1. Suppose that 3 balls are randomly selected from a box containing 3 red, 4 white, and 5 blue balls. If we let X and Y denote, respectively, the number of red and white balls chosen, then the joint probability mass function of X and Y, $p(0,1) = P(X=0, Y=1)$ is equal to

   A) $\frac{1}{22}$ B) $\frac{3}{22}$ C) $\frac{2}{11}$ D) $\frac{3}{55}$

2. Suppose that $p(x, y)$, the joint PMF of X and Y, is given by

Joint probability	$Y=0$	$Y=1$
$X=0$	$p(0,0)=0.4$	$p(0,1)=0.2$
$X=1$	$p(1,0)=0.1$	$p(1,1)=0.3$

   Calculate the conditional distributions of X given that $Y=1$.

   A) $P(X=0|Y=1)=0.2\,,\quad P(X=1|Y=1)=0.8$
   B) $P(X=0|Y=1)=0.3\,,\quad P(X=1|Y=1)=0.7$
   C) $P(X=0|Y=1)=0.5\,,\quad P(X=1|Y=1)=0.5$
   D) $P(X=0|Y=1)=0.4\,,\quad P(X=1|Y=1)=0.6$

3. Suppose that 15% of the families in a certain community have no children, 20% have 1 child, 35% have 2 children, and 30% have 3 children, and suppose that in each family each child is equally likely to be a boy or a girl. If a family is chosen from this community, then let B be the number of boys, and G be the number of girls, what is the value of the joint probability mass function $p(1,2) = P(B=1, G=2)$?

   A) 0.1125 B) 0.175 C) 0.0875 D) 0.0375

4. The joint PDF of X and Y is given by

$$f(x,y) = \begin{cases} 2e^{-x}e^{-2y}\,, & 0 < x < \infty\,, 0 < y < \infty \\ 0\,, & \text{otherwise} \end{cases}$$

   Compute

   i) $P(X > 1, Y < 1)$

   A) e^{-3} B) $e^{-2}(1-e^{-1})$ C) $e^{-2}(1+e^{-1})$ D) $e^{-1}(1-e^{-2})$

   ii) $P(X < Y)$

   A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) $\frac{1}{4}$

   iii) $P(X < a)$

   A) $e^{a}-1$ B) e^{-a} C) $1+e^{-a}$ D) $1-e^{-a}$

5. Let the random variable X denote the time until a server connects to your computer (in milliseconds), and let Y denote the time until the server authorizes you as a valid user (in milliseconds). Each of the these random variables measures the waiting time from a common starting time and $X < Y$. Assume that the joint PDF for X and Y is

$$f(x,y) = 6 \times 10^{-6} \times e^{-0.001x - 0.002y} \qquad \text{for} \qquad x < y$$

   Find the probability that $X \leq 1000$ and $Y \leq 2000$.

A) 0.368 B) 0.091 C) 0.915 D) 0.047

6. Consider a circle of radius R and suppose that a point within the circle is randomly chosen in such a manner that all regions within the circle of equal area are equally likely to contain the point, in other words, the point is uniformly distributed within the circle. If we define X and Y to be the coordinates of the point chosen, the joint PDF of X and Y is given by

$$f(x,y) = \begin{cases} c\,, & \text{if} \quad x^2+y^2 \le R^2 \\ 0\,, & \text{if} \quad x^2+y^2 > R^2 \end{cases}$$

for some value of c.

i) Determine c.

A) $c = \dfrac{1}{\pi R^2}$ B) $c = \pi R^2$ C) $c = 4\pi R$ D) $c = 4\pi R^2$

ii) Find the marginal density functions of X.
(Hint: since $x^2+y^2 \le R^2$, the limit of y lying between $-\sqrt{R^2-x^2} \le y \le \sqrt{R^2-x^2}$)

A) $f_X(x) = \begin{cases} \frac{1}{\pi R^2}\sqrt{R^2-x^2}\,, & x^2 \le R^2 \\ 0\,, & x^2 > R^2 \end{cases}$

C) $f_X(x) = \begin{cases} -\frac{2}{\pi R^2}\sqrt{R^2-x^2}\,, & x^2 \le R^2 \\ 0\,, & x^2 > R^2 \end{cases}$

B) $f_X(x) = \begin{cases} 0\,, & x^2 \le R^2 \\ \frac{2}{\pi R^2}\sqrt{R^2-x^2}\,, & x^2 > R^2 \end{cases}$

D) $f_X(x) = \begin{cases} \frac{2}{\pi R^2}\sqrt{R^2-x^2}\,, & x^2 \le R^2 \\ 0\,, & x^2 > R^2 \end{cases}$

iii) Compute the probability that the distance of the point selected from the origin of the circle is less than or equal to a.

A) $\dfrac{1}{\pi a^2}$ B) $\dfrac{a^2}{R^2}$ C) πa^2 D) $\sqrt{\dfrac{a}{R}}$