Q1 (a) $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$

Product Rule

Week 3

Integration by parts.

(b) $f(g(x))$

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x).$$

Chain Rule

Week 2

Integration by substitution

2(a) $f(x) = x^3 + 2x^2 - 6x$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$f'(x) = 3x^2 + 4x - 6$

(b) $f(x) = \cos(2x)$

$f'(x) = -\sin(2x) \cdot 2 = -2\sin(2x)$

(c) $g(t) = \tan^{2}(2t)$

$$g'(t) = 2\tan(2t)\cdot\sec^2(2t)\cdot 2$$

$$= 4\tan(2t)\cdot\sec^2(2t)$$

(d) $f(x) = \sqrt{x^2+2x}$

$$f'(x) = \frac{1}{2}(x^2+2x)^{-\frac{1}{2}}\cdot(2x+2)$$

$$= \frac{x+1}{\sqrt{x^2+2x}}.$$

(e) $g(n) = \dfrac{n}{n^2+1}$

$$g'(n) = \frac{(n^2+1)\cdot 1 - n(2n)}{(n^2+1)^2} = \frac{1-n^2}{(n^2+1)^2}$$

Quotient Rule

(f) $h(x) = \sec^2(x)$

$$h'(x) = 2\sec(x)\cdot\sec(x)\tan(x)$$

$$= 2\tan(x)\sec^2(x).$$

(g) $u(t) = \dfrac{\ln t}{t^2}$

$$u'(t) = \frac{t^2\cdot\frac{1}{t} - (\ln t)(2t)}{t^4}$$

$$= \frac{\tilde{t} - 2t\ln t}{t^4} = \frac{1-2\ln t}{t^3}.$$

(h) $v(t) = \sin(t) + \cos^2(2t)$

$$v'(t) = \cos(t) + 2\cos(2t)\cdot(-\sin(2t))\cdot 2$$

$$= \cos(t) - 4\sin(2t)\cos(2t)$$

$$= \cos(t) - 2\sin(4t).$$

(ii) $f(x) = \dfrac{1}{(1-x)^2} = (1-x)^{-2}$

$f'(x) = -2(1-x)^{-3} \cdot (-1)$

$= \dfrac{2}{(1-x)^3}$.

Q3 $f(x) = \dfrac{1}{x}$ $(0, \infty)$

$x > 0$

$f'(x) = -\dfrac{1}{x^2} < 0$ for $x \in (0, \infty)$

>0

$\therefore f$ is decreasing.

Q4 $f(x) = \dfrac{\ln x}{x} \Rightarrow f'(x) = \dfrac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$

(e, ∞)

$= \dfrac{1 - \ln x}{x^2}$

Need to check $1 - \ln x < 0$

$$\Rightarrow 1 < \ln x$$

$$\Rightarrow e < e^{\ln x} = x$$

$$\Rightarrow \underline{\underline{x > e}} \Leftrightarrow x \in (e, \infty).$$

f is decreasing on (e, ∞).

$$g(x) = -\frac{\ln x}{x} = -f(x)$$

$$g'(x) = -\underbrace{f'(x)}_{<0 \text{ on } (e,\infty)}$$

>0 on (e, ∞).

g is increasing on (e, ∞).

Q5

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

general pt x

$$\boxed{f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}}$$

(a) $\lim_{x\to 1} \frac{x^7 - 1}{x-1}$

f(x)

a

$a=1$ $\quad f(a) = 1^7 = 1$

$f(x) = x^7 \quad f'(x) = 7x^6$

$= f'(1) = 7.$

(b) $\lim_{x\to \frac{\pi}{2}} \frac{e^{4x}\sin(3x) + e^{2\pi}}{x-\frac{\pi}{2}}$

f(x)

a

$f(x) = e^{4x}\sin(3x) \Rightarrow f(\frac{\pi}{2})$

$= e^{4\cdot\frac{\pi}{2}} \sin\left(\frac{3\pi}{2}\right)$

$= -e^{2\pi}$

$$f'(x) = 4e^{4x}\sin(3x) + e^{4x}\cos(3x)\cdot 3$$

$$= \underline{4e^{4x}\sin(3x)} + 3e^{4x}\underline{\cos(3x)}$$

$$f'\left(\tfrac{\pi}{2}\right) = 4\cdot e^{4\cdot\frac{\pi}{2}}\sin\left(\tfrac{3\pi}{2}\right)$$

$$= -4e^{2\pi}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{e^{4x}\sin(3x)+e^{2\pi}}{x-\frac{\pi}{2}} = f'\left(\tfrac{\pi}{2}\right)$$

$$= -4e^{2\pi}$$

(c) By limit laws and part (b)

$$\lim_{x\to\frac{\pi}{2}} \frac{e^{4x}\sin(3x)+e^{2\pi}}{\sqrt{x}-\sqrt{\frac{\pi}{2}}} \cdot \frac{\left(\sqrt{x}+\sqrt{\frac{\pi}{2}}\right)}{\left(\sqrt{x}+\sqrt{\frac{\pi}{2}}\right)}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{e^{4x}\sin(3x)+e^{2\pi}}{x-\frac{\pi}{2}} \cdot \underline{\lim_{x\to\frac{\pi}{2}}\left(\sqrt{x}+\sqrt{\tfrac{\pi}{2}}\right)}$$

$$= -4e^{2\pi} \cdot 2\sqrt{\frac{\pi}{2}} \rightarrow \cancel{2}\frac{\sqrt{\pi}}{\sqrt{\cancel{2}}} = \sqrt{2}\sqrt{\pi}$$

$$= -4e^{2\pi} \cdot \sqrt{2\pi}$$

$$= -4\sqrt{2\pi}\, e^{2\pi}$$

Q6 (a)

$$y = \sin^{-1} x \qquad x \in [-1,1]$$

$$\sin y = x$$

Many intervals of y

$$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

Implicit differentiation

$$\cos y \cdot \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$$

Focus

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\cos x = \pm\sqrt{1 - \sin^2 x}$$

$$\cos y = \pm\sqrt{1 - \sin^2 y}$$

Since $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow \cos y \geq 0$

$$\therefore \cos y = \sqrt{1-\sin^2 y}$$

$$= \sqrt{1-x^2}$$

$x = \sin y$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}.$$

(b) $y = \tan^{-1} x \qquad x \in \mathbb{R}$

$\tan y = x$

$$\sec^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$= \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$