Week 10 tutorial.

Question 1:

expected score is $\mu = 72$, $\sigma = 12$.

CLT tells us that the class average can be approximated by a normal distribution.

$\boxed{S_{120} \sim N(\mu, \frac{\sigma^2}{n})} \sim N(72, \frac{12^2}{120}) = N(72, 1.2)$

Then the CDF for S_{120} is

$$\phi\left(\frac{\sqrt{n}(X-\mu)}{\sigma}\right) = \phi\left(\frac{\sqrt{120}(X-72)}{12}\right) = \phi\left(\frac{X-72}{\sqrt{1.2}}\right)$$

$$P(68 \le S_{120} \le 76) \approx \phi\left(\frac{76-72}{\sqrt{1.2}}\right) - \phi\left(\frac{68-72}{\sqrt{1.2}}\right)$$
$$= \phi\left(\frac{4}{\sqrt{1.2}}\right) - \phi\left(\frac{-4}{\sqrt{1.2}}\right).$$

Question 2:

# of mutants	0	1	2	3	4
probability	0.1	0.3	0.1	0.4	0.1

(a) Expectation for just 1 experiment.

$\mu = (0)(0.1) + (1)(0.3) + (2)(0.1) + (3)(0.4) + (4)(0.1) = 2.1$

(b) $Var(X) = E(X^2) - \mu^2 = 5.9 - 2.1^2$

$$E(X^2) = (0^2)(0.1) + (1)^2(0.3) + (2)^2(0.1) + (3)^2(0.4) + (4^2)(0.1)$$
$$= (0)(0.1) + (1)(0.3) + (4)(0.1) + (9)(0.4) + (16)(0.1)$$
$$= 5.9.$$

$\sigma = \sqrt{Var(X)} = \sqrt{5.9 - 2.1^2} \approx 1.22.$

(c). $\boxed{X_{40} \sim N(n\mu, n\sigma^2)} = N(40(2.1), 40(1.49))$

$= N(84, 59.6)$

$$P(81 \le X_{40} \le 87) = \phi\left(\frac{87-84}{\sqrt{59.6}}\right) - \phi\left(\frac{81-84}{\sqrt{59.6}}\right)$$
$$= \phi\left(\frac{3}{\sqrt{59.6}}\right) - \phi\left(\frac{-3}{\sqrt{59.6}}\right).$$

Question 3

$\mu = 1518$ and $\sigma = 325$

(a) $X \sim N(1518, 325^2)$

$$P(1440 \le X \le 1480) = P(X \le 1480) - P(X \le 1440)$$
$$= \Phi\left(\frac{1480 - 1518}{325}\right) - \Phi\left(\frac{1440 - 1518}{325}\right)$$
$$= \Phi(-0.1169) - \Phi(-0.24)$$

(b). distribution of sample mean of size 16.

$\mu = 1518 \quad \sigma = \frac{325}{\sqrt{n}} = 81.25.$

$S_{16} \sim N(1518, (81.25)^2)$

$P(1440 \le S_{16} \le 1480) = P(S_{16} \le 1480) - P(S_{16} \geq 1440)$

$= \Phi\left(\frac{1480-1518}{81.25}\right) - \Phi\left(\frac{1440-1518}{81.25}\right)$

$= \Phi(-0.4677) - \Phi(-0.96).$

(c). The CLT tells us about the distribution of sample mean tend towards a normal distribution, as ~~the~~ the sample size increases. In this case, the original population distribution was already normally distributed. so all of the distribution of sample mean must already be. normally distributed.

Problem 4. $X \sim N(268, 15^2)$.

(a). $P(X \le 260) = \Phi\left(\frac{260-268}{15}\right) = \Phi(-0.5333).$

(b) $\mu = 268, \quad \sigma = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{25}} = 3.$

$S_{25} \sim N(268, 3^2)$

$P(S_{25} \le 260) = \Phi\left(\frac{260-268}{3}\right) = \Phi\left(-\frac{8}{3}\right).$

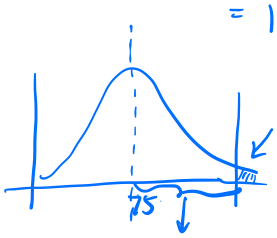
(c) Yes, the medical supervisors should be concerned.

Question 5: $N(75, 10^2)$

(a) $X \sim N(75, 100)$

$P(X > 100) = 1 - P(X \le 100).$

$= 1 - \Phi\left(\frac{100-75}{10}\right) = 1 - \Phi(2.5).$

(b) 

2.5% = 0.025. $Z = 1.96$

1.96σ to the right of the mean.

1.96·10. $\mu + 1.96\sigma = 75 + (1.96)(10) = 94.6.$

2.5% of the students scored more than 94.6.

(c) $P(X < 60) = \Phi\left(\frac{60-75}{10}\right) = \Phi(-1.5) = 0.0668.$

$0.0668 \times 100 = 6.68 \approx 7.$

(d) $\mu = 75$ $\sigma = \frac{100}{\sqrt{100}} = 1.$

$S_{100} \sim N(75, 1)$

$P(S_{100} < 70) = \Phi\left(\frac{70-75}{1}\right) = \Phi(-5)$