TUTORIAL 4

Question 1. Consider the function

$$f(x) = \begin{cases} ce^{-2x} & \text{if } x \geq 0, \\ 0 & \text{if } x < 0. \end{cases}$$

(a) Find c so that $f(x)$ is the PDF of a continuous random variable X.

(b) With the value c found in part (a), find the CDF $F(x)$ of X

(b) Compute $P(1 \leq X \leq 10)$.

Hint. A function $f(x)$ is the PDF of a continuous random variable if it satisfies three conditions (i)-(iii) from Lecture 4. In this case, use condition (ii).

Question 2. The height of adult male population of Singapore can be described with normal distribution with the mean value 165 cm and the standard deviation of 20 cm.

(a) What is the probability that a person randomly met on the street of Singapore is taller than 190cm?

(b) How much taller than 165 cm a random person should be so that the probability of meeting this person would be ≤ 0.05?

Hint for (b): You need to find the minimum height h such that $P(X > h) = 0.05$. Hence, the answer is "the random person needs to be at least $h - 165$ cm higher than 165 cm" so that the probability of meeting this person would be ≤ 0.05.

z	+ 0.00	+ 0.01	+ 0.02	+ 0.03	+ 0.04	+ 0.05	+ 0.06	+ 0.07	+ 0.08	+ 0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56360	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449

CDF of N(0,1)

Question 3. The height of adult male population of Singapore can be described by a normal distribution with the mean value $\mu = 165$cm and the standard deviation $\sigma = 20$cm.

(a) What is the proportion of the Singapore adult male population who are at least 175cm tall?

(b) The **median** of a population is the 50% upper percentage point of the population. Equivalently, this is the point m such that $P(X > m) = 0.5$. What is the median of the adult male population in Singapore?

(c) Find the upper 5% and 30% points of the Singapore adult male population.

Question 4. The following function is given

$$f(x) = \begin{cases} \frac{1+\alpha x}{2} & \text{if } -1 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) For which values of α is $f(x)$ a PDF of a continuous random variable X?

(b) Find the correspoding CDF $F(x)$ of X.

(c) The median of a distribution is defined as in Question 3 part b. Find the median of X.

Hint. For b, you need to consider 3 cases to find $F(x)$: $x < -1$, $-1 \le x \le 1$ and $x > 1$. For c, you need to find m such that $P(X > m) = 0.5$.

Question 5. Suppose that the lifetime T (in years) of an electronic component follows an exponential distribution with $\lambda = 0.1$, that is, $T \sim \text{Exp}(0.1)$.

(a) Find the probability that the lifetime is less than 10.

(b) Find the probability that the lifetime is between 5 and 15.

(c) Find t such that the probability that the lifetime is greater than t is 0.01.