CSD2259 Tutorial 4

Problem 1. Use truth tables to prove the validity of the following argument forms. In your truth table, indicate clearly the premises, the conclusion and the critical rows. Include a few words of explanation to support your answers.

(a) The following argument form is invalid (converse error)

$$p \to q$$
$$q$$
$$\therefore p$$

(b) The following argument form is invalid (inverse error)

$$p \to q$$
$$\neg p$$
$$\therefore \neg q$$

(c) The following argument form is valid (division into cases)

$$p \vee q$$
$$p \to r$$
$$q \to r$$
$$\therefore r$$

Problem 2. Use basic rules of inference (see last page) to deduce that the following argument form is valid. Supply a reason for each step.

(a) $\neg p \to r \wedge \neg s$

(b) $t \to s$

(c) $u \to \neg p$

(d) $\neg w$

(e) $u \vee w$

(f) $\therefore \neg t$

Problem 3. Let x be a variable and let $\mathbb{Z}$ be its domain. Define following predicates

$$\begin{aligned} O(x): &\quad x \text{ is odd,} \\ P(x): &\quad x \text{ is prime,} \\ S(x): &\quad x \text{ is a perfect square.} \end{aligned}$$

Rewrite the following sentences using $O(x), P(x), S(x)$ and suitable logical operators $(\forall, \exists, \neg, \wedge, \vee, \rightarrow, \leftrightarrow)$.

(a) There exists an integer which is a prime and not an odd number.

(b) If an integer is a prime, then it is not a perfect square.

(c) There are integers which are both odd numbers and perfect squares.

Problem 4. Let $P(x), Q(x)$ be predicates both having domain D.

(a) In the lectures, we studied that

$$\forall x(P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x)$$

Give a counterexample to show that this relation hold for the operator $\vee$, that is,

$$\forall x(P(x) \vee Q(x)) \not\equiv \forall x P(x) \vee \forall x Q(x).$$

(b) Show that $\exists x(P(x) \vee Q(x))$ and $\exists x P(x) \vee \exists x Q(x)$ are logically equivalent.

(c) Give a counterexample to show that

$$\exists x(P(x) \wedge Q(x)) \not\equiv \exists x P(x) \wedge \exists x Q(x).$$

Problem 5. Show that the following argument is valid. Support your answer by forming an argument form and proving the validity of that argument form (by inference rules or truth table)

1. It is not raining or Yvette has her umbrella
2. Yvette does not have her umbrella or she does not get wet
3. It is raining or Yvette does not get wet

$\therefore$ Yvette does not get wet.

Problem 6. Let $P(x), Q(x), R(x)$ be the statements "x is a clear explanation", "x is satisfactory", and "x is an excuse". Suppose that the domain for x consists of *all English text*. Express each of these statements using quantifiers, logical connectives, and $P(x), Q(x), R(x)$.
(a) All clear explanations are satisfactory.
(b) Some excuses are unsatisfactory.
(c) Some excuses are not clear explanations.
(d) Show that (c) follow from (a) and (b).

Hints and Instructions

2. Combining e and d, u is correct (elimination rule). Then continue from here.
3. Try it.
4a. Think of an example with $D = \mathbb{R}$ and $P(x) \wedge Q(x)$ true for all $x \in D$.
4b. You need to explain 2 things:

   1. If $\exists x(P(x) \vee Q(x))$ is true, then $\exists x P(x) \vee \exists x Q(x)$ is true.
   2. If $\exists x P(x) \vee \exists x Q(x)$ is true, then $\exists x(P(x) \vee Q(x))$ is true.

4c. Try it.
5. Define p: It is raining, q: Yvette has her umbrella, r: she gets wet.
6. Try it.

Basic Rules of Inference

Name	Argument form	Example
Generalization	p $\therefore p \vee q$	$x = 3$ $\therefore x = 3$ or $x = -3$
Specialization	$p \wedge q$ $\therefore p$	$y > 0$ and y is an integer $\therefore y > 0$
Elimination	$p \vee q$ $\neg q$ $\therefore p$	$x - 3 = 0$ or $x + 2 = 0$ $x \neq -2$ $\therefore x - 3 = 0$
Transitivity	$p \to q$ $q \to r$ $\therefore p \to r$	If $x > a$, then $x > b$ If $x > b$, then $x > c$ $\therefore$ if $x > a$, then $x > c$
Division into cases	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$	x is positive or x is negative If x is positive, then $x^2 > 0$ If x is negative, then $x^2 > 0$ $\therefore x^2 > 0$
Contradiction	$\neg p \to \mathbf{F}$ $\therefore p$	If everyone sleeps before 12am, then there is no Covid-19 $\therefore$ Not everyone sleeps before 12am