CSD3240/MAT340 TUTORIAL 3

Question 1. A multiple-choice test consists of 20 questions, each with four choices. A student is able to eliminate one of the choices on each question as incorrect and chooses randomly from the remaining three choices. The requirement for passing is ≥ 12 questions answered correctly

(a) Find the probability that the student passes.

(b) Answer part (a) again, assuming that the student can eliminate two choices for each question.

Question 2. Two dice are rolled 100 times. The number of double sixes X is counted.

(a) Show that X follows a binomial distribution, that is, $X \sim \text{Binomial}(n, p)$. Moreover, find n and p.

(b) Is it suitable to approximate the distribution of X by Poisson(pn)? Justify your answer.

(c) Find $P(X \geq 3)$ using $X \sim \text{Binomial}(n, p)$ and find $P(X \geq 3)$ using $X \sim \text{Poisson}(np)$.

Question 3. The Poisson distribution can be adapted if, instead of the average number of events λ, we are given the average rate r at which events occur. Then $\lambda = rt$ and

$$P(k \text{ events in interval } t = \frac{(rt)^k e^{-rt}}{k!}$$

Suppose the number of hits a web site receives in any time interval is a Poisson random variable. A particular site gets on average 5 hits per second.

(a) What is the probability that there will be no hits in an interval of two seconds?

(b) What is the probability that there is at least one hit in an interval of one second?

Question 4. The discrete random variable X is modelled as being geometrically distributed with parameter 0.2.

(a) State two conditions that must be satisfied by X, so that the geometric model is valid

(b) Showing full workings, where appropriate, calculate the value of

1. $P(X = 3)$
2. $P(X > 8)$
3. $P(5 \leq X < 13)$

Question 5. Find $\int_0^\infty e^{-x} dx$ (if it converges)

Question 6. Find $\int_{-\infty}^\infty \frac{1}{1+x^2} dx$ (if it converges)

Question 7. Find $\int 4x \cos(2 - 3x) dx$ (if it converges)