CSD2301 Lecture
1. Measurements & Vectors

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Outline

- Measurements
- Simple properties of vectors

Quantities and Units

- Experiments require measurements of physical quantities.
- Every measurement gives a number (value depends on units that goes with it.)
- SI (metric) vs Imperial – know how to convert especially when you work in different countries.
- Agree on **certain important basic** physical quantities and standard units.

7 Base Quantities

Quantity	Unit	Definition
Length	m	Distance light traveled in vacuum for 1/299792458 seconds
Mass	kg	Mass of a specific platinum-iridium alloy
Time	s	9192631770 cycles of radiation of cesium-133
Current	A	That flows in 2 parallel wires resulting in force of 2×10^{-7} N / m on the wires.
Temperature	K	1/273.16 of thermodynamic temperature of triple point of water
Amount of substance	mol	That contains equal number of fundamental entities as 0.012 kg of carbon-12
Luminous intensity	cd	Of a source that emits monochromatic radiation of frequency 540×10^{12} Hz and that has a radiant intensity of 1/683 watt/steradian.

Unit Prefixes and Scientific Notation

- Physics and Engineering deal with many orders of magnitude.
- **Prefixes** to some of the SI units simplifies our work.
- Common ones that are used in Physics / Engineering:
  - **kilo – 10^3** (e.g., km), **Mega – 10^6** (e.g., MHz), Giga – 10^9 (e.g., GByte), Tera – 10^{12} (e.g., TByte)
  - **milli – 10^{-3}** (e.g., mW); **micro – 10^{-6}** (e.g., μK), nano – 10^{-9} (e.g., nm), pico – 10^{-12} (e.g., pF), femto – 10^{-15} (e.g., fs).
  - deci – 10^{-1} (e.g., dB), **centi – 10^{-2}** (e.g., cm)

Scientific Notation

- **Scientific notation** may replace the use of prefixes
  - λ = 852 nm = 8.52 x 10^{-7} m
  - ρ = 13 600 kg/m^3 = 1.36 x 10^4 kg/m^3
- Also allows us to know the number of significant figures (S.F)
  - For example: 400 m – what is the precision?
    - If it is the 400 m athletics track : very accurate!
    - If describing distance to MRT station from house – maybe 5 m or even 49 m off.
  - Writing as 4.000 x 10^2 m, 4.00 x 10^2 m, 4.0 x 10^2 or 4 x 10^2 would be a better indication.

Uncertainty

- Uncertainty in measurement depends on
  - the quality of the apparatus
  - skill of the user
  - number of measurements taken
- **Accuracy** (closer to some agreed "true" value) versus **precision** (more significant figures)
- Systematic and Random Errors
  - Systematic: Repeatable with small deviations from reading to reading – Averaging with same instrument do not help!
  - Random: Deviations due to conditions that do not remain the same – Averaging helps!

Significant Figures (S.F)

- For multiplication & division
  - Number of S.F in answer = smallest number of S.F in the input
  - Example: A metre rule length measurements of (11.3 ± 0.1) cm and (6.3 ± 0.1) cm,

    Calculated area = (11.3 cm)(6.3 cm) = 71 cm^2 (not 71.19 cm^2)
- For addition and subtraction
  - Number of decimal places = smallest number of decimal places in the input
  - Example: 132 + 7.23 = 139 (not 139.23)

Significant Figures (S.F)

- Having 3 or 4 significant figures is usually a safe choice in most answers and lab work – except for very precise measurements.

Dimensional Analysis

- Very powerful tool.
- 'Dimension' denotes the physical nature of a quantity, e.g. length L. In Mechanics, almost all quantities can be reduced to combination of **Mass (M)**, **Length (L)** and **Time (T)**.
  - Velocity: $[\ v\ ] = L/T$
  - Acceleration : $[\ a\ ] = L/T^2$
  - Force: $[\ F\ ] = ML/T^2$
  - Energy: $[\ E\ ] = ML^2/T^2$
- Square brackets are used to denote dimensions.

Dimensional Analysis

- Often used to check formula – both sides of an equation must have the same dimensions
  - For example: $F = ma$
- Provide hints to formulate correct equations.

Example: Dimensional Analysis

The velocity v, in metres per second, of an object is given by the equation $v = X + Yt^2$, where t represents time in seconds. What are the SI units of X and Y, respectively?

Answer:

X: m/s

Y: m/s^3

Example: Dimensional Analysis

Deduce the relationship between period of pendulum t and its mass m, length of string l and acceleration due to gravity g.

$[t] = T$, $[l] = L$, $[m] = M$, $[g] = LT^{-2}$

Suppose $t = k\, m^{x}\, l^{y}\, g^{z}$ (k is a dimensionless constant)

Answer: Then $T = M^{x}\, L^{y}\, (LT^{-2})^{z}$

Equating indices:

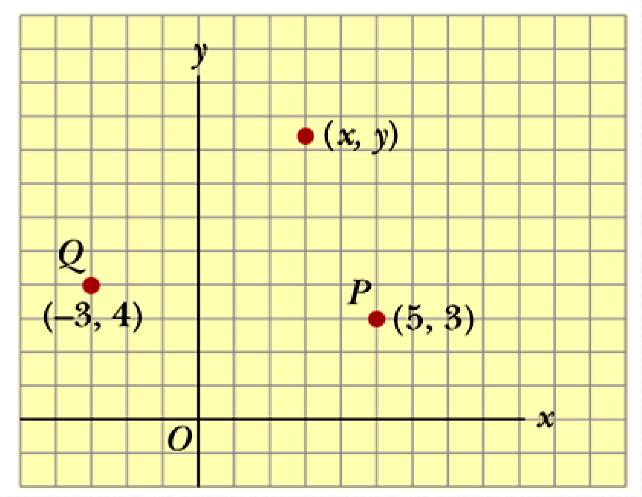
$$\begin{array}{lll} M:\; 0 = x & & x = 0 \\ T:\; 1 = -2z & \rightarrow & z = -1/2 \\ L:\; 0 = y + z & & y = 1/2 \end{array}$$

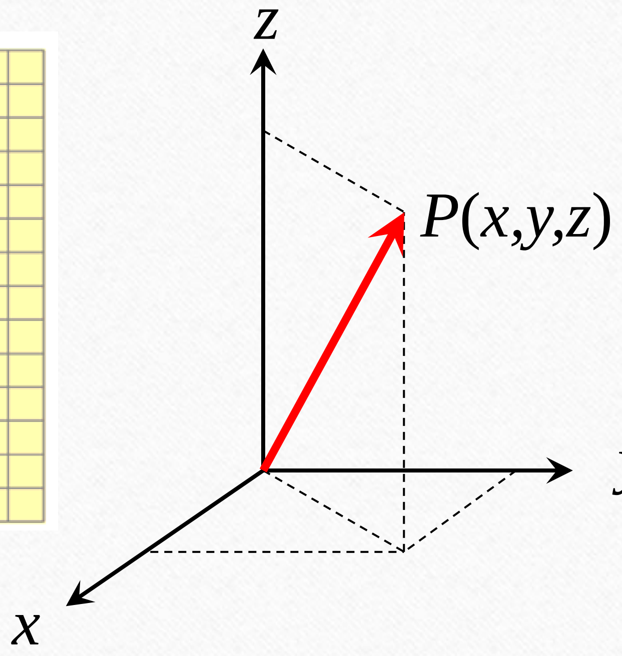
Coordinate Systems

- To describe the position of a point in space
  - 1D line requires **one** coordinate;
  - 2D plane requires **two** coordinates;
  - 3D space requires **three** coordinates
- A coordinate system consists of:
  - a fixed reference point O (origin)
  - a set of specified axes with scales and labels
  - instructions on how to label a point relative to the origin and axes

Cartesian Coordinate

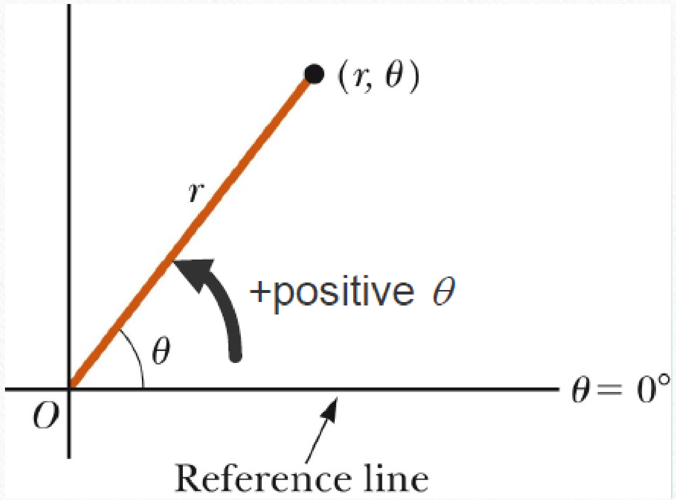
- Points are labelled (x,y).
- Use x and y axes to locate the position.
- Positive x is usually selected to be right of origin.
- Positive y is usually selected to be above the origin.





Plane Polar Coordinate

- Points are labelled (r,θ).
- Standard reference line is usually selected to be the positive x axis.
- Use radius (r) and angle (θ) to locate position.
  - Point is distance r from the origin in the direction of angle θ.
- Positive angles are measured counter-clockwise from reference line.

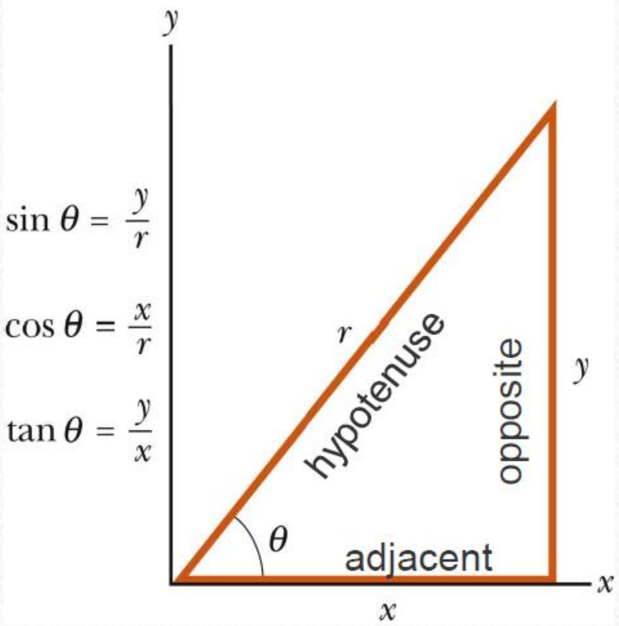


Trigonometry

$$\sin\theta = \frac{\textit{opposite side}}{\textit{hypotenuse}}$$

$$\cos\theta = \frac{\textit{adjacent side}}{\textit{hypotenuse}}$$

$$\tan\theta = \frac{\textit{opposite side}}{\textit{adjacent side}}$$



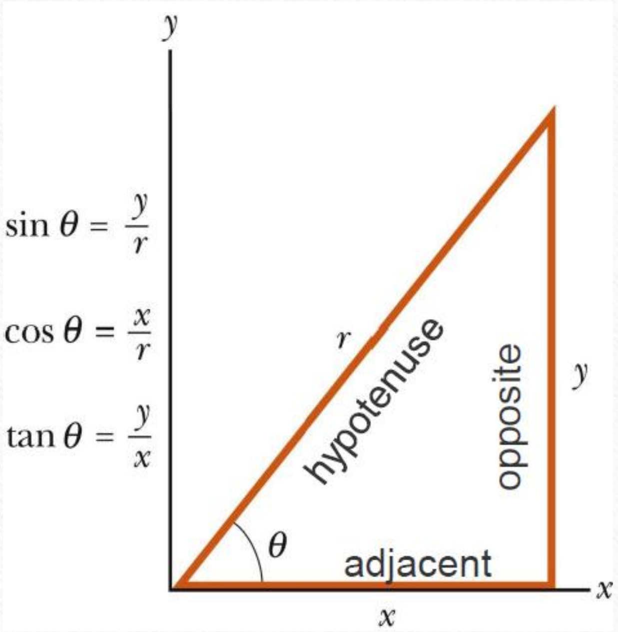
To find x and y if you have r and θ.

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

Trigonometry

- Pythagoras (Pythagorean) Theorem:
  - $r^2 = x^2 + y^2$
- To find θ, you need inverse trigonometry function and any 2 sides.
  - For e.g. $\theta = sin^{-1} \frac{opposite\ side}{hypotenuse}$

make sure calculator in correct "deg" or "rad" mode!



To find r if you have x and y.

$$r = \sqrt{x^2 + y^2}$$

Example: Cartesian to Polar

Given x = 5 and y =3, find r and θ.

Ans:

$r^2 = x^2 + y^2$

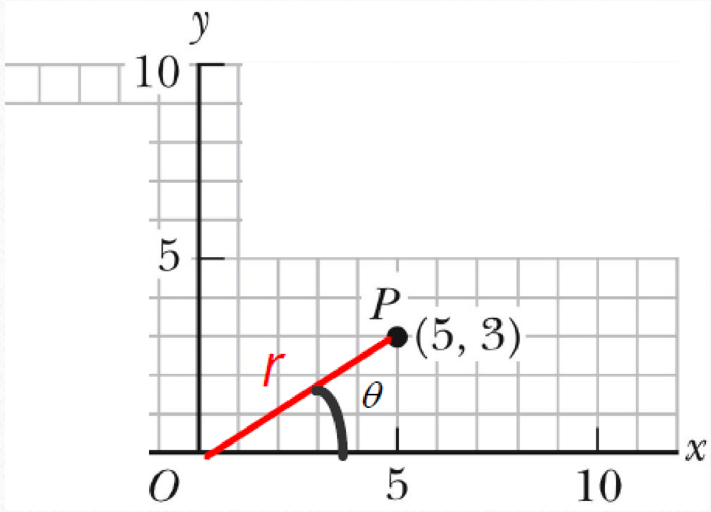
$r^2 = 5^2 + 3^2$

$r^2 = 34$

$r = \sqrt{34} = 5.83$ (ans)

$$tan\theta = \frac{y}{x} = \frac{3}{5}$$

$$\theta = tan^{-1}\frac{3}{5} = 30.96° \approx 31.0° \text{ (ans)}$$



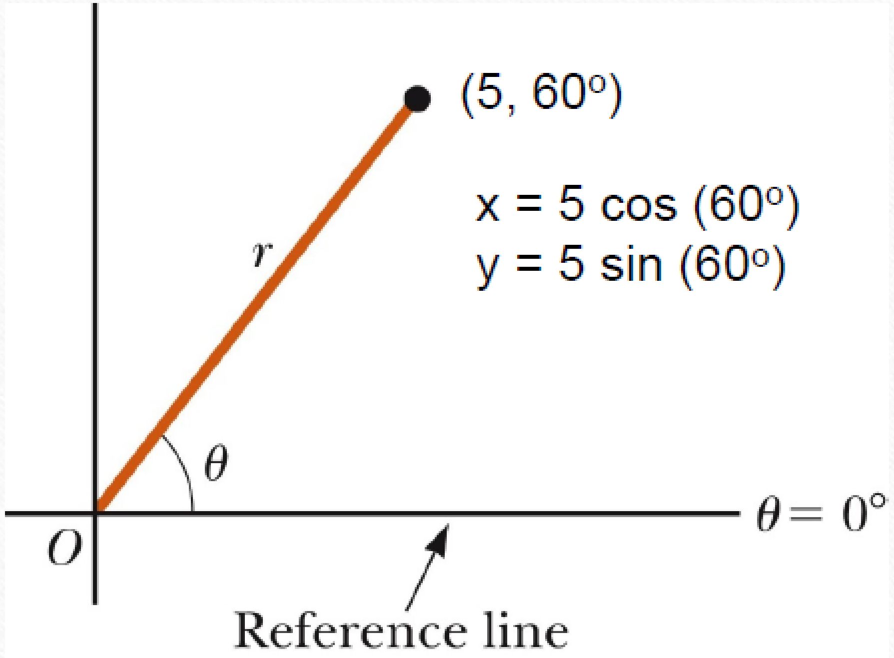
Example: Polar to Cartesian

Given r = 5 and θ = 60°, find x and y.

Ans:

$$x = r\ cos\theta = 5\ cos60° = 2.50$$

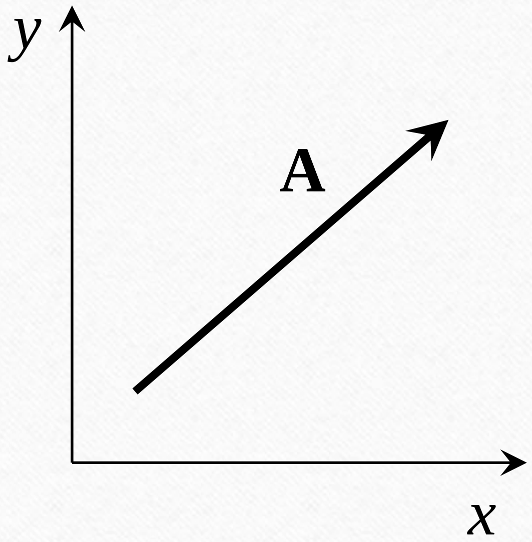
$$y = r\ sin\theta = 5\ sin60° = 4.33$$



Vectors

- Physical quantities can be **scalars** or **vectors**.
- A scalar quantity is specified by a single value with an appropriate unit and has no direction (temperature, mass, volume. Only has magnitude.
- A vector quantity has both magnitude and direction (displacement, velocity, acceleration, force, etc.). Vector quantities are specified by a number with appropriate units plus a direction.

Vectors



Magnitude ~ length of arrow
Direction ~ direction of arrow

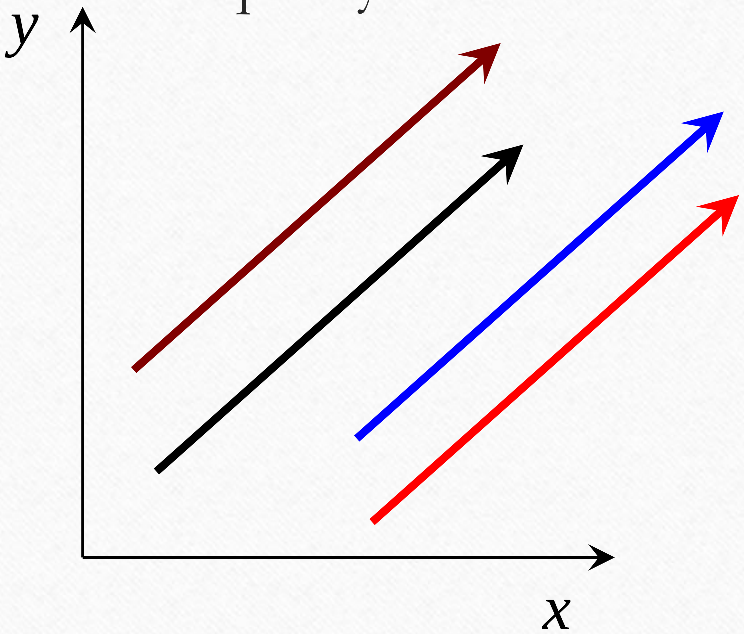
Symbols:

A (bold face, printed materials)
$\vec{A}$ (hand writing)

$|\mathbf{A}| = |\vec{A}| = A$
for magnitude

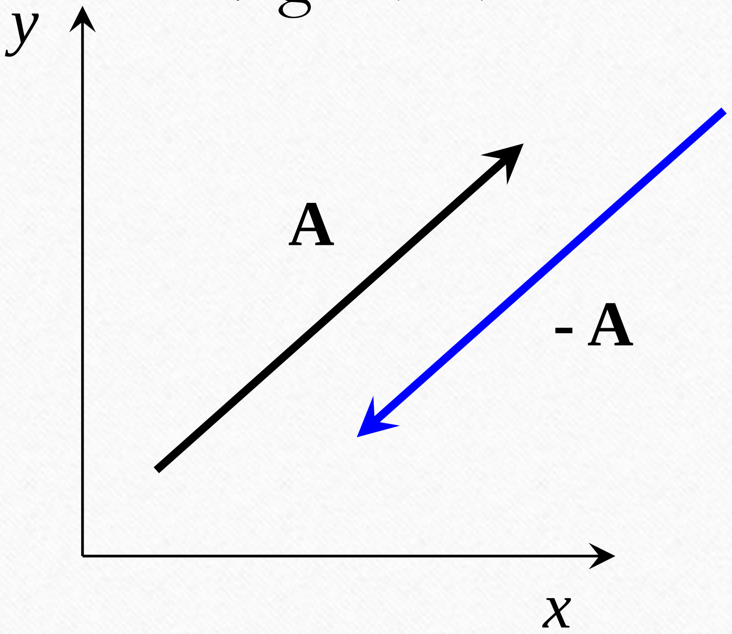
Vectors

Equality of vectors



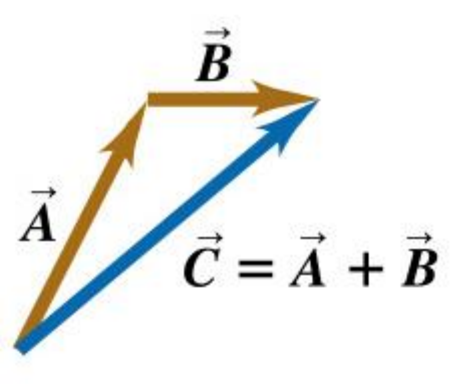
Same magnitudes and same direction.

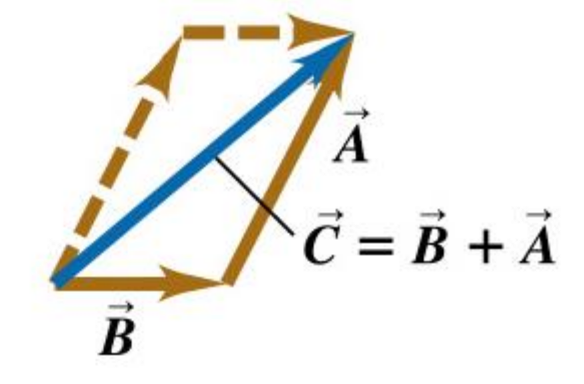
Negative vectors

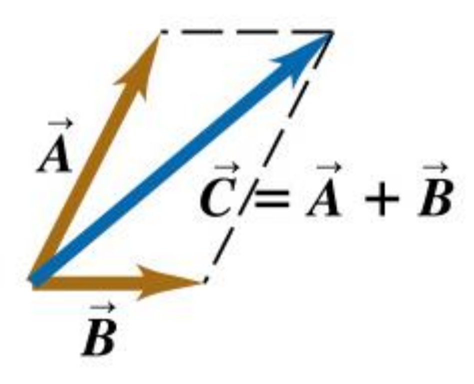


A – A = A + (-A) = 0

Vectors

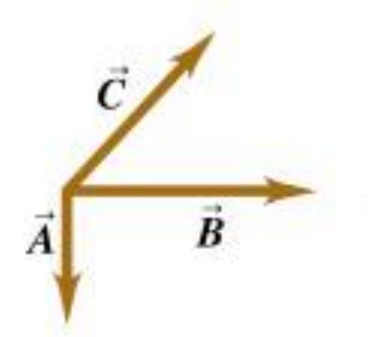


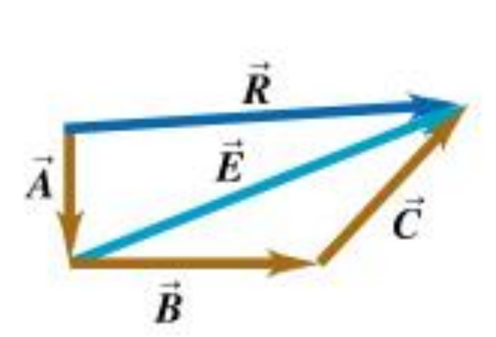


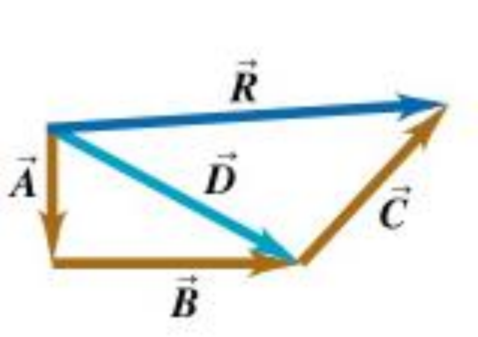


Commutative : A + B = B + A

Vectors



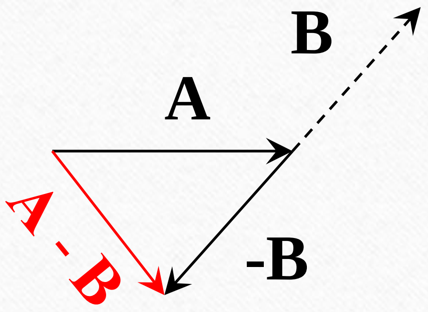




Associative: A + (B+C) = (A+B) + C

Vectors

- Subtracting Vectors: **A** – **B** = **A** + (-**B**)



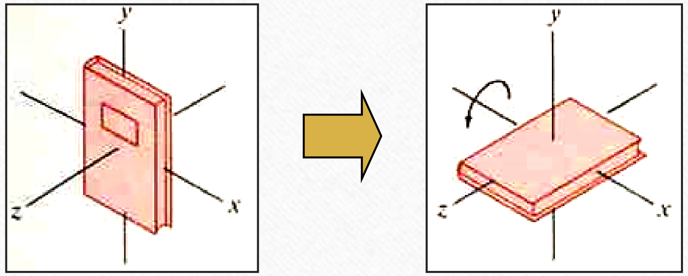
- Multiplying a Vector by a Scalar:
  - *m***A** is a vector that has the same direction as **A** and magnitude *mA*

Is Rotation a Vector?

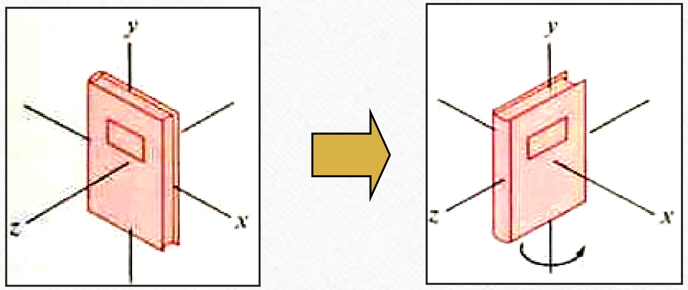
You have learnt that a vector is any quantity that has a magnitude and direction. Consider rotation through a finite angle in three dimensions about some axis. It has magnitude and direction. Is it a vector?

For example:

Rotate 90^{o} about x-axis



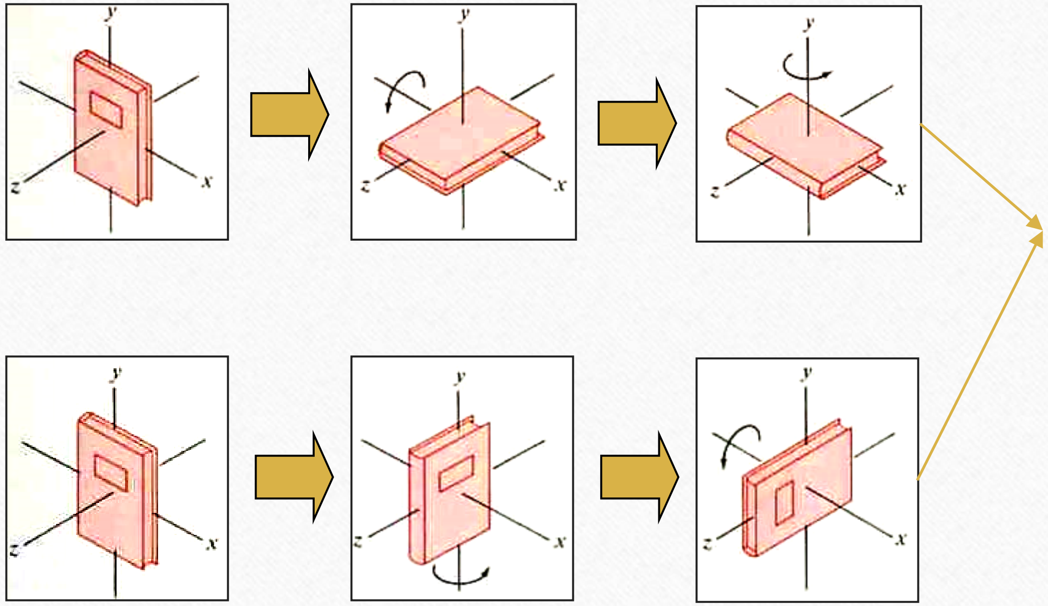
Rotate 90^{o} about y-axis



Is Rotation a Vector?

Rotation has magnitude and direction.

- But consider this:
  - rotate 90° about x-axis,
  - rotate 90° about y-axis.
- Now reverse the order:
  - rotate 90o about y-axis,
  - rotate 90o about x-axis.

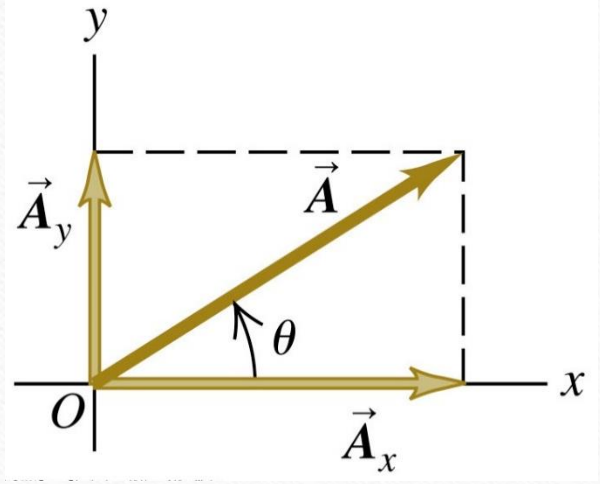


Rotation is **not commutative** under addition:

$$\mathbf{A} + \mathbf{B} \neq \mathbf{B} + \mathbf{A}$$

Components of a Vector

- Components are projections of a vector along coordinate axes.
- Any vector can be described by its components.



$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$$

$$\begin{aligned} A_x &= A\cos\theta \\ A_y &= A\sin\theta \end{aligned}$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

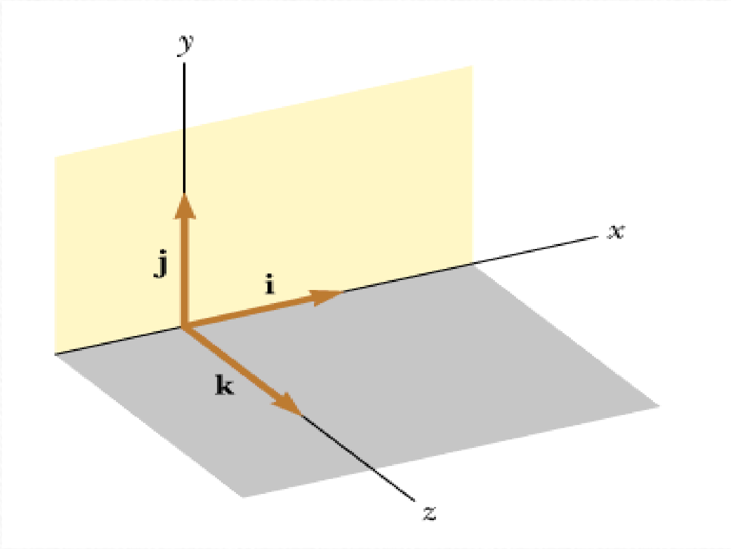
Unit Vectors

- A dimensionless vector having a magnitude of exactly one.
- Particularly useful in a Cartesian (rectangular) coordinate system to introduce unit vectors **i, j** and **k** pointing in the positive *x*, *y* and *z* directions respectively.

  $$|\mathbf{i}| = |\mathbf{j}| = |\mathbf{k}| = 1$$

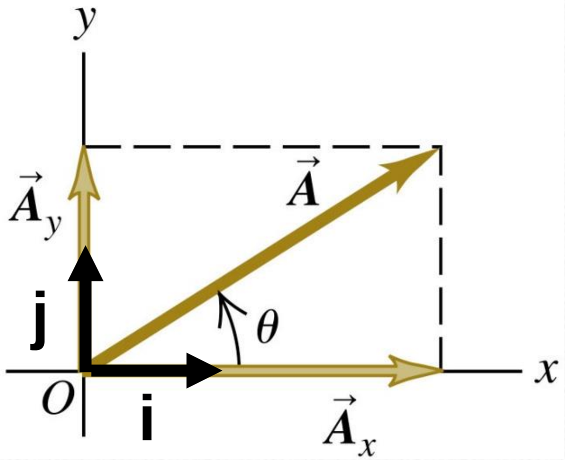
- **i**, **j** and **k** form a set of mutually perpendicular vectors in a right-handed coordinate system.

$$\mathbf{i} = \hat{\imath} = \underline{i}$$



Unit Vectors

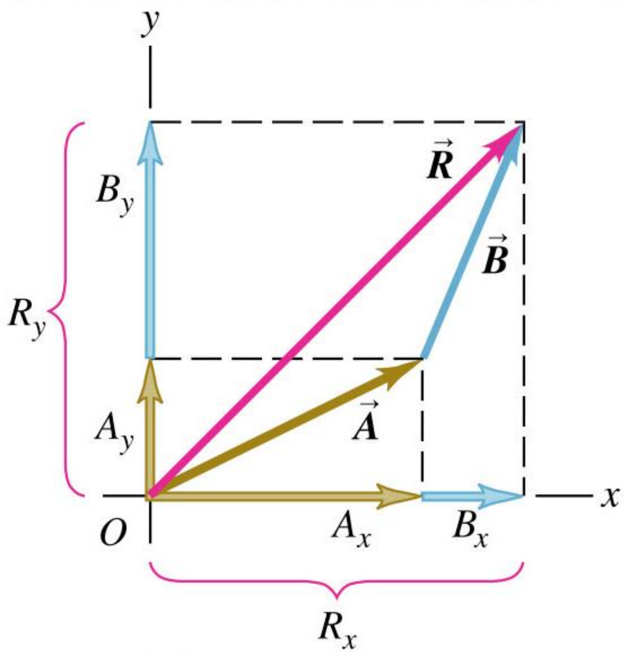
- For a 2D system, vector **A** can be written: $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y = A_x\mathbf{i} + A_y\mathbf{j}$



- Position vectors are vectors that start at the origin: $\vec{r} = x\hat{\imath} + y\hat{\jmath}$

Vector Addition

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$



$$\begin{aligned} R_x &= A_x + B_x \\ R_y &= A_y + B_y \end{aligned}$$

Vector Addition for 3D

$$\begin{aligned}\vec{A} &= A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \\ \vec{B} &= B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\end{aligned}$$

$$\begin{aligned}\vec{R} &= \vec{A} + \vec{B} \\ &= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}\end{aligned}$$

The End