# Analysis of Variance (ANOVA)

## Question:

A researcher wishes to try three different techniques to lower the blood pressure of individuals diagnosed with high blood pressure. The subjects are randomly assigned to three groups; the first group takes medication, the second group exercises, and the third group follows a special diet. After four weeks, the reduction in each person's blood pressure is recorded. At $\alpha = 0.05$, test the claim that there is no difference among the means. The data are shown.

| Medication | Exercise | Diet |
|---|---|---|
| 10 | 6 | 5 |
| 12 | 8 | 9 |
| 9 | 3 | 12 |
| 15 | 0 | 8 |
| 13 | 2 | 4 |

## Solution:

**Step 1** State the hypotheses and identify the claim.

$H_0$: $\mu_1 = \mu_2 = \mu_3$ (claim)

$H_1$: At least one mean is different from the others.

**Step 2** Find the critical value. Since $k = 3$ and $N = 15$,

d.f.N. $= k - 1 = 3 - 1 = 2$

d.f.D. $= N - k = 15 - 3 = 12$

At $\alpha = 0.05, v_1 = 2\ and\ v_1 = 12$ read the $f$ value from table, which is $f = 3.89$, for rejection region

**Step 3** Compute the test value, using the procedure outlined here.

| | Medication $y_{1j}$ | Exercise $y_{2j}$ | Diet $y_{3j}$ | $y_{1j} - \bar{y}_{..}$ | $y_{2j} - \bar{y}_{..}$ | $y_{2j} - \bar{y}_{..}$ |
|---|---|---|---|---|---|---|
| | 10 | 6 | 5 | 5.139289 | 3.003289 | 7.469289 |
| | 12 | 8 | 9 | 18.20729 | 0.071289 | 1.605289 |
| | 9 | 3 | 12 | 1.605289 | 22.40129 | 18.20729 |
| | 15 | 0 | 8 | 52.80929 | 59.79929 | 0.071289 |
| | 13 | 2 | 4 | 27.74129 | 32.86729 | 13.93529 |
| Sum $\sum y_{1j}$ | 59 | 19 | 38 | 105.5024 | 118.1424 | 41.28845 |
| Column Mean $\overline{y_{1.}}$ | $\frac{59}{5} = 11.8$ | $\frac{19}{5} = 3.8$ | $\frac{38}{5} = 7.6$ | | | |
| Mean of Means $\bar{y}_{..}$ | $\frac{11.8 + 3.8 + 7.6}{3} = 7.7333$ | | | | | |

$$SST = \sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2 = \text{total sum of squares},$$

$$SST = 105.5024 + 118.1424 + 41.28845 = 264.9333$$

$$SSA = n\sum_{i=1}^{k}(\bar{y}_{i.} - \bar{y}_{..})^2 = \text{treatment sum of squares},$$

$$SSA = 5(11.8 - 7.7333)^2 + 5(3.8 - 7.7333)^2 + 5(7.6 - 7.7333)^2 = 160$$

$$SSE = SST - SSA = 264.9333 - 160$$
$$SSE = 104.9333$$

$$(MSA)\ s_1^2 = \frac{SSA}{k-1} = \frac{160}{3-1} = 80$$

$$(MSE)s^2 = \frac{SSE}{k(n-1)} = \frac{104.9333}{3(5-1)} = 8.74$$

$$f = \frac{s_1^2}{s^2} = \frac{80}{8.74} = 9.15$$

**Step 4** Make the decision. The decision is to reject the null hypothesis, since 9.17 > 3.89.

**Step 5** Summarize the results. There is enough evidence to reject the claim and conclude that at least one mean is different from the others.

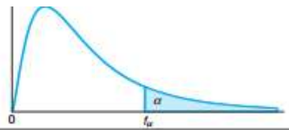

**Table A.6** Critical Values of the *F*-Distribution

$f_{0.05}(v_1, v_2)$

| $v_2$ \ $v_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| **1** | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 |
| **2** | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 |
| **3** | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 |
| **4** | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 |
| **5** | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 |
| **6** | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 |
| **7** | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 |
| **8** | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 |
| **9** | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 |
| **10** | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 |
| **11** | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 |
| **12** | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 |

## Question:

A state employee wishes to see if there is a significant difference in the number of employees at the interchanges of three state toll roads. The data are shown. At $\alpha = 0.05$, can it be concluded that there is a significant difference in the average number of employees at each interchange?

| Pennsylvania Turnpike | Greensburg Bypass/ Mon-Fayette Expressway | Beaver Valley Expressway |
|---|---|---|
| 7 | 10 | 1 |
| 14 | 1 | 12 |
| 32 | 1 | 1 |
| 19 | 0 | 9 |
| 10 | 11 | 1 |
| 11 | 1 | 11 |

## Solution:

**Step 1** State the hypotheses and identify the claim.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others (claim)

**Step 2** Find the critical value. Since $k = 3$, $N = 18$, and $\alpha = 0.05$,

d.f.N. $= k - 1 = 3 - 1 = 2$

d.f.D. $= N - k = 18 - 3 = 15$

The critical value is 3.68.

**Step 3** Compute the test value.

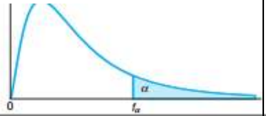

**Table A.6** Critical Values of the $F$-Distribution

| | $f_{0.05}(v_1, v_2)$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $v_1$ | | | | | | | |
| $v_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 |

| | Pennsylvania Turnpike $y_{1j}$ | Greensburg Bypass/ Mon-Fayette Expressway $y_{2j}$ | Beaver Valley Expressway $y_{3j}$ | $y_{1j}-\bar{y}_{..}$ | $y_{2j}-\bar{y}_{..}$ | $y_{2j}-\bar{y}_{..}$ |
|---|---|---|---|---|---|---|
| | 7 | 10 | 1 | 2.085136 | 2.421136 | 55.41314 |
| | 14 | 1 | 12 | 30.86914 | 55.41314 | 12.64514 |
| | 32 | 1 | 1 | 554.8851 | 55.41314 | 55.41314 |
| | 19 | 0 | 9 | 111.4291 | 71.30114 | 0.309136 |
| | 10 | 11 | 1 | 2.421136 | 6.533136 | 55.41314 |
| | 11 | 1 | 11 | 6.533136 | 55.41314 | 6.533136 |
| Sum $\sum y_{1j}$ | 93 | 24 | 35 | 708.2228 | 246.4948 | 185.7268 |
| Column Mean $\overline{y_{1.}}$ | 15.5 | 4 | 5.833333 | | | |
| Mean of Means $\bar{y}_{..}$ | 8.44 | | | | | |

$$SST = \sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij}-\bar{y}_{..})^2 = \text{total sum of squares,}$$

$$SST = 708.2228 + 246.4948 + 185.7268 = 1140.444$$

$$SSA = n\sum_{i=1}^{k}(\bar{y}_{i.}-\bar{y}_{..})^2 = \text{treatment sum of squares,}$$

$$SSA = 6(15.5-8.44)^2 + 6(4-8.44)^2 + 6(5.833-8.44)^2 = 458.121$$

$$SSE = SST - SSA = 1140.444 - 458.121$$
$$SSE = 682.322$$

$$(MSA)\ s_1^2 = \frac{SSA}{k-1} = \frac{458.121}{3-1} = 229.06$$
$$(MSE) s^2 = \frac{SSE}{k(n-1)} = \frac{682.322}{3(6-1)} = 45.488$$

$$f = \frac{s_1^2}{s^2} = \frac{229.06}{45.488} = 5.03$$

**Step 4** Make the decision.

Since $\mathbf{5.03 > 3.68}$, the decision is to reject the null hypothesis

**Step 5** Summarize the results.

There is enough evidence to support the claim that there is a difference among the means.

Questions Sets from neil a weiss

*In Exercises* ***16.42–16.47****, we provide data from independent simple random samples from several populations. In each case,*

*a. compute SST, SSTR, and SSE by using the computing formulas given in Formula 16.1 on page 726.*

*b. compare your results in part (a) for SSTR and SSE with those in Exercises 16.24–16.29, where you employed the defining formulas.*

*c. construct a one-way ANOVA table.*

*d. decide, at the 5% significance level, whether the data provide sufficient evidence to conclude that the means of the populations from which the samples were drawn are not all the same.*

**16.42**

| Sample 1 | Sample 2 | Sample 3 |
|---|---|---|
| 1 | 10 | 4 |
| 9 | 4 | 16 |
| | 8 | 10 |
| | 6 | |
| | 2 | |

**16.43**

| Sample 1 | Sample 2 | Sample 3 |
|---|---|---|
| 8 | 2 | 4 |
| 4 | 1 | 3 |
| 6 | 3 | 6 |
| | | 3 |

**16.44**

| Sample 1 | Sample 2 | Sample 3 | Sample 4 |
|---|---|---|---|
| 6 | 9 | 4 | 8 |
| 3 | 5 | 4 | 4 |
| 3 | 7 | 2 | 6 |
| | 8 | 2 | |
| | 6 | 3 | |

**16.45**

| Sample 1 | Sample 2 | Sample 3 | Sample 4 | Sample 5 |
|---|---|---|---|---|
| 7 | 5 | 6 | 3 | 7 |
| 4 | 9 | 7 | 7 | 9 |
| 5 | 4 | 5 | 7 | 11 |
| 4 | | 4 | 4 | |
| | | 8 | 4 | |

**16.46**

| Sample 1 | Sample 2 | Sample 3 | Sample 4 | Sample 5 |
|---|---|---|---|---|
| 4 | 8 | 9 | 4 | 3 |
| 2 | 5 | 6 | 0 | 6 |
| 3 | 5 | 9 | 2 | 9 |

**16.47**

| Sample 1 | Sample 2 | Sample 3 | Sample 4 |
|---|---|---|---|
| 11 | 9 | 16 | 5 |
| 6 | 2 | 10 | 1 |
| 7 | 4 | 10 | 3 |