2024 Fall CSD3240/41/SEM4108 - Probability and Statistics Homework 10 Solution

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1. Ans: B
2. Ans: C
3. Ans: A
4. Ans: D
5. Ans: B
6. Ans: A

$$\beta = 0.002$$

The power of the test is

$$1 - \beta = 0.998$$

7. Ans: C
8. Ans: D

$$H_0 : \mu = 120 \qquad H_a : \mu > 120$$

Let $\bar{X}$ be the random variable where $\bar{X} \sim N\left(120\,,\, \frac{32.17}{\sqrt{40}}\right)$. The significance level is $\alpha = 0.05$.

$$\begin{aligned} z &= \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \\ z &= \frac{105.37 - 120}{32.17/\sqrt{40}} \\ &= -2.8762 \\ \Phi(-2.8762) &= 0.003 \end{aligned}$$

Therefore,

$$p - value = P(-2.8762 < z) = 1 - 0.003 = 0.997$$

9. Ans: C

$$H_0 : \mu = 16.43 \qquad H_a : \mu < 16.43$$

Let $\bar{X}$ be random variable of mean time to swim the 50-meter freestyle, $\bar{X} \sim N\left(\mu\,,\, \frac{\sigma}{\sqrt{n}}\right) = \left(16.43\,,\, \frac{0.8}{\sqrt{20}}\right)$

$$\begin{aligned} p - value &= P(\bar{X} < 16) \\ &= P\left(z < \frac{16 - 16.43}{0.8/\sqrt{20}}\right) \\ &= P\left(z < -2.4038\right) \\ &= 0.0082 \end{aligned}$$

10. Ans: C

$$H_0 : \mu = 4.5 \qquad H_a : \mu > 4.5$$

Let $\bar{X}$ be the mean hours per week spent on phone, $\bar{X} \sim \left(4.5\,,\,\frac{2}{\sqrt{15}}\right)$. The significance level is $\alpha = 0.05$.

$$\begin{aligned} p - value &= P(\bar{X} > 4.5) \\ &= P\left(z > \frac{4.5 - 4.75}{2/\sqrt{15}}\right) \\ &= P(z > -0.484) \\ &= 0.3142 \end{aligned}$$

Decision: Do not reject the null hypothesis because the $p - value$ is greater than $\alpha = 0.05$.

Conclusion: At the 5% significance level, there is insufficient evidence to conclude that the mean number of hours is more than 4.5.