CSD1251/CSD1250 Week 3 Tutorial Problems

16 – 22 January 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Important: From now onwards, unless otherwise specified, if any midterm/final exam question or part of a question requires you to differentiate a function, you may use any of the methods we learnt in class.

Qn 1 (Definition of derivative at a point)

In slide 7 of this week's lecture, we have two versions of the definition of the derivative at a point, which were labelled as equations (1) and (2) respectively.

Use equation (1) to find the derivative of the function at a point for parts (a) and (b), and use equation (2) find the derivative of the function at a point for parts (c) and (d).

(a) $f(x) = \sqrt{4x+1},\ a = 6$

(b) $f(x) = 5x^4,\ a = -1$

(c) $f(x) = \dfrac{x^2}{x+6},\ a = 3$

(d) $f(x) = \dfrac{1}{\sqrt{2x+2}},\ a = 1$

Qn 2 (Modulus function)

We have previously defined the modulus function $f(x) = |x|$ in Tutorial 2 as

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

(a) Draw the graph of f.

(b) Using the definition of the derivative, show that f is not differentiable at the point 0.

Qn 3 (Derivative functions)

Using the definition of the derivative, find the derivative of the following functions. You may use either equation (1) or (2) on slide 14 of this week's lecture slides.

(a) $f(x) = 3$ (b) $f(x) = 6x - 4$ (c) $f(x) = 4 + 8x - 5x^2$

(d) $f(v) = \dfrac{v}{v+2}$ (e) $f(x) = \dfrac{1}{x^2 - 4}$ (f) $g(x) = \dfrac{1}{1+\sqrt{x}}$

Qn 4 ("Efficient" differentiation)

(a) The quotient rule for differentiation can be written as

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}.$$

Suppose $g(0) \neq 0$. Write down the quotient rule when $x = 0$.

(b) Let R be the function

$$R(x) = \frac{x - 3x^3 + 5x^5}{1 + 3x^3 + 6x^6 + 9x^9}.$$

Find $R'(0)$ in the following ways.

(1) Differentiate $R(x)$ using the quotient rule to get $R'(x)$, then substituting $x = 0$ into $R'(x)$ to get $R'(0)$.

(2) Set $f(x)$ and $g(x)$ to be the numerator and denominator of $R(x)$ respectively, then compute $f(0), f'(0), g(0), g'(0)$, and finally use part (a) to compute $R'(0)$.

(3) Which method is easier to evaluate, (1) or (2)?

Qn 5 (Differentiation rules)

Differentiate the following functions with respect to their variables.

(a) $f(x) = 10$ (b) $f(x) = 2x - 5$ (c) $g(x) = x^3 - x + 1$

(d) $F(\theta) = \cos\theta + \sec\theta$ (e) $h(x) = 7\sin x - 3\tan x$ (f) $R(t) = 4\sqrt[3]{t}$

(g) $u(x) = 3x\ln x + 2^x$ (h) $f(x) = \ln x \sin x$ (i) $g(t) = t^2 \cos t$

(j) $h(\theta) = e^{\theta}\left(\theta + \theta\sqrt{\theta}\right)$ (k) $h(x) = \dfrac{e^x \sin x}{x^2 + 1}$ (l) $g(u) = \dfrac{3\sin u + 1}{\cos u}$

(m) $f(x) = \dfrac{\tan x}{\sec x}$ (n) $g(t) = \dfrac{5^t \tan t}{t^5}$ (o) $r(\theta) = \dfrac{\theta^2 + 2}{\theta^3 + 4\theta}$