 - 2y\sin x.$

$N = 3xy^2 + 2y\cos x.$

$\frac{\partial N}{\partial x} = 3y^2 - 2y\sin x.$

- Given eqn is exact D.E.

$\frac{\partial f}{\partial x} = -x + y^3 - y^2 \sin x$

$\partial f = (-x + y^3 - y^2 \sin x)\,\partial x$

$f = -\frac{x^2}{2} + y^3 x + y^2 \cos x + h(x)$ ——— (i)

$$\frac{\partial f}{\partial y} = 3xy^2 + 2y\cos x$$

$$\frac{\partial}{\partial y}\left(-x^2/2 + y^3x + y^2\cos x + h(x)\right) = 3xy^2 + 2y\cos x$$

$$\cancel{3xy^2} + \cancel{2y\cos x} + h'(x) = \cancel{3xy^2} + \cancel{2y\cos x}$$

$$h'(x) = \cancel{3xy^2 - y^3}\ 0$$

~~integrating B.S.~~

$$\cancel{x = xy^3 - \frac{y^4}{4}}$$

(i) $\Rightarrow$ $$-\frac{x^2}{2} + y^3x + y^2\cos x + \underbrace{xy^3 - \frac{y^4}{4}}_{0} = C$$

$$-\frac{x^2}{2} + y^3x + y^2\cos x = C$$

~~Ans!~~

Q11 $(y\ln y - e^{-xy})dx + \left(\frac{1}{y} + x\ln y\right)dy = 0$

SOLUTION:-

let $M = y\ln y - e^{-xy}$.

$$\frac{\partial M}{\partial y} = 1 + \ln y - e^{-xy}\left(x + y\frac{\partial x}{\partial y}\right)$$

$$N = \frac{1}{y} + x\ln y.$$

$$\frac{\partial N}{\partial x} = \frac{x}{y} + \ln y$$

Given eqn is not an exact eqn.

~~Q13 $(x + ye^{y/x})dx - xe^{y/x}dy = 0$~~

Q13 $x\frac{dy}{dx} = 2xe^{x} - y + 6x^{2}$

SOLUTION:-

$(2xe^{x} - y + 6x^{2})\,dx - x\,dy = 0$

$(y - 2xe^{x} - 6x^{2})\,dx + x\,dy = 0$

Let $\frac{\partial M}{\partial y} = y - 2xe^{x} - 6x^{2}$, $\frac{\partial N}{\partial x} = x$

$\frac{\partial M}{\partial y} = 1$ $\frac{\partial N}{\partial x} = 1$

Given eq^{n} is exact D.E.

$\frac{\partial f}{\partial x} = y - 2xe^{x} - 6x^{2}$

$\int \partial f = \int (y - 2xe^{x} - 6x^{2})\,\partial x$

$f = xy - 2xe^{x} + 2e^{x} - 2x^{3} + h(y)$ ____ (i)

$\frac{\partial f}{\partial y} = x$

$\frac{\partial}{\partial y}(xy - 2xe^{x} + 2e^{x} - 2x^{3} + h(y)) = x$

$\cancel{x} + h'(y) = \cancel{x}$

$h'(y) = 0$

$y = 0$

(i) $\Rightarrow$ $xy - 2xe^{x} + 2e^{x} - 2x^{3}$ Ans!

Q15 $\left(x^2y^3 - \dfrac{1}{1+9x^2}\right)\dfrac{dx}{dy} + x^3y^2 = 0$

SOLUTION:-

$$\left(x^2y^3 - \frac{1}{1+9x^2}\right)dx + x^3y^2\,dy = 0$$

$$\frac{\partial M}{\partial y} = x^2y^3 - \frac{1}{1+9x^2}$$

$$\frac{\partial M}{\partial y} = 3x^2y^2$$

$$\frac{\partial N}{\partial x} = x^3y^2$$

$$\frac{\partial N}{\partial x} = 2x^2y^2$$

Given eq^n is Exact D.E.

$$\frac{\partial f}{\partial x} = x^2y^3 - \frac{1}{1+9x^2}$$

$$\int \partial f = \int\left(x^2y^3 - \frac{1}{1+9x^2}\right)\partial x$$

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$$f = \frac{1}{3}x^3y^3 - \tan^{-1}(3x) + h(y) \quad \text{---(i)}$$

$$\frac{\partial f}{\partial y} = x^3y^2$$

$$\frac{\partial}{\partial y}\left(\frac{1}{3}x^3y^3 - \tan^{-1}(3x) + h(y)\right) = x^3y^2$$

$$\cancel{\frac{3}{3}x^3y^2} + h'(y) = \cancel{x^3y^2}$$

$$h'(y) = 0$$

$$y = 0$$

$$\text{(i)} \Rightarrow \frac{1}{3}x^3y^3 - \tan^{-1}(3x) = c \quad \text{Ans!}$$

Q17 $(\tan x - \sin x \sin y)\,dx + \cos x \cos y\,dy = 0$

SOLUTION:-

$$\frac{\partial M}{\partial y} = \tan x - \sin x \sin y, \quad \frac{\partial N}{\partial x} = \cos x \cos y.$$

$$\frac{\partial M}{\partial y} = -\sin x \cos y \qquad \frac{\partial N}{\partial x} = -\cos y \sin x.$$

Given eq^n is Exact D.E.

$$\frac{\partial f}{\partial x} = \tan x - \sin x \sin y$$

$$\int \partial f = \int (\tan x - \sin x \sin y)\, \partial x$$

$$f = \ln|\sec x| + \sin y \cos x + h(y) \quad (i)$$

$$\frac{\partial f}{\partial y} = \cos x \cos y$$

$$\frac{\partial}{\partial y}\left[\ln|\sec x| + \sin y \cos x\right] + h(y) = \cos x \cos y$$

$$\cos x \cos y + h(y) = \cos x \cos y.$$

$$y = 0$$

$(i) \Rightarrow \quad \ln|\sec x| + \sin y \cos x = c$ Ans!

Q19 $(4t^3y - 15t^2 - y)\,dt + (t^4 + 3y^2 - t)\,dy = 0$

SOLUTION:-

$\frac{\partial M}{\partial y} = 4t^3y - 15t^2 - y$, $\frac{\partial N}{\partial t} = t^4 + 3y^2 - t$

$\frac{\partial M}{\partial y} = 4t^3 - 1$ $\qquad \frac{\partial N}{\partial t} = 4t^3 - 1$

Given eq^n is Exact D.E.

$$\frac{\partial f}{\partial t} = 4t^3y - 15t^2 - y$$

$$\int \partial f = \int (4t^3y - 15t^2 - y)\, \partial t$$

$$f = \frac{4t^4 y}{4} - \frac{15t^3}{3} - yt$$

$$f = t^4y - 5t^3 - yt + \underline{h(y)} \quad (i)$$

$$\frac{\partial f}{\partial y} = t^4 + 3y^2 - t$$

$$\frac{\partial (t^4y - 5t^3 - yt + h(y))}{\partial y} = t^4 + 3y^2 - t$$

$$\cancel{t^4} - \cancel{t} + h'(y) = \cancel{t^4} + 3y^2 - \cancel{t}$$

$$h'(y) = 3y^2$$

$$y = y^3$$

$$(i) \Rightarrow \quad t^4y - 5t^3 - yt + y^3 = c \quad \text{Ans!}$$

Q21 $(x+y)^2 dx + (2xy + x^2 - 1) dy = 0$

$y(1) = 1$

SOLUTION:-

$\frac{\partial M}{\partial y} = (x+y)^2$, $\frac{\partial N}{\partial x} = 2xy + x^2 - 1$.

$\frac{\partial M}{\partial y} = 2(x+y)$ $\frac{\partial N}{\partial x} = 2y + 2x$

Given eq^n is Exact D.E

$\frac{\partial f}{\partial x} = (x+y)^2$

$\int \partial f = \int (x^2 + 2xy + y^2)\, \partial x$

$f = \frac{x^3}{3} + \frac{\cancel{2}yx^2}{\cancel{2}} + xy^2$

$f = \frac{x^3}{3} + x^2y + xy^2 + \underline{h(y)}$. (i)

$\frac{\partial f}{\partial y} = 2xy + x^2 - 1$

$\frac{\partial}{\partial y}\left(x^3/3 + x^2y + xy^2 + h(y)\right) = 2xy + x^2 - 1$

$\cancel{x^2} + \cancel{2xy} + h'(y) = \cancel{2xy} + \cancel{x^2} - 1$

$h(y) = -y$.

(i) => $\frac{x^3}{3} + x^2y + xy^2 - y = c$ ___ (ii)

Given that $y(1) = 1$

$\frac{1}{3} + 1 + \cancel{1} - \cancel{1} = c$

$\frac{4}{3} = c$

(ii) => $\frac{x^3}{3} + x^2y + xy^2 - y = \frac{4}{3}$ Ans!

Q31. $(2y^2 + 3x)dx + 2xy\,dy = 0$

SOLUTION:-

$M = 2y^2 + 3x$ $\qquad N = 2xy$

$\frac{\partial M}{\partial y} = 4y$ $\qquad \frac{\partial N}{\partial x} = 2y$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

Eq^n is non exact

$\frac{M_y - N_x}{N} = \frac{4y - 2y}{2xy} = \frac{1}{x}$

integrating factor $= e^{\int 1/x\,dx} = x$

$M = 2xy^2 + 3x^2$ $\qquad$ $N = 2x^2y$.

$\frac{\partial M}{\partial y} = 4xy$ $\qquad$ $\frac{\partial N}{\partial x} = 4xy$.

eq^n is exact.

$\frac{\partial f}{\partial x} = 2xy^2 + 3x^2$.

$\int \partial f = x^2y^2 + x^3$

$f = x^2y^2 + x^3 + h(y)$ —— (i)

$\frac{\partial f}{\partial y} = 2x^2y$.

$\frac{\partial(x^2y^2 + x^3 + h(y))}{\partial y} = 2x^2y$.

$\cancel{2x^2y} + h'(y) = \cancel{2x^2y}$.

$h'(y) = 0$.

$y = 0$.

(i) $\Rightarrow$ $x^2y^2 + x^3 = c$ Ans.

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Q33 $6xy\,dx + (4y + 9x^2)\,dy = 0$

SOLUTION :-

$M = 6xy$ $\qquad$ $N = (4y + 9x^2)$

$\frac{\partial M}{\partial y} = 6x$ $\qquad$ $\frac{\partial N}{\partial x} = 18x$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{M_y - N_x}{N} = \frac{6x - 18x}{4y + 9x^2} = \frac{-12x}{4y + 9x^2}.$$

$$\frac{N_x - M_y}{M} = \frac{18 - 6x}{6xy} = \frac{\cancel{12}^{2}\cancel{x}}{\cancel{6x}y} = \frac{2}{y}$$

Integrating factor will be $= e^{\int \frac{2}{y} dy}$.

$= e^{2\ln|y|}$

$= e^{\ln y^2}$

$= y^2$

$M = 6xy^3$ $\qquad$ $N = 4y^3 + 9x^2y^2$

$\frac{\partial M}{\partial y} = 18xy^2$ $\qquad$ $\frac{\partial N}{\partial x} = 18xy^2$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

Eqn is become exact.

$$\frac{\partial f}{\partial x} = 6xy^3$$

$$\int \partial f = \int 6xy^3 \, \partial x$$

$$f = 3x^2y^3 - h(y) \quad \text{———} \quad (i)$$

$$\frac{\partial f}{\partial y} = 4y^3 + 9x^2y^2$$

$$\frac{\partial}{\partial y}\left(3x^2y^3 - h(y)\right) = 4y^3 + 9x^2y^2$$

$$\cancel{9x^2y^2} + h'(y) = 4y^3 + \cancel{9x^2y^2}$$

$$h'(y) = +4y^3$$

$$y = y^4$$

$$(i) \Rightarrow f = 3x^2y^3 - y^4$$ Ans!

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Q35 $(10 - 6y + e^{-3x})dx - 2dy = 0$

SOLUTION:-

Eq^n is non exact

$$\frac{M_y - N_x}{N} = \frac{-6-0}{-2} = 3$$

Integrating factor is $= e^{\int 3\,dx}$

e^{3x}

$M = 10e^{3x} - 6ye^{3x} + 1$, $N = -2e^{3x}$

$$\frac{\partial M}{\partial y} = \cancel{-18ye^{3x}} - 6e^{3x} \qquad \frac{\partial N}{\partial x} = -6e^{3x}$$

Eq^n is become exact

$$\frac{\partial f}{\partial x} = 10e^{3x} - 6ye^{3x} + 1$$

$$f = \frac{10e^{3x}}{3} - \frac{6ye^{3x}}{3}$$

$$f = \frac{10e^{3x}}{3} - \frac{2ye^{3x}}{3} + h(y) \text{ ——— (i)}$$

$$\frac{\partial f}{\partial y} = -2e^{3x}$$

$$\frac{1}{3}\cdot\frac{\partial}{\partial y}\left(10e^{3x} - 2ye^{3x} + h(y)\right) = -2e^{3x}$$

$$\frac{1}{3}(-2\cancel{e^{3x}}) + h'(y) = -2\cancel{e^{3x}}$$

$$h'(y) = 0$$

$$y = 0$$

$$C = \frac{10}{3} e^{3x} \cancel{+} - \frac{2}{3} y e^{3x} \quad \text{Ans.}$$

EXERCISE 2.5

Q. $(x-y)\,dx + x\,dy = 0$

SOLUTION:-

$(x-y)\,dx + x\,dy = 0$ ——— (A)

let $u = \dfrac{y}{x}$.

$ux = y$.

$\dfrac{dy}{dx} = u + x\dfrac{du}{dx}$.

$dy = u\,dx + x\,du$.

(A) $\Rightarrow$ $(x - ux)\,dx + x\,(u\,dx + x\,du) = 0$

$x\,dx - \cancel{ux\,dx} + \cancel{ux\,dx} + x^2\,du = 0$

$x\,dx + x^2\,du = 0$

$\displaystyle\int \frac{dx}{x} = -\int du$

$\ln|x| = -u$

$\ln|x| = -\dfrac{y}{x} + c$ Ans.

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Q2. $(x+y)dx + x\,dy = 0$ ——— (A)

SOLUTION:-

Since eq^n is homogenous.

let $u = \frac{y}{x}$,

$$y = ux.$$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$dy = u\,dx + x\,du.$$

(A) ⟹ $(x + ux)dx + x(u\,dx + x\,du) = 0$

$$x\,dx + ux\,dx + ux\,dx + x^2du = 0$$

$$x\,dx + 2ux\,dx + x^2du = 0.$$

$$x\,dx\,(1+2u) + x^2du = 0.$$

$$x^2du = -x\,dx\,(1+2u).$$

$$\frac{du}{1+2u} = -\frac{\cancel{x}dx}{x^{\cancel{2}}}.$$

$$\int \frac{du}{1+2u} = \int -\frac{dx}{x}$$

$$\frac{1}{2}\ln|1+2u| = -\ln|x| + c$$

$$\ln|x| + \frac{1}{2}\ln|1+2u| = c$$

$$\frac{2\ln|x|}{2} + \ln|1+2u| = c$$

$$\ln|x| + \ln|x| + \ln|1+2u| = c_1$$

$$\ln(x^2) + \ln|1+2u| = c_1$$

$$\ln\left[x^2\left(1+2u\right)\right] = c_1$$

$$x^2\left(1+\frac{2y}{x}\right) = c_1$$

$$x^2 + 2xy = c_1 \quad \underline{\text{Ans.}}$$

Q3 $x\,dx + (y-2x)\,dy = 0$

SOLUTION:-

$$x\,dx + (y-2x)\,dy = 0 \quad \text{——— (A)}$$

let $u = \frac{y}{x} \Rightarrow y = ux$

$$dy = u\,dx + x\,du.$$

(A) $\Rightarrow$ $x\,dx + (ux-2x)(u\,dx + x\,du) = 0$

$$\cancel{x\,dx + u^2x\,dx + ux^2\,du - 2xu\,dx - 2x^2\,du = 0}$$

$$x\,dx + x(u-2)(u\,dx + x\,du) = 0$$

$$dx + (u-2)(u\,dx + x\,du) = 0$$

$$dx + u^2\,dx + xu\,du - 2u\,dx - 2x\,du = 0$$

$$dx + u^2\,dx - 2u\,dx = 2x\,du - xu\,du.$$

$$(1 + u^2 - 2u)\, dx = x(2du - u\, du)$$

$$\frac{dx}{x} = \frac{(2-u)\, du}{(1 + u^2 - 2u)}$$

$$\int \frac{dx}{x} = \int \frac{(2-u)}{(1-u^2)^2}\, du$$

$$\ln x = \int \frac{1 + (1-u)}{(1-u)^2}\, du$$

$$\ln x = \int \frac{du}{(1-u)^2} + \int \frac{du}{1-u}$$

$$\ln x = \frac{-1}{1-u} + - \ln|1-u|$$

$$\ln x + \ln\left(1 - \frac{y}{x}\right) + \frac{1}{1-u}$$

$$\ln\left(\cancel{x} \times \frac{x-y}{\cancel{x}}\right) + \frac{1}{1 - y/x} = C$$

$$\ln|x-y| + \frac{x}{x-y} = C$$

$$(x-y)\ln|x-y| + x = (x-y)\, C.$$

Q_5. $(y^2 + yx)dx - x^2dy = 0$

SOLUTION:-

$$(y^2 + yx)dx - x^2dy = 0 \quad \text{——— (A)}$$

let $\quad u = \dfrac{y}{x}$

$$y = ux$$

$$dy = u\,dx + x\,du.$$

(A) $\Rightarrow$ $\quad (u^2x^2 + ux^2)dx - x^2(u\,dx + x\,du) = 0$

$$u^2x^2\,dx + ux^2\,dx - x^2u\,dx + x^3du = 0$$

$$u^2dx + \cancel{u\,dx} - \cancel{u\,dx} - x\,du = 0$$

$$u^2dx - x\,du = 0.$$

$$u^2dx = x\,du.$$

$$\int \frac{dx}{x} = \int \frac{du}{u^2}.$$

$$\ln|x| = -u^{-1} + c$$

$$\ln|x| = -\frac{1}{u} + c$$

$$\ln|x| + \frac{x}{y} = 0 + c.$$

$$y\ln|x| + x = cy \quad \text{A}$$

Q6. $(y^2 + yx)dx + x^2 dy = 0$

SOLUTION:-

$$(y^2 + yx)dx + x^2 dy = 0 \quad \text{——— (A)}$$

let $y = ux$, $u = \frac{y}{x}$.

$$dy = u dx + x du.$$

(A) $\Rightarrow$ $(u^2x^2 + ux^2)dx + x^2(u dx + x du) = 0$

$$x^2(u^2 + u)dx + x^2(u dx + x du) = 0$$

$$u^2 dx + u dx + u dx + x du = 0.$$

$$u^2 dx + 2u dx + x du = 0$$

$$(u^2 + 2u) dx = - x du.$$

$$\int \frac{dx}{x} = -\int \frac{du}{u^2 + 2u}$$

$$\ln|x| = - I_1 \quad \text{——— (a)}$$

$$I_1 = \int \frac{du}{u^2 + 2u}.$$

$$\frac{1}{u^2 + 2u} = \frac{A}{u} + \frac{B}{(u+2)}$$

$$1 = A(u+2) + Bu.$$

for B Put $u = -2$.

$$1 = 0 + (-2B).$$

$$-\frac{1}{2} = B$$

for A put $u = 1$ & $B = -\frac{1}{2}$.

$1 = 3A + (-1/2)$.

$1 + \frac{1}{2} = 3A$

$3/2 = 3A$

$\frac{1}{2} = A$

$I_1 = \frac{1}{2}\int \frac{du}{u} + \left(\frac{-1}{2}\right)\int \frac{du}{(u+2)}$

$= \frac{1}{2}\ln|u| - \frac{1}{2}\ln|u+2|$.

ⓐ $\Rightarrow$ $\ln|x| = -\left[\frac{1}{2}\ln|u| - \frac{1}{2}\ln|u+2|\right]$

$\ln|x| + \frac{1}{2}\ln|u| - \frac{1}{2}\ln|u+2| = c$.

$\ln\left(\frac{x^2 u}{u+2}\right) = c$

$\frac{x^2 u}{u+2} = c$

$x^2 \frac{y}{x} = c\left(\frac{y}{x} + 2\right)$

$xy = c\left(\frac{y}{x} + 2\right)$ Ans!

Q7. $\frac{dy}{dx} = \frac{y-x}{y+x}$

SOLUTION:-

$$\frac{dy}{dx} = \frac{y-x}{y+x} \quad \text{———} \quad (A)$$

let $u = \frac{y}{x}$ $\Rightarrow$ $y = ux$

$$dy = u\,dx + x\,du$$

(A) $\Rightarrow$ $$\frac{u\,dx + x\,du}{dx} = \frac{ux - x}{ux + x}.$$

$$u + \frac{x\,du}{dx} = \frac{u-1}{u+1}$$

$$\frac{x\,du}{dx} = \frac{u-1}{u+1} - u.$$

$$\frac{x\,du}{dx} = \frac{\cancel{u} - 1 - u^2 - \cancel{u}}{u+1}$$

$$\frac{x\,du}{dx} = \frac{-u^2 - 1}{u+1}$$

$$\int \frac{dx}{x} = -\int \frac{(u+1)}{(u^2+1)}\,du$$

$$\ln x = -\int \frac{u}{u^2+1}\,du - \int \frac{1}{u^2+1}\,du$$

$$\ln x = -\frac{1}{2}\ln|u^2+1| - \tan^{-1}u + c$$

$$\ln x + \frac{1}{2}\ln|u^2+1| + \tan^{-1}u = c$$

$$2\ln x + \ln|u^2+1| + 2\tan^{-1}u = c_1$$

$$\ln x^2 + \ln|u^2+1| + 2\tan^{-1}u = c_1$$

$$\ln\left(\cancel{x^2} \times \frac{y^2}{\cancel{x^2}} + 1\right) + 2\tan^{-1}u = c_1$$

$$\ln|y^2+x^2| + 2\tan^{-1}u = c_1$$

A.

Q4 $y\,dx = 2(x+y)\,dy$

SOLUTION:-

let $u = \frac{y}{x}$ $\Rightarrow$ $y = ux$

$$dy = u\,dx + x\,du.$$

$$\Rightarrow \quad ux\,dx = 2(x+ux)(u\,dx + x\,du).$$

$$ux\,dx = 2x(1+u)(u\,dx + x\,du).$$

$$u\,dx = 2(1+u)(u\,dx + x\,du)$$

$$u\,dx = 2u\,dx + 2x\,du + 2u^2\,dx + 2xu\,du$$

$$u\,dx + 2u^2\,dx + 2x\,du + 2xu\,du = 0$$

$$u\,dx + 2u^2\,dx + 2x(1+u)\,du = 0$$

$$(u+2u^2)\,dx + 2x(1+u)$$

Q9 $-y\,dx + (x + \sqrt{xy})\,dy = 0$

SOLUTION:-

$-y\,dx + (x + \sqrt{xy})\,dy = 0$ ——— (A)

let $u = \frac{y}{x} \Rightarrow y = ux$

$dy = u\,dx + x\,du$

(A)' $\Rightarrow$ $-ux\,dx + (x + \sqrt{xux})(u\,dx + x\,du) = 0$

$-ux\,dx + (x + x\sqrt{u})(u\,dx + x\,du) = 0$

$\cancel{-ux\,dx} + \cancel{xu\,dx} + x^2\,du + x\,u^{1/2}.u\,dx + x^2\sqrt{u}\,du = 0$

$x^2\,du + x\,u^{3/2}\,dx + x^2 u^{1/2}\,du = 0$

$(x^2 + x^2 u^{1/2})\,du + x\,u^{3/2}\,dx = 0$

$(1 + u^{1/2})\,du + \frac{u^{3/2}}{x}\,dx = 0$

$\int \left(u^{-3/2} + \frac{1}{u}\right) du + \int \frac{dx}{x} = 0$

$2u^{-1/2} + \ln|u| + \ln(x) = 0 + c$

$2\sqrt{x/y} + \ln|y/x| + \ln x = 0 + c$

$2\sqrt{x/y} + \ln|y| = c$ ✱

Q_{10} $x\frac{dy}{dx} = y + \sqrt{x^2-y^2}$, $x > 0$ ——— (A)

SOLUTION :-

let $u = \frac{y}{x} \Rightarrow y = ux$

$dy = u\,dx + x\,du$

(A) $\Rightarrow$ $\frac{x(u\,dx + x\,du)}{dx} = ux + \sqrt{x^2 - u^2x^2}$

$\cancel{xu} + \frac{x^2 du}{dx} = \cancel{ux} + x\sqrt{1-u^2}$

$\frac{x^{\cancel{2}} du}{dx} = \cancel{x}\sqrt{1-u^2}$

$\int \frac{du}{\sqrt{1-u^2}} = \int \frac{dx}{x}$

$\sin^{-1} u = \ln|x| + c$

$\sin^{-1} \frac{y}{x} = \ln|x| + c$

$\frac{y}{x} = \sin(\ln|x| + c)$

$y = x \sin(\ln|x| + c)$ A

Q8 $\frac{dy}{dx} = \frac{x+3y}{3x+y}$ ——— (A)

SOLUTION :-

let $u = \frac{y}{x}$ $\Rightarrow$ $y = ux$.

$dy = u\,dx + x\,du$.

(A) $\Rightarrow$ $\frac{u\,dx + x\,du}{dx} = \frac{x + 3ux}{3x + ux}$.

$u + \frac{x\,du}{dx} = \frac{\cancel{x}(1+3u)}{\cancel{x}(3+u)}$.

$\frac{x\,du}{dx} = \frac{1+3u}{3+u} - u$.

$\frac{x\,du}{dx} = \frac{1+\cancel{3u}-\cancel{3u}-u^2}{3+u}$.

$\frac{x\,du}{dx} = \frac{(1+u)(1-u)}{3+u}$.

$\int \frac{dx}{x} = \int \frac{(3+u)\,du}{(1+u)(1-u)}$.

$\ln|x| = I_1$ ——— (a)

$$I_1 = \int \frac{(3+u)\,du}{(1+u)(1-u)}$$

$$\frac{3+u}{(1+u)(1-u)} = \frac{A}{(1+u)} + \frac{B}{(1-u)}$$

$$3+u = A(1-u) + B(1+u) \quad \text{--- (i)}$$

for B put $u = 1$.

$$(i) \Rightarrow 3+1 = A(1-1) + B(1+1)$$

$$4 = 0 + 2B$$

$$\boxed{2 = B}$$

for A put $u = 0$ and $B = 2$.

$$(i) \Rightarrow 3+0 = A(1-0) + 2(1+0)$$

$$3 = A + 2$$

$$3-2 = A$$

$$\boxed{1 = A}$$

$$I_1 = \int \frac{1}{(1+u)}\,du + \int \frac{2}{1-u}\,du$$

$$I_1 = \ln|1+u| + 2\ln|1-u| + c$$

$$(a) \Rightarrow \ln|x| = \ln|1+u| + 2\ln|1-u| + c$$

$$\ln|x| = \ln\left[(1+u)(1-u)\right] + c$$

$$\ln|x| = \ln(1-u^2) + c$$

$$\ln\left(\frac{x}{1-u^2}\right) = c \Rightarrow x = c\left(1 - \frac{y^2}{x^2}\right)$$

Q11 $xy^2\frac{dy}{dx} = y^3 - x^3$; $y(1) = 2$.

SOLUTION:-

let $v = \frac{y}{x}$ => $y = vx$.

$dy = v\,dx + x\,dv$.

=> $x v^2 x^2 \frac{dy}{dx} = v^3 x^3 - x^3$.

$\frac{\cancel{x^3} v^2 (v\,dx + x\,dv)}{dx} = \cancel{x^3}(v^3 - 1)$.

$v^3 dx + v^2 x\,dv = (v^3 - 1)dx$

$\cancel{v^3 dx} + v^2 x\,dv = \cancel{v^3 dx} - dx$.

$v^2 x\,dv = -dx$.

$\int v^2 dv = \int -\frac{dx}{x}$.

$\frac{v^3}{3} = -\ln|x| + c$

$\frac{y^3}{x^3} = -3\ln|x| + c_1$.

$y^3 = -3x^3\ln|x| + c_1 x^3$ —— (B)

$(2)^3 = -3(1)^3 \ln|1| + c_1 x^3$; $y(1) = 2$.

$8 = 0 + c_1$

(B) => $y^3 = -3x^3\ln|x| + 8x^3$ A

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$Q_{12}.\ (x^2 + 2y^2)\dfrac{dx}{dy} = xy\ ,\ y(-1) = 1$

SOLUTION:-

let $\quad u = y/x \quad \Rightarrow y = ux.$

$$dy = u\,dx + x\,du.$$

$$\Rightarrow \frac{(x^2 + 2u^2x^2)\,dx}{(u\,dx + x\,du)} = x \cdot ux$$

$$x^2dx + 2u^2x^2dx = x^2u\,(u\,dx + x\,du).$$

$$x^2dx + 2u^2x^2dx = x^2u^2dx + x^3u\,du.$$

$$x^2dx + 2u^2x^2dx - x^2u^2dx = x^3u\,du.$$

$$x^2dx + u^2x^2dx = x^3u\,du.$$

$$dx + u^2dx = xu\,du$$

$$(1+u^2)\,dx = xu\,du.$$

$$\int\frac{dx}{x} = \int\frac{u\,du}{(1+u^2)}.$$

$$\ln|x| = \frac{1}{2}\ln|1+u^2| + c. \quad \text{(B)}$$

$$\ln|-1| = \frac{1}{2}\ln\left|1 + \frac{(1)^2}{(-1)^2}\right| + c \qquad \because y(-1) = 1$$

$$2\ln|-1| - \ln|1+1| = c_1$$

$$\ln(-1)^2 - \ln(2) = c_1.$$

$$\ln(2) = -c_1.$$

$$2 = e^{-c}.$$

$$\frac{1}{2} = c_1$$

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(B) => $\ln|x| = \frac{1}{2}\ln\left|1+\frac{y^2}{x^2}\right| + \frac{1}{2}$

$$2\ln|x| = \ln\left(\frac{x^2+y^2}{x^2}\right) + 1 \quad A$$

Q13. $\left(x + ye^{y/x}\right)dx - xe^{y/x}\,dy = 0$, $y(1)=0$

SOLUTION:-

Let $u = y/x \Rightarrow y = ux$.

$dy = u\,dx + x\,du$.

$\Rightarrow (x + uxe^u)\,dx - xe^u(u\,dx + x\,du) = 0$

$x\,dx + \cancel{uxe^u\,dx} - \cancel{xe^u u\,dx} - x^2e^u\,du = 0$.

$x\,dx - x^2e^u\,du = 0$.

$\cancel{x}\,dx = x^{\cancel{2}}e^u\,du$.

$\int \frac{dx}{x} = \int e^u\,du$.

$\ln|x| = e^u + c$.

$\ln|x| = e^{y/x} + c$. ——— (B).

$\ln|1| = e^{0/1} + c$. , $y(1)=0$.

$-1 = c$.

(B) => $\ln|x| = e^{y/x} - 1$ A

Q14. $y\,dx + x(\ln x - \ln y - 1)\,dy = 0, \; y(1) = e$

SOLUTION :-

let $v = y/x \Rightarrow y = vx$

$dy = v\,dx + x\,dv$

$\Rightarrow \quad vx\,dx + x(\ln x - \ln vx - 1)(v\,dx + x\,dv) = 0$

$v\,dx + (\ln x - \ln vx - 1)(v\,dx + x\,dv) = 0$

BERNOULLI EQUATION:-

Q15. $x\frac{dy}{dx} + y = \frac{1}{y^2}$

SOLUTION:-

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{xy^2}$$

$$y^2\frac{dy}{dx} + \frac{y^3}{x} = \frac{1}{x}.$$

Here

$P(x) = 1/x$, $Q(x) = 1/x$; $n = -2$.

let $z = y^{1-n}$.

$z = y^3$.

After the transformation.

$$\frac{dz}{dx} + (1+2)x^{-1}z = 3x^{-1} \text{ ——— }$$

$$\frac{dz}{dx} + \frac{3z}{x} = \frac{3}{x} \text{ ——— (i)}$$

Now $P(x) = 3/x$.

$\int P(x) = 3\ln x$.

$$I.F = e^{3\ln x} = x^3.$$

Question after the transformed and multiplying I.f becomes.

$$\text{(i)} \Rightarrow \left(\frac{dz}{dx} + \frac{3}{x}z\right)x^3 = \frac{3}{x}\cdot x^3$$

$$\frac{dz}{dx} \cdot x^3 = 3x^2$$

$$z \cdot x^3 = \int 3x^2$$

$$z \cdot x^3 = x^3$$

$$z = 1 + cx^{-3}.$$

$$\boxed{y^3 = 1 + cx^{-3}}$$

Q17. $\frac{dy}{dx} = y(xy^3 - 1)$

SOLUTION:-

$$\frac{dy}{dx} = xy^4 - y.$$

$$\frac{dy}{dx} + y = xy^4$$

$$y^4 \frac{dy}{dx} + \frac{1}{y^3} = x.$$

let $z = y^{1-n}.$

$$z = y^{1-4} = y^{-3}.$$

$$\therefore \frac{dz}{dx} + (1-n)P(x)z = (1-n)Q(x)$$

$$\frac{dz}{dx} + (1-4)(1)z = (1-4)(x)$$

$$\frac{dz}{dx} - 3z = -3x \quad \text{—— (i)}$$

Now $\quad p(x) = -3$

$\int p(x) = -3x$

$I.F = e^{-3x}$

$$(i) \Rightarrow \left(\frac{dz}{dx} - 3z\right)e^{-3x} = -3x\, e^{-3x}$$

$$\frac{dz}{dx}\, e^{-3x} = -3x\, e^{-3x}$$

$$z e^{-3x} = -3\int x e^{-3x} \quad \text{—— (a)}$$

let $\quad I = -3\int x e^{-3x}$

$u = x \qquad\qquad \int dv = \int e^{-3x}$

$du = dx \qquad\qquad v = -\frac{e^{-3x}}{3}$

$$= \int u\,dv = uv - \int v\,du$$

$$= x\left(-\frac{e^{-3x}}{3}\right) - \int -\frac{e^{-3x}}{3}\,dx$$

$$= -\frac{x}{3}e^{-3x} - \frac{1}{3\times 3}e^{-3x}$$

$$= -\frac{x}{3}e^{-3x} - \frac{1}{9}e^{-3x}$$

$$(a) \quad z e^{-3x} = -3\left(-\frac{x e^{-3x}}{3} - \frac{1}{9}e^{-3x}\right)$$

$$ze^{-3x} = xe^{-3x} + \frac{1}{3}e^{-3x} + C.$$

$$z = x + \frac{1}{3} + ce^{-3x}.$$

$$y^{-3} = x + \frac{1}{3} + ce^{-3x}$$ Ans!

Q19. $t^2 \frac{dy}{dt} + y^2 = ty$

SOLUTION:-

$$y^{-1}\frac{dy}{dt} + \frac{y}{t^2} = \frac{1}{t}$$

~~let $z = y^{1-n}$.~~

$$\frac{dy}{dt} + \frac{y^2}{t^2} = \frac{y}{t} \quad \text{——— (a)}$$

let $z = y^{(1-n)} = y^{(1-2)}$

$z = y^{-1}$

Question will become.

(a) $\rightarrow$ $\frac{dy}{dt} + (1-2)\frac{1}{t^2}z = \frac{1}{t}(1-2)$

$$\frac{dy}{dt} - \frac{z}{t^2} = \frac{-1}{t} \quad \text{——— (b)}$$

$P(x) = \frac{1}{t^2}$

$\int P(x) = -\frac{1}{t}$, $I.F = e^{-1/t}$

(b) => $\left(\frac{dy}{dt} - \frac{z}{t^2}\right) e^{-1/t} = \frac{-1}{t} e^{-1/t}$

$\frac{dy}{dt} e^{-1/t} = -\frac{e^{-1/t}}{t}$

let $u = -1/t$, $du = 1/t^2 \, dt$

=>

Q21. $x^2 \frac{dy}{dx} - 2xy = 3y^4$, $y(1) = \frac{1}{2}$

SOLUTION:-

$$y^{-4} \frac{dy}{dx} - \frac{2}{xy^3} = \frac{3}{x^2}.$$

$$\frac{1}{y^4} \frac{dy}{dx} - \frac{2}{xy^3} = \frac{3}{x^2}$$

$$\frac{dy}{dx} - \frac{2y}{x} = \frac{3y^4}{x^2}$$

Let $z = y^{1-n}$

$z = y^{1-4}$

$z = y^{-3}$

Now the transformation become.

$$\frac{dz}{dx} + (1-n)\frac{2}{x} z = (1-n)\frac{3}{x^2}$$

$$\frac{dz}{dx} + (-3)(2)\frac{z}{x} = -(3)(3)\frac{1}{x^2}.$$

$$\frac{dz}{dx} - 6\frac{z}{x} = \frac{-9}{x^2}. \quad \text{——— (a)}$$

Now, $P(x) = -6/x$.

$\int P(x) = -6 \ln x = \ln x^{-6}$

$I.F = e^{\ln x^{-6}} = \frac{1}{x^6}$

Ⓠ $\Rightarrow$ $$\left(\frac{dz}{dx} - \frac{6z}{x}\right)\frac{1}{x^6} = \frac{-9}{x^2}\cdot\frac{1}{x^6}$$

$$\frac{dz}{dx}\,\frac{1}{x^6} = \frac{-9}{x^8}$$

$$zx^{-6} = \frac{-9}{-7}\,\frac{1}{x^7} + C$$

$$z = \frac{9}{7}\,\frac{1}{x} + Cx^{-6}$$

$$y^{-3} = \frac{9}{7}x^{-1} + Cx^{-6}$$ ~~Ans~~! Ⓐ

$$\because\ y(1) = \frac{1}{2}$$

$$\left(\frac{1}{2}\right)^{-3} = \frac{9}{7}(1) + C(1)^{-6}$$

$$8 = \frac{9}{7} + C$$

$$\frac{47}{7} = C$$

Ⓐ $\Rightarrow$ $$y^{-3} = \frac{9}{7}x^{-1} + \frac{47}{7}x^{-6}$$ Ans!

Q23. $\frac{dy}{dx} = (x+y+1)^2$

SOLUTION:-

Let $v = x + y + 1$

$$\frac{dv}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dv}{dx} - 1 = \frac{dy}{dx}$$

Question now become.

$$\frac{dv}{dx} - 1 = v^2$$

$$\frac{dv}{1+v^2} = dx$$

$$\int \frac{dv}{1+v^2} = dx.$$

$$\tan^{-1} u = x + C.$$

$$v = \tan(x+c)$$

$$x + y + 1 = \tan(x+c)$$

$$y = \tan(x+c) - x - 1$$

Ans!

Q25. $\frac{dy}{dx} = \tan^2(x+y)$

let, $u = x+y$

$\frac{du}{dx} = 1 + \frac{dy}{dx}$.

$\frac{du}{dx} - 1 = \frac{dy}{dx}$

Question now become,

$\frac{du}{dx} - 1 = \tan^2 u$

$\frac{du}{1+\tan^2 u} = dx$.

$\frac{du}{\sec^2 u} = dx = \cos^2 u \, du$.

$\frac{1}{2}(1 + \cos 2u)\, du = dx$.

$\frac{1}{2} u + \frac{1}{4} \sin 2u = x + c$.

$\frac{1}{2}(x+y) + \frac{1}{4} \sin 2(x+y) = x + c$

Ans!

REDUCTION OF ORDER

EXERCISE 4.2

Q1 $y'' - 4y' + 4y = 0 \; ; \quad y_1 = e^{2x}$

SOLUTION:-

We know that

$$y_2 = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)}$$

We will find I.F first.

$$I.F = e^{-\int P(x)\,dx}.$$

$$= e^{-\int -4\,dx}$$

$$= e^{4x}.$$

$$y_2 = (e^{2x}) \int \frac{\cancel{e^{4x}}}{\cancel{(e^{2x})^2}}\,dx.$$

$$y_2 = e^{2x}x + c.$$

Q3 $y'' + 16y = 0 \; ; \quad y_1 = \cos 4x$

SOLUTION:-

$$y_2 = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)}$$

we will find I.F first

$$I.F = e^{-\int P(x)} = e^{-\int 0\,dx} = 1$$

$$y_2 = \cos 4x \int \frac{1}{\cos^2 4x}\,dx.$$

$$y_2 = \cos 4x \int \frac{1}{\cos^2 4x}\, dx$$

$$= \cos 4x \int \frac{1}{\left(\frac{1+\cos 8x}{2}\right)}\, dx$$

$$= \cos 4x \int \sec^2 4x \, dx$$

$$= \cos 4x \, \frac{\tan 4x}{4}$$

$$= \cos 4x \, \frac{\sin 4x}{4\cos 4x}$$

$$y_2 = \frac{1}{4} \sin 4x$$

Q5 $y'' - y = 0 \; ; \quad y_1 = \cosh x$

SOLUTION:-

$$\because \; y_2 = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{y_1^2(x)}$$

$$= \cosh(x) \int \frac{e^{-\int 0\,dx}}{\cosh x}$$

$$= \cosh x \int \frac{1}{\frac{e^x + e^{-x}}{2}}\, dx$$

$$= 2\cosh x \int \frac{e^x}{e^{2x} + 1}$$

$$y_2 = 2\cosh x \; I_1 \quad \text{———} \quad (A)$$

$$I_1 = \int \frac{e^x}{e^{2x}+1}\,dx \quad \text{———} \quad (i)$$

let $u = e^x$.

$du = e^x dx$.

$$(i) \Rightarrow \int \frac{du}{u^2+1} = \tan^{-1} u = \tan^{-1} e^x$$

$$(A)\; y_2 = 2\cosh x \tan^{-1} e^x.$$

Q7 $9y'' - 12y' + 4y = 0$; $y_1 = e^{2x/3}$

SOLUTION:-

$$\therefore \; y_2 = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)}$$

$P(x) = -12$.

$$y_2 = e^{2x/3} \int \frac{e^{-\int(-12)dx}}{e^{(2x/3)^2}}$$

$$= e^{2x/3} \int \frac{e^{12x}}{e^{4x/9}} dx$$

$$= e^{2x/3} \int e^{12} \cdot e^{-4x/9}\, dx$$

Q7 $9y'' - 12y' + 4y = 0 \; ; \; y_1 = e^{2x/3}$

SOLUTION:-

$$y_2 = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)}$$

$$9y'' - 12y' + 4y = 0$$

$$y'' - \frac{4}{3}y' + \frac{4}{9}y = 0$$

$$y_2 = e^{2x/3} \int \frac{e^{-\int(-4/3)dx}}{(e^{2x/3})^2} dx$$

$$= e^{2x/3} \int \frac{e^{4x/3}}{e^{4x/3}} dx.$$

$$y_2 = xe^{2x/3}$$

Q9 $x^2y'' - 7xy' + 16y = 0 \; ; \; y_1 = x^4$

SOLUTION:-

$$y'' - \frac{7}{x}y' + 16\frac{y}{x^2} = 0$$

$$P(x) = -7/x.$$

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2} = x^4 \int \frac{e^{-\int(-7/x)}}{x^8}$$

$y_2 =$

$y_2 =$

Q11 xy

SOLUTIO

$$y_2 = x^4 \int \frac{e^{7\ln x}}{x^8}\, dx$$

$$= x^4 \int \frac{x^7}{x^8}\, dx.$$

$$y_2 = x^4 \ln(x)$$

Q11 $xy'' - y' = 0$; $y_1 = \ln x$

SOLUTION.-

$$y'' - \frac{y'}{x} = 0$$

$$\therefore \quad y_2 = y_1(x) \int \frac{e^{-\int P(x)\, dx}}{y_1^2(x)}\, dx$$

$$= \ln x \int \frac{e^{-\int 1/x\, dx}}{\ln^2 x}$$

$$= \ln x \int \frac{1}{x \ln^2 x}\, dx.$$

$$= \ln x \cdot \frac{1}{\ln x}$$

$$y_2 = 1$$

Q13 $x^2y'' - xy' + 2y = 0$; $y = x\sin(\ln x)$

SOLUTION:-

$$y'' - \frac{y'}{x} + \frac{2y}{x^2} = 0$$

$$P(x) = -1/x.$$

$$y_2 = x\sin(\ln x)\int \frac{e^{-\int(-1/x)\,dx}}{\{x\sin(\ln x)\}^2}\,dx$$

$$= x\sin(\ln x)\int \frac{x}{x^2(\sin(\ln x))^2}$$

$$= x\sin(\ln x)\int \frac{dx}{x(\sin(\ln x))^2}$$

let $u = \ln x$, $du = dx/x$.

$$\Rightarrow x\sin u \int \frac{du}{\sin^2 u}$$

$$= x\sin u \int \csc^2(u)\,du$$

$$= x\sin u \cdot \cot(u)$$

$$= x\,\cancel{\sin u}\,\frac{\cos u}{\cancel{\sin u}}$$

$$= x\cos u$$

$$y_2 = x\cos(\ln(x))$$

Q15 $(1-2x-x^2)y'' + 2(1+x)y' - 2y = 0$

$y_1 = x+1$

SOLUTION:-

$$y'' + \frac{2(1+x)}{(1-2x-x^2)} y' - \frac{2y}{(1-2x-x^2)} = 0$$

$$y_2 = (x+1)\int \frac{e^{-\int \frac{(2+2x)\,dx}{(1-2x-x^2)}}}{(x+1)^2}\,dx$$

$$= (x+1)\int \frac{e^{A}}{(x+1)^2}$$

$$A = -\int \frac{(2+2x)\,dx}{-(x^2+2x-1)}$$

$$A = \ln(x^2+2x-1)$$

$$y_2 = (x+1)\int \frac{e^{\ln(x^2+2x-1)}}{(x+1)^2}\,dx$$

$$= (x+1)\int \frac{x^2+2x-1}{x^2+2x+1}\,dx$$

$$= (x+1)\int \frac{x^2+2x+1-1-1}{x^2+2x+1}\,dx$$

$$= (x+1)\int \frac{(x+1)^2-2}{(x+1)^2}\,dx$$

$$= (x+1)\left[\int \frac{(x+1)^2}{(x+1)^2}\,dx - \int \frac{2}{(x+1)^2}\,dx\right.$$

$$= (x+1)\left[x - \frac{2}{x+1} \right].$$

$$= x(x+1) - 2$$

$$y_2 = x^2 + x - 2$$

Q17 $y'' - 4y = 2$; (i) $y_1 = e^{-2x}$

SOLUTION:-

$Y = uy_1$ —— (A)

$Y = ue^{-2x}$

$y' = -2ue^{-2x} + e^{-2x}u'$

$y'' = -2(-2ue^{-2x} + e^{-2x}u') + e^{-2x}u''$ ~~$+u'-2$~~ $-2e^{-2x}u'$

$= +4ue^{-2x} - 2e^{-2x}u' + e^{-2x}u'' - 2e^{-2x}u'$

$y'' = 4ue^{-2x} - 4e^{-2x}u' + e^{-2x}u''$

(i) $\Rightarrow$ $4ue^{-2x} - \cancel{4e^{-2x}u'} + e^{-2x}u'' - 4(-2ue^{-2x})$ ~~$+4e^{-2x}u'$~~

$= 2.$

$12ue^{-2x} + e^{-2x}u'' = 2$ —— (2).

let $v = u'$

$v' = u''$

(2) $\Rightarrow$ 12

$(1) \Rightarrow \cancel{4ue^{-2x}} - 4e^{-2x}u' + e^{-2x}u'' - \cancel{4ue^{-2x}} = 2$

$e^{-2x}u'' - 4e^{-2x}u' = 2$ ——— (2)

let $v = u'$

$v' = u''$

$(2) \Rightarrow e^{-2x}v' - 4e^{-2x}v = 2$

$v' - 4v = \frac{2}{e^{-2x}}$ ——— (3)

comp. with

$y' - P(x)y = Q(x)$

$P(x) = -4$

$e^{\int P(x)} = e^{-4x} =$

$(3) \Rightarrow (v' - 4v)e^{-4x} = 2e^{2x} \cdot e^{-4x}$

$v'e^{-4x} = 2e^{-2x}$

$ve^{-4x} = -\cancel{4}e^{-2x} + ce^{4x}$

$v = -\cancel{4}e^{2x} + c_1e^{4x}$

$u' = v$

$u = -\frac{2e^{2x}}{4} + \frac{4c_1e^{4x}}{16}$

$= -\frac{1}{2}e^{2x} + \frac{1}{4}ce^{4x} + c_2$

dy.

(A) =>

$$y = \left(-\frac{1}{2} e^{2x} + \frac{1}{4} c_1 e^{4x} + c_2\right) e^{-2x}$$

$$y = -\frac{1}{2} + \frac{1}{4} c_1 e^{2x} + c_2 e^{-2x}$$

A

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EXERCISE 4.3

Q1 $4y'' + y' = 0$

SOLUTION:-

The auxilary eq^n of given eq^n is given by.

$4m^2 + m = 0$.

$m(4m + 1) = 0$.

$\boxed{m = 0}$ $\boxed{m = -\frac{1}{4}}$

$y(x) = c_1 e^0 + c_2 e^{-1/4 x}$

$y(x) = c_1 + c_2 e^{-1/4 x}$ Ans!

Q3. $y'' - y' - 6y = 0$

SOLUTION:-

Auxilary eq^n of given eq^n is

$m^2 - m - 6 = 0$.

$m^2 - 3m + 2m - 6 = 0$

$m(m-3) + 2(m-3) = 0$

$(m-3)(m+2) = 0$

$\boxed{m = 3}$ $\boxed{m = -2}$

$\boxed{y(x) = c_1 e^{3x} + c_2 e^{-2x}}$ Ans!

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(A)

Q5 $y'' + 8y' + 16y = 0$.

SOLUTION:-

Auxilary eqⁿ of given question is

$m^2 + 8m + 16 = 0$

$m^2 + 4m + 4m + 16 = 0$,

$m(m+4) + 4(m+4) = 0$,

$(m+4)^2 = 0$,

$m = -4$, $m = -4$

$y(x) = c_1 e^{-4x} + c_2 x e^{-4x}$. Ans!

Q7 $12y'' - 5y' - 2y = 0$

SOLUTION:-

$12m^2 - 5m - 2 = 0$,

$12m^2 - 8m + 3m - 2 = 0$.

$4m(3m-2) + (3m-2) = 0$

$(3m-2)(4m+1) = 0$

$m = 2/3$, $m = -1/4$

$y(x) = c_1 e^{2/3 x} + c_2 e^{-1/4 x}$ Ans!

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Q9 $y'' + 9y = 0$

SOLUTION:-

$m^2 + 9m = 0$.

$m(m+9) = 0$.

$m = 0$ $\quad$ $m = -9$.

$y(x) = c_1 + c_2 e^{-9x}$ Ans!

Q4 $y'' - 4y' + 5y = 0$

$m^2 - 4m + 5 = 0$

~~$m^2 + 5m$~~

$x = \frac{4 \pm \sqrt{16 - 4(1)(5)}}{2}$.

$= \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm 2i}{2}$.

$x = 2 \pm i$

$y(x) = e^{2x} [A \cos ix + B \sin 2ix]$.

Ans!

Q13 $3y'' + 2y' + y = 0$

SOLUTION:-

$3m^2 + 2m + \cancel{y}1 = 0$

$x = \dfrac{-2 \pm \sqrt{4-12}}{6}$

$= \dfrac{-2 \pm 2\sqrt{2}\, i}{6}$

$x = \dfrac{-1 \pm \sqrt{2}\, i}{3} = -\dfrac{1}{3} \pm \dfrac{\sqrt{2}\, i}{3}$

$y(x) = e^{\frac{-x}{3}} \left[A\cos \dfrac{\sqrt{2}}{3}x + B\sin \dfrac{\sqrt{2}}{3}x \right]$

Q29 $y'' + 16y = 0$, $y(0) = 2$, $y'(0) = -2$ A's

SOLUTION:-

$\cancel{m^2 + 16m = 0}$

$\cancel{m = 0, \quad m = -16}$

~~y(x) = C1~~ $\cancel{y(x) = C_1 + C_2 e^{-16x}}$

$m^2 + 16 = 0$

$m^2 = -16$

$m = \pm 4i$

$y(x) = e^{0(x)} \left[A\cos 4x + B\sin 4x \right]$

$y(x) = A\cos 4x + B\sin 4x$ ——— (i)

If $y(0) = 2$

$2 = A\cos(0) + B\sin(0)$

$\boxed{2 = A}$

(i) => $y'(x) = -\frac{A\sin 4x}{4} + \frac{B\cos 4x}{4}$.

$-2 = \frac{-4A\sin(0)}{\cancel{4}} + \frac{4B\cos(0)}{\cancel{4}}$.

$\frac{-2}{4} = B$.

$-\frac{1}{2} = B$

(i) => $y(x) = 2\cos 4x + \left(-\frac{1}{2}\right)\sin 4x$

Ans!

Q27 $\frac{d^5u}{dr^5} + \frac{5d^4u}{dr^4} - \frac{2d^3u}{dr^3} - \frac{10d^2u}{dr^2} + \frac{du}{r} + 5u = 0$

SOLUTION:-

The auxilary eq^n of the given question is,

$m^5 + 5m^4 - 2m^3 - 10m^2 + m + 5 = 0$.(i)

$\cancel{1} + 5 - \cancel{2} - 10 + \cancel{1} + \cancel{5} = 0$.

$\boxed{m = 1}$

$$\begin{array}{c|cccccc} 1 & 1 & +5 & -2 & -10 & +1 & +5 \\ & & 1 & 6 & 4 & -6 & -5 \\ \hline & 1 & 6 & 4 & -6 & -5 & 0 \end{array}$$

$m^4 + 6m^3 + 4m^2 - 6m - 5 = 0$.

$1 + \cancel{6} + 4 - \cancel{6} - \cancel{5} = 0$

$\boxed{m = 1}$

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$$\begin{array}{r|rrrrr} 1 & 1 & +6 & +4 & -6 & -5 \\ & & 1 & 7 & 11 & 5 \\ \hline & 1 & 7 & 11 & 5 & \lfloor 0 \end{array}$$

$$m^3 + 7m^2 + 11m + 5 = 0$$

$$-1 + 7 - 11 + 5 = 0$$

$$\boxed{m = -1}$$

$$\begin{array}{r|rrrr} -1 & 1 & 7 & +11 & +5 \\ & & -1 & -6 & -5 \\ \hline & 1 & 6 & 5 & \lfloor 0 \end{array}$$

$$m^2 + 6m + 5 = 0$$

$$m^2 + 5m + m + 5 = 0$$

$$m(m+5) + (m+5) = 0$$

$$(m+5)(m+1) = 0$$

$$\boxed{m = -5} \qquad \boxed{m = -1}$$

$$y(x) = c_1 e^{x} + c_2 x e^{x} + c_3 e^{-x} + c_4 x e^{-x} + c_5 e^{-5}$$

Q31 $\frac{d^2y}{dt^2} - \frac{4dy}{dt} - 5y = 0$; $y(1) = 0$; $y'(1) = 2$.

SOLUTION:-

$m^2 - 4m - 5 = 0$.

$m^2 - 5m + m - 5 = 0$.

$m(m-5) + (m-5) = 0$

$\boxed{m = 5}$ $\boxed{m = -1}$

$y(x) = c_1 e^{5t} + c_2 e^{-t}$ ——— (i)

$\because y(1) = 0$

$0 = c_1 e^{5} + c_2 e^{-1}$

$0 = \frac{c_1 e^5 \cdot e^1 + c_2}{e}$

$0 = c_1 e^6 + c_2$

$c_2 = -c_1 e^6$ ——— (a)

(i) $\Rightarrow$ $y'(x) = 5c_1 e^{5x} - c_2 e^{-x}$

$\because y'(1) = 2$.

$2 = 5c_1 e^5 - c_2 e^{-1}$

$2e^1 = 5c_1 e^6 - c_2$

$2e^1 = 5c_1 e^6 - (-c_1 e^6)$

$2e^1 = 6c_1 e^6$

$\frac{\cancel{2}e^1}{\cancel{6}_3 e^6} = c_1 = \frac{1}{3} e^{-5} = c_1$

$3e^{-6}$

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$\textcircled{a} \Rightarrow \quad C_2 = -\left(\frac{1}{3}\cancel{e^{-6}}\right) e^{+6}$

$C_2 = -\frac{1}{3}$

$\text{(i)} \Rightarrow \quad y(x) = \frac{1}{3} e^{-6} \cdot e^{5t} - \frac{1}{3} e^{-t}$

$= \frac{1}{3} e^{5t-6} - \frac{1}{3} e^{-t}$ Ans!

Q35 $y''' + 12y'' + 36y' = 0,$

$y(0) = 0, \quad y'(0) = 1, \quad y''(0) = -7.$

SOLUTION:-

$m^3 + 12m^2 + 36m = 0$

$\boxed{m = 0}$

$$\begin{array}{c|ccc} 0 & 1 & +12 & +36 \\ & & 0 & 0 \\ \hline & 1 & 12 & 36 \end{array}$$

m

Q_{35} $y''' + 12y'' + 36y' = 0$; $y(0) = 0$; $y'(0) = 1$

SOLUTION:-

$y''(0) = -7$

$m^3 + 12m^2 + 36m = 0$

$m(m^2 + 12m + 36) = 0$

$\boxed{m = 0}$

$m^2 + 12m + 36 = 0$

$m^2 + 6m + 6m + 36 = 0$

$m(m+6) + 6(m+6) = 0$

$(m+6)^2 = 0$

$\boxed{m = -6}$ $\boxed{m = -6}$

$y(x) = c_1 + C_2 e^{-6x} + C_3 x e^{-6x}$ ——— (i)

$\because y(0) = 0$.

$0 = C_1 + C_2 + C_3(0) = C_1 + C_2$.

$y'(x) = -6C_2 e^{-6x} + 6C_3 e^{-6x} x - C_3 e^{-6x}$

$\because y'(0) = 1$.

$1 = -6C_2 - C_3$.

$-1 = 6C_2 + C_3$

VARIATION OF PARAMETERS :-

EXERCISE 4.6

Q1 $y'' + y = \sec x$

SOLUTION:-

Then auxily eqn of given question is.

$m^2 + 1 = 0$.

$m = \pm i$

$y_c = C_1 \cos x + C_2 \sin x$.

$y_1 = \cos x$, $y_2 = \sin x$. , $f(x) = \sec x$

$$w = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$= \cos^2 x + \sin^2 x$

$w = 1$

$u_1' = \frac{-y_2 f(x)}{w}$, $u_2' = +\frac{y_1 f(x)}{w}$

$u_1' = \frac{-\sin x \sec x}{1}$, $u_2' = \frac{\cos x \sec x}{w}$.

$u_1' = -\tan x$, $u_2' = 1$

$u_1 = -\ln|\sec x|$, $u_2 = x$.

$u_1 = \ln|\cos x|$

$y_p = y_1 u_1 + y_2 u_2$

$y_p = \cos x \ln \cos x + x \sin x$.

$y = C_1 \cos x + C_2 \sin x + \cos x \ln x + x \sin x$.

Ans!

Q3 $y'' + y = \sin x$

SOLUTION:-

$m^2 + 1 = 0$

$m = \pm i$

$y_c = c_1 \cos x + c_2 \sin x$

$y_1 = \cos x$, $y_2 = \sin x$, $f(x) = \sin x$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$u_1' = -\dfrac{y_2 f(x)}{W}$ $\qquad$ $u_2' = \dfrac{y_1 f(x)}{W}$

$u_1' = -\dfrac{\sin x \sin x}{1}$ $\qquad$ $u_2' = \dfrac{\cos x \sin x}{1}$

$u_1' = -\left(\dfrac{1 - \cos 2x}{2}\right)$ $\qquad$ $u_2' = \sin x \cos x$

$u_1 = -\dfrac{1}{2}x + \dfrac{1}{2 \times 2} \cos 2x$, $u_2 = \dfrac{-1}{2}\cos^2 x$

$= -\dfrac{1}{2}x + \dfrac{1}{4}\sin 2x$ $\qquad$ $u_2 = -\dfrac{1}{2}\cos^2 x$

$u_1 = -\dfrac{1}{2}x + \dfrac{1}{2}\sin x \cos x$

$y_p = u_1 y_1 + u_2 y_2$

$= \left(\dfrac{-1}{2}x + \dfrac{1}{2}\sin x \cos x\right)\cos x + \left(\dfrac{-1}{2}\cos^2 x\right)\sin x$

$$= -\frac{1}{2}x\cos x + \frac{1}{2}\sin x\cos^2 x - \frac{1}{2}\sin x\cos^2 x$$

$$y_p = -\frac{1}{2}x\cos x$$

$$y = c_1\cos x + c_2\sin x - \frac{1}{2}x\cos x$$

Ans.

Q5. $y'' + y = \cos^2 x$.

SOLUTION:-

$$m^2 + 1 = 0.$$

$$m^2 = -1$$

$$m = \pm i$$

$$y_c = c_1\cos x + c_2\sin x.$$

$y_1 = \cos x$, $y_2 = \sin x$, $f(x) = \cos^2 x$.

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$u_1' = -\frac{y_2 f(x)}{W} \qquad u_2' = \frac{y_1 f(x)}{W}$$

$$u_1' = \frac{-\sin x\cos^2 x}{1} \qquad u_2' = \frac{\cos x \cdot \cos^2 x}{1}$$

$$u_1 = \frac{\cos^3 x}{3} \qquad u_2' = \cos x(1 - \sin^2 x)$$

$$u_2' = \cos x - \sin^2 x \cos x.$$

$$u_2 = \sin x - \frac{\sin^3 x}{3}.$$

$$y_p = u_1 y_1 + u_2 y_2.$$

$$= \left(\frac{\cos^3 x}{3}\right)\left(\cos x\right) + \left(\sin x - \frac{\sin^3 x}{3}\right)\sin x.$$

$$y_p = \frac{\cos^4 x}{3} + \sin^2 x - \frac{\sin^4 x}{3}.$$

$$= \frac{1}{3}\left(\cos^4 x - \sin^4 x\right) + \sin^2 x.$$

$$= \frac{1}{3}\left(\cos^2 x + \sin^2 x\right)\left(\cos^2 x - \sin^2 x\right) + \sin^2 x).$$

$$= \frac{1}{3}\cos^2 x - \frac{1}{3}\sin^2 x. + \sin^2 x.$$

$$y_p = \frac{1}{3}\cos^2 x + \frac{2}{3}\sin^2 x$$

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Q9 $y'' - 9y = \frac{9x}{e^{3x}}$

SOLUTION:-

$$m^2 - 9 = 0.$$

$$m = \pm 3.$$

$$y_c = c_1 e^{3x} + c_2 e^{-3x}.$$

$$y_1 = e^{3x}, \quad y_2 = e^{-3x}, \quad f(x) = 9x/e^{3x}$$

$$W = \begin{vmatrix} e^{3x} & e^{-3x} \\ 3e^{3x} & -3e^{-3x} \end{vmatrix} = -3\cancel{e^{6x}} - 3 = -6.$$

$$u_1' = -\frac{y_2 f(x)}{W}, \quad u_2' = \frac{y_1 f(x)}{W}.$$

$$\cancel{u_1' = \frac{-e^{-3x}(9x/e^{3x})}{-3e^{6x} - 3}}$$

$$\cancel{= -\frac{9x/e^{6x}}{-3e^{6x} - 3}}$$

$$\cancel{= \frac{9x}{e^{6x}(3e^{6x} - 3)}}$$

$$=$$

$$= \frac{e^{3x}(9x/e^{3x})}{-6}$$

$$u_2' = -\frac{1}{\cancel{6}_2} \frac{\cancel{9}^3 x}{\cancel{e}}$$

$$u_2 = \frac{-3}{2} \frac{x^2}{2}$$

$$u_2 = \frac{-3}{4} x^2.$$

$$u_1' = \frac{-e^{-3x} \cdot \frac{9x}{e^{3x}}}{-6}.$$

$$u_1' = \frac{1}{6} \frac{9x}{e^{6x}} = \frac{3}{2} x e^{-6x}$$

$$u = x \qquad \int dv = e^{-6x}$$

$$du = dx \qquad v = \frac{e^{-6x}}{-6}$$

$$= uv - \int v\,du$$

$$= x\frac{e^{-6x}}{-6} - \int \frac{e^{-6x}}{6}\,dx$$

$$u_1 = \frac{-1}{6}xe^{-6x} + \frac{1}{36}e^{-6x}$$

$$y_p = \left(\frac{-1}{6}xe^{-6x} + \frac{1}{36}e^{-6x}\right)e^{3x} + \left(\frac{-3}{4}x^2\right)e^{-3x}$$

$$y_p = \frac{-1}{6}xe^{-3x} + \frac{1}{36}e^{-3x} - \frac{3}{4}x^2e^{-3x}$$

Q4 $y'' + 3y' + 2y = \frac{1}{1+e^x}$

SOLUTION:-

$$m^2 + 3m + 2 = 0$$

$$m^2 + 2m + m + 2 = 0$$

$$m(m+2) + 1(m+2) = 0$$

$$m = -2, \quad m = -1$$

$$y_c = c_1e^{-x} + c_2e^{-2x}$$

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$y_1 = e^{-x}$, $y_2 = e^{-2x}$, $f(x) = \frac{1}{1+e^{x}}$.

$$W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -2e^{-3x} + e^{-3x} = -e^{-3x}.$$

$u_1' = \frac{-y_2 f(x)}{W}$, $u_2' = \frac{y_1 f(x)}{W}$.

$u_1' = \frac{-e^{-2x}\left(1/_{1+e^{x}}\right)}{-e^{-3x}}$ $\quad u_2' = \frac{e^{-x}\left(1/_{1+e^{x}}\right)}{-e^{-3x}}$.

$u_1' = \frac{1}{(1+e^{x})e^{-x}}$ $\quad = \frac{-1}{(1+e^{x})e^{-2x}}$

$= \frac{e^{x}}{1+e^{x}}$ $\quad = \frac{-e^{2x}}{1+e^{x}}$.

$u_1 = \ln|1+e^{x})$ $\quad = \frac{e^{x}}{1+e^{x}} - e^{x}$

$u_2 = \ln|1+e^{x}| - e^{x}$

$$y = c_1e^{-x} + c_2e^{-2x} + e^{-x}\ln|1+e^{x}| + e^{-2x}\ln|1+e^{x}| - e^{-x}$$

Ans.

Q15 $y'' + 2y' + y = e^{-t}\ln t$

$$m^2 + 2m + 1 = 0$$

$$m = -1, -1$$

$$y_c = c_1 e^{-t} + c_2 t e^{-t}$$

$$y_1 = e^{-t}, \quad y_2 = te^{-t}, \quad f(x) = e^{-t}\ln t$$

$$W = \begin{vmatrix} e^{-t} & te^{-t} \\ -e^{-t} & -te^{-t} + e^{-t} \end{vmatrix} = \cancel{-te^{-2t}} + e^{-2t} + \cancel{te^{-2t}} = e^{-2t}$$

$$u_1' = -\frac{te^{\cancel{-t}}\, e^{\cancel{-t}}\ln t}{\cancel{e^{-2t}}} \qquad u_2' = \frac{e^{\cancel{-t}}\, e^{\cancel{-t}}\ln t}{\cancel{e^{-2t}}}$$

$$u_1' = -t\ln t \qquad u_2' = \ln t$$

$$u_1 = -\frac{1}{2}t^2\ln t + \frac{1}{4}t^2 \qquad u_2 = t\ln t - t$$

$$y_p = \left(-\frac{1}{2}t^2\ln t + \frac{1}{4}t^2\right)\left(e^{-t}\right) + \left(t\ln t - t\right)\left(te^{-t}\right)$$

$$= \frac{1}{2}t^2\ln t\, e^{-t} + \frac{1}{4}t^2e^{-t} + t^2\ln t\, e^{-t} - t^2e^{-t}$$

$$y_p = \frac{3}{2}e^{-t}t^2\ln t - \frac{3}{4}t^2e^{-t}$$

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CAUCHY-EULER EQUATIONS

EXERCISE 4.7

Q1 $x^2y'' - 2y = 0$

Solution:-

let $x = e^t$, $t = \ln x$.

$D = \frac{d}{dx}$ $\Delta = \frac{dy}{dt}$,

The Question will become.

$x^2D^2y - 2y = 0$.

$\Delta(\Delta-1) - 2 = 0$,

$\Delta^2 - \Delta - 2 = 0$.

$\Delta^2 - 2\Delta + \Delta - 2 = 0$

$\Delta(\Delta-2) + 1(\Delta-2) = 0$

$(\Delta-2)(\Delta+1) = 0$

$\boxed{\Delta = 2}$, $\boxed{\Delta = -1}$

$y = c_1e^{-t} + c_2e^{2t}$

$y = c_1x^{-1} + c_2\ln x^2$ Ans!

Q3 $xy'' + y' = 0$

SOLUTION:-

$$xy'' + y' = 0$$
$$x^2y'' + xy' = 0.$$

let $e^t = x$, $t = \ln x$.

Question now become.

$$x^2D^2y + xDy = 0$$

$$\Delta(\Delta-1) + \Delta = 0.$$
$$\Delta^2 - \cancel{\Delta} + \cancel{\Delta} = 0.$$
$$\Delta^2 = 0,$$

$$y = c_1e^{0t} + c_2 t e^{0t}$$
$$= c_1 + c_2t$$
$$y = c_1 + c_2\ln x \quad \text{Ans!}$$

Q5 $x^2y'' + xy' + 4y = 0$

SOLUTION:-

$$x^2D^2y + xDy + 4y = 0$$

$$\Delta(\Delta-1) + \Delta + 4 = 0$$

$$\Delta^2 - \Delta + \Delta + 4 = 0$$

$$\Delta^2 = -4$$

$$\Delta = \pm 2i$$

$$y = e^{0t}(A\cos 2t + B\sin 2t)$$

$$y = x(A\cos(2\ln x) + B\sin(2\ln x))$$

Ans!

Q7 $x^2y'' - 3xy' - 2y = 0$

SOLUTION:-

$$x^2D^2y - 3xDy - 2y = 0$$

$$\Delta(\Delta-1) - 3\Delta - 2 = 0$$

$$\Delta^2 - \Delta - 3\Delta - 2 = 0$$

$$\Delta^2 - 4\Delta - 2 = 0$$

$$\Delta = \frac{4 \pm \sqrt{16+8}}{2} = \frac{4 \pm \sqrt{20}}{2}$$

$$\Delta = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$$y = c_1 e^{(2+\sqrt{5})t} + c_2 e^{(2-\sqrt{5})t}$$
$$= c_1 e^{(2+\sqrt{5})\ln x} + c_2 e^{(2-\sqrt{5})\ln x}$$
$$= c_1 x^{(2+\sqrt{5})} + c_2 e^{(2-\sqrt{5})} \quad \text{Ans!}$$

Q15 $x^3 y''' - 6y = 0$

SOLUTION:-

$$x^3 D^3 y - 6y = 0.$$

$$\Delta(\Delta-1)(\Delta-2) - 6 = 0.$$
$$(\Delta^2 - \Delta)(\Delta - 2) - 6 = 0$$
$$\Delta^3 - 2\Delta^2 - \Delta^2 + 2\Delta - 6 = 0$$
$$\Delta^3 - 3\Delta^2 + 2\Delta - 6 = 0$$
$$\Delta^2(\Delta - 3) + 2(\Delta - 3) = 0$$
$$(\Delta - 3)(\Delta^2 + 2) = 0.$$

$$\boxed{\Delta = 3} \qquad \Delta^2 = -2$$
$$\Delta = \pm\sqrt{2}\, i$$

$$y = \cancel{c_1 e^{3t} + c_2 t e^{\sqrt{2} i t} + c_3 t^2 e^{-\sqrt{2} i t}}$$
$$= \cancel{c_1 x^3 + c_2 \ln x \cdot x^{\sqrt{2}} + c_3 \ln x^2 \cdot x^{-\sqrt{2}}}$$

$$y = c_1 e^{3t} + e^{0t}\left(A\cos\sqrt{2}t + B\sin\sqrt{2}t\right)$$
$$= c_1 e^{3\ln x} + \left[A\cos(\sqrt{2}\ln x) + B\sin(\sqrt{2}\ln x)\right].$$
$$y = c_1 x^3 + A\cos(\sqrt{2}\ln x) + B\sin(\sqrt{2}\ln x)$$

Ans!

Q16 $x^3y''' + xy' - y = 0$

SOLUTION:-

$x^3D^3y + xDy - y = 0$

$\Delta(\Delta-1)(\Delta-2) + \Delta - 1 = 0$

$(\Delta^2 - \Delta)(\Delta-2) + \Delta - 1 = 0$

$\Delta^3 - 2\Delta^2 - \Delta^2 + 2\Delta + \Delta - 1 = 0$

$\Delta^3 - 3\Delta^2 + 3\Delta - 1 = 0$

Let $\Delta = 1$

$1 - 3 + 3 - 1 = 0$

$0 = 0$

$\boxed{\Delta = 1}$

1	1	-3	+3	-1
		1	-2	1
	1	-2	1	0

$\Delta^2 - 2\Delta + 1 = 0$

$\Delta^2 - \Delta - \Delta + 1 = 0$

$\Delta(\Delta-1) - 1(\Delta-1) = 0$

$\Delta^2 = 1$

$y = c_1e^t + c_2te^t + c_3t^2e^t$

$= c_1x + c_2\ln x \cdot x + c_3(\ln x)^2 \cdot x$. Ans!

EXERCISE 4.9

Q. $\frac{dx}{dt} = 2x - y$, $\frac{dy}{dt} = x$

SOLUTION:-

let $D = \frac{d}{dt}$

$Dx = 2x - y$ ——— (i)

$Dy = x$ ——— (ii)

(i) $\Rightarrow$

$Dx = \cancel{2x} - y.$

$-2Dy = \cancel{-2x}$

$Dx - 2Dy = -y$

(i) $\Rightarrow$ $Dx - 2x + y = 0$ $\Rightarrow$ $(D-2)x + y = 0$

(ii) $\Rightarrow$ $Dy - x = 0$, $\Rightarrow$ $Dy - x = 0$

$\cancel{(D-2)x} + y = 0$

$-\cancel{(D-2)x} + Dy(D-2) = 0$

$D^2y - 2Dy + y = 0$ ——— (iii)

The auxilary eq^n of (iii) is

$D^2 - 2D + 1 = 0.$

$D^2 - D - D + 1 = 0.$

$D(D-1) - 1(D-1) = 0$

$\boxed{D = 1}$ $\boxed{D = -1}$

$$\boxed{y_{(t)} = c_1 e^t + c_2 t e^t}$$

(i) => $D^2 x = 2Dx - Dy$

(ii) => $-Dy = -x$

$D^2 x = 2Dx - x$.

$D^2 x - 2Dx + x = 0$.

Auxilary eq^n will become

$D^2 - 2D + 1 = 0$.

$D^2 - D - D + 1 = 0$.

$\boxed{D = 1}$, $\boxed{D = 1}$

$x_t = c_3 e^t + c_4 t e^t$

Q3 $\frac{dx}{dt} = -y + t$, $\frac{dy}{dt} = x - t$

Solution:-

$Dx = -y + t \Rightarrow Dx + y = t$ ...(i)

$Dy = x - t \Rightarrow Dy - x = -t$ ...(ii)

x. eq^n (ii) b D Then add with (i)

$Dx + y = t$

$D^2 y - Dx = -Dt$.

$D^2 y + y = t - 1$ ——— (3)

Auxilary

m

m

$y_c =$

For y_p

f

f

=

=

=

=

y_p

$y =$

Auxiliary eq^n of (3) is

$$m^2 + 1 = 0$$

$$m^2 = -1$$

$$m = \pm i$$

$$y_c = c_1 \cos t + c_2 \sin t$$

For y_p:

$$f(D) = D^2 + 1$$

$$f(t) = t - 1$$

$$= \frac{1}{D^2+1} \; t - 1$$

$$\cancel{\frac{1}{(t-1)^2+1}} \; t - 1$$

$$= \frac{1}{1-(-D^2)} \; t - 1$$

$$= \{1 - (-D^2)\}^{-1} \; t - 1$$

$$= \{1 + (-D^2) + (-D^2)^2 + \cdots\} \; t - 1$$

$$y_p = t - 1$$

$$y = c_1 \cos t + c_2 \sin t + t - 1$$

for x_c

$\times$ eq^n (i) by $-D$.

$$-D^2x - Dy = 1 \quad \text{---} \quad (4)$$

$$Dy - x = -t \quad \text{---} \quad (5)$$

$$-D^2x - x = 1 - t.$$

$$D^2x + x = t - 1$$

Auxilary eq^n.

$$m^2 + 1 = 0.$$

$$m = \pm i$$

$$x_c = C_3 \cos t + C_4 \sin t.$$

for x_p.

$$f(D) = D^2 + 1$$

$$f(t) = t + 1$$

$$= \frac{1}{D^2+1} \; t + 1$$

$$= \frac{1}{1-(-D^2)} \; t + 1$$

$$= \{1 - (-D^2)\}^{-1} \; t + 1$$

$$= \{1 - (D^2)^2 - (D^2)^2\} \; t + 1$$

$$x_p = t + 1$$

$$x = c_3 \cos t + c_4 \sin t + t + 1$$

Put x and y in eq (i).

$$D\left[c_3 \cos t + c_4 \sin t + t + 1\right] + c_1 \cos t + c_2 \sin t + \cancel{t} - 1 = \cancel{t}.$$

$$-c_3 \sin t + c_4 \cos t + \cancel{1} + c_1 \cos t + c_2 \sin t - \cancel{1} = 0$$

$$(c_2 - c_3) \sin t + (c_1 + c_4) \cos t = 0 \quad (a)$$

Put x and y in (ii).

$$D\left[c_1 \cos t + c_2 \sin t + t - 1\right] - \left[c_3 \cos t + c_4 \sin t + t + 1\right] = -t.$$

$$-c_1 \sin t + c_2 \cos t + \cancel{1} - c_3 \cos t - c_4 \sin t - \cancel{t} - \cancel{1} = -\cancel{t}$$

$$(-c_4 - c_1) \sin t + (c_2 - c_3) \cos t = 0. \quad (b)$$

$$(c_2 - c_3) \sin t = (c_2 - c_3) \cos t.$$

$$c_2 \sin t - c_3 \sin t = c_2 \cos t - c_3 \cos t$$

$$c_2 (\sin t - \cos t) = c_3 (\sin t - \cos t)$$

$$\boxed{c_2 = c_3}$$

$(c_1 + c_4)\cos t = -(c_4 - c_1)\sin t$

$c_1 \cos t + c_4 \cos t = -c_4 \sin t - c_1 \sin t$.

$c_1(\cos t + \sin t) = c_4(-\sin t - \cos t)$.

$c_1 = -c_4$

$x = c_2 \cos t - c_1 \sin t + t + 1$

$y = c_1 \cos t + c_2 \sin t + t - 1$

Q5 $(D^2+5)x - 2y = 0$ —— (i)

$-2x + (D^2+2)y = 0$. —— (ii)

$\cancel{(D^2+5)x} - 2y = 0$

$\cancel{-(D^2+5)x} + \frac{(D^2+2)(D^2+5)}{\cancel{(D^2+5)}\,2} y = 0$. $\times\frac{(D^2+5)}{2}$

$-4y + (D^2+2)(D^2+5)y = 0$.

$-4 + D^4 + 5D^2 + 2D^2 + 10 = 0$.

$D^4 + 7D^2 + 6 = 0$.

$D^4 + 6D^2 + D^2 + 6 = 0$.

$D^2(D^2+6) + (D^2+6)$

$(D^2+6)(D^2+1)$.

$D^2 = -6$ $D^2 = -1$

$D = \pm\sqrt{6}\,i$ $D = \pm i$

$$y = c_1 \cos t + c_2 \sin t + c_3 \cos \sqrt{6}t + c_4 \sin \sqrt{6}t$$

$$\frac{(D^2+5)(D^2+2)x}{2} - \cancel{(D^2+2)y} = 0.$$

$$-2x \quad + \cancel{(D^2+2)y} = 0.$$

$$(D^4 + 2D^2 + 5D^2 + 10)x - 4x = 0.$$

$$D^4 + 7D^2 + 6 = 0.$$

$$D^4 + 6D^2 + D^2 + 6 = 0$$

$$D^2(D^2+6) + (D^2+6) = 0$$

$$(D^2+6)(D^2+1) = 0$$

$$D = \pm 6\sqrt{i} \qquad D = \pm i$$

$$x = c_5 \cos t + c_6 \sin t + c_7 \cos \sqrt{6}t + c_8 \sin \sqrt{6}t.$$

Put x and y in (i).

EXERCISE 6.2

Q7 $y'' + xy = 0$

SOLUTION:-

$$Y = \sum_{n=0}^{\infty} C_n x^n$$

$$Y' = \sum_{n=1}^{\infty} n C_n x^{n-1}$$

$$Y'' = \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2}.$$

Question now become.

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} + x \cdot \sum_{n=0}^{\infty} C_n x^n = 0.$$

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} + \sum_{n=0}^{\infty} C_n x^{n+1} = 0$$

Put $K = n-2$ in first series and $K = n+1$ in 2^{nd}.

$$\sum_{K=0}^{\infty} (K+2)(K+1) C_{K+2} x^K + \sum_{K=1}^{\infty} C_{K+1} x^K = 0$$

$$\sum_{K=1}^{\infty} (K+2)(K+1) C_{K+2} x^K + 2C_2 + \sum_{K=1}^{\infty} C_{K+1} x^K = 0.$$

$$\sum_{K=1}^{\infty} \left\{ (K+2)(K+1) C_{K+2} + C_{K+1} \right\} x^K + 2C_2 = 0.$$

$2C_2 = 0$

$\boxed{C_2 = 0}$

$(K+2)(K+1)C_{K+2} + C_{K-1} = 0$

$$C_{K+2} = \frac{-C_{K-1}}{(K+2)(K+1)}$$

Put $K=1$

$$C_3 = \frac{-C_0}{6}$$

Put $K=2$

$$C_4 = \frac{-C_1}{12}$$

$K=3$

$$C_5 = \frac{C_2}{5 \cdot 4} = 0$$

$K=4$

$$C_6 = \frac{-C_3}{6 \cdot 5} = \frac{C_0}{180}$$

$$y_1(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \cdots$$

$$= C_0 + C_1 x + 0 + \left(\frac{-C_0}{6}\right)x^3 + \frac{-C_1 x^4}{6} + 0 + \frac{C_0 x^6}{180}$$

$$y = C_0\left(1 - \frac{x^3}{6}\right) + C_1\left(x - \frac{x^4}{6}\right)$$

Q9 $y'' - 2xy' + y = 0$

Let $y = \sum_{n=0}^{\infty} C_n x^n$.

$y' = \sum_{n=1}^{\infty} n C_n x^{n-1}$

$y'' = \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2}$

$\Rightarrow \sum_{n=2}^{\infty} n(n-1)C_n x^{n-2} - 2\sum_{n=1}^{\infty} nC_n x^{n-1} + \sum_{n=0}^{\infty} C_n x^n$

$2(2-1)C_2 x^0 + \sum_{n=3}^{\infty} n(n-1)C_n x^{n-2} - 2\sum_{n=1}^{\infty} nC_n x^{n-1}$

$+ \sum_{n=0}^{\infty} C_n x^n$

$2C_2 + \sum_{n=3}^{\infty} n(n-1)C_n x^{n-2} - 2\sum_{n=1}^{\infty} nC_n x^{n} + \sum_{n=0}^{\infty} C_n x^n$

$K = n-2$ $\quad$ $K = n$ $\quad$ $K = n$.

$n = K+2$

$2C_2 + \sum_{K=1}^{\infty} (K+2)(K+1)C_{K+1} x^{K} - 2\sum_{K=0}^{\infty} (K)C_{K} x^{K} +$

$\sum_{K=0}^{\infty} C_K x^K$.

$2C_2 + \sum_{K=1}^{\infty} (K+2)(K+1)C_{K+1} x^K - 2\sum_{K=1}^{\infty} (K)C_{K} x^K$

$C_0 + \sum_{K=1}^{\infty} C_K x^K$.

$$2C_2 + \sum_{k=1}^{\infty} (k+2)(k+1) C_{k+2} x^k - 2\sum_{k=1}^{\infty} (k+1) C_{k+1} x^k$$

$$+ C_0 + \sum_{k=1}^{\infty} C_k x^k = 0.$$

$$2C_2 + C_0 + \sum_{k=1}^{\infty} \left[\overset{C_{k+2}}{(k+2)(k+1)} - 2(k+1)C_{k+1} + C_k \right] x^k = 0$$

$$2C_2 + C_0 = 0$$

$$\boxed{C_2 = \frac{-C_0}{2}}$$

$$C_{k+2}(k+2)(k+1) - 2C_k k + C_k = 0.$$

$$\boxed{C_{k+2} = \frac{2C_k k - C_k}{(k+2)(k+1)}}$$

Put $k = 1$

$$\boxed{C_3 = \frac{C_1}{C_0}}$$

Put $k = 0$.

$$C_2 = -\frac{C_0}{2}.$$

Put $k = 2$.

$$C_4 = \frac{3C_2}{12} = -\frac{\cancel{3}C_0}{4 \cdot \cancel{12} \cdot 2} = -\frac{C_0}{8}.$$

Put

Put

Put

y

$y =$

Put $K = 3$.

$$C_5 = \frac{\cancel{5}\, C_3}{20} = \frac{\cancel{5}\, C_1}{{}_4\cancel{20} \cdot 6} = \frac{C_1}{24}.$$

Put $K = 4$.

$$C_6 = \frac{7 C_4}{30} = \frac{-7 C_0}{30 \cdot 8}.$$

Put $K = 5$

$$C_7 = \frac{9 C_5}{42} = \frac{9 C_1}{42 \cdot 24}.$$

$$y = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6 + C_7 x^7$$

$$y = C_0 + C_1 x - \frac{C_0 x^2}{2} + \frac{C_1}{6} x^3 + \left(-\frac{C_0}{8}\right) x^4 + \left(\frac{C_1}{24}\right) x^5 + \left(\frac{-7 C_0}{30 \cdot 8}\right) x^6 + \left(\frac{9 C_1}{42 \cdot 24}\right)$$

A

EXERCISE 7.1

Q19 $f(t) = 2t^4$

SOLUTION:–

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$= 2\frac{4!}{s^{4+1}}$$

$$= 2\frac{4!}{s^5}$$ Ans!,

Q21 $f(t) = 4t - 10$

SOLUTION:–

$$\mathcal{L}\{t\} = \frac{4}{s^2} - \frac{10}{s}$$

$\because$ $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$

$\mathcal{L}\{1\} = \frac{1}{s}$.

Q23 $f(t) = t^2 + 6t - 3$

SOLUTION:–

$$\mathcal{L}\{t\} = \frac{2!}{s^3} + \frac{6}{s^2} - \frac{3}{s}.$$

Q25 $f(t) = (t+1)^3$

SOLUTION:-

$$f(t) = t^3 + 3t^2 + 3t + 1$$

$$\mathcal{L}\{(t+1)^3\} = \frac{6}{t^4} + \frac{3 \cdot 2!}{s^3} + \frac{3}{s^2} + \frac{1}{s}$$

Q27 $f(t) = 1 + e^{4t}$

SOLUTION:-

$$\mathcal{L}\{(1+e^{4t})\} = \frac{1}{s} + \frac{1}{s-4}$$

$$\because \mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

Q29 $f(t) = (1 + e^{2t})^2$

SOLUTION:-

$$f(t) = 1 + 2e^{2t} + e^{4t}$$

$$\mathcal{L}\{(1+e^{2t})^2\} = \frac{1}{s} + 2 \cdot \frac{1}{s-2} + \frac{1}{s-4}$$

Q31 $f(t) = 4t^2 - 5\sin 3t$

SOLUTION:-

$$\mathcal{L}\{4t^2 - 5\sin 3t\} = 4 \cdot \frac{2}{s^3} - 5\frac{3}{s^2+9}$$

Q33 $f(t) = \sinh kt$

SOLUTION:-

$$\mathcal{L}\{\sinh kt\} = \frac{k}{s^2 - k^2}.$$

Q35 $f(t) = e^t \sinh t$

SOLUTION:-

$$f(t) = e^t\left(\frac{e^t - e^{-t}}{2}\right)$$

$$= \frac{e^{2t} - e^0}{2} = \frac{e^{2t}}{2} - \frac{1}{2}$$

$$= \frac{1}{2(s-2)} - \frac{1}{2s}$$

Q39 $f(t) = \sin(4t+5)$

SOLUTION:-

$$f(t) = \sin 4t \cos 5 + \cos 4t \sin 5.$$

$$\mathcal{L}\{\sin(4t+5)\} = \cos 5 \, \mathcal{L}\{\sin 4t\} + \sin 5 \, \{\cos 4t\}$$

$$= \cos 5 \frac{4}{s^2+16} + \sin 5 \frac{s}{s^2+16}.$$

$$= \frac{4\cos 5 + s \sin 5}{s^2+16}.$$

Q37 $f(t) = \sin 2t \cos 2t$

SOLUTION:-

$$f(t) = \sin 2t \cos 2t$$

$$= \frac{1}{2}\left[\sin(2t+2t) + \sin(2t-2t)\right]$$

$$f(t) = \frac{1}{2} \sin 4t$$

$$\mathcal{L}\{\sin 2t \cos 2t\} = \frac{1}{2} \frac{4}{s^2+16}$$

$$= \frac{2}{s^2+16}$$

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→ INVERSE TRANSFORM:-

EXERCISE 7.2

Q1 $\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}$

SOLUTION:-

$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2!}{s^{2+1}}\right\}$

$= \frac{1}{2}t^2$ Ans!

Q3 $\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{48}{s^5}\right\}$

SOLUTION:-

$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \mathcal{L}^{-1}\left\{\frac{48}{s^5}\right\}$

$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \frac{48}{4!}\mathcal{L}^{-1}\left\{\frac{4!}{s^5}\right\}$

$t - \frac{48}{24}t^4$

$t - 2t^4$ Ans!

Q5 $\mathcal{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\}$

SOLUTION:-

$$\mathcal{L}^{-1}\left\{\frac{s^3+3s^2+3s+1}{s^4}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}+\frac{3}{s^2}+\frac{3}{s^3}+\frac{1}{s^4}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}+3\cdot\frac{1}{s^2}+\frac{3}{2!}\frac{2!}{s^3}+\frac{1}{3!}\frac{3!}{s^4}\right\}$$

$$1+3t+\frac{3}{2}t^2+\frac{1}{6}t^3$$ Ans!

Q7 $\mathcal{L}^{-1}\left\{\frac{1}{s^2}-\frac{1}{s}+\frac{1}{s-2}\right\}$

SOLUTION:-

$$t-1+e^{2t}$$ Ans!

Q9 $\mathcal{L}^{-1}\left\{\dfrac{1}{4s+1}\right\}$

SOLUTION:-

$\mathcal{L}^{-1}\left\{\dfrac{1}{4(s+1/4)}\right\}$

$\dfrac{1}{4}\mathcal{L}^{-1}\left\{\dfrac{1}{s+(-1/4)}\right\}$

$\dfrac{1}{4}e^{-1/4t}$ Ans!

Q11 $\mathcal{L}^{-1}\left\{\dfrac{5}{s^2+49}\right\}$

SOLUTION:-

$\dfrac{5}{7}\mathcal{L}^{-1}\left\{\dfrac{7}{s^2+49}\right\}$

$\dfrac{5}{7}\sin 7t$ Ans!

Q13. $\mathcal{L}^{-1}\left\{\dfrac{4s}{4s^2+1}\right\}$

SOLUTION:-

$$\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+(1/2)^2}\right\}$$

$$= \cos\frac{1}{2}t$$ Ans!

Q15 $\mathcal{L}^{-1}\left\{\dfrac{2s-6}{s^2+9}\right\}$

SOLUTION:-

$$2\,\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+9}\right\} - \mathcal{L}^{-1}\left\{\dfrac{6}{s^2+9}\right\}$$

$$2\cos 3t - 2\sin 3t$$ Ans!

Q17 $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+3s}\right\}$

SOLUTION:-

let $\dfrac{1}{s^2+3s} = \dfrac{A}{s} + \dfrac{B}{s+3}$ —— (A)

~~Put~~ $1 = A(s+3) + Bs$ —— (B)

put $s = -3$.

(B) $\Rightarrow$ $1 = -3B$, $-\dfrac{1}{3} = B$

Put $s = 0$ in eqⁿ (B).

$$1 = 3A$$

$$\frac{1}{3} = A.$$

(A) $\Rightarrow$ $$\frac{1}{s^2+3s} = \frac{1}{3s} + \left(\frac{-1}{3(s+3)}\right)$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{3s} - \frac{1}{3(s+3)}\right\}$$

$$= \frac{1}{3} - \frac{1}{3}e^{-3t} \quad \text{Ans!}$$

Q19 $$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s-3}\right\}$$

Solution:-

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+3s-s-3}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s(s+3)-1(s+3)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s+3)(s-1)}\right\} \quad \text{——— (1)}$$

Let $$\frac{s}{(s+3)(s-1)} = \frac{A}{s+3} + \frac{B}{s-1} \quad \text{——— (2)}$$

$$s = A(s-1) + B(s+3) \quad \text{——— (3)}$$

Put $s = 1$ in eq^n (3)

$$1 = 4B.$$

$$\frac{1}{4} = B.$$

Put $s = -3$ in eq^n (3)

$$-3 = A(-4)$$

$$\frac{3}{4} = A$$

Now eq^n (1) will be

$$\mathcal{L}^{-1}\left\{\frac{3}{4}\frac{1}{s+3} + \frac{1}{4}\frac{1}{s-1}\right\}$$

$$\frac{3}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-(-3)}\right\} + \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}$$

$$\frac{3}{4}e^{-3t} + \frac{1}{4}e^{t}.$$ Ans!

Put

Put

Questio

Q4 $\mathcal{L}^{-1}\left\{\dfrac{0.9s}{(s-0.1)(s+0.2)}\right\}$

Solution:-

Let $\dfrac{0.9s}{(s-0.1)(s+0.2)} = \dfrac{A}{(s-0.1)} + \dfrac{B}{(s+0.2)}$

$$0.9s = A(s+0.2) + B(s-0.1) \quad (1)$$

Put $s = -0.2$ in eq^n (1)

$$(0.9)(-0.2) = 0 + B(-0.2-0.1)$$

$$-0.18 = B(-0.3).$$

$$0.6 = B$$

Put $s = 0.1$ in eq^n (1).

$$0.09 = A(0.3).$$

$$0.3 = A.$$

Question now become.

$$\mathcal{L}^{-1}\left\{\frac{0.3}{(s-0.1)} + \frac{0.6}{(s+0.2)}\right\}$$

$$0.3\,e^{+0.1t} + 0.6\,e^{-0.2t}$$ Ans.

Q23 $\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s-3)(s-6)}\right\}$

SOLUTION:-

let $\frac{s}{(s-2)(s-3)(s-6)} = \frac{A}{s-2} + \frac{B}{s-3} + \frac{C}{s-6}$.

$s = A(s-3)(s-6) + B(s-2)(s-6) + C(s-2)(s-3)$ ——— (1).

Put $s = 3$ in eq^n (1).

$3 = B(1)(-3)$.

$-1 = B$.

Put $s = 6$. in eq^n (1).

$6 = C(4)(3)$.

$\frac{1}{2} = C$.

Put $s = 2$ in eq^n (1).

$2 = A(-1)(-4)$

$\frac{1}{2} = A$

Question will now become.

$$\mathcal{L}^{-1}\left\{\frac{1}{2(s-2)} + \left(\frac{-1}{s-3}\right) + \frac{1}{2(s-6)}\right\}$$

$$\frac{1}{2}e^{2t} + - e^{3t} + \frac{1}{2}e^{6t}$$ Ans!

Q25 $\mathcal{L}^{-1}\left\{\frac{1}{s^3+5s}\right\}$

SOLUTION:-

let $\frac{1}{s^3+5s} = \frac{A}{s} + \frac{Bs}{s^2+5}$

$1 = A(s^2+5) + Bs \cdot$ ——— (1)

Put $s = 0$ in (1)

$1 = 5A \Rightarrow A = 1/5$

Put $A = 1/5$ and $s = 1$ in (1).

$1 = 1/5\,(6) + B.$

$1 - 6/5 = B.$

$-1/5 = B.$

Question will become

$$\mathcal{L}^{-1}\left\{\frac{1}{5s} + \left[\frac{-s}{5(s^2+5)}\right]\right\}$$

$$\frac{1}{5} - \frac{1}{5}\cos\sqrt{5}t$$ Ans!

Q27 $$\mathcal{L}^{-1}\left\{\frac{2s-4}{(s^2+s)(s^2+1)}\right\}$$

SOLUTION:-

$$\mathcal{L}^{-1}\left\{\frac{2s-4}{s(s+1)(s^2+1)}\right\}$$

let $$\frac{2s-4}{s(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{Cs}{(s^2+1)}$$

$$2s-4 = A(s+1)(s^2+1) + Bs(s^2+1) + Cs^2(s+1) \quad \text{——— (1)}$$

Put $s = -1$ in eqn (1)

$$-6 = B(-1)(2)$$

$$\boxed{-3 = B}$$

Put $s = 0$ in eqn (1)

$$-4 = A(1)(1)$$

$$\boxed{-4 = A}$$

Put $s = 1$, $A = -4$ & $B = -3$ in eqⁿ (1)

$$-2 = -4(2)(2) + -3(1)(2) + C(1)(2).$$

$$-2 = -16 - 6 + 2C$$

$$\frac{-2+22}{2} = C = 10$$

Question now become,

$$\mathcal{L}^{-1}\left\{ \frac{-4}{s} + \frac{3}{s+1} + \frac{10s}{s^2+1} \right\}$$

$$-4 + 3e^{-t} + 10\cos t$$

Q29 $\mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)(s^2+4)}\right\}$

Solution:-

let $\frac{1}{(s^2+1)(s^2+4)} = \frac{A}{s^2+1} + \frac{B}{s^2+4}$.

$1 = A(s^2+4) + B(s^2+1)$. ——— (1).

Put $s = \sqrt{-4}$ in eqn (1).

$1 = 0 + B(-3)$.

$-1/3 = B$.

Put $s = \sqrt{-1}$ in eqn (1).

$1 = A(3) + 0$.

$1/3 = A$

Question now become.

$\mathcal{L}^{-1}\left\{\frac{1}{3}\cdot\frac{1}{(s^2+1)} + \left(\frac{-1}{3}\right)\cdot\frac{1}{s^2+4}\right\}$

$\frac{1}{3}\sin t - \frac{1}{6}\sin 2t$ Ans!

Q35 $\frac{dy}{dt} - y = 1, \quad y(0) = 0$

SOLUTION:-

$$\mathcal{L}\left\{\frac{dy}{dt} - y\right\} = \mathcal{L}\{1\}$$

$$\mathcal{L}\left\{\frac{dy}{dt}\right\} - \mathcal{L}\{y\} = \frac{1}{s}.$$

$$sY(s) - y(0) - Y(s) = \frac{1}{s}.$$

$$Y(s)(s-1) = \frac{1}{s}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\} \quad \text{—— (A)}$$

$$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$$1 = A(s-1) + Bs.$$

Putting $s = 0$.

$$1 = -A$$

$$\boxed{-1 = A}$$

Putting $s = 1$

$$\boxed{1 = B}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{-\frac{1}{s} + \frac{1}{s-1}\right\}$$

$$= -t^{0} + e^{t}.$$

$$= -1 + e^{t} \quad \text{Ans!}$$

Q37 $y' + 6y = e^{4t}, \quad y(0) = 2.$

SOLUTION:-

$$\mathcal{L}\{y'\} + 6\mathcal{L}\{y\} = \mathcal{L}\{e^{4t}\}$$

$$= \frac{1}{s-4}$$

$$sY(s) + 6Y(s) - y(0) = \frac{1}{s-4}$$

$$(s+6)Y(s) - 2 = \frac{1}{s-4}$$

$$(s+6)Y(s) = \frac{1+s-4}{s-4}$$

$$Y(s) = \frac{s-3}{(s+6)(s-4)} \quad \text{—— (A)}$$

let $\dfrac{s-3}{(s+6)(s-4)} = \dfrac{A}{(s+6)} + \dfrac{B}{(s-4)}.$

$$s-3 = (s-4)A + (s+6)B$$

Put $s = 4.$

$$1 = 10B. \implies \frac{1}{10} = B$$

Put $s = -6$.

$-9 = -10A$.

$\frac{9}{10} = A$.

Ⓐ $\Rightarrow$ $Y(s) = \frac{9}{10(s+6)} + \frac{1}{10(s-4)}$.

$Y(s) = \frac{9}{10} e^{-6t} + \frac{1}{10} e^{4t}$

Q38 $y' - y = 2\cos 5t$. $y(0) = 0$

SOLUTION:

$\mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{2\cos 5t\}$

$sY(s) - y(0) + Y(s) = \frac{2s}{s^2+25}$.

$sY(s) - Y(s) = \frac{2s}{s^2+25}$

$(s-1)\,Y(s) = \frac{2s}{s^2+25}$.

$Y(s) = \frac{2s}{(s^2+25)(s-1)}$.

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$$\frac{2s}{(s^2+25)(s-1)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+25}$$

$$2s = A(s^2+25) + Bs(s-1) + C(s-1)$$

Put $s = \cancel{25}\ 1$

$$2 = A + 25A$$

$$2 = 26A$$

$$\frac{1}{13} = A$$

Put $s = 0$ and $A = 1/13$

$$0 = \frac{25}{13} + (-C)$$

$$C = \frac{25}{13}$$

~~Put $s = 1$~~, $A = 1/13$ and $C = 25/13$

$$2 = \frac{26}{13} + B$$

Put $s = 2$

$$4 = \frac{29}{13} + 2B + \frac{25}{13}$$

$$-\frac{1}{13} = B$$

$$Y(s) = \frac{1}{13}\cdot\frac{1}{s-1} + \left(-\frac{1}{13}\right)\cdot\frac{s}{s^2+25} + \frac{5}{13}\,\frac{5}{s^2+25}$$

$$y(t) = \frac{1}{13}e^{t} - \frac{1}{13}\cos 5t + \frac{5}{13}\sin 5t$$

Ans!

Q40 $y'' - 4y' = 6e^{3t} - 3e^{-t}, \; y(0) = 1, \; y'(0) = -1$

SOLUTION:-

$$\mathcal{L}\{y''\} - \mathcal{L}\{-4y'\} = \mathcal{L}\{6e^{3t}\} - \mathcal{L}\{3e^{-t}\}$$

$$s^2Y(s) - sy(0) - y'(0) - 4[sY(s) - y(0)] = \frac{6}{s-3} - \frac{3}{s+1}$$

$$s^2Y(s) - 1s + 1 - 4sY(s) + 4 = \frac{6}{s-3} - \frac{3}{s+1}$$

$$s(s-4)Y(s) - 1s + 4 + 1 = \frac{6}{s-3} - \frac{3}{s+1}$$

$$(s^2 - 4s)Y_s - s + 5 = \frac{6}{s-3} - \frac{3}{s+1}$$

$$\cancel{Y(s)} = 6(s+1) - 3(s-3) +$$

$$(s^2 - 4s)Y_s = \frac{6}{s-3} - \frac{3}{s+1} + s - 5$$

$$Y(s) = \frac{6}{(s^2-4s)(s-3)} - \frac{3}{(s^2-4s)(s+1)} + \frac{s-5}{(s^2-4s)}$$

$$= \frac{6s \pm 6 + 3s - 9}{(s^2 - 4s)} + \frac{s - 5}{(s^2 - 4s)}$$

$$= \frac{9s - 3}{s^2 - 4s} + \frac{s - 5}{s^2 - 4s}$$

$$= \frac{9}{s - 4} - \frac{3}{s^2 - 4s} + \frac{s - 5}{s^2 - 4s}$$

$$= \frac{9}{s - 4} + \frac{-3 + s - 5}{s^2 - 4s}$$

$$= \frac{9}{s - 4} + \frac{s - 8}{s^2 - 4s}$$

$$= \frac{9}{s - 4} + \frac{1}{s - 4} - \frac{8}{s^2 - 4s}$$

$$= \frac{10}{s - 4} - \frac{8}{s^2 - 4s}$$

$$\frac{8}{s^2 - 4s} = \frac{A}{s} + \frac{B}{s - 4}$$

$$8 = (s - 4)A + Bs$$

Put $s = 4$

$$8 = 4B$$

$$2 = s$$

Put $s = 0$

$$8 = -4A$$

$$-2 = A$$

$$= \frac{10}{s-4} + \frac{2}{s} + \frac{2}{s-4}.$$

$$= 10e^{4t} + 2 + 2e^{4t}.$$

$$= 12e^{4t} + 2.$$ Ans!

EXERCISE 7.3

Q. $\mathcal{L}\{te^{10t}\}$

SOLUTION:-

$\mathcal{L}\{e^{10t}t\} = F(s-a)$

~~F(t)~~

$\mathcal{L}\{f(t)\}_{s \to a}$

$= F(s)\Big|_{s \to (s-a)}$

$= F(s-a).$

$a = 10 \quad , \quad f(t) = t.$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{t\} = \frac{1}{s^2}.$

$\mathcal{L}\{e^{at}f(t)\} = \left(\frac{1}{s^2}\right)_{s \to (s-a)}$

$= \frac{1}{(s-10)^2}$ Ans.

Q3 $\mathcal{L}\{t^3 e^{-2t}\}$

Solution.-

$\mathcal{L}\{e^{-2t} f(t)\}$

$\mathcal{L}\{e^{-2t} f(t)\} = \mathcal{L}\{f(t)\}_{s \to s-a}$

$= F(s-a)$.

$a = -2$, $f(t) = t^3$.

$\mathcal{L}\{f(t)\} = F(s)$.

$\mathcal{L}\{t^3\} = \frac{3!}{s^4}$

$\mathcal{L}\{t^3\} = \left[\frac{3!}{s^4}\right]_{s \to (s-a)}$

$\mathcal{L}\{t^3\} = \frac{3!}{(s+2)^4}$

Q5 $\mathcal{L}\{t(e^t + e^{2t})^2\}$

SOLUTION:-

$\mathcal{L}\{te^{2t} + 2te^{3t} + te^{4t}\}$.

$\mathcal{L}\{t\}_{s \to s-a}$

$= \frac{1}{s^2}$

$$= \frac{1}{(s-2)^2} + \frac{2}{(s-3)^2} + \frac{1}{(s-4)^2}$$

Q7 $\mathcal{L}\{e^t \sin 3t\}$

SOLUTION :-

$\mathcal{L}\{\sin 3t\}_{s \to s-a}$

$\mathcal{L}\{\sin 3t\}_{s \to s-1}$

$$\mathcal{L} = \left. \frac{3}{s^2+9} \right|_{s \to s-1}$$

$$= \frac{3}{(s-1)^2+9}.$$

~~Q9 $\mathcal{L}\{e^{3t}(9-$~~

Q19 $\mathcal{L}\{(1-e^t+3e^{-4t})\cos 5t\}$

SOLUTION :-

$\mathcal{L}\{\cos 5t - e^t \cos 5t + 3e^{-4t}\cos 5t\}$

$\mathcal{L}\{\cos 5t\} - \mathcal{L}\{\cos 5t\}_{s \to (s-1)} + 3\mathcal{L}\{\cos 5t\}_{s \to (s+4)}$

$$\frac{s}{s^2+25} - \frac{s-1}{(s-1)^2+25} + \frac{3(s+4)}{(s+4)^2+25} \text{ Ans.}$$

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Q_{11} $\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^3}\right\}$

SOLUTION:-

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}_{s \to (s-(-2))}$$

$$\frac{1}{2!}\mathcal{L}^{-1}\left\{\frac{2!}{s^3}\right\}_{s \to (s-(-2))}$$

$$\frac{1}{2} t^2 e^{-2t}$$

Ans!

Q_{13} $\mathcal{L}^{-1}\left\{\frac{1}{s^2-6s+10}\right\}$

SOLUTION:-

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2+10-9}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}_{s \to (s-3)}.$$

$$e^{3t} \sin t$$

Ans!

Q15 $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\}$

SOLUTION :-

$$\mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s+2-2}{(s+2)^2+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{2}{(s+2)^2+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}_{s\to(s+2)-(-2)} - \mathcal{L}^{-1}\left\{\frac{2}{s^2+1}\right\}_{s\to s-(-2)}$$

$$e^{-2t}\cos t - 2e^{-2t}\sin t$$

Q17 $\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2}\right\}$

SOLUTION :-

$$\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2} - \frac{1}{(s+1)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s^2}\right\}_{s\to s-(-1)}$$

$$e^{-t} - e^{-t}t \quad \text{Ans}$$

Q21 $y' + 4y = e^{-4t}$, $y(0) = 2$.

SOLUTION:-

$$\mathcal{L}\{y' + 4y\} = \mathcal{L}\{e^{-4t}\}$$

$$sY(s) + 4Y(s) - y(0) = \frac{1}{s+4}.$$

$$(s+4)Y(s) - 2 = \frac{1}{s+4}$$

$$(s+4)Y(s) = \frac{1+s+4}{s+4}$$

$$Y(s) = \frac{s+5}{(s+4)^2}$$

$$Y(s) = \frac{2s}{(s+4)^2} + \frac{5}{(s+4)^2}$$

$$Y(s) = \frac{s+4}{(s+4)^2} - \frac{4}{(s+4)^2} + \frac{5}{(s+4)^2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\}_{s\to s-(-4)} - 4\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}_{s\to s-(-4)} + 5\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}_{s\to s-(-4)}$$

$$= e^{-4t} - 4te^{-4t} + 5te^{-4t}.$$

$$= e^{-4t} + te^{-4t}.$$

Q21 $y' + 4y = e^{-4t}$, $y(0) = 2$

SOLUTION:-

$$\mathcal{L}\{y' + 4y\} = \mathcal{L}\{e^{-4t}\}$$

$$sY(s) - y(0) + 4Y(s) = \frac{1}{s+4}$$

$$(s+4)Y(s) - 2 = \frac{1}{s+4}$$

$$Y(s)(s+4) = \frac{1 + 2s + 8}{s+4}$$

$$Y(s) = \frac{2s+9}{(s+4)^2}$$

$$Y(s) = 2\left[\frac{s+4}{(s+4)^2} - \frac{4}{(s+4)^2}\right] + \frac{9}{(s+4)^2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = 2\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}_{s \to s-(-4)} - 8\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}_{s \to s-(-4)}$$

$$+ 9\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}_{s \to s-(-4)}$$

$$= 2e^{-4t} - 8e^{-4t}t + 9e^{-4t}t$$

$$= 2e^{-4t} + te^{-4t}$$

Ans!

27 $y'' - 6y' + 13y = 0 \quad , \quad y(0) = 0 \quad , \quad y'(0) = -3$

SOLUTION:-

$$s^2Y(s) - sy(0) - y'(0) - 6sY(s) + y(0) + 13Y(s) = 0$$

$$s^2Y(s) - 0 + 3 - 6sY(s) + 0 + 13Y(s) = 0$$

$$s^2Y(s) - 3Y(s) = -3.$$

$$(s^2 - 3)Y(s) = -3.$$

$$Y(s) = \frac{-3}{s^2 - 3}.$$

let $\dfrac{-3}{s^2-3} = \dfrac{A}{s} + \dfrac{B}{s^2-3}.$

$$\mathcal{L}^{-1}\{Y(s)\} =$$

$$(s^2 - 6s + 13)Y(s) = -3.$$

$$Y(s) = \frac{-3}{(s-3)^2 + 4}$$

$$Y(s) = \frac{-3}{2} \, \frac{2}{(s-3)^2 + 4}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \frac{-3}{2} \mathcal{L}^{-1}\left\{\frac{2}{(s-3)^2 + 2^2}\right\}$$

$$= \frac{-3}{2} e^{3t} \sin 2t.$$

Ans.

Q29 $y'' - y' = e^t \cos t, \quad y(0) = 0, \; y'(0) =$

SOLUTION:-

$\mathcal{L}\{y'' - y'\} = \mathcal{L}\{e^t \cos t\}$

$s^2 Y(s) - s\,y(0) - y'(0) - sY(s) + y(0) = \left.\dfrac{s}{s^2+1}\right|_{s \to s-}$

$s^2 Y(s) - 4 - sY(s) = \dfrac{s-1}{(s-1)^2+1}$

$\cancel{(s^2 - s - 4)\, Y(s) = \dfrac{s-1}{s^2-2+1+1}}$

$\cancel{Y(s) = \dfrac{s-1}{s^2(s^2-s-4)}} \quad \text{--- (A)}$

$\cancel{\text{let } \dfrac{s-1}{s^2(s^2-s-4)} = \dfrac{A}{s} + \dfrac{Bs+C}{(s^2-s-4)}}$

$(s^2 - s)\, Y(s) = \dfrac{s-1}{(s-1)^2+1} + 4$

$\cancel{= \dfrac{s-1}{s^2-2s+2+1} + 4}$

$=$

$$s^2 Y(s) - s Y(s) = \frac{s-1}{s^2 - 2s + 1 + 1}$$

$$(s^2 - s)(Y(s)) = \frac{s-1}{s^2 - 2s + 2}$$

$$Y(s) = \frac{\cancel{s-1}}{s\cancel{(s-1)}(s^2 - 2s + 2)}$$

$$Y(s) = \frac{1}{s(s^2 - 2s + 2)} \quad \text{——— (A)}$$

let $\frac{1}{s(s^2-2s+2)} = \frac{A}{s} + \frac{Bs + C}{(s^2 - 2s + 2)}$

$$1 = A(s^2 - 2s + 2) + (Bs + c)s$$

$$= As^2 - 2As + 2A + Bs^2 + cs$$

$$1 = (A+B)s^2 + (-2A + c)s + 2A$$

$A + B = 0$

$\frac{1}{2} + B = 0$

$\boxed{B = -1/2}$

$-2A + c = 0$

$-2(1/2) + c = 0$

$\boxed{c = 1}$

$2A = 1$

$\boxed{A = 1/2}$

(A) $\Rightarrow$ $Y(s) = \frac{1}{2s} + \frac{-(1/2)s + 1}{s^2 - 2s + 2}$

$$= \frac{1}{2s} - \frac{s}{2(s^2 - 2s + 2)} + \frac{1}{s^2 - 2s + 2}$$

$$= \frac{1}{2s} - \frac{(s-1)+1}{2[(s-1)^2 + 1]} + \frac{1}{(s-1)^2 + 1}$$

$$= \frac{1}{2s} - \frac{1}{2}\frac{(s-1)}{(s-1)^2+1} + \frac{1}{(s-1)^2+1} + \frac{1}{(s-1)^2+1}$$

$$= \frac{1}{2} - \frac{1}{2}e^{-t}\cos t + 2e^{-t}\sin t$$

Ans!

Q37 $\mathcal{L}\{(t-1)\,u(t-1)\}$

Solution:-

~~Here $f(t-1) = t-1$~~

~~$a = 1$.~~

$\mathcal{L}\{f(t-a)u(t-a)\}$

Here $a = 1$

$F(t-a) = t-1$.

$f(t) = t$.

$$\mathcal{L}\{(t-1)u(t-1)\} = e^{-s}\mathcal{L}\{f(t)\}$$

$$= e^{-s}\mathcal{L}\{t\}$$

$$= \frac{e^{-s}}{s^2} \quad \text{Ans!}$$

Q39 $\mathcal{L}\{t\, u(t-2)\}$

SOLUTION:-

$$\mathcal{L}\{f(t-a)\, u(t-a)\}$$

$a = 2$

$f(t) = t$

$t = t-2$

$$\mathcal{L}\{t\, u(t-2)\} = e^{-2s}\, \mathcal{L}\{t\}$$

$$= \frac{e^{-2s}}{s^2} \quad \text{Ans!}$$

Q41 $\mathcal{L}\{\cos 2t\, u(t-\pi)\}$

SOLUTION:-

$$\mathcal{L}\{f(t-a)\, u(t-a)\}$$

$a = \pi$

$f(t) = \cos 2t$

$$\mathcal{L}\{\cancel{f}\cos 2t\, u(t-\pi)\} = e^{-\pi s}\, \mathcal{L}\{\cos 2t\}$$

$$= \frac{e^{-\pi s}\, s}{s^2+4} \quad \text{Ans!}$$

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Q43 $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\}$

$\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2!}{s^{2+1}} e^{-2s}\right\}$

$\frac{1}{2}(t-2)\,u(t-2)$

Q45 $\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\}$

$\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\}$

$\sin 1(t-\pi)u(t-\pi)$ Ans!

Q47 $\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}$

$\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)} e^{-s}\right\}$

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}.$$

$$1 = A(s+1) + Bs.$$

at $s = 0$.

$$1 = A$$

at $s = -1$

$$1 = -B.$$

$$-1 = B.$$

$$\mathcal{L}^{-1}\left\{\left(\frac{1}{s} - \frac{1}{s+1}\right)e^{-s}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}e^{-s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}e^{-s}\right\}$$

$$1\,\mu(t-1) - e^{(t-1)}\,\mu(t-1)$$

Ans!

EXERCISE 7.4

– DERIVATIVE OF TRANSFORM :-

Q1. $\mathcal{L}\{te^{-10t}\}$

SOLUTION :-

$\mathcal{L}\{t^n f(t)\}$

Here $n = 1$

$f(t) = e^{-10t}$.

$$\mathcal{L}\{t^1 e^{-10t}\} = (-1)^1 \frac{d}{ds} \mathcal{L}\{e^{-10t}\}$$

$$= -\frac{d}{ds}\left\{\frac{1}{s+10}\right\}$$

$$= -\left\{\frac{(s+10)\,0 - (1)(1)}{(s+10)^2}\right\}$$

$$= +\frac{1}{(s+10)^2} \quad \text{Ans!}$$

Q3. $\mathcal{L}\{t\cos 2t\}$

SOLUTION :-

$\mathcal{L}\{t^n f(t)\}$

Here $n = 1$, $f(t) = \cos 2t$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d}{ds} \mathcal{L}\{\cos 2t\}$$

$$= -\frac{d}{ds}\left(\frac{s}{s^2+4}\right)$$

$$= -\frac{d}{ds}\left(\frac{s}{s^2+4}\right)$$

$$= -\left\{\frac{(s^2+4)(1) - s(2s)}{(s^2+4)^2}\right\}$$

$$= -\left\{\frac{s^2+4-2s^2}{(s^2+4)^2}\right\}$$

$$= -\frac{4-s^2}{(s^2+4)^2}$$

$$= \frac{s^2-4}{(s^2+4)^2}$$

Q5: $\mathcal{L}\{t^2 \sinh t\}$

SOLUTION:-

Here $n = 2$.

$f(t) = \sinh t$

$$\mathcal{L}\{t^2 \sinh t\} = (-1)^2 \frac{d^2}{ds^2} \mathcal{L}\{\sinh t\}$$

$$= \frac{d^2}{ds^2}\left\{\frac{1}{s^2-1}\right\}$$

$$= \frac{d}{ds}\left\{\frac{(s^2-1)(0) - (1)(2s)}{(s^2-1)^2}\right\}$$

$$= \frac{d}{ds}\left\{\frac{-2s}{(s^2-1)^2}\right\}$$

Q7 $\mathcal{L}\{$

SOLUTION

Here

$= ($

$= -$

$= -$

$= -$

$= -$

$$= \left\{ \frac{(s^2-1)^2(-2) - (-2s)(2)\cancel{(s^2-1)}(2s)}{(s^2-1)^4} \right\}$$

$$= \left\{ \frac{(s^2-1)(-2) + 8s^2}{(s^2-1)^3} \right\}$$

$$= \left\{ \frac{-2(s^2-1) + 8s^2}{(s^2-1)^3} \right\}$$

$$= \frac{6s^2 + 2}{(s^2-1)^3} \quad \text{Ans!}$$

Q7 $\mathcal{L}\{te^{2t}\sin 6t\}$

Solution:-

Here $n = 1$

$f(t) = e^{2t}\sin 6t$

$$= (-1)^1 \frac{d}{ds} \mathcal{L}\{e^{2t}\sin 6t\}$$

$$= -\frac{d}{ds}\left\{\frac{6}{(s-2)^2 - 36}\right\}$$

$$= -\frac{d}{ds}\left(\frac{6}{s^2-4s+4-36}\right)$$

$$= -\frac{d}{ds}\left(\frac{6}{s^2-4s-32}\right)$$

~~$= -\frac{d}{ds}\Big((s^2-4s-32)(0) - (s^2-4s-$~~

$$= -\frac{d}{ds}\left(\frac{6}{s^2-4s-32}\right)$$

$$= -\left\{\frac{(s^2-4s-32)(0)-6(2s-4)}{s^2-4s-32}\right\}$$

$$= \frac{12s-24}{s^2-4s-32} \quad \text{Ans!}$$

Q9 $y' + y = t\sin t, \quad y(0) = 0$

SOLUTION:-

$$sY(s) - y(0) + Y(s) = \mathcal{L}\{t\sin t\}$$

$$sY(s) - 0 + Y(s) = (-1)\frac{d}{ds}\frac{1}{s^2+1}$$

$$= (-1)\left[\frac{(s^2+1)(0)-(2s)(1)}{(s^2+1)^2}\right]$$

$$(s+1)\,Y(s) = \frac{2s}{(s^2+1)^2}$$

$$Y(s) = \frac{2s}{(s+1)(s^2+1)^2}$$

$$\frac{2s}{(s+1)(s^2+1)^2} = \frac{A}{s+1} + \frac{Bs+C}{s^2+1} + \frac{Ds+E}{(s^2+1)^2}$$

$$2s = A(s^2+1)^2 + (Bs+C)(s+1)(s^2+1) + (Ds+E)(s+1) \quad \text{(A)}$$

$$2s = As^4 + 2As^2 + A + (Bs+C)(s^3+s+s^2+1) +$$
$$(Ds^2 + Es + Ds + E).$$
$$= As^4 + 2As^2 + A + Bs^4 + Bs^2 + Bs^3 + Bs +$$
$$Cs^3 + Cs + Cs^2 + C + Ds^2 + Es + Ds + E$$
$$2s = (A+B)s^4 + (B+C)s^3 + (2A+B+C+D)s^2 +$$
$$(B+C+E+D)s + (A+C+E)$$

$A + B = 0$ ——— (i)

$B + C = 0$ ——— (ii).

$2A + B + C + D = 0$ ——— (iii).

$B + C + E + D = 2$ ——— (iv)

$A + C + E = 0$ ——— (v)

Put $s = -1$ in (A).

$-2 = A(4) + 0 + 0$.

$$\boxed{-\frac{1}{2} = A}$$

(i) => $\boxed{B = \frac{1}{2}}$ (ii) => $\boxed{C = -\frac{1}{2}}$

(iii) => $\boxed{D = 1}$

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(iv) $\Rightarrow$ $\boxed{E = 1}$

(v) $\Rightarrow$ $Y(s) = -\frac{1}{2(s+1)} + \frac{(1/2)s + (-1/2)}{s^2+1} + \frac{(1)s+1}{(s^2+1)^2}$

$Y(s) = -\frac{1}{2(s+1)} + \frac{s}{2(s^2+1)} - \frac{1}{2(s^2+1)} + \frac{s}{(s^2+1)^2} + \frac{1}{(s^2+1}$

Q. $y'' + 9y = \cos 3t$, $y(0) = 2$, $y'(0) = 5$

$s^2 Y(s) - s\,y(0) - y'(0) + 9Y(s) = \frac{s}{s^2+9}$

$s^2 Y(s) - 2s - 5 + 9Y(s) = \frac{s}{s^2+9}$

$Y_s(s^2+9) - 2s - 5 = \frac{s}{s^2+9}$

$Y_s(s^2+9) = \frac{s + 2s(s^2+9) + 5(s^2+9)}{s^2+9}$

$Y(s) = \frac{s + 2s^3 + 18s + 5s^2 + 45}{(s^2+9)^2}$

$= \frac{2s^3 + 19s + 5s^2 + 45}{(s^2+9)^2}$

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Q19. $f(t) = 4t$; $g(t) = 3t^2$

SOLUTION:-

$\mathcal{L}\{4t * 3t^2\}$

$4\mathcal{L}\{t\} * 3\mathcal{L}\{t^2\}$

$\frac{4}{s^2} * 3\frac{2!}{s^3}$

$\frac{24}{s^5}$ Ans!

Q21. $f(t) = e^{-t}$, $g(t) = e^{t}$

SOLUTION:-

$\mathcal{L}\{f * g\}$

$\mathcal{L}\{e^{-t}\} * \mathcal{L}\{e^{t}\}$

$\frac{1}{s+1} * \frac{1}{s-1}$

$\frac{1}{s^2-1}$ Ans!

Q23. $\mathcal{L}\{1 * t^3\}$

$\mathcal{L}\{1\} * \mathcal{L}\{t^3\}$

$\frac{1}{s} * \frac{3!}{s^4} = \frac{6}{s^5}$ Ans!

Q25 $\mathcal{L}\{e^{-t} * e^{t}\cos t\}$

Solution:-

$= \mathcal{L}\{e^{-t}\} * \mathcal{L}\{e^{t}\cos t\}$

$= \dfrac{1}{1+s} * \left.\dfrac{s}{s^2+1}\right|_{s \to (s-1)}$

$\dfrac{1}{s+1} \quad \dfrac{(s-1)}{(s-1)^2+1}$

$\dfrac{(s-1)}{(s-1)^2[(s+1)] + (s+1)}$

Q29 $\mathcal{L}\left\{\int_0^t e^{-\tau}\cos\tau \, d\tau\right\}$

Solution:-

$= \mathcal{L}\{e^{-\tau}\cos\tau\} \cdot \mathcal{L}\{1\}$

$= \dfrac{s}{(s+1)^2+1} \cdot \dfrac{1}{s}$ Ans!

Q27 $\mathcal{L}\left\{\int_0^t e^{\tau} \, d\tau\right\}$

Solution:-

$\mathcal{L}\{e^{\tau}\} \cdot \mathcal{L}\{1\}$

$\dfrac{1}{(s-1)s}$ Ans!

Q31. $\mathcal{L}\left\{\int_0^t \tau e^{t-\tau}\, d\tau\right\}$

Solution:-

$\mathcal{L}\{t\} \cdot \mathcal{L}\{e^t\}$

$\dfrac{1}{s^2(s-1)}$. Ans!

Q35 $\mathcal{L}^{-1}\left\{\dfrac{1}{s(s-1)}\right\}$

Solution:-

$\mathcal{L}^{-1}\left\{\dfrac{1}{s} \cdot \dfrac{1}{s-1}\right\}$

$1 * e^t$

$\int_0^t e^{\tau} \cdot 1\, d\tau$

$e^t - e^0$

$e^t - 1$ Ans!

Q37. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^3(s-1)}\right\}$

$\mathcal{L}^{-1}\left\{\dfrac{1}{s^3} \cdot \dfrac{1}{s-1}\right\}$

$\frac{1}{2}t^2 \cdot e^t$

$\dfrac{1}{2}\int_0^t \tau^2 \cdot e^{(t-\tau)}$.

c)

b)

EXERCISE 7.6

Q1 $\frac{dx}{dt} = -x + y$ ——— (i) ; $x(0) = 0$, $y(0) = 1$

$\frac{dy}{dt} = 2x$ ——— (ii) ;

SOLUTION:-

let $\mathcal{L}\{x(t)\} = X(s)$

$\mathcal{L}\{y(t)\} = Y(s)$.

(i) $\Rightarrow$ $\frac{dx}{dt} + x - y = 0$

(ii) $\Rightarrow$ $\frac{dy}{dt} - 2x = 0$.

$\mathcal{L}\{x'\} + \mathcal{L}\{x\} + \mathcal{L}\{y\} = 0$.

$sX(s) + x(0) + X(s) + Y(s) = 0$.

$sX(s) + X(s) + Y(s) = 0$ ——— (a)

$\mathcal{L}\{y'\} - 2\mathcal{L}\{x\} = 0$.

$sY(s) - y(0) - 2X(s) = 0$

$sY(s) - 2X(s) = 1$ ——— (b)

~~× eq (a) by 2 and add.~~

$$2SY(s) + 2x(s) + 2Y(s) = 0$$
$$SY(s) - 2x(s) = 1$$
$$2SY(s) + 2Y(s) + SY(s) = 1$$
$$(2S + 2 + S)\,Y(s) = 1$$
$$(3S + 2)\,Y(s) = 1$$
$$Y(s) = \frac{1}{3S + 2}$$

× eq^n (a) by s then add with (b)

$$s^2x(s) + sx(s) + \cancel{sy(s)} = 0$$
$$-\cancel{SY(s)} + 2x(s) = -1$$
$$s^2x + sx + 2x = -1$$
$$(s^2 + s + 2)\,x(s) = -1$$
$$x(s) = \frac{-1}{s^2 + s + 2}$$

$x(s) =$

Q1 $\frac{dx}{dt} = -x + y$ —— (i) , $\frac{dy}{dt} = 2x$ —— (ii)

$x(0) = 0$, $y(0) = 1$

SOLUTION:-

$x' + x - y = 0$

$\mathcal{L}\{x'\} + \mathcal{L}\{x\} - \mathcal{L}\{y\} = 0.$

$sx(s) + x(0) + x(s) - y(s) = 0.$

$sx(s) + x(s) - y(s) = 0$ —— (a)

$(s+1)\,x(s) - y(s) = 0.$

$y' - 2x = 0.$

$\mathcal{L}\{y'\} - 2\mathcal{L}\{x\} = 0.$

$sy(s) - y(0) - 2x(s) = 0.$

$sy(s) - 2x(s) = 1$ —— (b).

* eq (a) by s then add with (b).

$s^2 x(s) + s x(s) - \cancel{sy(s)} = 0.$

$-2x(s) + sy(s) = 1$

$s^2 x(s) + s x(s) - 2x(s) = 1.$

$(s^2 + s - 2)\,x(s) = 1$

$x(s) = \frac{1}{s^2 + s - 2}$

$= \frac{1}{s^2 + 2s + s - 2}.$

$= \frac{1}{s(s+2) + 1(s+2)} = \frac{1}{(s+2)(s-1)}$

$$x(s) = \frac{1}{(s+2)(s-1)}$$

let $1 = A(s-1) + B(s+2)$

Put $s = 1$

$1 = 3B$

$1/3 = B$

Put $s = -2$

$1 = -3A$

$-1/3 = A$

$$x(s) = \frac{1}{3} \cdot \frac{1}{s+2} + \frac{-1}{3} \cdot \frac{1}{s-1}$$

$$x(s) = \frac{1}{3} e^{-2t} - \frac{1}{3} e^{t}$$

$\times$ eqn (b) $\{(s+1)/2\}$ then add with (a)

(b) $\Rightarrow$ $\cancel{(s+1)\, x(s)} - y(s) = 0$

(a) $\Rightarrow$ $\frac{s(s+1)}{2} y(s) - \cancel{(s+1)\, x(s)} = \frac{s+1}{2}$

$$\frac{s(s+1)}{2} y(s) - y(s) = \frac{s+1}{2}$$

$$s(s+1) y(s) - 2 y(s) = [2(s+1)]/2$$

$$\{s(s+1) - 2\}\, y(s) = (s+1)$$

$$\{s^2 + s - 2\}\, y(s) = (s+1)$$

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$$y(s) = \frac{s+1}{(s+2)(s-1)}$$

$$\frac{s+1}{(s+2)(s-1)} = \frac{A}{(s+2)} + \frac{B}{s-1}$$

$$s+1 = A(s-1) + B(s+2)$$

put $s = 1$

$$2 = 3B$$

$$\boxed{2/3 = B}$$

Put $s = -2$

$$-1 = -3A$$

$$\boxed{1/3 = A}$$

$$y(s) = \frac{1}{3} \cdot \frac{1}{(s+2)} + \frac{2}{3} \frac{1}{(s-1)}$$

$$= \frac{1}{3} e^{-2t} + \frac{2}{3} e^{t}$$

Ans!

Q_2 $\frac{dx}{dt} = 2y + e^t$; $x(0) = 1$; $y(0) = 1$

$\frac{dy}{dt} = 8x - t$

$x' - 2y = e^t$

$sX(s) - x(0) - 2Y(s) = e^t$

$sX(s) - 2Y(s) = e^t + 1$ ——— (a)

$y' - 8x = -t$

$sy(s) - y(0) - 8x(s) = -t$

$sy(s) - 1 - 8x(s)$.

$sy(s) - 8x(s) = 1 - t$ ——— (b).

× eqn (b) by $s/8$ then add with (a)

(b) => $\frac{s^2 y(s)}{8} - \cancel{sx(s)} = \frac{(1-t)}{8} \cdot \frac{s}{8}$

(a) => $\cancel{sx(s)} - 2y(s) = e^t + 1$

$\frac{y(s)}{8} - 2y(s) = \frac{s(1-t)}{64} + \frac{e^t + 1}{}$

$s^2 y(s) - 16y(s) = \frac{s(1-t) + (8e^t + 8)8}{8 \cdot 8}$

$y(s) = \frac{s - st + 64e^t + 64}{64(s^2 - 1)}$.

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$\frac{dx}{dt} = 2y + e^t$, $\frac{dy}{dt} = 8x - t$

$x(0) = 1$; $y(0) = 1$

$x' - 2y = e^t$

$sX(s) - x(0) - 2Y(s) = \frac{1}{s-1}$

$sX(s) - 2Y(s) = \frac{1 + s - 1}{s-1} = \frac{s}{s-1}$ ——— (a)

$y' - 8x = -t$

$sy(s) - y(0) - 8\cancel{X}x(s) = \frac{-1}{s^2}$

$sy(s) - 8x(s) = \frac{s^2 - 1}{s^2}$ ——— (b)

× eqn (a) by s & eqn (b) by 2 then add

(a) ⇒ $s^2 x(s) - \cancel{2sY(s)} = \frac{s}{s-1}$

(b) ⇒ $\cancel{2sy(s)} - 16x(s) = \frac{2(s^2-1)}{s^2}$

$s^2 x(s) - 16x(s) = \frac{s}{s-1} + \frac{2(s^2-1)}{s^2}$

$(s^2 - 16)\, x(s) = \frac{s^3 + 2(s-1)(s+1)(s-1)}{s^2(s-1)}$

$$\cancel{s^3} \quad x(s)\,(s^2-16) = \frac{s^3 + (2s+2)(s^2-2s+1)}{s^2(s-1)}$$

$$= s^3 +$$

EXERCISE 9.1

Q1 $y' = 2x - 3y + 1$, $y(1) = 5$; $y(1.5)$

SOLUTION:-

We know that

$$y^*_{n+1} = y_n + h f(x_n, y_n).$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})\right]$$

for $h = 0.1$.

$x_0 = 1$, $y_0 = 5$.

$x_1 = 1.1$	$y_1^* = 3.8$	$y_1 = 3.99$
$x_2 = 1.2$	$y_2^* = 3.133$	$y_2 = 3.2515$
$x_3 = 1.3$	$y_3^* = 3.2615$	
$x_4 = 1.4$	$y_4^* =$	
$x_5 = 1.5$	$y_5^* =$	

$n = 0$

$$y^*_{0+1} = y_0 + h f(x_0, y_0)$$

$$= 5 + (0.1)\left(2(1) - 3(5) + 1\right)$$

$$y^*_1 = 3.8$$

~~$n = 1$~~

~~$y_{1+1} = y_1 + (0.1) f(x_1 + y_1)$.~~

~~$y_2 =$~~

$$y_{0+1} = y_0 + \frac{h}{2}\left[f(y_0 + x_0) + f(x_{n+1} + y^*_{n+1})\right]$$

$$= 5 + \frac{0.1}{2}\left[\{2(1) - 3(5) + 1\} + \{2(1.1) - 3(3.8) + 1\}\right]$$

$$\boxed{y_1 = 3.99}$$

for $n = 1$

$$y_{1+1} = y_1 + h\,(2(1.2) - 3(3.99) + 1)$$

$$= 3.99 + 0.1\,(2.4 - 11.97 + 1)$$

$$y^*_2 = 3.133$$

$$y_{1+1} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_{1+1} + y^*_{1+1})\right]$$

$$= 3.99 + \frac{0.1}{2}\left[\{2(1.1) - 3(3.99) + 1\} + \{2(1.2) - 3(3.133) + 1\}\right]$$

$$\boxed{y_2 = 3.2515}$$

for $n = 2$

$$y^*_{1+2} = y_2 + hf(x_2, y_2)$$

$$= 3.2515 + 0.1\,\{2(1.2) - 3(3.255) + 1\}$$

$$y_3 = 2.6185$$

Q3: $y' = 2x - 3y + 1$, $y(1) = 5$; $y(1.5)$.

We know that

$K_1 = hf(x_n, y_n)$

$K_2 = hf(x_n + h/2, y_n + K_1/2)$

$K_3 = hf(x_n + h/2, y_n + K_2/2)$.

$K_4 = hf(x_n + h, y_n + K_3)$.

$$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$Y_{n+1} = Y_n + K$.

$x_0 = 1$ $y_0 = 5$

$x_1 = 1.1$ $y_1 =$

$x_2 = 1.2$ $y_2 = 3.2079$.

$x_3 = 1.3$ $y_3 =$

$x_4 = 1.4$ $y_4 =$

$x_5 = 1.5$ $y_5 =$

For $n = 0$

$K_1 = hf(x_0, y_0) = (0.1)(2(1) - 3(5) + 1)$

$K_1 = -1.2$

$K_2 = hf(x_n + h/2, y_n + K_1/2)$

$= (0.1)(x_0 + h/2, y_0 + K_1/2)$

$= (0.1)\left(1 + \frac{0.1}{2}, 5 + \left(\frac{-1.2}{2}\right)\right) = 0.1(1.05, 4.4)$

$= (0.1)\{2\cdot(1.05) - 3(4.4) + 1\}$

$K_2 = -1.01$

$K_3 = hf\{x_0 + 0.1/2,\ y_0 + K_2/2\}$

$= 0.1 f\{1 + 0.1/2,\ 5 + -1.01/2\}$

$= 0.1 f\{1.05,\ 4.495\}$

$= 0.1\{(2)(1.05) - 3(4.495) + 1\}$

$K_3 = -1.0385$

$K_4 = hf\{x_0 + h,\ y_0 + K_3\}$

$= 0.1 f\{1 + 0.1,\ 5 + (-1.0385)\}$

$= 0.1 f\{1.1,\ 3.9615)$

$= 0.1\{2(1.1) - 3(3.9615) + 1\}$

$K_4 = -0.86845$

$K = \frac{1}{6}\{K_1 + 2K_2 + 2K_3 + K_4\}$

$= \frac{1}{6}\{(-1.2) + 2(-1.01) + 2(-1.0385) + -(\cancel{8.6845})$ 0.86845

$K = -\cancel{2.33025}\ -1.02765$

$Y_{n+1} = Y_n + K$

$Y_1 = 5 + (\cancel{-2.33025})(-1.02765)$

$Y_1 = \cancel{2.6697}\ 3.9723$

for $n = 1$

$K_1 = hf(x_1, y_1)$

$= hf(1.10, 3.9724)$

$= (0.1)\{2(1.10) - 3(3.9724) + 1\}$

$K_1 = -0.87172$

$K_2 = hf(x_1 + 0.1/2, y_1 + K_1/2)$

$= (0.1)f\{(1.10) + 0.1/2, 3.9724 + \frac{0.87172}{2}\}$

$= 0.1 f(1.15, 3.5368)$

$= 0.1\{2(1.15) - 3(3.5368) + 1\}$

$K_2 = -0.73104$

$K_3 = hf(x_1 + 0.1/2, y_1 + K_2/2)$

$= (0.1)\{1.10 + 0.1/2, 3.9724 - \frac{0.73104}{2}\}$

$= (0.1)\{1.15, 3.6068\}$

$= 0.1\{2(1.15) - 3(3.6068) + 1\}$

$K_3 = -0.75204$

$K_4 = 0.1 f\{1.10 + 0.1, 3.9724 + \frac{(-0.75024)}{2}\}$

$= 0.1 f(1.15, 3.59728)$

$= 0.1\{2(1.15) - 3(3.59728) + 1\}$

$= 0.7491$

$K = -0.7644$

$y_{n+1} = y_1 + (-0.7644) = 3.2079$