

CSD3240/3241/SEM4108 TUTORIAL 5

Question 1. Let $\alpha \in [-1, 1]$ and let X be a random variable with PDF

$$f(x) = \begin{cases} \frac{1+\alpha x}{2} & \text{if } -1 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(Note that this function appeared in Question 2 Tutorial 4)

- (a) Find $E(X)$.
- (b) Find $\text{Var}(X)$.

Question 2. Let $X \sim \text{Poisson}(\lambda)$, that is, its PMF is

$$p(x) = \frac{\lambda^x}{x!} e^{-\lambda}, \quad x = 0, 1, \dots$$

- (a) Prove that

$$\sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{\lambda}.$$

- (b) Find $E(X)$.

Question 3. Let the PDF given by (This is the old example we did in Week 5 Lecture)

$$f(x) = \begin{cases} \frac{6}{5}(x^2 + x) & \text{if } 0 \leq x \leq 1, \\ 0 & \text{else} \end{cases}$$

For the random variable X .

- (a) Find the mean of the distribution
- (b) Find the variance and standard deviation of X

Question 4. Let X be a continuous random variable with PDF

$$f(x) = \begin{cases} x + 0.5 & \text{if } 0 \leq x \leq 1, \\ 0 & \text{else} \end{cases}$$

Find $E(X^n)$ where $n \in \mathbb{N}$.

Question 5. Let X be a continuous random variable with PDF

$$f(x) = \begin{cases} 4x^3 & \text{if } 0 \leq x \leq 1, \\ 0 & \text{else} \end{cases}$$

Find $P(X \leq \frac{2}{3} | X > \frac{1}{3})$.

Question 6. Let X be a random variable that follows Binomial distribution with expectation $E(X) = 7$ and variance $V(x) = 6$. Then what is the probability of success p ?