2024 Fall CSD3240/3241/SEM4108 - Probability and Statistics Homework 2

P(A) = test positive
P(B) = Have disease
Find P(B|A)

1. A laboratory blood test is 95% effective in detecting a certain disease when it is in fact present. However, the test also yields a false positive result for 1% of the healthy persons test. It 0.5% of the population has the disease, what is the probability a person has the disease given that the test result is positive?

   A) 0.452 B) 0.116 C) 0.231 D) 0.323

2. In answering a question on a multiple-choice test, a student either knows the answer or guesses. The probability that the student knows the answer is 1/2. Assume that a student who guesses at the answer will be correct with probability 1/5. What is the probability that a student knew the answer to a question, given that he or she answered it correctly?

   P(A) = Know answer
   P(B) = Correct answer
   Find P(A|B)

   A) 5/6 B) 1/2 C) 2/5 D) 1/6

3. When coin A is flipped it comes up head with probability $\frac{1}{4}$, whereas when coin B is flipped it comes up head with probability $\frac{3}{4}$. Suppose that one of these coins is randomly chosen and is flipped twice. If both flips land heads, what is the probability that coin B was the one flipped? Conditional property and bayes therom

   Coin flipped twiced so need calculate 3/4 / 1/4 twice first before doing bayes therom

   ~~A) $\frac{9}{16}$~~ ~~B) $\frac{1}{16}$~~ C) $\frac{1}{10}$ D) $\frac{9}{10}$

   P(H) = Heads
   P(A) = Coin A
   Find P(A^c|H)

4. Three balls are to be randomly selected without replacement from the box containing 20 balls numbered 1 through 20. If we be that at least one of the drown balls has a number as large as or larger than 18, what is the probability that we win the bet? Easier to calculate event of opposite event. This case, find the probability of getting 17 and below

   Use condition since order dont matter = P(losing) = 17c3/20c3
   Then minus by 1 = 1 - p(losing)

   A) 0.508 B) 0.403 C) 0.284 ~~D) 0.150~~

5. Three balls are randomly chosen from the box containing 3 white, 3 red, and 5 black balls. Suppose that we win \$1 for each white ball selected and lose \$1 for each red selected. What is the probability that we win money?

   To win need more white ball than red or all white ball or more white ball than black ball.

   Possible combination:
   1W,0R,2B = (3C1)(3C0)(5C2)/(11C3)
   2W,0R,1B = (3C2)(3C0)(5C1)/(11C3)
   2W,1R,0B = (3C2)(3C1)(5C0)/(11C3)
   3W,0R,0B = (3C3)(3C0)(5C0)/(11C3)

   Then add them all together

   A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{13}{55}$ D) $\frac{1}{11}$

   Idea is just finding all possible probability to win an adding them together to see overall probability of us winning

6. An infinite sequence of independent trials is to be performed. Each trial results in a success with probability p and a failure with probability $1-p$. What is the probability that

   since at least 1 success is subset of all success, it be easier to find all failure and minus it off

   p(fail) = (1-p)^n

   p(success) = 1 - (1-p^n)

   i) at least 1 success occurs in the first n trials

      A) $(1-p)^n$ B) p^n C) $1-(1-p)^n$ D) $1-p^n$

   ii) exactly 5 successes occur in the first 10 trials if $p = 0.4$

      we want to find 5 success in 10 trails
      Can use binomial probability distribution formula

      A) 0.031 B) 0.001 C) 0.200 D) 0.313

7. The probability mass function (PMF) of a discrete random variable X is given by

$$p(x) = \frac{c\alpha^x}{x!}, \qquad x = 0, 1, 2, \cdots$$

   PMF => for a discrete random varaable, sum of probabilites over all possible values must = 1. This is a fundamental property of probability distributions
   Σ p(x) = 1

   where α is some positive value.

   i) What is the constant c?

      A) $c = \frac{2!}{\alpha^2}$ B) $c = \frac{1}{\alpha}$ ~~C) $c = e^{\alpha}$~~ D) $c = e^{-\alpha}$

   ii) Find $P(X > 1)$.

A) $P(X > 1) = \alpha e^{-\alpha}$

B) $P(X > 1) = (1 + \alpha)e^{\alpha}$

C) $P(X > 1) = 1 - (1 + \alpha)e^{-\alpha}$

D) $P(X > 1) = 1 - \alpha e^{\alpha}$

8. Given the cumulative distribution function

$$F(x) = \begin{cases} 0\,, & x < -2 \\ 0.2\,, & -2 \leq x < 0 \\ 0.7\,, & 0 \leq x < 2 \\ 1\,, & 2 \leq \end{cases}$$

Find the probability $P(X = 0)$.

A) 0.2

B) 0.5

C) 0.7

D) 0.45