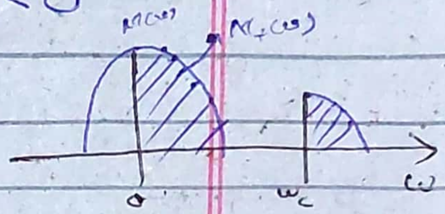
After mids

Communication Systems Lectures

H3 | R#10 4

Hilber transformation:

$$H(\omega) \begin{cases} 1e^{-j\pi/2} ; & \omega > 0 \\ 1e^{j\pi/2} ; & \omega < 0 \end{cases}$$



$$M_+(\omega) = M(\omega)U(\omega)$$

$$= M(\omega)\left(\frac{1+\mathrm{sgn}(\omega)}{2}\right)$$

$$\phi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t$$

tone modulation:

Let $m(t) = \cos\omega_m t$

then $m_h(t) = \cos(\omega_m t - \pi/2)$

$= \sin\omega_m t$

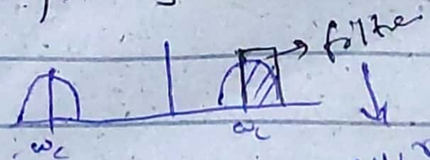
thin $\Rightarrow \phi_{SSB}(t) =$ ~~cos ωc t ± sin~~

$\cos\omega_m t\cos\omega_c t \mp \sin\omega_m t\sin\omega_c t$

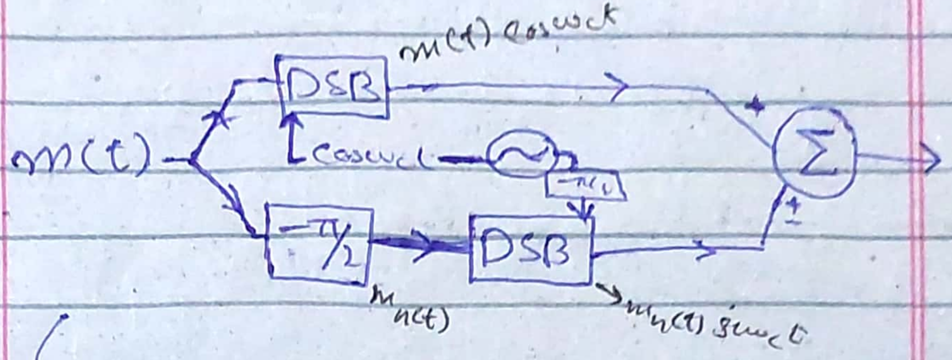
$= \cos(\omega_c \pm \omega_m)t$

Generation of SSB:

→ Selective filtering method:



→ phase shift method.



↳ unrealisible becaus we can't have amplitud response the same for all bands, secondly the phase shift is also not linear

LPF

Demodulation:

$$\left(m(t)\cos\omega_c t + m_h(t)\sin\omega_c t\right)\cos\omega_c t$$

$$= m(t)\cos^2\omega_c t + m_h(t)\sin\omega_c t\cos\omega_c t$$

Synchronous Detection

$$= m(t)\left(\frac{1+\cos 2\omega_c t}{2}\right) + m_h(t)\left(\sin 2\omega_c t\right)$$

$$= \frac{M(\omega)}{2} + \frac{M(\omega+2\omega_c)}{4} + \frac{M(\omega-2\omega_c)}{4}$$

Envelope Detection of SSB

$$\phi_{SSB}(t) = A\cos\omega_c t + m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

$$= [A + m(t)]\cos\omega_c t + m_h(t)\sin\omega_c t \quad -(1)$$

Let (1) $= E(t)\cos(\omega_c t + \theta)$.

which shows that

$$E(t) = \left[(A+m(t))^2 + m_h^2(t)\right]^{1/2}$$

$$= \left[A^2 + m^2(t) + 2Am(t) + m_h^2(t)\right]^{1/2}$$

$$= A\left[1 + \frac{2m(t)}{A} + \frac{m^2(t)}{A^2} + \frac{m_h^2(t)}{A^2}\right]^{1/2}$$

$$E(t) \cong A\left[1 + \frac{2m(t)}{A}\right]^{1/2}$$

By using binomial expansion

$$E(t) = A\left[1 + 2 \times \frac{1}{2}\frac{m(t)}{A} + \cdots\right]$$

$$= A\left[1 + \frac{m(t)}{A}\right]$$

$$= A + m(t)$$

A) $\mu = m_p / A$

If we increase A it will contain more power

$$\eta = \frac{\mu^2}{2+\mu^2}$$

(4) * carrier is also known as pilot signal.

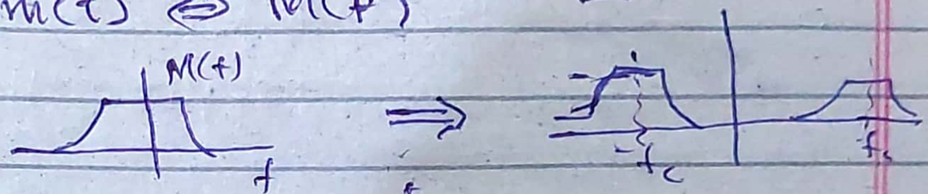
5/12/23 - Tu.

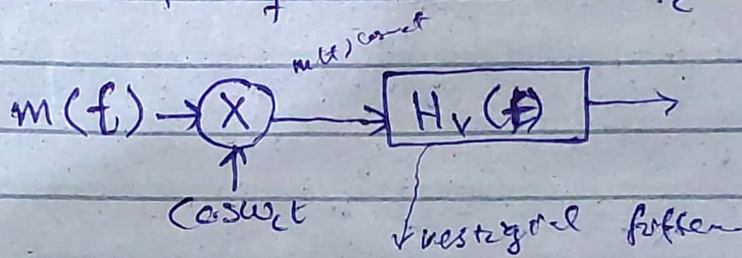
(VSB) Vestigeal side band modulation:

→ Compromise b/w DSB & SSB.
→ we transmit a single band + vestage of other band.

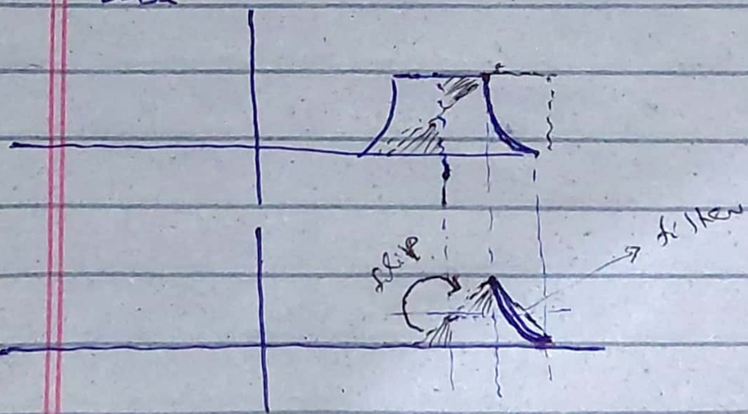
$\phi_{DSBSC}(t) = M(\omega)\cos\omega_c t$

$m(t) \Leftrightarrow M(f)$





$$\phi_{DSBSC}(f)\cdot H_v(f) = \{M(f+f_c) + M(f-f_c)\}\cdot H_v(f)$$



$$s(t) = \phi_{VSB}(t)\cos\omega_c t$$

$$S(f) = \phi_{VSB}(f+f_c) + \phi_{VSB}(f-f_c)$$

$$\Phi_{VSB}(f) = \{M(f+f_c) + M(f-f_c)\} H_v(f)$$

$$\Phi_{VSB}(f+f_c) = \{M(f+2f_c) + M(f)\} H_v(f+f_c)$$

$$\Phi_{VSB}(f-f_c) = \{M(f) + M(f-2f_c)\} H_v(f-f_c)$$

Hence $S(f) = M(f) \cdot \{H_v(f+f_c) + H_v(f-f_c)\}$
$+ M(f+2f_c) H_v(f+2f_c) + M(f-2f_c) H_v(f-2f_c)$.

We get $M(f)$ by passing it with LPF

$$M(f) = S(f) \cdot H_{LPF}(f)$$

$$\cancel{M(f)} = \cancel{M(f)} \{H_v(f+f_c) + H_v(f-f_c)\} H_{LPF}(f)$$

$$H_{LPF}(f) = \frac{1}{H_v(f+f_c) + H_v(f-f_c)}$$

Carrier Acquisition:

$m(t)$ Cos$\omega_c t$ $\cos\{(\omega_c + \Delta\omega)t + \delta\}$

$= m(t) \{\cos(2\omega_c t + \Delta\omega t + \delta) + \cos(\Delta\omega t + \delta)\}$

After LPF

$= m(t) \cos(\Delta\omega t + \delta)$.

(6)

* $f = \frac{d}{dt}\phi(t)$

* Case I: $\Delta\omega = 0$ & δ = const

$\Rightarrow m(t)\cos\delta$

$=$ const $\times$ $m(t)$

will not distort the signal.

* Case II: $\Delta\omega = 0$ δ = variable:

$\Rightarrow m(t)\cos\delta$

will distort the signal but will not be severe.

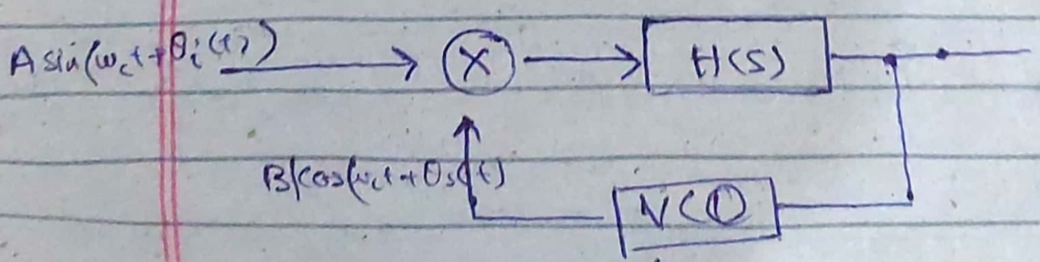
* Case III: $\Delta\omega$ = const $\delta = 0$

$\Rightarrow m(t)\cos\Delta\omega$

m(t) will be severly distorted and we can't recover the sig.

i) pilot Method is used for carrier acquisition when we send the carrier with baseband signal.

ii) PLL (phase loop lock) It is a feedback control circuit.



voltage control

CS lec 12/12/23 — Tu

Angle Modulation.

→ When we vary a frequency or phase is called angle modulation.

$\phi(t) = A_m \cos(\text{angle}(\theta))$ → (freq + phase).

$= A \cos(\omega_c t + \theta)$ → it shows a linear model

→ 'AM' is linear modulation.

- translated linearly
- shape isn't change.

$2B \le B_T \le B$

- We don't put additional frequencies.

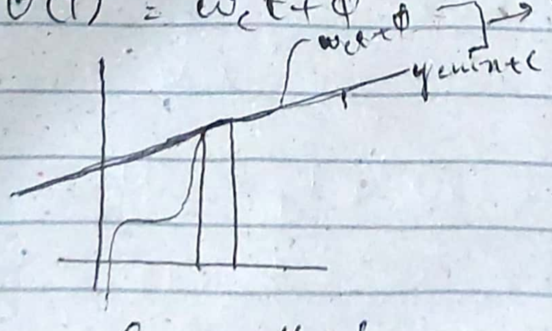
'FM' is non-linear modulation

- Shape is changed

$B_T \to \infty$

- Introduce some additional frequencies

Let $\theta(t) = \omega_c t + \phi$ → true for small interval but not generalise for all.



It is clear that $\omega_i = \frac{d}{dt}\theta(t)$.

Phase & Frequency Modulation

$\phi(t) = A\cos\theta(t)$

$= A\cos(\omega_c t + \theta_0)$

If we vary θ_0 (phase) of carrier we get phase modulated signal:

$\phi(t) = A\cos[\omega_c t + \theta_0 + k_p m(t)]$.

If $\theta_0 = 0$

$\phi(t) = A\cos(\omega_c t + k_p m(t))$.

$\theta(t) = \omega_c t + k_p m(t)$

$\frac{d}{dt}\theta(t) = \omega_c + k_p m'(t)$ — (1) $= \omega_i$

$f(t) = \frac{d}{dt}\theta(t)$.

This shows that changing phase also change instantaneous frequency.

Now if we vary frequency of carrier we get FM signal:

$\omega_i = \omega_c + k_f m(t)$

$\frac{d}{dt}\theta(t) = \omega_c + k_f m(t)$

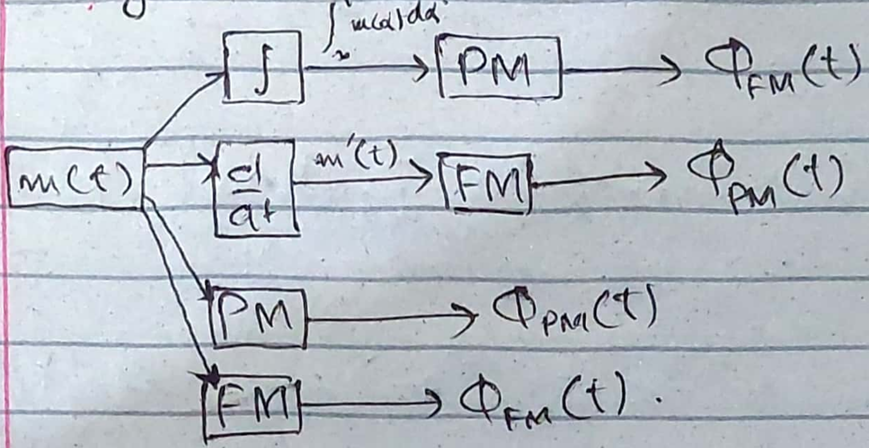
$\int d\theta(t) = \int(\omega_c + k_f m(t))dt$

$\theta(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha$

$$e^{j\theta} = \cos\theta + j\sin\theta).$$

$$\phi_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right) \quad (2)$$

We can see from (1) and (2) then with frequency is add an angle changed with derivative of integrator.



Example: — — - ?

P+F,

* Power of EM

$$\phi_{EM}(t) = A\cos\theta(t)$$

$$\mathcal{P}\{\phi_{EM}(t)\} = A^2/2.$$

Bandwidth of EM wave

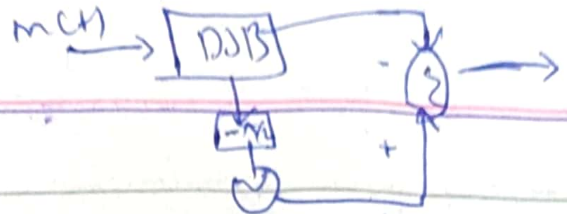
$$\phi_{EM}(t) = A\cos(\omega_c t + \theta_0)$$

$$= A e^{j(\omega_c t + \theta_0)}$$

let = a(α).

As $\phi_{FM}(t) = A\cos\left(\omega_c t + k_f \int^t m(\alpha)\, d\alpha\right.$

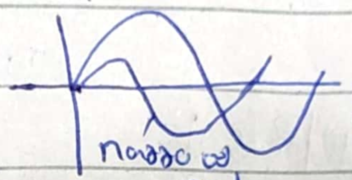
gewhen In Pm there will be no S-



- Wide BanD frequency Modulated Signal:-

if $|k_f a(t)| \ll 1$

$\varphi_{NBFM} =$



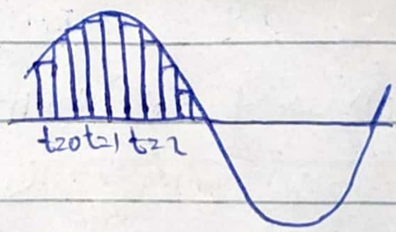
dB 2BHz

Bm = (Because equal to DSB)

- So only magnitude less then 1 will be considered.
- So by squaring and all we will decrease more.

Now for wide Bandr

2Bm (Double of the highest freq).



$w_i = w_c + k_P m(t)$

$f_i = f_c + \frac{k_f}{2\pi} m(t)$

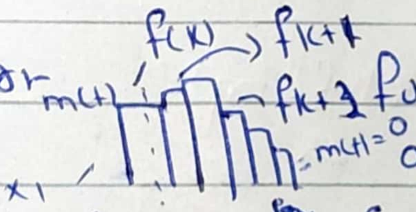
$f_i = f_c + \Delta f$

* Sampling theorem Varifyr

The Sampling we take should be equal to the Double of highest freqveng.
e.g. 4KHz, Samply Rate = 8 KHz.
exactly Double is nyquest criteria.

$$R_S = \frac{1}{T_S} \text{ (Samply Rate).}$$

for m(t) f(K) → fk+1 fk+2 for t=0 m(t) is amplitude of x1



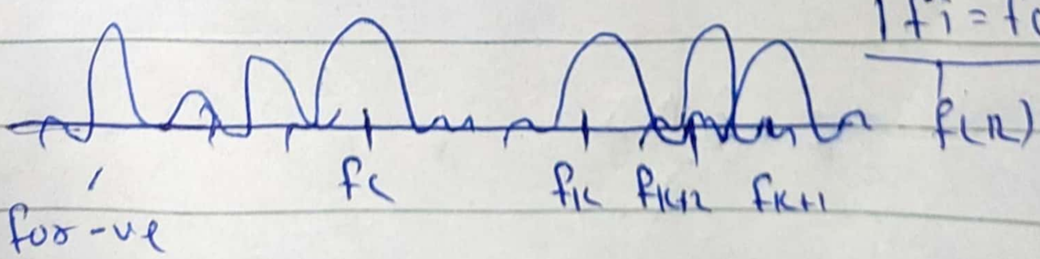
kf = const   t=0   & when we put it in eq

$$f_i = f_c + \frac{k_f}{2\pi} m(t)$$

So the graph will be Sinc in freq, because it is ⊓ Rect.

when m(t) = 0

$\boxed{f_i = f_c}$



when we will put m(t) values so it will inc(Towards) like Sinc in the graph, but when m(t) start Dec so it will come near to fc (0), when m(t) = 0, so exactly at fc.

Similarly at -ve rect freq.

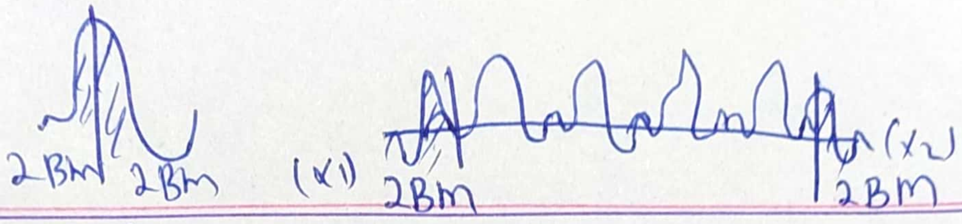
$$f_{i\,max} = f_c + \frac{K_f}{2\pi} m_p$$

$$f_{i\,min} = f_c - \frac{K_f}{2\pi} m_p$$

$$\text{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau \cdot \text{Sinc}\left(\frac{\omega\tau}{2}\right)$$

$$\frac{\omega\tau}{2} = \pm n\pi$$

$$\omega = \pm \frac{2n\pi}{\tau}$$



$= 4Bm$

(mp) (-mp)

$$B_{fm} = (f_{i\,max} - f_{i\,min}) + 2(2B_m)$$

we cannot ignore (x1) & (x2) Band so we will add em as well.

$$B_{fm} = \left(\cancel{f_c} + \frac{k_f}{2\pi} m_p - \cancel{f_c} + \frac{k_f}{2\pi} m_p\right) + 4B_m$$

$$B_{fm} = \left(\underbrace{2\frac{k_f}{2\pi} m_p}_{\Delta f}\right) + 4B_m$$

$$B_{fm} = 2\{\Delta f + 2B_m\}$$

$$\boxed{B_{fm} = 2\{\Delta f + B_m\}}$$ Generalize expression.

$$B_{fm} = 2B_m\left[\frac{\Delta f}{B_m} + 2\right]$$

$$B_{fm} = 2B_m[\beta + 2]$$ modulation index

$\therefore \mu = \frac{m_p}{A}$

This generalize expression is not equal to Narrow Band

$$B_{fm} = 2(\Delta f + 2B_m)$$

$\Delta f = 0$

$B_{fm} \neq 4B_m$: $2B_m$ (DSB).

So we will modify the generalize expression.

$$\boxed{B_{fm} = 2(\Delta f + B_m)}$$

lec:

$B_{FM} = 2(\Delta f + 2B_m)$

(Carson's Rule

$B_{FM} = 2(\Delta f + B_m)$

Lets Say tone Signal.

$m(t) = A \cos w_m t$

$\int_{-\infty}^{t} m(t)\,dt = \frac{A \cdot \sin w_m t}{w_m}$

$\varphi_{FM} = \cos\{w_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha$

$\varphi_{FM} = \cos\{w_c t + \frac{A k_f}{w_m} \cdot \sin w_m t]$

$\varphi_{FM}(t) = \cos(w_c t + \beta \sin w_m(t)]$

$\hat{\varphi}_{FM}(t) = e^{j w_c t} e^{\beta \sin w_m t}$

(only Real)

"Investigate periodic Signal $e^{\beta \sin w_m t}$."

Apply fourier Series.

$\beta = \frac{\Delta f}{B_m}$

$= \frac{k_f m_p}{2\pi B_m}$, A

$\frac{A k_p}{w_m}$

$w_m = 2\pi B_m$

where

Dn= Amplitude.

$$\sum_{n=-\infty}^{\infty} D_n e^{jn\omega_m t}$$

$$D_n = \frac{1}{T}\int_{-T/2}^{T/2} e^{jB\sin\omega_m t}\, e^{-jn\omega_m t}\, dt$$

(fourier Series).

$$D_n = \frac{1}{T}\int_{-T/2}^{T/2} e^{j\{B\sin\omega_m t - n\omega_m t\}}\, dt.$$

let $x = \omega_m t$

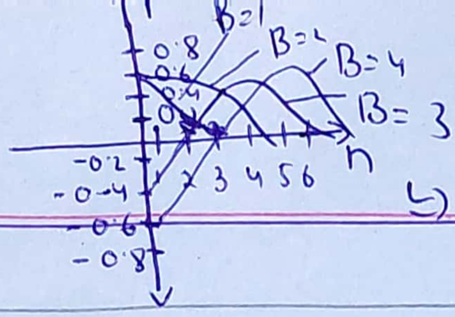
$$\therefore \frac{dx}{dt} = \omega_m \quad , \quad \omega_m = 2\pi f_m$$

$$\therefore \boxed{dt = \frac{dx}{\omega_m}} \qquad \omega_m = \frac{2\pi}{T}$$

$$\boxed{T = \frac{2\pi}{\omega_m}.}$$

$$D_n = \frac{\cancel{\omega_m}}{2\pi}\int_{-\pi/\omega_m}^{\pi/\cancel{\omega_m}} e^{j\{B\sin x - nx\}}\, \frac{dx}{\cancel{\omega_m}}$$

$$D_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\{B\sin x - nx\}}\, dx$$



↳ not prope graph.

$J_n(\beta)$ Bassel function — from graph $J_n(\beta) = 0$ when $n = \beta + 1$

when at 1, $J_n(\beta+1) = 0$ at 2.

we ignore & earlie

$$\hat{\varphi}_{Fm} = e^{jw_c t} \sum_{n=-\alpha}^{\alpha} j_n(\beta)\, e^{jnw_m t}$$

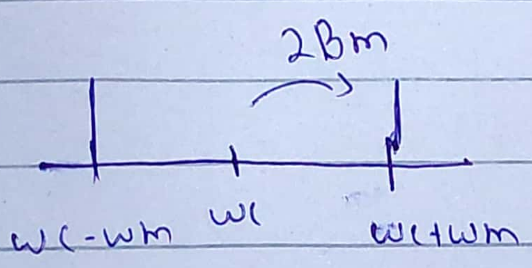
$$\varphi_{Fm}(t) = Re\{\hat{\varphi}_{Fm}(t)\}$$

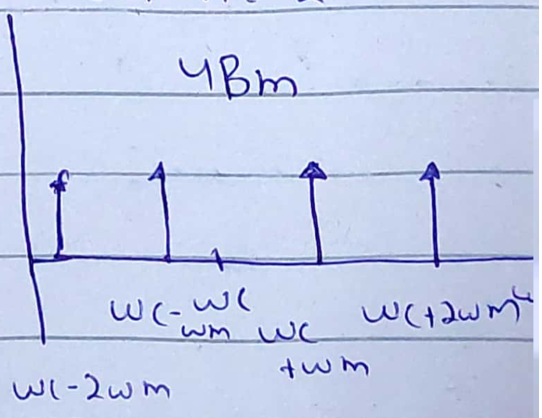
$$\varphi_{Fm}(t) = Re\{\hat{\varphi}_{Fm}(t)\} = \sum_{n=-\alpha}^{\alpha} j_n(\beta) \cos(w_c + nw_m)t$$

$$= \sum_{n=-\alpha}^{\alpha} j_n(\beta) \cos(w_c + nw_m)t$$

Graph it. at $n = 2$

at $n=1$





So $B_{Fm} = n\, 2B_m$

$B_{FM} = n\,2B_m$

$= (\beta+1)\,2B_m$

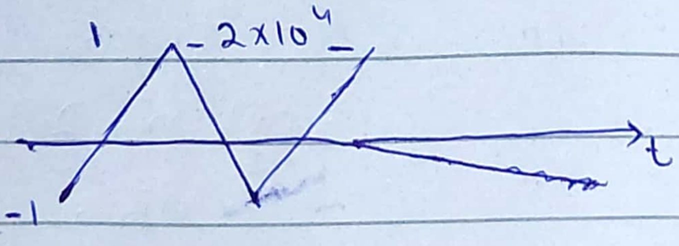
$= 2B_m\{\beta+1\}$

$= 2B_m\left\{\frac{\Delta f}{B_m}+1\right\}$

B_{FM} { $B_{PM} = ?$

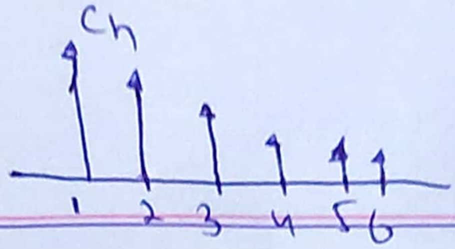
$K_f = 2\pi \times 10^5$

$K_p = 5\pi$



$B_{FM} = 2(\Delta f + B_m)$

$= 2\left\{\frac{K_f m_P}{2\pi} + B_m\right\}$ — B_m find by fourier series. $\hookrightarrow$ (ii)



magnitude is Reduced.

Fourier Series $= \sum_{n=-\infty}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$

$C_n = \dfrac{8}{n^2\pi^2}$ $\quad n = \text{odd}$

$C_n = \dfrac{8}{n\pi^2}$, $C_1 = \dfrac{8}{\pi^2}$, $P = \dfrac{C_1^2}{2}$

$C_3 = \dfrac{8}{9\pi^2}$, $P = \dfrac{C_3^2}{2}$

ignored.

$\left\{ C_5 = \dfrac{8}{25\pi^2} \right\}$, $P = \dfrac{C_4^2}{2}$

So we will ignore the C_5 because B.W & Power is reduced. | (Power is also Reducing). (in squaring term).

So

$= \begin{matrix} C_3 \cos 3\omega_m t \\ \cos 2\pi\ 3 f_m \end{matrix}$

$B_m = 3 f_m$

$B_m = 3\dfrac{1}{T}$, $3\left(\dfrac{1}{2\times10^{-4}}\right)$

$3(1\times10^4) = 3\times5\ \text{kHz} = 15\text{k}$

Put in (i)

$$= 2\left\{\frac{\cancel{2\pi}\times 10^5 m_p}{\cancel{2\pi}} + 15KHz\right\}$$

$$\phi_{pm} = 2\left\{\Delta f + B_m\right\}$$

$$\phi_{pm} = 2\left\{\frac{K_P m_p'}{2\pi} + B_m\right\}$$

$$m\dot{P} = \frac{2}{1\times 10^{-4}} = 20,000$$

$$\phi_{pm} = \cancel{2}\left\{\frac{5\cancel{\pi}(20,000)}{\cancel{2\pi}} + 15KHz\right\}$$

$$\phi_{pm} = \left\{5(20,000) + 15KHz\right\}$$

Q #.

$w_c = 2\pi \times 10^5$

$\varphi_{EM}(t) = 10\cos\{w_c t + 5\sin 3000t + 10\sin 2000\pi t\}$

$P = \frac{A^2}{2} = \frac{10^2}{2} = \frac{100}{2} = 50$ watt

$\Delta f = ?$

So;

$w_i = w_c + \Delta w$

$f_i = f_c + \Delta f$

we know

$\theta(t) = w_c t + 5\sin 3000t + 10\sin 2000\pi t\}$

$\frac{d\theta}{dt} = w_c + 15000 \cos 3000t + 20000\pi \cos 2000\pi t$

when $\cos 3000$ & $\cos 2000\pi$ become 1 at same time then

$\Delta f = \frac{15000 + 20000\pi}{2\pi}$

~~w_i ... Δf~~

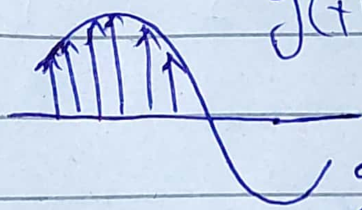
Chapter-

$g(t)\ \delta(t) = g(0)$

$g(t)\ \delta_{Ts}(t)$

$\bar{g}(t) = \frac{1}{Ts} \sum_{n} g(t)\, e^{jn\omega_s t}$

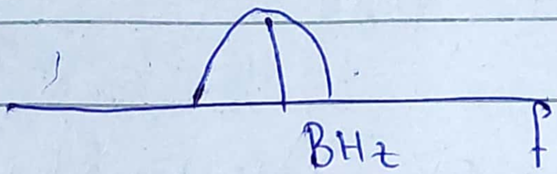
$g(t)$



$g(t) e^{j\omega_s t} \Leftrightarrow G(\omega - \omega_s)$

$g(t) e^{-j\omega_s t} \Leftrightarrow G(\omega + \omega_s)$

$fs \geq 2B$



$\frac{1}{Ts} \sum_{n=-\infty}^{\infty} G(f - nfs)$

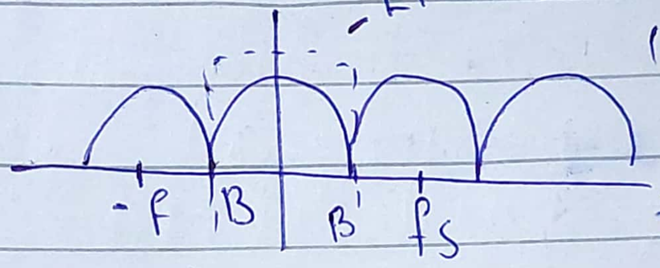
$fs = 2B$

$fs \geq 2B$

$\frac{1}{Ts} = 2B$

$Ts = 1/2B$

Signal Reconstruction using interpolation



$H(\omega) = T_s\, \text{rect}\,\frac{\omega}{2B}$ or $\frac{f}{2B}$

Property of LPF

↳ freq Domain

To find in time Domain

$\frac{\omega}{\pi} \sin \omega t \Leftrightarrow \text{rect}\,\frac{\omega}{2\omega}$

↳ $B \cdot \omega = 2\pi B$

So

$T_s \frac{2\pi B}{\pi} \sin 2\pi B t \Leftrightarrow T_s \text{rect}\,\frac{2\pi f}{2(2\pi B)}$

$2BT_s \;\text{sinc}\; 2\pi B t \Leftrightarrow T_s\, \text{rect}\,\frac{f}{B}$

$h(t) = 2BT_s \;\text{sinc}\, 2\pi B t$ → impulse Response

(Time Domain).

- The time Domain function will be xplied with samples.

$2BTs = 2B \times \frac{1}{2B}$

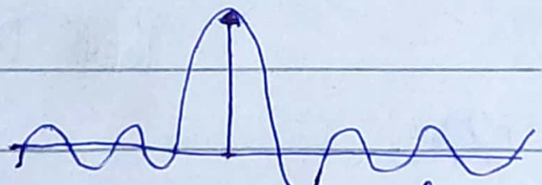
when impulse Response convolve with $g(kTs)$ it will give original signal.

for every individual impulse (train of impulses).

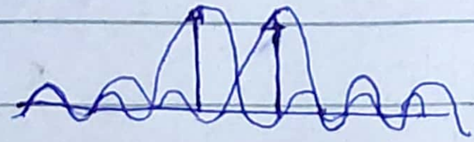
$$g(t) = \sum_{k=-\infty}^{\infty} g(kTs) \cdot h(t-kTs)$$

impulse multiply (gi) So out will be $h(t-kTs)$

output



if there's two impulse.



So

$$g(t) = \sum_{k=-\infty}^{\infty} g(kTs)\, 2BTs \,\mathrm{Sinc}\, 2\pi B\,(t-kTs)$$

$$g(t) = \sum_{k=-\infty}^{\infty} g(kTs)\, \mathrm{Sinc}\,(2\pi Bt - 2B\pi B\, kTs)$$

$$= \sum_{k=-\infty}^{\infty} g(kTs)\, \mathrm{Sinc}\,(2\pi B + k\pi)$$

$\frac{1}{T_s}$ is the Sampling Rate, how Samp are far (Separated).

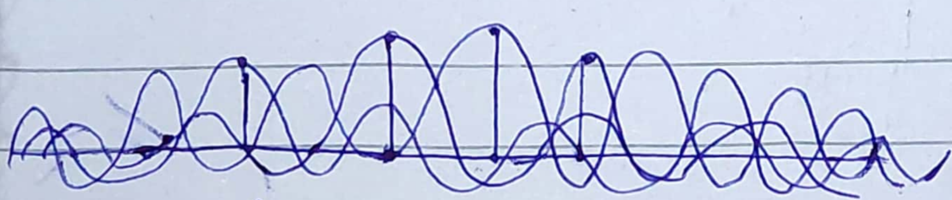
when Does Sinc function passes through zero.

zero passes

$$\text{Sinc}(2\pi B t) = \pm n\pi$$

$$\frac{2\pi B t}{2\pi B} = \pm \frac{n\pi}{2\pi B}$$

$$t = \pm \frac{n}{2B} = \boxed{t = \pm n T_s}$$



The Signal all the Signal with impulse $g(k+d)$ when xplied with Sinc.

(we Reconstruct our Signal from the Samples).

$f(T)$, be cle $g(t)$ is n. Periodic

$g(t) = g(t)\, \delta_{s}(t)$

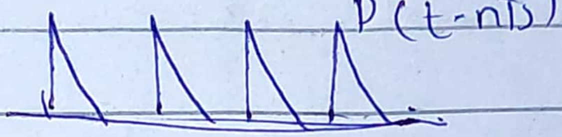
$= \frac{1}{T} \sum_{n=-\infty} g(nT)\, \delta(t - nT)$ $\Leftrightarrow$ $G(f) = \frac{1}{T_s} \sum_{n=\infty}$

B.w of the Samples are infinite

$\frac{\omega T}{2} = \pm n\pi$, $\omega = \frac{2\pi n}{T}$ $\omega = \infty$.

if $T \to 0$

• Practical Signal Reconstruction

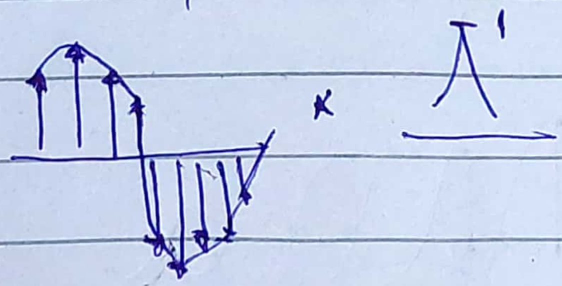


(Practical signal not impulse).

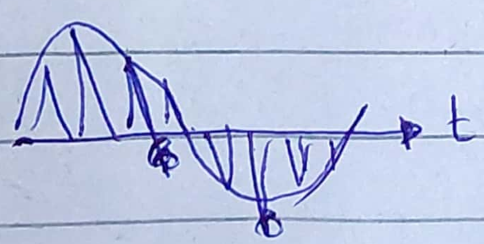
$$\bar{g}(t) = \sum_n g(nt_s)\, P(t - nT_s)$$

$$\bar{g}(t) = P(t) * \sum_{K=-\infty}^{\infty} g(KT_s)\, \delta(t - KT_s)$$

So the practical signal xplied with impulse



So the signal will be P(t) with magnitude of impulse



$$\tilde{G}(t) = P(t)\,\frac{1}{T_s}\sum_{k=-\infty}^{\infty} G(t-nf_s)$$

$$\tilde{G}(t) = \mathcal{E}(f)\,P(t)\,\frac{1}{T_s}\sum_{k=-\infty}^{\infty} G(t-nf_s)$$

Filter to extract orignl signl. (Equilizer)

$$\mathcal{E}(f)\,P(f) = T_s$$

$$\mathcal{E}(f)\,P(f) = T_s \quad \text{for } |f| \leq f_s - B$$

$$\mathcal{E}(f)\,P(f) = 0 \quad |f| > f_s - B$$

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