Quiz 04

A local college wants to compare the mean GPA for players on four of its sports teams: basketball, baseball, hockey, and lacrosse. A random sample of players was taken from each team and their GPA recorded in the table below.

Basketball	Baseball	Hockey	Lacrosse
3.6	2.1	4.0	2.0
2.9	2.6	2.0	3.6
2.5	3.9	2.6	3.9
3.3	3.1	3.2	2.7
3.8	3.4	3.2	2.5

Assume the populations are normally distributed and have equal variances. At the 5% significance level, is there a difference in the average GPA between the sports team.

Solution:

Let basketball be population 1, let baseball be population 2, let hockey be population 3, and let lacrosse be population 4. From the question we have the following information:

Basketball	Baseball	Hockey	Lacrosse
$n_1 = 5$	$n_2 = 5$	$n_3 = 5$	$n_4 = 5$
$\bar{x}_1 = 3.22$	$\bar{x}_2 = 3.02$	$\bar{x}_3 = 3$	$\bar{x}_4 = 2.94$
$s_1^2 = 0.277$	$s_2^2 = 0.487$	$s_3^2 = 0.56$	$s_4^2 = 0.613$

Previously, we found $k = 4, n = 20$, and $\bar{\bar{x}} = 3.045$.

Hypotheses:

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$
$$H_a : \text{at least one population mean is different from the others}$$

***p*-value:**

To calculate out the F-score, we need to find MST and MSE.

$$\begin{aligned} SST &= n_1 \times (\bar{x}_1 - \bar{\bar{x}})^2 + n_2 \times (\bar{x}_2 - \bar{\bar{x}})^2 + n_3 \times (\bar{x}_3 - \bar{\bar{x}})^2 + n_4 \times (\bar{x}_4 - \bar{\bar{x}})^2 \\ &= 5 \times (3.22 - 3.045)^2 + 5 \times (3.02 - 3.045)^2 + 5 \times (3 - 3.045)^2 \\ &\quad + 5 \times (2.94 - 3.045)^2 \\ &= 0.2215 \end{aligned}$$

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$$
\begin{aligned}
MST &= \frac{SST}{k-1} \\
&= \frac{0.2215}{4-1} \\
&= 0.0738\ldots
\end{aligned}
$$

$$
\begin{aligned}
SSE &= (n_1-1)\times s_1^2 + (n_2-1)\times s_2^2 + (n_3-1)\times s_3^2 + (n_4-1)\times s_4^2 \\
&= (5-1)\times 0.277 + (5-1)\times 0.487 + (5-1)\times 0.56 + (5-1)\times 0.623 \\
&= 7.788
\end{aligned}
$$

$$
\begin{aligned}
MSE &= \frac{SSE}{n-k} \\
&= \frac{7.788}{20-4} \\
&= 0.48675
\end{aligned}
$$

The *p*-value is the area in the right tail of the F -distribution. To use the **f.dist.rt** function, we need to calculate out the F -score and the degrees of freedom:

$$
\begin{aligned}
F &= \frac{MST}{MSE} \\
&= \frac{0.0738\ldots}{0.48675} \\
&= 0.15168\ldots
\end{aligned}
$$

$$
\begin{aligned}
df_1 &= k-1 \\
&= 4-1 \\
&= 3
\end{aligned}
$$

$$
\begin{aligned}
df_2 &= n-k \\
&= 20-4 \\
&= 16
\end{aligned}
$$

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The ANOVA summary table generated by Excel is shown below:

Anova: Single Factor						
SUMMARY						
Groups	***Count***	***Sum***	***Average***	***Variance***		
Basketball	5	16.1	3.22	0.277		
Baseball	5	15.1	3.02	0.487		
Hockey	5	15	3	0.56		
Lacrosse	5	14.7	2.94	0.623		
ANOVA						
Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	0.2215	3	0.073833	0.151686	0.927083	3.238872
Within Groups	7.788	16	0.48675			
Total	8.0095	19				

The p-value for the test is in the ***P*-value column** of the **between groups row**. So the p-value $= 0.9271$.

Conclusion:

Because p-value$= 0.9271 > 0.05 = \alpha$, we do not reject the null hypothesis. At the 5% significance level there is enough evidence to suggest that the mean GPA for the sports teams are the same.