Homework 3.

① C. $\lim_{x\to 2}\dfrac{x^2+3x+2}{x^3+6x}$ direct substitution.

$$\frac{2^2+3\cdot 2+2}{2^3+6\cdot 2} = \frac{4+6+2}{8+12} = \frac{12}{20} = \frac{3}{5}.$$

② B $\lim_{x\to -2}\dfrac{x^2+4x+4}{x^2+2x} = \lim_{x\to -2}\dfrac{(x+2)^2}{x(x+2)} = \lim_{x\to -2}\dfrac{x+2}{x} = \dfrac{0}{-2}$

③ E $\lim_{x\to 11}\dfrac{\sqrt{x-7}-2}{x^2-12x+11} = \lim_{x\to 11}\dfrac{(\sqrt{x-7}-2)(\sqrt{x-7}+2)}{(x-11)(x-1)(\sqrt{x-7}+2)}$

$$= \lim_{x\to 11}\frac{\cancel{x-7-4}}{\cancel{(x-11)}(x-1)(\sqrt{x-7}+2)} = \lim_{x\to 11}\frac{1}{(x-1)(\sqrt{x-7}+2)}$$

$$= \frac{1}{(11-1)(\sqrt{11-7}+2)} = \frac{1}{10\cdot 4} = \frac{1}{40}.$$

④ A $f(x) = \dfrac{3x^4}{x^2+4x}$ $f'(1)$

$$f'(x) = \frac{12x^3(x^2+4x) - 3x^4(2x+4)}{(x^2+4x)^2}$$

$$f'(1) = \frac{12(1+4) - 3(6)}{(1+4)^2} = \frac{60-18}{5^2} = \frac{42}{25}$$

⑤ C. $f(x) = x^2 \sin x$ find $f'(x)$.

$$f'(x) = 2x\sin x + x^2\cos x = x(x\cos x + 2\sin x)$$

⑥ B. $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$. $\Longleftrightarrow \lim_{y\to 0} \frac{e^{y+1}-e}{y}$.

$\lim_{h\to 0} \frac{f(h+1)-f(1)}{h} \Longleftrightarrow \lim_{y\to 0} \frac{e^{y+1}-e}{y}$.

$f(y+1) = e^{y+1}$ $\qquad$ $f(y+1) = e^{y+1}$

$\boxed{f(1) = e^1} \rightarrow \underline{f(x)} = \underline{e^x}$

⑦ A. $\boxed{\lim_{x\to\pi} \frac{\cos x + 1}{x-\pi}}$. $\Longleftrightarrow \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$. $= f'(\pi)$.

$$\begin{cases} f(x) = \cos x \\ f(\pi) = -1 \end{cases} \Rightarrow f'(x) = -\sin x. \quad \boxed{f'(\pi) = 0}$$