

# Lecture 9

Acceleration  
Deceleration  
Friction

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CSD1130  
Game  
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# 1. ASTEROIDS – Physics

## 1.1. Velocity – Moving Along Vectors

- In most games, sprite movement is done according to some vector and speed.
- The direction of the vector is usually determined by user input, collision reaction and reflection.
- The movement speed is determined by a “speed” variable and not by the direction vector's length.
- **Velocity = Normalized Direction \* speed**
- Moving a sprite is done by changing its current position (which will later result in a new translation matrix).
- Getting the next position of a sprite whose speed is S and direction vector is V:
- Next Position =  $S * V * \text{TimeStep} + \text{Current Position}$ 
  - The mathematical formula of the above equation is:
    - $\text{Pos} = V * t + \text{Pos}_0$ . The only difference is that in real time simulation,  $\text{Pos}_0$  represents the last frame's position.
- Example:
  - T is a 2D point located at (0;0)
  - Direction vector is (0.31 ; 0.95) (Normalized, explained later).
  - Its speed value is 2.
  - Assume the TimeStep is 1.0 second (In games, the TimeStep is the frame's frame time).
  - Frame 0:
    - $X_0 = 0$
    - $Y_0 = 0$
  - Frame 1:
    - $X_1 = \text{Direction}(X) * \text{speed} * \text{ts} + X_0 = 0.31 * 2 * 1 + 0 = 0.62$
    - $Y_1 = \text{Direction}(Y) * \text{speed} * \text{ts} + Y_0 = 0.95 * 2 * 1 + 0 = 1.9$
  - Frame 2:
    - $X_2 = \text{Direction}(X) * \text{speed} * \text{ts} + X_1 = 0.31 * 2 * 1 + 0.62 = 1.24$
    - $Y_2 = \text{Direction}(Y) * \text{speed} * \text{ts} + Y_1 = 0.95 * 2 * 1 + 1.9 = 3.8$
  - Frame 3:
    - $X_3 = \text{Direction}(X) * \text{speed} * \text{ts} + X_2 = 0.31 * 2 * 1 + 1.24 = 1.86$
    - $Y_3 = \text{Direction}(Y) * \text{speed} * \text{ts} + Y_2 = 0.95 * 2 * 1 + 3.8 = 5.7$

- As long as the direction vector isn't changed, the sprite will keep moving in the same direction (Unless its speed is set to 0).
- On the other hand, if the speed's value goes from being positive to negative, the sprite direction will become the opposite of its own direction vector.
- Note that in most games, the direction vector is normalized, and the object's movement speed is determined by the “speed” value.
- This normalization, which separates the object's direction from its speed, allows us to move an object in any direction at a constant speed.
  - The direction vector's only responsibility is to direct the object.
  - And the object's speed is controlled only by the speed value.
  - Keeping the speed value “2” for example will ensure that that particular object will move 2 units per second no matter what its direction vector is (as long as that direction vector is normalized).

## 1.2. Acceleration

- Object's velocity doesn't have to be constant.
- It can be altered by adding the acceleration element.
- The acceleration affects the velocity the same way the velocity affects the position.
- Getting the next position of an object whose velocity is  $V$  and acceleration is  $A$ :
  - Next Position =  $\frac{1}{2} A * \text{TimeStep} * \text{TimeStep} + V * \text{TimeStep} + \text{Current Position}$ 
    - The mathematical formulas of the above equation are:
      - $\text{Pos}_1 = \frac{1}{2} A * t * t + V_0 * t + \text{Pos}_0$ .  
The only difference is that in real time simulation,  $\text{Pos}_0$  represents last frame's position,  $V_0$  represents last frame's velocity and  $\text{Pos}_1$  represents the current frame's position.
      - $V_1 = A * t + V_0$ .  $V_1$  is the current frame's velocity and  $V_0$  is last frame's velocity.
  - Note that the speed value (used previously) doesn't exist anymore. This is because the change in velocity is affected by the acceleration, and not by a hard-coded “speed” value.

### 1.3. Acceleration – In Games (example Asteroids)

- Object's velocity doesn't have to be constant in games.
- It can be altered by adding the acceleration element.
- The acceleration affects the velocity the same way the velocity affects the position.
- In games, the player usually can control the acceleration of objects, which will implicitly change the objects' velocity and eventually position.
- Within one game loop (or one frame) the velocity is considered constant, and no acceleration is applied.
  - The acceleration will affect/change the velocity from one game loop to another, and not within/inside a game loop.
- Getting the next position of a sprite whose next velocity  $V_1$  is calculated based on the previous velocity  $V_0$ , and acceleration is  $A$ :
  - Next Velocity =  $V_1 = A * \text{TimeStep} + V_0$
  - Next Position =  $V_1 * \text{TimeStep} + \text{Current Position}$ 
    - The mathematical formulas of the above equations are:
      - $V_1 = A * t + V_0$ .  $V_1$  is the current frame's velocity and  $V_0$  is last frame's velocity.
      - $\text{Pos}_1 = V_1 * t + \text{Pos}_0$ .

The only difference is that in real time simulation,  $\text{Pos}_0$  represents last frame's position,  $V_0$  represents last frame's velocity and  $\text{Pos}_1$  represents the current frame's position.
- Example:
  - T is a 2D point located at (0;0),
  - Velocity is (0, 0) (The object is initially not moving)
  - Its acceleration is (3, 2) during frame 1 and frame 2, then it goes back to (0, 0) in frame 3. (This can be the result of the player pressing the “forward” button during frame 1 and 2)
  - Assume the TimeStep is 1.0 second.

- Frame 0:
  - $X_0 = 0$
  - $Y_0 = 0$
  
- Frame 1:
  - $V_{1x} = A_x * ts + V_{0x} = 3 * 1 + 0 = 3$
  - $V_{1y} = A_y * ts + V_{0y} = 2 * 1 + 0 = 2$
  
  - $X_1 = V_{1x} * ts + X_0 = 3 * 1 + 0 = 3$
  - $Y_1 = V_{1y} * ts + Y_0 = 2 * 1 + 0 = 2$
  
- Frame 2:
  - $V_{2x} = A_x * ts + V_{1x} = 3 * 1 + 3 = 6$
  - $V_{2y} = A_y * ts + V_{1y} = 2 * 1 + 2 = 4$
  
  - $X_2 = V_{2x} * ts + X_1 = 6 * 1 + 3 = 9$
  - $Y_2 = V_{2y} * ts + Y_1 = 4 * 1 + 2 = 6$
  
- Frame 3:
  - $V_{3x} = A_x * ts + V_{2x} = 0 * 1 + 6 = 6$
  - $V_{3y} = A_y * ts + V_{2y} = 0 * 1 + 4 = 4$
  
  - $X_3 = V_{2x} * ts + X_2 = 6 * 1 + 9 = 15$
  - $Y_3 = V_{2y} * ts + Y_2 = 4 * 1 + 6 = 10$