

# Week 1 Tutorial

**Problem 1.** Let  $P$  be a probability measure on a sample space  $\Omega$ , and let  $A, B, C$  be events.

- (a) Suppose that  $P(A) = 1/3$ ,  $P(B) = 1/4$ , and  $P(A \cup B) = 5/9$ . Find  $P(A \cap B)$ .
- (b) Suppose that  $P(A) = 1/2$ ,  $P(B) = 1/4$ ,  $P(A \cap B^c) = 3/8$ , and  $B \subset C$ . Find  $P(A \cup B^c \cup C^c)$ .
- (c) Someone claims that  $P(A) = P(B) = P(C) = 9/10$ ,  $P(A \cap B) = P(A \cap C) = P(B \cap C) = 7/10$ , and  $P(A \cap B \cap C) = 5/10$ . Can the claim be correct?

**Problem 2.** (a) What is the conditional probability that a family with two children has two boys, given they have at least one boy? Assume that each of the possibilities BB, BG, GB, and GG is equally likely, where B represents a boy and G represents a girl.

(b) A 0-1 bit string of length 4 is generated randomly so that each of 16 strings is equally likely. What is the probability that it contains at least two consecutive 0's, given that its first bit is 0? (we assume that 0 bits and 1 bits are equally likely).

**Problem 3.** (a) You know that a bridge players' hand of 13 cards contains at least one ace. What is the probability that it contains exactly 2 aces?

(b) You know that a bridge players' hand of 13 cards contains the ace of hearts. What is the probability that it contains exactly two aces.

**Problem 4.** You have a flight from Amsterdam to Sydney with a stopover in Dubai. The probabilities that a luggage is put on the wrong plane at different airports are

Amsterdam : 0.05, Dubai : 0.03.

What is the probability that your luggage does not reach Sydney with you?

**Hint:**  $A$ =event that the luggage is put on the correct plane at Amsterdam,  $B$ =event that the luggage is put in the correct plane at Dubai. The event that the luggage does not reach Sydney with you is  $(A \cap B)^c$ .