2024 Fall MAT340 - Probability and Statistics
Quiz 1 Solution

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1. Ans: A

   There are 10 women to be chosen as the outcome of the first selection, and the subsequent choice of one of her children as the outcome of the second selection, we have $10 \times 3 = 30$ possible choices.

2. Ans: D

   There are $9! = 362,880$ possible batting orders.

3. Ans: C

   To reach B from A we need to take 4 steps to the right and 3 steps upward. Let R be one step to the right and U be one step upward. To find the different ordered arrangements of four R and three U, the total possible arrangement is

   $$\frac{7!}{3!4!} = 35$$

4. Ans: B

   There are 6! possible rankings for the men and 4! possible rankings for the women, there are $6! \times 4! = 17,280$ possible rankings.

5. Ans: D

   There are 4!3!2!1! arrangements such that the mathematics books are first in line, then the physics books, then the chemistry books, and then the biology book. There are also 4! possible orderings of the subjects for each arrangement. The desired answer is $4!4!3!2!1! = 6912$.

6. Ans: C

   There are $\binom{4}{2}\binom{4}{3}$ different combinations, and because there are 13 different choices for the kind of pair and after a pair has been chosen, there are 12 other choices for the denomination of the remaining 3 cards, it follows that the probability of a full house is

   $$\frac{13 \times 12 \times \binom{4}{2}\binom{4}{3}}{\binom{52}{5}} = 0.0014$$

7. Ans: A

   Since

   $$A^C = \{c, d\},$$

   then

   $$A^C \cup B = \{b, c, d\},$$

   and then

   $$(A^C \cup B)^C = \{a\},$$

   and also

   $$C^C = \{a, b, c\}$$

   We have

   $$(A^C \cup B)^C \cap C^C = \{a\}$$

   The probability $P((A^C \cup B)^C \cap C^C) = 0.1$

8. Ans: B

   The remaining 26 cards, exactly 5 of them being clubs to be distributed among players C and D hands. The conditional probability that player C will have 3 clubs among his 13 cards is

$$\frac{\binom{5}{3}\binom{21}{10}}{\binom{26}{13}} = 0.339$$

9. Ans: C

   Let G denote the event that the marble selected is green, and let B^C denote the event that it is not blue. This is a conditional probability of G given B^C, that is

$$P(G|B^C) = \frac{P(G \cap B^C)}{P(B^C)} = \frac{\frac{5}{25}}{\frac{15}{25}} = \frac{1}{3}$$

10. Ans: C

    Let RR, BB and RB denote, respectively, the events that the chosen card is all red, all black, or red-black card. Let R be the event that the upturned side of the chosen card is red, the desired probability is given by

$$\begin{aligned} P(RB|R) &= \frac{P(RB \cap R)}{P(R)} \\ &= \frac{P(R|RB)P(RB)}{P(R|RR)P(RR) + P(R|RB)P(RB) + P(R|BB)P(BB)} \\ &= \frac{(\frac{1}{2})(\frac{1}{3})}{(1)(\frac{1}{3}) + (\frac{1}{2})(\frac{1}{3}) + (0)(\frac{1}{3})} \\ &= \frac{1}{3} \end{aligned}$$

11. Ans: D

    Let G denote the event that the suspect is guilty and C the event that he possesses the characteristic of the criminal. The desired probability is

$$\begin{aligned} P(G|C) &= \frac{P(G \cap C)}{P(C)} \\ &= \frac{P(C|G)P(G)}{P(C|G)P(G) + P(C|G^c)P(G^c)} \\ &= \frac{(1)(0.6)}{(1)(0.6) + (0.2)(0.4)} \\ &= 0.882 \end{aligned}$$

12. Ans: A

    Let D denote the event that you have the illness, and let S denote the event that the test signals positive. Since the probability that the test correctly signals someone without illness as negative is 0.95, then the probability of a positive test without illness is $P(S|D^c) = 0.05$. The probability that you have the illness when the test result is positive is

$$\begin{aligned} P(D|S) &= \frac{P(S|D)P(D)}{P(S|D)P(D) + P(S|D^c)P(D^c)} \\ &= \frac{0.99(0.0001)}{0.99(0.0001) + 0.05(1 - 0.0001)} \\ &= 0.002 \end{aligned}$$

13. Ans: C

$$\begin{aligned} P(X < 1.65) &= f(1.25) + f(1.5) \\ &= \frac{1}{8} + \frac{2}{8} \\ &= \frac{3}{8} \end{aligned}$$

14. Ans: C

$$\begin{aligned}
P(X > 2) &= 1 - P(X = 0) - P(X = 1) - P(X = 2) \\
&= 1 - f(0) - f(1) - f(2) \\
&= 1 - \frac{3}{4}\left(\frac{1}{4}\right)^0 - \frac{3}{4}\left(\frac{1}{4}\right)^1 - \frac{3}{4}\left(\frac{1}{4}\right)^2 \\
&= 0.0156
\end{aligned}$$

15. Ans: B

$$p(3) = \frac{7}{8} - \frac{3}{4} = \frac{1}{8}$$

16. Ans: A

$$\begin{aligned}
P(X = 1) &= \frac{800}{850} \times \frac{50}{849} + \frac{50}{850} \times \frac{800}{849} = 0.111 \\
k &= 0.886 + 0.111 = 0.997
\end{aligned}$$

17. Ans: C

Let X be the number of defective item produced by the machine, then $X \sim B(10, 0.1)$. The desired probability is

$$\binom{10}{0}(0.1)^0(0.9)^{10} + \binom{10}{1}(0.1)^1(0.9)^9 = 0.7361$$

18. Ans: B

Let X denote the number of particles in the area of a disk. The mean number of particles is $\lambda A = 0.1 \times 100 = 10$ particles. Therefore,

$$P(X = 12) = \frac{10^{12}e^{-10}}{12!} = 0.095$$

19. Ans: A

$$\begin{aligned}
\lambda \times 2 \textbf{ weeks} &= 4 \\
P(X \geq 3) &= 1 - P(X = 0) - P(X = 1) - P(X = 2) \\
&= 1 - \frac{4^0}{0!}e^{-4} - \frac{4^1}{1!}e^{-4} - \frac{4^2}{2!}e^{-4} \\
&= 1 - e^{-4} - 4e^{-4} - 8e^{-4} \\
&= 0.7619
\end{aligned}$$

20. Ans: D

The probability to obtain a black ball is $p = \frac{M}{M+N}$, the probability that at least k trials are necessary to obtain a success is equal to the probability that the first $k-1$ trials are all failures, that is, for a geometric random variable

$$P(X \geq k) = (1-p)^{k-1} = \left(1 - \frac{M}{M+N}\right)^{k-1} = \left(\frac{N}{M+N}\right)^{k-1}$$