CSD2301 Practice Solutions

12. **Rotation and Moment of Inertia Part 2**

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Practice Question 1

Calculate the moment of inertia of each of the following uniform objects about the axes indicated.

(a) A thin 2.50 kg rod of length 75.0 cm, about an axis perpendicular to it and passing through (i) one end and (ii) its center, and (iii) about an axis parallel to the rod and passing through it.

(b) A 3.00 kg sphere 38.0 cm in diameter, about an axis through its center, if the sphere is (i) solid and (ii) a thin-walled hollow shell.

(c) An 8.00 kg cylinder, of length 19.5 cm and diameter 12.0 cm, about the central axis of the cylinder, if the cylinder if (i) thin-walled and hollow, and (ii) solid.

Practice Question 1

SET UP: In each case express the mass in kg and the length in m, so the moment of inertia will be in $\text{kg}\cdot\text{m}^2$.

EXECUTE: **(a)** (i) $I = \frac{1}{3}ML^2 = \frac{1}{3}(2.50\ \text{kg})(0.750\ \text{m})^2 = 0.469\ \text{kg}\cdot\text{m}^2$.

(ii) $I = \frac{1}{12}ML^2 = \frac{1}{4}(0.469\ \text{kg}\cdot\text{m}^2) = 0.117\ \text{kg}\cdot\text{m}^2$. (iii) For a very thin rod, all of the mass is at the axis and $I = 0$.

(b) (i) $I = \frac{2}{5}MR^2 = \frac{2}{5}(3.00\ \text{kg})(0.190\ \text{m})^2 = 0.0433\ \text{kg}\cdot\text{m}^2$.

(ii) $I = \frac{2}{3}MR^2 = \frac{5}{3}(0.0433\ \text{kg}\cdot\text{m}^2) = 0.0722\ \text{kg}\cdot\text{m}^2$.

(c) (i) $I = MR^2 = (8.00\ \text{kg})(0.0600\ \text{m})^2 = 0.0288\ \text{kg}\cdot\text{m}^2$.

(ii) $I = \frac{1}{2}MR^2 = \frac{1}{2}(8.00\ \text{kg})(0.0600\ \text{m})^2 = 0.0144\ \text{kg}\cdot\text{m}^2$.

EVALUATE: I depends on how the mass of the object is distributed relative to the axis.

Practice Question 2

Energy is to be stored in a 70.0 kg flywheel in the shape of a uniform solid disk with radius R=1.20 m. To prevent structural failure of the flywheel, the maximum allowed radial acceleration of a point on its rim is 3500 m/s^2. What is the maximum kinetic energy that can be stored in the flywheel?

EXECUTE: $K = \frac{1}{2} I\omega^2$

$a_{rad} = R\omega^2$, so $\omega = \sqrt{a_{rad}/R} = \sqrt{(3500 \text{ m/s}^2)/1.20 \text{ m}} = 54.0$ rad/s

For a disk, $I = \frac{1}{2} MR^2 = \frac{1}{2}(70.0 \text{ kg})(1.20 \text{ m})^2 = 50.4 \text{ kg} \cdot \text{m}^2$

Thus $K = \frac{1}{2} I\omega^2 = \frac{1}{2}(50.4 \text{ kg} \cdot \text{m}^2)(54.0 \text{ rad/s})^2 = 7.35 \times 10^4$ J

EVALUATE: The limit on a_{rad} limits ω which in turn limits K.

Practice Question 3

About what axis will a uniform sphere have the same moment of inertia as does a thin-walled, hollow, lead sphere of the same mass and radius, with the axis along a diameter?

SET UP: For a thin-walled hollow sphere, axis along a diameter, $I = \frac{2}{3}MR^2$.

For a solid sphere with mass M and radius R, $I_{cm} = \frac{2}{5}MR^2$, for an axis along a diameter.

EXECUTE: Find d such that $I_P = I_{cm} + Md^2$ with $I_P = \frac{2}{3}MR^2$:

$$\frac{2}{3}MR^2 = \frac{2}{5}MR^2 + Md^2$$

The factors of M divide out and the equation becomes $\left(\frac{2}{3} - \frac{2}{5}\right)R^2 = d^2$

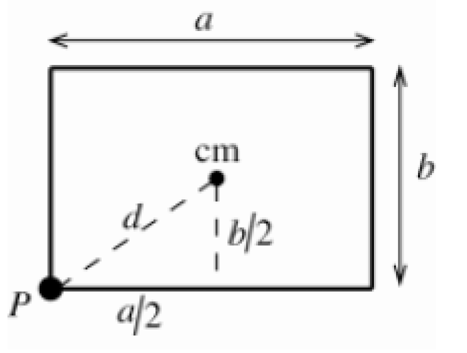
$$d = \sqrt{(10-6)/15}R = 2R/\sqrt{15} = 0.516R.$$

The axis is parallel to a diameter and is $0.516R$ from the center.

Practice Question 4

A thin, rectangular sheet of metal has mass M and sides of length a and b. Use the parallel-axis theorem to calculate the moment of Inertia of the sheet for an axis that is perpendicular to the plane of the sheet and that passes through one corner of the sheet.

EXECUTE: $I_P = I_{cm} + Md^2$.



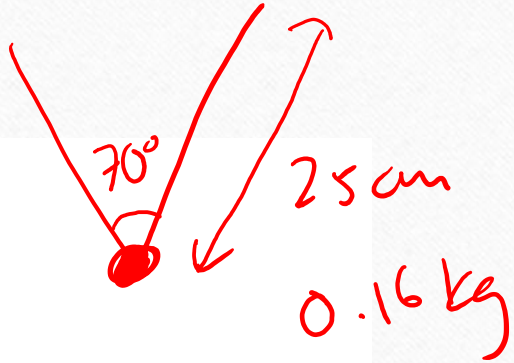
$I_{cm} = \frac{1}{12}M(a^2+b^2)$.
The distance d of P from the cm is

$d = \sqrt{(a/2)^2 + (b/2)^2}$.

Thus $I_P = I_{cm} + Md^2 = \frac{1}{12}M(a^2+b^2) + M\left(\frac{1}{4}a^2 + \frac{1}{4}b^2\right) = \left(\frac{1}{12}+\frac{1}{4}\right)M(a^2+b^2) =$
$\frac{1}{3}M(a^2+b^2)$

Practice Question 5

A thin uniform rod 50.0 cm long with mass 0.320 kg is bent at its center into a V shape, with a 70.0° angle at its vertex. Find the moment of inertia of this V-shaped object about an axis perpendicular to the plane of the V at its vertex.



IDENTIFY: Treat the V like two thin 0.160 kg bars, each 25 cm long.

SET UP: For a slender bar with the axis at one end, $I = \frac{1}{3}mL^2$.

EXECUTE: $I = 2\left(\frac{1}{3}mL^2\right) = 2\left(\frac{1}{3}\right)(0.160\text{ kg})(0.250\text{ m})^2 = 6.67\times10^{-3}\text{ kg}\cdot\text{m}^2$

EVALUATE: The value of I is independent of the angle between the two sides of the V; the angle 70.0° didn't enter into the calculation.

Practice Question 6

A passenger bus in Zurich, Switzerland, derived its motive power from the energy stored in a large flywheel. The wheel was brought up to speed periodically, when the bus stopped at a station, by an electric motor, which could then be attached to the electric power lines. The flywheel was a solid cylinder with mass 1000 kg and diameter 1.80 m. Its top angular speed was 3000 rev/min. (a) At this angular speed, what is the kinetic energy of the fly-wheel? (b) If the average power required to operate the bus is 1.86 x 10^4 W, how long could it operate between stops?

IDENTIFY: $K = \frac{1}{2}I\omega^2$, with ω in rad/s. $P = \dfrac{\text{energy}}{t}$

SET UP: For a solid cylinder, $I = \frac{1}{2}MR^2$. 1 rev/min $= (2\pi/60)$ rad/s

EXECUTE: **(a)** $\omega = 3000 \text{ rev/min} = 314 \text{ rad/s}$. $I = \frac{1}{2}(1000 \text{ kg})(0.900 \text{ m})^2 = 405 \text{ kg}\cdot\text{m}^2$

$K = \frac{1}{2}(405 \text{ kg}\cdot\text{m}^2)(314 \text{ rad/s})^2 = 2.00\times10^7 \text{ J}$.

(b) $t = \dfrac{K}{P} = \dfrac{2.00\times10^7 \text{ J}}{1.86\times10^4 \text{ W}} = 1.08\times10^3 \text{ s} = 17.9 \text{ min}$.

EVALUATE: In $K = \frac{1}{2}I\omega^2$, we must use ω in rad/s.