DAA

21-1-22

Complexity Eq

Bubble Sort

void main ( )			
{			
int a[5];	C_1	1	1
cout<<"plz enter number	C_2	1	1
for(int i=0;i<5;i++)	C_3	6	n
{			
cin>>a[i];	C_4	5	n
}			
for(int i=0;i<5;i++){	C_5	6	n
for(int j=0;j<4;j++){	C_6	25	n
if(a[j]>a[j>1]){	C_7	20	n
int t=a[j];	C_8	0/20	0/
a[j]=a[j+1];	C_9	0/20	0/
a[j+1]=t;	C_{10}	0/20	0/
}			
}			
}			

Best Case

$C_1(1) + C_2(1) + C_3(n+1) + C_4(n) + C_5(n$
$C_6(n^2) + C_7(n^2 - n) + C_8(0) + C_9(0) + C_{10}(0)$

$C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = C_9 = C_{10} = 1$

$(1)(1) + (1)(1) + (1)(n+1) + 1(n) + (1)(n$
$+1(n^2) + (1)(n^2 - n) + (1)(0) + (1)(0) + (1)(0)$

$1 + 1 + n + 1 + n + \cancel{n} + 1 + n^2 + n^2 - \cancel{n}$

$4 + 2n + 2n^2$

$\boxed{2n^2 + 2n + 4}$

Worst Case

$C_1(1) + C_2(1) + C_3(n+1) + C_4(n) +$
$C_5(n+1) + C_6(n^2) + C_7(n^2 - n) + C_8(n^2 - n$
$C_9(n^2 - n) + C_{10}(n^2 - n)$

$C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = C_9 = C_{10} = 1$

DAA 21-1-22

Complexity Eq

Bubble Sort

```
void main( )
{
int a[5];                          C1    1     1
cout<<"plz enter number            C2    1     1
for(int i=0;i<5;i++)               C3    6     n
{
cin>>a[i];                         C4    5     r
}
for(int i=0;i<5;i++){              C5    6     n+
for(int j=0;j<4;j++){              C6    25    r
 if(a[j]>a[j>1]){                  C7    20    n
     int t=a[j];                   C8    0/20  0/
     a[j]=a[j+1];                  C9    0/20  0/
     a[j+1]=t;                     C10   0/20  0/
}
}
}
```

Best Case

$= C_1(1) + C_2(1) + C_3(n+1) + C_4(n) + C_5(n+$
$C_6(n^2) + C_7(n^2-n) + C_8(0) + C_9(0) + C_{10}(0$

$= C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = C_9 = C_{10} = 1$

$= (1)(1) + (1)(1) + (1)(n+1) + 1(n) + (1)(n+$
$+1(n^2) + (1)(n^2-n) + (1)(0) + (1)(0) + (1)(0)$

$= 1 + 1 + n + 1 + n + \cancel{n} + 1 + n^2 + n^2 - \cancel{n}$

$= 4 + 2n + 2n^2$

$= \boxed{2n^2 + 2n + 4}$

Worst Case

$= C_1(1) + C_2(1) + C_3(n+1) + C_4(n) +$
$C_5(n+1) + C_6(n^2) + C_7(n^2-n) + C_8(n^2-n$
$C_9(n^2-n) + C_{10}(n^2-n)$

$= C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = C_9 = C_{10} = 1$

$(1)(1)+(1)(1)+(1)(n+1)+(1)(n)+(1)($
$(1)(n^2)+(1)(n^2-n)+(1)(n^2-n)+(1)(n^2-n)$

$1+1+\cancel{n}+1+\cancel{n}+\cancel{n}+1+n^2+n^2-\cancel{n}+n^2-$
$n^2-\cancel{n}+n^2-\cancel{n}$

$4+5n^2-n$

$\boxed{5n^2-n+4}$

$n^2-n+2n-2$

n^2+n-2

N

100

25+

n^2+

Selection Sort

void main( ){			
int a[5];	C_1	1	1
cout<<"plz enter number:";	C_2	1	1
for(int i=0; i<5;i++){	C_3	n+1	6
cin>>a[i];	C_4	n	5
}			
for(int i=0; i<4;i++){	C_5	n	5
for(int j=i+1;j<5; j++){	C_6		14
if (a[i]>a[j]){	C_7		10
int t=a[j];	C_8		
a[j]=a[j+1];	C_9		
}} a[j+1]=t;}	C_{10}		

$$\frac{6n+6+n^2+n-2+n^2-n}{2}$$

$$\frac{{}^2+6n+4}{2}$$

$$\frac{1}{2}(n^2+3n+2)$$

First term $\quad$ 2nd term

$$\frac{n(a_1+a_n)}{2}$$

Formula

$$\boxed{\frac{n(n+1)-1}{2}} \quad \boxed{\frac{n(n+1)-n}{2}}$$

nula $\hookrightarrow$ $\boxed{\frac{n^2+n-2n}{2}}$

$$\frac{100+10-20}{2} = \frac{90}{2} \boxed{= 45}$$

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Insertion Sort

insertion sort (A)
for (int j=2; j n+1; j++){ n C
Key A[j] } n-1 C
// insert A[j] into A{1,..j-1} n-1 C
i=j-1; } n-1 C

while (i>0 and A[i]>key){ - - C
A[i+1] = A[i]; - C
i=i-1;} } - C
A [i+1]=Key (n-1) C
}

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```
for(int i=0; i<4; i++){
    for(int j=i+1; j<5; j++){
        if(a[i] < a[j]){
            int temp= a[i];
            a[i] = a[j];
            a[j] = temp;
        }
```

6 5

15 14

20 10

0/20

0/20

0/20

i	j
0 →	1,2,3,4,5(F)
1 →	2,3,4,5(F)
2 →	3,4,5(F)
3 →	4,5(F)
4 →	~~5(F)~~
$\boxed{5(F)}$	
6	

$n=5$

$$\frac{n}{2(a+L)} = \frac{5}{2(2+10)} = \frac{5}{24}$$

Array $\boxed{10}$

$$n\frac{(n+1}{2} \qquad 10\frac{(10+1)}{2} - 1$$

Bubble Sort

n ← for(int i=0;i<9;i++){

+1)-1 / 2 ← for(int j+i+1;j<10;j++){

n+1)-n / 2 ← if(a[i] < a[j]){

int temp = a[i];

a[i]= a[j];

a[j]=tamp;

}

Insertion Dgsun

```
int i,j,key;
for(int i=1; i<5; i++) {
        key=array[i];
         j=i-1;
  while ( j>=0 && array [j] >key) {
        array[j+1] = array[j]
           j=j-1;
       }

       arr [j+1]= key;
    }
}
```

5   n

5   n

5   n

15    $\frac{n(n+1)}{2}-1$

15    $\frac{n(n+1)}{2}-1$

4    $n-1$

4-2-22

Big O

$f(n) \leq c \cdot g(n)$

$n > 0, c > 0$

$n_0 > n$

$$\underbrace{n^2 + 2n + 1}_{f(n)} = \underbrace{O(n^2)}_{g(n)}$$

$n^2 + 2n + 1 \leq Cn^2$ — (1)

$$\frac{n^2 + 2n + 1}{n^2} \leq C$$

put $n = 1$

$$\frac{(1)^2 + 2(1) + 1}{(1)^2} \leq C$$

$\boxed{4 = C}$

put in eq (1)

$n^2 + 2n + 1 \leq 4n^2$

put $n = 1$

$(1)^2 + 2(1) + 1 \leq 4(1)^2$

$\boxed{4 \leq 4}$ (T)

put $n = 2$

$(2)^2 + 2(2) + 1 \leq 4(2)^2$

$4+4+1 \leq 8$

$\boxed{9 \leq 16}$ (T)

put $\cancel{n} = 4$

$(4)^2+2(4)+1 \leq 4(4)^2$

$16+8+1 \leq 4(16)$

$\boxed{25 \leq 64}$ (T)

put $n=10$

$(10)^2+2(10)+1 \leq 4(10)^2$

$100+20+1 \leq 4(100)$

$\boxed{121 \leq 400}$ (T)

put $n=15$

$(15)^2+2(15)+1 \leq 4(15)^2$

$225+30+1 < 4(225)$

$\boxed{256 \leq 900}$ (T)

$\boxed{\begin{array}{l} C=4 \\ n_0=1 \end{array}}$

$3n+7$ is $O(n)$

$$\underbrace{3n+7}_{f(n)} = \underbrace{O(n)}_{g(n)}$$

$$3n+7 \leq Cn$$

$$\frac{3n+7}{n} \leq C$$

put $n=1$

$$\frac{3(1)+7}{1} = C$$

$$\boxed{C = 10}$$

$$3n+7 \leq 10n$$

put $n=1$

$$3(1)+7 \leq 10(1)$$

$$3+7 \leq 10$$

$$\boxed{10 \leq 10} \; (T)$$

put $n=4$

$$3(4)+7 \leq 10(4)$$

$$12+7 \leq 40$$

$$\boxed{19 \leq 4} \; (T)$$

put ~~x~~ $n=6$

$3n+7 \leq 10(n)$

$3(6)+7 \leq 10(6)$

$18+7 \leq 60$

$\boxed{25 \leq 60}$ (T)

$n^2-2n+1 \neq O(n)$

$\underbrace{n^2-2n+1}_{f(n)} \neq \underbrace{O(n)}_{g(n)}$

$n^2-2n+1 \leq Cn$ — (i)

$\frac{n^2-2n+1}{n} \leq C$ — (ii)

put $n=1$

$\frac{(1)^2-2(1)+1}{1} \neq C$

$1-2+1 \neq C$

$\boxed{C=0}$

put $n=2$ in eq (ii)

$\frac{(2)^2-2(2)+1}{2} = C$

$\frac{4-4+1}{2} = C$

$\boxed{C=\frac{1}{2}}$

$n^2-2n+1 \leq c(n)$

put $c=0.5$

$n^2-2n+1 \leq 0.5(n)$

put $n=1$

$(1)^2-2(1)+1 \leq 0.5(1)$

$1-2+1 \leq 0.5$

$\boxed{0 \leq 0.5}$ (F)

put $n=3$

$(3)^2-2(3)+1 \leq 0.5(3)$

$9-6+1 \leq 1.5$

$\boxed{4 \leq 1.5}$ (F)

put $n=7$

$(7)^2-2(7)+1 \leq 0.5(7)$

$49-14+1 \leq 3.5$

$\boxed{36 \leq 3.5}$ (F)

n^2-2n+1 is not $O(n)$

$f(n)$ is not $g(n)$

Omega

$\boxed{c.g(n) \le f(n)}$

$\underbrace{n^3+4n^2}_{f(n)} = \Omega(\underbrace{n^2}_{g(n)})$

$c\,\cancel{(n)}\,n^2 \le n^3+4n^2$

$c \le \frac{n^3+4n^2}{n^2}$

$c \le \frac{n^3}{n^2} + \frac{4n^2}{n^2}$

$c = n+4$

put $n=1$

$\boxed{c=5}$

$5n^2 \le n^3+4n^2$

put $n=2$

$5(2)^2 \le (2)^3+4(2)^2$

$5(4) \le 8+4(4)$

$20 \le 8+16$

$\boxed{20 \le 24}$ (T)

put $n=5$

$5(5)^2 \le (5)^3+4(5)^2$

$5(25) \le 125+4(25)$

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$\boxed{125 \leq 225}$ (T)

put $n=8$

$5(8)^2 \leq (8)^3 + 4(8)^2$

$5(64) \leq 512 + 4(64)$

$320 \leq 512 + 256$

$\boxed{320 \leq 768}$ (T)

$\boxed{C=5,\ n_0=1}$

Theata

~~Sigma~~ $c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n)$

$$\underbrace{n^2+5n+7}_{f(n)} = \Theta\ \underbrace{n^2}_{g(n)}$$

$$c_1 \cdot n^2 \le n^2+5n+7 \le c_2 \cdot n^2$$

$c_1 n^2 \le n^2+5n+7$ | $n^2+5n+7 \le c_2 \cdot n^2$

$c_1 \le \frac{n^2}{n^2} + \frac{5n}{n^2} + \frac{7}{n^2}$

$\le 1 + \frac{5}{n} + \frac{7}{n^2}$

$\boxed{c = 1}$

$n^2 \le n^2+5n+7$

put $n=1$

$(1)^2 \le (1)$

put $n=2$

$4 \le 4+10+7$

$4 \le 21$

put $n = 4$

$16 \le 16+20+7$

$16 \le 43$

$\frac{n^2+5n+7}{n^2} \le c_2$

$c_2 \ge \frac{n^2+5n+7}{n^2}$

put $n=1$

$\boxed{c_2 = 13}$

$n^2+5n+7 \le 13n^2$

put $n=1$

$(1)^2+5(1)+7 \le 13(1)^2$

$\boxed{13 \le 13}$ (T)

put $n=3$

$(3)^2+5(3)+7 \le 13(3)$

$\boxed{31 \le 117}$ (T)

put $n=5$

$(5)^2+5(5)+7 \le 13(5)^2$

$\boxed{57 \le 325}$ (T)

$\boxed{c_1 = 1, c_2 = 13, n = 1}$

Master Theorem

These are the different master theorem:

If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$

If $f(n) = \Theta(n^{\log_b a})$, then ~~T(n)~~=
$T(n) = \Theta(n^{\log_b a} \lg n)$.

If $f(n) = \Omega(n^{\log_b a + \epsilon})$ from some constant $\epsilon > 0$, and if $af(n/b) \leq cf$ for some constant $c < 1$ and all sufficiently large n, then
$T(n) = \Theta(f(n))$

$$T(n) = \underbrace{2}_{a}\,T\Big(\frac{n}{\underbrace{2}_{b}}\Big) + \underbrace{\sqrt{n}}_{f(n)}$$

1) $T(n) = 2T\left(\frac{n}{2}\right) + \sqrt{n}$

$n^{\log_b a} = n^{\log_2 2} = n^1 \boxed{= n}$

Ans Compare with $f(n)$

$n > \sqrt{n}$

Then $T(n) = \Theta\left(n^{\log_b a}\right)$

$= \Theta\left(n^{\log_2 2}\right)$

$\boxed{= \Theta(n)}$

$T(n) = 2T(n/2) + n$

Here $a = 2$, $b = 2$, $f(n) = n$

$= n^{\log_b a} = n^{\log_2 2} = n^1 \boxed{= n}$

Ans compare with $f(n)$

$n = n$

Then $T(n) = \Theta\left(n^{\log_b a} \lg n\right)$

$= \Theta\left(n^{\log_2 2} \lg n\right)$

$\boxed{= (n \lg n)}$

$T(n) = 2T(n/2) + n^2$

$a = 2,\ b = 2,\ f(n) = n^2$

$= n^{\log_2 2} = n^1 = n$

Ans compare with $f(n)$

then $T(n) = \Theta\, f(n)$

$= \Theta\, n^2$

D)

```
for(int i=0; i<n; i++){      -> n+1
    cout<<i;                 -> n
}

for(int i=0; i<=n; i++){     -> n+2
    cout<<i;                 -> n+1
}

for(int i=1 ; i<=n; i*2){    -> log_2 n
    cout<<i;
}
```

$\log_2(n) = \log_2 2^k$

$\log_2 n = K(1)$

$\boxed{\log_2 n = K}$

```
for(int i=1; i<=32; i*2){    -> 2K
    cout<<i;
}

for(int i=n; i>1; i=i/2){    logn
    cout<<i;
}
```

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Substitution

$T(n) = T(n-1) + 1$

$= \{T(n-2) + 1\} + 1$

$= T(n-2) + 2$

$= T(n-3) + 3$

$= T(n-4) + 4$

$\vdots \quad \vdots \quad \vdots$

$T(n-n) + n$

$T(0) + n$

$1 + n$

$\boxed{O(n)}$

$T(n) = T(n-1) + n$

put $n = n-1$

$T(n-1) = \{T(n-1-1) + n-1\}$

$T(n-2) = \{T(n-2) + n-1\}$

put $n = n-1$

$T(n-3) = \{T(n-1-2) + n-1-1\}$

$= \{T(n-3) + n-2\}$

put $n-1$

$T(n-4) = \{T(n-3-1) + n-1-2$

$\boxed{\{T(n-4) + n-3\}}$

$T(n) = T(n-1) + n$

$= \{T(n-2) + (n-1)\} + n$

$= T(n-2) + (n-1) + n$

$= \{T(n-3) + (n-2)\} + (n-1) + n$

$= T(n-3) + (n-2) + (n-1) + n$

$= T(n-n) + (n-(n-1)) + 2 + 3 + \dots + (n-3)$

$(n-2) + (n-1) + n$

$= 1 + 2 + 3 + 4 + 5 + \dots + (n)$

$\boxed{O(n^2)}$ $\Leftarrow \frac{n(n+1)}{2}$

$T(n) = T(n-1) + \lg n$

$T(n-1) = T(n-2) + \log(n-1) + \log n$

$= T(n-3) + \log(n-2) + \log(n-1) + \log($

$= T(n-n) + \log(n-(n-1)) + \log(n-(n-2)) +$

$- - - - - + \log(n-3) + \log(n-2) + \log(n-1) +$

$T(0) + \log(1) + \log(2) + \log(3) + \dots +$

$(n-1)(n)$

$\boxed{\log(n!) = n \log n}$