CSD2201/CSD2200 Week 12 Tutorial Problems

13th November – 19th November 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learned from the attempt as a means of consolidating what you have learned.

Question 1

Use convergence tests to establish the convergence/divergence of the following series.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{3n}{n^2-1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2}{n^3+n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^4+n^2}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^4+1}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{4n+3}$ (f) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\ln n}$ (g) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{3^n}$ (h) $\displaystyle\sum_{n=3}^{\infty} (-1)^n \tan^{-1} n$

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{2n+1}$ (j) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{23^n}$ (k) (*) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$ (l) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

(m) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{4^n}$ (n) (*) $\displaystyle\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$ (o) $\displaystyle\sum_{k=1}^{\infty} ke^{-k}$ (p) (*) $\displaystyle\sum_{n=1}^{\infty} \frac{(3n)!}{(n!)^3}$

Question 2

Determine if the following series is absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n^4}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{3^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{0.3}}$ (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2n^2+n}{n^2+1}$

Question 3

Find the radius of convergence for the following power series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} 3^n n^3 x^n$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{n\,9^n}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(3x+2)^n}{n^4+1}$

(e) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{2^n x^n}{n}$ (f) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ (g) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$

Final Answers (C means convergent, D means divergent):

Q1: (a) D, (b) D, (c) C, (d) D, (e) C, (f) C, (g) C, (h) D, (i) C, (j) D, (k) D, (l) AC, (m) AC, (n) D, (o) AC, (p) D.

Q2: (a) AC, (b) AC, (c) CC, (d) D.

Q3: (a) 2, (b) $\frac{1}{3}$, (c) 3, (d) $\frac{1}{3}$, (e) $\frac{1}{2}$, (f) ∞, (g) ∞.