CSD2301 Lecture

6. Application of Newton's Laws: Part 2

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Outline

- Dynamics of uniform circular motion
- Dynamics of non-uniform circular motion

Uniform Circular Motion

- A particle moving with uniform speed *v* in a circular path of radius *r* experiences an acceleration $\mathbf{a}_r$ that has a magnitude

$$a_r = \frac{v^2}{r}$$

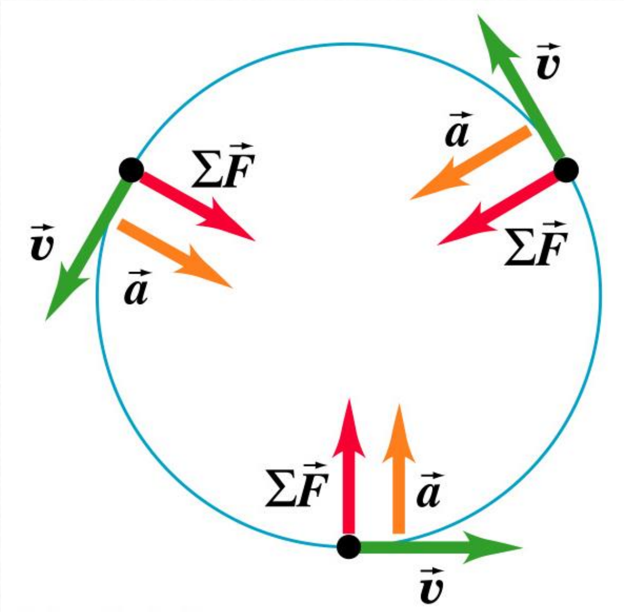
- $\mathbf{a}_r$ is directed toward the center of the circle (centripetal acceleration).
- $\mathbf{a}_r$ is always perpendicular to $\mathbf{v}$.

Uniform Circular Motion

- Apply Newton's 2nd law along the radial direction

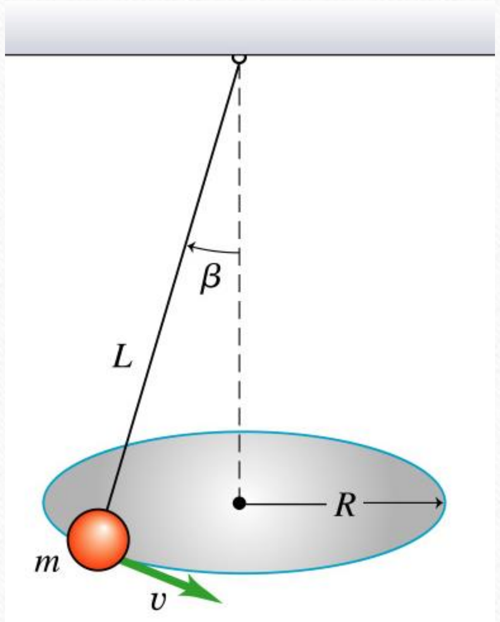
$$\boxed{\sum F_r = ma_r = m\frac{v^2}{r}}$$

- A force causing a centripetal acceleration acts toward the centre of the circular path and causes a change in the direction of the velocity vector (centripetal force).



Example: Conical Pendulum

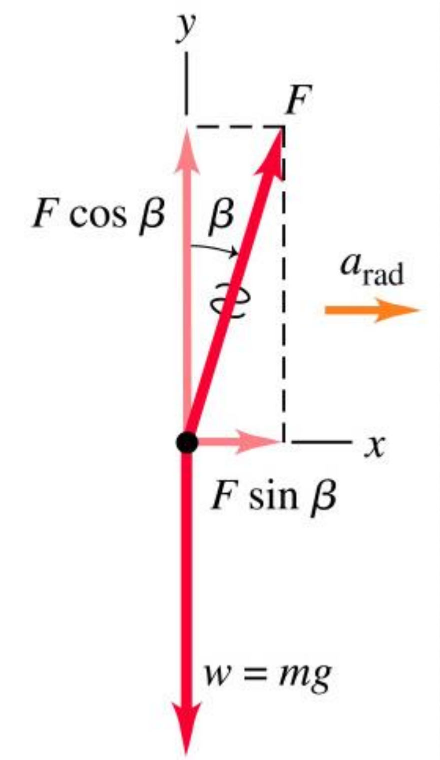
- A pendulum bob of mass m is suspended from a thin wire of length L. The bob moves in a horizontal circle with constant speed v, with the wire making a constant angle β with the vertical direction. Find the tension F in the wire and the period T (time for one revolution of the bob).



(a)

Example: Conical Pendulum

- A pendulum bob of mass *m* is suspended from a thin wire of length *L*. The bob moves in a horizontal circle with constant speed *v*, with the wire making a constant angle *β* with the vertical direction. Find the tension *F* in the wire and the period *T* (time for one revolution of the bob).



$$\sum F_x = F \sin\beta = ma_{\mathrm{rad}} = m\frac{v^2}{R}$$

$$\sum F_y = F\cos\beta + (-mg) = 0$$

Example: Conical Pendulum

sin/cos = tan

$$\sum F_x = F\sin\beta = ma_{\text{rad}} = m\frac{v^2}{R}$$

$$\sum F_y = F\cos\beta + (-mg) = 0$$

fx/fy

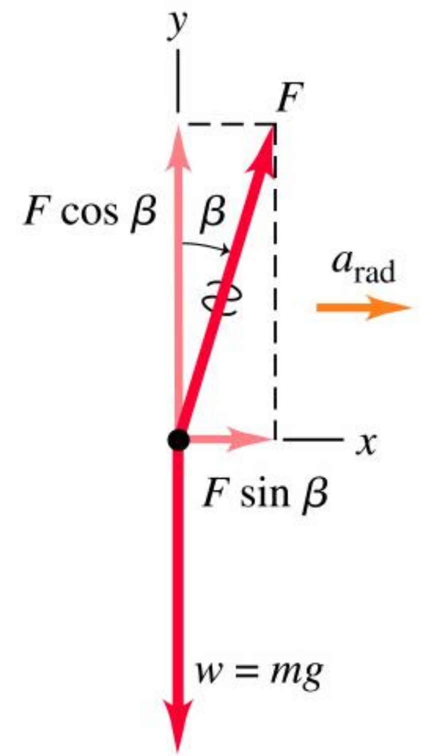
$$\tan\beta = \frac{v^2}{Rg} = \frac{v^2}{gL\sin\beta}$$

$$\boxed{F = \frac{mg}{\cos\beta}}$$

$$v = \sqrt{gL\sin\beta\tan\beta}$$

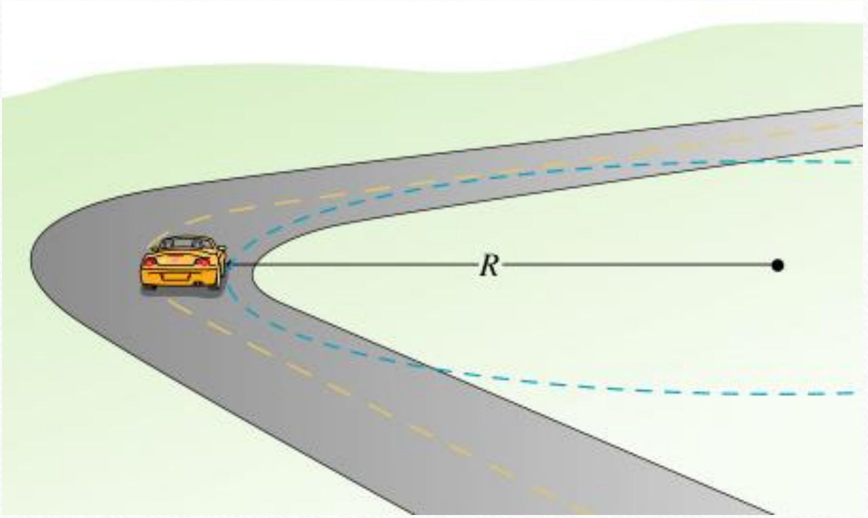
Since Period, $T = \frac{2\pi R}{v}$

$$\boxed{T = \frac{2\pi L\sin\beta}{\sqrt{gL\sin\beta\tan\beta}} = 2\pi\sqrt{\frac{L\cos\beta}{g}}}$$

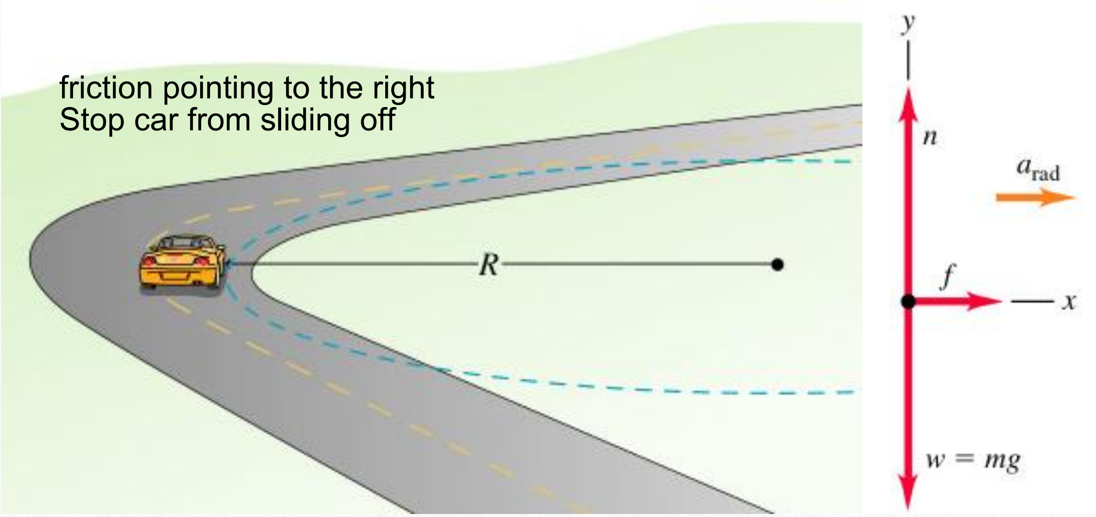


Example: Negotiating a Curve

- A car is rounding a flat unbanked curve with radius R. If the coefficient of friction between tires and road is μ_s, what is the maximum speed v_{max} at which the driver can take the curve without sliding?



Example: Negotiating a Curve



$$\sum F_y = n + (-mg) = 0$$

$$\sum F_x = f = ma_{\text{rad}} = m\frac{v^2}{R}$$

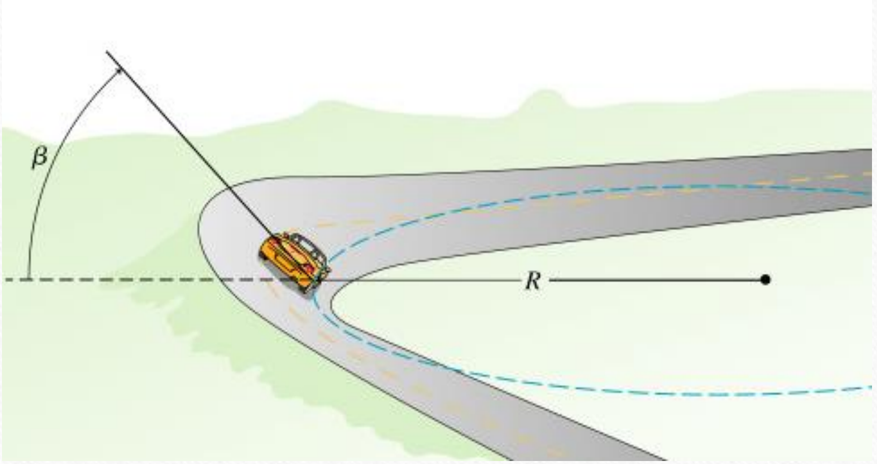
$$m\frac{v^2}{R} = f \le \mu_s mg$$

$$\boxed{v_{\text{max}} = \sqrt{\mu_s g R}}$$

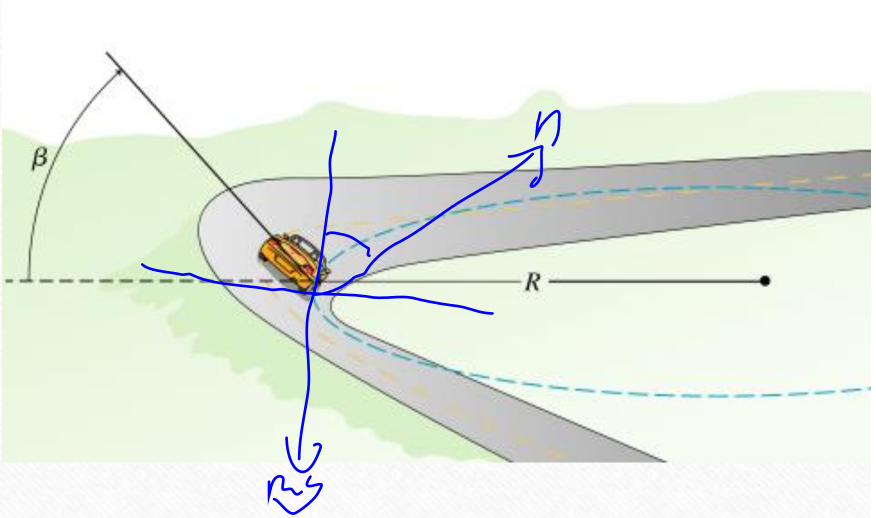
For example, if $\mu_s = 0.87$, and $R = 230$ m : $v_{\text{max}} = \sqrt{(0.87)(9.8)(230)} = 44\ \text{m/s}$

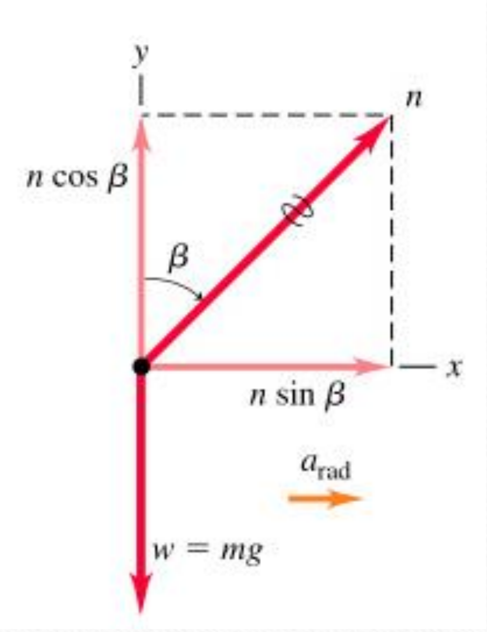
Example: Negotiating a Banked Curve

- A curved exit ramp is to be designed for a highway in such a way that a car will not have to rely on friction to round the curve without skidding. Such a ramp is usually banked. Suppose the designated speed for the ramp is to be 25 m/s and the radius of the curve is 230 m. At what angle should the curve be banked?



Example: Negotiating a Banked Curve





$$\sum F_x = n \sin \beta = m a_{\text{rad}} = m \frac{v^2}{R}$$

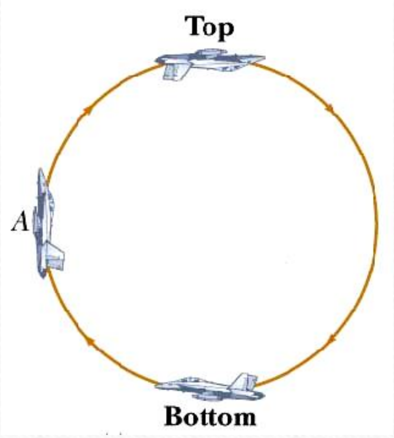
$$\sum F_y = n \cos \beta + (-mg) = 0$$

$$\boxed{\tan \beta = \frac{v^2}{gR}}$$

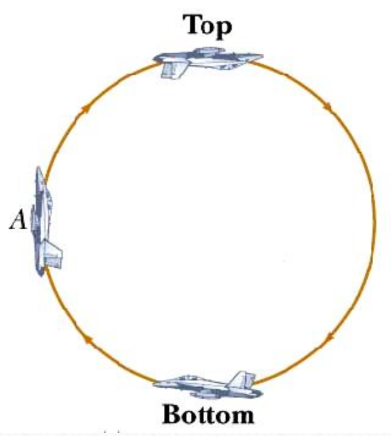
If v = 25 m/s, R = 230 m, then $\boxed{\beta = \tan^{-1}\left(\frac{25}{(9.8)(230)}\right) = 15^\circ}$

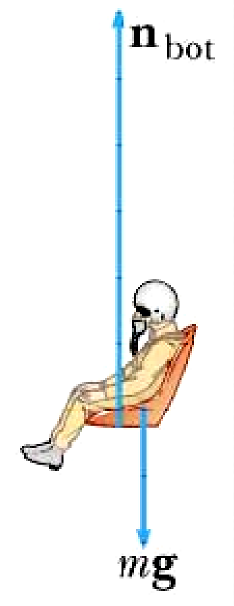
Example: Loop

- A pilot of mass *m* in a jet aircraft executes a loop-the-loop. In this maneuver, the aircraft moves in a vertical circle of radius 2.70 km at a constant speed of 225 m/s. Determine the force exerted by the seat on the pilot (a) at the bottom of the loop and (b) at the top of the loop.

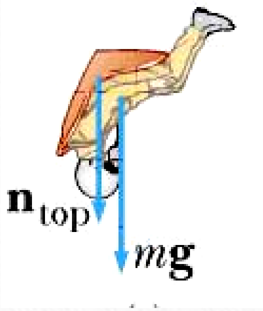


Example: Loop





At bottom



At top

$$n_{bot} - mg = m\frac{v^2}{r} \Rightarrow n_{bot} = mg\,(\frac{v^2}{rg} + 1)$$

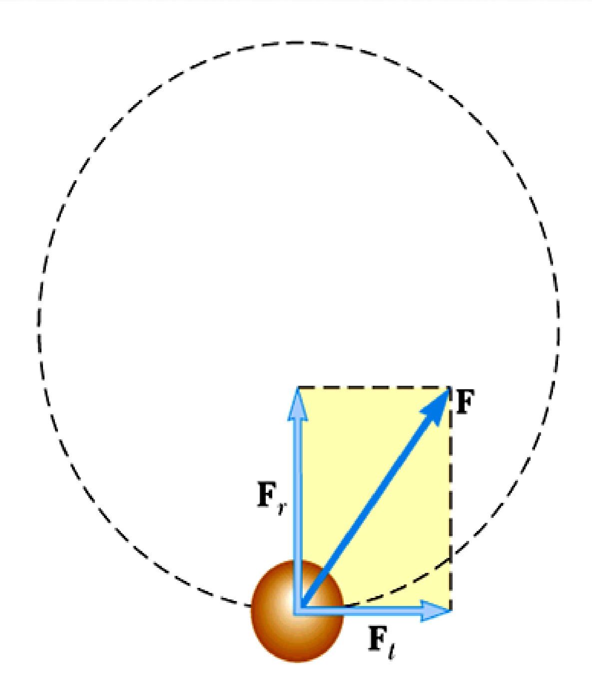
$$n_{top} + mg = m\frac{v^2}{r} \Rightarrow n_{top} = mg\,(\frac{v^2}{rg} - 1)$$

Non-uniform Circular Motion

- F_r is directed toward the centre of the circle and is responsible for the centripetal acceleration. F_t is tangent to the circle and is responsible for the tangential acceleration (change in speed).

$$\vec{F} = \vec{F}_r + \vec{F}_t$$

$$\vec{a} = \vec{a}_r + \vec{a}_t$$



Example: Mass round a Vertical Circle

non uniform circular motion

- A small sphere of mass *m* is attached to the end of a cord of length *R* and whirls in a vertical circle about a fixed point *O*. **Determine the tension in the cord** at any instant when the speed of the sphere is *v* and the cord makes an angle θ with the vertical.

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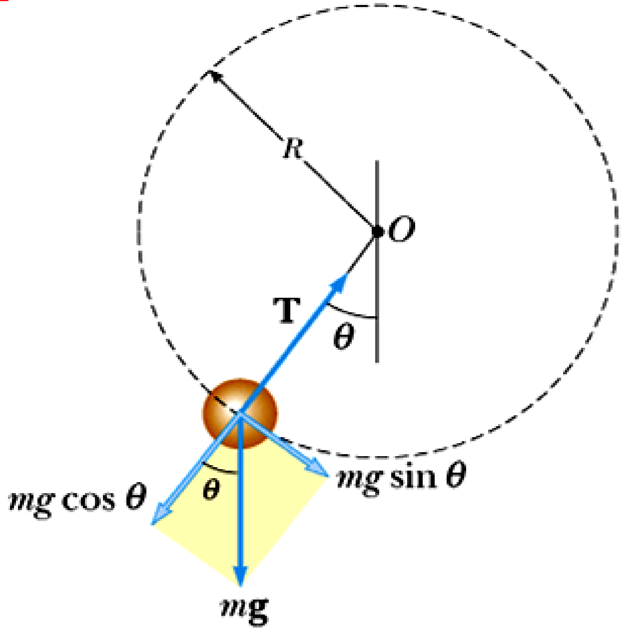
$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$

$$\sum F_t = mg\sin\theta = ma_t$$

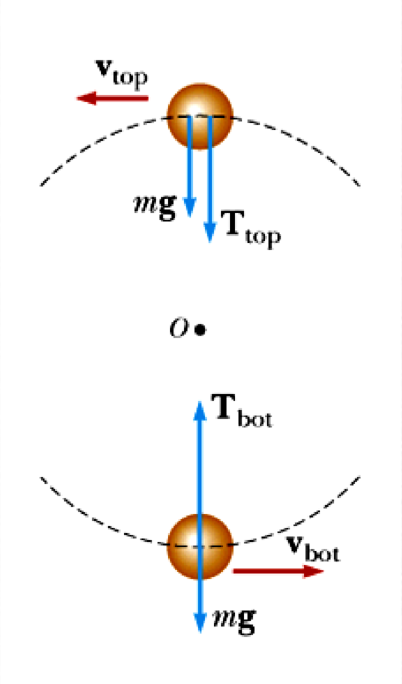
$$\boxed{T = m\left(\frac{v^2}{R} + g\cos\theta\right)}$$

Speed is **not constant**



Example: Mass round a Vertical Circle

- Interesting cases



$$\boxed{T = m\left(\frac{v^2}{R} + g\cos\theta\right)}$$

At the top (θ = 180 deg): $T_{\text{top}} = m\left(\frac{v_{\text{top}}^2}{R} - g\right)$

At the bottom (θ = 0 deg): $T_{\text{bot}} = m\left(\frac{v_{\text{bot}}^2}{R} + g\right)$

The End