CSD1251/CSD1250 Week 12 Tutorial Problems

19th March – 25th March 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Starred(*) questions are slightly more difficult.

Question 1

The sum of two positive numbers is 16. What is the smallest possible value for the sum of their squares?
Final answer: 128.

Question 2

If one side of a triangle has length a and another side has length $2a$, show that the largest possible area of the triangle is a^2.
Hint: The area of a triangle with sides of length a and b and included angle θ is $\frac{1}{2}ab\sin\theta$.

Question 3

The rate (in mg carbon/m³/h) at which photosynthesis takes place for a species of phytoplankton is modelled by the function

$$P = \frac{100I}{I^2 + I + 4}$$

where I is the light intensity (measured in thousands of foot-candles). For what light intensity is P a maximum?
Final answer: $I = 2$.

Question 4

Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

Final answer: $(\dfrac{5}{2}, \dfrac{\sqrt{10}}{2})$.

Question 5*

Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r.

Hint: The equation of a circle with radius r centered at the origin is $x^2 + y^2 = r^2$.

Final answer: $x = y = \dfrac{r}{\sqrt{2}}$ (when the rectangle is a square).

Question 6* (Calculations are tedious)

A cone-shaped paper drinking cup is to be made to hold 27 cm^3 of water. Find the height and radius of the cup that will use the smallest amount of paper.

Hint: The surface area of the cone without the base is $\pi r \sqrt{r^2 + h^2}$.

Final answer: $r = \dfrac{3^{4/3}}{\sqrt[6]{2\pi^2}}$, $h = \sqrt[3]{\dfrac{162}{\pi}}$.