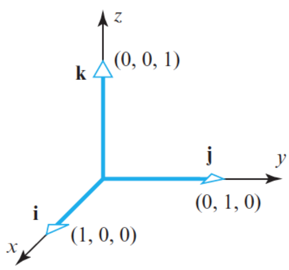
CSD1241 Tutorial 3

Problem 1. (Right-hand rule)
The xyz-space that we are familiar to uses the right-hand rule.

Let $\vec{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ = unit vectors along x-axis, y-axis, z-axis. The following equations were discussed in the lecture

$$\vec{i} \times \vec{j} = \vec{k}, \; \vec{j} \times \vec{k} = \vec{i}, \; \vec{k} \times \vec{i} = \vec{j}$$

Practice the right-hand rule for the above equation, that is, verify that $\vec{i} \times \vec{j}$ points to the same direction as $\vec{k}$, $\vec{j} \times \vec{k}$ points to the same direction as $\vec{i}$, and $\vec{k} \times \vec{i}$ points to the same direction as $\vec{j}$.



Problem 2. Find the area of the triangles with given vertices A, B, C.
(a) $A(2,6,1),\; B(1,1,1),\; C(-1,2,3)$.
(b) $A(2,0), B(3,5), C(-1,-2)$.

Problem 3. Find both the *parametric equation* and the *general equation* of the plane β containing three points P, Q, R in the following cases. Further, let $A = (1,2,3)$. find the point B on β which is at the closest distance to A.

(a) $P(3,-1,4),\; Q(6,0,2),\; R(5,1,1)$.
(b) $P(2,1,3),\; Q(1,3,4),\; R(-2,-1,-5)$

Problem 4. Find the intersection of the lines l_1 and l_2 (in $\mathbb{R}^2$) in following cases.

(a) $l_1 : \begin{cases} x = -3 + t \\ y = 1 - t \end{cases}$ and $l_2 : (x, y) = (7, 0) + s \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(b) $l_1 : x + 4y = 13$ and l_2 : go through $(4, 0)$ and $(5, -1)$.
(c) $l_1 : y - 1 = -(x + 3)$ and l_2 : go through $(4, 0)$ and perpendicular to $x + 4y = 13$.

Problem 5. Find the relative position (intersecting, parallel, skew) and the intersections between any two of the three lines k, l, m (in $\mathbb{R}^3$)

$$k : \begin{cases} x = 1 + 2r \\ y = -1 + 2r \\ z = 2 + 3r \end{cases} \qquad l : \begin{cases} x = -1 + 2t \\ y = t \\ z = 2 + 2t \end{cases} \qquad m : (x, y, z) = (2, 0, 1) + s \begin{bmatrix} 1 \\ 1 \\ 1/3 \end{bmatrix}$$

Problem 6. Let l be the line going through $P = (2, 3, 1)$ and $Q = (5, -3, 4)$. Let α be the plane going through $(0, 2, -1)$ with direction vectors $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \vec{v} = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}$.

(a) Find the intersection of l and α.

(b) Find the intersection of α and the plane β: through $(1, 2, 0)$ with normal $\vec{n} = \begin{bmatrix} 1 \\ -2 \\ 6 \end{bmatrix}$.