

CSD1100

Conversions Between DEC, BIN, OCT, and HEX

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BIN to DEC

BIN to DEC

- The position of a digit in a binary number determines its value.
- Starting from right to the left, the decimal value of the first digit is $2^0=1$ times the digit value.
- The value of the second digit is $2^1=2$ times the digit value.
- The value of the third digit is $2^2=4$ times the digit value.
- The value of the nth digit is 2^{n-1} times the digit value.
- In order to convert a binary number to decimal, the values of all the operations must be added.

BIN to DEC

Example1: 111_2 to Decimal

$$\begin{aligned} 111_2 &= (2^2 * 1) + (2^1 * 1) + (2^0 * 1) \\ &= 4 + 2 + 1 \\ &= 7_{10} \end{aligned}$$

BIN to DEC

Example2: 1001_2 to Decimal

$$\begin{aligned} 1001_2 &= (2^3 * 1) + (2^2 * 0) + (2^1 * 0) + (2^0 * 1) \\ &= 8 + 0 + 0 + 1 \\ &= 9_{10} \end{aligned}$$

BIN to DEC

Example3: 1010_2 to Decimal

$$\begin{aligned} 1010_2 &= (2^3 * 1) + (2^2 * 0) + (2^1 * 1) + (2^0 * 0) \\ &= 8 + 0 + 2 + 0 \\ &= 10_{10} \end{aligned}$$

BIN to DEC

- Another way to do a binary to decimal conversion is by using this template:

<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>					
x 128	x 64	x 32	x 16	x 8	x 4	x 2	x 1					
<input type="text"/>	+	<input type="text"/>	+	<input type="text"/>	+	<input type="text"/>	+	<input type="text"/>	+	<input type="text"/>	=	<input type="text"/>

BIN to DEC

- The upper part of the template is for writing binary numbers as for the lower part, is the result of every binary digit multiplied by its corresponding value. The results of all the multiplications are added together to get the final result in decimal.
- Here is an example of converting 10101101_2 to decimal:

<div>1</div>	<div>0</div>	<div>1</div>	<div>0</div>	<div>1</div>	<div>1</div>	<div>0</div>	<div>1</div>									
x 128	x 64	x 32	x 16	x 8	x 4	x 2	x 1									
<div>128</div>	+	<div>0</div>	+	<div>32</div>	+	<div>0</div>	+	<div>8</div>	+	<div>4</div>	+	<div>0</div>	+	<div>1</div>	=	<div>173</div>

DEC to BIN

DEC to BIN

- Decimal numbers have 10 digits. Binary numbers have 2 digits.
- The number must be converted from 10 digits to 2 digits representation.
- How?
 - Divide the decimal number by two.
 - Save the remainder.
 - If the result of the division is divisible by two, repeat the process.
- The binary number is formed by writing the remainders off all the divisions starting from the last.

Recal: Quotient And Remainder

- Given an integer A and a non-zero integer D .
- It can be shown that there exist unique integers q and r , such that $A = qD + r$ and $0 \leq r < |D|$.
- The number q is called the **quotient**, while r is called the **remainder**.
- Ex:

$$43 = 8 \times 5 + 3$$

DEC to BIN

Example1: 7_{10} to Binary

$7/2 = 3$ Remainder = 1

$3/2 = 1$ Remainder = 1

$1/2 = 0$ Remainder = 1 (last remainder)

Decimal $7_{10} = 111_2$ Binary

DEC to BIN

Example2: 9_{10} to Binary

$$9/2 = 4 \text{ Remainder} = 1$$

$$4/2 = 2 \text{ Remainder} = 0$$

$$2/2 = 1 \text{ Remainder} = 0$$

$$1/2 = 0 \text{ Remainder} = 1 \text{ (last remainder)}$$

$$\text{Decimal } 9_{10} = 1001_2 \text{ Binary}$$

DEC to BIN

Example3: 10_{10} to Binary

$$10/2 = 5 \quad \text{Remainder} = 0$$

$$5/2 = 2 \quad \text{Remainder} = 1$$

$$2/2 = 1 \quad \text{Remainder} = 0$$

$$1/2 = 0 \quad \text{Remainder} = 1 \quad (\text{last remainder})$$

Decimal $10_{10} = 1010_2$ binary

DEC to BIN

- Also a template is available to convert from decimal to binary:

<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
÷ 128	÷ 64	÷ 32	÷ 16	÷ 8	÷ 4	÷ 2	÷ 1
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

DEC to BIN

In this template the upper part is the decimal part and the lower one is binary. Here how it works:

- Start by the upper left square and write in it the decimal number that needs to be converted to binary.
- Divide the number by 128.
- Write the quotient in the lower left square. As for the remainder, write it in the next upper square.
- Repeat the same steps for the rest of the columns.
- The result is the lower part

DEC to BIN

- Here is an example of converting 173_{10} to binary using the template:

173	45	45	13	13	5	1	1
÷ 128	÷ 64	÷ 32	÷ 16	÷ 8	÷ 4	÷ 2	÷ 1
1	0	1	0	1	1	0	1

HEX to BIN

HEX to BIN

- Each digit of an hexadecimal number could be converted to a 4 bit binary number ($16=2^4$).

Binary	Octal	Hexadecimal	Decimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	10	8	8
1001	11	9	9
1010	12	A	10
1011	13	B	11
1100	14	C	12
1101	15	D	13
1110	16	E	14
1111	17	F	15

HEX to BIN

Binary	Octal	Hexadecimal	Decimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	10	8	8
1001	11	9	9
1010	12	A	10
1011	13	B	11
1100	14	C	12
1101	15	D	13
1110	16	E	14
1111	17	F	15

- Example 1: FE5₁₆ to Binary

Hex to Binary: FE5 ₁₆		
<i>F</i>	<i>E</i>	<i>5</i>
1 1 1 1	1 1 1 0	0 1 0 1

BIN to HEX

BIN to HEX

- In this case the conversion follows a reverse process than the one described in the previous slide.
- The binary number is converted 4 bit at a time starting from the LSB.

BIN to HEX

Binary	Octal	Hexadecimal	Decimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	10	8	8
1001	11	9	9
1010	12	A	10
1011	13	B	11
1100	14	C	12
1101	15	D	13
1110	16	E	14
1111	17	F	15

- Example 1: $111\ 1101\ 1001\ 1111_2$ to Hexadecimal

Binary to Hex:			
$0111\ 1101\ 1001\ 1111_2$			
0111	1101	1001	1111
7	D	9	F

OCT to BIN

OCT to BIN

- Each digit of an octal number could be converted to a 3 bit binary number ($8=2^3$).

Binary	Octal	Hexadecimal	Decimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	10	8	8
1001	11	9	9
1010	12	A	10
1011	13	B	11
1100	14	C	12
1101	15	D	13
1110	16	E	14
1111	17	F	15

OCT to BIN

Binary	Octal	Hexadecimal	Decimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	10	8	8
1001	11	9	9
1010	12	A	10
1011	13	B	11
1100	14	C	12
1101	15	D	13
1110	16	E	14
1111	17	F	15

- Example 1: 413_8 to Binary
 - $100\ 001\ 011_2$

BIN to OCT

BIN to OCT

- In this case the conversion follows a reverse process than the one described in the previous slide.
- The binary number is converted 3 bit at a time starting from the LSB.

BIN to OCT

Binary	Octal	Hexadecimal	Decimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	10	8	8
1001	11	9	9
1010	12	A	10
1011	13	B	11
1100	14	C	12
1101	15	D	13
1110	16	E	14
1111	17	F	15

- Example 1: 111110110011111_2 to Octal
 - $111\ 110\ 110\ 011\ 111_2 = 76637_8$

HEX to DEC

HEX to DEC

- The position of a digit in a hexadecimal number determines its value.
- Starting from right to the left, the decimal value of the first digit is $16^0=1$ times the digit value.
- The value of the second digit is $16^1=16$ times the digit value.
- The value of the third digit is $16^2=256$ times the digit value.
- The value of the nth digit is 16^{n-1} times the digit value.
- In order to convert a hexadecimal number to decimal, the values of all the operations must be added.

HEX to DEC

Example1: 7_{16} to Decimal

$$7_{16} = (16^0 * 7)$$

$$= 7$$

$$= 7_{10}$$

HEX to DEC

Example2: 10_{16} to Decimal

$$10_{16} = (16^1 * 1) + (16^0 * 0)$$

$$= 16 + 0$$

$$= 16_{10}$$

HEX to DEC

Example3: $A3C_{16}$ to Decimal

$$A3C_{16} = (16^2 * 10) + (16^1 * 3) + (16^0 * 12)$$

$$= 2560 + 48 + 12$$

$$= 2620_{10}$$

DEC to HEX

DEC to HEX

- Hexadecimal numbers have 16 digits. Decimal numbers have 10 digits.
- The number must be converted from 10 digits to 16 digits representation.
- How?
 - Divide the decimal number by 16.
 - Save the remainder.
 - If the result of the division is divisible by 16, repeat the process.
- The hexadecimal number is formed by writing the remainders off all the divisions starting from the last.

DEC to HEX

Example 1: 7_{10} to Hexadecimal.

$7/16 = 0$ Remainder = 7 (last remainder)

Decimal $7_{10} = 7_{16}$ Hexadecimal

DEC to HEX

Example2: 16_{10} to Hexadecimal

$16/16 = 1$ Remainder = 0

$1/16 = 0$ Remainder = 1 (last remainder)

Decimal $8_{10} = 10_{16}$ Hexadecimal

DEC to HEX

Example3: 2620_{10} to Hexadecimal

$2620/16 = 163$ Remainder = C

$163/16 = 10$ Remainder = 3

$10/16 = 0$ Remainder = A (last remainder)

Decimal $156_{10} = A3C_{16}$ Hexadecimal

Simplified HEX/DEC conversions

Simplified HEX/DEC conversions

- Conversions can be done by using intermediate binary representation.
- DEC to HEX:
 1. DEC to BIN (easy)
 2. BIN to HEX (super easy)
- HEX to DEC
 1. HEX to BIN (super easy)
 2. BIN to DEC (easy)

OCT to DEC

OCT to DEC

- The position of a digit in a octal number determines its value.
- Starting from right to the left, the decimal value of the first digit is $8^0=1$ times the digit value.
- The value of the second digit is $8^1=8$ times the digit value.
- The value of the third digit is $8^2=64$ times the digit value.
- The value of the nth digit is 8^{n-1} times the digit value.
- In order to convert a octal number to decimal, the values of all the operations must be added.

DEC to OCT

DEC to OCT

- Octal numbers have 8 digits. Decimal numbers have 10 digits.
- The number must be converted from 10 digits to 8 digits representation.
- How?
 - Divide the decimal number by 8.
 - Save the remainder.
 - If the result of the division is divisible by 8, repeat the process.
- The octal number is formed by writing the remainders off all the divisions starting from the last.

Simplified OCT/DEC conversions

Simplified OCT/DEC conversions

- Conversions can be done by using intermediate binary representation.
- DEC to OCT:
 1. DEC to BIN (easy)
 2. BIN to OCT (super easy)
- OCT to DEC
 1. OCT to BIN (super easy)
 2. BIN to DEC (easy)

OCT to HEX, HEX to OCT

OCT to HEX, HEX to OCT

- Conversions can be done by using intermediate binary representation.
- OCT to HEX:
 1. OCT to BIN (super easy)
 2. BIN to HEX (super easy)
- HEX to OCT
 1. HEX to BIN (super easy)
 2. BIN to OCT (super easy)