Game Probability

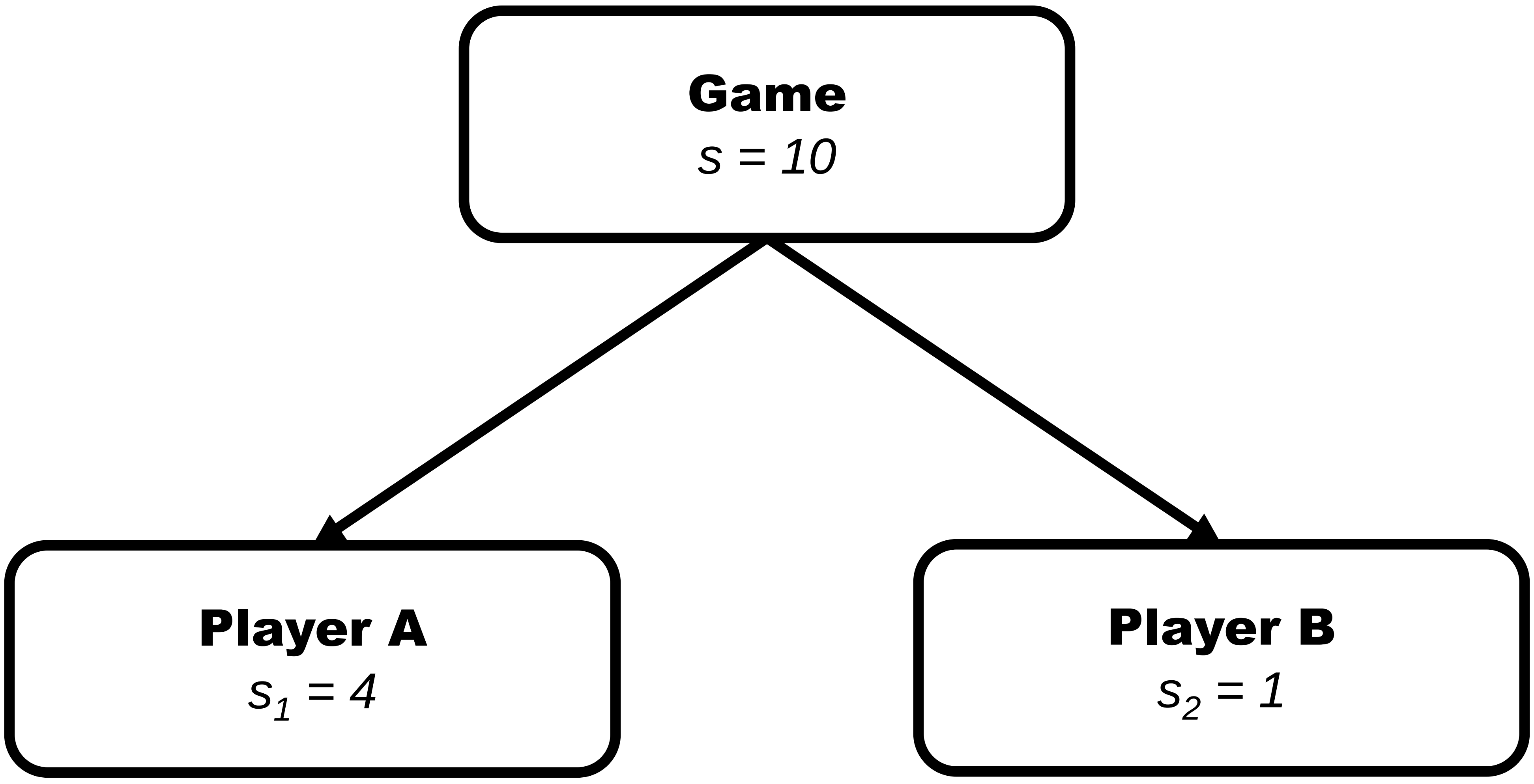
Questionnaire Results

John M. Quick

Problem of Points

"Two players, A and B, agree to play a series of fair games until one of them has won a specified number of games, s, say. For some accidental reason, the play is stopped when A has won s_1 and B s_2 games, s_1 and s_2 being smaller than s. How should the stakes be divided?"

Hald, *A History of Probability and Statistics*, 1990, p. 35


Game
$s = 10$
Player A
$s_1 = 4$
Player B
$s_2 = 1$

$S = 10$

$S_1 = 4$

$S_2 = 1$

$S_c = S - S_1 - S_2 = 5$

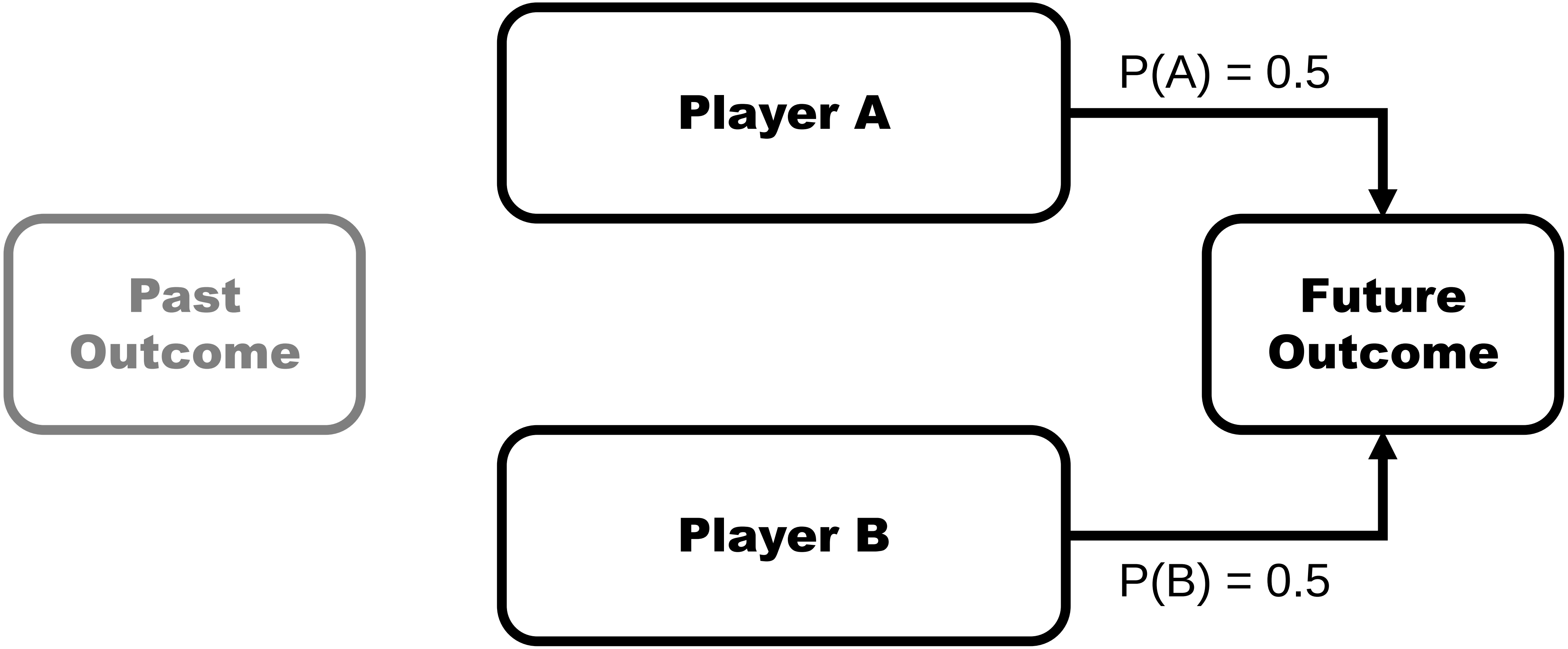
A: $(S_1/S_c * S)/S = (4/5 * 10)/10 = 8/10$ or 80% of the stakes

B: $(S_2/S_c * S)/S = (1/5 * 10)/10 = 2/10$ or 20% of the stakes

Is this fair?

Thirteen Correct Solutions to the "Problem of Points" and Their Histories

PRAKASH GORROOCHURN


Past
Outcome
Player A
P(A) = 0.5
Future
Outcome
Player B
P(B) = 0.5

$S = 10$

$S_1 = 4$

$S_2 = 1$

$S_c = S – S_1 – S_2 = 5$

A: $S_1/S + S_c /S/2 = 4/10 + 2.5/10 = 65\%$ of the stakes

B: $S_2/S + S_c /S/2 = 1/10 + 2.5/10 = 35\%$ of the stakes

Is this fair?

The game is invalid, because the conditions weren't fulfilled.
Give A and B back the stakes they put in at the start.

Is this fair?

Drawing 4 Cards from a 52-Card Deck

	Ordered	Unordered
With Replacement	Each time we draw a card, we <u>put it back</u>. We keep track of the <u>specific order</u> in each set.	Each time we draw a card, we <u>put it back</u>. We only count <u>unique sets</u>.
Without Replacement	Each time we draw a card, we <u>keep it out</u>. We keep track of the <u>specific order</u> in each set.	Each time we draw a card, we <u>keep it out</u>. We only count <u>unique sets</u>.

	Ordered	Unordered
With Replacement	$X = n^k$	$X = \frac{(n+k-1)!}{k!(n-1)!}$
Without Replacement	$P_{n,k} = \frac{n!}{(n-k)!}$	$C_{n,k} = \frac{n!}{k!(n-k)!}$

	Ordered	Unordered
With Replacement	7,311,616	341,055
Without Replacement	6,497,400	270,725

Design with Distributions

"A good time to use a **flat distribution** would be when all outcomes have relatively equal value. An example of this would be when a player gets a random starting item to start a game. All options will be relatively equal in overall power with some being better in certain contexts of events than others, but all having the same amount of usefulness."

Design Examples*

- Encountering a random enemy
- Randomized rewards or quests
- Distributing resources across a game world
- Deciding a random starting location or item

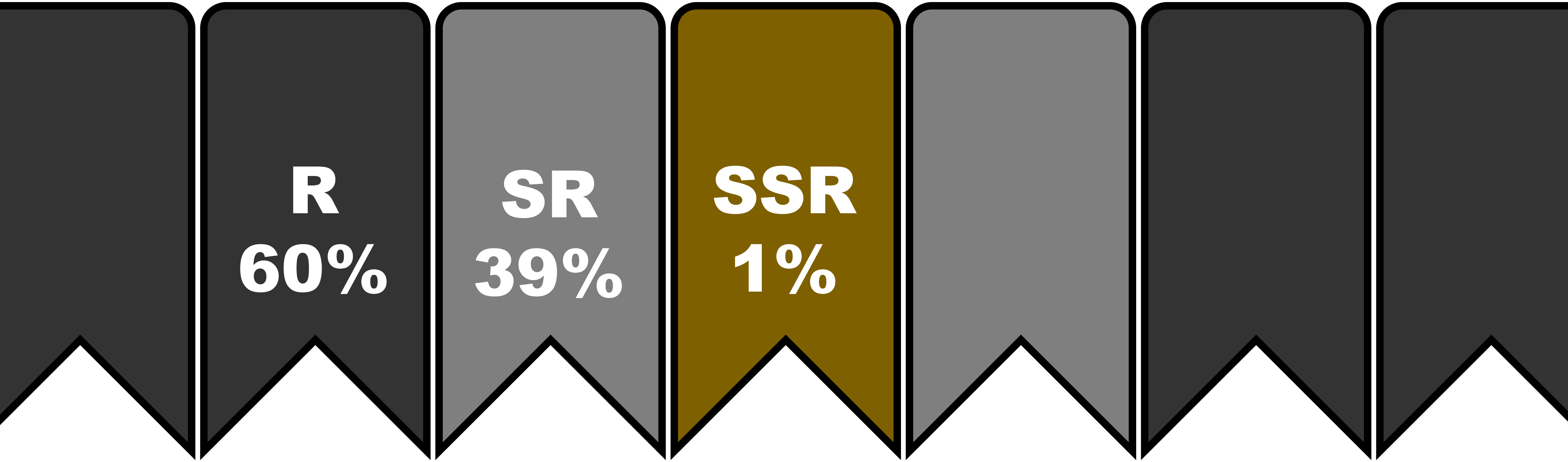
*Assuming the value of all possible outcomes is equal

"**Curved distributions**… can be useful in game design when you want to introduce variability and create different levels of rarity or probability for different outcomes. It can add depth and strategic decisions to gameplay."

Design Examples

- Variable rates for rewards of different rarities
- Variable rates for spawning common/special enemies
- Skill activation chance varies by level/investment
- Control the frequency of rare events/encounters
- Balancing the power of items/cards

Bonus Challenge


R
60%
SR
39%
SSR
1%

Expectations After N Pulls

- 50: 0, 0.5, or 1
- 100: 1
- 200: 2

Are these reasonable according to probability theory?

Expected Value

$$E(X) = \sum X_i \cdot P(X_i)$$

$\sum$ = Summation
X_i = Value of X
$P(X_i)$ = Probability of X

Brute Force Formula

$$P(A_N) = 1 - \left(1 - P(A)\right)^N \cdot 100$$

$P(A_N) =$ Probability of A after N attempts
$1 - P(A) =$ Complement of A
Based on intersection (and) of independent events

Brute Force Formula

$$P(A_N) = 1 - \left(1 - P(A)\right)^{N} \cdot 100$$

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N (Attempts)	P(A) @ 1%
1	1.00%
20	18.21%
50	38.50%
100	63.40%
200	86.60%
400	98.20%
600	99.76%
800	99.97%
1000	>99.99%
10000	>99.99%

N (Attempts)	Quantity SSR	Percent SSR
1	0	0.00%
20	1	5.00%
50	2	4.00%
100	2	2.00%
200	3	1.50%
400	3	0.75%
600	6	1.00%
800	8	1.00%
1000	9	0.90%
10000	103	1.03%

N (Attempts)	Quantity SSR	Percent SSR
1	0	0.00%
20	0	0.00%
50	0	0.00%
100	0	0.00%
200	2	1.00%
400	4	1.00%
600	6	1.00%
800	7	0.88%
1000	7	0.70%
10000	84	0.84%

References

Gorroochurn, Prakash. "Thirteen Correct Solutions to the 'Problem of Points' and Their Histories." *The Mathematical Intelligencer* 36, (2014): 56-64.

Hald, Anders. *A History of Probability and Statistics and Their Applications before 1750*. Hoboken: John Wiley & Sons, 1990.