CSD1251/CSD1250 Week 11 Tutorial Problems

13th March – 19th March 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Starred(*) questions are slightly more difficult.

Question 1

Find the global extreme values and points of f on the given interval.

(a) $f(x) = 12 + 4x - x^2$, $[0, 5]$

(b) $f(x) = x^3 - 6x^2 + 5$, $[-3, 5]$

(c) $f(x) = x + \dfrac{1}{x}$, $[0.2, 4]$

(d) $f(x) = \dfrac{x}{x^2 - x + 1}$, $[0, 3]$

(e) $f(x) = \ln(x^2 + x + 1)$, $[-1, 1]$

(f) $f(x) = \dfrac{\ln x}{x^2}$, $\left[\dfrac{1}{2}, 4\right]$

Question 2

After an alcoholic beverage is consumed, the concentration of alcohol in the bloodstream (blood alcohol concentration, or BAC) surges as the alcohol is absorbed, followed by a gradual decline as the alcohol is metabolized. The function

$$C(t) = 0.135te^{-2.802t}$$

models the average BAC, measured in g/dL (grams/deciliter), of a group of eight male subjects t hours after rapid consumption of 15 mL of ethanol (corresponding to one alcoholic drink). What is the maximum average BAC during the first 3 hours? For your reference, the legal blood alcohol limit for driving in Singapore is 0.08 g/dL.

Question 3

After an antibiotic tablet is taken, the concentration of the antibiotic in the bloodstream is modelled by the function

$$C(t) = 8(e^{-0.4t} - e^{-0.6t})$$

where the time t is measured in hours and C is measured in μg/mL. What is the maximum concentration of the antibiotic during the first 12 hours?

Question 4

A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 16 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

Question 5 (in preparation for Question 6)

Let $\mu = \tan\theta$, where $\mu > 0$.

(a) Show that $\sin\theta = \dfrac{\mu}{\sqrt{\mu^2+1}}$ with the aid of a right-angle triangle.

(b) Show that $\dfrac{\mu}{\sqrt{\mu^2+1}} \leq 1$, and $\dfrac{\mu}{\sqrt{\mu^2+1}} \leq \mu$.

Question 6*

An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F(\theta) = \frac{\mu W}{\mu\sin\theta + \cos\theta}$$

where μ is a positive constant called the *coefficient of friction*, and where $0 \leq \theta \leq \dfrac{\pi}{2}$.
Show that F is minimized when $\tan\theta = \mu$.
Hint: You may find that the result from Question 5(b) is helpful in showing that F is indeed minimized when $\mu = \tan\theta$.