CSD2301 Practice Solutions
10. **Momentum and Collisions Part 2**

LIN QINJIE

Practice Question 1

find center of mass

Three odd-shaped blocks of chocolate have the following masses and center-of-mass coordinates: (1) 0.300 kg, (0.200 m, 0.300 m); (2) 0.400 kg, (0.100 m, -0.400 m); (3) 0.200 kg, (-0.300 m, 0.600 m). Find the coordinates of the center of mass of the system of the three chocolate blocks.

$m_A = 0.300\text{ kg}\,,\ m_B = 0.400\text{ kg}\,,\ m_C = 0.200\text{ kg}\,.$

$$x_{\text{cm}} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C}\,.$$

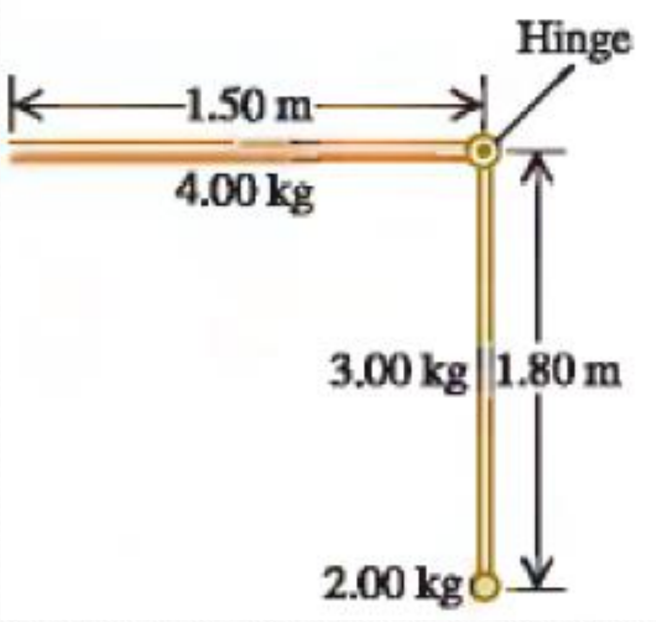
$$x_{\text{cm}} = \frac{(0.300\text{ kg})(0.200\text{ m}) + (0.400\text{ kg})(0.100\text{ m}) + (0.200\text{ kg})(-0.300\text{ m})}{0.300\text{ kg} + 0.400\text{ kg} + 0.200\text{ kg}} = 0.0444\text{ m}\,.$$

$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C}\,.$$

$$y_{\text{cm}} = \frac{(0.300\text{ kg})(0.300\text{ m}) + (0.400\text{ kg})(-0.400\text{ m}) + (0.200\text{ kg})(0.600\text{ m})}{0.300\text{ kg} + 0.400\text{ kg} + 0.200\text{ kg}} = 0.0556\text{ m}\,.$$

Practice Question 2

A machine part consists of a thin, uniform 4.00 kg bar that is 1.50 m long, hinged perpendicular to a similar vertical bar of mass 3.00 kg and length 1.80 m. The longer bar has a small but dense 2.00 kg ball at one end. By what distance will the center of mass of this part move horizontally and vertically if the vertical bar is pivoted counterclockwise through 90° to make the entire part horizontal?



Practice Question 2

+x

need set coordinates

+y

SET UP: Use coordinates with the axis at the hinge and the $+x$ and $+y$ axes along the horizontal and vertical bars in the figure in the problem. Let (x_i, y_i) and (x_f, y_f) be the coordinates of the bar before and after the vertical bar is pivoted. Let object 1 be the horizontal bar, object 2 be the vertical bar and 3 be the ball.

Middle

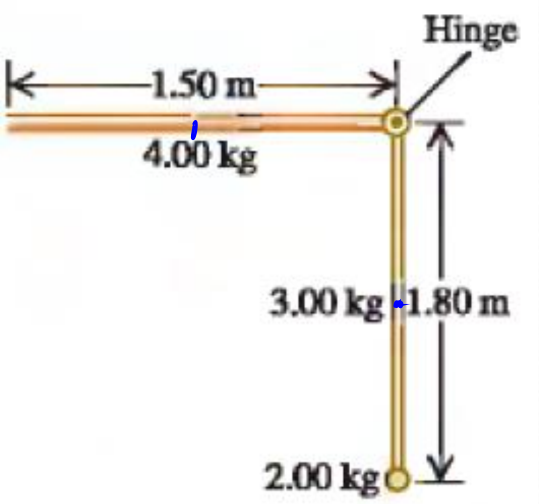
EXECUTE: $$x_i = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{(4.00\text{ kg})(0.750\text{ m}) + 0 + 0}{4.00\text{ kg} + 3.00\text{ kg} + 2.00\text{ kg}} = 0.333\text{ m}.$$

$$y_i = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{0 + (3.00\text{ kg})(0.900\text{ m}) + (2.00\text{ kg})(1.80\text{ m})}{9.00\text{ kg}} = 0.700\text{ m}.$$

$$x_f = \frac{(4.00\text{ kg})(0.750\text{ m}) + (3.00\text{ kg})(-0.900\text{ m}) + (2.00\text{ kg})(-1.80\text{ m})}{9.00\text{ kg}} = -0.366\text{ m}.$$

$y_f = 0$. $x_f - x_i = -0.700\text{ m}$ and $y_f - y_i = -0.700\text{ m}$. The center of mass moves 0.700 m to the right and 0.700 m upward.

EVALUATE: The vertical bar moves upward and to the right so it is sensible for the center of mass of the machine part to move in these directions.



Practice Question 3

A steel ball with mass 40.0 g is dropped from a height of 2.00 m onto a horizontal steel slab. The ball rebounds to a height of 1.60 m. (a) Calculate the impulse delivered to the ball during impact. (b) If the ball is in contact with the slab for 2.00 ms, find the average force on the ball during impact.

Set Up: Let $+y$ be downward.

Execute: **(a)** $\frac{1}{2}mv^2 = mgh$ so $v = \pm\sqrt{2gh}$.

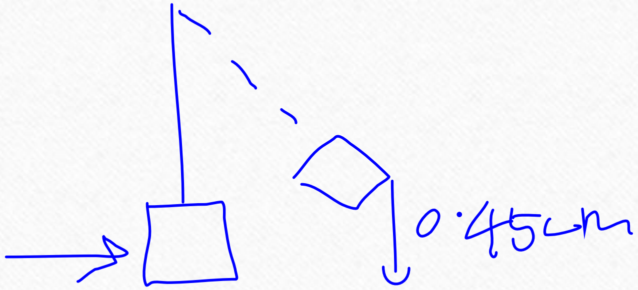
$v_{1y} = +\sqrt{2(9.80 \text{ m/s}^2)(2.00 \text{ m})} = 6.26 \text{ m/s}$. $v_{2y} = -\sqrt{2(9.80 \text{ m/s}^2)(1.60 \text{ m})} = -5.60 \text{ m/s}$.

$$J_y = \Delta p_y = m(v_{2y} - v_{1y}) = (40.0\times10^{-3} \text{ kg})(-5.60 \text{ m/s} - 6.26 \text{ m/s}) = -0.474 \text{ kg}\cdot\text{m/s}.$$

The impulse is $0.474 \text{ kg}\cdot\text{m/s}$, upward.

(b) $F_y = \dfrac{J_y}{\Delta t} = \dfrac{-0.474 \text{ kg}\cdot\text{m/s}}{2.00\times10^{-3} \text{ s}} = -237 \text{ N}$. The average force on the ball is 237 N, upward.

Evaluate: The upward force on the ball changes the direction of its momentum.



Practice Question 4

A 5.00 g bullet is shot *through* a 1.00 kg wood block suspended on a string 2.00 m long. The center of mass of the block rises a distance of 0.45 cm. Find the speed of the bullet as it emerges from the block if its initial speed is 450 m/s.

IDENTIFY: Apply conservation of momentum to the collision and conservation of energy to the motion of the block after the collision.

SET UP: Let $+x$ be to the right. Let the bullet be A and the block be B. Let V be the velocity of the block just after the collision.

EXECUTE: Motion of block after the collision: $K_1 = U_{\text{grav2}}$. $\frac{1}{2}m_B V^2 = m_B gh$.

$$V = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(0.450\times10^{-2}\ \text{m})} = 0.297\ \text{m/s}.$$

Collision: $v_{B2} = 0.297$ m/s. $P_{1x} = P_{2x}$ gives $m_A v_{A1} = m_A v_{A2} + m_B v_{B2}$.

$$v_{A2} = \frac{m_A v_{A1} - m_B v_{B2}}{m_A} = \frac{(5.00\times10^{-3}\ \text{kg})(450\ \text{m/s}) - (1.00\ \text{kg})(0.297\ \text{m/s})}{5.00\times10^{-3}\ \text{kg}} = 391\ \text{m/s}.$$

EVALUATE: We assume the block moves very little during the time it takes the bullet to pass through it.

Practice Question 5

A neutron at rest decays (breaks up) to a proton and an electron. Energy is released in the decay and appears as kinetic energy of the proton and electron. The mass of a proton is 1836 times the mass of an electron. What fraction of the total energy released goes into the kinetic energy of the proton?

IDENTIFY: Apply conservation of momentum to the system of the neutron and its decay products.

SET UP: Let the proton be moving in the $+x$ direction with speed v_p after the decay. The initial momentum of the neutron is zero, so to conserve momentum the electron must be moving in the $-x$ direction after the decay. Let the speed of the electron be v_e.

EXECUTE: $P_{1x} = P_{2x}$ gives $0 = m_p v_p - m_e v_e$ and $v_e = \left(\frac{m_p}{m_e}\right) v_p$. The total kinetic energy after the decay is

$$K_{tot} = \tfrac{1}{2} m_e v_e^2 + \tfrac{1}{2} m_p v_p^2 = \tfrac{1}{2} m_e \left(\frac{m_p}{m_e}\right)^2 v_p^2 + \tfrac{1}{2} m_p v_p^2 = \tfrac{1}{2} m_p v_p^2 \left(1 + \frac{m_p}{m_e}\right).$$

Thus, $\frac{K_p}{K_{tot}} = \frac{1}{1 + m_p / m_e} = \frac{1}{1 + 1836} = 5.44 \times 10^{-4} = 0.0544\%$.

EVALUATE: Most of the released energy goes to the electron, since it is much lighter than the proton.