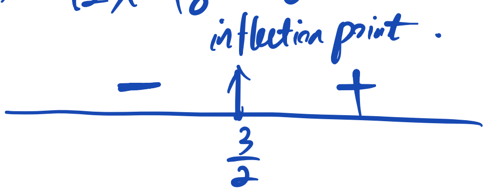
Question 2. inflection point.

(a) $f(x) = 2x^3 - 9x^2 + 12x - 3$.

$f'(x) = 6x^2 - 18x + 12$.

$f''(x) = 12x - 18 = 0$. $\quad 12x = 18 \Rightarrow x = \frac{18}{12} = \frac{3}{2}$.



(i) $x = \frac{3}{2}$

(ii). CU: $(\frac{3}{2}, \infty)$

CD: $(-\infty, \frac{3}{2})$.

$f''(0) = -18 < 0$

$f''(2) = 12 \cdot 2 - 18 > 0$

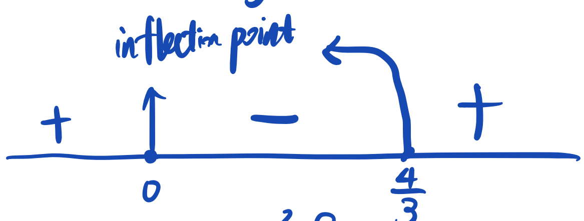
(b) $f(x) = 6x^4 - 16x^3 + 1$

$f'(x) = 24x^3 - 48x^2$

$f''(x) = 72x^2 - 96x = 0 \Rightarrow x(72x - 96) = 0$

$x = 0$ or $72x - 96 = 0 \Rightarrow x = \frac{96}{72} = \frac{4}{3}$.

(i) $x = 0, \frac{4}{3}$.



(ii) CU: $(-\infty, 0)$

and $(\frac{4}{3}, +\infty)$

CD: $(0, \frac{4}{3})$.

$f''(-1) = 72(-1)^2 - 96(-1) > 0$.

$f''(1) = 72(1)^2 - 96(1) < 0$

$f''(2) = 72(2)^2 - 96 \times 2 > 0$

(c) $f(x) = x^2 - x - \ln x$ $(0,\infty)$.

$f'(x) = 2x - 1 - \frac{1}{x}$

$f''(x) = \underbrace{2 + \frac{1}{x^2}} > 0$.

$(-\frac{1}{x})' = (-x^{-1})'$
$= (-1)(-1)(x^{-1-1})$
$= 1\, x^{-2} = \frac{1}{x^2}$.

(i) No inflection point.

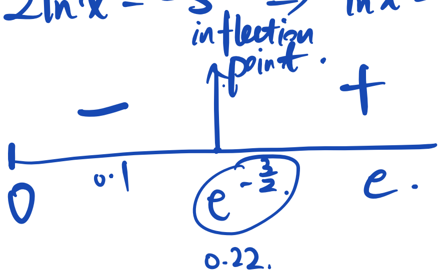
(ii) CU on the entire domain $(0,\infty)$.

(d). $f(x) = x^2 \ln x$

$f'(x) = 2x\ln x + x^2 \cdot \frac{1}{x} = \underline{2x\ln x + x}$.

$f''(x) = 2\ln x + 2x\frac{1}{x} + 1 = 2\ln x + 2 + 1 = 2\ln x + 3 = 0$

$2\ln x = -3 \Rightarrow \ln x = -\frac{3}{2}$. $\Rightarrow x = e^{-\frac{3}{2}}$.



(i) $x = e^{-\frac{3}{2}}$ inflection pt.

(ii) CU: $(e^{-\frac{3}{2}}, +\infty)$
CD: $(0, e^{-\frac{3}{2}})$

$f''(e) = 2\ln e + 3 = 5 > 0$

$f''(0.1) < 0$

(e) $f(x) = xe^{2x}$

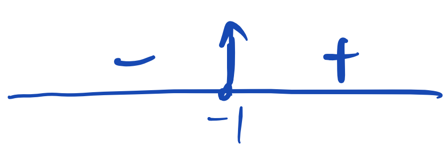
$f'(x) = e^{2x} + x \cdot 2e^{2x} = \underbrace{e^{2x}}\underbrace{(2x+1)}$

$f''(x) = 2e^{2x}(2x+1) + e^{2x} \cdot 2$.

$= e^{2x}(4x+2+2) = \underbrace{e^{2x}}_{+,\ \neq 0}(4x+4) = 0$

$4x+4 = 0 \Rightarrow x = -1$

(i) $x = -1$ is the inflection point



(ii). CU: $(-1,\infty)$

CD: $(-\infty,-1)$.

$f''(0)=e^{0}(4\cdot 0+4)=1(4)>0$

$f''(-2)=e^{-4}(-8+4)<0$

Question 3.

$$f(x)=x^4(x-1)^3.$$

(a) Find the critical points of $f(x)$

$f'(x)=4x^3(x-1)^3+x^4\cdot 3(x-1)^2.$

$=x^3(x-1)^2[4(x-1)+3x]=\boxed{x^3(x-1)^2(7x-4)}=0.$

$x=0,1,\frac{4}{7}.$

(b) Second Derivative Test.

$f''(x)=[\underbrace{\overbrace{x^3}^{f_1(x)}\overbrace{(x-1)^2}^{f_2(x)}}_{f(x)}\underbrace{(7x-4)}_{g(x)}]'=f'(x)g(x)+f(x)g'(x)$

$=[3x^2(x-1)^2+x^3\cdot 2(x-1)](7x-4)$

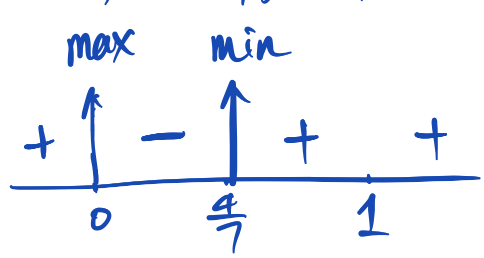
$+x^3(x-1)^2\cdot 7$

$f''(0)=0$ inconclusive.

$f''(1)=0$ inconclusive.

$f''(\frac{4}{7})=(\frac{4}{7})^3(\frac{4}{7}-1)^2\cdot 7>0.$ $x=\frac{4}{7}$ local min point

(c).



$f'(x)=x^3(x-1)^2(7x-4)$

$f'(-1)>0$

$f'(\frac{1}{2})<0$

$f'(\frac{6}{7})>0.$

$f'(2)>0$

$x=0$ is a local max

$x=\frac{4}{7}$ is a local min

$x=1$ is neither a local min nor a local max.

Question 4

(a). $f(x) = 2x^3 - 9x^2 + 12x - 3$

$f'(x) = 6x^2 - 18x + 12 = 0$ $\Rightarrow 6(x^2 - 3x + 2) = 0$

$6(x-2)(x-1) = 0$

$f''(x) = 12x - 18$ $\quad \underline{x=2}$ or $\underline{x=1}$

$f''(1) = -6 < 0$ local max

$f''(2) = 24 - 18 > 0$. local min.

(i) $x = 1, 2$.

(ii) $x=1$ is a local max point

$x=2$ is a local min point

(b). (i) $x = 0, 2$.

(ii) $x=2$. local min point.

$x=0$ inconclusive. } SDT

$x=0$ is neither a local max nor a local min.] FDT

(c). (i) $x = -\frac{1}{2}$, 1.

$f(-\frac{1}{2})$ not defined.

Cannot be a local max/min point.

(ii) $x=1$ local min point.

(d). (i) $x=0$, $x = \frac{1}{\sqrt{e}}$.

$f(0)$ not defined.

(ii) $x = \frac{1}{\sqrt{e}}$ is a local min point.

(e) (i) $x = -\frac{1}{2}$.

(ii) $x = -\frac{1}{2}$ is a local min point.

Question 5 .

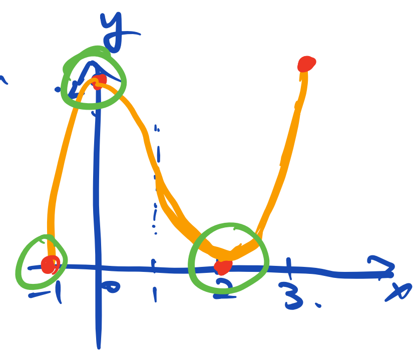
$f(x) = x^3 - 3x^2 + 4$ on $[-1, 3]$.

$f'(x) = 3x^2 - 6x = 0 \Rightarrow x = 0, \; x = 2$.

$\boxed{f''(x) = 6x - 6 = 0}$ $\quad x = 1$ inflection point .

$f''(0) = -6 < 0$ local max

$f''(2) > 0$. local min



$f(0) = 4$

$f(2) = 2^3 - 12 + 4 = 8 + 4 - 12 = 0$.

$f(-1) = -1 - 3 + 4 = 0$

$f(3) = 27 - 3 \cdot 9 + 4 = 4$.