CSD2301 Lecture
8. Work and Energy Part 2

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Outline

- Potential energy
- Conservative forces
- Conservation of energy
- Non-conservative force
- Potential energy function
- Energy diagrams & equilibrium

Potential Energy

- **Potential energy** is energy associated with the position (gravitational P.E.) or configuration (elastic P.E.) of an object.
- P.E. can be considered as stored energy that can be converted to K.E. or other forms of energy.

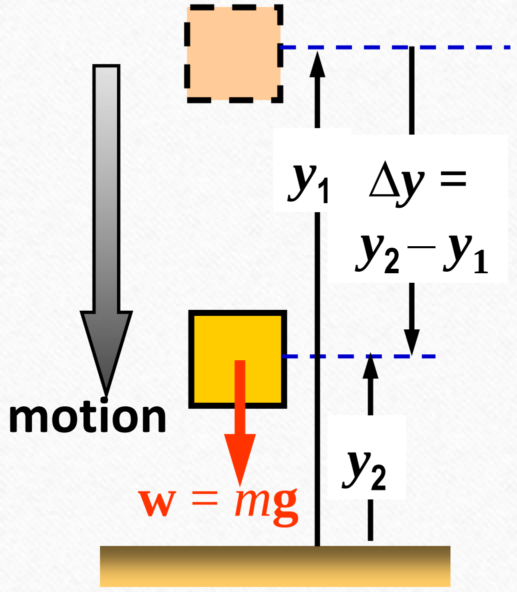
Gravitational Potential Energy

- Consider an object moving downwards from y_1 to y_2
- Work done by gravitational force is given by:

$$W_{grav} = \vec{F} \cdot \Delta\vec{y} = (-mg)\hat{\jmath} \cdot (y_2 - y_1)\hat{\jmath} = mgy_1 - mgy_2$$

- If m is moving upward from y_2 to y_1, the work done is given by the same formula (value is of course negative, since F is opposite to displacement)
- Define gravitational potential energy:

$$\boxed{U = mgy}$$



Gravitational Potential Energy

- We can write:

$$W_{tot} = W_{grav} = -\Delta U = U_1 - U_2$$

- From work-energy theorem:

$$W_{tot} = \Delta K = K_2 - K_1$$

$$\Downarrow$$

Kinetic Energy $K_2 - K_1 = U_1 - U_2$ potential energy

$$\Downarrow$$

$$K_1 + U_1 = K_2 + U_2 \Rightarrow \boxed{\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2}$$

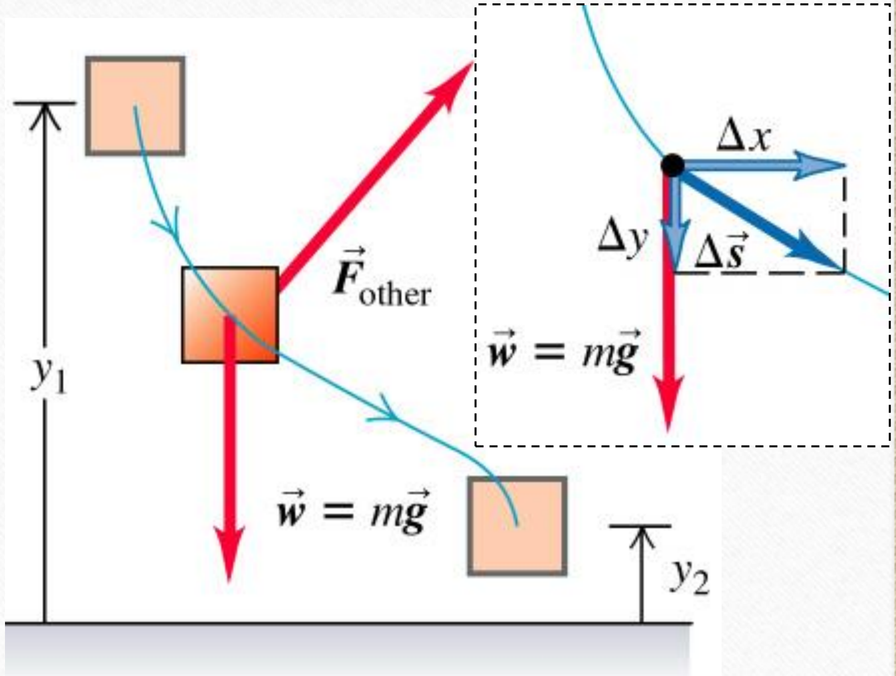
Gravitational PE along Curved Path

- If m travels along a curved path, we can resolve along the usual x-y direction to find the work done **by** gravitational force.

$$\vec{w} = m\vec{g} = -mg\hat{\jmath} \qquad \Delta\vec{s} = \Delta x\hat{\imath} + \Delta y\hat{\jmath}$$

$$\vec{w} \cdot \Delta\vec{s} = -mg\hat{\jmath} \cdot (\Delta x\hat{\imath} + \Delta y\hat{\jmath}) = -mg\Delta y$$

$$\boxed{W_{grav} = -mg(y_2 - y_1) = mgy_1 - mgy_2 = U_1 - U_2}$$



- Work is unaffected by any horizontal motion.
- Only the positions of initial and final points in motion important.

Gravitational Potential Energy

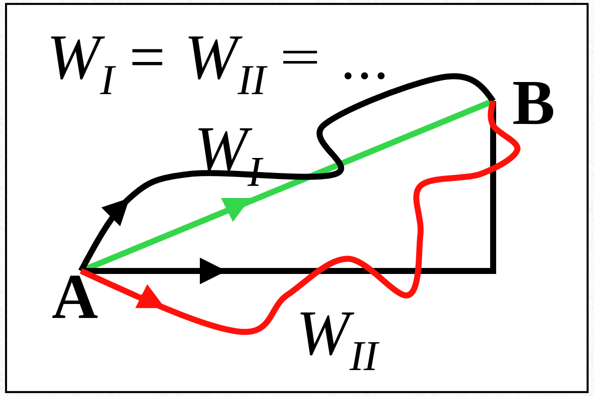
- Only the Δ GPE matters – Free to place the origin of coordinates (or zero of PE) in any convenient location.
- Horizontal motion does not affect the value of GPE.
- The unit of gravitational potential energy is same as that of work (joule). Potential energy is a scalar quantity.
- Work done by gravitational force depends only on its initial and final vertical coordinates (path independent).

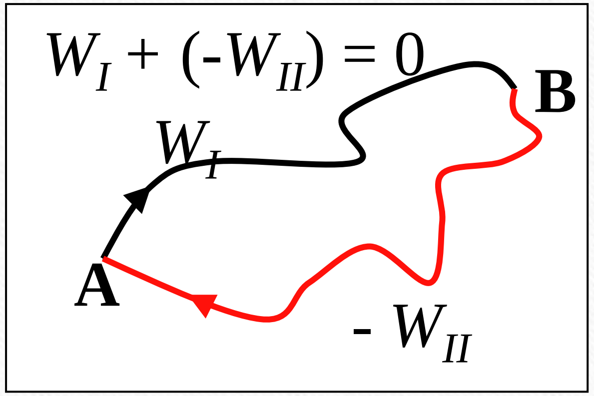
PE and Conservative Force

- Gravitational force and the force that the spring exerts are both **conservative** forces – the energy "stored" as PE can be converted back to kinetic energy.
- Some forces are **non-conservative**, e.g, friction and air resistance – these are dissipative forces (mechanical energy reduces when acted upon by these forces).

Properties of Conservative Forces

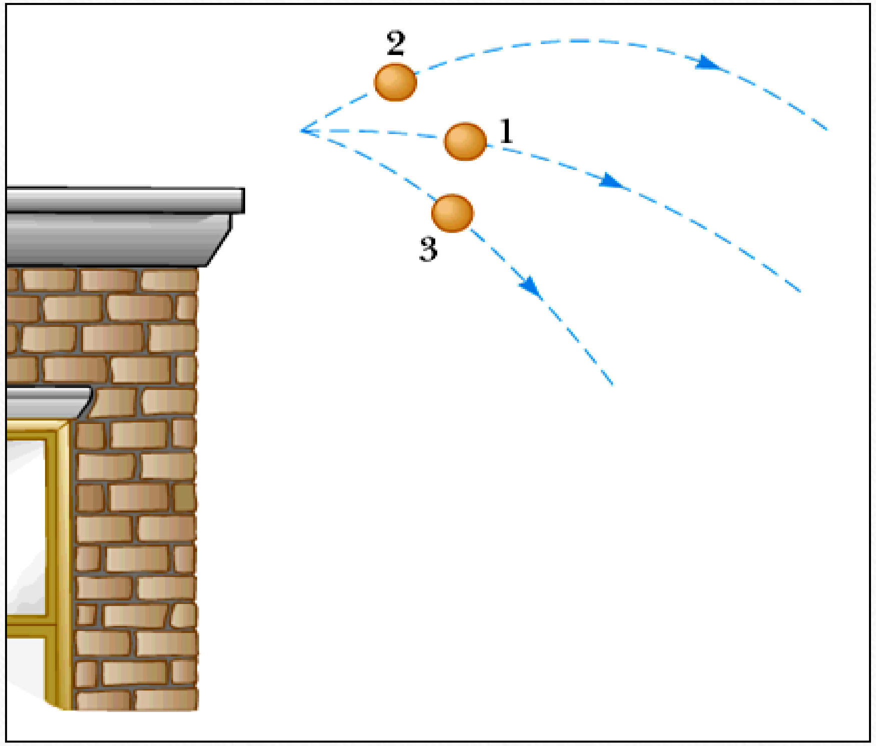
- A force is conservative if the work it does on a particle moving between any two points is independent of the path taken by the particle.
- Work done by a conservative force exerted on a particle moving through any closed path is zero.





Concept Question

- Three identical balls are thrown from the top of a building, all with the same initial speed. The 1st is thrown horizontally, the 2nd at some angle above the horizontal, and the 3rd at some angle below the horizontal. Neglect air resistance, rank the speeds of the balls at the instant each hits the ground.



Concept Question

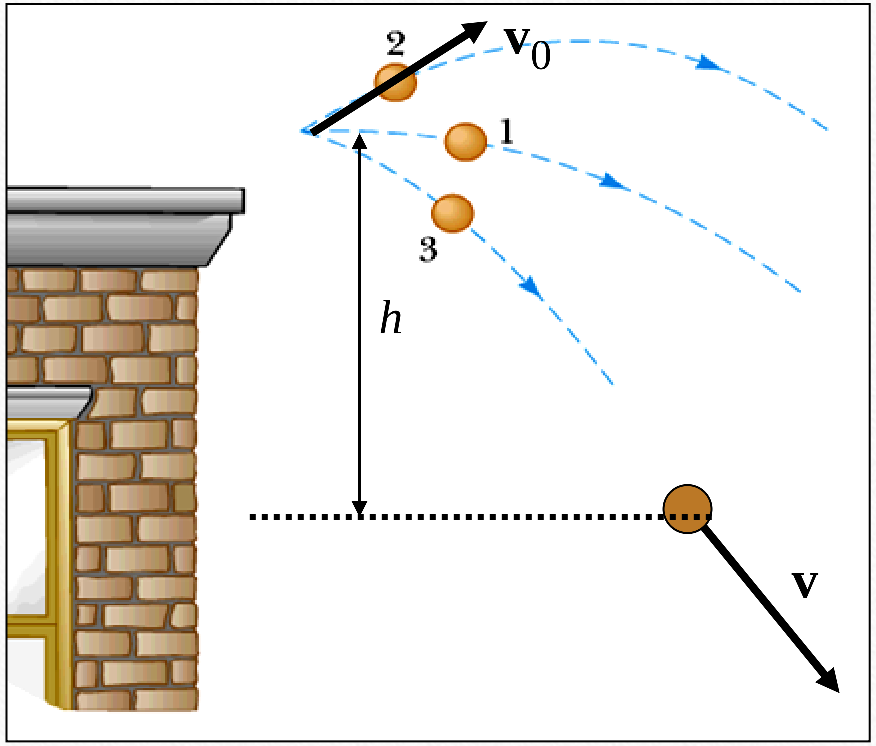
$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_0^2 + mg(h) = \frac{1}{2}mv^2 + mg(0)$$

$$v^2 = v_0^2 + 2gh$$

$$\boxed{v = \sqrt{v_0^2 + 2gh}}$$

Final speed will be same for any angle!



Example: Pulley

A man with mass 70.0 kg sits on a platform suspended from a movable pulley and raises himself at constant speed by a rope passing over a fixed pulley. The platform and the pulleys have negligible mass. Assume that there is no friction loss. (a) Find the force he must exert. (b) Find the increase in the energy of the system when he raises himself 1.20 m.

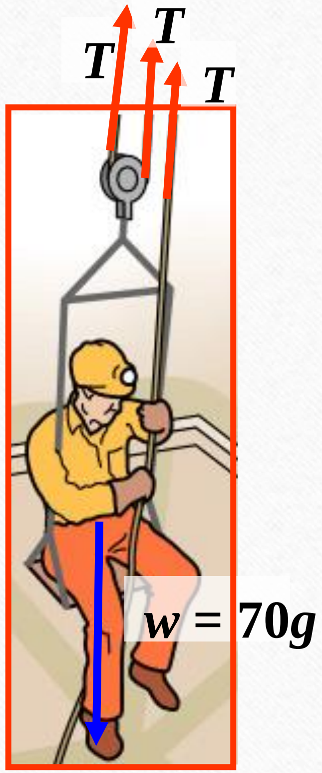
Example: Pulley

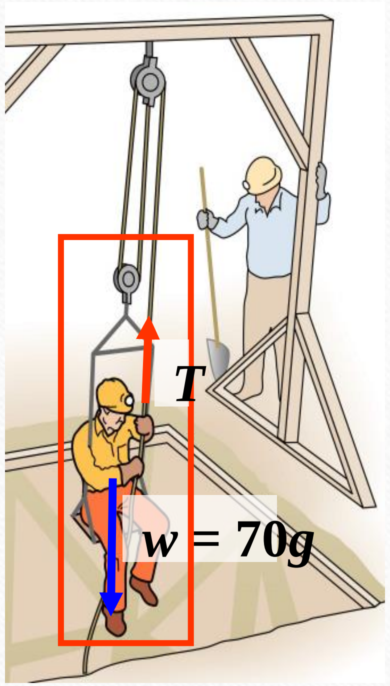
Note that the tension is the same for the whole rope if the mass of pulley is small and friction is negligible.

(a) Consider the system as man + platform + movable pulley:

$$\sum F_y = 3T - w = 0$$

$$\boxed{T = \frac{mg}{3} = \frac{70(9.8)}{3} = 229\ \text{N}}$$





Example: Pulley

(b) Since there is no change in speed,

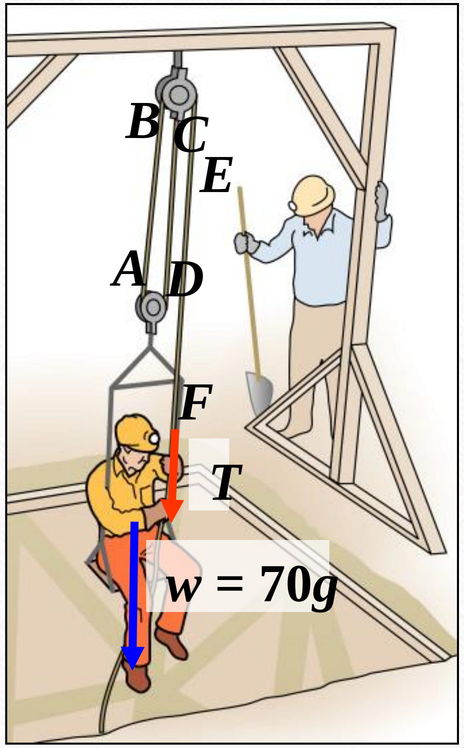
1st method: Energy

$$\Delta K = 0 \Rightarrow \Delta E = \Delta U = mg(y_2 - y_1)$$

$$\boxed{\Delta E = (70)(9.8)(1.2) = 823 \text{ J}}$$

2nd method: If the movable pulley moves up a distance s, then both rope segments AB and CD must each shorten by s. Any point on the rope segment EF will have to move down a distance $2s$. The man has also moved up a distance s. Displacement of man relative to rope he is pulling is $3s$. Therefore, work done by the force ($= T$ downwards) he exerts on rope:

$$\boxed{\Delta E = W_T = T \times 3(y_2 - y_1) = \frac{mg}{3}(3 \times 1.2) = 823 \text{ J}}$$



Elastic Potential Energy

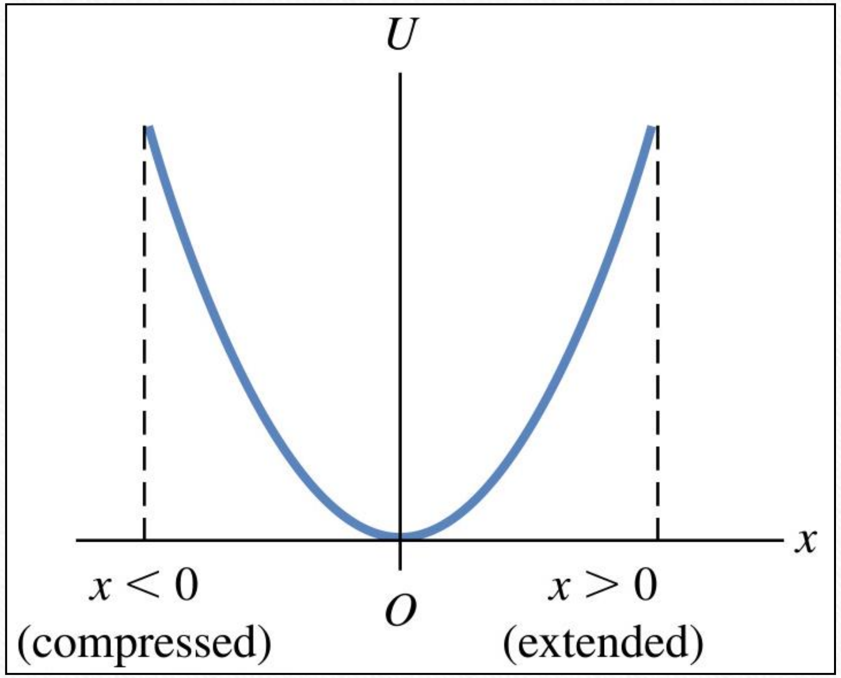
- Define **elastic potential energy**:

$$\boxed{U = \frac{1}{2}kx^2}$$

- No negative value

0 is only when spring is at rest

- Cannot arbitrarily assign zero value.
  - $x = 0$ is when spring is neither extended nor compressed.
- Will assume that spring constant is the same during compression and extension (unless stated otherwise).



Conservation of Energy

- Work done by spring:

$$W_{\mathrm{el}} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 \Rightarrow \boxed{W_{\mathrm{el}} = U_1 - U_2 = -\Delta U}$$

- If there are no other forces acting: $W_{\mathrm{el}} = K_2 - K_1$

$$\boxed{\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2}$$

Conservation of Energy

- Total mechanical energy: $\boxed{E = K + U}$

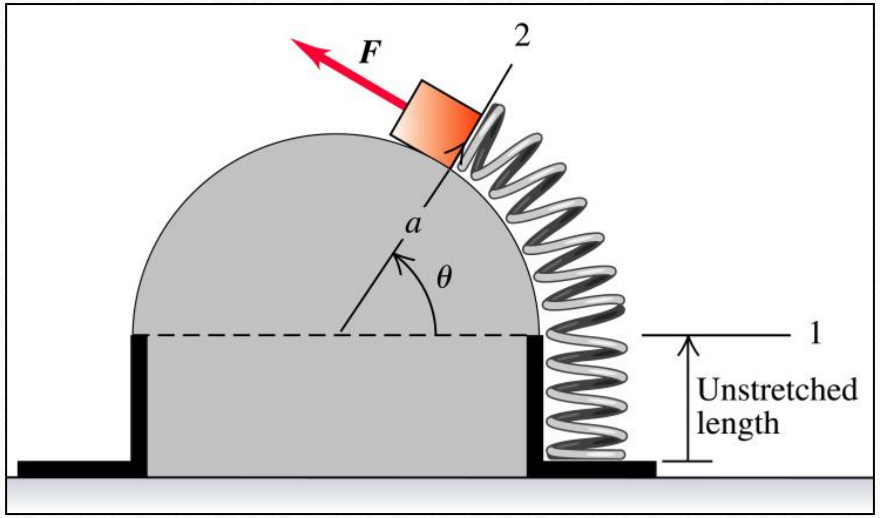
- In the absence of other (non-conservative) forces, such as friction, total mechanical energy of an isolated system of objects (that interact only through conservative forces), is constant:

$$\boxed{E_i = E_f} \Rightarrow \boxed{K_i + \sum U_i = K_f + \sum U_f}$$

Elastic + gravity

Example: Work Done by Variable Force

A variable force **F** is maintained tangent to a frictionless semicircular surface. By slowly varying the force, a block with weight w is moved, and the spring to which it is attached is stretched from position 1 to position 2. The spring has negligible mass and force constant k. The end of the spring moves in an arc of radius a. Calculate the work done by the force **F**.



Example: Work Done by Variable Force

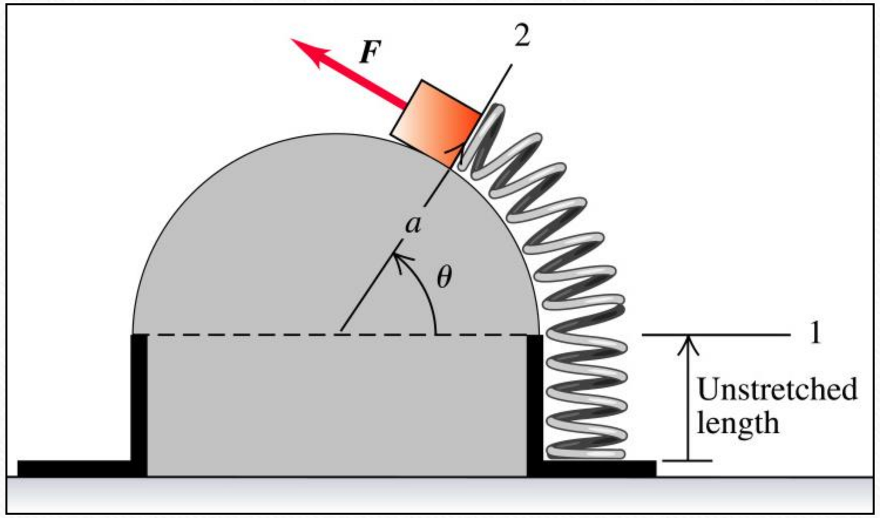
no sin/cos = In radian rad

The spring is stretched an extension:

$$\Delta s = a\theta \Rightarrow \Delta U_{el} = \frac{1}{2}k(a\theta)^2$$

The mass *m* is lifted up a distance of :

$$\Delta y = a\sin\theta \Rightarrow \Delta U_{grav} = mg(a\sin\theta)$$



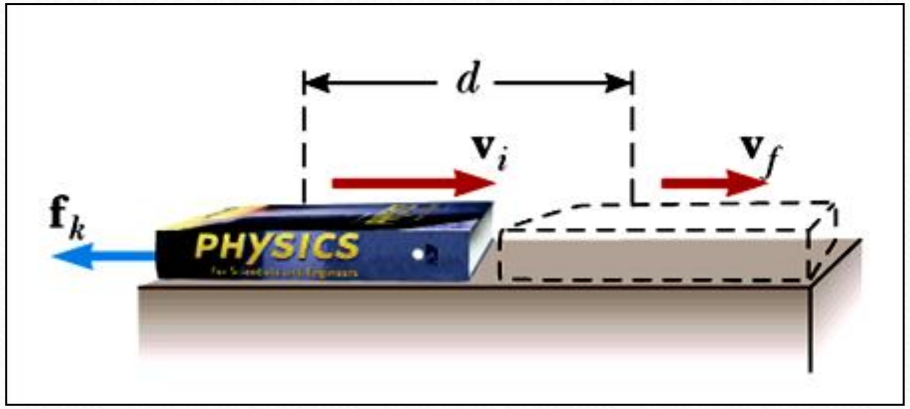
The velocity can be assumed to be constant. The work done by *F*:

$$W = \underbrace{\Delta U_{el}}_{+ve} + \underbrace{\Delta U_{grav}}_{+ve} + \Delta K \Rightarrow \boxed{W = \frac{1}{2}k(a\theta)^2 + mg(a\sin\theta)}$$

Non-conservative Force: Kinetic Friction

- Suppose an object given an initial velocity v_i slides on a rough horizontal surface for a distance d before reaching a final velocity v_f:

$$\boxed{\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -f_k d}$$



- Loss in KE is $f_k d$, which is the energy dissipated by the force of kinetic friction. Part of the energy is transferred to the internal energy of the block, and part to the surface.

Conservation of Energy in General

- Total mechanical energy is conserved when conservative forces act within the system
- Thermodynamics: Energy lost by non-conservative forces (e.g. friction) can be converted into internal energy in the body (rise in temperature) associated with atomic vibration
- Internal atomic motion has K.E. and P.E.

Potential Energy Function

- In general, since work done by a conservative force is a function only of a particle's initial and final coordinates, we can define a **potential energy function U**:

r = 3D

$$\boxed{U(\mathbf{r}) = -\int_{\mathbf{r}_0}^{\mathbf{r}} \vec{F} \cdot d\vec{s} + U_0}$$

$$\boxed{U(x) = -\int_{x_0}^{x} F_x \, dx + U_0}$$

(In 1D)

U_0 is often taken to be zero at some arbitrary (convenient) reference point. Only the change in potential energy is physically significant.

Potential Energy Function

- Conservative force in 1D: $\int F dx = -\Delta U$

- In differential form: $dU = -F dx$

- The conservative force is related to the potential energy function by:

$$\boxed{F = -\frac{dU}{dx}}$$

Force from Potential Energy

- Spring force:

$$\boxed{F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx}$$

- Gravitational force near surface of Earth:

$$\boxed{F_g = -\frac{dU_g}{dy} = -\frac{d}{dy}(mgy) = -mg}$$

- The conservative force is related to the potential energy function by:

$$\boxed{F_g = -\frac{dU_g}{dy} = -\frac{d}{dy}\left(-\frac{GMm}{r}\right) = -\frac{GMm}{r^2}}$$

Example: Spring Potential Energy Function

A certain spring is found not to obey Hooke's Law; it exerts a restoring force $F_x = -\alpha x - \beta x^2$ if it is stretched or compressed, where $\alpha = 60.0\ N/m$ and $\beta = 18.0\ N/m^2$. The mass of the spring is negligible.

(a) Calculate the potential energy function $U(x)$ for this spring. Let $U = 0$ when $x = 0$.

$$U(x) = -\int_0^x F_x\,dx + U_0 \Rightarrow U(x) = -\int_0^x (-\alpha x - \beta x^2)\,dx + 0$$

$$U(x) = \frac{\alpha}{2}x^2 + \frac{\beta}{3}x^3$$

$$\boxed{U(x) = 30.0x^2 + 6.00x^3}$$

Example: Spring Potential Energy Function

(b) An object with mass 0.900 kg on a frictionless, horizontal surface is attached to this spring, pulled a distance 1.00 m to the right (the $+x$ direction) to stretch the spring and released. What is the speed of the object when it is 0.50 m to the right of the $x = 0$ equilibrium position?

Using Principle of Conservation of energy:

$$K_1 + U_1 = K_2 + U_2$$

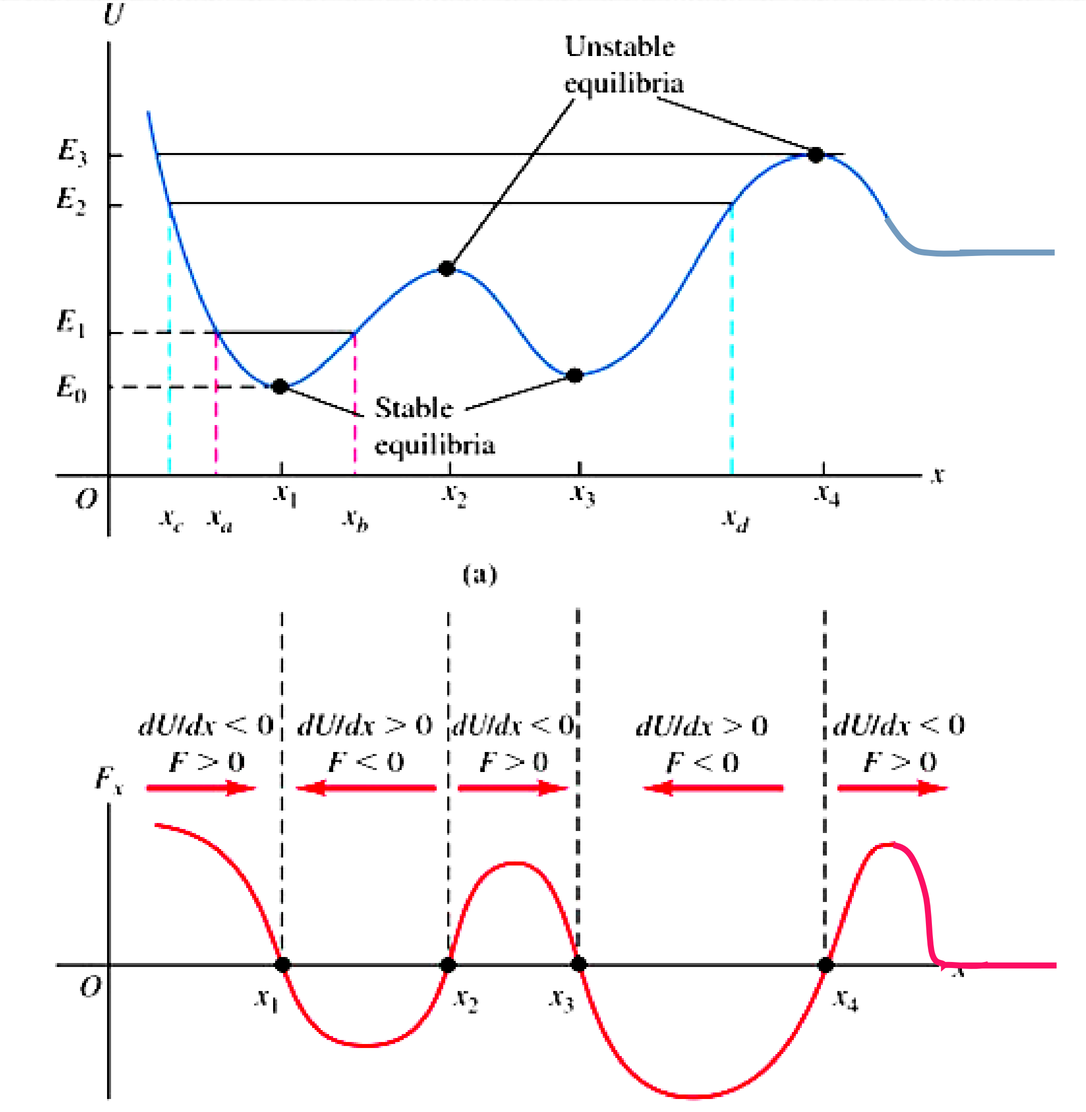
Rmb from part (a): $\boxed{U(x) = 30.0x^2 + 6.00x^3}$

$$0 + \left[30(1)^2 + 6(1)^3\right] = \frac{1}{2}(0.9)v_2^2 + \left[30(0.5)^2 + 6(0.5)^3\right]$$

$$\boxed{v_2 = \sqrt{\frac{2(36 - 8.25)}{0.9}} = 7.85 \text{ m/s}}$$

Energy Diagrams & Equilibrium

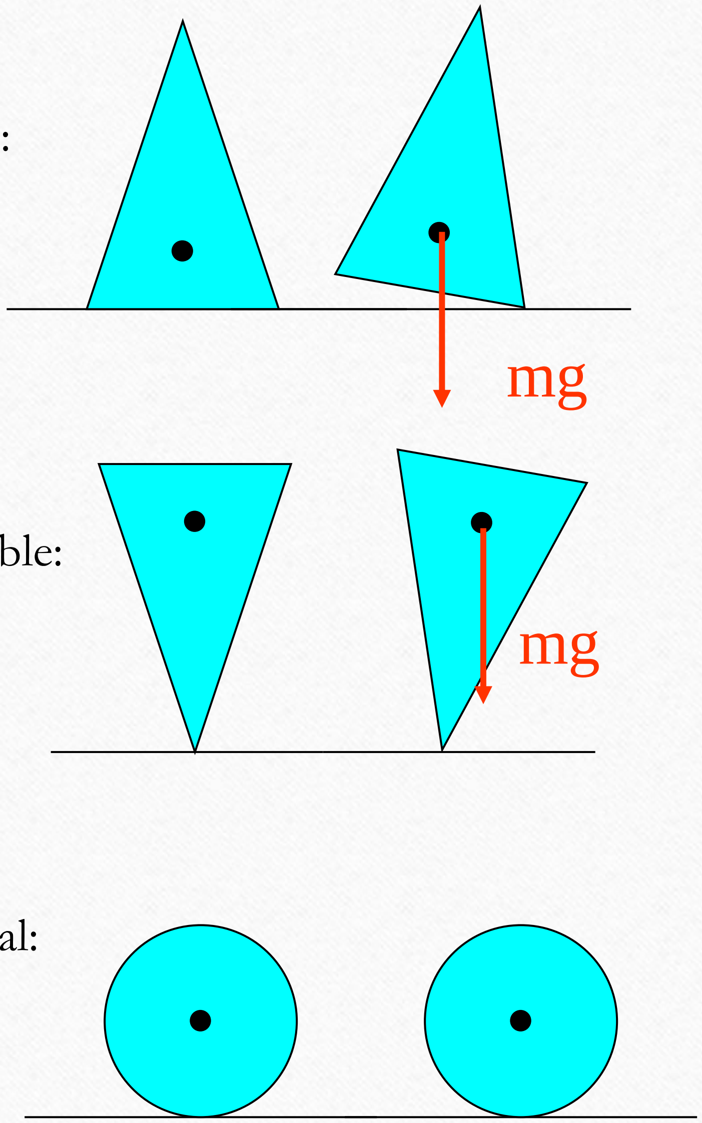
- The motion of a system can be understood qualitatively through its potential energy curve.
- There are three types of equilibrium when the force on the object is zero.
  - Positions of **stable equilibrium** correspond to those points for which U(x) has a minimum value.
  - Positions of **unstable equilibrium** correspond to those points for which U(x) has a maximum value
  - Positions of **neutral equilibrium** correspond to a region where U(x) is constant



Stable:

Unstable:

Neutral:



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