CSD2301 Lecture
14. Rolling Motion

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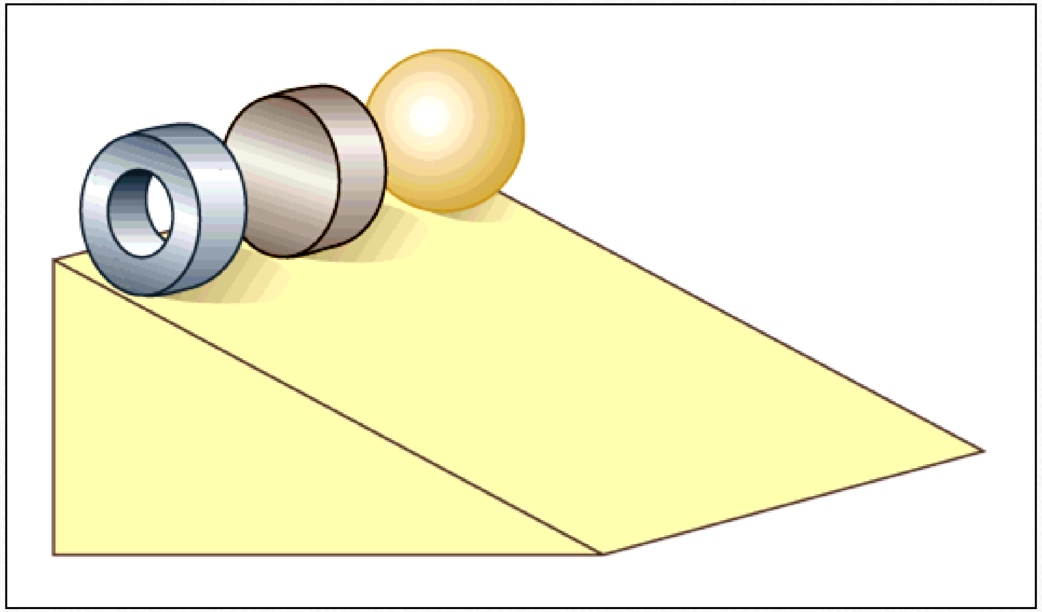
Outline

- Rolling motion
- Rolling friction
- Rolling with slipping

Rolling Motion

- The axis of rotation is not fixed in space.
- Assumptions:
  - Homogeneous rigid body
  - High degree of symmetry (e.g. cylinder, sphere etc.)
  - Rolling without slipping
  - Rolling motion along a flat surface



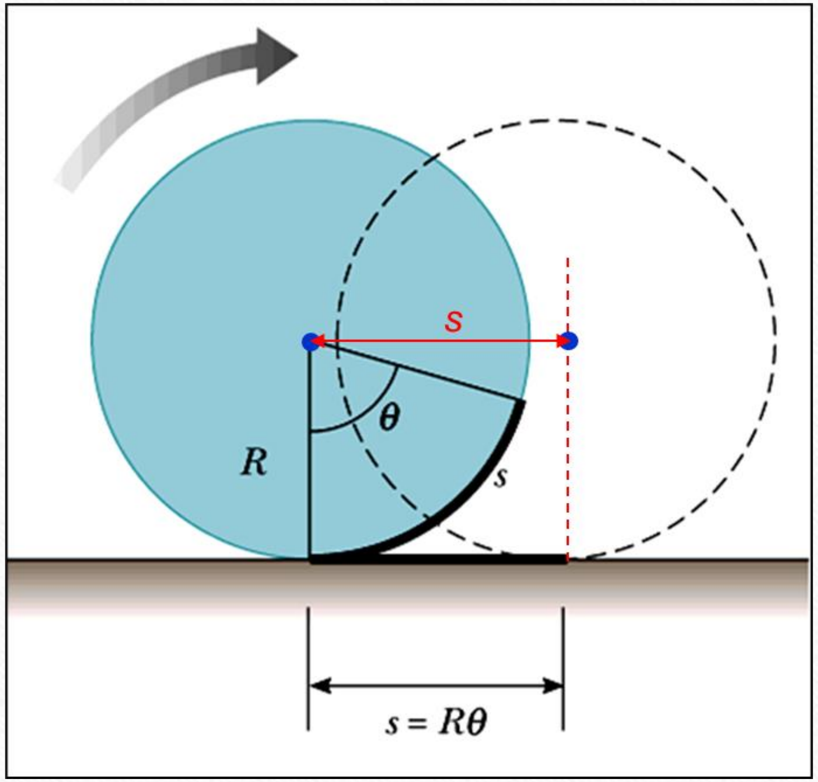
Rolling Motion

- When the cylinder rotates through an angle θ, its centre of mass moves a linear distance s.

$$\boxed{s = R\theta}$$

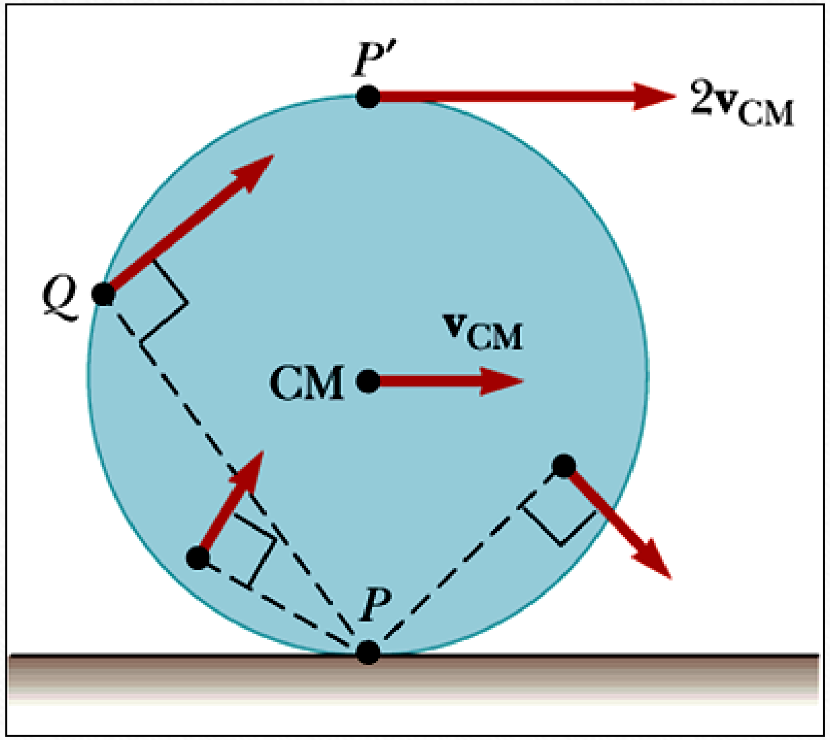
$$\boxed{v_{CM} = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega}$$

$$\boxed{a_{CM} = \frac{dv_{CM}}{dt} = R\frac{d\omega}{dt} = R\alpha}$$

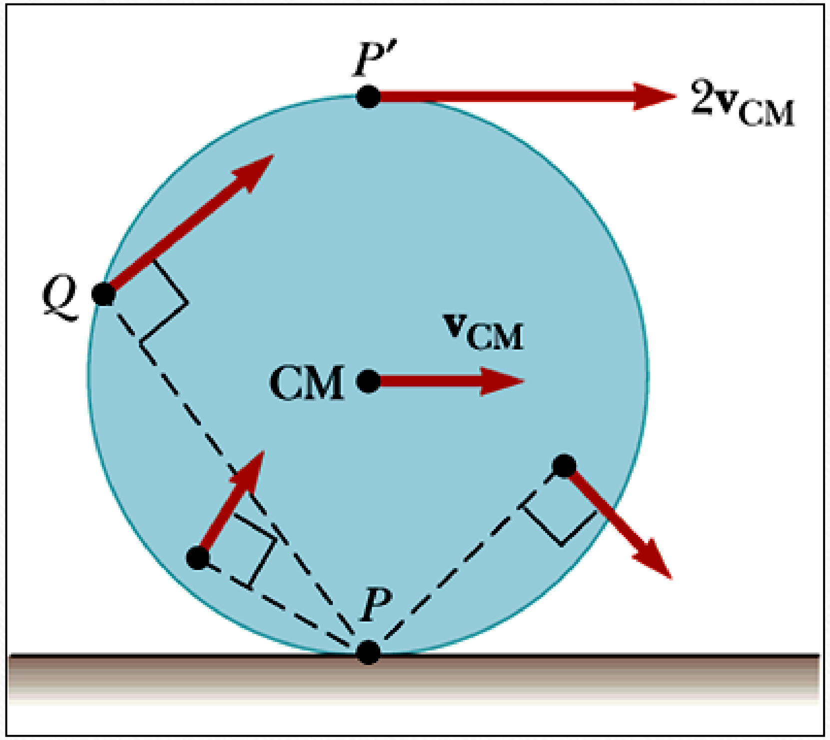


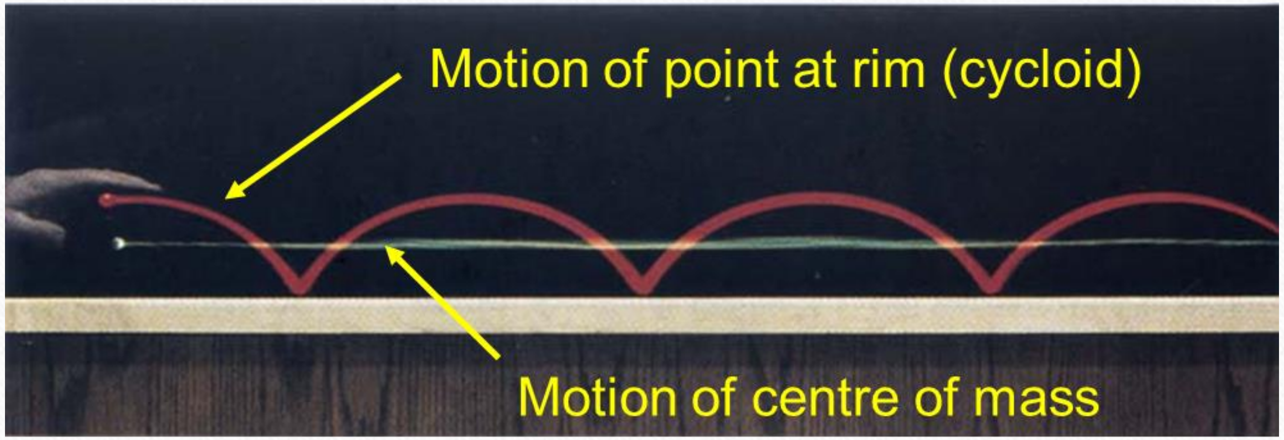
Rolling Motion

- All points on the cylinder have the same angular speed, but different linear velocity.
- The linear velocity of any point is in a direction perpendicular to the line from that point to the contact point P.
- At any instant, the part of the rim that is at point *P* is at rest relative to the surface because slipping does not occur.



Rolling Motion

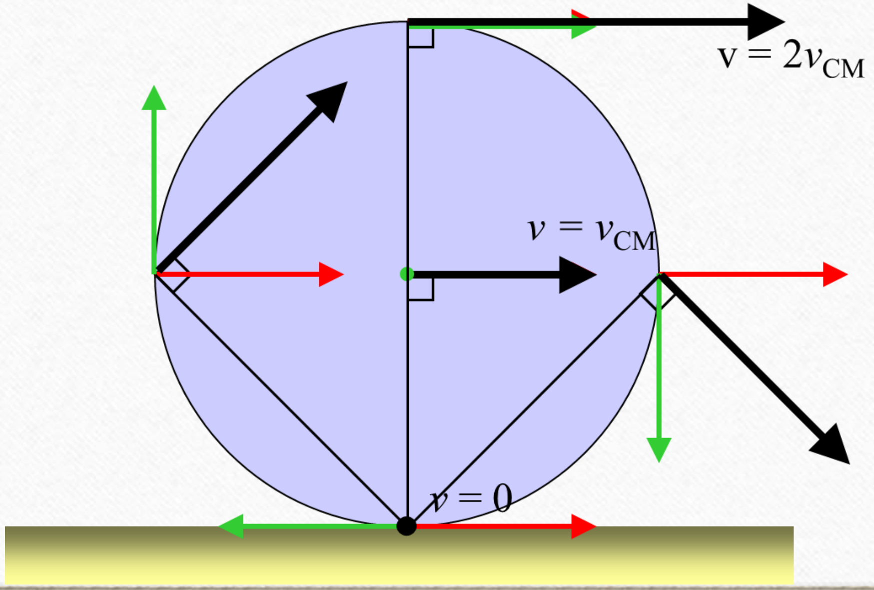


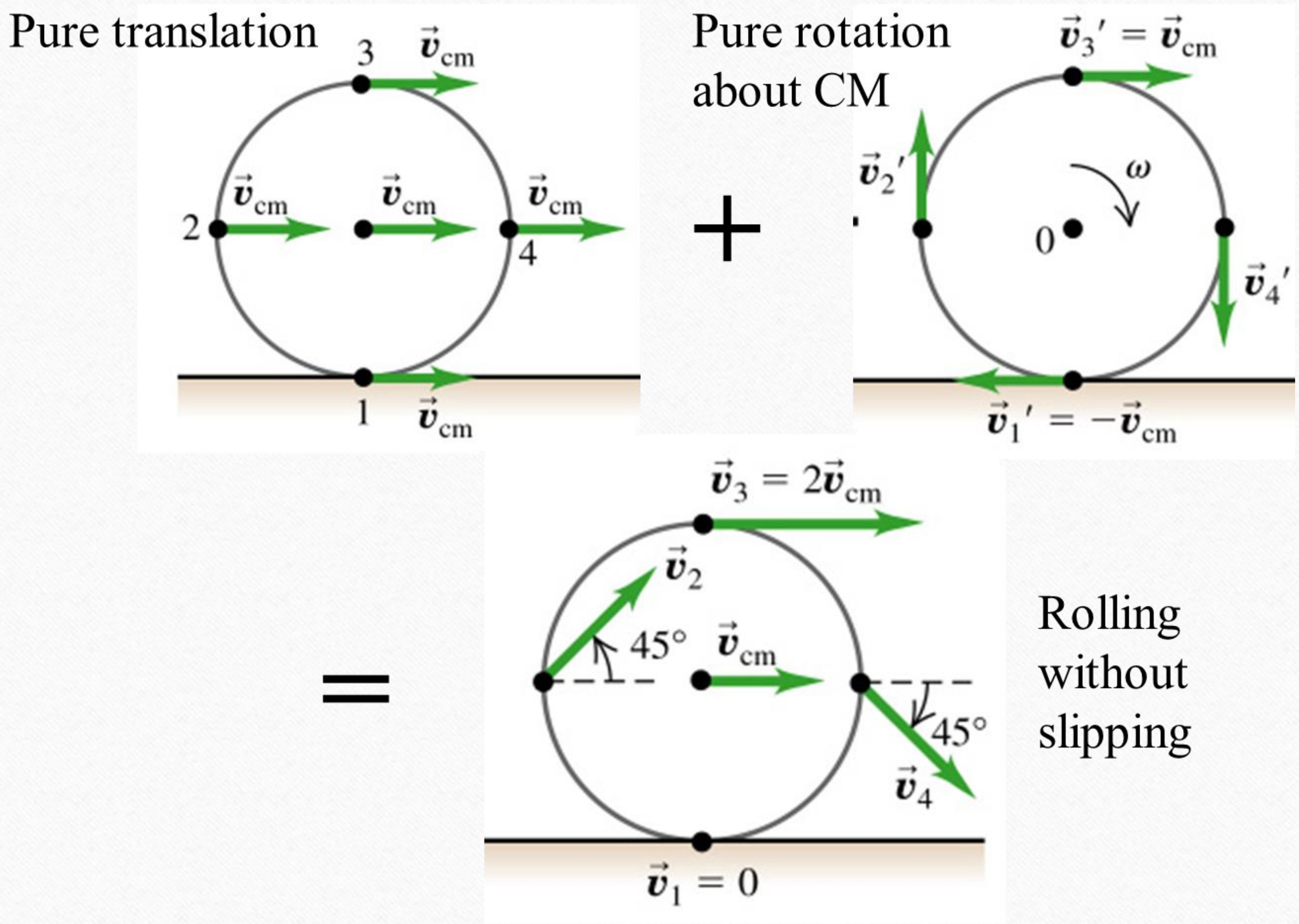


Rolling Motion

- Rolling motion with CM moving at v_{CM} =

  Pure translation with speed v_{CM} + Pure rotation with $\omega = v_{CM}/R$




Pure translation
3
$\vec{v}_{cm}$
2
$\vec{v}_{cm}$
$\vec{v}_{cm}$
$\vec{v}_{cm}$
4
1
$\vec{v}_{cm}$
+
Pure rotation
about CM
$\vec{v}_3{}' = \vec{v}_{cm}$
$\vec{v}_2{}'$
ω
0
$\vec{v}_4{}'$
$\vec{v}_1{}' = -\vec{v}_{cm}$
=
$\vec{v}_3 = 2\vec{v}_{cm}$
$\vec{v}_2$
45°
$\vec{v}_{cm}$
45°
$\vec{v}_4$
$\vec{v}_1 = 0$
Rolling
without
slipping

Rolling Motion

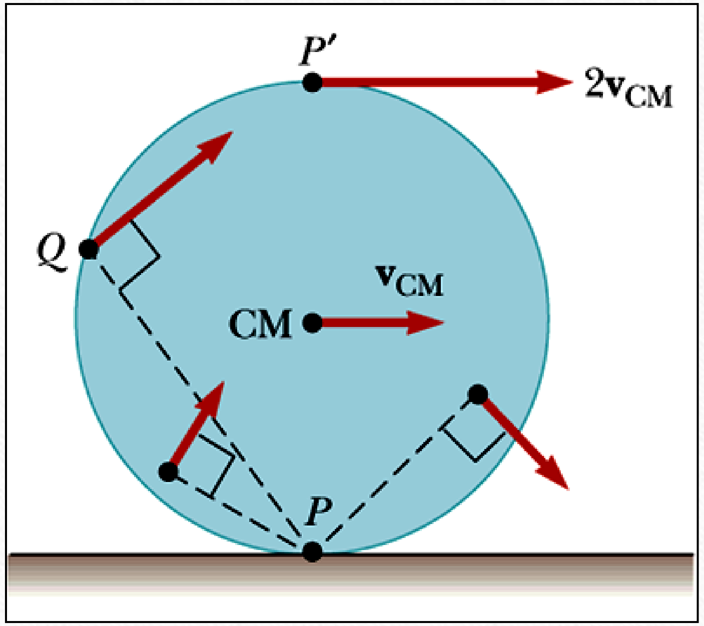
- Consider rotation about the contact point P.

$$\boxed{K = \frac{1}{2} I_P \omega^2}$$

Since $\boxed{I_P = I_{CM} + MR^2}$

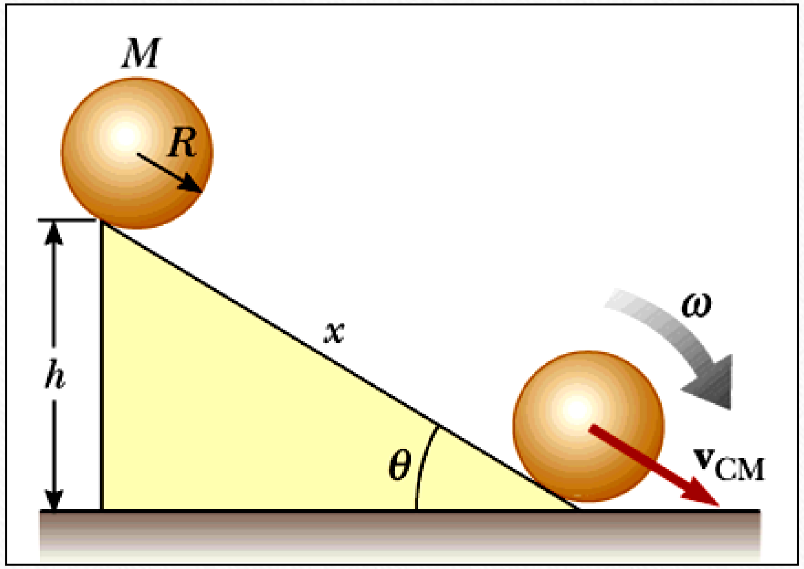
$$\boxed{K = \frac{1}{2} I_{CM} \omega^2 + \frac{1}{2} MR^2 \omega^2 = \frac{1}{2} I_{CM} \omega^2 + \frac{1}{2} M v_{CM}^2}$$

The total KE of an object undergoing rolling motion is the sum of the rotational KE about its CM and the translational KE of its CM.



Sphere on an Incline

- Accelerated rolling motion is possible only if a frictional force is present.
- But no loss of mechanical energy because the contact point is at rest at any instance.



Energy conservation ⇨ $$K = \frac{1}{2} I_{CM}\omega^2 + \frac{1}{2} M v_{CM}^2 = Mgh$$

Sphere on an Incline

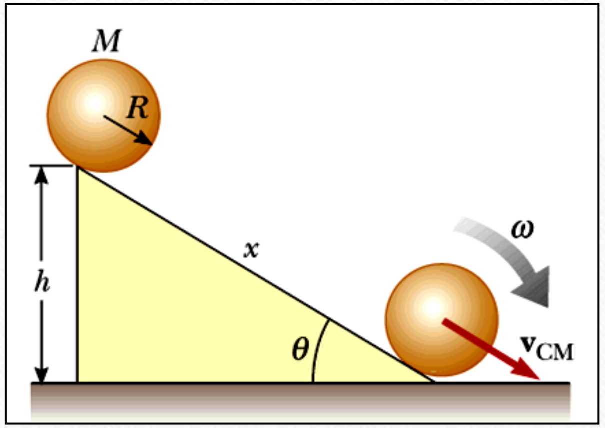
Energy conservation

$$K = \frac{1}{2} I_{CM}\omega^2 + \frac{1}{2} M v_{CM}^2 = Mgh$$

$$v_{CM} = R\omega$$

$$I_{CM}\frac{v_{CM}^2}{MR^2} + v_{CM}^2 = 2gh$$

$$\boxed{v_{CM} = \sqrt{\frac{2gh}{1 + I_{CM}/MR^2}}}$$

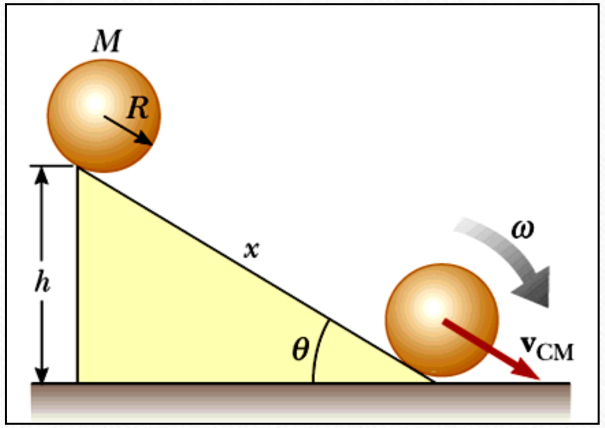


Sphere on an Incline

- Energy Method (to find a_{CM}):

$$v_{CM} = \sqrt{\frac{2gh}{1 + I_{CM}/MR^2}}$$

For sphere, $I_{CM} = \frac{2}{5}MR^2$



$$v_{CM} = \sqrt{\frac{10}{7}gh}$$

$h = x\sin\theta$

$$v_{CM}^2 = 2a_{CM}x$$

$$\boxed{a_{CM} = \frac{5}{7}g\sin\theta}$$

Sphere on an Incline

- Dynamics Method (to find a_{CM}):

$$\sum F_x = Mg\sin\theta - f = Ma_{CM}$$

$$\sum F_y = n - Mg\cos\theta = 0$$

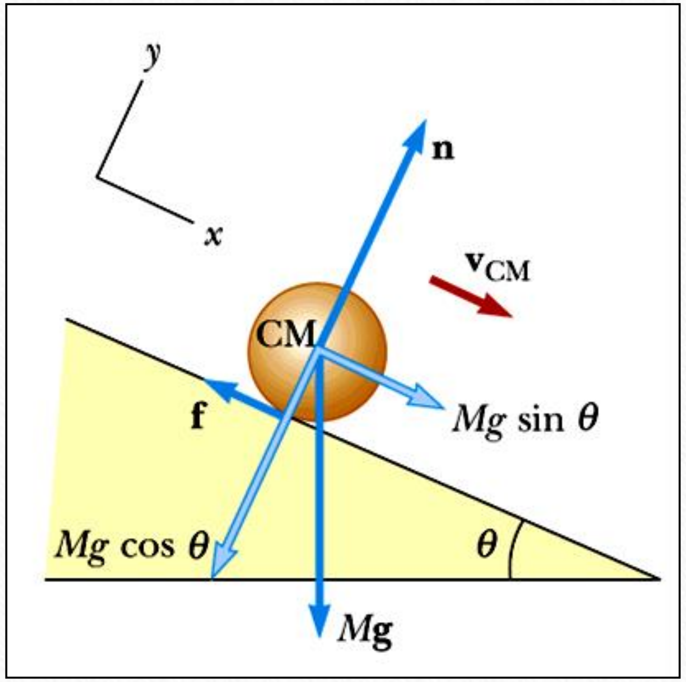
$$\tau_{CM} = fR = I_{CM}\alpha$$

$$a_{CM} = R\alpha$$

$$f = I_{CM}\frac{\alpha}{R} = I_{CM}\frac{a_{CM}}{R^2} = \frac{2}{5}MR^2\frac{a_{CM}}{R^2} = \frac{2}{5}Ma_{CM}$$

$$Mg\sin\theta - \frac{2}{5}Ma_{CM} = Ma_{CM}$$

$$\boxed{a_{CM} = \frac{5}{7}g\sin\theta}$$



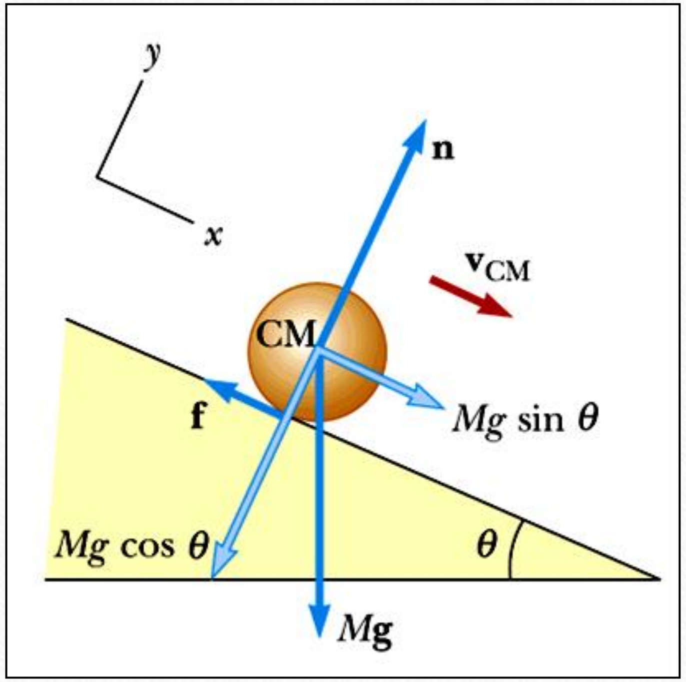
Questions Arising

1. What if θ is increased so that

$$f = \frac{2}{7} Mg\sin\theta > \mu_s n = \mu_s Mg\cos\theta$$

i.e., $\boxed{\tan\theta > \frac{7}{2}\mu_s}$?

It will slide and roll. $v, \omega \uparrow$, but $\boxed{v_{cm} \neq R\omega}$

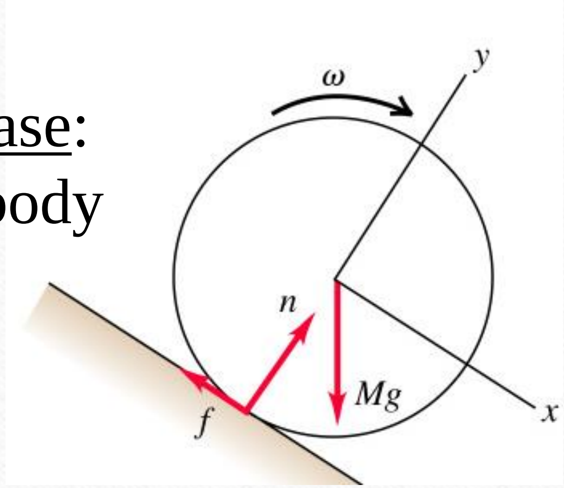


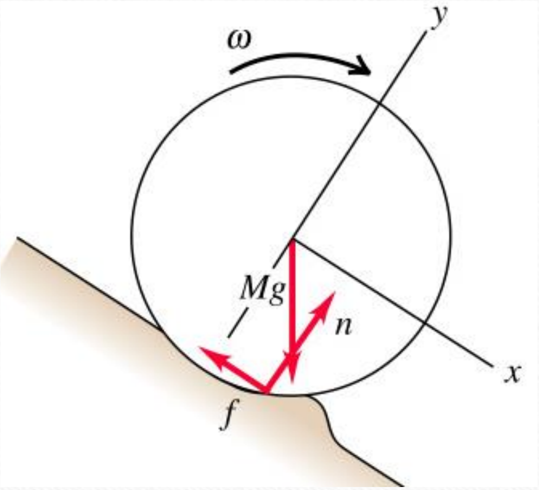
2. What happens when $\theta \to 0$? Will $f \to 0$? $\boxed{\text{Yes!}}$
   - If friction = 0 AND point of contact does not move, why does a rolling ball, say a basketball, ultimately stop? Due to presence of other resistive forces such as air resistance and "rolling friction".

Rolling Friction

- Rigid body is an idealization, especially for a ball or a rubber-tyred wheel. Deformation occurs, and the wheel rests not on a single point but on an area. Sliding occurs on this area of contact, and "normal force" has a small backward component to motion. We call this **rolling friction** or tractive resistance (by engineers).
- Coefficient of rolling friction (μ_r) is much smaller than those of static (μ_s) and kinetic (μ_k) friction. Typical value for μ_r is 0.002 to 0.003 for steel wheels on steel rails, and 0.01 – 0.02 for rubber tyres on concrete. (Neglect in our calculations unless stated.)

<u>Ideal case</u>:
Rigid body



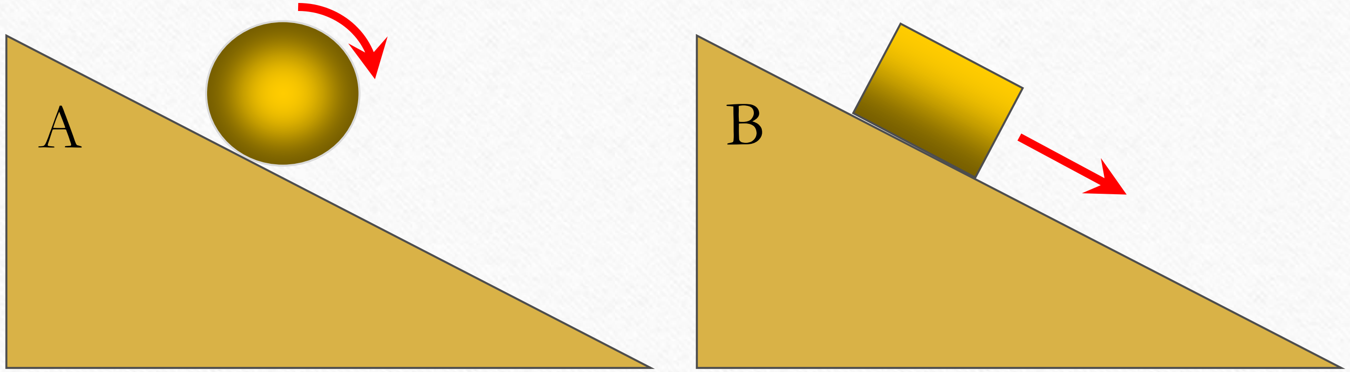


<u>In reality</u>:
Deformation occurs

$$\boxed{F_r = \mu_r N}$$

Racing down incline (I)

- A ball rolls without sliding down incline A. A box slides down a frictionless incline B. Assume A and B have some dimensions. Which gets to the bottom first?

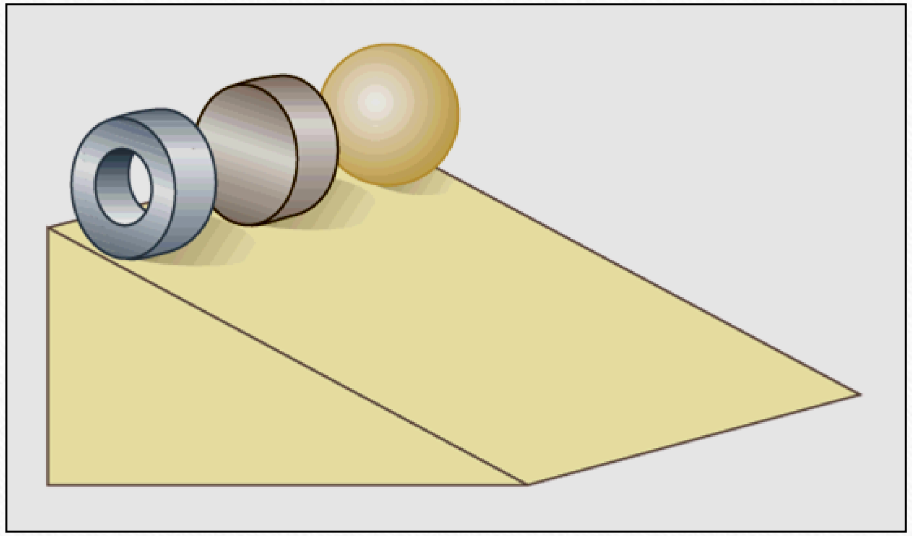


$$\boxed{a_{CM} = \frac{5}{7} g \sin\theta}$$

$$\boxed{a_{CM} = g \sin\theta}$$

Racing down incline (II)

- Three objects of uniform density - a solid sphere, a solid cylinder, and a hollow cylinder - are placed on top of an incline. They are all released from rest at the same elevation and roll without slipping. Which object will reach the bottom first, and which object will be last?



$$v_{CM} = \sqrt{\frac{2gh}{1 + I_{CM}/MR^2}}$$

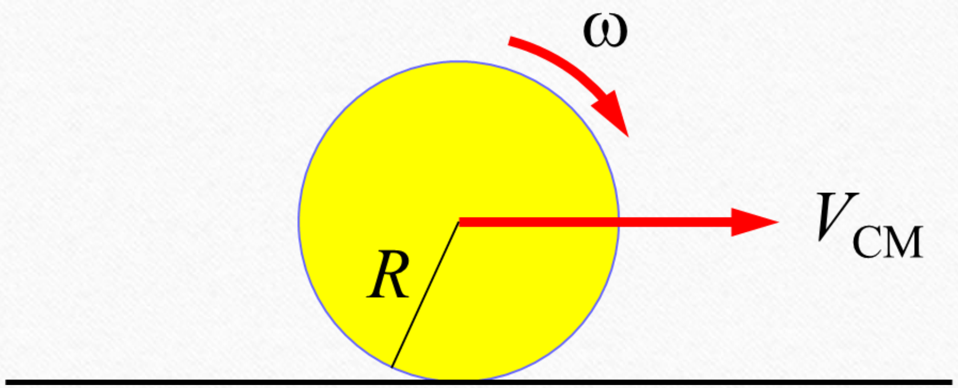
$$I_{cylinder} = \frac{1}{2}MR^2$$

$$I_{sphere} = \frac{2}{5}MR^2$$

$$I_{hol.\ cyl.} = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$$

Rolling with Slipping

- It is not necessary for $V_{CM} = R\omega$.
- Slipping can occur
- Kinetic friction acts

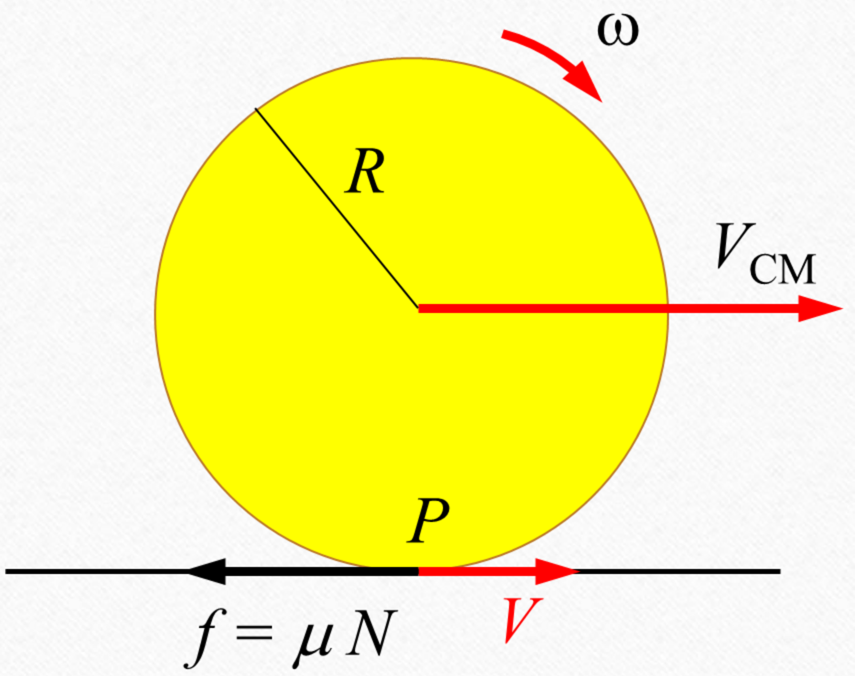


Rolling with Slipping

- For $V_{CM} > R\omega$,

$$M\frac{dV_{CM}}{dt} = -f \quad \text{and} \quad I\frac{d\omega}{dt} = fR$$

V_{CM} decreases, ω increases
Until $V_{CM} = Rw$

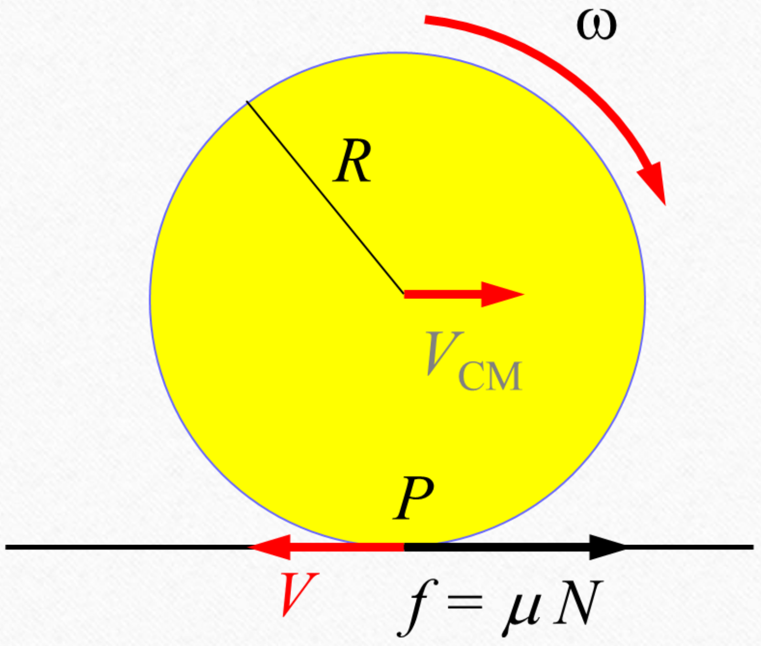


Rolling with Slipping

- For $V_{CM} < R\omega$,

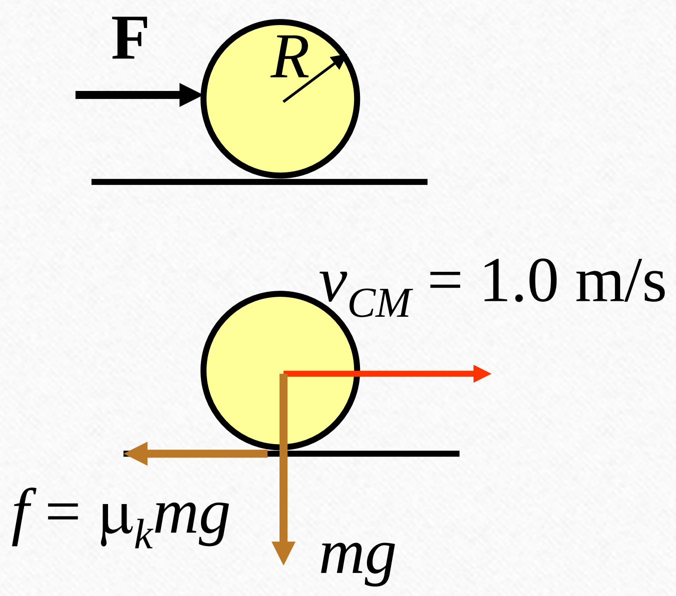
$$M\frac{dV_{CM}}{dt} = f \quad \text{and} \quad I\frac{d\omega}{dt} = -fR$$

V_{CM} increases, ω decreases
Until $V_{CM} = Rw$



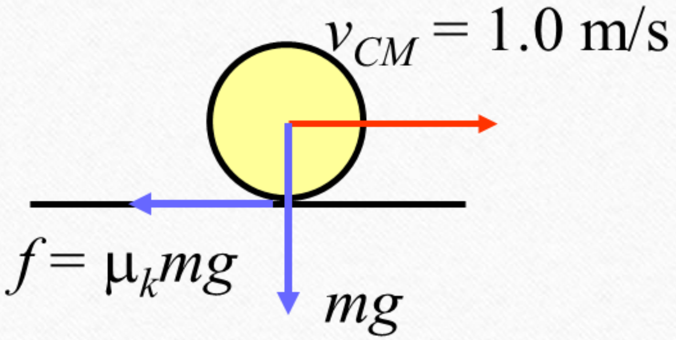
Example

- A uniform sphere rests initially on a level floor. An impulse, delivered horizontally at its centre, gives it a starting speed of 1.0 m/s and causes it to skid and roll on the floor. After 0.5 second, it stops skidding and turns to pure rolling. Calculate the coefficient of sliding friction due to contact with the floor.



Example

- Find μ_k if $v_o = 1.0$ m/s, and skidding → rolling in 0.5 s.



<u>2 motions</u>: 1) Rotation about CM.
2) Translation of CM.

During skidding,

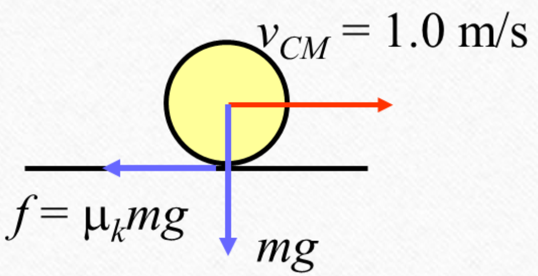
$$\boxed{v_{cm} \neq R\omega}$$

because the point of contact is moving.

Example

- Find μ_k if $v_o = 1.0$ m/s, and skidding $\rightarrow$ rolling in 0.5 s.

For translation motion: $v = v_o - \frac{\mu_k mg}{m}t = v_o - \mu_k gt$



For rotational motion: $\alpha = \frac{\tau}{I} = \frac{\mu_k mgR}{\frac{2}{5}mR^2}$ $\Rightarrow$ $\omega = \frac{5\mu_k g}{2R}t$

$\omega = \omega_0 + \alpha t$

After 0.5s, $\boxed{v_{cm} = R\omega}$

$v_o - \mu_k gt = R\frac{5\mu_k g}{2R}t$ $\Rightarrow$ $\mu_k = \frac{2v_o}{7gt} = \frac{(2)(1.0)}{7(9.8)(0.5)}$ $\Rightarrow$ $\boxed{\mu_k = 0.058}$

The End