18, Jan 2024 - Th.

CSE-305

Engineering Economics Notes

chapter # Application of Money-Time Relationships.

4.1 Introduction:

→ We have a proposed capital investment and its associated expenditure can be recovered by revenue (or savings) over time in addition to a return on the capital that is sufficiently attractive in view of risks involved and the potential alternative uses.

→ PW : present worth

→ FW : future worth

→ AW : annual worth

→ IRR : Internal rate of returns

→ B/C : Benift / Cost ratio

→ MARR : minimum attractive rate of returns.

→ Mostly (unless specified) end-of-period cash flow convention and discrete compounding of interest are utilized throughout this chapter.

4.2 Determining the minimum Attractive rate of return (MARR):

→ Among many some considerations are following:

1. The amount of money available for investment, and the sources and cost of these funds.
2. The number of good projects available for investment and their purpose
3. The amount of preceived risk that is associated with investeta opportunities available to the firm and the projected cost of administering projects over short planning horizons versus long planing horizons.
4. The type of organisation involved.

→ The MARR should be chosen to maximise the economic well being of an organisation.

→ One popular approach to establish a MARR involves the "opportunity cost" viewpoint, and its results from the phenomena of capital rationing.

4.3 The present worth (PW) method

→ In PW all cash inflows and outflows are discounted to the base point at an intrest rate i.e generally the MARR.

∴ Total PW = PW of cash receipts − PW of cash outlays.

→ To find the PW as a function of i% of a series of cash receipts and/or expenses, it is necessary to discount future amounts to present by using an interest rate for the appropriates study period.

i.e

$$PW(i\%) = \sum_{K=0}^{N} F_K (1+i)^{-K}$$

where;

K = index of each compounding period $0 \le K \le N$

F_K = future cash flow at end of period K.

→ The higher the interest rate and the further into the future a cash flow occurs, the lower is its PW.

→ As long as the PW, (i.e present equivalent of cash inflows minus cash outflows) is greater than or equal to zero, the project is economically justified; otherwise it is not acceptable.

4.3.1 Bond Value:-

→ A bond provides an excellent example of commercial value being the PW of the future net cash flows that are expected to be received throug

ownership of property.

→ The value of bond, at any time is the PW of future cash receipts.

→ Let for a bond;

Z = face, or pair value

C = redemption or disposal price (usually equal to Z)

r = bond rate per interest period.

V_n = Value of bond N periods prior to redemption

Then

$$V_n = C\,(P/F, i\%, N) + rZ\,(P/A, i\%, N).$$

4.4 The future worth (FW) method

→ With this method the future worth of an alternative can be calculated in view of the MARR and compared with do-nothing (0) option.

→ If $FW \geq 0$, the alternative will be recomended.

→ All cash inflows and outflows are compounded forward to a reference point in time called the future.

i.e

$$FW(i\%) = \sum_{k=0}^{N} F_k (1+i)^{N-k}$$

4.5 The Annual worth (AW) method:

→ An annual worth of a project is a uniform annual series of dollars amounts, for a stated study period, that is equivalent to the cash inflows (receipts or savings) and/or cash outflows (expenses) under consideration.

→ AW is function of i%, i.e.

$$AW(i\%) = R - E - CR(i\%)$$

where R = annual equivalent revenues
E = annual equivalent expenses
CR = annual equivalent capital recovery

→ As long as $AW \geq 0$, the project is economically attractive, otherwise it is not.

→ The CR cost for a project is the equivalent uniform annual cost of the capital invested. It is an annual amount that covers the following two items:

1. Loss in value of the asset.
2. Interest on invested capital (ie MARR).

→ To get CR, find annual equivalent of initial investment and then subtracting the annual equivalent of the salvage value. Thus

$$CR(i\%) = I(A/P, i\%, N) - S(A/F, i\%, N)$$

where I = intial investment of project
S = Salvage value at end of study
N = project study period.

→ Another way to calculate CR cost is to add an annual sinking fund amount (or deposit) to the interest on the original investment (sometimes called minimum required profit). Thus

$$CR(i\%) = (I-S)(A/F, i\%, N) + I(i\%).$$

→ Another formula is

$$CR(i\%) = (I-S)(A/P, i\%, N) + S(i\%).$$

→ The AW method is sometimes called the annual cost (AC) method when only costs are involved.

4.6 The Internal Rate of Return Method (IRR)

→ IRR is commonly called by several other names, such as "investor's method", discounted "cash flow method" and "profitability index".

→ This method solves for the interest rate that equates the equivalent worth of an alternative's cash

inflows (receipt, savings) to the equivalent worth of cash outflows (expenditure, including investments). the resultant interest rate is termed the "internal rate of return (IRR)".

→ For a single alternative, the IRR is not positive unless both receipts and expenses are present in the cashflow pattern and the sum of receipts exceed the sum of all cash outflows.

→ By using PW formulation, the IRR is the i'% at which.

$$\sum_{K=0}^{N} R_K (P/F, i'\%, K) = \sum_{K=0}^{N} E_K (P/F, i'\%, K).$$

where R_K = net revenue/saving for K^{th} year
E_K = net expenditure for K^{th} year
N = project life (or study period).

→ If $i' \geq$ MARR, the alternative is acceptable; otherwise, it is not.

→ Take common convention of "+" sign for cash inflows and "−" sign for cash outflows.

4.7 The Benifit/Cost ratio method:

→ The B/C ratio is defined as the ratio of the equivalent worth of benifits to the equivalent worth of costs.

→ It is also reffered to as "saving-~~ratio~~investment ratio" (SIR) by some governmental agencies.

→ $B/C = \frac{AW(\text{benefits of proposed project})}{AW(\text{total cost of proposed project})}$

$= \frac{B}{CR+(O\&M)}$

where B = equalent annual worth of benifits of proposed project

O&M = equivalent annual operating and maintainance expense of the proposed project.

→ The modified B/C ratio:

$B/C = \frac{B-(O\&M)}{CR}$

4.9 The Payback (payout) period method:

→ The payback method, which is often called the simple payout

method, mainly indicates a project's liquidity rather than its profitability.

→ liquidity deals with how fast an investment can be recovered.

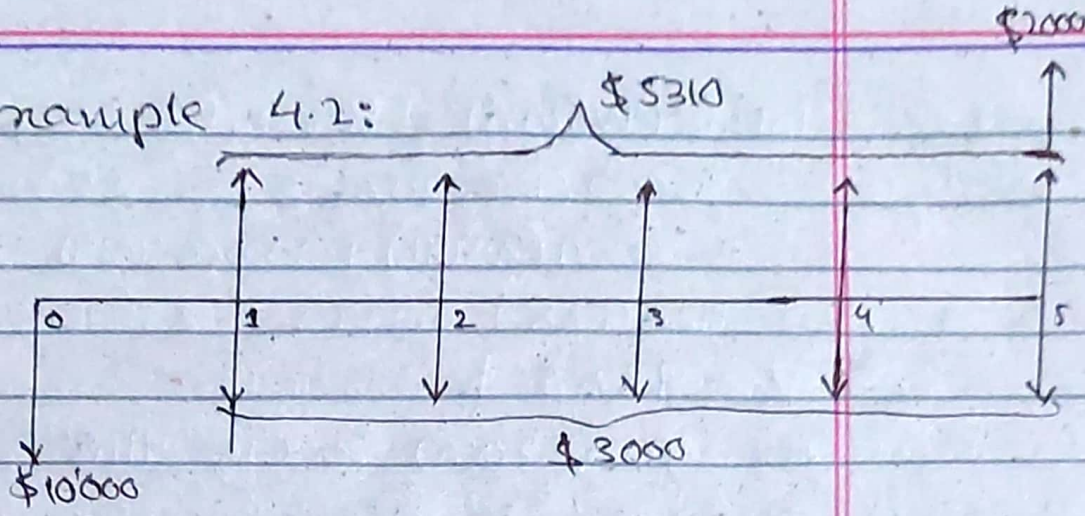
→ Simple payback period is the smallest value of θ ($\theta \leq N$) for which the relationship is satisfied under end-of-year cash flow convention

$$\sum_{k=0}^{N} (R_k - E_k) - I \geq 0$$

→ Sometime the discounted payback period θ' ($\theta' \leq N$), is calculated so that the time value of money is considered:

$$\sum_{k=0}^{N} (R_k - E_k)(P/F, i\%, k) - 1 \geq 0.$$

Example 4.2:



Sol.

Annual revenue = $5'310 (P/A, 10%, 5)

= 5310 (3.791) = $ 20'130.21

Salvage Value = $2000 (P/F, 10%, 5)

= $2000 (0.6209) = 1241.8

Investment = -$10'000

Annual expese = - $3000 (P/A, 10%, 5)

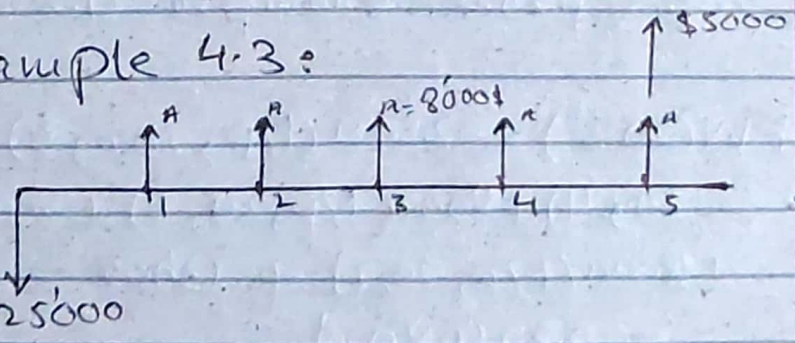
= -$3000 (3.791) = -11373 $

Total PW = 2013021 + 1241.8 - 10'000

- 11'373

PW(10%) = -1 hence inacceptable but in book it is show to be marginally acceptable.

Example 4.3:



Solution: total PW(i%)

= $PW_{receipt} - PW_{outflays}$

Total PW(20%) = 8000 (P/A, 20%, 5) + 5000 (P/F, 20%, 5) − 25000
= 8000 (2.991) + 5000 (0.4019) − 25000
= 23928 + 2009.5 − 25000
= 937.5 $

As PW(20%) ≥ 0, this equipment is economically justified.

Example 4.6: (semi annually)

N = 10 × 2 = 20 periods
r = 6%/2 = 3% per period
i = 10%/2 = 5% per period
C = Z = $1000.

Solution: As

$V_N = C(P/F, i\%, N) + rZ(P/A, i\%, N)$
= 1000 (P/F, 5%, 20) + (0.03)(1000)(P/A, 5, 20)
= 376.89 + 373.87
= $750.76.

Example 4.7:

Sol: From example 4.3

FW(20%) = −25000 (F/P, 20%, 3) + 8000 (F/P, 20%, 4) + 8000 (F/P, 20, 3) + 8000 (F/P, 20, 2) + 8000 (F/P, 20, 1) + 13000
= 2324.80 $

FW(20%) ≥ 0, hence shown again that it is a good investment.

The Annual Worth Method.

Problem

[redacted] (4.3) part b.

Solution:

formula for capital Recovery

$$CR(i\%) = I(A/P, i\%, N) - S(A/F, i\%, N)$$

$I = \$13{,}000 \quad S = \$3000$

$$CR(i\%) = 13000(A/P, 12\%, 15) - 3000(A/F, 12\%, 15)$$

$$CR(i\%) = 13000(0.1468) - 3000(0.0268)$$

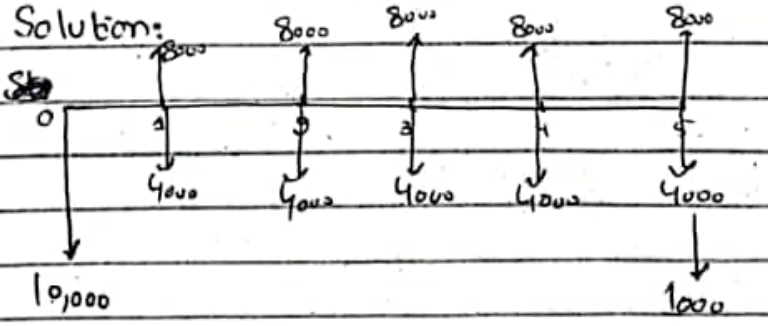
$$CR(i\%) = 1908.4 - 80.4$$

$$\boxed{CR(i\%) = \$1828}$$ Answer

Problem

[redacted] (4.4) part d:

Solution:



Step 1: finding the value of capital Recovery when

$I = 10{,}000 \quad S = -1{,}000$

$$CR(i\%) = I(A/P, 15\%, 5) - S(A/F, 15\%, 5)$$

$$CR(15\%) = 10{,}000(0.2983) - (-1000)(0.1483)$$

$$CR(15\%) = 2983 + 148 = \boxed{\$3131}$$

Step:

AW = I - O I = Inflows, O = Outflows

AW = Annual Receipts - Annual Expense - CR

AW = 8000 - 4000 - 3131

AW = $869

Problem

Example 4-19: part (a):

Given:

I = $8,000
N = 10 years
S = $0
Annual net receipts = 2400
MARR = 18%
AW = ?

Solution:

Step1: Find the value of capital Recovery

$CR(i\%) = I(A/P, i\%, N) - S(A/F, i\%, N)$

$CR(18\%) = 8000(A/P, 18\%, 10) - 0(A/F, i\%, N)$

$CR(18\%) = 8000(0.2225)$

$CR(18\%) = \$1780$

Step2: Find the value of AW

AW = net Receipts - CR

AW = 2400 - 1780

AW = $620

Part b:

Given:

$I = \$2500$

$N = 5$ years

$S = \$1,000$

$MARR = 10\%$

$CR(i\%) = ?$

Solution:

$$CR(i\%) = \$(A/P, 10\%, 5) - S(A/F, 10\%, 5)$$

$$CR(10\%) = 2500(0.2638) - 1000(0.1638)$$

$$CR(10\%) = 659.5 - 163.8$$

$$\boxed{CR(10\%) = \$ 496}$$

(Problem 4-20)

Given:

$I = \$100,000$

Initial Annual Cost $= \$5,000$ per year

roof repair Cost $= 9000$ at the end of 3rd year.

$$\text{Final Annual Cost} = 5000 + \frac{9000}{5} = 5000 + 1800$$

$$\boxed{= \$6800}$$

$S = \$120,000$

Annual Receipt $= \$10,000$

Annual expense $= \$5000$

$N = 5$ years

$MARR = 12\%$ per year

Solution:

Step1:

$CR(i\%) = T(A/P, i\%, N) - S(A/F, i\%, N)$

$CR(12\%) = \$100,000(A/P, 12\%, 5) - 120,000(A/F, 12\%, 5)$

$CR(12\%) = 100,000(0.2774) - 120,000(0.1574)$

$CR(12\%) = 27740 - 18888$

$\boxed{CR(12\%) = \$8852}$

Step2:

AW = Annual Receipt - Annual Expense - CR

$AW = 10,000 - 6800 - 8852$

$\boxed{AW = -5652}$ negative value.

Problem 4-24

Given:

Total Initial Investment = $100,000 + $15,000

= 115,000

Salvage = $15,000

N = 15 years

Total Receipts = Sales + Job's money

= 52000,00 + $40,000

= 92 40,000

(P.T.O)

Total expense = 40,000 + 70,000 + 6,000 + (0.1)(200,000) + 5000

= 145,000

Solution:

Step 1:

$CR(\text{total}) = I(A/P, 8\%, N) - S(A/F, 8\%, N)$

$CR(85\%) = 115{,}000(A/P, 8\%, 15) - 15{,}000(A/F, 8\%, 15)$

$CR(15\%) = 115{,}000(0.1710) - 15{,}000(0.0247)$

$CR(15\%) = 19665 - 315 = \boxed{\$19{,}350}$

Step 2:

Assuming Job's money is included in Receipts

AW = Net Receipts − Net expense − CR

AW = 240,000 − 145,000 − 19350

$\boxed{AW = \$75650}$

Assuming Job's money is not included.

AW = 200,000 − 145000 − 19350

$= \boxed{\$35650}$

Exam Problem 4-27

Given:

Initial Cost = 7000

Storage Shed Costs = 2000

I = 7000 + 2000 = 9000

Salvage = 0

Material Cost = $1.10 per unit

labor Cost = $2.40 per unit

Total Variable Cost = $3.50

Overhead = 2.40/2 = $1.20

There's annual charge of two percent of the first cost for taxes and insurance.

MARR = 15%

N = 5 years

Solution:

Step 1:

$CR(15\%) = I(A/P, 15\%, 5) - S(A/F, 15\%, 5)$

$CR(15\%) = 9000(A/P, 15\%, 5) - 0(A/F, 15\%, 5)$

$CR = 9000(0.2983)$

$CR = \$2685$

Step 2:

AW = Total Receipts - Total Expenses - CR

Total Expenses = (3.50 + 1.20)* 40,000 + (9000 * 0.2) = [redacted]

AW = 0 - 20600 - 9000 [redacted]

$-29600