#### CSD1100

# Boolean Expression Simplification

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#### Recap: Boolean Expression Simplification

- Digital computers contain circuits that implement Boolean logic.
- The simpler that we can make a Boolean expression, the smaller the circuit that will result.
- With this in mind, we always want to reduce our Boolean expressions to their simplest form.
- There are a number of Boolean identities that help us to do this.

#### **Boolean Identities: Trivial**

Logical Inverse	!0 = 1	!1 = 0
Involution	!!A = A	
Dominance	A+1=1	A 0=0
Identity	A+0=A	A 1=A
Idempotence	A+A=A	AA=A
Complementarity	A+!A=1	A !A=0
Commutativity	A+B=B+A	A B=B A
Associativity	(A+B)+C=A+(B+C)	(AB) C=A (BC)

#### **Boolean Identities: Non-Trivial**

Distributivity	A (B+C) = A B+A C	A+B C = (A+B) (A+C)
Absorption	A (A+B) = A	A+AB=A
DeMorgan's	!(A+B) = !A !B	!(A B) = !A+!B
Unnamed	A+!A B = A+B	
This one is usefull in assignment	X Y+!X Z+Y Z = X Y+!X Z	

#### Absorption 1

$$A + (A \cdot B) = (A \cdot 1) + (A \cdot B)$$
  
=  $A \cdot (1 + B)$   
=  $A \cdot 1$   
 $\therefore A + (A \cdot B) = A$ 

#### Absorption 2

$$A \cdot (A + B) = (A \cdot A) + (A \cdot B)$$

$$= A + (A \cdot B)$$

$$= (A \cdot 1) + (A \cdot B)$$

$$= A \cdot (1 + B)$$

$$= A \cdot 1$$

$$\therefore A \cdot (A + B) = A$$

#### Chain Of Absorptions

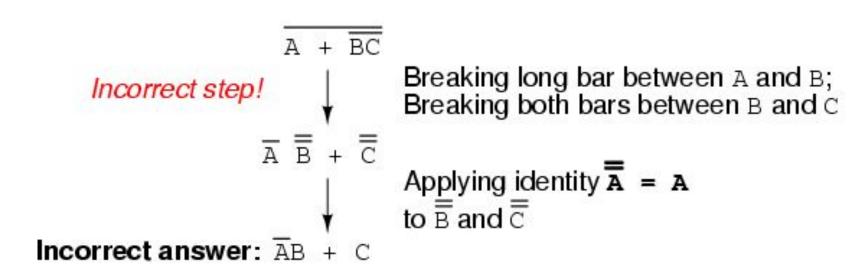
• A+AB+AC+AD+AE+ ... = A

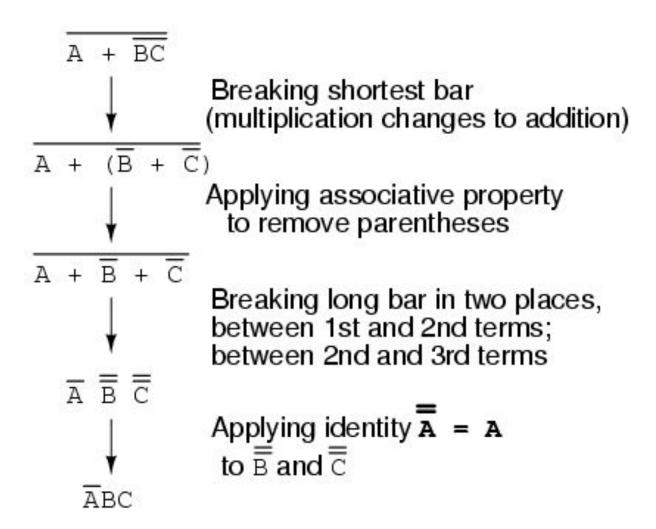
#### Let's prove last one

$$AB + BC(B + C)$$

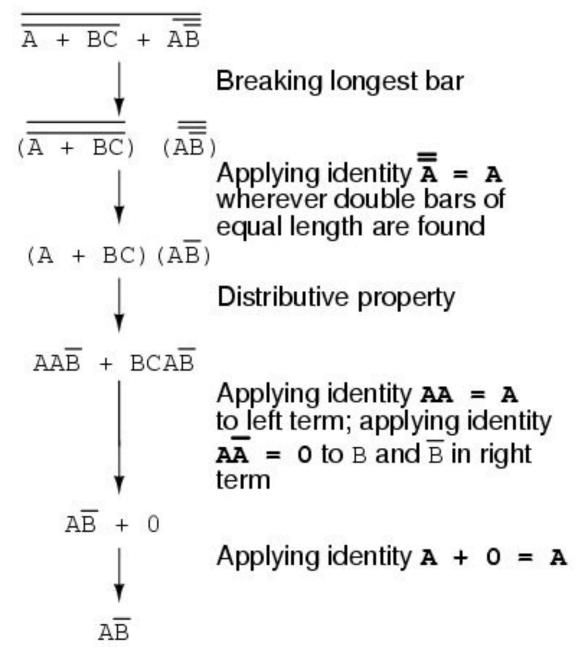
$$A + B(A + C) + AC$$

$$\overline{A + BC}$$





$$\overline{\overline{A} + BC} + \overline{\overline{AB}}$$



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$$\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$$\overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC}$$

$$Factoring BC out of 1^{st} and 4^{th} terms$$

$$BC(\overline{A} + \overline{A}) + \overline{ABC} + \overline{ABC}$$

$$Applying identity A + \overline{A} = 1$$

$$BC(1) + \overline{ABC} + \overline{ABC}$$

$$Applying identity 1A = A$$

$$BC + \overline{ABC} + \overline{ABC}$$

$$Factoring B out of 1^{st} and 3^{rd} terms$$

$$B(C + \overline{AC}) + \overline{ABC}$$

$$Applying rule A + \overline{AB} = A + B to the C + \overline{AC} term$$

#### References

 https://www.allaboutcircuits.com/textbook/digital/chpt-7/bo olean-algebraic-identities/