

Fakultät II: Informatik, Wirtschafts- und Rechtswissenschaften
Department für Informatik
Abteilung: Entwicklung korrekter Systeme

Transformational semantics of the combination π -OZ for mobile processes with data

Masterarbeit

- post version -

Name: Muhammad Ekbal Ahmad

E-Mail: muhammad.ekbal.ahmad@uni-oldenburg.de

Studiengang: Fach-Master Informatik

Erstgutachter: Prof. Dr. Ernst-Rüdiger Olderog

Zweitgutachter: M.Sc. Manuel Gieseking

Datum: 20. Januar 2020

Contents

Lis	st of	Figures	5														V
1	Intr	oductio	on														1
2	2.1	2.1.1 2.1.2 2.1.3 2.1.4 2.1.5 2.1.6	Intuition Syntax Semantic Visualize Mobility Strong s	oration control in the control in th	 	 · · · · · · · · · · · · · · · · · · ·		 	· · · · · · · · · · · · · · · · · · ·							· · · · · ·	3 4 6 8 9 10
3	2.2 Con		$^{ m Z}$ and futu			 	 •	 •		•	•	•	•	•	•		14 15
Bi	bliog	raphy															17

List of Figures

2.1	The transition rules [Mil99]	7
2.2	The inference tree [Mil99]	8
2.3	Stargazer code for the process P	8
2.4	The process P before reaction occurrence	8
2.5	The process P after reaction occurrence	8
2.6	Stargazer code for the process $\underline{\mathtt{new}}x,y\;(A\langle x,y\rangle\mid B\langle x\rangle)$	9
2.7	Before reaction occurrence	9
2.8	After reaction occurrence	9
2.9	Transition graphs	11
2.10	ABC code for P and Q	12
2.11	ABC output: check if Q strongly simulates P	13
2.12	ABC output: check if P strongly simulates Q	14

1 Introduction

2 Preliminaries

2.1 The π -calculus

The π -calculus is a process algebra that can be used to describe the behavior. This section introduces the pure polyadic version of the π -calculus as decriped in [Mil99].

2.1.1 Intuition

To explain the π -calculus inuitively we will use the ion example as in [Mil99]. Let us imagine a positive and a negative ion. When those two ions merge, we get a new construct. The merge operation is called a reaction, since an ion acts and the other reacts. This reaction can be seen as communication between two processes. The two processes communicate to share some information. One process is the sender and the other is the receiver. By doing the reaction both processes evoulte to some thing new. The reaction, information sharing and evolution concepts are the core of the π -calculus. Using those concepts we can understand the title of Milner's book communicating and mobile processes: the π -calculus [Mil99]. The word communicating refers to the reaction concept. The word mobile refers to the information sharing and evolution concepts, since the receiver process can use the received information to change its location as we will see in Section 2.1.5. Intuitively, the π -calculus consists of:

- a set of names starting with capital case letters like $P, P_1, Q, ... etc$ used to refer to a process directly.
- ullet a set of names starting with capital case letters like A,B,C,...etc used a process identifier. The process identifier will be used to define recursion with parameters.
- a set of names starting with lower case letter like a, b, x, y, ...etc used as a channel and message name. This set is denoted by \mathcal{N} .

- operators like:
 - Parallel composition operator: " | ".
 - Sequential composition operator: " . ".
 - Choice operator: " + ".
 - Scope restriction operator: " <u>new</u> ".

So a simple example of a process can be: $\overline{x}\langle y\rangle$.0 this process simply sends the message y via the channel x and stops. The full syntax of π -calculus process is given in Definition 2.1.1. In this thesis starting from this point, when we mention the word *names* we refer to \mathcal{N} . Furthermore, we shall often write \vec{y} for a sequence $y_1,, y_n$ of names.

2.1.2 Syntax

Definition 2.1.1 (Process syntax) The syntax of a π -calculus process P is defined by:

$$P ::= \sum_{i \in I} \pi_i.P_i \ \big| \ P_1 \mid P_2 \ \big| \ \underline{\mathrm{new}} \, \vec{y} \, P \ \big| \ A \langle \vec{v} \rangle$$

where:

- $\sum_{i \in I} \pi_i . P_i$ is the guarded sum.
- $P_1 \mid P_2$ is the parallel composition of processes.
- $\underline{\text{new}} \vec{y} P$ is the restriction of the scope of the names \vec{y} to the process P
- $A\langle \vec{v} \rangle$ is a process call.

Guarded sum:

Is the *choice* between multiple guarded processes. If the guard of one process took place, other guarded processes will be discarded. For example, the processes: $x().P_1 + y().P_2$ will evolve to the process P_1 if the guard x() occurred.

Furthermore, The process $\mathbf{0}$ is called the *stop process* or *inaction* and stands for the process that can do nothing. It can be omitted.

Guard:

Also called *action prefix* and denoted by π . It's syntax is defined by:

Definition 2.1.2 (Action prefix syntax)

$$\pi ::= \overline{x} \langle \vec{y} \rangle \mid x(\vec{y}) \mid \tau$$

where:

- $\overline{x}\langle \vec{y}\rangle^{-1}$ represents the action: send \vec{y} via the channel x.
- $x(\vec{y})^2$ represents the action: receive \vec{y} via the channel x.
- τ represents an internal non observable action.

The set of all *actions* is defined as $Act =_{def} Out \cup In \cup \{\tau\}$, where:

- Out is the set of all *output actions*, defined as Out $=_{\text{def}} \{ \overline{x} \langle \vec{y} \rangle \mid x \in \mathcal{N} \}$.
- In the set of all *input actions*, defined as $In =_{def} \{x(\vec{y}) \mid x \in \mathcal{N}\}.$

Parallel composition:

The parallel composition operator | represent the concept of concurrency in the π -calculus, where two processes can evolve in concurrent. It represents an interleaving behavior of the concurrency. For example let: $P =_{\text{def}} P_1 \mid (P_2 \mid P_3)$ where: $P_1 =_{\text{def}} x(y).Q_1$, $P_2 =_{\text{def}} \overline{x}\langle y\rangle.Q_2$ and $P_3 =_{\text{def}} x(y).Q_3$. So $P =_{\text{def}} x(y).Q_1 \mid (\overline{x}\langle y\rangle.Q_2 \mid x(y).Q_3)$. Possible evolution cases of P are:

- $P_1 \mid (Q_2 \mid Q_3)$. P_2 sends y via x to P_3 .
- $Q_1 \mid (Q_2 \mid P_3)$. P_2 sends y via x to P_1 .

The example above illustrated the privacy nature of the parallel operator in the π -calculus. A process can via a channel communicate with only one process pro time. P_2 cannot communicate with both P_1 , P_3 in the same time, while in Communicating Sequential Processes (CSP) a process can communicate with multiple processes in the same time via the same channel by sending multiple copies of the same message, in other words: in CSP the channel represents a Hub.

 $^{{}^1\}overline{x}\langle\rangle$ means: send a signal via x. $\overline{x}\langle y\rangle$ means: send the name y via x. $\overline{x}\langle \vec{y}\rangle$ means: send the sequence \vec{y} via x.

 $^{^2}x()$ means: receive a signal via x. x(y) means: receive any name y via x. $x(\vec{y})$ means: receive any sequence \vec{y} via x. "y here plays the role of parameter"

Restriction:

The expression $\underline{\text{new}} \vec{y} P$ binds the names \vec{y} to the process P. In other words: the visibility scope of the names \vec{y} is restricted to the process P. It is similar to declaring a private variable in programming languages. Thus the names \vec{y} are not visible outside P and P cannot use them to communicate with outside. For example, let $P =_{\text{def}} P_1 \mid P_2$ where: $P_1 =_{\text{def}} \underline{\text{new}} y \ \overline{y} \langle z \rangle$. Q_1 and $P_2 =_{\text{def}} y(z)$. Q_2 . The process P cannot evolute to $Q_1 \mid Q_2$, since the name y in P_1 is only visible inside it, i.e., from the P_2 's point of view P_1 doesn't have a channel called y. This takes us to the definition of the Bound and free names.

Definition 2.1.3 (Free names) are all the restricted names in a process.

Definition 2.1.4 (Bound names) are all the name that occur in a process except the bound names.

For example, let $P_1 =_{\text{def}} \underline{\text{new}} x \ \overline{x} \langle y \rangle . P_2$ where $P_2 =_{\text{def}} \underline{\text{new}} z \ \overline{x} \langle z \rangle . P_3$. The name x is bound in P_1 but free in P_2 .

Process call:

Let P be a process and let A be a process identifier. To be able to use the process P recursively we use the process identifier A as follow: $A(\vec{w}) =_{\text{def}} P$. Thus, when we write $A\langle \vec{v} \rangle$ we are using the identifier A to call the process P with replacing the names \vec{w} in P with the names \vec{v} . This replacement is called the α -conversion

For example, let $P =_{\text{def}} \overline{w}\langle y \rangle$. **0** and let $A(w) =_{\text{def}} P$ be the recursive definition of the process P, then the behavior of $A\langle v \rangle$ is equlivant to $\overline{v}\langle y \rangle$. **0**

2.1.3 Semantics

To understand the operational semantics of π -calculus we will use a labelled transition system LTS. Using this LTS we can investigate π -calculus process evolution. The definition of LTS is adapted from [Mil99] pages 39³, 91⁴, 132⁵ with some changes.

³Transition Rules: LTS for concurrent processes not for π -calculus processes.

⁴Reaction Rules: no labels and no LTS.

⁵Commitment Rules: abstractions and concretions are out of this thesis's scope.

Definition

Definition 2.1.5 (The LTS of π-calculus) The labelled transition system $(\mathcal{P}^{\pi}, \mathcal{T})$ of π-calculus processes over the action set Act has the process expressions \mathcal{P}^{π} as its states, and its transitions \mathcal{T} are those which can be inferred from the rules in Figure 2.1. The rule REACT is the most important one. It shows the process evolution when a reaction occurs. The reaction requires two complementary transitions $P \xrightarrow{\overline{x}\langle \overline{y}\rangle} P'$ and $Q \xrightarrow{x(\overline{z})} Q'$, we call them commitments. so the process P takes a commitment to take part in the reaction, and so does Q.

$$\underbrace{OUT}: \overline{x}\langle \overline{y}\rangle.P \xrightarrow{\overline{x}\langle \overline{y}\rangle} P \qquad \underline{IN}: x(\overline{y}).P \xrightarrow{x(\overline{y})} P$$

$$\underline{TAU}: \tau.P \xrightarrow{\tau} P \qquad \underline{SUM}: \alpha.P + \sum_{i \in I} \pi_i.P_i \xrightarrow{\alpha} P$$

$$\underline{L_{PAR}}: \frac{P \xrightarrow{\alpha} P'}{P \mid Q \xrightarrow{\alpha} P' \mid Q} \qquad \underline{R_{PAR}}: \frac{Q \xrightarrow{\alpha} Q'}{P \mid Q \xrightarrow{\alpha} P \mid Q'}$$

$$\underline{RESTRICTION}: \frac{P \xrightarrow{\alpha} P'}{\underline{new} x P \xrightarrow{\alpha} \underline{new} x P'} \quad \text{if } \alpha \notin \{\overline{x}, x\}$$

$$\underline{PROCESS_{CALL}}: \frac{\subseteq \overline{y} \overline{z} P \xrightarrow{\alpha} P'}{A\langle \overline{y}\rangle \xrightarrow{\alpha} P'} \quad \text{if } A(\overline{z}) =_{\operatorname{def}} P$$

$$\underline{REACT}: \frac{P \xrightarrow{\overline{x}\langle \overline{y}\rangle} P' \quad Q \xrightarrow{x(\overline{z})} Q'}{P \mid Q \xrightarrow{\tau} P' \mid \subseteq \overline{y} \overline{z} Q'} *$$

Figure 2.1: The transition rules [Mil99].

An example of using the transition rules of this LTS to infer a transition is: Let $P =_{\text{def}} \underline{\text{new}} x \ (A_1 \langle x \rangle \mid B_1 \langle x \rangle)$ where: $A_1(y) =_{\text{def}} \overline{y} \langle \rangle . A_2 \langle y \rangle$ and $B_1(z) =_{\text{def}} z() . B_2 \langle z \rangle$. the process P can do the transition $\underline{\text{new}} x \ (A_1 \langle x \rangle \mid B_1 \langle x \rangle) \xrightarrow{\tau} \underline{\text{new}} x \ (A_2 \langle x \rangle \mid B_2 \langle x \rangle)$, which is a reaction. The inference tree of this transition is shown in Figure 2.2. Thus, using the LTS we can enumerate sll possible transitions of a π -calculus process.

$$\frac{\overline{x}\langle \rangle. A_2 \langle x \rangle \xrightarrow{\overline{x}\langle \rangle} A_2 \langle x \rangle}{\overline{x}\langle \rangle. A_2 \langle x \rangle} \text{ by PROCESS CALL } \frac{x().B_2 \langle x \rangle \xrightarrow{x()} B_2 \langle x \rangle}{B_1 \langle x \rangle \xrightarrow{\overline{x}\langle \rangle} A_2 \langle x \rangle} \text{ by PROCESS CALL } \frac{A_1 \langle x \rangle \xrightarrow{\overline{x}\langle \rangle} A_2 \langle x \rangle}{B_1 \langle x \rangle \xrightarrow{\tau} A_2 \langle x \rangle \mid B_2 \langle x \rangle} \text{ by REACT } \frac{A_1 \langle x \rangle \mid B_1 \langle x \rangle \xrightarrow{\tau} A_2 \langle x \rangle \mid B_2 \langle x \rangle}{\underline{new} \, x \, (A_1 \langle x \rangle \mid B_1 \langle x \rangle) \xrightarrow{\tau} \underline{new} \, x \, (A_2 \langle x \rangle \mid B_2 \langle x \rangle)} \text{ by RESTRICTION }$$

Figure 2.2: The *inference tree* [Mil99].

2.1.4 Visualization

To gain more understanding of the π -calculus we will use Stargazer[Star]. Stargazer is a visual simulator for π -calculus. Figure 2.3 shows the code listing of the process $P =_{\text{def}} \underline{\text{new}} x \ (A_1 \langle x \rangle \mid B_1 \langle x \rangle)$ where: $A_1(y) =_{\text{def}} \overline{y} \langle \rangle . A_2 \langle y \rangle$ and $B_1(z) =_{\text{def}} z() . B_2 \langle z \rangle$ in Stargazer syntax.

```
new x. (A1[x] | B1[x])

A1[y] := y <> .A2[y]

B1[z] := z().B2[z]
```

Figure 2.3: Stargazer code for the process P .

Stargazer can visualize the reaction $\underline{\text{new}} x$ $(A_1\langle x \rangle \mid B_1\langle x \rangle) \xrightarrow{\tau} \underline{\text{new}} x$ $(A_2\langle x \rangle \mid B_2\langle x \rangle)$ as shown in Figure 2.4 and Figure 2.5.

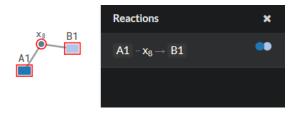


Figure 2.4: The process P before reaction occurrence.



Figure 2.5: The process P after reaction occurrence.

2.1.5 Mobility

As mentioned previously, the word mobile refers to the information sharing and evolution concepts, since the receiver process can use the received information to change its location. Let us take an example to illustrate the mobility. Let: $\underline{\text{new}} x, y \ (A\langle x, y \rangle \mid B\langle x \rangle)$ where:

- $A(a,b) =_{\text{def}} \overline{a} \langle b \rangle . A \langle a,b \rangle$
- $B(c) =_{\text{def}} c(d).B\langle d \rangle$

Figure 2.6 shows the stargazer code listing of the process $\underline{\text{new}} x, y \ (A\langle x, y \rangle \mid B\langle x \rangle)$, Figure 2.7 shows it's visualization before the interaction occurrence, and Figure 2.8 shows it's visualization after the interaction occurrence.

Figure 2.6: Stargazer code for the process $\underline{\mathtt{new}}\,x,y\;(A\langle x,y\rangle\mid B\langle x\rangle)$.



Figure 2.7: Before reaction occurrence.



Figure 2.8: After reaction occurrence.

Intuitively, The mobility can be noticed in Figure 2.7 and Figure 2.7, since B changed it's position in the connection topology. The following explains the mobility through interaction step by step:

- Initially the process $A\langle x,y\rangle$ has the channels x,y and the process $B\langle x\rangle$ has the channel x. Thus, $A\langle x,y\rangle$ and $B\langle x\rangle$ are connected via channel x.
- $A\langle x,y\rangle$ has commitment $\overline{x}\langle y\rangle$, i.e., send the channel name y via the channel x.
- $B\langle x\rangle$ has commitment x(d), i.e., receive a message d via x.
- That means: a reaction can occur between $A\langle x, y \rangle$ and $B\langle x \rangle$. This reaction is: $\underline{\text{new}} x, y \ (\overline{x}\langle y \rangle. A\langle x, y \rangle \mid x(d). B\langle d \rangle) \xrightarrow{\tau} \underline{\text{new}} x, y \ (A\langle x, y \rangle \mid B\langle y \rangle).$
- Information sharing: the process $A\langle x,y\rangle$ sends the name y to $B\langle x\rangle$ when the interaction occurs.
- Evolution: when interaction occurs $B\langle x\rangle$ knows about the channel y and uses it as parameter for the the process call $B\langle y\rangle$.i.e, The $B\langle y\rangle$ now has the channel y, and no more x.
- Finally, in other words:
 - before the reaction: B was connected to A via x as shown in Figure 2.7.
 - after the reaction: B is connected to A via y as shown in Figure 2.8.

2.1.6 Strong simulation

The strong simulation is comparison of processes based on their behavior. To understand this let us start with a simple example: Let $P =_{\text{def}} \tau.\tau.\mathbf{0}$ and $Q =_{\text{def}} \tau.\mathbf{0}$. We can notice that P can do two τ transitions, but Q can do only one. Thus Q doesn't strongly simulates P. The word strongly refers to the point that: the strong simulation comparison takes the internal transition τ into account. There is another kind of comparison called the weak simulation, which doesn't consider the internal transition τ , but this kind of comparison is not considered is this thesis. The formal definition of the strong simulation is given in Definition 2.1.6, which is adapted from [Gi14] page 32 with some changes.

Definition 2.1.6 (Strong simulation) A relation $S \subseteq \mathcal{P}^{\pi} \times \mathcal{P}^{\pi}$ is called a *strong simulation*, if $(P, Q) \in S$ implies that

if $P \xrightarrow{\alpha} P'$ then $Q' \in \mathcal{P}^{\pi}$ exists such that $Q \xrightarrow{\alpha} Q'$ and $(P', Q') \in \mathcal{S}$.

An example of checking the strong simulation is: Let

- $P =_{\text{def}} \underline{\text{new}} x (A_1 \langle x \rangle \mid B_1 \langle x \rangle)$
- $Q =_{\text{def}} \underline{\text{new}} x ((\overline{x}\langle \rangle.0 \mid x().0) + \tau.Q)$

where:

- $A_1(y) =_{\text{def}} \overline{y}\langle\rangle.\mathbf{0}$
- $B_1(z) =_{\text{def}} z().\mathbf{0}$

Intuitively, The behavior of P and Q can be illustrated using transition graphs as shown in Figure 2.9. Q's transition graph is the same as P's, except one thing: Q has a loop with label τ . This loop is due to the τ transition in Q's definition. Hence, we can notice that Q can do all the transitions that P can, plus an extra transition tau. In other words Q simulates P, but P doesn't simulate Q.

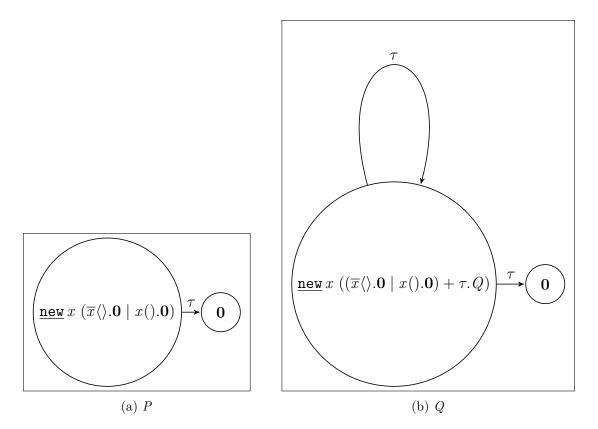


Figure 2.9: Transition graphs

To check the strong simulation we can use ABC (Another Bisimilarity Checker) [ABC]. ABC is a tool that checks simulation between π -calculus processes. Figure 2.10 shows the code listing of the process P and Q in ABC syntax.

```
agent P = (^x)( A_1 x | B_1 x)

agent A_1(y) = 'y.0

agent B_1(z) = z.0

agent Q = (^x)((A_1 x | B_1 x) + t.Q)

// check if Q strongly simulates P

lt P Q

// check if P strongly simulates Q

lt Q P
```

Figure 2.10: ABC code for P and Q.

Figure 2.11 and Figure 2.12 shows the result of running Figure 2.10, where x0 stands for x, since ABC renames the channels and messages names internally.

In Figure 2.11 we can see the result of the command $lt\ P\ Q$, which checks if Q strongly simulates P. The result is yes and the simulation relation is $S = \{(\mathbf{0},\mathbf{0}), (\underline{\mathtt{new}}\ x\ (A_1\langle x\rangle \mid B_1\langle x\rangle), \underline{\mathtt{new}}\ x\ ((\overline{x}\langle\rangle.\mathbf{0}\mid x().\mathbf{0}) + \tau.Q))\}$, where x0 stands for x. The simulation relation S says that:

- The state $\mathbf{0}$ of Q is powerful as $\mathbf{0}$ of P.
- The state $\underline{\text{new}} x ((\overline{x}\langle \rangle.\mathbf{0} \mid x().\mathbf{0}) + \tau.Q)$ of Q is powerful as $\underline{\text{new}} x (\overline{x}\langle \rangle.\mathbf{0} \mid x().\mathbf{0})$ of P.

Thus, Q strongly simulates the behavior of P.

Figure 2.11: ABC output: check if Q strongly simulates P.

In Figure 2.12 we can see the result of the command $lt\ Q\ P$, which checks if P strongly simulates Q. The result is no, since:

When:

- Q is in the state $\underline{\text{new}} x ((\overline{x}\langle \rangle.0 \mid x().0) + \tau.Q)$.
- P is in the state $\underline{\text{new}} x (\overline{x} \langle \rangle.0 \mid x().0)$.

, then:

- Q can do a τ transition ,which is the loop, to be in the state $\underline{\mathbf{new}} x$ (($\overline{x}\langle\rangle.\mathbf{0} \mid x().\mathbf{0}) + \tau.Q$).
- P can do a τ transition ,which is a reaction, to be in the state **0**.

, then:

- Q can do a τ transition, which is a reaction, to be in the state $\mathbf{0}$.
- P cannot go ahead, denoted by "*", since it is in the state $\mathbf{0}$.

Thus, P doesn't strongly simulates the behavior of Q.

```
The two agents are not strongly related (2).

Do you want to see some traces (yes/no) ? yes traces of

Q
P
-t->
-t->
(^x0)(('x0.0 | x0.0) + t.Q)
0
-t->
-t->
-t->
```

Figure 2.12: ABC output: check if P strongly simulates Q.

2.2 The OZ

The OZ

```
Shape \\ c: chan[bool] \\ Q =_{\text{def}} \underbrace{\text{new}}_{x} x \left( \left( \overline{x} \left\langle \right\rangle . \mathbf{0} \mid x (). \mathbf{0} \right) + \tau . Q \right) \\ \\ x, y : \mathbb{R} \\ \\ -I_{NIT} \\ x = y = 0 \\ \\ -T_{ranslate} \\ \\ \Delta(x, y) \\ dx?, dy? : \mathbb{R} \\ \\ x' = x + dx? \\ y' = y + dy? \\ \\ \\ \end{bmatrix}
```

3 Conclusion and future work

In this thesis ...

Bibliography

- [Mil99] R. Milner. Communicating and Mobile Systems: the π -Calculus. Cambridge University Press, Cambridge, England, 1999.
- [SW01] D. Sangiorgi and D. Walker. The π -calculus: a Theory of Mobile Processes. Cambridge University Press, Cambridge, England, 2001.
- [Gi14] M. Gieseking. Refinement of π -calculus processes. Master thesis, Carl von Ossietzky Universität Oldenburg, 2014.
- [Star] E. D'Osualdo. Stargazer: A π -calculus simulator. http://www.emanueledosualdo.com/stargazer/
- [ABC] S. Briais. Another Bisimilarity Checker. http://sbriais.free.fr/tools/abc/

Index

Symbols	G
α -conversion	I
π -calculus	K
A	L
action	M
internal	message
silent	N
B bisimulation strong	name
weak 32	substitution
\mathbf{C}	O
call	P
channel	parallel composition
commitments33	parameter
D	prefix
D.	operator
${f E}$	output
\mathbf{F}	silent
free name	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

choice
input10
output10
parallel
prefixg
restriction
stop
D
R
\mathbf{S}
silent
action
prefix
simulation
strong
weak
stop process
strong
simulation32
subject
sum
summation
T
transition system
\mathbf{U}
U
\mathbf{V}
V
\mathbf{W}

Erklärung

Hiermit versichere ich, dass ich diese Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe. Außerdem versichere ich, dass ich die allgemeinen Prinzipien wissenschaftlicher Arbeit und Veröffentlichung, wie sie in den Leitlinien guter wissenschaftlicher Praxis der Carl von Ossietzky Universität Oldenburg festgelegt sind, befolgt habe.

Oldenburg,	den	20.	Januar	2020
(Muhamma	d El	kbal	Ahmad	<u>l)</u>