CSD1251/CSD1250 Week 5 Tutorial Problems

30th January – 5th February 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Qn 1 (Second derivatives)

Let A and B be constants. Let y be the function

$$y = e^{2x}[A\cos(3x) + B\sin(3x)].$$

(a) Find the first and second derivatives y' and y''.

(b) Show that the derivatives in part (a) and y satisfy the *differential equation*

$$y'' - 4y' + 13y = 0.$$

Qn 2 (n-th derivatives)

Let $f(x) = \sin x$.

(a) (⋆) Deduce a general formula for the n-th derivative of $f(x)$.
Hint: Differentiate $\sin x$ for 7 times and observe if there are any repeating patterns.

(b) Find $f^{(6311)}(x)$ (the 6311st derivative of f).

Qn 3 (Increasing/Decreasing Test)

For each of the following functions f, find the intervals on which f is increasing or decreasing.

(a) $f(x) = 2x^3 - 15x^2 + 24x - 5$

(b) $f(x) = x^4 e^{-x}$

(c) $f(x) = x + \dfrac{4}{x^2}$