27/12/2023 — We.

Communication Systems Notes.

chapter # 5 : Angle {Exponential} Modulation.

→ A sinusoidal signal is described by amplitude and angle (frequency and phase). → $A \cos \theta$

→ In AM signal the information content of m(t) is in amplitude variation of carrier.

→ There exists a same possibility of carrying the same information by varying the angle of carrier.

→ 'AM' is a linear modulation.
- translated linearly
- Shape isn't change.
- We don't put additional frequencies.

→ 'FM' is non-linear modulation in which wave shape is changed and some additional frequencies are also introduced.

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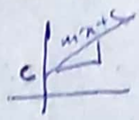
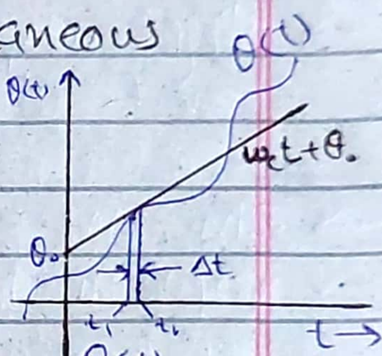
$\boxed{\omega(t) = \omega_c + K m(t)}$



→ If the peak value of m(t) is m_p, then max and min values of carrier frequency would be $\omega_c + K m_p$ and $\omega_c - K m_p$ respectively.

5.1 Concept of Instantaneous frequencies:-



→ The generalised sinusoidal signal is given by $\varphi(t) = A\cos\theta(t)$, where $\theta(t)$ is generalised angle and is function of 't'. $\theta(t) = \omega_c t + \theta_0$ which is a straight line of slope ω_c and intercept θ_0.

→ $\theta(t)$ is tangential to $\omega_c t + \theta_0$ at small interval $\Delta t \to 0$, hence for $t_1 < t < t_2$:
$\varphi(t) = A\cos\theta(t) = A\cos(\omega_c t + \theta_0)$.

→ Generalising the above concept we can say that at every every instant of t the instantaneous frequency ω_i is slope of $\theta(t)$ at t. i.e

$$\omega_i = \frac{d}{dt}\theta(t)$$

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\, d\alpha$$

→ A technique in which the angle of carrier is varied in some manner with a modulating signal $m(t)$ ar known as angle or exponential modulation (EM). It has two possibilities:

- Phase Modulation (PM)
- Frequency Modulation (FM).

→ In PM the instantaneous frequency ω_i varies linearly with derivative of modulating signal.

$$\theta(t) = \omega_c t + \theta_0 + K_p m(t)$$

angle carrier frequency phase constant info.

$$\theta(t) = \omega_c t + K_p m(t) \qquad \because \theta_0 = 0$$

$$\varphi_{PM}(t) = A \cos[\omega_c t + K_p m(t)].$$

$$\omega_i = \frac{d}{dt}\theta(t) = \omega_c + K_p \frac{d}{dt} m(t)$$

$$\omega_i = \omega_c + K_p m'(t)$$

→ In FM the instantaneous frequency ω_i is varied linearly with modulating signal. i.e

$$\omega_i(t) = \omega_c + K_f m(t).$$

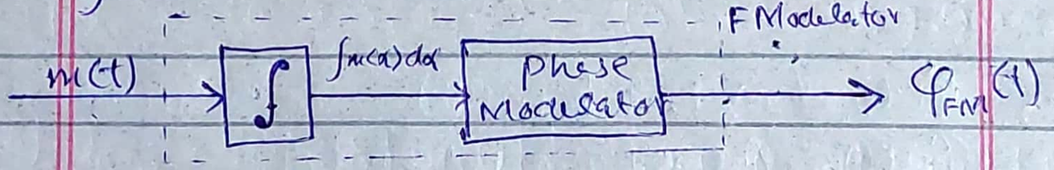
As $\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\,d\alpha = \int_{-\infty}^{t} [\omega_c + K_f m(\alpha)]\,d\alpha$

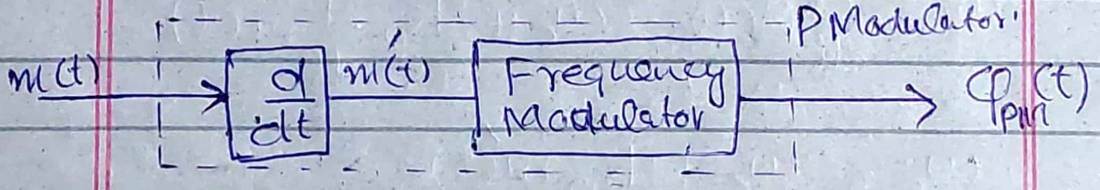
$$\theta(t) = \omega_c t + K_f \int_{-\infty}^{t} m(\alpha)\,d\alpha .$$

$$\varphi_{FM}(t) = A\cos\left[\omega_c t + K_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]$$

→ FM and PM are not very similar but are inseperable. Looking to EM signal we can't say wether it is FM or PM.

→ We can change a $\varphi_{PM}(t)$ signal to φ_{FM} by replacing $m(t)$ with $\int m(t)$ ~~dt~~. And an $\varphi_{FM}(t)$ can be corresponded to $\varphi_{PM}(t)$ by changing $\int m(\alpha)\,d\alpha$ to $m(t)$.





→ In EM angle of carrier is varied in ~~same~~ proportion to some measure of $m(t)$. In PM it is directly proportional to $m(t)$, whereas in FM, it is proportional to integral of $m(t)$.

→ The generalised angle-modulated carrier $\varphi_{EM}(t)$ can be expressed as:

$$\varphi_{EM}(t) = A\cos(\omega_c t + \psi(t))$$

$$= A\cos\left(\omega_c t + \int_{-\infty}^{t} m(\alpha)\, h(t-\alpha)\, d\alpha\right)$$

If $h(t) = K_p\,\delta(t)$ the equation reduce to that PM and when $h(t) = K_f u(t)$ it reduces to FM equation.

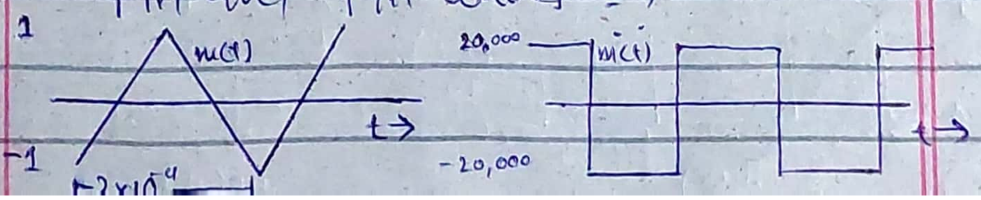
→ Bandwidth of FM is approximately $2K_f m_p$, where m_p is peak amplitude of $m(t)$.

→ Bandwidth of PM is approximately $2K_p m_p'$, where m_p' is peak amplitude of $m'(t)$.

Example 5.1:

$K_f = 2\pi \times 10^5$ & $K_p = 10\pi$, $f_c = 100\,MHz$

FM and PM waves = ?



for FM we have: $\omega_i = \omega_c + K_f\, m(t)$

$$f_i = f_c + \frac{K_f}{2\pi} m(t) \qquad \because \div 2\pi$$

$$= 10^8 + 10^5 m(t)$$

$$(f_i)_{min} = 10^8 + 10^5 [m(t)]_{min} = 10^8 + 10^5(-1) = 99.9\,\text{MHz}$$

$$(f_i)_{max} = 10^8 + 10^5 [m(t)]_{max} = 10^8 + 10^5(1) = 100.1\,\text{MHz}.$$

For PM we have: $\omega_i = \omega_c + K_p\, m'(t)$

$$f_i = f_c + \frac{K_p}{2\pi} m'(t) \qquad \because \div 2$$

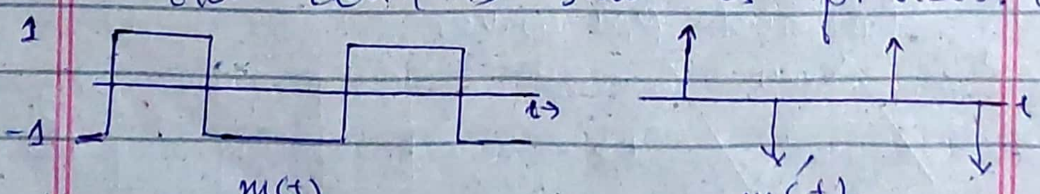
$$f_i = 10^8 + 5\,(m'(t))$$

$$(f_i)_{min} = 10^8 + 5\,(m'(t))_{min} = 10^8 - 10^5 = 99.9\,\text{MHz}$$

$$(f_i)_{max} = 10^8 + 5\,[m'(t)]_{max} = 10^8 + 5(20{,}000) = 100.1\,\text{MHz}.$$

Example 5.2:

Sketch FM and PM for digital modulating signal m(t). shown below other data is same as previous. $K_p = \pi/2$.



m(t) m'(t)

As for FM: $f_i = f_c + \frac{K_f}{2\pi} m(t) = 10^8 + 10^5 m(t)$

we will get answer same as previous.

for PM: $\omega_i = \omega_c + K_p m'(t)$

$$f_i = f_c + \frac{K_p}{2\pi} m'(t) = 100 + \frac{\frac{\pi}{2}}{2\pi} m'(t)$$

$$= 100 + \frac{1}{4} m'(t)$$

~~$(f_i)_{min} = 100 + \frac{1}{4} m'(t)_{min} = 100 + \frac{1}{4}$~~

By direct approach as derivative contain impulses, we have:

$$\varphi_{PM}(t) = A \cos[\omega_c t + K_p m(t)$$
$$= A \cos[\omega_c t + \frac{\pi}{2} m(t)]$$
$$= \begin{cases} A \sin \omega_c t ; & m(t) = -1 \\ -A \sin \omega_c t ; & m(t) = 1 \end{cases}$$

→ The scheme of carrier frequency modulation by a digital signal is called frequency-shift keying (FSK) because information digits are transmitted by shifting the carrier frequency.

⑧ $e^{x} = \sum_{n=0} \frac{x^{n}}{n!}$

→ The scheme of carrier PM by a digital signal is called phase-shift keying (PSK) because information digits are transmitted by shifting the carrier phase.

→ The amount of phase discontinuity in $\varphi_{PM}(t)$ at the instant where $m(t)$ is discontinuous is $K_p m_d$, where m_d is amount of discontinuity in $m(t)$ at that instant.

→ Althoug ω_i of FM and PM varies with time, the amplitude A always remain constant. Hence the Power of an EM wave is:

$P_{EM} = A^2/2$.

regardless of the value K_p or K_f.

5.2 Bandwidth of Angle Modulated Waves:-

Let $a(t) = \int_{-\infty}^{t} m(\alpha)\,d\alpha$

and

$$\hat{\varphi}_{FM}(t) = A e^{j[\omega_c t + K_f a(t)]}$$
$$= A e^{jK_f a(t)} e^{j\omega_c t}$$

It shows that

$$\varphi_{FM}(t) = Re\{\hat{\varphi}_{FM}(t)\}.$$

Now expanding $e^{jK_f a(t)}$ yields

$$\hat{\varphi}_{FM}(t) = A\left[1 + jK_f a(t) - \frac{K_f^2}{2!}a^2(t) + \cdots + j^n \frac{K_f^n}{n!}a^n(t) + \cdots\right] e^{j\omega_c t}$$

$\because$ using $e^x = \sum_0^\infty \frac{x^n}{n!}$

Then

$$\varphi_{FM}(t) = Re\{\hat{\varphi}_{FM}(t)\} = A\left[\cos\omega_c t - K_f a(t)\sin\omega_c t - \frac{K_f^2}{2!}a^2(t)\cos\omega_c t + \frac{K_f^3}{3!}a^3(t)\sin\omega_c t + \cdots\right]$$

→ If $M(\omega)$ is bandlimited to B, then $A(\omega)$ is also band-limited to B. Similarly the band of $a^2(t)$ is simply $A(\omega) * A(\omega)/2\pi$ and is band-limited to $2B$. Similarly of $a^n(t)$ is nB.

→ Although the theoretical bandwidth of an FM wave is infinite, most of the modulated-signal power resides in a finite bandwidth.

5.2.1 Narrow-band Angle Modulation

→ Unlike AM, FM is non-linear, The principle of superposition doesn't apply.

→ If K_f is verysmall i.e $|K_f a(t)| << 1$. The all the terms are negligible except first two

$$\varphi_{FM}(t) \simeq A[\cos\omega_c t - K_f a(t)\sin\omega_c t]$$

→ Here the bandwidth is 2B

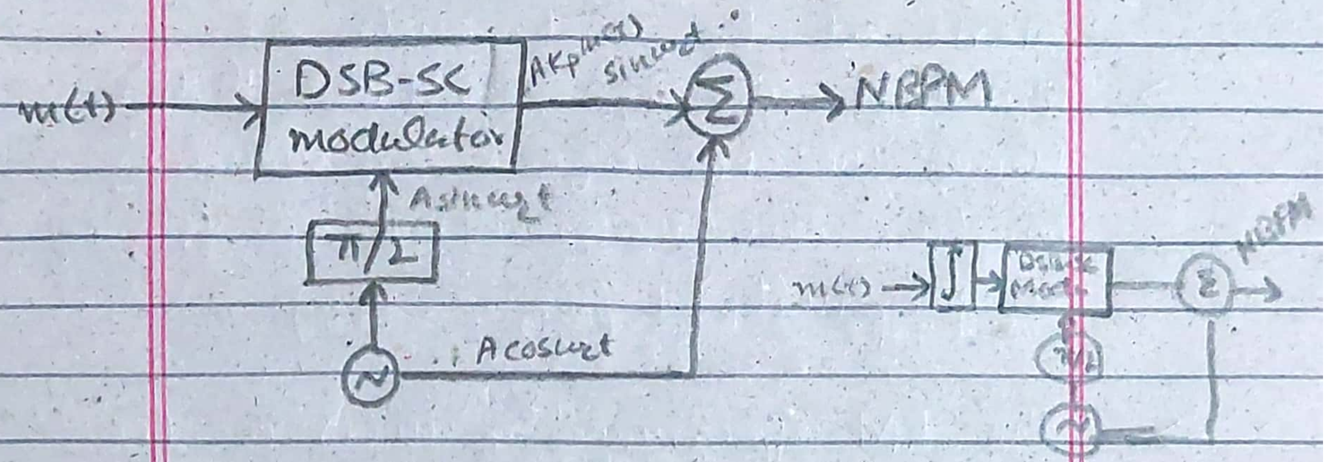
→ It is called Narrow-band FM (NBFM) because of $|K_f a(t)| << 1$.

→ Narrow band phase Modulation (NBPM) case is similarly given.

$$\varphi_{PM}(t) \simeq A[\cos\omega_c t - K_p m(t)\sin\omega_c t].$$

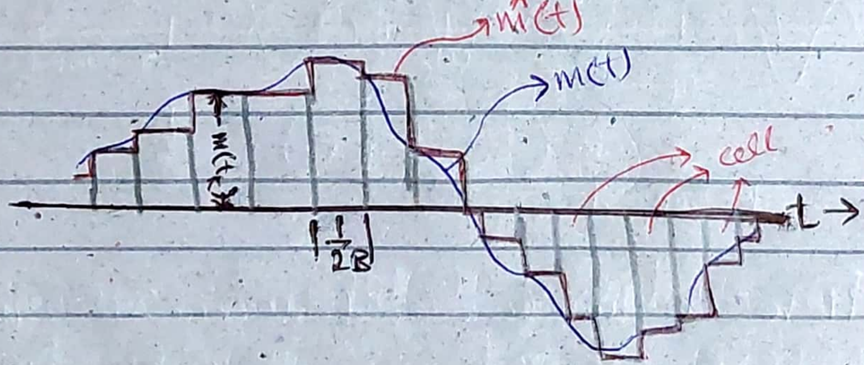
→ The sideband spectrum for FM has a phase shift of $\pi/2$ with respect to the carrier, wheares that of AM is in phase with carrier.

→ In AM the frequency is constant wherase the amplitude vary with time, while in FM the amplitude is constant and frequency varies with time.



Wide-Band FM (WBFM): The Fallacy Exposed:

→ If condition $|k_f a(t)| \ll 1$ is not satisfied, we can't ignore the higher order terms.



→ The signal of band 2B Hz is approximated by a staircase signal $\hat{m}(t)$ shown in red

→ Each cell has constant amplitudes and width must be no greater than Nyquist criteria / interval of 1/2B seconds.

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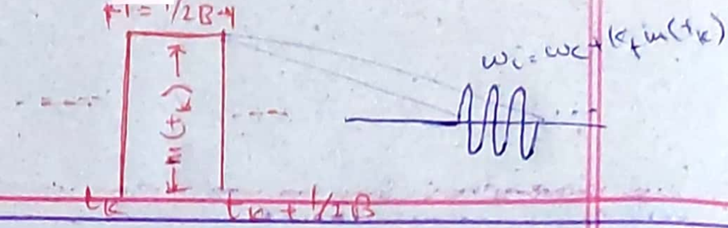
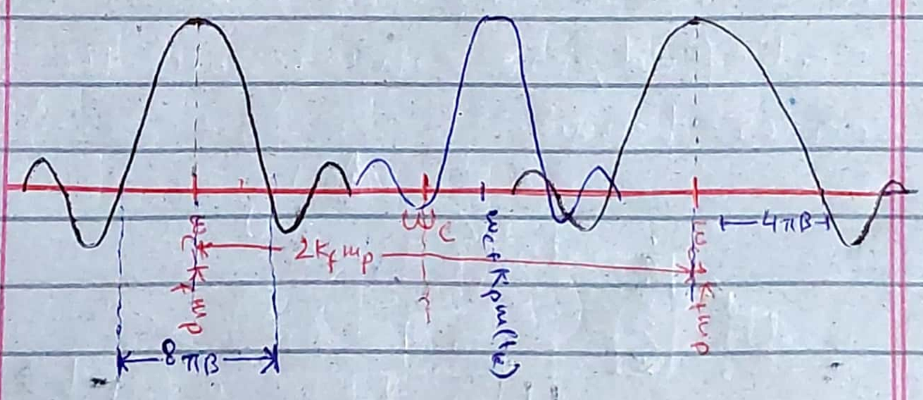


→ Consider a K_{th} cell of width $T = 1/2B$ seconds, which starts at t_k and ends at $t_k + 1/2B$ having amplitude $m(t_k)$. Hence the FM signal corresponding to this cell is sinusoid of frequency $\omega_c + K_f m(t_k)$.

→ The FM spectrum for $\hat{m}(t)$ is the sum of fourier transforms of these sinusoidal pulses is a sinc function.



Here the minimum and maximum amplitudes are $-m_p$ and $+m_p$ respectively. Hence the max and min frequencies are $\omega_c + K_f m_p$ and $\omega_c - K_f m_p$. The spectrum of each sinusoid is $4\pi B$ rad/s on either side. The maximum and minimum significant frequencies in spectrum are $\omega_c + K_f m_p + 4\pi B$ and $\omega_c - K_f m_p - 4\pi B$.

Hence the spectrum width is

$B = 2K_f m_p + 8\pi B$

→ The deviation of carrier frequency is $\pm K_f m_p$. ie

$$\Delta\omega = K_f m_p$$

$$2\pi \Delta f = K_f m_p$$

$$\Delta f = \frac{K_f m_p}{2\pi}$$

The estimated band can be thus

$$B_{FM} = \frac{1}{2\pi}\left(2K_f m_p + 8\pi B\right)$$

$$= \frac{1}{2\pi}\left(2(2\pi\Delta f) + \overset{4}{8}\pi B\right)$$

$$= 2\Delta f + 4B$$

$$B_{FM} = 2(\Delta f + 2B)$$

→ This estimated bandwidth is somewhat heigher than the actual value because this is the bandwidth corresponding stirese to $\hat{m}(t)$ not the actual $m(t)$.

→ Also we have fallacy because we thougt That the spectral component must lie in min and max $(\omega_c \pm K_p \omega_p)$ rang which is only true for everlasting sinusoid.

→ Now if we consider that K_f is very small then Δf is very small compared to B.

$$B_{FM} = \lim_{K_f \to 0} \frac{1}{2\pi}\left(2K_f m_p + 8\pi B\right)$$

$$= \frac{1}{2\pi}\left(0 + 8\pi B\right)$$

$$\approx 4B$$

But for narrow band the FM bandwidth is 2B. this indicates a better estimation:

$$B_{FM} = 2(\Delta f + B)$$

$$= 2\left(\frac{K_f m_p}{2\pi} + B\right)$$

→ For a truly wideband case where $\Delta f \gg B$, this equation can be approximated as:

$$B_{FM} \approx 2\Delta f \quad ; \quad \Delta f \gg B$$

and for narrow-band where $\Delta f \ll B$ we have

$$B_{FM} \approx 2B \quad ; \quad \Delta f \ll B$$

→ The deviation ratio (β) is define as:

$$\beta = \frac{\Delta f}{B}$$

→ Carson's rule can be expressed in term of β as

$$B_{FM} = 2(\Delta f + B)$$

$$= 2B\left(\frac{\Delta f}{B} + 1\right)$$

$$= 2B(\beta + 1)$$

→ Deviation ration controls the amount of modulation and consequently plays a role similar to modulation index (μ) in AM.

→ For a special case of tone modulated FM, the deviation ratio β is called modulation index.

→ For phase modulation all results of FM can be directly applied.

$$\omega_i' = \omega_c + K_p m'(t)$$

$$\Delta\omega = K_p m_p' \qquad \because m_p' = [m'(t)]_{max}$$

$$B_{PM} = 2(\Delta f + B)$$
$$= 2\left(\frac{k_p m_p'}{2\pi} + B\right).$$

→ $\Delta\omega = k_f m_p$ of FM depends only on peak value of $m(t)$ and is independent of the spectrum of $m(t)$.

→ $\Delta\omega = k_p m_p'$ of PM depends on peak value of $m'(t)$, but $m(t)$ depends strongly on frequency spectrum of $m(t)$.

→* Verification of FM bandwidth relationship.

Let $m(t)$ be a sinosoid so we can perform tone-modulation

$$m(t) = \alpha \cos \omega_m t$$

as $$a(t) = \int_{-\infty}^{t} m(\alpha)\, d\alpha$$
$$= \int_{-\infty}^{t} \alpha \cos \omega_m \alpha$$

This is const

$$= \frac{\alpha}{\omega_m} \sin \omega_m \alpha \Big|_{-\infty}^{t}$$

$$a(t) = \frac{\alpha}{\omega_m} \sin\omega_m t \quad -① \qquad \because \text{ let } a(-\infty) = 0$$

From page ⑧ we have

$$\hat{\varphi}_{FM}(t) = A e^{(j\omega_c t + K_f a(t))}$$

$$= A e^{\left(j\omega_c t + \frac{K_f \alpha}{\omega_m} \sin\omega_m t\right)}$$

as we know : $\Delta\omega = K_f m_p = \alpha K_f$
and the bandwidth of $m(t)$ is
$B = f_m$ then deviation ratio is

$$\beta = \frac{\Delta f}{B} = \frac{2\pi \Delta f}{2\pi f_m} = \frac{\Delta\omega}{\omega_m} = \frac{\alpha K_f}{\omega_m}$$

put $\beta = \alpha K_f / \omega_m$ in $\hat{\varphi}_{FM}(t)$

$$\hat{\varphi}_{FM}(t) = A e^{(j\omega_c t + \beta \sin\omega_m t)}$$

$$= A e^{j\omega_c t} \underline{e^{j\beta \sin\omega_m t}}$$

As the underline term is periodic signal with period $2\pi/\omega_m$. then by fourier series

$$e^{j\beta \sin\omega_m t} = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_m t}$$

where; $C_n = \frac{\omega_m}{2\pi} \int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin \omega_m t} \cdot e^{-jn\omega_m t} \, dt$

Let $x = \omega_m t \Rightarrow \frac{dx}{\omega_m} = dt$

then

limits

$x_l = \omega_m t_0$
$= \omega_m(-\pi/\omega_m)$
$= -\pi$

$x_u = \omega_m t_0$
$= \omega_m \cdot \pi/\omega_m$
$= \pi$

$$C_n = \frac{\cancel{\omega_m}}{2\pi} \int_{-\pi}^{\pi} e^{j\beta \sin x} \cdot e^{-jnx} \frac{dx}{\cancel{\omega_m}}$$

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \, dx$$

This integral is known as Bessel function and is denoted by $J_n(\beta)$. It is a Bessel funtion of first kind and n^{th} order.

hence

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega_m t}$$

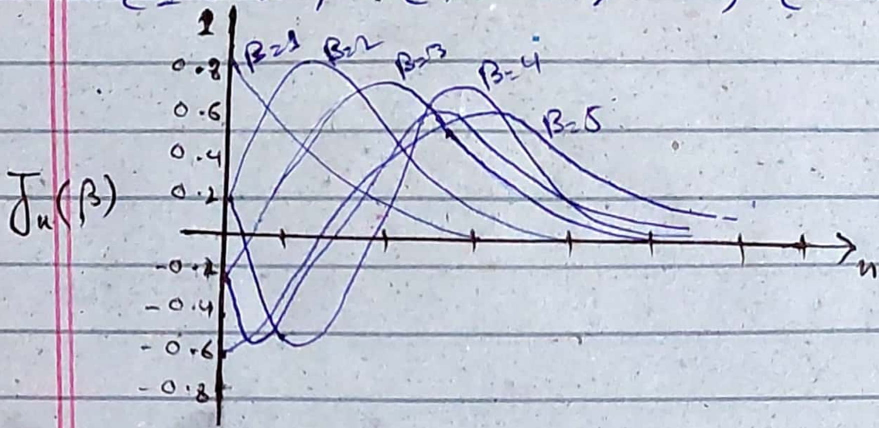
putting this in $\hat{\phi}_{FM}(t)$ we get

$$\hat{\phi}_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega_m t} \cdot e^{j\omega_c t}$$

$$= A \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j(\omega_c + n\omega_m)t}$$

$$\hat{\varphi}_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

This signal has a carrier component and and infinite number of sidebands of frequencies $\omega_c \pm \omega_m$, $\omega_c \pm 2\omega_m$, $\omega_c \pm 3\omega_m$, ---, $\omega_c \pm n\omega_m$, ---



From plots of $J_n(\beta)$ that for a given β, $J_n(\beta)$ decrease with n and negligible for sufficien large n. ie $n > \beta+1$, Hence the number of significant sidebands are $\beta+1$.

$$B_{FM} = 2nf_m = 2(\beta+1)f_m$$

$$= 2\left(\frac{\Delta f}{B} + 1\right) B \quad \because f_m = B \;\&\; \beta = \frac{\Delta f}{B}$$

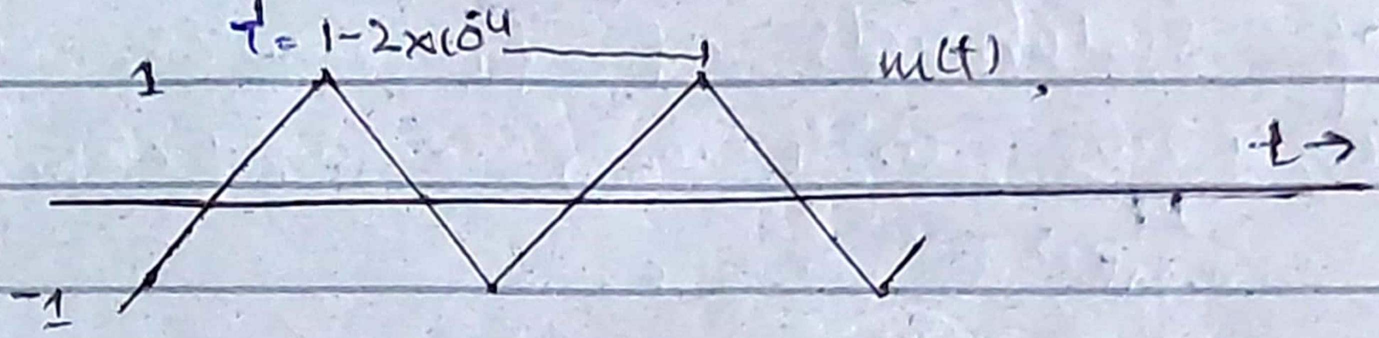
$$= 2\left(\frac{\Delta f}{B} B + B\right)$$

$$= 2(\Delta f + B)$$

This is verification, not a proof

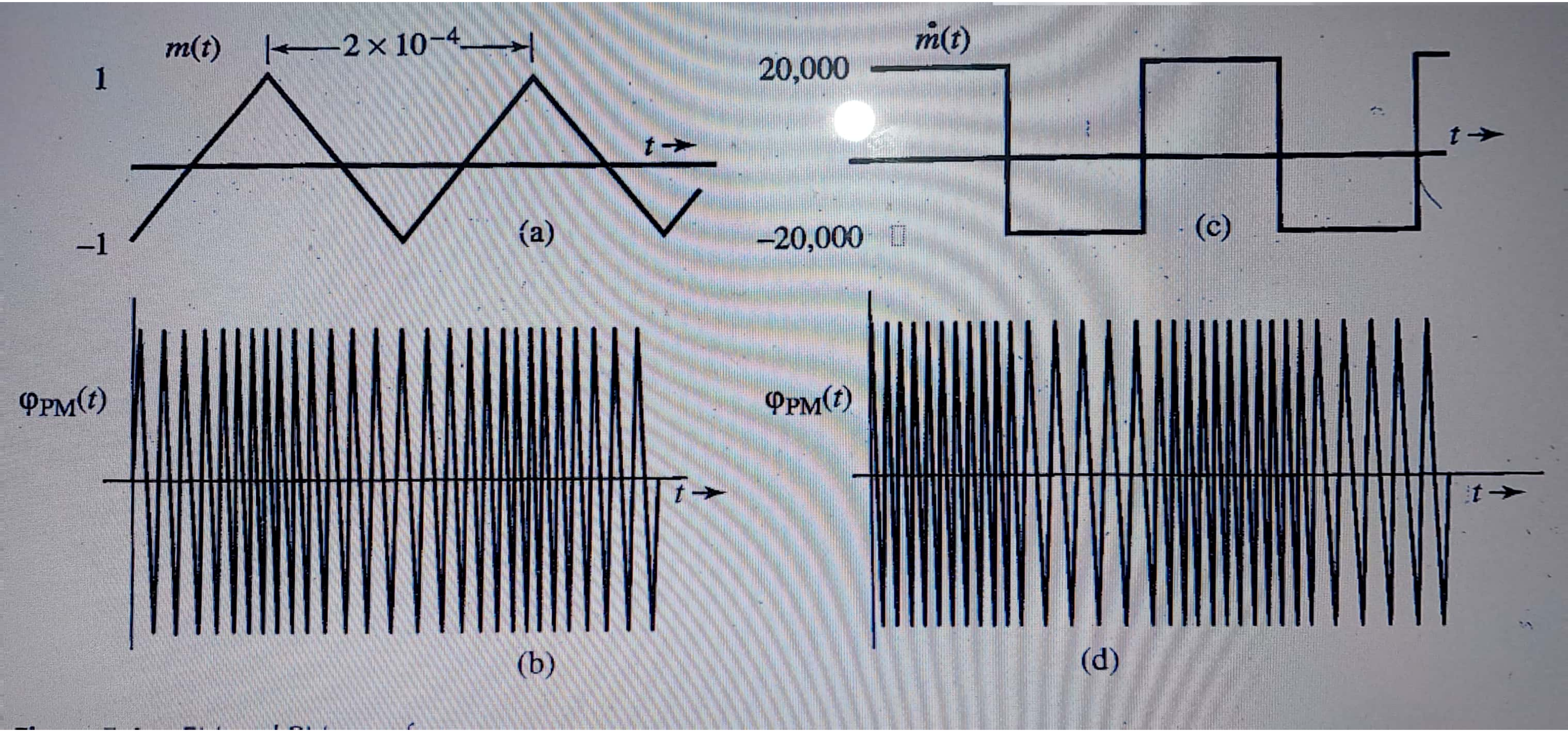
of Corson's formula

Example 5.3:



a) B_{FM} & B_{PM} = ? , $K_f = 2\pi \times 10^5$, $K_p = 5\pi$.

b) Repeat (a) if $m_2(t) = 2\, m(t)$



m(t)
2 × 10⁻⁴
1
−1
t
(a)
ṁ(t)
20,000
−20,000
t
(c)
φPM(t)
t
(b)
φPM(t)
t
(d)

Sol: By fourier series

$$m(t) = \sum C_n \cos n\omega_0 t$$

for $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\times10^{-4}} = 10^4\pi$.

where $C_n = \begin{cases} \frac{8}{n^2\pi^2} & \text{for odd } n \\ 0 & \text{for even } n. \end{cases}$

Now, as we can see that amplitudes 'C_n' decrease rapidly with n, which is negligible after fifth hormonic (only 4% power). hence for 3rd hormonic we have

$n=3$

$$\frac{8}{3^2\pi^2} \times \cos(3)2\pi f_0 t$$

$$= \frac{8}{9\pi^2} \times \cos 2\pi\, 3f_0\, t$$

$$B = 3f_0 = \frac{3}{T_0} = 3 \times \frac{1}{2\times10^{-4}}$$

$$B = \frac{3}{2}\times10^{4} = 15\,KHz$$

Now as $B_{FM} = 2\left(\frac{\overset{\Delta f}{K_f m_p'}}{2\pi} + B\right)$

$$B_{FM} = 2\left[\frac{(2\pi\times10^5)(1)}{2\pi} + 15'000\right]$$

$$= 2(100'000 + 15'000]$$

$$= 2(115'000\ Hz)$$

$$= 2(115\,kHz)$$

$$= 230\,KHz$$

Now for PM by using another formula $\beta = \frac{\Delta f}{B}$

$$B_{PM} = 2B(\beta + 1)$$

$$\Delta f = \frac{K_p m_p'}{2\pi} = \frac{5\pi \times 20'000}{2\pi} = 50\,KHz$$

$$= 2B\left(\frac{50\,KHz}{15\,KHz} + 1\right)$$

$$= 30\,KHz\left(\frac{10}{3} + 1\right)$$

$$= 130\,KHz.$$

b) Now for $m_p = 2$. its bandwidth is unchange $B = 15\,KHz$.

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for FM:

$$\Delta f = \frac{k_f m_p}{2\pi} = \frac{1}{\cancel{2\pi}}(\cancel{2\pi} \times 10^5)\, 2$$

$$= 200 \text{ kHz}$$

$$B_{FM} = 2(15 \text{ kHz})\left(\frac{200 \text{ kHz}}{15 \text{ kHz}} + 1\right)$$

$$= \cancel{30}\left(\frac{43}{\cancel{3}}\right) = 4\cancel{30} \text{ kHz}$$

for PM

$$\Delta f = \frac{k_p m_p'}{2\pi} = \frac{1}{\cancel{2\pi}}\, 5\cancel{\pi} \times \cancel{4}0'000$$

$$\Delta f = 100 \text{ kHz}$$

$$B_{PM} = 2(100 \text{ kHz} + 15 \text{ kHz})$$

$$= 2(115 \text{ kHz})$$

$$= 230 \text{ kHz}..$$

Example 5.4: Repeat 5.3
if $m(t)$ is time-expanded by 2 ...

Sol:

Time expansion of a signal by two reduce the signal bandwidth by a factor of 2.

we can see the $m_p = 1$ which is unchanged and $m_p' = 10,000$ which is changed. also $B = 7.5$ kHz

Hence

for FM:

$$\Delta f = k_f m_p / 2\pi = \frac{2\pi \times 10^5 \times 1}{2\pi} = 100 \text{ kHz}$$

$$B_{FM} = 2(100 \text{ kHz} + 7.5 \text{ kHz})$$
$$= 215 \text{ kHz}$$

for PM

$$\Delta f = k_p m_p' / 2\pi = \frac{5\pi \times 10,000}{2\pi}$$
$$= 25 \text{ kHz}$$

$$B_{PM} = 2(7.5)\left[\frac{25 \text{ kHz}}{7.5 \text{ k}} + 1\right]$$
$$= 65 \text{ kHz.}$$

It shows that $m(t)$ has very

little effect of FM bandwidth, but it halves PM bandwidth. This verifies that PM spectrum is strongly dependent on the spectrum of $m(t)$.

* Example 5.5:

$\omega_c = 2\pi \times 10^5$

$\phi_{EM}(t) = 10 \cos(\omega_c t + 5 \sin 300t + 10 \sin 2000\pi t)$.

~~a) P_{EM}~~ : Solution: the heighest frequency f_m in $m(t)$ or $m(t)'$ is its bandwidth hence

$B = 2000\pi / 2\pi = 1000 \text{ Hz} = \underline{1 \text{ KHz}}$

a) P_{EM}; as amplitude is 10 hence

$P_{EM} = \frac{A^2}{2} = \frac{10^2}{2} = 50$ watt

b) Δf:

As $\omega_i = \frac{d}{dt}\theta(t) = \frac{d}{dt}[\omega_c t + 5 \sin 300t + 10 \sin \ldots]$

$\omega_i = \omega_c + 5.(300)(+\cos 300t) + 10(2000\pi) \cos 2000\pi t$

$\omega_i = \omega_c + 1500 \cos 300t + 20000\pi \cos 2000\pi t$

as $\omega_i = \omega_c \pm \Delta\omega = \omega_c + 2\pi\Delta f$

Comparing both we get

$$2\pi \Delta f = 15'000 \cos 300t + 20'000\pi \sin 2000\pi t.$$

The maximum will occur at some point ~~when~~ i-e $15000 + 20000\pi$

hence

$$\Delta f = \frac{15000 + 20'000\pi}{2\pi}$$

$$= 12'387.3241$$

c) β:

As deviation ratio is given by $\beta = \frac{\Delta f}{B} = \frac{12'387.3241}{1000}$

$$\beta = 12.3873241$$

d) $\Delta\phi$:

As $\theta(t) = \omega t + (5 \sin 300t + 10 \sin 2000\pi t)$

~~The value in parenthesis is maximum~~

at same time when added in phase hence

$$\Delta\phi = 10 + 5 = 15 \text{ rad.}$$

e) B_{EM}:

As $B_{EM} = 2(\Delta f + B)$ $= \cancel{2B(\beta)}$

$= 2B(\beta + 1)$

$= 2(12387.3241 + 1000)$

$= 26774.65$ Hz

also $B_{EM} = 2(1000)(12.3873 + 1)$

$= 2000 \times 12.3873$

$= 26774.65$ Hz.

— END — of — CH# — 5 —