CSD2201/2200 Week 13 Homework

Due: 30th November 2023, 2359 HRS

For each question, key in **the** correct option into the homework into the "Week 13 Homework" option in the "20 November to 26 November" section in our **combined** CSD2201 and CSD2200 meta course page on Moodle.

Question 1

LCT $\frac{1}{n}$ div $\frac{3}{e} > 1 =$ div   div

Amongst the following series, which are absolutely convergent?

(A) $\displaystyle\sum_{n=2}^{\infty} \left|\frac{n^2+3}{2n^3+n-1}\right|$ (B) $\displaystyle\sum_{n=1}^{\infty} \left|\frac{n!\,3^n}{n^n}\right|$ (C) $\displaystyle\sum_{n=1}^{\infty} \left|(-1)^{n-1}\tan\left(\frac{1}{n}\right)\right|$ AS

(a) (A) only   (b) (B) only   (c) (C) only

(d) (A) and (C) only   (e) None of these

Question 2

ACT - F cond conv   AST, Ratio... $\frac{1}{n^2}$ AST

Amongst the following series, which are conditionally convergent?

(A) $\displaystyle\sum_{n=2}^{\infty} \left|(-1)^n \frac{n^2}{3n^3+1}\right|$ (B) $\displaystyle\sum_{n=1}^{\infty} \left|\frac{\cos(n\pi)}{n^2}\right|$ (C) $\displaystyle\sum_{n=1}^{\infty} \left|(-1)^n \sin\left(\frac{1}{n}\right)\right|$

(a) (A) only   (b) (A) and (C) only   (c) (B) and (C) only

(d) (A) and (B) only   (e) All of these

Question 3

Find the radius of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{3n}}{n\,27^n}$.

(a) ∞ (b) 3   (c) $\sqrt{27}$ (d) 27   (e) None of these

Question 4

Find the radius of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(4x-5)^{2n}}{\sqrt{n}\,25^n}$.

(a) ∞ (b) 25 (c) 5 (d) $\frac{5}{4}$ (e) None of these

Question 5

Find the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{(3n)!}{23^n(n!)^3}(7x-2)^n$.

(a) ∞ (b) $\frac{27}{23}$ (c) $\frac{23}{27}$ (d) $\frac{23}{189}$ (e) None of these

Question 6

Given that the n-th order derivatives of a function f at $a = 3$ are

$$f^{(n)}(3) = \frac{(2n+1)}{4^n},$$

find the radius of convergence of the Taylor series of f centered at a.

(a) ∞ (b) 4 (c) $\frac{4}{3}$ (d) 2 (e) None of these

$\hookrightarrow \frac{0}{1} = 0 \Rightarrow R = \infty$

Question 7

Given that the n-th order derivatives of a function f at $a = 4$ are

$$f^{(n)}(4) = \frac{(3n+1)!}{9^n},$$

find the radius of convergence of the Taylor series of f centered at a.

(a) 0 (b) 9 (c) 27 (d) $\frac{9}{4}$ (e) None of these

$\hookrightarrow \frac{27}{0} \Rightarrow L = \infty$
$R = 0$