CSD1241 Tutorial 6 Solutions

Remarks. The solution should only be used as guidance for your study. There is no guarantee on errors and typos. Would appreciate if you let me know the errors.

Problem 1. The **matrix representation** of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is an $m \times n$ matrix M such that $T(\vec{x}) = M\vec{x}$ for any $\vec{x} \in \mathbb{R}^n$.
Find the matrix representation of T in the following cases. In each case, find the points $\vec{x}$ that are fixed by T, that is, $T(\vec{x}) = \vec{x}$.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ with

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x + y \\ y \end{pmatrix}$$

(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$ with

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x + y - z \\ y + z \\ x - y - z \end{pmatrix}$$

(c) Based on the results from a,b, we see that the map T always fixes the origin $O = \vec{0}$. Show that this property is true for any linear map, that is, $T(\vec{0}) = \vec{0}$ whenever $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation.
(d) Let l be a line in $\mathbb{R}^2$ which doesn't go through the origin. Using the result in c, could you explain that the map $T : \mathbb{R}^2 \to \mathbb{R}^2$ which is the reflection through l is not a linear transformation?

Solution. The map $T : \mathbb{R}^n \to \mathbb{R}^m$ defined by $T\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{pmatrix}$
has matrix representation

$$M = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}.$$

We simply collect the coefficients which appear in the definition of T to form M.

(a) The matrix of T is

$$M = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$$

Let $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ be a fixed point of T. We have

$$T(\vec{x}) = \vec{x} \Leftrightarrow \begin{pmatrix} 2x + y \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Leftrightarrow y = -x$$

Therefore, all fixed points of T are $\begin{pmatrix} x \\ -x \end{pmatrix}$. These are all points on the line $y = -x$.

(b) The matrix of T is

$$M = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & -1 \end{pmatrix}$$

Let $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ be a fixed point of T. We have

$$T(\vec{x}) = \vec{x} \Leftrightarrow \begin{pmatrix} 2x + y - z \\ y + z \\ x - y - z \end{pmatrix} = \begin{cases} 2x + y - z = x \\ y + z = y \\ x - y - z = z \end{cases}$$

The second equation implies $z = 0$. Substituting $z = 0$ into the remaining equations, we obtain $x + y = 0$ and $x - y = 0$, which implies $x = y = 0$. Therefore, the only fixed point of T is the origin, that is, $O = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

(c) Since $T : \mathbb{R}^n \to \mathbb{R}^m$ is linear, it preserves addition. We have

$$T(\vec{0} + \vec{0}) = T(\vec{0}) + T(\vec{0}) \Rightarrow 2T(\vec{0}) = T(\vec{0}) \Rightarrow T(\vec{0}) = \vec{0}.$$

(d) Since l doesn't go through the origin, the image of the origin $O = \vec{0}$ is a point different from O, that is, $T(\vec{0}) \neq \vec{0}$. Therefore, T is not a linear transformation. □

Problem 2. Another way to find the matrix representation M of $T : \mathbb{R}^n \to \mathbb{R}^m$ is to use the standard unit vectors $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n$ of $\mathbb{R}^n$

$$M = [T(\vec{e}_1) \;\; T(\vec{e}_2) \;\; \cdots \;\; T(\vec{e}_n)]$$

In the following cases, find the matrix representation of the linear transformation T by the method described above. In each case, find the points $\vec{x}$ that are mapped to $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, that is, $T(\vec{x}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ with

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

(b) $T : \mathbb{R}^3 \to \mathbb{R}^2$ with

$$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \; T\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \; T\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

Solution. (a) Note that $\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Further $\vec{e}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. So

$$T(\vec{e}_2) = T\begin{pmatrix} 1 \\ 1 \end{pmatrix} - T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

We obtain

$$M = [T(\vec{e}_1)\ T(\vec{e}_2)] = \begin{pmatrix} 1 & 1 \\ -1 & -4 \end{pmatrix}.$$

Assume $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ is mapped to $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. We have

$$T(\vec{x}) = M\vec{x} = \begin{pmatrix} 1 & 1 \\ -1 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ -x-4y \end{pmatrix},$$

which implies

$$\begin{cases} x + y = 1 \\ -x - 4y = 1 \end{cases} \quad \Rightarrow x = \frac{5}{3},\ y = -\frac{2}{3}.$$

We obtain $\vec{x} = \begin{pmatrix} 5/3 \\ -2/3 \end{pmatrix}$.

(b) Note that $\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\vec{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Further $\vec{e}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and

$\vec{e}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. So

$$\begin{aligned} T(\vec{e}_2) &= T \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \\ T(\vec{e}_3) &= T \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - T \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \end{aligned}$$

We obtain

$$M = [T(\vec{e}_1)\ T(\vec{e}_2)\ T(\vec{e}_3)] = \begin{pmatrix} 1 & -2 & -3 \\ 1 & 1 & 2 \end{pmatrix}.$$

Assume $\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is mapped to $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. We have

$$T(\vec{x}) = M\vec{x} = \begin{pmatrix} 1 & -2 & -3 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x - 2y - 3z \\ x + y + 2z \end{pmatrix},$$

which implies

$$\begin{cases} x - 2y - 3z = 1 \\ x + y + 2z = 1 \end{cases} \Rightarrow \begin{cases} x - 2y - 3z = 1 \\ 3y + 5z = 0 \end{cases}$$

From the 2nd equation, we have $y = -\frac{5}{3}z$. Substituting this into the first equation, we obtain $x = 1 + 2y + 3z = 1 - \frac{1}{3}z$. The points $\vec{x}$ which are mapped to $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ have coordinates

$$\vec{x} = \begin{pmatrix} 1 - 1/3z \\ -5/3z \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{z}{3} \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix},$$

which is a line in $\mathbb{R}^3$ going through the point $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and having direction $\begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix}$. $\square$

Problem 3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the orthogonal projection onto the line $l : x - 2y = 0$.
(a) Find the matrix M of T.
(b) Find the image of the points $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$.
(c) Find the image of the line $m : x + y = 3$ under this map (find normal equation).
Hint. Use the vector equation of m to express its coordinates.
(d) Find the image of the line $n : 2x + y = 15$ under this map (find normal equation).

Solution. (a) The line l has direction $\vec{d} = \begin{bmatrix} -b \\ a \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. So the matrix of T is

$$M = \frac{1}{||\vec{d}||^2} \vec{d}\vec{d}^T = \frac{1}{5} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$$

(b) The images of $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$ are

$$\frac{1}{5} \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ and } \frac{1}{5} \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \begin{pmatrix} 5 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

(c) The line m goes through the point $\vec{x}_0 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and has direction $\vec{d} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. So it has vector equation

$$\vec{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Any point $\vec{x}$ on m is mapped to

$$\begin{aligned} M\vec{x} = M\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) &= \frac{1}{5} \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \frac{1}{5} \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \frac{t}{5} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \end{aligned}$$

where $s = t/5$. This is a point on the line $m' : \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Note that m' has normal equation $x - 2y = 0$, which is exactly the line l. This makes sense because we are projecting onto l.

(d) The line n goes through the point $\vec{x}_0 = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ and has direction $\vec{d} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. So it has vector equation

$$\vec{x} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Any point $\vec{x}$ on m is mapped to

$$M\vec{x} = M\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) \quad = \quad \frac{1}{5}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}\begin{pmatrix} 5 \\ 5 \end{pmatrix} + t\frac{1}{5}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

Therefore, the whole line n is mapped to the point $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$. This happens because n is perpendicular to l and the point $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ is the intersection of n and m. □

Problem 4. Redo Problem 3 with T be the skew projection onto $l : x - 2y = 0$ along the direction $\vec{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

Solution. (a) Note that l has normal $\vec{n} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. The matrix of T is

$$M = I_2 - \frac{1}{\vec{v}\cdot\vec{n}}\vec{v}\vec{n}^T = \frac{1}{4}\begin{pmatrix} 6 & -4 \\ 3 & -2 \end{pmatrix}$$

(b) The images of $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$ are

$$\frac{1}{4}\begin{pmatrix} 6 & -4 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ and } \frac{1}{4}\begin{pmatrix} 6 & -4 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 5 \\ 5 \end{pmatrix} = \begin{pmatrix} 2.5 \\ 1.25 \end{pmatrix}$$

(c) Any point $\vec{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ on the line m mapped to

$$\frac{1}{4}\begin{pmatrix} 6 & -4 \\ 3 & -2 \end{pmatrix}\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \end{pmatrix},$$

where $s = \frac{5t}{4}$. Similar to Problem 3c, this is exactly the line $l : x - 2y = 0$.

(d) Similar to c. The answer is the line l. □

Problem 5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the orthogonal reflection through the line $l : x - 2y = 0$.
(a) Find the matrix M of T.
(b) Find the image of the points $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.
(c) Find the image of the line $m : x + y = 3$ under this map (find normal equation).
(d) Find the image of the line $n : 2x + y = 5$ under this map (find normal equation).

Solution. (a) The line l has direction vector $\vec{d} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. The matrix of T is

$$M = \frac{2}{||\vec{d}||^2}\vec{d}\vec{d}^T - I_2 = \frac{1}{5}\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$$

(b) The images of $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ are

$$\frac{1}{5}\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ and } \frac{1}{5}\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

(c) The line m has vector equation $\vec{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. So any point $\vec{x}$ on m is mapped to

$$\frac{1}{5}\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + s\begin{pmatrix} -1 \\ 7 \end{pmatrix},$$

where $s = \frac{t}{5}$. This is a point on the line $m' : \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + s\begin{pmatrix} -1 \\ 7 \end{pmatrix}$, which is a line through the point $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and having normal $\vec{n}' = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$. The normal equation of m' is

$$7(x - 2) + 1(y - 1) = 0 \Leftrightarrow 7x + y = 15.$$

(d) The line n has vector equation $\vec{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 2 \end{pmatrix}$. So any point $\vec{x}$ on n is mapped to

$$\frac{1}{5}\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \end{pmatrix},$$

where $s = \dfrac{t}{5}$. This is a point on the line $n' : \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, which is a line containing the point $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and having normal vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Its normal equation is

$$2(x-2) + 1(y-1) = 0 \Leftrightarrow 2x + y = 5,$$

which is the line l itself. This makes sense because n is perpendicular to l, which implies that its orthogonal projection is itself. $\square$

Problem 6. Redo Problem 5 with T be the skew reflection through $l : x - 2y = 0$ along the direction $\vec{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

Solution. (a) Note that l has normal $\vec{n} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. The matrix of T is

$$M = I_2 - \frac{2}{\vec{v} \cdot \vec{n}} \vec{v}\vec{n}^T = \frac{1}{2} \begin{pmatrix} 4 & -4 \\ 3 & -4 \end{pmatrix}$$

(b) The images of $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ are

$$\frac{1}{2} \begin{pmatrix} 4 & -4 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ and } \frac{1}{2} \begin{pmatrix} 4 & -4 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2.5 \end{pmatrix}$$

(c) Any point $\vec{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ on the line m mapped to

$$\frac{1}{2} \begin{pmatrix} 4 & -4 \\ 3 & -4 \end{pmatrix} \left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

where $s = \dfrac{7t}{2}$. Note that the direction vector of the above line is the vertical vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (also the unit vector on y-axis). So the image of m is the vertical line $m' : x = 2$.

(d) Any point $\vec{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ on the line m mapped to

$$\frac{1}{2} \begin{pmatrix} 4 & -4 \\ 3 & -4 \end{pmatrix} \left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \end{pmatrix} \right) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + s \begin{pmatrix} 12 \\ 11 \end{pmatrix}.$$

This line has normal equation $11x - 12y = 10$. $\square$