CSD2201/CSD2200 Week 1 Tutorial Problems

28th August – 3rd September 2023

These are recap/revision questions from CSD1251/1250 Calculus I. Be sure to familiarize yourselves with the concepts focused in these questions, as they will be used actively in class, quizzes and examinations.

Question 1

Let f and g be differentiable functions, where the range of f is a subset of the domain of g. Name, and state the differentiation formula for

(a) $(fg)'(x)$

(b) $(f \circ g)'(x)$

Question 2

Differentiate the following functions with respect to their variables.

(a) $f(x) = x^3 + 2x^2 - 6x$ (b) $f(x) = \cos(2x)$ (c) $g(t) = \tan^2(2t)$

(d) $f(x) = \sqrt{x^2 + 2x}$ (e) $g(n) = \dfrac{n}{n^2+1}$ (f) $h(x) = \sec^2(x)$

(g) $u(t) = \dfrac{\ln t}{t^2}$ (h) $v(t) = \sin(t) + \cos^2(2t)$ (i) $f(x) = \dfrac{1}{(1-x)^2}$

Question 3

Show that the function $f(x) = \dfrac{1}{x}$ is decreasing on $(0, \infty)$.

Question 4

Show that the function $f(x) = \dfrac{\ln x}{x}$ is decreasing on the interval (e, ∞).

What can we say about the function $g(x) = -\dfrac{\ln x}{x}$?

Question 5

Compute the following limits.

(a) $\displaystyle\lim_{x\to 1} \frac{x^7 - 1}{x - 1}$ (b) $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{e^{4x}\sin(3x) + e^{2\pi}}{x - \frac{\pi}{2}}$ (c) $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{e^{4x}\sin(3x) + e^{2\pi}}{\sqrt{x} - \sqrt{\frac{\pi}{2}}}$

Question 6

The *inverse sine* function $y = \sin^{-1}(x)$ has domain $-1 \le x \le 1$ and range y such that

$$\sin(y) = x \quad \text{where} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}.$$

Meanwhile, the *inverse tangent* function $y = \tan^{-1}(x)$ has domain $x \in \mathbb{R}$ and range y such that

$$\tan(y) = x \quad \text{where} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

Using implicit differentiation, show that

(a) The derivative of $y = \sin^{-1}(x)$ is

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}.$$

(b) The derivative of $y = \tan^{-1}(x)$ is

$$\frac{dy}{dx} = \frac{1}{1 + x^2}.$$

You may need the following trigonometric identities here:

$$\sin^2 x + \cos^2 x = 1 \qquad \text{and} \qquad \tan^2 x + 1 = \sec^2 x.$$