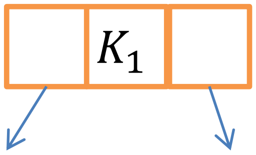
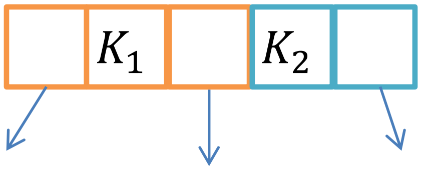
2-3 Search Tree

2-3 Search Trees

- Each node can contain 1 or 2 keys.
- Each node has 2 or 3 children, hence 2-3 trees.



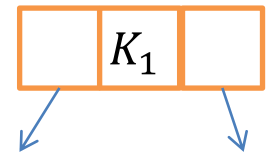
2-node



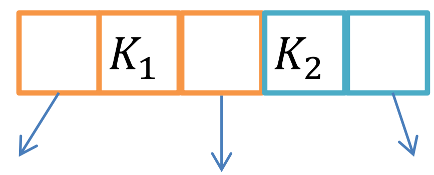
3-node

2-3 Search Trees

- Each node can contain 1 or 2 keys.
- Each node has 2 or 3 children, hence 2-3 trees.
- The keys in the nodes are ordered from **small to large**.



2-node

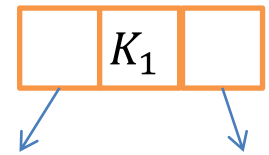


3-node

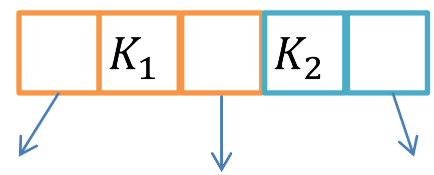
$K_1 < K_2$

2-3 Search Trees

- Each node can contain 1 or 2 keys.
- Each node has 2 or 3 children, hence 2-3 trees.
- The keys in the nodes are ordered from **small to large**.
- All leaves are at the same (bottom most) level, meaning **we always add at the bottom**.



2-node

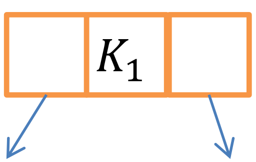


3-node

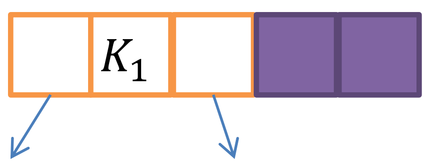
$K_1 < K_2$

2-3 Search Tree Node

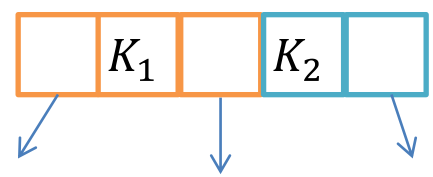
```
struct Node23{
  Node23 *left, *middle, *right;
  Key key1, key2;
};
```



2-node (not showing empty)



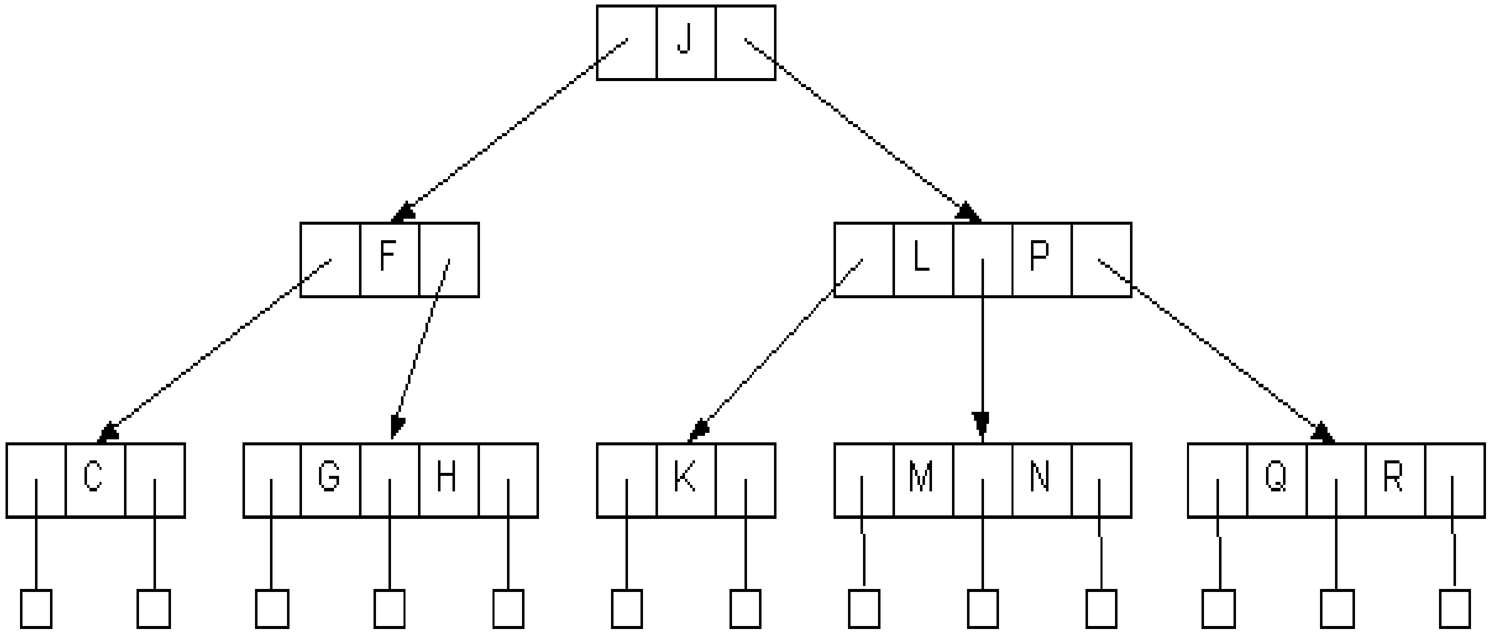
2-node (showing empty)



3-node

$$K_1 < K_2$$

Example

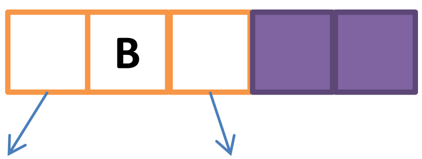


Insertion

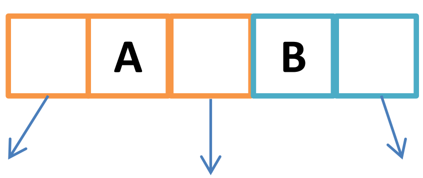
- Splitting this way is called bottom-up balancing
  - Insert the node at the bottom-most level at correct location.
  - If the node is a 3-node, split it and pass the middle key to the parent.
    - If the parent is also a 3-node, split the parent and pass the middle key up
      - Etc…
  - Eventually, the root will also be a 3-node and splitting it will grow the tree one level.

Insertion

- If the node you insert is a 2-node, simply grow the node to a 3-node

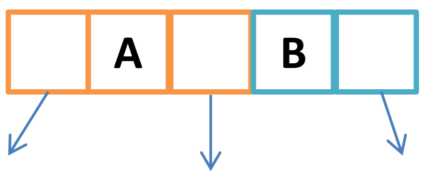


Inserting **'A'**

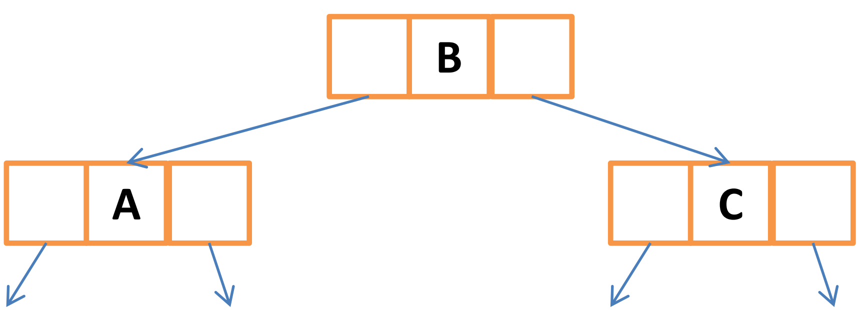


Insertion

- If the node you insert is a 3-node, we cannot grow the node more
  - We split it!

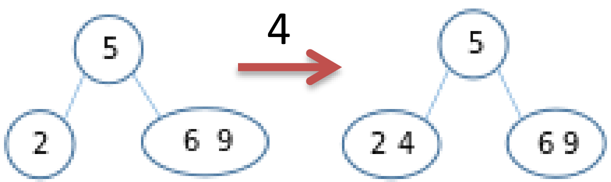


Inserting **'C'**

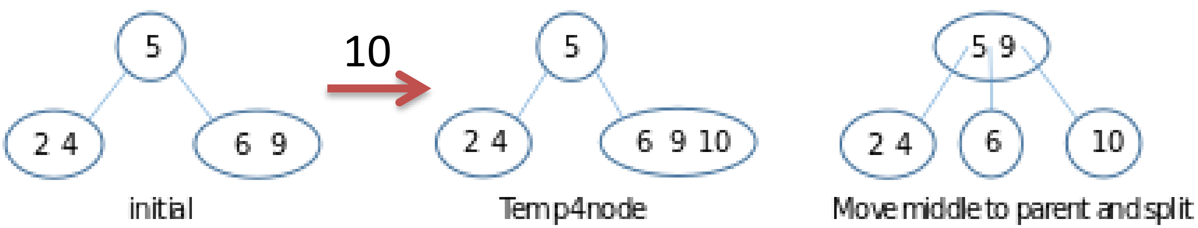


Insertion - Cases

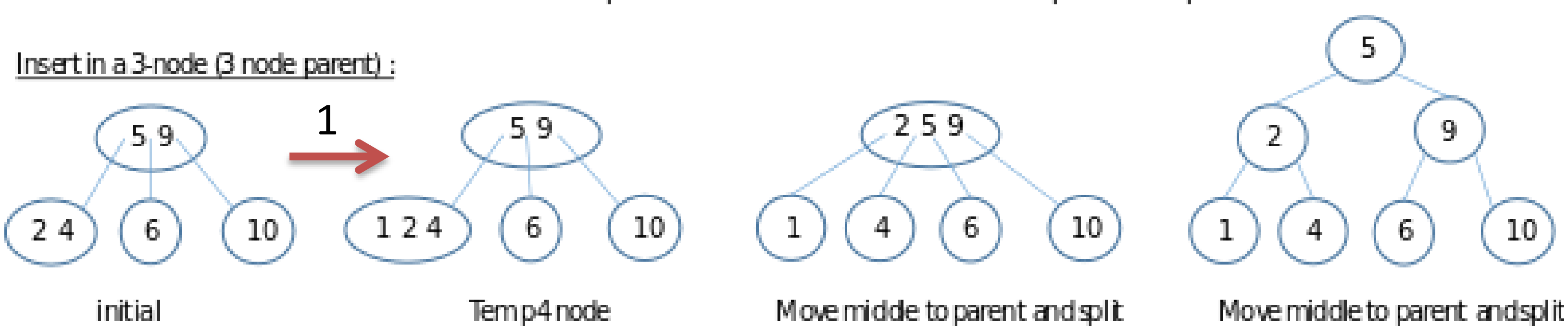
Insert in a 2-node :



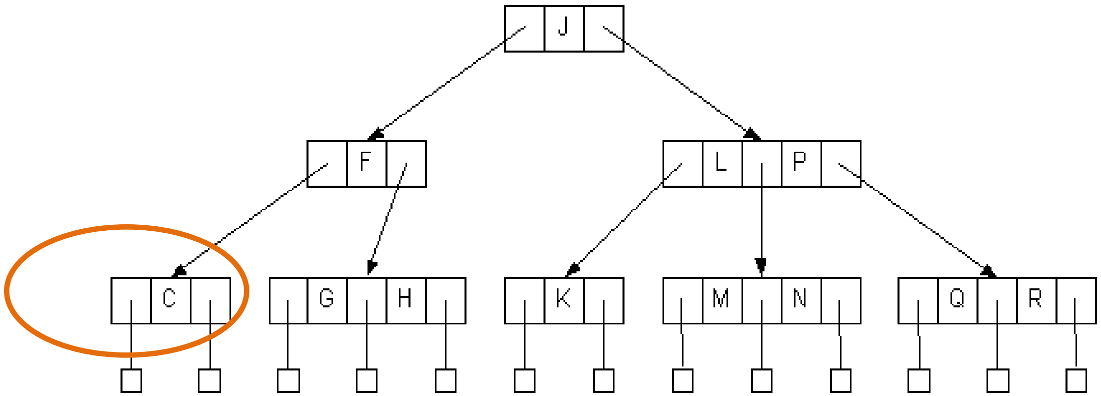
Insert in a 3-node (2 node parent) :

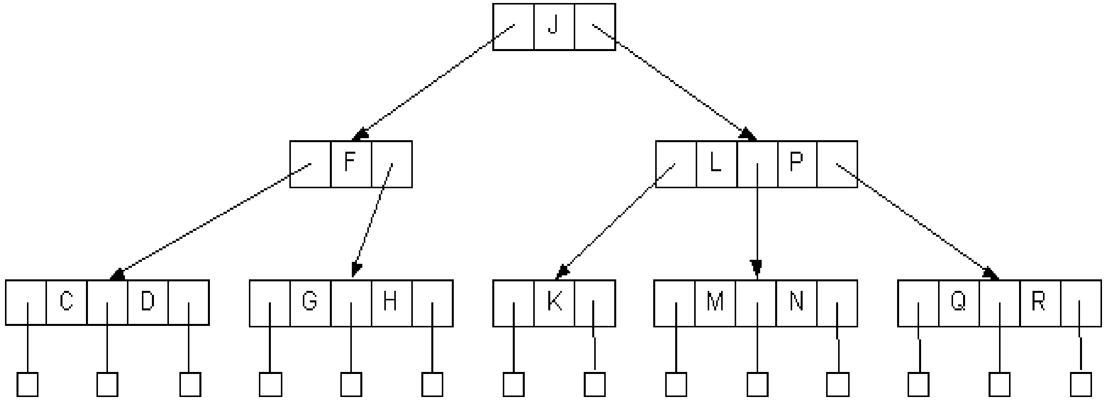


Insert in a 3-node (3 node parent) :

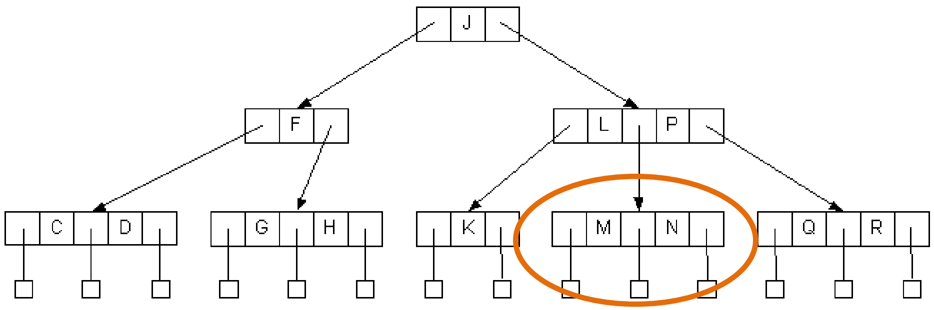


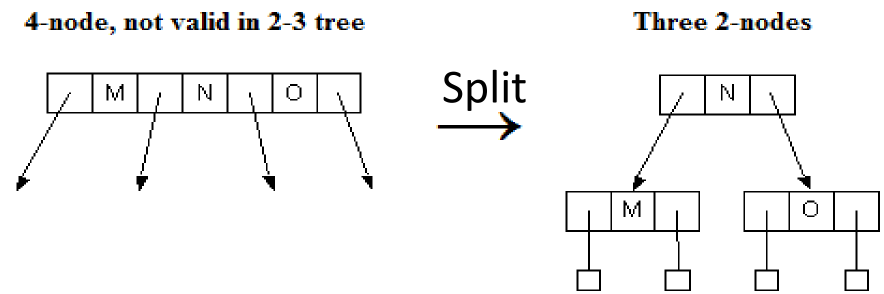
Example: Insert D

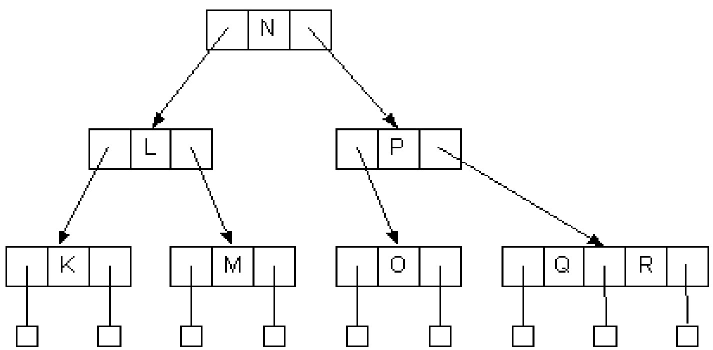


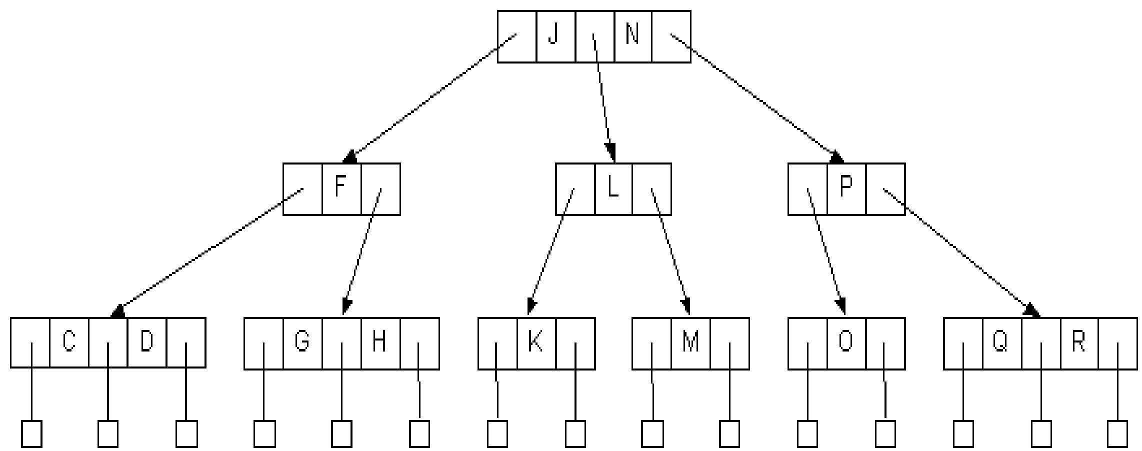


Insert O

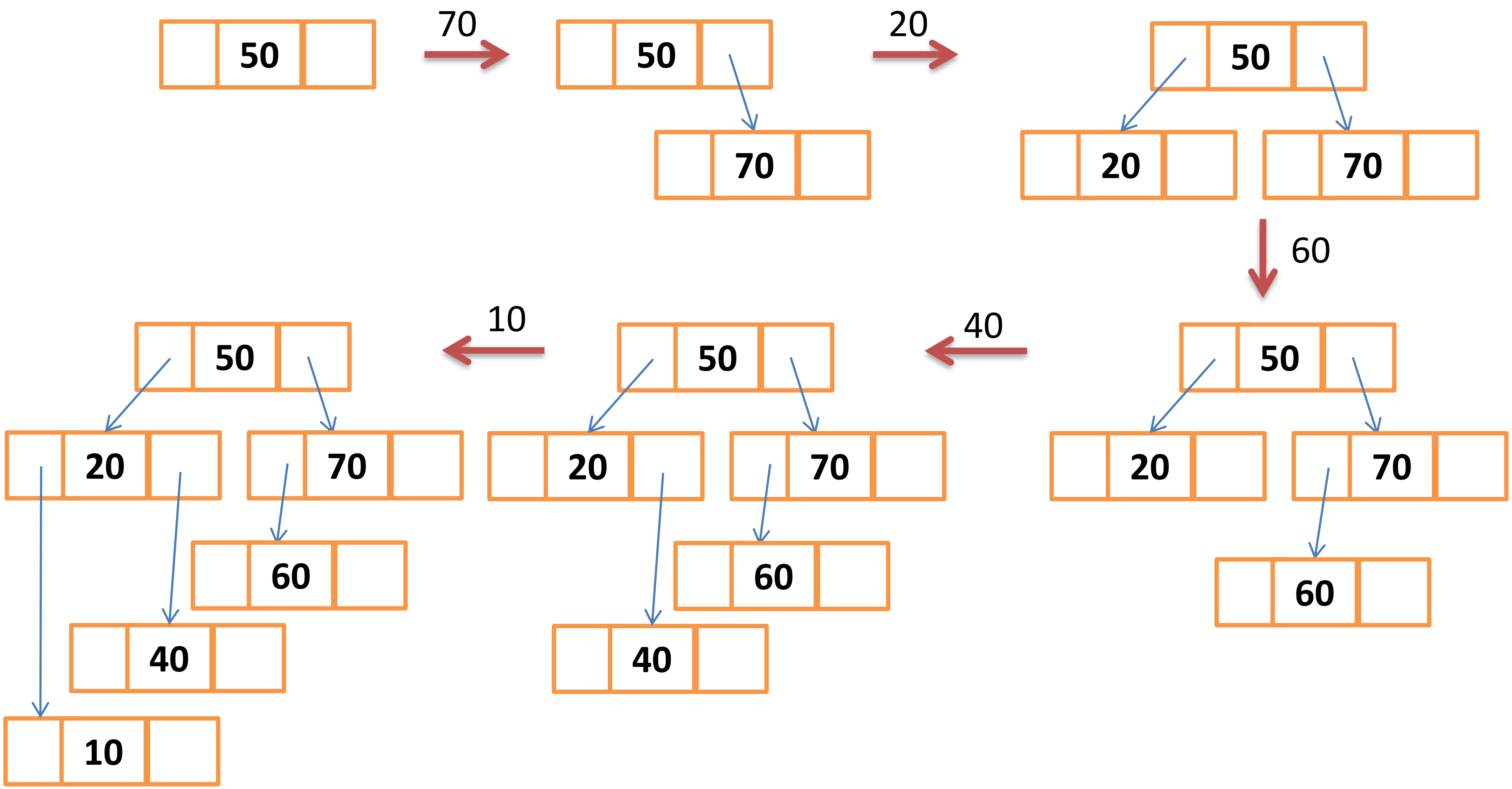

J
F
L
P
C
D
G
H
K
M
N
Q
R


4-node, not valid in 2-3 tree
M
N
O
Split
Three 2-nodes
N
M
O

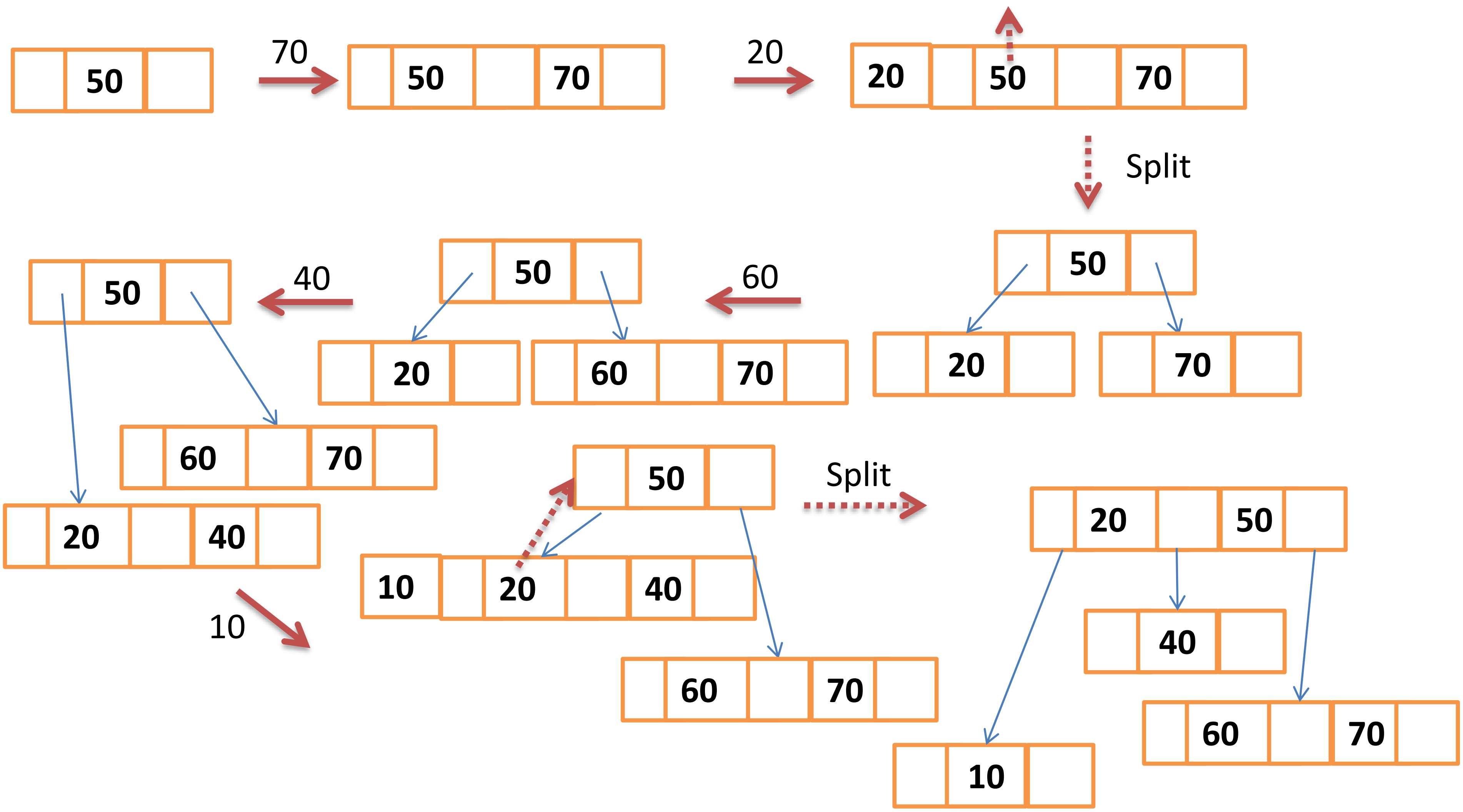

N
L
P
K
M
O
Q
R


J
N
F
L
P
C
D
G
H
K
M
O
Q
R

BST with 50, 70, 20, 60, 40, 10



2-3 Tree with 50, 70, 20, 60, 40, 10



Properties of 2-3 Search Trees

- 2-3 search trees guarantee to be balanced at all times.
- Searches are O(log N) in worst case.
- Balance is maintained during insertion
  - Splitting nodes, worst case O(log N), average case O(1)

Deletion

1. Find the key (X) that needs to be deleted
2. If X is in a non-leaf node

   2.1 Replace X with its predecessor

   2.2 If no predecessor replace with successor
3. If X is in leaf node

   3.1 if left/right sibling has enough keys

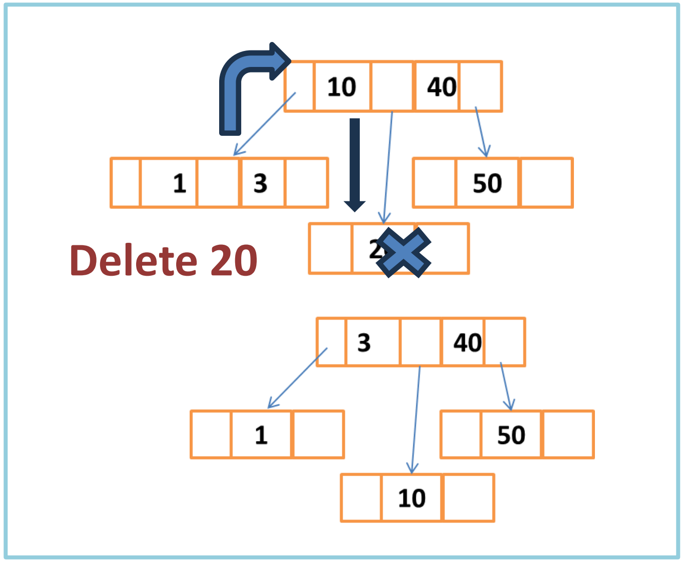
   (ie more than 1)

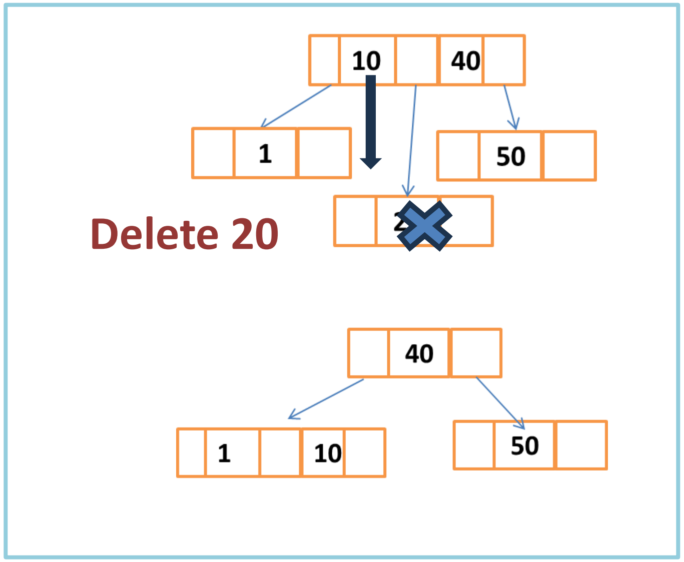
   3.1.1 donate by rotating with parent (may need to take over sibling's child)

   3.2 else

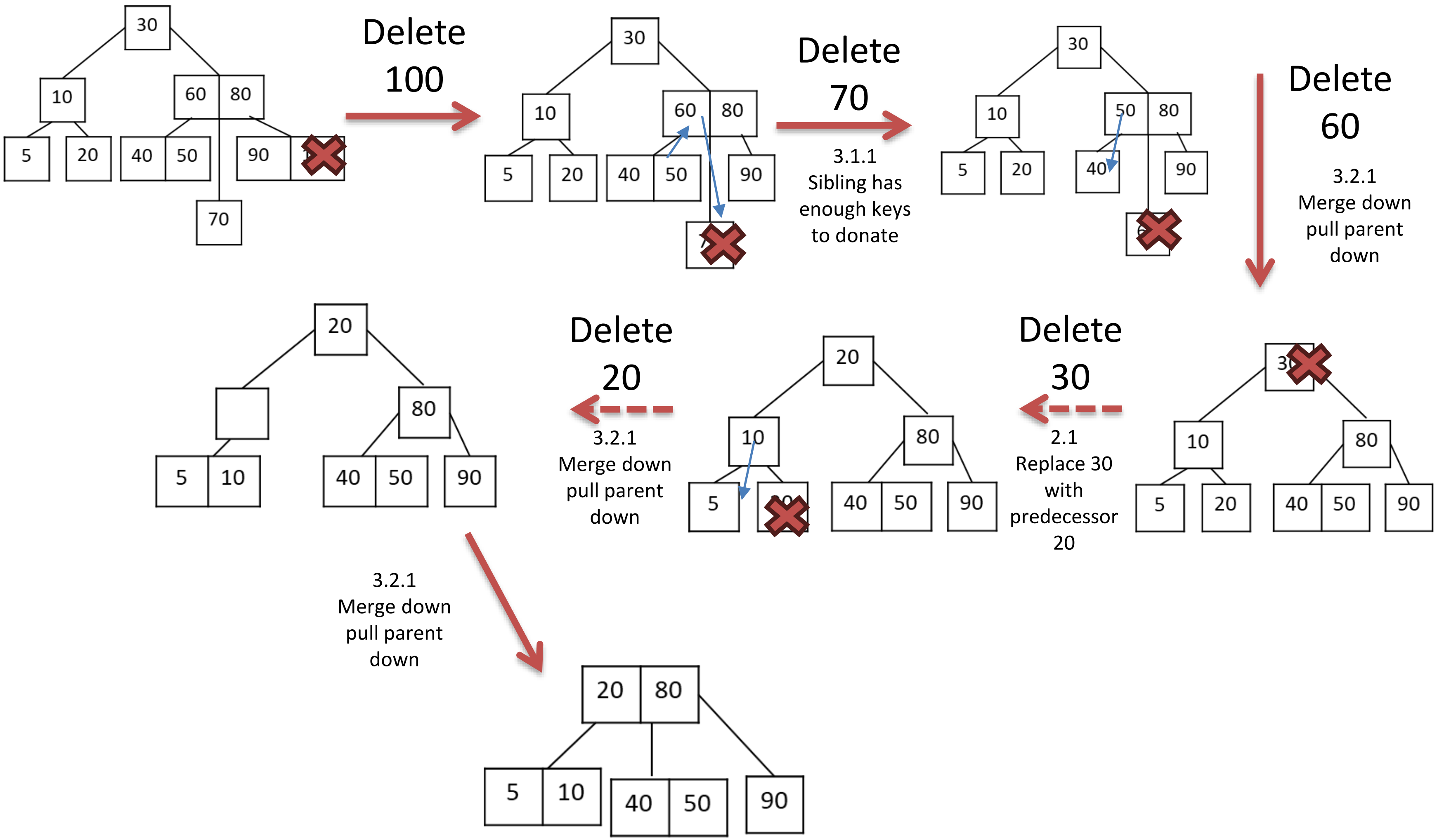
   3.2.1 Merge down (pull the parent down)

   If parent is underful repeat Step 3 recursively

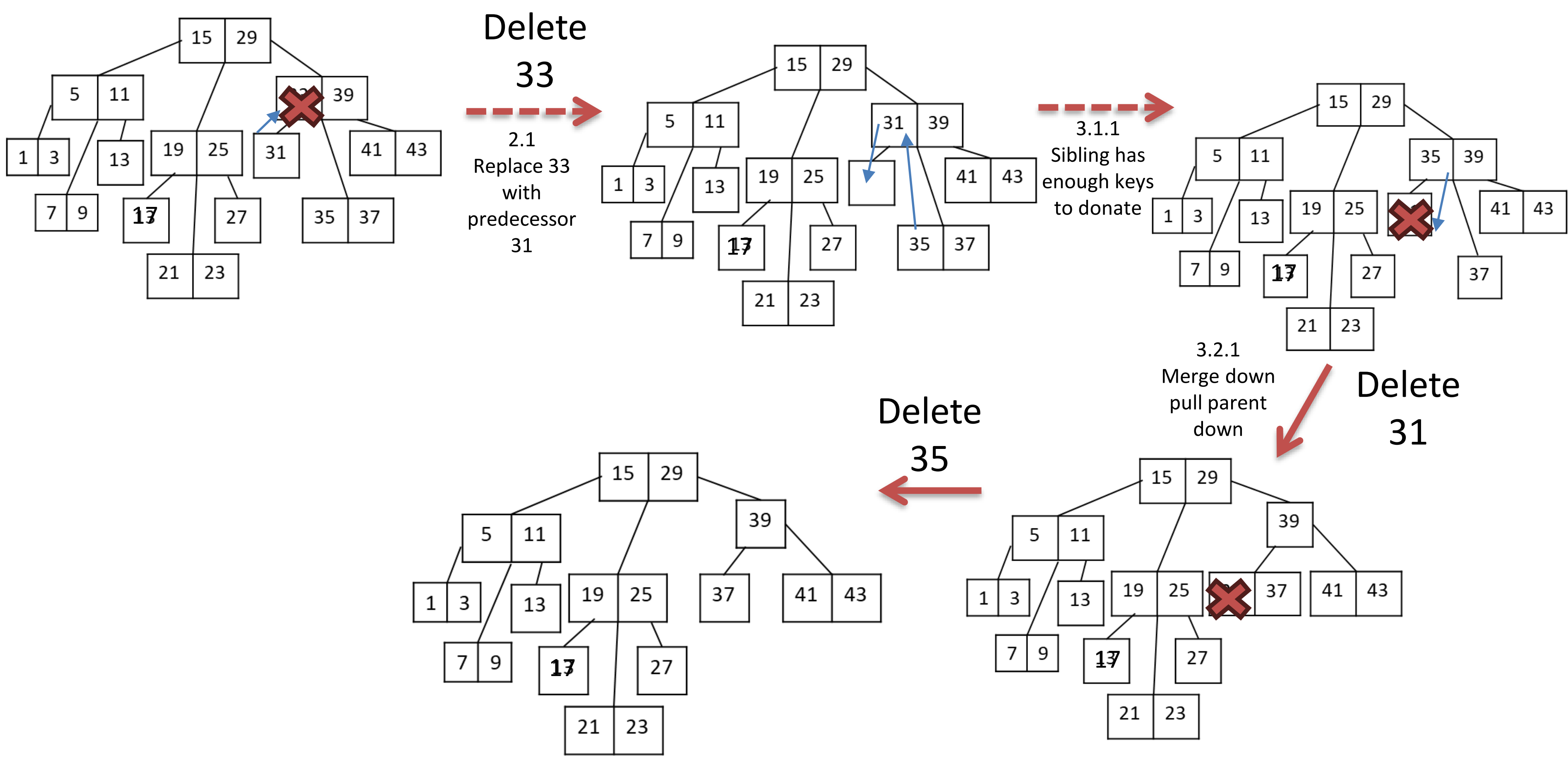




Example 1


30
10
60 80
5
20
40 50
90
70
Delete
100
30
10
60 80
5
20
40 50
90
Delete
70
3.1.1
Sibling has
enough keys
to donate
30
10
50 80
5
20
40
90
Delete
60
3.2.1
Merge down
pull parent
down
10
80
5
20
40 50
90
Delete
30
2.1
Replace 30
with
predecessor
20
20
10
80
5
40 50
90
Delete
20
3.2.1
Merge down
pull parent
down
20
80
5 10
40 50
90
3.2.1
Merge down
pull parent
down
20 80
5 10
40 50
90

Example 2


Delete 33
2.1
Replace 33 with predecessor 31
3.1.1
Sibling has enough keys to donate
Delete 31
3.2.1
Merge down pull parent down
Delete 35

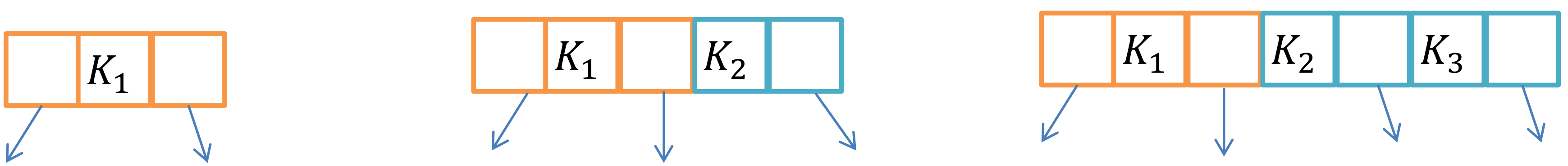
2-3-4 Search Tree

Basic Properties

- Similar to 2-3 trees
- Nodes can contain 1, 2, or 3 keys.
- Nodes can has 2, 3, 4 children, hence 2-3-4 tree.
  - Each can have at most 4 children.
- Similarly to 2-3 trees, 2-3-4 trees are guaranteed to be always balanced.
- Balancing algorithm also relies on Splitting nodes
- Number of splits in the worst-case is O(log N)
  - When is the worst-case?
- Average number of splits is very few.

2-3-4 Node

```
struct Node234
{
  Node23 *left, *midleft, *midright, *right;
  Key key1, key2, key3;
};
```



2-node

3-node

4-node

$$K_1 < K_2 < K_3$$

Balancing Algorithm

- Balancing also occurs on insertion.
- Modifying the algorithms for balancing can produce **better efficiency.**
- Previously, with 2-3 Trees, we have seen bottom-up balancing. O(log N) splits in worst case where splitting propagates up to the root
- We will see top-down balancing
  - As you go down the tree to insert a node, split any full node.
  - **A full node is a 4-node (3 keys/4 child node)**.

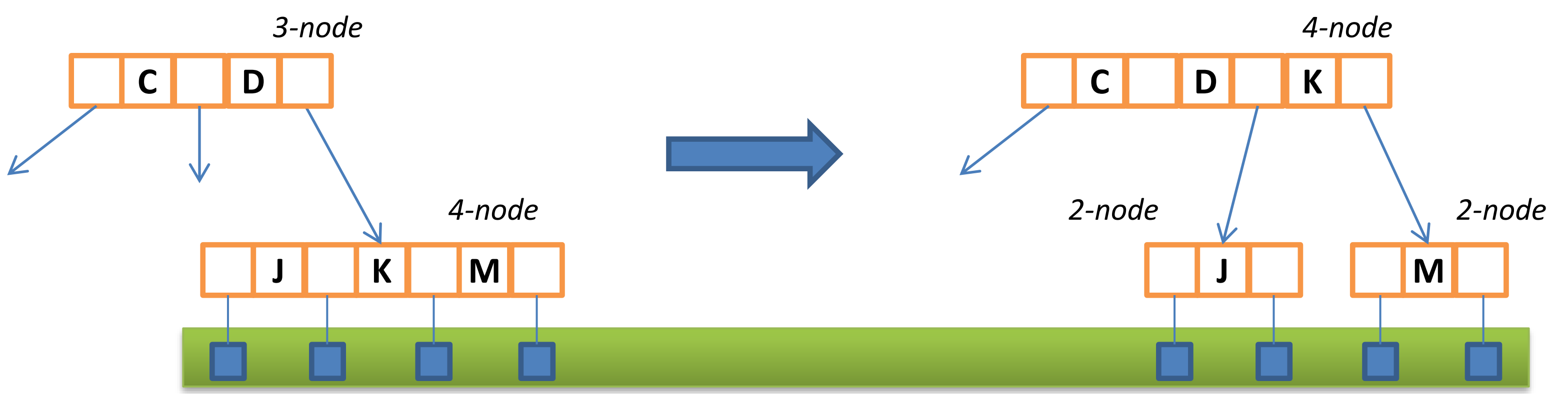
How to split a 2-3-4 node?

Insertion

- Insert at leaf position
- While going down the tree, if encounter a 4 – node
  - Split it, push the middle element to the parent
  - If it's the root, split and create a new root

Advantage of Splitting 2-3-4 Trees

- Splitting a node is cleaner.
- Splitting a 4-node into two 2-nodes preserves the number of child links.
- Changes do not have to be propagated. Change remains local to split.

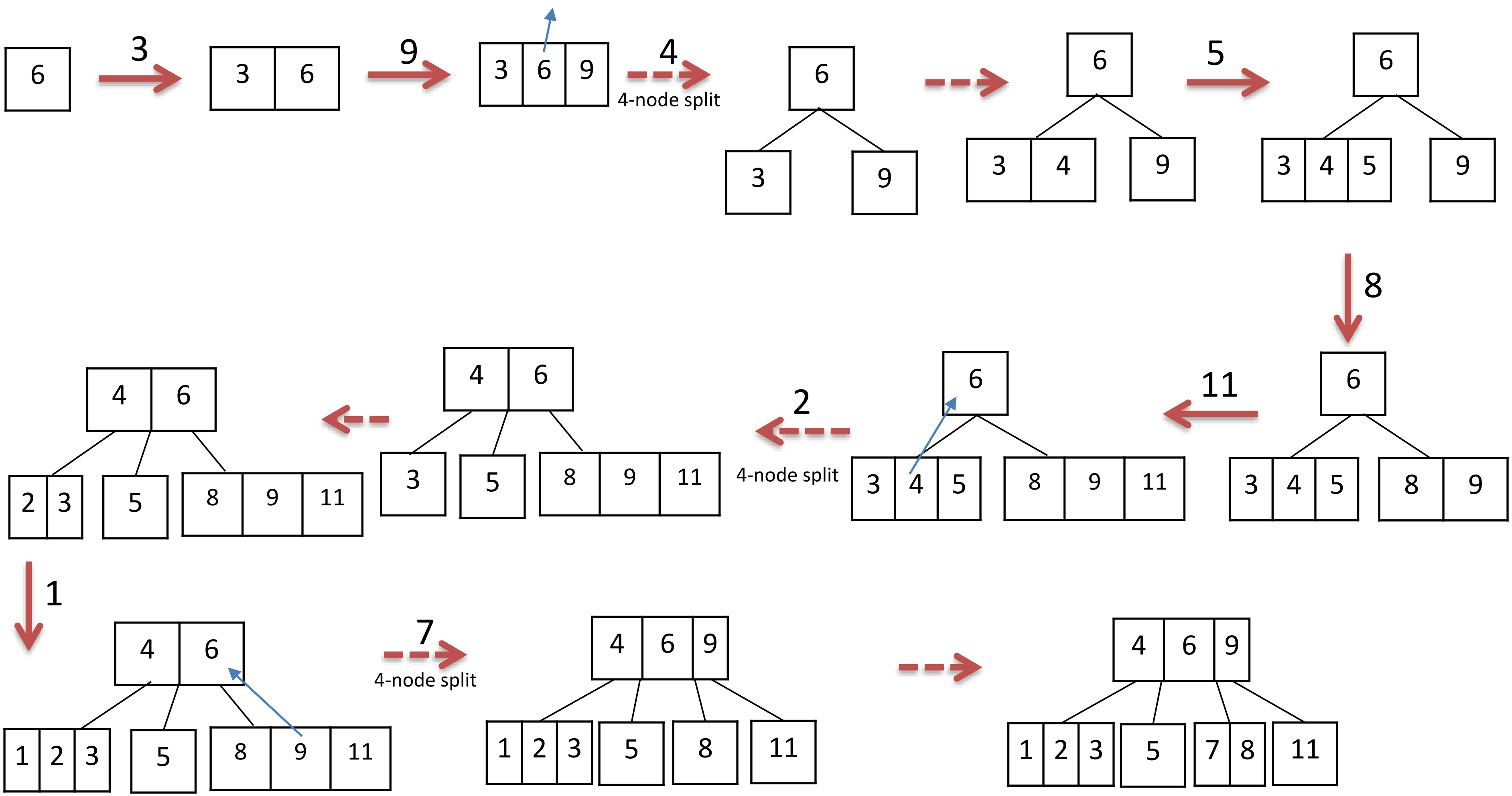


Top-Down Balancing

- Split nodes on the way down.
  - Guarantees that each node we pass through is not a 4-node.
  - When we reach the bottom, we will not be on a 4-node (think about it)

- This way, we only traverse the tree once, when inserting/balancing.

2-3-4 tree with 6,3,9,4,5,8,11,2,1,7

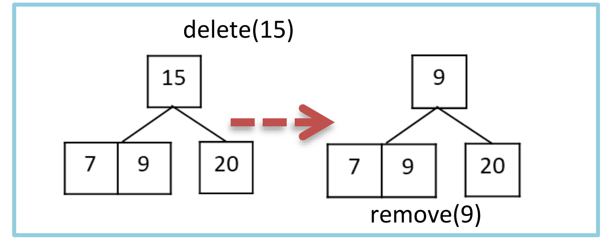


Deletion

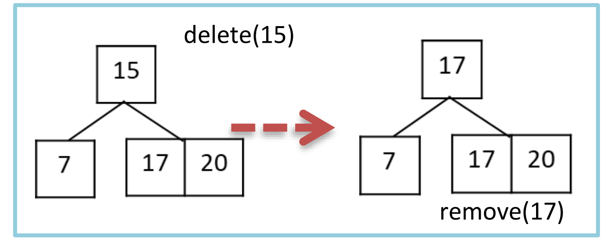
Find the key (X) that needs to be deleted

1. if (X is in a leaf node with atleast 2 keys)
    - 1.1 remove (X)
2. else if (X is in a non-leaf node)
    - 2.1 if (left_child of the node >= 2 keys)
        - 2.1.1 replace (X,predecessor)
        - 2.1.2 remove (predecessor)
    - 2.2 else if (right_child of the node >= 2 keys)
        - 2.2.1 replace (X,successor)
        - 2.2.2 remove (successor)
    - 2.3 else // both children have only 1 key each
        - 2.2.1 merge(left_child,X,right_child)
        - 2.2.2 remove (X)
3. else // ch is the node_visited_so_far
    - 3.1 if (ch has only 1 key)
        - 3.1.1 if (ch's sibling >=2 keys)
            - rotate a key into ch
        - 3.1.2 else if (ch's sibling has 1 key)
            - merge(ch,parent,sibling)

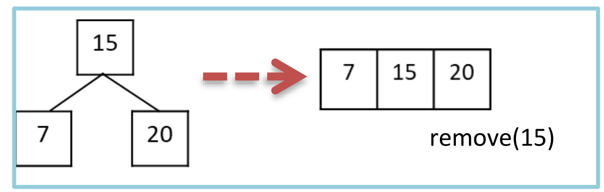
2.1



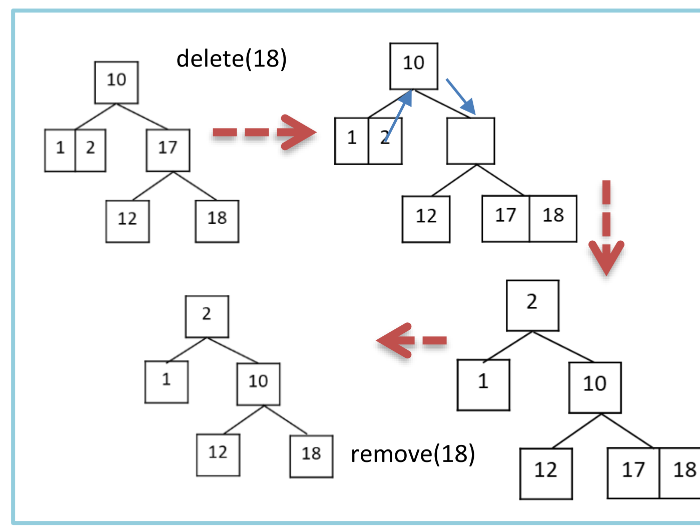
2.2



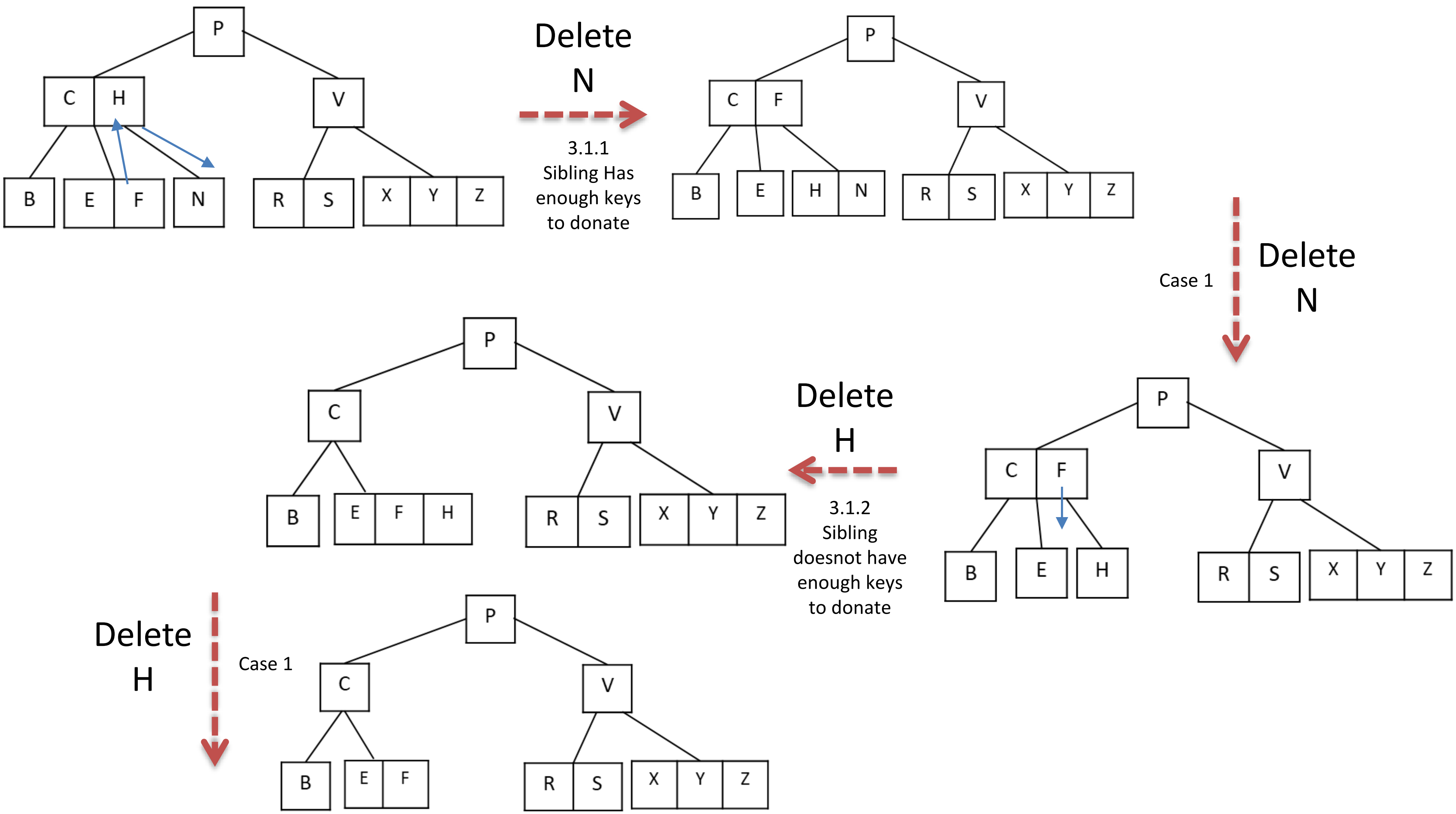
2.3



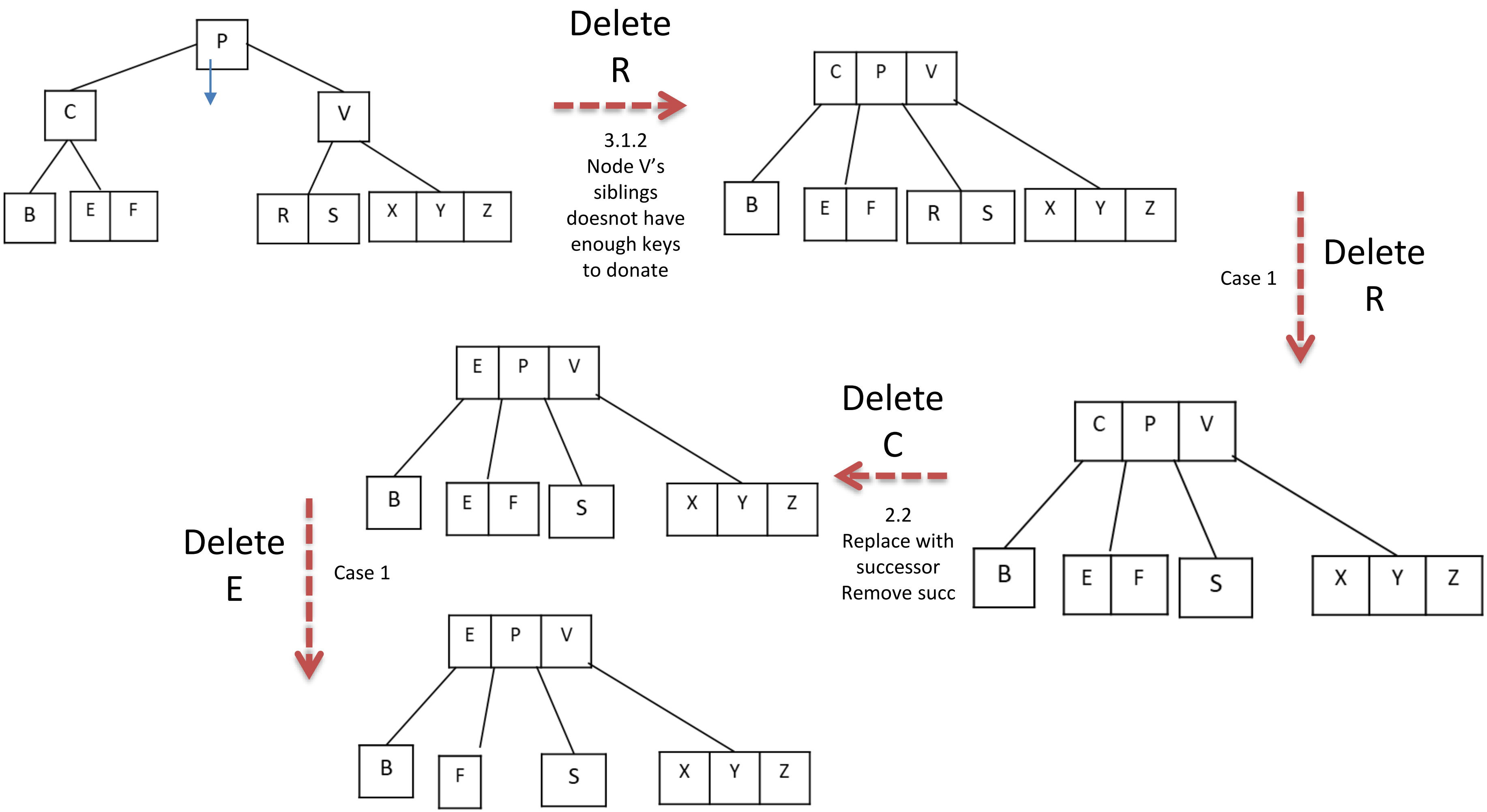
3



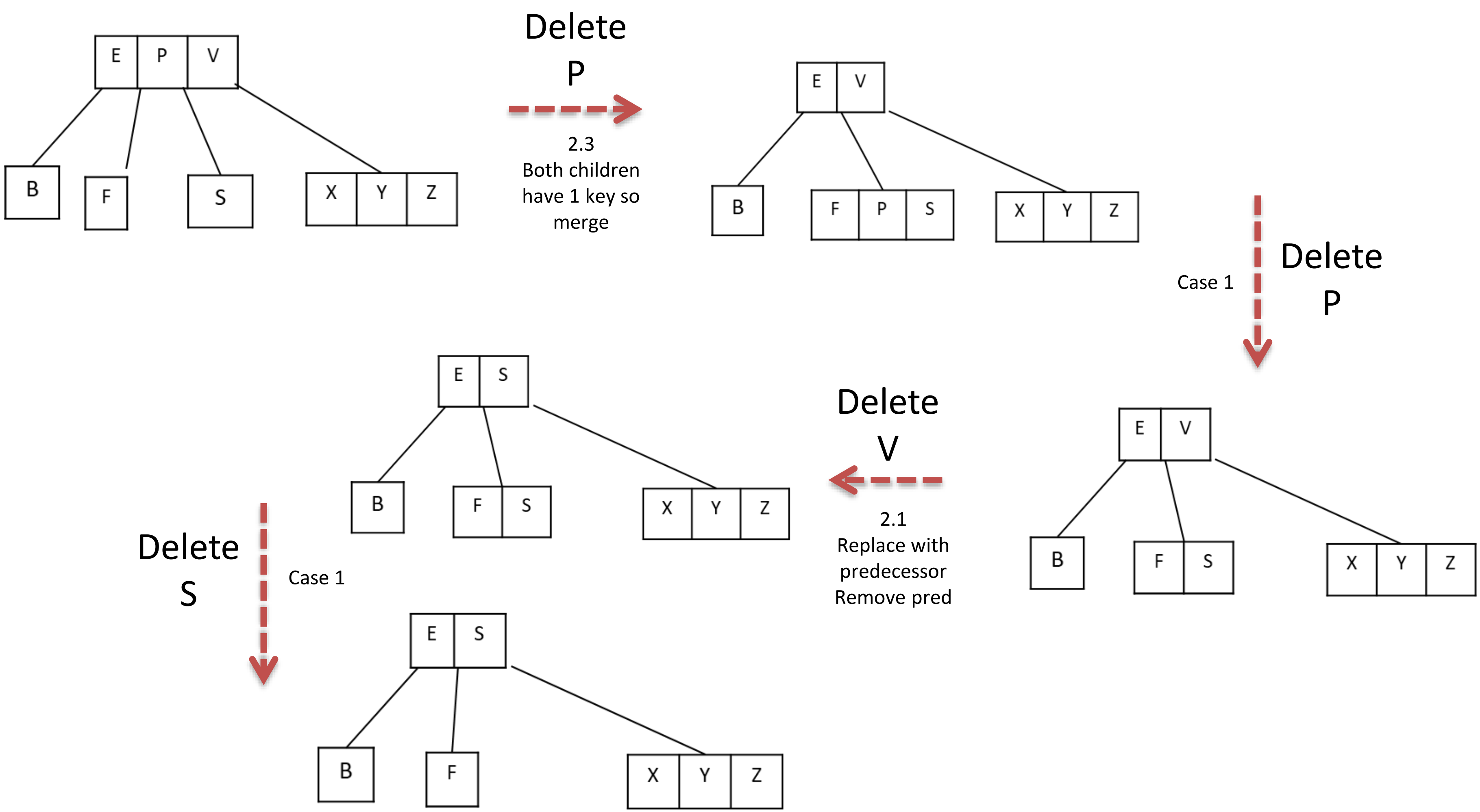
Delete N,H,R,C,P,V



Delete R,C,P,V


P
C
V
B
E F
R S
X Y Z
Delete R
3.1.2
Node V's
siblings
doesnot have
enough keys
to donate
C P V
B
E F
R S
X Y Z
Case 1
Delete R
C P V
B
E F
S
X Y Z
Delete C
2.2
Replace with
successor
Remove succ
E P V
B
E F
S
X Y Z
Delete E
Case 1
E P V
B
F
S
X Y Z

Delete P,V


E P V
B
F
S
X Y Z
Delete P
2.3
Both children have 1 key so merge
E V
B
F P S
X Y Z
Delete P
Case 1
E V
B
F S
X Y Z
Delete V
2.1
Replace with predecessor
Remove pred
E S
B
F S
X Y Z
Delete S
Case 1
E S
B
F
X Y Z