Second Midterm

Ec240a – Second Half, Fall 2021

Please read each question carefully. Start each question on a new bluebook page (or sheet of paper). The use of calculators and other computational aides is not allowed. Please write your name on the top of any loose sheets of paper and number then. Good luck!

[1] **[5 Points]** Please write your full name on this exam sheet and turn it in with your bluebook.

[2] **[15 Points]** Let $X_1, ..., X_K$ be a set of regressors with the property that $\mathbb{C}(X_k, X_l) = 0$ for all $k \neq l$. In this problem you will show that

$$\mathbb{E}^*[Y|X_1, \ldots, X_K] = \sum_{k=1}^{K} \mathbb{E}^*[Y|X_k] - (K-1)\mathbb{E}[Y].$$

[a] **[5 Points]** Describe in words what you are trying to accomplish in answering this problem **[2-4 sentences]**.

[b] **[5 Points]** First show that

$$\mathbb{E}^*[\mathbb{E}^*[Y|X_k]|X_l] = \mathbb{E}[Y]$$

for every $k \neq l$.

[c] **[5 Points]** Second verify the orthogonality conditions

$$\mathbb{E}[UX_l] = 0$$

for $U = \left(Y - \sum_{k=1}^{K} \mathbb{E}^*[Y|X_k] + (K-1)\mathbb{E}[Y]\right)$ and $l = 1, \ldots, K$.

[3] **[25 Points]** Let $X \in \{2, 4, 6\}$ and $Y \in \{2, 4, 6\}$. The probability of the event $X = x$ and $Y = y$ for all possible combinations of x and y is given in the following table:

X\Y	2	4	6
2	$\frac{1}{9}$	$\frac{3}{18}$	$\frac{1}{18}$
4	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
6	$\frac{1}{18}$	$\frac{3}{18}$	$\frac{1}{9}$

[a] **[5 Points]** Calculate $\mathbb{E}[Y]$ and $\mathbb{E}[Y|X = 2]$. Are X and Y independent?

[b] **[5 Points]** Calculate $\mathbb{E}[X]$, $\mathbb{E}[X^2]$ and hence $\mathbb{V}(X)$.

[c] **[7 Points]** Calculate $\mathbb{C}(X, Y)$ and also the coefficient on X in $\mathbb{E}^*[Y|X]$.

[d] **[3 Points]** Calculate the intercept of $\mathbb{E}^*[Y|X]$.

[e] **[5 Points]** Repeat [a] to [d] above for the following joint distribution

X\Y	2	4	6
2	$\frac{1}{9}$	0	0
4	0	$\frac{2}{3}$	0
6	0	0	$\frac{2}{9}$

[4] **[30 Points]** Let Y equal tons of banana's harvested in a given season for a randomly sampled Honduran banana planation. Output is produced using labor and land according to $Y = AL^{\alpha_0}D^{1-\alpha_0}$, where L is the number of employed workers and D is the size of the plantation in acres and we assume that $0 < \alpha_0 < 1$. The price of a unit of output is P, while that of a unit of labor is W. These prices may vary across plantations (e.g., due to transportation costs, labor market segmentation etc.). We will treat D as a fixed factor; A captures sources of plantation-level differences in farm productivity due to unobserved differences in, for example, soil quality and managerial capacity. Plantation owners choose the level of employed labor to maximize profits. The observed values of L are therefore solutions to the optimization problem:

$$L = \arg\max_{l} P \cdot Al^{\alpha_0}D^{1-\alpha_0} - W \cdot l.$$

[a] **[2 Points]** Show that the amount of employed labor is given by

$$L = \left\{\alpha_0 \frac{P}{W} A\right\}^{\frac{1}{1-\alpha_0}} D. \tag{1}$$

[b] **[3 Points]** Let $a_0 = \frac{1}{1-\alpha_0}\ln\alpha_0 + \frac{1}{1-\alpha_0}\mathbb{E}[\ln A]$, $b_0 = \frac{1}{1-\alpha_0}$, and $V = \frac{1}{1-\alpha_0}\{\ln A - \mathbb{E}[\ln A]\}$. Show that the log of the labor-land ratio is given by

$$\ln\left(\frac{L}{D}\right) = a_0 + b_0 \ln\left(\frac{P}{W}\right) + V \tag{2}$$

and that, letting $c_0 = \mathbb{E}[\ln A]$ and $U = \ln A - \mathbb{E}[\ln A]$, the log of planation yield (output per unit of land) is given by

$$\ln\left(\frac{Y}{D}\right) = c_0 + \alpha_0 \ln\left(\frac{L}{D}\right) + U. \tag{3}$$

[c] **[5 Points]** Briefly discuss the content and plausibility of the restriction **[2-4 sentences]**

$$\mathbb{E}[\ln A | \ln(P/W)] = \mathbb{E}[\ln A]. \tag{4}$$

[d] **[8 Points]** Using (2), (3) and (4) show that the coefficient on $\ln(L/D)$ in $\mathbb{E}^*[\ln(Y/D)|\ln(L/D)]$ equals

$$\alpha_0 + (1-\alpha_0)\frac{\mathbb{V}(\ln A)}{\mathbb{V}(\ln A) + \mathbb{V}(\ln(P/W))}.$$

Provide some economic intuition for this result **[3-6 sentences]**.

[e] **[6 Points]** Using (2), (3) and (4) show that the coefficient on $\ln(L/D)$ in $\mathbb{E}^*[\ln(Y/D)|\ln(L/D), V]$ equals α_0. Provide some economic intuition for this result **[2-4 sentences]**.

[f] **[6 Points]** Assume that all plantations face the same output price (P) and labor cost (W). What value does the coefficient on $\ln(L/D)$ in $\mathbb{E}^*[\ln(Y/D)|\ln(L/D)]$ equal now? Why? **[2-4 sentences]**.

[5] **[25 Points]** Consider the following statistical model for the logarithm of daily city-wide sales of Bob Dylan's landmark *Christmas in the Heart* album:

$$\ln S = \alpha_0 + \beta_0 R + \gamma_0 P + U,\ \mathbb{E}[U|R,P] = 0,$$

where R is the number of times a song from the album is played on KALX on the given day, and P is the

price of the album (which varies across your sample due to various (exogenous) record label promotions, holiday sales and so on). A friend estimates $\theta_0 = (\alpha_0, \beta_0, \gamma_0)'$ by the method of least squares. She claims that $\sqrt{N}\left(\hat{\theta} - \theta_0\right) \xrightarrow{D} \mathcal{N}(0, \Lambda_0)$ and reports the following:

$$\hat{\theta} = \begin{pmatrix} 1.0 \\ 0.01 \\ -0.51 \end{pmatrix}, \quad \frac{\hat{\Lambda}}{N} = \begin{pmatrix} 0.25 & -0.002 & 0.010 \\ -0.002 & 0.01 & 0.005 \\ 0.010 & 0.005 & 0.03 \end{pmatrix}.$$

[a] **[2 Points]** Calculate a 95 confidence interval for β_0.

[b] **[5 Points]** Your friend would like to test the hypothesis that "for Bob Dylan one song on the radio is as good as cutting record price by \$1" (a phrase used by her record store boss). Explain why this corresponds to:

$$H_0 : \beta_0 = -\gamma_0$$
$$H_1 : \beta_0 \neq -\gamma_0$$

[c] **[5 Points]** We can re-write H_0 as

$$H_0 : C\theta = c$$

Provide the appropriate forms for C and c.

[d] **[5 Points]** How many restrictions on θ does H_0 imposes?

[e] **[5 Points]** Calculate the Wald statistics for H_0. Can we reject with size $\alpha = 0.05$?

[f] **[8 Points]** Now formalize and test the hypothesis that "for Bob Dylan one song on the radio is as good as cutting record price by \$3".