Attendance : 13z78l

Q1, Q2, Q3 (e), (i), (l), (m)

Q4 (l), (m), (b), (e), (h)

<u>Riemann Sum</u> $f(x)$ and $[a,b] \rightarrow \Delta x = \frac{b-a}{n}$

n = no. of rectangles

$$\sum_{i=1}^{n} f(x_i^*) \Delta x$$

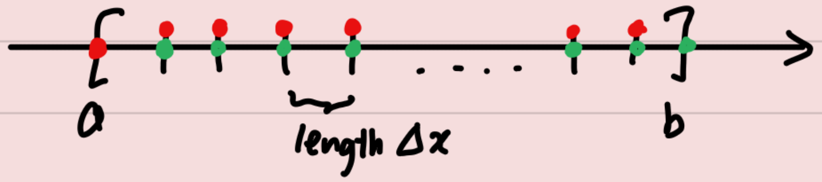
sigma

$\underline{a + i\Delta x}$

right endpoints • $i = 1, \ldots, n$

left endpoints • $i = 0, 1, \ldots, n-1$

```
S=0;
for i=1 to n
    S = S + f(x_i^*) * Δx;
end.
```



<u>Q1</u> $f(x) = \sqrt{x}$ $[0, 6]$ ($0 = a$, $6 = b$) $n = 6$ $\Delta x = \frac{6-0}{6} = 1$

(a) right endpoints: $0 + i\Delta x$ $i = 1, \ldots, 6$

$\Delta x, 2\Delta x, \ldots, 6\Delta x \Rightarrow \boxed{1, 2, \ldots, 6}$ right endpoints

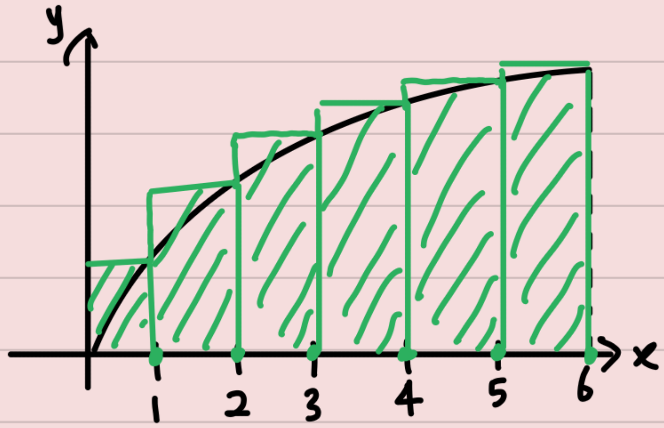
$$R_n = \sum_{i=1}^{n} f(i)\Delta x = \sqrt{1}\cdot 1 + \sqrt{2}\cdot 1 + \sqrt{3}\cdot 1 + \sqrt{4}\cdot 1 + \sqrt{5}\cdot 1 + \sqrt{6}\cdot 1$$

$$= 1 + \sqrt{2} + \sqrt{3} + 2 + \sqrt{5} + \sqrt{6}$$

$$\simeq 10.831822$$

$$L_n = \sum_{i=0}^{n-1} f(i)\Delta x = \sqrt{0} + \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5}$$
$$\approx 8.382332$$

Right endpoints:



Left endpoints:

(c) $$\int_0^6 \sqrt{x}\,dx = \int_0^6 x^{\frac{1}{2}}\,dx$$

$$= \left[\frac{x^{3/2}}{3/2}\right]_0^6$$ 9.797959 $\approx$

$$= \frac{2}{3}\left[x^{3/2}\right]_0^6 = \frac{2}{3}\left(6^{3/2} - 0^{3/2}\right) = \underline{\frac{2}{3}6^{3/2}}$$

$$L_n < 9.797959 < R_n$$

Q2 $n = 6$, $a = 0$, $b = \frac{3\pi}{2}$, $f(x) = \sin x$

$$\Delta x = \frac{\frac{3\pi}{2} - 0}{6} = \frac{\pi}{4}.$$

Right endpoints: $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}$

Left endpoints: $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}$

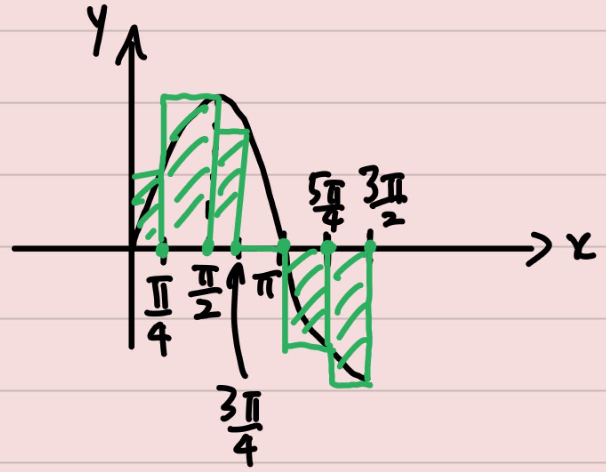
$$R_n = \frac{\pi}{4}\left(\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{2}\right) + \sin\left(\frac{3\pi}{4}\right) + \sin(\pi) + \sin\left(\frac{5\pi}{4}\right) + \sin\left(\frac{3\pi}{2}\right)\right)$$

$$= \frac{\pi}{4\sqrt{2}} \approx 0.555360$$

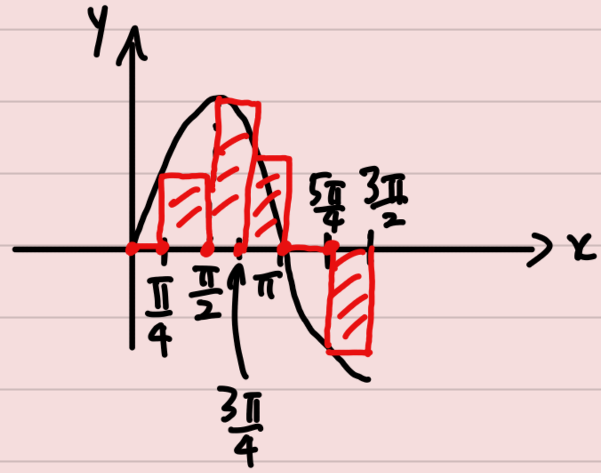
$$L_n = \frac{\pi}{4}\left[\sin(0) + \sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{2}\right) + \sin\left(\frac{3\pi}{4}\right) + \sin(\pi) + \sin\left(\frac{5\pi}{4}\right)\right]$$

$$= \frac{1}{4}\left(1 + \frac{1}{\sqrt{2}}\right)\pi \approx 1.3407759.$$

Right endpoints



Left endpoints



$$\int_0^{\frac{3\pi}{2}} \sin x \, dx = \left[-\cos x\right]_0^{\frac{3\pi}{2}}$$

$$= -\cos\frac{3\pi}{2} - (-\cos 0) = 1$$

3(e) $$\int 5x^{\frac{1}{2}} - \frac{3}{1+x^2}\, dx$$

$$= 5\int x^{\frac{1}{2}}\, dx - 3\int \frac{1}{1+x^2}\, dx$$

$$= 5\left(\frac{x^{3/2}}{3/2}\right) - 3\tan^{-1}x + C = \frac{10}{3}x^{3/2} - 3\tan^{-1}x + C$$

(i) $\int \frac{1}{\cos^2 x} - \frac{7}{\sqrt{1-x^2}}\,dx$

$= \int \sec^2 x - \frac{7}{\sqrt{1-x^2}}\,dx$

$= \tan x - 7\sin^{-1} x + C$

(l) $\int 8x^{3/5} - \frac{1}{\cos^2 x}\,dx$

$= \int 8x^{3/5} - \sec^2 x\,dx$

$= 8\int x^{3/5}\,dx - \int \sec^2 x\,dx$

$= 8\left[\frac{x^{8/5}}{8/5}\right] - \tan x + C$

$= 5x^{8/5} - \tan x + C.$

$\sin^2 x + \cos^2 x = 1$

Divide throughout by $\cos^2 x$

$\frac{\sin^2 x}{\cos^2 x} + 1 = \frac{1}{\cos^2 x}$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

(m) $\int x^2 + \tan^2 x\,dx$

$= \int x^2 + \sec^2 x - 1\,dx$

$= \frac{x^3}{3} + \tan x - x + C.$

4(b) $$\int_{-1}^{1} x^3\,dx = \left[\frac{x^4}{4}\right]_{-1}^{1}$$

odd function

$f(-x) = -f(x)$

$$= \frac{1^4}{4} - \frac{(-1)^4}{4} = \frac{1}{4} - \frac{1}{4} = 0$$

(e) $$\int_0^1 \frac{23}{1+x^2}\,dx = 23\int_0^1 \frac{1}{1+x^2}\,dx$$

$$= 23\left[\tan^{-1}x\right]_0^1$$

$$= 23(\tan^{-1}1 - \tan^{-1}0)$$

$$= 23\left(\frac{\pi}{4} - 0\right) = \frac{23\pi}{4}$$

(h) $$\int_{-1}^{1} 5x^4 + 2x^3\,dx$$

$$= 5\int_{-1}^{1} x^4\,dx + 2\int_{-1}^{1} x^3\,dx$$

part (a) (b)

$$= 5\cdot\frac{2}{5} + 2\cdot 0 = \frac{10}{5} = 2.$$

$$\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}} = \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

(l) $$\int_0^{\frac{\pi}{4}} 6e^x + \sec x\tan x\,dx$$

$$= \left[6e^x + \sec x\right]_0^{\frac{\pi}{4}} = \left[6e^{\frac{\pi}{4}} + \sqrt{2} - (\underbrace{6e^0}_{6} + 1)\right]$$

$$= 6e^{\frac{\pi}{4}} + \sqrt{2} - 7$$

(11) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 5\cos x + 8x^3 \, dx$

$= 5\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx + 8\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 \, dx.$ ✓

$\left(-\frac{\pi}{2}\right)^4 = (-1)^4\left(\frac{\pi}{2}\right)^4$

$= 5\left[\sin x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + 8\left[\frac{x^4}{4}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

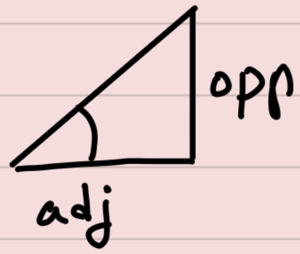
$= 5\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right] + 2\left[\left(\frac{\pi}{2}\right)^4 - \left(-\frac{\pi}{2}\right)^4\right]$

$= 5\left[1-(-1)\right] + 2\left[\left(\frac{\pi}{2}\right)^4 - \left(\frac{\pi}{2}\right)^4\right] = 10$

$\tan^{-1} x = y \iff x = \tan y \qquad -\frac{\pi}{2} < y < \frac{\pi}{2}$

$x = 1, \; y = ?$ $\qquad 1 = \tan y$

$y = \frac{\pi}{4}$



$\frac{\text{opp}}{\text{adj}} = 1$