problem 1

(a) $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{4}$. $P(A\cup B)=\frac{5}{9}$. $P(A\cap B)=?$

$$\underbrace{P(A\cup B)}_{\frac{5}{9}}=\underbrace{P(A)}_{\frac{1}{3}}+\underbrace{P(B)}_{\frac{1}{4}}-\underline{P(A\cap B)}$$

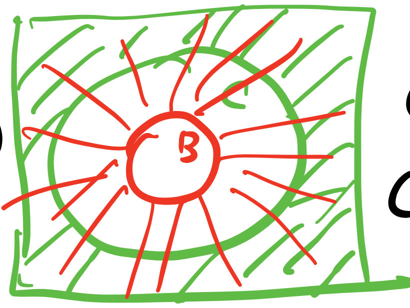
$$P(A\cap B)=\frac{1}{36}.$$

(b) $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{4}$, $P(A\cap B^c)=\frac{3}{8}$, $B\subset C$.

$P(A\cup \underbrace{B^c\cup C^c})$

$= P(A\cup B^c)=P(A)+P(B^c)-P(A\cap B^c)$

$= \frac{1}{2}+(1-\frac{1}{4})-\frac{3}{8}=\frac{7}{8}$



$C^c\subset B^c$

$C^c\cup B^c=B^c$

(c) $\begin{cases} P(A)=P(B)=P(C)=\frac{9}{10}. \\ P(A\cap B)=P(A\cap C)=P(B\cap C)=\frac{7}{10}. \\ P(A\cap B\cap C)=\frac{5}{10} \end{cases}$

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)$$
$$+P(A\cap B\cap C)$$

$$=3\times\frac{9}{10}-3\times\frac{7}{10}+\frac{5}{10}.=\frac{11}{10}>1$$

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problem 2:

(a). $\underbrace{BB, BG. GB}$, GG. $\frac{1}{3}$.

let E be the event that the family has 2 Boys let F . . . . at least 1 Boy.

$$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

(b). $\frac{0}{1}\ \frac{2}{}\times\frac{2}{}\times\frac{2}{}$

$$\begin{cases} 0\,0\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \\ 0\,0\,1\,1 \\ 0\,1\,0\,0 \end{cases}$$

$\frac{5}{8}$

let A. the string contains at least 2 consecutive 0's

let B . . . the first bit is 0.

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{5}{16}}{\frac{1}{2}} = \frac{5}{8}.$$

$$A\cap B = \left\{\{(0,0,c,d) : c,d\in\{0,1\}\} \cup \{(0,1,0,0)\}\right\}$$

$4+1=5$

problem 3

(a) let A be the event that 13 cards contain at least 1 Ace.

let B be the . . . . exactly 2 Aces.

$$P(A) = 1-P(A^c) = 1-\frac{\binom{48}{13}}{\binom{52}{13}}.$$

$$P(B) = \frac{\binom{4}{2}\binom{48}{11}}{\binom{52}{13}}$$

$\binom{4}{2}\binom{48}{11}\Big/\binom{52}{13}$

$$P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{P(B)}{P(A)} = \frac{[\text{illegible}]}{1-\binom{48}{13}/\binom{52}{13}} \approx 0.31.$$

(b) let A be the event that 13 cards contain the ace of heart

let B . . . . exactly 2 aces.

$$P(A) = \frac{\binom{1}{1}\binom{51}{12}}{\binom{52}{13}}$$

$$P(A\cap B) = \frac{\binom{3}{1}\binom{1}{1}\binom{48}{11}}{\binom{52}{13}}.$$

$$P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{\binom{3}{1}\binom{1}{1}\binom{48}{11}}{\binom{1}{1}\binom{51}{12}} \approx 0.43$$

problem 4.

let A be the event that the luggage is put on the correct plane at Amsterdam.

let B . . . . at Dubai

The Event that the luggage doesn't sydney is $(A\cap B)^c$.

$P(A) = 0.95$; $P(B) = 0.97$.

$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

$$P(A\cap B) = P(A)\,\underbrace{P(B|A)}_{=P(B)}$$

$$\begin{aligned} P(A\cap B) &= P(A)\,P(B|A) \\ &= P(A)\,P(B) \\ &= 0.95 \times 0.97 = 0.92115 \end{aligned}$$

luggage with you in Sydney

$$P((A \cap B)^c) = 1 - 0.92115 = 0.0785 = 7.85\%.$$