CSD2301 Practice Solutions
3. Kinematics in 2D and 3D

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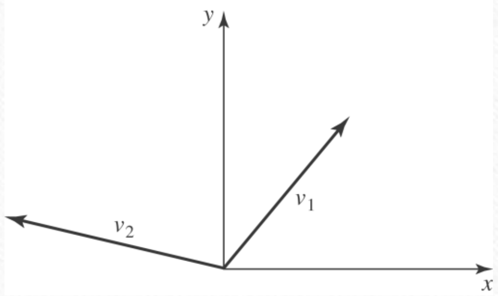
Practice Question 1

A jet plane is flying at a constant altitude. At time $t_1 = 0$ it has components of velocity $v_x = 90$ m/s, $v_y = 110$ m/s. At time $t_2 = 30.0$ s the components are $v_x = -170$ m/s, $v_y = 40$ m/s. a) Sketch the velocity vectors at t_1 and t_2. For this time interval calculate b) the components of the average acceleration; c) the magnitude and direction of the average acceleration



b) $$a_{x,\text{ave}} = \frac{(-170\text{ m/s}) - (90\text{ m/s})}{(30.0\text{ s})} = -8.7\text{ m/s}^2,$$

$$a_{y,\text{ave}} = \frac{(40\text{ m/s}) - (110\text{ m/s})}{(30.0\text{ s})} = -2.3\text{ m/s}^2.$$

c) $\sqrt{(-8.7\text{ m/s}^2)^2 + (-2.3\text{ m/s}^2)^2} = 9.0\text{ m/s}^2$, $\arctan\left(\frac{-2.3}{-8.7}\right) = 14.8° + 180° = 195°.$

Practice Question 2

Two crickets, Chirpy and Milada, jump from the top of a vertical cliff. Chirpy jumps horizontally and reaches the ground in 3.50s. Milada jumps with an initial velocity of 95.0 cm/s at an angle of 32.0° above the horizontal. How far from the base of the cliff will Milada hit the ground?

Take $+y$ to be upward.

Use Chirpy's motion to find the height of the cliff.

$$v_{0y} = 0,\ a_y = -9.80\ \text{m/s}^2,\ y - y_0 = -h,\ t = 3.50\,\text{s}$$

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 \text{ gives } h = 60.0\,\text{m}$$

Milada: Use vertical motion to find time in the air.

$$v_{0y} = v_0 \sin 32.0°,\ y - y_0 = -60.0\,\text{m},\ a_y = -9.80\ \text{m/s}^2,\ t = ?$$

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 \text{ gives } t = 3.55\,\text{s}$$

Then $v_{0x} = v_0 \cos 32.0°,\ a_x = 0,\ t = 3.55\,\text{s}$ gives $x - x_0 = 2.86\,\text{m}$.

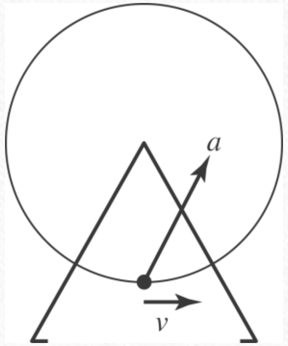
Practice Question 3

A Ferris wheel, which rotates counter-clockwise, is just starting up. At a given instant, a passenger on the rim of the wheel and passing through the lowest point of his circular motion is moving at 3.00 m/s and is gaining speed at a rate of 0.500 m/s^2. Radius of the wheel is 14 m. a) Find the magnitude and the direction of the passenger's acceleration at this instant. b) Sketch the Ferris wheel and passenger showing his velocity and acceleration vectors.

a) $a_{\text{rad}} = (3\,\text{m/s})^2/(14\,\text{m}) = 0.643\,\text{m/s}^2$, and $a_{\tan} = 0.5\,\text{m/s}^2$. So,

$$a = ((0.643\,\text{m/s}^2)^2 + (0.5\,\text{m/s}^2)^2)^{1/2} = 0.814\,\text{m/s}^2, 37.9° \text{ to the right of vertical.}$$

b)



Practice Question 4

A jungle veterinarian with a blow-gun loaded with a tranquilizer dart and a sly 1.5 kg monkey are each 25 m above the ground in trees which are 90 m apart. Just as the hunter shoots horizontally at the monkey, the monkey dops from the tree in a vain attempt to escape being hit. What must the minimum muzzle velocity of the dart have been for the hunter to hit the monkey before it reached the ground?

Take $+y$ to be downward.

Use the vertical motion to find the time in the air:

$$v_{0y} = 0,\ a_y = 9.80\ \text{m/s}^2,\ y - y_0 = 25\ \text{m},\ t = ?$$

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 \text{ gives } t = 2.259\ \text{s}$$

During this time the dart must travel 90 m horizontally, so the horizontal component of its velocity must be

$$v_{0x} = \frac{x - x_0}{t} = \frac{90\ \text{m}}{2.25\ \text{s}} = 40\ \text{m/s}$$

Practice Question 5

A swimmer dives off a cliff with a running horizontal leap, as shown in the figure. What must her minimum speed be just as she leaves the top of the cliff so that she will miss the ledge at the bottom, which is 1.75 m wide and 9.00 m below the top of the cliff?

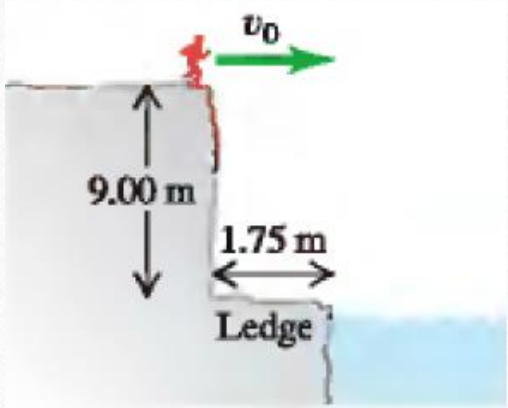
IDENTIFY: The person moves in projectile motion. She must travel 1.75 m horizontally during the time she falls 9.00 m vertically.

SET UP: Take $+y$ downward. $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

EXECUTE: Time to fall 9.00 m: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(9.00\ \text{m})}{9.80\ \text{m/s}^2}} = 1.36\ \text{s}$.

Speed needed to travel 1.75 m horizontally during this time: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$$v_0 = v_{0x} = \frac{x - x_0}{t} = \frac{1.75\ \text{m}}{1.36\ \text{s}} = 1.29\ \text{m/s}.$$



Practice Question 6

A small marble rolls horizontally with speed v_0 off the top of a platform which is 2.75 m tall. There is no air resistance and friction. On the ground, 2.00 m horizontally from the base of the platform, there is a gaping hole in the ground which is 1.50 m wide. For what range of marble speeds v_0 will the marble land in the hole?

IDENTIFY: The marble moves with projectile motion, with initial velocity that is horizontal and has magnitude v_0. Treat the horizontal and vertical motions separately. If v_0 is too small the marble will land to the left of the hole and if v_0 is too large the marble will land to the right of the hole.

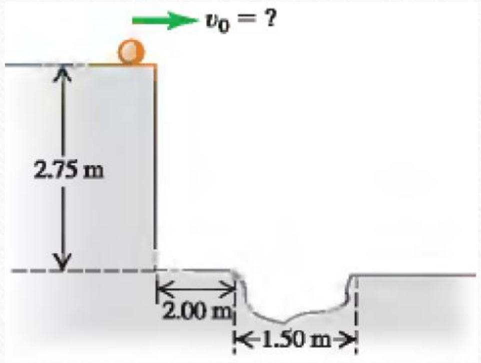
SET UP: Let $+x$ be horizontal to the right and let $+y$ be upward. $v_{0x} = v_0$, $v_{0y} = 0$, $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$

EXECUTE: Use the vertical motion to find the time it takes the marble to reach the height of the level ground; $y - y_0 = -2.75\ \text{m}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(-2.75\ \text{m})}{-9.80\ \text{m/s}^2}} = 0.749\ \text{s}$. The time does not depend on v_0.

Minimum v_0: $x - x_0 = 2.00\ \text{m}$, $t = 0.749\ \text{s}$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_0 = \dfrac{x-x_0}{t} = \dfrac{2.00\ \text{m}}{0.749\ \text{s}} = 2.67\ \text{m/s}$.

Maximum v_0: $x - x_0 = 3.50\ \text{m}$ and $v_0 = \dfrac{3.50\ \text{m}}{0.749\ \text{s}} = 4.67\ \text{m/s}$.

EVALUATE: The horizontal and vertical motions are independent and are treated separately. Their only connection is that the time is the same for both.



Practice Question 7

A young boy is oscillating back and forth at the playground swing. At the lowest point, his speed is 0.92 m/s. The distance between the boy and the swing hinge is 4.3 m. What is the magnitude of his acceleration at the lowest point?

At lowest point, there is only radial acceleration.

$$a = \frac{v^2}{r} = \frac{0.92^2}{4.3} = 0.197 \; or \; 0.20 \text{ m/s}^2$$

Practice Question 8

To triple the centripetal acceleration, what should you do to the period of rotation during a washer spin cycle?

IDENTIFY: Determine how a_{rad} depends on the rotational period T.

SET UP: $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$.

EXECUTE: For any item in the washer, the centripetal acceleration will be inversely proportional to the square of the rotational period; tripling the centripetal acceleration involves decreasing the period by a factor of $\sqrt{3}$, so that the new period T' is given in terms of the previous period T by $T' = T/\sqrt{3}$.

EVALUATE: The rotational period must be decreased in order to increase the rate of rotation and therefore increase the centripetal acceleration.