CSD2259 Tutorial 1

Problem 1. Let a, b be distinct elements.

(a) List all elements in the power set of $A = \{a, b\}$ and $B = \{\emptyset, \{\emptyset\}\}$.

(b) Which of the following sets can be a power set?

(A) $\emptyset$ (B) $\{\emptyset, \{a\}\}$ (C) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$ (D) $\{\emptyset, \{a\}, \{b\}, \{a, b\}$

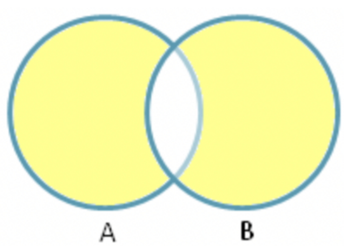
Problem 2. (a) How many positive integers not exceeding 100 are divisible either by 3 or 4 or 6?
(b) How many positive integers not exceeding 100 are divisible either by 4 or 5 or 6?

Problem 3. Three officers — a president, a treasurer, and a secretary — are to be chosen from among four people: Alice, Bob, Cyd, and Dan. Assume that each person can take only one position.
(a) How many ways can the officers be chosen?
(b) What if Bob is not qualified to be treasurer and Cyd is not qualified to be secretary?

Problem 4. Would you believe a market investigator's report that, of 1000 people, 818 like candy, 723 like ice cream, 645 cake, while 562 like both candy and ice cream, 463 like both candy and cake, 470 both ice cream and cake, and 310 like all three? State your reasons!

Problem 5. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in A_1, 1000 in A_2, and $10,000$ in A_3 if
(a) $A_1 \subset A_2$ and $A_2 \subset A_3$.
(b) the sets are pairwise disjoint.
(c) there are 2 common elements in each pair of sets and 1 element in all three sets.

Problem 6. The symmetric difference between A and B, denoted by $A \oplus B$, is the set of all elements which are in either A or B, but not in both A and B.



Suppose that A, B, C are three sets such that $A \oplus C = B \oplus C$. Prove that $A = B$.

Hint and Instruction

2. The number of integers between $[1, n]$ which are divisible by d is $\lfloor \frac{n}{d} \rfloor$.
3b. Define B and C = sets of choices for which Bob is a treasurer and Cyd is a secretary. You need to compute $24 - |B \cup C|$.
4. Answer: No. You should not believe the investigator.
5. Inclusion-exclusion principle.
6. You need to prove 2 things: $A \subset B$ and $B \subset A$. To prove $A \subset B$, consider an element $x \in A$ and you need to show $x \in B$. To do this, you should use the equation $A \oplus C = B \oplus C$ and consider 2 cases: $x \in C$ and $x \notin C$.