

# **Dynamic Programming and Applications**

## Heterogeneous Agents and Inequality

### Lecture 10

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# What this part of the course is about

- Understanding **cross-sectional household heterogeneity** (“inequality”) is one of the biggest questions in macroeconomics
- Vast literature studies and models inequality since at least 1980s
- First wave of research: document inequality and understand where it comes from
- Recent “boom” in **heterogeneous agent macro**: inequality shapes aggregate outcomes (business cycles, growth, policy transmission, ...)

# Heterogeneous agent models

- Studying inequality (both its origins and consequences) requires modeling **distributions**
- This is very difficult because distributions are large, high-dimensional objects
- But also opens door to vast number of interesting questions:
  - Why are income and wealth so unequally distributed? What explains changes over time?
  - Is there a trade-off between inequality and economic growth?
  - How does inequality vary over the business cycle?
  - Does inequality affect the transmission and effectiveness of monetary and fiscal policy?
- Main idea: solving heterogeneous agent models = solving PDEs  
This is the current frontier in macro. This is why we invested so much in studying differential equations!

# Solving HA models = solving PDEs

- More precisely: a system of two PDEs
  1. **Hamilton-Jacobi-Bellman** equation for individual choices
  2. **Kolmogorov Forward** equation for evolution of distribution
- Many well-developed methods for analyzing and solving these

<https://benjaminmoll.com/codes/>

<https://github.com/schaab-lab/SparseEcon>

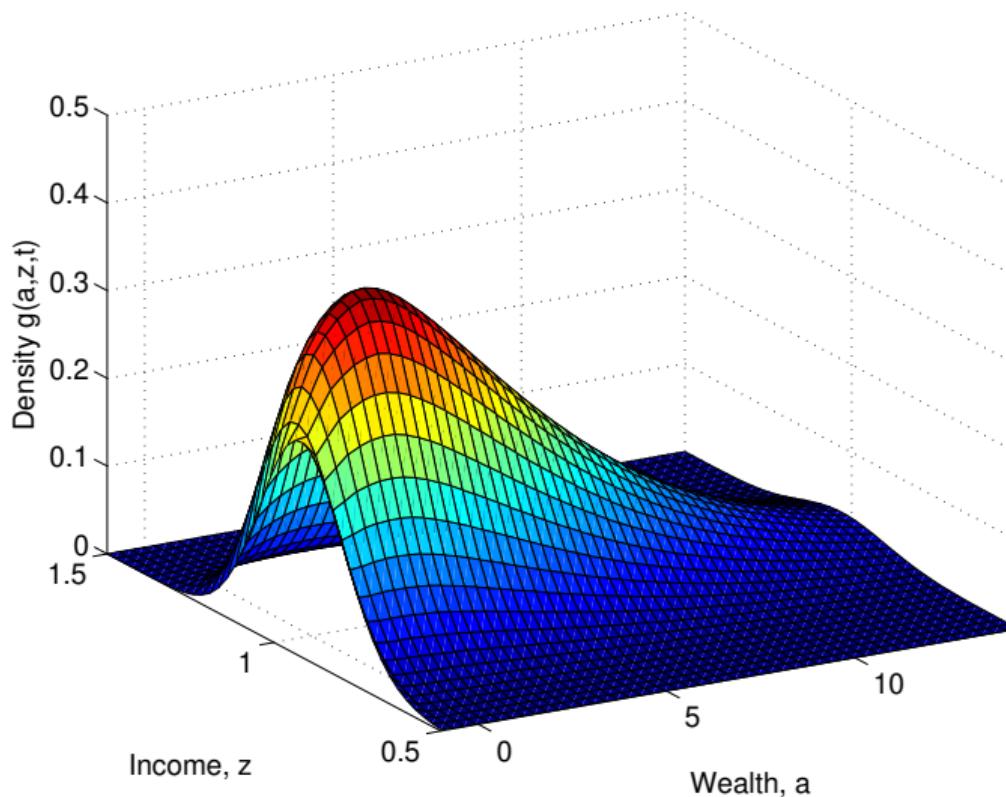
# Solving HA models = solving PDEs

- More precisely: a system of two PDEs
  1. **Hamilton-Jacobi-Bellman** equation for individual choices
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- Many well-developed methods for analyzing and solving these
  - <https://benjaminmoll.com/codes/>
  - <https://github.com/schaab-lab/SparseEcon>
- Apparatus is very general: applies to any heterogeneous agent model with continuum of agents:
  1. heterogeneous households, firms, banks, countries, ...
  2. multiple assets, complicated income dynamics, ...
  3. optimal stopping problems, discrete choice, migration / location decisions, ...
  4. non-convexities, kinks, ...

*Learning this will empower you to tackle whatever questions **you** are most interested in!*

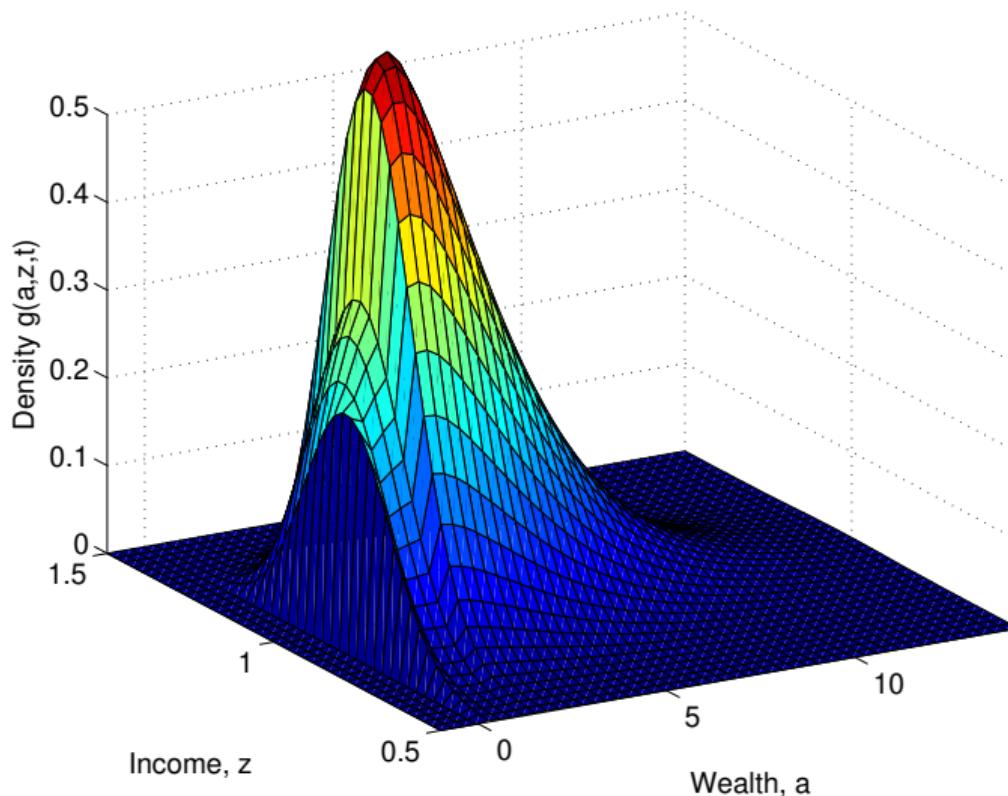
## What you'll be able to do at end of this course segment

- Joint distribution of income and wealth in Aiyagari model



## What you'll be able to do at end of this course segment

- Experiment: effect of one-time redistribution of wealth



## What you'll be able to do at end of this course segment

Video of convergence back to steady state

[https://www.dropbox.com/s/op5u2nlifmmer2o/distribution\\_tax.mp4?dl=0](https://www.dropbox.com/s/op5u2nlifmmer2o/distribution_tax.mp4?dl=0)

# Outline

Part 1: 30 empirical regularities about inequality

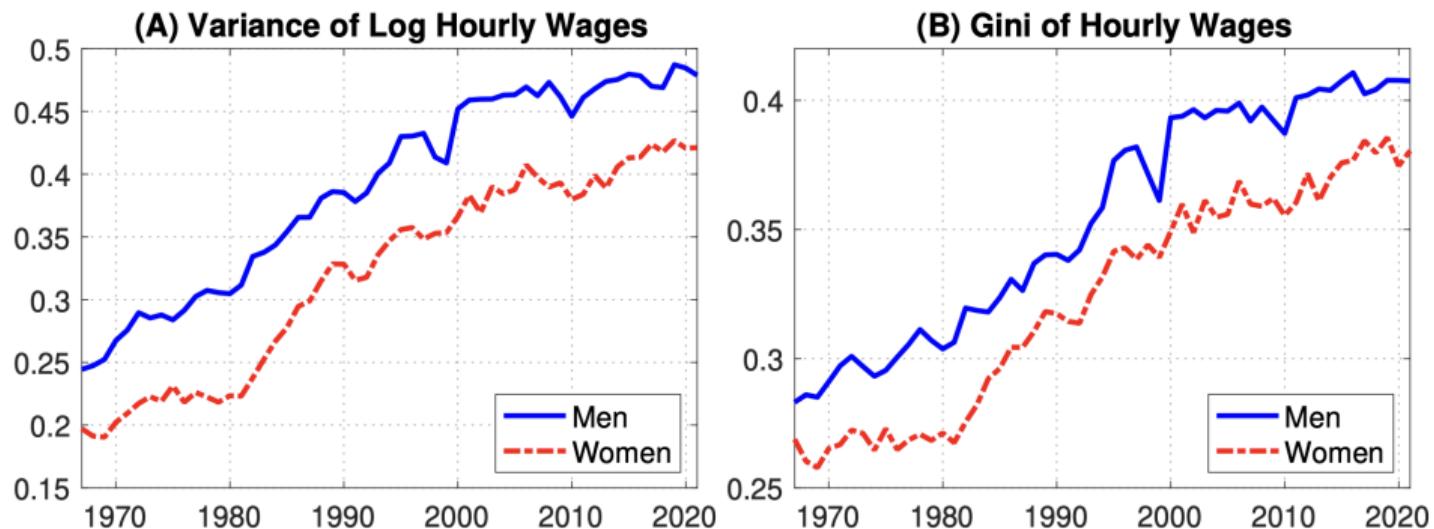
1. Wage inequality
2. Earnings inequality
3. Individual- versus household-level inequality
4. Government taxes and transfers
5. Consumption and wealth inequality

# Part 1: 30 Facts about Inequality

## Overview

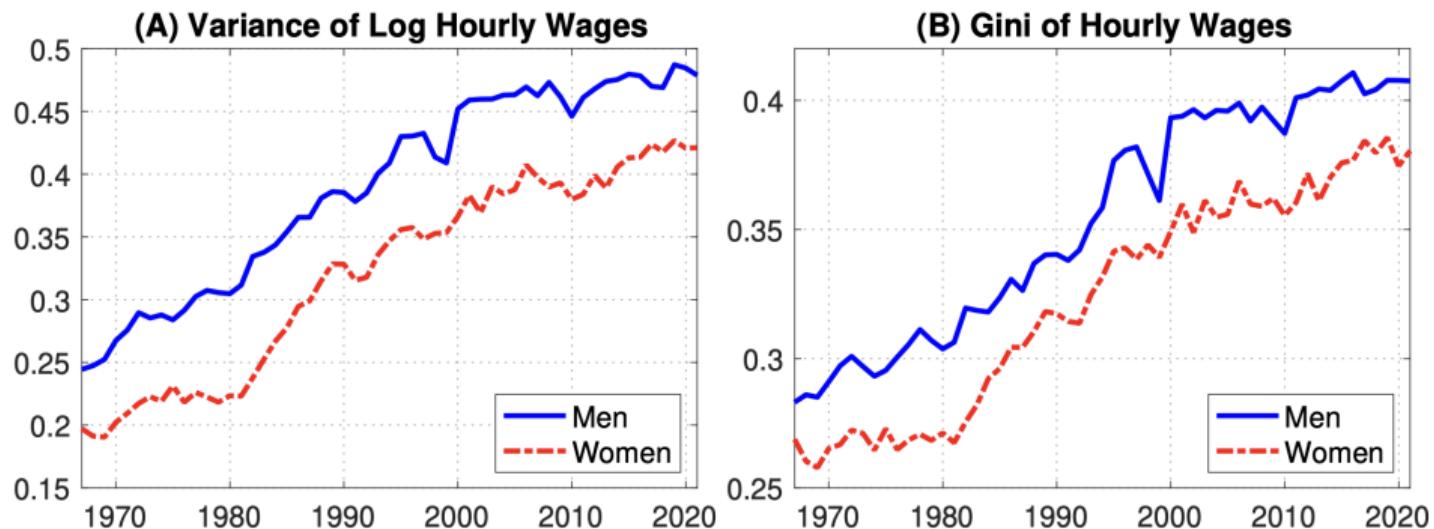
- Document 30 **empirical regularities** (ER) about inequality
- Knowing consensus ERs is critical for quantitative and applied theory research:
  1. Build theories (models) to explain ERs
  2. Models we use to study counter-factuals should be consistent with ERs
- Main reference: Heathcote-Perri-Violante-Zhang (2023) (**HPVZ**), also Acemoglu-Autor (2011) (**AA**)  
Also work by: Acemoglu, Autor, Picketty, Saez, ...
- Organizing framework: household budget constraint

$$c + s = \sum_i^N w_i h_i + d + T^p + T^g - \tau$$



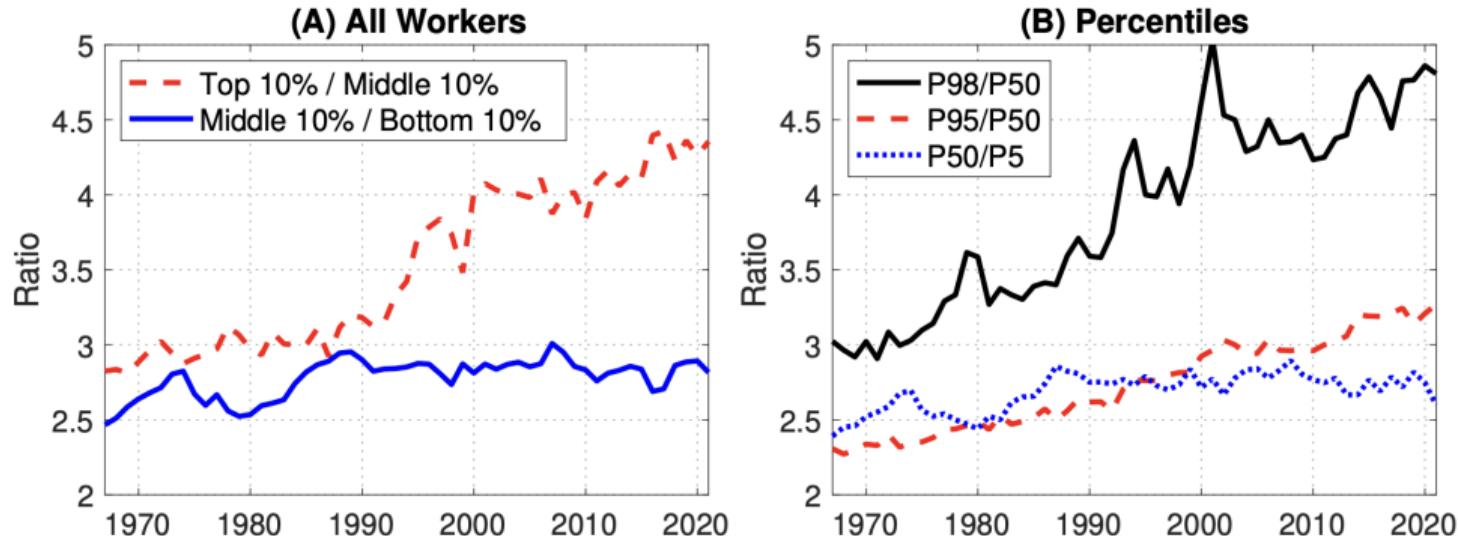
**Note:** This is Figure 6 in HPVZ. Sample: CPS, working-age individuals with minimum labor force attachment.

**ER1:** Wage dispersion has been rising steadily for men and women, but less so since 2000



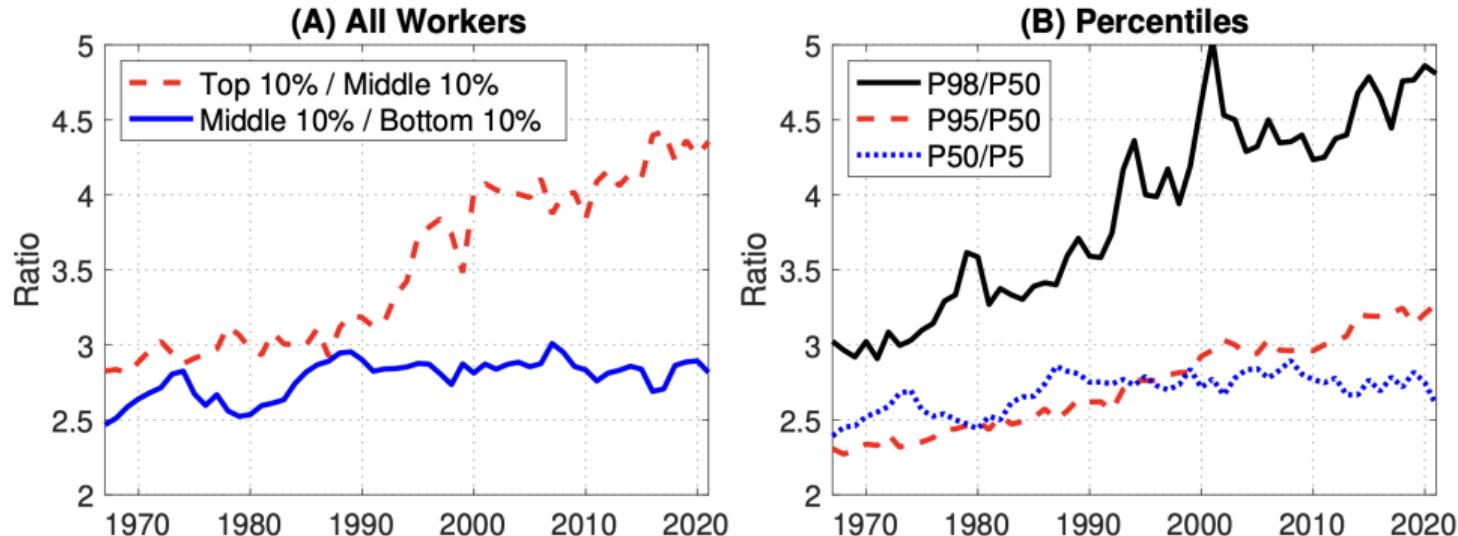
**Note:** This is Figure 6 in HPVZ. Sample: CPS, working-age individuals with minimum labor force attachment.

**ER2:** Wage inequality systematically lower for women, only started rising in 1980



**Note:** This is Figure 7 in HPVZ. Sample: CPS, working-age individuals with minimum labor force attachment.

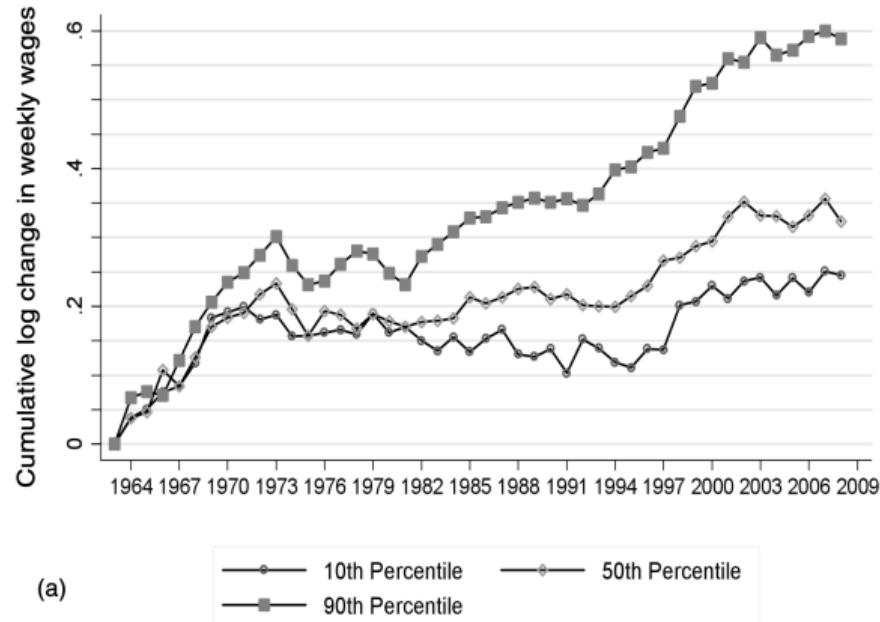
**ER3:** Rise in wage inequality comes from top of distribution, especially since 1990



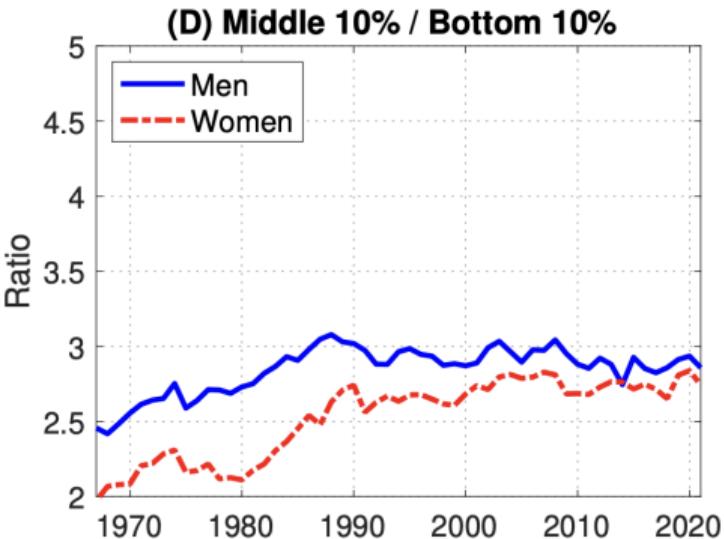
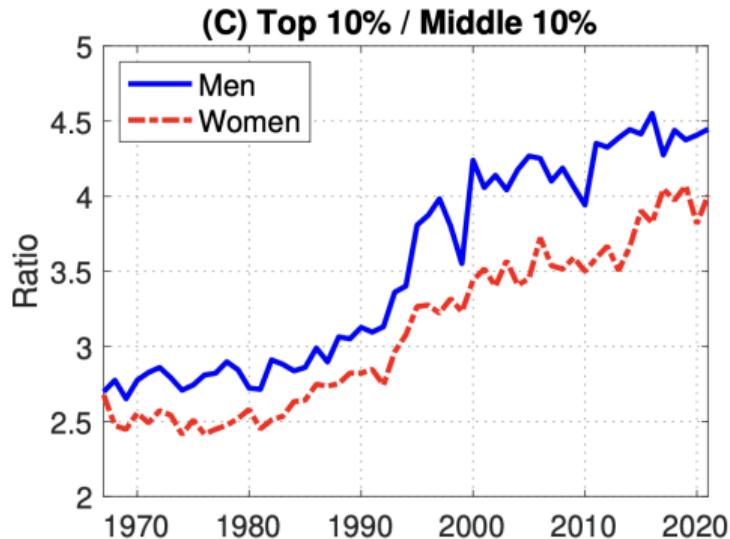
**Note:** This is Figure 7 in HPVZ. Sample: CPS, working-age individuals with minimum labor force attachment.

**ER4:** Little to no rise in wage dispersion in bottom half of distribution

Cumulative log change in real weekly earnings at the 90th, 50th and 10th  
wage percentiles  
1963-2008: full-time full-year males and females

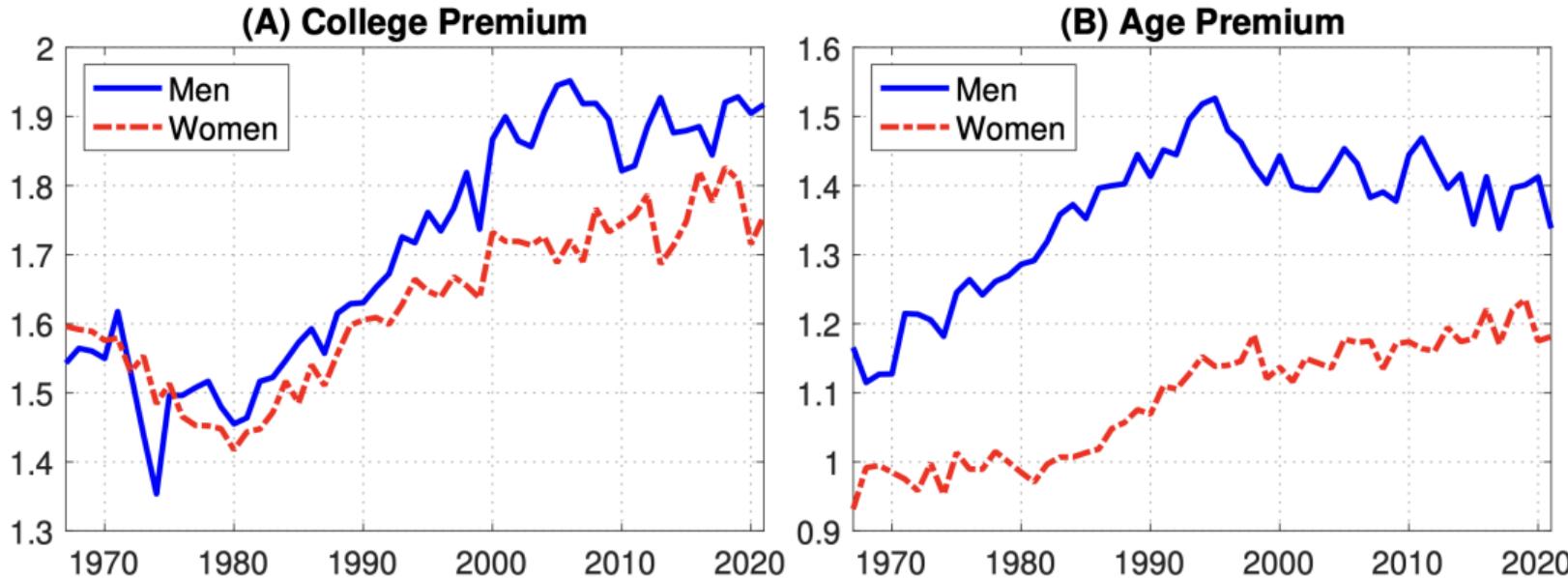


**Note:** This is Figure 7 in AA. Sample: CPS, full-time full-year workers.



**Note:** This is Figure 7 in HPVZ. Sample: CPS, working-age individuals with minimum labor force attachment.

**ER3** and **ER4** same pattern for men and women



**Note:** This is Figure 8 in HPVZ. Sample: CPS, working-age individuals with minimum labor force attachment.

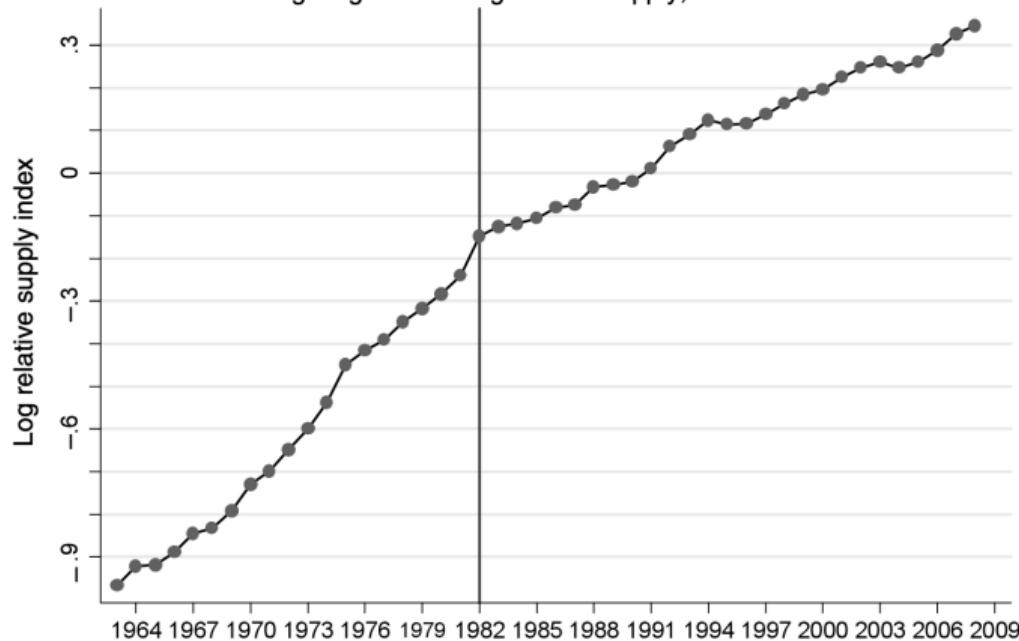
**ER5:** Large rise in ratio of average hourly wages with more / less than 16 years of schooling from 1980 until mid-2000s

Composition adjusted college/high-school log weekly wage ratio, 1963-2008

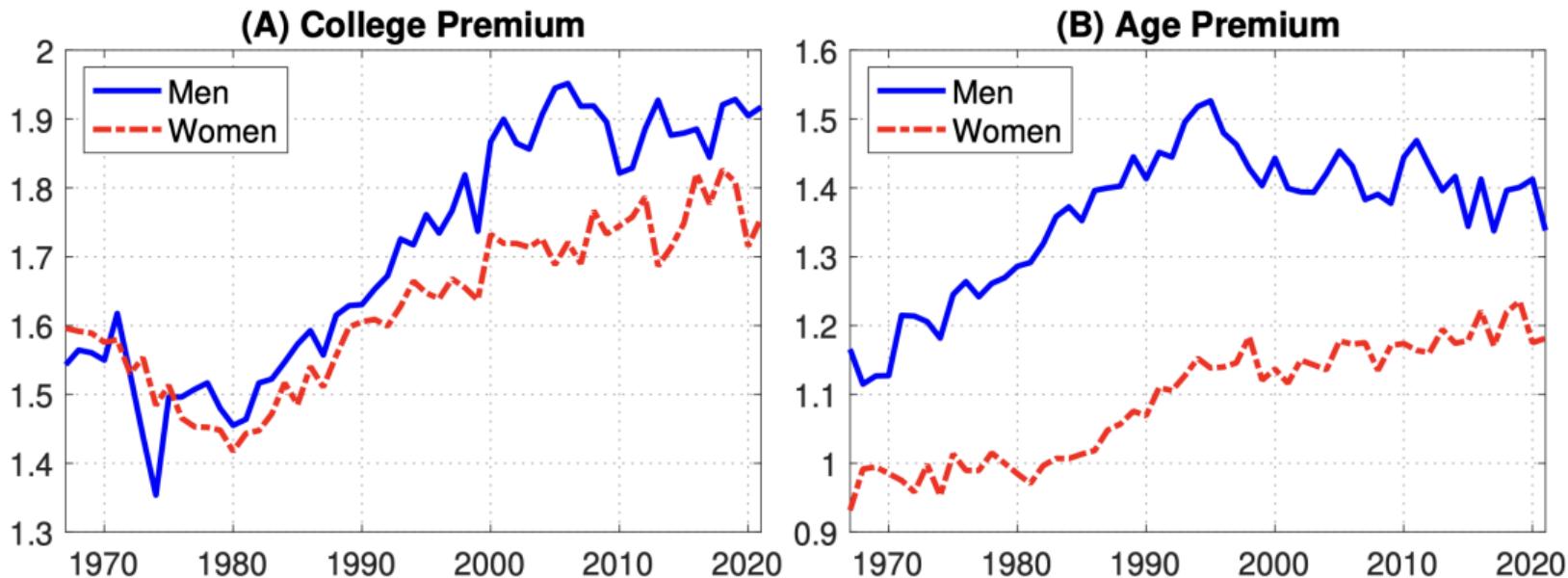


**Note:** This is Figure 1 in AA. Sample: March CPS, log weekly wages for full-time full-year workers.

College/high-school log relative supply, 1963-2008



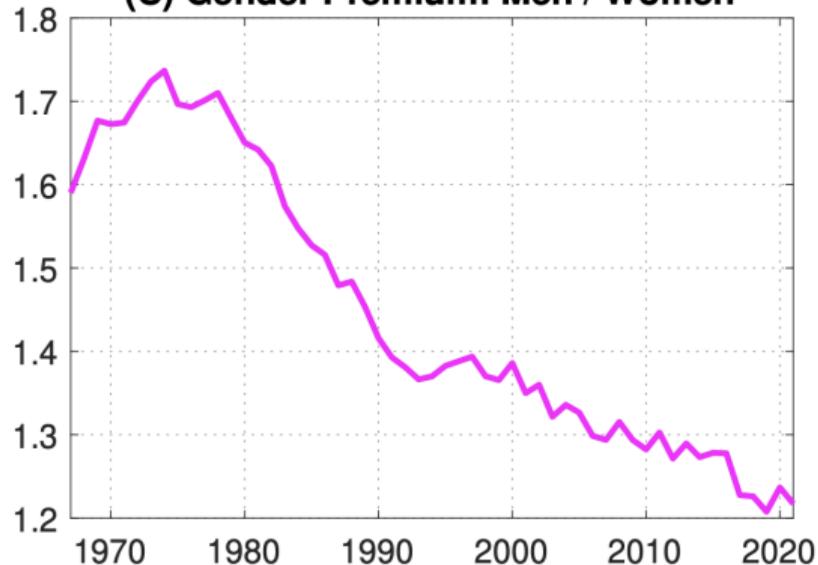
**Note:** This is Figure 2 in AA. Sample: March CPS, log weekly wages for full-time full-year workers.



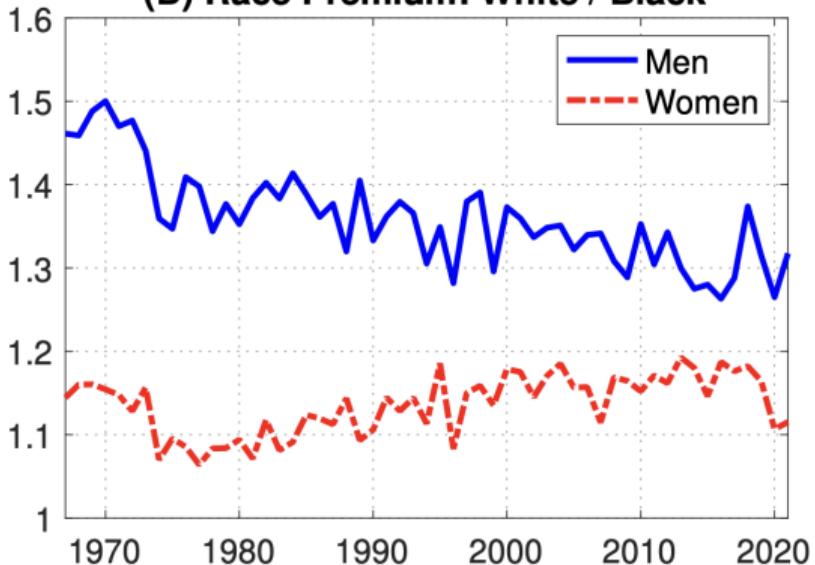
**Note:** This is Figure 8 in HPVZ. Sample: CPS, working-age individuals with minimum labor force attachment.

**ER6:** Ratio in average hourly wage of 45-55 year-olds vs. 25-35 year-olds rises steadily from mid-1970s to mid-1990s

**(C) Gender Premium: Men / Women**



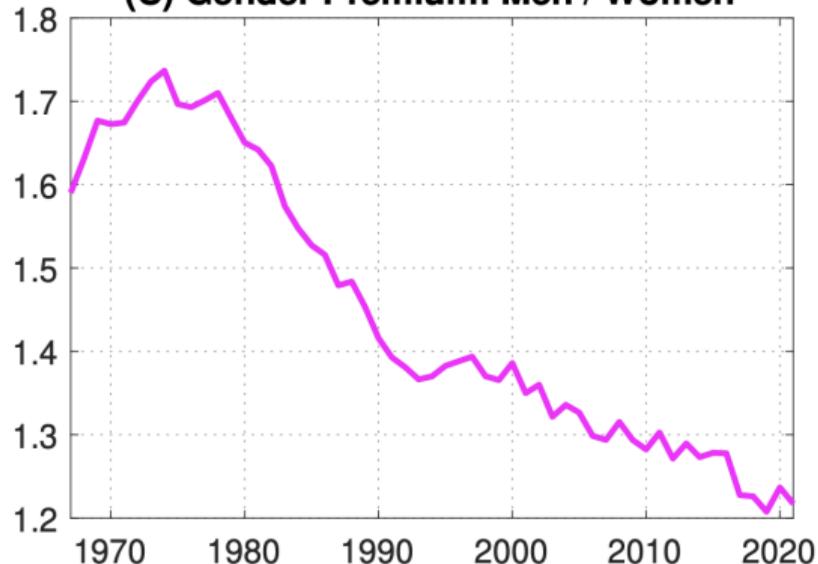
**(D) Race Premium: White / Black**



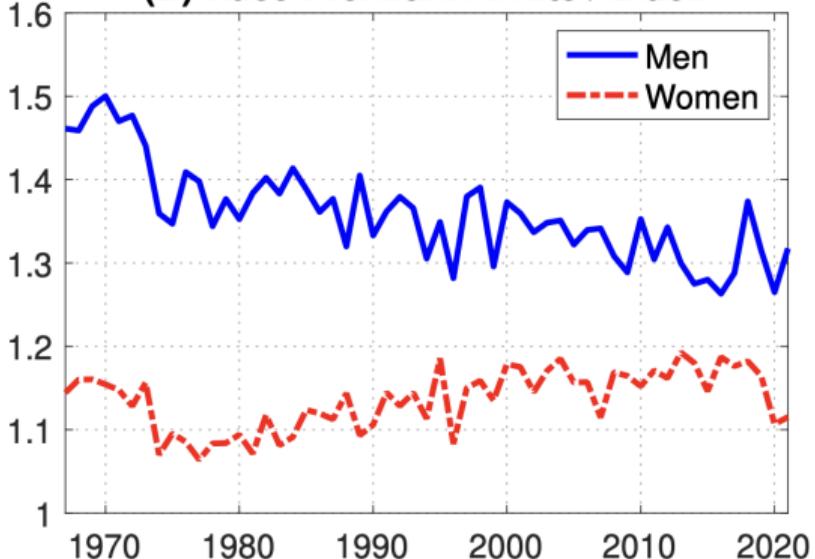
**Note:** This is Figure 8 in HPVZ. Sample: CPS, working-age individuals with minimum labor force attachment.

**ER7:** Men earned 40-70% more than women before 1990 but only 20% more in 2020

**(C) Gender Premium: Men / Women**



**(D) Race Premium: White / Black**



**Note:** This is Figure 8 in HPVZ. Sample: CPS, working-age individuals with minimum labor force attachment.

**ER8:** Ratio of average hourly wages for White and Black workers has fallen slowly

## Summary: wage inequality

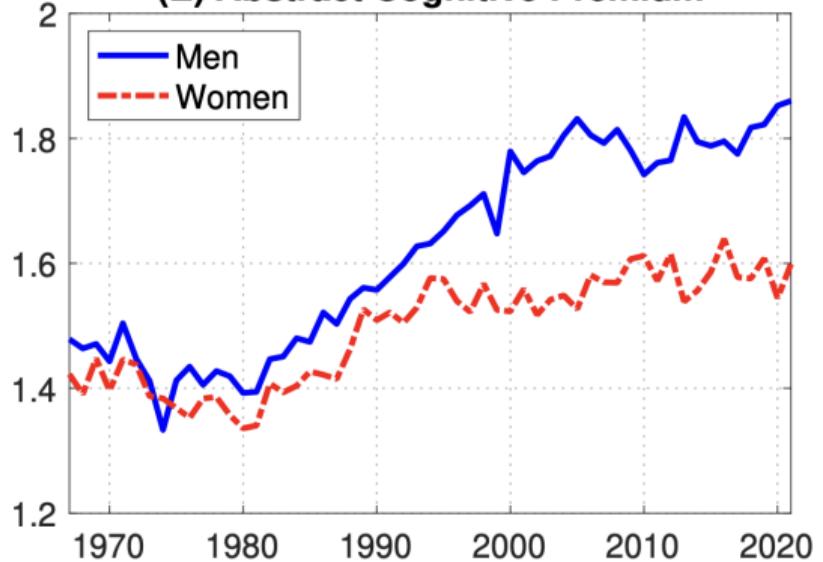
- Wage dispersion has been rising steadily for men and women, but less so since 2000
- Wage dispersion at top of distribution (above median) steadily rising since 1960, while relatively stable at bottom of distribution (below median) over last 20 years
- Gender wage gap continues to narrow, while racial wage gaps are falling slowly
- Rise of the college premium 1980 to mid-2000s (massive literature)
- Wage differentials by education, age, and race groups have stabilized since 2000



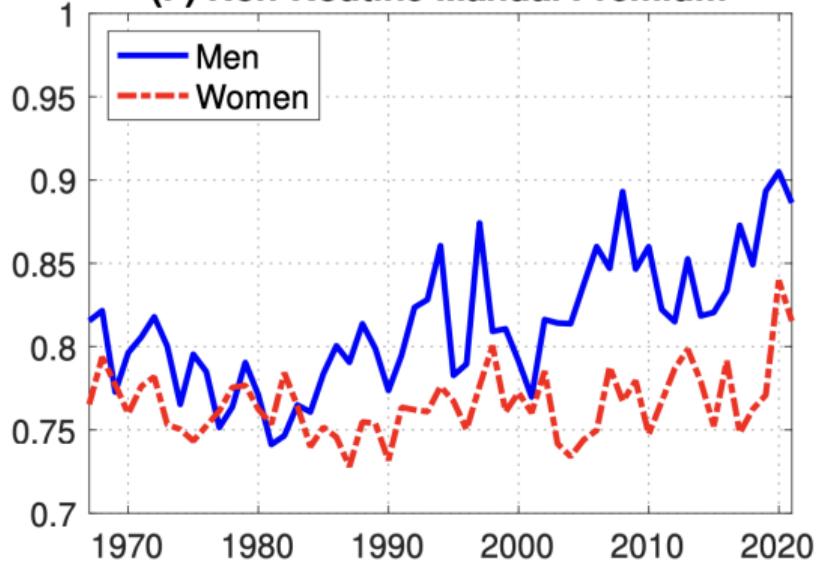
**Note:** This is Figure 9 in AA. Sample: May/ORG CPS, all workers excluding military and self-employed.

**ER9:** Wage growth was monotonic (along income distribution) until 1980s but has become U-shaped (“polarized”) since

**(E) Abstract Cognitive Premium**

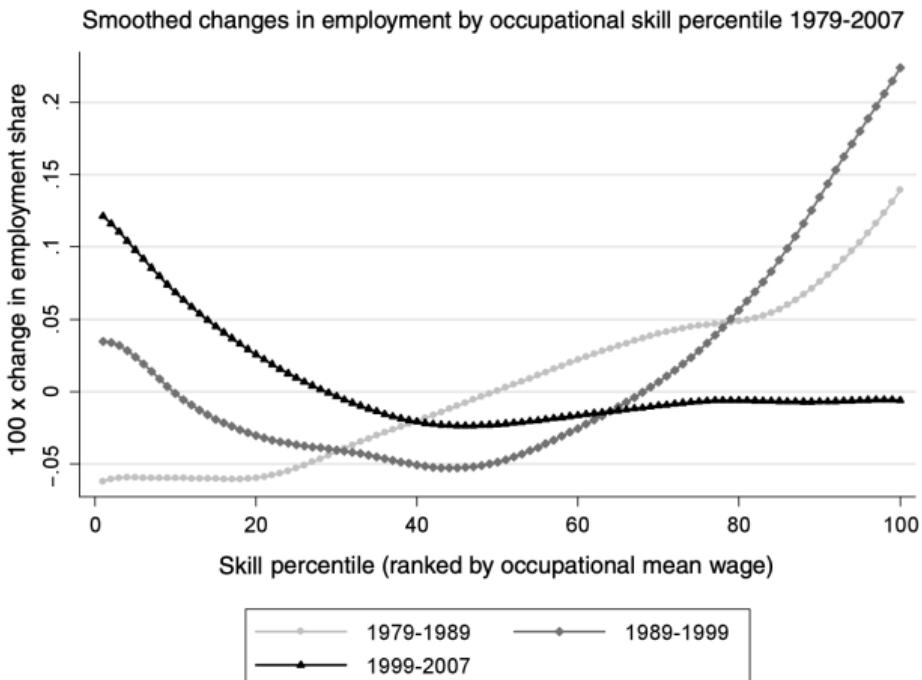


**(F) Non-Routine Manual Premium**



**Note:** This is Figure 8 in HPVZ. Sample: CPS, working-age individuals with minimum labor force attachment.

**ER9:** Since 1980 wages of abstract cognitive (highest paid) and non-routine manual (lowest-paid) have increased relative to routine occupations (**wage polarization**)



**Note:** This is Figure 10 in AA. Sample: Census IMPUS and Census ACS.

**ER10:** Simultaneous growth of employment in high-skill/wage and low-skill/wage occupations (**job polarization**)

## Summary: wage and job polarization

- Wage growth was monotonic along income distribution until 1980s but has become polarized since then
- At the same time, employment shares have risen the most for low- and high-wage occupations → hollowing out of employment distribution
- Readings:
  - Autor-Levy-Murnane (2003): routinization hypothesis
  - Acemoglu-Autor (2011) Handbook chapter
  - Autor-Dorn (2013): rise of low-skill service jobs

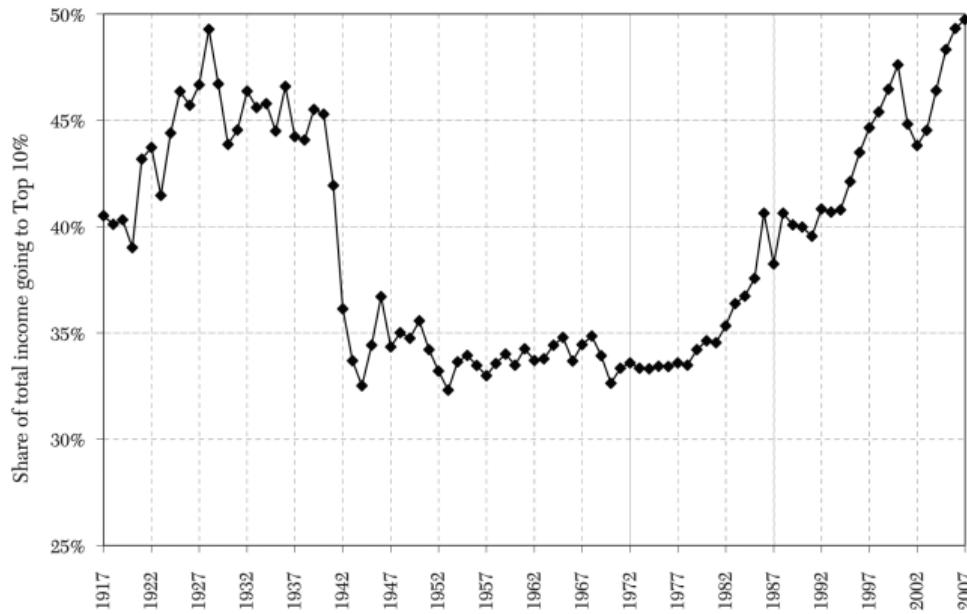


Figure 1. The Top Decile Income Share in the United States, 1917–2007.

Notes: Income is defined as market income including realized capital gains (excludes government transfers). In 2007, top decile includes all families with annual income above \$109,600.

Source: Piketty and Saez (2003), series updated to 2007.

**ER11:** Income share of top 10% was high before WW2, fell and remained low until 1980, rising strongly since

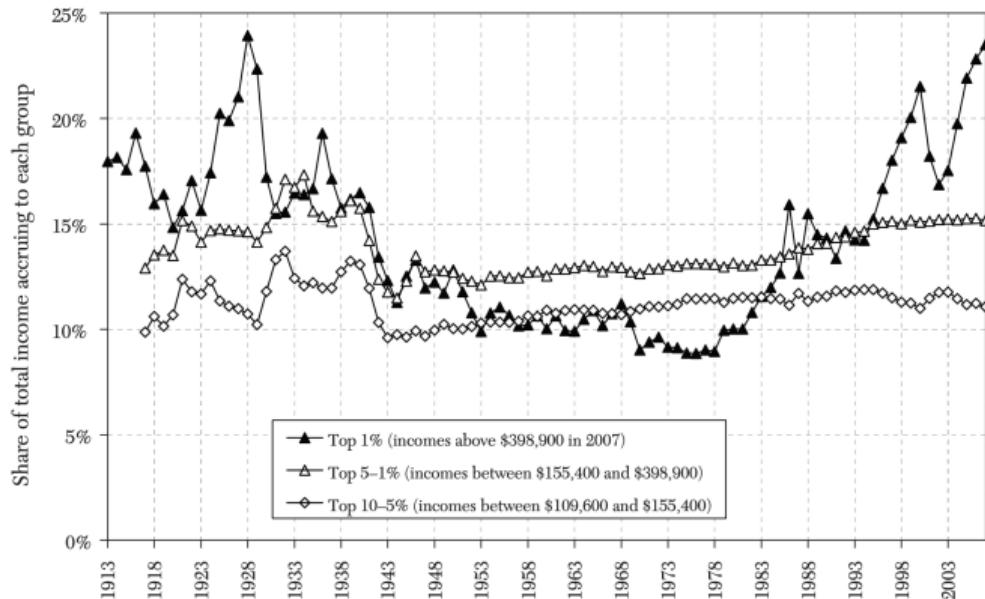


Figure 2. Decomposing the Top Decile US Income Share into three Groups, 1913–2007

Notes: Income is defined as market income including capital gains (excludes all government transfers).

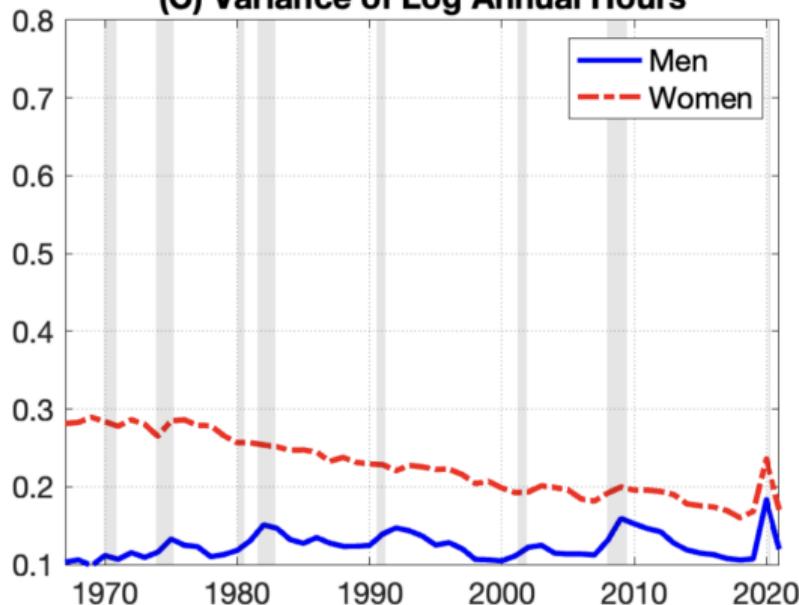
Top 1 percent denotes the top percentile (families with annual income above \$398,900 in 2007).

Top 5–1 percent denotes the next 4 percent (families with annual income between \$155,400 and \$398,900 in 2007).

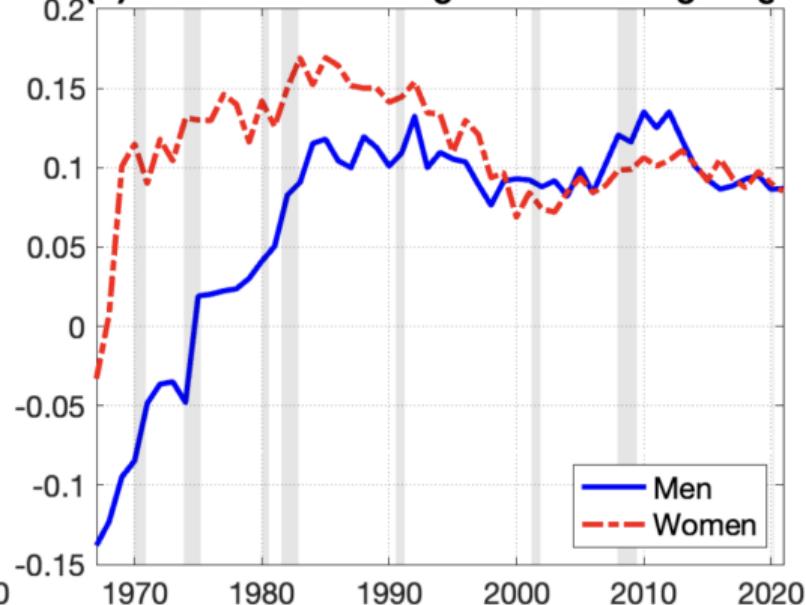
Top 10–5 percent denotes the next 5 percent (bottom half of the top decile, families with annual income between \$109,600 and \$155,400 in 2007).

**ER12:** Rise in income share since 1980 driven almost entirely by top 1%

**(C) Variance of Log Annual Hours**

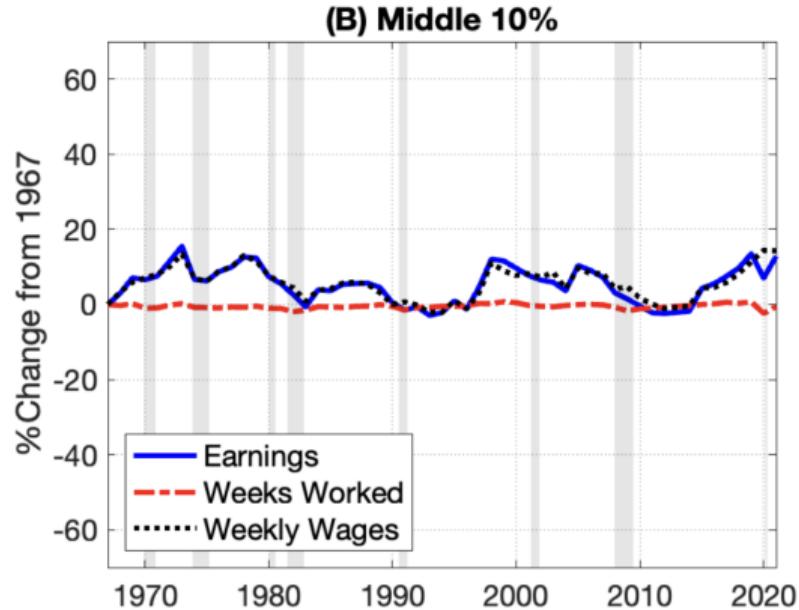


**(D) Correlation btw Log Hours and Log Wage**



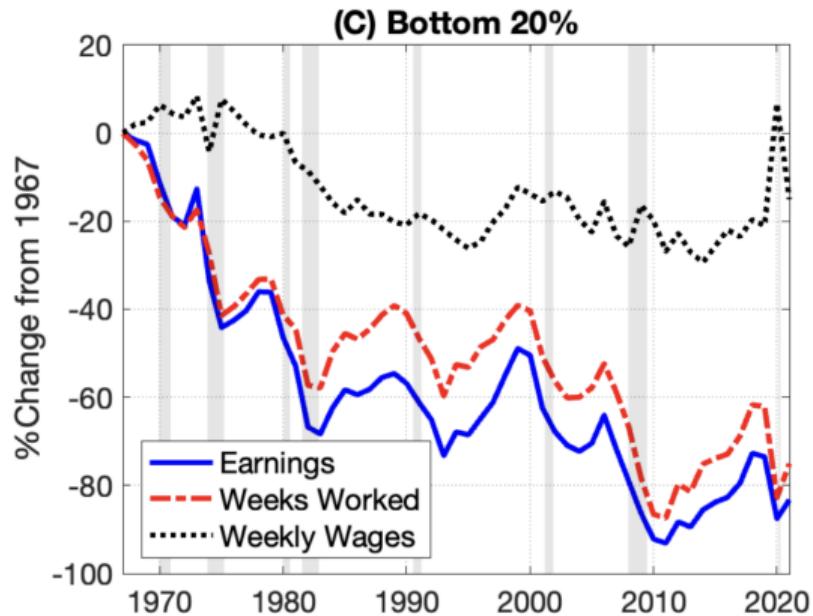
**Note:** This is Figure 10 in HPVZ. Sample: CPS, working-age individuals with minimum labor force attachment.

**ER13:** Correlation between hours and wages rose until 1980s, positive and stable since



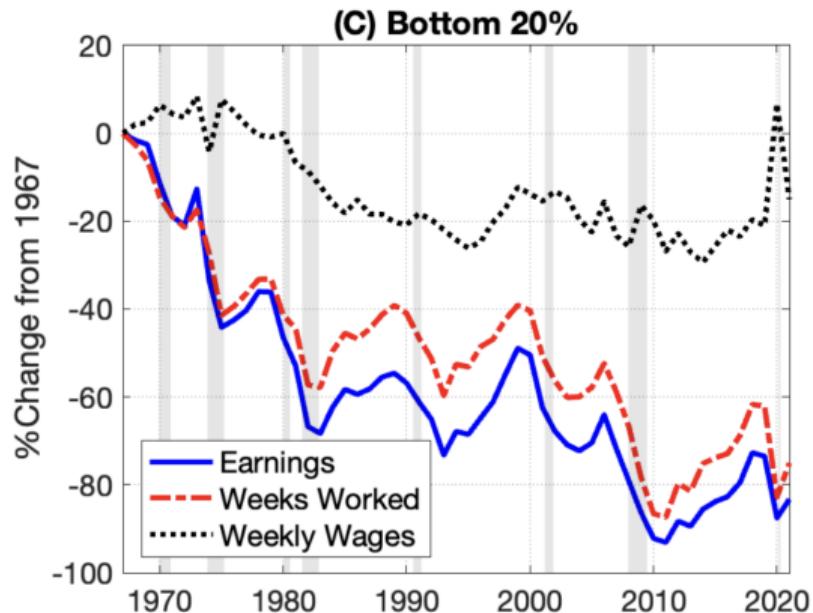
**Note:** This is Figure 11 in HPVZ. Sample: CPS, workers including those with little to no hours.

**ER14:** Stark increase in earnings inequality at the top driven by rising wage gap at the top. Top wages doubled last 50 years, middle wages flat. (Weeks worked flat.)



**Note:** This is Figure 11 in HPVZ. Sample: CPS, workers including those with little to no hours.

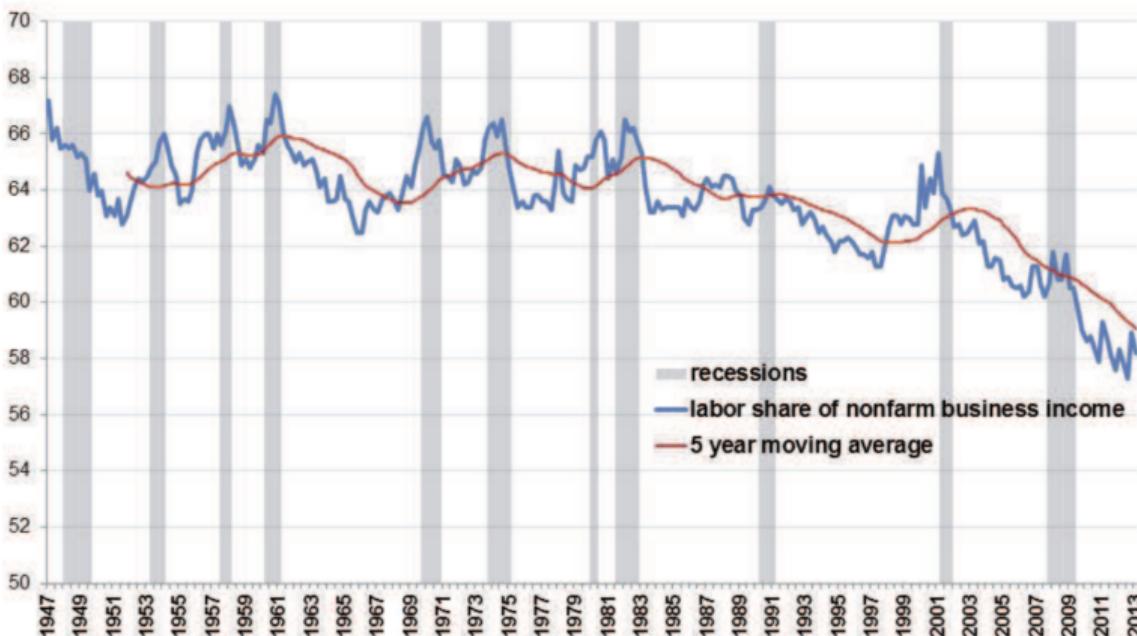
**ER15:** Earnings inequality at the bottom rises throughout sample period, driven by decline in weeks worked by low-earners



**Note:** This is Figure 11 in HPVZ. Sample: CPS, workers including those with little to no hours.

**ER16:** Weeks worked by low-earners very cyclical

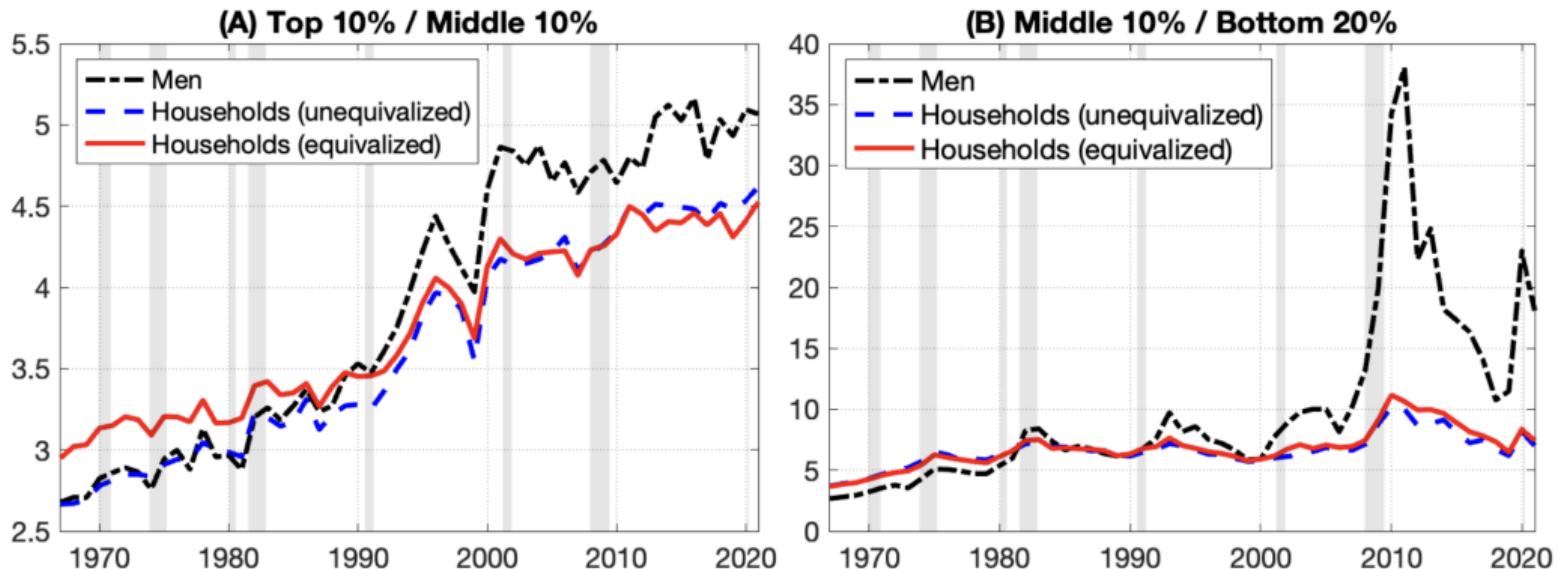
## Labor Share of National Income, 1947-2013



**ER17:** labor share (in aggregate income) has fallen since 1980

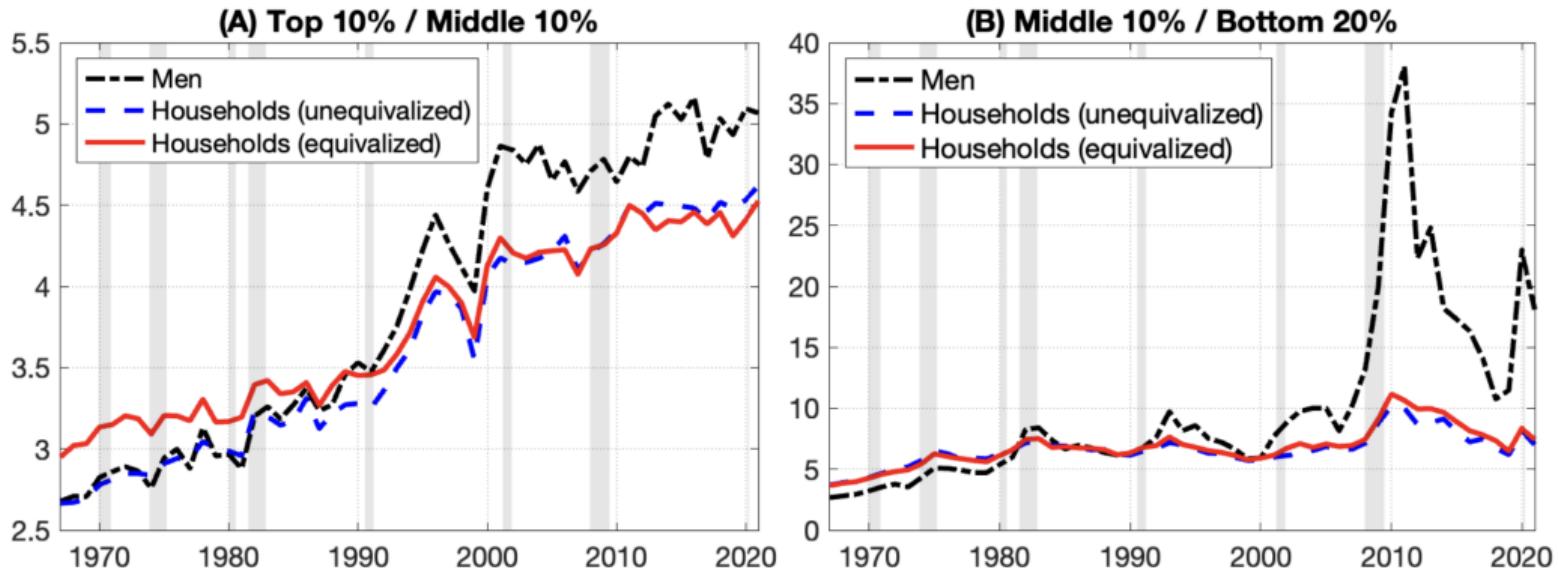
## Summary: earnings inequality

- Earnings dispersion at the top driven by long-run trends, while at the bottom strongly cyclical
- Earnings dispersion at the top is about changes in wages (hours stable)
- Earnings dispersion at the bottom is about hours (annual weeks worked)
- Labor share has fallen since 1980 (massive literature!)



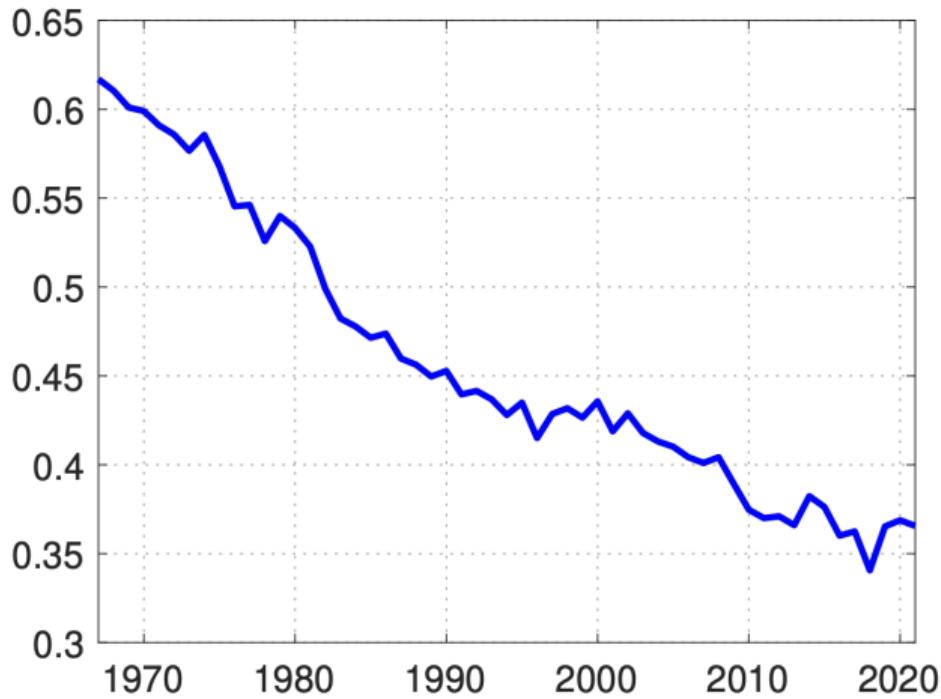
**Note:** This is Figure 12 in HPVZ. Sample: CPS, all households with one member aged 25-60.

**ER18:** Rising dispersion in household earnings at the top (slight attenuation compared to individual earnings)



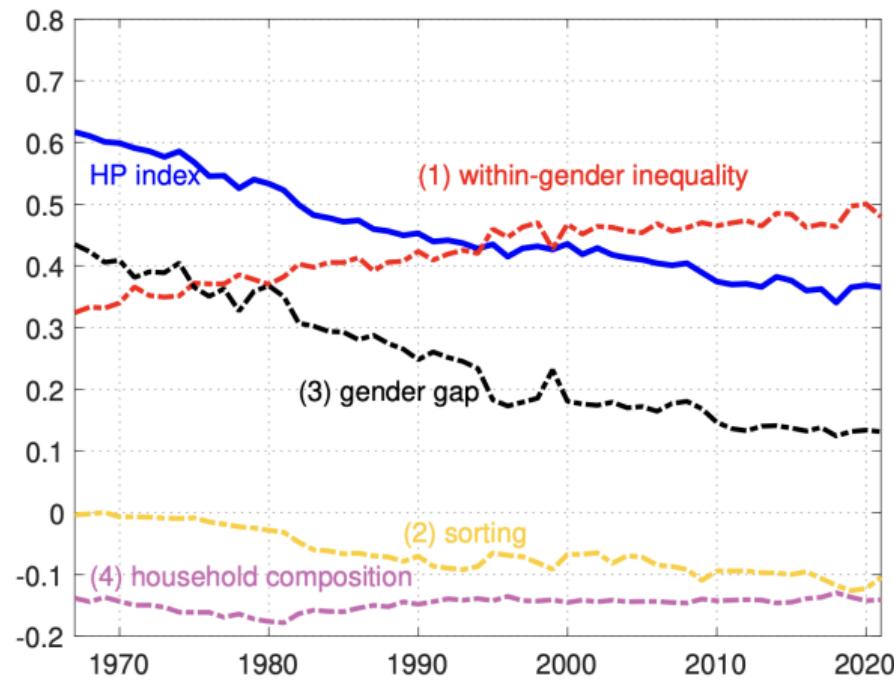
**Note:** This is Figure 12 in HPVZ. Sample: CPS, all households with one member aged 25-60.

**ER19:** Large spikes in male earnings dispersion during recent recessions strongly attenuated in household earnings dispersion



**Note:** This is Figure 13 in HPVZ. Sample: CPS, all households with one member aged 25-60.

**ER20:** Large decline in household income pooling. In 1970, income pooling reduced variance of individual earnings dispersion by 60%, down to 35% today

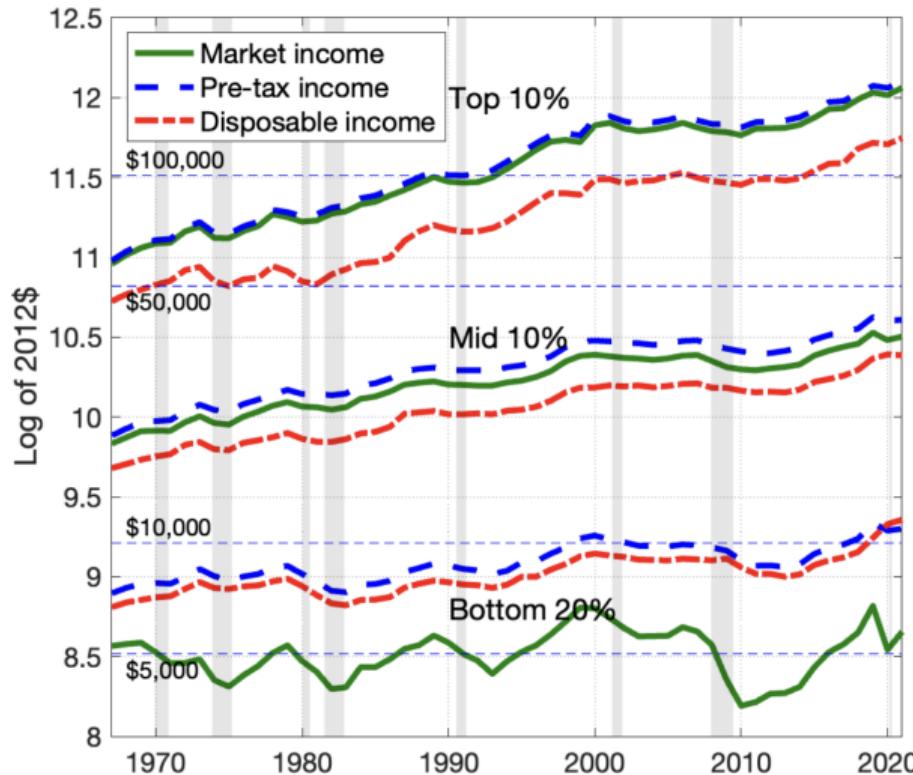


**Note:** This is Figure 13 in HPVZ. Sample: CPS, all households with one member aged 25-60.

**ER21:** Decline in gender gap and increased sorting main drivers of lower household pooling

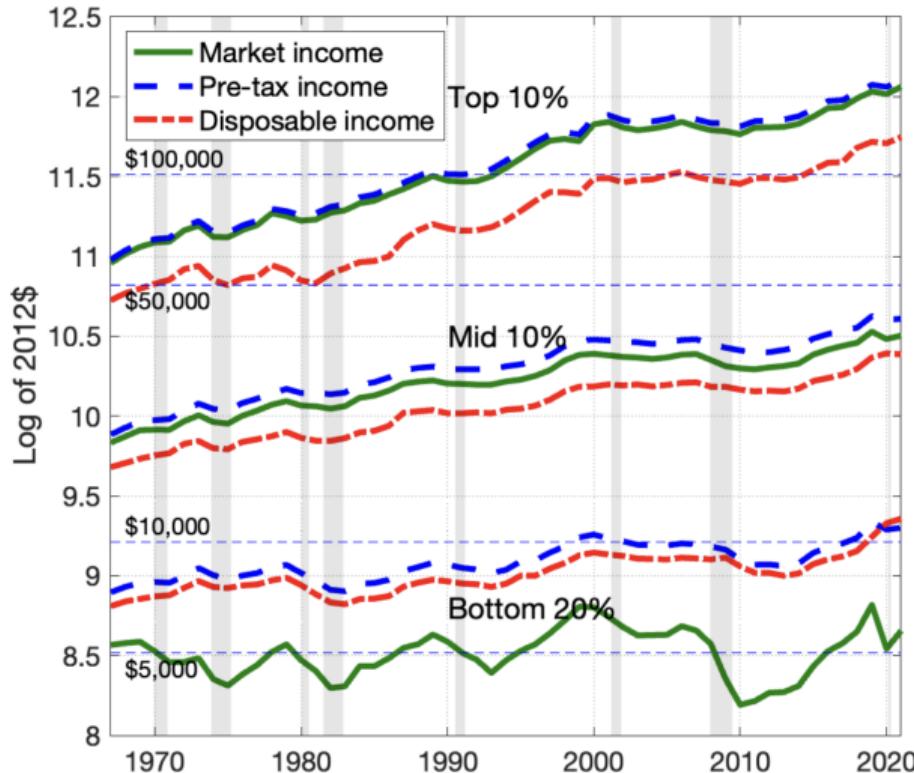
## Summary: individual- vs. household-level earnings inequality

- Household is potentially important source of insurance and redistribution due to income pooling
- Correlation between earnings within couples has increased (sorting)
- Gender gap in earnings has declined
- Consequently: within-household insurance and redistribution less important



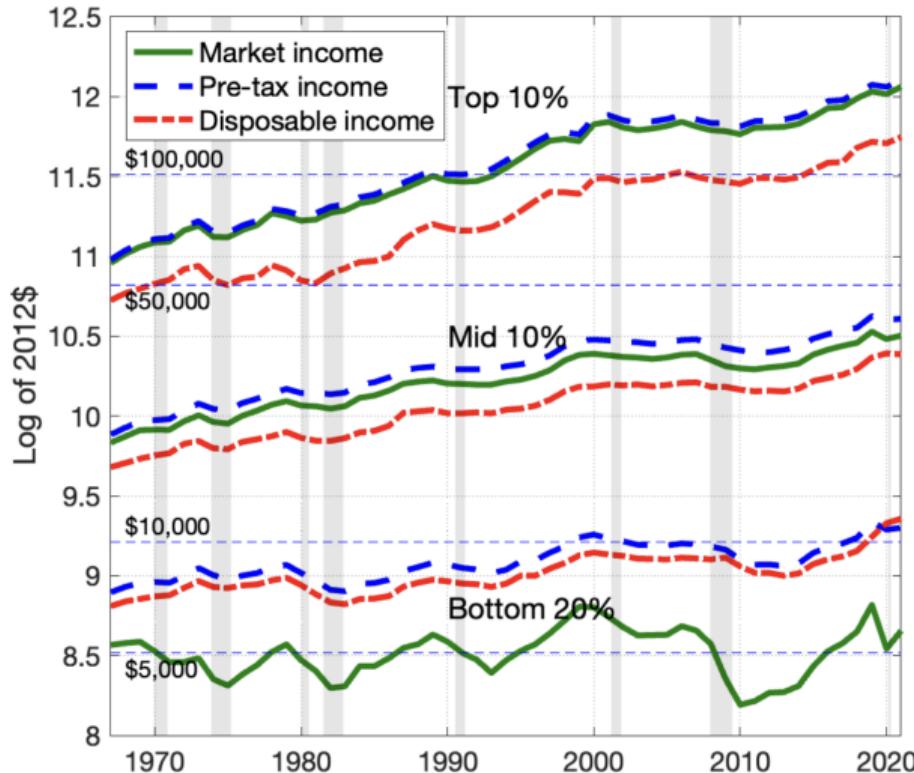
**Note:** This is Figure 16 in HPVZ. Sample: CPS, all households with one member aged 25-60.

**ER22:** Trend growth in market income at top and middle. No trend growth at bottom but large cyclical



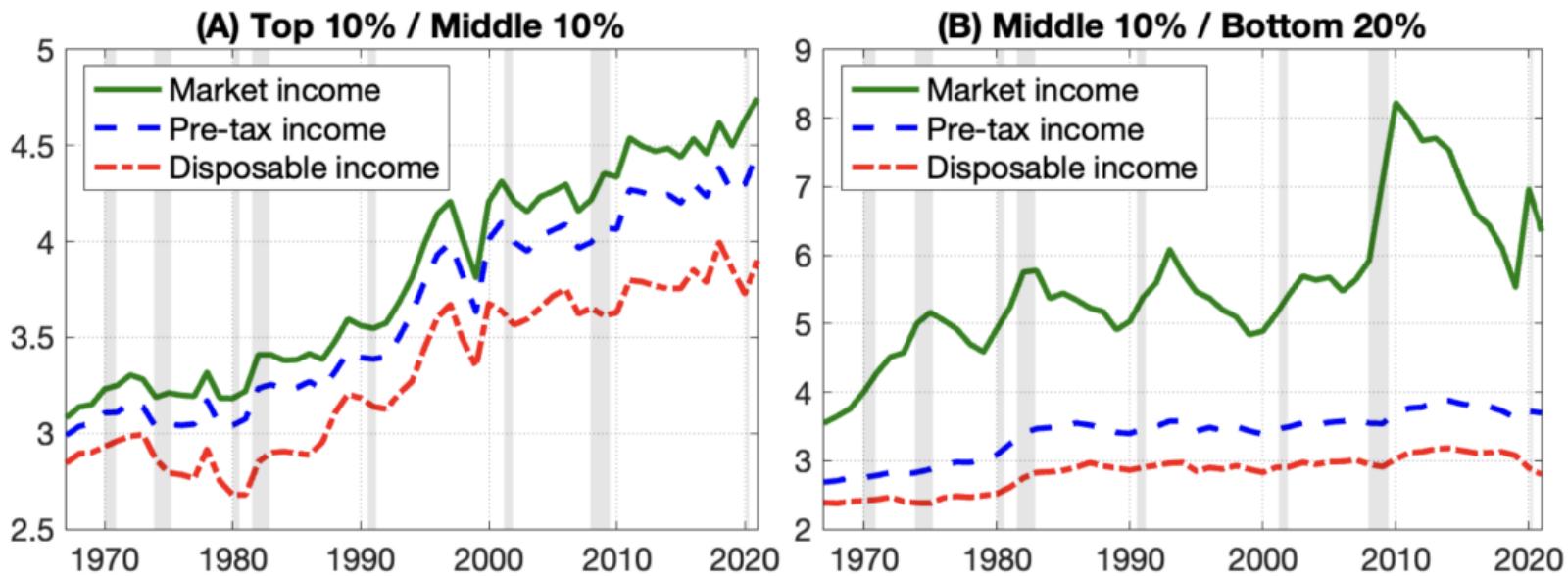
**Note:** This is Figure 16 in HPVZ. Sample: CPS, all households with one member aged 25-60.

**ER23:** Transfers large and rising at bottom, mute cyclicity in disposable income

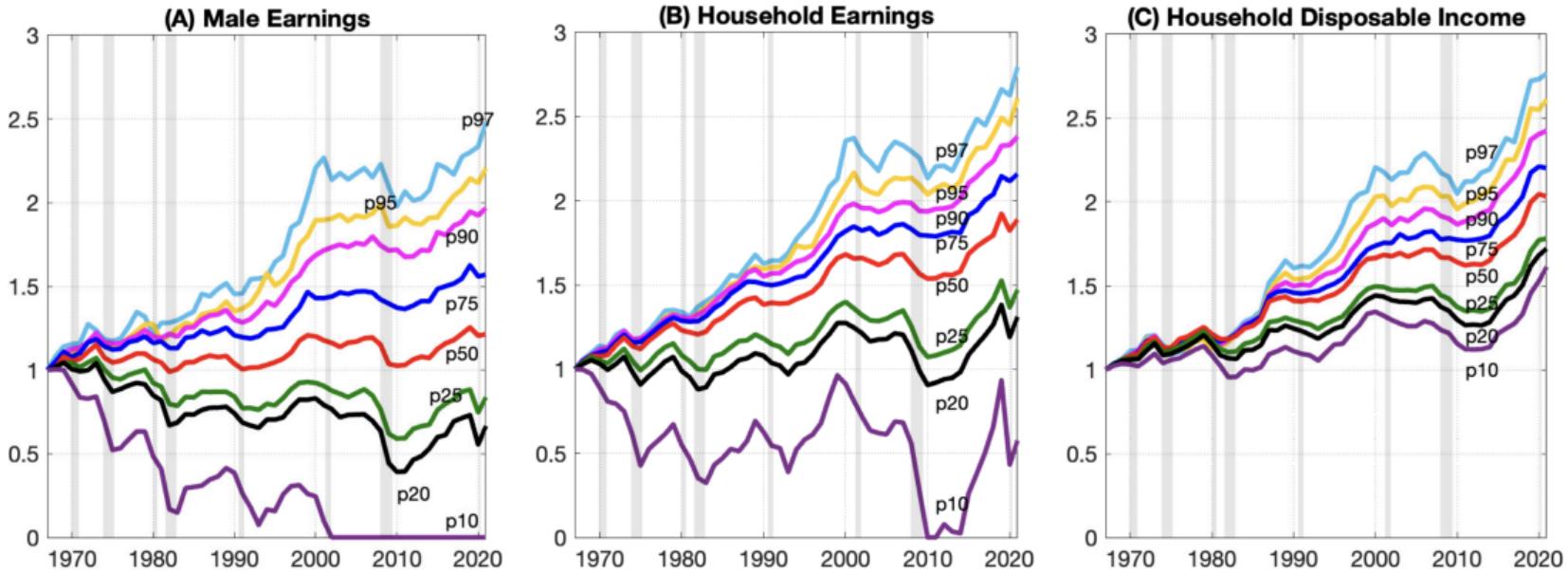


**Note:** This is Figure 16 in HPVZ. Sample: CPS, all households with one member aged 25-60.

**ER24:** Taxes mute rise in disposable income dispersion at the top

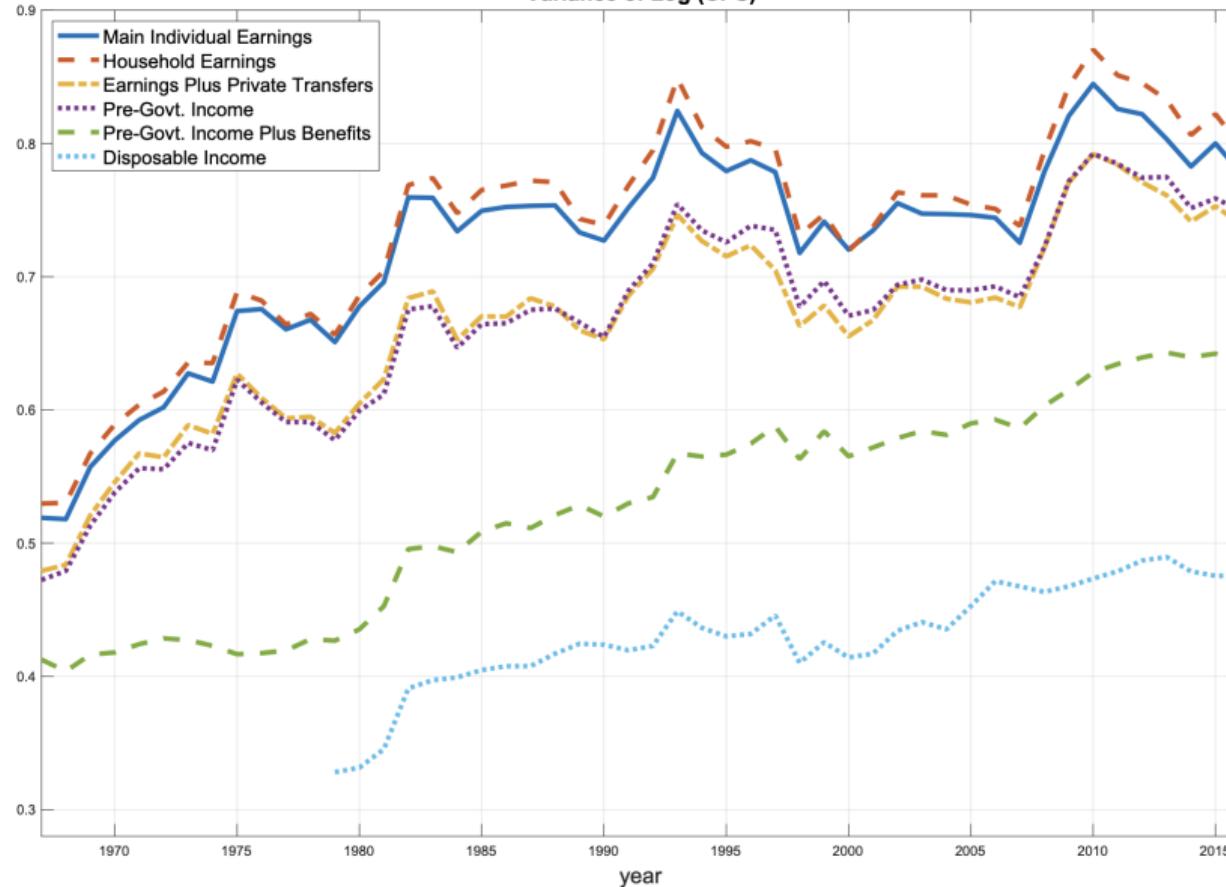


**Note:** This is Figure 17 in HPVZ. Sample: CPS, all households with one member aged 25-60.



**Note:** This is Figure 18 in HPVZ. Sample: CPS, all households with one member aged 25-60.

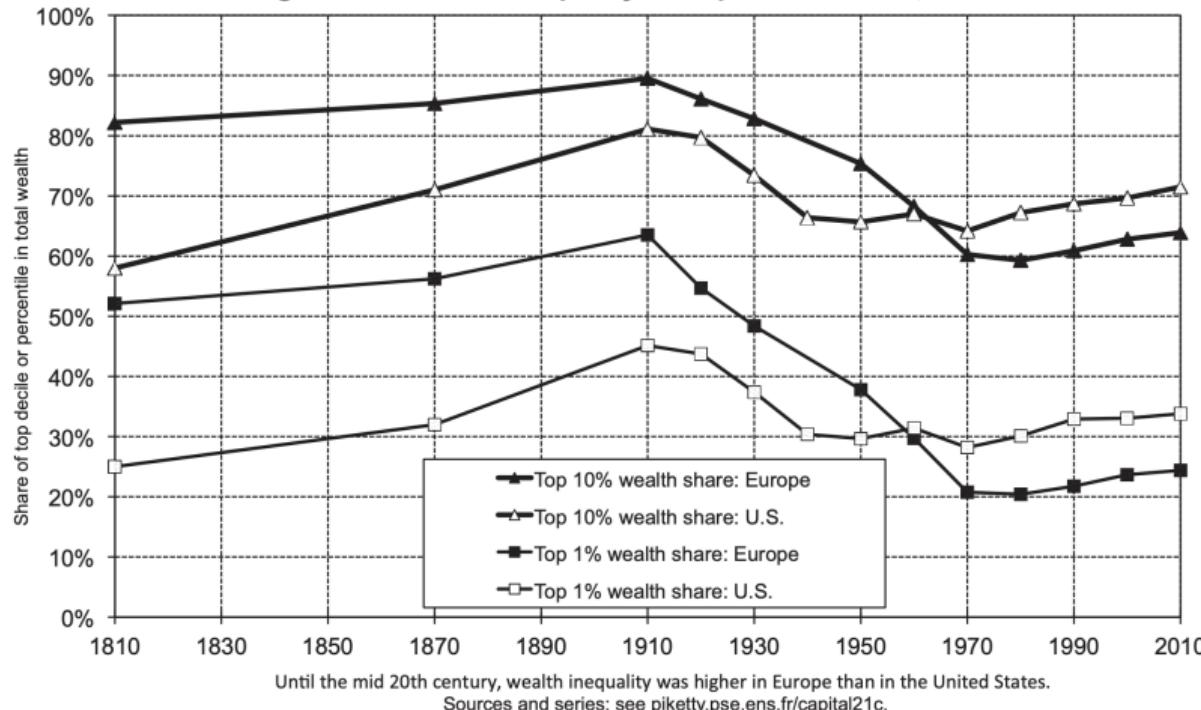
### Variance of Log (CPS)



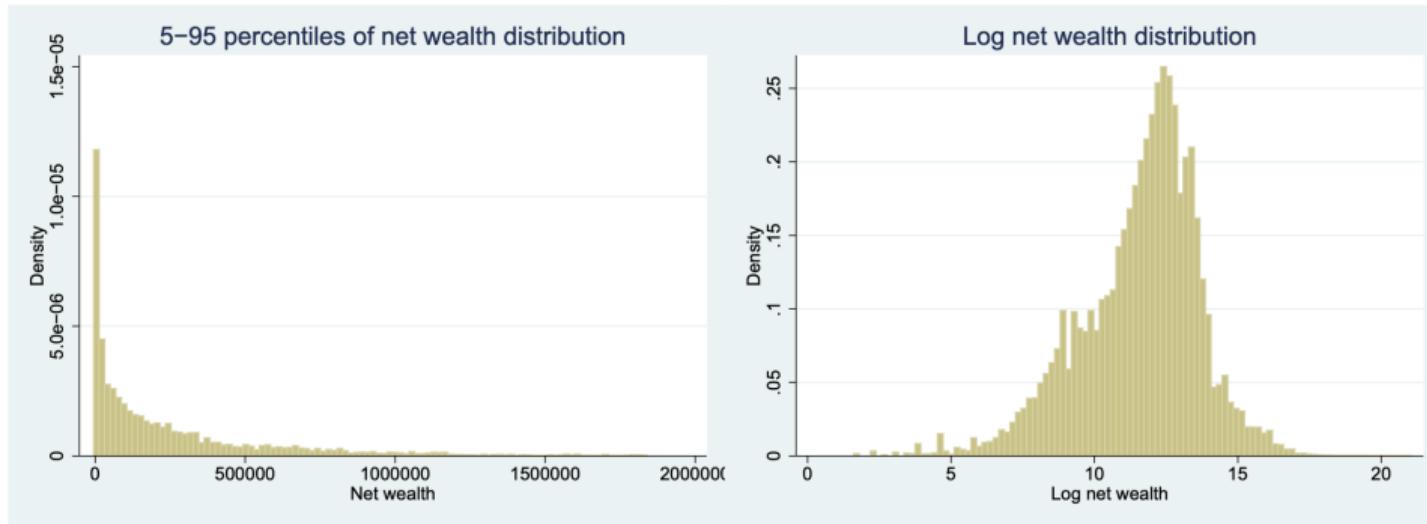
## Summary: insurance and redistribution

- Large rise in market income dispersion
- Income pooling reduces dispersion in household income dispersion, but increasingly less so
- At bottom (below median), inequality in market (pretax) income strongly counter-cyclical (unemployment)
- Transfers (and taxes to lesser extent) compress inequality and dampen cyclical
- Transfers and automatic stabilizers during Covid so large that disposable income inequality declined for first time across 8 last recessions

**Figure 10.6. Wealth inequality: Europe and the U.S., 1810-2010**



**ER25:** Wealth inequality fell sharply in first half of 20th century, rising steadily since 1970/80 (more in US than in Europe)

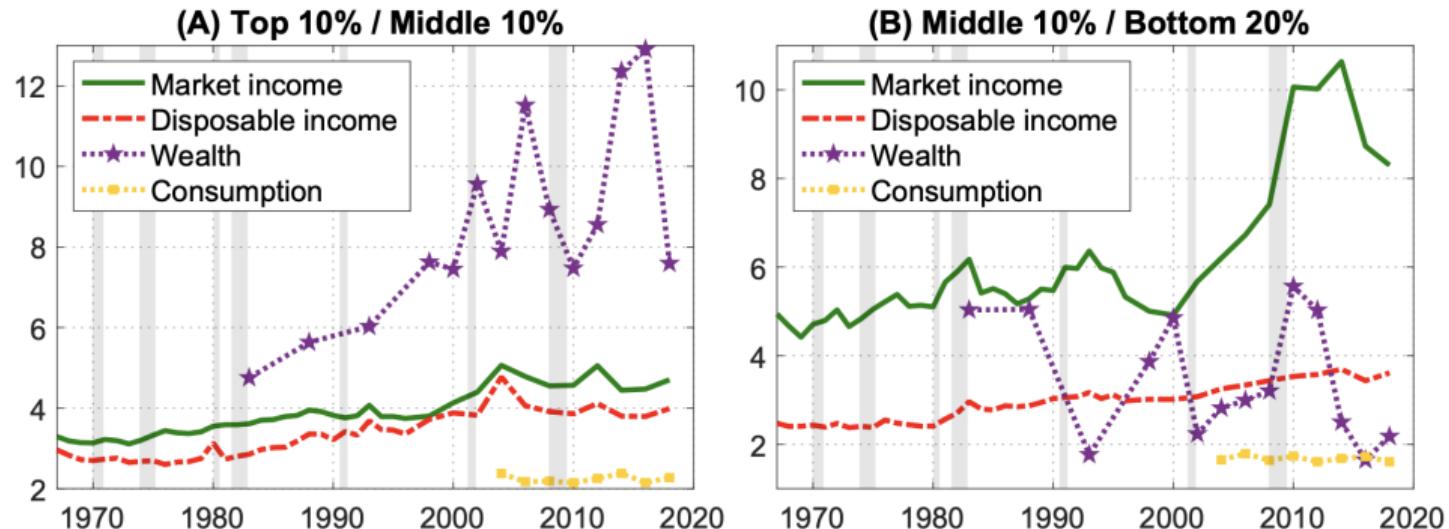


**ER26:** Wealth distribution is right-skewed and has a fat right tail (Pareto distributed)

U.S. SCF (2019). Values of earnings and wealth are in thousands of 2019 \$

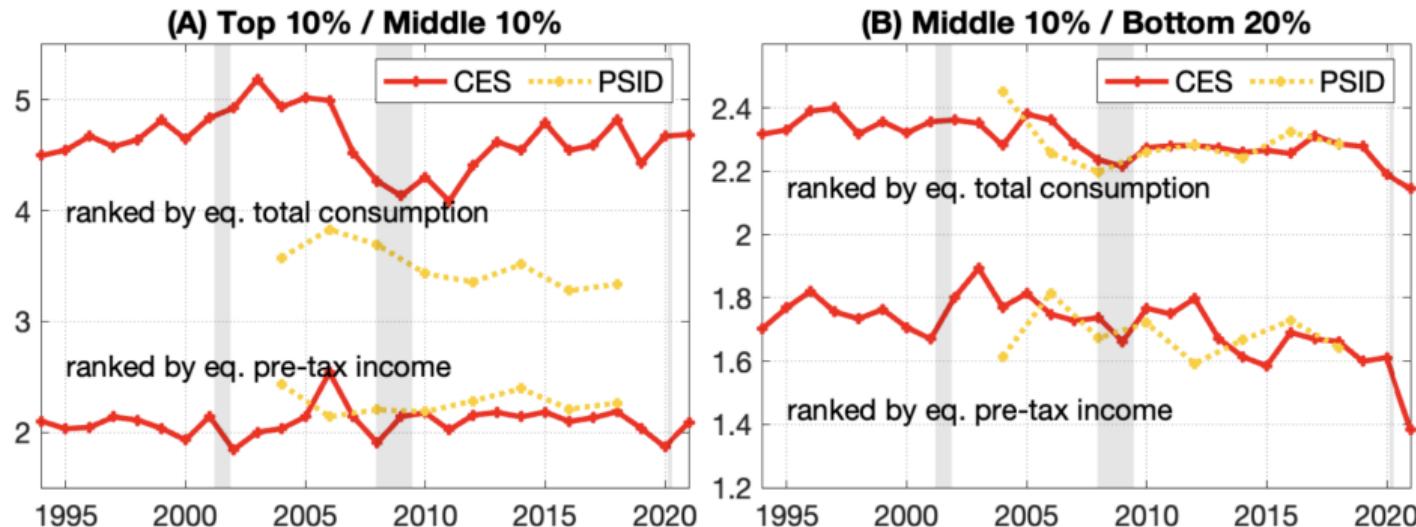
	P5	P20	P40	P80	P95	P99	Share of Top 1%
Earnings	0	0	29	107	244	536	17%
Net Worth	-18	6	68	557	2,575	11,121	37%
	Mean	Mean Median	Gini	CV			
Earnings	78	1.9	0.65	2.57			
Net Worth	746	6.3	0.85	7.53			

**ER27:** Wealth inequality is larger than earnings inequality



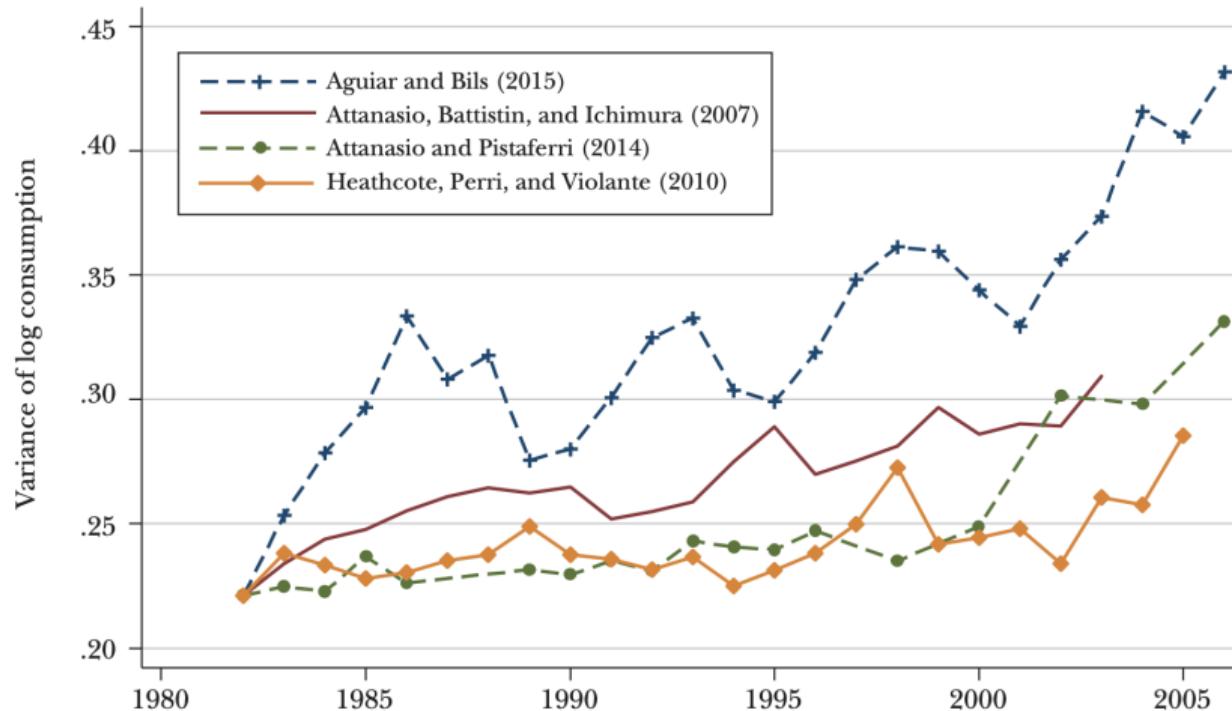
**Note:** This is Figure 24 in HPVZ. Sample: PSID, all households with one member aged 25-60.

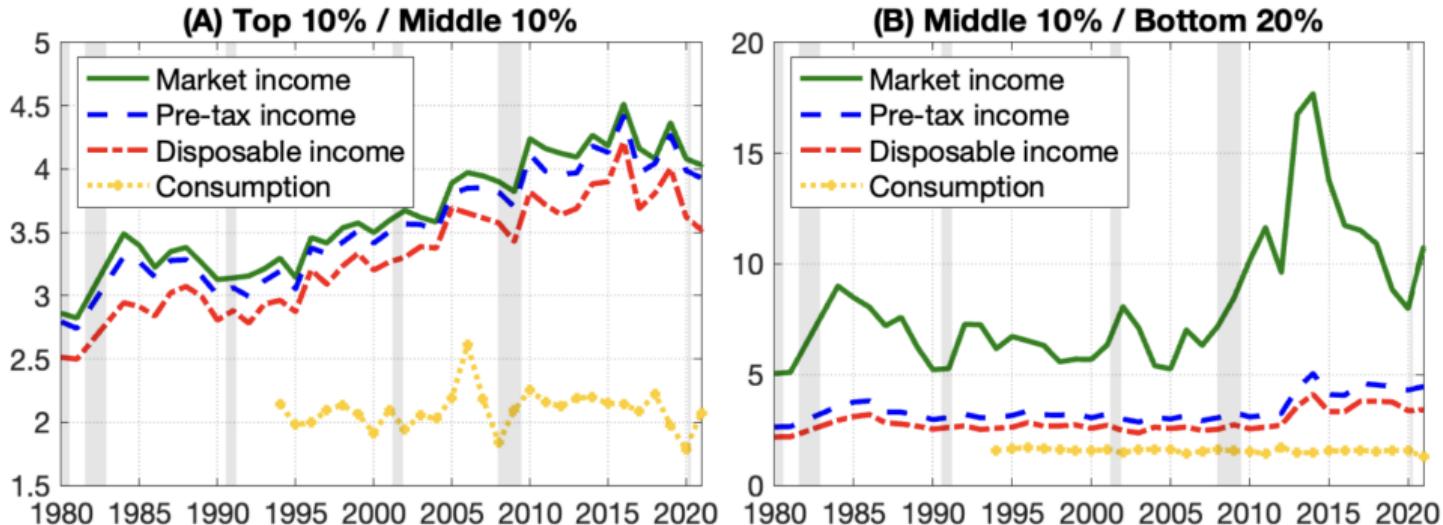
**ER28:** Wealth ratio below median is similar to income ratio, whereas above median it's much larger and growing



**Note:** This is Figure 22 in HPVZ. Sample: CES and PSID, all households with one member aged 25-60.

**ER29:** Consumption inequality is flat over time at top and bottom!!





**Note:** This is Figure 16 in HPVZ. Sample: CPS, all households with one member aged 25-60.

**ER30:** Consumption differentials small compared to income differentials (at bottom and top)

## **Summary: wealth and consumption inequality**

- Wealth inequality fell in first half of 20th century but is rising since 1980
- Wealth inequality > income inequality > consumption inequality
- Despite rising income and wealth inequality, consumption inequality has been relatively flat across entire distribution!