CSD2201/CSD2200 Week 6 Tutorial Problems

2nd October – 8th October 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Question 1

Evaluate the following integrals.

(a) $\int \frac{x+3}{x^3+3x}\,dx$ (b) $\int \frac{5}{x^3+5x}\,dx$ (c) $\int \frac{4x}{x^3+x^2+x+1}\,dx$

(d) $\int \frac{2}{x^3+2x^2+2x}\,dx$ (e) $\int \frac{x-1}{x^3+4x^2+5x}\,dx$ (f) $\int \frac{17x+34}{x^3+8x^2+17x}\,dx$

Question 2

(a) Find the approximations T_{10} and M_{10} for the integral $\int_0^1 \sin(x^2)\,dx$. Give your final answers in 6 decimal places.
(b) Find error bounds for the approximations in part (a).
(c) How large do we have to choose n so that the approximations T_n and M_n to the integral in part (a) is accurate to less than 10^{-5}?

Question 3

(a) Find the approximation S_8 for the integral $\int_0^1 e^{x^2}\,dx$. Give your final answer in 6 decimal places.
(b) Find error bounds for the approximations in part (a).
(c) How large do we have to choose n so that the approximation S_n to the integral in part (a) is accurate to within 0.0001?

Final Answers:

Q1: (a) $\ln|x| - \frac{1}{2}\ln(x^2+3) + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$, (b) $\ln|x| - \frac{1}{2}\ln(x^2+5) + C$,

(c) $-2\ln|x+1| + \ln(x^2+1) + 2\tan^{-1}x + C$, (d) $\ln|x| - \frac{1}{2}\ln(x^2+2x+2) - \tan^{-1}(x+1) + C$,

(e) $\frac{1}{10}\ln(x^2+4x+5) + \frac{7}{5}\tan^{-1}(x+2) - \frac{1}{5}\ln|x| + C$,

(f) $2\ln|x| - \ln(x^2+8x+17) + 9\tan^{-1}(x+4) + C$

Q2: (a) $M_{10} \approx 0.309816$, $T_{10} \approx 0.311171$, (b) $K = 6$, $|E_T| \leq \frac{1}{200}$, $|E_M| \leq \frac{1}{400}$,

(c) $n = 224$ for Trapezoidal Rule and $n = 159$ for Midpoint Rule

Q3: $S_8 \approx 1.462723$, (b) $K = 76e$, $|E_S| \leq 0.0179331$, (c) $n = 12$.