Probability on an Event

The **probability** of an event A is the sum of the weights of all sample points in A. Therefore,

$$0 \leq P(A) \leq 1, \quad P(\phi) = 0, \quad \text{and} \quad P(S) = 1.$$

Furthermore, if $A_1, A_2, A_3, \ldots$ is a sequence of mutually exclusive events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots.$$

If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of event A is

$$P(A) = \frac{n}{N}.$$

Addition Rule:

If A and B are two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

If A and B are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B).$$

If $A_1, A_2, \ldots, A_n$ are mutually exclusive, then

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n).$$

For three events A, B, and C,

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\ &\quad - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C). \end{aligned}$$

If A and A' are complementary events, then

$$P(A) + P(A') = 1.$$

Example:

A coin is tossed twice. What is the probability that at least 1 head occurs?
The sample space for this experiment is

$$S = \{HH, HT, TH, TT\}.$$

If the coin is balanced, each of these outcomes is equally likely to occur. Therefore, we assign a probability of ω to each sample point. Then $4\omega = 1$, or $\omega = 1/4$. If A represents the event of at least 1 head occurring, then

$$A = \{HH, HT, TH\} \text{ and } P(A) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}.$$

Example:

A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If E is the event that a number less than 4 occurs on a single toss of the die, find $P(E)$.

The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. We assign a probability of w to each odd number and a probability of $2w$ to each even number. Since the sum of the probabilities must be 1, we have $9w = 1$ or $w = 1/9$. Hence, probabilities of $1/9$ and $2/9$ are assigned to each odd and even number, respectively. Therefore,

$$E = \{1, 2, 3\} \text{ and } P(E) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}.$$

Example:

In Example 2.25, let A be the event that an even number turns up and let B be the event that a number divisible by 3 occurs. Find $P(A \cup B)$ and $P(A \cap B)$.
For the events $A = \{2, 4, 6\}$ and $B = \{3, 6\}$, we have

$$A \cup B = \{2, 3, 4, 6\} \text{ and } A \cap B = \{6\}.$$

By assigning a probability of $1/9$ to each odd number and $2/9$ to each even number, we have

$$P(A \cup B) = \frac{2}{9} + \frac{1}{9} + \frac{2}{9} + \frac{2}{9} = \frac{7}{9} \quad \text{and} \quad P(A \cap B) = \frac{2}{9}.$$

Example:

A statistics class for engineers consists of 25 industrial, 10 mechanical, 10 electrical, and 8 civil engineering students. If a person is randomly selected by the instructor to answer a question, find the probability that the student chosen is (a) an industrial engineering major and (b) a civil engineering or an electrical engineering major.

Denote by I, M, E, and C the students majoring in industrial, mechanical, electrical, and civil engineering, respectively. The total number of students in the class is 53, all of whom are equally likely to be selected.

(a) Since 25 of the 53 students are majoring in industrial engineering, the probability of event I, selecting an industrial engineering major at random, is

$$P(I) = \frac{25}{53}.$$

(b) Since 18 of the 53 students are civil or electrical engineering majors, it follows that

$$P(C \cup E) = \frac{18}{53}.$$

Example:

In a poker hand consisting of 5 cards, find the probability of holding 2 aces and 3 jacks.

The number of ways of being dealt 2 aces from 4 cards is

$$\binom{4}{2} = \frac{4!}{2!\ 2!} = 6,$$

and the number of ways of being dealt 3 jacks from 4 cards is

$$\binom{4}{3} = \frac{4!}{3!\ 1!} = 4.$$

By the multiplication rule (Rule 2.1), there are $n = (6)(4) = 24$ hands with 2 aces and 3 jacks. The total number of 5-card poker hands, all of which are equally likely, is

$$N = \binom{52}{5} = \frac{52!}{5!\ 47!} = 2{,}598{,}960.$$

Therefore, the probability of getting 2 aces and 3 jacks in a 5-card poker hand is

$$P(C) = \frac{24}{2,598,960} = 0.9 \times 10^{-5}.$$

Example:

John is going to graduate from an industrial engineering department in a university by the end of the semester. After being interviewed at two companies he likes, he assesses that his probability of getting an offer from company A is 0.8, and his probability of getting an offer from company B is 0.6. If he believes that the probability that he will get offers from both companies is 0.5, what is the probability that he will get at least one offer from these two companies?
Using the additive rule, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8 + 0.6 - 0.5 = 0.9.$$

Example:

What is the probability of getting a total of 7 or 11 when a pair of fair dice is tossed?
Let A be the event that 7 occurs and B the event that 11 comes up. Now, a total of 7 occurs for 6 of the 36 sample points, and a total of 11 occurs for only 2 of the sample points. Since all sample points are equally likely, we have $P(A) = 1/6$ and $P(B) = 1/18$. The events A and B are mutually exclusive, since a total of 7 and 11 cannot both occur on the same toss. Therefore,

$$P(A \cup B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{18} = \frac{2}{9}.$$

This result could also have been obtained by counting the total number of points for the event $A \cup B$, namely 8, and writing

$$P(A \cup B) = \frac{n}{N} = \frac{8}{36} = \frac{2}{9}.$$

Example:

If the probabilities are, respectively, 0.09, 0.15, 0.21, and 0.23 that a person purchasing a new automobile will choose the color green, white, red, or blue, what is the probability that a given buyer will purchase a new automobile that comes in one of those colors?
Let G, W, R, and B be the events that a buyer selects, respectively, a green, white, red, or blue automobile. Since these four events are mutually exclusive, the probability is

$$\begin{aligned} P(G \cup W \cup R \cup B) &= P(G) + P(W) + P(R) + P(B) \\ &= 0.09 + 0.15 + 0.21 + 0.23 = 0.68. \end{aligned}$$

Example:

If the probabilities that an automobile mechanic will service 3, 4, 5, 6, 7, or 8 or more cars on any given workday are, respectively, 0.12, 0.19, 0.28, 0.24, 0.10, and 0.07, what is the probability that he will service at least 5 cars on his next day at work?

Let E be the event that at least 5 cars are serviced. Now, $P(E) = 1 - P(E')$, where E' is the event that fewer than 5 cars are serviced. Since

$$P(E') = 0.12 + 0.19 = 0.31,$$

it follows from Theorem 2.9 that

$$P(E) = 1 - 0.31 = 0.69.$$

Example:

Suppose the manufacturer's specifications for the length of a certain type of computer cable are 2000 ± 10 millimeters. In this industry, it is known that small cable is just as likely to be defective (not meeting specifications) as large cable. That is, the probability of randomly producing a cable with length exceeding 2010 millimeters is equal to the probability of producing a cable with length smaller than 1990 millimeters. The probability that the production procedure meets specifications is known to be 0.99.

(a) What is the probability that a cable selected randomly is too large?

(b) What is the probability that a randomly selected cable is larger than 1990 millimeters?

Let M be the event that a cable meets specifications. Let S and L be the events that the cable is too small and too large, respectively. Then

(a) $P(M) = 0.99$ and $P(S) = P(L) = (1 - 0.99)/2 = 0.005$.

(b) Denoting by X the length of a randomly selected cable, we have

$$P(1990 \leq X \leq 2010) = P(M) = 0.99.$$

Since $P(X \geq 2010) = P(L) = 0.005$,

$$P(X \geq 1990) = P(M) + P(L) = 0.995.$$

This also can be solved by using Theorem 2.9:

$$P(X \geq 1990) + P(X < 1990) = 1.$$

Thus, $P(X \geq 1990) = 1 - P(S) = 1 - 0.005 = 0.995$.

Practice Problems

2.55 If each coded item in a catalog begins with 3 distinct letters followed by 4 distinct nonzero digits, find the probability of randomly selecting one of these coded items with the first letter a vowel and the last digit even.

2.54 From past experience, a stockbroker believes that under present economic conditions a customer will invest in tax-free bonds with a probability of 0.6, will invest in mutual funds with a probability of 0.3, and will invest in both tax-free bonds and mutual funds with a probability of 0.15. At this time, find the probability that a customer will invest

(a) in either tax-free bonds or mutual funds;

(b) in neither tax-free bonds nor mutual funds.

2.58 A pair of fair dice is tossed. Find the probability of getting

(a) a total of 8;

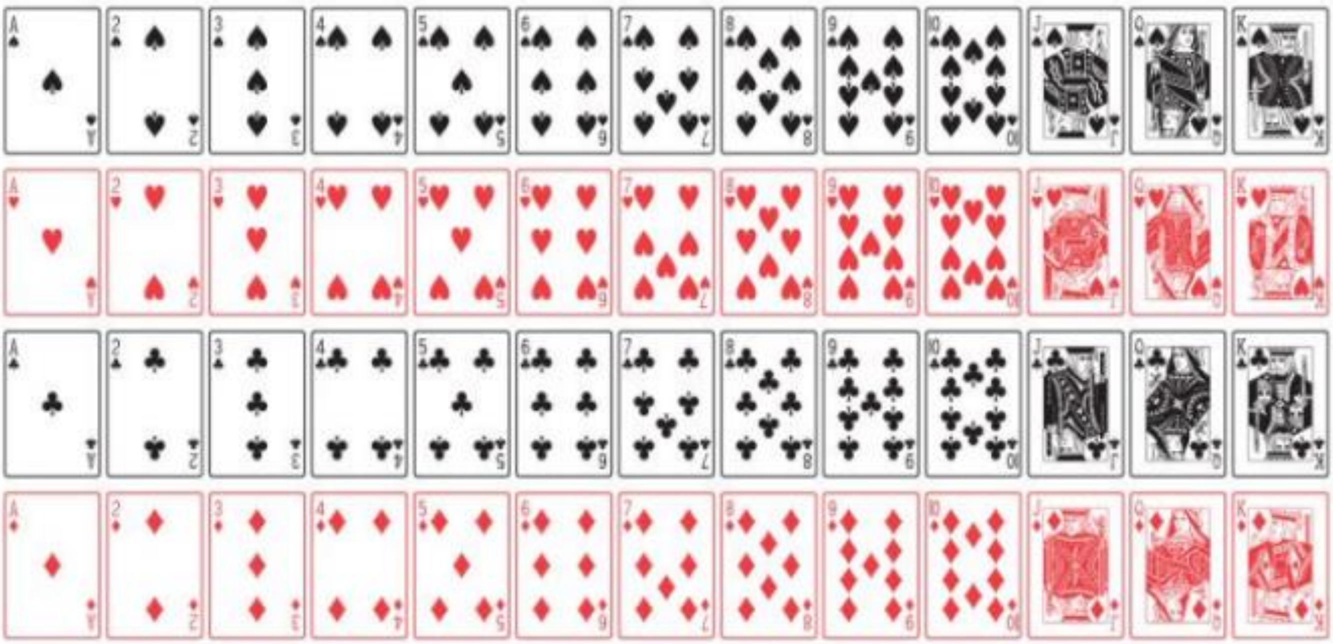
(b) at most a total of 5.

2.64 Interest centers around the life of an electronic component. Suppose it is known that the probability that the component survives for more than 6000 hours is 0.42. Suppose also that the probability that the component survives *no longer than* 4000 hours is 0.04.

(a) What is the probability that the life of the component is less than or equal to 6000 hours?

(b) What is the probability that the life is greater than 4000 hours?

A deck of playing Cards



A card is drawn at random from the well shuffled pack of 52 playing cards. Find the probability that the card:

(a) is a Jack
(b) is not a Jack
(c) Is an ace or a king?
(d) Is a heart or spade?
(e) Is a queen or black
(f) In a poker hand consisting of 5 cards, find the probability of holding 2 aces and 3 jacks.