CSD2301 Lecture
16. Static Equilibrium

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Outline

- Static equilibrium
- Solving rigid-body equilibrium problems

Static Equilibrium

- When a rigid body is at rest (no translation or rotation), it is said to be in static equilibrium.
- Two conditions for equilibrium:
  - **Net force = 0** so body at rest has no tendency to start moving

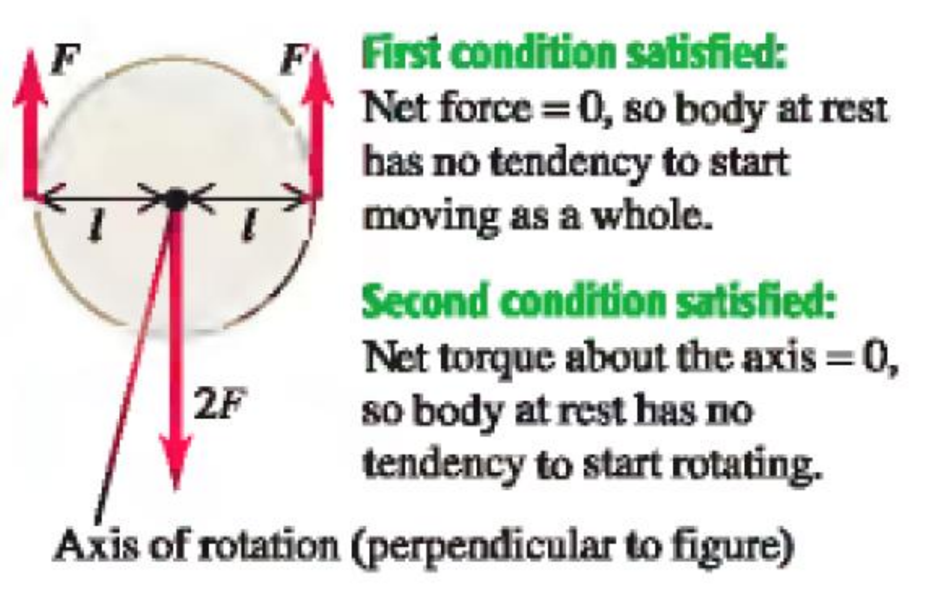
$$\sum \vec{F} = 0$$

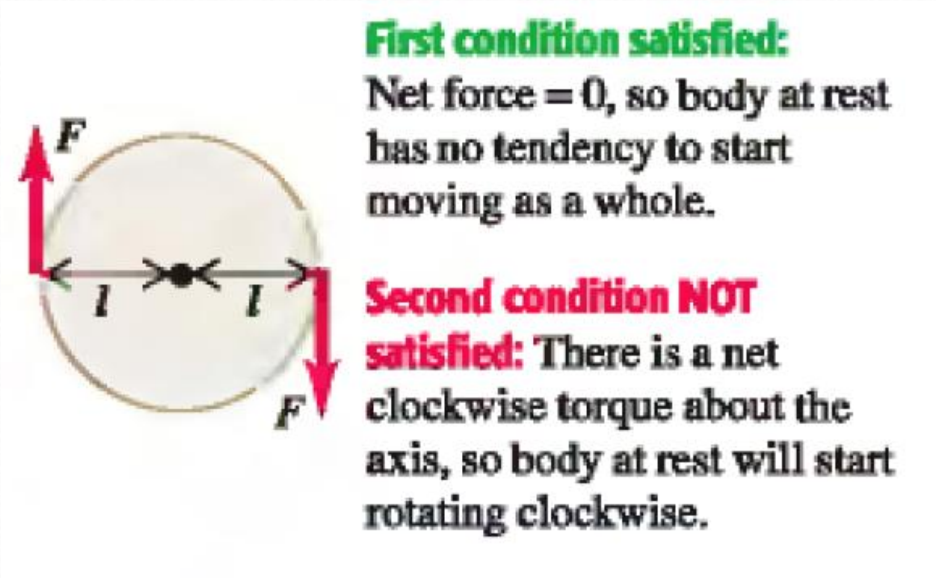
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

  - **Net torque about the axis = 0** so body at rest has no tendency to start rotating

$$\sum \vec{\tau} = 0 \quad \text{about any point}$$

Static Equilibrium





Object is in equilibrium

Object is not in equilibrium

Solving Rigid-body Equilibrium Problems

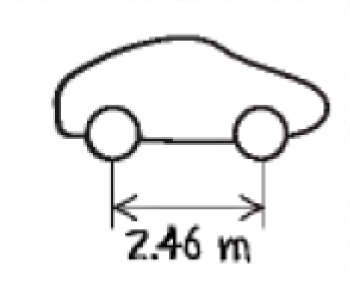
1. Draw a FBD showing forces acting on the body. (Very important step so must draw correctly.)
2. Choose axes and specify directions.
3. Take summations of forces $\sum F_x = 0$ and $\sum F_y = 0$
4. Choose a convenient point and take summation of torque $\sum \tau_z = 0$

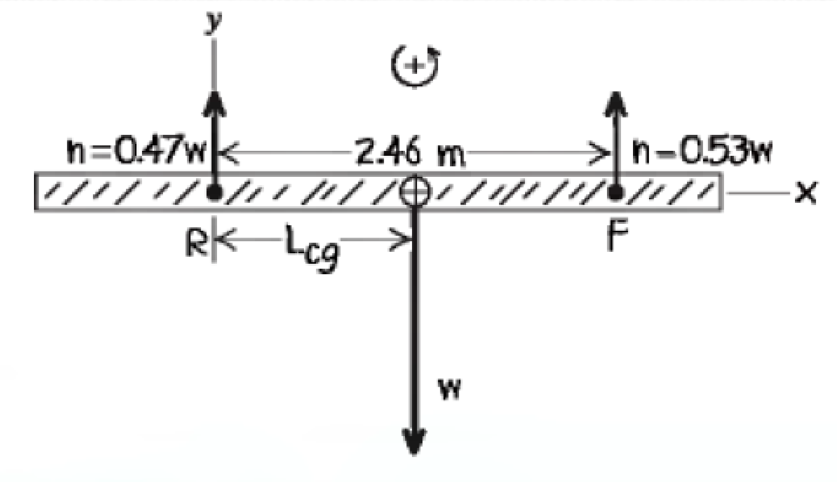
Solving Rigid-body Equilibrium Problems

- Points to note:
  - If a force has a line of action that goes through a point, the torque of the force w.r.t that point is 0 – which is very useful to eliminate unknown forces/components.
  - Need to draw force(s) at points where there is contact i.e. when movement is restricted. For example, at supports or hinges.
  - Try to first solve for unknown forces which are more straightforward.
  - Simultaneous equations may be required at times.

Example: Weight distribution for car

- A sports car has 53% of its weight on front wheels and 47% on its rear wheels. It has 2.46 m distance between front and rear wheels. How far in front of the rear axle is the car's center of gravity?



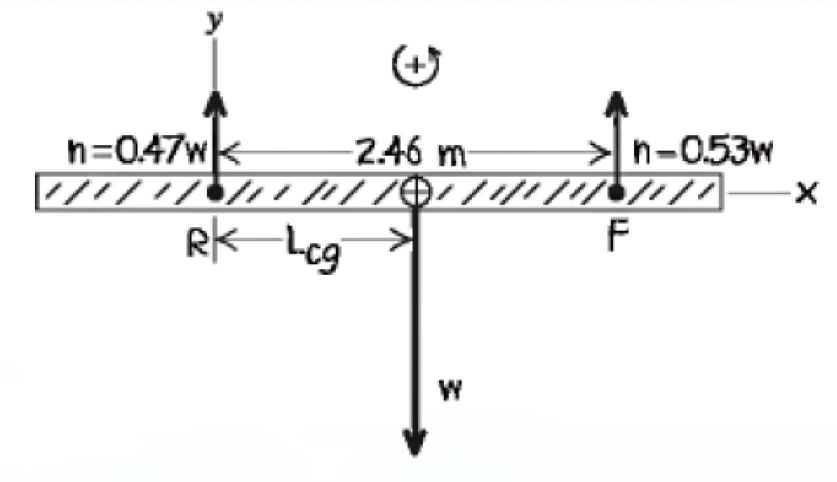


Example: Weight distribution for car

Taking moments about point R:

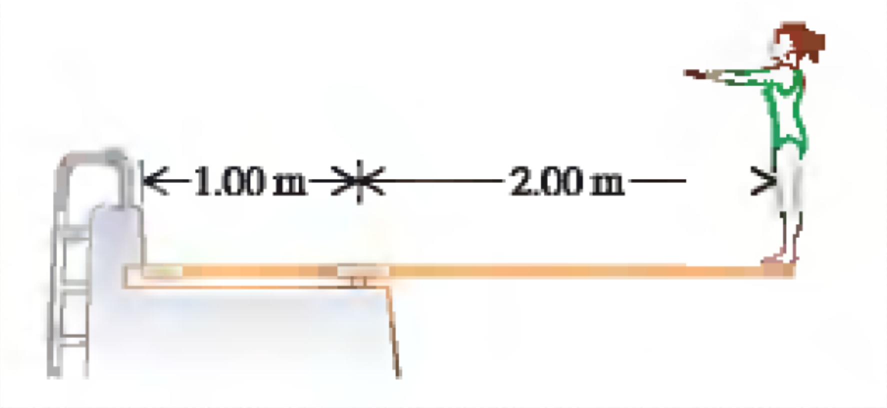
$\sum \tau_R = 0.47w\,(0) - wL_{cg} + 0.53w(2.46) = 0$

$L_{cg} = \underline{1.30\text{ m}}$



Example: Diving board

A diving board 3.00 m long is supported at a point 1.00 m from the end, and a diver weighing 500 N stands at the free end. The diving board is of uniform cross section and weighs 280 N. Find (a) the force at the support joint and (b) the force at the left-hand end.



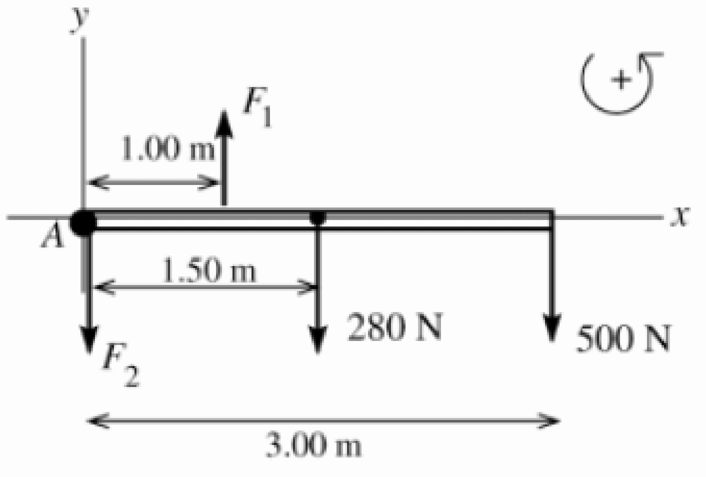


Figure 11.11

$\vec{F}_1$ is the force applied at the support point and $\vec{F}_2$ is the force at the end that is held down.

EXECUTE: $\sum \tau_A = 0$ gives $+F_1(1.0\text{ m}) - (500\text{ N})(3.00\text{ m}) - (280\text{ N})(1.50\text{ m}) = 0$

$$F_1 = \frac{(500\text{ N})(3.00\text{ m}) + (280\text{ N})(1.50\text{ m})}{1.00\text{ m}} = 1920\text{ N}$$

(b) $\sum F_y = ma_y$

$F_1 - F_2 - 280\text{ N} - 500\text{ N} = 0$

$F_2 = F_1 - 280\text{ N} - 500\text{ N} = 1920\text{ N} - 280\text{ N} - 500\text{ N} = 1140\text{ N}$

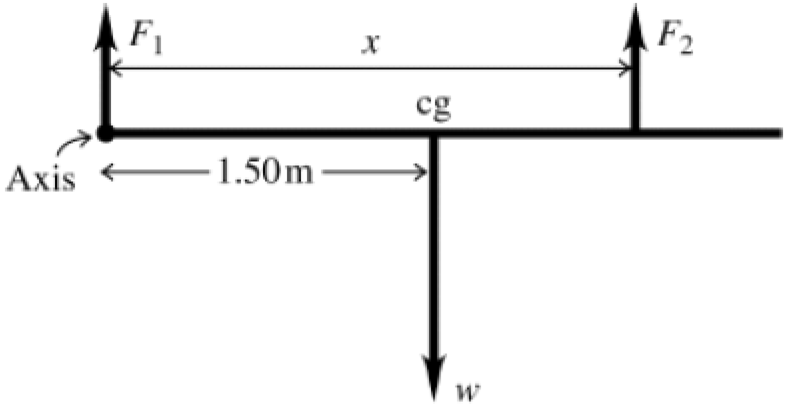
Example: Carrying wooden board

Two people are carrying a uniform wooden board that is 3.00 m long and weighs 160 N. If one person applies an upwards force equal to 60 N at one end, at what point does the other person lift? Begin with a free-body diagram of the board.

Summation of torque about left end

EXECUTE: $\sum F_y = 0$ gives $F_1 + F_2 - w = 0$ and $F_2 = w - F_1 = 160\text{ N} - 60\text{ N} = 100\text{ N}$. $\sum \tau = 0$ gives $F_2 x - w(1.50\text{ m}) = 0$ and $x = \left(\dfrac{w}{F_2}\right)(1.50\text{ m}) = \left(\dfrac{160\text{ N}}{100\text{ N}}\right)(1.50\text{ m}) = 2.40\text{ m}$. The other person lifts with a force of 100 N at a point 2.40 m from the end where the other person lifts.

EVALUATE: By considering the axis at the center of gravity we can see that a larger force is applied by the person who pushes closer to the center of gravity.



The End