Methods to test statistical Hypothesis

1. The traditional method (The Critical-Value Approach)
2. The *P*-value method
3. The confidence interval method

The Critical-Value Approach

Rejection Region, Nonrejection Region, and Critical Values

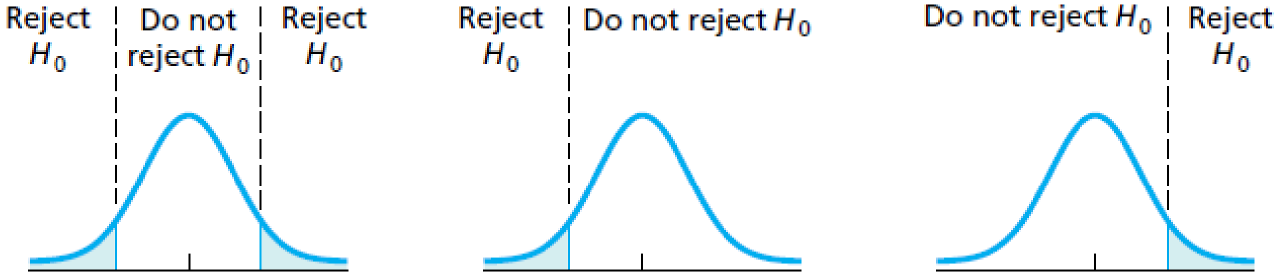
Rejection region: The set of values for the test statistic that leads to rejection of the null hypothesis.

Nonrejection region: The set of values for the test statistic that leads to nonrejection of the null hypothesis.

Critical value(s): The value or values of the test statistic that separate the rejection and nonrejection regions. A critical value is considered part of the rejection region.

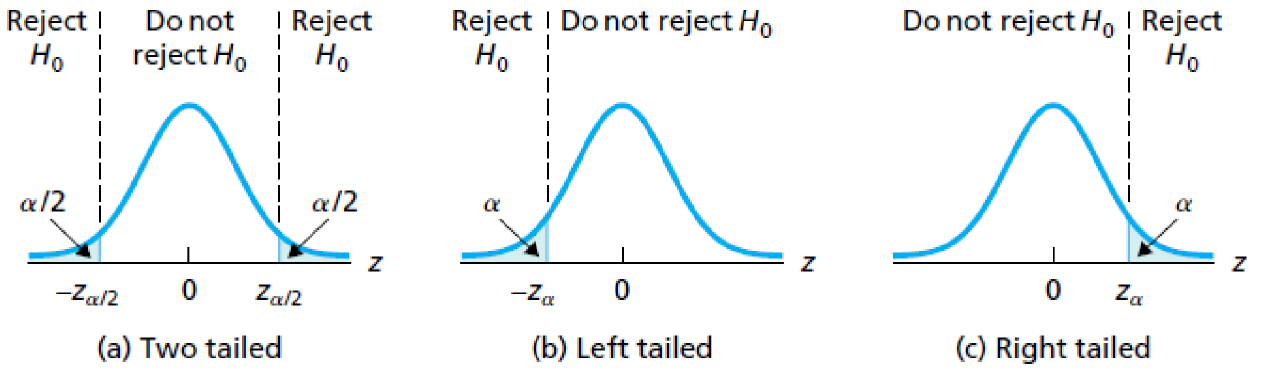
CRITICAL-VALUE APPROACH TO HYPOTHESIS TESTING

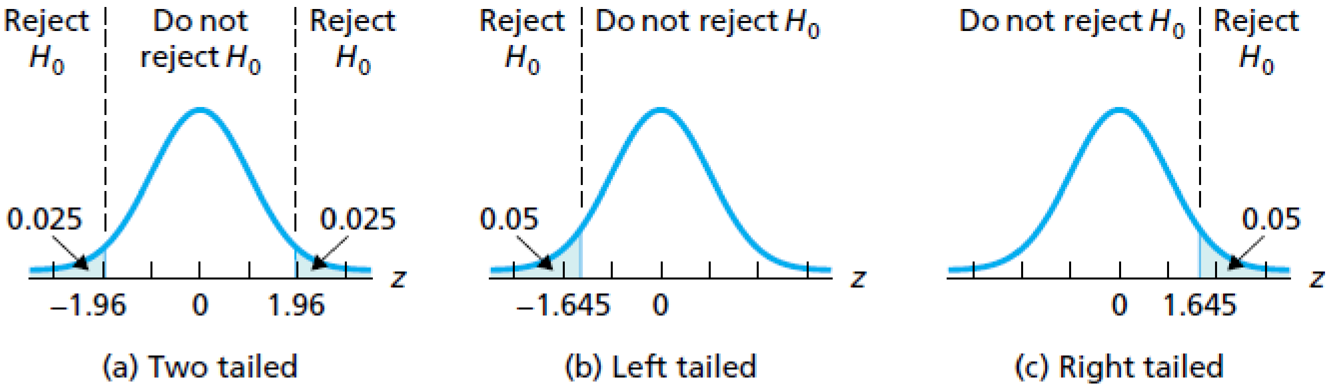
Step 1	State the null and alternative hypotheses.
Step 2	Decide on the significance level, α.
Step 3	Compute the value of the test statistic.
Step 4	Determine the critical value(s).
Step 5	If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.
Step 6	Interpret the result of the hypothesis test.



(a) Two tailed (b) Left tailed (c) Right tailed

	Two-tailed test	Left-tailed test	Right-tailed test
Sign in H_a	$\neq$	$<$	$>$
Rejection region	Both sides	Left side	Right side





The P-Value Approach

P-VALUE APPROACH TO HYPOTHESIS TESTING

Step 1 State the null and alternative hypotheses.

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic.

Step 4 Determine the P-value, P.

Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the result of the hypothesis test.

Decision Criterion for a Hypothesis Test Using the *P*-Value

If the P-value is less than or equal to the specified significance level, reject the null hypothesis; otherwise, do not reject the null hypothesis. In other words, if $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Hypothesis testing for one population Mean σ is known

Critical-Value and P-Value Approach

PROCEDURE 9.1 One-Mean z-Test

Purpose To perform a hypothesis test for a population mean, μ

Assumptions
1. Simple random sample
2. Normal population or large sample
3. σ known

Step 1 The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is

H_a: $\mu \neq \mu_0$ (Two tailed) or H_a: $\mu < \mu_0$ (Left tailed) or H_a: $\mu > \mu_0$ (Right tailed).

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

and denote that value z_0.

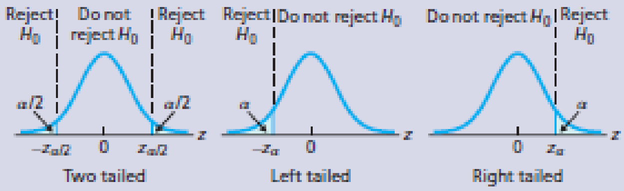
CRITICAL-VALUE APPROACH OR **P-VALUE APPROACH**

CRITICAL-VALUE APPROACH

Step 4 The critical value(s) are

$\pm z_{\alpha/2}$ (Two tailed) or $-z_\alpha$ (Left tailed) or z_α (Right tailed).

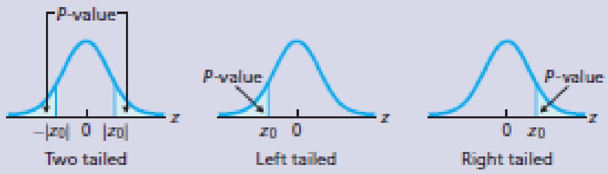
Use Table II to find the critical value(s).



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

P-VALUE APPROACH

Step 4 Use Table II to obtain the P-value.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Note: The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

When to Use the One-Mean z-Test†

- For small samples—say, of size less than 15—the z-test should be used only when the variable under consideration is normally distributed or very close to being so.
- For samples of moderate size—say, between 15 and 30—the z-test can be used unless the data contain outliers or the variable under consideration is far from being normally distributed.
- For large samples—say, of size 30 or more—the z-test can be used essentially without restriction. However, if outliers are present and their removal is not justified, you should perform the hypothesis test once with the outliers and once without them to see what effect the outliers have. If the conclusion is affected, use a different procedure or take another sample, if possible.
- If outliers are present but their removal is justified and results in a data set for which the z-test is appropriate (as previously stated), the procedure can be used.

CONFIDENCE INTERVAL APPROACH

Confidence Interval on μ, σ^2 Known

If $\bar{x}$ is the mean of a random sample of size n from a population with known variance σ^2, a $100(1-\alpha)\%$ confidence interval for μ is given by

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}},$$

where $z_{\alpha/2}$ is the z-value leaving an area of $\alpha/2$ to the right.

Hypothesis testing for one population Mean when σ is unknown

Critical-Value and P-Value Approach

PROCEDURE 9.2 One-Mean t-Test

Purpose To perform a hypothesis test for a population mean, μ

Assumptions

1. Simple random sample
2. Normal population or large sample
3. σ unknown

Step 1 **The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is**

$$H_a: \mu \neq \mu_0 \text{ (Two tailed)} \quad \text{or} \quad H_a: \mu < \mu_0 \text{ (Left tailed)} \quad \text{or} \quad H_a: \mu > \mu_0 \text{ (Right tailed)}.$$

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

and denote that value t_0.

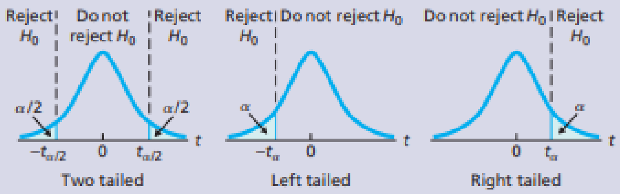
CRITICAL-VALUE APPROACH	OR	P-VALUE APPROACH

CRITICAL-VALUE APPROACH

Step 4 **The critical value(s) are**

$$\pm t_{\alpha/2} \text{ (Two tailed)} \quad \text{or} \quad -t_\alpha \text{ (Left tailed)} \quad \text{or} \quad t_\alpha \text{ (Right tailed)}$$

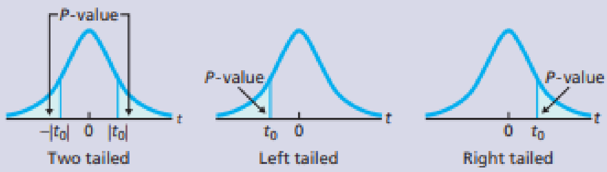
with df $= n - 1$. Use Table IV to find the critical value(s).



Step 5 **If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.**

P-VALUE APPROACH

Step 4 **The t-statistic has df $= n - 1$. Use Table IV to estimate the P-value, or obtain it exactly by using technology.**



Step 5 **If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.**

Step 6 **Interpret the results of the hypothesis test.**

Note: The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

CONFIDENCE INTERVAL APPROACH

Confidence Interval on μ, σ^2 Unknown

If $\bar{x}$ and s are the mean and standard deviation of a random sample from a normal population with unknown variance σ^2, a $100(1-\alpha)\%$ confidence interval for μ is

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}},$$

where $t_{\alpha/2}$ is the t-value with $v = n - 1$ degrees of freedom, leaving an area of $\alpha/2$ to the right.

Hypothesis Tests for the Means of Two Populations with Equal Standard Deviations, Using Independent Samples

Critical-Value and P-Value Approach

PROCEDURE 10.1 Pooled t-Test

Purpose To perform a hypothesis test to compare two population means, μ_1 and μ_2

Assumptions

1. Simple random samples
2. Independent samples
3. Normal populations or large samples
4. Equal population standard deviations

Step 1 The null hypothesis is H_0: $\mu_1 = \mu_2$, and the alternative hypothesis is

$$\begin{array}{ccccc} H_a: \mu_1 \neq \mu_2 & \text{or} & H_a: \mu_1 < \mu_2 & \text{or} & H_a: \mu_1 > \mu_2 \\ \text{(Two tailed)} & & \text{(Left tailed)} & & \text{(Right tailed)} \end{array}.$$

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{(1/n_1) + (1/n_2)}},$$

where

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}.$$

Denote the value of the test statistic t_0.

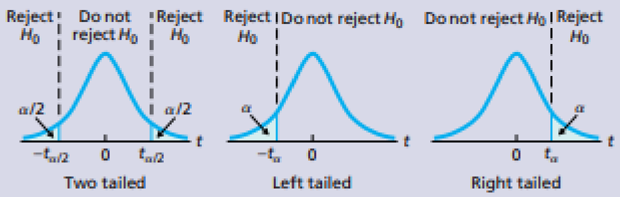
CRITICAL-VALUE APPROACH OR **P-VALUE APPROACH**

CRITICAL-VALUE APPROACH

Step 4 The critical value(s) are

$$\begin{array}{ccccc} \pm t_{\alpha/2} & \text{or} & -t_\alpha & \text{or} & t_\alpha \\ \text{(Two tailed)} & & \text{(Left tailed)} & & \text{(Right tailed)} \end{array}$$

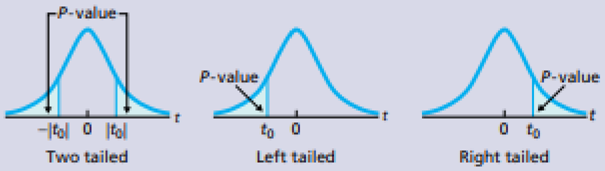
with df $= n_1 + n_2 - 2$. Use Table IV to find the critical value(s).



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

P-VALUE APPROACH

Step 4 The t-statistic has df $= n_1 + n_2 - 2$. Use Table IV to estimate the P-value, or obtain it exactly by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Note: The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

CONFIDENCE INTERVAL APPROACH

PROCEDURE 10.2 Pooled t-Interval Procedure

Purpose To find a confidence interval for the difference between two population means, μ_1 and μ_2

Assumptions

1. Simple random samples
2. Independent samples
3. Normal populations or large samples
4. Equal population standard deviations

Step 1 **For a confidence level of $1 - \alpha$, use Table IV to find $t_{\alpha/2}$ with $\mathrm{df} = n_1 + n_2 - 2$.**

Step 2 **The endpoints of the confidence interval for $\mu_1 - \mu_2$ are**

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \cdot s_p\sqrt{(1/n_1) + (1/n_2)},$$

where s_p is the pooled sample standard deviation.

Step 3 **Interpret the confidence interval.**

Note: The confidence interval is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Hypothesis Tests for the Means of Two Populations with un-equal Equal Standard Deviations Using Independent Samples

Critical-Value and P-Value Approach

PROCEDURE 10.3 Nonpooled t-Test

Purpose To perform a hypothesis test to compare two population means, μ_1 and μ_2

Assumptions

1. Simple random samples
2. Independent samples
3. Normal populations or large samples

Step 1 The null hypothesis is H_0: $\mu_1 = \mu_2$, and the alternative hypothesis is

$$\underset{\text{(Two tailed)}}{H_a\text{: } \mu_1 \neq \mu_2} \quad \text{or} \quad \underset{\text{(Left tailed)}}{H_a\text{: } \mu_1 < \mu_2} \quad \text{or} \quad \underset{\text{(Right tailed)}}{H_a\text{: } \mu_1 > \mu_2}.$$

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}.$$

Denote the value of the test statistic t_0.

CRITICAL-VALUE APPROACH	OR	P-VALUE APPROACH

CRITICAL-VALUE APPROACH

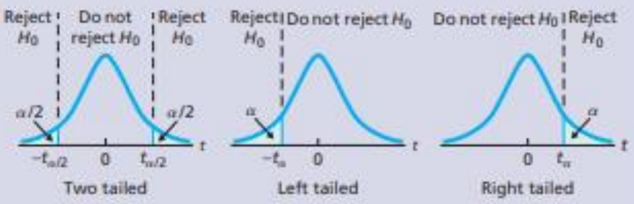
Step 4 The critical value(s) are

$$\underset{\text{(Two tailed)}}{\pm t_{\alpha/2}} \quad \text{or} \quad \underset{\text{(Left tailed)}}{-t_{\alpha}} \quad \text{or} \quad \underset{\text{(Right tailed)}}{t_{\alpha}}$$

with df = Δ, where

$$\Delta = \frac{\left[(s_1^2/n_1) + (s_2^2/n_2)\right]^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}}$$

rounded down to the nearest integer. Use Table IV to find the critical value(s).



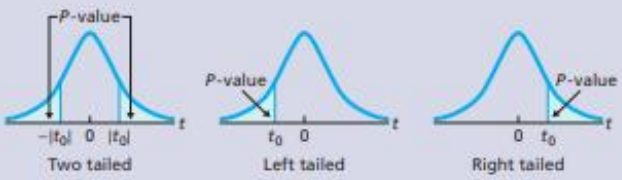
Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

P-VALUE APPROACH

Step 4 The t-statistic has df = Δ, where

$$\Delta = \frac{\left[(s_1^2/n_1) + (s_2^2/n_2)\right]^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}}$$

rounded down to the nearest integer. Use Table IV to estimate the P-value, or obtain it exactly by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

CONFIDENCE INTERVAL APPROACH

PROCEDURE 10.4 Nonpooled t-Interval Procedure

Purpose To find a confidence interval for the difference between two population means, μ_1 and μ_2

Assumptions

1. Simple random samples
2. Independent samples
3. Normal populations or large samples

Step 1 For a confidence level of $1-\alpha$, use Table IV to find $t_{\alpha/2}$ with df $= \Delta$, where

$$\Delta = \frac{\left[(s_1^2/n_1) + (s_2^2/n_2)\right]^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

rounded down to the nearest integer.

Step 2 The endpoints of the confidence interval for $\mu_1 - \mu_2$ are

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \cdot \sqrt{(s_1^2/n_1) + (s_2^2/n_2)}.$$

Step 3 Interpret the confidence interval.

Hypothesis Tests for the Means of Two Populations with σ_1 and σ_2 are known

Critical-Value and P-Value Approach

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}},$$

CONFIDENCE INTERVAL APPROACH

Confidence Interval for $\mu_1 - \mu_2$, σ_1^2 and σ_2^2 Known

If $\bar{x}_1$ and $\bar{x}_2$ are means of independent random samples of sizes n_1 and n_2 from populations with known variances σ_1^2 and σ_2^2, respectively, a $100(l-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$ is given by

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}},$$

where $z_{\alpha/2}$ is the z-value leaving an area of $\alpha/2$ to the right.