CSD1251/CSD1250 Week 2 Tutorial Problems

9 – 15 January 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Qn 1 (Meaning of the limit)

(a) Explain why the following is not true for <u>every</u> real number x.

$$\frac{x^2+x-6}{x-2} = x+3$$

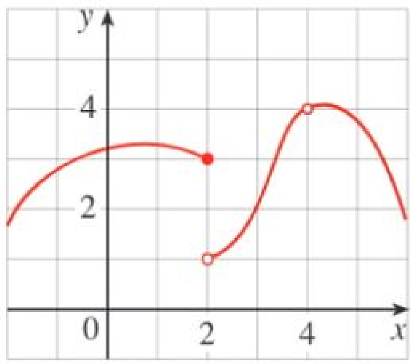
(b) Keeping part (a) in view, explain why

$$\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2}(x+3)$$

is still correct.

Qn 2 (Finding the limit using graphs)

The graph of the function f can be found below.



Find the following limits and function values, if it exists or is defined. If a limit does not exist or a function value is not defined, explain why.

(a) $\lim_{x\to 2^-} f(x)$ (b) $\lim_{x\to 2^+} f(x)$ (c) $\lim_{x\to 2} f(x)$ (d) $f(2)$

(e) $\lim_{x\to 4^-} f(x)$ (f) $\lim_{x\to 4^+} f(x)$ (g) $\lim_{x\to 4} f(x)$ (h) $f(4)$

Qn 3 (Limit Laws)

Evaluate the following limits. For each step, justify by indicating the appropriate Limit Law(s).

(a) $\lim_{x\to 5}(4x^2 - 5x)$ (b) $\lim_{x\to -3}(2x^3 + 6x^2 - 9)$ (c) $\lim_{v\to 2}(v^2 + 2v)(2v^3 - 5)$

(d) $\lim_{t\to 7} \dfrac{3t^2 + 1}{t^2 - 5t + 2}$ (e) $\lim_{t\to -1} \left(\dfrac{2t^5 - t^4}{5t^2 + 4}\right)^3$

Qn 4 (Left and right-handed limits)

Recall that the modulus function $f(x) = |x|$ is defined as

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Use left and right-handed limits to show that

$$\lim_{x \to 0} |x| = 0.$$

Qn 5 (Factorization)

Evaluate the following limits.

(a) $\displaystyle\lim_{h \to 0} \frac{(h-3)^2 - 9}{h}$ (b) $\displaystyle\lim_{t \to 4} \frac{t^2 - 2t - 8}{t - 4}$ (c) $\displaystyle\lim_{x \to -3} \frac{x^2 + 3x}{x^2 - x - 12}$

(d) $\displaystyle\lim_{x \to -2} \frac{x^2 - x - 6}{3x^2 + 5x - 2}$ (e) $\displaystyle\lim_{x \to -5} \frac{2x^2 + 9x - 5}{x^2 - 25}$

Qn 6 (Rationalization)

Evaluate the following limits.

(a) $\displaystyle\lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}$ (b) $\displaystyle\lim_{x \to 9} \frac{9 - x}{3 - \sqrt{x}}$ (c) $\displaystyle\lim_{x \to 2} \frac{2 - x}{\sqrt{x+2} - 2}$

(d) $\displaystyle\lim_{t \to 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t}$ (e) $\displaystyle\lim_{x \to -4} \frac{\sqrt{x^2+9} - 5}{x + 4}$