Measures of Position

A **measure of position** determines the position of a single value in relation to other values in a sample or a population data set. There are many measures of position; however, only quartiles, percentiles, and percentile rank are discussed in this section.

Percentiles and Percentile Rank:

Percentiles are the summary measures that divide a ranked data set into 100 equal parts. Each (ranked) data set has 99 percentiles that divide it into 100 equal parts. The data should be ranked in increasing order to compute percentiles. The kth percentile is denoted by P_k, where k is an integer in the range 1 to 99. For instance, the 25th percentile is denoted by P_{25}. Figure 3.12 shows the positions of the 99 percentiles.
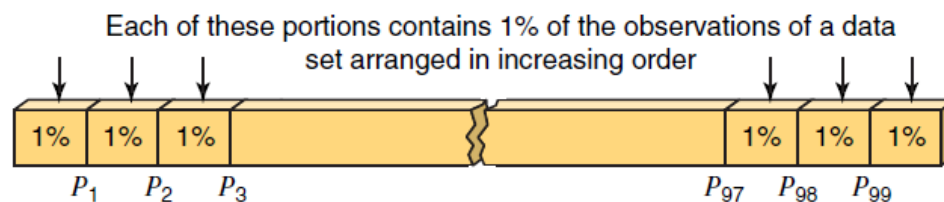

Figure 3.12 Percentiles.

Thus, the kth percentile, P_k, can be defined as a value in a data set such that about $k\%$ of the measurements are smaller than the value of P_k and about $(100 - k)\%$ of the measurements are greater than the value of P_k.

The approximate value of the kth percentile is determined as explained next.

Calculating Percentiles   The (approximate) value of the **kth percentile**, denoted by P_k, is

$$P_k = \text{Value of the } \left(\frac{k \times n}{100}\right)\text{th term in a ranked data set}$$

where k denotes the number of the percentile and n represents the sample size.

If the value of $\dfrac{k \times n}{100}$ is fractional, always round it up to the next higher whole number.

Thus, to calculate the kth percentile, first we rank the given data set in increasing order. Then, to find the kth percentile, we find the $(k \times n/100)$th term where n is the total number of data values in the given data set. If the value of $(k \times n/100)$ is fractional, we round it up to the next higher whole number. The value of this $(k \times n/100)$th term in the ranked data set gives the kth percentile, P_k.

We can calculate the **percentile rank** for a particular value x_i of a data set by using the formula given below. The percentile rank of x_i gives the percentage of values in the data set that are less than x_i. To find the percentile rank for a data value x_i, we first rank the given data set in increasing order. Then we find the number of data values that are less than x_i. Finally, we divide this number by the number of total values and multiply by 100. This gives the percentile rank for x_i.

Finding Percentile Rank of a Value

$$\text{Percentile rank of } x_i = \frac{\text{Number of values less than } x_i}{\text{Total number of values in the data set}} \times 100\%$$

Finding the percentile for a data set.
Example: Commuting Times for College Students

Refer to the data on one-way commuting times (in minutes) from home to college of 12 students given in Example 3–23, which is reproduced below.

$$29 \quad 14 \quad 39 \quad 17 \quad 7 \quad 47 \quad 63 \quad 37 \quad 42 \quad 18 \quad 24 \quad 55$$

Find the value of the 70$^{\text{th}}$ percentile. Give a brief interpretation of the 70$^{\text{th}}$ percentile.

Solution We perform the following three steps to find the 70$^{\text{th}}$ percentile for the given data.

Step 1. First we rank the given data in increasing order as follows:

$$7 \quad 14 \quad 17 \quad 18 \quad 24 \quad 29 \quad 37 \quad 39 \quad 42 \quad 47 \quad 55 \quad 63$$

Step 2. We find the $(k \times n/100)$th term. Here $n = 12$ and $k = 70$, as we are to find the 70$^{\text{th}}$ percentile.

$$\frac{k \times n}{100} = \frac{70 \times 12}{100} = 8.4 = 9^{\text{th}} \text{ term}$$

Thus, the 70$^{\text{th}}$ percentile, P_{70}, is given by the value of the 9$^{\text{th}}$ term in the ranked data set. Note that we rounded 8.4 up to 9, which is always the case when calculating a percentile.

Step 3. We find the value of the 9$^{\text{th}}$ term in the ranked data. This gives the value of the 70$^{\text{th}}$ percentile, P_{70}.

$$P_{70} = \text{Value of the 9}^{\text{th}} \text{ term} = \textbf{42 minutes}$$

Thus, we can state that approximately 70% of these 12 students commute for less than or equal to 42 minutes. ■

Finding the percentile rank for a data value.

Solution   We perform the following three steps to find the percentile rank of 42.

Step 1.   First we rank the given data in increasing order as follows:

$$7 \quad 14 \quad 17 \quad 18 \quad 24 \quad 29 \quad 37 \quad 39 \quad 42 \quad 47 \quad 55 \quad 63$$

Step 2.   Find how many data values are less than 42.

In the above ranked data, there are eight data values that are less than 42.

Step 3.   Find the percentile rank of 42 as follows given that 8 of the 12 values in the given data set are smaller than 42:

$$\text{Percentile rank of } 42 = \frac{8}{12} \times 100\% = \mathbf{66.67\%}$$

Rounding this answer to the nearest integral value, we can state that about 67% of the students in this sample commute for less than 42 minutes. ∎

3.69 The following data give the speeds of 13 cars (in mph) measured by radar, traveling on I-84.

73	75	69	68	78	69	74
76	72	79	68	77	71	

    a. Find the values of the three quartiles and the interquartile range.
    b. Calculate the (approximate) value of the 35th percentile.
    c. Compute the percentile rank of 71.

3.70 The following data give the total food expenditures (in dollars) for the past one month for a sample of 20 families.

1125	530	1234	595	427	872	1480	699	1274	1187
933	1127	716	1065	934	1630	1046	2199	1353	441

    a. Calculate the values of the three quartiles and the interquartile range.
    b. Find the approximate value of the 57th percentile.
    c. Calculate the percentile rank of 1046. Give a brief interpretation of this percentile rank.

3.71 The following data give the number of text messages sent by a high school student on 40 randomly selected days during 2015:

32	33	33	34	35	36	37	37	37	37
38	39	40	41	41	42	42	42	43	44
44	45	45	45	47	47	47	47	47	48
48	49	50	50	51	52	53	54	59	61

    a. Calculate the values of the three quartiles and the interquartile range. Where does the value 49 fall in relation to these quartiles?
    b. Determine the approximate value of the 91st percentile. Give a brief interpretation of this percentile.
    c. For what percentage of the days was the number of text messages sent 40 or higher? Answer by finding the percentile rank of 40.

3.72 The following data give the number of new cars sold at a dealership during a 20-day period.

8	5	12	3	9	10	6	12	8	8
4	16	10	11	7	7	3	5	9	11

a. Calculate the values of the three quartiles and the interquartile range. Where does the value of 4 lie in relation to these quartiles?
b. Find the (approximate) value of the 25th percentile. Give a brief interpretation of this percentile.
c. Find the percentile rank of 10. Give a brief interpretation of this percentile rank.

3.73 The following data give the annual salaries (in thousand dollars) of 20 randomly selected health care workers.

50	71	57	39	45	64	38	53	35	62
74	40	67	44	77	61	58	55	64	59

a. Calculate the values of the three quartiles and the interquartile range. Where does the value 57 fall in relation to these quartiles?
b. Find the approximate value of the 30th percentile. Give a brief interpretation of this percentile.
c. Calculate the percentile rank of 61. Give a brief interpretation of this percentile rank.