CSD1100

Boolean Expression Simplification

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Recap: Boolean Expression Simplification

- Digital computers contain circuits that implement Boolean logic.
- The simpler that we can make a Boolean expression, the smaller the circuit that will result.
- With this in mind, we always want to reduce our Boolean expressions to their simplest form.
- There are a number of Boolean identities that help us to do this.

Boolean Identities: Trivial

Logical Inverse	!0 = 1	!1 = 0
Involution	!!A = A	
Dominance	A+1=1	A 0=0
Identity	A+0=A	A 1=A
Idempotence	A+A=A	A A=A
Complementarity	A+!A=1	A !A=0
Commutativity	A+B=B+A	A B=B A
Associativity	(A+B)+C=A+(B+C)	(A B) C=A (B C)

Boolean Identities: Non-Trivial

Distributivity	A (B+C) = A B+A C	A+B C = (A+B) (A+C)
Absorption	A (A+B) = A	A+A B = A
DeMorgan's	!(A+B) = !A !B	!(A B) = !A+!B
Unnamed	A+!A B = A+B	
This one is usefull in assignment	X Y+!X Z+Y Z = X Y+!X Z	

Absorption 1

$$
\begin{aligned}
A + (A \cdot B) &= (A \cdot 1) + (A \cdot B) \\
&= A \cdot (1 + B) \\
&= A \cdot 1 \\
\therefore A + (A \cdot B) &= A
\end{aligned}
$$

Absorption 2

$$\begin{aligned} A \cdot (A + B) &= (A \cdot A) + (A \cdot B) \\ &= A + (A \cdot B) \\ &= (A \cdot 1) + (A \cdot B) \\ &= A \cdot (1 + B) \\ &= A \cdot 1 \\ \therefore A \cdot (A + B) &= A \end{aligned}$$

Chain Of Absorptions

- $A+AB+AC+AD+AE+ \ldots = A$

Let's prove last one

XY+!XZ+YZ = XY+!XZ

XY+!XZ+YZ =>

XY+!XZ+1YZ =>

XY+!XZ+(X+!X)YZ =>

XY+!XZ+XYZ+!XYZ =>

(XY+XYZ)+(!XZ+!XYZ) =>

XY(1+Z)+ !XZ(1+Y) => XY+!XZ

Example

$$AB + BC(B + C)$$

Example

$AB + BC(B + C)$

$\downarrow$ Distributing terms

$AB + BBC + BCC$

$\downarrow$ Applying identity $AA = A$ to 2nd and 3rd terms

$AB + BC + BC$

$\downarrow$ Applying identity $A + A = A$ to 2nd and 3rd terms

$AB + BC$

$\downarrow$ Factoring B out of terms

$B(A + C)$

Example

$$A + B(A + C) + AC$$

Example

`A + B(A + C) + AC`

↓ Distributing terms

`A + AB + BC + AC`

↓ Applying rule **`A + AB = A`** to 1st and 2nd terms

`A + BC + AC`

↓ Applying rule **`A + AB = A`** to 1st and 3rd terms

`A + BC`

Example

$$\overline{A + \overline{BC}}$$

Example

$$\overline{A + \overline{BC}}$$

$$\downarrow$$

Breaking longest bar
(addition changes to multiplication)

$$\overline{A}\ \overline{\overline{BC}}$$

$$\downarrow$$

Applying identity $\overline{\overline{A}} = A$
to $\overline{\overline{BC}}$

$$\overline{A}BC$$

Example

$$\overline{A + \overline{BC}}$$

Incorrect step!

Breaking long bar between A and B;
Breaking both bars between B and C

$$\overline{A}\;\overline{\overline{B}} + \overline{\overline{C}}$$

Applying identity $\overline{\overline{A}} = A$
to $\overline{\overline{B}}$ and $\overline{\overline{C}}$

Incorrect answer: $\overline{A}B + C$

Example

$$\overline{A + \overline{BC}}$$

$\downarrow$ Breaking shortest bar (multiplication changes to addition)

$$\overline{A + (\overline{B} + \overline{C})}$$

$\downarrow$ Applying associative property to remove parentheses

$$\overline{A + \overline{B} + \overline{C}}$$

$\downarrow$ Breaking long bar in two places, between 1st and 2nd terms; between 2nd and 3rd terms

$$\overline{A}\ \overline{\overline{B}}\ \overline{\overline{C}}$$

$\downarrow$ Applying identity $\overline{\overline{A}} = A$ to $\overline{\overline{B}}$ and $\overline{\overline{C}}$

$$\overline{A}BC$$

Example

$$\overline{\overline{A + BC} + \overline{A\overline{B}}}$$

Example

$$\overline{\overline{A + BC} + \overline{A\overline{B}}}$$

Breaking longest bar

$$(\overline{\overline{A + BC}})\ (\overline{\overline{A\overline{B}}})$$

Applying identity $\overline{\overline{A}} = A$ wherever double bars of equal length are found

$$(A + BC)(A\overline{B})$$

Distributive property

$$AA\overline{B} + BCA\overline{B}$$

Applying identity $AA = A$ to left term; applying identity $A\overline{A} = 0$ to B and $\overline{B}$ in right term

$$A\overline{B} + 0$$

Applying identity $A + 0 = A$

$$A\overline{B}$$

Example

$$\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

Example

$$\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$\downarrow$ Factoring **BC** out of 1st and 4th terms

$$BC(\overline{A} + A) + A\overline{B}C + AB\overline{C}$$

$\downarrow$ Applying identity $\mathbf{A + \overline{A} = 1}$

$$BC(1) + A\overline{B}C + AB\overline{C}$$

$\downarrow$ Applying identity $\mathbf{1A = A}$

$$BC + A\overline{B}C + AB\overline{C}$$

$\downarrow$ Factoring **B** out of 1st and 3rd terms

$$B(C + A\overline{C}) + A\overline{B}C$$

$\downarrow$ Applying rule $\mathbf{A + \overline{A}B = A + B}$ to the $C + A\overline{C}$ term

Example

$$B(C + A) + A\overline{B}C$$

$\downarrow$ Distributing terms

$$BC + AB + A\overline{B}C$$

$\downarrow$ Factoring **A** out of 2nd and 3rd terms

$$BC + A(B + \overline{B}C)$$

$\downarrow$ Applying rule $\mathbf{A + \overline{A}B = A + B}$ to the $B + \overline{B}C$ term

$$BC + A(B + C)$$

$\downarrow$ Distributing terms

$$BC + AB + AC$$

or Simplified result

$$AB + BC + AC$$

References

- https://www.allaboutcircuits.com/textbook/digital/chpt-7/boolean-algebraic-identities/