2024 Fall CSD3240/3241/SEM4108 - Probability and Statistics Homework 7

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1. Let X and Y are independent random variables with $E(X) = 0, Var(X) = 4, E(Y) = 10$ and $Var(Y) = 9$.

   i) Find $E(2X + 3Y)$

   A) 0 B) 10 C) 30 D) 20

   ii) Find $Var(2X + 3Y)$

   A) 16 B) 81 C) 97 D) 36

2. Suppose X and Y are random variables with $Var(X) = 1, Var(Y) = 4, Cov(X, Y) = 2$.

   i) Calculate $Cov(X + Y, 2X - 3Y)$

   A) 0 B) 12 C) 1 D) -12

   ii) Calculate the correlation, $\rho_{3X,2Y}$

   A) 0.2667 B) 0.0667 C) -0.0667 D) 0.1667

3. The mobile response time is the speed of page downloads. Let X denote the number of bars of signal strength, and let Y denote the response time (in second) for a particular user and site. The joint probability distribution of X and Y are shown in the table below:

Joint Probability	$X = 1$	$X = 2$	$X = 3$
$Y = 4$	0.15	0.1	0.05
$Y = 3$	0.02	0.1	0.05
$Y = 2$	0.02	0.03	0.2
$Y = 1$	0.01	0.02	0.25

   Calculate

   i) $Cov(X, Y)$

   A) -0.5815 B) 5.27 C) 2.35 D) 2.49

   ii) Correlation of X and Y

   A) 1.4099 B) 0.6275 C) -0.6573 D) -0.6182

4. The joint probability mass function is given by $p(x, y) = c(x + y)$ for $x = 1, 2, 3$ and $y = 1, 2, 3$.

   i) determine the value of c

   A) $\frac{1}{9}$ B) $\frac{1}{21}$ C) $\frac{1}{36}$ D) $\frac{1}{48}$

   ii) find the covariance $Cov(X, Y)$

   A) -0.0279 B) -2.16667 C) -4.6667 D) -3.6667

   iii) find the correlation $\rho_{X,Y}$

   A) 0.6389 B) -0.0435 C) 5.3333 D) -0.0279

5. Suppose that the correlation between X and Y is ρ. For constants a, b, c and d, what is the correlation between the random variables $U = aX + b$ and $V = cY + d$?

A) $\rho_{U,V} = ac\rho$

B) $\rho_{U,V} = ac\rho + bd$

C) $\rho_{U,V} = \rho$

D) $\rho_{U,V} = (a\rho + b)(c\rho + d)$