# CSD2258 Tutorial 9

*Problem* 1. Let $P$ be a probability measure on a sample space $\Omega$ and let $A, B, C$ be events.
(a) Suppose that $P(A) = 1/3, P(B) = 1/4$, and $P(A \cup B) = 5/9$. Find $P(A \cap B)$.
(b) Suppose that $P(A) = 1/2, P(B) = 1/4, P(A \cap \bar{B}) = 3/8$, and $B \subset C$. Find $P(A \cup \bar{B} \cup \bar{C})$.
(c) Someone claims that $P(A) = P(B) = P(C) = 9/10$, $P(A \cap B) = P(A \cap C) = P(B \cap C) = 7/10$, and $P(A \cap B \cap C) = 5/10$. Can the claim be correct?

*Problem* 2. (a) A family has 2 children. Assume that each of the possibilities BB, BG, GB, and GG is equally likely, where B = boy and G = girl. What is the conditional probability that the family has two boys, given that they have at least one boy?
(b) A 0-1 bit string of length 4 is generated randomly so that each of 16 strings is equally likely. What is the probability that it contains at least two consecutive 0's, given that its first bit is 0? (we assume that 0 bits and 1 bits are equally likely).

*Problem* 3. (a) You know that a bridge players' hand of 13 cards contains at least one ace. What is the probability that it contains exactly 2 aces?
(b) You know that a bridge players' hand of 13 cards contains the ace of hearts. What is the probability that it contains exactly two aces.

*Problem* 4. A 0-1 bit string of length 4 is generated randomly so that each of 16 strings is equally likely. Let $A$ be the event that the string starts with 1. Let $B$ be the event that the string contains an even number of 1's. Are $A$ and $B$ independent? Justify your answer.

*Problem* 5. Let $A$ and $B$ be independent events.
(a) Prove that the events $A$ and $\bar{B}$ are independent.
(b) Prove that $\bar{A}$ and $B$ are independent. Further, prove that $\bar{A}$ and $\bar{B}$ are also independent.

# Hints and Instructions

1b. $B \subset C \Rightarrow \bar{C} \subset \bar{B}$. So $A \cup \bar{B} \cup \bar{C} = A \cup \bar{B}$.
1c. Compute $P(A \cup B \cup C)$.

2a. The sample space is $\Omega$={BB,BG,GB,GG}. Let $E$ and $F$ be events that the family has two boys and the family has at least one boy, respectively. You need to compute $P(E|F)$.
2b. The sample space is $\Omega = \{(a, b, c, d) : a, b, c, d \in \{0, 1\}\}$. Let $A$ and $B$ be the events that the string contains $\geq 2$ consecutive 0's and the first bit is 0, respectively. You need to compute $P(A|B)$.

3a. Let $A$ and $B$ be the events that 13 cards contain at least one ace and 13 cards contain exactly two aces, respectively. You need to compute $P(B|A)$.
3b. Let $C$ and $D$ be the events that 13 cards contain the ace of hearts and 13 cards contain exactly two aces. You need to compute $P(D|C)$.

4. Compute $P(A), P(B), P(A \cap B)$ and check whether $P(A \cap B) = P(A)P(B)$.

5a. Use the equation $P(A) = P(A \cap B) + P(A \cap \bar{B})$ to show that $P(A \cap \bar{B}) = P(A)P(\bar{B})$.
5b. Use part a.