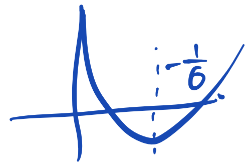
Question 2: critical pts & determine local max/min.

(a) $f(x) = 3x^2 + x - 2$.



$f'(x) = 6x + 1 = 0 \Rightarrow x = -\frac{1}{6}$

$f'(-1) = 6(-1) + 1 = -5 < 0$. $\downarrow$

$f'(0) = 6(0) + 1 = 1 > 0$. $\uparrow$

local min. $x = -\frac{1}{6}$.

(b). $g(v) = v^3 - 12v + 4$

$g'(v) = 3v^2 - 12 = 0 \Rightarrow 3v^2 = 12 \Rightarrow v^2 = 4 \Rightarrow v = \pm 2$.

max min.

$+$ $\quad -$ $\quad +$

-2 $\quad 2$.

$g'(-3) = 3(-3)^2 - 12 = 3 \cdot 9 - 12 > 0$.

$g'(0) = -12 < 0$

$g'(3) = 3(3)^2 - 12 > 0$

(c). $f(x) = 3x^4 + 8x^3$

$x = 0$ or $x = -2$

$f'(x) = 12x^3 + 24x^2 = 12x^2(x+2) = 0$.

min. neither.

$-$ $\quad +$ $\quad +$

-2 $\quad 0$

$x = -2$: local min.

$x = 0$: neither

$f'(-3) = 12(-3)^2(-3+2) < 0$

$f'(-1) = 12(-1)^2(-1+2) > 0$

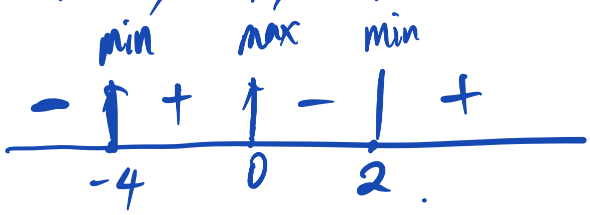
$f'(1) = 12(1)^2(1+2) > 0$

(d) $f(x) = 3x^4 + 8x^3 - 48x^2.$

$f'(x) = 12x^3 + 24x^2 - 96x = 12x(x^2+2x-8)$

$= 12x(x+4)(x-2) = 0.$

$x = 0, -4, 2.$



$f(-5) = 12(-5)(-5+4)(-5-2) < 0$

$f(-1) = 12\underset{-}{(-1)}\underset{+}{(-1+4)}\underset{-}{(-1-2)} > 0$

$f(1) = 12\underset{+}{(1)}\underset{+}{(1+4)}\underset{-}{(1-2)} < 0$

$f(3) = 12\underset{+}{(3)}\underset{+}{(3+4)}\underset{+}{(3-2)} > 0.$

$x = -4$ & $x = 2.$
local min points

$x = 0$
local max point.

(e) $g(t) = t^5 + 5t^3 + 50t.$

$g'(t) = 5t^4 + 15t^2 + 50 = 5(t^4 + 3t^2 + 10) = 0.$

$t^4 + 3t^2 + 10.$ let $p = t^2 \Rightarrow p^2 + 3p + 10$

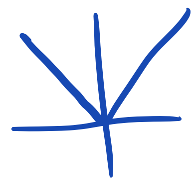
$a = 1$
$b = 3$
$c = 10$

$b^2 - 4ac < 0.$

$9 - 40 = -31 < 0.$

No critical point.

(f) $f(x) = |x|$



$x=0$ local min point.

$x=0$ Continuous but not differentiable

$$f(x) = \begin{cases} x & x>0. \\ 0 & x=0 \\ -x & x<0 \end{cases}.$$

(g) $h(y) = \dfrac{y-1}{y^2-y+1}$

$$h'(y) = \frac{(y-1)'(y^2-y+1) - (y-1)(y^2-y+1)'}{(y^2-y+1)^2}$$

$$= \frac{y^2-y+1 - (y-1)(2y-1)}{(y^2-y+1)^2}$$

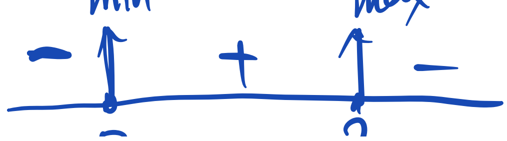
$$= \frac{y^2-y+1 - (2y^2-y-2y+1)}{(y^2-y+1)^2}$$

$$= \frac{y^2-y+1-2y^2+3y-1}{(y^2-y+1)^2} = \frac{-y^2+2y}{(y^2-y+1)^2}. = 0$$

$-y^2+2y=0 \Rightarrow y(-y+2)=0$

$y=0$ or $y=2$.

min max



$$h'(-1)=\frac{-(-1)^2+2(-1)}{[(-1)^2-(-1)+1]^2}<0$$

$y=0$ local min

$$h'(1)=\frac{-(1)^2+2(1)}{(1^2-1+1)^2}>0.$$

$y=2$ local max

$$h'(3)=\frac{-3^2+2\times 3}{3^2-3+1}<0.$$

(h). $$p(x)=\frac{x^2+2}{2x-1}$$

$$p'(x)=\frac{(x^2+2)'(2x-1)-(x^2+2)(2x-1)'}{(2x-1)^2}$$

$$=\frac{2x(2x-1)-(x^2+2)(2)}{(2x-1)^2}$$

$$=\frac{4x^2-2x-2x^2-4}{(2x-1)^2}=\frac{2x^2-2x-4}{(2x-1)^2}=0.$$

$$2(x^2-x-2)=0 \Rightarrow 2(x-2)(x+1)=0$$

$\boxed{x=2 \quad \text{or} \quad x=-1.}$ $\qquad 2x-1=0 \Rightarrow \boxed{x=\frac{1}{2}}$

max $\qquad$ min. $\qquad$ not a local extreme point

$+ \quad \uparrow \quad - \quad \uparrow \quad +$

$-1 \qquad\qquad 2$

$$p'(-2)=\frac{2(-2)^2-2(-2)-4}{(-5)^2}>0$$

$x=-1$.
local max point

$$p'(0)=\frac{-4}{1}<0.$$

$x=2$
local min point.

$$p'(3)=\frac{2(3)^2-2(3)-4}{5^2}>0.$$

(i). $f(x) = x + \frac{4}{x^2}$.

$f'(x) = 1 + (-2)\,4\,(x)^{-3}$

$= 1 - \frac{8}{x^3} = 0$. $x = 2$.

$x = 0$ is also a critical point.

min.

$-$ $\uparrow$ $+$

2

$x=2$ is a local min.

$x=0$ is not a local extreme point because $f(0)$ is not defined.

$f'(1) = 1 - \frac{8}{1} < 0$

$f'(3) = 1 - \frac{8}{3^3} = 1 - \frac{8}{27} > 0$

(j). $g(x) = \boxed{x^{\frac{4}{5}}}(x-4)^2$.

$g'(x) = \frac{4}{5}x^{-\frac{1}{5}}(x-4)^2 + x^{\frac{4}{5}}\,2(x-4)$.

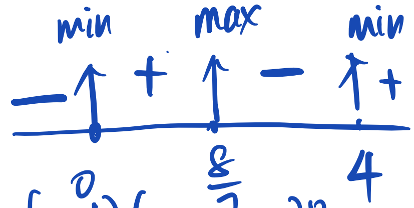
$= x^{-\frac{1}{5}}(x-4)\left[\frac{4}{5}(x-4) + x \cdot 2\right]$.

$= x^{-\frac{1}{5}}(x-4)\left[\frac{4}{5}x - \frac{16}{5} + 2x\right]$

$= \frac{1}{\sqrt[5]{x}}(x-4)\left(\frac{14x}{5} - \frac{16}{5}\right] = 0$.

$x = 4$ $x = \frac{8}{7}$

$x = 0$ is also a critical point.



$g'(-1) = (-1)(-5)(-\frac{30}{5}) < 0$

$g'(5) = \frac{1}{\sqrt[5]{5}}(5-4)(\frac{14\cdot 5}{5} - \frac{16}{5}) > 0$

$g'(1) = \frac{1}{\sqrt[5]{1}}(1-4)(\frac{14\cdot 1}{5} - \frac{16}{5}) > 0$

$g'(2) = \frac{1}{\sqrt[5]{2}}(2-4)(\frac{14\cdot 2}{5} - \frac{16}{5}) < 0$

$x=0$ and $x=4$: local min points.

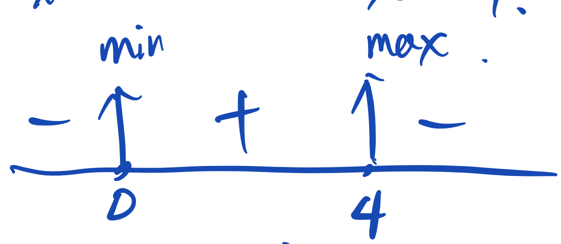
$x=\frac{8}{7}$: local max point.

(k). $f(x) = x^4 e^{-x}$

$f'(x) = 4x^3 e^{-x} + x^4 \cdot -e^{-x}$

$= x^3 \underbrace{e^{-x}}_{>0}(4-x) = 0.$

$x=0$ or $x=4$.



$f'(-1) = (-1)^3(e^1)(5) = -5e. < 0$

$f'(1) = (1)^3(e^{-1})(4-1) = 3\cdot e^{-1} > 0$

$f'(5) = (5)^3(e^{-5})(-1) < 0$

$x=0$ · local min point

$x=4$ local max point.

(2) $g(x)=x^2\ln x$.

$g'(x)=2x\ln x+x^2\cdot\frac{1}{x}=2x\ln x+x=x(2\ln x+1)$

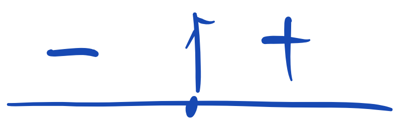
$x=0$ or $2\ln x=-1$

reject. $\ln x=-\frac{1}{2}$.

$x=e^{-\frac{1}{2}}=\frac{1}{\sqrt{e}}$.

$\boxed{\ln x}$ $x>0$.

min



$0.606\approx\frac{1}{\sqrt{e}}=e^{-\frac{1}{2}}$.

$g'(e)=e(2\ln e+1)>0$

$g'(0.1)=0.1(2\ln(0.1)+1)<0$

$x=\frac{1}{\sqrt{e}}$: local minimum point #