CSD2201/CSD2200 Week 10 Tutorial Problems

30th October – 5th November 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learned from the attempt as a means of consolidating what you have learned.

Starred (*) questions are slightly more conceptual/difficult.

Squeeze + Heron need help

Question 1

The **Fibonacci sequence** is defined by the recursive formula

$$a_n = a_{n-1} + a_{n-2} \quad \text{with} \quad a_1 = 1,\ a_2 = 1.$$

Find the first eight terms of the sequence (until a_8).

Question 2

For this question, you are recommended to write up a short code (which prints a_n after each iteration) to help make this exercise less tedious.

Find the first 30 terms (until a_{20}) of the sequence defined by

$$a_{n+1} = \begin{cases} \dfrac{a_n}{2} & \text{if } a_n \text{ is an even number} \\ 3a_n + 1 & \text{if } a_n \text{ is an odd number} \end{cases}$$

and $a_1 = 11$. Repeat the exercise above with $a_1 = 25$. Make a conjecture (guess/hypothesis) about this type of sequence.

Question 3

Show that the sequence $a_n = \cos\left(\dfrac{n\pi}{2}\right)$ is divergent.

Question 4*

We know from the lectures that the sequence $a_n = (-1)^n$ is divergent. Use the Squeeze Theorem to show that

$$\lim_{n\to\infty}(-1)^n\frac{1}{n}=0.$$

Question 5

Determine whether the limits below exist. If they exist, find the limit. Otherwise, explain why.

(a) $\lim\limits_{n\to\infty}\dfrac{3}{n^3}$ (b) $\lim\limits_{n\to\infty}\sqrt{n}$ (c) $\lim\limits_{n\to\infty}\dfrac{n^2-1}{2n^2+1}$ (d) $(*)$ $\lim\limits_{n\to\infty}\dfrac{\tan^{-1}n}{n}$

(e) $\lim\limits_{n\to\infty}\dfrac{(\ln n)^3}{n}$ (f) $\lim\limits_{n\to\infty}e^{\frac{1}{n}}$ (g) $\lim\limits_{n\to\infty}\dfrac{\sin n}{n^2}$ (h) $\lim\limits_{n\to\infty}\dfrac{6^n+5^n}{7^n+2^n}$

(i) $\lim\limits_{n\to\infty}\dfrac{n^3-n^2}{e^n}$ (j) $(*)$ $\lim\limits_{n\to\infty}\sqrt{n+1}-\sqrt{n}$ (k) $\lim\limits_{n\to\infty}\dfrac{n^3+n^2+1}{5n^3-1}$

(l) $\lim\limits_{n\to\infty}\cos\left(\dfrac{n\pi}{4n-3}\right)$ (m) $\lim\limits_{n\to\infty}\dfrac{3^n}{1+5^n}$ (n) $(*)$ $\lim\limits_{n\to\infty}\left(1+\dfrac{3}{n}\right)^n$

(o) $\lim\limits_{n\to\infty}\dfrac{2\sqrt{n}}{\sqrt{n}+3}$ (p) $\lim\limits_{n\to\infty}\dfrac{n^2\cos(n\pi)}{n^2+1}$ (q) $(**)$ $\lim\limits_{n\to\infty}\dfrac{7^n}{n!}$

Final Answers:

Q1: $1, 1, 2, 3, 5, 8, 13, 21$.

Q5: (a) 0, (b) ∞; Limit does not exist, (c) $\dfrac{1}{2}$, (d) 0, (e) 0, (f) 1, (g) 0, (h) 0, (i) 0, (j) 0, (k) $\dfrac{1}{5}$, (l) $\dfrac{1}{\sqrt{2}}$, (m) 0, (n) e^3, (o) 2, (p) Limit does not exist, (q) 0.