TOA : Assignment 02 23K-2001

Answer #01:

1. Fig. 3. Fig. 4

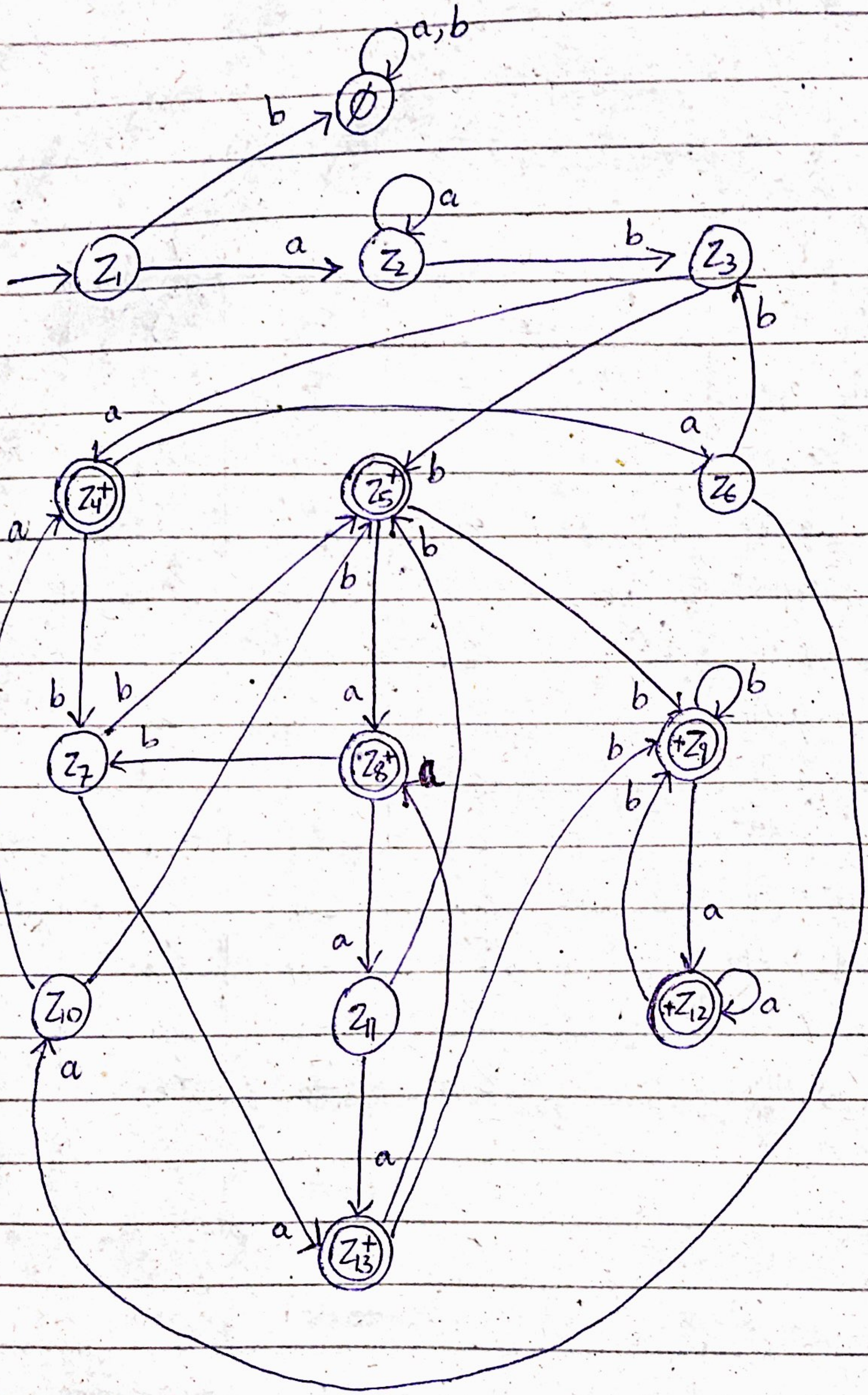
state	a	b
$\rightarrow Z_1 = q_0$	q_1	$\emptyset$
$Z_2 = q_1$	q_1	q_2A
$Z_3 = q_2A$	q_1B	q_2AB
$+ Z_4 = q_1B$	q_1C	q_2AC
$+ Z_5 = q_2AB$	q_1BC	q_2ABC
$Z_6 = q_1C$	q_1A	q_2A
$Z_7 = q_1AC$	q_1AB	q_2AB
$+ Z_8 = q_1BC$	q_1AC	q_2AC
$+ Z_9 = q_2ABC$	q_1ABC	q_2ABC
$Z_{10} = q_1A$	q_1B	q_2AB
$Z_{11} = q_1AC$	q_1AB	q_2AB
$+ Z_{12} = q_1ABC$	q_1ABC	q_2ABC
$+ Z_{13} = q_1AB$	q_1BC	q_2ABC

$\Rightarrow$

Z_i	a	b
$\rightarrow Z_1$	Z_2	$\emptyset$
Z_2	Z_2	Z_3
Z_3	Z_4	Z_5
$+ Z_4$	Z_6	Z_7
$+ Z_5$	Z_8	Z_9
Z_6	Z_{10}	Z_3
Z_7	Z_{13}	Z_5
$+ Z_8$	Z_{11}	Z_7
$+ Z_9$	Z_{12}	Z_9
Z_{10}	Z_4	Z_5
Z_{11}	Z_{13}	Z_5
$+ Z_{12}$	Z_{12}	Z_9
$+ Z_{13}$	Z_8	Z_9

(graph on next page)

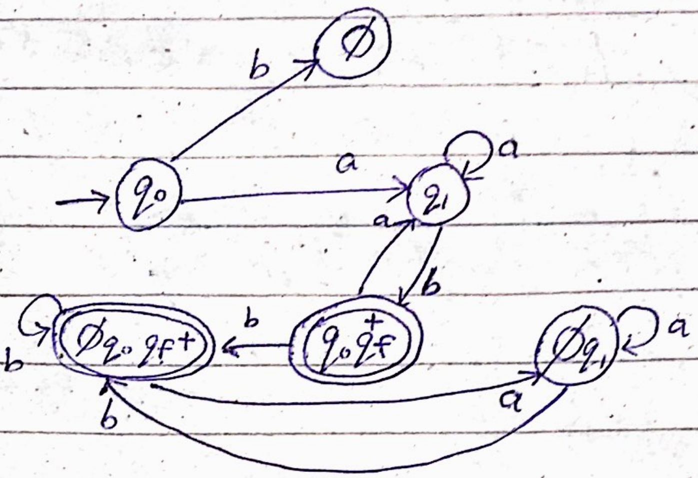
Fig .3. Fig. 4



a,b
∅
b
a
Z1
a
Z2
b
Z3
b
a
a
Z4+
Z5+
b
Z6
a
b
b
b
b
b
a
b
b
Z7
Z8+
a
b
+Z9
b
b
a
a
Z10
Z11
+Z12
a
a
a
a
Z13+

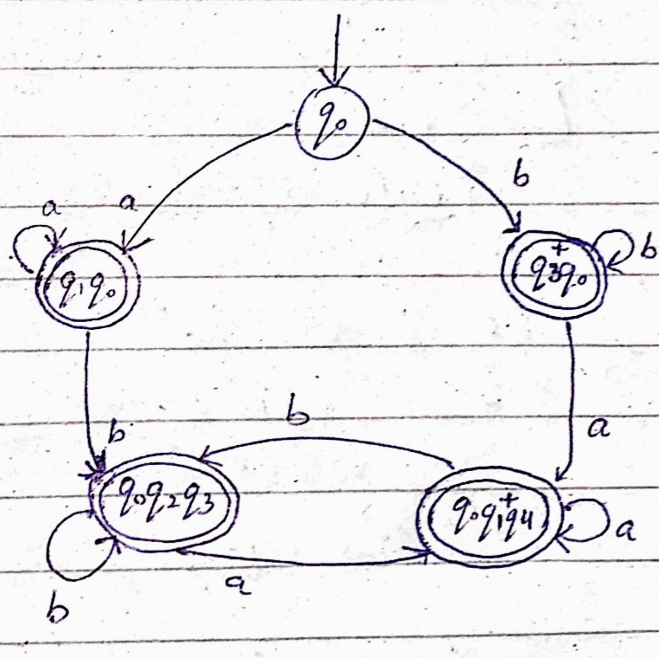
2. Kleene * Closure on Fig. 3:

Q	a	b
→ q_0	q_1	$\emptyset$
q_1	q_1	$q_f q_0$
+ $q_f q_0$	q_1	$\emptyset q_f q_0$
+ $\emptyset q_f q_0$	$\emptyset q_1$	$\emptyset q_f q_0$
$\emptyset q_1$	$\emptyset q_1$	$\emptyset q_f q_0$



3. Kleene * Closure on Fig. 5:

Q	a	b
→ q_0	$q_1 q_0$	$q_3 q_0$
+ $q_1 q_0$	$q_1 q_0$	$q_0 q_2 q_3$
+ $q_3 q_0$	$q_0 q_1 q_4$	$q_0 q_3$
+ $q_0 q_2 q_3$	$q_0 q_1 q_4$	$q_0 q_2 q_3$
+ $q_0 q_1 q_3$	$q_0 q_1 q_4$	$q_0 q_2 q_3$



Answer # 02:

i) $L_1 = \{0^n 1^m : n \leq m+3\}$

$L = \{0, 00, \varepsilon, 000, 0001, \underline{00001}, 0000011, \ldots\}$

$\underbrace{000}_{x}\underbrace{0}_{y}\underbrace{1}_{z}$ such that xy^iz

$i = 1 \Rightarrow 00001$

$i = 2 \Rightarrow 000001 \notin L_1$ as $n = 5, m = 1$

as given:

$L_1 \rightarrow n \leq m+3$

$5 \not\leq 4$

$5 \not\leq 1+3$

$5 \not\leq 4$

The given language is not regular. Ans.

ii) $L_2 = \{01(1100)^n 110(10)^n : n \geq 0\}$

$L = \{\underline{01110}, 0111001101 0, \ldots\}$

$\underbrace{01}_{x}\underbrace{1}_{y}\underbrace{1}_{z}0$ such that xy^iz

$i = 2 \Rightarrow 0111110 \notin L_2$

The language is not regular. Ans.

iii) $L_3 = \{a^i b^j c^k : i >= j >= k >= 1\}$

$L = \{\underline{abc}, aabbc, aaabc, \ldots\}$

$\underbrace{a}_{x}\underbrace{b}_{y}\underbrace{c}_{z}$ such that xy^iz

$i = 2 \Rightarrow abbc \notin L_3$ as $j = 2, i = k = 1$

as given: $i >= j > k >= 1$

The given language is not regular. Ans.

Answer#03:

Q	a	b
→ 0	1	9
1	8	2
2	3	2
+ 3	2	4
+ 4	5	8
5	4	5
6	7	5
7	6	5
+ 8	1	3
+ 9	7	8

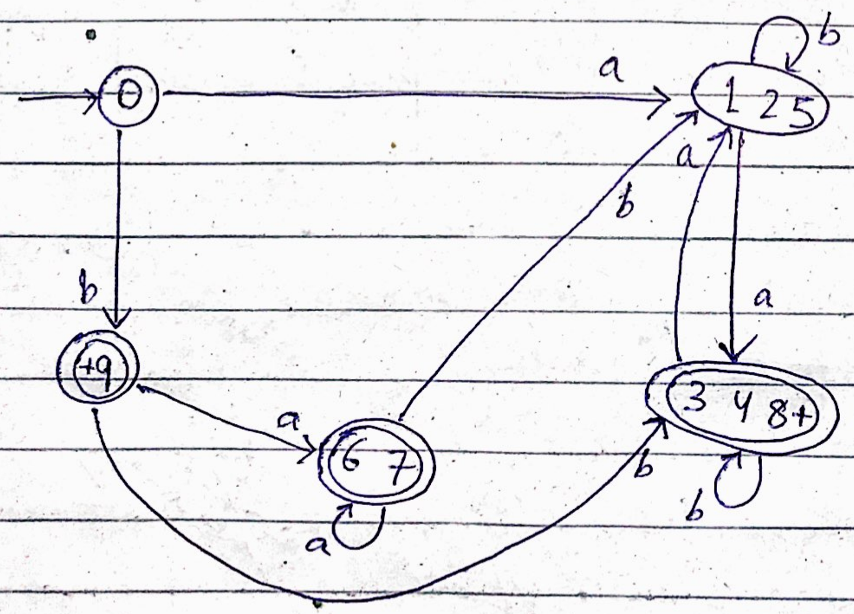
$\pi_0 = \{0,1,2,5,6,7\}\{3,4,8,9\}$

$\pi_1 = \{0\}\{1,2,5\}\{6,7\}\{3,4,8,9\}$

$\pi_2 - \{0\}\{1,2,5\}\{6,7\}\{3,4,8\}\{9\}$

(Final)

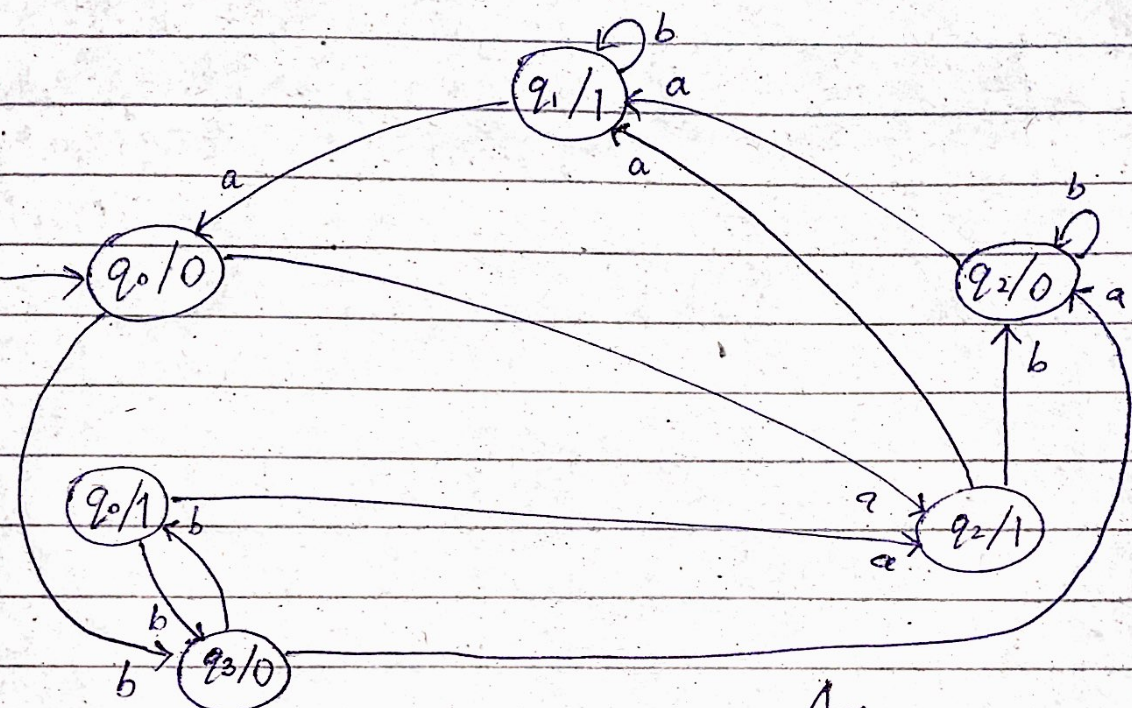
Minimized DFA:



Ans.

Answer#04:

a. Mealy to Moore Conversion:

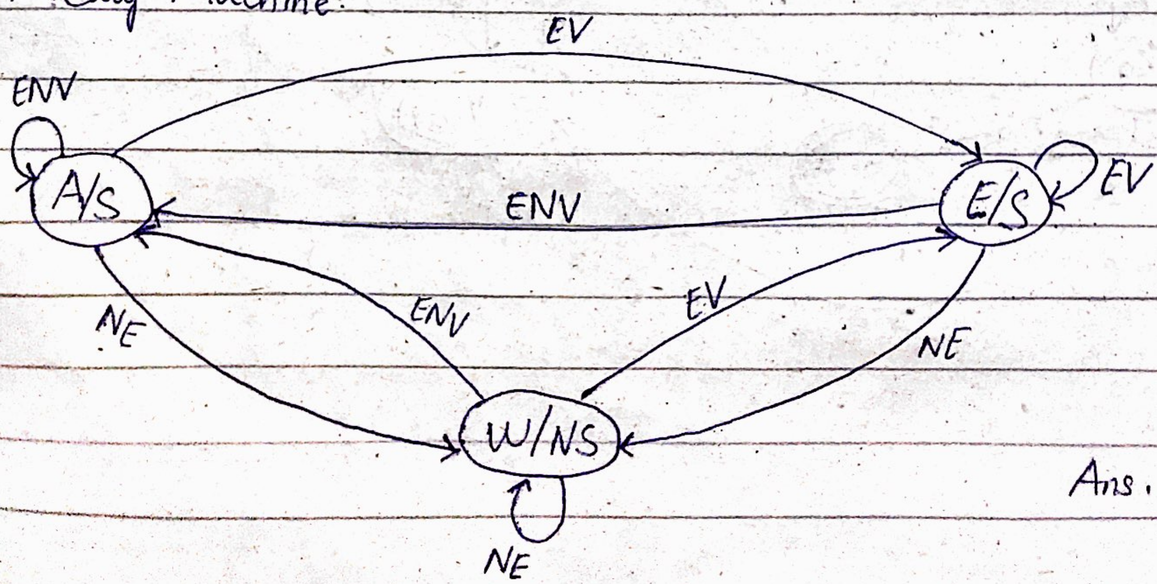


Ans.

b.

Q	NE	ENV	EV
E	W/NS	A/S	E/S
A	W/NS	A/S	E/S
W	W/NS	A/S	E/S

Mealy Machine:



Ans.

Answer #05:

a. i) $V = \{q_0, q_1, q_2, q_3, q_4\}$ or $\{A, B, C, D, E\}$

$T = \{a, b, c\}$

ii) $S = q_0$ or A, Final state $= \{q_2, q_3\}$ or $\{C, D\}$

iii)

δ_{DFA}	a	b	c
$q_0 = A$	q_1	q_1	q_4
$q_1 = B$	q_2	q_0	q_3
$q_2 = C$	q_3	q_4	q_2
$q_3 = D$	q_3	q_2	q_1
$q_4 = E$	q_2	q_4	q_3

iv) Regular grammar:

$A \rightarrow aB \mid bB \mid cE$

$B \rightarrow aC \mid a \mid bA \mid cC \mid c$

$C \rightarrow aD \mid a \mid bE \mid cC \mid c$

$D \rightarrow aD \mid a \mid bC \mid b \mid cB$

$E \rightarrow aC \mid a \mid bE \mid cA$

Ans.

b. $A \rightarrow aB \mid bC \mid \lambda \Rightarrow a(a+b)^* + ba^*b + \lambda$

$B \rightarrow aB \mid bB \mid \lambda \Rightarrow (a+b)^*$

$C \rightarrow aC \mid b \Rightarrow a^*b$

$RE \Rightarrow a(a+b)^* + ba^*b + \lambda$ Ans.

c. $\{b, abc, aabcc, aaabccc, \ldots a^n b c^n \ldots\}$

→ only one b

→ any number of a's, $n \geq 0$

→ same number of a's as c's

RG ⇒ $S \rightarrow aSc \mid b$ Ans.

d.

(i)

$L = A^*$ $A \rightarrow aA \mid \epsilon$

Proof:

→ generates λ, which $\in A^*$

→ Assume that A generates a^n, $n \geq 0$

So, $A \rightarrow aA$, we get:

$$a(a^n) = a^{n+1}$$

Thus, A can generate a^{n+1}

Conclusion: 'A' can generate any amount of a's i.e a^*

(ii)

$L1 = (a+b)^* a$, $L2 = b(a+b)^*$

RG_1 : $S_1 \rightarrow aS_1 \mid bS_1 \mid a$

RG_2 : $S_2 \rightarrow bS_2 \mid aS_2 \mid b$

Concatenation ⇒ $S_1 \rightarrow a$ becomes $S_1 \rightarrow aS_2$

RG_c : $S \rightarrow S_1$

$S_1 \rightarrow aS_1 \mid bS_1 \mid aS_2$ Ans.

$S_2 \rightarrow bS_2 \mid aS_2 \mid b$

(iii)

G_1, G_2

Proof:

G_1 generates a RL L_1 where $L_1 = \{0, 1\}$

G_2 generates a RL L_2 where $L_2 = \{0, 1\}$

→ Replacing every occurence of a terminal 0 in G_1 with L_2. Since RL are closed under substitution, the resulting language is regular.

Conclusion:

Substitution of a terminal in a regular grammar with another regular language preserves Regularity.

(iv)

Left-linear Grammar → L must be regular

Proof:

Every left-linear grammar is equivalent to its right-linear grammar, by reversing productions and swapping start and final symbols.

⇒ Since, right-linear grammar generates RL_1 left-linear grammar also generates RL_1.

Hence, left-linear grammar define languages that are Regular.