$$\frac{\dot{c}}{c} = \frac{r(t) - \rho}{\theta}$$

$$\dot{c}(t) = \frac{r(t) - \rho}{\theta} c(t)$$

$$\dot{c}(t) - \frac{r(t) - \rho}{\theta} c(t) = 0$$

$$\theta \rightarrow \frac{1}{\theta} \int_0^t (r(s) - \rho)\, ds$$

$$\Rightarrow c(t) = c_0 \, e^{\frac{1}{\theta}\int_0^t (r(s) - \rho) ds}$$

Using terminal condition,

$$2 = c(10) = c_0 \, e^{\frac{1}{\theta}\int_0^{10} (r(s) - \rho) ds}$$

$$r(s) = 0.05 + 0.01 t$$

$$\int_0^{10} (r(s) - \rho)\, ds = \int_0^{10} (0.02 + 0.01 s)\, ds$$

$$= 0.02(10) + 0.01\left(\frac{100}{2}\right)$$

$$= 0.7$$

$$\Rightarrow 2 = C_0 \times e^{0.7/2}$$

$$C_0 = 2 \times e^{-\frac{0.7}{2}} = 1.4094$$

$$C(t) = 1.4094\, e^{\frac{1}{2}\int_0^t (0.02 + 0.01s)\, ds}$$

$$= 1.4094\, e^{\frac{1}{2}\left[0.02t + \frac{0.01}{2} t^2\right]}$$