CSD1241 Tutorial 8

Problem 1. Let T be the orthogonal projection onto the plane $\alpha : x - 3y + 2z = 0$.
(a) Find the matrix representation of T.
(b) Find the images of the points

$$\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}3\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\8\end{pmatrix}, \begin{pmatrix}3\\-3\\1\end{pmatrix}$$

(c) Find all the points $\vec{x}$ that are fixed under this transformation, that is, $T(\vec{x}) = \vec{x}$.
(d) Find the image of the plane β under T with

$$\beta : \vec{x} = \begin{pmatrix}1\\1\\1\end{pmatrix} + s\begin{pmatrix}1\\-1\\0\end{pmatrix} + t\begin{pmatrix}-8\\0\\1\end{pmatrix}$$

(e) Find the image of $\gamma : x + y + z = 1$ under T.
(f) Find the image of the line $l : \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6\\-20\\9\end{pmatrix} + t\begin{pmatrix}3\\-3\\1\end{pmatrix}$ under T.

Problem 2. Let T be the skew projection onto the plane $\alpha : x - 3y + 2z = 0$ along the vector $\vec{v} = \begin{pmatrix}1\\1\\8\end{pmatrix}$. Redo (a,c,e,f) of Problem 1.

Problem 3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the orthogonal reflection through the line

$$l : \vec{x} = t\begin{pmatrix}1\\2\\-1\end{pmatrix}$$

(a) Find the matrix of T.
(b) Find the image of the line $k : \vec{x} = \begin{pmatrix}1\\2\\3\end{pmatrix} + t\begin{pmatrix}0\\1\\-1\end{pmatrix}$ under T.

Problem 4. Let T be the orthogonal projection onto the plane $\alpha : x - 2y + z = 0$.
(a) Find the matrix M of T.
(b) Find the images of the points $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -5 \end{pmatrix}$.
(c) Find the image of $\beta : x - z = 6$ under T.
(d) Find the image of $l : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ under T.
(e) Let Q be the intersection of β and l. Find the image of Q under T.

Problem 5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the reflection in the xz-plane, and let $S : \mathbb{R}^3 \to \mathbb{R}^3$ the reflection in the plane $x - y = 0$.
The composition $T \circ S : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T \circ S(\vec{x}) = T(S(\vec{x}))$.
(a) Find the matrix K of the composition $T \circ S$.
Hint: $M, N =$ matrices of $T, S \Rightarrow$ matrix of $T \circ S$ is $K = MN$.
(b) Find the matrix L of the composition $S \circ T$. (*Hint*: $L = NM$).
(c) Check that K and L are inverses of each other, that is,

$$KL = LK = I_3.$$