TUTORIAL 3 Solutions

1 a) $((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$

b) Write the negation of

p

This computer program has a logical error in the 1st ten lines or it is being run with an incomplete data set.

q

a) We'll show that this is a contradiction:

Case 1: $q = 1 \Rightarrow \neg q = 0$

$$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q = 0$$

Case 2: $q = 0 \Rightarrow \neg q = 1$

$$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q = (\neg p \wedge 0) \wedge (0 \wedge r)$$

$$= 0 \wedge 0 = 0$$

$\therefore ((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$ is a contradiction.

(b) $\neg (p \vee q) \equiv \neg p \wedge \neg q$

This computer program doesn't have any logical error in the 1st ten lines and it is being run with a complete data set.

2) Determine which following pairs are equivalent?

a) $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$

b) $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$

a) Equivalent by distributive law.

b) Not equivalent: We give an assignment of values for p, q, r for which

$(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$ have different truth values.

Assign $r = 0$: $(p \vee q) \wedge r = 0$

$$(p \vee q) \vee (p \wedge r) \overset{r=0}{=\!=} (p \vee q) \vee 0 = p \vee q \overset{p=q=1}{=\!=} 1$$

Hence for $p = q = 1$, $r = 0$

$$(p \vee q) \vee (p \wedge r) = 1 \neq (p \vee q) \wedge r = 0$$

$$\therefore (p \vee q) \vee (p \wedge r) \not\equiv (p \vee q) \wedge r$$

3) Prove the following by logical equivalence laws

a) $(p \vee \neg q) \wedge (\neg p \vee \neg q) \equiv \neg q$

b) $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$

a) $(p \vee \neg q) \wedge (\neg p \vee \neg q) \equiv (\neg q \vee p) \wedge (\neg q \vee \neg p)$

$\equiv \neg q \vee (p \wedge \neg p)$ (distributive law)

$\equiv \neg q \vee F$ (negation law)

$\equiv \neg q$ (identity law)

b) $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ (De Morgan)

$\equiv \neg p \wedge (q \vee \neg q)$ (distributive law)

$\equiv \neg p \wedge T$ (negation law)

$\equiv \neg p$ (identity law)

4, Consider $(p \to r) \leftrightarrow (q \to r)$ (1)

a) Using $p \to q \equiv \neg p \vee q$ (lecture example), rewrite (1) without $\to$ and $\leftrightarrow$.

b) Using $p \vee q \equiv \neg(\neg p \wedge \neg q)$ (De Morgan), rewrite (1) with only $\wedge$ and $\neg$

a) $p \to q \equiv \neg p \vee q$

$$(p \to r) \leftrightarrow (q \to r) \equiv (\neg p \vee r) \leftrightarrow (\neg q \vee r)$$
$$\equiv ((\neg p \vee r) \to (\neg q \vee r)) \wedge ((\neg q \vee r) \to (\neg p \vee r))$$
$$\equiv (\neg(\neg p \vee r) \vee (\neg q \vee r)) \wedge (\neg(\neg q \vee r) \vee (\neg p \vee r))$$
$$\equiv ((p \wedge \neg r) \vee (\neg q \vee r)) \wedge ((q \wedge \neg r) \vee (\neg p \vee r))$$

b) $p \vee q \equiv \neg(\neg p \wedge \neg q)$

$$(p \wedge \neg r) \vee (\neg q \vee r) \equiv \neg(\neg(p \wedge \neg r) \wedge \neg(\neg q \vee r))$$
$$\equiv \neg(\neg(p \wedge \neg r) \wedge (q \wedge \neg r))$$

$$(q \wedge \neg r) \vee (\neg p \vee r) \equiv \neg(\neg(q \wedge \neg r) \wedge \neg(\neg p \vee r))$$
$$\equiv \neg(\neg(q \wedge \neg r) \wedge (p \wedge \neg r))$$

$$\therefore \neg(\neg(p \wedge \neg r) \wedge (q \wedge \neg r)) \wedge \neg(\neg(q \wedge \neg r) \wedge (p \wedge \neg r)).$$