DSP Notes 14 Jan 2024/Su

Chapter # 3: The Z-Transform.

3.0 Introduction

→ The Z-transform for DTS is the counter part of Laplace transform for CTS, and they have a similar relationship to the corresponding Fourier transform.

→ Z-transform is used because FT doesn't converges for all sequences. secondly Z-transform notation is often more convenient than the FT notation

3.1 The Z-transform:

→ Z-transform of a sequence $x[n]$ is defined as:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

where $z \in \mathbb{C}$.

→ This equation, is in general, an infinite sum or infinite power series.

Z-transform = ZT

→ An operator that transform a sequence into a function is called the z-transform operator $Z\{\cdot\}$, defined as:

$$Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n} = X(z).$$

→ notation: $x[n] \xrightarrow{z} X(z)$

→ The z-transform, defined above is refered to as two-sided or bilateral z-transform.

→ The ~~uni~~ one-sided or unilateral ZT is defined as

$$X(z) = \sum_{n=0}^{\infty} x[n]\, z^{-n}.$$

→ The bilateral and unilateral ZT are identical if $x[n]=0,\ n<0$ but they differ otherwise

→ Fourier transform is simply $X(z)$ with $z = e^{j\omega}$.

→ The power series representing the F.T does'nt converges for all sequences i.e. the infinite sum may always be finite. Similarly ZT doesn't converges for all sequences or for all values of z.

→ For any given sequence the set of values of z for which ZT power series converges is called the region of convergence (ROC), of the ZT.

→ ROC will consist of a ring in z-plane centered about the origin. Its outer boundry will be a circle (may extended to infinity), and its inner boundry will be a circle (or it may extended to include origin.