

CSD2259 Homework 1

Due: Jan 21, 2024

The following problem set is used for the on-line homework 1 set up on Moodle. Please key in your answers on Moodle by the due date.

Feel free to let me know if you find typos and errors.

Question 1. Let X, Y, Z be defined as follows

- X = set of airline flights from Singapore to New York
- Y = set of non-stop (direct) flights from Singapore to New York
- Z = set of flights from Singapore to New York which stop over at Japan

Which of the following is the correct description between X, Y, Z ?

- ☒ (A) $X \subset (Y \cup Z)$ (B) $Y \subset Z$ (C) $X - Y \subset Z$ (D) $Z \subset X - Y$ (E) None of these

Question 2. Determine whether each following pair of sets are equal

- (a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}$ and $\{5, 3, 1\}$ (b) $\{\{1\}\}$ and $\{1, \{1\}\}$ (c) \emptyset and $\{\emptyset\}$

- ☒ (A) Only a (B) Only b (C) Only c (D) All of these (E) None of these

Question 3. Given 3 sets A, B, C with

$$A = \{1, 2, 3\}, B = \{2, \{2\}, \{2, \{2\}\}\}, C = \{1, 1, 2, 2, 3, 3\}$$

Which power set among $\mathcal{P}(A), \mathcal{P}(B), \mathcal{P}(C)$ has the largest size?

- (A) $\mathcal{P}(A)$ (B) $\mathcal{P}(B)$ (C) $\mathcal{P}(C)$ (D) Both $\mathcal{P}(B)$ and $\mathcal{P}(C)$ ☒ (E) All have same size

Question 4. Let a, b be distinct elements. Among 4 sets

$$A = \emptyset, B = \{\emptyset, \{a\}\}, C = \{\emptyset, \{a\}, \{\emptyset, a\}\}, D = \{\emptyset, \{a\}, \{b\}, \{a, b\}\},$$

which set could be a power set of some set?

- ☒ (A) Only A ☒ (B) Only B (C) Only C (D) Both B and D (E) Only D

Questions 5-7. Consider three sets

$$A = \emptyset, B = \{1, 2\}, C = \{1, 2, \{1, 2\}\}$$

Question 5. What is $|\mathcal{P}(A)|$?

- (A) 0 ☒ (B) 1 (C) 2 (D) 4 (E) None of these

Question 6. What is $|\mathcal{P}(B \cup C)|$?

- (A) 4 ☒ (B) 8 (C) 16 (D) 32 (E) None of these

Question 7. What is $|\mathcal{P}(B \times C)|$?

- (A) 4 (B) 8 (C) 16 ☒ (D) 32 (E) None of these

Questions 8-11. Let A be the set of sophomores at your school and B be the set of students in discrete mathematics at your school.

Question 8. Which of the following is the set of sophomores taking discrete mathematics in your school?

- ☒ (A) $A \cap B$ (B) $A \cup B$ (C) $A - B$ (D) $B - A$ (E) None of these

Question 9. Which of the following is the set of sophomores who are not taking discrete mathematics?

- (A) $A \cap B$ (B) $A \cup B$ (C) $A - B$ ☒ (D) $B - A$ (E) None of these

Question 10. Which of the following is the set of students at your school who are sophomores or are taking discrete mathematics?

- (A) $A \cap B$ ☒ (B) $A \cup B$ (C) $A - B$ (D) $B - A$ (E) None of these

Question 11. Which of the following is the set of students at your school who are not sophomores or are not taking discrete mathematics?

- (A) $A \cap B$ (B) $A \cup B$ (C) $A - B$ (D) $B - A$ ☒ (E) None of these

Questions 12-14. The symmetric difference of A and B, denoted by $A \oplus B$, is the set containing those elements in either A or B, but not in both A and B.

Let $U = \{1, 2, 3, 4, 5\}$ be the universal set and let $A = \{1, 2, 3\}$, $B = \{1, 3, 5\}$.

Question 12. What is $A \oplus B$?

- (A) A (B) B (C) $\{2, 5\}$ (D) $\{1, 3\}$ (E) None of these

Question 13. What is $|\mathcal{P}(A \oplus \bar{B})|$?

- (A) 1 (B) 2 (C) 4 (D) 8 (E) None of these

Question 14. Assume that C is a subset of U such that $A \oplus C = A$. Which of the following is the correct statement about C?

- (A) The only possibility for C is the empty set. ✓
(B) C can be any subset of A. ✗
(C) A is a subset of C. ✗
(D) C is the universal set U. ✗
(E) None of these is correct.

Questions 15-17. Let A_1, A_2, A_3 be three sets with 100 elements each.

Question 15. What is $|A_1 \cup A_2 \cup A_3|$ if A_1, A_2, A_3 are pairwise disjoint?

- (A) 0 (B) 100 (C) 200 (D) 300 (E) Not enough information

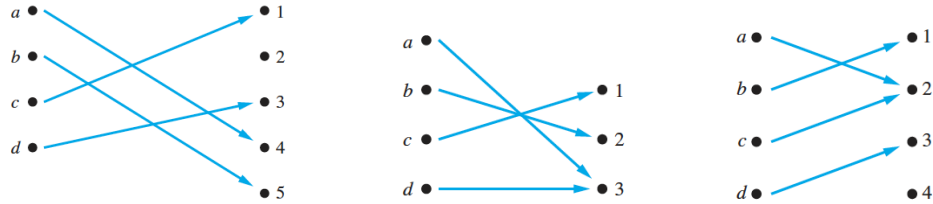
Question 16. What is $|A_1 \cup A_2 \cup A_3|$ if there are 50 common elements in each pair of sets and no elements in all three sets?

- (A) 100 (B) 150 (C) 250 (D) 300 (E) None of these

Question 17. What is $|A_1 \cup A_2 \cup A_3|$ if the three sets are equal (the same set)?

- (A) 100 (B) 150 (C) 250 (D) 300 (E) None of these

Questions 18-19. Three functions f, g, h are given by the following rules



Question 18. Which functions are 1-1?

- (A) Only f (B) Only g (C) Only h (D) All of them (E) None of them

Question 19. Which functions are onto?

- (A) Only f (B) Only g (C) Only h (D) All of them (E) None of them

Questions 20-24. Let $f, g, h : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = x^2, \quad g(x) = 2x + 5, \quad h(x) = x^3 + 1$$

Question 20. What is the correct formula for $(f + g)(x)$?

- (A) x^2 (B) $2x + 5$ (C) $x^2 + 2x + 5$ (D) $x^2 - 2x - 5$ (E) None of these

Question 21. What is the correct formula for $(f \circ h)(x)$?

- (A) $(x^3 + 1)^2$ (B) $x^6 + 1$ (C) $x^2(x^3 + 1)$ (D) $x^2 + x^3 + 1$ (E) None of these

Question 22. Which functions among f, g, h are increasing?

- (A) Only f (B) Only g (C) Only h (D) Both g and h (E) All of them

Question 23. What is the correct formula for $h^{-1}(x)$?

- (A) $x^3 + 1$ (B) $\sqrt[3]{x-1}$ (C) $\sqrt[3]{x}$ (D) $\sqrt[3]{x+1}$ (E) None of these

Question 24. What is the correct formula for $(h \circ g)^{-1}(x)$?

- (A) $\frac{x-5}{2}$ (B) $\sqrt[3]{x-1}$ (C) $(2x+1)^3 + 1$ (D) $2x^3 + 1$ (E) None of these