Limit Techniques we have learnt in class:

(1) Factorization, (2) Rationalization

Both techniques address the problem

$$\lim_{x \to a} \frac{p(x)}{q(x)} \quad \text{where} \quad \lim_{x \to a} p(x) = 0 \text{ and } \lim_{x \to a} q(x) = 0.$$

We refer to this as the "$\frac{0}{0}$" indeterminate form.

☆ General flow to evaluate limits of the form $\lim_{x \to a} \frac{p(x)}{q(x)}$ where p and q are continuous at a:

(1) Check if $\lim_{x \to a} q(x) \neq 0$. If it is, then evaluate by plugging $x=a$ into fraction to get limit.

(2) If $\lim_{x \to a} q(x) = 0$, then check if $\lim_{x \to a} p(x) = 0$. If it is, then we have a "$\frac{0}{0}$" indeterminate form. If p, q are polynomials, factor $(x-a)^n$ from p and q, then evaluate limit.

usually $n=1$, but can be $n=2$ if double root.

If p or q contains sums or differences of square roots (eg. $\sqrt{x}-\sqrt{2}$), then multiply numerator and denominator by its conjugate pair (eg. $\sqrt{x}+\sqrt{2}$). Simplify, then cancel common terms and evaluate.

③ If $\lim_{x\to a} p(x) \neq 0$, then we have one of these cases:

(a) $\lim_{x\to a} \frac{p(x)}{q(x)} = \infty$ (sign of $\frac{p(x)}{q(x)}$ is >0 as $x\to a$)

(b) $\lim_{x\to a} \frac{p(x)}{q(x)} = -\infty$ (sign of $\frac{p(x)}{q(x)}$ is <0 as $x\to a$)

(c) $\lim_{x\to a^-} \frac{p(x)}{q(x)}$ $\quad \lim_{x\to a^+} \frac{p(x)}{q(x)}$

one of them is $-\infty$, the other $+\infty$ or vice versa.

In either (a), (b) or (c), $\lim_{x\to a} \frac{p(x)}{q(x)}$ does not exist.

* we don't usually see ③ but occasionally it might come up.