CSD1241 Tutorial 9

Problem 1. Let T be the scaling given by the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

(a) Find the images of the points $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$.

(b) Find all points $\vec{x}$ that are fixed under this transformation, that is, $T(\vec{x}) = \vec{x}$.

(c) Find the image of the plane $\beta : 2x + 3y - 4z = 12$ under T.

(d) Find the image of the line $l : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ under T.

(e) Let Q be the intersection of β and l. Find the image of Q under T.

Problem 2. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the shear with respect to the xy-plane (equation $z = 0$) and shearing vector $\vec{v} = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$ (note that $\vec{v}$ is parallel to the xy-plane).

(a) Find the matrix of T and the images of the points $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$.

(b) Find all points $\vec{x}$ that are fixed by T, that is, $T(\vec{x}) = \vec{x}$.

(c) Find the image β' of the plane $\beta : x - 2y + 3z = 9$ under T.

(d) Find the image of the line $l : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 7 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ under T.

(e) Let Q be the point of intersection of β and l. Find the image of Q.

Problem 3. Let T be the rotation about the vector $\vec{v} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ over $60°$.

a) Find the matrix A of this transformation.

b) Find the images $\begin{pmatrix} 1 \\ -1 \\ -7 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$.

c) Find all points $\vec{x}$ that are fixed under this transformation, i.e. $T(\vec{x}) = \vec{x}$.
d) Find the image of the plane $\beta : 3x - 2y - z = 9$ under T.
e) Find the image of the line $l : \vec{x} = \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ under T.
f) Let Q be the point of intersection of β and l. Find the image of Q.

Problem 4. In this problem, we learn that the composition of two reflections is a rotation. The following maps S, T were used in Tutorial 8 (Problem 5).
Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the reflection through the xz-plane, and let $S : \mathbb{R}^3 \to \mathbb{R}^3$ be the reflection through the plane $x - y = 0$.
(a) Find the matrices M and N of the composition $T \circ S$ and $S \circ T$.
(b) Show that both $T \circ S$ and $S \circ T$ are rotations.