# CSD1251/CSD1250 Week 10 Tutorial Problems

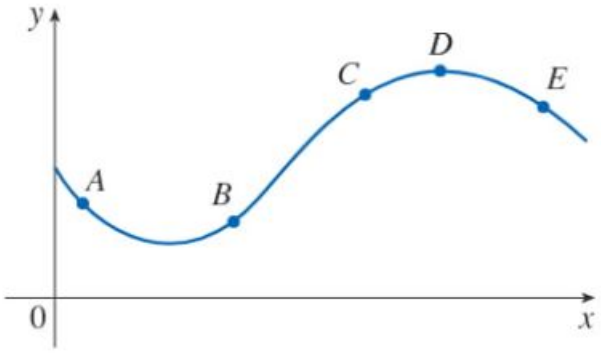
6th March – 12th March 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

## Question 1

The graph of a function $y = f(x)$ can be found below. At which point(s) are the following true? Explain your answer.




Answers are in red.

(a) $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are both positive.

(b) $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are both negative.

(c) $\frac{dy}{dx}$ is negative but $\frac{d^2y}{dx^2}$ is positive.

(d) $\frac{dy}{dx}$ is positive but $\frac{d^2y}{dx^2}$ is negative.

## Question 2

Find the (i) inflection points of the following functions, and (ii) determine the intervals for which $f$ is concave upwards or concave downwards.

(a) $f(x) = 2x^3 - 9x^2 + 12x - 3$ (b) $f(x) = 6x^4 - 16x^3 + 1$

(c) $f(x) = x^2 - x - \ln x$ (d) $f(x) = x^2 \ln x$ (e) $f(x) = xe^{2x}$

## Question 3

Let $f(x) = x^4(x-1)^3$.

(a) Find the critical points of $f$.

(b) What does the Second Derivative Test tell you about the critical points in part (a) being local extrema?

(c) Repeat part (b), but using the First Derivative Test instead of the Second Derivative Test.

## Question 4

For the functions $f$ in Question 2, (i) find the critical points of $f$, and (ii) use the Second Derivative Test to find local extrema of $f$. If the Second Derivative Test turns out to be inconclusive at a certain critical point, then use the First Derivative Test in place of the Second Derivative Test check if the critical point is a local extrema.

## Question 5 (Bonus: Graph Sketching)

Using the information from the following parts (a) to (c), sketch the graph of the function $f(x) = x^3 - 3x^2 + 4$ on $[-1, 3]$, labelling all $x$ and $y$-intercepts.

(a) Find intervals where $f$ is increasing or decreasing.

(b) Find the local extrema of $f$.

(c) Find the intervals where $f$ is CU or CD, and inflection points.

You may want to check a graphing software (e.g. www.desmos.com/calculator) to see if you have drawn correctly.