# CSD2259 Tutorial 10

*Problem* 1. Joe is a randomly chosen member of a large population in which 3% are heroin users. Joe went for a test that correctly identifies users 95% of the time and correctly identifies nonusers 90% of the time.
(a) Given that Joe is not a heroin user, find the probability that he tests positive.
(b) Given that Joe tested positive, find the probability that he is a heroin user.
(c) Given that Joe tested negative, find the probability that he is not a heroin user.

*Problem* 2. Suppose that one person in 10,000 people has a rare genetic disease. There is an excellent test for the disease: 99.9% of people with the disease test positive and only 0.02% who do not have the disease test positive.
a) What is the probability that someone who tests positive has the genetic disease?
b) What is the probability that someone who tests negative does not have the disease?

*Problem* 3. A fair dice is rolled repeatedly. Let $X$ be the number of rolls until a six shows up.
(a) Let $x$ be any positive integer. What is $P(X = x)$?
(b) What is the probability that at least 5, but not more than 10 rolls are required for the six to show up?

*Problem* 4. A hand of 5 cards is drawn randomly from a standard deck of 52 cards. Let $X$ be the number of hearts contained in the hand of 5 cards.
(a) What is the set of possible values of $X$?
(b) Find $P(X = x)$ and $P(X \leq x)$ for all possible values $x$ of $X$ found in (a).

*Problem* 5. A multiple-choice test consists of 10 questions, each with four choices. A student is able to eliminate one of the choices on each question as incorrect and chooses randomly from the remaining three choices. The requirement for passing is $\geq 6$ questions answered correctly.
(a) Find the probability that the student passes.
(b) Answer (a) again, assuming that the student can eliminate 2 choices for each question.

*Problem* 6. In this problem, we study about information transmission. Usually to transmit information through a noisy channel, the information is sent multiple times. The answer will be decoded as the majority of the information that is received.

One bit (0 or 1) is transmitted 9 times through a noisy channel. The probability of a failure in a single transmission is 0.05. The bit is decoded 1 if at least 5 ones are received, and 0 otherwise. Find the probability of incorrect decoding.

## Hints and Instructions

1. A = event that Joe is a heroin user. B = event that Joe tests positive for heroin. You are given $P(A) = 0.03, P(B|A) = 0.95, P(\bar{B}|\bar{A}) = 0.9$. You need to find $P(B|\bar{A})$ in a, $P(A|B)$ in b, and $P(\bar{A}|\bar{B})$ in c.

2. A = event that a randomly chosen person has the disease. B = event that a randomly chosen person tests positive. You are given $P(A) = 10^{-4}, P(B|A) = 0.999, P(B|\bar{A}) = 0.0002$.

3. Try it.

4. The set of possible values for $X$ is $\{0, 1, 2, 3, 4, 5\}$. You can give a table for both $P(X = x)$ and $P(X \leq x)$. Here is the table. You need to work out how to get these numbers.

| $x$ | $P(X = x)$ | $P(X \leq x)$ |
|---|---|---|
| 0 | 0.2215 | 0.2215 |
| 1 | 0.4114 | 0.6329 |
| 2 | 0.2743 | 0.9072 |
| 3 | 0.0815 | 0.9887 |
| 4 | 0.0107 | 0.9994 |
| 5 | 0.0005 | 0.9999 |

5. Define $X$ = number of questions answer correctly. You need to find $P(X \geq 6)$.

6. X= # failures in 9 transmissions. You need to find $P(X \geq 5)$.