TUTORIAL 8-C

1) Choose from 13 people of a softball team

(a) 10 players $\Rightarrow \binom{13}{10} = 286$ choices

(b) Assign 10 positions

Assign 1st position → 13 choices

Assign 2nd position → 12 choices

⋮

Assign 10th position → 4 choices

$\therefore$ total choices $= 13 \cdot 12 \cdots 4 = \frac{13!}{3!}$

(c) 13 people = 3 women + 10 men

choose 10 people with at least 1 woman

Solution 1: There are 3 possibilities:

① 1 woman + 9 men → $\binom{3}{1}\binom{10}{9} = 30$

② 2 women + 8 men → $\binom{3}{2}\binom{10}{8} = 135$ $\Rightarrow 30 + 135 + 120 = 285$

③ 3 women + 7 men → $\binom{3}{3}\binom{10}{7} = 120$

Solution 2:

10 men

choices = total − choices with no women $= \binom{13}{10} - \binom{10}{10} = 285$

2) Choose from 10 men + 10 women

(a) 6 people with 3 men & 3 women

$$\#\text{choices} = \binom{10}{3}\cdot\binom{10}{3} = \binom{10}{3}^2$$

(b) 6 people with more women than men

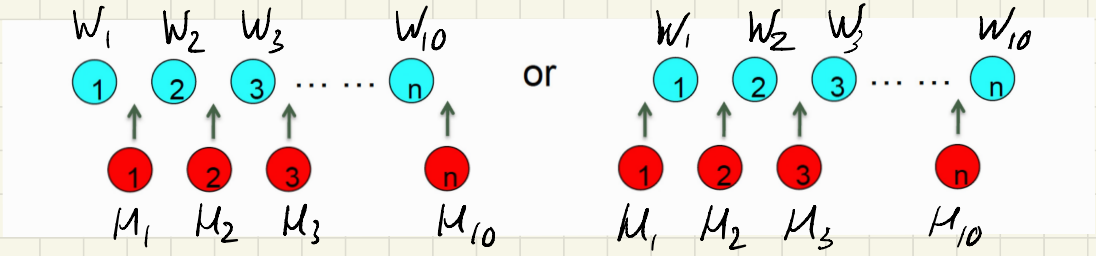
① 4 women + 2 men → $\binom{10}{4}\cdot\binom{10}{2}$

② 5 women + 1 man → $\binom{10}{5}\binom{10}{1}$

③ 6 women → $\binom{10}{6}$

$$\therefore \#\text{choices} = \binom{10}{4}\binom{10}{2} + \binom{10}{5}\binom{10}{1} + \binom{10}{6}$$

(c) Arrange them into a row: men & women are alternate



Arrangement 1 | Arrangement 2

1st: Arrange 10 women → 10! choices

2nd: Arrange 10 men → 10! choices

$\Rightarrow 10! \times 10! = (10!)^2$ choices

Arrangement 2: $(10!)^2$ choices

$\therefore \#\text{choices} = 2\cdot(10!)^2$

3, $S = \{1, 2, 3\}$

a) All 2-permutations with repetition allowed

$\{(a, b) : a, b \in S\} \longrightarrow (1,1), (1,2), (1,3), (2,1), (2,2), (2,3),$

$(3,1), (3,2), (3,3)$

b) All 2-combinations with repetition allowed

$\{\{a, b\} : a, b \in S\} \longrightarrow \{1,1\}, \{1,2\}, \{1,3\},$

list out sets: $a \leq b$ $\{2,2\}, \{2,3\}, \{3,3\}$

c) All 3-combinations with repetition allowed

$\{\{a, b, c\} : a, b, c \in S\} \rightarrow \{1,1,1\}, \{1,1,2\}, \{1,1,3\}, \{1,2,2\}, \{1,2,3\},$

list out sets: $a \leq b \leq c$ $\{1,3,3\}, \{2,2,2\}, \{2,2,3\}, \{2,3,3\}, \{3,3,3\}$

$\#$ r-combinations with rep. allowed $= \binom{r+n-1}{n-1}$ $\binom{3+3-1}{3-1} = 10$

d) All 4-combinations with repetition allowed

$\{\{a, b, c, d\} : a, b, c, d \in S\} \rightarrow \{1,1,1,1\}, \{1,1,1,2\}, \{1,1,1,3\}, \{1,1,2,2\}$

$a \leq b \leq c \leq d$ $\{1,1,2,3\}, \{1,1,3,3\}, \{1,2,2,2\}, \{1,2,2,3\}$

$\{1,2,3,3\}, \{1,3,3,3\},$

$\binom{r+n-1}{n-1} \overset{\substack{r=4 \\ n=3}}{=} \binom{4+3-1}{3-1} = 15$ $\{2,2,2,2\}, \{2,2,2,3\}, \{2,2,3,3\}, \{2,3,3,3\}$

$\{3,3,3,3\}$

Review on combination with repetition allowed

$S = \{1, 2, \ldots, n\}$

Question: # multisets $\{a_1, \ldots, a_r\} : a_i \in S \ \forall i = ?$

View S as follows: $S = \{\underbrace{\text{type } 1, \text{type } 2, \ldots, \text{type } n}\}$

↓ can be separated by $n-1$ bars

View r elements as r stars.

Each arrangement of r stars + $n-1$ bars give one r-combination

1 2 3

$** | *** | * | \quad \rightarrow \{1, 1, 2, 2, 2, 3\}$

$* | ** | *** | \quad \rightarrow \{1, 2, 2, 3, 3, 3\}$

$$\#\ r\text{-combinations} = \#\ \text{arrangements} = \binom{r+n-1}{n-1}$$

4/ S = 6 types of croissants

$\binom{r+n-1}{n-1}$, r = size of combin.
$n = |S| = 6$

$S = \{$plain, cherry, chocolate, almond, apple, broccoli$\}$

a) # ways to choose 12 croissants

$$\binom{r+n-1}{n-1} \overset{r=12}{=} \binom{12+6-1}{6-1} = \binom{17}{5}$$

b) # ways to choose 36 croissants

$$\binom{r+n-1}{n-1} \overset{r=36}{=} \binom{36+6-1}{6-1} = \binom{41}{5}$$

c) # ways to choose 24 croissants with ≥ 2 of each kind

Take out 2 of each kind (12 in total) → remain $24 - 12 = 12$

$$\binom{r+n-1}{n-1} \overset{r=12}{=} \binom{12+6-1}{6-1} = \binom{17}{5}$$

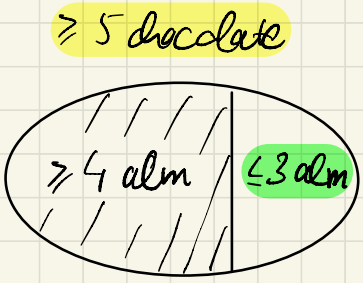
d) # ways to choose 24 croissants with ≤ 2 broccoli

≤ 2 broccoli = (total) − (≥ 3 broccoli) → take out 3 broccoli → remain 21

$$= \binom{24+6-1}{6-1} - \binom{21+6-1}{6-1}$$

$$= \binom{29}{5} - \binom{26}{5}.$$

e) # ways to choose 24 croissants with ≥ 5 chocolate & ≤ 3 almond



choices = (≥ 5 chocolate) − (≥ 5 chocolate & ≥ 4 almond)

$$= \binom{19+6-1}{6-1} - \binom{15+6-1}{6-1}$$

$$= \binom{24}{5} - \binom{20}{5}$$