

Tutorial 3

Question 1:

Find the computational complexity of the following piece of code using Big-oh notation:

```
for (int i = 1; i < n; i *= 2) {  
    for (int j = n; j > 0; j /= 2) {  
        for (int k = j; k < n; k += 2) {  
            sum += (i + j * k);  
        }  
    }  
}
```

Question 2:

Write a recursive function GCD(n,m) that returns the greatest common divisor of two integer n and m according to the following definition (recurrence relation):

$\text{GCD}(n,m) = \{$
 $m, \text{ if } m \leq n \text{ and } n \bmod m = 0 \{$
 $\text{GCD}(m,n), \text{ if } n < m \{$
 $\text{GCD}(m, n \bmod m), \text{ otherwise}$

Example:

Enter the first number: 54

Enter the second number: 24

The GCD of 54 and 24 is 6

Question 3:

Use the master method to give tight asymptotic bounds for the following recurrences (if the master method cannot be applied give your argument):

(a) $T(n) = 4T(n/2) + n$.

(b) $T(n) = 4T(n/2) + n^3$.

Question 4:

The following is the running time of a recursion merge sort algorithm:

$$T(n) = 2T(n/2) + O(n)$$

Using the substitution method, proof that the time complexity of this algorithm is $O(n \lg n)$. Verify your answer with the tree method and the master method.