CSD1241 Tutorial 5

Problem 1. Given $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \\ -5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -8 \\ 9 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & -1 \\ 6 & 9 & 10 \\ -2 & 10 & 5 \end{bmatrix}$. Find

(a) $A^T AB$ (b) $A - B$ (c) $3B - A^T A$ (d) $C^T CA$

Problem 2. In this problem, we will learn that in the matrix multiplication AB,

1. the jth column of AB is AB_j, where B_j is the jth column of B
2. the ith row of AB is $A_i B$, where A_i is the ith row of A

Consider two matrices

$$A = \begin{bmatrix} 1 & 2 & 0 & 6 \\ -2 & -1 & 5 & 0 \\ 7 & 8 & 1 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 0 \\ 0 & 7 & 1 \\ -1 & -2 & 5 \end{bmatrix}$$

Let A_1, A_2, A_3 be the rows of A and let B_1, B_2, B_3 be the columns of B.
(a) Compute AB.
(b) Verify that

$$\text{1st, 2nd, 3rd rows of } AB \text{ are } A_1B, A_2B, A_3B, \text{ and}$$
$$\text{1st, 2nd, 3rd columns of } AB \text{ are } AB_1, AB_2, AB_3.$$

Problem 3. Determine which of the following statements are true. Justify your answer (for false statements, you need to give counterexamples).
(a) The (i, j)-entry of AB can be computed by multiplying the ith row of A by the jth column of B.
(b) For every matrix A, it is true that $(A^T)^T = A$.
(c) If A and B are square matrices of the same order, then

$$AB = BA.$$

(d) If A is a 6×4 matrix and B is an $m \times n$ matrix such that $B^T A^T$ is a 2×6 matrix, then $m = 4$ and $n = 2$.
(e) If B has a column of zeros, then so does AB if this product is defined.
(f) If A has a row of zeros, then so does AB if this product is defined.

Problem 4. Find λ so that $\det(A) = 0$.

(a) $A = \begin{bmatrix} \lambda - 2 & 1 \\ -5 & \lambda + 4 \end{bmatrix}$ (b) $A = \begin{bmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{bmatrix}$ (c) $A = \begin{bmatrix} 1 & 1 & 2 \\ \lambda & -1 & -2 \\ 2 & 3 & 7 \end{bmatrix}$

Problem 5. Consider 3 vectors $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{v} = \begin{bmatrix} 0 \\ \lambda - 1 \\ 3 \end{bmatrix}, \vec{w} = \begin{bmatrix} 0 \\ -1 \\ \lambda + 1 \end{bmatrix}$. Let V be the volume of the parallelepiped formed by $\vec{u}, \vec{v}, \vec{w}$.

(a) What is V for $\lambda = 2$?

(b) What is the value of λ so that V has the smallest possible value?