23k-2001

PROB & STATS: Assignment 02 Date:

Answer#01:

$f(x,y) = 24xy \quad , \quad 0 \le x, y \le 1 \, , \; x+y \le 1$

X = Turkish

Y = domestic

a. $P(x > 1/2) = \int_{1/2}^{1} \int_{0}^{1-x} 24xy \, dy \, dx$

$= \int_{1/2}^{1} 24x \int_{0}^{1-x} y \, dy \, dx$

$= \int_{1/2}^{1} 24x \left[\frac{y^2}{2} \right]_0^{1-x} dx$

$= \int_{1/2}^{1} 24x \frac{(1-x)^2}{2} dx$

$= \int_{1/2}^{1} (12x - 24x^2 + 12x^3) dx$

$= \left[6x^2 - 8x^3 + 3x^4 \right]_{1/2}^{1}$

$P(x>1/2) = 6 - 8 + 3 - [6(1/4) - 8(1/8) + 3(1/16)]$

$P(x>1/2) = \frac{5}{16}$

$P(x>1/2) = 0.3125$ Ans.

b. $f_Y(y) = \int_0^{1-y} f(x,y)\,dx$

$= \int_0^{1-y} 24xy\,dx$

$= 24y \int_0^{1-y} x\,dx$

$= 12y\,[x^2]_0^{1-y}$

$= 12y\,(1-y)^2$

$f_Y(y) = 12y - 24y^2 + 12y^3$ Ans.

c. $P(X< 1/8 \mid y = 3/4) = \int_0^{1/8} f(x \mid y = 3/4)\,dx \div \int_0^{1-y} f(x,y)\,dx$

$= \int_0^{1/8} 24x\,(3/4)\,dx \div (12y[1-y]^2)$

$= \int_0^{1/8} 18x\,dx \div [12(3/4)[1-3/4]^2]$

$= 18 \left[\frac{x^2}{2}\right]_0^{1/8} \div \frac{9}{16}$

$= \frac{18}{2}(1/8)^2 \times \frac{16}{9}$

$P(X< 1/8 \mid y = 3/4) = 0.25$ Ans.

Date:

Answer #02:

$$f(x,y) = \begin{cases} ye^{-y(1+x)}, & x,y \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

a.

$$f_X(x) = \int_0^{\infty} f(x,y)\,dy = \int_0^{\infty} ye^{-y(1+x)}\,dy$$

$$f_X(x) = \frac{1}{(1+x)^2} \quad \text{Ans.}$$

$$f_Y(y) = \int_0^{\infty} f(x,y)\,dx$$

$$= \int_0^{\infty} ye^{-y(1+x)}\,dx = ye^{-y}\int_0^{\infty} e^{-yx}\,dx$$

$$= ye^{-y} \cdot \frac{1}{y}$$

$$f_Y(y) = e^{-y} \quad \text{Ans.}$$

b. $$P(x>2, y>2) = \int_2^{\infty}\int_2^{\infty} f(x,y)\,dx\,dy$$

$$= \int_2^{\infty}\int_2^{\infty} ye^{-y(1+x)}\,dx\,dy$$

$$= e^{-y}\int_2^{\infty} e^{-y(1+x)}\,dx = \int_2^{\infty} e^{-3y}\,dy$$

$$= \frac{e^{-6}}{3}$$

$$P(x>2, y>2) = 0.00083 \quad \text{Ans.}$$

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Date:

Answer #03:

$$f(x,y) = \begin{cases} \frac{3x-y}{9} &, 1<x<3,\ 1<y<2 \\ 0 &, \text{elsewhere} \end{cases}$$

a. $f_x(x) = \int_1^2 \frac{3x-y}{9}\,dy = \frac{3x}{9}\int_1^2 dy - \frac{1}{9}\int_1^2 y\,dy$

$= \frac{3x}{9}\,|y|_1^2 - \frac{|y^2|_1^2}{18}$

$= \frac{x}{3}(1) - \frac{(4-1)}{18}$

$f_x(x) = \frac{x}{3} - \frac{1}{6}$ Ans.

$f_y(y) = \int_1^3 \frac{3x-y}{9}\,dx = \int_1^3 \frac{x}{3}\,dx - \frac{y}{9}\int_1^3 dx$

$= \frac{|x^2|_1^3}{6} - \frac{y}{9}\,|x|_1^3$

$= (8/6) - 2y/9$

$f_y(y) = \frac{4}{3} - \frac{2y}{9}$ Ans.

b. $f(x,y) = f_x(x)\,f_y(y)$, if independent.

$f_x(x) = \frac{2x-1}{6}$, $f_y(y) = \frac{12-2y}{9}$

L.H.S $\neq$ R.H.S

→ Also prove by inserting values $x = 1.5$, $y = 1.25$

$$\frac{3(1.5)-1.25}{9} \neq \left[\frac{2(1.5)-1}{6}\right]\left[\frac{12-2(1.25)}{9}\right]$$

$0.361 \neq 0.351$ ⇒ Hence X, Y are NOT Independent

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c. $P(x>2) = \int_2^3 f_X(x)\,dx$

$= \int_2^3 \left(\frac{2x-1}{6}\right) dx$

$= \frac{1}{6}\int_2^3 2x\,dx - \frac{1}{6}\int_2^3 dx$

$= \frac{1}{3}\left[\frac{x^2}{2}\right]_2^3 - \frac{1}{6}[x]_2^3$

$= \frac{1}{6}(9-4) - \frac{1}{6}(3-2)$

$P(x>2) = 2/3$ Ans.

Answer#04:

a. PMF_X :?

$P(x=1) = 0.4 - 0 = 0.4$

$P(x=3) = 0.6 - 0.4 = 0.2$

$P(x=5) = 0.8 - 0.6 = 0.2$

$P(x=7) = 1.0 - 0.8 = 0.2$

b. $P(4 < X \leq 7)$:?

$\Rightarrow = P(X=5) + P(X=6) + P(X=7)$

$P(4 < x \leq 7) = 0.2 + 0 + 0.2 = 0.4$ Ans.

Date:

Answer # 05:

$$f(x,y) = \frac{9}{16} \cdot \frac{1}{4^{(x+y)}} \quad , \text{ for } x = 0,1,2.., \ y = 0,1,2,..$$

a. $$f_x(x) = \frac{9}{16} \cdot \frac{1}{4^x} \sum_0^{\infty} \frac{1}{4^y}$$

$$= \frac{9}{16} \cdot \frac{1}{4^x} \cdot \frac{4}{3}$$

$$f_x(x) = \frac{3}{4 \cdot 4^x}$$

$$f_y(y) = \frac{9}{16} \cdot \frac{1}{4^y} \sum_0^{\infty} \frac{1}{4^x}$$

$$= \frac{9}{16} \cdot \frac{1}{4^y} \cdot \frac{4}{3}$$

$$f_y(y) = \frac{3}{4 \cdot 4^y}$$

$$f(x,y) = f_x(x) \cdot f_y(y)$$

$$\frac{9}{16} \cdot \frac{1}{4^{(x+y)}} = \left(\frac{3}{4 \cdot 4^x}\right)\left(\frac{3}{4 \cdot 4^y}\right) \Rightarrow L.H.S = R.H.S$$

Hence, they are independent

b. $$P(x+y<4) = \sum_{x=0}^{3} \sum_{y=0}^{3-x} f(x,y)$$

$$\Rightarrow = \sum_{x=0}^{3} \sum_{y=0}^{3-x} \left(\frac{3}{4 \cdot 4^x}\right)\left(\frac{3}{4 \cdot 4^y}\right) = \frac{9}{16} \sum_{x=0}^{3} \sum_{y=0}^{3-x} \frac{1}{4^{x+y}}$$

$$= \frac{63}{64}$$

$$P(x+y<4) = 0.99 \quad \text{Ans.}$$

Date:

Answer#06:

$$f(y) = \begin{cases} 10(1-y)^9, & 0 \leq y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

a. i. $f(y) \geq 0$ for all values of y

$\Rightarrow 10(1-y)^9$ is always non-negative as $0 \leq y \leq 1$

$\rightarrow$ First condition is satisfied.

ii. $\int_y f(y) = 1$, $\int_0^1 f(y)dy = \int_0^1 10(1-y)^9 dy$

$$= \left[-(1-y)^{10}\right]_0^1 = (-(0)^{10}) - (-(1)^{10}) = 1$$

$\rightarrow$ Second condition satisfied.

b. $P(0.6 < y \leq 1) = \int_{0.6}^1 f(y)\,dy$

$$= \int_{0.6}^1 10(1-y)^9 dy$$

$$= \left[-(1-y)^{10}\right]_{0.6}^1$$

$$= -(0)^{10} - (-(1-0.6)^{10})$$

$P(0.6 < y \leq 1) = (0.4)^{10}$ Ans.

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Answer # 07:

$$E(x) = n\frac{k}{N} = 3 \times \frac{1}{5} = 0.6 \quad \text{Ans.}$$

$$var(x) = n\frac{k}{N} \times \frac{N.k}{N} \times \left(\frac{N-n}{n-1}\right)$$

$$= 3 \times \frac{1}{5} \times \frac{4}{5} \times \frac{2}{4}$$

$$var(x) = 0.24 \quad \text{Ans}$$

Answer # 08:

$$f(x,y) = \begin{cases} \frac{16y}{x^3} & , x > 2, 0 < y < 1 \\ 0 & , \text{elsewhere} \end{cases}$$

ρ_{xy} : ?

$$E(x^2) = \int_x \int_y x^2 f(x,y) = \int_2^\infty \int_0^1 x^2 . \frac{16y}{x^3} = \int_2^\infty \int_0^1 \frac{16y}{x} \, dy dx$$

$$= \int_2^\infty \left[\frac{8y^2}{x}\right]_0^1 dx = \int_2^\infty \frac{8}{x} \, dx = \left[8 \ln x\right]_2^\infty$$

$$= 8 \ln \infty - 8 \ln 2$$

$$= \infty$$

$\Rightarrow$ The mean of (x^2) is not finite, so variance of x does not exist, thus we cannot compute the correlation coefficient.

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Answer #09:

a. $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{8}{x} 0.6^x (0.4)^{8-x}$

$P(X=3) = \binom{8}{3}(0.6)^3 (0.4)^5$

$P(x=3 = 0.12386$ Ans.

b. If atleast 5 took for non-psychological problems, then at most 3 took for psychological.

$P(x \leq 3) = P(x=0) + p(x=1) + p(x=2) + P(x=3)$

$= (0.4)^8 + \binom{8}{1}(0.6)^1(0.4)^7$

$+ \binom{8}{2}(0.6)^2(0.4)^6$

$+ \binom{8}{3}(0.6)^3(0.4)^5$

$p(x \leq 3) = 0.1737$ Ans.

Answer #10:

a. $P(X=x) = \frac{e^{-\lambda}(\lambda)^x}{x!}$

$P(X=5) = \frac{e^{-3}(3)^5}{5!} = 0.1008$ Ans.

b. $P(x<3) = P(x=0) + P(x=1) + P(x=2)$

$= e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}\right)$

$P(x<3) = 0.4232$ Ans.

Date:

c. $P(x \geq 2) = 1 - P(x \leq 1)$

$= 1 - P(x=0) - P(x=1)$

$= 1 - \frac{e^{-3} \cdot 3^0}{0!} - \frac{e^{-3}\,3^1}{1!}$

$P(x \geq 2) = 0.8008$ Ans.

Answer # 11:

a. Since, mean time lies in the range when it is desirable, this shows that the resin performance is desirable more than it is undesirable. Ans.

b. $z = \frac{x - \mu}{\sigma}$

$z_1 = \frac{1-3}{0.5} = -4$, $z_2 = \frac{4-3}{0.5} = 2$

$P(1 < x < 4) = P(-4 < z < 2)$

$= P(z < 2) - P(z < -4)$

$= 0.9772 - 0.000032$

$P(1 < x < 4) = 0.977168$ Ans.

$P(\text{undesirable}) = 1 - p(1 < x < 4)$

$= 1 - 0.977168$

$= 0.0228$ or 2.28% Ans.

23K-2001

Date:

Answer # 12:

$P(x < 95) = P(x < 94.5)$, $z = \frac{x - \mu}{\sigma}$

$P(x < 95) = P(z < -1.71)$

$P(x < 95) = 0.0436$

$z = \frac{94.5 - 115}{12}$

$z = -1.71$

$E(x) = np = 600 \times 0.0436$

$E(x) = 26.16$

$E(x) \approx 26$ students

Ans.