CSD2301 Lecture
2. Kinematics in 1D

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Outline

- Kinematics
  - Position (and displacement), Velocity, and Acceleration
  - Average and Instantaneous
- Motion in one dimension (1D)

Kinematics

- **Kinematics** describes motion of object without consideration of the masses or the forces that may have caused the motion.
  - As opposed to **dynamics** – branch of physics involving the motion of an object under the action of forces.
- Three quantities to describe motion:
  - Position
  - Velocity
  - Acceleration

Kinematics

- Types of motion:
  - Translational
  - Rotational
  - Vibrational
- **Particle approximation**: Assume that the moving object is a point mass.
  - Valid when object is small relative to whole system; e.g. gas molecules in a box, planet around the sun, etc.

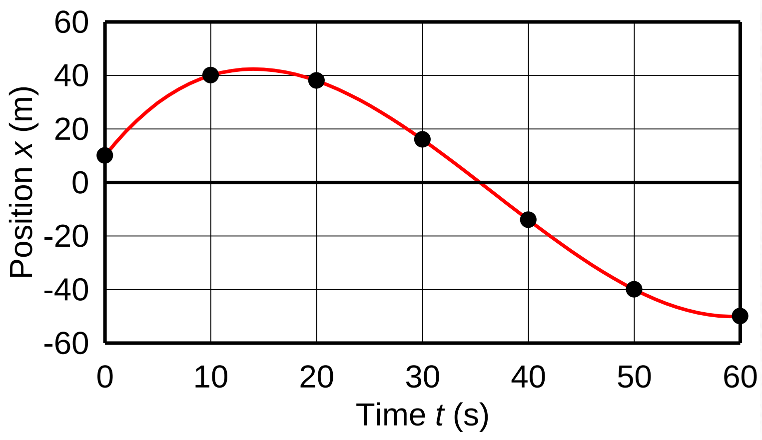
How to Define Position of an Object

- Use a convenient coordinate system and a specified origin.
  - Usually use cartesian coordinates.
- In this way, we can have a **frame of reference** to define a starting point for measuring the motion.
  - For e.g. “What is the distance to the canteen?” cannot be answered.
    - We need some reference.
    - Is it measured from the entrance of the school? Or is it measured from the general office?

Ways to Describe Motion

- Equations
- Graph
- Tables

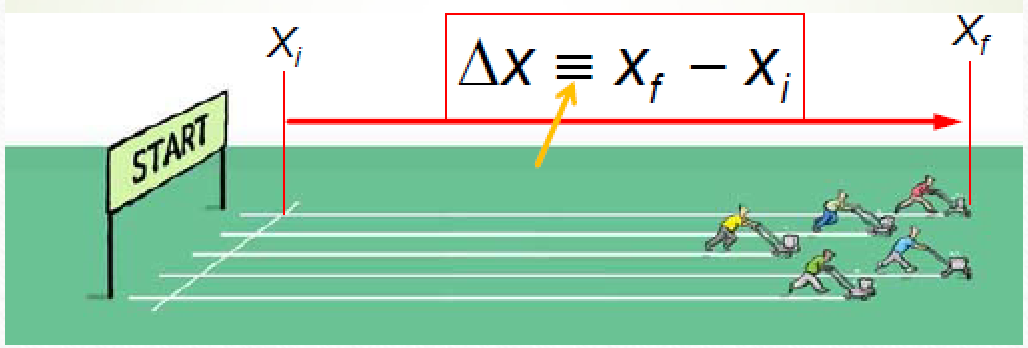
$$x(t) = 10 + 5t - 0.22t^2 + 0.002t^3$$



t (s)	0	10	20	30	40	50	60
x (m)	10	40	38	16	-14	-40	-50

Position, Distance and Displacement

- **Position:** Location of an object at any given time from a specific frame of reference
- **Distance:** Complete length of path between 2 points
- **Displacement**: Change in a particle's position from x_i to x_f
  - i.e. $\Delta x = x_f - x_i$
- SI units: meters (m)
- For motion in 1D, use only one axis.



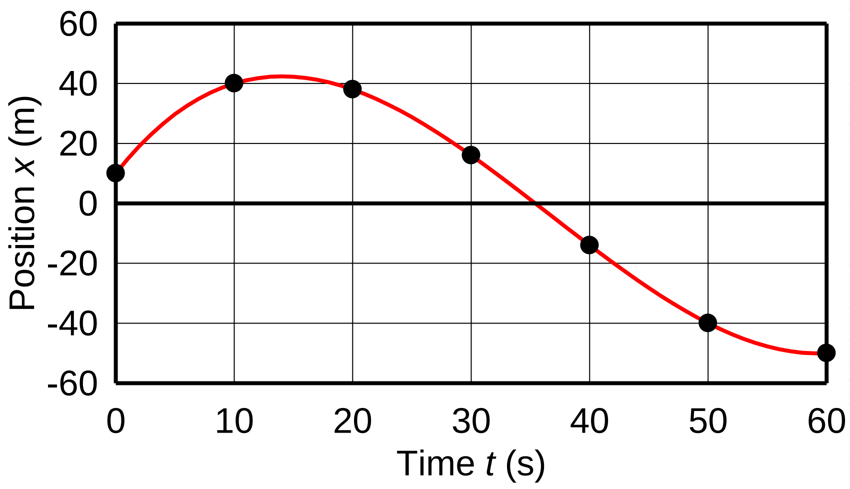
Where *i* represents initial, *f* represents final, Δ (delta) represents change.

Position, Distance and Displacement

- The motion of a particle is completely known if its position in space is known at all times, e.g. for 1D motion : $x(t)$
- Displacement can be +ve or -ve (**vector**).
- Distance only can be +ve (**scalar**).
- **Position-time graph**: Plot of $x(t)$ against t

Position, Distance and Displacement

- Example:



t (s)	0	10	20	30	40	50	60
x (m)	10	40	38	16	-14	-40	-50
Displacement (m)	0	30	28	6	-24	-50	-60
Distance (m)	0	30	37	59	89	115	125

Average Velocity and Speed

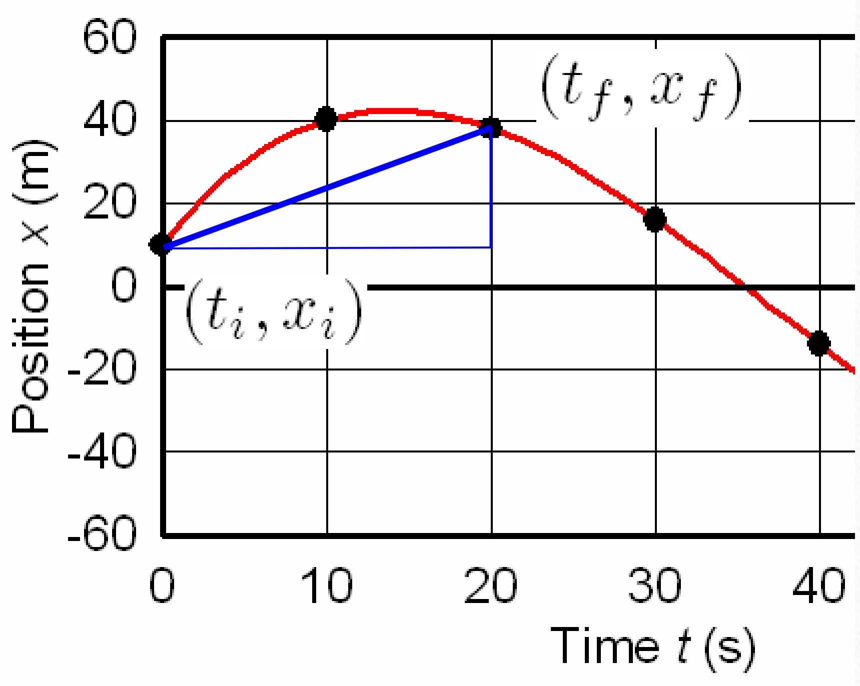
- Speed is a scalar
- Velocity is a vector
- Average velocity is defined as the ratio of its displacement Δx and the time interval Δt
- Average speed is defined as the ratio of total distance travelled to the total time taken

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$\text{Ave speed} = \frac{\text{total distance}}{\text{total time}}$$

Average Velocity and Speed

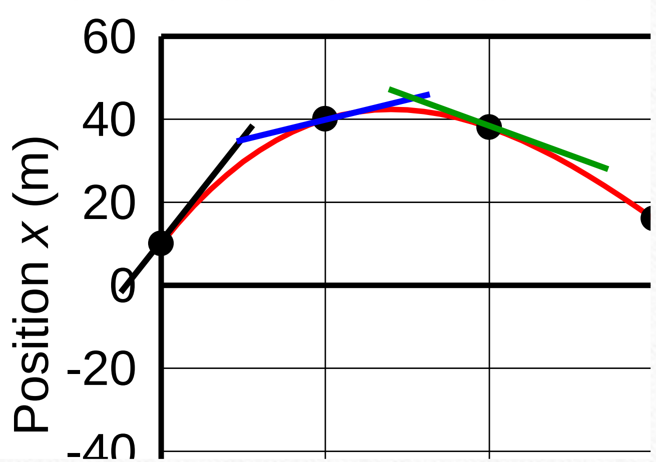
- For e.g. at $t = 20$s,
  - Average velocity = 28m/20s = 1.4 m/s
  - Average speed = 37m/20s = 1.9 m/s



Instantaneous Velocity and Speed

- Instantaneous velocity is the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero
- This is the slope of the position-time graph
- Instantaneous velocity is also simply referred to as velocity (a vector quantity)

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



Instantaneous Velocity and Speed

- Example:

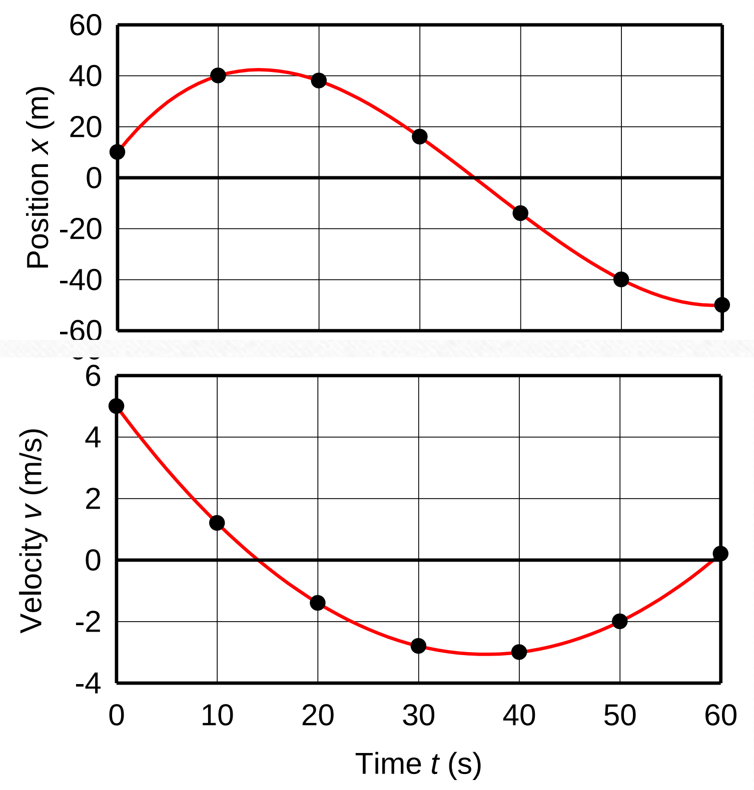
speed

$$x(t) = 10 + 5t - 0.22t^2 + 0.002t^3$$

$$v_x = \frac{dx}{dt}$$

velocity

$$v(t) = 5 - 0.44t + 0.006t^2$$



Acceleration

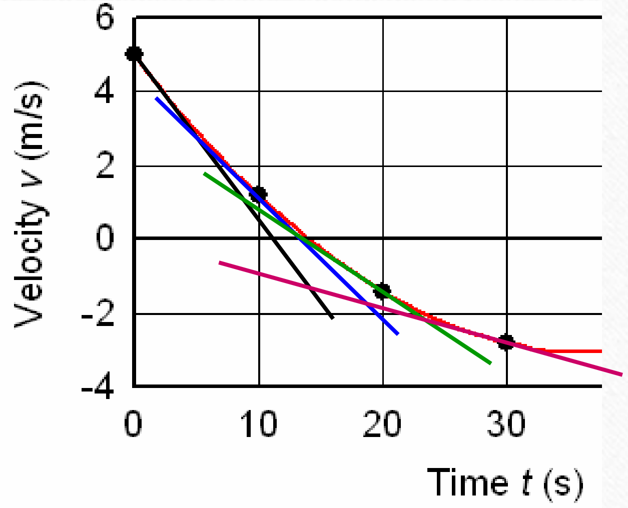
- A particle is said to be accelerating if the velocity changes with time.
- Average acceleration is defined as the change in velocity divided by the time interval during which that change occurred.

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

Instantaneous Acceleration

- The instantaneous acceleration is the derivative of the velocity with respect to time.
- Acceleration is the slope of the v-t graph. (Velocity is the slope of the x-t graph.)

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

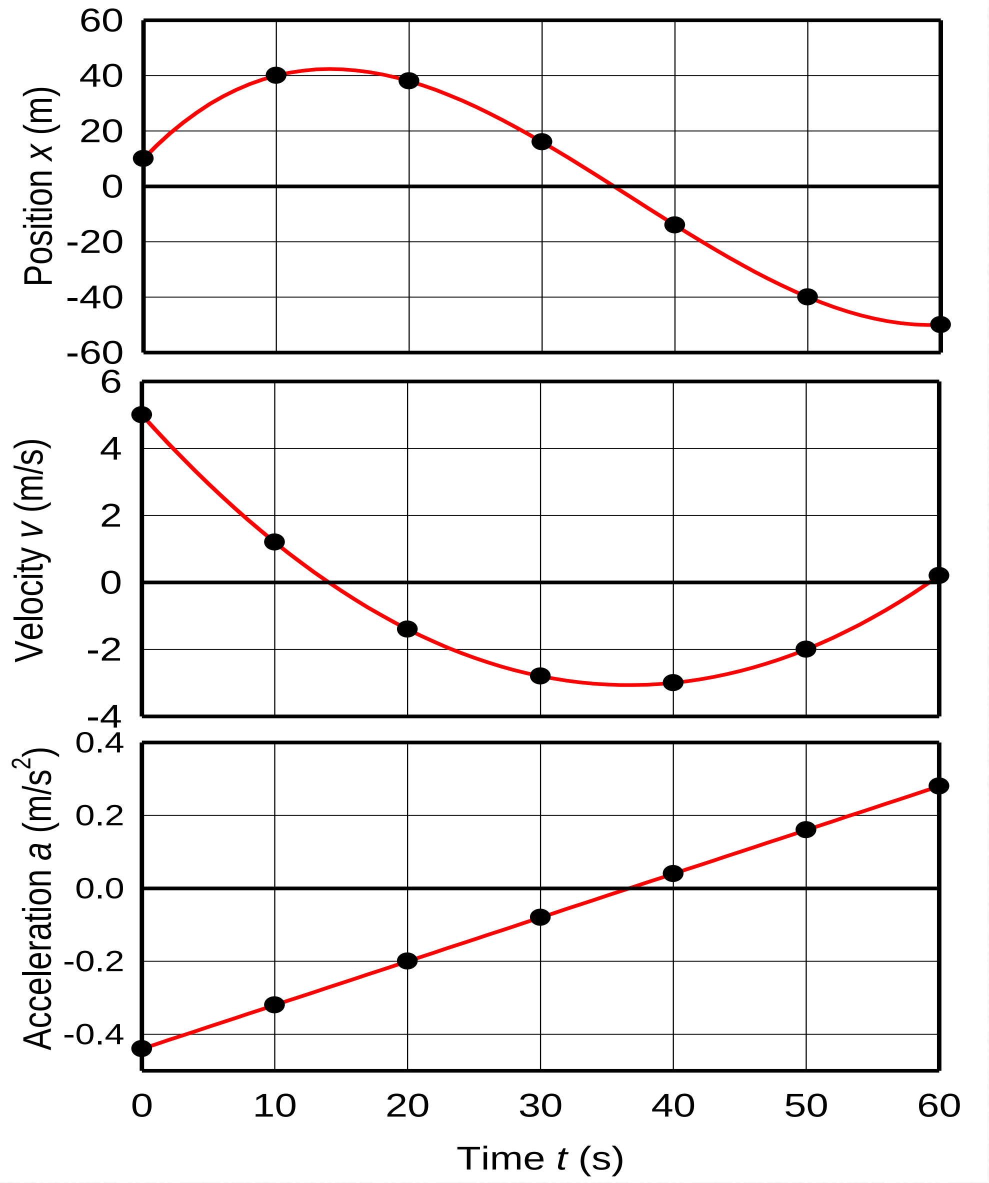


Instantaneous Acceleration

- Did you notice this?

$$a_x = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

- When you differentiate position-time 2 times, you get acceleration



speed

$$x(t) = 10 + 5t - 0.22t^2 + 0.002t^3$$

$$v_x = \frac{dx}{dt}$$

velocity

$$v(t) = 5 - 0.44t + 0.006t^2$$

$$a_x = \frac{dv_x}{dt}$$

acceleration

$$a(t) = -0.44 + 0.012t$$

Example

A car's velocity as a function of time is given by

$$v_x(t) = \alpha + \beta t^2$$

where $\alpha = 3.00$ m/s, and $\beta = 0.100$ m/s^3.

a) Calculate the average acceleration for the time interval $t = 0$ to $t = 5.00$ s.

b) Calculate the instantaneous acceleration for (i) $t = 0$ s, (ii) $t = 5.00$ s.

Example

- a) Calculate the average acceleration for the time interval $t = 0$ to $t = 5.00$ s.

(a) $v_x(t) = 3.00 + 0.100t^2$

$t = 0.00\ \mathrm{s} \Rightarrow v_{xi} = 3.00 + 0.100(0.00)^2 = 3.00\ \mathrm{m/s}$

$t = 5.00\ \mathrm{s} \Rightarrow v_{xf} = 3 + (0.100)(5.00)^2 = 5.50\ \mathrm{m/s}$

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{5.50 - 3.00}{5.00 - 0.00} = 0.500\ \mathrm{m/s^2}$$

Example

- b) Calculate the instantaneous acceleration for (i) $t = 0$ s, (ii) $t = 5.00$ s.

(b) $a_x(t) = \dfrac{dv_x}{dt} = 0.200t$

(i) $t = 0.00\ \mathrm{s}$ ⇨ $a_{xi} = 0.00\ \mathrm{m/s^2}$

(ii) $t = 5.00\ \mathrm{s}$ ⇨ $a_{xf} = (0.200)(5.00) = 1.00\ \mathrm{m/s^2}$

1-D Motion (constant acceleration)

If a is constant: $$a = \frac{dv}{dt}$$ ⇒ $$dv = a dt$$

Integrating: $$\int dv = \int a dt$$ ⇒ $$v = at + C$$

If $v = v_0$ at $t = 0$: $$\boxed{v = v_0 + at}$$

1-D Motion (constant acceleration)

- Similarly, for velocity,

$$v = \frac{dx}{dt} = v_0 + at \quad \Rightarrow \quad dx = v_0 dt + at dt$$

Integrating: $$\int dx = v_0 \int dt + a \int t dt$$

If $v = v_0$ and $x = x_0$ at $t = 0$: $$\boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2}$$

1-D Motion (constant acceleration)

- To eliminate t from $v = v_0 + at$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Square first equation and substitute in the second equation, we get $\boxed{v^2 = v_0^2 + 2a(x - x_0)}$

Eliminate a, we get $\boxed{x - x_0 = \frac{1}{2}(v + v_0)t}$

1-D Motion (constant acceleration)

- 4 very **important** equations for 1-D motion with constant acceleration:

$$\boxed{v = v_0 + at}$$

v = u + at

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$

s = ut + 1/2at^2

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

v^2 = u^2+2as

$$\boxed{x - x_0 = \frac{1}{2}(v + v_0)t}$$

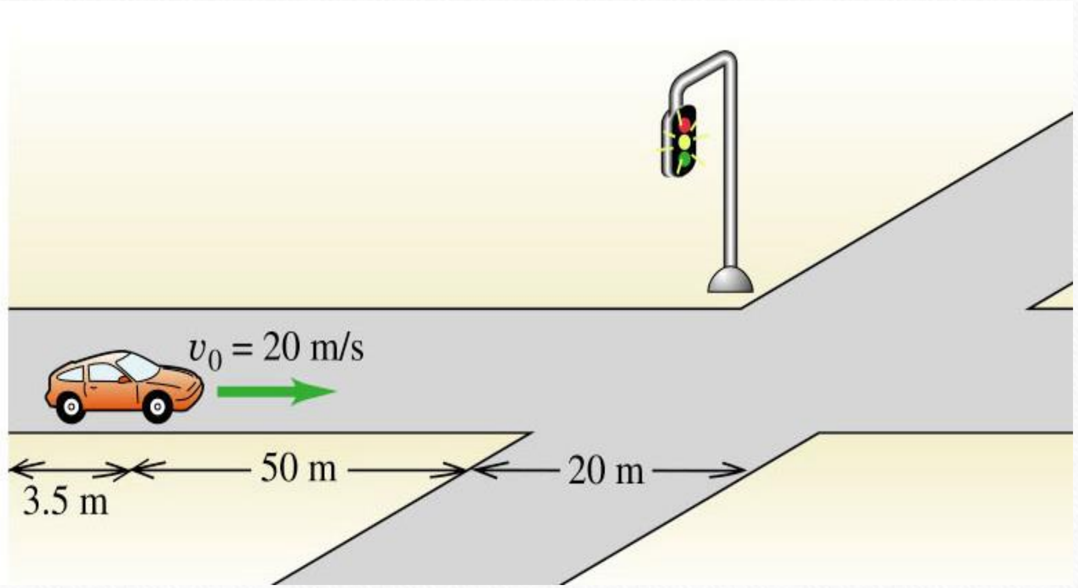
s = 1/2(v+u)t

Note: Valid only for <u>**constant acceleration**</u>

Example

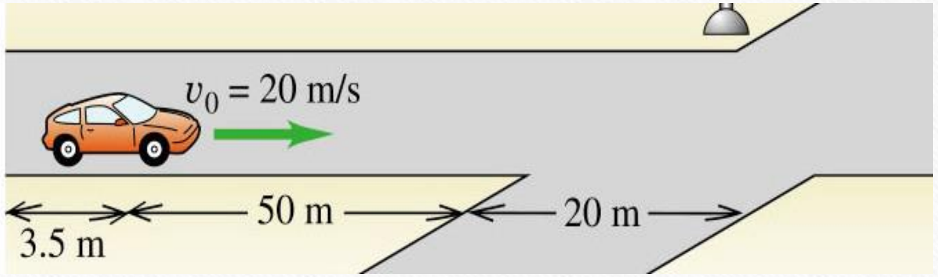
A car 3.5 m in length approaches an intersection that is 20 m wide. The light turns yellow when the car's speed is 20 m/s and the front of the car is 50 m from the beginning of the intersection. If the driver steps on the brake, the car slows at -3.8 m/s^2. If he instead steps on the gas pedal, the car accelerates at 2.3 m/s^2. The light stays yellow for 3.0 s.

Ignore the driver's reaction time. To avoid being in the intersection at the instant the light turns red, should the driver step on the brake or the gas?



Example

- $v_0 = 20$ m/s
- $a_1 = -3.8$ m/s^2
- $a_2 = +2.3$ m/s
- $t = 3.0$ s



If he presses on the brakes, the car would have **moved**:

$$x_1 = v_0 t + \frac{1}{2} a_1 t^2 = 20(3) + \frac{1}{2}(-3.8)(3)^2 = 42.9 \text{ m} < 50.0 \text{ m}$$

If he presses on the gas (accelerator), the car would have moved:

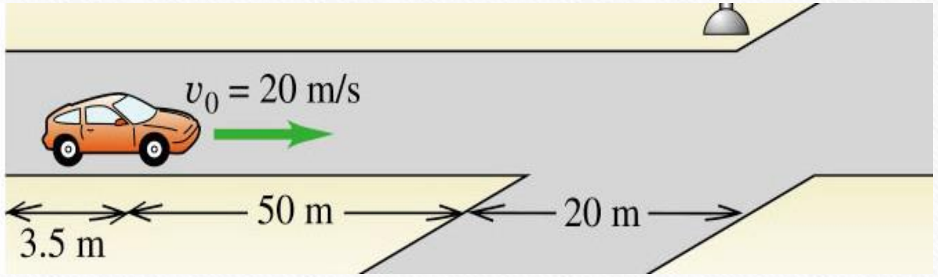
$$x_2 = v_0 t + \frac{1}{2} a_2 t^2 = 20(3) + \frac{1}{2}(2.3)(3)^2 = 70.4 \text{ m} < 73.5 \text{ m}$$

Parts of the car inside intersection!

Ans: He needs to step on the brake pedal.

Example

- $v_0 = 20$ m/s
- $a_1 = -3.8$ m/s^2
- $a_2 = +2.3$ m/s
- $t = 3.0$ s



BUT logically should he step on the brake?

IF he stepped on brake, at $t = 3.0$ s, the car has not stopped yet!

The car will only stop at t_1 where:

$$20 - 3.8t_1 = 0 \quad \Rightarrow \quad t_1 = 20/3.8 = 5.3 \text{ s}$$

$$x = 20(5.3) - \frac{1}{2}(3.8)(5.3)^2 = 52.6 \text{ m}$$

He will not be in the intersection when the light turns red BUT will be when the car stops.

Free Fall

- All objects when dropped, fall toward the Earth with nearly constant acceleration (height << Radius of earth)
- If air resistance can be neglected, then the motion is described as free fall
- Free fall acceleration or acceleration due to gravity:

$$g \approx 9.80 \text{ ms}^{-2}$$

- Whether an object is moving upward or downward, it will experience an acceleration downward of magnitude g if it is in free fall.
- 1D motion under constant acceleration.

Free Fall

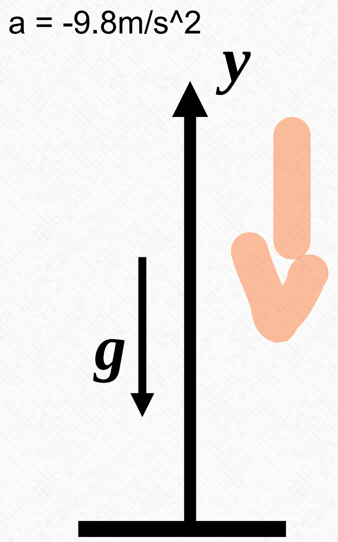
By convention, the vertical direction y is positive upward.

Hence, the acceleration due to gravity (downward):

$$a = -g$$

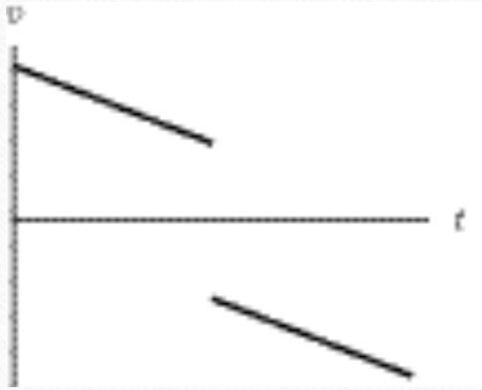
Corresponding kinematic equations:

$$v = v_0 - gt$$
$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$
$$v^2 = v_0^2 - 2g(y - y_0)$$



Concept Question

The graph shows the **velocity versus time** graph for a ball. Which explanation best fits the motion of the ball as shown by the graph?



A. The ball is rolling, stops, and then continues rolling.

B. The ball is rising, hits the ceiling, and falls down.

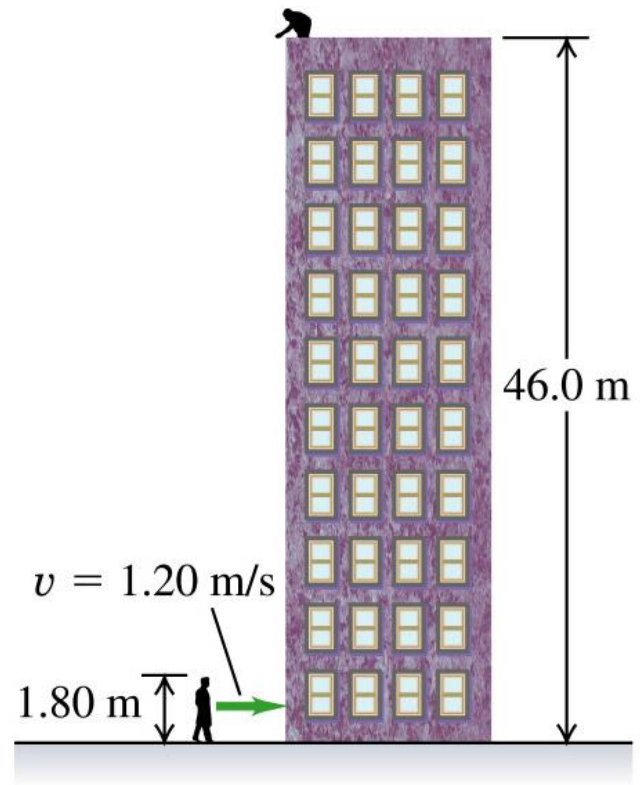
C. The ball is falling, hits the floor, and bounces up.

D. The ball is rising, is caught, and then is thrown down.

E. The ball is falling, is caught, and is thrown down with greater velocity.

Example

You are on the roof of a building, 46.0 m above the ground. Your most hated friend, who is 1.80 m tall, is walking alongside the building at a constant speed of 1.20 m/s. If you wish to drop an egg on his head, how far from the building should your friend be when you drop the egg? (Assume free fall.)



Example

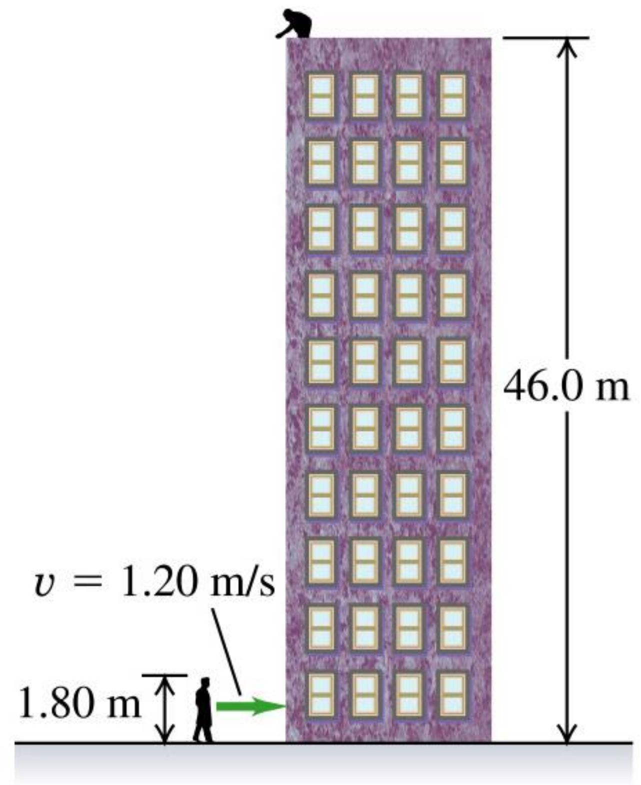
- To find the time taken, *t*, to reach the right height

$$y = y_0 + \cancel{v_0}^{\,0} t - \frac{1}{2} g t^2$$

$$1.80 = 46.0 - \frac{1}{2}(9.80)t^2$$

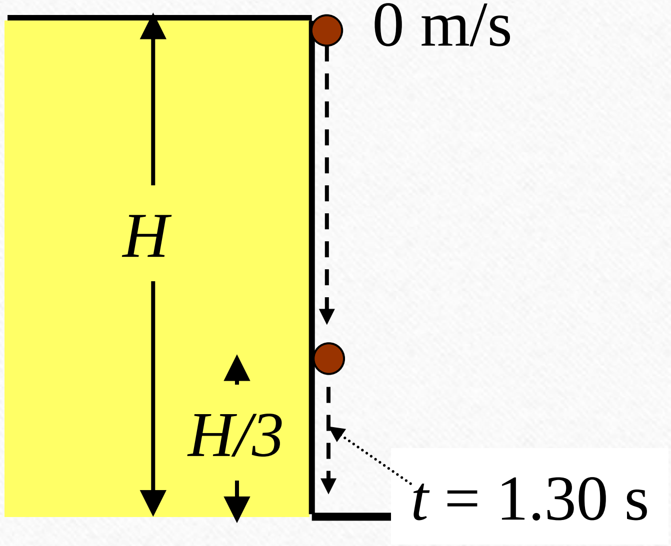
$$t = \sqrt{\frac{2 \times (46.0 - 1.80)}{9.80}} = 3.00 \text{ s}$$

$$x = v_p t = 1.20 \times 3.00 = 3.60 \text{ m}$$



Example

An alert hiker sees a boulder fall from the top of a distant cliff and notes that it takes 1.30 s for the boulder to fall from the last third of the way to the ground. What is the height of the cliff? (Ignore air resistance.)



Example

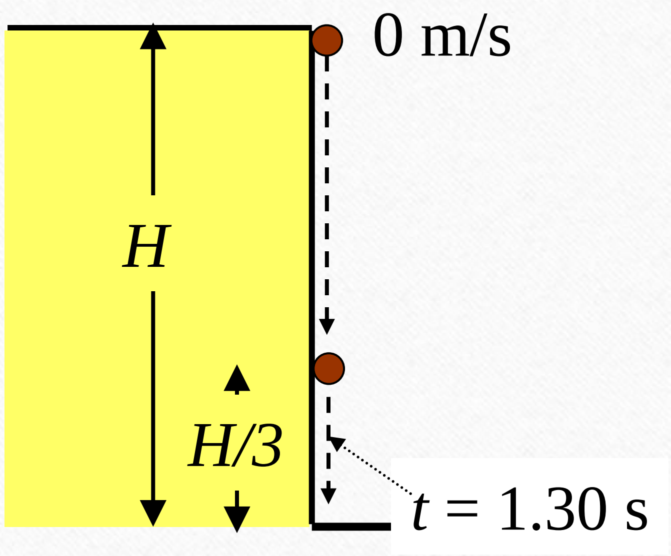
Let T be the time it takes to fall to the ground.

Then: $H = \frac{1}{2}gT^2$; $\frac{2}{3}H = \frac{1}{2}g(T-t)^2$

$$\frac{1}{3}gT^2 = \frac{1}{2}g(T-t)^2 \Rightarrow \frac{2}{3}T^2 = (T-t)^2$$

$$\Downarrow$$

$$\boxed{H = 246 \text{ m}} \Leftarrow T = \frac{t}{1-\sqrt{\frac{2}{3}}} = 7.08 \text{ s} \Leftarrow \sqrt{\frac{2}{3}}T = T - t$$



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