Tutorial 1

1. Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

Statement: Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S.

This statement is true because since m prefer w and vice versa, if m were to pair with another, it may incentives m to deviate from its current pair just to pair up with w thus leading to instability. Thus, by having m pair with w, this ensures stability

2. Let $M = m_1, m_2, m_3$ and $W = w_1, w_2, w_3$. Suppose that you are given the following preference lists:

$m_1 :< w_3, w_2, w_1 >; m_2 :< w_2, w_3, w_1 >; m_3 :< w_2, w_3, w_1 >;$

$w_1 :< m_3, m_1, m_2 >; w_2 :< m_1, m_3, m_2 >; w_3 :< m_3, m_1, m_2 >;$

Stable: (m1,w3),(m2,w2),(m3,w1)

1) Give a stable perfect matching, and an unstable perfect matching.

Unstable:(m1,w2),(m2,w3),(m3,w1)

2) Find the best valid partner for each member of sets M and W.

(m1,w3) - top choice
(m2,w2) - top choice
(m3,w1) - only choice left which ensure stability

3. List the following functions according to their order of growth from the lowest to the highest. (Hint: you could start with using basic asymptotic efficiency classes)

$$f_1(n) = n^{2.5}; \quad f_2(n) = \sqrt{2n}; \quad f_3(n) = n + 10$$
$$f_4(n) = 10^n; \quad f_5(n) = 100^n; \quad f_6(n) = n^2 \log n$$

f3,f2,f6,f1,f4,f5

f2,f3,f6,f1,f4,f5