CSD1100

Arithmetic Unit

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Introduction

- ALU is a combinational digital electronic circuit that performs arithmetic and bitwise operations on integer binary numbers.

- A central processing unit (CPU), floating-point unit (FPU) or graphics processing units (GPU) may contain multiple ALUs.

ALU

- Data: binary numbers as operands for operators
- Operators:
  - Arithmetic: add, subtract, increment, …
  - Bitwise logical
  - Bit shift
- Status: supplemental information about the result of an operation
  - Ex: Overflow, which indicates the result of an arithmetic operation has exceeded the numeric range of output.

Adder

- Addition is the most basic of all arithmetic operations.
  - As a matter of fact addition is the only operation the computer does. All other operations are reduced to addition. This is true for subtraction, multiplication, and division.
- Computers store numbers in binary format. Therefore, we need to focus on building a binary adding machine, known as a binary adder.
- A single binary adder adds 2 bits together. To add numbers more than 1 bit long several single binary adders are needed.

Review: Binary Addition

- Binary addition follows a similar procedure as decimal addition:
  - When adding numbers we add each column separately.
  - Then if the addition of the digits in the column generates a carry the carry is then added to the next column to the left.
- Thus, a column (other than the first one) may receive a carry from the column to its right and can generate a carry as well.

Review: Binary Addition

- The binary addition table showing the carry underlined:

+	0	1
0	$\underline{0}0$	$\underline{0}1$
1	$\underline{0}1$	$\underline{1}0$

- In order to build the 1 bit adder one has to take a closer look to the table of addition.
- The binary addition table showing the carry underlined can be consider to be the merger of two separate tables, one representing the sum and the second representing the carry

1 Bit Adder

+	0	1
0	00	01
1	01	10

=

sum	0	1
0	0	1
1	1	0

+

carry	0	1
0	0	0
1	0	1

1 Bit Adder

- The sum table is identical to the one generated by an XOR gate.

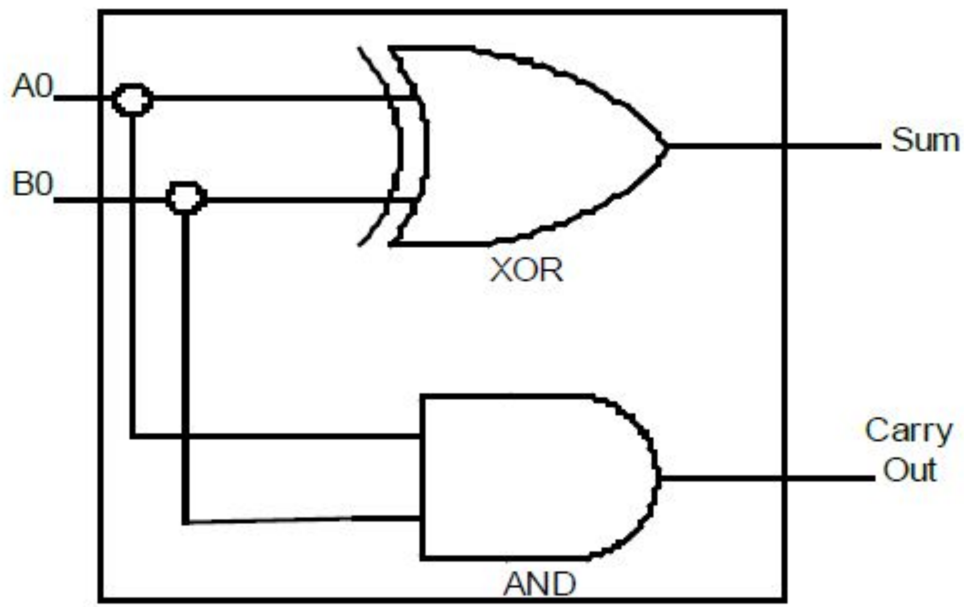
XOR	0	1
0	0	1
1	1	0

- And the carry table is identical to the one generated by an AND gate.

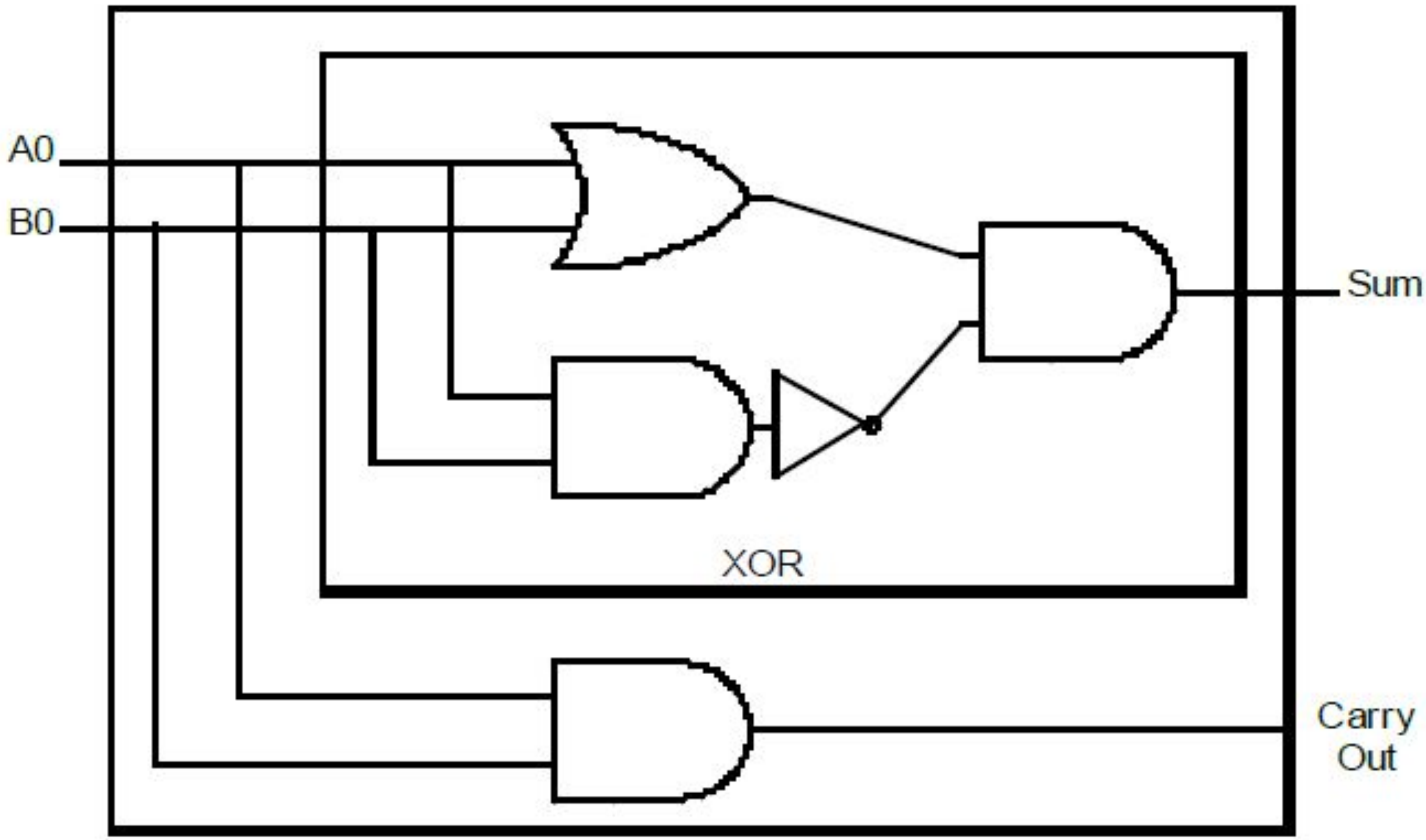
AND	0	1
0	0	0
1	0	1

1 Bit Adder

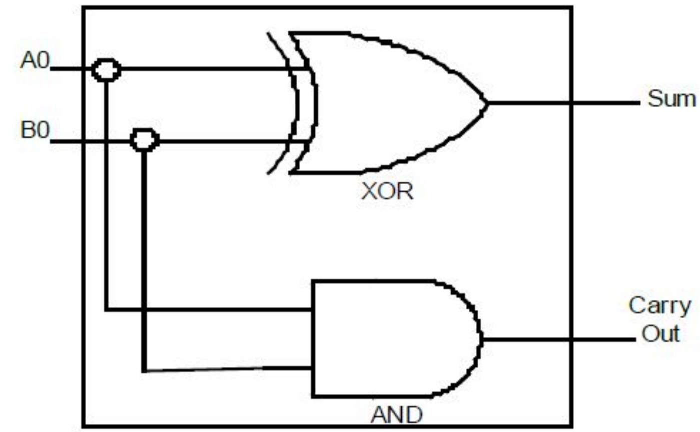
- The following diagram shows how the XOR and the AND gates are coupled together in order to generate the first version of the 2-bit adder we are trying to build.
- This early version is called a half adder:



Half Adder



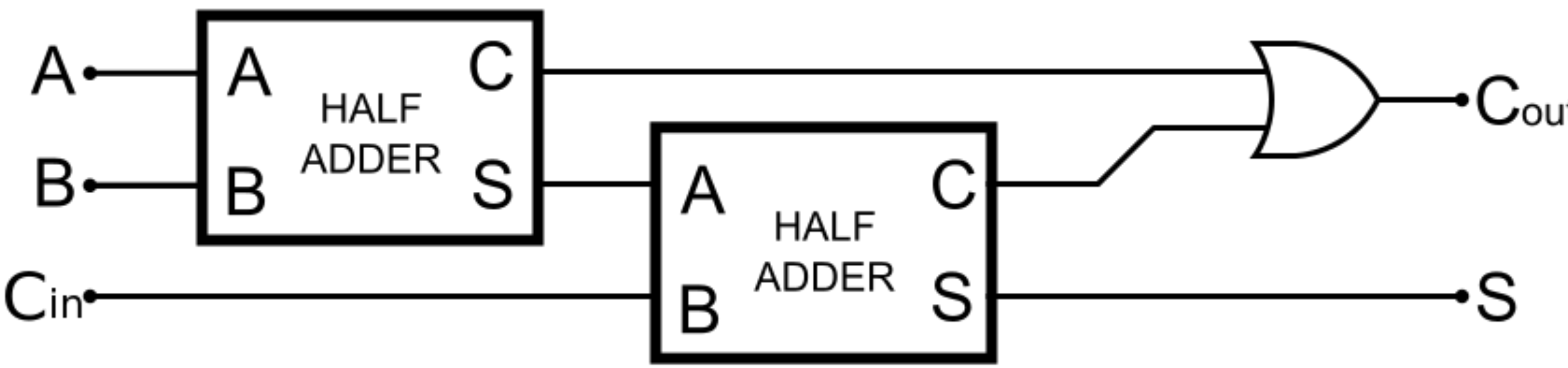
Half Adder



- The half adder has:
  - 2 inputs A0, B0.
  - These are the 2 bits to add.
- 2 outputs, one from the XOR which represents the sum and the other from the AND which represent the carry out.
- The half adder gets its name from the fact that it does half the job (2/3d maybe more accurate!).
- The half adder does not have a way to accept any carry in bit resulting from a previous carry out. The solution to this problem is simple and is described in the next section.

Full Adder

- The solution to the problem is obtained by using two half adders connected together with an OR gate as shown in the diagram below:



Full Adder

- The logic expressions for this full adder are:
  - $S = (A \text{ XOR } B) \text{ XOR } C_{in}$
  - $C_{out} = AB + C_{in}(A \text{ XOR } B)$

- By applications of boolean algebra, the second statement equivalent to
  - $C_{out} = AB + BC_{in} + AC_{in}$

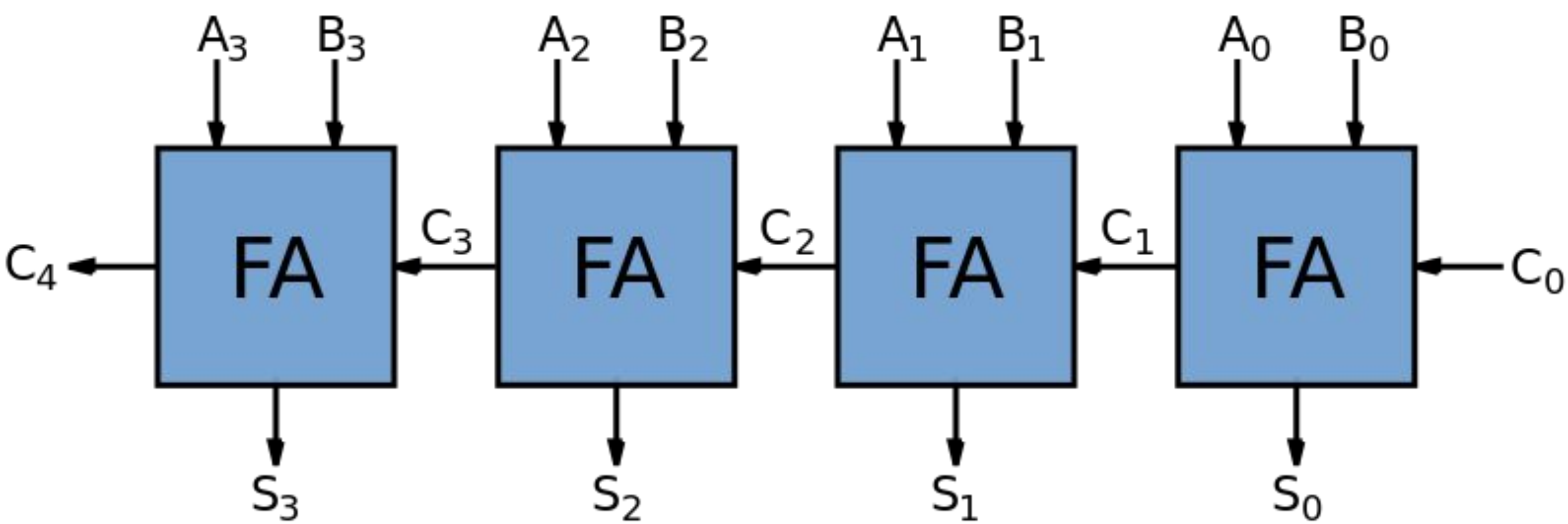
Truth Table For A Full Adder

A	B	C_{in}		C_{out}	S
0	0	0		0	0
0	0	1		0	1
0	1	0		0	1
0	1	1		1	0
1	0	0		0	1
1	0	1		1	0
1	1	0		1	0
1	1	1		1	1

Binary number of 1s in input

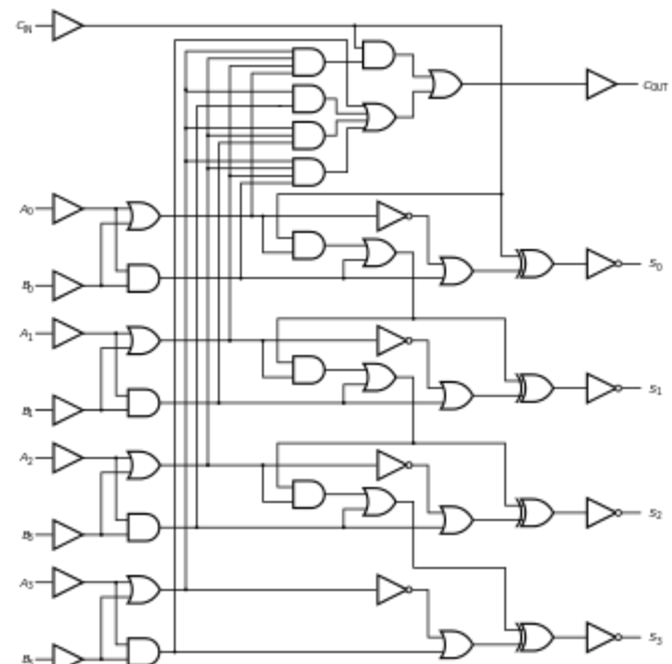
N-bit Adder

- Several full adders can be combined together to add numbers that are more than one bit wide.
- The following diagrams represent a 4-bit adder:

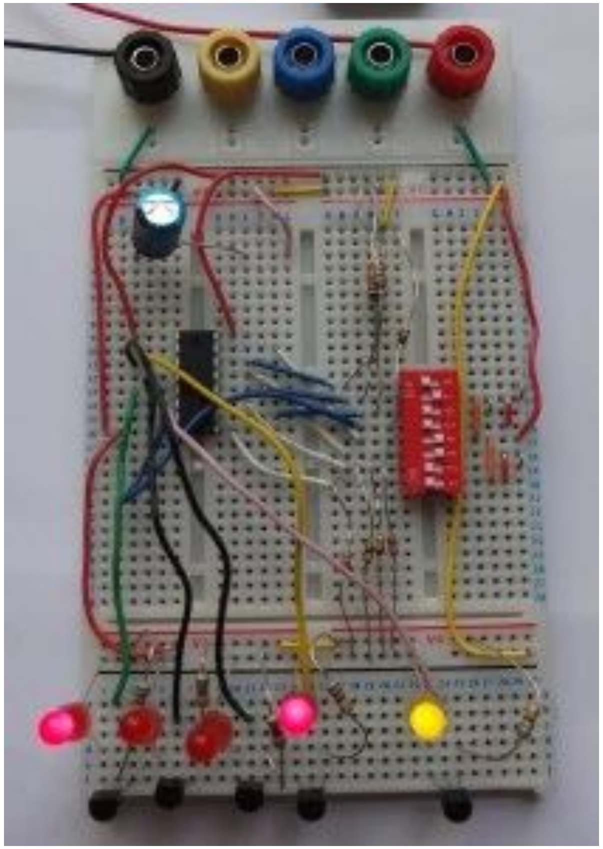


4008

- The 4008 is a 4-bit full adder that takes two 4-bit binary numbers, A[3:0] and B[3:0] and carry-in signal, C_{in} and adds them to produce a 5-bit result, made up of a sum, S[3:0] and a carry-out.



The 4008 On A Board



Source:
https://simplestcomputer.wordpress.com/route-map/stage-2-arithmetic-handling-numbers/full-adder/

References

- https://en.wikipedia.org/wiki/Arithmetic_logic_unit
- https://en.wikibooks.org/wiki/Practical_Electronics/Adders