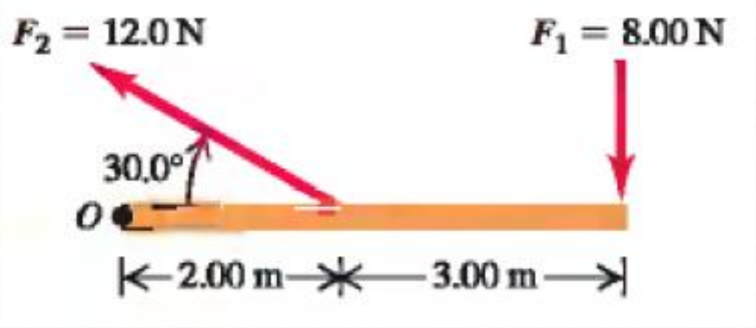
CSD2301 Practice Solutions
13. **Rotational Dynamics**

LIN QINJIE

Practice Question 1

Calculate the net torque about point O for the two forces applied as in the figure shown. The rod and both forces are in the plane of the page.



IDENTIFY: $\tau = Fl$ with $l = r\sin\phi$. Add the two torques to calculate the net torque.

SET UP: Let counterclockwise torques be positive.

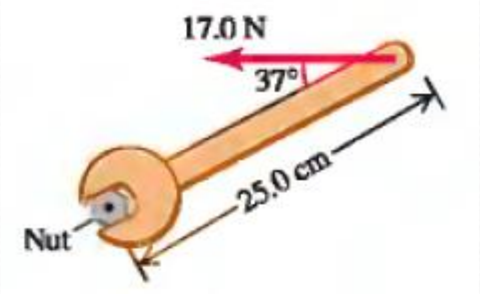
EXECUTE: $\tau_1 = -F_1 l_1 = -(8.00\text{ N})(5.00\text{ m}) = -40.0\text{ N}\cdot\text{m}$. $\tau_2 = +F_2 l_2 = (12.0\text{ N})(2.00\text{ m})\sin 30.0^\circ = +12.0\text{ N}\cdot\text{m}$.

$\sum\tau = \tau_1 + \tau_2 = -28.0\text{ N}\cdot\text{m}$. The net torque is $28.0\text{ N}\cdot\text{m}$, clockwise.

EVALUATE: Even though $F_1 < F_2$, the magnitude of τ_1 is greater than the magnitude of τ_2, because F_1 has a larger moment arm.

Practice Question 2

A machinist is using a wrench to loosen a nut. The wrench is 25.0 cm long, and he exerts a 17.0 N force at the end of the handle at 37° with the handle as shown in the figure. (a) What torque does the machinist exert about the center of the nut? (b) What is the maximum torque he could exert with this force, and how should the force be oriented?



EXECUTE: (a) $\tau = (17.0\ \text{N})(0.250\ \text{m})\sin 37^\circ = 2.56\ \text{N}\cdot\text{m}$. The torque is counterclockwise.

(b) The torque is maximum when $\phi = 90^\circ$ and the force is perpendicular to the wrench. This maximum torque is $(17.0\ \text{N})(0.250\ \text{m}) = 4.25\ \text{N}\cdot\text{m}$.

EVALUATE: If the force is directed along the handle then the torque is zero. The torque increases as the angle between the force and the handle increases.

Practice Question 3

The flywheel of an engine has moment of inertia 2.50 kgm^2 about its rotation axis. What constant torque is required to bring it up to an angular speed of 400 rev/min in 8.00 s, starting from rest?

IDENTIFY: Apply $\sum \tau_z = I\alpha_z$.

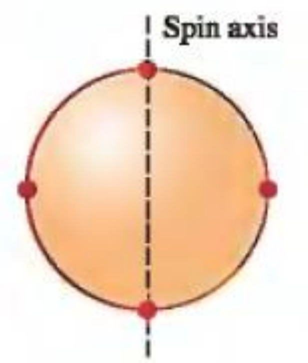
SET UP: $\omega_{0z} = 0$. $\omega_z = (400 \text{ rev/min})\left(\dfrac{2\pi \text{ rad/rev}}{60 \text{ s/min}}\right) = 41.9 \text{ rad/s}$

EXECUTE: $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t} = \left(2.50 \text{ kg}\cdot\text{m}^2\right)\dfrac{41.9 \text{ rad/s}}{8.00 \text{ s}} = 13.1 \text{ N}\cdot\text{m}.$

EVALUATE: In $\tau_z = I\alpha_z$, α_z must be in rad/s^2 .

Practice Question 4

A uniform, 8.40 kg, spherical shell 50.0 cm in diameter has four small 2.00 kg masses attached to its outer surface and equally spaced around it. This combination is spinning about an axis running through the center of the sphere and two of the small masses. What friction torque is needed to reduce its angular speed from 75.0 rpm to 50.0 rpm in 30.0 s?



IDENTIFY: Use a constant acceleration equation to calculate α_z and then apply $\sum \tau_z = I\alpha_z$.

SET UP: $I = \frac{2}{3}MR^2 + 2mR^2$, where $M = 8.40$ kg, $m = 2.00$ kg, so $I = 0.600\ \text{kg}\cdot\text{m}^2$.

$\omega_{0z} = 75.0\ \text{rpm} = 7.854\ \text{rad/s}$; $\omega_z = 50.0\ \text{rpm} = 5.236\ \text{rad/s}$; $t = 30.0\ \text{s}$.

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -0.08726\ \text{rad/s}^2$. $\tau_z = I\alpha_z = -0.0524\ \text{N}\cdot\text{m}$

EVALUATE: The torque is negative because its direction is opposite to the direction of rotation, which must be the case for the speed to decrease.

Practice Question 5

An electric motor consumes 9.00 kJ of electrical energy in 1.00 min. If one-third of this energy goes into heat and other forms of internal energy of the motor, with the rest going to the motor output, how much torque will this engine develop if you run it at 2500 rpm?

IDENTIFY: The power output of the motor is related to the torque it produces and to its angular velocity by $P = \tau_z \omega_z$, where ω_z must be in rad/s.

SET UP: The work output of the motor in 60.0 s is $\frac{2}{3}(9.00\text{ kJ}) = 6.00\text{ kJ}$, so $P = \frac{6.00\text{ kJ}}{60.0\text{ s}} = 100\text{ W}$.

$\omega_z = 2500\text{ rev/min} = 262\text{ rad/s}$.

EXECUTE: $\tau_z = \frac{P}{\omega_z} = \frac{100\text{ W}}{262\text{ rad/s}} = 0.382\text{ N}\cdot\text{m}$

EVALUATE: For a constant power output, the torque developed decreases and the rotation speed of the motor increases.

Practice Question 6

A 1.50 kg grinding wheel is in the form of a solid cylinder of radius 0.100 m. (a) What constant torque will bring it from rest to an angular speed of 1200 rev/min in 2.5 s? (b) Through what angle has it turned during that time? (c) Calculate the work done by the torque. (d) What is the wheel's kinetic energy when it is rotating at 1200 rev/min? Compare your answer to the result in part (c).

I for solid cylinder

EXECUTE: **(a)** $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t}$.

From $\theta = \frac{1}{2}(\omega_0 + \omega)t$

$$\tau_z = \frac{\left((1/2)(1.50\ \text{kg})(0.100\ \text{m})^2\right)(1200\ \text{rev/min})\left(\dfrac{\pi\quad \text{rad/s}}{30\ \text{rev/min}}\right)}{2.5\ \text{s}} = 0.377\ \text{N}\cdot\text{m}$$

(b) $\omega_{\text{av}}\Delta t = \dfrac{(600\ \text{rev/min})(2.5\ \text{s})}{60\ \text{s/min}} = 25.0\ \text{rev} = 157\ \text{rad}.$

(c) $W = \tau\Delta\theta = (0.377\ \text{N}\cdot\text{m})(157\ \text{rad}) = 59.2\ \text{J}$.

(d) $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\left((1/2)(1.5\ \text{kg})(0.100\ \text{m})^2\right)\left((1200\ \text{rev/min})\left(\dfrac{\pi\quad \text{rad/s}}{30\ \text{rev/min}}\right)\right)^2 = 59.2\ \text{J}$.

the same as in part (c).