CSD2301 Practice Solutions
17. **Gravitation**

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Practice Question 1

A typical adult human has a mass of about 70 kg. (a) What force does a full moon exert on such a human when it is directly overhead with its center 378,000 km away? (b) Compare this force with the force exerted on the human by the earth.

IDENTIFY: The force exerted by the moon is the gravitational force, $F_g = \dfrac{Gm_M m}{r^2}$. The force exerted on the person by the earth is $w = mg$.

SET UP: The mass of the moon is $m_M = 7.35\times10^{22}\ \text{kg}$. $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** $F_{\text{moon}} = F_g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(7.35\times10^{22}\ \text{kg})(70\ \text{kg})}{(3.78\times10^8\ \text{m})^2} = 2.4\times10^{-3}\ \text{N}$.

(b) $F_{\text{earth}} = w = (70\ \text{kg})(9.80\ \text{m/s}^2) = 690\ \text{N}$. $F_{\text{moon}}/F_{\text{earth}} = 3.5\times10^{-6}$.

EVALUATE: The force exerted by the earth is much greater than the force exerted by the moon. The mass of the moon is less than the mass of the earth and the center of the earth is much closer to the person than is the center of the moon.

Practice Question 2

An 8 kg point mass and a 15 kg point mass are held in place 50.0 cm apart. A particle of mass m is released from a point between the two masses 20.0 cm from the 8 kg mass along the line connecting the two fixed masses. Find the magnitude and direction of the acceleration of the particle.

SET UP: Each force is attractive. The particle (mass m) is a distance $r_1 = 0.200$ m from $m_1 = 8.00$ kg and therefore a distance $r_2 = 0.300$ m from $m_2 = 15.0$ kg. Let $+x$ be toward the 15.0 kg mass.

EXECUTE: $F_1 = \dfrac{Gm_1 m}{r_1^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(8.00\ \text{kg})m}{(0.200\ \text{m})^2} = (1.334\times10^{-8}\ \text{N/kg})m$, in the $-x$-direction.

$F_2 = \dfrac{Gm_2 m}{r_2^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(15.0\ \text{kg})m}{(0.300\ \text{m})^2} = (1.112\times10^{-8}\ \text{N/kg})m$, in the $+x$-direction. The net force is

$F_x = F_{1x} + F_{2x} = (-1.334\times10^{-8}\ \text{N/kg} + 1.112\times10^{-8}\ \text{N/kg})m = (-2.2\times10^{-9}\ \text{N/kg})m$. $a_x = \dfrac{F_x}{m} = -2.2\times10^{-9}\ \text{m/s}^2$. The acceleration is $2.2\times10^{-9}\ \text{m/s}^2$, toward the 8.00 kg mass.

EVALUATE: The smaller mass exerts the greater force, because the particle is closer to the smaller mass.

Practice Question 3

Ten days after it was launched toward Mars in December 1998, the Mars Climate Orbiter spacecraft (mass 629 kg) was 2.87×10^6 km from the earth and travelling at 1.20×10^4 km/h relative to the earth. At this time, what were (a) the spacecraft's kinetic energy relative to the earth and (b) the potential energy of the earth-spacecraft system? Mass of earth is 5.97×10^{24} kg

IDENTIFY: The kinetic energy is $K = \frac{1}{2}mv^2$ and the potential energy is $U = -\dfrac{GMm}{r}$

SET UP: The mass of the earth is $M_E = 5.97 \times 10^{24}$ kg.

EXECUTE: **(a)** $K = \frac{1}{2}(629\text{ kg})(3.33 \times 10^3\text{ m/s})^2 = 3.49 \times 10^9\text{ J}$

(b) $U = -\dfrac{GM_E m}{r} = -\dfrac{(6.673 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\text{ kg})(629\text{ kg})}{2.87 \times 10^9\text{ m}} = -8.73 \times 10^7\text{ J}$.

Practice Question 4

The International Space Station makes 15.65 revolutions per day in its orbit around the earth. Assuming a circular orbit, how high is this satellite above the surface of the earth? Mass of earth is 5.97×10^{24} kg and radius of earth is 6380 km.

EXECUTE: The satellite moves 15.65 revolutions in 8.64×10^4 s, so the time for 1.00 revolution is

$T = \dfrac{8.64 \times 10^4\,\text{s}}{15.65} = 5.52 \times 10^3$ s. $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ gives

$$r = \left(\frac{Gm_E T^2}{4\pi^2}\right)^{1/3} = \left(\frac{[6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2][5.97 \times 10^{24}\ \text{kg}][5.52 \times 10^3\,\text{s}]^2}{4\pi^2}\right)^{1/3}.$$

$r = 6.75 \times 10^6$ m and

$h = r - R_E = 3.7 \times 10^5\ \text{m} = 370\ \text{km}$.

Practice Question 5

NASA launched the Aura spacecraft to study the earth's climate and atmosphere. This satellite was injected into an orbit 705 km above the earth's surface, and we shall assume a circular orbit. (a) How many hours does it take this satellite to make one orbit? (b) How fast (in km/s) is the Aura spacecraft moving? Mass of earth is 5.97×10^{24} kg and radius of earth is 6380 km.

SET UP: If h is the height of the orbit above the earth's surface, the radius of the orbit is $r = h + R_E$.

$R_E = 6.38\times10^6$ m and $m_E = 5.97\times10^{24}$ kg.

EXECUTE: **(a)** $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}} = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.38\times10^6\ \text{m})^{3/2}}{\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}} = 5.94\times10^3\ \text{s} = 99.0\ \text{min}$

$v = \dfrac{2\pi r}{T}$

(b) $v = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.38\times10^6\ \text{m})}{5.94\times10^3\ \text{s}} = 7.49\times10^3\ \text{m/s} = 7.49\ \text{km/s}$