7. Work and Energy Part 1

Practice Question 7

A 20.0 kg rock is sliding on a rough, horizontal surface at 8.00 m/s and eventually stops due to friction. The coefficient of kinetic friction between the rock and the surface is 0.200. What average thermal power is produced as the rock stops?

friction

$\Sigma F = ma : \mu_k mg = ma$

a is the acceleration that friction force is providing. Which will be negative.

$a = \mu_k g = (0.200)(9.80\,\text{m/s}^2) = 1.96\,\text{m/s}^2$

$v = v_0 + at$

$0 = 8.00\,\text{m/s} - (1.96\,\text{m/s}^2)t$

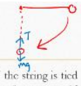
$t = 4.08\,\text{s}$

$P = \dfrac{KE}{t} = \dfrac{\frac{1}{2}mv^2}{t}$

$= \dfrac{\frac{1}{2}(20.0\,\text{kg})(8.00\,\text{m/s}^2)}{4.08\,\text{s}} = 157\,\text{W}$

8. Work and Energy Part 2

Practice Question 5



A 0.100 kg potato is tied to a string with length 2.50 m, and the other end of the string is tied to a rigid support. The potato is held straight out horizontally from the point of support, with the string pulled taut, and is then released. a) What is the speed of the potato at the lowest point of its motion? b) What is the tension in the string at this point?

a) The kinetic energy of the potato is the work done by gravity (or the potential energy lost), $\frac{1}{2}mv^2 = mgl$, or $v = \sqrt{2gl} = \sqrt{2(9.80\,\text{m/s}^2)(2.50\,\text{m})} = 7.00\,\text{m/s}$.

b)

$$T - mg = m\frac{v^2}{l} = 2mg,$$

so $T = 3mg = 3(0.100\,\text{kg})(9.80\,\text{m/s}^2) = 2.94\,\text{N}$.

On a horizontal surface, a crate with mass 50.0 kg is placed against a spring that stores 360 J of energy. The spring is released and the crate slides 5.60 m before coming to rest. What is the speed of the crate when it is 2.00 m from its initial position?

Work done by friction against the crate brings it to a halt:

$f_k x =$ potential energy of compressed spring

$f_k = \dfrac{360\,\text{J}}{5.60\,\text{m}} = 64.29\,\text{N}$

The friction force working over a 2.00-m distance does work $f_k x = (-64.29\,\text{N})(2.00\,\text{m}) = -128.6\,\text{J}$. The kinetic energy of the crate at this point is thus $360\,\text{J} - 128.6\,\text{J} = 231.4\,\text{J}$, and its speed is found from

$\dfrac{mv^2}{2} = 231.4\,\text{J}$

$v^2 = \dfrac{2(231.4\,\text{J})}{50.0\,\text{kg}} = 9.256\,\text{m}^2/\text{s}^2$

$v = 3.04\,\text{m/s}$

A 60.0-kg skier starts from rest at the top of a ski slope 65.0 m high. a) If frictional forces do -10.5 kJ of work on her as she descends, how fast is she going at the bottom of the slope? b) Now moving horizontally, the skier crosses a patch of soft snow, where coefficient of kinetic friction is 0.20. If the patch is 82.0 m wide and the average force of air resistance on the skier is 160 N, how fast is she going after crossing the patch? c) The skier hits a snowdrift and penetrates 2.5 m into it before coming to a stop. What is the average force exerted on her by the snowdrift as it stops her?

a) The skier's kinetic energy at the bottom can be found from the potential energy at the top minus the work done by friction,

$K_1 = mgh - W_F = (60.0\,\text{kg})(9.8\,\text{N/kg})(65.0\,\text{m}) - 10.500\,\text{J}$, or

$K_1 = 38{,}200\,\text{J} - 10{,}500\,\text{J} = 27{,}720\,\text{J}$. Then $v_1 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(27{,}720\,\text{J})}{60\,\text{kg}}} = 30.4\,\text{m/s}$.

b) $K_2 = K_1 - (W_F + W_A) = 27{,}720\,\text{J} - (\mu_k mgd + f_{air}d)$. $K_2 = 27{,}720\,\text{J} - [(.2)(588\,\text{N}) \times (82\,\text{m}) + (160\,\text{N})(82\,\text{m})]$, or $K_2 = 27{,}720\,\text{J} - 22{,}763\,\text{J} = 4957\,\text{J}$. Then,

$$v_2 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4957\,\text{J})}{60\,\text{kg}}} = 12.85\,\text{m/s} \approx 12.9\,\text{m/s}.$$

c) Use the Work-Energy Theorem to find the force. $W = \Delta KE$,

$F = KE/d = (4957\,\text{J})/(2.5\,\text{m}) = 1983\,\text{N} \approx 2000\,\text{N}$.

9. Momentum and Collisions Part 1

Practice Question 3

On a frictionless, horizontal air table, puck A (with mass 0.250 kg) is moving towards puck B (with mass 0.350 kg), that is initially at rest. After the collision, puck A has a velocity of 0.120 m/s to the left, and puck B has velocity 0.650 m/s to the right. a) What was the speed of puck A before the collision? b) Calculate the change in the total kinetic energy of the system that occurs during the collision.

a) The final momentum is

$(0.250\,\text{kg})(-0.120\,\text{m/s}) + (0.350)(0.650\,\text{m/s}) = 0.1975\,\text{kg}\cdot\text{m/s}$, taking positive directions to the right. a) Before the collision, puck B was at rest, so all of the momentum is due to puck A's motion, and

$v_{A1} = \dfrac{p}{m_A} = \dfrac{0.1975\,\text{kg}\cdot\text{m/s}}{0.250\,\text{kg}} = 0.790\,\text{m/s}.$

b) $\Delta K = K_2 - K_1 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 - \frac{1}{2}m_A v_{A1}^2$

$= \frac{1}{2}(0.250\,\text{kg})(-0.120\,\text{m/s})^2 + \frac{1}{2}(0.350\,\text{kg})(0.650\,\text{m/s})^2$

$- \frac{1}{2}(0.250\,\text{kg})(-0.7900\,\text{m/s})^2$

$= -0.0023\,\text{J}$

Practice Question 6

A ball with mass M, moving horizontally at 5.00 m/s, collides elastically with a block of mass 3M that is initially hanging at rest from the ceiling on the end of a 50.0 cm wire. Find the maximum angle through which the block swings after it is hit.

v_0 is initial velocity of ball.
v_1 is final velocity of ball.
v_2 is final velocity of block.

Collision: Momentum conservation gives

$mv_0 = mv_1 + (3m)v_2$

$v_0 = v_1 + 3v_2$ (1)

Energy Conservation:

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(3m)v_2^2$

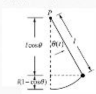
$v_0^2 = v_1^2 + 3v_2^2$ (2)

Solve (1) and (2) for v_2 : $v_2 = 2.50$ m/s

Energy conservation after collision:

$\frac{1}{2}(3m)v_2^2 = (3m)gh = (3m)gl(1-\cos\theta)$

Solve for θ : $\theta = 68.8°$



Practice Question 4

On a greasy, essentially frictionless lunch counter, a 0.500 kg submarine sandwich, moving 3.00 m/s to the left, collides with an 0.250 kg grilled cheese sandwich moving 1.20 m/s to the right. a) If the two sandwiches stick together, what is the final velocity? b) How much mechanical energy dissipates in the collision?

a) From $m_1v_1 + m_2v_2 = m_1v + m_2v = (m_1 + m_2)v$, $v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$. Taking positive velocities to the right, $v_1 = -3.00$ m/s and $v_2 = 1.20$ m/s, so $v = -1.60$ m/s.

If you get a +ve value that is also reasonable because question asks for energy dissipated. So question is abt motion.

But if question asks for change in energy, then it must be -ve value

b) $\Delta K = \frac{1}{2}(0.500\,\text{kg} + 0.250\,\text{kg})(-1.60\,\text{m/s})^2$

$- \frac{1}{2}(0.500\,\text{kg})(-3.00\,\text{m/s})^2 - \frac{1}{2}(0.250\,\text{kg})(1.20\,\text{m/s})^2$

$= -1.47\,\text{J}$.

A steel ball with mass 40.0 g is dropped from a height of 2.00 m onto a horizontal steel slab. The ball rebounds to a height of 1.60 m. (a) Calculate the impulse delivered to the ball during impact. (b) If the ball is in contact with the slab for 2.00 ms, find the average force on the ball during impact.

Set Up: Let $+y$ be downward.

Execute: **(a)** $\frac{1}{2}mv^2 = mgh$ so $v = \pm\sqrt{2gh}$.

$v_{1y} = +\sqrt{2(9.80\,\text{m/s}^2)(2.00\,\text{m})} = 6.26\,\text{m/s}$. $v_{2y} = -\sqrt{2(9.80\,\text{m/s}^2)(1.60\,\text{m})} = -5.60\,\text{m/s}$.

$J_y = \Delta p_y = m(v_{2y} - v_{1y}) = (40.0\times10^{-3}\,\text{kg})(-5.60\,\text{m/s} - 6.26\,\text{m/s}) = -0.474\,\text{kg}\cdot\text{m/s}$.

The impulse is $0.474\,\text{kg}\cdot\text{m/s}$, upward.

(b) $F_y = \dfrac{J_y}{\Delta t} = \dfrac{-0.474\,\text{kg}\cdot\text{m/s}}{2.00\times10^{-3}\,\text{s}} = -237\,\text{N}$. The average force on the ball is 237 N, upward.

Evaluate: The upward force on the ball changes the direction of its momentum.

10. Momentum and Collisions Part 2

Practice Question 2

+x

+y

need set coordinates

Set Up: Use coordinates with the axis at the hinge and the +x and +y axes along the horizontal and vertical bars in the figure in the problem. Let (x_i, y_i) and (x_f, y_f) be the coordinates of the bar before and after the vertical bar is pivoted. Let object 1 be the horizontal bar, object 2 be the vertical bar and 3 be the ball.

Execute: $x_i = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{(4.00\,\text{kg})(0.750\,\text{m}) + 0 + 0}{4.00\,\text{kg} + 3.00\,\text{kg} + 2.00\,\text{kg}} = 0.333\,\text{m}$.

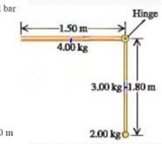
Middle

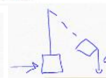
$y_i = \dfrac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \dfrac{0 + (3.00\,\text{kg})(0.900\,\text{m}) + (2.00\,\text{kg})(1.80\,\text{m})}{9.00\,\text{kg}} = 0.700\,\text{m}$.

$x_f = \dfrac{(4.00\,\text{kg})(0.750\,\text{m}) + (3.00\,\text{kg})(-0.900\,\text{m}) + (2.00\,\text{kg})(-1.80\,\text{m})}{9.00\,\text{kg}} = -0.366\,\text{m}$.

$y_f = 0$. $x_f - x_i = -0.700\,\text{m}$ and $y_f - y_i = -0.700\,\text{m}$. The center of mass moves 0.700 m to the right and 0.700 m upward.

Evaluate: The vertical bar moves upward and to the right so it is sensible for the center of mass of the machine part to move in these directions.





Practice Question 4

A 5.00 g bullet is shot *through* a 1.00 kg wood block suspended on a string 2.00 m long. The center of mass of the block rises a distance of 0.45 cm. Find the speed of the bullet as it emerges from the block if its initial speed is 450 m/s.

Identify: Apply conservation of momentum to the collision and conservation of energy to the motion of the block after the collision.

Set Up: Let +x be to the right. Let the bullet be A and the block be B. Let V be the velocity of the block just after the collision.

Execute: Motion of block after the collision: $K_1 = U_{grav2}$. $\frac{1}{2}m_BV^2 = m_Bgh$.

$$V = \sqrt{2gh} = \sqrt{2(9.80\,\text{m/s}^2)(0.450\times10^{-2}\,\text{m})} = 0.297\,\text{m/s}.$$

Collision: $v_{B2} = 0.297$ m/s. $P_{1x} = P_{2x}$ gives $m_Av_{A1} = m_Av_{A2} + m_Bv_{B2}$.

$$v_{A2} = \frac{m_Av_{A1} - m_Bv_{B2}}{m_A} = \frac{(5.00\times10^{-3}\,\text{kg})(450\,\text{m/s}) - (1.00\,\text{kg})(0.297\,\text{m/s})}{5.00\times10^{-3}\,\text{kg}} = 391\,\text{m/s}.$$

Evaluate: We assume the block moves very little during the time it takes the bullet to pass through it.

11. Rotation and Moment of Inertia Part 1

Practice Question 4

The rotating blade of a blender turns with constant angular acceleration 1.50 rad/s². (a) How much time does it take to reach an angular velocity of 36.0 rad/s, starting from rest? (b) Through how many revolutions does the blade turn in this time interval?

Identify: Apply the constant angular acceleration equations to the motion. The target variables are t and $\theta - \theta_0$.

Set Up: **(a)** $\alpha_z = 1.50\,\text{rad/s}^2$; $\omega_{0z} = 0$ (starts from rest); $\omega_z = 36.0\,\text{rad/s}$; $t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

Execute: $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{36.0\,\text{rad/s} - 0}{1.50\,\text{rad/s}^2} = 24.0\,\text{s}$

(b) $\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = 0 + \frac{1}{2}(1.50\,\text{rad/s}^2)(2.40\,\text{s})^2 = 432\,\text{rad}$

$\theta - \theta_0 = 432\,\text{rad}(1\,\text{rev}/2\pi\,\text{rad}) = 68.8\,\text{rev}$

Evaluate: We could use $\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$ to calculate $\theta - \theta_0 = \frac{1}{2}(0 + 36.0\,\text{rad/s})(24.0\,\text{s}) = 432\,\text{rad}$, which checks.

Practice Question 6

A high-speed flywheel in a motor is spinning at 500 rpm when a power failure suddenly occurs. The flywheel has mass 40.0 kg and diameter 75.0 cm. The power is off for 30.0 s, and during this time the flywheel slows due to friction in its axle bearings. During the time the power is off, the flywheel makes 200 complete revolutions. (a) At what rate is the flywheel spinning when the power comes back on? (b) How long after the beginning of the power failure would it have taken the flywheel to stop if the power had not come back on, and how many revolutions would the wheel have made during this time?

Identify: Apply constant angular acceleration equations.

Set Up: Let the direction the flywheel is rotating be positive.

$\theta - \theta_0 = 200\,\text{rev}$, $\omega_{0z} = 500\,\text{rev/min} = 8.333\,\text{rev/s}$, $t = 30.0\,\text{s}$.

Execute: **(a)** $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = 5.00\,\text{rev/s} = 300\,\text{rpm}$

(b) Use the information in part (a) to find α_z: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -0.1111\,\text{rev/s}^2$. Then $\omega_z = 0$, $\alpha_z = -0.1111\,\text{rev/s}^2$, $\omega_{0z} = 8.333\,\text{rev/s}$ in $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = 75.0\,\text{s}$ and $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\theta - \theta_0 = 312\,\text{rev}$.

Evaluate: The mass and diameter of the flywheel are not used in the calculation.

Practice Question 9

According to the shop manual, when drilling a 12.7-mm-diameter hole in wood, a drill should have a speed of 1250 rev/min. For a 12.7-mm-diameter drill bit turning at a constant 1250 rev/min, find (a) the maximum linear speed of any part of the bit and (b) the maximum radial acceleration of any part of the bit.

Identify: $v = r\omega$ and $a_{rad} = r\omega^2 = v^2/r$.

Set Up: 2π rad = 1 rev, so π rad/s = 30 rev/min.

Execute: **(a)** $\omega r = (1250\,\text{rev/min})\left(\dfrac{\pi\,\text{rad/s}}{30\,\text{rev/min}}\right)\left(\dfrac{12.7\times10^{-3}\,\text{m}}{2}\right) = 0.831\,\text{m/s}.$

(b) $\dfrac{v^2}{r} = \dfrac{(0.831\,\text{m/s})^2}{(12.7\times10^{-3}\,\text{m})/2} = 109\,\text{m/s}^2.$

Evaluate: In $v = r\omega$, ω must be in rad/s.

Practice Question 8

An advertisement claims that a centrifuge takes up only 0.127 m diameter of bench space but can produce a radial acceleration of 3000g (3000 times of gravitational acceleration) at 5000 rev/min. Calculate the required radius of the centrifuge. Is the claim realistic?

Set Up: $a_{rad} = r\omega^2$ so $r = a_{rad}/\omega^2$, where ω must be in rad/s.

Execute: $a_{rad} = 3000g = 3000(9.80\,\text{m/s}^2) = 29{,}400\,\text{m/s}^2$

$\omega = (5000\,\text{rev/min})\left(\dfrac{1\,\text{min}}{60\,\text{s}}\right)\left(\dfrac{2\pi\,\text{rad}}{1\,\text{rev}}\right) = 523.6\,\text{rad/s}$

Then $r = \dfrac{a_{rad}}{\omega^2} = \dfrac{29{,}400\,\text{m/s}^2}{(523.6\,\text{rad/s})^2} = 0.107\,\text{m}.$

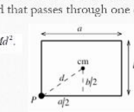
Evaluate: The diameter is then 0.214 m, which is larger than 0.127 m, so the claim is *not* realistic.

12. Rotation and Moment of Inertia Part 2

Practice Question 4

A thin, rectangular sheet of metal has mass M and sides of length a and b. Use the parallel-axis theorem to calculate the moment of Inertia of the sheet for an axis that is perpendicular to the plane of the sheet and that passes through one corner of the sheet.

Execute: $I_P = I_{cm} + Md^2$.



$I_{cm} = \frac{1}{12}M(a^2 + b^2)$.

The distance d of P from the cm is

$d = \sqrt{(a/2)^2 + (b/2)^2}$.

Thus $I_P = I_{cm} + Md^2 = \frac{1}{12}M(a^2 + b^2) + M\left(\frac{1}{4}a^2 + \frac{1}{4}b^2\right) = \left(\frac{1}{12} + \frac{1}{4}\right)M(a^2 + b^2) = \frac{1}{3}M(a^2 + b^2)$

A passenger bus in Zurich, Switzerland, derived its motive power from the energy stored in a large flywheel. The wheel was brought up to speed periodically, when the bus stopped at a station, by an electric motor, which could then be attached to the electric power lines. The flywheel was a solid cylinder with mass 1000 kg and diameter 1.80 m. Its top angular speed was 3000 rev/min. (a) At this angular speed, what is the kinetic energy of the fly-wheel? (b) If the average power required to operate the bus is 1.86 x 10^4 W, how long could it operate between stops?

Identify: $K = \frac{1}{2}I\omega^2$, with ω in rad/s. $P = \dfrac{\text{energy}}{t}$

Set Up: For a solid cylinder, $I = \frac{1}{2}MR^2$. 1 rev/min = $(2\pi/60)$ rad/s

Execute: **(a)** $\omega = 3000\,\text{rev/min} = 314\,\text{rad/s}$. $I = \frac{1}{2}(1000\,\text{kg})(0.900\,\text{m})^2 = 405\,\text{kg}\cdot\text{m}^2$

$K = \frac{1}{2}(405\,\text{kg}\cdot\text{m}^2)(314\,\text{rad/s})^2 = 2.00\times10^7\,\text{J}$.

(b) $t = \dfrac{K}{P} = \dfrac{2.00\times10^7\,\text{J}}{1.86\times10^4\,\text{W}} = 1.08\times10^3\,\text{s} = 17.9\,\text{min}$.

Evaluate: In $K = \frac{1}{2}I\omega^2$, we must use ω in rad/s.

About what axis will a uniform sphere have the same moment of inertia as does a thin-walled, hollow, lead sphere of the same mass and radius, with the axis along a diameter?

Set Up: For a thin-walled hollow sphere, axis along a diameter, $I = \frac{2}{3}MR^2$.

For a solid sphere with mass M and radius R, $I_{cm} = \frac{2}{5}MR^2$, for an axis along a diameter.

Execute: Find d such that $I_P = I_{cm} + Md^2$ with $I_P = \frac{2}{3}MR^2$:

$\frac{2}{3}MR^2 = \frac{2}{5}MR^2 + Md^2$

The factors of M divide out and the equation becomes $\left(\frac{2}{3} - \frac{2}{5}\right)R^2 = d^2$

$d = \sqrt{(10-6)/15}R = 2R/\sqrt{15} = 0.516R$.

The axis is parallel to a diameter and is 0.516R from the center.

Work Done

W = F . S(Constant)

W = KE = ½mv^2(Kinetic)

W = GPE = mg

h (Gravitational)

W = SE = ½kx^2(Spring)

W = KE = ½mv^2$_f$ – ½v^2$_i$

W = KE$_f$ + GPE$_f$ = KE$_i$ + GPE$_i$

(Same idea for GPE & SE when displacement occur)

Momentum from start and end is the same(conserved)

Scalar Product of 2 Vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$
$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$
$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

$\boxed{F_x = kx}$ (Hooke's Law)

Cannot arbitrarily assign zero value.

Power

P = mv(Momentum)

P_{av} = W/t

SI unit : Watt(W)

1 horsepower = 746W

1 kWh = 10^3 x 3600 Ws = 3.6 MJ

Reminder

F = μmg = ma = mg

J = F x T

Properties of conservative force

Force is independent of path.

Force at end = Force at start

Two cars, one a compact with mass 1200 kg and the other a large car with mass 3000 kg collide head-on at typical highway speeds. a) Which car has a greater magnitude of momentum change? Which car has a greater velocity change? Calculate the change in the velocity of the small car relative to that of the large car. b) Which car's occupants would you expect to sustain greater injuries? Explain.

(a) Momentum conservation tells us that both cars have the same change in momentum, but the smaller car has a greater velocity change because it has a smaller mass.

$$M\Delta V = m\Delta v$$
$$\Delta v\text{ (small car)} = \frac{M}{m}\Delta V\text{ (large car)}$$
$$= \frac{3000\text{ kg}}{1200\text{ kg}}\Delta V = 2.5\Delta V\text{ (large car)}$$

(b) The occupants of the small car experience 2.5 times the velocity change of those in the large car, so they also experience 2.5 times the acceleration. Therefore they feel 2.5 times the force, which causes whiplash and other serious injuries.

A ball with mass M, moving horizontally at 5.00 m/s, collides elastically with a block of mass 3M that is initially hanging at rest from the ceiling on the end of a 50.0 cm wire. Find the maximum angle through which the block swings after it is hit.

Collision: Momentum conservation gives

v_0 is initial velocity of ball.
v_1 is final velocity of ball.
v_2 is final velocity of block.

$$mv_0 = mv_1 + (3m)v_2$$
$$v_0 = v_1 + 3v_2 \quad (1)$$

Energy Conservation:

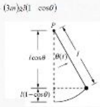
$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(3m)v_2^2 \quad (2)$$
$$v_0^2 = v_1^2 + 3v_2^2$$

Solve (1) and (2) for v_2: $v_2 = 2.50$ m/s

Energy conservation after collision:

$$\frac{1}{2}(3m)v_2^2 = (3m)gh = (3m)gl(1-\cos\theta)$$

Solve for θ. $\theta = 68.8°$



Moment of Inertia

Kinetic energy can be written as: $K_R = \sum_i K_i = \frac{1}{2}\sum_i m_i r_i^2\omega^2$ $\frac{1}{2}mv^2$

$$\boxed{K_R = \frac{1}{2}I\omega^2}$$

present for rotation only

Moment of inertia, I, is defined as: $\boxed{I = ½MR^2}$

For an extended rigid object (divide into small elements):

$$\boxed{I = \lim_{\Delta m_i \to 0}\sum_i \Delta m_i r_i^2 = \int r^2 dm = \int \rho r^2 dV} \quad m = \rho\cdot V$$

A measure of the resistance of an object to changes in its rotational motion, just as mass is a measure of the tendency of an object to resists changes in its linear motion.

Mass is an intrinsic property of an object, but I depends on the physical arrangement of that mass.

Also depends on the axis of rotation. Shape matters

Dimension: ML²; units kg.m²

Parallel axis theorem

The moment of inertia about any axis parallel to and at distance D away from the axis that passes through the centre of mass is:

$$\boxed{I = I_{CM} + MD^2}$$

Theorem works for any solids and shapes.

Momentum and Collision

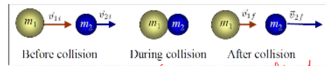
P = mv (Momentum)

P = KE = $\boxed{p = \sqrt{2mK}}$

$$\boxed{\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2 = \text{const.}}$$

Remember where all forces and momentum are present.

Elastic Collision



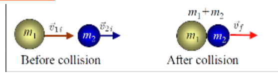
Momentum conservation: initial final $\boxed{m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}}$

Energy conservation: $\boxed{\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2}$

Final Velocity

$$\boxed{v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}} \quad \boxed{v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}}$$

Inelastic Collision



Momentum conservation: $\boxed{m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f}$

$$\boxed{\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2}}$$

Inelastic Collision

$$e = \frac{\text{Relative speed of separation}}{\text{Relative speed of approach}} = \frac{v_{2f} - v_{1f}}{v_{1i} - v_{2i}}$$

e	Type
0	Perfectly inelastic
<1	Inelastic
1	Elastic
>1	?

Momentum found at collision.

Angular Motion (Rotational motion of body)

Angular displacement (θ) is the angle through which a point revolves around a centre.

Angular velocity (ω) is the rate of change of angular displacement.

Angular acceleration (α) is the rate of change of angular velocity.

Arc length

$$\boxed{s = r\theta}$$

closer to pivot = slower rotation and vice versa

$$\boxed{v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega}$$

$$\boxed{a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha} \text{ tangent acceleration}$$

$$\boxed{a_r = \frac{v^2}{r} = r\omega^2} \text{ Radial acceleration}$$

Kinematics (Angular vs Linear)

Time present = use a_t
Else use a_r (if ask for centipedal, use a_r)

$\theta \Longleftrightarrow x$
$\omega \Longleftrightarrow v$
$\alpha \Longleftrightarrow a$

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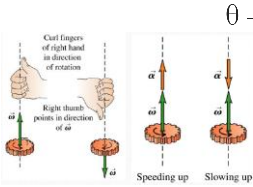
Linear		Angular
$v = v_0 + at$	⟺	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0t + \frac{1}{2}at^2$	⟺	$\theta = \theta_0 + \omega_0t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	⟺	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$$\theta - \theta_0 = 1/2(\omega_0 + \omega)\,t$$



The direction of ω is determined by right-hand rule.

θ => can be use to find m and angle/rad

$$\text{Radians} = \left(\frac{\pi}{180^0}\right) \times \text{degrees}$$

$$\text{Degrees} = \left(\frac{180^0}{\pi}\right) \times \text{radians}$$

circular Acceleration

centripetal acceleration: $\frac{V^2}{R} = \frac{4\pi^2 R}{T^2}$

a_r => radial acceleration

a_t => tangential acceleration: $\frac{dv}{dt}$

$T \Rightarrow \frac{2\pi R}{V}$

$V \Rightarrow \frac{2\pi R}{T}$

Projectile motion

Centre of Mass (Shaped object)

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

Centre of Mass (System of particles)

$$\boxed{\vec{r}_{CM} = \frac{\sum_i m_i\vec{r}_i}{M}}$$

Centre of Mass (Extended Object)

$$\boxed{\vec{r}_{CM} = \frac{1}{M}\int \vec{r}dm}$$

Characteristics of CM

For a **homogenous** (constant density) body that has a geometric centre, the CM is the geometric centre (eg, solid sphere, cube, and cylinder.)

The centre of mass of any symmetric object lies on an axis of symmetry and on any plane of symmetry.

If g is constant over the mass distribution, the centre of gravity coincides with the centre of mass. If an object is hung freely from any point, the vertical line through this point must pass through the centre of mass.

CM needs not be within the body itself.

Centre of mass (Right angle triangle)

$$\boxed{x_{CM} = \frac{2}{3}a} \quad \boxed{y_{CM} = \frac{1}{3}b}$$

Centre of mass (Cone)

$$\boxed{z_{CM} = \frac{3}{4}h} \quad \boxed{\begin{matrix} x_{CM} = 0 \\ y_{CM} = 0 \end{matrix}}$$

Motion of system of particles

$$\boxed{\vec{r}_{CM} = \frac{\sum_i m_i\vec{r}_i}{M}}$$

$\boxed{\vec{v}_{CM} = \frac{d\vec{r}_{CM}}{dt} = \frac{1}{M}\sum_i m_i\frac{d\vec{r}_i}{dt} = \frac{\sum_i m_i\vec{v}_i}{M}}$ ➡ Velocity of the CM

$\boxed{M\vec{v}_{CM} = \sum_i m_i\vec{v}_i = \sum_i \vec{p}_i = \vec{p}_{tot}}$ ➡ Total momentum of a system of particles

The total linear momentum of the system is equal to that of a single particle of mass M moving with a velocity $\mathbf{v}_{CM}$.

$\boxed{\vec{a}_{CM} = \frac{d\vec{v}_{CM}}{dt} = \frac{1}{M}\sum_i m_i\frac{d\vec{v}_i}{dt} = \frac{1}{M}\sum_i m_i\vec{a}_i}$ ➡ Acceleration of the CM

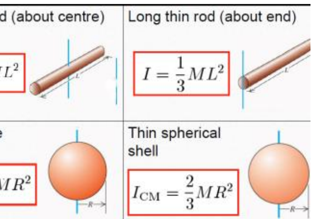
$\boxed{M\vec{a}_{CM} = \sum_i m_i\vec{a}_i = \sum_i \vec{F}_i}$ ➡ Newton's second law for a system of particles

$\boxed{\sum \vec{F}_{ext} = M\vec{a}_{CM} = \frac{d\vec{p}_{tot}}{dt}}$ ➡ F_i includes int. & ext. forces. Internal forces cancel out.

The CM moves like an imaginary particle of mass M under the influence of the external resultant force on the system.

Moment of Inertia for different objects

Long thin rod (about centre)	Long thin rod (about end)
$I_{CM} = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$
Solid sphere: $I_{CM} = \frac{2}{5}MR^2$	Thin spherical shell: $I_{CM} = \frac{2}{3}MR^2$



Hoop or thin cylindrical shell	Hollow cylinder
$I_{CM} = MR^2$	$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$
Solid cylinder or disk: $I_{CM} = \frac{1}{2}MR^2$	Rectangular plate: $I_{CM} = \frac{1}{12}M(a^2 + b^2)$