Tutorial 1

1. Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

Statement: Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S.

Answer:

True.

Proof: Contradiction

→ There are pairs (m, w') and (m', w) where w' ranked lower than w, and m' ranked lower than m

→ m prefers w to w' and w prefers m to m'

→ there is instability in the matching

→ contradict with S is a stable matching.

2. Let $M = m_1, m_2, m_3$ and $W = w_1, w_2, w_3$. Suppose that you are given the following preference lists:

$m_1 : < w_3, w_2, w_1 >$; $m_2 : < w_2, w_3, w_1 >$; $m_3 : < w_2, w_3, w_1 >$;

$w_1 : < m_3, m_1, m_2 >$; $w_2 : < m_1, m_3, m_2 >$; $w_3 : < m_3, m_1, m_2 >$;

1) Give a stable perfect matching, and an unstable perfect matching.
2) Find the best valid partner for each member of sets M and W.

Answer:

Stable matchings

{(m1 w3) (m2 w1) (m3 w2)} men propose.

{(m2 w1) (m1 w2) (m3 w3)} women propose.

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Unstable matching

{(m1 w1) (m2 w2) (m3 w3)}

{(m1 w1) (m2 w3) (m3 w2)}

{(m1 w2) (m2 w3) (m3 w1)}

{(m1 w3) (m2 w2) (m3 w1)}

m1	m2	m3	w1	w2	w3
w3	w2	w2	m3	m1	m3
w2	w3	w3	m1	m3	m1
w1	w1	w1	m2	m2	m2

m1 preference list is w3,w2,w1. he is assigned with w1 (red) which has lower priority than w2.

m1	w1
w3	m3
w2	m1
w1	m2

m2	w2
w2	m1
w3	m3
w1	m2

m3	w3
w2	m3
w3	m1
w1	m2

m1	w1
w3	m3
w2	m1
w1	m2

m1 and w2 prefer each other with higher priority than what is being assigned (instability)

m2 and w3 was assigned to their preferred choice

m3 assigned to his preferred choice. But w1 get a lower preference. Since only 1 side thus no instability.

m1	w1
w3	m3
w2	m1
w1	m2

m2	w3
w2	m3
w3	m1
w1	m2

m3	w2
w2	m1
w3	m3
w1	m2

m1	w1
w3	m3
w2	m1
w1	m2

m1 and w3 prefer each other but get someone with a lower priority (instability)

m2 didn't get his preferred choice but w2 get his preferred choice. Since only 1 sidethus no instability.

m3 get his preferred choice, but w1 didn't. Since only 1 side thus no instability.

m1	w2
w3	m1
w2	m3
w1	m2

m2	w3
w2	m3
w3	m1
w1	m2

m3	w1
w2	m3
w3	m1
w1	m2

m1	w2
w3	m1
w2	m3
w1	m2

m1 abd w3 prefer each other but get someone else with lower priority (instability)

m2 and w1 was assigned to their preferred choice.

m3 didn't get his preferrred choice, but w2 is ok.

m1	w3
w3	m3
w2	m1
w1	m2

m2	w2
w2	m1
w3	m3
w1	m2

m3	w1
w2	m3
w3	m1
w1	m2

m1	w3
w3	m3
w2	m1
w1	m2

m1 get his preferrred choice, but w2 didn't.

both m2 and w1 was assigned to their preferred choice.

m3 and w3 preferred each other but was assigned to some else with lower priority. Thus this is an instability.

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m1	m2	m3	w1	w2	w3
w3	w2	w2	m3	m1	m3
w2	w3	w3	m1	m3	m1
w1	w1	w1	m2	m2	m2

	m1	m2	m3	
w1	3,2	3,3	3,1	(m1,w2),(m3,w3),(m2,w1)
w2	2,1	1,3	1,2	(m1,w3),(m3,w2),(m2,w1)
w3	1,2	2,3	2,1	

3. List the following functions according to their order of growth from the lowest to the highest. (Hint: you could start with using basic asymptotic efficiency classes)

$$f_1(n) = n^{2.5}; \quad f_2(n) = \sqrt{2n}; \quad f_3(n) = n + 10$$

$$f_4(n) = 10^n; \quad f_5(n) = 100^n; \quad f_6(n) = n^2 \log n$$

Answer:

$f_2(n) = \sqrt{2n}$

$f_3(n) = n + 10$

$f_6(n) = n^2 \log n$

$f_1(n) = n^{2.5}$

$f_4(n) = 10^n$

$f_5(n) = 100^n$