

CSD3240/MAT340/SEM4108 Week 9

Tutorial

Question 1. Let X and Y be jointly distributed random variables whose joint PMF $p(x, y)$ is given in the follow table.

$X \setminus Y$	1	2	3	4
1	0.1	0.05	0.02	0.02
2	0.05	0.2	0.05	0.02
3	0.02	0.05	0.2	0.04
4	0.02	0.02	0.04	0.1

- (a) Write marginal PMFs $p_X(x)$ of X and $p_Y(y)$ of Y in the same table above.
- (b) Are X and Y independent?
- (c) Find $\text{Cov}(X, Y)$.

In Week 9 Lecture, we see that X, Y independent $\Rightarrow \text{Cov}(X, Y) = 0$. In the following question, we show that the reverse implication $\text{Cov}(X, Y) = 0 \Rightarrow X, Y$ independent is wrong.

Question 2. Let X be a random variable that takes on values $-2, -1, 0, 1, 2$, each with probability $1/5$. Let $Y = X^2$.

- (a) Express the values of $p(x, y)$, $p_X(x)$, $p_Y(y)$ in the joint probability table.
- (b) Show that $\text{Cov}(X, Y) = 0$.
- (c) Are X and Y independent?

Question 3. Let X and Y be jointly distributed variables with joint PDF

$$f(x, y) = \begin{cases} 2x + 2y - 4xy & \text{if } 0 \leq x, y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the marginal PDFs $f_X(x)$ of X and $f_Y(y)$ of Y .
- (b) Compute $E(X)$ and $E(Y)$.
- (c) Compute $\text{Cov}(X, Y)$.

Question 4. Let c be a real number and let $f(x, y)$ be the following function

$$f(x, y) = \begin{cases} c(x + y)^2 & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find c so that $f(x, y)$ is the joint PDF of two random variables X and Y .
- (b) Find the marginal PDFs $f_X(x)$ of X and $f_Y(y)$ of Y .
- (c) Are X and Y independent?
- (d) Compute $\text{Cov}(X, Y)$.