CSD2259 Homework 4

Due: Mar 31, 2024

The following problem set is used for the on-line homework 4 set up on Moodle. Please key in your answers on Moodle by the due date.
Highly appreciate if you could let me know typos and errors.

Questions 1-4. Consider the set $S = \{1, 2, 3, 4, 5\}$.

Question 1. How many 3-permutations does S have?
Remark: r-permutation means the normal permutation (without repetition). For permutation with repetition allowed, the phrase *repetition allowed* will always be mentioned.

(A) 6 (B) 20 (C) 24 (D) 60 (E) None of these

Question 2. How many 3-permutations of S start with 1?

(A) 6 (B) 12 (C) 24 (D) 60 (E) None of these

Question 3. How many 3-combinations does S have?
Remark: r-combination means the normal combination (without repetition). For combination with repetition allowed, the phrase *repetition allowed* will always be mentioned.

(A) 10 (B) 15 (C) 20 (D) 60 (E) None of these

Question 4. How many 3-combinations of S **not containing** 1?

(A) 5 (B) 10 (C) 15 (D) 20 (E) None of these

Questions 5-8. Consider binary strings of length 10.

Question 5. How many of them contain an equal number of 0's and 1's?

(A) 120 (B) 210 (C) 252 (D) 848 (E) None of these

Question 6. How many strings contain exactly four 1's?

(A) $\binom{10}{4}$ (B) $\binom{10}{6}$ (C) $\frac{10!}{4!6!}$ (D) All A,B,C (E) None of these

Question 7. How many of them contain at most ($\leq$) four 1's?

(A) 210 (B) 386 (C) 440 (D) 848 (E) None of these

Question 8. How many of them contain at least ($\geq$) five 1's?

(A) 210 (B) 386 (C) 440 (D) 638 (E) None of these

Questions 9-10. Consider a class with 20 primary school students, 10 boys and 10 girls.

Question 9. How many ways are there to form a group of 6 students so that the number of boys and the number of girls are different?

(A) $\binom{10}{3}^2$ (B) $\binom{10}{3}$ (C) $\binom{20}{6} - \binom{10}{3}$ (D) $\binom{20}{6} - \binom{10}{3}^2$ (E) None of these

Question 10. 2 students A and B insist that they are either both chosen in a group or both not chosen. How many ways are there to form a group of 6 with this property?

(A) $\binom{18}{6}$ (B) $\binom{18}{6} + \binom{18}{4}$ (C) $\binom{20}{6} - \binom{20}{2}$ (D) $\binom{20}{6} - \binom{18}{6}$ (E) None of these

Questions 11-15. A bagel shop sells 8 types of bagels:

onion, poppy seed, egg, salty, pumpernickel, sesame seed, raisin, and plain.

How many ways are there to choose

Question 11. six bagels?

(A) $\binom{13}{7}$ (B) $\binom{14}{7}$ (C) $\binom{8}{7}$ (D) $\binom{8}{6}$ (E) None of these

Question 12. a dozen bagels?

(C) $\binom{20}{12}$ (B) $\binom{20}{11}$ (C) $\binom{19}{7}$ (D) $\binom{19}{8}$ (E) None of these

Question 13. two dozen bagels?

(C) $\binom{32}{12}$ (B) $\binom{32}{11}$ (C) $\binom{31}{8}$ (D) $\binom{31}{7}$ (E) None of these

Question 14. a dozen bagels with at least one of each kind?

(A) $\binom{19}{7}$ (B) $\binom{11}{7}$ (C) $\binom{20}{7}$ (D) $\binom{20}{8}$ (E) None of these

Question 15. a dozen bagels with at least 3 egg bagels and at most 2 salty bagels?

(A) $\binom{16}{7}$ (B) $\binom{13}{7}$ (C) $\binom{16}{7} - \binom{13}{7}$ (D) $\binom{20}{7} - \binom{16}{7}$ (E) None of these