

## Tutorial 1

1. Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

Statement: Consider an instance of the Stable Matching Problem in which there exists a man  $m$  and a woman  $w$  such that  $m$  is ranked first on the preference list of  $w$  and  $w$  is ranked first on the preference list of  $m$ . Then in every stable matching  $S$  for this instance, the pair  $(m, w)$  belongs to  $S$ .

This statement is true because since  $m$  prefer  $w$  and vice versa, if  $m$  were to pair with another, it may incentives  $m$  to deviate from its current pair just to pair up with  $w$  thus leading to instability. Thus, by having  $m$  pair with  $w$ , this ensures stability

2. Let  $M = m_1, m_2, m_3$  and  $W = w_1, w_2, w_3$ . Suppose that you are given the following preference lists:

$m_1: < w_3, w_2, w_1 >; m_2: < w_2, w_3, w_1 >; m_3: < w_2, w_3, w_1 >;$

$w_1: < m_3, m_1, m_2 >; w_2: < m_1, m_3, m_2 >; w_3: < m_3, m_1, m_2 >;$

Stable:  $(m_1, w_3), (m_2, w_2), (m_3, w_1)$

Unstable:  $(m_1, w_2), (m_2, w_3), (m_3, w_1)$

1) Give a stable perfect matching, and an unstable perfect matching.

2) Find the best valid partner for each member of sets  $M$  and  $W$ .

$(m_1, w_3)$  - top choice

$(m_2, w_2)$  - top choice

$(m_3, w_1)$  - only choice left which ensure stability

3. List the following functions according to their order of growth from the lowest to the highest. (Hint: you could start with using basic asymptotic efficiency classes)

$$f_1(n) = n^{2.5}; \quad f_2(n) = \sqrt{2n}; \quad f_3(n) = n + 10$$

$$f_4(n) = 10^n; \quad f_5(n) = 100^n; \quad f_6(n) = n^2 \log n$$

$f_3, f_2, f_6, f_1, f_4, f_5$