1. Let X be a geometric random variable with parameter p. Find the maximum likelihood estimator of pbased on a random sample of size n.

$$\hat{p} = \frac{X_1 + X_2 + \dots + X_n}{n}$$

$$\hat{\mathbf{Q}}\hat{p} = \frac{n}{X_1 + X_2 + \dots + X_n}$$

B)
$$\hat{p} = n(X_1 + X_2 + \dots + X_n)$$

- D) $\hat{p} = X_1 + X_2 + \dots + X_n n$
- 2. Let X be random variable with probability density function

$$f(x) = \theta x^{\theta - 1}$$
, $0 < \theta < \infty$, $0 < x < 1$

A random sample of n is taken, find the maximum likelihood estimator for θ .

$$\hat{\theta} = -nX_1X_2\cdots X_n$$

C)
$$\hat{\theta} = \frac{1 \pm \sqrt{1 - 4nX_1X_2 \cdots X_n}}{2X_1X_2 \cdots X_n}$$
D)
$$\hat{\theta} = -\frac{n}{\ln(X_1X_2 \cdots X_n)}$$

$$B) \hat{\theta} = -\frac{n}{X_1 X_2 \cdots X_n}$$

$$\hat{\theta} = -\frac{n}{\ln(X_1 X_2 \cdots X_n)}$$

3. Let X be random variable with probability density function

$$f(x) = \frac{1}{\theta} x^{(1-\theta)/\theta}, \quad 0 < x < 1, \quad 0 < \theta < \infty$$

If the random sample of size n are taken, find the maximum likelihood estimator for θ .

A)
$$\hat{\theta} = -\frac{1}{n} \ln(X_1) \ln(X_2) \cdots \ln(X_n)$$
 C) $\hat{\theta} = -\frac{X_1 X_2 \cdots X_n}{n}$
B) $\hat{\theta} = -\frac{1}{n} [\ln(X_1) + \ln(X_2) + \cdots + \ln(X_n)]$ D) $\hat{\theta} = -\frac{1}{n} [\ln(X_1 + X_2 + \cdots + X_n)]$

C)
$$\hat{\theta} = -\frac{X_1 X_2 \cdots X_n}{n}$$

$$\mathbb{B}^{\hat{y}} \hat{y} = -\frac{1}{n} \left[\ln(X_1) + \ln(X_2) + \dots + \ln(X_n) \right]$$

D)
$$\hat{\theta} = -\frac{1}{n} \left[\ln(X_1 + X_2 + \dots + X_n) \right]$$

4. A random variable x has probability density function

$$f(x) = \frac{1}{2\lambda^3} x^2 e^{-x/\lambda}, \qquad 0 < x < \infty, \quad 0 < \lambda < \infty$$

If the random sample of size n are selected, find the maximum likelihood estimator for λ .

$$\hat{\lambda} = \frac{X_1 + X_2 + \dots + X_n}{3n}$$

$$\hat{\lambda} = \frac{\ln(X_1 + X_2 + \dots + X_n)}{3n}$$

$$C\hat{\lambda} = \frac{\ln(X_1) + \ln(X_2) + \dots + \ln(X_n)}{3n}$$

$$\hat{\lambda} = 3n \ln(X_1 + X_2 + \dots + X_n)$$

$$\hat{\lambda} = 3n \ln (X_1 + X_2 + \dots + X_n)$$

5. The Rayleigh distribution has probability density function

$$f(x) = \frac{x}{\theta} e^{-x^2/2\theta}, \quad x > 0, \quad 0 < \theta < \infty$$

i) Find the maximum likelihood estimator of θ .

$$\hat{\theta} = \frac{2(X_1^2 + X_2^2 + \dots + X_n^2)}{n}$$
B)
$$\hat{\theta} = \frac{X_1^2 + X_2^2 + \dots + X_n^2}{3n}$$

$$\hat{\theta} = \frac{X_1^2 + X_2^2 + \dots + X_n^2}{2n}$$

$$\hat{\theta} = \frac{X_1^2 + X_2^2 + \dots + X_n^2}{n}$$

ii) If the maximum likelihood estimator of $\hat{\theta}$ is an unbiased estimator, find $E(X^2)$.

$$A)E(X^2) = 2\theta$$

A)
$$E(X^2) = 2\theta$$
 B) $E(X^2) = \frac{1}{2}\theta$

C)
$$E(X^2) = \theta$$

D)
$$E(X^2) = 3\theta$$

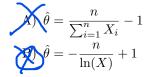
6. Let X denote the proportion of allotted time that a randomly selected student spends working on a certain aptitude test. Suppose the probability density function of X is

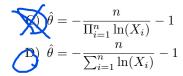
$$f(x) = \begin{cases} (\theta + 1)x^{\theta}, & 0 \le x \le 1\\ 0, & \text{otherwise} \end{cases}$$

where $-1 < \theta$. A random sample of ten students yields data

$$x_1 = 0.92$$
, $x_2 = 0.79$, $x_3 = 0.90$, $x_4 = 0.65$, $x_5 = 0.86$, $x_6 = 0.47$, $x_7 = 0.73$, $x_8 = 0.97$, $x_9 = 0.94$, $x_{10} = 0.77$

i) Find the maximum likelihood estimator of θ ,





ii) Compute the maximum likelihood estimate for the given data.

A) 3.00

C) 2.96

D) 2.54

7. For the standard normal distribution, $X \sim N(0,1)$, what is the range of z score for 95% confidence level?

A) -1.645 < z < 1.645 B) -1.96 < z < 1.96 C) -2.11 < z < 2.11 D) -2.576 < z < 2.576

8. In a random sample of 3534 participants attending the examination of the Offspring Heart Study it shows a point estimate for the mean of the population having systolic blood pressure is $\hat{X} = 127.3$ and standard deviation $\sigma = 19.0$. Find the 95% confidence interval.

(Hint: For the large sample n, the lower and higher interval limit can be calculated as $\hat{X} \pm \frac{z\sigma}{\sqrt{n}}$)

A) $p_l = 117.7, p_h = 120.7$ $p_l = 119.7, p_h = 122.9$

 $p_l = 122.7, p_h = 125.8$ $p_l = 126.7, p_h = 127.9$