CSD2301 Practice Solutions
8. **Work and Energy Part 2**

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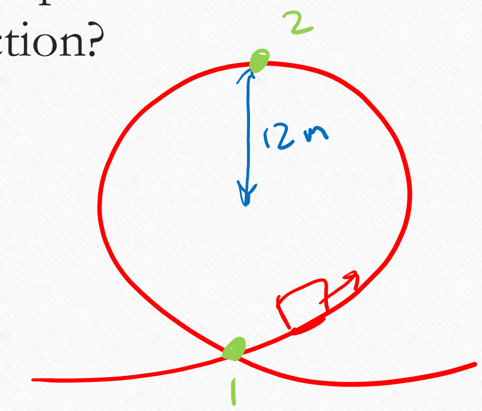
Practice Question 1

What is the potential energy of an 800 kg elevator at the top of a tower which is 440 m above the street level? Let the potential energy be zero at street level.

$$mgy = (800\,\mathrm{kg})\,(9.80\,\mathrm{m/s^2})\,(440\,\mathrm{m}) = 3.45\times10^6\ \mathrm{J} = 3.45\ \mathrm{MJ}.$$

Practice Question 2

You are testing a new amusement park roller coaster with an empty car with mass 120 kg. One part of the track is a vertical loop with radius 12.0 m. At the bottom of the loop (point 1), the car has a speed of 25.0 m/s and at the top of the loop (point 2), the car has a speed of 8.0 m/s. As the car rolls from point A to point B, how much work is done by friction?

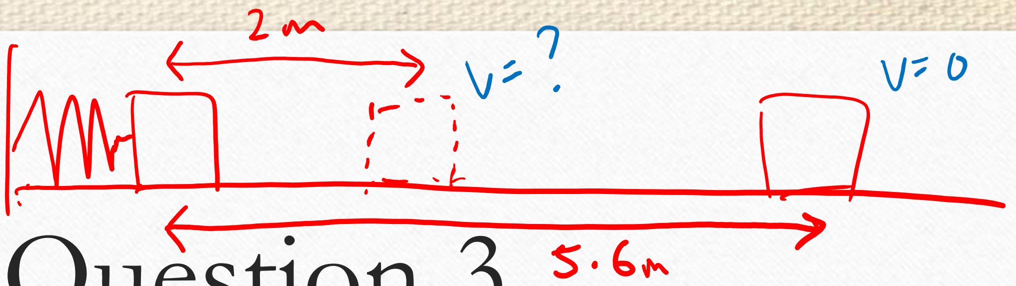


$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

$$U_1 = 0,\ U_2 = mg(2R) = 28{,}224\,\text{J}, W_{\text{other}} = W_f$$

$$K_1 = \frac{1}{2}mv_1^2 = 37{,}500\,\text{J}, K_2 = \tfrac{1}{2}mv_2^2 = 3840\,\text{J}$$

The work - energy relation then gives $W_f = K_2 + U_2 - K_1 = -5400\ \text{J}.$



Practice Question 3

On a horizontal surface, a crate with mass 50.0 kg is placed against a spring that stores 360 J of energy. The spring is released and the crate slides 5.60 m before coming to rest. What is the speed of the crate when it is 2.00 m from its initial position?

Work done by friction against the crate brings it to a halt:

$$f_k x = \text{potential energy of compressed spring}$$

$$f_k = \frac{360\,\text{J}}{5.60\,\text{m}} = 64.29\,\text{N}$$

The friction force working over a 2.00-m distance does work $f_k x = (-64.29\,\text{N})(2.00\,\text{m}) = -128.6\,\text{J}$. The kinetic energy of the crate at this point is thus $360\,\text{J} - 128.6\,\text{J} = 231.4\,\text{J}$, and its speed is found from

$$\frac{mv^2}{2} = 231.4\,\text{J}$$

$$v^2 = \frac{2(231.4\,\text{J})}{50.0\,\text{kg}} = 9.256\,\text{m}^2/\text{s}^2$$

$$v = 3.04\,\text{m/s}$$

Practice Question 4

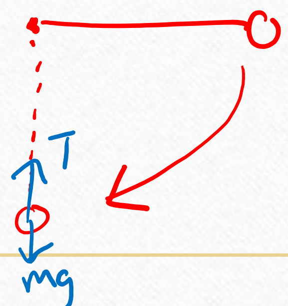
A 650-gram rubber ball is dropped from an initial height of 2.50m and on each bounce it returns to 75% of its previous height. a) What is the initial mechanical energy of the ball, just after it is released from its initial height? b) How much mechanical energy does the ball lose during its first bounce? c) How much mechanical energy is lost during the second bounce?

a) $mgh = (0.650\,\text{kg})(9.80\ \text{m/s}^2)(2.50\,\text{m}) = 15.9\,\text{J}$

b) The second height is $0.75(2.50\,\text{m}) = 1.875\,\text{m}$, so second $mgh = 11.9\,\text{J}$; loses $15.9\,\text{J} - 11.9\,\text{J} = 4.0\,\text{J}$ on first bounce. This energy is converted to thermal energy.

a) The third height is $0.75(1.875\,\text{m}) = 1.40\,\text{m}$,, so third $mgh = 8.9\,\text{J}$; loses $11.9\,\text{J} - 8.9\,\text{J} = 3.0\,\text{J}$ on second bounce.

Practice Question 5



A 0.100 kg potato is tied to a string with length 2.50 m, and the other end of the string is tied to a rigid support. The potato is held straight out horizontally from the point of support, with the string pulled taut, and is then released. a) What is the speed of the potato at the lowest point of its motion? b) What is the tension in the string at this point?

a) The kinetic energy of the potato is the work done by gravity (or the potential energy lost), $\frac{1}{2}mv^2 = mgl$, or $v = \sqrt{2gl} = \sqrt{2(9.80\ \text{m/s}^2)(2.50\ \text{m})} = 7.00\ \text{m/s}$.

b)

$$T - mg = m\frac{v^2}{l} = 2mg,$$

so $T = 3mg = 3(0.100\ \text{kg})(9.80\ \text{m/s}^2) = 2.94\ \text{N}$.

Practice Question 6

A 60.0-kg skier starts from rest at the top of a ski slope 65.0 m high. a) If frictional forces do -10.5 kJ of work on her as she descends, how fast is she going at the bottom of the slope? b) Now moving horizontally, the skier crosses a patch of soft snow, where coefficient of kinetic friction is 0.20. If the patch is 82.0 m wide and the average force of air resistance on the skier is 160 N, how fast is she going after crossing the patch? c) The skier hits a snowdrift and penetrates 2.5 m into it before coming to a stop. What is the average force exerted on her by the snowdrift as it stops her?

a) The skier's kinetic energy at the bottom can be found from the potential energy at the top minus the work done by friction,

$K_1 = mgh - W_\mathrm{F} = (60.0\ \mathrm{kg})(9.8\ \mathrm{N/kg})(65.0\ \mathrm{m}) - 10.500\ \mathrm{J}$, or

$K_1 = 38{,}200\ \mathrm{J} - 10{,}500\ \mathrm{J} = 27{,}720\ \mathrm{J}$. Then $v_1 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(27{,}720\ \mathrm{J})}{60\ \mathrm{kg}}} = 30.4\ \mathrm{m/s}$.

b) $K_2 = K_1 - (W_\mathrm{F} + W_\mathrm{A}) = 27{,}720\ \mathrm{J} - (\mu_\mathrm{k} mgd + f_\mathrm{air} d)$, $K_2 = 27{,}720\ \mathrm{J} - [(.2)(588\ \mathrm{N}) \times (82\ \mathrm{m}) + (160\ \mathrm{N})(82\ \mathrm{m})]$, , or $K_2 = 27{,}720\ \mathrm{J} - 22{,}763\ \mathrm{J} = 4957\ \mathrm{J}$. Then,

$$v_2 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4957\ \mathrm{J})}{60\ \mathrm{kg}}} = 12.85\ \mathrm{m/s} \approx 12.9\ \mathrm{m/s}.$$

c) Use the Work-Energy Theorem to find the force. $W = \Delta KE$,

$F = KE/d = (4957\ \mathrm{J})/(2.5\ \mathrm{m}) = 1983\ \mathrm{N} \approx 2000\ \mathrm{N}.$

The End