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# Transformational semantics of the combination $\pi$ -*OZ* for mobile processes with data

Masterarbeit

- *post version* -

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# 1 Introduction

In many cases of modern computing it is of interest to describe and model concurrency. Computers no longer just solve a problem by subsequently working off the single tasks of their own, but they decompose and concurrently calculate the problem even together in a network. The increase in the number of CPU cores and more heavily of GPU cores within one single computer convincingly demonstrates how fundamental concurrency is for modern computing. Moreover, the rapidly increasing spread of the Internet is one of the most common examples which shows the importance of networks.

This thesis is divided into five chapters. In Chapter 2 we briefly introduce sequences and properly investigate the  $\pi$ -calculus and its operational semantics (the *early transition system* [SW01]). Thereby, we investigate its properties and define the refinement based on the trace semantics. Finally, the conclusion in Chapter 3 gives a brief summary of our results and presents ideas for future work.





## 2 Preliminaries

At the heart of the refinement of  $\pi$ -calculus processes is the theory of *sequences*. Thus, in this chapter, we recall the model of sequences to gain a formal construct to handle ordered elements.

Furthermore, we introduce the  $\pi$ -calculus and investigate its behavior properly. In particular, we carefully explain the operational semantics of  $\pi$ -calculus processes, since its peculiarities induce the characteristics of the refinement and its properties. Moreover, we discuss why we choose this particular operational semantics for the following work in this thesis and compare it to other semantics.

The majority of those definitions and notions can, for example, be found in [Mil99, SW01].

As mathematical notations, we consider the natural numbers starting with zero ( $\mathbb{N} = \{0, 1, 2, \dots\}$ ) and use  $\circ$  as the composition of relations. Furthermore, we denote  $R^*$  as the reflexive and transitive closure of a relation  $R$ .

### 2.1 The $\pi$ -calculus

The  $\pi$ -calculus is a process algebra that can be used to describe the behavior. This section introduces the pure polyadic version of the  $\pi$ -calculus as depicted in [Mil99].

#### 2.1.1 Intuition

To explain the  $\pi$ -calculus intuitively we will use the ion example as in [Mil99]. Let us imagine a positive and a negative ion. When those two ions merge, we get a new construct. The merge operation is called a *reaction*, since an ion acts and the other reacts. This reaction can be seen as communication between two processes. The two processes communicate to share some information. One process is the sender and the other is the receiver. By doing the reaction both processes evolve to something new. The reaction, information sharing and evolution concepts are the core of the  $\pi$ -calculus. Using those concepts we can understand the title of

Milner's book *communicating and mobile processes: the  $\pi$ -calculus* [Mil99]. The word *communicating* refers to the *reaction* concept. The word *mobile* refers to the *information sharing and evolution* concepts, since the receiver process can use the received information to change its location as we will see in .. .

The  $\pi$ -calculus consists of:

- a set of names starting with capital case letters like  $P, P_1, Q, \dots$  etc used to refer to a process directly.
- a set of names starting with capital case letters like  $A, B, C, \dots$  etc used as a process identifier. The process identifier will be used to define recursion with parameters.
- a set of names starting with lower case letter like  $a, b, x, y, \dots$  etc used as a channel and message name. This set is denoted by  $\mathcal{N}$ .
- operators like:
  - Parallel composition operator: “  $|$  ”.
  - Sequential composition operator: “  $.$  ”.
  - Choice operator: “  $+$  ”.
  - Scope restriction operator: “ new ”.

So a simple example of a process can be:  $\bar{x}\langle y \rangle.0$  this process simply sends the message  $y$  via the channel  $x$  and stops. The full syntax of  $\pi$ -calculus process is given in Definition 2.1.1. In this thesis starting from this point, when we mention the word *names* we refer to  $\mathcal{N}$ . Furthermore, we shall often write  $\vec{y}$  for a sequence  $y_1, \dots, y_n$  of names.

### 2.1.2 Syntax

**Definition 2.1.1 (Process syntax)** The syntax of a  $\pi$ -calculus process  $P$  is defined by:

$$P ::= \sum_{i \in I} \pi_i.P_i \mid P_1 \mid P_2 \mid \text{new } \vec{y} P \mid A\langle \vec{v} \rangle$$

where:

- $\sum_{i \in I} \pi_i.P_i$  is the guarded sum.

- $P_1 \mid P_2$  is the parallel composition of processes.
- $\text{new } \vec{y} P$  is the restriction of the scope of the names  $\vec{y}$  to the process  $P$
- $A(\vec{v})$  is a process call. \*

**Guarded sum:**

Is the *choice* between multiple guarded processes. If the guard of one process took place, other guarded processes will be discarded. For example, the processes:  $x().P_1 + y().P_2$  will evolve to the process  $P_1$  if the guard  $x()$  occurred.

Furthermore, The process  $\mathbf{0}$  is called the *stop process* or *inaction* and stands for the process that can do nothing. It can be omitted.

**Guard:**

Also called *action prefix* and denoted by  $\pi$ . It's syntax is defined by:

**Definition 2.1.2 (Action prefix syntax)**

$$\pi ::= \bar{x}(\vec{y}) \mid x(\vec{y}) \mid \tau$$

where:

- $\bar{x}(\vec{y})$ <sup>1</sup> represents the action: send  $\vec{y}$  via the channel  $x$ .
- $x(\vec{y})$ <sup>2</sup> represents the action: receive  $\vec{y}$  via the channel  $x$ .
- $\tau$  represents an internal non observable action. \*

The set of all *actions* is defined as  $\mathbf{Act} =_{\text{def}} \mathbf{Out} \cup \mathbf{In} \cup \{\tau\}$ , where:

- $\mathbf{Out}$  is the set of all *output actions*, defined as  $\mathbf{Out} =_{\text{def}} \{\bar{x}(\vec{y}) \mid x \in \mathcal{N}\}$ .
- $\mathbf{In}$  is the set of all *input actions*, defined as  $\mathbf{In} =_{\text{def}} \{x(\vec{y}) \mid x \in \mathcal{N}\}$ .

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<sup>1</sup> $\bar{x}()$  means: send a signal via  $x$ .  $\bar{x}(y)$  means: send the name  $y$  via  $x$ .  $\bar{x}(\vec{y})$  means: send the sequence  $\vec{y}$  via  $x$ .

<sup>2</sup> $x()$  means: receive a signal via  $x$ .  $x(y)$  means: receive any name  $y$  via  $x$ .  $x(\vec{y})$  means: receive any sequence  $\vec{y}$  via  $x$ . “ $y$  here plays the role of parameter”

**Parallel composition:**

The parallel composition operator  $|$  represent the concept of concurrency in the  $\pi$ -calculus, where two processes can evolve in concurrent. It represents an interleaving behavior of the concurrency. For example let:  $P =_{\text{def}} P_1 | (P_2 | P_3)$  where:  $P_1 =_{\text{def}} x(y).Q_1$ ,  $P_2 =_{\text{def}} \bar{x}(y).Q_2$  and  $P_3 =_{\text{def}} x(y).Q_3$ . So  $P =_{\text{def}} x(y).Q_1 | (\bar{x}(y).Q_2 | x(y).Q_3)$ . Possible evolution cases of  $P$  are:

- $P_1 | (Q_2 | Q_3)$ .  $P_2$  sends  $y$  via  $x$  to  $P_3$ .
- $Q_1 | (Q_2 | P_3)$ .  $P_2$  sends  $y$  via  $x$  to  $P_1$ .

The example above illustrated the privacy nature of the parallel operator in the  $\pi$ -calculus. A process can via a channel communicate with only one process pro time.  $P_2$  cannot communicate with both  $P_1$ ,  $P_3$  in the same time, while in Communicating Sequential Processes (CSP) a process can communicate with multiple processes in the same time via the same channel by sending multiple copies of the same message, in other words: in CSP the channel represents a Hub.

**Restriction:**

The expression  $\text{new } \vec{y} P$  binds the names  $\vec{y}$  to the process  $P$ . In other words: the visibility scope of the names  $\vec{y}$  is restricted to the process  $P$ . It is similar to declaring a private variable in programming languages. Thus the names  $\vec{y}$  are not visible outside  $P$  and  $P$  cannot use them to communicate with outside. For example, let  $P =_{\text{def}} P_1 | P_2$  where:  $P_1 =_{\text{def}} \text{new } y \bar{y}(z).Q_1$  and  $P_2 =_{\text{def}} y(z).Q_2$ . The process  $P$  cannot evolve to  $Q_1 | Q_2$ , since the name  $y$  in  $P_1$  is only visible inside it, i.e., from the  $P_2$ 's point of view  $P_1$  doesn't have a channel called  $y$ . This takes us to the definition of the Bound and free names.

**Definition 2.1.3 (Free names)** are all the restricted names in a process. \*

**Definition 2.1.4 (Bound names)** are all the name that occur in a process except the bound names. \*

For example, let  $P_1 =_{\text{def}} \text{new } x \bar{x}(y).P_2$  where  $P_2 =_{\text{def}} \text{new } z \bar{x}(z).P_3$ . The name  $x$  is bound in  $P_1$  but free in  $P_2$ .

**Process call:**

Let  $P$  be a process and let  $A$  be a process identifier. To be able to use the process  $P$  recursively we use the process identifier  $A$  as follow:  $A(\vec{w}) =_{\text{def}} P$ . Thus, when we write  $A(\vec{v})$  we are using the identifier  $A$  to call the process  $P$  with replacing the names  $\vec{w}$  in  $P$  with the names  $\vec{v}$ . This replacement is called the  $\alpha$ -conversion

For example, let  $P =_{\text{def}} \bar{w}\langle y \rangle.0$  and let  $A(w) =_{\text{def}} P$  be the recursive definition of the process  $P$ , then the behavior of  $A(v)$  is equivalent to  $\bar{v}\langle y \rangle.0$

**2.1.3 Semantics**

We already explained the intuition of the semantics of the  $\pi$ -calculus in Section 2.1.2. In this section we formalize this intuition by presenting a definition of an *operational semantics* of the  $\pi$ -calculus. This definition yields a labeled transition system called the *early transition system* as presented in [SW01] in Table 1.5 on page 38. For this thesis, we applied only minor changes to the original presentation. In particular, we replace the replication rules by the  $E - \text{CALL}$  rule, which is also defined in [SW01], and omit the matching rule due to the definition of our syntax. Furthermore, the labels of the transitions are adapted to the notion of this thesis.

**Definition**

**Definition 2.1.5** The relation  $\left\{ \xrightarrow{\alpha} \mid \alpha \in \text{Act} \right\} \subseteq \mathcal{P}_{\alpha}^{\pi} \times \mathcal{P}_{\alpha}^{\pi}$  is called the *early transition system* and is defined by the rules in Figure 2.1 apart from the omission of the rules  $E - \text{SUM}_R$ ,  $E - \text{PAR}_R$ ,  $E - \text{COM}_R$  and  $E - \text{CLOSE}_R$  for a shorter presentation. They can be obtained from their related rules by interchanging the roles of  $P$  and  $Q$ . \*

$$\begin{aligned}
 \underline{TAU} : \tau.P + \sum_{i \in I} \pi_i.P_i &\xrightarrow{\tau} P \\
 \underline{REACT} : (\bar{x}(\vec{y}).P + \sum_{i \in I} \pi_i.P_i) \mid (x(\vec{z}).Q + \sum_{i \in I} \pi_i.P_i) &\xrightarrow{\tau} \{\bar{y}/\vec{z}\} P \mid Q \\
 \underline{PAR} : \frac{P \xrightarrow{\alpha} P'}{P \mid Q \xrightarrow{\alpha} P' \mid Q} \quad \alpha \in ACT &\quad \underline{RES} : \frac{P \xrightarrow{\alpha} P'}{\underline{\text{new}} x P \xrightarrow{\alpha} \underline{\text{new}} x P'} \text{ if } \alpha \notin \{\bar{x}, x\} \\
 \underline{STRUCT} : \frac{P \xrightarrow{\alpha} P'}{Q \xrightarrow{\alpha} Q'} \text{ if } P \equiv P' \text{ and } Q \equiv Q'
 \end{aligned}$$

Figure 2.1: The *early transition system* [SW01].

## 2.2 The OZ

The OZ

# **3 Conclusion and future work**

In this thesis ...





# Bibliography

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- [SW01] D. Sangiorgi and D. Walker. *The  $\pi$ -calculus: a Theory of Mobile Processes*. Cambridge University Press, Cambridge, England, 2001.

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## Erklärung

Hiermit versichere ich, dass ich diese Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe. Außerdem versichere ich, dass ich die allgemeinen Prinzipien wissenschaftlicher Arbeit und Veröffentlichung, wie sie in den Leitlinien guter wissenschaftlicher Praxis der Carl von Ossietzky Universität Oldenburg festgelegt sind, befolgt habe.

Oldenburg, den 18. Januar 2020

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(Muhammad Ekbal Ahmad)