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Add paths to the necessary functions

```
addpath(genpath('/Users/muhammadbashir/GitHub/MuhammadCourses/
DynamicProgramming2024/ProblemSetSolutions/PS4/GE'))
addpath(genpath('/Users/muhammadbashir/GitHub/MuhammadCourses/
DynamicProgramming2024/ProblemSetSolutions/PS4/Asset Supply'))
addpath(genpath('/Users/muhammadbashir/GitHub/MuhammadCourses/
DynamicProgramming2024/ProblemSetSolutions/PS4/KF'))
addpath(genpath('/Users/muhammadbashir/GitHub/MuhammadCourses/
DynamicProgramming2024/ProblemSetSolutions/PS4/HJB'))
```

1. DEFINE PARAMETERS

```
p = define_parameters_assetsupply();
```

2. INITIALIZE GRID POINTS

```
a = linspace(p.amin, p.amax, p.I)';
da = (p.amax-p.amin)/(p.I-1);

aa = [a, a]; % I*2 matrix

    % %% 2-2. INITIALIZE GRID POINTS FOR INTEREST RATES

    % rgrid = linspace(p.rmin, p.rmax, p.Ir)';
```

3. PRE-ITERATION INITIALIZATION

```
% 3-1. Construct the forward and backward differential operator
% Df such that Df*V=dVf and Db such that Db*V=dVb

    Df = zeros(p.I, p.I);
    for i = 1:p.I-1
    Df(i,i) = -1/da; Df(i,i+1) = 1/da;
    end
    Df = sparse(Df);

    Db = zeros(p.I, p.I);
```

```
    for i = 2:p.I
    Db(i,i-1) = -1/da; Db(i,i) = 1/da;
    end
    Db = sparse(Db);

% 3-2. Construct A_switch matrix

    A_switch = [speye(p.I).*(-p.lambda(1)), speye(p.I).*p.lambda(1);
        speye(p.I).*p.lambda(2), speye(p.I).*(-p.lambda(2))];

%A_switch = zeros(2*I, 2*I);
%for i=1:I
%    A_switch(i,i) = -lambda(1);
%    A_switch(i,i+I) = lambda(1);
%    A_switch(i+I,i) = lambda(2);
%    A_switch(i+I,i+I) = -lambda(2);
%end
```

3-3. Guess an initial value of the value function

```
        zz = ones(p.I, 1).*p.zz; % I*2 matrix
        % Define the function to find the zero
        f = @(r) Huggett_assetsupply(r, p, aa, zz, Df, Db, A_switch, da);

        % Initial guess for r
        r_guess = 0.03;

        % Tolerance and maximum iterations
        tol = 1e-6;
        max_iter = 100;

        % Newton's method
        for iter = 1:max_iter
        % Evaluate function and its derivative
        f_val = f(r_guess);
        f_prime = (f(r_guess + tol) - f_val) / tol;

        % Update r using Newton's method
        r_new = r_guess - f_val / f_prime;

        % Check for convergence
        if abs(r_new - r_guess) < tol
            fprintf('Converged to r = %.6f in %d iterations\n', r_new, iter);
            break;
        end

        % Update guess
        r_guess = r_new;
        end

        if iter == max_iter
        warning('Newton method did not converge within the maximum number of
iterations');
```

```
    end

    % Final value of r
    r = r_new;
```

Getting all variables at this interest rate

Initial guess for the value function

```
v0 = p.u(zz + max(r, 0.01) .* aa) ./ p.rho;
V = v0;

% 1. VALUE FUNCTION ITERATION
for n = 1:p.maxit

% 1-1. Compute derivatives of the value function
dVf = Df * V;
dVb = Db * V;

% 1-2. Boundary conditions
dVb(1, :) = p.mu(zz(1, :) + r .* aa(1, :));      % Enforce borrowing
constraint a >= a_min
dVf(end, :) = p.mu(zz(end, :) + r .* aa(end, :)); % Enforce upper bound a
<= a_max

% 1-3. Compute optimal consumption
cf = p.inv_mu(dVf);
cb = p.inv_mu(dVb);

% 1-4. Compute optimal savings
sf = zz + r .* aa - cf;
sb = zz + r .* aa - cb;

% 1-5. Upwind scheme
If = sf > 0;
Ib = sb < 0;
I0 = 1 - If - Ib;
dV_Upwind = If .* dVf + Ib .* dVb + I0 .* p.mu(zz + r .* aa);
c = p.inv_mu(dV_Upwind);
s = zz + r .* aa - c;

% 1-6. Construct the matrix for the linear system
c_stacked = c(:);
V_stacked = V(:);

% Drift terms
A1 = spdiags(If(:, 1) .* sf(:, 1), 0, p.I, p.I) * Df + spdiags(Ib(:,
1) .* sb(:, 1), 0, p.I, p.I) * Db;
A2 = spdiags(If(:, 2) .* sf(:, 2), 0, p.I, p.I) * Df + spdiags(Ib(:,
2) .* sb(:, 2), 0, p.I, p.I) * Db;
A = [A1, sparse(p.I, p.I); sparse(p.I, p.I), A2];

% Total generator
```

```
    P = A + A_switch;

    % Right-hand side and left-hand side for the linear system
    B = (p.rho + 1 / p.Delta) * speye(2 * p.I) - P;
    b = p.u(c_stacked) + V_stacked / p.Delta;

    % Solve for the updated value function
    V_new = B \ b;
    V_change = max(abs(V_new - V_stacked));

    % Update value function
    V = reshape(V_new, p.I, 2);

    % Check for convergence
    if V_change < p.tol
        break;
    end
    end

    % 2. KF EQUATION

    % Transpose of generator matrix
    P_transpose = P';

    % Solve P'*g = 0 with normalization condition
    g_stacked = zeros(2 * p.I, 1);
    i_fix = 1;
    P_transpose(i_fix, :) = zeros(1, 2 * p.I);
    P_transpose(i_fix, i_fix) = 1;
    g_stacked(i_fix) = 1;

    % Solve for the stationary distribution
    g = P_transpose \ g_stacked;

    % Normalize the distribution
    g = g / sum(g * da);

    % Reshape to original dimensions
    g = reshape(g, p.I, 2);

    % Compute asset supply S(r)
    S = sum(g(:, 1) .* aa(:, 1) * da) + sum(g(:, 2) .* aa(:, 2) * da);
```

Converged to r = 0.033935 in 4 iterations

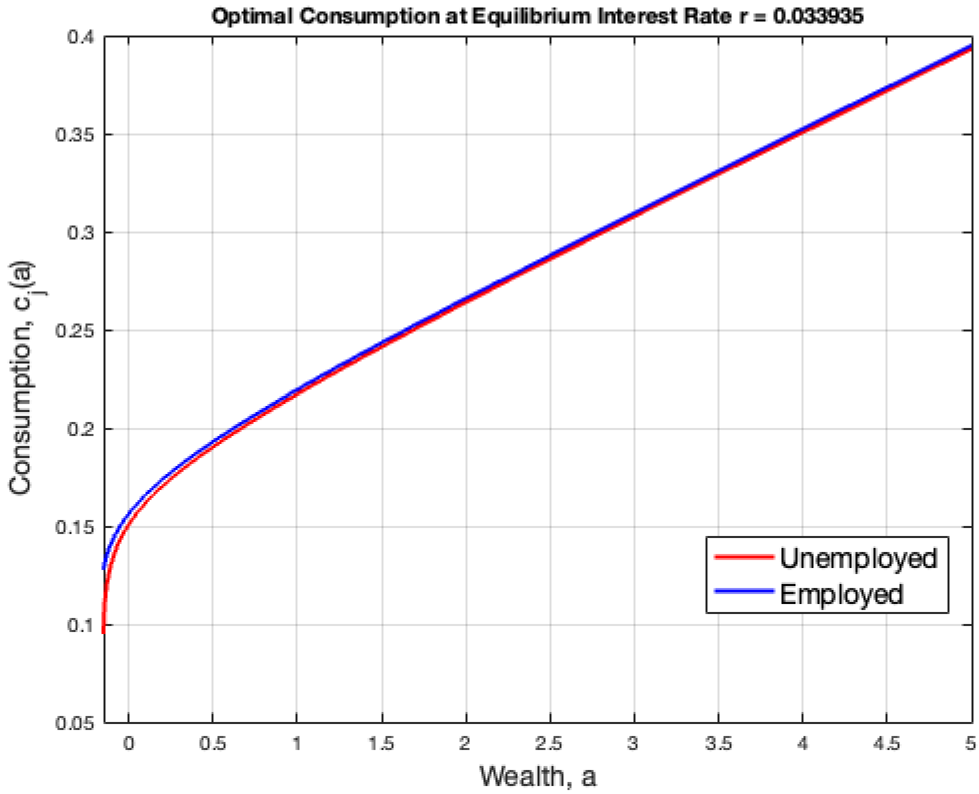
6. GRAPHS

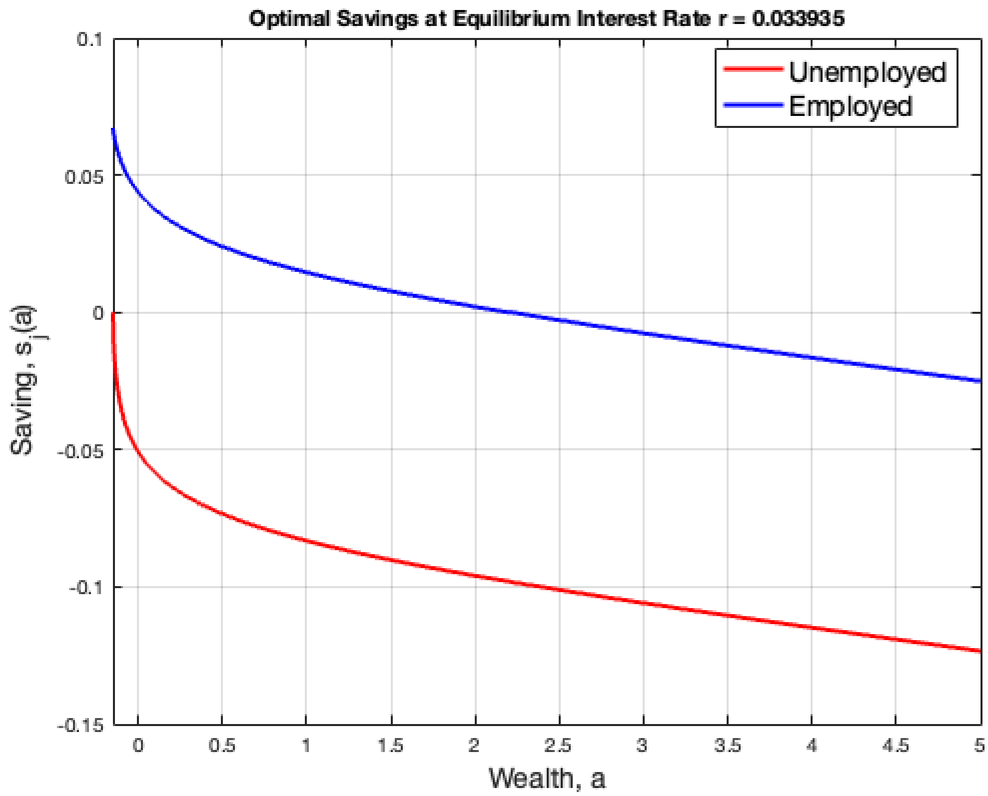
6-1. Optimal consumption

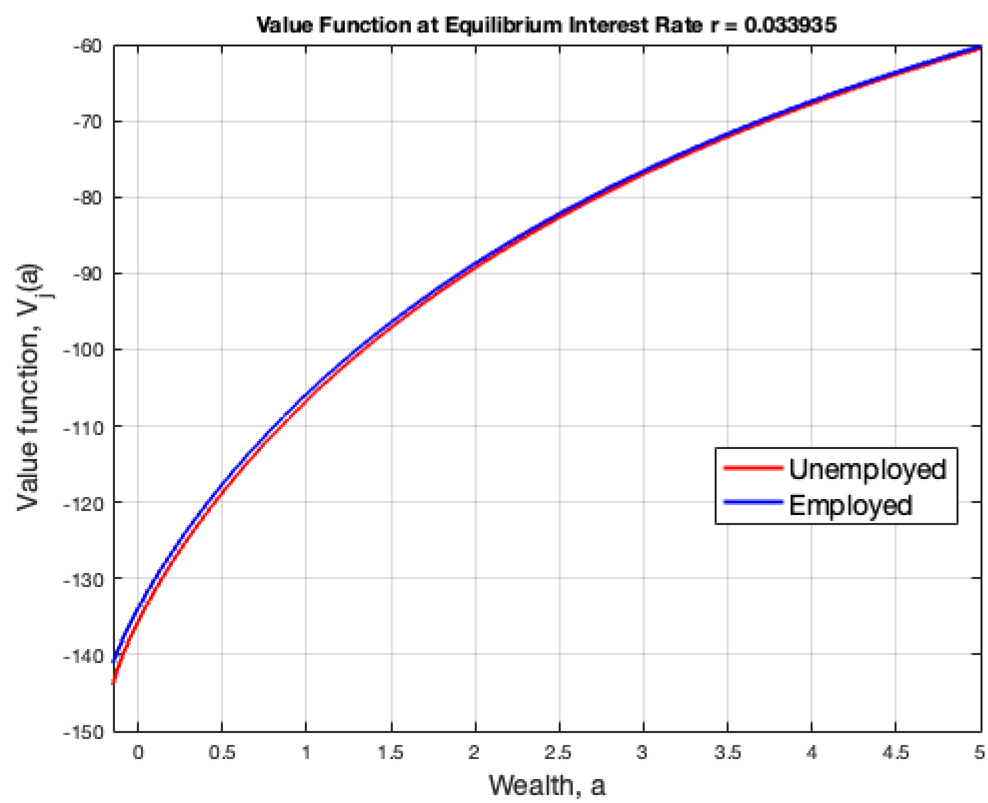
```
figure
set(gca, 'FontSize', 18)
plot(a, c(:,1), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'r')
hold on
plot(a, c(:,2), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'b')
```

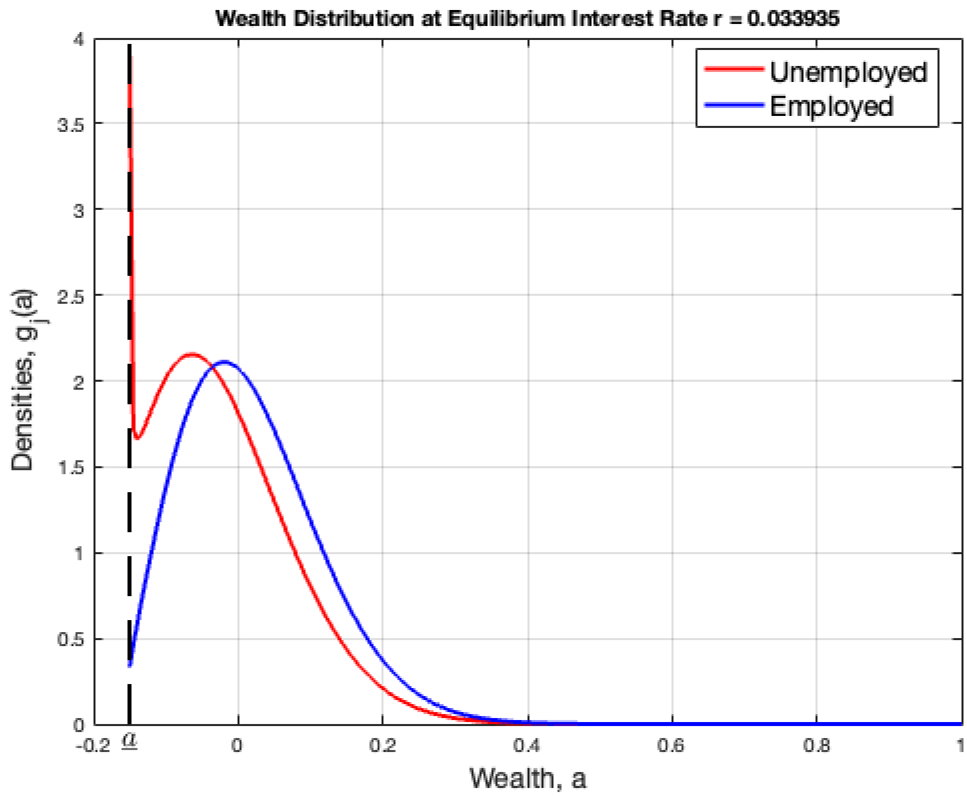
```
hold off
grid
xlabel('Wealth, a','FontSize', 14)
ylabel('Consumption, c_j(a)','FontSize', 14)
xlim([p.amin p.amax])
legend('Unemployed', 'Employed', 'Location', 'best', 'FontSize', 14)
title(sprintf('Optimal Consumption at Equilibrium Interest Rate r = %.6f', r))

% 6-2. Optimal savings
figure
set(gca, 'FontSize', 18)
plot(a, s(:,1), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'r')
hold on
plot(a, s(:,2), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'b')
hold off
grid
xlabel('Wealth, a', 'FontSize', 14)
ylabel('Saving, s_j(a)', 'FontSize', 14)
xlim([p.amin p.amax])
legend('Unemployed', 'Employed', 'Location', 'best', 'FontSize', 14)
title(sprintf('Optimal Savings at Equilibrium Interest Rate r = %.6f', r))

% 6-3. Value function
figure
set(gca, 'FontSize', 18)
plot(a, V(:,1), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'r')
hold on
plot(a, V(:,2), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'b')
hold off
grid
xlabel('Wealth, a', 'FontSize', 14)
ylabel('Value function, V_j(a)', 'FontSize', 14)
xlim([p.amin p.amax])
legend('Unemployed', 'Employed', 'Location', 'best', 'FontSize', 14)
title(sprintf('Value Function at Equilibrium Interest Rate r = %.6f', r))

% 6-4. Wealth distribution
figure
set(gca, 'FontSize', 14)
plot(a, g(:,1), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'r')
hold on
plot(a, g(:,2), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'b')
hold off
grid
xlabel('Wealth, a', 'FontSize', 14)
ylabel('Densities, g_j(a)', 'FontSize', 14)
yy = get(gca, 'yLim');
hold on
plot([p.amin, p.amin], yy, '--k', 'LineWidth', 2)
hold off
text(-0.15, yy(1)-0.02*(yy(2) - yy(1)), '$\underline{a}$',
'HorizontalAlignment', 'center', 'FontSize', 15, 'Interpreter', 'latex')
xlim([-0.2 1])
```

```
legend('Unemployed', 'Employed', 'Location', 'best', 'FontSize', 14)
title(sprintf('Wealth Distribution at Equilibrium Interest Rate r = %.6f', r))
```




Optimal Savings at Equilibrium Interest Rate r = 0.033935
Unemployed
Employed
Saving, $s_j(a)$
Wealth, a
0.1
0.05
0
-0.05
-0.1
-0.15
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Value Function at Equilibrium Interest Rate r = 0.033935
Unemployed
Employed
Value function, $V_j(a)$
Wealth, a
-60
-70
-80
-90
-100
-110
-120
-130
-140
-150
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



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