## Trigonometric Identities

### The six trigonometric functions/Reciprocals Identities

$$\sin x = \frac{opp}{hyp} = \frac{y}{r} = \frac{1}{\csc x}$$

$$\cos x = \frac{adj}{r} = \frac{x}{r} = \frac{1}{r}$$

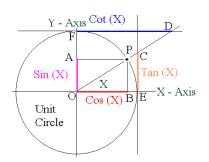
$$\csc x = \frac{hyp}{opp} = \frac{r}{y} = \frac{1}{\sin x}$$

$$\cos x = \frac{adj}{hyp} = \frac{x}{r} = \frac{1}{\sec x}$$

$$\sec x = \frac{hyp}{adj} = \frac{r}{x} = \frac{1}{\cos x}$$

$$tan x = \frac{opp}{adi} = \frac{y}{x} = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{adj}{opp} = \frac{x}{y} = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$



## Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
  
$$\tan^2 x - \sec^2 x = -1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

### Sum or difference of two angles

$$sin(a + b) = sina cosb + cosa sinb$$
  
 $sin(a - b) = sina cosb - cosa sinb$ 

$$cos(a + b) = cosa cosb - sina sinb$$
  
 $cos(a - b) = cosa cosb + sina sinb$ 

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

### <u>Double Angle Formulas</u>

#### $\cos 2x = 2\cos^2 x - 1$ $\cos 2x = 1 - 2\sin^2 x$

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2} \qquad \tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

$$\cos 2x = \cos^2 x - \sin^2 x \quad \sin 2x = 2\sin x \cos x$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \qquad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

### Reduction Formulas

#### $a^2 = b^2 + c^2 - 2bc \cos A$ Law of cosines

sin(-x) = -sinxcos(-x) = cosx $cos(x) = -cos(x - \pi)$  $sin(x) = -sin(x - \pi)$ tan(-x) = -tanx $tan(x) = tan(x - \pi)$ 

(where A is the angle of a scalene  $\triangle$  opposite side a)

# $\pm \sin x = \cos \left( x \pm \frac{\pi}{2} \right)$ $\pm \cos x = \sin \left( x \pm \frac{\pi}{2} \right)$

$$\frac{\text{Co-Functions}}{\sin x = \cos(90^{\circ} - x)}$$
 
$$\cos x = \sin(90^{\circ} - x)$$

$$tanx = cot(90^{\circ} - x)$$
 secx = csc(90° - x)

### Sum and Product Formulas

sina cosb = 
$$\frac{1}{2}[\sin(a + b) + \sin(a - b)]$$
  
cosa sinb =  $\frac{1}{2}[\sin(a + b) - \sin(a - b)]$ 

$$\cos a \cosh = \frac{1}{2}[\cos(a + b) + \cos(a - b)]$$
  
 $\sin a \sin b = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$ 

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y - 2\cos\left(\frac{y}{2}\right) \sin\left(\frac{y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$