CSD1130
Game Implementation Techniques

Lecture 20

Questions?

- Reflection
- Animated Circle to Line Segment

Overview

- Animated Circular Object and Stationary Circular Object
- Collision Response (Reflection)

Modeling Pinball Animation as Ray

- Located at center point B_s at beginning of current frame
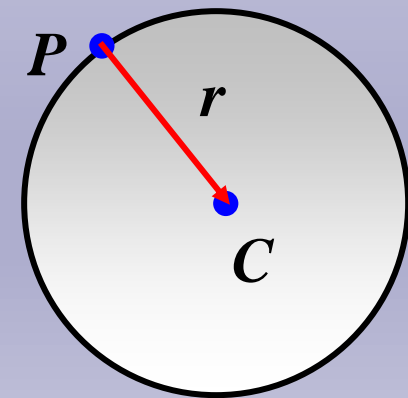- Moving in direction given by the vector v

$$B(t) = B_s + \vec{v}(t) \qquad t \in [0,1]$$

Circular Pillars

- Circular pillars are stationary and defined by center point C and radius r
- Boundary of circle defined as all points P whose distance from center C is equal to radius r

$$\left\| C - P \right\| = r \quad \Rightarrow \left\| C - P \right\|^2 = r^2$$
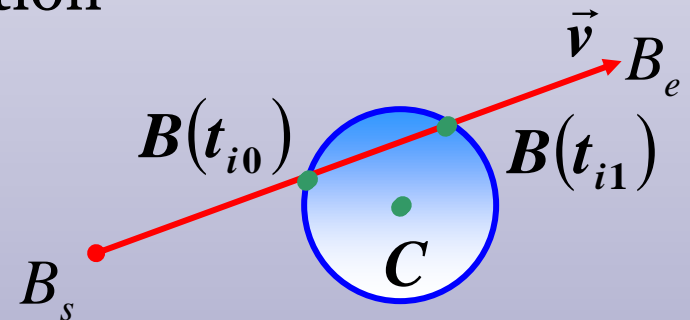
$$\Rightarrow \left(C - P \right) \bullet \left(C - P \right) = r^2$$

Ray-Circle Intersection (1/6)

- To solve for intersection between ray $B(t)$ and circle, replace P with $B(t_i)$ in circle equation

$$(C - B(t)) \bullet (C - B(t)) = r^2$$

$$\Rightarrow (C - B_s - t\vec{v}) \bullet (C - B_s - t\vec{v}) = r^2$$

$$\Rightarrow t^2(\vec{v} \bullet \vec{v}) - 2t(C - B_s) \bullet \vec{v} + (C - B_s) \bullet (C - B_s) - r^2 = 0$$
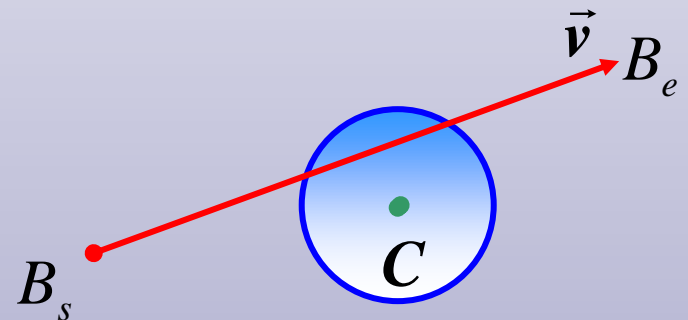
Ray-Circle Intersection (2/6)

$$\Rightarrow \mathrm{a}t^2 + \mathrm{b}t + \mathrm{c} = 0$$

where

$$\mathrm{a} = \vec{v} \bullet \vec{v}$$

$$\mathrm{b} = -2\left(\overrightarrow{B_sC}\right) \bullet \vec{v}$$

$$\mathrm{c} = \left(\overrightarrow{B_sC}\right) \bullet \left(\overrightarrow{B_sC}\right) - r^2$$
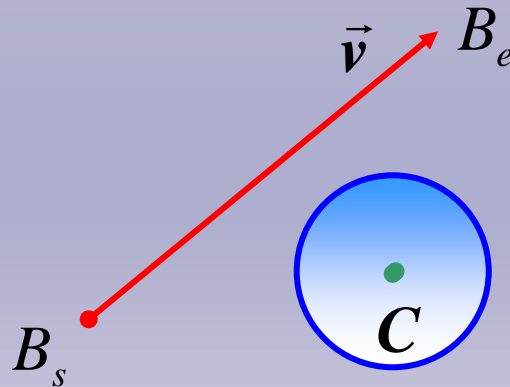
Ray–Circle Intersection (3/6)

Solve for t,

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ray-Circle Intersection (4/6)

$$\text{Given}: a = \vec{v} \bullet \vec{v}, b = -2\left(C - B_s\right) \bullet \vec{v}, \text{and}$$

$$c = \left(C - B_s\right) \bullet \left(C - B_s\right) - r^2$$

$$\text{If } b^2 - 4ac < 0 \Rightarrow \text{ray misses circle}$$

$$\vec{v} \quad B_e$$

$$B_s$$

$$C$$

Ray–Circle Intersection (5/6)

$$\text{If } b^2 - 4ac \equiv 0 \Rightarrow \text{ray grazes circle}$$

$$t_i = \frac{-b}{2a} \in [0,1]$$

$$\boldsymbol{B}_i = \boldsymbol{B}(t_i) = \boldsymbol{B}_s + \vec{v}t_i$$
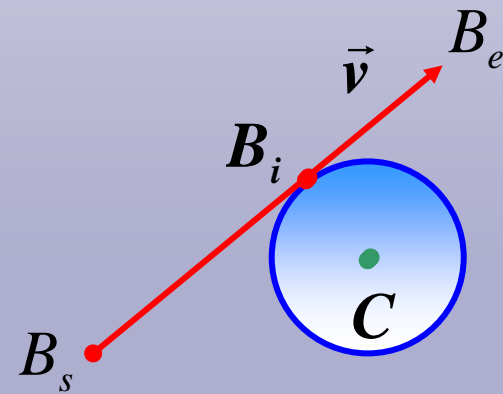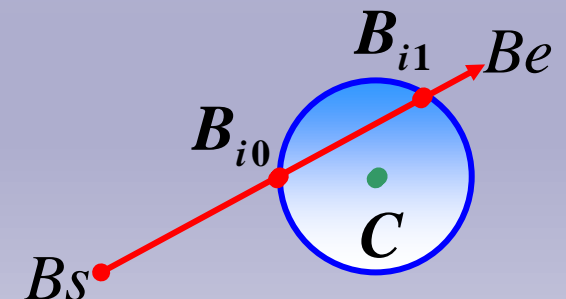
Ray-Circle Intersection (6/6)

If $b^2 - 4ac > 0 \Rightarrow$ ray intersects circle at B_{i0} and B_{i1}
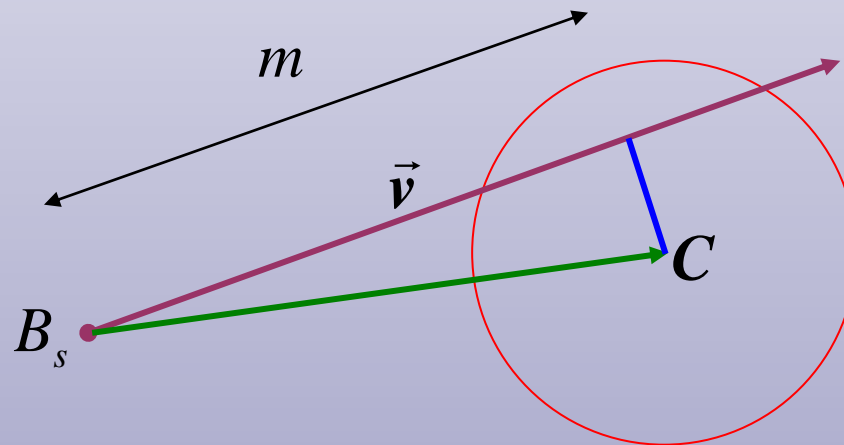
$$t_{i0} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$t_{i1} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$t_i = \min(t_{i0}, t_{i1}) \text{ and } t_i \in [0,1]$$
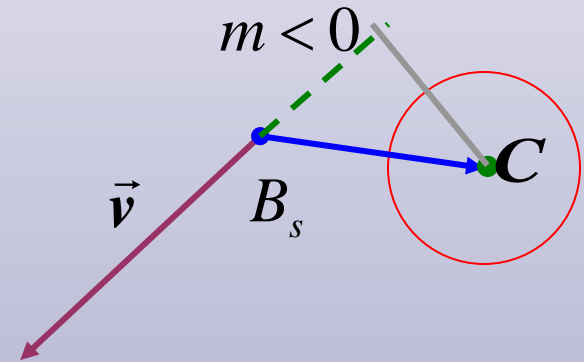
$$B(t_i) = B_i = B_s + \vec{v} t_i$$

Test for Non-Collision (1/3)

Compute projection of $\overrightarrow{B_sC}$ onto $\hat{v}$

Test for Non-Collision (2/3)

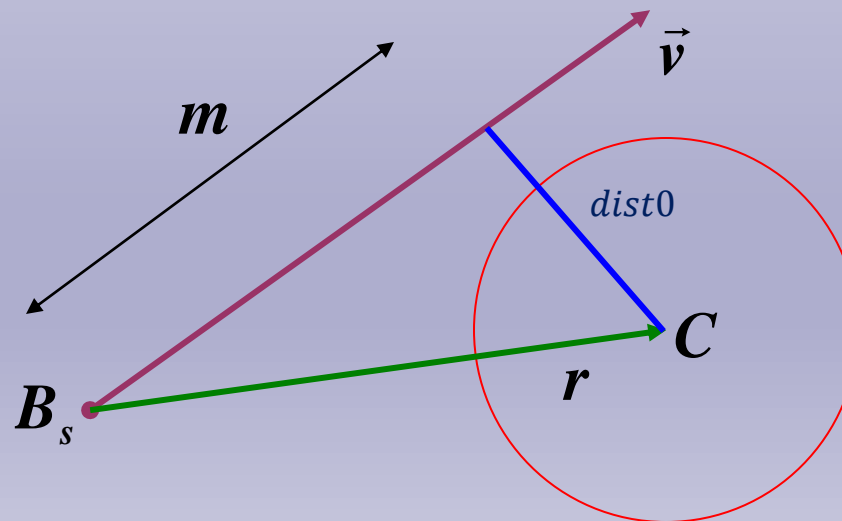$$m = \overrightarrow{B_s C} \bullet \frac{\vec{v}}{\|\vec{v}\|}$$

If $m < 0$ && B_s outside circle

$\Rightarrow$ **circle behind ray origin**

Test for Non-Collision (3/3)

Compute: $dist0^2 = \|B_sC\|^2 - m^2$

If $dist0^2 > r^2$ ray will miss the circle

… Otherwise: Compute t_i (1/2)

- There are two ways to compute the time of intersection:
  - Using the quadratic equation

$$t_{i0} = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \qquad\qquad t_{i1} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$t_i = \min(t_{i0}, t_{i1}) \text{ and } t_i \in [0,1]$$

(Or make sure that the
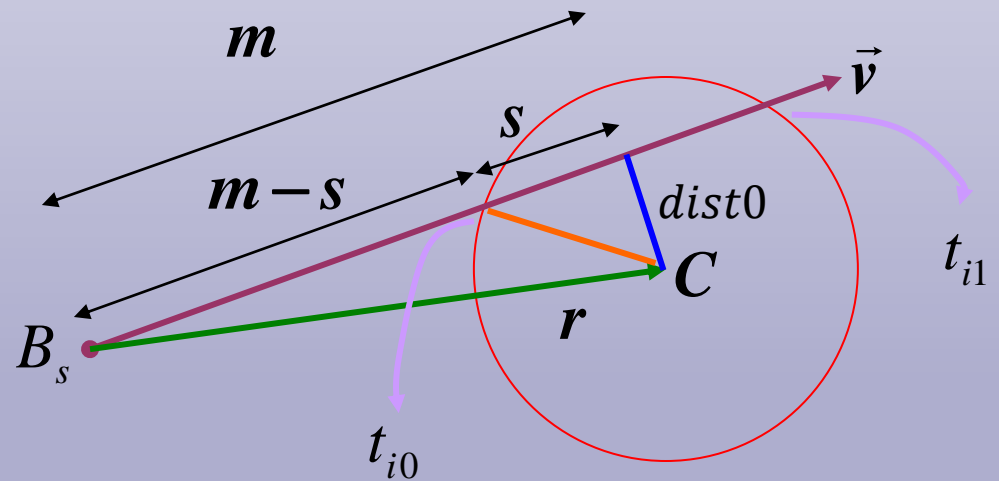intersection point is between B_s and B_e)

Compute t_i (2nd method) (2/2)

Compute: $s^2 = r^2 - dist0^2$

Since $dist0^2 \leq r^2 \Rightarrow s^2 \geq 0 \Rightarrow s \geq 0$

$$t_{i0} = \frac{m-s}{\|\vec{v}\|}$$

$$t_{i1} = \frac{m+s}{\|\vec{v}\|}$$

(Make sure that the
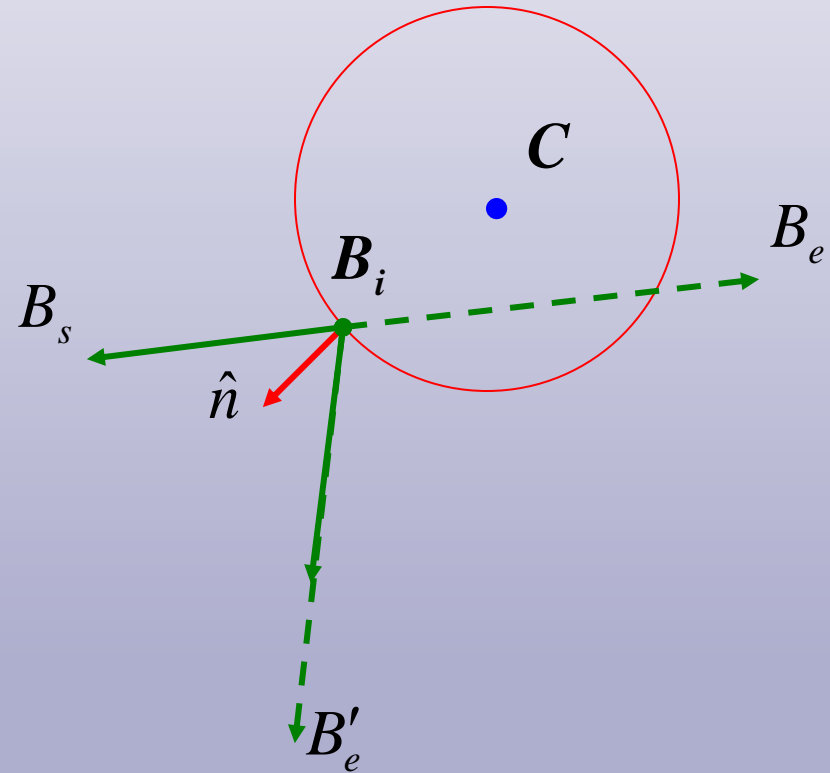intersection point is between B_s and B_e)

Overview

- Animated Circular Object and Stationary Circular Object
- **Collision Response (Reflection)**

Reflection (1/4) (skip this one – use previous method)

Compute : $\boldsymbol{B}_i = \boldsymbol{B}_s + \vec{\boldsymbol{v}}\boldsymbol{t}_i$

Compute : $\vec{n} = \overrightarrow{CB_i}$

$$\hat{n} = \frac{\overrightarrow{CB_i}}{\left\|\overrightarrow{CB_i}\right\|}$$

Reflection (2/4) (skip this one – use previous method)

$$\vec{m} = \overrightarrow{B_i B_s} = B_s - B_i$$
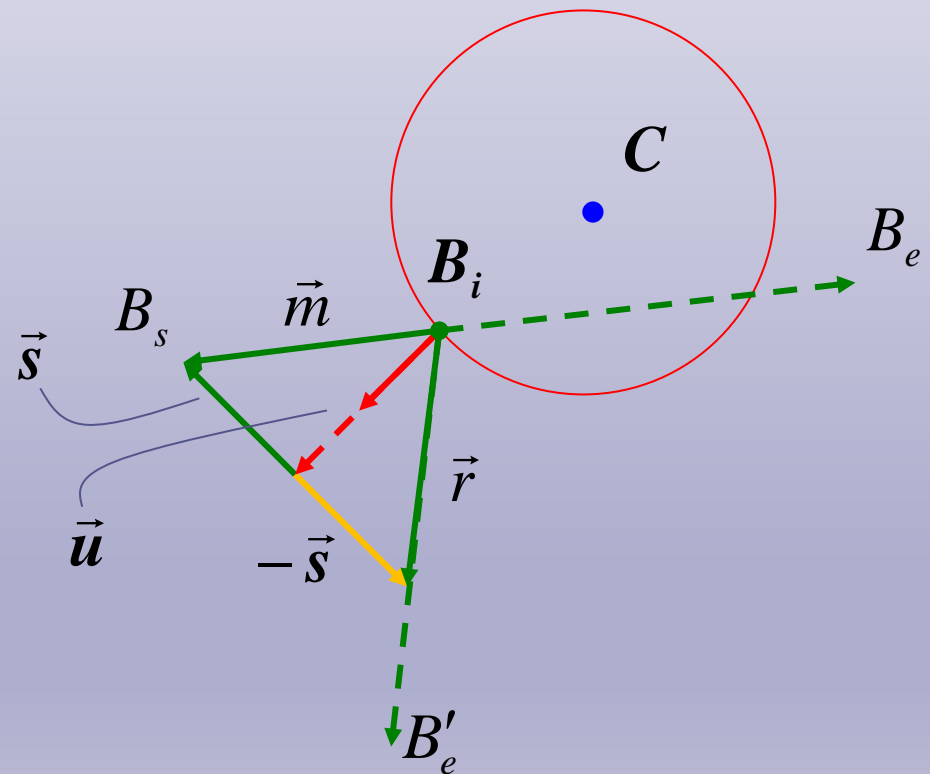
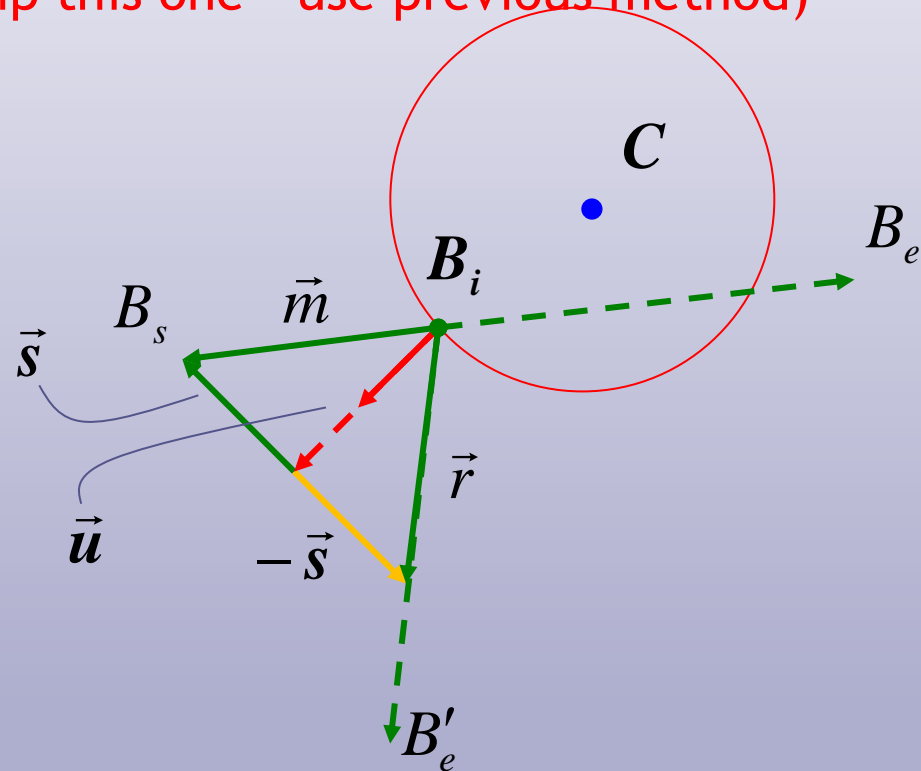# Reflection (3/4) (skip this one – use previous method)

$$\vec{u} + \vec{s} = \vec{m}$$

$$\vec{u} - \vec{s} = \vec{r}$$

$$2\vec{u} = \vec{m} + \vec{r} \implies \vec{r} = 2\vec{u} - \vec{m}$$

$$\vec{u} = (\vec{m} \bullet \hat{n})\hat{n}$$

$$\text{Reflection}: \vec{r} = 2(\vec{m} \bullet \hat{n})\hat{n} - \vec{m}$$

Reflection (4/4) (skip this one – use previous method)

$$\text{Given}: \boldsymbol{B}_s, \boldsymbol{B}_i, \boldsymbol{t}_i \text{ and } \vec{\boldsymbol{n}}$$

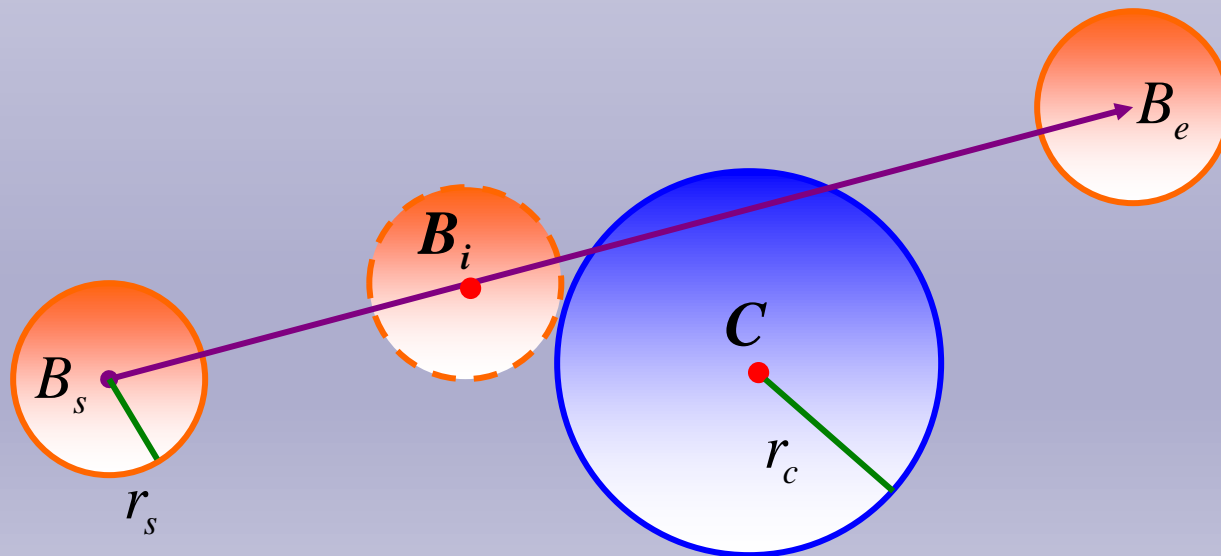$$\vec{m} = \overrightarrow{B_i B_s} = B_s - B_i$$

$$\vec{r} = 2(\vec{m} \bullet \hat{n})\hat{n} - \vec{m} \qquad \hat{r} = \frac{\vec{r}}{\|\vec{r}\|}$$

$$\Rightarrow B_e{}' = B_i + k\hat{r}(1 - t_i)$$

(k is the length of the original vector v. Refer to slide 4)

Pinball-Circular Pillar Collision (1/2)

- Animated pinball modeled by a circle with center B_s and radius r_s
- Stationary circular pillar defined by center point C and radius r_c

Pinball-Circular Pillar Collision (2/2)

- Similar to intersection tests between ray from B_s to B_e and circle of radius (r_s+r_c)

B_e

B_i

C r_c

B_s

r_s

$r = r_s + r_c$