CSD2301 Practice Solutions
11. **Rotation and Moment of Inertia Part 1**

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Practice Question 1

(a) What angle in radians is subtended by an arc 1.50 m long on the circumference of a circle of radius 2.50 m? What is this angle in degrees? (b) An arc 14.0 cm long on the circumference of a circle subtends an angle of 128°. What is the radius of the circle? (c) The angle between two radii of a circle with radius 1.50 m is 0.700 rad. What length of arc is intercepted on the circumference of the circle by the two radii?

IDENTIFY: $s = r\theta$, with θ in radians.

SET UP: $\pi \text{ rad} = 180°$.

EXECUTE: **(a)** $\theta = \dfrac{s}{r} = \dfrac{1.50 \text{ m}}{2.50 \text{ m}} = 0.600 \text{ rad} = 34.4°$

(b) $r = \dfrac{s}{\theta} = \dfrac{14.0 \text{ cm}}{(128°)(\pi \text{ rad}/180°)} = 6.27 \text{ cm}$

(c) $s = r\theta = (1.50 \text{ m})(0.700 \text{ rad}) = 1.05 \text{ m}$

EVALUATE: An angle is the ratio of two lengths and is dimensionless. But, when $s = r\theta$ is used, θ must be in radians. Or, if $\theta = s/r$ is used to calculate θ, the calculation gives θ in radians.

Practice Question 2

An airplane propeller is rotating at 1900 rpm (rev/min). (a) Compute the propeller's angular velocity in rad/s. (b) How many seconds does it take for the propeller to turn through 35˚?

IDENTIFY: $\theta - \theta_0 = \omega t$, since the angular velocity is constant.

SET UP: $1\text{ rpm} = (2\pi/60)\text{ rad/s}$.

EXECUTE: **(a)** $\omega = (1900)(2\pi\text{ rad}/60\text{ s}) = 199\text{ rad/s}$

(b) $35° = (35°)(\pi/180°) = 0.611\text{ rad}$. $t = \dfrac{\theta - \theta_0}{\omega} = \dfrac{0.611\text{ rad}}{199\text{ rad/s}} = 3.1\times 10^{-3}\text{ s}$

EVALUATE: In $t = \dfrac{\theta - \theta_0}{\omega}$ we must use the same angular measure (radians, degrees or revolutions) for both $\theta - \theta_0$ and ω.

Practice Question 3

A bicycle wheel has an initial angular velocity of 1.50 rad/s. (a) If its angular acceleration is constant and equal to 0.300 rad/s^2, what is its angular velocity at t = 2.50 s? (b) Through what angle has the wheel turned between t = 0 and t = 2.50 s?

IDENTIFY: Apply the constant angular acceleration equations.

SET UP: Let the direction the wheel is rotating be positive.

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t = 1.50\ \text{rad/s} + (0.300\ \text{rad/s}^2)(2.50\ \text{s}) = 2.25\ \ \text{rad/s}.$

(b) $\theta - \theta_0 = \omega_{0z} t + \frac{1}{2}\alpha_z t^2 = (1.50\ \text{rad/s})(2.50\ \text{s}) + \frac{1}{2}(0.300\ \text{rad/s}^2)(2.50\ \text{s})^2 = 4.69\ \text{rad}.$

EVALUATE: $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t = \left(\dfrac{1.50\ \text{rad/s} + 2.25\ \text{rad/s}}{2}\right)(2.50\ \text{s}) = 4.69\ \text{rad}$, the same as calculated with another equation in part (b).

Practice Question 4

The rotating blade of a blender turns with constant angular acceleration 1.50 rad/s^2. (a) How much time does it take to reach an angular velocity of 36.0 rad/s, starting from rest? (b) Through how many revolutions does the blade turn in this time interval?

IDENTIFY: Apply the constant angular acceleration equations to the motion. The target variables are t and $\theta - \theta_0$.

SET UP: **(a)** $\alpha_z = 1.50\ \text{rad/s}^2$; $\omega_{0z} = 0$ (starts from rest); $\omega_z = 36.0\ \text{rad/s}$; $t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{36.0\ \text{rad/s} - 0}{1.50\ \text{rad/s}^2} = 24.0\ \text{s}$

(b) $\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z} t + \frac{1}{2}\alpha_z t^2 = 0 + \frac{1}{2}(1.50\ \text{rad/s}^2)(2.40\ \text{s})^2 = 432\ \text{rad}$

$\theta - \theta_0 = 432\ \text{rad}(1\ \text{rev}/2\pi\ \text{rad}) = 68.8\ \text{rev}$

EVALUATE: We could use $\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$ to calculate $\theta - \theta_0 = \frac{1}{2}(0 + 36.0\ \text{rad/s})(24.0\ \text{s}) = 432\ \text{rad}$, which checks.

Practice Question 5

A circular saw blade 0.200 m in diameter starts from rest. In 6.00 s it accelerates with constant angular acceleration to an angular velocity of 140 rad/s. Find the angular acceleration and the angle through which the blade has turned.

IDENTIFY: Apply the constant angular acceleration equations.

SET UP: Let the direction of the rotation of the blade be positive. $\omega_{0z} = 0$.

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z$ gives $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{140\ \text{rad/s} - 0}{6.00\ \text{s}} = 23.3\ \text{rad/s}^2$.

$$(\theta - \theta_0) = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t = \left(\frac{0 + 140\ \text{rad/s}}{2}\right)(6.00\ \text{s}) = 420\ \text{rad}$$

EVALUATE: We could also use $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$. This equation gives $\theta - \theta_0 = \frac{1}{2}(23.3\ \text{rad/s}^2)(6.00\ \text{s})^2 = 419\ \text{rad}$, in agreement with the result obtained above.

Practice Question 6

A high-speed flywheel in a motor is spinning at 500 rpm when a power failure suddenly occurs. The flywheel has mass 40.0 kg and diameter 75.0 cm. The power is off for 30.0 s, and during this time the flywheel slows due to friction in its axle bearings. During the time the power is off, the flywheel makes 200 complete revolutions. (a) At what rate is the flywheel spinning when the power comes back on? (b) How long after the beginning of the power failure would it have taken the flywheel to stop if the power had not come back on, and how many revolutions would the wheel have made during this time?

IDENTIFY: Apply constant angular acceleration equations.

SET UP: Let the direction the flywheel is rotating be positive.

$\theta - \theta_0 = 200$ rev, $\omega_{0z} = 500$ rev/min $= 8.333$ rev/s, $t = 30.0$ s.

EXECUTE: **(a)** $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = 5.00$ rev/s $= 300$ rpm

(b) Use the information in part (a) to find α_z: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -0.1111$ rev/s^2. Then $\omega_z = 0$,

$\alpha_z = -0.1111$ rev/s^2, $\omega_{0z} = 8.333$ rev/s in $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = 75.0$ s and $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives

$\theta - \theta_0 = 312$ rev.

EVALUATE: The mass and diameter of the flywheel are not used in the calculation.

Practice Question 7

A wheel of diameter 40.0 cm starts from rest and rotates with a constant angular acceleration of 3.00 rad/s^2. At the instant the wheel has computed its second revolution, compute the radial acceleration of a point on the rim in two ways: (a) using the relationship $a_{rad} = r\omega^2$ and (b) from the relationship $a_{rad} = v^2/r$.

IDENTIFY: Use constant acceleration equations to calculate the angular velocity at the end of two revolutions. $v = r\omega$.

SET UP: 2 rev $= 4\pi$ rad. $r = 0.200$ m.

EXECUTE: **(a)** $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$. $\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(3.00\ \text{rad/s}^2)(4\pi\ \text{rad})} = 8.68\ \text{rad/s}$.

$a_{rad} = r\omega^2 = (0.200\ \text{m})(8.68\ \text{rad/s})^2 = 15.1\ \text{m/s}^2$.

(b) $v = r\omega = (0.200\ \text{m})(8.68\ \text{rad/s}) = 1.74\ \text{m/s}$. $a_{rad} = \dfrac{v^2}{r} = \dfrac{(1.74\ \text{m/s})^2}{0.200\ \text{m}} = 15.1\ \text{m/s}^2$.

EVALUATE: $r\omega^2$ and v^2/r are completely equivalent expressions for a_{rad}.

Practice Question 8

An advertisement claims that a centrifuge takes up only 0.127 m diameter of bench space but can produce a radial acceleration of 3000g (3000 times of gravitational acceleration) at 5000 rev/min. Calculate the required radius of the centrifuge. Is the claim realistic?

SET UP: $a_{\text{rad}} = r\omega^2$ so $r = a_{\text{rad}} / \omega^2$, where ω must be in rad/s

EXECUTE: $a_{\text{rad}} = 3000g = 3000(9.80 \text{ m/s}^2) = 29,400 \text{ m/s}^2$

$$\omega = (5000 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 523.6 \text{ rad/s}$$

Then $r = \dfrac{a_{\text{rad}}}{\omega^2} = \dfrac{29,400 \text{ m/s}^2}{(523.6 \text{ rad/s})^2} = 0.107 \text{ m}.$

EVALUATE: The diameter is then 0.214 m, which is larger than 0.127 m, so the claim is *not* realistic.

Practice Question 9

According to the shop manual, when drilling a 12.7-mm-diameter hole in wood, a drill should have a speed of 1250 rev/min. For a 12.7-mm-diameter drill bit turning at a constant 1250 rev/min, find (a) the maximum linear speed of any part of the bit and (b) the maximum radial acceleration of any part of the bit.

IDENTIFY: $v = r\omega$ and $a_{\text{rad}} = r\omega^2 = v^2/r$.

SET UP: 2π rad $= 1$ rev , so π rad/s $= 30$ rev/min .

EXECUTE: **(a)** $\omega r = (1250\ \text{rev/min})\left(\dfrac{\pi\ \text{rad/s}}{30\ \text{rev/min}}\right)\left(\dfrac{12.7\times 10^{-3}\,\text{m}}{2}\right) = 0.831\ \text{m/s}.$

(b) $\dfrac{v^2}{r} = \dfrac{(0.831\ \text{m/s})^2}{(12.7\times 10^{-3}\ \text{m})/2} = 109\ \text{m/s}^2.$

EVALUATE: In $v = r\omega$, ω must be in rad/s.