CSD1241 Tutorial 2

Problem 1. Consider two points

$$P = (9, -1),\ Q = (5, -3)$$

(a) Find the general equation, the vector equation and the parametric equation of the line l passing through P and Q.
(b) Find the condition for a, b, c so that the line $l' : ax + by + c = 0$ is parallel to l.
(c) Find the condition on d, e, f so that $l'' : dx + ey + f = 0$ is perpendicular to l.
Hint. l has direction $\overrightarrow{PQ}$, l' has direction $\vec{v} = \begin{bmatrix} -b \\ a \end{bmatrix}$ and l'' has direction $\vec{w} = \begin{bmatrix} -e \\ d \end{bmatrix}$.

Problem 2. Consider the point $P = (3, 2)$. In each of the following cases, find the distance from P to the given line l. Further, find the point Q on l which is at the shortest distance to P (Q is the orthogonal projection of P onto l). (a) l has general equation

$$x - y - 3 = 0.$$

(b) l has vector equation

$$(x, y) = (1, -1) + t \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

(c) l has parametric equation

$$\begin{cases} x = 3t \\ y = 1 - 2t \end{cases}$$

(d) l passes through $A(0, 5)$ and $B(10, 1)$.

Problem 3. Find the normal equation (form $ax + by + cz = d$) of the plane β in the following cases

(a) β goes through $P = (1, -1, 2)$ and has normal vector $\vec{n} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$.

(b) β goes through $S = (1, 2, 3)$ and parallel to the plane $\alpha : 3x - 2y + z = 7$.
(c) β goes through $S = (1, 2, 3)$ and perpendicular to the line

$$l : (x, y, z) = (1, -1, 2) + t \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Problem 4. Let α be the plane going through 3 points $P = (1, -1, 2), Q = (3, 1, 0), R = (2, 1, 1)$.
(a) Find the vector equation and the parametric equation of α.
(b) Find the general equation (form $ax + by + cz + d = 0$) of α.

Hint. Let $\vec{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ be a normal vector of α. Then $\vec{n} \cdot \overrightarrow{PQ} = 0$ and $\vec{n} \cdot \overrightarrow{PR} = 0$. You can find $\vec{n}$ from these 2 equations.

(c) Find the distance from the point $A = (1, 1, 1)$ to α.
(d) Find the point B on α which is at the closet distance to A (Hint. $B =$ orthogonal projection of A onto α).