CSD2301 Practice Solutions
14. **Rolling Motion**

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Practice Question 1

A string is wrapped several times around the rim of a small hoop with radius 8.00 cm and mass 0.180 kg. The free end of the string is held in place and the hoop is released from rest. After the hoop has descended 75.0 cm, calculate (a) the angular speed of the rotating hoop and (b) the speed of its center.

Practice Question 1

IDENTIFY: Only gravity does work, so $W_{\text{other}} = 0$ and conservation of energy gives $K_i + U_i = K_f + U_f$. $K_f = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$.

SET UP: Let $y_f = 0$, so $U_f = 0$ and $y_i = 0.750$ m. The hoop is released from rest so $K_i = 0$. $v_{\text{cm}} = R\omega$. For a hoop with an axis at its center, $I_{\text{cm}} = MR^2$.

$\frac{1}{2}mv^2$ $\frac{1}{2}I\omega^2$

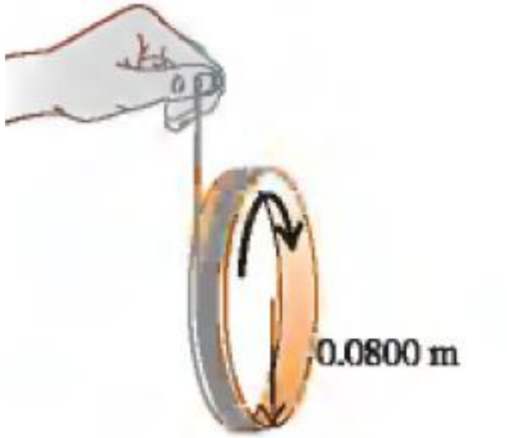
EXECUTE: **(a)** Conservation of energy gives $U_i = K_f$. $K_f = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2$, so $MR^2\omega^2 = Mgy_i$.

$$\omega = \frac{\sqrt{gy_i}}{R} = \frac{\sqrt{(9.80\ \text{m/s}^2)(0.750\ \text{m})}}{0.0800\ \text{m}} = 33.9\ \text{rad/s}.$$

Translational Rotational

(b) $v = R\omega = (0.0800\ \text{m})(33.9\ \text{rad/s}) = 2.71\ \text{m/s}$

EVALUATE: An object released from rest and falling in free-fall for 0.750 m attains a speed of $\sqrt{2g(0.750\ \text{m})} = 3.83\ \text{m/s}$. The final speed of the hoop is less than this because some of its energy is in kinetic energy of rotation. Or, equivalently, the upward tension causes the magnitude of the net force of the hoop to be less than its weight.



Practice Question 2

$$\frac{\text{rotational KE}}{\text{total KE}}$$

What fraction of the total kinetic energy is rotational for the following objects rolling without slipping on a horizontal surface? (a) a uniform solid cylinder; (b) a uniform sphere; (c) a thin-walled, hollow sphere; (d) a hollow cylinder with outer radius R and inner radius R/2.

SET UP: For an object that is rolling without slipping, $v_{cm} = R\omega$.

EXECUTE: The fraction of the total kinetic energy that is rotational is

$$\frac{(1/2)I_{cm}\omega^2}{(1/2)Mv_{cm}^2 + (1/2)I_{cm}\omega^2} = \frac{1}{1(M/I_{cm})v_{cm}^2/\omega^2} = \frac{1}{1 + (MR^2/I_{cm})}$$

(a) $I_{cm} = (1/2)MR^2$, so the above ratio is 1/3.

(b) $I_{cm} = (2/5)MR^2$ so the above ratio is $2/7$.

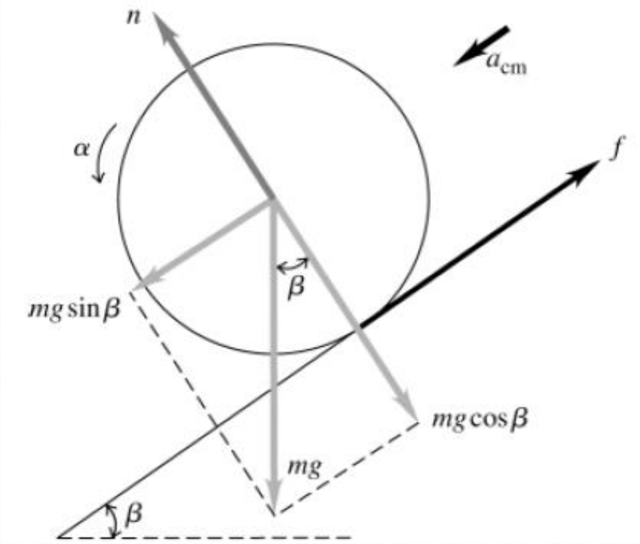
(c) $I_{cm} = (2/3)MR^2$ so the ratio is $2/5$.

(d) $I_{cm} = (5/8)MR^2$ so the ratio is $5/13$.

Practice Question 3

A hollow, spherical shell with mass 2.00 kg rolls without slipping down a 38.0° slope. (a) Find the acceleration, the friction force, and the minimum coefficient of friction force needed to prevent slipping. (b) How would your answers to part (a) change if the mass were double to 4.00 kg?

Firstly, draw the FBD:



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EXECUTE: $\sum F_x = ma_x$ gives $mg\sin\beta - f = ma_{\text{cm}}$. $\sum \tau_z = I\alpha_z$ gives $fR = (\frac{2}{3}mR^2)\alpha$. With $\alpha = a_{\text{cm}}/R$ this becomes $f = \frac{2}{3}ma_{\text{cm}}$. Combining the equations gives $mg\sin\beta - \frac{2}{3}ma_{\text{cm}} = ma_{\text{cm}}$ and

$a_{\text{cm}} = \dfrac{3g\sin\beta}{5} = \dfrac{3(9.80\text{ m/s}^2)(\sin 38.0°)}{5} = 3.62\text{ m/s}^2$. $f = \frac{2}{3}ma_{\text{cm}} = \frac{2}{3}(2.00\text{ kg})(3.62\text{ m/s}^2) = 4.83\text{ N}$. The friction is static since there is no slipping at the point of contact. $n = mg\cos\beta = 15.45\text{ N}$. $\mu_s = \dfrac{f}{n} = \dfrac{4.83\text{ N}}{15.45\text{ N}} = 0.313$.

(b) The acceleration is independent of m and doesn't change. The friction force is proportional to m so will double; $f = 9.66\text{ N}$. The normal force will also double, so the minimum μ_s required for no slipping wouldn't change.

Practice Question 4

A uniform marble rolls down a symmetric bowl, starting from rest at the top of the left side. The top of each side is a distance h above the bottom of the bowl. The left half of the bowl is rough enough to cause the marble to roll without slipping, but the right half has no friction because it is coated with oil. (a) How far up the smooth side will the marble go, measured vertically from the bottom? (b) How high would the marble go if both sides were as rough as the left side? (c) How do you account for the fact that the marble goes higher with friction on the right side than without friction?

Practice Question 4

EXECUTE: **(a)** Motion from the release point to the bottom of the bowl: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$ and $v = \sqrt{\frac{10}{7}gh}$.

Motion along the smooth side: The rotational kinetic energy does not change, since there is no friction torque on the marble, $\frac{1}{2}mv^2 + K_{\text{rot}} = mgh' + K_{\text{rot}}$. $h' = \frac{v^2}{2g} = \frac{\frac{10}{7}gh}{2g} = \frac{5}{7}h$

(b) $mgh = mgh'$ so $h' = h$.

EVALUATE: **(c)** With friction on both halves, all the initial potential energy gets converted back to potential energy. Without friction on the right half some of the energy is still in rotational kinetic energy when the marble is at its maximum height.