CSD2201/CSD2200 Week 4 Tutorial Problems

18th September – 24th September 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Starred (*) questions are slightly more conceptual/difficult.

Question 1

Evaluate the following integrals.

(a) $\int x \sec^2 x \, dx$ 　　　　 (b) (*) $\int x^3 \cos(x^2) \, dx$ 　　　　 (c) (*) $\int e^x \sin x \, dx$

(d) $\int t^3 e^t \, dt$ 　　　 (e) $\int \dfrac{\sin^{-1} x}{\sqrt{1 - x^2}} \, dx$ 　　　 (f) $\int (\ln x)^3 \, dx$ 　　　 (g) (*) $\int e^{\sqrt{x}} \, dx$

(h) $\int \sin^3 x \cos^4 x \, dx$ 　　　　 (i) $\int \sin^4 x \cos^3 \, dx$ 　　　　 (j) $\int \cos^2(2x) \, dx$

Question 2

Evaluate the following integrals.

(a) $\int_0^{2\pi} x^2 \sin(2x) \, dx$ 　　　　 (b) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos(x) \ln(\sin x) \, dx$ 　　　　 (c) (*) $\int_0^1 \dfrac{x^3}{\sqrt{4 + x^2}} \, dx$

(d) $\int_1^2 x^2 \ln x \, dx$ 　　　 (e) $\int_1^4 \dfrac{(\ln x)^2}{x^3} \, dx$ 　　　 (f) $\int_1^9 \dfrac{x}{e^x} \, dx$ 　　　 (g) $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \tan^{-1}\left(\dfrac{1}{x}\right) \, dx$

(h) $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^6 x \, dx$ 　　　　 (i) $\int_0^{\frac{\pi}{3}} \sin^5 x \cos^3 x \, dx$ 　　　　 (j) $\int_0^{\frac{\pi}{4}} \sin^4 x \, dx$

Question 3

Let n be a positive integer.

(a) Show that

$$\int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx.$$

(b) Use part (a) to find

$$\int x^4 e^x \, dx.$$

Final Answers:

Q1: (a) $x \tan x + \ln|\cos x| + C$, (b) $\frac{1}{2}[x^2 \sin(x^2) + \cos(x^2)] + C$, (c) $\frac{e^x}{2}(\sin x - \cos x) + C$,

(d) $e^t(t^3 - 3t^2 + 6t - 6) + C$, (e) $\frac{(\sin^{-1} x)^2}{2} + C$, (f) $x[(\ln x)^3 - 3(\ln x)^2 + 6\ln x - 6] + C$,

(g) $2\sqrt{x}\, e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$, (h) $\frac{\cos^7}{7} - \frac{\cos^5 x}{5} + C$, (i) $\frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$,

(j) $\frac{4x + \sin(4x)}{8} + C$

Q2: (a) $-2\pi^2$, (b) $\frac{\sqrt{3}}{2}\left(1 - \ln\left(\frac{\sqrt{3}}{2}\right)\right) - 1$, (c) $\frac{1}{3}(16 - 7\sqrt{5})$, (d) $\frac{8}{3}\ln 2 - \frac{7}{9}$,

(e) $-\frac{1}{32}(\ln 4)^2 - \frac{1}{32}\ln 4 + \frac{15}{64}$, (f) $2e^{-1} - 10e^{-9}$, (g) $\frac{\pi}{6\sqrt{3}} + \frac{1}{2}\ln 3$, (h) $\frac{2}{63}$, (i) $\frac{63}{2048}$,

(j) $\frac{3\pi}{32} - \frac{1}{4}$

Q4: (b) $e^x(x^4 - 4x^3 + 12x^2 - 24x + 24) + C$