National University of Computer and Emerging Sciences

Karachi Campus

# Theory of Automata (CS3005) Sessional-II Exam

Date: April 8th 2025

**Course Instructor(s)**

Dr. Nasir Uddin, Syed Faisal Ali, Ms. Shaharbano, Ms. Bakhtawar, Ms. Ms. Abeeha Sattar

| | |
|---|---|
| **Total Time (Hrs):** | **1** |
| **Total Marks:** | **15** |
| **Total Questions:** | **4** |

_____ _____ _____

Roll No Section Student Signature

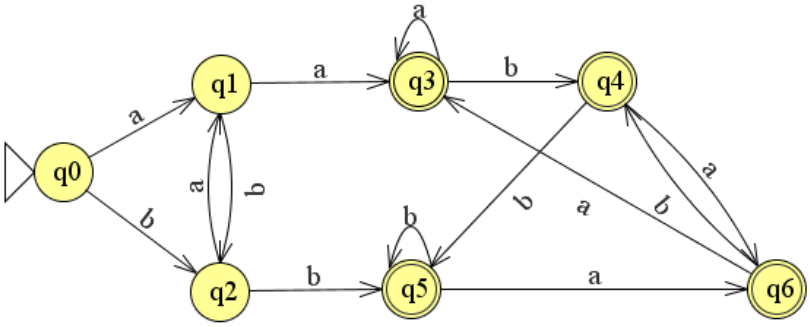
**Do not write below this line**

**Attempt all the questions.**

1. Do not waste pages by adding extra sheets.
2. Draw proper readable diagrams to get marks.
3. Solve the paper according to the question order.

***CLO #4:*** **DFA minimization**

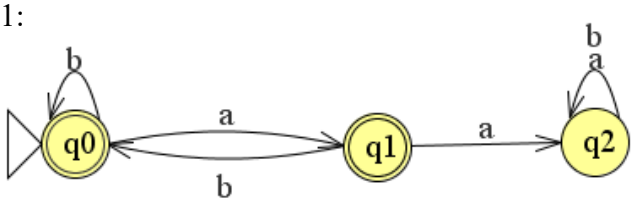
Question 1: DFA Minimization using any algorithm. [**Marks: 04**]



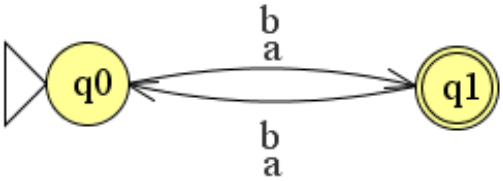

***CLO # 3:*** **Kleene's Properties**

**Question 2.** Kleene's Theorem **[Marks: 04]**

FA1:




FA2:

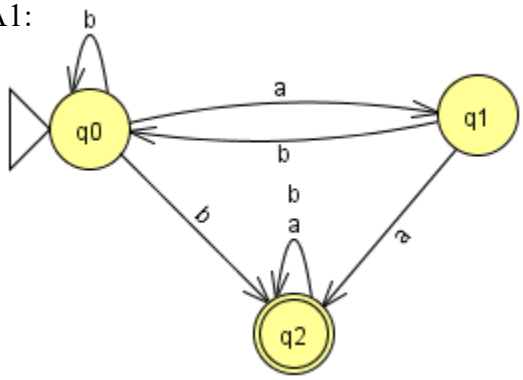



i. Concatenate FA1 with FA2 resulting FA1.FA2.

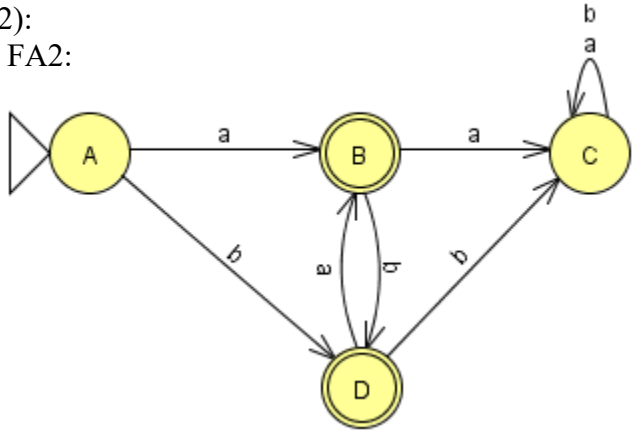
# National University of Computer and Emerging Sciences
Karachi Campus

Consider the following two DFAs (FA1 and FA2):

FA1:




FA2:




ii. What will be the resultant DFA (Union) and DFA (intersection) of the FA1 and FA2? Write two strings of each FA, and prove that the result of Union and Intersection support's your string answer.

Show proper steps for both parts including any diagrams or tables that you are using to reach the final DFA.

***CLO # 2:*** **Pumping Lemma**

Question 3: Use Pumping Lemma to prove the following language is non regular. **[Marks: 02]**

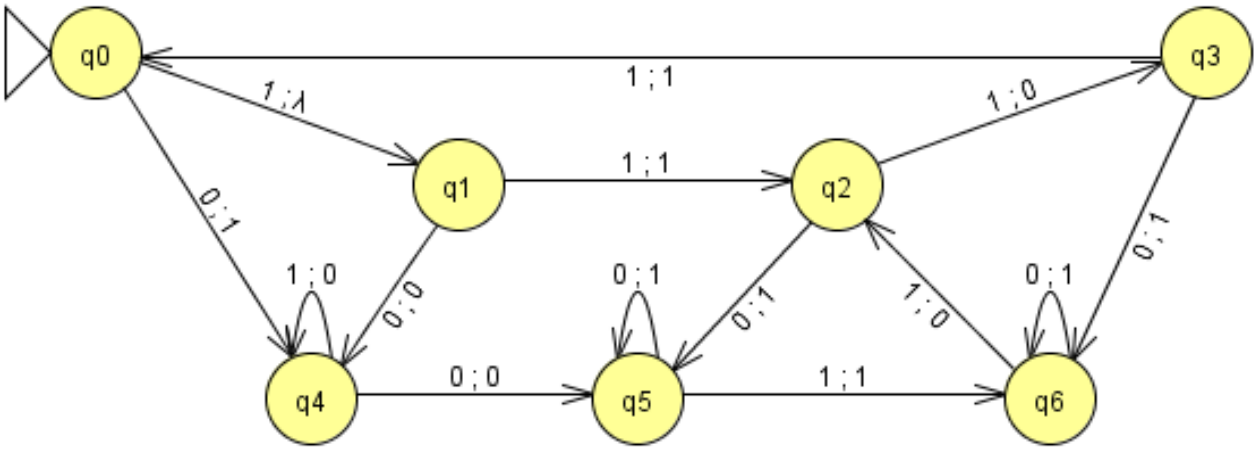
i. $L = \{ab^n ac^{n+2} | n \in N\}$

ii. Use pumping lemma to show that whether L= $a^i\ b\ 3^i\ \ i \geq 100\ and\ i \leq 500$ is non-regular or regular . show your steps against each of the pumping Lemma.

***CLO # 4:*** **Finite Automata with Outputs**

Question 4: Finite Automata with outputs.

i. Given the following Mealy Machine, create its equivalent Moore Machine. **[Marks: 02]**




ii. Create a Moore machine which provides the remainder as output when a number is divided by 5. Provided Σ = {0,1,2,3,4,5,6,7,8,9} **[Marks: 03]**

National University of Computer and Emerging Sciences
Karachi Campus

Solution:

Question 1 Time 10 Mins DFA Minimization

| State | a | b |
|---|---|---|
| $q_0$ | $q_1$ I, II | $q_2$ I, III |
| $q_1$ | $q_3$ II, IV | $q_2$ I, III |
| $q_2$ | $q_4$ I, II | $q_5$ II, III |
| $q_4$ | $q_6$ II, IV | $q_5$ II, IV |
| $q_3$ | $q_3$ II, IV | $q_4$ II, IV |
| $q_5$ | $q_6$ II, IV | $q_5$ II, IV |
| $q_6$ | $q_3$ II, IV | $q_4$ II, IV |

I II

$\{q_0, q_1, q_2\}$ $\{q_3, q_4, q_5, q_6\}$

I II III

$\{q_0, q_4, q_3, q_5\}$ $\{q_1\}$ $\{q_2\}$

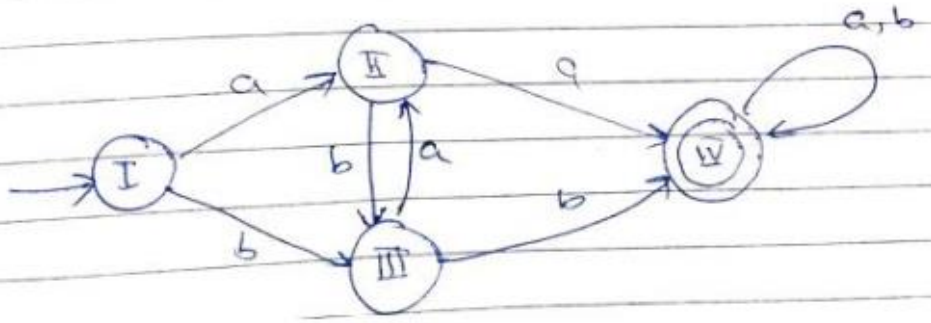
I II III IV

$\{q_0\}$ $\{q_1\}$ $\{q_2\}$ $\{q_3, q_4, q_5, q_6\}$

I II III IV

$\{q_0\}$ $\{q_1\}$ $\{q_2\}$ $\{q_3, q_4, q_5, q_6\}$

Hence the minimized DFA is

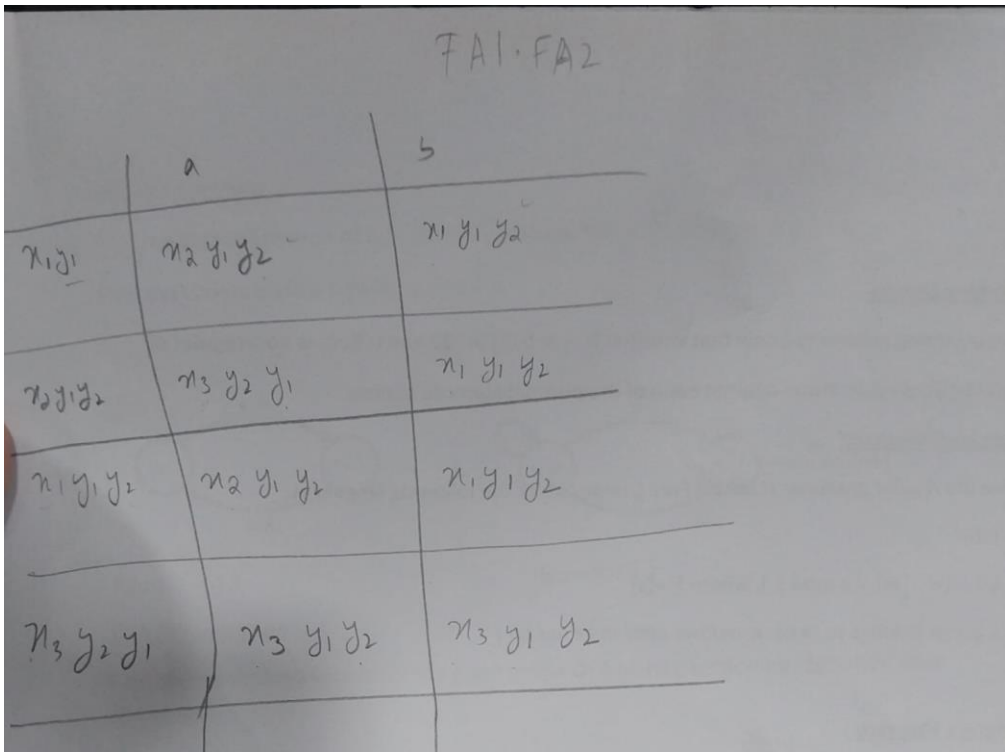


National University of Computer and Emerging Sciences

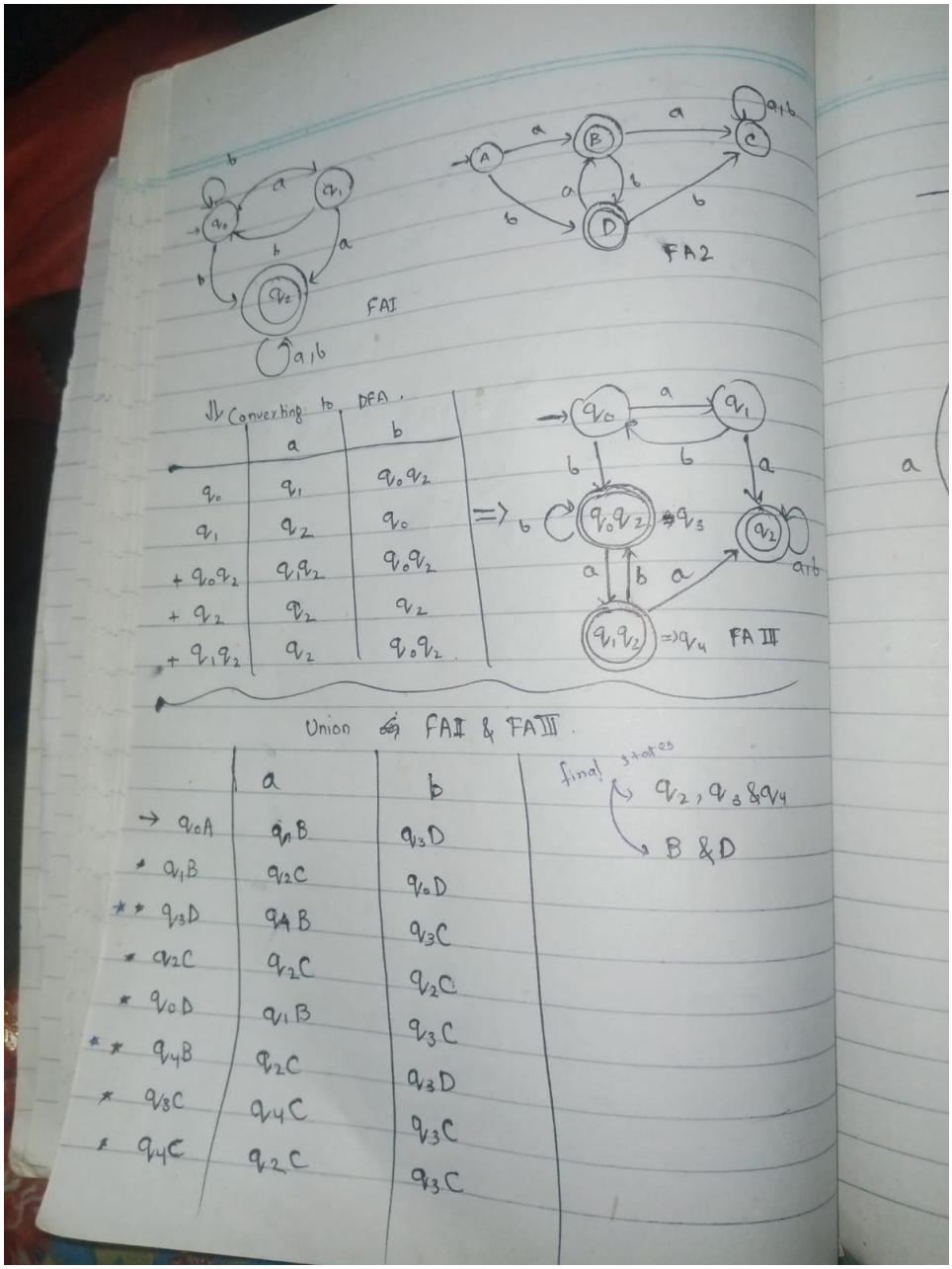
Karachi Campus

Question 2: Time 10 Mins Kleene's Theorem

Concat

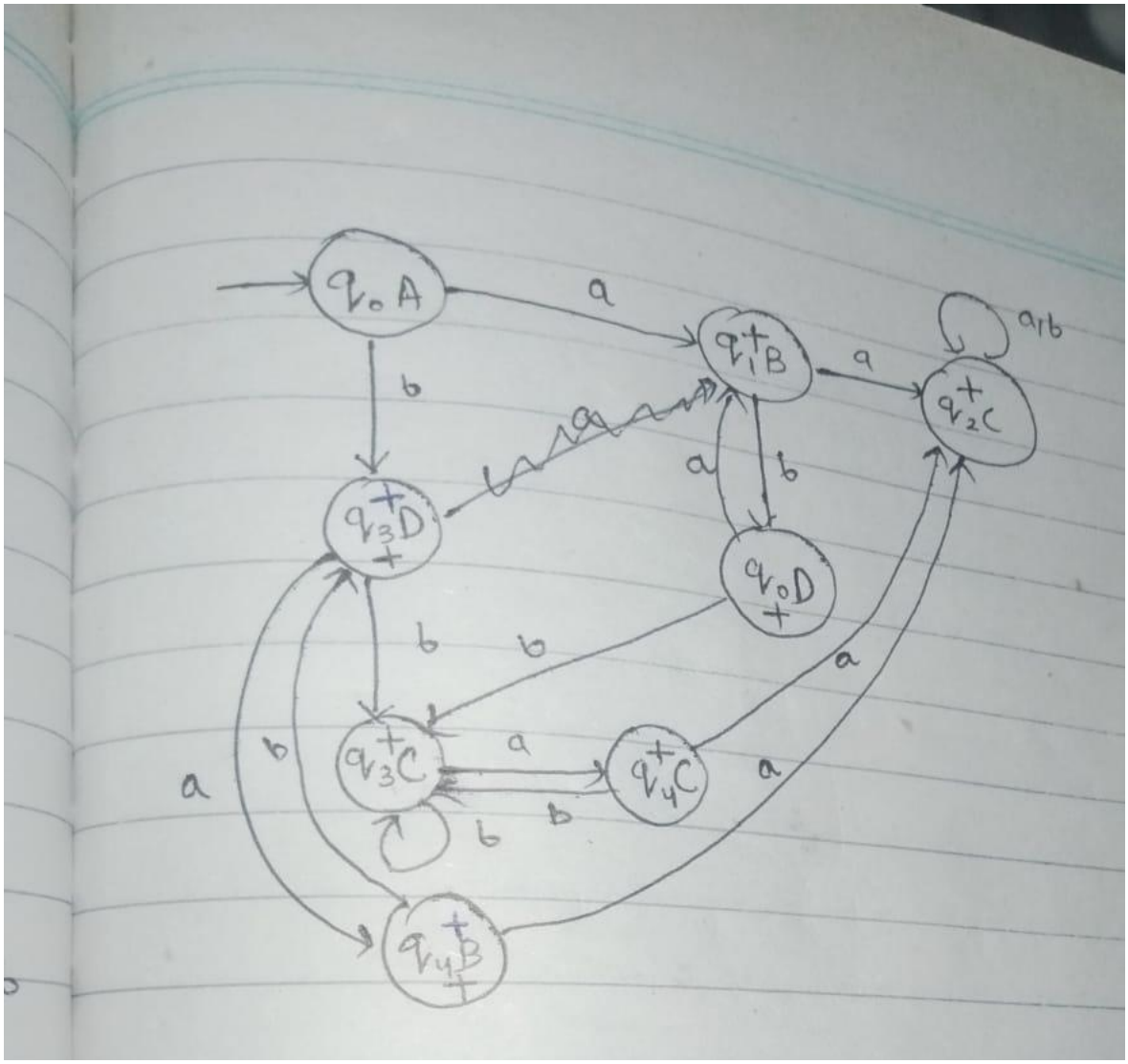


Union/Intersection (after converting FA1 to DFA)

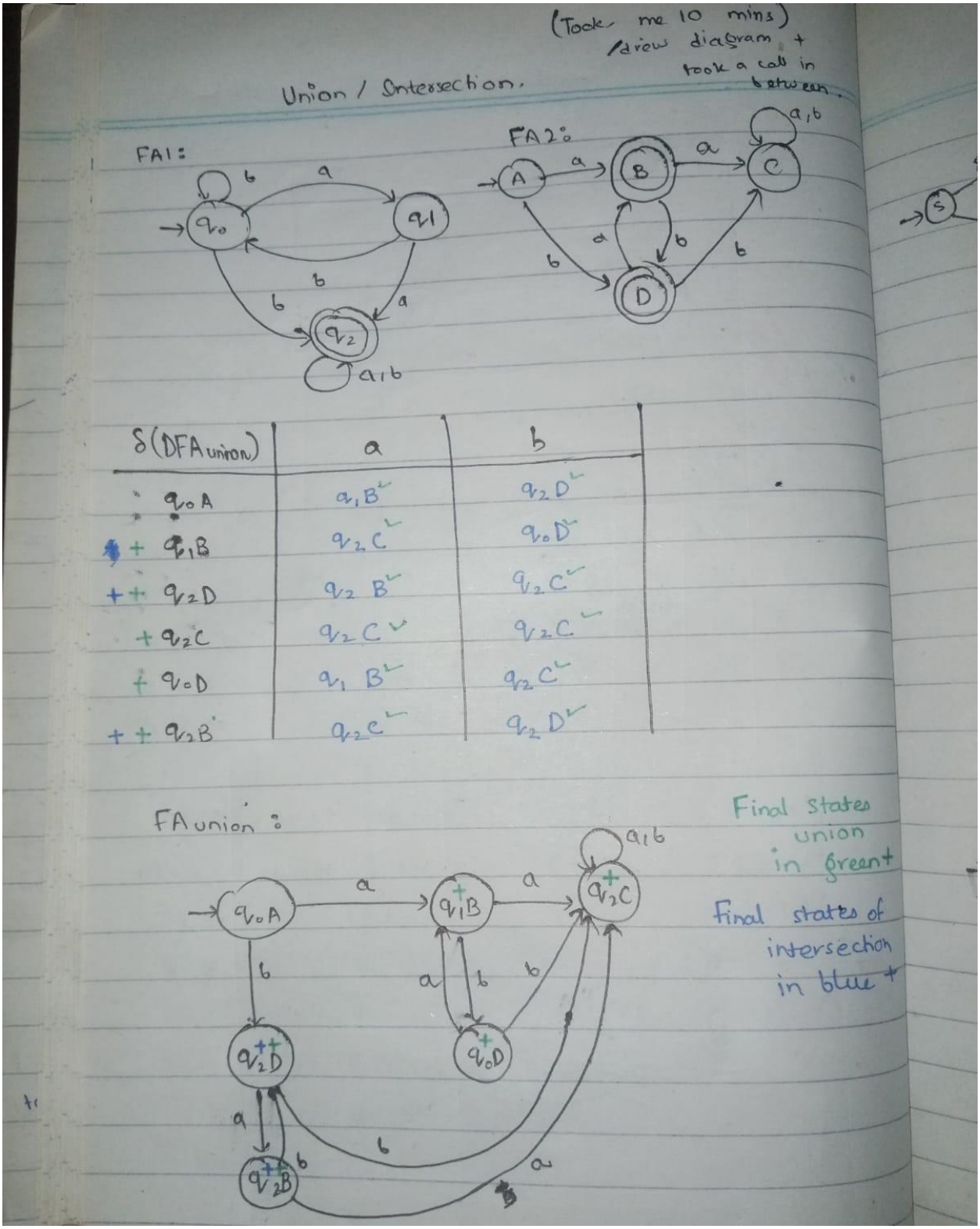


National University of Computer and Emerging Sciences

Karachi Campus



Union/Intersection (FA1 not converted to DFA)




| δ(DFA union) | a | b |
|---|---|---|
| q0A | q1B | q2D |
| + q1B | q2C | q0D |
| + + q2D | q2B | q2C |
| + q2C | q2C | q2C |
| + q0D | q1B | q2C |
| + + q2B | q2C | q2D |

Solution Question 3: Pumping Lemma Time (5 Mins)

Assume $L$ is regular. Then let $p$ be a pumping number for $L$.

The word $x = \mathrm{a}\mathrm{b}^p\mathrm{a}\mathrm{c}^{p+2}$ is in $L$ and has length $\geq p$.
Let $x = uvw$ be a split with the properties of the PL.

From $|uv| \leq p$ and $|v| \geq 1$ we know that $uv$ consists of one a followed by at most $p-1$ bs.

We distinguish two cases, $|u| = 0$ and $|u| > 0$.

---

If $|u| = 0$, then word $v$ starts with an a.
Hence, $uv^0w = \mathrm{b}^{p-|v|+1}\mathrm{a}\mathrm{c}^{p+2}$ does not start with symbol a and is therefore not in $L$. This is a contradiction to the PL.

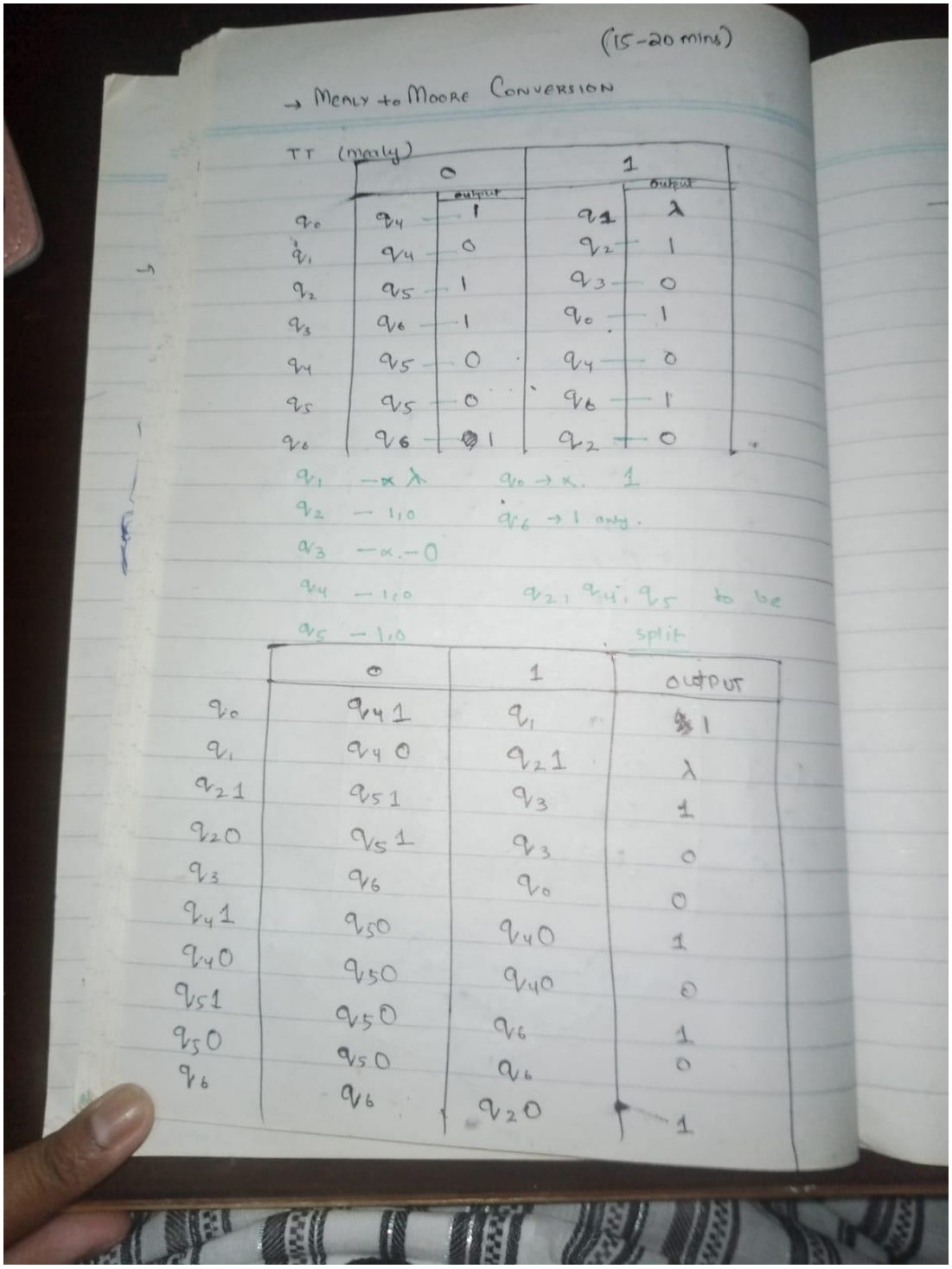
If $|u| > 0$, then word $v$ consists only of bs.
Consider $uv^0w = \mathrm{a}\mathrm{b}^{p-|v|}\mathrm{a}\mathrm{c}^{p+2}$. As $|v| \geq 1$, this word does not contain two more cs than bs and is therefore not in language $L$.
This is a contradiction to the PL.

We have in all cases a contradiction to the PL.
$\rightsquigarrow$ $L$ is not regular.

Question 4: i (Time 10 Mins)

National University of Computer and Emerging Sciences

Karachi Campus



(15-20 mins)

→ Mealy to Moore Conversion

TT (mealy)

| | 0 | | 1 | |
|---|---|---|---|---|
| | | output | | output |
| $q_0$ | $q_4$ | 1 | $q_1$ | λ |
| $q_1$ | $q_4$ | 0 | $q_2$ | 1 |
| $q_2$ | $q_5$ | 1 | $q_3$ | 0 |
| $q_3$ | $q_6$ | 1 | $q_0$ | 1 |
| $q_4$ | $q_5$ | 0 | $q_4$ | 0 |
| $q_5$ | $q_5$ | 0 | $q_6$ | 1 |
| $q_6$ | $q_6$ | 1 | $q_2$ | 0 |

$q_1$ — x λ  $q_0 \to x.\ 1$

$q_2$ — 1,0  $q_6 \to 1$ only.

$q_3$ — x.-0

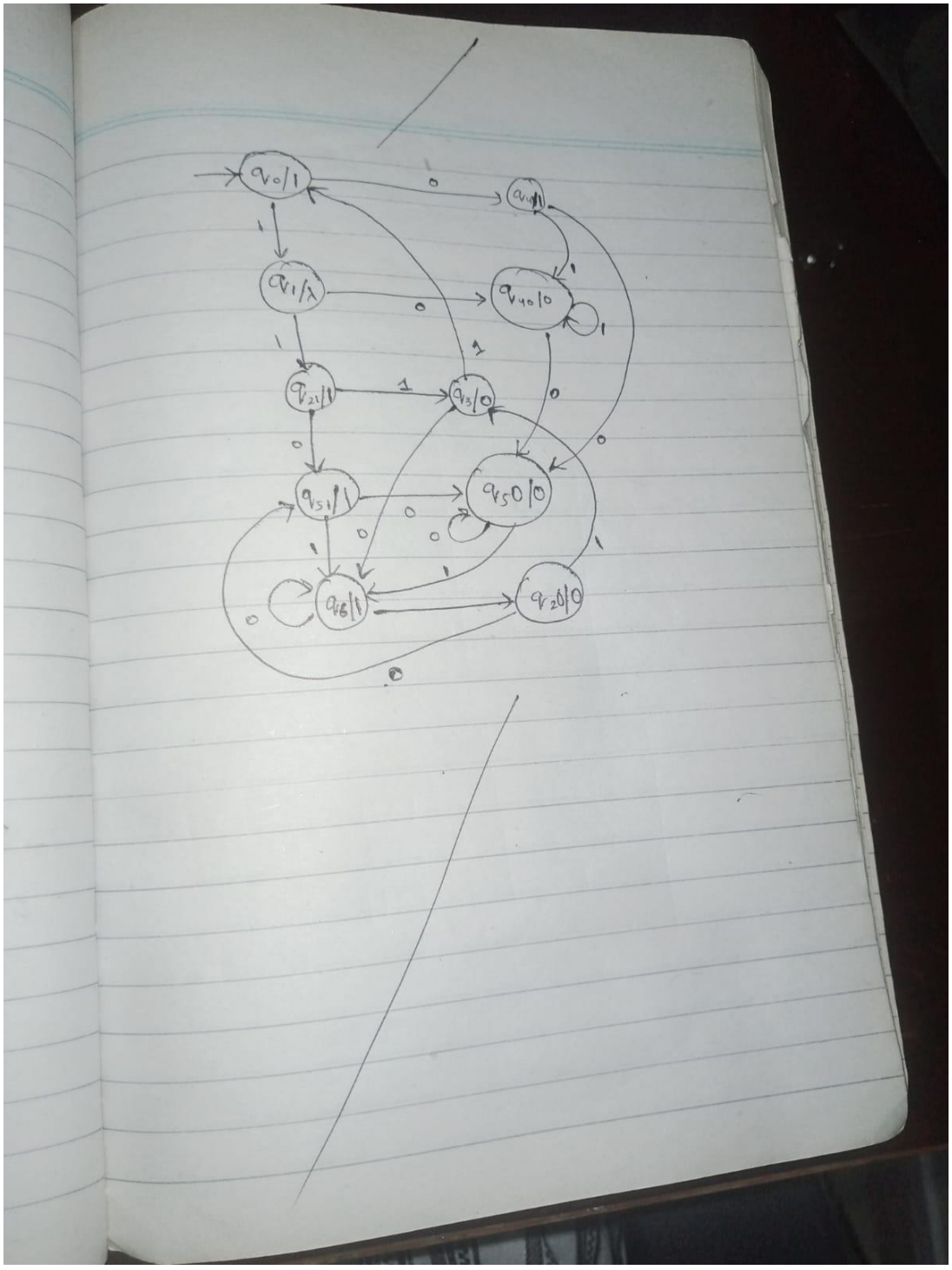
$q_4$ — 1,0  $q_2, q_4, q_5$ to be split

$q_5$ — 1,0

| | 0 | 1 | output |
|---|---|---|---|
| $q_0$ | $q_4$ 1 | $q_1$ | 1 |
| $q_1$ | $q_4$ 0 | $q_2$ 1 | λ |
| $q_2$ 1 | $q_5$ 1 | $q_3$ | 1 |
| $q_2$ 0 | $q_5$ 1 | $q_3$ | 0 |
| $q_3$ | $q_6$ | $q_0$ | 0 |
| $q_4$ 1 | $q_5$ 0 | $q_4$ 0 | 1 |
| $q_4$ 0 | $q_5$ 0 | $q_4$ 0 | 0 |
| $q_5$ 1 | $q_5$ 0 | $q_6$ | 1 |
| $q_5$ 0 | $q_5$ 0 | $q_6$ | 0 |
| $q_6$ | $q_6$ | $q_2$ 0 | 1 |

National University of Computer and Emerging Sciences

Karachi Campus




Question 4 ii. Time (15 mins)

National University of Computer and Emerging Sciences

Karachi Campus