

## CSD2259 Homework 1

Due: Jan 21, 2024

The following problem set is used for the on-line homework 1 set up on Moodle. Please key in your answers on Moodle by the due date.

Feel free to let me know if you find typos and errors.

*Question 1.* Let  $X, Y, Z$  be defined as follows

- $X$  = set of airline flights from Singapore to New York
- $Y$  = set of non-stop (direct) flights from Singapore to New York
- $Z$  = set of flights from Singapore to New York which stop over at Japan

Which of the following is the correct description between  $X, Y, Z$ ?

- ☒ (A)  $X \subset (Y \cup Z)$  (B)  $Y \subset Z$  (C)  $X - Y \subset Z$  (D)  $Z \subset X - Y$  (E) None of these

*Question 2.* Determine whether each following pair of sets are equal

- (a)  $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}$  and  $\{5, 3, 1\}$  (b)  $\{\{1\}\}$  and  $\{1, \{1\}\}$  (c)  $\emptyset$  and  $\{\emptyset\}$

- ☒ (A) Only a (B) Only b (C) Only c (D) All of these (E) None of these

*Question 3.* Given 3 sets  $A, B, C$  with

$$A = \{1, 2, 3\}, B = \{2, \{2\}, \{2, \{2\}\}\}, C = \{1, 1, 2, 2, 3, 3\}$$

Which power set among  $\mathcal{P}(A), \mathcal{P}(B), \mathcal{P}(C)$  has the largest size?

- (A)  $\mathcal{P}(A)$  (B)  $\mathcal{P}(B)$  (C)  $\mathcal{P}(C)$  (D) Both  $\mathcal{P}(B)$  and  $\mathcal{P}(C)$  ☒ (E) All have same size

*Question 4.* Let  $a, b$  be distinct elements. Among 4 sets

$$A = \emptyset, B = \{\emptyset, \{a\}\}, C = \{\emptyset, \{a\}, \{\emptyset, a\}\}, D = \{\emptyset, \{a\}, \{b\}, \{a, b\}\},$$

which set could be a power set of some set?

- ☒ (A) Only A ☒ (B) Only B (C) Only C (D) Both B and D (E) Only D

**Questions 5-7.** Consider three sets

$$A = \emptyset, B = \{1, 2\}, C = \{1, 2, \{1, 2\}\}$$

*Question 5.* What is  $|\mathcal{P}(A)|$ ?

- (A) 0    **(B) 1**    (C) 2    (D) 4    (E) None of these

*Question 6.* What is  $|\mathcal{P}(B \cup C)|$ ?

- (A) 4    **(B) 8**    (C) 16    (D) 32    (E) None of these

*Question 7.* What is  $|\mathcal{P}(B \times C)|$ ?

- (A) 4    (B) 8    (C) 16    ~~(D) 32~~    (E) None of these

**Questions 8-11.** Let  $A$  be the set of sophomores at your school and  $B$  be the set of students in discrete mathematics at your school.

*Question 8.* Which of the following is the set of sophomores taking discrete mathematics in your school?

- (A)  $A \cap B$**     (B)  $A \cup B$     (C)  $A - B$     (D)  $B - A$     (E) None of these

*Question 9.* Which of the following is the set of sophomores who are not taking discrete mathematics?

- (A)  $A \cap B$     (B)  $A \cup B$     (C)  $A - B$     ~~(D)  $B - A$~~     (E) None of these

*Question 10.* Which of the following is the set of students at your school who are sophomores or are taking discrete mathematics?

- (A)  $A \cap B$     **(B)  $A \cup B$**     (C)  $A - B$     (D)  $B - A$     (E) None of these

*Question 11.* Which of the following is the set of students at your school who are not sophomores or are not taking discrete mathematics?

- (A)  $A \cap B$     (B)  $A \cup B$     (C)  $A - B$     (D)  $B - A$     **(E) None of these**

**Questions 12-14.** The symmetric difference of A and B, denoted by  $A \oplus B$ , is the set containing those elements in either A or B, but not in both A and B.

Let  $U = \{1, 2, 3, 4, 5\}$  be the universal set and let  $A = \{1, 2, 3\}$ ,  $B = \{1, 3, 5\}$ .

*Question 12.* What is  $A \oplus B$ ?

- (A) A      (B) B      (C)  $\{2, 5\}$       (D)  $\{1, 3\}$       (E) None of these

*Question 13.* What is  $|\mathcal{P}(A \oplus \bar{B})|$ ?

- (A) 1      (B) 2      (C) 4      (D) 8      (E) None of these

*Question 14.* Assume that  $C$  is a subset of  $U$  such that  $A \oplus C = A$ . Which of the following is the correct statement about  $C$ ?

- (A) The only possibility for  $C$  is the empty set. ✓  
(B)  $C$  can be any subset of  $A$ . ✗  
(C)  $A$  is a subset of  $C$ . ✗  
(D)  $C$  is the universal set  $U$ . ✗  
(E) None of these is correct.

**Questions 15-17.** Let  $A_1, A_2, A_3$  be three sets with 100 elements each.

*Question 15.* What is  $|A_1 \cup A_2 \cup A_3|$  if  $A_1, A_2, A_3$  are pairwise disjoint?

- (A) 0      (B) 100      (C) 200      (D) 300      (E) Not enough information

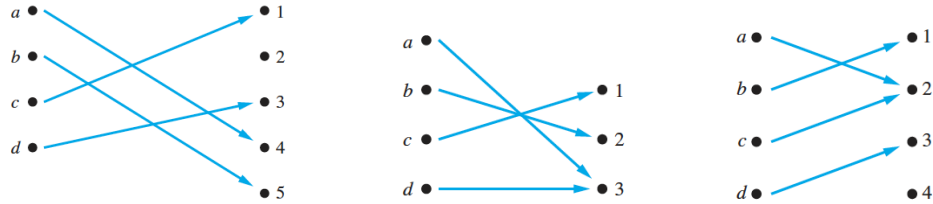
*Question 16.* What is  $|A_1 \cup A_2 \cup A_3|$  if there are 50 common elements in each pair of sets and no elements in all three sets?

- (A) 100      (B) 150      (C) 250      (D) 300      (E) None of these

*Question 17.* What is  $|A_1 \cup A_2 \cup A_3|$  if the three sets are equal (the same set)?

- (A) 100      (B) 150      (C) 250      (D) 300      (E) None of these

**Questions 18-19.** Three functions  $f, g, h$  are given by the following rules



*Question 18.* Which functions are 1-1?

- (A) Only  $f$  (B) Only  $g$  (C) Only  $h$  (D) All of them (E) None of them

*Question 19.* Which functions are onto?

- (A) Only  $f$  (B) Only  $g$  (C) Only  $h$  (D) All of them (E) None of them

**Questions 20-24.** Let  $f, g, h : \mathbb{R} \rightarrow \mathbb{R}$  be defined by

$$f(x) = x^2, \quad g(x) = 2x + 5, \quad h(x) = x^3 + 1$$

*Question 20.* What is the correct formula for  $(f + g)(x)$ ?

- (A)  $x^2$  (B)  $2x + 5$  (C)  $x^2 + 2x + 5$  (D)  $x^2 - 2x - 5$  (E) None of these

*Question 21.* What is the correct formula for  $(f \circ h)(x)$ ?

- (A)  $(x^3 + 1)^2$  (B)  $x^6 + 1$  (C)  $x^2(x^3 + 1)$  (D)  $x^2 + x^3 + 1$  (E) None of these

*Question 22.* Which functions among  $f, g, h$  are increasing?

- (A) Only  $f$  (B) Only  $g$  (C) Only  $h$  (D) Both  $g$  and  $h$  (E) All of them

*Question 23.* What is the correct formula for  $h^{-1}(x)$ ?

- (A)  $x^3 + 1$  (B)  $\sqrt[3]{x-1}$  (C)  $\sqrt[3]{x}$  (D)  $\sqrt[3]{x+1}$  (E) None of these

*Question 24.* What is the correct formula for  $(h \circ g)^{-1}(x)$ ?

- (A)  $\frac{x-5}{2}$  (B)  $\sqrt[3]{x-1}$  (C)  $(2x+1)^3 + 1$  (D)  $2x^3 + 1$  (E) None of these