Chap # 4   1/10/23

Chap # 4 : Amplitude (Linear) Modulation.

→ Modulation is a process that causes a shift in the range of frequencies in a signal.

→ Exchange of messages over a long distances (tele-communication) by using some electronic gats.

→ baseband communication doesn't use modulation

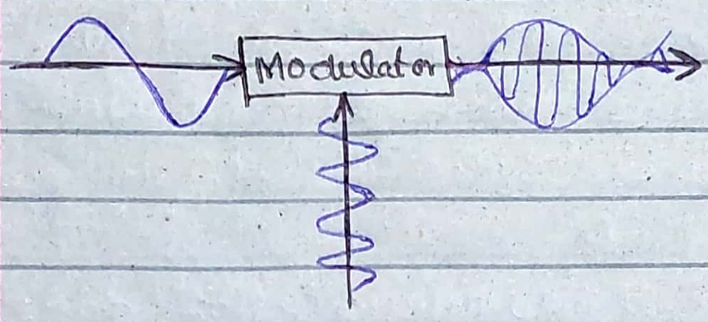
→ carrier communication uses modulation

→ Baseband refer to orginal frequency range of a transmission signal before it is converted or modulated to a different frequency range. These signals typically originates from transducers. These signals may be analog or digital.

→ band of voice signal (audio) is 0 - 3.5 kHz
→ band of video signal is 0 - 4.3 MHz.

→ PCM: pulse code modulation.
→ FDM: frequency division multiplexing.

→ Communication that uses modulation to shift the frequency spectrum of a signal is known as carrier communication.



4.1 Baseband and carrier communication:

→ The term baseband is used to designate the band of frequencies of the signal delivered by the source of input transducer.

→ In baseband Communication, baseband signals are transmitted without modulation.

→ By modulating several baseband signals and shifting their spectra to non-overlaping bands, one can use all the available bandwidth through FDM.

→ Some types of modulation are Amplitude modulation (AM), frequency modulation (FM) and phase Modulation (PM). The later two are similar and are known as angle modulation

4.2 Amplitud Modulation: Double Sideband (DSB):-

→ Amplitude modulation is characterised by the fact that the amplitude A of carrier signal $A\cos(\omega_c t+\theta_c)$ is varied in proportional to the baseband (message) $m(t)$, the modulating signal.

→ AM simply shifts the spectrum of $m(t)$ to the carrier frequency.

i.e if $m(t) \Leftrightarrow M(\omega)$

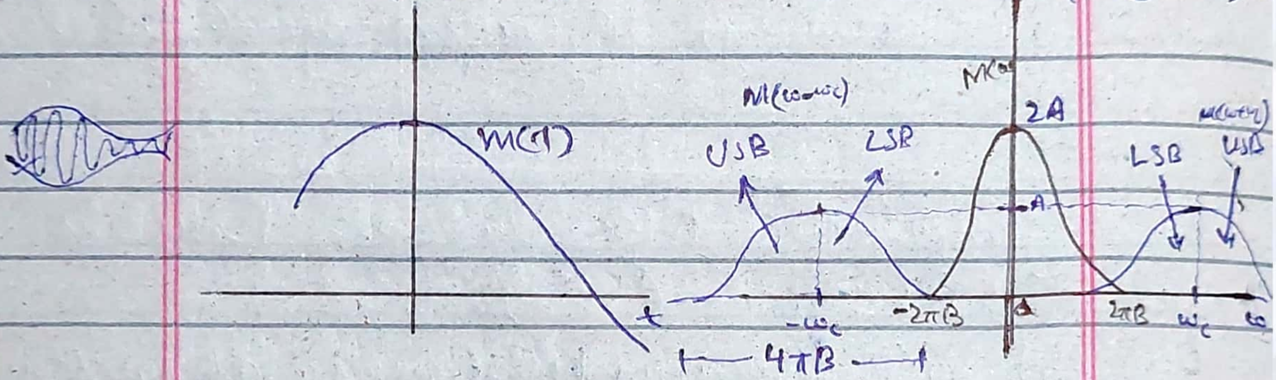
Then

$$m(t)\cos\omega_c t \Leftrightarrow \frac{1}{2}\left[M(\omega+\omega_c)+M(\omega-\omega_c)\right]$$

→ If bandwidth of $m(t)$ is B Hz that that of modulated signal is 2 B Hz.

→ modulated signal is centered at $\pm\omega_c$

→ The portion above ω_c is called uppersideband (USB) and below ω_c is called lower sideband (LSB).



→ The modulated signal in this scheme doeit contain a discrete compouent of corrier frequency ω_c. For this reason it is known as double-sideband supressed carrier (DSB-SC) modulation.

Demodulation

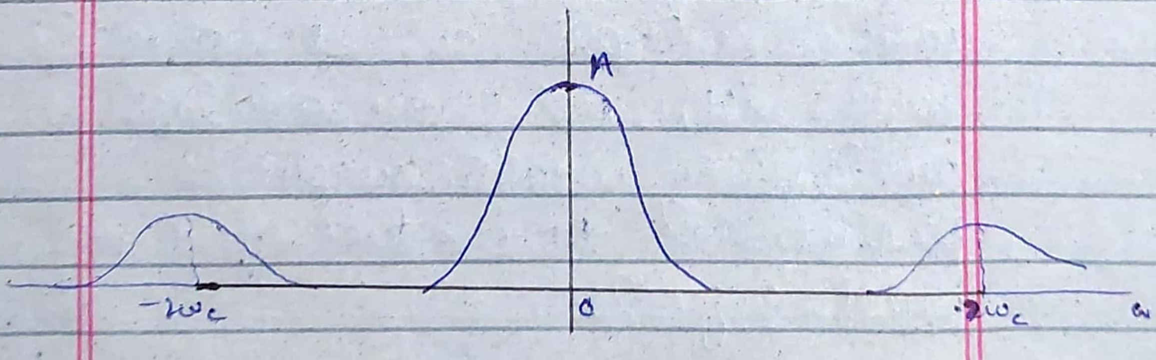
→ To recover orginal $m(t)$ from the modulated signal, it is necessary to retranslate the spectrum to its origional position, This process is called demodulation.

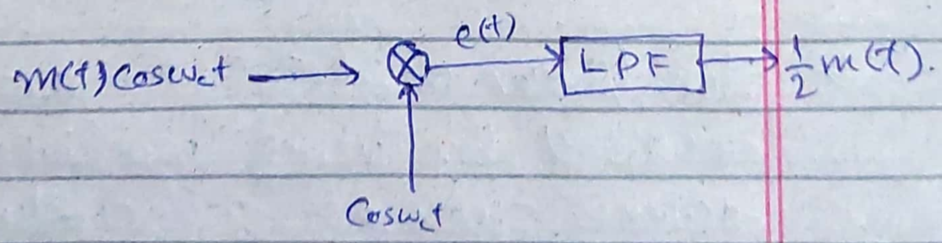
$\because \cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

→ demodulation consists of multipling the incoming modulated signal $m(t)\cos\omega_c t$ with a carrier signal $\cos\omega_c t$ followed by a low pass filter.

$$e(t) = m(t)\cos\omega_c t \times \cos\omega_c t$$
$$= m(t)\cos^2\omega_c t$$
$$= m(t)\left[\frac{1+\cos 2\omega_c t}{2}\right]$$
$$= \frac{1}{2}\left[m(t) + m(t)\cos 2\omega_c t\right]$$

$$E(\omega) = \frac{1}{2}M(\omega) + \frac{1}{2}\left[\frac{1}{2}\left(M(\omega+2\omega_c) + M(\omega-2\omega_c)\right)\right]$$
$$= \frac{1}{2}M(\omega) + \frac{1}{4}\left[M(\omega+2\omega_c) + M(\omega-2\omega_c)\right]$$





→ we can get $m(t)$ insted of $\frac{1}{2}m(t)$ by using (xing) with carrier $2\cos\omega_c t$.

→ This method of recovering the baseband signal is called synchronous / coherent detection where we use a carrier of exactly same frequey and phase as carrier used for modulation

→ When the modulating signal is a pure sinosoid then the modulation is refered to tone modulation

Modulators:-

→ Multiplier modulator are those which multiply $\cos\omega_c t$ directly with $m(t)$ using an analoge multiplier whose output is proportional to product of two input signals.

→ Non-linear modulators are acheived by using nonlinear devices, such as semiconductor diode or a transistor.

- Let non-linear elements be approximated by a power series:

$$y(t) = a\,x(t) + b\,x^2(t)$$

(output: $y(t)$; input: $x(t)$)

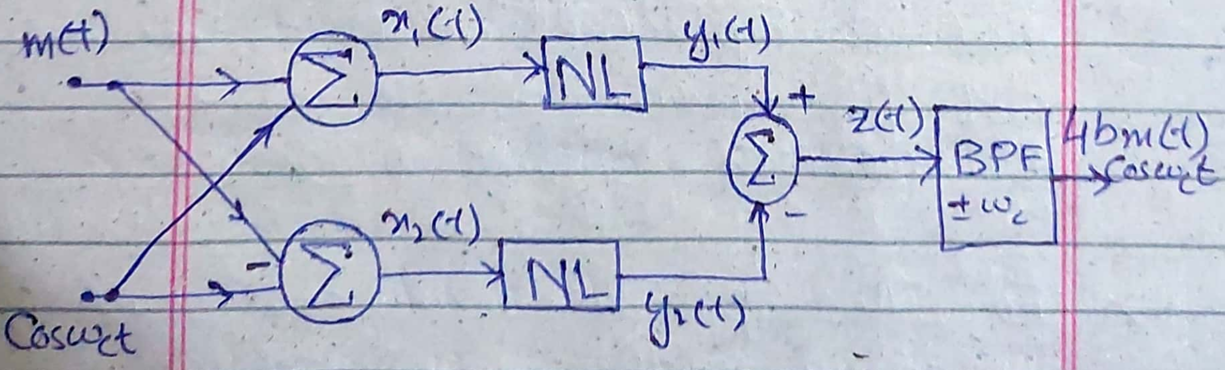
- The summer $z(t)$ is given by:

$$z(t) = y_1(t) - y_2(t)$$
$$= [a_1 x_1(t) + b_1 x_1^2(t)] - [a_2 x_2(t) + b_2 x_2^2(t)]$$

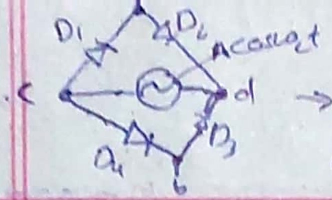
- Let the two inputs $x_1(t)$ and $x_2(t)$ be $\cos\omega_c t + m(t)$ and $\cos\omega_c t - m(t)$ respectively:

$$z(t) = 2a\,m(t) + 4b\,m(t)\cos\omega_c t$$

- When $z(t)$ is pass throug BPF of ω_c, the signal $a\,m(t)$ is suppressed and mudulated signal $4b\,m(t)\cos\omega_c t$ is passed throug unharmed.



Non-Linear DSB-SC Modulator.



→ Modulated signal can be obtained by multiplying $m(t)$ not only by pure sinosoid but by any periodic signal $\phi(t)$ of fundamental radian frequency ω_c. Thus modulation can be simply replaced by a switching operation.

- Such a signal represented by trigonometric fourier series is:

$$\phi(t) = \sum_{n=0}^{\infty} C_n \cos(n\omega_c t + \theta_n)$$

$$m(t)\,\phi(t) = \sum_{n=0}^{\infty} C_n\, m(t) \cos(n\omega_c t + \theta_n).$$

- The $m(t)\,\phi(t)$ is the spectrum $M(\omega)$ shifted $\pm\omega_c$, $\pm 2\omega_c$, + --- $\pm n\omega_c$ +---

4.3 Amplitude Modulation (AM)

→ As generation of same frequency and phase carrier at receiver ~~trans~~ end is inpractical used at transmitter end for modulation we can't demodulat a signal

→ For This problem & solve, we can send the carrier with modulated signal

* $\cos\alpha \cdot \cos\beta = \dfrac{\cos(\alpha+\beta) + \cos(\alpha-\beta)}{2}$ (9)

also to use its frequency and phase for demodulation at receiver end.

→ The cons of this system is that transmitter will use larger power, which make it expensive. 66.67% power is disipated while 33.33% is only used by $m(t)$ and is the useful power.

→ $\phi_{AM}(t) = A\cos\omega_c t + m(t)\cos\omega_c t$
$= [A + \cancel{\cos} m(t)]\cos\omega_c t$

$$\Phi_{AM}(\omega) = \frac{1}{2}\left[M(\omega+\omega_c) + M(\omega-\omega_c)\right] + \pi A[\delta(\omega+\omega_c) + \delta(\omega-\omega_c)].$$

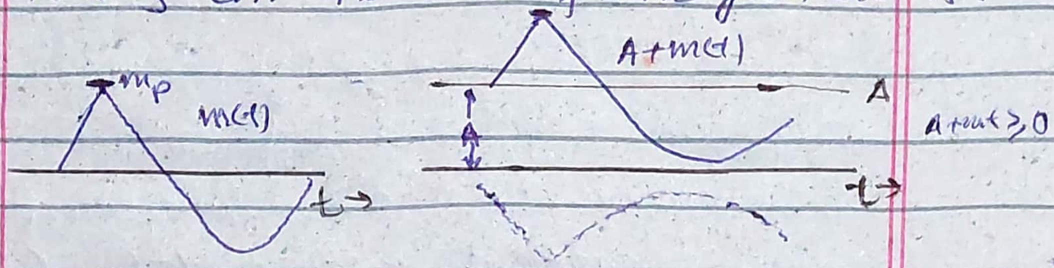
→ This modulation is similar to DSB-SC but instead of $m(t)$ we are modulating $A + m(t)$.

→ This modulation shall follow two conditions:

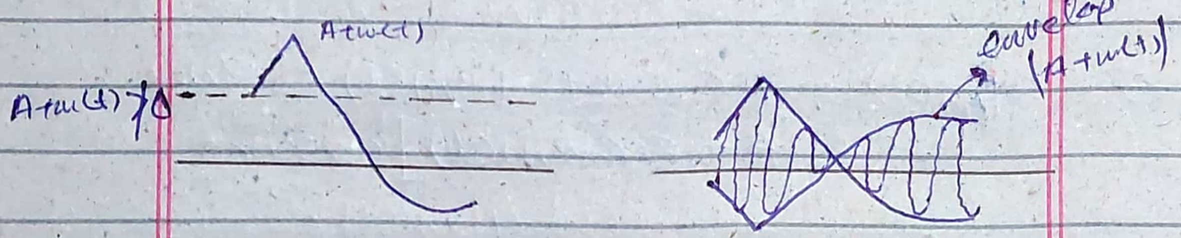
i) $A + m(t) \geq 0$. (non-negative).

ii) A is not large enough to satisfy (i).

→ In first case the envelop has the same shape as m(t), although riding on a dc of magnitude A.



→ In second case, the envelop shap is not m(t). because some parts get rectified



In this case we can't detect the envelop.

→ $A+m(t)$ is the envelop of $\varphi_{AM}(t)$ only if $A+m(t) \geq 0$. for all t.

→ We can detect m(t) only by detecting the envelop of $\varphi_{AM}(t)$ and do not need any cos$\omega_c t$ (carrier). of DSB-SC. if $A+m(t) \geq 0$. for all t.

→ Let m_p be peak amplitude (+ve or −ve) of $m(t)$. the from figure

$m(t) \geq -m_p$ then

$$A + m(t) \geq 0$$

$$A - m_p \geq 0$$

$$\boxed{A \geq m_p} \quad \text{for all } t$$

Thus minimum carrier amplitude for envelop detectio is $A \geq m_p$ for all t

→ We can define modulation index is

$$\mu = \overset{\text{peak of } m(t)}{m_p} / \underset{\text{carrier amplitude}}{A}$$

as $A \geq m_p \Rightarrow 0 \leq \mu \leq 1$

→ When $A < m_p \Rightarrow \mu > 1$ and is called over modulation. In This case the envelope detection is no longer viable. We then need to use synchronous demodulation.

Sideband and Carrier power:

→ $$\varphi_{AM}(t) = \underbrace{A\cos\omega_c t}_{\text{carrier}} + \underbrace{m(t)\cos\omega_c t}_{\text{sideband}}$$

→ In AM, the carrier doesn't carry

any information, and hence, the carrier power is wasted.

→ $P_\phi = P_c + P_s$
$= \frac{A^2}{2} + \frac{1}{2} m^2(t)$.

→ The power efficiency (η) is:

$$\eta = \frac{\text{useful power}}{\text{total power}} \times 100\% = \frac{P_s}{P_\phi} \times 100\%$$

$$= \frac{P_s}{P_c + P_s} \times 100\% = \frac{m^2(t)}{A^2 + m^2(t)} \times 100\%$$

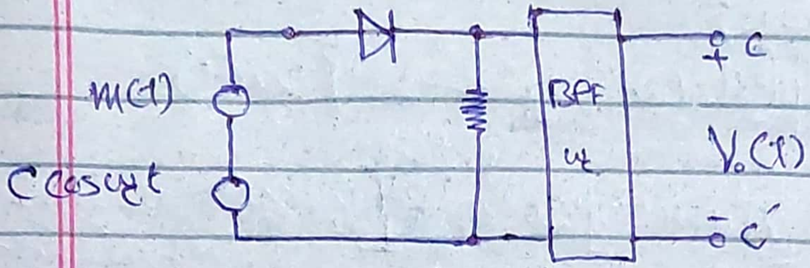
→ $\eta = \frac{\mu^2}{2+\mu^2} 100\%$ with the condition that $0 \le \mu \le 1$, η increases monotonically with μ and η_{max} occur at μ_{max}

i.e $\mu = 1$

$\eta_{max} = 33\%$.

Generation of AM Signals:

→ Can be generated by any DSB-SC modulator. As there is no need to supress carrier in output, the modulating circuits do not have to be balanced.



$$v_{bb'}(t) = [C\cos\omega_c t + m(t)]\,w(t)$$

$$= [C\cos\omega_c t + m(t)]\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \dots\right)\right]$$

$$= \underbrace{\frac{C}{2}\cos\omega_c t + \frac{2}{\pi} m(t)\cos\omega_c t}_{\text{AM}} + \underbrace{\text{other term}}_{\text{suppressed by BPF}}$$

Demodulation of AM Signals:

→ The two non-coherent methods of AM demodulation are:
→ rectifier detection
→ envelop detection.

i) If an AM signal is applied to a diode and a resistor circuit, the negative part of AM signal will be suppressed, the output accross the resistor is a half-wave rectified version of AM signal. The signal is multiplied by

(14) * Taylor Series: $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

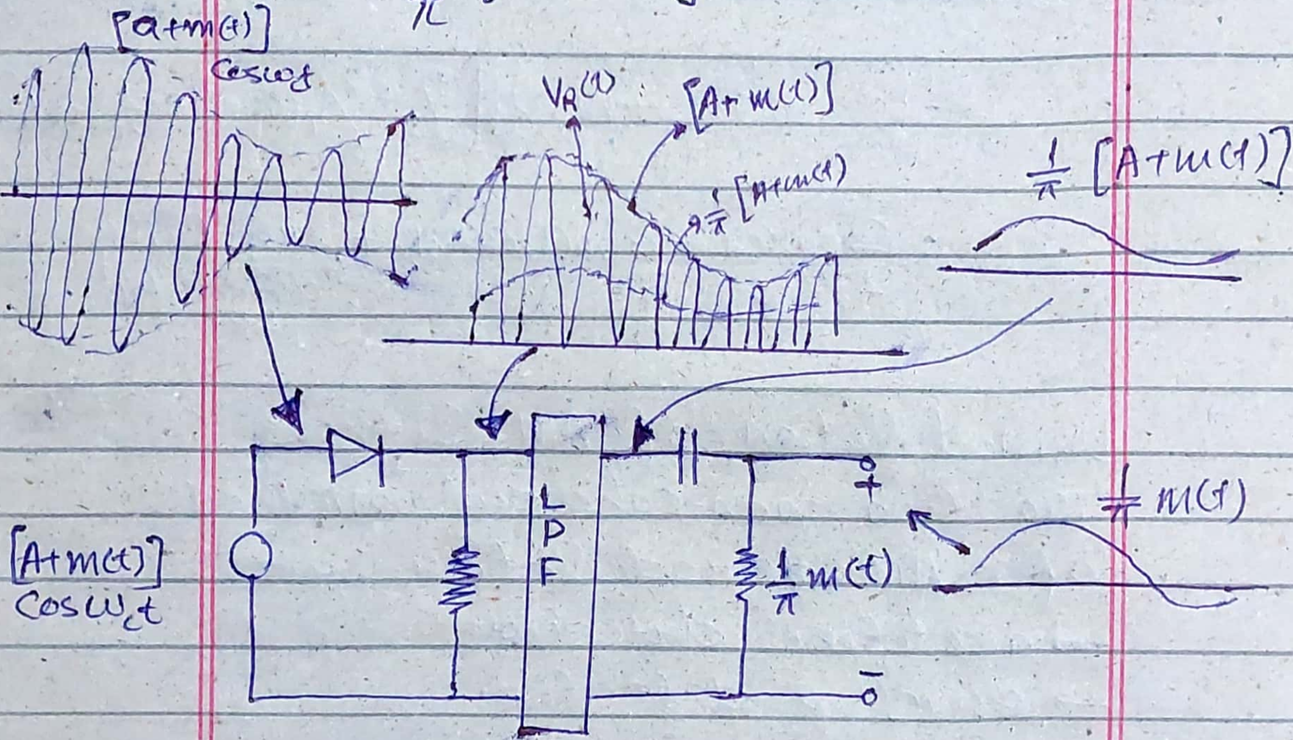
* M.S $(e^x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots \frac{x^n}{n!}$,

$w(t)$, Hence the rectified output V_R of Rectifier detector will be;

$$V_R = \{[A+m(t)]\cos\omega_c t\}\, w(t)$$

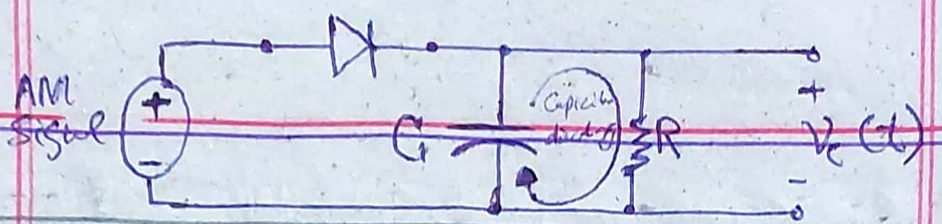
$$= [A+m(t)]\cos\omega_c t \left[\tfrac{1}{2} + \tfrac{2}{\pi}\left(\cos\omega_c t - \tfrac{1}{3}\cos 3\omega_c t + \tfrac{1}{5}\cos 5\omega_c t \cdots\right)\right]$$

$$= \frac{1}{\pi}[A+m(t)] + \text{other terms}$$



Rectifier Detector for AM.

ii) In envelope detector, the output of the detector follows the envelope of modulated signal.



→ During each positive cycle the capacitor charges up ~~and then~~ to the peak and then slowly decays until the next positive cycle.

→ Capacitor discharges causes riple of frequency ω_c in the output which can be decrease by increasing time constant RC. ($RC >> 1/\omega_c$).

→ the rectifier detector is a synchronously demodulator, while the envelope detection is a non-linear operation.

4.4 Quadrature Amplitud Modulation (QAM):

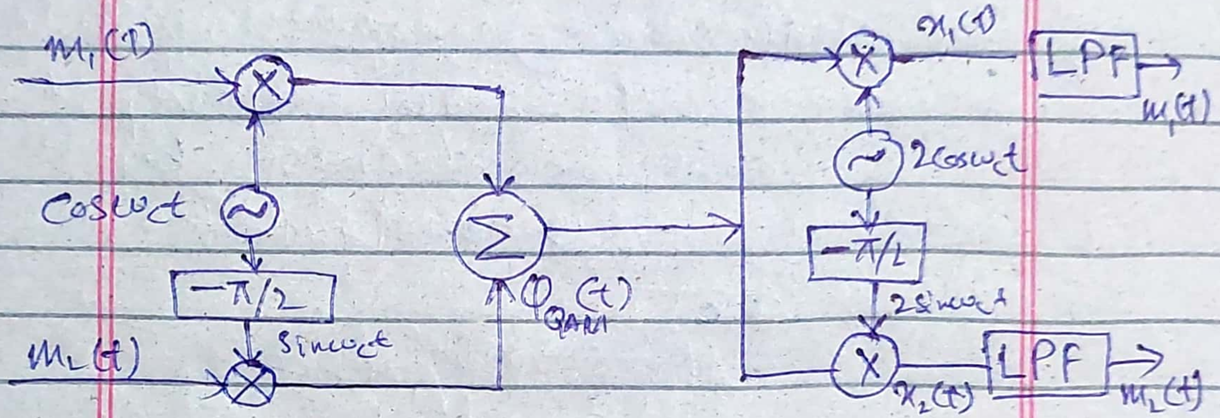
→ The DSB- signal occupy twice the bandwidth required for the baseband.

→ This disadvantage can be

(16)

overcome by transmitting two DSB signals using carrier of same frequency but ~~different~~ in phase quadrature.

→ $\varphi_{QAM}(t) = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$.



→ $x_1(t) = 2\varphi_{QAM}(t)\cos\omega_c t = 2[m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t]\cos\omega_c t$

$= m_1(t) + m_1(t)\cos 2\omega_c t + m_2(t)\sin 2\omega_c t$

→ The last two terms are suppresed by LPF. yeilding desired $m_1(t)$.

→ This scheme is known as quadrature amplitude ~~multiplexing~~ modulation (QAM) or quadrature multiplexing.

→ The upper channel is called in-phase channel (I) and the lower channel is Quadrature (Q) chanl.

→ A slight error in phase or frequency of carrier at demodulator in QAM will result in loss and distortion of signal, and interference b/w two channels.

→ QAM is used in color television.

4.5 Single Sideband Amplitude Modulation (SSB):

→ A scheme in which only one sideband (LSB or USB) is transmitted is known as single-sideband transmission, which requires only one-half of the DSB signal.

→ Demodulation SSB-SC is similar to that of DSB-SC.

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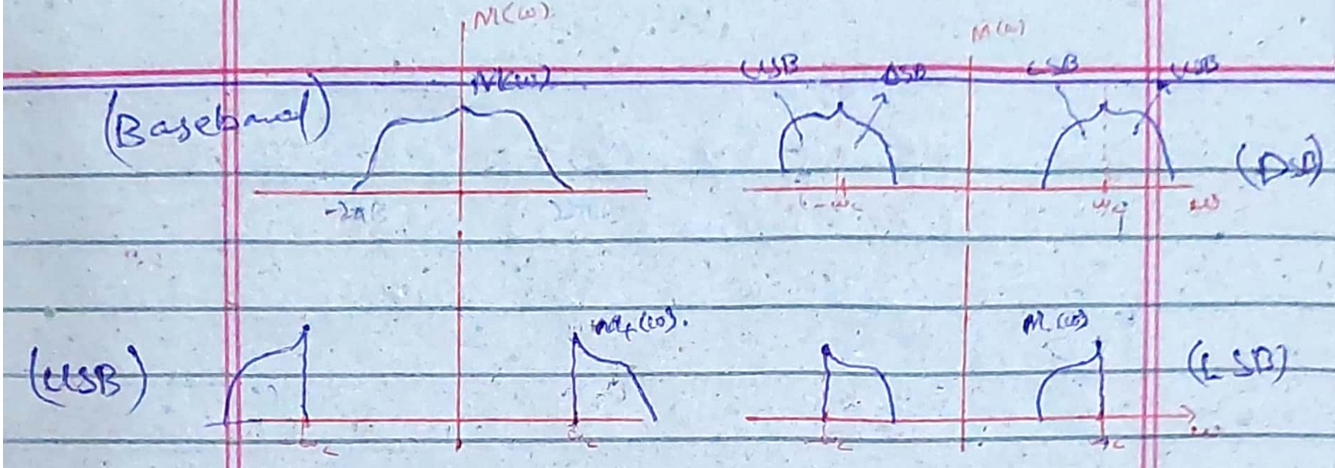
After mid course:



4.5.1 Time domain representation of SSB.

Let USB = $M_+(\omega)$ and
LSB = $M_-(\omega)$.

from above figure we can see
$M_+(\omega) = M(\omega)u(\omega)$ & $M_-(\omega) = M(\omega)u(-\omega)$

Let $\mathcal{F}\{M_+(\omega)\} = m_+(t)$
and $\mathcal{F}\{M_-(\omega)\} = m_-(t)$

As $M_+(-\omega) \neq M_+(\omega)$
and $M_-(-\omega) \neq M_-(\omega)$ } Not Even hence time domain will be a complex.

As $M_+(-\omega) = M_-(\omega)$
and $M_-(-\omega) = M_+(\omega)$ } Conjugates.

hence the time domain will be also a conjugate of each other i.e

$$M(\omega) = M_+(\omega) + M_-(\omega)$$

$$m(t) = m_+(t) + m_-(t)$$

$$= \frac{1}{2}\left[m(t) + j m_h(t)\right] + \frac{1}{2}\left[m(t) - j m_h(t)\right]$$

To determin $m_h(t)$:

$$M_+(\omega) = M(\omega)\, u(\omega)$$

$$= M(\omega)\left(\frac{1+\text{sgn}(\omega)}{2}\right)$$

$$= \frac{1}{2} M(\omega) + \frac{1}{2} M(\omega)\,\text{sgn}(\omega)$$

as $M(\omega) \Leftrightarrow m(t)$ & $M(\omega)\,\text{sgn}(\omega) \Leftrightarrow j m_h(t)$

from above equations.

$$\mathcal{F}\{m_h(t)\} = M_h(\omega)$$

$$\Rightarrow M_h(\omega) = -j\, M(\omega)\,\text{sgn}(\omega)$$

$$= M(\omega) \cdot -j\,\text{sgn}(\omega)$$

let $H(\omega) = -j\,\text{sgn}(\omega)$

which shows that

$$\mathcal{F}\{H(\omega)\} = \frac{1}{\pi t}$$

$$M_h(\omega) = M(\omega)\,H(\omega)$$

$$m_h(t) = m(t) * h(t)$$
$$= m(t) * \frac{1}{\pi t}$$
$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(t-\alpha)}{t-\alpha}\,d\alpha$$

which is Hilbert function.

Now as $H(\omega) = -j\,\mathrm{sgn}(\omega)$

$$H(\omega) = \begin{cases} -j = 1e^{-j\pi/2} ; & \omega > 0 \\ j = 1e^{j\pi/2} ; & \omega < 0 . \end{cases}$$

which shows that

$$|H(\omega)| = 1 \;\; \& \;\; \theta_h(\omega) = \begin{cases} -\pi/2 & \omega > 0 \\ +\pi/2 & \omega < 0 \end{cases}$$

→ A Hilbert transform is ideal phase shifter that shifts the phase of every spectral component by $-\pi/2$.

→ Now SSB can be expressed in term of $m(t)$ and $m_h(t)$.

$$\Phi_{USB}(\omega) = M_+(\omega-\omega_c) + M_-(\omega-\omega_c)$$

$$\Phi_{USB}(t) = m_+(t)e^{j\omega_c t} + m_-(t)e^{-j\omega_c t}$$

$$= \frac{1}{2}\left[m(t)+jm_h(t)\right]e^{j\omega_c t} + \frac{1}{2}\left[m(t)-jm_h(t)\right]e^{-j\omega_c t}$$

$$= \frac{1}{2}\left[m(t)e^{j\omega_c t}+jm_h(t)e^{j\omega_c t} + m(t)e^{-j\omega_c t} - jm_h(t)e^{-j\omega_c t}\right]$$

$$= \frac{1}{2}\left[m(t)\left(e^{j\omega_c t}+e^{-j\omega_c t}\right) + jm_h(t)\left(e^{j\omega_c t}-e^{-j\omega_c t}\right)\right]$$

$$= m(t)\left(\frac{e^{j\omega_c t}+e^{-j\omega_c t}}{2}\right) + jm_h(t)\left(\frac{e^{j\omega_c t}-e^{-j\omega_c t}}{2j}\right)j$$

$$= m(t)\cos\omega_c t + j^2 m_h(t)\sin\omega_c t$$

$$= m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$$

Similarly

$$\Phi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t.$$

* $\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$
* $\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$

Example 4.7:

$m(t) = \cos\omega_m t$, $\phi_{SSB}(t) = ?$

Sol: $m_h(t) = m(t - \pi/2)$

$= \cos(\omega_m t - \pi/2) = \sin\omega_m t$

$\phi_{SSB}(t) = \cos\omega_m t \cdot \cos\omega_c t \mp \sin\omega_m t \cdot \sin\omega_c t$

$= \cos t(\omega_m \pm \omega_c)$

$\phi_{USB}(t) = \cos(\omega_m + \omega_c)t$

$\phi_{LSB}(t) = \sin(\omega_m \mp \omega_c)t$

4.5.2 Generation of SSB signals:

→ Three methods are used for generating SSB signals:

i - Selective - Filtering Method

ii - Phase - Shift Method

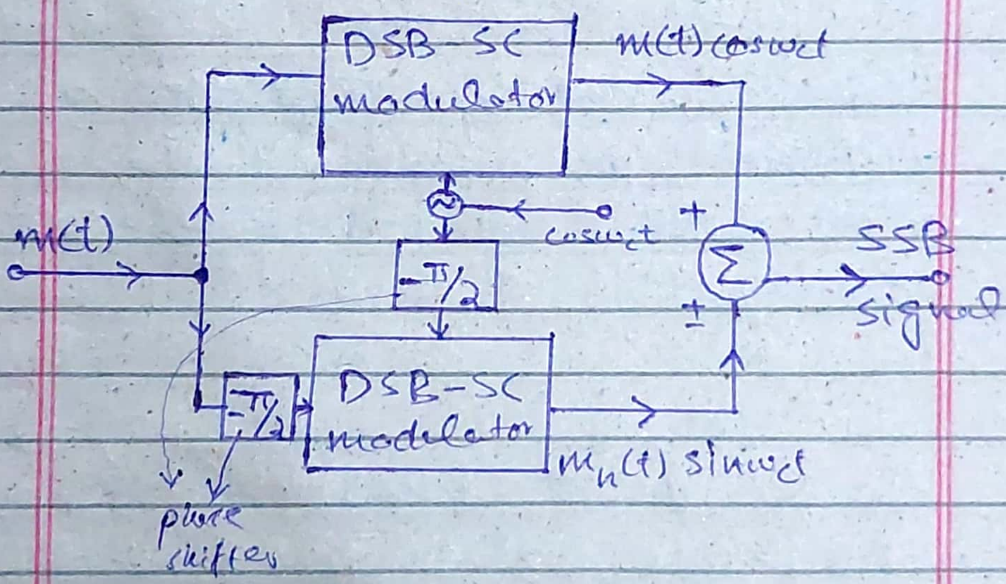
iii - Weaver's method

→ In (i) A DSB-SC signal is passed through a sharp cutoff filter to eliminate the undesired sideband.

→ Such operation ~~which~~ require an ideal filter which is unrealizable. It can be realizable if there is some seperation between the passband and stopband.

→ To minimize adjacent channel interference, the undesired sideband should be attenuated at least 40dB.

→ The equation $\phi_{SSB}(t) = m(t)\cos\omega_c t \pm m_h(t)\sin\omega_c t$ is the basis for second (ii) method.



→ An ideal phase shifter is also unrealizable.

→ In term of bandwidth requirements, SSB is similar to QAM but less exacting in term of carrier frequency and phase or requirement of a distortionless transmission medium.

4.5.3 Demodulation of SSB-SC Sig:

→ SSB-SC signals can be coherently demodulated:

$$\varphi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t$$

$$\varphi_{SSB}(t)\cos\omega_c t = \frac{1}{2}m(t)\left[1+\cos 2\omega_c t\right] \mp \frac{1}{2}m_h(t)\sin 2\omega_c t$$

$$= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t \mp \frac{1}{2}m_h(t)\sin 2\omega_c t$$

→ Thus product of φ_{SSB} with $\cos\omega_c t$ yields the baseband signal and another SSB signal with a carrier $2\omega_c$.

→ After passing it with LPF, the unwanted SSB terms are suppressed and gives the required baseband signal $m(t)/2$.

→ Any synchronous demodulator used for DSB-SC can be used to demodulate an SSB-SC signal.

→ Envelope detection of SSB signal can be done by sending carrier with SSB signal (SSB+C). ie.

$$\varphi_{SSB+C} = A\cos\omega_c t + [m(t)\cos\omega_c t + m_h(t)\sin\omega_c t]$$

$$= [A + m(t)]\cos\omega_c t + m_h(t)\sin\omega_c t$$

→ If A is large enough the m(t) can be recovered with envelop detection

$$\varphi_{SSB+C} = E(t)\cos(\omega_c t + \theta)$$

where $E(t) = \left\{[A+m(t)]^2 + m_h^2(t)\right\}^{1/2}$

$$= A\left[A + \frac{2m(t)}{A} + \frac{m^2(t)}{A^2} + \frac{m_h^2(t)}{A^2}\right]^{1/2}$$

Then by binomial theorm; if $A \gg |m(t)|$ and $A \gg |m_h(t)|$

$$E(t) \cong A\left[1 + \frac{2m(t)}{A}\right]^{1/2}$$

$$\cong A\left[1 + \frac{m(t)}{A}\right]$$

$$= A + m(t)$$

→ Thus envelop detection require the condition $A \geq |m(t)|$, where for

SSB+C, the condition is
$A >> |m(t)|$.

4.6 Vestigial sideband (VSB) Amplitude modulation:

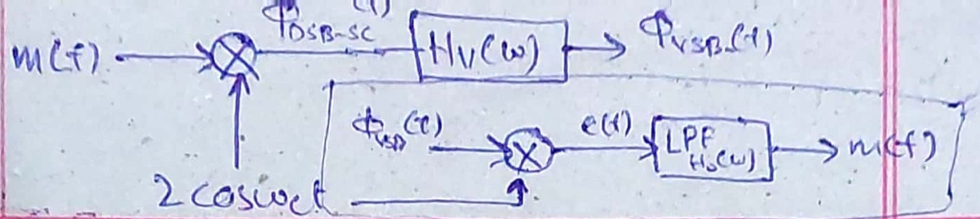
→ VSB also called asymmetric sideband system is a compromise between DSB and SSB. It inherits advantages of SSB and DSB but avoids its disadvantages at a small cost.

→ Bandwidth is typically 25% greater than that of SSB.

→ As $\phi_{DSB}(\omega) = M(\omega+\omega_c) + M(\omega-\omega_c)$
if $H_i(\omega)$ produce VSB from DSB
then

$$\phi_{VSB}(\omega) = [M(\omega+\omega_c) + M(\omega-\omega_c)] H_i(\omega)$$

→ $H_i(\omega)$ allows transmission of one side band, but suppresses the other sideband, not completely but gradually.



As DSB-SC is given

$\phi_{DSB-SC}(t) = m(t)\cos\omega_c t = M(\omega+\omega_c)+M(\omega-\omega_c)$

BPF ← Let when it is passed with $H_v(\omega)$, which allow transmission of one sideband gradually.

$\phi_{VSB}(t) = \phi_{DSB-SC}(t) \cdot H_v(\omega)$

$\phi_{VSB}(\omega) = \phi_{DSB-SC}(\omega) \cdot H_V(\omega)$

$\phi_{VSB}(\omega) = \left(M(\omega+\omega_c)+M(\omega-\omega_c)\right)H_V(\omega)$ — (1)

Let m(t) be recoverably by synchronous detection:

(ii)

$e(t) = \phi_{VSB}(t)\cdot 2\cos\omega_c t = \phi_{VSB}(\omega+\omega_c)+\phi_{VSB}(\omega-\omega_c)$

Now from (1) we get

$\phi_{VSB}(\omega+\omega_c) = \left[M(\omega)+M(\omega+2\omega_c)\right]H_V(\omega+\omega_c)$

and

$\phi_{VSB}(\omega-\omega_c) = \left[M(\omega-2\omega_c)+M(\omega)\right]H_V(\omega-\omega_c)$

putting these values in (ii) we get.

$$e(\omega) = M(\omega) H_v(\omega+\omega_c) + M(\omega) H_v(\omega-\omega_c) + M(\omega+2\omega_c) H_v(\omega+\omega_c) + M(\omega-2\omega_c) H_v(\omega-\omega_c)$$

$$e(\omega) = M(\omega)\left[H_v(\omega+\omega_c) + H_v(\omega-\omega_c)\right] + M(\omega+2\omega_c) H_v(\omega+\omega_c) + M(\omega-2\omega_c) H_v(\omega-\omega_c)$$

Now further passing with $H_o(\omega)$ which is a LPF to get $m(t)$

$$e(\omega)\cdot H_o(\omega) = \{ \cdots \} H_o(\omega)$$

$$\cancel{M(\omega)} = \cancel{M(\omega)}\left[H_v(\omega+\omega_c) + H_v(\omega-\omega_c)\right]\cdot H_o(\omega)$$

(others are suppressed)

$$H_o(\omega) = \frac{1}{H_v(\omega+\omega_c) + H_v(\omega-\omega_c)}$$

→ We know that $H_v(\omega)$ is BPF which contain low-pass components in term $H_v(\omega \pm \omega_c)$.

→ As SSB-C requires a much larger carrier than DSB+C for envelope detection. Because VSB+C is in between case, so the $C_{DSB} < C_{VSB} < C_{SSB}$.

→ The baseband video signal needs a bandwidth of 9 MHz in DSB and 4.5 MHz in SSB. While the ~~VSB~~ VSB occupies a bandwidth of 6 MHz.

→ In all types of modulation discussed so far, the modulated signal satisfies the principles of Superposition.

4.7. Carrier Acquisition:

→ In ~~the~~ the supressed-carrier AM systems (DSB-SC, SSB-SC, VSB-SC), one must generate a local carrier at receiver for the purpose of synchronous demodulation.

→ Local carrier must be in frequency and phase synchronism with the incoming carrier, else any discrepency will cause distortion.

→ Let the local carrier is:
$2\cos[(\omega_c+\Delta\omega)t+\delta]$, Then

$$e(t) = m(t)\cos\omega_c t \cdot 2\cos[(\omega_c+\Delta\omega)t+\delta]$$

$e(t) = m(t)\left[\cos\left((\Delta\omega)t + \delta\right) + \cos\left((2\omega_c + \Delta\omega)t + \delta\right)\right]$

After filtering we get

$e_o(t) = m(t)\cos\{(\Delta\omega)t + \delta\}$ — (1)

Here we have some cases:

* Case I: If $\Delta\omega$ & δ are both zero, then we have no frequency or phase errors:

$$e_o(t) = m(t) \cdot \cos(0+0) = m(t) = e(t).$$

* Case II: If $\Delta\omega = 0$, and δ is any constant then the output is maximum at $\delta = 0$ and minimum (zero) at $\delta = \pm\pi/2$. Thus phase error causes attenuation of output signal without causing distortion.

$$e_o(t) = m(t)\cos(0+\delta)$$
$$= m(t)\cos\delta$$

for $\delta = \pm\pi/2$ $e_o(t) = m(t) \times 0$

$\delta = 0$ $e_o(t) = m(t) \times 1$

* Case III: If $\delta = 0$ and $\Delta\omega \neq 0$

so $e_o(t) = m(t)\cos(\Delta\omega)t$

The output is attenuated as well as severly distorted

(31)

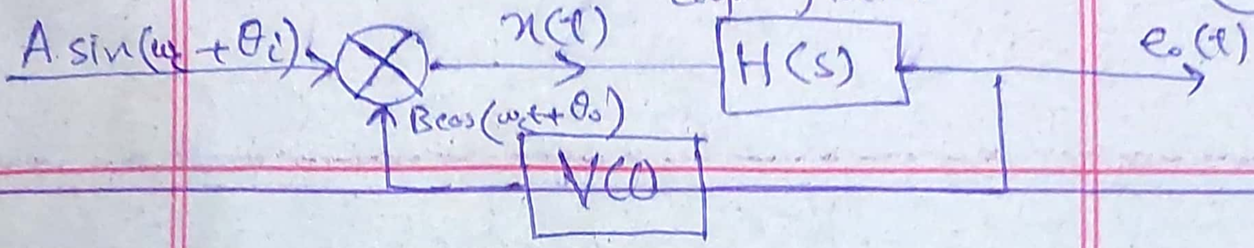
→ For carrier acquisition, we can use two method to solve above problem:

1. → Pilot Method: In such a case, a carrier or pilot is transmitted along with sidebands.

2. → Phase-Locked Loop (PLL) Method:
PLL is used to track the phase and frequency of carrier.
• Useful device for synchronous demodulation of AM signals with suppressed carrier or with a little carrier (the pilot).

• Under low SNR it can be used for the demodulation of Angle-modulated signals.

• It has three basic components
1. VCO (Voltage-controlled oscillator).
2. A multiplier, serving as phase detector or a phase comparator
3. A loop filter. H(s).



- Operation of PLL is similar to a feedback system in the figure above.

* Examples:

* E# 4.5: Sketch $\mu_1 = 0.5$, $\mu_2 = 1$
$m(t) = B\cos\omega_m t$.

Sol:- As we know $\mu = m_p/A$

here $m_p = B$

$$\mu = \frac{B}{A} \Rightarrow B = \mu A$$

The $m(t) = B\cos\omega_m t = \mu A\cos\omega_m t$

Therefor

$$\varphi_{AM}(t) = [A + m(t)]\cos\omega_c t$$
$$= [A + \mu A\cos\omega_m t]\cos\omega_c t$$
$$= [1 + \mu\cos\omega_m t]\,A\cos\omega_c t.$$

Hence for μ_1

$1 + 0.5\cos\omega_m t$

and for μ_2

$1 + \cos\omega_m t$

* figures in book (p# 164).

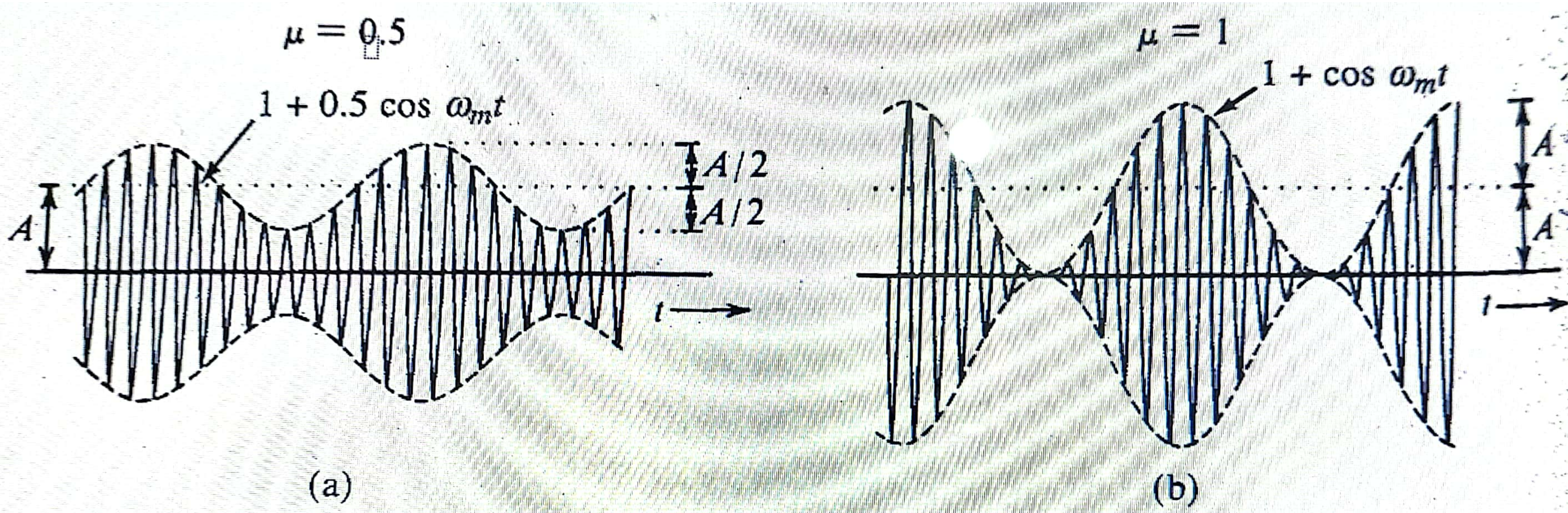


Figure 4.9 Tone-modulated AM. (a) $\mu = 0.5$. (b) $\mu = 1$.

* E # 4.8 : $\eta = ?$ for $\mu_1 = 0.5$ & $\mu_2 = 0.3$.

Salution:

As we know $\eta = \dfrac{\mu^2}{2 + \mu^2} \times 100\%$

for $\mu_1 = 0.5$

$$\eta_1 = \frac{(0.5)^2}{2+(0.5)^2} \times 100\% = 0.1111 \times 100\%$$

$$\eta_1 = 11.11\ \%$$

for $\mu_2 = 0.3$

$$\eta_2 = \frac{(0.3)^2}{2+(0.3)^2} \times 100\% = 0.0431 \times 100\%$$

$$\eta_2 = 4.31\ \%$$

* Ex # 4.6: For tone modulation:--

Sol:- The capicitor discharge from some peak E is given by

$$v_c = E\, e^{-t/RC}$$

By using Mclaren Series $e^{x} = \sum_{0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n = \frac{x^n}{n!}$

$= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$$e^{-t/RC} = \left(1 - \frac{t}{RC} + \frac{t^2}{2R^2C^2} + \cdots\right)$$

$$e^{-t/RC} \simeq (1 - t/RC)$$

hence

$$V_c \approx E(1 - t/RC)$$

Now finding its slope

$$\frac{d}{dt} V_c = \frac{d}{dt} E(1 - t/RC) = -E/RC$$

and $\left|\frac{d}{dt} V_c\right| = \frac{E}{RC}$

As for tone modulation the slope is

$$E(t) = A[1+\mu\cos\omega_m t]$$

$$\frac{d}{dt}E(t) = 0 - \mu A \sin\omega_m t \cdot \omega$$

$$= -\mu A\omega_m \sin\omega_m t$$

$$\left|\frac{d}{dt}E(t)\right| = \mu A\omega_m \sin\omega_m t$$

Now we have both slopes. But we know that RC discharge must be greater than slope of envelop E(t) then

$$\frac{E}{RC} \leftarrow \left|\frac{d}{dt}V_c\right| \geq \frac{d}{dt}E(t)$$

$$\frac{E}{RC} \geq \mu A\omega_m \sin\omega_m t$$

$$\frac{A[1+\mu\cos\omega_m t]}{RC} \geq \mu A\omega_m \sin\omega_m t \quad ; \text{for all } t.$$

$$\frac{RC}{A[1+\mu\cos\omega_m t]} \leq \frac{1}{\mu A\omega_m \sin\omega_m t}$$

$$RC \leq \frac{\cancel{A}[1+\mu\cos\omega_m t]}{\mu \cancel{A}\omega_m \sin\omega_m t}$$

$$RC \leq \frac{1+\mu\cos\omega_m t}{\mu\omega_m \sin\omega_m t}$$

The worst thing happen when R.H.S is minimum.

Hence

$$\frac{d}{dt}\left[\frac{1+\mu\cos\omega_m t}{\mu\omega_m \sin\omega_m t}\right] = 0$$

$$\frac{\mu\omega_m \sin\omega_m t\,(0-\mu\omega_m \sin\omega_m t) - [1+\mu\cos\omega_m t]\,\mu\omega_m^2 \cos\omega_m t}{\mu^2\omega^2 \sin^2\omega_m t} = 0$$

$$= \frac{-\mu^2\omega_m^2 \sin^2\omega_m t - \mu\omega_m^2 \cos\omega_m t - \mu^2\omega_m^2 \cos^2\omega_m t}{\mu^2\omega_m^2 \sin^2\omega_m t} = 0$$

$$= \frac{-\mu^2\omega_m^2 (\overbrace{\sin^2\omega_m t + \cos^2\omega_m t}^{1}) - \mu\omega_m^2 \cos\omega_m t}{\mu^2\omega_m^2 \sin^2\omega_m t} = 0$$

$$= \frac{\cancel{\mu\omega_m^2}(-\mu - \cos\omega_m t)}{\mu^2\cancel{\omega_m^2} \sin^2\omega_m t} = 0$$

$$\Rightarrow -\mu - \cos\omega_m t = 0$$

$$\Rightarrow \cos\omega_m t = -\mu$$

putting this in RHS we get

$$\frac{\sqrt{1-\mu^2}}{\mu\omega_m}$$

Hence

$$RC \le \frac{1}{\omega_m}\frac{\sqrt{1-\mu^2}}{\mu}$$

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* Ex # 4.7: $\phi_{SSB}(t) = ?$, $m(t) = \cos\omega_m t$.

Solution: As we know

$$\phi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t.$$

as $m_h(t)$ is Hilbert transform that delay the phase of each spectral component by $\pi/2$. hence

$$m_h(t) = m(t - \pi/2) = \cos\left(\omega_m - \frac{\pi}{2}\right)t$$

$$= \sin\omega_m t$$

$$\phi_{SSB}(t) = \cos\omega_m t \cdot \cos\omega_c t \mp \sin\omega_m t \cdot \sin\omega_c t$$

As $\cos(\alpha \mp \beta) = \cos\alpha \cdot \cos\beta \pm \sin\alpha \sin\beta$

hence $\phi_{SSB}(t) = \cos(\omega_m \pm \omega_c)t$

Thus $\phi_{USB}(t) = \cos(\omega_m + \omega_c)t$

$\phi_{LSB}(t) = \cos(\omega_m - \omega_c)t$.

* see figure 4.8 in book p#(175).

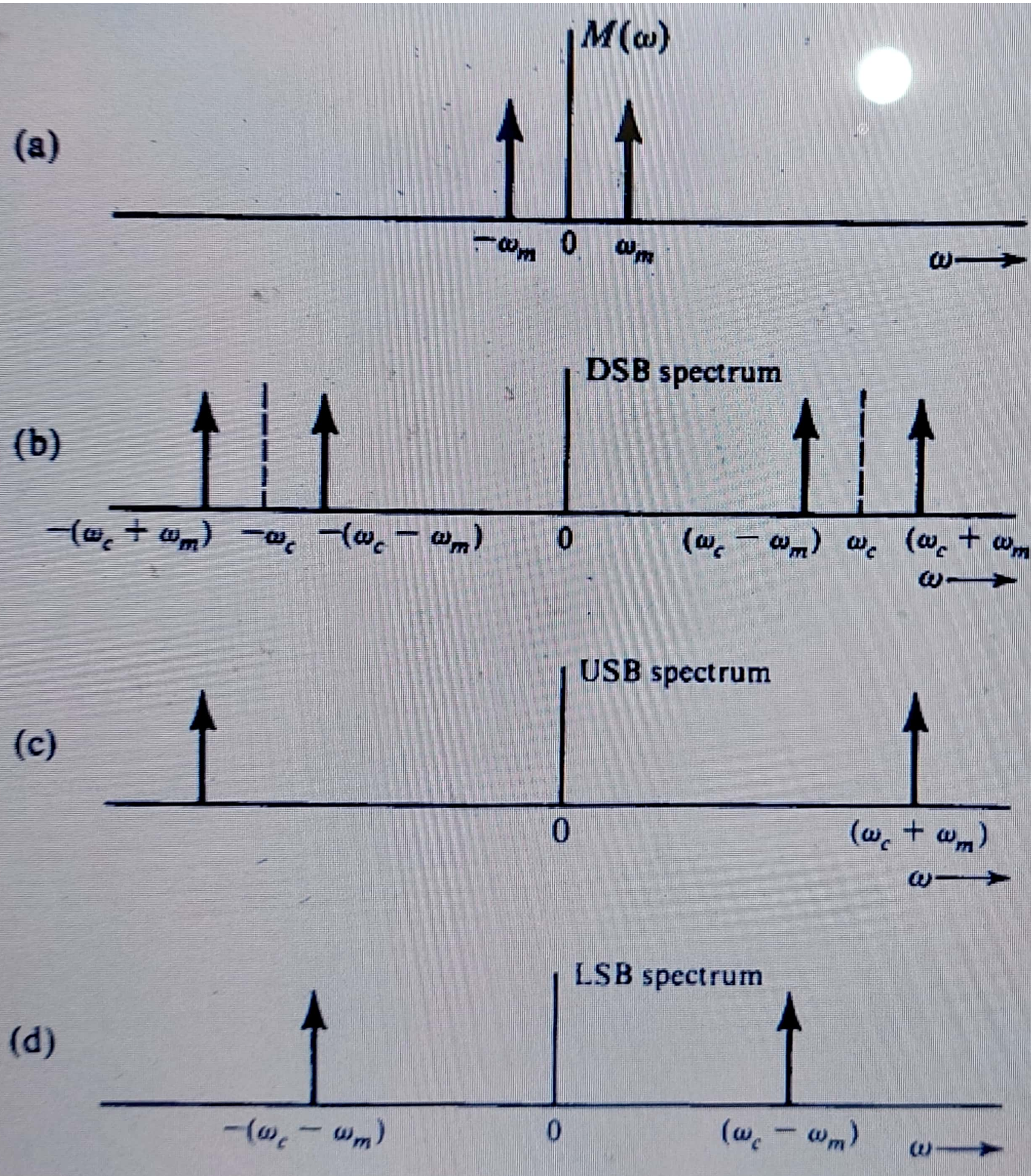

Figure 4.18 SSB spectra for tone modulation.