CSD2301 Lecture

# 5. Application of Newton's Laws: Part 1

LIN QINJIE

# Outline

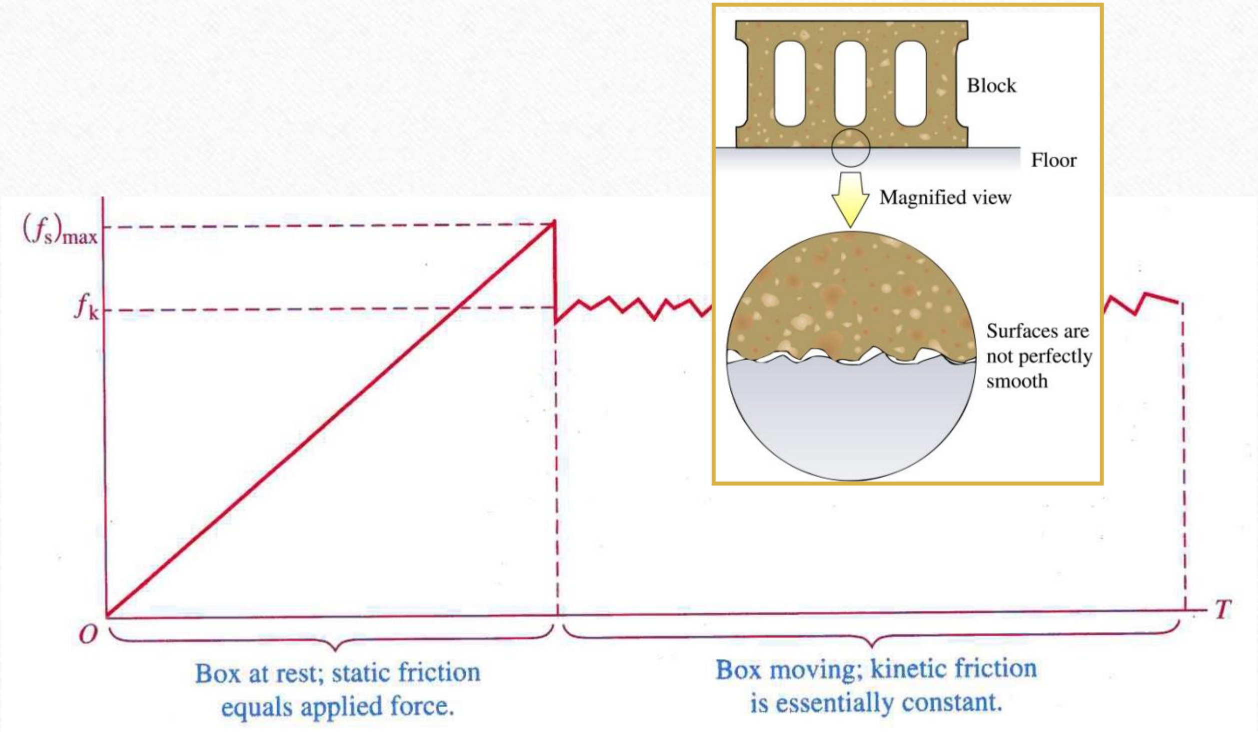
- Friction force
  - Coefficients of friction
- Resistive forces
- Terminal speed/velocity
- Archimedes' Principle

# Friction Force

- When a body moves on a surface or through a viscous medium (e.g. air, water), there are forces of friction because the body interacts with its surroundings.
- Force of **static** friction, $\mathbf{f}_s$ : the force that counteracts the applied force and keeps the object from moving
  - Present when object is not yet in motion
- Force of **kinetic** friction, $\mathbf{f}_k$ : the retarding frictional force on the object in motion
  - Present when object is in motion

# Concept of Friction Force

- $f_s$ increases as the magnitude of the applied force increases, keeping the object in place
- When it is on the verge of moving, $f_s$ is a maximum
- When the applied force exceeds $(f_s)_{max}$, the object accelerates
- Once the object is in motion, the frictional force ($f_k$) becomes less than $(f_s)_{max}$



# Empirical Laws of Friction

- The direction of the force of static friction between any two surfaces in contact is opposite the direction of any applied force and can have values:

$$f_s \leq \mu_s n$$ where $\mu_s$ is coefficient of static friction.

- The direction of the force of kinetic friction acting on an object is opposite the direction of its motion and is given by:

$$f_k = \mu_k n$$ where $\mu_k$ is coefficient of kinetic friction.

# Coefficients of Friction

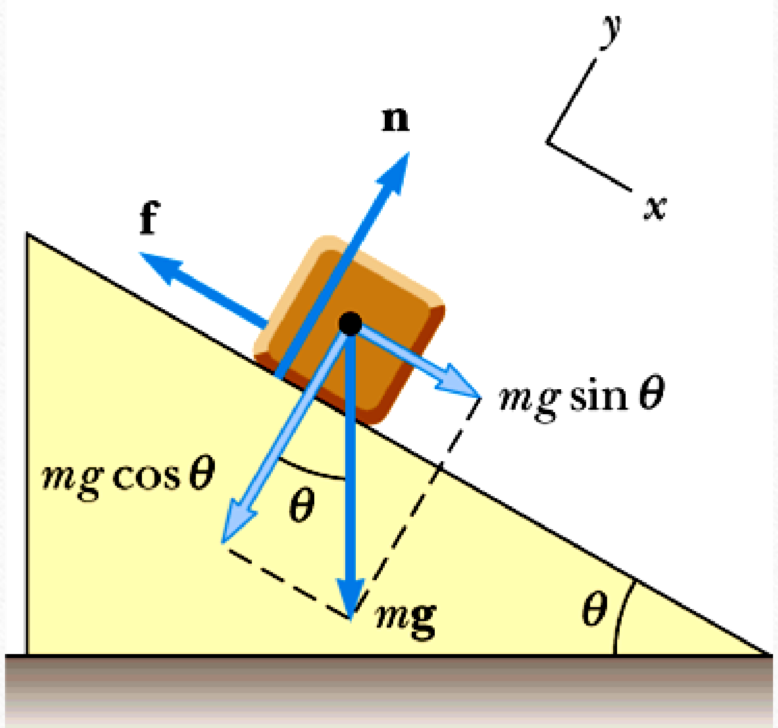
- The values of $\mu_s$ and $\mu_k$ depend on the nature of the surfaces (0.05-1.5), but $\mu_k$ is generally $< \mu_s$.
- The coefficients of friction are nearly independent of the area of contact between the surfaces.
- Although $\mu_k$ varies with $v$, we normally neglect this (stick-slip motion at low $v$).

# Coefficients of Friction

| | $\mu_s$ | $\mu_k$ |
|---|---|---|
| Steel on steel | 0.74 | 0.57 |
| Aluminum on steel | 0.61 | 0.47 |
| Copper on steel | 0.53 | 0.36 |
| Rubber on concrete (Dry) | 1.0 | 0.8 |
| Rubber on concrete (Wet) | 0.30 | 0.25 |
| Zinc on cast iron | 0.85 | 0.21 |
| Copper on cast iron | 1.05 | 0.29 |
| Glass on glass | 0.94 | 0.40 |
| Copper on Glass | 0.68 | 0.53 |
| Teflon on Teflon | 0.04 | 0.04 |
| Teflon on steel | 0.04 | 0.04 |
| Synovial joints in humans | 0.01 | 0.003 |

# Experimental Determination of Coefficients of Friction

- Suppose a block is placed on a rough surface inclined relative to the horizontal. The incline angle is increased until the block starts to move. By measuring the critical angle, θc at which this slipping just occurs, we can **obtain $\mu_s$**.
- Once the block starts to move, it accelerates down the incline. However, if θ is reduced to a value less than $\theta_c$, it may be possible to find an angle $\theta'_c$ such that the block moves down the incline with constant speed. We can then **obtain $\mu_k$**.



# Experimental Determination of Coefficients of Friction

$$\sum F_x = mg\sin\theta - f_s = ma_x = 0 \Rightarrow f_s = mg\sin\theta$$

$$\sum F_y = n - mg\cos\theta = ma_y = 0 \Rightarrow n = mg\cos\theta$$

$$\Downarrow$$

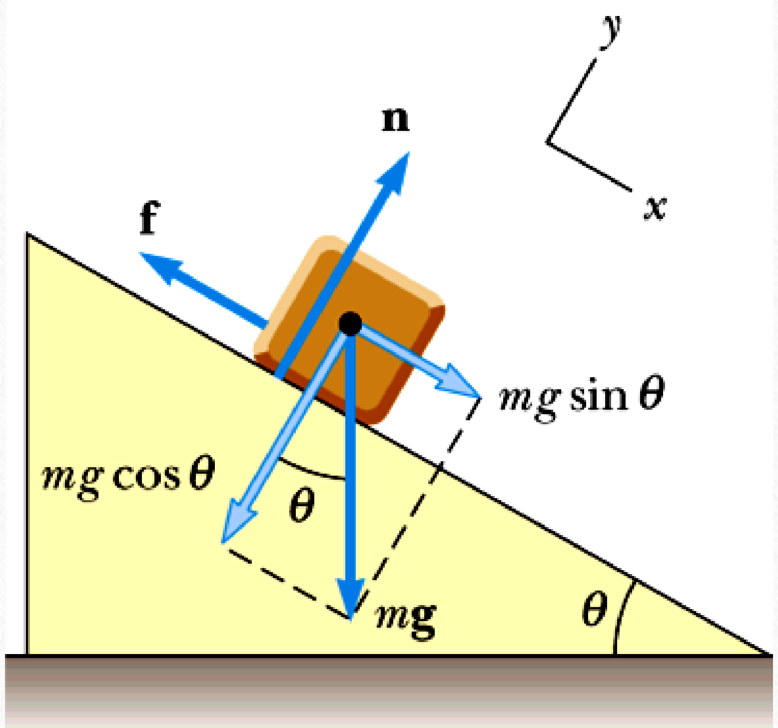
$$f_s = n\tan\theta$$

Since $f_s \leq \mu_s n$

$$\Downarrow$$

$$\boxed{\mu_s = \tan\theta_c}$$

If moving at constant **v**: $f_k = \mu_k n \Rightarrow \boxed{\mu_k = \tan\theta_c'}$

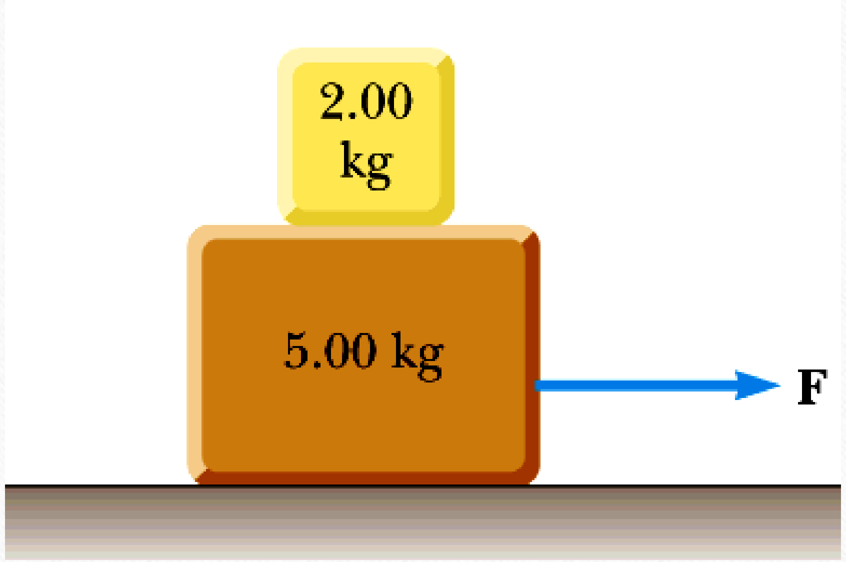


# Example

- A 2.00-kg block is placed on top of a 5.00-kg block. The coefficient of kinetic friction between the 5.00 kg block and the surface is 0.200.

a) Calculate the force needed to pull both blocks with an acceleration of 3.00 m/s$^2$.

b) Find the minimum coefficient of static friction between the blocks such that the upper block does not slip under this acceleration.



# Example

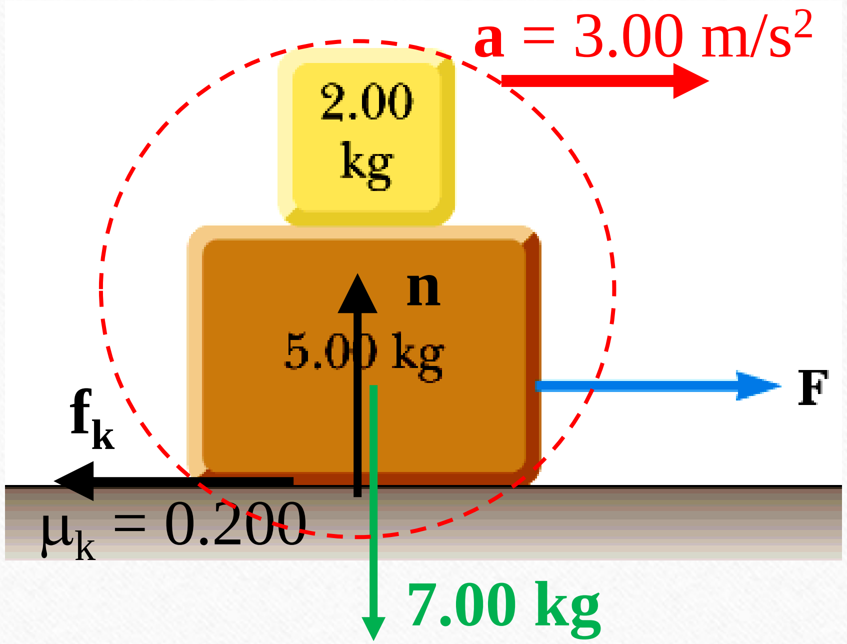
Part (a): Consider total mass of 7 kg,

$$n = 7.00(9.80) = 68.6 \text{ N}$$

$$f_k = \mu_k n = 0.200(68.6) = 13.7 \text{ N}$$

$$\sum F_x = F - f_k = (m_1 + m_2)a$$

$$\boxed{F = 7.00(3.00) + 13.7 = 34.7 \text{ N}}$$



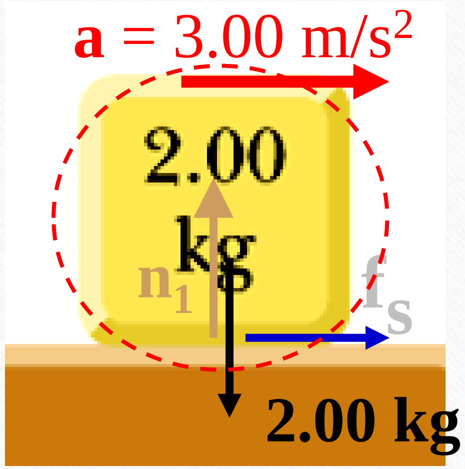
# Example

Part (b): Consider the mass of 2 kg alone,

$$\sum F_y = n_1 - m_1 g = 0$$

$$n_1 = 2.00(9.80) = 19.6 \text{ N}$$

$$f_s \leq \mu_s n_1 \Rightarrow \boxed{\mu_s \geq \frac{\overbrace{6.00}^{ma}}{19.6} = 0.306}$$



# Resistive Forces

- Interaction between moving object and medium (liquid, gas) sometimes cannot be neglected.
- The medium exerts a resistive force **f** on the object opposite to its direction of motion.

# Resistive Forces

- The magnitude of the resistive force, ***f***, generally increases with increasing speed; the actual dependence is complicated; common approximations:
  - For small objects at low speeds (e.g. dust in air):

$$\boxed{f = kv}$$

  - For large objects at high speeds (e.g. skydiver):

$$\boxed{f = Dv^2}$$

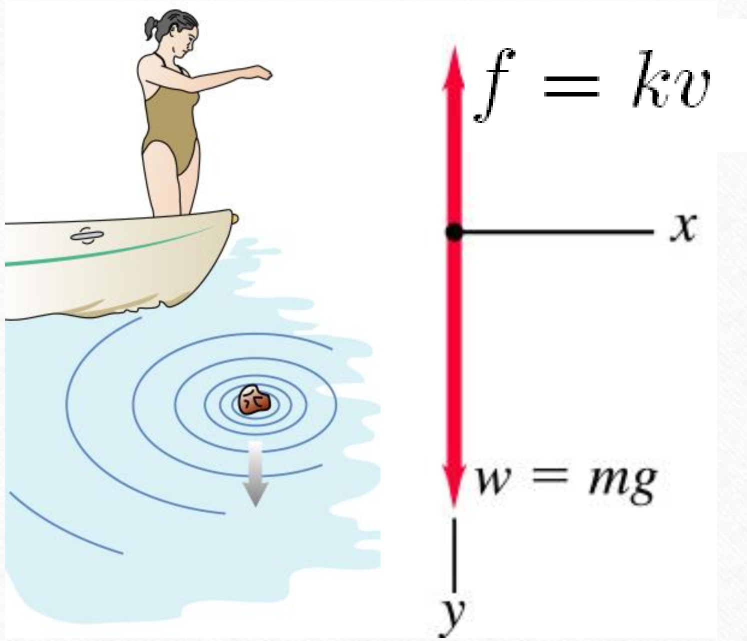
# Resistive Forces at Low Speeds

$$\sum F_y = ma = m\frac{dv}{dt}$$

$$f = kv$$

$$mg - kv = m\frac{dv}{dt}$$

$$\boxed{\frac{dv}{dt} = g - \frac{k}{m}v}$$

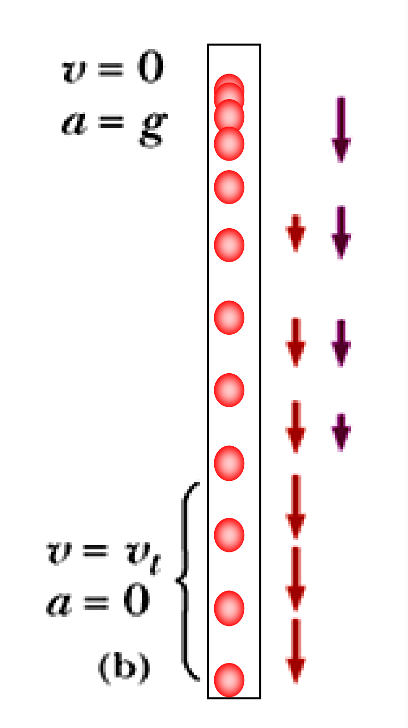



- k is a constant value which depends on medium and the object.

# Terminal Speed/Velocity

$$\boxed{\frac{dv}{dt} = g - \frac{k}{m}v}$$

- When $v = 0$, the resistive force $-kv$ is also zero and the acceleration is simply $g$
- As $t$ increases, the resistive force increases and the acceleration decreases.
- Eventually, the acceleration becomes zero and object moves with a constant speed $v_t$ which is the **terminal speed**.




$$mg - kv_t = 0$$

$$\boxed{v_t = \frac{mg}{k}}$$

# Terminal Speed/Velocity

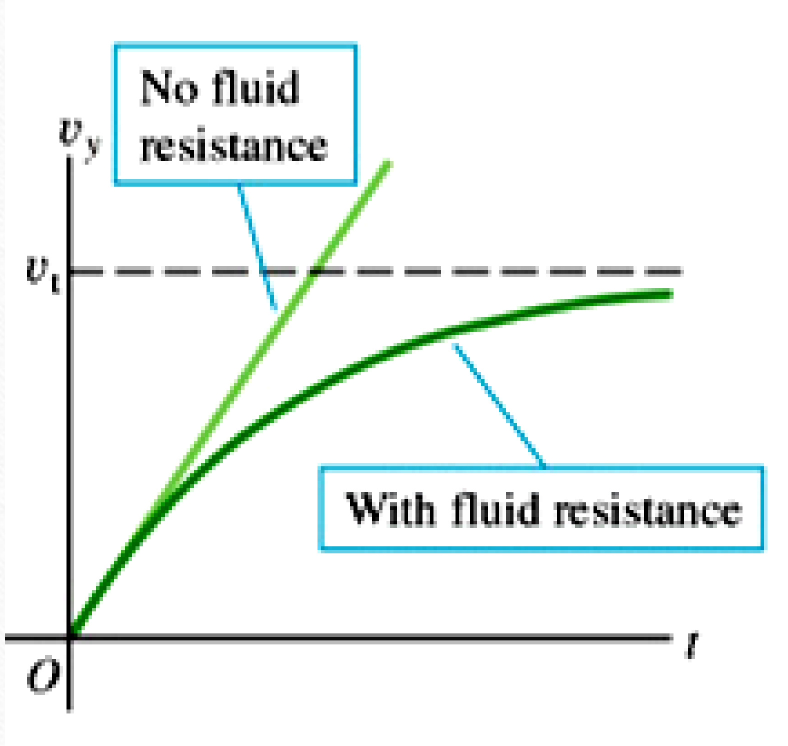
$$\frac{dv}{dt} = g - \frac{k}{m}v$$

First order ODE

$$v = \frac{mg}{k}\left(1 - e^{-kt/m}\right)$$

Define (time constant) $\tau = \frac{m}{k}$

$$\boxed{v = v_t\left(1 - e^{-t/\tau}\right)}$$



# Air Drag at High Speeds

- For objects moving at high speeds through air, the resistive force is $\propto$ square of the speed:

$$\boxed{f = Dv^2}$$ from (Young & Freedman)

- A more detailed formula is given in Serway & Jewett:

$$\boxed{f = \frac{1}{2} D\rho A v^2}$$

$\rho$ = density of air
$A$ = cross-sectional area of the falling object
(measured in a plane $\perp$ its motion)
$D$ = drag coefficient

# Air Drag at High Speeds

**Young & Freedman**

$$mg - Dv^2 = ma$$

$$a = g - \frac{D}{m}v^2$$

When a = 0

$$\boxed{v_t = \sqrt{\frac{mg}{D}}}$$

**Serway & Jewett**

$$mg - \frac{1}{2}D\rho A v^2 = ma$$

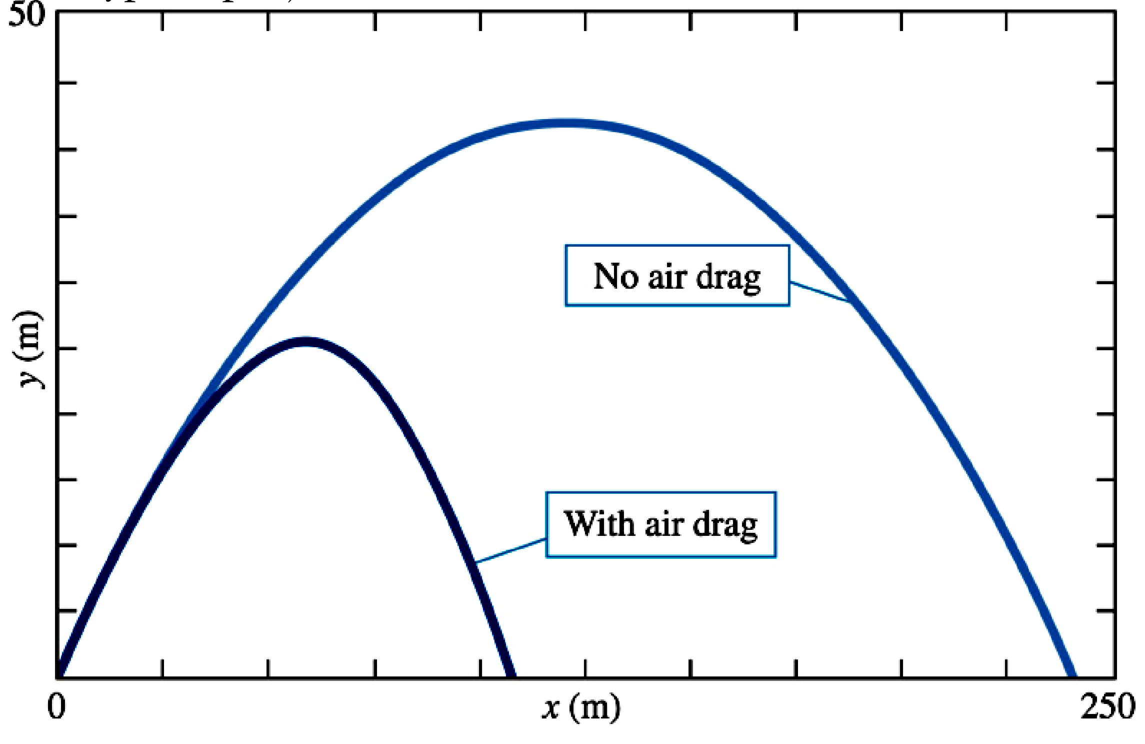
$$a = g - \left(\frac{D\rho A}{2m}\right)v^2$$

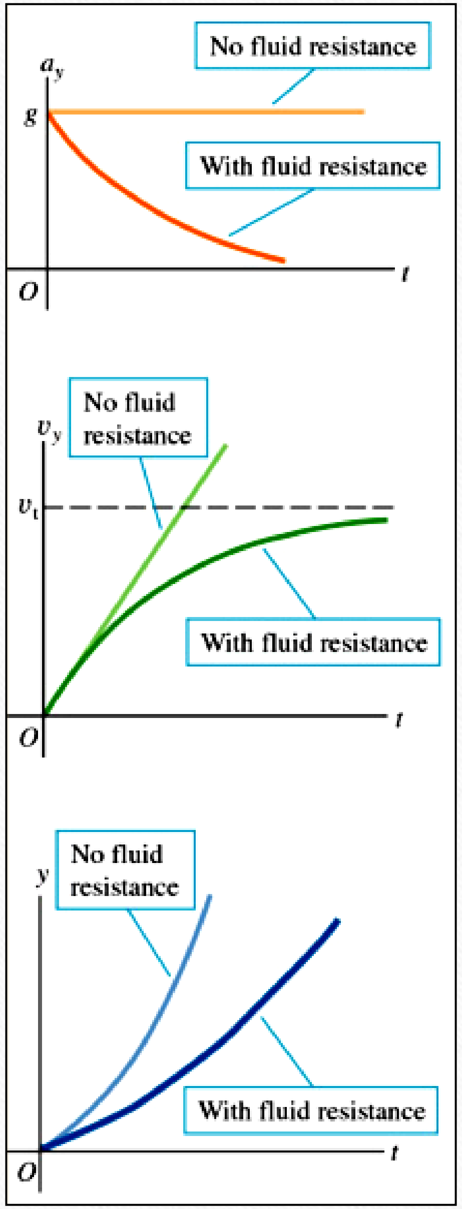
When a = 0

$$\boxed{v_t = \sqrt{\frac{2mg}{D\rho A}}}$$

# Comparing With/Without Resistive Forces

Typical projectile motion



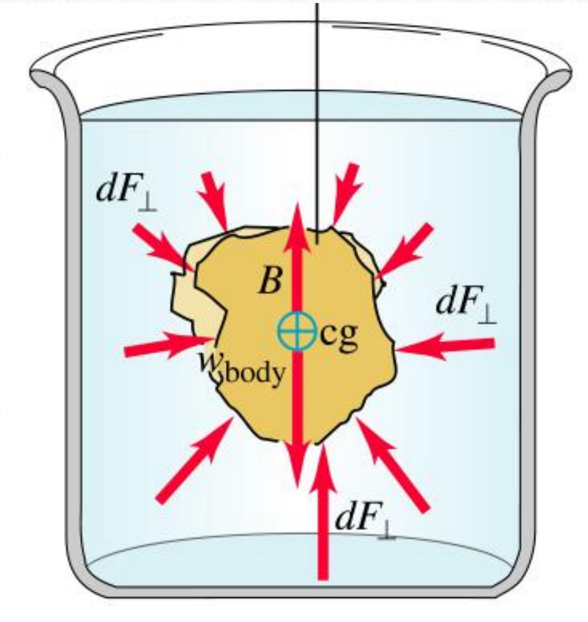




Typical free falling object

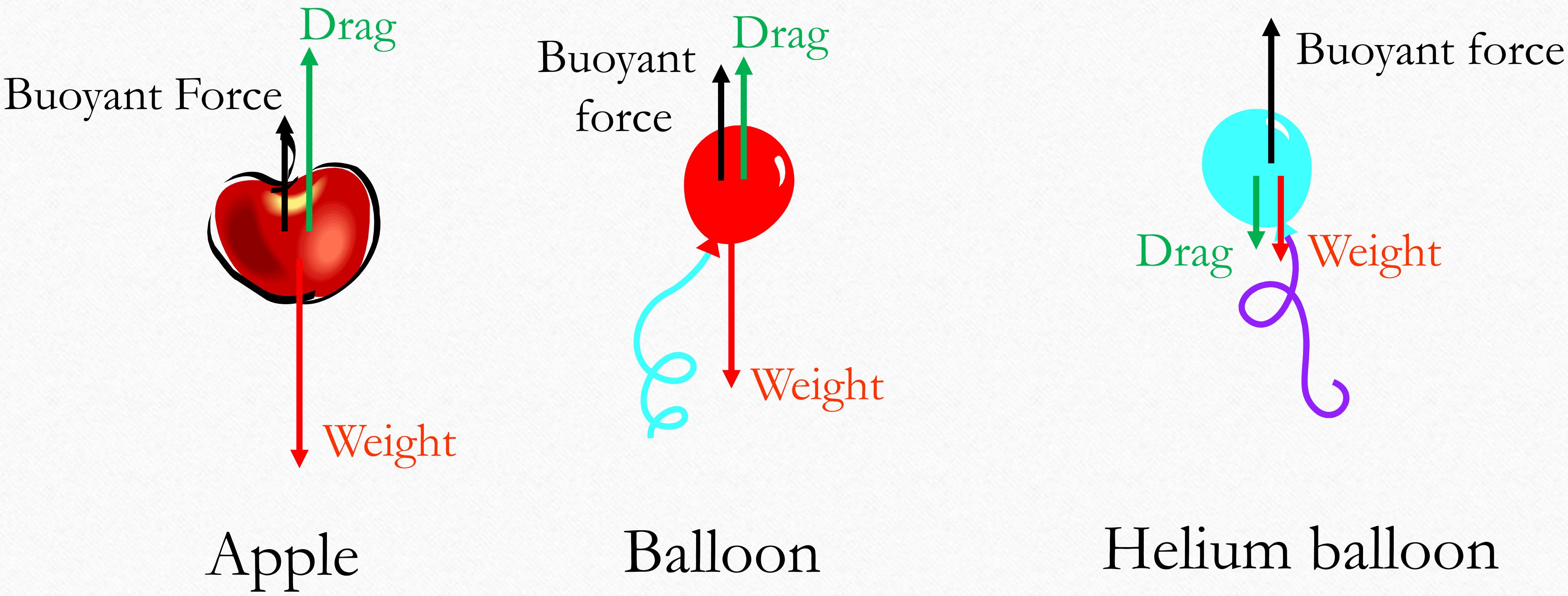
# Archimedes' Principle

- States that: When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.
- The upward force that a fluid exerts on any submerged object is called the buoyant force.

$$\boxed{B = W_{fluid}}$$



# Understanding Various Resistive Forces



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