J 5: Template (Complete this one)

Matrices

Activity	Links	Deadline
2* 2 ☐ A. Add, Subtract ☐ B. Multiply ☐ C. Transpose ☐ D. Determinant ☐ E. Inverse **3 * 3** ☐ A. Inverse ☐ B. Solve Linear Equations	JOURNAL	**Journal 5 : Matrices**	Upload pdf Tues 7th Dec 5pm Accuracy Check quiz Tues 7th Dec 5pm
	QUIZ (100%) 	**Mastery Quiz 5 (a)** [2 x 2] Matrices **Mastery Quiz 5 (b)** [3 x 3] Matrices	Friday 10th Dec 5pm

A. Addition and Subtraction

Question 1: Addition and Subtraction

(i) Calculate

$$\begin{bmatrix} 3 & -5 & 4 \\ -1 & 4 & 6 \end{bmatrix} + \begin{bmatrix} -1 & 4 & 2 \\ -5 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 6 \\ -6 & 2 & 9 \end{bmatrix}$$

Working out.

3+(-1)=2 **-5+4=-1** **4+2=6**
-1+(-5)=-6 4+(-2)=2 6+3=9

(ii) **Calculate**

$$\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix} - \begin{bmatrix} -1 & -5 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ -7 & 4 \end{bmatrix}$$

2-(-1)=3 -3-(-5)=2
-4-3=-7 2-(-2)=4

B. Multiplication

Question 2:

(i) What is **3A – 2B?**

If A= $\begin{bmatrix} -3 & 1 \\ -2 & 4 \\ 5 & -1 \end{bmatrix}$ **and If B=** $\begin{bmatrix} 4 & -3 \\ 0 & -2 \\ -2 & -4 \end{bmatrix}$

3A

$$3 * \begin{bmatrix} -3 & 1 \\ -2 & 4 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} -9 & 3 \\ -6 & 12 \\ 15 & -3 \end{bmatrix}$$

B= 2* $\begin{bmatrix} 4 & -3 \\ 0 & -2 \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} 8 & -6 \\ 0 & -4 \\ -2 & -8 \end{bmatrix}$

3A – 2B=

$$\begin{bmatrix} -9 & 3 \\ -6 & 12 \\ 15 & -3 \end{bmatrix} - \begin{bmatrix} 8 & -6 \\ 0 & -4 \\ -2 & -8 \end{bmatrix} = \begin{bmatrix} -17 & 9 \\ -6 & 16 \\ 17 & 5 \end{bmatrix}$$

(ii) Multiplication *Show calculations*

Let A and B be the following matrices
(matrices are really cool)

$$A = \begin{bmatrix} -4 & 0 \\ -4 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 1 & -2 \\ 3 & -5 & -5 \end{bmatrix}$$

Find the product of A.B

$$\begin{bmatrix} 8 & -4 & 8 \\ 5 & 1 & 13 \end{bmatrix}$$

Show Calculations

(-4*-2)+(0*3)=8
(-4*1)+(0*-5)=-4
(-4*-2)+(0*-5)=8

(-4*-2)+(-1*3)=5
(-4*1)+(-1*-5)=1
(-4*-2)+(-1*-5)=13

C. Transpose

Q3. Transpose

(i) Find $\mathbf{A^T}$

$$\text{If } A = \begin{bmatrix} 3 & -5 & 4 \\ -1 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ -5 & 4 \\ 4 & 6 \end{bmatrix}$$

(ii) Find $\mathbf{B^T}$

$$\text{If } B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

D. Determinants

Q4. Determinants

(i) **Find IAI**

If A = $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

(3*2)-(1*4)=2

(ii) **Find Det IAI**

A= $\begin{bmatrix} 12 & 20 \\ 15 & 7 \end{bmatrix}$

(12*7)-(20*15)=-216

E. Inverse

Q5. Inverse

(i)

Find $\mathbf{A^{-1}}$

If A = $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

(3*2)-(1*4)=2

1/2 $\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & -0.5 \\ -2 & 1.5 \end{bmatrix}$

(ii) Determine $\mathbf{A^{-1}}$ and identify the element $\mathbf{a_{12}}$

If A = $\begin{bmatrix} 11 & 5 \\ 3 & 11 \end{bmatrix}$ *and* A = $\begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$

(11*11)-(5*3)=106

$1/106 \begin{bmatrix} 11 & -5 \\ -3 & 11 \end{bmatrix}$
$\begin{bmatrix} 11/106 & -5/106 \\ -3/106 & 11/106 \end{bmatrix}$
a12= -0.04716

E. [3 *3]

Q6. Inverse

$$\text{If } A = \begin{bmatrix} 3 & 1 & 2 \\ 4 & -1 & 1 \\ 2 & 1 & -2 \end{bmatrix}$$

(a)

Find (i) **|A|**

Write matrix here

$$\det = \begin{bmatrix} 3 & 1 & 2 \\ 4 & -1 & 1 \\ 2 & 1 & -2 \end{bmatrix}$$

$\begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix}$		$\begin{vmatrix} 4 & 1 \\ 2 & -2 \end{vmatrix}$		$\begin{vmatrix} 4 & -1 \\ 2 & 1 \end{vmatrix}$
3*((-1*-2) - (1*1))	-	1*((4*-2)-(1*2))	+	2*((4*1) -(-1*2))
3	-	-10	+	12
Write Solution det =	=25			

Find (ii) **Adj A**

Step 1: Break into Minors Matrix

$$\begin{bmatrix} 3 & 1 & 2 \\ 4 & -1 & 1 \\ 2 & 1 & -2 \end{bmatrix}$$

ignore the values on the current row and column

$$\begin{bmatrix} \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} & \begin{vmatrix} 4 & 1 \\ 2 & -2 \end{vmatrix} & \begin{vmatrix} 4 & -1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 1 & -2 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 2 & -2 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 4 & -1 \end{vmatrix} \end{bmatrix}$$

calculate the determinant of the remaining values

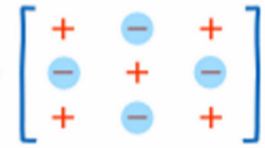
The determinant is defined as ***ad–bc***

Matrix of Minors

$$\begin{bmatrix} 1 & -10 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -10 & 6 \\ -4 & -10 & 1 \\ 3 & -5 & -7 \end{bmatrix}$$

Step 2: **Apply Signs**



Matrix of Minors	Co-Factors
$\begin{bmatrix} 1 & -10 & 6 \\ -4 & -10 & 1 \\ 3 & -5 & -7 \end{bmatrix}$	$\begin{bmatrix} 1 & 10 & 6 \\ 4 & -10 & -1 \\ 3 & 5 & -7 \end{bmatrix}$

Adjugate

Step 3: **Transpose** = **Adjugate**

Flip Row into columns

Co-Factors	Adjugate
$\begin{bmatrix} 1 & 10 & 6 \\ 4 & -10 & -1 \\ 3 & 5 & -7 \end{bmatrix}$	$\begin{bmatrix} 1 & 4 & 3 \\ 10 & -10 & 5 \\ 6 & -1 & -7 \end{bmatrix}$

Find (iii) $\mathbf{A^{-1}}$

Step 4: **Multiply by** $\frac{1}{Determinant}$

$$A^{-1} = \frac{1}{\det(A)} \times \text{adj}(A)$$

$$A^{-1} = \text{Adjugate} \times \frac{1}{\det 25} \begin{bmatrix} 1 & 4 & 3 \\ 10 & -10 & 5 \\ 6 & -1 & -7 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0.04 & 0.16 & 0.12 \\ 0.4 & -0.4 & 0.2 \\ 0.24 & -0.04 & -0.28 \end{bmatrix}$$

F. Solving Linear Equations

Q7. Solving Linear Equations

$$\text{If A} = \begin{bmatrix} 3 & 1 & 2 \\ 4 & -1 & 1 \\ 2 & 1 & -2 \end{bmatrix}$$

(no need to work out inverse again
use answer to question 6 (iii) for inverse)

(b) Use the **inverse matrix method** to solve the following system of equations for x, y and z.

$$\begin{aligned} 3x + y + 2z &= 15 \\ 4x - y + z &= 17 \\ 2x + y - 2z &= 14 \end{aligned}$$

Show Calculations

$$\frac{1}{\det 25}\begin{bmatrix} 1 & 4 & 3 \\ 10 & -10 & 5 \\ 6 & -1 & -7 \end{bmatrix}\begin{bmatrix} 15 \\ 17 \\ 14 \end{bmatrix}$$

(1*15)+(4*17)+(3*14)=125

(10*15)+(-10*17)+(5*14)=50

(6*15)+(-1*17)+(-7*14)=-25

$$\frac{1}{\det 25}\begin{bmatrix} 125 \\ 50 \\ -25 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$