CSD1241 Tutorial 10

Question 1. Determine which of the following maps are affine transformations. Further find the matrix A and the vector $\vec{b}$ of the affine transformations.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ by

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+1 \\ \sqrt{x}+y+1 \end{pmatrix}$$

(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$ by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y+1 \\ y \\ z-x-y \end{pmatrix}$$

(c) $T : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y+1 \\ z-x^2-y \end{pmatrix}$$

(d) $T : \mathbb{R}^2 \to \mathbb{R}^3$ by

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-y \\ 2y+2 \\ y+3x \end{pmatrix}$$

In the next problems, we use the 3-step approach

- Translating to the origin
- Performing the transformation around the origin (rotation, reflection, etc.)
- Translating back using the same vector

The 3 steps can be summarized by the formula

$$T(\vec{x}) = A(\vec{x} - \vec{x}_0) + \vec{x}_0$$

Question 2. Consider the line $l : x - 2y = 5$ in $\mathbb{R}^2$.

(a) Write the reflection T through l as an affine map

$$T(\vec{x}) = A\vec{x} + \vec{b}$$

Hint. Take $\vec{x}_0 = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ and $A =$ matrix of reflection through $l' : x - 2y = 0$, which is a line through O and parallel to l.

(b) Find the image of the points $\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}3\\1\end{pmatrix}, \begin{pmatrix}5\\-1\end{pmatrix}$ under T.

(c) Find the image of the line $m : x + y = 3$ under T.

Question 3. Consider the line $l : x + y = 3$ in $\mathbb{R}^2$.

(a) Write the projection T through l as an affine map

$$T(\vec{x}) = Ax + \vec{b}$$

(b) Find the image of the line $m : x - y = 5$ under T.

Question 4. (a) Write the rotation T around the point $Q = \begin{pmatrix}-1\\5\end{pmatrix}$ over $\theta = -45°$ as $T(\vec{x}) = A\vec{x} + \vec{b}$.

(b) Find the image of the line $m : x + y = 3$ under T.

Question 5. Write the shear parallel to the line $l : x+y = 3$ in the direction $\vec{v} = \begin{pmatrix}1\\-1\end{pmatrix}$ as an affine map $T(\vec{x}) = A\vec{x} + \vec{b}$.

Question 6. Consider the plane the plane $\alpha : 2x - 4y + 3z = 12$ in $\mathbb{R}^3$.

(a) Write the reflection T through α affine map $T(\vec{x}) = A\vec{x} + \vec{b}$ (Take $\vec{x_0} = \begin{pmatrix}6\\0\\0\end{pmatrix}$).

(b) Find the image of the points $\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}3\\1\\0\end{pmatrix}, \begin{pmatrix}5\\0\\-1\end{pmatrix}$ under T.

(c) Find the image of the plane $m : x + y + z = 3$ under T.