CSD1251/CSD1250 Week 4 Tutorial Problems

23 – 29 January 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Qn 1 (Chain Rule)

Differentiate the following functions with respect to their variables.

(a) $f(x) = (x^4+5)^{77}$ (b) $g(x) = e^{\sin x}$ (c) $h(\theta) = \ln(\tan(3\theta))$

(d) $f(x) = 5^{x^3}$ (e) $u(x) = \sin(\cos(\tan x))$ (f) $t(x) = \sqrt{4x-1}$

(g) $r(\theta) = \cos(\theta^2)$ (h) $f(t) = e^{2t}\sin(3t)$ (i) $G(z) = (1-4z)^2(z^2-2z+5)^4$

(j) $g(t) = \dfrac{1}{(2t+1)^3}$ (k) $f(x) = \dfrac{1}{\sqrt[3]{x^2-1}}$ (l) $s(t) = \sqrt{\dfrac{1+\sin t}{1+\cos t}}$

Final solutions:

(a) $f'(x) = 308x^3(x^4+5)^{76}$ (b) $g(x) = e^{\sin x}\cos x$

(c) $h'(\theta) = \dfrac{3\sec^2(3\theta)}{\tan(3\theta)}$ (d) $f'(x) = (3\ln 5)x^2 5^{x^3}$

(e) $u'(x) = -\cos(\cos(\tan x))\sin(\tan x)\sec^2 x$ (f) $t'(x) = \dfrac{2}{\sqrt{4x-1}}$

(g) $r'(\theta) = -2\theta\sin(\theta^2)$ (h) $f'(t) = e^{2t}(2\sin(3t)+3\cos(3t))$

(i) $G'(z) = -8(1-4z)(z^2-2z+5)^3(5z^2-7z+6)$ (j) $g'(t) = -\dfrac{6}{(2t+1)^4}$

(k) $f'(x) = -\dfrac{2x}{3(x^2-1)^{4/3}}$ (l) $s'(t) = \dfrac{1+\sin t+\cos t}{2\sqrt{1+\sin t}(1+\cos t)^{3/2}}$

Qn 2 (Proof of the Power Rule)

Using implicit differentiation, show that for any real number n, and $x > 0$,

$$\frac{d}{dx}x^n = nx^{n-1}.$$

Hint: Let $y = x^n$, then take the natural log on both sides of the equation, which will yield

$$\ln y = \ln(x^n),$$

then use implicit differentiation.

Qn 3 (Implicit Differentiation)

Find $\frac{dy}{dx}$ by implicit differentiation.

(a) $x^2 - 4xy + y^2 = 4$ (b) $x^4 + x^2y^2 + y^3 = 5$ (c) $x^3 - xy^2 + y^3 = 1$

(d) $\sin(x+y) = \cos x + \cos y$ (e) $\tan(x-y) = 2xy^3 + 1$ (f) $xy = \sqrt{x^2+y^2}$

Final solutions:

(a) $\frac{2y-x}{y-2x}$ (b) $-\frac{2x(2x^2+y^2)}{y(2x^2+3y)}$ (c) $\frac{y^2-3x^2}{y(3y-2x)}$

(d) $-\frac{\sin x + \cos(x+y)}{\sin y + \cos(x+y)}$ (e) $\frac{\sec^2(x-y) - 2y^3}{\sec^2(x-y) + 6xy^2}$ (f) $\frac{y\sqrt{x^2+y^2} - x}{y - x\sqrt{x^2+y^2}}$

Qn 4 (Tangent lines)

Find an equation of the tangent line to the curve at the given point.

(a) $y = 2^x$ at $(0,1)$ (b) $y = \sqrt{1+x^3}$ at $(2,3)$ (c) $y = \frac{2}{1+e^{-x}}$ at $(0,1)$

(d) $x^{2/3} + y^{2/3} = 4$ at $(-3\sqrt{3}, 1)$ (e) $x^2y^2 = (y+1)^2(4-y^2)$ at $(2\sqrt{3}, 1)$

Final solutions:

(a) $y = (\ln 2)x + 1$ (b) $y = 2x - 1$ (c) $y = \frac{1}{2}x + 1$

(d) $y = \frac{1}{\sqrt{3}}x + 4$ (e) $y = -\frac{\sqrt{3}}{5}x + \frac{11}{5}$