# CSD1130
# Game Implementation Techniques

Lecture 21

Animated Circular Objects

# Questions?

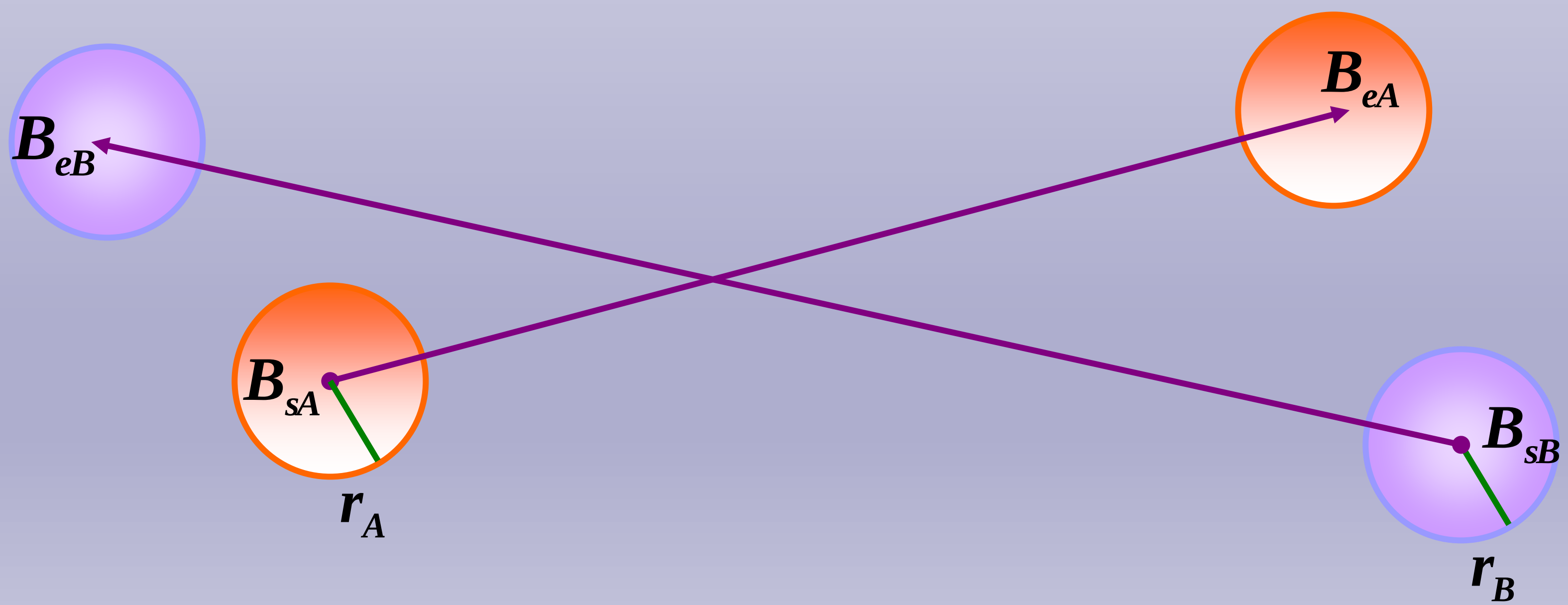
- Intersection Between Animated
  - Animated Circular Object and Stationary Circular Object
- Collision Response

# Overview

- Intersection Between Animated Circular Objects

# Pinball-Pinball Collision (1/6)

- Both objects involved are moving



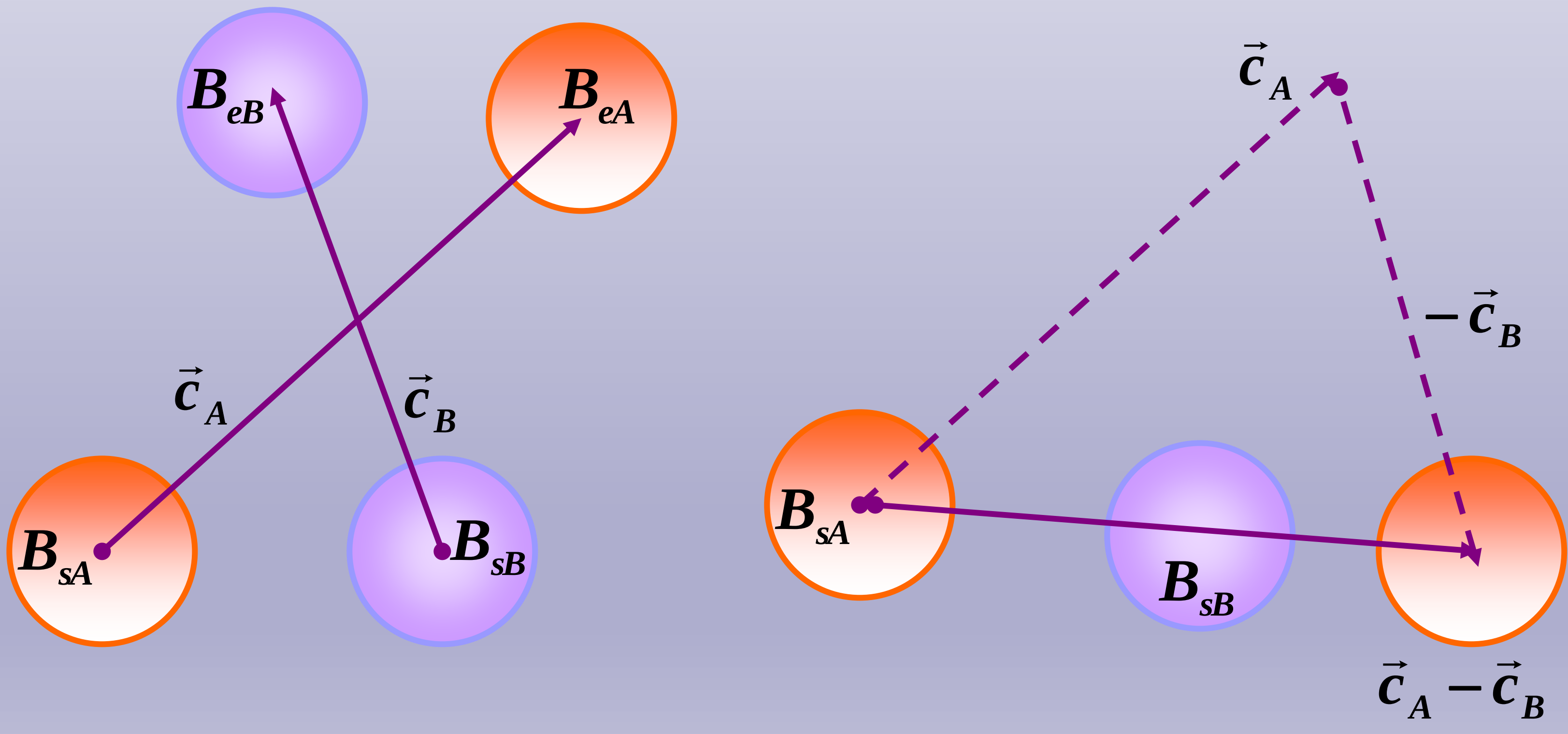
# Pinball-Pinball Collision (2/6)

# Pinball-Pinball Collision (3/6)

- Pinballs $A$ and $B$
  - Have radii $r_A$ and $r_B$, respectively
  - Are moving with velocities $c_A$ and $c_B$, respectively

- First, we simplify the velocity problem by assuming that $A$ is animated while $B$ is stationary
  - Subtract velocity of pinball $B$ from both pinballs
  - Velocity of pinball $A$ is: $c_{AB} = c_A - c_B$
  - Velocity of pinball $B$ is: $c_{BB} = c_B - c_B = 0$

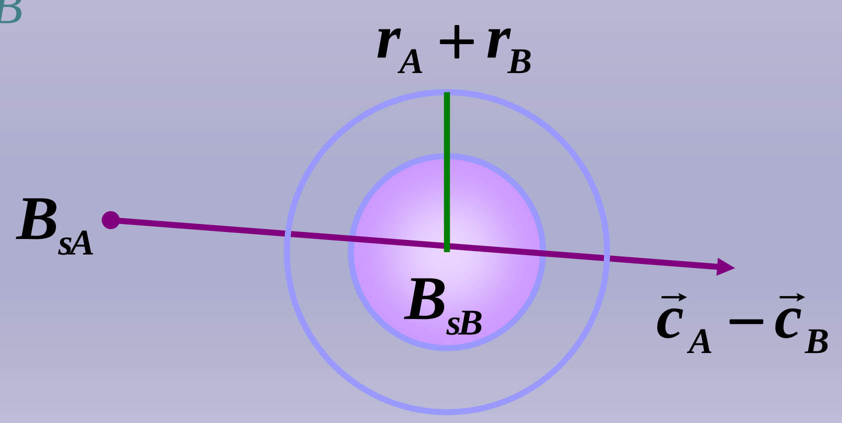
# Pinball-Pinball Collision (4/6)



# Pinball-Pinball Collision (5/6)

- Next, check for collision between pinball and circular pillar using ray-circle intersection tests
  - Move pinball *A* over surface of pinball *B* to create a new circular pillar whose radius is:

    $r = r_A + r_B$



# Pinball-Pinball Collision (6/6)

Using ray-circle intersection, compute $t_i \in [t_s, t_e]$

Compute position of pinball centers:

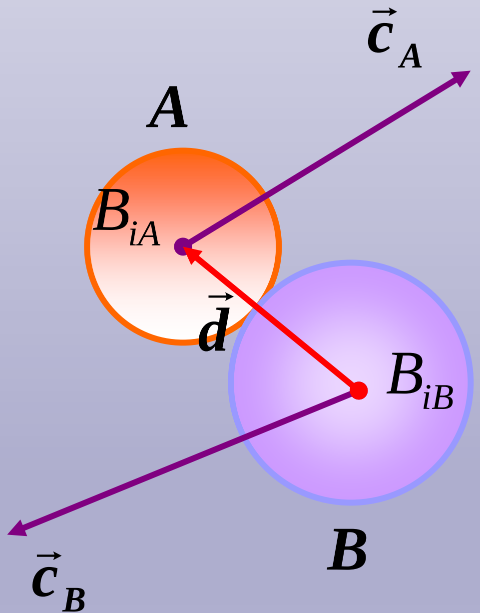
$$B_{iA} = B_A(t_i) = B_{sA} + \vec{c}_A t_i$$

$$B_{iB} = B_B(t_i) = B_{sB} + \vec{c}_B t_i$$

# Collision Response (1)

$$\vec{d} = B_{iA} - B_{iB}$$

$$\hat{\mathbf{d}} = \frac{\vec{\mathbf{d}}}{\left\|\vec{\mathbf{d}}\right\|}$$



# Conservation of Momentum

Courtesy Wikipedia

# Conservation of Momentum

- The momentum of a closed system remains constant.
- Closed system is defined as no external forces act on the objects.

# Collision Response (2)

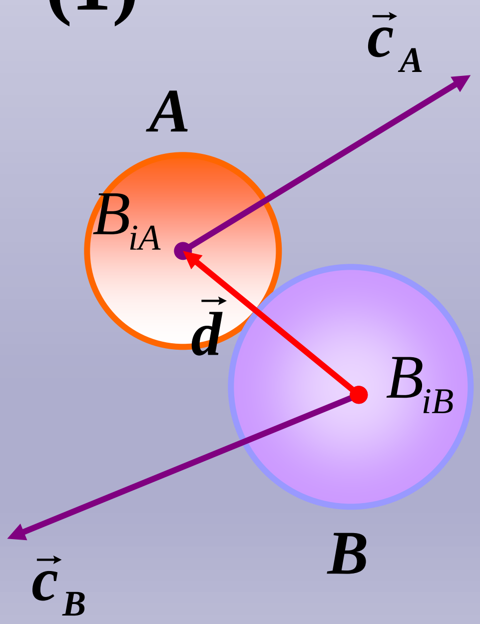
**Using law of conservation of momentum :**

$$m_A\vec{c}_A + m_B\vec{c}_B = m_A\vec{c}\,'_A + m_B\vec{c}\,'_B \quad (1)$$

$$m_A\vec{c}\,'_A = m_A\vec{c}_A - \vec{P} \quad (2)$$

$$m_B\vec{c}\,'_B = m_B\vec{c}_B + \vec{P} \quad (3)$$

$$\vec{P} = \left\|\vec{P}\right\|\hat{d} \quad (4)$$

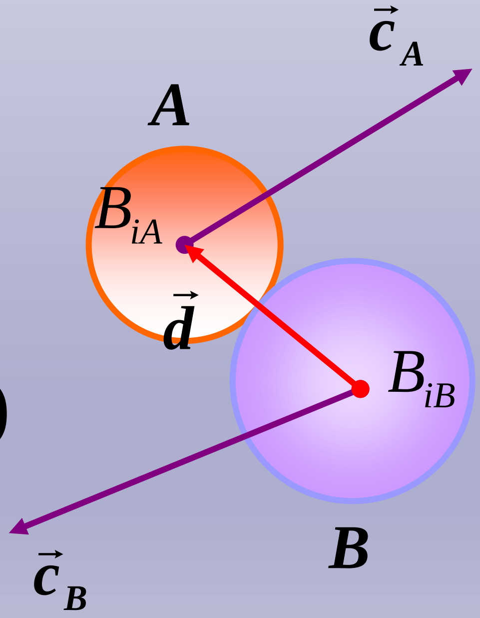


# Collision Response (3)

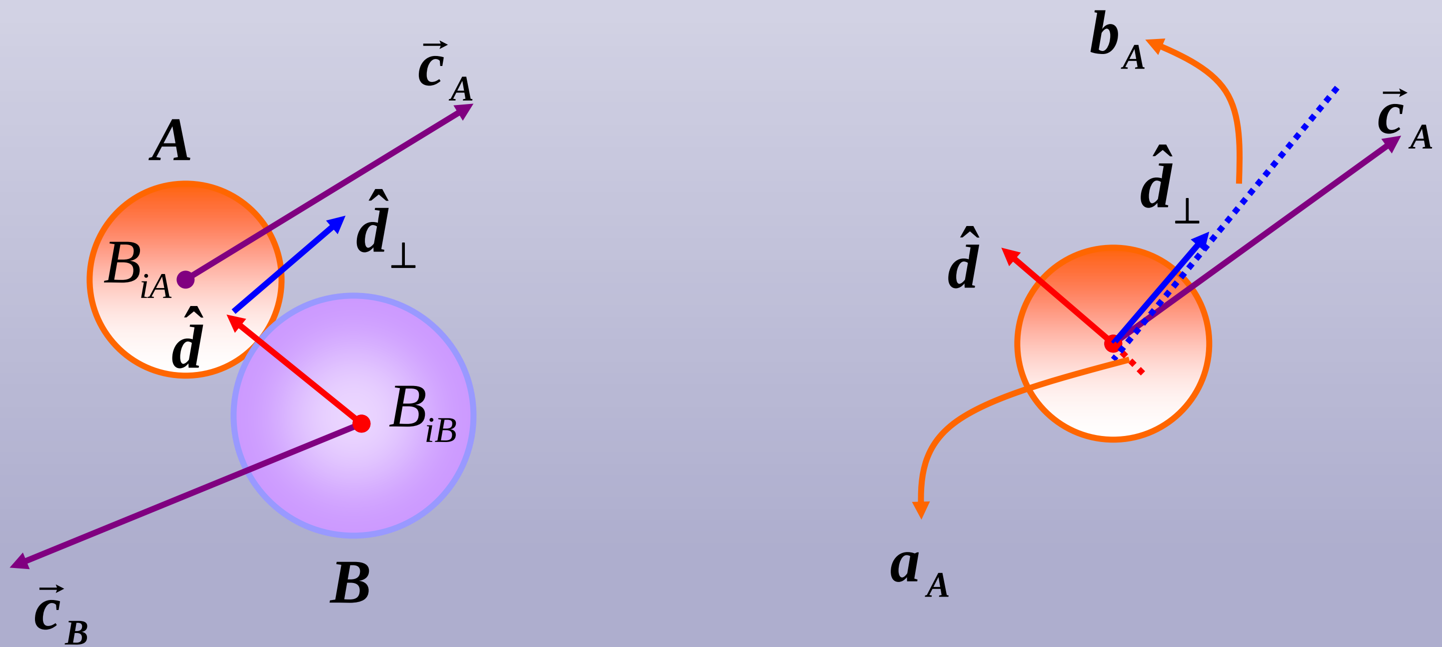
**Rewriting equations (2) and (3):**

$$\vec{c}\,'_A = \vec{c}_A - \left( \frac{\left\| \vec{P} \right\|}{m_A} \right) \hat{d} \quad (5)$$

$$\vec{c}\,'_B = \vec{c}_B + \left( \frac{\left\| \vec{P} \right\|}{m_B} \right) \hat{d} \quad (6)$$



# Collision Response (4)



# Collision Response (5)

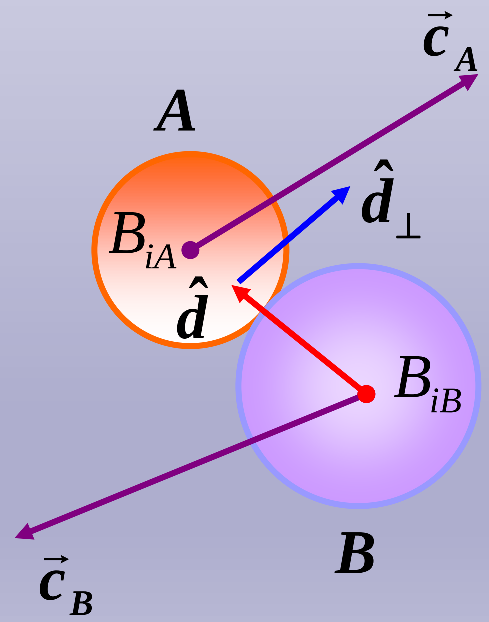
**Rewriting $\vec{c}_A, \vec{c}_B, \vec{c}'_A$, and $\vec{c}'_B$ in terms of $\hat{d}$ and $\hat{d}_\perp$ :**

$$\vec{c}_A = a_A\hat{d} + b_A\hat{d}_\perp \quad (7)$$

$$\vec{c}_B = a_B\hat{d} + b_B\hat{d}_\perp \quad (8)$$

$$\vec{c}'_A = a'_A\hat{d} + b'_A\hat{d}_\perp \quad (9)$$

$$\vec{c}'_B = a'_B\hat{d} + b'_B\hat{d}_\perp \quad (10)$$



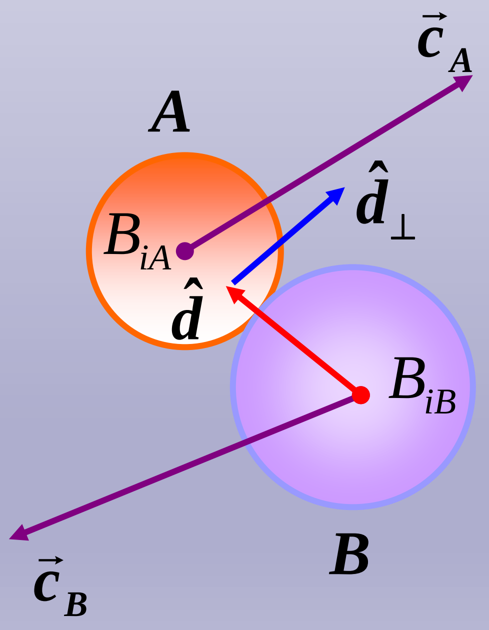
# Collision response (6)

**Substituti ng equation (7) into equation (5):**

$$\vec{c}\,'_A = \vec{c}_A - \left( \frac{\left\| \vec{P} \right\|}{m_A} \right) \hat{d} \qquad (5)$$

$$\vec{c}_A = a_A \hat{d} + b_A \hat{d}_\perp \qquad (7)$$

$$\Rightarrow \vec{c}\,'_A = \left( a_A - \frac{\left\| \vec{P} \right\|}{m_A} \right) \hat{d} + b_A \hat{d}_\perp \qquad (11)$$



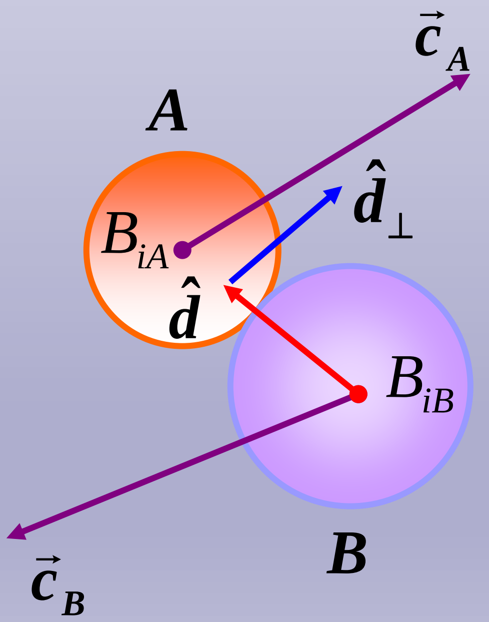
# Collision Response (7)

**Substituti ng equation (8) into equation (6):**

$$\vec{c}\,'_B = \vec{c}_B + \left( \frac{\|\vec{P}\|}{m_B} \right) \hat{d} \quad (6)$$

$$\vec{c}_B = a_B \hat{d} + b_B \hat{d}_{\perp} \quad (8)$$

$$\Rightarrow \vec{c}\,'_B = \left( a_B + \frac{\|\vec{P}\|}{m_B} \right) \hat{d} + b_B \hat{d}_{\perp} \quad (12)$$



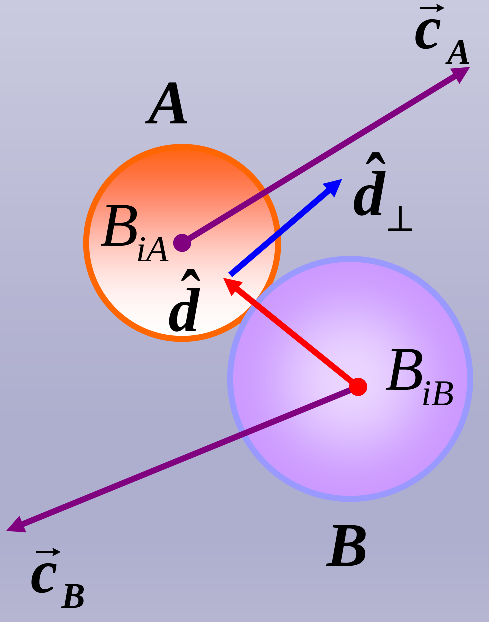
# Collision Response (8)

**Comparing equations (9) and (11):**

$$\vec{c}'_A = a'_A \hat{d} + b'_A \hat{d}_\perp \qquad (9)$$

$$\vec{c}'_A = \left( a_A - \frac{\left\|\vec{P}\right\|}{m_A} \right) \hat{d} + b_A \hat{d}_\perp \qquad (11)$$

$$\Rightarrow a'_A = a_A - \frac{\left\|\vec{P}\right\|}{m_A} \quad (13) \qquad \Rightarrow b'_A = b_A \; (14)$$



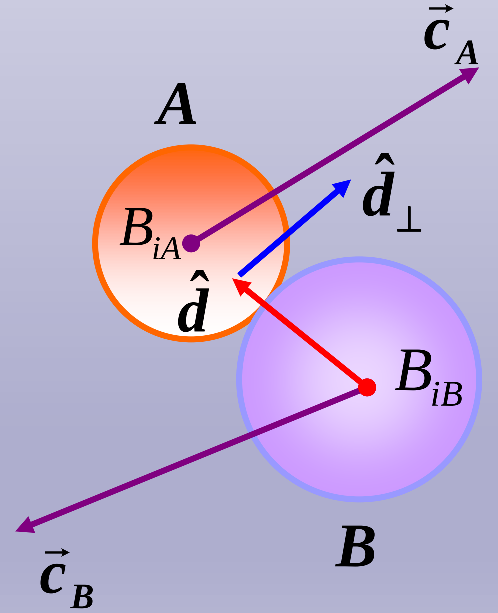
# Collision Response (9)

**Comparing equations (10) and (12):**

$$\vec{c}\,'_B = a'_B \hat{d} + b'_B \hat{d}_\perp \quad (10)$$

$$\vec{c}\,'_B = \left( a_B + \frac{\left\| \vec{P} \right\|}{m_B} \right) \hat{d} + b_B \hat{d}_\perp \quad (12)$$

$$\Rightarrow a'_B = a_B + \frac{\left\| \vec{P} \right\|}{m_B} \quad (15) \qquad \Rightarrow b'_B = b_B \ (16)$$



# Collision Response (10)

$$\textbf{Kineticenergy}: E = \tfrac{1}{2}\, mass * velocity^2$$

**Using law of conservation of kinetic energy :**

$$\tfrac{1}{2} m_A \|\vec{c}_A\|^2 + \tfrac{1}{2} m_B \|\vec{c}_B\|^2 = \tfrac{1}{2} m_A \|\vec{c}\,'_A\|^2 + \tfrac{1}{2} m_B \|\vec{c}\,'_B\|^2 \quad (17)$$

$$\|\vec{c}_A\|^2 = a_A{}^2 + b_A{}^2 \quad (18) \qquad \|\vec{c}_B\|^2 = a_B{}^2 + b_B{}^2 \quad (19)$$

$$\|\vec{c}\,'_A\|^2 = a'_A{}^2 + b'_A{}^2 \quad (20) \qquad \|\vec{c}\,'_B\|^2 = a'_B{}^2 + b'_B{}^2 \quad (21)$$

# Collision Response (11)

$$\textbf{Substitute eqns } 18 \rightarrow 21 \textbf{ into eqn } 17 \textbf{ and}$$

$$\textbf{using equations } 13 \rightarrow 16 \textbf{:}$$

$$\left\| \vec{P} \right\| = \frac{2 m_A m_B \left( a_A - a_B \right)}{m_A + m_B} \qquad (22)$$

# Collision Response (12)

**Substitute eqn 22 into eqn 5 :**

$$\vec{c}'_A = \vec{c}_A - \left( \frac{\left\| \vec{P} \right\|}{m_A} \right) \hat{d} \quad (5)$$

$$\left\| \vec{P} \right\| = \frac{2 m_A m_B (a_A - a_B)}{m_A + m_B} \quad (22)$$

$$\Rightarrow \vec{c}'_A = \vec{c}_A - \left( \frac{2(a_A - a_B)}{m_A + m_B} \right) m_B \hat{d} \quad (23)$$

# Collision Response (13)

**Substitute eqn 22 into eqn 6 :**

$$\vec{c}'_B = \vec{c}_B + \left( \frac{\left\| \vec{P} \right\|}{m_B} \right) \hat{d} \quad (6)$$

$$\left\| \vec{P} \right\| = \frac{2 m_A m_B (a_A - a_B)}{m_A + m_B} \quad (22)$$

$$\Rightarrow \vec{c}'_B = \vec{c}_B + \left( \frac{2(a_A - a_B)}{m_A + m_B} \right) m_A \hat{d} \quad (24)$$

# Collision Response (14)

**Using equations 23 and 24:**

$$\boldsymbol{k_A = \left\|\vec{c}'_A\right\| \quad \hat{c}'_A = \frac{\vec{c}'_A}{\left\|\vec{c}'_A\right\|}} \qquad \boldsymbol{k_B = \left\|\vec{c}'_B\right\| \quad \hat{c}'_B = \frac{\vec{c}'_B}{\left\|\vec{c}'_B\right\|}}$$

$$B_{eA} = B_A\left(t_e - t_i\right) = B_{iA} + k_A\hat{c}'_A\left(t_e - t_i\right)$$

$$B_{eB} = B_B\left(t_e - t_i\right) = B_{iB} + k_B\hat{c}'_B\left(t_e - t_i\right)$$