1. Ans: D

   Let D be the event that tested person has the disease and E the event that the test result is positive. The desired probability is

$$\begin{aligned}
P(D|E) &= \frac{P(D \cap E)}{P(E)} \\
&= \frac{P(E|D)P(D)}{P(E|D)P(D) + P(E|D^c)P(D^c)} \\
&\frac{(0.95)(0.005)}{(0.95)(0.005) + (0.01)(0.995)} \\
&= 0.323
\end{aligned}$$

2. Ans: A

   Let A and B denote, respectively, the events that the student answer the question correctly and the event that he or she knows the answer. The probability that a student knew the answer to a question, given that he or she answered it correctly is

$$\begin{aligned}
P(B|A) &= \frac{P(A \cap B)}{P(A)} \\
&= \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)} \\
&= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{5}(1 - \frac{1}{2})} \\
&= \frac{5}{6}
\end{aligned}$$

3. Ans: D

   Let B be the event that coin B was the one flipped. The probability that coin B was the one flipped given that both flips land heads is

$$P(B|HH) = \frac{P(B|H)P(B|H)}{P(B|H)P(B|H) + P(B^c|H)P(B^c|H)} = \frac{(\frac{3}{4})(\frac{3}{4})}{(\frac{3}{4})(\frac{3}{4}) + (\frac{1}{4})(\frac{1}{4})} = \frac{9}{10}$$

4. Ans: B

   Let X denote the largest number selected. Then X is a random variable taking on one of the values $3, 4, \cdots, 20$. The number selections that result in the event $X = x$ is the number selections that result in ball numbered x and two other balls numbered 1 through $x - 1$ being chosen. The probability

$$P(X = x) = \frac{\binom{x-1}{2}}{\binom{20}{3}}, \qquad x = 3, 4, \cdots, 20$$

   From this equation,

$$P(X = 18) = \frac{\binom{17}{2}}{\binom{20}{3}} = 0.119$$

$$P(X = 19) = \frac{\binom{18}{2}}{\binom{20}{3}} = 0.134$$

$$P(X = 20) = \frac{\binom{19}{2}}{\binom{20}{3}} = 0.150$$

The probability that at least one of the drown balls has a number as large as or larger than 18 is

$$P(X \geq 18) = 0.119 + 0.134 + 0.150 = 0.403$$

5. Ans: A

   Let X denote the winnings from the experiment, then X is a random variable taking on the possible values $0, \pm 1, \pm 2, \pm 3$ with respective probabilities

$$P(X = 0) = \frac{\binom{5}{3} + \binom{3}{1}\binom{3}{1}\binom{3}{1}}{\binom{11}{3}} = \frac{1}{3}$$

$$P(X = 1) = P(X = -1) = \frac{\binom{3}{1}\binom{5}{2} + \binom{3}{2}\binom{3}{1}}{\binom{11}{3}} = \frac{13}{55}$$

$$P(X = 2) = P(X = -2) = \frac{\binom{3}{2}\binom{5}{1}}{\binom{11}{3}} = \frac{1}{11}$$

$$P(X = 3) = P(X = -3) = \frac{\binom{3}{3}}{\binom{11}{3}} = \frac{1}{165}$$

   The probability that we win money is

$$\sum_{i=1}^{3} P(X = i) = P(X = 1) + P(X = 2) + P(X = 3) = \frac{55}{165} = \frac{1}{3}$$

6. a) Ans: C

   Let E denote the event of a success on the first n trials, then the probability of no successes is

$$P(E_1^c)P(E_2^c)\cdots P(E_n^c) = (1-p)^n$$

   The probability of at least 1 success occur in the first n trials is $1 - (1-p)^n$

   b) Ans: C

   Consider the first n trials containing 5 successes and 5 failures, the probability is $p^5(1-p)^5$. As there are $\binom{10}{5}$ such sequences, the desired probability is

$$P(\text{exactly 5 successes in 10 trials}) = \binom{10}{5}(0.4)^5(0.6)^5 = 0.2$$

7. i) Ans: D

   Since $\sum_{x=0}^{\infty} p(x) = 1$, we have

$$\begin{aligned}
\sum_{x=0}^{\infty} \frac{c\alpha^x}{x!} &= 1 \\
c\sum_{x=0}^{\infty} \frac{\alpha^x}{x!} &= 1 \\
ce^{\alpha} &= 1 \\
c &= e^{-\alpha}
\end{aligned}$$

ii) Ans: C

$$
\begin{aligned}
P(X > 1) &= 1 - P(X \leq 1) \\
&= 1 - P(X = 0) - P(X = 1) \\
&= 1 - \frac{e^{-\alpha}\alpha^0}{0!} - \frac{e^{-\alpha}\alpha^1}{1!} \\
&= 1 - (1 + \alpha)e^{-\alpha}
\end{aligned}
$$

8. Ans: B

$$
P(X = 0) = 0.7 - 0.2 = 0.5
$$