Problem 1:

$$\underline{2} \times \underline{2} \times \underline{2} \times \underline{2} = 16$$

A: starting with 1

B: even # of 1s.

A: $\{(1, a, b, c) : a, b, c \in \{0,1\}\}$. $|A| = 8$.

$P(A) = \frac{8}{16} = \frac{1}{2}$.

$\binom{4}{2} = 6$.

B: { no bit equal to 1 } ∪ { 2 bits equal to 1 }
∪ { 4 bits equal to 1 }.

$|B| = 1 + 6 + 1 = 8$

$P(B) = \frac{8}{16} = \frac{1}{2}$.

$A \cap B$ = { even # 1's with the first entry equal 1 }

$= \{(1,1,0,0), (1,0,0,1), (1,0,1,0), (1,1,1,1)\}$.

$|A \cap B| = 4$

$P(A \cap B) = \frac{4}{16} = \frac{1}{4}$.

$P(A \cap B) = P(A)P(A)$ A and B are independent.

Problem 2: A and B independent evens.
prove A and B^c are independent.

proof: $P(A \cap B) = P(A)P(B)$

$$P(A) = \underbrace{P(A\cap B)} + P(A\cap B^c)$$

$$P(A) = P(A)P(B) + P(A\cap B^c)$$

$$P(A\cap B^c) = P(A) - P(A)P(B)$$

$$= P(A)(\underbrace{1-P(B)}) = P(A)P(B^c)$$

Therefore A and B^c are independent.

problem 3:

$P(\text{six show up}) = \frac{1}{6}$.

$P(\text{other number show up}) = \frac{5}{6}$

$P(X=k) = \left(\frac{5}{6}\right)^{k-1}\cdot\frac{1}{6}$

$$P(5 \le X \le 10) = P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= \underbrace{\left(\frac{5}{6}\right)^{5-1}\cdot\frac{1}{6}} + \left(\frac{5}{6}\right)^{6-1}\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^{7-1}\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^{8-1}\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^{9-1}\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^{10-1}\cdot\frac{1}{6}.$$

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geometric sequence. $r = \frac{5}{6}$.

$$S_n = \frac{a(1-r^n)}{1-r}$$

a: first term

n: # of terms.

r: common ratio

$$= \frac{\left(\frac{5}{6}\right)^4\cdot\frac{1}{6}\left(1-\frac{5}{6}^6\right)}{1-\frac{5}{6}} \approx 32.07\%.$$

problem 4: (a) $S = \{0, 1, 2, 3, 4, 5\}$.

(b) $P(X=x)$ $\quad$ $P(X \le x)$

$$P(X=0) = \frac{\binom{13}{0}\binom{39}{5}}{\binom{52}{5}} = 0.2215 \qquad P(X \le 0) = 0.2215$$

$$P(X=1) = \frac{\binom{13}{1}\binom{39}{4}}{\binom{52}{5}} = 0.4114 \qquad P(X \le 1) = 0.6329.$$

$$P(X=2) = \frac{\binom{13}{2}\binom{39}{3}}{\binom{52}{5}} = 0.2743 \qquad P(X \le 2) = 0.9072.$$

$$P(X=3) = \frac{\binom{13}{3}\binom{39}{2}}{\binom{52}{5}} = 0.0815 \qquad P(X \le 3) = 0.9887$$

$$P(X=4) = \frac{\binom{13}{4}\binom{39}{1}}{\binom{52}{5}} = 0.0107 \qquad P(X \le 4) = 0.9994$$

$$P(X=5) = \frac{\binom{13}{5}\binom{39}{0}}{\binom{52}{5}} = 0.0005 \qquad P(X \le 5) = 1$$