CSD2258 Tutorial 3

Problem 1. (a) Use truth table to determine whether the following compound proposition is a tautology, a contradiction or a contingency.

$$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q.$$

(b) Use De Morgan's laws to write a negation for the statement
"This computer program has a logical error in the first ten lines or it is being run with an incomplete data set."

Problem 2. Using truth tables, determine which pair of statements is logically equivalent.

(a) $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$.

(b) $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$.

Problem 3. Prove the following equivalences using basic logical equivalence laws (see last page). Supply a reason for each step.

(a) $(p \vee \neg q) \wedge (\neg p \vee \neg q) \equiv \neg q$.

(b) $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$.

Problem 4. Let p, q, r be propositions. Consider the following statement

$$(p \to r) \leftrightarrow (q \to r) \tag{1}$$

(a) Use the logical equivalences $p \to q \equiv \neg p \vee q$ and $p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$ to rewrite (1) without the symbol $\to$ and $\leftrightarrow$.

(b) Use the equivalence $p \vee q \equiv \neg(\neg p \wedge \neg q)$ to rewrite (1) using only $\wedge$ and $\neg$.

Logical Equivalence Laws

Identity law	$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Negation law	$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$
Domination law	$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Commutativity	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative law	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$	Distributivity	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Double negation	$\neg(\neg p) \equiv p$	Conditional statement	$p \to q \equiv \neg q \to \neg p$

Hints and Instructions.

1a. Your truth table should have the following form.

p	q	r	$\neg p$	$\neg p \wedge q$	$q \wedge r$	$(\neg p \wedge q) \wedge (q \wedge r)$	$\neg q$	$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$
0	0	0						
0	0	1						
0	1	0						
0	1	1						
1	0	0						
1	0	1						
1	1	0						
1	1	1						

2a. This is the distributive law. The statements are equivalent.
2b. The statements are not equivalent.
3. Try it.

4. The answers are not beautiful.
a. $[(p \wedge \neg r) \vee (\neg q \vee r)] \wedge [(q \wedge \neg r) \vee (\neg p \vee r)]$
b. $\neg[\neg(p \wedge \neg r) \wedge (q \wedge \neg r)] \wedge \neg[\neg(q \wedge \neg r) \wedge (p \wedge \neg r)]$