CSD2301 Practice Solutions
1. Measurements & Vectors

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Practice Question 1

The density of lead is 11.3 g/cm^3. What is this value in kilograms per cubic metre?

Answer: $$11.3\ \frac{\text{g}}{\text{cm}^3}\times\left(\frac{1\text{ kg}}{1000\text{ g}}\right)\times\left(\frac{100\text{ cm}}{1\text{ m}}\right)^3=1.13\times10^4\ \frac{\text{kg}}{\text{m}^3}.$$

Practice Question 2

A line can be described by the equation $y = 4x + 7$. What is the polar coordinates of the point at $x = 5$?

Substitute x=5 into eqn to find y:

y = 4(5)+7 = 27

$r^2 = x^2 + y^2$

$r^2 = 5^2 + 27^2$

$r^2 = 754$

$r = \sqrt{754} = 27.5$

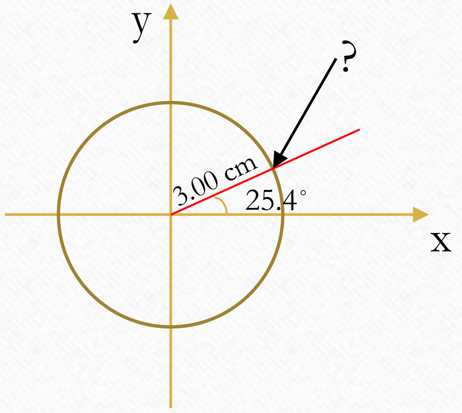
$$tan\theta = \frac{y}{x} = \frac{27}{5}$$

$$\theta = tan^{-1}\frac{27}{5} = 79.5°$$

Therefore, polar coordinates: (27.5,79.5°)

Practice Question 3

The centre of a circle is set as the origin of a coordinate system. The radius of the circle is 3.00 cm. A line which passes through the origin, intersects the circle. This line forms an angle of 25.4° with the horizontal axis. What is the cartesian coordinates at the point of intersection between the line and the circle?



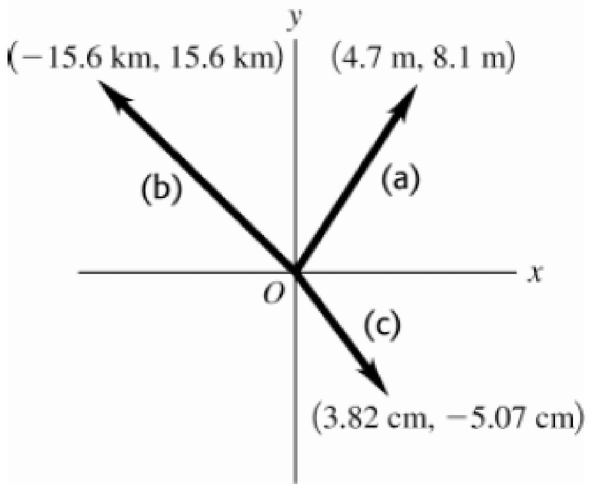
$$x = r\,cos\theta = 3.00\ cm\ cos25.4° = 2.71\ cm$$
$$y = r\,sin\theta = 3.00\ cm\ sin25.4° = 1.29\ cm$$

Therefore, cartesian coordinates: (2.71 cm, 1.29 cm)

Practice Question 4

Find the x- and y-components of the following vectors. For each vector the numbers given are the magnitude of the vector and the angle, measured in the sense from the +x-axis toward the +y-axis, that it makes with the +x-axis: (a) magnitude 9.30 m, angle 60.0°; (b) magnitude 22.0 km, angle 135°; (c) magnitude 6.35 cm, angle 307°

The signs of the components depend on the quadrant in which the vector lies.



Practice Question 5

Vector $\vec{A}$ has components $A_x = 1.30$ cm, $A_y = 2.25$ cm; vector $\vec{B}$ has components $B_x = 4.10$ cm, $B_y = -3.75$ cm. Find (a) the components of the vector sum $\vec{A} + \vec{B}$; (b) the magnitude and direction of $\vec{A} + \vec{B}$; (c) the components of the vector difference $\vec{B} - \vec{A}$; (d) the magnitude and direction of $\vec{B} - \vec{A}$.

(a) To find the components of the vector sum $\vec{A} + \vec{B}$:

- x-component of $\vec{A} + \vec{B} = A_x + B_x = 1.30\text{ cm} + 4.10\text{ cm} = 5.40\text{ cm}$
- y-component of $\vec{A} + \vec{B} = A_y + B_y = 2.25\text{ cm} - 3.75\text{ cm} = -1.50\text{ cm}$

(b) The magnitude and direction of $\vec{A} + \vec{B}$:

- Magnitude: $\sqrt{(1.30\text{ cm} + 4.10\text{ cm})^2 + (2.25\text{ cm} - 3.75\text{ cm})^2} = 5.60\text{ cm}$
- Direction: $-15.5°$ (measured counterclockwise from the positive x-axis)

(c) To find the components of the vector difference $\vec{B} - \vec{A}$:

- x-component of $\vec{B} - \vec{A} = B_x - A_x = 4.10\text{ cm} - 1.30\text{ cm} = 2.80\text{ cm}$
- y-component of $\vec{B} - \vec{A} = B_y - A_y = -3.75\text{ cm} - 2.25\text{ cm} = -6.00\text{ cm}$

(d) The magnitude and direction of $\vec{B} - \vec{A}$:

- Magnitude: $\sqrt{(4.10\text{ cm} - 1.30\text{ cm})^2 + (-3.75\text{ cm} - 2.25\text{ cm})^2} = 6.62\text{ cm}$
- Direction: $-65.0°$ (measured counterclockwise from the positive x-axis)

Practice Question 6

A rocket fires two engines simultaneously. One produces a thrust of 725 N directly forward while the other gives a 513 N thrust at 32.4° above the forward direction. Find the magnitude and direction (relative to the forward direction) of the resultant force which these engines exert on the rocket.

Answer: Take the +x-direction to be forward and the +y-direction to be upward. Then the second force has components $F_{2x} = F_2 \cos 32.4^\circ = 433\,\text{N}$ and $F_{2y} = F_2 \sin 32.4^\circ = 275\,\text{N}$.

The first force has components $F_{1x} = 725\,\text{N}$ and $F_{1y} = 0$.

$F_x = F_{1x} + F_{2x} = 1158\,\text{N}$ and $F_y = F_{1y} + F_{2y} = 275\,\text{N}$

The resultant force is 1190 N in the direction 13.4° above the forward direction.

Practice Question 7

Let the angle θ be the angle that the vector $\vec{A}$ makes with the +x-axis, measured counter-clockwise from that axis. Find the angle θ for a vector that has the following components: (a) A_x = 2.00 m, A_y = -1.00 m; (b) A_x = 2.00 m, A_y = 1.00 m; (c) A_x = -2.00 m, A_y = 1.00 m; (d) A_x = -2.00 m, A_y = -1.00 m.

IDENTIFY: $\tan\theta = \dfrac{A_y}{A_x}$, for θ measured counterclockwise from the $+x$-axis.

SET UP: A sketch of A_x, A_y and $\vec{A}$ tells us the quadrant in which $\vec{A}$ lies.

EXECUTE:

(a) $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{-1.00\text{ m}}{2.00\text{ m}} = -0.500$. $\theta = \tan^{-1}(-0.500) = 360° - 26.6° = 333°$.

(b) $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{1.00\text{ m}}{2.00\text{ m}} = 0.500$. $\theta = \tan^{-1}(0.500) = 26.6°$.

(c) $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{1.00\text{ m}}{-2.00\text{ m}} = -0.500$. $\theta = \tan^{-1}(-0.500) = 180° - 26.6° = 153°$.

(d) $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{-1.00\text{ m}}{-2.00\text{ m}} = 0.500$. $\theta = \tan^{-1}(0.500) = 180° + 26.6° = 207°$

Practice Question 8

Given two vectors $\vec{A} = 4.00\hat{\imath} + 3.00\hat{\jmath}$ and $\vec{B} = 5.00\hat{\imath} - 2.00\hat{\jmath}$, (a) find the magnitude of each vector; (b) write an expression for the vector difference $\vec{A} - \vec{B}$ using unit vectors; (c) find the magnitude and direction of the vector difference $\vec{A} - \vec{B}$.

EXECUTE: **(a)** $\vec{A} = 4.00\hat{\boldsymbol{i}} + 3.00\hat{\boldsymbol{j}}$; $A_x = +4.00$; $A_y = +3.00$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(4.00)^2 + (3.00)^2} = 5.00$$

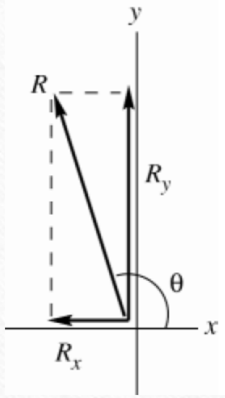
$\vec{\boldsymbol{B}} = 5.00\hat{\boldsymbol{i}} - 2.00\hat{\boldsymbol{j}}$; $B_x = +5.00$; $B_y = -2.00$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.00)^2 + (-2.00)^2} = 5.39$$

(b) $\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} = 4.00\hat{\boldsymbol{i}} + 3.00\hat{\boldsymbol{j}} - \left(5.00\hat{\boldsymbol{i}} - 2.00\hat{\boldsymbol{j}}\right) = (4.00 - 5.00)\hat{\boldsymbol{i}} + (3.00 + 2.00)\hat{\boldsymbol{j}}$

$$\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} = -1.00\hat{\boldsymbol{i}} + 5.00\hat{\boldsymbol{j}}$$

(c) Let $\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} = -1.00\hat{\boldsymbol{i}} + 5.00\hat{\boldsymbol{j}}$. Then $R_x = -1.00$, $R_y = 5.00$.



$$R = \sqrt{R_x^2 + R_y^2}$$

$$R = \sqrt{(-1.00)^2 + (5.00)^2} = 5.10.$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{5.00}{-1.00} = -5.00$$

$$\theta = -78.7^\circ + 180^\circ = 101.3^\circ.$$

The End