CSD2301 Lecture
7. Work and Energy Part 1

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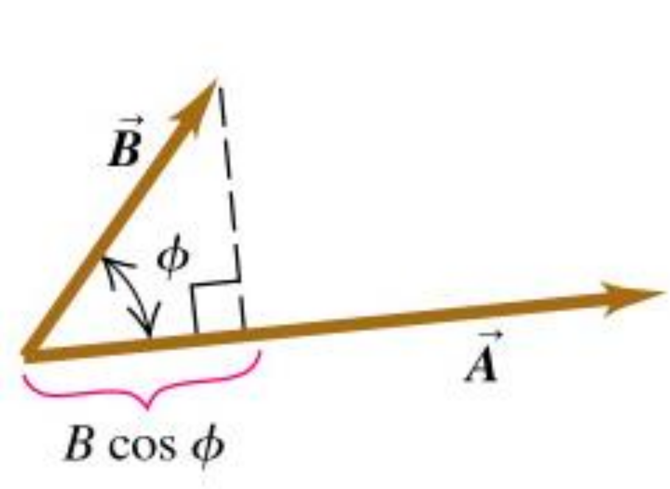
Outline

- Scalar Product
- Work done
- Work-Energy Theorem (Kinetic Energy)
- Power

Scalar Product of 2 Vectors

- The scalar product of any two vectors A and B, denoted by **A·B** is a scalar quantity equal to the product of the magnitude of the two vectors and the cosine of the angle ϕ between them.
- It is often called the dot product.
- **A·B** is the product of the magnitude of **A** and the projection of **B** onto **A**.

$$\boxed{\vec{A} \cdot \vec{B} = AB\cos\phi = \left|\vec{A}\right| \left|\vec{B}\right| \cos\phi}$$



Properties of Scalar Products

- Scalar product is commutative:

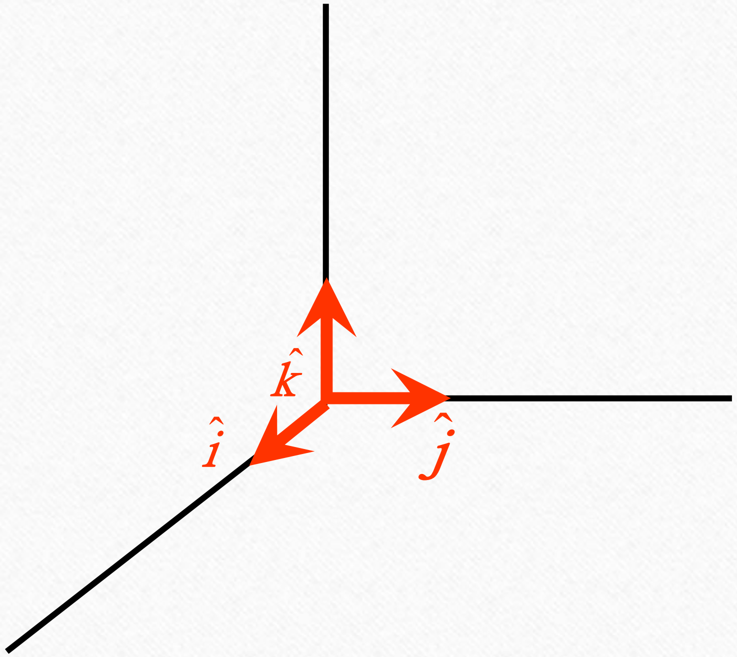
$$\boxed{\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}}$$

- Scalar product obeys the **distributive** law of multiplication:

$$\boxed{\vec{A} \cdot \left(\vec{B} + \vec{C}\right) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}}$$

- **A·B** = **0** if **A** is perpendicular to **B** ($\cos 90^\circ = 0$).
- **A·B** = AB if **A** is parallel to **B** ($\cos 0^\circ = 1$).

Scalar Products of Unit Vectors



$$\hat{\imath}\cdot\hat{\imath}=\hat{\jmath}\cdot\hat{\jmath}=\hat{k}\cdot\hat{k}=1$$

$$\hat{\imath}\cdot\hat{\jmath}=\hat{\jmath}\cdot\hat{k}=\hat{k}\cdot\hat{\imath}=0$$

Scalar Product of 2 Vectors

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

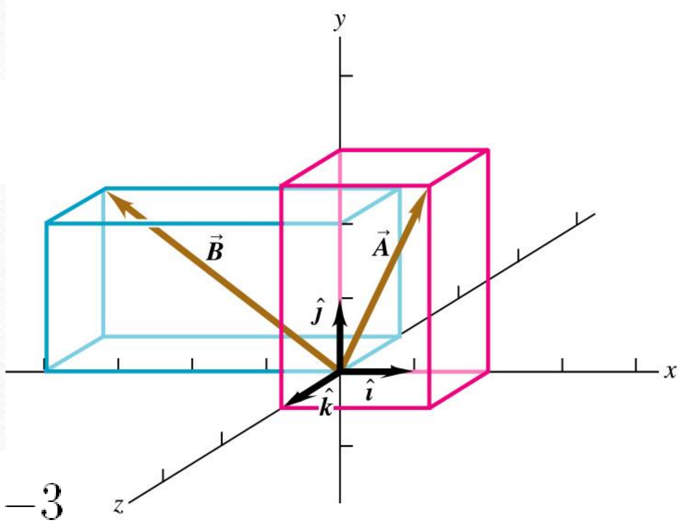
$$\vec{A}\cdot\vec{B} = \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right)\cdot\left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right)$$

$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

Example: Angle between Vectors

- Find the angle between 2 vectors. $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$

$$\vec{B} = -4\hat{\imath} + 2\hat{\jmath} - \hat{k}$$



$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$$

$$\left|\vec{B}\right| = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(-4)^2 + 2^2 + (-1)^2} = \sqrt{21}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = (2)(-4) + (3)(2) + (1)(-1) = -3$$

$$\vec{A} \cdot \vec{B} = \left|\vec{A}\right| \left|\vec{B}\right| \cos\phi \Rightarrow -3 = \sqrt{14}\sqrt{21}\cos\phi \Rightarrow \boxed{\phi = \cos^{-1}\left(\frac{-3}{\sqrt{14}\sqrt{21}}\right) = 100^\circ}$$

Work and Energy

- In classical mechanics, concepts of work and energy are critical.
- **Work-energy concepts** are based on Newton's laws.
  - Can be applied to the dynamics of a mechanical system
  - Useful in complex situations (for e.g. variable forces)
  - Applicable to a wide range of phenomena (for e.g. electro-magnetism, atomic physics etc.)

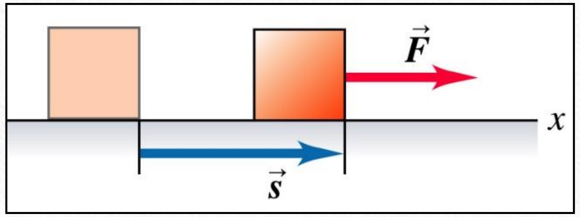
Energy

- Energy is present in various forms
  - For e.g. mechanical energy, electromagnetic energy, electrical energy, chemical energy, thermal energy, nuclear energy, etc.
- Energy cannot be created or destroyed, but can be converted from one form to another
- **Conservation of energy** – when energy is converted from one form to another, the total amount in the system remains the same
  - For e.g. car petrol: chemical → mechanical + electrical + heat

Work done by a Constant Force

- Work done by a force = force x displacement

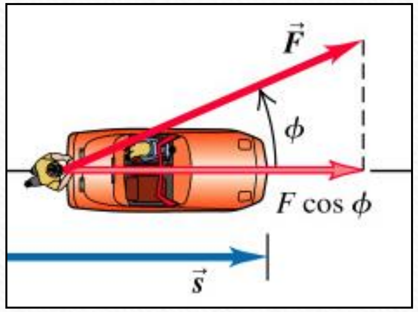
$$\boxed{W = Fs}$$ SI Unit : N·m = joule (J)



- **Work done** by an agent exerting a constant force is the product of the **component of the force in the direction of the displacement** and the **magnitude of the displacement of the force**

$$\boxed{W = Fs\cos\phi}$$

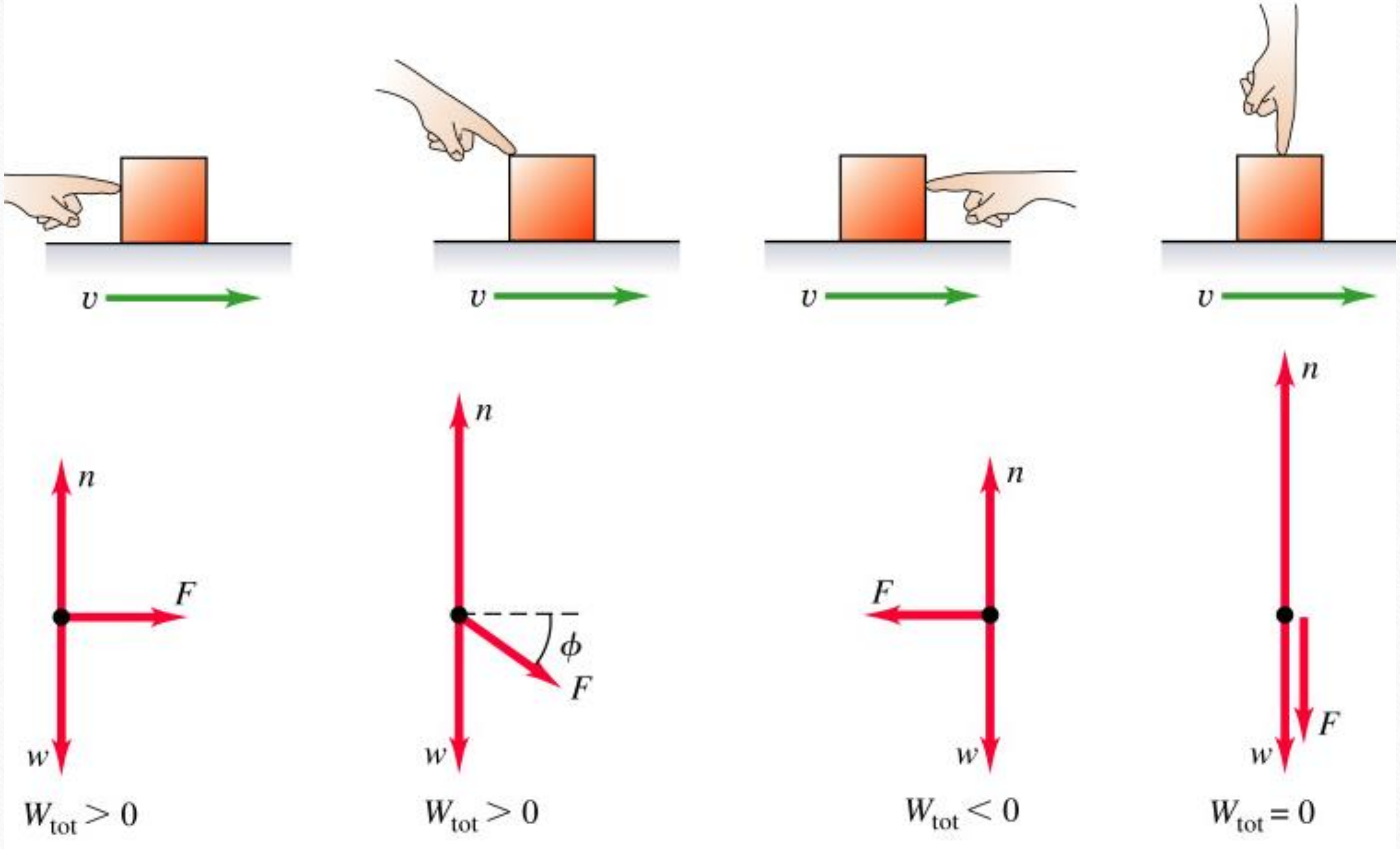
$$\boxed{W = \vec{F}\cdot\vec{s}}$$



Work done by a Constant Force

$$\boxed{W = \vec{F} \cdot \vec{s}}$$

- There is no work done if object doesn't move
- There is no work done if force applied is perpendicular to the displacement
- Work done can be positive or negative
  - For e.g. when object is lifted, work done by the applied force is positive as lifting force is upward. However, the work done by gravitational force is negative.



v
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F
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$W_{tot} > 0$
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n
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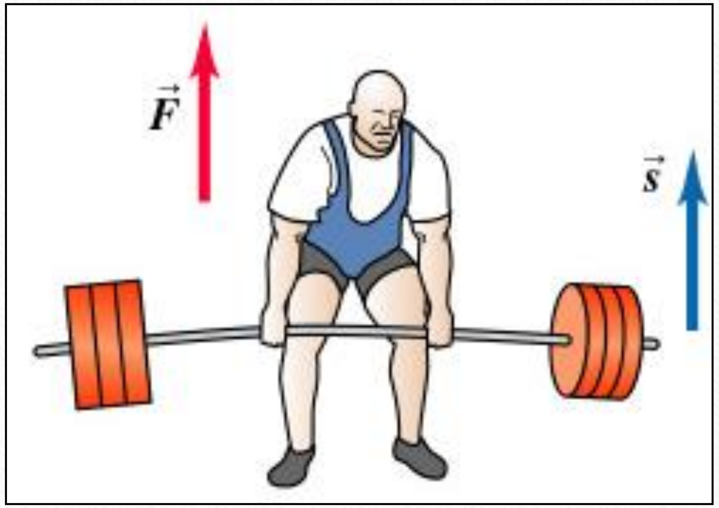
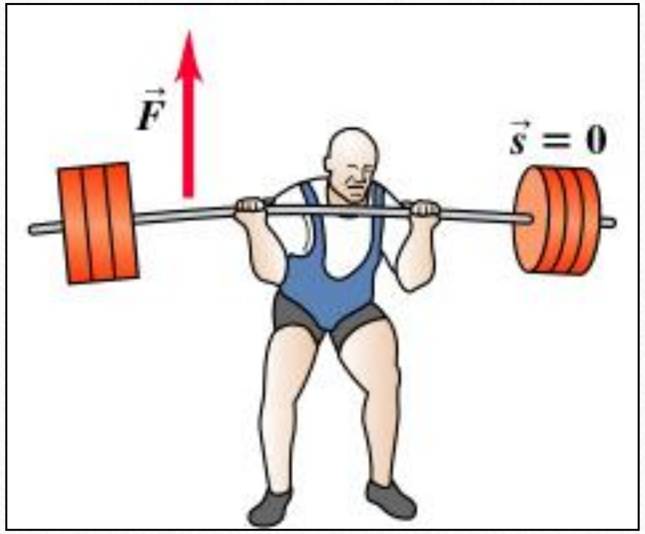
F
$W_{tot} = 0$

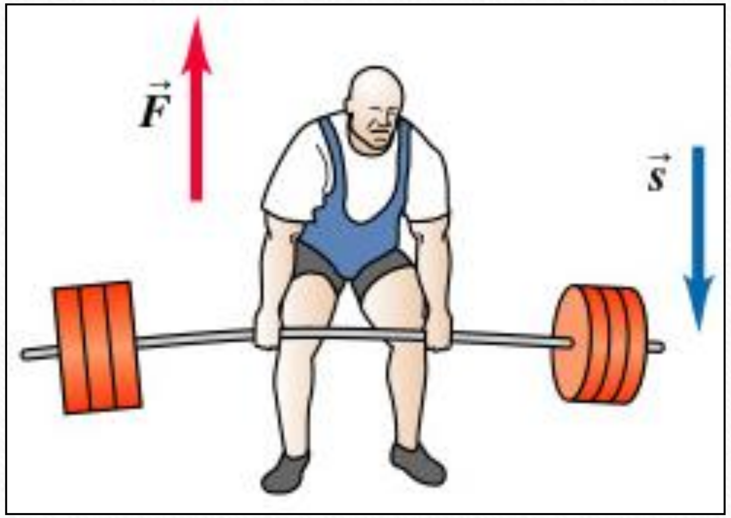
Illustration: Work done by Weight Lifter



+ve work done



No work done



-ve work done

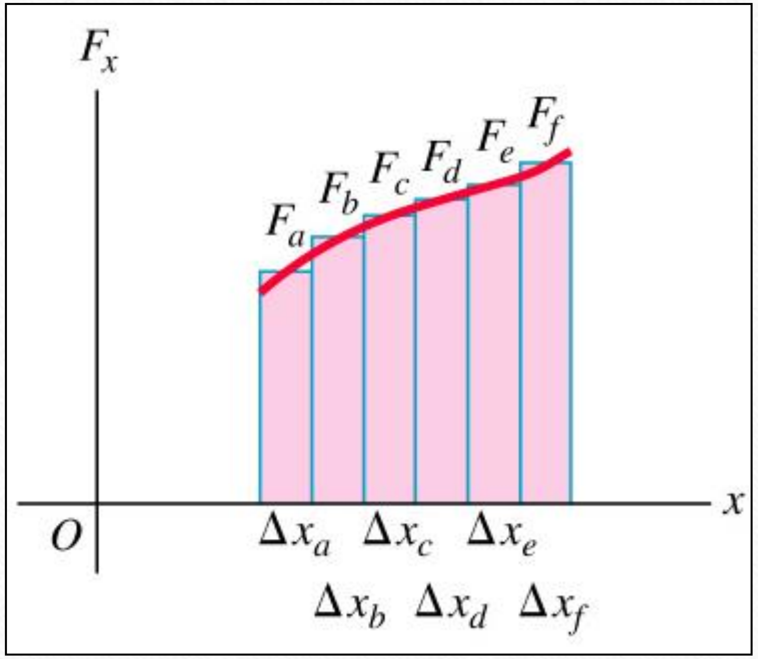
Work done by a Varying Force

- For a varying force **F**(x), consider the work done over a small displacement $\Delta \boldsymbol{x}_i$.

$$\Delta W = F_i \Delta x_i$$

- Total work done from the displacement from x_1 to x_2 is

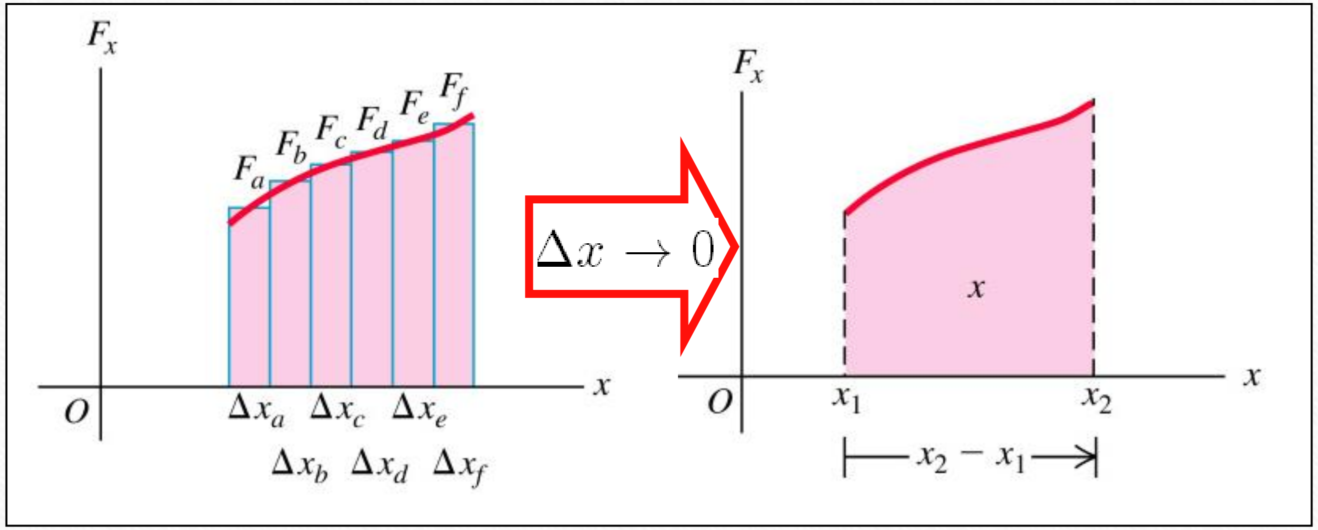
$$W \approx \sum F_i \Delta x_i$$



Work done by a Varying Force

- The work done by the component F_x of the varying force as the particle moves from x_1 to x_2 is equal to the area under the curve.

$\Delta x \to 0$ ⇒ $\boxed{W = \int_{x_1}^{x_2} F_x \, dx}$



Work done on a Spring

constant

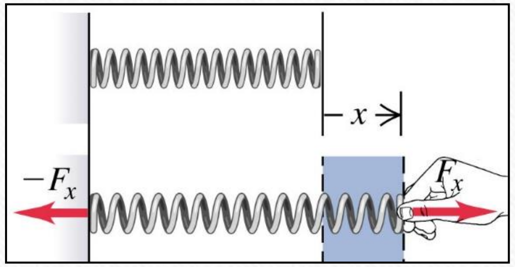
- Force required to stretch a spring: $\boxed{F_x = kx}$ (Hooke's Law)
- Work done in stretching spring from $x = 0$ to X

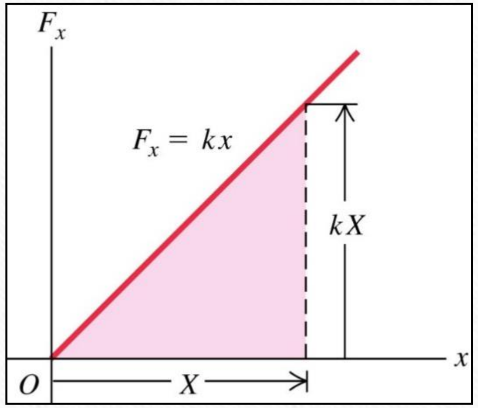
$$W = \int_0^X F_x\,dx = \int_0^X kx\,dx = \frac{1}{2}kX^2$$

elastic potential energy

- Work done in stretching spring from $x = x_1$ to $x = x_2$

$$\boxed{W = \int_{x_1}^{x_2} F_x\,dx = \int_{x_1}^{x_2} kx\,dx = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2}$$





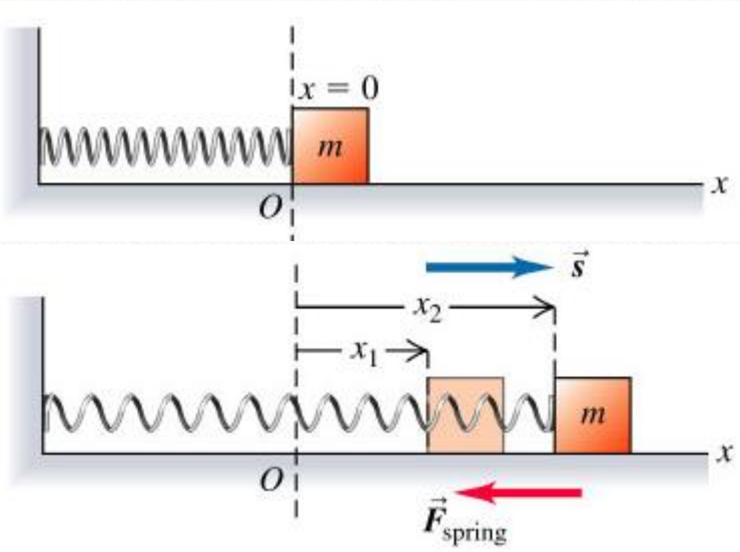
$$\boxed{F_x = kx}$$ (Hooke's Law)

- Work done **on the spring (by external force)** when the block is displaced from x_1 to x_2:

$$\boxed{W = \int_{x_1}^{x_2} kx\,dx = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2}$$

- Work done **by the spring** when the block is displaced from x_1 to x_2:

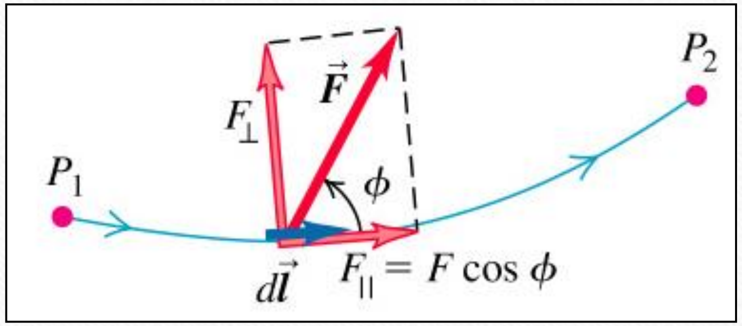
$$\boxed{W_{\text{el}} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2}$$



Work along a curved path

- In general, for a varying force along a curved path, small element of work done dW during displacement $d\boldsymbol{l}$ is

$$dW = F\cos\phi\, dl = F_{||}dl = \vec{F}\cdot\vec{dl}$$



- Total work done along path P_1 to P_2 is

$$\boxed{W = \int_{P_1}^{P_2} F\cos\phi\, dl = \int_{P_1}^{P_2} F_{||}dl = \int_{P_1}^{P_2} \vec{F}\cdot\vec{dl}}$$

- Note that the component of force perpendicular to path does no work
  - No work done for $F_\perp = F\sin\phi$

Work-energy Theorem (F constant)

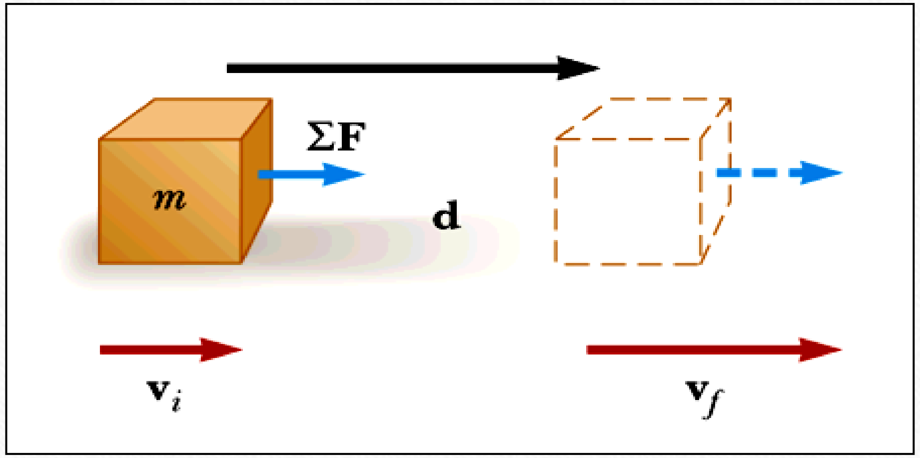
$$a = \frac{v_f - v_i}{t}$$

$$d = \frac{1}{2}(v_i + v_f)\,t$$

$$W = Fd = (ma)d$$

$$W = m\left(\frac{v_f - v_i}{t}\right)\frac{1}{2}(v_i + v_f)\,t$$

$$\boxed{W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$$



Kinetic Energy: $\boxed{K = \frac{1}{2}mv^2}$

Work-energy Theorem (General)

- Work done by an external force (can be varying) in moving an object from x_i to x_f.

$$W = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} madx$$

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

$$W = \int_{x_i}^{x_f} mv\frac{dv}{dx}dx = \int_{v_i}^{v_f} mvdv$$

$$\boxed{W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$$

Kinetic Energy

- Kinetic energy is associated with the motion of a body.
- Kinetic energy is a scalar and has the same units as work (J).
- Work done by a force F in displacing a particle is equal to the change in kinetic energy of the particle:

$$\boxed{W = K_f - K_i = \Delta K}$$

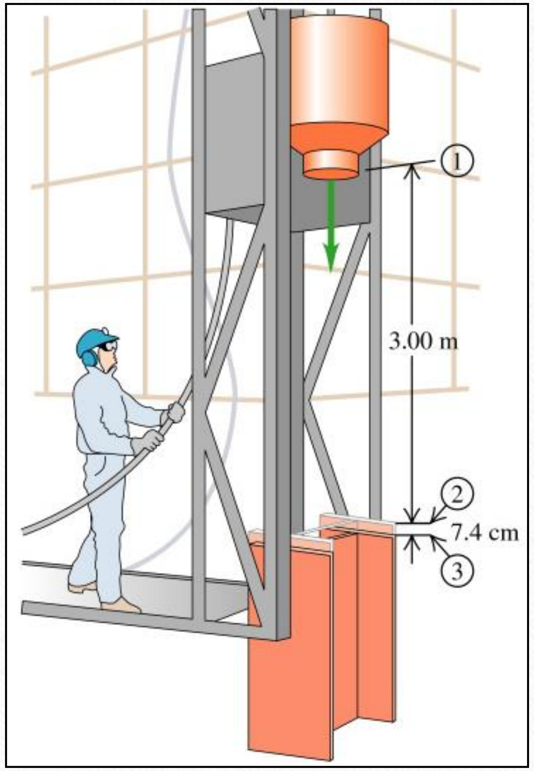
- $\Delta K > 0 \rightarrow$ speed increases ; $\Delta K < 0 \rightarrow$ speed decreases

Example: Hammerhead Forces

In a pile driver, a steel hammerhead with mass 200 kg is lifted 3.00 m above the top of a vertical I-beam being driven into the ground. The hammer is then dropped, driving the I-beam 7.4 cm further into the ground. The vertical rails that guide the hammerhead exert a constant 60-N friction force on the hammer head. Use work-energy theorem to find

a) the speed of the hammerhead just as it hits the I-beam.

b) the average force the hammerhead exerts on the I-beam.



Example: Hammerhead Forces

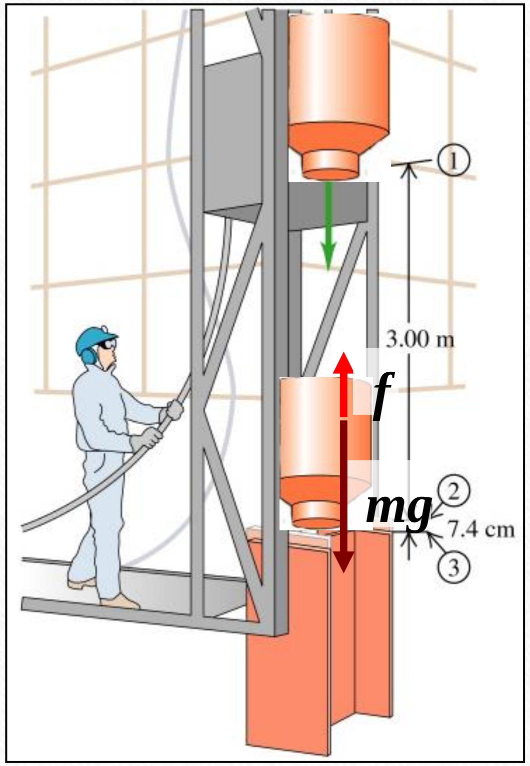
a) the speed of the hammerhead just as it hits the I-beam.

Net downward force = *mg - f*

$$W_{net} = K_2 - K_1$$

$$(mg - f)s_{12} = \frac{1}{2}mv_2^2 - 0$$

$$\boxed{v_2 = \sqrt{\frac{2[200(9.80) - 60](3)}{200}} = 7.55 \text{ m/s}}$$



Example: Hammerhead Forces

b) the average force the hammerhead exerts on the I-beam.

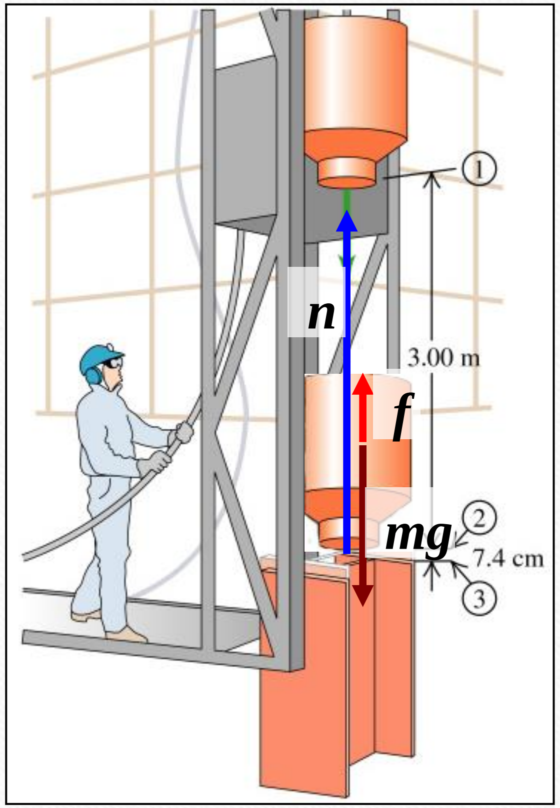
Net downward force = *mg* - *f* - *n*

$$W_{net,2\to 3} = K_3 - K_2$$

$$(mg - f - n)s_{23} = 0 - \frac{1}{2}mv_2^2$$

$$[200(9.8) - 60 - n](0.074) = 0 - \frac{1}{2}200(7.55^2)$$

$$\boxed{n = 1960 - 60 + \frac{200(7.55^2)}{2(0.074)} = 7.9 \times 10^4 \text{ N}}$$



Power

- Power is the rate of doing work.
- Average power: $\boxed{P_{av} = \dfrac{\Delta W}{\Delta t}}$
- For instantaneous power:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \Rightarrow \boxed{P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v}}$$

- Units: Watt (W)
- 1 horsepower = 746 W
- 1 kWh = 10^3 x 3600 Ws = 3.6 MJ (This is unit for Energy)

Example: Power of the Heart

- The human heart is a powerful and extremely reliable pump. Each day it takes in and discharges about 7500 L of blood. Assume that the work done by the heart is equal to the work required to lift this amount of blood a height equal to that of the average American female (1.63 m). The density of blood is 1.05 x 10^3 kg/m^3.

a) How much work does the heart do in a day?

$$m = (7500\,\mathrm{L})(0.001\,\mathrm{m}^3/\mathrm{L})(1.05 \times 10^3\ \mathrm{kg/m}^3) = 7880\ \mathrm{kg}$$

$$W = (mg)y = 7880(9.80)(1.63) = \boxed{1.26 \times 10^5\ \mathrm{J.}}$$

b) What is its power output in watts?

$$P_{AV} = \frac{W}{t} = \frac{1.26 \times 10^5}{(24)(3600)} = \boxed{1.46\mathrm{W}}$$

The End