CSD1241 Extra Exercises

Problem 1. Consider two vectors $\vec{u} = \begin{bmatrix} 2 \\ -4 \\ c \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ c \\ -1 \end{bmatrix}$. Find c such that

(a) $\vec{u}$ is parallel to $\vec{v}$.
(b) $\vec{u}$ is perpendicular to $\vec{v}$.
(c) In each case in part a, b, find the area of the parallelogram formed by $\vec{u}, \vec{v}$ if c exists.

Problem 2. Given 3 points $A = (1,2,0), B = (0,3,1), C = (-1,0,1)$ and the line $l : (x,y,z) = (3,5,-1) + t \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$.
(a) Let m be the line containing A, B. Find the intersection, the angle and the distance between l and m.
(b) Let α be the plane through A, B, C. Find the intersection and the angle between l and α.

Problem 3. In this problem, we find the angle between 2 planes by definition.

Consider 2 planes $\alpha : x - y + 2z = 1$ and $\beta : 2x - y = 0$.
(a) Find the line l which is the intersection of α and β.
(b) Find the plane γ containing the point $(1,3,5)$ and perpendicular to l.
(c) Let l_1 be the intersection between γ and α. Let l_2 be the intersection between γ and β. Find the direction vectors $\vec{d_1}, \vec{d_2}$ of l_1, l_2.
(d) Find the angle a between l_1 and l_2. Verify that the value of a is the same as the value of the angle between α and β which is computed by $\angle(\alpha, \beta) = \cos^{-1}\left(\frac{|\vec{n}_\alpha \cdot \vec{n}_\beta|}{||\vec{n}_\alpha||||\vec{n}_\beta||}\right)$.

Problem 4. The parallelogram formed by two vectors $\vec{u}, \vec{v} \in \mathbb{R}^2$ of equal length is called a **rhombus**. Show that the diagonals of this rhombus are perpendicular.

Problem 5. (a) Given 3 points A, B, C in $\mathbb{R}^2$. Find a method to determine whether they lie on a line? Further, give two examples of 3 points in $\mathbb{R}^2$ (one for 3 points on a line, and one for 3 points not a line) and illustrate your method on these examples.
(b) Given 4 points A, B, C, D in $\mathbb{R}^3$. Find a method to determine whether they lie on a plane. Further, give two examples of 4 points in $\mathbb{R}^3$ (one for 4 points on a plane, and one for 4 points not on a plane) and illustrate your method on these examples.