Midterm Exam Revision

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Composition of functions (Variant 1: 29%)

For functions $f(x) = \dfrac{x^2 - 1}{x^6}$ and $g(x) = \sec(x)$, find $(f \circ g)(x)$.

Composition of functions (Variant 2: 44%)

For functions $f(x) = \dfrac{1 - x^2}{x^4}$ and $g(x) = \cos(x)$, find $(f \circ g)(x)$.

Inverse functions (48%)

Let $f(x) = x^2 + 6x + 4$ for $x \leq -3$. Find $f^{-1}(x)$.

Limit Techniques: Factorization (Variant 1: 51%)

Evaluate $\lim_{x \to -1} \dfrac{x^2 + 3x + 2}{x^3 + x^2 + x + 1}$.

Limit Techniques: Factorization (Variant 2: 52%)

Evaluate $\lim_{x \to -\frac{1}{3}} \dfrac{3x^2 + 4x + 1}{3x^3 + x^2 + 3x + 1}$.

Limits/defn of derivative (Variant 1: 44%)

Evaluate $\lim_{h \to 0} \dfrac{(h+2)^6 - 64}{h}$.

Limits/defn of derivative (Variant 2: 46%)

Evaluate $\lim_{h \to 0} \dfrac{(h+2)^6 - 64}{2h}$.

Limits/defn of derivative (Variant 3: 38%)

Evaluate $\lim_{h \to 0} \dfrac{(h+2)^7 - 128}{h}$.

Differentiability (Variant 1: 48%)

Let $f(x) = |x|$. Find $f'(0)$, if it exists.

Differentiability (Variant 2: 37%)

Let $f(x) = x|x|$. Find $f'(0)$, if it exists.

Question 1

Differentiate $f(x) = e^{\sin^2(x^2)}$.

Question 2

Find an equation of the tangent line to the function $f(x) = 10xe^{-x^2}$ at the point $(0, 0)$.

Question 3

Find an equation of the tangent line to the graph of $y^2 = x^3 + 3x^2$ at the point $(1, -2)$.

Question 4

Find $\frac{dy}{dx}$ for the following equation.

$$\cos(x^2 + 2y) + xe^{y^2} = 1$$

Question 5

There is only one critical point c of the function $f(x) = x^2 + x$. Find c.

Question 6

For the function f in Question 5, find **an** interval where f is increasing.

(a) $(-1, \infty)$ (b) $(-\infty, 0)$ (c) $(0, 1)$

(d) $(-2, \infty)$ (e) None of the above

Question 7

For the function f in Question 5, find **an** interval where f is decreasing.

(a) $(1, \infty)$ (b) $(-\infty, -1)$ (c) $(0, 1)$

(d) $(0, \infty)$ (e) None of the above