CSD2301 Lecture

11. Rotation and Moment of Inertia Part 1

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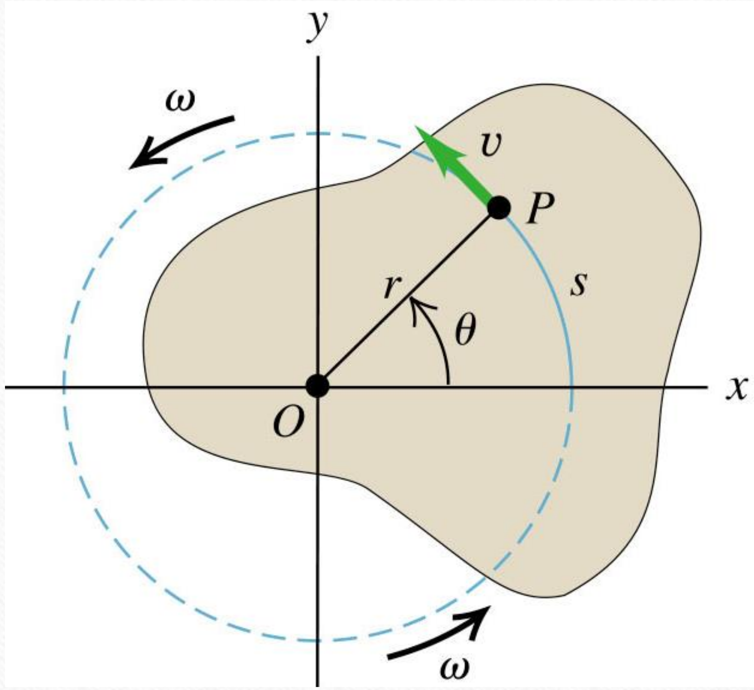
Outline

- Angular motion
- Angular velocity and acceleration
- Rigid body rotation
- Rotational kinematics

Angular Motion

- **Angular motion** refers to the rotational motion of a body.
  - **Angular displacement (θ)** is the angle through which a point revolves around a centre.
  - **Angular velocity (ω)** is the rate of change of angular displacement.
  - **Angular acceleration (α)** is the rate of change of angular velocity.

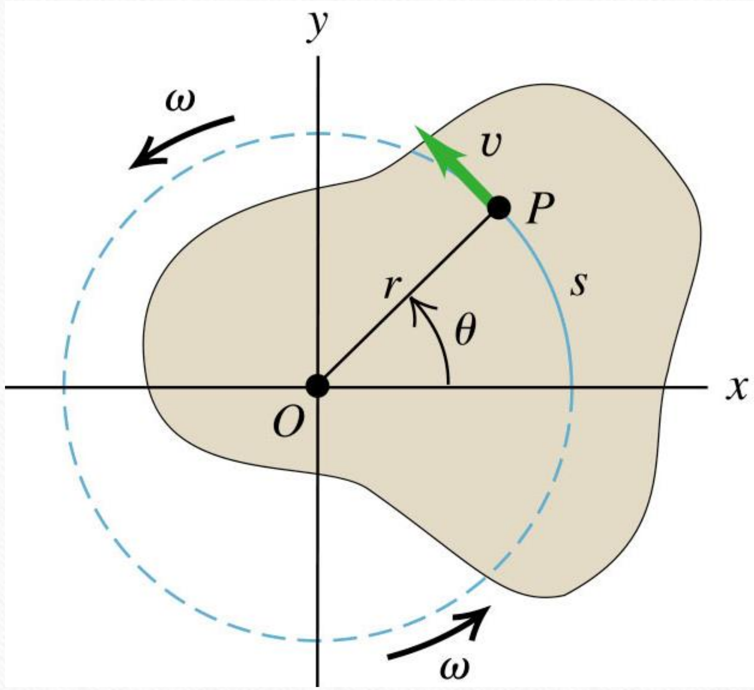


Angular Velocity and Acceleration

- Average angular velocity: $\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$
- Instantaneous angular velocity: $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$
- Average angular acceleration: $\bar{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$
- Instantaneous angular acceleration: $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$

Rigid Body Rotation

- Rotation of an extended object
  - Different parts of an object have different linear velocities and acceleration
- Assume the object is rigid
  - Means non-deformable, internal motion neglected
- Pure rotation motion
  - Rotation about a fixed axis
- Every point has the same angular speed and angular acceleration



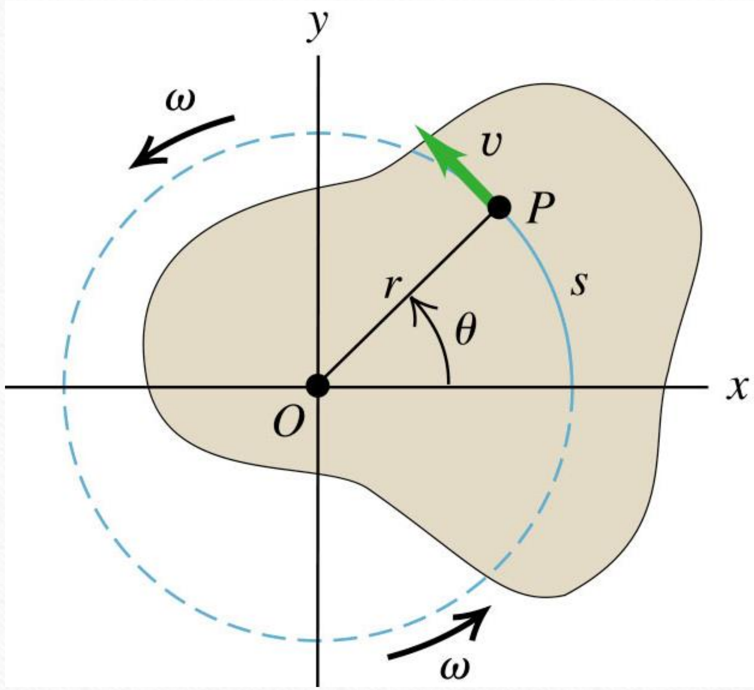
Rigid Body Rotation

- More convenient to use polar coordinates as only θ changes. Then:

$$s = r\theta \quad \text{or} \quad \theta = \frac{s}{r}$$

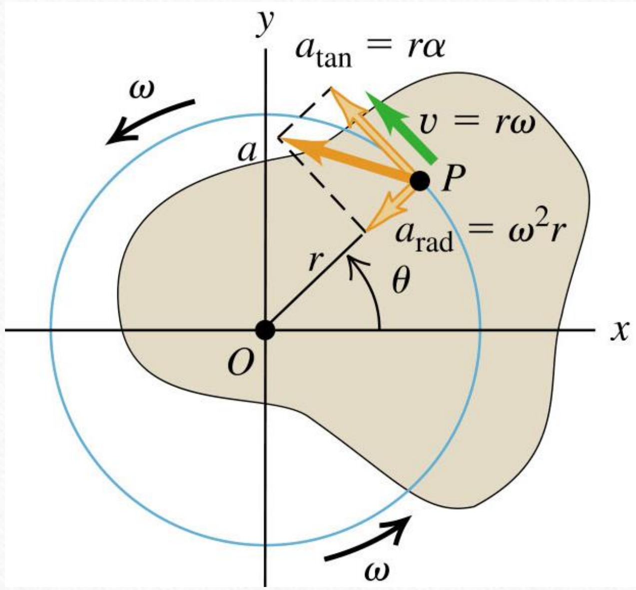
- The unit of θ is radian (rad).
    - One radian is the angle subtended by an arc length equal to the radius of the arc.

$$360^\circ = 2\pi \text{ rad} \Rightarrow \boxed{1 \text{ rad} = \frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi} \approx 57.30^\circ}$$



Angular & Linear Quantities

- When a rigid body rotates about a fixed axis, every particle in the body moves with the same angular speed ω.



$$\boxed{s = r\theta}$$

$$\boxed{v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega}$$

$$\boxed{a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha}$$

$$\boxed{a_r = \frac{v^2}{r} = r\omega^2}$$

closer to pivot = slower rotation
and vice versa

Rotational Kinematics for constant α

$$\alpha = \frac{d\omega}{dt} = \text{const.}$$

$$d\omega = \alpha dt$$

Integrating both sides and take

$\omega = \omega_0$ at $t = 0$:

$$\boxed{\omega = \omega_0 + \alpha t}$$

v = ut+at

$$\omega = \frac{d\theta}{dt}$$

$$d\theta = \omega dt = (\omega_0 + \alpha t)dt$$

Integrating both sides and take

$\theta = \theta_0$ at $t = 0$

$$\boxed{\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2}$$

Eliminating t, $\boxed{\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)}$

Kinematics: Angular vs Linear

- There is a one-to-one correspondence between:

$$\theta \Longleftrightarrow x$$
$$\omega \Longleftrightarrow v$$
$$\alpha \Longleftrightarrow a$$

- For constant a and α, we can write analogous equations for rotational motion as in linear motion.

Acceleration must be constant

$$\boxed{v = v_0 + at} \Longleftrightarrow \boxed{\omega = \omega_0 + \alpha t}$$

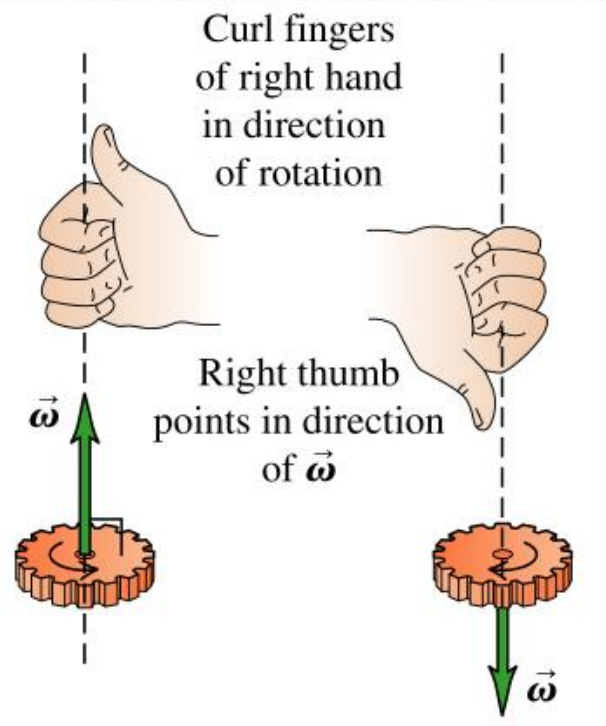
$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2} \Longleftrightarrow \boxed{\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2}$$

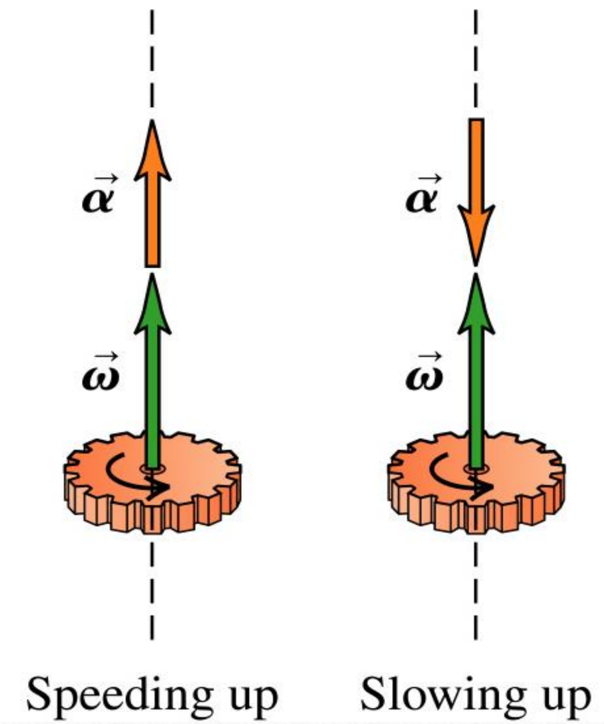
$$\boxed{v^2 = v_0^2 + 2a(x - x_0)} \Longleftrightarrow \boxed{\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)}$$

$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$

Vectors ω and α

- omega ω and alpha α are vectors
- The direction of ω is determined by right-hand rule.
- The direction of α follows from its definition dω/dt.





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