

## 2.1 Introduction

Accomplishing engineering design to meet economic needs and to achieve competitive operations in private and public sector organizations depends on a prudent balance between what is technically feasible and what is economically acceptable. Unfortunately, there is no shortcut method available to reach this balance between technical and economic feasibility. Thus, engineering economic analysis concepts and methods should be used to provide results that will help to attain an acceptable balance.

The word *cost* has meanings that vary in usage.\* Since concepts are idea generalized from particular instances or situations, the *cost concepts* used in engineering economy study will depend on the problem or situation and the decision to be made. Such studies involve an integration of cost concept with the basic principles and the analysis procedure discussed in Chapter 1. Consequently, the content of Chapter 2, which involves selected cost concepts and their use, is important to the methodology and applications covered in subsequent chapters of the book.

In Section 2.2, selected cost terms used extensively in engineering economy are defined and illustrated. Applications of some general economic concepts selected because of their importance, are discussed in Section 2.3. In Section 2.4, the important concept of present value studies is covered, and its role in engineering economic analysis is illustrated.

## 2.2 Cost Terminology

When accomplishing engineering economy studies and communicating results it is important to use consistent definitions for cost terms. Otherwise, common understanding is jeopardized. The definitions for selected terms provided in this section will help achieve that goal.

### 2.2.1 Fixed, Variable, and Incremental Costs

*Fixed costs* are those that are unaffected by changes in activity level over feasible range of operations for the capacity or capability available. Typical fixed

\*For purposes of this book, the words *cost* and *expense* are used interchangeably.

costs include insurance and taxes on facilities, general management and administrative salaries, license fees, and interest costs on borrowed capital.

Of course, any cost is subject to change, but fixed costs tend to remain constant over a specific range of operating conditions. When large changes in usage of resources occur, or when plant expansion or shutdown is involved, you should expect fixed costs to be affected.

Variable costs are those associated with an operation that will vary in total with the quantity of output or other measures of activity level. If you were making an engineering economic analysis of a proposed change to an existing operation, the variable costs may be the primary part of the prospective differences between the present and changed operations as long as the range of activities is not significantly changed. For example, the costs of material and labor used in a product or service are variable costs, since they vary in total with the number of output units, even though the costs per unit stay the same.

An incremental cost, or an incremental revenue, refers to the additional cost, or revenue, that will result from increasing the output of a system by one or more units. Reference is frequently made to the incremental cost associated with "go/no go" decisions that involve a limited change in output or activity level. For instance, the incremental cost per mile for driving an automobile may be \$0.27 but this depends on several considerations, such as total mileage driven during the year (normal operating range), the mileage expected for the next major trip, and the age of the automobile. Also, it is common to read of the "incremental cost of producing a barrel of oil" and the "incremental cost to the state for educating a student." As these examples indicate, the incremental cost (or revenue) is often quite difficult to determine in practice.

### EXAMPLE 2-1

In connection with surfacing a new highway, the contractor has a choice of two sites on which to set up the asphalt mixing plant equipment. The contractor estimates that it will cost \$1.15 per cubic yard per mile ( $\text{yd}^3\text{-mile}$ ) to haul the asphalt paving material from the mixing plant to the job site. Factors relating to the two site alternatives are as follows (production costs at each site are the same):

Cost Factor	Site A	Site B
Average hauling distance	6 miles	4.3 miles
Monthly rental of site	\$1,000	\$5,000
Cost to set up and remove equipment*	\$15,000	\$25,000
Hauling cost	\$1.15/ $\text{yd}^3\text{-mile}$	\$1.15/ $\text{yd}^3\text{-mile}$

If site B is selected, there will be an added charge of \$96 per day for a flagman.

The job involves 50,000 cubic yards of mixed asphalt paving material. It is estimated that 4 months (17 weeks of 5 working days per week) will be required for the job. Compare the two sites in terms of their fixed, variable, and total costs. Which is the better site? For the selected site, how many cubic yards of paving material does the contractor have to deliver before starting to make a profit if paid \$8.05 per cubic yard delivered to the job site?

Solution The fixed and variable costs for this job are indicated in the following table. Site rental and setup/removal costs, and the cost of the flagman at Site B, would be constant for the total job, but the hauling cost would vary in total amount with the distance and thus with the total output quantity of  $\text{yd}^3$ -miles.

Cost	Fixed	Variable	Site A		Site B	
Rent	✓		= \$ 4,000		= \$ 20,000	
Setup/removal	✓		= 15,000		= 25,000	
Flagman	✓		= 0	5(17)(\$96)	= 8,160	
Hauling	✓	6(50,000)(\$1.15)	= 345,000	4.3(50,000)(\$1.15)	= 247,250	
			Total: \$364,000			\$300,410

Thus Site B, which has the largest fixed costs, has the smallest total cost for the job.

The contractor will begin to make a profit at the point where total revenue equals total cost as a function of the cubic yards of asphalt pavement mix delivered. Based on Site B, we have:

$$4.3(\$1.15) = \$4.95 \text{ in variable cost per } \text{yd}^3 \text{ delivered}$$

$$\text{Total cost} = \text{total revenue}$$

$$\underline{\$53,160 + \$4.95x = \$8.05x}$$

$$x = 17,149 \text{ yd}^3 \text{ delivered}$$

Therefore, by using Site B, the contractor will begin to make a profit on the job after delivering 17,149 cubic yards of material.

### EXAMPLE 2-2

Four college students who live in the same geographical area intend to go home for Christmas vacation (a distance of 400 miles each way). One of the students has an automobile and agrees to take the other three if they will pay the cost of operating the automobile for the trip. When they return from the trip, the owner presents each of them with a bill for \$102.40, stating that she has kept careful records of the cost of operating the car and that based on an annual average of 15,000 miles, the cost per mile is \$0.384. The three others feel that the charge is too high and ask to see the cost figures on which it is based. The owner shows them the following list:

Cost Element	Cost per Mile
Gasoline	\$0.120
Oil and lubrication	0.021
Tires	0.027
Depreciation	0.150
Insurance and taxes	0.024
Repairs	0.030
Garage	0.012
Total	\$0.384

The three riders, after reflecting on the situation, form the opinion that only the costs for gasoline, oil and lubrication, tires, and repairs are a function of mileage driven (variable costs) and thus could be caused by the trip. Since these four costs total only \$0.198 per mile, and thus \$158.40 for the 800-mile trip, the share for each student would be  $\$158.40/3 = \$52.80$ . Obviously, the opposing views are substantially different. Which, if either, is correct? What are the consequences of the two different viewpoints in this matter, and what should be the decision-making criterion?

**Solution** In this instance, assume that the owner of the automobile agreed to accept \$52.80 per person from the three riders, based on the variable costs that were purely incremental for the Christmas trip versus the owner's average annual mileage. That is, the \$52.80 per person is the "with a trip" cost relative to the "without" alternative.

Now, what would the situation be if the three students, because of the low cost, returned and proposed another 800-mile trip the following weekend? And what if there were several more such trips on subsequent weekends? Quite clearly, what started out to be a small marginal (and temporary) change in operating conditions—from 15,000 miles per year to 15,800 miles—soon would become a normal operating condition of 18,000 or 20,000 miles per year. On this basis it would not be valid to compute the extra cost per mile as \$0.198.

Since the normal operating range would be changed, the fixed costs would also have to be considered. A more valid incremental cost would be obtained by computing the total annual cost if the car were driven, say, 18,000 miles, then subtracting the total cost for 15,000 miles of operation, and thereby determining the cost of the 3,000 additional miles of operation. From this difference, the cost per mile for the additional mileage could be obtained. In this instance, the total cost for 15,000 miles of driving per year was  $15,000 \times \$0.384 = \$5,760$ . If the cost of 18,000 miles per year of service, due to increased depreciation, repairs, and so forth, turned out to be \$6,570, it is evident that the cost of the additional 3,000 miles is \$810. Then the corresponding incremental cost per mile due to the increase in the operating range would be \$0.27. Therefore, if several weekend trips were expected to become normal operation, the owner would be on more reasonable economic ground to quote an incremental cost of \$0.27 per mile for even the first trip.

### Recurring and Nonrecurring Costs

These two general cost terms are often used to describe various types of expenditures. Recurring costs are those that are repetitive and occur when an organization produces similar goods or services on a continuing basis. Variable costs are also recurring costs since they repeat with each unit of output. But recurring costs are not limited to variable costs. A fixed cost that is paid on a repeatable basis is a recurring cost. For example, in an organization providing architectural and engineering services, office space rental, which is a fixed cost, is also a recurring cost.

**Nonrecurring costs**, then, are those that are not repetitive even though the expenditure may be cumulative over a relatively short period of time. Typically, nonrecurring costs involve developing or establishing a capability or capacity to operate. For example, the purchase cost for real estate upon which a plant will be built is a nonrecurring cost, as is the cost of constructing the plant itself.

### 2.2.3 Direct, Indirect, and Overhead Costs

These frequently encountered cost terms involve most of the cost elements that also fit into the previous overlapping categories of fixed and variable costs. Recurring and nonrecurring costs. **Direct costs** are those that can be reasonably measured and allocated to a specific output or work activity. The labor and material costs directly associated with a product, service, or construction activity are direct costs. Materials needed to make a pair of scissors typify a direct cost.

**Indirect costs** are those that are difficult to attribute or allocate to a specific output or work activity. The term is normally used for those cost elements closely linked to the support of operations that would involve too much effort to allocate directly to a specific output. In this usage, they are costs allocated through a selected formula (such as proportional to direct labor hours, direct labor dollars, or direct material dollars) to the outputs or work activities. For example, the cost of common tools, general supplies, and equipment maintenance at a plant are treated as an indirect cost.

Overhead consists of plant operating costs that are not direct labor or direct material costs. In this book the terms *indirect costs*, *overhead*, and *burden* are used interchangeably. Examples of overhead include electricity, general repairs, property taxes, and supervision. Administrative and selling expenses are usually added to direct costs and overhead costs to arrive at a unit selling price of a product or service. (Appendix A provides a more detailed discussion of accounting principles.)

Different methods are used to allocate overhead costs among products, services, or activities. The most commonly used methods involve allocation in proportion to direct labor costs, direct labor hours, direct materials costs, or sum of direct labor and direct materials costs (referred to as *prime cost* in a manufacturing operation), or machine hours. In each of these methods it is necessary to know what the total overhead costs have been, or are estimated to be, for a time period (typically a year) to allocate them to the production (or service delivery) outputs. Also, total overhead costs are associated with a certain level of production. This is an important condition that should be remembered when dealing with unit cost data (see Section 2.4.2).

We can illustrate direct, indirect, and overhead costs using a typical project situation such as the construction of an addition to an existing plant. The work would be planned, scheduled, and controlled—including cost control—of defined activities. Costs of labor and material for each activity are direct costs; that is, they are directly charged to each activity as they are used in accomplishing the work. Then there are other project costs associated with accomplishing the work that would be very difficult to allocate directly to each construction activity.

For example, consider the costs of operating a tool crib, distributing construction material, providing miscellaneous material, supplying compressed air to the site, and so on. All such indirect costs would be allocated to the construction activities. Also, the costs associated with central purchasing, payroll administration, general project management, insurance taxes, and so on, which are general and administrative overhead costs, would be allocated among the work activities, but probably at a more macro level.

### Standard Costs

**Standard costs** are representative costs per unit of output that are established in advance of actual production or service delivery. They are developed from the direct labor hours, materials, and support functions (with their established costs per unit) planned for the production or delivery process. For example, a standard cost for manufacturing one unit of an automotive part such as a starter would be developed as follows:

Standard Cost Element	Sources of Data for Standard Costs
Direct labor	Process routing sheets, standard times, standard labor rates
+ Direct material	Material quantities per unit, standard unit material costs
+ Factory overhead costs	Total factory overhead costs allocated based on prime costs (direct labor plus direct material costs)
<b>= Standard cost (per unit)</b>	

Standard costs play an important role in cost control and other management functions. Some representative uses are the following:

1. Estimating future manufacturing or service delivery costs.
2. Measuring operating performance by comparing actual cost per unit with the standard unit cost.
3. Preparing bids on products or services requested by customers.
4. Establishing the value of work-in-process and finished inventories.

### Cash Cost Versus Book Cost

A cost that involves payment of cash is called a *cash cost* (and results in a cash flow) to distinguish it from one that does not involve a cash transaction but is reflected only in the accounting system as a *noncash cost*. This noncash cost is often referred to as a *book cost*. Cash costs are estimated from the perspective established for the analysis (Principle 3, Section 1.3) and are the future expenses incurred for the alternatives being analyzed. Book costs are costs that do not

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involve cash payments, but rather represent the recovery of past expenditures over a fixed period of time. The most common example of book costs is a charge called *depreciation* for the use of assets such as plant and equipment. In engineering economic analysis, only those costs that are cash flows or potential cash flows from the defined perspective for the analysis need to be considered. *Depreciation, for example, is not a cash flow* and is important in an analysis only because it affects income taxes, which are cash flows. We discuss the topics of depreciation and income taxes in Chapters 6 and 7, respectively.

### 2.2.6 Sunk Cost

A *sunk cost* is one that has occurred in the past and has no relevance to estimates of future costs and revenues related to an alternative course of action. Thus, a sunk cost is common to all alternatives, is not part of the future (prospective) cash flows, and can be disregarded in an engineering economic analysis. We need to be able to recognize sunk costs and then handle them properly in an analysis. Specifically, we need to be alert in any situation that involves a past expenditure that cannot be recovered, or capital that has already been invested and cannot be retrieved, for the possible existence of sunk costs.

The concept of sunk cost may be illustrated by the following simple example. Suppose that Joe College finds a motorcycle he likes and pays \$40 as a down payment, which will be applied to the \$1,300 purchase price but which must be forfeited if he decides not to take the cycle. Over the weekend, Joe finds another motorcycle he considers equally desirable for a purchase price of \$1,230. For the purpose of deciding which cycle to purchase, the \$40 is a sunk cost and thus would not enter into the decision except that it lowers the remaining cost of the first cycle. The decision then is between paying \$1,260 ( $\$1,300 - \$40$ ) for the first motorcycle versus \$1,230 for the second motorcycle.

In summary, sunk costs result from past decisions and therefore are irrelevant in the analysis and comparison of alternatives that affect the future. Even though it is sometimes emotionally difficult to do, sunk costs should be ignored, except possibly to the extent that their existence assists you to anticipate better what will happen in the future.

#### EXAMPLE 2-3

A classic example of sunk cost occurs in the replacement of assets. Suppose that your firm is considering the replacement of a piece of equipment. It originally cost \$50,000, is presently shown on the company records with a value of \$20,000, and can be sold for an estimated \$5,000. For purposes of replacement analysis the \$50,000 is a sunk cost. However, one view is that the sunk cost should be considered as the difference between the value shown in the company records and the present realizable selling price. According to this viewpoint, the sunk cost is \$20,000 minus \$5,000, or \$15,000. Neither the \$50,000 nor the \$15,000, however, should be considered in an engineering economic analysis except

for the manner in which the \$15,000 may affect income taxes, which will be discussed in Chapter 7.

### 2.2.7 Opportunity Cost

An opportunity cost is incurred because of the use of limited resources such that the opportunity to use those resources to monetary advantage in an alternative use is forgone. Thus, it is the cost of the best rejected (i.e., forgone) opportunity and is often hidden or implied.

As an example, suppose that a project involves the use of vacant warehouse space presently owned by a company. The cost for that space to the project should be the income or savings that possible alternative uses of the space may bring to the firm. In other words, the opportunity cost for the warehouse space should be the income derived from the best alternative use of the space. This may be more than or less than the average cost of that space obtained from the accounting records of the company.

As another illustration, consider a student who could earn \$20,000 for working during a year and who chooses instead to go to school for a year and spend \$5,000 to do so. The opportunity cost of going to school for that year is \$25,000: \$5,000 cash outlay and \$20,000 for income forgone. (This neglects the influence of income taxes and assumes that the student has no earning capability while in school.)

#### EXAMPLE 2-4

The concept of an opportunity cost is often encountered when the replacement of a piece of equipment or other capital asset is being analyzed. Let us reconsider Example 2-3, where your firm is considering the replacement of an existing piece of equipment that originally cost \$50,000, is presently shown on the company records with a value of \$20,000, but has a present market value of only \$5,000. For purposes of an engineering economic analysis of whether to replace the equipment, the present investment in that equipment should be considered as \$5,000, for by keeping the equipment, the firm is giving up the opportunity to obtain \$5,000 from its disposal. Thus, the \$5,000 immediate selling price is really the investment cost of not replacing the equipment and is based on the opportunity cost concept.

### 2.2.8 Life-Cycle Cost

In engineering practice, the term life-cycle cost is often encountered. This term refers to a summation of all the costs, both recurring and nonrecurring, related to a product, structure, system, or service during its life span. The life cycle is illustrated in Figure 2-1. It begins with identification of the economic need or want (the requirement) and ends with retirement and disposal activities. It is a time horizon that has to be defined in the context of the specific situation, whether it is a highway bridge, a jet engine for commercial aircraft, or an automated flexible manufacturing cell for a factory. The end of the life cycle may

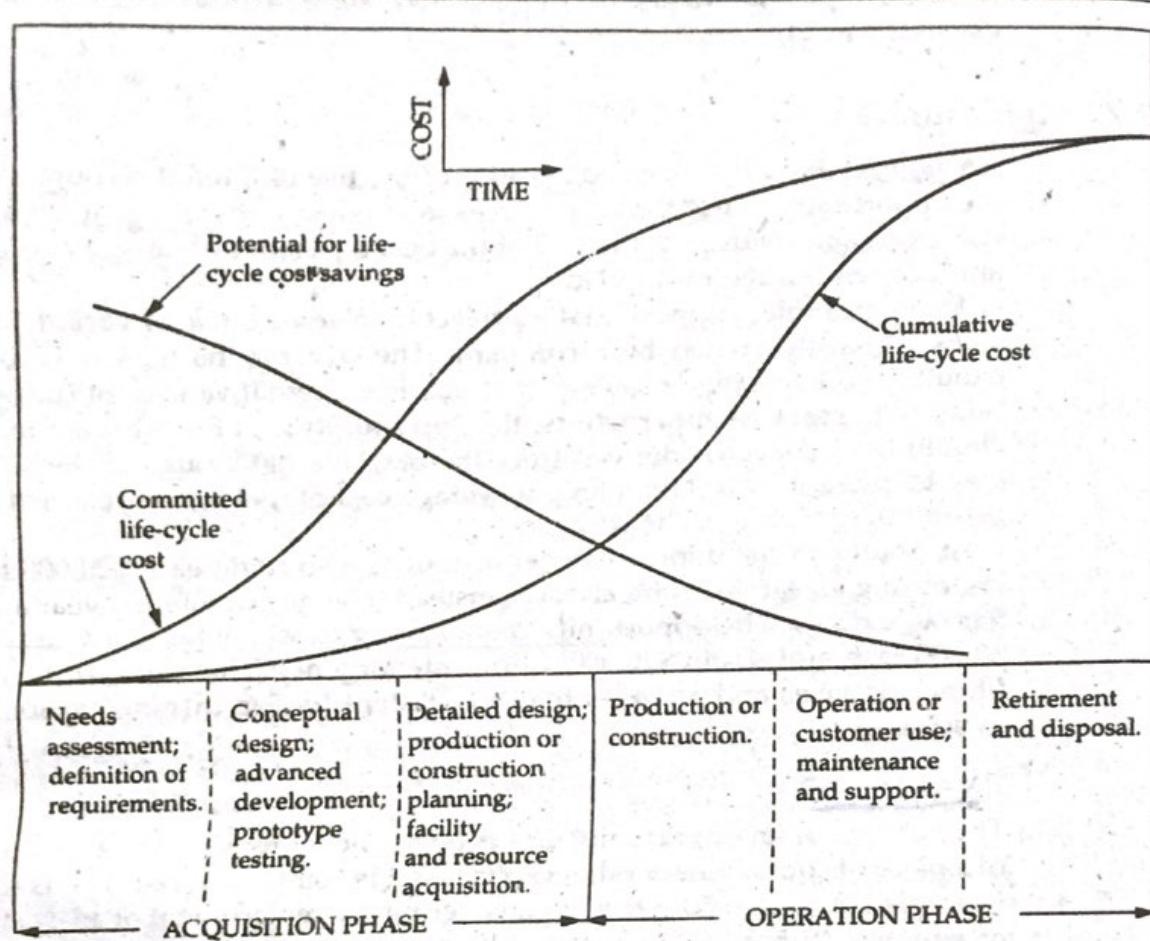


FIGURE 2-1 Phases of the Life Cycle and Their Relative Cost

be projected on a functional or an economic basis. For example, the time that a piece of equipment or a structure is able to perform economically may be shorter than that permitted by its physical capability. Changes in the design efficiency of a boiler represent this situation. The old boiler may be able to produce steam required but not economically enough for the intended use.

The life cycle may be divided into two general time periods, the acquisition phase and the operation phase. As shown in Figure 2-1, each of these phases is further subdivided into interrelated but different activity periods.

The acquisition phase begins with an analysis of the economic need or want—the analysis necessary to make explicit the requirement for the product, structure, system, or service. Then, with the requirement explicitly defined, the other activities in the acquisition phase can proceed in a logical sequence. The conceptual design activities translate the defined technical and operational requirements into a preferred preliminary design. Included in these activities

development of the feasible alternatives and engineering economic analyses to assist in selection of the preferred preliminary design. Also, advanced development and prototype testing activities to support the preliminary design work occur during this period.

The next group of activities in the acquisition phase involves detailed design and the planning for production or construction. This is followed by the activities necessary to prepare, acquire, and make ready for operation the facilities and other resources needed for the production, delivery, or construction of the product, structure, system, or service involved. Again, engineering economy studies are an essential part of the design process to analyze and compare alternatives and to assist in determining the final detailed design.

In the operation phase, the production, delivery, or construction of the end item(s) or service and their operation or customer use occur. This phase ends with retirement from active operation or use and, often, disposal of the physical assets involved. The priorities for engineering economy studies during the operation phase are achieving efficient and effective support to operations, determining whether (and when) replacement of assets should occur, and projecting the timing of retirement and disposal activities.

In Figure 2-1, relative cost profiles for the life cycle are shown. The greatest potential for achieving life-cycle cost savings is during the acquisition phase. How much of the life-cycle costs for the product, structure, system, or service can be saved is dependent on many factors. However, effective engineering design and economic analysis during this phase are critical in maximizing potential savings.

One aspect of cost-effective engineering design is minimizing the impact of design changes during the steps in the life cycle. In general, the cost of a design change increases by a multiple of approximately 10 with each step, as illustrated in Figure 2-2. Thus, there is a large savings incentive to have a sound preliminary design on which to base the detailed design and to prevent any changes occurring during the production/construction and operation parts of the life cycle.

The committed life-cycle cost curve increases rapidly during the acquisition phase. In general, approximately 80% of life-cycle costs are "locked in" at the end of this phase by the decisions made during requirements analysis and conceptual and detailed design. In contrast, as reflected by the cumulative life-cycle cost curve, only about 20% of actual costs occur during the acquisition phase, with about 80% being incurred during the operation phase.

Thus, the purpose of the life-cycle concept is to make explicit the interrelated effects of costs over the total life span for a product, structure, system, or service. The objective of the design process is to minimize the life-cycle cost, while meeting other performance requirements, by making the right trade-offs

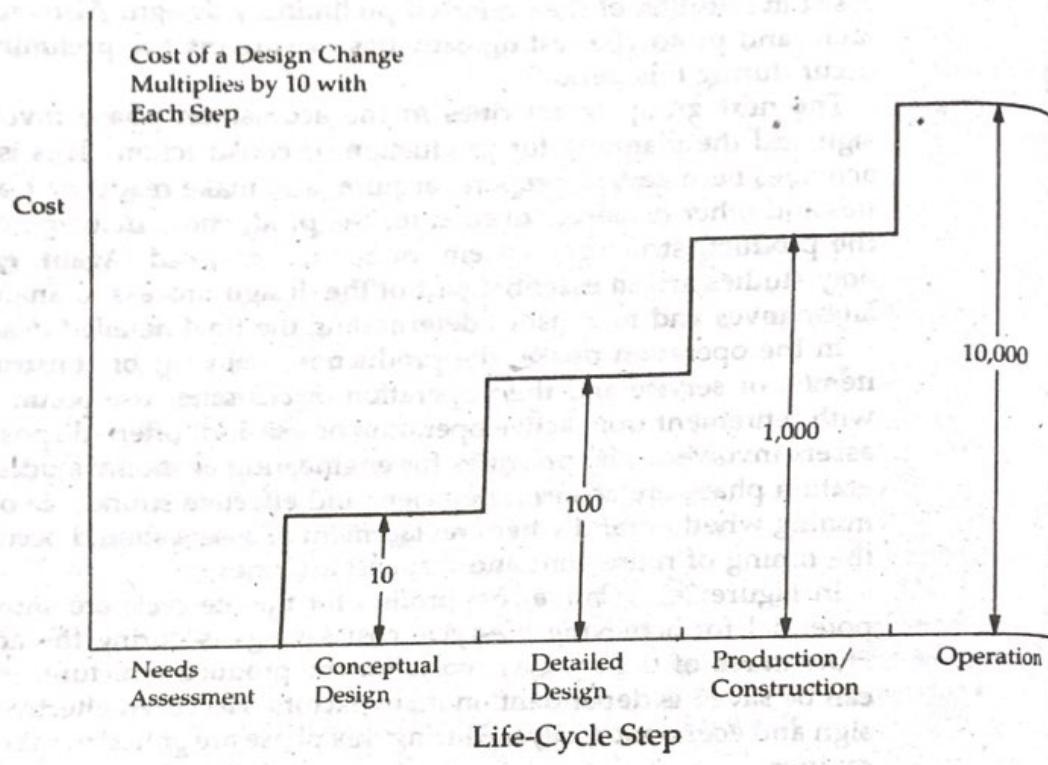


FIGURE 2-2 Costs of Design Changes Are Significant

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between prospective costs during the acquisition phase and those during operation phase.

The cost elements of the life cycle that need to be considered will vary with the situation. Because of their common use, however, several basic life-cycle cost categories will now be defined.

The investment cost, or *first cost*, is the capital required for most of the activities in the acquisition phase. In simple cases, such as acquiring specific equipment, a first cost may be incurred as a single expenditure. On a large, complex construction project, however, a series of expenditures over an extended period could be incurred.

#### EXAMPLE 2-5

Consider the situation where the equipment and related support for a computer-aided design/computer-aided manufacturing (CAD/CAM) workstation are being acquired for the engineering department that you work in. The applicable cost elements and estimated expenditures are as follows:

Cost Element	Cost
Install a leased telephone line for communication	\$ 1,100/month
Lease CAD/CAM software (includes installation and debugging)	550/month
Purchase hardware (CAD/CAM workstation)	20,000
Purchase a 9600-baud modem	2,500
Purchase a high-speed printer	1,500
Purchase a four-color plotter	10,000
Shipping costs	500
Initial training (in house) to gain proficiency with CAD/CAM software	6,000

What is the investment cost of this CAD/CAM system?

Solution The investment cost in this example is the sum of all the cost elements except the two monthly lease expenditures: specifically, the sum of the initial costs for the CAD/CAM workstation, modem, printer, and plotter (\$34,000); shipping cost (\$500); and the initial training cost (\$6,000). These cost elements result in a total investment cost of \$40,500. The two cost elements that involve lease payments on a monthly basis (telephone line and CAD/CAM software) are part of the recurring costs in the operation phase. ■

The term *working capital* refers to the funds required for current assets (i.e., other than fixed assets such as equipment, facilities, etc.) that are needed for the start and subsequent support of operation activities. For example, products cannot be made or services delivered without having some materials available in inventory. Functions such as maintenance cannot be supported without a minimum level of spare parts, tools, trained personnel, and other resources. Also, some cash must be available to pay employee salaries and the other immediate expenses of operation. The amount of working capital needed will vary with the project involved, and some or all of the investment in working capital is usually recovered at the end of a project's life.

The *operation and maintenance cost* category contains most of the recurring cost elements associated with the operation phase of the life cycle. The direct and indirect costs of operation associated with the five primary resource areas—people, machines, materials, energy, and information—are a major part of the costs in this category. Additional costs in the overhead classification, not already included in the indirect costs of the five resource areas, are another significant part of the operation and maintenance costs.

The *disposal cost* category includes those nonrecurring costs of shutting down the operation and the retirement and disposal of assets at the end of the life cycle. Normally, costs associated with personnel, materials, transportation, and one-time special activities can be expected. These costs, in some instances, will be partially offset by receipts from the sale of assets with remaining market value. A classical example of a disposal cost is that associated with cleaning up a site where a chemical processing plant had been located.

## 2.3 The General Economic Environment

There are certain general economic principles that frequently must be taken into account in economy studies. In broad terms, economics deals with interactions between people and wealth. Because people, as individuals, are not all alike as to their reactions, the subject of economics necessarily must deal with these interactions in generalized terms. The purpose of this section is to examine briefly some of these basic economic concepts and to indicate how they may be factors for consideration in making engineering economy studies and managerial decisions.

### 2.3.1 Consumer and Producer Goods and Services

The goods and services that are produced and utilized may be conveniently divided into two classes. Consumer goods and services are those products or services that are directly used by people to satisfy their wants. Food, clothing, homes, cars, television sets, haircuts, opera, and medical services are examples. The producers of consumer goods and services must be aware of, and are subject to, the changing wants of the people to whom their products are sold. At the same time, the demand for such goods and services is directly related to people and in many cases, as will be discussed later, may be determined with considerable certainty.

Producer goods and services are used to produce consumer goods and services or other producer goods. Machine tools, factory buildings, buses, and farm machinery are examples. Although, in the long run, producer goods serve to satisfy human wants, they are the means to that end. Thus the amount of producer goods needed is determined indirectly by the amount of consumer goods or services that are demanded by people. However, because the relationship is much less direct than for consumer goods and services, the demand for, and production of, producer goods may greatly precede or lag behind the demand for the consumer goods that they will produce.

### 2.3.2 Measures of Economic Worth

Goods and services are produced, and desired, because directly or indirectly they have utility—the power to satisfy human wants and needs. Thus they may be used or consumed directly, or they may be used to produce other goods or services that may, in turn, be used directly. Utility most commonly is measured in terms of value, expressed in some medium of exchange as the price that must be paid to obtain the particular item.

Much of our business activity, including engineering, focuses on increasing the utility (value) of materials and products by changing their form or location. Thus iron ore, worth only a few dollars per ton, may be increased in value to several dollars a pound by processing it and combining it with suitable alloying elements and converting it into razor blades. Similarly, snow, worth almost nothing when high in distant mountains, can be made quite valuable when it is delivered in melted form several hundred miles away to dry southern California.

### 2.3.3 Necessities, Luxuries, and Price Demand

Goods and services may be divided into two types, *necessities* and *luxuries*. Obviously, these terms are relative, because for most goods and services, what one person may consider to be a necessity may be considered by another to be a luxury. Economic status is an important factor in one's views regarding luxuries and necessities. Other factors also may be determining. For example, a person living in one community may find that an automobile is a necessity to get to and from work. If the same person lived and worked in a different city, adequate public transportation might be available, and an automobile would be a luxury. Also, the classification of goods and services into luxuries and necessities is less easy for producer goods than for consumer goods. However, for all goods and services, there is a relationship between the price that must be paid and the quantity that will be demanded or purchased.

This general relationship is depicted in Figure 2-3. As the selling price per unit ( $p$ ) is increased, there will be less demand ( $D$ ) for the product, and as the selling price is decreased, the demand will increase. The relationship between price and demand can be expressed as a linear function:

$$p = a - bD \quad \text{for } 0 \leq D \leq \frac{a}{b} \quad (2-1)$$

where  $a$  is the intercept on the price axis and  $-b$  is the slope. Thus  $b$  is the amount by which demand increases for each unit/decrease in  $p$ . Both  $a$  and  $b$  are constants. It follows, of course, that

$$D = \frac{a - p}{b} \quad (2-2)$$

Although Figure 2-3 illustrates the general relationship between price and demand, this relationship would probably be different for necessities and luxuries. Consumers can readily forgo the consumption of luxuries if the price is greatly increased, but they find it more difficult to reduce their consumption of true necessities. Also, they will use the money saved by not buying luxuries to pay the increased cost of necessities. Figure 2-4 shows how the demand curves for luxuries and necessities might differ.

The extent to which price changes influence demand varies according to the elasticity of the demand. The demand for products is said to be *elastic* when a decrease in the selling price results in a considerable increase in sales. On

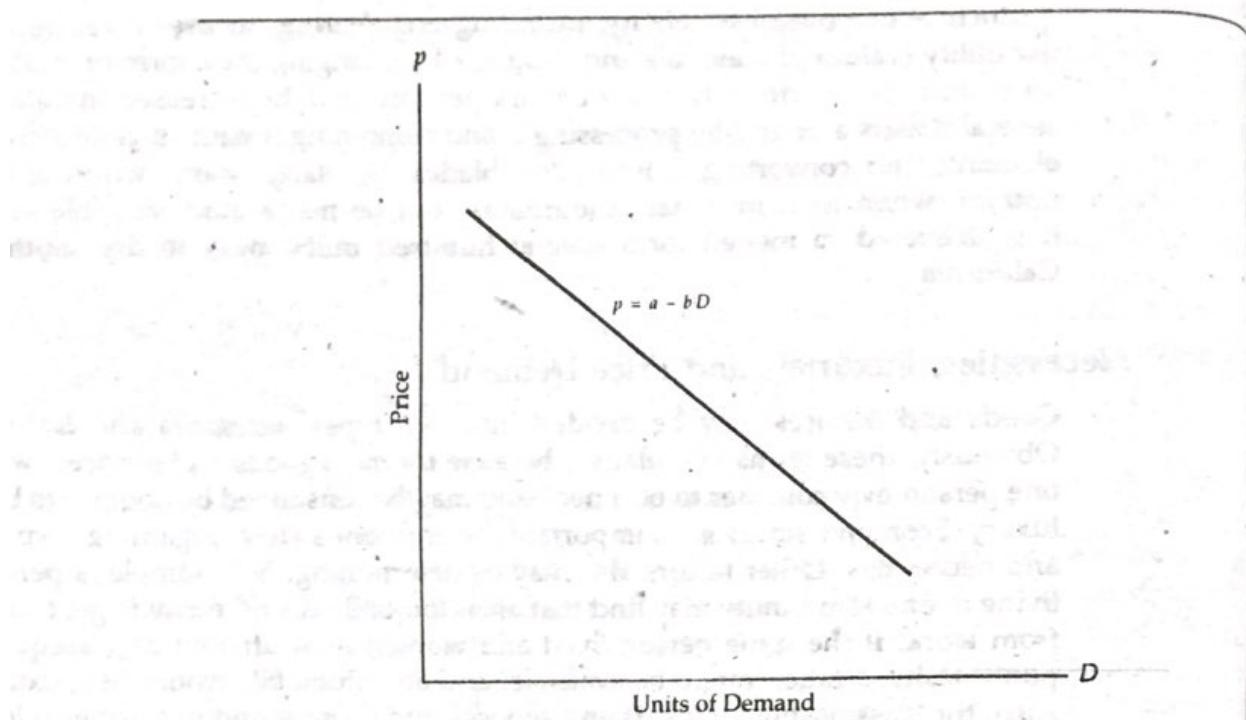


FIGURE 2-3 General Price-Demand Relationship

(Note that price is considered to be the independent variable but is shown as the vertical axis. This convention is commonly used by economists.)

the other hand, if a change in selling price produces little or no effect on demand, the demand is said to be *inelastic*. It is clear, from Figure 2-4, that luxury items have greater elasticity of demand than do necessities.

### 2.3.4 Competition

Because economic laws are general statements regarding the interaction of people and wealth, they will be affected by the economic environment in which the people and the wealth exist. Most general economic principles are stated for situations in which perfect competition exists.

Perfect competition occurs in a situation in which any given product is supplied by a large number of vendors and there is no restriction on additional vendors entering the market. Under such conditions, there is assurance of complete freedom on the part of both buyer and seller. Actually, of course, perfect competition may never exist because of a multitude of factors that impose some degree of limitation upon the actions of buyers or sellers, or both. However, with conditions of perfect competition assumed, it is easier to formulate general economic laws. When deviations from perfect competition are known to exist, their probable economic effects can be taken into account, at least approximately.

The existing competitive situation is an important factor in most engineering economy studies. It will have a very real effect upon decisions that are made.

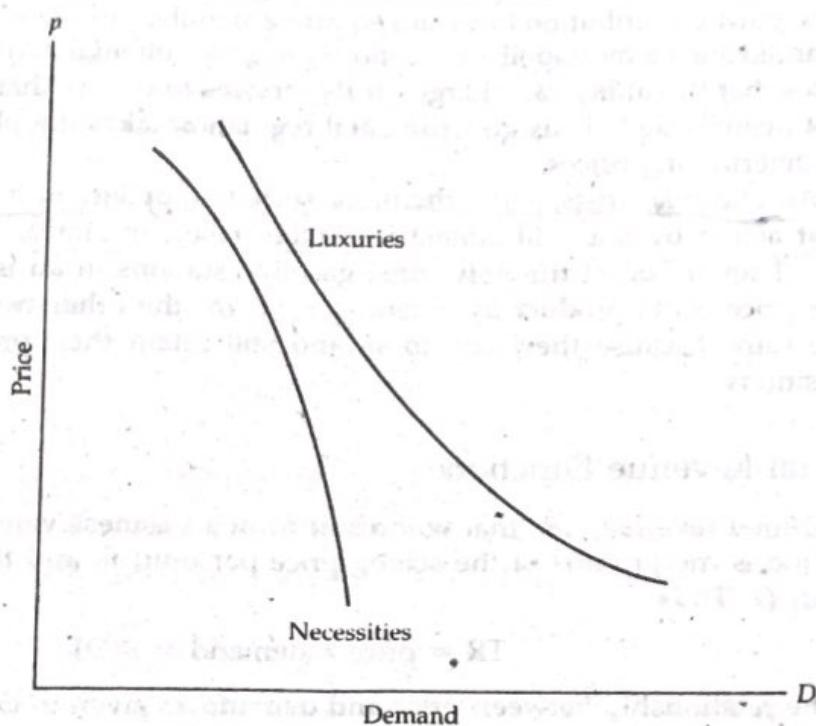


FIGURE 2-4 Generalized Price-Demand Relationship for Luxuries and Necessities

(Note that price is considered to be the independent variable but is shown as the vertical axis.)

unless information is available to the contrary, it should be assumed that competitors do or will exist and that they produce a quality product or service, and the resulting effects should be taken into account.

Monopoly is at the opposite pole from perfect competition. A perfect monopoly exists when a unique product or service is available from only a single vendor and that vendor can prevent the entry of all others into the market. Under such conditions, the buyer is at the complete mercy of the vendor in terms of the availability and price of the product. Actually, there seldom is a perfect monopoly. This is due to the fact that few products are so unique that substitutes cannot be used satisfactorily, or to the fact that governmental regulations prohibit monopolies if they are unduly restrictive.

A monopoly may be of great benefit to a producer in that it may permit control of the supply and the price to provide maximum profit. Although this might result in high prices for the product, higher total profits may be obtained from lower prices and wider distribution. Under some conditions, a monopoly may avoid costly duplication of facilities and thus make possible lower prices for products and services. This situation is recognized by governing bodies in granting public utilities exclusive rights to render service in a given territory.

Such a practice, for example, avoids having two electric power companies duplicate power-distribution lines and equipment in the same city. When vendors are granted such a monopolistic position, the governmental body also regulates the rates that the utility can charge for its services to ensure that the customers are not overcharged. Thus governmental regulation takes the place of competition in determining prices.

An oligopoly exists when there are so few suppliers of a product or service that action by one will almost inevitably result in similar action by the others. Thus if one of the only three gasoline stations in an isolated town raises the price of its product by 1 cent per gallon, the other two will probably do the same because they can do so and still retain their previous competitive positions.

### 2.3.5 The Total Revenue Function

The total revenue, TR, that will result from a business venture during a given period is the product of the selling price per unit,  $p$ , and the number of units sold,  $D$ . Thus

$$TR = \text{price} \times \text{demand} = p(D) \quad (2-3)$$

If the relationship between price and demand as given in Equation 2-1 is used,

$$TR = (a - bD)D = aD - bD^2 \quad \text{for } 0 \leq D \leq \frac{a}{b} \quad (2-4)$$

The relationship between total revenue and demand for the condition expressed in Equation 2-4 may be represented by the curve shown in Figure 2-5. Under these conditions, the maximum total revenue also would produce maximum total profit. From calculus the demand,  $\hat{D}$ , that will produce maximum total revenue can be obtained by solving

$$\frac{d\text{TR}}{dD} = a - 2bD = 0 \quad (2-5)$$

Thus

$$\hat{D} = \frac{a}{2b} \quad (2-6)$$

For example, if the equation for price is given by  $50,000 - 200D$ , the demand,  $\hat{D}$ , that maximizes total revenue is equal to  $50,000/400 = 125$  units. It must be emphasized that, because of cost-volume relationships discussed in the following section, most businesses would not obtain maximum profits by maximizing revenue. Thus the cost-volume relationship must be considered and related to revenue.

At this point, attention is called to the derivative of the total revenue with respect to volume (demand),  $d\text{TR}/dD$ , which is called the incremental or marginal revenue.

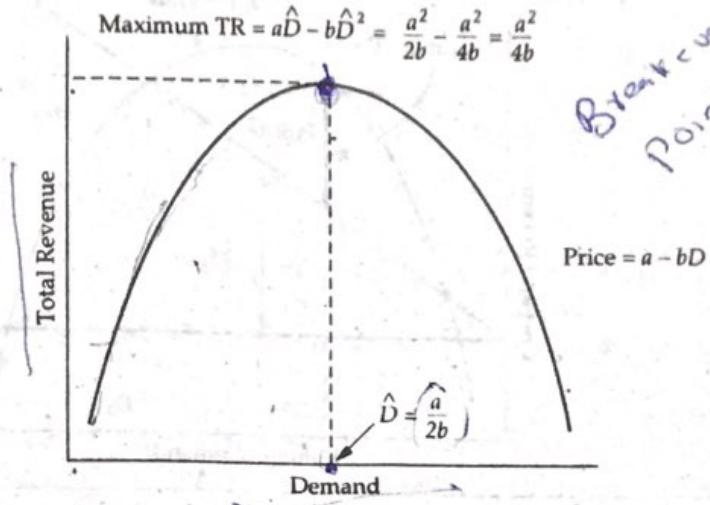


FIGURE 2-5 Total Revenue Function as a Function of Demand

### 2.3.6 Cost, Volume, and Breakeven Point Relationships

Fixed costs remain constant over a wide range of activities as long as the business does not permanently discontinue operations, but variable costs vary in total with the volume of output (Section 2.2.1). Thus, at any demand,  $D$ , total cost is

$$C_T = C_F + C_V \quad (2-7)$$

where  $C_F$  and  $C_V$  denote fixed and variable costs, respectively. For the linear relationship assumed here,

$$C_V = (c_v)(D) \quad (2-8)$$

where  $c_v$  is the variable cost per unit.

When total revenue, as depicted in Figure 2-5, and total cost, as given by Equations 2-7 and 2-8, are combined, the typical results as a function of demand are depicted in Figure 2-6. At breakeven point  $D'_1$ , total revenue is equal to total cost, and an increase in demand will result in a profit for the operation. Then at optimal demand,  $D^*$ , profit is maximized (Equation 2-10). At breakeven point  $D'_2$ , total revenue and total cost are again equal, but additional volume will result in an operating loss instead of a profit. Obviously, the conditions for which breakeven and maximum profit occurs are our primary interest. First, at any volume (demand),  $D$ ,

$$\begin{aligned} \text{Profit (loss)} &= \text{total revenue} - \text{total costs} \\ &= (aD - bD^2) - (C_F + c_vD) \\ &= -C_F + (a - c_v)D - bD^2 \quad \text{for } 0 \leq D \leq \frac{a}{b} \end{aligned} \quad (2-9)$$

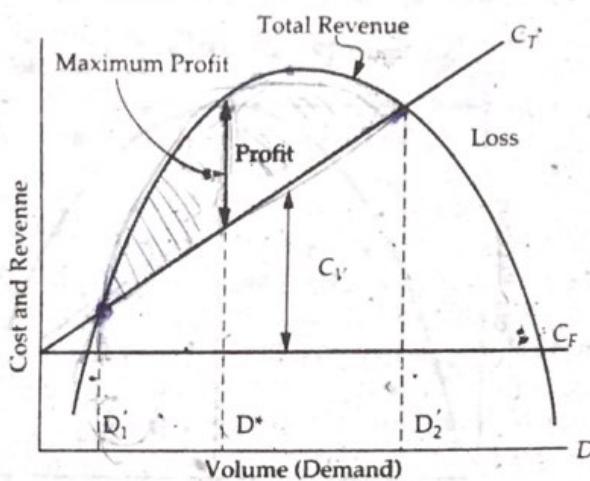


FIGURE 2-6 Combined Cost and Revenue Functions, and Breakeven Points, as Functions of Volume, and Their Effect on Typical Profit

In order for a profit to occur, based on Equation 2-9, and to achieve the typical results depicted in Figure 2-6, two conditions must be met:

1.  $(a - c_v) > 0$ ; that is, the price per unit that will result in no demand has to be greater than the variable cost per unit.
2. Total revenue (TR) must exceed total cost ( $C_T$ ) for the period involved.

If these conditions are met, we can find the optimal demand at which maximum profit will occur by taking the first derivative of Equation 2-9 with respect to  $D$  and setting it equal to zero:

$$\frac{d(\text{profit})}{dD} = a - c_v - 2bD = 0 \Rightarrow a - c_v = 2bD \quad D^* = \frac{a - c_v}{2b}$$

The optimal value of  $D$  that maximizes profit is

$$D^* = \frac{a - c_v}{2b} \quad (2-10)$$

An economic breakeven point for an operation is reached when total revenue equals total cost. Then for total revenue and total cost, as used in the development of Equations 2-9 and 2-10 and at any demand,  $D$ ,

$$\begin{aligned} \text{Total revenue} &= \text{total cost} \quad (\text{breakeven point}) \\ aD - bD^2 &= C_F + c_v D \\ -bD^2 + (a - c_v)D - C_F &= 0 \end{aligned} \quad (2-11)$$

Since Equation 2-11 is a quadratic equation with one unknown ( $D$ ), we can solve for the breakeven points  $D'_1$  and  $D'_2$  (the roots of the equation).

$$D' = \frac{-(a - c_v) \pm [(a - c_v)^2 - 4(-b)(-C_F)]^{1/2}}{2(-b)} \quad (2-12)$$

With the conditions for a profit satisfied (Equation 2-9), the quantity in the brackets of the numerator (the discriminant) in Equation 2-12 will be greater than zero. This will ensure that  $D'_1$  and  $D'_2$  have real positive, and unequal, values.

### EXAMPLE 2-6 ✓

A company produces an electronic timing switch that is used in consumer and commercial products made by several other manufacturing firms. The fixed cost ( $C_F$ ) is \$73,000 per month, and the variable cost ( $c_v$ ) is \$83 per unit. The selling price per unit is  $p = \$180 - 0.02(D)$ , based on Equation 2-1. For this situation, (a) determine the optimal volume for this product, and confirm that a profit occurs (instead of a loss) at this demand, and (b) find the volumes at which breakeven occurs; that is, what is the range of profitable demand?

Solution

$$(a) D^* = \frac{a - c_v}{2b} = \frac{\$180 - 83}{2(0.02)} = 2,425 \text{ units per month} \quad (\text{Equation 2-10})$$

Is  $(a - c_v) > 0$ ?

$$(\$180 - 83) = \$97, \text{ which is greater than } 0.$$

And is  $(\text{Total revenue} - \text{total cost}) > 0$  for  $D^* = 2,425$  units per month?

$$[\$180(2,425) - 0.02(2,425)^2] - [\$73,000 + 83(2,425)] = \$44,612$$

Thus, a demand of  $D^* = 2,425$  units per month results in a maximum profit of \$44,612 per month.

(b) Total revenue = total cost (breakeven point)

$$-bD^2 + (a - c_v)D - C_F = 0 \quad (\text{Equation 2-11})$$

$$-0.02D^2 + (\$180 - \$83)D - \$73,000 = 0$$

$$-0.02D^2 + 97D - 73,000 = 0$$

And, from Equation 2-12,

$$D' = \frac{-97 \pm [(97)^2 - 4(-0.02)(-73,000)]^{0.5}}{2(-0.02)}$$

$$D'_1 = \frac{-97 + 59.74}{-0.04} = 932 \text{ units per month}$$

$$D'_2 = \frac{-97 - 59.74}{-0.04} = 3,918 \text{ units per month}$$

Thus, the range of profitable demand is 932 to 3,918 units per month. ■

When the price per unit ( $p$ ) for a product or service can be represented as constant over a range of demand (versus being a linear function of demand, as

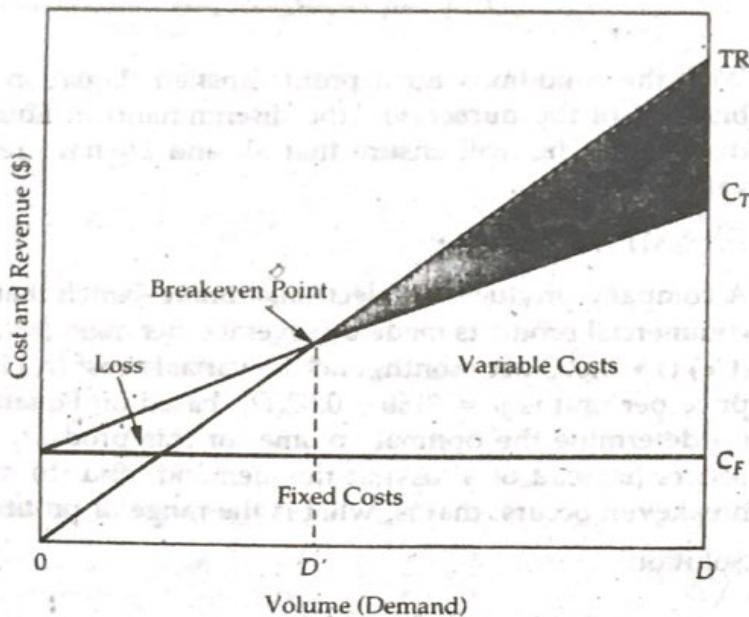


FIGURE 2-7 Typical Breakeven Chart with Price ( $p$ ) a Constant

assumed in Equation 2-1), and is greater than the variable cost per unit ( $c_v$ ), a single breakeven point results. Then under the assumption that demand is immediately met, total revenue ( $TR = p(D)$ ). If the linear relationship for costs in Equations 2-7 and 2-8 is also used in the model, the typical situation is depicted in Figure 2-7.

#### EXAMPLE 2-7

An engineering consulting firm measures its output in a standard service hour unit, which is a function of the personnel grade levels in the professional staff. The variable cost ( $c_v$ ) is \$62 per standard service hour. The charge-out-rate, i.e., selling price ( $p$ ), is  $1.38(c_v) = \$85.56$  per hour. The maximum output of the firm is 160,000 hours per year, and its fixed cost ( $C_F$ ) is \$2,024,000 per year. For this firm, (a) what is the breakeven point in standard service hours and in percentage of total capacity, and (b) what is the percentage reduction in the breakeven point (sensitivity), if fixed costs are reduced 10%; if variable cost per hour is reduced 10%; if both costs are reduced 10%; and if the selling price per unit is increased by 10%?

Solution

(a)

$$\text{Total revenue} = \text{total cost} \quad (\text{breakeven point})$$

$$pD' = C_F + c_v D'$$

$$D' = \frac{C_F}{(p - c_v)} \quad (2-1)$$

and  $D' = \frac{\$2,024,000}{(\$85.56 - \$62)} = 85,908 \text{ hours per year}$

*Cv/hour < Cv/stand*  
 $D' = \frac{85,908}{160,000} = 0.537, \text{ or } 53.7\% \text{ of capacity}$

(b) 10% reduction in  $C_F$ : *→ O/P of firm*

$$D' = \frac{0.9(\$2,024,000)}{(\$85.56 - \$62)} = 77,318 \text{ hours per year}$$

and  $\frac{85,908 - 77,318}{85,908} = 0.10, \text{ or a } 10\% \text{ reduction in } D'$

10% reduction in  $c_v$ :

$$D' = \frac{\$2,024,000}{[\$85.56 - 0.9(\$62)]} = 68,011 \text{ hours per year}$$

and  $\frac{85,908 - 68,011}{85,908} = 0.208, \text{ or a } 20.8\% \text{ reduction in } D'$

10% reduction in both  $C_F$  and  $c_v$ :

$$D' = \frac{0.9(\$2,024,000)}{[\$85.56 - 0.9(\$62)]} = 61,210 \text{ hours per year}$$

and  $\frac{85,908 - 61,210}{85,908} = 0.287, \text{ or a } 28.7\% \text{ reduction in } D'$

10% increase in  $p$ :

$$D' = \frac{\$2,024,000}{[1.1(\$85.56) - \$62]} = 63,021 \text{ hours per year}$$

and  $\frac{85,908 - 63,021}{85,908} = 0.266, \text{ or a } 26.6\% \text{ reduction in } D'$

Thus, the breakeven point is more sensitive to a reduction in variable cost per hour than to the same percentage reduction in the fixed cost, but reduced costs in both areas should be sought. However, the breakeven point is the most sensitive in this example to the selling price per unit,  $p$ . These results are summarized:

Change in Factor Value(s)	Decrease in Breakeven Point
10% reduction in $C_F$	10.0%
10% reduction in $c_v$	20.8
10% reduction in $C_F$ and in $c_v$	28.7
10% increase in $p$	26.6

The breakeven point for an operating situation can be determined in units of output, percentage utilization of capacity, or sales volume (demand). In Example 2-7 (part a), the breakeven point ( $D'$ ) was calculated in units of output (85,908 standard service hours) and then, using the total capacity figure (160,000 hours per year), it was also expressed as percentage utilization of capacity (53.7%). In terms of sales volume, the breakeven point in Example 2-7 is  $\$85.56(85,908) = \$7,350,288$ .

Market competition often creates pressure to lower the breakeven point of an operation; the lower the breakeven point, the less likely that a loss will occur during market fluctuations. Also, if the selling price remains constant, a larger profit will be achieved at any level of operation above the reduced breakeven point.

### 2.3.7 Average Unit Cost Function

Most engineering projects and business operations are designed to operate more efficiently at a certain level of capacity utilization. Deviations from this level may affect the variable cost of operation and possibly the fixed cost, and will influence the *average unit cost* of the product or service. Any impacts will, in turn, affect the total profit.

The average unit cost ( $C_U$ ) at any volume (demand,  $D$ ), within the capacity related to the fixed cost ( $C_F$ ), is simply the total cost ( $C_T$ ) at that volume divided by  $D$ .

$$C_U = \frac{C_T}{D} = \frac{C_F + C_V}{D} \quad (2-14)$$

When total variable costs are a linear function of demand,  $C_V = c_v D$  (Equation 2-8), the average unit cost is

$$C_U = \frac{C_F}{D} + c_v \quad (2-15)$$

Applying Equation 2-15 to Example 2-7, the average unit cost is

$$C_U = \frac{\$2,024,000}{D} + \$85.56$$

and the result for the engineering consulting firm is shown in Figure 2-8. Thus, in the linear model situation, the minimum average cost occurs at the maximum output level that is feasible without affecting fixed costs.

When the variable cost per unit does not remain constant over the capacity range, but begins to increase at a certain demand level, say  $D = K$ , then the

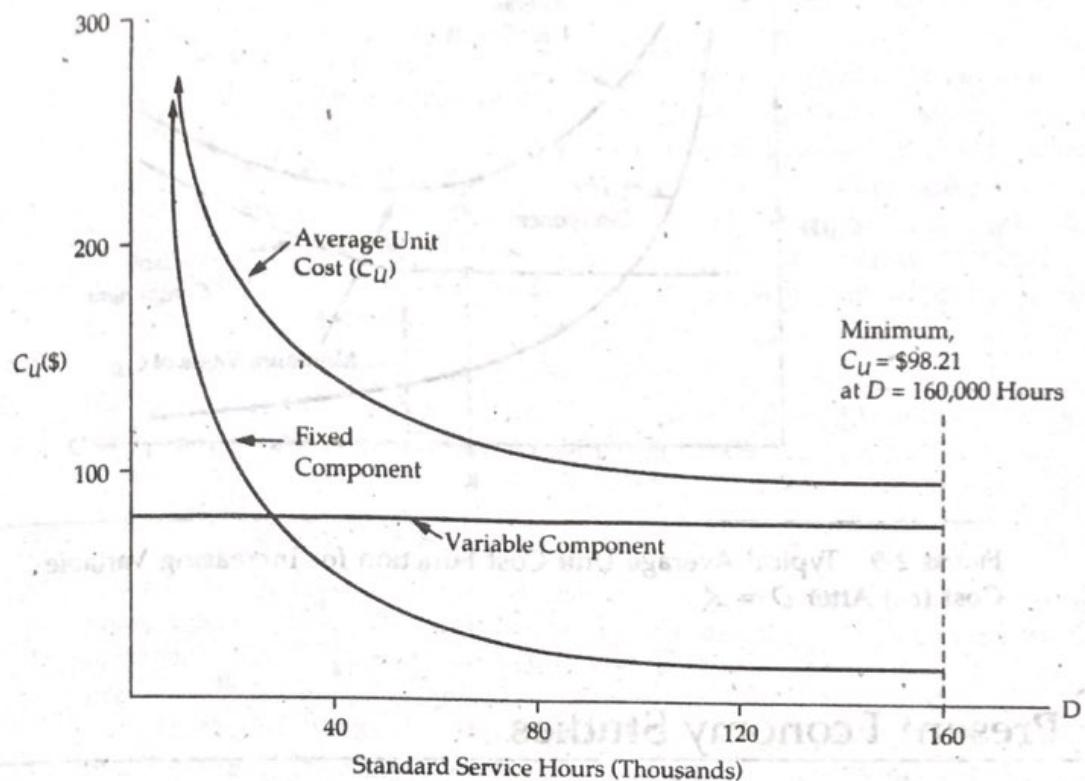


FIGURE 2-8 Average Unit Cost Function for Example 2-7

minimum average unit cost occurs at, or after, demand  $K$ . The location of the minimum value of  $C_U$ , between  $D = K$  and the upper limit of the capacity range, depends upon the rate at which the average variable unit cost increases versus the rate at which the average fixed unit cost decreases. Increasing the variable unit cost, after a certain demand level has been reached, is common in many operations. This is caused by personnel overtime costs, increased maintenance expenses, and so on, which are often required to reach maximum capacity output level. A typical average unit cost function is shown in Figure 2-9, where the average variable unit cost is increasing at a rate greater than the decrease in the average fixed unit cost at some demand after  $D = K$ . Otherwise, the minimum value of  $C_U$  occurs at the maximum capacity output level. The mathematical expression for  $C_U$  in this situation is

$$C_U = \frac{C_F}{D} + c_v \quad \text{when } D \leq K \quad (2-16)$$

$$C_U = \frac{C_F}{D} + \frac{c_v(K) + f(c_v, K, D)}{D} \quad \text{when } K < D \leq \text{capacity}$$

where  $f(c_v, K, D)$  is a nonlinear variable cost function. This situation is the subject of Problem 2-23 in Section 2.7.

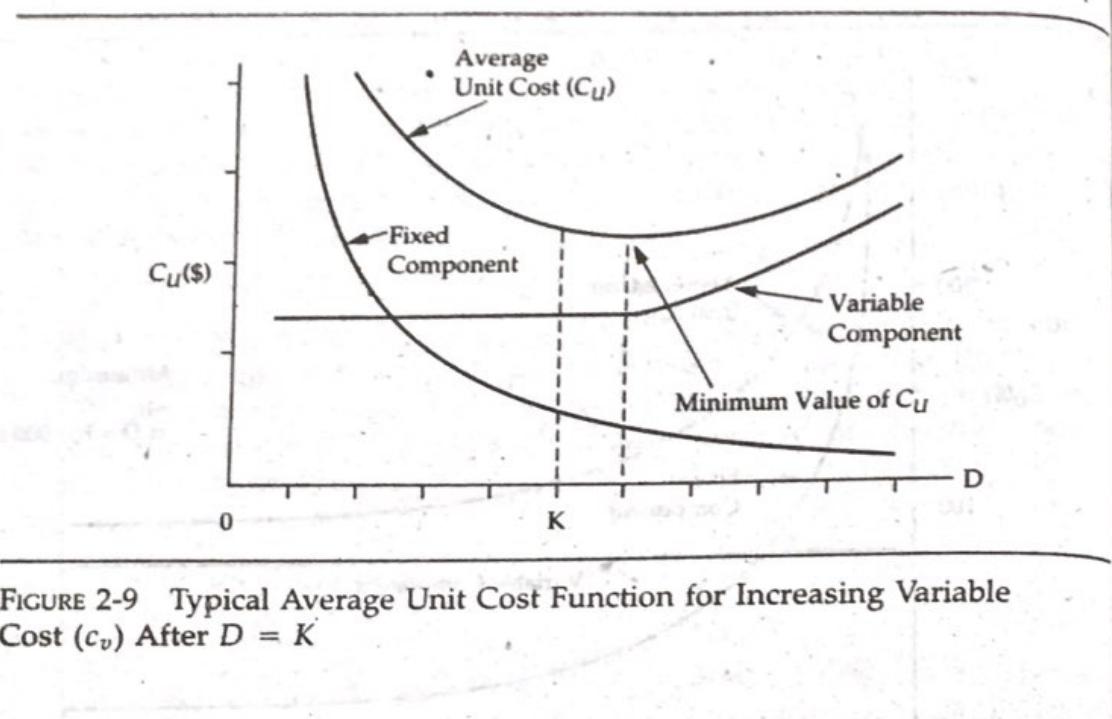


FIGURE 2-9 Typical Average Unit Cost Function for Increasing Variable Cost ( $c_v$ ) After  $D = K$

## 2.4 Present Economy Studies

When the influence of time on money is not a significant consideration, cost analyses are usually called *present economy studies*.

Typical situations involving present economy studies are as follows:

1. There is no initial investment of capital; only immediate operating costs and other factors are involved. As an example, assume that you are employed by Company A and are making plans for a business trip. You can travel by commercial aircraft, which will require 3 hours of travel time and the rental of a car at your destination. The other alternative is to travel by automobile, which will take 7 hours. Here the basic considerations are the immediate costs, the value of your time, and nonmonetary factors (e.g., fatigue).
2. There is an initial investment of capital, but after this first cost the remaining life-cycle cost is estimated to be the same, or directly proportional to the initial investment. Thus, the alternative with the lowest first cost will be the most economical. As an illustration, consider the construction of an interstate highway bridge overpass. Whether a longitudinally reinforced concrete slab or a precast (prestressed) concrete design is used, the maintenance and other life-cycle costs of the two designs would be proportionally the same. (However, if one or more steel design alternatives were also being considered, a present economy study probably would not be appropriate. Maintenance and other related costs would be expected to vary among the

alternatives, and the cost analysis should be based on the life cycle of the structure.)

3. The differences in revenues and costs among the alternatives all occur within a limited time period (1 year or less is a general guideline), or any future differences are estimated to remain proportional to those in the first time period. This is often the case when the decision is between alternative materials in manufacturing. For example, if using low-alloy/high-yield strength steel in a particular application is estimated to give better revenue and cost results than low-carbon steel, this relative advantage would be expected to remain in future time periods.

In engineering practice, situations that give rise to present economy studies are quite common, and five typical situations will be discussed. Recognizing these situations will often save considerable analysis effort.

#### 2.4.1 Total Cost in Material Selection

In a large proportion of cases, economic selection among materials cannot be based solely on the costs of the materials. Frequently, a change in materials will affect the processing costs, and shipping costs may also be altered. A good example of this is the product illustrated in Figure 2-10. The part was produced in considerable quantities on a high-speed turret lathe, using 1112 screw-machine steel costing \$0.30 per pound. A study was made to determine whether it might be cheaper to use brass screw stock, costing \$1.40 per pound. Since the weight of steel required per piece was 0.0353 pound and that of brass was 0.0384 pound, the material cost per piece was \$0.0106 for steel and \$0.0538 for brass. However, when the manufacturing and standards departments were consulted, it was found that, although 57.1 parts per hour were being produced using steel, the output would be 102.9 parts per hour if brass were used. Inasmuch as the machine operator was paid \$7.50 per hour and the overhead cost for the turret lathe was \$10.00 per hour, the total-cost comparison for the two materials was as follows.

	1112 Steel	Brass
Material	$\$0.30 \times 0.0353 = \$0.0106$	$\$1.40 \times 0.0384 = \$0.0538$
Labor	$\$7.50/57.1 = 0.1313$	$\$7.50/102.9 = 0.0729$
Overhead <sup>a</sup>	$\$10.00/57.1 = 0.1751$	$\$10.00/102.9 = 0.0972$
Total cost per piece	$\$0.3170$	$\$0.2239$

$$\text{Saving per piece by use of brass} = \$0.3170 - \$0.2239 = \$0.0931$$

<sup>a</sup>A given overhead rate applied to different alternatives without modification may be invalid for economic analyses even though useful in after-the-fact accounting allocations for whatever alternative is used.

Because a large number of parts were made each year, the saving of \$93.10 per thousand was a substantial amount. It is also clear that costs other than the cost of material were of basic importance in the economy study.

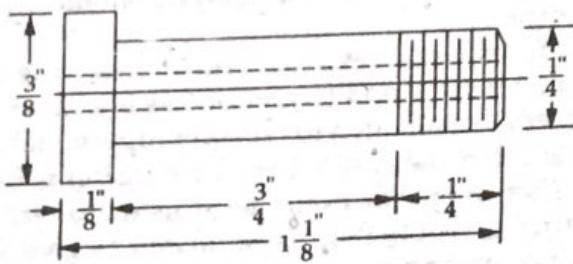


FIGURE 2-10 Small Screw Machine Product

The later history of this same product illustrates that shipping costs also must often be considered in selecting between materials. After the part had been made from brass stock for several years, it was found desirable to supply the domestic and foreign assembly plants of the company by using air freight for shipping. This led to a study of the possible use of a heat-treated aluminum alloy. This material cost \$0.85 per pound and the cost of heat-treating each part, at an outside plant, was \$0.018. Production studies indicated that the aluminum alloy could be machined at the same speeds as the brass stock.

The specific gravities of the brass and aluminum alloy are 8.7 and 2.75, respectively, and the raw and finished weights of the parts were as follows:

	Brass (lb)	Aluminum Alloy (lb)
Raw material	0.0384	$(0.0384)(2.75/8.7) = 0.01213$
Finished part	0.0150	$(0.0150)(2.75/8.7) = 0.00474$

Consequently, the comparative costs, including shipping at \$3.00 per pound of finished part, were as follows.

	Brass	Aluminum Alloy
Material	\$0.0538	\$0.0103
Labor	0.0729	0.0729
Heat treatment	—	0.0180
Overhead	0.0972	0.0972
Shipping	0.0450	0.0142
Total cost per piece	0.2689	0.2126

A decision was made to use the aluminum alloy for the parts that were to be shipped by air to the subsidiary plants and for the parts that were consumed at the local plant. There was no advantage to using brass even when shipping costs could be omitted.

Care should be taken in making economic selections between materials to ensure that any differences in yields or resulting scrap are taken into account. Commonly, alternative materials do not come in the same stock sizes, such as sheet sizes and bar lengths. This may considerably affect the yield obtained from a given weight of material; similarly, the resulting scrap may differ for different materials. This factor can have serious economic implications when one of the materials is considerably more costly than another. Determination of these effects is an illustration of where experience may be most helpful.

#### 2.4.2 Make Versus Purchase Studies

It is likely that more mistakes have been made in make versus purchase economic decisions, because of the improper use of standard or unit costs based on accounting records, than those due to any other single cause. Here the relationship of fixed costs, incremental costs, and unit costs is of the utmost importance. As was pointed out previously in this chapter, all standard costs and unit costs are based on some level of activity, and a number of arbitrary cost allocations are used in their determination, such as in the allocation of direct and other overhead costs. If operations are to be carried out at a different level, the unit costs are bound to be inaccurate. Since engineering economy studies often involve changes from existing conditions, it is apparent that the use of standard or unit costs may lead to considerable error. This is illustrated in Example 2-8.

#### EXAMPLE 2-8

A manufacturing plant consists of three departments: A, B, and C. Department A occupies 100 square meters in one corner of the plant. Product X is one of several products being produced in Department A. The daily production rate of X is 576 pieces. The cost accounting records show the following average daily production costs for Product X:

Direct labor	(1 operator working 4 hours per day at \$22.50/hr, inclusive of fringe benefits, plus a part-time foreman at \$30/day)	\$120.00
Material		86.40
Overhead	(at \$0.82 per square meter of floor area)	82.00
Total daily cost =		\$288.40

The department foreman has recently learned about an outside company that sells Product X at \$0.35 per piece. Accordingly, the foreman figured a daily cost of  $\$0.35(576) = \$201.60$ , resulting in a daily savings of  $\$288.40 - \$201.60 = \$86.80$ . Therefore, a proposal was submitted to the plant manager for shutting down the production line of Product X and buying it from the outside company.

However, the plant manager decided not to accept the foreman's proposal based on Product X's unit cost after examining each cost component separately.

1. *Direct labor:* Since the foreman was supervising the manufacture of other products in Department A in addition to Product X, the only possible savings in labor would occur if the operator working 4 hours per day on Product X were not reassigned after this line is shut down. That is, a maximum saving of \$90.00 per day would result.
2. *Materials:* The maximum savings on material will be \$86.40. However, this figure could be lower if some of the material for Product X is obtained from scrap of another product.
3. *Overhead:* Since other products are made in Department A, no reduction in total floor space requirements will probably occur. Therefore, no reduction in overhead costs will result from discontinuing Product X. It has been estimated that there will be daily savings in overhead of about \$3.00 due to a reduction in power costs and in insurance premiums.

Thus, the firm will save at most \$90.00 in direct labor, \$86.40 in materials, and \$3.00 in overhead, which totals \$179.40 per day. This daily savings estimate would not exceed the \$201.60 to be paid to the outside company if Product X is purchased. For this reason, the plant manager rejected the proposal of the foreman and continued the manufacture of Product X.

In conclusion, Example 2-8 shows how an erroneous decision might be made by using the unit cost of Product X without detailed analysis. The fixed cost portion of Product X's unit cost, which is present even if the manufacture of Product X is discontinued, was not properly accounted for in the original analysis by the foreman.

#### 2.4.3 Alternative Machine Speeds

Machines frequently can be operated at various speeds, resulting in different rates of product output. However, this usually results in different frequencies of machine downtime to permit servicing or maintaining the machine, such as resharpening or adjusting tooling. Such situations lead to present economy studies to determine the optimum or preferred operating speed.

A simple example of this type involved the planing of lumber. Lumber put through the planer increased in value by \$0.10 per board foot. When the planer was operated at a cutting speed of 5,000 feet per minute, the blades had to be sharpened after 2 hours of operation, and lumber could be planed at the rate of 1,000 board feet per hour. When the machine was operated at 6,000 feet per minute, the blades had to be sharpened after  $1\frac{1}{2}$  hours of operation, and the rate of planing was 1,200 board feet per hour. Each time the blades were changed, the machine had to be shut down for 15 minutes. The blades, unsharpened, cost \$50 per set and could be sharpened 10 times before having to be discarded. Sharpening cost \$10 per set. The crew that operated the planer changed and reset the blades. At what speed should the planer be operated?

Because the labor cost for the crew would be the same for either speed of operation, and because there was no discernible difference in wear upon the planer, these factors did not have to be included in the study.

*In problems of this type, the operating time plus the delay time due to the necessity for tool changes constitute a cycle time that determines the output from the machine. The time required for a complete cycle determines the number of cycles that can be completed in a period of available time—for example, 1 day—and a certain portion of each complete cycle is productive. The actual productive time will be the product of the productive time per cycle and the number of cycles per day.*

	Value per Day
At 5,000 feet per minute	
Cycle time = 2 hours + 0.25 hour = 2.25 hours	
Cycles per day = $8 \div 2.25 = 3.555$	
Value added by planing = $1,000 \times 3.555 \times 2 \times \$0.10 =$	$\$711.00$
Cost of resharpening blades = $3.555 \times \$10 = \$35.55$	
Cost of blades = $3.555 \times \$50/10 = 17.78$	
Total cost	$-53.33$
Net increase in value per day	$\$657.67$
At 6,000 feet per minute	
Cycle time = 1.5 hours + 0.25 hour = 1.75 hours	
Cycles per day = $8 \div 1.75 = 4.57$	
Value added by planing = $4.57 \times 1.5 \times 1,200 \times \$0.10 =$	$\$822.60$
Cost of resharpening blades = $4.57 \times \$10 = \$45.70$	
Cost of blades = $4.57 \times \$50/10 = 22.85$	
Total cost	$-68.55$
Net increase in value per day	$\$754.05$

Thus it was more economical to operate at the higher speed, in spite of the more frequent sharpening of blades that was required.

It should be noted that this analysis assumes that the added production can be used. If, for example, the maximum production needed is equal to or less than that obtained by the slower machine speed ( $1,000 \times 3.555$  cycles  $\times 2$  hours = 7,110 board feet per day), then the value added would be the same for each speed, and the decision then should be based on which speed minimizes total cost.

This type of study is also of great importance in connection with metal-cutting machine tool operations. Changes of cutting speeds can have a great effect on tool life. In addition, because the cost of machine tools and wage rates has increased, it is important that productivity be maintained at as high a level as possible. Under these conditions, it has frequently been found that increased cutting speeds give greater overall economy, even though the cutting-tool life is considerably less than was accepted practice in former years. This is particularly true if rapid means can be devised for changing tools when required.

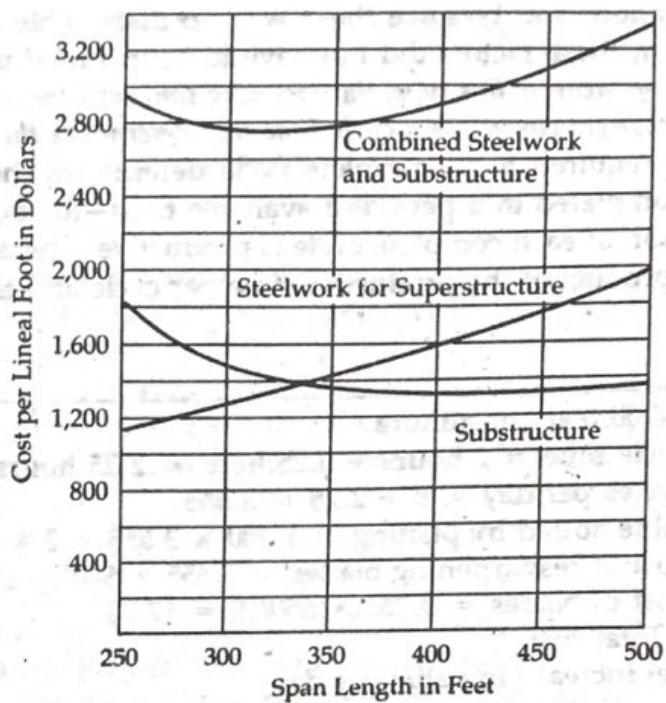


FIGURE 2-11 Costs per Lineal Foot of Structure for Low-Level Combined Bridges on Sand Foundations 200 Feet Deep

#### 2.4.4 Economic Span Length for Bridges

The design of a multiple-span bridge can be used to illustrate (1) the effect of design on costs, (2) the use of approximate cost curves, and (3) some of the limitations of such curves.

In many cases of design selection, some of the cost factors increase as certain design parameters are changed, and others decrease. The problem is one of selecting the design that affords the lowest total cost. Figure 2-11 shows the curves relating to such a condition. These cost curves show the cost per foot of structure for low-level combined bridges on sand foundations 200 feet deep. As might be expected, the cost of the substructure decreases somewhat as the span length increases. On the other hand, the cost of the steelwork increases quite rapidly as the span length increases. The greatest economy would be achieved by using span lengths of about 325 feet.

#### EXAMPLE 2-9

Now consider the application of these cost curves to the design of a bridge having a total length of 1,600 feet and the added requirement that at or adjacent to the center of the bridge there must be one span of at least 400 feet. Two designs are being considered: (1) four equal spans of 400-foot length and (2) one center span of 400 feet and two 300-foot spans on each side of the center span.

Solution Directly applying the cost data contained in Figure 2-11, we obtain the following figures:

Alternative	Length of Spans (ft)	Number of Spans	Cost per Foot	Cost
1	400	4	\$2,880	\$4,608,000
2	400	1	2,880	1,152,000
	300	4	2,760	3,312,000
Total cost				<u><u>\$4,464,000</u></u>

It thus appears that Alternative 2 will be more economical. However, before coming to a final decision, we should consider two factors related to the use of the cost curves of Figure 2-11. First, these cost curves assume that *all* spans are of equal length. Thus, when these curves are used in estimating the cost for Alternative 2, some error would be introduced. Second, it should be asked whether the construction methods and materials used in connection with determining the cost relationships portrayed in Figure 2-11 were the same as would be involved in building the bridge being considered. The effect of changed materials and methods could be considerable. Therefore, when using historical-cost data, either in tabular or curve form, considerable care and judgment must be exercised. For preliminary economy studies such data may be quite adequate; for final studies upon which important decisions are to be based, they may not be sufficiently accurate.

#### 4.5 The Relation of Design Tolerances and Quality to Production Cost

Design alternatives commonly must be considered from the viewpoint of economy of manufacture. Some designs contain features that are inherently more costly to produce than others. Unnecessarily tight dimensional tolerances are prime culprits in this respect. Not only does increased accuracy cost money, but the relaxation of accuracy requirements may make it possible to use different and less expensive processes. Figure 2-12 shows a set of curves relating tolerance and increased cost for several casting methods.

An example of how relaxation of tolerances can affect costs is found in the case of a small sand-cast part that required considerable machining so that the specified tolerances could be obtained. The cost for the finished part was as follows:

Cost of casting	\$ 9.50
Tooling cost, per part	0.50
Machining	8.25
Total	\$18.25

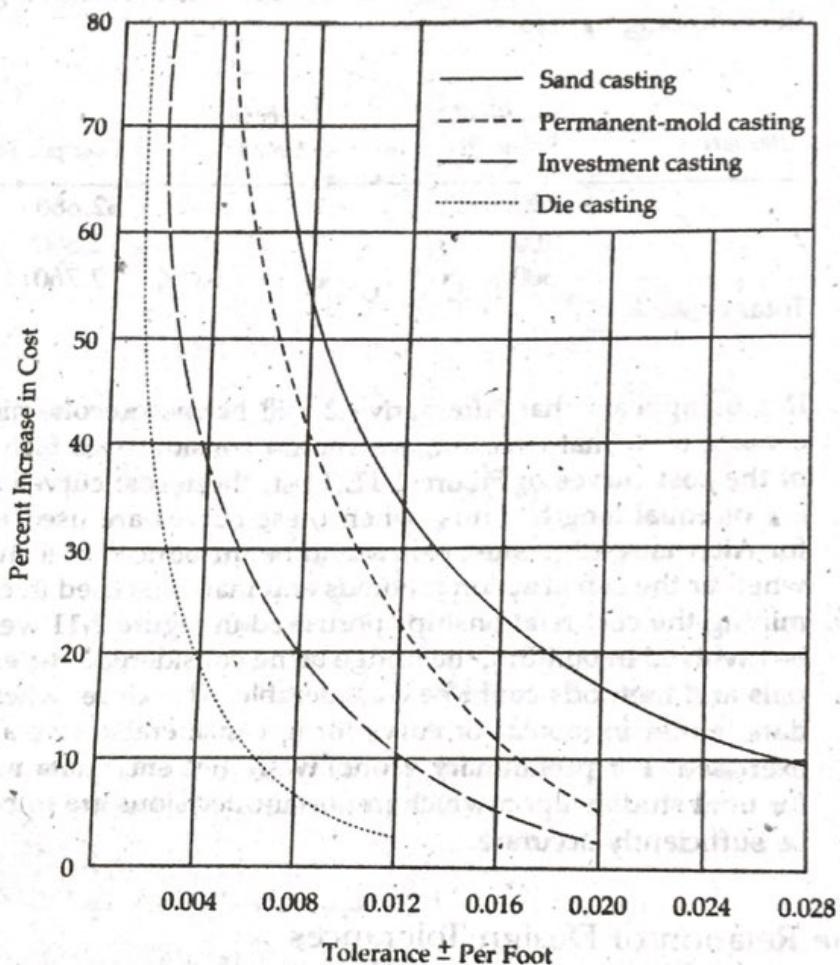


FIGURE 2-12 Relative Costs for Maintaining Specified Tolerances with Various Casting Processes

When the tolerance requirements were reconsidered, it was found that by relaxing the tolerances the part could be used as cast if it were made by die casting except for tapping some holes. An economy study of the die cast part showed the cost to be as follows:

Cost of casting	\$2.25
Tooling cost, per part	1.75
Machining	3.10
Total	\$7.10

Thus, in making design studies and economy studies of alternative designs we should not overlook the possibility of making more economical production methods possible by changes in design.

## 2.5 Summary

In this chapter we have discussed selected cost terminology and concepts important in engineering economic analysis. A listing of important abbreviations and notation, by chapter, is provided in Appendix B. It is important that the meaning and use of various cost terms are understood in order to communicate effectively.

Several general economic concepts were discussed and illustrated. First, the ideas of consumer and producer goods and services, measures of economic growth, competition, and necessities and luxuries were covered. Then, some relationships among costs, price, and volume (demand) were discussed and illustrated. Included were the concepts of optimal volume, breakeven points, and the average unit cost function.

The use of present economy studies in noncomplex engineering decision making can provide satisfactory results and save considerable analysis effort. When an adequate engineering economic analysis can be accomplished by considering the various monetary consequences that occur in a short time period (usually 1 year or less), then a present economy study should be used.

## 2.6 References

- BIERMAN, H., and S. SMIDT. *The Capital Budgeting Decision: Economic Analysis of Investment Projects*, 7th ed. New York: Macmillan Publishing Co., 1988.
- BLECKE, CURTIS J. *Financial Analysis for Decision Making*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1980.
- SCHWEYER, HERBERT E. *Analytic Models for Managerial and Engineering Economics*. New York: Reinhold Publishing Corp., 1964.

## 2.7 Problems

- 2-1. A company in the process industry produces a chemical compound that is sold to manufacturers for use in the production of certain plastic products. The plant that produces the compound employs approximately 300 people. Develop a list of six different cost elements that would be *fixed* and a similar list of six cost elements that would be *variable*. (2.2)
- 2-2. Refer to Problem 2-1 and your answer to it. (2.2)
- a. Develop a table that shows the cost elements you defined and classified as fixed and variable. Indicate which of these costs are also *recurring, nonrecurring, direct, or indirect*.
- b. Identify one additional cost element for each of the cost categories: recurring, nonrecurring, direct, and indirect.
- 2-3. Within the context of the plant operation used in Problem 2-1, describe and illustrate an *incremental cost* situation. (2.2)
- 2-4. Classify each of the following cost items as mostly fixed or variable: (2.2)
- Raw materials  
Direct labor  
Depreciation  
Supplies  
Utilities

Property taxes  
 Administrative salaries  
 Payroll taxes  
 Insurance (building and equipment)  
 Clerical salaries  
 Sales commissions  
 Rent  
 Interest on borrowed money

- 2-5. What is the relationship between the term *cash cost* and the *cash flow concept*? (2.2)
- 2-6. In your own words, describe the life-cycle cost concept. Why is the potential for achieving life-cycle cost savings greatest in the acquisition phase of the life cycle? (2.2)
- 2-7. Discuss the concept of economic utility. How does the utility of necessities differ from the utility of luxuries for low-income groups in this country? (2.3)
- 2-8. Explain why the elasticity of demand for a luxury product (e.g., tennis rackets) may be relevant in an economic study of an investment in proposed equipment for manufacturing this product. (2.3)
- 2-9. Explain why perfect competition is an ideal that is difficult to attain in the United States. List several business situations in which perfect competition is approached. (2.3)
- 2-10. Differentiate between monopoly, oligopoly, and perfect competition. For which situation are most general economic principles stated? Is a monopoly ever desirable for the economic welfare of the public? (2.3)
- 2-11. A company has established that the relationship between the sales price for one of its products and the quantity sold per month is approximately  $D = 780 - 10p$  units ( $D$  is the demand or quantity sold per month, and  $p$  is the price in dollars). The fixed cost is \$800 per month, and the variable cost is \$30 per unit produced. What number of units,  $D^*$ , should be produced per month and sold to maximize net profit? What is the maximum profit per month related to the product? (2.3)
- 2-12. A company estimates that the relationship between unit price and demand per month for a potential new product is approximated by  $p = \$100.00 - \$0.10D$ . The company can produce the product by increasing fixed costs \$17,500 per month, and the estimated variable cost is \$40 per unit. What is the optimal demand,  $D^*$ , and based on this demand, should the company produce the new product? Why? (2.3)
- a. Work out the complete solution by differential calculus, starting with the formula for profit or loss per month.
- b. Solve graphically for an approximate answer.
- 2-13. Refer to Problem 2-11. What is the range of profitable demand and production in number of units per month? (2.3)
- 2-14. A company produces and sells a consumer product, and thus far has been able to control the volume of the product by varying the selling price. The company is seeking to maximize its net profit. It has been concluded that the relationship between price and demand, per month, is approximately  $D = 500 - 5p$ , where  $p$  is the price per unit in dollars. The fixed cost is \$1,000 per month, and the variable cost is \$20 per unit. Obtain the answer, both mathematically and graphically, to the following questions (2.3)
- a. What is the optimal number of units that should be produced and sold per month?
- b. What is the maximum profit per month?
- c. What are the breakeven sales quantities (range of profitable demand volume)?
- 2-15. A company estimates that as it increases its sales volume by decreasing the selling price of its product, revenue =  $aD - bD^2$  (where  $D$  represents the units of demand per month with  $0 \leq D \leq a/b$ ). The fixed cost is \$1,000 per month, and the variable cost is \$4 per unit. If  $a = \$6$  and  $b = \$0.001$ , determine the sale volume for maximum profit, and the maximum profit per month. (2.3)
- 2-16. A plant has a capacity of 4,100 hydraulic pumps per month. The fixed cost is \$504,000 per month. The variable cost is \$166 per pump and the sales price is \$328 per pump (assume that sales equal output volume). What is the breakeven point in number of pumps per month? What percentage reduction will occur in the breakeven point if fixed costs were reduced by 18% and unit variable costs by 6%? (2.3)
- 2-17. Suppose that the ABC Corporation has a production (and sales) capacity of \$1,000,000

per month. Its fixed costs are \$350,000 per month, and the variable costs—over a considerable range of volume—are \$0.50 per dollar of sales. (2.3)

- What is the annual breakeven point volume ( $D'$ )? Develop (graph) the breakeven chart.
  - What would be the effect on  $D'$  of decreasing the variable cost per unit by 25% if the fixed costs thereby increased by 10%?
  - What would be the effect on  $D'$  if the fixed costs were decreased by 10% and the variable cost per unit increased by the same percentage?
- 2-18. The annual fixed costs for a plant are \$100,000, and the variable costs are \$140,000 at 70% utilization of available capacity with net sales of \$280,000. What is the breakeven point in units of production if the selling price per unit is \$40? (2.3)

- 2-19. Refer to Problem 2-17. Graph the average unit cost function for this situation as originally given for part (a). At what annual output within the present annual production (and sales) capacity of \$12,000,000 does the minimum average cost per unit ( $C_U$ ) occur? Why? (2.3)

- 2-20. The fixed cost related to the production of a product is \$500,000 per year. Assume that the variable cost is \$20,000 and the selling price is \$30,000 for each percentage point of annual output capacity (which equals sales demand). Thus, the maximum sales per year are \$3,000,000 (at 100% of output capacity), and we have: (2.3)

$$\begin{aligned} C_F &= \$500,000 \text{ per year} && \text{(Fixed cost)} \\ c_v &\approx \$20,000/\% \text{ of annual} && \text{(Variable} \\ &\quad \text{output capacity} && \text{cost/unit)} \\ p &= \$30,000/\% \text{ of annual} && \text{(Selling} \\ &\quad \text{output capacity} && \text{price/unit)} \end{aligned}$$

- Determine the breakeven point for this situation.
- Develop the mathematical expression for profit or loss in this situation as a function of demand,  $D$ .

- 2-21. A plant operation has fixed costs of \$2,000,000 per year, and its output capacity is 100,000 electrical appliances per year. The variable cost is \$40 per unit, and the product sells for \$90 per unit.

- Construct the economic breakeven chart.
- Compare annual profit when the plant is operating at 90% of capacity with the plant operation at 100% capacity. Assume that the first 90% of capacity output is sold at \$90 per unit, and the remaining 10% of production is sold at \$70 per unit. (2.3)

- 2-22. A large semiconductor plant has approximately 95% of sales due to a single circuit design. The plant can therefore be considered a single-product plant and has the capacity to produce 3,000,000 printed circuit boards (PCBs) per year. Presently, the plant is operating at 60% of capacity. The selling price of the PCB is  $p = \$19.25 - (10^{-6})D$ , and the variable cost per PCB is \$15.75. At zero output, the plant's annual fixed costs are \$1,000,000 and are approximately constant up to the maximum production quantity per year. (2.3)

- What is the present expected annual profit or loss (60% of capacity)?
- What is the percentage of production capacity that will result in optimal operation? What is the maximum profit or minimum loss at this optimal volume (demand)?
- Determine at what demand(s) breakeven occurs in the operation.
- If variable cost per unit were reduced 10% and fixed costs were reduced 15%, what would be the effect on the demand(s) at which breakeven operation occurs?

- 2-23. Refer to Problem 2-20, where for the production of a product, the fixed costs were \$500,000 per year, and the variable cost and selling price were \$20,000 and \$30,000, respectively, per 1% of annual output capacity. Assume that the variable costs become nonlinear after demand ( $D$ ) reaches 60% of output capacity. Specifically, the total variable cost,  $C_V$ , is

$$C_V = c_v(D) \quad \text{when } D \leq K = 60$$

$$C_V = c_v(K) + f(c_v, K, D) \quad \text{when } 60 < D \leq 100$$

$$\text{where } f(c_v, K, D) = \int_{X=K=60}^{D} c_v(1.0375)^{X-60} dx$$

$$c_v = \$20,000, \text{ and } K = 60 \quad (\text{percent of capacity}).$$

Thus, after 60% capacity is used, the variable cost per unit increases continuously at a rate of 3.75% for each additional 1% of capacity used. For this situation, plot the average unit cost ( $C_U$ ), and its fixed and variable components, over the range of capacity utilization up to 100%. (2.3)

- 2-24. A company is analyzing a make versus purchase situation for a component used in several products, and the engineering department has developed these data:

Option A: Purchase 10,000 items per year at a fixed price of \$8.50 per item. The cost of placing the order is negligible according to the present cost accounting procedure.

Option B: Manufacture 10,000 items per year using available capacity in the factory. Cost estimates are direct materials = \$5.00 per item and direct labor = \$1.50 per item. Overhead is allocated at 200% of direct labor (= \$3.00/item).

The engineering department has determined that fixed overhead (depreciation on plant and equipment, insurance, property taxes, etc.) is approximately \$2.00 per item, and variable overhead, which tends to be incremental to the manufacture of this item, is about \$1.00 per item. (2.4)

- a. Based on these data, should the item be purchased or manufactured?
- b. If manufacturing overhead can be traced directly to this item, thus avoiding the 200% overhead rate, and it amounts to \$2.45 per unit, what should be recommended? (Traceable overhead is possible with an activity-based cost accounting procedure, is incremental to the manufacture of the part, and consists of such cost elements as employee training, material handling, quality control, supervision, and utilities.) Traceable overhead associated with purchasing this item (vendor certification, benchmarking, etc.) is \$0.30 per item.

- 2-25. A company finds that, on the average, two of its engineers, always traveling together, spend 60 hours each month in flying time on commercial airlines in making service calls to customers' plants. Also, the cost for airline tickets, airport buses, car rental, and so on is

approximately \$2,000 per person per month. An air service offers to supply a small business jet and pilot on 24-hour notice at a cost of \$1,200 per month plus \$125 per hour of flying time and \$25 per hour for waiting time on the ground at the destination. It states that experience with similar situations has shown that using the charter service will reduce total travel time by 50%. The company estimates that the cost of car rental at destinations probably would amount to about \$250 per month if the charter service is used, and the average waiting time will be about 40 hours per month. It also estimates that each engineer's time is worth \$40 per hour to the company. Should the charter service be used? (2.4)

- 2-26. A certain item can be readily purchased from a local vendor for \$0.50 per unit. The shop foreman in your company has proposed manufacturing the item in an idle part of the production area. He has computed that labor, materials, and overhead per unit would be \$0.15, \$0.20, and \$0.15, respectively. However, he contends that overhead should not be included in the manufactured cost, so that it is less expensive to make the item compared to purchasing it. Do you agree with the foreman's analysis? What other factors might have a bearing on this decision? (2.4)

- 2-27. In the design of an automobile engine part, an engineer has a choice of either a steel casting or an aluminum alloy casting. Either material provides the same service. However, the steel casting weighs 8 ounces, compared with 5 ounces for the aluminum casting. Every pound of extra weight in the automobile has been assigned a penalty of \$6 to account for increased fuel consumption during the life cycle of the car. The steel casting costs \$3.20 per pound, while the aluminum alloy can be cast for \$7.40 per pound. Machining costs per casting are \$5.00 for steel and \$4.20 for aluminum. Which material should the engineer select, and what is the difference in unit costs? (2.4)

- 2-28. Fiber X has 40% greater heat insulating value than fiber Y. A particular design calls for a 6-inch thickness of fiber X or its equivalent. If fiber X costs \$7.50 per cubic foot and fiber Y costs \$5.00 per cubic foot, which is more economical? Re-

member to make the comparison on an equivalent basis. (2.4)

2-29. One method for developing a mine will result in the recovery of 62% of the available ore deposit and will cost \$23 per ton of material removed. A second method of development will recover only 50% of the ore deposit, but it will cost only \$15 per ton of material removed. Subsequent processing of the removed ore recovers 300 pounds of metal from each ton of processed ore and costs \$40 per ton of ore processed. The ore contains 300 pounds of metal per ton, which can be sold for \$0.80 per pound. Which method for developing the mine should be used? (2.4)

2-30. Which of the following statements are true, and which are false? (all sections)

- Working capital is a variable cost.
- The greatest potential for cost savings occurs in the operation phase of the life cycle.
- If the capacity of an operation is significantly changed (e.g., a manufacturing plant), the fixed costs will also change.
- The initial investment cost for a project is a nonrecurring cost.
- Variable costs per output unit are a recurring cost.
- A noncash cost is a cash flow.
- Goods and services have utility because they have the power to satisfy human wants and needs.
- The demand for necessities is more inelastic than the demand for luxuries.
- Indirect costs can normally be allocated to a specific output or work activity.
- Present economy studies are often done when the time value of money is not a significant factor in the situation.
- Overhead costs normally include all costs that are not direct costs.
- Optimal volume (demand) occurs when total costs equal total revenues.
- Standard costs per unit of output are established in advance of actual production or service delivery.
- A related sunk cost will normally affect the prospective cash flows associated with a situation.

- The life cycle needs to be defined within the context of the specific situation.
- The greatest commitment of costs occurs in the acquisition phase of the life cycle.
- The average unit cost function is a linear function of demand.

2-31. A manufacturing company leases for \$100,000 per year a building that houses its manufacturing facilities. In addition, the machinery in the building is being paid for in installments of \$20,000 per year. Each unit of the product produced costs \$15 in labor and \$10 in materials and can be sold for \$40. (2.3)

- How many units per year must be sold for the company to break even (choose the closest answer)?
  - 4,800
  - 3,000
  - 8,000
  - 6,667
  - 4,000
- If 10,000 units per year are sold, what is the annual profit (closest answer)?
  - \$280,000
  - \$50,000
  - \$150,000
  - \$50,000
  - \$30,000
- If the selling price is lowered to \$35 per unit, how many units must be sold each year for the company to earn a profit of \$60,000 per year?
  - 12,000
  - 10,000
  - 16,000
  - 18,000
  - 5,143

2-32. Refer to Figure 2-11. Apply these cost curves to the design of a bridge having a total length of 2,000 feet, with the requirement that at least one span must be 500 feet in length. Formulate all alternative designs that use 300-, 375-, 450-, and 500-foot span lengths and determine which design is most economical. (2.4)

2-33. A farmer estimates that if he harvests his soybean crop now, he will obtain 1,000 bushels, which he can sell at \$3.00 per bushel. However, he estimates that this crop will increase by an additional 1,200 bushels of soybeans for each week he delays harvesting, but the price will drop at a rate of 50 cents per bushel per week; in addition, it is likely that he will experience spoilage of approximately 200 bushels per week for each week he delays harvesting. When should he harvest his crop to obtain the largest total cash return, and how much will be received for his crop at that time? (2.4)

**2-34.** A recent engineering graduate was given the job of determining the best production rate for a new type of casting in a foundry. After experimenting with many combinations of hourly production rates and total production cost per hour, he summarized his findings in Table I below. The engineer then talked to the firm's marketing specialist, who provided these estimates of selling price per casting as a function of production output (see Table II below).

- a. What production rate would you recommend to maximize total profits? (2.4)
- b. How sensitive is the rate in part (a) to changes in total production cost per hour?

**2-35. Brain Teaser**

The student chapter of the American Society of Mechanical Engineers is planning a 6-day trip to the national conference in Albany, New York. For transportation the group will rent a car from either the State Tech Motor Pool or a local car dealer. The Motor Pool charges \$0.26 per mile, has no daily fee, and the motor pool pays for

the gas. The car dealer charges \$25 per day, \$0.14 per mile, but the group must pay for gas. The car's fuel rating is 20 miles per gallon and the price of gas is estimated to be \$1.20/gallon. (2.3)

- a. At what point, in miles, is the cost of both options equal?
- b. The car dealer has offered a special student discount and will give the students 100 free miles per day. What is the new break-even point?
- c. Suppose now that the Motor Pool reduces its all-inclusive rate to \$0.23 per mile and the car dealer increases its rate to \$25 per day and \$0.21 per mile. In this case, the car dealer wants to encourage student business, so he offers 1,000 free miles for the entire 6-day trip. He claims that if more than 882 miles are driven, students will come out ahead with one of his rental cars. If the students anticipate driving 1,600 miles (total), from whom should they rent a car? Is the car dealer's claim entirely correct?

I.	Total cost/hour	\$1,000	\$2,600	\$3,200	\$3,900	\$4,700
	Castings produced/hour	100	200	300	400	500
II.	Selling price/casting	\$20.00	\$17.00	\$16.00	\$15.00	\$14.50
	Castings produced/hour	100	200	300	400	500