CSD2201/CSD2200 Week 5 Tutorial Problems

25th September – 1st October 2023

It is recommended to treat the attempt of these problems seriously, even though they are not graded. You may refer to the lecture slides if you are unsure of any concepts.

After attempting each problem, think about what you have learnt from the attempt as a means of consolidating what you have learnt.

Starred (*) questions are slightly more conceptual/difficult.

Question 1

Let $n \geq 2$ be an integer.

(a) Show that

$$\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx.$$

(b) Using part (a), evaluate

$$\int \tan^2 x \sec^3 x \, dx.$$

Question 2

Determine if the following rational functions are proper or improper. If they are improper, perform long division.

(a) $\dfrac{3x-2}{x+1}$ (b) $\dfrac{x^3-4x+1}{x^2-3x+2}$ (c) $\dfrac{x^5+x-1}{x^3+1}$ (d) $\dfrac{x^3-1}{x-1}$

(e) $\dfrac{5}{x^4-1}$ (f) $\dfrac{x^2}{x^2+x-6}$ (g) $\dfrac{x^6}{x^2-4}$ (h) $\dfrac{x+4}{x^2+2x+5}$

Question 3

Evaluate the following integrals.

(a) $\int \tan^4 x \sec^4 x \, dx$ (b) $\int \tan^3 x \sec^3 x \, dx$ (c) $\int \sec^5 x \, dx$

(d) $\int \tan^4 x \sec^6 x \, dx$ (e) $\int \frac{1}{x^2 - 2x - 15} \, dx$ (f) $\int \frac{x}{2x^2 + 7x - 4} \, dx$

(g) $\int \frac{x^2}{x - 1} \, dx$ (h) $\int \frac{1}{x^4 - 2x^2 + 1} \, dx$ (i) $\int_1^2 \frac{x^3 + 4x^2 + x - 1}{x^3 + x^2} \, dx$

(j) $\int_0^1 \frac{x^2 + x + 1}{(x + 2)(x + 1)^2} \, dx$ (k) $(*) \int \frac{\sqrt{x + 4}}{x} \, dx$ (l) $(*) \int \frac{1}{2\sqrt{x + 3} + x} \, dx$

Final Answers:

Q1: (b) $\frac{\sec^3 x \tan x}{4} - \frac{1}{8}(\sec x \tan x + \ln|\sec x + \tan x|) + C$

Q2: (a) Improper, $3 - \frac{5}{x + 1}$, (b) Improper, $x + 3 + \frac{3x - 5}{x^2 - 3x + 2}$,

(c) Improper, $x^2 - \frac{x^2 - x + 1}{x^3 + 1}$, (d) Improper, $x^2 + x + 1$, (e) Proper, (f) $1 + \frac{6 - x}{x^2 + x - 6}$,

(g) Improper, $x^4 + 4x^2 + 16 + \frac{64}{x^2 - 4}$, (h) Proper

Q3: (a) $\frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C$, (b) $\frac{\sec^5}{5} - \frac{\sec^3 x}{3} + C$,

(c) $\frac{\sec^3 \tan x}{4} + \frac{3}{8}(\sec x \tan x + \ln|\sec x + \tan x|) + C$, (d) $\frac{\tan^9 x}{9} + \frac{2\tan^7 x}{7} + \frac{\tan^5 x}{5} + C$,

(e) $\frac{1}{8} \ln\left|\frac{x - 5}{x + 3}\right| + C$, (f) $\frac{1}{18} \ln|2x - 1| + \frac{4}{9} \ln|x + 4| + C$, (g) $\frac{x^2}{2} + x + \ln|x - 1| + C$,

(h) $\frac{1}{4} \ln\left|\frac{x + 1}{x - 1}\right| - \frac{1}{4}\left(\frac{1}{x + 1} + \frac{1}{x - 1}\right) + C$, (i) $\frac{1}{2} \ln 6$, (j) $\frac{1}{2} - \ln\left(\frac{32}{27}\right)$,

(k) $2(\sqrt{x + 4} + \ln|\sqrt{x + 4} - 2| - \ln|\sqrt{x + 4} + 2|) + C$,

(l) $\frac{1}{2}(3 \ln|\sqrt{x + 3} + 3| + \ln|\sqrt{x + 3} - 1|) + C$