Counting Sample Points

Formulas:

If an operation can be performed in n_1 ways, and if for each of these ways a second operation can be performed in n_2 ways, then the two operations can be performed together in $n_1 n_2$ ways.

If an operation can be performed in n_1 ways, and if for each of these a second operation can be performed in n_2 ways, and for each of the first two a third operation can be performed in n_3 ways, and so forth, then the sequence of k operations can be performed in $n_1 n_2 \cdots n_k$ ways.

For any non-negative integer n, $n!$, called "n factorial," is defined as

$$n! = n(n-1)\cdots(2)(1),$$

with special case $0! = 1$.

The number of permutations of n distinct objects taken r at a time is

$${}_nP_r = \frac{n!}{(n-r)!}.$$

The number of permutations of n objects arranged in a circle is $(n-1)!$.

The number of distinct permutations of n things of which n_1 are of one kind, n_2 of a second kind, ..., n_k of a kth kind is

$$\frac{n!}{n_1!n_2!\cdots n_k!}.$$

The number of combinations of n distinct objects taken r at a time is

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Question Set

Race cars

In a fuel economy study, each of 3 race cars is tested using 5 different brands of gasoline at 7 test sites located in different regions of the country. If 2 drivers are used in the study, and test runs are made once under each distinct set of conditions, how many test runs are needed?

Multiple-choice test

If a multiple-choice test consists of 5 questions, each with 4 possible answers of which only 1 is correct, (a) in how many different ways can a student check off one answer to each question? (b) in how many ways can a student check off one answer to each question and get all the answers wrong?

Spelling bee

In a regional spelling bee, the 8 finalists consist of 3 boys and 5 girls. Find the number of sample points in the sample space S for the number of possible orders at the conclusion of the contest for (a) all 8 finalists; (b) the first 3 positions.

Television News Stories

A television news director wishes to use 3 news stories on an evening show. One story will be the lead story, one will be the second story, and the last will be a closing story. If the director has a total of 8 stories to choose from, how many possible ways can the program be set up?

School Musical Plays

A school musical director can select 2 musical plays to present next year. One will be presented in the fall, and one will be presented in the spring. If she has 9 to pick from, how many different possibilities are there?

Book Reviews

A newspaper editor has received 8 books to review. He decides that he can use 3 reviews in his newspaper. How many different ways can these 3 reviews be selected?

Committee Selection

In a club there are 7 women and 5 men. A committee of 3 women and 2 men is to be chosen. How many different possibilities are there?

ID Cards

How many different ID cards can be made if there are 6 digits on a card and no digit can be used more than once?

Ticket Selection

How many different ways can 4 tickets be selected from 50 tickets if each ticket wins a different prize?

Task Assignments

How many ways can an adviser choose 4 students from a class of 12 if they are all assigned the same task? How many ways can the students be chosen if they are each given a different task?

Caravan of covered wagons

In how many ways can a caravan of 8 covered wagons from Arizona be arranged in a circle?

INFINITY

How many distinct permutations can be made from the letters of the word INFINITY?