Week 12 Tutorial.

Question 1: x, y.

$x+y=16$. $\Big\}$ $x^2+y^2 \to \boxed{\text{small}}$

example: if $x=1$, $y=15$ $\quad x^2+y^2=1^2+225=\underline{226}$.

if $x=6$ $y=10$. $\quad x^2+y^2=36+100=\underline{136}$.

$x,\ 16-x.$

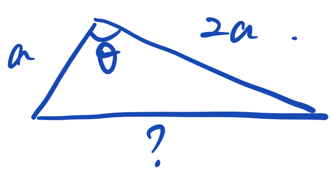
$$f(x)=x^2+(16-x)^2=x^2+256+x^2-32x.$$

$$=2x^2-32x+256.$$

$$f'(x)=4x-32=0 \qquad x=8$$

$$f(8)=8^2+(16-8)^2=64+64=128$$

Question 2:



$$A=\frac{1}{2}\overset{a}{a}\overset{2a}{b}\sin\theta.$$

$$=\frac{1}{2}\cdot a\cdot 2a\cdot\sin\theta.$$

$$=a^2\sin\theta.$$

$$A'(\theta)=a^2\cos\theta=0 \Rightarrow \cos\theta=0$$

$$\theta=90^\circ.$$

$$A=a^2\sin(90^\circ)=a^2\cdot 1=a^2.$$

Thus the largest possible area of the triangle is a^2.

Question 3

$$P = \frac{100I}{I^2+I+4}$$

$$P'(I) = \frac{(100I)'(I^2+I+4)-(100I)(I^2+I+4)'}{(I^2+I+4)^2}$$

$$= \frac{100(I^2+I+4)-(100I)(2I+1)}{(I^2+I+4)^2} = 0$$

$$100I^2 + \cancel{100I} + 400 - 200I^2 - \cancel{100I} = 0.$$

$$400 - 100I^2 = 0$$

$$100I^2 = 400 \Rightarrow I^2 = 4$$

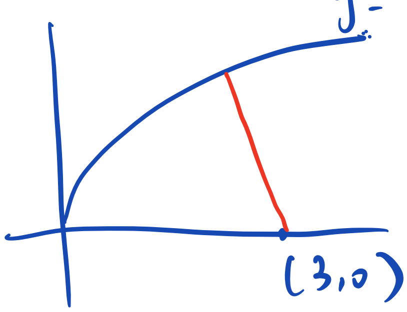
$I = \pm 2$ .. I can only be positive.

$\boxed{I = 2.\ P \text{ is maximized.}}$

Question 4

distance of two points.

$y = \sqrt{x}$



(x_1, y_1) & (x_2, y_2)

$$d = \underbrace{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}$$

$$d^2 = (x_1-x_2)^2+(y_1-y_2)^2.$$

$(3,0)$ & $(x, \sqrt{x})$.

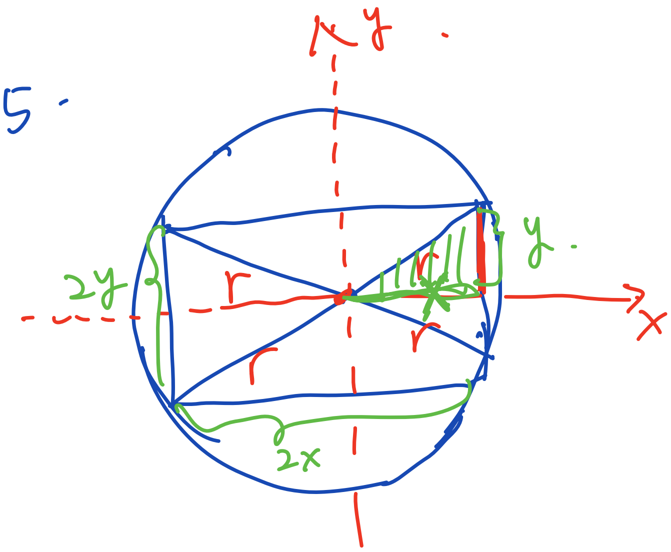
$$f(x) = d^2 = (3-x)^2 + (0-\sqrt{x})^2.$$

$$= 9 + x^2 - 6x + x = x^2 - 5x + 9$$

$$f'(x) = 2x - 5 = 0 \qquad x = \frac{5}{2}.$$

$$\sqrt{\frac{5}{2}} = \frac{\sqrt{10}}{2}, \quad \text{point } \left(\frac{5}{2}, \frac{\sqrt{10}}{2}\right).$$

Question 5.



$$x^2 + y^2 = r^2$$

$$A = 4xy$$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2}$$

$$A = 4 \cdot x \cdot \sqrt{r^2 - x^2}.$$

$$A'(x) = 4\sqrt{r^2-x^2} + \overset{2}{\cancel{4}}x \cdot \frac{1}{\cancel{2}}(r^2-x^2)^{-\frac{1}{2}} \cdot (r^2-x^2)'$$

$$= 4\sqrt{r^2-x^2} + \frac{2x}{\sqrt{r^2-x^2}} \cdot (-2x)$$

$$= 4\sqrt{r^2-x^2} - \frac{4x^2}{\sqrt{r^2-x^2}}$$

$$= \frac{4(r^2-x^2)-4x^2}{\sqrt{r^2-x^2}} = \frac{4r^2-8x^2}{\sqrt{r^2-x^2}} = 0$$

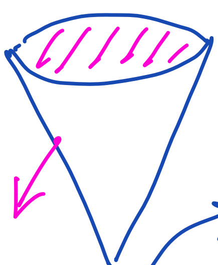
$$r^2 = 2x^2 \Rightarrow r = \sqrt{2}\,x \Rightarrow x = \frac{r}{\sqrt{2}}.$$

$$y = \sqrt{r^2-x^2} = \sqrt{r^2-\frac{r^2}{2}} = \sqrt{\frac{r^2}{2}} = \frac{r}{\sqrt{2}}.$$

$x = y = \frac{r}{\sqrt{2}}$. is the dimension of the rectangle of the largest area that can be inscribed in a circle of radius r.

Question 6 :

$\boxed{27\,cm^3} =$ Volume.



$$A = \pi r\sqrt{r^2+h^2}.$$

Volume of a Cone.

$$V = \frac{1}{3}bh = \frac{1}{3}\pi r^2 h = 27$$

$$h = \frac{27\times 3}{\pi r^2} = \frac{81}{\pi r^2}.$$

$$A = \pi \cdot r \sqrt{r^2 + \frac{81^2}{\pi^2 r^4}} = \pi \, r \sqrt{\frac{r^6\pi^2 + 81^2}{\pi^2 r^4}}$$

$$= \sqrt{\cancel{\pi^2}\cancel{r^2} \cdot \frac{r^6\pi^2 + 81^2}{\cancel{\pi^2}\cancel{r^4} \cdot r^2}} = \frac{\sqrt{r^6\pi^2 + 81^2}}{r}$$

$$A'(r) = \frac{\left[\left(r^6\pi^2 + 81^2\right)^{\frac{1}{2}}\right]' r - \sqrt{r^6\pi^2 + 81^2}}{r^2}$$

$$= \frac{\frac{1}{2}\left(r^6\pi^2 + 81^2\right)^{-\frac{1}{2}} r \cdot \pi^2 6 r^5 - \sqrt{r^6\pi^2 + 81^2}}{r^2} = 0$$

$$= \frac{\cancel{\frac{1}{2}}\frac{\overset{3}{\cancel{6}}\, r^6\pi^2}{\sqrt{r^6\pi^2 + 81^2}} - \sqrt{r^6\pi^2 + 81^2}}{r^2} = 0$$

$$= \frac{\frac{3r^6\pi^2 - r^6\pi^2 - 81^2}{\sqrt{r^6\pi^2 + 81^2}}}{r^2} = 0$$

$$= \frac{3r^6\pi^2 - r^6\pi^2 - 81^2}{r^2\sqrt{r^6\pi^2 + 81^2}} = 0$$

$$2r^6\pi^2 = 81^2$$

$$r^6 = \frac{81^2}{2\pi^2}$$

$$r = \sqrt[6]{\frac{81^2}{2\pi^2}} = \frac{\sqrt[6]{3^{4\cdot 2}}}{\sqrt[6]{2\pi^2}} = \frac{3^{4/3}}{\sqrt[6]{2\pi^2}}$$

$$h = \frac{81}{\pi r^2} = \frac{81}{\pi \cdot \dfrac{3^{8/3}}{\sqrt[3]{2\pi^2}}} = \frac{3^4}{\pi \cdot \dfrac{3^{8/3}}{\sqrt[3]{2} \cdot \pi^{2/3}}}$$

$$= \frac{3^{4/3}}{\dfrac{\pi^{1/3}}{\sqrt[3]{2}}} = \boxed{\sqrt[3]{\frac{162}{\pi}}}$$