

CSD2201/CSD2200 Week 2 Tutorial Problems

4th September – 10th September 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Question 1

- (a) Estimate the area under the graph of $f(x) = \sqrt{x}$ from $a = 0$ to $b = 6$ using a Riemann sum, with $n = 6$ rectangles, and using right endpoints as sample points. Sketch the graph and the rectangles.
- (b) Repeat part (a) using left endpoints as sample points.
- (c) Compute the integral $\int_0^6 \sqrt{x} dx$, and compare this value with the values obtained in part (a) and (b).

Question 2

- (a) Estimate the area under the graph of $f(x) = \sin x$ from $a = 0$ to $b = \frac{3\pi}{2}$ using a Riemann sum, with $n = 6$ rectangles, and using right endpoints as sample points. Sketch the graph and the rectangles.
- (b) Repeat part (a) using left endpoints as sample points.
- (c) Compute the integral $\int_0^{\frac{3\pi}{2}} \sin(x) dx$.

Question 3

Evaluate the following indefinite integrals.

- (a) $\int 3x^2 dx$ (b) $\int 4x^{\frac{3}{2}} dx$ (c) $\int \frac{5}{x^2} dx$ (d) $\int \frac{8}{x} dx$
- (e) $\int 5x^{\frac{1}{2}} - \frac{3}{1+x^2} dx$ (f) $\int 8e^x + \frac{1}{x^3} dx$ (g) $\int 3x^5 + 4x^2 + x dx$
- (h) $\int \frac{\sin^2 x}{\cos^2 x} + 1 dx$ (i) $\int \frac{1}{\cos^2 x} - \frac{7}{\sqrt{1-x^2}} dx$ (j) $\int \frac{\tan x}{\cos x} dx$
- (k) $\int \frac{1}{x^5} + 4 \cos x dx$ (l) $\int 8x^{\frac{3}{5}} - \frac{1}{\cos^2 x} dx$ (m) $\int x^2 + \tan^2 x dx$

Question 4

Evaluate the following definite integrals.

- (a) $\int_{-1}^1 x^4 dx$ (b) $\int_{-1}^1 x^3 dx$ (c) $\int_{-1}^0 x^2 dx$ (d) $\int_{-2}^{-1} \frac{3}{x} dx$
- (e) $\int_0^1 \frac{23}{1+x^2} dx$ (f) $\int_{-1}^1 \frac{24}{\sqrt{1-x^2}} dx$ (g) $\int_{-1}^2 x^3 - 2x^2 + 4x dx$
- (h) $\int_{-1}^1 5x^4 + 2x^3 dx$ (i) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x dx$ (j) $\int_0^2 \frac{4}{5}x^3 - \frac{3}{4}x^2 + \frac{2}{5}x dx$
- (k) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 7 \sec^2 x dx$ (l) $\int_0^{\frac{\pi}{4}} 6e^x + \sec x \tan x dx$ (m) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 5 \cos x + 8x^3 dx$

Final answers (approximations in 6 d.p.):

Q1: (a) 10.831822, (b) 8.382332, (c) $\frac{2}{3}6^{\frac{3}{2}} \approx 9.797959$.

Q2: (a) 0.555360, (b) 1.340759, (c) 1.

Q3: (a) $x^3 + C$, (b) $\frac{8}{5}x^{\frac{5}{2}} + C$, (c) $-\frac{5}{x} + C$, (d) $8 \ln x + C$, (e) $\frac{10}{3}x^{\frac{3}{2}} - 3 \tan^{-1} x + C$

(f) $8e^x - \frac{1}{2x^2} + C$, (g) $\frac{x^6}{2} + \frac{4x^3}{3} + \frac{x^2}{2} + C$, (h) $\tan x + C$, (i) $\tan x - 7 \sin^{-1} x + C$

(j) $\sec x + C$, (k) $4 \sin x - \frac{1}{4x^4} + C$, (l) $5x^{\frac{8}{5}} - \tan x + C$, (m) $\frac{x^3}{3} + \tan x - x + C$

Q4: (a) $\frac{2}{5}$, (b) 0, (c) $\frac{1}{3}$, (d) $-3 \ln 2$, (e) $\frac{23\pi}{4}$, (f) 24π , (g) $\frac{15}{4}$, (h) 2, (i) 0, (j) 2,

(k) 14, (l) $6e^{\frac{\pi}{4}} + \sqrt{2} - 7$, (m) 10