CSD1241 Tutorial 4 Answer Keys

Problem 1. In each of the following cases, find the intersection between 2 planes α, β and find the angle between them.

(a) $\alpha : x + 2y + 6z = 5$ and $\beta : 2x - 3y + 5z = 3$.
Hint. When solving the common points on α and β, you need to solve a system of 2 equations. To solve these equations, solve x, y in terms of z.

(b) $\alpha : x + y + z - 1 = 0$ and $(x, y, z) = (1, 2, 3) + s \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$.

Solution. (a) α and β intersect at the line $(x, y, z) = (3, 1, 0) + z \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix}$. The angle between α and β is $a \approx 48.70°$.

(a) α and β intersect at the line $(x, y, z) = (6, -8, 3) + t \begin{bmatrix} 12 \\ -17 \\ 7 \end{bmatrix}$. The angle between α and β is $a \approx 70.09°$. □

Problem 2. Find the distance and the angle between l_1 and l_2 in following cases.

(a) $l_1 : (x, y, z) = (2, 0, 3) + t \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ and $l_2 : (x, y, z) = (1, 0, -2) + s \begin{bmatrix} 1 \\ -4 \\ 5 \end{bmatrix}$

(b) $l : (x, y, z) = (1, 5, 3) + t \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$ and $l_2 : \begin{cases} x = 1 + 4s \\ y = 2s \\ z = -2 - 6s \end{cases}$

(c) $l_1 : (x, y, z) = (7, 1, 0) + t \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ and $l_2 : \begin{cases} x = 5 + s \\ y = 2 + 2s \\ z = 8 + s \end{cases}$

Solution. (a) $d(l_1, l_2) \approx 3.14$ and $\angle(l_1, l_2) \approx 24.87°$.

(b) $d(l_1, l_2) \approx 6.55$ and $\angle(l_1, l_2) = 0°$.

(c) $d(l_1, l_2) \approx 4.28$ and $\angle(l_1, l_2) = 54.74°$. □

Problem 3. Find the distance between the planes α and β in following cases.

(a) $\alpha : x + y - z - 1 = 0$ and $\beta : 2x + 2y + 2z - 3 = 0$

(b) $\alpha : 3x + 4y - 5z = 2$ and $\beta : 6x + 8y - 10z = 2$

(c) $\alpha : (x, y, z) = (1, 2, 1) + s \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$ and $\beta : \begin{cases} x = 2 + 2s + t \\ y = 3 - t \\ z = 4 + 3s + t \end{cases}$

Hint. First, you need to check whether α and β are parallel by checking if $\vec{n}_\alpha$ is parallel to $\vec{n}_\beta$. If they are not parallel, then $d(\alpha, \beta) = 0$.

Solution. (a) $d(\alpha, \beta) = 0$.

(b) $d(\alpha, \beta) = \sqrt{2}/10$.

(c) $d(\alpha, \beta) = 2/\sqrt{14}$. □

Problem 4. Consider the plane $\alpha : 2x + 3y - z = 5$. In each of the following cases, find the intersection between α and the line l. Further, find the angle between α and l.

(a) $l : (x, y, z) = (15, 7, 11) + t \begin{bmatrix} 7 \\ 2 \\ 3 \end{bmatrix}$

(b) l : through $P = (1, 2, 3)$ and $Q = (1, -1, 1)$.

Solution. (a) l intersects α at the point $\left(\frac{10}{17}, \frac{49}{17}, \frac{82}{17} \right)$ and $\angle(l, \alpha) \approx 35.24°$.

(b) l intersects α at the point $P = (1, 2, 3)$ and $\angle(l, \alpha) \approx 31.26°$.