CSD1241 Tutorial 1

Problem 1. Given 3 vectors $\vec{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

(a) Find the coordinates of the vectors $\vec{u} + \vec{v}$, $\vec{v} - \vec{u}$, $-3\vec{w}$ and $\vec{u} + \vec{v} - 3\vec{w}$.

(b) Graph in one picture the vectors $\vec{u} + \vec{v}, \vec{v} - \vec{u}, 2\vec{w}$ and $\vec{v} - \vec{u} + 2\vec{w}$.

(c) Compute the dot products $\vec{u} \cdot \vec{w}, (\vec{u} + \vec{v}) \cdot (-3\vec{w})$ and $(\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v} - 3\vec{w})$.

(d) Find the angle between the vectors $\vec{u}$ and $\vec{v}$.

(e) Find the angle between the vectors $\vec{v} - \vec{u}$ and $\vec{w} - \vec{u}$.

Problem 2. Consider the points $P = (2, 5), Q = (4, -1), R = (5, 2)$.

(a) Find the midpoints M_{PQ} and M_{PR} of the line segments PQ and PR.

(b) Find the midpoint M of the line segment $M_{PQ}M_{PR}$.

(c) Find real numbers a, b such that

$$\overrightarrow{PM} = a\overrightarrow{PQ} + b\overrightarrow{PR}.$$

Problem 3. (a) Graph in one picture the points $P = (3, 2), Q = (5, 0)$ and $R = (2, -1)$.

(b) Compute the distances $d(P, Q), d(P, R), d(Q, R)$.

(c) Compute all three angles of the triangle $\triangle PQR$.

(d) Compute the area of $\triangle PQR$.

Hint for d. Area($\triangle PQR$)=$\frac{1}{2}PQ \times PR \times \sin(\angle P)$

Problem 4. Consider three points $A = (2, 3), B = (-2, 4)$, $C = (-3, -2)$.

(a) Find all the lengths of the sides of $\triangle ABC$.

(b) Find all three angles of $\triangle ABC$.

(c) Compute the area of $\triangle ABC$.

(d) From C, draw vertically to AB and let H be the intercept of the vertical line with AB. Find the coordinates of H.

Problem 5. Consider three points $A = (1, 1, 2), B = (0, 1, 4), C = (2, 3, 5)$.

(a) Find the projection of $\overrightarrow{AC}$ onto $\overrightarrow{AB}$, that is, $\text{proj}_{\overrightarrow{AB}}(\overrightarrow{AC})$.

(b) Find the orthogonal complement of $\overrightarrow{AC}$ on $\overrightarrow{AB}$, that is,

$$\overrightarrow{AC}^{\perp} = \overrightarrow{AC} - \text{proj}_{\overrightarrow{AB}}(\overrightarrow{AC}).$$

Further, check that $\overrightarrow{AC}^{\perp}$ and $\overrightarrow{AB}$ are orthogonal.

(c) Let D be another point such that $ABCD$ is a parallelogram. Find D.

Hint: $\overrightarrow{AD} = \overrightarrow{BC}$.

Problem 6. (a) Find the condition for the coordinates a, b of $\vec{x} = \begin{bmatrix} a \\ b \end{bmatrix}$ such that $\vec{x}$ is orthogonal to $\vec{u} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$. Could you give 3 examples of such vectors $\vec{y}$? Could you give a geometric interpretation for all vectors which are orthogonal to $\vec{u}$?

(b) Find the condition for the coordinates a, b, c of $\vec{y} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ such that $\vec{y}$ is orthogonal to $\vec{v} = \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix}$. Could you give 3 examples of such vectors $\vec{y}$? Do you know the geometric description for all these vectors $\vec{y}$?