Some illustrations will improve the understanding of the concept.

Example 1

A bag I contains 4 white and 6 black balls while another Bag II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags, and it is found to be black. Find the probability that it was drawn from Bag I.

Solution:

Let E_1 be the event of choosing bag I, E_2 the event of choosing bag II, and A be the event of drawing a black ball. Then,

$$P(E_1) = P(E_2) = \frac{1}{2}$$

Also, $P(A \mid E_1) = P($ drawing a black ball from Bag $I) = 6/10 = 3/5$
$P(A \mid E_2) = P($ drawing a black ball from Bag $II) = 3/7$
By using Bayes' theorem, the probability of drawing a black ball from bag I out of two bags,

$$P(E_1 \mid A) = \frac{P(E_1)\,P(A \mid E_1)}{P(E_1)\,P(A \mid E_1) + P(E_2)\,P(A \mid E_2)} = \frac{\frac{1}{2} \times \frac{3}{5}}{\frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{3}{7}} = \frac{7}{12}$$

Example 2

A man is known to speak the truth 2 out of 3 times. He throws a die and reports that the number obtained is a four. Find the probability that the number obtained is actually a four.

Solution:

Let A be the event that the man reports that number four is obtained. Let E_1 be the event that four is obtained and E_2 be its complementary event. Then, $P(E_1) =$ Probability that four occurs $= 1/6$. $P(E_2) =$ Probability that four does not occur $= 1 - P(E_1) = 1 - (1/6) = 5/6$. Also, $P(A \mid E_1) =$ Probability that man reports four and it is actually a four $= 2/3$ $P(A \mid E_2) =$ Probability that man reports four and it is not a four $= 1/3$. By using Bayes' theorem, probability that number obtained is actually a four,

$$P(E_1 \mid A) = \frac{P(E_1)\,P(A \mid E_1)}{P(E_1)\,P(A \mid E_1) + P(E_2)\,P(A \mid E_2)} = \frac{\frac{1}{6} \times \frac{2}{3}}{\frac{1}{6} \times \frac{2}{3} + \frac{5}{6} \times \frac{1}{3}} = \frac{2}{7}$$

Example 3

. It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email?

Example 4

. Among a group of male pensioners, 10% are smokers and 90% are nonsmokers. The probability of a smoker dying in the next year is 0.05 while the probability for a nonsmoker is 0.005. Given one of these pensioners dies in the next year, what is the probability that he is a smoker?