CSD2301 Practice Solutions
15. **Angular Momentum**

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Practice Question 1

A woman with mass 50 kg is standing on the rim of a large disk that is rotating at 0.50 rev/s about an axis through its center. The disk has a mass 110 kg and radius 4.0 m. Calculate the magnitude of the total angular momentum of the woman-plus-disk system. Assume that you can treat the person as a point in your calculation.

IDENTIFY: $L = I\omega$ and $I = I_{\text{disk}} + I_{\text{woman}}$.

SET UP: $\omega = 0.50\ \text{rev/s} = 3.14\ \text{rad/s}$. $I_{\text{disk}} = \frac{1}{2} m_{\text{disk}} R^2$ and $I_{\text{woman}} = m_{\text{woman}} R^2$.

EXECUTE: $I = (55\ \text{kg} + 50.0\ \text{kg})(4.0\ \text{m})^2 = 1680\ \text{kg} \cdot \text{m}^2$. $L = (1680\ \text{kg} \cdot \text{m}^2)(3.14\ \text{rad/s}) = 5.28 \times 10^3\ \text{kg} \cdot \text{m}^2/\text{s}$

EVALUATE: The disk and the woman have similar values of I, even though the disk has twice the mass.

$\frac{1}{2} m_{disk}$ m_{woman}

Practice Question 2

Find the magnitude of the angular momentum of the second hand on a clock about an axis through the center of the clock face. The clock hand has a length of 15.0 cm and a mass of 6.00 g. Take the second hand to be a slender rod rotating with constant angular velocity about one end.

IDENTIFY and SET UP: Use $L = I\omega$

EXECUTE: The second hand makes 1 revolution in 1 minute, so

$$\omega = (1.00 \text{ rev/min})(2\pi \text{ rad/1 rev})(1 \text{ min/60 s}) = 0.1047 \text{ rad/s}$$

For a slender rod, with the axis about one end,

$$I = \tfrac{1}{3}ML^2 = \tfrac{1}{3}(6.00\times10^{-3} \text{ kg})(0.150 \text{ m})^2 = 4.50\times10^{-5} \text{ kg}\cdot\text{m}^2$$

Then $L = I\omega = (4.50\times10^{-5} \text{ kg}\cdot\text{m}^2)(0.1047 \text{ rad/s}) = 4.71\times10^{-6} \text{ kg}\cdot\text{m}^2/\text{s}$.

EVALUATE: $\vec{L}$ is clockwise.

Solution takes clockwise to be +ve. So if answer of L is +ve, means it is clockwise

Practice Question 3

A diver jumps off a board with arms straight up and legs straight down, giving her a moment of inertia about her rotation axis of 18 kgm^2. She then tucks into a small ball, decreasing this moment of inertia to 3.6 kgm^2. While tucked, she makes two complete revolutions in 1.0 s. If she hadn't tucked at all, how many revolutions would she have made in the 1.5 s from board to water?

IDENTIFY: Apply conservation of angular momentum to the diver. $I_i\omega_i = I_f\omega_f$

SET UP: The number of revolutions she makes in a certain time is proportional to her angular velocity. The ratio of her untucked to tucked angular velocity $(\frac{\omega_i}{\omega_f})$ is $(\frac{I_f}{I_i} = \frac{3.6\ kg \cdot m_2}{18\ kg \cdot m_2})$.

EXECUTE: If she had untucked, she would have made (2 rev $\times \frac{3.6\ kg \cdot m_2}{18\ kg \cdot m_2}$ = 0.40 rev) in the last 1.0 s, so she would have made (0.40 rev)(1.5/1.0) = 0.60 rev in the total 1.5 s.

Practice Question 4

A hollow thin walled sphere of mass 12.0 kg and diameter 48.0 cm is rotating about an axle through its center. The angle (in radians) through which it turns as a function of time (in seconds) is given by $\theta(t) = At^2 + Bt^4$, where A has a numerical value of 1.50 and B has a numerical value of 1.10. (a) What are the units of the constants A and B? (b) At the time 3.00 s find (i) the angular momentum of the sphere and (ii) the net torque on the sphere.

IDENTIFY: $\omega_z = d\theta/dt$. $L_z = I\omega_z$ and $\tau_z = dL_z dt$.

SET UP: For a hollow, thin-walled sphere rolling about an axis through its center, $I = \frac{2}{3}MR^2$. $R = 0.240$ m.

EXECUTE: **(a)** $A = 1.50\ \text{rad/s}^2$ and $B = 1.10\ \text{rad/s}^4$, so that $\theta(t)$ will have units of radians.

(b) (i) $\omega_z = \dfrac{d\theta}{dt} = 2At + 4Bt^3$. At $t = 3.00$ s, $\omega_z = 2(1.50\ \text{rad/s}^2)(3.00\ \text{s}) + 4(1.10\ \text{rad/s}^4)(3.00\ \text{s})^3 = 128\ \text{rad/s}$.

$L_z = (\frac{2}{3}MR^2)\omega_z = \frac{2}{3}(12.0\ \text{kg})(0.240\ \text{m})^2(128\ \text{rad/s}) = 59.0\ \text{kg}\cdot\text{m}^2/\text{s}$.

Differentiate ω to get α. Because τ = Iα

(ii) $\tau_z = \dfrac{dL_z}{dt} = I\dfrac{d\omega_z}{dt} = I(2A + 12Bt^2)$ and

$\tau_z = \frac{2}{3}(12.0\ \text{kg})(0.240\ \text{m})^2(2[1.50\ \text{rad/s}^2] + 12[1.10\ \text{rad/s}^4][3.00\ \text{s}]^2) = 56.1\ \text{N}\cdot\text{m}$.

Practice Question 5

A large wooden turntable in the shape of a flat uniform disk has a radius of 2.00 m and a total mass of 120 kg. The turntable is initially rotating at 3.00 rad/s about a vertical axis through its center. Suddenly, a 70.0 kg parachutist makes a soft landing on the turntable at a point near the outer edge. (a) Find the angular speed of the turntable after the parachutist lands. (Assume that you can treat the parachutist as a particle.) (b) Compute the kinetic energy of the system before and after the parachutist lands. Why are these kinetic energies not equal?

EXECUTE: **(a)** $L_1 = L_2$ gives $I_1\omega_1 = I_2\omega_2$, so $\omega_2 = (I_1/I_2)\omega_1$

$$I_1 = I_{tt} = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(120\text{ kg})(2.00\text{ m})^2 = 240\text{ kg}\cdot\text{m}^2$$

$$I_2 = I_{tt} + I_p = 240\text{ kg}\cdot\text{m}^2 + mR^2 = 240\text{ kg}\cdot\text{m}^2 + (70\text{ kg})(2.00\text{ m})^2 = 520\text{ kg}\cdot\text{m}^2$$

$$\omega_2 = (I_1/I_2)\omega_1 = (240\text{ kg}\cdot\text{m}^2/520\text{ kg}\cdot\text{m}^2)(3.00\text{ rad/s}) = 1.38\text{ rad/s}$$

(b) $K_1 = \tfrac{1}{2}I_1\omega_1^2 = \tfrac{1}{2}(240\text{ kg}\cdot\text{m}^2)(3.00\text{ rad/s})^2 = 1080\text{ J}$

$$K_2 = \tfrac{1}{2}I_2\omega_2^2 = \tfrac{1}{2}(520\text{ kg}\cdot\text{m}^2)(1.38\text{ rad/s})^2 = 495\text{ J}$$

EVALUATE: The kinetic energy decreases because of the negative work done on the turntable and the parachutist by the friction force between these two objects.