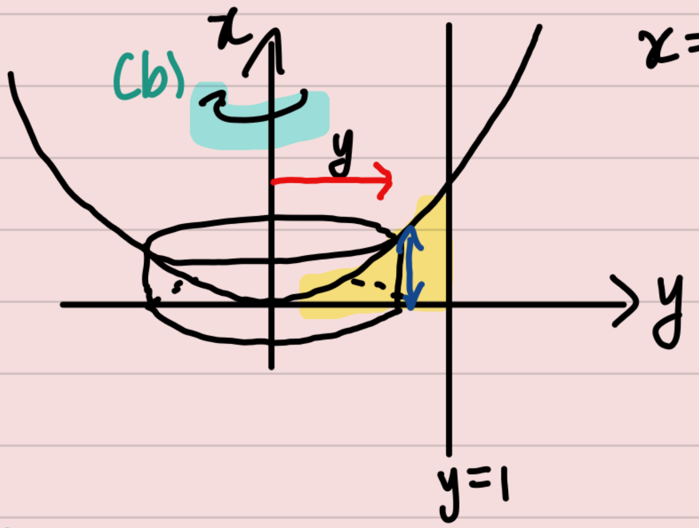
✓ ✓
Q2, 3, 6, 5, 7

Q2 (a) $x=\sqrt{5}y^2$, $y=1$, $x=0$. ↙ y-axis



$x=\sqrt{5}y^2$

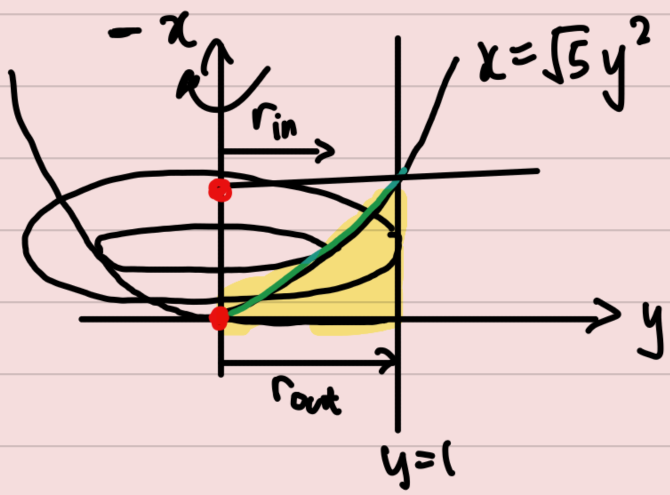
- Cross-sectional method
  axis of revolution
  ↔ same variable
- cylindrical shells method
  axis of revolution
  ↔ use the other variable

(b) Radius $= y$
(shells) Height $= \sqrt{5}\,y^2$

$$\text{Volume} = \int_0^1 2\pi \underbrace{y}_{\text{radius}} \underbrace{\sqrt{5}y^2}_{\text{height}}\,dy$$ ↔ Later compare w cross-sectional method

$$= 2\sqrt{5}\pi \int_0^1 y^3\,dy = 2\sqrt{5}\pi\left[\frac{y^4}{4}\right]_0^1 = \frac{\sqrt{5}\pi}{2}.$$ ←

Cross-sectional method (Alternative)



$x=\sqrt{5}y^2 \Rightarrow y^2 = \frac{x}{\sqrt{5}}$

$\Rightarrow y = \frac{\sqrt{x}}{5^{\frac{1}{4}}}$

$r_{out} = 1$,

$r_{in} = \frac{\sqrt{x}}{5^{\frac{1}{4}}}$

Cross-sectional area $= A(x) = \pi\left(r_{out}^2 - r_{in}^2\right)$

(Area of washer)

$$= \pi\left(1^2 - \left(\frac{\sqrt{x}}{5^{\frac{1}{4}}}\right)^2\right) = \pi\left(1 - \frac{x}{\sqrt{5}}\right)$$

$$\text{Volume} = \int_0^{\sqrt{5}} A(x)\,dx$$

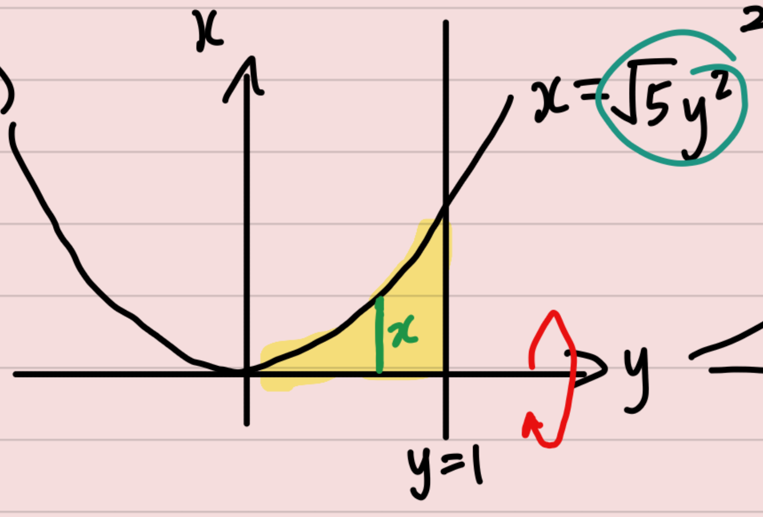
$$= \int_0^{\sqrt{5}} \pi\left(1 - \frac{x}{\sqrt{5}}\right) dx$$

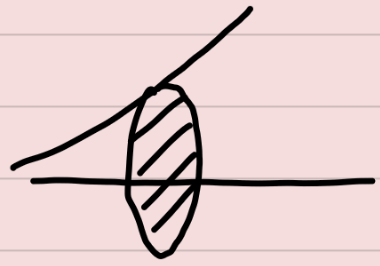
$$= \pi\left[x - \frac{x^2}{2\sqrt{5}}\right]_0^{\sqrt{5}} = \pi\left[\sqrt{5} - \frac{5}{2\sqrt{5}}\right]$$

$$= \pi\left[\sqrt{5} - \frac{\sqrt{5}}{2}\right]$$

$$= \frac{\sqrt{5}\pi}{2}.$$

(c)





<u>Cross-sectional Method</u> (variable y)

Radius $= \sqrt{5}y^2$,

Cross-sectional area $= A(y) = \pi\left(\sqrt{5}y^2\right)^2 = 5\pi y^4$

$$\text{Volume} = \int_0^1 A(y)\,dy = \underline{\int_0^1 5\pi y^4\,dy} = 5\pi\left[\frac{y^5}{5}\right]_0^1 = \pi.$$

Cylindrical Shells Method

Radius $= x$

Height $= 1 - \dfrac{\sqrt{x}}{5^{\frac{1}{4}}}$

$$\text{Volume} = \int_0^{\sqrt{5}} 2\pi x \left(1 - \frac{\sqrt{x}}{5^{1/4}}\right) dx$$

$$= 2\pi \int_0^{\sqrt{5}} x - \frac{x^{3/2}}{5^{\frac{1}{4}}}\, dx = 2\pi \left[\frac{x^2}{2} - \frac{1}{5^{\frac{1}{4}}} \frac{x^{5/2}}{5/2}\right]_0^{\sqrt{5}}$$

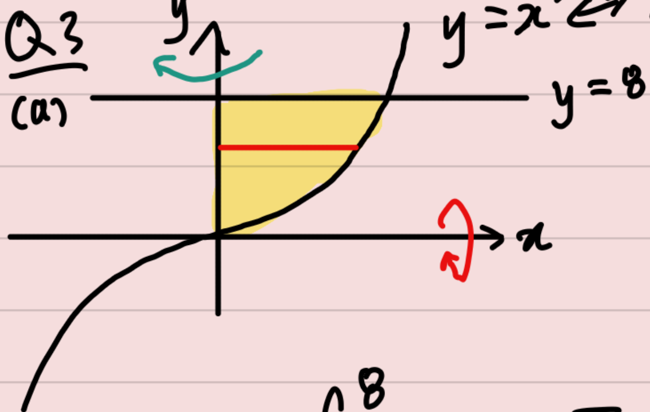
$$= 2\pi\left(\frac{(\sqrt{5})^2}{2} - \frac{1}{5^{\frac{1}{4}}}\frac{(\sqrt{5})^{5/2}}{5/2}\right)$$

$$= 2\pi\left(\frac{5}{2} - \frac{1}{\cancel{5^{\frac{1}{4}}}} \cdot \frac{2}{\cancel{5}}\, \underbrace{5^{5/4}}_{\cancel{5}\cdot\cancel{5^{\frac{1}{4}}}}\right)$$

$$= 2\pi\left(\frac{5}{2} - 2\right) = 2\pi \cdot \frac{1}{2} = \pi$$

Q3

(a)



$y = x^3 \leftrightarrow x = \sqrt[3]{y}$

(b) Cshells

Radius $= y$

Height $= \sqrt[3]{y}$

2^7

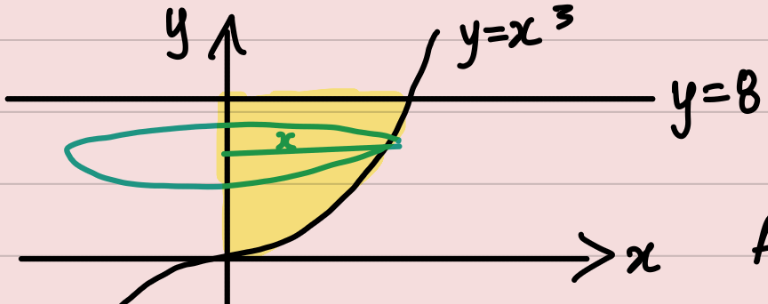
$\downarrow$

$8^{7/3}$

$$\text{Volume} = \int_0^8 2\pi \underline{y} \cdot \underline{\sqrt[3]{y}}\, dy$$

$$= 2\pi \int_0^8 y^{\frac{4}{3}}\, dy = 2\pi \left[\frac{y^{7/3}}{7/3}\right]_0^8 = \frac{6\pi}{7} \cdot 128 = \frac{768\pi}{7}.$$

(c) Cross-Sectional Method



Variable in y

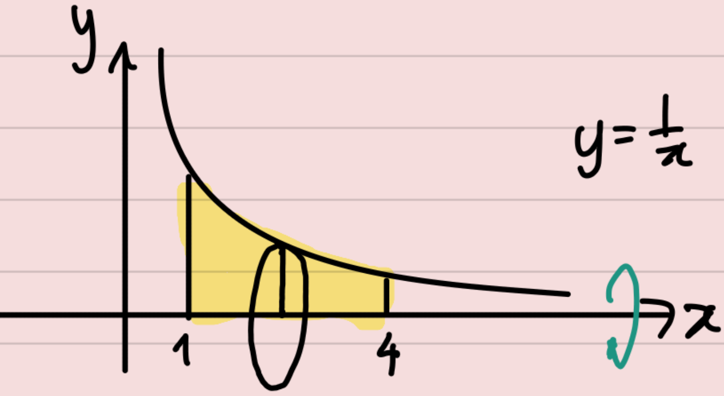
Radius $= \sqrt[3]{y}$

$A(y) = \pi\left(\sqrt[3]{y}\right)^2 = \pi y^{2/3}$

$$\text{Volume} = \int_0^8 \pi y^{2/3}\,dy = \pi\left[\frac{y^{5/3}}{5/3}\right]_0^8$$

$$= \frac{3\pi}{5}\cdot\underbrace{\left[8^{5/3}\right]}_{32} = \frac{96\pi}{5}.$$

Q5



(b) Cross-sectional Method Radius $= \frac{1}{x}$

Cross-sectional area $= A(x) = \pi\left(\frac{1}{x}\right)^2 = \pi\frac{1}{x^2}$

$$\text{Volume} = \int_1^4 \pi\frac{1}{x^2}\,dx = \pi\int_1^4 \underbrace{\frac{1}{x^2}}_{x^{-2}}\,dx = \pi\left[-\frac{1}{x}\right]_1^4$$

$$= \pi\left[-\frac{1}{4}+1\right] = \frac{3\pi}{4}$$

(c)

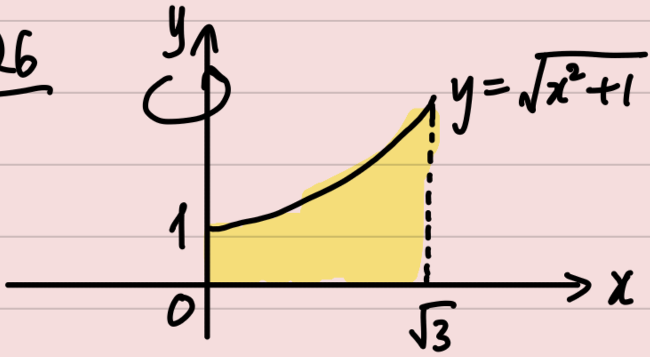
Cross-sectional method is tedious.

Cshells Radius $= x$

Height $= \frac{1}{x}$

$$\text{Volume} = \int_1^4 2\pi x\cdot\frac{1}{x}\,dx = 6\pi.$$

Q6



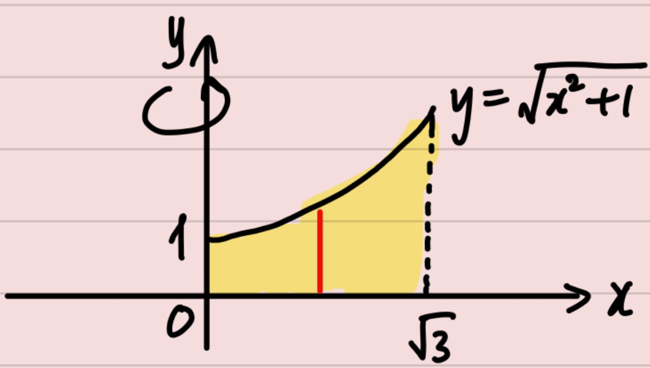
(a) Cross-sectional method

Radius $= \sqrt{x^2+1}$

$$A(x) = \pi\left(\sqrt{x^2+1}\right)^2 = \pi(x^2+1)$$

$$\text{Volume} = \int_0^{\sqrt{3}} \pi(x^2+1)\,dx = \pi\left[\frac{x^3}{3} + x\right]_0^{\sqrt{3}} = \pi\left[\frac{3^{3/2}}{3^1} + \sqrt{3}\right] = \pi\left[\sqrt{3} + \sqrt{3}\right] = 2\sqrt{3}\,\pi$$

(c) Cshells



Radius $= x$

Height $= \sqrt{x^2+1}$

$$\text{Volume} = \int_0^{\sqrt{3}} 2\pi\, x\sqrt{x^2+1}\,dx$$

$u = \sqrt{x^2+1}$ $\quad du = \dfrac{2x}{2\sqrt{x^2+1}}\,dx = \dfrac{x}{\sqrt{x^2+1}}\,dx$

$\sqrt{x^2+1}\,du = x\,dx$ ← insert $\sqrt{x^2+1}$

$\sqrt{x^2+1}\,\sqrt{x^2+1}\,du = x\sqrt{x^2+1}\,dx$

$\sqrt{x^2+1}\,\sqrt{x^2+1}\,du = u^2\,du$

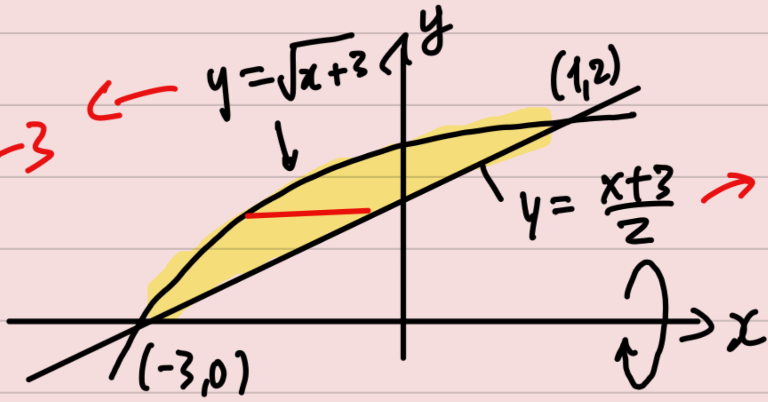
$x = 0 \rightarrow u = 1$

$x = \sqrt{3} \rightarrow u = 2$

$$= \int_1^2 2\pi u^2\,du = 2\pi \underbrace{\left[\frac{u^3}{3}\right]_1^2}_{7/3} = \frac{14\pi}{3}$$

Q7

$x = y^2 - 3$



$x = 2y - 3$

Cross-Sectional

$r_{out} = \sqrt{x+3} \quad r_{in} = \frac{x+3}{2}$

$A(x) = \pi\left(\left(\sqrt{x+3}\right)^2 - \left(\frac{x+3}{2}\right)^2\right)$

$= \pi\left(x+3 - \frac{(x+3)^2}{4}\right)$

$V = \int_{-3}^{1} \pi\left(x+3 - \frac{(x+3)^2}{4}\right) dx$

C Shells

Height $= (2y-3) - (y^2-3)$

$= 2y - y^2$

Radius $= y$

$V = \int_0^2 2\pi y\,(2y - y^2)\,dy$