after mids course ↓

Engineering Economics Notes

Chapter # 3: Principles of Money-Time Relationships.

3.1 Introduction:

→ Capital refers to wealth in form of money or property that is capable of being used to produce more wealth.

→ Money has a time value.

3.2 Why Consider return to Capital:

→ Equity Capital is that wealth owned by the individuals that are invested in a business or project or venture in the hope of profit.

→ Debt/Borrowed Capital is that money or wealth borrowed from others that are invested in a business or project or venture in hope of profit (intrest).

→ Interest and profit are payments for the risk the invester takes in permitting another person, or an organisation to use his or her capital.

3.3 The origins of interest:

→ Idea of interest became so well established that a firm of international banker existed in 570 B.C, with home officies in Babylon.

3.4 Simple Interest:

→ When total interest earned or changed is directly proportional to the initial amount of loan (principal), the interest rate and the number of interest periods for which the ~~interest~~ principal is commited, the interest and interest rate are said to be simple.

→ The total Interest, I, earned or paid may be computed in the formula:

$$I = (P)(N)(i)$$

where,

P = principal amount lent or borrowed

N = number of interest periods

i = interest rate per interest period.

F = Total amount accumulated at end of period N. = $P \times (1 + (N)(i))$

→ Simple Interest is not used frequently in commercial practices in moderen times.

→ Simple interest is a linear function of time.

3.5 Compound Interest:

→ Whenever the interest charge for any interest period is based on the remaining principal amount plus any accumulated interest charges up to the begining of that period, the interest is said to be Compound.

→ The difference is due to compounding, which essentially is the calculation of interest on previously earned interest.

→ Compound interest is much more common and used frequently in moderen times.

$$F = P(1+i)^N$$

3.6 The concept of Equivalence:

→ equivalence is established when total interest paid, divided by dollar-years of borrowing, is a constant ratio among financing plans (i.e alternatives).

3.7 Notations and Cash Flow Diagrams / Tables.

→ i = effective interest rate per interest period.

N = number of compounding periods

P = present sum of money

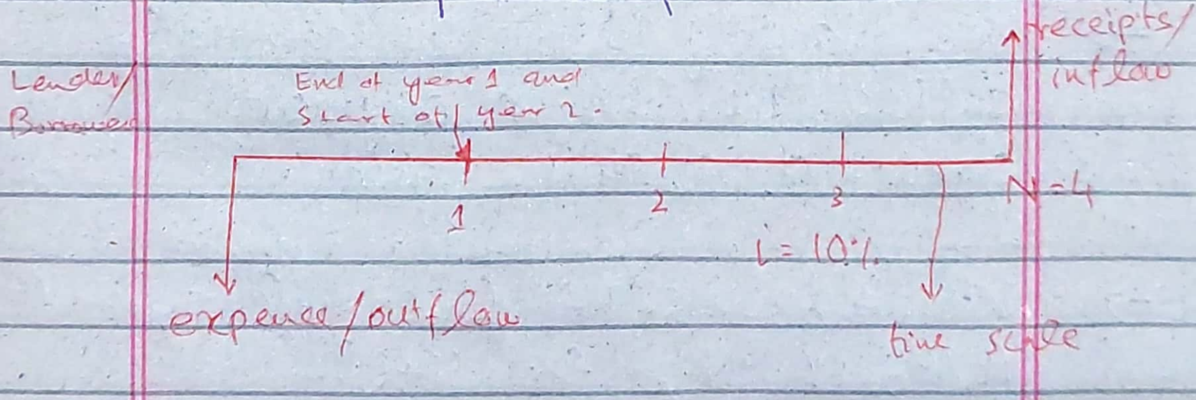
F = Future sum of money

A = end-of-period cash flows in a uniform series continuing for a specified number of periods, starting at the end of first period and continuing through the last period.

→ The difference between total cash inflows (receipts) and cash outflows (expenditures) for a specified period of time is the net cash flow for the period.

→ Cash flow diagram employs several conventions.

- The horizontal line is a time-scale, with progression of time moving from left to right.
- The arrows signifies cash flows and are placed at the end of period. Downward arrows represent expense (outflows / negative) and upward arrows represent receipts (inflows / positive).
- Cash flow diagram is dependent on the point of view.



3.8 Interest formulas relating Present and future Equivalent values of single cash flows:

3.8.1 Finding F when Given P:

→ If an amount of P dollars exists at a point in time and i% is

⑥

the interest (profit/growth) rate per period, the amount will grow to future amount of $F = P(1+i)^N$ by the end of N periods. i.e

$$F = P(1+i)^N.$$

*E Example 3.3:

$P = 8000\$$, $i = 10\%$ \$/year, $N = 4$ years.

Salution:

$F = (8000)(0.1+1)^4 = 8000(1.1)^4 = 11730$

→ The quantity $(1+i)^N$ is commonly called the 'single payment compound amount factor'.

→ We also use this functional symbal $(F/P, i\%, N)$ for $(1+i)^N$.

Hence the above formula can be

$$F = P\underbrace{(F/P, i\%, N)}.$$

→ Can be read as "find F given P at i% interest per period for N interest period.

→ This equation can be interpreted as that the calculated amount, F, at the point in time at which it occurs, is equivalent to (i.e can be traded for) the known value P, at the point in time at which it occurs, for given interest or profit rate i.

3.8.2 Finding P when Given F.

→ As $F = P(1+i)^N$

$$\Rightarrow P = \frac{F}{(1+i)^N} = F(1+i)^{-N}$$

→ The quantity $(1+i)^{-N}$ is called single payment present worth factor. We can use this symbol for this factor: $(P/F, i\%, N)$. hence:

$$P = F\,(P/F,\ i\%,\ N).$$

*E Example 3.4:

$F = 10000\,\$$, $i = 8\%$, $N = 6$ years.

$P = ?$

Sol: $P = 10000(1+0.08)^{-6} = 6301.696$

$= 6302\,\$.$

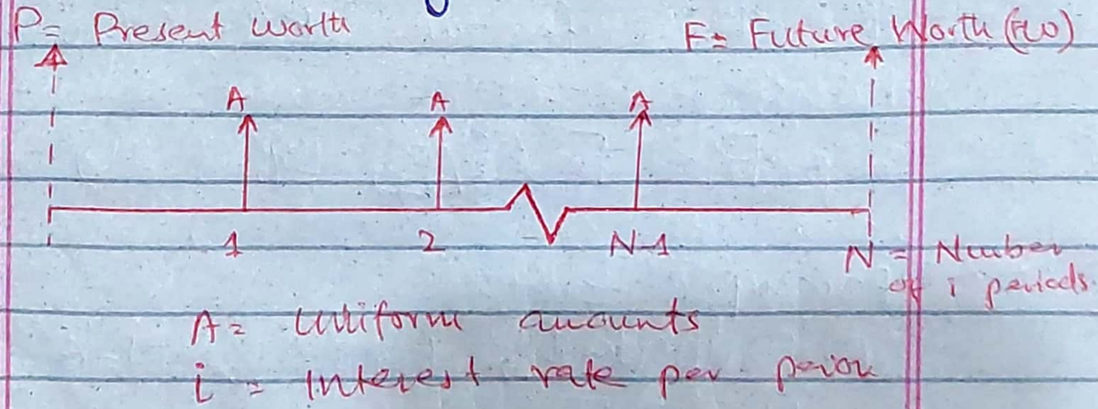
Also solving from tabel

$P = F(P/F, 8\%, 6)$

$= 10000\ (0.6302) = 6302\ \$$

3.9 Interest formulas relating a uniform series (Annuity) to its present and future equivalent value:

→ Below figure shows a general cash flow diagram of uniform receipts, each of amount A, occuring at the end of each period for N periods with interest at i% per period. Such a unifor series is often called annuity.



→ P (present worth, PW) occur one interest period before the first

* sum of N terms is geometric sequence:

$$S_N = \frac{a_1 - b a_N}{1-b} \quad \because b \neq 1.$$

A (uniform payment).

→ F (future worth, FW) accurs at the same time as the last A, and N periods after P.

→ A (annual worth, AW) occurs at the end of periods 1 through N, inclusive

3.9.1 Finding F when given A.

→ A cash flow that occur in the amount of A dollars at the end of each period for N period and i% is the interest rate per period, the FW at end of N period is:

$$FW = A(F/P, i\%, N-1) + A(P/F, i\%, N-2) + A(F/P, i\%, N-3) + \cdots + A(P/F, i\%, 0)$$

$$= A\left[(1+i)^{N-1} + (1+i)^{N-2} + \cdots + (1+i)^1 + (1+i)^0\right]$$

this shows a geometric series havin common ration $r = (1+i)^{N-1}/(1+i)^{N-1} = (1+i)^{-1}$ hence its sum is

$$F = A\left[\frac{(1+i)^{N-1} - \frac{1}{(1+i)}(1+i)^0}{1 - \frac{1}{(1+i)}}\right]$$

$$= A\left[\frac{(1+i)^N - 1}{i}\right]$$

→ The quantity $\{[(1+i)^N - 1]/i\}$ is called the "uniform series compound amount factor". We shall use the symbol (F/A, i%, N) for this factor.
hence

$$F = A\,(F/A,\ i\%,\ N).$$

*E Example 3.5:

F = 100'000 \$, i = 6% , N = 25
A = ?

Sol: $A = F\,(A/F,\ i\%,\ N)$
$= 100'000\,(A/F,\ 6\%,\ 25)$
$= 100'000\,(0.0182)$
$= 1820$ \$

3.9.2 Finding P when given A:

→ As we know $F = P(1+i)^N$
and also

$$F = A\left[\frac{(1+i)^N - 1}{i}\right]$$

$$\Rightarrow P(1+i)^N = A\left[\frac{(1+i)^N - 1}{i}\right]$$

$$P = A\left[\frac{(1+i)^N - 1}{i\,(1+i)^N}\right]$$

→ This equation is the relation for finding PW of a uniform series of end-of-period cash flows of amount A for N periods.

→ The quantity in brackets is called the "uniform series present worth factor". We shall use the functional symbol (P/A, i%, N) for this factor hence

$$P = A\,(P/A;\ i\%,\ N).$$

3.9.3 Finding A when given F.

→ As $F = A\left[\frac{(1+i)^N - 1}{i}\right]$

$$A = F\left[\frac{i}{(1+i)^N - 1}\right]$$

→ A relation for finding the amount A, of a uniform series of cash flows occurring at the end of N interest periods that would be equivalent to (have the same value as ~~to~~ its future worth F.

occuring at the end of the last period.

→ The quantity in the brackets is called "sinking fund factor". We shall use the functional symbol $(A/F, i\%, N)$ for this factor, hence:

$$A = F(A/F, i\%, N).$$

3.9.4 Finding A when given P:

→ As $P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$

$\Rightarrow \quad Pi(1+i)^N = A[(1+i)^N - 1]$

$\Rightarrow \quad A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$

→ It is the relationship for finding the amount A, of a uniform series of cash flows occuring at the end of each N interest periods that would be equivalent to, or could be traded for, the present worth P, occuring at the begining of the first period.

→ The quantity in brackets is called the "capital recovery factor". we shall use the functional symbol (A/P, i%, N) hence

$$A = P(A/P, i\%, N).$$

→ The capital recovery factor can be simplified to

$$\frac{i(1+i)^N}{(1+i)^N - 1} = \frac{i}{(1+i)^{-N}\left[(1+i)^N - 1\right]}$$

$$= \frac{i}{1-(1+i)^{-N}}.$$

3.9.5 Interest factor relationships:

→ $(A/P, i\%, N) = 1/(P/A, i\%, N)$

→ $(A/F, i\%, N) = 1/(F/A, i\%, N)$

→ $(F/A, i\%, N) = (P/A, i\%, N)\ (F/P, i\%, N)$

→ $(P/A, i\%, N) = \sum_{K=1}^{N} (P/F, i\%, K)$

→ $(F/A, i\%, N) = \sum_{K=1}^{N} (F/P, i\%, N-K)$

→ $(A/F, i\%, N) = (A/P, i\%, N) - i$

3.10 → Interest formulas for discrete compounding and discrete cash flows:

#	To find	Given	Factor	Factor Name	Symbol
	For single cash flows:				
1	F	P	$(1+i)^N$	Single Payment compound amount	$(F/P, i\%, N)$
2	P	F	$\frac{1}{(1+i)^N}$	single payment present worth	$(P/F, i\%, N)$
	For uniform series annuities:				
3	F	A	$[(1+i)^N - 1]/i$	uniform series compound amount	$(F/A, i\%, N)$
4	P	A	$\frac{(1+i)^N - 1}{i(1+i)^N}$	uniform series present worth	$(P/A, i\%, N)$
5	A	F	$\frac{i}{(1+i)^N - 1}$	sinking fund	$(A/F, i\%, N)$
6	A	P	$\frac{i(1+i)^N}{(1+i)^N - 1}$	Capital recovery	$(A/P, i\%, N)$

→ Above table is a summary of discrete compound interest factor.

→ Discrete compounding mean that the interest is compound at the end of each finite length period (month, year).

→ These formula assume that i remains constant during the N compounding periods and also cash flows are spaced at equal interval of time.

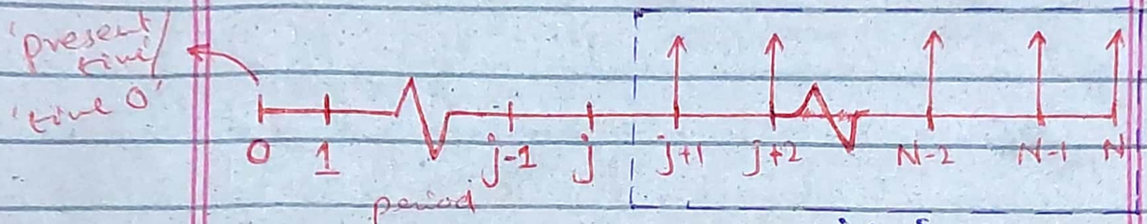
3.11 Deffered Annuities (Uniform Series):

→ When the first cash flow is being made at the end of the first period, they are called ordinary annuities.

→ If the cash flow does't begin untill some later date,

(16)

the annuity is known as, a deferred annuity.



$i = \%$

→ The present worth at the end of period J of an annuity with cash flows of amount A is from equation $A(P/A, i\%, N-J)$.

→ The present worth of the single amount $A(P/A, i\%, N-J)$ as of time 0 will then be

$$A(P/A, i\%, N-J)(P/F, i\%, J).$$

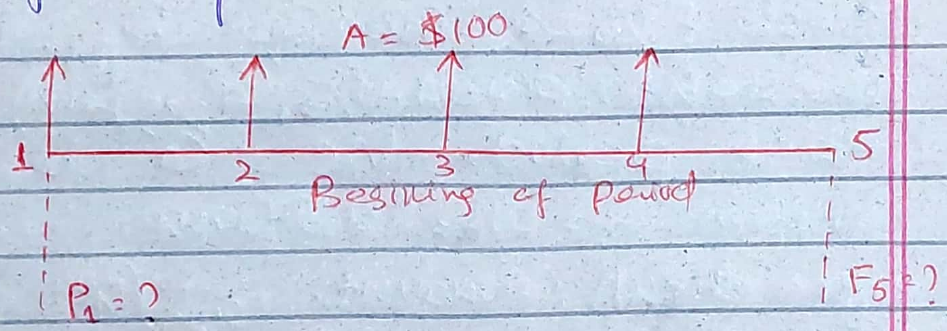
3.12 Uniform series with beginning-of-period cash flow:

→ Begining-of-period cash flows exists merely by remembering that

- P occurs one interest peiod before the first A.
- F occurs at the same time

as the last A and N periods after P.

→ On the begining ~~cash~~ of-period cash flow diagram, all cash flows that occur during a time period are placed at the point designated as the begining of the period.



3.13 Equivalence Calculations involving multiple interest formulas:

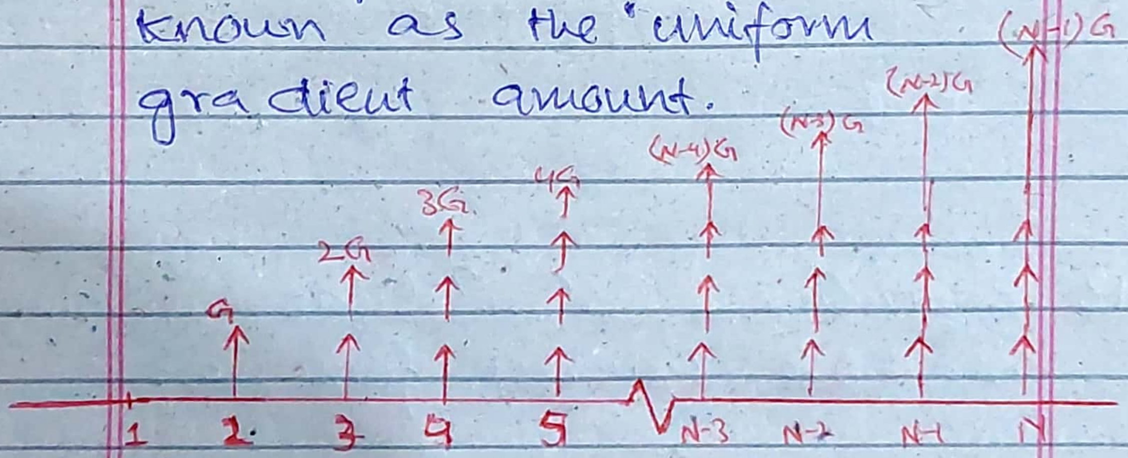
→ See example 3.10 and 3.11 in book to understand this topic.

3.14 Interest formulas relating a uniform Gradient of cash flows to its annual & Present worths:

18

→ Some economic analysis involve receipt → and expenses that are projected to increase or decrease by a uniform amount each time/period, thus constituting arithmetic sequence of cash flows.

→ Below cash flow diagram of sequence of end-of-period cash flows increase by a constant amount, 'G', in each period. The 'G' is known as the 'uniform gradient amount.



3.14.1 Finding F when give G

→ As future worth, F, of arithmetic sequence can be shown as

$$F = G(F/A, i\%, N-1) + G(F/A, i\%, N-2) + \cdots$$

$$\cdots + G(F/A, i\%, 2) + G(F/A, i\%, 1)$$

$$F = G\left[(F/A, i\%, N-1) + \cdots + (F/A, i\%, 1)\right]$$

$$F = G\left[\frac{(1+i)^{N-1}-1}{i} + \cdots + \frac{(1+i)^{1}-1}{i}\right]$$

$$= \frac{G}{i}\left[(1+i)^{N-1} - 1 + (1+i)^{N-2} - 1 + \cdots + (1+i)^{1} - 1\right]$$

as -1 occurs $(N-1)$ times by adding 1 then it occurs N times hence

$$= \frac{G}{i}\left[(1+i)^{N-1} + (1+i)^{N-2} + \cdots + (1+i)^{1} + 1\right]$$

$$= \frac{G}{i}\left[(1+i)^{N-1} + \cdots + (1+i)^{1} + (1+i)^{0}\right] - \frac{NG}{i}$$

$$= \frac{G}{i}\left[\sum_{k=0}^{N-1}(1+i)^{k}\right] - \frac{NG}{i}$$

Hence

$$F = \frac{G}{i}\left[\sum_{k=0}^{N-1}(1+i)^{k}\right] - \frac{NG}{i}$$

$$F = \frac{G}{i}(F/A, i\%, N) - \frac{NG}{i}$$

3.14.2 Finding A when given G.

→ As we know that from page 11

$$A = F(A/F, i\%, N) = F\left[\frac{i}{(1+i)^N - 1}\right]$$

put F from above formula

hence

$$A = \left\{\frac{G}{i}\left[\sum_{k=0}^{N-1}(1+i)^k\right] - \frac{NG}{i}\right\}\left[\frac{i}{(1+i)^N - 1}\right]$$

$$= \left\{\frac{G}{i}\left[\cancel{\sum_{k=0}^{N-1}(1+i)^k}\right]\frac{i}{\cancel{(1+i)^N - 1}}\right\} - \frac{NG}{i}\left[\frac{i}{(1+i)^N - 1}\right]$$

$$= \frac{G}{i} - \frac{NG}{i}\left[\frac{i}{(1+i)^N - 1}\right]$$

$$A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$

→ The term in brackets is called "gradient to uniform series conversion factor". We shall use this symbol for this factor (A/G, i, N)

hence

$$A = G(A/G, i\%, N).$$

3.14.3 Finding P When given G.

→ As we know $P = A\,(P/A, i, N)$

$$P = G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

$$= G\left[\frac{(1+i)^N - 1 - Ni}{i^2 (1+i)^N}\right]$$

$$P = G\left\{\frac{1}{i}\left[\frac{(1+i)^N - 1}{i(1+i)^N} - \frac{N}{(1+i)^N}\right]\right\}$$

→ The term in braces is called "gradient to present worth conversion factor". It can also be expressed as: $(1/i)\,[(P/A, i, N) - N(P/F, i, N)]$

→ We shall use the functional symbal $(P/G, i\%, N)$. hence

$$P = G\,(P/G, i\%, N).$$

3.14.4 Computations Using G

→ Be sure that the direct usage of gradient conversion factor applies when there is no cash flow at the end of period 1.

*E Example 3.12

$N_2 = \$1K$, $N_3 = \$2K$, $N_4 = \$3K$

$i = 15\%$, $PW_0 = ?$, $AW_4 = ?$

Solution

a) $P_0 = G(P/G, 15\%, 4)$

$= 1000(3.79) = 3790\ \$$

b) $A = G(A/G, 15\%, 4)$

$= 1000(1.3263) = \$1326.30$

also $A = P_0 (A/P, 15\%, 4)$

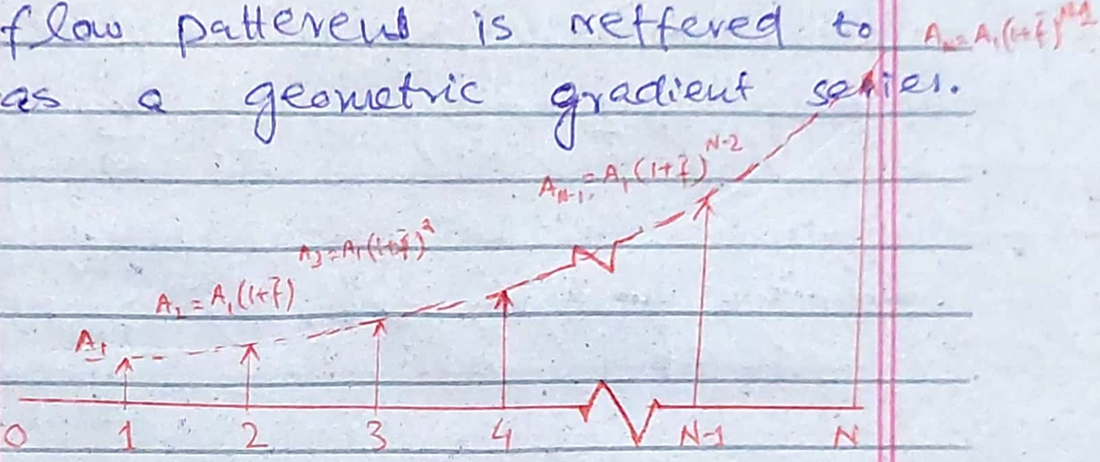
$= 3790(0.3503) = \$1326.30$

3.15 Interest formulas relating a geometric sequence of cash flow to its present and annual worths.

→ A fixed amount of commodity that inflates in price at a constant rate each year is a typical situation that can be model with geometric sequence.

→ The cash flow patterns increase at an average rate '$\bar{f}$', each period.

→ The resultant end-of-period cash flow pattern is refferred to as a geometric gradient series.



→ In this series; the initial cash flow, A_1, occur at end of period 1 and the k^{th} term is $A_K = A_1(1+\bar{f})^{K-1}$ wher $2 \le K \le N$. and n^{th} term is $A_N = A_1(1+\bar{f})^{N-1}$ with common ratio $\bar{f} = [(A_K - A_{K-1})/(A_{K-1})]$

→ A compact expression for P at interest rate i per period is:

$$P = \sum_{K=1}^{N} A_K(1+i)^{-K}$$

$$= \sum_{K=1}^{N} A_1(1+\bar{f})^{K-1} \cdot (1+i)^{-K}$$

$$P = \frac{A_1}{1+\bar{f}} \sum_{K=1}^{N} \left[\frac{1+\bar{f}}{1+i}\right]^K$$

→ When $i \neq \bar{f}$; a 'convenience rate' i_{CR} is defined as

$$i_{CR} = \frac{1+i}{1+\bar{f}} - 1 = \frac{i-\bar{f}}{1+\bar{f}}$$

Thus the above equation becomes

$$P = \frac{A_1}{1+\bar{f}} \sum_{K=1}^{N} \left(\frac{1+i}{1+\bar{f}}\right)^{-K} = \frac{A_1}{1+\bar{f}} \sum_{K=1}^{N} (1+i_{CR})^{-K}$$

$$P = \frac{A_1}{1+\bar{f}} (P/A, i_{CR}\%, N)$$

$\bar{f} > i \Rightarrow i_{CR} = -\text{ve}$

only valid when $(N < \infty)$ N is finite.

13 Jan 2024 / Sa.

Continuation Sheet

UNIVERSITY OF ENGINEERING & TECHNOLOGY, PESHAWAR

Serial No. 11343

Reg. / Roll No.

Engineering Economics chapter # 3
Problems by Aimal Khan in sequence
$5n + 5, \; 0 \leq n \leq 21.$

n=0

Problem # 3.5:

Sol: This problem is continuation of P # 3.4.

$P = \$ 2'000$, $i = 10\%$, $N = 6$ years

The total lump-sum amount will be

$$F = P(1+i)^N = 2'000 (1.1)^6 = \$ 3'543.12$$

n=1

Problem # 3.10:

$P = \$ 1'500$, $N = 8$ years, $i = 12\%$

$F = ?$

2

Solution:

$$F = P(F/P, i\%, N) = 1500(F/P, \overset{2.476}{\uparrow}12\%, 8)$$

$$= 1500(1.12)^8 = \$3713.945$$

Problem # 3.15:

Continuation Sheet
UNIVERSITY OF ENGINEERING & TECHNOLOGY, PESHAWAR
Serial No. 11344
Reg. / Roll No

n= 3

Problem # 3.20: A proposed product ....

$P = \$14'000$, $N = 4$ years , $i = 10\%$

$A = ?$

Sol:

$A = P(A/P, i\%, N)$

$= 14'000\ (A/P, 10\%, 4)$

$= 14'000\ (0.3155)$

$= 4417\ \$$

$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$

$= 14'000\left[\frac{0.1 \times 1.1^4}{1.1^4 - 1}\right]$

$= 14'000\ (0.3154)$

$= 4416.59\ \$$.

n= 4

Problem # 3.25: What must be the prospec.....

$P = \$2'250$, $N = 6$ years., $i = 12\%$

Sol:

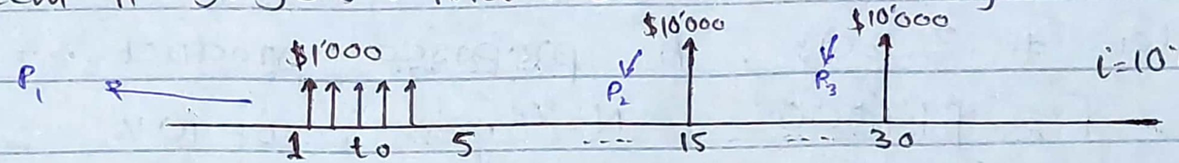
$F = P(F/P, i, N) = 2250\ (F/P, 12\%, 6)$

$= 2250\ (1.974) = 4441.5\ \$$

4

n= 5:

Problem # 3.30: . Maintenance cost for a new ....



Solution: First finding present values of these components

$$P_1 = A\,(P/A,\, i,\, N) = 1'000\,(P/A,\, 10\%,\, 5)$$

$$= 1'000\,(3.791) = 3'791\ \$$$

$$P_2 = F\,(P/F,\, i,\, N) = 10'000\,(P/F,\, 10\%,\, 15)$$

$$= 10'000\,(0.2394) = \cancel{10'000}\ 2394\ \$$$

$$P_3 = 10'000\,(1+i)^{-N} = 10'000\,(1.1)^{-30} = 573.083\ \$$$

Now the total present maintenance cost will be

$$P = P_1 + P_2 + P_3 = 3791 + 2394 + 573$$

$P = 6758\ \$$.

Now to find EUAC over the entire period.

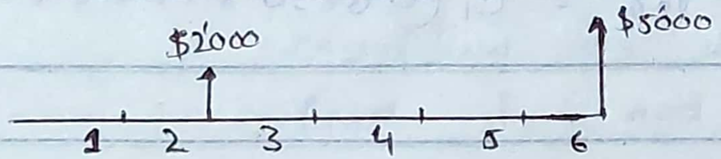
$$A = P\,(A/P,\ i\%,\ N)$$
$$= 6758\,(A/P,\ 10\%,\ 50)$$
$$= 6758 \times 0.1009$$
$$= 681.882\ \$$$

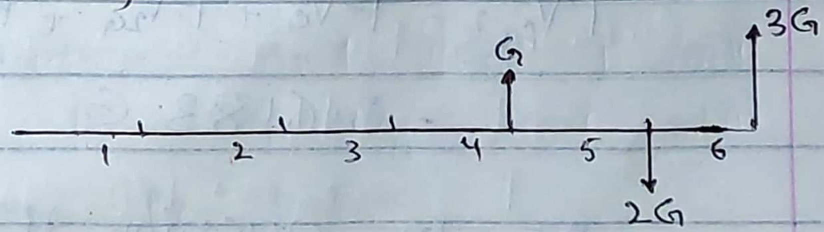
(or)

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$
$$A = 6758\left[\frac{0.1 \times 1.1^{50}}{1.1^{50} - 1}\right]$$
$$A = 6758 \times 0.1008591$$
$$= 681.606\ \$$$

n=6

Problem # 3.35 : Solve for the value of G in ...





Solution: First we have to calculate the present or future value of these cash flows.

(6)

For figure 1st converting the values to PV_0.

$PV_2 = 2000\,(P/F, 10\%, 2) = 2000(0.8264) = 1652.8$

$PV_6 = 5000\,(P/F, 10\%, 6) = 5000(0.5645) = 2822.5$

$PV_{fig1} = PV_2 + PV_6 = 1652.8 + 2822.5 = 4475.3$.

Now same procedure for figure 2 we get

$PV_G = G\,(P/F, 10, 4) = G(0.6830) = 0.6830\,G$

$PV_{2G} = 2G\,(P/F, 10, 5) = 2G(0.6209) = 1.2418\,G$

$PV_{3G} = 3G\,(P/F, 10, 6) = 3G\,(0.5645) = 1.6935\,G$

$PV_{fig\,2} = PV_G + PV_{2G} + PV_{3G} = G\,(0.6830 + 1.2418 + 1.6935)$

$= 3.6183\,G$

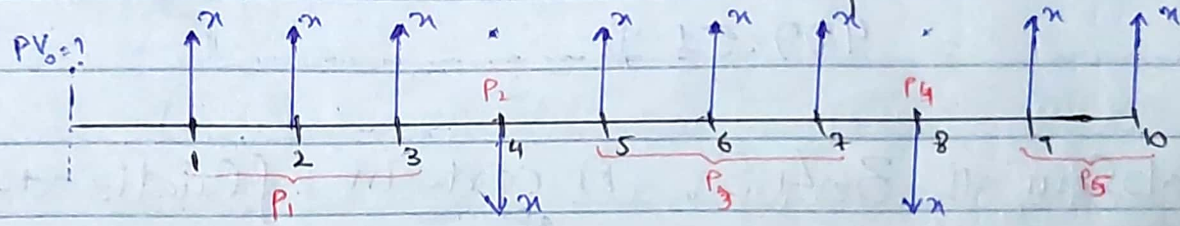
As in question it is shown that both figure will be same, hence

$PV_{fig1} = PV_{fig2}$

$$4475.3 = 3.6183\ G$$

$$G = \frac{4475.3}{3.6183} = \$\,1236.85156.$$

n=7 Problem # 3.40: Determine the present equivalent...



$i = 10\%$

$x = 100\$$

Solution: As simply I can calculate 5 different factors to find PV of given flow. to minimize it further. I will treat The overall cash flow as uniform and then I will find P_2 and P_4 seperatly and will subtract it from the value

$$P_{uniform} = A(P/A, 10\%, N) = 100(P/A, 10\%, 10)$$
$$= 100(6.145) = 614.5\,\$$$

8

Now P_2 and P_4 are outflows so

$P_2 = 100(P/F, 10, 4) = 100(0.6834) = 68.34$

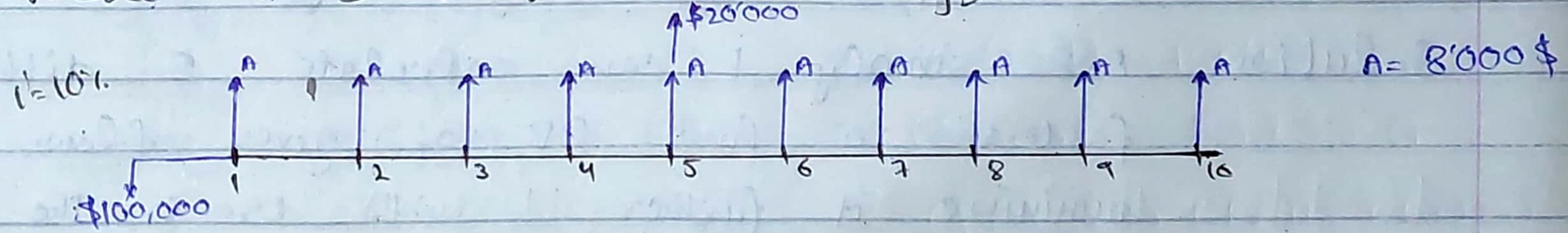
$P_4 = 100(P/F, 10, 8) = 100(0.4665) = 46.65$

$P_0 = P_{uniform} - (P_2 + P_4) = 614.5 - (68.34 + 46.65)$

$= 499.51\ \$$

n=8

Problem # 3.45: A certain fludized-bed com...



Solr Salution, First finding the PW of all costs. hence

$P_0 = \$100'000$

$P_{annual} = 8000(P/A, 15, 10) = 8000 \times 6.145 = 49160\ \$$

$P_{relining} = 20'000 \times 1.15^{-5} = 9943.$

The lump-sum equivalent cost is

$= P_0 - P_{annual} - P_{relining}$

$= 100000 - 49160 - 9943$

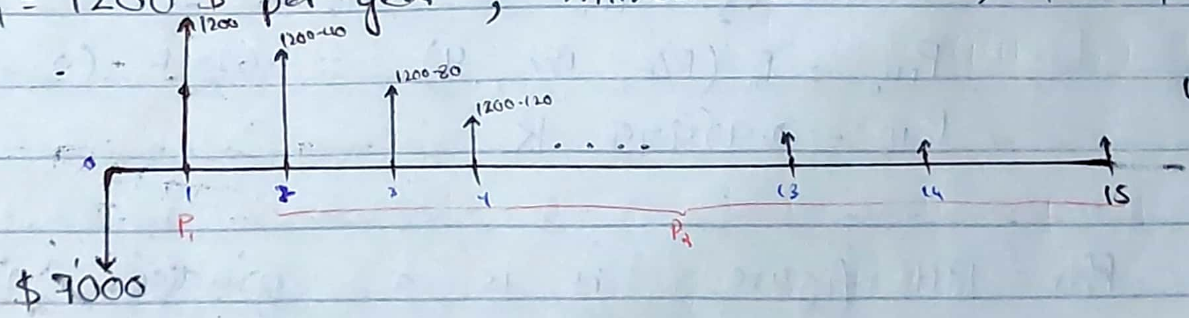
$= 40897$ \$

$n = 8$

Problem # 3.50: Suppose that annual income ....

Initial invesment, $I = \$7000$, $i = 10\%$ per year, $N = 15$ years

$A_1 = 1200$ \$ per year, Annual decrease, $d = \$40$.



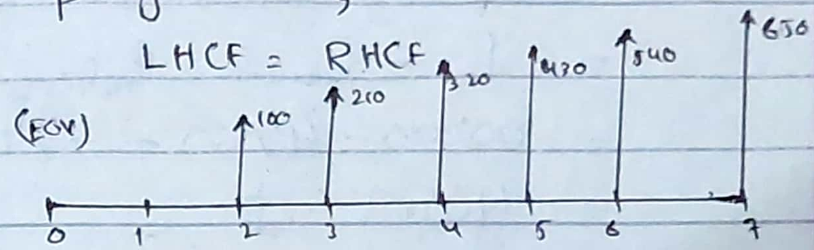
n:10

Problem # 3.55: On page 123, what is the ...

$i = 15\%$, $K = ?$

LHCF = RHCF

⇔



Sol: First converting it to PW or FW.

For LH figure.

$$P_{LH_4} = A\,(P/A,\ 15\%,\ 2) = K\,(1.626) = 1.626\,K$$

Now, converting P_{LH_4} to P_{LH_0}

$$P_{LH_0} = F_4\,(P/F,\ 15\%,\ 4) = 1.626\,K\,(0.5718)$$

$$P_{LH_0} = 0.92975\ K$$

For RH figure it is a gradient hence

$$G = 210 - 100 = 110$$

$$P_{RH} = G\,(P/G,\ 15\%,\ 7) = 110\,(10.192)$$

$$= 1121.12$$

As both cash flows are equivalent hence.

$P_{LH_0} = P_{RH} \Rightarrow 0.92975\,K = 1121.12$

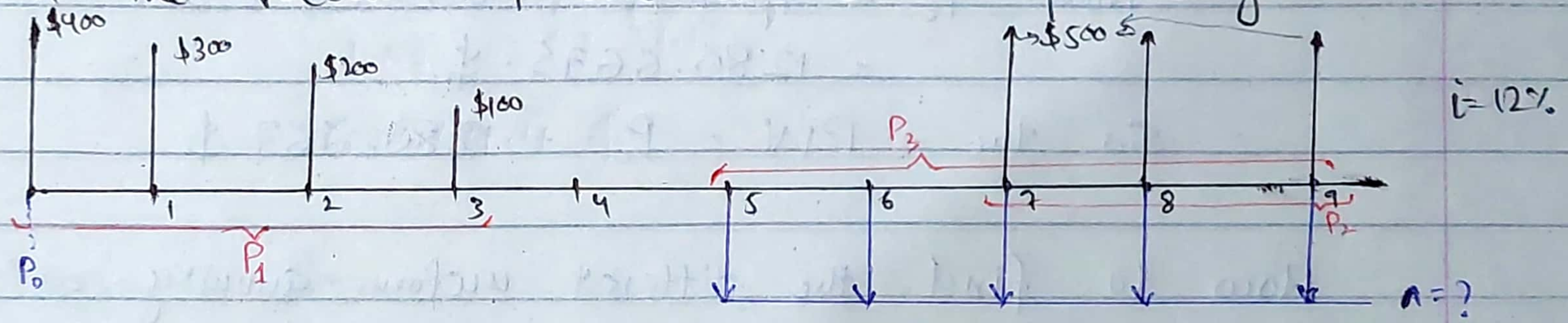
$K = 1121.12 / 0.92975$

$= 1205\ \$$

Problem # 3.60:

n=11

Find the value of the unknown quantity ...



Sol: First converting the all values to P_0 at $t=0$

$P_1 = 400 + 300(1.12)^{-1} + 200(1.12)^{-2} + 100(1.12)^{-3}$

$\because$ By using $F(1+i)^{-N} = P$

$= 898.474\ \$$

12

Now as P_2 is differed annuity where $j = 7$

hence $P_2 = A(P/A, i\%, N-j) \; (P/F, i\%, j)$

$= 500\,(P/A, 12\%, 2) \; (P/F, 12\%, 7)$

$= 500\,(1.690) \; (0.4523)$

$= 382.1935$

Now $P_0 = P_1 + P_2 = 898.474 + 382.1935$

$= 1280.6675 \, \$$

So the $PW = P_0 = 1280.667 \, \$$

Now to find the differed uniform annuity at $t = 5$ to 9.

$A = PW \, (A/P, i\%, 5) \; (F/P, i\%, 5)$

$= 1280.667 \; (A/P, 12\%, 5) \; (F/P, 12\%, 5)$

$= 1280.667 \; (0.2774) \; (1.762)$

$A = 625.963 \, \$$ ans.

n=12 Problem # 3.65: You are the manager of a large...

$A_1 = 75000\ \$$, $\bar{f} = 8\%$, $N = 5$ years, $i = 18\%$

Solution:

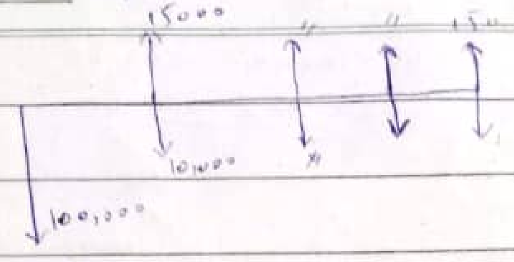
$$P = \frac{A_1}{1+\bar{f}} \sum_{k=1}^{N} \left(\frac{1+\bar{f}}{1+i}\right)^k$$

$$= \frac{75000}{1.08} \sum_{k=1}^{5} \left(\frac{1.08}{1.18}\right)^k$$

$$= 69444.\bar{4}\ \$$$

n=13 Problem # 3.

Day: Q-3-21 Date:



Q - 3-26.

$P = 10{,}000$

$i = 0.15,\ 4.$

$F = 10{,}000 \times (1+0.15)^{4} \times 4.$

$F = 17500 + 3000 = 20500$

Amount remain to be paid = 4450

Q - 31

$P = 0.10,\ F = 0.29,\ n = 20$

$F = P(1+i)^n$

$\frac{0.10}{0.29} = (1+i)^n$

$0.345 = (1+i)^{20}$

~~n log(1+i)~~

$1+i = (0.345)^{1/20}$

~~i = 571 × 10 − 1~~

~~i = −665 × 0.665~~

$F = \cdot i = 0.948 = -0.052$

Q = 36

$P = 100{,}000 \quad , i = 8$

$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$

$= P \times 0.0888$

$A = 8880$

$F_8 = 2A \times \left[\frac{(i+1)^4 - 1}{i}\right]$

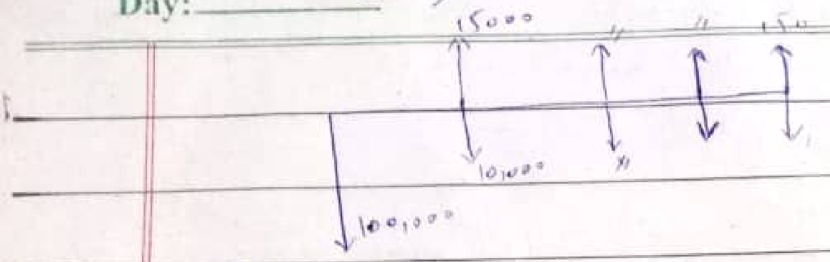
$F_8 = 94500$

~~$F_{12} = P_8$~~ $F_{12} =$

$F_{12} = 2A \times \left[\frac{(i+1)^4 - 1}{i}\right]$

$F_{12} = 80{,}000$

Day: Q-3-21. Date:



Q - 3-26.

$P = 10{,}000$

$i = 0.15 \quad 4.$

$F = 10{,}000 \times (1 + 0.15)^{8} \times 4.$

$F = 17500 + 3000 = 20500$

Amount remain to be paid = 4500

Q - 31

$P = 0.10, \quad F = 0.29, \quad n = 20$

$F = P(1+i)^{n}$

$\frac{0.10}{0.29} = (1+i)^{n}$

$0.345 = (1+i)^{20}$

~~n log (1+i)~~

$1 + i = (0.345)^{1/20}$

~~$i = 511 \times 10 - 1$~~

~~$i = -665 \times 0.665$~~

$F = \quad i = 0.948 = -0.052$

Q - 36.

$P = 100{,}000 \qquad i = 8\cdot$

$A = P\left[\frac{i(1+i)^{n}}{(1+i)^{n} - 1}\right]$

$= P \times 0.0888$

$A = 8880$

$F_8 = 2A \times \left[\frac{(i+1)^{4} - 1}{i \cancel{(1+i)^{n} - 1}}\right]$

$F_8 = 94500$

~~$F_{12} = P_8$~~ $F_{12} =$

$F_{12} = 2A \times \left[\frac{(1+i)^{4} - 1}{i}\right]$

$F_{12} = 80{,}000$

7.59

Day: Date:

Q-3-41

$F_5 = 100\left[\frac{(1.10)^4 - 1}{0.10}\right] \times 1.10 + 100 \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

$= 511 + 249$

$F_5 = 760$

Q-3-46

$A = 400, \quad i = 10\%$

$n = 12$

$F_n = A\left[\frac{(1+i)^n - 1}{i}\right]$

$F_{12} = \$8550$

Q-3-56

$P_0 = \$100(P/A, 10\%, 4) + \$100(P/G, 10\%, 4) + \$100(P/G, 10\%, 3)$

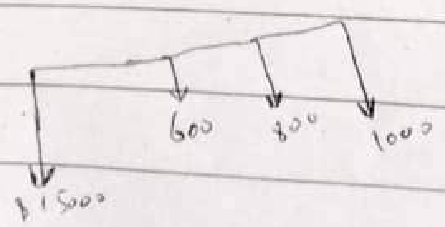
Q-3-61

if installation is not done

$i = 12\%, \quad A = 300, \quad G = 200$

$F_{10} = A(F/A, 12\%, 10) + G(F/G, 12\%, 10)$

If installation is done.



$F_{10} = A(F/A, 12\%, 10) + G(F/G, 12\%, 10)$

Q-3-41

$$F_5 = 100\left[\frac{(1+0.1)^4 - 1}{0.10}\right] \times 1.10 + 100 \times \left[\frac{(1+i)^n - 1}{i \times (1+i)^2}\right]$$

$$= 511 + 249$$

$$F_5 = 760$$

Q-3-46

$A = 400$, $i = 10\%$

$n = 12$

$$F_n = A\left[\frac{(1+i)^n - 1}{i}\right]$$

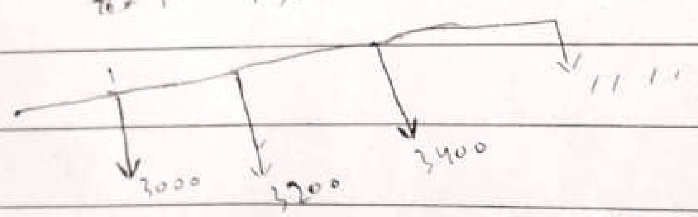
$$F_{12} = \$8550$$

Q-3-~~8~~6

$$P_0 = \$100(P/A, 10\%, 4) + \$100(P/G, 10\%, 4) + \$100(P/G, 10\%, 3)$$

Q-3-61

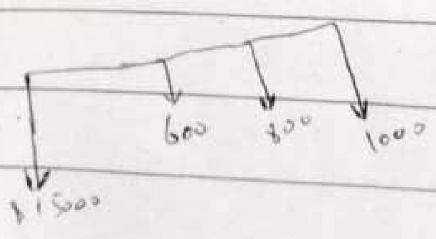
if installation is not done



$i = 12\%$, $A = 300$, $G = 200$

$$F_{10} = A(F/A, 12\%, 10) + G(F/G, 12\%, 10)$$

If ~~installeant~~ is done



$$F_{10} = A(F/A, 12\%, 10) + G(F/G, 12\%, 10)$$

$= 1150\ \$$

Question # 3.4.

Solution:

For first three years:

$P = 2000\ \$$, $N = 3$ years

$i = 0.1\ \$/\text{year}$

$$I_1 = 2000 \times 0.1 \times 3.$$

$$= 600\ \$$$

For Last three years: $P = 1000\ \$$

$$I_2 = 1000 \times 0.1 \times 3$$

$$= 300\ \$$$

Total Interes $I = I_1 + I_2 = 900\ \$$

Question # 3.5:

Solution:

Intrest, $d = I_c - I_s = 3993 - 900$

$= 3093\ \$$

Question # 3.9

Solution:

To calculate the annual payment needed to reach a future value, $F = 10{,}000\ \$$ with an annual intrest rate of $i = 5\%$ over $N = 15$ years, we can use the below formula.

$$F = P\left(\frac{(1+i)^N - 1}{i}\right)$$

$$P = \frac{F \cdot i}{(1+i)^N - 1}$$

$$= \frac{10{,}000 \times 0.05}{(1.05)^{15} - 1}$$

$$= 463.4229\ \$$$

CamScanner

Question # 3.14:

Solution:

Using formula for Compound Interest:

$$F_v = P_v \times (1+i)^N$$

$$\frac{F_v}{P_v} = (1+i)^N \quad = \quad 1+i = \sqrt[N]{\frac{F_v}{P_v}}$$

$$i = \sqrt[N]{\frac{F_v}{P_v}} - 1$$

$$i = \sqrt[8]{\frac{1000}{350}} - 1$$

$$= 0.1402$$

$$i = 14.02\%$$

19) A = \$150
i = 8%
N = 18.
P = ?

Solution:

find P given A.

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

$$\Rightarrow 150\left[\frac{(1+0.08)^{18} - 1}{0.08(1+0.08)^{18}}\right]$$

$$\boxed{P = \$1405.78}$$

from table:

$P = 150 \times 9.372 = 1405.8$

24) Finding $A/P = \dfrac{i(1+i)^N}{(1+i)^N - 1}$

Find P/F $\dfrac{1}{(1+i)^N}$

a)

$\Rightarrow (A/P, i, N) = \dfrac{i}{1-(P/F, i, N)}$

$$\frac{i(1+i)^N}{(1+i)^N - 1} = \frac{i}{1 - \left(\frac{1}{(1+i)^N}\right)}$$

$$= \frac{i}{\frac{(1+i)^N}{(1+i)^N} - \frac{1}{(1+i)^N}}$$

$$= \frac{i}{\frac{(1+i)^N - 1}{(1+i)^N}}$$

$$\frac{i(1+i)^N}{(1+i)^N - 1} = \frac{i(1+i)^N}{(1+i)^N - 1}$$ Hence prooved

29)

$$\frac{A}{P} = \frac{i(1+i)^N}{(1+i)^N - 1}$$

$$A(1+i)^N - A = Pi(1+i)^N$$

$$A(1+i)^N - Pi(1+i)^N - A = 0$$

$$(1+i)^N (A - Pi) = A$$

$$(1+i)^N = \frac{A}{A-Pi}$$

$$\log(1+i)^N = \log\left(\frac{A}{A-Pi}\right)$$

$$N \log(1+i) = \log A - \log(A-Pi)$$

$$N = \frac{\log A - \log(A-Pi)}{\log(1+i)}$$

putting values.

$$= \frac{\log(263.8) - \log(263.8 - 100)}{\log(1.1)}$$

$$= 4.9999 = 5 \text{ years.}$$

$?\, 0 - 1 - 2 \rightarrow 11 - 71 \rightarrow 72 \rightarrow 73$

34) $A = 500$ per month.
$i = 1\%$ per month.
$N = 5$ year $= 60$ months.
but withdrawl starts after 6 years of deposition (72 months).

Solution:

As we know that first withdrawl will be start at the end of 72th month and up to 132th month.
So 72th is present for 132 month with $A = 500$.

so we use.

$$P_o = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

$$= 500\left[\frac{(1+0.01)^{61} - 1}{0.01(1+0.01)^{61}}\right]$$

$$P_o = 22750$$

But this P_o is the future value fo today. Now we find P_2.

$$P = F\left(\frac{1}{(1+i)^N}\right)$$

$$= 22750\left(\frac{1}{(1+0.01)^{71}}\right)$$

$$\boxed{P = 11224.47}$$

39) ~~P~~ $P = \$4{,}000$

$N = 8$ years

$i = 12\%$

A_1 is half of the principal loan over first 4-years.

A_2 is the other half of the principal loan over second 4-years.

So

for $A_1 \Rightarrow P = \$2000$

$i = 12\%$

$N = 4$

$$A_1 = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

$$= P(0.3292)$$

$$= 2000(0.3292)$$

$$\boxed{A_1 = 658.4}$$

Now for A_2: the principal amount will be greater tha 2000 because of the First four years.

So

$$\cancel{P = F\left(\frac{1}{(1+i)^N}\right)}$$

$$F = P(1+i)^N$$

$$F = 2000(1.574) = 3148$$

CamScanner

Now $F = \$3148$ is the principal amount for A_2.

So

$P = \$3148$

$i = 12\%$

$N = 4$

$A_2 = ?$

$$A_2 = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

$= \cancel{P(\ldots)}$

$A_2 = 3148\,(0.3292)$

$$\boxed{A_2 = 1036.3}$$

44)

Repair cost today = \$10,000

Repair cost after 4 years = \$25,000

Interest rate = 20%

So

$$F = P(1+i)^N$$

$$= 10{,}000\,(2.074)$$

$$F = 20740\ \$$$

So The immediate repair is economic. The company should repair today.

49)

$G = 600 - 500 = 100.$

$N = 10$

$i = 8\%.$

$P : ?$

Solution:

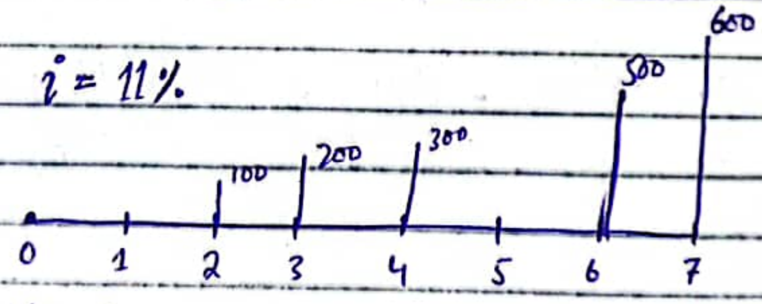
$$P = G(25.977).$$

$$= 100(25.977)$$

$$\boxed{P = 2597.7} \text{ Ans}$$

54)

$i = 11\%.$



$N = 7.$

$G = 200 - 100 = 100$

$$P_0 = G(P/G, 11\%, 7)$$

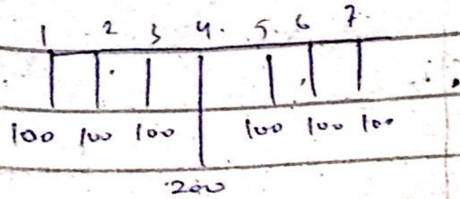
$$= 100(12.20)$$

$$\boxed{P_0 = 1220.35}$$

5n+3.

~~63: N=? i=15%~~

58: $i=12\%$: $N=7$



$P_0 = 100(P/A, 12\%, 3)$
$+ 200(P/F, 12\%, 4)$
$+ 100(P/A, 12\%, 3)(P/F, 12\%, 4)$

$= 100(2.402) + 200(\cancel{0.7118})$ 0.6355
$+ 100(2.402)(0.6355)$
$= 519.9471$

$A = 519.9471(A/P_0, 12\%, 7)$
$= 113.920$ Am

53, $F = 10k$, $G = 600$, $N = 6$
$i = ?$

$F = \frac{G}{i}(F/A, i\%, N) - \frac{NG}{i}$

$= \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$

$= \frac{600}{i}\left[\frac{(1+i)^6 - 1}{i}\right] - \frac{3600}{i}$

$10k = \frac{600[(1+i)^6 - 1]}{i^2} - \frac{3600}{i}$

$10k = \frac{600[(1+i)^6 - 1] - 3600i}{i^2}$

$(10k \times i^2) + 3600i = 600(1+i)^6 - 600$

$10000i^2 + 3600i = 600(1+i)^6 - 600$

باقی ان solve کرنی

$i = \ldots$

b, $F = 10{,}000$, $G = 600$, $i = 5\%$.
$N = ?$

$10k = \frac{600}{0.05}\left[\frac{(1.05)^N - 1}{0.05}\right] - \frac{600N}{0.05}$

$500 = \frac{600(1.05^N - 1) - 600N}{0.05}$

$25 = 600(1.05^N - 1) - 600N$.

~~25 let $1.05^N = x$.~~

~~$25 = 600x - 600$~~

$\ln 1.05^{N \ln} = x$

$N\, 0.04 = \ln x$

N,

$25 = 600 \frac{(1.05^{N} - 1)}{G} - 600N$

$0.041 = 1.05^{N} - 1 - 600N$

$1.041 = 1.05^{N} - 600N$

$N = ?$

c, $G = 1k,\ N = 12\ \&\ i = 10\%$

$F = ?$

$F = \frac{1000}{0.1}(F/A, 10\%, 12) - \frac{12k}{0.1}$

$= \frac{1000}{0.1}(21.384) - \frac{12k}{0.1}$

$= 93840$

d, $F = 8k,\ N = 6,\ i = 10\%$

$G = ?$

$8000 = \frac{G}{0.1}(F/A, 10\%, 6) - \frac{6G}{0.1}$

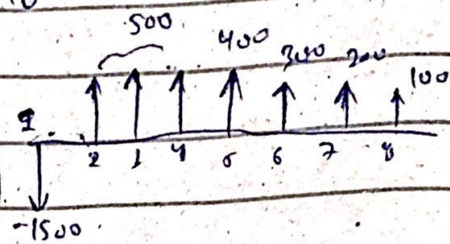
$8000 = \frac{G}{0.1}(7.716) - \frac{6G}{0.1}$

$8000 = 77.16G - 60G$

$8000 = 17.16$

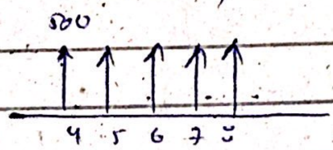
$G = 466.200$ Ar

48, $i = 8\%$

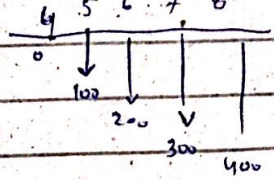


sol $P_0 = ?$

$P_0 = \cancel{-1500(P/F, 8\%, 1) + 500(P/A, 8\%, 2)(P/F, 8\%, 1) + 400(P/F, 8\%, 5) + 300}$



+



$P_0 = -1500(P/F, 8\%, 1) + 500(P/A, 8\%, 2)(P/F, 8\%, 1) + 500(P/A, 8\%, 5) - 100(P/G, 8\%, 5)$

43

0 1

2

2

$P_0 =$

$P_0 =$

$P_0 = 1$

By Hamza (1990)

$P_o = 695.885$

Annual eq = ?

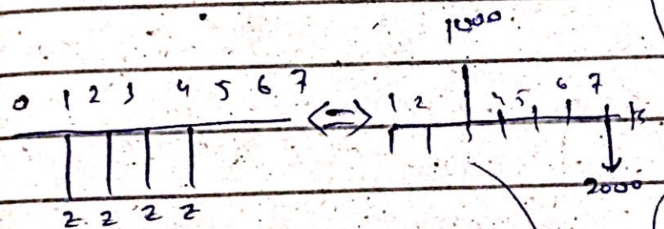
$A = 695\,(A/P, 8\%, 8)$

$= 121.08399$ Ans.

Review this above question.

Q3, ~~A = ? i = 15%~~

$Z = ?$ $i = 12\%$



$Z = ?$

$P_o = Z\,(P/A, 12\%, 4)$

$P_o = 3.037Z \rightarrow$ (i)

$P_o = 1000\,(P/F, 12\%, 3) - 2000\,(P/F, 12, 7)$

$(P/G, 8\%, 5)$

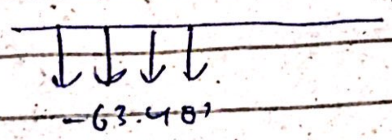
$P = -192.8 \rightarrow$ (ii)

Compare (i) & (ii)

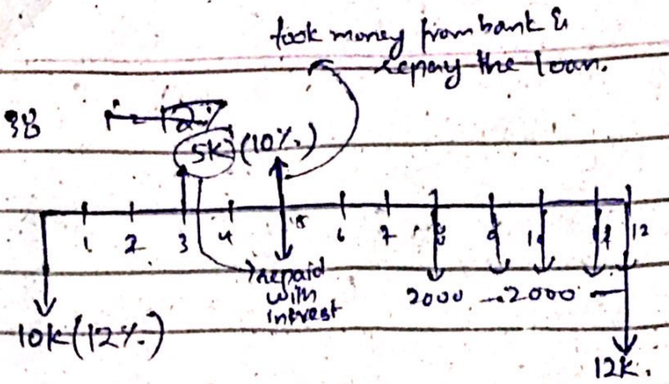
$3.037z = -192.8$

$z = -63.483$

Ans



38



How much remaining FW = ?

~~$P_0 = 10k$~~

At year = 3 total W is

$F_3 = 10k\,(F/p, 12\%, 3)$
$= 14050$

if you loaned 5000 &
added to fund so

$F_3 = 19050$

21336
23896

Now find F_5

$F_5 = P_3\,(F/p, 12\%, 2)$
$= \cancel{33566.1}$
$= 23888.7$

Interest on 5k loan
after 2 years.

$F_5' = 5k\,(F/p, 10\%, 2)$
$= 6050$

You repaid it
so given worth.

$F_5 = 23888.7 - 6050$

$F_5 = 17838.7$

$F_7 = F_5\,(F/p, 12, 2)$
$F_7 = 22369.72$

~~$F_{12} = 2000$~~

$= 12706$

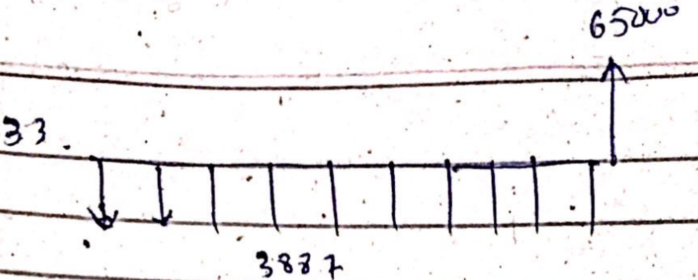
$F_8 = 25062.39$
$- 2000$
$+ 23062$

$F_9 - 2000$
$F_{10} - 2000$
$F_{11} - 2000$
$F_{12} - 12000 = FW$

If any other way
from $F_9 - F_{12}$ show plz

Ans

33.



$N = 10$

$A = 3887 \quad F = 65k$

$i = ?$

$F = A(F/A, 10, i)$

$65k = 3887\left[\frac{(1+i)^{10} - 1}{i}\right]$

$16.722 = \frac{(1+i)^{10} - 1}{i}$

~~test~~

$16.722 i = (1+i)^{10} - 1$

$\cancel{16.777(x-1) = x^{10} - 1}$

$\boxed{i = 0.1099}$

Kindly show how its done

$1+i = x$

$i = x$

$1+i = x$

$x =$

28, $P = 10,000$

$N = ? \quad i = 10\%$

$F = 1000,000$

$F = P(F/P, 10\%, N)$

$F = (1+i)^N$

$1000,000 = 10,000\,(1.1)^N$

$\ln(100) = \ln(\;)$

$\cancel{13.815} = N(0.095)$

$4.605 = \;"$

$N = 48.47$ years.