Fakultät II: Informatik, Wirtschafts- und Rechtswissenschaften
Department für Informatik
Abteilung: Entwicklung korrekter Systeme

Transformational semantics of the combination π-OZ for mobile processes with data

Masterarbeit

Name:	Muhammad Ekbal Ahmad
Straße:	Brunsbrok 4
Wohnort:	26127 Oldenburg
Geburtstag:	05.10.1985
Matrikelnr.:	4565577
E-Mail:	muhammad.ekbal.ahmad@uni-oldenburg.de
Studiengang:	Fach-Master Informatik
Erstgutachter:	Prof. Dr. Ernst-Rüdiger Olderog
Zweitgutachter:	Manuel Gieseking M. Sc.
Datum:	18. Juli 2020

Contents

List of Figures

1 Introduction

Modeling distributed computing systems exhibit various aspects such as modeling the components of the system and modeling the behavior and communication between the concurrent mobile components. Formal specification techniques for such systems have to be able to describe all these aspects. Unfortunately, a single specification technique that is well suited for all these aspects is not yet available. Instead, one finds techniques that are very good at describing individual aspects of system. This observation has led to research into the combination and semantic integration of specification techniques. In this thesis we research combining the two specification techniques: π-calculus and Object-Z.

The π-calculus is a process algebra originally introduced by Robin Milner, Joachim Parrow, and David Walker in [MPW92]. The central concepts of π-calculus are communication via channels between different processes, creating new channels, parallel composition, and the mobility of channels, which we will use to model the mobility of components. Tool support comes through the π-calculus visualizer Stargazer and the bisimulation checker ABC.

Object-Z [Smi00] is an object-oriented extension of the Z formal specification language. It represents a set-theoretic and predicate language for the specification of data. It extends Z by the addition of language constructs resembling the object-oriented paradigm, most notably, classes to encapsulate a state schema, an initial state schema and operation schemas. The state schema specifies the set of the legal states. The initial state schema specifies the set of the legal initial states. The operation schemas specify the legal ways of moving from one state to another.

The main contribution of this thesis is to develop the combination π-OZ and to transform it into a π-calculus process, similarly to the approach for CSP-OZ in [Old19]. π-OZ is a new combination of formal techniques for the specification of components and their behavior. The basic idea is to use a π-calculus process to specify the behavior of an Object-Z class. This enables the mobility of Object-Z classes through the use of the state pattern. Syntactically, the π-OZ specification is divided into an OZ part specifying the data and the possible state transitions, and a π part specifying the behavior. This combination is illustrated by the example of a mobile vending machine and two shops, which alternate their behavior when they connect to the mobile vending machine.

Transforming the combination π-OZ into a π-calculus process enables checking the simulation of mobile processes with data. The transformation gives the combination a π-calculus semantics. The idea is that this semantics comes from the parallel composition of the π part and a π process, representing the OZ part transformed syntactically into a π-calculus process.

Furthermore, we investigate a failure-model and a success-model for the π-calculus processes and show their relation to the simulation and point out its limitations.

This thesis is divided into five chapters. Chapter 2 gives an overview of the π-calculus and Object-Z. Thereby, Dynamic OZ, a new extension of Object-Z is introduced to model mobile components with alternating behavior. Chapter 3 proceeds by transforming OZ class into a π-calculus process. Subsequently, Chapter 4 introduces the combination π-OZ and its transformation into the π-calculus. Chapter 5 introduces the failure and the success refinement of the π-calculus and their relation to the simulation. Finally, the conclusion in Chapter 6 summarizes our results and presents future directions of our research.

2 Preliminaries

2.1 The π-calculus

The π-*calculus* is a process algebra that can be used to describe a behavior of communicating mobile systems. This section introduces the pure polyadic version of the π-calculus as described in [Mil99].

2.1.1 Intuition

To explain the π-calculus intuitively we will use the ion example as in [Mil99]. Let us imagine a positive and a negative ion. When those two ions merge, we get a new construct. The merge operation is called a reaction, since an ion acts and the other reacts. This reaction can be seen as communication between two processes. The two processes communicate to share some information. One process is the sender and the other is the receiver. By doing the reaction both processes evolve. The reaction, information sharing, and evolution concepts are the core of the π-calculus. Using those concepts we can understand the title of Milner's book *communicating and mobile processes*: the π-calculus [Mil99]. The word *communicating* refers to the *reaction* concept. The word *mobile* refers to the *information sharing and evolution* concepts, since the receiver process can use the received information to change its location as we will see in Section 2.1.5.
Intuitively, the π-calculus consists of:

- a set of names starting with capital case letters like $P, P_1, Q, \ldots$ used to refer to a process directly.
- a set of names starting with capital case letters like $A, B, C, \ldots$ used a process identifier. The process identifier will be used to define recursion with parameters.
- a set of names starting with lower case letter like $a, b, x, y, \ldots$ used as a channel and message name. This set is denoted by $\mathcal{N}$.

- operators like:
    - Parallel composition operator: "|".
    - Sequential composition operator: ".".
    - Choice operator: "+".
    - Scope restriction operator: "new".

So a simple example of a process can be: $\overline{x}\langle y\rangle.\mathbf{0}$ this process simply sends the message y via the channel x and stops. The full syntax of π-calculus process is given in Definition 2.1.1. In this thesis starting from this point, when we mention the word *names* we refer to $\mathcal{N}$. Furthermore, we shall often write $\vec{y}$ for a *sequence* $y_1, ..., y_n$ of names.

2.1.2 Syntax

Definition 2.1.1 (Process syntax) The syntax of a π-calculus process P is defined by:

$$P ::= \sum_{i \in I} \pi_i.P_i \;\Big|\; P_1 \mid P_2 \;\Big|\; \underline{\texttt{new}}\,\vec{y}\;P \;\Big|\; A\langle\vec{v}\rangle$$

where:

- $\sum_{i \in I} \pi_i.P_i$ is the guarded sum.
- $P_1 \mid P_2$ is the parallel composition of processes.
- $\underline{\texttt{new}}\,\vec{y}\;P$ is the restriction of the scope of the names $\vec{y}$ to the process P
- $A\langle\vec{v}\rangle$ is a process call. $\triangle$

***Guarded sum*:**

The guarded sum is the choice between multiple guarded processes. If the guard of one process took place, other guarded processes will be discarded. For example, the processes: $x().P_1 + y().P_2$ will evolve to the process P_1 if the guard $x()$ occurred.

Furthermore, The process $\mathbf{0}$ is called the *stop process* stands for the process that can do nothing. It can be omitted.

Guard:

The guard is also called *action prefix* and denoted by π. Its syntax is defined by:

Definition 2.1.2 (Action prefix syntax)

$$\pi ::= \overline{x}\langle\vec{y}\rangle \;\Big|\; x(\vec{y}) \;\Big|\; \tau$$

where:

- $\overline{x}\langle\vec{y}\rangle$[1] represents the action: send $\vec{y}$ via the channel x.
- $x(\vec{y})$[2] represents the action: receive $\vec{y}$ via the channel x.
- τ represents an internal non-observable action. $\triangle$

We use the symbols α or β to denote an action. Furthermore, we call x the subject and $\vec{y}$ the object of a an action $\overline{x}\langle\vec{y}\rangle$ or $x(\vec{y})$. The functions $\mathtt{sub}(\alpha)$ and $\mathtt{sub}(\alpha)$ returns the subject and object of an action α. More formally:

$\mathtt{sub}(\alpha)$: returns the channel name through which the exchange occurs.

$\mathtt{obj}(\alpha)$: return the exchanged names across the channel.

The set of all actions is defined as $\mathtt{Act} =_{\text{def}} \mathtt{Out} \cup \mathtt{In} \cup \{\tau\}$, Where:

- $\mathtt{Out}$ is the set of output actions, defined as $\mathtt{Out} =_{\text{def}} \{\overline{x}\langle\vec{y}\rangle \mid x, y_1, \ldots, y_n \in \mathcal{N}\}$.
- $\mathtt{In}$ is the set of input actions, defined as $\mathtt{In} =_{\text{def}} \{x(\vec{y}) \mid x, y_1, \ldots, y_n \in \mathcal{N}\}$.

Parallel composition:

The parallel composition operator $|$ represents the concept of concurrency in the π-calculus, where two processes can evolve concurrently. It represents an interleaving behavior of the concurrency. For example let: $P =_{\text{def}} P_1 \mid (P_2 \mid P_3)$ where: $P_1 =_{\text{def}} x(y).Q_1$, $P_2 =_{\text{def}} \overline{x}\langle y\rangle.Q_2$ and $P_3 =_{\text{def}} x(y).Q_3$. So $P =_{\text{def}} x(y).Q_1 \mid (\overline{x}\langle y\rangle.Q_2 \mid x(y).Q_3)$. Possible evolution cases of P are:

- $P_1 \mid (Q_2 \mid Q_3)$. P_2 sends y via x to P_3.

[1] $\overline{x}\langle\rangle$ means: send a signal via x. $\overline{x}\langle y\rangle$ means: send the name y via x. $\overline{x}\langle\vec{y}\rangle$ means: send the sequence $\vec{y}$ via x.

[2] $x()$ means: receive a signal via x. $x(y)$ means: receive any name y via x. $x(\vec{y})$ means: receive any sequence $\vec{y}$ via x. "y here plays the role of parameter"

- $Q_1 \mid (Q_2 \mid P_3)$. P_2 sends y via x to P_1.

The example above illustrates the privacy nature of the parallel operator in the π-calculus. A process can communicate via a channel with only one process per time, i.e., a binary synchronization occurs via the channel. P_2 cannot communicate with both P_1, P_3 at the same time, while in Communicating Sequential Processes (CSP) a process can communicate with multiple processes at the same time via the same channel by sending multiple copies of the same message, i.e., in CSP a multiple synchronization occurs via the channel.

***Restriction*:**

The expression $\underline{\texttt{new}}\,\vec{y}\ P$ binds the names $\vec{y}$ to the process P. In other words: the visibility scope of the names $\vec{y}$ is restricted to the process P. It is similar to declaring a private variable in programming languages. Thus, the names $\vec{y}$ are not visible outside P and P cannot use them to communicate with the outside. For example, let $P =_{\text{def}} P_1 \mid P_2$ where: $P_1 =_{\text{def}} \underline{\texttt{new}}\,y\ \overline{y}\langle z\rangle.Q_1$ and $P_2 =_{\text{def}} y(z).Q_2$. The process P cannot evolve to $Q_1 \mid Q_2$, since the name y in P_1 is only visible inside P_1, i.e., from the P_2's point of view P_1 does not have a channel called y. This takes us to the definition of the *bound names* and *free names*.

Definition 2.1.3 (Bound names) the bound names are all the restricted names in a process. $\triangle$

Definition 2.1.4 (Free names) the free names are all the names that occur in a process except the bound names. $\triangle$

For example, let $P_1 =_{\text{def}} \underline{\texttt{new}}\,x\ (\overline{x}\langle y\rangle.P_2)$ where $P_2 =_{\text{def}} \underline{\texttt{new}}\,z\ (\overline{x}\langle z\rangle.P_3)$. The name x is bound in P_1 but free in P_2.

***Process call*:**

Let P be a process and let A be a process identifier. To be able to use the process P recursively we use the process identifier A as follows: $A(\vec{w}) =_{\text{def}} P$. Thus, when we write $A\langle\vec{v}\rangle$ we are using the identifier A to call the process P with replacing the names $\vec{w}$ in P with the names $\vec{v}$. This replacement is called the α-conversion.

For example, let $A(w) =_{\text{def}} \overline{w}\langle y\rangle.A(w)$, then the behavior of $A\langle v\rangle$ is equivalent to $\overline{v}\langle y\rangle.A\langle v\rangle$.

2.1.3 Semantics

To understand the operational semantics of π-calculus we will use a labelled transition system LTS. Using this LTS we can investigate the π-calculus process evolution. The definition of LTS is adapted from [Mil99] pages 39[3], 91[4], 132[5] with some changes.

Definition 2.1.5 (LTS of π-calculus) The *labelled transition system* $(\mathcal{P}^\pi, \mathcal{T})$ of π-calculus processes over the action set `Act` has the process expressions $\mathcal{P}^\pi$ as its states, and its transitions $\mathcal{T}$ are those which can be inferred from the rules in Figure 2.1. The rule REACT is the most important one. It shows the process evolution when a reaction occurs. The reaction requires two complementary transitions $P \xrightarrow{\overline{x}\langle\vec{y}\rangle} P'$ and $Q \xrightarrow{x(\vec{z})} Q'$, we call them commitments. So the process P takes a commitment to take part in the reaction, and so does Q.

$$\underline{\mathit{OUT}} : \overline{x}\langle\vec{y}\rangle.P \xrightarrow{\overline{x}\langle\vec{y}\rangle} P \qquad \underline{\mathit{IN}} : x(\vec{y}).P \xrightarrow{x(\vec{y})} P$$

$$\underline{\mathit{TAU}} : \tau.P \xrightarrow{\tau} P \qquad \underline{\mathit{SUM}} : \alpha.P + \sum_{i \in I} \pi_i.P_i \xrightarrow{\alpha} P$$

$$\underline{\mathit{L\_PAR}} : \frac{P \xrightarrow{\alpha} P'}{P \mid Q \xrightarrow{\alpha} P' \mid Q} \qquad \underline{\mathit{R\_PAR}} : \frac{Q \xrightarrow{\alpha} Q'}{P \mid Q \xrightarrow{\alpha} P \mid Q'}$$

$$\underline{\mathit{RESTRICTION}} : \frac{P \xrightarrow{\alpha} P'}{\underline{\mathtt{new}}\, x\ P \xrightarrow{\alpha} \underline{\mathtt{new}}\, x\ P'} \quad \mathit{if}\ \mathtt{sub}(\alpha) \notin \{\overline{x}, x\}$$

$$\underline{\mathit{PROCESS\_CALL}} : \frac{\{\vec{y}/\vec{z}\}\, P \xrightarrow{\alpha} P'}{A\langle\vec{y}\rangle \xrightarrow{\alpha} P'} \quad \mathit{if}\ \ A(\vec{z}) =_{\mathrm{def}} P$$

$$\underline{\mathit{REACT}} : \frac{P \xrightarrow{\overline{x}\langle\vec{y}\rangle} P' \quad Q \xrightarrow{x(\vec{z})} Q'}{P \mid Q \xrightarrow{\tau} P' \mid \{\vec{y}/\vec{z}\}\, Q'} \qquad \triangle$$

Figure 2.1: The transition rules [Mil99].

[3] Transition Rules: LTS for concurrent processes not for π-calculus processes.
[4] Reaction Rules: no labels and no LTS.
[5] Commitment Rules: abstractions and concretions are out of this thesis's scope.

An example of using the transition rules of this LTS to infer a transition is: Let $P =_{\text{def}} \underline{\texttt{new}}\, x\ (A_1\langle x\rangle \mid B_1\langle x\rangle)$, where: $A_1(y) =_{\text{def}} \overline{y}\langle\rangle.A_2\langle y\rangle$ and $B_1(z) =_{\text{def}} z().B_2\langle z\rangle$. The process P can do the transition $\underline{\texttt{new}}\, x\ (A_1\langle x\rangle \mid B_1\langle x\rangle) \xrightarrow{\tau} \underline{\texttt{new}}\, x\ (A_2\langle x\rangle \mid B_2\langle x\rangle)$, which is a reaction. The inference tree of this transition is shown in Figure 2.2. Thus, using the LTS we can enumerate all possible transitions of a π-calculus process.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\overline{x}\langle\rangle.A_2\langle x\rangle \xrightarrow{\overline{x}\langle\rangle} A_2\langle x\rangle}\text{ by OUT}}{A_1\langle x\rangle \xrightarrow{\overline{x}\langle\rangle} A_2\langle x\rangle}\text{ by PROCESS CALL} \qquad \dfrac{\dfrac{}{x().B_2\langle x\rangle \xrightarrow{x()} B_2\langle x\rangle}\text{ by IN}}{B_1\langle x\rangle \xrightarrow{x()} B_2\langle x\rangle}\text{ by PROCESS CALL}}{A_1\langle x\rangle \mid B_1\langle x\rangle \xrightarrow{\tau} A_2\langle x\rangle \mid B_2\langle x\rangle}\text{ by REACT}}{\underline{\texttt{new}}\, x\ (A_1\langle x\rangle \mid B_1\langle x\rangle) \xrightarrow{\tau} \underline{\texttt{new}}\, x\ (A_2\langle x\rangle \mid B_2\langle x\rangle)}\text{ by RESTRICTION}$$

Figure 2.2: The inference tree of the reaction of the process P.

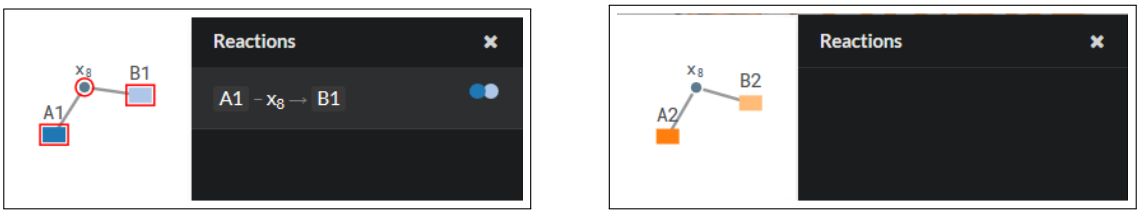
2.1.4 Visualization

To gain more understanding of the π-calculus we will use *Stargazer* [D'O]. Stargazer is a visual simulator for the π-calculus. Listing 2.1 shows the code of the process $P =_{\text{def}} \underline{\texttt{new}}\, x\ (A_1\langle x\rangle \mid B_1\langle x\rangle)$ where: $A_1(y) =_{\text{def}} \overline{y}\langle\rangle.A_2\langle y\rangle$ and $B_1(z) =_{\text{def}} z().B_2\langle z\rangle$ in stargazer syntax.

```
new x. (A1[x] | B1[x])
A1[y] := y<>.A2[y]
B1[z] := z().B2[z]
```

Listing 2.1: Stargazer code for the process P.

Stargazer can visualize the reaction $\underline{\texttt{new}}\, x\ (A_1\langle x\rangle \mid B_1\langle x\rangle) \xrightarrow{\tau} \underline{\texttt{new}}\, x\ (A_2\langle x\rangle \mid B_2\langle x\rangle)$ as shown in Figure 2.3.



(a) Before reaction occurrence. (b) After reaction occurrence.

Figure 2.3: The reaction of the process P.

2.1.5 Mobility

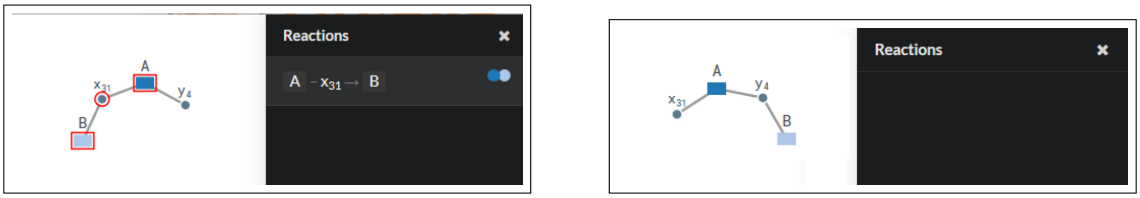
As mentioned previously, the word *mobile* refers to the information sharing and evolution, since the receiver process may use the received information to change its location. For example, consider the process: $Q =_{\text{def}} \underline{\texttt{new}}\, x, y\ (A\langle x, y\rangle \mid B\langle x\rangle)$ where:

- $A(a, b) =_{\text{def}} \overline{a}\langle b\rangle.A\langle a, b\rangle$
- $B(c) =_{\text{def}} c(d).B\langle d\rangle$

Listing 2.2 shows the stargazer code of Q. Figure 2.4 shows visualization of Q before and after the interaction occurrence.

```
new x,y. (A[x,y] | B[x])
A[a,b] := a<b>.A[a,b]
B[c] := c(d).B[d]
```

Listing 2.2: Stargazer code for the process Q.



(a) Before reaction occurrence. (b) After reaction occurrence.

Figure 2.4: Mobility reaction

Intuitively, the mobility can be noticed in Figure 2.4, since B changed its position in the connection topology. The following explains the mobility through interaction step by step:

- Initially the process $A\langle x, y\rangle$ has the channels x, y and the process $B\langle x\rangle$ has the channel x. Thus, $A\langle x, y\rangle$ and $B\langle x\rangle$ are connected via channel x.
- $A\langle x, y\rangle$ has the action $\overline{x}\langle y\rangle$, i.e., it sends the channel name y via the channel x.
- $B\langle x\rangle$ has the action $x(d)$, i.e., it receives a message d via x.
- That means: a reaction can occur between $A\langle x, y\rangle$ and $B\langle x\rangle$. This reaction is: $\underline{\texttt{new}}\, x, y\ (\overline{x}\langle y\rangle.A\langle x, y\rangle \mid x(d).B\langle d\rangle) \xrightarrow{\tau} \underline{\texttt{new}}\, x, y\ (A\langle x, y\rangle \mid B\langle y\rangle)$.

- Information sharing: the process $A\langle x, y\rangle$ sends the name y to $B\langle x\rangle$ when the interaction occurs.
- Evolution: when the interaction occurs $B\langle x\rangle$ knows about the channel y and uses it as parameter for the process call $B\langle y\rangle$, i.e, the process $B\langle y\rangle$ now has the channel y, and no more x.
- Finally, in other words:
  - before the reaction: B was connected to A via x as shown in Figure 2.4.
  - after the reaction: B is connected to A via y as shown in Figure 2.4.

2.1.6 Strong simulation

The *strong simulation* is a comparison of processes based on their behavior. To understand this let us start with a simple example: Let $P =_{\text{def}} \tau.\tau.\mathbf{0}$ and $Q =_{\text{def}} \tau.\mathbf{0}$. We can notice that P can do two τ transitions, but Q can do only one. Thus Q does not strongly simulate P. The word *strongly* refers to the point that: the strong simulation comparison takes the internal transition τ into account. There is another kind of comparison called the *weak simulation*, which does not consider the internal transition τ, but this kind of comparison is not considered in this thesis. The formal definition of the strong simulation is given in Definition 2.1.6.

Definition 2.1.6 (Strong simulation) A relation $\mathcal{S} \subseteq \mathcal{P}^\pi \times \mathcal{P}^\pi$ is called a *strong simulation*, if $(P, Q) \in \mathcal{S}$ implies that

$$\text{if } P \xrightarrow{\alpha} P' \text{ then } Q' \in \mathcal{P}^\pi \text{ exists such that } Q \xrightarrow{\alpha} Q' \text{ and } (P', Q') \in \mathcal{S}.$$

where α is an input, output or τ action. △

An example of checking the strong simulation is:
Let

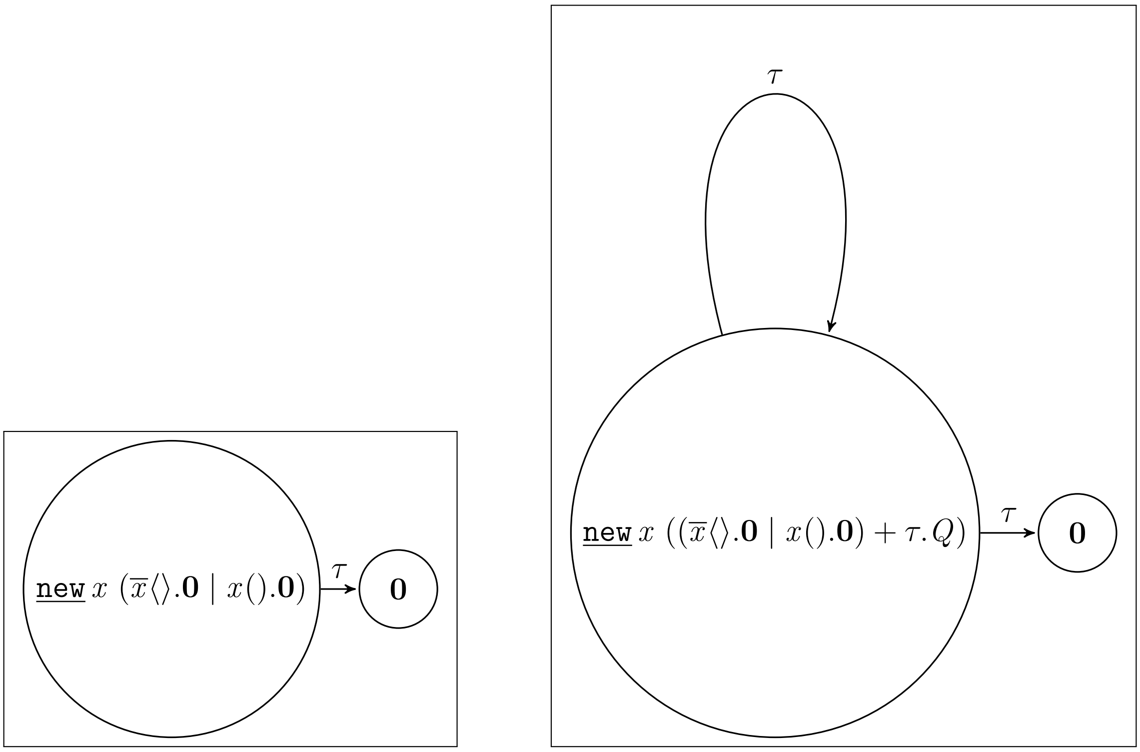
- $P =_{\text{def}} \underline{\texttt{new}}\, x\ (A_1\langle x\rangle \mid B_1\langle x\rangle)$
- $Q =_{\text{def}} \underline{\texttt{new}}\, x\ ((A_1\langle x\rangle \mid B_1\langle x\rangle) + \tau.Q)$

where:

- $A_1(y) =_{\text{def}} \overline{y}\langle\rangle.\mathbf{0}$

- $B_1(z) =_{\text{def}} z().\mathbf{0}$

Intuitively, the behavior of P and Q can be illustrated using transition graphs as shown in Figure 2.5. Q's transition graph is the same as P's, except of one thing: Q has a loop with label τ. This loop is due to the τ transition in Q's definition. Hence, we can notice that Q can do all the transitions that P can, plus an extra transition τ. In other words Q simulates P, but P does not simulate Q.



(a) P (b) Q

Figure 2.5: Transition graphs.

To check the strong simulation we can use *ABC (Another Bisimulation Checker)* [Bri]. ABC is a tool that checks simulation between π-calculus processes. Listing 2.3 shows the code of the process P and Q in ABC syntax.

```
agent P = (^x)( A_1 x | B_1 x)
agent A_1(y) = 'y.0
agent B_1(z) = z.0
agent Q = (^x)((A_1 x | B_1 x) + t.Q)
// check if Q strongly simulates P
lt P Q
```

```
// check if P strongly simulates Q
lt Q P
```

Listing 2.3: ABC code for P and Q.

Listing 2.4 and Listing 2.5 shows the result of running Listing 2.3, where x0 stands for x, since ABC renames the channels and messages names internally.

In Listing 2.4 we see the result of the command *lt P Q*, which checks if Q strongly simulates P. The result is *yes* and the simulation relation is shown, where x0 stands for x. In Figure 2.4 we see the two pairs of the simulation relation, where:

- (0 { } 0) stands for the pair $(\mathbf{0}, \mathbf{0})$, which means: The state $\mathbf{0}$ of Q is as powerful as $\mathbf{0}$ of P.

- ((^x0)('x0.0 | x0.0) { } (^x0)(('x0.0 | x0.0) + t.Q)) stands for the pair $(\underline{\texttt{new}}\, x\ (A_1\langle x\rangle \mid B_1\langle x\rangle), \underline{\texttt{new}}\, x\ ((A_1\langle x\rangle \mid B_1\langle x\rangle) + \tau.Q))$, which means: The state $\underline{\texttt{new}}\, x\ ((\overline{x}\langle\rangle.\mathbf{0} \mid x().\mathbf{0}) + \tau.Q)$ of Q is as powerful as $\underline{\texttt{new}}\, x\ (\overline{x}\langle\rangle.\mathbf{0} \mid x().\mathbf{0})$ of P.

Thus, Q strongly simulates the behavior of P and the simulation relation is:

$$\mathcal{S} = \{(\mathbf{0}, \mathbf{0}), (\underline{\texttt{new}}\, x\ (A_1\langle x\rangle \mid B_1\langle x\rangle), \underline{\texttt{new}}\, x\ ((A_1\langle x\rangle \mid B_1\langle x\rangle) + \tau.Q))\}$$

```
The two agents are strongly related (2).
Do you want to see the core of the simulation (yes/no) ? yes
{
  (
    0
    { }
    0
  )


  (
    (^x0)('x0.0 | x0.0)
    { }
    (^x0)(('x0.0 | x0.0) + t.Q)
  )
}
```

Listing 2.4: ABC output: check if Q strongly simulates P.

In Listing 2.5 we can see the result of the command *lt Q P*, which checks if P strongly simulates Q. The result is *no*, since:

- when:
  - Q is in the state $\underline{\texttt{new}}\, x\ ((\overline{x}\langle\rangle.\mathbf{0} \mid x().\mathbf{0}) + \tau.Q)$.
  - P is in the state $\underline{\texttt{new}}\, x\ (\overline{x}\langle\rangle.\mathbf{0} \mid x().\mathbf{0})$.
- then:
  - Q can do a τ transition, which is the loop, to the state $\underline{\texttt{new}}\, x\ ((\overline{x}\langle\rangle.\mathbf{0} \mid x().\mathbf{0}) + \tau.Q)$.
  - P can do a τ transition, which is a reaction, to the state $\mathbf{0}$.
- then:
  - Q can do a τ transition, which is a reaction, to the state $\mathbf{0}$.
  - P cannot go ahead, denoted by "$*$", since it is in the state $\mathbf{0}$.

Thus, P does not strongly simulate the behavior of Q.

```
The two agents are not strongly related (2).
Do you want to see some traces (yes/no) ? yes
traces of

Q
P

-t->
-t->

(^x0)(('x0.0 | x0.0) + t.Q)
0

-t->
-t->
```

```
0
*
```

Listing 2.5: ABC output: check if P strongly simulates Q.

2.2 The Object-Z

The Object-Z, shortly OZ, is a specification language used to describe an entity through specifying its data, operations and the effects of those operations on the data. This section introduces the Object-Z as described in [Old19].

2.2.1 Intuition

To explain the OZ intuitively, we will start by examining the vending machine example, then we will explain how to build a set mathematically, finally we will explain the main concepts in OZ.

Vending Machine:

Let us imagine that we have the task: specifying a vending machine.

- Let cv be the amount of coffee, and tv be the amount of tea.
- Let *coffee* be the selling coffee operation, and *tea* be the selling tea operation.

The specifications are:

- It sells coffee and tea, and the maximum amount for each of them is 3.
- Its initial state is $cv = 3$ and $tv = 3$.
- When the operation *coffee* or *tea*, then the amount should be decreased by one.

The state space of the vending machine can be visualized as shown in Figure 2.6, where we see the initial state $VM(3, 3)$. The arrow indicates a state transition decrementing the amount of coffee. Later in **Main concepts of OZ** we will learn how to write the specifications using OZ language notations.

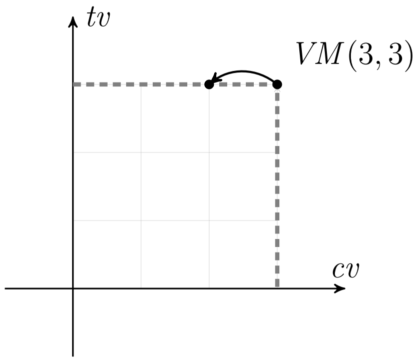


Figure 2.6: The state space of VM.

Set building:

A set is a collection of things. For example: $\{5, 7, 11\}$ is a set. But we can also build a set by describing what is in it using the following notation:

$$\{Deklaration \mid predicate \bullet expression\}$$

For example: $\{x : \mathbb{Z} \mid x \geq 0 \bullet x^2\}$ means *the set of all squared x's, such that x is an integer and greater than 0.*

Main concepts of OZ:

The main concepts of OZ are:

- *Schema*: it can be seen as a set [KSw88].
- *Class*: it can be seen as a grouping of a *state schema*, *initial state schema* and *operation schemas* [TDC04]. It represents the object oriented approach.

To illustrate those main concepts, consider the vending machine example denoted by VM:

- *Class*: To model the vending machine we need to define a class VM. Syntactically, in OZ a class definition is a named box as shown in Figure 2.7, where the dots "…" refer to details explained next.

VM

$\ldots$

Figure 2.7: VM class.

- *State space*: The state space of our vending machine can be seen as a set of all valid states. The set of all valid states is:
  - In mathematics:

$$\begin{aligned} State\_Space &= \{cv, tv : \mathbb{Z} \mid (0 \leq cv \leq 3) \wedge (0 \leq tv \leq 3) \bullet (cv, tv)\} \\ &= \{(0,0), \ldots, (3,3)\} \end{aligned}$$

  - In OZ: The set *State_Space* can be described using a *state schema*, which is a box without name added to the class box as shown in Figure 2.8.

VM

$cv, tv : \mathbb{Z}$

$0 \leq cv \leq 3$
$0 \leq tv \leq 3$

Figure 2.8: VM class: state schema.

- *Initial state*: Our vending machine has an initial state with $cv = 3$ and $tv = 3$. The set of all possible initial states, that respects those conditions is:
  - In mathematics:

$$\begin{aligned} Initial\_States = \{cv, tv : \mathbb{Z} \mid\ & (0 \leq cv \leq 3) \wedge (0 \leq tv \leq 3) \\ & \wedge (cv = 3) \wedge (tv = 3) \bullet (cv, tv)\} \\ =\ & \{(3,3)\} \end{aligned}$$

  - In OZ: the set *Initial_States* can be described using a *initial state schema*, which is a box named *Init* added to the class box as shown in Figure 2.9.

VM

$cv, tv : \mathbb{Z}$

$0 \leq cv \leq 3$
$0 \leq tv \leq 3$

INIT

$cv = 3$
$tv = 3$

Figure 2.9: VM class, initial state schema.

- *State transition*: When the vending machine sells a coffee, the amount of coffee should be decreased by one. This is a state transition. The set of all possible state transitions when the selling coffee operation occurs is:
  - In mathematics:

$$\begin{aligned} \mathit{coffee} = \{&cv, tv, cv', tv' : \mathbb{Z} \mid (0 \leq cv \leq 3) \wedge (0 \leq tv \leq 3) \\ &\wedge (0 \leq cv' \leq 3) \wedge (0 \leq tv' \leq 3) \wedge (tv' = tv) \\ &\wedge (cv' = cv - 1) \bullet ((cv, tv), (cv', tv'))\} \\ &= \{((3,3),(2,3)), \ldots, ((1,0),(0,0))\} \end{aligned}$$

    where (cv, tv) represents the *pre-state* and (cv', tv') represents the *post-state* of a state transition.
  - In OZ: the set *coffee* can be described using an *operation schema*, which is a box named with the operation name added to the class box as shown in Figure 2.10 left.

VM

$cv, tv : \mathbb{Z}$

$0 \leq cv \leq 3$
$0 \leq tv \leq 3$

Init
$cv = 3$
$tv = 3$

coffee
$cv, tv, cv', tv' : \mathbb{Z}$

$0 \leq cv \leq 3$
$0 \leq tv \leq 3$
$0 \leq cv' \leq 3$
$0 \leq tv' \leq 3$
$tv' = tv$
$cv' = cv - 1$

VM

$cv, tv : \mathbb{Z}$

$0 \leq cv \leq 3$
$0 \leq tv \leq 3$

Init
$cv = 3$
$tv = 3$

coffee
$\Delta(cv)$

$cv' = cv - 1$

VM

$cv, tv : \mathbb{Z}$

$0 \leq cv \leq 3$
$0 \leq tv \leq 3$

Init
$cv = 3$
$tv = 3$

coffee
$\Delta(cv)$

$cv' = cv - 1$

tea
$\Delta(tv)$

$tv' = tv - 1$

Figure 2.10: VM class, operation schema.

OZ offers a nicer way to write the operation schema using Δ-list. In OZ:

- Operation schema has a Δ-list of state variables whose values may change. By convention, no Δ-list means no attribute changes value.
- Operation schema implicitly includes the state schema and a primed version of it.

Thus, since the schema operation *coffee* specifies changes on the *coffee* value only, we can write it as shown in Figure 2.10 middle. Similarly, the operation schema *tea* is shown in Figure 2.10 right.

***Operation's input and output parameters*:**

Some operations can have input and output parameters, just like a method in a programming language, where the method's parameters represent the *input*, and the returned values represent the *output*. To illustrate the idea let us extend our vending machine. The new *VM* can talk to a shop sending a message to it. So it has a new operation *talk* and a state variable m representing the message to be sent.

The set of all possible state transitions when the *talk* operation occurs is:

- In mathematics:

$$\begin{aligned} talk = \{&cv, tv, message, cv', tv', message', y : \mathbb{Z} \mid (0 \leq cv \leq 3) \\ &\wedge (0 \leq tv \leq 3) \wedge (0 \leq cv' \leq 3) \wedge (0 \leq tv' \leq 3) \wedge (tv' = tv) \\ &\wedge (cv' = cv) \wedge (message' = message) \wedge (y = message) \bullet \\ &((cv, tv, message), (cv', tv', message'))\} \\ &= \{((3, 3, 1), (3, 3, 1)), \ldots, ((0, 0, 1), (0, 0, 1))\} \end{aligned}$$

- In OZ: the set *talk* can be described using an operation schema, as shown in Figure 2.11. We can notice that this operation does not change any state variable's value, it just says that the value of the output parameter y, written as $y!$, must be equal to the value of the state variable *message*. For input parameter we use the ? symbol.

VM

$cv, tv, message : \mathbb{Z}$

$0 \leq cv \leq 3$
$0 \leq tv \leq 3$

Init

$cv = 3$
$tv = 3$
$message = 1$

coffee

$\Delta(cv)$

$cv' = cv - 1$

tea

$\Delta(tv)$

$tv' = tv - 1$

talk

$y! : \mathbb{Z}$

$y! = message$

Figure 2.11: VM class, *talk* operation with output parameter.

***Instance reference*:**

OZ is an object oriented approach, thus every instance of a class needs a unique identifier, i.e., a reference name to refer to it. In OZ this can be seen simply as state constant *self* initialized with some *id* when the instance is created. Furthermore, operations can share the instance identity through output or input the reference name *self* as shown in Figure 2.12 in the operation *talk*.

$VM(id : \mathbb{Z})$

$self, cv, tv, message : \mathbb{Z}$

$0 \leq cv \leq 3$
$0 \leq tv \leq 3$

Init

$self = id$
$cv = 3$
$tv = 3$
$message = 1$

coffee

$\Delta(cv)$

$cv' = cv - 1$

tea

$\Delta(tv)$

$tv' = tv - 1$

talk

$y! : \mathbb{Z}$
$z! : \mathbb{Z}$

$y! = message$
$z! = self$

Figure 2.12: VM class, instance reference.

2.2.2 Semantics

To understand the operational semantics of OZ we will use a labelled transition system LTS. Using this LTS we can investigate the state evolution of a OZ object. The definition of this LTS is adapted from Definition 2.1.5 with some changes.

Definition 2.2.1 (LTS of OZ) The *labelled transition system* $(\mathcal{S}^{OZ}, \mathcal{T})$ of OZ class states over the set of operations, has the valid states $\mathcal{S}^{OZ}$ as its states and its transitions $\mathcal{T}$ are those which can be inferred from the following rule:
$\underline{\textit{OPER}} : PRE\_STATE \xrightarrow{operation} POST\_STATE.$ $\triangle$

An example of using the transition rule of this LTS is: drawing the transition graph of the vending machine shown in Figure 2.10. The transition graph is shown in Figure 2.13, where we show only a small part of it. The transitions *coffee* and *tea* refer to the operation schema *coffee* and *tea*. Thus, using the LTS we can enumerate all possible states transitions of an OZ state.

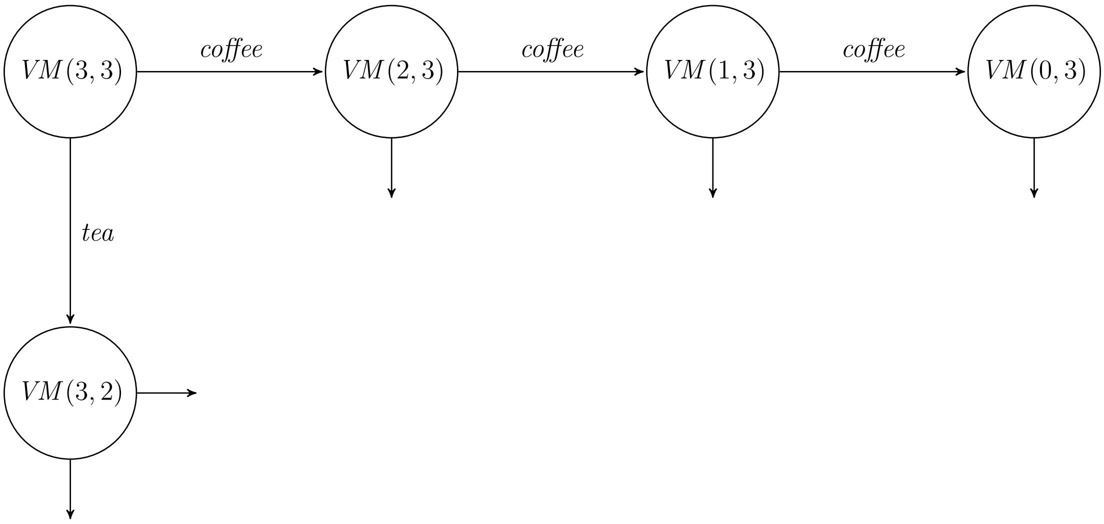


Figure 2.13: VM transition graph.

2.2.3 Dynamic OZ

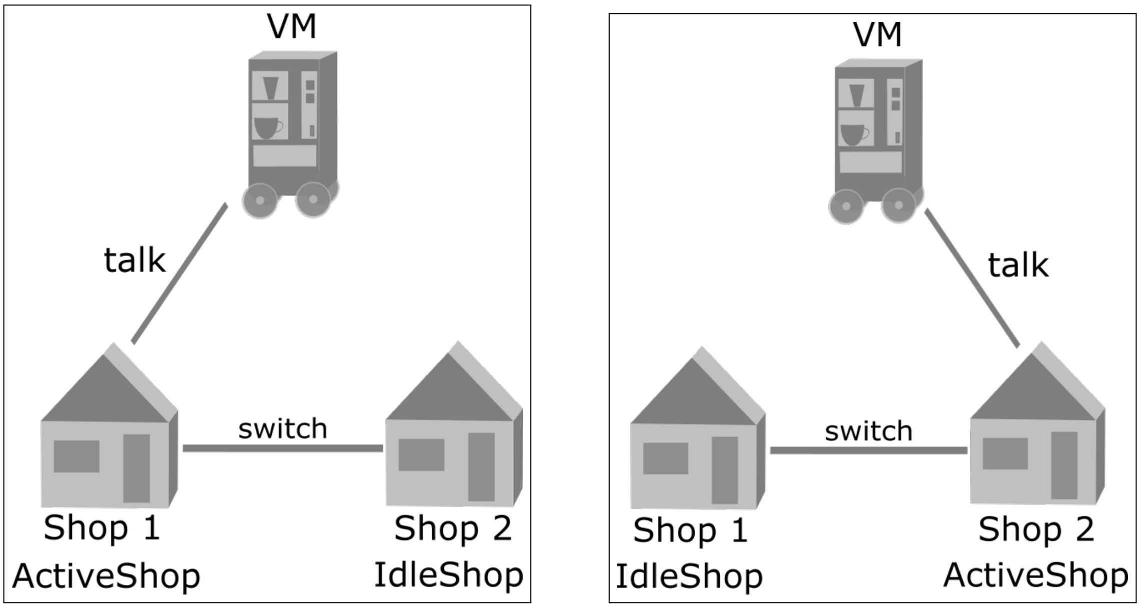
OZ can be used to model an entity with unchanged behavior, but sometimes we need to model an entity that changes its behavior. We introduce dynamic OZ, which is a version of OZ that uses the state pattern to model an entity with varying behavior.

OZ and state pattern

The state pattern is a behavioral software design pattern that allows an object to alter its behavior when its state changes. This pattern is close to the concept of finite-state machines. Changing behavior can be seen as that the object has changed its class. To illustrate the idea imagine that our vending machine *VM* is a mobile vending machine and that it is connected by a wireless link *talk* to a shop *Shop1*. On signal fading, *Shop1* decides to send the link *talk* to another shop *Shop2* through the link *switch* as shown in Figure 2.14. *Shop1* and *Shop2* change their behavior after switching. This varying behavior of shop can be handled through using two classes *ActiveShop* and *IdleShop*. A shop changes its class when *switch* occurs:

- *Shop1* sends *talk* via *switch* and changes its class from *ActiveShop* to *IdleShop* as shown in Figure 2.15.
- *Shop2* receives *talk* via *switch* and changes its class from *IdleShop* to *ActiveShop* as shown in Figure 2.15.

Notice, when an object changes its class it keeps its state variables and skips the *Init* schema of the new class. Dynamic OZ is based on the Agent-Place model used by MobileOZ described in [TD02]. MobileOZ has two essential entities, agents and places. The main difference in the roles of these entities is that agents can move around the network, while places cannot. Dynamic OZ takes another approach by allowing places transferring, as shown in Figure 2.14, where the link *talk* can be transferred from *Shop*1 to *Shop*2. In Dynamic OZ, mobility is achieved by attaching a distinguished variable *transferableOperation* for storing names of locations. Location transferring is mimicked by assigning a new location to that variable.



(a) Before switching (b) After switching

Figure 2.14: Mobile vending machine and shops.

$ActiveShop(id : \mathbb{Z})$

$self, vmId, message : \mathbb{Z}$
$transferableOperation : nil \mid talk$

$\textsc{Init}$
$self = id$
$transferableOperation = talk$

$switch \,\_\_\, then\ IdleShop$
$x! : nil \mid talk$
$x! = transferableOperation$
$transferableOperation' = nil$

$talk$
$\Delta(vmId, message)$
$y?, z? : \mathbb{Z}$
$y? = message'$
$z? = vmId'$

$IdleShop(id : \mathbb{Z})$

$self, vmId, message : \mathbb{Z}$
$transferableOperation : nil \mid talk$

$\textsc{Init}$
$self = id$
$transferableOperation = nil$

$switch \,\_\_\, then\ ActiveShop$
$\Delta(transferableOperation)$
$x? : nil \mid talk$
$x? = transferableOperation'$

Figure 2.15: Active and idle shop.

Restriction

In this work when we use OZ to model an operation, we restrict ourself to use only one type of parameters in the operation schema. Either input or output. This can be noticed in the operation schema *talk* in:

- In Figure 2.12 all the parameters of the operation schema *talk* are output parameters.
- In Figure 2.15 all the parameters of the operation schema *talk* are input parameters.

Why this restriction? Because a channel in the π-calculus is *unidirectional* pro reaction. In the next chapter we will map the OZ class constructs to π-calculus constructs, so we will map an OZ operation to an π-calculus name, i.e., channel. In π-calculus a process can send or receive over a channel per reaction, but not the both together.

3 Transformational semantics of OZ

This chapter studies the syntactic transformation of an OZ class into an π-calculus process. The resulting process is intuitively defined as follows:

$$P_{OZ\_PI} = \sum_{v\_st \in Init} \tau . Q(v\_st, v\_self)$$

$$Q(v\_st, v\_self) = (\sum_{c \in \mathrm{In}} c(v\_in_c) \; + \sum_{c \in \mathrm{Out}} \tau . \overline{c}\langle v\_out_c \rangle) \; . \\ \sum_{v\_st'} \tau . Q(v\_st', v\_self)$$

Where:

- $\mathtt{In}$ the set of all input actions, defined as $\mathtt{In} =_{\mathrm{def}} \{x(\vec{y}) \mid x \in \mathcal{N}\}$.
- $\mathtt{Out}$ is the set of all output actions, defined as $\mathtt{Out} =_{\mathrm{def}} \{\overline{x}\langle\vec{y}\rangle \mid x \in \mathcal{N}\}$.
- c refers to an operation.
- v_st refers to the variables of the current state.
- v_st' refers to the variables of the successor state.
- v_self refers to the instance reference.
- $c.v\_in_c$ refers to the occurrence of the operation c, where v_in_c represents the values of the input parameters of c.
- $c.v\_out_c$ refers to the occurrence of the operation c, where v_out_c represents the values of the output parameters of c.

What is the benefit of transforming an OZ class into a π-calculus process? The main advantage is that it can be combined using the parallel operator with a second explicit π-calculus process that represents the desired sequencing of the operations of the OZ class. This will enable us to study the behavior of an entity as will be shown later in the next chapter.

To transform an OZ class into a π-calculus process we need to remember that the π-calculus has only names and processes, and nothing else. A *name* in π-calculus can be seen as a *channel* or a *memory location*. Thus, in this work when we use the word *channel* we refer to a π-calculus *name*. We need to use the names and processes to represent: value, state variable, state schema, initial state schema and operation schema in π-calculus.

3.1 Mapping values

We consider a finite set of natural numbers represented as binary numbers shown in Table 3.1. A value can be mapped to a π-calculus process. Listing 3.1 shows the π-calculus implementation of the values 0,1,2,3 in ABC syntax. The keyword *agent* defines a new process. The process *Zero* is modeled using alternative choice: it either receives a signal via the channel a and switch off, or it receives two channels *tt,ff* via a, then it sends two signals via the channel *ff*.

Decimal	Binary
0	00
1	01
2	10
3	11

Table 3.1: Two bits binary numbers.

```
agent Zero(a) = a(tt,ff).'ff.'ff.Zero(a) + a.0
agent One(a) = a(tt,ff).'tt.'ff.One(a) + a.0
agent Two(a) = a(tt,ff).'ff.'tt.Two(a) + a.0
agent Three(a) = a(tt,ff).'tt.'tt.Three(a) + a.0
```

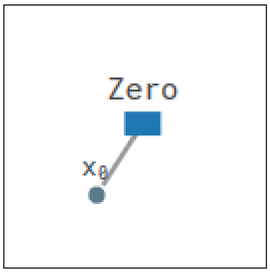
Listing 3.1: 0,1,2,3 as π-calculus processes.

3.2 Mapping state variables

A variable can be mapped to a channel. Creating a variable x and initializing it with the value 0 (int x = 0;) is mapped to creating a new channel x and initialize the process Zero with the channel x as shown in Figure 3.1. The wide hat refers to creating a new channel.

(^x) (Zero(x))

(a) ABC code.



(b) visualization.

Figure 3.1: Variable as a channel.

Thus, we map the state variables *self*, *cv*, *tv*, *message* of Figure 2.12 to π-calculus channels *self*, *cv*, *tv*, *message* as shown in Figure 3.2.

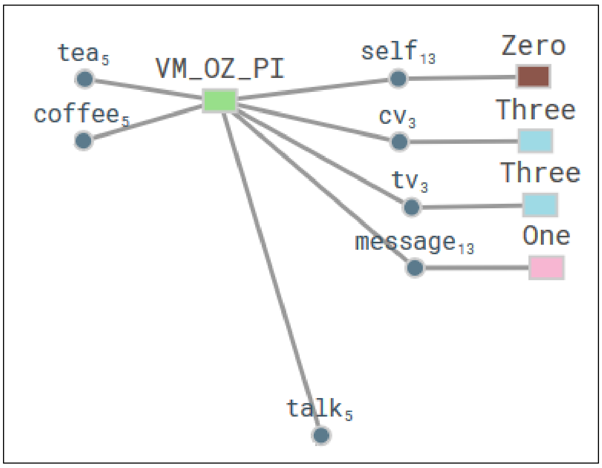


Figure 3.2: VM as a π-calculus process VM_OZ.

3.3 Mapping operations

We map OZ class operations to π-calculus channels as we did with state variables. That is, we map the operations *coffee*, *tea*, *talk* of Figure 2.12 to π-calculus channels *coffee*, *tea*, *talk* as shown in Figure 3.2

3.4 Mapping data Types

For simplicity, we do not implement any kind of type checking, but we deal with types by representing the value of the type by corresponding process. For example $cv : \{0, 1, 2, 3\}$, means that the allowed processes to be initialized with cv are: $Zero$, One, Two, $Three$.

3.5 Mapping mathematical operators

Addition:

To add two numbers we need an addition processes that mimics the behavior of arithmetic circuits for adding two bits binary numbers shown in Figure 3.3. Figure 3.4 shows the visualization of the addition process and the ABC code. The full implementation of *Add* processes can be found in the appendix.

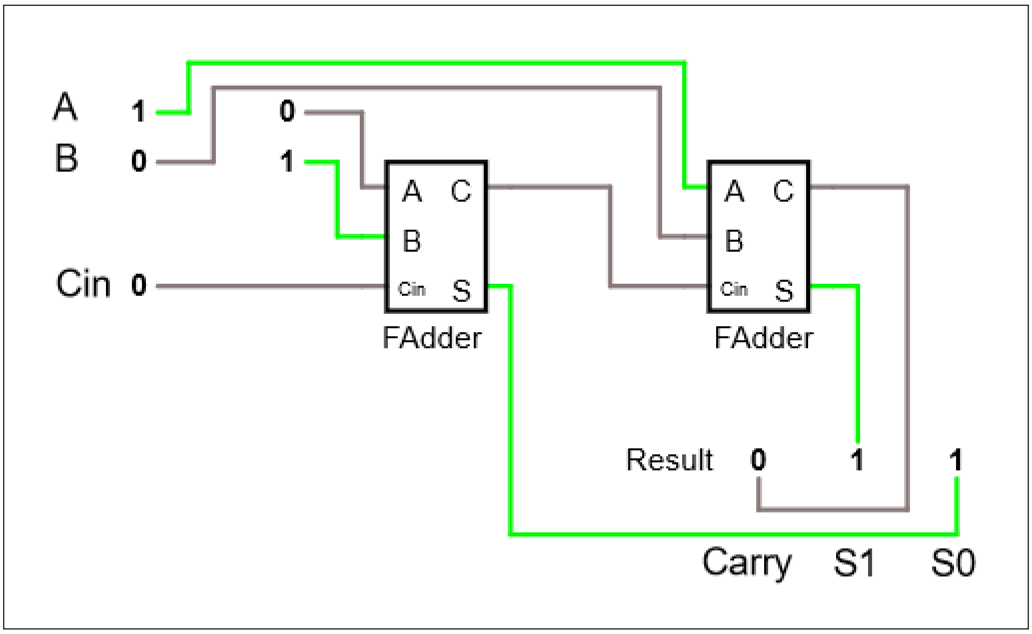
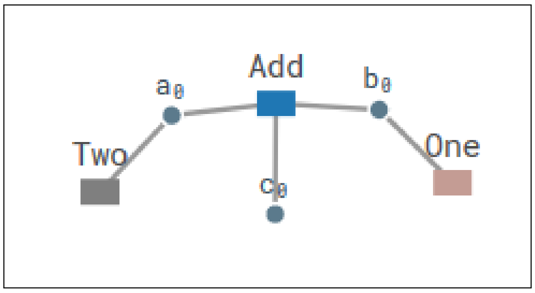
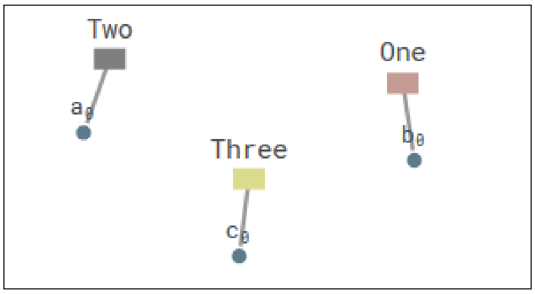


Figure 3.3: Adder circuit.



(a) Before addition.



(b) After addition.

(^a,b,c) (Two(a) | One(b) | Add(a,b,c))

(c) Abc code.

Figure 3.4: Addition as a process.

Subtraction:

To subtract two numbers we use a subtraction process that mimics the behavior of arithmetic circuits for subtracting two bits binary numbers shown in Figure 3.5. Figure 3.6 shows the visualization of the subtraction process and the ABC code. The full implementation of *Sub* processes can be found in the appendix.

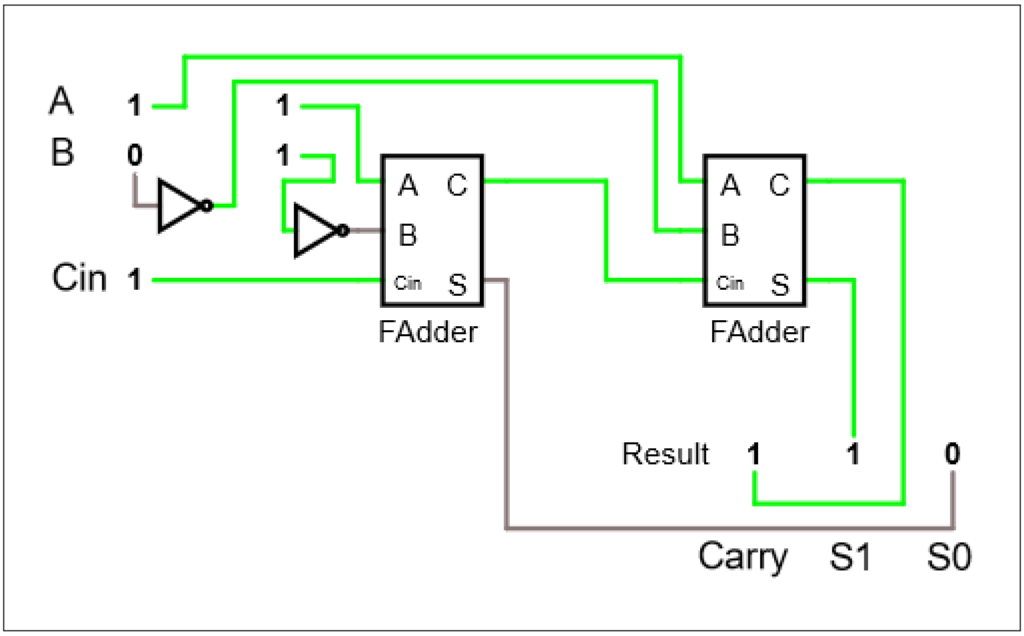
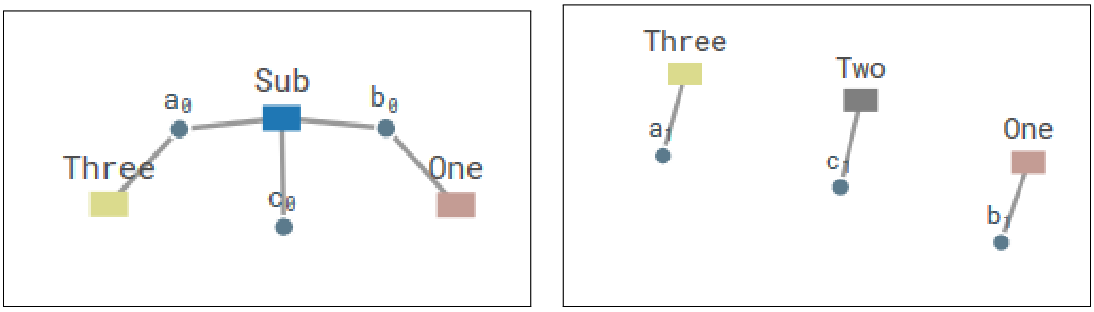


Figure 3.5: Subtractor circuit.



(a) Before subtraction.   (b) After subtraction.

```
(^a,b,c ) ( Three(a) | One(b) | Sub(a,b,c) )
```

(c) ABC code.

Figure 3.6: Subtraction as a process.

Comparation

To compare two numbers we use a process that mimics the behavior of arithmetic circuits for comparing two bits binary numbers shown in Figure 3.7. Figure 3.8 shows the visualization and ABC code of the comparator process and a simple if-else statement. The full implementation of *Compare* processes can be found in the appendix.

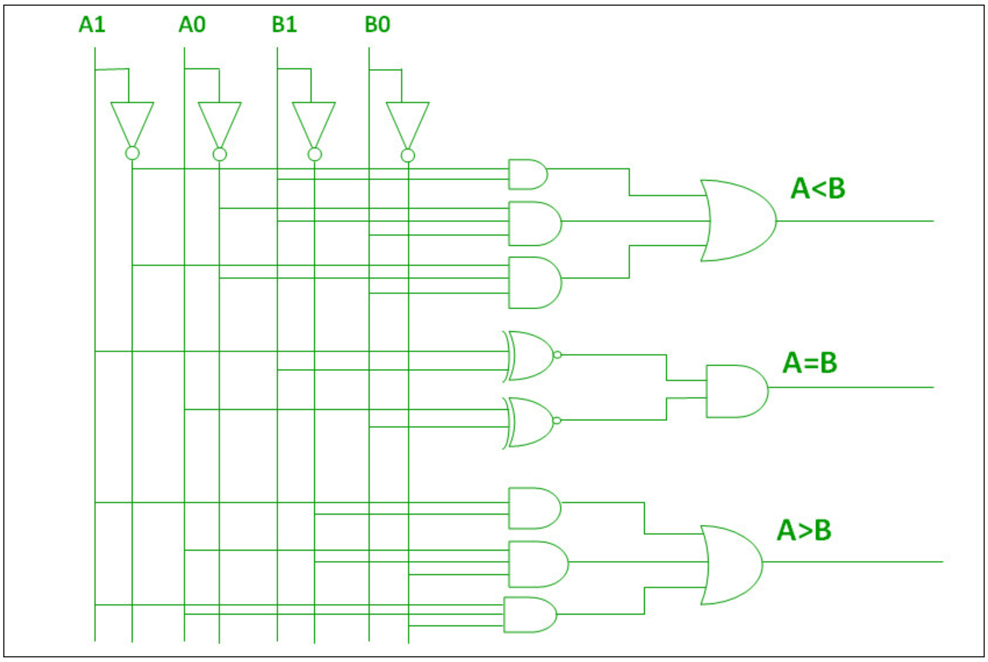
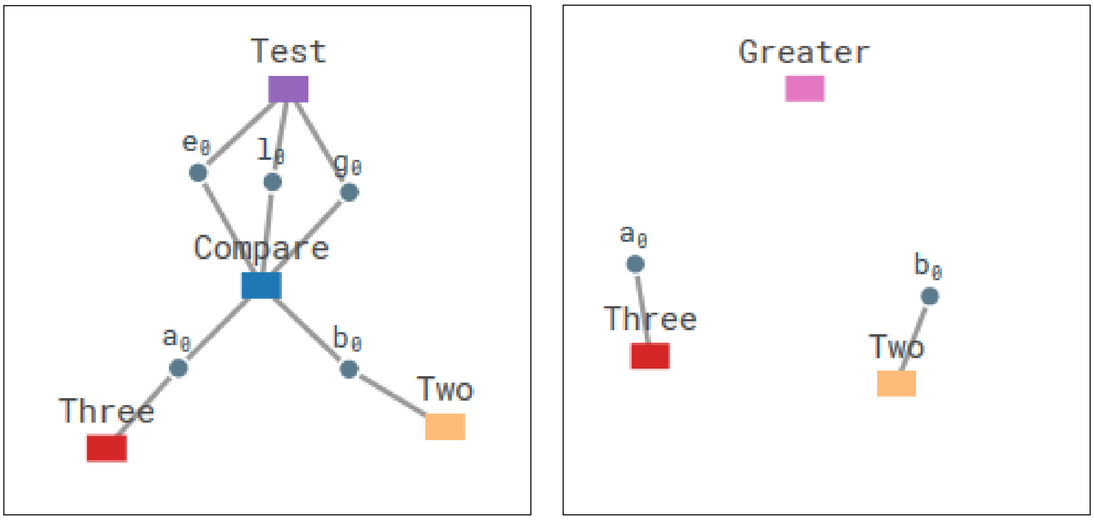


Figure 3.7: Comparator circuit.



(a) Before comparation. (b) After comparation.

(⌒a,b,g,e,l) (Three(a) | Two(b) | Compare(a,b,g,e,l) | If_Else(g,e,l))

(c) ABC code.

Figure 3.8: Comparation as a process.

Set union and subtraction:

The implementation of *set union and subtraction* processes can be found in the appendix.

3.6 Mapping OZ class

The class *VM* shown in Figure 3.9 is mapped to a π-calculus process *VM_OZ_PI* shown in Listing 3.2. The processes **VM_OZ** has six parameters

- *self, message, cv* and *tv* represent the state variables.
- *coffee, tea* and *talk* represent the operations.

The processes *VM_OZ_PI* mimics the behaviour of *VM*:

- On receiving a signal via *coffee*, then *VM_Condition_IF_Else_coffee* checks if the condition *VM_Condition_coffee* is fulfilled. If it is fulfilled it makes a state transition *VM_State_Transition_coffee* to decrease the value of *cv* by one *One(b) | Sub(cv,b,c,done)*.

- The same goes for *tea*.
- *VM_OZ_PI* can send a copy of the value of *self,message* via *talk*

The processes *VM_OZ_PI_Init* creates an instance of *VM_OZ_PI* and initialize its state variables *self, cv, tv* and *message* with the values *Zero, Three, Three, One*. The full implementation can be found in the appendix.

$VM(id : \mathbb{Z})$

$self, cv, tv, message : \mathbb{Z}$

$0 \leq cv \leq 3$
$0 \leq tv \leq 3$

INIT

$self = id$
$cv = 3$
$tv = 3$
$message = 1$

coffee

$\Delta(cv)$

$cv' = cv - 1$

tea

$\Delta(tv)$

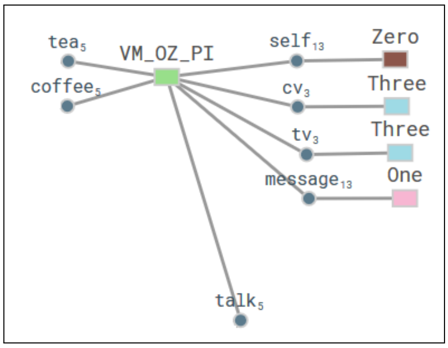
$tv' = tv - 1$

talk

$y! : \mathbb{Z}$
$z! : \mathbb{Z}$

$y! = message$
$z! = self$

(a) VM class in OZ.



(b) Stargazer visualization.

Figure 3.9: Transforming VM into π-calculus process VM_OZ_PI.

```
/* The process VM_OZ_PI_Init mimics the intitial state schema
   of the class VM.*/
```

```
agent VM_OZ_PI_Init(coffee,tea,talk) = (^self,cv,tv,message)
    (VM_OZ_PI(self,coffee,tea,talk,cv,tv,message) | Zero(
    self) | Three(cv) | Three(tv) | One(message))

// The process VM_OZ_PI mimic the behaviour of the class VM.
agent VM_OZ_PI(self,coffee,tea,talk,cv,tv,message) =
coffee.(^res_t,res_f) (VM_Condition_coffee(self,coffee,tea,
    talk,cv,tv,message,res_t,res_f) |
    VM_Condition_IF_Else_coffee(self,coffee,tea,talk,cv,tv,
    message,res_t,res_f)) \
+ tea.(^res_t,res_f) (VM_Condition_tea(self,coffee,tea,talk,
    cv,tv,message,res_t,res_f) | VM_Condition_IF_Else_tea(
    self,coffee,tea,talk,cv,tv,message,res_t,res_f)) \
+ (^m_c,m_done,r_c,r_done) ( (m_done.r_done.'talk<r_c,m_c>.
    VM_OZ_PI(self,coffee,tea,talk,cv,tv,message)) | Copy(
    message,m_c,m_done) | Copy(self,r_c,r_done))

/* The processes VM_Condition_coffee and VM_Condition_IF_Else_coffee
     mimic checking if the values of the state variables
     fullfils the pre and post condtions of the operation
     schema coffee.*/
agent VM_Condition_coffee(self,coffee,tea,talk,cv,tv,message
    ,res_t,res_f) = (^b,g,e,l) (Zero(b) | Compare(cv,b,g,e,l)
     | CleanAndTF(g,e,l,res_t,res_f,b))
agent VM_Condition_IF_Else_coffee(self,coffee,tea,talk,cv,tv
    ,message,res_t,res_f) = res_t.(VM_State_Transition_coffee
    (self,coffee,tea,talk,cv,tv,message)) + res_f.
    VM_PleaseFillMe

/* The process VM_State_Transition_coffee to mimic the state
     transtion to decrease the amount of coffee by one.*/
agent VM_State_Transition_coffee(self,coffee,tea,talk,cv,tv,
    message) = (^sub_done, b,c,done) ((One(b) | Sub(cv,b,c,
    done) | ClearThenCopy(cv,b,c,done,sub_done)) | (sub_done
    .'coffee.VM_OZ_PI(self,coffee,tea,talk,cv,tv,message)) )
```

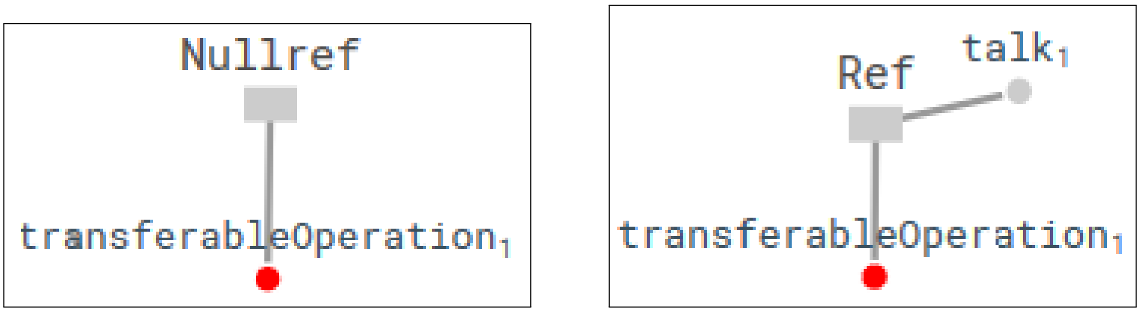
Listing 3.2: the process VM_OZ_PI in ABC code.

3.7 Mapping transferable operation's variable

As mentioned in Section 2.2.3, the mobility in Dynamic OZ is achieved by attaching a distinguished variable transferableOperation for location. Location transferring is mimicked by assigning a new location to that variable. To translate the variable *transferableOperation* into the π-calculus we cannot use π-calculus channel as in Section 3.2, since the value of *transferableOperation* will be a channel name and not a process representing a value like *Zero*. Thus, we map the variable transferableOperation to a channel named transferableOperation, where:

- transferableOperation = nil is mapped to Nullref(transferableOperation).
- transferableOperation = talk is mapped to Ref(transferableOperation,talk).

as shown in Figure 3.10 and Listing 3.3.



(a) transferableOperation = nil. (b) transferableOperation = talk.

Figure 3.10: Mapping transferable operation's variable.

```
agent Nullref(r) = r(n,c).('n.Nullref(r) + c(m).Ref(r,m) + n
    .Nullref(r))
agent Ref(r,v) = r(n,c).('c<v>.Ref(r,v) + c(m).Ref(r,m) + 'n
    .Nullref(r))
```

Listing 3.3: Nullref and Ref processes in ABC code.

Thus, using the concept of transferable operation's variable we can now transform the classes *IdleShop* and *ActiveShop* as shown in Figure 3.11, Listing 3.4 and Figure 3.12, Listing 3.5.

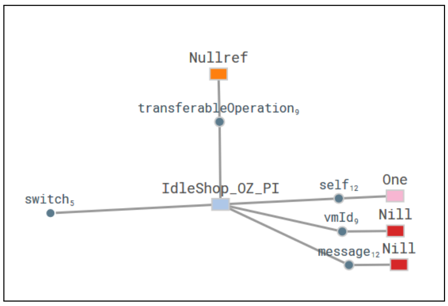
$IdleShop(id : \mathbb{Z})$

$self, vmId, message : \mathbb{Z}$
$transferableOperation : nil \mid talk$

$\textsc{Init}$
$self = id$
$transferableOperation = nil$

$switch$ then $ActiveShop$
$\Delta(transferableOperation)$
$x? : nil \mid talk$
$x? = transferableOperation'$

(a) IdleShop class in OZ



(b) stargazer visualization

Figure 3.11: Transforming IdleShop into π-calculus process IdleShop_OZ_PI.

```
/* The process IdleShop_OZ_PI mimic the behaviour of the class
    IdleShop.*/
agent IdleShop__OZ__PI(self,switch,transferableOperation,vmId,
   message) = switch(talk__new).((^n,c) ('
   transferableOperation<n,c>.'c<talk__new>.ActiveShop__OZ(
   self,switch,transferableOperation,talk__new,vmId,message))
   )

/* The process IdleShop_OZ_PI_Init_Null mimics the intitial state
    schema of the class IdleShop.*/
agent IdleShop__OZ__PI__Init__Null(switch) = (^self,
   transferableOperation,vmId,message) (IdleShop__OZ__PI(self,
   switch,transferableOperation,vmId,message) | One(self) |
   Nullref(transferableOperation) | Nill(vmId) | Nill(
   message))
```

Listing 3.4: The process IdleShop_OZ_PI in ABC code.

ActiveShop$(id : \mathbb{Z})$

$self, vmId, message : \mathbb{Z}$
$transferableOperation : nil \mid talk$

INIT

$self = id$
$transferableOperation = talk$

switch then *IdleShop*

$x! : nil \mid talk$

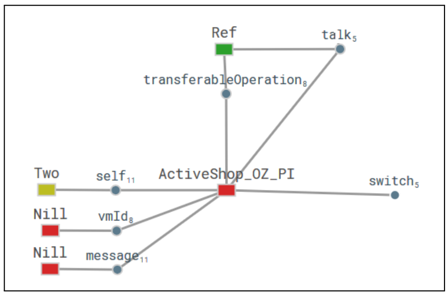
$x! = transferableOperation$
$transferableOperation' = nil$

talk

$\Delta(vmId, message)$
$y?, z? : \mathbb{Z}$

$y? = message'$
$z? = vmId'$

(a) ActiveShop class in OZ.



(b) stargazer visualization.

Figure 3.12: Transforming ActiveShop into π-calculus process ActiveShop_OZ_PI.

```
/* The process ActiveShop_OZ_PI mimic the behaviour of the
    class ActiveShop.*/
agent ActiveShop_OZ_PI(self,switch,transferableOperation,
   talk_current,vmId,message) =
'switch<talk_current>.( ((^n,c) ('transferableOperation<n,c
   >.'n.IdleShop_OZ(self,switch,transferableOperation,vmId,
   message))) | KillAndSetNilIfNotNill(vmId) |
   KillAndSetNilIfNotNill(message)) \
+ talk_current(vmId_new,m_new).(^done_ref,done_m) ( (
   done_ref.done_m.ActiveShop_OZ_PI(self,switch,
```

```
    transferableOperation,talk_current,vmId,message)) |
    KillThenCopyValueThenKillTemp(vmId_new,vmId,done_ref) |
    KillThenCopyValueThenKillTemp(m_new,message,done_m))

/* The process ActiveShop_OZ_PI_Init_Talk mimics the intitial
    state schema of the class ActiveShop.*/
agent ActiveShop_OZ_PI_Init_Talk(switch,talk_current) = (^
    self,transferableOperation,vmId,message) (
    ActiveShop_OZ_PI(self,switch,transferableOperation,
    talk_current,vmId,message) | Two(self) | Ref(
    transferableOperation,talk_current) | Nill(vmId) | Nill(
    message))
```

Listing 3.5: The process ActiveShop_OZ_PI in ABC code.

Finally, Figure 3.13, Figure 3.14 and Listing 3.6 show the big picture of a system consisting of two shops, a vending machine and a customer. The full implementation can be found in the appendix.

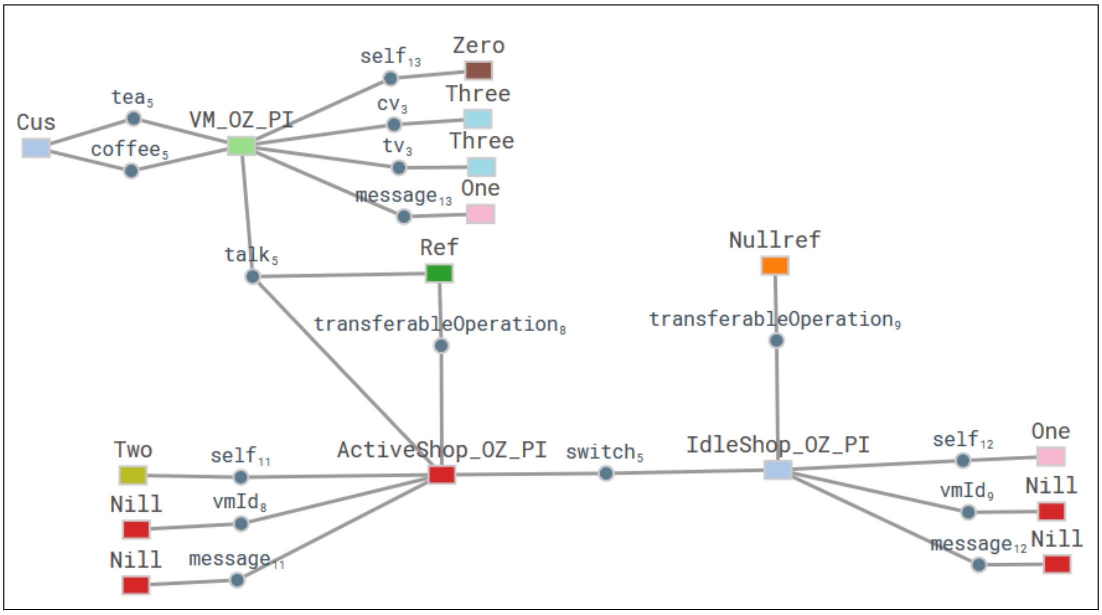


Figure 3.13: System before switching.

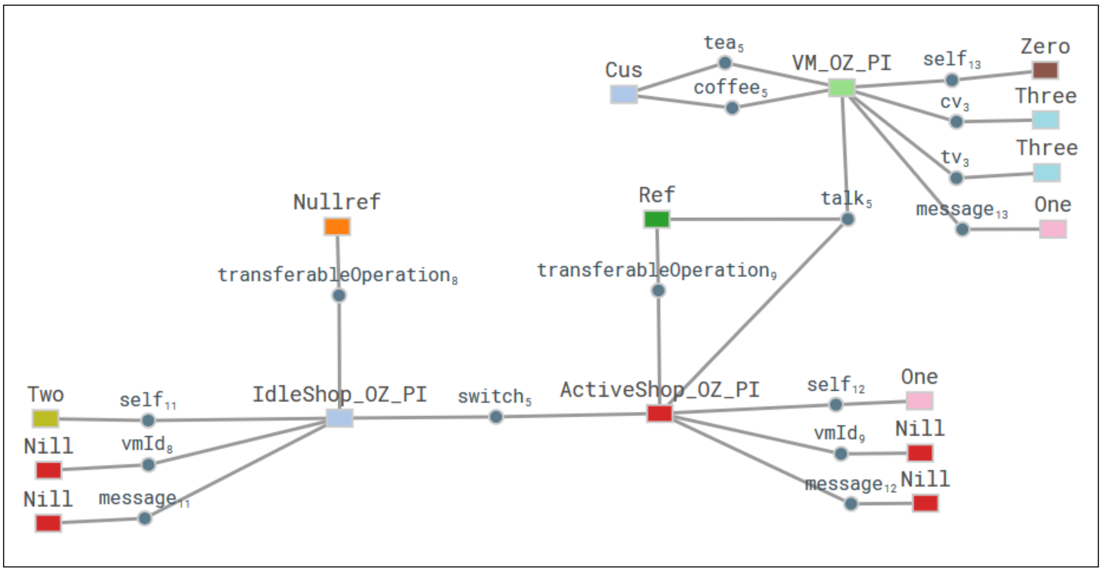


Figure 3.14: System after switching.

```
agent System = (^coffee ,tea ,switch ,talk ) ( VM_OZ_PI_Init(
    coffee ,tea ,talk ) | Cus(coffee ,tea ) |
    IdleShop_OZ_Init_Null(switch ) |
    ActiveShop_OZ_PI_Init_Talk(switch ,talk ))
```

Listing 3.6: The system consisting of: two shops, vending machine and a customer in ABC code.

4 The combination π-OZ

In this chapter we will combine the specifications languages OZ and π-calculus into the combination π-OZ, and we will study its transformational semantics.

4.1 Syntax

Syntactically the π-OZ specification is divided into an interface, a π part and an OZ part as shown in Figure 4.1. The idea of the combination is that communication in the π part has effects on the state space of the OZ part as specified in its operation schema. Figure 4.2 shows the π-OZ specification of our vending machine VM. In the interface, all channels are declared with the associated types. The π part is a system of recursive equations written according to the π-calculus syntax represents the sequencing of operations as shown in VM_PI. The state space, the initial schema and the operation schemas are encapsulated in the OZ part, where the operation schemes defines the effect of communications on the specified channels.

S

$Interface$

S_{π_part}

S_{OZ_part}

Figure 4.1: π-OZ specification of an entity S.

$VM(id : \mathbb{Z})$

$chan\ coffee, tea$
$chan\ talk : \mathbb{Z} \times \mathbb{Z}$

$$VM\_PI = coffee().VM\_PI + tea().VM\_PI$$
$$+\overline{talk}\langle self, message\rangle.VM\_PI$$

$self, cv, tv, message : \mathbb{Z}$

$0 \leq cv \leq 3$
$0 \leq tv \leq 3$

INIT

$self = id$
$cv = 3$
$tv = 3$
$message = 1$

coffee

$\Delta(cv)$

$cv' = cv - 1$

tea

$\Delta(tv)$

$tv' = tv - 1$

talk

$y! : \mathbb{Z}$
$z! : \mathbb{Z}$

$y! = message$
$z! = self$

Figure 4.2: π-OZ specification of the VM.

Figure 4.3 shows the π-OZ specification of the active and idle shop.

$ActiveShop(id : \mathbb{Z})$

$chan\ switch : nil \mid talk$
$chan\ talk : \mathbb{Z} \times \mathbb{Z}$

$ActiveShop\_PI =$
$\quad talk(vmId, message)$
$\quad\quad .ActiveShop\_PI$
$\quad +\overline{switch}\langle talk \rangle$
$\quad\quad .IdleShop\_PI$

$self, vmId, message : \mathbb{Z}$
$transferableOperation : nil \mid talk$

$\textsc{Init}$
$self = id$
$transferableOperation = talk$

$switch$ ___ $then\ IdleShop$
$x! : nil \mid talk$

$x! = transferableOperation$
$transferableOperation' = nil$

$talk$
$\Delta(vmId, message)$
$y?, z? : \mathbb{Z}$

$y? = message'$
$z? = vmId'$

$IdleShop(id : \mathbb{Z})$

$chan\ switch : nil \mid talk$

$IdleShop\_PI =$
$\quad switch(transferableOperation)$
$\quad\quad .ActiveShop\_PI$

$self, vmId, message : \mathbb{Z}$
$transferableOperation : nil \mid talk$

$\textsc{Init}$
$self = id$
$transferableOperation = nil$

$switch$ ___ $then\ ActiveShop$
$\Delta(transferableOperation)$
$x? : nil \mid talk$

$x? = transferableOperation'$

Figure 4.3: π-OZ specification of the active and idle shop.

4.2 Transformational semantics

The semantics of the combination π-OZ specification S can then be described by the π-calculus process $S_{OZ\_part_\pi} \mid S_{\pi\_part}$, where $S_{OZ\_part_\pi}$ is the syntactic transformation of OZ part into π-calculus process. For example, the semantics of Figure 4.2 is $VM\_OZ\_PI \mid VM\_PI$, where VM_OZ_PI is as described in Listing 3.2. Unfortunately, this will not work well, since the parallel operator "|" only allows the binary synchronization via a channel, not like in *CSP* where the parallel operator "||" allows the multiple synchronization via a channel. That will be problematic when we try to combine the π-OZ specification of an entity S with a π-OZ specification of another entity R in parallel. To solve this problem we can use **broadcast channel** or **non-atomic reaction** concept as follows:

***Shared channel*:**

To allow the multiple synchronization via a channel in π-calculus, we use the concept of the shared channel. [EM00] introduces the bπ, which is an extension of π-calculus implementing broadcast communications. Additionally, the UPPAAL model checker introduces the broadcast channel too [OD08]. For simplicity, we use the broadcast channel from UPPAAL with a little change. On a shared channel one sender synchronizes with at least one receiver. Thus, like binary synchronization, a shared channel blocks the sender if there are no receivers. Furthermore, we can send and receive on a shared channel. We extend the transition rules of π-calculus defined in Definition 2.1.5, with an additional rule:

$$\underline{\mathit{Shared\_Chan\_PAR}} : \frac{P \xrightarrow{\overline{x}\langle\vec{y}\rangle} P' \quad Q \xrightarrow{x(\vec{y})} Q' \quad R \xrightarrow{x(\vec{z})} R'}{P \mid Q \mid R \xrightarrow{\tau} P' \mid \{\vec{y}/\vec{z}\}\, Q' \mid \{\vec{y}/\vec{z}\}\, R'} \quad \mathit{if}\ x : \mathit{Shared}$$

Figure 4.4: Transition rule for shared channel.

Figure 4.5 shows the π-OZ specification of our *VM* using shared channels. The combination's process $S_{OZ\_part_\pi} \mid S_{\pi\_part}$ for *VM* is $VM\_OZ\_PI \mid VM\_PI$. The main advantage of the shared channels in *VM* is that, if we combine the combination's processes with a third processes *Cus* representing a customer which issues a signal on the *coffee* channel, this will enforce both *VM_OZ_PI* and *VM_PI* to evolve, since they are listening on coffee, which is a shared channel in *Cus*, *VM_OZ_PI* and *VM_PI*. The behavior of *VM* can be seen as the intersection of the behavior of *VM_OZ_PI* and *VM_PI*, i.e., the intersection of the transition graphs, i.e., the automates. Unfortunately, our tools do not support the shared channel, thus we will not proceed with this approach.

$VM(id : \mathbb{Z})$

$shared\ chan\ coffee, tea$
$shared\ chan\ talk : \mathbb{Z} \times \mathbb{Z}$

$$VM\_PI = coffee().VM\_PI + tea().VM\_PI + \overline{talk}\langle self, message\rangle.VM\_PI$$

$self, cv, tv, message : \mathbb{Z}$

$0 \leq cv \leq 3$
$0 \leq tv \leq 3$

INIT
$self = id$
$cv = 3$
$tv = 3$
$message = 1$

$coffee$
$\Delta(cv)$

$cv' = cv - 1$

tea
$\Delta(tv)$

$tv' = tv - 1$

$talk$
$y! : \mathbb{Z}$
$z! : \mathbb{Z}$

$y! = message$
$z! = self$

Figure 4.5: π-OZ specification of the VM using broadcast channels.

***Non-atomic reaction*:**

Let us examine the process $Cus \mid VM\_OZ\_PI \mid VM\_PI$ shown in Figure 4.6. When Cus issues a signal on the $coffee$ channel, it is required that VM_OZ_PI and VM_PI receives the signal and evolve together. This is not possible, since the π-calculus communications are binary, so either VM_OZ_PI or VM_PI will evolve and the other will not. To solve this problem using binary communications we propose to break the channel *coffee* down into two channels: *ex_coffee* and *in_coffee* as shown in Figure 4.7. The channel *ex_coffee* is for the external, outside VM, communication between Cus and VM_PI. The channel *in_coffee* is for the internal, inside VM, communication between VM_PI and VM_OZ_PI. In Figure 4.7 the numbered arrows represent the communication flow from VM_PI's point of view. VM_PI receives a signal via *ex_coffee* and re-sends it via *in_coffee*. When VM_OZ_PI ends its processing it sends a done signal via *done_in_coffee* to VM_PI which re-sends the done signal to Cus via *done_ex_coffee*. This way the combination's process $S_{OZ\_part_\pi} \mid S_{\pi\_part}$, i.e., $VM\_OZ\_PI \mid VM\_PI$, behaves as a one processes from the view point of its environment, i.e., Cus, by breaking down the channel, reproducing the signal, and using the done signal. All that makes the reaction *ordering a coffee* a non-atomic reaction. Furthermore, we can notice that the non-atomic reaction concept is overburdening, since we now have four channels *ex_coffee, in_coffee, done_in_coffee, done_ex_coffee* instead of having one channel for *coffee*.

Figure 4.8 shows how the π-OZ specification of VM implements the non-atomic reaction concept. In the interface part it defines the needed channels. For *coffee* four channels: one external, one internal and two for done signaling. The internal channels *in_coffee*, *done_in_coffee* are invisible outside VM. Thus, we need to extend OZ with a new construct *chan local* to define local channels. The local channel is like the *new* operator in the π-calculus, i.e., restriction as follows: $VM = new\ in\_coffee, done\_in\_coffee...(VM\_PI \mid VM\_OZ\_PI)$. For *tea* and *talk* the same is done like *coffee*. The behavior sequence part $VM\_PI = ex\_coffee().\overline{in\_coffee}\langle\rangle$ $.done\_in\_coffee()....$ reflects exactly the numbered arrows shown in Figure 4.7.

The π-OZ specification of VM reads: the combination is ready to participate in an *ex_coffee* action issued by the environment. On receiving a signal via *ex_coffee*, the π part will make a transition and issue a signal via *in_coffee* enforcing the OZ part to make a transition specified with the operation schema *in_coffee*. When the OZ part ends its transition it sends a signal via *done_in_coffee* enforcing the π part to make a transition, and finally the π part issues a done signal via *done_ex_coffee* to the environment declaring that ordering a coffee has done successfully. Notice that the specification has a schema for *in_coffee* which represents the conditions on the data, and there no schemes for *ex_coffee*, *done_ex_coffee* and *done_in_coffee*, since they serve for orchestrating.

Figure 4.9 and Figure 4.10 show the π-OZ specification of *ActiveShop* and *IdleShop* respectively, using the non-atomic reaction concept. Figure 4.11 shows a big picture of a system consisting of a customer, vending machine and two shops.

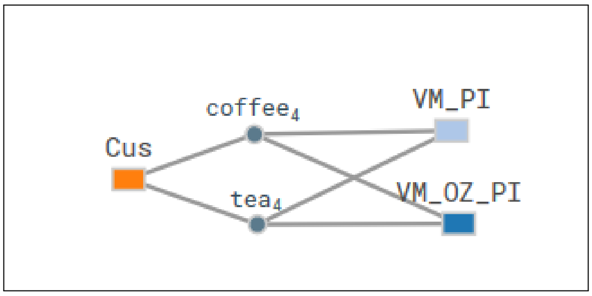


Figure 4.6: The process $Cus \mid VM\_OZ\_PI \mid VM\_PI$.

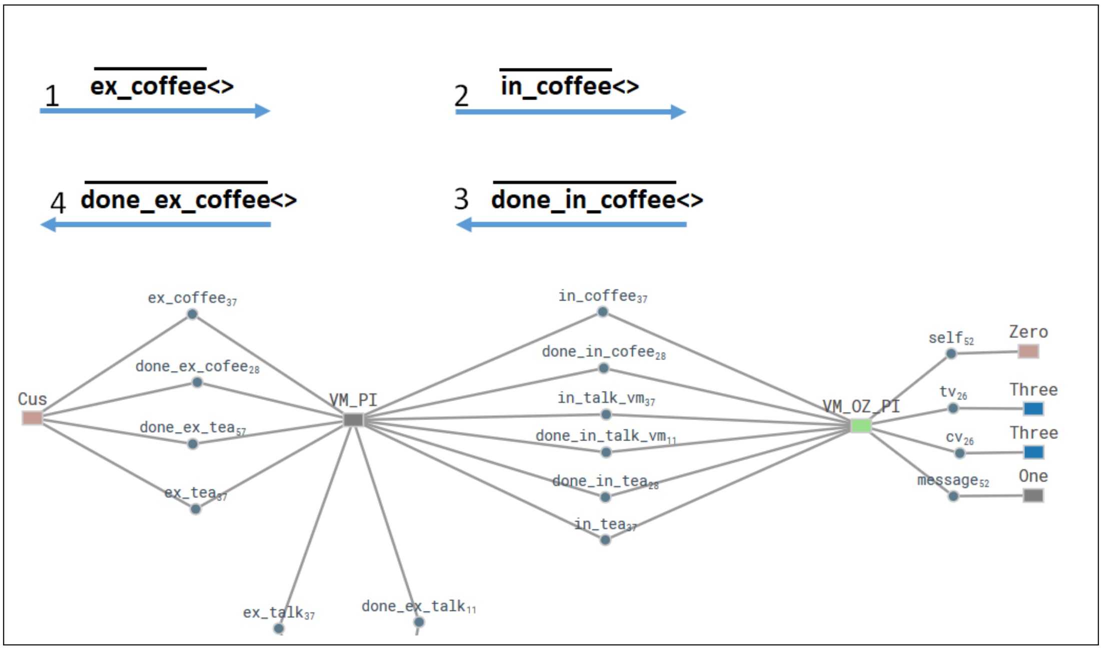


Figure 4.7: Action reproducing and non-atomic reaction.

$VM(id : \mathbb{Z})$

$$
\begin{array}{l}
chan\ ex\_coffee, done\_ex\_coffee, ex\_tea, done\_ex\_tea \\
chan\ local\ in\_coffee, done\_in\_coffee, in\_tea, done\_in\_tea \\
chan\ ex\_talk : \mathbb{Z} \times \mathbb{Z} \\
chan\ local\ in\_talk : \mathbb{Z} \times \mathbb{Z} \\
chan\ done\_ex\_talk \\
chan\ local\ done\_in\_talk
\end{array}
$$

$$
\begin{array}{rl}
VM\_PI = & ex\_coffee().\overline{in\_coffee}\langle\rangle \\
& \quad .done\_in\_coffee().\overline{done\_ex\_coffee}\langle\rangle.VM\_PI \\
+ & ex\_tea().\overline{in\_tea}\langle\rangle \\
& \quad .done\_in\_tea().\overline{done\_ex\_tea}\langle\rangle.VM\_PI \\
+ & in\_talk(y, z).\overline{ex\_talk}\langle y, z\rangle \\
& \quad .done\_ex\_talk().\overline{done\_in\_talk}\langle\rangle.VM\_PI
\end{array}
$$

$$
\begin{array}{l}
self, cv, tv, message : \mathbb{Z} \\
\hline
0 \leq cv \leq 3 \\
0 \leq tv \leq 3
\end{array}
$$

INIT

$$
\begin{array}{l}
self = id \\
cv = 3 \\
tv = 3 \\
message = 1
\end{array}
$$

in_coffee

$$
\begin{array}{l}
\Delta(cv) \\
\hline
cv' = cv - 1
\end{array}
$$

in_tea

$$
\begin{array}{l}
\Delta(tv) \\
\hline
tv' = tv - 1
\end{array}
$$

in_talk

$$
\begin{array}{l}
y! : \mathbb{Z} \\
z! : \mathbb{Z} \\
\hline
y! = message \\
z! = self
\end{array}
$$

Figure 4.8: π-OZ specification of the VM using non-atomic reaction.

$ActiveShop(id : \mathbb{Z})$

$chan\ ex\_switch : nil \mid in\_talk$
$chan\ local\ in\_switch : nil \mid in\_talk$
$chan\ ex\_talk : \mathbb{Z} \times \mathbb{Z}$
$chan\ local\ in\_talk : \mathbb{Z} \times \mathbb{Z}$
$chan\ done\_ex\_talk$
$chan\ local\ done\_in\_talk$

$$
\begin{aligned}
&ActiveShop\_PI = \\
&\quad ex\_talk(y, z).\overline{in\_talk}\langle y, z\rangle \\
&\qquad\qquad .done\_in\_talk().\overline{done\_ex\_talk}\langle\rangle.ActiveShop\_PI \\
&\quad + in\_switch(x).\overline{ex\_switch}\langle x, ex\_talk\rangle.IdleShop\_PI
\end{aligned}
$$

$self, vmId, message : \mathbb{Z}$
$transferableOperation : nil \mid in\_talk$

$\textsc{Init}$
$self = id$
$transferableOperation = in\_talk$

in_switch $then\ IdleShop$
$x! : nil \mid talk$
$x! = transferableOperation$
$transferableOperation' = nil$

in_talk
$\Delta(vmId, message)$
$y?, z? : \mathbb{Z}$
$y? = message'$
$z? = vmId'$

Figure 4.9: π-OZ specification of the *ActiveShop* using non-atomic reaction.

$$
\begin{array}{|l}
\hline
\multicolumn{1}{l}{\textit{IdleShop}(id : \mathbb{Z})} \\
\\
chan\ ex\_switch : nil \mid talk \\
chan\ local\ in\_switch : nil \mid talk \\
\\
IdleShop\_PI = \\
\qquad ex\_switch(x, ex\_talk).\overline{in\_switch}\langle x \rangle.ActiveShop\_PI \\
\\
\quad \begin{array}{|l}
\hline
self, vmId, message : \mathbb{Z} \\
transferableOperation : nil \mid talk \\
\hline
\end{array} \\
\\
\quad \begin{array}{|l}
\multicolumn{1}{l}{\textsc{Init}} \\
\hline
self = id \\
transferableOperation = nil \\
\hline
\end{array} \\
\\
\quad \begin{array}{|l}
\multicolumn{1}{l}{in\_switch \_\_\ then\ ActiveShop} \\
\hline
\Delta(transferableOperation) \\
x? : nil \mid talk \\
\hline
x? = transferableOperation' \\
\hline
\end{array} \\
\hline
\end{array}
$$

Figure 4.10: π-OZ specification of the *IdleShop* using non-atomic reaction.

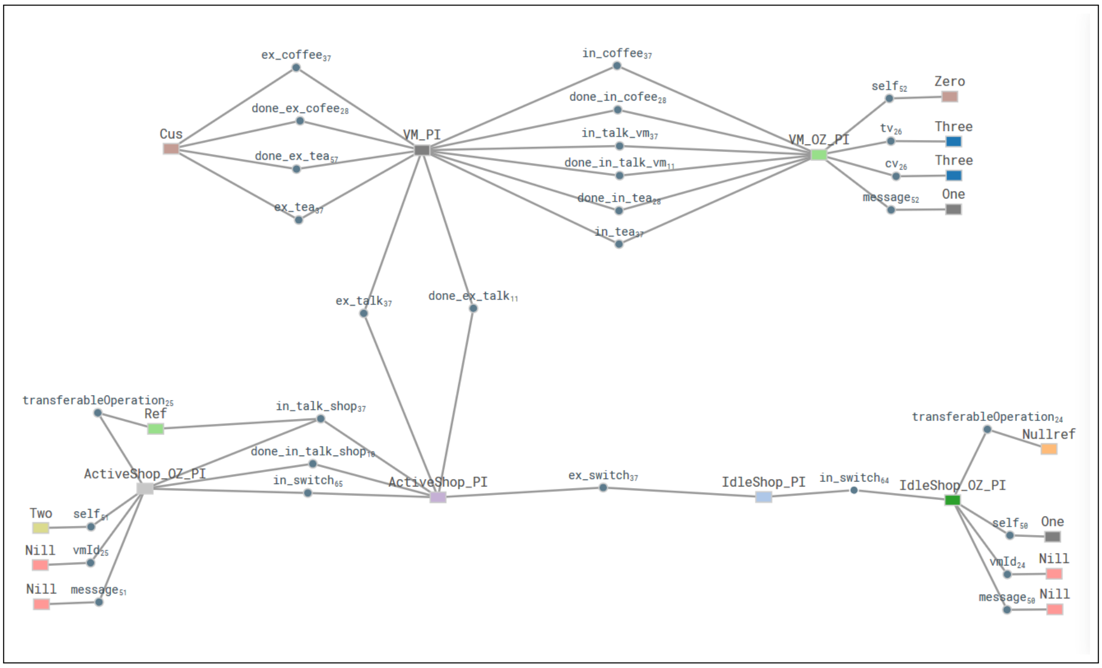


Figure 4.11: System consisting of a customer, vending machine and two shops.

Additionally, Listing 4.1 shows the direct implementation of the π-part of *VM* specification shown in Figure 4.8 using ABC code. We can notice that: ex_coffee.'in_coffee.done_in_coffee.'done_ex_coffee.VM_PI... exactly reflects the π-part of Figure 4.8, where the parameters are removed for clarity.

```
// The process VM_PI represents the π−part of VM.
agent VM_PI(ex_coffee ,in_coffee ,ex_tea ,in_tea ,done_ex_cofee ,
   done_ex_tea ,done_in_cofee ,done_in_tea ,ex_talk ,in_talk ,
   done_ex_talk ,done_in_talk) =

 ex_coffee.'in_coffee.done_in_cofee.'done_ex_cofee.VM_PI(
    ex_coffee ,in_coffee ,ex_tea ,in_tea ,done_ex_cofee ,
    done_ex_tea ,done_in_cofee ,done_in_tea ,ex_talk ,in_talk ,
    done_ex_talk ,done_in_talk)

+  ex_tea.'in_tea.done_in_tea.'done_ex_tea.VM_PI(ex_coffee ,
   in_coffee ,ex_tea ,in_tea ,done_ex_cofee ,done_ex_tea ,
   done_in_cofee ,done_in_tea ,ex_talk ,in_talk ,done_ex_talk ,
   done_in_talk)
```

```
+   in_talk(r_c,m_c).'ex_talk<r_c,m_c>.done_ex_talk.'
    done_in_talk.VM_PI(ex_coffee,in_coffee,ex_tea,in_tea,
    done_ex_cofee,done_ex_tea,done_in_cofee,done_in_tea,
    ex_talk,in_talk,done_ex_talk,done_in_talk)
```

Listing 4.1: VM (π-part) in ABC code.

Listing 4.2 shows the direct implementation of the OZ-part of *VM* specification shown in Figure 4.8 using ABC code.

```
/* The process VM_OZ_PI_Init mimics the intitial state schema
     of the OZ-part of VM.*/
agent VM_OZ_PI_Init(in_coffee,in_tea,done_in_cofee,
    done_in_tea,in_talk,done_in_talk) =
  (^self,cv,tv,message) (VM_OZ_PI(self,in_coffee,in_tea,
      done_in_cofee,done_in_tea,in_talk,done_in_talk,cv,tv,
      message) | Zero(self) | Three(cv) | Three(tv) | One(
      message))

/* The process VM_OZ_PI mimic the behaviour of the OZ-part
     of VM.*/
agent VM_OZ_PI(self,in_coffee,in_tea,done_in_cofee,
    done_in_tea,in_talk,done_in_talk,cv,tv,message) =

  in_coffee.(^res_t,res_f) (VM_Condition_coffee(self,
      in_coffee,in_tea,in_talk,cv,tv,message,res_t,res_f) |
      VM_Condition_IF_Else_coffee(self,in_coffee,in_tea,
      done_in_cofee,done_in_tea,in_talk,done_in_talk,cv,tv,
      message,res_t,res_f))

 + in_tea.(^res_t,res_f) (VM_Condition_tea(self,in_coffee,
      in_tea,in_talk,cv,tv,message,res_t,res_f) |
      VM_Condition_IF_Else_tea(self,in_coffee,in_tea,
      done_in_cofee,done_in_tea,in_talk,done_in_talk,cv,tv,
      message,res_t,res_f))

 + (^m_c,m_done,r_c,r_done) ( (m_done.r_done.'in_talk<r_c,
```

```
m_c>.done_in_talk.VM_OZ_PI(self,in_coffee,in_tea,
 done_in_cofee,done_in_tea,in_talk,done_in_talk,cv,tv,
 message)) | Copy(message,m_c,m_done) | Copy(self,r_c,
 r_done))
```

Listing 4.2: VM (OZ-part) in ABC code.

Listing 4.3 shows the direct implementation of the π-part | OZ-part of *VM* specification shown in Figure 4.8 using ABC code.

```
// The process VM represents the combination π–part | OZ–part
agent VM(ex_coffee,ex_tea,done_ex_cofee,done_ex_tea,ex_talk,
    done_ex_talk) =
(^in_coffee,in_tea,done_in_cofee,done_in_tea,in_talk,
    done_in_talk)
(
 VM_OZ_PI_Init(in_coffee,in_tea,done_in_cofee,done_in_tea,
     in_talk,done_in_talk)
|
 VM_PI(ex_coffee,in_coffee,ex_tea,in_tea,done_ex_cofee,
     done_ex_tea,done_in_cofee,done_in_tea,ex_talk,in_talk,
     done_ex_talk,done_in_talk)
)
```

Listing 4.3: the combination π-OZ of VM in ABC code.

Listing 4.4 shows a part of the direct implementation of the system consisting of a customer, vending machine, and two shops shown in Figure 4.11 using ABC code. For the full code please see the appendix.

```
agent System = (^ex_coffee,done_ex_cofee,in_coffee,
    done_in_cofee,ex_tea,done_ex_tea,in_tea,done_ex_tea,
    done_in_tea,ex_switch,in_switch,ex_talk,in_talk_vm,
    in_talk_shop,done_ex_talk,done_in_talk_vm,
    done_in_talk_shop)


(
VM(ex_coffee,ex_tea,done_ex_cofee,done_ex_tea,ex_talk,
    done_ex_talk)
```

```
|
Cus(ex_coffee,ex_tea,done_ex_cofee,done_ex_tea)
|
IdleShop(ex_switch)
|
ActiveShop(ex_switch,ex_talk,done_ex_talk)
)
```

Listing 4.4: the system consisting of: customer, vending machine and two shops in ABC code.

5 Refinement

Due to performance issues, we will limit the refinement check to π processes without data. Additionally, we only consider processes without the *new* operator[1]. To study the refinement of π-calculus processes we will use the results of the big-step trace semantics defined in [Gie14], with some simplifications. The main result in [Gie14] is the following property:

$$([P], [Q]) \in \mathcal{S} \Rightarrow [Q] \sqsubseteq_{\mathcal{T}} [P] \tag{5.1}$$

We ignore the concept of equivalence classes. Thus:

$$(P, Q) \in \mathcal{S} \Rightarrow Q \sqsubseteq_{\mathcal{T}} P \tag{5.2}$$

Property 5.2 reads: Q strongly simulates P implies P refines Q in trace model, where:

$$Q \sqsubseteq_{\mathcal{T}} P \Leftrightarrow \mathcal{T}(P) \subseteq \mathcal{T}(Q)$$

$$\mathcal{T}(P) =_{\text{def}} \{t \in \texttt{Traces} \mid \exists\, Q \in \mathcal{P}^{\pi} : P \overset{t}{\Rightarrow} Q\}$$

$$\texttt{Traces} =_{\text{def}} \texttt{seq}(\texttt{Act} \setminus \{\tau\})$$

$$\texttt{Act} =_{\text{def}} \texttt{Out} \cup \texttt{In} \cup \{\tau\}$$

$$\texttt{Out} =_{\text{def}} \{\overline{x}\langle\vec{y}\rangle \mid x \in \mathcal{N}\}$$

$$\texttt{In} =_{\text{def}} \{x(\vec{y}) \mid x \in \mathcal{N}\}$$

However, the work in [Gie14] was limited to recursion-free processes: “The limitation to recursion-free processes depends on the circumstances that we neither have any fix-point algorithm up to now nor showed that one exists”[2]. In this work we assume the existence of a fix-point algorithm and that Property 5.1 also applies to recursive processes.

[1] Processes transparent to the environment.
[2] [Gie14], page 80.

Since the trace refinement does not say too much about the behavior of processes, we propose to use the Failure-Refinement model, originally introduced for CSP. In the next section we will define the failure-refinement for π-calculus processes and show that the strong simulation does not imply the failure-refinement. Thus, later in Section 5.2 we will introduce the Success-Refinement model for π-calculus processes and show that the strong simulation implies the success-refinement.

5.1 Failure-Refinement

We checked whether the strong simulation implies failure-refinement. We found that this is not the case. We start by defining the failure of a process. The pair (t, X) is called a *failure pair*, where t is a trace and X is a set of refused next actions after t. Formally, $P \overset{t}{\Rightarrow} Q$ and Q ref X. Any process P is assigned a set of failures $\mathcal{F}$. Formally, this means:

$$\mathcal{F}(P) =_{\text{def}} \{(t, X) \mid \exists\, Q \in \mathcal{P}^{\pi} : P \overset{t}{\Rightarrow} Q \wedge Q \text{ ref } X \wedge X \in \mathit{Refusals}\} \tag{5.3}$$

where: $\mathit{Refusals} =_{\text{def}} \mathbb{P}(\texttt{Act})$.
The special inclusion of failures sets is defined as follows:

Definition 5.1.1 (special inclusion of failures set) Let $P, Q \in \mathcal{P}^{\pi}$, then:

$$\begin{aligned} \mathcal{F}(P)\ddot{\subseteq}\mathcal{F}(Q) \Leftrightarrow\ & \forall (t_P, Y_P) \in \mathcal{F}(P) \text{ then} \\ & \exists (t_Q, Y_Q) \in \mathcal{F}(Q) : t_p = t_Q \wedge Y_P \subseteq Y_Q \end{aligned}$$

△

Notice, we are neither focusing on how to calculate the $\mathcal{T}(P)$, since it has been studied in [Gie14] nor on how to calculate the refusal set X of a trace t, since the failure-refinement is not useful as we will show later in Remark 5.1.1. Later in Section 5.2 we will focus on calculating the acceptance set of a trace. For now, let us go ahead and define the failure-refinement of π-calculus processes as follows:

Definition 5.1.2 (Failure refinement) Let $P, Q \in \mathcal{P}^{\pi}$, then P is a *failure refinement* of Q iff the inverse set inclusion of traces and failures holds:

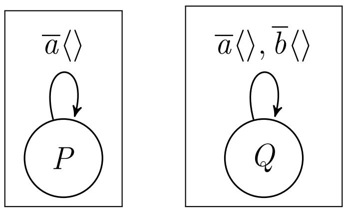
$$Q \sqsubseteq_{\mathcal{F}} P \Leftrightarrow \mathcal{T}(P) \subseteq \mathcal{T}(Q) \wedge \mathcal{F}(P)\ddot{\subseteq}\mathcal{F}(Q)$$ △

From Property 5.2 and Definition 5.1.2 we can drive the following remark:

Remark 5.1.1 (Simulation and Failure refinement) *Let $P, Q \in \mathcal{P}^{\pi}$. If Q strongly simulates P, this does not imply that P refines Q in Failure-Refinement model. Formally written:*

$$(P, Q) \in \mathcal{S} \not\Rightarrow Q \sqsubseteq_{\mathcal{F}} P \qquad \square$$

Proof: by counter-example. Assume that $(P, Q) \in \mathcal{S} \Rightarrow Q \sqsubseteq_{\mathcal{F}} P$ and let $P =_{\text{def}} \overline{a}\langle\rangle.P$ and $Q =_{\text{def}} \overline{a}\langle\rangle.Q + \overline{b}\langle\rangle.Q$, shown in Figure 5.1.



(a) (b)

Figure 5.1: P and Q.

It is clear that Q strongly simulates P. This should imply, according to our assumption, that P refines Q in the failure model, thus we need to show that $\mathcal{T}(P) \subseteq \mathcal{T}(Q) \wedge \mathcal{F}(P) \ddot{\subseteq} \mathcal{F}(Q)$.

- For $\mathcal{T}(P) \subseteq \mathcal{T}(Q)$:

$$\mathcal{T}(P) =_{\text{def}} \{a()\}^*$$

$$\mathcal{T}(Q) =_{\text{def}} \{a(), b()\}^*$$

It is clear that $\mathcal{T}(P) \subseteq \mathcal{T}(Q)$ holds.

- For $\mathcal{F}(P) \ddot{\subseteq} \mathcal{F}(Q)$:

$$\mathcal{F}(P) =_{\text{def}} \{(\langle\rangle, \{b()\}), \dots\}$$

$$\mathcal{F}(Q) =_{\text{def}} \{(\langle\rangle, \{\}), \dots\}$$

It is clear that $\mathcal{F}(P) \ddot{\not\subseteq} \mathcal{F}(Q)$, since $\{b()\} \not\subseteq \{\}$ for $\langle\rangle$. Thus $Q \not\sqsubseteq_{\mathcal{F}} P$. $\blacksquare$

So, strong simulation does not imply failure-refinement. Thus, in the next section we will introduce the Success-Refinement model and use it instead of the Failure-Refinement model.

5.2 Success-Refinement

To compare π-calculus processes we define the *s*uccess refinement and relate it to the strong simulation. We start by defining the *success pair* of a process.

The pair (t, Y) is called a *success pair*, where t is a trace and Y is a set of acceptable next actions after t. Formally, $P \stackrel{t}{\Rightarrow} Q$ and Q acp Y. Any process P is assigned a set of success pairs $\mathcal{S}uc$. Formally, this means:

$$\mathcal{S}uc(P) =_{\text{def}} \{(t, Y) \mid \exists\, Q \in \mathcal{P}^{\pi} : P \stackrel{t}{\Rightarrow} Q \wedge Q \text{ acp } Y \wedge Y \in \mathit{Acceptances}\} \tag{5.4}$$

where: $\mathit{Acceptances} =_{\text{def}} \mathbb{P}(\mathtt{Act})$.
The special inclusion of success sets is defined as follows:

Definition 5.2.1 (special inclusion of success set) Let $P, Q \in \mathcal{P}^{\pi}$, then:

$$\begin{aligned} \mathcal{S}uc(P)\ddot{\subseteq}\mathcal{S}uc(Q) \Leftrightarrow \forall(t_P, Y_P) \in \mathcal{S}uc(P) \ \mathit{then} \\ \exists(t_Q, Y_Q) \in \mathcal{S}uc(Q) : t_p = t_Q \wedge Y_P \subseteq Y_Q \end{aligned}$$

△

To determine the acceptance set Y we need to collect all the possible output, input, and τ actions after the substitution of the bound names of the input actions. That requires collecting the available actions before the substitution of the bound names of the input actions.

To collect the available actions before the substitution of the bound names of the input actions we need to collect the available output actions $Av_{\mathtt{Out}}$, collect the available input actions $Av_{\mathtt{In}}$, and the available τ actions Av_{τ} as follows:

$$Av_{\mathtt{Out}} =_{\text{def}} \{\alpha \in \mathtt{Out} \mid \alpha \text{ is a syntatically possible next action in P}\}$$

$$Av_{\mathtt{In}} =_{\text{def}} \{\alpha \in \mathtt{In} \mid \alpha \text{ is a syntatically possible next action in P}\}$$

$$Av_{\tau} =_{\text{def}} \{\tau \mid \tau \text{ is a syntatically possible next action in P}\}$$

Now, we can collect all the possible actions after the substitution of bound names of the input actions, where $Acc_{\mathtt{Out}}$ is the set of the possible output actions, $Acc_{\mathtt{In}}$ is the set of the possible input actions, and Acc_{τ} is the set of the possible the τ actions defined as follows:

$$Acc_{\mathtt{Out}} =_{\text{def}} Av_{\mathtt{Out}}$$

$$\begin{aligned} Acc_{\texttt{In}} =_{\text{def}} \{\alpha \in \texttt{In} \mid \exists\, \beta \in Av_{\texttt{In}} \wedge \texttt{sub}(\alpha) = \texttt{sub}(\beta) \wedge \alpha = \{\vec{y}/\vec{z}\}\, \beta \\ \text{where } \vec{z} = \texttt{obj}(\beta) \wedge \vec{y} = \texttt{obj}(\alpha)\} \end{aligned}$$

$$\begin{aligned} Acc_{\tau} =_{\text{def}} \{\tau \mid \forall\, \alpha \in Acc_{\texttt{Out}} \text{ if } \exists\, \beta \in Acc_{\texttt{In}} : \texttt{sub}(\alpha) = \texttt{sub}(\beta) \wedge \texttt{obj}(\alpha) = \texttt{obj}(\beta)\} \\ \cup\, Av_{\tau} \end{aligned}$$

$\texttt{sub}(\alpha)$: returns the channel name through which the exchange occurs

$\texttt{obj}(\alpha)$: return the exchanged names across the channel

Finally, we can determine Y as follows:

$$Y =_{\text{def}} Acc_{\tau} \cup Acc_{\texttt{Out}} \cup Acc_{\texttt{In}}$$

Let us take an example to illustrate our approach of calculating the acceptance set of a trace. Let $P =_{\text{def}} (a(x).\overline{x}\langle a\rangle \mid b(y)).P$. Figure 5.2 shows $\mathcal{T}(P)$ and how to calculate the acceptance set Y of the empty trace $\langle\rangle$. P has only two syntactically available actions $a(x)$ and $b(d)$ as shown in the set $Av_{\texttt{In}}$. Once one of them occurs, its bound name must be substituted with the received name. This substitution is shown in $Acc_{\texttt{In}}$ for all possible received names. Finally, the acceptance set Y of $\langle\rangle$ is the union of $Acc_{\texttt{Out}}$, $Acc_{\texttt{In}}$ and Acc_{τ}. The sets $Av_{\texttt{Out}}$ and Av_{τ} are empty, because P can neither make an output action nor τ action. Thus, $Acc_{\texttt{Out}}$ and Acc_{τ} are empty too.

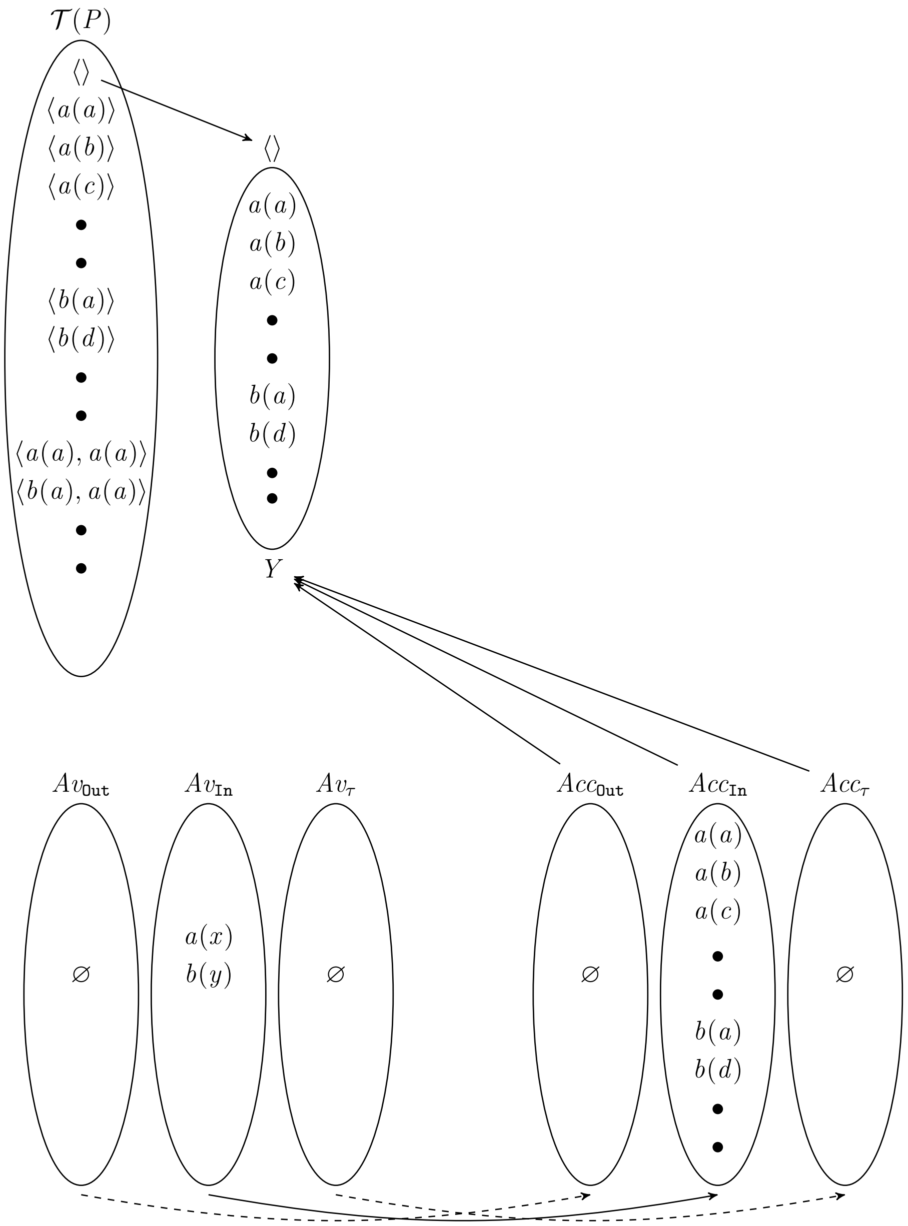


Figure 5.2: Calculating the acceptance set Y of the trace $\langle\rangle$ of the processes P.

We can now define the success-refinement of π-calculus processes as follows:

Definition 5.2.2 (Success refinement) Let $P, Q \in \mathcal{P}^{\pi}$, then P is a success refinement of Q iff the inverse set inclusion of traces and success holds:

$$Q \sqsubseteq_{\mathcal{S}uc} P \Leftrightarrow \mathcal{T}(P) \subseteq \mathcal{T}(Q) \wedge \mathcal{S}uc(P) \ddot{\subseteq} \mathcal{S}uc(Q) \qquad \triangle$$

From Property 5.2 and Definition 5.2.2 we can drive the following Corollary:

Corollary 5.2.1 (Simulation and Success refinement) *Let $P, Q \in \mathcal{P}^{\pi}$. If Q strongly simulates P, then P refines Q in Success-Refinement model. Formally written:*

$$(P, Q) \in \mathcal{S} \Rightarrow Q \sqsubseteq_{\mathcal{S}uc} P$$

holds. □

Proof: [3] Let $(P, Q) \in \mathcal{S}_v$, then $\mathcal{T}(P) \subseteq \mathcal{T}(Q) \wedge \mathcal{S}uc(P) \subseteq \mathcal{S}uc(Q)$ holds, Since:

- For $\mathcal{T}(P) \subseteq \mathcal{T}(Q)$: it holds using Property 5.2.
- For $\mathcal{S}uc(P) \ddot{\subseteq} \mathcal{S}uc(Q)$: we need to show that,
  $\forall (t_P, Y_P) \in \mathcal{S}uc(P)$ *then* $\exists (t_Q, Y_Q) \in \mathcal{S}uc(Q) : t_p = t_Q \wedge Y_P \subseteq Y_Q$
  - $t_p = t_Q$ holds using Property 5.2.
  - $Y_P \subseteq Y_Q$ holds, since Q strongly simulates P means that Q can do any the actions that P can do respecting the use of the same free and bound names, what implies that Q can do any trace t that P does. This implies that after any trace t, Q accepts all the actions that P accepts. This means that the acceptance set of P is included in the acceptance set of Q after any trace t. Formally, $Y_P \subseteq Y_Q$ for any trace t. ■

Success-Refinement use case

Let us take an example to inspect if Corollary 5.2.1 and our approach of calculating the acceptance set works. Let:

$$P =_{\text{def}} (a(x).\overline{x}\langle a \rangle \mid b(y)).P$$

$$Q =_{\text{def}} (a(x).\overline{x}\langle a \rangle \mid b(y)).Q + \tau.Q$$

It is clear that Q strongly simulates P. This result implies, according to Corollary 5.2.1, that P refines Q in the success-refinement model. In other words, after a common trace t, the acceptance set Y_P of the process P is included in the acceptance set Y_Q of the process Q. More formally:

$$\forall (t, Y_P) \in \mathcal{S}uc(P) \text{ then } \exists (t, Y_Q) \in \mathcal{S}uc(Q) : Y_P \subseteq Y_Q$$

[3] Our proof is not complete and can be enhanced in later works.

We will show that this holds for the empty trace $\langle\rangle$ using our approach of determining the acceptance set Y. This can be seen in Figure 5.2 and Figure 5.3. We can notice that there are no differences between $\mathcal{T}(P)$ and $\mathcal{T}(Q)$, since the τ action that Q can do, cannot appear in the trace. But τ appears in the acceptance set Y of the empty trace $\langle\rangle$ of Q shown in Figure 5.3. In the same manner, we can determine Y_P and Y_Q for any trace t and see that $Y_P \subseteq Y_Q$ holds. Thus, $\mathcal{S}uc(P)\ddot{\subseteq}\mathcal{S}uc(Q)$ holds and so $Q \sqsubseteq_{\mathcal{S}uc} P$.

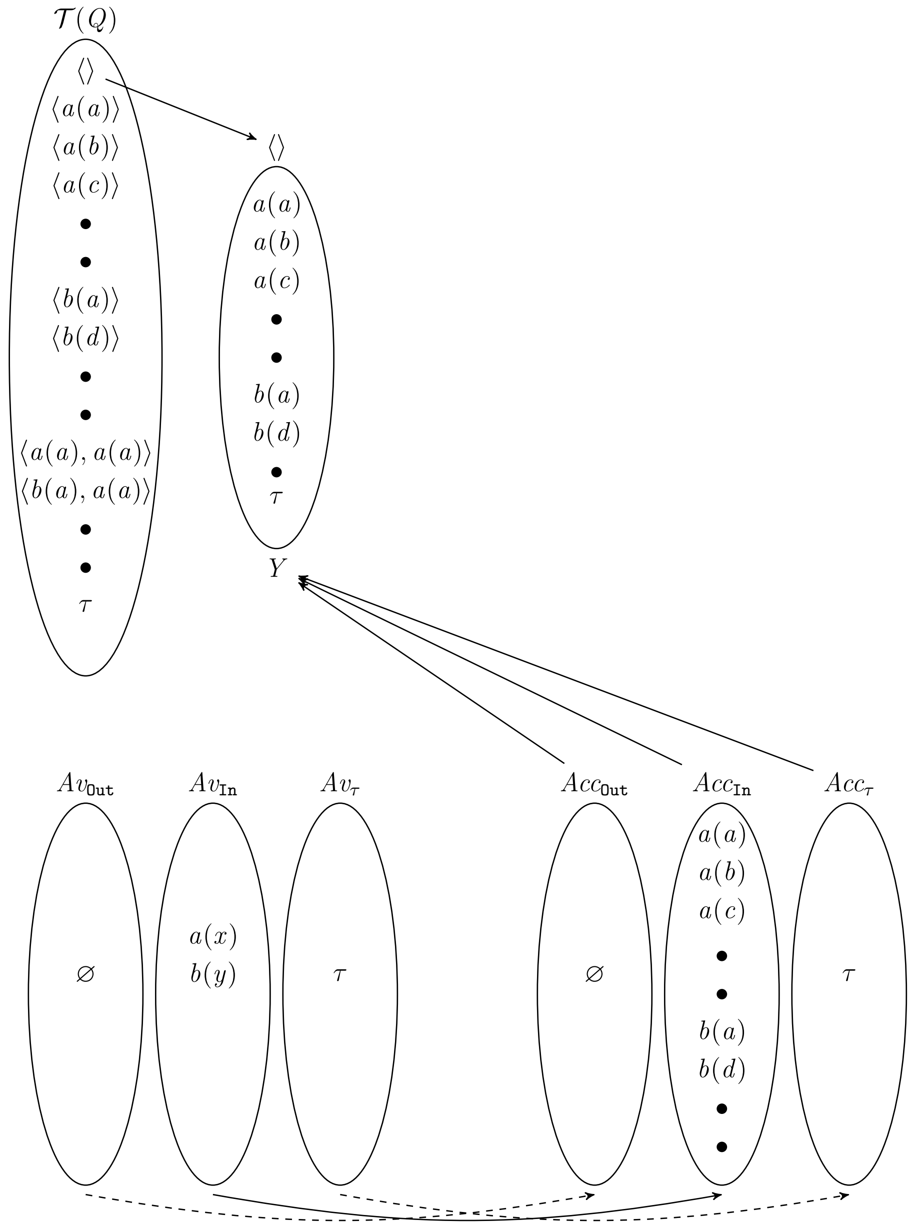


Figure 5.3: Calculating the acceptance set Y of the trace $\langle\rangle$ of the processes Q.

6 Conclusion and future work

In this thesis we investigated the transformational semantics of the combination π-OZ for mobile processes with data.

The aim of this work is to combine the OZ specification with π-calculus specification, and to transform the combination into a π-calculus processes, similarly to the CSP-OZ [Old19] approach. Unfortunately, we found out that the transformation is cumbersome, since the π-calculus has only elementary constructs and is not suitable to express complex constructs like OZ class constructs. On the one hand, we showed how to integrate a π-calculus process, describing the desired sequence of behavior, into an OZ class to form the π-OZ combination. On the other hand, we explained how to transform the combination π-OZ into a π-calculus process through transforming OZ class constructs value, state variable, state schema, initial state schema and operation schema into π-calculus processes and names accompanied by the processes of the desired sequence of behavior using the parallel operator. In spite of that, we introduced the Failure-Refinement model for π-calculus and showed that the strong simulation does not imply the failure-refinement. Finally, we introduced the Success-Refinement model and showed that the strong simulation implies success-refinement.

The elementary nature of the π-calculus is a good start for future work through introducing a state-full π-calculus, which is an extension of the π-calculus to support data, data types, variable and transition conditions. This will ease the mapping between OZ and π-calculus constructs. Additionally, a tool can be developed to visualize the state-full π-calculus in a similar way to Stargazer [D'O]. Furthermore, it would be good to extend the ABC simulation-checker [Bri] to support simulation checking and success-refinement verification of the state-full π-calculus. Finally, it would be interesting to extend [Gie14] trace semantics to support recursive processes through developing a fixed-point algorithm for the recursive processes. This will facilitate the study of the composition properties of the Success-Refinement model, which will permit to study the possibility of introducing the Success-Divergence model and to explore its composition properties.

7 Appendix

7.1 Addition

```
agent Zero(a) = a(tt,ff).'ff.'ff.Zero(a) + a.0
agent One(a) = a(tt,ff).'tt.'ff.One(a) + a.0
agent Two(a) = a(tt,ff).'ff.'tt.Two(a) + a.0
agent Three(a) = a(tt,ff).'tt.'tt.Three(a) + a.0
```

Listing 7.1: 0,1,2,3 as π-calculus processes.

```
agent FullAdderWait(t1,f1,t2,f2,cin_t,cin_f,cout3_t,cout3_f,
    s3_t,s3_f) = \
cin_t.('cin_t | FullAdder(t1,f1,t2,f2,cin_t,cin_f,cout3_t,
    cout3_f,s3_t,s3_f)) \
+ cin_f.('cin_f | FullAdder(t1,f1,t2,f2,cin_t,cin_f,cout3_t,
    cout3_f,s3_t,s3_f))

agent FullAdder(t1,f1,t2,f2,cin_t,cin_f,cout_t,cout_f,s2_t,
    s2_f) = \
(^t1a,f1a,t1b,f1b,t2a,f2a,t2b,f2b,c1_t,c1_f,s1_t,s1_f,c2_t,
    c2_f,s1_ta,s1_fa,s1_tb,s1_fb,cin_ta,cin_fa,cin_tb,cin_fb)
     \
( \
HalfAdder(t1,f1,t1a,f1a,t1b,f1b,t2,f2,t2a,f2a,t2b,f2b,c1_t,
    c1_f,s1_t,s1_f) \
| HalfAdder(s1_t,s1_f,s1_ta,s1_fa,s1_tb,s1_fb,cin_t,cin_f,
    cin_ta,cin_fa,cin_tb,cin_fb,c2_t,c2_f,s2_t,s2_f) \
| OR(c1_t,c1_f,c2_t,c2_f,cout_t,cout_f) \
)
```

```
agent HalfAdder(t1,f1,t1a,f1a,t1b,f1b,t2,f2,t2a,f2a,t2b,f2b,
    c_t,c_f,s_t,s_f) = (Repeate(t1,f1,t1a,f1a,t1b,f1b) |
    Repeate(t2,f2,t2a,f2a,t2b,f2b) | AND(t1a,f1a,t2a,f2a,c_t,
    c_f) | XOR(t1b,f1b,t2b,f2b,s_t,s_f))

agent AND(t1,f1,t2,f2,o_t,o_f) = f1.(f2.'o_f + t2.'o_f) + t1
    .(f2.'o_f + t2.'o_t)
agent NAND(t1,f1,t2,f2,o_t,o_f) = f1.(f2.'o_t + t2.'o_t) +
    t1.(f2.'o_t + t2.'o_f)
agent OR(t1,f1,t2,f2,o_t,o_f) = f1.(f2.'o_f + t2.'o_t) + t1
    .(f2.'o_t + t2.'o_t)
agent NOR(t1,f1,t2,f2,o_t,o_f) = f1.(f2.'o_t + t2.'o_f) + t1
    .(f2.'o_f + t2.'o_f)
agent XOR(t1,f1,t2,f2,o_t,o_f) = f1.(f2.'o_f + t2.'o_t) + t1
    .(f2.'o_t + t2.'o_f)
agent XNOR(t1,f1,t2,f2,o_t,o_f) = f1.(f2.'o_t + t2.'o_f) +
    t1.(f2.'o_f + t2.'o_t)

agent Send(a) = 'a
agent Neg(tt,ff) = tt.'ff + ff.'tt
agent Repeate(tt,ff,ta,fa,tb,fb) = tt.('ta | 'tb) + ff.('fa
    | 'fb)
```

Listing 7.2: Gates.

```
agent Example = (^a,b,c) (Two(a) | One(b) | Add(a,b,c))

agent Add(a,b,c)= (^t1,f1,t2,f2) ('a<t1,f1>.'b<t2,f2>.(^
    cin_t,cin_f,cout_t,cout_f,s2_t,s2_f,cout3_t,cout3_f,s3_t,
    s3_f) ( \
FullAdderWait(t1,f1,t2,f2,cin_t,cin_f,cout_t,cout_f,s2_t,
    s2_f) \
| 'cin_f \
| FullAdderWait(t1,f1,t2,f2,cout_t,cout_f,cout3_t,cout3_f,
    s3_t,s3_f) \
| Result(s2_t,s2_f,s3_t,s3_f,cout3_t,cout3_f,c) \
```

```
))

agent Result(s0_t,s0_f,s1_t,s1_f,c_t,c_f,res) = \
s0_f.(s1_f.(c_f.Zero(res) + c_t.OverFlow) + s1_t.(c_f.Two(
    res)+ c_t.OverFlow)) \
+ s0_t.(s1_f.(c_f.One(res) + c_t.OverFlow) + s1_t.(c_f.Three
    (res)+ c_t.OverFlow))

agent OverFlow = 0
```

Listing 7.3: Adder.

```
agent Example = (^a,b,c) (Three(a) | One(b) | Sub(a,b,c))

(* the trick is to invert t2,f2 places to mimic the Inverter
     *)
agent Sub(a,b,c) = (^t1,f1,t2,f2) ('a<t1,f1>.'b<t2,f2>.(^
    cin_t,cin_f,cout_t,cout_f,s2_t,s2_f,cout3_t,cout3_f,s3_t,
    s3_f) ( \
FullAdderWait(t1,f1,f2,t2,cin_t,cin_f,cout_t,cout_f,s2_t,
    s2_f) \
| 'cin_t \
| FullAdderWait(t1,f1,f2,t2,cout_t,cout_f,cout3_t,cout3_f,
    s3_t,s3_f) \
| Result_S(s2_t,s2_f,s3_t,s3_f,cout3_t,cout3_f,c) \
))

agent Result_S(s0_t,s0_f,s1_t,s1_f,c_t,c_f,res) = \
s0_f.(s1_f.(c_t.Zero(res) + c_f.ErrNegResult ) + s1_t.(c_t.
    Two(res) + c_f.ErrNegResult )) \
+ s0_t.(s1_f.(c_t.One(res) + c_f.ErrNegResult ) + s1_t.(c_t.
    Three(res) + c_f.ErrNegResult))



agent ErrNegResult = 0
```

```
agent FullAdderWait(t1,f1,t2,f2,cin_t,cin_f,cout3_t,cout3_f,
    s3_t,s3_f) = \
cin_t.('cin_t | FullAdder(t1,f1,t2,f2,cin_t,cin_f,cout3_t,
    cout3_f,s3_t,s3_f)) \
+ cin_f.('cin_f | FullAdder(t1,f1,t2,f2,cin_t,cin_f,cout3_t,
    cout3_f,s3_t,s3_f))



agent FullAdder(t1,f1,t2,f2,cin_t,cin_f,cout_t,cout_f,s2_t,
    s2_f) = \
(^t1a,f1a,t1b,f1b,t2a,f2a,t2b,f2b,c1_t,c1_f,s1_t,s1_f,c2_t,
    c2_f,s1_ta,s1_fa,s1_tb,s1_fb,cin_ta,cin_fa,cin_tb,cin_fb)
     \
( \
HalfAdder(t1,f1,t1a,f1a,t1b,f1b,t2,f2,t2a,f2a,t2b,f2b,c1_t,
    c1_f,s1_t,s1_f) \
| HalfAdder(s1_t,s1_f,s1_ta,s1_fa,s1_tb,s1_fb,cin_t,cin_f,
    cin_ta,cin_fa,cin_tb,cin_fb,c2_t,c2_f,s2_t,s2_f) \
| OR(c1_t,c1_f,c2_t,c2_f,cout_t,cout_f) \
)

agent HalfAdder(t1,f1,t1a,f1a,t1b,f1b,t2,f2,t2a,f2a,t2b,f2b,
    c_t,c_f,s_t,s_f) = (Repeate(t1,f1,t1a,f1a,t1b,f1b) |
    Repeate(t2,f2,t2a,f2a,t2b,f2b) | AND(t1a,f1a,t2a,f2a,c_t,
    c_f) | XOR(t1b,f1b,t2b,f2b,s_t,s_f))
```

Listing 7.4: Subtractor.

```

agent Example = (^a,b,g,e,l) (Three(a) | Two(b) | Compare(a,
    b,g,e,l) | If_Else(g,e,l))

agent If_Else(g,e,l) = g.Greater + e.Equal + l.Less

agent Greater = 0
agent Equal = 0
```

```
agent Less = 0

agent Compare(a,b,g,e,l) = (^ta,fa,tb,fb,l_t,l_f,e_t,e_f,g_t
    ,g_f) ('a<ta,fa>.'b<tb,fb>.(^tb1,fb1,tb2,fb2,o_xor_t,
    o_xor_f,o_xor_1t,o_xor_1f,o_xor_2t,o_xor_2f,o_nand_1_t,
    o_nand_1_f,o_nand_1_1t,o_nand_1_1f,o_nand_1_2t,
    o_nand_1_2f)( \
Repeate(tb,fb,tb1,fb1,tb2,fb2) \
| XOR(ta,fa,tb1,fb1,o_xor_t,o_xor_f) \
| Repeate(o_xor_t,o_xor_f,o_xor_1t,o_xor_1f,o_xor_2t,
    o_xor_2f) \
| NAND(tb2,fb2,o_xor_2t,o_xor_2f,o_nand_1_t,o_nand_1_f) \
| Repeate(o_nand_1_t,o_nand_1_f,o_nand_1_1t,o_nand_1_1f,
    o_nand_1_2t,o_nand_1_2f) \
| Compare_3(a,b,l_t,l_f,e_t,e_f,g_t,g_f,ta,fa,tb,fb,
    o_nand_1_1t,o_nand_1_1f,o_nand_1_2t,o_nand_1_2f,o_xor_1t,
    o_xor_1f,g,e,l) \
))


agent Compare_3(a,b,l_t,l_f,e_t,e_f,g_t,g_f,ta,fa,tb,fb,
    o_nand_1_1t,o_nand_1_1f,o_nand_1_2t,o_nand_1_2f,o_xor_1t,
    o_xor_1f,g,e,l) = \
o_nand_1_1t.Compare_4(a,b,l_t,l_f,e_t,e_f,g_t,g_f,ta,fa,tb,
    fb,o_nand_1_2t,o_nand_1_2f,o_xor_1t,o_xor_1f,g,e,l) \
+ o_nand_1_1f.Compare_4(a,b,l_t,l_f,e_t,e_f,g_t,g_f,ta,fa,tb
    ,fb,o_nand_1_2t,o_nand_1_2f,o_xor_1t,o_xor_1f,g,e,l)


agent Compare_4(a,b,l_t,l_f,e_t,e_f,g_t,g_f,ta,fa,tb,fb,
    o_nand_1_2t,o_nand_1_2f,o_xor_1t,o_xor_1f,g,e,l) = (^tb1,
    fb1,tb2,fb2,o_xnor_t,o_xnor_f,o_xnor_1t,o_xnor_1f,
    o_xnor_2t,o_xnor_2f,o_nor_1_t,o_nor_1_f,o_nand_2_t,
    o_nand_2_f,o_nand_2_1t,o_nand_2_1f,o_nand_2_2t,
    o_nand_2_2f,o_nor_2_t,o_nor_2_f,o_xor_2_t,o_xor_2_f,
```

```
    o_nor_2_1t,o_nor_2_1f,o_nor_2_2t,o_nor_2_2f,o_xor_2_1t,
    o_xor_2_1f,o_xor_2_2t,o_xor_2_2f,o_nor_3_t,o_nor_3_f,e2t,
    e2f,l2t,l2f)(\
Repeate(tb,fb,tb1,fb1,tb2,fb2) \
| XNOR(ta,fa,tb1,fb1,o_xnor_t,o_xnor_f) \
| Repeate(o_xnor_t,o_xnor_f,o_xnor_1t,o_xnor_1f,o_xnor_2t,
    o_xnor_2f) \
| NOR(tb2,fb2,o_xnor_1t,o_xnor_1f,o_nor_1_t,o_nor_1_f) \
| NAND(o_xnor_2t,o_xnor_2f,o_nand_1_2t,o_nand_1_2f,
    o_nand_2_t,o_nand_2_f) \
| Repeate(o_nand_2_t,o_nand_2_f,o_nand_2_1t,o_nand_2_1f,
    o_nand_2_2t,o_nand_2_2f) \
| NOR(o_nand_2_1t,o_nand_2_1f,o_xor_1t,o_xor_1f,o_nor_2_t,
    o_nor_2_f) \
| XOR(o_nor_1_t,o_nor_1_f,o_nand_2_2t,o_nand_2_2f,o_xor_2_t,
    o_xor_2_f) \
| Repeate(o_nor_2_t,o_nor_2_f,e_t,e_f,e2t,e2f) \
| Repeate(o_xor_2_t,o_xor_2_f,l_t,l_f,l2t,l2f) \
| NOR(e2t,e2f,l2t,l2f,g_t,g_f) \
| Compare_5(l_t,l_f,e_t,e_f,g_t,g_f,g,e,l) \
)
agent Compare_5(l_t,l_f,e_t,e_f,g_t,g_f,g,e,l) = g_t.
    Compare_6(g,e_f,l_f) + e_t.Compare_6(e,l_f,g_f)+ l_t.
    Compare_6(l,e_f,g_f)

agent Compare_6(a,b,c) = b.c.'a
```

Listing 7.5: Comparator.

```
agent Example = (^a1,a3) (List1(a1) | List2(a3) | Union(a1,
    a3))

agent List1(a1) = (^a0,b0,c0,d0,b1,c1) (Node(a1,b1,a0) | Ref
    (b1,c1) | Three(c1) | Node(a0,b0,d0) | Ref(b0,c0) | Zero(
    c0) | Nil(d0))
agent List2(a3) = (^a2,b2,c2,d1,b3,c3) (Node(a3,b3,a2) | Ref
```

```
    (b3,c3) | Three(c3) | Node(a2,b2,d1) | Ref(b2,c2) | Two(
    c2) | Nil(d1))

agent Union(a1,a3) = AddElement(a1,a3)
agent AddElement(a1,a3) = (^res_t,res_f,o) ( GetValue(a3,o)
    | CheckValue(a1,o,a3))
agent GetValue(r,o) = (^n,c) ('r<n,c>.(c(rv,l).'rv<n,c>.c(v)
    .'o<v,l> + n.'o))
agent CheckValue(k1,o,k2) =  o(v,l).(^res_t,res_f) (In(v,k1,
    res_t,res_f) | Append(v,k1,l,res_t,res_f)) + o
agent Append(v,k1,l,res_t,res_f) = res_t.AppendNo(k1,l) +
    res_f.AppendYes(v,k1,l)
agent AppendNo(a,l) = AddElement(a,l)
agent AppendYes(v,k1,l) = (^a,b,c) (Copy(v,c) | Node(a,b,k1)
     | Ref(b,c) | AddElement(a,l))
agent Copy(a,b) = (^tt,ff) ('a<tt,ff>.(ff.(ff.Zero(b) + tt.
    Two(b)) + tt.(tt.Three(b)+ ff.One(b))))
agent In(a,b,res_t,res_f) = (^n,c) (In_1(a,b,res_t,res_f,n,c
    ))
agent In_1(a,b,res_t,res_f,n,c) = 'b<n,c>.(c(r,l).'r<n,c>.
    In_4(a,b,res_t,res_f,n,c,r,l)  + n.'res_f)
agent In_4(a,b,res_t,res_f,n,c,r,l) = c(v).(^out_t,out_f) (
    IsEqual(a,v,out_t,out_f) | In_5(a,b,res_t,res_f,n,c,r,l,
    out_t,out_f))
agent In_5(a,b,res_t,res_f,n,c,r,l,out_t,out_f) = out_t.'
    res_t + out_f.In_1(a,l,res_t,res_f,n,c)
agent IsEqual(a,b,out_t,out_f) = (^t1,f1,t2,f2) ('a<t1,f1>.'
    b<t2,f2>.(^o_t,o_f) (IsEqual_4(a,b,out_t,out_f,t1,f1,t2,
    f2,o_t,o_f) | CompareBit( t1,f1,t2,f2,o_t,o_f)))
agent IsEqual_4(a,b,out_t,out_f,t1,f1,t2,f2,o_t,o_f) = o_t.(
    IsEqual_5(a,b,out_t,out_f,t1,f1,t2,f2,o_t,o_f) |
    CompareBit( t1,f1,t2,f2,o_t,o_f)) + o_f.IsEqualPassBit( a
    ,b,out_t,out_f,t1,f1,t2,f2)
agent IsEqualPassBit(a,b,out_t,out_f,t1,f1,t2,f2) = f1.(f2.'
    out_f + t2.'out_f) + t1.(f2.'out_f + t2.'out_f)
```

```
agent IsEqual_5(a,b,out_t,out_f,t1,f1,t2,f2,o_t,o_f) = o_t.'
    out_t + o_f.'out_f

agent CompareBit(t1,f1,t2,f2,o_t,o_f) = f1.(f2.'o_t + t2.'
    o_f) + t1.(t2.'o_t + f2.'o_f)

agent Nullref(r) = r(n,c).('n.Nullref(r) + c(m).Ref(r,m) + n
    .Nullref(r))
agent Ref(r,v) = r(n,c).('c<v>.Ref(r,v) + c(m).Ref(r,m) + 'n
    .Nullref(r))
agent Nil(k) = k(n,c).'n.Nil(k)
agent Node(k,v,l) = k(n,c).'c<v,l>.Node(k,v,l)
```

Listing 7.6: Set union.

```
agent Example = (^a1,a5) (List1(a1) | List2(a5) | Subtract(
    a1,a5))

agent List1(a1) = (^a0,b0,c0,d0,b1,c1) (Node(a1,b1,a0) | Ref
    (b1,c1) | Three(c1) | Node(a0,b0,d0) | Ref(b0,c0) | Zero(
    c0) | Nil(d0))
agent List2(a5) = (^a2,b2,c2,d1,a3,b3,c3,a4,b4,c4,b5,c5) (
    Node(a5,b5,a4) | Ref(b5,c5) | Zero(c5) | Node(a4,b4,a3) |
     Ref(b4,c4) | One(c4) | Node(a3,b3,a2) | Ref(b3,c3) |
    Three(c3) | Node(a2,b2,d1) | Ref(b2,c2) | One(c2) | Nil(
    d1))

agent Subtract(a1,a3) = SubtElement(a1,a3)
agent SubtElement(a1,a3) = (^res_t,res_f,o) ( GetValue(a3,o)
     | CheckValue_S(a1,o,a3))

agent GetValue(r,o) = (^n,c) ('r<n,c>.(c(rv,l).'rv<n,c>.c(v)
    .'o<v,l> + n.'o))

agent CheckValue_S(k1,o,k2) =  o(v,l).(^res_t,res_f) (In(v,
    k1,res_t,res_f) | Check_In(v,k1,k2,l,res_t,res_f)) + o
```

```
agent Append(v,k1,l,res_t,res_f) = res_t.AppendNo(k1,l) +
    res_f.AppendYes(v,k1,l)
agent AppendNo(a,l) = AddElement(a,l)
agent AppendYes(v,k1,l) = (^a,b,c) (Copy(v,c) | Node(a,b,k1)
    | Ref(b,c) | AddElement(a,l))

agent Check_In(v,k1,k2,l,res_t,res_f) = res_t.RemoveYes(k1,
    k2,l) + res_f.RemoveNo(k1,l)
agent RemoveNo(k,l) = SubtElement(k,l)
agent RemoveYes(k1,k2,l) = 'k2.RemoveYes_1(k1,k2,l)
agent RemoveYes_1(k1,k2,l) = (^o) (GetValue(l,o) | FixIndex(
    o,k1,k2,l))
agent FixIndex(o,k1,k2,l) = o(v,l1).(^a,b,c,done) (Copy_S(v,
    c,done) | Node(k2,b,l1) | Ref(b,c) | done.'l.SubtElement(
    k1,l1))

agent Copy_S(a,b,done) = (^tt,ff) ('a<tt,ff>.(ff.(ff.(Zero(b
    ) | 'done) + tt.(Two(b) | 'done)) + tt.(tt.(Three(b) | '
    done)+ ff.(One(b) | 'done))))

agent In(a,b,res_t,res_f) = (^n,c) (In_1(a,b,res_t,res_f,n,c
    ))
agent In_1(a,b,res_t,res_f,n,c) = 'b<n,c>.(c(r,l).'r<n,c>.
    In_4(a,b,res_t,res_f,n,c,r,l) + n.'res_f)

agent In_4(a,b,res_t,res_f,n,c,r,l) = c(v).(^out_t,out_f) (
    IsEqual(a,v,out_t,out_f) | In_5(a,b,res_t,res_f,n,c,r,l,
    out_t,out_f))
agent In_5(a,b,res_t,res_f,n,c,r,l,out_t,out_f) = out_t.'
    res_t + out_f.In_1(a,l,res_t,res_f,n,c)

agent IsEqual(a,b,out_t,out_f) = (^t1,f1,t2,f2) ('a<t1,f1>.'
    b<t2,f2>.(^o_t,o_f) (IsEqual_4(a,b,out_t,out_f,t1,f1,t2,
    f2,o_t,o_f) | CompareBit( t1,f1,t2,f2,o_t,o_f)))
```

```
agent IsEqual_4(a,b,out_t,out_f,t1,f1,t2,f2,o_t,o_f) = o_t.(
    IsEqual_5(a,b,out_t,out_f,t1,f1,t2,f2,o_t,o_f) |
    CompareBit( t1,f1,t2,f2,o_t,o_f)) + o_f.IsEqualPassBit( a
    ,b,out_t,out_f,t1,f1,t2,f2)

agent IsEqualPassBit(a,b,out_t,out_f,t1,f1,t2,f2) = f1.(f2.'
    out_f + t2.'out_f) + t1.(f2.'out_f + t2.'out_f)
agent IsEqual_5(a,b,out_t,out_f,t1,f1,t2,f2,o_t,o_f) = o_t.'
    out_t + o_f.'out_f

agent CompareBit(t1,f1,t2,f2,o_t,o_f) = f1.(f2.'o_t + t2.'
    o_f) + t1.(t2.'o_t + f2.'o_f)

agent Nullref(r) = r(n,c).('n.Nullref(r) + c(m).Ref(r,m) + n
    .Nullref(r))
agent Ref(r,v) = r(n,c).('c<v>.Ref(r,v) + c(m).Ref(r,m) + 'n
    .Nullref(r))
agent Nil(k) = k(n,c).'n.Nil(k)
agent Node(k,v,l) = k(n,c).'c<v,l>.Node(k,v,l)
```

Listing 7.7: Set subtraction.

Bibliography

[Bri] Sebastien Briais. Abc: Another bisimilarity checker. http://sbriais.free.fr/tools/abc/.

[D'O] Emanuele D'Osualdo. Stargazer: A π-calculus simulator. http://www.emanueledosualdo.com/stargazer/.

[EM00] Cristian Ene and Traian Muntean. Expressiveness of point-to-point versus broadcast communications. 07 2000.

[Gie14] Manuel Gieseking. *Refinement of π-calculus processes*. PhD thesis, 03 2014.

[KSw88] Steve King, Ib Holm Sorensen, and Jim woodcock. Z: Grammar and concrete and abstract syntaxes (version 2.0). Technical Report PRG68, OUCL, July 1988.

[Mil99] R Milner. *Communicating and mobile systems : the π-calculus*. Cambridge University Press, Cambridge England New York, 1999.

[MPW92] Robin Milner, Joachim Parrow, and David Walker. A calculus of mobile processes, i. *Information and computation*, 100(1):1–40, 1992.

[OD08] Ernst-Rüdiger Olderog and Henning Dierks. *Real-Time Systems: Formal Specification and Automatic Verification*. 01 2008.

[Old19] Ernst-Rüdiger Olderog. *Kombination von Spezifikationstechniken*. Carl von Ossietzky Universität, Oldenburg, 2018/19.

[Smi00] Graeme Smith. *The Object-Z Specification Language*. Springer US, Boston, MA, 2000.

[TD02] Kenji Taguchi and Jin Dong. An overview of mobile object-z. pages 144–155, 10 2002.

[TDC04] Kenji Taguchi, Jin Dong, and Gabriel Ciobanu. Relating pi-calculus to object-z. pages 97–106, 01 2004.

Index

Acknowledgment

Special thanks goes to:

- Prof. Dr. Ernst-Rüdiger Olderog for suggesting the topic of the thesis and for the continuous support.
- M.Sc. Manuel Gieseking for support with π-calculus and providing the latex template.
- Dr. Emanuele D'Osualdo for support with π-calculus.

Erklärung

Hiermit versichere ich, dass ich diese Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe. Außerdem versichere ich, dass ich die allgemeinen Prinzipien wissenschaftlicher Arbeit und Veröffentlichung, wie sie in den Leitlinien guter wissenschaftlicher Praxis der Carl von Ossietzky Universität Oldenburg festgelegt sind, befolgt habe.

Oldenburg, den 18. Juli 2020

____

(Muhammad Ekbal Ahmad)