CSD2301 Lecture
9. Momentum and Collisions Part 1

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Outline

- Linear momentum, impulse and impulsive force
- Conservation of linear momentum
- Collision in 1D
- Elastic, inelastic and perfectly inelastic collisions
- Collision in 2D

Linear Momentum

- Linear momentum provides information about the object and its motion.
- The linear momentum of a particle of mass m moving with velocity **v** is defined to be: $\boxed{\vec{p} = m\vec{v}}$ ⇨ $\boxed{p_x = mv_x, \quad p_y = mv_y, \quad p_z = mv_z}$

  p = momentum
- Very useful in treating problems involving collisions and for analysing rocket propulsion
- Momentum is a vector, and its direction is along **v**
- Dimension ML/T, Unit: kg.m/s

Newton's 2^{nd} Law (again)

Rmb the definition? It states that: Time rate of change of the momentum of a body is equal in both **magnitude** and **direction** to the force imposed on it.

- In terms of linear momentum, Newton's 2^{nd} law can be written as: $$\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$
- If m is constant: $$\boxed{\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}}$$

Impulse and Momentum

- Assume that a single force **F** acts on a particle and that this force varies with time.

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}} \Rightarrow \boxed{d\vec{p} = \vec{F}dt}$$

$$\boxed{\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}dt}$$

Impulse and Momentum

- The **impulse (J)** of a force **F**(t) acting on a particle from time t_i to t_f is:

$$\boxed{\vec{J} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}\,dt}$$

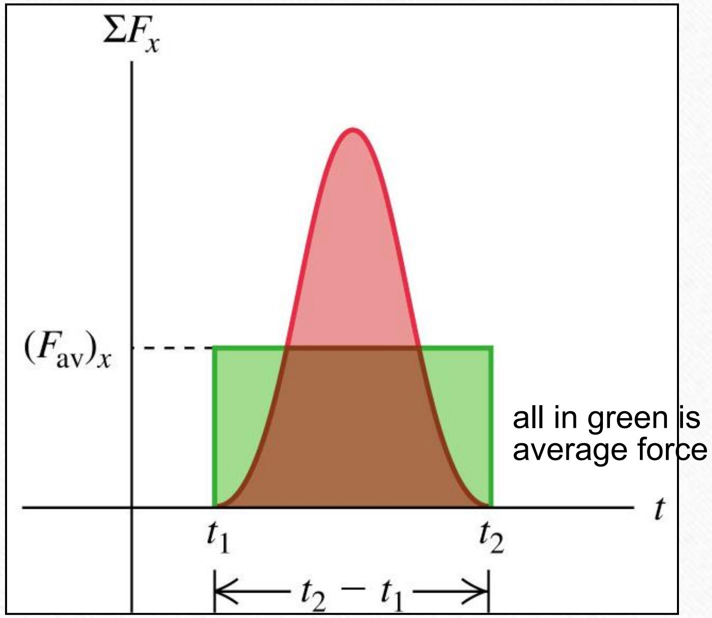
area below f-t curve

- Impulse of a force acting on a particle equals the change in momentum of the particle caused by that force
- Impulse is a vector; and has the same dimensions as momentum (ML/T)

Impulsive Force

$$(\vec{F}_{av}) = \frac{1}{\Delta t}\int_{t_1}^{t_2}\left(\sum \vec{F}\right)dt$$

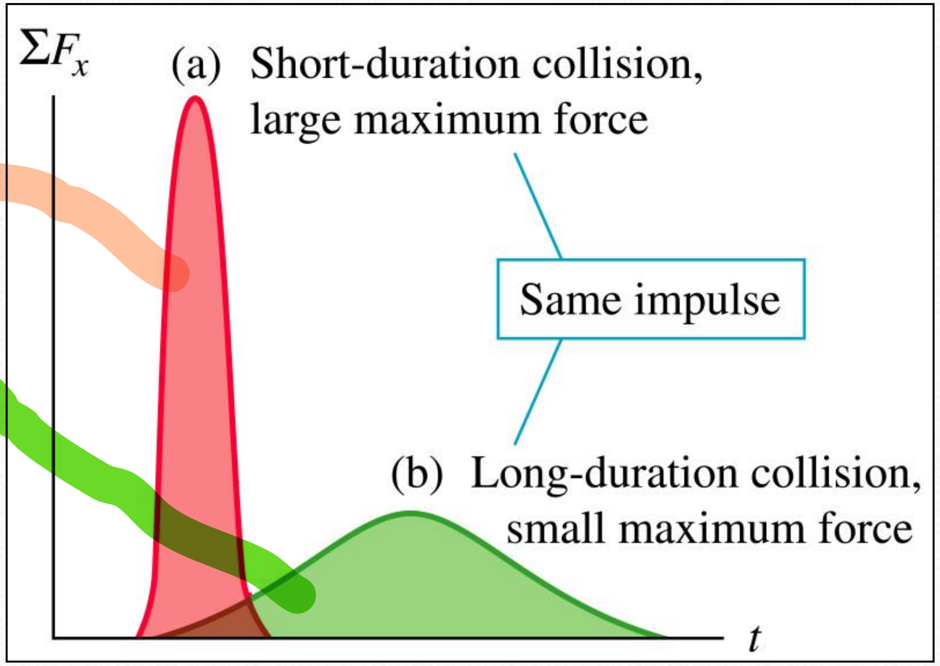
$$\vec{J} = \Delta\vec{p} = (\vec{F}_{av})(t_2 - t_1) = \vec{F}_{av}\Delta t$$



- **Impulse approximation** assumes that the impulsive force acts for a short time but is much larger than any other force present; very little motion takes place during this time. (Usually also neglect effects of external forces during this time.)

Impulse

- $p_f - p_i$ (impulse) is the same if the area under the F-t curves are the same. The impulsive force is bigger if it acts over a shorter time (e.g., a golf ball hit by the golf club) and is smaller if it acts over a longer time (e.g., a tennis ball hit by a racket).



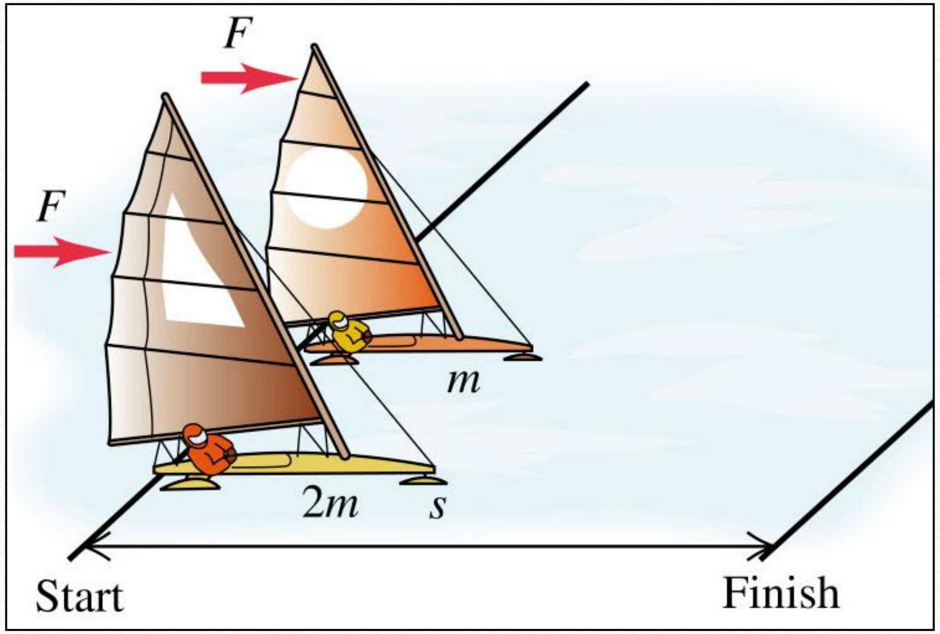
Momentum vs Kinetic Energy

- Consider two iceboats A and B of mass m and $2m$ respectively. Both are stationary at the Start line subjected to the same force F until they cross the Finish line.

a) Which iceboat reaches the Finish line first?

b) Which iceboat has the higher kinetic energy when it crosses the Finish line?

c) Which iceboat has the greater momentum when it crosses the Finish line?



Momentum vs Kinetic Energy

kinetic and momentum energy are different

since b is x2 heavier

a) $a_A = \frac{F}{m} = 2\frac{F}{2m} = 2a_B$ ⇨ $\boxed{t_A < t_B}$

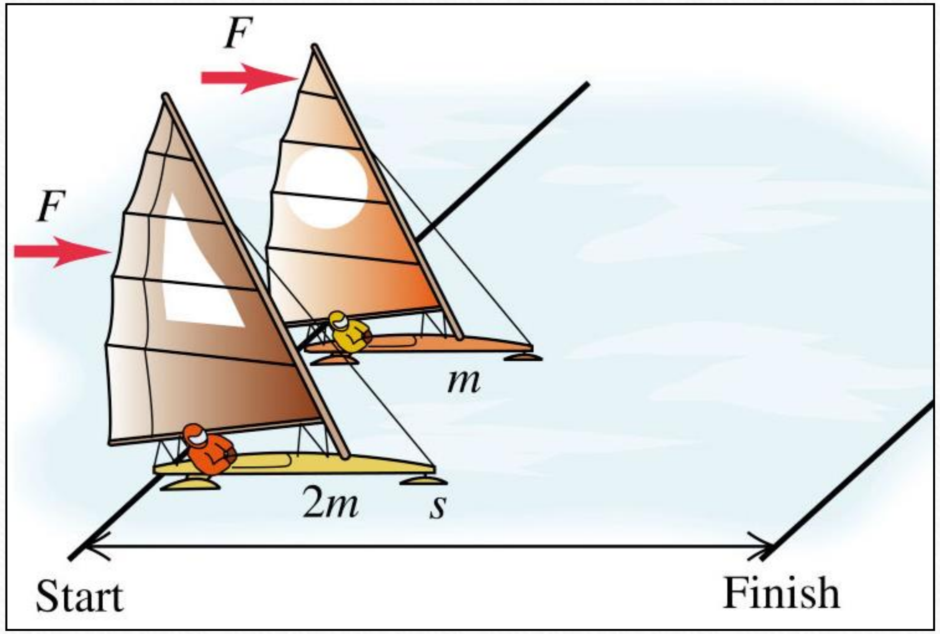
Fds = force x distance

b) $K_A - 0 = \int_0^s F\,ds' = K_B - 0$ ⇨ $\boxed{K_A = K_B}$

same k
different v

c) $p_A = \int_0^{t_A} F\,dt = Ft_A$ $<$ $p_B = \int_0^{t_B} F\,dt = Ft_B$

Since $t_A < t_B$



Momentum vs Kinetic Energy

- From the definition of kinetic energy and linear momentum, we can also write down the direct relationship between them:

kinetic

momentum

$$\boxed{K = \frac{1}{2}mv^2} \quad \& \quad \boxed{p = mv}$$

$$\boxed{K = \frac{(mv)^2}{2m} = \frac{p^2}{2m}} \Rightarrow \boxed{p^2 = 2mK} \Rightarrow \boxed{p = \sqrt{2mK}}$$

Conservation of Linear Momentum

- If there are no external forces acting on an isolated system of particles:

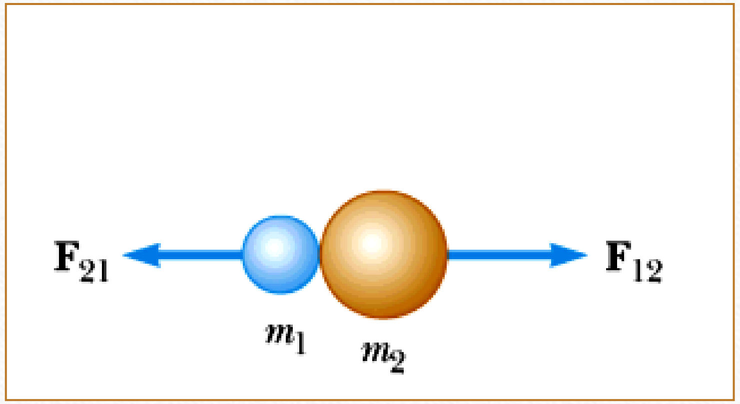
start and the end, always same momentum

$$\sum \vec{F} = 0 \Rightarrow \sum \frac{d\vec{p}}{dt} = 0 \Rightarrow \sum \vec{p} = \text{const.} \Rightarrow \boxed{\sum \vec{p}_i = \sum \vec{p}_f}$$

- The total momentum of an isolated system at all times equals its initial momentum.
- The total momentum of an isolated system remains constant if there are no external forces present.
- A convenient way to calculate some parameters, such as final velocities, in collisions without the need to know the exact forms of the forces involved.

Collisions

- Two particles come together for a short time and thereby producing impulsive forces on each other.



Collisions

- From Newton's 3rd law:

$$\boxed{\Delta\vec{p}_1 = -\Delta\vec{p}_2}$$

$$\boxed{\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0}$$

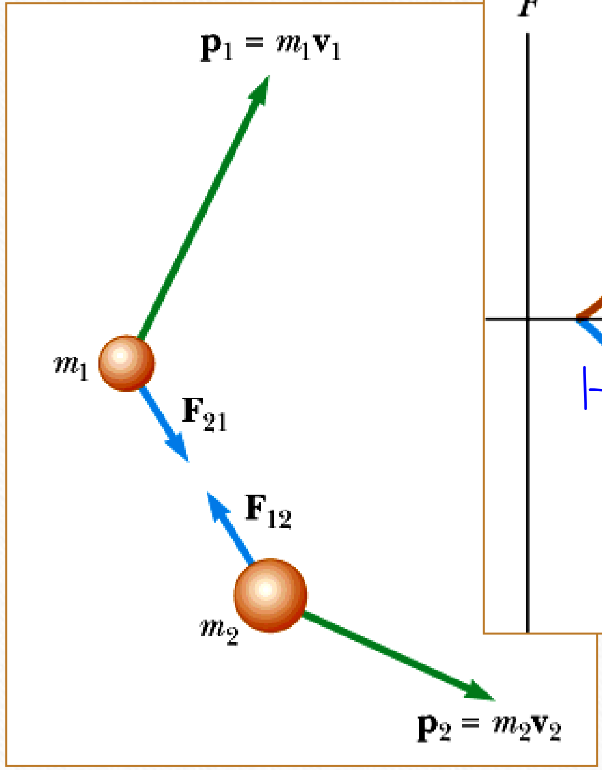
$$\boxed{\vec{p}_{\text{tot}} = \vec{p}_1 + \vec{p}_2 = \text{const.}}$$

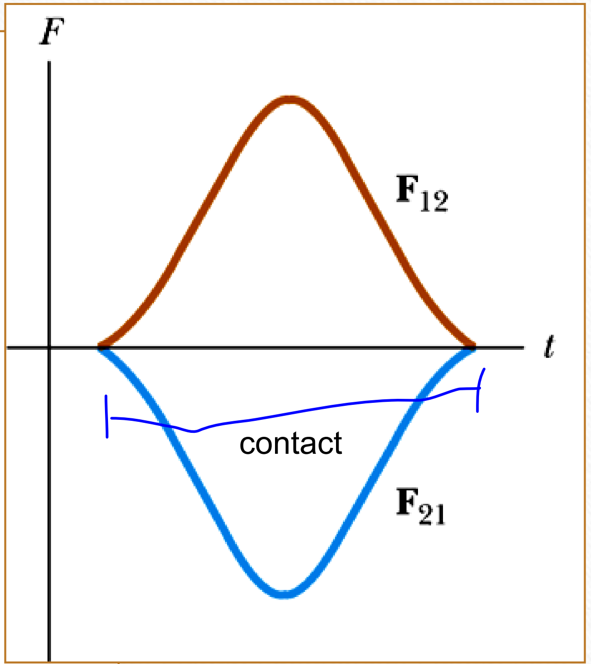
For m_1:

$$\boxed{\Delta\vec{p}_1 = \int_{t_i}^{t_f} \vec{F}_{21}dt}$$

For m_2:

$$\boxed{\Delta\vec{p}_2 = \int_{t_i}^{t_f} \vec{F}_{12}dt}$$

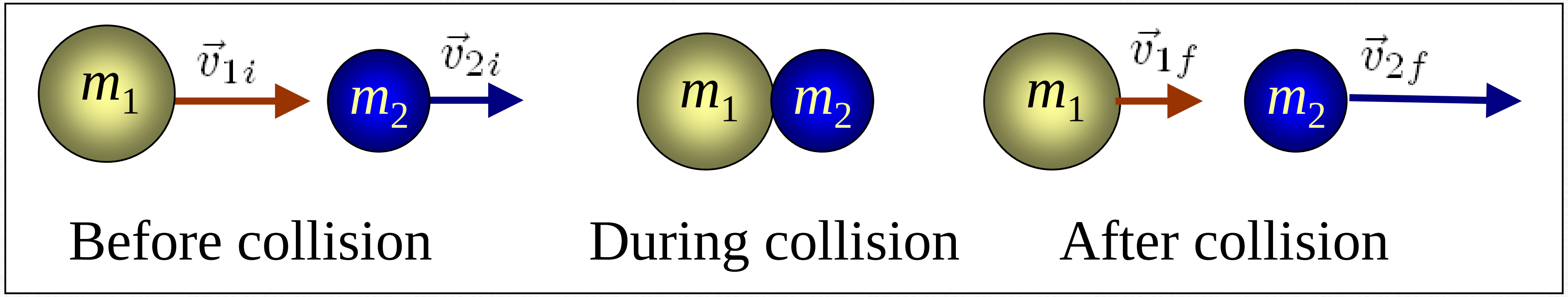




Elastic & Inelastic Collisions

- Momentum is always conserved.
- If the total kinetic energy is also **the same** before and after the collision, the collision is **elastic**.
- If the total kinetic energy is **not the same** before and after the collision, the collision is **inelastic**

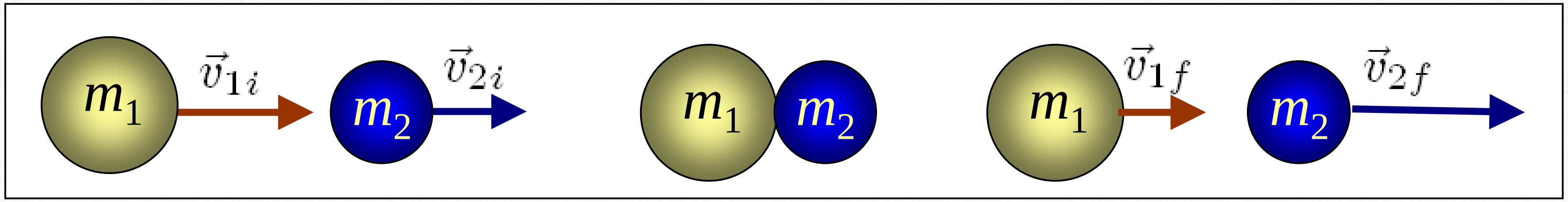
Elastic Collision in 1D



initial final

Momentum conservation: $\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$

Energy conservation: $\boxed{\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2}$



$$\boxed{\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2}$$ KE Constant

$$\boxed{m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)}$$

$$\boxed{m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i})} \quad (1)$$

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$ Conservation of momentum

$$\boxed{m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})} \quad (2)$$

Taking (1) / (2): $\boxed{v_{1i} + v_{1f} = v_{2f} + v_{2i}} \Rightarrow \boxed{v_{1i} - v_{2i} = v_{2f} - v_{1f}}$

Approach speed = Separation speed

To find final velocities in terms of initial velocities

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

$$v_{2f} = v_{1i} + v_{1f} - v_{2i}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 (v_{1i} + v_{1f} - v_{2i})$$

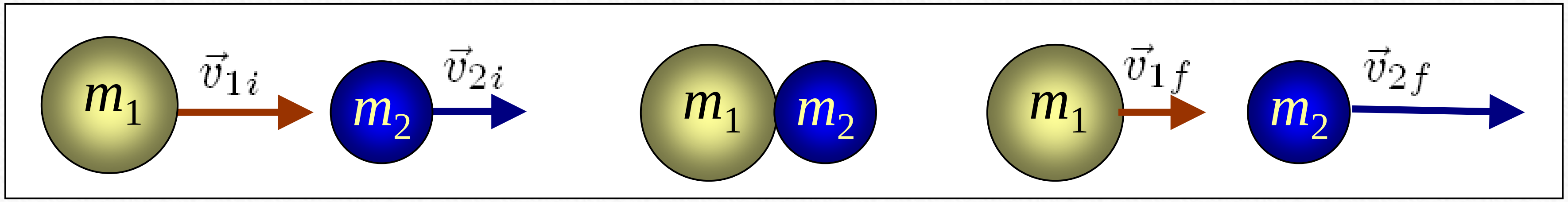
$$(m_1 - m_2) v_{1i} + (m_2 + m_2) v_{2i} = (m_1 + m_2) v_{1f}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

got intial, can find final

Similarly:

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$



$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

Case I : If $m_1 = m_2$, then $\boxed{v_{1f} = v_{2i}}$ & $\boxed{v_{2f} = v_{1i}}$

The particles exchange velocities

10ms 5ms =

collide and switch velocity

Case II : If m_2 is initially at rest ($v_{2i} = 0$), then $\boxed{v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i}}$ & $\boxed{v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}}$

If $\boxed{m_1 \gg m_2}$ ⇒ $\boxed{v_{1f} \approx v_{1i}}$ & $\boxed{v_{2f} \approx 2v_{1i}}$

think tennis ball hit ping pong ball
after hit, ping pong will move faster

If $\boxed{m_2 \gg m_1}$ ⇒ $\boxed{v_{1f} \approx -v_{1i}}$ & $\boxed{v_{2f} \approx 0}$

Transfer of KE during Collision

- Often we want to know how much KE is transferred to a stationary target, ie, with $v_{2i} = 0$ as before,

$$\boxed{v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}} \Rightarrow \boxed{\mathrm{KE}_{2f} = \frac{1}{2} m_2 \left(\frac{2m_1}{m_1 + m_2}\right)^2 v_{1i}^2 = \frac{4m_2 m_1}{(m_1 + m_2)^2}\ \mathrm{KE}_{1i}}$$

$m_2 \gg m_1$

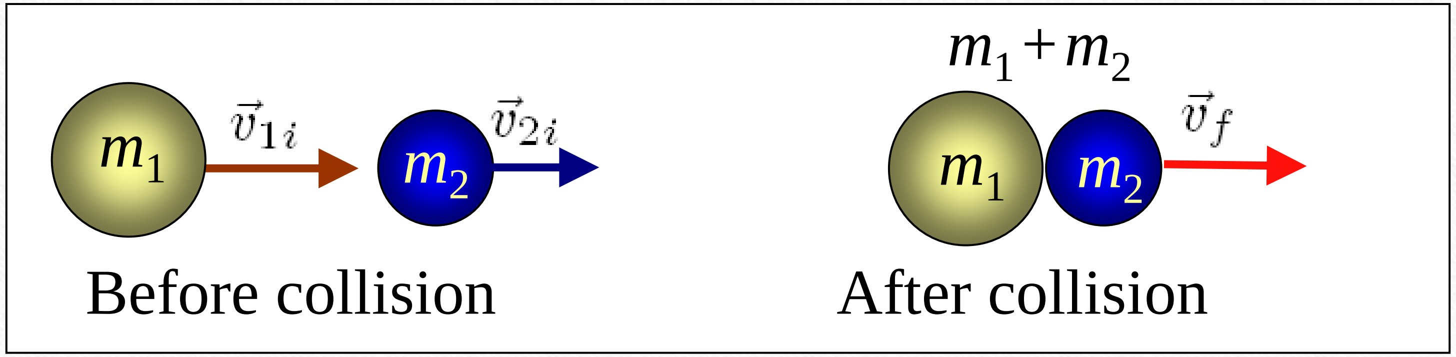
$$\boxed{\mathrm{KE}_{2f} \approx 4\frac{m_1}{m_2}\ \mathrm{KE}_{1i}}$$

$m_1 \gg m_2$

$$\boxed{\mathrm{KE}_{2f} \approx 4\frac{m_2}{m_1}\ \mathrm{KE}_{1i}}$$

Maximum transfer occurs when $m_1 = m_2$.

Perfectly (Totally) Inelastic Collisions



- Two particles m_1 and m_2 collide head on and stick together moving with some common velocity v_f after the collision.
- Momentum conservation: $\boxed{m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f}$

$$\boxed{\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2}}$$

Coefficient of Restitution

- Defined as:

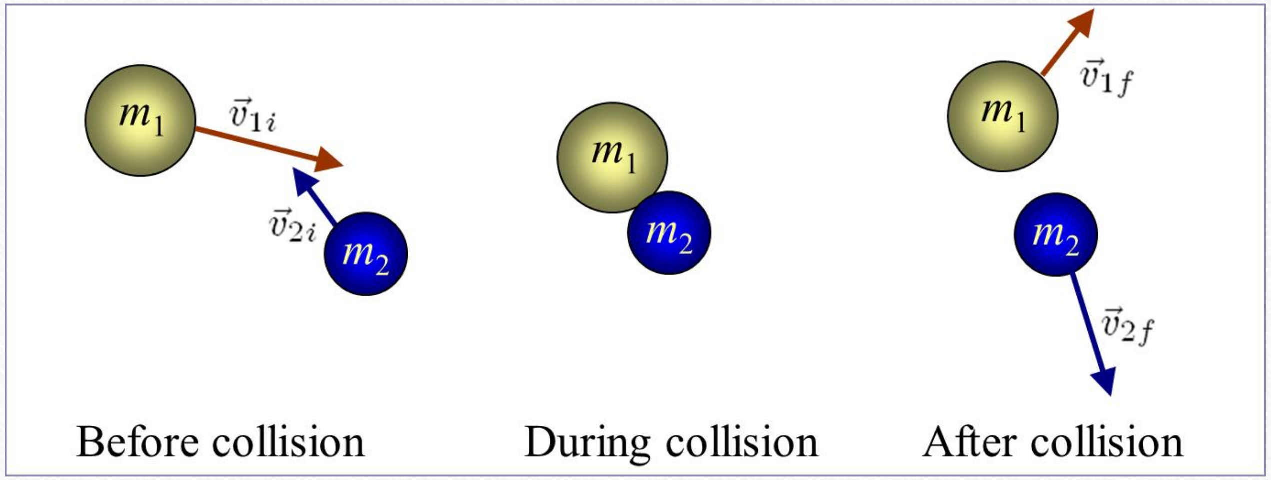
$$e = \frac{\text{Relative speed of separation}}{\text{Relative speed of approach}}$$

$$= \frac{v_{2f} - v_{1f}}{v_{1i} - v_{2i}}$$

e	Type
0	Perfectly inelastic
<1	Inelastic
1	Elastic
>1	? Explosion

can be more than 1 if there is an energy gain during the collision

2D Collisions

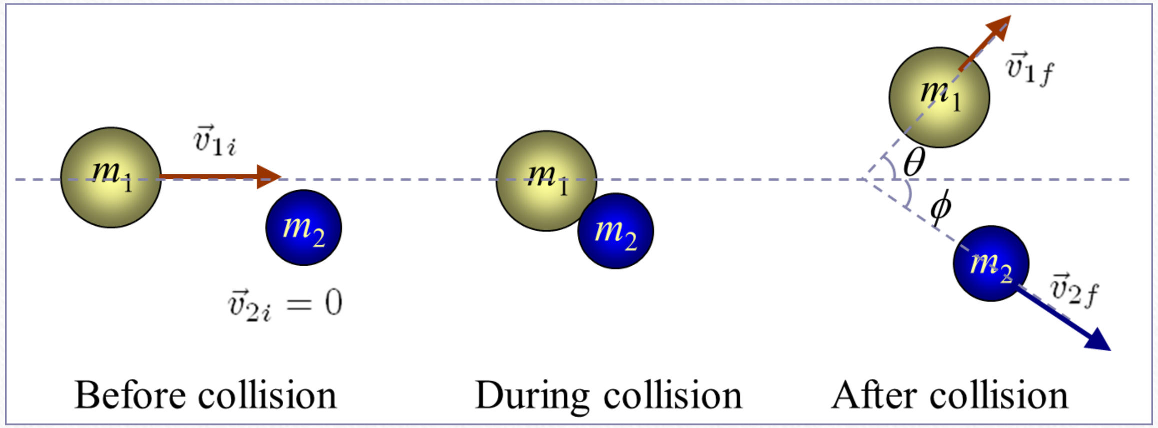


$$\boxed{m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}}$$

$$\boxed{m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}}$$

Momentum in each direction is conserved!

Elastic Glancing Collision



Conservation of momentum:

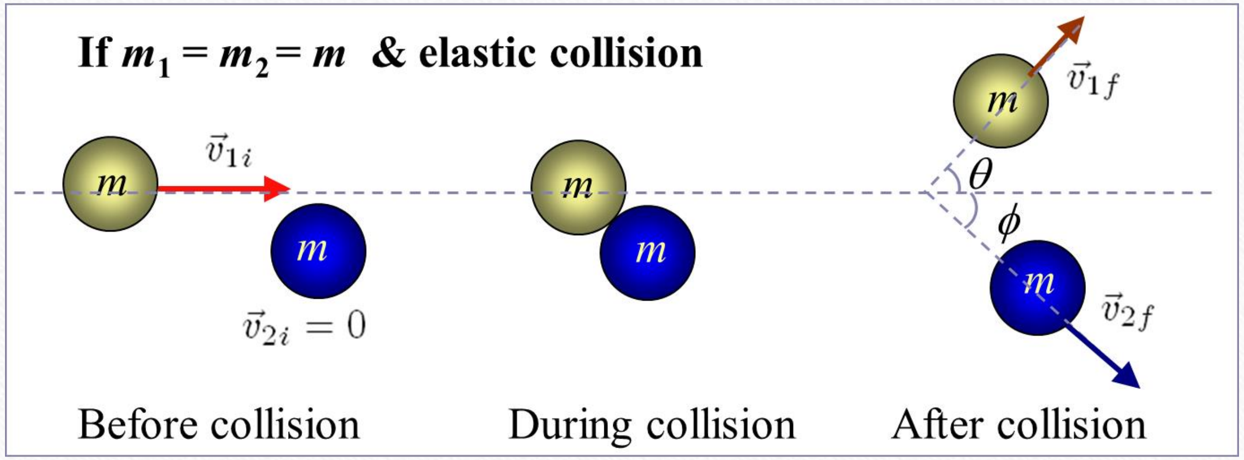
$$\boxed{m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi} \quad \text{x}$$

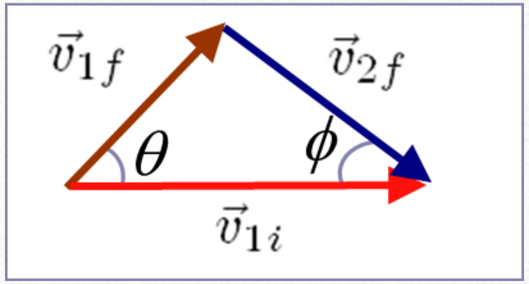
$$\boxed{0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi} \quad \text{y}$$

If elastic collision:

$$\boxed{\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2}$$

Elastic Glancing Collision



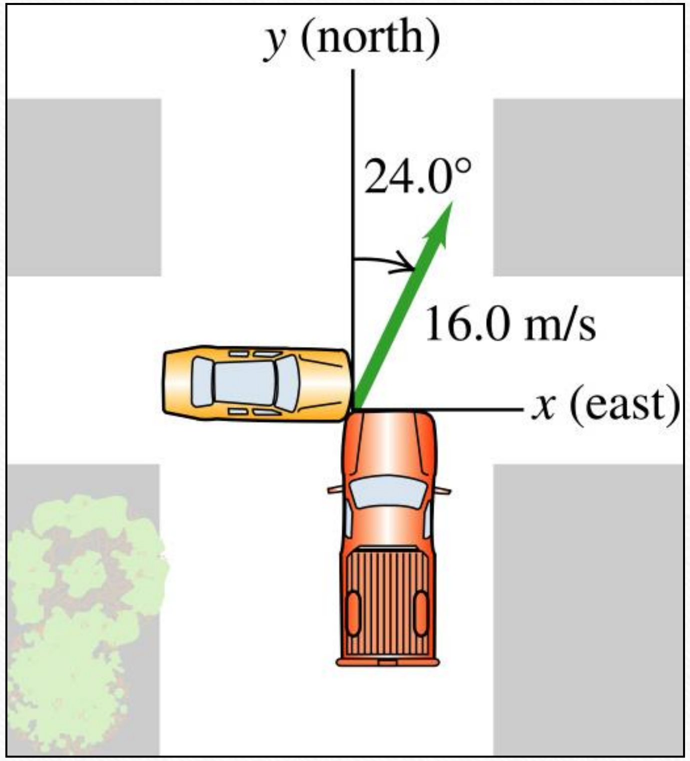


Conservation of momentum: $\boxed{\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}}$

If elastic collision: $\boxed{v_{1i}^2 = v_{1f}^2 + v_{2f}^2}$ $\Rightarrow$ $\boxed{\theta + \phi = 90^\circ}$

Example: Car Accident

- A 950 kg car traveling east collides at an intersection with a 1900 kg pickup traveling north. The two vehicles stick together as a result of the collision, and, immediately after the collision, the wreckage is sliding at 16.0 m/s in the direction 24.0° east of north. Calculate the speed of each vehicle before the collision.



Example: Car Accident

Using conservation of linear momentum:

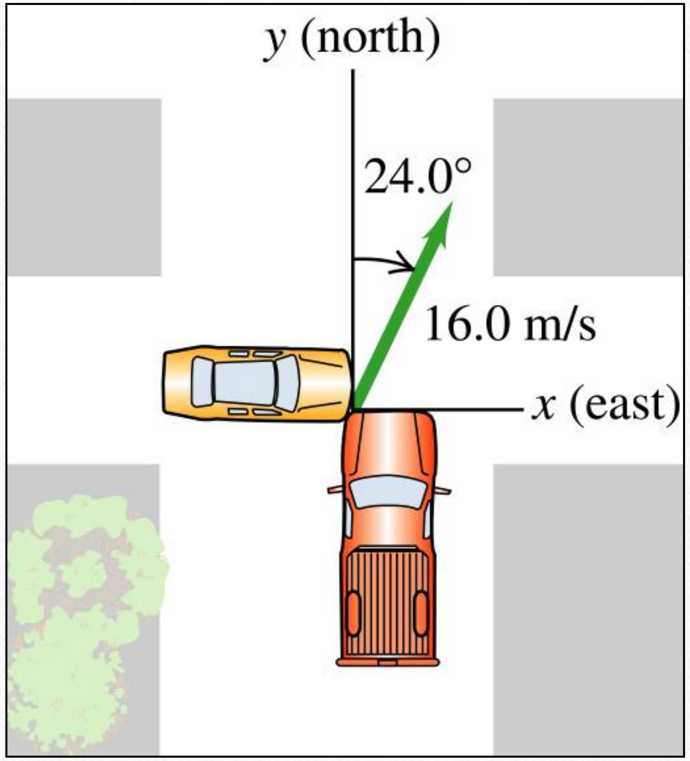
In the x-direction, $\boxed{m_c v_{ci} = (m_c + m_p) v_f \sin\theta}$

$$\boxed{(950) v_{ci} = (950 + 1900)(16) \sin 24^\circ}$$

$$\boxed{v_{ci} = 19.5 \text{ m/s}}$$

In the y-direction, $\boxed{m_p v_{pi} = (m_c + m_p) v_f \cos\theta}$

$$\boxed{v_{pi} = \frac{(950 + 1900)(16) \cos 24^\circ}{1900} = 21.9 \text{ m/s}}$$



The End