4

Applications of Money–Time Relationships

The two primary objectives of this chapter are (1) to illustrate the mechanics of several basic methods for making engineering economy studies considering the time value of money and (2) to describe briefiy the underlying assumptions and interrelationships of these methods. The principal topics discussed in Chapter 4 are as follows:

- Determining the minimum attractive rate of return
- The present worth method
- The annual worth method
- The internal rate of return method
- The benefit/cost ratio method
- A computer program for selected methods
- The payback (payout) period method
- Case study examples

4.1 Introduction

All engineering economy studies of capital projects should consider the return that a given project will or should produce. A basic question this book addresses is whether a proposed capital investment and its associated expenditures can be recovered by revenue (or savings) over time *in addition to* a return on the capital that is sufficiently attractive in view of the risks involved and the potential alternative uses. The interest and money–time relationships discussed in Chapter 3 emerge as essential ingredients in answering this question, and they are applied to many different types of problems in this chapter.

Because patterns of capital investment, revenue (or savings) cash flows, and disbursement cash flows can be quite different in various projects, there is no single method for performing engineering economic analyses that is ideal for all cases. Consequently, several methods are commonly used.

In this chapter we concentrate on the correct use of five methods for evaluating the economic profitability of a single proposed problem solution (i.e., *alternative*). Later, in Chapter 5, multiple alternatives are evaluated. The five methods described in Chapter 4 are present worth (PW), annual worth (AW), future worth (FW), internal rate of return (IRR), and benefit/cost ratio (B/C). All these methods, except IRR, convert cash flows resulting from a proposed solution into their equivalent worth at some point (or points) in time by using an interest rate known as the *minimum attractive rate of return (MARR)*. The concept of a MARR, as well as the determination of its value, are discussed in the following section. The IRR method produces an annual rate of profit, or return, resulting from an investment and is compared to the MARR.

A sixth method, the payback period, is also discussed briefly in this chapter. The payback period is a measure of the *speed* with which an investment is recovered by the profits it produces. This measure, in its most common form, ignores time value of money principles. For this reason, the payback method is often used to supplement information produced by the five primary methods featured in this chapter.

> Unless otherwise specified, the end-of-period cash flow convention and discrete compounding of interest are utilized throughout this and subsequent chapters. A planning horizon, or study (analysis) period, of N compounding periods (usually years) is used to evaluate prospective investments throughout the remainder of the book.

[4-]2 Determining the Minimum Attractive Rate of Return

The minimum attractive rate of return (MARR) is usually a policy issue resolved by the top management of an organization in view of numerous considerations. Among these considerations are the following:

1. The amount of money available for investment, and the source and cost of these funds (i.e., equity funds or borrowed funds).
2. The number of good projects available for investment and their purpose (i.e., whether they sustain present operations and are *essential*, or expand on present operations and are *elective*).
3. The amount of perceived risk that is associated with investment opportunities available to the firm and the projected cost of administering projects over short planning horizons versus long planning horizons.
4. The type of organization involved (i.e., government, public utility, or competitive industry).

In theory the MARR should be chosen to maximize the economic well-being of an organization, subject to the types of considerations just listed. How an individual firm accomplishes this in practice is far from clear-cut and is frequently open to criticism. One popular approach to establishing a MARR involves the *opportunity cost* viewpoint described in Chapter 2, and it results from the phenomenon of *capital rationing*.

Rationing of capital is necessary when the amount of available capital is insufficient to sponsor all worthy investment opportunities. A simple example of capital rationing is given in Figure 4-1, where the cumulative investment requirements of seven independent projects are plotted against the prospective annual rate of profit of each. Figure 4-1 shows a limit of $6 million on available capital. In view of this limitation the last funded project would be E, with a prospective rate of profit, or *hurdle rate*, of 19%, and the best *rejected* project is project F. In this case the MARR by the opportunity cost principle would be 16%. By *not* being able to invest in project F, the firm would presumably be forgoing the chance to realize a 16% return. As the amount of investment capital and opportunities available change over time, the firm's MARR will also change.

Superimposed on Figure 4-1 is the approximate cost of obtaining the $6 million to illustrate that project E is acceptable only as long as its annual rate of profit exceeds the cost of raising the last $1 million. As shown in Figure 4-1, the cost of capital will tend to increase gradually as larger sums of money are acquired through increased borrowing and/or new issuances of common stock (equity). One last observation in connection with Figure 4-1 is that the perceived risk associated with financing and undertaking the seven projects has been determined by top management to be acceptable.

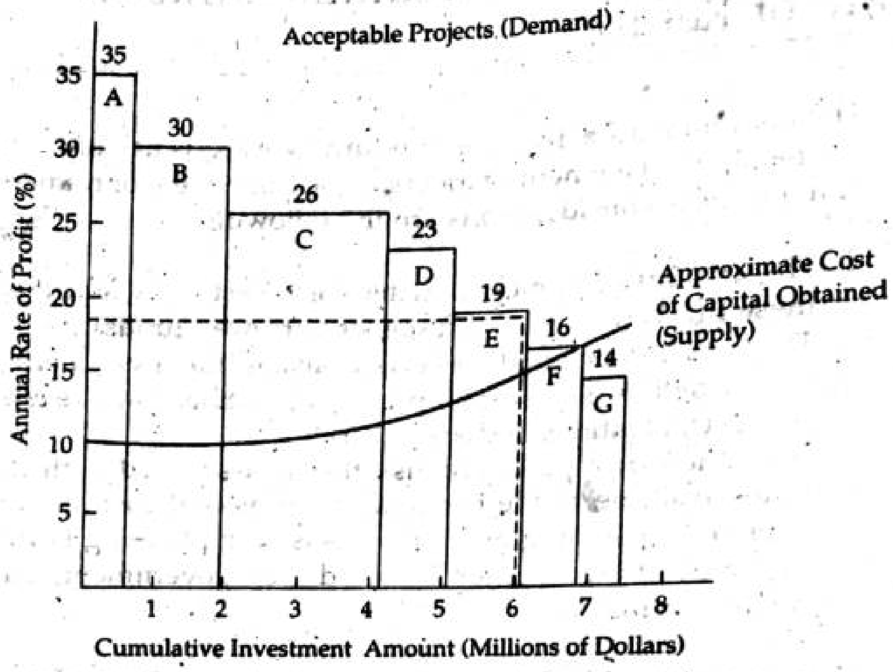


FIGURE 4-1 Determination of the MARR Based on the Opportunity Cost Viewpoint. A popular measure of annual rate of profit is "internal rate of return." (Discussed later in this chapter.)

EXAMPLE 4-1

Consider the following schedule, which shows prospective annual rates of profit for a company's portfolio of capital investment projects (this is the *demand* for capital):

Expected Annual Rate of Profit	*Investment Requirements (Thousands of Dollars)*	*Cumulative Investment*
40% and over	$ 2,200	$ 2,200
30–39.9%	3,400	5,600
20–29.9%	6,800	12,400
10–19.9%	14,200	26,600
Below 10%	22,800	49,400

If the supply of capital obtained from internal and external sources has a cost of 15% for the first $5,000,000 invested and then increases 1% for every $5,000,000 thereafter, what is this company's MARR when using an opportunity cost viewpoint?

Solution Cumulative capital demand versus supply can be plotted against prospective annual rate of profit, as shown in Figure 4-2. The point of in-

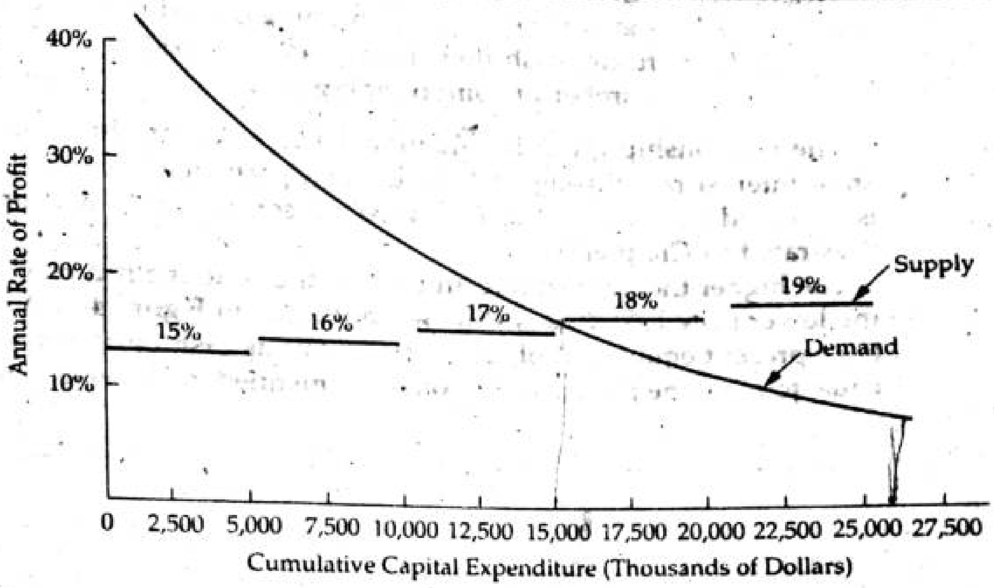


FIGURE 4-2 Solution Graph for Example 4-1

tersection is approximately 17%, which represents a realistic estimate of this company's MARR using the opportunity cost viewpoint.

4.3 The Present Worth Method

The present worth (PW) method is based on the concept of equivalent worth of all cash flows relative to some base or beginning point in time called the present. That is, all cash inflows and outflows are discounted to the base point at an interest rate that is generally the MARR.

The PW of an investment alternative is a measure of how much money an individual or a firm could afford to pay for the investment in excess of its cost. Or, stated differently, a positive PW for an investment project is a dollar amount of profit over the minimum amount required by investors. It is assumed that cash generated by the alternative is placed in reserve and earns interest at a rate equal to the MARR.

To find the PW as a function of i% of a series of cash receipts and/or expenses, it is necessary to discount future amounts to the present by using an interest rate for the appropriate study period (years, for example) in the following manner:

$$\begin{aligned} \text{PW}(i\%) &= F_0(1+i)^0 + F_1(1+i)^{-1} + F_2(1+i)^{-2} + \cdots \\ &\quad + F_k(1+i)^{-k} + \cdots + F_N(1+i)^{-N} \\ &= \sum_{k=0}^{N} F_k(1+i)^{-k} \end{aligned} \tag{4-1}$$

where i = effective interest rate, or MARR, per compounding period
k = index for each compounding period $(0 \leq k \leq N)$
F_k = future cash flow at the end of period k
N = number of compounding periods in the planning horizon

The relationship given in Equation 4-1 is based on the assumption of a constant interest rate throughout the life of a particular project. If the interest rate is assumed to change, the PW must be computed in two or more steps, as illustrated in Chapter 3.

The higher the interest rate and the further into the future a cash flow occurs, the lower is its PW. This is shown graphically in Figure 4-3. As long as the PW, (i.e., present equivalent of cash inflows minus cash outflows) is greater than or equal to zero, the project is economically justified; otherwise, it is not acceptable.

high i
& F 加
PW ↓↓

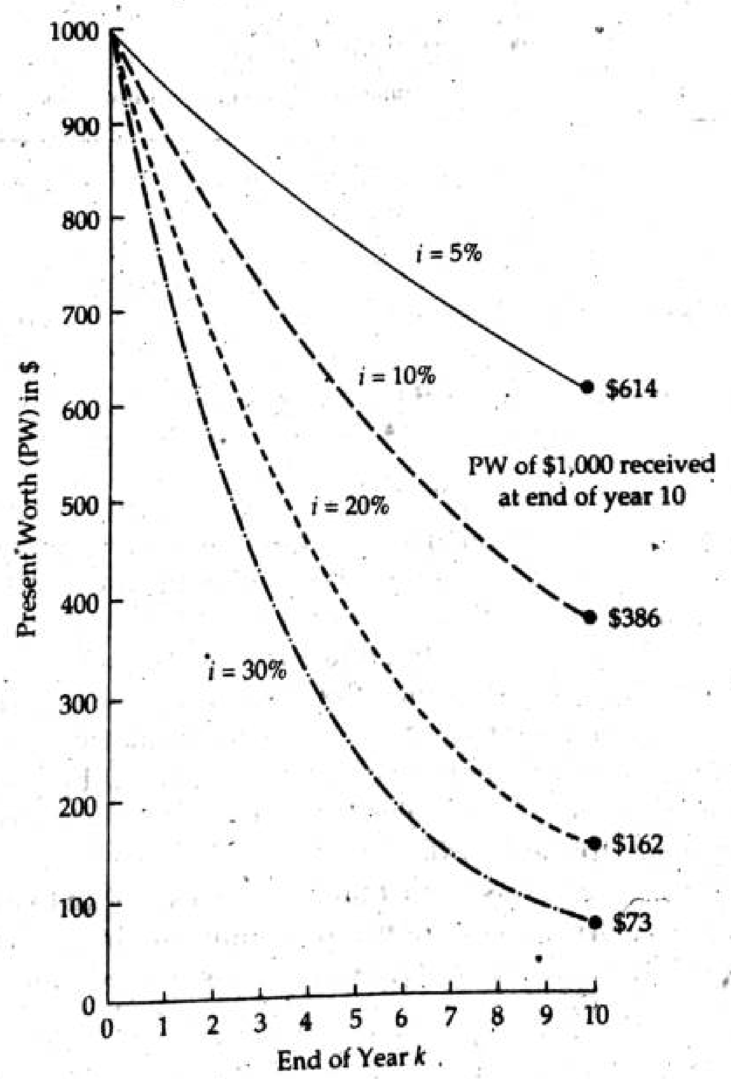


FIGURE 4-3 PW of $1,000 Received at the End of Year k at an Interest Rate of i%

If receipts or savings are not known, so that only cash outflows (expenses) are relevant, the method is characterized by negative-valued PW, and then it is often referred to as the *present worth–cost (PW–C)* method.

EXAMPLE 4-2

An investment of \$10,000 can be made in a project that will produce a uniform annual revenue of \$5,310 for 5 years and then have a salvage value of \$2,000. Annual expenses will be \$3,000 each year for operation and maintenance. The company is willing to accept any project that will earn an annual return of 10% or more, before income taxes, on all invested capital. Show whether this is a desirable investment by using the PW method.

Solution

slage = P

	PW Outflows	PW Inflows
Annual revenue: \$5,310(*P/A*, 10%, 5)		\$20,125
Salvage value: \$2,000(*P/F*, 10%, 5)		1,245
Investment	−\$10,000	
Annual expenses: \$3,000(*P/A*, 10%, 5)	− 11,370	
Total	−\$21,370	\$21,370
Total PW		\$ 0

A

Since total PW(10%) = \$0, the project is shown to be marginally acceptable. ■

EXAMPLE 4-3

A piece of new equipment has been proposed by engineers to increase the productivity of a certain manual welding operation. The initial investment (first cost) is \$25,000, and the equipment will have a salvage value of \$5,000 at the end of a study period of 5 years. Increased productivity attributable to the equipment will amount to \$8,000 per year after extra operating costs have been subtracted from the value of the additional production. Thus, it is obvious that the do-nothing alternative has been considered in estimating these cash flows. A cash flow diagram for this equipment is given in Figure 4-4. If the firm's MARR (before income taxes) is 20% per year, is this proposal a sound one? Use the PW method.

Solution

annual expenses

$$\text{Total PW} = \text{PW of cash receipts} - \text{PW of cash outlays}$$

or

annual ~~expens~~ revenue   slage

$$\begin{aligned}\text{Total PW}(20\%) &= \$8{,}000(P/A, 20\%, 5) + \$5{,}000(P/F, 20\%, 5) - \$25{,}000\\ &= \$934.29\end{aligned}$$

Because PW(20%) > 0, this equipment is economically justified. ■

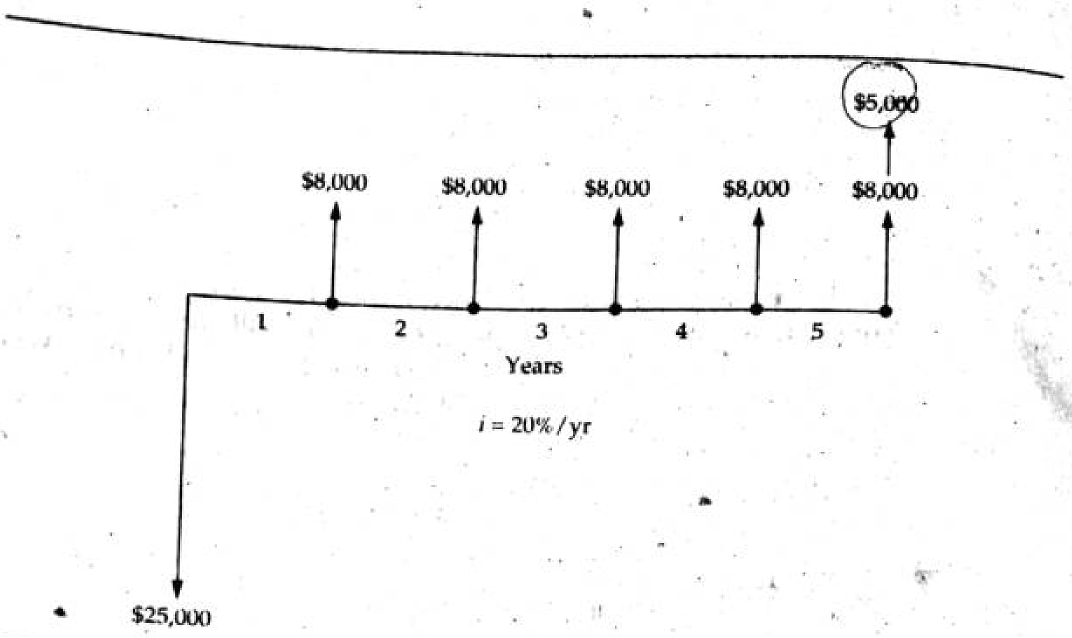


FIGURE 4-4 Cash Flow Diagram for Example 4-3

Based on Example 4-3, Table 4-1 can be utilized to plot the cumulative PW of cash flows through year k. The graphs of cumulative PW shown in Figure 4-5 are plotted from columns (C) and (D) of Table 4-1.

The MARR in Example 4-3 (and in other examples throughout this chapter) is to be interpreted as an effective interest rate (i). Here $i = 20\%$ per year. Cash flows are discrete, end-of-year amounts. If *continuous compounding* had been specified for a nominal interest rate (r) of 20%, the PW would have been calculated by using the interest factors presented in Appendix D:

$$\begin{aligned} \text{PW}(\underline{r} = 20\%) &= -\$25{,}000 + \$8{,}000(P/A, \underline{r} = 20\%, 5) \\ &\quad + \$5{,}000(P/F, \underline{r} = 20\%, 5) \\ &= -\$25{,}000 + \$8{,}000(2.8551) + \$5{,}000(0.3679) \\ &= -\$319.60 \end{aligned}$$

Consequently, with continuous compounding, the equipment would not be economically justifiable. The reason is that the higher effective interest rate ($e^{0.20} - 1 = 0.2214$) reduces the PW of future positive cash flows but does not affect the PW of the capital invested at the beginning of year 1.

4.3.1 Bond Value

A bond provides an excellent example of commercial value being the PW of the future net cash flows that are expected to be received through ownership of property. Thus, the value of a bond, at any time, is the PW of future cash receipts. For a bond, let

TABLE 4-1 Cumulative PW Calculations for Example 4-3

End of Year k	(A) Cash Flow	(B) Present Worth of Cash Flow at i = 20%	(C) Cumulative PW at i = 20% through Year k	(D) Cumulative PW at i = 0% through Year k
0	−$25,000	−$25,000	−$25,000	−$25,000
1	8,000	6,667	− 18,333	− 17,000
2	8,000	5,556	− 12,777	− 9,000
3	8,000	4,630	− 8,147	− 1,000
4	8,000	3,858	− 4,289	7,000
5	13,000	5,223	− 934	20,000

Z = face, or par, value
C = redemption or disposal price (usually equal to Z)
r = bond rate (nominal interest) per interest period
N = number of periods before redemption
i = bond *yield* rate per period
V_N = value (price) of the bond N periods prior to redemption

$i = (1+\frac{r}{M})^M - 1$

The owner of a bond is paid two types of payments by the borrower. The first consists of the series of periodic interest payments he or she will receive until the bond is retired. There will be N such payments, each amounting to rZ.

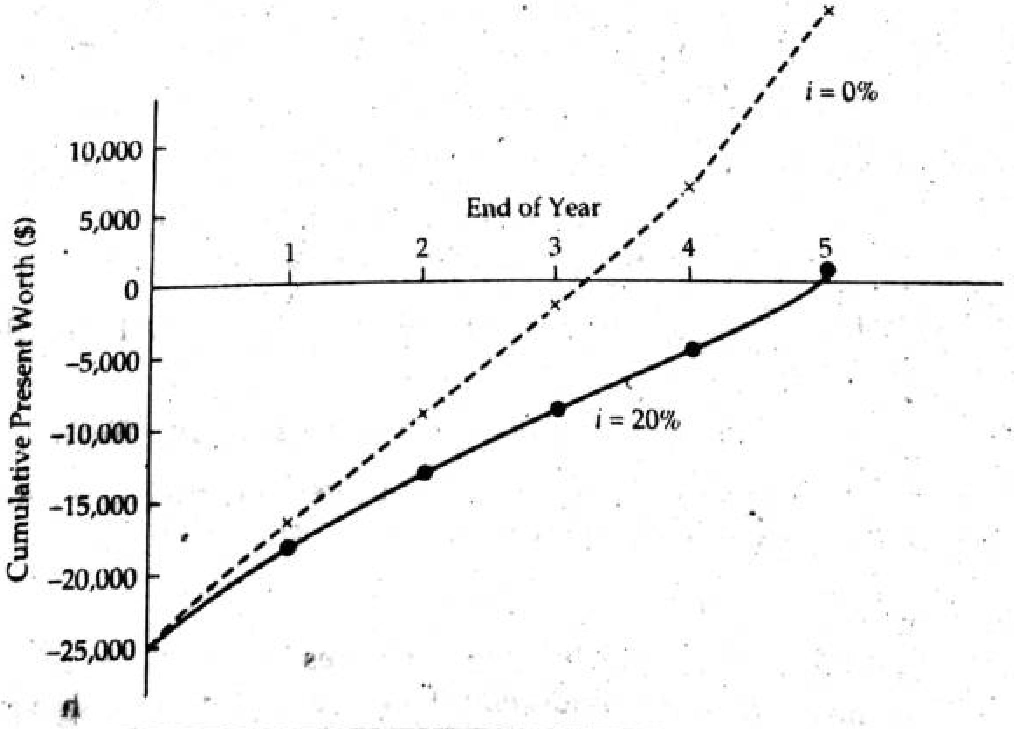


FIGURE 4-5 Graph of Cumulative PW for Example 4-3

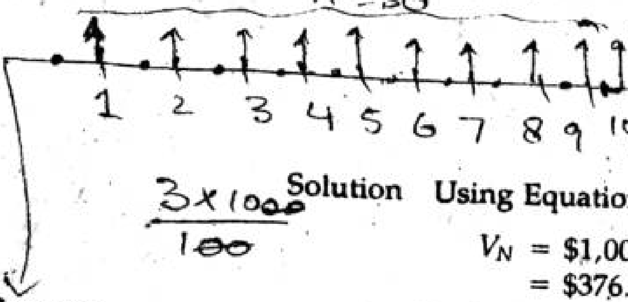
These constitute an annuity of N payments. In addition, when the bond is retired or sold, the bondholder will receive a single payment equal in amount to C. The PW of the bond is the sum of present worths of these two types of payments at the bond's yield rate:

$$\boxed{V_N = C(P/F, i\%, N) + rZ(P/A, i\%, N)} \tag{4-2}$$

EXAMPLE 4-4

Find the current price (PW) of a 10-year 6% bond, interest payable semiannually, that is redeemable at par value, if bought to yield 10% per year. The face value of the bond is $1,000.

A = 30



$N = 10 \times 2 = 20$ periods
$r = 6\%/2 = 3\%$ per period = 0.03
$i = 10\%/2 = 5\%$ per period 0.05
$C = Z = \$1,000$

$i = \left(1 + \frac{r}{m}\right)^m - 1$

$\frac{3 \times 1000}{100}$

1000

Solution Using Equation 4-2, we obtain

$$\begin{aligned} V_N &= \$1,000(P/F, 5\%, 20) + \$1,000(0.03)(P/A, 5\%, 20) \\ &= \$376.89 + \$373.87 = \$750.76 \end{aligned}$$

= 4.9%

EXAMPLE 4-5

A bond with a face value of $5,000 pays interest of 8% annually. This bond will be redeemed at par value at the end of its 20-year life, and the first interest payment is due 1 year from now.

(a) How much should be paid now for this bond in order to receive a yield of 10% per year on the investment?

(b) If this bond is purchased now for $4,600, what yield would the buyer receive?

Solution

(a) By using Equation 4-2, the value of V_N can be determined:

$$\begin{aligned} V_N &= \$5,000(P/F, 10\%, 20) + \$5,000(0.08)(P/A, 10\%, 20) \\ &= \$743.00 + \$3,405.44 = \$4,148.44 \end{aligned}$$

(b) Here we are given $V_N = \$4,600$ and we must find the value of $i\%$ in Equation 4-2:

$$\$4,600 = \$5,000(P/F, i'\%, 20) + \$5,000(0.08)(P/A, i'\%, 20)$$

To solve for $i'\%$, we can resort to an iterative trial-and-error procedure (e.g., try 8.5%, 9.0%, etc.), to determine that $i'\% = 8.9\%$.

EXAMPLE 4-6

A certain U.S. Treasury bond that matures in 8 years has a face value of $10,000. This means that the bondholder will receive $10,000 cash when the bond's maturity date is reached. The bond stipulates a fixed interest rate of 8% per year, but interest payments are made to the bondholder every 3 months and amount to 2% of the face value.

i

A prospective buyer of this bond would like to earn 10% nominal interest on his investment because interest rates in the economy have risen since the bond was issued. How much should this buyer be willing to pay for the bond?

Solution To establish the value of this bond in view of stated conditions, the PW of future cash flows during the next 8 years (the study period) must be evaluated. Interest payments are quarterly so the PW is computed at 10%/4 = 2.5% per quarter for the remaining 8(4) = 32 quarters of the bond's life:

(0.02)(10,000)

$$V_N = \frac{\$200}{\text{qtr}}\left(P/A, \frac{2.5\%}{\text{qtr}}, 32 \text{ qtrs}\right) + \$10{,}000\left(P/F, \frac{2.5\%}{\text{qtr}}, 32 \text{ qtrs}\right)$$
$$= \$4{,}369.84 + \$4{,}537.71$$
$$= \$8{,}907.55$$

calcutu.

Thus, the buyer should pay no more than $8,907.55 when 10% nominal interest is desired. ∎

4.4 The Future Worth Method

Because a primary objective of all time value of money methods is to maximize the future wealth of the owners of a firm, the future worth (FW) criterion has become increasingly popular in recent years. With this method, the future worth of an alternative can be calculated in view of the MARR and compared with the do-nothing option. If FW ≥ 0, the alternative would be recommended.

The FW method is exactly comparable to the PW method except that all cash inflows and outflows are compounded forward to a reference point in time called the future. Equation 4-3 summarizes the general calculations necessary to determine a project's future worth:

$$\text{FW}(i\%) = F_0(1+i)^N + F_1(1+i)^{N-1} + \cdots + F_N(1+i)^0 = \sum_{k=0}^{N} F_k(1+i)^{N-k} \tag{4-3}$$

EXAMPLE 4-7

Evaluate the FW of the equipment described in Example 4-3. Show the relationship between FW and PW for this example.

Solution

2500

$$\begin{aligned}\text{FW}(20\%) = &-\$25{,}000(F/P, 20\%, 5) \\ &+ \$8{,}000(F/P, 20\%, 4) + \$8{,}000(F/P, 20\%, 3) \\ &+ \$8{,}000(F/P, 20\%, 2) + \$8{,}000(F/P, 20\%, 1) \\ &+ \$13{,}000 \\ = &\ \$2{,}324.80\end{aligned}$$

? 8000+5000

Again, the equipment is shown to be a good investment. The FW is larger than the corresponding PW, and it is equivalent:

$$\text{PW}(20\%) = \$2{,}324.80(P/F, 20\%, 5) = \$934.29$$ ∎

To this point the PW and FW methods have utilized a known and constant MARR over the study period. Each method produces a measure of merit expressed in dollars and is equivalent to the other.

4.5 The Annual Worth Method

The annual worth (AW) of a project is a uniform annual series of dollar amounts, for a stated study period, that is *equivalent* to the cash inflows (receipts or savings) and/or cash outflows (expenses) under consideration. Hence, the AW of a project is not a cash flow per se. In other words, the AW is the annual equivalent revenues (or savings) ($\underline{R}$) minus annual equivalent expenses ($\underline{E}$), less its annual equivalent capital recovery (CR) cost which is defined in Equation 4-5. An annual equivalent value of $\underline{R}$, $\underline{E}$, and CR is computed using the MARR. The study period is denoted by N, which is usually years. In equation form the AW, which is a function of $i\%$, is

$$\text{AW}(i\%) = \underline{R} - \underline{E} - \text{CR}(i\%) \tag{4-4}$$

As long as the AW is greater than or equal to zero, the project is economically attractive; otherwise, it is not. An AW of zero means that an annual return exactly equal to the MARR has been earned.

The capital recovery (CR) cost for a project is the equivalent uniform annual cost of the capital invested. It is an annual amount that covers the following two items:

1. Loss in value of the asset.
2. Interest on invested capital (i.e., the MARR). def

As an example, consider a machine or other asset that will cost \$10,000, last 5 years, and have a salvage value of \$2,000. Further, the MARR is 10%.

It can be shown that no matter which method of calculating an asset's loss in value over time is used, the equivalent annual CR cost is the same. For example, if uniform loss in value is assumed, the equivalent annual CR cost is calculated to be \$2,310, as shown in Table 4-2.

There are several convenient formulas by which CR cost may be calculated to obtain the result in Table 4-2. Probably the easiest formula to understand involves finding the annual equivalent of the initial investment and then subtracting the annual equivalent of the salvage value. Thus

$$\text{CR}(i\%) = I(A/P, i\%, N) - S(A/F, i\%, N) \tag{4-5}$$

where I = initial investment for the project*
S = salvage (residual) value at the end of the study period
N = project study period

*In some cases the investment will be spread over several periods. In such situations, I is the PW of all investment amounts.

capital recovry

TABLE 4-2 Calculation of Equivalent Annual CR Cost

Year	Value of Investment at Beginning of Year[a]	Uniform Loss in Value	Interest on Beginning-of-Year Investment at $i = 10\%$	CR Cost Each Year	PW of CR Cost at $i = 10\%$
1	\$10,000	\$1,600 +	\$1,000	\$2,600	\$2,600(P/F, 10%, 1) = \$2,364
2	8,400	1,600	840	2,440	2,440(P/F, 10%, 2) = \$2,016
3	6,800	1,600	680	2,280	2,280(P/F, 10%, 3) = \$1,713
4	5,200	1,600	520	2,120	2,120(P/F, 10%, 4) = \$1,448
5	3,600	1,600	360	1,960	1,960(P/F, 10%, 5) = \$1,217
					\$8,758

Total CR cost = \$8,758(*A*/*P*, 10%, 5) = \$2,310

[a]This is also referred to later as the *beginning-of-year unrecovered investment*.

When Equation 4-5 is applied to the example in Table 4-2, the CR amount is

$$\begin{aligned} CR(10\%) &= \$10{,}000(A/P, 10\%, 5) - \$2{,}000(A/F, 10\%, 5) \\ &= \$10{,}000(0.2638) - 2{,}000(0.1638) = \$2{,}310 \end{aligned}$$

Another way to calculate the CR cost is to add an annual sinking fund amount (or deposit) to the interest on the original investment (sometimes called *minimum required profit*). Thus

$$CR(i\%) = (I - S)(A/F, i\%, N) + I(i\%) \tag{4-6}$$

When Equation 4-6 is applied to the example in Table 4-2, the CR amount is

$$\begin{aligned} CR(10\%) &= (\$10{,}000 - \$2{,}000)(A/F, 10\%, 5) + \$10{,}000(10\%) \\ &= \$8{,}000(0.1638) + \$10{,}000(0.10) = \$2{,}310 \end{aligned}$$

Yet another very popular way to calculate the CR cost is to add the equivalent annual cost of the uniform loss in value of the investment and the interest on the salvage value.

$$CR(i\%) = (I - S)(A/P, i\%, N) + S(i\%) \tag{4-7}$$

Applied to the example used previously,

$$\begin{aligned} CR(10\%) &= (\$10{,}000 - \$2{,}000)(A/P, 10\%, 5) + \$2{,}000(10\%) \\ &= \$8{,}000(0.2638) + \$2{,}000(0.10) = \$2{,}310 \end{aligned}$$

EXAMPLE 4-8

Considering the same project as in Example 4-2, show whether it is justified using the AW method (i.e., Equation 4-4).

Solution

	Annual Worth	
	Outflows	*Inflows*
Annual revenue		\$5,310
Annual expenses	−\$3,000	
CR cost[a] = (\$10,000 − \$2,000)(A/P, 10%, 5) +\$2,000(10%)	−\$2,310	
Total	−\$5,310	\$5,310
Total AW(10%)		\$ 0

[a]Uses Equation 4-7.

Since total AW(10%) = \$0, the project earns exactly 10% and is thus minimally acceptable. Of course, nonmonetary considerations would probably sway the decision to accept or reject the project. ■

EXAMPLE 4-9

By using the AW method and Equation 4-4, determine whether the equipment described in Example 4-3 should be recommended.

Solution The AW method applied to Example 4-3 yields the following:

$$\begin{aligned}\text{AW}(20\%) &= \overbrace{\$8000}^{\underline{R}-\underline{E}} - \overbrace{[\$25,000(A/P, 20\%, 5) - \$5000(A/F, 20\%, 5)]}^{\text{CR amount (Eq. 4-5)}} \\ &= \$8000 - (\$8359.49 - \$671.90) \\ &= \$312.41\end{aligned}$$

Because its AW(20%) is positive, the equipment more than pays for itself over a period of 5 years while earning a 20% return per year on the unrecovered investment. In fact, the annual equivalent "surplus" is \$312.41, which means that the equipment provided more than a 20% return on beginning-of-year unrecovered investment. This piece of equipment should be recommended as an attractive investment opportunity. Also notice that the AW(20%) in Example 4-9 is equivalent to PW(20%) = \$934.29 in Example 4-3 and FW(20%) = \$2,324.80 in Example 4-7. ■

EXAMPLE 4-10 ✓ old paper

An investment company is considering building a 25-unit apartment complex in a growing town. Because of the long-term growth potential of the town, it is felt that the company could average 90% of full rent for the whole complex each year. If the following items are reasonably accurate predictions, what is the minimum monthly rent that should be charged if a 12% MARR is desired? Use the AW method.

Land investment	\$50,000
Building investment	\$225,000
Study period, N	20 years
Rent per unit per month	?
Upkeep per unit per month	\$35
Property taxes and insurance per year	10% of total investment

Solution The procedure for solving this problem is first to determine the equivalent AW of all costs at the MARR of 12% per year. To earn exactly 12% on this project, the annual rental income, adjusted for 90% occupancy, must equal the AW of costs:

$$\begin{aligned}\text{Initial investment} &= \$50{,}000 + \$225{,}000 = \$275{,}000\\ \text{Taxes and insurance/year} &= 0.1(\$275{,}000) = \$27{,}500\\ \text{Upkeep/year} &= \$35(12 \times 25)(0.9) = \$9{,}450\\ \text{CR cost/year} &= \$275{,}000(A/P, 12\%, 20)\\ &\quad - \$50{,}000(A/F, 12\%, 20) = \$36{,}123\end{aligned}$$

(We assume that investment in land is recovered at the end of year 20 and that annual upkeep is directly proportional to the occupancy rate.)

$$\text{Equivalent AW of costs} = \$27{,}500 + \$9{,}450 + \$36{,}123 = \$73{,}073$$

Minimum *annual* rental required = \$73,073 and with annual compounding ($M = 1$) the monthly rental amount, $\hat{R}$, is

$$\hat{R} = \frac{\$73{,}073}{(12 \times 25)(0.9)} = \$270.64 \qquad \blacksquare$$

The AW method is sometimes called the *annual cost (AC)* method when only costs are involved. The goal (when the do-nothing alternative is not an option) is to select the least negative AC when several alternatives are being compared.

Many decision makers prefer the AW method because it is relatively easy to interpret when one is accustomed to working with annual income statements and cash flow summaries.

4.6 The Internal Rate of Return Method

The internal rate of return (IRR) method is the most widely used rate of return method for performing engineering economic analyses. It commonly is called by several other names, such as *investor's method, discounted cash flow method,* and *profitability index*.

This method solves for the interest rate that equates the equivalent worth of an alternative's cash inflows (receipts or savings) to the equivalent worth of cash outflows (expenditures, including investments). Equivalent worth may be computed with any of the three methods discussed earlier. The resultant interest rate is termed the *internal rate of return (IRR)*. For a single alternative, the IRR is not positive unless both receipts and expenses are present in the cash flow pattern and the sum of receipts exceeds the sum of all cash outflows.

By using a PW formulation, the IRR is the $i'\%$* at which

$$\sum_{k=0}^{N} R_k(P/F, i'\%, k) = \sum_{k=0}^{N} E_k(P/F, i'\%, k) \tag{4-8}$$

where R_k = net revenues or savings for the kth year
E_k = net expenditures including investments for the kth year
N = project life (or study period)

Once i' has been calculated, it is compared with the MARR to assess whether the alternative in question is acceptable. If $i' \geq$ MARR, the alternative is acceptable; otherwise, it is not.

A popular variation of Equation 4-8 for computing the IRR for an alternative is to determine the i' at which its *net* PW is zero. In equation form, the IRR is the value of i' at which

$$\sum_{k=0}^{N} R_k(P/F, i'\%, k) - \sum_{k=0}^{N} E_k(P/F, i'\%, k) = 0 \tag{4-9}$$

For an alternative with a single investment at the present time ($k = 0$) followed by a series of positive cash flows over N, a graph of net PW versus the interest rate typically has the general form shown in Figure 4-6. The point at which PW = 0 in Figure 4-6 defines i', which is the project's IRR.

The value of i' can also be determined as the interest rate at which FW = 0 or AW = 0. For example, by setting net FW equal to zero, Equation 4-10 would result:

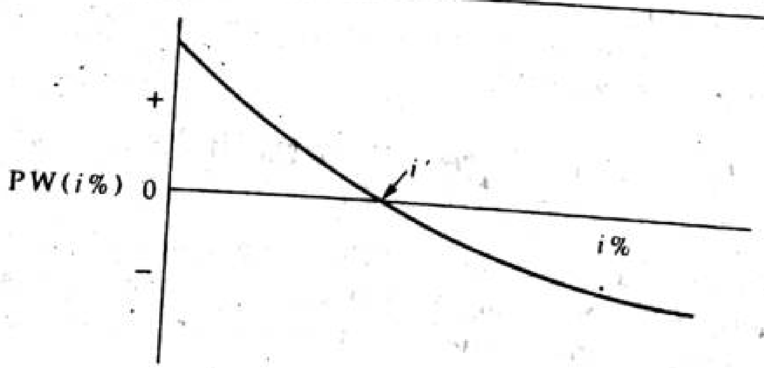


FIGURE 4-6 Plot of PW Versus Interest Rate

*i' is often used in place of i to mean the interest rate that is to be determined.

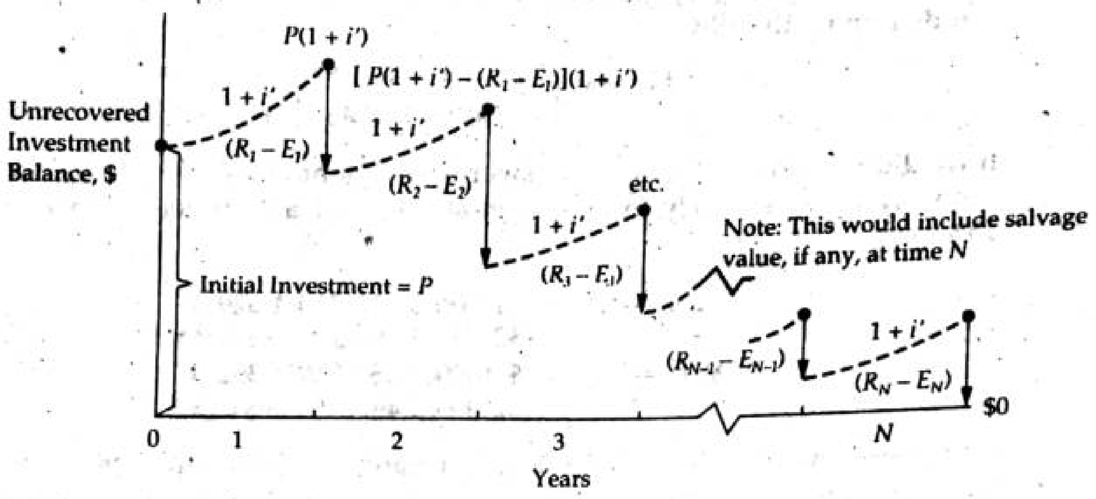


FIGURE 4-7 Unrecovered Investment Balance Diagram That Illustrates IRR

$$\sum_{k=0}^{N} R_k(F/P, i'\%, N-k) - \sum_{k=0}^{N} E_k(F/P, i'\%, N-k) = 0 \qquad (4\text{-}10)$$

Another way to interpret the IRR is through an *unrecovered investment balance diagram*. Figure 4-7 shows how much of the original investment in an alternative is still to be recovered as a function of time. The downward arrows in Figure 4-7 represent annual returns, $(R_k - E_k)$ for $1 \le k \le N$, against the unrecovered investment and the dashed lines indicate the opportunity cost of interest, or profit, on the beginning-of-year investment balance. The IRR is the value of i' in Figure 4-7 that causes the unrecovered investment balance to exactly equal 0 at the end of the study period (year N) and thus represents the *internal* earning rate of a project. It is important to notice that i' is calculated on the *beginning-of-year unrecovered* investment through the life of a project rather than on the total initial investment.

The method of solving Equations 4-8 to 4-10 normally involves trial-and-error calculations until the $i'\%$ is converged upon or can be interpolated. Example 4-11 presents a typical solution using the common convention of "+" signs for cash inflows and "−" signs for cash outflows.

EXAMPLE 4-11

An investment of $10,000 can be made in a project that will produce a uniform annual revenue of $5,310 for 5 years and then have a salvage value of $2,000. Annual expenses will be $3,000 each year for operation and maintenance costs. The company is willing to accept any project that will earn at least 10% per year, before income taxes, on all invested capital. Determine whether it is acceptable by using the IRR method.

Solution By writing an equation for net PW and setting it equal to zero, we can determine the IRR.

$$0 = -\$10{,}000 + (\$5{,}310 - \$3{,}000)(P/A, i'\%, 5) + \$2{,}000(P/F, i'\%, 5);\ i'\% = ?$$

If we did not already know the answer from Example 4-2 ($i' = 10\%$), we would probably try a relatively low i', such as 5%, and a relatively high i', such as 15%.

$$\begin{aligned} \text{At } i' = 5\%&: -\$10{,}000 + \$2{,}310(4.3295) \\ &\quad + \$2{,}000(0.7835) = +\$1{,}568 \\ \text{At } i' = 15\%&: -\$10{,}000 + \$2{,}310(3.3522) \\ &\quad + \$2{,}000(0.4972) = -\$1{,}262 \end{aligned}$$

Since we have both a positive and a negative PW, the answer is bracketed. Linear interpolation can be used to find an approximation of the unknown $i'\%$, as shown in Figure 4-8. (The dashed curve in Figure 4-8 is what we are linearly approximating.) The answer, $i'\%$, can be obtained graphically to be approximately 11%, which is the interest rate at which the PW = \$0.

Linear interpolation for the answer, $i'\%$, can be accomplished by using the similar triangles dashed in Figure 4-8.

$$\frac{15\% - 5\%}{\$1{,}568 - (-\$1{,}262)} = \frac{i'\% - 5\%}{\$1{,}568 - \$0}$$

or

$$\begin{aligned} i'\% &= 5\% + \frac{\$1{,}568}{\$1{,}568 - (-\$1{,}262)}(15\% - 5\%) \\ &= 5\% + 5.5\% = 10.5\% \end{aligned}$$

This approximate solution illustrates the trial-and-error process, together with linear interpolation. The error in this answer is due to nonlinearity of the PW

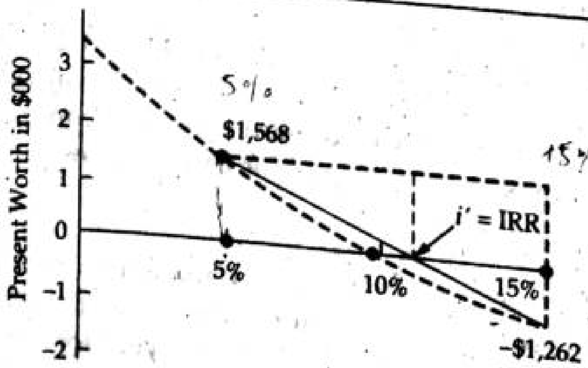


FIGURE 4-8 Use of Linear Interpolation to Find the Approximation of IRR for Example 4-11

function and would be less if the range of interest rates used in the interpolation had been smaller. ■

From the results of Examples 4-2 and 4-8, we already know that the project is minimally acceptable and that $i' = \text{MARR} = 10\%$. We can confirm this by substituting $i = 10\%$ in the PW equation as follows:

$$\text{At } i = 10\%: -\$10{,}000 + (\$5{,}310 - \$3{,}000)(P/A, 10\%, 5) + \$2{,}000(P/F, 10\%, 5) = 0$$

EXAMPLE 4-12

A piece of new equipment has been proposed by engineers to increase the productivity of a certain manual welding operation. The initial investment (first cost) is \$25,000, and the equipment will have a salvage value of \$5,000 at the end of its expected life of 5 years. Increased productivity attributable to the equipment will amount to \$8,000 per year after extra operating costs have been subtracted from the value of the additional production. A cash flow diagram for this equipment is given in Figure 4-4. Evaluate the IRR of the proposed equipment. Is the investment a good one? Recall that the MARR is 20%.

Solution By utilizing Equation 4-9, the following expression is obtained:

$$\$8{,}000(P/A, i'\%, 5) + \$5{,}000(P/F, i'\%, 5) - \$25{,}000 \doteq 0 \tag{4-11}$$

To initiate solving Equation 4-11 by trial and error, Table 4-3 is helpful. The PW computations in Table 4-3 are illustrated in Figure 4-9.

By inspection, the value of i' where PW = 0 is about 22%. For most applications an i' value of 22% is accurate enough since our major concern is whether i' exceeds the MARR. A more precise value of i' can be determined by directly solving Equation 4-11 with repeated trial-and-error calculations ($i' = 21.577\%$) or by using the computer program in Appendix E. Clearly, this is economically attractive because $21.577\% > 20\%$.

A final point needs to be illustrated for Example 4-12. The unrecovered investment balance diagram is provided in Figure 4-10, and the reader should notice that $i' = 21.577\%$ is a rate of return calculated on the beginning-of-year unrecovered investment. The IRR is *not* an average return each year based on the total investment of \$25,000. ■

A rather common application of the IRR method is in so-called *installment financing* types of problems. These problems are associated with financing arrangements for purchasing merchandise "on time." An interest, or finance,

TABLE 4-3 Computation of Selected PW (i) in Example 4-12

i'	$PW(i'\%)$	
0.00	\$8,000(5) + \$5,000(1) − \$25,000 =	\$20,000
0.10	8,000(3.7908) + 5,000(0.6209) − 25,000 =	8,430.90
0.20	8,000(2.9906) + 5,000(0.4019) − 25,000 =	934.30
0.25	8,000(2.6893) + 5,000(0.3277) − 25,000 =	−1,847.10

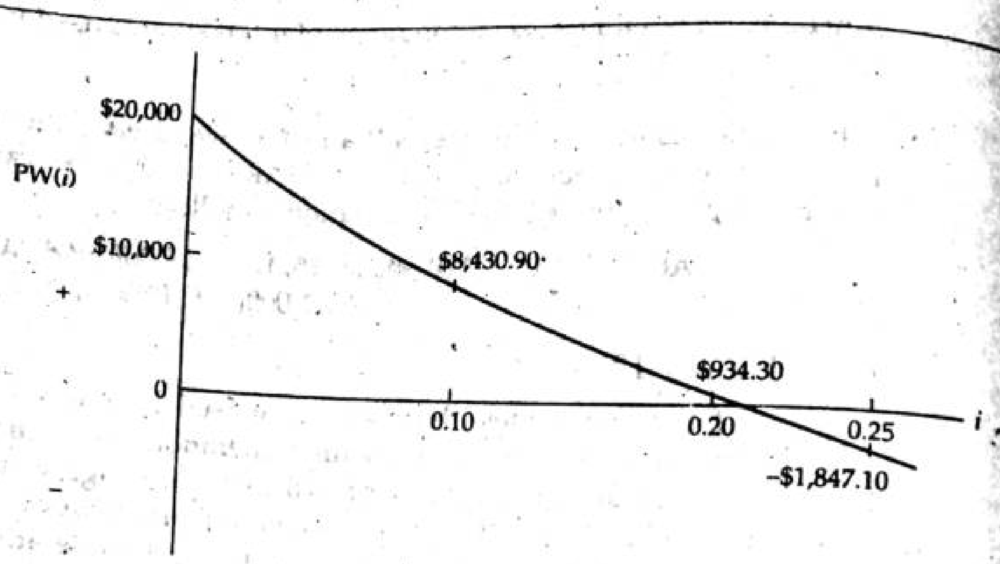


FIGURE 4-9 PW Plotted Against i for Example 4-12

charge is typically based on the total amount of money borrowed and is paid by the borrower at the beginning of the loan instead of on the unpaid loan balance, as illustrated by Figure 4-10. Such a finance charge is, of course, not in accord with the true definition of interest. To determine the true interest rate being charged in such cases, the IRR method is frequently employed. Examples 4-13 and 4-14 are representative installment financing problems.

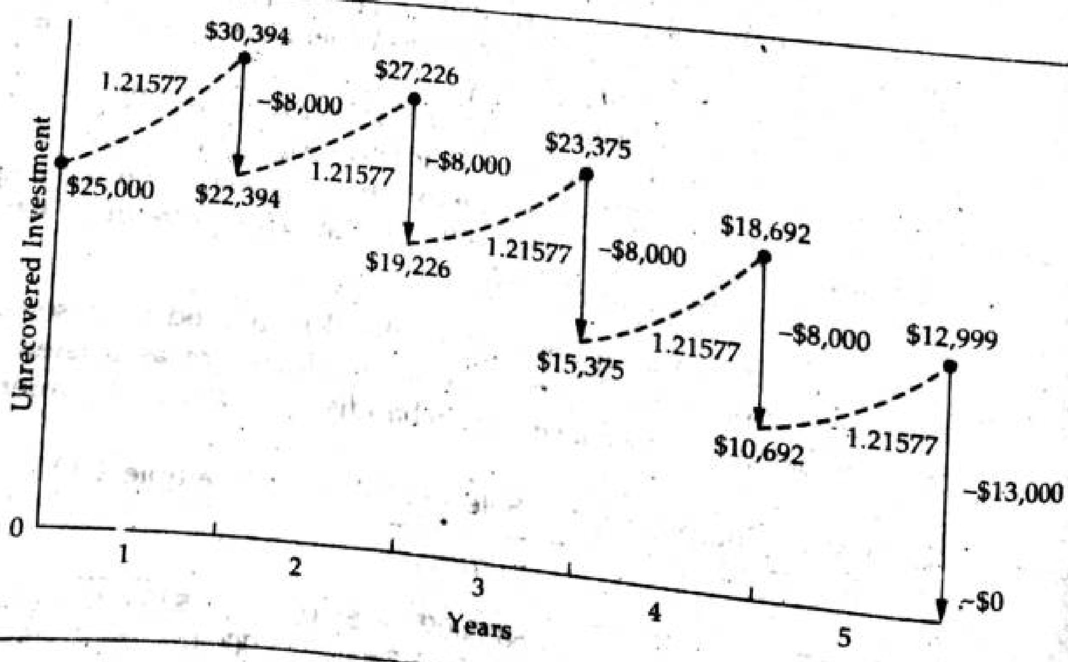


FIGURE 4-10 Unrecovered Investment Balance Diagram for Example 4-12

EXAMPLE 4-13

The Fly-by-Night finance company advertises a "bargain 6% plan" for financing the purchase of automobiles. To the amount of the loan being financed, 6% is added for each year money is owed. This total is then divided by the number of months over which the payments are to be made, and the result is the amount of the monthly payments. For example, a woman purchases a \$10,000 automobile under this plan and makes an initial cash payment of \$2,500. She wishes to pay the balance in 24 monthly payments.

Purchase price	= \$10,000
− Initial payment	= 2,500
= Balance due, (P)	= 7,500
+ 6% finance charge = 0.06 × 2 years × \$7,500	= 900
= Total to be paid	= 8,400
∴ Monthly payments = \$8,400/24	= \$ 350

What rate of interest does she actually pay?

Solution Because there are to be 24 payments of \$350 each, made at the end of each month, these constitute an annuity (A) at some unknown rate of interest, $i'\%$. that should be computed only upon the unpaid balance. In this example, the amount owed on the automobile (i.e., the unpaid balance) is \$7,500. Letting P_0 = \$7,500, the following equivalence expression is utilized to compute the unknown monthly interest rate:

$$P_0 = A(P/A, i'\%, N)$$
$$\$7{,}500 = \$350(P/A, i'\%, 24)$$
$$(P/A, i'\%, 24) = \frac{\$7{,}500}{\$350} = 21.43$$

By examination of the interest tables for P/A factors at $N = 24$ that come closest to 21.43, one finds that $(P/A, 3/4\%, 24) = 21.8891$ and $(P/A, 1\%, 24) = 21.2434$. Linear interpolation gives

$$i'\% = 1\% - \frac{21.43 - 21.2434}{21.8891 - 21.2434}(1\% - 3/4\%) = 0.93\%$$

Since payments are monthly, 0.93% is the interest rate being charged per month. The nominal rate paid on the borrowed money is 0.93%(12) = 11.16% compounded monthly. This corresponds to an effective annual interest rate of $[(1 + 0.0093)^{12} - 1] \times 100 \cong 12\%$. What appeared at first to be a real bargain turns out to involve effective annual interest at recent typical rates for automobile loans. ■

EXAMPLE 4-14

A small company needs to borrow \$160,000. The local (and only) banker makes this statement: "We can loan you \$160,000 at a very favorable rate of 12% per

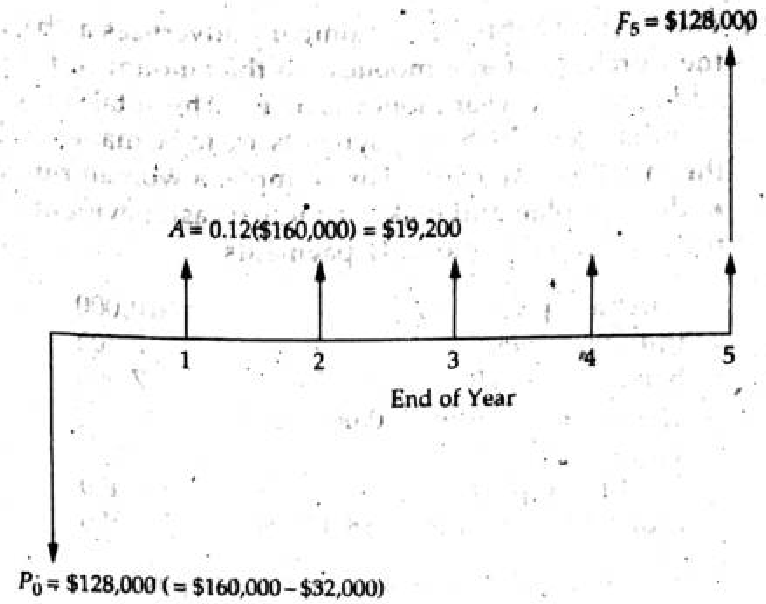


FIGURE 4-11 Cash Flow Diagram for Example 4-14

year for a 5-year loan. However, to secure this loan you must agree to establish a checking account (with no interest) in which the *minimum* average balance is $32,000. In addition, your interest payments are due at the end of each year and the principal will be repaid in a lump-sum amount at the end of year 5." What is the true effective interest rate being charged?

Solution The cash flow diagram from the banker's viewpoint appears in Figure 4-11. When solving for an unknown interest rate, it is good practice to draw a *cash flow diagram* prior to writing an equivalence relationship. The interest rate (IRR) that establishes equivalence of positive and negative cash flows can now easily be computed:

$$P_0 = F_5(P/F, i'\%, 5) + A(P/A, i'\%, 5)$$
$$\$128{,}000 = \$128{,}000(P/F, i'\%, 5) + \$19{,}200(P/A, i'\%, 5)$$

If we try $i' = 15\%$, we discover that $128,000 = $128,000. Therefore, the true effective interest rate is 15% per year. ■

4.6.1 Difficulties Associated with the IRR Method

The PW, AW, and FW methods utilize the assumption that net receipts less expenses (positive recovered funds) each time period are reinvested at the MARR during the study period, N. However, the IRR method is based on the assumption that recovered funds, if not consumed in each time period, are reinvested at $i'\%$ rather than at the MARR. This latter assumption may not mirror reality in some problems, thus making IRR an unacceptable method for analyzing engineering alternatives.*

Other difficulties with the IRR method include its computational intractability and the occurrence of multiple IRRs in some types of problems. A computational aid for determining the IRR for a single alternative having both positive and negative cash flows or for cash flow differences between two alternatives is given in Appendix 4-A. Two procedures for dealing with multiple rates of return are discussed and demonstrated in Appendix 4-B. Generally speaking, multiple rates are not meaningful for decision-making purposes, and another method of evaluation must be utilized.

Another possible drawback to the IRR method is that it must be carefully applied and interpreted in the analysis of two or more alternatives when only one of them is to be selected (i.e., mutually exclusive alternatives). This is discussed further in Chapter 5. The key advantage of the method is its widespread acceptance by industry, where various types of rates of return and ratios are routinely used in making project selections.

4.7 The Benefit/Cost Ratio Method

The benefit/cost (B/C) ratio method of economic evaluation was made popular by the Flood Control Act passed by Congress in 1936. Historically, the B/C ratio method has been widely used to assess large-scale public projects, but this method is also employed in the private sector.

Basically, an engineering economy study of a public project is no different from any other economy study. Any of the methods discussed in Chapter 4 could be used for such a study. Because profit almost never is involved and most public projects have multiple benefits, some of which cannot be measured precisely in terms of dollars, economy studies of public projects tend to be made by determining the ratio of the equivalent worth of benefits to the equivalent worth of costs at an effective interest rate of i% per period. Accordingly, the B/C ratio is defined as the ratio of the equivalent worth of benefits to the equivalent worth of costs. The equivalent worth utilized is customarily present worth (PW) or annual worth (AW), but it can also be future worth (FW). The B/C ratio is also referred to as the *savings–investment ratio* (*SIR*) by some governmental agencies.

Two commonly used formulations of the B/C ratio (here expressed in terms of AW) are as follows:

*See H. Bierman and S. Smidt, *The Capital Budgeting Decision: Economic Analysis of Investment Projects* (New York: Macmillan Publishing Company, 1984). The term *internal* rate of return means that the value of this measure depends only on the cash flows from an investment and not on any assumptions about reinvestment rates: "One does not need to know the reinvestment rates to compute the internal rate of return. However, one may need to know the reinvestment rates to compare alternatives." (p. 60).

1. Conventional B/C ratio:

$$B/C = \frac{\text{AW (benefits of the proposed project)}}{\text{AW (total costs of the proposed project)}} = \frac{\underline{B}}{\text{CR} + (\text{O\&M})} \qquad (4\text{-}12)$$

where AW(·) = annual worth of (·)

$\underline{B}$ = annual equivalent worth of benefits of the proposed project

CR = capital recovery cost (i.e., the equivalent annual cost of the initial investment, I, including an allowance for salvage value, if any).

O&M = equivalent annual operating and maintenance expenses of the proposed project

2. Modified B/C ratio:

$$B/C = \frac{\underline{B} - (\text{O\&M})}{\text{CR}} \qquad (4\text{-}13)$$

The numerator of the modified B/C ratio expresses the equivalent worth of the benefits minus the equivalent annual operating and maintenance costs; the denominator includes only the annual equivalent investment (i.e., CR) costs. A project is acceptable when the B/C ratios as defined in Equations 4-12 and 4-13 are greater than or equal to 1.0.

Both B/C methods give consistent answers to the question of whether the ratio is greater than or equal to 1.0, and both yield the same recommended choice when comparing *mutually exclusive* investment alternatives. As discussed in Chapter 14, an incremental procedure is required when comparing mutually exclusive alternatives with the B/C ratio method. (The reader is now in a position to read Chapter 14.)

EXAMPLE 4-15

Determine the B/C ratio for a pollution-control project with the following data.

First cost of stack gas scrubber	\$20,000
Project life	5 years
Salvage value	\$ 4,000
Annual benefits (reduced emissions)	\$10,000
Annual O&M expenses	\$ 4,400
Interest rate	8%

Solution A cash flow diagram, showing benefits for this project as upward arrows and costs as downward arrows, is provided in Figure 4-12. The conventional B/C ratio and modified B/C ratio, based on AW, are computed as follows:

$$\text{CR} = (\$20{,}000 - \$4{,}000)(A/P, 8\%, 5) + \$4{,}000(0.08) = \$4{,}327$$

$$\text{Conventional B/C ratio} = \frac{\underline{B}}{\text{CR} + (\text{O\&M})} = \frac{\$10{,}000}{\$4{,}327 + \$4{,}440} = 1.146 > 1.0$$

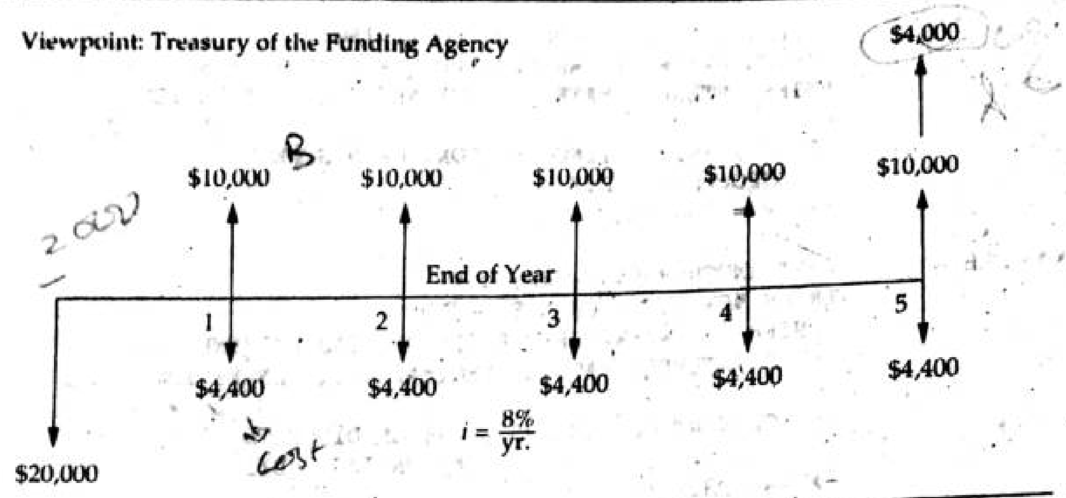


FIGURE 4-12 Cash Flow Diagram for Example 4-15

$$\text{Modified B/C ratio} = \frac{B - (\text{O\&M})}{\text{CR}} = \frac{\$10{,}000 - \$4{,}400}{\$4{,}327}$$
$$= 1.294 > 1.0$$

Since B/C > 1.0 (by either ratio), the stack gas scrubber should be installed. ■

4.8 A Computer Program for Selected Methods

To assist with the solution of engineering economy problems by methods discussed in this and subsequent chapters, a computer program called BTAX is provided in Appendix E. The program is written in BASIC and is suitable for most IBM or compatible personal computers. The results of running this computer program for Examples 4-3, 4-9, and 4-12 are shown here. Other capabilities of BTAX are described in Appendix E.

EXAMPLE 4-3

```
HOW MANY DIFFERENT CASH FLOW VALUES ARE THERE ? 3
ENTER CASH FLOW, FIRST PERIOD, LAST PERIOD
(SEPARATE BY COMMA)
? -25000,0,0
? 8000,1,4
? 13000,5,5

YEAR     CASH FLOW
----     ----------
0        -25,000.00
1          8,000.00
2          8,000.00
3          8,000.00
4          8,000.00
5         13,000.00
```

```
WANT TO MAKE ANY CORRECTIONS (Y/N) ? N

SELECT A METHOD FROM THE FOLLOWING:
(ENTER PW/FW/AW/IRR/ERR/SPP/DPP/TABLE/ ?/END):PW
ENTER INTEREST RATE AS DECIMAL (e.g.  ENTER 10%
                                      AS .1) = .2

DO YOU WANT CONTINUOUS (CON) OR DISCRETE (DIS)
  COMPOUNDING? DIS
-> PW = 934.2832
```

EXAMPLE 4-9

```
WANT ANOTHER RUN (Y/N) ? Y
ENTER NEW/OLD/?/CHANGE ? OLD
(ENTER PW/FW/AW/IRR/ERR/SPP/DPP/TABLE/?/END):AW
ENTER INTEREST RATE AS DECIMAL (e.g. ENTER 10%
                                     AS .1) = .2

DO YOU WANT CONTINUOUS (CON) OR DISCRETE (DIS)
                        COMPOUNDING ? DIS

-> AW = 312.405
```

EXAMPLE 4-12

```
WANT ANOTHER RUN (Y/N) ? Y
ENTER NEW/OLD/?/CHANGE ? OLD
(ENTER PW/FW/AW/IRR/ERR/SPP/DPP/TABLE/ ?/END):IRR
DO YOU WANT DISCRETE (DIS) OR CONTINUOUS (CON)
                        INTEREST RATE ? DIS

-> IRR IS BETWEEN 21.577 AND 21.578 %
```

4.9 The Payback (Payout) Period Method

All methods presented thus far reflect the *profitability* of a proposed alternative for a study period of N. The payback method, which is often called the *simple payout method*, mainly indicates a project's *liquidity* rather than its profitability. Historically, the payback method has been used as a measure of a project's riskiness, since liquidity deals with how fast an investment can be recovered. A low-valued payback period is considered desirable. Quite simply, the payback method calculates the number of years required for positive cash flows to just equal the total investment, I. Hence the simple payback period is the *smallest* value of θ ($\theta \leq N$) for which this relationship is satisfied under end-of-year cash flow convention:

$$\sum_{k=1}^{\theta}(R_k - E_k) - I \geq 0 \tag{4-14}$$

The *simple* payback period, θ, ignores the time value of money and all cash flows that occur after θ. If this method is applied to the investment project in Example 4-11, the number of years for payout can be calculated to be

$$\sum_{k=1}^{\theta=4.33}(\$5{,}310 - \$3{,}000)_k - \$10{,}000 \geq 0$$

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When $\theta = N$ (the last time period in the planning horizon), the salvage value is included in the determination of a payback period. As can be seen from Equation 4-14, the payback period does not indicate anything about project desirability except the speed with which the investment will be recovered. The payback period can produce misleading results, and it is definitely not recommended except as supplemental information in conjunction with one of the five methods previously discussed.

It is informative to plot undiscounted cash flows against time in order to see the simple payback period, θ. For Example 4-3, this is shown in Figure 4-5 as the 0% plot.

Sometimes the *discounted* payback period, $\theta'(\theta' \leq N)$, is calculated so that the time value of money is considered:

$$\sum_{k=1}^{\theta'} (R_k - E_k)(P/F, i\%, k) - I \geq 0 \tag{4-15}$$

a project in view of the time value of money. However, neither payback period calculation includes cash flows occurring after θ (or θ'). This means that they do not take into consideration the economic life of physical assets. Thus, these methods will be misleading if one alternative that has a longer (less desirable) payout period than another produces a higher rate of return (or net PW) on the invested capital.

The use of the payout period for making investment decisions should be avoided except as a measure of how quickly invested capital will be recovered, which is an indicator of project risk. For instance, the breakeven life of the investment described in Table 4-1 is 5 years (in column C), which corresponds to the discounted payback period at $i = 20\%$. Payback periods of 3 years or less are often desired in U.S. industry, so under this criterion the equipment in Example 4-3 would be regarded as too risky even though its profitability is acceptable.

EXAMPLE 4-16

Considering the alternative presented in Example 4-12, calculate the simple payback period and the discounted payback period. In view of a minimum acceptable payback period of 5 years, is this alternative attractive?

Solution

Payback period:

$$\left[\sum_{k=1}^{\theta=3.13} (\$8{,}000)_k\right] - \$25{,}000 \approx 0$$

Thus $\theta = 3.13$ years and the alternative is acceptable. (With the end-of-year cash flow convention, this value of θ would be rounded up to 4 years.) Notice that the $5,000 salvage value was not considered in Equation 4-14 to determine the simple payback period.

Discounted payback period:

$$\left[\sum_{k=1}^{\theta'=5} (\$8{,}000)_k(P/F, 20\%, k)\right] + \$5{,}000(P/F, 20\%, 5) - \$25{,}000 \geq 0$$

or

$$\$8{,}000(P/A, 20\%, 5) + \$5{,}000(P/F, 20\%, 5) - \$25{,}000 > 0$$

at the project's expected life of $N = 5$ years. The alternative is minimally acceptable with the discounted payback method since $\theta' = 5$ years. In this case, the salvage value was considered in determining the value of θ'. Observe that when $\theta' = 4$, Equation 4-15 is not satisfied. ■

4.10 An Example of a Proposed Investment to Reduce Costs

Many engineering projects are aimed at reducing costs rather than enhancing revenues. Example 4-17 illustrates a cost reduction opportunity, and Example 4-18 describes a large industrial revenue creation proposal.

EXAMPLE 4-17

A manufacturer of jewelry is contemplating the installation of a system that will recover a larger proportion of the fine particles of gold and platinum that result from the various manufacturing operations. At the present time a little over $45,000 worth of these metals is being lost per year, and it is anticipated that, because of the growth of the company, this amount will increase by $5,000 each year for the next 10 years (the study period). The proposed system, involving a network of exhaust ducts and separators, will recover approximately two-thirds of the gold and platinum that otherwise would be lost. The complete installation would cost $140,000. The best estimates for the operating costs of the system, obtained from operations of similar systems, are $10,000 per year for operating expense, $1,800 per year for maintenance and repairs, and 2% of the first cost annually for property taxes and insurance. The company would require the investment to be recovered with interest in 10 years. The MARR of the company, before taxes, is 15%. Should the recovery system be installed?

Solution Such an investment is made to reduce some of the operating expenses, in this case the cost of the material used. Thus, the saving (income) to be obtained by making an investment is almost entirely within the control of the investors. The company knows exactly what expenses have been. If the efficiency of the proposed equipment is known, the only factors that should affect the saving are the variation of production, operation, and maintenance expenses of the proposed equipment. In most cases of this type, these items are known or may be predicted quite accurately. The company would have a good idea of how its volume might vary. Operation and maintenance expenses can usually be estimated accurately, especially if historical data are available on the proposed equipment.

Using the data given, we find that the PW and IRR calculations would be as shown in Table 4-4. In deciding whether or not the calculated internal rate of return of 16.8% is sufficient to justify the investment, each factor that might contribute to the risk must be examined so that a measure of the total risk may be obtained. In this case it appears that the factors are quite well controlled or known. There is little reason to believe that much more risk would be involved than is present in all the normal operations of the company. Thus, the company can use its own experience as a basis of comparison. If the company is sound and its business is quite stable, a return of 16.8% coupled with a simple payback period of 6 years should be satisfactory. ■

It may be seen that when capital is invested in a going concern in order to reduce costs, the risk is usually easier to assess and is often much less than when entirely new enterprises are involved. As a result, the rate of return required for such investments is often lower.

EXAMPLE 4-17A

Using the cash flow data of Example 4-17, determine the (a) IRR, (b) PW at 15%, (c) simple payback period, and (d) discounted payback period at 15% with the computer program (BTAX) in Appendix E.

Solution

```
PLEASE ANSWER THE FOLLOWING QUESTIONS:

HOW MANY DIFFERENT CASH FLOW VALUES ARE THERE? 11
ENTER CASH FLOW, FIRST PERIOD, LAST PERIOD
(SEPARATE BY COMMA)
? -140000,0,0
? 18733,1,1
? 22067,2,2
? 25400,3,3
? 28733,4,4
? 32067,5,5
? 35400,6,6
? 38733,7,7
? 42067,8,8
? 45400,9,9
? 48733,10,10

WANT TO MAKE ANY CORRECTIONS ON CASH FLOW
                                  (Y/N) ? N
```

Table 4-4 Tabular Determination of PW and IRR[a] for the Proposed System in Example 4-17

Year End, N	Investment	Recovery	Costs	Net Cash Flow	(P/F, 15%, N)	Present Worth at i = 15%	(P/F, 20%, N)	Present Worth at i = 20%
0	−$140,000	—	—	−$140,000	1.000	−$140,000	1.0000	−$140,000
1	—	$33,333	−$14,600	18,733	0.8696	16,290	0.8333	15,610
2	—	36,667	− 14,600	22,067	0.7561	16,685	0.6944	15,323
3	—	40,000	− 14,600	25,400	0.6575	16,701	0.5787	14,699
4	—	43,333	− 14,600	28,733	0.5718	16,430	0.4823	13,858
5	—	46,667	− 14,600	32,067	0.4972	15,944	0.4019	12,888
6	—	50,000	− 14,600	35,400	0.4323	15,303	0.3349	11,855
7	—	53,333	− 14,600	38,733	0.3759	14,560	0.2791	10,810
8	—	56,667	− 14,600	42,067	0.3269	13,752	0.2326	9,785
9	—	60,000	− 14,600	45,400	0.2843	12,907	0.1938	8,799
10	—	63,333	− 14,600	48,733	0.2472	12,047	0.1615	7,870
Total				$197,333		$ 10,619		−$ 18,503

[a] IRR $\cong$ 15% + [$10,619/(10,619 + 18,503)] × (20% − 15%) $\cong$ 16.8%.

(A)

```
SELECT A METHOD FROM THE FOLLOWING:
(ENTER PW/FW/AW/IRR/ERR/SPP/DPP/TABLE/
                                        ?/END):IRR
DO YOU WANT DISCRETE (DIS) OR CONTINUOUS (CON)
                               INTEREST RATE? DIS
-> IRR IS BETWEEN 16.64 AND 16.641 %
```

(B)

```
WANT ANOTHER RUN (Y/N) ? Y
ENTER NEW/OLD/?/CHANGE ? OLD
(ENTER PW/FW/AW/IRR/ERR/SPP/DPP/TABLE/
                                        ?/END):PW
ENTER INTEREST RATE AS DECIMAL (e.g. ENTER 10%
                                   AS .1) = .15
DO YOU WANT CONTINUOUS (CON) OR DISCRETE (DIS)
                                 COMPOUNDING? DIS
-> PW = 10616.36
```

(C)

```
WANT ANOTHER RUN (Y/N) ? Y
ENTER NEW/OLD/?/CHANGE ? OLD
(ENTER PW/FW/AW/IRR/ERR/SPP/DPP/TABLE/
                                        ?/END):SPP
-> SIMPLE PAYBACK PERIOD = YEAR 6
```

(D)

```
WANT ANOTHER RUN (Y/N) ? Y
ENTER NEW/OLD/?/CHANGE / OLD
(ENTER PW/FW/AW/IRR/ERR/SPP/DPP/TABLE/
                                        ?/END):DPP
ENTER INTEREST RATE AS DECIMAL (e.g. ENTER 10%
                                   AS .1) = .15
DO YOU WANT CONTINUOUS (CON) OR DISCRETE (DIS)
                                 COMPOUNDING? DIS
-> DISCOUNTED PAYBACK PERIOD = YEAR 10

WANT ANOTHER RUN (Y/N) ? N
```

■

4.11 An Example of a Large Industrial Investment Opportunity

The following example illustrates the before-tax cash flow analysis of a typical industrial plant expansion problem. It involves many types of cash flows that are summarized in a large cash flow table. Particular attention is directed to variable, fixed, and sunk costs that are present in addition to expenditures that are *capitalized* (e.g., investments) versus those such as disbursements for materials that are *expensed*.

EXAMPLE 4-18

1. PROBLEM STATEMENT Product X-21, a food preservative, is manufactured in a facility with a nominal capacity of 180,000 pounds per year. The ultimate capacity of the facility can be increased, at some loss in cost efficiency, by process modifications and the use of extensive amounts of overtime, to 195,000 pounds per year. With the present marketing strategy, sales are expected to level off at about 190,000 pounds per year in 1995 and to remain at that volume. Therefore, the existing capacity will be sufficient under the present marketing strategy.

However, an opportunity has become available to enter a new market for an X-21 product modified slightly to give it better warm weather stability. A German firm has a trade secret on the modifying process but is willing to enter into a nondisclosure agreement for a one-time fee of $75,000. A proposal has been submitted to management to expand the existing X-21 plant to permit the company to enter this new market. *Management has requested an economic analysis of this proposal and a recommendation.* Your assignment is to provide this recommendation.

II. GENERAL BACKGROUND INFORMATION

A. ***Demand schedule:*** The demand schedule for X-21 is as follows:

Year	*Total Demand (lb)*	*Incremental Sales from This Project (lb)*
1994	175,000	0
1995	190,000	0
1996	220,000	30,000
1997	250,000	60,000
1998–2010	260,000	70,000

Both the modified and regular grades of X-21 will be sold at the same price of $39.50 per pound. All demand over 190,000 pounds per year is attributable to the new market.

B. ***Existing manufacturing facility:*** The existing facility consists of two production lines, each with an ultimate capacity of 97,500 pounds per year. A new line of the same capacity can be added in an existing building, which because of federal restrictions and its unique layout, cannot be used for any other purpose. That is, no cost need be allocated for the building space. The new line can be used for both the regular X-21 and the modified X-21.

C. ***Expansion plan:*** The total installed cost of the new line is estimated at $430,000, assuming project operation by January 1, 1996. Of this total, engineering expenditures of $40,000 would be required in 1994 and all other capital costs would be incurred in 1995.

D. ***Project life:*** The company considers all projects of this type and degree of risk to have a 15-year life. Thus, commercial operation of the new production line would begin in early 1996 and terminate at the end of year 2010.

E. ***Inflation:*** The analysis will ignore the effects of inflation (in Chapter 9, this topic is considered).

F. ***Federal and state income taxes:*** The analysis will ignore all income taxes (income taxes are considered in Chapter 7).

G. ***Interest rate and financing:*** The minimum acceptable before-tax rate of return for projects with this degree of risk is 20% per year (effective). (The financing decision for large projects of this type is discussed in Chapter 13.)

H. ***Other profitable company operations:*** These will offset any short-term negative cash flows.

I. ***Expenditures:*** Those occurring throughout a year will be assumed for cash flow purposes to occur at the end of the year.

III. CASH FLOW DATA

A. ***Capitalizable project expenditures:*** A total of \$430,000 will be required to install the new production line. In 1994, \$40,000 is needed, and the remainder of \$390,000 would be expected in 1995. (Capitalizable expenditures are those expenditures that must be depreciated using the methods discussed in Chapter 6.)

B. ***Project expenditures chargeable to operations:*** Certain expenditures will result from the dismantlement and rearrangement of special equipment that will be required to fit the new line into the existing building. Estimated before-tax cost for this work is \$48,600.

C. ***Related projects and work orders:*** Various types of auxiliary equipment will have to be replaced at 5-year intervals. Therefore, equipment replacement expenditures of \$120,000 are planned for 2000 and 2005.

D. ***Net change in working capital:*** All firms require capital for day-to-day operations, including funds to support prepaid expenses, inventories, and bank balances. Such *working capital* has associated with it an opportunity cost that in many cases is a substantial item in an engineering economy study. Working capital caused by a project is normally treated as an asset whose first cost and salvage value are equal. Thus, the annual opportunity cost (i) of working capital in year k, WC_k, is iWC_k.

The change in working capital should be roughly proportional to the change in sales. Working capital in 1993 was approximately \$200,000 when sales were 164,000 pounds. The following *net changes in working capital* are computed using this proportionality:

Year	*Net Change from Previous Year*
1994	\$13,400
1995	18,300
1996	36,600
1997	36,600
1998	12,200
1999–2010	0

Only changes in 1996 and later are pertinent to the analysis.

E. ***Prepaid know-how:*** A single payment of \$75,000 to the German firm is required before startup. This payment will be made at the end of 1995.

F. ***Pretax cash flows from commercial operation (incremental due to the new line):***

1. ***Fixed costs:*** Fixed costs for supervisory salaries, general plant overhead, insurance, and property taxes will increase by $750,000 per year if this project is implemented. This increase will remain constant over the 15-year life of the project.
2. ***Variable costs:*** These costs consist primarily of raw materials costs, direct labor, utilities, and the variable portion of maintenance costs. Variable costs are expected to be $9.18 per pound in 1996. As operating experience is gained, yields should increase, resulting in variable costs of $8.83 per pound in 1997 and $7.56 per pound in 1998 and thereafter.
3. ***General and administrative (G and A) costs:*** G and A costs are budgeted for $900,000 in 1996 when the product is first introduced. After 1996, G and A costs are budgeted for $1,100,000 per year. This amounts to about 40% of sales revenue after sales have leveled out.

The following table summarizes the annual before-tax cash flows in the years 1996–2010 attributable to this proposed project.

Year	(1) *Fixed Cost (thousands)*	(2) *Variable Cost (thousands)*	(3) *G and A Costs (thousands)*	*Sales Revenues (thousands)*	*Sales Revenue Less Costs in Columns 1–3 (thousands)*
1996	$750	$275.4	$ 900	$1,185	−$740.4
1997	750	529.8	1,100	2,370	− 9.8
1998–2010	750	529.2	1,100	2,765	385.8

G. ***Introductory costs:*** Introductory costs will be relatively small. The only significant introductory cost is training personnel in modifications to the process and technical assistance during the startup. The total introductory cost in 1995 is estimated at $8,500.

H. ***Sales Revenues:*** During 1996, sales revenues are expected to be $39.50/pound × 30,000 pounds/year = $1,185,000. They level out to $39.50 × 70,000 pounds/year = $2,765,000 from 1998 through 2010.

IV. ANALYSIS RESULTS AND RECOMMENDATIONS

A cash flow worksheet for this example is given in Table 4-5 (pp. 169–70). From the before-tax cash flows in column (9), the project's IRR can be calculated based on Equation 4-9.

$$0 = -\$40{,}000 - \$522{,}100(P/F, i'\%, 1) - \$777{,}000(P/F, i'\%, 2) - \cdots + \$471{,}200(P/F, i'\%, 16)$$

By trial and error, $i' = 18.3\% <$ MARR. Because this determination is quite tedious, it is suggested that the BTAX computer program in Appendix E be employed to verify the project's IRR.

The before-tax MARR is 20% and the PW at the end of 1994 is −$87,137. This may also be confirmed by utilizing BTAX in Appendix E. Thus, the IRR and

Table 4-5 Cash Flow Worksheet for Example 4-18

End of Year	(1) Capitalizable Project Expenditures	(2) Expenditures Chargeable to Operations	(3) Related Projects and Work Orders	(4) Net Change in Working Capital	(5) Prepaid Know-How
1994	−\$ 40,000				
1995	− 390,000	−\$48,600			−\$75,000
1996				−\$36,600	
1997				− 36,600	
1998				− 12,200	
1999					
2000			−\$120,000		
2001					
2002					
2003					
2004					
2005			− 120,000		
2006					
2007					
2008					
2009					
2010				+ 85,400[a]	

[a]This is return of working capital at the end of the project.

(Continued)

PW measures of profitability indicate that the company should *not* enter the new market for a modified X-21 product. Even if these criteria had signaled a favorable project (e.g., IRR ≥ 20%), a detailed analysis of uncertainty should be performed and nonmonetary considerations analyzed before a decision is made. These topics are addressed in Chapters 10 and 11, respectively.

4.12 Summary

Throughout this chapter we have examined five basic methods for evaluating the financial *profitability* of a single project: present worth, annual worth, future worth, internal rate of return, and benefit/cost ratio. Two supplemental methods for assessing a project's *liquidity* were also presented: the simple payback period and the discounted payback period. Computational procedures, assumptions, and acceptance criteria for all methods were discussed and illustrated with examples. Appendix B provides a listing of new abbreviations and notation that have been introduced in this chapter.

TABLE 4-5 (Continued)

End of Year	(6) Total Investment Cash Flow: Cols. 1+2+3 +4+5	(7) Before-Tax Cash Flows During Commercial Operation	(8) Introductory Costs	(9) Before-Tax Cash Flows for the Entire Project: Cols. 6+7+8
				−$ 40,000
1994	−$ 40,000		−$8,500	− 522,100
1995	− 513,600			− 777,000
1996	− 36,600	−$740,400		− 46,400
1997	− 36,600	− 9,800		373,600
1998	− 12,200	385,800		385,800
1999		385,800		265,800
2000	− 120,000	385,800		385,800
2001		385,800		385,800
2002		385,800		385,800
2003		385,800		385,800
2004		385,800		265,800
2005	− 120,000	385,800		385,800
2006		385,800		385,800
2007		385,800		385,800
2008		385,800		385,800
2009		385,800		385,800
2010	+ 85,400[a]	− 385,800		− 471,200

[a]This is return of working capital at the end of the project.

4.13 References

CANADA, J. R., and J. A. WHITE. *Capital Investment Decision Analysis for Management and Engineering*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1980.

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THUESEN, G. J., and W. J. FABRYCKY. *Engineering Economy*, 7th ed. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1989.

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4.14 Problems

Unless stated otherwise, discrete compounding of interest and end-of-period cash flows should be assumed in all problem exercises in the remainder of the book. The number in parentheses at the end of a problem refers to the chapter section most closely related to the problem.

4-1. In your own words, what is the meaning of
- **a.** present worth? (4.3)
- **b.** capital recovery cost? (4.5)
- **c.** minimum attractive rate of return? (4.2)

4-2. You are faced with a decision on an investment proposal. Specifically, the estimated additional income from the investment is \$180,000 per year; the initial investment costs are \$640,000; and the estimated annual costs are \$44,000, which begin decreasing by \$4,000 per year starting at the end of the third year. Assume an 8-year analysis period, no salvage value, and MARR = 15%. (4.3, 4.6)
- **a.** What is the PW of this proposal?
- **b.** What is the IRR of this proposal?

4-3.
- **a.** Evaluate machine XYZ on the basis of the PW method when the MARR is 12%. Pertinent cost data are as follows: (4.3)

	Machine XYZ
First cost	\$13,000
Useful life	15 years
Salvage value	\$ 3,000
Annual operating costs	100
Overhaul—end of fifth year	200
Overhaul—end of tenth year	550

- **b.** Determine the capital recovery cost of machine XYZ by all three formulas presented in the text. (4.4)

4-4.
- **a.** Determine the PW and AW of the following proposal when the MARR is 15%. (4.3, 4.4)

	Proposal A
First cost	\$10,000
Expected life	5 years
Salvage value[a]	−\$ 1,000
Annual receipts	8,000
Annual expenses	4,000

[a] A negative salvage value means that there is a net cost to dispose of an asset.

- **b.** Determine the IRR for proposal A. Is it acceptable? (4.6)
- **c.** What is the B/C ratio for this proposal? (4.7)

4-5. A company is considering constructing a plant to manufacture a proposed new product. The land costs \$300,000, the building costs \$600,000, the equipment costs \$250,000, and \$100,000 working capital is required. It is expected that the product will result in sales of \$750,000 per year for 10 years, at which time the land can be sold for \$400,000, the building for \$350,000, and the equipment for \$50,000 and all of the working capital recovered. The annual out-of-pocket expenses for labor, materials, and all other items are estimated to total \$475,000. If the company requires an MARR of 25% on projects of comparable risk, determine if it should invest in the new product line. Use the PW method. (4.3)

4-6. Uncle Wilbur's trout ranch is now for sale for \$40,000. Annual property taxes, maintenance, supplies, and so on are estimated to continue to be \$3,000 per year. Revenues from the ranch are expected to be \$10,000 next year and then to decline by \$500 per year thereafter through the tenth year. If you bought the ranch, you would plan to keep it for only 5 years and at that time to sell it for the value of the land, which is \$15,000. If your desired annual rate of return is 12%, should you become a trout rancher? Use the PW method. (4.3)

4-7. A friend of yours just purchased a \$10,000 bond that was discounted to sell for \$7,500. It is a 6% bond with interest payable annually that matures in 7 years. In exactly 3 years, your friend plans to sell the bond at a price that gives the buyer a 12% interest rate compounded annually. For how much will your friend sell the bond? (4.3)

4-8. How much can be paid a \$5,000, 14% bond, with interest paid semiannually, if the bond matures 12 years hence? Assume that the purchaser will be satisfied with 9% nominal interest compounded semiannually. (4.3)

4-9. A 20-year bond with a face value of \$5,000 is offered for sale at \$3,800. The rate of interest on the bond is 7%, paid semiannually. This bond is now 10 years old (i.e., the owner has received 20 semiannual interest payments). If the bond is purchased for \$3,800, what effective rate of interest would be realized on this investment opportunity? (4.3)

4-10.
- **a.** A company has issued 10-year bonds, with a face value of \$1,000,000, in \$1,000 units.

Interest at 8% is paid quarterly. If an investor desires to earn 10% nominal interest on $10,000 worth of these bonds, what would the selling price have to be?

b. If the company plans to redeem these bonds in total at the end of 10 years and establishes a sinking fund that earns 8%, compounded semiannually, for this purpose, what is the *annual* cost of interest and redemption? (4.3)

4-11. You bought a $1,000 bond at par that paid interest at the rate of 7%, payable semiannually, and held it for 10 years. You then sold it at a price that resulted in a yield of 10% nominal on your capital. What was the selling price? (4.3)

4-12. Based on a desired nominal rate of return (yield) of 6%, a bond is purchased for $5,000. The bond matures in 2 years and has a stated rate of 12% nominal interest. What is the face value of the bond? Assume that face value and redemption value are equal. (4.3)

a. $5,000
b. $4,500
c. $4,876
d. $1,652
e. $2,640

4-13. A small company bought BMI bonds at their face value on January 1, 1981. These bonds pay nominal interest of 7.25% every 6 months (14.5% per year). The face value of the bonds is $100,000, and they mature on December 31, 1996. On January 1, 1991, these bonds were sold for $120,000. What IRR (per 6 months) was earned by the company on the BMI bonds? (4.3)

4-14. Susie Queue has a $100,000 mortgage on her deluxe townhouse in urban Philadelphia. She makes monthly payments on a 10% (nominal interest per year) loan and has a 30-year mortgage. Home mortgages are presently available at an 8% nominal interest on a 30-year loan. Susie has lived in the townhouse for only 2 years, and she is considering refinancing her mortgage at 8% nominal interest. The mortgage company informs her that the one-time cost to refinance the present mortgage is $4,500.

How many months must Susie continue to live in her townhouse to make the decision to refinance a good one? Her MARR is the return she can earn on a 30-month certificate of deposit that pays ½% per month (6% nominal interest). (4.3, 4.5)

4-15. On January 1, 1990, your brother bought a used car for $8,200, and he agreed to make a down payment of $1,500 and repay the balance in 36 equal payments, with the first payment due February 1. The interest rate is 13.8%, compounded monthly. During the summer your brother made enough money so that he decided to repay the entire balance due on the car as of September 1. How much did he repay on September 1? (4.3)

4-16. List the advantages and disadvantages of each of these five basic methods for performing engineering economy studies (PW, AW, FW, IRR, B/C).

4-17. The Anirup Food Processing Company is presently using an outdated method for filling 25-pound sacks of dry dog food. To compensate for weighing inaccuracies inherent to this packaging method, the process engineer at the plant has estimated that each sack is overfilled by ⅛ pound on the average. A modern method of packaging is now available that would eliminate overfilling (and underfilling). The production quota for the plant is 300,000 sacks per year for the next 6 years, and a pound of dog food costs this plant $0.15 to produce. The present system has no salvage value and will last another 4 years, and the modern method has an estimated life of 4 years with a salvage value equal to 10% of its investment cost, I. It is also estimated that the present packaging operation will cost $2,100 per year more to maintain than the modern method. If the MARR is 12% for this company, what amount, I, could be justified for the purchase of the modern packaging method? (4.3)

4-18. Fill in the table at the top of page 173 when $P = \$10{,}000$, $S = \$2{,}000$, and $i = 15\%/\text{yr}$. Complete the table and show that the equivalent uniform CR cost equals $3,102.12. (4.5)

4-19.

a. A certain service can be performed satisfactorily by process R, which has a first cost of $8,000, an estimated life of 10 years, no salvage value, and annual net receipts (revenues − expenses) of $2,400. Assuming a MARR of 18% before income taxes, find the AW of this process and specify whether you would recommend it. (4.5)

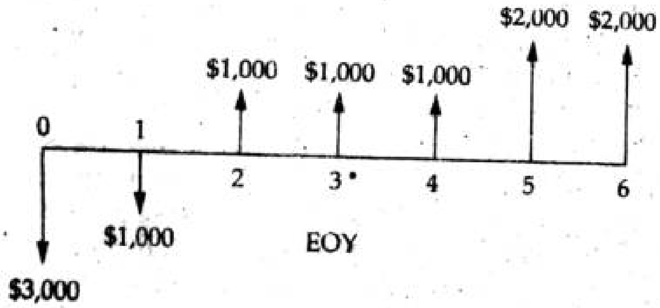
Year	*Investment at Beginning of Year*	*Opportunity Cost of Interest (i = 15%)*	*Loss in Value of Asset During Year*	*Capital Recovery Cost*
1	$10,000		$8000	
2			$2000	
3			$2000	
4				

(Pr. 4-18).

b. A compressor that costs $2,500 has a 5-year useful life and a salvage value of $1,000 after 5 years. At nominal interest of 10%, compounded quarterly, what is the *annual* CR cost of the compressor? (4.5)

4-20. You purchased a building 5 years ago for $100,000. Its annual maintenance cost has been $5,000 per year. At the end of 3 years, you have spent $9,000 on roof repairs. At the end of 5 years, you sell the building for $120,000. During the period of ownership, you rented the building for $10,000 per year paid at the *beginning* of each year. Use the annual worth method to evaluate this investment when your MARR is 12% per year. (4.5)

4-21. Based on the following cash flow diagram, answer the following questions (4.3, 4.5, 4.9)



a. As $i \to \infty$, the PW equals ________.

b. The discounted payback period is ________ years. Let MARR = 15%.

c. If the cash flow at the end of year 6 had been −$2,000 instead of +$2,000, AW (0%) = ________.

4-22. A manufacturing firm has considerable excess capacity in its plant and is seeking ways to utilize it. The firm has been invited to submit a bid to become a subcontractor on a product that is not competitive with the one it produces but that, with the addition of $75,000 in new equipment, could readily be produced in its plant. The contract would be for 5 years at an annual output of 20,000 units.

In analyzing probable costs, direct labor is estimated at $1.00 per unit and new materials at $0.75 per unit. In addition, it is discovered that in each new unit one pound of scrap material can be used from the present operation, which is now selling for $0.30 per pound of scrap. The firm has been charging overhead at 150% of prime cost, but it is believed that for this new operation the incremental overhead, above depreciation, maintenance, taxes, and insurance on the new equipment, would not exceed 60% of the direct labor cost. The firm estimates that the maintenance expenses on this equipment would not exceed $2,000 per year, and annual taxes and insurance would average 5% of the first cost. (*Note:* Prime cost = direct labor + direct materials cost).

While the firm can see no clear use for the equipment beyond the 5 years of the proposed contract, the owner believes it could be scrapped for $3,000 at that time. He estimates that the project will tie up $15,000 in working capital, and he wants to earn at least a 20% before-tax rate of return on all capital utilized. (4.3, 4.5)

a. What unit price should be bid?

b. Suppose that the purchaser of the product wants to sell it at a price that will result in a profit of 20% of the selling price. What should be the selling price?

4-23. Suppose that you borrow $1,000 from the Easy Credit Company with the agreement to repay it over a 5-year period. Their stated interest rate is 9% per year. They show you the following items in determining the monthly payment: (4.6)

Principal	$1,000
Total interest: 0.09 (5 years) ($1,000)	$ 450

They ask you to pay 20% of the interest immediately, so you leave with $1,000 − $90 = $910 in your pocket. Your monthly payment is calculated as follows:

$$\frac{\$1{,}000 + \$450}{60} = \$24.17/\text{month}$$

a. Draw a cash flow diagram of this transaction.

b. Determine the effective *annual* interest rate.

4-24. Joe Roe is considering establishing a company to produce impellers for water pumps. An investment of $100,000 will be required for the plant and equipment, and $15,000 will be required for working capital. It is expected that the property will last for 15 years, at which time only the working capital part of the investment can be recovered. It is estimated that sales will be $200,000 per year and that operating expenses will be as follows:

Materials	$40,000 per year
Labor	$70,000 per year
Overhead	$10,000 + 10% of sales per year
Selling expense	$5,000 per year

Joe has a regular job paying $40,000 per year, but he will keep that job even if he establishes this company. If Joe expects to earn at least 15% on his capital, should this investment be made? Use the AW method. (4.5)

4-25. A meat-packing company is considering producing a new product, the entire domestic supply of which is now manufactured by three large companies. A new plant would be required that would cost $200,000 and be built on land that now is owned by the company but not used. The plant would have an expected life of 20 years. Annual costs for labor would be $60,000 and for material $110,000. These would provide for an annual output of 600,000 pounds. This would constitute 25% of the present domestic consumption and would be 80% of the plant capacity. Advertising and other overhead expenses would amount to $50,000 per year. Taxes and insurance would total 3% of the value of the plant. The product is a very stable one and at present is selling for 55 cents per pound. Over a period of years the price has varied by ±15%.

a. Would you recommend that the company go ahead on this project? Capital is available and earns not less than 15%.

b. What is the minimum selling price for the product that would justify the investment? (4.5)

4-26. A machine that is not equipped with a brake "coasts" 30 seconds after the power is turned off upon completion of each workpiece, thus preventing removal of the work from the machine. The time per piece, exclusive of this stopping time, is 2 minutes. The machine is used to produce 40,000 pieces per year. The operator receives $6.50 per hour, and the machine overhead rate is $4.00 per hour. How much could the company afford to pay for a brake that would reduce the stopping time to 3 seconds if it had a life of 5 years? Assume zero salvage value, a cost of capital of 10%, and repairs and maintenance on the brake totaling no more than $250 per year. (4.3)

4-27. The ABC Company recently started producing a new product, in addition to two others it has been producing for several years. One of the major parts of this new product now is being purchased from another company at a cost of $7.00 per unit. One of the officers of the ABC Company believes it would be advisable to purchase the required equipment, at a cost of $7,000, that would permit making this component. This equipment would have a capacity of 7,000 units per year and a useful life of at least 5 years. It could be installed in the existing plant if a small storage shed was built at a cost of $2,000 to make the necessary floor space available. Material costs would be $1.10 per unit and direct labor costs $2.40 per unit. Incremental overhead is 50% of the direct labor cost. There would also have to be an added annual charge of 2% of the first cost of the storage shed to cover taxes and insurance. The company now is purchasing 4,000 of the parts per year.

a. If capital is worth 15%, should the part be purchased or made in the plant?

b. What volume would be required to justify purchasing the equipment? Use the AW method. (4.5)

4-28. Your boss has just presented you with the summary (below) of projected costs and annual receipts (before taxes) for a new product line. He asks you to calculate the before-tax IRR for this investment opportunity. What would you present to your boss, and how would you explain the results of your analysis? (It is widely known that the boss likes to see graphs of present worth versus interest rate for this type of problem.) The company's MARR is 10% per year. (4.6)

End of Year	*Net Cash Flow*
0	−$450,000
1	− 42,500
2	+ 92,800
3	+ 386,000
4	+ 614,600
5	−$202,200

4-29. Rework Problem 4-28 with the B/C method. Should the answers to the two problems be the same? (4.7)

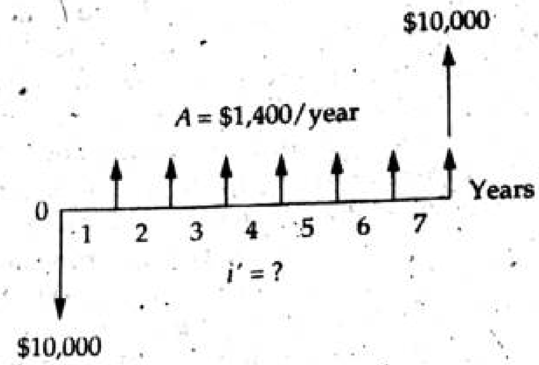
4-30. In order to enter the market to produce a new toy for children, a manufacturer will have to make an immediate investment of $60,000 and additional investments of $5,000 at the end of 1 year and $3,000 more at the end of 2 years. Competing toys are now being produced by two large manufacturers. From a fairly extensive study of the market, it is believed that sufficient sales can be achieved to produce year-end, before-tax net cash flows as follows (investment capital not included):

Year	*Cash Flow*	*Year*	*Cash Flow*
1	−$10,000	6	$26,000
2	5,000	7	26,000
3	5,000	8	20,000
4	20,000	9	15,000
5	21,000	10	7,000

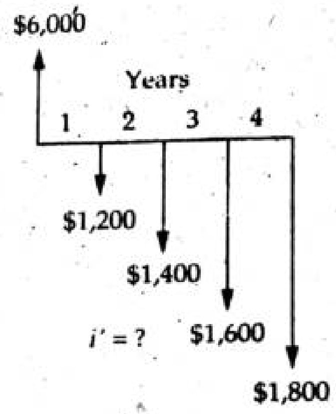
In addition, while it is believed that after 10 years the demand for the toy will no longer be sufficient to justify production, it is estimated that the physical assets would have a scrap value of about $8,000. If capital is worth not less than 12% before taxes, would you recommend undertaking the project? Make a recommendation with each of these methods: (a) FW and (b) discounted payback with a maximum acceptable value of 5 years. (4.4, 4.9)

4-31. Find the IRR in each of these situations: (4.6)

a.



b.



c. You purchased a used car for $4,200. After you make a $1,000 down payment on the car, the salesperson looks in her *Interest Calculations Made Simple* handbook and announces: "The monthly payments will be $160 for the next 24 months and the first payment is due 1 month from now." (Draw a cash flow diagram.)

4-32. Rework part (a) of Problem 4-31 by using the B/C method when MARR = 8% per year. (4.7)

4-33. Plot the PW of part (a) of Problem 4-31 as a function of the interest rate. The MARR is equal to 8%. (4.3)

4-34. Draw an unrecovered investment balance diagram for part (a) of Problem 4-31 using the IRR determined in that problem. (4.6)

4-35. As often quoted, the prepaid premium of insurance policies covering natural disasters (floods, etc.) during a 3-year period is 2.4 times the cost for a single year of coverage. What annual rate of interest is being earned on the extra investment if a 3-year policy is purchased now rather than three 1-year policies at the beginning of each of the years? (4.6)

4-36. A zero-coupon certificate involves payment of a fixed sum of money now with future lump-sum withdrawal of an accumulated amount. Earned interest is not paid out periodically, but instead compounds to become the major component of the accumulated amount paid when the zero-coupon certificate matures. Consider a certain zero-coupon certificate that was issued on March 25, 1993, and matures on January 30, 2010. A person who purchases a certificate for $13,500 will receive a check for $54,000 when the certificate matures. What is the interest rate (yield) that will be earned on this certificate? (4.3)

4-37. A company that sells and installs residential heat pumps has made an estimate of $2,500 for a certain job. Rather than immediately paying cash for the heat pump, the homeowner has the option of making a down payment of $500 and then paying $100 per month for 25 months. If the homeowner elects to use the company's finance plan, what annual effective rate of interest is being paid compared to a cash payment of $2,500? State any assumptions that you make. (4.6)

4-38. An individual approaches the Loan Shark Agency for $1,000 to be repaid in 24 monthly installments. The agency advertises an interest rate of 1.5% per month. They proceed to calculate a monthly payment in the following manner:

Amount requested	$1,000
Credit investigation	25
Credit risk insurance	5
Total	$1,030

Interest: ($1030)(24)(0.015) = $371

Total owed: $1,030 + $371 = $1,401

$$\text{Payment: } \frac{\$1{,}401}{24} = \$58.50$$

What effective annual interest rate is the individual paying? (4.6)

4-39. A small company purchased now for $23,000 will lose $1,200 each year the first 4 years. An additional $8,000 invested in the company during the fourth year will result in a profit of $5,500 each year from the fifth year through the fifteenth year. At the end of 15 years the company can be sold for $33,000.

a. Determine the IRR. (4.6)
b. Calculate the FW if MARR = 12%. (4.4)
c. Calculate the B/C when MARR = 12%. (4.7)

4-40.

a. Monthly amounts of $200 each are deposited into an account that earns 12% nominal interest, compounded quarterly. After 48 deposits of $200 each, what is the *future equivalent* worth of the account? State your assumptions. (4.4)
b. A "Christmas Plan" requires deposits of $10 per week for 52 weeks each year. The stated nominal interest rate is 20%, compounded semiannually. What is the *present equivalent worth* of the plan (beginning of week 1)? (4.3)

4-41. In a certain foreign country, a man who wanted to borrow $10,000 for 1 year was informed that he would have to pay only $2,000 in interest (i.e., a 20% interest rate). The lender stated that the total owed to him, $12,000, would be repaid at the rate of $1,000 per month for 12 months. What was the true effective interest being charged in this transaction? Why is it greater than the apparent rate of 20%? (4.6)

4-42. Determine the effective interest rate per annum being charged in any one of the following loan situations. (4.6)

Monthly Payments Review for John Doe

Amount of Loan	No. of Monthly Payments 24	36	48
$3000	$160.10	$119.27	$ 99.46
3500	186.79	139.15	116.04
4000	213.47	159.02	132.61
4500	240.15	178.90	149.19
5000	266.84	198.78	165.76

4-43. Suppose that you borrow $500 from the Easy Credit Company with the agreement to repay it over a 3-year period. Their stated interest rate is 9% per year. They show you the following items in determining the monthly payment:

Principal	$500
Total interest: 0.09(3 years)($500)	135
Loan application fee	15

They ask you to pay the interest immediately, so you leave with $365 in your pocket. Your monthly payment is calculated as follows:

$$\frac{\$500 + \$15}{36} = \$14.30/\text{month}$$

What are the nominal and effective interest rates per year? (4.6)

4-44. On January 1, 1990, a government bond was purchased for $9,400. The face value of the bond is $10,000, and 8% nominal interest is paid on the face value four times each year. Thus, every 3 months the bondholder receives $200 as a cash payment. When the bond matures on January 1, 2000, $10,000 is paid to the bondholder. What is the effective rate of interest being earned in this situation? (4.6)

4-45. A $20,000 ordinary life insurance policy for a 22-year-old female can be obtained for annual premiums of approximately $250. This type of policy (ordinary life) would pay a death benefit of $20,000 in exchange for annual premiums of $250 that are paid during the lifetime of the insured person. If the average life expectancy of a 22-year-old female is 77 years, what interest rate establishes equivalence between cash outflows and inflows for this type of insurance policy? Assume that all premiums are paid on a beginning-of-year basis and that the last premium is paid on the female's 76th birthday. (4.6)

4-46. Evaluate the acceptability of the following project with all methods discussed in Chapter 4. Let MARR = 15%, maximum acceptable $\theta = 5$ years, and maximum acceptable $\theta' = 6$ years.

Project: R137-A
Title: Syn-Tree Fabrication
Description: Establish a production facility to manufacture synthetic palm trees for sale to resort areas in Alaska

Cash Flow Estimates:

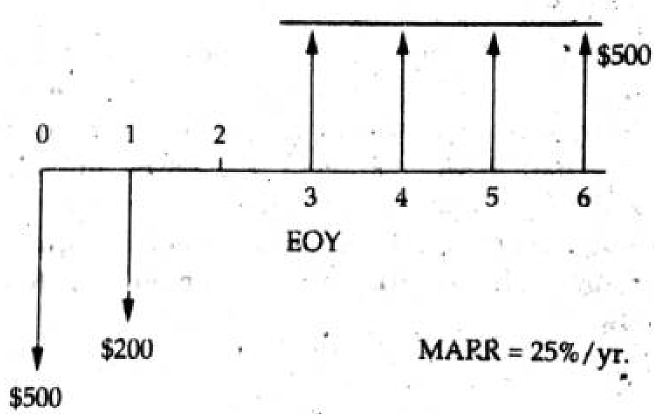
Year	*Amount (thousands)*
0	−$1,500
1	200
2	400
3	450
4	450
5	600
6	900
7	1,100

4-47. The equipment in one department of a company is operating at only 75% of capacity because the painting booths are overloaded. By building and equipping a new painting shed on adjoining land that is owned by the company, at a total cost of $14,000, the output could be stepped up to 100% (4,000 units per year). It is estimated that the useful life of the new paint shed and equipment would be 10 years. The old paint booths could be utilized for 10 more years. To operate at full capacity would require employing two more machinists and two painters at monthly salaries of $1,200 each. Indirect labor costs would be 10% of direct labor costs. Operation and maintenance costs per month on the new equipment are estimated to be $50 per 1,000 units processed. The cost for materials is $50 per unit. Annual expenses for taxes and insurance on the new facilities would be 8% of the first cost. The sales department assures management that the full output of 4,000 units per year can be sold at the present selling price of $200 per unit. Capital earns an average of 20% before income taxes. What would you recommend? (4.5)

4-48. A homeowner purchased a split-level dwelling for $50,000 in 1970. The monthly payments on her $30,000 loan (30-year mortgage) have been $200.00 at 7% nominal interest. It is now 1990, and the homeowner sold the dwelling for $100,000. Her before-tax rate of return, as an annual percentage, is closest to which of the following answers? (4.4)

a. 3.6%
b. 8.5%
c. 5.3%
d. 1.5%

4-49. With reference to the following cash flow diagram:



a. What is the breakeven life [θ'] of this project? (4.9)

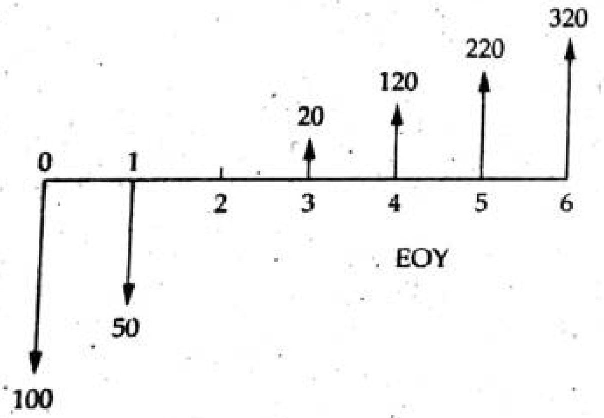
b. What is the breakeven interest rate (i')? (4.6)

4-50. The Going Aircraft Corporation is manually producing a certain subassembly at a direct labor cost of $100,000 per year. This manual work can be totally automated so that $80,000 in direct labor and $20,000 in indirect labor and overhead will be saved each year. Annual maintenance for the automated system will be $10,000, and its salvage (scrap) value will be $7,000 at any time in the future. The system's useful life is 5 to 10 years, inclusive.

a. If the firm's MARR is 15% per year, develop a graph that shows how much money can be spent on the automated equipment. (*Hint:* Plot the PW of positive cash flows versus the useful life). (4.3)

b. When $N = 6$ years and $P = \$344,000$, what is the simple payback period? (4.9)

4-51. Consider this cash flow diagram:



a. If the MARR is 15% per year, is this project financially profitable? (4.3)

b. Calculate the simple payback period, θ. (4.9)

c. Calculate the discounted payback period, θ'. (4.9)

4-52. A state Resources Development Department has proposed building a dam and a hydroelectric project that will remedy a flood situation on a mountain river, generate power, provide water for irrigation and domestic use, and provide certain recreational facilities for boating and fishing. The construction costs would be as follows:

Dam	$40,000,000
Access roads	2,000,000
Power plant	4,000,000
Transmission lines	1,500,000
Fish ladders and elevators	800,000
Irrigation and water canals	3,000,000

It is proposed to finance the project by issuing 8%, tax-exempt, 40-year bonds.

It is estimated that the annual operation and maintenance costs will be $1,250,000 for the power generating and distributing facilities and $750,000 for all other portions of the project. In addition, the state will pay $400,000 annually to the county where the project is located in lieu of property taxes. Estimates of the annual benefits and revenues are as follows:

Flood control	$ 900,000
Sale of power	2,700,000
Sale of water	1,600,000
Recreation benefits	1,800,000
Income from sports concessions	200,000

a. Determine the B/C ratio for the project, using the estimated values of the benefits as stated. (4.7)

b. Would the elimination of the benefits that you consider to be intangible leave the project justified economically? (4.7)

4-53. To reduce the damage from floods at a point where a river makes a bend through a certain town, the federal government is studying the possibility of realigning an earth dike. It is estimated that over the next 30 years the total cost of maintaining the present dike would be $84,000 to the federal government and $5,000 to the local interests. If the dike is realigned, the total cost over the same period would be $333,000 to the federal government and $14,000 to the local interests. If the dike is not realigned, it is estimated that the losses over the 30-year period probably will amount to $320,000, whereas the realignment should reduce the loss to not more than $50,000. (All costs and losses include interest at 4%.) Compute the B/C ratio for this project. (4.7)

4-54. It is proposed to finance and build a 20-mile highway around a small city so as to speed the flow of traffic. It will cost $20,000,000 and be financed by 20-year, 5% bonds. It is estimated that the new road will increase the distance by 5 miles, but will reduce travel time for passenger cars by 15 minutes and for trucks by 20 minutes. A traffic check shows that passenger vehicles would make 2,000,000 trips over the road each year and trucks 750,000 trips. The incremental operating cost of passenger cars is estimated to be $0.04 per mile and that of trucks $0.10, plus a time charge of $1.00 per hour for all passenger car drivers and $4.00 for truck drivers. It is estimated that the maintenance cost of the highway, although paid by a different governmental agency, would not exceed that for the city streets being used at present. However, building the new highway would remove property from the tax rolls that currently produces $100,000 per year in property taxes. Make an economy study of the proposed highway, determining the B/C ratio, and make a recommendation. (4.7)

4-55. A company has the opportunity to take over a redevelopment project in an industrial area of a city. No immediate investment is required, but it must raze the existing buildings over a 4-year period and at the end of the fourth year invest $2,400,000 for new construction. It will collect all revenues and pay all costs for a period of 10 years, at which time the entire project, and properties thereon, will revert to the city. The net cash flow is estimated to be as follows:

Year End	*Net Cash Flow*
1	$ 500,000
2	300,000
3	100,000
4	− 2,400,000
5	150,000
6	200,000
7	250,000
8	300,000
9	350,000
10	400,000

Tabulate the PW versus the interest rate and determine whether multiple IRRs exist. If so, use the Appendix 4-A method (ERR) when ϵ = 8% to determine a rate of return. (4.3 and App. 4-A)

4-56. A certain project has net receipts equaling $1,000 now, has costs of $5,000 at the end of the first year, and earns $6,000 at the end of the second year.

a. Show that multiple rates of return exist for this problem when using the IRR method (i' = 100%, 200%). (Appendix 4-A)

b. If an external reinvestment rate of 10% is available, what is the rate of return for this project using the ERR method of Appendix 4-A?

4-57. The prospective exploration for oil in the outer continental shelf by a small, independent drilling company has produced a rather curious pattern of cash flows, as follows:

End of Year	*Net Cash Flow*
0	−$ 520,000
1–10	+ 200,000
10	− 1,500,000

The $1,500,000 expense at the end of year 10 will be incurred by the company in dismantling the drilling rig.

a. Over the 10-year period, plot PW versus the interest rate (i) in an attempt to discover whether multiple rates of return exist. (4.6)

b. Based on the projected net cash flows and results in part (a), what would you recommend regarding the pursuit of this project? Customarily, the company expects to earn at least 20% on invested capital before taxes. Use the ERR method. (Appendix 4-A)

4-58. *Brain Teaser*

In 1992 interest rates in the United States were declining, while interest rates in Germany were increasing. Suppose that interest rates in Germany turn out to be twice those in the United States. Develop a graphical model to explain how the ratio of CR costs in Germany to CR costs in the United States *varies* with (a) the magnitude of the interest rates in the United States, (b) the length of the study period, and (c) the balance between initial investment and terminal salvage value. What general observations can you make regarding how reduced interest rates and longer study periods affect the opportunity for productivity growth and increased competitiveness in the United States? (4.5)

Appendix 4-A: The Multiple Rate of Return Problem with the IRR Method

Whenever the IRR method is used and the cash flows reverse sign (from net outflow to net inflow or the opposite) more than once over the period of study, one should be alert to the rather remote possibility that either no interest rate or multiple interest rates may exist. Actually, the *maximum* number of possible rates of return in the $(-1, \infty)$ interval for any given project is equal to the number of cash flow reversals during the study period. As an example, consider the following project for which the IRR is desired.

EXAMPLE 4-A-1

Determine the IRR of the following cash flows.

Year, k	*Net Cash Flow*
0	$ 500
1	− 1,000
2	0
3	250
4	250
5	250

Solution

		PW at 35%		*PW at 63%*	
Year	*Net Cash Flow*	*Factor*	*Amount*	*Factor*	*Amount*
0	$ 500	1.3500	$ 675	1.6300	$ 815
1	− 1,000	1.0000	− 1,000	1.0000	− 1,000
2	0				
3	250	0.5487	137	0.3764	94
4	250	0.4064	102	0.2309	58
5	250	0.3011	75	0.1417	35
PW			$\Sigma =$ 11		$\Sigma =$ 2

Thus, the PW of the net cash flows equals zero for interest rates of about 35% and 63%. Whenever multiple answers such as these exist, it is likely that none is correct.

One effective way to overcome this difficulty and obtain a plausible answer is to manipulate cash flows as little as necessary so that there is only one reversal of the cash flows over time. This can be done by using the MARR or an external reinvestment rate to manipulate the funds, and then solving for the rate of return by using Equation 4-6 or 4-7. For example, if the MARR is 10%, the +\$500 at year 0 can be compounded to year 1 to be \$500$(F/P, 10\%, 1)$ = +\$550. This, added to the −\$1,000 at year 1, equals −\$450. The −\$450, together with the remaining cash flows, which are all positive, now fit the condition that there be only one reversal in the cash flows over time. The *return on invested capital* (*RIC*) at which the PW of the cash flows equals 0 can now be shown to be 19%, per the following table:

Year	*Cash Flows*	*PW at 19%* Factor	*Amount*
1	−$450	1.0000	−$450
3	250	0.7062	177
4	250	0.5934	148
5	250	0.4987	125
PW			$\Sigma = 0$

It should be noted that whenever a manipulation of cash flows such as this one is done, the calculated rate of return will vary according to what cash flows are manipulated and at what interest rate.

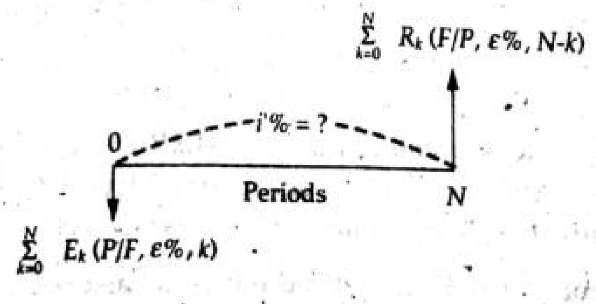
Another, more objective, way to deal with multiple IRRs is to resort to a rate of return method known as *external rate of return* (*ERR*) method. This method takes into account the external interest rate (ϵ) at which net cash flows generated (or required) by a project can be reinvested (or borrowed) outside the firm. If this external reinvestment rate happens to equal the project's IRR, then the ERR method produces results identical to those of the IRR method.

In general, all cash outflows are discounted to period 0 (the present) at ϵ% per compounding period, while all cash inflows are compounded to period N at ϵ%. The ERR is then the interest rate that establishes equivalence between the two quantities. In equation form, the ERR is the i'% at which

$$\sum_{k=0}^{N} E_k(P/F, \epsilon\%, k)(F/P, i'\%, N) = \sum_{k=0}^{N} R_k(F/P, \epsilon\%, N-k)$$

where R_k = excess of receipts over expenditures in period k
E_k = excess of expenditures over receipts in period k
N = project life or number of periods for the study
ϵ = external reinvestment rate per period

Graphically, we have the following:



A project is acceptable when the i'% of the ERR method is greater than or equal to the firm's MARR.

For Example 4-A-1 the ERR is 12.4% when the reinvestment rate (ϵ) is 10%:

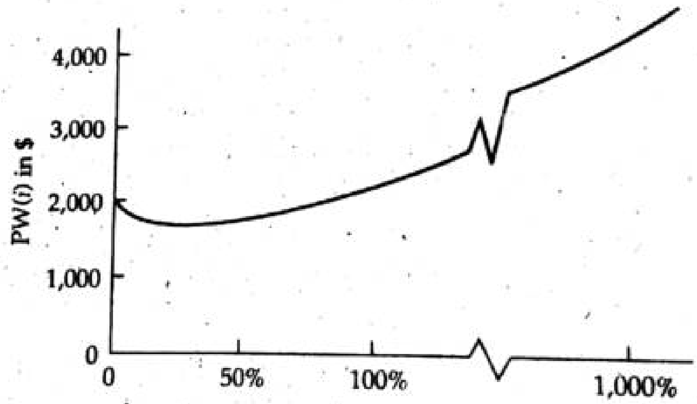
$$\$1{,}000(P/F, 10\%, 1)(F/P, i'\%, 5) = \$500(F/P, 10\%, 5) + \$250(F/A, 10\%, 3)$$
$$(F/P, i', 5) = 1.632$$
$$i' = 0.124(12.4\%)$$

This differs from the 19% return in the preceding table because the ERR method makes the assumption that all positive cash flows are reinvested at 10%. The RIC method assumes that funds can be reinvested at 19%. ▪

EXAMPLE 4-A-2

Use the ERR method to analyze the cash flow pattern shown in the following table. The IRR is indeterminant (none exists), so the IRR is not a workable procedure. The external reinvestment rate is 12% and MARR equals 15%.

Year	*Cash Flows*
0	\$5,000
1	− 7,000
2	2,000
3	2,000



Solution The ERR method provides these results:

$$\$7{,}000(P/F, 12\%, 1)(F/P, i'\%, 3) = \$5{,}000(F/P, 12\%, 3) + \$2{,}000(F/P, 12\%, 1) + \$2{,}000$$
$$(F/P, i', 3) = 1.802$$
$$i' = 21.7\%$$

Thus, the ERR is greater than the MARR. Apparently, the project having this cash flow pattern would be acceptable. The PW at 15% is equal to \$1,740.36, which confirms the acceptability of this project. ▪

In summary, the ERR method has two basic advantages over the IRR method:

1. It usually can be solved for directly rather than by trial and error.
2. It is not subject to the possibility of multiple rates of return.