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```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% MATLAB Code: HJB_Huggett_GE
%
% Author: Muhammad (Modified section code from Kiyea)
% Date: Nov 7, 2024
%
% Description:
% This MATLAB script solves the general equilibrium of the Huggett model,
% finding the equilibrium interest rate that clears the bond market.
%
% Reference: Huggett_equilibrium_iterate.m by Benjamin Moll
%
% Notes:
% - CRRA utility function: U(c) = (c^(1-sigma))/(1-sigma)
% - Elasticity of intertemporal substitution (sigma): 2
% - Discount rate (rho): 0.05
% - Income: z = [z_u, z_e] = [0.1, 0.2];
% - Lambda: la = [la_u, la_e] = [1.2, 1.2];
% - Discrete grid of asset levels (a): -0.15 to 5
% - Borrowing constraint: a>=-0.15
% - Delta = 1000; (Can be arbitrarily large in implicit method)
%
% Code Structure:
% 1. DEFINE PARAMETERS
% 2. INITIALIZE GRID POINTS
% 3. PRE-ITERATION INITIALIZATION
% 4. VALUE FUNCTION ITERATION
% 5. KF EQUATION
% 6. GRAPHS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all;
close all;
clc;
```

1. DEFINE PARAMETERS

```
p = define_parameters_GE();
```

2. INITIALIZE GRID POINTS

```
a = linspace(p.amin, p.amax, p.I)';
da = (p.amax-p.amin)/(p.I-1);

aa = [a, a]; % I*2 matrix

        % %% 2-2. INITIALIZE GRID POINTS FOR INTEREST RATES
        %
        % rgrid = linspace(p.rmin, p.rmax, p.Ir)';
```

3. PRE-ITERATION INITIALIZATION

```
% 3-1. Construct the forward and backward differential operator
% Df such that Df*V=dVf and Db such that Db*V=dVb

    Df = zeros(p.I, p.I);
    for i = 1:p.I-1
        Df(i,i) = -1/da; Df(i,i+1) = 1/da;
    end
    Df = sparse(Df);

    Db = zeros(p.I, p.I);
    for i = 2:p.I
        Db(i,i-1) = -1/da; Db(i,i) = 1/da;
    end
    Db = sparse(Db);

% 3-2. Construct A_switch matrix

    A_switch = [speye(p.I).*(-p.lambda(1)), speye(p.I).*p.lambda(1);
                speye(p.I).*p.lambda(2), speye(p.I).*(-p.lambda(2))];

%A_switch = zeros(2*I, 2*I);
%for i=1:I
%    A_switch(i,i) = -lambda(1);
%    A_switch(i,i+I) = lambda(1);
%    A_switch(i+I,i) = lambda(2);
%    A_switch(i+I,i+I) = -lambda(2);
%end
```

3-3. Guess an initial value of the interest rate

```
            r0 = 0.03;
            r_min = 0.01;
            r_max = 0.04;
```

3-4. Guess an initial value of the value function

```
zz = ones(p.I, 1).*p.zz; % I*2 matrix

% The value function of "staying put"
r = r0;

v0 = p.u(zz + r.*aa)./p.rho;
V = v0;
```

4. VALUE FUNCTION ITERATION

```
    for nr=1:p.Nr

        r_r(nr) = r;
        rmin_r(nr) = r_min;
        rmax_r(nr) = r_max;

        % Use the value function solution from the previous interest rate
iteration
        % as the initial guess for the next iteration
        if nr>1
           v0 = V_r(:,:,nr-1);
           V = v0;
        end
```

4. VALUE FUNCTION ITERATION

```
for n=1:p.maxit

    % 4-1. Compute the derivative of the value function
    dVf = Df*V;
    dVb = Db*V;

    % 4-2. Boundary conditions
    dVb(1,:) = p.mu(zz(1,:) + r.*aa(1,:)); % a>=a_min is enforced (borrowing
constraint)
    dVf(end,:) = p.mu(zz(end,:) + r.*aa(end,:)); % a<=a_max is enforced which
helps stability of the algorithm

    I_concave = dVb > dVf; % indicator whether value function is concave
(problems arise if this is not the case)

    % 4-3. Compute the optimal consumption
    cf = p.inv_mu(dVf);
    cb = p.inv_mu(dVb);

    % 4-4. Compute the optimal savings
    sf = zz + r.*aa - cf;
    sb = zz + r.*aa - cb;

    % 4-5. Upwind scheme
```

```
    If = sf>0;
    Ib = sb<0;
    I0 = 1-If-Ib;
    dV0 = p.mu(zz + r.*aa); % If sf<=0<=sb, set s=0

    dV_upwind = If.*dVf + Ib.*dVb + I0.*dV0;

    c = p.inv_mu(dV_upwind);

    % 4-6. Update value function:
    % Vj^(n+1) = [(rho + 1/Delta)*I - (Sj^n*Dj^n+A_switch)]^(-1)*[u(cj^n) +
1/Delta*Vj^n]

    V_stacked = V(:); % 2I*1 matrix
    c_stacked = c(:); % 2I*1 matrix

    % A = SD
    SD_u = spdiags(If(:,1).*sf(:,1), 0, p.I, p.I)*Df +
spdiags(Ib(:,1).*sb(:,1), 0, p.I, p.I)*Db; % I*I matrix
    SD_e = spdiags(If(:,2).*sf(:,2), 0, p.I, p.I)*Df +
spdiags(Ib(:,2).*sb(:,2), 0, p.I, p.I)*Db; % I*I matrix
    SD = [SD_u, sparse(p.I, p.I);
         sparse(p.I, p.I), SD_e]; % 2I*2I matrix

    % P = A + A_switch
    P = SD + A_switch;

    % B = [(rho + 1/Delta)*I - P]
    B = (p.rho + 1/p.Delta)*speye(2*p.I) - P;

    % b = u(c) + 1/Delta*V
    b = p.u(c_stacked) + (1/p.Delta)*V_stacked;

    % V = B\b;
    V_update = B\b; % 2I*1 matrix
    V_change = V_update - V_stacked;
    V = reshape(V_update, p.I, 2); % I*2 matrix

    % 3-6. Convergence criterion
    dist(n) = max(abs(V_change));
    if dist(n)<p.tol
       disp('Value function converged. Iteration = ')
       disp(n)
       break
    end
end

toc;
```

```
Value function converged. Iteration =
     9

Elapsed time is 289.363564 seconds.
```

```
Value function converged. Iteration =
     5

Elapsed time is 289.397454 seconds.

Value function converged. Iteration =
     5

Elapsed time is 289.445028 seconds.

Value function converged. Iteration =
     5

Elapsed time is 289.471892 seconds.

Value function converged. Iteration =
     5

Elapsed time is 289.498808 seconds.

Value function converged. Iteration =
     4

Elapsed time is 289.516747 seconds.

Value function converged. Iteration =
     4

Elapsed time is 289.529866 seconds.

Value function converged. Iteration =
     4

Elapsed time is 289.542925 seconds.

Value function converged. Iteration =
     4

Elapsed time is 289.555628 seconds.

Value function converged. Iteration =
     4

Elapsed time is 289.575480 seconds.

Value function converged. Iteration =
     4

Elapsed time is 289.594777 seconds.

Value function converged. Iteration =
     3

Elapsed time is 289.608361 seconds.

Value function converged. Iteration =
     3
```

```
Elapsed time is 289.621413 seconds.

Value function converged. Iteration =
     3

Elapsed time is 289.633536 seconds.

Value function converged. Iteration =
     3

Elapsed time is 289.643795 seconds.
```

5. KF EQUATION

```
% 5-1. Solve for 0=gdot=P'*g

PT = P';
gdot_stacked = zeros(2*p.I,1);

% need to fix one value, otherwise matrix is singular
i_fix = 1;
gdot_stacked(i_fix)=.1;

row_fix = [zeros(1,i_fix-1),1,zeros(1,2*p.I-i_fix)];
AT(i_fix,:) = row_fix;

g_stacked = PT\gdot_stacked;

% 5-2. Normalization

g_sum = g_stacked'*ones(2*p.I,1)*da;
g_stacked = g_stacked./g_sum;

% 5-3. Reshape

gg = reshape(g_stacked, p.I, 2);
```

5-4. COMPUTE VARIABLES FOR A GIVEN r_r(nr)

Notes: Each matrix has dimensions p.I*2(u,e)*nr

```
            g_r(:,:,nr) = gg;
            adot(:,:,nr) = zz + r.*aa - c;
            V_r(:,:,nr) = V;
            dV_r(:,:,nr) = dV_upwind;
            c_r(:,:,nr) = c;

            S(nr) = gg(:,1)'*a*da + gg(:,2)'*a*da;
```

5-5. UPDATE INTEREST RATE

```
        if S(nr)>p.tol_S
            disp('Excess Supply')
            % Decrease r whenever S(r)>0
            r_max = r;
            r = 0.5*(r_min+r_max);
        elseif S(nr)<-p.tol_S
            disp('Excess Demand')
            % Increase r whenever S(r)<0
            r_min = r;
            r = 0.5*(r_min+r_max);
        elseif abs(S(nr))<p.tol_S
            disp('Equilibrium Found, Interest rate =')
            disp(r)
            break
        end
```

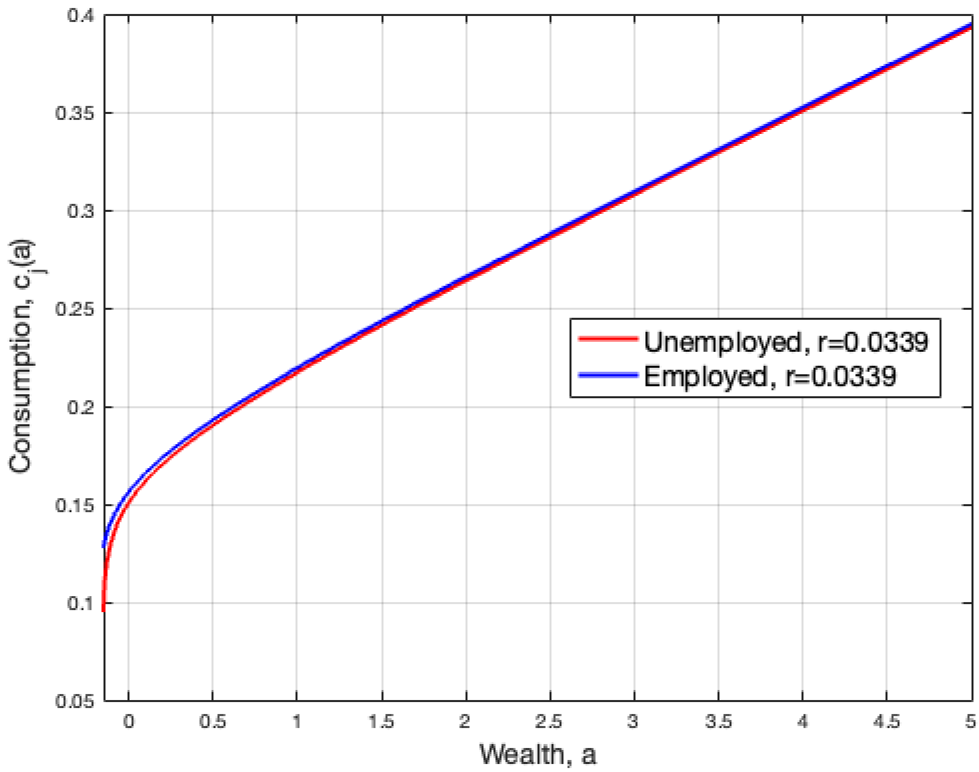
```
Excess Demand

Excess Supply

Excess Demand

Excess Demand

Excess Supply

Excess Supply

Excess Demand

Excess Supply

Excess Supply

Excess Demand

Excess Supply

Excess Demand

Excess Demand

Excess Demand

Equilibrium Found, Interest rate =
    0.0339

Algorithm converged
```

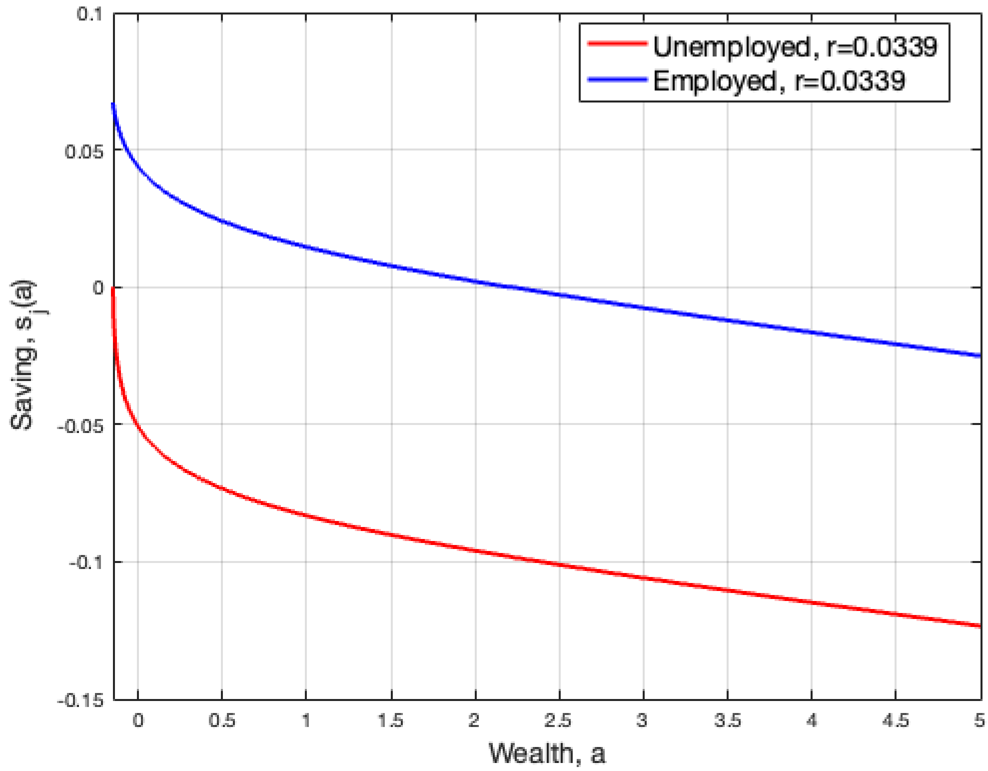
```
end

disp("Algorithm converged")
```

6. GRAPHS

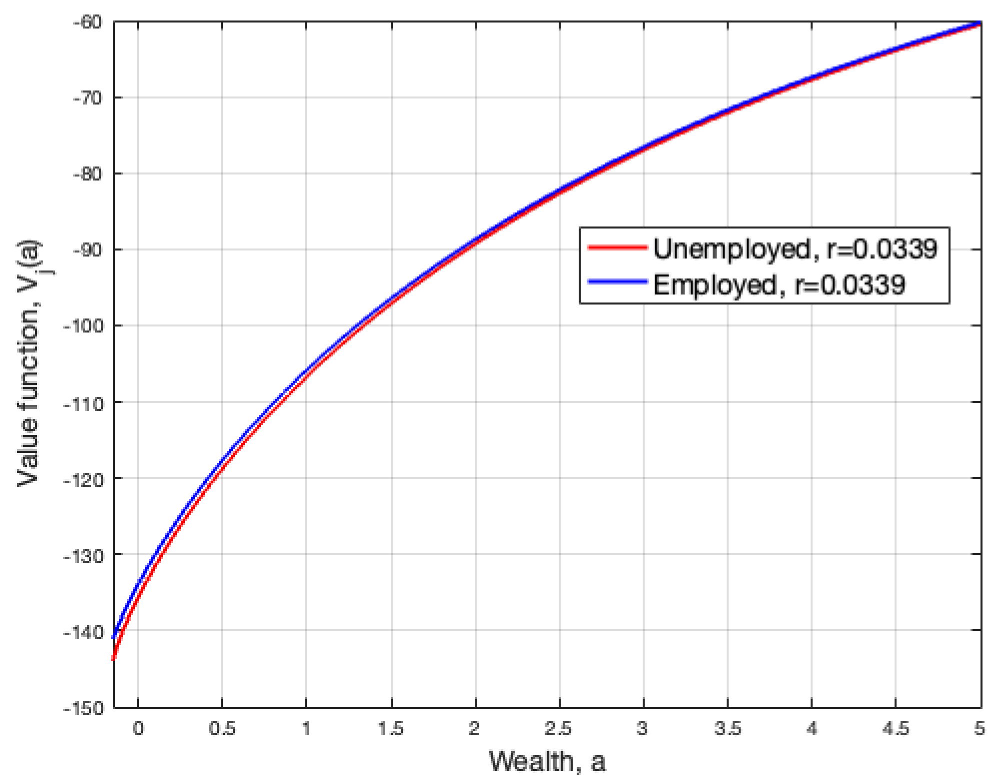
```
% 6-1. Optimal consumption
figure;
set(gca, 'FontSize', 18)
plot(a, c_r(:,1,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'r')
hold on
plot(a, c_r(:,2,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'b')
hold off
grid
xlabel('Wealth, a','FontSize', 14)
ylabel('Consumption, c_j(a)','FontSize', 14)
xlim([p.amin p.amax])
legend(sprintf('Unemployed, r=%.4f', r), ...
       sprintf('Employed, r=%.4f', r), 'Location', 'best', 'FontSize', 14)

% 6-2. Optimal savings
figure;

set(gca, 'FontSize', 18)
plot(a, adot(:,1,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'r')
hold on
plot(a, adot(:,2,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'b')
hold off
grid
xlabel('Wealth, a', 'FontSize', 14)
ylabel('Saving, s_j(a)', 'FontSize', 14)
xlim([p.amin p.amax])
legend(sprintf('Unemployed, r=%.4f', r), ...
       sprintf('Employed, r=%.4f', r), 'Location', 'best', 'FontSize', 14)

% 6-3. Value function
figure;

set(gca, 'FontSize', 18)
plot(a, V_r(:,1,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'r')
hold on
plot(a, V_r(:,2,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'b')
hold off
grid
xlabel('Wealth, a', 'FontSize', 14)
ylabel('Value function, V_j(a)', 'FontSize', 14)
xlim([p.amin p.amax])
legend(sprintf('Unemployed, r=%.4f', r), ...
       sprintf('Employed, r=%.4f', r), 'Location', 'best', 'FontSize', 14)

% 6-4. Wealth distribution
figure;

set(gca, 'FontSize', 14)
plot(a, g_r(:,1,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'r')
hold on
plot(a, g_r(:,2,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', 'b')
```

```
hold off
grid
xlabel('Wealth, a', 'FontSize', 14)
ylabel('Densities, g_j(a)', 'FontSize', 14)
yy = get(gca, 'yLim');
hold on
plot([p.amin, p.amin], yy, '--k', 'LineWidth', 2)
hold off
text(-0.15, yy(1)-0.02*(yy(2) - yy(1)), '$\underline{a}$',
'HorizontalAlignment', 'center', 'FontSize', 15, 'Interpreter', 'latex')
xlim([-0.2 1])
legend(sprintf('Unemployed, r=%.4f', r), ...
       sprintf('Employed, r=%.4f', r), 'Location', 'best', 'FontSize', 14)
```

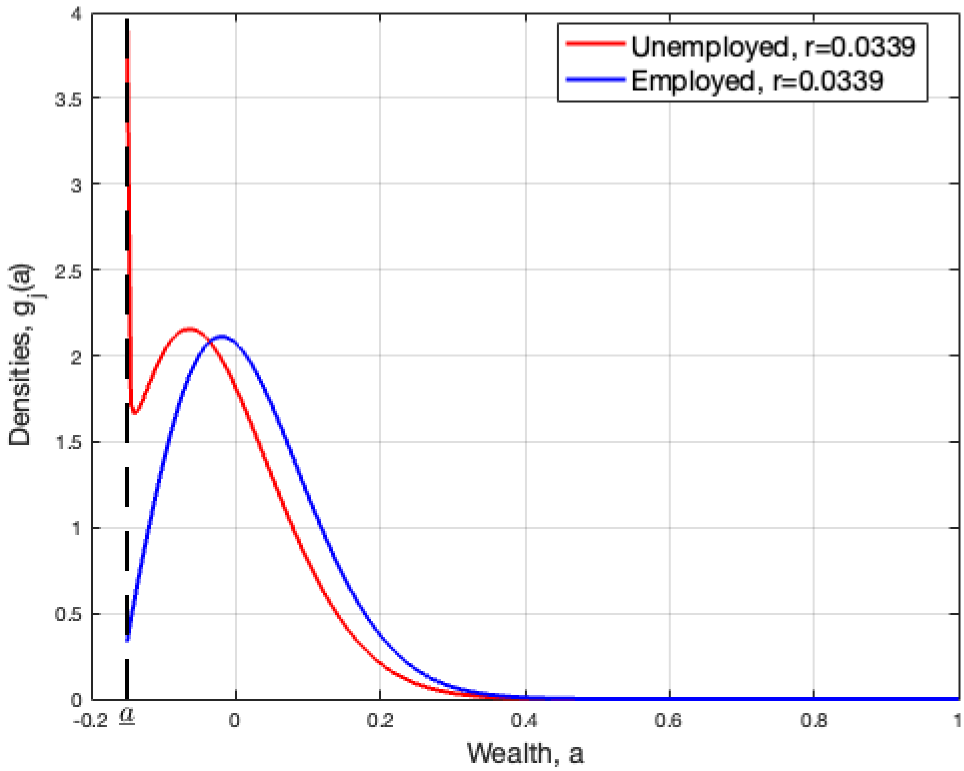




Unemployed, r=0.0339
Employed, r=0.0339
Saving, $s_j(a)$
Wealth, a



Unemployed, r=0.0339
Employed, r=0.0339
Value function, $V_j(a)$
Wealth, a



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