Quiz 2

Q:1

Two components of a laptop computer have the following joint probability density function for their useful lifetimes X and Y (in years):

$$f(x, y) = \{xe^{-x(1+y)}\ x \geq 0, y \geq 0\ 0\ otherwise$$

i. Find the marginal probability density function of X
ii. Find the marginal probability density function of Y
iii. What is the probability that the lifetime of at least one component exceeds 1 year (when the manufacturer's warranty expires)?

Q:2

There are four projects being considered and the payoffs for the four options are modeled as a discrete distribution with probability distribution as follows

Payoff (Rs. 000)	Probability
0	0.50
10	0.25
20	0.15
30	0.10

i. Find the expected value of the option payoff
ii. Describe what this expected value represents
iii. Find the standard deviation of the option payoff
iv. Find the probability that the option will pay at least Rs. 20.

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$$f(x,y) = \begin{cases} x e^{-x(1+y)} & x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

i) $g(X)$:

$$g(X) = \int_0^\infty x e^{-x(1+y)}\, dy$$

$$g(X) = \lim_{a\to\infty} \int_0^a x e^{-x(1+y)}\, dy$$

$$= \lim_{a\to\infty} \left[\frac{\cancel{x} e^{-x(1+y)}}{-\cancel{x}} \right]_0^a$$

$$= \lim_{a\to\infty} \left(-e^{-x(1+y)} \right)_0^a$$

$$= \lim_{a\to\infty} \left(-e^{-x(1+a)} + e^{-x} \right)$$

$$= -e^{-x(1+\infty)} + e^{0-x}$$

$$= -e^{-\infty} + e^{0-x}$$

$$\cancel{= 0 + 1}$$

$$\boxed{g(X) = e^{-x}}$$

x	$+$	$e^{-(1+y)x}$
1	$-$	$\frac{-e^{-(1+y)x}}{(1+y)}$
0		$\frac{e^{-(1+y)x}}{(1+y)^2}$

ii) $h(y)$:

$$h(y) = \int_0^\infty x e^{-x(1+y)}\, dx$$

$$= \lim_{a\to\infty} \int_0^a x e^{-x(1+y)}\, dx.$$

$$h(y) = \lim_{a \to \infty} \left[\frac{-x e^{-(1+y)x}}{(1+y)} - \frac{e^{-(1+y)x}}{(1+y)^2} \right]_0^a$$

$$= \lim_{a \to \infty} \left(\frac{-a\left(e^{-(1+y)a}\right)}{(1+y)} - \frac{e^{-(1+y)a}}{(1+y)^2} + \frac{0\left(e^{-(1+y)0}\right)}{(1+y)} + \frac{e^{-(1+y)0}}{(1+y)^2} \right)$$

$$= \frac{-\infty e^{-\infty}}{(1+y)} - \frac{e^{-\infty}}{(1+y)^2} + 0 + \frac{\cancel{e^0}\, 1}{(1+y)^2}$$

$$\boxed{h(y) = \frac{1}{(1+y)^2}}$$

(iii) $P(X>1 \;\|\; Y>1) = 1 - P(X<1 \;\&\&\; Y<1)$

$$P(X<1 \;\&\&\; Y<1) = \int_0^1 \int_0^1 x e^{-x(1+y)}\, dy\, dx$$

$$= \int_0^1 \left[\frac{-\cancel{x} e^{-x(1+y)}}{\cancel{x}} \right]_0^1 dx$$

$$= \int_0^1 \left(-e^{-2x} + e^{-x}\right) dx$$

$$= \left[\frac{e^{-2x}}{2} - e^{-x} \right]_0^1$$

$$= \frac{e^{-2}}{2} - \frac{e^0}{2} - e^{-1} + e^0$$

$$= \frac{1}{2e^2} + \frac{1}{2} - \frac{1}{e}$$

$$P(X<1 \;\&\&\; Y<1) = 0.1998$$

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$$P(X>1 \;\|\; Y>1) = 1 - 0.1998$$

$$\boxed{P(X>1 \;\|\; Y>1) \approx 0.8}$$

(i) Expected value of payoff:

$$E(P) = (0\times0.5)+(10\times0.25)+(20\times0.15)$$
$$(30\times0.1)$$
$$= 2.5 + 3 + 3$$
$$E(P) = 8.5$$

(ii) It represents that the projects will be expected to get payoff of Rs 8.5 has a great chance lies b/w 0 and 10

(iii) $E(P^2) = (0^2\times0.5)+(10^2\times0.25)+(20^2\times0.15)+$
$$(30^2\times0.1)$$
$$= 25 + 60 + 90$$
$$= 175$$

$$\sigma = \sqrt{175-(8.5)^2}$$

~~6.83~~ $\sigma = 10.13$

(iv) $P(X \geq 20) = P(20) + P(30)$
$$= 0.15 + 0.1$$
$$P(X \geq 20) = 0.25$$