CSD2201/CSD2200 Week 3 Tutorial Problems

11th September – 17th September 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Starred (*) questions are slightly more conceptual/difficult.

Question 1

For a general definite integral

$$\int_a^b f(x)\ dx$$

the *variable of integration* x is **irrelevant** to the value of this definite integral, i.e.

$$\int_a^b f(x)\ dx = \int_a^b f(y)\ dy = \int_a^b f(\odot)\ d\odot.$$

Essentially, we can replace the variable of integration with **any symbol that we want**; the integral will still give us the same value. Given this information, and if

$$\int_0^4 f(x)\ dx = 10,$$

find the value of the integral

$$\int_0^2 f(2x)\ dx.$$

Question 2*

Let a be a positive constant. We note that in Question 1, the variable of integration is irrelevant to the value of any definite integral. Prove, using appropriate substitutions, the following:

(a) A function f is **even** if for all x, $f(-x) = f(x)$. Show that if f is even, then

$$\int_{-a}^{a} f(x) \, dx = 2 \int_{0}^{a} f(x) \, dx.$$

(b) A function f is **odd** if for all x, $f(-x) = -f(x)$. Show that if f is odd, then

$$\int_{-a}^{a} f(x) \, dx = 0.$$

Question 3

Evaluate the following indefinite integrals.

(a) $\displaystyle\int 4x^3 (x^4 + 5)^3 \, dx$

(b) $\displaystyle\int x^5 \cos(x^6 + 17) \, dx$

(c) $\displaystyle\int \frac{1}{2x(\ln x)^4} \, dx$

(d) $\displaystyle\int 2x^4 (x^5 - 4)^2 \, dx$

(e) $\displaystyle\int 2x^3 e^{x^4} \, dx$

(f) $\displaystyle\int \sec^2 x \tan x \, dx$

(g) $\displaystyle\int \sqrt{3x + 2} \, dx$

(h) $\displaystyle\int \frac{x^2}{\sqrt{1 - 2x^3}} \, dx$

(i) (*) $\displaystyle\int \sqrt{1 + x^2} \, x^5 \, dx$

(j) $\displaystyle\int \frac{1}{\sqrt{x}(1 + \sqrt{x})^4} \, dx$

(k) $\displaystyle\int \tan x \, dx$

(l) $\displaystyle\int \frac{\sin\left(\frac{1}{x^2}\right)}{x^3} \, dx$

Question 4

Evaluate the following definite integrals.

(a) $\displaystyle\int_{-1}^{2} 6x^5(x^6 - 2)^3\, dx$

(b) $\displaystyle\int_{0}^{\frac{\sqrt{\pi}}{\sqrt{3}}} x\sin(x^2)\, dx$

(c) $\displaystyle\int_{2}^{e} \frac{1}{5x(\ln x)^3}\, dx$

(d) $\displaystyle\int_{0}^{2} 5x^2(x^3 - 6)^3\, dx$

(e) $\displaystyle\int_{1}^{2} 7x^4 e^{x^5}\, dx$

(f) $\displaystyle\int_{0}^{\frac{\pi}{4}} \sin^5 x \cos x\, dx$

(g) $\displaystyle\int_{1}^{3} \sqrt{5x - 4}\, dx$

(h) $\displaystyle\int_{0}^{1} \frac{x^3}{\sqrt{1 - x^8}}\, dx$

(i) (*) $\displaystyle\int_{4}^{5} x\sqrt{x - 2}\, dx$

(j) $\displaystyle\int_{1}^{2} \frac{e^{\frac{1}{x}}}{x^2}\, dx$

(k) $\displaystyle\int_{0}^{1} (5x - 3)^{42}\, dx$

(l) $\displaystyle\int_{0}^{1} \frac{3x^3 + 4x}{1 + x^4}\, dx$

Final Answers:

Q1: 5

Q3: (a) $\dfrac{(x^4 + 5)^4}{4} + C$, (b) $\dfrac{\sin(x^6 + 17)}{6} + C$, (c) $-\dfrac{1}{6(\ln x)^3} + C$, (d) $\dfrac{2(x^5 - 4)^3}{15} + C$,

(e) $\dfrac{1}{2}e^{x^4} + C$, (f) $\dfrac{\sec^2 x}{2} + C$, (g) $\dfrac{2(3x + 2)^{\frac{3}{2}}}{9} + C$, (h) $-\dfrac{(1 - 2x^3)^{\frac{1}{2}}}{3} + C$,

(i) $\dfrac{(x^2 + 1)^{\frac{7}{2}}}{7} - \dfrac{2(x^2 + 1)^{\frac{5}{2}}}{5} + \dfrac{(x^2 + 1)^{\frac{3}{2}}}{3} + C$, (j) $-\dfrac{2}{3(1 + \sqrt{x})^3} + C$,

(k) $\ln|\sec x| + C$, (l) $\dfrac{\cos(\frac{1}{x^2})}{2} + C$

Q4: (a) $\dfrac{14,776,335}{4}$, (b) $\dfrac{1}{4}$, (c) $\dfrac{1}{10}\left[\dfrac{1}{(\ln 2)^2} - 1\right]$, (d) $-\dfrac{1600}{3}$, (e) $\dfrac{7}{5}(e^{32} - e)$, (f) $\dfrac{1}{48}$,

(g) $\dfrac{2}{15}(11^{\frac{3}{2}} - 1)$, (h) $\dfrac{\pi}{8}$, (i) $\dfrac{114\sqrt{3} - 64\sqrt{2}}{15}$, (j) $e - \sqrt{e}$, (k) $\dfrac{1}{215}(2^{43} + 3^{43})$, (l) $\dfrac{3}{4}\ln 2 + \dfrac{\pi}{2}$