CSD1251/CSD1250 Week 13 Tutorial Problems

27th March – 2nd April 2023

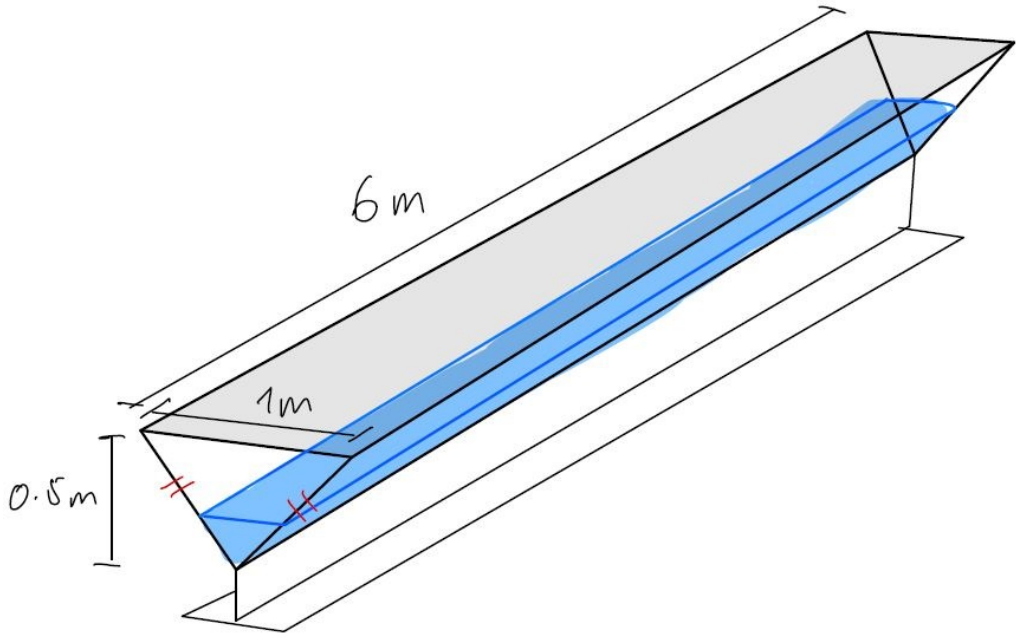
It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Starred(*) questions are slightly more difficult.

Question 1

A trough (see figure below for an illustration) is 6 m long and its cross-section has the shape of a isoceles triangle (triangle with two equal sides) that are 1 m across at the top and have a height of 0.5 m. If the trough is being filled with water at a rate of 1.2 m^3/min, how fast is the water level rising when the water is 0.3 m deep?



Question 2

Answer all of the following.

(a) Suppose the derivative of a function f is

$$f'(x) = x^2(x+3)^4(x-6)^5.$$

On what interval(s) is f increasing? Decreasing? Explain your answer.

(b) Suppose the second derivative of a function g is

$$g''(x) = (x+2)^2(x-4)^3.$$

What are the inflection points of g?

Question 3

Find all local extreme point(s) for the following functions. Explain your answer.

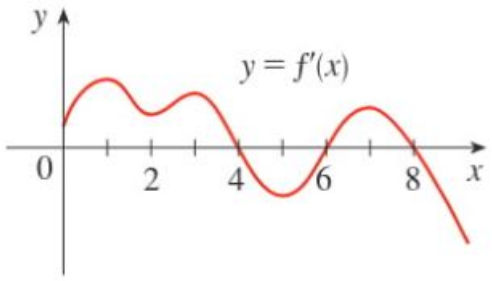
(a) $f(x) = x^2 + 1$ (b) $f(x) = \ln(x^2 + 9)$ (c) $f(x) = \dfrac{x^2 - 24}{x - 5}$

Question 4

A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle (a triangle with 3 equal sides). How should the wire be cut so that the total area enclosed is a maximum? A minimum? *Hint:* Use the area of a triangle $A = \dfrac{1}{2}ab\sin\theta$ to find the formula for the area of an equilateral triangle.

Question 5

The graph of the first derivative f' of a function f on $[0, 9]$ is shown below.



In all parts, explain how you arrived at your answer.

(a) On what intervals is f increasing? Decreasing?
(b) At what x does f have a local maximum or minimum?
(c) On what intervals is f CU? CD?
(d) What are the inflection points of f?

Question 6*

What is the minimum vertical distance between the parabolas $y = x^2 + 1$ and $y = x - x^2$? *Hint:* Draw the two graphs to visualize the problem. You may need to use the techniques we have developed so far to draw the graph of $y = x - x^2$.