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| Course Code: CS301 | Course Name: Theory of Automata |
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| Student Roll No: | Section No: |

Instructions:

- Attempt all questions.
- All questions carry different marks. But equally distributed in all parts.
- It is advisable to go through the paper once before starting with the first question.
- Exam is a closed books and closed notes.
- Don't use pencil , write in dark blue or black pen.

Q 1a) Decide if the following statements are TRUE or FALSE:

[7.5 points = 15*0.5 points]

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| 1) A production of the form non-terminal-> non-terminal is called a dead Production. | T/F |
| 2) Semi-word is a string having some terminals and one non-terminal at the right of string. | T/F |
| 3) Two FAs are equivalent if they have same no. of states. | T/F |
| 4) There exist exactly two different derivations in an ambiguous CFG for a word. | T/F |
| 5) Regular languages are closed under Union, Concatenation and Kleene star. | T/F |
| 6) CFG may also represent a regular language. | T/F |
| 7) PDA is stronger than FA. | T/F |
| 8) There always exist an FA for each PDA. | T/F |
| 9) If, two strings x and y, defined over Σ , are run over an FA accepting the language L, then x and y are said to belong to the same class if they end in the same state, no matter that state is final or not. | T/F |
| 10) The context free grammar $S \rightarrow a/ab/SS/Sb$ is ambiguous. | T/F |
| 11) The class of non-regular languages is closed under complementation. | T/F |
| 12) The concatenation of the two CFGs is not context free. | T/F |
| 13) The class of the non-context free languages is closed under complementation. | T/F |
| 14) If L_1 and L_2 are context free, then the language $L_1 - L_2$ must be context free. | T/F |
| 15) If L_1 is context free and L_2 is regular then the language $L_1 - L_2$ must be context free. | T/F |

Q1b) Choose the best option in each of the following statements:**[7.5 points = 15*0.5 points]**

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| <p>1) Grammatical rules which do not involve the meaning of words are called -----</p> <p>A. Semantics B. Syntactic C. Both a and b D. None of given</p> | <p>2) Grammatical rules which involve the meaning of words are called -----</p> <p>A. Semantics B. Syntactic C. Both a and b D. None of given</p> |
| <p>3) The PDA is called non-deterministic PDA when there are more than one out going edges from..... state</p> <p>A. START or READ B. POP or REJECT C. READ or POP D. PUSH or POP</p> | <p>4) $S \rightarrow aXb b, \quad XaX \rightarrow aX bX \Lambda$ The given CFG generates the language in English _____</p> <p>A. Beginning and ending in different letters B. Beginning and ending in same letter C. Having even-even language D. None of given</p> |
| <p>5) The symbols that can't be replaced by anything are called -----</p> <p>A. Productions B. Terminals C. Non-terminals D. All of above</p> | <p>6) The symbols that must be replaced by other things are called _____</p> <p>A. Productions B. Terminals C. Non-terminals D. None of given</p> |
| <p>7) The grammatical rules are often called _____</p> <p>A. Productions B. Terminals C. Non-terminals D. None of given</p> | <p>8) The language generated by that CFG is regular if _____</p> <p>A. No terminal \rightarrow semi word B. No terminal \rightarrow word C. Both a and b D. None of given</p> |
| <p>9) The terminals are designated by _____ letters, while the non-terminals are designated by _____ letters.</p> <p>A. Capital, bold B. Small, capital C. Capital, small D. Small, bold</p> | <p>10) The language generated by _____ is called Context Free Language (CFL).</p> <p>A. FA B. TG C. CFG D. TGT</p> |
| <p>11) Identify the TRUE statement:</p> <p>A. A PDA is non-deterministic, if there are more than one READ states in PDA B. A PDA is never non-deterministic C. Like TG, A PDA can also be non-deterministic D. A PDA is non-deterministic, if there are more than one REJECT states in PDA</p> | <p>12) Which statement is true?</p> <p>A. The tape of turing machine is infinite. B. The tape of turing machine is finite. C. The tape of turing machine is infinite when the language is regular D. The tape of turing machine is finite when the language is nonregular.</p> |
| <p>13) The productions of the form nonterminal \rightarrow one nonterminal, is called _____</p> <p>A. Null production B. Unit production C. Null able production D. None of given</p> | <p>14) For language L defined over {a, b}, then L partitions $\{a, b\}^*$ into classes</p> <p>A. Infinite B. Finite C. Distinct D. Non-distinct</p> |

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|---|--|
| 15) $\Sigma = \{a,b\}$ Productions $S \rightarrow XaaX$ $X \rightarrow aX$ $X \rightarrow bX$ $X \rightarrow \Lambda$ This grammar defines the language expressed by_____ A. $(a+b)^*aa(a+b)^*$ B. $(a+b)^*a(a+b)^*a$ C. $(a+b)^*aa(a+b)^*aa$ D. $(a+b)^*aba+b)^*$ | |
|---|--|

Q2) Provide short answers to each of the following questions:

[10 points = 5*2 points]

- a) Every subset of a regular language is regular.
- b) Let $L_4 = L_1L_2L_3$. If L_1 and L_2 are regular and L_3 is not regular, it is possible that L_4 is regular.
- c) Let $L_1 = L_2 \cap L_3$. Show values for L_1 , L_2 , and L_3 , such that L_1 is context-free but neither L_2 nor L_3 is.
- d) Let $L_1 = L_2 \cap L_3$. Show values for L_1 , L_2 , and L_3 , such that L_1 is context-free but neither L_2 nor L_3 is.
- e) Let $L_4 = L_1L_2L_3$. If L_1 and L_2 are regular and L_3 is not regular, it is possible that L_4 is regular.

Q3) Following problems are related to CFG & CNF:

[10 points=5*2 points]

- a) Show a context-free grammar that generates $L = \{w \in \{a, b\}^* : \text{the first, middle, and last characters of } w \text{ are identical}\}$.
- b) Convert the following grammar (over the alphabet $\{a, b, c, d\}$) to the Chomsky normal form.

$S \rightarrow aSd \mid T$

$T \rightarrow bTc \mid \epsilon$.

- c) Consider the following grammar G :

$S \rightarrow 1S1 \mid T$

$T \rightarrow 1X1 \mid X$

$X \rightarrow 0X0 \mid 1$

- (i) What are the first four strings in the lexicographic enumeration of $L(G)$?
- (ii) Show that G is ambiguous.

d) Let G be the context free grammar:

$S \rightarrow ASB \mid \varepsilon \quad A \rightarrow S \mid aAS \mid \varepsilon \quad B \rightarrow SbS \mid A \mid bb$

(i) Find a grammar G_1 which has no ε -rule and $L(G_1) = L(G) - \{\varepsilon\}$. [5pts]

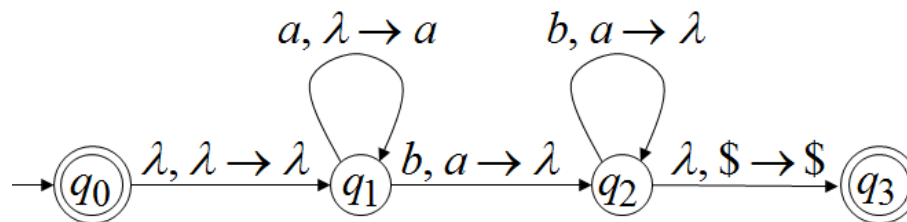
(ii) Find a grammar G_2 which is equivalent to G_1 and has no unit productions

e) Define components of CFG

Q4) All of the following problems are related to PDA:

[10 points=5*2 points]

a) Process the string **aaabbb** and fill the table with all possible values of State, STACK and Tape using following NPDA:



| STATE | STACK | TAPE |
|-------|-------|--------|
| q_0 | \$ | aaabbb |
| | | |
| | | |
| | | |

Note: Highlights the current tape symbol with underline

b) Suppose the PDA

$P = (\{q, p\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\})$

has the following transition function:

- $\delta(q, 0, Z_0) = \{(q, XZ_0)\}$.
- $\delta(q, 0, X) = \{(q, XX)\}$.
- $\delta(q, 1, X) = \{(q, X)\}$.
- $\delta(q, \epsilon, X) = \{(p, \epsilon)\}$.
- $\delta(p, \epsilon, X) = \{(p, \epsilon)\}$.
- $\delta(p, 1, X) = \{(p, XX)\}$.
- $\delta(p, 1, Z_0) = \{(p, \epsilon)\}$.

Starting from the initial ID (q, w, Z_0) , show all the reachable ID's when the input w is:

i) 01

ii) 0011

c): Convert the following expression grammar into a PDA:

$I \rightarrow a / b / Ia / Ib / IO / II$

$E \rightarrow I / E * E / E + E / (E)$

d) Construct PDA of the given language:

$$L(M) = \{a^n b^n : n \geq 0\}$$

e) Write down the capabilities of PDA which cannot be achieved by CFG.

Q5) Attempt all following related to TM:

[10 points = 2 * 5 points]

a): Prove that the following function is computable.

$$f(n) = n + 2.$$

We know that if any function is computable, then there exists a Turing Machine for it. So, it will be sufficient to construct a TM to prove any function is computable.

TM behaves as follows:

$q_0 I I I B \vdash^* I I I q_0 B \vdash I I I q_1 B \vdash^* I I I I B h B$

b) Construct a Turing Machine accepting a language of palindrome over $\{a,b\}^*$ with each string of even length.