Dynamic Programming with Applications

Instructors:
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Prerequisites: If you have had no exposure to any of the following, you should try to brush up

- Constrained optimization: Lagrange multipliers, necessary and sufficient conditions for optimality, Karush-Kuhn-Tucker conditions
- Difference equations
- Ordinary differential equations (we will also encounter partial differential equations)
- Probability theory: Markov chains, stochastic processes, Brownian motion, stochastic differential equations (we will also encounter stochastic calculus and Ito's lemma)

Textbooks and References

Required:

- LeVeque [**LV**]: Finite difference methods for ordinary and partial differential equations

The most relevant (but optional) textbooks for this course would be:

- Stokey and Lucas [**SL**]: Recursive methods in economic dynamics
- Ljungqvist and Sargent [**LS**]: Recursive macroeconomic theory
- Kamien and Schwartz [**KS**]: Dynamic optimization: the calculus of variations and optimal control
- Oksendal: Stochastic differential equations
- Stokey: The economics of inaction
- Acemoglu [**A**]: Introduction to modern economic growth
- Barro and Sala-i-Martin [**BS**]: Economic growth
- Romer: Advanced macro

Other useful references:

- Blitzstein: Introduction to probability
- Steele: Stochastic calculus and financial applications
- Durrett: Probability: theory and examples
- Graduate-level probability theory (Harvard Stats 210): https://www.ekzhang.com/assets/pdf/Stat_210_Notes.pdf
- Stachurski: Economic dynamics
- Miao: Economic dynamics in discrete time

Discrete-time Markov chains and (stochastic) difference equations:

- Stachurski chapters 4, 5, 6, 8, 11
- Miao chapters 1, 2, 3
- LS chapters 2
- SL chapters 6, 8

Differential equations:
- A appendix B
- BS appendix A.1 through A.3
- BS chapters 2, 3
- LeVeque (entire book)
- KS (entire book on calc of variations and optimal control theory)

Integration, measure, and probability theory:
- Blitzstein (this is a fantastic undergraduate-level intro to probability theory)
- Durrett (entire book on probability theory)
- Stachurski chapters 7, 9
- SL chapter 7
- Miao appendix D

Stochastic processes, continuous-time Markov chains, stochastic differential equations:
- Durrett (entire book)
- Steele (entire book)
- Oksendal (entire book)
- Stokey chapters 2, 3

Grading and Homework

Homework is optional and the final exam counts for 100% of the grade. However, handing in the problem sets earns you extra credit, up to a total of 60% of the grade. To illustrate: Suppose you score 75/100 on the final exam but fully complete and hand in all 6 problem sets. Then your final grade will be 60 + 0.4*75 = 90/100. If you complete half of each problem set, then your final grade would be 30 + 0.7*75 = 82.5/100.

Office hours

Andreas: by appointment
Kiyea: TBD

Part I: Dynamic Programming

Lectures 1-2: dynamic programming in discrete time

- Neoclassical growth model
- Sequence problem
- Dynamic programming: the principle of optimality
- Bellman equation
- Solving the Bellman equation via guess-and-verify (closed-form solution of growth model)
- Bellman operator
- Contraction mapping
- Gentle introduction to measure theory: Expectation, conditional expectation, law of iterated expectations,
- Adding uncertainty: the Bellman equation with stochastic dynamics

Lectures 3-4: deterministic dynamic programming in continuous time

- Ordinary differential equations (ODEs)
- Boundary conditions
- ODEs with closed-form solutions: population growth, half-life formula, AR(1) process, general homogeneous solution, …
- Partial differential equations (PDEs)
- Neoclassical growth model in continuous time
- Sequence problem
- Calculus of variations
- Optimal control theory
- Deriving the Hamilton-Jacobi-Bellman (HJB) equation (Stokey derivation)
- First-order condition for consumption
- Envelope condition and Euler equation
- Connection between calculus of variations / optimal control and HJBs
- Boundary conditions: no-borrowing in the wealth / capital dimension

Lecture 5-6: stochastic dynamic programming in continuous time

- Stochastic processes
- Brownian motion
- Stochastic differential equations (SDEs)
- What is the generator of a stochastic process?
- Neoclassical growth model with diffusion process
- Neoclassical growth model with Poisson process
- Examples

Part II: Applications

Lectures 7-8: consumption

- Permanent income hypothesis
- Lifetime budget constraint
- Certainty equivalence and consumption as a martingale (quadratic preferences)
- Eat-the-pie problem (CRRA preferences)
- Closed-form examples
- GHH preferences
- Income fluctuations and precautionary savings
- Linearization of consumption Euler equation: deterministic and stochastic
- Liquidity constraints: the buffer stock model
- Consumption over the life cycle

Lecture 9: investment

- When to shut down a plant?
- Optimal stopping problems
- Option value
- Tobin's Q

Lecture 10-11: inequality

Lecture 12-13: welfare