1

Circuit Variables

Assessment Problems

AP 1.1 Use a product of ratios to convert two-thirds the speed of light from meters per second to miles per second:

$$\left(\frac{2}{3}\right)\frac{3\times10^8\text{ m}}{1\text{ s}}\cdot\frac{100\text{ cm}}{1\text{ m}}\cdot\frac{1\text{ in}}{2.54\text{ cm}}\cdot\frac{1\text{ ft}}{12\text{ in}}\cdot\frac{1\text{ mile}}{5280\text{ feet}}=\frac{124{,}274.24\text{ miles}}{1\text{ s}}$$

Now set up a proportion to determine how long it takes this signal to travel 1100 miles:

$$\frac{124{,}274.24\text{ miles}}{1\text{ s}}=\frac{1100\text{ miles}}{x\text{ s}}$$

Therefore,

$$x=\frac{1100}{124{,}274.24}=0.00885=8.85\times10^{-3}\text{ s}=8.85\text{ ms}$$

AP 1.2 To solve this problem we use a product of ratios to change units from dollars/year to dollars/millisecond. We begin by expressing \$10 billion in scientific notation:

$$\$100\text{ billion }=\$100\times10^9$$

Now we determine the number of milliseconds in one year, again using a product of ratios:

$$\frac{1\text{ year}}{365.25\text{ days}}\cdot\frac{1\text{ day}}{24\text{ hours}}\cdot\frac{1\text{ hour}}{60\text{ mins}}\cdot\frac{1\text{ min}}{60\text{ secs}}\cdot\frac{1\text{ sec}}{1000\text{ ms}}=\frac{1\text{ year}}{31.5576\times10^9\text{ ms}}$$

Now we can convert from dollars/year to dollars/millisecond, again with a product of ratios:

$$\frac{\$100\times10^9}{1\text{ year}}\cdot\frac{1\text{ year}}{31.5576\times10^9\text{ ms}}=\frac{100}{31.5576}=\$3.17/\text{ms}$$

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AP 1.3 Remember from Eq. (1.2), current is the time rate of change of charge, or $i = \frac{dq}{dt}$ In this problem, we are given the current and asked to find the total charge. To do this, we must integrate Eq. (1.2) to find an expression for charge in terms of current:

$$q(t) = \int_0^t i(x)\,dx$$

We are given the expression for current, i, which can be substituted into the above expression. To find the total charge, we let $t \to \infty$ in the integral. Thus we have

$$q_{\text{total}} = \int_0^\infty 20e^{-5000x}\,dx = \frac{20}{-5000}e^{-5000x}\Big|_0^\infty = \frac{20}{-5000}(e^{-\infty} - e^0)$$

$$= \frac{20}{-5000}(0-1) = \frac{20}{5000} = 0.004 \text{ C} = 4000\,\mu\text{C}$$

AP 1.4 Recall from Eq. (1.2) that current is the time rate of change of charge, or $i = \frac{dq}{dt}$. In this problem we are given an expression for the charge, and asked to find the maximum current. First we will find an expression for the current using Eq. (1.2):

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[\frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-\alpha t}\right]$$

$$= \frac{d}{dt}\left(\frac{1}{\alpha^2}\right) - \frac{d}{dt}\left(\frac{t}{\alpha}e^{-\alpha t}\right) - \frac{d}{dt}\left(\frac{1}{\alpha^2}e^{-\alpha t}\right)$$

$$= 0 - \left(\frac{1}{\alpha}e^{-\alpha t} - \alpha\frac{t}{\alpha}e^{-\alpha t}\right) - \left(-\alpha\frac{1}{\alpha^2}e^{-\alpha t}\right)$$

$$= \left(-\frac{1}{\alpha} + t + \frac{1}{\alpha}\right)e^{-\alpha t}$$

$$= te^{-\alpha t}$$

Now that we have an expression for the current, we can find the maximum value of the current by setting the first derivative of the current to zero and solving for t:

$$\frac{di}{dt} = \frac{d}{dt}(te^{-\alpha t}) = e^{-\alpha t} + t(-\alpha)e^{\alpha t} = (1-\alpha t)e^{-\alpha t} = 0$$

Since $e^{-\alpha t}$ never equals 0 for a finite value of t, the expression equals 0 only when $(1-\alpha t) = 0$. Thus, $t = 1/\alpha$ will cause the current to be maximum. For this value of t, the current is

$$i = \frac{1}{\alpha}e^{-\alpha/\alpha} = \frac{1}{\alpha}e^{-1}$$

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Remember in the problem statement, $\alpha = 0.03679$. Using this value for α,

$$i = \frac{1}{0.03679}e^{-1} \cong 10 \text{ A}$$

AP 1.5 Start by drawing a picture of the circuit described in the problem statement:

Also sketch the four figures from Fig. 1.6:

[a] Now we have to match the voltage and current shown in the first figure with the polarities shown in Fig. 1.6. Remember that 4A of current entering Terminal 2 is the same as 4A of current leaving Terminal 1. We get

(a) $v = -20\,\text{V}, \quad i = -4\,\text{A}$; (b) $v = -20\,\text{V}, \quad i = 4\,\text{A}$

(c) $v = 20\,\text{V}, \quad i = -4\,\text{A}$; (d) $v = 20\,\text{V}, \quad i = 4\,\text{A}$

[b] Using the reference system in Fig. 1.6(a) and the passive sign convention, $p = vi = (-20)(-4) = 80\,\text{W}$. Since the power is greater than 0, the box is absorbing power.

[c] From the calculation in part (b), the box is absorbing 80 W.

AP 1.6 **[a]** Applying the passive sign convention to the power equation using the voltage and current polarities shown in Fig. 1.5, $p = vi$. To find the time at which the power is maximum, find the first derivative of the power with respect to time, set the resulting expression equal to zero, and solve for time:

$$p = (80{,}000te^{-500t})(15te^{-500t}) = 120 \times 10^4 t^2 e^{-1000t}$$

$$\frac{dp}{dt} = 240 \times 10^4 te^{-1000t} - 120 \times 10^7 t^2 e^{-1000t} = 0$$

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Therefore,

$$240 \times 10^4 - 120 \times 10^7 t = 0$$

Solving,

$$t = \frac{240 \times 10^4}{120 \times 10^7} = 2 \times 10^{-3} = 2 \text{ ms}$$

[b] The maximum power occurs at 2 ms, so find the value of the power at 2 ms:

$$p(0.002) = 120 \times 10^4 (0.002)^2 e^{-2} = 649.6 \text{ mW}$$

[c] From Eq. (1.3), we know that power is the time rate of change of energy, or $p = dw/dt$. If we know the power, we can find the energy by integrating Eq. (1.3). To find the total energy, the upper limit of the integral is infinity:

$$\begin{aligned} w_{\text{total}} &= \int_0^\infty 120 \times 10^4 x^2 e^{-1000x}\, dx \\ &= \frac{120 \times 10^4}{(-1000)^3} e^{-1000x}[(-1000)^2 x^2 - 2(-1000)x + 2)\Big|_0^\infty \\ &= 0 - \frac{120 \times 10^4}{(-1000)^3} e^0 (0 - 0 + 2) = 2.4 \text{ mJ} \end{aligned}$$

AP 1.7 At the Oregon end of the line the current is leaving the upper terminal, and thus entering the lower terminal where the polarity marking of the voltage is negative. Thus, using the passive sign convention, $p = -vi$. Substituting the values of voltage and current given in the figure,

$$p = -(800 \times 10^3)(1.8 \times 10^3) = -1440 \times 10^6 = -1440 \text{ MW}$$

Thus, because the power associated with the Oregon end of the line is negative, power is being generated at the Oregon end of the line and transmitted by the line to be delivered to the California end of the line.

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Chapter Problems

P 1.1 $\dfrac{(260 \times 10^6)(540)}{10^9} = 104.4$ gigawatt-hours

P 1.2 $\dfrac{(480)(320)\text{ pixels}}{1\text{ frame}} \cdot \dfrac{2\text{ bytes}}{1\text{ pixel}} \cdot \dfrac{30\text{ frames}}{1\text{ sec}} = 9.216 \times 10^6$ bytes/sec

$(9.216 \times 10^6 \text{ bytes/sec})(x \text{ secs}) = 32 \times 2^{30}$ bytes

$x = \dfrac{32 \times 2^{30}}{9.216 \times 10^6} = 3728 \text{ sec} = 62 \text{ min} \approx 1$ hour of video

P 1.3 **[a]** $\dfrac{20{,}000\text{ photos}}{(11)(15)(1)\text{ mm}^3} = \dfrac{x\text{ photos}}{1\text{ mm}^3}$

$x = \dfrac{(20{,}000)(1)}{(11)(15)(1)} = 121$ photos

[b] $\dfrac{16 \times 2^{30}\text{ bytes}}{(11)(15)(1)\text{ mm}^3} = \dfrac{x\text{ bytes}}{(0.2)^3\text{ mm}^3}$

$x = \dfrac{(16 \times 2^{30})(0.008)}{(11)(15)(1)} = 832{,}963$ bytes

P 1.4 $(4\text{ cond.}) \cdot (845\text{ mi}) \cdot \dfrac{5280\text{ ft}}{1\text{ mi}} \cdot \dfrac{2526\text{ lb}}{1000\text{ ft}} \cdot \dfrac{1\text{ kg}}{2.2\text{ lb}} = 20.5 \times 10^6$ kg

P 1.5 Volume = area × thickness

Convert values to millimeters, noting that $10\text{ m}^2 = 10^6\text{ mm}^2$

$10^6 = (10 \times 10^6)(\text{thickness})$

$\Rightarrow \text{ thickness } = \dfrac{10^6}{10 \times 10^6} = 0.10$ mm

P 1.6 **[a]** We can set up a ratio to determine how long it takes the bamboo to grow $10\,\mu$m First, recall that $1\text{ mm} = 10^3\mu\text{m}$. Let's also express the rate of growth of bamboo using the units mm/s instead of mm/day. Use a product of ratios to perform this conversion:

$$\frac{250\text{ mm}}{1\text{ day}} \cdot \frac{1\text{ day}}{24\text{ hours}} \cdot \frac{1\text{ hour}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}} = \frac{250}{(24)(60)(60)} = \frac{10}{3456}\text{ mm/s}$$

Use a ratio to determine the time it takes for the bamboo to grow $10\,\mu$m:

$$\frac{10/3456 \times 10^{-3}\text{ m}}{1\text{ s}} = \frac{10 \times 10^{-6}\text{ m}}{x\text{ s}} \qquad \text{so} \qquad x = \frac{10 \times 10^{-6}}{10/3456 \times 10^{-3}} = 3.456\text{ s}$$

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[b] $\dfrac{1 \text{ cell length}}{3.456 \text{ s}} \cdot \dfrac{3600 \text{ s}}{1 \text{ hr}} \cdot \dfrac{(24)(7) \text{ hr}}{1 \text{ week}} = 175{,}000 \text{ cell lengths/week}$

P 1.7 **[a]** First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 0.125e^{-2500t}$$

Therefore, $dq = 0.125e^{-2500t}\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 0.125 \int_0^t e^{-2500y}\,dy$$

We solve the integral and make the substitutions for the limits of the integral:

$$q(t) - q(0) = 0.125\frac{e^{-2500y}}{-2500}\bigg|_0^t = 50 \times 10^{-6}(1 - e^{-2500t})$$

But $q(0) = 0$ by hypothesis, so

$$q(t) = 50(1 - e^{-2500t})\,\mu\text{C}$$

[b] As $t \to \infty$, $q_T = 50\,\mu$C.

[c] $q(0.5 \times 10^{-3}) = (50 \times 10^{-6})(1 - e^{(-2500)(0.0005)}) = 35.675\,\mu$C.

P 1.8 First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 20\cos 5000t$$

Therefore, $dq = 20\cos 5000t\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 20\int_0^t \cos 5000y\,dy$$

We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 20\frac{\sin 5000y}{5000}\bigg|_0^t = \frac{20}{5000}\sin 5000t - \frac{20}{5000}\sin 5000(0) = \frac{20}{5000}\sin 5000t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 4 \times 10^{-3}\sin 5000t\,\text{C} = 4\sin 5000t\,\text{mC}$

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P 1.9 **[a]** First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 40te^{-500t}$$

Therefore, $dq = 40te^{-500t}\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 40\int_0^t ye^{-500y}\,dy$$

We solve the integral and make the substitutions for the limits of the integral:

$$q(t) - q(0) = 40\frac{e^{-500y}}{(-500)^2}(-500y - 1)\Big|_0^t = 160 \times 10^{-6}e^{-500t}(-500t - 1) + 160 \times 10^{-6}$$

$$= 160 \times 10^{-6}(1 - 500te^{-500t} - e^{-500t})$$

But $q(0) = 0$ by hypothesis, so

$$q(t) = 160(1 - 500te^{-500t} - e^{-500t})\,\mu\text{C}$$

[b] $q(0.001) = (160)[1 - 500(0.001)e^{-500(0.001)} - e^{-500(0.001)} = 14.4\,\mu\text{C}$.

P 1.10 $n = \dfrac{35 \times 10^{-6}\ \text{C/s}}{1.6022 \times 10^{-19}\ \text{C/elec}} = 2.18 \times 10^{14}\ \text{elec/s}$

P 1.11 $w = qV = (1.6022 \times 10^{-19})(6) = 9.61 \times 10^{-19} = 0.961\ \text{aJ}$

P 1.12 **[a]**

$p = vi = (40)(-10) = -400$ W
Power is being delivered by the box.

[b] Entering

[c] Gaining

P 1.13 **[a]** $p = vi = (-60)(-10) = 600$ W, so power is being absorbed by the box.

[b] Entering

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[c] Losing

P 1.14 Assume we are standing at box A looking toward box B. Use the passive sign convention to get $p = vi$, since the current i is flowing into the + terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

[a] $p = (30)(6) = 180$ W 180 W from A to B

[b] $p = (-20)(-8) = 160$ W 160 W from A to B

[c] $p = (-60)(4) = -240$ W 240 W from B to A

[d] $p = (40)(-9) = -360$ W 360 W from B to A

P 1.15 **[a]** In Car A, the current i is in the direction of the voltage drop across the 12 V battery(the current i flows into the + terminal of the battery of Car A). Therefore using the passive sign convention,
$p = vi = (30)(12) = 360$ W.
Since the power is positive, the battery in Car A is absorbing power, so Car A must have the "dead" battery.

[b] $w(t) = \int_0^t p\,dx;$ 1 min = 60 s

$$w(60) = \int_0^{60} 360\,dx$$

$$w = 360(60 - 0) = 360(60) = 21{,}600 \text{ J} = 21.6 \text{ kJ}$$

P 1.16 $p = vi;$ $w = \int_0^t p\,dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.

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Note that in constructing the plot above, we used the fact that 40 hr = 144,000 s = 144 ks

$p(0) = (1.5)(9 \times 10^{-3}) = 13.5 \times 10^{-3}$ W

$p(144 \text{ ks}) = (1)(9 \times 10^{-3}) = 9 \times 10^{-3}$ W

$$w = (9 \times 10^{-3})(144 \times 10^3) + \frac{1}{2}(13.5 \times 10^{-3} - 9 \times 10^{-3})(144 \times 10^3) = 1620 \text{ J}$$

P 1.17 $p = (12)(100 \times 10^{-3}) = 1.2$ W; $\quad 4 \text{ hr} \cdot \dfrac{3600 \text{ s}}{1 \text{ hr}} = 14{,}400$ s

$$w(t) = \int_0^t p\,dt \qquad w(14{,}400) = \int_0^{14{,}400} 1.2\,dt = 1.2(14{,}400) = 17.28 \text{ kJ}$$

P 1.18 **[a]** $p = vi = (15e^{-250t})(0.04e^{-250t}) = 0.6e^{-500t}$ W

$$p(0.01) = 0.6e^{-500(0.01)} = 0.6e^{-5} = 0.00404 = 4.04 \text{ mW}$$

[b] $$w_{\text{total}} = \int_0^\infty p(x)\,dx = \int_0^\infty 0.6e^{-500x}\,dx = \frac{0.6}{-500}e^{-500x}\Big|_0^\infty$$

$$= -0.0012(e^{-\infty} - e^0) = 0.0012 = 1.2 \text{ mJ}$$

P 1.19 **[a]** $p = vi = (0.05e^{-1000t})(75 - 75e^{-1000t}) = (3.75e^{-1000t} - 3.75e^{-2000t})$ W

$$\frac{dp}{dt} = -3750e^{-1000t} + 7500e^{-2000t} = 0 \qquad \text{so} \qquad 2e^{-2000t} = e^{-1000t}$$

$$2 = e^{1000t} \qquad \text{so} \qquad \ln 2 = 1000t \qquad \text{thus} \qquad p \text{ is maximum at } t = 693.15\,\mu\text{s}$$

$$p_{\max} = p(693.15\,\mu\text{s}) = 937.5 \text{ mW}$$

[b] $$w = \int_0^\infty [3.75e^{-1000t} - 3.75e^{-2000t}]\,dt = \left[\frac{3.75}{-1000}e^{-1000t} - \frac{3.75}{-2000}e^{-2000t}\Big|_0^\infty\right]$$

$$= \frac{3.75}{1000} - \frac{3.75}{2000} = 1.875 \text{ mJ}$$

P 1.20 **[a]** $p = vi = 0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t}$

$p(625\,\mu\text{s}) = 42.2$ mW

[b]
$$\begin{aligned} w(t) &= \int_0^t (0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t}) \\ &= 140.625 - 78.125e^{-3200t} + 250e^{-2000t} - 312.5e^{-800t}\,\mu\text{J} \\ w(625\,\mu\text{s}) &= 12.14\,\mu\text{J} \end{aligned}$$

[c] $w_{\text{total}} = 140.625\,\mu\text{J}$

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P 1.21 **[a]**

$$\begin{aligned} p &= vi \\ &= [(1500t+1)e^{-750t}](0.04e^{-750t}) \\ &= (60t+0.04)e^{-1500t} \\ \frac{dp}{dt} &= 60e^{-1500t} - 1500e^{-1500t}(60t+0.04) \\ &= -90{,}000te^{-1500t} \end{aligned}$$

Therefore, $\dfrac{dp}{dt} = 0$ when $t = 0$
so $p_{\max}$ occurs at $t = 0$.

[b]

$$\begin{aligned} p_{\max} &= [(60)(0)+0.04]e^0 = 0.04 \\ &= 40 \text{ mW} \end{aligned}$$

[c]

$$\begin{aligned} w &= \int_0^t p\,dx \\ w &= \int_0^t 60xe^{-1500x}\,dx + \int_0^t 0.04e^{-1500x}\,dx \\ &= \frac{60e^{-1500x}}{(-1500)^2}(-1500x-1)\bigg|_0^t + 0.04\frac{e^{-1500x}}{-1500}\bigg|_0^t \end{aligned}$$

When $t = \infty$ all the upper limits evaluate to zero, hence
$w = \dfrac{60}{225\times 10^4} + \dfrac{0.04}{1500} = 53.33\,\mu\text{J}$.

P 1.22 **[a]**

$$\begin{aligned} p &= vi \\ &= [(3200t+3.2)e^{-1000t}][(160t+0.16)e^{-1000t}] \\ &= e^{-2000t}[512{,}000t^2+1024t+0.512] \\ \frac{dp}{dt} &= e^{-2000t}[1{,}024{,}000t+1024] - 2000e^{-2000t}[512{,}000t^2+1024t+0.512] \\ &= -e^{-2000t}[1024\times 10^6t^2+1{,}024{,}000t] \end{aligned}$$

Therefore, $\dfrac{dp}{dt} = 0$ when $t = 0$
so $p_{\max}$ occurs at $t = 0$.

[b]

$$\begin{aligned} p_{\max} &= e^{-0}[0+0+0.512] \\ &= 512 \text{ mW} \end{aligned}$$

[c]

$$\begin{aligned} w &= \int_0^t p\,dx \\ w &= \int_0^t 512{,}000x^2e^{-2000x}\,dx + \int_0^t 1024xe^{-2000x}\,dx + \int_0^t 0.512e^{-2000x}\,dx \\ &= \frac{512{,}000e^{-2000x}}{-8\times 10^9}[4\times 10^6x^2+4000x+2]\bigg|_0^t + \\ &\quad \frac{1024e^{-2000x}}{4\times 10^6}(-2000x-1)\bigg|_0^t + \frac{0.512e^{-2000x}}{-2000}\bigg|_0^t \end{aligned}$$

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When $t \to \infty$ all the upper limits evaluate to zero, hence

$$w = \frac{(512{,}000)(2)}{8 \times 10^9} + \frac{1024}{4 \times 10^6} + \frac{0.512}{2000}$$

$$w = 128 \times 10^{-6} + 256 \times 10^{-6} + 256 \times 10^{-6} = 640\,\mu\text{J}.$$

P 1.23 **[a]** We can find the time at which the power is a maximum by writing an expression for $p(t) = v(t)i(t)$, taking the first derivative of $p(t)$ and setting it to zero, then solving for t. The calculations are shown below:

$$\begin{aligned}
p &= 0 \quad t < 0, \qquad p = 0 \quad t > 40 \text{ s} \\
p &= vi = t(1 - 0.025t)(4 - 0.2t) = 4t - 0.3t^2 + 0.005t^3 \text{ W} \qquad 0 \le t \le 40 \text{ s} \\
\frac{dp}{dt} &= 4 - 0.6t + 0.015t^2 = 0.015(t^2 - 40t + 266.67) \\
\frac{dp}{dt} &= 0 \qquad \text{when } t^2 - 40t + 266.67 = 0 \\
t_1 &= 8.453 \text{ s}; \qquad t_2 = 31.547 \text{ s} \\
& \quad \text{(using the polynomial solver on your calculator)} \\
p(t_1) &= 4(8.453) - 0.3(8.453)^2 + 0.005(8.453)^3 = 15.396 \text{ W} \\
p(t_2) &= 4(31.547) - 0.3(31.547)^2 + 0.005(31.547)^3 = -15.396 \text{ W}
\end{aligned}$$

Therefore, maximum power is being delivered at $t = 8.453$ s.

[b] The maximum power was calculated in part (a) to determine the time at which the power is maximum: $p_{\max} = 15.396$ W (delivered)

[c] As we saw in part (a), the other "maximum" power is actually a minimum, or the maximum negative power. As we calculated in part (a), maximum power is being extracted at $t = 31.547$ s.

[d] This maximum extracted power was calculated in part (a) to determine the time at which power is maximum: $p_{\max} = 15.396$ W (extracted)

[e] $w = \int_0^t p\,dx = \int_0^t (4x - 0.3x^2 + 0.005x^3)dx = 2t^2 - 0.1t^3 + 0.00125t^4$

$$\begin{aligned}
w(0) &= 0 \text{ J} & w(30) &= 112.5 \text{ J} \\
w(10) &= 112.5 \text{ J} & w(40) &= 0 \text{ J} \\
w(20) &= 200 \text{ J}
\end{aligned}$$

To give you a feel for the quantities of voltage, current, power, and energy and their relationships among one another, they are plotted below:

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P 1.24 **[a]** $v(10\text{ ms}) = 400e^{-1}\sin 2 = 133.8\text{ V}$
$i(10\text{ ms}) = 5e^{-1}\sin 2 = 1.67\text{ A}$
$p(10\text{ ms}) = vi = 223.80\text{ W}$

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[b]
$$\begin{aligned}
p &= vi = 2000e^{-200t}\sin^2 200t \\
&= 2000e^{-200t}\left[\frac{1}{2} - \frac{1}{2}\cos 400t\right] \\
&= 1000e^{-200t} - 1000e^{-200t}\cos 400t \\
w &= \int_0^\infty 1000e^{-200t}\,dt - \int_0^\infty 1000e^{-200t}\cos 400t\,dt \\
&= 1000\left.\frac{e^{-200t}}{-200}\right|_0^\infty \\
&\quad -1000\left.\left\{\frac{e^{-200t}}{(200)^2+(400)^2}\left[-200\cos 400t + 400\sin 400t\right]\right\}\right|_0^\infty \\
&= 5 - 1000\left[\frac{200}{4\times 10^4 + 16\times 10^4}\right] = 5 - 1 \\
w &= 4 \text{ J}
\end{aligned}$$

P 1.25 **[a]** $p = vi = 2000\cos(800\pi t)\sin(800\pi t) = 1000\sin(1600\pi t)$ W

Therefore, $p_{\max} = 1000$ W

[b] $p_{\max}(\text{extracting}) = 1000$ W

[c]
$$\begin{aligned}
p_{\text{avg}} &= \frac{1}{2.5\times 10^{-3}}\int_0^{2.5\times 10^{-3}} 1000\sin(1600\pi t)\,dt \\
&= 4\times 10^5\left[\frac{-\cos 1600\pi t}{1600\pi}\right]_0^{2.5\times 10^{-3}} = \frac{250}{\pi}[1-\cos 4\pi] = 0
\end{aligned}$$

[d]
$$\begin{aligned}
p_{\text{avg}} &= \frac{1}{15.625\times 10^{-3}}\int_0^{15.625\times 10^{-3}} 1000\sin(1600\pi t)\,dt \\
&= 64\times 10^3\left[\frac{-\cos 1600\pi t}{1600\pi}\right]_0^{15.625\times 10^{-3}} = \frac{40}{\pi}[1-\cos 25\pi] = 25.46 \text{ W}
\end{aligned}$$

P 1.26 **[a]**
$$\begin{aligned}
q &= \text{area under } i \text{ vs. } t \text{ plot} \\
&= \tfrac{1}{2}(8)(12{,}000) + (16)(12{,}000) + \tfrac{1}{2}(16)(4000) \\
&= 48{,}000 + 192{,}000 + 32{,}000 = 272{,}000 \text{ C}
\end{aligned}$$

[b]
$$\begin{aligned}
w &= \int p\,dt = \int vi\,dt \\
v &= 250\times 10^{-6}t + 8 \qquad 0 \le t \le 16 \text{ ks}
\end{aligned}$$

$0 \le t \le 12{,}000s$:
$$\begin{aligned}
i &= 24 - 666.67\times 10^{-6}t \\
p &= 192 + 666.67\times 10^{-6}t - 166.67\times 10^{-9}t^2 \\
w_1 &= \int_0^{12{,}000}(192 + 666.67\times 10^{-6}t - 166.67\times 10^{-9}t^2)\,dt \\
&= (2304 + 48 - 96)10^3 = 2256 \text{ kJ}
\end{aligned}$$

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12,000 s $\leq t \leq$ 16,000 s:

$$\begin{aligned} i &= 64 - 4 \times 10^{-3}t \\ p &= 512 - 16 \times 10^{-3}t - 10^{-6}t^2 \\ w_2 &= \int_{12,000}^{16,000} (512 - 16 \times 10^{-3}t - 10^{-6}t^2)\, dt \\ &= (2048 - 896 - 789.33)10^3 = 362.667 \text{ kJ} \\ w_T &= w_1 + w_2 = 2256 + 362.667 = 2618.667 \text{ kJ} \end{aligned}$$

P 1.27 **[a]** 0 s $\leq t <$ 10 ms:

$v = 8$ V; $i = 25t$ A; $p = 200t$ W

10 ms $< t \leq$ 30 ms:

$v = -8$ V; $i = 0.5 - 25t$ A; $p = 200t - 4$ W

30 ms $\leq t <$ 40 ms:

$v = 0$ V; $i = -250$ mA; $p = 0$ W

40 ms $< t \leq$ 60 ms:

$v = 8$ V; $i = 25t - 1.25$ A; $p = 200t - 10$ W

$t >$ 60 ms:

$v = 0$ V; $i = 250$ mA; $p = 0$ W

[b] Calculate the area under the curve from zero up to the desired time:

$$\begin{aligned} w(0.01) &= \tfrac{1}{2}(2)(0.01) = 10 \text{ mJ} \\ w(0.03) &= w(0.01) - \tfrac{1}{2}(2)(0.01) + \tfrac{1}{2}(2)(0.01) = 10 \text{ mJ} \\ w(0.08) &= w(0.03) - \tfrac{1}{2}(2)(0.01) + \tfrac{1}{2}(2)(0.01) = 10 \text{ mJ} \end{aligned}$$

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P 1.28 **[a]**

[b]

$$
\begin{aligned}
i(t) &= 10 + 0.5 \times 10^{-3} t \text{ mA}, & 0 \le t \le 10 \text{ ks} \\
i(t) &= 15 \text{ mA}, & 10 \text{ ks} \le t \le 20 \text{ ks} \\
i(t) &= 25 - 0.5 \times 10^{-3} t \text{ mA}, & 20 \text{ ks} \le t \le 30 \text{ ks} \\
i(t) &= 0, & t > 30 \text{ ks}
\end{aligned}
$$

$p = vi = 120i$ so

$$
\begin{aligned}
p(t) &= 1200 + 0.06t \text{ mW}, & 0 \le t \le 10 \text{ ks} \\
p(t) &= 1800 \text{ mW}, & 10 \text{ ks} \le t \le 20 \text{ ks} \\
p(t) &= 3000 - 0.06t \text{ mW}, & 20 \text{ ks} \le t \le 30 \text{ ks} \\
p(t) &= 0, & t > 30 \text{ ks}
\end{aligned}
$$

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[c] To find the energy, calculate the area under the plot of the power:

$$w(10\text{ ks}) = \frac{1}{2}(0.6)(10{,}000) + (1.2)(10{,}000) = 15\text{ kJ}$$

$$w(20\text{ ks}) = w(10\text{ ks}) + (1.8)(10{,}000) = 33\text{ kJ}$$

$$w(10\text{ ks}) = w(20\text{ ks}) + \frac{1}{2}(0.6)(10{,}000) + (1.2)(10{,}000) = 48\text{ kJ}$$

P 1.29 We use the passive sign convention to determine whether the power equation is $p = vi$ or $p = -vi$ and substitute into the power equation the values for v and i, as shown below:

$$\begin{aligned}
p_\text{a} &= -v_\text{a}i_\text{a} = -(40)(-4\times10^{-3}) = 160\text{ mW}\\
p_\text{b} &= v_\text{b}i_\text{b} = (-24)(-4\times10^{-3}) = 96\text{ mW}\\
p_\text{c} &= -v_\text{c}i_\text{c} = -(-16)(4\times10^{-3}) = 64\text{ mW}\\
p_\text{d} &= -v_\text{d}i_\text{d} = -(-80)(-1.5\times10^{-3}) = -120\text{ mW}\\
p_\text{e} &= v_\text{e}i_\text{e} = (40)(2.5\times10^{-3}) = 100\text{ mW}\\
p_\text{f} &= v_\text{f}i_\text{f} = (120)(-2.5\times10^{-3}) = -300\text{ mW}
\end{aligned}$$

Remember that if the power is positive, the circuit element is absorbing power, whereas is the power is negative, the circuit element is developing power. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal:
$\sum P_\text{dev} = 120 + 300 = 420$ mW;
$\sum P_\text{abs} = 160 + 96 + 64 + 100 = 420$ mW
Thus, the power balances and the total power absorbed in the circuit is 420 mW.

P 1.30

$$\begin{aligned}
p_\text{a} &= -v_\text{a}i_\text{a} = -(-3000)(-250\times10^{-6}) = -0.75\text{ W}\\
p_\text{b} &= -v_\text{b}i_\text{b} = -(4000)(-400\times10^{-6}) = 1.6\text{ W}\\
p_\text{c} &= -v_\text{c}i_\text{c} = -(1000)(400\times10^{-6}) = -0.4\text{ W}\\
p_\text{d} &= v_\text{d}i_\text{d} = (1000)(150\times10^{-6}) = 0.15\text{ W}\\
p_\text{e} &= v_\text{e}i_\text{e} = (-4000)(200\times10^{-6}) = -0.8\text{ W}\\
p_\text{f} &= v_\text{f}i_\text{f} = (4000)(50\times10^{-6}) = 0.2\text{ W}
\end{aligned}$$

Therefore,

$$\sum P_\text{abs} = 1.6 + 0.15 + 0.2 = 1.95\text{ W}$$

$$\sum P_\text{del} = 0.75 + 0.4 + 0.8 = 1.95\text{ W} = \sum P_\text{abs}$$

Thus, the interconnection does satisfy the power check.

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P 1.31 **[a]** From the diagram and the table we have

$$\begin{aligned}
p_a &= -v_a i_a = -(46.16)(6) = -276.96 \text{ W} \\
p_b &= v_b i_b = (14.16)(4.72) = 66.8352 \text{ W} \\
p_c &= v_c i_c = (-32)(-6.4) = 204.8 \text{ W} \\
p_d &= -v_d i_d = -(22)(1.28) = -28.16 \text{ W} \\
p_e &= v_e i_e = (-33.6)(-1.68) = 56.448 \text{ W} \\
p_f &= v_f i_f = (66)(0.4) = 26.4 \text{ W} \\
p_g &= v_g i_g = (2.56)(1.28) = 3.2768 \text{ W} \\
p_h &= -v_h i_h = -(-0.4)(0.4) = 0.16 \text{ W} \\
\sum P_{del} &= 276.96 + 28.16 = 305.12 \text{ W} \\
\sum P_{abs} &= 66.8352 + 204.8 + 56.448 + 26.4 + 3.2768 + 0.16 = 357.92 \text{ W}
\end{aligned}$$

Therefore, $\sum P_{del} \neq \sum P_{abs}$ and the subordinate engineer is correct.

[b] The difference between the power delivered to the circuit and the power absorbed by the circuit is

$$-305.12 + 357.92 = 52.8 \text{ W}$$

One-half of this difference is 26.4 W, so it is likely that p_f is in error. Either the voltage or the current probably has the wrong sign. (In Chapter 2, we will discover that using KCL at the node connecting components f and h, the current i_f should be -0.4 A, not 0.4 A!) If the sign of p_f is changed from negative to positive, we can recalculate the power delivered and the power absorbed as follows:

$$\begin{aligned}
\sum P_{del} &= 276.96 + 28.16 + 26.4 = 331.52 \text{ W} \\
\sum P_{abs} &= 66.8352 + 204.8 + 56.448 + 3.2768 + 0.16 = 331.52 \text{ W}
\end{aligned}$$

Now the power delivered equals the power absorbed and the power balances for the circuit.

P 1.32 **[a]** Remember that if the circuit element is absorbing power, the power is positive, whereas if the circuit element is supplying power, the power is negative. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal: $\sum P_{sup} = 600 + 50 + 600 + 1250 = 2500$ W;
$\sum P_{abs} = 400 + 100 + 2000 = 2500$ W
Thus, the power balances.

[b] The current can be calculated using $i = p/v$ or $i = -p/v$, with proper application of the passive sign convention:

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$$
\begin{aligned}
i_a &= -p_a/v_a = -(-600)/(400) = 1.5 \text{ A} \\
i_b &= p_b/v_b = (-50)/(-100) = 0.5 \text{ A} \\
i_c &= p_c/v_c = (400)/(200) = 2.0 \text{ A} \\
i_d &= p_d/v_d = (-600)/(300) = -2.0 \text{ A} \\
i_e &= p_e/v_e = (100)/(-200) = -0.5 \text{ A} \\
i_f &= -p_f/v_f = -(2000)/(500) = -4.0 \text{ A} \\
i_g &= p_g/v_g = (-1250)/(-500) = 2.5 \text{ A}
\end{aligned}
$$

P 1.33 **[a]** If the power balances, the sum of the power values should be zero:

$$p_{\text{total}} = 0.175 + 0.375 + 0.150 - 0.320 + 0.160 + 0.120 - 0.660 = 0$$

Thus, the power balances.

[b] When the power is positive, the element is absorbing power. Since elements a, b, c, e, and f have positive power, these elements are absorbing power.

[c] The voltage can be calculated using $v = p/i$ or $v = -p/i$, with proper application of the passive sign convention:

$$
\begin{aligned}
v_a &= p_a/i_a = (0.175)/(0.025) = 7 \text{ V} \\
v_b &= p_b/i_b = (0.375)/(0.075) = 5 \text{ V} \\
v_c &= -p_c/i_c = -(0.150)/(-0.05) = 3 \text{ V} \\
v_d &= p_d/i_d = (-0.320)/(0.04) = -8 \text{ V} \\
v_e &= -p_e/i_e = -(0.160)/(0.02) = -8 \text{ V} \\
v_f &= p_f/i_f = (0.120)/(-0.03) = -4 \text{ V} \\
v_g &= p_g/i_g = (-0.66)/(0.055) = -12 \text{ V}
\end{aligned}
$$

P 1.34

$$
\begin{aligned}
p_a &= v_a i_a = (120)(-10) = -1200 \text{ W} \\
p_b &= -v_b i_b = -(120)(9) = -1080 \text{ W} \\
p_c &= v_c i_c = (10)(10) = 100 \text{ W} \\
p_d &= -v_d i_d = -(10)(-1) = 10 \text{ W} \\
p_e &= v_e i_e = (-10)(-9) = 90 \text{ W} \\
p_f &= -v_f i_f = -(-100)(5) = 500 \text{ W} \\
p_g &= v_g i_g = (120)(4) = 480 \text{ W} \\
p_h &= v_h i_h = (-220)(-5) = 1100 \text{ W}
\end{aligned}
$$

$$\sum P_{\text{del}} = 1200 + 1080 = 2280 \text{ W}$$

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$\sum P_{\text{abs}} = 100 + 10 + 90 + 500 + 480 + 1100 = 2280 \text{ W}$
Therefore, $\sum P_{\text{del}} = \sum P_{\text{abs}} = 2280 \text{ W}$

Thus, the interconnection now satisfies the power check.

P 1.35 **[a]** The revised circuit model is shown below:

[b] The expression for the total power in this circuit is

$$v_a i_a - v_b i_b - v_f i_f + v_g i_g + v_h i_h$$

$$= (120)(-10) - (120)(10) - (-120)(3) + 120 i_g + (-240)(-7) = 0$$

Therefore,

$$120 i_g = 1200 + 1200 - 360 - 1680 = 360$$

so

$$i_g = \frac{360}{120} = 3 \text{ A}$$

Thus, if the power in the modified circuit is balanced the current in component g is 3 A.

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2

Circuit Elements

Assessment Problems

AP 2.1

[a] Note that the current i_b is in the same circuit branch as the 8 A current source; however, i_b is defined in the opposite direction of the current source. Therefore,

$$i_b = -8\,\text{A}$$

Next, note that the dependent voltage source and the independent voltage source are in parallel with the same polarity. Therefore, their voltages are equal, and

$$v_g = \frac{i_b}{4} = \frac{-8}{4} = -2\,\text{V}$$

[b] To find the power associated with the 8 A source, we need to find the voltage drop across the source, v_i. Note that the two independent sources are in parallel, and that the voltages v_g and v_1 have the same polarities, so these voltages are equal:

$$v_i = v_g = -2\,\text{V}$$

Using the passive sign convention,

$$p_s = (8\,\text{A})(v_i) = (8\,\text{A})(-2\,\text{V}) = -16\,\text{W}$$

Thus the current source generated 16 W of power.

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AP 2.2

[a] Note from the circuit that $v_x = -25$ V. To find α note that the two current sources are in the same branch of the circuit but their currents flow in opposite directions. Therefore

$$\alpha v_x = -15\,\text{A}$$

Solve the above equation for α and substitute for v_x,

$$\alpha = \frac{-15\,\text{A}}{v_x} = \frac{-15\,\text{A}}{-25\,\text{V}} = 0.6\ \text{A/V}$$

[b] To find the power associated with the voltage source we need to know the current, i_v. Note that this current is in the same branch of the circuit as the dependent current source and these two currents flow in the same direction. Therefore, the current i_v is the same as the current of the dependent source:

$$i_v = \alpha v_x = (0.6)(-25) = -15\,\text{A}$$

Using the passive sign convention,

$$p_s = -(i_v)(25\,\text{V}) = -(-15\,\text{A})(25\,\text{V}) = 375\,\text{W}.$$

Thus the voltage source dissipates 375 W.

AP 2.3

[a] The resistor and the voltage source are in parallel and the resistor voltage and the voltage source have the same polarities. Therefore these two voltages are the same:

$$v_R = v_g = 1\,\text{kV}$$

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Note from the circuit that the current through the resistor is $i_g = 5$ mA. Use Ohm's law to calculate the value of the resistor:

$$R = \frac{v_R}{i_g} = \frac{1\,\text{kV}}{5\,\text{mA}} = 200\,\text{k}\Omega$$

Using the passive sign convention to calculate the power in the resistor,

$$p_R = (v_R)(i_g) = (1\,\text{kV})(5\,\text{mA}) = 5\,\text{W}$$

The resistor is dissipating 5 W of power.

[b] Note from part (a) the $v_R = v_g$ and $i_R = i_g$. The power delivered by the source is thus

$$p_{\text{source}} = -v_g i_g \qquad \text{so} \qquad v_g = -\frac{p_{\text{source}}}{i_g} = -\frac{-3\,\text{W}}{75\,\text{mA}} = 40\,\text{V}$$

Since we now have the value of both the voltage and the current for the resistor, we can use Ohm's law to calculate the resistor value:

$$R = \frac{v_g}{i_g} = \frac{40\,\text{V}}{75\,\text{mA}} = 533.33\,\Omega$$

The power absorbed by the resistor must equal the power generated by the source. Thus,

$$p_R = -p_{\text{source}} = -(-3\,\text{W}) = 3\,\text{W}$$

[c] Again, note the $i_R = i_g$. The power dissipated by the resistor can be determined from the resistor's current:

$$p_R = R(i_R)^2 = R(i_g)^2$$

Solving for i_g,

$$i_g^2 = \frac{p_r}{R} = \frac{480\,\text{mW}}{300\,\Omega} = 0.0016 \qquad \text{so} \qquad i_g = \sqrt{0.0016} = 0.04\,\text{A} = 40\,\text{mA}$$

Then, since $v_R = v_g$

$$v_R = Ri_R = Ri_g = (300\,\Omega)(40\,\text{mA}) = 12\,\text{V} \qquad \text{so} \qquad v_g = 12\,\text{V}$$

AP 2.4

[a] Note from the circuit that the current through the conductance G is i_g, flowing from top to bottom, because the current source and the

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conductance are in the same branch of the circuit so must have the same current. The voltage drop across the current source is v_g, positive at the top, because the current source and the conductance are also in parallel so must have the same voltage. From a version of Ohm's law,

$$v_g = \frac{i_g}{G} = \frac{0.5\,\text{A}}{50\,\text{mS}} = 10\,\text{V}$$

Now that we know the voltage drop across the current source, we can find the power delivered by this source:

$$p_{\text{source}} = -v_g i_g = -(10)(0.5) = -5\,\text{W}$$

Thus the current source delivers 5 W to the circuit.

[b] We can find the value of the conductance using the power, and the value of the current using Ohm's law and the conductance value:

$$p_g = Gv_g^2 \qquad \text{so} \qquad G = \frac{p_g}{v_g^2} = \frac{9}{15^2} = 0.04\,\text{S} = 40\,\text{mS}$$

$$i_g = Gv_g = (40\,\text{mS})(15\,\text{V}) = 0.6\,\text{A}$$

[c] We can find the voltage from the power and the conductance, and then use the voltage value in Ohm's law to find the current:

$$p_g = Gv_g^2 \qquad \text{so} \qquad v_g^2 = \frac{p_g}{G} = \frac{8\,\text{W}}{200\,\mu\text{S}} = 40{,}000$$

Thus $\qquad v_g = \sqrt{40{,}000} = 200\,\text{V}$

$$i_g = Gv_g = (200\,\mu\text{S})(200\,\text{V}) = 0.04\,\text{A} = 40\,\text{mA}$$

AP 2.5 **[a]** Redraw the circuit with all of the voltages and currents labeled for every circuit element.

Write a KVL equation clockwise around the circuit, starting below the voltage source:

$$-24\,\text{V} + v_2 + v_5 - v_1 = 0$$

Next, use Ohm's law to calculate the three unknown voltages from the three currents:

$$v_2 = 3i_2; \qquad v_5 = 7i_5; \qquad v_1 = 2i_1$$

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A KCL equation at the upper right node gives $i_2 = i_5$; a KCL equation at the bottom right node gives $i_5 = -i_1$; a KCL equation at the upper left node gives $i_s = -i_2$. Now replace the currents i_1 and i_2 in the Ohm's law equations with i_5:

$$v_2 = 3i_2 = 3i_5; \qquad v_5 = 7i_5; \qquad v_1 = 2i_1 = -2i_5$$

Now substitute these expressions for the three voltages into the first equation:

$$24 = v_2 + v_5 - v_1 = 3i_5 + 7i_5 - (-2i_5) = 12i_5$$

Therefore $i_5 = 24/12 = 2$ A

[b] $v_1 = -2i_5 = -2(2) = -4\,\text{V}$

[c] $v_2 = 3i_5 = 3(2) = 6\,\text{V}$

[d] $v_5 = 7i_5 = 7(2) = 14\,\text{V}$

[e] A KCL equation at the lower left node gives $i_s = i_1$. Since $i_1 = -i_5$, $i_s = -2\,\text{A}$. We can now compute the power associated with the voltage source:

$$p_{24} = (24)i_s = (24)(-2) = -48\,\text{W}$$

Therefore 24 V source is delivering 48 W.

AP 2.6 Redraw the circuit labeling all voltages and currents:

We can find the value of the unknown resistor if we can find the value of its voltage and its current. To start, write a KVL equation clockwise around the right loop, starting below the $24\,\Omega$ resistor:

$$-120\,\text{V} + v_3 = 0$$

Use Ohm's law to calculate the voltage across the $8\,\Omega$ resistor in terms of its current:

$$v_3 = 8i_3$$

Substitute the expression for v_3 into the first equation:

$$-120\,\text{V} + 8i_3 = 0 \qquad \text{so} \qquad i_3 = \frac{120}{8} = 15\,\text{A}$$

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Also use Ohm's law to calculate the value of the current through the 24 Ω resistor:

$$i_2 = \frac{120\,\text{V}}{24\,\Omega} = 5\,\text{A}$$

Now write a KCL equation at the top middle node, summing the currents leaving:

$$-i_1 + i_2 + i_3 = 0 \qquad \text{so} \qquad i_1 = i_2 + i_3 = 5 + 15 = 20\,\text{A}$$

Write a KVL equation clockwise around the left loop, starting below the voltage source:

$$-200\,\text{V} + v_1 + 120\,\text{V} = 0 \qquad \text{so} \qquad v_1 = 200 - 120 = 80\,\text{V}$$

Now that we know the values of both the voltage and the current for the unknown resistor, we can use Ohm's law to calculate the resistance:

$$\text{R} = \frac{v_1}{i_1} = \frac{80}{20} = 4\,\Omega$$

AP 2.7 **[a]** Plotting a graph of v_t versus i_t gives

Note that when $i_t = 0$, $v_t = 25$ V; therefore the voltage source must be 25 V. Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25 - 0}{0.25 - 0} = \frac{25}{0.25} = 100\,\Omega$$

A circuit model having the same $v - i$ characteristic is a 25 V source in series with a 100Ω resistor, as shown below:

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[b] Draw the circuit model from part (a) and attach a 25 Ω resistor:

To find the power delivered to the 25 Ω resistor we must calculate the current through the 25 Ω resistor. Do this by first using KCL to recognize that the current in each of the components is i_t, flowing in a clockwise direction. Write a KVL equation in the clockwise direction, starting below the voltage source, and using Ohm's law to express the voltage drop across the resistors in the direction of the current i_t flowing through the resistors:

$$-25\,\text{V} + 100i_t + 25i_t = 0 \qquad \text{so} \qquad 125i_t = 25 \qquad \text{so} \qquad i_t = \frac{25}{125} = 0.2\,\text{A}$$

Thus, the power delivered to the 25 Ω resistor is

$$p_{25} = (25)i_t^2 = (25)(0.2)^2 = 1\,\text{W}.$$

AP 2.8 **[a]** From the graph in Assessment Problem 2.7(a), we see that when $v_t = 0$, $i_t = 0.25\,\text{A}$. Therefore the current source must be 0.25 A. Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25 - 0}{0.25 - 0} = \frac{25}{0.25} = 100\,\Omega$$

A circuit model having the same $v - i$ characteristic is a 0.25 A current source in parallel with a 100Ω resistor, as shown below:

[b] Draw the circuit model from part (a) and attach a 25 Ω resistor:

Note that by writing a KVL equation around the right loop we see that the voltage drop across both resistors is v_t. Write a KCL equation at the top center node, summing the currents leaving the node. Use Ohm's law

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to specify the currents through the resistors in terms of the voltage drop across the resistors and the value of the resistors.

$$-0.25 + \frac{v_t}{100} + \frac{v_t}{25} = 0, \qquad \text{so} \qquad 5v_t = 25, \qquad \text{thus} \qquad v_t = 5\,\text{V}$$

$$p_{25} = \frac{v_t^2}{25} = 1\,\text{W}.$$

AP 2.9 First note that we know the current through all elements in the circuit except the 6 kΩ resistor (the current in the three elements to the left of the 6 kΩ resistor is i_1; the current in the three elements to the right of the 6 kΩ resistor is $30i_1$). To find the current in the 6 kΩ resistor, write a KCL equation at the top node:

$$i_1 + 30i_1 = i_{6\text{k}} = 31i_1$$

We can then use Ohm's law to find the voltages across each resistor in terms of i_1. The results are shown in the figure below:

[a] To find i_1, write a KVL equation around the left-hand loop, summing voltages in a clockwise direction starting below the 5V source:

$$-5\,\text{V} + 54{,}000i_1 - 1\,\text{V} + 186{,}000i_1 = 0$$

Solving for i_1

$$54{,}000i_1 + 186{,}000i_1 = 6\,\text{V} \qquad \text{so} \qquad 240{,}000i_1 = 6\,\text{V}$$

Thus,

$$i_1 = \frac{6}{240{,}000} = 25\,\mu\text{A}$$

[b] Now that we have the value of i_1, we can calculate the voltage for each component except the dependent source. Then we can write a KVL equation for the right-hand loop to find the voltage v of the dependent source. Sum the voltages in the clockwise direction, starting to the left of the dependent source:

$$+v - 54{,}000i_1 + 8\,\text{V} - 186{,}000i_1 = 0$$

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Thus,

$v = 240{,}000 i_1 - 8\,\text{V} = 240{,}000(25 \times 10^{-6}) - 8\,\text{V} = 6\,\text{V} - 8\,\text{V} = -2\,\text{V}$

We now know the values of voltage and current for every circuit element. Let's construct a power table:

Element	**Current (μA)**	**Voltage (V)**	**Power Equation**	**Power (μW)**
5 V	25	5	$p = -vi$	−125
54 kΩ	25	1.35	$p = Ri^2$	33.75
1 V	25	1	$p = -vi$	−25
6 kΩ	775	4.65	$p = Ri^2$	3603.75
Dep. source	750	−2	$p = -vi$	1500
1.8 kΩ	750	1.35	$p = Ri^2$	1012.5
8 V	750	8	$p = -vi$	−6000

[c] The total power generated in the circuit is the sum of the negative power values in the power table:

$-125\,\mu\text{W} + -25\,\mu\text{W} + -6000\,\mu\text{W} = -6150\,\mu\text{W}$

Thus, the total power generated in the circuit is $6150\,\mu$W.

[d] The total power absorbed in the circuit is the sum of the positive power values in the power table:

$33.75\,\mu\text{W} + 3603.75\,\mu\text{W} + 1500\,\mu\text{W} + 1012.5\,\mu\text{W} = 6150\,\mu\text{W}$

Thus, the total power absorbed in the circuit is $6150\,\mu$W.

AP 2.10 Given that $i_\phi = 2\,\text{A}$, we know the current in the dependent source is $2i_\phi = 4\,\text{A}$. We can write a KCL equation at the left node to find the current in the $10\,\Omega$ resistor. Summing the currents leaving the node,

$$-5\,\text{A} + 2\,\text{A} + 4\,\text{A} + i_{10\Omega} = 0 \qquad \text{so} \qquad i_{10\Omega} = 5\,\text{A} - 2\,\text{A} - 4\,\text{A} = -1\,\text{A}$$

Thus, the current in the $10\,\Omega$ resistor is 1 A, flowing right to left, as seen in the circuit below.

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[a] To find v_s, write a KVL equation, summing the voltages counter-clockwise around the lower right loop. Start below the voltage source.

$$-v_s + (1\,\text{A})(10\,\Omega) + (2\,\text{A})(30\,\Omega) = 0 \qquad \text{so} \qquad v_s = 10\,\text{V} + 60\,\text{V} = 70\,\text{V}$$

[b] The current in the voltage source can be found by writing a KCL equation at the right-hand node. Sum the currents leaving the node

$$-4\,\text{A} + 1\,\text{A} + i_v = 0 \qquad \text{so} \qquad i_v = 4\,\text{A} - 1\,\text{A} = 3\,\text{A}$$

The current in the voltage source is 3 A, flowing top to bottom. The power associated with this source is

$$p = vi = (70\,\text{V})(3\,\text{A}) = 210\,\text{W}$$

Thus, 210 W are absorbed by the voltage source.

[c] The voltage drop across the independent current source can be found by writing a KVL equation around the left loop in a clockwise direction:

$$-v_{5A} + (2\,\text{A})(30\,\Omega) = 0 \qquad \text{so} \qquad v_{5A} = 60\,\text{V}$$

The power associated with this source is

$$p = -v_{5A}i = -(60\,\text{V})(5\,\text{A}) = -300\,\text{W}$$

This source thus delivers 300 W of power to the circuit.

[d] The voltage across the controlled current source can be found by writing a KVL equation around the upper right loop in a clockwise direction:

$$+v_{4A} + (10\,\Omega)(1\,\text{A}) = 0 \qquad \text{so} \qquad v_{4A} = -10\,\text{V}$$

The power associated with this source is

$$p = v_{4A}i = (-10\,\text{V})(4\,\text{A}) = -40\,\text{W}$$

This source thus delivers 40 W of power to the circuit.

[e] The total power dissipated by the resistors is given by

$$(i_{30\Omega})^2(30\,\Omega) + (i_{10\Omega})^2(10\,\Omega) = (2)^2(30\,\Omega) + (1)^2(10\,\Omega) = 120 + 10 = 130\,\text{W}$$

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Problems

P 2.1 **[a]** Yes, independent voltage sources can carry the 5 A current required by the connection; independent current source can support any voltage required by the connection, in this case 5 V, positive at the bottom.

[b] 20 V source: absorbing

15 V source: developing (delivering)

5 A source: developing (delivering)

[c]
$$P_{20\text{V}} = (20)(5) = 100 \text{ W} \quad \text{(abs)}$$
$$P_{15\text{V}} = -(15)(5) = -75 \text{ W} \quad \text{(dev/del)}$$
$$P_{5\text{A}} = -(5)(5) = -25 \text{ W} \quad \text{(dev/del)}$$
$$\sum P_{\text{abs}} = \sum P_{\text{del}} = 100 \text{ W}$$

[d] The interconnection is valid, but in this circuit the voltage drop across the 5 A current source is 35 V, positive at the top; 20 V source is developing (delivering), the 15 V source is developing (delivering), and the 5 A source is absorbing:
$$P_{20\text{V}} = -(20)(5) = -100 \text{ W} \quad \text{(dev/del)}$$
$$P_{15\text{V}} = -(15)(5) = -75 \text{ W} \quad \text{(dev/del)}$$
$$P_{5\text{A}} = (35)(5) = 175 \text{ W} \quad \text{(abs)}$$
$$\sum P_{\text{abs}} = \sum P_{\text{del}} = 175 \text{ W}$$

P 2.2 The interconnect is valid since the voltage sources can all carry 5 A of current supplied by the current source, and the current source can carry the voltage drop required by the interconnection. Note that the branch containing the 10 V, 40 V, and 5 A sources must have the same voltage drop as the branch containing the 50 V source, so the 5 A current source must have a voltage drop of 20 V, positive at the right. The voltages and currents are summarize in the circuit below:

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$$P_{50\text{V}} = (50)(5) = 250 \text{ W} \quad \text{(abs)}$$
$$P_{10\text{V}} = (10)(5) = 50 \text{ W} \quad \text{(abs)}$$
$$P_{40\text{V}} = -(40)(5) = -200 \text{ W} \quad \text{(dev)}$$
$$P_{5\text{A}} = -(20)(5) = -100 \text{ W} \quad \text{(dev)}$$
$$\sum P_{\text{dev}} = 300 \text{ W}$$

P 2.3 The interconnection is valid. The 10 A current source has a voltage drop of 100 V, positive at the top, because the 100 V source supplies its voltage drop across a pair of terminals shared by the 10 A current source. The right hand branch of the circuit must also have a voltage drop of 100 V from the left terminal of the 40 V source to the bottom terminal of the 5 A current source, because this branch shares the same terminals as the 100 V source. This means that the voltage drop across the 5 A current source is 140 V, positive at the top. Also, the two voltage sources can carry the current required of the interconnection. This is summarized in the figure below:

From the values of voltage and current in the figure, the power supplied by the current sources is calculated as follows:

$$P_{10\text{A}} = -(100)(10) = -1000 \text{ W} \quad \text{(1000 W supplied)}$$
$$P_{5\text{A}} = -(140)(5) = -700 \text{ W} \quad \text{(700 W supplied)}$$
$$\sum P_{\text{dev}} = 1700 \text{ W}$$

P 2.4 The interconnection is not valid. Note that the 3 A and 4 A sources are both connected in the same branch of the circuit. A valid interconnection would require these two current sources to supply the same current in the same direction, which they do not.

P 2.5 The interconnection is valid, since the voltage sources can carry the currents supplied by the 2 A and 3 A current sources, and the current sources can carry whatever voltage drop from the top node to the bottom node is required by the interconnection. In particular, note the the voltage drop between the top and bottom nodes in the right hand branch must be the same as the voltage drop between the top and bottom nodes in the left hand branch. In particular, this means that

$$-v_1 + 8 \text{ V} = 12 \text{ V} + v_2$$

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Hence any combination of v_1 and v_2 such that $v_1 + v_2 = -4\,\text{V}$ is a valid solution.

P 2.6 **[a]** Because both current sources are in the same branch of the circuit, their values must be the same. Therefore,

$$\frac{v_1}{50} = 0.4 \quad \rightarrow \quad v_1 = 0.4(50) = 20 \text{ V}$$

[b] $p = v_1(0.4) = (20)(0.4) = 8$ W (absorbed)

P 2.7 **[a]** The voltage drop from the top node to the bottom node in this circuit must be the same for every path from the top to the bottom. Therefore, the voltages of the two voltage sources are equal:

$$-\alpha i_\Delta = 6$$

Also, the current i_Δ is in the same branch as the 15 mA current source, but in the opposite direction, so

$$i_\Delta = -0.015$$

Substituting,

$$-\alpha(-0.015) = 6 \quad \rightarrow \quad \alpha = \frac{6}{0.015} = 400$$

The interconnection is valid if $\alpha = 400$ V/A.

[b] The voltage across the current source must equal the voltage across the 6 V source, since both are connected between the top and bottom nodes. Using the passive sign convention,

$$p = vi = (6)(0.015) = 0.09 = 90 \text{ mW}$$

[c] Since the power is positive, the current source is absorbing power.

P 2.8 **[a]** Yes, each of the voltage sources can carry the current required by the interconnection, and each of the current sources can carry the voltage drop required by the interconnection. (Note that $i_1 = 50$ mA.)

[b] No, because the voltage drop between the top terminal and the bottom terminal cannot be determined. For example, define v_1, v_2, and v_3 as shown:

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The voltage drop across the left branch, the center branch, and the right branch must be the same, since these branches are connected at the same two terminals. This requires that

$v_1 - 20 = v_2 = v_3 + 30$

But this equation has three unknown voltages, so the individual voltages cannot be determined, and thus the power of the sources cannot be determined.

P 2.9 The interconnection is invalid. In the middle branch, the value of the current i_x must be 50 mA, since the 50 mA current source supplies current in this branch in the same direction as the current i_x. Therefore, the voltage supplied by the dependent voltage source in the right hand branch is $1800(0.05) = 90$ V. This gives a voltage drop from the top terminal to the bottom terminal in the right hand branch of $90 + 60 = 150$ V. But the voltage drop between these same terminals in the left hand branch is 30 V, due to the voltage source in that branch. Therefore, the interconnection is invalid.

P 2.10

First, $10v_a = 5$ V, so $v_a = 0.5$ V. Then recognize that each of the three branches is connected between the same two nodes, so each of these branches must have the same voltage drop. The voltage drop across the middle branch is 5 V, and since $v_a = 0.5$ V, $v_g = 0.5 - 5 = -4.5$ V. Also, the voltage drop across the left branch is 5 V, so $20 + v_{9A} = 5$ V, and $v_{9A} = -15$ V, where v_{9A} is positive at the top. Note that the current through the 20 V source must be 9 A, flowing from top to bottom, and the current through the v_g is 6 A flowing from top to bottom. Let's find the power associated with the left and middle branches:

$p_{9A} = (9)(-15) = -135\,\text{W}$
$p_{20V} = (9)(20) = 180\,\text{W}$
$p_{v_g} = -(6)(-4.5) = 27\,\text{W}$
$p_{6A} = (6)(0.5) = 3\,\text{W}$

Since there is only one component left, we can find the total power:

$p_{\text{total}} = -135 + 180 + 27 + 3 + p_{\text{ds}} = 75 + p_{\text{ds}} = 0$

so p_{ds} must equal -75 W.

Therefore,

$$\sum P_{\text{dev}} = \sum P_{\text{abs}} = 210\,\text{W}$$

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P 2.11 **[a]** Using the passive sign convention and Ohm's law,

$$v = Ri = (3000)(0.015) = 45 \text{ V}$$

[b] $P_{\text{R}} = \dfrac{v^2}{R} = \dfrac{45^2}{3000} = 0.675 = 675 \text{ mW}$

[c] Using the passive sign convention with the current direction reversed,

$$v = -Ri = -(3000)(0.015) = -45 \text{ V}$$

$$P_{\text{R}} = \frac{v^2}{R} = \frac{-45^2}{3000} = 0.675 = 675 \text{ mW}$$

P 2.12 **[a]** Using the passive sign convention and Ohm's law,

$$i = -\frac{v}{R} = -\frac{40}{2500} = -0.016 = -16 \text{ mA}$$

[b] $P_{\text{R}} = Ri^2 = (2500)(-0.016)^2 = 0.64 = 640 \text{ mW}$

[c] Using the passive sign convention with the voltage polarity reversed,

$$i = \frac{v}{R} = \frac{40}{2500} = 0.016 = 16 \text{ mA}$$

$$P_{\text{R}} = Ri^2 = (2500)(0.016)^2 = 0.64 = 640 \text{ mW}$$

P 2.13 **[a]**

[b]

V_{bb}	=	no-load voltage of battery
R_{bb}	=	internal resistance of battery
R_x	=	resistance of wire between battery and switch
R_y	=	resistance of wire between switch and lamp A
R_{a}	=	resistance of lamp A
R_{b}	=	resistance of lamp B
R_w	=	resistance of wire between lamp A and lamp B
R_{g1}	=	resistance of frame between battery and lamp A
R_{g2}	=	resistance of frame between lamp A and lamp B
S	=	switch

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P 2.14 Since we know the device is a resistor, we can use Ohm's law to calculate the resistance. From Fig. P2.14(a),

$$v = Ri \qquad \text{so} \qquad R = \frac{v}{i}$$

Using the values in the table of Fig. P2.14(b),

$$R = \frac{-7200}{-6} = \frac{-3600}{-3} = \frac{3600}{3} = \frac{7200}{6} = \frac{10{,}800}{9} = 1.2\,\text{k}\Omega$$

Note that this value is found in Appendix H.

P 2.15 Since we know the device is a resistor, we can use the power equation. From Fig. P2.15(a),

$$p = vi = \frac{v^2}{R} \qquad \text{so} \qquad R = \frac{v^2}{p}$$

Using the values in the table of Fig. P2.13(b)

$$R = \frac{(-8)^2}{640 \times 10^{-3}} = \frac{(-4)^2}{160 \times 10^{-3}} = \frac{(4)^2}{160 \times 10^{-3}} = \frac{(8)^2}{640 \times 10^{-3}}$$

$$= \frac{(12)^2}{1440 \times 10^{-3}} = \frac{(16)^2}{2560 \times 10^{-3}} = 100\,\Omega$$

Note that this value is found in Appendix H.

P 2.16 The resistor value is the ratio of the power to the square of the current: $R = \frac{p}{i^2}$. Using the values for power and current in Fig. P2.16(b),

$$\frac{8.25 \times 10^{-3}}{(0.5 \times 10^{-3})^2} = \frac{33 \times 10^{-3}}{(1 \times 10^{-3})^2} = \frac{74.25 \times 10^{-3}}{(1.5 \times 10^{-3})^2} = \frac{132 \times 10^{-3}}{(2 \times 10^{-3})^2}$$

$$= \frac{206.25 \times 10^{-3}}{(2.5 \times 10^{-3})^2} = \frac{297 \times 10^{-3}}{(3 \times 10^{-3})^2} = 33\ \text{k}\Omega$$

Note that this is a value from Appendix H.

P 2.17 Label the unknown resistor currents and voltages:

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[a] KCL at the top node: $0.02 = i_1 + i_2$

KVL around the right loop: $-v_o + v_2 - 5 = 0$

Use Ohm's law to write the resistor voltages in the previous equation in terms of the resistor currents:

$$-5000i_1 + 2000i_2 - 5 = 0 \qquad \rightarrow \qquad -5000i_1 + 2000i_2 = 5$$

Multiply the KCL equation by -2000 and add it to the KVL equation to eliminate i_2:

$$-2000(i_1 + i_2) + (-5000i_1 + 2000i_2) = -2000(0.02) + 5 \quad \rightarrow \quad -7000i_1 = -35$$

Solving,

$$i_1 = \frac{-35}{-7000} = 0.005 = 5 \text{ mA}$$

Therefore,

$$v_o = Ri_1 = (5000)(0.005) = 25 \text{ V}$$

[b] $p_{20\text{mA}} = -(0.02)v_o = -(0.02)(25) = -0.5 \text{ W}$

$$i_2 = 0.02 - i_1 = 0.02 - 0.005 = 0.015 \text{ A}$$

$$p_{5\text{V}} = -(5)i_2 = -(5)(0.015) = -0.075 \text{ W}$$

$$p_{5\text{k}} = 5000i_1^2 = 5000(0.005)^2 = 0.125 \text{ W}$$

$$p_{2\text{k}} = 2000i_2^2 = 2000(0.015)^2 = 0.45 \text{ W}$$

$$p_{\text{total}} = p_{20\text{mA}} + p_{5\text{V}} + p_{5\text{k}} + p_{2\text{k}} = -0.5 - 0.075 + 0.125 + 0.45 = 0$$

Thus the power in the circuit balances.

P 2.18 **[a]**

$$\begin{aligned} 20i_\text{a} &= 80i_\text{b} \qquad i_g = i_\text{a} + i_\text{b} = 5i_\text{b} \\ i_\text{a} &= 4i_\text{b} \\ 50 &= 4i_g + 80i_\text{b} = 20i_\text{b} + 80i_\text{b} = 100i_\text{b} \\ i_\text{b} &= 0.5 \text{ A, therefore, } i_\text{a} = 2 \text{ A} \quad \text{and} \quad i_g = 2.5 \text{ A} \end{aligned}$$

[b] $i_\text{b} = 0.5 \text{ A}$

[c] $v_o = 80i_\text{b} = 40 \text{ V}$

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[d] $p_{4\Omega} = i_g^2(4) = 6.25(4) = 25$ W

$p_{20\Omega} = i_a^2(20) = (4)(20) = 80$ W

$p_{80\Omega} = i_b^2(80) = 0.25(80) = 20$ W

[e] $p_{50\text{V}}$ (delivered) $= 50i_g = 125$ W

Check:

$\sum P_{\text{dis}} = 25 + 80 + 20 = 125\,\text{W}$

$\sum P_{\text{del}} = 125\,\text{W}$

P 2.19

[a] Write a KCL equation at the top node:

$-1.5 + i_1 + i_2 = 0 \qquad \text{so} \qquad i_1 + i_2 = 1.5$

Write a KVL equation around the right loop:

$-v_1 + v_2 + v_3 = 0$

From Ohm's law,

$v_1 = 100i_1, \qquad v_2 = 150i_2, \qquad v_3 = 250i_2$

Substituting,

$-100i_1 + 150i_2 + 250i_2 = 0 \qquad \text{so} \qquad -100i_1 + 400i_2 = 0$

Solving the two equations for i_1 and i_2 simultaneously,

$i_1 = 1.2\,\text{A} \qquad \text{and} \qquad i_2 = 0.3\,\text{A}$

[b] Write a KVL equation clockwise around the left loop:

$-v_o + v_1 = 0 \qquad \text{but} \qquad v_1 = 100i_1 = 100(1.2) = 120\,\text{V}$

So $\quad v_o = v_1 = 120\,\text{V}$

[c] Calculate power using $p = vi$ for the source and $p = Ri^2$ for the resistors:

$p_{\text{source}} = -v_o(1.5) = -(120)(1.5) = -180\,\text{W}$

$p_{100\Omega} = 1.2^2(100) = 144\,\text{W}$

$p_{150\Omega} = 0.3^2(150) = 13.5\,\text{W}$

$p_{250\Omega} = 0.3^2(250) = 22.5\,\text{W}$

$\sum P_{\text{dev}} = 180\,\text{W} \qquad \sum P_{\text{abs}} = 144 + 13.5 + 22.5 = 180\,\text{W}$

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P 2.20 Label the unknown resistor voltages and currents:

[a] $i_a = \dfrac{3.5}{175} = 0.02\,\text{A}$ (Ohm's law)

$i_1 = i_a = 0.02\,\text{A}$ (KCL)

[b] $v_b = 200 i_1 = 200(0.02) = 4\,\text{V}$ (Ohm's law)

$-v_1 + v_b + 3.5 = 0$ so $v_1 = 3.5 + v_b = 3.5 + 4 = 7.5\,\text{V}$ (KVL)

[c] $v_a = 0.05(50) = 2.5\,\text{V}$ (Ohm's law)

$-v_g + v_a + v_1 = 0$ so $v_g = v_a + v_1 = 2.5 + 7.5 = 10\,\text{V}$ (KVL)

[d] $p_g = v_g(0.05) = 10(0.05) = 0.5\,\text{W}$

P 2.21 **[a]** Use KVL for the right loop to calculate the voltage drop across the right-hand branch v_o. This is also the voltage drop across the middle branch, so once v_o is known, use Ohm's law to calculate i_o:

$$\begin{aligned} v_o &= 1000 i_a + 4000 i_a + 3000 i_a = 8000 i_a = 8000(0.002) = 16 \text{ V} \\ 16 &= 2000 i_o \\ i_o &= \frac{16}{2000} = 8 \text{ mA} \end{aligned}$$

[b] KCL at the top node: $i_g = i_a + i_o = 0.002 + 0.008 = 0.010 \text{ A} = 10 \text{ mA}$.

[c] The voltage drop across the source is v_0, seen by writing a KVL equation for the left loop. Thus,
$p_g = -v_o i_g = -(16)(0.01) = -0.160 \text{ W} = -160 \text{ mW}$.
Thus the source delivers 160 mW.

P 2.22 **[a]**

$v_2 = 150 - 50(1) = 100\text{V}$

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$$i_2 = \frac{v_2}{25} = 4\text{A}$$

$$i_3 + 1 = i_2, \qquad i_3 = 4 - 1 = 3\text{A}$$

$$v_1 = 10i_3 + 25i_2 = 10(3) + 25(4) = 130\text{V}$$

$$i_1 = \frac{v_1}{65} = \frac{130}{65} = 2\text{A}$$

Note also that

$$i_4 = i_1 + i_3 = 2 + 3 = 5\,\text{A}$$

$$i_g = i_4 + i_o = 5 + 1 = 6\,\text{A}$$

[b]

$$\begin{aligned} p_{4\Omega} &= 5^2(4) = 100\ \text{W} \\ p_{50\Omega} &= 1^2(50) = 50\ \text{W} \\ p_{65\Omega} &= 2^2(65) = 260\ \text{W} \\ p_{10\Omega} &= 3^2(10) = 90\ \text{W} \\ p_{25\Omega} &= 4^2(25) = 400\ \text{W} \end{aligned}$$

[c] $\sum P_{\text{dis}} = 100 + 50 + 260 + 90 + 400 = 900\,\text{W}$

$$P_{\text{dev}} = 150 i_g = 150(6) = 900\,\text{W}$$

P 2.23 Label all unknown resistor voltages and currents:

Ohms' law for 5 kΩ resistor: $v_1 = (0.01)(5000) = 50$ V
KVL for lower left loop: $-80 + v_2 + 50 = 0 \quad \rightarrow \quad v_2 = 80 - 50 = 30$ V
Ohm's law for 1.5 kΩ resistor: $i_2 = v_2/1500 = 30/1500 = 20$ mA
KCL at center node:
$i_2 = i_3 + 0.01 \quad \rightarrow \quad i_3 = i_2 - 0.01 = 0.02 - 0.01 = 0.01 = 10$ mA
Ohm's law for 3 kΩ resistor $v_3 = 3000 i_3 = 3000(0.01) = 30$ V
KVL for lower right loop:
$-v_1 + v_3 + v_4 = 0 \quad \rightarrow \quad v_4 = v_1 - v_3 = 50 - 30 = 20$ V

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Ohm's law for $500\,\Omega$ resistor: $i_4 = v_4/500 = 20/500 = 0.04 = 40$ mA
KCL for right node:
$i_3 + i_{\mathrm{R}} = i_4 \quad \rightarrow \quad i_{\mathrm{R}} = i_4 - i_3 = 0.04 - 0.01 = 0.03 = 30$ mA
KVL for outer loop:
$-80 + v_{\mathrm{R}} + v_4 = 0 \quad \rightarrow \quad v_{\mathrm{R}} = 80 - v_4 = 80 - 20 = 60$ V
Therefore,

$$R = \frac{v_{\mathrm{R}}}{i_{\mathrm{R}}} = \frac{60}{0.03} = 2000 = 2\ \mathrm{k}\Omega$$

P 2.24 **[a]**

$v_a = (5 + 10)(4) = 60\,\mathrm{V}$
$-240 + v_a + v_b = 0 \qquad \text{so} \qquad v_b = 240 - v_a = 240 - 60 = 180\,\mathrm{V}$
$i_e = v_b/(14 + 6) = 180/20 = 9\,\mathrm{A}$
$i_d = i_e - 4 = 9 - 4 = 5\,\mathrm{A}$
$v_c = 4i_d + v_b = 4(5) + 180 = 200\,\mathrm{V}$
$i_c = v_c/10 = 200/10 = 20\,\mathrm{A}$
$v_d = 240 - v_c = 240 - 200 = 40\,\mathrm{V}$
$i_a = i_d + i_c = 5 + 20 = 25\,\mathrm{A}$
$R = v_d/i_a = 40/25 = 1.6\,\Omega$

[b] $i_g = i_a + 4 = 25 + 4 = 29$ A
p_g (supplied) $= (240)(29) = 6960$ W

P 2.25 **[a]**

$i_{\mathrm{cd}} = 80/16 = 5\,\mathrm{A}$

$v_{\mathrm{ac}} = 125 - 80 = 45 \qquad \text{so} \qquad i_{\mathrm{ac}} = 45/15 = 3\,\mathrm{A}$

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$i_{ac} + i_{bc} = i_{cd}$ so $i_{bc} = 5 - 3 = 2\,\text{A}$

$v_{ab} = 15i_{ac} - 5i_{bc} = 15(3) - 5(2) = 35\,\text{V}$ so $i_{ab} = 35/7 = 5\,\text{A}$

$i_{bd} = i_{ab} - i_{bc} = 5 - 2 = 3\,\text{A}$

Calculate the power dissipated by the resistors using the equation $p_R = Ri_R^2$:

$p_{7\Omega} = (7)(5)^2 = 175\,\text{W}$ $\quad p_{30\Omega} = (30)(3)^2 = 270\,\text{W}$

$p_{15\Omega} = (15)(3)^2 = 135\,\text{W}$ $\quad p_{16\Omega} = (16)(5)^2 = 400\,\text{W}$

$p_{5\Omega} = (5)(2)^2 = 20\,\text{W}$

[b] Calculate the current through the voltage source:

$i_{ad} = -i_{ab} - i_{ac} = -5 - 3 = -8\,\text{A}$

Now that we have both the voltage and the current for the source, we can calculate the power supplied by the source:

$p_g = 125(-8) = -1000\,\text{W}$ thus p_g (supplied) $= 1000\,\text{W}$

[c] $\sum P_{\text{dis}} = 175 + 270 + 135 + 400 + 20 = 1000\,\text{W}$

Therefore,

$\sum P_{\text{supp}} = \sum P_{\text{dis}}$

P 2.26 **[a]**

$v_2 = 100 + 4(15) = 160\,\text{V}$; $\quad v_1 = 160 - (9 + 11 + 10)(2) = 100\,\text{V}$

$i_1 = \dfrac{v_1}{4 + 16} = \dfrac{100}{20} = 5\,\text{A}$; $\quad i_3 = i_1 - 2 = 5 - 2 = 3\,\text{A}$

$v_g = v_1 + 30i_3 = 100 + 30(3) = 190\,\text{V}$

$i_4 = 2 + 4 = 6\,\text{A}$

$i_g = -i_4 - i_3 = -6 - 3 = -9\,\text{A}$

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[b] Calculate power using the formula $p = Ri^2$:

$p_{9\,\Omega} = (9)(2)^2 = 36\,\text{W}$; $\quad p_{11\,\Omega} = (11)(2)^2 = 44\,\text{W}$

$p_{10\,\Omega} = (10)(2)^2 = 40\,\text{W}$; $\quad p_{5\,\Omega} = (5)(6)^2 = 180\,\text{W}$

$p_{30\,\Omega} = (30)(3)^2 = 270\,\text{W}$; $\quad p_{4\,\Omega} = (4)(5)^2 = 100\,\text{W}$

$p_{16\,\Omega} = (16)(5)^2 = 400\,\text{W}$; $\quad p_{15\,\Omega} = (15)(4)^2 = 240\,\text{W}$

[c] $v_g = 190\,\text{V}$

[d] Sum the power dissipated by the resistors:

$$\sum p_{\text{diss}} = 36 + 44 + 40 + 180 + 270 + 100 + 400 + 240 = 1310\,\text{W}$$

The power associated with the sources is

$$p_{\text{volt-source}} = (100)(4) = 400\,\text{W}$$

$$p_{\text{curr-source}} = v_g i_g = (190)(-9) = -1710\,\text{W}$$

Thus the total power dissipated is $1310 + 400 = 1710$ W and the total power developed is 1710 W, so the power balances.

P 2.27 **[a]** Start by calculating the voltage drops due to the currents i_1 and i_2. Then use KVL to calculate the voltage drop across and $35\,\Omega$ resistor, and Ohm's law to find the current in the $35\,\Omega$ resistor. Finally, KCL at each of the middle three nodes yields the currents in the two sources and the current in the middle $2\,\Omega$ resistor. These calculations are summarized in the figure below:

$$p_{147(\text{top})} = -(147)(28) = -4116\ \text{W}$$

$$p_{147(\text{bottom})} = -(147)(21) = -3087\ \text{W}$$

Therefore the top source supplies 4116 W of power and the bottom source supplies 3087 W of power.

[b]

$$\begin{aligned}\sum P_{\text{dis}} &= (28)^2(1) + (7)^2(2) + (21)^2(1) + (21)^2(5) + (14)^2(10) + (7)^2(35)\\ &= 784 + 98 + 441 + 2205 + 1960 + 1715 = 7203\ \text{W}\\ \sum P_{\text{sup}} &= 4116 + 3087 = 7203\ \text{W}\end{aligned}$$

Therefore, $\sum P_{\text{dis}} = \sum P_{\text{sup}} = 7203$ W

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P 2.28 **[a]** Plot the $v - i$ characteristic

From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(130 - 50)}{(10 - 0)} = 8\,\Omega$$

When $i_t = 0$, $v_t = 50$ V; therefore the ideal voltage source has a voltage of 50 V.

[b]

When $v_t = 0$, $\qquad i_t = \dfrac{-50}{8} = -6.25\text{A}$

Note that this result can also be obtained by extrapolating the $v - i$ characteristic to $v_t = 0$.

P 2.29 **[a]** Plot the v—i characteristic:

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From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(180 - 100)}{(16 - 0)} = 5\,\Omega$$

When $i_t = 0$, $v_t = 100$ V; therefore the ideal current source must have a current of $100/5 = 20$ A

[b] We attach a $20\,\Omega$ resistor to the device model developed in part (a):

Write a KCL equation at the top node:

$$20 + i_t = i_1$$

Write a KVL equation for the right loop, in the direction of the two currents, using Ohm's law:

$$5i_1 + 20i_t = 0$$

Combining the two equations and solving,

$$5(20 + i_t) + 20i_t = 0 \qquad \text{so} \qquad 25i_t = -100; \qquad \text{thus} \qquad i_t = -4\,\text{A}$$

Now calculate the power dissipated by the resistor:

$$p_{20\,\Omega} = 20i_t^2 = 20(-4)^2 = 320\,\text{W}$$

P 2.30 **[a]**

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[b] $\Delta v = 25\text{V}; \quad \Delta i = 2.5 \text{ mA}; \quad R = \dfrac{\Delta v}{\Delta i} = 10 \text{ k}\Omega$

[c] $10{,}000 i_1 = 2500 i_s, \qquad i_1 = 0.25 i_s$

$0.02 = i_1 + i_s = 1.25 i_s, \qquad i_s = 16 \text{ mA}$

[d] $v_s(\text{open circuit}) = (20 \times 10^{-3})(10 \times 10^3) = 200 \text{ V}$

[e] The open circuit voltage can be found in the table of values (or from the plot) as the value of the voltage v_s when the current $i_s = 0$. Thus, $v_s(\text{open circuit}) = 140$ V (from the table)

[f] Linear model cannot predict the nonlinear behavior of the practical current source.

P 2.31 **[a]** Begin by constructing a plot of voltage versus current:

[b] Since the plot is linear for $0 \le i_s \le 24$ mA amd since $R = \Delta v/\Delta i$, we can calculate R from the plotted values as follows:

$$R = \frac{\Delta v}{\Delta i} = \frac{24 - 18}{0.024 - 0} = \frac{6}{0.024} = 250\,\Omega$$

We can determine the value of the ideal voltage source by considering the value of v_s when $i_s = 0$. When there is no current, there is no voltage drop across the resistor, so all of the voltage drop at the output is due to the voltage source. Thus the value of the voltage source must be 24 V. The model, valid for $0 \le i_s \le 24$ mA, is shown below:

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[**c**] The circuit is shown below:

Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i:

$$-24\,\text{V} + 250i + 1000i = 0 \qquad \text{so} \qquad 1250i = 24\,\text{V}$$

Thus, $$i = \frac{24\,\text{V}}{1250\,\Omega} = 19.2\,\text{mA}$$

[**d**] The circuit is shown below:

Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i:

$$-24\,\text{V} + 250i = 0 \qquad \text{so} \qquad 250i = 24\,\text{V}$$

Thus, $$i = \frac{24\,\text{V}}{250\,\Omega} = 96\,\text{mA}$$

[**e**] The short circuit current can be found in the table of values (or from the plot) as the value of the current i_s when the voltage $v_s = 0$. Thus,

$$i_{sc} = 48\,\text{mA} \qquad \text{(from table)}$$

[**f**] The plot of voltage versus current constructed in part (a) is not linear (it is piecewise linear, but not linear for all values of i_s). Since the proposed circuit model is a linear model, it cannot be used to predict the nonlinear behavior exhibited by the plotted data.

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P 2.32 Label unknown voltage and current:

$-v_x + v_o + 2i_x = 0$ (KVL)

$v_x = 6i_x$ (Ohm's law)

Therefore

$-6i_x + v_o + 2i_x = 0$ so $v_o = 4i_x$

Thus

$$i_x = \frac{v_o}{4}$$

Also,

$$i_1 = \frac{v_o}{2} \quad \text{(Ohm's law)}$$

$45 = i_x + i_1$ (KCL)

Substituting for the currents i_x and i_1:

$$45 = \frac{v_o}{4} + \frac{v_o}{2} = \frac{3v_o}{4}$$

Thus

$$v_o = 45\left(\frac{4}{3}\right) = 60\,\text{V}$$

The only two circuit elements that could supply power are the two sources, so calculate the power for each source:

$$v_x = 6i_x = 6\frac{v_o}{4} = 6(60/4) = 90\,\text{V}$$

$$p_{45\text{V}} = -45v_x = -45(90) = -4050\,\text{W}$$

$$p_{\text{d.s.}} = (2i_x)i_1 = 2(v_o/4)(v_o/2) = 2(60/4)(60/2) = 900\,\text{W}$$

Only the independent voltage source is supplying power, so the total power supplied is 4050 W.

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P 2.33 Label unknown current:

$-20 + 450i + 150i = 0$ (KVL and Ohm's law)

so $600i = 20 \quad \rightarrow \quad i = 33.33\,\text{mA}$

$v_x = 150i = 150(0.0333) = 5\,\text{V}$ (Ohm's law)

$v_o = 300\left(\dfrac{v_x}{100}\right) = 300(5/100) = 15\,\text{V}$ (Ohm's law)

Calculate the power for all components:

$p_{20\text{V}} = -20i = -20(0.0333) = -0.667\,\text{W}$

$p_{\text{d.s.}} = -v_o\left(\dfrac{v_x}{100}\right) = -(15)(5/100) = -0.75\,\text{W}$

$p_{450} = 450i^2 = 450(0.033)^2 = 0.5\,\text{W}$

$p_{150} = 150i^2 = 150(0.033)^2 = 0.1667\,\text{W}$

$p_{300} = \dfrac{v_o^2}{300} = \dfrac{15^2}{300} = 0.75\,\text{W}$

Thus the total power absorbed is

$p_{\text{abs}} = 0.5 + 0.1667 + 0.75 = 1.4167\,\text{W}$

P 2.34 The circuit:

$v_1 = (4000)(0.01) = 40\,\text{V}$ (Ohm's law)

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$$\frac{v_1}{2} = 2000i_o + 6000i_o = 8000i_o \qquad \text{(KVL)}$$

Thus,

$$i_o = \frac{v_1/2}{8000} = \frac{40/2}{8000} = 0.0025 = 2.5\,\text{mA}$$

Calculate the power for all components:

$$p_{10\text{mA}} = -(0.01)v_1 = -(0.01)(40) = -0.4\,\text{W}$$

$$p_{\text{d.s.}} = -(v_1/2)i_o = -(40/2)(2.5 \times 10^{-3}) = -0.05\,\text{W}$$

$$p_{4\text{k}} = \frac{v_1^2}{4000} = \frac{40^2}{4000} = 0.4\,\text{W}$$

$$p_{2\text{k}} = 2000i_o^2 = 2000(2.5 \times 10^{-3})^2 = 0.0125\,\text{W}$$

$$p_{6\text{k}} = 6000i_o^2 = 6000(2.5 \times 10^{-3})^2 = 0.0375\,\text{W}$$

Therefore,

$$p_{\text{total}} = -0.4 - 0.05 + 0.4 + 0.0125 + 0.0375 = 0$$

Thus the power in the circuit balances.

P 2.35 **[a]** $i_o = 0$ because no current can exist in a single conductor connecting two parts of a circuit.

[b]

$$18 = (12 + 6)i_g \qquad i_g = 1 \text{ A}$$

$$v_\Delta = 6i_g = 6\text{V} \qquad v_\Delta/2 = 3 \text{ A}$$

$$10i_1 = 5i_2, \text{ so } i_1 + 2i_1 = -3 \text{ A; therefore, } i_1 = -1 \text{ A}$$

[c] $i_2 = 2i_1 = -2$ A.

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P 2.36 **[a]** $-50 - 20i_\sigma + 18i_\Delta = 0$

$-18i_\Delta + 5i_\sigma + 40i_\sigma = 0 \quad \text{so} \quad 18i_\Delta = 45i_\sigma$

Therefore, $\quad -50 - 20i_\sigma + 45i_\sigma = 0, \quad \text{so} \quad i_\sigma = 2\,\text{A}$

$18i_\Delta = 45i_\sigma = 90;\ \text{so}\ i_\Delta = 5\,\text{A}$

$v_o = 40i_\sigma = 80\,\text{V}$

[b] i_g = current out of the positive terminal of the 50 V source
v_d = voltage drop across the $8i_\Delta$ source

$i_g = i_\Delta + i_\sigma + 8i_\Delta = 9i_\Delta + i_\sigma = 47\,\text{A}$

$v_d = 80 - 20 = 60\,\text{V}$

$$\begin{aligned}
\sum P_\text{gen} &= 50i_g + 20i_\sigma i_g = 50(47) + 20(2)(47) = 4230\ \text{W} \\
\sum P_\text{diss} &= 18i_\Delta^2 + 5i_\sigma(i_g - i_\Delta) + 40i_\sigma^2 + 8i_\Delta v_d + 8i_\Delta(20) \\
&= (18)(25) + 10(47-5) + 4(40) + 40(60) + 40(20) \\
&= 4230\ \text{W; Therefore,} \\
\sum P_\text{gen} &= \sum P_\text{diss} = 4230\ \text{W}
\end{aligned}$$

P 2.37 $40i_2 + \dfrac{5}{40} + \dfrac{5}{10} = 0; \quad i_2 = -15.625\ \text{mA}$

$v_1 = 80i_2 = -1.25\ \text{V}$

$25i_1 + \dfrac{(-1.25)}{20} + (-0.015625) = 0; \quad i_1 = 3.125\ \text{mA}$

$v_g = 60i_1 + 260i_1 = 320i_1$

Therefore, $v_g = 1$ V.

P 2.38 $i_E - i_B - i_C = 0$

$i_C = \beta i_B \qquad \text{therefore}\ i_E = (1+\beta)i_B$

$i_2 = -i_B + i_1$

$V_o + i_E R_E - (i_1 - i_B)R_2 = 0$

$-i_1 R_1 + V_{CC} - (i_1 - i_B)R_2 = 0 \quad \text{or} \quad i_1 = \dfrac{V_{CC} + i_B R_2}{R_1 + R_2}$

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$$V_o + i_E R_E + i_B R_2 - \frac{V_{CC} + i_B R_2}{R_1 + R_2} R_2 = 0$$

Now replace i_E by $(1+\beta)i_B$ and solve for i_B. Thus

$$i_B = \frac{[V_{CC} R_2/(R_1 + R_2)] - V_o}{(1+\beta)R_E + R_1 R_2/(R_1 + R_2)}$$

P 2.39 Here is Equation 2.25:

$$i_{\mathrm{B}} = \frac{(V_{\mathrm{CC}} R_2)/(R_1 + R_2) - V_0}{(R_1 R_2)/(R_1 + R_2) + (1+\beta)R_{\mathrm{E}}}$$

$$\frac{V_{CC} R_2}{R_1 + R_2} = \frac{(10)(60{,}000)}{100{,}000} = 6\mathrm{V}$$

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{(40{,}000)(60{,}000)}{100{,}000} = 24\ \mathrm{k}\Omega$$

$$i_B = \frac{6 - 0.6}{24{,}000 + 50(120)} = \frac{5.4}{30{,}000} = 0.18\ \mathrm{mA}$$

$$i_C = \beta i_B = (49)(0.18) = 8.82\ \mathrm{mA}$$

$$i_E = i_C + i_B = 8.82 + 0.18 = 9\ \mathrm{mA}$$

$$v_{3d} = (0.009)(120) = 1.08\mathrm{V}$$

$$v_{bd} = V_o + v_{3d} = 1.68\mathrm{V}$$

$$i_2 = \frac{v_{bd}}{R_2} = \frac{1.68}{60{,}000} = 28\,\mu\mathrm{A}$$

$$i_1 = i_2 + i_B = 28 + 180 = 208\,\mu\mathrm{A}$$

$$v_{\mathrm{ab}} = 40{,}000(208 \times 10^{-6}) = 8.32\,\mathrm{V}$$

$$i_{CC} = i_C + i_1 = 8.82 + 0.208 = 9.028\ \mathrm{mA}$$

$$v_{13} + (8.82 \times 10^{-3})(750) + 1.08 = 10\ \mathrm{V}$$

$$v_{13} = 2.305\,\mathrm{V}$$

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P 2.40 **[a]**

[b]

P 2.41 Each radiator is modeled as a $48\,\Omega$ resistor:

Write a KVL equation for each of the three loops:

$$-240 + 48i_1 = 0 \quad \rightarrow \quad i_1 = \frac{240}{48} = 5\,\text{A}$$

$$-48i_1 + 48i_2 = 0 \quad \rightarrow \quad i_2 = i_1 = 5\,\text{A}$$

$$-48i_2 + 48i_3 = 0 \quad \rightarrow \quad i_3 = i_2 = 5\,\text{A}$$

Therefore, the current through each radiator is 5 A and the power for each radiator is

$$p_{\text{rad}} = Ri^2 = 48(5)^2 = 1200\,\text{W}$$

There are three radiators, so the total power for this heating system is

$$p_{\text{total}} = 3p_{\text{rad}} = 3(1200) = 3600\,\text{W}$$

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P 2.42 Each radiator is modeled as a 48 Ω resistor:

Write a KVL equation for the left and right loops:

$$-240 + 48i_1 = 0 \quad \rightarrow \quad i_1 = \frac{240}{48} = 5\,\text{A}$$

$$-48i_1 + 48i_2 + 48i_2 = 0 \quad \rightarrow \quad i_2 = \frac{i_1}{2} = \frac{5}{2} = 2.5\,\text{A}$$

The power for the center radiator is

$$p_{\text{cen}} = 48i_1^2 = 48(5)^2 = 1200\,\text{W}$$

The power for each of the radiators on the right is

$$p_{\text{right}} = 48i_2^2 = 48(2.5)^2 = 300\,\text{W}$$

Thus the total power for this heating system is

$$p_{\text{total}} = p_{\text{cen}} + 2p_{\text{right}} = 1200 + 2(300) = 1800\,\text{W}$$

The center radiator produces 1200 W, just like the three radiators in Problem 2.41. But the other two radiators produce only 300 W each, which is 1/4th of the power of the radiators in Problem 2.41. The total power of this configuration is 1/2 of the total power in Fig. 2.41.

P 2.43 Each radiator is modeled as a 48 Ω resistor:

Write a KVL equation for the left and right loops:

$$-240 + 48i_1 + 48i_2 = 0$$

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$-48i_2 + 48i_3 = 0 \quad \rightarrow \quad i_2 = i_3$

Write a KCL equation at the top node:

$i_1 = i_2 + i_3 \quad \rightarrow \quad i_1 = i_2 + i_2 = 2i_2$

Substituting into the first KVL equation gives

$$-240 + 48(2i_2) + 48i_2 = 0 \quad \rightarrow \quad i_2 = \frac{240}{3(48)} = 1.67\,\text{A}$$

Solve for the currents i_1 and i_3:

$i_3 = i_2 = 1.67\,\text{A}; \qquad i_1 = 2i_2 = 2(1.67) = 3.33\,\text{A}$

Calculate the power for each radiator using the current for each radiator:

$p_{\text{left}} = 48i_1^2 = 48(3.33)^2 = 533.33\,\text{W}$

$p_{\text{middle}} = p_{\text{right}} = 48i_2^2 = 48(1.67)^2 = 133.33\,\text{W}$

Thus the total power for this heating system is

$p_{\text{total}} = p_{\text{left}} + p_{\text{middle}} + p_{\text{right}} = 533.33 + 133.33 + 133.33 = 800\,\text{W}$

All radiators in this configuration have much less power than their counterparts in Fig. 2.41. The total power for this configuration is only 22.2% of the total power for the heating system in Fig. 2.41.

P 2.44 Each radiator is modeled as a 48 Ω resistor:

Write a KVL equation for this loop:

$$-240 + 48i + 48i + 48i = 0 \quad \rightarrow \quad i = \frac{240}{3(48)} = 1.67\,\text{A}$$

Calculate the power for each radiator:

$p_{\text{rad}} = 48i^2 = 48(1.67)^2 = 133.33\,\text{W}$

Calculate the total power for this heating system:

$p_{\text{total}} = 3p_{\text{rad}} = 3(133.33) = 400\,\text{W}$

Each radiator has much less power than the radiators in Fig. 2.41, and the total power of this configuration is just 1/9th of the total power in Fig. 2.41.

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3

Simple Resistive Circuits

Assessment Problems

AP 3.1

Start from the right hand side of the circuit and make series and parallel combinations of the resistors until one equivalent resistor remains. Begin by combining the $6\,\Omega$ resistor and the $10\,\Omega$ resistor in series:

$$6\,\Omega + 10\,\Omega = 16\,\Omega$$

Now combine this $16\,\Omega$ resistor in parallel with the $64\,\Omega$ resistor:

$$16\,\Omega \| 64\,\Omega = \frac{(16)(64)}{16+64} = \frac{1024}{80} = 12.8\,\Omega$$

This equivalent $12.8\,\Omega$ resistor is in series with the $7.2\,\Omega$ resistor:

$$12.8\,\Omega + 7.2\,\Omega = 20\,\Omega$$

Finally, this equivalent $20\,\Omega$ resistor is in parallel with the $30\,\Omega$ resistor:

$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = \frac{600}{50} = 12\,\Omega$$

Thus, the simplified circuit is as shown:

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[a] With the simplified circuit we can use Ohm's law to find the voltage across both the current source and the $12\,\Omega$ equivalent resistor:

$$v = (12\,\Omega)(5\text{ A}) = 60\text{ V}$$

[b] Now that we know the value of the voltage drop across the current source, we can use the formula $p = -vi$ to find the power associated with the source:

$$p = -(60\text{ V})(5\text{ A}) = -300\text{ W}$$

Thus, the source delivers 300 W of power to the circuit.

[c] We now can return to the original circuit, shown in the first figure. In this circuit, $v = 60$ V, as calculated in part (a). This is also the voltage drop across the $30\,\Omega$ resistor, so we can use Ohm's law to calculate the current through this resistor:

$$i_A = \frac{60\text{ V}}{30\,\Omega} = 2\text{ A}$$

Now write a KCL equation at the upper left node to find the current i_B:

$$-5\text{ A} + i_A + i_B = 0 \qquad \text{so} \qquad i_B = 5\text{ A} - i_A = 5\text{ A} - 2\text{ A} = 3\text{ A}$$

Next, write a KVL equation around the outer loop of the circuit, using Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:

$$-v + 7.2i_B + 6i_C + 10i_C = 0$$

So $\quad 16i_C = v - 7.2i_B = 60\text{ V} - (7.2)(3) = 38.4\text{ V}$

Thus $\quad i_C = \dfrac{38.4}{16} = 2.4\text{ A}$

Now that we have the current through the $10\,\Omega$ resistor we can use the formula $p = Ri^2$ to find the power:

$$p_{10\,\Omega} = (10)(2.4)^2 = 57.6\text{ W}$$

AP 3.2

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[a] We can use voltage division to calculate the voltage v_o across the 75 kΩ resistor:

$$v_o(\text{no load}) = \frac{75{,}000}{75{,}000 + 25{,}000}(200\text{ V}) = 150\text{ V}$$

[b] When we have a load resistance of 150 kΩ then the voltage v_o is across the parallel combination of the 75 kΩ resistor and the 150 kΩ resistor. First, calculate the equivalent resistance of the parallel combination:

$$75\text{ k}\Omega \| 150\text{ k}\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50{,}000\,\Omega = 50\text{ k}\Omega$$

Now use voltage division to find v_o across this equivalent resistance:

$$v_o = \frac{50{,}000}{50{,}000 + 25{,}000}(200\text{ V}) = 133.3\text{ V}$$

[c] If the load terminals are short-circuited, the 75 kΩ resistor is effectively removed from the circuit, leaving only the voltage source and the 25 kΩ resistor. We can calculate the current in the resistor using Ohm's law:

$$i = \frac{200\text{ V}}{25\text{ k}\Omega} = 8\text{ mA}$$

Now we can use the formula $p = Ri^2$ to find the power dissipated in the 25 kΩ resistor:

$$p_{25k} = (25{,}000)(0.008)^2 = 1.6\text{ W}$$

[d] The power dissipated in the 75 kΩ resistor will be maximum at no load since v_o is maximum. In part (a) we determined that the no-load voltage is 150 V, so be can use the formula $p = v^2/R$ to calculate the power:

$$p_{75k}(\text{max}) = \frac{(150)^2}{75{,}000} = 0.3\text{ W}$$

AP 3.3

[a] We will write a current division equation for the current throught the 80Ω resistor and use this equation to solve for R:

$$i_{80\Omega} = \frac{R}{R + 40\,\Omega + 80\,\Omega}(20\text{ A}) = 4\text{ A} \qquad \text{so} \qquad 20R = 4(R + 120)$$

$$\text{Thus} \qquad 16R = 480 \qquad \text{and} \qquad R = \frac{480}{16} = 30\,\Omega$$

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[b] With $R = 30\,\Omega$ we can calculate the current through R using current division, and then use this current to find the power dissipated by R, using the formula $p = Ri^2$:

$$i_R = \frac{40+80}{40+80+30}(20\text{ A}) = 16\text{ A} \qquad \text{so} \qquad p_R = (30)(16)^2 = 7680\text{ W}$$

[c] Write a KVL equation around the outer loop to solve for the voltage v, and then use the formula $p = -vi$ to calculate the power delivered by the current source:

$$-v + (60\,\Omega)(20\text{ A}) + (30\,\Omega)(16\text{ A}) = 0 \qquad \text{so} \qquad v = 1200 + 480 = 1680\text{ V}$$

Thus, $\quad p_{\text{source}} = -(1680\text{ V})(20\text{ A}) = -33{,}600\text{ W}$

Thus, the current source generates 33,600 W of power.

AP 3.4

[a] First we need to determine the equivalent resistance to the right of the $40\,\Omega$ and $70\,\Omega$ resistors:

$$R_{\text{eq}} = 20\,\Omega \| 30\,\Omega \| (50\,\Omega + 10\,\Omega) \qquad \text{so} \qquad \frac{1}{R_{\text{eq}}} = \frac{1}{20\,\Omega} + \frac{1}{30\,\Omega} + \frac{1}{60\,\Omega} = \frac{1}{10\,\Omega}$$

Thus, $\quad R_{\text{eq}} = 10\,\Omega$

Now we can use voltage division to find the voltage v_o:

$$v_o = \frac{40}{40+10+70}(60\text{ V}) = 20\text{ V}$$

[b] The current through the $40\,\Omega$ resistor can be found using Ohm's law:

$$i_{40\Omega} = \frac{v_o}{40} = \frac{20\text{ V}}{40\,\Omega} = 0.5\text{ A}$$

This current flows from left to right through the $40\,\Omega$ resistor. To use current division, we need to find the equivalent resistance of the two parallel branches containing the $20\,\Omega$ resistor and the $50\,\Omega$ and $10\,\Omega$ resistors:

$$20\,\Omega \| (50\,\Omega + 10\,\Omega) = \frac{(20)(60)}{20+60} = 15\,\Omega$$

Now we use current division to find the current in the $30\,\Omega$ branch:

$$i_{30\Omega} = \frac{15}{15+30}(0.5\text{ A}) = 0.16667\text{ A} = 166.67\text{ mA}$$

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[c] We can find the power dissipated by the $50\,\Omega$ resistor if we can find the current in this resistor. We can use current division to find this current from the current in the $40\,\Omega$ resistor, but first we need to calculate the equivalent resistance of the $20\,\Omega$ branch and the $30\,\Omega$ branch:

$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = 12\,\Omega$$

Current division gives:

$$i_{50\Omega} = \frac{12}{12+50+10}(0.5\text{ A}) = 0.08333\text{ A}$$

Thus, $\quad p_{50\Omega} = (50)(0.08333)^2 = 0.34722\text{ W} = 347.22\text{ mW}$

AP 3.5 **[a]**

We can find the current i using Ohm's law:

$$i = \frac{1\text{ V}}{100\,\Omega} = 0.01\text{ A} = 10\text{ mA}$$

[b]

$$R_m = 50\,\Omega \| 5.555\,\Omega = 5\,\Omega$$

We can use the meter resistance to find the current using Ohm's law:

$$i_{\text{meas}} = \frac{1\text{ V}}{100\,\Omega + 5\,\Omega} = 0.009524 = 9.524\text{ mA}$$

AP 3.6 **[a]**

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Use voltage division to find the voltage v:

$$v = \frac{75{,}000}{75{,}000 + 15{,}000}(60\text{ V}) = 50\text{ V}$$

[b]

The meter resistance is a series combination of resistances:

$$R_m = 149{,}950 + 50 = 150{,}000\,\Omega$$

We can use voltage division to find v, but first we must calculate the equivalent resistance of the parallel combination of the 75 kΩ resistor and the voltmeter:

$$75{,}000\,\Omega \| 150{,}000\,\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50\text{ k}\Omega$$

Thus, $$v_{\text{meas}} = \frac{50{,}000}{50{,}000 + 15{,}000}(60\text{ V}) = 46.15\text{ V}$$

AP 3.7 **[a]** Using the condition for a balanced bridge, the products of the opposite resistors must be equal. Therefore,

$$100R_x = (1000)(150) \qquad \text{so} \qquad R_x = \frac{(1000)(150)}{100} = 1500\,\Omega = 1.5\text{ k}\Omega$$

[b] When the bridge is balanced, there is no current flowing through the meter, so the meter acts like an open circuit. This places the following branches in parallel: The branch with the voltage source, the branch with the series combination R_1 and R_3 and the branch with the series combination of R_2 and R_x. We can find the current in the latter two branches using Ohm's law:

$$i_{R_1,R_3} = \frac{5\text{ V}}{100\,\Omega + 150\,\Omega} = 20\text{ mA}; \qquad i_{R_2,R_x} = \frac{5\text{ V}}{1000 + 1500} = 2\text{ mA}$$

We can calculate the power dissipated by each resistor using the formula $p = Ri^2$:

$$p_{100\Omega} = (100\,\Omega)(0.02\text{ A})^2 = 40\text{ mW}$$

$$p_{150\Omega} = (150\,\Omega)(0.02\text{ A})^2 = 60\text{ mW}$$

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$$p_{1000\Omega} = (1000\,\Omega)(0.002\ \text{A})^2 = 4\ \text{mW}$$

$$p_{1500\Omega} = (1500\,\Omega)(0.002\ \text{A})^2 = 6\ \text{mW}$$

Since none of the power dissipation values exceeds 250 mW, the bridge can be left in the balanced state without exceeding the power-dissipating capacity of the resistors.

AP 3.8 Convert the three Y-connected resistors, $20\,\Omega$, $10\,\Omega$, and $5\,\Omega$ to three Δ-connected resistors R_a, R_b, and R_c. To assist you the figure below has both the Y-connected resistors and the Δ-connected resistors

$$R_\text{a} = \frac{(5)(10) + (5)(20) + (10)(20)}{20} = 17.5\,\Omega$$

$$R_\text{b} = \frac{(5)(10) + (5)(20) + (10)(20)}{10} = 35\,\Omega$$

$$R_\text{c} = \frac{(5)(10) + (5)(20) + (10)(20)}{5} = 70\,\Omega$$

The circuit with these new Δ-connected resistors is shown below:

From this circuit we see that the $70\,\Omega$ resistor is parallel to the $28\,\Omega$ resistor:

$$70\,\Omega \| 28\,\Omega = \frac{(70)(28)}{70 + 28} = 20\,\Omega$$

Also, the $17.5\,\Omega$ resistor is parallel to the $105\,\Omega$ resistor:

$$17.5\,\Omega \| 105\,\Omega = \frac{(17.5)(105)}{17.5 + 105} = 15\,\Omega$$

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Once the parallel combinations are made, we can see that the equivalent $20\,\Omega$ resistor is in series with the equivalent $15\,\Omega$ resistor, giving an equivalent resistance of $20\,\Omega + 15\,\Omega = 35\,\Omega$. Finally, this equivalent $35\,\Omega$ resistor is in parallel with the other $35\,\Omega$ resistor:

$$35\,\Omega \| 35\,\Omega = \frac{(35)(35)}{35+35} = 17.5\,\Omega$$

Thus, the resistance seen by the 2 A source is $17.5\,\Omega$, and the voltage can be calculated using Ohm's law:

$$v = (17.5\,\Omega)(2\text{ A}) = 35\text{ V}$$

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Problems

P 3.1 **[a]** From Ex. 3-1: $i_1 = 4$ A, $i_2 = 8$ A, $i_s = 12$ A
at node b: $-12 + 4 + 8 = 0$, at node d: $12 - 4 - 8 = 0$

[b] $v_1 = 4i_s = 48$ V $\quad v_3 = 3i_2 = 24$ V

$v_2 = 18i_1 = 72$ V $\quad v_4 = 6i_2 = 48$ V

loop abda: $-120 + 48 + 72 = 0$,
loop bcdb: $-72 + 24 + 48 = 0$,
loop abcda: $-120 + 48 + 24 + 48 = 0$

P 3.2 **[a]** $p_{4\Omega} = i_s^2 4 = (12)^2 4 = 576$ W $\quad p_{18\Omega} = (4)^2 18 = 288$ W

$p_{3\Omega} = (8)^2 3 = 192$ W $\quad p_{6\Omega} = (8)^2 6 = 384$ W

[b] $p_{120\text{V}}(\text{delivered}) = 120 i_s = 120(12) = 1440$ W

[c] $p_{\text{diss}} = 576 + 288 + 192 + 384 = 1440$ W

P 3.3 **[a]** The 5 kΩ and 7 kΩ resistors are in series. The simplified circuit is shown below:

[b] The 800 Ω and 1200 Ω resistors are in series, as are the 300 Ω and 200 Ω resistors. The simplified circuit is shown below:

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[**c**] The 35 Ω, 15 Ω, and 25 Ω resistors are in series. as are the 10 Ω and 40 Ω resistors. The simplified circuit is shown below:

[**d**] The 50 Ω and 90 Ω resistors are in series, as are the 80 Ω and 70 Ω resistors. The simplified circuit is shown below:

P 3.4 [**a**] The 36 Ω and 18 Ω resistors are in parallel. The simplified circuit is shown below:

[**b**] The 200 Ω and 120 Ω resistors are in parallel, as are the 210 Ω and 280 Ω resistors. The simplified circuit is shown below:

[**c**] The 100 kΩ, 150 kΩ, and 60 kΩ resistors are in parallel, as are the 75 kΩ and 50 kΩ resistors. The simplified circuit is shown below:

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[d] The 750 Ω and 500 Ω resistors are in parallel, as are the 1.5 kΩ and 3 kΩ resistors. The simplified circuit is shown below:

P 3.5 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[a] Circuit in Fig. P3.3(a):

$$R_{\text{eq}} = [(7000 + 5000)\|6000] + 8000 = 12{,}000\|6000 + 8000$$

$$= 4000 + 8000 = 12 \text{ k}\Omega$$

Circuit in Fig. P3.3(b):

$$R_{\text{eq}} = [500\|(800 + 1200)] + 300 + 200 = (500\|2000) + 300 + 200$$

$$= 400 + 300 + 200 = 900\,\Omega$$

Circuit in Fig. P3.3(c):

$$R_{\text{eq}} = (35 + 15 + 25)\|(10 + 40) = 75\|50 = 30\,\Omega$$

Circuit in Fig. P3.3(d):

$$R_{\text{eq}} = ([(70 + 80)\|100] + 50 + 90)\|300 = [(150\|100) + 50 + 90]\|300$$

$$= (60 + 50 + 90)\|300 = 200\|300 = 120\,\Omega$$

[b] Note that in every case, the power delivered by the source must equal the power absorbed by the equivalent resistance in the circuit. For the circuit in Fig. P3.3(a):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{18^2}{12{,}000} = 0.027 = 27 \text{ mW}$$

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For the circuit in Fig. P3.3(b):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{27^2}{900} = 0.81 = 810 \text{ mW}$$

For the circuit in Fig. P3.3(c):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{90^2}{30} = 270 \text{ W}$$

For the circuit in Fig. P3.3(d):

$$P = I_s^2(R_{\text{eq}}) = (0.03)^2(120) = 0.108 = 108 \text{ mW}$$

P 3.6 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[a] Circuit in Fig. P3.4(a):

$$R_{\text{eq}} = (36\|18) + 24 = 12 + 24 = 36\,\Omega$$

Circuit in Fig. P3.4(b):

$$R_{\text{eq}} = 200\|120\|[(210\|280) + 180] = 200\|120\|(120 + 180) = 200\|120\|300 = 60\,\Omega$$

Circuit in Fig. P3.4(c):

$$R_{\text{eq}} = (75 \text{ k}\|50 \text{ k}) + (100 \text{ k}\|150 \text{ k}\|60 \text{ k}) + 90 \text{ k} = 30 \text{ k} + 30 \text{ k} + 90 \text{ k} = 150 \text{ k}\Omega$$

Circuit in Fig. P3.4(d):

$$R_{\text{eq}} = [(600 + 900)\|750\|500] + (1500\|3000) + 2000 = (1500\|750\|500) + 1000 + 2000$$

$$= 250 + 1000 + 2000 = 3250 = 3.25 \text{ k}\Omega$$

[b] Note that in every case, the power delivered by the source must equal the power absorbed by the equivalent resistance in the circuit. For the circuit in Fig. P3.4(a):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{18^2}{36} = 9 \text{ W}$$

For the circuit in Fig. P3.4(b):

$$P = I_s^2(R_{\text{eq}}) = (0.03)^2(60) = 0.054 = 54 \text{ mW}$$

For the circuit in Fig. P3.4(c):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{60^2}{150{,}000} = 0.024 = 24 \text{ mW}$$

For the circuit in Fig. P3.4(d):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{65^2}{3250} = 1.3 \text{ W}$$

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P 3.7 **[a]** Circuit in Fig. P3.7(a):

$$R_{\text{eq}} = ([(15\|60) + (30\|45) + 20]\|50) + 25 + 10 = [(12 + 18 + 20)\|50] + 25 + 10$$

$$= (50\|50) + 25 + 10 = 25 + 25 + 10 = 60\,\Omega$$

Circuit in Fig. P3.7(b) – begin by simplifying the $75\,\Omega$ resistor and all resistors to its right:

$$[(18 + 12)\|60 + 30]\|75 = (30\|60 + 30)\|75 = (20 + 30)\|75 = 50\|75 = 30\,\Omega$$

Now simplify the remainder of the circuit:

$$R_{\text{eq}} = ([(30 + 20)\|50] + (20\|60))\|40 = [(50\|50) + 15]\|40 = (25 + 15)\|40$$

$$= 40\|40 = 20\,\Omega$$

Circuit in Fig. P3.7(c) – begin by simplifying the left and right sides of the circuit:

$$R_{\text{left}} = [(1800 + 1200)\|2000] + 300 = (3000\|2000) + 300 = 1200 + 300 = 1500\,\Omega$$

$$R_{\text{right}} = [(500 + 2500)\|1000] + 750 = (3000\|1000) + 750 = 750 + 750 = 1500\,\Omega$$

Now find the equivalent resistance seen by the source:

$$R_{\text{eq}} = (R_{\text{left}}\|R_{\text{right}}) + 250 + 3000 = (1500\|1500) + 250 + 3000$$

$$= 750 + 250 + 3000 = 4000 = 4\ \text{k}\Omega$$

Circuit in Fig. P3.7(d):

$$R_{\text{eq}} = ([(750 + 250)\|1000] + 100)\|([(150 + 600)\|500] + 300)$$

$$= [(1000\|1000) + 100]\|[(750\|500) + 300] = (500 + 100)\|(300 + 300)$$

$$= 600\|600 = 300\,\Omega$$

[b] Note that in every case, the power delivered by the source must equal the power absorbed by the equivalent resistance in the circuit. For the circuit in Fig. P3.7(a):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{30^2}{60} = 15\ \text{W}$$

For the circuit in Fig. P3.7(b):

$$P = I_s^2(R_{\text{eq}}) = (0.08)^2(20) = 0.128 = 128\ \text{mW}$$

For the circuit in Fig. P3.7(c):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{20^2}{4000} = 0.1 = 100\ \text{mW}$$

For the circuit in Fig. P3.7(d):

$$P = I_s^2(R_{\text{eq}}) = (0.05)^2(300) = 0.75 = 750\ \text{mW}$$

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P 3.8 **[a]** $R_{\text{ab}} = 24 + (90\|60) + 12 = 24 + 36 + 12 = 72\,\Omega$

[b] $R_{\text{ab}} = [(4\text{ k} + 6\text{ k} + 2\text{ k})\|8\text{ k}] + 5.2\text{ k} = (12\text{ k}\|8\text{ k}) + 5.2\text{ k} = 4.8\text{ k} + 5.2\text{ k} = 10\text{ k}\Omega$

[c] $R_{\text{ab}} = 1200\|720\|(320 + 480) = 1200\|720\|800 = 288\,\Omega$

P 3.9 Write an expression for the resistors in series and parallel from the right side of the circuit to the left. Then simplify the resulting expression from left to right to find the equivalent resistance.

[a] $R_{\text{ab}} = [(26 + 10)\|18 + 6]\|36 = (36\|18 + 6)\|36 = (12 + 6)\|36 = 18\|36 = 12\,\Omega$

[b] $R_{\text{ab}} = [(12 + 18)\|10\|15\|20 + 16]\|30 + 4 + 14 = (30\|10\|15\|20 + 16)\|30 + 4 + 14$

$= (4 + 16)\|30 + 4 + 14 = 20\|30 + 4 + 14 = 12 + 4 + 14 = 30\,\Omega$

[c] $R_{\text{ab}} = (500\|1500\|750 + 250)\|2000 + 1000 = (250 + 250)\|2000 + 1000$

$= 500\|2000 + 1000 = 400 + 1000 = 1400\,\Omega$

[d] Note that the wire on the far right of the circuit effectively removes the $60\,\Omega$ resistor!

$$\begin{aligned}
R_{\text{ab}} &= [([(30 + 18)\|16 + 28]\|40 + 20)\|24 + 25 + 10]\|50 \\
&= ([(48\|16 + 28)\|40 + 20]\|24 + 25 + 10)\|50 \\
&= ([(12 + 28)\|40 + 20]\|24 + 25 + 10)\|50 = [(40\|40 + 20)\|24 + 25 + 10]\|50 \\
&= [(20 + 20)\|24 + 25 + 10]\|50 = (40\|24 + 25 + 10)\|50 = (15 + 25 + 10)\|50 \\
&= 50\|50 = 25\,\Omega
\end{aligned}$$

P 3.10 **[a]** $R + R = 2R$

[b] $R + R + R + \cdots + R = nR$

[c] $R + R = 2R = 3000 \quad \text{so} \quad R = 1500 = 1.5\text{ k}\Omega$
This is a resistor from Appendix H.

[d] $nR = 4000; \qquad \text{so if } n = 4, \quad R = 1\text{ k}\Omega$
This is a resistor from Appendix H.

P 3.11 **[a]** $R_{\text{eq}} = R\|R = \dfrac{R^2}{2R} = \dfrac{R}{2}$

[b]
$$\begin{aligned}
R_{\text{eq}} &= R\|R\|R\|\cdots\|R \qquad (n\ R\text{'s}) \\
&= R\|\frac{R}{n-1} \\
&= \frac{R^2/(n-1)}{R + R/(n-1)} = \frac{R^2}{nR} = \frac{R}{n}
\end{aligned}$$

[c] $\dfrac{R}{2} = 5000 \quad \text{so} \quad R = 10\text{ k}\Omega$
This is a resistor from Appendix H.

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[d] $\dfrac{R}{n} = 4000 \quad \text{so} \quad R = 4000n$

If $n = 3 \qquad r = 4000(3) = 12\ \text{k}\Omega$

This is a resistor from Appendix H. So put three 12k resistors in parallel to get 4kΩ.

P 3.12 **[a]** $v_o = \dfrac{160(3300)}{(4700 + 3300)} = 66\ \text{V}$

[b] $i = 160/8000 = 20\ \text{mA}$

$$P_{R_1} = (400 \times 10^{-6})(4.7 \times 10^3) = 1.88\ \text{W}$$

$$P_{R_2} = (400 \times 10^{-6})(3.3 \times 10^3) = 1.32\ \text{W}$$

[c] Since R_1 and R_2 carry the same current and $R_1 > R_2$ to satisfy the voltage requirement, first pick R_1 to meet the 0.5 W specification

$$i_{R_1} = \frac{160 - 66}{R_1}, \qquad \text{Therefore,}\ \left(\frac{94}{R_1}\right)^2 R_1 \le 0.5$$

$$\text{Thus,}\ R_1 \ge \frac{94^2}{0.5} \qquad \text{or} \qquad R_1 \ge 17{,}672\,\Omega$$

Now use the voltage specification:

$$\frac{R_2}{R_2 + 17{,}672}(160) = 66$$

$$\text{Thus,}\ R_2 = 12{,}408\,\Omega$$

P 3.13 $4 = \dfrac{20R_2}{R_2 + 40} \quad \text{so} \quad R_2 = 10\,\Omega$

$$3 = \frac{20R_\text{e}}{40 + R_\text{e}} \quad \text{so} \quad R_\text{e} = \frac{120}{17}\,\Omega$$

$$\text{Thus,} \quad \frac{120}{17} = \frac{10R_\text{L}}{10 + R_\text{L}} \quad \text{so} \quad R_\text{L} = 24\,\Omega$$

P 3.14 **[a]** $v_o = \dfrac{40R_2}{R_1 + R_2} = 8 \quad \text{so} \quad R_1 = 4R_2$

$$\text{Let}\ R_\text{e} = R_2 \| R_\text{L} = \frac{R_2 R_\text{L}}{R_2 + R_\text{L}}$$

$$v_o = \frac{40R_\text{e}}{R_1 + R_\text{e}} = 7.5 \quad \text{so} \quad R_1 = 4.33R_\text{e}$$

$$\text{Then,}\ 4R_2 = 4.33R_\text{e} = \frac{4.33(3600R_2)}{3600 + R_2}$$

$$\text{Thus,}\ R_2 = 300\,\Omega \quad \text{and} \quad R_1 = 4(300) = 1200\,\Omega$$

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[b] The resistor that must dissipate the most power is R_1, as it has the largest resistance and carries the same current as the parallel combination of R_2 and the load resistor. The power dissipated in R_1 will be maximum when the voltage across R_1 is maximum. This will occur when the voltage divider has a resistive load. Thus,

$$v_{R_1} = 40 - 7.5 = 32.5 \text{ V}$$

$$p_{R_1} = \frac{32.5^2}{1200} = 880.2 \text{ m W}$$

Thus the minimum power rating for all resistors should be 1 W.

P 3.15 Refer to the solution to Problem 3.16. The voltage divider will reach the maximum power it can safely dissipate when the power dissipated in R_1 equals 1 W. Thus,

$$\frac{v_{R_1}^2}{1200} = 1 \quad \text{so} \quad v_{R_1} = 34.64 \text{ V}$$

$$v_o = 40 - 34.64 = 5.36 \text{ V}$$

$$\text{So, } \frac{40R_\text{e}}{1200 + R_\text{e}} = 5.36 \quad \text{and} \quad R_\text{e} = 185.68\,\Omega$$

$$\text{Thus, } \frac{(300)R_\text{L}}{300 + R_\text{L}} = 185.68 \quad \text{and} \quad R_\text{L} = 487.26\,\Omega$$

The minimum value for R_L from Appendix H is $560\,\Omega$.

P 3.16 $R_\text{eq} = 10\|[6 + 5\|(8 + 12)] = 10\|(6 + 5\|20) = 10\|(6 + 4) = 5\,\Omega$

$$v_{10\text{A}} = v_{10\Omega} = (10 \text{ A})(5\,\Omega) = 50 \text{ V}$$

Using voltage division:

$$v_{5\Omega} = \frac{5\|(8 + 12)}{6 + 5\|(8 + 12)}(50) = \frac{4}{6 + 4}(50) = 20 \text{ V}$$

$$\text{Thus, } p_{5\Omega} = \frac{v_{5\Omega}^2}{5} = \frac{20^2}{5} = 80 \text{ W}$$

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P 3.17 **[a]**

$$R_{\text{eq}} = (10+20)\|[12+(90\|10)] = 30\|15 = 10\,\Omega$$

$$v_{2.4\text{A}} = 10(2.4) = 24 \text{ V}$$

$$v_o = v_{20\Omega} = \frac{20}{10+20}(24) = 16 \text{ V}$$

$$v_{90\Omega} = \frac{90\|10}{6+(90\|10)}(24) = \frac{9}{15}(24) = 14.4 \text{ V}$$

$$i_o = \frac{14.4}{90} = 0.16 \text{ A}$$

[b] $p_{6\Omega} = \dfrac{(v_{2.4\text{A}} - v_{90\Omega})^2}{6} = \dfrac{(24-14.4)^2}{6} = 15.36 \text{ W}$

[c] $p_{2.4\text{A}} = -(2.4)(24) = -57.6 \text{ W}$

Thus the power developed by the current source is 57.6 W.

P 3.18 Begin by using KCL at the top node to relate the branch currents to the current supplied by the source. Then use the relationships among the branch currents to express every term in the KCL equation using just i_2:

$$0.05 = i_1 + i_2 + i_3 + i_4 = 0.6i_2 + i_2 + 2i_2 + 4i_1 = 0.6i_2 + i_2 + 2i_2 + 4(0.6i_2) = 6i_2$$

Therefore,

$$i_2 = 0.05/6 = 0.00833 = 8.33 \text{ mA}$$

Find the remaining currents using the value of i_2:

$$i_1 = 0.6i_2 = 0.6(0.00833) = 0.005 = 5 \text{ mA}$$

$$i_3 = 2i_2 = 2(0.00833) = 0.01667 = 16.67 \text{ mA}$$

$$i_4 = 4i_1 = 4(0.005) = 0.02 = 20 \text{ mA}$$

Since the resistors are in parallel, the same voltage, 25 V, appears across each of them. We know the current and the voltage for every resistor so we can use Ohm's law to calculate the values of the resistors:

$$R_1 = 25/i_1 = 25/0.005 = 5000 = 5 \text{ k}\Omega$$

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$R_2 = 25/i_2 = 25/0.00833 = 3000 = 3\ \text{k}\Omega$

$R_3 = 25/i_3 = 25/0.01667 = 1500 = 1.5\ \text{k}\Omega$

$R_4 = 25/i_4 = 25/0.02 = 1250 = 1.25\ \text{k}\Omega$

The resulting circuit is shown below:

P 3.19 $\dfrac{(24)^2}{R_1 + R_2 + R_3} = 80,$ Therefore, $R_1 + R_2 + R_3 = 7.2\,\Omega$

$$\frac{(R_1 + R_2)24}{(R_1 + R_2 + R_3)} = 12$$

Therefore, $2(R_1 + R_2) = R_1 + R_2 + R_3$

Thus, $R_1 + R_2 = R_3;$ $2R_3 = 7.2;$ $R_3 = 3.6\,\Omega$

$$\frac{R_2(24)}{R_1 + R_2 + R_3} = 5$$

$4.8R_2 = R_1 + R_2 + 3.6 = 7.2$

Thus, $R_2 = 1.5\,\Omega;$ $R_1 = 7.2 - R_2 - R_3 = 2.1\,\Omega$

P 3.20 [**a**]

$20\ \text{k}\Omega + 40\ \text{k}\Omega = 60\ \text{k}\Omega$

$30\ \text{k}\Omega \| 60\ \text{k}\Omega = 20\ \text{k}\Omega$

$$v_{o1} = \frac{20{,}000}{(10{,}000 + 20{,}000)}(180) = 120\ \text{V}$$

$$v_o = \frac{40{,}000}{60{,}000}(v_{o1}) = 80\ \text{V}$$

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[b]

$$i = \frac{180}{40{,}000} = 4.5 \text{ mA}$$

$$30{,}000i = 135 \text{ V}$$

$$v_o = \frac{40{,}000}{60{,}000}(135) = 90 \text{ V}$$

[c] It removes the loading effect of the second voltage divider on the first voltage divider. Observe that the open circuit voltage of the first divider is

$$v'_{o1} = \frac{30{,}000}{40{,}000}(180) = 135 \text{ V}$$

Now note this is the input voltage to the second voltage divider when the current-controlled voltage source is used.

P 3.21 **[a]** At no load: $v_o = kv_s = \dfrac{R_2}{R_1 + R_2} v_s.$

At full load: $v_o = \alpha v_s = \dfrac{R_\text{e}}{R_1 + R_\text{e}} v_s,$ where $R_\text{e} = \dfrac{R_o R_2}{R_o + R_2}$

$$\text{Therefore} \quad k = \frac{R_2}{R_1 + R_2} \quad \text{and} \quad R_1 = \frac{(1-k)}{k} R_2$$

$$\alpha = \frac{R_\text{e}}{R_1 + R_\text{e}} \quad \text{and} \quad R_1 = \frac{(1-\alpha)}{\alpha} R_\text{e}$$

Thus $\left(\dfrac{1-\alpha}{\alpha}\right)\left[\dfrac{R_2 R_o}{R_o + R_2}\right] = \dfrac{(1-k)}{k} R_2$

Solving for R_2 yields $R_2 = \dfrac{(k-\alpha)}{\alpha(1-k)} R_o$

Also, $R_1 = \dfrac{(1-k)}{k} R_2 \quad \therefore \quad R_1 = \dfrac{(k-\alpha)}{\alpha k} R_o$

[b] $R_1 = \left(\dfrac{0.05}{0.68}\right) R_o = 2.5 \text{ k}\Omega$

$R_2 = \left(\dfrac{0.05}{0.12}\right) R_o = 14.167 \text{ k}\Omega$

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[c]

Maximum dissipation in R_2 occurs at no load, therefore,

$$P_{R_2(\max)} = \frac{[(60)(0.85)]^2}{14{,}167} = 183.6 \text{ mW}$$

Maximum dissipation in R_1 occurs at full load.

$$P_{R_1(\max)} = \frac{[60 - 0.80(60)]^2}{2500} = 57.60 \text{ mW}$$

[d]

$$P_{R_1} = \frac{(60)^2}{2500} = 1.44 \text{ W} = 1440 \text{ mW}$$

$$P_{R_2} = \frac{(0)^2}{14{,}167} = 0 \text{ W}$$

P 3.22 **[a]** Let v_o be the voltage across the parallel branches, positive at the upper terminal, then

$$i_g = v_oG_1 + v_oG_2 + \cdots + v_oG_N = v_o(G_1 + G_2 + \cdots + G_N)$$

It follows that $\quad v_o = \dfrac{i_g}{(G_1 + G_2 + \cdots + G_N)}$

The current in the kth branch is $\quad i_k = v_oG_k; \qquad$ Thus,

$$i_k = \frac{i_gG_k}{[G_1 + G_2 + \cdots + G_N]}$$

[b] $i_5 = \dfrac{40(0.2)}{2 + 0.2 + 0.125 + 0.1 + 0.05 + 0.025} = 3.2 \text{ A}$

P 3.23 **[a]** The equivalent resistance of the 6 kΩ resistor and the resistors to its right is

$$6 \text{ k}\|(5 \text{ k} + 7 \text{ k}) = 6 \text{ k}\|12 \text{ k} = 4 \text{ k}\Omega$$

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Using voltage division,

$$v_{6\text{k}} = \frac{4000}{8000 + 4000}(18) = 6 \text{ V}$$

[b] $v_{5\text{k}} = \dfrac{5000}{5000 + 7000}(6) = 2.5 \text{ V}$

P 3.24 **[a]** The equivalent resistance of the $100\,\Omega$ resistor and the resistors to its right is

$$100\|(80 + 70) = 100\|150 = 60\,\Omega$$

Using current division,

$$i_{50} = \frac{(50 + 90 + 60)\|300}{50 + 90 + 60}(0.03) = \frac{120}{200}(0.03) = 0.018 = 18 \text{ mA}$$

[b] $v_{70} = \dfrac{(80 + 70)\|100}{80 + 70}(0.018) = \dfrac{60}{150}(0.018) = 0.0072 = 7.2 \text{ mA}$

P 3.25 **[a]** The equivalent resistance of the circuit to the right of, and including, the $50\,\Omega$ resistor is

$$[(60\|15) + (45\|30) + 20]\|50 = 25\,\Omega$$

Thus by voltage division,

$$v_{25} = \frac{25}{25 + 25 + 10}(30) = 12.5 \text{ V}$$

[b] The current in the $25\,\Omega$ resistor can be found from its voltage using Ohm's law:

$$i_{25} = \frac{12.5}{25} = 0.5 \text{ A}$$

[c] The current in the $25\,\Omega$ resistor divides between two branches – one containing $50\,\Omega$ and one containing $(45\|30) + (15\|60) + 20 = 50\,\Omega$. Using current division,

$$i_{50} = \frac{50\|50}{50}(i_{25}) = \frac{25}{50}(0.5) = 0.25 \text{ A}$$

[d] The voltage drop across the $50\,\Omega$ resistor can be found using Ohm's law:

$$v_{50} = 50 i_{50} = 50(0.25) = 12.5 \text{ V}$$

[e] The voltage v_{50} divides across the equivalent resistance $(45\|30)\,\Omega$, the equivalent resistance $(15\|60)\,\Omega$, and the $20\,\Omega$ resistor. Using voltage division,

$$v_{60} = v_{15\|60} = \frac{15\|60}{(15\|60) + (30\|45) + 20}(12.5) = \frac{12}{12 + 18 + 20}(12.5) = 3 \text{ V}$$

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P 3.26 **[a]** The equivalent resistance to the right of the $36\,\Omega$ resistor is

$$6 + [18\|(26 + 10)] = 18\,\Omega$$

By current division,

$$i_{36} = \frac{36\|18}{36}(0.45) = 0.15 = 150 \text{ mA}$$

[b] Using Ohm's law,

$$v_{36} = 36i_{36} = 36(0.15) = 5.4 \text{ V}$$

[c] Before using voltage division, find the equivalent resistance of the $18\,\Omega$ resistor and the resistors to its right:

$$18\|(26 + 10) = 12\,\Omega$$

Now use voltage division:

$$v_{18} = \frac{12}{12 + 6}(5.4) = 3.6 \text{ V}$$

[d] $v_{10} = \dfrac{10}{10 + 26}(3.6) = 1 \text{ V}$

P 3.27 **[a]** Begin by finding the equivalent resistance of the $30\,\Omega$ resistor and all resistors to its right:

$$([(12 + 18)\|10\|15\|20] + 16)\|30 = 12\,\Omega$$

Now use voltage division to find the voltage across the $4\,\Omega$ resistor:

$$v_4 = \frac{4}{4 + 12 + 14}(6) = 0.8 \text{ V}$$

[b] Use Ohm's law to find the current in the $4\,\Omega$ resistor:

$$i_4 = v_4/4 = 0.8/4 = 0.2 \text{ A}$$

[c] Begin by finding the equivalent resistance of all resistors to the right of the $30\,\Omega$ resistor:

$$[(12 + 18)\|10\|15\|20] + 16 = 20\,\Omega$$

Now use current division:

$$i_{16} = \frac{30\|20}{20}(0.2) = 0.12 = 120 \text{ mA}$$

[d] Note that the current in the $16\,\Omega$ resistor divides among four branches – $20\,\Omega, 15\,\Omega, 10\,\Omega,$ and $(12 + 18)\,\Omega$:

$$i_{10} = \frac{20\|15\|10\|(12 + 18)}{10}(0.12) = 0.048 = 48 \text{ mA}$$

[e] Use Ohm's law to find the voltage across the $10\,\Omega$ resistor:

$$v_{10} = 10i_{10} = 10(0.048) = 0.48 \text{ V}$$

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[f] $v_{18} = \dfrac{18}{12+18}(0.48) = 0.288 = 288 \text{ mV}$

P 3.28 **[a]** $v_{6\text{k}} = \dfrac{6}{6+2}(18) = 13.5 \text{ V}$

$$v_{3\text{k}} = \frac{3}{3+9}(18) = 4.5 \text{ V}$$

$$v_x = v_{6\text{k}} - v_{3\text{k}} = 13.5 - 4.5 = 9 \text{ V}$$

[b] $v_{6\text{k}} = \dfrac{6}{8}(V_s) = 0.75V_s$

$$v_{3\text{k}} = \frac{3}{12}(V_s) = 0.25V_s$$

$$v_x = (0.75V_s) - (0.25V_s) = 0.5V_s$$

P 3.29 Use current division to find the current in the branch containing the 10 k and 15 k resistors, from bottom to top

$$i_{10\text{k}+15\text{k}} = \frac{(10 \text{ k} + 15 \text{ k})\|(3 \text{ k} + 12 \text{ k})}{10 \text{ k} + 15 \text{ k}}(18) = 6.75 \text{ mA}$$

Use Ohm's law to find the voltage drop across the 15 k resistor, positive at the top:

$$v_{15\text{k}} = -(6.75 \text{ m})(15 \text{ k}) = -101.25 \text{ V}$$

Find the current in the branch containing the 3 k and 12 k resistors, from bottom to top

$$i_{10\text{k}+15\text{k}} = \frac{(10 \text{ k} + 15 \text{ k})\|(3 \text{ k} + 12 \text{ k})}{3 \text{ k} + 12 \text{ k}}(18) = 11.25 \text{ mA}$$

Use Ohm's law to find the voltage drop across the 12 k resistor, positive at the top:

$$v_{12\text{k}} = -(12 \text{ k})(11.25 \text{ m}) = -135 \text{ V}$$

$$v_o = v_{15\text{k}} - v_{12\text{k}} = -101.25 - (-135) = 33.75 \text{ V}$$

P 3.30 The equivalent resistance of the circuit to the right of the $90\,\Omega$ resistor is

$$R_{\text{eq}} = [(150\|75) + 40]\|(30 + 60) = 90\|90 = 45\,\Omega$$

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Use voltage division to find the voltage drop between the top and bottom nodes:

$$v_{\text{Req}} = \frac{45}{45+90}(3) = 1 \text{ V}$$

Use voltage division again to find v_1 from v_{Req}:

$$v_1 = \frac{150\|75}{150\|75+40}(1) = \frac{50}{90}(1) = \frac{5}{9} \text{ V}$$

Use voltage division one more time to find v_2 from v_{Req}:

$$v_2 = \frac{30}{30+60}(1) = \frac{1}{3} \text{ V}$$

P 3.31 Find the equivalent resistance of all the resistors except the $2\,\Omega$:

$$5\,\Omega\|20\,\Omega = 4\,\Omega; \qquad 4\,\Omega + 6\,\Omega = 10\,\Omega; \qquad 10\|(15+12+13) = 8\,\Omega = R_{\text{eq}}$$

Use Ohm's law to find the current i_g:

$$i_g = \frac{125}{2+R_{\text{eq}}} = \frac{125}{2+8} = 12.5 \text{ A}$$

Use current division to find the current in the $6\,\Omega$ resistor:

$$i_{6\Omega} = \frac{8}{6+4}(12.5) = 10 \text{ A}$$

Use current division again to find i_o:

$$i_o = \frac{5\|20}{20} i_{6\Omega} = \frac{5\|20}{20}(10) = 2 \text{ A}$$

P 3.32 Use current division to find the current in the $8\,\Omega$ resistor. Begin by finding the equivalent resistance of the $8\,\Omega$ resistor and all resistors to its right:

$$R_{\text{eq}} = ([(20\|80)+4]\|30) + 8 = 20\,\Omega$$

$$i_8 = \frac{60\|R_{\text{eq}}}{R_{\text{eq}}}(0.25) = \frac{60\|20}{20}(0.25) = 0.1875 = 187.5 \text{ mA}$$

Use current division to find i_1 from i_8:

$$i_1 = \frac{30\|[4+(80\|20)]}{30}(i_8) = \frac{30\|20}{30}(0.1875) = 0.075 = 75 \text{ mA}$$

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Use current division to find $i_{4\Omega}$ from i_8:

$$i_{4\Omega} = \frac{30\|[4 + (80\|20)]}{4 + (80\|20)}(i_8) = \frac{30\|20}{20}(0.1875) = 0.1125 = 112.5 \text{ mA}$$

Finally, use current division to find i_2 from $i_{4\Omega}$:

$$i_2 = \frac{80\|20}{20}(i_{4\Omega}) = \frac{80\|20}{20}(0.1125) = 0.09 = 90 \text{ mA}$$

P 3.33 The current in the shunt resistor at full-scale deflection is $i_{\text{A}} = i_{\text{fullscale}} - 3 \times 10^{-3}$ A. The voltage across R_{A} at full-scale deflection is always 150 mV; therefore,

$$R_{\text{A}} = \frac{150 \times 10^{-3}}{i_{\text{fullscale}} - 3 \times 10^{-3}} = \frac{150}{1000 i_{\text{fullscale}} - 3}$$

[a] $R_{\text{A}} = \dfrac{150}{5000 - 3} = 30.018 \text{ m}\Omega$

[b] Let R_m be the equivalent ammeter resistance:

$$R_m = \frac{0.15}{5} = 0.03 = 30 \text{ m}\Omega$$

[c] $R_{\text{A}} = \dfrac{150}{100 - 3} = 1.546\,\Omega$

[d] $R_m = \dfrac{0.15}{0.1} = 1.5\,\Omega$

P 3.34

Original meter: $R_{\text{e}} = \dfrac{50 \times 10^{-3}}{5} = 0.01\,\Omega$

Modified meter: $R_{\text{e}} = \dfrac{(0.02)(0.01)}{0.03} = 0.00667\,\Omega$

$\therefore \quad (I_{\text{fs}})(0.00667) = 50 \times 10^{-3}$

$\therefore \quad I_{\text{fs}} = 7.5 \text{ A}$

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P 3.35 At full scale the voltage across the shunt resistor will be 200 mV; therefore the power dissipated will be

$$P_{\mathrm{A}} = \frac{(200 \times 10^{-3})^2}{R_{\mathrm{A}}}$$

Therefore $R_{\mathrm{A}} \geq \dfrac{(200 \times 10^{-3})^2}{1.0} = 40 \text{ m}\Omega$

Otherwise the power dissipated in R_{A} will exceed its power rating of 1 W
When $R_{\mathrm{A}} = 40 \text{ m}\Omega$, the shunt current will be

$$i_{\mathrm{A}} = \frac{200 \times 10^{-3}}{40 \times 10^{-3}} = 5 \text{ A}$$

The measured current will be $i_{\mathrm{meas}} = 5 + 0.002 = 5.002$ A
$\therefore$ Full-scale reading for practical purposes is 5 A.

P 3.36 **[a]** The model of the ammeter is an ideal ammeter in parallel with a resistor whose resistance is given by

$$R_m = \frac{100\,\mathrm{mV}}{2\,\mathrm{mA}} = 50\,\Omega.$$

We can calculate the current through the real meter using current division:

$$i_m = \frac{(25/12)}{50 + (25/12)}(i_{\mathrm{meas}}) = \frac{25}{625}(i_{\mathrm{meas}}) = \frac{1}{25} i_{\mathrm{meas}}$$

[b] At full scale, $i_{\mathrm{meas}} = 5$ A and $i_{\mathrm{m}} = 2$ mA so $5 - 0.002 = 4998$ mA flows throught the resistor R_{A}:

$$R_{\mathrm{A}} = \frac{100 \text{ mV}}{4998 \text{ mA}} = \frac{100}{4998}\,\Omega$$

$$i_m = \frac{(100/4998)}{50 + (100/4998)}(i_{\mathrm{meas}}) = \frac{1}{2500}(i_{\mathrm{meas}})$$

[c] Yes

P 3.37 For all full-scale readings the total resistance is

$$R_V + R_{\mathrm{movement}} = \frac{\text{full-scale reading}}{10^{-3}}$$

We can calculate the resistance of the movement as follows:

$$R_{\mathrm{movement}} = \frac{20 \text{ mV}}{1 \text{ mA}} = 20\,\Omega$$

Therefore, $\quad R_V = 1000 \text{ (full-scale reading)} - 20$

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[a] $R_V = 1000(50) - 20 = 49,980\,\Omega$

[b] $R_V = 1000(5) - 20 = 4980\,\Omega$

[c] $R_V = 1000(0.25) - 20 = 230\,\Omega$

[d] $R_V = 1000(0.025) - 20 = 5\,\Omega$

P 3.38 **[a]** $v_{\text{meas}} = (50 \times 10^{-3})[15\|45\|(4980 + 20)] = 0.5612 \text{ V}$

[b] $v_{\text{true}} = (50 \times 10^{-3})(15\|45) = 0.5625 \text{ V}$

$$\%\text{ error } = \left(\frac{0.5612}{0.5625} - 1\right) \times 100 = -0.224\%$$

P 3.39 The measured value is $60\|20.1 = 15.05618\,\Omega$.

$$i_g = \frac{50}{(15.05618 + 10)} = 1.995526 \text{ A}; \qquad i_{\text{meas}} = \frac{60}{80.1}(1.996) = 1.494768 \text{ A}$$

The true value is $60\|20 = 15\,\Omega$.

$$i_g = \frac{50}{(15 + 10)} = 2 \text{ A}; \qquad i_{\text{true}} = \frac{60}{80}(2) = 1.5 \text{ A}$$

$$\%\text{error } = \left[\frac{1.494768}{1.5} - 1\right] \times 100 = -0.34878\% \approx -0.35\%$$

P 3.40 Begin by using current division to find the actual value of the current i_o:

$$i_{\text{true}} = \frac{15}{15 + 45}(50 \text{ mA}) = 12.5 \text{ mA}$$

$$i_{\text{meas}} = \frac{15}{15 + 45 + 0.1}(50 \text{ mA}) = 12.4792 \text{ mA}$$

$$\%\text{ error } = \left[\frac{12.4792}{12.5} - 1\right] 100 = -0.166389\% \approx -0.17\%$$

P 3.41 **[a]**

$$20 \times 10^3 i_1 + 80 \times 10^3 (i_1 - i_{\text{B}}) = 7.5$$

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$$80 \times 10^3(i_1 - i_B) = 0.6 + 40i_B(0.2 \times 10^3)$$

$$\therefore \quad 100i_1 - 80i_B = 7.5 \times 10^{-3}$$

$$80i_1 - 88i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 225\,\mu\text{A}$

[b] With the insertion of the ammeter the equations become

$$100i_1 - 80i_B = 7.5 \times 10^{-3} \quad \text{(no change)}$$

$$80 \times 10^3(i_1 - i_B) = 10^3 i_B + 0.6 + 40i_B(200)$$

$$80i_1 - 89i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 216\,\mu\text{A}$

[c] $\%\text{ error } = \left(\dfrac{216}{225} - 1\right)100 = -4\%$

P 3.42 **[a]** Since the unknown voltage is greater than either voltmeter's maximum reading, the only possible way to use the voltmeters would be to connect them in series.

[b]

$$R_{m1} = (300)(900) = 270\text{ k}\Omega; \qquad R_{m2} = (150)(1200) = 180\text{ k}\Omega$$

$$\therefore \quad R_{m1} + R_{m2} = 450\text{ k}\Omega$$

$$i_{1\text{ max}} = \frac{300}{270} \times 10^{-3} = 1.11\text{ mA}; \qquad i_{2\text{ max}} = \frac{150}{180} \times 10^{-3} = 0.833\text{ mA}$$

$$\therefore \quad i_{\max} = 0.833\text{ mA since meters are in series}$$

$$v_{\max} = (0.833 \times 10^{-3})(270 + 180)10^3 = 375\text{ V}$$

Thus the meters can be used to measure the voltage.

[c] $i_m = \dfrac{320}{450 \times 10^3} = 0.711\text{ mA}$

$$v_{m1} = (0.711)(270) = 192\text{ V}; \qquad v_{m2} = (0.711)(180) = 128\text{ V}$$

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P 3.43 The current in the series-connected voltmeters is

$$i_m = \frac{205.2}{270{,}000} = \frac{136.8}{180{,}000} = 0.76 \text{ mA}$$

$$v_{50\text{ k}\Omega} = (0.76 \times 10^{-3})(50{,}000) = 38 \text{ V}$$

$$V_{\text{power supply}} = 205.2 + 136.8 + 38 = 380 \text{ V}$$

P 3.44 $R_{\text{meter}} = R_m + R_{\text{movement}} = \dfrac{500 \text{ V}}{1 \text{ mA}} = 1000 \text{ k}\Omega$

$$v_{\text{meas}} = (50 \text{ k}\Omega \| 250 \text{ k}\Omega \| 1000 \text{ k}\Omega)(10 \text{ mA}) = (40 \text{ k}\Omega)(10 \text{ mA}) = 400 \text{ V}$$

$$v_{\text{true}} = (50 \text{ k}\Omega \| 250 \text{ k}\Omega)(10 \text{ mA}) = (41.67 \text{ k}\Omega)(10 \text{ mA}) = 416.67 \text{ V}$$

$$\%\text{ error } = \left(\frac{400}{416.67} - 1\right) 100 = -4\%$$

P 3.45 **[a]** $v_{\text{meter}} = 180 \text{ V}$

[b] $R_{\text{meter}} = (100)(200) = 20 \text{ k}\Omega$

$$20 \| 70 = 15.555556 \text{ k}\Omega$$

$$v_{\text{meter}} = \frac{180}{35.555556} \times 15.555556 = 78.75 \text{ V}$$

[c] $20 \| 20 = 10 \text{ k}\Omega$

$$v_{\text{meter}} = \frac{180}{80}(10) = 22.5 \text{ V}$$

[d] $v_{\text{meter a}} = 180 \text{ V}$

$$v_{\text{meter b}} + v_{\text{meter c}} = 101.26 \text{ V}$$

No, because of the loading effect.

P 3.46 **[a]**

$$R_1 = (100/2)10^3 = 50 \text{ k}\Omega$$

$$R_2 = (10/2)10^3 = 5 \text{ k}\Omega$$

$$R_3 = (1/2)10^3 = 500\,\Omega$$

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[b] Let i_a = actual current in the movement

i_d = design current in the movement

Then % error $= \left(\dfrac{i_a}{i_d} - 1\right) 100$

For the 100 V scale:

$$i_a = \frac{100}{50{,}000 + 25} = \frac{100}{50{,}025}, \qquad i_d = \frac{100}{50{,}000}$$

$$\frac{i_a}{i_d} = \frac{50{,}000}{50{,}025} = 0.9995 \qquad \%\text{ error } = (0.9995 - 1)100 = -0.05\%$$

For the 10 V scale:

$$\frac{i_a}{i_d} = \frac{5000}{5025} = 0.995 \qquad \%\text{ error } = (0.995 - 1.0)100 = -0.4975\%$$

For the 1 V scale:

$$\frac{i_a}{i_d} = \frac{500}{525} = 0.9524 \qquad \%\text{ error } = (0.9524 - 1.0)100 = -4.76\%$$

P 3.47 From the problem statement we have

$$50 = \frac{V_s(10)}{10 + R_s} \qquad (1) \quad V_s \text{ in mV; } R_s \text{ in M}\Omega$$

$$48.75 = \frac{V_s(6)}{6 + R_s} \qquad (2)$$

[a] From Eq (1) $\quad 10 + R_s = 0.2V_s$

$\therefore \quad R_s = 0.2V_s - 10$

Substituting into Eq (2) yields

$$48.75 = \frac{6V_s}{0.2V_s - 4} \qquad \text{or} \qquad V_s = 52 \text{ mV}$$

[b] From Eq (1)

$$50 = \frac{520}{10 + R_s} \qquad \text{or} \qquad 50R_s = 20$$

So $R_s = 400$ kΩ

P 3.48 **[a]** $R_{\text{movement}} = 50\,\Omega$

$$R_1 + R_{\text{movement}} = \frac{30}{1 \times 10^{-3}} = 30 \text{ k}\Omega \qquad \therefore \quad R_1 = 29{,}950\,\Omega$$

$$R_2 + R_1 + R_{\text{movement}} = \frac{150}{1 \times 10^{-3}} = 150 \text{ k}\Omega \qquad \therefore \quad R_2 = 120 \text{ k}\Omega$$

$$R_3 + R_2 + R_1 + R_{\text{movement}} = \frac{300}{1 \times 10^{-3}} = 300 \text{ k}\Omega$$

$\therefore \quad R_3 = 150$ kΩ

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[b]

$$v_1 = (0.96 \text{ m})(150 \text{ k}) = 144 \text{ V}$$

$$i_{\text{move}} = \frac{144}{120 + 29.95 + 0.05} = 0.96 \text{ mA}$$

$$i_1 = \frac{144}{750 \text{ k}} = 0.192 \text{ mA}$$

$$i_2 = i_{\text{move}} + i_1 = 0.96 \text{ m} + 0.192 \text{ m} = 1.152 \text{ mA}$$

$$v_{\text{meas}} = v_x = 144 + 150 i_2 = 316.8 \text{ V}$$

[c] $v_1 = 150$ V; $\qquad i_2 = 1 \text{ m} + 0.20 \text{ m} = 1.20 \text{ mA}$

$$i_1 = 150/750{,}000 = 0.20 \text{ mA}$$

$$\therefore \quad v_{\text{meas}} = v_x = 150 + (150 \text{ k})(1.20 \text{ m}) = 330 \text{ V}$$

P 3.49 **[a]** $R_{\text{meter}} = 300 \text{ k}\Omega + 600 \text{ k}\Omega \| 200 \text{ k}\Omega = 450 \text{ k}\Omega$

$$450 \| 360 = 200 \text{ k}\Omega$$

$$V_{\text{meter}} = \frac{200}{240}(600) = 500 \text{ V}$$

[b] What is the percent error in the measured voltage?

$$\text{True value } = \frac{360}{400}(600) = 540 \text{ V}$$

$$\%\text{ error } = \left(\frac{500}{540} - 1\right) 100 = -7.41\%$$

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P 3.50 Since the bridge is balanced, we can remove the detector without disturbing the voltages and currents in the circuit.

It follows that

$$i_1 = \frac{i_g(R_2 + R_x)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_2 + R_x)}{\sum R}$$

$$i_2 = \frac{i_g(R_1 + R_3)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_1 + R_3)}{\sum R}$$

$$v_3 = R_3 i_1 = v_x = i_2 R_x$$

$$\therefore \quad \frac{R_3 i_g(R_2 + R_x)}{\sum R} = \frac{R_x i_g(R_1 + R_3)}{\sum R}$$

$$\therefore \quad R_3(R_2 + R_x) = R_x(R_1 + R_3)$$

From which $R_x = \dfrac{R_2 R_3}{R_1}$

P 3.51 **[a]**

The condition for a balanced bridge is that the product of the opposite resistors must be equal:

$$(500)(R_x) = (1000)(750) \qquad \text{so} \qquad R_x = \frac{(1000)(750)}{500} = 1500\,\Omega$$

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[b] The source current is the sum of the two branch currents. Each branch current can be determined using Ohm's law, since the resistors in each branch are in series and the voltage drop across each branch is 24 V:

$$i_s = \frac{24\text{ V}}{500\,\Omega + 750\,\Omega} + \frac{24\text{ V}}{1000\,\Omega + 1500\,\Omega} = 28.8\text{ mA}$$

[c] We can use Ohm's law to find the current in each branch:

$$i_{\text{left}} = \frac{24}{500 + 750} = 19.2\text{ mA}$$

$$i_{\text{right}} = \frac{24}{1000 + 1500} = 9.6\text{ mA}$$

Now we can use the formula $p = Ri^2$ to find the power dissipated by each resistor:

$$p_{500} = (500)(0.0192)^2 = 184.32\text{ mW} \qquad p_{750} = (750)(0.0192)^2 = 276.18\text{ mW}$$

$$p_{1000} = (1000)(0.0096)^2 = 92.16\text{ mW} \qquad p_{1500} = (1500)(0.0096)^2 = 138.24\text{ mW}$$

Thus, the $750\,\Omega$ resistor absorbs the most power; it absorbs 276.48 mW of power.

[d] From the analysis in part (c), the $1000\,\Omega$ resistor absorbs the least power; it absorbs 92.16 mW of power.

P 3.52 Note the bridge structure is balanced, that is $15 \times 5 = 3 \times 25$, hence there is no current in the 5 kΩ resistor. It follows that the equivalent resistance of the circuit is

$$R_{\text{eq}} = 750 + (15{,}000 + 3000)\|(25{,}000 + 5000) = 750 + 11{,}250 = 12\ \text{k}\Omega$$

The source current is $192/12{,}000 = 16$ mA.
The current down through the branch containing the 15 kΩ and 3 kΩ resistors is

$$i_{3\text{k}} = \frac{11{,}250}{18{,}000}(0.016) = 10\text{ mA}$$

$$\therefore\quad p_{3\text{k}} = 3000(0.01)^2 = 0.3\ \text{W}$$

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P 3.53 Redraw the circuit, replacing the detector branch with a short circuit.

$6\ \text{k}\Omega \| 30\ \text{k}\Omega = 5\ \text{k}\Omega$

$12\ \text{k}\Omega \| 20\ \text{k}\Omega = 7.5\ \text{k}\Omega$

$$i_s = \frac{75}{12{,}500} = 6\ \text{mA}$$

$$v_1 = 0.006(5000) = 30\ \text{V}$$

$$v_2 = 0.006(7500) = 45\ \text{V}$$

$$i_1 = \frac{30}{6000} = 5\ \text{mA}$$

$$i_2 = \frac{45}{12{,}000} = 3.75\ \text{mA}$$

$$i_\text{d} = i_1 - i_2 = 1.25\ \text{mA}$$

P 3.54 In order that all four decades (1, 10, 100, 1000) that are used to set R_3 contribute to the balance of the bridge, the ratio R_2/R_1 should be set to 0.001.

P 3.55 Use the figure below to transform the Δ to an equivalent Y:

$$R_1 = \frac{(40)(25)}{40 + 25 + 37.5} = 9.756\,\Omega$$

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$$R_2 = \frac{(25)(37.5)}{40 + 25 + 37.5} = 9.1463\,\Omega$$

$$R_3 = \frac{(40)(37.5)}{40 + 25 + 37.5} = 14.634\,\Omega$$

Replace the Δ with its equivalent Y in the circuit to get the figure below:

Find the equivalent resistance to the right of the $5\,\Omega$ resistor:

$$(100 + 9.756)\|(125 + 9.1463) + 14.634 = 75\,\Omega$$

The equivalent resistance seen by the source is thus $5 + 75 = 80\,\Omega$. Use Ohm's law to find the current provided by the source:

$$i_{\mathrm{s}} = \frac{40}{80} = 0.5 \text{ A}$$

Thus, the power associated with the source is

$$P_{\mathrm{s}} = -(40)(0.5) = -20 \text{ W}$$

P 3.56 Use the figure below to transform the Y to an equivalent Δ:

$$R_{\mathrm{a}} = \frac{(25)(100) + (25)(40) + (40)(100)}{25} = \frac{7500}{25} = 300\,\Omega$$

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$$R_b = \frac{(25)(100) + (25)(40) + (40)(100)}{40} = \frac{7500}{40} = 187.5\,\Omega$$

$$R_c = \frac{(25)(100) + (25)(40) + (40)(100)}{100} = \frac{7500}{100} = 75\,\Omega$$

Replace the Y with its equivalent Δ in the circuit to get the figure below:

Find the equivalent resistance to the right of the $5\,\Omega$ resistor:

$300 \| [(125 \| 187.5) + (37.5 \| 75)] = 75\,\Omega$

The equivalent resistance seen by the source is thus $5 + 75 = 80\,\Omega$. Use Ohm's law to find the current provided by the source:

$$i_s = \frac{40}{80} = 0.5 \text{ A}$$

Thus, the power associated with the source is

$$P_s = -(40)(0.5) = -20 \text{ W}$$

P 3.57 Use the figure below to transform the Y to an equivalent Δ:

$$R_a = \frac{(25)(125) + (25)(37.5) + (37.5)(125)}{37.5} = \frac{8750}{37.5} = 233.33\,\Omega$$

$$R_b = \frac{(25)(125) + (25)(37.5) + (37.5)(125)}{25} = \frac{8750}{25} = 350\,\Omega$$

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$$R_c = \frac{(25)(125) + (25)(37.5) + (37.5)(125)}{125} = \frac{8750}{125} = 70\,\Omega$$

Replace the Y with its equivalent Δ in the circuit to get the figure below:

Find the equivalent resistance to the right of the $5\,\Omega$ resistor:

$$350 \| [(100 \| 233.33) + (40 \| 70)] = 75\,\Omega$$

The equivalent resistance seen by the source is thus $5 + 75 = 80\,\Omega$. Use Ohm's law to find the current provided by the source:

$$i_s = \frac{40}{80} = 0.5 \text{ A}$$

Thus, the power associated with the source is

$$P_s = -(40)(0.5) = -20 \text{ W}$$

P 3.58 **[a]** Use the figure below to transform the Y to an equivalent Δ:

$$R_a = \frac{(25)(30) + (25)(50) + (30)(50)}{30} = \frac{3500}{30} = 116.67\,\Omega$$

$$R_b = \frac{(25)(30) + (25)(50) + (30)(50)}{50} = \frac{3500}{50} = 70\,\Omega$$

$$R_c = \frac{(25)(30) + (25)(50) + (30)(50)}{25} = \frac{3500}{25} = 140\,\Omega$$

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Replace the Y with its equivalent Δ in the circuit to get the figure below:

Find the equivalent resistance to the right of the $13\,\Omega$ and $7\,\Omega$ resistors:

$70\|[(50\|116.67) + (20\|140)] = 30\,\Omega$

Thus, the equivalent resistance seen from the terminals a-b is:

$R_{\text{ab}} = 13 + 30 + 7 = 50\,\Omega$

[b] Use the figure below to transform the Δ to an equivalent Y:

$$R_1 = \frac{(50)(20)}{50 + 20 + 30} = 10\,\Omega$$

$$R_2 = \frac{(50)(30)}{50 + 20 + 30} = 15\,\Omega$$

$$R_3 = \frac{(20)(30)}{50 + 20 + 30} = 6\,\Omega$$

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Replace the Δ with its equivalent Y in the circuit to get the figure below:

Find the equivalent resistance to the right of the $13\,\Omega$ and $7\,\Omega$ resistors:

$(50+10)\|(25+15)+6 = 30\,\Omega$

Thus, the equivalent resistance seen from the terminals a-b is:

$R_{\text{ab}} = 13 + 30 + 7 = 50\,\Omega$

[c] Convert the delta connection R_1—R_2—R_3 to its equivalent wye. Convert the wye connection R_1—R_3—R_4 to its equivalent delta.

P 3.59 Begin by transforming the Δ-connected resistors $(10\,\Omega, 40\,\Omega, 50\,\Omega)$ to Y-connected resistors. Both the Y-connected and Δ-connected resistors are shown below to assist in using Eqs. 3.44 – 3.46:

Now use Eqs. 3.44 – 3.46 to calculate the values of the Y-connected resistors:

$$R_1 = \frac{(40)(10)}{10+40+50} = 4\,\Omega; \quad R_2 = \frac{(10)(50)}{10+40+50} = 5\,\Omega; \quad R_3 = \frac{(40)(50)}{10+40+50} = 20\,\Omega$$

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The transformed circuit is shown below:

The equivalent resistance seen by the 24 V source can be calculated by making series and parallel combinations of the resistors to the right of the 24 V source:

$$R_{\text{eq}} = (15+5)\|(1+4) + 20 = 20\|5 + 20 = 4 + 20 = 24\,\Omega$$

Therefore, the current i in the 24 V source is given by

$$i = \frac{24\text{ V}}{24\,\Omega} = 1\text{ A}$$

Use current division to calculate the currents i_1 and i_2. Note that the current i_1 flows in the branch containing the $15\,\Omega$ and $5\,\Omega$ series connected resistors, while the current i_2 flows in the parallel branch that contains the series connection of the $1\,\Omega$ and $4\,\Omega$ resistors:

$$i_1 = \frac{4}{15+5}(i) = \frac{4}{20}(1\text{ A}) = 0.2\text{ A}, \qquad \text{and} \qquad i_2 = 1\text{ A} - 0.2\text{ A} = 0.8\text{ A}$$

Now use KVL and Ohm's law to calculate v_1. Note that v_1 is the sum of the voltage drop across the $4\,\Omega$ resistor, $4i_2$, and the voltage drop across the $20\,\Omega$ resistor, $20i$:

$$v_1 = 4i_2 + 20i = 4(0.8\text{ A}) + 20(1\text{ A}) = 3.2 + 20 = 23.2\text{ V}$$

Finally, use KVL and Ohm's law to calculate v_2. Note that v_2 is the sum of the voltage drop across the $5\,\Omega$ resistor, $5i_1$, and the voltage drop across the $20\,\Omega$ resistor, $20i$:

$$v_2 = 5i_1 + 20i = 5(0.2\text{ A}) + 20(1\text{ A}) = 1 + 20 = 21\text{ V}$$

P 3.60 **[a]** Convert the upper delta to a wye.

$$R_1 = \frac{(50)(50)}{200} = 12.5\,\Omega$$

$$R_2 = \frac{(50)(100)}{200} = 25\,\Omega$$

$$R_3 = \frac{(100)(50)}{200} = 25\,\Omega$$

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Convert the lower delta to a wye.

$$R_4 = \frac{(60)(80)}{200} = 24\,\Omega$$

$$R_5 = \frac{(60)(60)}{200} = 18\,\Omega$$

$$R_6 = \frac{(80)(60)}{200} = 24\,\Omega$$

Now redraw the circuit using the wye equivalents.

$$R_{\text{ab}} = 1.5 + 12.5 + \frac{(120)(80)}{200} + 18 = 14 + 48 + 18 = 80\,\Omega$$

[b] When $v_{\text{ab}} = 400$ V

$$i_g = \frac{400}{80} = 5 \text{ A}$$

$$i_{31} = \frac{48}{80}(5) = 3 \text{ A}$$

$$p_{31\Omega} = (31)(3)^2 = 279 \text{ W}$$

P 3.61 **[a]** After the $20\,\Omega$—$100\,\Omega$—$50\,\Omega$ wye is replaced by its equivalent delta, the circuit reduces to

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Now the circuit can be reduced to

$$i = \frac{96}{400}(1000) = 240 \text{ mA}$$

$$i_o = \frac{400}{1000}(240) = 96 \text{ mA}$$

[b] $i_1 = \dfrac{80}{400}(240) = 48 \text{ mA}$

[c] Now that i_o and i_1 are known return to the original circuit

$$v_2 = (50)(0.048) + (600)(0.096) = 60 \text{ V}$$

$$i_2 = \frac{v_2}{100} = \frac{60}{100} = 600 \text{ mA}$$

[d] $v_g = v_2 + 20(0.6 + 0.048) = 60 + 12.96 = 72.96 \text{ V}$

$$p_g = -(v_g)(1) = -72.96 \text{ W}$$

Thus the current source delivers 72.96 W.

P 3.62 $8 + 12 = 20\,\Omega$

$20 \| 60 = 15\,\Omega$

$15 + 20 = 35\,\Omega$

$35 \| 140 = 28\,\Omega$

$28 + 22 = 50\,\Omega$

$50 \| 75 = 30\,\Omega$

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$30 + 10 = 40\,\Omega$

$i_g = 240/40 = 6$ A

$i_o = (6)(50)/125 = 2.4$ A

$i_{140\Omega} = (6 - 2.4)(35)/175 = 0.72$ A

$p_{140\Omega} = (0.72)^2(140) = 72.576$ W

P 3.63 **[a]** Replace the 60—120—20 Ω delta with a wye equivalent to get

$$i_s = \frac{750}{5 + (24+36)\|(14+6) + 12 + 43} = \frac{750}{75} = 10 \text{ A}$$

$$i_1 = \frac{(24+36)\|(14+6)}{24+36}(10) = \frac{15}{60}(10) = 2.5 \text{ A}$$

[b] $i_o = 10 - 2.5 = 7.5$ A

$v = 36i_1 - 6i_o = 36(2.5) - 6(7.5) = 45$ V

[c] $i_2 = i_o + \dfrac{v}{60} = 7.5 + \dfrac{45}{60} = 8.25$ A

[d] $P_{\text{supplied}} = (750)(10) = 7500$ W

P 3.64
$$\begin{aligned} G_{\text{a}} &= \frac{1}{R_{\text{a}}} = \frac{R_1}{R_1R_2 + R_2R_3 + R_3R_1} \\ &= \frac{1/G_1}{(1/G_1)(1/G_2) + (1/G_2)(1/G_3) + (1/G_3)(1/G_1)} \\ &= \frac{(1/G_1)(G_1G_2G_3)}{G_1 + G_2 + G_3} = \frac{G_2G_3}{G_1 + G_2 + G_3} \end{aligned}$$

Similar manipulations generate the expressions for G_{b} and G_{c}.

P 3.65 **[a]** Subtracting Eq. 3.42 from Eq. 3.43 gives

$R_1 - R_2 = (R_{\text{c}}R_{\text{b}} - R_{\text{c}}R_{\text{a}})/(R_{\text{a}} + R_{\text{b}} + R_{\text{c}})$.

Adding this expression to Eq. 3.41 and solving for R_1 gives

$R_1 = R_{\text{c}}R_{\text{b}}/(R_{\text{a}} + R_{\text{b}} + R_{\text{c}})$.

To find R_2, subtract Eq. 3.43 from Eq. 3.41 and add this result to Eq. 3.42. To find R_3, subtract Eq. 3.41 from Eq. 3.42 and add this result to Eq. 3.43.

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[b] Using the hint, Eq. 3.43 becomes

$$R_1 + R_3 = \frac{R_b[(R_2/R_3)R_b + (R_2/R_1)R_b]}{(R_2/R_1)R_b + R_b + (R_2/R_3)R_b} = \frac{R_b(R_1 + R_3)R_2}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

Solving for R_b gives $R_b = (R_1R_2 + R_2R_3 + R_3R_1)/R_2$. To find R_a: First use Eqs. 3.44–3.46 to obtain the ratios $(R_1/R_3) = (R_c/R_a)$ or $R_c = (R_1/R_3)R_a$ and $(R_1/R_2) = (R_b/R_a)$ or $R_b = (R_1/R_2)R_a$. Now use these relationships to eliminate R_b and R_c from Eq. 3.42. To find R_c, use Eqs. 3.44–3.46 to obtain the ratios $R_b = (R_3/R_2)R_c$ and $R_a = (R_3/R_1)R_c$. Now use the relationships to eliminate R_b and R_a from Eq. 3.41.

P 3.66 **[a]** $R_{ab} = 2R_1 + \dfrac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = R_L$

Therefore $\quad 2R_1 - R_L + \dfrac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = 0$

Thus $\quad R_L^2 = 4R_1^2 + 4R_1R_2 = 4R_1(R_1 + R_2)$

When $R_{ab} = R_L$, the current into terminal a of the attenuator will be v_i/R_L.
Using current division, the current in the R_L branch will be

$$\frac{v_i}{R_L} \cdot \frac{R_2}{2R_1 + R_2 + R_L}$$

Therefore $\quad v_o = \dfrac{v_i}{R_L} \cdot \dfrac{R_2}{2R_1 + R_2 + R_L} R_L$

and $\quad \dfrac{v_o}{v_i} = \dfrac{R_2}{2R_1 + R_2 + R_L}$

[b] $(300)^2 = 4(R_1 + R_2)R_1$

$22{,}500 = R_1^2 + R_1R_2$

$\dfrac{v_o}{v_i} = 0.5 = \dfrac{R_2}{2R_1 + R_2 + 300}$

$\therefore \quad R_1 + 0.5R_2 + 150 = R_2$

$0.5R_2 = R_1 + 150$

$R_2 = 2R_1 + 300$

$\therefore \quad 22{,}500 = R_1^2 + R_1(2R_1 + 300) = 3R_1^2 + 300R_1$

$\therefore \quad R_1^2 + 100R_1 - 7500 = 0$

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Solving,

$R_1 = 50\,\Omega$

$R_2 = 2(50) + 300 = 400\,\Omega$

[c] From Appendix H, choose $R_1 = 47\,\Omega$ and $R_2 = 390\,\Omega$. For these values, $R_{\text{ab}} \neq R_{\text{L}}$, so the equations given in part (a) cannot be used. Instead

$$R_{\text{ab}} = 2R_1 + [R_2 \| (2R_1 + R_{\text{L}})] = 2(47) + 390 \| (2(47) + 300)$$

$$= 94 + 390 \| 394 = 290\,\Omega$$

$$\%\text{ error} = \left(\frac{290}{300} - 1\right) 100 = -3.33\%$$

Now calculate the ratio of the output voltage to the input voltage. Begin by finding the current through the top left R_1 resistor, called i_{a}:

$$i_{\text{a}} = \frac{v_i}{R_{\text{ab}}}$$

Now use current division to find the current through the R_{L} resistor, called i_{L}:

$$i_{\text{L}} = \frac{R_2}{R_2 + 2R_1 + R_{\text{L}}} i_{\text{a}}$$

Therefore, the output voltage, v_o, is equal to $R_{\text{L}} i_{\text{L}}$:

$$v_o = \frac{R_2 R_{\text{L}} v_i}{R_{\text{ab}}(R_2 + 2R_1 + R_{\text{L}})}$$

Thus,

$$\frac{v_o}{v_i} = \frac{R_2 R_{\text{L}}}{R_{\text{ab}}(R_2 + 2R_1 + R_{\text{L}})} = \frac{390(300)}{290(390 + 2(47) + 300)} = 0.5146$$

$$\%\text{ error} = \left(\frac{0.5146}{0.5} - 1\right) 100 = 2.92\%$$

P 3.67 **[a]** After making the Y-to-Δ transformation, the circuit reduces to

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Combining the parallel resistors reduces the circuit to

Now note: $\quad 0.75R + \dfrac{3RR_{\mathrm{L}}}{3R + R_{\mathrm{L}}} = \dfrac{2.25R^2 + 3.75RR_{\mathrm{L}}}{3R + R_{\mathrm{L}}}$

Therefore $\quad R_{\mathrm{ab}} = \dfrac{3R\left(\dfrac{2.25R^2 + 3.75RR_{\mathrm{L}}}{3R + R_{\mathrm{L}}}\right)}{3R + \left(\dfrac{2.25R^2 + 3.75RR_{\mathrm{L}}}{3R + R_{\mathrm{L}}}\right)} = \dfrac{3R(3R + 5R_{\mathrm{L}})}{15R + 9R_{\mathrm{L}}}$

If $R = R_{\mathrm{L}}$, we have $\quad R_{\mathrm{ab}} = \dfrac{3R_{\mathrm{L}}(8R_{\mathrm{L}})}{24R_{\mathrm{L}}} = R_{\mathrm{L}}$

Therefore $\quad R_{\mathrm{ab}} = R_{\mathrm{L}}$

[b] When $R = R_{\mathrm{L}}$, the circuit reduces to

$$i_o = \frac{i_i(3R_{\mathrm{L}})}{4.5R_{\mathrm{L}}} = \frac{1}{1.5}i_i = \frac{1}{1.5}\frac{v_i}{R_{\mathrm{L}}}, \qquad v_o = 0.75R_{\mathrm{L}}i_o = \frac{1}{2}v_i,$$

Therefore $\quad \dfrac{v_o}{v_i} = 0.5$

P 3.68 **[a]** $3.5(3R - R_{\mathrm{L}}) = 3R + R_{\mathrm{L}}$

$10.5R - 1050 = 3R + 300$

$7.5R = 1350, \qquad R = 180\,\Omega$

$$R_2 = \frac{2(180)(300)^2}{3(180)^2 - (300)^2} = 4500\,\Omega$$

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[**b**]

$$v_o = \frac{v_i}{3.5} = \frac{42}{3.5} = 12 \text{ V}$$

$$i_o = \frac{12}{300} = 40 \text{ mA}$$

$$i_1 = \frac{42 - 12}{4500} = \frac{30}{4500} = 6.67 \text{ mA}$$

$$i_g = \frac{42}{300} = 140 \text{ mA}$$

$$i_2 = 140 - 6.67 = 133.33 \text{ mA}$$

$$i_3 = 40 - 6.67 = 33.33 \text{ mA}$$

$$i_4 = 133.33 - 33.33 = 100 \text{ mA}$$

$$p_{4500 \text{ top}} = (6.67 \times 10^{-3})^2(4500) = 0.2 \text{ W}$$

$$p_{180 \text{ left}} = (133.33 \times 10^{-3})^2(180) = 3.2 \text{ W}$$

$$p_{180 \text{ right}} = (33.33 \times 10^{-3})^2(180) = 0.2 \text{ W}$$

$$p_{180 \text{ vertical}} = (100 \times 10^{-3})^2(180) = 0.48 \text{ W}$$

$$p_{300 \text{ load}} = (40 \times 10^{-3})^2(300) = 0.48 \text{ W}$$

The 180 Ω resistor carrying i_2

[**c**] $p_{180 \text{ left}} = 3.2$ W

[**d**] Two resistors dissipate minimum power – the 4500 Ω resistor and the 180 Ω resistor carrying i_3.

[**e**] They both dissipate 0.2 W.

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P 3.69 **[a]**

$$v_{\mathrm{a}} = \frac{v_{\mathrm{in}} R_4}{R_o + R_4 + \Delta R}$$

$$v_{\mathrm{b}} = \frac{R_3}{R_2 + R_3} v_{\mathrm{in}}$$

$$v_o = v_{\mathrm{a}} - v_{\mathrm{b}} = \frac{R_4 v_{\mathrm{in}}}{R_o + R_4 + \Delta R} - \frac{R_3}{R_2 + R_3} v_{\mathrm{in}}$$

When the bridge is balanced,

$$\frac{R_4}{R_o + R_4} v_{\mathrm{in}} = \frac{R_3}{R_2 + R_3} v_{\mathrm{in}}$$

$$\therefore \quad \frac{R_4}{R_o + R_4} = \frac{R_3}{R_2 + R_3}$$

Thus,

$$\begin{aligned} v_o &= \frac{R_4 v_{\mathrm{in}}}{R_o + R_4 + \Delta R} - \frac{R_4 v_{\mathrm{in}}}{R_o + R_4} \\ &= R_4 v_{\mathrm{in}} \left[\frac{1}{R_o + R_4 + \Delta R} - \frac{1}{R_o + R_4} \right] \\ &= \frac{R_4 v_{\mathrm{in}} (-\Delta R)}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &\approx \frac{-(\Delta R) R_4 v_{\mathrm{in}}}{(R_o + R_4)^2}, \qquad \text{since } \Delta R << R_4 \end{aligned}$$

[b] $\Delta R = 0.03 R_o$

$$R_o = \frac{R_2 R_4}{R_3} = \frac{(1000)(5000)}{500} = 10{,}000\,\Omega$$

$$\Delta R = (0.03)(10^4) = 300\,\Omega$$

$$\therefore \quad v_o \approx \frac{-300(5000)(6)}{(15{,}000)^2} = -40 \text{ mV}$$

[c]

$$\begin{aligned} v_o &= \frac{-(\Delta R) R_4 v_{\mathrm{in}}}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &= \frac{-300(5000)(6)}{(15{,}300)(15{,}000)} \\ &= -39.2157 \text{ mV} \end{aligned}$$

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P 3.70 **[a]** approx value $= \dfrac{-(\Delta R)R_4 v_{\text{in}}}{(R_o + R_4)^2}$

$$\text{true value} = \frac{-(\Delta R)R_4 v_{\text{in}}}{(R_o + R_4 + \Delta R)(R_o + R_4)}$$

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{(R_o + R_4 + \Delta R)}{(R_o + R_4)}$$

$$\therefore \quad \%\ \text{error} = \left[\frac{R_o + R_4}{R_o + R_4 + \Delta R} - 1\right] \times 100 = \frac{-\Delta R}{R_o + R_4} \times 100$$

Note that in the above expression, we take the ratio of the true value to the approximate value because both values are negative.

But $R_o = \dfrac{R_2 R_4}{R_3}$

$$\therefore \quad \%\ \text{error} = \frac{-R_3 \Delta R}{R_4(R_2 + R_3)}$$

[b] $\%\ \text{error} = \dfrac{-(500)(300)}{(5000)(1500)} \times 100 = -2\%$

P 3.71 $\dfrac{\Delta R(R_3)(100)}{(R_2 + R_3)R_4} = 0.5$

$$\frac{\Delta R(500)(100)}{(1500)(5000)} = 0.5$$

$$\therefore \quad \Delta R = 75\,\Omega$$

$$\%\ \text{change} = \frac{75}{10{,}000} \times 100 = 0.75\%$$

P 3.72 **[a]** Using the equation for voltage division,

$$V_y = \frac{\beta R_y}{\beta R_y + (1-\beta)R_y} V_S = \frac{\beta R_y}{R_y} V_S = \beta V_S$$

[b] Since β represents the touch point with respect to the bottom of the screen, $(1-\beta)$ represents the location of the touch point with respect to the top of the screen. Therefore, the y-coordinate of the pixel corresponding to the touch point is

$$y = (1-\beta)p_y$$

Remember that the value of y is capped at $(p_y - 1)$.

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P 3.73 **[a]** Use the equations developed in the Practical Perspective and in Problem 3.72:

$$V_x = \alpha V_S \qquad \text{so} \qquad \alpha = \frac{V_x}{V_S} = \frac{1}{5} = 0.2$$

$$V_y = \beta V_S \qquad \text{so} \qquad \beta = \frac{V_y}{V_S} = \frac{3.75}{5} = 0.75$$

[b] Use the equations developed in the Practical Perspective and in Problem 3.72:

$$x = (1-\alpha)p_x = (1-0.2)(480) = 384$$

$$y = (1-\beta)p_y = (1-0.75)(800) = 200$$

Therefore, the touch occurred in the upper right corner of the screen.

P 3.74 Use the equations developed in the Practical Perspective and in Problem 3.72:

$$x = (1-\alpha)p_x \qquad \text{so} \qquad \alpha = 1 - \frac{x}{p_x} = 1 - \frac{480}{640} = 0.25$$

$$V_x = \alpha V_S = (0.25)(8) = 2 \text{ V}$$

$$y = (1-\beta)p_y \qquad \text{so} \qquad \beta = 1 - \frac{y}{p_y} = 1 - \frac{192}{1024} = 0.8125$$

$$V_y = \beta V_S = (0.8125)(8) = 6.5 \text{ V}$$

P 3.75 From the results of Problem 3.74, the voltages corresponding to the touch point $(480, 192)$ are

$$V_{x1} = 2 \text{ V}; \qquad V_{y1} = 6.5 \text{ V}$$

Now calculate the voltages corresponding to the touch point $(240, 384)$:

$$x = (1-\alpha)p_x \qquad \text{so} \qquad \alpha = 1 - \frac{x}{p_x} = 1 - \frac{240}{640} = 0.625$$

$$V_{x2} = \alpha V_S = (0.625)(8) = 5 \text{ V}$$

$$y = (1-\beta)p_y \qquad \text{so} \qquad \beta = 1 - \frac{y}{p_y} = 1 - \frac{384}{1024} = 0.625$$

$$V_{y2} = \beta V_S = (0.625)(8) = 5 \text{ V}$$

When the screen is touched at two points simultaneously, only the smaller of the two voltages in the x direction is sensed. The same is true in the y direction. Therefore, the voltages actually sensed are

$$V_x = 2 \text{ V}; \qquad V_y = 5 \text{ V}$$

These two voltages identify the touch point as $(480, 384)$, which does not correspond to either of the points actually touched! Therefore, the resistive touch screen is appropriate only for single point touches.

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4

Techniques of Circuit Analysis

Assessment Problems

AP 4.1 **[a]** Redraw the circuit, labeling the reference node and the two node voltages:

The two node voltage equations are

$$-15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0$$

$$5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{60} + \frac{1}{15} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) = 15$$

$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{2} + \frac{1}{5}\right) = -5$$

Solving, $v_1 = 60$ V and $v_2 = 10$ V;
Therefore, $i_1 = (v_1 - v_2)/5 = 10$ A

[b] $p_{15\text{A}} = -(15\,\text{A})v_1 = -(15\,\text{A})(60\,\text{V}) = -900$ W $= 900$ W(delivered)

[c] $p_{5\text{A}} = (5\,\text{A})v_2 = (5\,\text{A})(10\,\text{V}) = 50$ W$= -50$ W(delivered)

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AP 4.2 Redraw the circuit, choosing the node voltages and reference node as shown:

The two node voltage equations are:

$$-4.5 + \frac{v_1}{1} + \frac{v_1 - v_2}{6+2} = 0$$

$$\frac{v_2}{12} + \frac{v_2 - v_1}{6+2} + \frac{v_2 - 30}{4} = 0$$

Place these equations in standard form:

$$v_1\left(1 + \frac{1}{8}\right) + v_2\left(-\frac{1}{8}\right) = 4.5$$

$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) = 7.5$$

Solving, $v_1 = 6$ V $\quad v_2 = 18$ V

To find the voltage v, first find the current i through the series-connected $6\,\Omega$ and $2\,\Omega$ resistors:

$$i = \frac{v_1 - v_2}{6+2} = \frac{6-18}{8} = -1.5\,\text{A}$$

Using a KVL equation, calculate v:

$$v = 2i + v_2 = 2(-1.5) + 18 = 15\,\text{V}$$

AP 4.3 **[a]** Redraw the circuit, choosing the node voltages and reference node as shown:

The node voltage equations are:

$$\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0$$

$$-5 + \frac{v_2}{4} + \frac{v_2 - v_1}{2} + 3i_1 = 0$$

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The dependent source requires the following constraint equation:

$$i_1 = \frac{50 - v_1}{6}$$

Place these equations in standard form:

$$\begin{array}{lclclcl} v_1\left(\frac{1}{6}+\frac{1}{8}+\frac{1}{2}\right) & + & v_2\left(-\frac{1}{2}\right) & + & i_1(-3) & = & \frac{50}{6} \\ v_1\left(-\frac{1}{2}\right) & + & v_2\left(\frac{1}{4}+\frac{1}{2}\right) & + & i_1(3) & = & 5 \\ v_1\left(\frac{1}{6}\right) & + & v_2(0) & + & i_1(1) & = & \frac{50}{6} \end{array}$$

Solving, $v_1 = 32$ V; $\quad v_2 = 16$ V; $\quad i_1 = 3$ A

Using these values to calculate the power associated with each source:

$$\begin{aligned} p_{50\text{V}} = -50i_1 &= -150 \text{ W} \\ p_{5\text{A}} = -5(v_2) &= -80 \text{ W} \\ p_{3i_1} = 3i_1(v_2 - v_1) &= -144 \text{ W} \end{aligned}$$

[b] All three sources are delivering power to the circuit because the power computed in (a) for each of the sources is negative.

AP 4.4 Redraw the circuit and label the reference node and the node at which the node voltage equation will be written:

The node voltage equation is

$$\frac{v_o}{40} + \frac{v_o - 10}{10} + \frac{v_o + 20i_\Delta}{20} = 0$$

The constraint equation required by the dependent source is

$$i_\Delta = i_{10\Omega} + i_{30\Omega} = \frac{10 - v_o}{10} + \frac{10 + 20i_\Delta}{30}$$

Place these equations in standard form:

$$\begin{array}{lclcl} v_o\left(\frac{1}{40}+\frac{1}{10}+\frac{1}{20}\right) & + & i_\Delta(1) & = & 1 \\ v_o\left(\frac{1}{10}\right) & + & i_\Delta\left(1-\frac{20}{30}\right) & = & 1+\frac{10}{30} \end{array}$$

Solving, $\quad i_\Delta = -3.2$ A $\quad$ and $\quad v_o = 24$ V

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AP 4.5 Redraw the circuit identifying the three node voltages and the reference node:

Note that the dependent voltage source and the node voltages v and v_2 form a supernode. The v_1 node voltage equation is

$$\frac{v_1}{7.5} + \frac{v_1 - v}{2.5} - 4.8 = 0$$

The supernode equation is

$$\frac{v - v_1}{2.5} + \frac{v}{10} + \frac{v_2}{2.5} + \frac{v_2 - 12}{1} = 0$$

The constraint equation due to the dependent source is

$$i_x = \frac{v_1}{7.5}$$

The constraint equation due to the supernode is

$$v + i_x = v_2$$

Place this set of equations in standard form:

$$\begin{array}{lclclclcl}
v_1\left(\frac{1}{7.5}+\frac{1}{2.5}\right) & + & v\left(-\frac{1}{2.5}\right) & + & v_2(0) & + & i_x(0) & = & 4.8 \\
v_1\left(-\frac{1}{2.5}\right) & + & v\left(\frac{1}{2.5}+\frac{1}{10}\right) & + & v_2\left(\frac{1}{2.5}+1\right) & + & i_x(0) & = & 12 \\
v_1\left(-\frac{1}{7.5}\right) & + & v(0) & + & v_2(0) & + & i_x(1) & = & 0 \\
v_1(0) & + & v(1) & + & v_2(-1) & + & i_x(1) & = & 0
\end{array}$$

Solving this set of equations gives $v_1 = 15$ V, $v_2 = 10$ V, $i_x = 2$ A, and $v = 8$ V.

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AP 4.6 Redraw the circuit identifying the reference node and the two unknown node voltages. Note that the right-most node voltage is the sum of the 60 V source and the dependent source voltage.

The node voltage equation at v_1 is

$$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - (60 + 6i_\phi)}{3} = 0$$

The constraint equation due to the dependent source is

$$i_\phi = \frac{60 + 6i_\phi - v_1}{3}$$

Place these two equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{2} + \frac{1}{24} + \frac{1}{3}\right) &+ i_\phi(-2) &= 30 + 20 \\ v_1\left(\frac{1}{3}\right) &+ i_\phi(1-2) &= 20 \end{aligned}$$

Solving, $\quad i_\phi = -4$ A $\quad$ and $\quad v_1 = 48$ V

AP 4.7 [**a**] Redraw the circuit identifying the three mesh currents:

The mesh current equations are:

$$\begin{aligned} -80 + 5(i_1 - i_2) + 26(i_1 - i_3) &= 0 \\ 30i_2 + 90(i_2 - i_3) + 5(i_2 - i_1) &= 0 \\ 8i_3 + 26(i_3 - i_1) + 90(i_3 - i_2) &= 0 \end{aligned}$$

Place these equations in standard form:

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$$\begin{aligned} 31i_1 - 5i_2 - 26i_3 &= 80 \\ -5i_1 + 125i_2 - 90i_3 &= 0 \\ -26i_1 - 90i_2 + 124i_3 &= 0 \end{aligned}$$

Solving,

$i_1 = 5\,\text{A}; \qquad i_2 = 2\,\text{A}; \qquad i_3 = 2.5\,\text{A}$

$p_{80V} = -(80)i_1 = -(80)(5) = -400\,\text{W}$

Therefore the 80 V source is delivering 400 W to the circuit.

[b] $p_{8\Omega} = (8)i_3^2 = 8(2.5)^2 = 50$ W, so the $8\,\Omega$ resistor dissipates 50 W.

AP 4.8 **[a]** $b = 8, \qquad n = 6, \qquad b - n + 1 = 3$

[b] Redraw the circuit identifying the three mesh currents:

The three mesh-current equations are

$$\begin{aligned} -25 + 2(i_1 - i_2) + 5(i_1 - i_3) + 10 &= 0 \\ -(-3v_\phi) + 14i_2 + 3(i_2 - i_3) + 2(i_2 - i_1) &= 0 \\ 1i_3 - 10 + 5(i_3 - i_1) + 3(i_3 - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is

$v_\phi = 3(i_3 - i_2)$

Place these four equations in standard form:

$$\begin{aligned} 7i_1 - 2i_2 - 5i_3 + 0v_\phi &= 15 \\ -2i_1 + 19i_2 - 3i_3 + 3v_\phi &= 0 \\ -5i_1 - 3i_2 + 9i_3 + 0v_\phi &= 10 \\ 0i_1 + 3i_2 - 3i_3 + 1v_\phi &= 0 \end{aligned}$$

Solving

$i_1 = 4\,\text{A}; \qquad i_2 = -1\,\text{A}; \qquad i_3 = 3\,\text{A}; \qquad v_\phi = 12\,\text{V}$

$p_{\text{ds}} = -(-3v_\phi)i_2 = 3(12)(-1) = -36\,\text{W}$

Thus, the dependent source is delivering 36 W, or absorbing −36 W.

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AP 4.9 Redraw the circuit identifying the three mesh currents:

The mesh current equations are:

$$\begin{aligned} -25 + 6(i_\mathrm{a} - i_\mathrm{b}) + 8(i_\mathrm{a} - i_\mathrm{c}) &= 0 \\ 2i_\mathrm{b} + 8(i_\mathrm{b} - i_\mathrm{c}) + 6(i_\mathrm{b} - i_\mathrm{a}) &= 0 \\ 5i_\phi + 8(i_\mathrm{c} - i_\mathrm{a}) + 8(i_\mathrm{c} - i_\mathrm{b}) &= 0 \end{aligned}$$

The dependent source constraint equation is $i_\phi = i_\mathrm{a}$. We can substitute this simple expression for i_ϕ into the third mesh equation and place the equations in standard form:

$$\begin{aligned} 14i_\mathrm{a} - 6i_\mathrm{b} - 8i_\mathrm{c} &= 25 \\ -6i_\mathrm{a} + 16i_\mathrm{b} - 8i_\mathrm{c} &= 0 \\ -3i_\mathrm{a} - 8i_\mathrm{b} + 16i_\mathrm{c} &= 0 \end{aligned}$$

Solving,

$$i_\mathrm{a} = 4\,\mathrm{A}; \qquad i_\mathrm{b} = 2.5\,\mathrm{A}; \qquad i_\mathrm{c} = 2\,\mathrm{A}$$

Thus,

$$v_o = 8(i_\mathrm{a} - i_\mathrm{c}) = 8(4 - 2) = 16\,\mathrm{V}$$

AP 4.10 Redraw the circuit identifying the mesh currents:

Since there is a current source on the perimeter of the i_3 mesh, we know that $i_3 = -16$ A. The remaining two mesh equations are

$$\begin{aligned} -30 + 3i_1 + 2(i_1 - i_2) + 6i_1 &= 0 \\ 8i_2 + 5(i_2 + 16) + 4i_2 + 2(i_2 - i_1) &= 0 \end{aligned}$$

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Place these equations in standard form:

$$\begin{aligned} 11i_1 - 2i_2 &= 30 \\ -2i_1 + 19i_2 &= -80 \end{aligned}$$

Solving: $i_1 = 2\,\text{A}$, $i_2 = -4\,\text{A}$, $i_3 = -16\,\text{A}$
The current in the $2\,\Omega$ resistor is $i_1 - i_2 = 6\,\text{A}$ $\therefore$ $p_{2\,\Omega} = (6)^2(2) = 72\,\text{W}$
Thus, the $2\,\Omega$ resistors dissipates 72 W.

AP 4.11 Redraw the circuit and identify the mesh currents:

There are current sources on the perimeters of both the i_b mesh and the i_c mesh, so we know that

$$i_\text{b} = -10\,\text{A}; \qquad i_\text{c} = \frac{2v_\phi}{5}$$

The remaining mesh current equation is

$$-75 + 2(i_\text{a} + 10) + 5(i_\text{a} - 0.4v_\phi) = 0$$

The dependent source requires the following constraint equation:

$$v_\phi = 5(i_\text{a} - i_\text{c}) = 5(i_\text{a} - 0.4v_\phi)$$

Place the mesh current equation and the dependent source equation is standard form:

$$\begin{aligned} 7i_\text{a} - 2v_\phi &= 55 \\ 5i_\text{a} - 3v_\phi &= 0 \end{aligned}$$

Solving: $i_\text{a} = 15\,\text{A}$; $i_\text{b} = -10\,\text{A}$; $i_\text{c} = 10\,\text{A}$; $v_\phi = 25\,\text{V}$
Thus, $i_\text{a} = 15$ A.

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AP 4.12 Redraw the circuit and identify the mesh currents:

The 2 A current source is shared by the meshes i_a and i_b. Thus we combine these meshes to form a supermesh and write the following equation:

$$-10 + 2i_b + 2(i_b - i_c) + 2(i_a - i_c) = 0$$

The other mesh current equation is

$$-6 + 1i_c + 2(i_c - i_a) + 2(i_c - i_b) = 0$$

The supermesh constraint equation is

$$i_a - i_b = 2$$

Place these three equations in standard form:

$$\begin{aligned} 2i_a + 4i_b - 4i_c &= 10 \\ -2i_a - 2i_b + 5i_c &= 6 \\ i_a - i_b + 0i_c &= 2 \end{aligned}$$

Solving, $i_a = 7\,\text{A}$; $i_b = 5\,\text{A}$; $i_c = 6\,\text{A}$
Thus, $p_{1\,\Omega} = i_c^2(1) = (6)^2(1) = 36\,\text{W}$

AP 4.13 Redraw the circuit and identify the reference node and the node voltage v_1:

The node voltage equation is

$$\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0$$

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Rearranging and solving,

$$v_1\left(\frac{1}{15}+\frac{1}{10}\right)=2+\frac{20}{15}+\frac{25}{10} \qquad \therefore \quad v_1=35\,\text{V}$$

$$p_{2A}=-35(2)=-70\,\text{W}$$

Thus the 2 A current source delivers 70 W.

AP 4.14 Redraw the circuit and identify the mesh currents:

There is a current source on the perimeter of the i_3 mesh, so $i_3 = 4$ A. The other two mesh current equations are

$$\begin{aligned} -128+4(i_1-4)+6(i_1-i_2)+2i_1 &= 0 \\ 30i_x+5i_2+6(i_2-i_1)+3(i_2-4) &= 0 \end{aligned}$$

The constraint equation due to the dependent source is

$$i_x=i_1-i_3=i_1-4$$

Substitute the constraint equation into the second mesh equation and place the resulting two mesh equations in standard form:

$$\begin{aligned} 12i_1-6i_2 &= 144 \\ 24i_1+14i_2 &= 132 \end{aligned}$$

Solving,

$$i_1=9\,\text{A}; \qquad i_2=-6\,\text{A}; \qquad i_3=4\,\text{A}; \qquad i_x=9-4=5\,\text{A}$$

$$\therefore \quad v_{4\text{A}}=3(i_3-i_2)-4i_x=10\,\text{V}$$

$$p_{4A}=-v_{4\text{A}}(4)=-(10)(4)=-40\,\text{W}$$

Thus, the 2 A current source delivers 40 W.

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AP 4.15 **[a]** Redraw the circuit with a helpful voltage and current labeled:

Transform the 120 V source in series with the 20 Ω resistor into a 6 A source in parallel with the 20 Ω resistor. Also transform the −60 V source in series with the 5 Ω resistor into a −12 A source in parallel with the 5 Ω resistor. The result is the following circuit:

Combine the three current sources into a single current source, using KCL, and combine the 20 Ω, 5 Ω, and 6 Ω resistors in parallel. The resulting circuit is shown on the left. To simplify the circuit further, transform the resulting 30 A source in parallel with the 2.4 Ω resistor into a 72 V source in series with the 2.4 Ω resistor. Combine the 2.4 Ω resistor in series with the 1.6 Ω resisor to get a very simple circuit that still maintains the voltage v. The resulting circuit is on the right.

Use voltage division in the circuit on the right to calculate v as follows:

$$v = \frac{8}{12}(72) = 48\,\text{V}$$

[b] Calculate i in the circuit on the right using Ohm's law:

$$i = \frac{v}{8} = \frac{48}{8} = 6\,\text{A}$$

Now use i to calculate v_a in the circuit on the left:

$$v_\text{a} = 6(1.6 + 8) = 57.6\,\text{V}$$

Returning back to the original circuit, note that the voltage v_a is also the voltage drop across the series combination of the 120 V source and 20 Ω

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resistor. Use this fact to calculate the current in the 120 V source, i_a:

$$i_a = \frac{120 - v_a}{20} = \frac{120 - 57.6}{20} = 3.12\,\text{A}$$

$$p_{120V} = -(120)i_a = -(120)(3.12) = -374.40\,\text{W}$$

Thus, the 120 V source delivers 374.4 W.

AP 4.16 To find R_{Th}, replace the 72 V source with a short circuit:

Note that the $5\,\Omega$ and $20\,\Omega$ resistors are in parallel, with an equivalent resistance of $5\|20 = 4\,\Omega$. The equivalent $4\,\Omega$ resistance is in series with the $8\,\Omega$ resistor for an equivalent resistance of $4 + 8 = 12\,\Omega$. Finally, the $12\,\Omega$ equivalent resistance is in parallel with the $12\,\Omega$ resistor, so $R_{\text{Th}} = 12\|12 = 6\,\Omega$.

Use node voltage analysis to find v_{Th}. Begin by redrawing the circuit and labeling the node voltages:

The node voltage equations are

$$\begin{aligned} \frac{v_1 - 72}{5} + \frac{v_1}{20} + \frac{v_1 - v_{\text{Th}}}{8} &= 0 \\ \frac{v_{\text{Th}} - v_1}{8} + \frac{v_{\text{Th}} - 72}{12} &= 0 \end{aligned}$$

Place these equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{5} + \frac{1}{20} + \frac{1}{8}\right) &+ v_{\text{Th}}\left(-\frac{1}{8}\right) &= \frac{72}{5} \\ v_1\left(-\frac{1}{8}\right) &+ v_{\text{Th}}\left(\frac{1}{8} + \frac{1}{12}\right) &= 6 \end{aligned}$$

Solving, $v_1 = 60$ V and $v_{\text{Th}} = 64.8$ V. Therefore, the Thévenin equivalent circuit is a 64.8 V source in series with a $6\,\Omega$ resistor.

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AP 4.17 We begin by performing a source transformation, turning the parallel combination of the 15 A source and 8 Ω resistor into a series combination of a 120 V source and an 8 Ω resistor, as shown in the figure on the left. Next, combine the 2 Ω, 8 Ω and 10 Ω resistors in series to give an equivalent 20 Ω resistance. Then transform the series combination of the 120 V source and the 20 Ω equivalent resistance into a parallel combination of a 6 A source and a 20 Ω resistor, as shown in the figure on the right.

Finally, combine the 20 Ω and 12 Ω parallel resistors to give $R_N = 20\|12 = 7.5\,\Omega$. Thus, the Norton equivalent circuit is the parallel combination of a 6 A source and a 7.5 Ω resistor.

AP 4.18 Find the Thévenin equivalent with respect to A, B using source transformations. To begin, convert the series combination of the −36 V source and 12 kΩ resistor into a parallel combination of a −3 mA source and 12 kΩ resistor. The resulting circuit is shown below:

Now combine the two parallel current sources and the two parallel resistors to give a $-3 + 18 = 15$ mA source in parallel with a 12 k‖60 k= 10 kΩ resistor. Then transform the 15 mA source in parallel with the 10 kΩ resistor into a 150 V source in series with a 10 kΩ resistor, and combine this 10 kΩ resistor in series with the 15 kΩ resistor. The Thévenin equivalent is thus a 150 V source in series with a 25 kΩ resistor, as seen to the left of the terminals A,B in the circuit below.

Now attach the voltmeter, modeled as a 100 kΩ resistor, to the Thévenin equivalent and use voltage division to calculate the meter reading v_{AB}:

$$v_{AB} = \frac{100{,}000}{125{,}000}(150) = 120\,\text{V}$$

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AP 4.19 Begin by calculating the open circuit voltage, which is also v_{Th}, from the circuit below:

Summing the currents away from the node labeled v_{Th} We have

$$\frac{v_{\text{Th}}}{8} + 4 + 3i_x + \frac{v_{\text{Th}} - 24}{2} = 0$$

Also, using Ohm's law for the $8\,\Omega$ resistor,

$$i_x = \frac{v_{\text{Th}}}{8}$$

Substituting the second equation into the first and solving for v_{Th} yields $v_{\text{Th}} = 8\,\text{V}$.

Now calculate R_{Th}. To do this, we use the test source method. Replace the voltage source with a short circuit, the current source with an open circuit, and apply the test voltage v_{T}, as shown in the circuit below:

Write a KCL equation at the middle node:

$$i_{\text{T}} = i_x + 3i_x + v_{\text{T}}/2 = 4i_x + v_{\text{T}}/2$$

Use Ohm's law to determine i_x as a function of v_{T}:

$$i_x = v_{\text{T}}/8$$

Substitute the second equation into the first equation:

$$i_{\text{T}} = 4(v_{\text{T}}/8) + v_{\text{T}}/2 = v_{\text{T}}$$

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Thus,

$$R_{\text{Th}} = v_{\text{T}}/i_{\text{T}} = 1\,\Omega$$

The Thévenin equivalent is an 8 V source in series with a $1\,\Omega$ resistor.

AP 4.20 Begin by calculating the open circuit voltage, which is also v_{Th}, using the node voltage method in the circuit below:

The node voltage equations are

$$\begin{aligned} \frac{v}{60} + \frac{v - (v_{\text{Th}} + 160 i_\Delta)}{20} - 4 &= 0, \\ \frac{v_{\text{Th}}}{40} + \frac{v_{\text{Th}}}{80} + \frac{v_{\text{Th}} + 160 i_\Delta - v}{20} &= 0 \end{aligned}$$

The dependent source constraint equation is

$$i_\Delta = \frac{v_{\text{Th}}}{40}$$

Substitute the constraint equation into the node voltage equations and put the two equations in standard form:

$$\begin{aligned} v\left(\frac{1}{60} + \frac{1}{20}\right) &+ v_{\text{Th}}\left(-\frac{5}{20}\right) &= 4 \\ v\left(-\frac{1}{20}\right) &+ v_{\text{Th}}\left(\frac{1}{40} + \frac{1}{80} + \frac{5}{20}\right) &= 0 \end{aligned}$$

Solving, $v = 172.5$ V and $v_{\text{Th}} = 30$ V.

Now use the test source method to calculate the test current and thus R_{Th}. Replace the current source with a short circuit and apply the test source to get the following circuit:

Write a KCL equation at the rightmost node:

$$i_{\text{T}} = \frac{v_{\text{T}}}{80} + \frac{v_{\text{T}}}{40} + \frac{v_{\text{T}} + 160 i_\Delta}{80}$$

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The dependent source constraint equation is

$$i_\Delta = \frac{v_\mathrm{T}}{40}$$

Substitute the constraint equation into the KCL equation and simplify the right-hand side:

$$i_\mathrm{T} = \frac{v_\mathrm{T}}{10}$$

Therefore,

$$R_\mathrm{Th} = \frac{v_\mathrm{T}}{i_\mathrm{T}} = 10\,\Omega$$

Thus, the Thévenin equivalent is a 30 V source in series with a $10\,\Omega$ resistor.

AP 4.21 First find the Thévenin equivalent circuit. To find v_Th, create an open circuit between nodes a and b and use the node voltage method with the circuit below:

The node voltage equations are:

$$\begin{aligned}
\frac{v_\mathrm{Th} - (100 + v_\phi)}{4} + \frac{v_\mathrm{Th} - v_1}{4} &= 0 \\
\frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1 - v_\mathrm{Th}}{4} &= 0
\end{aligned}$$

The dependent source constraint equation is

$$v_\phi = v_1 - 20$$

Place these three equations in standard form:

$$\begin{aligned}
v_\mathrm{Th}\left(\frac{1}{4} + \frac{1}{4}\right) &+ v_1\left(-\frac{1}{4}\right) &&+ v_\phi\left(-\frac{1}{4}\right) &= 25 \\
v_\mathrm{Th}\left(-\frac{1}{4}\right) &+ v_1\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) &&+ v_\phi\,(0) &= 30 \\
v_\mathrm{Th}\,(0) &+ v_1\,(1) &&+ v_\phi\,(-1) &= 20
\end{aligned}$$

Solving, $v_\mathrm{Th} = 120$ V, $v_1 = 80$ V, and $v_\phi = 60$ V.

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Now create a short circuit between nodes a and b and use the mesh current method with the circuit below:

The mesh current equations are

$$\begin{aligned} -100 + 4(i_1 - i_2) + v_\phi + 20 &= 0 \\ -v_\phi + 4i_2 + 4(i_2 - i_{\text{sc}}) + 4(i_2 - i_1) &= 0 \\ -20 - v_\phi + 4(i_{\text{sc}} - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_{\text{sc}})$$

Place these four equations in standard form:

$$\begin{aligned} 4i_1 - 4i_2 + 0i_{\text{sc}} + v_\phi &= 80 \\ -4i_1 + 12i_2 - 4i_{\text{sc}} - v_\phi &= 0 \\ 0i_1 - 4i_2 + 4i_{\text{sc}} - v_\phi &= 20 \\ 4i_1 + 0i_2 - 4i_{\text{sc}} - v_\phi &= 0 \end{aligned}$$

Solving, $i_1 = 45$ A, $i_2 = 30$ A, $i_{\text{sc}} = 40$ A, and $v_\phi = 20$ V. Thus,

$$R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{\text{sc}}} = \frac{120}{40} = 3\,\Omega$$

[a] For maximum power transfer, $R = R_{\text{Th}} = 3\,\Omega$

[b] The Thévenin voltage, $v_{\text{Th}} = 120$ V, splits equally between the Thévenin resistance and the load resistance, so

$$v_{\text{load}} = \frac{120}{2} = 60\,\text{V}$$

Therefore,

$$p_{\max} = \frac{v_{\text{load}}^2}{R_{\text{load}}} = \frac{60^2}{3} = 1200\,\text{W}$$

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AP 4.22 Sustituting the value $R = 3\,\Omega$ into the circuit and identifying three mesh currents we have the circuit below:

The mesh current equations are:

$$\begin{aligned} -100 + 4(i_1 - i_2) + v_\phi + 20 &= 0 \\ -v_\phi + 4i_2 + 4(i_2 - i_3) + 4(i_2 - i_1) &= 0 \\ -20 - v_\phi + 4(i_3 - i_2) + 3i_3 &= 0 \end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_3)$$

Place these four equations in standard form:

$$\begin{aligned} 4i_1 - 4i_2 + 0i_3 + v_\phi &= 80 \\ -4i_1 + 12i_2 - 4i_3 - v_\phi &= 0 \\ 0i_1 - 4i_2 + 7i_3 - v_\phi &= 20 \\ 4i_1 + 0i_2 - 4i_3 - v_\phi &= 0 \end{aligned}$$

Solving, $i_1 = 30$ A, $i_2 = 20$ A, $i_3 = 20$ A, and $v_\phi = 40$ V.

[a] $p_{100\text{V}} = -(100)i_1 = -(100)(30) = -3000$ W. Thus, the 100 V source is delivering 3000 W.

[b] $p_{\text{depsource}} = -v_\phi i_2 = -(40)(20) = -800$ W. Thus, the dependent source is delivering 800 W.

[c] From Assessment Problem 4.21(b), the power delivered to the load resistor is 1200 W, so the load power is $(1200/3800)100 = 31.58\%$ of the combined power generated by the 100 V source and the dependent source.

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Problems

P 4.1

[**a**] 12 branches, 8 branches with resistors, 2 branches with independent sources, 2 branches with dependent sources.

[**b**] The current is unknown in every branch except the one containing the 25 mA current source, so the current is unknown in 11 branches.

[**c**] 10 essential branches – $R_1 - R_2$ forms an essential branch as does $R_8 - 2$ V. The remaining eight branches are essential branches that contain a single element.

[**d**] The current is known only in the essential branch containing the current source, and is unknown in the remaining 9 essential branches

[**e**] From the figure there are 7 nodes – three identified by rectangular boxes, two identified by triangles, and two identified by diamonds.

[**f**] There are 5 essential nodes, three identified with rectangular boxes and two identified with triangles.

[**g**] A mesh is like a window pane, and as can be seen from the figure there are 6 window panes or meshes.

P 4.2 [**a**] From Problem 4.1(d) there are 9 essential branches where the current is unknown, so we need 9 simultaneous equations to describe the circuit.

[**b**] From Problem 4.1(f), there are 5 essential nodes, so we can apply KCL at $(5 - 1) = 4$ of these essential nodes. There would also be a dependent source constraint equation.

[**c**] The remaining 4 equations needed to describe the circuit will be derived from KVL equations.

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[**d**] We must avoid using the bottom left-most mesh, since it contains a current source, and we have no way of determining the voltage drop across a current source. The two meshes on the bottom that share the dependent source must be handled in a special way.

P 4.3 [**a**] There are eight circuit components, seven resistors and the voltage source. Therefore there are **eight** unknown currents. However, the voltage source and the R_1 resistor are in series, so have the same current. The R_4 and R_6 resistors are also in series, so have the same current. The R_5 and R_7 resistors are in series, so have the same current. Therefore, we only need 5 equations to find the 5 distinct currents in this circuit.

[**b**]

There are three essential nodes in this circuit, identified by the boxes. At two of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the third node would be dependent on the first two. Therefore there are **two** independent KCL equations.

[**c**] Sum the currents at any two of the three essential nodes a, b, and c. Using nodes a and c we get

$$-i_1 + i_2 + i_4 = 0$$

$$i_1 - i_3 + i_5 = 0$$

[**d**] There are three meshes in this circuit: one on the left with the components v_s, R_1, R_2 and R_3; one on the top right with components R_2, R_4, and R_6; and one on the bottom right with components R_3, R_5, and R_7. We can write KVL equations for all three meshes, giving a total of **three** independent KVL equations.

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[**e**] $-v_s + R_1 i_1 + R_2 i_2 + R_3 i_3 = 0$

$R_4 i_4 + R_6 i_4 - R_2 i_2 = 0$

$R_3 i_3 + R_5 i_5 + R_7 i_5 = 0$

P 4.4

[**a**] At node a: $-i_1 + i_2 + i_4 = 0$

At node b: $-i_2 + i_3 - i_4 - i_5 = 0$

At node c: $i_1 - i_3 + i_5 = 0$

[**b**] There are many possible solutions. For example, adding the equations at nodes a and c gives the equation at node b:

$(-i_1 + i_2 + i_4) + (i_1 - i_3 + i_5) = 0$ so $i_2 - i_3 + i_4 + i_5 = 0$

This is the equation at node b with both sides multiplied by -1.

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P 4.5

[**a**] As can be seen from the figure, the circuit has 2 separate parts.

[**b**] There are 5 nodes – the four black dots and the node betweem the voltage source and the resistor R_3.

[**c**] There are 7 branches, each containing one of the seven circuit components.

[**d**] When a conductor joins the lower nodes of the two separate parts, there is now only a single part in the circuit. There would now be 4 nodes, because the two lower nodes are now joined as a single node. The number of branches remains at 7, where each branch contains one of the seven individual circuit components.

P 4.6 Use the lower terminal of the 25 Ω resistor as the reference node.

$$\frac{v_o - 24}{20 + 80} + \frac{v_o}{25} + 0.04 = 0$$

Solving, $\quad v_o = 4\,\text{V}$

P 4.7 [**a**] From the solution to Problem 4.6 we know $v_o = 4$ V, therefore

$$p_{40\text{mA}} = 0.04 v_o = 0.16\,\text{W}$$

$$\therefore \quad p_{40\text{mA}} \text{ (developed)} = -160 \text{ mW}$$

[**b**] The current into the negative terminal of the 24 V source is

$$i_g = \frac{24 - 4}{20 + 80} = 0.2\,\text{A}$$

$$p_{24\text{V}} = -24(0.2) = -4.8\,\text{W}$$

$$\therefore \quad p_{24\text{V}} \text{ (developed)} = 4800 \text{ mW}$$

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[c] $p_{20\Omega} = (0.2)^2(20) = 800 \text{ mW}$

$p_{80\Omega} = (0.2)^2(80) = 3200 \text{ mW}$

$p_{25\Omega} = (4)^2/25 = 640 \text{ mW}$

$\sum p_{\text{dev}} = 4800 \text{ mW}$

$\sum p_{\text{dis}} = 160 + 800 + 3200 + 640 = 4800 \text{ mW}$

P 4.8 **[a]** $\dfrac{v_0 - 24}{20 + 80} + \dfrac{v_o}{25} + 0.04 = 0; \quad v_o = 4\,\text{V}$

[b] Let v_x = voltage drop across 40 mA source

$v_x = v_o - (50)(0.04) = 2\,\text{V}$

$p_{40\text{mA}} = (2)(0.04) = 80 \text{ mW} \quad \text{so} \quad p_{40\text{mA}} \text{ (developed)} = -80 \text{ mW}$

[c] Let i_g = be the current into the positive terminal of the 24 V source

$i_g = (4 - 24)/100 = -0.2\,\text{A}$

$p_{24\text{V}} = (-0.2)(24) = -4800 \text{ mW} \quad \text{so} \quad p_{24\text{V}} \text{ (developed)} = 4800 \text{ mW}$

[d] $\sum p_{\text{dis}} = (0.2)^2(20) + (0.2)^2(80) + (4)^2/25 + (0.04)^2(50) + 0.08$

$= 4800 \text{ mW}$

[e] v_o is independent of any finite resistance connected in series with the 40 mA current source

P 4.9

$$-2 + \frac{v_o}{50} + \frac{v_o - 45}{1 + 4} = 0$$

$v_o = 50\,\text{V}$

$p_{2\text{A}} = -(50)(2) = -100\,\text{W} \quad \text{(delivering)}$

The 2 A source extracts −100 W from the circuit, because it delivers 100 W to the circuit.

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P 4.10 **[a]** $\dfrac{v_o - v_1}{R} + \dfrac{v_o - v_2}{R} + \dfrac{v_o - v_3}{R} + \cdots + \dfrac{v_o - v_n}{R} = 0$

$$\therefore \quad nv_o = v_1 + v_2 + v_3 + \cdots + v_n$$

$$\therefore \quad v_o = \frac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \frac{1}{n}\sum\nolimits_{k=1}^{n} v_k$$

[b] $v_o = \dfrac{1}{3}(100 + 80 - 60) = 40\,\text{V}$

P 4.11 **[a]**

$$\frac{v_1 - 128}{5} + \frac{v_1}{60} + \frac{v_1 - v_2}{4} = 0$$

$$\frac{v_2 - v_1}{4} + \frac{v_2}{80} + \frac{v_2 - 320}{10} = 0$$

In standard form,

$$v_1\left(\frac{1}{5} + \frac{1}{60} + \frac{1}{4}\right) + v_2\left(-\frac{1}{4}\right) = \frac{128}{5}$$

$$v_1\left(-\frac{1}{4}\right) + v_2\left(\frac{1}{4} + \frac{1}{80} + \frac{1}{10}\right) = \frac{320}{10}$$

Solving, $v_1 = 162\,\text{V}$; $\quad v_2 = 200\,\text{V}$

$$i_\text{a} = \frac{128 - 162}{5} = -6.8\,\text{A}$$

$$i_\text{b} = \frac{162}{60} = 2.7\,\text{A}$$

$$i_\text{c} = \frac{162 - 200}{4} = -9.5\,\text{A}$$

$$i_\text{d} = \frac{200}{80} = 2.5\,\text{A}$$

$$i_\text{e} = \frac{200 - 320}{10} = -12\,\text{A}$$

[b] $p_{128V} = -(128)(-6.8) = 870.4$ W (abs)
$p_{320V} = (320)(-12) = -3840$ W (dev)
Therefore, the total power developed is 3840 W.

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P 4.12

$$\frac{v_1 - 144}{4} + \frac{v_1}{10} + \frac{v_1 - v_2}{80} = 0 \qquad \text{so} \qquad 29v_1 - v_2 = 2880$$

$$-3 + \frac{v_2 - v_1}{80} + \frac{v_2}{5} = 0 \qquad \text{so} \qquad -v_1 + 17v_2 = 240$$

Solving, $v_1 = 100\,\text{V}$; $\quad v_2 = 20\,\text{V}$

P 4.13 $-6 + \dfrac{v_1}{40} + \dfrac{v_1 - v_2}{8} = 0$

$$\frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 = 0$$

Solving, $v_1 = 120$ V; $\quad v_2 = 96\,\text{V}$
CHECK:

$$p_{40\Omega} = \frac{(120)^2}{40} = 360\,\text{W}$$

$$p_{8\Omega} = \frac{(120 - 96)^2}{8} = 72\,\text{W}$$

$$p_{80\Omega} = \frac{(96)^2}{80} = 115.2\,\text{W}$$

$$p_{120\Omega} = \frac{(96)^2}{120} = 76.8\,\text{W}$$

$$p_{6\text{A}} = -(6)(120) = -720\,\text{W}$$

$$p_{1\text{A}} = (1)(96) = 96\,\text{W}$$

$$\sum p_{\text{abs}} = 360 + 72 + 115.2 + 76.8 + 96 = 720\,\text{W}$$

$$\sum p_{\text{dev}} = 720\,\text{W} \quad \text{(CHECKS)}$$

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P 4.14 **[a]**

$$\frac{v_1}{40} + \frac{v_1 - 40}{4} + \frac{v_1 - v_2}{2} = 0 \qquad \text{so} \qquad 31v_1 - 20v_2 + 0v_3 = 400$$

$$\frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{4} - 28 = 0 \qquad \text{so} \qquad -2v_1 + 3v_2 - v_3 = 112$$

$$\frac{v_3}{2} + \frac{v_3 - v_2}{4} + 28 = 0 \qquad \text{so} \qquad 0v_1 - v_2 + 3v_3 = -112$$

Solving, $v_1 = 60\,\text{V}$; $\quad v_2 = 73\,\text{V}$; $\quad v_3 = -13\,\text{V}$,

[b] $i_g = \dfrac{40 - 60}{4} = -5\,\text{A}$

$p_g = (40)(-5) = -200\,\text{W}$

Thus the 40 V source delivers 200 W of power.

P 4.15 **[a]**

$$\frac{v_1 - 125}{1} + \frac{v_1 - v_2}{6} + \frac{v_1 - v_3}{24} = 0$$

$$\frac{v_2 - v_1}{6} + \frac{v_2}{2} + \frac{v_2 - v_3}{12} = 0$$

$$\frac{v_3 + 125}{1} + \frac{v_3 - v_2}{12} + \frac{v_3 - v_1}{24} = 0$$

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In standard form:

$$
\begin{aligned}
v_1\left(\frac{1}{1}+\frac{1}{6}+\frac{1}{24}\right)+v_2\left(-\frac{1}{6}\right)+v_3\left(-\frac{1}{24}\right) &= 125\\
v_1\left(-\frac{1}{6}\right)+v_2\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{12}\right)+v_3\left(-\frac{1}{12}\right) &= 0\\
v_1\left(-\frac{1}{24}\right)+v_2\left(-\frac{1}{12}\right)+v_3\left(\frac{1}{1}+\frac{1}{12}+\frac{1}{24}\right) &= -125
\end{aligned}
$$

Solving, $v_1 = 101.24\,\text{V}$; $\quad v_2 = 10.66\,\text{V}$; $\quad v_3 = -106.57\,\text{V}$

Thus, $\quad i_1 = \dfrac{125 - v_1}{1} = 23.76\ \text{A} \qquad i_4 = \dfrac{v_1 - v_2}{6} = 15.10\,\text{A}$

$$i_2 = \frac{v_2}{2} = 5.33\ \text{A} \qquad i_5 = \frac{v_2 - v_3}{12} = 9.77\,\text{A}$$

$$i_3 = \frac{v_3 + 125}{1} = 18.43\ \text{A} \qquad i_6 = \frac{v_1 - v_3}{24} = 8.66\,\text{A}$$

[b] $\sum P_{\text{dev}} = 125 i_1 + 125 i_3 = 5273.09\,\text{W}$

$$\sum P_{\text{dis}} = i_1^2(1) + i_2^2(2) + i_3^2(1) + i_4^2(6) + i_5^2(12) + i_6^2(24) = 5273.09\,\text{W}$$

P 4.16

$$\frac{v_1 + 40}{12} + \frac{v_1}{25} + \frac{v_1 - v_2}{20} + 5 = 0$$

$$\left[\frac{v_2 - v_1}{20}\right] - 5 + \frac{v_2 - v_1}{40} + -7.5 = 0$$

$$\frac{v_3}{40} + \frac{v_3 - v_2}{40} + 7.5 = 0$$

Solving, $v_1 = -10\,\text{V}$; $\quad v_2 = 132\,\text{V}$; $\quad v_3 = -84\,\text{V}$; $\quad i_{40\text{V}} = \dfrac{-10 + 40}{12} = 2.5\,\text{A}$

$$p_{5\text{A}} = 5(v_1 - v_2) = 5(-10 - 132) = -710\,\text{W} \quad \text{(del)}$$

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$$p_{7.5\text{A}} = (-84 - 132)(7.5) = -1620\,\text{W} \quad \text{(del)}$$

$$p_{40\text{V}} = -(40)(2.5) = -100\,\text{W} \quad \text{(del)}$$

$$p_{12\Omega} = (2.5)^2(12) = 75\,\text{W}$$

$$p_{25\Omega} = \frac{v_1^2}{25} = \frac{10^2}{25} = 4\,\text{W}$$

$$p_{20\Omega} = \frac{(v_1 - v_2)^2}{20} = \frac{142^2}{20} = 1008.2\,\text{W}$$

$$p_{40\Omega}(\text{lower}) = \frac{(v_3)^2}{40} = \frac{84^2}{40} = 176.4\,\text{W}$$

$$p_{40\Omega}(\text{right}) = \frac{(v_2 - v_3)^2}{40} = \frac{216^2}{40} = 1166.4\,\text{W}$$

$$\sum p_{\text{diss}} = 75 + 4 + 1008.2 + 176.4 + 1166.4 = 2430\,\text{W}$$

$$\sum p_{\text{dev}} = 710 + 1620 + 100 = 2430\,\text{W} \qquad \text{(CHECKS)}$$

The total power dissipated in the circuit is 2430 W.

P 4.17 $-3 + \dfrac{v_o}{200} + \dfrac{v_o + 5i_\Delta}{10} + \dfrac{v_o - 80}{20} = 0; \quad i_\Delta = \dfrac{v_o - 80}{20}$

[a] Solving, $v_o = 50$ V

[b] $i_{\text{ds}} = \dfrac{v_o + 5i_\Delta}{10}$

$$i_\Delta = (50 - 80)/20 = -1.5\,\text{A}$$

$$\therefore \quad i_{\text{ds}} = 4.25\,\text{A}; \quad 5i_\Delta = -7.5\,\text{V}: \quad p_{\text{ds}} = (-5i_\Delta)(i_{\text{ds}}) = 31.875\,\text{W}$$

[c] $p_{3\text{A}} = -3v_o = -3(50) = -150\,\text{W} \quad \text{(del)}$

$$p_{80\text{V}} = 80i_\Delta = 80(-1.5) = -120\,\text{W} \quad \text{(del)}$$

$$\sum p_{\text{del}} = 150 + 120 = 270\,\text{W}$$

CHECK:

$$p_{200\Omega} = 2500/200 = 12.5\,\text{W}$$

$$p_{20\Omega} = (80 - 50)^2/20 = 900/20 = 45\,\text{W}$$

$$p_{10\Omega} = (4.25)^2(10) = 180.625\,\text{W}$$

$$\sum p_{\text{diss}} = 31.875 + 180.625 + 12.5 + 45 = 270\,\text{W}$$

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P 4.18

$$\frac{v_o - 160}{10} + \frac{v_o}{100} + \frac{v_o - 150i_\sigma}{50} = 0; \quad i_\sigma = -\frac{v_o}{100}$$

Solving, $v_o = 100\,\text{V}; \qquad i_\sigma = -1\,\text{A}$

$$i_o = \frac{100 - (150)(-1)}{50} = 5\,\text{A}$$

$$p_{150i_\sigma} = 150 i_\sigma i_o = -750\,\text{W}$$

$\therefore$ The dependent voltage source delivers 750 W to the circuit.

P 4.19

[a] $0.02 + \dfrac{v_1}{1000} + \dfrac{v_1 - v_2}{1250} = 0$

$$\frac{v_2 - v_1}{1250} + \frac{v_2}{4000} + \frac{v_2}{2000} + \frac{v_2 - 2500i_\Delta}{200} = 0$$

$$i_\Delta = \frac{v_2 - v_1}{1250}$$

Solving,

$v_1 = 60\,\text{V}; \qquad v_2 = 160\,\text{V}; \qquad i_\Delta = 80\text{ mA}$

$$P_{20\text{mA}} = (0.02)v_1 = (0.02)(60) = 1.2\,\text{W (absorbed)}$$

$$i_{\text{ds}} = \frac{v_2 - 2500i_\Delta}{200} = \frac{160 - (2500)(0.08)}{200} = -0.2\,\text{A}$$

$$P_{\text{ds}} = (2500i_\Delta)i_{\text{ds}} = 2500(0.08)(-0.2) = -40\,\text{W (40 W developed)}$$

$$P_{\text{developed}} = 40\,\text{W}$$

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[b] $P_{1\text{k}} = \dfrac{v_1^2}{1000} = \dfrac{60^2}{1000} = 3.6\,\text{W}$

$P_{1250} = 1250 i_\Delta^2 = 1250(0.08)^2 = 8\,\text{W}$

$P_{4\text{k}} = \dfrac{v_2^2}{4000} = \dfrac{160^2}{4000} = 6.4\,\text{W}$

$P_{2\text{k}} = \dfrac{v_2^2}{2000} = \dfrac{160^2}{2000} = 12.8\,\text{W}$

$P_{200} = 200 i_{\text{ds}}^2 = 200(-0.2)^2 = 8\,\text{W}$

$P_{\text{absorbed}} = P_{20\text{mA}} + P_{1\text{k}} + P_{1250} + P_{4\text{k}} + P_{2\text{k}} + P_{200}$

$= 1.2 + 3.6 + 8 + 6.4 + 12.8 + 8 = 40\,\text{W}$ (check)

P 4.20

[a] $\dfrac{v_\Delta - 50}{500} + \dfrac{v_\Delta}{1000} + \dfrac{v_\Delta - v_o}{2000} = 0$

$\dfrac{v_o - v_\Delta}{2000} - \dfrac{v_\Delta}{750} + \dfrac{v_o}{200} = 0$

Solving,

$v_\Delta = 30\,\text{V}; \qquad v_o = 10\,\text{V}$

[b] $i_{50\text{V}} = \dfrac{v_\Delta - 50}{500} = \dfrac{30 - 50}{500} = -0.04\,\text{A}$

$P_{50\text{V}} = 50 i_{50\text{V}} = 50(-0.04) = -2\,\text{W}$ (2 W supplied)

$P_{\text{ds}} = -v_o\left(\dfrac{v_\Delta}{750}\right) = -(10)(30/750) = -0.4\,\text{W}$ (0.4 W supplied)

$P_{\text{total}} = 2 + 0.4 = 2.4\,\text{W}$ supplied

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P 4.21 **[a]**

$$i_o = \frac{v_2}{40}$$

$$-5i_o + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0 \qquad \text{so} \qquad 10v_1 - 13v_2 + 0v_3 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} \qquad \text{so} \qquad -8v_1 + 13v_2 - 4v_3 = 0$$

$$\frac{v_3 - v_2}{10} + \frac{v_3 - 11.5i_o}{5} + \frac{v_3 - 96}{4} = 0 \qquad \text{so} \qquad 0v_1 - 63v_2 + 220v_3 = 9600$$

Solving, $v_1 = 156\,\text{V}; \quad v_2 = 120\,\text{V}; \quad v_3 = 78\,\text{V}$

[b] $i_o = \dfrac{v_2}{40} = \dfrac{120}{40} = 3\,\text{A}$

$$i_3 = \frac{v_3 - 11.5i_o}{5} = \frac{78 - 11.5(3)}{5} = 8.7\,\text{A}$$

$$i_g = \frac{78 - 96}{4} = -4.5\,\text{A}$$

$$p_{5i_o} = -5i_o v_1 = -5(3)(156) = -2340\,\text{W(dev)}$$

$$p_{11.5i_o} = 11.5i_o i_3 = 11.5(3)(8.7) = 300.15\,\text{W(abs)}$$

$$p_{96V} = 96(-4.5) = -432\,\text{W(dev)}$$

$$\sum p_{\text{dev}} = 2340 + 432 = 2772\,\text{W}$$

CHECK

$$\sum p_{\text{dis}} = \frac{156^2}{20} + \frac{(156-120)^2}{5} + \frac{120^2}{40} + \frac{(120-78)^2}{50}$$
$$+(8.7)^2(5) + (4.5)^2(4) + 300.15 = 2772\,\text{W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{dis}} = 2772\,\text{W}$$

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P 4.22 **[a]**

This circuit has a supernode includes the nodes v_1, v_2 and the 25 V source. The supernode equation is

$$2 + \frac{v_1}{50} + \frac{v_2}{150} + \frac{v_2}{75} = 0$$

The supernode constraint equation is

$$v_1 - v_2 = 25$$

Place these two equations in standard form:

$$\begin{array}{lcl} v_1\left(\frac{1}{50}\right) & + & v_2\left(\frac{1}{150}+\frac{1}{75}\right) \quad = \quad -2 \\ v_1(1) & + & v_2(-1) \quad = \quad 25 \end{array}$$

Solving, $v_1 = -37.5$ V and $v_2 = -62.5$ V, so $v_o = v_1 = -37.5$ V.

$$p_{2\text{A}} = (2)v_o = (2)(-37.5) = -75\,\text{W}$$

The 2 A source delivers 75 W.

[b]

This circuit now has only one non-reference essential node where the voltage is not known – note that it is not a supernode. The KCL equation at v_1 is

$$-2 + \frac{v_1}{50} + \frac{v_1+25}{150} + \frac{v_1+25}{75} = 0$$

Solving, $v_1 = 37.5$ V so $v_0 = -v_1 = -37.5$ V.

$$p_{2\text{A}} = (2)v_o = (2)(-37.5) = -75\,\text{W}$$

The 2 A source delivers 75 W.

[c] The choice of a reference node in part (b) resulted in one simple KCL equation, while the choice of a reference node in part (a) resulted in a supernode KCL equation and a second supernode constraint equation. Both methods give the same result but the choice of reference node in part (b) yielded fewer equations to solve, so is the preferred method.

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P 4.23

The two node voltage equations are:

$$\frac{v_1 - 50}{80} + \frac{v_1}{50} + \frac{v_1 - v_2}{40} = 0$$

$$\frac{v_2 - v_1}{40} - 0.75 + \frac{v_2}{200} + \frac{v_2 - 50}{800} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{80} + \frac{1}{50} + \frac{1}{40}\right) + v_2\left(-\frac{1}{40}\right) = \frac{50}{80}$$

$$v_1\left(-\frac{1}{40}\right) + v_2\left(\frac{1}{40} + \frac{1}{200} + \frac{1}{800}\right) = 0.75 + \frac{50}{800}$$

Solving, $v_1 = 34\,\text{V}$; $v_2 = 53.2\,\text{V}$.
Thus, $v_o = v_2 - 50 = 53.2 - 50 = 3.2\,\text{V}$.
POWER CHECK:

$$i_g = (50-34)/80 + (50-53.2)/800 = 196\,\text{mA}$$

$$p_{50\text{V}} = -(50)(0.196) = -9.8\,\text{W}$$

$$p_{80\Omega} = (50-34)^2/80 = 3.2\,\text{W}$$

$$p_{800\Omega} = (50-53.2)^2/800 = 12.8\,\text{mW}$$

$$p_{40\Omega} = (53.2-34)^2/40 = 9.216\,\text{W}$$

$$p_{50\Omega} = 34^2/50 = 23.12\,\text{W}$$

$$p_{200\Omega} = 53.2^2/200 = 14.1512\,\text{W}$$

$$p_{0.75\text{A}} = -(53.2)(0.75) = -39.9\,\text{W}$$

$$\sum p_{\text{abs}} = 3.2 + .0128 + 9.216 + 23.12 + 14.1512 = 49.7\,\text{W}$$

$$\sum p_{\text{del}} = 9.8 + 39.9 = 49.7 \text{ (check)}$$

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P 4.24

$$\frac{v_1}{30{,}000} + \frac{v_1 - v_2}{5000} + \frac{v_1 - 20}{2000} = 0 \qquad \text{so} \qquad 22v_1 - 6v_2 = 300$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{5000} + \frac{v_2 - 20}{5000} = 0 \qquad \text{so} \qquad -v_1 + 7v_2 = 20$$

Solving, $v_1 = 15\,\text{V}; \qquad v_2 = 5\,\text{V}$

Thus, $i_o = \dfrac{v_1 - v_2}{5000} = 2\ \text{mA}$

P 4.25 **[a]**

$$\frac{v_2 - 230}{1} + \frac{v_2 - v_4}{1} + \frac{v_2 - v_3}{1} = 0 \qquad \text{so} \qquad 3v_2 - 1v_3 - 1v_4 + 0v_5 = 230$$

$$\frac{v_3 - v_2}{1} + \frac{v_3}{1} + \frac{v_3 - v_5}{1} = 0 \qquad \text{so} \qquad -1v_2 + 3v_3 + 0v_4 - 1v_5 = 0$$

$$\frac{v_4 - v_2}{1} + \frac{v_4 - 230}{6} + \frac{v_4 - v_5}{2} = 0 \qquad \text{so} \qquad -12v_2 + 0v_3 + 20v_4 - 6v_5 = 460$$

$$\frac{v_5 - v_3}{1} + \frac{v_5}{6} + \frac{v_5 - v_4}{2} = 0 \qquad \text{so} \qquad 0v_2 - 12v_3 - 6v_4 + 20v_5 = 0$$

Solving, $v_2 = 150\,\text{V}; \quad v_3 = 80\,\text{V}; \quad v_4 = 140\,\text{V}; \quad v_5 = 90\,\text{V}$

$$i_{2\Omega} = \frac{v_4 - v_5}{2} = \frac{140 - 90}{2} = 25\,\text{A}$$

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$$p_{2\Omega} = (25)^2(2) = 1250\,\text{W}$$

[b]
$$\begin{aligned} i_{230\text{V}} &= \frac{v_1 - v_2}{1} + \frac{v_1 - v_4}{6} \\ &= \frac{230 - 150}{1} + \frac{230 - 140}{6} = 80 + 15 = 95\ \text{A} \end{aligned}$$

$$p_{230\text{V}} = (230)(95) = 21{,}850\,\text{W}$$

Check:

$$\begin{aligned} \sum P_{\text{dis}} &= (80)^2(1) + (70)^2(1) + (80)^2(1) + (15)^2(6) + (10)^2(1) \\ &\quad +(10)^2(1) + (25)^2(2) + (15)^2(6) = 21{,}850\,\text{W} \end{aligned}$$

P 4.26 Place $5v_\Delta$ inside a supernode and use the lower node as a reference. Then

$$\frac{v_\Delta - 15}{10} + \frac{v_\Delta}{2} + \frac{v_\Delta - 5v_\Delta}{20} + \frac{v_\Delta - 5v_\Delta}{40} = 0$$

$$12v_\Delta = 60; \qquad v_\Delta = 5\,\text{V}$$

$$v_o = v_\Delta - 5v_\Delta = -4(5) = -20\,\text{V}$$

P 4.27 **[a]**

There is only one node voltage equation:

$$\frac{v_\text{a} + 30}{5000} + \frac{v_\text{a}}{500} + \frac{v_\text{a} - 80}{1000} + 0.01 = 0$$

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Solving,
$v_a + 30 + 10v_a + 5v_a - 400 + 50 = 0 \qquad \text{so} \qquad 16v_a = 320$
$\therefore \qquad v_a = 20\,\text{V}$
Calculate the currents:

$$\begin{aligned} i_1 &= (-30-20)/5000 = -10 \text{ mA} \\ i_2 &= 20/500 = 40 \text{ mA} \\ i_4 &= 80/4000 = 20 \text{ mA} \\ i_5 &= (80-20)/1000 = 60 \text{ mA} \end{aligned}$$

$i_3 + i_4 + i_5 - 10 \text{ mA} = 0 \quad \text{so} \quad i_3 = 0.01 - 0.02 - 0.06 = -0.07 = -70 \text{ mA}$

[b]

$$\begin{aligned} p_{30\text{V}} &= (30)(-0.01) = -0.3 \text{ W} \\ p_{10\text{mA}} &= (20-80)(0.01) = -0.6 \text{ W} \\ p_{80\text{V}} &= (80)(-0.07) = -5.6 \text{ W} \\ p_{5\text{k}} &= (-0.01)^2(5000) = 0.5 \text{ W} \\ p_{500\Omega} &= (0.04)^2(500) = 0.8 \text{ W} \\ p_{1\text{k}} &= (80-20)^2/(1000) = 3.6 \text{ W} \\ p_{4\text{k}} &= (80)^2/(4000) = 1.6 \text{ W} \end{aligned}$$

$\sum p_{\text{abs}} = 0.5 + 0.8 + 3.6 + 1.6 = 6.5\,\text{W}$
$\sum p_{\text{del}} = 0.3 + 0.6 + 5.6 = 6.5 \text{ W (checks!)}$

P 4.28

The two node voltage equations are:

$$\begin{aligned} 7 + \frac{v_b}{3} + \frac{v_b - v_c}{1} &= 0 \\ -2v_x + \frac{v_c - v_b}{1} + \frac{v_c - 4}{2} &= 0 \end{aligned}$$

The constraint equation for the dependent source is:
$v_x = v_c - 4$

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Place these equations in standard form:

$$\begin{array}{lclclcl} v_{\rm b}\left(\frac{1}{3}+1\right) & + & v_{\rm c}(-1) & + & v_x(0) & = & -7 \\ v_{\rm b}(-1) & + & v_{\rm c}\left(1+\frac{1}{2}\right) & + & v_x(-2) & = & \frac{4}{2} \\ v_{\rm b}(0) & + & v_{\rm c}(1) & + & v_x(-1) & = & 4 \end{array}$$

Solving, $v_{\rm c} = 9\,{\rm V}, v_x = 5\,{\rm V},$ and $v_o = v_{\rm b} = 1.5\,{\rm V}$

P 4.29

[a] The left-most node voltage is $75 - 1250i_\phi$. The right-most node voltage is 75 V. Write KCL equations at the essential nodes labeled 1 and 2.

[b] From the values given,

$$i_\phi = \frac{v_1 - v_2}{500} = \frac{105-85}{500} = 0.04\,{\rm A}$$

$$v_\Delta = v_2 - 75 = 85 - 75 = 10\ {\rm V}$$

$$v_{\rm L} = 75 - (1250)(0.04) = 25\ {\rm V}$$

$$i_{\rm x} = \frac{v_{\rm L} - v_1}{1000} + \frac{v_{\rm L}}{2500} = \frac{25-105}{1000} + \frac{25}{2500} = -0.07\,{\rm A}$$

$$i_{\rm y} = \frac{v_2 - 75}{500} - i_{\rm x} = \frac{85-75}{500} + 0.07 = 0.09\,{\rm A}$$

Calculate the total power:

$$P_{\rm dstop} = 1250 i_\phi(i_{\rm x}) = 1250(0.04)(-0.07) = -3.5\,{\rm W}$$

$$P_{\rm dsbot} = -v_1(12\times 10^{-3} v_\Delta) = -(105)(12\times 10^{-3})(10) = -12.6\,{\rm W}$$

$$P_{75{\rm V}} = 75 i_{\rm y} = 75(0.09) = 6.75\,{\rm W}$$

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$$P_{1\text{k}} = \frac{(v_{\text{L}} - v_1)^2}{1000} = \frac{(25-105)^2}{1000} = 6.4\,\text{W}$$

$$P_{2.5\text{k}} = \frac{v_{\text{L}}^2}{2500} = \frac{25^2}{2500} = 0.25\,\text{W}$$

$$P_{500\text{mid}} = 500 i_\phi^2 = 500(0.04)^2 = 0.8\,\text{W}$$

$$P_{500\text{right}} = \frac{v_\Delta^2}{500} = \frac{10^2}{500} = 0.2\,\text{W}$$

$$P_{4.25\text{k}} = \frac{v_2^2}{4250} = \frac{85^2}{4250} = 1.7\,\text{W}$$

[c] $P_{\text{supplied}} = 3.5 + 12.6 = 16.1\,\text{W}$

$$P_{\text{absorbed}} = 6.75 + 6.4 + 0.25 + 0.8 + 0.2 + 1.7 = 16.1\,\text{W} = P_{\text{supplied}}$$

Therefore the analyst is correct.

P 4.30

Node equations:

$$\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_3 - v_2}{4} + \frac{v_3}{80} + 3.125 v_\Delta = 0$$

$$\frac{v_2}{40} + \frac{v_2 - v_3}{4} + \frac{v_2 - 20}{1} = 0$$

Constraint equations:

$$v_\Delta = 20 - v_2$$

$$v_1 - 35 i_\phi = v_3$$

$$i_\phi = v_2/40$$

Solving, $v_1 = -20.25\,\text{V}$; $\quad v_2 = 10\,\text{V}$; $\quad v_3 = -29\,\text{V}$

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Let i_g be the current delivered by the 20 V source, then

$$i_g = \frac{20-(20.25)}{2} + \frac{20-10}{1} = 30.125\,\text{A}$$

$$p_g \text{ (delivered) } = 20(30.125) = 602.5\,\text{W}$$

P 4.31 From Eq. 4.16, $\quad i_B = v_c/(1+\beta)R_E$

From Eq. 4.17, $\quad i_B = (v_b - V_o)/(1+\beta)R_E$

From Eq. 4.19,

$$\begin{aligned} i_B &= \frac{1}{(1+\beta)R_E}\left[\frac{V_{CC}(1+\beta)R_E R_2 + V_o R_1 R_2}{R_1R_2 + (1+\beta)R_E(R_1+R_2)} - V_o\right] \\ &= \frac{V_{CC}R_2 - V_o(R_1+R_2)}{R_1R_2 + (1+\beta)R_E(R_1+R_2)} = \frac{[V_{CC}R_2/(R_1+R_2)] - V_o}{[R_1R_2/(R_1+R_2)] + (1+\beta)R_E} \end{aligned}$$

P 4.32 **[a]**

$$80 = 400i_1 - 200i_2$$

$$-140 = -200i_1 + 600i_2$$

Solving, $i_1 = 0.1\,\text{A}; \quad i_2 = -0.2\,\text{A}$

$i_\text{a} = i_1 = 0.1\,\text{A}; \quad i_\text{b} = i_1 - i_2 = 0.3\,\text{A}; \quad i_\text{c} = -i_2 = 0.2\,\text{A}$

[b] If the polarity of the 140 V source is reversed, we have

$$80 = 400i_1 - 200i_2$$

$$140 = -200i_1 + 600i_2$$

$i_1 = 0.38\,\text{A} \quad \text{and} \quad i_2 = 0.36\,\text{A}$

$i_\text{a} = i_1 = 0.38\,\text{A}; \quad i_\text{b} = i_1 - i_2 = 0.02\,\text{A}; \quad i_\text{c} = -i_2 = -0.36\,\text{A}$

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P 4.33 **[a]**

The three mesh current equations are:

$$-128 + 5i_a + 60(i_a - i_c) = 0$$

$$4i_c + 80(i_c - i_e) + 60(i_c - i_a) = 0$$

$$320 + 80(i_e - i_c) + 10i_e = 0$$

Place these equations in standard form:

$$i_a(5 + 60) + i_c(-60) + i_e(0) = 128$$

$$i_a(-60) + i_c(4 + 80 + 60) + i_e(-80) = 0$$

$$i_a(0) + i_c(-80) + i_e(80 + 10) = -320$$

Solving, $i_a = -6.8$ A; $i_c = -9.5$ A; $i_e = -12$ A
Now calculate the remaining branch currents:

$$i_b = i_a - i_c = 2.7\,\text{A}$$

$$i_d = i_v - i_e = 2.5\,\text{A}$$

[b] $p_{128V} = -(128)(-6.8) = 870.4$ W (abs)
$p_{320V} = (320)(-12) = -3840$ W (dev)
Therefore, the total power developed is 3840 W.

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P 4.34 **[a]**

The three mesh current equations are:

$$-125 + 1i_1 + 6(i_1 - i_6) + 2(i_1 - i_3) = 0$$

$$24i_6 + 12(i_6 - i_3) + 6(i_6 - i_1) = 0$$

$$-125 + 2(i_3 - i_1) + 12(i_3 - i_6) + 1i_3 = 0$$

Place these equations in standard form:

$$i_1(1 + 6 + 2) + i_3(-2) + i_6(-6) = 125$$

$$i_1(-6) + i_3(-12) + i_6(24 + 12 + 6) = 0$$

$$i_1(-2) + i_3(2 + 12 + 1) + i_6(-12) = 125$$

Solving, $i_1 = 23.76$ A; $i_3 = 18.43$ A; $i_6 = 8.66$ A
Now calculate the remaining branch currents:

$$i_2 = i_1 - i_3 = 5.33\,\text{A}$$

$$i_4 = i_1 - i_6 = 15.10\,\text{A}$$

$$i_5 = i_3 - i_6 = 9.77\,\text{A}$$

[b] $p_{\text{sources}} = p_{\text{top}} + p_{\text{bottom}} = -(125)(23.76) - (125)(18.43)$

$$= -2969.58 - 2303.51 = -5273\,\text{W}$$

Thus, the power developed in the circuit is 5273 W.
Now calculate the power absorbed by the resistors:

$$p_{1\text{top}} = (23.76)^2(1) = 564.39\,\text{W}$$

$$p_2 = (5.33)^2(2) = 56.79\,\text{W}$$

$$p_{1\text{bot}} = (18.43)^2(1) = 339.59\,\text{W}$$

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$p_6 = (15.10)^2(6) = 1367.64\,\text{W}$

$p_{12} = (9.77)^2(12) = 1145.22\,\text{W}$

$p_{24} = (8.66)^2(24) = 1799.47\,\text{W}$

The power absorbed by the resistors is
$564.39 + 56.79 + 339.59 + 1367.64 + 1145.22 + 1799.47 = 5273$ W so the power balances.

P 4.35

The three mesh current equations are:

$$-20 + 2000(i_1 - i_2) + 30{,}000(i_1 - i_3) = 0$$

$$5000i_2 + 5000(i_2 - i_3) + 2000(i_2 - i_1) = 0$$

$$1000i_3 + 30{,}000(i_3 - i_1) + 5000(i_3 - i_2) = 0$$

Place these equations in standard form:

$$i_1(32{,}000) + i_2(-2000) + i_3(-30{,}000) = 20$$

$$i_1(-2000) + i_2(12{,}000) + i_3(-5000) = 0$$

$$i_1(-30{,}000) + i_2(-5000) + i_3(36{,}000) = 0$$

Solving, $i_1 = 5.5$ mA; $i_2 = 3$ mA; $i_3 = 5$ mA
Thus, $i_o = i_3 - i_2 = 2$ mA.

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P 4.36 **[a]**

$$40 + 10(i_1 - i_2) + 5(i_1 - i_3) + 90 + 30i_1 = 0$$

$$20i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

$$196 + 2i_3 - 90 + 5(i_3 - i_1) + 4(i_3 - i_2) = 0$$

Solving, $i_1 = -5\,\text{A}; \quad i_2 = -3\,\text{A}; \quad i_3 = -13\,\text{A}$

$$p_{40} = 40i_1 = -200\,\text{W (del)}$$

$$p_{90} = 90(i_1 - i_3) = 720\,\text{W (abs)}$$

$$p_{196} = 196i_3 = -2548\,\text{W (del)}$$

$$\therefore \quad \sum p_{\text{dev}} = 2748\,\text{W}$$

[b] $p_{20\Omega} = (-3)^2(20) = 180\,\text{W}$

$$p_{10\Omega} = (2)^2(10) = 40\,\text{W}$$

$$p_{4\Omega} = (10)^2(4) = 400\,\text{W}$$

$$p_{5\Omega} = (8)^2(5) = 320\,\text{W}$$

$$p_{30\Omega} = (-5)^2(30) = 750\,\text{W}$$

$$p_{2\Omega} = (-13)^2(2) = 338\,\text{W}$$

$$\therefore \quad \sum p_{\text{abs}} = 720 + 180 + 40 + 400 + 320 + 750 + 338 = 2748\,\text{W}$$

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P 4.37 **[a]**

The four mesh current equations are:

$$-230 + 1(i_1 - i_2) + 1(i_1 - i_3) + 1(i_1 - i_4) = 0$$

$$6i_2 + 1(i_2 - i_3) + 1(i_2 - i_1) = 0$$

$$2i_3 + 1(i_3 - i_4) + 1(i_3 - i_1) + 1(i_3 - i_2) = 0$$

$$6i_4 + 1(i_4 - i_1) + 1(i_4 - i_3) = 0$$

Place these equations in standard form:

$$i_1(3) + i_2(-1) + i_3(-1) + i_4(-1) = 230$$

$$i_1(-1) + i_2(8) + i_3(-1) + i_4(0) = 0$$

$$i_1(-1) + i_2(-1) + i_3(5) + i_4(-1) = 0$$

$$i_1(-1) + i_2(0) + i_3(-1) + i_4(8) = 0$$

Solving, $i_1 = 95$ A; $i_2 = 15$ A; $i_3 = 25$ A; $i_4 = 15$ A
The power absorbed by the $5\,\Omega$ resistor is

$$p_5 = i_3^2(2) = (25)^2(2) = 1250\,\text{W}$$

[b] $p_{230} = -(230)i_1 = -(230)(95) = -21{,}850\,\text{W}$

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P 4.38

$$-160 + 10i_1 + 100(i_1 - i_o) = 0$$

$$30i_o + 150i_\sigma + 20i_o + 100i_\sigma = 0$$

$$i_\sigma = i_o - i_1$$

Solving,

$$i_1 = 6\,\text{A}; \qquad i_o = 5\,\text{A}; \qquad i_\sigma = -1\,\text{A}$$

$$P_{\text{ds}} = (150i_\sigma)i_o = 150(-1)(5) = -750\,\text{W}$$

Thus, 750 W is delivered by the dependent source.

P 4.39

$$-65 + 4i_1 + 5(i_1 - i_2) + 6i_1 = 0$$

$$8i_2 + 3v_\Delta + 15i_2 + 5(i_2 - i_1) = 0$$

$$v_\Delta = 4i_1$$

Solving, $i_1 = 4\,\text{A} \qquad i_2 = -1\,\text{A} \qquad v_\Delta = 16\,\text{V}$

$$p_{15\Omega} = (-1)^2(15) = 15\,\text{W}$$

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P 4.40

$660 = 30i_1 - 10i_2 - 15i_3$

$20i_\phi = -10i_1 + 60i_2 - 50i_3$

$0 = -15i_1 - 50i_2 + 90i_3$

$i_\phi = i_2 - i_3$

Solving, $i_1 = 42$ A; $\quad i_2 = 27$ A; $\quad i_3 = 22$ A; $\quad i_\phi = 5$ A

$20i_\phi = 100\,\text{V}$

$p_{20i_\phi} = -100i_2 = -100(27) = -2700\,\text{W}$

$\therefore \quad p_{20i_\phi}$ (developed) $= 2700\,\text{W}$

CHECK:

$p_{660\text{V}} = -660(42) = -27{,}720\,\text{W}$ (dev)

$$\begin{aligned} \therefore \quad \sum P_{\text{dev}} &= 27{,}720 + 2700 = 30{,}420\,\text{W} \\ \sum P_{\text{dis}} &= (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) + \\ &\quad (15)^2(10) \\ &= 30{,}420\,\text{W} \end{aligned}$$

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P 4.41 [**a**]

$$40 = 125i_1 + 300(i_1 - i_\Delta)$$

$$0 = 75i_\Delta + 25i_\Delta \ + 300(i_\Delta - i_1)$$

$$0 = 50i_2 + 200i_2 - 500i_\Delta$$

Solving, $i_1 = 0.2$ A; $\quad i_2 = 0.3\,\text{A}$; $\quad i_\Delta = 0.15\,\text{A}$

$$v_o = 200i_2 = 200(0.3) = 60\,\text{V}$$

[**b**] $p_{\text{ds}} = -(500i_\Delta)i_2 = -500(0.15)(0.3) = -22.5\,\text{W}$

$\therefore \quad p_{\text{ds}}$ (delivered) $= 22.5\,\text{W}$

P 4.42

Mesh equations:

$$53i_\Delta + 8i_1 - 3i_2 - 5i_3 = 0$$

$$0i_\Delta - 3i_1 + 30i_2 - 20i_3 = 30$$

$$0i_\Delta - 5i_1 - 20i_2 + 27i_3 = 30$$

Constraint equations:

$$i_\Delta = i_2 - i_3$$

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Solving, $i_1 = 110$ A; $\quad i_2 = 52$ A; $\quad i_3 = 60$ A; $\quad i_\Delta = -8$ A

$p_{\text{depsource}} = 53 i_\Delta i_1 = (53)(-8)(110) = -46{,}640\,\text{W}$

Therefore, the dependent source is developing 46,640 W.
CHECK:

$p_{30\text{V}} = -30 i_2 = -1560\,\text{W}$ (left source)

$p_{30\text{V}} = -30 i_3 = -1800\,\text{W}$ (right source)

$\sum p_{\text{dev}} = 46{,}640 + 1560 + 1800 = 50\,\text{kW}$

$p_{3\Omega} = (110 - 52)^2(3) = 10{,}092\,\text{W}$

$p_{5\Omega} = (110 - 60)^2(5) = 12{,}500\,\text{W}$

$p_{20\Omega} = (-8)^2(20) = 1280\,\text{W}$

$p_{7\Omega} = (52)^2(7) = 18{,}928\,\text{W}$

$p_{2\Omega} = (60)^2(2) = 7200\,\text{W}$

$\sum p_{\text{diss}} = 10{,}092 + 12{,}500 + 1280 + 18{,}928 + 7200 = 50$ kW

P 4.43 **[a]**

The mesh current equation for the right mesh is:
$5400(i_1 - 0.005) + 3700 i_1 - 150(0.005 - i_1) = 0$
Solving, $\quad 9250 i_1 = 27.75 \quad \therefore \quad i_1 = 3$ mA
Then, $\quad i_\Delta = 5 - 3 = 2$ mA

[b] $v_o = (0.005)(10{,}000) + (5400)(0.002) = 60.8\,\text{V}$
$p_{5\text{mA}} = -(60.8)(0.005) = -304\,\text{mW}$
Thus, the 5 mA source delivers 304 mW

[c] $p_{\text{dep source}} = -150 i_\Delta i_1 = (-150)(0.002)(0.003) = -0.9\,\text{mW}$
The dependent source delivers 0.9 mW.

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P 4.44

Mesh equations:

$128i_1 - 80i_2 = 240$

$-80i_1 + 200i_2 = 120$

Solving,

$i_1 = 3\,\text{A}; \qquad i_2 = 1.8\,\text{A}$

Therefore,

$v_1 = 40(6-3) = 120\,\text{V}; \qquad v_2 = 120(1.8-1) = 96\,\text{V}$

P 4.45 **[a]**

Mesh equations:

$65i_1 - 40i_2 + 0i_3 - 100i_o = 0$

$-40i_1 + 55i_2 - 5i_3 + 11.5i_o = 0$

$0i_1 - 5i_2 + 9i_3 - 11.5i_o = 0$

$-1i_1 + 1i_2 + 0i_3 + 1i_o = 0$

Solving,

$i_1 = 7.2\,\text{A}; \qquad i_2 = 4.2\,\text{A}; \qquad i_3 = -4.5\,\text{A}; \qquad i_o = 3\,\text{A}$

Therefore,

$v_1 = 20[5(3) - 7.2] = 156\,\text{V}; \qquad v_2 = 40(7.2 - 4.2) = 120\,\text{V}$

$v_3 = 5(4.2 + 4.5) + 11.5(3) = 78\,\text{V}$

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[b] $p_{5i_o} = -5i_o v_1 = -5(3)(156) = -2340\,\text{W}$

$p_{11.5i_o} = 11.5i_o(i_2 - i_3) = 11.5(3)(4.2 + 4.5) = 300.15\,\text{W}$

$p_{96\text{V}} = 96i_3 = 96(-4.5) = -432\,\text{W}$

Thus, the total power dissipated in the circuit, which equals the total power developed in the circuit is $2340 + 432 = 2772$ W.

P 4.46

Mesh equations:

$10i_\Delta - 4i_1 = 0$

$-4i_\Delta + 24i_1 + 6.5i_\Delta = 400$

Solving, $i_1 = 15$ A; $\quad i_\Delta = 16$ A

$v_{20\text{A}} = 1i_\Delta + 6.5i_\Delta = 7.5(16) = 120\,\text{V}$

$p_{20\text{A}} = -20v_{20\text{A}} = -(20)(120) = -2400\,\text{W}$ (del)

$p_{6.5i_\Delta} = 6.5i_\Delta i_1 = (6.5)(16)(15) = 1560\,\text{W}$ (abs)

Therefore, the independent source is developing 2400 W, all other elements are absorbing power, and the total power developed is thus 2400 W.
CHECK:

$p_{1\Omega} = (16)^2(1) = 256\,\text{W}$

$p_{5\Omega} = (20 - 16)^2(5) = 80\,\text{W}$

$p_{4\Omega} = (1)^2(4) = 4\,\text{W}$

$p_{20\Omega} = (20 - 15)^2(20) = 500\,\text{W}$

$\sum p_{\text{abs}} = 1560 + 256 + 80 + 4 + 500 = 2400\,\text{W}$ (CHECKS)

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P 4.47

$$5 + 38(i_1 - 5) + 30(i_1 - i_2) + 12i_1 = 0$$

$$67 + 40i_2 + 30(i_2 - i_1) + 6(i_2 - 5) = 0$$

Solving, $i_1 = 2.5\,\text{A}; \quad i_2 = 0.5\,\text{A}$

[a] $v_{5\text{A}} = 38(2.5 - 5) + 6(0.5 - 5)$

$= -122\,\text{V}$

$p_{5\text{A}} = 5v_{5\text{A}} = 5(-122) = -610\,\text{W}$

Therefore, the 5 A source delivers 610 W.

[b] $p_{5\text{V}} = 5(2.5) = 12.5\,\text{W}$

$p_{67\text{V}} = 67(0.5) = 33.5\,\text{W}$

Therefore, only the current source delivers power and the total power delivered is 610 W.

[c] $\sum p_{\text{resistors}} = (2.5)^2(38) + (4.5)^2(6) + (2)^2(30) + (2.5)^2(12) + (0.5)^2(40)$

$= 564\,\text{W}$

$\sum p_{\text{abs}} = 564 + 12.5 + 33.5 = 610\,\text{W} = \sum p_{\text{del}}$ (CHECKS)

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P 4.48 **[a]**

Mesh equations:

$-50 + 6i_1 - 4i_2 + 9i_\Delta = 0$

$-9i_\Delta - 4i_1 + 29i_2 - 20i_3 = 0$

Constraint equations:

$i_\Delta = i_2; \qquad i_3 = -1.7v_\Delta; \qquad v_\Delta = 2i_1$

Solving, $i_1 = -5$ A; $\quad i_2 = 16$ A; $\quad i_3 = 17$ A; $\quad v_\Delta = -10$ V

$9i_\Delta = 9(16) = 144\,\text{V}$

$i_\text{a} = i_2 - i_1 = 21\,\text{A}$

$i_\text{b} = i_2 - i_3 = -1\,\text{A}$

$v_\text{b} = 20i_\text{b} = -20\,\text{V}$

$p_{50\text{V}} = -50i_1 = 250\,\text{W}$ (absorbing)

$p_{9i_\Delta} = -i_\text{a}(9i_\Delta) = -(21)(144) = -3024\,\text{W}$ (delivering)

$p_{1.7\text{V}} = -1.7v_\Delta v_\text{b} = i_3 v_\text{b} = (17)(-20) = -340\,\text{W}$ (delivering)

[b] $\sum P_\text{dev} = 3024 + 340 = 3364\,\text{W}$

$$\sum P_\text{dis} = 250 + (-5)^2(2) + (21)^2(4) + (16)^2(5) + (-1)^2(20)$$
$$= 3364\,\text{W}$$

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P 4.49

$$-100 + 6i_1 + 20i_2 - 25 + 30i_2 + 9i_1 = 0; \quad i_1 - i_2 = 4$$

Solving, $i_1 = 5\,\text{A}; \quad i_2 = 1\,\text{A}$

$p_{100\text{V}} = -100i_1 = -500\,\text{W}$ (delivered)

$p_{6\,\Omega} = (5)^2(6) = 150\,\text{W}$

$p_{9\,\Omega} = (5)^2(9) = 225\,\text{W}$

$p_{20\,\Omega} = (1)^2(20) = 20\,\text{W}$

$p_{30\,\Omega} = (1)^2(30) = 30\,\text{W}$

$v_o = 20(1) - 25 + 30(1) = 25\,\text{V}$

$p_{4\text{A}} = 4v_o = 100\,\text{W}$

$p_{25\text{V}} = -25i_2 = -25\,\text{W}$ (delivered)

$\sum p_{\text{dev}} = 150 + 225 + 20 + 30 + 100 = 525\,\text{W}$

$\sum p_{\text{diss}} = -500 - 25 = -525\,\text{W}$

Thus the total power dissipated is 525 W.

P 4.50 **[a]** Summing around the supermesh used in the solution to Problem 4.49 gives

$$-67.5 + 6i_1 + 20i_2 - 25 + 30i_2 + 9i_1 = 0; \qquad i_1 - i_2 = 4$$

$\therefore \quad i_1 = 4.5\,\text{A}; \qquad i_2 = 0.5\,\text{A}$

$p_{67.5\text{V}} = -67.5(4.5) = -303.75\,\text{W}$ (del)

$v_o = 20(0.5) - 25 + 30(0.5) = 0\,\text{V}$

$p_{4\text{A}} = 4v_o = 0\,\text{W}$

$p_{25\text{V}} = -25i_2 = -12.5\,\text{W}$ (del)

$\sum p_{\text{diss}} = (4.5)^2(6 + 9) + (0.5)^2(20 + 30) = 316.25\,\text{W}$

$\sum p_{\text{dev}} = 303.75 + 0 + 12.5 = 316.25\,\text{W} = \sum p_{\text{diss}}$

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[b] With 4 A current source replaced with a short circuit

$15i_1 = 67.5; \qquad 50i_2 = 25$

Solving,

$i_1 = 4.5\,\text{A}, \qquad i_2 = 0.5\,\text{A}$

$\therefore \quad \sum P_{\text{sources}} = -(67.5)(4.5) - (25)(0.5) = -316.25\,\text{W}$

[c] A 4 A source with zero terminal voltage is equivalent to a short circuit carrying 4 A.

[d] With the new value of the right-hand source, we want $v_o = 0$ but the current in the middle branch must still equal 4 A.
KVL left:

$-100 + 6i_1 + 0 + 9i_1 = 0 \qquad \text{so} \qquad i_1 = 6.667\,\text{A}$

$i_1 - i_2 = 4 \qquad \text{so} \qquad i_2 = i_1 - 4 = 2.667\,\text{A}$

KVL right:

$20i_2 - V_2 + 30i_2 + 0 = 0 \qquad \text{so} \qquad V_2 = 50i_2 = 133.333\,\text{V}$

To check these results, sum around the supermesh with the value of the source on the right as 133.333 V,

$-100 + 6i_1 + 20i_2 - 133.333 + 30i_2 + 9i_1 = 0; \qquad i_1 - i_2 = 4$

Solving,

$i_1 = 6.667\,\text{A}; \qquad i_2 = 2.667\,\text{A}$

Therefore,

$v_o = 20i_2 - 133.333 + 30i_2 = 0$

Thus,

$P_{4\text{A}} = 4v_o = 0$ (checks)

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P 4.51 **[a]**

Supermesh equations:

$1000i_b + 4000(i_c - i_d) + 500(i_c - i_a) = 0$

$i_c - i_b = 0.01$

Two remaining mesh equations:

$5500i_a - 500i_c = -30$

$4000i_d - 4000i_c = -80$

In standard form,

$-500i_a + 1000i_b + 4500i_c - 4000i_d = 0$

$0i_a - 1i_b + 1i_c + 0i_d = 0.01$

$5500i_a + 0i_b - 500i_c + 0i_d = -30$

$0i_a + 0i_b - 4000i_c + 4000i_d = -80$

Solving:

$i_a = -10\,\text{mA}; \qquad i_b = -60\,\text{mA}; \qquad i_c = -50\,\text{mA}; \qquad i_d = -70\,\text{mA}$

Then,

$i_1 = i_a = -10\,\text{mA}; \qquad i_2 = i_a - i_c = 40\,\text{mA}; \qquad i_3 = i_d = -70\,\text{mA}$

[b] $p_{\text{sources}} = 30(-0.01) + [1000(-0.06)](0.01) + 80(-0.07) = -6.5\,\text{W}$

$$p_{\text{resistors}} = 1000(0.06)^2 + 5000(0.01)^2 + 500(0.04)^2$$
$$+4000(-0.05 + 0.07)^2 = 6.5\,\text{W}$$

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P 4.52 **[a]**

$$200 = 85i_1 - 25i_2 - 50i_3$$

$$0 = -75i_1 + 35i_2 + 150i_3 \qquad \text{(supermesh)}$$

$$i_3 - i_2 = 4.3(i_1 - i_2)$$

Solving, $i_1 = 4.6$ A; $\quad i_2 = 5.7$ A; $\quad i_3 = 0.97$ A

$i_\mathrm{a} = i_2 = 5.7\,\mathrm{A}$; $\quad i_\mathrm{b} = i_1 = 4.6\,\mathrm{A}$

$i_\mathrm{c} = i_3 = 0.97\,\mathrm{A}$; $\quad i_\mathrm{d} = i_1 - i_2 = -1.1\,\mathrm{A}$

$i_\mathrm{e} = i_1 - i_3 = 3.63\,\mathrm{A}$

[b] $10i_2 + v_o + 25(i_2 - i_1) = 0$

$\therefore \quad v_o = -57 - 27.5 = -84.5\,\mathrm{V}$

$p_{4.3i_\mathrm{d}} = -v_o(4.3i_\mathrm{d}) = -(-84.5)(4.3)(-1.1) = -399.685\,\mathrm{W}$ (dev)

$p_{200\mathrm{V}} = -200(4.6) = -920\,\mathrm{W}$ (dev)

$$\begin{aligned}
\sum P_\mathrm{dev} &= 1319.685\,\mathrm{W} \\
\sum P_\mathrm{dis} &= (5.7)^2 10 + (1.1)^2(25) + (0.97)^2 100 + (4.6)^2(10) + \\
&\quad (3.63)^2(50) \\
&= 1319.685\,\mathrm{W}
\end{aligned}$$

$\therefore \quad \sum P_\mathrm{dev} = \sum P_\mathrm{dis} = 1319.685\,\mathrm{W}$

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P 4.53 **[a]**

$$-4i_d + 10(i_e - i_d) + 5(i_e - i_x) = 0$$

$$5(i_x - i_e) + 10(i_d - i_e) - 240 + 40(i_x - 19) = 0$$

$$i_d - i_x = 2i_b = 2(i_e - i_x)$$

Solving, $i_d = 10$ A; $\quad i_e = 18$ A; $\quad i_x = 26$ A

$i_a = 19 - i_x = -7\,\text{A}$; $\quad i_b = i_e - i_x = -8\,\text{A}$; $\quad i_c = i_e - i_d = 8\,\text{A}$;

[b] $v_\text{a} = 40i_\text{a} = -280\,\text{V}$; $\qquad v_\text{b} = 5i_b + 40i_a = -320\,\text{V}$

$$p_{19\text{A}} = -19v_a = 5320\,\text{W}$$

$$p_{4i_d} = -4i_d i_e = -720\,\text{W}$$

$$p_{2i_a} = -2i_b v_b = -5120\,\text{W}$$

$$p_{240\text{V}} = -240i_d = -2400\,\text{W}$$

$$p_{40\Omega} = (7)^2(40) = 1960\,\text{W} =$$

$$p_{5\Omega} = (8)^2(5) = 320\,\text{W}$$

$$p_{10\Omega} = (8)^2(10) = 640\,\text{W}$$

$$\sum P_\text{gen} = 720 + 5120 + 2400 = 8240\,\text{W}$$

$$\sum P_\text{diss} = 5320 + 1960 + 320 + 640 = 8240\,\text{W}$$

P 4.54 **[a]** There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

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[b]

The mesh current equations:

$$2500(i_1 - 0.01) + 2000i_1 + 1000(i_1 - i_2) = 0$$

$$5000(i_2 - 0.01) + 1000(i_2 - i_1) + 1000i_2 = 0$$

Place the equations in standard form:

$$i_1(2500 + 2000 + 1000) + i_2(-1000) = 25$$

$$i_1(-1000) + i_2(5000 + 1000 + 1000) = 50$$

Solving, $i_1 = 6$ mA; $\quad i_2 = 8$ mA

Find the power in the 1 kΩ resistor:

$i_{1\text{k}} = i_1 - i_2 = -2\,\text{mA}$

$p_{1\text{k}} = (-0.002)^2(1000) = 4$ mW

[c] No, the voltage across the 10 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

[d] $v_g = 2000i_1 + 1000i_2 = 12 + 8 = 20\,\text{V}$

$p_{10\text{mA}} = -(20)(0.01) = -200\,\text{mW}$

Thus the 10 mA source develops 200 mW.

P 4.55 **[a]** There are three unknown node voltages and three unknown mesh currents, so the number of simultaneous equations required is the same for both methods. The node voltage method has the advantage of having to solve the three simultaneous equations for one unknown voltage provided the connection at either the top or bottom of the circuit is used as the reference node. Therefore recommend the node voltage method.

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[b]

The node voltage equations are:

$$\frac{v_1}{5000} + \frac{v_1 - v_2}{2500} + \frac{v_1 - v_3}{1000} = 0$$

$$-0.01 + \frac{v_2}{4000} + \frac{v_2 - v_1}{2500} + \frac{v_2 - v_3}{2000} = 0$$

$$\frac{v_3 - v_1}{1000} + \frac{v_3 - v_2}{2000} + \frac{v_3}{1000} = 0$$

Put the equations in standard form:

$$v_1\left(\frac{1}{5000} + \frac{1}{2500} + \frac{1}{1000}\right) + v_2\left(-\frac{1}{2500}\right) + v_3\left(-\frac{1}{1000}\right) = 0$$

$$v_1\left(-\frac{1}{2500}\right) + v_2\left(\frac{1}{4000} + \frac{1}{2500} + \frac{1}{2000}\right) + v_3\left(-\frac{1}{2000}\right) = 0.01$$

$$v_1\left(-\frac{1}{1000}\right) + v_2\left(-\frac{1}{2000}\right) + v_3\left(\frac{1}{2000} + \frac{1}{1000} + \frac{1}{1000}\right) = 0$$

Solving, $v_1 = 6.67\,\text{V}$; $v_2 = 13.33\,\text{V}$; $v_3 = 5.33\,\text{V}$

$p_{10\text{m}} = -(13.33)(0.01) = -133.33\,\text{mW}$

Therefore, the 10 mA source is developing 133.33 mW

P 4.56 **[a]** The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 20 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 20 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node

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voltage method, it is the preferred approach.

[b]

Node voltage equations:

$$\frac{v_1}{100} + 0.003v_\Delta + \frac{v_2}{250} - 0.2 = 0$$

$$0.2 + \frac{v_3}{100} + \frac{v_4}{200} - 0.003v_\Delta = 0$$

Constraints:

$$v_2 = v_a; \qquad v_3 = v_\Delta; \qquad v_4 - v_3 = 0.4v_a; \qquad v_2 - v_1 = 20$$

Solving, $v_1 = 24\,\text{V}$; $v_2 = 44\,\text{V}$; $v_3 = -72\,\text{V}$; $v_4 = -54\,\text{V}$.

$$i_o = 0.2 - \frac{v_2}{250} = 24\,\text{mA}$$

$$p_{20\text{V}} = 20(0.024) = 480\,\text{mW}$$

Thus, the 20 V source absorbs 480 mW.

P 4.57 **[a]** The mesh-current method does not directly involve the voltage drop across the 40 mA source. Instead, use the node-voltage method and choose the reference node so that a node voltage is identical to the voltage across the 40 mA source.

[b]

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Since the 40 mA source is developing 0 W, v_1 must be 0 V.

Since v_1 is known, we can sum the currents away from node 1 to find v_2; thus:

$$\frac{0-(70+v_2)}{500}+\frac{0-v_2}{250}+\frac{0}{750}-0.04=0$$

$$\therefore \quad v_2=-30\,\text{V}$$

Now that we know v_2 we sum the currents away from node 2 to find v_3; thus:

$$\frac{v_2+70-0}{500}+\frac{v_2-0}{250}+\frac{v_2-v_3}{1250}=0$$

$$\therefore \quad v_3=-80\,\text{V}$$

Now that we know v_3 we sum the currents away from node 3 to find i_{dc}; thus:

$$\frac{v_3}{1000}+\frac{v_3-v_2}{1250}+i_{\text{dc}}=0$$

$$\therefore \quad i_{\text{dc}}=0.12=120\text{ mA}$$

P 4.58 **[a]** If the mesh-current method is used, then the value of the lower left mesh current is $i_o=0$. This shortcut will simplify the set of KVL equations. The node-voltage method has no equivalent simplifying shortcut, so the mesh-current method is preferred.

[b]

Write the mesh current equations. Note that if $i_o=0$, then $i_1=0$:

$$\begin{aligned}
-23+5(-i_2)+10(-i_3)+46 &= 0\\
30i_2+15(i_2-i_3)+5i_2 &= 0\\
V_{\text{dc}}+25i_3-46+10i_3+15(i_3-i_2) &= 0
\end{aligned}$$

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Place the equations in standard form:

$$\begin{aligned} i_2(-5) + i_3(-10) + V_{\text{dc}}(0) &= -23 \\ i_2(30+15+5) + i_3(-15) + V_{\text{dc}}(0) &= 0 \\ i_2(-15) + i_3(25+10+15) + V_{\text{dc}}(1) &= 46 \end{aligned}$$

Solving, $i_2 = 0.6\,\text{A}$; $i_3 = 2\,\text{A}$; $V_{\text{dc}} = -45\,\text{V}$

Thus, the value of V_{dc} required to make $i_o = 0$ is -45 V.

[c] Calculate the power:

$$\begin{aligned} p_{23\text{V}} &= -(23)(0) = 0\,\text{W} \\ p_{46\text{V}} &= -(46)(2) = -92\,\text{W} \\ p_{\text{Vdc}} &= (-45)(2) = -90\,\text{W} \\ p_{30\Omega} &= (30)(0.6)^2 = 10.8\,\text{W} \\ p_{5\Omega} &= (5)(0.6)^2 = 1.8\,\text{W} \\ p_{15\Omega} &= (15)(2-0.6)^2 = 29.4\,\text{W} \\ p_{10\Omega} &= (10)(2)^2 = 40\,\text{W} \\ p_{20\Omega} &= (20)(0)^2 = 0\,\text{W} \\ p_{25\Omega} &= (25)(2)^2 = 100\,\text{W} \end{aligned}$$

$$\sum p_{\text{dev}} = 92 + 90 = 182\,\text{W}$$

$$\sum p_{\text{dis}} = 10.8 + 1.8 + 29.4 + 40 + 0 + 100 = 182\,\text{W}(\text{checks})$$

P 4.59 **[a]**

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$v_o = (12{,}500)(0.001) = 12.5\,\text{V}$

[b]

$5000i_1 + 40{,}000i_2 - 30{,}000i_3 = 35$

$i_2 - i_1 = 0.008$

$-30{,}000i_2 + 70{,}000i_3 = 25$

Solving,

$i_1 = -5.33\,\text{mA}; \qquad i_2 = 2.667\,\text{mA}; \qquad i_3 = 1.5\,\text{mA}$

$v_o = (25{,}000)(i_3 - 0.001) = (25{,}000)(0.0005) = 12.5\,\text{V}$

P 4.60 **[a]**

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$$i_o = \frac{7.5}{25{,}000} = 0.3\,\text{mA}$$

[b]

$$\begin{aligned}
v_{\text{a}} &= (15{,}000)(0.0003) = 4.5\,\text{V} \\
i_{\text{a}} &= \frac{v_{\text{a}}}{20{,}000} = 225\,\mu\text{A} \\
i_{\text{b}} &= 1 + 0.225 + 0.3 = 1.525\,\text{mA} \\
v_{\text{b}} &= 5000(0.525 \times 10^{-3}) + 4.5 = 7.125\,\text{V} \\
v_{\text{c}} &= 3000(1.525 \times 10^{-3}) + 7.125 + 6000(1.525 \times 10^{-3}) = 20.85\,\text{V}
\end{aligned}$$

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$$
\begin{aligned}
i_g &= \frac{20.85 - 50}{10{,}000} = -2.915\,\text{mA} \\
p_{50\text{V}} &= (50)(-2.915 \times 10^{-3}) = -145.75\,\text{mW}
\end{aligned}
$$

Check:

$$
\begin{aligned}
p_{1\text{mA}} &= (7.125)(10^{-3}) = 7.125\,\text{mW} \\
\sum P_{\text{dev}} &= 145.75\,\text{mW} \\
\sum P_{\text{dis}} &= (10{,}000)(2.915 \times 10^{-3})^2 + (20.85)^2/15{,}000 + (9000)(1.525 \times 10^{-3})^2 \\
&\quad + (5000)(0.525 \times 10^{-3})^2 + (20{,}000)(0.225 \times 10^{-3})^2 \\
&\quad + (15{,}000)(0.3 \times 10^{-3})^2 + 7.125 \times 10^{-3} \\
&= 145.75\,\text{mW}
\end{aligned}
$$

P 4.61 **[a]** Apply source transformations to both current sources to get

$$
i_o = \frac{(6.75 + 8.25)}{330 + 150 + 270} = 20\,\text{mA}
$$

[b]

The node voltage equations:

$$
\begin{aligned}
0.025 + \frac{v_1}{330} + \frac{v_1 - v_2}{150} &= 0 \\
\frac{v_2}{270} + \frac{v_2 - v_1}{150} - 0.025 &= 0
\end{aligned}
$$

Place these equations in standard form:

$$
\begin{aligned}
v_1\left(\frac{1}{330} + \frac{1}{150}\right) + v_2\left(-\frac{1}{150}\right) &= -0.025 \\
v_1\left(-\frac{1}{150}\right) + v_2\left(\frac{1}{270} + \frac{1}{150}\right) &= 0.025
\end{aligned}
$$

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Solving, $v_1 = -1.65$ V; $\quad v_2 = 1.35\,\text{V}$

$$\therefore \quad i_o = \frac{v_2 - v_1}{150} = 20\,\text{mA}$$

P 4.62 **[a]** Applying a source transformation to each current source yields

Now combine the 12 V and 5 V sources into a single voltage source and the 6 Ω, 6 Ω and 5 Ω resistors into a single resistor to get

Now use a source transformation on each voltage source, thus

which can be reduced to

$$\therefore \quad i_o = -\frac{8.5}{10}(1) = -0.85\,\text{A}$$

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[b]

$34i_\mathrm{a} - 17i_\mathrm{b} = 12 + 5 + 34 = 51$

$-17i_\mathrm{a} + 18.5i_\mathrm{b} = -34$

Solving, $i_\mathrm{b} = -0.85\ \mathrm{A} = i_o$

P 4.63 **[a]** First remove the 16 Ω and 260 Ω resistors:

Next use a source transformation to convert the 1 A current source and 40 Ω resistor:

which simplifies to

$$\therefore \quad v_o = \frac{250}{300}(480) = 400\,\mathrm{V}$$

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[b] Return to the original circuit with $v_o = 400$ V:

$$i_g = \frac{520}{260} + 1.6 = 3.6\,\text{A}$$

$$p_{520\text{V}} = -(520)(3.6) = -1872\,\text{W}$$

Therefore, the 520 V source is developing 1872 W.

[c] $v_1 = -520 + 1.6(4 + 250 + 6) = -104\,\text{V}$

$$v_g = v_1 - 1(16) = -104 - 16 = -120\,\text{V}$$

$$p_{1\text{A}} = (1)(-120) = -120\,\text{W}$$

Therefore the 1 A source is developing 120 W.

[d] $\sum p_{\text{dev}} = 1872 + 120 = 1992\,\text{W}$

$$\sum p_{\text{diss}} = (1)^2(16) + \frac{(104)^2}{40} + \frac{(520)^2}{260} + (1.6)^2(260) = 1992\,\text{W}$$

$$\therefore \quad \sum p_{\text{diss}} = \sum p_{\text{dev}}$$

P 4.64 $v_{\text{Th}} = \dfrac{30}{40}(80) = 60\,\text{V}$

$$R_{\text{Th}} = 2.5 + \frac{(30)(10)}{40} = 10\,\Omega$$

P 4.65 Find the open-circuit voltage:

$$-0.075 + \frac{v_1}{4000} + \frac{v_1}{5000} = 0$$

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$$v_1 = 166.67\,\text{V}; \qquad \text{so} \qquad v_{\text{oc}} = \frac{3000}{5000} v_1 = 100\,\text{V}$$

Find the short-circuit current:

$$i_{\text{sc}} = \frac{4000 \| 2000}{2000}(0.075) = 50\,\text{mA}$$

Thus,

$$I_{\text{N}} = i_{\text{sc}} = 50\,\text{mA}; \qquad R_{\text{N}} = \frac{v_{\text{oc}}}{i_{\text{sc}}} = \frac{100}{0.05} = 2\,\text{k}\Omega$$

P 4.66

$$50i_1 - 40i_{\text{sc}} = 60 + 40$$

$$-40i_1 + 48i_{\text{scs}} = 32$$

Solving, $\quad i_{\text{sc}} = 7\,\text{A}$

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$$R_{\text{Th}} = 8 + \frac{(10)(40)}{50} = 16\,\Omega$$

P 4.67 After making a source transformation the circuit becomes

$$500 = 20i_1 - 8i_2$$

$$300 = -8i_1 + 43.2i_2$$

$$\therefore \quad i_1 = 30\,\text{A and } i_2 = 12.5\,\text{A}$$

$$v_{\text{Th}} = 12i_1 + 5.2i_2 = 425\,\text{V}$$

$$R_{\text{Th}} = (8\|12 + 5.2)\|30 = 7.5\,\Omega$$

P 4.68 First we make the observation that the 10 mA current source and the 10 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to

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or

Therefore the Norton equivalent is

P 4.69

$i_1 = 100/20 = 5\,\text{A}$

$100 = v_{\text{Th}} - 5R_{\text{Th}}, \qquad v_{\text{Th}} = 100 + 5R_{\text{Th}}$

$i_2 = 200/50 = 4\,\text{A}$

$200 = v_{\text{Th}} - 4R_{\text{Th}}, \qquad v_{\text{Th}} = 200 + 4R_{\text{Th}}$

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$\therefore \quad 100 + 5R_{\text{Th}} = 200 + 4R_{\text{Th}} \qquad \text{so} \qquad R_{\text{Th}} = 100\Omega$

$v_{\text{Th}} = 100 + 500 = 600\,\text{V}$

P 4.70

$12.5 = v_{\text{Th}} - 2R_{\text{Th}}$

$11.7 = v_{\text{Th}} - 18R_{\text{Th}}$

Solving the above equations for V_{Th} and R_{Th} yields

$v_{\text{Th}} = 12.6\,\text{V}, \qquad R_{\text{Th}} = 50\,\text{m}\Omega$

$\therefore \quad I_N = 252\,\text{A}, \qquad R_N = 50\,\text{m}\Omega$

P 4.71 First, find the Thévenin equivalent with respect to R_o.

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$R_o(\Omega)$	i_o(mA)	v_o(V)
100	13.64	1.364
120	12.5	1.5
150	11.11	1.667
180	10	1.8

P 4.72 **[a]** First, find the Thévenin equivalent with respect to a,b using a succession of source transformations.

$\therefore \quad v_{\text{Th}} = 54\,\text{V} \qquad R_{\text{Th}} = 4.5\,\text{k}\Omega$

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$$v_{\text{meas}} = \frac{54}{90}(85.5) = 51.3\,\text{V}$$

[b] $$\%\text{error} = \left(\frac{51.3 - 54}{54}\right) \times 100 = -5\%$$

P 4.73

$$v_1 = \frac{200}{700}(18) = 5.143\,\text{V}$$

$$v_2 = \frac{1200}{4203}(18) = 5.139\,\text{V}$$

$$v_{\text{Th}} = v_1 - v_2 = 5.143 - 5.139 = 3.67\,\text{mV}$$

$$R_{\text{Th}} = \frac{(500)(200)}{700} + \frac{(3003)(1200)}{4203} = 1000.24\,\Omega$$

$$i_{\text{gal}} = \frac{3.67 \times 10^{-3}}{1050.24} = 3.5\,\mu\text{A}$$

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P 4.74

OPEN CIRCUIT

$$100 = 2500i_1 + 625(i_1 + 10^{-3}v_2)$$

$$v_2 = \frac{6000}{10{,}000}(5000i_1)$$

Solving,

$$i_1 = 0.02\,\text{A}; \qquad v_2 = v_{\text{oc}} = 60\,\text{V}$$

SHORT CIRCUIT

$$v_2 = 0; \qquad i_{\text{sc}} = \frac{5000}{4000}i_1$$

$$i_1 = \frac{100}{2500 + 625} = 0.032\,\text{A}$$

Thus,

$$i_{\text{sc}} = \frac{5}{4}i_1 = 0.04\,\text{A}$$

$$R_{\text{Th}} = \frac{60}{0.04} = 1.5\,\text{k}\Omega$$

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P 4.75

The node voltage equations and dependant source equation are:

$$\frac{v_1 - 280}{2000} + \frac{v_1}{2000} + \frac{v_1 - v_2}{2000} + 0.2i_\Delta = 0$$

$$\frac{v_2 - v_1}{2000} + \frac{v_2}{5600} - 0.2i_\Delta = 0$$

$$i_\Delta = \frac{280 - v_1}{2000}$$

In standard form:

$$v_1\left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{2000}\right) + v_2\left(-\frac{1}{2000}\right) + i_\Delta(0.2) = \frac{280}{2000}$$

$$v_1\left(-\frac{1}{2000}\right) + v_2\left(\frac{1}{2000} + \frac{1}{5600}\right) + i_\Delta(-0.2) = 0$$

$$v_1\left(\frac{1}{2000}\right) + v_2(0) + i_\Delta(1) = \frac{280}{2000}$$

Solving, $v_1 = 120\,\text{V}$; $v_2 = 112\,\text{V}$; $i_\Delta = 0.08\,\text{A}$

$V_{\text{Th}} = v_2 = 112\,\text{V}$

The mesh current equations are:

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$$-280 + 2000i_\Delta + 2000(i_\Delta - i_{sc}) = 0$$

$$2000(i_{sc} - 0.2i_\Delta) + 2000(i_{sc} - i_\Delta) = 0$$

Put these equations in standard form:

$$i_\Delta(4000) + i_{sc}(-2000) = 280$$

$$i_\Delta(-2400) + i_{sc}(4000) = 0$$

Solving, $i_\Delta = 0.1\,\text{A}$; $\quad i_{sc} = 0.06\,\text{A}$

$$R_{Th} = \frac{112}{0.06} = 1866.67\,\Omega$$

P 4.76 **[a]** Find the Thévenin equivalent with respect to the terminals of the ammeter. This is most easily done by first finding the Thévenin with respect to the terminals of the $4.8\,\Omega$ resistor.

Thévenin voltage: note i_ϕ is zero.

$$\frac{v_{Th}}{100} + \frac{v_{Th}}{25} + \frac{v_{Th}}{20} + \frac{v_{Th} - 16}{2} = 0$$

Solving, $v_{Th} = 20\,\text{V}$.

Short-circuit current:

$$i_{sc} = 12 + 2i_{sc}, \qquad \therefore \quad i_{sc} = -12\,\text{A}$$

$$R_{Th} = \frac{20}{-12} = -(5/3)\,\Omega$$

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$$R_{\text{total}} = \frac{20}{6} = 3.33\,\Omega$$

$$R_{\text{meter}} = 3.33 - 3.13 = 0.2\,\Omega$$

[b] Actual current:

$$i_{\text{actual}} = \frac{20}{3.13} = 6.38\,\text{A}$$

$$\%\ \text{error} = \frac{6 - 6.38}{6.38} \times 100 = -6\%$$

P 4.77 **[a]** Replace the voltage source with a short circuit and find the equivalent resistance from the terminals a,b:

$$R_{\text{Th}} = 10\|30 + 2.5 = 10\,\Omega$$

[b] Replace the current source with an open circuit and the voltage source with a short circuit. Find the equivalent resistance from the terminals a,b:

$$R_{\text{Th}} = 10\|40 + 8 = 16\,\Omega$$

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P 4.78 [**a**] Open circuit:

$$\frac{v_2 - 9}{20} + \frac{v_2}{70} - 1.8 = 0$$

$$v_2 = 35\,\text{V}$$

$$v_{\text{Th}} = \frac{60}{70} v_2 = 30\,\text{V}$$

Short circuit:

$$\frac{v_2 - 9}{20} + \frac{v_2}{10} - 1.8 = 0$$

$$\therefore \quad v_2 = 15\,\text{V}$$

$$i_{\text{a}} = \frac{9 - 15}{20} = -0.3\,\text{A}$$

$$i_{\text{sc}} = 1.8 - 0.3 = 1.5\,\text{A}$$

$$R_{\text{Th}} = \frac{30}{1.5} = 20\,\Omega$$

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[b]

$R_{\text{Th}} = (20 + 10\|60 = 20\,\Omega$ (CHECKS)

P 4.79 $V_{\text{Th}} = 0$, since circuit contains no independent sources.

$$\frac{v_1}{100} + \frac{v_1 - 250i_\Delta}{200} + \frac{v_1 - v_t}{150} = 0$$

$$\frac{v_t - v_1}{150} + \frac{v_t - 250i_\Delta}{50} - 1 = 0$$

$$i_\Delta = \frac{v_t - v_1}{150}$$

In standard form:

$$v_1\left(\frac{1}{100} + \frac{1}{200} + \frac{1}{150}\right) + v_t\left(-\frac{1}{150}\right) + i_\Delta\left(-\frac{250}{200}\right) = 0$$

$$v_1\left(-\frac{1}{150}\right) + v_t\left(\frac{1}{150} + \frac{1}{50}\right) + i_\Delta\left(-\frac{250}{50}\right) = 1$$

$$v_1\left(-\frac{1}{150}\right) + v_t\left(\frac{1}{150}\right) + i_\Delta(-1) = 0$$

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Solving,

$v_1 = 75\,\text{V}; \qquad v_\text{t} = 150\,\text{V}; \qquad i_\Delta = 0.5\,\text{A}$

$\therefore \quad R_\text{Th} = \dfrac{v_\text{t}}{1\,\text{A}} = 150\,\Omega$

P 4.80 Since there is no independent source, $V_\text{Th} = 0$. Now apply a test source at the terminals a,b to find the Thévenin equivalent resistance:

$$\frac{v_1}{100} + \frac{v_1 + 13i_x}{10} + \frac{v_1 - v_2}{20} = 0$$

$$\frac{v_2 - v_1}{20} + \frac{v_2}{50} - 1 = 0$$

$$i_x = \frac{v_2}{50}$$

Solving,

$v_1 = 2.4\,\text{V}; \qquad v_2 = 16\,\text{V}; \qquad i_x = 0.32\,\text{A}$

$v_\text{t} - 24(1) = v_2; \qquad \text{so} \qquad v_\text{t} = 16 + 24 = 40\,\text{V}$

$R_\text{Th} = \dfrac{v_\text{t}}{1\,\text{A}} = 40\,\Omega$

The Thévenin equivalent is simply a $40\,\Omega$ resistor.

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P 4.81 $V_{\text{Th}} = 0$ since there are no independent sources in the circuit. Thus we need only find R_{Th}.

$$\frac{v - 250i_x}{500} - 1.5i_x + \frac{v}{750} - 1 = 0$$

$$i_x = \frac{v}{750}$$

Solving,

$$v = 1500\,\text{V}; \qquad i_x = 2\,\text{A}$$

$$R_{\text{Th}} = \frac{v}{1\,\text{A}} = 1500 = 1.5\,\text{k}\Omega$$

P 4.82 [**a**]

$$R_{\text{Th}} = 5000\|(1600 + 2400\|4800 + 1800) = 2.5\,\text{k}\Omega$$

$$R_o = R_{\text{Th}} = 2.5\,\text{k}\Omega$$

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[b]

$7200i_1 - 4800i_2 = 60$

$-4800i_1 + 4800i_2 + 8400i_3 = 0$

$i_2 - i_3 = 0.015$

Solving,

$i_1 = 19.4\,\text{mA}; \qquad i_2 = 16.6\,\text{mA}; \qquad i_3 = 1.6\,\text{mA}$

$v_{\text{oc}} = 5000i_3 = 8\,\text{V}$

$p_{\max} = (1.6 \times 10^{-3})^2(2500) = 6.4\,\text{mW}$

[c] The resistor closest to 2.5 kΩ from Appendix H has a value of 2.7 kΩ. Use voltage division to find the voltage drop across this load resistor, and use the voltage to find the power delivered to it:

$$v_{2.7\text{k}} = \frac{2700}{2700 + 2500}(8) = 4.154\,\text{V}$$

$$p_{2.7\text{k}} = \frac{(4.154)^2}{2700} = 6.391\,\text{mW}$$

The percent error between the maximum power and the power delivered to the best resistor from Appendix H is

$$\%\ \text{error} = \left(\frac{6.391}{6.4} - 1\right)(100) = -0.1\%$$

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P 4.83 Write KVL equations for the left mesh and the supermesh, place them in standard form, and solve:

At i_1: $\quad -60 + 2400i_1 + 4800(i_1 - i_2) = 0$

Supermesh: $\quad 4800(i_2 - i_1) + 1600i_3 + (5000\|2500)i_3 + 1800i_3 = 0$

Constraint: $\quad i_2 - i_3 = 0.015 = 0$

Standard form:

$i_1(7200) + i_2(-4800) + i_3(0) = 60$

$i_1(-4800) + i_2(4800) + i_3(5066.67) = 0$

$i_1(0) + i_2(1) + i_3(-1) = 0.015$

Calculator solution:

$i_1 = 19.933\,\text{mA}; \qquad i_2 = 17.4\,\text{mA}; \qquad i_3 = 2.4\,\text{mA}$

Calculate voltage across the current source:

$v_{15\text{mA}} = 4800(i_1 - i_2) = 12.16\,\text{V}$

Calculate power delivered by the sources:

$p_{15\text{mA}} = (0.015)(12.16) = 182.4\text{ mW (abs)}$

$p_{60\text{V}} = -60i_1 = -60(0.019933) = -1.196\text{ W (del)}$

$p_{\text{delivered}} = 1.196\,\text{W}$

Calculate power absorbed by the 2.5 kΩ resistor and the percentage power:

$$i_{2.5\text{k}} = \frac{5000\|2500}{2500} i_3 = 1.6\,\text{mA}$$

$p_{2.5\text{k}} = (0.0016)^2(2500) = 6.4\text{ mW}$

% delivered to R_o: $\quad \dfrac{0.0064}{1.196}(100) = 0.535\%$

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P 4.84 **[a]** From the solution to Problem 4.71 we have

$R_o(\Omega)$	p_o(mW)
100	18.595
120	18.75
150	18.52
180	18

The $120\,\Omega$ resistor dissipates the most power, because its value is equal to the Thévenin equivalent resistance of the circuit.

[b]

[c] $R_o = 120\,\Omega, \qquad p_o = 18.75$ mW, which is the maximum power that can be delivered to a load resistor.

P 4.85 **[a]** Since $0 \le R_o \le \infty$ maximum power will be delivered to the 6 Ω resistor when $R_o = 0$.

[b] $P = \dfrac{30^2}{6} = 150\,\text{W}$

P 4.86 **[a]** From the solution of Problem 4.75 we have $R_{\text{Th}} = 1866.67\,\Omega$ and $V_{\text{Th}} = 112$ V. Therefore

$$R_o = R_{\text{Th}} = 1866.67\,\Omega$$

[b] $p = \dfrac{(56)^2}{1866.67} = 1.68\,\text{W}$

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[c]

The node voltage equations are:

$$\frac{v_1 - 280}{2000} + \frac{v_1}{2000} + \frac{v_1 - v_2}{2000} + 0.2i_\Delta = 0$$

$$\frac{v_2 - v_1}{2000} + \frac{v_2}{5600} + \frac{v_2}{1866.67} - 0.2i_\Delta = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{280 - v_1}{2000}$$

Place these equations in standard form:

$$v_1\left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{2000}\right) + v_2\left(-\frac{1}{2000}\right) + i_\Delta(0.2) = \frac{280}{2000}$$

$$v_1\left(-\frac{1}{2000}\right) + v_2\left(\frac{1}{2000} + \frac{1}{5600} + \frac{1}{1866.67}\right) + i_\Delta(-0.2) = 0$$

$$v_1(1) + v_2(0) + i_\Delta(2000) = 280$$

Solving, $v_1 = 100\,\text{V}$; $v_2 = 56\,\text{V}$; $i_\Delta = 90$ mA

Calculate the power:

$p_{280\text{V}} = -(280)(0.09) = -25.2$ W

$p_{\text{dep source}} = (v_1 - v_2)(0.2i_\Delta) = 0.792$ W

$\sum p_{\text{dev}} = 25.2\,\text{W}$

$$\%\text{ delivered } = \frac{1.68}{25.2} \times 100 = 6.67\%$$

[d] The 1.8 kΩ resistor in Appendix H is closest to the Thévenin equivalent resistance.

[e] Substitute the 1.8 kΩ resistor into the original circuit and calculate the power developed by the sources in this circuit:

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The node voltage equations are:

$$\frac{v_1 - 280}{2000} + \frac{v_1}{2000} + \frac{v_1 - v_2}{2000} + 0.2i_\Delta = 0$$

$$\frac{v_2 - v_1}{2000} + \frac{v_2}{5600} + \frac{v_2}{1800} - 0.2i_\Delta = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{280 - v_1}{2000}$$

Place these equations in standard form:

$$v_1\left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{2000}\right) + v_2\left(-\frac{1}{2000}\right) + i_\Delta(0.2) = \frac{280}{2000}$$

$$v_1\left(-\frac{1}{2000}\right) + v_2\left(\frac{1}{2000} + \frac{1}{5600} + \frac{1}{1800}\right) + i_\Delta(-0.2) = 0$$

$$v_1(1) + v_2(0) + i_\Delta(2000) = 280$$

Solving, $v_1 = 99.64\,\text{V}$; $v_2 = 54.98\,\text{V}$; $i_\Delta = 90.18$ mA

Calculate the power:

$p_{280\text{V}} = -(280)(0.09018) = -25.25$ W

$\sum p_{\text{dev}} = 25.25\,\text{mW}$

$$p_L = (54.98)^2/1800 = 1.68\,\text{W} \text{ \% delivered } = \frac{1.68}{25.25} \times 100 = 6.65\%$$

P 4.87 [**a**] Open circuit voltage

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Node voltage equations:

$$\frac{v_1 - 110}{15} - 0.1v_\Delta + \frac{v_1 - v_2}{5} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2 + 3i_\Delta}{8} = 0$$

Constraint equations:

$$i_\Delta = \frac{v_2 - v_1}{5}; \qquad v_\Delta = 110 - v_1$$

Solving, $v_2 = 55$ V $= v_{\text{Th}}$
Thévenin resistance using a test source:

$$\frac{v_1}{15} - 0.1v_\Delta + \frac{v_1 - v_t}{5} = 0$$

$$\frac{v_t - v_1}{5} + \frac{v_t + 3i_\Delta}{8} - 1 = 0$$

$$i_\Delta = \frac{v_t - v_1}{5}; \qquad v_\Delta = -v_1$$

Solving, $v_t = 4$ V.

$$R_{\text{Th}} = \frac{v_t}{1} = 4\,\Omega$$

$$\therefore \quad R_o = R_{\text{Th}} = 4\,\Omega$$

[b]

$$p_{\max} = \frac{(27.5)^2}{4} = 189.0625\,\text{W}$$

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[c]

$$\frac{v_1 - 110}{15} - 0.1v_\Delta + \frac{v_1 - 27.5}{5} = 0$$

$$v_\Delta = 110 - v_1$$

Solving, $v_1 = 65$ V,

$$i_{110\text{V}} = \frac{65 - 110}{15} = -3\,\text{A}$$

$$p_{110\text{V}} = 110(-3) = -330\,\text{W}$$

$$i_\Delta = \frac{27.5 - 65}{5} = -7.5\,\text{A}$$

$$i_{\text{CCVS}} = \frac{27.5 + 3i_\Delta}{8} = 0.625\,\text{A}$$

$$p_{\text{CCVS}} = -[3(-7.5)](0.625) = 14.0625\,\text{W}$$

$$p_{\text{VCCS}} = -[0.1(45)](65) = -292.5\,\text{W}$$

$$\sum p_{\text{dev}} = 330 + 292.5 = 622.5\,\text{W}$$

$$\%\text{ delivered } = \frac{189.0625}{622.5} \times 100 = 30.37\%$$

P 4.88 We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to

$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2\,\text{A}$$

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$V_{\text{Th}} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100\,\text{V}$

Using the test-source method to find the Thévenin resistance gives

$$i_\text{T} = \frac{v_\text{T}}{30} + \frac{v_\text{T} - 30(-v_\text{T}/30)}{20}$$

$$\frac{i_\text{T}}{v_\text{T}} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_\text{Th} = \frac{v_\text{T}}{i_\text{T}} = \frac{15}{2} = 7.5\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.

$$p = \left(\frac{100}{7.5 + R_o}\right)^2 R_o = 250$$

$$\frac{10^4}{R_o^2 + 15R_o + 56.25} R_o = 250$$

$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$

$$40R_o = R_o^2 + 15R_o + 56.25$$

$$R_o^2 - 25R_o + 56.25 = 0$$

$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

$$R_o = 22.5\,\Omega$$

$$R_o = 2.5\,\Omega$$

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P 4.89 [**a**] First find the Thévenin equivalent with respect to R_o.

Open circuit voltage: $i_\phi = 0$; $184\phi = 0$

$$\frac{v_1}{16} + \frac{v_1 - 180}{20} + \frac{v_1 - 180}{10} + \frac{v_1}{10} - 0.1v_\Delta = 0$$

$$v_\Delta = \frac{v_1 - 180}{10}(2) = 0.2v_1 - 36$$

$$v_1 = 80\,\text{V}; \qquad v_\Delta = -20\,\text{V}$$

$$V_{\text{Th}} = 180 + v_\Delta = 180 - 20 = 160\,\text{V}$$

Short circuit current

$$\frac{v_1}{16} + \frac{v_1 - 180}{20} + \frac{v_2}{8} + \frac{v_2}{10} - 0.1(-180) = 0$$

$$v_2 + 184i_\phi = v_1$$

$$i_\phi = \frac{180}{2} + \frac{v_2}{8} = 90 + 0.125v_2$$

$$v_2 = -640\,\text{V}; \qquad v_1 = 1200\,\text{V}$$

$$i_\phi = i_{\text{sc}} = 10\,\text{A}$$

$$R_{\text{Th}} = V_{\text{Th}}/i_{\text{sc}} = 160/10 = 16\,\Omega$$

$$\therefore \quad R_o = 16\,\Omega$$

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[b]

$p_{\max} = (80)^2/16 = 400\,\text{W}$

[c]

$$\frac{v_1}{16} + \frac{v_1 - 180}{20} + \frac{v_2 - 80}{8} + \frac{v_2}{10} - 0.1(80 - 180) = 0$$

$$v_2 + 184 i_\phi = v_1; \qquad i_\phi = 80/16 = 5\,\text{A}$$

Therefore, $v_1 = 640\,\text{V}$ and $v_2 = -280$ V; thus,

$$i_g = \frac{180 - 80}{2} + \frac{180 - 640}{20} = 27\,\text{A}$$

$$p_{180\text{V}}\ (\text{dev}) = (180)(27) = 4860\,\text{W}$$

P 4.90 **[a]** Find the Thévenin equivalent with respect to the terminals of R_L.
Open circuit voltage:

The mesh current equations are:

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$$
\begin{aligned}
-3600 + 45(i_1 - i_2) + 300(i_1 - i_3) + 30i_1 &= 0 \\
30i_2 + 60(i_2 - i_3) + 45(i_2 - i_1) &= 0 \\
-150i_\beta + 15i_3 + 300(i_3 - i_1) + 60(i_3 - i_2) &= 0
\end{aligned}
$$

The dependent source constraint equation is:
$i_\beta = i_1 - i_2$

Place these equations in standard form:

$$
\begin{aligned}
i_1(45 + 300 + 30) + i_2(-45) + i_3(-300) + i_\beta(0) &= 3600 \\
i_1(-45) + i_2(30 + 60 + 45) + i_3(-60) + i_\beta(0) &= 0 \\
i_1(-300) + i_2(-60) + i_3(15 + 300 + 60) + i_\beta(-150) &= 0 \\
i_1(1) + i_2(-1) + i_3(0) + i_\beta(-1) &= 0
\end{aligned}
$$

Solving, $i_1 = 99.6\,\text{A}$; $i_2 = 78\,\text{A}$; $i_3 = 100.8\,\text{A}$; $i_\beta = 21.6\,\text{A}$
$V_{\text{Th}} = 300(i_1 - i_3) = -360\,\text{V}$
Short-circuit current:

The mesh current equations are:

$$
\begin{aligned}
-3600 + 45(i_1 - i_2) + 30i_1 &= 0 \\
30i_2 + 60(i_2 - i_3) + 45(i_2 - i_1) &= 0 \\
-150i_\beta + 15i_3 + 60(i_3 - i_2) &= 0
\end{aligned}
$$

The dependent source constraint equation is:
$i_\beta = i_1 - i_2$

Place these equations in standard form:

$$
\begin{aligned}
i_1(45 + 30) + i_2(-45) + i_3(0) + i_\beta(0) &= 3600 \\
i_1(-45) + i_2(30 + 60 + 45) + i_3(-60) + i_\beta(0) &= 0 \\
i_1(0) + i_2(-60) + i_3(60 + 15) + i_\beta(-150) &= 0 \\
i_1(1) + i_2(-1) + i_3(0) + i_\beta(-1) &= 0
\end{aligned}
$$

Solving, $i_1 = 92\,\text{A}$; $i_2 = 73.33\,\text{A}$; $i_3 = 96\,\text{A}$; $i_\beta = 18.67\,\text{A}$

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$$i_{sc} = i_1 - i_3 = -4\,\text{A}; \qquad R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-360}{-4} = 90\,\Omega$$

$$R_L = R_{Th} = 90\,\Omega$$

[b] $p_{max} = \dfrac{180^2}{90} = 360\,\text{W}$

P 4.91 **[a]** We begin by finding the Thévenin equivalent with respect to the terminals of R_o.

Open circuit voltage

The mesh current equations are:

$$\begin{aligned}
260i_1 - 16i_2 - 180i_3 &= -400 \\
-16i_1 + 48i_2 - 32i_3 + 316(i_1 - i_3) &= 0 \\
-180i_1 - 32i_2 + 260i_3 &= 200
\end{aligned}$$

Solving, $i_1 = 3$ A; $\quad i_2 = 17.5\,\text{A}$; $\quad i_3 = 5\,\text{A}$; $\quad i_\Delta = i_1 - i_3 = -2\,\text{A}$

Also, $\quad V_{Th} = v_{oc} = 180 i_\Delta = -360\,\text{V}$

Now find the short-circuit current.

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Note with the short circuit from a to b that i_Δ is zero, hence $316i_\Delta$ is also zero.

The mesh currents are:

$$\begin{aligned} 80i_1 - 16i_2 + 0i_3 &= -400 \\ -16i_1 + 48i_2 - 32i_3 &= 0 \\ 0i_1 - 32i_2 + 80i_3 &= 200 \end{aligned}$$

Solving, $\quad i_1 = -5\,\text{A}; \quad i_2 = 0\,\text{A}; \quad i_3 = 2.5\,\text{A}$

Then, $\quad i_{\text{sc}} = i_1 - i_3 = -7.5\,\text{A}$

$$R_{\text{Th}} = \frac{-360}{-7.5} = 48\,\Omega$$

For maximum power transfer $\quad R_o = R_{\text{Th}} = 48\,\Omega$

[b] $p_{\max} = \dfrac{180^2}{48} = 675\,\text{W}$

[c] The problem reduces to the analysis of the following circuit. In constructing the circuit we have used the fact that i_Δ is -1 A, and hence $316i_\Delta$ is -316 V.

Using the node voltage method to find v_1 and v_2 yields

$$\frac{v_1 + 400}{64} + \frac{v_1 + 180}{16} + \frac{v_2 + 180}{32} + \frac{v_2 + 200}{48} = 0$$

$$v_2 - v_1 = 316$$

Solving, $v_1 = -336$ V; $\qquad v_2 = -20\,\text{V}$.

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It follows that

$$
\begin{aligned}
i_{g_1} &= \frac{-336+400}{64} = 1\,\text{A} \\
i_{g_2} &= \frac{-20+200}{48} = 3.75\,\text{A} \\
i_2 &= \frac{-20+180}{32} = 5\,\text{A} \\
i_{\text{ds}} &= -5-3.75 = -8.75\,\text{A} \\
p_{400\text{V}} &= -400 i_{g_1} = -400\,\text{W} \\
p_{200\text{V}} &= -200 i_{g_2} = -750\,\text{W} \\
p_{\text{ds}} &= 316 i_{\text{ds}} = -2765\,\text{W}
\end{aligned}
$$

$$\therefore \quad \sum p_{\text{dev}} = 400+750+2765 = 3915\,\text{W}$$

$$\therefore \quad \%\text{ delivered } = \frac{675}{3915}(100) = 17.24\%$$

$\therefore$ 17.24% of developed power is delivered to load

[d] The resistor from Appendix H that is closest to the Thévenin resistance is 47 Ω. To calculate the power delivered to a 47 Ω load resistor, calculate the current using the Thévenin circuit and use it to find the power delivered to the load resistor:

$$i_{47} = \frac{-360}{47+48} = 3.7895\,\text{A}$$

$$p_{47} = 47(3.7895)^2 = 674.9\,\text{W}$$

Thus, using a 47 Ω resistor selected from Appendix H will cause 674.9 W of power to be delivered to the load, compared to the maximum power of 675 W that will be delivered if a 48 Ω resistor is used.

P 4.92 **[a]** By hypothesis $i_o' + i_o'' = 3$ mA.

$$i_o''' = -5\frac{(2)}{(8)} = -1.25\,\text{mA}; \qquad \therefore \quad i_o = 3.5 - 1.25 = 2.25\,\text{mA}$$

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[b] With all three sources in the circuit write a single node voltage equation.

$$\frac{v_b}{6} + \frac{v_b - 8}{2} + 5 - 10 = 0$$

$$\therefore \quad v_b = 13.5\,\text{V}$$

$$i_o = \frac{v_b}{6} = 2.25\,\text{mA}$$

P 4.93 **[a]** 110 V source acting alone:

$$R_e = \frac{10(14)}{24} = \frac{35}{6}\,\Omega$$

$$i' = \frac{110}{5 + 35/6} = \frac{132}{13}\,\text{A}$$

$$v' = \left(\frac{35}{6}\right)\left(\frac{132}{13}\right) = \frac{770}{13}\,\text{V} = 59.231\,\text{V}$$

4 A source acting alone:

$$5\,\Omega \| 10\,\Omega = 50/15 = 10/3\,\Omega$$

$$10/3 + 2 = 16/3\,\Omega$$

$$16/3 \| 12 = 48/13\,\Omega$$

Hence our circuit reduces to:

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It follows that

$$v_a'' = 4(48/13) = (192/13)\,\text{V}$$

and

$$v'' = \frac{-v_a''}{(16/3)}(10/3) = -\frac{5}{8}v_a'' = -(120/13)\,\text{V} = -9.231\,\text{V}$$

$$\therefore \qquad v = v' + v'' = \frac{770}{13} - \frac{120}{13} = 50\,\text{V}$$

[b] $p = \dfrac{v^2}{10} = 250\,\text{W}$

P 4.94 Voltage source acting alone:

$$i_{o1} = \frac{10}{45 + (5+5)\|10} = \frac{10}{45+5} = 0.2\,\text{A}$$

$$v_{o1} = \frac{20}{20+60}(-10) = -2.5\,\text{V}$$

Current source acting alone:

$$\frac{v_2}{5} + 2 + \frac{v_2 - v_3}{5} = 0$$

$$\frac{v_3}{10} + \frac{v_3 - v_2}{5} + \frac{v_3}{45} = 0$$

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Solving, $v_2 = -7.25\text{ V} = v_{o2}$; $\qquad v_3 = -4.5\,\text{V}$

$$i_{o2} = -\frac{v_3}{45} = -0.1\,\text{A}$$

$$i_{20} = \frac{60\|20}{20}(2) = 1.5\,\text{A}$$

$$v_{o2} = -20i_{20} = -20(1.5) = -30\,\text{V}$$

$$\therefore \quad v_o = v_{o1} + v_{o2} = -2.5 - 30 = -32.5\,\text{V}$$

$$i_o = i_{o1} + i_{o2} = 0.2 + 0.1 = 0.3\,\text{A}$$

P 4.95 6 A source:

$$30\,\Omega\|5\,\Omega\|60\,\Omega = 4\,\Omega$$

$$\therefore \quad i_{o1} = \frac{20}{20+5}(6) = 4.8\,\text{A}$$

10 A source:

$$i_{o2} = \frac{4}{25}(10) = 1.6\,\text{A}$$

75 V source:

$$i_{o3} = -\frac{4}{25}(15) = -2.4\,\text{A}$$

$$i_o = i_{o1} + i_{o2} + i_{o3} = 4.8 + 1.6 - 2.4 = 4\,\text{A}$$

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P 4.96 240 V source acting alone:

$$v_{o1} = \frac{20\|5}{5+7+20\|5}(240) = 60\,\text{V}$$

84 V source acting alone:

$$v_{o2} = \frac{20\|12}{1+4+20\|12}(-84) = -50.4\,\text{V}$$

16 A current source acting alone:

$$\frac{v_1 - v_2}{5} + \frac{v_1}{7} - 16 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{20} + \frac{v_2 - v_3}{4} = 0$$

$$\frac{v_3 - v_2}{4} + \frac{v_3}{1} + 16 = 0$$

Solving, $v_2 = 18.4\,\text{V} = v_{o3}$. Therefore,

$$v_o = v_{o1} + v_{o2} + v_{o3} = 60 - 50.4 + 18.4 = 28\,\text{V}$$

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P 4.97 Voltage source acting alone:

$$\frac{v_{o1} - 25}{4000} + \frac{v_{o1}}{20{,}000} - 2.2\left(\frac{v_{o1} - 25}{4000}\right) = 0$$

Simplifying $\quad 5v_{o1} - 125 + v_{o1} - 11v_{o1} + 275 = 0$

$\therefore \quad v_{o1} = 30\,\text{V}$

Current source acting alone:

$$\frac{v_{o2}}{4000} + \frac{v_{o2}}{20{,}000} + 0.005 - 2.2\left(\frac{v_{o2}}{4000}\right) = 0$$

Simplifying $\quad 5v_{o2} + v_{o2} + 100 - 11v_{o2} = 0$

$\therefore \quad v_{o2} = 20\,\text{V}$

$v_o = v_{o1} + v_{o2} = 30 + 20 = 50\,\text{V}$

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P 4.98 90-V source acting alone:

$$2000(i_1 - i_2) + 2.5v_b = 90$$

$$-2000i_1 + 7000i_2 - 4000i_3 = 0$$

$$-4000i_2 + 6000i_3 - 2.5v_b = 0$$

$$v_b = 1000i_2$$

Solving,

$$i_1 = 37.895\,\text{mA}; \qquad i_3 = 30.789\,\text{mA}; \qquad i' = i_1 - i_3 = 7.105\,\text{mA}$$

40-V source acting alone:

$$2000(i_1 - i_2) + 2.5v_b = 0$$

$$-2000i_1 + 7000i_2 - 4000i_3 = 0$$

$$-4000i_2 + 6000i_3 - 2.5v_b = -40$$

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$v_b = 1000 i_2$

Solving,

$i_1 = 2.105\,\text{mA}; \qquad i_3 = -15.789\,\text{mA}; \qquad i'' = i_1 - i_3 = 17.895\,\text{mA}$

Hence, $\quad i = i' + i'' = 7.105 + 17.895 = 25\,\text{mA}$

P 4.99 **[a]** In studying the circuit in Fig. P4.99 we note it contains six meshes and six essential nodes. Further study shows that by replacing the parallel resistors with their equivalent values the circuit reduces to four meshes and four essential nodes as shown in the following diagram.

The node-voltage approach will require solving three node voltage equations along with equations involving v_x, v_y, and i_x.

The mesh-current approach will require writing one mesh equation and one supermesh equation plus five constraint equations involving the five sources. Thus at the outset we know the supermesh equation can be reduced to a single unknown current. Since we are interested in the power developed by the 50 V source, we will retain the mesh current i_b and eliminate the mesh currents i_a, i_c and i_d.

The supermesh is denoted by the dashed line in the following figure.

[b] Summing the voltages around the supermesh yields

$$-1.25v_x + (100/3)i_a + (200/3)i_b + 50 + 50i_b + 250(i_c - i_d) + 50i_c = 0$$

The remaining mesh equation is

$$50i_x + 350i_d - 250i_c = 0$$

The constraint equations are

$$\frac{v_y}{25} = i_b - i_c; \qquad 0.9 = i_c - i_a; \qquad v_x = -(200/3)i_b$$

$$v_y = 50i_b; \qquad i_x = i_c - i_d$$

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Solving,

$i_a = -0.3\,\text{A}; \qquad i_b = -0.6\,\text{A}; \qquad i_c = 0.6\,\text{A}; \qquad i_d = 0.4\,\text{A}$

Finally,

$p_{50\text{V}} = 50 i_b = -30\,\text{W}$

The 50 V source delivers 30 W of power.

P 4.100

At v_1: $\dfrac{v_1 - 120}{10} + \dfrac{v_1 - v_2}{20} + \dfrac{v_1 - v_3}{10} = 0$

At v_2: $\dfrac{v_2 - 120}{20} + \dfrac{v_2 - 120}{20} + \dfrac{v_2 - v_1}{20} + \dfrac{v_1 - v_3}{20} + \dfrac{v_2 - v_4}{20} = 0$

At v_3: $\dfrac{v_3 - v_1}{10} + \dfrac{v_3 - v_2}{20} + \dfrac{v_3 - v_4}{20} + \dfrac{v_3}{10} + \dfrac{v_3 - v_5}{10} = 0$

At v_4: $\dfrac{v_4 - v_2}{20} + \dfrac{v_4 - v_3}{20} + \dfrac{v_4 - v_5}{10} = 0$

At v_5: $\dfrac{v_5 - v_4}{10} + \dfrac{v_5 - v_3}{10} + \dfrac{v_5}{5} = 0$

A calculator solution yields

$v_1 = 80$ A; $\qquad v_2 = 80$ A;

$v_3 = 40$ A; $\qquad v_4 = 40$ A;

$v_5 = 20$ A.

$\therefore \quad i_1 = \dfrac{v_2 - v_1}{20} = 0\,\text{A}; \qquad i_1 = \dfrac{v_3 - v_4}{20} = 0\,\text{A}$

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P 4.101

The mesh equations are:

$$
\begin{aligned}
i_1(36.3) + i_2(-0.2) + i_3(-36) + i_4(0) + i_5(0) &= 120 \\
i_1(-0.2) + i_2(45.3) + i_3(0) + i_4(-45) + i_5(0) &= 120 \\
i_1(-36) + i_2(0) + i_3(72.3) + i_4(-0.2) + i_5(-36) &= 0 \\
i_1(0) + i_2(-45) + i_3(-0.2) + i_4(90.3) + i_5(-45) &= 0 \\
i_1(0) + i_2(0) + i_3(-36) + i_4(-45) + i_5(108) &= 0
\end{aligned}
$$

Solving,
$i_1 = 15.226\,\text{A}$; $i_2 = 13.953\,\text{A}$; $i_3 = 11.942\,\text{A}$; $i_4 = 11.314\,\text{A}$; $i_5 = 8.695\,\text{A}$
Find the requested voltages:
$v_1 = 36(i_3 - i_5) = 118.6\,\text{V}$
$v_2 = 45(i_4 - i_5) = 117.8\,\text{V}$
$v_3 = 27 i_5 = 234.8\,\text{V}$

P 4.102 **[a]**

$$\frac{v - v_1}{2xr} + \frac{v}{R} + \frac{v - v_2}{2r(L-x)} = 0$$

$$v\left[\frac{1}{2xr} + \frac{1}{R} + \frac{1}{2r(L-x)}\right] = \frac{v_1}{2xr} + \frac{v_2}{2r(L-x)}$$

$$v = \frac{v_1 RL + xR(v_2 - v_1)}{RL + 2rLx - 2rx^2}$$

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[b] Let $D = RL + 2rLx - 2rx^2$

$$\frac{dv}{dx} = \frac{(RL + 2rLx - 2rx^2)R(v_2 - v_1) - [v_1RL + xR(v_2 - v_1)]2r(L - 2x)}{D^2}$$

$$\frac{dv}{dx} = 0 \quad \text{when numerator is zero.}$$

The numerator simplifies to

$$x^2 + \frac{2Lv_1}{(v_2 - v_1)}x + \frac{RL(v_2 - v_1) - 2rv_1L^2}{2r(v_2 - v_1)} = 0$$

Solving for the roots of the quadratic yields

$$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1v_2 - \frac{R}{2rL}(v_2 - v_1)^2}\right\}$$

[c] $$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1v_2 - \frac{R}{2rL}(v_1 - v_2)^2}\right\}$$

$$v_2 = 1200\,\text{V}, \qquad v_1 = 1000\,\text{V}, \qquad L = 16\ \text{km}$$

$$r = 5 \times 10^{-5}\,\Omega/m; \qquad R = 3.9\,\Omega$$

$$\frac{L}{v_2 - v_1} = \frac{16{,}000}{1200 - 1000} = 80; \qquad v_1v_2 = 1.2 \times 10^6$$

$$\frac{R}{2rL}(v_1 - v_2)^2 = \frac{3.9(-200)^2}{(10 \times 10^{-5})(16 \times 10^3)} = 0.975 \times 10^5$$

$$x = 80\{-1000 \pm \sqrt{1.2 \times 10^6 - 0.0975 \times 10^6}\}$$

$$= 80\{-1000 \pm 1050\} = 80(50) = 4000\ \text{m}$$

[d]

$$\begin{aligned} v_{\min} &= \frac{v_1RL + R(v_2 - v_1)x}{RL + 2rLx - 2rx^2} \\ &= \frac{(1000)(3.9)(16 \times 10^3) + 3.9(200)(4000)}{(3.9)(16{,}000) + 10 \times 10^{-5}(16{,}000)(4000) - 10 \times 10^{-5}(16 \times 10^6)} \\ &= 975\,\text{V} \end{aligned}$$

P 4.103 **[a]**

$$v_{\text{oc}} = V_{\text{Th}} = 75\,\text{V}; \qquad i_L = \frac{60}{20} = 3\,\text{A}; \qquad i_L = \frac{75 - 60}{R_{\text{Th}}} = \frac{15}{R_{\text{Th}}}$$

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Therefore $\quad R_{\text{Th}} = \dfrac{15}{3} = 5\,\Omega$

[b] $i_L = \dfrac{v_o}{R_L} = \dfrac{V_{\text{Th}} - v_o}{R_{\text{Th}}}$

Therefore $\quad R_{\text{Th}} = \dfrac{V_{\text{Th}} - v_o}{v_o/R_L} = \left(\dfrac{V_{\text{Th}}}{v_o} - 1\right) R_L$

P 4.104 $$\frac{dv_1}{dI_{g1}} = \frac{-R_1[R_2(R_3+R_4)+R_3R_4]}{(R_1+R_2)(R_3+R_4)+R_3R_4}$$

$$\frac{dv_1}{dI_{g2}} = \frac{R_1R_3R_4}{(R_1+R_2)(R_3+R_4)+R_3R_4}$$

$$\frac{dv_2}{dI_{g1}} + \frac{-R_1R_3R_4}{(R_1+R_2)(R_3+R_4)+R_3R_4}$$

$$\frac{dv_2}{dI_{g2}} = \frac{R_3R_4(R_1+R_2)}{(R_1+R_2)(R_3+R_4)+R_3R_4}$$

P 4.105 From the solution to Problem 4.104 we have

$$\frac{dv_1}{dI_{g1}} = \frac{-25[5(125)+3750]}{30(125)+3750} = -\frac{175}{12}\ \text{V/A} = -14.5833\ \text{V/A}$$

and

$$\frac{dv_2}{dI_{g1}} = \frac{-(25)(50)(75)}{30(125)+3750} = -12.5\ \text{V/A}$$

By hypothesis, $\Delta I_{g1} = 11 - 12 = -1\,\text{A}$

$$\therefore\quad \Delta v_1 = (-\frac{175}{12})(-1) = \frac{175}{12} = 14.583\,\text{V}$$

Thus, $v_1 = 25 + 14.583 = 39.583\,\text{V}$
Also,

$$\Delta v_2 = (-12.5)(-1) = 12.5\,\text{V}$$

Thus, $v_2 = 90 + 12.5 = 102.5\,\text{V}$
The PSpice solution is

$$v_1 = 39.583\,\text{V}$$

and

$$v_2 = 102.5\,\text{V}$$

These values are in agreement with our predicted values.

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P 4.106 From the solution to Problem 4.104 we have

$$\frac{dv_1}{dI_{g2}} = \frac{(25)(50)(75)}{30(125) + 3750} = 12.5 \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g2}} = \frac{(50)(75)(30)}{30(125) + 3750} = 15 \text{ V/A}$$

By hypothesis, $\Delta I_{g2} = 17 - 16 = 1\,\text{A}$

$$\therefore \quad \Delta v_1 = (12.5)(1) = 12.5\,\text{V}$$

Thus, $v_1 = 25 + 12.5 = 37.5\,\text{V}$
Also,

$$\Delta v_2 = (15)(1) = 15\,\text{V}$$

Thus, $v_2 = 90 + 15 = 105\,\text{V}$
The PSpice solution is

$$v_1 = 37.5\,\text{V}$$

and

$$v_2 = 105\,\text{V}$$

These values are in agreement with our predicted values.

P 4.107 From the solutions to Problems 4.104 — 4.106 we have

$$\frac{dv_1}{dI_{g1}} = -\frac{175}{12} \text{ V/A}; \qquad \frac{dv_1}{dI_{g2}} = 12.5 \text{ V/A}$$

$$\frac{dv_2}{dI_{g1}} = -12.5 \text{ V/A}; \qquad \frac{dv_2}{dI_{g2}} = 15 \text{ V/A}$$

By hypothesis,

$$\Delta I_{g1} = 11 - 12 = -1\,\text{A}$$

$$\Delta I_{g2} = 17 - 16 = 1\,\text{A}$$

Therefore,

$$\Delta v_1 = \frac{175}{12} + 12.5 = 27.0833\,\text{V}$$

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$\Delta v_2 = 12.5 + 15 = 27.5\,\text{V}$

Hence

$v_1 = 25 + 27.0833 = 52.0833\,\text{V}$

$v_2 = 90 + 27.5 = 117.5\,\text{V}$

The PSpice solution is

$v_1 = 52.0830\,\text{V}$

and

$v_2 = 117.5\,\text{V}$

These values are in agreement with our predicted values.

P 4.108 By hypothesis,

$\Delta R_1 = 27.5 - 25 = 2.5\,\Omega$

$\Delta R_2 = 4.5 - 5 = -0.5\,\Omega$

$\Delta R_3 = 55 - 50 = 5\,\Omega$

$\Delta R_4 = 67.5 - 75 = -7.5\,\Omega$

So

$\Delta v_1 = 0.5833(2.5) - 5.417(-0.5) + 0.45(5) + 0.2(-7.5) = 4.9168\,\text{V}$

$\therefore \quad v_1 = 25 + 4.9168 = 29.9168\,\text{V}$

$\Delta v_2 = 0.5(2.5) + 6.5(-0.5) + 0.54(5) + 0.24(-7.5) = -1.1\,\text{V}$

$\therefore \quad v_2 = 90 - 1.1 = 88.9\,\text{V}$

The PSpice solution is

$v_1 = 29.6710\,\text{V}$

and

$v_2 = 88.5260\,\text{V}$

Note our predicted values are within a fraction of a volt of the actual values.

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5

The Operational Amplifier

Assessment Problems

AP 5.1 **[a]** This is an inverting amplifier, so

$$v_o = (-R_f/R_i)v_s = (-80/16)v_s, \qquad \text{so} \qquad v_o = -5v_s$$

v_s(V)	0.4	2.0	3.5	−0.6	−1.6	−2.4
v_o(V)	−2.0	−10.0	−15.0	3.0	8.0	10.0

Two of the values, 3.5 V and −2.4 V, cause the op amp to saturate.

[b] Use the negative power supply value to determine the largest input voltage:

$$-15 = -5v_s, \quad v_s = 3 \text{ V}$$

Use the positive power supply value to determine the smallest input voltage:

$$10 = -5v_s, \qquad v_s = -2 \text{ V}$$

Therefore $\quad -2 \le v_s \le 3$ V

AP 5.2 From Assessment Problem 5.1

$$v_o = (-R_f/R_i)v_s = (-R_x/16{,}000)v_s = (-R_x/16{,}000)(-0.640)$$

$$= 0.64R_x/16{,}000 = 4 \times 10^{-5}R_x$$

Use the negative power supply value to determine one limit on the value of R_x:

$$4 \times 10^{-5}R_x = -15 \qquad \text{so} \qquad R_x = -15/4 \times 10^{-5} = -375\,\text{k}\Omega$$

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Since we cannot have negative resistor values, the lower limit for R_x is 0. Now use the positive power supply value to determine the upper limit on the value of R_x:

$$4 \times 10^{-5} R_x = 10 \qquad \text{so} \qquad R_x = 10/4 \times 10^{-5} = 250\,\text{k}\Omega$$

Therefore,

$$0 \le R_x \le 250\,\text{k}\Omega$$

AP 5.3 **[a]** This is an inverting summing amplifier so

$$v_o = (-R_f/R_\text{a})v_\text{a} + (-R_f/R_\text{b})v_\text{b} = -(250/5)v_\text{a} - (250/25)v_\text{b} = -50v_\text{a} - 10v_\text{b}$$

Substituting the values for v_a and v_b:

$$v_o = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$$

[b] Substitute the value for v_b into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50v_\text{a} - 10(0.25) = -50v_\text{a} - 2.5 = -10 \text{ V}$$

$$\text{Therefore} \quad 50v_\text{a} = 7.5, \quad \text{so} \quad v_\text{a} = 0.15 \text{ V}$$

[c] Substitute the value for v_a into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50(0.10) - 10v_\text{b} = -5 - 10v_\text{b} = -10 \text{ V};$$

$$\text{Therefore} \quad 10v_\text{b} = 5, \quad \text{so} \quad v_\text{b} = 0.5 \text{ V}$$

[d] The effect of reversing polarity is to change the sign on the v_b term in each equation from negative to positive.
Repeat part (a):

$$v_o = -50v_\text{a} + 10v_\text{b} = -5 + 2.5 = -2.5 \text{ V}$$

Repeat part (b):

$$v_o = -50v_\text{a} + 2.5 = -10 \text{ V}; \qquad 50v_\text{a} = 12.5, \quad v_\text{a} = 0.25 \text{ V}$$

Repeat part (c), using the value of the positive power supply:

$$v_o = -5 + 10v_\text{b} = 15 \text{ V}; \qquad 10v_\text{b} = 20; \qquad v_\text{b} = 2.0 \text{ V}$$

AP 5.4 **[a]** Write a node voltage equation at v_n; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v_n}{4500} + \frac{v_n - v_o}{63{,}000} = 0$$

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Solve for v_o in terms of v_n by multiplying both sides by 63,000 and collecting terms:

$$14v_n + v_n - v_o = 0 \qquad \text{so} \qquad v_o = 15v_n$$

Now use voltage division to calculate v_p. We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 kΩ resistor and the R_x resistor:

$$v_p = \frac{R_x}{15{,}000 + R_x}(0.400)$$

Now substitute the value $R_x = 60$ kΩ:

$$v_p = \frac{60{,}000}{15{,}000 + 60{,}000}(0.400) = 0.32 \text{ V}$$

Finally, remember that for an ideal op amp, $v_n = v_p$, so substitute the value of v_p into the equation for v_0

$$v_o = 15v_n = 15v_p = 15(0.32) = 4.8 \text{ V}$$

[b] Substitute the expression for v_p into the equation for v_o and set the resulting equation equal to the positive power supply value:

$$v_o = 15\left(\frac{0.4R_x}{15{,}000 + R_x}\right) = 5$$

$$15(0.4R_x) = 5(15{,}000 + R_x) \quad \text{so} \quad R_x = 75\,\text{k}\Omega$$

AP 5.5 **[a]** Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. 5.22:

$$v_o = \frac{20(60)}{10(24)}v_\text{b} - \frac{50}{10}v_\text{a}$$

Simplify this expression and subsitute in the value for v_b:

$$v_o = 5(v_\text{b} - v_\text{a}) = 20 - 5v_\text{a}$$

Set this expression for v_o to the positive power supply value:

$$20 - 5v_\text{a} = 10 \text{ V} \quad \text{so} \quad v_\text{a} = 2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$20 - 5v_\text{a} = -10 \text{ V} \quad \text{so} \quad v_\text{a} = 6 \text{ V}$$

Therefore $\quad 2 \le v_\text{a} \le 6 \text{ V}$

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[b] Begin as before by substituting the appropriate values into Eq. 5.22:

$$v_o = \frac{8(60)}{10(12)}v_\mathrm{b} - 5v_\mathrm{a} = 4v_\mathrm{b} - 5v_\mathrm{a}$$

Now substitute the value for v_b:

$$v_o = 4(4) - 5v_\mathrm{a} = 16 - 5v_\mathrm{a}$$

Set this expression for v_o to the positive power supply value:

$$16 - 5v_\mathrm{a} = 10 \text{ V} \quad \text{so} \quad v_\mathrm{a} = 1.2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$16 - 5v_\mathrm{a} = -10 \text{ V} \quad \text{so} \quad v_\mathrm{a} = 5.2 \text{ V}$$

Therefore $\quad 1.2 \le v_\mathrm{a} \le 5.2 \text{ V}$

AP 5.6 **[a]** Replace the op amp with the more realistic model of the op amp from Fig. 5.15:

Write the node voltage equation at the left hand node:

$$\frac{v_n}{500{,}000} + \frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} = 0$$

Multiply both sides by 500,000 and simplify:

$$v_n + 100v_n - 100v_g + 5v_n - 5v_0 = 0 \quad \text{so} \quad 21.2v_n - v_o = 20v_g$$

Write the node voltage equation at the right hand node:

$$\frac{v_o - 300{,}000(-v_n)}{5000} + \frac{v_o - v_n}{100{,}000} = 0$$

Multiply through by 100,000 and simplify:

$$20v_o + 6 \times 10^6 v_n + v_o - v_n = 0 \quad \text{so} \quad 6 \times 10^6 v_n + 21v_o = 0$$

Use Cramer's method to solve for v_o:

$$\Delta = \begin{vmatrix} 21.2 & -1 \\ 6 \times 10^6 & 21 \end{vmatrix} = 6{,}000{,}445.2$$

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$$N_o = \begin{vmatrix} 21.2 & 20v_g \\ 6 \times 10^6 & 0 \end{vmatrix} = -120 \times 10^6 v_g$$

$$v_o = \frac{N_o}{\Delta} = -19.9985 v_g; \qquad \text{so } \frac{v_o}{v_g} = -19.9985$$

[b] Use Cramer's method again to solve for v_n:

$$N_1 = \begin{vmatrix} 20v_g & -1 \\ 0 & 21 \end{vmatrix} = 420 v_g$$

$$v_n = \frac{N_1}{\Delta} = 6.9995 \times 10^{-5} v_g$$

$$v_g = 1 \text{ V}, \qquad v_n = 69.995\,\mu \text{ V}$$

[c] The resistance seen at the input to the op amp is the ratio of the input voltage to the input current, so calculate the input current as a function of the input voltage:

$$i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 6.9995 \times 10^{-5} v_g}{5000}$$

Solve for the ratio of v_g to i_g to get the input resistance:

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 6.9995 \times 10^{-5}} = 5000.35\,\Omega$$

[d] This is a simple inverting amplifier configuration, so the voltage gain is the ratio of the feedback resistance to the input resistance:

$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

Since this is now an ideal op amp, the voltage difference between the two input terminals is zero; since $v_p = 0$, $v_n = 0$
Since there is no current into the inputs of an ideal op amp, the resistance seen by the input voltage source is the input resistance:

$$R_g = 5000\,\Omega$$

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Problems

P 5.1 **[a]** The five terminals of the op amp are identified as follows:

[b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i_n = 0$ A.

[c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v_p - v_n) = 0$.

[d] Write a node voltage equation at v_n:

$$\frac{v_n - 2}{4000} + \frac{v_n - v_o}{12{,}000} = 0$$

But $v_p = 0$ and $v_n = v_p = 0$. Thus,

$$\frac{-2}{4000} - \frac{v_o}{12{,}000} = 0 \quad \text{so} \quad v_o = -6 \text{ V}$$

P 5.2 **[a]** Let the value of the voltage source be v_s:

$$\frac{v_n - v_s}{4000} + \frac{v_n - v_o}{12{,}000} = 0$$

But $v_n = v_p = 0$. Therefore,

$$v_o = -\frac{12{,}000}{4000} v_s = -3v_s$$

When $v_s = -6$ V, $v_o = -3(-6) = 18$ V; saturates at $v_o = 15$ V.
When $v_s = -3.5$ V, $v_o = -3(-3.5) = 10.5$ V.
When $v_s = -1.25$ V, $v_o = -3(-1.25) = 3.75$ V.
When $v_s = 2.4$ V, $v_o = -3(2.4) = -7.2$ V.
When $v_s = 4.5$ V, $v_o = -3(4.5) = -13.5$ V.
When $v_s = 5.4$ V, $v_o = -3(5.4) = -16.2$ V; saturates at $v_o = -15$ V.

[b] $-3v_s = 15 \qquad \text{so} \qquad v_s = \dfrac{15}{-3} = -5 \text{ V}$

$$-3v_s = -15 \qquad \text{so} \qquad v_s = \frac{-15}{-3} = 5 \text{ V}$$

The range of source voltages that avoids saturation is $-5 \text{ V} \le v_s \le 5$ V.

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P 5.3 $v_p = \dfrac{5000}{5000 + 10{,}000}(6) = 2 \text{ V} = v_n$

$$\frac{v_n + 5}{3000} + \frac{v_n - v_o}{6000} = 0$$

$$2(2 + 5) + (2 - v_o) = 0$$

$$v_o = 16 \text{ V}$$

$$i_{\text{L}} = \frac{v_o}{8000} = \frac{16}{8000} = 2000 \times 10^{-6}$$

$$i_{\text{L}} = 2 \text{ mA}$$

P 5.4 $\dfrac{v_{\text{b}} - v_{\text{a}}}{5000} + \dfrac{v_{\text{b}} - v_o}{40{,}000} = 0,$ therefore $v_o = 9v_{\text{b}} - 8v_{\text{a}}$

[a] $v_{\text{a}} = 1.5 \text{ V}, \quad v_{\text{b}} = 0 \text{ V}, \quad v_o = -12 \text{ V}$

[b] $v_{\text{a}} = -0.5 \text{ V}, \quad v_{\text{b}} = 0 \text{ V}, \quad v_o = 4 \text{ V}$

[c] $v_{\text{a}} = 1 \text{ V}, \quad v_{\text{b}} = 2.5 \text{ V}, \quad v_o = 14.5 \text{ V}$

[d] $v_{\text{a}} = 2.5 \text{ V}, \quad v_{\text{b}} = 1 \text{ V}, \quad v_o = -11 \text{ V}$

[e] $v_{\text{a}} = 2.5 \text{ V}, \quad v_{\text{b}} = 0 \text{ V}, \quad v_o = -20 \text{ V}$ (saturates at -16 V)

[f] If $v_{\text{b}} = 2 \text{ V}, \quad v_o = 18 - 8v_{\text{a}} = \pm 16$

$$\therefore \quad 0.25 \text{ V} \le v_{\text{a}} \le 4.25 \text{ V}$$

P 5.5 $v_o = -(0.5 \times 10^{-3})(10{,}000) = -5 \text{ V}$

$$\therefore \quad i_o = \frac{v_o}{5000} = \frac{-5}{5000} = -1 \,\text{mA}$$

P 5.6 **[a]** $i_{\text{a}} = \dfrac{240 \times 10^{-3}}{8000} = 30\,\mu\text{A}$

$$v_1 = -40 \times 10^3 i_2 = -3 \,\text{V}$$

[b] $\dfrac{0 - v_{\text{a}}}{60{,}000} = i_{\text{a}}$ so $v_{\text{a}} = -60{,}000 i_{\text{a}} = -1.8 \text{ V}$

[c] $\dfrac{v_{\text{a}}}{60{,}000} + \dfrac{v_{\text{a}}}{40{,}000} + \dfrac{v_{\text{a}} - v_{\text{o}}}{30{,}000} = 0$

$$\therefore \quad v_{\text{o}} = 2.25 v_{\text{a}} = -4.05 \text{ V}$$

[d] $i_{\text{o}} = \dfrac{-v_{\text{o}}}{20{,}000} + \dfrac{v_{\text{a}} - v_{\text{o}}}{30{,}000} = 277.5\,\mu\text{A}$

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P 5.7 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the 2.2 MΩ resistor is $(2.2 \times 10^6)(3.5 \times 10^{-6})$ or 7.7 V. Therefore the voltmeter reads 7.7 V.

P 5.8 **[a]** $\dfrac{30{,}000}{R_{\text{in}}} = 4$ so $R_{\text{in}} = \dfrac{30{,}000}{4} = 7500 = 7.5\,\text{k}\Omega$

[b] $-4v_{\text{in}} = 12$ so $v_{\text{in}} = \dfrac{12}{-4} = -3$ V

$-4v_{\text{in}} = -12$ so $v_{\text{in}} = \dfrac{12}{4} = 3$ V

$\therefore \quad -3 \text{ V} \le v_{\text{in}} \le 3 \text{ V}$

[c] $-\dfrac{R_{\text{f}}}{7500}(2) = -12$ so $R_{\text{f}} = 45\,\text{k}\Omega$

$$\left|\frac{v_{\text{o}}}{v_{\text{in}}}\right| = \frac{R_{\text{f}}}{R_{\text{in}}} = \frac{45{,}000}{7500} = 6$$

The amplifier has a gain of 6.

P 5.9 **[a]** The gain of an inverting amplifier is the negative of the ratio of the feedback resistor to the input resistor. If the gain of the inverting amplifier is to be 2.5, the feedback resistor must be 2.5 times as large as the input resistor. There are many possible designs that use a resistor value chosen from Appendix H. We present one here that uses 3.3 kΩ resistors. Use a single 3.3 kΩ resistor as the input resistor. Then construct a network of 3.3 kΩ resistors whose equivalent resistance is 2.5(3.3) = 8.25 kΩ by connecting two resistors in parallel and connecting the parallel resistors in series with two other resistors. The resulting circuit is shown here:

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[**b**] To amplify signals in the range −2 V to 3 V without saturating the op amp, the power supply voltages must be greater than or equal to the product of the input voltage and the amplifier gain.

$$-2.5(-2) = 5 \text{ V} \qquad \text{and} \qquad -2.5(3) = -7.5 \text{ V}$$

Thus, the power supplies should have values of −7.5 V and 5 V.

P 5.10 [**a**] Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{2000} + \frac{0 - v_\Delta}{50{,}000} = 0$$

$$-25v_g - v_\Delta = 0, \qquad \therefore \quad v_\Delta = -25(40 \times 10^{-3}) = -1 \text{ V}$$

$$\frac{v_\Delta}{\alpha R_\Delta} + \frac{v_\Delta - 0}{50{,}000} + \frac{v_\Delta - v_o}{(1-\alpha)R_\Delta} = 0$$

$$\frac{v_\Delta}{\alpha} + 2v_\Delta + \frac{v_\Delta - v_o}{1-\alpha} = 0$$

$$v_\Delta\left(\frac{1}{\alpha} + 2 + \frac{1}{1-\alpha}\right) = \frac{v_o}{1-\alpha}$$

$$\therefore \quad v_o = -1\left[1 + 2(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right]$$

When $\alpha = 0.2, \qquad v_o = -1(1 + 1.6 + 4) = -6.6 \text{ V}$

When $\alpha = 1, \qquad v_o = -1(1 + 0 + 0) = -1 \text{ V}$

$$\therefore \quad -6.6 \text{ V} \le v_o \le -1 \text{ V}$$

[**b**] $$-1\left[1 + 2(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right] = -7$$

$$\alpha + 2\alpha(1-\alpha) + (1-\alpha) = 7\alpha$$

$$\alpha + 2\alpha - 2\alpha^2 + 1 - \alpha = 7\alpha$$

$$\therefore \quad 2\alpha^2 + 5\alpha - 1 = 0 \quad \text{so} \quad \alpha \cong 0.186$$

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P 5.11 **[a]** Replace the combination of v_g, 1.6 kΩ, and the 6.4 kΩ resistors with its Thévenin equivalent.

Then $\quad v_o = \dfrac{-[12+\sigma 50]}{1.28}(0.20)$

At saturation $\quad v_o = -5$ V; $\qquad$ therefore

$$-\left(\frac{12+\sigma 50}{1.28}\right)(0.2) = -5, \quad \text{or} \quad \sigma = 0.4$$

Thus for $0 \le \sigma \le 0.40$ the operational amplifier will not saturate.

[b] When $\quad \sigma = 0.272, \qquad v_o = \dfrac{-(12+13.6)}{1.28}(0.20) = -4$ V

Also $\quad \dfrac{v_o}{10} + \dfrac{v_o}{25.6} + i_o = 0$

$$\therefore \quad i_o = -\frac{v_o}{10} - \frac{v_o}{25.6} = \frac{4}{10} + \frac{4}{25.6}\,\text{mA} = 556.25\,\mu\text{A}$$

P 5.12 **[a]** This circuit is an example of an inverting summing amplifier.

[b] $v_o = -\dfrac{220}{44}v_\text{a} - \dfrac{220}{27.5}v_\text{b} - \dfrac{220}{80}v_\text{c} = -5 - 12 + 11 = -6$ V

[c] $v_o = -6 - 8v_\text{b} = \pm 10$

$\therefore \quad v_\text{b} = -0.5$ V $\quad$ when $\quad v_o = 10$ V;

$v_\text{b} = 2$ V $\quad$ when $\quad v_o = -10$ V

$\therefore \quad -0.5 \text{ V} \le v_\text{b} \le 2$ V

P 5.13 $\quad v_o = -\left[\dfrac{R_\text{f}}{4000}(0.2) + \dfrac{R_\text{f}}{5000}(0.15) + \dfrac{R_\text{f}}{20{,}000}(0.4)\right]$

$$-6 = -0.1 \times 10^{-3} R_\text{f}; \qquad R_\text{f} = 60\,\text{k}\Omega; \qquad \therefore \quad 0 \le R_\text{f} \le 60\,\text{k}\Omega$$

P 5.14 **[a]** Write a KCL equation at the inverting input to the op amp:

$$\frac{v_\text{d} - v_\text{a}}{60{,}000} + \frac{v_\text{d} - v_\text{b}}{20{,}000} + \frac{v_\text{d} - v_\text{c}}{36{,}000} + \frac{v_\text{d}}{270{,}000} + \frac{v_\text{d} - v_o}{180{,}000} = 0$$

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Multiply through by 180,000, plug in the values of input voltages, and rearrange to solve for v_o:

$$v_o = 180{,}000\left(\frac{3}{60{,}000} + \frac{-3}{20{,}000} + \frac{1}{36{,}000} + \frac{6}{270{,}000} + \frac{6}{180{,}000}\right)$$

$$= -3 \text{ V}$$

[b] Write a KCL equation at the inverting input to the op amp. Use the given values of input voltages in the equation:

$$\frac{6-3}{60{,}000} + \frac{6-9}{20{,}000} + \frac{6-v_c}{36{,}000} + \frac{6}{270{,}000} + \frac{6-v_o}{180{,}000} = 0$$

Simplify and solve for v_o:

$$9 - 27 + 30 - 5v_c + 4 + 6 - v_o = 0 \quad \text{so} \quad v_o = 22 - 5v_c$$

Set v_o to the positive power supply voltage and solve for v_c:

$$22 - 5v_c = 10 \quad \therefore \quad v_c = 2.4 \text{ V}$$

Set v_o to the negative power supply voltage and solve for v_c:

$$22 - 5v_c = -10 \quad \therefore \quad v_c = 6.4 \text{ V}$$

Therefore,

$$2.4 \text{ V} \le v_c \le 6.4 \text{ V}$$

P 5.15 **[a]** $$\frac{6-3}{60{,}000} + \frac{6-9}{20{,}000} + \frac{6-5}{36{,}000} + \frac{6}{270{,}000} + \frac{6-v_o}{R_f} = 0$$

$$\frac{6-v_o}{R_f} = -5 \times 10^{-5} \quad \text{so} \quad R_f = \frac{6-v_o}{-5 \times 10^{-5}}$$

For $v_o = 10$ V, $R_f = 80\,\text{k}\Omega$

For $v_o = -10$ V, $R_f < 0$ so this solution is not possible.

[b] $$i_o = -(i_f + i_{16\text{k}}) = -\left[\frac{10-6}{80{,}000} + \frac{10}{16{,}000}\right] = -675\,\mu\text{A}$$

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P 5.16 **[a]**

$$\frac{120{,}000}{R_\mathrm{a}} = 8 \qquad \text{so} \qquad R_\mathrm{a} = \frac{120{,}000}{8} = 15\,\mathrm{k}\Omega$$

$$\frac{120{,}000}{R_\mathrm{b}} = 5 \qquad \text{so} \qquad R_\mathrm{b} = \frac{120{,}000}{5} = 24\,\mathrm{k}\Omega$$

$$\frac{120{,}000}{R_\mathrm{c}} = 12 \qquad \text{so} \qquad R_\mathrm{c} = \frac{120{,}000}{12} = 10\,\mathrm{k}\Omega$$

[b] $-[8(2) + 5v_\mathrm{b} + 12(-1)] = -4 - 5v_\mathrm{b}$

$$-4 - 5v_\mathrm{b} = -15 \qquad \text{so} \qquad 5v_\mathrm{b} = 11 \qquad \text{thus} \qquad v_\mathrm{b} = \frac{11}{5} = 2.2\ \mathrm{V}$$

$$-4 - 5v_\mathrm{b} = 15 \qquad \text{so} \qquad -5v_\mathrm{b} = 19 \qquad \text{thus} \qquad v_\mathrm{b} = \frac{-19}{5} = -3.8\ \mathrm{V}$$

Thus,

$-3.8\ \mathrm{V} \le v_b \le 2.2\ \mathrm{V}$

P 5.17 We want the following expression for the output voltage:

$$v_o = -(8v_a + 4v_b + 10v_c + 6v_d)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage. Pick a feedback resistor with divisors of 8, 4, 10, and 6 – say 120 kΩ:

$$v_o = -\left[\frac{120\mathrm{k}}{R_\mathrm{a}}v_\mathrm{a} + \frac{120\mathrm{k}}{R_\mathrm{b}}v_\mathrm{b} + \frac{120\mathrm{k}}{R_\mathrm{c}}v_\mathrm{c} + \frac{120\mathrm{k}}{R_\mathrm{d}}v_\mathrm{d}\right]$$

Solve for each input resistance value to yield the desired gain:

$$\therefore \quad R_\mathrm{a} = 120{,}000/8 = 15\,\mathrm{k}\Omega \qquad R_\mathrm{c} = 120{,}000/10 = 12\,\mathrm{k}\Omega$$

$$R_\mathrm{b} = 120{,}000/4 = 30\,\mathrm{k}\Omega \qquad R_\mathrm{d} = 120{,}000/6 = 20\,\mathrm{k}\Omega$$

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Now create the 5 resistor values needed from the realistic resistor values in Appendix H. Note that $R_f = 120\,\text{k}\Omega$, $R_a = 15\,\text{k}\Omega$, and $R_c = 12\,\text{k}\Omega$ are already values from Appendix H. Create $R_b = 30\,\text{k}\Omega$ by combining two $15\,\text{k}\Omega$ resistors in series. Create $R_d = 20\,\text{k}\Omega$ by combining two $10\,\text{k}\Omega$ resistors in series. Of course there are many other acceptable possibilities. The final circuit is shown here:

P 5.18 **[a]** The circuit shown is a non-inverting amplifier.

[b] We assume the op amp to be ideal, so $v_n = v_p = 2$ V. Write a KCL equation at v_n:

$$\frac{2}{25{,}000} + \frac{2 - v_o}{150{,}000} = 0$$

Solving,

$$v_o = 14 \text{ V}.$$

P 5.19 **[a]** This circuit is an example of the non-inverting amplifier.

[b] Use voltage division to calculate v_p:

$$v_p = \frac{8000}{8000 + 32{,}000} v_s = \frac{v_s}{5}$$

Write a KCL equation at $v_n = v_p = v_s/5$:

$$\frac{v_s/5}{7000} + \frac{v_s/5 - v_o}{56{,}000} = 0$$

Solving,

$$v_o = 8v_s/5 + v_s/5 = 1.8v_s$$

[c] $1.8v_s = 12 \qquad \text{so} \qquad v_s = 6.67 \text{ V}$

$1.8v_s = -15 \qquad \text{so} \qquad v_s = -8.33 \text{ V}$

Thus, $-8.33 \text{ V} \le v_s \le 6.67 \text{ V}$.

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P 5.20 **[a]** $v_p = v_n = \dfrac{68}{80}v_g = 0.85v_g$

$$\therefore \quad \frac{0.85v_g}{30{,}000} + \frac{0.85v_g - v_o}{63{,}000} = 0;$$

$$\therefore \quad v_o = 2.635v_g = 2.635(4), \qquad v_o = 10.54 \text{ V}$$

[b] $v_o = 2.635v_g = \pm 12$

$$v_g = \pm 4.55 \text{ V}, \qquad -4.55 \le v_g \le 4.55 \text{ V}$$

[c] $\dfrac{0.85v_g}{30{,}000} + \dfrac{0.85v_g - v_o}{R_\text{f}} = 0$

$$\left(\frac{0.85R_\text{f}}{30{,}000} + 0.85\right) v_g = v_o = \pm 12$$

$$\therefore \quad 1.7R_\text{f} + 51 = \pm 360; \qquad 1.7R_\text{f} = 360 - 51; \qquad R_\text{f} = 181.76\,\text{k}\Omega$$

P 5.21 **[a]** From Eq. 5.18,

$$v_\text{o} = \frac{R_\text{s} + R_\text{f}}{R_\text{s}} v_\text{g} \qquad \text{so} \qquad \frac{v_\text{o}}{v_\text{g}} = 1 + \frac{R_\text{f}}{R_\text{s}} = 6$$

So,

$$\frac{R_\text{f}}{R_\text{s}} = 5$$

Thus,

$$R_\text{s} = \frac{R_\text{f}}{5} = \frac{75{,}000}{5} = 15\,\text{k}\Omega$$

[b] $v_\text{o} = 6v_\text{g}$

When $v_\text{g} = -2.5$ V, $v_\text{o} = 6(-2.5) = -15$ V.
When $v_\text{g} = 1.5$ V, $v_\text{o} = 6(1.5) = 9$ V.
The power supplies can be set at 9 V and −15 V.

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P 5.22 **[a]** From the equation for the non-inverting amplifier,

$$\frac{R_s + R_f}{R_s} = 2.5 \qquad \text{so} \qquad R_s + R_f = 2.5R_s \qquad \text{and therefore} \qquad R_f = 1.5R_s$$

Choose $R_s = 22\,\text{k}\Omega$, which is a component in Appendix H. Then $R_f = (1.5)(22) = 33\,\text{k}\Omega$, which is also a resistor value in Appendix H. The resulting non-inverting amplifier circuit is shown here:

[b] $v_o = 2.5v_g = 16 \qquad \text{so} \qquad v_g = 6.4\ \text{V}$

$v_o = 2.5v_g = -16 \qquad \text{so} \qquad v_g = -6.4\ \text{V}$

Therefore,

$-6.4\ \text{V} \le v_g \le 6.4\ \text{V}$

P 5.23 **[a]** This circuit is an example of a non-inverting summing amplifier.

[b] Write a KCL equation at v_p and solve for v_p in terms of v_s:

$$\frac{v_p - 5}{16{,}000} + \frac{v_p - v_s}{24{,}000} = 0$$

$$3v_p - 15 + 2v_p - 2v_s = 0 \qquad \text{so} \qquad v_p = 2v_s/5 + 3$$

Now write a KCL equation at v_n and solve for v_o:

$$\frac{v_n}{24{,}000} + \frac{v_n - v_o}{96{,}000} = 0 \qquad \text{so} \qquad v_o = 5v_n$$

Since we assume the op amp is ideal, $v_n = v_p$. Thus,

$$v_o = 5(2v_s/5 + 3) = 2v_s + 15$$

[c] $2v_s + 15 = 10 \qquad \text{so} \qquad v_s = -2.5\ \text{V}$

$2v_s + 15 = -10 \qquad \text{so} \qquad v_s = -12.5\ \text{V}$

Thus, $-12.5\ \text{V} \le v_s \le -2.5\ \text{V}$.

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P 5.24 **[a]** $\dfrac{v_p - v_\mathrm{a}}{R_\mathrm{a}} + \dfrac{v_p - v_\mathrm{b}}{R_\mathrm{b}} + \dfrac{v_p - v_\mathrm{c}}{R_\mathrm{c}} = 0$

$$\therefore \quad v_p = \frac{R_\mathrm{b}R_\mathrm{c}}{D}v_\mathrm{a} + \frac{R_\mathrm{a}R_\mathrm{c}}{D}v_\mathrm{b} + \frac{R_\mathrm{a}R_\mathrm{b}}{D}v_\mathrm{c}$$

where $\quad D = R_\mathrm{b}R_\mathrm{c} + R_\mathrm{a}R_\mathrm{c} + R_\mathrm{a}R_\mathrm{b}$

$$\frac{v_n}{20{,}000} + \frac{v_n - v_o}{100{,}000} = 0$$

$$\left(\frac{100{,}000}{20{,}000} + 1\right) v_n = 6v_n = v_o$$

$$\therefore \quad v_o = \frac{6R_\mathrm{b}R_\mathrm{c}}{D}v_\mathrm{a} + \frac{6R_\mathrm{a}R_\mathrm{c}}{D}v_\mathrm{b} + \frac{6R_\mathrm{a}R_\mathrm{b}}{D}v_\mathrm{c}$$

By hypothesis,

$$\frac{6R_\mathrm{b}R_\mathrm{c}}{D} = 1; \qquad \frac{6R_\mathrm{a}R_\mathrm{c}}{D} = 2; \qquad \frac{6R_\mathrm{a}R_\mathrm{b}}{D} = 3$$

Then

$$\frac{6R_\mathrm{a}R_\mathrm{b}/D}{6R_\mathrm{a}R_\mathrm{c}/D} = \frac{3}{2} \qquad \text{so} \qquad R_\mathrm{b} = 1.5R_\mathrm{c}$$

But from the circuit

$$R_\mathrm{b} = 15\,\mathrm{k\Omega} \qquad \text{so} \qquad R_\mathrm{c} = 10\,\mathrm{k\Omega}$$

Similarly,

$$\frac{6R_\mathrm{b}R_\mathrm{c}/D}{6R_\mathrm{a}R_\mathrm{b}/D} = \frac{1}{3} \qquad \text{so} \qquad 3R_\mathrm{c} = R_\mathrm{a}$$

Thus,

$$R_\mathrm{a} = 30\,\mathrm{k\Omega}$$

[b] $v_o = 1(0.7) + 2(0.4) + 3(1.1) = 4.8\ \mathrm{V}$

$$v_n = v_o/6 = 0.8\ \mathrm{V} = v_p$$

$$i_\mathrm{a} = \frac{v_\mathrm{a} - v_p}{30{,}000} = \frac{0.7 - 0.8}{30{,}000} = -3.33\,\mu\mathrm{A}$$

$$i_\mathrm{b} = \frac{v_\mathrm{b} - v_p}{15{,}000} = \frac{0.4 - 0.8}{15{,}000} = -26.67\,\mu\mathrm{A}$$

$$i_\mathrm{c} = \frac{v_\mathrm{c} - v_p}{10{,}000} = \frac{1.1 - 0.8}{10{,}000} = 30\,\mu\mathrm{A}$$

Check:

$$i_\mathrm{a} + i_\mathrm{b} + i_\mathrm{c} = 0? \qquad -3.33 - 26.67 + 30 = 0 \text{ (checks)}$$

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P 5.25 **[a]** Assume v_a is acting alone. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_\mathrm{a}}{R_\mathrm{a}} + \frac{0 - v_o'}{R_\mathrm{b}} + i_n = 0, \qquad i_n = 0$$

Therefore

$$\frac{v_o'}{R_\mathrm{b}} = -\frac{v_\mathrm{a}}{R_\mathrm{a}}, \qquad v_o' = -\frac{R_\mathrm{b}}{R_\mathrm{a}} v_\mathrm{a}$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_\mathrm{b} R_\mathrm{d}}{R_\mathrm{c} + R_\mathrm{d}}$$

$$\frac{v_n}{R_\mathrm{a}} + \frac{v_n - v_o''}{R_\mathrm{b}} + i_n = 0, \qquad i_n = 0$$

$$\left(\frac{1}{R_\mathrm{a}} + \frac{1}{R_\mathrm{b}}\right)\left(\frac{R_\mathrm{d}}{R_\mathrm{c} + R_\mathrm{d}}\right) v_\mathrm{b} - \frac{v_o''}{R_\mathrm{b}} = 0$$

$$v_o'' = \left(\frac{R_\mathrm{b}}{R_\mathrm{a}} + 1\right)\left(\frac{R_\mathrm{d}}{R_\mathrm{c} + R_\mathrm{d}}\right) v_\mathrm{b} = \frac{R_\mathrm{d}}{R_\mathrm{a}}\left(\frac{R_\mathrm{a} + R_\mathrm{b}}{R_\mathrm{c} + R_\mathrm{d}}\right) v_\mathrm{b}$$

$$v_o = v_o' + v_o'' = \frac{R_\mathrm{d}}{R_\mathrm{a}}\left(\frac{R_\mathrm{a} + R_\mathrm{b}}{R_\mathrm{c} + R_\mathrm{d}}\right) v_\mathrm{b} - \frac{R_\mathrm{b}}{R_\mathrm{a}} v_\mathrm{a}$$

[b] $$\frac{R_\mathrm{d}}{R_\mathrm{a}}\left(\frac{R_\mathrm{a} + R_\mathrm{b}}{R_\mathrm{c} + R_\mathrm{d}}\right) = \frac{R_\mathrm{b}}{R_\mathrm{a}}, \qquad \text{therefore} \quad R_\mathrm{d}(R_\mathrm{a} + R_\mathrm{b}) = R_\mathrm{b}(R_\mathrm{c} + R_\mathrm{d})$$

$$R_\mathrm{d} R_\mathrm{a} = R_\mathrm{b} R_\mathrm{c}, \qquad \text{therefore} \quad \frac{R_\mathrm{a}}{R_\mathrm{b}} = \frac{R_\mathrm{c}}{R_\mathrm{d}}$$

When $$\frac{R_\mathrm{d}}{R_\mathrm{a}}\left(\frac{R_\mathrm{a} + R_\mathrm{b}}{R_\mathrm{c} + R_\mathrm{d}}\right) = \frac{R_\mathrm{b}}{R_\mathrm{a}}$$

Eq. (5.22) reduces to $$v_o = \frac{R_\mathrm{b}}{R_\mathrm{a}} v_\mathrm{b} - \frac{R_\mathrm{b}}{R_\mathrm{a}} v_\mathrm{a} = \frac{R_\mathrm{b}}{R_\mathrm{a}}(v_\mathrm{b} - v_\mathrm{a}).$$

P 5.26 **[a]** This is a difference amplifier circuit.

[b] Use Eq. 5.22 with $R_\mathrm{a} = 5\,\mathrm{k\Omega}$, $R_\mathrm{b} = 20\,\mathrm{k\Omega}$, $R_\mathrm{c} = 8\,\mathrm{k\Omega}$, $R_\mathrm{d} = 2\,\mathrm{k\Omega}$, and $v_\mathrm{b} = 5$ V:

$$v_\mathrm{o} = \frac{R_\mathrm{d}(R_\mathrm{a} + R_\mathrm{b})}{R_\mathrm{a}(R_\mathrm{c} + R_\mathrm{d})} v_\mathrm{b} - \frac{R_\mathrm{b}}{R_\mathrm{a}} v_\mathrm{a} = \frac{2(5 + 20)}{5(8 + 2)}(5) - \frac{20}{5} v_\mathrm{a} = 5 - 4v_\mathrm{a}$$

[c] $$\frac{2000(5000 + R_\mathrm{f})}{5000(8000 + 2000)}(5) - \frac{R_\mathrm{f}}{5000}(2) = \frac{5000 + R_\mathrm{f}}{5000} - \frac{2R_\mathrm{f}}{5000} = 1 - \frac{R_\mathrm{f}}{5000}$$

$$1 - \frac{R_\mathrm{f}}{5000} = 10 \qquad \text{so} \qquad R_\mathrm{f} < 0 \text{ which is not a possible solution.}$$

$$1 - \frac{R_\mathrm{f}}{5000} = -10 \qquad \text{so} \qquad R_\mathrm{f} = 5000(11) = 55\,\mathrm{k\Omega}$$

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P 5.27 **[a]** $v_o = \dfrac{R_\mathrm{d}(R_\mathrm{a}+R_\mathrm{b})}{R_\mathrm{a}(R_\mathrm{c}+R_\mathrm{d})}v_\mathrm{b} - \dfrac{R_\mathrm{b}}{R_\mathrm{a}}v_\mathrm{a} = \dfrac{120(24+75)}{24(130+120)}(5) - \dfrac{75}{24}(8)$

$$v_o = 9.9 - 25 = -15.1 \text{ V}$$

[b] $\dfrac{v_1 - 8}{24{,}000} + \dfrac{v_1 + 15.1}{75{,}000} = 0 \qquad \text{so} \qquad v_1 = 2.4 \text{ V}$

$$i_\mathrm{a} = \frac{8 - 2.4}{24{,}000} = 233\,\mu \text{ A}$$

$$R_\mathrm{ina} = \frac{v_\mathrm{a}}{i_\mathrm{a}} = \frac{8}{233 \times 10^{-6}} = 34.3\,\mathrm{k}\Omega$$

[c] $R_\mathrm{in\,b} = R_\mathrm{c} + R_\mathrm{d} = 250\,\mathrm{k}\Omega$

P 5.28 $v_p = \dfrac{1500}{9000}(-18) = -3 \text{ V} = v_n$

$$\frac{-3+18}{1600} + \frac{-3 - v_o}{R_\mathrm{f}} = 0$$

$$\therefore \quad v_o = 0.009375R_\mathrm{f} - 3$$

$$v_o = 9 \text{ V}; \qquad R_\mathrm{f} = 1280\,\Omega$$

$$v_o = -9 \text{ V}; \qquad R_\mathrm{f} = -640\,\Omega$$

But $\quad R_\mathrm{f} \geq 0, \qquad \therefore \quad R_\mathrm{f} = 1.28\,\mathrm{k}\Omega$

P 5.29 $v_o = \dfrac{R_\mathrm{d}(R_\mathrm{a}+R_\mathrm{b})}{R_\mathrm{a}(R_\mathrm{c}+R_\mathrm{d})}v_\mathrm{b} - \dfrac{R_\mathrm{b}}{R_\mathrm{a}}v_\mathrm{a}$

By hypothesis: $\quad R_\mathrm{b}/R_\mathrm{a} = 4; \quad R_\mathrm{c} + R_\mathrm{d} = 470\,\mathrm{k}\Omega; \quad \dfrac{R_\mathrm{d}(R_\mathrm{a}+R_\mathrm{b})}{R_\mathrm{a}(R_\mathrm{c}+R_\mathrm{d})} = 3$

$$\therefore \quad \frac{R_\mathrm{d}}{R_\mathrm{a}}\frac{(R_\mathrm{a}+4R_\mathrm{a})}{470{,}000} = 3 \quad \text{so} \quad R_\mathrm{d} = 282\,\mathrm{k}\Omega; \quad R_\mathrm{c} = 188\,\mathrm{k}\Omega$$

Create $R_\mathrm{d} = 282\,\mathrm{k}\Omega$ by combining a $270\,\mathrm{k}\Omega$ resistor and a $12\,\mathrm{k}\Omega$ resistor in series. Create $R_\mathrm{c} = 188\,\mathrm{k}\Omega$ by combining a $120\,\mathrm{k}\Omega$ resistor and a $68\,\mathrm{k}\Omega$ resistor in series. Also, when $v_o = 0$ we have

$$\frac{v_n - v_\mathrm{a}}{R_\mathrm{a}} + \frac{v_n}{R_\mathrm{b}} = 0$$

$$\therefore \quad v_n\left(1 + \frac{R_\mathrm{a}}{R_\mathrm{b}}\right) = v_\mathrm{a}; \qquad v_n = 0.8v_\mathrm{a}$$

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$$i_{\mathrm{a}} = \frac{v_{\mathrm{a}} - 0.8v_{\mathrm{a}}}{R_{\mathrm{a}}} = 0.2\frac{v_{\mathrm{a}}}{R_{\mathrm{a}}}; \qquad R_{\mathrm{in}} = \frac{v_{\mathrm{a}}}{i_{\mathrm{a}}} = 5R_{\mathrm{a}} = 22\,\mathrm{k}\Omega$$

$$\therefore \quad R_{\mathrm{a}} = 4.4\,\mathrm{k}\Omega; \qquad R_{\mathrm{b}} = 17.6\,\mathrm{k}\Omega$$

Create $R_{\mathrm{a}} = 4.4\,\mathrm{k}\Omega$ by combining two $2.2\,\mathrm{k}\Omega$ resistors in series. Create $R_{\mathrm{b}} = 17.6\,\mathrm{k}\Omega$ by combining a $12\,\mathrm{k}\Omega$ resistor and a $5.6\,\mathrm{k}\Omega$ resistor in series.

P 5.30 **[a]**

$$\frac{v_p}{20{,}000} + \frac{v_p - v_{\mathrm{c}}}{30{,}000} + \frac{v_p - v_{\mathrm{d}}}{20{,}000} = 0$$

$$\therefore \quad 8v_p = 2v_{\mathrm{c}} + 3v_{\mathrm{d}} = 8v_n$$

$$\frac{v_n - v_{\mathrm{a}}}{20{,}000} + \frac{v_n - v_{\mathrm{b}}}{18{,}000} + \frac{v_n - v_o}{180{,}000} = 0$$

$$\begin{aligned} \therefore \quad v_o &= 20v_n - 9v_{\mathrm{a}} - 10v_{\mathrm{b}} \\ &= 20[(1/4)v_{\mathrm{c}} + (3/8)v_{\mathrm{d}}] - 9v_{\mathrm{a}} - 10v_{\mathrm{b}} \\ &= 20(0.75 + 1.5) - 9(1) - 10(2) = 16 \text{ V} \end{aligned}$$

[b] $v_o = 5v_{\mathrm{c}} + 30 - 9 - 20 = 5v_{\mathrm{c}} + 1$

$$\pm 20 = 5v_{\mathrm{c}} + 1$$

$$\therefore \quad v_{\mathrm{b}} = -4.2 \text{ V} \qquad \text{and} \qquad v_{\mathrm{b}} = 3.8 \text{ V}$$

$$\therefore \quad -4.2 \text{ V} \le v_{\mathrm{b}} \le 3.8 \text{ V}$$

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P 5.31 $v_p = R_\mathrm{b} i_\mathrm{b} = v_n$

$$\frac{R_\mathrm{b} i_\mathrm{b}}{2000} + \frac{R_\mathrm{b} i_\mathrm{b} - v_o}{R_f} - i_\mathrm{a} = 0$$

$$\therefore \qquad R_\mathrm{b} i_\mathrm{b}\left(\frac{1}{2000} + \frac{1}{R_f}\right) - i_\mathrm{a} = \frac{v_o}{R_f}$$

$$\therefore \qquad R_\mathrm{b} i_\mathrm{b}\left(1 + \frac{R_f}{2000}\right) - R_f i_\mathrm{a} = v_o$$

By hypopthesis, $v_o = 8000(i_\mathrm{b} - i_\mathrm{a})$. Therefore,

$R_f = 8\,\mathrm{k}\Omega$ (use $3.3\,\mathrm{k}\Omega$ and $4.7\,\mathrm{k}\Omega$ resistors in series)

$$R_\mathrm{b}\left(1 + \frac{8000}{2000}\right) = 8000 \qquad \text{so} \qquad R_\mathrm{b} = 1.6\,\mathrm{k}\Omega$$

To construct the $1.6\,\mathrm{k}\Omega$ resistor, combine $270\,\Omega$, $330\,\Omega$, and $1\,\mathrm{k}\Omega$ resistors in series.

P 5.32 **[a]**

$$v_p = \frac{\alpha R_\mathrm{g}}{\alpha R_\mathrm{g} + (R_\mathrm{g} - \alpha R_\mathrm{g})} v_\mathrm{g}$$

$$v_n = v_p = \alpha v_g$$

$$\frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_\mathrm{f}} = 0$$

$$(v_n - v_g)\frac{R_\mathrm{f}}{R_1} + v_n - v_o = 0$$

$$\begin{aligned} v_o &= \left(1 + \frac{R_\mathrm{f}}{R_1}\right)\alpha v_\mathrm{g} - \frac{R_\mathrm{f}}{R_1} v_\mathrm{g} \\ &= (\alpha v_g - v_g)4 + \alpha v_g \\ &= [(\alpha - 1)4 + \alpha] v_g \\ &= (5\alpha - 4) v_g \\ &= (5\alpha - 4)(2) = 10\alpha - 8 \end{aligned}$$

α	v_o	α	v_o	α	v_o
0.0	−8 V	0.4	−4 V	0.8	0 V
0.1	−7 V	0.5	−3 V	0.9	1 V
0.2	−6 V	0.6	−2 V	1.0	2 V
0.3	−5 V	0.7	−1 V		

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[b] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1\right) v_g \alpha + -\left(\frac{R_f}{R_1}\right) v_g$$

Therefore,

$$\text{slope } = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad \text{intercept } = -\left(\frac{R_f}{R_1}\right) v_g$$

[c] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad 4 = -\left(\frac{R_f}{R_1}\right) v_g$$

Solving,

$$v_g = -2 \text{ V}; \qquad \frac{R_f}{R_1} = 2$$

P 5.33 **[a]** $A_{\text{dm}} = \dfrac{(24)(26) + (25)(25)}{(2)(1)(25)} = 24.98$

[b] $A_{\text{cm}} = \dfrac{(1)(24) - 25(1)}{1(25)} = -0.04$

[c] $\text{CMRR} = \left|\dfrac{24.98}{0.04}\right| = 624.50$

P 5.34 $A_{\text{cm}} = \dfrac{(3000)(6000) - (6000)R_x}{3000(6000 + R_x)}$

$$A_{\text{dm}} = \frac{6000(3000 + 6000) + 6000(6000 + R_x)}{2(3000)(6000 + R_x)}$$

$$\frac{A_{\text{dm}}}{A_{\text{cm}}} = \frac{R_x + 15{,}000}{2(3000 - R_x)}$$

$$\therefore \quad \frac{R_x + 15{,}000}{2(3000 - R_x)} = \pm 1500 \quad \text{for the limits on the value of } R_x$$

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If we use $+1500 \quad R_x = 2994\,\Omega$

If we use $-1500 \quad R_x = 3006\,\Omega$

$2994\,\Omega \le R_x \le 3006\,\Omega$

P 5.35 It follows directly from the circuit that $v_o = -(75/15)v_g = -5v_g$
From the plot of v_g we have $\quad v_g = 0, \quad t < 0$

$$\begin{aligned}
v_g &= t & 0 \le t \le 2 \\
v_g &= 4 - t & 2 \le t \le 6 \\
v_g &= t - 8 & 6 \le t \le 10 \\
v_g &= 12 - t & 10 \le t \le 14 \\
v_g &= t - 16 & 14 \le t \le 18, \quad \text{etc.}
\end{aligned}$$

Therefore

$$\begin{aligned}
v_o &= -5t & 0 \le t \le 2 \\
v_o &= 5t - 20 & 2 \le t \le 6 \\
v_o &= 40 - 5t & 6 \le t \le 10 \\
v_o &= 5t - 60 & 10 \le t \le 14 \\
v_o &= 80 - 5t & 14 \le t \le 18, \quad \text{etc.}
\end{aligned}$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ± 9, the output is clipped at ± 9. The plot is shown below.

P 5.36 $\quad v_p = \dfrac{5.4}{7.2}v_g = 0.75v_g = 3\cos(\pi/4)t \ \text{ V}$

$$\frac{v_n}{20{,}000} + \frac{v_n - v_o}{60{,}000} = 0$$

$4v_n = v_o; \qquad v_n = v_p$

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$$\therefore \quad v_o = 12\cos(\pi/4)t \ \text{ V} \qquad 0 \le t \le \infty$$

but saturation occurs at $v_o = \pm 10$ V

P 5.37 [**a**]

$$\frac{v_n - v_a}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_a = v_o$$

$$\frac{v_a}{R_a} + \frac{v_a - v_n}{R} + \frac{v_a - v_o}{R} = 0$$

$$v_a\left[\frac{1}{R_a} + \frac{2}{R}\right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_a\left(2 + \frac{R}{R_a}\right) - v_n = v_o$$

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$$v_n = v_p = v_\mathrm{a} + v_g$$

$$\therefore \quad 2v_n - v_\mathrm{a} = 2v_\mathrm{a} + 2v_g - v_\mathrm{a} = v_\mathrm{a} + 2v_g$$

$$\therefore \quad v_\mathrm{a} - v_o = -2v_g \qquad (1)$$

$$2v_\mathrm{a} + v_\mathrm{a}\left(\frac{R}{R_\mathrm{a}}\right) - v_\mathrm{a} - v_g = v_o$$

$$\therefore \quad v_\mathrm{a}\left(1 + \frac{R}{R_\mathrm{a}}\right) - v_o = v_g \qquad (2)$$

Now combining equations (1) and (2) yields

$$-v_\mathrm{a}\frac{R}{R_\mathrm{a}} = -3v_g$$

$$\text{or} \quad v_\mathrm{a} = 3v_g\frac{R_\mathrm{a}}{R}$$

$$\text{Hence} \quad i_\mathrm{a} = \frac{v_\mathrm{a}}{R_\mathrm{a}} = \frac{3v_g}{R} \qquad \text{Q.E.D.}$$

[b] At saturation $v_o = \pm\ \mathrm{V_{cc}}$

$$\therefore \quad v_\mathrm{a} = \pm\ \mathrm{V_{cc}} - 2v_g \qquad (3)$$

and

$$\therefore \quad v_\mathrm{a}\left(1 + \frac{R}{R_\mathrm{a}}\right) = \pm\ \mathrm{V_{cc}} + v_g \qquad (4)$$

Dividing Eq (4) by Eq (3) gives

$$1 + \frac{R}{R_\mathrm{a}} = \frac{\pm\ \mathrm{V_{cc}} + v_g}{\pm\ \mathrm{V_{cc}} - 2v_g}$$

$$\therefore \quad \frac{R}{R_\mathrm{a}} = \frac{\pm\ \mathrm{V_{cc}} + v_g}{\pm\ \mathrm{V_{cc}} - 2v_g} - 1 = \frac{3v_g}{\pm\ \mathrm{V_{cc}} - 2v_g}$$

$$\text{or} \quad R_\mathrm{a} = \frac{(\pm\ \mathrm{V_{cc}} - 2v_g)}{3v_g}R \qquad \text{Q.E.D.}$$

P 5.38 **[a]** $v_p = v_s, \qquad v_n = \dfrac{R_1 v_o}{R_1 + R_2}, \qquad v_n = v_p$

$$\text{Therefore} \quad v_o = \left(\frac{R_1 + R_2}{R_1}\right)v_s = \left(1 + \frac{R_2}{R_1}\right)v_s$$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

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P 5.39

$$i_1 = \frac{15 - 10}{5000} = 1\,\text{mA}$$

$$i_2 + i_1 + 0 = 10\,\text{mA}; \qquad i_2 = 9\,\text{mA}$$

$$v_{o2} = 10 + (400)(9) \times 10^{-3} = 13.6 \ \text{V}$$

$$i_3 = \frac{15 - 13.6}{2000} = 0.7\,\text{mA}$$

$$i_4 = i_3 + i_1 = 1.7\,\text{mA}$$

$$v_{o1} = 15 + 1.7(0.5) = 15.85 \ \text{V}$$

P 5.40 **[a]** $p_{16\,\text{k}\Omega} = \dfrac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4\,\mu\text{W}$

[b] $v_{16\,\text{k}\Omega} = \left(\dfrac{16}{64}\right)(320) = 80\,\text{mV}$

$$p_{16\,\text{k}\Omega} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4\,\mu\text{W}$$

[c] $\dfrac{p_\text{a}}{p_\text{b}} = \dfrac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

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- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification*.
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 5.41 **[a]** Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-47}{10}(1) = -4.7 \text{ V}; \qquad v_{o2} = \frac{-220}{33}(-0.15) = 1.0 \text{ V}$$

$$i_{\text{a}} = \frac{v_{o2} - v_{o1}}{1000} = 5.7\,\text{mA}$$

[b] $i_{\text{a}} = 0$ when $v_{o1} = v_{o2}$ so from (a) $v_{o2} = 1$ V

Thus

$$\frac{-47}{10}(v_{\text{L}}) = 1$$

$$v_{\text{L}} = -\frac{10}{47} = -212.77 \text{ mV}$$

P 5.42 **[a]** Assume the op-amp is operating within its linear range, then

$$i_L = \frac{8}{4000} = 2\,\text{mA}$$

For $R_L = 4\,\text{k}\Omega$ $\qquad v_o = (4+4)(2) = 16$ V

Now since $v_o < 20$ V our assumption of linear operation is correct, therefore

$$i_L = 2\,\text{mA}$$

[b] $20 = 2(4 + R_L); \qquad R_L = 6\,\text{k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L. From (b) we found the op-amp is operating in its linear region as long as $R_L \le 6\,\text{k}\Omega$. Therefore when $R_L = 6\,\text{k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 20/(4000 + 16{,}000) = 1\,\text{mA}$. To justify neglecting the current into the op-amp assume the drop across the 50 kΩ resistor is negligible, since the input resistance to the op-amp is at least $500\,\text{k}\Omega$. Then $i_p = i_n = (8-4)/(500 \times 10^3) = 8\,\mu\text{A}$. But $8\,\mu\text{A} \ll 1\,\text{mA}$, hence our assumption is reasonable.

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[d]

P 5.43 From Eq. 5.57,

$$\frac{v_{\text{ref}}}{R+\Delta R} = v_n\left(\frac{1}{R+\Delta R}+\frac{1}{R-\Delta R}+\frac{1}{R_f}\right)-\frac{v_o}{R_f}$$

Substituting Eq. 5.59 for $v_p = v_n$:

$$\frac{v_{\text{ref}}}{R+\Delta R} = \frac{v_{\text{ref}}\left(\frac{1}{R+\Delta R}+\frac{1}{R-\Delta R}+\frac{1}{R_f}\right)}{(R-\Delta R)\left(\frac{1}{R+\Delta R}+\frac{1}{R-\Delta R}+\frac{1}{R_f}\right)}-\frac{v_o}{R_f}$$

Rearranging,

$$\frac{v_o}{R_f} = v_{\text{ref}}\left(\frac{1}{R-\Delta R}-\frac{1}{R+\Delta R}\right)$$

Thus,

$$v_o = v_{\text{ref}}\left(\frac{2\Delta R}{R^2-(\Delta R)^2}\right)R_f$$

P 5.44 **[a]** Replace the op amp with the model from Fig. 5.15:

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Write two node voltage equations, one at the left node, the other at the right node:

$$\frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} + \frac{v_n}{500{,}000} = 0$$

$$\frac{v_o + 3 \times 10^5 v_n}{5000} + \frac{v_o - v_n}{100{,}000} + \frac{v_o}{500} = 0$$

Simplify and place in standard form:

$$106v_n - 5v_o = 100v_g$$

$$(6 \times 10^6 - 1)v_n + 221v_o = 0$$

Let $v_g = 1$ V and solve the two simultaneous equations:

$$v_o = -19.9844 \text{ V}; \qquad v_n = 736.1\,\mu\text{V}$$

Thus the voltage gain is $v_o/v_g = -19.9844$.

[b] From the solution in part (a), $v_n = 736.1\,\mu$V.

[c] $i_g = \dfrac{v_g - v_n}{5000} = \dfrac{v_g - 736.1 \times 10^{-6} v_g}{5000}$

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 736.1 \times 10^{-6}} = 5003.68\,\Omega$$

[d] For an ideal op amp, the voltage gain is the ratio between the feedback resistor and the input resistor:

$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

For an ideal op amp, the difference between the voltages at the input terminals is zero, and the input resistance of the op amp is infinite. Therefore,

$$v_n = v_p = 0 \text{ V}; \qquad R_g = 5000\,\Omega$$

P 5.45 **[a]**

$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500{,}000} + \frac{v_n - v_{\text{Th}}}{24{,}000} = 0$$

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$$\frac{v_{\text{Th}} + 10^5 v_n}{2000} + \frac{v_{\text{Th}} - v_n}{24{,}000} = 0$$

Solving, $v_{\text{Th}} = -13.198$ V

Short-circuit current calculation:

$$\frac{v_n}{500{,}000} + \frac{v_n - 0.88}{1600} + \frac{v_n - 0}{24{,}000} = 0$$

$\therefore \quad v_n = 0.8225$ V

$$i_{\text{sc}} = \frac{v_n}{24{,}000} - \frac{10^5}{2000} v_n = -41.13 \text{ A}$$

$$R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{\text{sc}}} = 320.9\,\text{m}\Omega$$

[b] The output resistance of the inverting amplifier is the same as the Thévenin resistance, i.e.,

$R_o = R_{\text{Th}} = 320.9\,\text{m}\Omega$

[c]

$$v_o = \left(\frac{330}{330.3209}\right)(-13.2) = -13.18 \text{ V}$$

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$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500{,}000} + \frac{v_n + 13.18}{24{,}000} = 0$$

$$\therefore \quad v_n = 942\,\mu\text{V}$$

$$i_g = \frac{0.88 - 942 \times 10^{-6}}{1600} = 549.41\,\mu\text{A}$$

$$R_g = \frac{0.88}{i_g} = 1601.71\,\Omega$$

P 5.46 **[a]** $v_{\text{Th}} = -\dfrac{24{,}000}{1600}(0.88) = -13.2 \text{ V}$

$R_{\text{Th}} = 0,$ since op-amp is ideal

[b] $R_o = R_{\text{Th}} = 0\,\Omega$

[c] $R_g = 1.6\,\text{k}\Omega$ since $v_n = 0$

P 5.47 **[a]**

$$\frac{v_n - v_g}{2000} + \frac{v_n - v_o}{10{,}000} = 0$$

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$\therefore \quad v_o = 6v_n - 5v_g$

Also $\quad v_o = A(v_p - v_n) = -Av_n$

$\therefore \quad v_n = \dfrac{-v_o}{A}$

$\therefore \quad v_o\left(1 + \dfrac{6}{A}\right) = -5v_g$

$v_o = \dfrac{-5A}{(6+A)}v_g$

[b] $v_o = \dfrac{-5(150)(1)}{156} = -4.81 \text{ V}$

[c] $v_o = \dfrac{-5}{1+(6/A)}(1) = -5 \text{ V}$

[d] $\dfrac{-5A}{A+6}(1) = -0.99(5) \qquad \text{so} \qquad -5A = -4.95(A+6)$

$\therefore \quad -0.05A = -29.7 \qquad \text{so} \qquad A = 594$

P 5.48 **[a]** $\dfrac{v_n}{16{,}000} + \dfrac{v_n - v_g}{800{,}000} + \dfrac{v_n - v_o}{200{,}000} = 0 \quad \text{or} \quad 55v_n - 4v_o = v_g \qquad \text{Eq (1)}$

$\dfrac{v_o}{20{,}000} + \dfrac{v_o - v_n}{200{,}000} + \dfrac{v_o - 50{,}000(v_p - v_n)}{8000} = 0$

$36v_o - v_n - 125 \times 10^4(v_p - v_n) = 0$

$v_p = v_g + \dfrac{(v_n - v_g)(240)}{800} = (0.7)v_g + (0.3)v_n$

$36v_o - v_n - 125 \times 10^4[(0.7)v_g - (0.7)v_n] = 0$

$36v_o + 874{,}999v_n = 875{,}000v_g \qquad \text{Eq (2)}$

Let $v_g = 1$ V and solve Eqs. (1) and (2) simultaneously:

$v_n = 999.446\,\text{mV} \qquad \text{and} \qquad v_o = 13.49 \text{ V}$

$\therefore \qquad \dfrac{v_o}{v_g} = 13.49$

[b] From part (a), $v_n = 999.446$ mV.

$v_p = (0.7)(1000) + (0.3)(999.446) = 999.834\,\text{mV}$

[c] $v_p - v_n = 387.78\,\mu\text{V}$

[d] $i_g = \dfrac{(1000 - 999.83)10^{-3}}{24 \times 10^3} = 692.47\,\text{pA}$

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[e] $\dfrac{v_g}{16{,}000} + \dfrac{v_g - v_o}{200{,}000} = 0, \qquad \text{since } v_n = v_p = v_g$

$$\therefore \quad v_o = 13.5 v_g, \qquad \frac{v_o}{v_g} = 13.5$$

$$v_n = v_p = 1 \text{ V}; \qquad v_p - v_n = 0 \text{ V}; \qquad i_g = 0 \text{ A}$$

P 5.49 **[a]** Use Eq. 5.61 to solve for R_f; note that since we are using 1% strain gages, $\Delta = 0.01$:

$$R_f = \frac{v_o R}{2\Delta v_{\text{ref}}} = \frac{(5)(120)}{(2)(0.01)(15)} = 2\,\text{k}\Omega$$

[b] Now solve for Δ given $v_o = 50$ mV:

$$\Delta = \frac{v_o R}{2R_f v_{\text{ref}}} = \frac{(0.05)(120)}{2(2000)(15)} = 100 \times 10^{-6}$$

The change in strain gage resistance that corresponds to a 50 mV change in output voltage is thus

$$\Delta R = \Delta R = (100 \times 10^{-6})(120) = 12 \text{ m}\Omega$$

P 5.50 **[a]**

Let $R_1 = R + \Delta R$

$$\frac{v_p}{R_f} + \frac{v_p}{R} + \frac{v_p - v_{\text{in}}}{R_1} = 0$$

$$\therefore \quad v_p \left[\frac{1}{R_f} + \frac{1}{R} + \frac{1}{R_1}\right] = \frac{v_{\text{in}}}{R_1}$$

$$\therefore \quad v_p = \frac{R R_f v_{\text{in}}}{R R_1 + R_f R_1 + R_f R} = v_n$$

$$\frac{v_n}{R} + \frac{v_n - v_{\text{in}}}{R} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n \left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R_f}\right] - \frac{v_o}{R_f} = \frac{v_{\text{in}}}{R}$$

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$$\therefore \quad v_n\left[\frac{R+2R_f}{RR_f}\right]-\frac{v_{\text{in}}}{R}=\frac{v_o}{R_f}$$

$$\therefore \quad \frac{v_o}{R_f}=\left[\frac{R+2R_f}{RR_f}\right]\left[\frac{RR_fv_{\text{in}}}{RR_1+R_fR_1+R_fR}\right]-\frac{v_{\text{in}}}{R}$$

$$\therefore \quad \frac{v_o}{R_f}=\left[\frac{R+2R_f}{RR_1+R_fR_1+R_fR}-\frac{1}{R}\right]v_{\text{in}}$$

$$\therefore \quad v_o=\frac{[R^2+2RR_f-R_1(R+R_f)-RR_f]R_f}{R[R_1(R+R_f)+RR_f]}v_{\text{in}}$$

Now substitute $R_1=R+\Delta R$ and get

$$v_o=\frac{-\Delta R(R+R_f)R_fv_{\text{in}}}{R[(R+\Delta R)(R+R_f)+RR_f]}$$

If $\Delta R \ll R$

$$v_o\approx\frac{(R+R_f)R_f(-\Delta R)v_{\text{in}}}{R^2(R+2R_f)}$$

[b] $v_o\approx\dfrac{47\times10^4(48\times10^4)(-95)15}{10^8(95\times10^4)}\approx-3.384\ \text{V}$

[c] $v_o=\dfrac{-95(48\times10^4)(47\times10^4)15}{10^4[(1.0095)10^4(48\times10^4)+47\times10^8]}=-3.368\ \text{V}$

P 5.51 **[a]** $v_o\approx\dfrac{(R+R_f)R_f(-\Delta R)v_{\text{in}}}{R^2(R+2R_f)}$

$$v_o=\frac{(R+R_f)(-\Delta R)R_fv_{\text{in}}}{R[(R+\Delta R)(R+R_f)+RR_f]}$$

$$\therefore \quad \frac{\text{approx value}}{\text{true value}}=\frac{R[(R+\Delta R)(R+R_f)+RR_f]}{R^2(R+2R_f)}$$

$$\therefore \quad \text{Error}\ =\frac{R[(R+\Delta R)(R+R_f)+RR_f]-R^2(R+2R_f)}{R^2(R+2R_f)}$$

$$=\frac{\Delta R}{R}\frac{(R+R_f)}{(R+2R_f)}$$

$$\therefore \quad \%\ \text{error}\ =\frac{\Delta R(R+R_f)}{R(R+2R_f)}\times100$$

[b] $\%\ \text{error}=\dfrac{95(48\times10^4)\times100}{10^4(95\times10^4)}=0.48\%$

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P 5.52 $$1 = \frac{\Delta R(48 \times 10^4)}{10^4(95 \times 10^4)} \times 100$$

$$\therefore \quad \Delta R = \frac{9500}{48} = 197.91667\,\Omega$$

$$\therefore \quad \%\text{ change in } R = \frac{197.19667}{10^4} \times 100 \approx 1.98\%$$

P 5.53 **[a]** It follows directly from the solution to Problem 5.50 that

$$v_o = \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f v_{\text{in}}}{R[R_1(R + R_f) + RR_f]}$$

Now $R_1 = R - \Delta R$. Substituting into the expression gives

$$v_o = \frac{(R + R_f)R_f(\Delta R)v_{\text{in}}}{R[(R - \Delta R)(R + R_f) + RR_f]}$$

Now let $\Delta R \ll R$ and get

$$v_o \approx \frac{(R + R_f)R_f \Delta R v_{\text{in}}}{R^2(R + 2R_f)}$$

[b] It follows directly from the solution to Problem 5.50 that

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{R[(R - \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore \quad \text{Error } = \frac{(R - \Delta R)(R + R_f) + RR_f - R(R + 2R_f)}{R(R + 2R_f)}$$

$$= \frac{-\Delta R(R + R_f)}{R(R + 2R_f)}$$

$$\therefore \quad \%\text{ error } = \frac{-\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

[c] $R - \Delta R = 9810\,\Omega \qquad \therefore \quad \Delta R = 10{,}000 - 9810 = 190\,\Omega$

$$\therefore \quad v_o \approx \frac{(48 \times 10^4)(47 \times 10^4)(190)(15)}{10^8(95 \times 10^4)} \approx 6.768 \text{ V}$$

[d] $$\%\text{ error } = \frac{-190(48 \times 10^4)(100)}{10^4(95 \times 10^4)} = -0.96\%$$

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6

Inductance, Capacitance, and Mutual Inductance

Assessment Problems

AP 6.1 **[a]** $i_g = 8e^{-300t} - 8e^{-1200t}\text{A}$

$$v = L\frac{di_g}{dt} = -9.6e^{-300t} + 38.4e^{-1200t}\text{V}, \qquad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8\,\text{V}$$

[b] $v = 0 \quad \text{when} \quad 38.4e^{-1200t} = 9.6e^{-300t} \quad \text{or} \quad t = (\ln 4)/900 = 1.54\,\text{ms}$

[c] $p = vi = 384e^{-1500t} - 76.8e^{-600t} - 307.2e^{-2400t}\,\text{W}$

[d] $\dfrac{dp}{dt} = 0 \quad \text{when} \quad e^{1800t} - 12.5e^{900t} + 16 = 0$

Let $x = e^{900t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.44766, \qquad t = \frac{\ln 1.45}{900} = 411.05\,\mu\text{s}$$

$$x = 11.0523, \qquad t = \frac{\ln 11.05}{900} = 2.67\,\text{ms}$$

p is maximum at $t = 411.05\,\mu\text{s}$

[e] $p_{\max} = 384e^{-1.5(0.41105)} - 76.8e^{-0.6(0.41105)} - 307.2e^{-2.4(0.41105)} = 32.72\,\text{W}$

[f] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54\,\text{ms}$.

[g] $i_{\max} = 8[e^{-0.3(1.54)} - e^{-1.2(1.54)}] = 3.78\,\text{A}$

$$w_{\max} = (1/2)(4 \times 10^{-3})(3.78)^2 = 28.6\,\text{mJ}$$

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AP 6.2 **[a]** $i = C\dfrac{dv}{dt} = 24 \times 10^{-6}\dfrac{d}{dt}[e^{-15{,}000t}\sin 30{,}000t]$

$$= [0.72\cos 30{,}000t - 0.36\sin 30{,}000t]e^{-15{,}000t}\,\text{A}, \qquad i(0^+) = 0.72\,\text{A}$$

[b] $i\left(\dfrac{\pi}{80}\,\text{ms}\right) = -31.66\,\text{mA}, \quad v\left(\dfrac{\pi}{80}\,\text{ms}\right) = 20.505\,\text{V},$

$$p = vi = -649.23\,\text{mW}$$

[c] $w = \left(\dfrac{1}{2}\right)Cv^2 = 126.13\,\mu\text{J}$

AP 6.3 **[a]** $v = \left(\dfrac{1}{C}\right)\displaystyle\int_{0^-}^{t} i\,dx + v(0^-)$

$$= \frac{1}{0.6 \times 10^{-6}}\int_{0^-}^{t} 3\cos 50{,}000x\,dx = 100\sin 50{,}000t\,\text{V}$$

[b] $p(t) = vi = [300\cos 50{,}000t]\sin 50{,}000t$

$$= 150\sin 100{,}000t\,\text{W}, \qquad p_{(\max)} = 150\,\text{W}$$

[c] $w_{(\max)} = \left(\dfrac{1}{2}\right)Cv_{\max}^2 = 0.30(100)^2 = 3000\,\mu\text{J} = 3\,\text{mJ}$

AP 6.4 **[a]** $L_{\text{eq}} = \dfrac{60(240)}{300} = 48\,\text{mH}$

[b] $i(0^+) = 3 + -5 = -2\,\text{A}$

[c] $i = \dfrac{125}{6}\displaystyle\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 2 = 0.125e^{-5t} - 2.125\,\text{A}$

[d] $i_1 = \dfrac{50}{3}\displaystyle\int_{0^+}^{t}(-0.03e^{-5x})\,dx + 3 = 0.1e^{-5t} + 2.9\,\text{A}$

$$i_2 = \frac{25}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 5 = 0.025e^{-5t} - 5.025\,\text{A}$$

$$i_1 + i_2 = i$$

AP 6.5 $v_1 = 0.5 \times 10^6\displaystyle\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 10 = -12e^{-10t} + 2\,\text{V}$

$$v_2 = 0.125 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 5 = -3e^{-10t} - 2\,\text{V}$$

$$v_1(\infty) = 2\,\text{V}, \qquad v_2(\infty) = -2\,\text{V}$$

$$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right] \times 10^{-6} = 20\,\mu\text{J}$$

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AP 6.6 **[a]** Summing the voltages around mesh 1 yields

$$4\frac{di_1}{dt} + 8\frac{d(i_2 + i_g)}{dt} + 20(i_1 - i_2) + 5(i_1 + i_g) = 0$$

or

$$4\frac{di_1}{dt} + 25i_1 + 8\frac{di_2}{dt} - 20i_2 = -\left(5i_g + 8\frac{di_g}{dt}\right)$$

Summing the voltages around mesh 2 yields

$$16\frac{d(i_2 + i_g)}{dt} + 8\frac{di_1}{dt} + 20(i_2 - i_1) + 780i_2 = 0$$

or

$$8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 800i_2 = -16\frac{di_g}{dt}$$

[b] From the solutions given in part (b)

$$i_1(0) = -0.4 - 11.6 + 12 = 0; \qquad i_2(0) = -0.01 - 0.99 + 1 = 0$$

These values agree with zero initial energy in the circuit. At infinity,

$$i_1(\infty) = -0.4\text{A}; \qquad i_2(\infty) = -0.01\text{A}$$

When $t = \infty$ the circuit reduces to

$$\therefore \quad i_1(\infty) = -\left(\frac{7.8}{20} + \frac{7.8}{780}\right) = -0.4\text{A}; \quad i_2(\infty) = -\frac{7.8}{780} = -0.01\text{A}$$

From the solutions for i_1 and i_2 we have

$$\frac{di_1}{dt} = 46.40e^{-4t} - 60e^{-5t}$$

$$\frac{di_2}{dt} = 3.96e^{-4t} - 5e^{-5t}$$

Also, $\dfrac{di_g}{dt} = 7.84e^{-4t}$

Thus

$$4\frac{di_1}{dt} = 185.60e^{-4t} - 240e^{-5t}$$

$$25i_1 = -10 - 290e^{-4t} + 300e^{-5t}$$

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$$8\frac{di_2}{dt} = 31.68e^{-4t} - 40e^{-5t}$$

$$20i_2 = -0.20 - 19.80e^{-4t} + 20e^{-5t}$$

$$5i_g = 9.8 - 9.8e^{-4t}$$

$$8\frac{di_g}{dt} = 62.72e^{-4t}$$

Test:

$$185.60e^{-4t} - 240e^{-5t} - 10 - 290e^{-4t} + 300e^{-5t} + 31.68e^{-4t} - 40e^{-5t}$$

$$+0.20 + 19.80e^{-4t} - 20e^{-5t} \stackrel{?}{=} -[9.8 - 9.8e^{-4t} + 62.72e^{-4t}]$$

$$-9.8 + (300 - 240 - 40 - 20)e^{-5t}$$

$$+(185.60 - 290 + 31.68 + 19.80)e^{-4t} \stackrel{?}{=} -(9.8 + 52.92e^{-4t})$$

$$-9.8 + 0e^{-5t} + (237.08 - 290)e^{-4t} \stackrel{?}{=} -9.8 - 52.92e^{-4t}$$

$$-9.8 - 52.92e^{-4t} = -9.8 - 52.92e^{-4t} \quad \text{(OK)}$$

Also,

$$8\frac{di_1}{dt} = 371.20e^{-4t} - 480e^{-5t}$$

$$20i_1 = -8 - 232e^{-4t} + 240e^{-5t}$$

$$16\frac{di_2}{dt} = 63.36e^{-4t} - 80e^{-5t}$$

$$800i_2 = -8 - 792e^{-4t} + 800e^{-5t}$$

$$16\frac{di_g}{dt} = 125.44e^{-4t}$$

Test:

$$371.20e^{-4t} - 480e^{-5t} + 8 + 232e^{-4t} - 240e^{-5t} + 63.36e^{-4t} - 80e^{-5t}$$

$$-8 - 792e^{-4t} + 800e^{-5t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(8 - 8) + (800 - 480 - 240 - 80)e^{-5t}$$

$$+(371.20 + 232 + 63.36 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(800 - 800)e^{-5t} + (666.56 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$-125.44e^{-4t} = -125.44e^{-4t} \quad \text{(OK)}$$

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Problems

P 6.1 **[a]** $v = L\dfrac{di}{dt}$

$$= (150 \times 10^{-6})(25)[e^{-500t} - 500te^{-500t}] = 3.75e^{-500t}(1 - 500t)\,\text{mV}$$

[b] $i(5\,\text{ms}) = 25(0.005)(e^{-2.5}) = 10.26\,\text{mA}$

$$v(5\,\text{ms}) = 0.00375(e^{-2.5})(1 - 2.5) = -461.73\,\mu\text{V}$$

$$p(5\,\text{ms}) = vi = (10.26 \times 10^{-3})(-461.73 \times 10^{-6}) = -4.74\,\mu\text{W}$$

[c] delivering $4.74\,\mu\text{W}$

[d] $i(5\,\text{ms}) = 10.26\,\text{mA}$ (from part [b])

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(150 \times 10^{-6})(0.01026)^2 = 7.9\,\text{nJ}$$

[e] The energy is a maximum where the current is a maximum:

$$\frac{di_L}{dt} = 0 \quad \text{when} \quad 1 - 500t = 0 \quad \text{or} \quad t = 2\,\text{ms}$$

$$i_{\max} = 25(0.002)e^{-1} = 18.39\,\text{mA}$$

$$w_{\max} = \frac{1}{2}(150 \times 10^{-6})(0.01839)^2 = 25.38\,\text{nJ}$$

P 6.2 **[a]**

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 4t\,\text{A} && 0 \le t \le 25\,\text{ms} \\ i &= 0.2 - 4t\,\text{A} && 25 \le t \le 50\,\text{ms} \\ i &= 0 && 50\,\text{ms} < t \end{aligned}$$

[b] $v = L\dfrac{di}{dt} = 500 \times 10^{-3}(4) = 2\,\text{V} \qquad 0 \le t \le 25\,\text{ms}$

$$v = 500 \times 10^{-3}(-4) = -2\,\text{V} \qquad 25 \le t \le 50\,\text{ms}$$

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 2\,\text{V} && 0 < t < 25\,\text{ms} \\ v &= -2\,\text{V} && 25 < t < 50\,\text{ms} \\ v &= 0 && 50\,\text{ms} < t \end{aligned}$$

$$p = vi$$

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$$
\begin{aligned}
p &= 0 && t < 0 \\
p &= (4t)(2) = 8t\,\mathrm{W} && 0 < t < 25\,\mathrm{ms} \\
p &= (0.2 - 4t)(-2) = 8t - 0.4\,\mathrm{W} && 25 < t < 50\,\mathrm{ms} \\
p &= 0 && 50\,\mathrm{ms} < t \\
w &= 0 && t < 0 \\
w &= \int_0^t (8x)\,dx = 8\frac{x^2}{2}\Big|_0^t = 4t^2\,\mathrm{J} && 0 < t < 25\,\mathrm{ms} \\
w &= \int_{0.025}^t (8x - 0.4)\,dx + 2.5 \times 10^{-3} \\
&= 4x^2 - 0.4x\Big|_{0.025}^t + 2.5 \times 10^{-3} \\
&= 4t^2 - 0.4t + 10 \times 10^{-3}\,\mathrm{J} && 25 < t < 50\,\mathrm{ms} \\
w &= 0 && 10\,\mathrm{ms} < t
\end{aligned}
$$

P 6.3 **[a]** $i(0) = A_1 + A_2 = 0.12$

$$\frac{di}{dt} = -500A_1e^{-500t} - 2000A_2e^{-2000t}$$

$$v = -25A_1e^{-500t} - 100A_2e^{-2000t}\,\mathrm{V}$$

$$v(0) = -25A_1 - 100A_2 = 3$$

Solving, $A_1 = 0.2$ and $A_2 = -0.08$

Thus,

$$i = 200e^{-500t} - 80e^{-2000t}\,\mathrm{mA} \qquad t \geq 0$$

$$v = -5e^{-500t} + 8e^{-2000t}\,\mathrm{V}, \qquad t \geq 0$$

[b] $i = 0$ when $200e^{-500t} = 80e^{-2000t}$

Therefore

$e^{1500t} = 0.4$ so $t = -610.86\,\mu\mathrm{s}$ which is not possible!

$v = 0$ when $5e^{-500t} = 8e^{-2000t}$

Therefore

$e^{1500t} = 1.6$ so $t = 313.34\,\mu\mathrm{s}$

Thus the power is zero at $t = 313.34\,\mu\mathrm{s}$.

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P 6.4 **[a]** From Problem 6.3 we have

$$i = A_1e^{-500t} + A_2e^{-2000t}\,\text{A}$$

$$v = -25A_1e^{-500t} - 100A_2e^{-2000t}\,\text{V}$$

$$i(0) = A_1 + A_2 = 0.12$$

$$v(0) = -25A_1 - 100A_2 = -18$$

Solving, $\quad A_1 = -0.08; \quad A_2 = 0.2$
Thus,

$$i = -80e^{-500t} + 200e^{-2000t}\,\text{mA} \quad t \geq 0$$

$$v = 2e^{-500t} - 20e^{-2000t}\,\text{V} \quad t \geq 0$$

[b] $i = 0$ when $80e^{-500t} = 200e^{-2000t}$

$$\therefore \quad e^{1500t} = 2.5 \quad \text{so} \quad t = 610.86\,\mu\text{s}$$

Thus,

$$i > 0 \quad \text{for} \quad 0 \leq t < 610.86\,\mu\text{s} \qquad \text{and} \qquad i < 0 \quad \text{for} \quad 610.86\,\mu\text{s} < t < \infty$$

$$v = 0 \quad \text{when} \quad 2e^{-500t} = 20e^{-2000t}$$

$$\therefore \quad e^{1500t} = 10 \quad \text{so} \quad t = 1535.06\,\mu\text{s}$$

Thus,

$$v < 0 \quad \text{for} \quad 0 \leq t < 1535.06\,\mu\text{s} \qquad \text{and} \qquad v > 0 \quad \text{for} \quad 1535.06\,\mu\text{s} < t < \infty$$

Therefore,

$$p < 0 \quad \text{for} \quad 0 \leq t < 610.86\,\mu\text{s} \qquad \text{and} \qquad 1535.06\,\mu\text{s} < t < \infty$$

(inductor delivers energy)

$$p > 0 \quad \text{for} \quad 610.86\,\mu\text{s} < t < 1535.06\,\mu\text{s} \quad \text{(inductor stores energy)}$$

[c] The energy stored at $t = 0$ is

$$w(0) = \frac{1}{2}L[i(0)]^2 = \frac{1}{2}(0.05)(0.12)^2 = 360\,\mu\text{J}$$

$$p = vi = -0.16e^{-1000t} + 2e^{-2500t} - 4e^{-4000t}\,\text{W}$$

For $t > 0$:

$$w = \int_0^\infty -0.16e^{-1000t}\,dt + \int_0^\infty 2e^{-2500t}\,dt - \int_0^\infty 4e^{-4000t}\,dt$$

$$= \frac{-0.16e^{-1000t}}{-1000}\Bigg|_0^\infty + \frac{2e^{-2500t}}{-2500}\Bigg|_0^\infty - \frac{4e^{-4000t}}{-4000}\Bigg|_0^\infty$$

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$$= (-1.6 + 8 - 10) \times 10^{-4}$$

$$= -360\,\mu\text{J}$$

Thus, the energy stored equals the energy extracted.

P 6.5 $i = (B_1 \cos 200t + B_2 \sin 200t)e^{-50t}$

$$i(0) = B_1 = 75\,\text{mA}$$

$$\frac{di}{dt} = (B_1 \cos 200t + B_2 \sin 200t)(-50e^{-50t}) + e^{-50t}(-200B_1 \sin 200t + 200B_2 \cos 200t)$$

$$= [(200B_2 - 50B_1)\cos 200t - (200B_1 + 50B_2)\sin 200t]e^{-50t}$$

$$v = 0.2\frac{di}{dt} = [(40B_2 - 10B_1)\cos 200t - (40B_1 + 10B_2)\sin 200t]e^{-50t}$$

$$v(0) = 4.25 = 40B_2 - 10B_1 = 40B_2 - 0.75 \quad \therefore \quad B_2 = 125\,\text{mA}$$

Thus,

$$i = (75\cos 200t + 125\sin 200t)e^{-50t}\,\text{mA}, \qquad t \geq 0$$

$$v = (4.25\cos 200t - 4.25\sin 200t)e^{-50t}\,\text{V}, \qquad t \geq 0$$

$$i(0.025) = -28.25\,\text{mA}; \qquad v(0.025) = 1.513\,\text{V}$$

$$p(0.025) = (-28.25)(1.513) = -42.7\,\text{mW delivering}$$

P 6.6 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

$$\text{W} = \int_0^\infty p\,dx = \int_0^\infty 40x[e^{-10x} - 10xe^{-20x} - e^{-20x}]\,dx = 0.2\,\text{J}$$

This is energy stored in the inductor at $t = \infty$.

P 6.7 **[a]** $0 \leq t \leq 50\,\text{ms}$:

$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{10^6}{750}\int_0^t 0.15\,dx + 0$$

$$= 200x\Big|_0^t = 200t\,\text{A}$$

$$i(0.05) = 200(0.05) = 10\,\text{A}$$

$$t \geq 50\,\text{ms}: \qquad i = \frac{10^6}{750}\int_{50\times10^{-3}}^t (0)\,dx + 10 = 10\,\text{A}$$

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[b] $i = 200t\,\text{A}, \quad 0 \le t \le 50\,\text{ms}; \qquad i = 10\,\text{A}, \quad t \ge 50\,\text{ms}$

P 6.8 $\quad 0 \le t \le 100\,\text{ms}:$

$$i_L = \frac{10^3}{50}\int_0^t 2e^{-100x}\,dx + 0.1 = 40\frac{e^{-100x}}{-100}\Big|_0^t + 0.1$$

$$= -0.4e^{-100t} + 0.5\,\text{A}$$

$$i_L(0.1) = -0.4e^{-10} + 0.5 = 0.5\,\text{A}$$

$t \ge 100\,\text{ms}:$

$$i_L = \frac{10^3}{50}\int_{0.1}^t -2e^{-100(x-0.1)}\,dx + 0.5 = -40\frac{e^{-100(x-0.1)}}{-100}\Big|_{0.1}^t + 0.5$$

$$= 0.4e^{-100(t-0.1)} + 0.1\,\text{A}$$

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P 6.9 **[a]** $0 \le t \le 25\,\text{ms}$:

$v = 800t$

$i = \dfrac{1}{10}\displaystyle\int_0^t 800x\,dx + 0 = 80\frac{x^2}{2}\Big|_0^t$

$i = 40t^2\ \text{A}$

$25\,\text{ms} \le t \le 75\,\text{ms}$:

$v = 20$

$i(0.025) = 25\,\text{mA}$

$$\begin{aligned} \therefore\quad i &= \frac{1}{10}\int_{0.025}^t 20\,dx + 0.025 \\ &= 2x\Big|_{0.025}^t + 0.025 \\ &= 2t - 0.025\,\text{A} \end{aligned}$$

$75\,\text{ms} \le t \le 125\,\text{ms}$:

$v = 80 - 800t\,\text{V}$

$i(0.075) = 2(0.075) - 0.025 = 0.125\,\text{A}$

$$\begin{aligned} i &= \frac{1}{10}\int_{0.075}^t (80 - 800x)\,dx + 0.125 \\ &= \left(8x - \frac{80x^2}{2}\right)\Big|_{0.075}^t + 0.125 \\ &= 8t - 40t^2 - 0.25\,\text{A} \end{aligned}$$

$125\,\text{ms} \le t \le 150\,\text{ms}$:

$v = 800t - 120$

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$$i(0.125) = 8(0.125) - 40(0.125)^2 - 0.25 = 0.125\,\text{A}$$

$$\begin{aligned} i &= \frac{1}{10}\int_{0.125}^{t}(800x - 120)\,dx + 0.125 \\ &= \left(\frac{80x^2}{2} - 12x\right)\Big|_{0.125}^{t} + 0.125 \\ &= 40t^2 - 12t + 1\,\text{A} \end{aligned}$$

$t \geq 150\,\text{ms}$:

$v = 0$

$$i(0.150) = 40(0.15)^2 - 12(0.15) + 1 = 0.1\,\text{A}$$

$$\begin{aligned} i &= \frac{1}{10}\int_{0.15}^{t} 0\,dx + 0.1 \\ &= 0.1\,\text{A} \end{aligned}$$

[b] $v = 0$ at $t = 100\,\text{ms}$ and $t = 150\,\text{ms}$

$$i(0.1) = 8(0.1) - 40(0.1)^2 - 0.25 = 0.15\,\text{A}$$

$$i(0.15) = 0.1\,\text{A}$$

[c]

P 6.10 **[a]**

$$\begin{aligned} i &= \frac{1}{0.1}\int_{0}^{t} 20\cos 80x\,dx \\ &= 200\frac{\sin 80x}{80}\Big|_{0}^{t} \\ &= 2.5\sin 80t\,\text{A} \end{aligned}$$

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[b]

$$\begin{aligned} p &= vi = (20\cos 80t)(2.5\sin 80t) \\ &= 50\cos 80t \sin 80t \\ p &= 25\sin 160t \text{ W} \\ w &= \frac{1}{2}Li^2 \\ &= \frac{1}{2}(0.1)(2.5\sin 80t)^2 \\ &= 312.5\sin^2 80t \text{ mJ} \\ w &= (156.25 - 156.25\cos 160t) \text{ mJ} \end{aligned}$$

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[c] Absorbing power: $0 \le t \le \pi/160\,\text{s}$, $\pi/80 \le t \le 3\pi/160\,\text{s}$

Delivering power: $\pi/160 \le t \le \pi/80\,\text{s}$, $3\pi/160 \le t \le \pi/40\,\text{s}$

P 6.11 **[a]** $v = L\dfrac{di}{dt}$

$$v = -25 \times 10^{-3}\frac{d}{dt}[10\cos 400t + 5\sin 400t]e^{-200t}$$

$$= -25 \times 10^{-3}(-200e^{-200t}[10\cos 400t + 5\sin 400t]$$

$$+e^{-200t}[-4000\sin 400t + 2000\cos 400t])$$

$$v = -25 \times 10^{-3}e^{-200t}(-1000\sin 400t - 4000\sin 400t)$$

$$= -25 \times 10^{-3}e^{-200t}(-5000\sin 400t)$$

$$= 125e^{-200t}\sin 400t\ \text{V}$$

$$\frac{dv}{dt} = 125(e^{-200t}(400)\cos 400t - 200e^{-200t}\sin 400t)$$

$$= 25{,}000e^{-200t}(2\cos 400t - \sin 400t)\ \text{V/s}$$

$$\frac{dv}{dt} = 0 \qquad \text{when} \qquad 2\cos 400t = \sin 400t$$

$$\therefore\ \tan 400t = 2, \qquad 400t = 1.11; \qquad t = 2.77\,\text{ms}$$

[b] $v(2.77\ \text{ms}) = 125e^{-0.55}\sin 1.11 = 64.27\,\text{V}$

P 6.12 For $0 \le t \le 1.6\,\text{s}$:

$$i_L = \frac{1}{5}\int_0^t 3 \times 10^{-3}\,dx + 0 = 0.6 \times 10^{-3}t$$

$$i_L(1.6\,\text{s}) = (0.6 \times 10^{-3})(1.6) = 0.96\,\text{mA}$$

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$R_m = (20)(1000) = 20\,\text{k}\Omega$

$v_m(1.6\,\text{s}) = (0.96 \times 10^{-3})(20 \times 10^3) = 19.2\,\text{V}$

P 6.13 **[a]** $i = C\dfrac{dv}{dt} = (5 \times 10^{-6})[500t(-2500)e^{-2500t} + 500e^{-2500t}]$

$$= 2.5 \times 10^{-3}e^{-2500t}(1 - 2500t)\,\text{A}$$

[b] $v(100\,\mu) = 500(100 \times 10^{-6})e^{-0.25} = 38.94\,\text{mV}$

$i(100\,\mu) = (2.5 \times 10^{-3})e^{-0.25}(1 - 0.25) = 1.46\,\text{mA}$

$p(100\,\mu) = vi = (38.94 \times 10^{-3})(1.46 \times 10^{-3}) = 56.86\,\mu\text{W}$

[c] $p > 0$, so the capacitor is absorbing power.

[d] $v(100\,\mu) = 38.94\,\text{mV}$

$w = \dfrac{1}{2}Cv^2 = \dfrac{1}{2}(5 \times 10^{-6})(38.94 \times 10^{-3})^2 = 3.79\,\text{nJ}$

[e] The energy is maximum when the voltage is maximum:

$\dfrac{dv}{dt} = 0$ when $(1 - 2500t) = 0$ or $t = 0.4\,\text{ms}$

$v_{\max} = 500(0.4 \times 10^{-3})^2e^{-1} = 73.58\,\text{mV}$

$p_{\max} = \dfrac{1}{2}Cv_{\max}^2 = 13.53\,\text{nJ}$

P 6.14 **[a]**

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 10t\,\text{A} && 0 \le t \le 2\,\text{s} \\ v &= 40 - 10t\,\text{A} && 2 \le t \le 6\,\text{s} \\ v &= 10t - 80\,\text{A} && 6 \le t \le 8\,\text{s} \\ v &= 0 && 8\,\text{s} < t \end{aligned}$$

[b] $i = C\dfrac{dv}{dt}$

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 2\,\text{mA} && 0 < t < 2\,\text{s} \\ i &= -2\,\text{mA} && 2 < t < 6\,\text{s} \\ i &= 2\,\text{mA} && 6 < t < 8\,\text{s} \\ i &= 0 && 8\,\text{s} < t \end{aligned}$$

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$p = vi$

$$
\begin{aligned}
p &= 0 && t < 0 \\
p &= (10t)(0.002) = 0.02t\,\mathrm{W} && 0 < t < 2\,\mathrm{s} \\
p &= (40 - 10t)(-0.002) = 0.02t - 0.08\,\mathrm{W} && 2 < t < 6\,\mathrm{s} \\
p &= (10t - 80)(0.002) = 0.02t - 0.16\,\mathrm{W} && 6 < t < 8\,\mathrm{s} \\
p &= 0 && 8\,\mathrm{s} < t
\end{aligned}
$$

$w = \int p\,dx$

$$
\begin{aligned}
w &= 0 && t < 0 \\
w &= \int_0^t (0.02x)\,dx = 0.01x^2 \Big|_0^t = 0.01t^2\,\mathrm{J} && 0 < t < 2\,\mathrm{s} \\
w &= \int_2^t (0.02x - 0.08)\,dx + 0.04 \\
&= (0.01x^2 - 0.08x)\Big|_2^t + 0.04 \\
&= 0.01t^2 - 0.08t + 0.16\,\mathrm{J} && 2 < t < 6\,\mathrm{s} \\
w &= \int_6^t (0.02x - 0.16)\,dx + \ 0.04 \\
&= (0.01x^2 - 0.16x)\Big|_6^t + 0.04 \\
&= 0.01t^2 - 0.16t + 0.64\,\mathrm{J} && 6 < t < 8\,\mathrm{s} \\
w &= 0 && 8\,\mathrm{s} < t
\end{aligned}
$$

[c]

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From the plot of power above, it is clear that power is being absorbed for $0 < t < 2$ s and for $4\text{ s} < t < 6$ s, because $p > 0$. Likewise, power is being delivered for $2\text{ s} < t < 4$ s and $6\text{ s} < t < 8$ s, because $p < 0$.

P 6.15 **[a]** $w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(5 \times 10^{-6})(60)^2 = 9\,\text{mJ}$

[b] $v = (A_1 + A_2 t)e^{-1500t}$

$$v(0) = A_1 = 60\,\text{V}$$

$$\begin{aligned}\frac{dv}{dt} &= -1500e^{-1500t}(A_1 + A_2 t) + e^{-1500t}(A_2) \\ &= (-1500A_2 t - 1500A_1 + A_2)e^{-1500t}\end{aligned}$$

$$\frac{dv(0)}{dt} = A_2 - 1500A_1$$

$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore \quad \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{100 \times 10^{-3}}{5 \times 10^{-6}} = 20 \times 10^3$$

$$\therefore \quad 20 \times 10^3 = A_2 - 1500(60)$$

Thus, $A_2 = 20 \times 10^3 + 90 \times 10^3 = 110 \times 10^3\,\frac{\text{V}}{\text{s}}$

[c] $v = (60 + 110 \times 10^3 t)e^{-1500t}$

$$i = C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(60 + 110 \times 10^3 t)e^{-1500t}$$

$$\begin{aligned} i &= (5 \times 10^{-6})[110{,}000e^{-1500t} - 1500(60 + 110{,}000t)e^{-1500t}] \\ &= (0.1 - 825t)e^{-1500t}\,\text{A}, \qquad t \geq 0\end{aligned}$$

P 6.16 $i_C = C(dv/dt)$

$$0 < t < 2\,\text{s}: \qquad i_C = 100 \times 10^{-9}(15)t^2 = 1.5 \times 10^{-6}t^2\,\text{A}$$

$$2 < t < 4\,\text{s}: \qquad i_C = 100 \times 10^{-9}(-15)(t-4)^2 = -1.5 \times 10^{-6}(t-4)^2\,\text{A}$$

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P 6.17 **[a]** $i = C\dfrac{dv}{dt} = 0, \quad t < 0$

[b] $i = C\dfrac{dv}{dt} = 120 \times 10^{-6}\dfrac{d}{dt}[30 + 5e^{-500t}(6\cos 2000t + \sin 2000t)]$

$$= 120 \times 10^{-6}[5(-500)e^{-500t}(6\cos 2000t + \sin 2000t)$$

$$+5(2000)e^{-500t}(-6\sin 2000t + \cos 2000t)]$$

$$= -0.6e^{-500t}[\cos 2000t + 12.5\sin 2000t]\ \text{A}, \quad t \geq 0$$

[c] no, $v(0^-) = 60$ V
$v(0^+) = 30 + 5(6) = 60$ V

[d] yes, $i(0^-) = 0$ A
$i(0^+) = -0.6$ A

[e] $v(\infty) = 30\,\text{V}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(120 \times 10^{-6})(30)^2 = 54\,\text{mJ}$$

P 6.18 **[a]**

$$\begin{aligned} v(20\,\mu\text{s}) &= 12.5 \times 10^9(20 \times 10^{-6})^2 = 5\,\text{V} \quad \text{(end of first interval)} \\ v(20\,\mu\text{s}) &= 10^6(20 \times 10^{-6}) - (12.5)(400) \times 10^{-3} - 10 \\ &= 5\,\text{V (start of second interval)} \\ v(40\,\mu\text{s}) &= 10^6(40 \times 10^{-6}) - (12.5)(1600) \times 10^{-3} - 10 \\ &= 10\,\text{V (end of second interval)} \end{aligned}$$

[b] $p(10\mu\text{s}) = 62.5 \times 10^{12}(10^{-5})^3 = 62.5\,\text{mW}, \qquad v(10\,\mu\text{s}) = 1.25\,\text{V},$

$i(10\mu\text{s}) = 50\,\text{mA}, \qquad p(10\,\mu\text{s}) = vi = (1.25)(50\,\text{m}) = 62.5\,\text{mW}$ (checks)

$p(30\,\mu\text{s}) = 437.50\,\text{mW}, \qquad v(30\,\mu\text{s}) = 8.75\,\text{V}, \qquad i(30\,\mu\text{s}) = 0.05\,\text{A}$

$p(30\,\mu\text{s}) = vi = (8.75)(0.05) = 62.5\,\text{mW}$ (checks)

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[c] $w(10\,\mu\text{s}) = 15.625 \times 10^{12}(10 \times 10^{-6})^4 = 0.15625\,\mu\text{J}$

$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 7.65625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625\,\mu\text{J}$

P 6.19 **[a]**

$$\begin{aligned} v &= \frac{1}{0.5 \times 10^{-6}} \int_0^{500\times10^{-6}} 50 \times 10^{-3} e^{-2000t}\,dt - 20 \\ &= 100 \times 10^3 \frac{e^{-2000t}}{-2000}\Big|_0^{500\times10^{-6}} - 20 \\ &= 50(1 - e^{-1}) - 20 = 11.61\,\text{V} \\ w &= \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.5)(10^{-6})(11.61)^2 = 33.7\,\mu\text{J} \end{aligned}$$

[b] $v(\infty) = 50 - 20 = 30\text{V}$

$$w(\infty) = \frac{1}{2}(0.5 \times 10^{-6})(30)^2 = 225\,\mu\text{J}$$

P 6.20 **[a]** $i = \dfrac{5}{2 \times 10^{-3}} t = 2500t \qquad 0 \le t \le 2\,\text{ms}$

$$i = \frac{-10}{4 \times 10^{-3}} t + 10 = 10 - 2500t \qquad 2 \le t \le 6\,\text{ms}$$

$$i = \frac{10}{4 \times 10^{-3}} t - 20 = 2500t - 20 \qquad 6 \le t \le 10\,\text{ms}$$

$$i = \frac{-5}{2 \times 10^{-3}} t + 30 = 30 - 2500t \qquad 10 \le t \le 12\,\text{ms}$$

$$\begin{aligned} q &= \int_0^{0.002} 2500t\,dt + \int_{0.002}^{0.006} (10 - 2500t)\,dt \\ &= \frac{2500t^2}{2}\Big|_0^{0.002} + \left(10t - \frac{2500t^2}{2}\right)\Big|_{0.002}^{0.006} \\ &= 0.005 - 0 + (0.06 - 0.045) - (0.02 - 0.005) \\ &= 5\,\text{mC} \end{aligned}$$

[b] $v = 0.5 \times 10^6 \displaystyle\int_0^{0.002} 2500x\,dx + 0.5 \times 10^6 \int_{0.002}^{0.006} (10 - 2500x)\,dx$

$$+ 0.5 \times 10^6 \int_{0.006}^{0.01} (2500x - 20)\,dx$$

$$= 0.5 \times 10^6 \left[\frac{2500x^2}{2}\Big|_0^{0.002} + 10x\Big|_{0.002}^{0.006} - \frac{2500x^2}{2}\Big|_{0.002}^{0.006} + \frac{2500x^2}{2}\Big|_{0.006}^{0.01} - 20x\Big|_{0.006}^{0.01} \right]$$

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$$= 0.5 \times 10^6[(0.005 - 0) + (0.06 - 0.02) - (0.045 - 0.005)$$

$$+ (0.125 - 0.045) - (0.2 - 0.12)]$$

$$= 2500\,\text{V}$$

$$v(10\,\text{ms}) = 2500\,\text{V}$$

[c] $v(12\,\text{ms}) = v(10\,\text{ms}) + 0.5 \times 10^6 \int_{0.01}^{0.012} (30 - 2500x)\,dx$

$$= 2500 + 0.5 \times 10^6 \left(30x - \frac{2500x^2}{2}\right)\Big|_{0.01}^{0.012}$$

$$= 2500 + 0.5 \times 10^6(0.36 - 0.18 - 0.3 + 0.125)$$

$$= 2500 + 2500 = 5000\,\text{V}$$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(2 \times 10^{-6})(5000)^2 = 25\,\text{J}$$

P 6.21 **[a]** $0 \le t \le 10\,\mu\text{s}$

$$C = 0.1\,\mu\text{F} \qquad \frac{1}{C} = 10 \times 10^6$$

$$v = 10 \times 10^6 \int_0^t -0.05\,dx + 15$$

$$v = -50 \times 10^4 t + 15\,\text{V} \qquad 0 \le t \le 10\,\mu\text{s}$$

$$v(10\,\mu\text{s}) = -5 + 15 = 10\,\text{V}$$

[b] $10\,\mu\text{s} \le t \le 20\,\mu\text{s}$

$$v = 10 \times 10^6 \int_{10\times10^{-6}}^t 0.1\,dx + 10 = 10^6 t - 10 + 10$$

$$v = 10^6 t\,\text{V} \qquad 10 \le t \le 20\,\mu\text{s}$$

$$v(20\,\mu\text{s}) = 10^6(20 \times 10^{-6}) = 20\,\text{V}$$

[c] $20\,\mu\text{s} \le t \le 40\,\mu\text{s}$

$$v = 10 \times 10^6 \int_{20\times10^{-6}}^t 1.6\,dx + 20 = 1.6 \times 10^6 t - 32 + 20$$

$$v = 1.6 \times 10^6 t - 12\,\text{V}, \qquad 20\,\mu\text{s} \le t \le 40\,\mu\text{s}$$

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[d] $40\,\mu\text{s} \le t < \infty$

$v(40\,\mu\text{s}) = 64 - 12 = 52\,\text{V} \qquad 40\,\mu\text{s} \le t < \infty$

P 6.22 **[a]** $15\|30 = 10\,\text{mH}$

$10 + 10 = 20\,\text{mH}$

$20\|20 = 10\,\text{mH}$

$12\|24 = 8\,\text{mH}$

$10 + 8 = 18\,\text{mH}$

$18\|9 = 6\,\text{mH}$

$L_{\text{ab}} = 6 + 8 = 14\,\text{mH}$

[b] $12 + 18 = 30\,\mu\text{H}$

$30\|20 = 12\,\mu\text{H}$

$12 + 38 = 50\,\mu\text{H}$

$30\|75\|50 = 15\,\mu\text{H}$

$15 + 15 = 30\,\mu\text{H}$

$30\|60 = 20\,\mu\text{H}$

$L_{\text{ab}} = 20 + 25 = 45\,\mu\text{H}$

P 6.23 **[a]** Combine two 10 mH inductors in parallel to get a 5 mH equivalent inductor. Then combine this parallel pair in series with three 1 mH inductors:

$10\,\text{m}\|10\,\text{m} + 1\,\text{m} + 1\,\text{m} + 1\,\text{m} = 8\,\text{mH}$

[b] Combine two $10\,\mu$H inductors in parallel to get a $5\,\mu$H inductor. Then combine this parallel pair in series with four more $10\,\mu$H inductors:

$10\,\mu\|10\,\mu + 10\,\mu + 10\,\mu + 10\,\mu + 10\,\mu = 45\,\mu\text{H}$

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[**c**] Combine two $100\,\mu$H inductors in parallel to get a $50\,\mu$H inductor. Then combine this parallel pair with a $100\,\mu$H inductor and three $10\,\mu$H inductors in series:

$$100\,\mu \| 100\,\mu + 100\,\mu + 10\,\mu + 10\,\mu + 10\,\mu = 180\,\mu\text{H}$$

P 6.24 [**a**]

$$3.2\frac{di}{dt} = 64e^{-4t} \qquad \text{so} \qquad \frac{di}{dt} = 20e^{-4t}$$

$$\begin{aligned} i(t) &= 20\int_0^t e^{-4x}\,dx - 5 \\ &= 20\frac{e^{-4x}}{-4}\Big|_0^t - 5 \\ i(t) &= -5e^{-4t}\,\text{A} \end{aligned}$$

[**b**] $4\dfrac{di_1}{dt} = 64e^{-4t}$

$$\begin{aligned} i_1(t) &= 16\int_0^t e^{-4x}\,dx - 10 \\ &= 16\frac{e^{-4x}}{-4}\Big|_0^t - 10 \\ i_1(t) &= -4e^{-4t} - 6\,\text{A} \end{aligned}$$

[**c**] $16\dfrac{di_2}{dt} = 64e^{-4t} \qquad \text{so} \qquad \dfrac{di_2}{dt} = 4e^{-4t}$

$$\begin{aligned} i_2(t) &= 4\int_0^t e^{-4x}\,dx + 5 \\ &= 4\frac{e^{-4x}}{-4}\Big|_0^t + 5 \\ i_2(t) &= -e^{-4t} + 6\,\text{A} \end{aligned}$$

[**d**] $p = -vi = (-64e^{-4t})(-5e^{-4t}) = 320e^{-8t}\,\text{W}$

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$$\begin{aligned} w &= \int_0^\infty p\,dt = \int_0^\infty 320e^{-8t}\,dt \\ &= 320\frac{e^{-8t}}{-8}\Big|_0^\infty \\ &= 40\,\text{J} \end{aligned}$$

[e] $w = \frac{1}{2}(4)(-10)^2 + \frac{1}{2}(16)(5)^2 = 400\,\text{J}$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 400 - 40 = 360\,\text{J}$

[g] $w_{\text{trapped}} = \frac{1}{2}(4)(-6)^2 + \frac{1}{2}(16)(6)^2 = 360\,\text{J}$ checks

P 6.25 **[a]** $i_o(0) = -i_1(0) - i_2(0) = 6 - 1 = 5\,\text{A}$

[b]

$$\begin{aligned} i_o &= -\frac{1}{4}\int_0^t 2000e^{-100x}\,dx + 5 = -500\,\frac{e^{-100x}}{-100}\Big|_0^t + 5 \\ &= 5(e^{-100t} - 1) + 5 = 5e^{-100t}\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[c]

$$v_a = 3.2(-500e^{-100t}) = -1600e^{-100t}\,\text{V}$$

$$\begin{aligned} v_c &= v_a + v_b = -1600e^{-100t} + 2000e^{-100t} \\ &= 400e^{-100t}\,\text{V} \\ i_1 &= \frac{1}{1}\int_0^t 400e^{-100x}\,dx - 6 \\ &= -4e^{-100t} + 4 - 6 \\ i_1 &= -4e^{-100t} - 2\,\text{A} \qquad t \geq 0 \end{aligned}$$

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[d] $i_2 = \frac{1}{4}\int_0^t 400e^{-100x}\,dx + 1$

$= -e^{-100t} + 2\,\text{A}, \qquad t \geq 0$

[e] $w(0) = \frac{1}{2}(1)(6)^2 + \frac{1}{2}(4)(1)^2 + \frac{1}{2}(3.2)(5)^2 = 60\,\text{J}$

[f] $w_{\text{del}} = \frac{1}{2}(4)(5)^2 = 50\,\text{J}$

[g] $w_{\text{trapped}} = 60 - 50 = 10\,\text{J}$

or $\qquad w_{\text{trapped}} = \frac{1}{2}(1)(2)^2 + \frac{1}{2}(4)(2)^2 + 10\,\text{J}$ (check)

P 6.26 $v_{\text{b}} = 2000e^{-100t}\,\text{V}$

$i_o = 5e^{-100t}\,\text{A}$

$p = 10{,}000e^{-200t}\,\text{W}$

$w = \int_0^t 10^4 e^{-200x}\,dx = 10{,}000\frac{e^{-200x}}{-200}\Big|_0^t = 50(1 - e^{-200t})\,\text{W}$

$w_{\text{total}} = 50\,\text{J}$

$80\%w_{\text{total}} = 40\,\text{J}$

Thus,

$50 - 50e^{-200t} = 40; \qquad e^{200t} = 5; \qquad \therefore \quad t = 8.05\,\text{ms}$

P 6.27 **[a]** $\frac{1}{C_1} = \frac{1}{48} + \frac{1}{24} = \frac{1}{16}; \qquad C_1 = 16\,\text{nF}$

$C_2 = 4 + 16 = 20\,\text{nF}$

$\frac{1}{C_3} = \frac{1}{20} + \frac{1}{30} = \frac{1}{12}; \qquad C_3 = 12\,\text{nF}$

$C_4 = 12 + 8 = 20\,\text{nF}$

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$$\frac{1}{C_5} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{1}{5}; \qquad C_5 = 5\,\text{nF}$$

Equivalent capacitance is 5 nF with an initial voltage drop of +15 V.

[b] $\frac{1}{36} + \frac{1}{18} + \frac{1}{12} = \frac{1}{6} \quad \therefore \quad C_{\text{eq}} = 6\,\mu\text{F} \qquad 24 + 6 = 30\,\mu\text{F}$

$25 + 5 = 30\,\mu\text{F}$

$$\frac{1}{30} + \frac{1}{30} + \frac{1}{30} = \frac{3}{30} \quad \therefore \quad C_{\text{eq}} = 10\,\mu\text{F}$$

Equivalent capacitance is $10\,\mu$F with an initial voltage drop of +25 V.

P 6.28 **[a]** Combine a 470 pF capacitor and a 10 pF capacitor in parallel to get a 480 pF capacitor:

(470 p) in parallel with (10 p) = 470 p + 10 p = 480 pF

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[b] Create a 1200 nF capacitor as follows:

$(1\,\mu)$ in parallel with $(0.1\,\mu)$ in parallel with $(0.1\,\mu)$

$$= 1000\,\text{n} + 100\,\text{n} + 100\,\text{n} = 1200\,\text{nF}$$

Create a second 1200 nF capacitor using the same three resistors. Place these two 1200 nF in series:

$$(1200\,\text{n}) \text{ in series with } (1200\,\text{n}) = \frac{(1200\,\text{n})(1200\,\text{n})}{1200\,\text{n} + 1200\,\text{n}} = 600\,\text{nF}$$

[a] Combine two $220\,\mu$F capacitors in series to get a $110\,\mu$F capacitor. Then combine the series pair in parallel with a $10\,\mu$F capacitor to get $120\,\mu$F:

$[(220\,\mu)$ in series with $(220\,\mu)]$ in parallel with $(10\,\mu)$

$$= \frac{(220\,\mu)(220\,\mu)}{220\,\mu + 220\,\mu} + 10\,\mu = 120\,\mu\text{F}$$

P 6.29 From Figure 6.17(a) we have

$$v = \frac{1}{C_1}\int_0^t i\,dx + v_1(0) + \frac{1}{C_2}\int_0^t i\,dx + v_2(0) + \cdots$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]\int_0^t i\,dx + v_1(0) + v_2(0) + \cdots$$

Therefore $\dfrac{1}{C_{\text{eq}}} = \left[\dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots\right], \qquad v_{\text{eq}}(0) = v_1(0) + v_2(0) + \cdots$

P 6.30 From Fig. 6.18(a)

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots = [C_1 + C_2 + \cdots]\frac{dv}{dt}$$

Therefore $C_{\text{eq}} = C_1 + C_2 + \cdots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on C_{eq}.

P 6.31 **[a]**

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$$\begin{aligned} v_o &= \frac{10^6}{1.6}\int_0^t 800\times 10^{-6}e^{-25x}\,dx - 20 \\ &= 500\frac{e^{-25x}}{-25}\Big|_0^t - 20 \\ &= -20e^{-25t}\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[b] $$\begin{aligned} v_1 &= \frac{10^6}{2}(800\times 10^{-6})\frac{e^{-25x}}{-25}\Big|_0^t + 5 \\ &= -16e^{-25t} + 21\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[c] $$\begin{aligned} v_2 &= \frac{10^6}{8}(800\times 10^{-6})\frac{e^{-25x}}{-25}\Big|_0^t - 25 \\ &= -4e^{-25t} - 21\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[d] $$\begin{aligned} p &= -vi = -(-20e^{-25t})(800\times 10^{-6})e^{-25t} \\ &= 16\times 10^{-3}e^{-50t} \\ w &= \int_0^\infty 16\times 10^{-3}e^{-50t}\,dt \\ &= 16\times 10^{-3}\frac{e^{-50t}}{-50}\Big|_0^\infty \\ &= -0.32\times 10^{-3}(0-1) = 320\,\mu\text{J} \end{aligned}$$

[e] $$\begin{aligned} w &= \tfrac{1}{2}(2\times 10^{-6})(5)^2 + \tfrac{1}{2}(8\times 10^{-6})(25)^2 \\ &= 2525\,\mu\text{J} \end{aligned}$$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 2525 - 320 = 2205\,\mu\text{J}$

[g] $$\begin{aligned} w_{\text{trapped}} &= \tfrac{1}{2}(2\times 10^{-6})(21)^2 + \tfrac{1}{2}(8\times 10^{-6})(-21)^2 \\ &= 2205\,\mu\text{J} \end{aligned}$$

P 6.32 $$\frac{1}{C_e} = \frac{1}{1} + \frac{1}{5} + \frac{1}{1.25} = \frac{10}{5} = 2$$

$\therefore \quad C_2 = 0.5\,\mu\text{F}$

$v_\text{b} = 20 - 250 + 30 = -200\,\text{V}$

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[a]

$$v_{\rm b} = -\frac{10^6}{0.5}\int_0^t -5\times10^{-3}e^{-50x}\,dx - 200$$

$$= 10{,}000\frac{e^{-50x}}{-50}\Big|_0^t - 200$$

$$= -200e^{-50t}\,\text{V}$$

[b] $$v_{\rm a} = -\frac{10^6}{0.5}\int_0^t -5\times10^{-3}e^{-50x}\,dx - 20$$

$$= 20(e^{-50t}-1) - 20$$

$$= 20e^{-50t} - 40\,\text{V}$$

[c] $$v_{\rm c} = \frac{10^6}{1.25}\int_0^t -5\times10^{-3}e^{-50x}\,dx - 30$$

$$= 80(e^{-50t}-1) - 30$$

$$= 80e^{-50t} - 110\,\text{V}$$

[d] $$v_{\rm d} = 10^6\int_0^t -5\times10^{-3}e^{-50x}\,dx + 250$$

$$= 100(e^{-50t}-1) + 250$$

$$= 100e^{-50t} + 150\,\text{V}$$

CHECK: $v_{\rm b} = -v_{\rm c} - v_{\rm d} - v_{\rm a}$

$= -200e^{-50t}\,\text{V}$ (checks)

[e] $$i_1 = 0.2\times10^{-6}\frac{d}{dt}\left[100e^{-50t}+150\right]$$

$$= 0.2\times10^{-6}(-5000e^{-50t}) = -e^{-50t}\,\text{mA}$$

[f] $$i_2 = 0.8\times10^{-6}\frac{d}{dt}\left[100e^{-50t}+150\right] = -4e^{-50t}\,\text{mA}$$

CHECK: $i_{\rm b} = i_1 + i_2 = -5e^{-50t}\,\text{mA}$ (OK)

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P 6.33 **[a]**

$$\begin{aligned} w(0) &= \tfrac{1}{2}(0.2\times10^{-6})(250)^2 + \tfrac{1}{2}(0.8\times10^{-6})(250)^2 + \tfrac{1}{2}(5\times10^{-6})(20)^2 \\ &\quad + \tfrac{1}{2}(1.25\times10^{-6})(30)^2 \\ &= 32{,}812.5\,\mu\text{J} \end{aligned}$$

[b]

$$\begin{aligned} w(\infty) &= \tfrac{1}{2}(5\times10^{-6})(40)^2 + \tfrac{1}{2}(1.25\times10^{-6})(110)^2 + \tfrac{1}{2}(0.2\times10^{-6})(150)^2 \\ &\quad + \tfrac{1}{2}(0.8\times10^{-6})(150)^2 \\ &= 22{,}812.5\,\mu\text{J} \end{aligned}$$

[c] $w = \dfrac{1}{2}(0.5\times10^{-6})(200)^2 = 10{,}000\,\mu\text{J}$

CHECK: $32{,}812.5 - 22{,}812.5 = 10{,}000\,\mu\text{J}$

[d] $\%\text{ delivered } = \dfrac{10{,}000}{32{,}812.5}\times100 = 30.48\%$

[e]

$$\begin{aligned} w &= \int_0^t (-0.005e^{-50x})(-200e^{-50x})\,dx = \int_0^t e^{-100x}\,dx \\ &= 10(1-e^{-100t})\,\text{mJ} \end{aligned}$$

$\therefore\ 10^{-2}(1-e^{-100t}) = 7.5\times10^{-3}; \qquad e^{-100t} = 0.25$

Thus, $t = \dfrac{\ln 4}{100} = 13.86\,\text{ms}.$

P 6.34

$$\begin{aligned} v_c &= -\frac{1}{10\times10^{-6}}\left(\int_0^t 0.2e^{-800x}\,dx - \int_0^t 0.04e^{-200x}\,dx\right) + 5 \\ &= 25(e^{-800t}-1) - 20(e^{-200t}-1) + 5 \\ &= 25e^{-800t} - 20e^{-200t}\ \text{V} \\ v_L &= 150\times10^{-3}\frac{di_o}{dt} \\ &= 150\times10^{-3}(-160e^{-800t} + 8e^{-200t}) \\ &= -24e^{-800t} + 1.2e^{-200t}\ \text{V} \\ v_o &= v_c - v_L \\ &= (25e^{-800t} - 20e^{-200t}) - (-24e^{-800t} + 1.2e^{-200t}) \\ &= 49e^{-800t} - 21.2e^{-200t}\ \text{V},\ t > 0 \end{aligned}$$

P 6.35

$$\begin{aligned} \frac{di_o}{dt} &= (2)\{e^{-5000t}[-1000\sin 1000t + 5000\cos 1000t] \\ &\quad +(-5000e^{-5000t})[\cos 1000t + 5\sin 1000t]\} \\ &= e^{-5000t}\{-52{,}000\sin 1000t\}\,\text{V} \end{aligned}$$

$$\frac{di_o}{dt}(0^+) = (1)[\sin(0)] = 0$$

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$$\therefore \qquad 50 \times 10^{-3}\frac{di_o}{dt}(0^+) = 0 \qquad \text{so} \qquad v_2(0^+) = 0$$

$$v_1(0^+) = 25i_o(0^+) + v_2(0^+) = 25(2) + 0 = 50\,\text{V}$$

P 6.36 **[a]** Rearrange by organizing the equations by di_1/dt, i_1, di_2/dt, i_2 and transfer the i_g terms to the right hand side of the equations. We get

$$4\frac{di_1}{dt} + 25i_1 - 8\frac{di_2}{dt} - 20i_2 = 5i_g - 8\frac{di_g}{dt}$$

$$-8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 80i_2 = 16\frac{di_g}{dt}$$

[b] From the given solutions we have

$$\frac{di_1}{dt} = -320e^{-5t} + 272e^{-4t}$$

$$\frac{di_2}{dt} = 260e^{-5t} - 204e^{-4t}$$

Thus,

$$4\frac{di_1}{dt} = -1280e^{-5t} + 1088e^{-4t}$$

$$25i_1 = 100 + 1600e^{-5t} - 1700e^{-4t}$$

$$8\frac{di_2}{dt} = 2080e^{-5t} - 1632e^{-4t}$$

$$20i_2 = 20 - 1040e^{-5t} + 1020e^{-4t}$$

$$5i_g = 80 - 80e^{-5t}$$

$$8\frac{di_g}{dt} = 640e^{-5t}$$

Thus,

$$-1280e^{-5t} + 1088e^{-4t} + 100 + 1600e^{-5t} - 1700e^{-4t} - 2080e^{-5t}$$

$$+1632e^{-4t} - 20 + 1040e^{-5t} - 1020e^{-4t} \stackrel{?}{=} 80 - 80e^{-5t} - 640e^{-5t}$$

$$80 + (1088 - 1700 + 1632 - 1020)e^{-4t}$$

$$+(1600 - 1280 - 2080 + 1040)e^{-5t} \stackrel{?}{=} 80 - 720e^{-5t}$$

$$80 + (2720 - 2720)e^{-4t} + (2640 - 3360)e^{-5t} = 80 - 720e^{-5t} \quad \text{(OK)}$$

$$8\frac{di_1}{dt} = -2560e^{-5t} + 2176e^{-4t}$$

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$$20i_1 = 80 + 1280e^{-5t} - 1360e^{-4t}$$

$$16\frac{di_2}{dt} = 4160e^{-5t} - 3264e^{-4t}$$

$$80i_2 = 80 - 4160e^{-5t} + 4080e^{-4t}$$

$$16\frac{di_g}{dt} = 1280e^{-5t}$$

$$2560e^{-5t} - 2176e^{-4t} - 80 - 1280e^{-5t} + 1360e^{-4t} + 4160e^{-5t} - 3264e^{-4t}$$
$$+80 - 4160e^{-5t} + 4080e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$(-80 + 80) + (2560 - 1280 + 4160 - 4160)e^{-5t}$$
$$+(1360 - 2176 - 3264 + 4080)e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$0 + 1280e^{-5t} + 0e^{-4t} = 1280e^{-5t} \quad \text{(OK)}$$

P 6.37 **[a]** Yes, using KVL around the lower right loop

$$v_o = v_{20\Omega} + v_{60\Omega} = 20(i_2 - i_1) + 60i_2$$

[b]
$$\begin{aligned} v_o &= 20(1 - 52e^{-5t} + 51e^{-4t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 60(1 - 52e^{-5t} + 51e^{-4t}) \\ &= 20(-3 - 116e^{-5t} + 119e^{-4t}) + 60 - 3120e^{-5t} + 3060e^{-4t} \\ v_o &= -5440e^{-5t} + 5440e^{-4t}\,\text{V} \end{aligned}$$

[c]
$$\begin{aligned} v_o &= L_2\frac{d}{dt}(i_g - i_2) + M\frac{di_1}{dt} \\ &= 16\frac{d}{dt}(15 + 36e^{-5t} - 51e^{-4t}) + 8\frac{d}{dt}(4 + 64e^{-5t} - 68e^{-4t}) \\ &= -2880e^{-5t} + 3264e^{-4t} - 2560e^{-5t} + 2176e^{-4t} \\ v_o &= -5440e^{-5t} + 5440e^{-4t}\,\text{V} \end{aligned}$$

P 6.38 **[a]**
$$\begin{aligned} v_g &= 5(i_g - i_1) + 20(i_2 - i_1) + 60i_2 \\ &= 5(16 - 16e^{-5t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 20(1 - 52e^{-5t} + 51e^{-4t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 60(1 - 52e^{-5t} + 51e^{-4t}) \\ &= 60 + 5780e^{-4t} - 5840e^{-5t}\,\text{V} \end{aligned}$$

[b] $v_g(0) = 60 + 5780 - 5840 = 0\,\text{V}$

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[c] $$\begin{aligned} p_{\text{dev}} &= v_g i_g \\ &= 960 + 92{,}480e^{-4t} - 94{,}400e^{-5t} - 92{,}480e^{-9t} + \\ &\quad 93{,}440e^{-10t}\text{W} \end{aligned}$$

[d] $p_{\text{dev}}(\infty) = 960\,\text{W}$

[e] $i_1(\infty) = 4\,\text{A}; \quad i_2(\infty) = 1\,\text{A}; \quad i_g(\infty) = 16\,\text{A};$

$$p_{5\Omega} = (16-4)^2(5) = 720\,\text{W}$$

$$p_{20\Omega} = 3^2(20) = 180\,\text{W}$$

$$p_{60\Omega} = 1^2(60) = 60\,\text{W}$$

$$\sum p_{\text{abs}} = 720 + 180 + 60 = 960\,\text{W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{abs}} = 960\,\text{W}$$

P 6.39 **[a]** $0.5\dfrac{di_g}{dt} + 0.2\dfrac{di_2}{dt} + 10i_2 = 0$

$$0.2\frac{di_2}{dt} + 10i_2 = -0.5\frac{di_g}{dt}$$

[b] $i_2 = 625e^{-10t} - 250e^{-50t}\,\text{mA}$

$$\frac{di_2}{dt} = -6.25e^{-10t} + 12.5e^{-50t}\,\text{A/s}$$

$$i_g = e^{-10t} - 10\,\text{A}$$

$$\frac{di_g}{dt} = -10e^{-10t}\,\text{A/s}$$

$$0.2\frac{di_2}{dt} + 10i_2 = 5e^{-10t} \qquad \text{and} \qquad -0.5\frac{di_g}{dt} = 5e^{-10t}$$

[c] $$\begin{aligned} v_1 &= 5\frac{di_g}{dt} + 0.5\frac{di_2}{dt} \\ &= 5(-10e^{-10t}) + 0.5(-6.25e^{-10t} + 12.5e^{-50t}) \\ &= -53.125e^{-10t} + 6.25e^{-50t}\,\text{V}, \qquad t > 0 \end{aligned}$$

[d] $v_1(0) = -53.125 + 6.25 = -46.875\,\text{V}; \qquad$ Also

$$\begin{aligned} v_1(0) &= 5\frac{di_g}{dt}(0) + 0.5\frac{di_2}{dt}(0) \\ &= 5(-10) + 0.5(-6.25 + 12.5) = -46.875\,\text{V} \end{aligned}$$

Yes, the initial value of v_1 is consistent with known circuit behavior.

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P 6.40 **[a]** $v_{\text{ab}} = L_1\dfrac{di}{dt} + L_2\dfrac{di}{dt} + M\dfrac{di}{dt} + M\dfrac{di}{dt} = (L_1 + L_2 + 2M)\dfrac{di}{dt}$

It follows that $L_{\text{ab}} = (L_1 + L_2 + 2M)$

[b] $v_{\text{ab}} = L_1\dfrac{di}{dt} - M\dfrac{di}{dt} + L_2\dfrac{di}{dt} - M\dfrac{di}{dt} = (L_1 + L_2 - 2M)\dfrac{di}{dt}$

Therefore $L_{\text{ab}} = (L_1 + L_2 - 2M)$

P 6.41 **[a]** $v_{\text{ab}} = L_1\dfrac{d(i_1 - i_2)}{dt} + M\dfrac{di_2}{dt}$

$$0 = L_1\frac{d(i_2 - i_1)}{dt} - M\frac{di_2}{dt} + M\frac{d(i_1 - i_2)}{dt} + L_2\frac{di_2}{dt}$$

Collecting coefficients of $[di_1/dt]$ and $[di_2/dt]$, the two mesh-current equations become

$$v_{\text{ab}} = L_1\frac{di_1}{dt} + (M - L_1)\frac{di_2}{dt}$$

and

$$0 = (M - L_1)\frac{di_1}{dt} + (L_1 + L_2 - 2M)\frac{di_2}{dt}$$

Solving for $[di_1/dt]$ gives

$$\frac{di_1}{dt} = \frac{L_1 + L_2 - 2M}{L_1L_2 - M^2}v_{\text{ab}}$$

from which we have

$$v_{\text{ab}} = \left(\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right)\left(\frac{di_1}{dt}\right)$$

$$\therefore \quad L_{\text{ab}} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

[b] If the magnetic polarity of coil 2 is reversed, the sign of M reverses, therefore

$$L_{\text{ab}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

P 6.42 When the switch is opened the induced voltage is negative at the dotted terminal. Since the voltmeter kicks downscale, the induced voltage across the voltmeter must be negative at its positive terminal. Therefore, the voltage is negative at the positive terminal of the voltmeter.

Thus, the upper terminal of the unmarked coil has the same instantaneous polarity as the dotted terminal. Therefore, place a dot on the upper terminal of the unmarked coil.

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P 6.43 **[a]** Dot terminal 1; the flux is up in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

[b] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[c] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[d] Dot terminal 1; the flux is down in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

P 6.44 **[a]** $\dfrac{1}{k^2} = \left(1 + \dfrac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right) = \left(1 + \dfrac{\mathcal{P}_{11}}{\mathcal{P}_{21}}\right)\left(1 + \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)$

Therefore

$$k^2 = \frac{\mathcal{P}_{12}\mathcal{P}_{21}}{(\mathcal{P}_{21} + \mathcal{P}_{11})(\mathcal{P}_{12} + \mathcal{P}_{22})}$$

Now note that

$$\phi_1 = \phi_{11} + \phi_{21} = \mathcal{P}_{11}N_1i_1 + \mathcal{P}_{21}N_1i_1 = N_1i_1(\mathcal{P}_{11} + \mathcal{P}_{21})$$

and similarly

$$\phi_2 = N_2i_2(\mathcal{P}_{22} + \mathcal{P}_{12})$$

It follows that

$$(\mathcal{P}_{11} + \mathcal{P}_{21}) = \frac{\phi_1}{N_1i_1}$$

and

$$(\mathcal{P}_{22} + \mathcal{P}_{12}) = \left(\frac{\phi_2}{N_2i_2}\right)$$

Therefore

$$k^2 = \frac{(\phi_{12}/N_2i_2)(\phi_{21}/N_1i_1)}{(\phi_1/N_1i_1)(\phi_2/N_2i_2)} = \frac{\phi_{12}\phi_{21}}{\phi_1\phi_2}$$

or

$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right)\left(\frac{\phi_{12}}{\phi_2}\right)}$$

[b] The fractions (ϕ_{21}/ϕ_1) and (ϕ_{12}/ϕ_2) are by definition less than 1.0, therefore $k < 1$.

P 6.45 **[a]** $k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{22.8}{\sqrt{576}} = 0.95$

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[b] $M_{\max} = \sqrt{576} = 24\,\text{mH}$

[c] $\dfrac{L_1}{L_2} = \dfrac{N_1^2\mathcal{P}_1}{N_2^2\mathcal{P}_2} = \left(\dfrac{N_1}{N_2}\right)^2$

$$\therefore \quad \left(\frac{N_1}{N_2}\right)^2 = \frac{60}{9.6} = 6.25$$

$$\frac{N_1}{N_2} = \sqrt{6.25} = 2.5$$

P 6.46 **[a]** $L_2 = \left(\dfrac{M^2}{k^2L_1}\right) = \dfrac{(0.09)^2}{(0.75)^2(0.288)} = 50\,\text{mH}$

$$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{288}{50}} = 2.4$$

[b] $\mathcal{P}_1 = \dfrac{L_1}{N_1^2} = \dfrac{0.288}{(1200)^2} = 0.2 \times 10^{-6}\ \text{Wb/A}$

$$\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{0.05}{(500)^2} = 0.2 \times 10^{-6}\ \text{Wb/A}$$

P 6.47 **[a]** $W = (0.5)L_1i_1^2 + (0.5)L_2i_2^2 + Mi_1i_2$

$$M = 0.85\sqrt{(18)(32)} = 20.4\,\text{mH}$$

$$W = [9(36) + 16(81) + 20.4(54)] = 2721.6\,\text{mJ}$$

[b] $W = [324 + 1296 + 1101.6] = 2721.6\,\text{mJ}$

[c] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

[d] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

P 6.48 **[a]** $M = 1.0\sqrt{(18)(32)} = 24\,\text{mH}, \qquad i_1 = 6\,\text{A}$

Therefore $\quad 16i_2^2 + 144i_2 + 324 = 0, \qquad i_2^2 + 9i_2 + 20.25 = 0$

Therefore $\quad i_2 = -\left(\dfrac{9}{2}\right) \pm \sqrt{\left(\dfrac{9}{2}\right)^2 - 20.25} = -4.5 \pm \sqrt{0}$

Therefore $\quad i_2 = -4.5\,\text{A}$

[b] No, setting W equal to a negative value will make the quantity under the square root sign negative.

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P 6.49 **[a]** $L_1 = N_1^2\mathcal{P}_1; \qquad \mathcal{P}_1 = \dfrac{72 \times 10^{-3}}{6.25 \times 10^4} = 1152 \text{ nWb/A}$

$$\frac{d\phi_{11}}{d\phi_{21}} = \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}} = 0.2; \qquad \mathcal{P}_{21} = 2\mathcal{P}_{11}$$

$$\therefore \quad 1152 \times 10^{-9} = \mathcal{P}_{11} + \mathcal{P}_{21} = 3\mathcal{P}_{11}$$

$$\mathcal{P}_{11} = 192 \text{ nWb/A}; \qquad \mathcal{P}_{21} = 960 \text{ nWb/A}$$

$$M = k\sqrt{L_1L_2} = (2/3)\sqrt{(0.072)(0.0405)} = 36\,\text{mH}$$

$$N_2 = \frac{M}{N_1\mathcal{P}_{21}} = \frac{36 \times 10^{-3}}{(250)(960 \times 10^{-9})} = 150 \text{ turns}$$

[b] $\mathcal{P}_2 = \dfrac{L_2}{N_2^2} = \dfrac{40.5 \times 10^{-3}}{(150)^2} = 1800 \text{ nWb/A}$

[c] $\mathcal{P}_{11} = 192 \text{ nWb/A}$ [see part (a)]

[d] $\dfrac{\phi_{22}}{\phi_{12}} = \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2 - \mathcal{P}_{12}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2}{\mathcal{P}_{12}} - 1$

$$\mathcal{P}_{21} = \mathcal{P}_{21} = 960 \text{ nWb/A}; \qquad \mathcal{P}_2 = 1800 \text{ nWb/A}$$

$$\frac{\phi_{22}}{\phi_{12}} = \frac{1800}{960} - 1 = 0.875$$

P 6.50 $\mathcal{P}_1 = \dfrac{L_1}{N_1^2} = 2 \text{ nWb/A}; \quad \mathcal{P}_2 = \dfrac{L_2}{N_2^2} = 2 \text{ nWb/A}; \quad M = k\sqrt{L_1L_2} = 180\,\mu\text{H}$

$$\mathcal{P}_{12} = \mathcal{P}_{21} = \frac{M}{N_1N_2} = 1.2 \text{ nWb/A}$$

$$\mathcal{P}_{11} = \mathcal{P}_1 - \mathcal{P}_{21} = 0.8 \text{ nWb/A}$$

P 6.51 When the touchscreen in the mutual-capacitance design is touched at the point x, y, the touch capacitance C_t is present in series with the mutual capacitance at the touch point, C_{mxy}. Remember that capacitances combine in series the way that resistances combine in parallel. The resulting mutual capacitance is

$$C'_{mxy} = \frac{C_{mxy}C_t}{C_{mxy} + C_t}$$

P 6.52 **[a]** The self-capacitance and the touch capacitance are effectively connected in parallel. Therefore, the capacitance at the x-grid electrode closest to the touch point with respect to ground is

$$C_x = C_p + C_t = 30\,\text{pF} + 15\,\text{pF} = 45\,\text{pF}.$$

The same capacitance exists at the y-grid electrode closest to the touch point with respect to ground.

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[b] The mutual-capacitance and the touch capacitance are effectively connected in series. Therefore, the mutual capacitance between the x-grid and y-grid electrodes closest to the touch point is

$$C'_{mxy} = \frac{C_{mxy}C_t}{C_{mxy} + C_t} = \frac{(30)(15)}{30 + 15} = 10\,\text{pF}.$$

[c] In the self-capacitance design, touching the screen increases the capacitance being measured at the point of touch. For example, in part (a) the measured capacitance before the touch is 30 pF and after the touch is 45 pF. In the mutual-capacitance design, touching the screen decreases the capacitance being measured at the point of touch. For example, in part (b) the measured capacitance before the touch is 30 pF and after the touch is 10 pF.

P 6.53 **[a]** The four touch points identified are the two actual touch points and two ghost touch points. Their coordinates, in inches from the upper left corner of the screen, are

$(2.1, 4.3)$; $(3.2, 2.5)$; $(2.1, 2.5)$; and $(3.2, 4.3)$

These four coordinates identify a rectangle within the screen, shown below.

[b] The touch points identified at time t_1 are those listed in part (a). The touch points recognized at time t_2 are

$(1.8, 4.9)$; $(3.9, 1.8)$; $(1.8, 1.8)$; and $(3.9, 4.9)$

The first two coordinates are the actual touch points and the last two coordinates are the associated ghost points. Again, the four coordinates identify a rectangle at time t_2, as shown here:

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Note that the rectangle at time t_2 is larger than the rectangle at time t_1, so the software would recognize the two fingers are moving toward the edges of the screen. This pinch gesture thus specifies a zoom-in for the screen.

[**c**] The touch points identified at time t_1 are those listed in part (a). The touch points recognized at time t_2 are

$(2.8, 3.9)$; $(3.0, 2.8)$; $(2.8, 2.8)$; and $(3.0, 3.9)$

The first two coordinates are the actual touch points and the last two coordinates are the associated ghost points. Again, the four coordinates identify a rectangle at time t_2, as shown here:

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Here, the rectangle at time t_2 is smaller than the rectangle at time t_1, so the software would recognize the two fingers are moving toward the middle of the screen. This pinch gesture thus specifies a zoom-out for the screen.

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7

Response of First-Order RL and RC Circuits

Assessment Problems

AP 7.1 **[a]** The circuit for $t < 0$ is shown below. Note that the inductor behaves like a short circuit, effectively eliminating the $2\,\Omega$ resistor from the circuit.

First combine the $30\,\Omega$ and $6\,\Omega$ resistors in parallel:
$30\|6 = 5\,\Omega$
Use voltage division to find the voltage drop across the parallel resistors:
$v = \dfrac{5}{5+3}(120) = 75\,\text{V}$
Now find the current using Ohm's law:
$i(0^-) = -\dfrac{v}{6} = -\dfrac{75}{6} = -12.5\,\text{A}$

[b] $w(0) = \dfrac{1}{2}Li^2(0) = \dfrac{1}{2}(8\times10^{-3})(12.5)^2 = 625\,\text{mJ}$

[c] To find the time constant, we need to find the equivalent resistance seen by the inductor for $t > 0$. When the switch opens, only the $2\,\Omega$ resistor remains connected to the inductor. Thus,
$\tau = \dfrac{L}{R} = \dfrac{8\times10^{-3}}{2} = 4\,\text{ms}$

[d] $i(t) = i(0^-)e^{t/\tau} = -12.5e^{-t/0.004} = -12.5e^{-250t}\,\text{A}, \qquad t \geq 0$

[e] $i(5\,\text{ms}) = -12.5e^{-250(0.005)} = -12.5e^{-1.25} = -3.58\,\text{A}$
So $\quad w\,(5\,\text{ms}) = \frac{1}{2}Li^2(5\,\text{ms}) = \frac{1}{2}(8)\times10^{-3}(3.58)^2 = 51.3\,\text{mJ}$

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$w\,(\text{dis}) = 625 - 51.3 = 573.7\,\text{mJ}$

$\%\ \text{dissipated} = \left(\dfrac{573.7}{625}\right)100 = 91.8\%$

AP 7.2 **[a]** First, use the circuit for $t < 0$ to find the initial current in the inductor:

Using current division,

$i(0^-) = \dfrac{10}{10+6}(6.4) = 4\,\text{A}$

Now use the circuit for $t > 0$ to find the equivalent resistance seen by the inductor, and use this value to find the time constant:

$R_{\text{eq}} = 4\|(6+10) = 3.2\,\Omega, \quad \therefore \quad \tau = \dfrac{L}{R_{\text{eq}}} = \dfrac{0.32}{3.2} = 0.1\,\text{s}$

Use the initial inductor current and the time constant to find the current in the inductor:

$i(t) = i(0^-)e^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t}\,\text{A}, \quad t \geq 0$

Use current division to find the current in the $10\,\Omega$ resistor:

$$i_o(t) = \frac{4}{4+10+6}(-i) = \frac{4}{20}(-4e^{-10t}) = -0.8e^{-10t}\,\text{A}, \quad t \geq 0^+$$

Finally, use Ohm's law to find the voltage drop across the $10\,\Omega$ resistor:

$v_o(t) = 10i_o = 10(-0.8e^{-10t}) = -8e^{-10t}\,\text{V}, \quad t \geq 0^+$

[b] The initial energy stored in the inductor is

$$w(0) = \frac{1}{2}Li^2(0^-) = \frac{1}{2}(0.32)(4)^2 = 2.56\,\text{J}$$

Find the energy dissipated in the $4\,\Omega$ resistor by integrating the power over all time:

$$v_{4\Omega}(t) = L\frac{di}{dt} = 0.32(-10)(4e^{-10t}) = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

$$p_{4\Omega}(t) = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0^+$$

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$$w_{4\Omega}(t) = \int_0^\infty 40.96e^{-20t}dt = 2.048\,\text{J}$$

Find the percentage of the initial energy in the inductor dissipated in the $4\,\Omega$ resistor:

$$\%\text{ dissipated} = \left(\frac{2.048}{2.56}\right)100 = 80\%$$

AP 7.3 **[a]** The circuit for $t < 0$ is shown below. Note that the capacitor behaves like an open circuit.

Find the voltage drop across the open circuit by finding the voltage drop across the $50\,\text{k}\Omega$ resistor. First use current division to find the current through the $50\,\text{k}\Omega$ resistor:

$$i_{50\text{k}} = \frac{80 \times 10^3}{80 \times 10^3 + 20 \times 10^3 + 50 \times 10^3}(7.5 \times 10^{-3}) = 4\,\text{mA}$$

Use Ohm's law to find the voltage drop:

$v(0^-) = (50 \times 10^3)i_{50\text{k}} = (50 \times 10^3)(0.004) = 200\,\text{V}$

[b] To find the time constant, we need to find the equivalent resistance seen by the capacitor for $t > 0$. When the switch opens, only the $50\,\text{k}\Omega$ resistor remains connected to the capacitor. Thus,

$\tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20\,\text{ms}$

[c] $v(t) = v(0^-)e^{-t/\tau} = 200e^{-t/0.02} = 200e^{-50t}\,\text{V}, \quad t \geq 0$

[d] $w(0) = \dfrac{1}{2}Cv^2 = \dfrac{1}{2}(0.4 \times 10^{-6})(200)^2 = 8\,\text{mJ}$

[e] $w(t) = \dfrac{1}{2}Cv^2(t) = \dfrac{1}{2}(0.4 \times 10^{-6})(200e^{-50t})^2 = 8e^{-100t}\,\text{mJ}$

The initial energy is 8 mJ, so when 75% is dissipated, 2 mJ remains:

$$8 \times 10^{-3}e^{-100t} = 2 \times 10^{-3}, \qquad e^{100t} = 4, \qquad t = (\ln 4)/100 = 13.86\,\text{ms}$$

AP 7.4 **[a]** This circuit is actually two RC circuits in series, and the requested voltage, v_o, is the sum of the voltage drops for the two RC circuits. The circuit for $t < 0$ is shown below:

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Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:

$$i = \frac{15}{75{,}000} = 0.2\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$$

There are two time constants in the circuit, one for each RC subcircuit. τ_5 is the time constant for the $5\,\mu\text{F} - 20\,\text{k}\Omega$ subcircuit, and τ_1 is the time constant for the $1\,\mu\text{F} - 40\,\text{k}\Omega$ subcircuit:

$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}; \qquad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}$

Therefore,

$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t}\,\text{V}, \quad t \geq 0$

$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t}\,\text{V}, \quad t \geq 0$

Finally,

$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$

[b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:

$v_1(60\,\text{ms}) = 8e^{-25(0.06)} \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) = 4e^{-10(0.06)} \cong 2.20\,\text{V}$

$w_1(60\,\text{ms}) = \frac{1}{2}Cv_1^2(60\,\text{ms}) = \frac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59\,\mu\text{J}$

$w_5(60\,\text{ms}) = \frac{1}{2}Cv_5^2(60\,\text{ms}) = \frac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05\,\mu\text{J}$

$w(60\,\text{ms}) = 1.59 + 12.05 = 13.64\,\mu\text{J}$

Find the initial energy from the initial voltage:

$w(0) = w_1(0) + w_2(0) = \frac{1}{2}(1 \times 10^{-6})(8)^2 + \frac{1}{2}(5 \times 10^{-6})(4)^2 = 72\,\mu\text{J}$

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:

$w_{\text{diss}} = w(0) - w(60\,\text{ms}) = 72 - 13.64 = 58.36\,\mu\text{J}$

$\%\ \text{dissipated} = (58.36 \times 10^{-6}/72 \times 10^{-6})(100) = 81.05\,\%$

AP 7.5 **[a]** Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:

$i(0^-) = 24/2 = 12\,\text{A} = i(0^+)$

Note that $i(0^-) = i(0^+)$ because the current in an inductor is continuous.

[b] Use the circuit at $t = 0^+$, shown below, to calculate the voltage drop across the inductor at 0^+. Note that this is the same as the voltage drop across the $10\,\Omega$ resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.

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$v(0^+) = -10(8 + 12) = -200\,\text{V}$

[c] To calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the $10\,\Omega$ resistor is connected to the inductor for $t > 0$. Thus,
$\tau = L/R = (200 \times 10^{-3}/10) = 20\,\text{ms}$

[d] To find $i(t)$, we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:

Note that the inductor behaves as a short circuit and all of the current from the 8 A source flows through the short circuit. Thus,
$i_f = -8\,\text{A}$
Now,
$i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02}$
$\quad = -8 + 20e^{-50t}\,\text{A}, \quad t \geq 0$

[e] To find $v(t)$, use the relationship between voltage and current for an inductor:

$$v(t) = L\frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$$

AP 7.6 **[a]**

From Example 7.6,

$v_o(t) = -60 + 90e^{-100t}\,\text{V}$

Write a KCL equation at the top node and use it to find the relationship between v_o and v_A:

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160{,}000} + \frac{v_A + 75}{40{,}000} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

$$v_A = 0.8v_o - 12$$

Use the above equation for v_A in terms of v_o to find the expression for v_A:

$$v_A(t) = 0.8(-60 + 90e^{-100t}) - 12 = -60 + 72e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

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[b] $t \geq 0^+$, since there is no requirement that the voltage be continuous in a resistor.

AP 7.7 **[a]** Use the circuit shown below, for $t < 0$, to calculate the initial voltage drop across the capacitor:

$$i = \left(\frac{40 \times 10^3}{125 \times 10^3}\right)(10 \times 10^{-3}) = 3.2\,\text{mA}$$

$$v_c(0^-) = (3.2 \times 10^{-3})(25 \times 10^3) = 80\,\text{V} \quad \text{so} \quad v_c(0^+) = 80\,\text{V}$$

Now use the next circuit, valid for $0 \leq t \leq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:

For $\quad 0 \leq t \leq 100\,\text{ms}$:

$$\tau = RC = (25 \times 10^3)(1 \times 10^{-6}) = 25\,\text{ms}$$

$$v_c(t) = v_c(0^-)e^{t/\tau} = 80e^{-40t}\,\text{V} \quad 0 \leq t \leq 10\,\text{ms}$$

[b] Calculate the starting capacitor voltage in the interval $t \geq 10\,\text{ms}$, using the capacitor voltage from the previous interval:
$v_c(0.01) = 80e^{-40(0.01)} = 53.63\,\text{V}$
Now use the next circuit, valid for $t \geq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:

For $\quad t \geq 10\,\text{ms}$:

$$R_{\text{eq}} = 25\,\text{k}\Omega \| 100\,\text{k}\Omega = 20\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (20 \times 10^3)(1 \times 10^{-6}) = 0.02\,\text{s}$$

Therefore $\quad v_c(t) = v_c(0.01^+)e^{-(t-0.01)/\tau} = 53.63e^{-50(t-0.01)}\,\text{V}, \qquad t \geq 0.01\,\text{s}$

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[c] To calculate the energy dissipated in the 25 kΩ resistor, integrate the power absorbed by the resistor over all time. Use the expression $p = v^2/R$ to calculate the power absorbed by the resistor.

$$w_{25\,\mathrm{k}} = \int_0^{0.01} \frac{[80e^{-40t}]^2}{25{,}000}dt + \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{25{,}000}dt = 2.91\,\mathrm{mJ}$$

[d] Repeat the process in part (c), but recognize that the voltage across this resistor is non-zero only for the second interval:

$$w_{100\,\mathrm{k\Omega}} = \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{100{,}000}dt = 0.29\,\mathrm{mJ}$$

We can check our answers by calculating the initial energy stored in the capacitor. All of this energy must eventually be dissipated by the 25 kΩ resistor and the 100 kΩ resistor.

Check: $w_{\mathrm{stored}} = (1/2)(1 \times 10^{-6})(80)^2 = 3.2\,\mathrm{mJ}$

$$w_{\mathrm{diss}} = 2.91 + 0.29 = 3.2\,\mathrm{mJ}$$

AP 7.8 **[a]** Prior to switch a closing at $t = 0$, there are no sources connected to the inductor; thus, $i(0^-) = 0$.
At the instant A is closed, $i(0^+) = 0$.
For $0 \le t \le 1\,\mathrm{s}$,

The equivalent resistance seen by the 10 V source is $2 + (3\|0.8)$. The current leaving the 10 V source is

$$\frac{10}{2 + (3\|0.8)} = 3.8\,\mathrm{A}$$

The final current in the inductor, which is equal to the current in the 0.8 Ω resistor is

$$I_{\mathrm{F}} = \frac{3}{3 + 0.8}(3.8) = 3\,\mathrm{A}$$

The resistance seen by the inductor is calculated to find the time constant:

$$[(2\|3) + 0.8]\|3\|6 = 1\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{1} = 2\,\mathrm{s}$$

Therefore,

$$i = i_{\mathrm{F}} + [i(0^+) - i_{\mathrm{F}}]e^{-t/\tau} = 3 - 3e^{-0.5t}\,\mathrm{A}, \quad 0 \le t \le 1\,\mathrm{s}$$

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For part (b) we need the value of $i(t)$ at $t = 1$ s:

$i(1) = 3 - 3e^{-0.5} = 1.18\,\text{A}$

.

[b] For $t > 1$ s

Use current division to find the final value of the current:

$$i = \frac{9}{9+6}(-8) = -4.8\,\text{A}$$

The equivalent resistance seen by the inductor is used to calculate the time constant:

$$3\|(9+6) = 2.5\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2.5} = 0.8\,\text{s}$$

Therefore,

$$i = i_\text{F} + [i(1^+) - i_\text{F}]e^{-(t-1)/\tau}$$

$$= -4.8 + 5.98e^{-1.25(t-1)}\,\text{A}, \quad t \geq 1\,\text{s}$$

AP 7.9 $0 \leq t \leq 32\,\text{ms}$:

$$v_o = -\frac{1}{RC_f}\int_0^{32\times10^{-3}} -10\,dt + 0 = -\frac{1}{RC_f}(-10t)\Big|_0^{32\times10^{-3}} = -\frac{1}{RC_f}(-320\times10^{-3})$$

$$RC_f = (200\times10^3)(0.2\times10^{-6}) = 40\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 25$$

$$v_o = -25(-320\times10^{-3}) = 8\,\text{V}$$

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$t \geq 32\,\text{ms}$:

$$v_o = -\frac{1}{RC_f}\int_{32\times10^{-3}}^{t} 5\,dy + 8 = -\frac{1}{RC_f}(5y)\Big|_{32\times10^{-3}}^{t} + 8 = -\frac{1}{RC_f}5(t - 32\times10^{-3}) + 8$$

$$RC_f = (250\times10^3)(0.2\times10^{-6}) = 50\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 20$$

$$v_o = -20(5)(t - 32\times10^{-3}) + 8 = -100t + 11.2$$

The output will saturate at the negative power supply value:

$$-15 = -100t + 11.2 \quad \therefore \quad t = 262\,\text{ms}$$

AP 7.10 **[a]** Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (0+2)e^{-t/\tau}$$

$$\tau = (160\times10^3)(10\times10^{-9}) = 10^{-3}; \qquad 1/\tau = 625$$

$$v_p = -2 + 2e^{-625t}\,\text{V}; \qquad v_n = v_p$$

Write a KVL equation at the inverting input, and use it to determine v_o:

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0$$

$$\therefore \quad v_o = 5v_n = 5v_p = -10 + 10e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 10e^{-625t} = -5; \quad e^{-625t} = 1/2; \quad t = \ln 2/625 = 1.11\,\text{ms}$$

[b] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (1+2)e^{-625t} = -2 + 3e^{-625t}\,\text{V}$$

The analysis for v_o is the same as in part (a):

$$v_o = 5v_p = -10 + 15e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 15e^{-625t} = -5; \qquad e^{-625t} = 1/3; \quad t = \ln 3/625 = 1.76\,\text{ms}$$

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Problems

P 7.1 **[a]** $i_o(0) = \dfrac{20}{16+12+4+8} = \dfrac{20}{40} = 0.5\,\text{A}$

$i_o(\infty) = 0\,\text{A}$

[b] $i_o = 0.5e^{-t/\tau}; \qquad \tau = \dfrac{L}{R} = \dfrac{80\times 10^{-3}}{12+8} = 4\,\text{ms}$

$i_o = 0.5e^{-250t}\,\text{A}, \quad t \geq 0$

[c] $0.5e^{-250t} = 0.1$

$e^{250t} = 5 \qquad \therefore \quad t = 6.44\,\text{ms}$

P 7.2 **[a]** For $t < 0$

$i_g = \dfrac{50}{20+(75\|50)} = \dfrac{50}{50} = 1\,\text{A}$

$i_o(0^-) = \dfrac{50}{75+50}(1) = 0.4\,\text{A} = i_o(0^+)$

For $t > 0$

$i_o(t) = i_o(0^+)e^{-t/\tau}\,\text{A}, \qquad t \geq 0$

$\tau = \dfrac{L}{R} = \dfrac{0.02}{3+60\|15} = 1.33\,\text{ms}; \qquad \dfrac{1}{\tau} = 750$

$i_o(t) = 0.4e^{-750t}\,\text{A}, \qquad t \geq 0$

[b] $v_\text{L} = L\dfrac{di_o}{dt} = 0.02(-750)(0.4e^{-750t}) = -6e^{-750t}\,\text{V}$

$v_o = \dfrac{60\|15}{3+60\|15}v_\text{L} = \dfrac{12}{15}(-6e^{-750t}) = -4.8e^{-750t}\,\text{V} \qquad t \geq 0^+$

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P 7.3 **[a]** $i(0) = \dfrac{60}{120} = 0.5\,\text{A}$

[b] $\tau = \dfrac{L}{R} = \dfrac{0.32}{160} = 2\,\text{ms}$

[c] $i = 0.5e^{-500t}\,\text{A}, \qquad t \geq 0$

$$v_1 = L\frac{d}{dt}(0.5e^{-500t}) = -80e^{-500t}\,\text{V} \qquad t \geq 0^+$$

$$v_2 = -70i = -35e^{-500t}\,\text{V} \qquad t \geq 0^+$$

[d] $w(0) = \dfrac{1}{2}(0.32)(0.5)^2 = 40\,\text{mJ}$

$$w_{90\Omega} = \int_0^t 90(0.25e^{-1000x})\,dx = 22.5\frac{e^{-1000x}}{-1000}\Bigg|_0^t = 22.5(1 - e^{-1000t})\,\text{mJ}$$

$$w_{90\Omega}(1\,\text{ms}) = 0.0225(1 - e^{-1}) = 14.22\,\text{mJ}$$

$$\%\text{ dissipated } = \frac{14.22}{40}(100) = 35.6\%$$

P 7.4 $t < 0$:

$$i_g = \frac{90}{13 + 12 + 6\|30} = 3\,\text{A}$$

$$i_L(0^-) = \frac{30}{36}(3) = 2.5\,\text{A}$$

$t > 0$:

$$R_e = 6 + 30\|(8 + 12) = 6 + 12 = 18\,\Omega$$

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$$\tau = \frac{L}{R_e} = \frac{36 \times 10^{-3}}{18} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$\therefore \quad i_L = 2.5e^{-500t}\,\text{A}$$

$$v_o = 6i_o = 15e^{-500t}\,\text{V}, \quad t \geq 0^+$$

P 7.5 $$p_{6\Omega} = \frac{v_o^2}{6} = \frac{(15)^2}{6}e^{-1000t} = 37.5e^{-1000t}\,\text{W}$$

$$w_{6\Omega} = \int_0^\infty 37.5e^{-1000t}\,dt = 37.5\frac{e^{-1000t}}{-1000}\bigg|_0^\infty = 37.5\,\text{mJ}$$

$$w(0) = \frac{1}{2}(36 \times 10^{-3})(2.5)^2 = 112.5\,\text{mJ}$$

$$\%\ \text{diss} = \frac{37.5}{112.5}(100) = 33.33\%$$

P 7.6 **[a]** $t < 0$

Simplify this circuit by creating a Thévenin equivalant to the left of the inductor and an equivalent resistance to the right of the inductor:

$$1\,\text{k}\Omega \| 4\,\text{k}\Omega = 0.8\,\text{k}\Omega$$

$$20\,\text{k}\Omega \| 80\,\text{k}\Omega = 16\,\text{k}\Omega$$

$$(105 \times 10^{-3})(0.8 \times 10^3) = 84\,\text{V}$$

$$i_L(0^-) = \frac{84}{16{,}800} = 5\,\text{mA}$$

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$t > 0$

$$\tau = \frac{L}{R} = \frac{6}{24} \times 10^{-3} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L(t) = 5e^{-4000t}\,\text{mA}, \qquad t \geq 0$$

$$p_{4\text{k}} = 25 \times 10^{-6} e^{-8000t}(4000) = 0.10e^{-8000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^t 0.10e^{-8000x}\,dx = 12.5 \times 10^{-6}[1 - e^{-8000t}]\,\text{J}$$

$$w(0) = \frac{1}{2}(6)(25 \times 10^{-6}) = 75\,\mu\text{J}$$

$$0.10w(0) = 7.5\,\mu\text{J}$$

$$12.5(1 - e^{-8000t}) = 7.5; \qquad \therefore \quad e^{8000t} = 2.5$$

$$t = \frac{\ln 2.5}{8000} = 114.54\,\mu\text{s}$$

[b] $w_{\text{diss}}(\text{total}) = 75(1 - e^{-8000t})\,\mu\text{J}$

$$w_{\text{diss}}(114.54\,\mu\text{s}) = 45\,\mu\text{J}$$

$$\% = (45/75)(100) = 60\%$$

P 7.7 **[a]** $v_o(t) = v_o(0^+)e^{-t/\tau}$

$$\therefore \quad v_o(0^+)e^{-10^{-3}/\tau} = 0.5v_o(0^+)$$

$$\therefore \quad e^{10^{-3}/\tau} = 2$$

$$\therefore \quad \tau = \frac{L}{R} = \frac{10^{-3}}{\ln 2}$$

$$\therefore \quad L = \frac{10 \times 10^{-3}}{\ln 2} = 14.43\,\text{mH}$$

[b] $v_o(0^+) = -10i_L(0^+) = -10(1/10)(30 \times 10^{-3}) = -30\,\text{mV}$

$$v_o(t) = -0.03e^{-t/\tau}\,\text{V}$$

$$p_{10\Omega} = \frac{v_o^2}{10} = 9 \times 10^{-5}e^{-2t/\tau}$$

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$$w_{10\Omega} = \int_0^{10^{-3}} 9 \times 10^{-5} e^{-2t/\tau}\, dt = 4.5\tau \times 10^{-5}(1 - e^{-2\times 10^{-3}/\tau})$$

$$\tau = \frac{1}{1000 \ln 2} \qquad \therefore \qquad w_{10\Omega} = 48.69\,\text{nJ}$$

$$w_L(0) = \frac{1}{2} L i_L^2(0) = \frac{1}{2}(14.43 \times 10^{-3})(3 \times 10^{-3})^2 = 64.92\,\text{nJ}$$

$$\%\ \text{diss in 1 ms} = \frac{48.69}{64.92} \times 100 = 75\%$$

P 7.8 **[a]** $t < 0$

$$4\,\text{k}\Omega \| 12\,\text{k}\Omega = 3\,\text{k}\Omega$$

Find the current from the voltage source by combining the resistors in series and parallel and using Ohm's law:

$$i_g(0^-) = \frac{80}{(2000 + 3000)} = 16\,\text{mA}$$

Find the branch currents using current division:

$$i_1(0^-) = \frac{3000}{12{,}000}(0.016) = 4\,\text{mA}$$

$$i_2(0^-) = \frac{3000}{4000}(0.016) = 12\,\text{mA}$$

[b] The current in an inductor is continuous. Therefore,

$$i_1(0^+) = i_1(0^-) = 4\,\text{mA}$$

$$i_2(0^+) = -i_1(0^+) = -4\,\text{mA} \qquad \text{(when switch is open)}$$

[c] $\tau = \dfrac{L}{R} = \dfrac{0.64 \times 10^{-3}}{16 \times 10^3} = 4 \times 10^{-5}\,\text{s}; \qquad \dfrac{1}{\tau} = 25{,}000$

$$i_1(t) = i_1(0^+)e^{-t/\tau} = 4e^{-25{,}000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2(t) = -i_1(t) \qquad \text{when} \quad t \geq 0^+$

$$\therefore \quad i_2(t) = -4e^{-25{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

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[**e**] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal 12 mA and $i_2(0^+) = -4\,\text{mA}$.

P 7.9 [**a**] For $t = 0^-$ the circuit is:

$i_o(0^-) = 0$ since the switch is open

$$i_\text{L}(0^-) = \frac{25}{250} = 0.1 = 100\,\text{mA}$$

$v_\text{L}(0^-) = 0$ since the inductor behaves like a short circuit

[**b**] For $t = 0^+$ the circuit is:

$i_\text{L}(0^+) = i_\text{L}(0^-) = 100\,\text{mA}$

$$i_\text{g} = \frac{25}{50} = 0.5 = 500\,\text{mA}$$

$i_o(0^+) = i_\text{g} - i_\text{L}(0^+) = 500 - 100 = 400\,\text{mA}$

$200 i_\text{L}(0^+) + v_\text{L}(0^+) = 0 \qquad \therefore \qquad v_\text{L}(0^+) = -200 i_\text{L}(0^+) = -20\,\text{V}$

[**c**] As $t \to \infty$ the circuit is:

$i_\text{L}(\infty) = 0; \qquad v_\text{L}(\infty) = 0$

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$$i_o(\infty) = \frac{25}{50} = 500\,\text{mA}$$

[d] $\tau = \dfrac{L}{R} = \dfrac{0.05}{200} = 0.25\,\text{ms}$

$$i_\text{L}(t) = 0 + (0.1 - 0)e^{-4000t} = 0.1e^{-4000t}\,\text{A}$$

[e] $i_o(t) = i_\text{g} - i_\text{L} = 0.5 - 0.1e^{-4000t}\,\text{A}$

[f] $v_\text{L}(t) = L\dfrac{di_\text{L}}{dt} = 0.05(-4000)(0.1)e^{-4000t} = -20e^{-4000t}\,\text{V}$

P 7.10 $w(0) = \dfrac{1}{2}(10 \times 10^{-3})(5)^2 = 125\,\text{mJ}$

$$0.9w(0) = 112.5\,\text{mJ}$$

$$w(t) = \frac{1}{2}(10 \times 10^{-3})i(t)^2, \qquad i(t) = 5e^{-t/\tau}\,\text{A}$$

$$\therefore \quad w(t) = 0.005(25e^{-2t/\tau}) = 125e^{-2t/\tau})\,\text{mJ}$$

$$w(10\,\mu\text{s}) = 125e^{-20\times10^{-6}/\tau}\,\text{mJ}$$

$$\therefore \quad 125e^{-20\times10^{-6}/\tau} = 112.5 \qquad \text{so} \qquad e^{20\times10^{-6}/\tau} = \frac{10}{9}$$

$$\tau = \frac{20 \times 10^{-6}}{\ln(10/9)} = \frac{L}{R}$$

$$R = \frac{10 \times 10^{-3}\ln(10/9)}{20 \times 10^{-6}} = 52.68\,\Omega$$

P 7.11 **[a]** $w(0) = \dfrac{1}{2}LI_g^2$

$$w_\text{diss} = \int_0^{t_o} I_g^2 R e^{-2t/\tau}\,dt = I_g^2 R \frac{e^{-2t/\tau}}{(-2/\tau)}\bigg|_0^{t_o}$$
$$= \frac{1}{2}I_g^2 R\tau(1 - e^{-2t_o/\tau}) = \frac{1}{2}I_g^2 L(1 - e^{-2t_o/\tau})$$

$$w_\text{diss} = \sigma w(0)$$

$$\therefore \quad \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \sigma\left(\frac{1}{2}LI_g^2\right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \qquad e^{2t_o/\tau} = \frac{1}{(1-\sigma)}$$

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$$\frac{2t_o}{\tau} = \ln\left[\frac{1}{(1-\sigma)}\right]; \qquad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L\ \ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \dfrac{(10\times 10^{-3})\ln[1/0.9]}{20\times 10^{-6}}$

$$R = 52.68\,\Omega$$

P 7.12 **[a]** $R = \dfrac{v}{i} = 25\,\Omega$

[b] $\tau = \dfrac{1}{10} = 100\,\text{ms}$

[c] $\tau = \dfrac{L}{R} = 0.1$

$$L = (0.1)(25) = 2.5\,\text{H}$$

[d] $w(0) = \dfrac{1}{2}L[i(0)]^2 = \dfrac{1}{2}(2.5)(6.4)^2 = 51.2\,\text{J}$

[e] $w_{\text{diss}} = \displaystyle\int_0^t 1024e^{-20x}\,dx = 1024\frac{e^{-20x}}{-20}\Big|_0^t = 51.2(1-e^{-20t})\,\text{J}$

$$\%\,\text{dissipated} = \frac{51.2(1-e^{-20t})}{51.2}(100) = 100(1-e^{-20t})$$

$$\therefore\quad 100(1-e^{-20t}) = 60 \qquad \text{so} \qquad e^{-20t} = 0.4$$

Therefore $\quad t = \dfrac{1}{20}\ln 2.5 = 45.81\,\text{ms}$

P 7.13 **[a]** Note that there are several different possible solutions to this problem, and the answer to part (c) depends on the value of inductance chosen.

$$R = \frac{L}{\tau}$$

Choose a 10 mH inductor from Appendix H. Then,

$R = \dfrac{0.01}{0.001} = 10\,\Omega$ which is a resistor value from Appendix H.

[b] $i(t) = I_o e^{-t/\tau} = 10e^{-1000t}\,\text{mA}, \qquad t \geq 0$

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[c] $w(0) = \frac{1}{2}LI_o^2 = \frac{1}{2}(0.01)(0.01)^2 = 0.5\,\mu\text{J}$

$$w(t) = \frac{1}{2}(0.01)(0.01e^{-1000t})^2 = 0.5 \times 10^{-6}e^{-2000t}$$

So $\quad 0.5 \times 10^{-6}e^{-2000t} = \frac{1}{2}w(0) = 0.25 \times 10^{-6}$

$e^{-2000t} = 0.5 \qquad$ then $\qquad e^{2000t} = 2$

$\therefore \quad t = \frac{\ln 2}{2000} = 346.57\,\mu\text{s} \qquad$ (for a 10 mH inductor)

P 7.14 $\quad t < 0$

$i_L(0^-) = i_L(0^+) = 12\,\text{A}$

$t > 0$

Find Thévenin resistance seen by inductor:

$i_T = 2.5v_T; \qquad \frac{v_T}{i_T} = R_{\text{Th}} = \frac{1}{2.5} = 0.4\,\Omega$

$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{0.4} = 50\,\text{ms}; \qquad 1/\tau = 20$

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$$i_o = 12e^{-20t}\,\text{A}, \qquad t \geq 0$$

$$v_o = L\frac{di_o}{dt} = (20 \times 10^{-3})(-240e^{-20t}) = -4.8e^{-20t}\,\text{V}, \quad t \geq 0^+$$

P 7.15 **[a]** $t < 0$:

$$i_L(0) = -\frac{72}{24+6} = -2.4\,\text{A}$$

$t > 0$:

$$i_\Delta = -\frac{100}{160}i_T = -\frac{5}{8}i_T$$

$$v_T = 20i_\Delta + i_T\frac{(100)(60)}{160} = -12.5i_T + 37.5i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = -12.5 + 37.5 = 25\,\Omega$$

$$\tau = \frac{L}{R} = \frac{250 \times 10^{-3}}{25} \qquad \frac{1}{\tau} = 100$$

$$i_L = -2.4e^{-100t}\,\text{A}, \qquad t \geq 0$$

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[b] $v_L = 250 \times 10^{-3}(240e^{-100t}) = 60e^{-100t}\,\text{V}, \quad t \geq 0^+$

[c] $i_\Delta = 0.625 i_L = -1.5e^{-100t}\,\text{A} \qquad t \geq 0^+$

P 7.16 $w(0) = \dfrac{1}{2}(250 \times 10^{-3})(-2.4)^2 = 720\,\text{mJ}$

$$p_{60\Omega} = 60(-1.5e^{-100t})^2 = 135e^{-200t}\,\text{W}$$

$$w_{60\Omega} = \int_0^\infty 135e^{-200t}\,dt = 135\frac{e^{-200t}}{-200}\bigg|_0^\infty = 675\,\text{mJ}$$

$$\%\text{ dissipated } = \frac{675}{720}(100) = 93.75\%$$

P 7.17 **[a]** $t > 0$:

$$L_{\text{eq}} = 1.25 + \frac{60}{16} = 5\,\text{H}$$

$$i_L(t) = i_L(0)e^{-t/\tau}\,\text{mA}; \qquad i_L(0) = 2\,\text{A}; \qquad \frac{1}{\tau} = \frac{R}{L} = \frac{7500}{5} = 1500$$

$$i_L(t) = 2e^{-1500t}\,\text{A}, \qquad t \geq 0$$

$$v_R(t) = Ri_L(t) = (7500)(2e^{-1500t}) = 15{,}000e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

$$v_o = -3.75\frac{di_L}{dt} = 11{,}250e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_o = \dfrac{-1}{6}\displaystyle\int_0^t 11{,}250e^{-1500x}\,dx + 0 = 1.25e^{-1500t} - 1.25\,\text{A}$

P 7.18 **[a]** From the solution to Problem 7.17,

$$w(0) = \frac{1}{2}L_{\text{eq}}[i_L(0)]^2 = \frac{1}{2}(5)(2)^2 = 10\,\text{J}$$

[b] $w_{\text{trapped}} = \dfrac{1}{2}(10)(1.25)^2 + \dfrac{1}{2}(6)(1.25)^2 = 12.5\,\text{J}$

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P 7.19 **[a]** $t < 0$

$$i_L(0^-) = \frac{150}{180}(12) = 10\,\text{A}$$

$t \geq 0$

$$\tau = \frac{1.6 \times 10^{-3}}{8} = 200 \times 10^{-6}; \qquad 1/\tau = 5000$$

$$i_o = -10e^{-5000t}\,\text{A} \quad t \geq 0$$

[b] $w_{\text{del}} = \dfrac{1}{2}(1.6 \times 10^{-3})(10)^2 = 80\,\text{mJ}$

[c] $0.95 w_{\text{del}} = 76\,\text{mJ}$

$$\therefore \quad 76 \times 10^{-3} = \int_0^{t_o} 8(100e^{-10{,}000t})\,dt$$

$$\therefore \quad 76 \times 10^{-3} = -80 \times 10^{-3} e^{-10{,}000t}\Big|_0^{t_o} = 80 \times 10^{-3}(1 - e^{-10{,}000t_o})$$

$$\therefore \quad e^{-10{,}000t_o} = 0.05 \quad \text{so} \quad t_o = 299.57\,\mu\text{s}$$

$$\therefore \quad \frac{t_o}{\tau} = \frac{299.57 \times 10^{-6}}{200 \times 10^{-6}} = 1.498 \quad \text{so} \quad t_o \approx 1.498\tau$$

P 7.20 **[a]** $t < 0$:

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$t = 0^+$:

$$120 = i_{\text{ab}} + 18 + 12, \qquad i_{\text{ab}} = 90\,\text{A}, \quad t = 0^+$$

[b] At $t = \infty$:

$$i_{\text{ab}} = 240/2 = 120\,\text{A}, \quad t = \infty$$

[c] $i_1(0) = 18, \qquad \tau_1 = \dfrac{2 \times 10^{-3}}{10} = 0.2\,\text{ms}$

$$i_2(0) = 12, \qquad \tau_2 = \frac{6 \times 10^{-3}}{15} = 0.4\,\text{ms}$$

$$i_1(t) = 18e^{-5000t}\,\text{A}, \quad t \geq 0$$

$$i_2(t) = 12e^{-2500t}\,\text{A}, \quad t \geq 0$$

$$i_{\text{ab}} = 120 - 18e^{-5000t} - 12e^{-2500t}\,\text{A}, \quad t \geq 0$$

$$120 - 18e^{-5000t} - 12e^{-2500t} = 114$$

$$6 = 18e^{-5000t} + 12e^{-2500t}$$

Let $\quad x = e^{-2500t} \qquad$ so $\qquad 6 = 18x^2 + 12x$

Solving $\quad x = \dfrac{1}{3} = e^{-2500t}$

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$$\therefore \quad e^{2500t} = 3 \qquad \text{and} \qquad t = \frac{\ln 3}{2500} = 439.44\,\mu\text{s}$$

P 7.21 **[a]** For $t < 0$:

$$v(0) = 20{,}000(0.01) = 200\,\text{V}$$

[b] $w(0) = \dfrac{1}{2}Cv(0)^2 = \dfrac{1}{2}(400 \times 10^{-9})(200)^2 = 8\,\text{mJ}$

[c] For $t > 0$:

$$R_{\text{eq}} = 10{,}000 + 50{,}000\|75{,}000 = 40\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (40{,}000)(400 \times 10^{-9}) = 16\,\text{ms}$$

[d] $v(t) = v(0)e^{-t/\tau} = 200e^{-62.5t}\,\text{V} \qquad t \geq 0$

P 7.22 For $t < 0$:

$$V_o = (20{,}000\|60{,}000)(20 \times 10^{-3}) = 300\,\text{V}$$

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For $t \geq 0$:

$$R_{\text{eq}} = 10{,}000 + (20{,}000 \| 60{,}000) = 25\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (25{,}000)(40 \times 10^{-9}) = 1\,\text{ms}$$

$$v(t) = V_o e^{-t/\tau} = 300e^{-1000t}\,\text{V} \qquad t \geq 0$$

P 7.23 **[a]** For $t < 0$:

$$V_o = \frac{10{,}000}{15{,}000}(120) = 80\,\text{V}$$

For $t \geq 0$:

$$R_{\text{eq}} = 25{,}000 \| (40{,}000 + 10{,}000) = 16.67\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (16{,}666/67)(160 \times 10^{-9}) = 2.67\,\text{ms}$$

$$v(t) = V_o e^{-t/\tau} = 80e^{-375t}\,\text{V} \qquad t \geq 0$$

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[b] For $t \geq 0$:

$$v_{\mathrm{R}}(t) = \frac{40{,}000}{50{,}000}(80e^{-375t}) = 64e^{-375t}\,\mathrm{V}$$

$$i(t) = \frac{v_{\mathrm{R}}}{40{,}000} = 1.6e^{-375t}\,\mathrm{mA}, \quad t \geq 0^+$$

P 7.24 Using the results of Problem 7.23:

$$w(0) = \frac{1}{2}CV_o^2 = \frac{1}{2}(160 \times 10^{-9})(80)^2 = 512\,\mu\mathrm{J}$$

$$p_{40\mathrm{k}} = Ri^2 = (40{,}000)(1.6 \times 10^{-3}e^{-375t})^2 = 0.1024e^{-750t}$$

$$w_{40\mathrm{k}} = \int_0^\infty p_{40k}\,dt = \int_0^\infty 0.1024e^{-750t}\,dt = \frac{0.1024e^{-750t}}{-750}\bigg|_0^\infty = 136.53\,\mu\mathrm{J}$$

$$\text{percent} = \frac{136.53}{512}(100) = 26.67\%$$

P 7.25 **[a]** $v_1(0^-) = v_1(0^+) = (0.006)(5000) = 30\,\mathrm{V}$ $\qquad v_2(0^+) = 0$

$$C_{\mathrm{eq}} = (30)(40)/90 = 20\,\mu\mathrm{F}$$

$$\tau = (2.5 \times 10^3)(20 \times 10^{-6}) = 50\mathrm{ms}; \qquad \frac{1}{\tau} = 20$$

$$i = \frac{30}{2500}e^{-20t} = 12e^{-20t}\,\mathrm{mA}, \qquad t \geq 0^+$$

$$v_1 = \frac{-1}{30^{-6}}\int_0^t 12 \times 10^{-3}e^{-20x}\,dx + 30 = 20e^{-20t} + 10\,\mathrm{V}, \qquad t \geq 0$$

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$$v_2 = \frac{1}{60 \times 10^{-6}} \int_0^t 12 \times 10^{-3} e^{-20x}\, dx + 0 = -10e^{-20t} + 10\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(30 \times 10^{-6})(30)^2 = 13.5\,\text{mJ}$

[c] $w_{\text{trapped}} = \frac{1}{2}(30 \times 10^{-6})(10)^2 + \frac{1}{2}(60 \times 10^{-6})(10)^2 = 4.5\,\text{mJ}.$

The energy dissipated by the 2.5 kΩ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors:

$$w_{\text{diss}} = \frac{1}{2}(20 \times 10^{-6})(30)^2 = 9\,\text{mJ}.$$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 4.5 + 9 = 13.5\,\text{mJ}; \qquad w(0) = 13.5\,\text{mJ}.$

P 7.26 **[a]** $t < 0$:

$$R_{\text{eq}} = 12\,\text{k} \| 8\,\text{k} = 10.2\,\text{k}\Omega$$

$$v_o(0) = \frac{10{,}200}{10{,}200 + 1800}(-120) = -102\,\text{V}$$

$t > 0$:

$$\tau = [(10/3) \times 10^{-6})(12{,}000) = 40\,\text{ms}; \qquad \frac{1}{\tau} = 25$$

$$v_o = -102e^{-25t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12{,}000} = 867 \times 10^{-3} e^{-50t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{12\times10^{-3}} 867 \times 10^{-3} e^{-50t}\, dt$$
$$= 17.34 \times 10^{-3}(1 - e^{-50(12\times10^{-3})}) = 7824\,\mu\text{J}$$

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[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{10}{3}\right)(102)^2 \times 10^{-6} = 17.34\,\text{mJ}$

$0.75w(0) = 13\,\text{mJ}$

$\int_0^{t_o} 867 \times 10^{-3} e^{-50x}\,dx = 13 \times 10^{-3}$

$\therefore \quad 1 - e^{-50t_o} = 0.75; \qquad e^{50t_o} = 4; \quad \text{so} \quad t_o = 27.73\,\text{ms}$

P 7.27 **[a]** $t < 0$:

$i_1(0^-) = i_2(0^-) = \frac{3}{30} = 100\,\text{mA}$

[b] $t > 0$:

$i_1(0^+) = \frac{0.2}{2} = 100\,\text{mA}$

$i_2(0^+) = \frac{-0.2}{8} = -25\,\text{mA}$

[c] Capacitor voltage cannot change instantaneously, therefore,

$i_1(0^-) = i_1(0^+) = 100\,\text{mA}$

[d] Switching can cause an instantaneous change in the current in a resistive branch. In this circuit

$i_2(0^-) = 100\,\text{mA} \quad \text{and} \quad i_2(0^+) = 25\,\text{mA}$

[e] $v_c = 0.2e^{-t/\tau}\,\text{V}, \qquad t \geq 0$

$\tau = R_eC = 1.6(2 \times 10^{-6}) = 3.2\,\mu\text{s}; \qquad \frac{1}{\tau} = 312{,}500$

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$$v_c = 0.2e^{-312{,}000t}\,\text{V}, \qquad t \geq 0$$

$$i_1 = \frac{v_c}{2} = 0.1e^{-312{,}000t}\,\text{A}, \qquad t \geq 0$$

[f] $i_2 = \dfrac{-v_c}{8} = -25e^{-312{,}000t}\,\text{mA}, \qquad t \geq 0^+$

P 7.28 $t < 0$

$t > 0$

$$v_T = -5i_o - 15i_o = -20i_o = 20i_T \quad \therefore \quad R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\Omega$$

$$\tau = RC = 40\,\mu\text{s}; \qquad \frac{1}{\tau} = 25{,}000$$

$$v_o = 15e^{-25{,}000t}\,\text{V}, \qquad t \geq 0$$

$$i_o = -\frac{v_o}{20} = -0.75e^{-25{,}000t}\,\text{A}, \qquad t \geq 0^+$$

P 7.29 **[a]** $R = \dfrac{v}{i} = 8\,\text{k}\Omega$

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[b] $\dfrac{1}{\tau} = \dfrac{1}{RC} = 500; \qquad C = \dfrac{1}{(500)(8000)} = 0.25\,\mu\text{F}$

[c] $\tau = \dfrac{1}{500} = 2\,\text{ms}$

[d] $w(0) = \dfrac{1}{2}(0.25 \times 10^{-6})(72)^2 = 648\,\mu\text{J}$

[e] $w_{\text{diss}} = \displaystyle\int_0^{t_o} \frac{(72)^2 e^{-1000t}}{(800)} dt$

$$= 0.648 \frac{e^{-1000t}}{-1000}\Bigg|_0^{t_o} = 648(1 - e^{-1000t_o})\,\mu\text{J}$$

$$\%\text{diss} = 100(1 - e^{-1000t_o}) = 68 \qquad \text{so} \qquad e^{1000t_o} = 3.125$$

$$\therefore \quad t = \frac{\ln 3.125}{1000} = 1139\,\mu\text{s}$$

P 7.30 **[a]** Note that there are many different possible correct solutions to this problem.

$R = \dfrac{\tau}{C}$

Choose a $100\,\mu$F capacitor from Appendix H. Then,

$R = \dfrac{0.05}{100 \times 10^{-6}} = 500\,\Omega$

Construct a $500\,\Omega$ resistor by combining two $1\,\text{k}\Omega$ resistors in parallel:

[b] $v(t) = V_o e^{-t/\tau} = 50e^{-20t}\,\text{V}, \qquad t \geq 0$

[c] $50e^{-20t} = 10 \qquad \text{so} \qquad e^{20t} = 5$

$$\therefore \quad t = \frac{\ln 5}{20} = 80.47\,\text{ms}$$

P 7.31 **[a]**

$$v_T = 20 \times 10^3(i_T + \alpha v_\Delta) + 5 \times 10^3 i_T$$

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$$v_\Delta = 5 \times 10^3 i_T$$

$$v_T = 25 \times 10^3 i_T + 20 \times 10^3 \alpha (5 \times 10^3 i_T)$$

$$R_{\text{Th}} = 25{,}000 + 100 \times 10^6 \alpha$$

$$\tau = R_{\text{Th}} C = 40 \times 10^{-3} = R_{\text{Th}}(0.8 \times 10^{-6})$$

$$R_{\text{Th}} = 50\,\text{k}\Omega = 25{,}000 + 100 \times 10^6 \alpha$$

$$\alpha = \frac{25{,}000}{100 \times 10^6} = 2.5 \times 10^{-4}\ \text{A/V}$$

[b] $v_o(0) = (-5 \times 10^{-3})(3600) = -18\,\text{V} \qquad t < 0$

$t > 0$:

$$v_o = -18e^{-25t}\,\text{V}, \quad t \geq 0$$

$$\frac{v_\Delta}{5000} + \frac{v_\Delta - v_o}{20{,}000} + 2.5 \times 10^{-4} v_\Delta = 0$$

$$4v_\Delta + v_\Delta - v_o + 5v_\Delta = 0$$

$$\therefore \quad v_\Delta = \frac{v_o}{10} = -1.8e^{-25t}\,\text{V}, \quad t \geq 0^+$$

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P 7.32 **[a]**

$$p_{ds} = (16.2e^{-25t})(-450 \times 10^{-6}e^{-25t}) = -7290 \times 10^{-6}e^{-50t}\,\text{W}$$

$$w_{ds} = \int_0^\infty p_{ds}\,dt = -145.8\,\mu\text{J}.$$

$\therefore$ dependent source is delivering $145.8\,\mu$J.

[b] $$w_{5\text{k}} = \int_0^\infty (5000)(0.36 \times 10^{-3}e^{-25t})^2\,dt = 648 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 12.96\,\mu\text{J}$$

$$w_{20\text{k}} = \int_0^\infty \frac{(16.2e^{-25t})^2}{20{,}000}\,dt = 13{,}122 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 262.44\,\mu\text{J}$$

$$w_c(0) = \frac{1}{2}(0.8 \times 10^{-6})(18)^2 = 129.6\,\mu\text{J}$$

$$\sum w_{\text{diss}} = 12.96 + 262.44 = 275.4\,\mu\text{J}$$

$$\sum w_{\text{dev}} = 145.8 + 129.6 = 275.4\,\mu\text{J}.$$

P 7.33 **[a]** At $t = 0^-$ the voltage on each capacitor will be $6\,\text{V}\,(0.075 \times 80)$, positive at the upper terminal. Hence at $t \geq 0^+$ we have

$$\therefore\quad i_{sd}(0^+) = 0.075 + \frac{6}{200} + \frac{6}{400} = 120\,\text{mA}$$

At $t = \infty$, both capacitors will have completely discharged.

$$\therefore\quad i_{sd}(\infty) = 75\,\text{mA}$$

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[b] $i_{sd}(t) = 0.075 + i_1(t) + i_2(t)$

$$\tau_1 = 200(25 \times 10^{-6}) = 5\,\text{ms}$$

$$\tau_2 = 400(50 \times 10^{-6}) = 20\,\text{ms}$$

$$\therefore \quad i_1(t) = 30e^{-200t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_2(t) = 15e^{-50t}\,\text{mA}, \qquad t \geq 0$$

$$\therefore \quad i_{sd} = 75 + 30e^{-200t} + 15e^{-50t}\,\text{mA}, \qquad t \geq 0^+$$

P 7.34 **[a]** The equivalent circuit for $t > 0$:

$$\tau = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o = 10e^{-500t}\,\text{V}, \qquad t \geq 0$$

$$i_o = e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{24\text{k}\Omega} = e^{-500t}\left(\frac{16}{40}\right) = 0.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$p_{24\text{k}\Omega} = (0.16 \times 10^{-6}e^{-1000t})(24{,}000) = 3.84e^{-1000t}\,\text{mW}$$

$$w_{24\text{k}\Omega} = \int_0^\infty 3.84 \times 10^{-3}e^{-1000t}\,dt = -3.84 \times 10^{-6}(0-1) = 3.84\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.25 \times 10^{-6})(40)^2 + \frac{1}{2}(1 \times 10^{-6})(50)^2 = 1.45\,\text{mJ}$$

$$\%\ \text{diss}\,(24\,\text{k}\Omega) = \frac{3.84 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.26\%$$

[b] $p_{400\Omega} = 400(1 \times 10^{-3}e^{-500t})^2 = 0.4 \times 10^{-3}e^{-1000t}$

$$w_{400\Omega} = \int_0^\infty p_{400}\,dt = 0.40\,\mu\text{J}$$

$$\%\ \text{diss}\,(400\,\Omega) = \frac{0.4 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.03\%$$

$$i_{16k\Omega} = e^{-500t}\left(\frac{24}{40}\right) = 0.6e^{-500t}\,\text{mA}, \quad t \geq 0^+$$

$$p_{16k\Omega} = (0.6 \times 10^{-3}e^{-500t})^2(16{,}000) = 5.76 \times 10^{-3}e^{-1000t}\,\text{W}$$

$$w_{16k\Omega} = \int_0^\infty 5.76 \times 10^{-3}e^{-1000t}\,dt = 5.76\,\mu\text{J}$$

$$\%\ \text{diss}\,(16\,\text{k}\Omega) = 0.4\%$$

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[c] $\sum w_{\text{diss}} = 3.84 + 5.76 + 0.4 = 10\,\mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 1.45 \times 10^{-3} - 10 \times 10^{-6} = 1.44\,\text{mJ}$$

$$\%\ \text{trapped} = \frac{1.44}{1.45} \times 100 = 99.31\%$$

Check: $0.26 + 0.03 + 0.4 + 99.31 = 100\%$

P 7.35 **[a]** For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 200 mH inductor. We get

$$i(0^-) = \frac{30 - 250}{25\,\text{k} + 30\,\text{k}} = -4\,\text{mA}$$

$$i(0^-) = i(0^+) = -4\,\text{mA}$$

[b] For $t > 0$, the circuit reduces to

Therefore $i(\infty) = 30/30{,}000 = 1\,\text{mA}$

[c] $\tau = \dfrac{L}{R} = \dfrac{200 \times 10^{-3}}{30{,}000} = 6.67\,\mu\text{s}$

[d] $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

$$= 0.001 + [-0.004 - 0.001]e^{-150{,}000t} = 1 - 5e^{-150{,}000t}\,\text{mA}, \qquad t \geq 0$$

P 7.36 **[a]** $t < 0$

$$i_L(0^-) = \frac{32}{20} = 1.6\,\text{A}$$

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$t > 0$

$$i_L(\infty) = \frac{32-48}{12+8} = -0.8\,\text{A}$$

$$\tau = \frac{L}{R} = \frac{5\times 10^{-3}}{12+8} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$= -0.8 + (1.6+0.8)e^{-4000t} = -0.8 + 2.4e^{-4000t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 8i_L + 48 = 8(-0.8 + 2.4e^{-4000t}) + 48 = 41.6 + 19.2e^{-4000t}\,\text{V}, \qquad t \geq 0$$

[b] $v_L = L\dfrac{di_L}{dt} = 5\times 10^{-3}(-4000)[2.4e^{-4000t}] = -48e^{-4000t}\,\text{V}, \qquad t \geq 0^+$

$v_L(0^+) = -48\,\text{V}$

From part (a) $\quad v_o(0^+) = 0\,\text{V}$

Check: at $t = 0^+$ the circuit is:

$v_o(0^+) = 48 + (8\,\Omega)(1.6\,\text{A}) = 60.8\,\text{V}; \qquad v_\text{L}(0^+) + v_o(0^+) = 12(-1.6) + 32$

$\therefore \qquad v_\text{L}(0^+) = -19.2 + 32 - 60.8 = -48\,\text{V}$

P 7.37 **[a]** $t < 0$

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KVL equation at the top node:

$$\frac{v_o - 240}{60} + \frac{v_o}{20} + \frac{v_o}{5} = 0$$

Multiply by 60 and solve:

$$240 = (3 + 1 + 12)v_o; \qquad v_o = 15\,\text{V}$$

$$\therefore \quad i_o(0^-) = \frac{v_o}{5} = 15/5 = 3\,\text{A}$$

$t > 0$

Use voltage division to find the Thévenin voltage:

$$V_{\text{Th}} = v_o = \frac{20}{20 + 5}(225) = 180\,\text{V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{\text{Th}} = 5 + 20\|5 = 5 + 4 = 9\,\Omega$$

The simplified circuit is:

$$\tau = \frac{L}{R} = \frac{10 \times 10^{-3}}{9} = 1.11\,\text{ms}; \qquad \frac{1}{\tau} = 900$$

$$i_o(\infty) = \frac{190}{9} = 20\,\text{A}$$

$$\begin{aligned} \therefore \quad i_o &= i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau} \\ &= 20 + (3 - 20)e^{-900t} = 20 - 17e^{-900t}\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[b]

$$\begin{aligned} v_o &= 5i_o + L\frac{di_o}{dt} \\ &= 5(20 - 17e^{-900t}) + 0.01(-900)(17e^{-900t}) \\ &= 100 - 85e^{-900t} + 153e^{-900t} \\ v_o &= 100 + 68e^{-900t}\,\text{V}, \qquad t \geq 0^+ \end{aligned}$$

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P 7.38 **[a]** $t < 0$

$$i_g = \frac{225}{5 + 20\|5} = \frac{225}{9} = 25\,\text{A}$$

$$\therefore \quad i_o(0^-) = \frac{20\|5}{5}(25) = 20\,\text{A}$$

$t > 0$

$$i_g(\infty) = \frac{240}{60 + 20\|5} = \frac{240}{64} = 3.75\,\text{A}$$

$$i_o(\infty) = \frac{20\|5}{5} i_g(\infty) = 3\,\text{A}$$

$$R_{\text{eq}} = 5 + 20\|60 = 3 + 15 = 20\,\Omega$$

$$\tau = \frac{L}{R_{\text{eq}}} = \frac{10 \times 10^{-3}}{20} = 0.5\,\text{ms}; \qquad \frac{1}{\tau} = 2000$$

$$\therefore \quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 3 + (20 - 3)e^{-2000t} = 3 + 17e^{-2000t}\,\text{A}, \qquad t \geq 0$$

[b]

$$\begin{aligned} v_o &= 5i_o + L\frac{di_o}{dt} \\ &= 5(3 + 17e^{-2000t}) + 0.01(-2000)(17e^{-2000t}) \\ &= 15 + 65e^{-2000t} - 340e^{-2000t} \\ v_o &= 15 - 255e^{-2000t}\,\text{V}, \qquad t \geq 0^+ \end{aligned}$$

P 7.39 **[a]** From Eqs. (7.35) and (7.42)

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right) e^{-(R/L)t}$$

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$$v = (V_s - I_oR)e^{-(R/L)t}$$

$$\therefore \quad \frac{V_s}{R} = 4; \qquad I_o - \frac{V_s}{R} = 4$$

$$V_s - I_oR = -80; \qquad \frac{R}{L} = 40$$

$$\therefore \quad I_o = 4 + \frac{V_s}{R} = 8\,\text{A}$$

Now since $V_s = 4R$ we have

$$4R - 8R = -80; \qquad R = 20\,\Omega$$

$$V_s = 80\,\text{V}; \qquad L = \frac{R}{40} = 0.5\,\text{H}$$

[b] $i = 4 + 4e^{-40t}; \qquad i^2 = 16 + 32e^{-40t} + 16e^{-80t}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.5)[16 + 32e^{-40t} + 16e^{-80t}] = 4 + 8e^{-40t} + 4e^{-80t}$$

$$\therefore \quad 4 + 8e^{-40t} + 4e^{-80t} = 9 \quad \text{or} \quad e^{-80t} + 2e^{-40t} - 1.25 = 0$$

Let $x = e^{-40t}$:

$x^2 + 2x - 1.25 = 0;$ Solving, $x = 0.5;$ $x = -2.5$

But $x \geq 0$ for all t. Thus,

$$e^{-40t} = 0.5; \qquad e^{40t} = 2; \qquad t = 25\ln 2 = 17.33\,\text{ms}$$

P 7.40 **[a]** Note that there are many different possible solutions to this problem.

$$R = \frac{L}{\tau}$$

Choose a 1 mH inductor from Appendix H. Then,

$$R = \frac{0.001}{8 \times 10^{-6}} = 125\,\Omega$$

Construct the resistance needed by combining $100\,\Omega$, $10\,\Omega$, and $15\,\Omega$ resistors in series:

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[b] $i(t) = I_f + (I_o - I_f)e^{-t/\tau}$

$$I_o = 0\,\text{A}; \qquad I_f = \frac{V_f}{R} = \frac{25}{125} = 200\,\text{mA}$$

$$\therefore \quad i(t) = 200 + (0 - 200)e^{-125{,}000t}\,\text{mA} = 200 - 200e^{-125{,}000t}\,\text{mA}, \qquad t \geq 0$$

[c] $i(t) = 0.2 - 0.2e^{-125{,}000t} = (0.75)(0.2) = 0.15$

$$e^{-125{,}000t} = 0.25 \qquad \text{so} \qquad e^{125{,}000t} = 4$$

$$\therefore \quad t = \frac{\ln 4}{125{,}000} = 11.09\,\mu\text{s}$$

P 7.41 **[a]** $v_o(0^+) = -I_g R_2; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_g R_2 e^{-[(R_1+R_2)/L]t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_o(0^+) \to \infty$, and the duration of $v_o(t) \to$ zero

[c] $v_{sw} = R_2 i_o; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g \frac{R_1}{R_1 + R_2}$$

Therefore $\quad i_o(t) = \frac{I_g R_1}{R_1+R_2} + \left[I_g - \frac{I_g R_1}{R_1+R_2}\right] e^{-[(R_1+R_2)/L]t}$

$$i_o(t) = \frac{R_1 I_g}{(R_1+R_2)} + \frac{R_2 I_g}{(R_1+R_2)} e^{-[(R_1+R_2)/L]t}$$

Therefore $\quad v_{sw} = \frac{R_1 I_g}{(1+R_1/R_2)} + \frac{R_2 I_g}{(1+R_1/R_2)} e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0^+$

[d] $|v_{sw}(0^+)| \to \infty; \qquad$ duration $\to 0$

P 7.42 Opening the inductive circuit causes a very large voltage to be induced across the inductor L. This voltage also appears across the switch (part [d] of Problem 7.41), causing the switch to arc over. At the same time, the large voltage across L damages the meter movement.

P 7.43 **[a]**

$$-\frac{V_s}{R} + \frac{v}{R} + \frac{1}{L}\int_0^t v\,dt + I_o = 0$$

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Differentiating both sides,

$$\frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

$$\therefore \qquad \frac{dv}{dt} + \frac{R}{L}v = 0$$

[b] $\dfrac{dv}{dt} = -\dfrac{R}{L}v$

$$\frac{dv}{dt}dt = -\frac{R}{L}v\,dt \qquad \text{so} \qquad dv = -\frac{R}{L}v\,dt$$

$$\frac{dv}{v} = -\frac{R}{L}dt$$

$$\int_{V_o}^{v(t)} \frac{dx}{x} = -\frac{R}{L}\int_0^t dy$$

$$\ln\frac{v(t)}{V_o} = -\frac{R}{L}t$$

$$\therefore \qquad v(t) = V_o e^{-(R/L)t} = (V_\text{s} - RI_o)e^{-(R/L)t}$$

P 7.44 For $t < 0$

$$\frac{v_x}{50} - 0.1v_\phi + \frac{v_x - 150}{75} = 0$$

$$v_\phi = \frac{40}{75}(v_x - 150)$$

Solving,

$$v_x = 300\,\text{V}; \qquad\qquad i_o(0^-) = \frac{v_x}{50} = 6\,\text{A}$$

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$t > 0$

Find Thévenin equivalent with respect to a, b. Use a test source to find the Thévenin equivalent resistance:

$$-1 - 0.1v_\phi + \frac{v_\mathrm{T} - v_x}{20} = 0$$

$$\frac{v_x - v_\mathrm{T}}{20} + \frac{v_x}{10} + \frac{v_x}{55} = 0$$

$$v_\phi = \frac{40}{55} v_x$$

Solving,

$$v_\mathrm{T} = 74\,\mathrm{V} \qquad \text{so} \qquad R_\mathrm{Th} = \frac{v_\mathrm{T}}{1\,\mathrm{A}} = 74\,\Omega$$

Find the open circuit voltage with respect to a, b:

$$-0.1v_\phi + \frac{v_\mathrm{Th} - v_x}{20} = 0$$

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$$\frac{v_x - v_{\text{Th}}}{20} + \frac{v_x - 140}{10} + \frac{v_x - 150}{55} = 0$$

$$v_\phi = \frac{40}{55}(v_x - 150)$$

Solving,

$$v_{\text{Th}} = 96\,\text{V}$$

$$i_o(\infty) = 96/124 = 0.774\,\text{A}$$

$$\tau = \frac{40 \times 10^{-3}}{124} = 0.3226\,\text{ms}; \qquad 1/\tau = 3100$$

$$i_o = 0.774 + (6 - 0.774)e^{-3100t} = 0.774 + 5.226e^{-3100t}\,\text{A}, \qquad t \geq 0$$

P 7.45 $t > 0$; calculate $v_o(0^+)$

$$\frac{v_\text{a}}{15} + \frac{v_\text{a} - v_o(0^+)}{5} = 20 \times 10^{-3}$$

$$\therefore \quad v_\text{a} = 0.75v_o(0^+) + 75 \times 10^{-3}$$

$$15 \times 10^{-3} + \frac{v_o(0^+) - v_\text{a}}{5} + \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3} = 0$$

$$13v_o(0^+) - 8v_\text{a} - 360i_\Delta = -2600 \times 10^{-3}$$

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$$i_\Delta = \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3}$$

$$\therefore \quad i_\Delta = \frac{v_o(0^+)}{80} + 5 \times 10^{-3}$$

$$\therefore \quad 360 i_\Delta = 4.5 v_o(0^+) + 1800 \times 10^{-3}$$

$$8 v_{\mathrm{a}} = 6 v_o(0^+) + 600 \times 10^{-3}$$

$$\therefore \quad 13 v_o(0^+) - 6 v_o(0^+) - 600 \times 10^{-3} - 4.5 v_o(0^+) -$$

$$1800 \times 10^{-3} = -2600 \times 10^{-3}$$

$$2.5 v_o(0^+) = -200 \times 10^{-3}; \qquad v_o(0^+) = -80\,\mathrm{mV}$$

$$v_o(\infty) = 0$$

Find the Thévenin resistance seen by the 4 mH inductor:

$$i_T = \frac{v_T}{20} + \frac{v_T}{8} - 9i_\Delta$$

$$i_\Delta = \frac{v_T}{8} - 9i_\Delta \qquad \therefore \quad 10 i_\Delta = \frac{v_T}{8}; \qquad i_\Delta = \frac{v_T}{80}$$

$$i_T = \frac{v_T}{20} + \frac{10 v_T}{80} - \frac{9 v_T}{80}$$

$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{80} = \frac{5}{80} = \frac{1}{16}\,\mathrm{S}$$

$$\therefore \quad R_{\mathrm{Th}} = 16\Omega$$

$$\tau = \frac{4 \times 10^{-3}}{16} = 0.25\,\mathrm{ms}; \qquad 1/\tau = 4000$$

$$\therefore \quad v_o = 0 + (-80 - 0)e^{-4000t} = -80 e^{-4000t}\,\mathrm{mV}, \qquad t \geq 0^+$$

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P 7.46 For $t < 0$, $\quad i_{45\text{mH}}(0) = 80\,\text{V}/2000\,\Omega = 40\,\text{mA}$

For $t > 0$, after making a Thévenin equivalent of the circuit to the right of the inductors we have

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right) e^{-t/\tau}$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{1200}{60 \times 10^{-3}} = 20{,}000$$

$$I_o = 40\,\text{mA}; \qquad I_f = \frac{V_s}{R} = \frac{60}{1200} = 50\,\text{mA}$$

$$i = 0.05 + (0.04 - 0.05)e^{-20{,}000t} = 50 - 10e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0$$

$$v_o = 0.045\frac{di}{dt} = 0.045(-0.01)(-20{,}000e^{-20{,}000t}) = 9e^{-20{,}000t}\,\text{V}, \qquad t \geq 0^+$$

P 7.47 $t > 0$

$$\tau = \frac{1}{40}$$

$$i_o = 5e^{-40t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 40i_o = 200e^{-40t}\,\text{V}, \qquad t > 0^+$$

$$200e^{-40t} = 100; \qquad e^{40t} = 2$$

$$\therefore \quad t = \frac{1}{40}\ln 2 = 17.33\,\text{ms}$$

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P 7.48 **[a]** $w_{\text{diss}} = \dfrac{1}{2}L_e i^2(0) = \dfrac{1}{2}(1)(5)^2 = 12.5\,\text{J}$

[b] $i_{3H} = \dfrac{1}{3}\displaystyle\int_0^t (200)e^{-40x}\,dx - 5$

$$= 1.67(1 - e^{-40t}) - 5 = -1.67e^{-40t} - 3.33\,\text{A}$$

$$i_{1.5H} = \frac{1}{1.5}\int_0^t (200)e^{-40x}\,dx + 0$$

$$= -3.33e^{-40t} + 3.33\,\text{A}$$

$$w_{\text{trapped}} = \frac{1}{2}(4.5)(3.33)^2 = 25\,\text{J}$$

[c] $w(0) = \dfrac{1}{2}(3)(5)^2 = 37.5\,\text{J}$

P 7.49 **[a]** $t < 0$

$t > 0$

$$i_L(0^-) = i_L(0^+) = 25\,\text{mA}; \qquad \tau = \frac{24 \times 10^{-3}}{120} = 0.2\,\text{ms}; \qquad \frac{1}{\tau} = 5000$$

$$i_L(\infty) = -50\,\text{mA}$$

$$i_L = -50 + (25 + 50)e^{-5000t} = -50 + 75e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

$$v_o = -120[75 \times 10^{-3}e^{-5000t}] = -9e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_1 = \dfrac{1}{60 \times 10^{-3}}\displaystyle\int_0^t -9e^{-5000x}\,dx + 10 \times 10^{-3} = (30e^{-5000t} - 20)\,\text{mA}, \qquad t \geq 0$

[c] $i_2 = \dfrac{1}{40 \times 10^{-3}}\displaystyle\int_0^t -9e^{-5000x}\,dx + 15 \times 10^{-3} = (45e^{-5000t} - 30)\,\text{mA}, \qquad t \geq 0$

P 7.50 **[a]** Let v be the voltage drop across the parallel branches, positive at the top node, then

$$-I_g + \frac{v}{R_g} + \frac{1}{L_1}\int_0^t v\,dx + \frac{1}{L_2}\int_0^t v\,dx = 0$$

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$$\frac{v}{R_g} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_0^t v\,dx = I_g$$

$$\frac{v}{R_g} + \frac{1}{L_e}\int_0^t v\,dx = I_g$$

$$\frac{1}{R_g}\frac{dv}{dt} + \frac{v}{L_e} = 0$$

$$\frac{dv}{dt} + \frac{R_g}{L_e}v = 0$$

Therefore $\quad v = I_g R_g e^{-t/\tau}; \qquad \tau = L_e/R_g$

Thus

$$i_1 = \frac{1}{L_1}\int_0^t I_g R_g e^{-x/\tau}\,dx = \frac{I_g R_g}{L_1}\frac{e^{-x/\tau}}{(-1/\tau)}\Big|_0^t = \frac{I_g L_e}{L_1}(1 - e^{-t/\tau})$$

$$i_1 = \frac{I_g L_2}{L_1 + L_2}(1 - e^{-t/\tau}) \quad \text{and} \quad i_2 = \frac{I_g L_1}{L_1 + L_2}(1 - e^{-t/\tau})$$

[b] $i_1(\infty) = \dfrac{L_2}{L_1 + L_2}I_g; \qquad i_2(\infty) = \dfrac{L_1}{L_1 + L_2}I_g$

P 7.51 **[a]** $v_c(0^+) = -120\,\text{V}$

[b] Use voltage division to find the final value of voltage:

$$v_c(\infty) = \frac{150{,}000}{200{,}000}(200) = 150\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = 150\,\text{V}, \qquad R_{\text{Th}} = 2500 + 150\,\text{k}\|50\,\text{k} = 40\,\text{k}\Omega,$$

Therefore $\quad \tau = R_{\text{eq}}C = (40{,}000)(25 \times 10^{-9}) = 1\,\text{ms}$

The simplified circuit for $t > 0$ is:

[d] $i(0^+) = \dfrac{150 - (-120)}{40{,}000} = 6.75\,\text{mA}$

[e] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= 150 + (-120 - 150)e^{-t/\tau} = 150 - 270e^{-1000t}\,\text{V}, \qquad t \geq 0$$

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[f] $i = C\dfrac{dv_c}{dt} = (25 \times 10^{-9})(-1000)(-270e^{-1000t}) = 6.75e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$

P 7.52 **[a]** for $t < 0$:

$v_c(0) = 400(0.015) = 6\,\text{V}$

For $t \geq 0$:

$v_c(\infty) = 10\,\text{V}$

$R_{\text{eq}} = 20\,\Omega \qquad \text{so} \qquad \tau = R_{\text{eq}}C = 250(25 \times 10^{-6}) = 6.25\,\text{ms}$

$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/\tau} = 10 + (6 - 10)e^{-160t} = 10 - 4e^{-160t}\,\text{V}$

[b] For $t < 0$:

$v_c(0) = 10\,\text{V}$

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For $t \geq 0$:

$v_c(\infty) = 400(0.015) = 6\,\text{V}$

$R_{\text{eq}} = 100 + 400 = 500\,\Omega \qquad \text{so} \qquad \tau = R_{\text{eq}}C = 500(25 \times 10^{-6}) = 12.5\,\text{ms}$

$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/\tau} = 6 + (10 - 6)e^{-80t} = 6 + 4e^{-80t}\,\text{V}$

P 7.53 **[a]** Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{9k} = \frac{9\,\text{k}}{9\,\text{k} + 3\,\text{k}}(120) = 90\,\text{V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{40k} = -(1.5 \times 10^{-3})(40 \times 10^3) = -60\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$V_{\text{Th}} = -60\,\text{V}, \qquad R_{\text{Th}} = 10\,\text{k} + 40\,\text{k} = 50\,\text{k}\Omega$

$\tau = R_{\text{Th}}C = 1\,\text{ms} \ = 1000\,\mu\text{s}$

[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -60 + (90 + 60)e^{-1000t} = -60 + 150e^{-1000t}\,\text{V}, \quad t \geq 0$$

We want $v_c = -60 + 150e^{-1000t} = 0$:

Therefore $\quad t = \dfrac{\ln(150/60)}{1000} = 916.3\,\mu\text{s}$

P 7.54 **[a]** For $t < 0$:

$$v_o(0) = \frac{10{,}000}{15{,}000}(75) = 50\,\text{V}$$

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For $t \geq 0$:

$$v_o(\infty) = \frac{40{,}000}{50{,}000}(-100) = -80\,\text{V}$$

$$R_{\text{eq}} = 40\,\text{k}\|10\,\text{k} = 8\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (8000)(40 \times 10^{-9}) = 0.32\,\text{ms}$$

$$v_o(t) = v_o(\infty) + (v_o(0) - v_o(\infty))e^{-t/\tau} = -80 + (50 + 80)e^{-3125t}$$

$$= -80 + 130e^{-3125t}\,\text{V}$$

[b] For $t \geq 0$:

$$i_o = \frac{130e^{-3125t} - 80 + 100}{10{,}000} = 13e^{-3125t} + 2\,\text{mA}$$

P 7.55 $t < 0$:

$$i_o(0^-) = \frac{20}{100}(10 \times 10^{-3}) = 2\,\text{mA}; \qquad v_o(0^-) = (2 \times 10^{-3})(50{,}000) = 100\,\text{V}$$

$t = \infty$:

$$i_o(\infty) = -5 \times 10^{-3}\left(\frac{20}{100}\right) = -1\,\text{mA}; \qquad v_o(\infty) = i_o(\infty)(50{,}000) = -50\,\text{V}$$

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$R_{\text{Th}} = 50\,\text{k}\Omega \| 50\,\text{k}\Omega = 25\,\text{k}\Omega; \qquad C = 16\,\text{nF}$

$\tau = (25{,}000)(16 \times 10^{-9}) = 0.4\,\text{ms}; \qquad \dfrac{1}{\tau} = 2500$

$\therefore \quad v_o(t) = -50 + 150e^{-2500t}\,\text{V}, \qquad t \geq 0$

$i_c = C\dfrac{dv_o}{dt} = -6e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$i_{50k} = \dfrac{v_o}{50{,}000} = -1 + 3e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$i_o = i_c + i_{50k} = -(1 + 3e^{-2500t})\,\text{mA}, \qquad t \geq 0^+$

P 7.56 For $t < 0$

Simplify the circuit:

$80/10{,}000 = 8\,\text{mA}, \qquad 10\,\text{k}\Omega \| 40\,\text{k}\Omega \| 24\,\text{k}\Omega = 6\,\text{k}\Omega$

$8\,\text{mA} - 3\,\text{mA} = 5\,\text{mA}$

$5\,\text{mA} \times 6\,\text{k}\Omega = 30\,\text{V}$

Thus, for $t < 0$

$\therefore \quad v_o(0^-) = v_o(0^+) = 30\,\text{V}$

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$t > 0$

Simplify the circuit:

$8\,\text{mA} + 2\,\text{mA} = 10\,\text{mA}$

$10\,\text{k}\|40\,\text{k}\|24\,\text{k} = 6\,\text{k}\Omega$

$(10\,\text{mA})(6\,\text{k}\Omega) = 60\,\text{V}$

Thus, for $t > 0$

$v_o(\infty) = -10 \times 10^{-3}(6 \times 10^3) = -60\,\text{V}$

$\tau = RC = (10\,\text{k})(0.05\,\mu) = 0.5\,\text{ms}; \qquad \dfrac{1}{\tau} = 2000$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = -60 + [30 - (-60)]e^{-2000t}$

$\quad = -60 + 90e^{-2000t}\,\text{V} \qquad t \geq 0$

P 7.57 Use voltage division to find the initial voltage:

$v_o(0) = \dfrac{60}{40 + 60}(50) = 30\,\text{V}$

Use Ohm's law to find the final value of voltage:

$v_o(\infty) = (-5\,\text{mA})(20\,\text{k}\Omega) = -100\,\text{V}$

$\tau = RC = (20 \times 10^3)(250 \times 10^{-9}) = 5\,\text{ms}; \qquad \dfrac{1}{\tau} = 200$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau}$

$\quad = -100 + (30 + 100)e^{-200t} = -100 + 130e^{-200t}\,\text{V}, \qquad t \geq 0$

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P 7.58 **[a]** $v = I_sR + (V_o - I_sR)e^{-t/RC} \qquad i = \left(I_s - \dfrac{V_o}{R}\right)e^{-t/RC}$

$$\therefore \quad I_sR = 40, \qquad V_o - I_sR = -24$$

$$\therefore \quad V_o = 16\,\text{V}$$

$$I_s - \frac{V_o}{R} = 3 \times 10^{-3}; \qquad I_s - \frac{16}{R} = 3 \times 10^{-3}; \qquad R = \frac{40}{I_s}$$

$$\therefore \quad I_s - 0.4I_s = 3 \times 10^{-3}; \qquad I_s = 5\,\text{mA}$$

$$R = \frac{40}{5} \times 10^3 = 8\,\text{k}\Omega$$

$$\frac{1}{RC} = 2500; \quad C = \frac{1}{2500R} = \frac{10^{-3}}{20 \times 10^3} = 50\,\text{nF}; \quad \tau = RC = \frac{1}{2500} = 400\,\mu\text{s}$$

[b] $v(\infty) = 40\,\text{V}$

$$w(\infty) = \frac{1}{2}(50 \times 10^{-9})(1600) = 40\,\mu\text{J}$$

$$0.81w(\infty) = 32.4\,\mu\text{J}$$

$$v^2(t_o) = \frac{32.4 \times 10^{-6}}{25 \times 10^{-9}} = 1296; \qquad v(t_o) = 36\,\text{V}$$

$$40 - 24e^{-2500t_o} = 36; \qquad e^{2500t_o} = 6; \qquad \therefore \quad t_o = 716.70\,\mu\text{s}$$

P 7.59 **[a]** Note that there are many different possible solutions to this problem.

$$R = \frac{\tau}{C}$$

Choose a $10\,\mu$H capacitor from Appendix H. Then,

$$R = \frac{0.25}{10 \times 10^{-6}} = 25\,\text{k}\Omega$$

Construct the resistance needed by combining $10\,\text{k}\Omega$ and $15\,\text{k}\Omega$ resistors in series:

[b] $v(t) = V_f + (V_o - V_f)e^{-t/\tau}$

$$V_o = 100\,\text{V}; \qquad V_f = (I_f)(R) = (1 \times 10^{-3})(25 \times 10^3) = 25\,\text{V}$$

$$\therefore \quad v(t) = 25 + (100 - 25)e^{-4t}\,\text{V} = 25 + 75e^{-4t}\,\text{V}, \qquad t \geq 0$$

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[c] $v(t) = 25 + 75e^{-4t} = 50 \qquad \text{so} \qquad e^{-4t} = \dfrac{1}{3}$

$$\therefore \quad t = \frac{\ln 3}{4} = 274.65\,\text{ms}$$

P 7.60 For $t > 0$

$$V_{\text{Th}} = (-25)(16{,}000)i_{\text{b}} = -400 \times 10^3 i_{\text{b}}$$

$$i_{\text{b}} = \frac{33{,}000}{80{,}000}(120 \times 10^{-6}) = 49.5\,\mu\text{A}$$

$$V_{\text{Th}} = -400 \times 10^3(49.5 \times 10^{-6}) = -19.8\,\text{V}$$

$$R_{\text{Th}} = 16\,\text{k}\Omega$$

$$v_o(\infty) = -19.8\,\text{V}; \qquad v_o(0^+) = 0$$

$$\tau = (16,000)(0.25 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$v_o = -19.8 + 19.8e^{-250t}\,\text{V}, \qquad t \geq 0$$

$$w(t) = \frac{1}{2}(0.25 \times 10^{-6})v_o^2 = w(\infty)(1 - e^{-250t})^2\,\text{J}$$

$$(1 - e^{-250t})^2 = \frac{0.36w(\infty)}{w(\infty)} = 0.36$$

$$1 - e^{-250t} = 0.6$$

$$e^{-250t} = 0.4 \quad \therefore \quad t = 3.67\,\text{ms}$$

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P 7.61 For $t < 0$, $v_o(0) = 90\,\text{V}$

$t > 0$:

$$v_{\text{Th}} = -10 \times 10^3 i_\Delta + (30/120)(300) = -10 \times 10^3 \left(\frac{-300}{120 \times 10^3}\right) + 75 = 100\,\text{V}$$

$$v_T = -10 \times 10^3 i_\Delta + 22.5 \times 10^3 i_T = -10 \times 10^3 (30/120) i_T + 22.5 \times 10^3 i_T$$

$$= 20 \times 10^3 i_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\text{k}\Omega$$

$t > 0$

$$v_o = 100 + (90 - 100)e^{-t/\tau}$$

$$\tau = RC = (20 \times 10^3)(25 \times 10^{-9}) = 500 \times 10^{-6}; \qquad \frac{1}{\tau} = 2000$$

$$v_o = 100 - 10e^{-2000t}\,\text{V}, \quad t \geq 0$$

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P 7.62 From Problem 7.61,

$$v_o(0) = 100\,\text{V}; \qquad v_o(\infty) = 90\,\text{V}$$

$$R_{\text{Th}} = 40\,\text{k}\Omega$$

$$\tau = (40)(25 \times 10^{-6}) = 10^{-3}; \qquad \frac{1}{\tau} = 1000$$

$$v = 90 + (100 - 90)e^{-1000t} = 90 + 10e^{-1000t}\,\text{V}, \quad t \geq 0$$

P 7.63 **[a]**

$$I_sR = Ri + \frac{1}{C}\int_{0^+}^{t} i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \quad \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $\dfrac{di}{dt} = -\dfrac{i}{RC}; \qquad \dfrac{di}{i} = -\dfrac{dt}{RC}$

$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0^+}^{t} dx$$

$$\ln\frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_sR - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore \quad i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$$

P 7.64 **[a]** For $t > 0$:

$$v(\infty) = \frac{60}{180}(90) = 30\,\text{V}$$

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$$R_{\text{eq}} = 60\,\text{k}\|120\,\text{k} = 40\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (40 \times 10^3)(20 \times 10^{-9}) = 0.8\,\text{ms}; \qquad \frac{1}{\tau} = 1250$$

$$v_o = 30 + (120 - 30)e^{-1250t} = 30 + 90e^{-1250t}\,\text{V}, \qquad t \geq 0^+$$

[b] $$i_o = \frac{v_o}{60{,}000}\,\underline{v_o}\,90120{,}000 = \frac{30 + 90e^{-1250t}}{60{,}000} + \frac{30 + 90e^{-1250t} - 90}{120{,}000}$$

$$= 2.25e^{-1250t}\,\text{mA}$$

$$v_1 = \frac{1}{60 \times 10^{-9}} \times 2.25 \times 10^{-3} \int_0^t e^{-1250x}\,dx = -30e^{-1250t} + 30\,\text{V}, \quad t \geq 0$$

P 7.65 **[a]** $t < 0$

$t > 0$

$$v_o(0^-) = v_o(0^+) = 40\,\text{V}$$

$$v_o(\infty) = 80\,\text{V}$$

$$\tau = (0.16 \times 10^{-6})(6.25 \times 10^3) = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 80 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $$i_o = -C\frac{dv_o}{dt} = -0.16 \times 10^{-6}[40{,}000e^{-1000t}]$$

$$= -6.4e^{-1000t}\,\text{mA}; \qquad t \geq 0^+$$

[c] $$v_1 = \frac{-1}{0.2 \times 10^{-6}} \int_0^t -6.4 \times 10^{-3}e^{-1000x}\,dx + 32$$

$$= 64 - 32e^{-1000t}\,\text{V}, \qquad t \geq 0$$

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[d] $v_2 = \dfrac{-1}{0.8 \times 10^{-6}} \displaystyle\int_0^t -6.4 \times 10^{-3} e^{-1000x}\, dx + 8$

$= 16 - 8e^{-1000t}\,\text{V}, \qquad t \geq 0$

[e] $w_{\text{trapped}} = \dfrac{1}{2}(0.2 \times 10^{-6})(64)^2 + \dfrac{1}{2}(0.8 \times 10^{-6})(16)^2 = 512\,\mu\text{J}.$

P 7.66 **[a]** Let i be the current in the clockwise direction around the circuit. Then

$$\begin{aligned} V_g &= iR_g + \frac{1}{C_1}\int_0^t i\,dx + \frac{1}{C_2}\int_0^t i\,dx \\ &= iR_g + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i\,dx = iR_g + \frac{1}{C_e}\int_0^t i\,dx \end{aligned}$$

Now differentiate the equation

$$0 = R_g\frac{di}{dt} + \frac{i}{C_e} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{R_gC_e}i = 0$$

Therefore $\quad i = \dfrac{V_g}{R_g}e^{-t/R_gC_e} = \dfrac{V_g}{R_g}e^{-t/\tau}; \qquad \tau = R_gC_e$

$$v_1(t) = \frac{1}{C_1}\int_0^t \frac{V_g}{R_g}e^{-x/\tau}\,dx = \frac{V_g}{R_gC_1}\,\frac{e^{-x/\tau}}{-1/\tau}\Bigg|_0^t = -\frac{V_gC_e}{C_1}(e^{-t/\tau} - 1)$$

$$v_1(t) = \frac{V_gC_2}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

$$v_2(t) = \frac{V_gC_1}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

[b] $v_1(\infty) = \dfrac{C_2}{C_1 + C_2}V_g; \qquad v_2(\infty) = \dfrac{C_1}{C_1 + C_2}V_g$

P 7.67 **[a]** $L_{\text{eq}} = \dfrac{(3)(15)}{3 + 15} = 2.5\,\text{H}$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{2.5}{7.5} = \frac{1}{3}\,\text{s}$$

$$i_o(0) = 0; \qquad i_o(\infty) = \frac{120}{7.5} = 16\,\text{A}$$

$$\therefore \quad i_o = 16 - 16e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 120 - 7.5i_o = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

$$i_1 = \frac{1}{3}\int_0^t 120e^{-3x}\,dx = \frac{40}{3} - \frac{40}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$i_2 = i_o - i_1 = \frac{8}{3} - \frac{8}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

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[b] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with initial conditions.
$v_o(0^+) = 120$ V, consistent with $i_o(0) = 0$.

$$v_o = 3\frac{di_1}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

or

$$v_o = 15\frac{di_2}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

The voltage solution is consistent with the current solutions.

$$\lambda_1 = 3i_1 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$$\lambda_2 = 15i_2 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$\therefore \quad \lambda_1 = \lambda_2$ as it must, since

$$v_o = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt}$$

$$\lambda_1(\infty) = \lambda_2(\infty) = 40 \text{ Wb-turns}$$

$$\lambda_1(\infty) = 3i_1(\infty) = 3(40/3) = 40 \text{ Wb-turns}$$

$$\lambda_2(\infty) = 15i_2(\infty) = 15(8/3) = 40 \text{ Wb-turns}$$

$\therefore \quad i_1(\infty)$ and $i_2(\infty)$ are consistent with $\lambda_1(\infty)$ and $\lambda_2(\infty)$.

P 7.68 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{0.125 - 0.0625}{0.75 + 0.5} = 50\,\text{mH}$$

$$\tau = \frac{L}{R} = \frac{1}{5000}; \qquad \frac{1}{\tau} = 5000$$

$$\therefore \quad i_o(t) = 40 - 40e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[b] $v_o = 10 - 250i_o = 10 - 250(0.04 + 0.04e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 0.5\dfrac{di_1}{dt} - 0.25\dfrac{di_2}{dt} = 10e^{-5000t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 200e^{-5000t} \text{ A/s}$$

$$\therefore \quad \frac{di_2}{dt} = 200e^{-5000t} - \frac{di_1}{dt}$$

$$\therefore \quad 10e^{-5000t} = 0.5\frac{di_1}{dt} - 50e^{-5000t} + 0.25\frac{di_1}{dt}$$

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$$\therefore \quad 0.75\frac{di_1}{dt} = 60e^{-5000t}; \qquad di_1 = 80e^{-5000t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 80e^{-5000y}\,dy$$

$$i_1 = \frac{80}{-5000}e^{-5000y}\Big|_0^t = 16 - 16e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 40 - 40e^{-5000t} - 16 + 16e^{-5000t}$

$$= 24 - 24e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = (0.05)(200)e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 0.5\frac{di_1}{dt} - 0.25\frac{di_2}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 0.25\frac{di_2}{dt} - 0.25\frac{di_1}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 10\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

$$i_o(\infty) = 40\,\text{mA}; \qquad i_o(\infty)L_{\text{eq}} = (0.04)(0.05) = 2 \text{ mWb-turns}$$

$$i_1(\infty)L_1 + i_2(\infty)M = (16\,\text{m})(500) + (24\,\text{m})(-250) = 2 \text{ mWb-turns (ok)}$$

$$i_2(\infty)L_2 + i_1(\infty)M = (24\,\text{m})(250) + (16\,\text{m})(-250) = 2 \text{ mWb-turns (ok)}$$

Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.69 **[a]** $L_{\text{eq}} = 0.02 + 0.04 + 2(0.015) = 0.09 = 90\,\text{mH}$

$$\tau = \frac{L}{R} = \frac{0.09}{4500} = 20\,\mu\text{s}; \qquad \frac{1}{\tau} = 50{,}000$$

$$i = 20 - 20e^{-50{,}000t}\,\text{mA}, \quad t \geq 0$$

[b] $v_1(t) = 0.02\frac{di}{dt} + 0.015\frac{di}{dt} = 0.035\frac{di}{dt} = 0.035(1000e^{-50{,}000t}) = 35e^{-50{,}000t}\,\text{V}, \quad t \geq 0^+$

[c] $v_2(t) = 0.04\frac{di}{dt} + 0.015\frac{di}{dt} = 0.055\frac{di}{dt} = 0.055(1000e^{-50{,}000t}) = 55e^{-50{,}000t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 0.02 - 0.02 = 0$, which agrees with initial conditions.

$$90 = 4500i + v_1 + v_2 = 4500(0.02 - 0.02e^{-50{,}000t}) + 35e^{-50{,}000t} + 55e^{-50{,}000t} = 90\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

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P 7.70 **[a]** $L_{\text{eq}} = 0.02 + 0.04 - 2(0.015) = 0.03 = 30\,\text{mH}$

$$\tau = \frac{L}{R} = \frac{0.03}{4500} = 6.67\,\mu\text{s}; \qquad \frac{1}{\tau} = 150{,}000$$

$$i = 0.02 - 0.02e^{-150{,}000t}\,\text{A}, \quad t \geq 0$$

[b] $v_1(t) = 0.02\dfrac{di}{dt} - 0.015\dfrac{di}{dt} = 0.005\dfrac{di}{dt} = 0.005(3000e^{-150{,}000t})$

$$= 15e^{-150{,}000t}\,\text{V}, \quad t \geq 0^+$$

[c] $v_2(t) = 0.04\dfrac{di}{dt} - 0.015\dfrac{di}{dt} = 0.025\dfrac{di}{dt} = 0.025(3000e^{-150{,}000t})$

$$= 75e^{-150{,}000t}\,\text{V}, \quad t \geq 0^+$$

[d] $i(0) = 0$, which agrees with initial conditions.

$$90 = 4500i_1 + v_1 + v_2 = 4500(0.02 - 0.02e^{-150{,}000t}) + 15e^{-150{,}000t}$$
$$+ 75e^{-150{,}000t} = 90\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.71 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{50 - 25}{15 + 10} = 1\,\text{H}$$

$$\tau = \frac{L}{R} = \frac{1}{20}; \qquad \frac{1}{\tau} = 20$$

$$\therefore \quad i_o(t) = 4 - 4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[b] $v_o = 80 - 20i_o = 80 - 80 + 80e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 5\dfrac{di_1}{dt} - 5\dfrac{di_2}{dt} = 80e^{-20t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 80e^{-20t}\ \text{A/s}$$

$$\therefore \quad \frac{di_2}{dt} = 80e^{-20t} - \frac{di_1}{dt}$$

$$\therefore \quad 80e^{-20t} = 5\frac{di_1}{dt} - 400e^{-20t} + 5\frac{di_1}{dt}$$

$$\therefore \quad 10\frac{di_1}{dt} = 480e^{-20t}; \qquad di_1 = 48e^{-20t}\,dt$$

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$$\int_0^{t_1} dx = \int_0^t 48e^{-20y}\,dy$$

$$i_1 = \frac{48}{-20}e^{-20y}\Big|_0^t = 2.4 - 2.4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 4 - 4e^{-20t} - 2.4 + 2.4e^{-20t}$

$$= 1.6 - 1.6e^{-20t}\,\text{A}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = 1(80)e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 5\frac{di_1}{dt} - 5\frac{di_2}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 10\frac{di_2}{dt} - 5\frac{di_1}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 80\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

$i_o(\infty) = 4\,\text{A}; \qquad i_o(\infty)L_{\text{eq}} = (4)(1) = 4$ Wb-turns

$i_1(\infty)L_1 + i_2(\infty)M = (2.4)(5) + (1.6)(-5) = 4$ Wb-turns (ok)

$i_2(\infty)L_2 + i_1(\infty)M = (1.6)(10) + (2.4)(-5) = 4$ Wb-turns (ok)

Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.72 For $t < 0$:

$$i(0) = \frac{10}{15}(15) = 10\,\text{A}$$

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$0 \le t \le 10\,\text{ms}$:

$i = 10e^{-100t}\,\text{A}$

$i(10\,\text{ms}) = 10e^{-1} = 3.68\,\text{A}$

$10\,\text{ms} \le t \le 20\,\text{ms}$:

$$R_{\text{eq}} = \frac{(5)(20)}{25} = 4\,\Omega$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{4}{50 \times 10^{-3}} = 80$$

$i = 3.68e^{-80(t-0.01)}\,\text{A}$

$20\,\text{ms} \le t < \infty$:

$i(20\,\text{ms}) = 3.68e^{-80(0.02-0.01)} = 1.65\,\text{A}$

$i = 1.65e^{-100(t-0.02)}\,\text{A}$

$$v_o = L\frac{di}{dt}; \qquad L = 50\,\text{mH}$$

$$\frac{di}{dt} = 1.65(-100)e^{-100(t-0.02)} = -165e^{-100(t-0.02)}$$

$$v_o = (50 \times 10^{-3})(-165)e^{-100(t-0.02)}$$

$$= -8.26e^{-100(t-0.02)}\,\text{V}, \qquad t > 20^+\,\text{ms}$$

$v_o(25\,\text{ms}) = -8.26e^{-100(0.025-0.02)} = -5.013\,\text{V}$

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P 7.73 From the solution to Problem 7.72, the initial energy is

$$w(0) = \frac{1}{2}(50\,\text{mH})(10\,\text{A})^2 = 2.5\,\text{J}$$

$$0.04w(0) = 0.1\,\text{J}$$

$$\therefore \quad \frac{1}{2}(50 \times 10^{-3})i_L^2 = 0.1 \quad \text{so} \quad i_L = 2\,\text{A}$$

Again, from the solution to Problem 7.73, t must be between 10 ms and 20 ms since

$$i(10\,\text{ms}) = 3.68\,\text{A} \quad \text{and} \quad i(20\,\text{ms}) = 1.65\,\text{A}$$

For $10\,\text{ms} \le t \le 20\,\text{ms}$:

$$i = 3.68e^{-80(t-0.01)} = 2$$

$$e^{80(t-0.01)} = \frac{3.68}{2} \quad \text{so} \quad t - 0.01 = 0.0076 \quad \therefore \quad t = 17.6\,\text{ms}$$

P 7.74 $t < 0$:

$$i_L(0^-) = 75\,\text{mA} = i_L(0^+)$$

$0 \le t \le 25\,\text{ms}$:

$$\tau = 0.01/0 = \infty$$

$$i_L(t) = 0.075e^{-t/\infty} = 0.075e^{-0} = 75\,\text{mA}$$

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$25\,\text{ms} \le t$:

$$\tau = \frac{0.01}{500} = 20\,\mu\text{s}; \qquad 1/\tau = 50{,}000$$

$$i_L(t) = 75e^{-50{,}000(t-0.025)}\,\text{mA}, \quad t \ge 25\,\text{ms}$$

P 7.75 **[a]** $t < 0$:

Using Ohm's law,

$$i_g = \frac{800}{40 + 60\|40} = 12.5\,\text{A}$$

Using current division,

$$i(0^-) = \frac{60}{60+40}(12.5) = 7.5\,\text{A} = i(0^+)$$

[b] $0 \le t \le 1\,\text{ms}$:

$$i = i(0^+)e^{-t/\tau} = 7.5e^{-t/\tau}$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{40 + 120\|60}{80 \times 10^{-3}} = 1000$$

$$i = 7.5e^{-1000t}$$

$$i(200\mu\text{s}) = 7.5e^{-10^3(200\times10^{-6})} = 7.5e^{-0.2} = 6.14\,\text{A}$$

[c] $i(1\,\text{ms}) = 7.5e^{-1} = 2.7591\,\text{A}$

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$1\,\text{ms} \le t$:

$$\frac{1}{\tau} = \frac{R}{L} = \frac{40}{80 \times 10^{-3}} = 500$$

$$i = i(1\,\text{ms})e^{-(t-1\,ms)/\tau} = 2.7591e^{-500(t-0.001)}\,\text{A}$$

$$i(6\text{ms}) = 2.7591e^{-500(0.005)} = 2.7591e^{-2.5} = 226.48\,\text{mA}$$

[d] $0 \le t \le 1\,\text{ms}$:

$$i = 7.5e^{-1000t}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-1000)(7.5e^{-1000t}) = -600e^{-1000t}\,\text{V}$$

$$v(1^{-}\text{ms}) = -600e^{-1} = -220.73\,\text{V}$$

[e] $1\,\text{ms} \le t \le \infty$:

$$i = 2.759e^{-500(t-0.001)}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-500)(2.759e^{-500(t-0.001)})$$

$$= -110.4e^{-500(t-0.001)}\,\text{V}$$

$$v(1^{+}\text{ms}) = -110.4\,\text{V}$$

P 7.76 $0 \le t \le 200\,\mu\text{s}$;

$$R_e = 150\|100 = 60\,\text{k}\Omega; \qquad \tau = \left(\frac{10}{3} \times 10^{-9}\right)(60{,}000) = 200\,\mu\text{s}$$

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$v_c = 300e^{-5000t}\,\text{V}$

$v_c(200\,\mu\text{s}) = 300e^{-1} = 110.36\,\text{V}$

$200\,\mu\text{s} \le t < \infty$:

$R_e = 30\|60 + 120\|40 = 20 + 30 = 50\,\text{k}\Omega$

$\tau = \left(\dfrac{10}{3} \times 10^{-9}\right)(50{,}000) = 166.67\,\mu\text{s}; \qquad \dfrac{1}{\tau} = 6000$

$v_c = 110.36e^{-6000(t\,-\,200\,\mu s)}\,\text{V}$

$v_c(300\,\mu\text{s}) = 110.36e^{-6000(100\,\mu\text{S})} = 60.57\,\text{V}$

$i_o(300\,\mu\text{s}) = \dfrac{60.57}{50{,}000} = 1.21\,\text{mA}$

$i_1 = \dfrac{60}{90}i_o = \dfrac{2}{3}i_o; \qquad i_2 = \dfrac{40}{160}i_o = \dfrac{1}{4}i_o$

$i_{\text{sw}} = i_1 - i_2 = \dfrac{2}{3}i_o - \dfrac{1}{4}i_o = \dfrac{5}{12}i_o = \dfrac{5}{12}(1.21 \times 10^{-3}) = 0.50\,\text{mA}$

P 7.77 $0 \le t \le 2.5\,\text{ms}$:

$\tau = RC = (500)(8 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$

$v_o(0) = 0\,\text{V}; \qquad v_o(\infty) = -20\,\text{V}$

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$v_o = -20 + 20e^{-250t}\,\text{V} \qquad 0 \le t \le 2.5\,\text{ms}$

$2.5\,\text{ms} \le t$:

$t \to \infty$:

$$i = \frac{-70\,\text{V}}{2.5\,\text{k}\Omega} = -28\,\text{mA}$$

$$v_o(\infty) = (-28 \times 10^{-3})(2000) + 50 = -6\,\text{V}$$

$$v_o(0.0025) = -20 + 20e^{-0.625} = -9.29\,\text{V}$$

$$v_o = -6 + (-9.29 + 6)e^{-(t\,-0.0025)/\tau}$$

$$R_{\text{Th}} = 2000\|500 = 400\,\Omega$$

$$\tau = (400)(8 \times 10^{-6}) = 3.2\,\text{ms}; \qquad 1/\tau = 312.5$$

$$v_o = -6 - 3.29e^{-312.5(t\,-\,0.0025)} \qquad 2.5\,\text{ms} \le t$$

P 7.78 Note that for $t > 0$, $\quad v_o = (10/15)v_c$, where v_c is the voltage across the 25 nF capacitor. Thus we will find v_c first.

$t < 0$

$$v_c(0) = \frac{30}{40}(10) = 7.5\,\text{V}$$

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$0 \le t \le 0.2\,\text{ms}$:

$\tau = R_e C, \qquad R_e = 15{,}000 \| 30{,}000 = 10\,\text{k}\Omega$

$\tau = (10 \times 10^3)(25 \times 10^{-9}) = 0.25\,\text{ms}, \qquad \dfrac{1}{\tau} = 4000$

$v_\text{c} = 7.5e^{-4000t}\,\text{V}, \qquad t \ge 0$

$v_\text{c}(0.2\,\text{ms}) = 7.5e^{-0.8} = 3.37\,\text{V}$

$0.2\,\text{ms} \le t \le 0.8\,\text{ms}$:

$\tau = (15 \times 10^3)(2.5 \times 10^{-9}) = 375\,\mu\text{s}, \qquad \dfrac{1}{\tau} = 2666.67$

$v_\text{c} = 3.37e^{-2666.67(t-200\times10^{-6})}\,\text{V}$

$0.8\,\text{ms} \le t <$:

$\tau = 0.25\,\text{ms}, \qquad \dfrac{1}{\tau} = 4000$

$v_\text{c}(0.8\,\text{ms}) = 3.37e^{-2666.67(800-200)10^{-6}} = 3.37e^{-1.6} = 0.68\,\text{V}$

$v_\text{c} = 0.68e^{-4000(t-0.8\times10^{-3})}\,\text{V}$

$v_\text{c}(1\,\text{ms}) = 0.68e^{-4000(1-0.8)10^{-3}} = 0.68e^{-0.8} = 0.306\,\text{V}$

$v_o = (10/15)(0.306) = 0.204\,\text{V}$

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P 7.79 $w(0) = \frac{1}{2}(25 \times 10^{-9})(7.5)^2 = 703.125\,\text{nJ}$

$0 \le t \le 200\,\mu\text{s}$:

$v_\text{c} = 7.5e^{-4000t}; \qquad v_\text{c}^2 = 56.25e^{-8000t}$

$p_{30\text{k}} = 1.875e^{-8000t}\,\text{mW}$

$$\begin{aligned} w_{30\text{k}} &= \int_0^{200\times10^{-6}} 1.875 \times 10^{-3} e^{-8000t}\,dt \\ &= 1.875 \times 10^{-3} \frac{e^{-8000t}}{-8000}\Bigg|_0^{200\times10^{-6}} \\ &= -234.375 \times 10^{-9}(e^{-1.6} - 1) = 187.1\,\text{nJ} \end{aligned}$$

$0.8\,\text{ms} \le t$:

$v_\text{c} = 0.68e^{-4000(t-0.8\times10^{-3})}\,\text{V}; \qquad v_\text{c}^2 = 0.46e^{-8000(t-0.8\times10^{-3})}$

$p_{30\text{k}} = 15.33e^{-8000(t-0.8\times10^{-3})}\,\mu\text{W}$

$$\begin{aligned} w_{30\text{k}} &= \int_{0.8\times10^{-3}}^{\infty} 15.33 \times 10^{-6} e^{-8000(t-0.8\times10^{-3})}\,dt \\ &= 15.33 \times 10^{-6} \frac{e^{-8000(t-0.8\times10^{-3})}}{-8000}\Bigg|_{0.8\times10^{-3}}^{\infty} \\ &= -1.9 \times 10^{-9}(0 - 1) = 1.9\,\text{nJ} \end{aligned}$$

$w_{30\text{k}} = 187.1 + 1.9 = 189\,\text{nJ}$

$\% = \frac{189}{703.125}(100) = 26.88\%$

P 7.80 $t < 0$:

$v_c(0^-) = -(5)(1000) \times 10^{-3} = -5\,\text{V} = v_c(0^+)$

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$0 \le t \le 5\,\text{s}$:

$\tau = \infty; \qquad 1/\tau = 0; \qquad v_o = -5e^{-0} = -5\,\text{V}$

$5\,\text{s} \le t < \infty$:

$\tau = (100)(0.1) = 10\,\text{s}; \qquad 1/\tau = 0.1; \qquad v_o = -5e^{-0.1(t-5)}\,\text{V}$

Summary:

$v_o = -5\,\text{V}, \qquad 0 \le t \le 5\,\text{s}$

$v_o = -5e^{-0.1(t-5)}\,\text{V}, \qquad 5\,\text{s} \le t < \infty$

P 7.81 **[a]** $i_o(0) = 0; \qquad i_o(\infty) = 50\,\text{mA}$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{3000}{75} \times 10^3 = 40{,}000$$

$$i_o = (50 - 50e^{-40{,}000t})\,\text{mA}, \qquad 0 \le t \le 25\,\mu\text{s}$$

$$v_o = 0.075\frac{di_o}{dt} = 150e^{-40{,}000t}\,\text{V}, \qquad 0 \le t \le 25\,\mu s$$

$25\,\mu\text{s} \le t$:

$$i_o(25\mu\text{s}) = 50 - 50e^{-1} = 31.6\,\text{mA}; \qquad i_o(\infty) = 0$$

$$i_o = 31.6e^{-40{,}000(t-25\times10^{-6})}\,\text{mA}$$

$$v_o = 0.075\frac{di_o}{dt} = -94.82e^{-40{,}000(t-25\mu s)}$$

$$\begin{aligned} \therefore\ t < 0: \qquad v_o &= 0 \\ 0 \le t \le 25\,\mu\text{s}: \qquad v_o &= 150e^{-40{,}000t}\,\text{V} \\ 25\,\mu\text{s} \le t: \qquad v_o &= -94.82e^{-40{,}000(t-25\mu s)}\,\text{V} \end{aligned}$$

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[b] $v_o(25^-\mu\text{s}) = 150e^{-1} = 55.18\,\text{V}$

$v_o(25^+\mu\text{s}) = -94.82\,\text{V}$

[c] $i_o(25^-\mu\text{s}) = i_o(25^+\mu\text{s}) = 31.6\,\text{mA}$

P 7.82 **[a]** $0 \le t \le 2.5\,\text{ms}$

$v_o(0^+) = 80\,\text{V}; \qquad v_o(\infty) = 0$

$\tau = \dfrac{L}{R} = 2\,\text{ms}; \qquad 1/\tau = 500$

$v_o(t) = 80e^{-500t}\,\text{V}, \qquad 0^+ \le t \le 2.5^-\,\text{ms}$

$v_o(2.5^-\,\text{ms}) = 80e^{-1.25} = 22.92\,\text{V}$

$i_o(2.5^-\,\text{ms}) = \dfrac{(80 - 22.92)}{20} = 2.85\,\text{A}$

$v_o(2.5^+\,\text{ms}) = -20(2.85) = -57.08\,\text{V}$

$v_o(\infty) = 0; \qquad \tau = 2\,\text{ms}; \qquad 1/\tau = 500$

$v_o = -57.08e^{-500(t\,-\,0.0025)}\,\text{V} \qquad t \ge 2.5^+\,\text{ms}$

[b]

[c] $v_o(5\,\text{ms}) = -16.35\,\text{V}$

$i_o = \dfrac{+16.35}{20} = 817.68\,\text{mA}$

P 7.83 **[a]** $t < 0; \qquad v_o = 0$

$0 \le t \le 25\,\mu\text{s}$:

$\tau = (4000)(50 \times 10^{-9}) = 0.2\,\text{ms}; \qquad 1/\tau = 5000$

$v_o = 10 - 10e^{-5000t}\,\text{V}, \qquad 0 \le t \le 25\,\mu\text{s}$

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$v_o(25\,\mu\text{s}) = 10(1 - e^{-0.125}) = 1.175\,\text{V}$

$25\,\mu\text{s} \le t \le 50\,\mu\text{s}$:

$v_o = -10 + 11.175e^{-5000(t-25\times10^{-6})}\,\text{V}, \quad 25\,\mu\text{s} \le t \le 50\,\mu\text{s}$

$v_o(50\,\mu\text{s}) = -10 + 11.175e^{-0.125} = -0.138\,\text{V}$

$t \ge 50\,\mu\text{s}$:

$v_o = -0.138e^{-5000(t-50\times10^{-6})}\,\text{V}, \quad t \ge 50\,\mu\text{s}$

[b]

[c] $t \le 0: \qquad v_o = 0$

$0 \le t \le 25\,\mu\text{s}$:

$\tau = (800)(50 \times 10^{-9}) = 40\,\mu\text{s} \quad 1/\tau = 25{,}000$

$v_o = 10 - 10e^{-25{,}000t}\,\text{V}, \qquad 0 \le t \le 25\,\mu\text{s}$

$v_o(25\,\mu\text{s}) = 10 - 10e^{-0.625} = 4.65\,\text{V}$

$25\,\mu\text{s} \le t \le 50\,\mu\text{s}$:

$v_o = -10 + 14.65e^{-25{,}000(t-25\times10^{-6})}\,\text{V}, \qquad 25\,\mu\text{s} \le t \le 50\,\mu\text{s}$

$v_o(50\,\mu\text{s}) = -10 + 14.65e^{-0.625} = -2.16\,\text{V}$

$t \ge 50\,\mu\text{s}$:

$v_o = -2.16e^{-25{,}000(t-50\times10^{-6})}\,\text{V}, \quad t \ge 50\,\mu\text{s}$

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P 7.84 **[a]** $0 \le t \le 1\,\text{ms}$:

$$v_c(0^+) = 0; \qquad v_c(\infty) = 50\,\text{V};$$

$$RC = 400 \times 10^3(0.01 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/RC = 250$$

$$v_c = 50 - 50e^{-250t}$$

$$v_o = 50 - 50 + 50e^{-250t} = 50e^{-250t}\,\text{V}, \qquad 0 \le t \le 1\,\text{ms}$$

$1\,\text{ms} \le t < \infty$:

$$v_c(1\,\text{ms}) = 50 - 50e^{-0.25} = 11.06\,\text{V}$$

$$v_c(\infty) = 0\,\text{V}$$

$$\tau = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$v_c = 11.06e^{-250(t-0.001)}\,\text{V}$$

$$v_o = -v_c = -11.06e^{-250(t-0.001)}\,\text{V}, \qquad t \ge 1\,\text{ms}$$

[b]

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P 7.85

$$v_T = 2000i_T + 4000(i_T - 2\times 10^{-3}v_\phi) = 6000i_T - 8v_\phi$$

$$= 6000i_T - 8(2000i_T)$$

$$\frac{v_T}{i_T} = -10{,}000$$

$$\tau = \frac{10}{-10{,}000} = -1\,\text{ms}; \qquad 1/\tau = -1000$$

$$i = 25e^{1000t}\,\text{mA}$$

$$\therefore\quad 25e^{1000t}\times 10^{-3} = 5; \qquad t = \frac{\ln 200}{1000} = 5.3\,\text{ms}$$

P 7.86 **[a]**

Using Ohm's law,

$v_T = 4000i_\sigma$

Using current division,

$$i_\sigma = \frac{12{,}000}{12{,}000 + 4000}(i_T + \beta i_\sigma) = 0.75i_T + 0.75\beta i_\sigma$$

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Solve for i_σ:

$$i_\sigma(1 - 0.75\beta) = 0.75 i_T$$

$$i_\sigma = \frac{0.75 i_T}{1 - 0.75\beta}; \qquad v_T = 4000 i_\sigma = \frac{3000 i_T}{(1 - 0.75\beta)}$$

Find β such that $R_{\text{Th}} = -4\,\text{k}\Omega$:

$$R_{\text{Th}} = \frac{v_T}{i_T} = \frac{3000}{1 - 0.75\beta} = -4000$$

$$1 - 0.75\beta = -0.75 \qquad \therefore \quad \beta = 2.33$$

[b] Find V_{Th};

Write a KCL equation at the top node:

$$\frac{V_{\text{Th}} - 30}{4000} + \frac{V_{\text{Th}}}{12{,}000} - 2.33 i_\sigma = 0$$

The constraint equation is:

$$i_\sigma = \frac{(V_{\text{Th}} - 30)}{4000} = 0$$

Solving,

$$V_{\text{Th}} = 40\,\text{V}$$

Write a KVL equation around the loop:

$$40 = -4000 i + 0.08 \frac{di}{dt}$$

Rearranging:

$$\frac{di}{dt} = 500 + 50{,}000 i = 25{,}000(i + 0.01)$$

Separate the variables and integrate to find i;

$$\frac{di}{i + 0.01} = 50{,}000\, dt$$

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$$\int_0^i \frac{dx}{x+0.01} = \int_0^t 50{,}000\,dx$$

$$\therefore \quad i = -10 + 10e^{50{,}000t}\,\text{mA}$$

$$\frac{di}{dt} = (10\times 10^{-3})(50{,}000)e^{50{,}000t} = 500e^{50{,}000t}$$

Solve for the arc time:

$$v = 0.08\frac{di}{dt} = 40e^{50{,}000t} = 30{,}000; \qquad e^{50{,}000t} = 750$$

$$\therefore \quad t = \frac{\ln 750}{50{,}000} = 132.4\,\mu\text{s}$$

P 7.87 $t > 0$:

$$v_T = 12\times 10^4 i_\Delta + 16\times 10^3 i_T$$

$$i_\Delta = -\frac{20}{100}i_T = -0.2i_T$$

$$\therefore \quad v_T = -24\times 10^3 i_T + 16\times 10^3 i_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = -8\,\text{k}\Omega$$

$$\tau = RC = (-8\times 10^3)(2.5\times 10^{-6}) = -0.02 \quad 1/\tau = -50$$

$$v_{\text{c}} = 20e^{50t}\,\text{V}; \qquad 20e^{50t} = 20{,}000$$

$$50t = \ln 1000 \quad \therefore \quad t = 138.16\,\text{ms}$$

P 7.88 Find the Thévenin equivalent with respect to the terminals of the capacitor. R_{Th} calculation:

$$i_T = \frac{v_T}{2000} + \frac{v_T}{5000} - 4\frac{v_T}{5000}$$

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$$\therefore \quad \frac{i_T}{v_T} = \frac{5+2-8}{10{,}000} = -\frac{1}{10{,}000}$$

$$\frac{v_T}{i_T} = -\frac{10{,}000}{1} = -10\,\text{k}\Omega$$

Open circuit voltage calculation:

The node voltage equations:

$$\frac{v_{\text{oc}}}{2000} + \frac{v_{\text{oc}} - v_1}{1000} - 4i_\Delta = 0$$

$$\frac{v_1 - v_{\text{oc}}}{1000} + \frac{v_1}{4000} - 5 \times 10^{-3} = 0$$

The constraint equation:

$$i_\Delta = \frac{v_1}{4000}$$

Solving, $v_{\text{oc}} = -80\,\text{V}, \quad v_1 = -60\,\text{V}$

$$v_{\text{c}}(0) = 0; \qquad v_{\text{c}}(\infty) = -80\,\text{V}$$

$$\tau = RC = (-10{,}000)(1.6 \times 10^{-6}) = -16\,\text{ms}; \qquad \frac{1}{\tau} = -62.5$$

$$v_{\text{c}} = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau} = -80 + 80e^{62.5t} = 14{,}400$$

Solve for the time of the maximum voltage rating:

$$e^{62.5t} = 181; \qquad 62.5t = \ln 181; \qquad t = 83.09\,\text{ms}$$

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P 7.89 **[a]**

$\tau = (25)(2) \times 10^{-3} = 50\,\text{ms}; \qquad 1/\tau = 20$

$v_c(0^+) = 80\,\text{V}; \qquad v_c(\infty) = 0$

$v_c = 80e^{-20t}\,\text{V}$

$\therefore \quad 80e^{-20t} = 5; \qquad e^{20t} = 16; \qquad t = \dfrac{\ln 16}{20} = 138.63\,\text{ms}$

[b] $0^+ \le t \le 138.63^-$ ms:

$i = (2 \times 10^{-6})(-1600e^{-20t}) = -3.2e^{-20t}\,\text{mA}$

$t \ge 138.63^+$ ms:

$\tau = (2)(4) \times 10^{-3} = 8\,\text{ms}; \qquad 1/\tau = 125$

$v_c(138.63^+\,\text{ms}) = 5\,\text{V}; \qquad v_c(\infty) = 80\,\text{V}$

$v_c = 80 - 75e^{-125(t-0.13863)}\,\text{V}, \qquad t \ge 138.63\,\text{ms}$

$$i = 2 \times 10^{-6}(9375)e^{-125(t-0.13863)}$$
$$= 18.75e^{-125(t-0.13863)}\,\text{mA}, \qquad t \ge 138.63^+\,\text{ms}$$

[c] $80 - 75e^{-125\Delta t} = 0.85(80) = 68$

$80 - 68 = 75e^{-125\Delta t} = 12$

$e^{125\Delta t} = 6.25; \qquad \Delta t = \dfrac{\ln 6.25}{125} \cong 14.66\,\text{ms}$

P 7.90 $\quad v_o = -\dfrac{1}{R(0.5 \times 10^{-6})}\displaystyle\int_0^t 4\,dx + 0 = \dfrac{-4t}{R(0.5 \times 10^{-6})}$

$$\frac{-4(15 \times 10^{-3})}{R(0.5 \times 10^{-6})} = -10$$

$$\therefore \qquad R = \frac{-4(15 \times 10^{-3})}{-10(0.5 \times 10^{-6})} = 12\,\text{k}\Omega$$

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P 7.91 $v_o = \dfrac{-4t}{R(0.5 \times 10^{-6})} + 6 = \dfrac{-4(40 \times 10^{-3})}{R(0.5 \times 10^{-6})} + 6 = -10$

$$\therefore \qquad R = \frac{-4(40 \times 10^{-3})}{-16(0.5 \times 10^{-6})} = 20\,\text{k}\Omega$$

P 7.92 $RC = (80 \times 10^3)(250 \times 10^{-9}) = 20\,\text{ms}; \qquad \dfrac{1}{RC} = 50$

[a] $t < 0: \qquad v_o = 0$

[b] $0 \le t \le 2\,\text{s}:$

$$v_o = -50 \int_0^t 0.075\,dx = -3.75t\,\text{V}$$

[c] $2\,\text{s} \le t \le 4\,\text{s};$

$$v_o(2) = -3.75(2) = -7.5\,\text{V}$$

$$v_o(t) = -50 \int_2^t -0.075\,dx - 7.5 = 3.75(t-2) - 7.5 = 3.75t - 15\,\text{V}$$

[d] $t \ge 4\,\text{s}:$

$$v_o(4) = 15 - 15 = 0\,\text{V}$$

$$v_o(t) = 0\,\text{V}$$

P 7.93 Write a KCL equation at the inverting input to the op amp, where the voltage is 0:

$$\frac{0 - v_g}{R_i} + \frac{0 - v_o}{R_f} + C_f \frac{d}{dt}(0 - v_o) = 0$$

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$$\therefore \qquad \frac{dv_o}{dt} + \frac{1}{R_f C_f} v_o = -\frac{v_g}{R_i C_f}$$

Note that this first-order differential equation is in the same form as Eq. 7.50 if $I_s = -v_g/R_i$. Therefore, its solution is the same as Eq. 7.51:

$$v_o = \frac{-v_g R_f}{R_i} + \left(V_o - \frac{-v_g R_f}{R_i}\right) e^{-t/R_f C_f}$$

[a] $v_o = 0, \qquad t < 0$

[b] $R_f C_f = (4 \times 10^6)(250 \times 10^{-9}) = 1; \qquad \dfrac{1}{R_f C_f} = 1$

$$\frac{-v_g R_f}{R_i} = \frac{-(0.075)(4 \times 10^6)}{80{,}000} = -3.75$$

$$V_o = v_o(0) = 0$$

$$\therefore \qquad v_o = -3.75 + (0 + 3.75)e^{-t} = -3.75(1 - e^{-t})\,\text{V}, \qquad 0 \le t \le 2\,\text{s}$$

[c] $\dfrac{1}{R_f C_f} = 1$

$$\frac{-v_g R_f}{R_i} = \frac{-(-0.075)(4 \times 10^6)}{80{,}000} = 3.75$$

$$V_o = v_o(2) = -3.75(1 - e^{-2}) = -3.24\,\text{V}$$

$$\therefore \qquad v_o = 3.75 + [-3.24 - 3.75]e^{-(t-2)}$$

$$= 3.75 - 6.99e^{-(t-2)}\,\text{V}, \quad 2\,\text{s} \le t \le 4\,\text{s}$$

[d] $\dfrac{1}{R_f C_f} = 1$

$$\frac{-v_g R_f}{R_i} = 0$$

$$V_o = v_o(4) = 3.75 - 6.99e^{-2} = 2.8\,\text{V}$$

$$v_o = 0 + (2.8 - 0)e^{-(t-4)} = 2.8e^{-(t-4)}\,\text{V}, \qquad 4\,\text{s} \le t$$

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P 7.94 **[a]** $\dfrac{Cdv_p}{dt} + \dfrac{v_p - v_\mathrm{b}}{R} = 0;$ therefore $\dfrac{dv_p}{dt} + \dfrac{1}{RC}v_p = \dfrac{v_\mathrm{b}}{RC}$

$$\frac{v_n - v_\mathrm{a}}{R} + C\frac{d(v_n - v_o)}{dt} = 0;$$

$$\text{therefore} \quad \frac{dv_o}{dt} = \frac{dv_n}{dt} + \frac{v_n}{RC} - \frac{v_\mathrm{a}}{RC}$$

But $v_n = v_p$

$$\text{Therefore} \quad \frac{dv_n}{dt} + \frac{v_n}{RC} = \frac{dv_p}{dt} + \frac{v_p}{RC} = \frac{v_\mathrm{b}}{RC}$$

$$\text{Therefore} \quad \frac{dv_o}{dt} = \frac{1}{RC}(v_\mathrm{b} - v_\mathrm{a}); \qquad v_o = \frac{1}{RC}\int_0^t (v_\mathrm{b} - v_\mathrm{a})\,dy$$

[b] The output is the integral of the difference between v_b and v_a and then scaled by a factor of $1/RC$.

[c] $v_o = \dfrac{1}{RC}\displaystyle\int_0^t (v_\mathrm{b} - v_\mathrm{a})\,dx$

$$RC = (50 \times 10^3)(10 \times 10^{-9}) = 0.5\,\mathrm{ms}$$

$$v_\mathrm{b} - v_\mathrm{a} = -25\,\mathrm{mV}$$

$$v_o = \frac{1}{0.0005}\int_0^t -25 \times 10^{-3}dx = -50t$$

$$-50t_\mathrm{sat} = -6; \qquad t_\mathrm{sat} = 120\,\mathrm{ms}$$

P 7.95 Use voltage division to find the voltage at the non-inverting terminal:

$$v_p = \frac{80}{100}(-45) = -36\,\mathrm{V} = v_n$$

Write a KCL equation at the inverting terminal:

$$\frac{-36 - 14}{80{,}000} + 2.5 \times 10^{-6}\frac{d}{dt}(-36 - v_o) = 0$$

$$\therefore \qquad 2.5 \times 10^{-6}\frac{dv_o}{dt} = \frac{-50}{80{,}000}$$

Separate the variables and integrate:

$$\frac{dv_o}{dt} = -250 \quad \therefore \quad dv_o = -250dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = -250\int_0^t dy \quad \therefore \quad v_o(t) - v_o(0) = -250t$$

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$v_o(0) = -36 + 56 = 20\,\text{V}$

$v_o(t) = -250t + 20$

Find the time when the voltage reaches 0:

$$0 = -250t + 20 \quad \therefore \quad t = \frac{20}{250} = 80\,\text{ms}$$

P 7.96 **[a]** $RC = (1000)(800 \times 10^{-12}) = 800 \times 10^{-9}; \qquad \dfrac{1}{RC} = 1{,}250{,}000$

$0 \le t \le 1\,\mu\text{s}$:

$$v_g = 2 \times 10^6 t$$

$$v_o = -1.25 \times 10^6 \int_0^t 2 \times 10^6 x\,dx + 0$$

$$= -2.5 \times 10^{12} \frac{x^2}{2}\bigg|_0^t = -125 \times 10^{10} t^2\,\text{V}, \quad 0 \le t \le 1\,\mu\text{s}$$

$$v_o(1\,\mu\text{s}) = -125 \times 10^{10}(1 \times 10^{-6})^2 = -1.25\,\text{V}$$

$1\,\mu\text{s} \le t \le 3\,\mu\text{s}$:

$$v_g = 4 - 2 \times 10^6 t$$

$$v_o = -125 \times 10^4 \int_{1\times10^{-6}}^t (4 - 2 \times 10^6 x)\,dx - 1.25$$

$$= -125 \times 10^4 \left[4x\bigg|_{1\times10^{-6}}^t - 2 \times 10^6 \frac{x^2}{2}\bigg|_{1\times10^{-6}}^t\right] - 1.25$$

$$= -5 \times 10^6 t + 5 + 125 \times 10^{10} t^2 - 1.25 - 1.25$$

$$= 125 \times 10^{10} t^2 - 5 \times 10^6 t + 2.5\,\text{V}, \quad 1\,\mu\text{s} \le t \le 3\,\mu\text{s}$$

$$v_o(3\,\mu\text{s}) = 125 \times 10^{10}(3 \times 10^{-6})^2 - 5 \times 10^6(3 \times 10^{-6}) + 2.5$$

$$= -1.25$$

$3\,\mu\text{s} \le t \le 4\,\mu\text{s}$:

$$v_g = -8 + 2 \times 10^6 t$$

$$v_o = -125 \times 10^4 \int_{3\times10^{-6}}^t (-8 + 2 \times 10^6 x)\,dx - 1.25$$

$$= -125 \times 10^4 \left[-8x\bigg|_{3\times10^{-6}}^t + 2 \times 10^6 \frac{x^2}{2}\bigg|_{3\times10^{-6}}^t\right] - 1.25$$

$$= 10^7 t - 30 - 125 \times 10^{10} t^2 + 11.25 - 1.25$$

$$= -125 \times 10^{10} t^2 + 10^7 t - 20\,\text{V}, \quad 3\,\mu\text{s} \le t \le 4\,\mu\text{s}$$

$$v_o(4\,\mu\text{s}) = -125 \times 10^{10}(4 \times 10^{-6})^2 + 10^7(4 \times 10^{-6}) - 20 = 0$$

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[b]

[c] The output voltage will also repeat. This follows from the observation that at $t = 4\,\mu$s the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 4\,\mu$s as it was at $t = 0$, thus as v_g repeats itself, so will v_o.

P 7.97 [a] T_2 is normally ON since its base current i_{b2} is greater than zero, i.e., $i_{\text{b2}} = V_{CC}/R$ when T_2 is ON. When T_2 is ON, $v_{\text{ce2}} = 0$, therefore $i_{\text{b1}} = 0$. When $i_{\text{b1}} = 0$, T_1 is OFF. When T_1 is OFF and T_2 is ON, the capacitor C is charged to V_{CC}, positive at the left terminal. This is a stable state; there is nothing to disturb this condition if the circuit is left to itself.

[b] When S is closed momentarily, v_{be2} is changed to $-V_{CC}$ and T_2 snaps OFF. The instant T_2 turns OFF, v_{ce2} jumps to $V_{CC}R_1/(R_1 + R_{\text{L}})$ and i_{b1} jumps to $V_{CC}/(R_1 + R_{\text{L}})$, which turns T_1 ON.

[c] As soon as T_1 turns ON, the charge on C starts to reverse polarity. Since v_{be2} is the same as the voltage across C, it starts to increase from $-V_{CC}$ toward $+V_{CC}$. However, T_2 turns ON as soon as $v_{\text{be2}} = 0$. The equation for v_{be2} is $v_{\text{be2}} = V_{CC} - 2V_{CC}e^{-t/RC}$. $v_{\text{be2}} = 0$ when $t = RC\,\ln 2$, therefore T_2 stays OFF for $RC\,\ln 2$ seconds.

P 7.98 [a] For $t < 0$, $v_{\text{ce2}} = 0$. When the switch is momentarily closed, v_{ce2} jumps to

$$v_{\text{ce2}} = \left(\frac{V_{CC}}{R_1 + R_{\text{L}}}\right) R_1 = \frac{6(5)}{25} = 1.2\,\text{V}$$

T_2 remains open for $(23{,}083)(250) \times 10^{-12}\,\ln 2 \cong 4\,\mu$s.

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[b] $i_{\mathrm{b2}} = \dfrac{V_{CC}}{R} = 259.93\,\mu\mathrm{A}, \qquad -5 \le t \le 0\,\mu\mathrm{s}$

$i_{\mathrm{b2}} = 0, \qquad 0 < t < RC\,\ln 2$

$$\begin{aligned} i_{\mathrm{b2}} &= \frac{V_{CC}}{R} + \frac{V_{CC}}{R_{\mathrm{L}}} e^{-(t - RC\,\ln 2)/R_{\mathrm{L}}C} \\ &= 259.93 + 300 e^{-0.2\times 10^6 (t - 4\times 10^{-6})}\,\mu\mathrm{A}, \qquad RC\,\ln 2 < t \end{aligned}$$

P 7.99 **[a]** While T_2 has been ON, C_2 is charged to V_{CC}, positive on the left terminal. At the instant T_1 turns ON the capacitor C_2 is connected across $b_2 - e_2$, thus $v_{\mathrm{be2}} = -V_{CC}$. This negative voltage snaps T_2 OFF. Now the polarity of the voltage on C_2 starts to reverse, that is, the right-hand terminal of C_2 starts to charge toward $+V_{CC}$. At the same time, C_1 is charging toward V_{CC}, positive on the right. At the instant the charge on C_2 reaches zero, v_{be2} is zero, T_2 turns ON. This makes $v_{\mathrm{be1}} = -V_{CC}$ and T_1 snaps OFF. Now the capacitors C_1 and C_2 start to charge with the polarities to turn T_1 ON and T_2 OFF. This switching action repeats itself over and over as long as the circuit is energized. At the instant T_1 turns ON, the voltage controlling the state of T_2 is governed by the following circuit:

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It follows that $v_{\text{be2}} = V_{CC} - 2V_{CC}e^{-t/R_2C_2}$.

[b] While T_2 is OFF and T_1 is ON, the output voltage v_{ce2} is the same as the voltage across C_1, thus

It follows that $v_{\text{ce2}} = V_{CC} - V_{CC}e^{-t/R_{\text{L}}C_1}$.

[c] T_2 will be OFF until v_{be2} reaches zero. As soon as v_{be2} is zero, i_{b2} will become positive and turn T_2 ON. $v_{\text{be2}} = 0$ when $V_{CC} - 2V_{CC}e^{-t/R_2C_2} = 0$, or when $t = R_2C_2 \ln 2$.

[d] When $t = R_2C_2 \ln 2$, we have

$$v_{\text{ce2}} = V_{CC} - V_{CC}e^{-[(R_2C_2 \ln 2)/(R_{\text{L}}C_1)]} = V_{CC} - V_{CC}e^{-10 \ln 2} \cong V_{CC}$$

[e] Before T_1 turns ON, i_{b1} is zero. At the instant T_1 turns ON, we have

$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_{\text{L}}}e^{-t/R_{\text{L}}C_1}$$

[f] At the instant T_2 turns back ON, $t = R_2C_2 \ln 2$; therefore

$$i_{\text{b1}} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_{\text{L}}}e^{-10 \ln 2} \cong \frac{V_{CC}}{R_1}$$

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[g]

[h]

P 7.100 **[a]** $t_{\text{OFF2}} = R_2C_2 \ln 2 = 18 \times 10^3(2 \times 10^{-9}) \ln 2 \cong 25\,\mu\text{s}$

[b] $t_{\text{ON2}} = R_1C_1 \ln 2 \cong 25\,\mu\text{s}$

[c] $t_{\text{OFF1}} = R_1C_1 \ln 2 \cong 25\,\mu\text{s}$

[d] $t_{\text{ON1}} = R_2C_2 \ln 2 \cong 25\,\mu\text{s}$

[e] $i_{\text{b1}} = \dfrac{9}{3} + \dfrac{9}{18} = 3.5\,\text{mA}$

[f] $i_{\text{b1}} = \dfrac{9}{18} + \dfrac{9}{3}e^{-6\ln 2} \cong 0.5469\,\text{mA}$

[g] $v_{\text{ce2}} = 9 - 9e^{-6\ln 2} \cong 8.86\,\text{V}$

P 7.101 **[a]** $t_{\text{OFF2}} = R_2C_2 \ln 2 = (18 \times 10^3)(2.8 \times 10^{-9}) \ln 2 \cong 35\,\mu\text{s}$

[b] $t_{\text{ON2}} = R_1C_1 \ln 2 \cong 37.4\,\mu\text{s}$

[c] $t_{\text{OFF1}} = R_1C_1 \ln 2 \cong 37.4\,\mu\text{s}$

[d] $t_{\text{ON1}} = R_2C_2 \ln 2 = 35\,\mu\text{s}$

[e] $i_{\text{b1}} = 3.5\,\text{mA}$

[f] $i_{\text{b1}} = \dfrac{9}{18} + 3e^{-5.6\ln 2} \cong 0.562\,\text{mA}$

[g] $v_{\text{ce2}} = 9 - 9e^{-5.6\ln 2} \cong 8.81\,\text{V}$

Note in this circuit T_2 is OFF $35\,\mu\text{s}$ and ON $37.4\,\mu\text{s}$ of every cycle, whereas T_1 is ON $35\,\mu\text{s}$ and OFF $37.4\,\mu\text{s}$ every cycle.

P 7.102 If $R_1 = R_2 = 50R_{\text{L}} = 100\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{100 \times 10^3 \ln 2} = 692.49\,\text{pF}; \qquad C_2 = \frac{36 \times 10^{-6}}{100 \times 10^3 \ln 2} = 519.37\,\text{pF}$$

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If $R_1 = R_2 = 6R_L = 12\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{12 \times 10^3 \ \ln 2} = 5.77\,\text{nF}; \qquad C_2 = \frac{36 \times 10^{-6}}{12 \times 10^3 \ \ln 2} = 4.33\,\text{nF}$$

Therefore $692.49\,\text{pF} \le C_1 \le 5.77\,\text{nF}$ and $519.37\,\text{pF} \le C_2 \le 4.33\,\text{nF}$

P 7.103 **[a]** $0 \le t \le 0.5$:

$$i = \frac{21}{60} + \left(\frac{30}{60} - \frac{21}{60}\right) e^{-t/\tau} \qquad \text{where } \tau = L/R.$$

$$i = 0.35 + 0.15e^{-60t/L}$$

$$i(0.5) = 0.35 + 0.15e^{-30/L} = 0.40$$

$$\therefore \quad e^{30/L} = 3; \qquad L = \frac{30}{\ln 3} = 27.31\,\text{H}$$

[b] $0 \le t \le t_r$, where t_r is the time the relay releases:

$$i = 0 + \left(\frac{30}{60} - 0\right) e^{-60t/L} = 0.5e^{-60t/L}$$

$$\therefore \quad 0.4 = 0.5e^{-60t_r/L}; \qquad e^{60t_r/L} = 1.25$$

$$t_r = \frac{27.31 \ln 1.25}{60} \cong 0.10\,\text{s}$$

P 7.104 From the Practical Perspective,

$$v_C(t) = 0.75V_S = V_S(1 - e^{-t/RC})$$

$$0.25 = e^{-t/RC} \qquad \text{so} \qquad t = -RC \ln 0.25$$

In the above equation, t is the number of seconds it takes to charge the capacitor to $0.75V_S$, so it is a period. We want to calculate the heart rate, which is a frequency in beats per minute, so $H = 60/t$. Thus,

$$H = \frac{60}{-RC \ln 0.25}$$

P 7.105 In this problem, $V_{max} = 0.6V_S$, so the equation for heart rate in beats per minute is

$$H = \frac{60}{-RC \ln 0.4}$$

Given $R = 150\,\text{k}\Omega$ and $C = 6\,\mu\text{F}$,

$$H = \frac{60}{-(150{,}000)(6 \times 10^{-6}) \ln 0.4} = 72.76$$

Therefore, the heart rate is about 73 beats per minute.

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P 7.106 From the Practical Perspective,

$$v_C(t) = V_{max} = V_S(1 - e^{-t/RC})$$

Solve this equation for the resistance R:

$$\frac{V_{max}}{V_S} = 1 - e^{-t/RC} \qquad \text{so} \qquad e^{-t/RC} = 1 - \frac{V_{max}}{V_S}$$

Then, $$\frac{-t}{RC} = \ln\left(1 - \frac{V_{max}}{V_S}\right)$$

$$\therefore \qquad R = \frac{-t}{C\ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

In the above equation, t is the time it takes to charge the capacitor to a voltage of V_{max}. But t and the heart rate H are related as follows:

$$H = \frac{60}{t}$$

Therefore,

$$R = \frac{-60}{HC\ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

P 7.107 From Problem 7.106,

$$R = \frac{-60}{HC\ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

Note that from the problem statement,

$$\frac{V_{max}}{V_S} = 0.68$$

Therefore,

$$R = \frac{-60}{(70)(2.5 \times 10^{-6})\ln(1 - 0.68)} = 301\,\text{k}\Omega$$

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8

Natural and Step Responses of RLC Circuits

Assessment Problems

AP 8.1 **[a]** $\dfrac{1}{(2RC)^2} = \dfrac{1}{LC}, \qquad \text{therefore} \quad C = 500\,\text{nF}$

[b] $\alpha = 5000 = \dfrac{1}{2RC}, \qquad \text{therefore} \quad C = 1\,\mu\text{F}$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - \frac{(10^3)(10^6)}{20}} = (-5000 \pm j5000)\ \text{rad/s}$$

[c] $\dfrac{1}{\sqrt{LC}} = 20{,}000, \qquad \text{therefore} \quad C = 125\,\text{nF}$

$$s_{1,2} = \left[-40 \pm \sqrt{(40)^2 - 20^2}\right] 10^3,$$

$$s_1 = -5.36\,\text{krad/s}, \qquad s_2 = -74.64\,\text{krad/s}$$

AP 8.2
$$\begin{aligned} i_\text{L} &= \frac{1}{50 \times 10^{-3}} \int_0^t [-14e^{-5000x} + 26e^{-20{,}000x}]\,dx + 30 \times 10^{-3} \\ &= 20\left\{ \frac{-14e^{-5000x}}{-5000}\bigg|_0^t + \frac{26e^{-20{,}000t}}{-20{,}000}\bigg|_0^t \right\} + 30 \times 10^{-3} \\ &= 56 \times 10^{-3}(e^{-5000t} - 1) - 26 \times 10^{-3}(e^{-20{,}000t} - 1) + 30 \times 10^{-3} \\ &= [56e^{-5000t} - 56 - 26e^{-20{,}000t} + 26 + 30]\,\text{mA} \\ &= 56e^{-5000t} - 26e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0 \end{aligned}$$

AP 8.3 From the given values of R, L, and C, $s_1 = -10\,\text{krad/s}$ and $s_2 = -40$ krad/s.

[a] $v(0^-) = v(0^+) = 0, \qquad \text{therefore} \quad i_\text{R}(0^+) = 0$

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[b] $i_{\mathrm{C}}(0^+) = -(i_L(0^+) + i_R(0^+)) = -(-4+0) = 4\,\mathrm{A}$

[c] $C\dfrac{dv_c(0^+)}{dt} = i_c(0^+) = 4, \qquad \text{therefore} \quad \dfrac{dv_c(0^+)}{dt} = \dfrac{4}{C} = 4 \times 10^8\,\mathrm{V/s}$

[d] $v = [A_1 e^{-10{,}000t} + A_2 e^{-40{,}000t}]\,\mathrm{V}, \qquad t \geq 0^+$

$$v(0^+) = A_1 + A_2, \qquad \frac{dv(0^+)}{dt} = -10{,}000A_1 - 40{,}000A_2$$

$$\text{Therefore} \quad A_1 + A_2 = 0, \qquad -A_1 - 4A_2 = 40{,}000; \qquad A_1 = 40{,}000/3\,\mathrm{V}$$

[e] $A_2 = -40{,}000/3\,\mathrm{V}$

[f] $v = [40{,}000/3][e^{-10{,}000t} - e^{-40{,}000t}]\,\mathrm{V}, \qquad t \geq 0$

AP 8.4 **[a]** $\dfrac{1}{2RC} = 8000, \qquad \text{therefore} \quad R = 62.5\,\Omega$

[b] $i_{\mathrm{R}}(0^+) = \dfrac{10\,\mathrm{V}}{62.5\,\Omega} = 160\,\mathrm{mA}$

$$i_{\mathrm{C}}(0^+) = -(i_L(0^+) + i_R(0^+)) = -80 - 160 = -240\,\mathrm{mA} = C\frac{dv(0^+)}{dt}$$

$$\text{Therefore} \quad \frac{dv(0^+)}{dt} = \frac{-240\,\mathrm{m}}{C} = -240\,\mathrm{kV/s}$$

[c] $B_1 = v(0^+) = 10\,\mathrm{V}, \qquad \dfrac{dv_c(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

$$\text{Therefore} \quad 6000B_2 - 8000B_1 = -240{,}000, \qquad B_2 = (-80/3)\,\mathrm{V}$$

[d] $i_{\mathrm{L}} = -(i_{\mathrm{R}} + i_{\mathrm{C}}); \qquad i_{\mathrm{R}} = v/R; \qquad i_{\mathrm{C}} = C\dfrac{dv}{dt}$

$$v = e^{-8000t}[10\cos 6000t - \frac{80}{3}\sin 6000t]\,\mathrm{V}$$

$$\text{Therefore} \quad i_{\mathrm{R}} = e^{-8000t}[160\cos 6000t - \frac{1280}{3}\sin 6000t]\,\mathrm{mA}$$

$$i_{\mathrm{C}} = e^{-8000t}[-240\cos 6000t + \frac{460}{3}\sin 6000t]\,\mathrm{mA}$$

$$i_{\mathrm{L}} = 10e^{-8000t}[8\cos 6000t + \frac{82}{3}\sin 6000t]\,\mathrm{mA}, \qquad t \geq 0$$

AP 8.5 **[a]** $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = \dfrac{10^6}{4}, \qquad \text{therefore} \quad \dfrac{1}{2RC} = 500, \quad R = 100\,\Omega$

[b] $0.5CV_0^2 = 12.5 \times 10^{-3}, \qquad \text{therefore} \quad V_0 = 50\,\mathrm{V}$

[c] $0.5LI_0^2 = 12.5 \times 10^{-3}, \qquad I_0 = 250\,\mathrm{mA}$

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[d] $D_2 = v(0^+) = 50, \qquad \dfrac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$i_{\mathrm{R}}(0^+) = \dfrac{50}{100} = 500\,\mathrm{mA}$

Therefore $\quad i_{\mathrm{C}}(0^+) = -(500 + 250) = -750\,\mathrm{mA}$

Therefore $\quad \dfrac{dv(0^+)}{dt} = -750 \times \dfrac{10^{-3}}{C} = -75{,}000\,\mathrm{V/s}$

Therefore $\quad D_1 - \alpha D_2 = -75{,}000; \qquad \alpha = \dfrac{1}{2RC} = 500, \quad D_1 = -50{,}000\ \mathrm{V/s}$

[e] $v = [50e^{-500t} - 50{,}000te^{-500t}]\,\mathrm{V}$

$i_{\mathrm{R}} = \dfrac{v}{R} = [0.5e^{-500t} - 500te^{-500t}]\,\mathrm{A}, \qquad t \geq 0^+$

AP 8.6 **[a]** $i_{\mathrm{R}}(0^+) = \dfrac{V_0}{R} = \dfrac{40}{500} = 0.08\,\mathrm{A}$

[b] $i_{\mathrm{C}}(0^+) = I - i_{\mathrm{R}}(0^+) - i_{\mathrm{L}}(0^+) = -1 - 0.08 - 0.5 = -1.58\,\mathrm{A}$

[c] $\dfrac{di_{\mathrm{L}}(0^+)}{dt} = \dfrac{V_o}{L} = \dfrac{40}{0.64} = 62.5\,\mathrm{A/s}$

[d] $\alpha = \dfrac{1}{2RC} = 1000; \qquad \dfrac{1}{LC} = 1{,}562{,}500; \qquad s_{1,2} = -1000 \pm j750\ \mathrm{rad/s}$

[e] $i_{\mathrm{L}} = i_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t, \qquad i_f = I = -1\,\mathrm{A}$

$i_{\mathrm{L}}(0^+) = 0.5 = i_f + B_1', \qquad$ therefore $\quad B_1' = 1.5\,\mathrm{A}$

$\dfrac{di_{\mathrm{L}}(0^+)}{dt} = 62.5 = -\alpha B_1' + \omega_d B_2', \qquad$ therefore $\quad B_2' = (25/12)\,\mathrm{A}$

Therefore $\quad i_{\mathrm{L}}(t) = -1 + e^{-1000t}[1.5\cos 750t + (25/12)\sin 750t]\,\mathrm{A}, \qquad t \geq 0$

[f] $v(t) = \dfrac{\text{Ł}di_{\mathrm{L}}}{dt} = 40e^{-1000t}[\cos 750t - (154/3)\sin 750t]V \qquad t \geq 0$

AP 8.7 **[a]** $i(0^+) = 0$, since there is no source connected to L for $t < 0$.

[b] $v_c(0^+) = v_{\mathrm{C}}(0^-) = \left(\dfrac{15\,\mathrm{k}}{15\,\mathrm{k} + 9\,\mathrm{k}}\right)(80) = 50\,\mathrm{V}$

[c] $50 + 80i(0^+) + L\dfrac{di(0^+)}{dt} = 100, \qquad \dfrac{di(0^+)}{dt} = 10{,}000\,\mathrm{A/s}$

[d] $\alpha = 8000; \qquad \dfrac{1}{LC} = 100 \times 10^6; \qquad s_{1,2} = -8000 \pm j6000\ \mathrm{rad/s}$

[e] $i = i_f + e^{-\alpha t}[B_1'\cos\omega_d t + B_2'\sin\omega_d t]; \qquad i_f = 0, \quad i(0^+) = 0$

Therefore $\quad B_1' = 0; \qquad \dfrac{di(0^+)}{dt} = 10{,}000 = -\alpha B_1' + \omega_d B_2'$

Therefore $\quad B_2' = 1.67\,\mathrm{A}; \qquad i = 1.67e^{-8000t}\sin 6000t\,\mathrm{A}, \qquad t \geq 0$

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AP 8.8 $v_c(t) = v_f + e^{-\alpha t}[B_1' \cos \omega_d t + B_2' \sin \omega_d t], \qquad v_f = 100\,\text{V}$

$$v_c(0^+) = 50\,\text{V}; \qquad \frac{dv_c(0^+)}{dt} = 0; \qquad \text{therefore} \quad 50 = 100 + B_1'$$

$$B_1' = -50\,\text{V}; \qquad 0 = -\alpha B_1' + \omega_d B_2'$$

$$\text{Therefore} \quad B_2' = \frac{\alpha}{\omega_d} B_1' = \left(\frac{8000}{6000}\right)(-50) = -66.67\,\text{V}$$

$$\text{Therefore} \quad v_c(t) = 100 - e^{-8000t}[50 \cos 6000t + 66.67 \sin 6000t]\,\text{V}, \qquad t \geq 0$$

Problems

P 8.1 **[a]** $i_\text{R}(0) = \dfrac{25}{125} = 200\,\text{mA}$

$$i_\text{L}(0) = -300\,\text{mA}$$

$$i_\text{C}(0) = -i_\text{L}(0) - i_\text{R}(0) = 300 - 200 = 100\,\text{mA}$$

[b] $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(125)(5 \times 10^{-6})} = 800$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200 \times 10^{-3})(8 \times 10^{-6})}} = 1000$$

$\alpha^2 < \omega_0^2$ The response is underdamped

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{1000^2 - 800^2} = 600$$

$$v = B_1 e^{-800t} \cos 600t + B_2 e^{-800t} \sin 600t$$

$$v(0) = B_1 = 25$$

$$\frac{dv}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{1}{C} i_\text{C}(0)$$

$$\text{So,} \quad -800(25) + 600B_2 = \frac{1}{5 \times 10^{-6}}(0.1) = 20{,}000$$

$$\therefore \quad B_2 = \frac{20{,}000 + 800(25)}{600} = 66.67$$

$$v = 25e^{-800t} \cos 600t + 66.67e^{-800t} \sin 600t\,\text{V}, \qquad t \geq 0$$

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$$[\mathbf{c}] \quad i_{\mathrm{C}} = C\frac{dv}{dt}$$

$$= 5 \times 10^{-6}[20{,}000e^{-800t}\cos 600t - 68{,}333.33e^{-800t}\sin 600t]$$

$$= 100e^{-800t}\cos 600t - 341.67e^{-800t}\sin 600t\,\mathrm{mA}$$

$$i_{\mathrm{R}} = \frac{v}{R} = 200e^{-800t}\cos 600t + 533.36e^{-800t}\sin 600t\,\mathrm{mA}$$

$$i_{\mathrm{L}} = -i_{\mathrm{C}} - i_{\mathrm{R}} = -300e^{-800t}\cos 600t - 191.7e^{-800t}\sin 600t\,\mathrm{mA}, \quad t \geq 0$$

P 8.2 $$\frac{1}{2RC} = \frac{1}{2(100)(5 \times 10^{-6})} = 1000$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200 \times 10^{-3})(5 \times 10^{-6})}} = 1000$$

$\alpha^2 = \omega_0^2$ So the response is critically damped

$$v(t) = D_1 te^{-1000t} + D_2 e^{-1000t}$$

$$v(0^+) = 25\,\mathrm{V} = D_2$$

$$\frac{dv(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

So, $$D_1 - 1000(25) = \frac{1}{5 \times 10^{-6}}\left(0.3 - \frac{25}{100}\right)$$

$$\therefore \qquad D_1 = 35{,}000$$

$$v(t) = 35{,}000e^{-1000t} + 25e^{-1000t}\,\mathrm{V}, \qquad t \geq 0$$

P 8.3 $$\alpha = \frac{1}{2RC} = \frac{1}{2(80)(5 \times 10^{-6})} = 1250$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200 \times 10^{-3})(5 \times 10^{-6})}} = 1000$$

$\alpha^2 > \omega_0^2$ So the response is overdamped

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 1000^2} = -500, -2000$$

$$v(t) = A_1 e^{-500t} + A_2 e^{-2000t}$$

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$v(0) = A_1 + A_2 = 25$

$\frac{dv}{dt}(0) = -500A_1 - 2000A_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right) = \frac{1}{5 \times 10^{-6}}\left(0.3 - \frac{25}{80}\right) = -2500$

Solving, $\quad A_1 = 31.67, \quad A_2 = -6.67$

$v(t) = 31.67e^{-500t} - 6.67e^{-2000t}\,\text{V}, \qquad t \geq 0$

P 8.4 **[a]** $\alpha = \frac{1}{2RC} = \frac{10^{12}}{(4000)(10)} = 25{,}000$

$\omega_o^2 = \frac{1}{LC} = \frac{10^{12}}{(250)(10)} = 4 \times 10^8$

$s_{1,2} = -25{,}000 \pm \sqrt{625 \times 10^6 - 400 \times 10^6} = -25{,}000 \pm 15{,}000$

$s_1 = -10{,}000$ rad/s

$s_2 = -40{,}000$ rad/s

[b] overdamped

[c] $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 4 \times 10^8 - 144 \times 10^6 = 256 \times 10^6$

$\alpha = 16 \times 10^3 = 16{,}000$

$\frac{1}{2RC} = 16{,}000; \qquad \therefore \quad R = \frac{10^9}{(32{,}000)(10)} = 3125\,\Omega$

[d] $s_1 = -16{,}000 + j12{,}000$ rad/s; $\qquad s_2 = -16{,}000 - j12{,}000$ rad/s

[e] $\alpha = 4 \times 10^4 = \frac{1}{2RC}; \qquad \therefore \quad R = \frac{1}{2C(4 \times 10^4)} = 2500\,\Omega$

P 8.5 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -1000$

$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -4000$

Adding the above equations, $\qquad -2\alpha = -5000$

$\alpha = 2500\,\text{rad/s}$

$-2500 \pm \sqrt{2500^2 - \omega_0^2} = -1000 \qquad \text{so} \qquad \sqrt{2500^2 - \omega_0^2} = 1500$

$\therefore \qquad -\omega_0^2 = 1500^2 - 2500^2 \qquad \text{thus} \qquad \omega_0 = 2000$

$\omega_0^2 = \frac{1}{LC} = \frac{1}{(0.01)C} = 2000^2 \qquad \text{so} \qquad C = \frac{1}{(0.01)2000^2} = 25\,\mu\text{F}$

$\alpha = \frac{1}{2RC} = \frac{1}{2R(25 \times 10^{-6})} = 2500 \qquad \text{so} \qquad R = \frac{1}{2(25 \times 10^{-6})(2500)} = 8\,\Omega$

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[b] $i_{\mathrm{R}} = \dfrac{v(t)}{R} = 5e^{-1000t} - 11.25e^{-4000t}\,\mathrm{A}, \qquad t \geq 0^+$

$i_{\mathrm{C}} = C\dfrac{dv(t)}{dt} = 9e^{-4000t} - e^{-1000t}\,\mathrm{A}, \qquad t \geq 0^+$

$i_{\mathrm{L}} = -(i_{\mathrm{R}} + i_{\mathrm{C}}) = 2.25e^{-4000t} - 4e^{-1000t}\,\mathrm{A}, \qquad t \geq 0$

P 8.6 **[a]** $\alpha = 400; \qquad \omega_d = 300$

$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 9 \times 10^4 + 16 \times 10^4 = 25 \times 10^4$

$\dfrac{1}{LC} = 25 \times 10^4$

$L = \dfrac{1}{(25 \times 10^4)(250 \times 10^{-6})} = 16\,\mathrm{mH}$

[b] $\alpha = \dfrac{1}{2RC}$

$\therefore \quad R = \dfrac{1}{2\alpha C} = \dfrac{1}{(800)(250 \times 10^{-6})} = 5\,\Omega$

[c] $V_o = v(0) = 120\,\mathrm{V}$

[d] $I_o = i_{\mathrm{L}}(0) = -i_{\mathrm{R}}(0) - i_{\mathrm{C}}(0)$

$i_{\mathrm{R}}(0) = \dfrac{120}{5} = 24\,\mathrm{A}$

$i_{\mathrm{C}}(0) = C\dfrac{dv}{dt}(0) = 250 \times 10^{-6}[-400(120) + 300(80)] = -6\,\mathrm{A}$

$\therefore \quad I_o = -24 + 6 = -18\,\mathrm{A}$

[e] $i_{\mathrm{C}}(t) = 250 \times 10^{-6}\dfrac{dv(t)}{dt} = e^{-400t}(-17\sin 300t - 6\cos 300t)\,\mathrm{A}$

$i_{\mathrm{R}}(t) = \dfrac{v(t)}{5} = e^{-400t}(24\cos 300t + 16\sin 300t)\,\mathrm{A}$

$i_{\mathrm{L}}(t) = -i_{\mathrm{R}}(t) - i_{\mathrm{C}}(t)$

$\qquad = e^{-400t}(-18\cos 300t + \sin 300t)\,\mathrm{A}, \quad t \geq 0$

Check:

$L\dfrac{di_{\mathrm{L}}}{dt} = 16 \times 10^{-3}e^{-400t}[7500\cos 300t + 5000\sin 300t]$

$v(t) = e^{-400t}[120\cos 300t + 80\sin 300t]\,\mathrm{V}$

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P 8.7 **[a]** $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = (80)^2$

$$\therefore \quad C = \frac{1}{2(50)(80)} = 125\,\mu\text{F}$$

$$\frac{1}{LC} = 80^2$$

$$\therefore \quad L = \frac{1}{80^2(125 \times 10^{-6})} = 1.25\,\text{H}$$

$$v(0) = D_2 = 5\,\text{V}$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad D_1 - 80(5) = \frac{1}{125 \times 10^{-6}}\left(0.025 - \frac{5}{50}\right) \qquad \text{so} \qquad D_1 = -200$$

[b] $v = -200te^{-80t} + 5e^{-80t}\,\text{V}, \quad t \geq 0$

$$\frac{dv}{dt} = [16{,}000t - 600]e^{-80t}$$

$$i_\text{C} = C\frac{dv}{dt} = (2000t - 75)e^{-80t}\,\text{mA}, \quad t \geq 0^+$$

P 8.8 **[a]** $\omega_o = \sqrt{\dfrac{1}{LC}} = \sqrt{\dfrac{1}{(20 \times 10^{-3})(500 \times 10^{-9})}} = 10{,}000$

$$\alpha = \frac{1}{2RC} = 10{,}000; \qquad R = \frac{1}{2(10{,}000)(500 \times 10^{-9})} = 100\,\Omega$$

[b] $v(t) = D_1te^{-10{,}000t} + D_2e^{-10{,}000t}$

$$v(0) = 40\,\text{V} = D_2$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad D_1 - 10{,}000D_2 = \frac{1}{500 \times 10^{-9}}\left(-0.12 - \frac{40}{100}\right) \qquad \text{so} \qquad D_1 = 640{,}000$$

$$\therefore \quad v(t) = (40 - 640{,}000t)e^{-10{,}000t}\,\text{V}, \qquad t \geq 0$$

[c] $i_\text{C}(t) = 0$ when $\dfrac{dv}{dt}(t) = 0$

$$\frac{dv}{dt} = (64 \times 10^8 t - 1040 \times 10^3)e^{-10{,}000t}$$

$$\frac{dv}{dt} = 0 \text{ when } 640 \times 10^8 t_1 = 1040 \times 10^3; \qquad \therefore \quad t_1 = 162.5\,\mu\text{s}$$

$$v(162.5\mu\text{s}) = (-64)e^{-1.625} = -12.6\,\text{V}$$

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[d] $w(0) = \frac{1}{2}(500 \times 10^{-9})(40)^2 + \frac{1}{2}(0.02)(0.012)^2 = 544\,\mu\text{J}$

$$w(162.5\,\mu\text{s}) = \frac{1}{2}(500 \times 10^{-9})(12.6)^2 + \frac{1}{2}(0.02)\left(\frac{12.6}{100}\right)^2 = 198.5\,\mu\text{J}$$

$$\%\text{ remaining } = \frac{198.5}{544}(100) = 36.5\%$$

P 8.9 $\alpha = 500/2 = 250$

$$R = \frac{1}{2\alpha C} = \frac{10^6}{(500)(18)} = 1000\,\Omega$$

$$v(0^+) = -11 + 20 = 9\,\text{V}$$

$$i_\text{R}(0^+) = \frac{9}{1000} = 9\,\text{mA}$$

$$\frac{dv}{dt} = 1100e^{-100t} - 8000e^{-400t}$$

$$\frac{dv(0^+)}{dt} = 1100 - 8000 = -6900\,\text{V/s}$$

$$i_\text{C}(0^+) = 2 \times 10^{-6}(-6900) = -13.8\,\text{mA}$$

$$i_\text{L}(0^+) = -[i_\text{R}(0^+) + i_\text{C}(0^+)] = -[9 - 13.8] = 4.8\,\text{mA}$$

P 8.10 **[a]** $\alpha = \frac{1}{2RC} = 1250, \qquad \omega_o = 10^3, \qquad$ therefore overdamped

$s_1 = -500, \qquad s_2 = -2000$

therefore $\quad v = A_1e^{-500t} + A_2e^{-2000t}$

$v(0^+) = 0 = A_1 + A_2; \qquad \left[\frac{dv(0^+)}{dt}\right] = \frac{i_\text{C}(0^+)}{C} = 98{,}000\,\text{V/s}$

Therefore $\quad -500A_1 - 2000A_2 = 98{,}000$

$A_1 = \frac{+980}{15}, \quad A_2 = \frac{-980}{15}$

$v(t) = \left[\frac{980}{15}\right][e^{-500t} - e^{-2000t}]\,\text{V}, \qquad t \geq 0$

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[b]

Example 8.4: $v_{\max} \cong 74.1\,\text{V}$ at 1.4 ms

Example 8.5: $v_{\max} \cong 36.1\,\text{V}$ at 1.0 ms

Problem 8.17: $v_{\max} \cong 30.9$ at 0.92 ms

P 8.11 $t < 0:$ $V_o = 100\,\text{V}$, $I_o = 5\,\text{A}$

$t > 0$:

$$\alpha = \frac{1}{2RC} = \frac{1}{2(50)(25 \times 10^{-6})} = 400 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(160 \times 10^{-3})(25 \times 10^{-6})}} = 500$$

$\alpha^2 < \omega_0^2$ Response is underdamped

$$\omega_d = \sqrt{500^2 - 400^2} = 300$$

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$$\therefore \quad v_o = B_1 e^{-400t}\cos 300t + B_2 e^{-400t}\sin 300t$$

$$v_o(0) = B_1 = 100$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad -(400)(100) + 300B_2 = \frac{1}{25\times 10^{-6}}\left(-5 - \frac{100}{50}\right) \qquad \text{so} \qquad B_2 = -800$$

$$\therefore \quad v_o = 100e^{-400t}\cos 300t - 800e^{-400t}\sin 300t\,\text{V}, \qquad t \geq 0$$

P 8.12 $$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(160\times 10^{-3})(25\times 10^{-6})}} = 500$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(40)(25\times 10^{-6})} = 500$$

$$\alpha^2 = \omega_0^2 \qquad \text{Response is critically damped}$$

$$v_o(t) = D_1 t e^{-500t} = D_2 e^{-500t}$$

$$v_o(0) = D_2 = 100\,\text{V}$$

$$\frac{dv_o}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad D_1 - 500(100) = \frac{1}{25\times 10^{-6}}\left(-5 - \frac{100}{40}\right) \qquad \text{so} \qquad D_1 = -250{,}000$$

$$v_o(t) = -250{,}000te^{-500t} + 100e^{-500t}\,\text{V}, \qquad t \geq 0$$

P 8.13 $$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(160\times 10^{-3})(25\times 10^{-6})}} = 500$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(32)(25\times 10^{-6})} = 625$$

$$\therefore \quad \alpha^2 > \omega_o^2 \text{ (overdamped)}$$

$$s_{1,2} = -625 \pm \sqrt{625^2 - 500^2} = -250, -1000$$

$$v_o(t) = A_1 e^{-250t} + A_2 e^{-1000t}$$

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$$v_o(0) = A_1 + A_2 = 100\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -250A_1 - 1000A_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad -250A_1 - 1000A_2 = \frac{1}{25 \times 10^{-6}}\left(-5 - \frac{100}{32}\right) = -325{,}000$$

Solving, $\quad A_1 = -300, \qquad A_2 = 400$

$$v_o(t) = -300e^{-250t} + 400e^{-1000t}\,\text{V}, \qquad t \geq 0$$

P 8.14

$$v_T = -10i_\phi + i_T\left(\frac{(150)(60)}{210}\right) = -10\frac{-i_T(150)}{210} + i_t\frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500 + 9000}{210} = 50\,\Omega$$

$$V_o = \frac{4000}{10{,}000}(50) = 20\,\text{V}; \qquad I_o = 0$$

$$i_\text{C}(0) = -i_R(0) - i_\text{L}(0) = -\frac{20}{50} = -0.4\,\text{A}$$

$$\frac{i_\text{C}(0)}{C} = \frac{-0.4}{8 \times 10^{-6}} = -50{,}000$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(51.2 \times 10^{-3})(8 \times 10^{-6})}} = 1562.5 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(50)(8 \times 10^{-6})} = 1250 \text{ rad/s}$$

$\alpha^2 < \omega_0^2 \qquad$ so the response is underdamped

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$$\omega_d = \sqrt{1562.5^2 - 1250^2} = 937.5$$

$$v_o = B_1 e^{-1250t}\cos 937.5t + B_2 e^{-1250t}\sin 937.5t$$

$$v_o(0) = B_1 20\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{i_C(0)}{C}$$

$$\therefore \qquad -1250(20) + 937.5B_2 = -50{,}000 \qquad \text{so} \qquad B_2 = -26.67$$

$$v_o = 20e^{-1250t}\cos 937.5t - 26.67e^{-1250t}\sin 937.5t\,\text{V}, \qquad t \geq 0$$

P 8.15

$$v_T = -10i_\phi + i_T\left(\frac{(150)(60)}{210}\right) = -10\frac{-i_T(150)}{210} + i_t\frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500 + 9000}{210} = 50\,\Omega$$

$$V_o = \frac{4000}{10{,}000}(50) = 20\,\text{V}; \qquad I_o = 0$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(80\times 10^{-3})(8\times 10^{-6})}} = 1250\ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(50)(8\times 10^{-6})} = 1250\ \text{rad/s}$$

$$\alpha^2 = \omega_0^2 \qquad \text{so the response is critically damped}$$

$$v_o = D_1 te^{-1250t} + D_2 e^{-1250t}$$

$$v_o(0) = D_2 = 20\,\text{V}$$

$$\frac{dv_o}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad D_1 - 1250(20) = \frac{1}{8\times 10^{-6}}\left(0 - \frac{20}{50}\right) \qquad \text{so} \qquad D_1 = -25{,}000$$

$$v_o = -25{,}000te^{-1250t} + 20e^{-1250t}\,\text{V}, \qquad t \geq 0$$

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P 8.16

$$v_T = -10i_\phi + i_T\left(\frac{(150)(60)}{210}\right) = -10\frac{-i_T(150)}{210} + i_t\frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500+9000}{210} = 50\,\Omega$$

$$V_o = \frac{4000}{10{,}000}(50) = 20\,\text{V}; \qquad I_o = 0$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(125\times10^{-3})(8\times10^{-6})}} = 1000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(50)(8\times10^{-6})} = 1250 \text{ rad/s}$$

$\alpha^2 > \omega_0^2$ so the response is overdamped

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 1000^2} = -500, -2000$$

$$v_o = A_1e^{-500t} + A_2e^{-1000t}$$

$$v_o(0) = A_1 + A_2 = 20\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -500A_1 - 2000A_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad -500A_1 - 2000A_2 = \frac{1}{8\times10^{-6}}\left(0 - \frac{20}{50}\right) = -50{,}000$$

Solving, $A_1 = -6.67$, $A_2 = 26.67$

$$v_o = -6.67e^{-500t} + 26.67e^{-2000t}\,\text{V}, \qquad t \geq 0$$

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P 8.17 **[a]** $\dfrac{1}{LC} = 5000^2$

There are many possible solutions. This one begins by choosing $L = 10\,\text{mH}$. Then,

$$C = \frac{1}{(10 \times 10^{-3})(5000)^2} = 4\,\mu\text{F}$$

We can achieve this capacitor value using components from Appendix H by combining four $1\,\mu$F capacitors in parallel.

Critically damped: $\alpha = \omega_0 = 5000$ so $\dfrac{1}{2RC} = 5000$

$$\therefore \quad R = \frac{1}{2(4 \times 10^{-6})(5000)} = 25\,\Omega$$

We can create this resistor value using components from Appendix H by combining a $10\,\Omega$ resistor and a $15\,\Omega$ resistor in series. The final circuit:

[b] $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5000 \pm 0$

Therefore there are two repeated real roots at -5000 rad/s.

P 8.18 **[a]** Underdamped response:

$\alpha < \omega_0$ so $\alpha < 5000$

Therefore we choose a larger resistor value than the one used in Problem 8.17. Choose $R = 100\,\Omega$:

$$\alpha = \frac{1}{2(100)(4 \times 10^{-6})} = 1250$$

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 5000^2} = -1250 \pm j4841.23 \text{ rad/s}$$

[b] Overdamped response:

$\alpha > \omega_0$ so $\alpha > 5000$

Therefore we choose a smaller resistor value than the one used in Problem 8.17. Choose $R = 20\,\Omega$:

$$\alpha = \frac{1}{2(20)(4 \times 10^{-6})} = 6250$$

$$s_{1,2} = -1250 \pm \sqrt{6250^2 - 5000^2} = -1250 \pm 3750$$

$$= -2500 \text{ rad/s}; \qquad \text{and} \qquad -10{,}000 \text{ rad/s}$$

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P 8.19 **[a]** $\alpha = \dfrac{1}{2RC} = 800 \text{ rad/s}$

$$\omega_o^2 = \frac{1}{LC} = 10^6$$

$$\omega_d = \sqrt{10^6 - 800^2} = 600 \text{ rad/s}$$

$$\therefore \quad v = B_1 e^{-800t}\cos 600t + B_2 e^{-800t}\sin 600t$$

$$v(0) = B_1 = 30$$

$$i_R(0^+) = \frac{30}{5000} = 6\,\text{mA}; \qquad i_C(0^+) = -12\,\text{mA}$$

$$\therefore \quad \frac{dv}{dt}(0^+) = \frac{-0.012}{125 \times 10^{-9}} = -96{,}000 \text{ V/s}$$

$$-96{,}000 = -\alpha B_1 + \omega_d B_2 = -(800)(30) + 600B_2$$

$$\therefore \quad B_2 = -120$$

$$\therefore \quad v = 30e^{-800t}\cos 600t - 120e^{-800t}\sin 600t\,\text{V}, \qquad t \geq 0$$

[b] $\dfrac{dv}{dt} = 6000e^{-800t}(13\sin 600t - 16\cos 600t)$

$$\frac{dv}{dt} = 0 \quad \text{when} \quad 16\cos 600t = 13\sin 600t \quad \text{or} \quad \tan 600t = \frac{16}{13}$$

$$\therefore \quad 600t_1 = 0.8885, \qquad t_1 = 1.48\,\text{ms}$$

$$600t_2 = 0.8885 + \pi, \qquad t_2 = 6.72\,\text{ms}$$

$$600t_3 = 0.8885 + 2\pi, \qquad t_3 = 11.95\,\text{ms}$$

[c] $t_3 - t_1 = 10.47\,\text{ms}; \qquad T_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{600} = 10.47\,\text{ms}$

[d] $t_2 - t_1 = 5.24\,\text{ms}; \qquad \dfrac{T_d}{2} = \dfrac{10.48}{2} = 5.24\,\text{ms}$

[e] $v(t_1) = 30e^{-(1.184)}(\cos 0.8885 - 4\sin 0.8885) = -22.7\,\text{V}$

$$v(t_2) = 30e^{-(5.376)}(\cos 4.032 - 4\sin 4.032) = 0.334\,\text{V}$$

$$v(t_3) = 30e^{-(9.56)}(\cos 7.17 - 4\sin 7.17) = -5.22\,\text{mV}$$

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[f]

P 8.20 [a] $\alpha = 0; \qquad \omega_d = \omega_o = \sqrt{10^6} = 1000\,\text{rad/s}$

$$v = B_1 \cos \omega_o t + B_2 \sin \omega_o t; \qquad v(0) = B_1 = 30$$

$$C\frac{dv}{dt}(0) = -i_{\text{L}}(0) = -0.006$$

$$-48{,}000 = -\alpha B_1 + \omega_d B_2 = -0 + 1000 B_2$$

$$\therefore \quad B_2 = \frac{-48{,}000}{1000} = -48\,\text{V}$$

$$v = 30 \cos 1000t - 48 \sin 1000t\,\text{V}, \qquad t \geq 0$$

[b] $2\pi f = 1000; \qquad f = \dfrac{1000}{2\pi} \cong 159.15\,\text{Hz}$

[c] $\sqrt{30^2 + 48^2} = 56.6\,\text{V}$

P 8.21 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2, respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$$

The characteristic determinant is

$$\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$$

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The numerator determinants are

$$N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$$

and $$N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$$

It follows that $$A_1 = \frac{N_1}{\Delta} = \frac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$$

and $$A_2 = \frac{N_2}{\Delta} = \frac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$$

We see from these expressions that $A_1 = A_2^*$.

P 8.22 By definition, $B_1 = A_1 + A_2$. From the solution to Problem 8.21 we have

$$A_1 + A_2 = \frac{2\omega_d K_1}{2\omega_d} = K_1$$

But K_1 is $v(0)$, therefore, $B_1 = v(0)$, which is identical to Eq. (8.30).
By definition, $B_2 = j(A_1 - A_2)$. From Problem 8.12 we have

$$B_2 = j(A_1 - A_2) = \frac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \frac{\alpha K_1 + K_2}{\omega_d}$$

It follows that

$$K_2 = -\alpha K_1 + \omega_d B_2, \quad \text{but} \quad K_2 = \frac{dv(0^+)}{dt} \quad \text{and} \quad K_1 = B_1.$$

Thus we have

$$\frac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2,$$

which is identical to Eq. (8.31).

P 8.23 **[a]** $2\alpha = 1000; \qquad \alpha = 500\,\text{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 600; \qquad \omega_o = 400\,\text{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{2(500)(250)} = 4\,\mu F$$

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$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(400)^2(4 \times 10^{-6})} = 1.5625\,\text{H}$$

$$i_\text{C}(0^+) = A_1 + A_2 = 45\,\text{mA}$$

$$\frac{di_\text{C}}{dt} + \frac{di_\text{L}}{dt} + \frac{di_\text{R}}{dt} = 0$$

$$\frac{di_\text{C}(0)}{dt} = -\frac{di_\text{L}(0)}{dt} - \frac{di_\text{R}(0)}{dt}$$

$$\frac{di_\text{L}(0)}{dt} = \frac{0}{1.5625} = 0\,\text{A/s}$$

$$\frac{di_\text{R}(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_\text{C}(0)}{C} = \frac{45 \times 10^{-3}}{(250)(4 \times 10^{-6})} = 45\,\text{A/s}$$

$$\therefore \quad \frac{di_\text{C}(0)}{dt} = 0 - 45 = -45\,\text{A/s}$$

$$\therefore \quad 200A_1 + 800A_2 = 45; \qquad A_1 + A_2 = 0.045$$

$$\text{Solving,} \quad A_1 = -15\,\text{mA}; \qquad A_2 = 60\,\text{mA}$$

$$\therefore \quad i_\text{C} = -15e^{-200t} + 60e^{-800t}\,\text{mA}, \qquad t \geq 0^+$$

[b] By hypothesis

$$v = A_3e^{-200t} + A_4e^{-800t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 0$$

$$\frac{dv(0)}{dt} = \frac{45 \times 10^{-3}}{4 \times 10^{-6}} = 11{,}250\,\text{V/s}$$

$$-200A_3 - 800A_4 = 11{,}250; \qquad \therefore \quad A_3 = 18.75\,\text{V}; \qquad A_4 = -18.75\,\text{V}$$

$$v = 18.75e^{-200t} - 18.75e^{-800t}\,\text{V}, \qquad t \geq 0$$

[c] $i_\text{R}(t) = \dfrac{v}{250} = 75e^{-200t} - 75e^{-800t}\,\text{mA}, \qquad t \geq 0^+$

[d] $i_\text{L} = -i_\text{R} - i_\text{C}$

$$i_\text{L} = -60e^{-200t} + 15e^{-800t}\,\text{mA}, \qquad t \geq 0$$

P 8.24 **[a]** $v = L\left(\dfrac{di_\text{L}}{dt}\right) = 16[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{V}, \qquad t \geq 0$

[b] $i_\text{R} = \dfrac{v}{R} = 40[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

[c] $i_\text{C} = I - i_\text{L} - i_\text{R} = [-8e^{-20{,}000t} + 32e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

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P 8.25 **[a]** $v = L\left(\dfrac{di_{\mathrm{L}}}{dt}\right) = 40e^{-32{,}000t}\sin 24{,}000t\,\mathrm{V}, \qquad t \geq 0$

[b] $i_{\mathrm{C}}(t) = I - i_{\mathrm{R}} - i_{\mathrm{L}} = 24 \times 10^{-3} - \dfrac{v}{625} - i_{\mathrm{L}}$

$$= [24e^{-32{,}000t}\cos 24{,}000t - 32e^{-32{,}000t}\sin 24{,}000t]\,\mathrm{mA}, \qquad t \geq 0^+$$

P 8.26 $v = L\left(\dfrac{di_{\mathrm{L}}}{dt}\right) = 960{,}000te^{-40{,}000t}\,\mathrm{V}, \qquad t \geq 0$

P 8.27 $\omega_o = \sqrt{\dfrac{1}{LC}} = \sqrt{\dfrac{1}{(25 \times 10^{-3})(62.5 \times 10^{-6})}} = 800 \text{ rad/s}$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(12.5)(62.5 \times 10^{-6})} = 640 \text{ rad/s} \qquad \therefore \quad \text{underdamped}$$

$$\omega_d = \sqrt{800^2 - 640^2} = 480$$

$$I_f = 2\,\mathrm{A}$$

$$i_L = 2 + B_1'e^{-640t}\cos 480t + B_2'e^{-640t}\sin 480t$$

$$i_L(0) = 2 + B_1' = 1 \qquad \text{so} \qquad B_1' = -1$$

$$\frac{di_L}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \frac{V_0}{L}$$

$$\therefore \qquad -640(-1) + 480B_2' = \frac{50}{25 \times 10^{-3}} \qquad \text{so} \qquad B_2' = 2.83$$

$$i_L(t) = 2 - e^{-640t}\cos 480t + 2.83e^{-640t}\sin 480t\,\mathrm{A}, \quad t \geq 0$$

P 8.28 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(8)(62.5 \times 10^{-6})} = 1000 \text{ rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(25 \times 10^{-3})(62.5 \times 10^{-6})}} = 800 \text{ rad/s}$$

Overdamped: $\quad s_{1,2} = -1000 \pm \sqrt{1000^2 - 800^2} = -400, -1600 \text{ rad/s}$

$$I_f = 2\,\mathrm{A}$$

$$i_L = 2 + A_1'e^{-400t} + A_2'e^{-1600t}$$

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$$i_L(0) = 2 + A_1' + A_2' = 1 \qquad \text{so} \qquad A_1' + A_2' = -1$$

$$\frac{di_L}{dt}(0) = -400A_1' - 1600A_2' = \frac{V_0}{L} = \frac{50}{25 \times 10^{-3}} = 2000$$

Solving, $\quad A_1' = \dfrac{1}{3}, \qquad A_2' = -\dfrac{4}{3}$

$$i_L(t) = 2 + \frac{1}{3}e^{-400t} - \frac{4}{3}e^{-1600t}\,\text{A}, \quad t \geq 0$$

P 8.29 $\quad \alpha = \dfrac{1}{2RC} = \dfrac{1}{2(10)(62.5 \times 10^{-6})} = 800$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(25 \times 10^{-3})(62.5 \times 10^{-6})}} = 800 \text{ rad/s}$$

$\alpha^2 = \omega_0^2 \qquad$ Critically damped

$I_f = 2\,\text{A}$

$$i_L = 2 + D_1' te^{-800t} + D_2' e^{-800t}$$

$$i_L(0) = 2 + D_2' = 1; \qquad \therefore \quad D_2' = -1\,\text{A}$$

$$\frac{di_L}{dt}(0) = D_1' - \alpha D_2' = \frac{V_0}{L}$$

$$\therefore \qquad D_1' - 800(-1) = \frac{50}{25 \times 10^{-3}} \qquad \text{so} \qquad D_1' = 1200$$

$$i_L = 2 + 1200te^{-800t} - e^{-800t}\,\text{A}, \quad t \geq 0$$

P 8.30 $\quad i_{\text{L}}(0^-) = i_{\text{L}}(0^+) = \dfrac{4}{16} = 0.25\,\text{A}$

For $t > 0$

$$\alpha = \frac{1}{2RC} = 100\,\text{rad/s}; \qquad \omega_o^2 = \frac{1}{LC} = 80^2 \quad \text{so} \quad \omega_o = 80 \text{ rad/s}$$

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$s_{1,2} = -100 \pm \sqrt{100^2 - 80^2} = -40, -160 \text{ rad/s}$

$v_o(\infty) = 0 = V_f; \qquad A_1' + A_2' = v(0) = 0$

$v_o = A_1' e^{-40t} + A_2' e^{-160t}$

$i_C(0^+) = -0.25 + 0.25 + 0 = 0$

$\therefore \quad \dfrac{dv_o}{dt} = 0 = -40A_1' - 160A_2'$

Solving, $\quad A_1' = 0; \quad A_2' = 0$

$\therefore \quad v_o = 0 \text{ for } t \geq 0$

Note: $\quad v_o(0) = 0; \quad v_o(\infty) = 0; \quad \dfrac{dv_o(0)}{dt} = 0$

Hence, the 0.25 A current circulates between the current source and the ideal inductor in the equivalent circuit. In the original circuit, the 4 V source sustains a current of 0.25 A in the inductor. This is an example of a circuit going directly into steady state when the switch is closed. There is no transient period, or interval.

P 8.31 $\quad \alpha = \dfrac{1}{2RC} = \dfrac{1}{2(20)(31.25 \times 10^{-6})} = 800 \text{ rad/s}$

$\omega_o = \sqrt{\dfrac{1}{LC}} = \sqrt{\dfrac{1}{(50 \times 10^{-3})(31.25 \times 10^{-6})}} = 800 \text{ rad/s}$

$\alpha^2 = \omega_0^2 \qquad$ Critically damped

$V_0 = v_C(0) = 60\,\text{V}; \qquad I_0 = i_o(0) = 0; \qquad I_f = i_o(\infty) = \dfrac{60}{20} = 3\,\text{A}$

$i_o = 3 + D_1' t e^{-800t} + D_2' e^{-800t}$

$i_o(0) = 3 + D_2' = 0 \qquad \text{so} \qquad D_2' = -3$

$\dfrac{di_o}{dt}(0) = D_1' - \alpha D_2' = \dfrac{V_0}{L} \qquad \text{so} \qquad D_1' - 800(-3) = \dfrac{60}{50 \times 10^{-3}}$

Solving, $\quad D_1' = -1200$

$i_o(t) = 3 - 1200te^{-800t} - 3e^{-800t}\,\text{A}, \qquad t \geq 0$

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P 8.32 **[a]** $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(20)(31.25 \times 10^{-6})} = 800 \text{ rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(50 \times 10^{-3})(31.25 \times 10^{-6})}} = 800 \text{ rad/s}$$

$\alpha^2 = \omega_0^2$ Critically damped

$V_f = v_C(\infty) = 0$

$v_o = D_1' t e^{-800t} + D_2' e^{-800t}$

$v_o(0) = D_2' = 60$

$\dfrac{v_o}{dt}(0) = D_1' - \alpha D_2' = 0$ so $D_1' = (800)(60) = 48{,}000$

$\therefore \quad v_o(t) = 48{,}000te^{-800t} + 60e^{-800t}\,\text{V}, \quad t \geq 0$

[b] $v_o = L\dfrac{di_o}{dt} = (50 \times 10^{-3})\dfrac{d}{dt}(3 - 1200te^{-800t} - 3e^{-800t})$

$v_o(t) = (50 \times 10^{-3})(960{,}000te^{-800t} + 1200e^{-800t})$

$= 48{,}000te^{-800t} + 60e^{-800t}\,\text{V}, \quad t \geq 0$

Thus the solutions for $i_o(t)$ and $v_o(t)$ agree.

P 8.33 For $t > 0$

$\alpha = \dfrac{1}{2RC} = 640; \qquad \dfrac{1}{LC} = 64 \times 10^4$

$\omega_d = \sqrt{800^2 - 640^2} = 480$

$i_o = I_f + B_1' e^{-640t}\cos 480t + B_2' e^{-640t}\sin 480t$

$I_f = \dfrac{25}{125} = 0.2\,\text{A}$

$i_o(0) = 0.2 + B_1' = 0$ so $B_1' = -0.2$

$\dfrac{di_o}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \dfrac{V_0}{L}$ so $-640(-0.2) + 480B_2' = 0$

Solving, $B_2' = -0.267$

$i_o(t) = 0.2 - 0.2e^{-640t}\cos 480t - 0.267e^{-640t}\sin 480t\,\text{A}, \quad t \geq 0$

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P 8.34 **[a]** $\alpha = \dfrac{1}{2RC} = 640; \qquad \dfrac{1}{LC} = 64 \times 10^4$

$$\omega_d = \sqrt{800^2 - 640^2} = 480$$

$$v_o = V_f + B_1' e^{-640t}\cos 480t + B_2' e^{-640t}\sin 480t$$

$$V_f = 0; \qquad V_o(0^+) = 0; \qquad i_C(0^+) = 0.2\,\text{A}$$

$$v_o(0) = 0 + B_1' = 0 \qquad \text{so} \qquad B_1' = 0$$

$$\frac{dv_o}{dt}(0) = -640(0) + 480B_2' = \frac{i_C(0^+)}{6.25 \times 10^{-6}} = 32{,}000 \qquad \text{so} \qquad B_2' = 66.67$$

$$\therefore \qquad v_o(t) = 66.67e^{-640t}\sin 480t\,\text{V}, \quad t \geq 0$$

[b] From the solution to Problem 8.33,

$$i_o(t) = 0.2 - 0.2e^{-640t}\cos 480t - 0.267e^{-640t}\sin 480t\,\text{A}$$

$$v_o = L\frac{di_o}{dt} = (0.25)(266.67e^{-640t}\sin 480t) = 66.67e^{-640t}\sin 480t\,\text{V}, \quad t \geq 0$$

Thus the solutions to Problems 8.33 and 8.34 are consistent.

P 8.35 $t < 0$:

$$V_0 = v_o(0^-) = v_o(0^+) = \frac{3000}{4000}(100) = 75\,\text{V}$$

$$I_0 = i_\text{L}(0^-) = i_\text{L}(0^+) = 100\,\text{mA}$$

$t > 0$:

$$\alpha = \frac{1}{2RC} = \frac{1}{2(40)(25 \times 10^{-6})} = 500\,\text{rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250 \times 10^{-3})(25 \times 10^{-6})}} = 400$$

$$\therefore \quad \alpha^2 > \omega_o^2 \quad \text{overdamped}$$

$$s_{1,2} = -500 \pm \sqrt{500^2 - 400^2} = -200, -800$$

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[a] $i_L = I_f + A_1 e^{-200t} + A_2 e^{-800t}$

$I_f = 100\,\text{mA}$

$i_L(0) = 0.1 + A_1 + A_2 = 0.1 \qquad \text{so} \qquad A_1 + A_2 = 0$

$\dfrac{di_L}{dt}(0) = -200A_1 - 800A_2 = \dfrac{V_0}{L} = \dfrac{75}{0.25} = 300$

Solving, $\qquad A_1 = 0.5, \qquad A_2 = -0.5$

$\therefore \qquad i_L(t) = 0.1 + 0.5e^{-200t} - 0.5e^{-800t}\,\text{A}$

[b] $v_C(t) = v_L(t) = L\dfrac{di_L}{dt} = (0.25)(-100e^{-200t} + 400e^{-800t})$

$= -25e^{-200t} + 100e^{-800t}\,\text{V}, \qquad t \geq 0$

P 8.36 **[a]** $w_\text{L} = \displaystyle\int_0^\infty p dt = \int_0^\infty v_o i_\text{L}\, dt$

$v_o = -25e^{-200t} + 100e^{-800t}\,\text{V}$

$i_\text{L} = 0.1 + 0.5e^{-200t} - 0.5e^{-800t}\,\text{A}$

$p = -2.5e^{-200t} - 12.5e^{-400t} + 10e^{-800t} + 62.5e^{-1000t} - 50e^{-1600t}\,\text{W}$

$$w_\text{L} = -2.5\int_0^\infty e^{-200t}\,dt - 12.5\int_0^\infty e^{-400t}\,dt + 10\int_0^\infty e^{-800t}\,dt$$

$$+62.5\int_0^\infty e^{-1000t}\,dt - 50\int_0^\infty e^{-1600t}\,dt$$

$$= -2.5\frac{e^{-200t}}{-200}\bigg|_0^\infty + -12.5\frac{e^{-400t}}{-400}\bigg|_0^\infty$$

$$= 10\frac{e^{-800t}}{-800}\bigg|_0^\infty + 62.5\frac{e^{-1000t}}{-1000}\bigg|_0^\infty$$

$$- \ 50\frac{e^{-1600t}}{-1600}\bigg|_0^\infty$$

All the upper limits evaluate to zero hence

$$w_\text{L} = \frac{-2.5}{200} - \frac{-12.5}{400} + \frac{10}{800} + \frac{62.5}{1000} - \frac{50}{1600} = 0\,\text{J}$$

Since the initial and final values of the current in the inductor are the same, the initial and final values of the energy in the inductor are the same. Thus, there is no net energy delivered to the inductor.

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[b] $v_o = -25e^{-200t} + 100e^{-800t}\,\text{V}$

$$i_\text{R} = \frac{v_o}{40} = -0.625e^{-200t} + 2.5e^{-800t}\,\text{A}$$

$$p_\text{R} = v_o i_\text{R} = 15.625e^{-400t} - 125e^{-1000t} + 250e^{-1600t}\,\text{W}$$

$$w_\text{R} = \int_0^\infty p_\text{R}\,dt$$

$$= 15.625\int_0^\infty e^{-400t}\,dt - 125\int_0^\infty e^{-1000t}\,dt + 250\int_0^\infty e^{-1600t}\,dt$$

$$= 15.625\frac{e^{-400t}}{-400}\Bigg|_0^\infty - 125\frac{e^{-1000t}}{-1000}\Bigg|_0^\infty + 250\frac{e^{-1600t}}{-1600}\Bigg|_0^\infty$$

Since all the upper limits evaluate to zero we have

$$w_\text{R} = \frac{15.625}{400} - \frac{125}{400} + \frac{250}{1600} = 70.3125\,\text{mJ}$$

[c] $0.1 = i_\text{R} + i_\text{C} + i_\text{L} \quad (\text{mA})$

$$i_\text{C} = 0.1 - (-0.625e^{-200t} + 2.5e^{-800t}) - (0.1 + 0.5e^{-200t} - 0.5e^{-800t})$$

$$= 0.125e^{-200t} - 2e^{-800t}\,\text{A}$$

$$p_\text{C} = v_o i_\text{C} = [-25e^{-200t} + 100e^{-800t}][0.125e^{-200t} - 2e^{-800t}]$$

$$= -3.125e^{-400t} + 62.5e^{-1000t} - 200e^{-1600t}\,\text{W}$$

$$w_\text{C} = -3.125\int_0^\infty e^{-400t}\,dt + 62.5\int_0^\infty e^{-1000t}\,dt - 200\int_0^\infty e^{-1600t}\,dt$$

$$= -3.125\frac{e^{-400t}}{-400}\Bigg|_0^\infty + 62.5\frac{e^{-1000t}}{-1000}\Bigg|_0^\infty - 200\frac{e^{-1600t}}{-1600}\Bigg|_0^\infty$$

Since all upper limits evaluate to zero we have

$$w_\text{C} = \frac{-3.125}{400} + \frac{62.5}{1000} - \frac{200}{1600} = -70.3125\,\text{mJ}$$

[d] $i_s = 100\,\text{mA}$

$$p_s(\text{del}) = 0.1v = -2.5e^{-200t} + 10e^{-800t}\,\text{W}$$

$$w_s = -2.5\int_0^\infty e^{-200t}\,dt + 10\int_0^\infty e^{-800t}\,dt$$

$$= -2.5\frac{e^{-200t}}{-200}\Bigg|_0^\infty + 10\frac{e^{-800t}}{-800}\Bigg|_0^\infty = \frac{-2.5}{200} + \frac{10}{800} = 0$$

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[e] $w_{\mathrm{L}} = 0\,\mathrm{J}$

$w_{\mathrm{R}} = 70.3125\,\mathrm{mJ}$ (absorbed)

$w_{\mathrm{C}} = 70.3125\,\mathrm{mJ}$ (delivered)

$w_S = 0\,\mathrm{mJ}$

$\sum w_{\mathrm{del}} = w_{\mathrm{abs}} = 70.3125\,\mathrm{mJ}.$

P 8.37 $t < 0:$ $\quad i_{\mathrm{L}}(0^-) = \dfrac{36}{300} = 0.12\,\mathrm{A};$ $\quad v_{\mathrm{C}}(0^-) = 0\,\mathrm{V}$

The circuit reduces to:

$i_{\mathrm{L}}(\infty) = 0.1\,\mathrm{A}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(20\times 10^{-3})(500\times 10^{-9})}} = 10{,}000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(100)(500\times 10^{-9})} = 10{,}000 \text{ rad/s}$$

Critically damped:

$$i_L = 0.1 + D_1' t e^{-10{,}000t} + D_2' e^{-10{,}000}$$

$$i_L(0) = 0.1 + D_2' = 0.12 \qquad \text{so} \qquad D_2' = 0.02$$

$$\frac{di_L}{dt}(0) = D_1' - \alpha D_2' = \frac{V_0}{L} \qquad \text{so} \qquad D_1' - (10{,}000)(0.02) = 0$$

Solving, $\quad D_1' = 200$

$$i_L(t) = 0.1 + 200te^{-10{,}000t} + 0.02e^{-10{,}000t}\,\mathrm{A}, \qquad t \geq 0$$

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P 8.38 $v_C(0^+) = \frac{1}{2}(80) = 40\,\text{V}$

$$i_L(0^+) = 10\,\text{A}; \qquad i_L(\infty) = \frac{80}{10} = 8\,\text{A}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(5)(250 \times 10^{-6})} = 400 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(16 \times 10^{-3})(250 \times 10^{-6})}} = 500 \text{ rad/s}$$

$$\alpha^2 < \omega_o^2; \qquad \therefore \quad \text{underdamped}$$

$$\omega_d = \sqrt{500^2 - 400^2} = 300 \text{ rad/s}$$

$$i_L = 8 + B_1' e^{-400t}\cos 300t + B_2' e^{-400t}\sin 300t$$

$$i_L(0) = 8 + B_1' = 10; \qquad B_1' = 10 - 8 = 2\,\text{A}$$

$$\frac{di_L}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \frac{V_0}{L}$$

$$\therefore \quad -400(2) + 300B_2' = \frac{40}{0.016} \qquad \text{so} \qquad B_2' = 11\,\text{A}$$

$$\therefore \quad i_L = 8 + 2e^{-400t}\cos 300t + 11e^{-400t}\sin 300t\,\text{A}, \qquad t \geq 0$$

P 8.39 $\alpha = 2000\,\text{rad/s}; \qquad \omega_d = 1500\,\text{rad/s}$

$$\omega_o^2 - \alpha^2 = 225 \times 10^4; \qquad \omega_o^2 = 625 \times 10^4; \quad w_o = 25{,}000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 2000; \qquad R = 4000L$$

$$\frac{1}{LC} = 625 \times 10^4; \qquad L = \frac{1}{(625 \times 10^4)(80 \times 10^{-9})} = 2\,\text{H}$$

$$\therefore \quad R = 8\,\text{k}\Omega$$

$$i(0^+) = B_1 = 7.5\,\text{mA}; \qquad \text{at } t = 0^+$$

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$$60 + v_{\mathrm{L}}(0^+) - 30 = 0; \qquad \therefore \qquad v_{\mathrm{L}}(0^+) = -30\,\mathrm{V}$$

$$\frac{di(0^+)}{dt} = \frac{-30}{2} = -15\,\mathrm{A/s}$$

$$\therefore \quad \frac{di(0^+)}{dt} = 1500B_2 - 2000B_1 = -15$$

$$\therefore \quad 1500B_2 = 2000(7.5 \times 10^{-3}) - 15; \qquad \therefore \quad B_2 = 0\,\mathrm{A}$$

$$\therefore \quad i = 7.5e^{-2000t}\cos 1500t\,\mathrm{mA}, \quad t \geq 0$$

P 8.40 From Prob. 8.39 we know v_c will be of the form

$$v_c = B_3e^{-2000t}\cos 1500t + B_4e^{-2000t}\sin 1500t$$

From Prob. 8.39 we have

$$v_c(0) = -30\,\mathrm{V} = B_3$$

and

$$\frac{dv_c(0)}{dt} = \frac{i_{\mathrm{C}}(0)}{C} = \frac{7.5 \times 10^{-3}}{80 \times 10^{-9}} = 93.75 \times 10^3$$

$$\frac{dv_c(0)}{dt} = 1500B_4 - 2000B_3 = 93{,}750$$

$$\therefore \quad 1500B_4 = 2000(-30) + 93{,}750; \qquad B_4 = 22.5\,\mathrm{V}$$

$$v_c(t) = -30e^{-2000t}\cos 1500t + 22.5e^{-2000t}\sin 1500t\,\mathrm{V} \qquad t \geq 0$$

P 8.41 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_0^2} = -4000; \qquad -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -16{,}000$

$$\therefore \quad \alpha = 10{,}000\ \mathrm{rad/s}, \qquad \omega_0^2 = 64 \times 10^6$$

$$\alpha = \frac{R}{2L} = 10{,}000; \qquad R = 20{,}000L$$

$$\omega_o^2 = \frac{1}{LC} = 64 \times 10^6; \qquad L = \frac{10^9}{64 \times 10^6(31.25)} = 0.5\,\mathrm{H}$$

$$R = 10{,}000\ \Omega$$

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[b] $i(0) = 0$

$$L\frac{di(0)}{dt} = v_c(0); \qquad \frac{1}{2}(31.25) \times 10^{-9} v_c^2(0) = 9 \times 10^{-6}$$

$$\therefore \quad v_c^2(0) = 576; \qquad v_c(0) = 24\,\text{V}$$

$$\frac{di(0)}{dt} = \frac{24}{0.5} = 48\,\text{A/s}$$

[c] $i(t) = A_1 e^{-4000t} + A_2 e^{-16{,}000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -4000A_1 - 16{,}000A_2 = 48$$

Solving,

$$\therefore \quad A_1 = 4\,\text{mA}; \qquad A_2 = -4\,\text{mA}$$

$$i(t) = 4e^{-4000t} - 4e^{-16{,}000t}\,\text{mA}, \qquad t \geq 0$$

[d] $\dfrac{di(t)}{dt} = -16e^{-4000t} + 64e^{-16{,}000t}$

$$\frac{di}{dt} = 0 \text{ when } 64e^{-16{,}000t} = 16e^{-4000t}$$

$$\text{or } e^{12{,}000t} = 4$$

$$\therefore \quad t = \frac{\ln 4}{12{,}000} = 115.52\,\mu\text{s}$$

[e] $i_{\max} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89\,\text{mA}$

[f] $v_L(t) = 0.5\dfrac{di}{dt} = [-8e^{-1000t} + 32e^{-4000t}]\,\text{V}, \quad t \geq 0^+$

P 8.42 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{(125)(0.32)} = 25 \times 10^6$

$$\alpha = \frac{R}{2L} = \omega_o = 5000\,\text{rad/s}$$

$$\therefore \quad R = (5000)(2)L = 1250\,\Omega$$

[b] $i(0) = i_{\text{L}}(0) = 6\,\text{mA}$

$$v_{\text{L}}(0) = 15 - (0.006)(1250) = 7.5\,\text{V}$$

$$\frac{di}{dt}(0) = \frac{7.5}{0.125} = 60\ \text{A/s}$$

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[c] $v_C = D_1 t e^{-5000t} + D_2 e^{-5000t}$

$$v_C(0) = D_2 = 15\,\text{V}$$

$$\frac{dv_C}{dt}(0) = D_1 - 5000D_2 = \frac{i_C(0)}{C} = \frac{-i_L(0)}{C} = -18{,}750$$

$$\therefore \quad D_1 = 56{,}250 \text{ V/s}$$

$$v_C = 56{,}250te^{-5000t} + 15e^{-5000t}\,\text{V}, \qquad t \geq 0$$

P 8.43 **[a]** $\dfrac{1}{LC} = 20{,}000^2$

There are many possible solutions. This one begins by choosing $L = 1\,\text{mH}$. Then,

$$C = \frac{1}{(1 \times 10^{-3})(20{,}000)^2} = 2.5\,\mu\text{F}$$

We can achieve this capacitor value using components from Appendix H by combining four $10\,\mu$F capacitors in series.

Critically damped: $\quad \alpha = \omega_0 = 20{,}000 \quad$ so $\quad \dfrac{R}{2L} = 20{,}000$

$$\therefore \quad R = 2(10^{-3})(20{,}000) = 40\,\Omega$$

We can create this resistor value using components from Appendix H by combining a $10\,\Omega$ resistor and two $15\,\Omega$ resistors in series. The final circuit:

[b] $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -20{,}000 \pm 0$

Therefore there are two repeated real roots at $-20{,}000$ rad/s.

P 8.44 **[a]** Underdamped response:

$$\alpha < \omega_0 \qquad \text{so} \qquad \alpha < 20{,}000$$

Therefore we choose a larger resistor value than the one used in Problem 8.40 to give a smaller value of α. For convenience, pick $\alpha = 16{,}000$ rad/s:

$$\alpha = \frac{R}{2L} = 16{,}000 \qquad \text{so} \qquad R = 2(16{,}000)(10^{-3}) = 32\,\Omega$$

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We can create a $32\,\Omega$ resistance by combining a $10\,\Omega$ resistor and a $22\,\Omega$ resistor in series.

$$s_{1,2} = -16{,}000 \pm \sqrt{16{,}000^2 - 20{,}000^2} = -16{,}000 \pm j12{,}000 \text{ rad/s}$$

[b] Overdamped response:

$$\alpha > \omega_0 \qquad \text{so} \qquad \alpha > 20{,}000$$

Therefore we choose a smaller resistor value than the one used in Problem 8.40. Choose $R = 50\,\Omega$, which can be created by combining two $100\,\Omega$ resistors in parallel:

$$\alpha = \frac{R}{2L} = 25{,}000$$

$$s_{1,2} = -25{,}000 \pm \sqrt{25{,}000^2 - 20{,}000^2} = -25{,}000 \pm 15{,}000$$

$$= -10{,}000 \text{ rad/s}; \qquad \text{and} \qquad -40{,}000 \text{ rad/s}$$

P 8.45 **[a]** $t < 0$:

$$i_o = \frac{100}{50} = 2\,\text{A}; \qquad v_o = -4(100) = -400\,\text{V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{500}{2(0.4)} = 625 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.4)(10 \times 10^{-6})}} = 500 \text{ rad/s}$$

$$\alpha^2 < \omega_o^2 \quad \therefore \quad \text{overdamped}$$

$$s_{1,2} = -625 \pm \sqrt{625^2 - 500^2} = -250, -1000 \text{ rad/s}$$

$$i_o = A_1 e^{-250t} + A_2 e^{-1000t}$$

$$i_o(0) = A_1 + A_2 = 2$$

$$\frac{di_o}{dt}(0) = -250A_1 - 1000A_2 = \frac{1}{L}(-V_0 - RI_0) = -1500$$

Solving, $\quad A_1 = \dfrac{2}{3}; \qquad A_2 = \dfrac{4}{3}$

$$\therefore \qquad i_o(t) = \frac{2}{3}e^{-250t} + \frac{4}{3}e^{-1000t}\,\text{A}, \quad t \geq 0$$

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[b] $v_o(t) = \dfrac{1}{10 \times 10^{-6}} \displaystyle\int_0^t i_o(x)\,dx - 400$

$$= 10^5 \left(\int_0^t \frac{2}{3} e^{-250x}\,dx + \int_0^t \frac{4}{3} e^{-1000x}\,dx \right) - 400$$

$$= 10^5 \left(\frac{(2/3)e^{-250x}}{-250} \bigg|_0^t + \frac{(4/3)e^{-1000x}}{-1000} \bigg|_0^t \right) - 400$$

$$= -266.67e^{-250t} - 133.33e^{-1000t}\,\text{V}, \quad t \geq 0$$

P 8.46 $t < 0: \qquad I_0 = -75\,\text{mA}; \qquad V_0 = 0$

$t > 0:$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(80 \times 10^{-3})(200 \times 10^{-6})}} = 250$$

$$\alpha = \frac{R}{2L} = \frac{40}{2(80 \times 10^{-3})} = 250$$

Critically damped:

$$i = D_1 t e^{-250t} + D_2 e^{-250t}$$

$$i(0) = D_2 = I_0 = -0.075$$

$$\frac{di}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{L}(-V_0 - RI_0)$$

So $\qquad D_1 - (250)(-0.075) = \dfrac{1}{80 \times 10^{-3}}(0 - (40)(-0.075))$

Solving, $\qquad D_1 = 18.75$

$\therefore \qquad i(t) = 18{,}750te^{-250t} - 75e^{-250t}\,\text{mA}, \quad t \geq 0$

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P 8.47 $t < 0$:

$500i_1 - 100i_o = 0; \qquad -100i_1 + 120i_o = 100$

Solving, $\quad i_1 = 0.2\,\text{A}; \qquad i_o = 1\,\text{A}; \qquad V_o = -100 + 20i_o + 100i_1 = -60\,\text{V}$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{200}{2(0.2)} = 500$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.2)(36.25 \times 10^{-6})}} = 400$$

$\alpha^2 > \omega_o^2 \quad$ overdamped

$$s_{1,2} = -100 \pm \sqrt{500^2 - 400^2} = -200, -800 \ \text{rad/s}$$

$$i_o = A_1 e^{-200t} + A_2 e^{-800t}$$

$$i_o(0) = A_1 + A_2 = 1$$

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$$\frac{di_o}{dt}(0) = -200A_1 - 800A_2 = \frac{1}{L}(-V_0 - RI_0) = -700$$

Solving, $\quad A_1 = 166.67\,\text{mA}; \qquad A_2 = 833.33\,\text{mA}$

$$\therefore \quad i_o(t) = 166.67e^{-200t} + 833.33e^{-800t}\,\text{mA}, \quad t \geq 0$$

$$v_o(t) = L\frac{di_o}{dt} = (0.2)[(-200)0.16667e^{-200t} + (-800)0.83333e^{-800t}]$$

$$= -6.67e^{-200t} - 133.33e^{-800t}\,\text{V}, \quad t \geq 0$$

P 8.48 **[a]** For $t > 0$:

Since $i(0^-) = i(0^+) = 0$

$v_a(0^+) = -300\,\text{V}$

[b] $v_a = 20i + \dfrac{1}{5 \times 10^{-6}}\displaystyle\int_0^t i\,dx - 300$

$$\frac{dv_a}{dt} = 20\frac{di}{dt} + 200{,}000i$$

$$\frac{dv_a(0^+)}{dt} = 20\frac{di(0^+)}{dt} + 200{,}000i(0^+) = 20\frac{di(0^+)}{dt}$$

$$\frac{di(0^+)}{dt} = \frac{1}{L}(-V_0 - RI_0) = \frac{1}{2 \times 10^{-3}}(300) = 150{,}000$$

$$\therefore \quad \frac{dv_a(0^+)}{dt} = 3 \times 10^6\,\text{V/s}$$

[c] $\alpha = \dfrac{R}{2L} = \dfrac{32}{2(2 \times 10^{-3}} = 8000\,\text{rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(2 \times 10^{-3})(5 \times 10^{-6})}} = 10{,}000$$

Underdamped:

$$\omega_d = \sqrt{10{,}000^2 - 8000^2} = 6000$$

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$$v_a = B_1 e^{-8000t}\cos 6000t + B_2 e^{-8000t}\sin 6000t$$

$$v_a(0) = B_1 = -300\,\text{V}$$

$$\frac{dv_a(0)}{dt} = -8000B_1 + 6000B_2 = 3\times 10^6; \qquad \therefore \quad B_2 = 100\,\text{V}$$

$$v_a(t) = -300e^{-8000t}\cos 6000t + 100e^{-8000t}\sin 6000t\,\text{V}, \quad t \geq 0^+$$

P 8.49 $\alpha = \dfrac{R}{2L} = \dfrac{200}{2(0.025)} = 400\,\text{rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250\times 10^{-3})(16\times 10^{-6})}} = 500\,\text{rad/s}$$

$\alpha^2 < \omega_0^2$: underdamped

$$\omega_d = \sqrt{500^2 - 400^2} = 300\,\text{rad/s}$$

$$v_o = V_f + B_1' e^{-400t}\cos 300t + B_2' e^{-400t}\sin 300t$$

$$v_o(\infty) = 200(0.08) = 16\,\text{V}$$

$$v_o(0) = 0 = V_f + B_1' = 0 \qquad \text{so} \qquad B_1' = -16\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 0 = -400B_1' + 300B_2' \qquad \text{so} \qquad B_2' = -21.33\,\text{V}$$

$$\therefore \quad v_o(t) = 16 - 16e^{-400t}\cos 300t - 21.33e^{-400t}\sin 300t\,\text{V}, \quad t \geq 0$$

P 8.50 $\alpha = \dfrac{R}{2L} = \dfrac{250}{2(0.025)} = 500\,\text{rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250\times 10^{-3})(16\times 10^{-6})}} = 500\,\text{rad/s}$$

$\alpha^2 = \omega_0^2$: critically damped

$$v_o = V_f + D_1' te^{-500t} + D_2' e^{-500t}$$

$$v_o(0) = 0 = V_f + D_2'$$

$$v_o(\infty) = (250)(0.08) = 20\,\text{V}; \qquad \therefore \quad D_2' = -20\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 0 = D_1' - \alpha D_2' \qquad \text{so} \qquad D_1' = (500)(-20) = -10{,}000\ \text{V/s}$$

$$\therefore \qquad v_o(t) = 20 - 10{,}000te^{-500t} - 20e^{-500t}\,\text{V}, \quad t \geq 0$$

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P 8.51 $\alpha = \dfrac{R}{2L} = \dfrac{312.5}{2(0.025)} = 625\,\text{rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250\times 10^{-3})(16\times 10^{-6})}} = 500\,\text{rad/s}$$

$\alpha^2 > \omega_0^2$: overdamped

$$s_{1,2} = -625 \pm \sqrt{625^2 - 500^2} = -250, -1000\,\text{rad/s}$$

$$v_o = V_f + A_1' e^{-250t} + A_2' e^{-1000t}$$

$$v_o(0) = 0 = V_f + A_1' + A_2'$$

$$v_o(\infty) = (312.5)(08) = 25\,\text{V}; \qquad \therefore \quad A_1' + A_2' = -25\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 0 = -250A_1' - 1000A_2'$$

Solving, $A_1' = -33.33\,\text{V}$; $A_2' = 8.33\,\text{V}$

$$v_o(t) = 25 - 33.33e^{-250t} + 8.33e^{-1000t}\,\text{V}, \quad t \geq 0$$

P 8.52 $t < 0$:

$$i_o(0^-) = \frac{20+28}{160+480} = 75\,\text{mA}$$

$$v_o(0^-) = 20 - 480(0.075) = -16\,\text{V}$$

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As $t \to \infty$, $V_f = 20$ V.

$R_{\text{eq}} = 960 \| 480 = 320\,\Omega$

$$\alpha = \frac{R_{\text{eq}}}{2L} = \frac{320}{2(0.5 \times 10^{-3})} = 320{,}000 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.5 \times 10^{-3})(12.5 \times 10^{-9})}} = 400{,}000\,\text{rad/s}$$

$\alpha^2 < \omega_0^2$: underdamped

$$\omega_d = \sqrt{400{,}000^2 - 320{,}000^2} = 240{,}000 \text{ rad/s}$$

$$v_o = 20 + B_1' e^{-320{,}000t} \cos 240{,}000t + B_2' e^{-320{,}000t} \sin 240{,}000t$$

$v_o(0) = 20 + B_1' = -16$ so $B_1' = -36\,\text{V}$

$\dfrac{dv_o}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \dfrac{I_0}{C}$ so $-320{,}000(-36) + 240{,}000 B_2' = \dfrac{75 \times 10^{-3}}{12.5 \times 10^{-9}}$

solving, $B_2' = -23$

$\therefore$ $v_o(t) = 20 - 36e^{-320{,}000t} \cos 240{,}000t - 23e^{-320{,}000t} \sin 240{,}000t\,\text{V} \quad t \geq 0$

P 8.53 $i_{\text{C}}(0) = 0$; $v_o(0) = 200\,\text{V}$

$$\alpha = \frac{R}{2L} = \frac{25}{2(0.25)} = 50\,\text{rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250 \times 10^{-3})(2.5 \times 10^{-3})}} = 40 \text{ rad/s}$$

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$\therefore \quad \alpha^2 > \omega_o^2; \qquad$ overdamped

$s_{1,2} = -50 \pm \sqrt{50^2 - 40^2} = -20, -80 \text{ rad/s}$

$v_o = V_f + A_1' e^{-20t} + A_2' e^{-80t}$

$V_f = 150\,\text{V}; \qquad v_o(0) = 150 + A_1' + A_2' = 200 \qquad \text{so} \qquad A_1' + A_2' = 50$

$\dfrac{v_o}{dt}(0) = -20A_1' - 80A_2' = \dfrac{I_0}{C} = 0$

Solving, $\qquad A_1' = 66.67; \qquad A_2' = -16.67$

$\therefore \qquad v_o(t) = 150 + 66.67e^{-20t} - 16.67e^{-80t}\,\text{V}, \quad t \geq 0$

P 8.54 $\quad t < 0$:

$i_{\text{L}}(0) = \dfrac{-150}{30} = -5\,\text{A}$

$v_{\text{C}}(0) = 18 i_{\text{L}}(0) = -90\,\text{V}$

$t > 0$:

$\alpha = \dfrac{R}{2L} = \dfrac{10}{2(0.1)} = 50 \text{ rad/s}$

$\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(0.1)(2 \times 10^{-3})} = 5000$

$\omega_o > \alpha^2 \quad \therefore \qquad$ underdamped

$s_{1,2} = -50 \pm \sqrt{50^2 - 5000} = -50 \pm j50$

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$$v_c = 60 + B_1'e^{-50t}\cos 50t + B_2'e^{-50t}\sin 50t$$

$$v_c(0) = -90 = 60 + B_1' \quad \therefore \quad B_1' = -150$$

$$C\frac{dv_c}{dt}(0) = -5; \qquad \frac{dv_c}{dt}(0) = \frac{-5}{2\times 10^{-3}} = -2500$$

$$\frac{dv_c}{dt}(0) = -50B_1' + 50B_2 = -2500 \quad \therefore \quad B_2' = -200$$

$$v_c = 60 - 150e^{-50t}\cos 50t - 200e^{-50t}\sin 50t\,\text{V}, \quad t \geq 0$$

P 8.55 **[a]** Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $\quad i = B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$

$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = \left[(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t\right]e^{-\alpha t}$$

Therefore $\quad \omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \dfrac{\alpha}{\omega_d}B_1' = \dfrac{\alpha}{\omega_d}\dfrac{V_g}{R}$

Therefore

$$v_o = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\frac{V_g L}{R}\left(\frac{\alpha^2 + \omega_d^2}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R}\left(\frac{\omega_o^2}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R\omega_d}\left(\frac{1}{LC}\right)e^{-\alpha t}\sin\omega_d t$$

$$v_o = -\frac{V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t\,\text{V}, \quad t \geq 0$$

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[b] $\dfrac{dv_o}{dt} = -\dfrac{V_g}{\omega_d RC}\{\omega_d \cos\omega_d t - \alpha \sin\omega_d t\}e^{-\alpha t}$

$$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}$$

Therefore $\quad \omega_d t = \tan^{-1}(\omega_d/\alpha) \quad$ (smallest t)

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

P 8.56 **[a]** From Problem 8.55 we have

$$v_o = \frac{-V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t$$

$$\alpha = \frac{R}{2L} = \frac{480}{2(8\times 10^{-3})} = 30{,}000\,\text{rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(8\times 10^{-3})(50\times 10^{-9})}} = 50{,}000\,\text{rad/s}$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 40{,}000\,\text{rad/s}$$

$$\frac{-V_g}{RC\omega_d} = \frac{-(-24)}{(480)(50\times 10^{-9})(40\times 10^3)} = 25$$

$$\therefore \quad v_o = 25e^{-30{,}000t}\sin 40{,}000t\,\text{V}$$

[b] From Problem 8.55

$$t_d = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = \frac{1}{40{,}000}\tan^{-1}\left(\frac{40{,}000}{30{,}000}\right)$$

$$t_d = 23.18\,\mu\text{s}$$

[c] $v_{\max} = 25e^{-0.03(23.18)}\sin[(0.04)(23.18)] = 9.98\,\text{V}$

[d] $R = 96\,\Omega; \qquad \alpha = 6000\,\text{rad/s}$

$$\omega_d = 49{,}638.7\,\text{rad/s}$$

$$v_o = 100.73e^{-6000t}\sin 49{,}638.7t\,\text{V}, \quad t \geq 0$$

$$t_d = 29.22\,\mu\text{s}$$

$$v_{\max} = 83.92\,\text{V}$$

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P 8.57 **[a]** $v_c = V_f + [B_1' \cos\omega_d t + B_2' \sin\omega_d t]\, e^{-\alpha t}$

$$\frac{dv_c}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]e^{-\alpha t}$$

Since the initial stored energy is zero,

$$v_c(0^+) = 0 \quad \text{and} \quad \frac{dv_c(0^+)}{dt} = 0$$

It follows that $\quad B_1' = -V_f \quad$ and $\quad B_2' = \dfrac{\alpha B_1'}{\omega_d}$

When these values are substituted into the expression for $[dv_c/dt]$, we get

$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{\omega_d} + \omega_d\right) V_f e^{-\alpha t}\sin\omega_d t$$

But $\quad V_f = V \quad$ and $\quad \dfrac{\alpha^2}{\omega_d} + \omega_d = \dfrac{\alpha^2 + \omega_d^2}{\omega_d} = \dfrac{\omega_o^2}{\omega_d}$

Therefore $\quad \dfrac{dv_c}{dt} = \left(\dfrac{\omega_o^2}{\omega_d}\right) V e^{-\alpha t}\sin\omega_d t$

[b] $\dfrac{dv_c}{dt} = 0 \quad$ when $\quad \sin\omega_d t = 0, \quad$ or $\quad \omega_d t = n\pi$

where $\quad n = 0, 1, 2, 3, \ldots$

Therefore $\quad t = \dfrac{n\pi}{\omega_d}$

[c] When $\quad t_n = \dfrac{n\pi}{\omega_d}, \quad \cos\omega_d t_n = \cos n\pi = (-1)^n$

and $\quad \sin\omega_d t_n = \sin n\pi = 0$

Therefore $\quad v_c(t_n) = V[1 - (-1)^n e^{-\alpha n\pi/\omega_d}]$

[d] It follows from [c] that

$$v(t_1) = V + Ve^{-(\alpha\pi/\omega_d)} \quad \text{and} \quad v_c(t_3) = V + Ve^{-(3\alpha\pi/\omega_d)}$$

Therefore $\quad \dfrac{v_c(t_1) - V}{v_c(t_3) - V} = \dfrac{e^{-(\alpha\pi/\omega_d)}}{e^{-(3\alpha\pi/\omega_d)}} = e^{(2\alpha\pi/\omega_d)}$

But $\quad \dfrac{2\pi}{\omega_d} = t_3 - t_1 = T_d, \quad$ thus $\quad \alpha = \dfrac{1}{T_d}\ln\dfrac{[v_c(t_1) - V]}{[v_c(t_3) - V]}$

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P 8.58 $\dfrac{1}{T_d}\ln\left\{\dfrac{v_c(t_1)-V}{v_c(t_3)-V}\right\}; \qquad T_d = t_3 - t_1 = \dfrac{3\pi}{7} - \dfrac{\pi}{7} = \dfrac{2\pi}{7}\,\text{ms}$

$$\alpha = \frac{7000}{2\pi}\ln\left[\frac{63.84}{26.02}\right] = 1000; \qquad \omega_d = \frac{2\pi}{T_d} = 7000\,\text{rad/s}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = 49\times 10^6 + 10^6 = 50\times 10^6$$

$$L = \frac{1}{(50\times 10^6)(0.1\times 10^{-6})} = 200\,\text{mH}; \qquad R = 2\alpha L = 400\,\Omega$$

P 8.59 At $t = 0$ the voltage across each capacitor is zero. It follows that since the operational amplifiers are ideal, the current in the $500\,\text{k}\Omega$ is zero. Therefore there cannot be an instantaneous change in the current in the $1\,\mu$F capacitor. Since the capacitor current equals $C(dv_o/dt)$, the derivative must be zero.

P 8.60 **[a]** From Example 8.13 $\dfrac{d^2v_o}{dt^2} = 2$

therefore $\dfrac{dg(t)}{dt} = 2, \quad g(t) = \dfrac{dv_o}{dt}$

$$g(t) - g(0) = 2t; \qquad g(t) = 2t + g(0); \qquad g(0) = \frac{dv_o(0)}{dt}$$

$$i_{\text{R}} = \frac{5}{500}\times 10^{-3} = 10\,\mu\text{A} = i_{\text{C}} = -C\frac{dv_o(0)}{dt}$$

$$\frac{dv_o(0)}{dt} = \frac{-10\times 10^{-6}}{1\times 10^{-6}} = -10 = g(0)$$

$$\frac{dv_o}{dt} = 2t - 10$$

$$dv_o = 2t\,dt - 10\,dt$$

$$v_o - v_o(0) = t^2 - 10t; \qquad v_o(0) = 8\,\text{V}$$

$$v_o = t^2 - 10t + 8, \qquad 0 \le t \le t_{\text{sat}}$$

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[b] $t^2 - 10t + 8 = -9$

$t^2 - 10t + 17 = 0$

$t \cong 2.17\,\text{s}$

P 8.61 Part (1) — Example 8.14, with R_1 and R_2 removed:

[a] $R_\text{a} = 100\,\text{k}\Omega; \qquad C_1 = 0.1\,\mu\text{F}; \qquad R_\text{b} = 25\,\text{k}\Omega; \qquad C_2 = 1\,\mu\text{F}$

$$\frac{d^2v_o}{dt^2} = \left(\frac{1}{R_\text{a}C_1}\right)\left(\frac{1}{R_\text{b}C_2}\right)v_g; \qquad \frac{1}{R_\text{a}C_1} = 100 \quad \frac{1}{R_\text{b}C_2} = 40$$

$$v_g = 250 \times 10^{-3}; \qquad \text{therefore} \quad \frac{d^2v_o}{dt^2} = 1000$$

[b] Since $v_o(0) = 0 = \dfrac{dv_o(0)}{dt}$, our solution is $v_o = 500t^2$

The second op-amp will saturate when

$$v_o = 6\,\text{V}, \quad \text{or} \quad t_\text{sat} = \sqrt{6/500} \cong 0.1095\,\text{s}$$

[c] $\dfrac{dv_{o1}}{dt} = -\dfrac{1}{R_\text{a}C_1}v_g = -25$

[d] Since $v_{o1}(0) = 0, \quad v_{o1} = -25t\,\text{V}$

At $t = 0.1095\,\text{s}, \quad v_{o1} \cong -2.74\,\text{V}$

Therefore the second amplifier saturates before the first amplifier saturates. Our expressions are valid for $0 \le t \le 0.1095\,\text{s}$. Once the second op-amp saturates, our linear model is no longer valid.

Part (2) — Example 8.14 with $v_{o1}(0) = -2\,\text{V}$ and $v_o(0) = 4\,\text{V}$:

[a] Initial conditions will not change the differential equation; hence the equation is the same as Example 8.14.

[b] $v_o = 5 + A_1'e^{-10t} + A_2'e^{-20t}$ (from Example 8.14)

$v_o(0) = 4 = 5 + A_1' + A_2'$

$$\frac{4}{100} + i_\text{C}(0^+) - \frac{2}{25} = 0$$

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$$i_C(0^+) = \frac{4}{100}\,\text{mA} = C\frac{dv_o(0^+)}{dt}$$

$$\frac{dv_o(0^+)}{dt} = \frac{0.04 \times 10^{-3}}{10^{-6}} = 40\,\text{V/s}$$

$$\frac{dv_o}{dt} = -10A_1'e^{-10t} - 20A_2'e^{-20t}$$

$$\frac{dv_o}{dt}(0^+) = -10A_1' - 20A_2' = 40$$

Therefore $-A_1' - 2A_2' = 4$ and $A_1' + A_2' = -1$
Thus, $A_1' = 2$ and $A_2' = -3$

$$v_o = 5 + 2e^{-10t} - 3e^{-20t}\,\text{V}$$

[c] Same as Example 8.14:

$$\frac{dv_{o1}}{dt} + 20v_{o1} = -25$$

[d] From Example 8.14:

$$v_{o1}(\infty) = -1.25\,\text{V}; \qquad v_1(0) = -2\,\text{V} \quad \text{(given)}$$

Therefore

$$v_{o1} = -1.25 + (-2 + 1.25)e^{-20t} = -1.25 - 0.75e^{-20t}\,\text{V}$$

P 8.62 **[a]**

$$2C\frac{dv_a}{dt} + \frac{v_a - v_g}{R} + \frac{v_a}{R} = 0$$

(1) Therefore $\dfrac{dv_a}{dt} + \dfrac{v_a}{RC} = \dfrac{v_g}{2RC}$

$$\frac{0 - v_a}{R} + C\frac{d(0 - v_b)}{dt} = 0$$

(2) Therefore $\dfrac{dv_b}{dt} + \dfrac{v_a}{RC} = 0, \quad v_a = -RC\dfrac{dv_b}{dt}$

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$$\frac{2v_\mathrm{b}}{R} + C\frac{dv_\mathrm{b}}{dt} + C\frac{d(v_\mathrm{b} - v_o)}{dt} = 0$$

(3) Therefore $\dfrac{dv_\mathrm{b}}{dt} + \dfrac{v_\mathrm{b}}{RC} = \dfrac{1}{2}\dfrac{dv_o}{dt}$

From (2) we have $\dfrac{dv_\mathrm{a}}{dt} = -RC\dfrac{d^2v_\mathrm{b}}{dt^2}$ and $v_\mathrm{a} = -RC\dfrac{dv_\mathrm{b}}{dt}$

When these are substituted into (1) we get

(4) $-RC\dfrac{d^2v_\mathrm{b}}{dt^2} - \dfrac{dv_\mathrm{b}}{dt} = \dfrac{v_g}{2RC}$

Now differentiate (3) to get

(5) $\dfrac{d^2v_\mathrm{b}}{dt^2} + \dfrac{1}{RC}\dfrac{dv_\mathrm{b}}{dt} = \dfrac{1}{2}\dfrac{d^2v_o}{dt^2}$

But from (4) we have

(6) $\dfrac{d^2v_\mathrm{b}}{dt^2} + \dfrac{1}{RC}\dfrac{dv_\mathrm{b}}{dt} = -\dfrac{v_g}{2R^2C^2}$

Now substitute (6) into (5)

$$\frac{d^2v_o}{dt^2} = -\frac{v_g}{R^2C^2}$$

[b] When $R_1C_1 = R_2C_2 = RC$: $\dfrac{d^2v_o}{dt^2} = \dfrac{v_g}{R^2C^2}$

The two equations are the same except for a reversal in algebraic sign.

[c] Two integrations of the input signal with one operational amplifier.

P 8.63 **[a]** $\dfrac{d^2v_o}{dt^2} = \dfrac{1}{R_1C_1R_2C_2}v_g$

$$\frac{1}{R_1C_1R_2C_2} = \frac{10^{-6}}{(100)(400)(0.5)(0.2)\times 10^{-6}\times 10^{-6}} = 250$$

$$\therefore\ \frac{d^2v_o}{dt^2} = 250v_g$$

$0 \le t \le 0.5^-$:

$v_g = 80\,\mathrm{mV}$

$$\frac{d^2v_o}{dt^2} = 20$$

Let $g(t) = \dfrac{dv_o}{dt}$, then $\dfrac{dg}{dt} = 20$ or $dg = 20\,dt$

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$$\int_{g(0)}^{g(t)} dx = 20\int_0^t dy$$

$$g(t) - g(0) = 20t, \quad g(0) = \frac{dv_o}{dt}(0) = 0$$

$$g(t) = \frac{dv_o}{dt} = 20t$$

$$dv_o = 20t\,dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = 20\int_0^t x\,dx; \qquad v_o(t) - v_o(0) = 10t^2, \quad v_o(0) = 0$$

$$v_o(t) = 10t^2\,\text{V}, \quad 0 \le t \le 0.5^-$$

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1C_1}v_g = -20v_g = -1.6$$

$$dv_{o1} = -1.6\,dt$$

$$\int_{v_{o1}(0)}^{v_{o1}(t)} dx = -1.6\int_0^t dy$$

$$v_{o1}(t) - v_{o1}(0) = -1.6t, \qquad v_{o1}(0) = 0$$

$$v_{o1}(t) = -1.6t\,\text{V}, \qquad 0 \le t \le 0.5^-$$

$0.5^+ \le t \le t_{\text{sat}}$:

$$\frac{d^2v_o}{dt^2} = -10, \qquad \text{let} \quad g(t) = \frac{dv_o}{dt}$$

$$\frac{dg(t)}{dt} = -10; \qquad dg(t) = -10\,dt$$

$$\int_{g(0.5^+)}^{g(t)} dx = -10\int_{0.5}^t dy$$

$$g(t) - g(0.5^+) = -10(t - 0.5) = -10t + 5$$

$$g(0.5^+) = \frac{dv_o(0.5^+)}{dt}$$

$$C\frac{dv_o}{dt}(0.5^+) = \frac{0 - v_{o1}(0.5^+)}{400 \times 10^3}$$

$$v_{o1}(0.5^+) = v_o(0.5^-) = -1.6(0.5) = -0.80\,\text{V}$$

$$\therefore \quad C\frac{dv_{o1}(0.5^+)}{dt} = \frac{0.80}{0.4 \times 10^3} = 2\,\mu\text{A}$$

$$\frac{dv_{o1}}{dt}(0.5^+) = \frac{2 \times 10^{-6}}{0.2 \times 10^{-6}} = 10\,\text{V/s}$$

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$$\therefore \quad g(t) = -10t + 5 + 10 = -10t + 15 = \frac{dv_o}{dt}$$

$$\therefore \quad dv_o = -10t\,dt + 15\,dt$$

$$\int_{v_o(0.5^+)}^{v_o(t)} dx = \int_{0.5^+}^{t} -10y\,dy + \int_{0.5^+}^{t} 15\,dy$$

$$v_o(t) - v_o(0.5^+) = -5y^2 \Big|_{0.5}^{t} + 15y \Big|_{0.5}^{t}$$

$$v_o(t) = v_o(0.5^+) - 5t^2 + 1.25 + 15t - 7.5$$

$$v_o(0.5^+) = v_o(0.5^-) = 2.5\,\text{V}$$

$$\therefore \quad v_o(t) = -5t^2 + 15t - 3.75\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$\frac{dv_{o1}}{dt} = -20(-0.04) = 0.8, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$dv_{o1} = 0.8\,dt; \qquad \int_{v_{o1}(0.5^+)}^{v_{o1}(t)} dx = 0.8\int_{0.5^+}^{t} dy$$

$$v_{o1}(t) - v_{o1}(0.5^+) = 0.8t - 0.4; \qquad v_{o1}(0.5^+) = v_{o1}(0.5^-) = -0.8\,\text{V}$$

$$\therefore \quad v_{o1}(t) = 0.8t - 1.2\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

Summary:

$$0 \le t \le 0.5^-\text{s}: \qquad v_{o1} = -1.6t\,\text{V}, \quad v_o = 10t^2\,\text{V}$$

$$0.5^+\text{s} \le t \le t_{\text{sat}}: \qquad v_{o1} = 0.8t - 1.2\,\text{V}, \quad v_o = -5t^2 + 15t - 3.75\,\text{V}$$

[b] $-12.5 = -5t_{\text{sat}}^2 + 15t_{\text{sat}} - 3.75$

$$\therefore \quad 5t_{\text{sat}}^2 - 15t_{\text{sat}} - 8.75 = 0$$

Solving, $\quad t_{\text{sat}} = 3.5\,\text{sec}$

$$v_{o1}(t_{\text{sat}}) = 0.8(3.5) - 1.2 = 1.6\,\text{V}$$

P 8.64 $\quad \tau_1 = (10^6)(0.5 \times 10^{-6}) = 0.50\,\text{s}$

$$\frac{1}{\tau_1} = 2; \qquad \tau_2 = (5 \times 10^6)(0.2 \times 10^{-6}) = 1\,\text{s}; \qquad \therefore \quad \frac{1}{\tau_2} = 1$$

$$\therefore \quad \frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = 20$$

$$s^2 + 3s + 2 = 0$$

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$(s+1)(s+2) = 0; \qquad s_1 = -1, \quad s_2 = -2$

$v_o = V_f + A_1' e^{-t} + A_2' e^{-2t}; \qquad V_f = \dfrac{20}{2} = 10\,\text{V}$

$v_o = 10 + A_1' e^{-t} + A_2' e^{-2t}$

$v_o(0) = 0 = 10 + A_1' + A_2'; \qquad \dfrac{dv_o}{dt}(0) = 0 = -A_1' - 2A_2'$

$\therefore \quad A_1' = -20, \qquad A_2' = 10\,\text{V}$

$v_o(t) = 10 - 20e^{-t} + 10e^{-2t}\,\text{V}, \qquad 0 \le t \le 0.5\,\text{s}$

$\dfrac{dv_{o1}}{dt} + 2v_{o1} = -1.6; \qquad \therefore \quad v_{o1} = -0.8 + 0.8e^{-2t}\,\text{V}, \quad 0 \le t \le 0.5\,\text{s}$

$v_o(0.5) = 10 - 20e^{-0.5} + 10e^{-1} = 1.55\,\text{V}$

$v_{o1}(0.5) = -0.8 + 0.8e^{-1} = -0.51\,\text{V}$

At $\quad t = 0.5\,\text{s}$

$i_\text{C} = \dfrac{0 + 0.51}{400 \times 10^3} = 1.26\,\mu\text{A}$

$C\dfrac{dv_o}{dt} = 1.26\,\mu\text{A}; \qquad \dfrac{dv_o}{dt} = \dfrac{1.26}{0.2} = 6.32\,\text{V/s}$

$0.5\,\text{s} \le t \le \infty$:

$\dfrac{d^2v_o}{dt^2} + 3\dfrac{dv_o}{dt} + 2 = -10$

$v_o(\infty) = -5$

$\therefore \quad v_o = -5 + A_1' e^{-(t-0.5)} + A_2' e^{-2(t-0.5)}$

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$$1.55 = -5 + A_1' + A_2'$$

$$\frac{dv_o}{dt}(0.5) = 6.32 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' + A_2' = 6.55; \qquad -A_1' - 2A_2' = 6.32$$

Solving,

$$A_1' = 19.42; \qquad A_2' = -12.87$$

$$\therefore \quad v_o = -5 + 19.42e^{-(t-0.5)} - 12.87e^{-2(t-0.5)}\,\text{V}, \qquad 0.5 \le t \le \infty$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = 0.8$$

$$\therefore \quad v_{o1} = 0.4 + (-0.51 - 0.4)e^{-2(t-0.5)} = 0.4 - 0.91e^{-2(t-0.5)}\,\text{V}, \qquad 0.5 \le t \le \infty$$

P 8.65 **[a]** $f(t) = \text{inertial force} + \text{frictional force} + \text{spring force}$

$$= M[d^2x/dt^2] + D[dx/dt] + Kx$$

[b] $$\frac{d^2x}{dt^2} = \frac{f}{M} - \left(\frac{D}{M}\right)\left(\frac{dx}{dt}\right) - \left(\frac{K}{M}\right)x$$

Given $v_A = \dfrac{d^2x}{dt^2}$, then

$$v_B = -\frac{1}{R_1C_1}\int_0^t \left(\frac{d^2x}{dy^2}\right) dy = -\frac{1}{R_1C_1}\frac{dx}{dt}$$

$$v_C = -\frac{1}{R_2C_2}\int_0^t v_B\, dy = \frac{1}{R_1R_2C_1C_2}x$$

$$v_D = -\frac{R_3}{R_4}\cdot v_B = \frac{R_3}{R_4R_1C_1}\frac{dx}{dt}$$

$$v_E = \left[\frac{R_5+R_6}{R_6}\right]v_C = \left[\frac{R_5+R_6}{R_6}\right]\cdot\frac{1}{R_1R_2C_1C_2}\cdot x$$

$$v_F = \left[\frac{-R_8}{R_7}\right]f(t), \qquad v_A = -(v_D + v_E + v_F)$$

Therefore $$\frac{d^2x}{dt^2} = \left[\frac{R_8}{R_7}\right]f(t) - \left[\frac{R_3}{R_4R_1C_1}\right]\frac{dx}{dt} - \left[\frac{R_5+R_6}{R_6R_1R_2C_1C_2}\right]x$$

Therefore $M = \dfrac{R_7}{R_8}$, $\quad D = \dfrac{R_3R_7}{R_8R_4R_1C_1}$ and $K = \dfrac{R_7(R_5+R_6)}{R_8R_6R_1R_2C_1C_2}$

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Box Number	Function
1	inverting and scaling
2	summing and inverting
3	integrating and scaling
4	integrating and scaling
5	inverting and scaling
6	noninverting and scaling

P 8.66 **[a]** $\omega_0 = \sqrt{\dfrac{1}{LC}} = \sqrt{\dfrac{1}{(5\times 10^{-9})(2\times 10^{-12})}} = 10^{10}$ rad/sec

$$\therefore \qquad f_0 = \frac{\omega_0}{2\pi} = \frac{10^{10}}{2\pi} = 1.59\times 10^9 \text{ Hz} = 1.59 \text{ GHz}$$

[b] $I_0 = \dfrac{10}{25} = 0.4\,\text{A}$

$$w(0) = \frac{1}{2}LI_0^2 = \frac{1}{2}(5\times 10^{-9})(0.4)^2 = 4\times 10^{-10}\text{ J} = 0.4 \text{ nJ}$$

[c] Because the inductor and capacitor are assumed to be ideal, none of the initial energy will ever be dissipated, so for all $t \geq 0$ the 0.4 nJ will be continually exchanged between the inductor and capacitor.

P 8.67 **[a]** $\omega_0 = 2\pi f_0 = 2\pi(2\times 10^9) = 4\pi\times 10^9 \text{rad/s}$

$$\omega_0^2 = \frac{1}{LC}$$

$$\therefore \qquad C = \frac{1}{L\omega_0^2} = \frac{1}{(10^{-9})(4\pi\times 10^9)^2} = 6.33\times 10^{-12} = 6.33 \text{ pF}$$

[b] $v_o(t) = \dfrac{V}{\omega_0 RC}\sin\omega_0 t = \dfrac{4}{(4\pi\times 10^9)(10)(6.33\times 10^{-12})}\sin 4\pi\times 10^9 t$

$$= 5.03\sin 4\pi\times 10^9 t\,\text{V}, \quad t \geq 0$$

P 8.68 **[a]** $\alpha = \dfrac{R}{2L} = \dfrac{0.01}{2(5\times 10^{-9})} = 10^6$ rad/s

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(5\times 10^{-9})(2\times 10^{-12})}} = 10^{10} \text{ rad/s}$$

[b] $\omega_0^2 > \alpha^2$ so the response is underdamped

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[c] $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(10^{10})^2 - (10^6)^2} \approx 10^{10}$ rad/s

$$f_0 = \frac{\omega_0}{2\pi} = \frac{10^{10}}{2\pi} = 1.59 \times 10^9 \text{ Hz} = 1.59 \text{ GHz}$$

Therefore the addition of the 10 mΩ resistance does not change the frequency of oscillation.

[d] Because of the added resistance, the oscillation now occurs within a decaying exponential envelope (see Fig. 8.9). The form of the exponential envelope is $e^{-\alpha t} = e^{-10^6 t}$. Let's assume that the oscillation is described by the function $Ke^{-10^6 t}\cos 10^{10} t$, where the maximum magnitude, K, exists at $t = 0$. How long does it take before the magnitude of the oscillation has decayed to $0.01K$?

$$Ke^{-10^6 t} = 0.01K \qquad \text{so} \qquad e^{-10^6 t} = 0.01$$

$$\therefore \qquad t = \frac{\ln 0.01}{-10^6} = 4.6 \times 10^{-6}\,\text{s} = 4.6\,\mu\text{s}$$

Therefore, the oscillations will persist for only 4.6 μs, due to the presence of a small amount of resistance in the circuit. This is why an LC oscillator is not used in the clock generator circuit.

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9

Sinusoidal Steady State Analysis

Assessment Problems

AP 9.1 **[a]** $\mathbf{V} = 170\underline{/-40^\circ}\ \text{V}$

[b] $10\sin(1000t + 20^\circ) = 10\cos(1000t - 70^\circ)$

$$\therefore \qquad \mathbf{I} = 10\underline{/-70^\circ}\ \text{A}$$

[c] $\mathbf{I} = 5\underline{/36.87^\circ} + 10\underline{/-53.13^\circ}$

$$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\underline{/-26.57^\circ}\ \text{A}$$

[d] $\sin(20{,}000\pi t + 30^\circ) = \cos(20{,}000\pi t - 60^\circ)$

Thus,

$$\mathbf{V} = 300\underline{/45^\circ} - 100\underline{/-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$$

$$= 162.13 + j298.73 = 339.90\underline{/61.51^\circ}\ \text{mV}$$

AP 9.2 **[a]** $v = 18.6\cos(\omega t - 54^\circ)\ \text{V}$

[b] $\mathbf{I} = 20\underline{/45^\circ} - 50\underline{/-30^\circ} = 14.14 + j14.14 - 43.3 + j25$

$$= -29.16 + j39.14 = 48.81\underline{/126.68^\circ}$$

Therefore $\quad i = 48.81\cos(\omega t + 126.68^\circ)\ \text{mA}$

[c] $\mathbf{V} = 20 + j80 - 30\underline{/15^\circ} = 20 + j80 - 28.98 - j7.76$

$$= -8.98 + j72.24 = 72.79\underline{/97.08^\circ}$$

$$v = 72.79\cos(\omega t + 97.08^\circ)\ \text{V}$$

AP 9.3 **[a]** $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

[b] $Z_L = j\omega L = j200\,\Omega$

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[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\underline{/30^\circ})(200\underline{/90^\circ}) \times 10^{-3} = 2\underline{/120^\circ}\,\text{V}$

[d] $v_L = 2\cos(10{,}000t + 120^\circ)\,\text{V}$

AP 9.4 **[a]** $X_C = \dfrac{-1}{\omega C} = \dfrac{-1}{4000(5 \times 10^{-6})} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{30\underline{/25^\circ}}{50\underline{/-90^\circ}} = 0.6\underline{/115^\circ}\,\text{A}$

[d] $i = 0.6\cos(4000t + 115^\circ)\,\text{A}$

AP 9.5 $\mathbf{I}_1 = 100\underline{/25^\circ} = 90.63 + j42.26$

$\mathbf{I}_2 = 100\underline{/145^\circ} = -81.92 + j57.36$

$\mathbf{I}_3 = 100\underline{/-95^\circ} = -8.72 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A}, \qquad \text{therefore} \quad i_4 = 0\,\text{A}$

AP 9.6 **[a]** $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{|Z|\underline{/\theta_z}} = \dfrac{125}{|Z|}\underline{/(-60 - \theta_Z)^\circ}$

But $-60 - \theta_Z = -105^\circ \qquad \therefore \quad \theta_Z = 45^\circ$

$Z = 90 + j160 + jX_C$

$\therefore \quad X_C = -70\,\Omega; \qquad X_C = -\dfrac{1}{\omega C} = -70$

$\therefore \quad C = \dfrac{1}{(70)(5000)} = 2.86\,\mu\text{F}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}_s}{Z} = \dfrac{125\underline{/-60^\circ}}{(90 + j90)} = 0.982\underline{/-105^\circ}A; \qquad \therefore \quad |\mathbf{I}| = 0.982\,\text{A}$

AP 9.7 **[a]**

$\omega = 2000\,\text{rad/s}$

$\omega L = 10\,\Omega, \qquad \dfrac{-1}{\omega C} = -20\,\Omega$

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$$Z_{xy} = 20\|j10 + 5 + j20 = \frac{20(j10)}{(20 + j10)} + 5 - j20$$

$$= 4 + j8 + 5 - j20 = (9 - j12)\,\Omega$$

[b] $\omega L = 40\,\Omega, \qquad \dfrac{-1}{\omega C} = -5\,\Omega$

$$Z_{xy} = 5 - j5 + 20\|j40 = 5 - j5 + \left[\frac{(20)(j40)}{20 + j40}\right]$$

$$= 5 - j5 + 16 + j8 = (21 + j3)\,\Omega$$

[c] $Z_{xy} = \left[\dfrac{20(j\omega L)}{20 + j\omega L}\right] + \left(5 - \dfrac{j10^6}{25\omega}\right)$

$$= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for ω yields $\omega = 4000\,\text{rad/s}$.

[d] $Z_{xy} = \dfrac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15\,\Omega$

AP 9.8 The frequency 4000 rad/s was found to give $Z_{\text{xy}} = 15\,\Omega$ in Assessment Problem 9.7. Thus,

$$\mathbf{V} = 150\underline{/0^\circ}, \qquad \mathbf{I}_s = \frac{\mathbf{V}}{Z_{\text{xy}}} = \frac{150\underline{/0^\circ}}{15} = 10\underline{/0^\circ}\,\text{A}$$

Using current division,

$$\mathbf{I}_L = \frac{20}{20 + j20}(10) = 5 - j5 = 7.07\underline{/-45^\circ}\,\text{A}$$

$$i_L = 7.07\cos(4000t - 45^\circ)\,\text{A}, \qquad I_m = 7.07\,\text{A}$$

AP 9.9 After replacing the delta made up of the $50\,\Omega$, $40\,\Omega$, and $10\,\Omega$ resistors with its equivalent wye, the circuit becomes

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The circuit is further simplified by combining the parallel branches,

$$(20 + j40)\|(5 - j15) = (12 - j16)\,\Omega$$

Therefore $\mathbf{I} = \dfrac{136\underline{/0^\circ}}{14 + 12 - j16 + 4} = 4\underline{/28.07^\circ}\,\text{A}$

AP 9.10

$$\mathbf{V}_1 = 240\underline{/53.13^\circ} = 144 + j192\,\text{V}$$

$$\mathbf{V}_2 = 96\underline{/-90^\circ} = -j96\,\text{V}$$

$$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$$

$$\frac{1}{j\omega C} = -j\frac{6 \times 10^6}{(4000)(25)} = -j60\,\Omega$$

Perform a source transformation:

$$\frac{\mathbf{V}_1}{j60} = \frac{144 + j192}{j60} = 3.2 - j2.4\,\text{A}$$

$$\frac{\mathbf{V}_2}{20} = -j\frac{96}{20} = -j4.8\,\text{A}$$

Combine the parallel impedances:

$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

$$Z = \frac{1}{Y} = 12\,\Omega$$

$$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8\,\text{V} = 48\underline{/36.87^\circ}\,\text{V}$$

$$v_o = 48\cos(4000t + 36.87^\circ)\,\text{V}$$

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AP 9.11 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the $20\,\Omega$ resistor and $\mathbf{V}_{\text{Th}}$ = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2\underline{/45^\circ} + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\text{Th}} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for $\mathbf{V}_{\text{Th}}$ gives $\mathbf{V}_{\text{Th}} = 10\underline{/45^\circ}V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus

It follows from the circuit that

$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\text{Th}} = (5 - j5)\,\Omega$$

AP 9.12 The phasor domain circuit is as shown in the following diagram:

The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{\mathbf{V}}{-j(20/9)} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\underline{/-90^\circ}}{20} = 0$$

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Therefore $\mathbf{V} = 10 - j30 = 31.62\underline{/-71.57^\circ}$

Therefore $v = 31.62\cos(50{,}000t - 71.57^\circ)\,\text{V}$

AP 9.13 Let $\mathbf{I}_\text{a}$, $\mathbf{I}_\text{b}$, and $\mathbf{I}_\text{c}$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_\text{a} + (3 - j5)(\mathbf{I}_\text{a} - \mathbf{I}_\text{b})$$

and

$$0 = (3 - j5)(\mathbf{I}_\text{b} - \mathbf{I}_\text{a}) + 2(\mathbf{I}_\text{b} - \mathbf{I}_\text{c}).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_\text{a} - \mathbf{I}_\text{b}),$$

therefore

$$\mathbf{I}_\text{c} = -0.75[-j5(\mathbf{I}_\text{a} - \mathbf{I}_\text{b})].$$

Solving for $\mathbf{I} = \mathbf{I}_\text{a} = 29 + j2 = 29.07\underline{/3.95^\circ}\,\text{A}$.

AP 9.14 **[a]** $M = 0.4\sqrt{0.0625} = 0.1\,\text{H}, \qquad \omega M = 80\,\Omega$

$$Z_{22} = 40 + j800(0.125) + 360 + j800(0.25) = (400 + j300)\,\Omega$$

Therefore $|Z_{22}| = 500\,\Omega, \qquad Z_{22}^* = (400 - j300)\,\Omega$

$$Z_\tau = \left(\frac{80}{500}\right)^2(400 - j300) = (10.24 - j7.68)\,\Omega$$

[b] $\mathbf{I}_1 = \dfrac{245.20}{184 + 100 + j400 + Z_\tau} = 0.50\underline{/-53.13^\circ}\,\text{A}$

$$i_1 = 0.5\cos(800t - 53.13^\circ)\,\text{A}$$

[c] $\mathbf{I}_2 = \left(\dfrac{j\omega M}{Z_{22}}\right)\mathbf{I}_1 = \dfrac{j80}{500\underline{/36.87^\circ}}(0.5\underline{/-53.13^\circ}) = 0.08\underline{/0^\circ}\,\text{A}$

$$i_2 = 80\cos 800t\,\text{mA}$$

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AP 9.15

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_1 + 2s^2 Z_2} = \frac{25 \times 10^3 \underline{/0^\circ}}{1500 + j6000 + (25)^2(4 - j14.4)}$$

$$= 4 + j3 = 5\underline{/36.87^\circ}\ \text{A}$$

$$\mathbf{V}_1 = \mathbf{V}_s - Z_1\mathbf{I}_1 = 25{,}000\underline{/0^\circ} - (4 + j3)(1500 + j6000)$$

$$= 37{,}000 - j28{,}500$$

$$\mathbf{V}_2 = -\frac{1}{25}\mathbf{V}_1 = -1480 + j1140 = 1868.15\underline{/142.39^\circ}\ \text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{1868.15\underline{/142.39^\circ}}{4 - j14.4} = 125\underline{/216.87^\circ}\ \text{A}$$

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Problems

P 9.1 **[a]** $\omega = 2\pi f = 800\,\text{rad/s}, \qquad f = \dfrac{\omega}{2\pi} = 127.32\,\text{Hz}$

[b] $T = 1/f = 7.85\,\text{ms}$

[c] $I_m = 125\,\text{mA}$

[d] $i(0) = 125\cos(36.87°) = 100\,\text{mA}$

[e] $\phi = 36.87°; \qquad \phi = \dfrac{36.87°(2\pi)}{360°} = 0.6435\ \text{rad}$

[f] $i = 0$ when $800t + 36.87° = 90°$. Now resolve the units:

$$(800\ \text{rad/s})t = \frac{53.13°}{57.3°/\text{rad}} = 0.927\,\text{rad}, \qquad t = 1.16\,\text{ms}$$

[g] $(di/dt) = (-0.125)800\sin(800t + 36.87°)$

$$(di/dt) = 0 \quad \text{when} \quad 800t + 36.87° = 180°$$

$$\text{or} \qquad 800t = \frac{143.13°}{57.3°/\text{rad}} = 2.498\,\text{rad}$$

Therefore $\quad t = 3.12\,\text{ms}$

P 9.2

[a] Right as ϕ becomes more negative

[b] Left

P 9.3 **[a]** $25\,\text{V}$

[b] $2\pi f = 400\pi; \qquad f = 200\,\text{Hz}$

[c] $\omega = 400\pi = 1256.64\ \text{rad/s}$

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[d] $\theta(\text{rad}) = 60^\circ \left(\dfrac{\pi}{180^\circ}\right) = \dfrac{\pi}{3} = 1.0472 \text{ rad}$

[e] $\theta = 60^\circ$

[f] $T = \dfrac{1}{f} = \dfrac{1}{200} = 5\,\text{ms}$

[g] $400\pi t + \dfrac{\pi}{3} = \dfrac{\pi}{2}; \qquad \therefore \quad 400\pi t = \dfrac{\pi}{6}$

$$\therefore \qquad t = \frac{1}{2400} = 416.67\,\mu\text{s}$$

[h] $v = 25\cos\left[400\pi\left(t - \dfrac{0.005}{6}\right) + \dfrac{\pi}{3}\right]$

$$= 25\cos[400\pi t - (\pi/3) + (\pi/3)]$$

$$= 25\cos 400\pi t\,\text{V}$$

[i] $400\pi(t + t_o) + (\pi/3) = 400\pi t + (3\pi/2)$

$$\therefore \quad 400\pi t_o = \frac{7\pi}{6}; \qquad t_o = \frac{7}{2400} = 2.92\,\text{ms}$$

P 9.4 **[a]** By hypothesis

$$v = 50\cos(\omega t + \theta)$$

$$\frac{dv}{dt} = -50\omega\sin(\omega t + \theta)$$

$$\therefore \quad 50\omega = 750\pi; \qquad \omega = 15\pi\,\text{rad/s}$$

[b] At $t = (40/3)$ ms, the argument of the cosine function must equal 90°. Remember that we must convert ωt to degrees:

$$15\pi\left(\frac{0.04}{3}\right)\frac{360^\circ}{2\pi} + \theta = 90^\circ$$

Solving,

$$\theta = 90^\circ - 36^\circ = 54^\circ$$

The problem description says that the voltage is increasing, but the derivative calculated in part (a) is negative. Therefore we need to shift the phase angle by 180° to $54 - 180 = -126^\circ$, effectively multiplying the expression for the voltage by -1, so that the expression for the voltage is increasing.

$$\therefore \quad v = 50\cos(15\pi t - 126^\circ)\,\text{V}$$

P 9.5 **[a]** $\dfrac{T}{2} = 25 - 5 = 20\,\text{ms}; \qquad T = 40\,\text{ms}$

$$f = \frac{1}{T} = \frac{1}{40 \times 10^{-3}} = 25\,\text{Hz}$$

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[b] $i = I_m \sin(\omega t + \theta)$

$\omega = 2\pi f = 50\pi \text{ rad/s}$

$50\pi(5 \times 10^{-3}) + \theta = 0; \qquad \therefore \quad \theta = \dfrac{-\pi}{4} \text{ rad} = -45°$

$i = I_m \sin[50\pi t - 45°]$

$0.5 = I_m \sin -45°; \qquad I_m = -70.71 \text{ mA}$

$i = -70.71 \sin[50\pi t - 45°] = 70.71 \cos[50\pi t + 45°] \text{ mA}$

P 9.6 $\quad V_m = \sqrt{2} V_{\text{rms}} = \sqrt{2}(240) = 339.41 \text{ V}$

P 9.7 $\quad V_{\text{rms}} = \sqrt{\dfrac{1}{T}\displaystyle\int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T} t \, dt}$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}t\right) dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right) dt = \frac{V_m^2 T}{4}$$

Therefore $\quad V_{\text{rms}} = \sqrt{\dfrac{1}{T}\dfrac{V_m^2 T}{4}} = \dfrac{V_m}{2}$

P 9.8
$$\begin{aligned}
\int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi)\, dt &= V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\, dt \\
&= \frac{V_m^2}{2}\left\{\int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi)\, dt\right\} \\
&= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t + 2\phi)\,\Big|_{t_o}^{t_o+T}\right]\right\} \\
&= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi)\right]\right\} \\
&= V_m^2\left(\frac{T}{2}\right) + \frac{1}{2\omega}(0) = V_m^2\left(\frac{T}{2}\right)
\end{aligned}$$

P 9.9 **[a]** The numerical values of the terms in Eq. 9.8 are

$V_m = 75, \qquad R/L = 5333.33, \qquad \omega L = 300$

$\sqrt{R^2 + \omega^2 L^2} = 500$

$\phi = -60°, \qquad \theta = \tan^{-1} 300/400, \qquad \theta = 36.87°$

Substitute these values into Equation 9.9:

$i = \left[-17.94 e^{-5333.33t} + 150\cos(4000t - 96.87°)\right] \text{ mA}, \qquad t \geq 0$

[b] Transient component $= -17.94 e^{-5333.33t} \text{ mA}$
Steady-state component $= 150\cos(4000t - 96.87°) \text{ mA}$

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[c] By direct substitution into Eq 9.9 in part (a), $i(750\,\mu\text{s}) = 38.44\,\text{mA}$

[d] $150\,\text{mA}, \quad 4000\,\text{rad/s}, \quad -96.87°$

[e] The current lags the voltage by 36.87°.

P 9.10 **[a]** From Eq. 9.9 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi-\theta)e^{-(R/L)t}}{\sqrt{R^2+\omega^2L^2}} + \frac{V_m R\cos(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$L\frac{di}{dt} + Ri = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$

But

$$\frac{R}{\sqrt{R^2+\omega^2L^2}} = \cos\theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2+\omega^2L^2}} = \sin\theta$$

Therefore the right-hand side reduces to

$$V_m\cos(\omega t+\phi)$$

At $t = 0$, Eq. 9.9 reduces to

$$i(0) = \frac{-V_m\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}} + \frac{V_m\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}} = 0$$

[b] $i_{ss} = \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$

Therefore

$$L\frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t+\phi-\theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$$

$$L\frac{di_{ss}}{dt} + Ri_{ss} = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$

$$= V_m\cos(\omega t+\phi)$$

P 9.11 **[a]** $\mathbf{Y} = 30\underline{/-160°} + 15\underline{/70°} = 29.38\underline{/170.56°}$

$$y = 28.38\cos(200t + 170.56°)$$

[b] $\mathbf{Y} = 90\underline{/-110°} + 60\underline{/-70°} = 141.33\underline{/-94.16°}$

$$y = 141.33\cos(50t - 94.16°)$$

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[c] $\mathbf{Y} = 50\underline{/-60^\circ} + 25\underline{/20^\circ} - 75\underline{/-30^\circ} = 16.7\underline{/170.52^\circ}$

$y = 16.7\cos(5000t + 170.52^\circ)$

[d] $\mathbf{Y} = 10\underline{/30^\circ} + 10\underline{/-90^\circ} + 10\underline{/150^\circ} = 0$

$y = 0$

P 9.12 **[a]** 400 Hz

[b] $\theta_v = 0^\circ$

$$\mathbf{I} = \frac{100\underline{/0^\circ}}{j\omega L} = \frac{100}{\omega L}\underline{/-90^\circ}; \qquad \theta_i = -90^\circ$$

[c] $\dfrac{100}{\omega L} = 20; \qquad \omega L = 5\,\Omega$

[d] $L = \dfrac{5}{800\pi} = 1.99\,\text{mH}$

[e] $Z_L = j\omega L = j5\,\Omega$

P 9.13 **[a]** $\omega = 2\pi f = 160\pi \times 10^3 = 502.65\,\text{krad/s} = 502{,}654.82\,\text{rad/s}$

[b] $\mathbf{I} = \dfrac{25 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(25 \times 10^{-3})\underline{/0^\circ} = 25 \times 10^{-3}\omega C\underline{/90^\circ}$

$\therefore \quad \theta_i = 90^\circ$

[c] $628.32 \times 10^{-6} = 25 \times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{25 \times 10^{-3}}{628.32 \times 10^{-6}} = 39.79\,\Omega, \quad \therefore \quad X_\text{C} = -39.79\,\Omega$$

[d] $C = \dfrac{1}{39.79(\omega)} = \dfrac{1}{(39.79)(160\pi \times 10^3)}$

$C = 0.05 \times 10^{-6} = 0.05\,\mu\text{F}$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j39.79\,\Omega$

P 9.14 **[a]** $\mathbf{V}_g = 300\underline{/78^\circ}; \qquad \mathbf{I}_g = 6\underline{/33^\circ}$

$$\therefore \quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = \frac{300\underline{/78^\circ}}{6\underline{/33^\circ}} = 50\underline{/45^\circ}\,\Omega$$

[b] i_g lags v_g by 45°:

$2\pi f = 5000\pi; \qquad f = 2500\,\text{Hz}; \qquad T = 1/f = 400\,\mu\text{s}$

$$\therefore \quad i_g \text{ lags } v_g \text{ by } \frac{45^\circ}{360^\circ}(400\,\mu\text{s}) = 50\,\mu\text{s}$$

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P 9.15 **[a]** $Z_L = j(500)(50 \times 10^{-3}) = j25\,\Omega$

$$Z_C = \frac{-j}{(500)(32 \times 10^{-6})} = -j62.5\,\Omega$$

[b] $\mathbf{I} = \dfrac{25\underline{/-60^\circ}}{25 + j25 - j62.5} = 554.7\underline{/-3.69^\circ}\,\text{mA}$

[c] $i = 554.7\cos(500t - 3.69^\circ)\,\text{mA}$

P 9.16 **[a]** $j\omega L = j(2500)(0.01) = j25\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{1}{(2500)(10 \times 10^{-6})} = -j40\,\Omega; \qquad \mathbf{I}_g = 125\underline{/-30^\circ}\,\text{A}$$

[b] $\mathbf{V}_o = 125\underline{/-30^\circ}Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{25} + \frac{1}{j25} + \frac{1}{30 - j40}$$

$$Y_e = 52 - j24\,\text{mS}$$

$$Z_e = \frac{1}{0.052 - j0.024} = 17.46\underline{/24.78^\circ}\,\Omega$$

$$\mathbf{V}_o = (125\underline{/-30^\circ})(17.46\underline{/24.78^\circ}) = 2182.6\underline{/-5.22^\circ}\,\text{V}$$

[c] $v_o = 2182.6\cos(2500t - 5.22^\circ)\,\text{V}$

P 9.17 **[a]** $Y = \dfrac{1}{3 + j4} + \dfrac{1}{16 - j12} + \dfrac{1}{-j4}$

$$= 0.12 - j0.16 + 0.04 + j0.03 + j0.25$$

$$= 160 + j120 = 200\underline{/36.87^\circ}\,\text{mS}$$

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[b] $G = 160\,\text{mS}$

[c] $B = 120\,\text{mS}$

[d] $\mathbf{I} = 8\underline{/0^\circ}\,\text{A}, \qquad \mathbf{V} = \dfrac{\mathbf{I}}{Y} = \dfrac{8}{0.2\underline{/36.87^\circ}} = 40\underline{/-36.87^\circ}\,\text{V}$

$$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40\underline{/-36.87^\circ}}{4\underline{/-90^\circ}} = 10\underline{/53.13^\circ}\,\text{A}$$

$$i_C = 10\cos(\omega t + 53.13^\circ)\,\text{A}, \qquad I_m = 10\,\text{A}$$

P 9.18 **[a]** $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[b] $R_1 = \dfrac{(4000)^2(1.25)^2(5000)}{5000^2 + 4000^2(1.25)^2} = 2500\,\Omega$

$$L_1 = \frac{(5000)^2(1.25)}{5000^2 + 4000^2(1.25)^2} = 625\,\text{mH}$$

P 9.19 **[a]** $Y_2 = \dfrac{1}{R_2} - \dfrac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $\quad Y_2 = Y_1 \quad$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $R_2 = \dfrac{8000^2 + 1000^2(4)^2}{8000} = 10\,\text{k}\Omega$

$$L_2 = \frac{8000^2 + 1000^2(4)^2}{1000^2(4)} = 20\,\text{H}$$

P 9.20 **[a]** $Z_1 = R_1 - j\dfrac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and}$$

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

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[b] $R_1 = \dfrac{1000}{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-4})^2} = 200\,\Omega$

$$C_1 = \frac{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-9})^2}{(40 \times 10^3)^2(1000)^2(50 \times 10^{-9})} = 62.5\,\text{nF}$$

P 9.21 **[a]** $Y_2 = \dfrac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $\quad Y_1 = Y_2 \quad$ when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[b] $R_2 = \dfrac{1 + (50 \times 10^3)^2(1000)^2(40 \times 10^{-9})^2}{(50 \times 10^3)^2(1000)(40 \times 10^{-9})^2} = 1250\,\Omega$

$$C_2 = \frac{40 \times 10^{-9}}{1 + (50 \times 10^3)^2(1000)^2(40 \times 10^{-9})^2} = 8\,\text{nF}$$

P 9.22 $Z_{\text{ab}} = 5 + j8 + 10\| - j20 + (8 + j16)\|(40 - j80)$

$$= 5 + j8 + 8 - j4 + 12 + j16 = 25 + j20\,\Omega = 32.02\underline{/38.66^\circ}\,\Omega$$

P 9.23 First find the admittance of the parallel branches

$$Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125\,\text{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8\,\Omega$$

$$Z_{\text{ab}} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12\,\Omega$$

$$Y_{\text{ab}} = \frac{1}{Z_{\text{ab}}} = \frac{1}{16 - j12} = 0.04 + j0.03\,\text{S}$$

$$= 40 + j30\,\text{mS} = 50\underline{/36.87^\circ}\,\text{mS}$$

P 9.24 **[a]** $\dfrac{1}{j\omega C} + R\|j\omega L = \dfrac{1}{j\omega C} + \dfrac{j\omega RL}{j\omega L + R}$

$$= \frac{j\omega L + R - \omega^2 RLC}{j\omega C(j\omega L + R)}$$

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$$= \frac{(R - \omega^2 RLC + j\omega L)(-\omega^2 LC - j\omega RC)}{(-\omega^2 LC + j\omega RC)(-\omega^2 LC - j\omega RC)}$$

The denominator in the expression above is purely real; set the imaginary part of the numerator in the above expression equal to zero and solve for ω:

$$-\omega^3 L^2 C - \omega R^2 C + \omega^3 R^2 C^2 L = 0$$

$$-\omega^2 L^2 - R^2 + \omega^2 R^2 LC = 0$$

$$\omega^2 = \frac{R^2}{R^2 LC - L^2} = \frac{200^2}{200^2(0.4)(20 \times 10^{-6}) - (0.4)^2} = 250{,}000$$

$$\therefore \qquad \omega = 500 \text{ rad/s}$$

[b] $Z_{\text{ab}}(500) = -j100 + \dfrac{(200)(j200)}{200 + j200} = 100\,\Omega$

P 9.25 **[a]** $R = 300\,\Omega = 120\,\Omega + 180\,\Omega$

$$\omega L - \frac{1}{\omega C} = -400 \quad \text{so} \quad 10{,}000L - \frac{1}{10{,}000C} = -400$$

Choose L = 10 mH. Then,

$$\frac{1}{10{,}000C} = 100 + 400 \quad \text{so} \quad C = \frac{1}{10{,}000(500)} = 0.2\,\mu\text{F}$$

We can achieve the desired capacitance by combining two $0.1\,\mu$F capacitors in parallel. The final circuit is shown here:

[b] $0.01\omega = \dfrac{1}{\omega(0.2 \times 10^{-6})}$ so $\omega^2 = \dfrac{1}{0.01(0.2 \times 10^{-6})} = 5 \times 10^8$

$$\therefore \quad \omega = 22{,}360.7 \text{ rad/s}$$

P 9.26 **[a]** Using the notation and results from Problem 9.19:

$$R \| L = 40 + j20 \quad \text{so} \quad R_1 = 40, \quad L_1 = \frac{20}{5000} = 4\,\text{mH}$$

$$R_2 = \frac{40^2 + 5000^2(0.004)^2}{40} = 50\,\Omega$$

$$L_2 = \frac{40^2 + 5000^2(0.004)^2}{5000^2(0.004)} = 20\,\text{mH}$$

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$R_2 \| j\omega L_2 = 50 \| j100 = 40 + j20\,\Omega$ (checks)

The circuit, using combinations of components from Appendix H, is shown here:

[b] Using the notation and results from Problem 9.21:

$R \| C = 40 - j20$ so $R_1 = 40$, $C_1 = 10\,\mu\text{F}$

$$R_2 = \frac{1 + 5000^2(40)^2(10\,\mu)^2}{5000^2(40)(10\,\mu)^2} = 50\,\Omega$$

$$C_2 = \frac{10\,\mu}{1 + 5000^2(40)^2(10\,\mu)^2} = 2\,\mu\text{F}$$

$R_2 \| (-j/\omega C_2) = 50 \| (-j100) = 40 - j20\,\Omega$ (checks)

The circuit, using combinations of components from Appendix H, is shown here:

P 9.27 **[a]** $(40 + j20) \| (-j/\omega C) = 50 \| j100 \| (-j/\omega C)$

To cancel out the $j100\,\Omega$ impedance, the capacitive impedance must be $-j100\,\Omega$:

$$\frac{-j}{5000C} = -j100 \quad \text{so} \quad C = \frac{1}{(100)(5000)} = 2\,\mu\text{F}$$

Check:

$R \| j\omega L \| (-j/\omega C) = 50 \| j100 \| (-j100) = 50\,\Omega$

Create the equivalent of a $2\,\mu$F capacitor from components in Appendix H by combining two $1\,\mu$F capacitors in parallel.

[b] $(40 - j20) \| (j\omega L) = 50 \| (-j100) \| (j\omega L)$

To cancel out the $-j100\,\Omega$ impedance, the inductive impedance must be $j100\,\Omega$:

$$j5000L = j100 \quad \text{so} \quad L = \frac{100}{5000} = 20\,\text{mH}$$

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Check:

$R\|j\omega L\|(-j/\omega C) = 50\|j100\|(-j100) = 50\,\Omega$

Create the equivalent of a 20 mH inductor from components in Appendix H by combining two 10 mH inductors in series.

P 9.28 $$Z = 4000 + j(2000)(0.5) - j\frac{1}{(2000)(100\times 10^{-9})}$$

$$= 3000 + j1000 - j5000 = 3000 - j4000\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{Z} = \frac{80\underline{/0^\circ}}{3000 - j4000} = 9.6 + j12.8 = 16\underline{/53.13^\circ}\,\text{mA}$$

$$i_o(t) = 16\cos(2000t + 53.13^\circ)\,\text{mA}$$

P 9.29 $$\frac{1}{j\omega C} = \frac{1}{j(5\times 10^{-6})(8000)} = -j25\,\Omega$$

$$j\omega L = j8000(3.125\times 10^{-3}) = j25\,\Omega$$

$$\mathbf{V}_g = 60\underline{/-90^\circ}\,\text{V}$$

$$Z_e = j25 + (50\| - j25) = 10 + j5\,\Omega$$

$$\mathbf{I}_g = \frac{60\underline{/-90^\circ}}{10 + j5} = -j2.4 - j4.8\,\text{A}$$

$$\mathbf{V}_o = (50\| - j25)\mathbf{I}_g = (10 - j20)(-j2.4 - j4.8) = -120\,\text{V}$$

$$v_o = -120\cos 8000t\,\text{V}$$

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P 9.30 $\mathbf{V}_s = 25\underline{/-90^\circ}\,\text{V}$

$$\frac{1}{j\omega C} = -j20\,\Omega$$

$$j\omega L = j10\,\Omega$$

$$Z_{\text{eq}} = 5 + j10\|(10 + 20\| - j20) = 10 + j10\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{Z_{\text{eq}}} = \frac{25\underline{/-90^\circ}}{10 + j10} = -1.25 - j1.25 = 1.77\underline{/-135^\circ}\,\text{A}$$

$$i_o = 1.77\cos(4000t - 135^\circ)\,\text{A}$$

P 9.31 **[a]** $\dfrac{1}{j\omega C} = -j250\,\Omega$

$$j\omega L = j200\,\Omega$$

$$Z_e = j200\|(100 + 500\| - j250) = 200 + j200\,\Omega$$

$$\mathbf{I}_g = 0.025\underline{/0^\circ}$$

$$\mathbf{V}_g = \mathbf{I}_g Z_e = 0.025(200 + j200) = 5 + j5\,\text{V}$$

$$\mathbf{V}_o = \frac{100 - j200}{200 - j200}(5 + j5) = 5 + j2.5 = 5.59\underline{/26.57^\circ}\,\text{V}$$

$$v_o = 5.59\cos(50{,}000t + 26.57^\circ)\,\text{V}$$

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[b] $\omega = 2\pi f = 50{,}000; \qquad f = \dfrac{25{,}000}{\pi}$

$$T = \frac{1}{f} = \frac{\pi}{25{,}000} = 40\pi\ \mu\text{s}$$

$$\therefore \quad \frac{26.57}{360}(40\pi\ \mu\text{s}) = 9.27\,\mu\text{s}$$

$\therefore \quad v_o$ leads i_g by $9.27\,\mu$s.

P 9.32

$$\mathbf{V}_1 = j5(-j2) = 10\,\text{V}$$

$$-25 + 10 + (4 - j3)\mathbf{I}_1 = 0 \quad \therefore \quad \mathbf{I}_1 = \frac{15}{4 - j3} = 2.4 + j1.8\,\text{A}$$

$$\mathbf{I}_\text{b} = \mathbf{I}_1 - j5 = (2.4 + j1.8) - j5 = 2.4 - j3.2\,\text{A}$$

$$\mathbf{V}_Z = -j5\mathbf{I}_2 + (4 - j3)\mathbf{I}_1 = -j5(2.4 - j3.2) + (4 - j3)(2.4 + j1.8) = -1 - j12\,\text{V}$$

$$-25 + (1 + j3)\mathbf{I}_3 + (-1 - j12) = 0 \quad \therefore \quad \mathbf{I}_3 = 6.2 - j6.6\,\text{A}$$

$$\mathbf{I}_Z = \mathbf{I}_3 - \mathbf{I}_2 = (6.2 - j6.6) - (2.4 - j3.2) = 3.8 - j3.4\,\text{A}$$

$$Z = \frac{\mathbf{V}_Z}{\mathbf{I}_Z} = \frac{-1 - j12}{3.8 - j3.4} = 1.42 - j1.88\,\Omega$$

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P 9.33

$$\frac{\mathbf{V}_\mathrm{a} - (100 - j50)}{20} + \frac{\mathbf{V}_\mathrm{a}}{j5} + \frac{\mathbf{V}_\mathrm{a} - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{V}_\mathrm{a} = 40 + j30\,\mathrm{V}$$

$$\mathbf{I}_Z + (30 + j20) - \frac{140 + j30}{-j10} + \frac{(40 + j30) - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{I}_Z = -30 - j10\,\mathrm{A}$$

$$Z = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = 2 + j2\,\Omega$$

P 9.34 $Z_L = j(10{,}000)(10 \times 10^{-3}) = j100\,\Omega$

$$Z_C = \frac{-j}{(10{,}000)(2 \times 10^{-6})} = -j50\,\Omega$$

Construct the phasor domain equivalent circuit:

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Using current division:

$$\mathbf{I} = \frac{(50 + j100)}{50 + j100 + 100 - j50}(0.06) = 30 + j30\,\text{mA}$$

$$\mathbf{V}_o = 100\mathbf{I} = 3 + j3 = 4.24\underline{/45^\circ}$$

$$v_o = 4.24\cos(10{,}000t + 45^\circ)\,\text{V}$$

P 9.35 $\mathbf{V}_g = 40\underline{/-15^\circ}\,\text{V}; \qquad \mathbf{I}_g = 40\underline{/-68.13^\circ}\,\text{mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 1000\underline{/53.13^\circ}\,\Omega = 600 + j800\,\Omega$$

$$Z = 600 + j\left(3.2\omega - \frac{0.4 \times 10^6}{\omega}\right)$$

$$\therefore \quad 3.2\omega - \frac{0.4 \times 10^6}{\omega} = 800$$

$$\therefore \quad \omega^2 - 250\omega - 125{,}000 = 0$$

Solving,

$$\omega = 500\,\text{rad/s}$$

P 9.36 **[a]**

$$\mathbf{V}_\text{b} = (2000 - j1000)(0.025) = 50 - j25\,\text{V}$$

$$\mathbf{I}_\text{a} = \frac{50 - j25}{500 + j250} = 60 - j80\,\text{mA} = 100\underline{/-53.13^\circ}\,\text{mA}$$

$$\mathbf{I}_\text{c} = \frac{50 - j25 + j50}{1000} = 50 + j25\,\text{mA} = 55.9\underline{/26.57^\circ}\,\text{mA}$$

$$\mathbf{I}_g = \mathbf{I}_\text{a} + \mathbf{I}_\text{b} + \mathbf{I}_\text{c} = 135 - j55\,\text{mA} = 145.77\underline{/-22.17^\circ}\,\text{mA}$$

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[b] $i_a = 100\cos(1500t - 53.13°)\,\text{mA}$

$i_c = 55.9\cos(1500t + 26.57°)\,\text{mA}$

$i_g = 145.77\cos(1500t - 22.17°)\,\text{mA}$

P 9.37 **[a]** In order for v_g and i_g to be in phase, the impedance to the right of the $500\,\Omega$ resistor must be purely real:

$$Z_{eq} = j\omega L \| (R + 1/\omega C) = \frac{j\omega L(R + 1/j\omega C)}{j\omega L + R + 1/j\omega C}$$

$$= \frac{j\omega L(j\omega RC + 1)}{j\omega RC - \omega^2 LC + 1}$$

$$= \frac{(-\omega^2 RLC + j\omega L)(1 - \omega^2 LC - j\omega RC)}{(1 - \omega^2 LC + j\omega RC)(1 - \omega^2 LC - j\omega RC)}$$

The denominator of the above expression is purely real. Now set the imaginary part of the numerator in that expression to zero and solve for ω:

$$\omega L(1 - \omega^2 LC) + \omega^3 R^2 lC^2 = 0$$

So $\quad \omega^2 = \dfrac{1}{LC - R^2C^2} = \dfrac{1}{(0.2)(10^{-6}) - 200^2(10^{-6})^2} = 6{,}250{,}000$

$\therefore \quad \omega = 2500 \text{ rad/s} \quad$ and $\quad f = 397.9 \text{ Hz}$

[b] $Z_{eq} = 500 + j500 \| (200 - j400) = 1500\,\Omega$

$$\mathbf{I}_g = \frac{90\underline{/0°}}{1500} = 60\underline{/0°}\,\text{mA}$$

$$i_g(t) = 60\cos 2500t\,\text{mA}$$

P 9.38 **[a]** For i_g and v_g to be in phase, the impedance to the right of the $480\,\Omega$ resistor must be purely real.

$$j\omega L + \frac{(1/j\omega C)(200}{(1/j\omega C) + 200} = \jmath\omega L + \frac{200}{1 + 200j\omega C}$$

$$= j\omega L + \frac{200(1 - 200j\omega C)}{1 + 200^2\omega^2C^2}$$

$$\frac{j\omega L(1 + 200^2\omega^2C^2) + 200(1 - 200j\omega C)}{1 + 200^2\omega^2C^2}$$

In the above expression the denominator is purely real. So set the imaginary part of the numerator to zero and solve for ω:

$$\omega L(1 + 200^2\omega^2c^2) - 200^2\omega C = 0$$

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$$\omega^2 = \frac{200^2C - L}{200^2C} = \frac{200^2(3.125 \times 10^{-6}) - 0.1}{200^2(3.125 \times 10^{-6})^2} = 64{,}000$$

$$\therefore \qquad \omega = 800 \text{ rad/s}$$

[b] When $\omega = 800\,\text{rad/s}$

$$Z_g = 480\|[j80 + (200\| - j400)] = 120\,\Omega$$

$$\therefore \quad \mathbf{V}_g = Z_g\mathbf{I}_g = 120(0.06) = 7.2\,\text{V}$$

$$v_o = 7.2\cos 800t\,\text{V}$$

P 9.39 **[a]** The voltage and current are in phase when the impedance to the left of the 1 kω resistor is purely real:

$$Z_{\text{eq}} = \frac{1}{j\omega C}\|(R + j\omega L) = \frac{R + j\omega L}{1 + j\omega RC - \omega^2 LC}$$

$$= \frac{(R + j\omega L)(1 - \omega^2 LC - j\omega RC)}{(1 + \omega^2 LC)^2 + \omega^2 R^2 C^2}$$

The denominator in the above expression is purely real, so set the imaginary part of the expression's numerator to zero and solve for ω:

$$-\omega R^2 C + \omega L - \omega^3 L^2 C = 0$$

$$\omega^2 = \frac{L - R^2C}{L^2C} = \frac{(0.01) - 240^2(62.5 \times 10^{-9})}{(0.01)^2(62.5 \times 10^{-9})^2} = 1024 \times 10^6$$

$$\therefore \qquad \omega = 32{,}000 \text{ rad/s}$$

[b] $Z_t = 1000 + (-j500)\|(240 + j320) = 1666.67\,\Omega$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{Z_T} = \frac{15\underline{/0^\circ}}{1666.67} = 9\underline{/0^\circ}\,\text{mA}$$

$$i_o = 9\cos 32{,}000t\,\text{mA}$$

P 9.40 **[a]** $Z_C = \dfrac{-j}{(1000)(10^{-6})} = -j1000\,\Omega$

$$Z_1 = 2500\|j\omega L = \frac{2500j\omega L}{2500 + j\omega L} = \frac{j2500\omega L(2500 - j\omega L}{2500^2 + \omega^2 L^2}$$

$$Z_T = 500 + Z_C + Z_1 = 500 - j1000 + \frac{2500\omega^2 L^2 + 2500^2 j\omega L}{2500^2 + \omega^2 L^2}$$

Z_T is resistive when

$$\frac{2500^2\omega L}{2500^2 + \omega^2 L^2} = 1000 \qquad \text{or}$$

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$$L^2(1000\omega^2) - L(2500^2\omega) + 1000(2500^2) = 0$$

For $\omega = 1000$:

$$L^2 - 6.25L + 6.25 = 0$$

Solving, $L_1 = 5$ H and $L_2 = 1.25$ H.

[b] When $L = 5$ H:

$$Z_T = 500 - j1000 + 2500\|j5000 = 2500\,\Omega$$

$$\mathbf{I}_g = \frac{40\underline{/0^\circ}}{2500} = 16\underline{/0^\circ}\,\text{mA}$$

$$i_g = 16\cos 1000t\,\text{mA}$$

When $L = 1.25$ H:

$$Z_T = 500 - j1000 + 2500\|j1250 = 1000\,\Omega$$

$$\mathbf{I}_g = \frac{40\underline{/0^\circ}}{1000} = 40\underline{/0^\circ}\,\text{mA}$$

$$i_g = 40\cos 1000t\,\text{mA}$$

P 9.41 **[a]** $$Z_p = \frac{\dfrac{R}{j\omega C}}{R + (1/j\omega C)} = \frac{R}{1 + j\omega RC}$$

$$= \frac{10{,}000}{1 + j(5000)(10{,}000)C} = \frac{10{,}000}{1 + j50 \times 10^6 C}$$

$$= \frac{10{,}000(1 - j50 \times 10^6 C)}{1 + 25 \times 10^{14} C^2}$$

$$= \frac{10{,}000}{1 + 25 \times 10^{14} C^2} - j\frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$$j\omega L = j5000(0.8) = j4000$$

$$\therefore \quad 4000 = \frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$$\therefore \quad 10^{14} C^2 - 125 \times 10^6 C + 1 = 0$$

$$\therefore \quad C^2 - 5 \times 10^{-8} C + 4 \times 10^{-16} = 0$$

Solving,

$C_1 = 40\,\text{nF}$ $\qquad$ $C_2 = 10\,\text{nF}$

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[b] $R_e = \dfrac{10{,}000}{1 + 25 \times 10^{14} C^2}$

When $C = 40\,\text{nF}$ $\quad R_e = 2000\,\Omega$;

$$\mathbf{I}_g = \frac{80\underline{/0^\circ}}{2000} = 40\underline{/0^\circ}\,\text{mA}; \qquad i_g = 40\cos 5000t\,\text{mA}$$

When $C = 10\,\text{nF}$ $\quad R_e = 8000\,\Omega$;

$$\mathbf{I}_g = \frac{80\underline{/0^\circ}}{8000} = 10\underline{/0^\circ}\,\text{mA}; \qquad i_g = 10\cos 5000t\,\text{mA}$$

P 9.42 Simplify the top triangle using series and parallel combinations:

$(1 + j1)\|(1 - j1) = 1\,\Omega$

Convert the lower left delta to a wye:

$$Z_1 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

$$Z_2 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_3 = \frac{(j1)(-j1)}{1 + j1 - j1} = 1\,\Omega$$

Convert the lower right delta to a wye:

$$Z_4 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_5 = \frac{(-j1)(j1)}{1 + j1 - j1} = 1\,\Omega$$

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$$Z_6 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

The resulting circuit is shown below:

Simplify the middle portion of the circuit by making series and parallel combinations:

$$(1 + j1 - j1)\|(1 + 1) = 1\|2 = 2/3\,\Omega$$

$$Z_{\text{ab}} = -j1 + 2/3 + j1 = 2/3\,\Omega$$

P 9.43 **[a]** $j\omega L = j(400)(400) \times 10^{-3} = j160\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(400)(31.25 \times 10^{-6})} = -j80\,\Omega$$

Using voltage division,

$$\mathbf{V}_{\text{ab}} = \frac{-j80\|j160}{320 + (-j80\|j160)}(-j50) = -20 - j10\,\text{V}$$

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{ab}} = -20 - j10\,\text{V}$$

[b] Remove the voltage source and combine impedances in parallel to find $Z_{\text{Th}} = Z_{\text{ab}}$:

$$Z_{\text{Th}} = Z_{\text{ab}} = 320\| - j80\|j160 = 64 - j128\,\Omega$$

[c]

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P 9.44 Step 1 to Step 2:

$$\frac{180\underline{/90^\circ}}{j30} = 6\underline{/0^\circ}\,\text{A}$$

Step 2 to Step 3:

$$(j30)\|15 = 12 + j6\,\Omega; \qquad (6\underline{/0^\circ})(12 + j6) = 72 + j36\,\text{V}$$

Step 3 to Step 4:

$$Z_N = 12 + j6 - j30 = 12 - j24\,\Omega; \qquad \mathbf{I}_N = \frac{72 + j36}{12 - j24} = j3\,\text{A}$$

P 9.45 Step 1 to Step 2:

$$(0.12\underline{/0^\circ})(250) = 30\underline{/0^\circ}\,\text{V}$$

Step 2 to Step 3:

$$250 - j400 + j150 = 250 - j250\,\Omega; \qquad \frac{30\underline{/0^\circ}}{250 - j250} = 60 - j60\,\text{mA}$$

Step 3 to Step 4:

$$(250 - j250)\|500 = 200 - j100\,\Omega; \qquad (200 - j100)(0.06 - j0.06) = 18 - j6\,\text{V}$$

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P 9.46 Open circuit voltage:

$$-j20\mathbf{I}_\mathrm{a} + 40\mathbf{I}_\mathrm{a} + 20(\mathbf{I}_\mathrm{a} - 0.4 - j0.2) = 0$$

Solving,

$$\mathbf{I}_\mathrm{a} = \frac{20(0.4 + j0.2)}{60 - j20} = 0.1 + j0.1\,\mathrm{A}$$

$$\mathbf{V}_\mathrm{oc} = 40\mathbf{I}_\mathrm{a} + j16(0.4 + j0.2) = 0.8 + j10.4\,\mathrm{V}$$

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Short circuit current:

$$-j20\mathbf{I}_{\mathrm{a}} + 40(\mathbf{I}_{\mathrm{a}} - \mathbf{I}_{\mathrm{sc}}) + 20(\mathbf{I}_{\mathrm{a}} - 0.4 - j0.2) = 0$$

$$40(\mathbf{I}_{\mathrm{sc}} - \mathbf{I}_{\mathrm{a}}) + j16(\mathbf{I}_{\mathrm{sc}} - 0.4 - j0.2) = 0$$

Solving,

$$\mathbf{I}_{\mathrm{sc}} = 0.3 + j0.5\,\mathrm{A}$$

$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{0.8 + j10.4}{0.3 + j0.5} = 16 + j8\,\Omega$$

P 9.47

$$\mathbf{V}_{\mathrm{Th}} = (-0.15 + j0.15)Z_{\mathrm{Th}} + (-30 + j30)$$

$$\frac{j200}{Z_{\mathrm{Th}} + j200}\mathbf{V}_{\mathrm{Th}} = -40 - j40 \qquad \text{so} \qquad -j200\mathbf{V}_{\mathrm{Th}} + (-40 - j40)(Z_{\mathrm{Th}} + j200)$$

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Place the two equations in standard form:

$$\mathbf{V}_{\text{Th}} + (0.15 - j0.15)Z_{\text{Th}} = -30 - j30$$

$$j200\mathbf{V}_{\text{Th}} + (40 + j40)C_{\text{Th}} = (-40 - j40)(j200)$$

Solving,

$$\mathbf{V}_{\text{Th}} = j240\,\text{V}; \qquad Z_{\text{Th}} = 600 - j800\,\Omega$$

P 9.48 Short circuit current

$$\mathbf{V}_x - j10(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) = -40 + j40$$

$$-5\mathbf{V}_x - j10(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) = 0$$

$$\mathbf{V}_x = 10\mathbf{I}_1$$

Solving,

$$\mathbf{I}_N = \mathbf{I}_{\text{sc}} = 6 + j4\,\text{A}$$

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Open circuit voltage

$$\mathbf{I} = \frac{-40 + j40}{10 - j10} = -4\,\text{A}$$

$$\mathbf{V}_x = 10\mathbf{I} = -40\,\text{V}$$

$$\mathbf{V}_{\text{oc}} = 5\mathbf{V}_x - j10\mathbf{I} = -200 + j40\,\text{V}$$

$$Z_N = \frac{-200 + j40}{6 + j4} = -20 + j20\,\Omega$$

P 9.49 Open circuit voltage:

$$\frac{\mathbf{V}_1 - 250}{20 + j10} - 0.03\mathbf{V}_o + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\therefore \quad \mathbf{V}_o = \frac{-j100}{50 - j100}\mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20 + j10} + \frac{j3\mathbf{V}_1}{50 - j100} + \frac{\mathbf{V}_1}{50 - j100} = \frac{250}{20 + j10}$$

$$\mathbf{V}_1 = 500 - j250\,\text{V}; \qquad \mathbf{V}_o = 300 - j400\,\text{V} = \mathbf{V}_{\text{Th}}$$

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Short circuit current:

$$\mathbf{I}_{\text{sc}} = \frac{250\underline{/0^\circ}}{70 + j10} = 3.5 - j0.5\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{300 - j400}{3.5 - j0.5} = 100 - j100\,\Omega$$

The Thévenin equivalent circuit:

P 9.50 Open circuit voltage:

$$\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - \frac{1}{5}\mathbf{V}_2}{-j50} = 0$$

$$\mathbf{I}_\phi = \frac{5 - (\mathbf{V}_2/5)}{200}$$

Solving,

$$\mathbf{V}_2 = -66 + j88 = 110\underline{/126.87^\circ}\,\text{V} = \mathbf{V}_{\text{Th}}$$

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Find the Thévenin equivalent impedance using a test source:

$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\mathbf{I}_\phi + \frac{0.8\mathbf{V}_t}{-j50}$$

$$\mathbf{I}_\phi = \frac{-\mathbf{V}_T/5}{200}$$

$$\mathbf{I}_T = \mathbf{V}_T\left(\frac{1}{10} - 88\frac{\mathbf{V}_T/5}{200} + \frac{0.8}{-j50}\right)$$

$$\therefore \qquad \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j40 = Z_{\text{Th}}$$

$$\mathbf{I}_{\text{N}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}}} = \frac{-66 + j88}{30 - j40} = -2.2 + j0\,\text{A}$$

The Norton equivalent circuit:

P 9.51 [**a**]

$$\mathbf{I}_T = \frac{\mathbf{V}_T}{1000} + \frac{\mathbf{V}_T - \alpha\mathbf{V}_T}{-j1000}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{1000} - \frac{(1-\alpha)}{j1000} = \frac{j - 1 + \alpha}{j1000}$$

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$$\therefore \quad Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{j1000}{\alpha - 1 + j}$$

Z_{Th} is real when $\alpha = 1$.

[b] $Z_{\text{Th}} = 1000\,\Omega$

[c] $Z_{\text{Th}} = 500 - j500 = \dfrac{j1000}{\alpha - 1 + j}$

$$= \frac{1000}{(\alpha-1)^2+1} + j\frac{1000(\alpha-1)}{(\alpha-1)^2+1}$$

Equate the real parts:

$$\frac{1000}{(\alpha-1)^2+1} = 500 \quad \therefore \quad (\alpha-1)^2 + 1 = 2$$

$$\therefore \quad (\alpha-1)^2 = 1 \quad \text{so} \quad \alpha = 0$$

Check the imaginary parts:

$$\left.\frac{(\alpha-1)1000}{(\alpha-1)^2+1}\right|_{\alpha=1} = -500$$

Thus, $\alpha = 0$.

[d] $Z_{\text{Th}} = \dfrac{1000}{(\alpha-1)^2+1} + j\dfrac{1000(\alpha-1)}{(\alpha-1)^2+1}$

For $\mathbf{Im}(Z_{\text{Th}}) > 0$, α must be greater than 1. So Z_{Th} is inductive for $1 < \alpha \le 10$.

P 9.52 $j\omega L = j100 \times 10^3(0.6 \times 10^{-3}) = j60\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(100 \times 10^3)(0.4 \times 10^{-6})} = -j25\,\Omega$$

$$\mathbf{V}_T = -j25\mathbf{I}_T + 5\mathbf{I}_\Delta - 30\mathbf{I}_\Delta$$

$$\mathbf{I}_\Delta = \frac{-j60}{30 + j60}\mathbf{I}_T$$

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$$\mathbf{V}_T = -j25\mathbf{I}_T + 25\frac{j60}{30+j60}\mathbf{I}_T$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{\text{ab}} = 20 - j15 = 25\underline{/-36.87^\circ}\,\Omega$$

P 9.53 $$\frac{1}{\omega C_1} = \frac{10^9}{50{,}000(2.5)} = 8\,\text{k}\Omega$$

$$\frac{1}{\omega C_2} = \frac{10^9}{50{,}000(5)} = 4\,\text{k}\Omega$$

$$\mathbf{V}_T = (2400 - j8000)\mathbf{I}_T + 40\mathbf{I}_T(90)$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 6000 - j8000\,\Omega$$

P 9.54

$$\frac{\mathbf{V}_1 - 240}{j10} + \frac{\mathbf{V}_1}{50} + \frac{\mathbf{V}_1}{30+j10} = 0$$

Solving for $\mathbf{V}_1$ yields

$$\mathbf{V}_1 = 198.63\underline{/-24.44^\circ}\,\text{V}$$

$$\mathbf{V}_o = \frac{30}{30+j10}(\mathbf{V}_1) = 188.43\underline{/-42.88^\circ}\,\text{V}$$

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P 9.55 Set up the frequency domain circuit to use the node voltage method:

At $\mathbf{V}_1$: $$-5\underline{/0^\circ} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\underline{/90^\circ}}{-j4} = 0$$

At $\mathbf{V}_2$: $$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\underline{/90^\circ}}{12} = 0$$

In standard form:

$$\mathbf{V}_1\left(\frac{1}{-j8} + \frac{1}{-j4}\right) + \mathbf{V}_2\left(-\frac{1}{-j8}\right) = 5\underline{/0^\circ} + \frac{20\underline{/90^\circ}}{-j4}$$

$$\mathbf{V}_1\left(-\frac{1}{-j8}\right) + \mathbf{V}_2\left(\frac{1}{-j8} + \frac{1}{j4} + \frac{1}{12}\right) = \frac{20\underline{/90^\circ}}{12}$$

Solving on a calculator:

$$\mathbf{V}_1 = -\frac{8}{3} + j\frac{4}{3} \qquad \mathbf{V}_2 = -8 + j4$$

Thus

$$\mathbf{V}_g = \mathbf{V}_1 - 20\underline{/90^\circ} = -\frac{8}{3} - j\frac{56}{3}\,\text{V}$$

P 9.56 $j\omega L = j(2500)(1.6 \times 10^{-3}) = j4\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(2500)(100 \times 10^{-6})} = -j4\,\Omega$$

$$\mathbf{I}_g = 5\underline{/0^\circ}\,\text{A}$$

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$\mathbf{V}_g = 20\underline{/90^\circ}\,\text{V}$

$$-5 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - j20}{-j4} = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - j20}{12} = 0$$

Solving,

$$\mathbf{V}_2 = -8 + j4\,\text{V}; \qquad \mathbf{I}_o = \frac{\mathbf{V}_2}{j4} = 1 + j2 = 2.24\underline{/63.43^\circ}\,\text{A}$$

$$i_o = 2.24\cos(2500t + 63.43^\circ)\,\text{A}$$

P 9.57 $j\omega L = j(400)(50 \times 10^{-3}) = j20\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(400)(50 \times 10^{-6})} = -j50\,\Omega$$

$$\mathbf{V}_{g1} = 25\underline{/53.13^\circ} = 15 + j20\,\text{V}$$

$$\mathbf{V}_{g2} = 18.03\underline{/33.69^\circ} = 15 + j10\,\text{V}$$

$$\frac{\mathbf{V}_o - (15 + j20)}{-j50} + \frac{\mathbf{V}_o}{150} + \frac{\mathbf{V}_o - (15 + j10)}{j20} = 0$$

Solving,

$$\mathbf{V}_o = 15\underline{/0^\circ}$$

$$v_o(t) = 15\cos 400t\,\text{V}$$

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P 9.58 Write a KCL equation at the top node:

$$\frac{\mathbf{V}_o}{-j8} + \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} + \frac{\mathbf{V}_o}{5} - (10 + j10) = 0$$

The constraint equation is:

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8}$$

Solving,

$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\ \text{V}$$

P 9.59

$$\frac{\mathbf{V}_o}{50} + \frac{\mathbf{V}_o}{-j25} + 20\mathbf{I}_o = 0$$

$$(2 + j4)\mathbf{V}_o = -2000\mathbf{I}_o$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/10)}{j25}$$

$$\therefore \quad \mathbf{V}_1 = (-20 + j65)\mathbf{I}_o$$

$$0.006 + j0.013 = \frac{\mathbf{V}_1}{50} + \mathbf{I}_o = (-0.4 + j1.3)\mathbf{I}_o + \mathbf{I}_o = (0.6 + j1.3)\mathbf{I}_o$$

$$\therefore \quad \mathbf{I}_o = \frac{0.6 + j1.3(10 \times 10^{-3})}{(0.6 + j1.3)} = 10\underline{/0^\circ}\ \text{mA}$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o = -2 + j4 = 4.47\underline{/116.57^\circ}\ \text{V}$$

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P 9.60

$$(12 - j12)\mathbf{I}_a - 12\mathbf{I}_g - 5(-j8) = 0$$

$$-12\mathbf{I}_a + (12 + j4)\mathbf{I}_g + j20 - 5(j4) = 0$$

Solving,

$$\mathbf{I}_g = 4 - j2 = 4.47\underline{/-26.57^\circ}\ \text{A}$$

P 9.61 The circuit with the mesh currents identified is shown below:

The mesh current equations are:

$$-(15 + j20) - j50\mathbf{I}_1 + 150(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$15 + j10 + 150(\mathbf{I}_2 - \mathbf{I}_1) + j20\mathbf{I}_2 = 0$$

In standard form:

$$\mathbf{I}_1(150 - j50) + \mathbf{I}_2(-150) = 15 + j20$$

$$\mathbf{I}_1(-150) + \mathbf{I}_2(150 + j20) = -(15 + j10)$$

Solving on a calculator yields:

$$\mathbf{I}_1 = -0.4\,\text{A}; \qquad \mathbf{I}_2 = -0.5\,\text{A}$$

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Thus,

$$\mathbf{V}_o = 150(\mathbf{I}_1 - \mathbf{I}_2) = 15\,\text{V}$$

and

$$v_o(t) = 15\cos 400t\,\text{V}$$

P 9.62

$$10\underline{/0^\circ} = (1 - j1)\mathbf{I}_1 - 1\mathbf{I}_2 + j1\mathbf{I}_3$$

$$-5\underline{/0^\circ} = -1\mathbf{I}_1 + (1 + j1)\mathbf{I}_2 - j1\mathbf{I}_3$$

$$1 = j1\mathbf{I}_1 - j1\mathbf{I}_2 + \mathbf{I}_3$$

Solving,

$$\mathbf{I}_1 = 11 + j10\,\text{A}; \qquad \mathbf{I}_2 = 11 + j5\,\text{A}; \qquad \mathbf{I}_3 = 6\,\text{A}$$

$$\mathbf{I}_\text{a} = \mathbf{I}_3 - 1 = 5\,\text{A}$$

$$\mathbf{I}_\text{b} = \mathbf{I}_1 - \mathbf{I}_3 = 5 + j10\,\text{A}$$

$$\mathbf{I}_\text{c} = \mathbf{I}_2 - \mathbf{I}_3 = 5 + j5\,\text{A}$$

$$\mathbf{I}_\text{d} = \mathbf{I}_1 - \mathbf{I}_2 = j5\,\text{A}$$

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P 9.63 $\mathbf{V}_\mathrm{a} = -j18\,\mathrm{V}; \qquad \mathbf{V}_\mathrm{b} = 12\,\mathrm{V}$

$$j\omega L = j(4000)(25 \times 10^{-3}) = j100\Omega$$

$$\frac{-j}{\omega C} = \frac{-j}{4000(625 \times 10^{-6})} = -j400\,\Omega$$

$$-j18 = -j300\mathbf{I}_\mathrm{a} - j100\mathbf{I}_\mathrm{b}$$

$$-12 = -j100\mathbf{I}_\mathrm{a} + (400 + j100)\mathbf{I}_\mathrm{b}$$

Solving,

$$\mathbf{I}_\mathrm{a} = 67.5 - j7.5\,\mathrm{mA}; \qquad \mathbf{I}_\mathrm{b} = -22.5 + j22.5\,\mathrm{mA}$$

$$\mathbf{V}_o = j100(\mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b}) = 3 + j9 = 9.49\underline{/71.57^\circ}\,\mathrm{A}$$

$$v_o(t) = 9.49\cos(4000t + 71.57^\circ)\,\mathrm{A}$$

P 9.64 $j\omega L_1 = j5000(4 \times 10^{-3}) = j20\,\Omega; \qquad j\omega L_2 = j5000(110 \times 10^{-3}) = j550\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(5000)(4 \times 10^{-6})} = -j50\,\Omega$$

$$75\underline{/0^\circ} = j500\mathbf{I}_\Delta - 100\mathbf{I}_\Delta - j550\mathbf{I}_\mathrm{a}$$

$$0 = (10 + j20)\mathbf{I}_\mathrm{a} + 100\mathbf{I}_\Delta + j550(\mathbf{I}_\mathrm{a} - \mathbf{I}_\Delta)$$

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Solving,

$\mathbf{I}_{\mathrm{a}} = j2.5\,\mathrm{A}$

$\mathbf{V}_o = 10\mathbf{I}_{\mathrm{a}} = j25 = 25\underline{/90^\circ}$

$v_o = 25\cos(5000t - 90^\circ) = 25\sin 5000t\,\mathrm{V}$

P 9.65 $\dfrac{1}{j\omega c} = \dfrac{-j}{(100{,}000)(3.125 \times 10^{-9})} = -j3200\,\Omega$

$j\omega L = j(100{,}000)(80 \times 10^{-3}) - j8000\,\Omega$

Let

$Z_1 = 1200 - j3200\,\Omega; \qquad Z_2 = 2400 + j8000\,\Omega$

$$\mathbf{V}_o = \frac{Z_2}{Z_1 + Z_2}\mathbf{V}_g = \frac{2400 + j8000}{(3600 + j4800}(120) = 156.8 + j57.6 = 167.045\underline{/20.17^\circ}$$

$\therefore \qquad v_o = 167.045\cos(100{,}000t + 20.17^\circ)\,\mathrm{V}$

P 9.66 $\dfrac{1}{j\omega C} = \dfrac{-j}{(250)(100 \times 10^{-6})} = -j40\,\Omega$

$j\omega L = j(250)(0.8) = j200\,\Omega$

Let $Z_1 = 20 - j40\,\Omega; \qquad Z_2 = 100 + j200\,\Omega$

$\mathbf{I}_g = 60\underline{/0^\circ}\,\mathrm{mA}$

$$\mathbf{I}_o = \frac{\mathbf{I}_g Z_1}{Z_1 + Z_2} = \frac{(0.06)(20 - j40)}{(120 + j160)}$$

$$= -6 - j12\,\mathrm{mA} = 13.42\underline{/-116.57^\circ}\,\mathrm{mA}$$

$i_o = 13.42\cos(250t - 116.57^\circ)\,\mathrm{mA}$

P 9.67 **[a]** Superposition must be used because the frequencies of the two sources are different.

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[b] For $\omega = 2000$ rad/s:

$10\| - j5 = 2 - j4\,\Omega$

so $\quad \mathbf{V}_{o1} = \dfrac{2 - j4}{2 - j4 + j2}(20\underline{/-36.87^\circ}) = 31.62\underline{/-55.3^\circ}\,\text{V}$

For $\omega = 5000$ rad/s:

$j5\|10 = 2 + j4\,\Omega$

$\mathbf{V}_{o2} = \dfrac{2 + j4}{2 + j4 - j2}(10\underline{/16.26^\circ}) = 15.81\underline{/34.69^\circ}\,\text{V}$

Thus,

$v_o(t) = [31.62\cos(2000t - 55.3^\circ) + 15.81\cos(5000t + 34.69^\circ)]\,\text{V}$

P 9.68 **[a]** Superposition must be used because the frequencies of the two sources are different.

[b] For $\omega = 16{,}000$ rad/s:

$$\frac{\mathbf{V}_o' - 10}{-j100} + \frac{\mathbf{V}_o'}{j400} + \frac{\mathbf{V}_o'}{400} = 0$$

$$\mathbf{V}_o'\left(\frac{1}{-j100} + \frac{1}{j400} + \frac{1}{400}\right) = \frac{10}{-j100}$$

$\therefore \quad \mathbf{V}_o' = 12 + j4\,\text{V}$

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$$\mathbf{I}'_o = \frac{\mathbf{V}'_o}{400} = 10 - j30\,\text{mA} = 31.62\underline{/-71.57^\circ}\,\text{mA}$$

For $\omega = 4000$ rad/s:

$$\frac{\mathbf{V}''_o}{-j400} + \frac{\mathbf{V}''_o}{j100} + \frac{\mathbf{V}''_o - 20}{400} = 0$$

$$\mathbf{V}''_o(j - j4 + 1) = 20 \qquad \text{so} \qquad \mathbf{V}''_o = \frac{20}{1 - j3} = 2 + j6\,\text{V}$$

$$\therefore \quad \mathbf{I}''_o = \frac{\mathbf{V}''}{j100} = \frac{2 + j6}{-j100} = 60 - j20\,\text{mA} = 63.25\underline{/-18.43^\circ}\,\text{mA}$$

Thus,

$$i_o(t) = [31.26\cos(16{,}000t - 71.57^\circ) + 63.25\cos(4000t - 18.43^\circ)]\,\text{mA}$$

P 9.69 $\mathbf{V}_g = 20\underline{/0^\circ}\,\text{V}; \qquad \dfrac{1}{j\omega C} = -j400\,\Omega$

Let $\mathbf{V}_\text{a}$ = voltage across the capacitor, positive at upper terminal
Then:

$$\frac{\mathbf{V}_\text{a} - 20\underline{/0^\circ}}{400} + \frac{\mathbf{V}_\text{a}}{-j400} + \frac{\mathbf{V}_\text{a}}{400} = 0; \qquad \therefore \quad \mathbf{V}_\text{a} = (8 - j4)\,\text{V}$$

$$\frac{0 - \mathbf{V}_\text{a}}{400} + \frac{0 - \mathbf{V}_o}{200} = 0; \qquad \mathbf{V}_o = -\frac{\mathbf{V}_\text{a}}{2}$$

$$\therefore \quad \mathbf{V}_o = -4 + j2 = 4.47\underline{/153.43^\circ}\,\text{V}$$

$$v_o = 4.47\cos(5000t + 153.43^\circ)\,\text{V}$$

P 9.70 **[a]**

$$\frac{\mathbf{V}_\text{a} - 20\underline{/0^\circ}}{400} + j\omega C_o\mathbf{V}_\text{a} + \frac{\mathbf{V}_\text{a}}{400} = 0$$

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$$\mathbf{V}_{\mathrm{a}} = \frac{20}{2 + j400\omega C_o}$$

$$\mathbf{V}_o = -\frac{\mathbf{V}_{\mathrm{a}}}{2}$$

$$\mathbf{V}_o = \frac{-10}{2 + j2 \times 10^6 C_o} = \frac{10\underline{/180^\circ}}{2 + j2 \times 10^6 C_o}$$

$\therefore$ denominator angle $= 45^\circ$

so $2 \times 10^6 C_o = 2 \quad \therefore \quad C = 1\,\mu\mathrm{F}$

[b] $\mathbf{V}_o = \dfrac{10\underline{/180^\circ}}{2 + j2} = 3.54\underline{/135^\circ}\,\mathrm{V}$

$$v_o = 3.54\cos(5000t + 135^\circ)\,\mathrm{V}$$

P 9.71 **[a]** $\mathbf{V}_g = 25\underline{/0^\circ}\,\mathrm{V}$

$$\mathbf{V}_{\mathrm{p}} = \frac{20}{100}\mathbf{V}_g = 5\underline{/0^\circ}; \qquad \mathbf{V}_{\mathrm{n}} = \mathbf{V}_{\mathrm{p}} = 5\underline{/0^\circ}\,\mathrm{V}$$

$$\frac{5}{80{,}000} + \frac{5 - \mathbf{V}_o}{Z_{\mathrm{p}}} = 0$$

$$Z_{\mathrm{p}} = -j80{,}000 \| 40{,}000 = 32{,}000 - j16{,}000\,\Omega$$

$$\mathbf{V}_o = \frac{5Z_{\mathrm{p}}}{80{,}000} + 5 = 7 - j = 7.07\underline{/-8.13^\circ}$$

$$v_o = 7.07\cos(50{,}000t - 8.13^\circ)\,\mathrm{V}$$

[b] $\mathbf{V}_{\mathrm{p}} = 0.2V_m\underline{/0^\circ}; \qquad \mathbf{V}_{\mathrm{n}} = \mathbf{V}_{\mathrm{p}} = 0.2V_m\underline{/0^\circ}$

$$\frac{0.2V_m}{80{,}000} + \frac{0.2V_m - \mathbf{V}_o}{32{,}000 - j16{,}000} = 0$$

$$\therefore \quad \mathbf{V}_o = 0.2V_m + \frac{32{,}000 - j16{,}000}{80{,}000}V_m(0.2) = V_m(0.28 - j0.04)$$

$$\therefore \quad |V_m(0.28 - j0.04)| \le 10$$

$$\therefore \quad V_m \le 35.36\,\mathrm{V}$$

P 9.72 $\dfrac{1}{j\omega C_1} = -j10\,\mathrm{k}\Omega$

$$\frac{1}{j\omega C_2} = -j100\,\mathrm{k}\Omega$$

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$$\frac{\mathbf{V}_\mathrm{a}-2}{5000}+\frac{\mathbf{V}_\mathrm{a}}{-j10{,}000}+\frac{\mathbf{V}_\mathrm{a}}{20{,}000}+\frac{\mathbf{V}_\mathrm{a}-\mathbf{V}_o}{100{,}000}=0$$

$$20\mathbf{V}_\mathrm{a}-40+j10\mathbf{V}_\mathrm{a}+5\mathbf{V}_\mathrm{a}+\mathbf{V}_\mathrm{a}-\mathbf{V}_o=0$$

$$\therefore\qquad (26+j10)\mathbf{V}_\mathrm{a}-\mathbf{V}_o=40$$

$$\frac{0-\mathbf{V}_\mathrm{a}}{20{,}000}+\frac{0-\mathbf{V}_o}{-j100{,}000}=0$$

$$j5\mathbf{V}_\mathrm{a}-\mathbf{V}_o=0$$

Solving,

$$\mathbf{V}_o=1.43+j7.42=7.56\underline{/79.09^\circ}\ \mathrm{V}$$

$$v_o(t)=7.56\cos(10^6t+79.09^\circ)\ \mathrm{V}$$

P 9.73 **[a]** $\dfrac{1}{j\omega C}=\dfrac{-j10^9}{(10^6)(10)}=-j100\,\Omega$

$$\mathbf{V}_g=30\underline{/0^\circ}\ \mathrm{V}$$

$$\mathbf{V}_\mathrm{p}=\frac{\mathbf{V}_g(1/j\omega C_o)}{25+(1/j\omega C_o)}=\frac{30\underline{/0^\circ}}{1+j25\omega C_o}=\mathbf{V}_\mathrm{n}$$

$$\frac{\mathbf{V}_\mathrm{n}}{100}+\frac{\mathbf{V}_\mathrm{n}-\mathbf{V}_o}{-j100}=0$$

$$\mathbf{V}_o=\frac{1+j1}{j}\mathbf{V}_\mathrm{n}=(1-j1)\mathbf{V}_\mathrm{n}=\frac{30(1-j1)}{1+j25\omega C_o}$$

$$|\mathbf{V}_o|=\frac{30\sqrt{2}}{\sqrt{1+625\omega^2C_o^2}}=6$$

Solving,

$$C_o=280\,\mathrm{nF}$$

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[b] $\mathbf{V}_o = \dfrac{30(1-j1)}{1+j7} = 6\underline{/-126.87^\circ}$

$$v_o = 6\cos(10^6 t - 126.87^\circ)\,\text{V}$$

P 9.74 $j\omega L_1 = j50\,\Omega$

$$j\omega L_2 = j32\,\Omega$$

$$\frac{1}{j\omega C} = -j20\,\Omega$$

$$j\omega M = j(4\times 10^3)k\sqrt{(12.5)(8)}\times 10^{-3} = j40k\,\Omega$$

$$Z_{22} = 5 + j32 - j20 = 5 + j12\,\Omega$$

$$Z_{22}^* = 5 - j12\,\Omega$$

$$Z_r = \left[\frac{40k}{|5+j12|}\right]^2 (5-j12) = 47.337k^2 - j113.609k^2$$

$$Z_{\text{ab}} = 20 + j50 + 47.337k^2 - j113.609k^2 = (20 + 47.337k^2) + j(50 - 113.609k^2)$$

Z_{ab} is resistive when

$$50 - 113.609k^2 = 0 \quad \text{or} \quad k^2 = 0.44 \quad \text{so} \quad k = 0.66$$

$$\therefore \quad Z_{\text{ab}} = 20 + (47.337)(0.44) = 40.83\,\Omega$$

P 9.75 Remove the voltage source to find the equivalent impedance:

$$Z_{\text{Th}} = 45 + j125 + \left(\frac{20}{|5+j5|}\right)^2 (5-j5) = 85 + j85\,\Omega$$

Using voltage division:

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{cd}} = j20\mathbf{I}_1 = j20\left(\frac{425}{5+j5}\right) = 850 + j850\,\text{V}$$

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P 9.76 **[a]** $j\omega L_1 = j(800)(100 \times 10^{-3}) = j80\,\Omega$

$j\omega L_2 = j(800)(400 \times 10^{-3}) = j320\,\Omega$

$j\omega M = j80\,\Omega$

$168 = (80 + j80)\mathbf{I}_g + j80\mathbf{I}_L$

$0 = j80\mathbf{I}_g + (240 + j320)\mathbf{I}_L$

Solving,

$\mathbf{I}_g = 1.2 - j0.9\,\text{A}; \qquad \mathbf{I}_L = -0.3\,\text{A}$

$i_g = 1.5\cos(800t - 36.87°)\,\text{A}$

$i_L = 0.3\cos(5000t - 180°)\,\text{A}$

[b] $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{0.1}{\sqrt{(0.1)(0.4)}} = 0.5$

[c] When $t = 625\pi\ \mu\text{s}$,

$800t = (800)(625\pi) \times 10^{-6} = 0.5\pi = \pi/2\,\text{rad} = 90°$

$i_g(625\pi\mu s) = 1.5\cos(53.13°) = 0.9\,\text{A}$

$i_L(625\pi\mu s) = 0.3\cos(-90°) = 0\,\text{A}$

$w = \dfrac{1}{2}L_1 i_g^2 + \dfrac{1}{2}L_2 i_L^2 + M i_g i_L = \dfrac{1}{2}(100 \times 10^{-3})(0.81) + 0 + 0 = 40.5\,\text{mJ}$

When $t = 1250\pi\ \mu\text{s}$,

$800t = \pi\,\text{rad} = 180°$

$i_g(1250\pi\mu s) = 1.5\cos(180 - 36.87) = -1.2\,\text{A}$

$i_L(1250\pi\mu s) = 0.3\cos(180 - 180) = 0.3\,\text{A}$

$w = \dfrac{1}{2}(100 \times 10^{-3})(1.44) + \dfrac{1}{2}(400 \times 10^{-3})(0.09)$

$\quad +100 \times 10^{-3}(-1.2)(0.3) = 54\,\text{mJ}$

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P 9.77 **[a]** $j\omega L_1 = j(200 \times 10^3)(10^{-3}) = j200\,\Omega$

$$j\omega L_2 = j(200 \times 10^3)(4 \times 10^{-3}) = j800\,\Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{(200 \times 10^3)(12.5 \times 10^{-9})} = -j400\,\Omega$$

$$\therefore \quad Z_{22} = 100 + 200 + j800 - j400 = 300 + j400\,\Omega$$

$$\therefore \quad Z_{22}^* = 300 - j400\,\Omega$$

$$M = k\sqrt{L_1 L_2} = 2k \times 10^{-3}$$

$$\omega M = (200 \times 10^3)(2k \times 10^{-3}) = 400k$$

$$Z_r = \left[\frac{400k}{500}\right]^2 (300 - j400) = k^2(192 - j256)\,\Omega$$

$$Z_{\text{in}} = 200 + j200 + 192k^2 - j256k^2$$

$$|Z_{\text{in}}| = [(200 + 192k)^2 + (200 - 256k)^2]^{\frac{1}{2}}$$

$$\frac{d|Z_{\text{in}}|}{dk} = \frac{1}{2}[(200 + 192k)^2 + (200 - 256k)^2]^{-\frac{1}{2}} \times$$

$$[2(200 + 192k^2)384k + 2(200 - 256k^2)(-512k)]$$

$$\frac{d|Z_{\text{in}}|}{dk} = 0 \text{ when}$$

$$768k(200 + 192k^2) - 1024k(200 - 256k^2) = 0$$

$$\therefore \quad k^2 = 0.125; \qquad \therefore \quad k = \sqrt{0.125} = 0.3536$$

[b] Z_{in} (min) $= 200 + 192(0.125) + j[200 - 0.125(256)]$

$$= 224 + j168 = 280\underline{/36.87^\circ}\,\Omega$$

$$\mathbf{I}_1 \text{ (max)} = \frac{560\underline{/0^\circ}}{224 + j168} = 2\underline{/-36.87^\circ}\,\text{A}$$

$$\therefore \quad i_1 \text{ (peak)} = 2\,\text{A}$$

Note — You can test that the k value obtained from setting $d|Z_{\text{in}}|/dt = 0$ leads to a minimum by noting $0 \le k \le 1$. If $k = 1$,

$$Z_{\text{in}} = 392 - j56 = 395.98\underline{/-8.13^\circ}\,\Omega$$

Thus,

$$|Z_{\text{in}}|_{k=1} > |Z_{\text{in}}|_{k=\sqrt{0.125}}$$

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If $k = 0$,

$Z_{\text{in}} = 200 + j200 = 282.84\underline{/45^\circ}\,\Omega$

Thus,

$|Z_{\text{in}}|_{k=0} > |Z_{\text{in}}|_{k=\sqrt{0.125}}$

P 9.78 **[a]** $j\omega L_L = j20\,\Omega$

$j\omega L_2 = j100\,\Omega$

$Z_{22} = 100 + 60 + j20 + j100 = 160 + j120\,\Omega$

$Z_{22}^* = 160 - j120\,\Omega$

$\omega M = 54\,\Omega$

$$Z_r = \left(\frac{54}{200}\right)^2 [160 - j120] = 11.66 - j8.75\,\Omega$$

[b] $Z_{\text{ab}} = R_1 + j\omega L_1 + Z_r = 8.34 + j36 + 11.66 - j8.75 = 20 + j27.25\,\Omega$

P 9.79 In Eq. 9.69 replace $\omega^2 M^2$ with $k^2\omega^2 L_1 L_2$ and then write X_{ab} as

$$X_{\text{ab}} = \omega L_1 - \frac{k^2\omega^2 L_1 L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}$$

$$= \omega L_1 \left\{1 - \frac{k^2\omega L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}\right\}$$

For X_{ab} to be negative requires

$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 < k^2\omega L_2(\omega L_2 + \omega L_L)$$

or

$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 - k^2\omega L_2(\omega L_2 + \omega L_L) < 0$$

which reduces to

$$R_{22}^2 + \omega^2 L_2^2(1 - k^2) + \omega L_2 \omega L_L(2 - k^2) + \omega^2 L_L^2 < 0$$

But $k \le 1$, so it is impossible to satisfy the inequality. Therefore X_{ab} can never be negative if X_L is an inductive reactance.

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P 9.80 **[a]**

$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_1} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_2 = \frac{N_2}{N_1}\mathbf{V}_1$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\mathbf{V}_2 = (\mathbf{I}_1 + \mathbf{I}_2)Z_L = \mathbf{I}_1\left(1 + \frac{N_1}{N_2}\right)Z_L$$

$$\mathbf{V}_1 + \mathbf{V}_2 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2 = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\text{ab}} = \frac{(1 + N_1/N_2)^2 Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L \quad \text{Q.E.D.}$$

[b] Assume dot on N_2 is moved to the lower terminal, then

$$\frac{\mathbf{V}_1}{N_1} = \frac{-\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_1 = \frac{-N_1}{N_2}\mathbf{V}_2$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{-N_1}{N_2}\mathbf{I}_1$$

As in part [a]

$$\mathbf{V}_2 = (\mathbf{I}_2 + \mathbf{I}_1)Z_L \quad \text{and} \quad Z_{\text{ab}} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = \frac{(1 - N_1/N_2)\mathbf{V}_2}{\mathbf{I}_1} = \frac{(1 - N_1/N_2)(1 - N_1/N_2)Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = [1 - (N_1/N_2)]^2 Z_L \quad \text{Q.E.D.}$$

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P 9.81 **[a]**

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{(1 + N_1/N_2)\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_2}, \qquad \mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2$$

$$\mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2$$

$$Z_{\text{ab}} = \frac{\mathbf{I}_1 Z_L}{(N_1/N_2 + 1)(1 + N_1/N_2)\mathbf{I}_1}$$

$$\therefore \quad Z_{\text{ab}} = \frac{Z_L}{[1 + (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

[b] Assume dot on the N_2 coil is moved to the lower terminal. Then

$$\mathbf{V}_1 = -\frac{N_1}{N_2}\mathbf{V}_2 \quad \text{and} \quad \mathbf{I}_2 = -\frac{N_1}{N_2}\mathbf{I}_1$$

As before

$$Z_{\text{ab}} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} \quad \text{and} \quad \mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\text{ab}} = \frac{\mathbf{V}_2}{(1 - N_1/N_2)\mathbf{I}_1} = \frac{Z_L\mathbf{I}_1}{[1 - (N_1/N_2)]^2\mathbf{I}_1}$$

$$Z_{\text{ab}} = \frac{Z_L}{[1 - (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

P 9.82

$$Z_L = \frac{\mathbf{V}_3}{\mathbf{I}_3}$$

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$$\frac{\mathbf{V}_2}{1} = \frac{-\mathbf{V}_3}{20}; \qquad 1\mathbf{I}_2 = -20\mathbf{I}_3$$

$$\frac{\mathbf{V}_1}{50} = \frac{\mathbf{V}_2}{1}; \qquad 50\mathbf{I}_1 = 1\mathbf{I}_2$$

$$Z_{\text{ab}} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

Substituting,

$$Z_{\text{ab}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{50\mathbf{V}_2}{\mathbf{I}_2/50} = \frac{50^2\mathbf{V}_2}{\mathbf{I}_2}$$

$$= \frac{50^2(-\mathbf{V}_3/20)}{-20\mathbf{I}_3} = \frac{(50)^2\mathbf{V}_3}{(20)^2\mathbf{I}_3} = 6.25Z_L = 6.25(200\underline{/-45^\circ}) = 1250\underline{/-45^\circ}\,\Omega$$

P 9.83 The phasor domain equivalent circuit is

$$V_o = \frac{V_m}{2} - \mathbf{I}R_x; \qquad \mathbf{I} = \frac{V_m}{R_x - jX_C}$$

As R_x varies from 0 to ∞, the amplitude of v_o remains constant and its phase angle increases from 0° to -180°, as shown in the following phasor diagram:

P 9.84 [a] $\mathbf{I} = \dfrac{240}{24} + \dfrac{240}{j32} = (10 - j7.5)\text{ A}$

$$\mathbf{V}_s = 240\underline{/0^\circ} + (0.1 + j0.8)(10 - j7.5) = 247 + j7.25 = 247.11\underline{/1.68^\circ}\text{ V}$$

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[b] Use the capacitor to eliminate the j component of $\mathbf{I}$, therefore

$$\mathbf{I}_c = j7.5\,\text{A}, \qquad Z_c = \frac{240}{j7.5} = -j32\,\Omega$$

$$\mathbf{V}_s = 240 + (0.1 + j0.8)10 = 241 + j8 = 241.13\underline{/1.90^\circ}\,\text{V}$$

[c] Let I_c denote the magnitude of the current in the capacitor branch. Then

$$\mathbf{I} = (10 - j7.5 + jI_c) = 10 + j(I_c - 7.5)\,\text{A}$$

$$\begin{aligned}\mathbf{V}_s &= 240\underline{/\alpha} = 240 + (0.1 + j0.8)[10 + j(I_c - 7.5)] \\ &= (247 - 0.8I_c) + j(7.25 + 0.1I_c)\end{aligned}$$

It follows that

$$240\cos\alpha = (247 - 0.8I_c) \quad \text{and} \quad 240\sin\alpha = (7.25 + 0.1I_c)$$

Now square each term and then add to generate the quadratic equation

$$I_c^2 - 605.77I_c + 5325.48 = 0; \qquad I_c = 302.88 \pm 293.96$$

Therefore

$I_c = 8.92\,\text{A}$ (smallest value) and $Z_c = 240/j8.92 = -j26.90\,\Omega$.

Therefore, the capacitive reactance is $-26.90\,\Omega$.

P 9.85 **[a]**

$$\mathbf{I}_\ell = \frac{240}{8} + \frac{240}{j6} = 30 - j40\,\text{A}$$

$$\mathbf{V}_\ell = (0.1 + j0.8)(30 - j40) = 35 + j20 = 40.31\underline{/29.74^\circ}\,\text{V}$$

$$\mathbf{V}_s = 240\underline{/0^\circ} + \mathbf{V}_\ell = 275 + j20 = 275.73\underline{/4.16^\circ}\,\text{V}$$

[b]

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[c] $\mathbf{I}_\ell = 30 - j40 + \dfrac{240}{-j5} = 30 + j8\,\text{A}$

$$\mathbf{V}_\ell = (0.1 + j0.8)(30 + j8) = -3.4 + j24.8 = 25.03\underline{/97.81^\circ}$$

$$\mathbf{V}_\text{s} = 240\underline{/0^\circ} + \mathbf{V}_\ell = 236.6 + j24.8 = 237.9\underline{/5.98^\circ}$$

P 9.86 **[a]** $\mathbf{I}_1 = \dfrac{120}{24} + \dfrac{240}{8.4 + j6.3} = 23.29 - j13.71 = 27.02\underline{/-30.5^\circ}\,\text{A}$

$$\mathbf{I}_2 = \frac{120}{12} - \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_3 = \frac{120}{12} + \frac{240}{8.4 + j6.3} = 28.29 - j13.71 = 31.44\underline{/-25.87^\circ}\,\text{A}$$

$$\mathbf{I}_4 = \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_5 = \frac{120}{12} = 10\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_6 = \frac{240}{8.4 + j6.3} = 18.29 - j13.71 = 22.86\underline{/-36.87^\circ}\,\text{A}$$

[b] When fuse A is interrupted,

$\mathbf{I}_1 = 0$ $\qquad \mathbf{I}_3 = 15\,\text{A}$ $\qquad \mathbf{I}_5 = 10\,\text{A}$

$\mathbf{I}_2 = 10 + 5 = 15\,\text{A}$ $\qquad \mathbf{I}_4 = -5\,\text{A}$ $\qquad \mathbf{I}_6 = 5\,\text{A}$

[c] The clock and television set were fed from the uninterrupted side of the circuit, that is, the $12\,\Omega$ load includes the clock and the TV set.

[d] No, the motor current drops to 5 A, well below its normal running value of 22.86 A.

[e] After fuse A opens, the current in fuse B is only 15 A.

P 9.87 [a] The circuit is redrawn, with mesh currents identified:

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The mesh current equations are:

$$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120\underline{/0^\circ} = -2\mathbf{I}_a + 43\mathbf{I}_b - 40\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 40\mathbf{I}_b + 70\mathbf{I}_c$$

Solving,

$$\mathbf{I}_a = 24\underline{/0^\circ}\text{ A} \qquad \mathbf{I}_b = 21.96\underline{/0^\circ}\text{ A} \qquad \mathbf{I}_c = 19.40\underline{/0^\circ}\text{ A}$$

The branch currents are:

$$\mathbf{I}_1 = \mathbf{I}_a = 24\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 2.04\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_3 = \mathbf{I}_b = 21.96\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_4 = \mathbf{I}_c = 19.40\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_5 = \mathbf{I}_a - \mathbf{I}_c = 4.6\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_6 = \mathbf{I}_b - \mathbf{I}_c = 2.55\underline{/0^\circ}\text{ A}$$

[b] Let N_1 be the number of turns on the primary winding; because the secondary winding is center-tapped, let $2N_2$ be the total turns on the secondary. From Fig. 9.58,

$$\frac{13{,}200}{N_1} = \frac{240}{2N_2} \qquad \text{or} \qquad \frac{N_2}{N_1} = \frac{1}{110}$$

The ampere turn balance requires

$$N_1\mathbf{I}_p = N_2\mathbf{I}_1 + N_2\mathbf{I}_3$$

Therefore,

$$\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{1}{110}(24 + 21.96) = 0.42\underline{/0^\circ}\text{ A}$$

P 9.88 **[a]**

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The three mesh current equations are

$$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120\underline{/0^\circ} = -2\mathbf{I}_a + 23\mathbf{I}_b - 20\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 20\mathbf{I}_b + 50\mathbf{I}_c$$

Solving,

$$\mathbf{I}_a = 24\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_b = 24\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_c = 19.2\underline{/0^\circ}\,\text{A}$$

$$\therefore \quad \mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 0\,\text{A}$$

[b] $$\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{N_2}{N_1}(\mathbf{I}_a + \mathbf{I}_b)$$

$$= \frac{1}{110}(24 + 24) = 0.436\underline{/0^\circ}\,\text{A}$$

[c] Yes; when the two 120 V loads are equal, there is no current in the "neutral" line, so no power is lost to this line. Since you pay for power, the cost is lower when the loads are equal.

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P 9.89 **[a]**

$$125 = (R + 0.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - R\mathbf{I}_3$$

$$125 = -(0.03 + j0.03)\mathbf{I}_1 + (R + 0.05 + j0.05)\mathbf{I}_2 - R\mathbf{I}_3$$

Subtracting the above two equations gives

$$0 = (R + 0.08 + j0.08)\mathbf{I}_1 - (R + 0.08 + j0.08)\mathbf{I}_2$$

$$\therefore \quad \mathbf{I}_1 = \mathbf{I}_2 \quad \text{so} \quad \mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = 0\,\text{A}$$

[b] $\mathbf{V}_1 = R(\mathbf{I}_1 - \mathbf{I}_3); \qquad \mathbf{V}_2 = R(\mathbf{I}_2 - \mathbf{I}_3)$

Since $\mathbf{I}_1 = \mathbf{I}_2$ (from part [a]) $\mathbf{V}_1 = \mathbf{V}_2$

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[c]

$250 = (440.04 + j0.04)\mathbf{I}_\mathrm{a} - 440\mathbf{I}_\mathrm{b}$

$0 = -440\mathbf{I}_\mathrm{a} + 448\mathbf{I}_\mathrm{b}$

Solving,

$\mathbf{I}_\mathrm{a} = 31.656207 - j0.160343\,\mathrm{A}$

$\mathbf{I}_\mathrm{b} = 31.090917 - j0.157479\,\mathrm{A}$

$\mathbf{I}_1 = \mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b} = 0.56529 - j0.002864\,\mathrm{A}$

$\mathbf{V}_1 = 40\mathbf{I}_1 = 22.612 - j0.11456 = 22.612\underline{/-0.290282^\circ}\,\mathrm{V}$

$\mathbf{V}_2 = 400\mathbf{I}_1 = 226.116 - j1.1456 = 226.1189\underline{/-0.290282^\circ}\,\mathrm{V}$

[d]

$125 = (40.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - 40\mathbf{I}_3$

$125 = -(0.03 + j0.03)\mathbf{I}_1 + (400.05 + j0.05)\mathbf{I}_2 - 400\mathbf{I}_3$

$0 = -40\mathbf{I}_1 - 400\mathbf{I}_2 + 448\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 34.19 - j0.182\,\mathrm{A}$

$\mathbf{I}_2 = 31.396 - j0.164\,\mathrm{A}$

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$$\mathbf{I}_3 = 31.085 - j0.163\,\text{A}$$

$$\mathbf{V}_1 = 40(\mathbf{I}_1 - \mathbf{I}_3) = 124.2\underline{/-0.35^\circ}\,\text{V}$$

$$\mathbf{V}_2 = 400(\mathbf{I}_2 - \mathbf{I}_3) = 124.4\underline{/-0.18^\circ}\,\text{V}$$

[e] Because an open neutral can result in severely unbalanced voltages across the 125 V loads.

P 9.90 **[a]** Let N_1 = primary winding turns and $2N_2$ = secondary winding turns. Then

$$\frac{14{,}000}{N_1} = \frac{250}{2N_2}; \qquad \therefore \quad \frac{N_2}{N_1} = \frac{1}{112} = a$$

In part c),

$$\mathbf{I}_\text{p} = 2a\mathbf{I}_\text{a}$$

$$\therefore \quad \mathbf{I}_\text{p} = \frac{2N_2\mathbf{I}_\text{a}}{N_1} = \frac{1}{56}\mathbf{I}_\text{a}$$

$$= \frac{1}{56}(31.656 - j0.16)$$

$$\mathbf{I}_\text{p} = 565.3 - j2.9\,\text{mA}$$

In part d),

$$\mathbf{I}_\text{p}N_1 = \mathbf{I}_1N_2 + \mathbf{I}_2N_2$$

$$\therefore \quad \mathbf{I}_\text{p} = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_2)$$

$$= \frac{1}{112}(34.19 - j0.182 + 31.396 - j0.164)$$

$$= \frac{1}{112}(65.586 - j0.346)$$

$$\mathbf{I}_\text{p} = 585.6 - j3.1\,\text{mA}$$

[b] Yes, because the neutral conductor carries non-zero current whenever the load is not balanced.

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10

Sinusoidal Steady State Power Calculations

Assessment Problems

AP 10.1 [a] $\mathbf{V} = 100\underline{/-45^\circ}\,\text{V}, \qquad \mathbf{I} = 20\underline{/15^\circ}\,\text{A}$

Therefore

$$P = \frac{1}{2}(100)(20)\cos[-45-(15)] = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin -60^\circ = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/165^\circ}$

$$P = 1000\cos(-210^\circ) = -866.03\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-210^\circ) = 500\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/-105^\circ}$

$$P = 1000\cos(60^\circ) = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin(60^\circ) = 866.03\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[d] $\mathbf{V} = 100\underline{/0^\circ}, \qquad \mathbf{I} = 20\underline{/120^\circ}$

$$P = 1000\cos(-120^\circ) = -500\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-120^\circ) = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

AP 10.2

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos[15-(75)] = \cos(-60^\circ) = 0.5 \text{ leading}$$

$$\text{rf} = \sin(\theta_v - \theta_i) = \sin(-60^\circ) = -0.866$$

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AP 10.3

From Ex. 9.4 $\quad I_{\text{eff}} = \dfrac{I_\rho}{\sqrt{3}} = \dfrac{0.18}{\sqrt{3}}\,\text{A}$

$$P = I_{\text{eff}}^2 R = \left(\frac{0.0324}{3}\right)(5000) = 54\,\text{W}$$

AP 10.4 **[a]** $Z = (39 + j26)\|(-j52) = 48 - j20 = 52\underline{/-22.62^\circ}\,\Omega$

Therefore $\quad \mathbf{I}_\ell = \dfrac{250\underline{/0^\circ}}{48 - j20 + 1 + j4} = 4.85\underline{/18.08^\circ}\,\text{A (rms)}$

$$\mathbf{V}_\text{L} = Z\mathbf{I}_\ell = (52\underline{/-22.62^\circ})(4.85\underline{/18.08^\circ}) = 252.20\underline{/-4.54^\circ}\,\text{V (rms)}$$

$$\mathbf{I}_\text{L} = \frac{\mathbf{V}_\text{L}}{39 + j26} = 5.38\underline{/-38.23^\circ}\,\text{A (rms)}$$

[b] $S_\text{L} = \mathbf{V}_L\mathbf{I}_L^* = (252.20\underline{/-4.54^\circ})(5.38\underline{/+38.23^\circ}) = 1357\underline{/33.69^\circ}$

$= (1129.09 + j752.73)\,\text{VA}$

$P_\text{L} = 1129.09\,\text{W}; \qquad Q_\text{L} = 752.73\ \text{VAR}$

[c] $P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52\,\text{W}; \qquad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09\ \text{VAR}$

[d] $S_g(\text{delivering}) = 250\mathbf{I}_\ell^* = (1152.62 - j376.36)\,\text{VA}$

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

[e] $Q_\text{cap} = \dfrac{|\mathbf{V}_\text{L}|^2}{-52} = \dfrac{(252.20)^2}{-52} = -1223.18\ \text{VAR}$

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

Check: $\quad 94.09 + 752.73 + 376.36 = 1223.18\ \text{VAR}$ and

$1129.09 + 23.52 = 1152.62\,\text{W}$

AP 10.5 Series circuit derivation:

$$S = 250\mathbf{I}^* = (40{,}000 - j30{,}000)$$

Therefore $\quad \mathbf{I}^* = 160 - j120 = 200\underline{/-36.87^\circ}\,\text{A (rms)}$

$$\mathbf{I} = 200\underline{/36.87^\circ}\,\text{A (rms)}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{250}{200\underline{/36.87^\circ}} = 1.25\underline{/-36.87^\circ} = (1 - j0.75)\,\Omega$$

Therefore $\quad R = 1\,\Omega, \qquad X_\text{C} = -0.75\,\Omega$

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Parallel circuit derivation

$$P = \frac{(250)^2}{R}; \qquad \text{therefore} \quad R = \frac{(250)^2}{40{,}000} = 1.5625\,\Omega$$

$$Q = \frac{(250)^2}{X_{\mathrm{C}}}; \qquad \text{therefore} \quad X_{\mathrm{C}} = \frac{(250)^2}{-30{,}000} = -2.083\,\Omega$$

AP 10.6

$$S_1 = 15{,}000(0.6) + j15{,}000(0.8) = 9000 + j12{,}000\,\mathrm{VA}$$

$$S_2 = 6000(0.8) - j6000(0.6) = 4800 - j3600\,\mathrm{VA}$$

$$S_T = S_1 + S_2 = 13{,}800 + j8400\,\mathrm{VA}$$

$$S_T = 200\mathbf{I}^*; \qquad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \qquad \mathbf{I} = 69 - j42\,\mathrm{A}$$

$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64\underline{/15.91^\circ}\,\mathrm{V\ (rms)}$$

AP 10.7 **[a]** The phasor domain equivalent circuit and the Thévenin equivalent are shown below:

Phasor domain equivalent circuit:

Thévenin equivalent:

$$\mathbf{V}_{\mathrm{Th}} = 3\frac{-j800}{20 - j40} = 48 - j24 = 53.67\underline{/-26.57^\circ}\,\mathrm{V}$$

$$Z_{\mathrm{Th}} = 4 + j18 + \frac{-j800}{20 - j40} = 20 + j10 = 22.36\underline{/26.57^\circ}\,\Omega$$

For maximum power transfer, $\quad Z_{\mathrm{L}} = (20 - j10)\,\Omega$

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[b] $\mathbf{I} = \dfrac{53.67\underline{/-26.57^\circ}}{40} = 1.34\underline{/-26.57^\circ}\,\text{A}$

Therefore $\quad P = \left(\dfrac{1.34}{\sqrt{2}}\right)^2 20 = 17.96\,\text{W}$

[c] $R_\text{L} = |Z_\text{Th}| = 22.36\,\Omega$

[d] $\mathbf{I} = \dfrac{53.67\underline{/-26.57^\circ}}{42.36 + j10} = 1.23\underline{/-39.85^\circ}\,\text{A}$

Therefore $\quad P = \left(\dfrac{1.23}{\sqrt{2}}\right)^2 (22.36) = 17\,\text{W}$

AP 10.8

Mesh current equations:

$$660 = (34 + j50)\mathbf{I}_1 + j100(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = j100(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 + 100\mathbf{I}_2$$

Solving,

$$\mathbf{I}_2 = 3.5\underline{/0^\circ}\,\text{A}; \qquad \therefore \quad P = \frac{1}{2}(3.5)^2(100) = 612.50\,\text{W}$$

AP 10.9 **[a]**

$$248 = j400\mathbf{I}_1 - j500\mathbf{I}_2 + 375(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = 375(\mathbf{I}_2 - \mathbf{I}_1) + j1000\mathbf{I}_2 - j500\mathbf{I}_1 + 400\mathbf{I}_2$$

Solving,

$$\mathbf{I}_1 = 0.80 - j0.62\,\text{A}; \qquad \mathbf{I}_2 = 0.4 - j0.3 = 0.5\underline{/-36.87^\circ}$$

$$\therefore \quad P = \frac{1}{2}(0.25)(400) = 50\,\text{W}$$

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[b] $\mathbf{I}_1 - \mathbf{I}_2 = 0.4 - j0.32\,\text{A}$

$$P_{375} = \frac{1}{2}|\mathbf{I}_1 - \mathbf{I}_2|^2(375) = 49.20\,\text{W}$$

[c] $P_g = \frac{1}{2}(248)(0.8) = 99.20\,\text{W}$

$$\sum P_{\text{abs}} = 50 + 49.2 = 99.20\,\text{W} \quad \text{(checks)}$$

AP 10.10 **[a]** $V_{\text{Th}} = 210\,\text{V}; \qquad \mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_1 = \frac{1}{4}\mathbf{I}_2$

Short circuit equations:

$$840 = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = 14\,\text{A}; \qquad R_{\text{Th}} = \frac{210}{14} = 15\,\Omega$$

[b] $P_{\max} = \left(\frac{210}{30}\right)^2 15 = 735\,\text{W}$

AP 10.11 **[a]** $\mathbf{V}_{\text{Th}} = -4(146\underline{/0^\circ}) = -584\underline{/0^\circ}\,\text{V (rms)}$

$\mathbf{V}_2 = 4\mathbf{V}_1; \qquad \mathbf{I}_1 = -4\mathbf{I}_2$

Short circuit equations:

$$146\underline{/0^\circ} = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = -146/365 = -0.40\,\text{A}; \qquad R_{\text{Th}} = \frac{-584}{-0.4} = 1460\,\Omega$$

[b] $P = \left(\frac{-584}{2920}\right)^2 1460 = 58.40\,\text{W}$

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Problems

P 10.1 **[a]** $P = \frac{1}{2}(250)(4)\cos(45+30) = 500\cos 75^\circ = 129.41\,\text{W}$ (abs)

$Q = 500\sin 75^\circ = 482.96\ \text{VAR}$ (abs)

[b] $P = \frac{1}{2}(18)(5)\cos(30+75) = 45\cos(105^\circ) = -11.65\,\text{W}$ (del)

$Q = 45\sin(105^\circ) = 43.47\ \text{VAR}$ (abs)

[c] $P = \frac{1}{2}(150)(2)\cos(-65-50) = 150\cos(-115^\circ) = -63.39\,\text{W}$ (del)

$Q = 150\sin(-115^\circ) = -135.95\ \text{VAR}$ (del)

[d] $P = \frac{1}{2}(80)(10)\cos(120-170) = 400\cos(-50^\circ) = 257.12\,\text{W}$ (abs)

$Q = 400\sin(-50^\circ) = -306.42\ \text{VAR}$ (del)

P 10.2 **[a]** coffee maker = 1200 W frying pan = 1196 W

microwave oven = 1450 W toaster = 1146 W

$\sum P = 4992\,\text{W}$

Therefore $I_{\text{eff}} = \frac{4992}{120} = 41.6\,\text{A}$

The breaker will not trip.

[b] $\sum P = 4992 + 860 + 630 = 6482\,\text{W}$; $I_{\text{eff}} = \frac{6482}{120} = 54.02\,\text{A}$

The breaker will trip because the current is greater than 20 A.

P 10.3 $p = P + P\cos 2\omega t - Q\sin 2\omega t$; $\frac{dp}{dt} = -2\omega P\sin 2\omega t - 2\omega Q\cos 2\omega t$

$\frac{dp}{dt} = 0$ when $-2\omega P\sin 2\omega t = 2\omega Q\cos 2\omega t$ or $\tan 2\omega t = -\frac{Q}{P}$

$\cos 2\omega t = \frac{P}{\sqrt{P^2+Q^2}}$; $\sin 2\omega t = -\frac{Q}{\sqrt{P^2+Q^2}}$

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Let $\theta = \tan^{-1}(-Q/P)$, then p is maximum when $2\omega t = \theta$ and p is minimum when $2\omega t = (\theta + \pi)$.

Therefore $\quad p_{\max} = P + P \cdot \dfrac{P}{\sqrt{P^2+Q^2}} - \dfrac{Q(-Q)}{\sqrt{P^2+Q^2}} = P + \sqrt{P^2+Q^2}$

and $\quad p_{\min} = P - P \cdot \dfrac{P}{\sqrt{P^2+Q^2}} - Q \cdot \dfrac{Q}{\sqrt{P^2+Q^2}} = P - \sqrt{P^2+Q^2}$

P 10.4 **[a]** $P = \dfrac{1}{2}\dfrac{(240)^2}{480} = 60\,\text{W}$

$$-\frac{1}{\omega C} = \frac{-9\times 10^6}{(5000)(5)} = -360\,\Omega$$

$$Q = \frac{1}{2}\frac{(240)^2}{(-360)} = -80\,\text{VAR}$$

$$p_{\max} = P + \sqrt{P^2+Q^2} = 60 + \sqrt{(60)^2+(80)^2} = 160\,\text{W (del)}$$

[b] $p_{\min} = 60 - \sqrt{60^2+80^2} = -40\,\text{W (abs)}$

[c] $P = 60\,\text{W}$ from (a)

[d] $Q = -80\,\text{VAR}$ from (a)

[e] generates, because $Q < 0$

[f] pf $= \cos(\theta_v - \theta_i)$

$$\mathbf{I} = \frac{240}{480} + \frac{240}{-j360} = 0.5 + j0.67 = 0.83\underline{/53.13^\circ}\,\text{A}$$

$\therefore$ pf $= \cos(0 - 53.13^\circ) = 0.6$ leading

[g] rf $= \sin(-53.13^\circ) = -0.8$

P 10.5 $\mathbf{I}_g = 4\underline{/0^\circ}\,\text{mA}; \qquad \dfrac{1}{j\omega C} = -j1250\,\Omega; \qquad j\omega L = j500\,\Omega$

$$Z_{\text{eq}} = 500 + [-j1250 \| (1000 + j500)] = 1500 - j500\,\Omega$$

$$P_g = -\frac{1}{2}|I|^2\text{Re}\{Z_{\text{eq}}\} = -\frac{1}{2}(0.004)^2(1500) = -12\,\text{mW}$$

The source delivers 12 mW of power to the circuit.

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P 10.6 $j\omega L = j20{,}000(0.5 \times 10^{-3}) = j10\,\Omega; \qquad \dfrac{1}{j\omega C} = \dfrac{10^6}{j20{,}000(1.25)} = -j40\,\Omega$

$$-6 + \frac{\mathbf{V}_o}{j10} + \frac{\mathbf{V}_o - 30(\mathbf{V}_o/j10)}{30 - j40} = 0$$

$$\therefore \quad \mathbf{V}_o\left[\frac{1}{j10} + \frac{1 + j3}{30 - j40}\right] = 6$$

$$\therefore \quad \mathbf{V}_o = 100\underline{/126.87^\circ}\ \text{V}$$

$$\therefore \quad \mathbf{I}_\Delta = \frac{\mathbf{V}_o}{j10} = 10\underline{/36.87^\circ}\ \text{A}$$

$$\mathbf{I}_o = 6\underline{/0^\circ} - \mathbf{I}_\Delta = 6 - 8 - j6 = -2 - j6 = 6.32\underline{/-108.43^\circ}\ \text{A}$$

$$P_{30\Omega} = \frac{1}{2}|\mathbf{I}_o|^2 30 = 600\,\text{W}$$

P 10.7 $Z_\text{f} = -j10{,}000 \| 20{,}000 = 4000 - j8000\,\Omega$

$$Z_\text{i} = 2000 - j2000\,\Omega$$

$$\therefore \quad \frac{Z_\text{f}}{Z_\text{i}} = \frac{4000 - j8000}{2000 - j2000} = 3 - j1$$

$$\mathbf{V}_o = -\frac{Z_\text{f}}{Z_\text{i}}\mathbf{V}_g; \qquad \mathbf{V}_g = 1\underline{/0^\circ}\ \text{V}$$

$$\mathbf{V}_o = -(3 - j1)(1) = -3 + j1 = 3.16\underline{/161.57^\circ}\ \text{V}$$

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\frac{(10)}{1000} = 5 \times 10^{-3} = 5\,\text{mW}$$

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P 10.8 **[a]** From the solution to Problem 9.63 we have:

$$\mathbf{I}_a = 67.5 - j7.5\,\text{mA} \qquad \text{and} \qquad \mathbf{I}_b = -22.5 + j22.5\,\text{mA}$$

$$S_{-j18\text{V}} = -\frac{1}{2}(-j18)(0.0675 + j0.0075) = -67.5 + j607.5 \text{ mVA}$$

$$P_{-j18\text{V}} = -67.5 \text{ mW}; \qquad \text{and} \qquad Q_{-j18\text{V}} = 607.5 \text{ mVAR}$$

$$S_{12\text{V}} = \frac{1}{2}(12)(-0.0225 - j0.0225) = -135 - j135 \text{ mVA}$$

$$P_{12\text{V}} = -135 \text{ mW}; \qquad \text{and} \qquad Q_{12V} = -135 \text{ mVAR}$$

$$P_{-j400\Omega} = 0\,\text{W}; \qquad \text{and} \qquad Q_{-j400\Omega} = -\frac{1}{2}(400)|\mathbf{I}_a|^2 = -922.5 \text{ mVAR}$$

$$P_{400\Omega} = \frac{1}{2}(400)|\mathbf{I}_b|^2 = 202.5\,\text{mW}; \qquad \text{and} \qquad Q_{400\Omega} = 0 \text{ mVAR}$$

$$P_{j100\Omega} = 0\,\text{W}; \qquad \text{and} \qquad Q_{j100\Omega} = \frac{1}{2}(100)|\mathbf{I}_a - \mathbf{I}_b|^2 = 450 \text{ mVAR}$$

[b] $\sum P_{\text{gen}} = 67.5 + 135 = 202.5\,\text{mW} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{gen}} = 135 + 922.5 = 1057.5\,\text{mVAR}$

$$\sum Q_{\text{abs}} = 607.5 + 450 = 1057.5\,\text{mVAR (check)}$$

P 10.9 **[a]** From the solution to Problem 9.64 we have

$$\mathbf{I}_\Delta = 0.5 + j2.5\,\text{A}; \qquad \text{and} \qquad \mathbf{I}_a = j2.5\,\text{A}$$

$$S_{75\text{V}} = -\frac{1}{2}(75)(0.5 - j2.5) = -18.75 + j93.75 \text{ VA}$$

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$$P_{75\text{V}} = -18.75\,\text{W}; \qquad \text{and} \qquad Q_{75\text{V}} = 93.75\ \text{VAR}$$

$$S_{\text{dep.source}} = \frac{1}{2}(100\mathbf{I}_\Delta)(\mathbf{I}_a - \mathbf{I}_\Delta)^* = -12.5 - j62.5\ \text{VA}$$

$$P_{\text{dep.source}} = -12.5\,\text{W}; \qquad \text{and} \qquad Q_{\text{dep.source}} = -62.5\ \text{VAR}$$

$$P_{-j50\Omega} = 0\,\text{W}; \qquad \text{and} \qquad Q_{-j50\Omega} = -\frac{1}{2}(50)|\mathbf{I}_\Delta|^2 = -162.5\ \text{VAR}$$

$$P_{j20\Omega} = 0\,\text{W}; \qquad \text{and} \qquad Q_{j20\Omega} = \frac{1}{2}(20)|\mathbf{I}_a|^2 = 62.5\ \text{VAR}$$

$$P_{10\Omega} = \frac{1}{2}(10)|\mathbf{I}_a|^2 = 31.25\,\text{W}; \qquad \text{and} \qquad Q_{10\Omega} = 0\ \text{VAR}$$

$$P_{j550\Omega} = 0,\text{W}; \qquad \text{and} \qquad Q_{j550\Omega} = \frac{1}{2}(550)|\mathbf{I}_\Delta - \mathbf{I}_a|^2 = 68.75\ \text{VAR}$$

[b] $\sum P_{\text{dev}} = 18.75 + 12.5 = 31.25\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{dev}} = 162.5 + 62.5 = 225\,\text{VAR}$

$$\sum Q_{\text{abs}} = 93.75 + 62.5 + 68.5 = 225\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.10 **[a]** line loss $= 8000 - 6000 = 2000\,\text{W}$

line loss $= |\mathbf{I}_g|^2 25 \qquad \therefore \quad |\mathbf{I}_g|^2 = 80$

$$|\mathbf{I}_g| = \sqrt{80}\,\text{A}$$

$$|\mathbf{I}_g|^2 R_{\text{L}} = 6000 \qquad \therefore \quad R_{\text{L}} = 75\,\Omega$$

$$|\mathbf{I}_g|^2 X_{\text{L}} = -8000 \qquad \therefore \quad X_{\text{L}} = -100\,\Omega$$

Thus,

$$|Z| = \sqrt{(100)^2 + (X_\ell - 100)^2} \qquad |\mathbf{I}_g| = \frac{1000}{\sqrt{10{,}000 + (X_\ell - 100)^2}}$$

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$$\therefore \quad 10{,}000 + (X_\ell - 100)^2 = \frac{10^6}{80} = 12{,}500$$

Solving, $\quad (X_\ell - 100) = \pm 50.$

Thus, $\quad X_\ell = 150\,\Omega \qquad$ or $\qquad X_\ell = 50\,\Omega$

[b] If $X_\ell = 150\,\Omega$:

$$\mathbf{I}_g = \frac{1000}{100 + j50} = 8 - j4\,\text{A}$$

$$S_g = -1000\mathbf{I}_g^* = -8000 - j4000\,\text{VA}$$

Thus, the voltage source is delivering 8 kW and 4 kvars.

$$Q_{j150} = |\mathbf{I}_g|^2 X_\ell = 80(150) = 12 \text{ kvars}$$

Therefore the line reactance is absorbing 12 kvars.

$$Q_{-j100} = |\mathbf{I}_g|^2 X_\text{L} = 80(-100) = -8\,\text{kvars}$$

Therefore the load reactance is generating 8 kvars.

$$\sum Q_\text{gen} = 12\,\text{kvars} = \sum Q_\text{abs}$$

If $X_\ell = 50\,\Omega$:

$$\mathbf{I}_g = \frac{1000}{100 - j50} = 8 + j4\,\text{A}$$

$$S_g = -1000\mathbf{I}_g^* = -8000 + j4000\,\text{VA}$$

Thus, the voltage source is delivering 8 kW and absorbing 4 kvars.

$$Q_{j50} = |\mathbf{I}_g|^2(50) = 80(50) = 4\,\text{kvars}$$

Therefore the line reactance is absorbing 4 kvars. The load continues to deliver 8 kvars.

$$\sum Q_\text{gen} = 8\,\text{kvars} = \sum Q_\text{abs}$$

P 10.11 **[a]** $I_\text{eff} = 40/115 \cong 0.35\,\text{A}$

[b] $I_\text{eff} = 130/115 \cong 1.13\,\text{A}$

P 10.12 $i(t) = 250t \qquad 0 \le t \le 80\,\text{ms}$

$$i(t) = 100 - 1000t \qquad 80\,\text{ms} \le t \le 100\,\text{ms}$$

$$\therefore \quad I_\text{rms} = \sqrt{\frac{1}{0.1}\left\{\int_0^{0.08}(250)^2t^2\,dt + \int_{0.08}^{0.1}(100 - 1000t)^2\,dt\right\}}$$

$$\int_0^{0.08}(250)^2t^2\,dt = (250)^2\frac{t^3}{3}\bigg|_0^{0.08} = \frac{32}{3}$$

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$$(100 - 1000t)^2 = 10^4 - 2 \times 10^5 t + 10^6 t^2$$

$$\int_{0.08}^{0.1} 10^4\, dt = 200$$

$$\int_{0.08}^{0.1} 2 \times 10^5 t\, dt = 10^5 t^2 \Big|_{0.08}^{0.1} = 360$$

$$10^6 \int_{0.08}^{0.1} t^2\, dt = \frac{10^6}{3} t^3 \Big|_{0.08}^{0.1} = \frac{488}{3}$$

$$\therefore \quad I_{\text{rms}} = \sqrt{10\{(32/3) + 225 - 360 + (488/3)\}} = 11.55\,\text{A}$$

P 10.13 $P = I_{\text{rms}}^2 R \qquad \therefore \quad R = \dfrac{1280}{(11.55)^2} = 9.6\,\Omega$

P 10.14 **[a]** $A = 40^2(5) + (-40)^2)(5) = 16{,}000$

$$\text{mean value} = \frac{A}{20} = 800$$

$$V_{\text{rms}} = \sqrt{800} = 28.28\ \text{V(rms)}$$

[b] $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{800}{40} = 20\,\text{W}$

[c] $R = \dfrac{V_{\text{rms}}^2}{P} = \dfrac{800}{0.01} = 80\,\text{k}\Omega$

P 10.15 **[a]** Area under one cycle of v_g^2:

$$A = (100)(25 \times 10^{-6}) + 400(25 \times 10^{-6}) + 400(25 \times 10^{-6}) + 100(25 \times 10^{-6})$$
$$= 1000(25 \times 10^{-6})$$

Mean value of v_g^2:

$$\text{M.V.} = \frac{A}{100 \times 10^{-6}} = \frac{1000(25 \times 10^{-6})}{100 \times 10^{-6}} = 250$$

$$\therefore \quad V_{\text{rms}} = \sqrt{250} = 15.81\,\text{V (rms)}$$

[b] $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{250}{4} = 62.5\,\text{W}$

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P 10.16 $W_{\text{dc}} = \dfrac{V_{\text{dc}}^2}{R}T; \qquad W_s = \displaystyle\int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$

$$\therefore \quad \frac{V_{\text{dc}}^2}{R}T = \int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$$

$$V_{\text{dc}}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt$$

$$V_{\text{dc}} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt} = V_{\text{rms}} = V_{\text{eff}}$$

P 10.17 [**a**]

$$Z_{\text{eq}} = 50\|(25 - j25) = 20 - j10\,\Omega$$

$$\therefore \qquad \mathbf{V}_o = 0.05Z_{\text{eq}} = 1 - j0.5 \text{ V(rms)}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{25 - j25} = 30 + j10 \text{ mA(rms)}$$

$$S_g = -\mathbf{V}_g\mathbf{I}_g^* = -(1 - j0.5)(0.05)$$
$$= -50 + j25\,\text{mVA}$$

[**b**] Source is delivering 50 mW.

[**c**] Source is absorbing 25 mVAR.

[**d**] $Q_{\text{cap}} = -|\mathbf{I}_1|^2(75) = -75$ mVAR

$$P_{50\Omega} = \frac{|\mathbf{V}_o|^2}{50} = 25\,\text{mW}$$

$$P_{25\Omega} = |\mathbf{I}_1|^2(25) = 25\,\text{mW}$$

$$Q_{\text{ind}} = |\mathbf{I}_1|^2(50) = 50 \text{ mVAR}$$

$P_{\text{middle branch}} = 25\,\text{mW}; \qquad Q_{\text{middle branch}} = 0$

$P_{\text{right branch}} = 25\,\text{mW}; \qquad Q_{\text{right branch}} = -75 + 50 = -25$ mVAR

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[e] $\sum P_{\text{del}} = 50\,\text{mW}$

$$\sum P_{\text{diss}} = 25 + 25 = 50\,\text{mW} = \sum P_{\text{del}}$$

[f] $\sum Q_{\text{abs}} = 25 + 50 = 75\,\text{mVAR} = \sum Q_{\text{dev}}$

P 10.18 $j\omega L = j25\,\Omega; \qquad \dfrac{1}{j\omega C} = -j75\,\Omega$

$$\mathbf{I}_o = \frac{j150}{50 - j25} = 2.4 + j1.2\,\text{A}$$

$$P = \frac{1}{2}|\mathbf{I}_o|^2(50) = \frac{1}{2}(7.2)(50) = 180\,\text{W}$$

$$Q = \frac{1}{2}|\mathbf{I}_o|^2(25) = 90\,\text{VAR}$$

$$S = P + jQ = 180 + j90\,\text{VA}$$

$$|S| = 201.25\,\text{VA}$$

P 10.19 **[a]** Let $\mathbf{V}_{\text{L}} = V_m\underline{/0^\circ}$:

$$S_{\text{L}} = 2500(0.8 + j0.6) = 2000 + j1500\,\text{VA}$$

$$\mathbf{I}_\ell^* = \frac{2000}{V_m} + j\frac{1500}{V_m}; \qquad \mathbf{I}_\ell = \frac{2000}{V_m} - j\frac{1500}{V_m}$$

$$250\underline{/\theta} = V_m + \left(\frac{2000}{V_m} - j\frac{1500}{V_m}\right)(1 + j2)$$

$$250V_m\underline{/\theta} = V_m^2 + (2000 - j1500)(1 + j2) = V_m^2 + 5000 + j2500$$

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$$250V_m \cos\theta = V_m^2 + 5000; \qquad 250V_m \sin\theta = 2500$$

$$(250)^2 V_m^2 = (V_m^2 + 5000)^2 + 2500^2$$

$$62{,}500V_m^2 = V_m^4 + 10{,}000V_m^2 + 31.25 \times 10^6$$

or

$$V_m^4 - 52{,}500V_m^2 + 31.25 \times 10^6 = 0$$

Solving,

$$V_m^2 = 26{,}250 \pm 25{,}647.86; \qquad V_m = 227.81\,\text{V} \text{ and } V_m = 24.54\,\text{V}$$

If $V_m = 227.81$ V:

$$\sin\theta = \frac{2500}{(227.81)(250)} = 0.044; \qquad \therefore \quad \theta = 2.52^\circ$$

If $V_m = 24.54$ V:

$$\sin\theta = \frac{2500}{(24.54)(250)} = 0.4075; \qquad \therefore \quad \theta = 24.05^\circ$$

[b]

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P 10.20 **[a]** $\dfrac{1}{j\omega C} = -j40\,\Omega; \qquad j\omega L = j80\,\Omega$

$$Z_{\text{eq}} = 40\|-j40 + j80 + 60 = 80 + j60\,\Omega$$

$$\mathbf{I}_g = \frac{40\underline{/0^\circ}}{80 + j60} = 0.32 - j0.24\,\text{A}$$

$$S_g = -\frac{1}{2}\mathbf{V}_g\mathbf{I}_g^* = -\frac{1}{2}40(0.32 + j0.24) = -6.4 - j4.8\,\text{VA}$$

$$P = 6.4\,\text{W (del)}; \qquad Q = 4.8\,\text{VAR (del)}$$

$$|S| = |S_g| = 8\,\text{VA}$$

[b] $\mathbf{I}_1 = \dfrac{-j40}{40 - j40}\mathbf{I}_g = 0.04 - j0.28\,\text{A}$

$$P_{40\Omega} = \frac{1}{2}|\mathbf{I}_1|^2(40) = 1.6\,\text{W}$$

$$P_{60\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(60) = 4.8\,\text{W}$$

$$\sum P_{\text{diss}} = 1.6 + 4.8 = 6.4\,\text{W} = \sum P_{\text{dev}}$$

[c] $\mathbf{I}_{-j40\Omega} = \mathbf{I}_g - \mathbf{I}_1 = 0.28 + j0.04\,\text{A}$

$$Q_{-j40\Omega} = \frac{1}{2}|\mathbf{I}_{-j40\Omega}|^2(-40) = -1.6\,\text{VAR (del)}$$

$$Q_{j80\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(80) = 6.4\,\text{VAR (abs)}$$

$$\sum Q_{\text{abs}} = 6.4 - 1.6 = 4.8\,\text{VAR} = \sum Q_{\text{dev}}$$

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P 10.21 $S_{\mathrm{T}} = 40{,}800 + j30{,}600\,\mathrm{VA}$

$$S_1 = 20{,}000(0.96 - j0.28) = 19{,}200 - j5600\,\mathrm{VA}$$

$$S_2 = S_{\mathrm{T}} - S_1 = 21{,}600 + j36{,}200 = 42{,}154.48\underline{/59.176^\circ}\,\mathrm{VA}$$

$$\mathrm{rf} = \sin(59.176^\circ) = 0.8587$$

$$\mathrm{pf} = \cos(59.176^\circ) = 0.5124 \text{ lagging}$$

P 10.22 **[a]** $S_1 = 10{,}000 - j4000\,\mathrm{VA}$

$$S_2 = \frac{|\mathbf{V}_{\mathrm{L}}|^2}{Z_2^*} = \frac{(1000)^2}{60 - j80} = 6 + j8\,\mathrm{kVA}$$

$$S_1 + S_2 = 16 + j4\,\mathrm{kVA}$$

$$1000\mathbf{I}_{\mathrm{L}}^* = 16{,}000 + j4000; \qquad \therefore \quad \mathbf{I}_{\mathrm{L}} = 16 - j4\,\mathrm{A(rms)}$$

$$\mathbf{V}_g = \mathbf{V}_{\mathrm{L}} + \mathbf{I}_{\mathrm{L}}(0.5 + j0.05) = 1000 + (16 - j4)(0.5 + j0.05)$$
$$= 1008.2 - j1.2 = 1008.2\underline{/-0.0682^\circ}\,\mathrm{Vrms}$$

[b] $T = \dfrac{1}{f} = \dfrac{1}{50} = 20\,\mathrm{ms}$

$$\frac{-0.0682^\circ}{360^\circ} = \frac{t}{20\text{ ms}}; \qquad \therefore \quad t = -3.79\,\mu\mathrm{s}$$

[c] $\mathbf{V}_{\mathrm{L}}$ leads $\mathbf{V}_g$ by 0.0682° or $3.79\,\mu\mathrm{s}$

P 10.23 **[a]**

$$\mathbf{I}_1 = \frac{6000 - j3000}{150} = 40 - j20\,\mathrm{A}\ \text{(rms)}$$

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$$\mathbf{I}_2 = \frac{7500 + j4500}{150} = 50 + j30\,\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{12{,}000 - j9000}{300} = 40 - j30\,\text{A (rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 80 - j50\,\text{A (rms)}$$

$$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = -10 - j50\,\text{A (rms)}$$

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 90 + j0\,\text{A}$$

$$\mathbf{V}_{g1} = 0.1\mathbf{I}_{g1} + 150 + 0.2\mathbf{I}_n = 156 - j15\,\text{V(rms)}$$

$$\mathbf{V}_{g2} = -0.2\mathbf{I}_n + 150 + 0.1\mathbf{I}_{g2} = 161 + j10\,\text{V(rms)}$$

$$S_{g1} = -[(156 - j15)(80 + j50)] = -[13{,}230 + j6600]\,\text{VA}$$

$$S_{g2} = -[(161 + j10)(90 + j0)] = -[14{,}490 + j900]\,\text{VA}$$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

[b] $P_{0.1} = |\mathbf{I}_{g1}|^2(0.1) = 890\,\text{W}$

$$P_{0.2} = |\mathbf{I}_n|^2(0.2) = 520\,\text{W}$$

$$P_{0.1} = |\mathbf{I}_{g2}|^2(0.1) = 810\,\text{W}$$

$$\sum P_{\text{dis}} = 890 + 520 + 810 + 6000 + 7500 + 12{,}000 = 27{,}720\,\text{W}$$

$$\sum P_{\text{dev}} = 13{,}230 + 14{,}490 = 27{,}720\,\text{W} = \sum P_{\text{dis}}$$

$$\sum Q_{\text{abs}} = 3000 + 9000 = 12{,}000\,\text{VAR}$$

$$\sum Q_{\text{del}} = 4500 + 6600 + 900 = 12{,}000\,\text{VAR} = \sum Q_{\text{abs}}$$

P 10.24 **[a]**

$$120\mathbf{I}_1^* = 4800 - j2400; \qquad \therefore \quad \mathbf{I}_1 = 40 + j20\,\text{A(rms)}$$

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$$120\mathbf{I}_2^* = 4800 + j3600; \qquad \therefore \quad \mathbf{I}_2 = 40 - j30\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{240}{24} + \frac{240}{j6} = 10 - j40\,\text{A(rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 50 - j20\,\text{A}$$

$$S_{g1} = -120(50 + j20) = -6000 - j2400\,\text{VA}$$

Thus the $\mathbf{V}_{g1}$ source is delivering 6 kW and 2.4 kvars.

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 50 - j70\,\text{A(rms)}$$

$$S_{g2} = -120(50 + j70) = -6000 - j8400\,\text{VA}$$

Thus the $\mathbf{V}_{g2}$ source is delivering 6 kW and 8.4 kvars.

[b] $\sum P_{\text{gen}} = 6 + 6 = 12\,\text{kW}$

$$\sum P_{\text{abs}} = 4800 + 4800 + \frac{(240)^2}{24} = 12\,\text{kW} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 2400 + 8400 + 2400 = 13.2\,\text{kVAR}$$

$$\sum Q_{\text{abs}} = 3600 + \frac{(240)^2}{6} = 13.2\,\text{kVAR} = \sum Q_{\text{del}}$$

P 10.25 $S_1 = 1146 + 1200 + 1450 = 3796 + j0\,\text{VA}$

$$\therefore \quad \mathbf{I}_1 = \frac{3796}{110} = 34.51\,\text{A}$$

$$S_2 = 145 + 630 + 1322 = 2097 + j0\,\text{VA}$$

$$\therefore \quad \mathbf{I}_2 = \frac{2097}{1!0} = 19.064\,\text{A}$$

$$S_3 = 512 + 4856 = 5368 + j0\,\text{VA}$$

$$\therefore \quad \mathbf{I}_3 = \frac{5368}{220} = 24.4\,\text{A}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 58.91\,\text{A}$$

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 43.464\,\text{A}$$

The breaker protecting the upper service conductor will trip because its feeder current exceeds 50 A. The breaker protecting the lower service conductor will not trip because its feeder current is less than 50 A. Thus, service to Loads 1 and 3 will be interrupted.

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P 10.26

$$250\mathbf{I}_1^* = 6000 - j8000$$

$$\mathbf{I}_1^* = 24 - j32; \qquad \therefore \quad \mathbf{I}_1 = 24 + j32\,\text{A(rms)}$$

$$250\mathbf{I}_2^* = 9000 + j3000$$

$$\mathbf{I}_2^* = 36 + j12; \qquad \therefore \quad \mathbf{I}_2 = 36 - j12\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{250\underline{/0^\circ}}{25} = 10 + j0\,\text{A}; \qquad \mathbf{I}_4 = \frac{250\underline{/0^\circ}}{-j5} = 0 + j50\,\text{A}$$

$$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 70 + j70\,\text{A}$$

$$\mathbf{V}_g = 250 + (70 + j70)(j0.1) = 243 + j7 = 243.1\underline{/1.65^\circ}\,\text{V (rms)}$$

P 10.27 **[a]** From Problem 9.78,

$$Z_{\text{ab}} = 20 + j27.25 \quad \text{so}$$

$$\mathbf{I}_1 = \frac{75}{10 + j12.75 + 20 + 27.25} = \frac{75}{30 + j40} = 0.9 - j1.2\,\text{A}$$

$$\mathbf{I}_2 = \frac{j\omega M}{Z_{22}}\mathbf{I}_1 = \frac{j54}{160 + j120}(0.9 - j1.2) = 405 + j0\,\text{mA}$$

$$\mathbf{V}_L = (60 + j20)(0.405 + j0) = 24.3 + j8.1$$

$$|\mathbf{V}_L| = 25.61\,\text{V}$$

[b] $P_g(\text{ideal}) = 75(0.9) = 67.5\,\text{W}$

$$P_g(\text{practical}) = 67.5 - |\mathbf{I}_1|^2(10) = 45\,\text{W}$$

$$P_{\text{L}} = |\mathbf{I}_2|^2(60) = 9.8415\,\text{W}$$

$$\%\ \text{delivered} = \frac{9.8415}{45}(100) = 21.87\%$$

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P 10.28 **[a]** $S_1 = 3 + j0\,\text{kVA}; \qquad S_2 = 4 - j3\,\text{kVA}; \qquad S_3 = 5 - j6\,\text{kVA}$

$$S_\text{T} = S_1 + S_2 + S_3 = 12 - j9\,\text{kVA}$$

$$300\mathbf{I}^* = (12 - j9) \times 10^3; \qquad \therefore \quad \mathbf{I} = 40 + j30\,\text{A}$$

$$Z = \frac{300}{40 + j30} = 4.8 - j3.6\,\Omega = 6\underline{/-36.87^\circ}\,\Omega$$

[b] $\text{pf} = \cos(-36.87^\circ) = 0.8$ leading

P 10.29 **[a]** From the solution to Problem 10.28 we have

$$\mathbf{I}_\text{L} = 40 + j30\,\text{A (rms)}$$

$$\therefore \quad \mathbf{V}_s = 300\underline{/0^\circ} + (40 + j30)(0.2 + j0.05) = 306.5 + j8$$

$$= 306.6\underline{/1.5^\circ}\,\text{V (rms)}$$

[b] $|\mathbf{I}_\text{L}| = \sqrt{16{,}000}$

$$P_\ell = (2500)(0.2) = 500\,\text{W} \qquad Q_\ell = (2500)(0.05) = 125\,\text{VAR}$$

[c] $P_s = 12{,}000 + 500 = 12.5\,\text{kW} \qquad Q_s = -9000 + 125 = -8.875\,\text{kVAR}$

[d] $\eta = \dfrac{12}{12.5}(100) = 96\%$

P 10.30 **[a]** $Z_1 = 240 + j70 = 250\underline{/16.26^\circ}\,\Omega$

$$\text{pf} = \cos(16.26^\circ) = 0.96 \text{ lagging}$$

$$\text{rf} = \sin(16.26^\circ) = 0.28$$

$$Z_2 = 160 - j120 = 200\underline{/-36.87^\circ}\,\Omega$$

$$\text{pf} = \cos(-36.87^\circ) = 0.8 \text{ leading}$$

$$\text{rf} = \sin(-36.87^\circ) = -0.6$$

$$Z_3 = 30 - j40 = 50\underline{/-53.13^\circ}\,\Omega$$

$$\text{pf} = \cos(-53.13^\circ) = 0.6 \text{ leading}$$

$$\text{rf} = \sin(-53.13^\circ) = -0.8$$

[b] $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{250\underline{/16.26^\circ}}; \qquad Y_2 = \frac{1}{200\underline{/-36.87^\circ}}; \qquad Y_3 = \frac{1}{50\underline{/-53.13^\circ}}$$

$$Y = 19.84 + j17.88\,\text{mS}$$

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$$Z = \frac{1}{Y} = 37.44\underline{/-42.03^\circ}\,\Omega$$

$$\text{pf} = \cos(-42.03^\circ) = 0.74 \text{ leading}$$

$$\text{rf} = \sin(-42.03^\circ) = -0.67$$

P 10.31 **[a]** $\mathbf{I} = \dfrac{270\underline{/0^\circ}}{36 + j48} = 2.7 - j3.6 = 4.5\underline{/-53.13^\circ}\,\text{A(rms)}$

$$P = (4.5)^2(6) = 121.5\,\text{W}$$

[b] $Y_\text{L} = \dfrac{1}{30 + j40} = 12 - j16 \text{ mS}$

$$\therefore \quad X_\text{C} = \frac{1}{-16 \times 10^{-3}} = -62.5\,\Omega$$

[c] $Z_\text{L} = \dfrac{1}{12 \times 10^{-3}} = 83.33\,\Omega$

[d] $\mathbf{I} = \dfrac{270\underline{/0^\circ}}{89.33 + j8} = 3.01\underline{/-5.12^\circ}\,\text{A}$

$$P = (3.01)^2(6) = 54.37\,\text{W}$$

[e] $\% = \dfrac{54.37}{121.5}(100) = 44.75\%$

Thus the power loss after the capacitor is added is 44.75% of the power loss before the capacitor is added.

P 10.32

$$\mathbf{I}_\text{L} = \frac{120{,}000 - j90{,}000}{4800} = 25 - j18.75\,\text{A (rms)}$$

$$\mathbf{I}_\text{C} = \frac{4800}{jX_\text{C}} = -j\frac{4800}{X_\text{C}} = jI_\text{C}$$

$$\mathbf{I}_\ell = 25 - j18.75 + jI_\text{C} = 25 + j(I_\text{C} - 18.75)$$

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$$\mathbf{V}_s = 4800 + (10 + j5)[25 + j(I_\mathrm{C} - 18.75)]$$
$$= (5143.75 - 5I_\mathrm{C}) + j(-62.5 + 10I_\mathrm{C})$$

$$|\mathbf{V}_s|^2 = (5143.75 - 5I_\mathrm{C})^2 + (-62.5 + 10I_\mathrm{C})^2 = (4800)^2$$

$$\therefore \quad 125I_\mathrm{C}^2 - 52{,}687.5I_\mathrm{C} + 3{,}422{,}070.313 = 0$$

$$I_\mathrm{C} = 341.284\,\mathrm{A(rms)} \quad \text{and} \quad I_\mathrm{C} = 80.2165\,\mathrm{A(rms)}^*$$

*Select the smaller value of I_C to minimize the magnitude of I_ℓ.

$$\therefore \quad X_\mathrm{C} = -\frac{4800}{80.2165} = -59.838$$

$$\therefore \quad C = \frac{1}{(59.838)(120\pi)} = 44.43\,\mu\mathrm{F}$$

P 10.33 **[a]** $S_\mathrm{L} = 20{,}000(0.85 + j0.53) = 17{,}000 + j10{,}535.65\,\mathrm{VA}$

$$125\mathbf{I}_\mathrm{L}^* = (17{,}000 + j10{,}535.65); \quad \mathbf{I}_\mathrm{L}^* = 136 + j84.29\,\mathrm{A(rms)}$$

$$\therefore \quad \mathbf{I}_\mathrm{L} = 136 - j84.29\,\mathrm{A(rms)}$$

$$\mathbf{V}_s = 125 + (136 - j84.29)(0.01 + j0.08) = 133.10 + j10.04$$
$$= 133.48\underline{/4.31^\circ}\,\mathrm{V(rms)}$$

$$|\mathbf{V}_s| = 133.48\,\mathrm{V(rms)}$$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.01) = (160)^2(0.01) = 256\,\mathrm{W}$

[c] $\dfrac{(125)^2}{X_\mathrm{C}} = -10{,}535.65; \qquad X_\mathrm{C} = -1.48306\,\Omega$

$$-\frac{1}{\omega C} = -1.48306; \qquad C = \frac{1}{(1.48306)(120\pi)} = 1788.59\,\mu\mathrm{F}$$

[d] $\mathbf{I}_\ell = 136 + j0\,\mathrm{A(rms)}$

$$\mathbf{V}_s = 125 + 136(0.01 + j0.08) = 126.36 + j10.88$$
$$= 126.83\underline{/4.92^\circ}\,\mathrm{V(rms)}$$

$$|\mathbf{V}_s| = 126.83\,\mathrm{V(rms)}$$

[e] $P_\ell = (136)^2(0.01) = 184.96\,\mathrm{W}$

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P 10.34 **[a]** $S_o =$ original load $= 1600 + j\dfrac{1600}{0.8}(0.6) = 1600 + j1200\,\text{kVA}$

$S_f =$ final load $= 1920 + j\dfrac{1920}{0.96}(0.28) = 1920 + j560\,\text{kVA}$

$\therefore \quad Q_{\text{added}} = 560 - 1200 = -640\,\text{kVAR}$

[b] deliver

[c] $S_{\text{a}} =$ added load $= 320 - j640 = 715.54\underline{/-63.43^\circ}\,\text{kVA}$

pf $= \cos(-63.43) = 0.447$ leading

[d] $\mathbf{I}_{\text{L}}^* = \dfrac{(1600 + j1200) \times 10^3}{2400} = 666.67 + j500\,\text{A}$

$\mathbf{I}_{\text{L}} = 666.67 - j500 = 833.33\underline{/-36.87^\circ}\,\text{A(rms)}$

$|\mathbf{I}_{\text{L}}| = 833.33\,\text{A(rms)}$

[e] $\mathbf{I}_{\text{L}}^* = \dfrac{(1920 + j560) \times 10^3}{2400} = 800 + j233.33$

$\mathbf{I}_{\text{L}} = 800 - j233.33 = 833.33\underline{/-16.26^\circ}\,\text{A(rms)}$

$|\mathbf{I}_{\text{L}}| = 833.33\,\text{A(rms)}$

P 10.35 **[a]** $P_{\text{before}} = P_{\text{after}} = (833.33)^2(0.25) = 173.611\,\text{kW}$

[b] $\mathbf{V}_s(\text{before}) = 2400 + (666.67 - j500)(0.25 + j0.1)$

$= 2616.67 - j58.33 = 2617.32\underline{/-1.28^\circ}\,\text{V(rms)}$

$|\mathbf{V}_s(\text{before})| = 2617.32\,\text{V(rms)}$

$\mathbf{V}_s(\text{after}) = 2400 + (800 - j233.33)(0.25 + j0.1)$

$= 2623.33 + j21.67 = 2623.42\underline{/0.47^\circ}\,\text{V(rms)}$

$|\mathbf{V}_s(\text{after})| = 2623.42\,\text{V(rms)}$

P 10.36 **[a]**

$50 = j10(\mathbf{I}_1 - \mathbf{I}_2) + j10(\mathbf{I}_3 - \mathbf{I}_2) + 10(\mathbf{I}_1 - \mathbf{I}_3)$

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$$0 = 10\mathbf{I}_2 + j20(\mathbf{I}_2 - \mathbf{I}_3) + j10(\mathbf{I}_2 - \mathbf{I}_1) + j10(\mathbf{I}_2 - \mathbf{I}_1) + j10(\mathbf{I}_2 - \mathbf{I}_3)$$

$$0 = -j10\mathbf{I}_3 + 10(\mathbf{I}_3 - \mathbf{I}_1) + j20(\mathbf{I}_3 - \mathbf{I}_2) + j10(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

$$\mathbf{I}_1 = 5.5 + j0.5\,\text{A(rms)}; \quad \mathbf{I}_2 = 3 + j2\,\text{A(rms)}; \quad \mathbf{I}_3 = 2 + j2\,\text{A(rms)}$$

$$\mathbf{I}_\text{a} = \mathbf{I}_1 = 5.5 + j0.5\,\text{A} \qquad \mathbf{I}_\text{b} = \mathbf{I}_1 - \mathbf{I}_2 = 2.5 - j1.5\,\text{A}$$

$$\mathbf{I}_\text{c} = \mathbf{I}_2 = 3 + j2\,\text{A} \qquad \mathbf{I}_\text{d} = \mathbf{I}_3 - \mathbf{I}_2 = -1\,\text{A}$$

$$\mathbf{I}_\text{e} = \mathbf{I}_1 - \mathbf{I}_3 = 3.5 - j1.5\,\text{A} \qquad \mathbf{I}_\text{f} = \mathbf{I}_3 = 2 + j2\,\text{A}$$

[b]

$$\mathbf{V}_\text{a} = 50\,\text{V} \qquad \mathbf{V}_\text{b} = j10\mathbf{I}_\text{b} + j10\mathbf{I}_\text{d} = 15 + j15\,\text{V}$$

$$\mathbf{V}_\text{c} = 10\mathbf{I}_\text{c} = 30 + j20\,\text{V} \qquad \mathbf{V}_\text{d} = j20\mathbf{I}_\text{d} + j10\mathbf{I}_\text{b} = 15 + j5\,\text{V}$$

$$\mathbf{V}_\text{e} = 10\mathbf{I}_\text{e} = 35 - j15\,\text{V} \qquad \mathbf{V}_\text{f} = -j10\mathbf{I}_\text{f} = 20 - j20\,\text{V}$$

$$S_\text{a} = -50\mathbf{I}_\text{a}^* = -275 + j25\,\text{VA}$$

$$S_\text{b} = \mathbf{V}_\text{b}\mathbf{I}_\text{b}^* = 15 + j60\,\text{VA}$$

$$S_\text{c} = \mathbf{V}_\text{c}\mathbf{I}_\text{c}^* = 130 + j0\,\text{VA}$$

$$S_\text{d} = \mathbf{V}_\text{d}\mathbf{I}_\text{d}^* = -15 - j5\,\text{VA}$$

$$S_\text{e} = \mathbf{V}_\text{e}\mathbf{I}_\text{e}^* = 145 - j0\,\text{VA}$$

$$S_\text{f} = \mathbf{V}_\text{f}\mathbf{I}_\text{f}^* = 0 - j80\,\text{VA}$$

[c] $\sum P_\text{dev} = 275 + 15 = 290\,\text{W}$

$$\sum P_\text{abs} = 15 + 130 + 145 = 290\,\text{W}$$

Note that the total power absorbed by the coupled coils is zero:
$15 - 15 = 0 = P_\text{b} + P_\text{d}$

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[d] $\sum Q_{\text{dev}} = 5 + 80 = 85\,\text{VAR}$

$\sum Q_{\text{abs}} = 25 + 60 = 85\,\text{VAR}$

$\sum Q$ absorbed by the coupled coils is $Q_{\text{b}} + Q_{\text{d}} = 55$

P 10.37 **[a]**

$$272\underline{/0^\circ} = 2\mathbf{I}_g + j10\mathbf{I}_g + j14(\mathbf{I}_g - \mathbf{I}_2) - j6\mathbf{I}_2$$
$$+j14\mathbf{I}_g - j8\mathbf{I}_2 + j20(\mathbf{I}_g - \mathbf{I}_2)$$
$$0 = j20(\mathbf{I}_2 - \mathbf{I}_g) - j14\mathbf{I}_g + j8\mathbf{I}_2 + j4\mathbf{I}_2$$
$$+j8(\mathbf{I}_2 - \mathbf{I}_g) - j6\mathbf{I}_g + 8\mathbf{I}_2$$

Solving,

$I_g = 20 - j4\,\text{A(rms)}; \qquad \mathbf{I}_2 = 24\underline{/0^\circ}\,\text{A(rms)}$

$P_{8\Omega} = (24)^2(8) = 4608\,\text{W}$

[b] $P_g(\text{developed}) = (272)(20) = 5440\,\text{W}$

[c] $Z_{\text{ab}} = \dfrac{\mathbf{V}_g}{\mathbf{I}_g} - 2 = \dfrac{272}{20 - j4} - 2 = 11.08 + j2.62 = 11.38\underline{/13.28^\circ}\,\Omega$

[d] $P_{2\Omega} = |I_g|^2(2) = 832\,\text{W}$

$\sum P_{\text{diss}} = 832 + 4608 = 5440\,\text{W} = \sum P_{\text{dev}}$

P 10.38 **[a]**

$300 = 60\mathbf{I}_1 + \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2)$

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$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{V}_2 + 40\mathbf{I}_2$$

$$\mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_2 = -4\mathbf{I}_1$$

Solving,

$$\mathbf{V}_1 = 260\,\text{V (rms)}; \qquad \mathbf{V}_2 = 65\,\text{V (rms)}$$

$$\mathbf{I}_1 = 0.25\,\text{A (rms)}; \qquad \mathbf{I}_2 = -1.0\,\text{A (rms)}$$

$$\mathbf{V}_{5\text{A}} = \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2) = 285\,\text{V (rms)}$$

$$\therefore \quad P = -(285)(5) = -1425\,\text{W}$$

Thus 1425 W is delivered by the current source to the circuit.

[b] $\mathbf{I}_{20\Omega} = \mathbf{I}_1 - \mathbf{I}_2 = 1.25\,\text{A(rms)}$

$$\therefore \quad P_{20\Omega} = (1.25)^2(20) = 31.25\,\text{W}$$

P 10.39 **[a]**

$$(30 - j40)\mathbf{I}_1 + \mathbf{V}_1 + \mathbf{V}_2 = 250$$

$$(5 + j10)\mathbf{I}_2 - \mathbf{V}_2 = 0$$

$$\frac{\mathbf{V}_1}{900} = \frac{-\mathbf{V}_2}{300}$$

$$900\mathbf{I}_1 = 300(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

$$\mathbf{V}_1 = 150 + j300\ \text{V(rms)}; \qquad \mathbf{V}_2 = -50 - j100\ \text{V(rms)}$$

$$\mathbf{I}_1 = 5 + j0\ \text{A(rms)}; \qquad \mathbf{I}_2 = -10 + j0\ \text{A(rms)}$$

$$P_{30\Omega} = (5)^2(30) = 750\,\text{W}; \qquad \text{and} \qquad P_{5\Omega} = (10)^2(5) = 500\,\text{W}$$

[b] $P_g = -250(5\underline{/0^\circ}) = -1250\,\text{W}$

$$\sum P_{\text{abs}} = 750 + 500 = 1250\,\text{W} = \sum P_{\text{dev}}$$

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P 10.40 **[a]** $25a_1^2 + 4a_2^2 = 500$

$\mathbf{I}_{25} = a_1\mathbf{I}; \qquad P_{25} = a_1^2\mathbf{I}^2(25)$

$\mathbf{I}_4 = a_2\mathbf{I}; \qquad P_4 = a_2^2\mathbf{I}^2(4)$

$P_4 = 4P_{25}; \qquad a_2^2\mathbf{I}^2 4 = 100a_1^2\mathbf{I}^2$

$\therefore \qquad 100a_1^2 = 4a_2^2$

$25a_1^2 + 100a_1^2 = 500; \qquad a_1 = 2$

$25(4) + 4a_2^2 = 500; \qquad a_2 = 10$

[b] $\mathbf{I} = \dfrac{2000\underline{/0^\circ}}{500 + 500} = 2\underline{/0^\circ}\,\text{A(rms)}$

$\mathbf{I}_{25} = a_1\mathbf{I} = 4\,\text{A}$

$P_{25\Omega} = (16)(25) = 400\,\text{W}$

[c] $\mathbf{I}_4 = a_2\mathbf{I} = 10(2) = 20\,\text{A(rms)}$

$\mathbf{V}_4 = (20)(4) = 80\underline{/0^\circ}\,\text{V(rms)}$

P 10.41 **[a]** $Z_{\text{Th}} = j4000 + \dfrac{(4000)(-j4000)}{4000 - j4000} = 2000 + j2000\,\Omega$

$\therefore \quad Z_{\text{L}} = Z_{\text{Th}}^* = 2000 - j2000\,\Omega$

[b] $\mathbf{V}_{\text{Th}} = \dfrac{10\underline{/0^\circ}(4000)}{4000 - j4000} = 5 + j5 = 5\sqrt{2}\underline{/45^\circ}\,\text{V}$

$\mathbf{I} = \dfrac{5\sqrt{2}\underline{/45^\circ}}{4000} = 1.25\sqrt{2}\underline{/45^\circ}\,\text{mA}$

$|\mathbf{I}_{\text{rms}}| = 1.25\,\text{mA}$

$P_{\text{load}} = (0.00125)^2(2000) = 3.125\,\text{mW}$

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[**c**] The closest resistor values from Appendix H are 1.8 kΩ and 2.2 kΩ. Find the capacitor value:

$$\frac{1}{8000C} = 2000 \qquad \text{so} \qquad C = 62.5\,\text{nF}$$

The closest capacitor value is 47 nF. Choose $R = 1.8\,\text{k}\Omega$:

$$\mathbf{I} = \frac{5\underline{/45^\circ}}{2000 + j2000 + 1800 - j2659.57} = 0.7462 + j1.06 \text{ mA(rms)}$$

$$= 1.3\underline{/54.85^\circ} \text{ mA(rms)}$$

$$P_{\text{load}} = (0.0013)^2(1800) = 3.03\,\text{mW} \qquad \text{(instead of 3.125 mW)}$$

Therefore, the load impedance is $\quad Z_{\text{L}} = 1800 - j2659.57\,\Omega.$

P 10.42 [**a**] From Problem 9.75, $Z_{\text{Th}} = 85 + j85\,\Omega$ and $\mathbf{V}_{\text{Th}} = 850 + j850\,\text{V}$. Thus, for maximum power transfer, $Z_{\text{L}} = Z^*_{\text{Th}} = 85 - j85\,\Omega$:

$$\mathbf{I}_2 = \frac{850 + j850}{170} = 5 + j5\,\text{A}$$

$$425\underline{/0^\circ} = (5 + j5)\mathbf{I}_1 - j20(5 + j5)$$

$$\therefore \quad \mathbf{I}_1 = \frac{325 + j100}{5 + j5} = 42.5 - j22.5\,\text{A}$$

$$S_g(\text{del}) = 425(42.5 + j22.5) = 18{,}062.5 + j9562.5\,\text{VA}$$

$$P_g = 18{,}062.5\,\text{W}$$

[**b**] $P_{\text{loss}} = |\mathbf{I}_1|^2(5) + |\mathbf{I}_2|^2(45) = 11{,}562.5 + 2250 = 13{,}812.5\,\text{W}$

$$\%\text{ loss in transformer} = \frac{13{,}812.5}{18{,}062.5}(100) = 76.47\%$$

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P 10.43 **[a]** $\dfrac{156 - j42 - 300}{Z_{\text{Th}}} + \dfrac{156 - j42}{200 - j500} = 0$

$$\therefore \quad Z_{\text{Th}} = \frac{300 - 156 + j42}{0.18 + j0.24} = 400 - j300\,\Omega$$

$$\therefore \quad Z_{\text{L}} = 400 + j300\,\Omega$$

[b] $\mathbf{I} = \dfrac{300\underline{/0^\circ}}{800\underline{/0^\circ}} = 0.375\underline{/0^\circ}\,\text{A(rms)}$

$$P = (0.375)^2(400) = 56.25\,\text{W}$$

[c] Let $R = 180\,\Omega + 220\,\Omega = 400\,\Omega$

$$2\pi(50)L = 300 \qquad \text{so} \qquad L = \frac{300}{100\pi} = 95.5\,\text{mH}$$

Use 9 series-connected 10 mH inductors to get 90 mH. Use 2 parallel-connected 10 mH inductors to get 5 mH. Use 2 parallel-connected 1 mH inductors to get 0.5 mH. Combine the 90 mH, the 5 mH, and the 0.5 mH in series to get 95.5 mH.

P 10.44 **[a]** Open circuit voltage:

$$\mathbf{V}_1 = 5\mathbf{I}\phi = 5\frac{100 - 5\mathbf{I}_\phi}{25 + j10}$$

$$(25 + j10)\mathbf{I}_\phi = 100 - 5\mathbf{I}\phi$$

$$\mathbf{I}_\phi = \frac{100}{30 + j10} = 3 - j\,\text{A}$$

$$\mathbf{V}_{\text{Th}} = \frac{j3}{1 + j3}(5\mathbf{I}_\phi) = 15\,\text{V}$$

Short circuit current:

$$\mathbf{V}_2 = 5\mathbf{I}_\phi = \frac{100 - 5\mathbf{I}_\phi}{25 + j10}$$

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$$\mathbf{I}_\phi = 3 - j1\,\text{A}$$

$$\mathbf{I}_{\text{sc}} = \frac{5\mathbf{I}_\phi}{1} = 15 - j5\,\text{A}$$

$$Z_{\text{Th}} = \frac{15}{15 - j5} = 0.9 + j0.3\,\Omega$$

$$Z_L = Z_{\text{Th}}^* = 0.9 - j0.3\,\Omega$$

$$\mathbf{I}_{\text{L}} = \frac{0.3}{1.8} = 8.33\,\text{A(rms)}$$

$$P = |\mathbf{I}_L|^2(0.9) = 62.5\,\text{W}$$

[b] $\mathbf{V}_{\text{L}} = (0.9 - j0.3)(8.33) = 7.5 - j2.5\,\text{V(rms)}$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{\text{L}}}{j3} = -0.833 - j2.5\,\text{A(rms)}$$

$$\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_{\text{L}} = 7.5 - j2.5\,\text{A(rms)}$$

$$5\mathbf{I}_\phi = \mathbf{I}_2 + \mathbf{V}_L \quad \therefore \quad \mathbf{I}_\phi = 3 - j1\,\text{A}$$

$$\mathbf{I}_{\text{d.s.}} = \mathbf{I}_\phi - \mathbf{I}_2 = -4.5 + j1.5\,\text{A}$$

$$S_g = -100(3 + j1) = -300 - j100\,\text{VA}$$

$$S_{\text{d.s.}} = 5(3 - j1)(-4.5 - j1.5) = -75 + j0\,\text{VA}$$

$$P_{\text{dev}} = 300 + 75 = 375\,\text{W}$$

$$\%\text{ developed} = \frac{62.5}{375}(100) = 16.67\%$$

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Checks:

$P_{25\Omega} = (10)(25) = 250\,\text{W}$

$P_{1\Omega} = (67.5)(1) = 67.5\,\text{W}$

$P_{0.9\Omega} = 62.5\,\text{W}$

$\sum P_{\text{abs}} = 230 + 62.5 + 67.5 = 375 = \sum P_{\text{dev}}$

$Q_{j10} = (10)(10) = 100\,\text{VAR}$

$Q_{j3} = (6.94)(3) = 20.82\,\text{VAR}$

$Q_{-j0.3} = (69.4)(-0.3) = -20.82\,\text{VAR}$

$Q_{\text{source}} = -100\,\text{VAR}$

$\sum Q = 100 + 20.82 - 20.82 - 100 = 0$

P 10.45 $Z_{\text{L}} = |Z_{\text{L}}|\underline{/\theta^\circ} = |Z_{\text{L}}|\cos\theta^\circ + j|Z_{\text{L}}|\sin\theta^\circ$

Thus $\quad |\mathbf{I}| = \dfrac{|\mathbf{V}_{\text{Th}}|}{\sqrt{(R_{\text{Th}} + |Z_{\text{L}}|\cos\theta)^2 + (X_{\text{Th}} + |Z_{\text{L}}|\sin\theta)^2}}$

Therefore $\quad P = \dfrac{0.5|\mathbf{V}_{\text{Th}}|^2|Z_{\text{L}}|\cos\theta}{(R_{\text{Th}} + |Z_{\text{L}}|\cos\theta)^2 + (X_{\text{Th}} + |Z_{\text{L}}|\sin\theta)^2}$

Let D = demoninator in the expression for P, then

$$\frac{dP}{d|Z_{\text{L}}|} = \frac{(0.5|\mathbf{V}_{\text{Th}}|^2\cos\theta)(D\cdot 1 - |Z_{\text{L}}|dD/d|Z_{\text{L}}|)}{D^2}$$

$$\frac{dD}{d|Z_{\text{L}}|} = 2(R_{\text{Th}} + |Z_{\text{L}}|\cos\theta)\cos\theta + 2(X_{\text{Th}} + |Z_{\text{L}}|\sin\theta)\sin\theta$$

$$\frac{dP}{d|Z_{\text{L}}|} = 0 \quad \text{when} \quad D = |Z_{\text{L}}|\left(\frac{dD}{d|Z_{\text{L}}|}\right)$$

Substituting the expressions for D and $(dD/d|Z_{\text{L}}|)$ into this equation gives us the relationship $R_{\text{Th}}^2 + X_{\text{Th}}^2 = |Z_{\text{L}}|^2$ or $|Z_{\text{Th}}| = |Z_{\text{L}}|$.

P 10.46 **[a]** $Z_{\text{Th}} = 200 - j480 + \dfrac{(j200)(500 + j300)}{500 + j500} = 240 - j320 = 400\underline{/-53.13^\circ}\,\Omega$

$\therefore \quad R = |Z_{\text{Th}}| = 400\,\Omega$

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[b] $\mathbf{V}_{\text{Th}} = \dfrac{j200}{500 + j300 + j200}(300\underline{/0^\circ}) = 60 + j60\,\text{V(rms)}$

$$\mathbf{I} = \frac{60 + j60}{640 - j320} = 37.5 + j112.5 \text{ mA(rms)} = 118.59\underline{/71.57^\circ} \text{ mA(rms)}$$

$$P = (0.11859)^2(400) = 5.625\,\text{W}$$

[c] Pick the $390\,\Omega$ resistor from Appendix H for the closest match:

$$\mathbf{I} = \frac{60 + j60}{630 - j320} = 120.084\underline{/71.93^\circ} \text{ mA(rms)}$$

$$P = (0.120084)^2(390) = 5.624\,\text{W}$$

P 10.47 **[a]** Open circuit voltage:

$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} - 0.1\mathbf{V}_\phi = 0$$

$$\therefore \quad \mathbf{V}_\phi = 40 + j80\,\text{V(rms)}$$

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_\phi + 0.1\mathbf{V}_\phi(-j5) = \mathbf{V}_\phi(1 - j0.5) = 80 + j60\,\text{V(rms)}$$

Short circuit current:

$$\mathbf{I}_{\text{sc}} = 0.1\mathbf{V}_\phi + \frac{\mathbf{V}_\phi}{-j5} = (0.1 + j0.2)\mathbf{V}_\phi$$

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$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_\phi}{-j5} = 0$$

$$\therefore \quad \mathbf{V}_\phi = 100\,\text{V(rms)}$$

$$\mathbf{I}_{\text{sc}} = (0.1 + j0.2)(100) = 10 + j20\,\text{A(rms)}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{80 + j60}{10 + j20} = 4 - j2\,\Omega$$

$$\therefore \quad R_o = |Z_{\text{Th}}| = 4.47\,\Omega$$

[b]

$$\mathbf{I} = \frac{80 + j60}{4 + \sqrt{20} - j2} = 7.36 + j8.82\,\text{A (rms)}$$

$$P = (11.49)^2(\sqrt{20}) = 590.17\,\text{W}$$

[c]

$$\mathbf{I} = \frac{80 + j60}{8} = 10 + j7.5\,\text{A (rms)}$$

$$P = (10^2 + 7.5^2)(4) = 625\,\text{W}$$

[d]

$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_o - (25 + j50)}{-j5} = 0$$

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$$\mathbf{V}_\phi = 50 + j25\,\text{V (rms)}$$

$$0.1\mathbf{V}_\phi = 5 + j2.5\,\text{V (rms)}$$

$$5 + j2.5 + \mathbf{I}_C = 10 + j7.5$$

$$\mathbf{I}_C = 5 + j5\,\text{A (rms)}$$

$$\mathbf{I}_L = \frac{\mathbf{V}_\phi}{j5} = 5 - j10\,\text{A (rms)}$$

$$\mathbf{I}_R = \mathbf{I}_C + \mathbf{I}_L = 10 - j5\,\text{A (rms)}$$

$$\mathbf{I}_g = \mathbf{I}_R + 0.1\mathbf{V}_\phi = 15 - j2.5\,\text{A (rms)}$$

$$S_g = -100\mathbf{I}_g^* = -1500 - j250\,\text{VA}$$

$$100 = 5(5 + j2.5) + \mathbf{V}_{\text{cs}} + 25 + j50 \quad \therefore \quad \mathbf{V}_{\text{cs}} = 50 - j62.5\,\text{V (rms)}$$

$$S_{\text{cs}} = (50 - j62.5)(5 - j2.5) = 93.75 - j437.5\,\text{VA}$$

Thus,

$$\sum P_{\text{dev}} = 1500$$

$$\%\text{ delivered to } R_o = \frac{625}{1500}(100) = 41.67\%$$

P 10.48 **[a]** First find the Thévenin equivalent:

$$j\omega L = j3000\,\Omega$$

$$Z_{\text{Th}} = 6000\|12{,}000 + j3000 = 4000 + j3000\,\Omega$$

$$\mathbf{V}_{\text{Th}} = \frac{12{,}000}{6000 + 12{,}000}(180) = 120\,\text{V}$$

$$\frac{-j}{\omega C} = -j1000\,\Omega$$

$$\mathbf{I} = \frac{120}{6000 + j2000} = 18 - j6\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(2000) = 360\,\text{mW}$$

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[b] Set $C_o = 0.1\,\mu\text{F}$ so $-j/\omega C = -j2000\,\Omega$
Set R_o as close as possible to

$$R_o = \sqrt{4000^2 + (3000 - 2000)^2} = 4123.1\,\Omega$$

$$\therefore \quad R_o = 4000\,\Omega$$

[c] $\mathbf{I} = \dfrac{120}{8000 + j1000} = 14.77 - j1.85\,\text{mA}$

$$P = \frac{1}{2}|\mathbf{I}|^2(4000) = 443.1\,\text{mW}$$

Yes; $443.1\,\text{mW} > 360\,\text{mW}$

[d] $\mathbf{I} = \dfrac{120}{8000} = 15\,\text{mA}$

$$P = \frac{1}{2}(0.015)^2(4000) = 450\,\text{mW}$$

[e] $R_o = 4000\,\Omega$; $C_o = 66.67\,\text{nF}$

[f] Yes; $450\,\text{mW} > 443.1\,\text{mW}$

P 10.49 **[a]** Set $C_o = 0.1\,\mu\text{F}$, so $-j/\omega C = -j2000\,\Omega$; also set $R_o = 4123.1\,\Omega$

$$\mathbf{I} = \frac{120}{8123.1 + j1000} = 14.55 - j1.79\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(4123.1) = 443.18\,\text{mW}$$

[b] Yes; $443.18\,\text{mW} > 360\,\text{mW}$

[c] Yes; $443.18\,\text{mW} < 450\,\text{mW}$

P 10.50 **[a]** $\dfrac{1}{\omega C} = 100\,\Omega$; $C = \dfrac{1}{(60)(200\pi)} = 26.53\,\mu\text{F}$

[b] $\mathbf{V}_{\text{swo}} = 4000 + (40)(1.25 + j10) = 4050 + j400$

$$= 4069.71\underline{/5.64^\circ}\,\text{V(rms)}$$

$$\mathbf{V}_{\text{sw}} = 4000 + (40 - j40)(1.25 + j10) = 4450 + j350 = 4463.73\underline{/4.50^\circ}\,\text{V(rms)}$$

$$\%\text{ increase } = \left(\frac{4463.73}{4069.71} - 1\right)(100) = 9.68\%$$

[c] $P_{\ell\text{wo}} = (40\sqrt{2})^2(1.25) = 4000\,\text{W}$

$$P_{\ell\text{w}} = 40^2(1.25) = 2000\,\text{W}$$

$$\%\text{ increase } = \left(\frac{4000}{2000} - 1\right)(100) = 100\%$$

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P 10.51 **[a]**

$$5120 = 30\mathbf{I}_1 + j100\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2) + j64(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1$$

$$0 = 160\mathbf{I}_2 + j64(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1$$

Solving,

$$\mathbf{I}_1 = 8 - j20\,\text{A(rms)}; \qquad \mathbf{I}_2 = 13\underline{/0^\circ}\,\text{A(rms)}$$

$$\therefore \quad \mathbf{V}_o = (13)(160) = 2080\,\text{V(rms)}$$

[b] $P = (13)^2(160) = 27{,}040\,\text{W}$

[c] $S_g = -(5120)(8 + j20) = -40{,}960 - j102{,}400\,\text{VA} \quad \therefore \quad P_g = -40{,}960\,\text{W}$

$$\%\text{ delivered } = \frac{27{,}040}{40{,}960}(100) = 66\%$$

P 10.52 **[a]** Open circuit voltage:

$$5120 = 30\mathbf{I}_1 + j100\mathbf{I}_1 + j40\mathbf{I}_1 + j64\mathbf{I}_1 + j40\mathbf{I}_1$$

$$\therefore \quad \mathbf{I}_1 = \frac{5120}{30 + j244} = 2.54 - j20.67\,\text{A(rms)}$$

$$\mathbf{V}_{\text{Th}} = j64\mathbf{I}_1 + j40\mathbf{I}_1 = j104\mathbf{I}_1 = 2149.8 + j264.32\,\text{V}$$

Short circuit current:

$$5120 = 30\mathbf{I}_1 + j100\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) + j64(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) + j40\mathbf{I}_1$$

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$$0 = j64(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) - j40\mathbf{I}_1$$

Solving,

$$\mathbf{I}_{\text{sc}} = 38.25 - j95.63\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{2149.8 + j264.32}{38.25 - j95.63} = 5.37 + j20.33\,\Omega$$

$$\mathbf{I}_{\text{L}} = \frac{2149.8 + j264.32}{10.74} = 200.17 + j24.61 = 201.67\underline{/7.01^\circ}\,\text{A}$$

$$P_{\text{L}} = (201.67)^2(5.37) = 218.4\,\text{kW}$$

[b] $Z_o\mathbf{I}_2 + j64(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 = 0$ so $\mathbf{I}_1 = \dfrac{Z_o + j64}{j104}\mathbf{I}_2$

$$\therefore \qquad \mathbf{I}_1 = \frac{5.37 - j20.33 + j64}{j104}(200.17 + j24.61) = 85.32\underline{/0^\circ}\,\text{A(rms)}$$

$$P_{\text{dev}} = (5120)(85.32) = 436.8\,\text{kW}$$

[c] Begin by choosing the capacitor value from Appendix H that is closest to the required reactive impedance, assuming the frequency of the source is 60 Hz:

$$20.33 = \frac{1}{2\pi(60)C} \qquad \text{so} \qquad C = \frac{1}{2\pi(60)(20.33)} = 130.48\,\mu\text{F}$$

Choose the capacitor value closest to this capacitance from Appendix H, which is $100\,\mu$F. Then,

$$X_{\text{L}} = -\frac{1}{2\pi(60)(100 \times 10^{-6})} = -26.5258\,\Omega$$

Now set R_{L} as close as possible to $\sqrt{R_{\text{Th}}^2 + (X_{\text{L}} + X_{\text{Th}})^2}$:

$$R_{\text{L}} = \sqrt{5.37^2 + (20.33 - 26.5258)^2} = 8.2\,\Omega$$

The closest single resistor value from Appendix H is $10\,\Omega$. The resulting real power developed by the source is calculated below, using the Thévenin equivalent circuit:

$$\mathbf{I} = \frac{2149.8 + j264.32}{5.37 + j20.33 + 10 - j26.5258} = 130.7\underline{/28.96^\circ}$$

$$P = |2149.8 + j264.32|(130.7) = 283.1\,\text{kW} \qquad \text{(instead of 436.8 kW)}$$

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P 10.53 Open circuit voltage:

$$\mathbf{I}_1 = \frac{30\underline{/0^\circ}}{15 + j30} = 0.4 - j0.8\,\text{A}$$

$$\mathbf{V}_{\text{Th}} = j30\mathbf{I}_1 - j18\mathbf{I}_1 = j12\mathbf{I}_1 = 9.6 + j4.8 = 10.73\underline{/26.57^\circ}$$

Short circuit current:

$$30\underline{/0^\circ} = 15\mathbf{I}_1 + j30(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) + j18\mathbf{I}_{\text{sc}}$$

$$0 = j15\mathbf{I}_{\text{sc}} - j18(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) + j30(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) - j18\mathbf{I}_{\text{sc}}$$

Solving,

$$\mathbf{I}_{\text{sc}} = 1.95\underline{/-43.025^\circ}\,\text{A}$$

$$Z_{\text{Th}} = \frac{9.6 + j4.8}{1.95\underline{/-43.025^\circ}} = 1.92 + j5.16\,\Omega$$

$$\therefore \quad \mathbf{I}_2 = \frac{9.6 + j4.8}{3.84} = 2.795\underline{/26.57^\circ}\,\text{A}$$

$$30\underline{/0^\circ} = 15\mathbf{I}_1 + j30(\mathbf{I}_1 i\mathbf{I}_2) + j18\mathbf{I}_2$$

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$$\therefore \quad \mathbf{I}_1 = \frac{30 + j12\mathbf{I}_2}{15 + j30} = \frac{30 + j12(2.795\underline{/26.57^\circ})}{15 + j30} = 1\,\text{A}$$

$$Z_g = \frac{30\underline{/0^\circ}}{1} = 30 + j0 = 30\underline{/0^\circ}\,\Omega$$

P 10.54 **[a]**

Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{150}{120 + j160}(j100) = 60 + j45\,\text{V}$$

Short circuit:

$$(120 + j160)\mathbf{I}_1 - j100\mathbf{I}_{\text{sc}} = 150$$

$$-j100\mathbf{I}_1 + (30 + j160)\mathbf{I}_{\text{sc}} = 0$$

Solving,

$$\mathbf{I}_{\text{sc}} = 0.5 - j2.5\,\text{A}$$

$$Z_{\text{Th}} = \frac{60 + j45}{0.5 - j2.5} = 64 + j112\,\Omega$$

$$\therefore \quad Z_{\text{L}} = Z_{\text{Th}}^* = 64 - j112\,\Omega$$

$$\mathbf{I}_{\text{L}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}} + Z_L} = \frac{60 + j45}{128} = 0.586\underline{/36.87^\circ}\,\text{A(rms)}$$

$$P_{\text{L}} = |\mathbf{I}_{\text{L}}|^2(64) = 21.98\,\text{W}$$

[b] $$\mathbf{I}_1 = \frac{Z_{22}\mathbf{I}_2}{j\omega M} = \frac{94 + j48}{j100}(0.586\underline{/36.87^\circ}) = 0.62\underline{/-26.08^\circ}\,\text{A (rms)}$$

$$P_{\text{transformer}} = (150)(0.62)\cos(-26.08^\circ) - (0.62)^2(40) = 68.15\,\text{W}$$

$$\%\text{ delivered } = \frac{21.98}{68.15}(100) = 32.25\%$$

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P 10.55 **[a]** $j\omega L_1 = j\omega L_2 = j(400)(625 \times 10^{-3}) = j250\,\Omega$

$j\omega M = j(400)(312.5 \times 10^{-3}) = j125\,\Omega$

$400 = (125 + j250)\mathbf{I}_g - j125\mathbf{I}_\mathrm{L}$

$0 = -j125\mathbf{I}_g + (375 + j250)\mathbf{I}_\mathrm{L}$

Solving,

$\mathbf{I}_g = 0.8 - j1.2\,\mathrm{A}; \qquad \mathbf{I}_\mathrm{L} = 0.4\,\mathrm{A}$

Thus,

$i_g = 1.44\cos(400t - 56.31^\circ)\,\mathrm{A}$

$i_\mathrm{L} = 0.4\cos 400t\,\mathrm{A}$

[b] $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{0.3125}{0.625} = 0.5$

[c] When $t = 1.25\pi$ ms:

$400t = (400)(1.25\pi) \times 10^{-3} = 0.5\pi \text{ rad } = 90^\circ$

$i_g(1.25\pi\,\mathrm{ms}) = 1.44\cos(90^\circ - 56.31^\circ) = 1.2\,\mathrm{A}$

$i_\mathrm{L}(1.25\pi\,\mathrm{ms}) = 0.4\cos(90^\circ) = 0\,\mathrm{A}$

$w = \dfrac{1}{2}L_1 i_1^2 + \dfrac{1}{2}L_2 i_2^2 - M i_1 i_2 = \dfrac{1}{2}(0.625)(1.2)^2 + 0 - 0 = 450\,\mathrm{mJ}$

When $t = 2.5\pi$ ms:

$400t = (400)(2.5\pi) \times 10^{-3} = \pi = 180^\circ$

$i_g(2.5\pi\,\mathrm{ms}) = 1.44\cos(180 - 56.31^\circ) = -0.8\,\mathrm{A}$

$i_\mathrm{L}(2.5\pi\,\mathrm{ms}) = 0.4\cos(180) = -0.4\,\mathrm{A}$

$w = \dfrac{1}{2}(0.625)(0.8)^2 + \dfrac{1}{2}(0.625)(-0.4)^2 - (0.3125)(-0.8)(-0.4) = 150\,\mathrm{mJ}$

[d] From (a), $I_L = 0.4$ A,

$\therefore \quad P = \dfrac{1}{2}(0.4)^2(375) = 30\,\mathrm{W}$

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[e] Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{400}{125 + j250}(j125) = 160 + j80\,\text{V}$$

Short circuit:

$$400 = (125 + j250)\mathbf{I}_1 - j125\mathbf{I}_{\text{sc}}$$

$$0 = -j125\mathbf{I}_1 + j250\mathbf{I}_{\text{sc}}$$

Solving,

$$\mathbf{I}_{\text{sc}} = 0.4923 - j0.7385$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{160 + j80}{0.4923 - j0.7385} = 25 + j200\,\Omega$$

$$\therefore \quad R_{\text{L}} = 201.56\,\Omega$$

[f]

$$\mathbf{I} = \frac{160 + j80}{226.56 + j200} = 0.592\underline{/-14.87^\circ}\,\text{A}$$

$$P = \frac{1}{2}(0.592)^2(201.56) = 35.3\,\text{W}$$

[g] $Z_{\text{L}} = Z_{\text{Th}}^* = 25 - j200\,\Omega$

[h] $\mathbf{I} = \dfrac{160 + j80}{50} = 3.58\underline{/26.57^\circ}$

$$P = \frac{1}{2}(3.58)^2(25) = 160\,\text{W}$$

P 10.56 **[a]**

$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j3\mathbf{I}_2$$

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$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j3\mathbf{I}_2 + j8\mathbf{I}_2 + j3(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

$$\mathbf{I}_1 = 12 - j21\,\text{A (rms)}; \qquad \mathbf{I}_2 = -3\,\text{A (rms)}$$

$$\mathbf{V}_o = 7\mathbf{I}_2 = -21\underline{/0^\circ}\,\text{V(rms)}$$

[b] $P = |\mathbf{I}_2|^2(7) = 63\,\text{W}$

[c] $P_g = (54)(12) = 648\,\text{W}$

$$\%\text{ delivered } = \frac{63}{648}(100) = 9.72\%$$

P 10.57 **[a]**

$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j4k\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j4k\mathbf{I}_2 + j8\mathbf{I}_2 + j4k(\mathbf{I}_1 - \mathbf{I}_2)$$

Place the equations in standard form:

$$54 = (1 + j2)\mathbf{I}_1 + j(4k - 2)\mathbf{I}_2$$

$$0 = j(4k - 2)\mathbf{I}_1 + [7 + j(10 - 8k)]\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{54 - \mathbf{I}_2 j(4k - 2)}{(1 + j2)}$$

Substituting,

$$\mathbf{I}_2 = \frac{j54(4k - 2)}{[7 + j(10 - 8k)](1 + j2) - (4k - 2)}$$

For $\mathbf{V}_o = 0, \mathbf{I}_2 = 0$, so if $4k - 2 = 0$, then $k = 0.5$.

[b] When $\mathbf{I}_2 = 0$

$$\mathbf{I}_1 = \frac{54}{1 + j2} = 10.8 - j21.6\,\text{A(rms)}$$

$$P_g = (54)(10.8) = 583.2\,\text{W}$$

Check:

$$P_{\text{loss}} = |\mathbf{I}_1|^2(1) = 583.2\,\text{W}$$

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P 10.58 **[a]**

Open circuit:

$\mathbf{V}_{\text{Th}} = -j3\mathbf{I}_1 + j2\mathbf{I}_1 = -j\mathbf{I}_1$

$\mathbf{I}_1 = \dfrac{54}{1+j2} = 10.8 - j21.6$

$\mathbf{V}_{\text{Th}} = -21.6 - j10.8\,\text{V}$

Short circuit:

$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) + j3\mathbf{I}_{\text{sc}}$

$0 = j2(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) - j3\mathbf{I}_{\text{sc}} + j8\mathbf{I}_{\text{sc}} + j3(\mathbf{I}_1 - \mathbf{I}_{\text{sc}})$

Solving,

$\mathbf{I}_{\text{sc}} = -3.32 + j5.82$

$Z_{\text{Th}} = \dfrac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \dfrac{-21.6 - j10.8}{-3.32 + j5.82} = 0.2 + j3.6 = 3.6\underline{/86.86^\circ}\,\Omega$

$\therefore \quad R_{\text{L}} = |Z_{\text{Th}}| = 3.6\,\Omega$

[b]

$\mathbf{I} = \dfrac{-21.6 - j10.8}{3.8 + j3.6} = 4.614\underline{/163.1^\circ}$

$P = |\mathbf{I}|^2(3.6) = 76.6\,\text{W}, \quad$ which is greater than when $R_L = 7\,\Omega$

P 10.59 **[a]** $Z_{\text{ab}} = 50 - j400 = \left(1 - \dfrac{N_1}{N_2}\right)^2 Z_{\text{L}}$

$\therefore \quad Z_{\text{L}} = \dfrac{1}{(1-6)^2}(50 - j400) = 2 - j16\,\Omega$

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[b]

$$\mathbf{I}_1 = \frac{24}{100} = 240\underline{/0^\circ}\,\text{mA}$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

$$\mathbf{I}_2 = -6\mathbf{I}_1 = -1.44\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_\text{L} = \mathbf{I}_1 + \mathbf{I}_2 = -1.68\underline{/0^\circ}\,\text{A}$$

$$\mathbf{V}_\text{L} = (2 - j16)\mathbf{I}_\text{L} = -3.36 + j26.88 = 27.1\underline{/97.13^\circ}\,\text{V(rms)}$$

P 10.60 **[a]** Replace the circuit to the left of the primary winding with a Thévenin equivalent:

$$\mathbf{V}_\text{Th} = (15)(20\|j10) = 60 + j120\,\text{V}$$

$$Z_\text{Th} = 2 + 20\|j10 = 6 + j8\,\Omega$$

Transfer the secondary impedance to the primary side:

$$Z_p = \frac{1}{25}(100 - jX_\text{C}) = 4 - j\frac{X_\text{C}}{25}\,\Omega$$

Now maximize $\mathbf{I}$ by setting $(X_\text{C}/25) = 8\,\Omega$:

$$\therefore\quad C = \frac{1}{200(20 \times 10^3)} = 0.25\,\mu\text{F}$$

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[b] $\mathbf{I} = \dfrac{60 + j120}{10} = 6 + j12\,\text{A}$

$P = |\mathbf{I}|^2(4) = 720\,\text{W}$

[c] $\dfrac{R_o}{25} = 6\,\Omega; \qquad \therefore \quad R_o = 150\,\Omega$

[d] $\mathbf{I} = \dfrac{60 + j120}{12} = 5 + j10\,\text{A}$

$P = |\mathbf{I}|^2(6) = 750\,\text{W}$

P 10.61

$$\frac{\mathbf{V}_a}{1} = \frac{-\mathbf{V}_o}{50}; \qquad 1\mathbf{I}_a = -50\mathbf{I}_o$$

$$\therefore \qquad \frac{\mathbf{V}_a}{\mathbf{I}_a} = \frac{-\mathbf{V}_o/50}{-50\mathbf{I}_o} = \frac{\mathbf{V}_o/\mathbf{I}_o}{50^2} = \frac{5000}{50^2} = 2\,\Omega$$

$$\frac{\mathbf{V}_b}{25} = \frac{\mathbf{V}_a}{1}; \qquad 25\mathbf{I}_b = 1\mathbf{I}_a$$

$$\therefore \qquad \frac{\mathbf{V}_b/25}{25\mathbf{I}_b} = \frac{\mathbf{V}_b/\mathbf{I}_b}{25^2} = \frac{\mathbf{V}_a}{\mathbf{I}_a}$$

$$\therefore \qquad \frac{\mathbf{V}_b}{\mathbf{I}_b} = 25^2\frac{\mathbf{V}_a}{\mathbf{I}_a} = 25^2(2) = 1250\,\Omega$$

Thus $\mathbf{I}_\text{b} = [145/(200 + 1250)] = 100\,\text{mA}$ (rms); since the ideal transformers are lossless, $P_{5\text{k}\Omega} = P_{1250\Omega}$, and the power delivered to the $1250\,\Omega$ resistor is $(0.1)^2(1250)$ or $12.5\,\text{W}$.

P 10.62 **[a]** $\dfrac{\mathbf{V}_\text{b}}{\mathbf{I}_\text{b}} = \dfrac{25^2(5000)}{a^2} = 200\,\Omega; \qquad$ therefore $\quad a^2 = 15{,}625, \qquad a = 125$

[b] $\mathbf{I}_\text{b} = \dfrac{145}{400} = 362.5\,\text{mA}; \qquad P = (0.3625)^2(200) = 26.28125\,\text{W}$

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P 10.63 **[a]**

For maximum power transfer, $Z_{\text{ab}} = 90\,\text{k}\Omega$

$$Z_{\text{ab}} = \left(1 - \frac{N_1}{N_2}\right)^2 Z_{\text{L}}$$

$$\therefore \quad \left(1 - \frac{N_1}{N_2}\right)^2 = \frac{90{,}000}{400} = 225$$

$$1 - \frac{N_1}{N_2} = \pm 15; \qquad \frac{N_1}{N_2} = 15 + 1 = 16$$

[b] $P = |\mathbf{I}_i|^2(90{,}000) = \left(\dfrac{180}{180{,}000}\right)^2 (90{,}000) = 90\,\text{mW}$

[c] $\mathbf{V}_1 = R_i\mathbf{I}_i = (90{,}000)\left(\dfrac{180}{180{,}000}\right) = 90\,\text{V}$

[d]

$$\mathbf{V}_g = (2.25 \times 10^{-3})(100{,}000\|80{,}000) = 100\,\text{V}$$

$$P_g(\text{del}) = (2.25 \times 10^{-3})(100) = 225\,\text{mW}$$

$$\%\ \text{delivered} = \frac{90}{225}(100) = 40\%$$

P 10.64 **[a]** $Z_{\text{Th}} = 720 + j1500 + \left(\dfrac{200}{50}\right)^2 (40 - j30) = 1360 + j1020 = 1700\underline{/36.87^\circ}\,\Omega$

$$\therefore \quad Z_{\text{ab}} = 1700\,\Omega$$

$$Z_{\text{ab}} = \frac{Z_{\text{L}}}{(1 + N_1/N_2)^2}$$

$$(1 + N_1/N_2)^2 = 6800/1700 = 4$$

$$\therefore \quad N_1/N_2 = 1 \quad \text{or} \quad N_2 = N_1 = 1000 \text{ turns}$$

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[b] $\mathbf{V}_{\text{Th}} = \dfrac{255\underline{/0^\circ}}{40 + j30}(j200) = 1020\underline{/53.13^\circ}\,\text{V}$

$$\mathbf{I}_L = \frac{1020\underline{/53.13^\circ}}{3060 + j1020} = 0.316\underline{/34.7^\circ}\,\text{A(rms)}$$

Since the transformer is ideal, $P_{6800} = P_{1700}$.

$P = |\mathbf{I}|^2(1700) = 170\,\text{W}$

[c]

$255\underline{/0^\circ} = (40 + j30)\mathbf{I}_1 - j200(0.26 + j0.18)$

$\therefore \quad \mathbf{I}_1 = 4.13 - j1.80\,\text{A(rms)}$

$P_{\text{gen}} = (255)(4.13) = 1053\,\text{W}$

$P_{\text{diss}} = 1053 - 170 = 883\,\text{W}$

$\%\text{ dissipated} = \dfrac{883}{1053}(100) = 83.85\%$

P 10.65 **[a]** Open circuit voltage:

$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + \mathbf{V}_{\text{Th}}$

$\dfrac{\mathbf{I}_1}{4} = -\mathbf{I}_3; \qquad \mathbf{I}_1 = -4\mathbf{I}_3$

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Solving,

$\mathbf{V}_{\text{Th}} = 40\underline{/0^\circ}\,\text{V}$

Short circuit current:

$40\underline{/0^\circ} = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$

$4\mathbf{V}_1 = 16(\mathbf{I}_1/4) = 4\mathbf{I}_1; \qquad \therefore \quad \mathbf{V}_1 = \mathbf{I}_1$

$\therefore \quad 40\underline{/0^\circ} = 6\mathbf{I}_1 + 4\mathbf{I}_3$

Also,

$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 6\,\text{A}; \qquad \mathbf{I}_3 = 1\,\text{A}; \qquad \mathbf{I}_{\text{sc}} = \mathbf{I}_1/4 + \mathbf{I}_3 = 2.5\,\text{A}$

$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{40}{2.5} = 16\,\Omega$$

$$\mathbf{I} = \frac{40\underline{/0^\circ}}{32} = 1.25\underline{/0^\circ}\,\text{A(rms)}$$

$P = (1.25)^2(16) = 25\,\text{W}$

[b]

$40 = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + 20$

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$$4\mathbf{V}_1 = 4\mathbf{I}_1 + 16(\mathbf{I}_1/4 + \mathbf{I}_3); \qquad \therefore \quad \mathbf{V}_1 = 2\mathbf{I}_1 + 4\mathbf{I}_3$$

$$40 = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$$

$$\therefore \quad \mathbf{I}_1 = 6\,\text{A}; \qquad \mathbf{I}_3 = -0.25\,\text{A}; \qquad \mathbf{I}_1 + \mathbf{I}_3 = 5.75\underline{/0^\circ}\,\text{A}$$

$$P_{40\text{V}}(\text{developed}) = 40(5.75) = 230\,\text{W}$$

$$\therefore \quad \%\text{ delivered } = \frac{25}{230}(100) = 10.87\%$$

[**c**] $P_{R_L} = 25\,\text{W}; \qquad P_{16\Omega} = (1.5)^2(16) = 36\,\text{W}$

$$P_{4\Omega} = (5.75)^2(4) = 132.25\,\text{W}; \qquad P_{1\Omega} = (6)^2(1) = 36\,\text{W}$$

$$P_{12\Omega} = (-0.25)^2(12) = 0.75\,\text{W}$$

$$\sum P_{\text{abs}} = 25 + 36 + 132.25 + 36 + 0.75 = 230\,\text{W} = \sum P_{\text{dev}}$$

P 10.66 [**a**] Open circuit voltage:

$$500 = 100\mathbf{I}_1 + \mathbf{V}_1$$

$$\mathbf{V}_2 = 400\mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{1} = \frac{\mathbf{V}_2}{2} \qquad \therefore \qquad \mathbf{V}_2 = 2\mathbf{V}_1$$

$$\mathbf{I}_1 = 2\mathbf{I}_2$$

Substitute and solve:

$$2\mathbf{V}_1 = 400\mathbf{I}_1/2 = 200\mathbf{I}_1 \qquad \therefore \qquad \mathbf{V}_1 = 100\mathbf{I}_1$$

$$500 = 100\mathbf{I}_1 + 100\mathbf{I}_1 \qquad \therefore \qquad \mathbf{I}_1 = 500/200 = 2.5\,\text{A}$$

$$\therefore \qquad \mathbf{I}_2 = \frac{1}{2}\mathbf{I}_1 = 1.25\,\text{A}$$

$$\mathbf{V}_1 = 100(2.5) = 250\,\text{V}; \qquad \mathbf{V}_2 = 2\mathbf{V}_1 = 500\,\text{V}$$

$$V_{\text{Th}} = 20\mathbf{I}_1 + \mathbf{V}_1 - \mathbf{V}_2 + 40\mathbf{I}_2 = -150\,\text{V(rms)}$$

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Short circuit current:

$$500 = 80(\mathbf{I}_{\text{sc}} + \mathbf{I}_1) + 360(\mathbf{I}_{\text{sc}} + 0.5\mathbf{I}_1)$$

$$2\mathbf{V}_1 = 40\frac{\mathbf{I}_1}{2} + 360(\mathbf{I}_{\text{sc}} + 0.5\mathbf{I}_1)$$

$$500 = 80(\mathbf{I}_1 + \mathbf{I}_{\text{sc}}) + 20\mathbf{I}_1 + \mathbf{V}_1$$

Solving,

$$\mathbf{I}_{\text{sc}} = -1.47\,\text{A}$$

$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{-150}{-1.47} = 102\,\Omega$$

$$P = \frac{75^2}{102} = 55.15\,\text{W}$$

[b]

$$500 = 80[\mathbf{I}_1 - (75/102)] - 75 + 360[\mathbf{I}_2 - (75/102)]$$

$$575 + \frac{6000}{102} + \frac{27{,}000}{102} = 80\mathbf{I}_1 + 180\mathbf{I}_2$$

$$\therefore \qquad \mathbf{I}_1 = 3.456\,\text{A}$$

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$$P_{\text{source}} = (500)[3.456 - (75/102)] = 1360.35\,\text{W}$$

$$\%\ \text{delivered} = \frac{55.15}{1360.35}(100) = 4.05\%$$

[c] $P_{80\Omega} = 80(\mathbf{I}_1 + \mathbf{I}_L)^2 = 592.13\,\text{W}$

$$P_{20\Omega} = 20\mathbf{I}_1^2 = 238.86\,\text{W}$$

$$P_{40\Omega} = 40\mathbf{I}_2^2 = 119.43\,\text{W}$$

$$P_{102\Omega} = 102\mathbf{I}_L^2 = 55.15\,\text{W}$$

$$P_{360\Omega} = 360(\mathbf{I}_2 + \mathbf{I}_L)^2 = 354.73\,\text{W}$$

$$\sum P_{\text{abs}} = 592.13 + 238.86 + 119.43 + 55.15 + 354.73 = 1360.3\,\text{W} = \sum P_{\text{dev}}$$

P 10.67 **[a]** $\dfrac{30[5(44.28) + 19(15.77)]}{1000} = 15.63\ \text{kWh}$

[b] $\dfrac{30[5(44.28) + 19(8.9)]}{1000} = 11.72\ \text{kWh}$

[c] $\dfrac{30[5(44.28) + 19(4.42)]}{1000} = 9.16\ \text{kWh}$

[d] $\dfrac{30[5(44.28) + 19(0)]}{1000} = 6.64\ \text{kWh}$

Note that this is about 40 % of the amount of total power consumed in part (a).

P 10.68 **[a]** $\dfrac{30[0.2(1433) + 23.8(3.08)]}{1000} = 10.8\ \text{kWh}$

[b] The standby power consumed in one month by the microwave oven when in the ready state is

$$\frac{30[23.8(3.08)]}{1000} = 2.2\ \text{kWh}$$

This is $(2.2/10.8) * 100 = 20.4\%$ of the total power consumed by the microwave in one month. Since it is not practical to unplug the microwave when you are not using it, this is the cost associated with having a microwave oven.

P 10.69 $j\omega L_1 = j(2\pi)(60)(0.25) = j94.25\,\Omega$

$$\mathbf{I} = \frac{120}{5 + j94.25} = 1.27\underline{/-86.96^\circ}\ \text{A(rms)}$$

$$P = R_1|\mathbf{I}|^2 = 5(1.27)^2 = 8.06\,\text{W}$$

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P 10.70 $j\omega L_1 = j(2\pi)(60)(0.25) = j94.25\,\Omega$

$$\mathbf{I} = \frac{120}{0.05 + j94.25} = 1.27\underline{/-86.97^\circ}\ \text{A(rms)}$$

$$P = R_1|\mathbf{I}|^2 = 0.05(1.27)^2 = 81.1\,\text{mW}$$

Note that while the current supplied by the voltage source is virtually identical to that calculated in Problem 10.69, the much smaller value of transformer resistance results in a much smaller value of real power consumed by the transformer.

P 10.71 An ideal transformer has no resistance, so consumes no real power. This is one of the important characteristics of ideal transformers.

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11

Balanced Three-Phase Circuits

Assessment Problems

AP 11.1 Make a sketch:

We know $\mathbf{V}_{AN}$ and wish to find $\mathbf{V}_{BC}$. To do this, write a KVL equation to find $\mathbf{V}_{AB}$, and use the known phase angle relationship between $\mathbf{V}_{AB}$ and $\mathbf{V}_{BC}$ to find $\mathbf{V}_{BC}$.

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} + \mathbf{V}_{NB} = \mathbf{V}_{AN} - \mathbf{V}_{BN}$$

Since $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$ form a balanced set, and $\mathbf{V}_{AN} = 240\underline{/-30^\circ}$V, and the phase sequence is positive,

$$\mathbf{V}_{BN} = |\mathbf{V}_{AN}|\underline{/\angle\mathbf{V}_{AN} - 120^\circ} = 240\underline{/-30^\circ - 120^\circ} = 240\underline{/-150^\circ}\ \text{V}$$

Then,

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = (240\underline{/-30^\circ}) - (240\underline{/-150^\circ}) = 415.46\underline{/0^\circ}\ \text{V}$$

Since $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ form a balanced set with a positive phase sequence, we can find $\mathbf{V}_{BC}$ from $\mathbf{V}_{AB}$:

$$\mathbf{V}_{BC} = |\mathbf{V}_{AB}|\underline{/(\angle\mathbf{V}_{AB} - 120^\circ)} = 415.69\underline{/0^\circ - 120^\circ} = 415.69\underline{/-120^\circ}\ \text{V}$$

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Thus,

$$\mathbf{V}_{\text{BC}} = 415.69\underline{/-120^\circ}\ \text{V}$$

AP 11.2 Make a sketch:

We know $\mathbf{V}_{\text{CN}}$ and wish to find $\mathbf{V}_{\text{AB}}$. To do this, write a KVL equation to find $\mathbf{V}_{\text{BC}}$, and use the known phase angle relationship between $\mathbf{V}_{\text{AB}}$ and $\mathbf{V}_{\text{BC}}$ to find $\mathbf{V}_{\text{AB}}$.

$$\mathbf{V}_{\text{BC}} = \mathbf{V}_{\text{BN}} + \mathbf{V}_{\text{NC}} = \mathbf{V}_{\text{BN}} - \mathbf{V}_{\text{CN}}$$

Since $\mathbf{V}_{\text{AN}}$, $\mathbf{V}_{\text{BN}}$, and $\mathbf{V}_{\text{CN}}$ form a balanced set, and $\mathbf{V}_{\text{CN}} = 450\underline{/-25^\circ}$ V, and the phase sequence is negative,

$$\mathbf{V}_{\text{BN}} = |\mathbf{V}_{\text{CN}}|\underline{/\underline{/\mathbf{V}_{\text{CN}}} - 120^\circ} = 450\underline{/-23^\circ - 120^\circ} = 450\underline{/-145^\circ}\ \text{V}$$

Then,

$$\mathbf{V}_{\text{BC}} = \mathbf{V}_{\text{BN}} - \mathbf{V}_{\text{CN}} = (450\underline{/-145^\circ}) - (450\underline{/-25^\circ}) = 779.42\underline{/-175^\circ}\ \text{V}$$

Since $\mathbf{V}_{\text{AB}}$, $\mathbf{V}_{\text{BC}}$, and $\mathbf{V}_{\text{CA}}$ form a balanced set with a negative phase sequence, we can find $\mathbf{V}_{\text{AB}}$ from $\mathbf{V}_{\text{BC}}$:

$$\mathbf{V}_{\text{AB}} = |\mathbf{V}_{\text{BC}}|\underline{/\underline{/\mathbf{V}_{\text{BC}}} - 120^\circ} = 779.42\underline{/-295^\circ}\ \text{V}$$

But we normally want phase angle values between $+180^\circ$ and -180°. We add 360° to the phase angle computed above. Thus,

$$\mathbf{V}_{\text{AB}} = 779.42\underline{/65^\circ}\ \text{V}$$

AP 11.3 Sketch the a-phase circuit:

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[**a**] We can find the line current using Ohm's law, since the a-phase line current is the current in the a-phase load. Then we can use the fact that $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$ form a balanced set to find the remaining line currents. Note that since we were not given any phase angles in the problem statement, we can assume that the phase voltage given, $\mathbf{V}_{AN}$, has a phase angle of 0°.

$$2400\underline{/0^\circ} = \mathbf{I}_{aA}(16 + j12)$$

so

$$\mathbf{I}_{aA} = \frac{2400\underline{/0^\circ}}{16 + j12} = 96 - j72 = 120\underline{/-36.87^\circ}\ \text{A}$$

With an acb phase sequence,

$$\underline{/\mathbf{I}_{bB}} = \underline{/\mathbf{I}_{aA}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{I}_{cC}} = \underline{/\mathbf{I}_{aA}} - 120^\circ$$

so

$$\mathbf{I}_{aA} = 120\underline{/-36.87^\circ}\ \text{A}$$

$$\mathbf{I}_{bB} = 120\underline{/83.13^\circ}\ \text{A}$$

$$\mathbf{I}_{cC} = 120\underline{/-156.87^\circ}\ \text{A}$$

[**b**] The line voltages at the source are $\mathbf{V}_{ab}$ $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$. They form a balanced set. To find $\mathbf{V}_{ab}$, use the a-phase circuit to find $\mathbf{V}_{AN}$, and use the relationship between phase voltages and line voltages for a y-connection (see Fig. 11.9[b]). From the a-phase circuit, use KVL:

$$\mathbf{V}_{an} = \mathbf{V}_{aA} + \mathbf{V}_{AN} = (0.1 + j0.8)\mathbf{I}_{aA} + 2400\underline{/0^\circ}$$

$$= (0.1 + j0.8)(96 - j72) + 2400\underline{/0^\circ} = 2467.2 + j69.6$$

$$2468.18\underline{/1.62^\circ}\ \text{V}$$

From Fig. 11.9(b),

$$\mathbf{V}_{ab} = \mathbf{V}_{an}(\sqrt{3}\underline{/-30^\circ}) = 4275.02\underline{/-28.38^\circ}\ \text{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{bc}} = \underline{/\mathbf{V}_{ab}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{ca}} = \underline{/\mathbf{V}_{ab}} - 120^\circ$$

so

$$\mathbf{V}_{ab} = 4275.02\underline{/-28.38^\circ}\ \text{V}$$

$$\mathbf{V}_{bc} = 4275.02\underline{/91.62^\circ}\ \text{V}$$

$$\mathbf{V}_{ca} = 4275.02\underline{/-148.38^\circ}\ \text{V}$$

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[c] Using KVL on the a-phase circuit

$$\mathbf{V}_{a'n} = \mathbf{V}_{a'a} + \mathbf{V}_{an} = (0.2 + j0.16)\mathbf{I}_{aA} + \mathbf{V}_{an}$$

$$= (0.02 + j0.16)(96 - j72) + (2467.2 + j69.9)$$

$$= 2480.64 + j83.52 = 2482.05\underline{/1.93^\circ}\ \text{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{b'n}} = \underline{/\mathbf{V}_{a'n}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{c'n}} = \underline{/\mathbf{V}_{a'n}} - 120^\circ$$

so

$$\mathbf{V}_{a'n} = 2482.05\underline{/1.93^\circ}\ \text{V}$$

$$\mathbf{V}_{b'n} = 2482.05\underline{/121.93^\circ}\ \text{V}$$

$$\mathbf{V}_{c'n} = 2482.05\underline{/-118.07^\circ}\ \text{V}$$

AP 11.4

$$\mathbf{I}_{cC} = (\sqrt{3}\underline{/-30^\circ})\mathbf{I}_{CA} = (\sqrt{3}\underline{/-30^\circ}) \cdot 8\underline{/-15^\circ} = 13.86\underline{/-45^\circ}\ \text{A}$$

AP 11.5

$$\mathbf{I}_{aA} = 12\underline{/(65^\circ - 120^\circ)} = 12\underline{/-55^\circ}$$

$$\mathbf{I}_{AB} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right]\mathbf{I}_{aA} = \left(\frac{\underline{/-30^\circ}}{\sqrt{3}}\right) \cdot 12\underline{/-55^\circ}$$

$$= 6.93\underline{/-85^\circ}\ \text{A}$$

AP 11.6 **[a]** $\mathbf{I}_{AB} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/20^\circ}\ \text{A}$

Therefore $\quad Z_\phi = \dfrac{4160\underline{/0^\circ}}{40\underline{/20^\circ}} = 104\underline{/-20^\circ}\ \Omega$

[b] $\mathbf{I}_{AB} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/-40^\circ}\ \text{A}$

Therefore $\quad Z_\phi = 104\underline{/40^\circ}\ \Omega$

AP 11.7

$$\mathbf{I}_\phi = \frac{110}{3.667} + \frac{110}{j2.75} = 30 - j40 = 50\underline{/-53.13^\circ}\ \text{A}$$

Therefore $\quad |\mathbf{I}_{aA}| = \sqrt{3}\mathbf{I}_\phi = \sqrt{3}(50) = 86.60\ \text{A}$

AP 11.8 **[a]** $|S| = \sqrt{3}(208)(73.8) = 26{,}587.67\ \text{VA}$

$$Q = \sqrt{(26{,}587.67)^2 - (22{,}659)^2} = 13{,}909.50\ \text{VAR}$$

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[b] $\text{pf} = \dfrac{22{,}659}{26{,}587.67} = 0.8522 \quad \text{lagging}$

AP 11.9 **[a]** $\mathbf{V}_{\text{AN}} = \left(\dfrac{2450}{\sqrt{3}}\right)\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{\text{AN}}\mathbf{I}^*_{\text{aA}} = S_\phi = 144 + j192\,\text{kVA}$

Therefore

$$\mathbf{I}^*_{\text{aA}} = \frac{(144 + j192)1000}{2450/\sqrt{3}} = (101.8 + j135.7)\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 101.8 - j135.7 = 169.67\underline{/-53.13^\circ}\,\text{A}$$

$$|\mathbf{I}_{\text{aA}}| = 169.67\,\text{A}$$

[b] $P = \dfrac{(2450)^2}{R}; \qquad \text{therefore} \quad R = \dfrac{(2450)^2}{144{,}000} = 41.68\,\Omega$

$Q = \dfrac{(2450)^2}{X}; \qquad \text{therefore} \quad X = \dfrac{(2450)^2}{192{,}000} = 31.26\,\Omega$

[c] $Z_\phi = \dfrac{\mathbf{V}_{\text{AN}}}{\mathbf{I}_{\text{aA}}} = \dfrac{2450/\sqrt{3}}{169.67\underline{/-53.13^\circ}} = 8.34\underline{/53.13^\circ} = (5 + j6.67)\,\Omega$

$\therefore \quad R = 5\,\Omega, \qquad X = 6.67\,\Omega$

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Problems

P 11.1 **[a]** First, convert the cosine waveforms to phasors:

$\mathbf{V}_a = 137\underline{/63^\circ}$; $\mathbf{V}_b = 137\underline{/-57^\circ}$; $\mathbf{V}_c = 137\underline{/183^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_a} = 63^\circ - 63^\circ = 0^\circ$

$\underline{/\mathbf{V}'_b} = -57^\circ - 63^\circ = -120^\circ$

$\underline{/\mathbf{V}'_c} = 183^\circ - 63^\circ = 120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore abc

[b] First, convert the cosine waveforms to phasors, making sure that all waveforms are represented as cosines:

$\mathbf{V}_a = 820\underline{/-36^\circ}$; $\mathbf{V}_b = 820\underline{/84^\circ}$; $\mathbf{V}_c = 820\underline{/-156^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_a} = -36^\circ + 36^\circ = 0^\circ$

$\underline{/\mathbf{V}'_b} = 84^\circ + 36^\circ = 120^\circ$

$\underline{/\mathbf{V}'_c} = -156^\circ + 36^\circ = -120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore acb

P 11.2 **[a]** $\mathbf{V}_a = 48\underline{/-45^\circ}$ V

$\mathbf{V}_b = 48\underline{/-165^\circ}$ V

$\mathbf{V}_c = 48\underline{/75^\circ}$ V

Balanced, positive phase sequence

[b] $\mathbf{V}_a = 188\underline{/60^\circ}$ V

$\mathbf{V}_b = -188\underline{/0^\circ}$ V $= 188\underline{/180^\circ}$ V

$\mathbf{V}_c = 188\underline{/-60^\circ}$ V

Balanced, negative phase sequence

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[c] $\mathbf{V}_a = 426\underline{/0^\circ}$ V

$\mathbf{V}_b = 462\underline{/120^\circ}$ V

$\mathbf{V}_c = 426\underline{/-120^\circ}$ V

Unbalanced due to unequal amplitudes

[d] $\mathbf{V}_a = 1121\underline{/-20^\circ}$ V

$\mathbf{V}_b = 1121\underline{/-140^\circ}$ V

$\mathbf{V}_c = 1121\underline{/100^\circ}$ V

Balanced, positive phase sequence

[e] $\mathbf{V}_a = 540\underline{/-90^\circ}$ V

$\mathbf{V}_b = 540\underline{/-120^\circ}$ V

$\mathbf{V}_c = 540\underline{/120^\circ}$ V

Unbalanced due to unequal phase separation

[f] $\mathbf{V}_a = 144\underline{/80^\circ}$ V

$\mathbf{V}_b = 144\underline{/-160^\circ}$ V

$\mathbf{V}_c = 144\underline{/-40^\circ}$ V

Balanced, negative phase sequence

P 11.3 $\mathbf{V}_a = V_m\underline{/0^\circ} = V_m + j0$

$$\mathbf{V}_b = V_m\underline{/-120^\circ} = -V_m(0.5 + j0.866)$$

$$\mathbf{V}_c = V_m\underline{/120^\circ} = V_m(-0.5 + j0.866)$$

$$\begin{aligned}\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c &= (V_m)(1 + j0 - 0.5 - j0.866 - 0.5 + j0.866)\\ &= V_m(0) = 0\end{aligned}$$

P 11.4 $$\mathbf{I} = \frac{188\underline{/60^\circ} + 188\underline{/180^\circ} + 188\underline{/-60^\circ}}{3(R_W + jX_W)} = 0$$

P 11.5 $$\mathbf{I} = \frac{426\underline{/0^\circ} + 462\underline{/120^\circ} + 426\underline{/-120^\circ}}{3(R_W + jX_W)} = \frac{36\underline{/120^\circ}}{3(R_W + jX_W)}$$

P 11.6 [a] The voltage sources form a balanced set, the source impedances are equal and the line impedances are equal. But the load impedances are not equal. Therefore, the circuit is unbalanced. Also,

$$\mathbf{I}_{aA} = \frac{110}{32 - j24} = 2.75\underline{/36.87^\circ}\text{ A (rms)}$$

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$$\mathbf{I}_{\mathrm{bB}} = \frac{110\underline{/-120^\circ}}{6+j8} = 11\underline{/-173.13^\circ}\ \mathrm{A\ (rms)}$$

$$\mathbf{I}_{\mathrm{cC}} = \frac{110\underline{/120^\circ}}{40+j30} = 2.2\underline{/83.13^\circ}\ \mathrm{A\ (rms)}$$

The magnitudes are unequal and the phase angles are not 120° apart, so the currents are not balanced and thus the circuit is not balanced.

b] $\mathbf{I}_o = \mathbf{I}_{\mathrm{aA}} + \mathbf{I}_{\mathrm{bB}} + \mathbf{I}_{\mathrm{cC}} = 11.79\underline{/67.58^\circ}\ \mathrm{A\ (rms)}$

P 11.7 **[a]** $\mathbf{I}_{\mathrm{aA}} = \dfrac{277\underline{/0^\circ}}{80+j60} = 2.77\underline{/-36.87^\circ}\ \mathrm{A\ (rms)}$

$$\mathbf{I}_{\mathrm{bB}} = \frac{277\underline{/-120^\circ}}{80+j60} = 2.77\underline{/-156.87^\circ}\ \mathrm{A\ (rms)}$$

$$\mathbf{I}_{\mathrm{cC}} = \frac{277\underline{/120^\circ}}{80+j60} = 2.77\underline{/83.13^\circ}\ \mathrm{A\ (rms)}$$

$$\mathbf{I}_o = \mathbf{I}_{\mathrm{aA}} + \mathbf{I}_{\mathrm{bB}} + \mathbf{I}_{\mathrm{cC}} = 0$$

[b] $\mathbf{V}_{\mathrm{AN}} = (78+j54)\mathbf{I}_{\mathrm{aA}} = 262.79\underline{/-2.17^\circ}\ \mathrm{V\ (rms)}$

[c] $\mathbf{V}_{\mathrm{AB}} = \mathbf{V}_{\mathrm{AN}} - \mathbf{V}_{\mathrm{BN}}$

$$\mathbf{V}_{\mathrm{BN}} = (77+j56)\mathbf{I}_{\mathrm{bB}} = 263.73\underline{/-120.84^\circ}\ \mathrm{V\ (rms)}$$

$$\mathbf{V}_{\mathrm{AB}} = 262.79\underline{/-2.17^\circ} - 263.73\underline{/-120.84^\circ} = 452.89\underline{/28.55^\circ}\ \mathrm{V\ (rms)}$$

[d] Unbalanced — see conditions for a balanced circuit in the text

P 11.8 $Z_{ga} + Z_{la} + Z_{La} = 60 + j80\,\Omega$

$$Z_{gb} + Z_{lb} + Z_{Lb} = 90 + j120\,\Omega$$

$$Z_{gc} + Z_{lc} + Z_{Lc} = 30 + j40\,\Omega$$

$$\frac{\mathbf{V}_N - 320}{60+j80} + \frac{\mathbf{V}_N - 320\underline{/-120^\circ}}{90+j120} + \frac{\mathbf{V}_N - 320\underline{/120^\circ}}{30+j40} + \frac{\mathbf{V}_N}{20} = 0$$

Solving for $\mathbf{V}_N$ yields

$$\mathbf{V}_N = 49.47\underline{/75.14^\circ}\ \mathrm{V\ (rms)}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_N}{20} = 2.47\underline{/75.14^\circ}\ \mathrm{A\ (rms)}$$

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P 11.9 $\mathbf{V}_{\text{AN}} = 285\underline{/-45^\circ}\,\text{V}$

$\mathbf{V}_{\text{BN}} = 285\underline{/-165^\circ}\,\text{V}$

$\mathbf{V}_{\text{CN}} = 285\underline{/75^\circ}\,\text{V}$

$\mathbf{V}_{\text{AB}} = \mathbf{V}_{\text{AN}} - \mathbf{V}_{\text{BN}} = 498.83\underline{/-15^\circ}\,\text{V}$

$\mathbf{V}_{\text{BC}} = \mathbf{V}_{\text{BN}} - \mathbf{V}_{\text{CN}} = 498.83\underline{/-135^\circ}\,\text{V}$

$\mathbf{V}_{\text{CA}} = \mathbf{V}_{\text{CN}} - \mathbf{V}_{\text{AN}} = 498.83\underline{/105^\circ}\,\text{V}$

$v_{\text{AB}} = 498.83\cos(\omega t - 15^\circ)\,\text{V}$

$v_{\text{BC}} = 498.83\cos(\omega t - 135^\circ)\,\text{V}$

$v_{\text{CA}} = 498.83\cos(\omega t + 105^\circ)\,\text{V}$

P 11.10 **[a]** $\mathbf{V}_{\text{an}} = 1/\sqrt{3}\underline{/-30^\circ}\mathbf{V}_{\text{ab}} = 110\underline{/-90^\circ}\,\text{V (rms)}$

The a-phase circuit is

[b] $\mathbf{I}_{\text{aA}} = \dfrac{110\underline{/-90^\circ}}{40 + j30} = 2.2\underline{/-126.87^\circ}\,\text{A (rms)}$

[c] $\mathbf{V}_{\text{AN}} = (37 + j28)\mathbf{I}_{\text{aA}} = 102.08\underline{/-89.75^\circ}\,\text{V (rms)}$

$\mathbf{V}_{\text{AB}} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{\text{AN}} = 176.81\underline{/-59.75^\circ}\,\text{A (rms)}$

P 11.11 **[a]**

$$\mathbf{I}_{\text{aA}} = \frac{6600}{\sqrt{3}(240 - j70)} = 15.24\underline{/16.26^\circ}\,\text{A (rms)}$$

$$|\mathbf{I}_{\text{aA}}| = |\mathbf{I}_{\text{L}}| = 15.24\,\text{A (rms)}$$

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[b] $\mathbf{V}_{\text{an}} = (15.24\underline{/16.26^\circ})(240 - j66) = 3801.24\underline{/0.91^\circ}$

$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(3801.24) = 6583.94\,\text{V (rms)}$

P 11.12 Make a sketch of the a-phase:

[a] Find the a-phase line current from the a-phase circuit:

$$\mathbf{I}_{\text{aA}} = \frac{125\underline{/0^\circ}}{0.1 + j0.8 + 19.9 + j14.2} = \frac{125\underline{/0^\circ}}{20 + j15}$$

$$= 4 - j3 = 5\underline{/-36.87^\circ}\,\text{A (rms)}$$

Find the other line currents using the acb phase sequence:

$\mathbf{I}_{\text{bB}} = 5\underline{/-36.87^\circ + 120^\circ} = 5\underline{/83.13^\circ}\,\text{A (rms)}$

$\mathbf{I}_{\text{cC}} = 5\underline{/-36.87^\circ - 120^\circ} = 5\underline{/-156.87^\circ}\,\text{A (rms)}$

[b] The phase voltage at the source is $\mathbf{V}_{\text{an}} = 125\underline{/0^\circ}$ V. Use Fig. 11.9(b) to find the line voltage, $\mathbf{V}_{\text{an}}$, from the phase voltage:

$\mathbf{V}_{\text{ab}} = \mathbf{V}_{\text{an}}(\sqrt{3}\underline{/-30^\circ}) = 216.51\underline{/-30^\circ}\,\text{V (rms)}$

Find the other line voltages using the acb phase sequence:

$\mathbf{V}_{\text{bc}} = 216.51\underline{/-30^\circ + 120^\circ} = 216.51\underline{/90^\circ}\,\text{V (rms)}$

$\mathbf{V}_{\text{ca}} = 216.51\underline{/-30^\circ - 120^\circ} = 216.51\underline{/-150^\circ}\,\text{V (rms)}$

[c] The phase voltage at the load in the a-phase is $\mathbf{V}_{\text{AN}}$. Calculate its value using $\mathbf{I}_{\text{aA}}$ and the load impedance:

$\mathbf{V}_{\text{AN}} = \mathbf{I}_{\text{aA}} Z_{\text{L}} = (4 - j3)(19.9 + j14.2) = 122.2 - j2.9 = 122.23\underline{/-1.36^\circ}\,\text{V (rms)}$

Find the phase voltage at the load for the b- and c-phases using the acb sequence:

$\mathbf{V}_{\text{BN}} = 122.23\underline{/-1.36^\circ + 120^\circ} = 122.23\underline{/118.64^\circ}\,\text{V (rms)}$

$\mathbf{V}_{\text{CN}} = 122.23\underline{/-1.36^\circ - 120^\circ} = 122.23\underline{/-121.36^\circ}\,\text{V (rms)}$

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[d] The line voltage at the load in the a-phase is $\mathbf{V}_{\text{AB}}$. Find this line voltage from the phase voltage at the load in the a-phase, $\mathbf{V}_{\text{AN}}$, using Fig, 11.9(b):

$$\mathbf{V}_{\text{AB}} = \mathbf{V}_{\text{AN}}(\sqrt{3}\underline{/-30^\circ}) = 211.72\underline{/-31.36^\circ}\ \text{V (rms)}$$

Find the line voltage at the load for the b- and c-phases using the acb sequence:

$$\mathbf{V}_{\text{BC}} = 211.72\underline{/-31.36^\circ + 120^\circ} = 211.72\underline{/88.64^\circ}\ \text{V (rms)}$$

$$\mathbf{V}_{\text{CA}} = 211.72\underline{/-31.36^\circ - 120^\circ} = 211.72\underline{/-151.36^\circ}\ \text{V (rms)}$$

P 11.13 **[a]** $\mathbf{I}_{\text{AB}} = \dfrac{7200}{216 - j288} = 20\underline{/53.13^\circ}\ \text{A (rms)}$

$$\mathbf{I}_{\text{BC}} = 20\underline{/173.13^\circ}\ \text{A (rms)}$$

$$\mathbf{I}_{\text{CA}} = 20\underline{/-66.87^\circ}\ \text{A (rms)}$$

[b] $\mathbf{I}_{\text{aA}} = \sqrt{3}\underline{/30^\circ}\mathbf{I}_{\text{AB}} = 34.64\underline{/83.13^\circ}\ \text{A (rms)}$

$$\mathbf{I}_{\text{bB}} = 34.64\underline{/-156.87^\circ}\ \text{A (rms)}$$

$$\mathbf{I}_{\text{cC}} = 34.64\underline{/-36.87^\circ}\ \text{A (rms)}$$

[c]

$$\mathbf{V}_{\text{an}} = \frac{7200}{\sqrt{3}}\underline{/30^\circ} + (3 + j5)(34.64\underline{/83.13^\circ})$$
$$= 4085\underline{/32.62^\circ}\ \text{V (rms)}$$

$$\mathbf{V}_{\text{ab}} = \sqrt{3}\underline{/-30^\circ}\mathbf{V}_{\text{an}} = 7075.43\underline{/2.62^\circ}\ \text{V (rms)}$$

$$\mathbf{V}_{\text{bc}} = 7075.43\underline{/122.62^\circ}\ \text{V (rms)}$$

$$\mathbf{V}_{\text{ca}} = 7075.43\underline{/-117.38^\circ}\ \text{V (rms)}$$

P 11.14 **[a]** $\mathbf{V}_{\text{an}} = \mathbf{V}_{\text{bn}} - 1\underline{/120^\circ} = 150\underline{/15^\circ}\ \text{V (rms)}$

$$Z_y = Z_\Delta/3 = 43 + j57\,\Omega$$

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The a-phase circuit is

[**b**] $\mathbf{I}_{\text{aA}} = \dfrac{150\underline{/15^\circ}}{45 + j60} = 2\underline{/-38.13^\circ}$ A (rms)

[**c**] $\mathbf{V}_{\text{AN}} = (43 + j57)\mathbf{I}_{\text{aA}} = 142.8\underline{/14.84^\circ}$ V (rms)

$\mathbf{V}_{\text{AB}} = \sqrt{3}\underline{/-30^\circ}\mathbf{V}_{\text{AN}} = 247.34\underline{/-15.16^\circ}$ A (rms)

P 11.15 $Z_y = Z_\Delta/3 = 4 + j3\,\Omega$

The a-phase circuit is

$$\mathbf{I}_{\text{aA}} = \frac{240\underline{/-170^\circ}}{(1 + j1) + (4 + j3)} = 37.48\underline{/151.34^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{\text{AB}} = \frac{1}{\sqrt{3}}\underline{/-30^\circ}\mathbf{I}_{\text{aA}} = 21.64\underline{/121.34^\circ}\text{ A (rms)}$$

P 11.16 $\mathbf{V}_{\text{an}} = 1/\sqrt{3}\underline{/-30^\circ}\mathbf{V}_{\text{ab}} = \dfrac{208}{\sqrt{3}}\underline{/20^\circ}$ V (rms)

$Z_y = Z_\Delta/3 = 1 - j3\,\Omega$

The a-phase circuit is

$Z_{\text{eq}} = (4 + j3)\|(1 - j3) = 2.6 - j1.8\,\Omega$

$$\mathbf{V}_{\text{AN}} = \frac{2.6 - j1.8}{(1.4 + j0.8) + (2.6 - j1.8)}\left(\frac{208}{\sqrt{3}}\right)\underline{/20^\circ} = 92.1\underline{/-0.66^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{\text{AB}} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{\text{AN}} = 159.5\underline{/29.34^\circ}\text{ V (rms)}$$

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P 11.17 **[a]**

$$\mathbf{I}_{\text{aA}} = \frac{300}{60 - j45} + \frac{300}{30 + j30} = 8.6\underline{/-17.59^\circ}\,\text{A (rms)}$$

$$|\mathbf{I}_{\text{aA}}| = 8.6\,\text{A (rms)}$$

[b] $\mathbf{I}_{\text{AB}} = \dfrac{300\sqrt{3}\underline{/30^\circ}}{90 + j90} = 4.08\underline{/-15^\circ}\,\text{A (rms)}$

$$|\mathbf{I}_{\text{AB}}| = 4.08\,\text{A (rms)}$$

[c] $\mathbf{I}_{\text{AN}} = \dfrac{300\underline{/0^\circ}}{60 - j45} = 4\underline{/36.87^\circ}\,\text{A (rms)}$

$$|\mathbf{I}_{\text{AN}}| = 4\,\text{A (rms)}$$

[d] $\mathbf{V}_{\text{an}} = (8.6\underline{/-17.59^\circ})(2 + j2) + 300\underline{/0^\circ} = 321.79\underline{/1.99^\circ}\,\text{V (rms)}$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(321.79) = 557.37\,\text{V (rms)}$$

P 11.18 **[a]**

[b] $\mathbf{I}_{\text{aA}} = \dfrac{13{,}800}{\sqrt{3}(2.375 + j1.349)} = 2917\underline{/-29.6^\circ}\,\text{A (rms)}$

$$|\mathbf{I}_{\text{aA}}| = 2917\,\text{A (rms)}$$

[c] $\mathbf{V}_{\text{AN}} = (2.352 + j1.139)(2917\underline{/-29.6^\circ}) = 7622.93\underline{/-3.76^\circ}\,\text{V (rms)}$

$$|\mathbf{V}_{\text{AB}}| = \sqrt{3}|\mathbf{V}_{\text{AN}}| = 13{,}203.31\,\text{V (rms)}$$

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[d] $\mathbf{V}_{\text{an}} = (2.372 + j1.319)(2917\underline{/-29.6^\circ}) = 7616.93\underline{/-0.52^\circ}\,\text{V (rms)}$

$|\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 13{,}712.52\,\text{V (rms)}$

[e] $|\mathbf{I}_{\text{AB}}| = \dfrac{|\mathbf{I}_{\text{aA}}|}{\sqrt{3}} = 1684.13\,\text{A (rms)}$

[f] $|\mathbf{I}_{\text{ab}}| = |\mathbf{I}_{\text{AB}}| = 1684.13\,\text{A (rms)}$

P 11.19 [a] $\mathbf{I}_{\text{AB}} = \dfrac{13{,}200\underline{/0^\circ}}{100 - j75} = 105.6\underline{/36.87^\circ}\,\text{A (rms)}$

$\mathbf{I}_{\text{BC}} = 105.6\underline{/156.87^\circ}\,\text{A (rms)}$

$\mathbf{I}_{\text{CA}} = 105.6\underline{/-83.13^\circ}\,\text{A (rms)}$

[b] $\mathbf{I}_{\text{aA}} = \sqrt{3}\underline{/-30^\circ}\mathbf{I}_{\text{AB}} = 182.9\underline{/66.87^\circ}\,\text{A (rms)}$

$\mathbf{I}_{\text{bB}} = 182.9\underline{/-173.13^\circ}\,\text{A (rms)}$

$\mathbf{I}_{\text{cC}} = 182.9\underline{/-53.13^\circ}\,\text{A (rms)}$

[c] $\mathbf{I}_{\text{ba}} = \mathbf{I}_{\text{AB}} = 105.6\underline{/36.87^\circ}\,\text{A (rms)}$

$\mathbf{I}_{\text{cb}} = \mathbf{I}_{\text{BC}} = 105.6\underline{/156.87^\circ}\,\text{A (rms)}$

$\mathbf{I}_{\text{ac}} = \mathbf{I}_{\text{CA}} = 105.6\underline{/-83.13^\circ}\,\text{A (rms)}$

P 11.20 [a] $\mathbf{I}_{\text{AB}} = \dfrac{480\underline{/0^\circ}}{2.4 - j0.7} = 192\underline{/16.26^\circ}\,\text{A (rms)}$

$\mathbf{I}_{\text{BC}} = \dfrac{480\underline{/120^\circ}}{8 + j6} = 48\underline{/83.13^\circ}\,\text{A (rms)}$

$\mathbf{I}_{\text{CA}} = \dfrac{480\underline{/-120^\circ}}{20} = 24\underline{/-120^\circ}\,\text{A (rms)}$

[b] $\mathbf{I}_{\text{aA}} = \mathbf{I}_{\text{AB}} - \mathbf{I}_{\text{CA}}$

$= 210\underline{/20.79^\circ}$

$\mathbf{I}_{\text{bB}} = \mathbf{I}_{\text{BC}} - \mathbf{I}_{\text{AB}}$

$= 178.68\underline{/-178.04^\circ}$

$\mathbf{I}_{\text{cC}} = \mathbf{I}_{\text{CA}} - \mathbf{I}_{\text{BC}}$

$= 70.7\underline{/-104.53^\circ}$

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P 11.21 **[a]** Since the phase sequence is abc (positive) we have:

$$\mathbf{V}_{\text{an}} = 498.83\underline{/-30^\circ}\ \text{V (rms)}$$

$$\mathbf{V}_{\text{bn}} = 498.83\underline{/-150^\circ}\ \text{V (rms)}$$

$$\mathbf{V}_{\text{cn}} = 498.83\underline{/90^\circ}\ \text{V (rms)}$$

$$Z_Y = \frac{1}{3}Z_\Delta = 1.5 + j1\,\Omega/\phi$$

[b] $\mathbf{V}_{\text{ab}} = 498.83\underline{/-30^\circ} - 498.83\underline{/-150^\circ} = 498.83\sqrt{3}\underline{/0^\circ} = 864\underline{/0^\circ}\ \text{V (rms)}$

Since the phase sequence is positive, it follows that

$$\mathbf{V}_{\text{bc}} = 864\underline{/-120^\circ}\ \text{V (rms)}$$

$$\mathbf{V}_{\text{ca}} = 864\underline{/120^\circ}\ \text{V (rms)}$$

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[c]

$$\mathbf{I}_{\text{ba}} = \frac{864}{4.5 + j3} = 159.75\underline{/-33.69^\circ}\ \text{A (rms)}$$

$$\mathbf{I}_{\text{ac}} = 159.75\underline{/86.31^\circ}\ \text{A (rms)}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_{\text{ba}} - \mathbf{I}_{\text{ac}} = 276.70\underline{/-63.69^\circ}\ \text{A (rms)}$$

Since we have a balanced three-phase circuit and a positive phase sequence we have:

$$\mathbf{I}_{\text{bB}} = 276.70\underline{/176.31^\circ}\ \text{A (rms)}$$

$$\mathbf{I}_{\text{cC}} = 276.70\underline{/-56.31^\circ}\ \text{A (rms)}$$

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[d]

$$\mathbf{I}_{\mathrm{aA}} = \frac{498.83\underline{/30^\circ}}{1.5 + j1} = 276.70\underline{/-63.69^\circ}\ \mathrm{A\ (rms)}$$

Since we have a balanced three-phase circuit and a positive phase sequence we have:

$$\mathbf{I}_{\mathrm{bB}} = 276.70\underline{/176.31^\circ}\ \mathrm{A\ (rms)}$$

$$\mathbf{I}_{\mathrm{cC}} = 276.70\underline{/56.31^\circ}\ \mathrm{A\ (rms)}$$

P 11.22 **[a]**

[b] $$\mathbf{I}_{\mathrm{aA}} = \frac{498.83\underline{/-30^\circ}}{1200 + j1600} = 249.42\underline{/-83.13^\circ}\ \mathrm{mA\ (rms)}$$

$$\mathbf{V}_{\mathrm{AN}} = (1192 + j1584)(0.24942\underline{/-83.13^\circ}) = 494.45\underline{/-30.09^\circ}\ \mathrm{V\ (rms)}$$

$$|\mathbf{V}_{\mathrm{AB}}| = \sqrt{3}(494.45) = 856.41\ \mathrm{V\ (rms)}$$

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[c] $|\mathbf{I}_{\text{ab}}| = \dfrac{0.24942}{\sqrt{3}} = 144\,\text{mA (rms)}$

[d] $\mathbf{V}_{\text{an}} = (1198.5 + j1599)(0.24942\underline{/-83.13^\circ}) = 498.42\underline{/-29.98^\circ}\,\text{V (rms)}$

$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(498.42) = 863.29\,\text{V (rms)}$

P 11.23 **[a]**

$$\mathbf{I}_{\text{aA}} = \frac{1365\underline{/0^\circ}}{30 + j40} = 27.3\underline{/-53.13^\circ}\,\text{A (rms)}$$

$$\mathbf{I}_{\text{CA}} = \frac{\mathbf{I}_{\text{aA}}}{\sqrt{3}}\underline{/150^\circ} = 15.76\underline{/96.87^\circ}\,\text{A (rms)}$$

[b] $S_{g/\phi} = -1365\mathbf{I}^*_{\text{aA}} = -22{,}358.75 - j29{,}811.56\,\text{VA}$

$\therefore\quad P_{\text{developed/phase}} = 22.359\,\text{kW}$

$P_{\text{absorbed/phase}} = |\mathbf{I}_{\text{aA}}|^2 28.5 = 21.241\,\text{kW}$

$\%\text{ delivered } = \dfrac{21.241}{22.359}(100) = 95\%$

P 11.24 The complex power of the source per phase is $S_s = 20{,}000\underline{/(\cos^{-1}0.6)} = 20{,}000\underline{/53.13^\circ} = 12{,}000 + j16{,}000$ kVA. This complex power per phase must equal the sum of the per-phase impedances of the two loads:

$S_s = S_1 + S_2 \quad \text{so} \quad 12{,}000 + j16{,}000 = 10{,}000 + S_2$

$\therefore\quad S_2 = 2000 + j16{,}000\text{ VA}$

Also, $\quad S_2 = \dfrac{|V_{\text{rms}}|^2}{Z_2^*}$

$|V_{\text{rms}}| = \dfrac{|\text{V}_{\text{load}}|}{\sqrt{3}} = 120\text{ V (rms)}$

Thus, $\quad Z_2^* = \dfrac{|V_{\text{rms}}|^2}{S_2} = \dfrac{(120)^2}{2000 + j16{,}000} = 0.11 - j0.89\,\Omega$

$\therefore\quad Z_2 = 0.11 + j0.89\,\Omega$

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P 11.25 The a-phase of the circuit is shown below:

$$\mathbf{I}_1 = \frac{120\underline{/20^\circ}}{8+j6} = 12\underline{/-16.87^\circ}\text{A (rms)}$$

$$\mathbf{I}_2^* = \frac{600\underline{/36^\circ}}{120\underline{/20^\circ}} = 5\underline{/16^\circ}\text{A (rms)}$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 12\underline{/-16.87^\circ} + 5\underline{/-16^\circ} = 17\underline{/-16.61^\circ}\text{ A (rms)}$$

$$S_\text{a} = \mathbf{V}\mathbf{I}^* = (120\underline{/20^\circ})(17\underline{/16.61^\circ}) = 2040\underline{/36.61^\circ}\text{ VA}$$

$$S_\text{T} = 3S_\text{a} = 6120\underline{/36.61^\circ}\text{ VA}$$

P 11.26 **[a]** $\mathbf{I}_\text{aA}^* = \dfrac{(128+j96)10^3}{1600} = 80 + j60$

$$\mathbf{I}_\text{aA} = 80 - j60\,\text{A (rms)}$$

$$\mathbf{V}_\text{an} = 1600 + (80 - j60)(0.2 + j0.8) = 1664 + j52\,\text{V (rms)}$$

$$\mathbf{I}_\text{C} = \frac{1664 + j52}{-j25} = -2.08 + j66.56\,\text{A (rms)}$$

$$\mathbf{I}_\text{na} = \mathbf{I}_\text{aA} + \mathbf{I}_\text{C} = 77.92 + j6.56 = 78.2\underline{/4.81^\circ}\,\text{A (rms)}$$

[b] $S_{g/\phi} = -(1664 + j52)(77.92 - j6.56) = -130{,}000 + j6864\,\text{VA}$

$$S_{gT} = 3S_{g/\phi} = -390{,}000 + j20{,}592\,\text{VA}$$

Therefore, the source is delivering 390 kW and absorbing 20.592 kvars.

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[c] $P_{\text{del}} = 390\,\text{kW}$

$$P_{\text{abs}} = 3(128{,}000) + 3|\mathbf{I}_{\text{aA}}|^2(0.2)$$
$$= 390\,\text{kW} = P_{\text{del}}$$

[d] $Q_{\text{del}} = 3|\mathbf{I}_{\text{C}}|^2(25) = 332{,}592\,\text{VAR}$

$$Q_{\text{abs}} = 3(96{,}000) + 3|\mathbf{I}_{\text{aA}}|^2(0.8) + 20{,}592$$
$$= 332{,}592\,\text{VAR} = Q_{\text{del}}$$

P 11.27 **[a]** $S_{T\Delta} = 14{,}000\underline{/41.41^\circ} - 9000\underline{/53.13^\circ} = 5.5\underline{/22^\circ}$ kVA

$$S_\Delta = S_{T\Delta}/3 = 1833.46\underline{/22^\circ}\ \text{VA}$$

[b] $|\mathbf{V}_{\text{an}}| = \left|\dfrac{3000\underline{/53.13^\circ}}{10\underline{/-30^\circ}}\right| = 300$ V (rms)

$$|\mathbf{V}_{\text{line}}| = |\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 300\sqrt{3} = 519.62\ \text{V (rms)}$$

P 11.28 **[a]** $S_{1/\phi} = 60{,}000(0.866) + j60{,}000(0.5) = 51{,}960 + j30{,}000$ VA

$$S_{2/\phi} = 50{,}000(0.28) - j50{,}000(0.96) = 14{,}000 - j48{,}000\ \text{VA}$$

$$S_{3/\phi} = 24{,}040\ \text{VA}$$

$$S_{T/\phi} = S_1 + S_2 + S_3 = 90{,}000 - j18{,}000\ \text{VA}$$

$$\therefore\quad \mathbf{I}^*_{\text{aA}} = \frac{90{,}000 - j18{,}000}{1800} = 50 - j10$$

$$\therefore\quad \mathbf{I}_{\text{aA}} = 50 + j10\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 1800 + (50 + j10)(5 + j10) = 1950 + j550 = 2026.08\underline{/15.75^\circ}\,\text{V (rms)}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(2026.08) = 3509.27\,\text{V (rms)}$$

[b] $S_{g/\phi} = (1950 + j550)(50 - j10) = 103 + j0.8\,\text{kVA}$

$$\%\ \text{efficiency} = \frac{90{,}000}{103{,}000}(100) = 87.38\%$$

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P 11.29 **[a]** $S_1 = 10{,}200(0.87) + j10{,}200(0.493) = 8874 + j5029.13\,\text{VA}$

$S_2 = 4200 + j1913.6\,\text{VA}$

$\sqrt{3}V_\text{L}I_\text{L}\sin\theta_3 = 7250; \qquad \sin\theta_3 = \dfrac{7250}{\sqrt{3}(220)(36.8)} = 0.517$

Therefore $\quad \cos\theta_3 = 0.856$

Therefore

$P_3 = \dfrac{7250}{0.517} \times 0.856 = 12{,}003.9\,\text{W}$

$S_3 = 12{,}003.9 + j7250\,\text{VA}$

$S_T = S_1 + S_2 + S_3 = 25.078 + j14.192\,\text{kVA}$

$S_{T/\phi} = \dfrac{1}{3}S_T = 8359.3 + j4730.7\,\text{VA}$

$\dfrac{220}{\sqrt{3}}\mathbf{I}^*_\text{aA} = (8359.3 + j4730.7); \qquad \mathbf{I}^*_\text{aA} = 65.81 + j37.24\,\text{A}$

$\mathbf{I}_\text{aA} = 65.81 - j37.24 = 75.62\underline{/-29.51^\circ}\,\text{A} \qquad \text{(rms)}$

[b] pf $= \cos(0^\circ - 29.51^\circ) = 0.87$ lagging

P 11.30 From the solution to Problem 11.18 we have:

$S_\text{AB} = (480\underline{/0^\circ})(192\underline{/-16.26^\circ}) = 88{,}473.7 - j25{,}804.5\,\text{VA}$

$S_\text{BC} = (480\underline{/120^\circ})(48\underline{/-83.13^\circ}) = 18{,}431.98 + j13{,}824.03\,\text{VA}$

$S_\text{CA} = (480\underline{/-120^\circ})(24\underline{/120^\circ}) = 11{,}520 + j0\,\text{VA}$

P 11.31 Let p_a, p_b, and p_c represent the instantaneous power of phases a, b, and c, respectively. Then assuming a positive phase sequence, we have

$p_\text{a} = v_\text{an}i_\text{aA} = [V_m\cos\omega t][I_m\cos(\omega t - \theta_\phi)]$

$p_\text{b} = v_\text{bn}i_\text{bB} = [V_m\cos(\omega t - 120^\circ)][I_m\cos(\omega t - \theta_\phi - 120^\circ)]$

$p_\text{c} = v_\text{cn}i_\text{cC} = [V_m\cos(\omega t + 120^\circ)][I_m\cos(\omega t - \theta_\phi + 120^\circ)]$

The total instantaneous power is $p_T = p_\text{a} + p_\text{b} + p_\text{c}$, so

$p_T = V_mI_m[\cos\omega t\cos(\omega t - \theta_\phi) + \cos(\omega t - 120^\circ)\cos(\omega t - \theta_\phi - 120^\circ)$

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$$+\cos(\omega t + 120^\circ)\cos(\omega t - \theta_\phi + 120^\circ)]$$

Now simplify using trigonometric identities. In simplifying, collect the coefficients of $\cos(\omega t - \theta_\phi)$ and $\sin(\omega t - \theta_\phi)$. We get

$$\begin{aligned} p_T &= V_m I_m[\cos\omega t(1 + 2\cos^2 120^\circ)\cos(\omega t - \theta_\phi) \\ &\quad +2\sin\omega t \sin^2 120^\circ \sin(\omega t - \theta_\phi)] \\ &= 1.5 V_m I_m[\cos\omega t\cos(\omega t - \theta_\phi) + \sin\omega t\sin(\omega t - \theta_\phi)] \\ &= 1.5 V_m I_m \cos\theta_\phi \end{aligned}$$

P 11.32 $|I_{\text{line}}| = \dfrac{1600}{240/\sqrt{3}} = 11.547$ A (rms)

$$|Z_y| = \frac{|V|}{|I|} = \frac{240/\sqrt{3}}{11.547} = 12$$

$$Z_y = 12\underline{/-50^\circ}\,\Omega$$

$$Z_\Delta = 3Z_y = 36\underline{/-50^\circ} = 23.14 - j27.58\,\Omega/\phi$$

P 11.33 Assume a Δ-connected load (series):

$$S_\phi = \frac{1}{3}(150 \times 10^3)(0.96 - j0.28) = 48{,}000 - j14{,}000\,\text{VA}$$

$$Z^*_{\Delta\phi} = \frac{|600|^2}{48{,}000 - j14{,}000} = 6.912 + j2.016\,\Omega/\phi$$

$$Z_{\Delta\phi} = 6.912 - j2.016\,\Omega$$

Now assume a Y-connected load (series):

$$Z_{Y\phi} = \frac{1}{3}Z_{\Delta\phi} = 2.304 - j0.672\,\Omega/\phi$$

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Now assume a Δ-connected load (parallel):

$$P_\phi = \frac{|600|^2}{R_\Delta}$$

$$R_{\Delta\phi} = \frac{|600|^2}{48{,}000} = 7.5\,\Omega$$

$$Q_\phi = \frac{|600|^2}{X_\Delta}$$

$$X_\Delta\phi = \frac{|600|^2}{-14{,}000} = -25.714\,\Omega$$

Now assume a Y-connected load (parallel):

$$R_{Y\phi} = \frac{1}{3}R_{\Delta\phi} = 2.5\,\Omega$$

$$X_{Y\phi} = \frac{1}{3}X_{\Delta\phi} = -8.571\,\Omega$$

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P 11.34 **[a]** $P_{\text{OUT}} = 746 \times 100 = 74{,}600\,\text{W}$

$$P_{\text{IN}} = 74{,}600/(0.97) = 76{,}907.22\,\text{W}$$

$$\sqrt{3}V_L I_L \cos\theta = 76{,}907.22$$

$$I_L = \frac{76{,}907.22}{\sqrt{3}(208)(0.88)} = 242.58\,\text{A (rms)}$$

[b] $Q = \sqrt{3}V_L I_L \sin\phi = \sqrt{3}(208)(242.58)(0.475) = 41{,}511.90\,\text{VAR}$

P 11.35

$$4000\mathbf{I}_1^* = (210 + j280)10^3$$

$$\mathbf{I}_1^* = \frac{210}{4} + j\frac{280}{4} = 52.5 + j70\,\text{A (rms)}$$

$$\mathbf{I}_1 = 52.5 - j70\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{4000\underline{/0^\circ}}{15.36 - j4.48} = 240 + j70\,\text{A (rms)}$$

$$\therefore\quad \mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 = 292.5 + j0\,\text{A (rms)}$$

$$\mathbf{V}_{\text{an}} = 4000 + j0 + 292.5(0.1 + j0.8) = 4036.04\underline{/3.32^\circ}\,\text{V (rms)}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 6990.62\,\text{V (rms)}$$

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P 11.36 **[a]**

$$\mathbf{I}_1 = \frac{24{,}000\sqrt{3}\underline{/0^\circ}}{400 + j300} = 66.5 - j49.9\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{24{,}000\sqrt{3}\underline{/0^\circ}}{800 - j600} = 33.3 + j24.9\,\text{A (rms)}$$

$$\mathbf{I}_3^* = \frac{57{,}600 + j734{,}400}{24{,}000\sqrt{3}} = 1.4 + j17.7$$

$$\mathbf{I}_3 = 1.4 - j17.7\,\text{A (rms)}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 101.2 - j42.7\,\text{A} = 109.8\underline{/-22.9^\circ}\,\text{A (rms)}$$

$$\mathbf{V}_{\text{an}} = (2 + j16)(101.2 - j42.7) + 24{,}000\sqrt{3} = 42{,}454.8 + j1533.8\,\text{V (rms)}$$

$$S_\phi = \mathbf{V}_{\text{an}}\mathbf{I}_{\text{aA}}^* = (42{,}454.8 + j1533.8)(101.2 + j42.7)$$

$$= 4{,}230{,}932.5 + j1{,}968{,}040.5\,\text{VA}$$

$$S_T = 3S_\phi = 12{,}692.8 + j5904.1\,\text{kVA}$$

[b] $S_{1/\phi} = 24{,}000\sqrt{3}(66.5 + j49.9) = 2764.4 + j2074.3\,\text{kVA}$

$$S_{2/\phi} = 24{,}000\sqrt{3}(33.3 - j24.9) = 1384.3 - j1035.1\,\text{kVA}$$

$$S_{3/\phi} = 57.6 + j734.4\,\text{kVA}$$

$$S_\phi(\text{load}) = 4206.3 + j1773.6\,\text{kVA}$$

$$\%\ \text{delivered} = \left(\frac{4206.3}{4230.9}\right)(100) = 99.4\%$$

P 11.37 **[a]** $S_{g/\phi} = \frac{1}{3}(41.6)(0.707 + j0.707) \times 10^3 = 9803.73 + j9803.73\,\text{VA}$

$$\mathbf{I}_{\text{aA}}^* = \frac{9803.73 + j9803.73}{240/\sqrt{3}} = 70.76 + j70.76\,\text{A (rms)}$$

$$\mathbf{I}_{\text{aA}} = 70.76 - j70.76\,\text{A (rms)}$$

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$$\mathbf{V}_{\text{AN}} = \frac{240}{\sqrt{3}} - (0.04 + j0.03)(70.76 - j70.76)$$
$$= 133.61 + j0.71 = 133.61\underline{/0.30^\circ}\,\text{V (rms)}$$

$$|\mathbf{V}_{\text{AB}}| = \sqrt{3}(133.61) = 231.42\,\text{V (rms)}$$

[b] $S_{L/\phi} = (133.61 + j0.71)(70.76 + j70.76) = 9404 + j9504.5\,\text{VA}$

$S_L = 3S_{L/\phi} = 28{,}212 + j28{,}513\,\text{VA}$

Check:

$S_g = 41{,}600(0.7071 + j0.7071) = 29{,}415 + j29{,}415\,\text{VA}$

$P_\ell = 3|\mathbf{I}_{\text{aA}}|^2(0.04) = 1202\,\text{W}$

$P_g = P_L + P_\ell = 28{,}212 + 1202 = 29{,}414\,\text{W}$ (checks)

$Q_\ell = 3|\mathbf{I}_{\text{aA}}|^2(0.03) = 901\,\text{VAR}$

$Q_g = Q_L + Q_\ell = 28{,}513 + 901 = 29{,}414\,\text{VAR}$ (checks)

P 11.38 **[a]**

$$S_g = \frac{1}{3}(540)(0.96 + j0.28) = 172.8 + j50.4\,\text{kVA}$$

$$S_1 = \frac{1}{3}(28.4 - j208.8) = 12.8 - j69.6\,\text{kVA}$$

$$S_2 = S_g - S_1 = 160 + j120\,\text{kVA}$$

$$\therefore\quad \mathbf{I}^*_{L2} = \frac{(160 + j120)10^3}{1600} = 100 + j75$$

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$$\mathbf{I}_{L2} = 100 - j75\,\text{A (rms)}$$

$$Z_Y = \frac{1600}{100 - j75} = 10.24 + j7.68\,\Omega$$

$$Z_\Delta = 3Z_Y = 30.72 + j23.04\,\Omega$$

[b] $R = \dfrac{(1600)^2}{160 \times 10^3} = 16\,\Omega \quad \rightarrow \quad R_\Delta = 3R = 48\,\Omega$

$$X_\text{L} = \frac{(1600)^2}{120 \times 10^3} = 21.33\,\Omega \quad \rightarrow \quad X_{\text{L}\Delta} = 3X_\text{L} = 64\,\Omega$$

P 11.39 **[a]**

$$S_{L/\phi} = \frac{1}{3}\left[900 + j\frac{900}{0.6}(0.8)\right]10^3 = 300{,}000 + j400{,}000\,\text{VA}$$

$$\mathbf{I}^*_{\text{aA}} = \frac{300{,}000 + j400{,}000}{2500} = 120 + j160\,\text{A (rms)}$$

$$\mathbf{I}_{\text{aA}} = 120 - j160\,\text{A (rms)}$$

$$\mathbf{V}_{\text{an}} = 2500 + (1 + j3)(120 - j160)$$

$$= 3100 + j200 = 3106.44\underline{/3.69^\circ}\,\text{V (rms)}$$

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$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(3106.44) = 5380.5\,\text{V (rms)}$$

[b]

$$\mathbf{I}_1 = 120 - j160\,\text{A} \qquad \text{(from part [a])}$$

$$S_2 = 0 - j\frac{1}{3}(1125) \times 10^3 = -j375{,}000\,\text{VAR}$$

$$\mathbf{I}_2^* = \frac{-j375{,}000}{2500} = -j150\,\text{A (rms)}$$

$$\therefore \quad \mathbf{I}_2 = j150\,\text{A (rms)}$$

$$\mathbf{I}_{\text{aA}} = 120 - j160 + j150 = 120 - j10\,\text{A (rms)}$$

$$\begin{aligned}\mathbf{V}_{\text{an}} &= 2500 + (120 - j10)(1 + j3)\\ &= 2650 + j350 = 2673.01\underline{/7.52^\circ}\,\text{V (rms)}\end{aligned}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(2673.01) = 4629.8\,\text{V (rms)}$$

[c] $|\mathbf{I}_{\text{aA}}| = 200\,\text{A (rms)}$

$$P_{\text{loss}/\phi} = (200)^2(1) = 40\,\text{kW}$$

$$P_{g/\phi} = 300{,}000 + 40{,}000 = 340\,\text{kW}$$

$$\%\,\eta = \frac{300}{340}(100) = 88.2\%$$

[d] $|\mathbf{I}_{\text{aA}}| = 120.416\,\text{A (rms)}$

$$P_{\ell/\phi} = (120.416)^2(1) = 14{,}500\,\text{W}$$

$$\%\,\eta = \frac{300{,}000}{314{,}500}(100) = 95.4\%$$

[e] $Z_{\text{cap/Y}} = \dfrac{2500^2}{j375{,}000} = -j16.67\,\Omega$

$$\therefore \quad \frac{1}{\omega C} = 16.67; \qquad C = \frac{1}{(16.67)(120\pi)} = 159.155\,\mu\text{F}$$

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P 11.40 **[a]** From Assessment Problem 11.9, $\mathbf{I}_{\text{aA}} = (101.8 - j135.7)\,\text{A (rms)}$

Therefore $\mathbf{I}_{\text{cap}} = j135.7\,\text{A (rms)}$

Therefore $Z_{CY} = \dfrac{2450/\sqrt{3}}{j135.7} = -j10.42\,\Omega$

Therefore $C_Y = \dfrac{1}{(10.42)(2\pi)(60)} = 254.5\,\mu\text{F}$

$Z_{C\Delta} = (-j10.42)(3) = -j31.26\,\Omega$

Therefore $C_\Delta = \dfrac{254.5}{3} = 84.84\,\mu\text{F}$

[b] $C_Y = 254.5\,\mu\text{F}$

[c] $|\mathbf{I}_{\text{aA}}| = 101.8\,\text{A (rms)}$

P 11.41 $W_{m1} = |\mathbf{V}_{\text{AB}}||\mathbf{I}_{\text{aA}}|\cos(\underline{/\mathbf{V}_{\text{AB}}} - \underline{/\mathbf{I}_{\text{aA}}}) = (199.58)(2.4)\cos(65.68^\circ) = 197.26\,\text{W}$

$W_{m2} = |\mathbf{V}_{\text{CB}}||\mathbf{I}_{\text{cC}}|\cos(\underline{/\mathbf{V}_{\text{CB}}} - \underline{/\mathbf{I}_{\text{cC}}}) = (199.58)(2.4)\cos(5.68^\circ) = 476.64\,\text{W}$

CHECK: $W_1 + W_2 = 673.9 = (2.4)^2(39)(3) = 673.9\,\text{W}$

P 11.42 $\tan\phi = \dfrac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} = 0.75$

$\therefore\quad \phi = 36.87^\circ$

$\therefore\quad 2400\sqrt{3}|\mathbf{I}_{\text{L}}|\cos 66.87^\circ = 40{,}823.09$

$|\mathbf{I}_{\text{L}}| = 25\,\text{A}$

$|Z| = \dfrac{2400}{25} = 96\,\Omega \qquad \therefore\quad Z = 96\underline{/36.87^\circ}\,\Omega$

P 11.43 $\mathbf{I}_{\text{aA}} = \dfrac{\mathbf{V}_{\text{AN}}}{Z_\phi} = |\mathbf{I}_L|\underline{/-\theta_\phi}\,\text{A},$

$Z_\phi = |Z|\underline{/\theta_\phi}, \qquad \mathbf{V}_{\text{BC}} = |\mathbf{V}_L|\underline{/-90^\circ}\,\text{V},$

$$\begin{aligned} W_m &= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos[-90^\circ - (-\theta_\phi)] \\ &= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos(\theta_\phi - 90^\circ) \\ &= |\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi, \end{aligned}$$

therefore $\sqrt{3}W_m = \sqrt{3}|\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi = Q_{\text{total}}$

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P 11.44 **[a]** $Z = 16 - j12 = 20\underline{/-36.87^\circ}\,\Omega$

$\mathbf{V}_{\mathrm{AN}} = 680\underline{/0^\circ}\,\mathrm{V}; \qquad \therefore \quad \mathbf{I}_{\mathrm{aA}} = 34\underline{/36.87^\circ}\,\mathrm{A}$

$\mathbf{V}_{\mathrm{BC}} = \mathbf{V}_{\mathrm{BN}} - \mathbf{V}_{\mathrm{CN}} = 680\sqrt{3}\underline{/-90^\circ}\,\mathrm{V}$

$W_m = (680\sqrt{3})(34)\cos(-90 - 36.87^\circ) = -24{,}027.07\,\mathrm{W}$

$\sqrt{3}W_m = -41{,}616.1\,\mathrm{W}$

[b] $Q_\phi = (34^2)(-12) = -13{,}872\,\mathrm{VAR}$

$Q_T = 3Q_\phi = -41{,}616\,\mathrm{VAR} = \sqrt{3}W_m$

P 11.45 **[a]**
$$\begin{aligned} W_2 - W_1 &= V_L I_L[\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)] \\ &= V_L I_L[\cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ \\ &\qquad - \cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ] \\ &= 2V_L I_L \sin\theta\sin 30^\circ = V_L I_L \sin\theta, \end{aligned}$$

therefore $\sqrt{3}(W_2 - W_1) = \sqrt{3}V_L I_L \sin\theta = Q_T$

[b] $Z_\phi = (8 + j6)\,\Omega$

$Q_T = \sqrt{3}[2476.25 - 979.75] = 2592\,\mathrm{VAR},$

$Q_T = 3(12)^2(6) = 2592\,\mathrm{VAR};$

$Z_\phi = (8 - j6)\,\Omega$

$Q_T = \sqrt{3}[979.75 - 2476.25] = -2592\,\mathrm{VAR},$

$Q_T = 3(12)^2(-6) = -2592\,\mathrm{VAR};$

$Z_\phi = 5(1 + j\sqrt{3})\,\Omega$

$Q_T = \sqrt{3}[2160 - 0] = 3741.23\,\mathrm{VAR},$

$Q_T = 3(12)^2(5\sqrt{3}) = 3741.23\,\mathrm{VAR};$

$Z_\phi = 10\underline{/75^\circ}\,\Omega$

$Q_T = \sqrt{3}[-645.53 - 1763.63] = -4172.80\,\mathrm{VAR},$

$Q_T = 3(12)^2[-10\sin 75^\circ] = -4172.80\,\mathrm{VAR}$

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P 11.46 $Z_\phi = |Z|\underline{/\theta} = \dfrac{\mathbf{V}_{\mathrm{AN}}}{\mathbf{I}_{\mathrm{aA}}}$

$$\theta = \underline{/\mathbf{V}_{\mathrm{AN}}} - \underline{/\mathbf{I}_{\mathrm{aA}}}$$

$$\theta_1 = \underline{/\mathbf{V}_{\mathrm{AB}}} - \underline{/\mathbf{I}_{\mathrm{aA}}}$$

For a positive phase sequence,

$$\underline{/\mathbf{V}_{\mathrm{AB}}} = \underline{/\mathbf{V}_{\mathrm{AN}}} + 30^\circ$$

Thus,

$$\theta_1 = \underline{/\mathbf{V}_{\mathrm{AN}}} + 30^\circ - \underline{/\mathbf{I}_{\mathrm{aA}}} = \theta + 30^\circ$$

Similarly,

$$Z_\phi = |Z|\underline{/\theta} = \frac{\mathbf{V}_{\mathrm{CN}}}{\mathbf{I}_{\mathrm{cC}}}$$

$$\theta = \underline{/\mathbf{V}_{\mathrm{CN}}} - \underline{/\mathbf{I}_{\mathrm{cC}}}$$

$$\theta_2 = \underline{/\mathbf{V}_{\mathrm{CB}}} - \underline{/\mathbf{I}_{\mathrm{cC}}}$$

For a positive phase sequence,

$$\underline{/\mathbf{V}_{\mathrm{CB}}} = \underline{/\mathbf{V}_{\mathrm{BA}}} - 120^\circ = \underline{/\mathbf{V}_{\mathrm{AB}}} + 60^\circ$$

$$\underline{/\mathbf{I}_{\mathrm{cC}}} = \underline{/\mathbf{I}_{\mathrm{aA}}} + 120^\circ$$

Thus,

$$\begin{aligned}\theta_2 &= \underline{/\mathbf{V}_{\mathrm{AB}}} + 60^\circ - (\underline{/\mathbf{I}_{\mathrm{aA}}} + 120^\circ) = \theta_1 - 60^\circ \\ &= \theta + 30^\circ - 60^\circ = \theta - 30^\circ\end{aligned}$$

P 11.47 **[a]** $Z_\phi = 100 - j75 = 125\underline{/-36.87^\circ}\,\Omega$

$$S_\phi = \frac{(13{,}200)^2}{125\underline{/36.87^\circ}} = 1{,}115{,}136 + j836{,}352\,\mathrm{VA}$$

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[b] $\dfrac{13{,}200}{\sqrt{3}}\underline{/30^\circ}\mathbf{I}^*_{\text{aA}} = S_\phi$ so $\mathbf{I}_{\text{aA}} = 182.9\underline{/66.87^\circ}$

$W_{m1} = (13{,}200)(182.9)\cos(0 - 66.87^\circ) = 948{,}401.92\,\text{W}$

$W_{m2} = (13{,}200)(182.9)\cos(-60^\circ + 53.13^\circ) = 2{,}397{,}006.08\,\text{W}$

Check: $P_T = 3(1{,}115{,}136)\,\text{W} = W_{m1} + W_{m2}$.

P 11.48 From the solution to Prob. 11.20 we have

$\mathbf{I}_{\text{aA}} = 210\underline{/20.79^\circ}\,\text{A}$ and $\mathbf{I}_{\text{bB}} = 178.68\underline{/-178.04^\circ}\,\text{A}$

[a] $W_1 = |\mathbf{V}_{\text{ac}}|\,|\mathbf{I}_{\text{aA}}|\cos(\theta_{\text{ac}} - \theta_{\text{aA}})$

$= 480(210)\cos(60^\circ - 20.79^\circ) = 78{,}103.2\,\text{W}$

[b] $W_2 = |\mathbf{V}_{\text{bc}}|\,|\mathbf{I}_{\text{bB}}|\cos(\theta_{\text{bc}} - \theta_{\text{bB}})$

$= 480(178.68)\cos(120^\circ + 178.04^\circ) = 40{,}317.7\,\text{W}$

[c] $W_1 + W_2 = 118{,}421\,\text{W}$

$P_{\text{AB}} = (192)^2(2.4) = 88{,}473.6\,\text{W}$

$P_{\text{BC}} = (48)^2(8) = 18{,}432\,\text{W}$

$P_{\text{CA}} = (24)^2(20) = 11{,}520\,\text{W}$

$P_{\text{AB}} + P_{\text{BC}} + P_{\text{CA}} = 118{,}425.7$

therefore $W_1 + W_2 \approx P_{\text{total}}$ (round-off differences)

P 11.49 [a] $\mathbf{I}^*_{\text{aA}} = \dfrac{144(0.96 - j0.28)10^3}{7200} = 20\underline{/-16.26^\circ}\,\text{A}$

$\mathbf{V}_{\text{BN}} = 7200\underline{/-120^\circ}\,\text{V}$; $\mathbf{V}_{\text{CN}} = 7200\underline{/120^\circ}\,\text{V}$

$\mathbf{V}_{\text{BC}} = \mathbf{V}_{\text{BN}} - \mathbf{V}_{\text{CN}} = 7200\sqrt{3}\underline{/-90^\circ}\,\text{V}$

$\mathbf{I}_{\text{bB}} = 20\underline{/-103.74^\circ}\,\text{A}$

$W_{\text{m1}} = (7200\sqrt{3})(20)\cos(-90^\circ + 103.74^\circ) = 242{,}278.14\,\text{W}$

[b] Current coil in line aA, measure $\mathbf{I}_{\text{aA}}$.
Voltage coil across AC, measure $\mathbf{V}_{\text{AC}}$.

[c] $I_{\text{aA}} = 20\underline{/16.76^\circ}\,\text{A}$

$\mathbf{V}_{\text{CA}} = \mathbf{V}_{\text{AN}} - \mathbf{V}_{\text{CN}} = 7200\sqrt{3}\underline{/-30^\circ}\,\text{V}$

$W_{\text{m2}} = (7200\sqrt{3})(20)\cos(-30^\circ - 16.26^\circ) = 172{,}441.86\,\text{W}$

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[d] $W_{\mathrm{m1}} + W_{\mathrm{m2}} = 414.72\mathrm{kW}$

$$P_T = 432{,}000(0.96) = 414.72\,\mathrm{kW} = W_{\mathrm{m1}} + W_{\mathrm{m2}}$$

P 11.50 **[a]** $W_1 = |\mathbf{V}_{\mathrm{BA}}||\mathbf{I}_{\mathrm{bB}}|\cos\theta$

Negative phase sequence:

$$\mathbf{V}_{\mathrm{BA}} = 240\sqrt{3}\underline{/150^\circ}\,\mathrm{V}$$

$$\mathbf{I}_{\mathrm{aA}} = \frac{240\underline{/0^\circ}}{13.33\underline{/-30^\circ}} = 18\underline{/30^\circ}\,\mathrm{A}$$

$$\mathbf{I}_{\mathrm{bB}} = 18\underline{/150^\circ}\,\mathrm{A}$$

$$W_1 = (18)(240)\sqrt{3}\cos 0^\circ = 7482.46\,\mathrm{W}$$

$$W_2 = |\mathbf{V}_{\mathrm{CA}}||\mathbf{I}_{\mathrm{cC}}|\cos\theta$$

$$\mathbf{V}_{\mathrm{CA}} = 240\sqrt{3}\underline{/-150^\circ}\,\mathrm{V}$$

$$\mathbf{I}_{\mathrm{cC}} = 18\underline{/-90^\circ}\,\mathrm{A}$$

$$W_2 = (18)(240)\sqrt{3}\cos(-60^\circ) = 3741.23\,\mathrm{W}$$

[b] $P_\phi = (18)^2(40/3)\cos(-30^\circ) = 3741.23\,\mathrm{W}$

$$P_T = 3P_\phi = 11{,}223.69\,\mathrm{W}$$

$$W_1 + W_2 = 7482.46 + 3741.23 = 11{,}223.69\,\mathrm{W}$$

$$\therefore\quad W_1 + W_2 = P_T \qquad \text{(checks)}$$

P 11.51 **[a]** $Z = \dfrac{1}{3}Z_\Delta = 4.48 + j15.36 = 16\underline{/73.74^\circ}\,\Omega$

$$\mathbf{I}_{\mathrm{aA}} = \frac{600\underline{/0^\circ}}{16\underline{/73.74^\circ}} = 37.5\underline{/-73.74^\circ}\,\mathrm{A}$$

$$\mathbf{I}_{\mathrm{bB}} = 37.5\underline{/-193.74^\circ}\,\mathrm{A}$$

$$\mathbf{V}_{\mathrm{AC}} = 600\sqrt{3}\underline{/-30^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{BC}} = 600\sqrt{3}\underline{/-90^\circ}\,\mathrm{V}$$

$$W_1 = (600\sqrt{3})(37.5)\cos(-30 + 73.74^\circ) = 28{,}156.15\,\mathrm{W}$$

$$W_2 = (600\sqrt{3})(37.5)\cos(-90 + 193.74^\circ) = -9256.15\,\mathrm{W}$$

[b] $W_1 + W_2 = 18{,}900\,\mathrm{W}$

$$P_T = 3(37.5)^2(13.44/3) = 18{,}900\,\mathrm{W}$$

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[c] $\sqrt{3}(W_1 - W_2) = 64{,}800\,\text{VAR}$

$$Q_T = 3(37.5)^2(46.08/3) = 64{,}800\,\text{VAR}$$

P 11.52 **[a]** Negative phase sequence:

$$\mathbf{V}_{\text{AB}} = 240\sqrt{3}\underline{/-30^\circ}\,\text{V}$$

$$\mathbf{V}_{\text{BC}} = 240\sqrt{3}\underline{/90^\circ}\,\text{V}$$

$$\mathbf{V}_{\text{CA}} = 240\sqrt{3}\underline{/-150^\circ}\,\text{V}$$

$$\mathbf{I}_{\text{AB}} = \frac{240\sqrt{3}\underline{/-30^\circ}}{20\underline{/30^\circ}} = 20.78\underline{/-60^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{BC}} = \frac{240\sqrt{3}\underline{/90^\circ}}{60\underline{/0^\circ}} = 6.93\underline{/90^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{CA}} = \frac{240\sqrt{3}\underline{/-150^\circ}}{40\underline{/-30^\circ}} = 10.39\underline{/-120^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_{\text{AB}} + \mathbf{I}_{\text{AC}} = 18\underline{/-30^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{cC}} = \mathbf{I}_{\text{CB}} + \mathbf{I}_{\text{CA}} = \mathbf{I}_{\text{CA}} + \mathbf{I}_{\text{BC}} = 16.75\underline{/-108.06^\circ}$$

$$W_{m1} = 240\sqrt{3}(18)\cos(-30 + 30^\circ) = 7482.46\,\text{W}$$

$$W_{m2} = 240\sqrt{3}(16.75)\cos(-90 + 108.07^\circ) = 6621.23\,\text{W}$$

[b] $W_{m1} + W_{m2} = 14{,}103.69\,\text{W}$

$$P_{\text{A}} = (12\sqrt{3})^2(20\cos 30^\circ) = 7482.46\,\text{W}$$

$$P_{\text{B}} = (4\sqrt{3})^2(60) = 2880\,\text{W}$$

$$P_{\text{C}} = (6\sqrt{3})^2[40\cos(-30^\circ)] = 3741.23\,\text{W}$$

$$P_{\text{A}} + P_{\text{B}} + P_{\text{C}} = 14{,}103.69 = W_{m1} + W_{m2}$$

P 11.53 **[a]**

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[b]

[c]

[d]

P 11.54 **[a]** $Q = \dfrac{|\mathbf{V}|^2}{X_\mathrm{C}}$

$$\therefore \quad |X_\mathrm{C}| = \frac{(13{,}800)^2}{1.2 \times 10^6} = 158.70\,\Omega$$

$$\therefore \quad \frac{1}{\omega C} = 158.70; \qquad C = \frac{1}{2\pi(60)(158.70)} = 16.71\,\mu\mathrm{F}$$

[b] $|X_\mathrm{C}| = \dfrac{(13{,}800/\sqrt{3})^2}{1.2 \times 10^6} = \dfrac{1}{3}(158.70)$

$$\therefore \quad C = 3(16.71) = 50.14\,\mu\mathrm{F}$$

P 11.55 **[a]** The capacitor from Appendix H whose value is closest to $50.14\,\mu\mathrm{F}$ is $47\,\mu\mathrm{F}$.

$$|X_C| = \frac{1}{\omega C} = \frac{1}{2\pi(60)(47 \times 10^{-6})} = 56.4\,\Omega$$

$$Q = \frac{|V|^2}{3X_C} = \frac{(13{,}800)^2}{3(56.4)} = 1{,}124{,}775.6\,\mathrm{VAR}$$

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[b] $\mathbf{I}^*_{\text{aA}} = \dfrac{1{,}200{,}000 + j75{,}224}{13{,}800/\sqrt{3}} = 150.6 + j9.4\,\text{A}$

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(150.6 - j9.4) = 8134.8\underline{/5.06^\circ}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(8134.8) = 14{,}089.9\,\text{V}$$

This voltage falls within the allowable range of 13 kV to 14.6 kV.

P 11.56 **[a]** The capacitor from Appendix H whose value is closest to $16.71\,\mu$F is $22\,\mu$F.

$$|X_C| = \frac{1}{\omega C} = \frac{1}{2\pi(60)(22 \times 10^{-6})} = 120.57\,\Omega$$

$$Q = \frac{|V|^2}{X_C} = \frac{(13{,}800)^2}{120.57} = 1{,}579{,}497\,\text{VAR}/\phi$$

[b] $\mathbf{I}^*_{\text{aA}} = \dfrac{1{,}200{,}000 - j379{,}497}{13{,}800/\sqrt{3}} = 50.2 - j15.9\,\text{A}$

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(50.2 + j15.9) = 7897.8\underline{/1.76^\circ}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(7897.8) = 13{,}679.4\,\text{V}$$

This voltage falls within the allowable range of 13 kV to 14.6 kV.

P 11.57 If the capacitors remain connected when the substation drops its load, the expression for the line current becomes

$$\frac{13{,}800}{\sqrt{3}}\mathbf{I}^*_{\text{aA}} = -j1.2 \times 10^6$$

or $\quad \mathbf{I}^*_{\text{aA}} = -j150.61\,\text{A}$

Hence $\quad \mathbf{I}_{\text{aA}} = j150.61\,\text{A}$

Now,

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(j150.61) = 7244.49 + j90.37 = 7245.05\underline{/0.71^\circ}\,\text{V}$$

The magnitude of the line-to-line voltage at the generating plant is

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(7245.05) = 12{,}548.80\,\text{V}.$$

This is a problem because the voltage is below the acceptable minimum of 13 kV. Thus when the load at the substation drops off, the capacitors must be switched off.

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P 11.58 Before the capacitors are added the total line loss is

$$P_{\mathrm{L}} = 3|150.61 + j150.61|^2(0.6) = 81.66\,\mathrm{kW}$$

After the capacitors are added the total line loss is

$$P_{\mathrm{L}} = 3|150.61|^2(0.6) = 40.83\,\mathrm{kW}$$

Note that adding the capacitors to control the voltage level also reduces the amount of power loss in the lines, which in this example is cut in half.

P 11.59 **[a]** $\dfrac{13{,}800}{\sqrt{3}}\mathbf{I}^*_{\mathrm{aA}} = 60 \times 10^3 + j160 \times 10^3 - j1200 \times 10^3$

$$\mathbf{I}^*_{\mathrm{aA}} = \frac{60\sqrt{3} - j1040\sqrt{3}}{13.8} = 7.53 - j130.53\,\mathrm{A}$$

$$\therefore\quad \mathbf{I}_{\mathrm{aA}} = 7.53 + j130.53\,\mathrm{A}$$

$$\mathbf{V}_{\mathrm{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(7.53 + j130.53)$$

$$= 7345.41 + j114.46 = 7346.3\underline{/0.89^\circ}\,\mathrm{V}$$

$$\therefore\quad |\mathbf{V}_{\mathrm{ab}}| = \sqrt{3}(7346.3) = 12{,}724.16\,\mathrm{V}$$

[b] Yes, the magnitude of the line-to-line voltage at the power plant is less than the allowable minimum of 13 kV.

P 11.60 **[a]** $\dfrac{13{,}800}{\sqrt{3}}\mathbf{I}^*_{\mathrm{aA}} = (60 + j160) \times 10^3$

$$\mathbf{I}^*_{\mathrm{aA}} = \frac{60\sqrt{3} + j160\sqrt{3}}{13.8} = 7.53 + j20.08\,\mathrm{A}$$

$$\therefore\quad \mathbf{I}_{\mathrm{aA}} = 7.53 - j20.08\,\mathrm{A}$$

$$\mathbf{V}_{\mathrm{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(7.53 - j20.08)$$

$$= 8068.34 + j24.10 = 8068.38\underline{/0.17^\circ}\,\mathrm{V}$$

$$\therefore\quad |\mathbf{V}_{\mathrm{ab}}| = \sqrt{3}(8068.38) = 13{,}974.77\,\mathrm{V}$$

[b] Yes: $13\,\mathrm{kV} < 13{,}974.77 < 14.6\,\mathrm{kV}$

[c] $P_{\mathrm{loss}} = 3|7.53 + j130.53|^2(0.6) = 30.77\,\mathrm{kW}$

[d] $P_{\mathrm{loss}} = 3|7.53 - j20.08|^2(0.6) = 0.83\,\mathrm{kW}$

[e] Yes, the voltage at the generating plant is at an acceptable level and the line loss is greatly reduced.

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12

Introduction to the Laplace Transform

Assessment Problems

AP 12.1 **[a]** $\cosh \beta t = \dfrac{e^{\beta t} + e^{-\beta t}}{2}$

Therefore,

$$\mathcal{L}\{\cosh \beta t\} = \frac{1}{2}\int_{0^-}^{\infty} [e^{(s-\beta)t} + e^{-(s-\beta)t}]dt$$

$$= \frac{1}{2}\left[\frac{e^{-(s-\beta)t}}{-(s-\beta)}\Bigg|_{0^-}^{\infty} + \frac{e^{-(s+\beta)t}}{-(s+\beta)}\Bigg|_{0^-}^{\infty}\right]$$

$$= \frac{1}{2}\left(\frac{1}{s-\beta} + \frac{1}{s+\beta}\right) = \frac{s}{s^2-\beta^2}$$

[b] $\sinh \beta t = \dfrac{e^{\beta t} - e^{-\beta t}}{2}$

Therefore,

$$\mathcal{L}\{\sinh \beta t\} = \frac{1}{2}\int_{0^-}^{\infty} \left[e^{-(s-\beta)t} - e^{-(s+\beta)t}\right] dt$$

$$= \frac{1}{2}\left[\frac{e^{-(s-\beta)t}}{-(s-\beta)}\right]_{0^-}^{\infty} - \frac{1}{2}\left[\frac{e^{-(s+\beta)t}}{-(s+\beta)}\right]_{0^-}^{\infty}$$

$$= \frac{1}{2}\left(\frac{1}{s-\beta} - \frac{1}{s+\beta}\right) = \frac{\beta}{(s^2-\beta^2)}$$

AP 12.2 **[a]** Let $f(t) = te^{-at}$:

$$F(s) = \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

Now, $\quad \mathcal{L}\{tf(t)\} = -\dfrac{dF(s)}{ds}$

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$$\text{So,} \quad \mathcal{L}\{t \cdot te^{-at}\} = -\frac{d}{ds}\left[\frac{1}{(s+a)^2}\right] = \frac{2}{(s+a)^3}$$

[b] Let $f(t) = e^{-at}\sinh\beta t$, then

$$\mathcal{L}\{f(t)\} = F(s) = \frac{\beta}{(s+a)^2 - \beta^2}$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-) = \frac{s(\beta)}{(s+a)^2 - \beta^2} - 0 = \frac{\beta s}{(s+a)^2 - \beta^2}$$

[c] Let $f(t) = \cos\omega t$. Then

$$F(s) = \frac{s}{(s^2+\omega^2)} \quad \text{and} \quad \frac{dF(s)}{ds} = \frac{-(s^2-\omega^2)}{(s^2+\omega^2)^2}$$

$$\text{Therefore} \quad \mathcal{L}\{t\cos\omega t\} = -\frac{dF(s)}{ds} = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$$

AP 12.3

$$F(s) = \frac{6s^2+26s+26}{(s+1)(s+2)(s+3)} = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

$$K_1 = \frac{6-26+26}{(1)(2)} = 3; \qquad K_2 = \frac{24-52+26}{(-1)(1)} = 2$$

$$K_3 = \frac{54-78+26}{(-2)(-1)} = 1$$

$$\text{Therefore} \quad f(t) = \left[3e^{-t} + 2e^{-2t} + e^{-3t}\right]u(t)$$

AP 12.4

$$F(s) = \frac{7s^2+63s+134}{(s+3)(s+4)(s+5)} = \frac{K_1}{s+3} + \frac{K_2}{s+4} + \frac{K_3}{s+5}$$

$$K_1 = \frac{63-189-134}{1(2)} = 4; \qquad K_2 = \frac{112-252+134}{(-1)(1)} = 6$$

$$K_3 = \frac{175-315+134}{(-2)(-1)} = -3$$

$$f(t) = [4e^{-3t} + 6e^{-4t} - 3e^{-5t}]u(t)$$

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AP 12.5

$$F(s) = \frac{10(s^2+119)}{(s+5)(s^2+10s+169)}$$

$$s_{1,2} = -5 \pm \sqrt{25-169} = -5 \pm j12$$

$$F(s) = \frac{K_1}{s+5} + \frac{K_2}{s+5-j12} + \frac{K_2^*}{s+5+j12}$$

$$K_1 = \frac{10(25+119)}{25-50+169} = 10$$

$$K_2 = \frac{10[(-5+j12)^2+119]}{(j12)(j24)} = j4.17 = 4.17\underline{/90^\circ}$$

Therefore

$$\begin{aligned} f(t) &= [10e^{-5t} + 8.33e^{-5t}\cos(12t+90^\circ)]\,u(t) \\ &= [10e^{-5t} - 8.33e^{-5t}\sin 12t]\,u(t) \end{aligned}$$

AP 12.6

$$F(s) = \frac{4s^2+7s+1}{s(s+1)^2} = \frac{K_0}{s} + \frac{K_1}{(s+1)^2} + \frac{K_2}{s+1}$$

$$K_0 = \frac{1}{(1)^2} = 1; \qquad K_1 = \frac{4-7+1}{-1} = 2$$

$$\begin{aligned} K_2 &= \frac{d}{ds}\left[\frac{4s^2+7s+1}{s}\right]_{s=-1} = \left.\frac{s(8s+7)-(4s^2+7s+1)}{s^2}\right|_{s=-1} \\ &= \frac{1+2}{1} = 3 \end{aligned}$$

Therefore $\quad f(t) = [1 + 2te^{-t} + 3e^{-t}]\,u(t)$

AP 12.7

$$\begin{aligned} F(s) &= \frac{40}{(s^2+4s+5)^2} = \frac{40}{(s+2-j1)^2(s+2+j1)^2} \\ &= \frac{K_1}{(s+2-j1)^2} + \frac{K_2}{(s+2-j1)} + \frac{K_1^*}{(s+2+j1)^2} \\ &\quad + \frac{K_2^*}{(s+2+j1)} \end{aligned}$$

$$K_1 = \frac{40}{(j2)^2} = -10 = 10\underline{/180^\circ} \quad \text{and} \quad K_1^* = -10$$

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$$K_2 = \frac{d}{ds}\left[\frac{40}{(s+2+j1)^2}\right]_{s=-2+j1} = \frac{-80(j2)}{(j2)^4} = -j10 = 10\underline{/-90^\circ}$$

$$K_2^* = j10$$

Therefore

$$\begin{aligned} f(t) &= [20te^{-2t}\cos(t+180^\circ) + 20e^{-2t}\cos(t-90^\circ)]\,u(t) \\ &= 20e^{-2t}[\sin t - t\cos t]\,u(t) \end{aligned}$$

AP 12.8

$$F(s) = \frac{5s^2+29s+32}{(s+2)(s+4)} = \frac{5s^2+29s+32}{s^2+6s+8} = 5 - \frac{s+8}{(s+2)(s+4)}$$

$$\frac{s+8}{(s+2)(s+4)} = \frac{K_1}{s+2} + \frac{K_2}{s+4}$$

$$K_1 = \frac{-2+8}{2} = 3; \qquad K_2 = \frac{-4+8}{-2} = -2$$

Therefore,

$$F(s) = 5 - \frac{3}{s+2} + \frac{2}{s+4}$$

$$f(t) = 5\delta(t) + [-3e^{-2t} + 2e^{-4t}]u(t)$$

AP 12.9

$$F(s) = \frac{2s^3+8s^2+2s-4}{s^2+5s+4} = 2s - 2 + \frac{4(s+1)}{(s+1)(s+4)} = 2s - 2 + \frac{4}{s+4}$$

$$f(t) = 2\frac{d\delta(t)}{dt} - 2\delta(t) + 4e^{-4t}\,u(t)$$

AP 12.10

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{7s^3[1+(9/s)+(134/(7s^2))]}{s^3[1+(3/s)][1+(4/s)][1+(5/s)]}\right] = 7$$

$$\therefore \quad f(0^+) = 7$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{7s^3+63s^2+134s}{(s+3)(s+4)(s+5)}\right] = 0$$

$$\therefore \quad f(\infty) = 0$$

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$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{s^3[4+(7/s)+(1/s)^2]}{s^3[1+(1/s)]^2}\right] = 4$$

$$\therefore \quad f(0^+) = 4$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{4s^2+7s+1}{(s+1)^2}\right] = 1$$

$$\therefore \quad f(\infty) = 1$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{40s}{s^4[1+(4/s)+(5/s^2)]^2}\right] = 0$$

$$\therefore \quad f(0^+) = 0$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{40s}{(s^2+4s+5)^2}\right] = 0$$

$$\therefore \quad f(\infty) = 0$$

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Problems

P 12.1 **[a]**

[b] $f(t) = 5t[u(t) - u(t-10)] + (100 - 5t)[u(t-10) - u(t-30)]$

$-50[u(t-30) - u(t-40)]$

$+(2.5t - 150)[u(t-40) - u(t-60)]$

P 12.2

P 12.3 **[a]** $(-3t - 15)[u(t+5) - u(t)] + (-3t + 15)[u(t) - u(t-5)]$

$= -3(t+5)u(t+5) + 30u(t) + 3(t-5)u(t-5)$

[b] $(5t + 20)[u(t+4) - u(t+2)] - 5t[u(t+2) - u(t-2)]$

$+(5t - 20)[u(t-2) - u(t-4)]$

$= 5(t+4)u(t+4) - 10(t+2)u(t+2) + 10(t-2)u(t-2)$

$-5(t-4)u(t-4)$

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P 12.4 **[a]** $6.25t[u(t) - u(t-8)] + 50[u(t-8) - u(t-12)]$

$$+(125 - 6.25t)[u(t-12) - u(t-20)]$$

[b] $25e^{-t}[u(t) - u(t-2)]$

[c] $(30 - 2t)[u(t) - u(t-5)] + 20[u(t-5) - u(t-15)]$

$$+(50 - 2t)[u(t-15) - u(t-25)]$$

P 12.5 As $\varepsilon \to 0$ the amplitude $\to \infty$; the duration $\to 0$; and the area is independent of ε, i.e.,

$$A = \int_{-\infty}^{\infty} \frac{\varepsilon}{\pi} \frac{1}{\varepsilon^2 + t^2}\, dt = 1$$

P 12.6 **[a]** $A = \left(\frac{1}{2}\right) bh = \left(\frac{1}{2}\right)(2\varepsilon)\left(\frac{1}{\varepsilon}\right) = 1$

[b] 0; **[c]** ∞

P 12.7

$$F(s) = \int_{-\varepsilon}^{-\varepsilon/2} \frac{4}{\varepsilon^3} e^{-st}\, dt + \int_{-\varepsilon/2}^{\varepsilon/2} \left(\frac{-4}{\varepsilon^3}\right) e^{-st}\, dt + \int_{\varepsilon/2}^{\varepsilon} \frac{4}{\varepsilon^3} e^{-st}\, dt$$

Therefore $\quad F(s) = \dfrac{4}{s\varepsilon^3}[e^{s\varepsilon} - 2e^{s\varepsilon/2} + 2e^{-s\varepsilon/2} - e^{-s\varepsilon}]$

$$\mathcal{L}\{\delta''(t)\} = \lim_{\varepsilon \to 0} F(s)$$

After applying L'Hopital's rule three times, we have

$$\lim_{\varepsilon \to 0} \frac{2s}{3}\left[se^{s\varepsilon} - \frac{s}{4}e^{s\varepsilon/2} - \frac{s}{4}e^{-s\varepsilon/2} + se^{-s\varepsilon}\right] = \frac{2s}{3}\left(\frac{3s}{2}\right)$$

Therefore $\quad \mathcal{L}\{\delta''(t)\} = s^2$

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P 12.8 **[a]** $I = \int_{-1}^{3} (t^3+2)\delta(t)\,dt + \int_{-1}^{3} 8(t^3+2)\delta(t-1)\,dt$

$$= (0^3+2) + 8(1^3+2) = 2 + 8(3) = 26$$

[b] $I = \int_{-2}^{2} t^2\delta(t)\,dt + \int_{-2}^{2} t^2\delta(t+1.5)\,dt + \int_{-2}^{2} \delta(t-3)\,dt$

$$= 0^2 + (-1.5)^2 + 0 = 2.25$$

P 12.9 $F(s) = \int_{-\varepsilon}^{\varepsilon} \frac{1}{2\varepsilon} e^{-st}\,dt = \frac{e^{s\varepsilon} - e^{-s\varepsilon}}{2\varepsilon s}$

$$F(s) = \frac{1}{2s} \lim_{\varepsilon \to 0} \left[\frac{se^{s\varepsilon} + se^{-s\varepsilon}}{1} \right] = \frac{1}{2s} \cdot \frac{2s}{1} = 1$$

P 12.10 $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{(4+j\omega)}{(9+j\omega)} \cdot \pi\delta(\omega) \cdot e^{jt\omega}\,d\omega = \left(\frac{1}{2\pi}\right)\left(\frac{4+j0}{9+j0}\pi e^{-jt0}\right) = \frac{2}{9}$

P 12.11 $\mathcal{L}\left\{\frac{d^n f(t)}{dt^n}\right\} = s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \cdots,$

Therefore

$$\mathcal{L}\{\delta^n(t)\} = s^n(1) - s^{n-1}\delta(0^-) - s^{n-2}\delta'(0^-) - s^{n-3}\delta''(0^-) - \cdots = s^n$$

P 12.12 **[a]** Let $dv = \delta'(t-a)\,dt, \qquad v = \delta(t-a)$

$u = f(t), \qquad du = f'(t)\,dt$

Therefore

$$\int_{-\infty}^{\infty} f(t)\delta'(t-a)\,dt = f(t)\delta(t-a)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \delta(t-a)f'(t)\,dt$$

$$= 0 - f'(a)$$

[b] $\mathcal{L}\{\delta'(t)\} = \int_{0^-}^{\infty} \delta'(t)e^{-st}\,dt = -\left[\frac{d(e^{-st})}{dt}\right]_{t=0} = -\left[-se^{-st}\right]_{t=0} = s$

P 12.13 **[a]** $\mathcal{L}\{20e^{-500(t-10)}u(t-10)\} = \frac{20e^{-10s}}{(s+500)}$

[b] First rewrite $f(t)$ as

$$\begin{aligned} f(t) &= (5t+20)u(t+4) - (10t+20)u(t+2) \\ &\quad +(10t-20)u(t-2) - (5t-20)u(t-4) \\ &= 5(t+4)u(t+4) - 10(t+2)u(t+2) \\ &\quad +10(t-2)u(t-2) - 5(t-4)u(t-4) \end{aligned}$$

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$$\therefore \quad F(s) = \frac{5[e^{4s} - 2e^{2s} + 2e^{-2s} - e^{-4s}]}{s^2}$$

P 12.14 **[a]**

$$\begin{aligned} f(t) &= (-8t - 80)[u(t+10) - u(t+5)] \\ &\quad +8t[u(t+5) - u(t-5)] \\ &\quad +(-8t + 80)[u(t-5) - u(t-10)] \\ &= -8(t+10)u(t+10) + 16(t+5)u(t+5) \\ &\quad -16(t-5)u(t-5) + 8(t-10)u(t-10) \end{aligned}$$

$$\therefore \quad F(s) = \frac{8[-e^{10s} + 2e^{5s} - 2e^{-5s} + e^{-10s}]}{s^2}$$

[b]

$$\begin{aligned} f'(t) &= -8[u(t+10) - u(t+5)] + 8[u(t+5) - u(t-5)] \\ &\quad +(-8)[u(t-5) - u(t-10)] \\ &= -8u(t+10) + 16u(t+5) - 16u(t-5) + 8u(t-10) \end{aligned}$$

$$\mathcal{L}\{f'(t)\} = \frac{8[-e^{10s} + 2e^{5s} - 2e^{-5s} + e^{-10s}]}{s}$$

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[c]

$$f''(t) = -8\delta(t+10) + 16\delta(t+5) - 16\delta(t-5) + 8\delta(t-10)$$

$$\mathcal{L}\{f''(t)\} = 8[-e^{10s} + 2e^{5s} - 2e^{-5s} + e^{-10s}]$$

P 12.15 $\mathcal{L}\{e^{-at}f(t)\} = \int_{0^-}^{\infty} [e^{-at}f(t)]e^{-st}\,dt = \int_{0^-}^{\infty} f(t)e^{-(s+a)t}\,dt = F(s+a)$

P 12.16 $\mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(at)e^{-st}\,dt$

Let $u = at, \qquad du = a\,dt, \qquad u = 0^- \quad \text{when} \quad t = 0^-$

and $u = \infty \quad \text{when} \quad t = \infty$

Therefore $\mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(u)e^{-(u/a)s}\dfrac{du}{a} = \dfrac{1}{a}F(s/a)$

P 12.17 **[a]** $\mathcal{L}\{te^{-at}\} = \int_{0^-}^{\infty} te^{-(s+a)t}\,dt$

$$= \frac{e^{-(s+a)t}}{(s+a)^2}\Big[\ -(s+a)t - 1\Big]_{0^-}^{\infty}$$

$$= 0 + \frac{1}{(s+a)^2}$$

$$\therefore \quad \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

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[b] $\mathcal{L}\left\{\dfrac{d}{dt}(te^{-at})u(t)\right\} = \dfrac{s}{(s+a)^2} - 0$

$$\mathcal{L}\left\{\frac{d}{dt}(te^{-at})u(t)\right\} = \frac{s}{(s+a)^2}$$

[c] $\dfrac{d}{dt}(te^{-at}) = -ate^{-at} + e^{-at}$

$$\mathcal{L}\{-ate^{-at} + e^{-at}\} = \frac{-a}{(s+a)^2} + \frac{1}{(s+a)} = \frac{-a}{(s+a)^2} + \frac{s+a}{(s+a)^2}$$

$$\therefore \quad \mathcal{L}\left\{\frac{d}{dt}(te^{-at})\right\} = \frac{s}{(s+a)^2} \qquad \text{CHECKS}$$

P 12.18 **[a]** $\mathcal{L}\left\{\displaystyle\int_{0^-}^{t} e^{-ax}\,dx\right\} = \dfrac{F(s)}{s} = \dfrac{1}{s(s+a)}$

[b] $\displaystyle\int_{0^-}^{t} e^{-ax}\,dx = \frac{1}{a} - \frac{e^{-at}}{a}$

$$\mathcal{L}\left\{\frac{1}{a} - \frac{e^{-at}}{a}\right\} = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{s+a}\right] = \frac{1}{s(s+a)}$$

P 12.19 **[a]** $\displaystyle\int_{0^-}^{t} x\,dx = \frac{t^2}{2}$

$$\mathcal{L}\left\{\frac{t^2}{2}\right\} = \frac{1}{2}\int_{0^-}^{\infty} t^2 e^{-st}\,dt$$

$$= \frac{1}{2}\left[\frac{e^{-st}}{-s^3}(s^2t^2 + 2st + 2)\Big|_{0^-}^{\infty}\right]$$

$$= \frac{1}{2s^3}(2) = \frac{1}{s^3}$$

$$\therefore \quad \mathcal{L}\left\{\int_{0^-}^{t} x\,dx\right\} = \frac{1}{s^3}$$

[b] $\mathcal{L}\left\{\displaystyle\int_{0^-}^{t} x\,dx\right\} = \dfrac{\mathcal{L}\{t\}}{s} = \dfrac{1/s^2}{s} = \dfrac{1}{s^3}$

$$\therefore \quad \mathcal{L}\left\{\int_{0^-}^{t} x\,dx\right\} = \frac{1}{s^3} \qquad \text{CHECKS}$$

P 12.20 **[a]** $\mathcal{L}\{t\} = \dfrac{1}{s^2}$; therefore $\mathcal{L}\{te^{-at}\} = \dfrac{1}{(s+a)^2}$

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[b] $\sin\omega t = \dfrac{e^{j\omega t} - e^{-j\omega t}}{j2}$

Therefore

$$\mathcal{L}\{\sin\omega t\} = \left(\frac{1}{j2}\right)\left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right) = \left(\frac{1}{j2}\right)\left(\frac{2j\omega}{s^2+\omega^2}\right)$$

$$= \frac{\omega}{s^2+\omega^2}$$

[c] $\sin(\omega t+\theta) = (\sin\omega t\cos\theta + \cos\omega t\sin\theta)$

Therefore

$$\mathcal{L}\{\sin(\omega t+\theta)\} = \cos\theta\mathcal{L}\{\sin\omega t\} + \sin\theta\mathcal{L}\{\cos\omega t\}$$

$$= \frac{\omega\cos\theta + s\sin\theta}{s^2+\omega^2}$$

[d] $\mathcal{L}\{t\} = \displaystyle\int_0^\infty te^{-st}\,dt = \frac{e^{-st}}{s^2}(-st-1)\Big|_0^\infty = 0 - \frac{1}{s^2}(0-1) = \frac{1}{s^2}$

[e] $f(t) = \cosh t\cosh\theta + \sinh t\sinh\theta$

From Assessment Problem 12.1(a)

$$\mathcal{L}\{\cosh t\} = \frac{s}{s^2-1}$$

From Assessment Problem 12.1(b)

$$\mathcal{L}\{\sinh t\} = \frac{1}{s^2-1}$$

$$\therefore\quad \mathcal{L}\{\cosh(t+\theta)\} = \cosh\theta\left[\frac{s}{(s^2-1)}\right] + \sinh\theta\left[\frac{1}{s^2-1}\right]$$

$$= \frac{\sinh\theta + s[\cosh\theta]}{(s^2-1)}$$

P 12.21 **[a]** $\dfrac{dF(s)}{ds} = \dfrac{d}{ds}\left[\displaystyle\int_{0^-}^\infty f(t)e^{-st}\,dt\right] = -\displaystyle\int_{0^-}^\infty tf(t)e^{-st}\,dt$

Therefore $\quad \mathcal{L}\{tf(t)\} = -\dfrac{dF(s)}{ds}$

[b] $\dfrac{d^2F(s)}{ds^2} = \displaystyle\int_{0^-}^\infty t^2f(t)e^{-st}\,dt; \qquad \dfrac{d^3F(s)}{ds^3} = \displaystyle\int_{0^-}^\infty -t^3f(t)e^{-st}\,dt$

Therefore $\quad \dfrac{d^nF(s)}{ds^n} = (-1)^n\displaystyle\int_{0^-}^\infty t^nf(t)e^{-st}\,dt = (-1)^n\mathcal{L}\{t^nf(t)\}$

[c] $\mathcal{L}\{t^5\} = \mathcal{L}\{t^4t\} = (-1)^4\dfrac{d^4}{ds^4}\left(\dfrac{1}{s^2}\right) = \dfrac{120}{s^6}$

$$\mathcal{L}\{t\sin\beta t\} = (-1)^1\frac{d}{ds}\left(\frac{\beta}{s^2+\beta^2}\right) = \frac{2\beta s}{(s^2+\beta^2)^2}$$

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$\mathcal{L}\{te^{-t}\cosh t\}$:

From Assessment Problem 12.1(a),

$$F(s) = \mathcal{L}\{\cosh t\} = \frac{s}{s^2-1}$$

$$\frac{dF}{ds} = \frac{(s^2-1)1 - s(2s)}{(s^2-1)^2} = -\frac{s^2+1}{(s^2-1)^2}$$

Therefore $\quad -\dfrac{dF}{ds} = \dfrac{s^2+1}{(s^2-1)^2}$

Thus

$$\mathcal{L}\{t\cosh t\} = \frac{s^2+1}{(s^2-1)^2}$$

$$\mathcal{L}\{e^{-t}t\cosh t\} = \frac{(s+1)^2+1}{[(s+1)^2-1]^2} = \frac{s^2+2s+2}{s^2(s+2)^2}$$

P 12.22 **[a]** $\mathcal{L}\left\{\dfrac{d\sin\omega t}{dt}u(t)\right\} = \dfrac{s\omega}{s^2+\omega^2} - \sin(0) = \dfrac{s\omega}{s^2+\omega^2}$

[b] $\mathcal{L}\left\{\dfrac{d\cos\omega t}{dt}u(t)\right\} = \dfrac{s^2}{s^2+\omega^2} - \cos(0) = \dfrac{s^2}{s^2+\omega^2} - 1 = \dfrac{-\omega^2}{s^2+\omega^2}$

[c] $\mathcal{L}\left\{\dfrac{d^3(t^2)}{dt^3}u(t)\right\} = s^3\left(\dfrac{2}{s^3}\right) - s^2(0) - s(0) - 2(0) = 2$

[d] $\dfrac{d\sin\omega t}{dt} = (\cos\omega t)\cdot\omega, \qquad \mathcal{L}\{\omega\cos\omega t\} = \dfrac{\omega s}{s^2+\omega^2}$

$$\frac{d\cos\omega t}{dt} = -\omega\sin\omega t$$

$$\mathcal{L}\{-\omega\sin\omega t\} = -\frac{\omega^2}{s^2+\omega^2}$$

$$\frac{d^3(t^2u(t))}{dt^3} = 2\delta(t); \qquad \mathcal{L}\{2\delta(t)\} = 2$$

P 12.23 **[a]** $\mathcal{L}\{f'(t)\} = \displaystyle\int_{-\varepsilon}^{\varepsilon}\frac{e^{-st}}{2\varepsilon}\,dt + \int_{\varepsilon}^{\infty} -ae^{-a(t-\varepsilon)}e^{-st}\,dt$

$$= \frac{1}{2s\varepsilon}(e^{s\varepsilon} - e^{-s\varepsilon}) - \left(\frac{a}{s+a}\right)e^{-s\varepsilon} = F(s)$$

$$\lim_{\varepsilon\to 0} F(s) = 1 - \frac{a}{s+a} = \frac{s}{s+a}$$

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[b] $\mathcal{L}\{e^{-at}\} = \dfrac{1}{s+a}$

Therefore $\quad \mathcal{L}\{f'(t)\} = sF(s) - f(0^-) = \dfrac{s}{s+a} - 0 = \dfrac{s}{s+a}$

P 12.24 **[a]** $f_1(t) = e^{-at}\cos\omega t; \qquad F_1(s) = \dfrac{s+a}{(s+a)^2+\omega^2}$

$$F(s) = sF_1(s) - f_1(0^-) = \frac{s(s+a)}{(s+a)^2+\omega^2} - 1 = \frac{-a^2 - sa - \omega^2}{(s+a)^2+\omega^2}$$

[b] $f_1(t) = e^{-at}\sin\omega t; \qquad F_1(s) = \dfrac{\omega}{(s+a)^2+\omega^2}$

$$F(s) = \frac{F_1(s)}{s} = \frac{\omega}{s[(s+a)^2+\omega^2]}$$

[c] $\dfrac{d}{dt}[e^{-at}\cos\omega t] = -\omega e^{-at}\sin\omega t - ae^{-at}\cos\omega t$

Therefore $\quad F(s) = \dfrac{-\omega^2 - a(s+a)}{(s+a)^2+\omega^2} = \dfrac{-a^2 - sa - \omega^2}{(s+a)^2+\omega^2}$

$$\int_{0^-}^{t} e^{-ax}\sin\omega x\,dx = \frac{-ae^{-at}\sin\omega t - \omega e^{-at}\cos\omega t + \omega}{a^2+\omega^2}$$

Therefore

$$F(s) = \frac{1}{a^2+\omega^2}\left[\frac{-a\omega}{(s+a)^2+\omega^2} - \frac{\omega(s+a)}{(s+a)^2+\omega^2} + \frac{\omega}{s}\right]$$

$$= \frac{\omega}{s[(s+a)^2+\omega^2]}$$

P 12.25 **[a]** $\displaystyle\int_s^\infty F(u)du = \int_s^\infty \left[\int_{0^-}^\infty f(t)e^{-ut}\,dt\right]du = \int_{0^-}^\infty \left[\int_s^\infty f(t)e^{-ut}\,du\right]dt$

$$= \int_{0^-}^\infty f(t)\int_s^\infty e^{-ut}\,du\,dt = \int_{0^-}^\infty f(t)\left[\frac{e^{-tu}}{-t}\Big|_s^\infty\right]dt$$

$$= \int_{0^-}^\infty f(t)\left[\frac{-e^{-st}}{-t}\right]dt = \mathcal{L}\left\{\frac{f(t)}{t}\right\}$$

[b] $\mathcal{L}\{t\sin\beta t\} = \dfrac{2\beta s}{(s^2+\beta^2)^2}$

therefore $\quad \mathcal{L}\left\{\dfrac{t\sin\beta t}{t}\right\} = \displaystyle\int_s^\infty \left[\frac{2\beta u}{(u^2+\beta^2)^2}\right]du$

Let $\omega = u^2+\beta^2$, then $\omega = s^2+\beta^2$ when $u = s$, and $\omega = \infty$ when $u = \infty$; also $d\omega = 2u\,du$, thus

$$\mathcal{L}\left\{\frac{t\sin\beta t}{t}\right\} = \beta\int_{s^2+\beta^2}^\infty \left[\frac{d\omega}{\omega^2}\right] = \beta\left(\frac{-1}{\omega}\right)\Big|_{s^2+\beta^2}^\infty = \frac{\beta}{s^2+\beta^2}$$

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P 12.26 $I_g(s) = \dfrac{5s}{s^2+400}; \qquad \dfrac{1}{RC} = 16; \qquad \dfrac{1}{LC} = 100; \qquad \dfrac{1}{C} = 20$

$$\frac{V(s)}{R} + \frac{1}{L}\frac{V(s)}{s} + C[sV(s) - v(0^-)] = I_g(s)$$

$$V(s)\left[\frac{1}{R} + \frac{1}{Ls} + sC\right] = I_g(s)$$

$$V(s) = \frac{I_g(s)}{\frac{1}{R} + \frac{1}{Ls} + sC} = \frac{LsI_g(s)}{\frac{sL}{R} + 1 + s^2LC} = \frac{\frac{1}{C}sI_g(s)}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

$$= \frac{(20)(5)s^2}{(s^2+16s+100)(s^2+400)} = \frac{100s^2}{(s^2+16s+100)(s^2+400)}$$

P 12.27 **[a]** $I_{\text{dc}} = \dfrac{1}{L}\displaystyle\int_0^t v_o\,dx + \frac{v_o}{R} + C\frac{dv_o}{dt}$

[b] $\dfrac{I_{\text{dc}}}{s} = \dfrac{V_o(s)}{sL} + \dfrac{V_o(s)}{R} + sCV_o(s)$

$$\therefore \quad V_o(s) = \frac{I_{\text{dc}}/C}{s^2 + (1/RC)s + (1/LC)}$$

[c] $i_o = C\dfrac{dv_o}{dt}$

$$\therefore \quad I_o(s) = sCV_o(s) = \frac{sI_{\text{dc}}}{s^2 + (1/RC)s + (1/LC)}$$

P 12.28 **[a]** For $t \geq 0^+$:

$$\frac{v_o}{R} + C\frac{dv_o}{dt} + i_o = 0$$

$$v_o = L\frac{di_o}{dt}; \qquad \frac{dv_o}{dt} = L\frac{d^2i_o}{dt^2}$$

$$\therefore \qquad \frac{L}{R}\frac{di_o}{dt} + LC\frac{d^2i_o}{dt^2} + i_o = 0$$

$$\text{or} \quad \frac{d^2i_o}{dt^2} + \frac{1}{RC}\frac{di_o}{dt} + \frac{1}{LC}i_o = 0$$

[b] $s^2I_o(s) - sI_{\text{dc}} - 0 + \dfrac{1}{RC}[sI_o(s) - I_{\text{dc}}] + \dfrac{1}{LC}I_o(s) = 0$

$$I_o(s)\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right] = I_{\text{dc}}(s + 1/RC)$$

$$I_o(s) = \frac{I_{\text{dc}}[s + (1/RC)]}{[s^2 + (1/RC)s + (1/LC)]}$$

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P 12.29 **[a]** For $t \geq 0^+$:

$$Ri_o + L\frac{di_o}{dt} + v_o = 0$$

$$i_o = C\frac{dv_o}{dt} \qquad \frac{di_o}{dt} = C\frac{d^2v_o}{dt^2}$$

$$\therefore \quad RC\frac{dv_o}{dt} + LC\frac{d^2v_o}{dt^2} + v_o = 0$$

or

$$\frac{d^2v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC}v_o = 0$$

[b] $s^2V_o(s) - sV_{\text{dc}} - 0 + \dfrac{R}{L}[sV_o(s) - V_{\text{dc}}] + \dfrac{1}{LC}V_o(s) = 0$

$$V_o(s)\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right] = V_{\text{dc}}(s + R/L)$$

$$V_o(s) = \frac{V_{\text{dc}}[s + (R/L)]}{[s^2 + (R/L)s + (1/LC)]}$$

P 12.30 **[a]** $\dfrac{v_o - V_{\text{dc}}}{R} + \dfrac{1}{L}\displaystyle\int_0^t v_o\,dx + C\frac{dv_o}{dt} = 0$

$$\therefore \qquad v_o + \frac{R}{L}\int_0^t v_o\,dx + RC\frac{dv_o}{dt} = V_{\text{dc}}$$

[b] $V_o + \dfrac{R}{L}\dfrac{V_o}{s} + RCsV_o = \dfrac{V_{\text{dc}}}{s}$

$$\therefore \qquad sLV_o + RV_o + RCLs^2V_o = LV_{\text{dc}}$$

$$\therefore \qquad V_o(s) = \frac{(1/RC)V_{\text{dc}}}{s^2 + (1/RC)s + (1/LC)}$$

[c] $i_o = \dfrac{1}{L}\displaystyle\int_0^t v_o\,dx$

$$I_o(s) = \frac{V_o}{sL} = \frac{V_{\text{dc}}/RLC}{s[s^2 + (1/RC)s + (1/LC)]}$$

P 12.31 **[a]** $\dfrac{1}{L}\displaystyle\int_0^t v_1\,d\tau + \frac{v_1 - v_2}{R} = i_g u(t)$

and

$$C\frac{dv_2}{dt} + \frac{v_2}{R} - \frac{v_1}{R} = 0$$

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[b] $\dfrac{V_1}{sL} + \dfrac{V_1 - V_2}{R} = I_g$

$$\frac{V_2 - V_1}{R} + sCV_2 = 0$$

or

$$(R + sL)V_1(s) - sLV_2(s) = RLsI_g(s)$$

$$-V_1(s) + (RCs + 1)V_2(s) = 0$$

Solving,

$$V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}$$

P 12.32 **[a]** $625 = 150i_1 + 62.5\dfrac{di_1}{dt} + 25\dfrac{d}{dt}(i_2 - i_1) + 12.5\dfrac{d}{dt}(i_1 - i_2) - 25\dfrac{di_1}{dt}$

$$0 = 12.5\frac{d}{dt}(i_2 - i_1) + 25\frac{di_1}{dt} + 100i_2$$

Simplifying the above equations gives:

$$625 = 150i_1 + 25\frac{di_1}{dt} + 12.5\frac{di_2}{dt}$$

$$0 = 100i_2 + 12.5\frac{di_1}{dt} + 12.5\frac{di_2}{dt}$$

[b] $\dfrac{625}{s} = (25s + 150)I_1(s) + 12.5sI_2(s)$

$$0 = 12.5sI_1(s) + (12.5s + 100)I_2(s)$$

[c] Solving the equations in (b),

$$I_1(s) = \frac{50(s + 8)}{s(s + 4)(s + 24)}$$

$$I_2(s) = \frac{-50}{(s + 4)(s + 24)}$$

P 12.33 From Problem 12.26:

$$V(s) = \frac{100s^2}{(s^2 + 16s + 100)(s^2 + 400)}$$

$$s^2 + 16s + 100 = (s + 8 + j6)(s + 8 - j6); \qquad s^2 + 400 = (s - j20)(s + j20)$$

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Therefore

$$V(s) = \frac{100s^2}{(s+8+j6)(s+8-j6)(s-j20)(s+j20)}$$
$$= \frac{K_1}{s+8-j6} + \frac{K_1^*}{s+8+j6} + \frac{K_2}{s-j20} + \frac{K_2^*}{s+j20}$$

$$K_1 = \frac{100s^2}{(s+8+j6)(s^2+400)}\bigg|_{s=-8+j6} = 1.9\underline{/-151.1^\circ}$$

$$K_2 = \frac{100s^2}{(s+j20)(s^2+16s+100)}\bigg|_{s=j20} = 2.28\underline{/-43.15^\circ}$$

Therefore

$$v(t) = [3.8e^{-8t}\cos(6t-151.1^\circ) + 4.56\cos(20t-43.15^\circ)]u(t)\,\text{V}$$

P 12.34 **[a]** $\dfrac{1}{RC} = \dfrac{1}{(20)(20\times10^{-6})} = 2500$

$$\frac{1}{LC} = \frac{1}{(0.05)(20\times10^{-6})} = 10^6; \qquad \frac{1}{C} = 50{,}000$$

$$V_o(s) = \frac{50{,}000I_{\text{dc}}}{s+2500s+10^6}$$
$$= \frac{50{,}000I_{\text{dc}}}{(s+500)(s+2000)}$$
$$= \frac{3750}{(s+500)(s+2000)}$$
$$= \frac{K_1}{s+500} + \frac{K_2}{s+2000}$$

$$K_1 = \frac{3750}{1500} = 2.5; \qquad K_2 = \frac{3750}{-1500} = -2.5$$

$$V_o(s) = \frac{2.5}{s+500} - \frac{2.5}{s+2000}$$

$$v_o(t) = [2.5e^{-500t} - 2.5e^{-2000t}]u(t)\,\text{V}$$

[b] $I_o(s) = \dfrac{0.075s}{(s+500)(s+2000)}$

$$= \frac{K_1}{s+500} + \frac{K_2}{s+2000}$$

$$K_1 = \frac{0.075(-500)}{1500} = -0.025$$

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$$K_2 = \frac{0.075(-2000)}{-1500} = 0.1$$

$$I_o(s) = \frac{-0.025}{s+500} + \frac{0.1}{s+2000}$$

$$i_o(t) = (-25e^{-500t} + 100e^{-2000t})u(t)\,\text{mA}$$

[c] $i_o(0) = 100 - 25 = 75\,\text{mA}$

Yes. The initial inductor current is zero by hypothesis, the initial resistor current is zero because the initial capacitor voltage is zero by hypothesis. Thus at $t = 0$ the source current appears in the capacitor.

P 12.35 $\dfrac{1}{RC} = 200{,}000; \qquad \dfrac{1}{LC} = 10^{10}$

$$I_o(s) = \frac{0.04(s+200{,}000)}{s^2 + 200{,}000s + 10^{10}}$$

$$s_{1,2} = -100{,}000$$

$$I_o(s) = \frac{0.04(s+200{,}000)}{(s+100{,}000)^2} = \frac{K_1}{(s+100{,}000)^2} + \frac{K_2}{s+100{,}000}$$

$$K_1 = 0.04(s+200{,}000)\Big|_{s=-100{,}000} = 4000$$

$$K_2 = \frac{d}{ds}[0.04(s+200{,}000)]_{s=-100{,}000} = 0.04$$

$$I_o(s) = \frac{4000}{(s+100{,}000)^2} + \frac{0.04}{s+100{,}000}$$

$$i_o(t) = [4000te^{-100{,}000t} + 0.04e^{-100{,}000t}]u(t)\,\text{V}$$

P 12.36 $\dfrac{R}{L} = 10{,}000; \qquad \dfrac{1}{LC} = 16 \times 10^6$

$$V_o(s) = \frac{120(s+10{,}000)}{s^2 + 10{,}000s + 16 \times 10^6}$$

$$= \frac{120(s+10{,}000)}{(s+2000)(s+8000)} = \frac{K_1}{s+2000} + \frac{K_2}{s+8000}$$

$$K_1 = \frac{120(8000)}{6000} = 160\,\text{V}; \qquad K_2 = \frac{120(2000)}{-6000} = -40\,\text{V}$$

$$V_o(s) = \frac{160}{s+2000} - \frac{40}{s+8000}$$

$$v_o(t) = [160e^{-2000t} - 40e^{-8000t}]u(t)\,\text{V}$$

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P 12.37 **[a]** $\dfrac{1}{LC} = \dfrac{1}{(1.6)(5\times 10^{-6})} = 125\times 10^3$

$$\frac{1}{RC} = \frac{1}{(2000)(5\times 10^{-6})} = 100$$

$$V_o(s) = \frac{5600}{s^2 + 100s + 125\times 10^3}$$

$$s_{1,2} = -50 \pm j350 \text{ rad/s}$$

$$V_o(s) = \frac{5600}{(s+50-j350)(s+50+j350)}$$

$$= \frac{K_1}{s+50-j350} + \frac{K_1^*}{s+50+j350}$$

$$K_1 = \frac{5600}{j700} = 8\underline{/-90^\circ}$$

$$v_o(t) = 16e^{-50t}\cos(350t - 90^\circ)]u(t)\,\text{V}$$

$$= [16e^{-50t}\sin 350t]u(t)\,\text{V}$$

[b] $I_o(s) = \dfrac{56(62.5)}{s(s+50-j350)(s+50+j350)}$

$$= \frac{K_1}{s} + \frac{K_2}{s+50-j350} + \frac{K_2^*}{s+50+j350}$$

$$K_1 = \frac{3500}{125\times 10^3} = 28\,\text{mA}$$

$$K_2 = \frac{3500}{(-50+j350)(j700)} = 14.14\underline{/171.87^\circ}\,\text{mA}$$

$$i_o(t) = [28 + 28.28e^{-50t}\cos(350t + 171.87^\circ)]u(t)\,\text{mA}$$

P 12.38 $\dfrac{1}{C} = 64\times 10^6;\qquad \dfrac{1}{LC} = 1600\times 10^6;\qquad \dfrac{R}{L} = 100{,}000;\qquad I_g = \dfrac{0.15}{s}$

$$V_2(s) = \frac{96\times 10^5}{s^2 + 10^5 s + 1600\times 10^6}$$

$$s_1 = -20{,}000;\qquad s_2 = -80{,}000$$

$$V_2(s) = \frac{96\times 10^5}{(s+20{,}000)(s+80{,}000)}$$

$$= \frac{160}{s+20{,}000} - \frac{160}{s+80{,}000}$$

$$v_2(t) = [160e^{-20{,}000t} - 160e^{-80{,}000t}]u(t)\,\text{V}$$

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P 12.39 **[a]** $I_1(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s+4} + \dfrac{K_3}{s+24}$

$$K_1 = \frac{(50)(8)}{(4)(24)} = 4.167; \qquad K_2 = \frac{(50)(4)}{(-4)(20)} = -2.5$$

$$K_3 = \frac{(50)(-16)}{(-24)(-20)} = -1.667$$

$$I_1(s) = \left(\frac{4.167}{s} - \frac{2.5}{s+4} - \frac{1.667}{s+24}\right)$$

$$i_1(t) = (4.167 - 2.5e^{-4t} - 1.667e^{-24t})u(t)\,\text{A}$$

$$I_2(s) = \frac{K_1}{s+4} + \frac{K_2}{s+24}$$

$$K_1 = \frac{-50}{20} = -2.5; \qquad K_2 = \frac{-50}{-20} = 2.5$$

$$I_2(s) = \left(\frac{-2.5}{s+4} + \frac{2.5}{s+24}\right)$$

$$i_2(t) = (2.5e^{-24t} - 2.5e^{-4t})u(t)\,\text{A}$$

[b] $i_1(\infty) = 4.167\,\text{A}; \qquad i_2(\infty) = 0\,\text{A}$

[c] Yes, at $t = \infty$

$$i_1 = \frac{625}{150} = 4.167\,\text{A}$$

Since i_1 is a dc current at $t = \infty$ there is no voltage induced in the 12.5 H inductor; hence, $i_2 = 0$. Also note that $i_1(0) = 0$ and $i_2(0) = 0$. Thus our solutions satisfy the condition of no initial energy stored in the circuit.

P 12.40 **[a]** $F(s) = \dfrac{K_1}{s+5} + \dfrac{K_2}{s+8}$

$$K_1 = \frac{6(s+10)}{(s+8)}\bigg|_{s=-5} = 10$$

$$K_2 = \frac{6(s+10)}{(s+5)}\bigg|_{s=-8} = -4$$

$$f(t) = [10e^{-5t} - 4e^{-8t}]u(t)$$

[b] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s+3} + \dfrac{K_3}{s+7}$

$$K_1 = \frac{20s^2 + 141s + 315}{(s+3)(s+7)}\bigg|_{s=0} = 15$$

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$$K_2 = \frac{20s^2 + 141s + 315}{s(s+7)}\bigg|_{s=-3} = -6$$

$$K_3 = \frac{20s^2 + 141s + 315}{s(s+3)}\bigg|_{s=-7} = 11$$

$$f(t) = [15 - 6e^{-3t} + 11e^{-7t}]u(t)$$

[c] $F(s) = \dfrac{K_1}{s+2} + \dfrac{K_2}{s+4} + \dfrac{K_3}{s+6}$

$$K_1 = \frac{15s^2 + 112s + 228}{(s+4)(s+6)}\bigg|_{s=-2} = 8$$

$$K_2 = \frac{15s^2 + 112s + 228}{(s+2)(s+6)}\bigg|_{s=-4} = -5$$

$$K_3 = \frac{15s^2 + 112s + 228}{(s+2)(s+4)}\bigg|_{s=-6} = 12$$

$$f(t) = [8e^{-2t} - 5e^{-4t} + 12e^{-6t}]u(t)$$

[d] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s+1} + \dfrac{K_3}{s+2} + \dfrac{K_3}{s+3}$

$$K_1 = \frac{2s^3 + 33s^2 + 93s + 54}{(s+1)(s+2)(s+3)}\bigg|_{s=0} = 9$$

$$K_2 = \frac{2s^3 + 33s^2 + 93s + 54}{s(s+2)(s+3)}\bigg|_{s=-1} = 4$$

$$K_3 = \frac{2s^3 + 33s^2 + 93s + 54}{s(s+1)(s+3)}\bigg|_{s=-2} = -8$$

$$K_4 = \frac{2s^3 + 33s^2 + 93s + 54}{s(s+1)(s+2)}\bigg|_{s=-3} = -3$$

$$f(t) = [9 + 4e^{-t} - 8e^{-2t} - 3e^{-3t}]u(t)$$

P 12.41 **[a]** $F(s) = \dfrac{K_1}{s+7-j14} + \dfrac{K_1^*}{s+7+j14}$

$$K_1 = \frac{280}{s+7+j14}\bigg|_{s=-7+j14} = -j10 = 10\underline{/-90^\circ}$$

$$f(t) = [20e^{-7t}\cos(14t - 90^\circ)]u(t) = [20e^{-7t}\sin 14t]u(t)$$

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[b] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s+5-j8} + \dfrac{K_2^*}{s+5+j8}$

$$K_1 = \frac{-s^2+52s+445}{s^2+10s+89}\bigg|_{s=0} = 5$$

$$K_2 = \frac{-s^2+52s+445}{s(s+5+j8)}\bigg|_{s=-5+j8} = -3-j2 = 3.6\underline{/-146.31^\circ}$$

$$f(t) = [5 + 7.2e^{-5t}\cos(8t - 146.31^\circ)]u(t)$$

[c] $F(s) = \dfrac{K_1}{s+6} + \dfrac{K_2}{s+2-j4} + \dfrac{K_2^*}{s+2+j4}$

$$K_1 = \frac{14s^2+56s+152}{s^2+4s+20}\bigg|_{s=-6} = 10$$

$$K_2 = \frac{14s^2+56s+152}{(s+6)(s+2+j4)}\bigg|_{s=-2+j4} = 2+j2 = 2.83\underline{/45^\circ}$$

$$f(t) = [10e^{-6t} + 5.66e^{-2t}\cos(4t + 45^\circ)]u(t)$$

[d] $F(s) = \dfrac{K_1}{s+5-j3} + \dfrac{K_1^*}{s+5+j3} + \dfrac{K_2}{s+4-j2} + \dfrac{K_2^*}{s+4+j2}$

$$K_1 = \frac{8(s+1)^2}{(s+5+j3)(s^2+8s+20)}\bigg|_{s=-5+j3} = 4.62\underline{/-40.04^\circ}$$

$$K_2 = \frac{8(s+1)^2}{(s+4+j2)(s^2+10s+34)}\bigg|_{s=-4+j2} = 3.61\underline{/168.93^\circ}$$

$$f(t) = [9.25e^{-5t}\cos(3t - 40.05^\circ) + 7.21e^{-4t}\cos(2t + 168.93^\circ)]u(t)$$

P 12.42 **[a]** $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s+8}$

$$K_1 = \frac{320}{s+8}\bigg|_{s=0} = 40$$

$$K_2 = \frac{d}{ds}\left[\frac{320}{s+8}\right] = \left[\frac{-320}{(s+8)^2}\right]_{s=0} = -5$$

$$K_3 = \frac{320}{s^2}\bigg|_{s=-8} = 5$$

$$f(t) = [40t - 5 + 5e^{-8t}]u(t)$$

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[b] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+2)^2} + \dfrac{K_3}{s+2}$

$$K_1 = \frac{80(s+3)}{(s+2)^2}\bigg|_{s=0} = 60$$

$$K_2 = \frac{80(s+3)}{s}\bigg|_{s=-2} = -40$$

$$K_3 = \frac{d}{ds}\left[\frac{80(s+3)}{s}\right] = \left[\frac{80}{s} - \frac{80(s+3)}{s^2}\right]_{s=-2} = -60$$

$$f(t) = [60 - 40te^{-2t} - 60e^{-2t}]u(t)$$

[c] $F(s) = \dfrac{K_1}{(s+1)^2} + \dfrac{K_2}{s+1} + \dfrac{K_3}{s+3-j4} + \dfrac{K_3^*}{s+3+j4}$

$$K_1 = \frac{60(s+5)}{s^2+6s+25}\bigg|_{s=-1} = 12$$

$$K_2 = \frac{d}{ds}\left[\frac{60(s+5)}{s^2+6s+25}\right] = \left[\frac{60}{s^2+6s+25} - \frac{60(s+5)(2s+6)}{(s^2+6s+25)^2}\right]_{s=-1} = 0.6$$

$$K_3 = \frac{60(s+5)}{(s+1)^2(s+3+j4)}\bigg|_{s=-3+j4} = 1.68\underline{/100.305^\circ}$$

$$f(t) = [12te^{-t} + 0.6e^{-t} + 3.35e^{-3t}\cos(4t + 100.305^\circ)]u(t)$$

[d] $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{(s+5)^2} + \dfrac{K_4}{s+5}$

$$K_1 = \frac{25(s+4)^2}{(s+5)^2}\bigg|_{s=0} = 16$$

$$K_2 = \frac{d}{ds}\left[\frac{25(s+4)^2}{(s+5)^2}\right] = \left[\frac{25(2)(s+4)}{(s+5)^2} - \frac{25(2)(s+4)^2}{(s+5)^3}\right]_{s=0} = 1.6$$

$$K_3 = \frac{25(s+4)^2}{s^2}\bigg|_{s=-5} = 1$$

$$K_4 = \frac{d}{ds}\left[\frac{25(s+4)^2}{s^2}\right] = \left[\frac{25(2)(s+4)}{s^2} - \frac{25(2)(s+4)^2}{s^3}\right]_{s=-5} = -1.6$$

$$f(t) = [16t + 1.6 + te^{-5t} - 1.6e^{-5t}]u(t)$$

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P 12.43 **[a]** $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+3)^3} + \dfrac{K_3}{(s+3)^2} + \dfrac{K_4}{s+3}$

$$K_1 = \frac{135}{(s+3)^3}\bigg|_{s=0} = 5$$

$$K_2 = \frac{135}{s}\bigg|_{s=-3} = -45$$

$$K_3 = \frac{d}{ds}\left[\frac{135}{s}\right] = \left[\frac{-135}{s^2}\right]_{s=-3} = -15$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{-135}{s^2}\right] = \left[\frac{1}{2}(-2)\left(\frac{-135}{s^3}\right)\right]_{s=-3} = -5$$

$$f(t) = [5 - 22.5t^2e^{-3t} - 15te^{-3t} - 5e^{-3t}]u(t)$$

[b] $F(s) = \dfrac{K_1}{(s+1-j1)^2} + \dfrac{K_1^*}{(s+1+j1)^2} + \dfrac{K_2}{s+1-j1} + \dfrac{K_2^*}{s+1+j1}$

$$K_1 = \frac{10(s+2)^2}{(s+1+j1)^2}\bigg|_{s=-1+j1} = -j5 = 5\underline{/-90^\circ}$$

$$K_2 = \frac{d}{ds}\left[\frac{10(s+2)^2}{(s+1+j1)^2}\right] = \left[\frac{10(2)(s+2)}{(s+1+j1)^2} - \frac{10(2)(s+2)^2}{(s+1+j1)^3}\right]_{s=0}$$

$$= -j5 = 5\underline{/-90^\circ}$$

$$f(t) = [10te^{-t}\cos(t-90^\circ) + 10e^{-t}\cos(t-90^\circ)]u(t)$$

[c]

$$F(s) = \begin{array}{r|r} & 25 \\ \hline s^2+15s+54 & 25s^2+395s+1494 \\ & 25s^2+375s+1350 \\ \hline & 20s+144 \end{array}$$

$$F(s) = 25 + \frac{20s+144}{s^2+15s+54} = 25 + \frac{K_1}{s+6} + \frac{K_2}{s+9}$$

$$K_1 = \frac{20s+144}{s+9}\bigg|_{s=-6} = 8$$

$$K_2 = \frac{20s+144}{s+6}\bigg|_{s=-9} = 12$$

$$f(t) = 25\delta(t) + [8e^{-6t} + 12e^{-9t}]u(t)$$

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[d]

$$F(s) = \begin{array}{r|l} & \quad 5s - 15 \\ \hline s^2 + 7s + 10 & 5s^3 + 20s^2 - 49s - 108 \\ & 5s^3 + 35s^2 + 50s \\ \hline & \quad -15s^2 - 99s - 108 \\ & \quad -15s^2 - 105s - 150 \\ \hline & \qquad\qquad 6s + 42 \end{array}$$

$$F(s) = 5s - 15 + \frac{K_1}{s+2} + \frac{K_2}{s+5}$$

$$K_1 = \left.\frac{6s+42}{s+5}\right|_{s=-2} = 10$$

$$K_2 = \left.\frac{6s+42}{s+2}\right|_{s=-5} = -4$$

$$f(t) = 5\delta'(t) - 15\delta(t) + [10e^{-2t} - 4e^{-5t}]u(t)$$

P 12.44 $f(t) = \mathcal{L}^{-1}\left\{\dfrac{K}{s+\alpha-j\beta} + \dfrac{K^*}{s+\alpha+j\beta}\right\}$

$$= Ke^{-\alpha t}e^{j\beta t} + K^*e^{-\alpha t}e^{-j\beta t}$$

$$= |K|e^{-\alpha t}[e^{j\theta}e^{j\beta t} + e^{-j\theta}e^{-j\beta t}]$$

$$= |K|e^{-\alpha t}[e^{j(\beta t+\theta)} + e^{-j(\beta t+\theta)}]$$

$$= 2|K|e^{-\alpha t}\cos(\beta t + \theta)$$

P 12.45 **[a]** $\mathcal{L}\{t^n f(t)\} = (-1)^n\left[\dfrac{d^nF(s)}{ds^n}\right]$

Let $f(t) = 1$, then $F(s) = \dfrac{1}{s}$, thus $\dfrac{d^nF(s)}{ds^n} = \dfrac{(-1)^n n!}{s^{(n+1)}}$

Therefore $\mathcal{L}\{t^n\} = (-1)^n\left[\dfrac{(-1)^n n!}{s^{(n+1)}}\right] = \dfrac{n!}{s^{(n+1)}}$

It follows that $\mathcal{L}\{t^{(r-1)}\} = \dfrac{(r-1)!}{s^r}$

and $\mathcal{L}\{t^{(r-1)}e^{-at}\} = \dfrac{(r-1)!}{(s+a)^r}$

Therefore $\dfrac{K}{(r-1)!}\mathcal{L}\{t^{r-1}e^{-at}\} = \dfrac{K}{(s+a)^r} = \mathcal{L}\left\{\dfrac{Kt^{r-1}e^{-at}}{(r-1)!}\right\}$

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[b] $f(t) = \mathcal{L}^{-1}\left\{\dfrac{K}{(s+\alpha-j\beta)^r} + \dfrac{K^*}{(s+\alpha+j\beta)^r}\right\}$

Therefore

$$f(t) = \frac{Kt^{r-1}}{(r-1)!}e^{-(\alpha-j\beta)t} + \frac{K^*t^{r-1}}{(r-1)!}e^{-(\alpha+j\beta)t}$$

$$= \frac{|K|t^{r-1}e^{-\alpha t}}{(r-1)!}\left[e^{j\theta}e^{j\beta t} + e^{-j\theta}e^{-j\beta t}\right]$$

$$= \left[\frac{2|K|t^{r-1}e^{-\alpha t}}{(r-1)!}\right]\cos(\beta t + \theta)$$

P 12.46 **[a]** $\lim\limits_{s\to\infty} sV(s) = \lim\limits_{s\to\infty}\left[\dfrac{1.92s^3}{s^4[1+(1.6/s)+(1/s^2)][1+(1/s^2)]}\right] = 0$

Therefore $v(0^+) = 0$

[b] No, V has a pair of poles on the imaginary axis.

P 12.47 $sV_o(s) = \dfrac{(I_{\text{dc}}/C)s}{s^2+(1/RC)s+(1/LC)}$

$\lim\limits_{s\to 0} sV_o(s) = 0, \qquad \therefore \quad v_o(\infty) = 0$

$\lim\limits_{s\to\infty} sV_o(s) = 0, \qquad \therefore \quad v_o(0^+) = 0$

$sI_o(s) = \dfrac{s^2 I_{\text{dc}}}{s^2+(1/RC)s+(1/LC)}$

$\lim\limits_{s\to 0} sI_o(s) = 0, \qquad \therefore \quad i_o(\infty) = 0$

$\lim\limits_{s\to\infty} sI_o(s) = I_{\text{dc}}, \qquad \therefore \quad i_o(0^+) = I_{\text{dc}}$

P 12.48 $sI_o(s) = \dfrac{I_{\text{dc}}s[s+(1/RC)]}{s^2+(1/RC)s+(1/LC)}$

$\lim\limits_{s\to 0} sI_o(s) = 0, \qquad \therefore \quad i_o(\infty) = 0$

$\lim\limits_{s\to\infty} sI_o(s) = I_{\text{dc}}, \qquad \therefore \quad i_o(0^+) = I_{\text{dc}}$

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P 12.49 $sV_o(s) = \dfrac{sV_{\text{dc}}/RC}{s^2 + (1/RC)s + (1/LC)}$

$$\lim_{s\to 0} sV_o(s) = 0, \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to \infty} sV_o(s) = 0, \qquad \therefore \quad v_o(0^+) = 0$$

$$sI_o(s) = \frac{V_{\text{dc}}/RC)}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s\to 0} sI_o(s) = \frac{V_{\text{dc}}/RLC}{1/LC} = \frac{V_{\text{dc}}}{R}, \qquad \therefore \quad i_o(\infty) = \frac{V_{\text{dc}}}{R}$$

$$\lim_{s\to \infty} sI_o(s) = 0, \qquad \therefore \quad i_o(0^+) = 0$$

P 12.50 **[a]** $sF(s) = \dfrac{6s^2 + 60s}{(s+5)(s+8)}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to \infty} sF(s) = 6, \qquad \therefore \quad f(0^+) = 6$$

[b] $sF(s) = \dfrac{20s^2 + 141s + 315}{(s^2 + 10s + 21)}$

$$\lim_{s\to 0} sF(s) = 15; \qquad \therefore \quad f(\infty) = 15$$

$$\lim_{s\to \infty} sF(s) = 20, \qquad \therefore \quad f(0^+) = 20$$

[c] $sF(s) = \dfrac{15s^3 + 112s^2 + 228s}{(s+2)(s+4)(s+6)}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to \infty} sF(s) = 15, \qquad \therefore \quad f(0^+) = 15$$

[d] $sF(s) = \dfrac{2s^3 + 33s^2 + 93s + 54}{(s+1)(s^2 + 5s + 6)}$

$$\lim_{s\to 0} sF(s) = 9, \qquad \therefore \quad f(\infty) = 9$$

$$\lim_{s\to \infty} sF(s) = 2, \qquad \therefore \quad f(0^+) = 2$$

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P 12.51 **[a]** $sF(s) = \dfrac{280s}{s^2 + 14s + 245}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

[b] $sF(s) = \dfrac{-s^2 + 52s + 445}{s^2 + 10s + 89}$

$$\lim_{s\to 0} sF(s) = 5, \qquad \therefore \quad f(\infty) = 5$$

$$\lim_{s\to \infty} sF(s) = -1, \qquad \therefore \quad f(0^+) = -1$$

[c] $sF(s) = \dfrac{14s^3 + 56s^2 + 152s}{(s+6)(s^2 + 4s + 20)}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to \infty} sF(s) = 14, \qquad \therefore \quad f(0^+) = 14$$

[d] $sF(s) = \dfrac{8s(s+1)^2}{(s^2 + 10s + 34)(s^2 + 8s + 20)}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

P 12.52 **[a]** $sF(s) = \dfrac{320}{s(s+8)}$

$F(s)$ has a second-order pole at the origin so we cannot use the final value theorem here.

$$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

[b] $sF(s) = \dfrac{80(s+3)}{(s+2)^2}$

$$\lim_{s\to 0} sF(s) = 60, \qquad \therefore \quad f(\infty) = 60$$

$$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

[c] $sF(s) = \dfrac{60s(s+5)}{(s+1)^2(s^2 + 6s + 25)}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

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[d] $sF(s) = \dfrac{25(s+4)^2}{s(s+5)^2}$

$F(s)$ has a second-order pole at the origin so we cannot use the final value theorem here.

$$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

P 12.53 **[a]** $sF(s) = \dfrac{135}{(s+3)^3}$

$$\lim_{s\to 0} sF(s) = 5, \qquad \therefore \quad f(\infty) = 5$$

$$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

[b] $sF(s) = \dfrac{10s(s+2)^2}{(s^2+2s+2)^2}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

[c] This $F(s)$ function is an improper rational function, and thus the corresponding $f(t)$ function contains impulses ($\delta(t)$). Neither the initial value theorem nor the final value theorem may be applied to this $F(s)$ function!

[d] This $F(s)$ function is an improper rational function, and thus the corresponding $f(t)$ function contains impulses ($\delta(t)$). Neither the initial value theorem nor the final value theorem may be applied to this $F(s)$ function!

P 12.54 **[a]** $Z_L = j120\pi(0.01) = j3.77\,\Omega; \qquad Z_C = \dfrac{-j}{120\pi(100\times 10^{-6})} = -j26.526\,\Omega$

The phasor-transformed circuit is

$$\mathbf{I}_L = \frac{1}{15 + j3.77 - j26.526} = 36.69\underline{/56.61^\circ}\ \text{mA}$$

$$\therefore \quad i_{L-ss}(t) = 36.69\cos(120\pi t + 56.61^\circ)\ \text{mA}$$

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[b] The steady-state response is the second term in Eq. 12.109, which matches the steady-state response just derived in part (a).

P 12.55 The transient and steady-state components are both proportional to the magnitude of the input voltage. Therefore,

$$K = \frac{40}{42.26} = 0.947$$

So if we make the amplitude of the sinusoidal source 0.947 instead of 1, the current will not exceed the 40 mA limit. A plot of the current through the inductor is shown below with the amplitude of the sinusoidal source set at 0.947.

P 12.56 We begin by using Eq. 12.105, and changing the right-hand side so it is the Laplace transform of Kte^{-100t}:

$$15I_L(s) + 0.01sI_L(s) + 10^4\frac{I_L(s)}{s} = \frac{A}{(s+100)^2}$$

Solving for $I_L(s)$,

$$I_L(s) = \frac{100Ks}{(s^2+1500s+10^6)(s+100)^2} = \frac{K_1}{s+750-j661.44} + \frac{K_1^*}{s+750+j661.44}$$

$$+\frac{K_2}{(s+100)^2} + \frac{K_3}{s+100}$$

$$K_1 = \frac{100Ks}{(s+750+j661.44)(s+100)^2}\bigg|_{s=-750+j661.44} = 87.9K\underline{/139.59^\circ}\ \mu\text{A}$$

$$K_2 = \frac{100Ks}{(s^2+1500s+10^6)}\bigg|_{s=-100} = -11.63K\ \text{mA}$$

$$K_3 = \frac{d}{ds}\left[\frac{100Ks}{(s^2+1500s+10^6)}\right]_{s=-100} = 133.86K\ \mu\text{A}$$

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Therefore,

$$i_L(t) = K[0.176e^{-750t}\cos(661.44t + 139.59°) - 11.63te^{-100t} + 0.134e^{-100t}]u(t)\,\text{mA}$$

Plot the expression above with $K = 1$:

The maximum value of the inductor current is $0.068K$ mA. Therefore,

$$K = \frac{40}{0.068} = 588$$

So the inductor current rating will not be exceeded if the input to the RLC circuit is $588te^{-100t}$ V.

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13

The Laplace Transform in Circuit Analysis

Assessment Problems

AP 13.1 **[a]** $Y = \dfrac{1}{R} + \dfrac{1}{sL} + sC = \dfrac{C[s^2 + (1/RC)s + (1/LC)}{s}$

$$\frac{1}{RC} = \frac{10^6}{(500)(0.025)} = 80{,}000; \qquad \frac{1}{LC} = 25 \times 10^8$$

$$\text{Therefore} \quad Y = \frac{25 \times 10^{-9}(s^2 + 80{,}000s + 25 \times 10^8)}{s}$$

[b] $z_{1,2} = -40{,}000 \pm \sqrt{16 \times 10^8 - 25 \times 10^8} = -40{,}000 \pm j30{,}000 \text{ rad/s}$

$$-z_1 = -40{,}000 - j30{,}000 \text{ rad/s}$$

$$-z_2 = -40{,}000 + j30{,}000 \text{ rad/s}$$

$$p_1 = 0 \text{ rad/s}$$

AP 13.2 **[a]** $Z = 2000 + \dfrac{1}{Y} = 2000 + \dfrac{4 \times 10^7 s}{s^2 + 80{,}000s + 25 \times 10^8}$

$$= \frac{2000(s^2 + 10^5 s + 25 \times 10^8)}{s^2 + 80{,}000s + 25 \times 10^8} = \frac{2000(s + 50{,}000)^2}{s^2 + 80{,}000s + 25 \times 10^8}$$

[b] $-z_1 = -z_2 = -50{,}000 \text{ rad/s}$

$$-p_1 = -40{,}000 - j30{,}000 \text{ rad/s}$$

$$-p_2 = -40{,}000 + j30{,}000 \text{ rad/s}$$

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AP 13.3 **[a]** At $t = 0^-$, $\quad 0.2v_1 = (0.8)v_2; \quad v_1 = 4v_2; \quad v_1 + v_2 = 100\,\text{V}$

Therefore $\quad v_1(0^-) = 80V = v_1(0^+); \quad v_2(0^-) = 20V = v_2(0^+)$

$$I = \frac{(80/s) + (20/s)}{5000 + [(5 \times 10^6)/s] + (1.25 \times 10^6/s)} = \frac{20 \times 10^{-3}}{s + 1250}$$

$$V_1 = \frac{80}{s} - \frac{5 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{80}{s + 1250}$$

$$V_2 = \frac{20}{s} - \frac{1.25 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{20}{s + 1250}$$

[b] $i = 20e^{-1250t}u(t)\,\text{mA}; \qquad v_1 = 80e^{-1250t}u(t)\,\text{V}$

$v_2 = 20e^{-1250t}u(t)\,\text{V}$

AP 13.4 **[a]**

$$I = \frac{V_{\text{dc}}/s}{R + sL + (1/sC)} = \frac{V_{\text{dc}}/L}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{V_{\text{dc}}}{L} = 40; \qquad \frac{R}{L} = 1.2; \qquad \frac{1}{LC} = 1.0$$

$$I = \frac{40}{(s + 0.6 - j0.8)(s + 0.6 + j0.8)} = \frac{K_1}{s + 0.6 - j0.8} + \frac{K_1^*}{s + 0.6 + j0.8}$$

$$K_1 = \frac{40}{j1.6} = -j25 = 25\underline{/-90^\circ}; \qquad K_1^* = 25\underline{/90^\circ}$$

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[b] $i = 50e^{-0.6t}\cos(0.8t - 90°) = [50e^{-0.6t}\sin 0.8t]u(t)\,\text{A}$

[c] $$V = sLI = \frac{160s}{(s+0.6-j0.8)(s+0.6+j0.8)}$$

$$= \frac{K_1}{s+0.6-j0.8} + \frac{K_1^*}{s+0.6+j0.8}$$

$$K_1 = \frac{160(-0.6+j0.8)}{j1.6} = 100\underline{/36.87°}$$

[d] $v(t) = [200e^{-0.6t}\cos(0.8t + 36.87°)]u(t)\,\text{V}$

AP 13.5 **[a]**

The two node voltage equations are

$$\frac{V_1 - V_2}{s} + V_1 s = \frac{5}{s} \quad \text{and} \quad \frac{V_2}{3} + \frac{V_2 - V_1}{s} + \frac{V_2 - (15/s)}{15} = 0$$

Solving for V_1 and V_2 yields

$$V_1 = \frac{5(s+3)}{s(s^2+2.5s+1)}, \qquad V_2 = \frac{2.5(s^2+6)}{s(s^2+2.5s+1)}$$

[b] The partial fraction expansions of V_1 and V_2 are

$$V_1 = \frac{15}{s} - \frac{50/3}{s+0.5} + \frac{5/3}{s+2} \quad \text{and} \quad V_2 = \frac{15}{s} - \frac{125/6}{s+0.5} + \frac{25/3}{s+2}$$

It follows that

$$v_1(t) = \left[15 - \frac{50}{3}e^{-0.5t} + \frac{5}{3}e^{-2t}\right]u(t)\,\text{V} \quad \text{and}$$

$$v_2(t) = \left[15 - \frac{125}{6}e^{-0.5t} + \frac{25}{3}e^{-2t}\right]u(t)\,\text{V}$$

[c] $$v_1(0^+) = 15 - \frac{50}{3} + \frac{5}{3} = 0$$

$$v_2(0^+) = 15 - \frac{125}{6} + \frac{25}{3} = 2.5\,\text{V}$$

[d] $v_1(\infty) = 15\,\text{V}; \qquad v_2(\infty) = 15\,\text{V}$

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AP 13.6 **[a]**

With no load across terminals $a - b$ $\quad V_x = 20/s$:

$$\frac{1}{2}\left[\frac{20}{s} - V_{\text{Th}}\right]s + \left[1.2\left(\frac{20}{s}\right) - V_{\text{Th}}\right] = 0$$

therefore $\quad V_{\text{Th}} = \dfrac{20(s+2.4)}{s(s+2)}$

$V_x = 5I_T \quad$ and $\quad Z_{\text{Th}} = \dfrac{V_T}{I_T}$

Solving for I_T gives

$$I_T = \frac{(V_T - 5I_T)s}{2} + V_T - 6I_T$$

Therefore

$$14I_T = V_T s + 5sI_T + 2V_T; \qquad \text{therefore} \quad Z_{\text{Th}} = \frac{5(s+2.8)}{s+2}$$

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[b]

$$I = \frac{V_{\text{Th}}}{Z_{\text{Th}} + 2 + s} = \frac{20(s+2.4)}{s(s+3)(s+6)}$$

AP 13.7 **[a]** $i_2 = 1.25e^{-t} - 1.25e^{-3t}$; therefore $\dfrac{di_2}{dt} = -1.25e^{-t} + 3.75e^{-3t}$

Therefore $\dfrac{di_2}{dt} = 0$ when

$1.25e^{-t} = 3.75e^{-3t}$ or $e^{2t} = 3$, $t = 0.5(\ln 3) = 549.31\,\text{ms}$

$i_2(\text{max}) = 1.25[e^{-0.549} - e^{-3(0.549)}] = 481.13\,\text{mA}$

[b] From Eqs. 13.68 and 13.69, we have

$\Delta = 12(s^2 + 4s + 3) = 12(s+1)(s+3)$ and $N_1 = 60(s+2)$

Therefore $I_1 = \dfrac{N_1}{\Delta} = \dfrac{5(s+2)}{(s+1)(s+3)}$

A partial fraction expansion leads to the expression

$$I_1 = \frac{2.5}{s+1} + \frac{2.5}{s+3}$$

Therefore we get

$i_1 = 2.5[e^{-t} + e^{-3t}]u(t)\,\text{A}$

[c] $\dfrac{di_1}{dt} = -2.5[e^{-t} + 3e^{-3t}]$; $\dfrac{di_1(0.54931)}{dt} = -2.89\,\text{A/s}$

[d] When i_2 is at its peak value,

$$\frac{di_2}{dt} = 0$$

Therefore $L_2\left(\dfrac{di_2}{dt}\right) = 0$ and $i_2 = -\left(\dfrac{M}{12}\right)\left(\dfrac{di_1}{dt}\right)$

[e] $i_2(\text{max}) = \dfrac{-2(-2.89)}{12} = 481.13\,\text{mA}$ (checks)

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AP 13.8 **[a]** The s-domain circuit with the voltage source acting alone is

$$\frac{V' - (20/s)}{2} + \frac{V'}{1.25s} + \frac{V's}{20} = 0$$

$$V' = \frac{200}{(s+2)(s+8)} = \frac{100/3}{s+2} - \frac{100/3}{s+8}$$

$$v' = \frac{100}{3}[e^{-2t} - e^{-8t}]u(t)\,\text{V}$$

[b] With the current source acting alone,

$$\frac{V''}{2} + \frac{V''}{1.25s} + \frac{V''s}{20} = \frac{5}{s}$$

$$V'' = \frac{100}{(s+2)(s+8)} = \frac{50/3}{s+2} - \frac{50/3}{s+8}$$

$$v'' = \frac{50}{3}[e^{-2t} - e^{-8t}]u(t)\,\text{V}$$

[c] $v = v' + v'' = [50e^{-2t} - 50e^{-8t}]u(t)\,\text{V}$

AP 13.9 **[a]** $\dfrac{V_o}{s+2} + \dfrac{V_o s}{10} = I_g;$ therefore $\dfrac{V_o}{I_g} = H(s) = \dfrac{10(s+2)}{s^2+2s+10}$

[b] $-z_1 = -2$ rad/s; $-p_1 = -1 + j3$ rad/s; $-p_2 = -1 - j3$ rad/s

AP 13.10 **[a]**

$$V_o = \frac{10(s+2)}{s^2+2s+10} \cdot \frac{1}{s} = \frac{K_o}{s} + \frac{K_1}{s+1-j3} + \frac{K_1^*}{s+1+j3}$$

$$K_o = 2; \qquad K_1 = 5/3\underline{/-126.87^\circ}; \qquad K_1^* = 5/3\underline{/126.87^\circ}$$

$$v_o = [2 + (10/3)e^{-t}\cos(3t - 126.87^\circ)]u(t)\,\text{V}$$

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[b] $V_o = \dfrac{10(s+2)}{s^2+2s+10} \cdot 1 = \dfrac{K_2}{s+1-j3} + \dfrac{K_2^*}{s+1+j3}$

$K_2 = 5.27\underline{/-18.43^\circ}; \qquad K_2^* = 5.27\underline{/18.43^\circ}$

$v_o = [10.54e^{-t}\cos(3t - 18.43^\circ)]u(t)\,\text{V}$

AP 13.11 **[a]**

$H(s) = \mathcal{L}\{h(t)\} = \mathcal{L}\{v_o(t)\}$

$$v_o(t) = 10{,}000\cos\theta e^{-70t}\cos 240t - 10{,}000\sin\theta e^{-70t}\sin 240t$$
$$= 9600e^{-70t}\cos 240t - 2800e^{-70t}\sin 240t$$

Therefore $\quad H(s) = \dfrac{9600(s+70)}{(s+70)^2+(240)^2} - \dfrac{2800(240)}{(s+70)^2+(240)^2}$

$$= \frac{9600s}{s^2+140s+62{,}500}$$

[b] $V_o(s) = H(s)\cdot\dfrac{1}{s} = \dfrac{9600}{s^2+140s+62{,}500}$

$$= \frac{K_1}{s+70-j240} + \frac{K_1^*}{s+70+j240}$$

$K_1 = \dfrac{9600}{j480} = -j20 = 20\underline{/-90^\circ}$

Therefore

$v_o(t) = [40e^{-70t}\cos(240t - 90^\circ)]u(t)\,\text{V} = [40e^{-70t}\sin 240t]u(t)\,\text{V}$

AP 13.12 From Assessment Problem 13.9:

$$H(s) = \frac{10(s+2)}{s^2+2s+10}$$

Therefore $\quad H(j4) = \dfrac{10(2+j4)}{10-16+j8} = 4.47\underline{/-63.43^\circ}$

Thus,

$v_o = (10)(4.47)\cos(4t - 63.43^\circ) = 44.7\cos(4t - 63.43^\circ)\,\text{V}$

AP 13.13 **[a]**

Let $\quad R_1 = 10\,\text{k}\Omega, \quad R_2 = 50\,\text{k}\Omega, \quad C = 400\ \text{pF}, \quad R_2C = 2\times 10^{-5}$

then $\quad V_1 = V_2 = \dfrac{V_gR_2}{R_2 + (1/sC)}$

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Also $\dfrac{V_1 - V_g}{R_1} + \dfrac{V_1 - V_o}{R_1} = 0$

therefore $V_o = 2V_1 - V_g$

Now solving for V_o/V_g, we get $H(s) = \dfrac{R_2Cs - 1}{R_2Cs + 1}$

It follows that $H(j50{,}000) = \dfrac{j-1}{j+1} = j1 = 1\underline{/90^\circ}$

Therefore $v_o = 10\cos(50{,}000t + 90^\circ)\,\text{V}$

[b] Replacing R_2 by R_x gives us $H(s) = \dfrac{R_xCs - 1}{R_xCs + 1}$

Therefore

$$H(j50{,}000) = \frac{j20 \times 10^{-6}R_x - 1}{j20 \times 10^{-6}R_x + 1} = \frac{R_x + j50{,}000}{R_x - j50{,}000}$$

Thus,

$$\frac{50{,}000}{R_x} = \tan 60^\circ = 1.7321, \qquad R_x = 28{,}867.51\,\Omega$$

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Problems

P 13.1 $i = \dfrac{1}{L}\displaystyle\int_{0^-}^{t} v\,d\tau + I_0;$ therefore $I = \left(\dfrac{1}{L}\right)\left(\dfrac{V}{s}\right) + \dfrac{I_0}{s} = \dfrac{V}{sL} + \dfrac{I_0}{s}$

P 13.2 $V_{\text{Th}} = V_{\text{ab}} = CV_0\left(\dfrac{1}{sC}\right) = \dfrac{V_0}{s};$ $Z_{\text{Th}} = \dfrac{1}{sC}$

P 13.3 $I_{sc_{\text{ab}}} = I_{\text{N}} = \dfrac{-LI_0}{sL} = \dfrac{-I_0}{s};$ $Z_{\text{N}} = sL$

Therefore, the Norton equivalent is the same as the circuit in Fig. 13.4.

P 13.4 **[a]** $Z = R + sL + \dfrac{1}{sC} = \dfrac{L[s^2 + (R/L)s + (1/LC)]}{s}$

$$= \frac{[s^2 + 32{,}000s + 4 \times 10^8]}{s}$$

[b] $s_{1,2} = -16{,}000 \pm j12{,}000$ rad/s

Zeros at $-16{,}000 + j12{,}000$ rad/s and $-16{,}000 - j12{,}000$ rad/s

Pole at 0.

P 13.5 **[a]** $Y = \dfrac{1}{R} + \dfrac{1}{sL} + sC = \dfrac{C[s^2 + (1/RC)s + (1/LC)]}{s}$

$$Z = \frac{1}{Y} = \frac{s/C}{s^2 + (1/RC)s + (1/LC)} = \frac{16 \times 10^9 s}{s^2 + 2 \times 10^6 s + 64 \times 10^10}$$

[b] zero at $z_1 = 0$

poles at $-p_1 = -400$ krad/s and $-p_2 = -1600$ krad/s

P 13.6 **[a]**

$$Z = \frac{(R + 1/sC)(sL)}{R + sL + (1/sC)} = \frac{(Rs)(s + 1/RC)}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{R}{L} = 500; \qquad \frac{1}{RC} = 80; \qquad \frac{1}{LC} = 40 \times 10^3$$

$$Z = \frac{200s(s + 80)}{s^2 + 500s + 40 \times 10^3}$$

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[b] $Z = \dfrac{200s(s+80)}{(s+100)(s+400)}$

$z_1 = 0; \qquad -z_2 = -80 \text{ rad/s}$

$-p_1 = -100 \text{ rad/s}; \qquad -p_2 = -400 \text{ rad/s}$

P 13.7 $\left(1\|s + \dfrac{1}{s}\right)\|1 = \left(\dfrac{s}{s+1} + \dfrac{1}{s}\right)\|1 = \dfrac{s^2+s+1}{s(s+1)}\|1$

$$= \frac{\dfrac{s^2+s+1}{s(s+1)}}{\dfrac{s^2+s+1}{s(s+1)}+1} = \frac{s^2+s+1}{2s^2+2s+1} = \frac{0.5(s^2+s+1)}{s^2+s+0.5}$$

$-z_1 = -0.5 + j0.866 \text{ rad/s}; \qquad -z_2 = -0.5 - j0.866 \text{ rad/s}$

$-p_1 = -0.5 + j0.5 \text{ rad/s}; \qquad -p_2 = -0.5 - j0.5 \text{ rad/s}$

P 13.8 Transform the Y-connection of the two resistors and the capacitor into the equivalent delta-connection:

where

$$Z_{\text{a}} = \frac{(1/s)(1) + (1)(1/s) + (1)(1)}{(1/s)} = s + 2$$

$$Z_{\text{b}} = Z_{\text{c}} = \frac{(1/s)(1) + (1)(1/s) + (1)(1)}{1} = \frac{s+2}{s}$$

Then

$$Z_{\text{ab}} = Z_{\text{a}}\|[(s\|Z_{\text{c}}) + (s\|Z_{\text{b}})] = Z_{\text{a}}\|2(s\|Z_{\text{b}})$$

$$s\|Z_{\text{b}} = \frac{s[(s+2)/s]}{s + [(s+2)/s]} = \frac{s(s+2)}{s^2+s+2}$$

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$$Z_{\text{ab}} = (s+2)\|\frac{2s(s+2)}{s^2+s+2} = \frac{\dfrac{2s(s+2)^2}{s^2+s+2}}{s+2+\dfrac{2s(s+2)}{s^2+s+2}}$$

$$= \frac{2s(s+2)^2}{(s+2)(s^2+s+2)+2s(s+2)} = \frac{2s(s+2)}{s^2+3s+2}$$

Zeros at 0 and -2 rad/s; poles at -1 rad/s and -2 rad/s.

P 13.9

$$V_o = \frac{(90/s)(8\times 10^5/s)}{140+0.005s+(8\times 10^5/s)}$$

$$= \frac{144\times 10^8}{s(s^2+28{,}000s+16\times 10^7)}$$

$$= \frac{144\times 10^8}{s(s+8000)(s+20{,}000)}$$

$$= \frac{K_1}{s}+\frac{K_2}{s+8000}+\frac{K_3}{s+20{,}000}$$

$$K_1 = \frac{144\times 10^8}{16\times 10^7} = 90$$

$$K_2 = \frac{144\times 10^8}{(-8000)(12{,}000)} = -150$$

$$K_3 = \frac{144\times 10^8}{(-12{,}000)(-20{,}000)} = 60$$

$$V_o = \frac{90}{s}-\frac{150}{s+8000}+\frac{60}{s+20{,}000}$$

$$v_o(t) = [90-150e^{-8000t}+60e^{-20{,}000t}]u(t)\,\text{V}$$

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P 13.10 With a non-zero initial voltage on the capacitor, the s-domain circuit becomes:

$$\frac{V_o - 90/s}{0.005s + 140} + \frac{(V_o - 150/s)s}{8 \times 10^5} = 0$$

$$V_o\left[\frac{200}{s + 28{,}000} + \frac{s}{8 \times 10^5}\right] = \frac{150}{8 \times 10^5} + \frac{18{,}000}{s(s + 28{,}000)}$$

$$\therefore \quad V_o = \frac{150(s^2 + 28{,}000s + 96 \times 10^6}{s(s + 8000)(s + 20{,}000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 8000} + \frac{K_3}{s + 20{,}000}$$

$$K_1 = \frac{144 \times 10^8}{160 \times 10^6} = 90$$

$$K_2 = \frac{150(s^2 + 28{,}000s + 96 \times 10^6}{s(s + 20{,}000)}\Bigg|_{s=-8000} = 100$$

$$K_3 = \frac{150(s^2 + 28{,}000s + 96 \times 10^6}{s(s + 8000)}\Bigg|_{s=-20{,}000} = -40$$

$$V_o = \frac{90}{s} + \frac{100}{s + 8000} - \frac{40}{s + 20{,}000}$$

$$v_o(t) = [90 + 100e^{-8000t} - 40e^{-20{,}000t}]u(t)\ \text{V}$$

P 13.11 **[a]** For $t > 0$:

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[b] $I_o = \dfrac{75/s}{(3200/s) + 100 + 0.5s}$

$$= \frac{75}{0.5s^2 + 100s + 3200}$$

$$= \frac{150}{(s^2 + 200s + 6400)} = \frac{150}{(s+40)(s+160)}$$

[c] $I_o = \dfrac{K_1}{s+40} + \dfrac{K_2}{s+160}$

$$K_1 = \frac{150}{s+160}\bigg|_{s=-40} = 1.25$$

$$K_2 = \frac{150}{s+40}\bigg|_{s=-160} = -1.25$$

$$I_o = \frac{1.25}{s+40} - \frac{1.25}{s+160}$$

$$i_o(t) = (1.25e^{-40t} - 1.25e^{-160t})u(t)\,\text{A}$$

P 13.12 **[a]** $i_o(0^-) = \dfrac{20}{4000} = 5\,\text{mA}$

$$I_o = \frac{80/s + L\rho}{R + sL + 1/sC} = \frac{sC(80/s + L\rho)}{s^2LC + RsC + 1}$$

$$= \frac{80/L + s\rho}{s^2 + sR/L + 1/LC} = \frac{20{,}000 + s(0.1)}{s^2 + 200{,}000s + 10^{10}}$$

$$= \frac{0.1(s + 200{,}000)}{s^2 + 200{,}000s + 10^{10}} = \frac{K_1}{(s+100{,}000)^2} + \frac{K_2}{s+100{,}000}$$

$K_1 = 10{,}000;\qquad K_2 = 0.1$

$$i_o(t) = [10{,}000te^{-100{,}000t} + 0.1e^{-100{,}000t}]u(t)\,\text{A}$$

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[b] $V_o = (R + sL)I_o - L\rho = \dfrac{(800 + 0.004s)(0.1s + 20{,}000)}{s^2 + 200{,}000s + 10^{10}} - 4 \times 10^{-4}$

$$= \frac{80(s + 150{,}000)}{(s + 100{,}000)^2} = \frac{K_1}{(s + 100{,}000)^2} + \frac{K_2}{s + 100{,}000}$$

$K_1 = 4 \times 10^6 \qquad K_2 = 80$

$v_o(t) = [4 \times 10^6 t e^{-100{,}000t} + 80e^{-100{,}000t}]u(t)\,\text{A}$

P 13.13 **[a]** For $t < 0$:

$$\frac{V_c - 100}{80} + \frac{V_c}{240} + \frac{V_c - 275}{100} = 0$$

$$V_c\left(\frac{1}{80} + \frac{1}{240} + \frac{1}{100}\right) = \frac{100}{80} + \frac{275}{100}$$

$V_c = 150\,\text{V}$

$$i_L(0^-) = \frac{150 - 275}{100} = -1.25\,\text{A}$$

For $t > 0$:

[b] $V_o = \dfrac{12{,}500}{s}I + \dfrac{150}{s}$

$$0 = -\frac{275}{s} + 20I + \frac{12{,}500}{s}I + \frac{150}{s} - 20 \times 10^{-3} + 0.016sI$$

$$I\left(20 + \frac{12{,}500}{s} + 0.016s\right) = \frac{125}{s} + 20 \times 10^{-3}$$

$$\therefore \quad I = \frac{7812.5 + 1.25s}{s^2 + 1250s + 781{,}250}$$

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$$V_o = \frac{12{,}500}{s}\left(\frac{7812.5 + 1.25s}{s^2 + 1250s + 781{,}250}\right) + \frac{150}{s}$$

$$= \frac{150s^2 + 203{,}125s + 214{,}843{,}750}{s(s^2 + 1250s + 781{,}250)}$$

[c] $V_o = \dfrac{K_1}{s} + \dfrac{K_2}{s + 625 - j625} + \dfrac{K_2^*}{s + 625 + j625}$

$$K_1 = \left.\frac{150s^2 + 203{,}125s + 214{,}843{,}750}{s^2 + 1250s + 781{,}250}\right|_{s=0} = 275$$

$$K_2 = \left.\frac{150s^2 + 203{,}125s + 214{,}843{,}750}{s(s + 625 + j625)}\right|_{s=-625+j625} = 80.04\underline{/141.34^\circ}$$

$$v_o(t) = [2755 + 160.08e^{-625t}\cos(625t + 141.34^\circ)]u(t)\,\text{V}$$

P 13.14 **[a]** For $t < 0$:

$$v_1 = \frac{4000\|1000}{400 + 4000\|1000}(360) = 240\,\text{V}$$

$$i_\text{L}(0^-) = \frac{240}{1000} = 0.24\,\text{A}$$

$$v_\text{C}(0^-) = -v_1 = -240\,\text{V}$$

For $t = 0^+$:

s-domain circuit:

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where

$R = 1\,\text{k}\Omega; \qquad C = 6.25\,\text{nF}; \qquad \gamma = -240\,\text{V};$

$L = 16\,\text{mH}; \qquad \text{and} \qquad \rho = -0.24\,\text{A}$

[b] $\dfrac{V_o}{R} + V_o sC - \gamma C + \dfrac{V_o}{sL} - \dfrac{\rho}{s} = 0$

$$\therefore \quad V_o = \frac{\gamma[s + (\rho/\gamma C)]}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{\rho}{\gamma C} = \frac{-0.24}{(-240)(6.25 \times 10^{-9})} = 160{,}000$$

$$\frac{1}{RC} = \frac{1}{(1000)(6.25 \times 10^{-9})} = 160{,}000$$

$$\frac{1}{LC} = \frac{1}{(16 \times 10^{-3})(6.25 \times 10^{-9})} = 10^{10}$$

$$V_o = \frac{-240(s + 160{,}000)}{s^2 + 160{,}000s + 10^{10}}$$

[c] $I_\text{L} = \dfrac{V_o}{sL} - \dfrac{\rho}{s} = \dfrac{V_o}{0.016s} + \dfrac{0.24}{s}$

$$= \frac{-15{,}000(s + 160{,}000)}{s(s^2 + 160{,}000s + 10^{10})} + \frac{0.24}{s} = \frac{0.24(s + 97{,}500)}{(s^2 + 160{,}000s + 10^{10})}$$

[d] $V_o = \dfrac{-240(s + 160{,}000)}{s^2 + 160{,}000s + 10^{10}}$

$$= \frac{K_1}{s + 80{,}000 - j60{,}000} + \frac{K_1^*}{s + 80{,}000 + j60{,}000}$$

$$K_1 = \left.\frac{-240(s + 160{,}000)}{s + 80{,}000 + j60{,}000}\right|_{s=-80{,}000+j60{,}000} = 200\underline{/126.87^\circ}$$

$$v_o(t) = [400e^{-80{,}000t}\cos(60{,}000t + 126.87^\circ)]u(t)\,\text{V}$$

[e] $I_\text{L} = \dfrac{0.24(s + 97{,}500)}{s^2 + 160{,}000s + 10^{10}}$

$$= \frac{K_1}{s + 80{,}000 - j60{,}000} + \frac{K_1^*}{s + 80{,}000 + j60{,}000}$$

$$K_1 = \left.\frac{0.24(s + 97{,}500)}{s + 80{,}000 + j60{,}000}\right|_{s=-80{,}000+j60{,}000} = 0.125\underline{/-16.26^\circ}$$

$$i_\text{L}(t) = [0.5e^{-80{,}000t}\cos(60{,}000t - 16.26^\circ)]u(t)\,\text{A}$$

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P 13.15 **[a]** For $t < 0$:

$$i_{\mathrm{L}}(0^-) = \frac{40}{32 + 80\|320 + 64} = \frac{40}{160} = 0.25\,\mathrm{A}$$

$$i_1 = \frac{80}{400}(-0.25) = -0.05\,\mathrm{A}$$

$$v_{\mathrm{C}}(0^-) = 80(-0.05) + 32(-0.25) + 40 = 28\,\mathrm{V}$$

For $t > 0$:

[b] $(160 + 0.2s + 40{,}000/s)I = 0.05 + \dfrac{28}{s}$

$$\therefore \qquad I = \frac{0.25(s + 560)}{s^2 + 800s + 200{,}000}$$

$$= \frac{K_1}{s + 400 - j200} + \frac{K_1^*}{s + 400 + j200}$$

$$K_1 = \frac{0.25(s + 560)}{s + 400 + j200}\bigg|_{s=-400+j200} = \underline{0.16/-38.66^\circ}$$

[c] $i_o(t) = 0.32e^{-400t}\cos(200t - 38.66^\circ)u(t)\,\mathrm{A}$

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P 13.16 For $t < 0$:

$$v_o(0^-) = (100\|500\|2000)(0.005) = 0.4\,\text{V}$$

$$i_L(0^-) = \frac{v_o(0^-)}{500} = 0.8\,\text{mA}$$

For $t > 0$:

$$\frac{V_o + 0.8\times10^{-3}}{500+s} + \frac{V_o}{2000} + \frac{V_o - (0.4/s)}{10^6/s} = 0$$

$$V_o\left(\frac{1}{500+s} + \frac{1}{2000} + \frac{s}{10^6}\right) = 0.4\times10^{-6} - \frac{0.8\times10^{-3}}{500+s}$$

$$\therefore \qquad V_o = \frac{0.4(s-1500)}{s^2 + 1000s + 125\times10^4}$$

$$= \frac{K_1}{s+500-j1000} + \frac{K_1^*}{s+500+j1000}$$

$$K_1 = \frac{0.4(s-1500)}{s+500+j1000}\bigg|_{s=-500+j1000} = 0.447\underline{/63.43^\circ}$$

$$v_o(t) = [0.894e^{-500t}\cos(1000t + 63.43^\circ)]u(t)\,\text{V}$$

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P 13.17 **[a]**

$$V_o = \frac{sL(I_g/sC)}{R + sL + (1/sC)} = \frac{I_g/C}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{I_g}{C} = \frac{0.06}{0.001} = 60$$

$$\frac{R}{L} = 140; \qquad \frac{1}{LC} = 4000$$

$$V_o = \frac{60}{s^2 + 140s + 4000}$$

[b] $$sV_o = \frac{60s}{s^2 + 140s + 4000}$$

$$\lim_{s\to 0} sV_o = 0; \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to \infty} sV_o = 0; \qquad \therefore \quad v_o(0^+) = 0$$

[c] $$V_o = \frac{60}{(s+40)(s+100)} = \frac{1}{s+40} + \frac{-1}{s+100}$$

$$v_o = [e^{-40t} - e^{-100t}]u(t)\,\text{V}$$

P 13.18 $$I_C = \frac{I_g}{s} - \frac{V_o}{sL} = \frac{0.06}{s} - \frac{240}{s(s^2 + 140s + 4000)}$$

$$= \frac{0.06}{s} - \frac{0.06}{s} + \frac{0.1}{s+40} - \frac{0.04}{s+100}$$

$$\therefore \qquad i_C(t) = [100e^{-40t} - 40e^{-100t}]u(t)\,\text{mA}$$

Check:

$$i_C(0^+) = 60\,\text{mA (ok)}; \qquad i_C(\infty) = 0 \text{ (ok)}$$

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P 13.19 **[a]**

$$200i_1 + 0.4s(I_1 - I_2) = \frac{200}{s}; \qquad 200I_2 + \frac{10^5}{s}I_2 + 0.4s(I_2 - I_1) = 0$$

Solving the second equation for I_1:

$$I_1 = \frac{s^2 + 500s + 25 \times 10^4}{s^2} I_2$$

Substituting into the first equation and solving for I_2:

$$(0.4s + 200)\frac{s^2 + 500s + 25 \times 10^4}{s^2} - 0.4s = \frac{200}{s}$$

$$\therefore \qquad I_2 = \frac{0.5s}{s^2 + 500s + 125{,}000}$$

$$\therefore \qquad I_1 = \frac{s^2 + 500s + 25 \times 10^4}{s^2} \cdot \frac{0.5s}{s^2 + 500s + 125{,}000}$$

$$= \frac{0.5(s^2 + 500s + 25 \times 10^4)}{s(s^2 + 500s + 125{,}000)}$$

$$I_o = I_1 - I_2 = \frac{0.5(s^2 + 500s + 25 \times 10^4)}{s(s^2 + 500s + 125{,}000)} - \frac{0.5s}{s^2 + 500s + 125{,}000}$$

$$= \frac{250(s + 500)}{s(s^2 + 500s + 125{,}000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 250 - j250} + \frac{K_2^*}{s + 250 + j250}$$

$$K_1 = 1; \qquad K_2 = 0.5\underline{/-180^\circ} = -0.5$$

$$\therefore \qquad i_o(t) = [1 - 1e^{-250t}\cos 250t]u(t)\,\text{A}$$

[b] $$V_o = 0.4sI_o = \frac{100(s + 500)}{s^2 + 500s + 125{,}000} = \frac{K_1}{s + 250 - j250} + \frac{K_1^*}{s + 250 + j250}$$

$$K_1 = 70.71\underline{/-45^\circ}$$

$$\therefore \qquad v_o(t) = 141.42e^{-250t}\cos(250t - 45^\circ)u(t)\,\text{V}$$

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[**c**] At $t = 0^+$ the circuit is

$$\therefore \quad v_o(0^+) = 100\,\text{V} = 141.42\cos(-45°); \qquad I_o(0^+) = 0$$

Both values agree with our solutions for v_o and i_o.
At $t = \infty$ the circuit is

$$\therefore \quad v_o(\infty) = 0; \qquad i_o(\infty) = 1\,\text{A}$$

Both values agree with our solutions for v_o and i_o.

P 13.20 [**a**]

$$\frac{V_1 - 325/s}{250} + \frac{V_1}{500} + \frac{V_1 - V_2}{250} = 0$$

$$\frac{V_2}{0.625s} + \frac{V_2 - V_1}{250} + \frac{(V_2 - 325/s)s}{125 \times 10^4} = 0$$

Thus,

$$5V_1 - 2V_2 = \frac{650}{s}$$

$$-5000sV_1 + (s^2 + 5000s + 2 \times 10^6)V_2 = 325s$$

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$$\Delta = \begin{vmatrix} 5 & -2 \\ -5000s & s^2 + 5000s + 2 \times 10^6 \end{vmatrix} = 5(s+1000)(s+2000)$$

$$N_2 = \begin{vmatrix} 5 & 650/s \\ -5000s & 325s \end{vmatrix} = 1625(s+2000)$$

$$V_2 = \frac{N_2}{\Delta} = \frac{1625(s+2000)}{5(s+1000)(s+2000)} = \frac{325}{s+1000}$$

$$V_o = \frac{325}{s} - \frac{325}{s+1000} = \frac{325{,}000}{s(s+1000)}$$

$$I_o = \frac{V_2}{0.625s} = \frac{520}{s(s+1000)} = \frac{0.52}{s} - \frac{0.52}{s+1000}$$

[b] $v_o(t) = (325 - 325e^{-1000t})u(t)\,\text{V}$

$i_o(t) = (520 - 520e^{-1000t})u(t)\,\text{mA}$

[c] At $t = 0^+$ the circuit is

$v_o(0^+) = 0; \qquad i_o(0^+) = 0 \qquad$ Checks

At $t = \infty$ the circuit is

$v_o(\infty) = 325\,\text{V}; \qquad i_o(\infty) = \dfrac{325}{250 + (500\|250)} \cdot \dfrac{500}{750} = 0.52\,\text{A} \quad$ Checks

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P 13.21 **[a]**

$$0 = 0.5s(I_1 - 30/s) + \frac{2500}{s}(I_1 - I_2) + 100I_1$$

$$\frac{-375}{s} = \frac{2500}{s}(I_2 - I_1) + 50(I_2 - 30/s)$$

or

$$(s^2 + 200s + 5000)I_1 - 5000I_2 = 30s$$

$$-50I_1 + (s + 50)I_2 = 22.5$$

$$\Delta = \begin{vmatrix} (s^2 + 200s + 5000) & -5000 \\ -50 & (s + 50) \end{vmatrix} = s(s + 100)(s + 150)$$

$$N_1 = \begin{vmatrix} 30s & -5000 \\ 22.5 & (s + 50) \end{vmatrix} = 30(s^2 + 50s + 3750)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{30(s^2 + 50s + 3750)}{s(s + 100)(s + 150)}$$

$$N_2 = \begin{vmatrix} (s^2 + 200s + 5000) & 30s \\ -50 & 22.5 \end{vmatrix} = 22.5s^2 + 6000s + 112{,}500$$

$$I_2 = \frac{N_2}{\Delta} = \frac{22.5s^2 + 6000s + 112{,}500}{s(s + 100)(s + 150)}$$

[b] $sI_1 = \dfrac{30(s^2 + 50s + 3750)}{(s + 100)(s + 150)}$

$$\lim_{s\to\infty} sI_1 = i_1(0^+) = 30\,\text{A}; \qquad \lim_{s\to 0} sI_1 = i_1(\infty) = 7.5\,\text{A}$$

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$$sI_2 = \frac{22.5s^2 + 6000s + 112{,}500}{(s+100)(s+150)}$$

$$\lim_{s\to\infty} sI_2 = i_2(0^+) = 22.5\,\text{A}; \qquad \lim_{s\to 0} sI_2 = i_2(\infty) = 7.5\,\text{A}$$

[c] $$I_1 = \frac{30(s^2+50s+3750)}{s(s+100)(s+150)} = \frac{K_1}{s} + \frac{K_2}{s+100} + \frac{K_3}{s+150}$$

$$K_1 = 7.5; \qquad K_2 = -52.5; \qquad K_3 = 75$$

$$i_1(t) = [7.5 - 52.5e^{-100t} + 75e^{-150t}]u(t)\,\text{A}$$

$$I_2 = \frac{22.5s^2 + 6000s + 112{,}500}{s(s+100)(s+150)} = \frac{K_1}{s} + \frac{K_2}{s+100} + \frac{K_3}{s+150}$$

$$K_1 = 7.5; \qquad K_2 = 52.5; \qquad K_3 = -37.5$$

$$i_2(t) = [7.5 + 52.5e^{-100t} - 37.5e^{-250t}]u(t)\,\text{A}$$

P 13.22 **[a]**

$$\frac{V_1}{50} + \frac{V_1 - 250/s}{6250/s} + \frac{V_1 - V_o}{0.4s} = 0$$

$$\frac{-25}{s} + \frac{V_o - V_1}{0.4s} + \frac{V_o - 250/s}{150} = 0$$

Simplfying,

$$(s^2 + 125s + 15{,}625)V_1 - 15{,}625V_o = 250s$$

$$-375V_1 + (s+375)V_o = 4000$$

$$\Delta = \begin{vmatrix} (s^2+125s+15{,}625) & -15{,}625 \\ -375 & (s+375) \end{vmatrix} = s(s+250)^2$$

$$N_o = \begin{vmatrix} (s^2+125s+15{,}625) & 250s \\ -375 & 4000 \end{vmatrix} = 4000s^2 + 593{,}750s + 625\times 10^5$$

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$$V_o = \frac{N_o}{\Delta} = \frac{4000s^2 + 593{,}750s + 625 \times 10^5}{s(s+250)^2} = \frac{K_1}{s} + \frac{K_2}{(s+250)^2} + \frac{K_3}{s+250}$$

$$K_1 = 1000; \quad K_2 = -656{,}250$$

$$K_3 = 25\frac{d}{ds}\left[\frac{4000s^2 + 593{,}750s + 625 \times 10^5}{s}\right]_{s=-250} = 3000$$

$$\therefore \quad V_o = \frac{1000}{s} - \frac{656{,}250}{(s+250)^2} + \frac{3000}{s+250}$$

[b] $v_o(t) = [1 - 656.25te^{-250t} + 3e^{-250t}]u(t)\,\text{kV}$

[c] At $t = 0^+$:

$$v_o(0^+) = 250 + 3750 = 4\,\text{kV(checks)}$$

At $t = \infty$:

$$\frac{v_o(\infty)}{50} - 25 + \frac{v_o(\infty) - 250}{150} = 0$$

$$\therefore \quad 3v_o(\infty) - 3750 + v_o(\infty) - 250 = 0; \qquad \therefore \quad 4v_o(\infty) = 4000$$

$$\therefore \qquad v_o(\infty) = 1\,\text{kV(checks)}$$

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P 13.23

$$\frac{20\times10^{-3}}{s} = \frac{V_o}{50+10^4/s} + 15\times10^{-3}V_\phi + \frac{V_o}{s}$$

$$V_\phi = \frac{10^4/s}{50+10^4/s}V_o = \frac{10^4V_o}{50s+10^4}$$

$$\therefore \qquad \frac{20\times10^{-3}}{s} = \frac{V_os}{50s+10^4} + \frac{150V_o}{50s+10^4} + \frac{V_o}{s}$$

$$\therefore \quad V_o = \frac{s+200}{s^2+200s+10^4} = \frac{K_1}{(s+100)^2} + \frac{K_2}{s+100}$$

$$K_1 = 100; \qquad K_2 = 1$$

$$V_o = \frac{100}{(s+100)^2} + \frac{1}{s+100}$$

$$v_o(t) = [100te^{-100t} + e^{-100t}]u(t)\,\text{V}$$

P 13.24 $v_\text{C}(0^-) = v_\text{C}(0^+) = 0$

$$\frac{0.05}{s} = \frac{V_o}{1000} + \frac{V_o}{100} + \frac{V_os}{5\times10^6} - \frac{0.3V_o}{100}$$

$$\frac{250\times10^3}{s} = (5000 + 50{,}000 + s - 15{,}000)V_o$$

$$V_o = \frac{250\times10^3}{s(s+40{,}000)} = \frac{K_1}{s} + \frac{K_2}{s+40{,}000}$$

$$= \frac{6.25}{s} - \frac{6.25}{s+40{,}000}$$

$$v_o(t) = [6.25 - 6.25e^{-40{,}000t}]u(t)\,\text{V}$$

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P 13.25 **[a]**

$$\frac{V_o - 35/s}{2} + 0.4V_\Delta + \frac{V_o - 8I_\phi}{s + (250/s)} = 0$$

$$V_\Delta = \left[\frac{V_o - 8I_\phi}{s + (250/s)}\right] s; \qquad I_\phi = \frac{(35/s) - V_o}{2}$$

Solving for V_o yields:

$$V_o = \frac{29.4s^2 + 56s + 1750}{s(s^2 + 2s + 50)} = \frac{29.4s^2 + 56s + 1750}{s(s + 1 - j7)(s + 1 + j7)}$$

$$V_o = \frac{K_1}{s} + \frac{K_2}{s + 1 - j7} + \frac{K_2^*}{s + 1 + j7}$$

$$K_1 = \frac{29.4s^2 + 56s + 1750}{s^2 + 2s + 50}\bigg|_{s=0} = 35$$

$$K_2 = \frac{29.4s^2 + 56s + 1750}{s(s + 1 + j7)}\bigg|_{s=-1+j7}$$

$$= -2.8 + j0.6 = 2.86\underline{/167.91^\circ}$$

$$\therefore \quad v_o(t) = [35 + 5.73e^{-t}\cos(7t + 167.91^\circ)]u(t)\,\text{V}$$

[b] At $t = 0^+$ $\qquad v_o = 35 + 5.73\cos(167.91^\circ) = 29.4\,\text{V}$

$$\frac{v_o - 35}{2} + 0.4v_\Delta = 0; \qquad v_o - 35 + 0.8v_\Delta = 0$$

$$v_o = v_\Delta + 8i_\phi = v_\Delta + 8(0.4v_\Delta) = 4.2\,\text{V}$$

$$v_o + (0.8)\frac{v_o}{4.2} = 35; \qquad \therefore \quad v_o(0^+) = 29.4\,\text{V(checks)}$$

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At $t = \infty$, the circuit is

$v_\Delta = 0, \quad i_\phi = 0 \qquad \therefore \quad v_o = 35\,\text{V(checks)}$

P 13.26 **[a]**

$$\frac{V_\phi}{10/s} + \frac{V_\phi - (600/s)}{10} + \frac{V_\phi - V_o}{20s} = 0$$

$$\frac{V_o}{140} + \frac{V_o - V_\phi}{20s} + \frac{V_\phi}{4} = 0$$

Simplfying,

$$(2s^2 + 2s + 1)V_\phi - V_o = 1200$$

$$(35s - 7)V_\phi + (s + 7)V_o = 0$$

$$\Delta = \begin{vmatrix} 2s^2 + 2s + 1 & -1 \\ 35s - 7 & s + 7 \end{vmatrix} = 2s(s^2 + 8s + 25)$$

$$N_2 = \begin{vmatrix} 2s^2 + 2s + 1 & 1200 \\ 35s - 7 & 0 \end{vmatrix} = -42{,}000s + 8400$$

$$V_o = \frac{N_2}{\Delta} = \frac{-21{,}000s + 4200}{s(s^2 + 8s + 25)} = \frac{-4200(5s - 1)}{s(s^2 + 8s + 25)}$$

[b] $v_o(0^+) = \lim_{s \to \infty} sV_o = 0$

$$v_o(\infty) = \lim_{s \to 0} sV_o = \frac{4200}{25} = 168$$

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[**c**] At $t = 0^+$ the circuit is

At $t = \infty$ the circuit is

$$\frac{V_\phi - 600}{10} + \frac{V_\phi}{140} + \frac{V_\phi}{4} = 0$$

$$\therefore \quad V_\phi = 168\,\text{V} = V_o(\infty) \qquad \text{(checks)}$$

[**d**] $$V_o = \frac{N_2}{\Delta} = \frac{-21{,}000s + 4200}{s(s^2 + 8s + 25)} = \frac{K_1}{s} + \frac{K_2}{s + 4 - j3} + \frac{K_2^*}{s + 4 + j3}$$

$$K_1 = \frac{4200}{25} = 168$$

$$K_2 = \frac{-21{,}000(-4 + j3) + 4200}{(-4 + j3)(j6)} = -84 + j3612 = 3612.98\underline{/91.33^\circ}$$

$$v_o(t) = [168 + 7225.95e^{-4t}\cos(3t + 91.33^\circ)]u(t)\,\text{V}$$

Check: $\quad v_o(0^+) = 0\,\text{V}; \qquad v_o(\infty) = 168\,\text{V}$

P 13.27 [**a**]

$$20I_\phi + 25s(I_o - I_\phi) + 25(I_o - I_1) = 0$$

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$$25s(I_\phi - I_o) + \frac{50}{s}I_\phi + 5I_1 + 25(I_1 - I_o) = 0$$

$$I_\phi - I_1 = \frac{100}{s}; \qquad \therefore \quad I_1 = I_\phi - \frac{100}{s}$$

Simplifying,

$$(-5s-1)I_\phi + (5s+5)I_o = -500/s$$

$$(5s^2+6s+10)I_\phi + (-5s^2-5s)I_o = 600$$

$$\Delta = \begin{vmatrix} -5s-1 & 5s+5 \\ 5s^2+6s+10 & -5s^2-5s \end{vmatrix} = -25(s^2+3s+2)$$

$$N_2 = \begin{vmatrix} -5s-1 & -500/s \\ 5s^2+6s+10 & 600 \end{vmatrix} = -\frac{500}{s}(s^2-4.8s-10)$$

$$I_o = \frac{N_2}{\Delta} = \frac{20s^2-96s-200}{s(s+1)(s+2)}$$

[b] $i_o(0^+) = \lim_{s\to\infty} sI_o = 20\,\text{A}$

$$i_o(\infty) = \lim_{s\to 0} sI_o = \frac{-200}{2} = -100\,\text{A}$$

[c] At $t = 0^+$ the circuit is

$$20I_\phi + 5I_1 = 0; \quad I_\phi - I_1 = 100$$

$$\therefore \quad 20I_\phi + 5(I_\phi - 100) = 0; \qquad 25I_\phi = 500$$

$$\therefore \quad I_\phi = I_o(0^+) = 20\,\text{A}(\text{checks})$$

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At $t + \infty$ the circuit is

$I_o(\infty) = -100\,\text{A}(\text{checks})$

[d] $I_o = \dfrac{20s^2 - 96s - 200}{s(s+1)(s+2)} = \dfrac{K_1}{s} + \dfrac{K_2}{s+1} + \dfrac{K_3}{s+2}$

$K_1 = \dfrac{-200}{(1)(2)} = -100; \qquad K_2 = \dfrac{20 + 96 - 200}{(-1)(1)} = 84$

$K_3 = \dfrac{80 + 192 - 200}{(-2)(-1)} = 36$

$I_o = \dfrac{-100}{s} + \dfrac{84}{s+1} + \dfrac{36}{s+2}$

$i_o(t) = (-100 + 84e^{-t} + 36e^{-2t})u(t)\,\text{A}$

$i_o(\infty) = -100\,\text{A}(\text{checks})$

$i_o(0^+) = -100 + 84 + 36 = 20\,\text{A}(\text{checks})$

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P 13.28 **[a]** $i_L(0^-) = i_L(0^+) = \dfrac{24}{3} = 8\,\text{A}$ directed upward

$$V_T = 25I_\phi + \left[\frac{20(10/s)}{20 + (10/s)}\right] I_T = \frac{25I_T(10/s)}{20 + (10/s)} + \left(\frac{200}{10 + 20s}\right) I_T$$

$$\frac{V_T}{I_T} = Z = \frac{250 + 200}{20s + 10} = \frac{45}{2s + 1}$$

$$\frac{V_o}{5} + \frac{V_o(2s + 1)}{45} + \frac{V_o}{5.625s} = \frac{8}{s}$$

$$\frac{[9s + (2s + 1)s + 8]V_o}{45s} = \frac{8}{s}$$

$$V_o[2s^2 + 10s + 8] = 360$$

$$V_o = \frac{360}{2s^2 + 10s + 8} = \frac{180}{s^2 + 5s + 4}$$

[b] $V_o = \dfrac{180}{(s + 1)(s + 4)} = \dfrac{K_1}{s + 1} + \dfrac{K_2}{s + 4}$

$$K_1 = \frac{180}{3} = 60; \qquad K_2 = \frac{180}{-3} = -60$$

$$V_o = \frac{60}{s + 1} - \frac{60}{s + 4}$$

$$v_o(t) = [60e^{-t} - 60e^{-4t}]u(t)\,\text{V}$$

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P 13.29 **[a]** For $t < 0$:

$$V_2 = \frac{10}{10+40}(450) = 90\,\text{V}$$

[b] $$V_1 = \frac{25(450/s)}{(125{,}000/s) + 25 + 1.25 \times 10^{-3}s}$$

$$= \frac{9 \times 10^6}{s^2 + 20,000s + 10^8} = \frac{9 \times 10^6}{(s + 10{,}000)^2}$$

$$v_1(t) = (9 \times 10^6 te^{-10{,}000t})u(t)\,\text{V}$$

[c] $$V_2 = \frac{90}{s} - \frac{(25{,}000/s)(450/s)}{(125{,}000/s) + 1.25 \times 10^{-3}s + 25}$$

$$= \frac{90(s + 20{,}000)}{s^2 + 20{,}000s + 10^8}$$

$$= \frac{900{,}000}{(s + 10{,}000)^2} + \frac{90}{s + 10{,}000}$$

$$v_2(t) = [9 \times 10^5 te^{-10{,}000t} + 90e^{-10{,}000t}]u(t)\,\text{V}$$

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P 13.30 **[a]**

[b] $$Z_{\text{eq}} = 50{,}000 + \frac{10^7}{3s} + \frac{20 \times 10^{12}/s^2}{12 \times 10^6/s}$$

$$= 50{,}000 + \frac{10^7}{3s} + \frac{20 \times 10^{12}}{12 \times 10^6 s}$$

$$= \frac{100{,}000s + 10^7}{2s}$$

$$I_1 = \frac{20/s}{Z_{\text{eq}}} = \frac{0.4 \times 10^{-3}}{s + 100}$$

$$V_1 = \frac{10^7}{3s} I_1 = \frac{4000/3}{s(s + 100)}$$

$$V_2 = \frac{10^7}{6s} \cdot \frac{0.4 \times 10^{-4}}{s + 100} = \frac{2000/3}{s(s + 100)}$$

[c] $i_1(t) = 0.4e^{-100t}u(t)\,\text{mA}$

$$V_1 = \frac{40/3}{s} - \frac{40/3}{s + 100}; \qquad v_1(t) = (40/3)(1 - 1e^{-100t})u(t)\,\text{V}$$

$$V_2 = \frac{20/3}{s} - \frac{20/3}{s + 100}; \qquad v_2(t) = (20/3)(1 - 1e^{-100t})u(t)\,\text{V}$$

[d] $i_1(0^+) = 0.4\,\text{mA}$

$$i_1(0^+) = \frac{20}{50} \times 10^{-3} = 0.44\,\text{mA}(\text{checks})$$

$$v_1(0^+) = 0; \qquad v_2(0^+) = 0(\text{checks})$$

$$v_1(\infty) = 40/3\,\text{V}; \qquad v_2(\infty) = 20/3\,\text{V}(\text{checks})$$

$$v_1(\infty) + v_2(\infty) = 20\,\text{V}(\text{checks})$$

$$(0.3 \times 10^{-6})v_1(\infty) = 4\,\mu\text{C}$$

$$(0.6 \times 10^{-6})v_2(\infty) = 4\,\mu\text{C}(\text{checks})$$

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P 13.31 [**a**]

$$\frac{10}{s}I_1 + \frac{10}{s}(I_1 - I_2) + 10(I_1 - 9/s) = 0$$

$$\frac{10}{s}(I_2 - 9/s) + \frac{10}{s}(I_2 - I_1) + 10I_2 = 0$$

Simplifying,

$$(s+2)I_1 - I_2 = 9$$

$$-I_1 + (s+2)I_2 = \frac{9}{s}$$

$$\Delta = \begin{vmatrix} (s+2) & -1 \\ -1 & (s+2) \end{vmatrix} = s^2 + 4s + 3 = (s+1)(s+3)$$

$$N_1 = \begin{vmatrix} 9 & -1 \\ 9/s & (s+2) \end{vmatrix} = \frac{9s^2 + 18s + 9}{s} = \frac{9}{s}(s+1)^2$$

$$I_1 = \frac{N_1}{\Delta} = \frac{9}{s}\left[\frac{(s+1)^2}{(s+1)(s+3)}\right] = \frac{9(s+1)}{s(s+3)}$$

$$N_2 = \begin{vmatrix} (s+2) & 9 \\ -1 & 9/s \end{vmatrix} = \frac{18}{s}(s+1)$$

$$I_2 = \frac{N_2}{\Delta} = \frac{18(s+1)}{s(s+1)(s+3)} = \frac{18}{s(s+3)}$$

$$I_{\mathrm{a}} = I_1 = \frac{9(s+1)}{s(s+3)} = \frac{3}{s} + \frac{6}{s+3}$$

$$I_{\mathrm{b}} = \frac{9}{s} - I_1 = \frac{9}{s} - \frac{9(s+1)}{s(s+3)} = \frac{6}{s} - \frac{6}{s+3}$$

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[b] $i_a(t) = 3(1 + 2e^{-3t})u(t)\,\text{A}$

$i_b(t) = 6(1 - e^{-3t})u(t)\,\text{A}$

[c] $V_a = \dfrac{10}{s} I_b = \dfrac{10}{s}\left(\dfrac{3}{s} + \dfrac{6}{s+3}\right)$

$$= \frac{30}{s^2} + \frac{60}{s(s+3)} = \frac{30}{s^2} + \frac{20}{s} - \frac{20}{s+3}$$

$$V_b = \frac{10}{s}(I_2 - I_1) = \frac{10}{s}\left[\left(\frac{6}{s} - \frac{6}{s+3}\right) - \left(\frac{3}{s} + \frac{6}{s+3}\right)\right]$$

$$= \frac{10}{s}\left[\frac{3}{s} - \frac{12}{s+3}\right] = \frac{30}{s^2} - \frac{40}{s} + \frac{40}{s+3}$$

$$V_c = \frac{10}{s}(9/s - I_2) = \frac{10}{s}\left(\frac{9}{s} - \frac{6}{s} + \frac{6}{s+3}\right)$$

$$= \frac{30}{s^2} + \frac{20}{s} - \frac{20}{s+3}$$

[d] $v_a(t) = [30t + 20 - 20e^{-3t}]u(t)\,\text{V}$

$v_b(t) = [30t - 40 + 40e^{-3t}]u(t)\,\text{V}$

$v_c(t) = [30t + 20 - 20e^{-3t}]u(t)\,\text{V}$

[e] Calculating the time when the capacitor voltage drop first reaches 1000 V:

$30t + 20 - 20e^{-3t} = 1000 \quad \text{or} \quad 30t - 40 + 40e^{-3t} = 1000$

Note that in either of these expressions the exponential tem is negligible when compared to the other terms. Thus,

$30t + 20 = 1000 \quad \text{or} \quad 30t - 40 = 1000$

Thus,

$t = \dfrac{980}{30} = 32.67\,\text{s} \quad \text{or} \quad t = \dfrac{1040}{30} = 34.67\,\text{s}$

Therefore, the breakdown will occur at $t = 32.67$ s.

P 13.32 **[a]** The s-domain equivalent circuit is

$$I = \frac{V_g}{R + sL} = \frac{V_g/L}{s + (R/L)}, \qquad V_g = \frac{V_m(\omega\cos\phi + s\sin\phi)}{s^2 + \omega^2}$$

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$$I = \frac{K_0}{s + R/L} + \frac{K_1}{s - j\omega} + \frac{K_1^*}{s + j\omega}$$

$$K_0 = \frac{V_m(\omega L \cos\phi - R\sin\phi)}{R^2 + \omega^2 L^2}, \qquad K_1 = \frac{V_m \underline{/\phi - 90^\circ - \theta(\omega)}}{2\sqrt{R^2 + \omega^2 L^2}}$$

where $\tan\theta(\omega) = \omega L/R$. Therefore, we have

$$i(t) = \frac{V_m(\omega L \cos\phi - R\sin\phi)}{R^2 + \omega^2 L^2} e^{-(R/L)t} + \frac{V_m \sin[\omega t + \phi - \theta(\omega)]}{\sqrt{R^2 + \omega^2 L^2}}$$

[b] $i_{ss}(t) = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin[\omega t + \phi - \theta(\omega)]$

[c] $i_{\text{tr}} = \dfrac{V_m(\omega L \cos\phi - R\sin\phi)}{R^2 + \omega^2 L^2} e^{-(R/L)t}$

[d] $\mathbf{I} = \dfrac{\mathbf{V}_g}{R + j\omega L}, \qquad \mathbf{V}_g = V_m \underline{/\phi - 90^\circ}$

Therefore $\quad \mathbf{I} = \dfrac{V_m \underline{/\phi - 90^\circ}}{\sqrt{R^2 + \omega^2 L^2} \underline{/\theta(\omega)}} = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \underline{/\phi - \theta(\omega) - 90^\circ}$

Therefore $\quad i_{ss} = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin[\omega t + \phi - \theta(\omega)]$

[e] The transient component vanishes when

$$\omega L \cos\phi = R\sin\phi \quad \text{or} \quad \tan\phi = \frac{\omega L}{R} \quad \text{or} \quad \phi = \theta(\omega)$$

P 13.33 $v_C = 12 \times 10^5 t e^{-5000t}\,\text{V}, \quad C = 5\,\mu\text{F}; \qquad$ therefore

$$i_C = C\left(\frac{dv_C}{dt}\right) = 6e^{-5000t}(1 - 5000t)\,\text{A}$$

$i_C > 0 \quad$ when $\quad 1 > 5000t \quad$ or $\quad i_C > 0 \quad$ when $\quad 0 < t < 200\,\mu\text{s}$

and $\quad i_C < 0 \quad$ when $\quad t > 200\,\mu\text{s}$

$i_C = 0 \quad$ when $\quad 1 - 5000t = 0, \quad$ or $\quad t = 200\,\mu\text{s}$

$$\frac{dv_C}{dt} = 12 \times 10^5 e^{-5000t}[1 - 5000t]$$

$$\therefore \quad i_C = 0 \quad \text{when} \quad \frac{dv_C}{dt} = 0$$

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P 13.34 **[a]**

$$200\|4s = \frac{800s}{4s+200} = \frac{200s}{s+50}$$

$$V_o = \frac{200s}{s+50}\left[\frac{25}{(s+75)^2}\right] = \frac{5000s}{(s+50)(s+75)^2}$$

$$I_o = \frac{V_o}{200} = \frac{25s}{(s+50)(s+75)^2}$$

$$I_L = \frac{V_o}{4s} = \frac{1250}{(s+50)(s+75)^2}$$

[b] $$V_o = \frac{K_1}{s+50} + \frac{K_2}{(s+75)^2} + \frac{K_3}{s+75}$$

$$K_1 = \frac{5000s}{(s+75)^2}\bigg|_{s=-50} = -400$$

$$K_2 = \frac{5000s}{(s+50)}\bigg|_{s=-75} = 15{,}000$$

$$K_3 = \frac{d}{ds}\left[\frac{5000s}{s+50}\right]_{s=-75} = \left[\frac{5000}{s+50} - \frac{5000s}{(s+50)^2}\right]_{s=-75} = 400$$

$$v_o(t) = [-400e^{-50t} + 15{,}000te^{-75t} + 400e^{-75t}]u(t)\,\text{V}$$

$$I_o = \frac{K_1}{s+50} + \frac{K_2}{(s+75)^2} + \frac{K_3}{s+75}$$

$$K_1 = \frac{25s}{(s+75)^2}\bigg|_{s=-50} = -2$$

$$K_2 = \frac{25s}{(s+50)}\bigg|_{s=-75} = 75$$

$$K_3 = \frac{d}{ds}\left[\frac{25s}{s+50}\right]_{s=-75} = \left[\frac{25}{s+50} - \frac{25s}{(s+50)^2}\right]_{s=-75} = 2$$

$$i_o(t) = [-2e^{-50t} + 75te^{-75t} + 2e^{-75t}]u(t)\,\text{V}$$

$$I_L = \frac{K_1}{s+50} + \frac{K_2}{(s+75)^2} + \frac{K_3}{s+75}$$

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$$K_1 = \frac{1250}{(s+75)^2}\bigg|_{s=-50} = 2$$

$$K_2 = \frac{1250}{(s+50)}\bigg|_{s=-75} = -50$$

$$K_3 = \frac{d}{ds}\left[\frac{1250}{s+50}\right]_{s=-75} = \left[-\frac{1250}{(s+50)^2}\right]_{s=-75} = -2$$

$$i_L(t) = [2e^{-50t} - 50te^{-75t} - 2e^{-75t}]u(t)\,\text{V}$$

P 13.35

$$V_{\text{Th}} = \frac{10s}{10s+1000}\cdot\frac{40}{s} = \frac{400}{10s+1000} = \frac{40}{s+100}$$

$$Z_{\text{Th}} = 1000 + 1000\|10s = 1000 + \frac{10{,}000s}{10s+1000} = \frac{2000(s+50)}{s+100}$$

$$I = \frac{40/(s+100)}{(5\times10^5)/s + 2000(s+50)/(s+100)} = \frac{40s}{2000s^2 + 600{,}000s + 5\times10^7}$$

$$= \frac{0.02s}{s^2+300s+25{,}000} = \frac{K_1}{s+150-j50} + \frac{K_1^*}{s+150+j50}$$

$$K_1 = \frac{0.02s}{s+150+j50}\bigg|_{s=-150+j50} = 31.62\times10^{-3}\underline{/71.57^\circ}$$

$$i(t) = 63.25e^{-150t}\cos(50t+71.57^\circ)u(t)\,\text{mA}$$

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P 13.36 **[a]**

$$\frac{150}{s} = (100 + 0.1875s)I_1 + 0.125sI_2$$

$$0 = 0.125sI_1 + (200 + 0.25s)I_1$$

$$\Delta = \begin{vmatrix} 0.1875s + 100 & 0.125s \\ 0.125s & 0.25s + 200 \end{vmatrix} = 0.03125(s + 400)(s + 1600)$$

$$N_1 = \begin{vmatrix} 150/s & 0.125s \\ 0 & 0.25s + 200 \end{vmatrix} = \frac{37.5(s + 800)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{1200(s + 800)}{s(s + 400)(s + 1600)}$$

[b] $sI_1 = \dfrac{1200(s + 800)}{(s + 400)(s + 1600)}$

$$\lim_{s\to 0} sI_1 = i_1(\infty) = 1.5\,\text{A}$$

$$\lim_{s\to\infty} sI_1 = i_1(0) = 0$$

[c] $I_1 = \dfrac{K_1}{s} + \dfrac{K_2}{s + 400} + \dfrac{K_3}{s + 1600}$

$$K_1 = 1.5; \qquad K_2 = -1; \qquad K_3 = -0.5$$

$$i_1(t) = (1.5 - e^{-400t} - 0.5e^{-1600t})u(t)\,\text{A}$$

P 13.37 **[a]** From the solution to Problem 13.36 we have

$$N_2 = \begin{vmatrix} 0.1875s + 100 & 150/s \\ 0.125s & 0 \end{vmatrix} = -18.75$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-600}{(s + 400)(s + 1600)} = \frac{K_1}{s + 400} + \frac{K_2}{s + 1600}$$

$$K_1 = \frac{-600}{1200} = -0.5; \qquad K_2 = \frac{-600}{-1200} = 0.5$$

$$i_2(t) = (-0.5e^{-40t} + 0.5e^{-1600t})u(t)\,\text{A}$$

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[b] Reversing the dot on the 250 mH coil will reverse the sign of M, thus the circuit becomes

The two simulanteous equations are

$$\frac{150}{s} = (100 + 0.1875s)I_1 - 0.125sI_2$$

$$0 = -0.125sI_1 + (0.25s + 200)I_2$$

When these equations are compared to those derived in Problem 13.36 we see the only difference is the algebraic sign of the $0.125s$ term. Thus reversing the dot will have no effect on I_1 and will reverse the sign of I_2. Hence,

$$i_2(t) = (0.5e^{-400t} - 0.5e^{-1600t})u(t)\,\text{A}$$

P 13.38 **[a]** $W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$

$$W = 0.16(300)^2 + 0.36(200)^2 + 0.24(200)(300) = 43.2\,\text{kJ}$$

[b] $240i_1 + 0.32\frac{di_1}{dt} - 0.24\frac{di_2}{dt} = 0$

$$540i_2 + 0.72\frac{di_2}{dt} - 0.24\frac{di_1}{dt} = 0$$

Laplace transform the equations to get

$$240I_1 + 0.32(sI_1 - 300) - 0.24(sI_2 + 200) = 0$$

$$540I_2 + 0.72(sI_2 + 200) - 0.24(sI_1 - 300) = 0$$

In standard form,

$$(0.32s + 240)I_1 - 0.24sI_2 = 144$$

$$-0.24sI_1 + (0.72s + 540)I_2 = -216$$

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$$\Delta = \begin{vmatrix} 0.32s + 240 & -0.24s \\ -0.24s & 0.72s + 540 \end{vmatrix} = 0.1728(s + 500)(s + 1500)$$

$$N_1 = \begin{vmatrix} 144 & -0.24s \\ -216 & 0.72s + 540 \end{vmatrix} = 51.84(s + 1500)$$

$$N_2 = \begin{vmatrix} 0.32s + 240 & 144 \\ -0.24s & -216 \end{vmatrix} = -103.68(s + 1500)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{51.84(s + 1500)}{0.1728(s + 500)(s + 1500)} = \frac{300}{s + 500}$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-103.68(s + 1500)}{0.1728(s + 500)(s + 1500)} = \frac{-200}{s + 500}$$

[c] $i_1(t) = 300e^{-500t}u(t)\,\text{A}; \qquad i_2(t) = -200e^{-500t}u(t)\,\text{A}$

[d] $W_{240\Omega} = \int_0^\infty (9 \times 10^4 e^{-1000t})(240)\,dt = 216 \times 10^5 \frac{e^{-1000t}}{-1000}\Big|_0^\infty = 21{,}600\,\text{J}$

$$W_{540\Omega} = \int_0^\infty (4 \times 10^4 e^{-1000t})(540)\,dt = 216 \times 10^5 \frac{e^{-1000t}}{-1000}\Big|_0^\infty = 21{,}600\,\text{J}$$

$$W_{240\Omega} + W_{540\Omega} = 43.2\,\text{kJ}$$

[e] $W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = 14{,}400 + 14{,}400 - 14{,}400 = 14.4\,\text{kJ}$

With the dot reversed the s-domain equations are

$$(0.32s + 240)I_1 + 0.24sI_2 = 48$$

$$0.24sI_1 + (0.72s + 540)I_2 = -72$$

As before, $\Delta = 0.1728(s + 500)(s + 1500)$. Now,

$$N_1 = \begin{vmatrix} 48 & -0.24s \\ -72 & 0.72s + 540 \end{vmatrix} = 51.84(s + 500)$$

$$N_2 = \begin{vmatrix} 0.32s + 240 & 48 \\ 0.24s & -72 \end{vmatrix} = -34.56(s + 500)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{300}{s + 1500}; \qquad I_2 = \frac{N_2}{\Delta} = \frac{-200}{s + 1500}$$

$$i_1(t) = 300e^{-1500t}u(t)\,\text{A}; \qquad i_2(t) = -200e^{-1500t}u(t)\,\text{A}$$

$$W_{240\Omega} = \int_0^\infty (9 \times 10^4 e^{-3000t})(240)\,dt = 7200\,\text{J}$$

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$$W_{540\Omega} = \int_0^\infty (4 \times 10^4 e^{-3000t})(540)\,dt = 7200\,\text{J}$$

$$W_{120\Omega} + W_{270\Omega} = 14.4\,\text{kJ}$$

P 13.39 For $t < 0$:

For $t > 0^+$:

Note that because of the dot locations on the coils, the sign of the mutual inductance is negative! (See Example C.1 in Appendix C.)

$L_1 - M = 3 + 1 = 4\,\text{H}; \qquad L_2 - M = 2 + 1 = 3\,\text{H}$

$18 \times 4 = 72; \qquad 18 \times 3 = 54$

$$\frac{V - 72}{4s + 20} + \frac{V}{-s + 10} + \frac{V + 54}{3s} = 0$$

$$V\left(\frac{1}{4s + 20} + \frac{1}{-s + 10} + \frac{1}{3s}\right) = \frac{72}{4s + 20} - \frac{54}{3s}$$

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$$V\left[\frac{3s(-s+10)+3s(4s+20)+(4s+20)(-s+10)}{3s(-s+10)(4s+20)}\right]=\frac{72(3s)-54(4s+20)}{3s(4s+20)}$$

$$V=\frac{[72(3s)-54(4s+20)](-s+10)}{5s^2+110s+200}$$

$$I_o=\frac{V}{-s+10}=\frac{-108}{(s+2)(s+20)}=\frac{-1.2}{s+2}+\frac{1.2}{s+20}$$

$$i_o(t)=1.2[e^{-20t}-e^{-2t}]u(t)\,\text{A}$$

P 13.40 The s-domain equivalent circuit is

$$\frac{V_1-48/s}{4+(100/s)}+\frac{V_1+9.6}{0.8s}+\frac{V_1}{0.8s+20}=0$$

$$V_1=\frac{-1200}{s^2+10s+125}$$

$$V_o=\frac{20}{0.8s+20}V_1=\frac{-30{,}000}{(s+25)(s+5-j10)(s+5+j10)}$$

$$=\frac{K_1}{s+25}+\frac{K_2}{s+5-j10}+\frac{K_2^*}{s+5+j10}$$

$$K_1=\frac{-30{,}000}{s^2+10s+125}\bigg|_{s=-25}=-60$$

$$K_2=\frac{-30{,}000}{(s+25)(s+5+j10)}\bigg|_{s=-5+j10}=67.08\underline{/63.43^\circ}$$

$$v_o(t)=[-60e^{-25t}+134.16e^{-5t}\cos(10t+63.43^\circ)]u(t)\,\text{V}$$

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P 13.41 **[a]** s-domain equivalent circuit is

Note: $\quad i_2(0^+) = -\dfrac{20}{10} = -2 \text{ A}$

[b] $\dfrac{24}{s} = (120 + 3s)I_1 + 3sI_2 + 6$

$0 = -6 + 3sI_1 + (360 + 15s)I_2 + 36$

In standard form,

$(s + 40)I_1 + sI_2 = (8/s) - 2$

$sI_1 + (5s + 120)I_2 = -10$

$$\Delta = \begin{vmatrix} s + 40 & s \\ s & 5s + 120 \end{vmatrix} = 4(s + 20)(s + 60)$$

$$N_1 = \begin{vmatrix} (8/s) - 2 & s \\ -10 & 5s + 120 \end{vmatrix} = \frac{-200(s - 4.8)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{-50(s - 4.8)}{s(s + 20)(s + 60)}$$

[c] $sI_1 = \dfrac{-50(s - 4.8)}{(s + 20)(s + 60)}$

$\lim\limits_{s\to\infty} sI_1 = i_1(0^+) = 0 \text{ A}$

$\lim\limits_{s\to 0} sI_1 = i_1(\infty) = \dfrac{(-50)(-4.8)}{(20)(60)} = 0.2 \text{ A}$

[d] $I_1 = \dfrac{K_1}{s} + \dfrac{K_2}{s + 20} + \dfrac{K_3}{s + 60}$

$K_1 = \dfrac{240}{1200} = 0.2; \qquad K_2 = \dfrac{-50(-20) + 240}{(-20)(40)} = -1.55$

$K_3 = \dfrac{-50(-60) + 240}{(-60)(-40)} = 1.35$

$i_1(t) = [0.2 - 1.55e^{-20t} + 1.35e^{-60t}]u(t) \text{ A}$

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P 13.42 $\Delta = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{vmatrix} = Y_{11}Y_{22} - Y_{12}^2$

$$N_2 = \begin{vmatrix} Y_{11} & [(V_g/R_1) + \gamma C - (\rho/s)] \\ Y_{12} & (I_g - \gamma C) \end{vmatrix}$$

$$V_2 = \frac{N_2}{\Delta}$$

Substitution and simplification lead directly to Eq. 13.90.

P 13.43 **[a]** Voltage source acting alone:

$$\frac{V_{o1} - 400/s}{40} + \frac{V_{01}s}{32{,}000} + \frac{V_{01}}{80 + 0.4s} = 0$$

$$\therefore \quad V_{01} = \frac{32 \times 10^4(s + 200)}{s(s + 400)(s + 600)}$$

$$\frac{V_{o2}}{40} + \frac{V_{02}s}{32{,}000} + \frac{V_{02} - 480/s}{0.4s + 80} = 0$$

$$\therefore \quad V_{02} = \frac{38.4 \times 10^6}{s(s + 400)(s + 600)}$$

$$V_o = V_{o1} + V_{o2} = \frac{32 \times 10^4(s + 200) + 38.4 \times 10^6}{s(s + 400)(s + 600)} = \frac{32 \times 10^4(s + 320)}{s(s + 400)(s + 600)}$$

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[b] $V_o = \dfrac{K_1}{s} + \dfrac{K_2}{s+400} + \dfrac{K_3}{s+600}$

$$K_1 = \frac{(32\times 10^4)(320)}{(400)(600)} = 426.67; \qquad K_2 = \frac{32\times 10^4(-80)}{(-400)(200)} = 320;$$

$$K_3 = \frac{32\times 10^4(-280)}{(-600)(-200)} = -746.67$$

$$v_o(t) = [426.67 + 320e^{-400t} - 746.67e^{-600t}]u(t)\,\text{V}$$

P 13.44 **[a]** $V_o = -\dfrac{Z_f}{Z_i}V_g$

$$Z_f = \frac{2000(1.6\times 10^6/s)}{2000 + 1.6\times 10^6/s} = \frac{1.6\times 10^6}{s+800}$$

$$Z_i = 800 + \frac{10^6}{3.125s} = \frac{800(s+400)}{s}$$

$$V_g = \frac{5000}{s^2}$$

$$\therefore \quad V_o = \frac{-10^7}{s(s+400)(s+800)}$$

[b] $V_o = \dfrac{K_1}{s} + \dfrac{K_2}{s+400} + \dfrac{K_3}{s+800}$

$$K_1 = \frac{-10^7}{(400)(800)} = -31.25$$

$$K_2 = \frac{-10^7}{(-400)(400)} = 62.5$$

$$K_3 = \frac{-10^7}{(-800)(-400)} = -31.25$$

$$\therefore \quad v_o(t) = (-31.25 + 62.5e^{-400t} - 31.25e^{-800t})u(t)\,\text{V}$$

[c] $-31.25 + 62.5e^{-400t_s} - 31.25e^{-800t_s} = -10$

$$\therefore \quad 62.5e^{-400t_s} - 31.25e^{-800t_s} = 21.25$$

Let $x = e^{-400t_s}$. Then

$62.5x - 31.25x^2 = 21.25; \qquad$ or $x^2 - 2x + 0.68 = 0$

Solving,

$x = 0.4343$

$\therefore \quad e^{-400t_s} = 0.4343; \qquad \therefore \quad t_s = 2.1\,\text{ms}$

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[d] $v_g = m\,tu(t); \qquad V_g = \dfrac{m}{s^2}$

$$V_o = \frac{-2000m}{s(s+400)(s+800)}$$

$$K_1 = \frac{-2000m}{(400)(800)} = \frac{-2000m}{32\times 10^4}$$

$$\therefore \quad -10 = \frac{-2000m}{32\times 10^4} \qquad \therefore \quad m = 1600\,\text{V/s}$$

Thus, m must be less than or equal to 1600 V/s to avoid saturation.

P 13.45

$$\frac{V_\text{a} - 0.016/s}{2000} + \frac{V_\text{a}s}{50\times 10^6} + \frac{(V_\text{a} - V_o)s}{50\times 10^6} = 0$$

$$\frac{(0 - V_\text{a})s}{50\times 10^6} + \frac{(0 - V_o)}{10{,}000} = 0$$

$$V_\text{a} = \frac{-5000V_o}{s}$$

$$\therefore \quad \frac{-5000V_o}{s}(2s + 25{,}000) - sV_o = 25{,}000\left(\frac{0.016}{s}\right)$$

$$V_o = \frac{-4000}{(s + 5000 - j10{,}000)(s + 5000 + j10{,}000)}$$

$$K_1 = \frac{-400}{j10{,}000} = j0.02 = 0.02\underline{/90^\circ}$$

$$v_o(t) = 40e^{-5000t}\cos(10{,}000t + 90^\circ) = -40e^{-5000t}\sin(10{,}000t)u(t)\,\text{mV}$$

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P 13.46 **[a]**

$$V_p = \frac{50/s}{5 + 50/s} V_{g2} = \frac{50}{5s + 50} V_{g2}$$

$$\frac{V_p - 40/s}{20} + \frac{V_p - V_o}{5} + \frac{V_p - V_o}{100/s} = 0$$

$$V_p \left(\frac{1}{20} + \frac{1}{5} + \frac{s}{100} \right) - V_o \left(\frac{1}{5} + \frac{s}{100} \right) = \frac{2}{s}$$

$$\frac{s + 25}{100} \left(\frac{50}{5s + 50} \right) \frac{16}{s} - \frac{2}{s} = V_o \left(\frac{1}{5} + \frac{s}{100} \right) = V_o \left(\frac{s + 20}{100} \right)$$

$$V_o = \frac{100}{s + 20} \left[\frac{16(s + 25)}{10(s + 10)(s)} - \frac{2}{s} \right] = \frac{-40s + 2000}{s(s + 10)(s + 20)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 10} + \frac{K_3}{s + 20}$$

$$K_1 = 10; \qquad K_2 = -24; \qquad K_3 = 14$$

$$\therefore \qquad v_o(t) = [10 - 24e^{-10t} + 14e^{-20t}]u(t)\,\text{V}$$

[b] $10 - 24x + 14x^2 = 5$

$$14x^2 - 24x + 5 = 0$$

$$x = 0 \quad \text{or} \quad 0.242691$$

$$e^{-10t} = 0.242691 \qquad \therefore \qquad t = 141.60\,\text{ms}$$

P 13.47 **[a]** Let v_a be the voltage across the $0.5\,\mu$F capacitor, positive at the upper terminal.
Let v_b be the voltage across the $100\,\text{k}\Omega$ resistor, positive at the upper terminal.

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Also note

$$\frac{10^6}{0.5s} = \frac{2\times 10^6}{s} \quad \text{and} \quad \frac{10^6}{0.25s} = \frac{4\times 10^6}{s}; \qquad V_g = \frac{0.5}{s}$$

$$\frac{sV_\text{a}}{s\times 10^6} + \frac{V_\text{a} - (0.5/s)}{200{,}000} + \frac{V_\text{a}}{200{,}000} = 0$$

$$sV_\text{a} + 10V_\text{a} - \frac{5}{s} + 10V_\text{a} = 0$$

$$V_\text{a} = \frac{5}{s(s+20)}$$

$$\frac{0 - V_\text{a}}{200{,}000} + \frac{(0 - V_\text{b})s}{4\times 10^6} = 0$$

$$\therefore \qquad V_\text{b} = -\frac{20}{s}V_\text{a} = \frac{-100}{s^2(s+20)}$$

$$\frac{V_\text{b}}{100{,}000} + \frac{(V_\text{b} - 0)s}{4\times 10^6} + \frac{(V_\text{b} - V_o)s}{4\times 10^6} = 0$$

$$40V_\text{b} + sV_\text{b} + sV_\text{b} = sV_o$$

$$\therefore \qquad V_o = \frac{2(s+20)V_\text{b}}{s}; \qquad V_o = 2\left(\frac{-100}{s^3}\right) = \frac{-200}{s^3}$$

[b] $v_o(t) = -100t^2u(t)\,\text{V}$

[c] $-100t^2 = -4; \qquad t = 0.2\,\text{s} = 200\,\text{ms}$

P 13.48 **[a]** $\dfrac{1/sC}{R + 1/sC} = \dfrac{1}{RsC + 1} = \dfrac{1/RC}{s + 1/RC}$

There are no zeros, and a single pole at $-1/RC$ rad/sec.

[b] $\dfrac{R}{R + sL} = \dfrac{R/L}{s + R/L}$

There are no zeros, and a single pole at $-R/L$ rad/sec.

[c] There are several possible solutions. One is

$$R = 10\,\Omega; \qquad L = 10\,\text{mH}; \qquad C = 100\,\mu\text{F}$$

P 13.49 **[a]** $\dfrac{R}{R + 1/sC} = \dfrac{RsC}{RsC + 1} = \dfrac{s}{s + 1/RC}$

There is a single zero at 0 rad/sec, and a single pole at $-1/RC$ rad/sec.

[b] $\dfrac{sL}{R + sL} = \dfrac{s}{s + R/L}$

There is a single zero at 0 rad/sec, and a single pole at $-R/L$ rad/sec.

[c] There are several possible solutions. One is

$$R = 100\,\Omega; \qquad L = 10\,\text{mH}; \qquad C = 1\,\mu\text{F}$$

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P 13.50 **[a]** $\dfrac{R}{1/sC + sL + R} = \dfrac{(R/L)s}{s^2 + (R/L)s + 1/LC}$

There is a single zero at 0 rad/sec, and two poles:

$$p_1 = -(R/2L) + \sqrt{(R/2L)^2 - (1/LC)}; \qquad p_2 = -(R/2L) - \sqrt{(R/2L)^2 - (1/LC)}$$

[b] There are several possible solutions. One is

$R = 250\,\Omega; \quad L = 10\,\text{mH}; \quad C = 1\,\mu\text{F}$

These component values yield the following poles:

$-p_1 = -5000$ rad/sec and $-p_2 = -20{,}000$ rad/sec

[c] There are several possible solutions. One is

$R = 200\,\Omega; \quad L = 10\,\text{mH}; \quad C = 1\,\mu\text{F}$

These component values yield the following poles:

$-p_1 = -10{,}000$ rad/sec and $-p_2 = -10{,}000$ rad/sec

[d] There are several possible solutions. One is

$R = 120\,\Omega; \quad L = 10\,\text{mH}; \quad C = 1\,\mu\text{F}$

These component values yield the following poles:

$-p_1 = -6000 + j8000$ rad/sec and $-p_2 = -6000 - j8000$ rad/sec

P 13.51 **[a]**

$$\frac{100}{500}V_i = V_o + \frac{2500V_i}{2500 + (2 \times 10^6/s)}$$

$$0.2V_i - \frac{sV_i}{s + 800} = V_o$$

$$\therefore \quad \frac{V_o}{V_i} = H(s) = \frac{-0.8(s - 200)}{(s + 800)}$$

[b] $-z_1 = 200\,\text{rad/s}$

$-p_1 = -800\,\text{rad/s}$

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P 13.52 **[a]** $\dfrac{V_o}{V_i} = \dfrac{1/sC}{R + 1/sC} = \dfrac{1}{RCs + 1}$

$$H(s) = \frac{(1/RC)}{s + (1/RC)} = \frac{25}{s + 25}; \qquad -p_1 = -25\,\text{rad/s}$$

[b] $\dfrac{V_o}{V_i} = \dfrac{R}{R + 1/sC} = \dfrac{RCs}{RCs + 1} = \dfrac{s}{s + (1/RC)}$

$$= \frac{s}{s + 25}; \qquad z_1 = 0, \quad -p_1 = -25\,\text{rad/s}$$

[c] $\dfrac{V_o}{V_i} = \dfrac{sL}{R + sL} = \dfrac{s}{s + R/L} = \dfrac{s}{s + 2000}$

$$z_1 = 0; \qquad -p_1 = -2000\,\text{rad/s}$$

[d] $\dfrac{V_o}{V_i} = \dfrac{R}{R + sL} = \dfrac{R/L}{s + (R/L)} = \dfrac{2000}{s + 2000}$

$$-p_1 = -2000\,\text{rad/s}$$

[e]

$$\frac{V_o}{0.05s} + \frac{V_o}{200} + \frac{V_o - V_i}{800} = 0$$

$$sV_o + 4sV_o + 16{,}000V_o = sV_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{0.2s}{s + 3200}$$

$$z_1 = 0; \qquad -p_1 = -3200\,\text{rad/s}$$

P 13.53 **[a]** Let $R_1 = 250\,\text{k}\Omega$; $R_2 = 125\,\text{k}\Omega$; $C_2 = 1.6\,\text{nF}$; and $C_f = 0.4\,\text{nF}$. Then

$$Z_f = \frac{(R_2 + 1/sC_2)1/sC_f}{\left(R_2 + \frac{1}{sC_2} + \frac{1}{sC_f}\right)} = \frac{(s + 1/R_2C_2)}{C_f s\left(s + \frac{C_2 + C_f}{C_2C_fR_2}\right)}$$

$$\frac{1}{C_f} = 2.5 \times 10^9$$

$$\frac{1}{R_2C_2} = \frac{62.5 \times 10^7}{125 \times 10^3} = 5000\,\text{rad/s}$$

$$\frac{C_2 + C_f}{C_2C_fR_2} = \frac{2 \times 10^{-9}}{(0.64 \times 10^{-18})(125 \times 10^3)} = 25{,}000\,\text{rad/s}$$

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$$\therefore \quad Z_f = \frac{2.5 \times 10^9(s+5000)}{s(s+25{,}000)}\Omega$$

$$Z_i = R_1 = 250 \times 10^3\,\Omega$$

$$H(s) = \frac{V_o}{V_g} = \frac{-Z_f}{Z_i} = \frac{-10^4(s+5000)}{s(s+25{,}000)}$$

[b] $-z_1 = -5000\,\text{rad/s}$

$$-p_1 = 0; \qquad -p_2 = -25{,}000\,\text{rad/s}$$

P 13.54 **[a]** $Z_i = 250 + \dfrac{31{,}250}{s} = \dfrac{250(s+125)}{s}$

$$Z_f = \frac{25 \times 10^4}{s} \| 10{,}000 = \frac{25 \times 10^4}{s+25}$$

$$H(s) = -\frac{Z_f}{Z_i} = \frac{-25 \times 10^4/(s+25)}{250(s+125)/s} = \frac{-1000s}{(s+25)(s+125)}$$

[b] Zero at $z_1 = 0$; Poles at $-p_1 = -25$ rad/s and $-p_2 = -125$ rad/s

P 13.55 **[a]**

$$\frac{V_\text{a} - V_g}{1000} + \frac{sV_\text{a}}{5 \times 10^6} + \frac{(V_\text{a} - V_o)s}{5 \times 10^6} = 0$$

$$5000V_\text{a} - 5000V_g + 2sV_\text{a} - sV_o = 0$$

$$(5000 + 2s)V_\text{a} - sV_o = 5000V_g$$

$$\frac{(0 - V_\text{a})s}{5 \times 10^6} + \frac{0 - V_o}{5000} = 0$$

$$-sV_\text{a} - 1000V_o = 0; \quad \therefore \quad V_\text{a} - \frac{-1000}{s}V_o$$

$$(2s + 5000)\left(\frac{-1000}{s}\right)V_o - sV_o = 5000V_g$$

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$$1000V_o(2s+5000)+s^2V_o=-5000sV_g$$

$$V_o(s^2+2000s+5\times10^6)=-5000sV_g$$

$$\frac{V_o}{V_g}=\frac{-5000s}{s^2+2000s+5\times10^6}$$

$$s_{1,2}=-1000\pm\sqrt{10^6-5\times10^6}=-1000\pm j2000$$

$$\frac{V_o}{V_g}=\frac{-5000s}{(s+1000-j2000)(s+1000+j2000)}$$

[b] $z_1=0;\qquad -p_1=-1000+j2000;\qquad -p_2=-1000-j2000$

P 13.56 **[a]**

$$\frac{V_o}{50}+\frac{V_o}{s}+\frac{V_os}{2000}=I_g$$

$$V_o=\frac{2000s}{s^2+40s+2000}\cdot I_g\qquad\text{and}\qquad I_o=5\times10^{-4}sV_o$$

$$\therefore\quad H(s)=\frac{I_o}{I_g}=\frac{s^2}{s^2+40s+2000}$$

[b] $I_g=\dfrac{0.025s}{s^2+40{,}000}\qquad\text{so}\qquad I_o=\dfrac{(s^2)(0.025s)}{(s+20-j40)(s+20+j40)(s^2+40{,}000)}$

$$I_o=\frac{0.025s^3}{(s+20-j40)(s+20+j40)(s+j200)(s-j200)}$$

[c] Damped sinusoid of the form

$$Me^{-20t}\cos(40t+\theta_1)$$

[d] Steady-state sinusoid of the form

$$N\cos(200t+\theta_2)$$

[e] $I_o=\dfrac{K_1}{s+20-j40}+\dfrac{K_1^*}{s+20+j40}+\dfrac{K_2}{s-j200}+\dfrac{K_2^*}{s+j200}$

$$K_1=\frac{0.025(-20+j40)^3}{(j80)(-20-j160)(-20+j240)}=719.77\times10^{-6}\underline{/-97.94^\circ}$$

$$K_2=\frac{0.025(j200)^3}{(j400)(20+j160)(20+j240)}=12.88\times10^{-3}\underline{/11.89^\circ}$$

$$i_o(t)=[1.44e^{-20t}\cos(40t-97.94^\circ)+25.76\cos(200t+11.89^\circ)]\,\text{mA}$$

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P 13.57 **[a]**

$$2000(I_o - I_g) + 8000I_o + \mu(I_g - I_o)(2000) + 2sI_o = 0$$

$$\therefore \quad I_o = \frac{1000(1-\mu)}{s + 1000(5-\mu)} I_g$$

$$\therefore \quad H(s) = \frac{1000(1-\mu)}{s + 1000(5-\mu)}$$

[b] $\mu < 5$

[c]

μ	$H(s)$	I_o
-3	$4000/(s+8000)$	$20{,}000/s(s+8000)$
0	$1000/(s+5000)$	$5000/s(s+5000)$
4	$-3000/(s+1000)$	$-15{,}000/s(s+1000)$
5	$-4000/s$	$-20{,}000/s^2$
6	$-5000/(s-1000)$	$-25{,}000/s(s-1000)$

$\mu = -3$:

$$I_o = \frac{2.5}{s} - \frac{2.5}{(s+8000)}; \qquad i_o = [2.5 - 2.5e^{-8000t}]u(t)\ \text{A}$$

$\mu = 0$:

$$I_o = \frac{1}{s} - \frac{1}{s+5000}; \qquad i_o = [1 - e^{-5000t}]u(t)\ \text{A}$$

$\mu = 4$:

$$I_o = \frac{-15}{s} - \frac{15}{s+1000}; \qquad i_o = [-15 + 15e^{-1000t}]u(t)\ \text{A}$$

$\mu = 5$:

$$I_o = \frac{-20{,}000}{s^2}; \qquad i_o = -20{,}000t\, u(t)\ \text{A}$$

$\mu = 6$:

$$I_o = \frac{25}{s} - \frac{25}{s-1000}; \qquad i_o = 25[1 - e^{1000t}]u(t)\ \text{A}$$

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P 13.58

$$V_g = 25sI_1 - 35sI_2$$

$$0 = -35sI_1 + \left(50s + 10{,}000 + \frac{16 \times 10^6}{s}\right) I_2$$

$$\Delta = \begin{vmatrix} 25s & -35s \\ -35s & 50s + 10{,}000 + 16 \times 10^6/s \end{vmatrix} = 25(s + 2000)(s + 8000)$$

$$N_2 = \begin{vmatrix} 25s & V_g \\ -35s & 0 \end{vmatrix} = 35sV_g$$

$$I_2 = \frac{N_2}{\Delta} = \frac{35sV_g}{25(s + 2000)(s + 8000)}$$

$$H(s) = \frac{I_2}{V_g} = \frac{1.4s}{(s + 2000)(s + 8000)}$$

$$\therefore \qquad z_1 = 0; \qquad -p_1 = -2000 \text{ rad/s}; \qquad -p_2 = -8000 \text{ rad/s}$$

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P 13.59 $H(s) = \dfrac{V_o}{V_i} = \dfrac{1}{s+1}; \qquad h(t) = e^{-t}$

For $0 \le t \le 1$:

$$v_o = \int_0^t e^{-\lambda}\, d\lambda = (1 - e^{-t})\,\mathrm{V}$$

For $1 \le t \le \infty$:

$$v_o = \int_{t-1}^t e^{-\lambda}\, d\lambda = (e-1)e^{-t}\,\mathrm{V}$$

P 13.60 $H(s) = \dfrac{V_o}{V_i} = \dfrac{s}{s+1} = 1 - \dfrac{1}{s+1}; \qquad h(t) = \delta(t) - e^{-t}$

$$h(\lambda) = \delta(\lambda) - e^{-\lambda}$$

For $0 \le t \le 1$:

$$v_o = \int_0^t [\delta(\lambda) - e^{-\lambda}]\, d\lambda = [1 + e^{-\lambda}]\,\Big|_0^t = e^{-t}\,\mathrm{V}$$

For $1 \le t \le \infty$:

$$v_o = \int_{t-1}^t (-e^{-\lambda})\, d\lambda = e^{-\lambda}\,\Big|_{t-1}^t = (1-e)e^{-t}\,\mathrm{V}$$

P 13.61 **[a]** From Problem 13.52(a)

$$H(s) = \frac{25}{s+25}$$

$$h(\lambda) = 25e^{-25\lambda}$$

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$0 \le t \le 40\,\text{ms}$:

$$v_o = \int_0^t 10(25)e^{-25\lambda}\,d\lambda = 10(1 - e^{-25t})\,\text{V}$$

$40\,\text{ms} \le t \le \infty$:

$$v_o = \int_{t-0.04}^t 10(25)e^{-25\lambda}\,d\lambda = 10(e-1)e^{-25t}\,\text{V}$$

[b]

P 13.62 **[a]** $H(s) = \dfrac{250}{s+250} \qquad \therefore \quad h(\lambda) = 250e^{-250\lambda}$

$0 \le t \le 40\,\text{ms}$:

$$v_o = \int_0^t 10(250)e^{-250\lambda}\,d\lambda = 10(1 - e^{-250t})\,\text{V}$$

$40\,\text{ms} \le t \le \infty$:

$$v_o = \int_{t-0.04}^t 10(250)e^{-250\lambda}\,d\lambda = 10(e^{10} - 1)e^{-250t}\,\text{V}$$

[b] decrease

[c] The circuit with $R = 200\,\Omega$.

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P 13.63 **[a]** $0 \le t \le 40$:

$$y(t) = \int_0^t (10)(1)(d\lambda) = 10\lambda \Big|_0^t = 10t$$

$40 \le t \le 80$:

$$y(t) = \int_{t-40}^{40} (10)(1)(d\lambda) = 10\lambda \Big|_{t-40}^{40} = 10(80 - t)$$

$t \ge 80: \qquad y(t) = 0$

[b] $0 \le t \le 10$:

$$y(t) = \int_0^t 40\, d\lambda = 40\lambda \Big|_0^t = 40t$$

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$10 \le t \le 40$:

$$y(t) = \int_{t-10}^{t} 40\,d\lambda = 40\lambda \Big|_{t-10}^{t} = 400$$

$40 \le t \le 50$:

$$y(t) = \int_{t-10}^{40} 40\,d\lambda = 40\lambda \Big|_{t-10}^{40} = 40(50-t)$$

$t \ge 50: \qquad y(t) = 0$

[c] The expressions are

$$0 \le t \le 1: \qquad y(t) = \int_{0}^{t} 400\,d\lambda = 400\lambda \Big|_{0}^{t} = 400t$$

$$1 \le t \le 40: \qquad y(t) = \int_{t-1}^{t} 400\,d\lambda = 400\lambda \Big|_{t-1}^{t} = 400$$

$$40 \le t \le 41: \qquad y(t) = \int_{t-1}^{40} 400\,d\lambda = 400\lambda \Big|_{t-1}^{40} = 400(41-t)$$

$$41 \le t < \infty: \qquad y(t) = 0$$

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[d]

[e] Yes, note that $h(t)$ is approaching $40\delta(t)$, therefore $y(t)$ must approach $40x(t)$, i.e.

$$y(t) = \int_0^t h(t-\lambda)x(\lambda)\,d\lambda \to \int_0^t 40\delta(t-\lambda)x(\lambda)\,d\lambda$$

$$\to 40x(t)$$

This can be seen in the plot, e.g., in part (c), $y(t) \cong 40x(t)$.

P 13.64 **[a]**

$y(t) = 0 \qquad t < 0$

$0 \le t \le 10: \qquad y(t) = \int_0^t 625\,d\lambda = 625t$

$10 \le t \le 20: \qquad y(t) = \int_{t-10}^{10} 625\,d\lambda = 625(10 - t + 10) = 625(20 - t)$

$20 \le t < \infty: \qquad y(t) = 0$

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[b]

$y(t) = 0 \qquad t < 0$

$0 \le t \le 10: \qquad y(t) = \int_0^t 312.5\, d\lambda = 312.5t$

$10 \le t \le 20: \qquad y(t) = \int_0^{10} 312.5\, d\lambda = 3125$

$20 \le t \le 30: \qquad y(t) = \int_{t-20}^{10} 312.5\, d\lambda = 312.5(30 - t)$

$30 \le t < \infty: \qquad y(t) = 0$

[c]

$y(t) = 0 \qquad t < 0$

$0 \le t \le 1: \qquad y(t) = \int_0^t 625\, d\lambda = 625t$

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$$1 \le t \le 10: \qquad y(t) = \int_0^1 625\,d\lambda = 625$$

$$10 \le t \le 11: \qquad y(t) = \int_{t-10}^1 625\,d\lambda = 625(11-t)$$

$$11 \le t < \infty: \qquad y(t) = 0$$

P 13.65 **[a]** $-1 \le t \le 4$:

$$v_o = \int_0^{t+1} 10\lambda\,d\lambda = 5\lambda^2\Big|_0^{t+1} = 5t^2 + 10t + 5\,\text{V}$$

$4 \le t \le 9$:

$$v_o = \int_{t-4}^{t+1} 10\lambda\,d\lambda = 5\lambda^2\Big|_{t-4}^{t+1} = 50t - 75\,\text{V}$$

$9 \le t \le 14$:

$$v_o = 10\int_{t-4}^{10} \lambda\,d\lambda + 10\int_{10}^{t+1} 10\,d\lambda$$

$$= 5\lambda^2\Big|_{t-4}^{10} + 100\lambda\Big|_{10}^{t+1} = -5t^2 + 140t - 480\,\text{V}$$

$14 \le t \le 19$:

$$v_o = 100\int_{t-4}^{t+1} d\lambda = 500\,\text{V}$$

$19 \le t \le 24$:

$$v_o = \int_{t-4}^{20} 100\lambda\,d\lambda + \int_{20}^{t+2} 10(30-\lambda)\,d\lambda$$

$$= 100\lambda\Big|_{t-2}^{20} + 300\lambda\Big|_{20}^{t+1} - 5\lambda^2\Big|_{20}^{t+2}$$

$$= -5t^2 + 190t - 1305\,\text{V}$$

$24 \le t \le 29$:

$$v_o = 10\int_{t-4}^{t+1} (30-\lambda)\,d\lambda = 300\lambda\Big|_{t-4}^{t+1} - 5\lambda^2\Big|_{t-4}^{t+1}$$

$$= 1575 - 50t\,\text{V}$$

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$29 \le t \le 34$:

$$v_o = 10\int_{t-4}^{30}(30-\lambda)\,d\lambda = 300\lambda\,\Big|_{t-4}^{30} - 5\lambda^2\,\Big|_{t-2}^{30}$$

$$= 5t^2 - 340t + 5780\,\text{V}$$

Summary:

$v_o = 0 \qquad -\infty \le t \le -1$

$v_o = 5t^2 + 10t + 5\,\text{V} \qquad -1 \le t \le 4$

$v_o = 50t - 75\,\text{V} \qquad 4 \le t \le 9$

$v_o = -5t^2 + 140t - 480\,\text{V} \qquad 9 \le t \le 14$

$v_o = 500\,\text{V} \qquad 14 \le t \le 19$

$v_o = -5t^2 + 190t - 1305\,\text{V} \qquad 19 \le t \le 24$

$v_o = 1575 - 50t\,\text{V} \qquad 24 \le t \le 29$

$v_o = 5t^2 - 340t + 5780\,\text{V} \qquad 29 \le t \le 34$

$v_o = 0 \qquad 34 \le t \le \infty$

[b]

P 13.66 **[a]** $h(\lambda) = \dfrac{2}{5}\lambda \qquad 0 \le \lambda \le 5$

$$h(\lambda) = \left(4 - \frac{2}{5}\lambda\right) \qquad 5 \le \lambda \le 10$$

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$0 \le t \le 5$:

$$v_o = 10\int_0^t \frac{2}{5}\lambda\, d\lambda = 2t^2$$

$5 \le t \le 10$:

$$v_o = 10\int_0^5 \frac{2}{5}\lambda\, d\lambda + 10\int_5^t \left(4 - \frac{2}{5}\lambda\right)\, d\lambda$$

$$= \frac{4\lambda^2}{2}\Big|_0^5 + 40\lambda\Big|_5^t - \frac{4\lambda^2}{2}\Big|_5^t$$

$$= -100 + 40t - 2t^2$$

$10 \le t \le \infty$:

$$v_o = 10\int_0^5 \frac{2}{5}\lambda\, d\lambda + 10\int_5^{10} \left(4 - \frac{2}{5}\lambda\right)\, d\lambda$$

$$= \frac{4\lambda^2}{2}\Big|_0^5 + 40\lambda\Big|_5^{10} - \frac{4\lambda^2}{2}\Big|_5^{10}$$

$$= 50 + 200 - 150 = 100$$

$v_o = 2t^2\,\text{V} \qquad 0 \le t \le 5$

$v_o = 40t - 100 - 2t^2\,\text{V} \qquad 5 \le t \le 10$

$v_o = 100\,\text{V} \qquad 10 \le t \le \infty$

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[b]

[c] Area $= \frac{1}{2}(10)(2) = 10 \quad \therefore \quad \frac{1}{2}(4)h = 10 \quad \text{so} \quad h = 5$

$$h(\lambda) = \frac{5}{2}\lambda \qquad 0 \le \lambda \le 2$$

$$h(\lambda) = \left(10 - \frac{5}{2}\lambda\right) \qquad 2 \le \lambda \le 4$$

$0 \le t \le 2$:

$$v_o = 10\int_0^t \frac{5}{2}\lambda\, d\lambda = 12.5t^2$$

$2 \le t \le 4$:

$$v_o = 10\int_0^2 \frac{5}{2}\lambda\, d\lambda + 10\int_2^t \left(10 - \frac{5}{2}\lambda\right)\, d\lambda$$

$$= \frac{25\lambda^2}{2}\Big|_0^2 + 100\lambda\Big|_2^t - \frac{25\lambda^2}{2}\Big|_2^t$$

$$= -100 + 100t - 12.5t^2$$

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$4 \le t \le \infty$:

$$v_o = 10\int_0^2 \frac{5}{2}\lambda\, d\lambda + 10\int_2^4 \left(10 - \frac{5}{2}\lambda\right)\, d\lambda$$

$$= \frac{25\lambda^2}{2}\Big|_0^2 + 100\lambda\Big|_2^4 - \frac{25\lambda^2}{2}\Big|_2^4$$

$$= 50 + 200 - 150 = 100$$

$$v_o = 12.5t^2\,\text{V} \qquad 0 \le t \le 2$$

$$v_o = 100t - 100 - 12.5t^2\,\text{V} \qquad 2 \le t \le 4$$

$$v_o = 100\,\text{V} \qquad 4 \le t \le \infty$$

[d] The waveform in part (c) is closer to replicating the input waveform because in part (c) $h(\lambda)$ is closer to being an ideal impulse response. That is, the area was preserved as the base was shortened.

P 13.67 **[a]**

$$v_o = \int_0^t 10(10e^{-4\lambda})\, d\lambda$$

$$= 100\frac{e^{-4\lambda}}{-4}\Big|_0^t = -25[e^{-4t} - 1]$$

$$= 25(1 - e^{-4t})\,\text{V}, \qquad 0 \le t \le \infty$$

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[b]

$0 \le t \le 0.5$:

$$v_o = \int_0^t 100(1-2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^t = 100t(1-t)$$

$0.5 \le t \le \infty$:

$$v_o = \int_0^{0.5} 100(1-2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^{0.5} = 25$$

[c]

P 13.68 $H(s) = \dfrac{16s}{40+4s+16s} = \dfrac{0.8s}{s+2} = 0.8\left(1 - \dfrac{2}{s+2}\right) = 0.8 - \dfrac{1.6}{s+2}$

$$h(\lambda) = 0.8\delta(\lambda) - 1.6e^{-2\lambda}u(\lambda)$$

$$v_o = \int_0^t 75[0.8\delta(\lambda) - 1.6e^{-2\lambda}]\,d\lambda = \int_0^t 60\delta(\lambda)\,d\lambda - 120\int_0^t e^{-2\lambda}\,d\lambda$$

$$= 60 - 120\frac{e^{-2\lambda}}{-2}\Big|_0^t = 60 + 60(e^{-2t} - 1)$$

$$= 60e^{-2t}u(t)\,\text{V}$$

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P 13.69 **[a]** $I_o = \dfrac{V_o}{10^5} + \dfrac{V_o s}{5 \times 10^6} = \dfrac{V_o(s+50)}{5 \times 10^6}$

$$\frac{V_o}{I_g} = H(s) = \frac{5 \times 10^6}{s+50}$$

$$h(\lambda) = 5 \times 10^6 e^{-50\lambda} u(\lambda)$$

$0 \le t \le 0.1\,\text{s}$:

$$v_o = \int_0^t (50 \times 10^{-6})(5 \times 10^6) e^{-50\lambda}\, d\lambda = 250 \frac{e^{-50\lambda}}{-50} \Big|_0^t = 5(1 - e^{-50t})\,\text{V}$$

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$0.1\,\text{s} \le t \le 0.2\,\text{s}$:

$$v_o = \int_0^{t-0.1} (-50 \times 10^{-6})(5 \times 10^6 e^{-50\lambda}\, d\lambda)$$

$$+\int_{t-0.1}^{t} (50 \times 10^{-6})(5 \times 10^6 e^{-50\lambda}\, d\lambda)$$

$$= -250\frac{e^{-50\lambda}}{-50}\Bigg|_0^{t-0.1} + 250\frac{e^{-50\lambda}}{-50}\Bigg|_{t-0.1}^{t}$$

$$= 5\left[e^{-50(t-0.1)} - 1\right] - 5\left[e^{-50t} - e^{-50(t-0.1)}\right]$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50t} - 5]\,\text{V}$$

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$0.2\,\text{s} \le t \le \infty$:

$$v_o = \int_{t-0.2}^{t-0.1} -250e^{-50\lambda}\,d\lambda + \int_{t-0.1}^{t} 250e^{-50\lambda}\,d\lambda$$

$$= \left[5e^{-50\lambda}\Big|_{t-0.2}^{t-0.1} - 5e^{-50\lambda}\Big|_{t-0.1}^{t}\right]$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50(t-0.2)} - 5e^{-50t}]\,\text{V}$$

[b] $I_o = \dfrac{V_o s}{5\times 10^6} = \dfrac{s}{5\times 10^6}\cdot\dfrac{5\times 10^6 I_g}{s+50}$

$$\frac{I_o}{I_g} = H(s) = \frac{s}{s+50} = 1 - \frac{50}{s+50}$$

$$h(\lambda) = \delta(\lambda) - 50e^{-50\lambda}$$

$0 < t < 0.1\,\text{s}$:

$$i_o = \int_0^t (50\times 10^{-6})[\delta(\lambda) - 50e^{-50\lambda}]\,d\lambda$$

$$= 50\times 10^{-6} - 25\times 10^{-3}\frac{e^{-50\lambda}}{-50}\Big|_0^t$$

$$= 50\times 10^{-6} + 50\times 10^{-6}[e^{-50t} - 1] = 50e^{-50t}\,\mu\text{A}$$

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$0.1\,\text{s} < t < 0.2\,\text{s}$:

$$i_o = \int_0^{t-0.1} (-50 \times 10^{-6})[\delta(\lambda) - 50e^{-50\lambda}]\, d\lambda$$

$$+\int_{t-0.1}^{t} (50 \times 10^{-6})(-50e^{-50\lambda})\, d\lambda$$

$$= -50 \times 10^{-6} + 2.5 \times 10^{-3} \frac{e^{-50\lambda}}{-50}\Bigg|_0^{t-0.1} - 2.5 \times 10^{-3} \frac{e^{-50\lambda}}{-50}\Bigg|_{t-0.1}^{t}$$

$$= -50 \times 10^{-6} - 50 \times 10^{-6} e^{-50(t-0.1)} + 50 \times 10^{-6}$$

$$+50 \times 10^{-6} e^{-50t} - 50 \times 10^{-6} e^{-50(t-0.1)}$$

$$= 50e^{-50t} - 100e^{-50(t-0.1)}\, \mu\text{A}$$

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$0.2\,\text{s} < t < \infty$:

$$i_o = \int_{t-0.2}^{t-0.1} (-50 \times 10^{-6})(-50e^{-50\lambda})\, d\lambda$$

$$+\int_{t-0.1}^{t} (50 \times 10^{-6})(-50e^{-50\lambda})\, d\lambda$$

$$= 50e^{-50t} - 100e^{-50(t-0.1)} + 50e^{-50(t-0.2)}\,\mu\text{A}$$

[**c**] At $t = 0.1^-$:

$$v_o = 5(1 - e^{-5}) = 4.97\,\text{V}; \qquad i_{100\text{k}\Omega} = \frac{4.97}{0.1} = 49.66\,\mu\text{A}$$

$$\therefore \quad i_o = 50 - 49.66 = 0.34\,\mu\text{A}$$

From the solution for i_o we have

$i_o(0.1^-) = 50e^{-5} = 0.34\,\mu\text{A}$ (checks)

At $t = 0.1^+$:

$v_o(0.1^+) = v_o(0.1^-) = 4.97\,\text{V}$

$i_{100\text{k}\Omega} = 49.66\,\mu\text{A}$

$\therefore \quad i_o(0.1^+) = -(50 + 49.66) = -99.66\,\mu\text{A}$

From the solution for i_o we have

$i_o(0.1^+) = 50e^{-5} - 100 = 99.66\,\mu\text{A}$ (checks)

At $t = 0.2^-$:

$v_o = 10e^{-5} - 5e^{-10} - 5 = -4.93\,\text{V}$

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$i_{100\text{k}\Omega} = 49.33\,\mu\text{A}$

$i_o = -50 + 49.33 = -0.67\,\mu\text{A}$

From the solution for i_o,

$v_o(0.2^-) = 50e^{-10} - 100e^{-5} = -0.67\,\mu\text{A} \quad \text{(checks)}$

At $\quad t = 0.2^+$:

$v_o(0.2^+) = v_o(0.2^-) = -4.93\,\text{V}; \qquad i_{100\text{k}\Omega} = -49.33\,\mu\text{A}$

$i_o = 0 + 49.33 = 49.33\,\mu\text{A}$

From the solution for i_o,

$i_o(0.2^+) = 50e^{-10} - 100e^{-5} + 50 = 49.33\,\mu\text{A}\,\text{(checks)}$

P 13.70 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{1/LC}{s^2 + (R/L)s + (1/LC)}$

$$= \frac{100}{s^2 + 20s + 100} = \frac{100}{(s+10)^2}$$

$h(\lambda) = 100\lambda e^{-10\lambda} u(\lambda)$

$0 \le t \le 0.5$:

$$v_o = 500\int_0^t \lambda e^{-10\lambda}\,d\lambda$$

$$= 500\left\{\frac{e^{-10\lambda}}{100}(-10\lambda - 1)\,\Big|_0^t\right\}$$

$$= 5[1 - e^{-10t}(10t + 1)]$$

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$0.5 \le t \le \infty$:

$$v_o = 500\int_{t-0.5}^{t} \lambda e^{-10\lambda}\, d\lambda$$

$$= 500\left\{\frac{e^{-10\lambda}}{100}(-10\lambda - 1)\,\Big|_{t-0.5}^{t}\right\}$$

$$= 5e^{-10t}[e^5(10t-4) - 10t - 1]$$

[b]

P 13.71 **[a]** $V_o = \dfrac{16}{20}V_g$

$$\therefore \qquad H(s) = \frac{V_o}{V_g} = \frac{4}{5}$$

$$h(\lambda) = 0.8\delta(\lambda)$$

[b]

$$0 < t < 0.5\,\text{s}: \qquad v_o = \int_0^t 75[0.8\delta(\lambda)]\, d\lambda = 60\,\text{V}$$

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$0.5\,\text{s} \le t \le 1.0\,\text{s}$:

$$v_o = \int_0^{t-0.5} -75[0.8\delta(\lambda)]\,d\lambda = -60\,\text{V}$$

$$1\,\text{s} < t < \infty: \qquad v_o = 0$$

[c]

Yes, because the circuit has no memory.

P 13.72 **[a]**

$$\frac{V_o - V_g}{5} + \frac{V_o s}{4} + \frac{V_o}{20} = 0$$

$$(5s+5)V_o = 4V_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{0.8}{s+1}; \qquad h(\lambda) = 0.8e^{-\lambda}u(\lambda)$$

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[b]

$0 \le t \le 0.5\,\text{s}$;

$$v_o = \int_0^t 75(0.8e^{-\lambda})\,d\lambda = 60\frac{e^{-\lambda}}{-1}\Big|_0^t$$

$$v_o = 60 - 60e^{-t}\,\text{V}, \qquad 0 \le t \le 0.5\,\text{s}$$

$0.5\,\text{s} \le t \le 1\,\text{s}$:

$$v_o = \int_0^{t-0.5}(-75)(0.8e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 75(0.8e^{-\lambda})\,d\lambda$$

$$= -60\frac{e^{-\lambda}}{-1}\Big|_0^{t-0.5} + 60\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

$$= 120e^{-(t-0.5)} - 60e^{-t} - 60\,\text{V}, \qquad 0.5\,\text{s} \le t \le 1\,\text{s}$$

$1\,\text{s} \le t \le \infty$;

$$v_o = \int_{t-1}^{t-0.5}(-75)(0.8e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 75(0.8e^{-\lambda})\,d\lambda$$

$$= -60\frac{e^{-\lambda}}{-1}\Big|_{t-1}^{t-0.5} + 60\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

$$= 120e^{-(t-0.5)} - 60e^{-(t-1)} - 60e^{-t}\,\text{V}, \qquad 1\,\text{s} \le t \le \infty$$

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[c]

[d] No, the circuit has memory because of the capacitive storage element.

P 13.73

$$V_o = \frac{5 \times 10^3 I_g}{25 \times 10^3 + 2.5 \times 10^6/s}(20 \times 10^3)$$

$$\frac{V_o}{I_g} = H(s) = \frac{4000s}{s + 100}$$

$$H(s) = 4000\left[1 - \frac{100}{s + 100}\right] = 4000 - \frac{4 \times 10^5}{s + 100}$$

$$h(t) = 4000\delta(t) - 4 \times 10^5 e^{-100t}$$

$$v_o = \int_0^{10^{-3}} (-20 \times 10^{-3})[4000\delta(\lambda) - 4 \times 10^5 e^{-100\lambda}]\, d\lambda$$

$$+\int_{10^{-3}}^{5\times10^{-3}} (10 \times 10^{-3})[-4 \times 10^5 e^{-100\lambda}]\, d\lambda$$

$$= -80 + 8000\int_0^{10^{-3}} e^{-100\lambda}\, d\lambda - 4000\int_{10^{-3}}^{5\times10^{-3}} e^{-100\lambda}\, d\lambda$$

$$= -80 - 80(e^{-0.1} - 1) + 40(e^{-0.5} - e^{-0.1})$$

$$= 40e^{-0.5} - 120e^{-0.1} = -84.32\,\text{V}$$

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Alternate:

$$I_g = \int_0^{4\times10^{-3}} (10\times10^{-3})e^{-st}\,dt + \int_{4\times10^{-3}}^{6\times10^{-3}} (-20\times10^{-3})e^{-st}\,dt$$

$$= \left[\frac{10}{s} - \frac{30}{s}e^{-4\times10^{-3}s} + \frac{20}{s}e^{-6\times10^{-3}s}\right]\times10^{-3}$$

$$V_o = I_gH(s) = \frac{40}{s+100}[1 - 3e^{-4\times10^{-3}s} + 2e^{-6\times10^{-3}s}]$$

$$= \frac{40}{s+100} - \frac{120e^{-4\times10^{-3}s}}{s+100} + \frac{80e^{-6\times10^{-3}s}}{s+100}]$$

$$v_o(t) = 40e^{-100t} - 120e^{-100(t-4\times10^{-3})}u(t-4\times10^{-3})$$
$$+80e^{-100(t-6\times10^{-3})}u(t-6\times10^{-3})$$

$$v_o(5\times10^{-3}) = 40e^{-0.5} - 120e^{-0.1} + 80(0) = -84.32\,\text{V} \quad \text{(checks)}$$

P 13.74 $v_i = 25\,\sin 10\lambda\,[u(\lambda) - u(\lambda - \pi/10)]$

$$H(s) = \frac{32}{s+32}$$

$$h(\lambda) = 32e^{-32\lambda}$$

$$h(t-\lambda) = 32e^{-32(t-\lambda)} = 32e^{-32t}e^{32\lambda}$$

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$$v_o = 800e^{-32t}\int_0^t e^{32\lambda}\sin 10\lambda\, d\lambda$$

$$= 800e^{-32t}\left[\frac{e^{32\lambda}}{32^2+10^2}(32\sin 10\lambda - 10\cos 10\lambda\,\Big|_0^t\right]$$

$$= \frac{800e^{-32t}}{1124}[e^{32t}(32\sin 10t - 10\cos 10t) + 10]$$

$$= \frac{800}{1124}[32\sin 10t - 10\cos 10t + 10e^{-32t}]$$

$$v_o(0.075) = 10.96\,\text{V}$$

P 13.75 **[a]** $Y(s) = \int_0^\infty y(t)e^{-st}\,dt$

$$Y(s) = \int_0^\infty e^{-st}\left[\int_0^\infty h(\lambda)x(t-\lambda)\,d\lambda\right]dt$$

$$= \int_0^\infty\int_0^\infty e^{-st}h(\lambda)x(t-\lambda)\,d\lambda\,dt$$

$$= \int_0^\infty h(\lambda)\int_0^\infty e^{-st}x(t-\lambda)\,dt\,d\lambda$$

But $x(t-\lambda) = 0$ when $t < \lambda$.

Therefore $Y(s) = \int_0^\infty h(\lambda)\int_\lambda^\infty e^{-st}x(t-\lambda)\,dt\,d\lambda$

Let $u = t-\lambda$; $du = dt$; $u = 0$ when $t = \lambda$; $u = \infty$ when $t = \infty$.

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$$Y(s) = \int_0^\infty h(\lambda) \int_0^\infty e^{-s(u+\lambda)} x(u)\, du\, d\lambda$$

$$= \int_0^\infty h(\lambda) e^{-s\lambda} \int_0^\infty e^{-su} x(u)\, du\, d\lambda$$

$$= \int_0^\infty h(\lambda) e^{-s\lambda} X(s)\, d\lambda = H(s)\, X(s)$$

Note on $\quad x(t-\lambda) = 0, \qquad t < \lambda$

We are using one-sided Laplace transforms; therefore $h(t)$ and $x(t)$ are assumed zero for $t < 0$.

[b] $F(s) = \dfrac{a}{s(s+a)^2} = \dfrac{1}{s} \cdot \dfrac{a}{(s+a)^2} = H(s)X(s)$

$\therefore \quad h(t) = u(t), \qquad x(t) = at\, e^{-at} u(t)$

$$\therefore \quad f(t) = \int_0^t (1) a\lambda e^{-a\lambda}\, d\lambda = a \left[\frac{e^{-a\lambda}}{a^2}(-a\lambda - 1) \right]_0^t$$

$$= \frac{1}{a}[e^{-at}(-at-1) - 1(-1)] = \frac{1}{a}[1 - e^{-at} - ate^{-at}]$$

$$= \left[\frac{1}{a} - \frac{1}{a} e^{-at} - te^{-at} \right] u(t)$$

Check:

$$F(s) = \frac{a}{s(s+a)^2} = \frac{K_0}{s} + \frac{K_1}{(s+a)^2} + \frac{K_2}{s+a}$$

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$$K_0 = \frac{1}{a}; \qquad K_1 = -1; \qquad K_2 = \frac{d}{ds}\left(\frac{a}{s}\right)_{s=-a} = -\frac{1}{a}$$

$$f(t) = \left[\frac{1}{a} - te^{-at} - \frac{1}{a}e^{-at}\right]u(t)$$

P 13.76 **[a]** $H(s) = \dfrac{-Z_f}{Z_i}$

$$Z_f = \frac{(1/C_f)}{s + (1/R_fC_f)} = \frac{10^7}{s + 5000}$$

$$Z_i = \frac{R_i[s + (1/R_iC_i)]}{s} = \frac{200(s + 2000)}{s}$$

$$H(s) = \frac{-50{,}000s}{(s + 2000)(s + 5000)}$$

[b] $H(j3000) = \dfrac{-50{,}000(j3000)}{(2000 + j3000)(5000 + j3000)} = 7.135\underline{/-177.27^\circ}$

$$v_o(t) = 7.135\cos(3000t - 177.27^\circ)\ \text{V}$$

P 13.77 **[a]**

$$V_p = \frac{0.01s}{80 + 0.01s}V_g = \frac{s}{s + 8000}V_g$$

$$V_n = V_p$$

$$\frac{V_n}{5000} + \frac{V_n - V_o}{25{,}000} + (V_n - V_o)8 \times 10^{-9}s = 0$$

$$5V_n + V_n - V_o + (V_n - V_o)2 \times 10^{-4}s = 0$$

$$6V_n + 2 \times 10^{-4}sV_n = V_o + 2 \times 10^{-4}sV_o$$

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$$2 \times 10^{-4} V_n(s + 30{,}000) = 2 \times 10^{-4} V_o(s + 5000)$$

$$V_o = \frac{s + 30{,}000}{s + 5000} V_i = \left(\frac{s + 30{,}000}{s + 5000}\right)\left(\frac{sV_g}{s + 8000}\right)$$

$$H(s) = \frac{V_o}{V_g} = \frac{s(s + 30{,}000)}{(s + 5000)(s + 8000)}$$

[b] $v_g = 0.6u(t); \qquad V_g = \dfrac{0.6}{s}$

$$V_o = \frac{0.6(s + 30{,}000)}{(s + 5000)(s + 8000)} = \frac{K_1}{s + 5000} + \frac{K_2}{s + 8000}$$

$$K_1 = \frac{0.6(25{,}000)}{3000} = 5; \qquad K_2 = \frac{0.6(22{,}000)}{-3000} = -4.4$$

$$\therefore \quad v_o(t) = (5e^{-5000t} - 4.4e^{-8000t})u(t)\ \text{V}$$

[c] $V_g = 2\cos 10{,}000t\ \text{V}$

$$H(j\omega) = \frac{j10{,}000(30{,}000 + j10{,}000)}{(5000 + j10{,}000)(8000 + j10{,}000)} = 2.21\underline{/-6.34^\circ}$$

$$\therefore \qquad v_o = 4.42\cos(10{,}000t - 6.34^\circ)\ \text{V}$$

P 13.78 $H(j20) = \dfrac{25(8 + j20)}{-400 + j1200 + 150} = 0.44\underline{/-33.57^\circ}$

$$\therefore \quad v_o(t) = 4.4\cos(20t - 33.57^\circ)\ \text{V}$$

P 13.79 $H(j500) = \dfrac{125(400 + j500)}{(j500)(10^4 - 500^2 + j10^5)} = 0.6157 \times 10^{-3}\underline{/163.96^\circ}$

$$\therefore \quad i_o(t) = 49.3\cos(500t + 163.96^\circ)\ \text{mA}$$

P 13.80 $V_o = \dfrac{50}{s + 8000} - \dfrac{20}{s + 5000} = \dfrac{30(s + 3000)}{(s + 5000)(s + 8000)}$

$$V_o = H(s)V_g = H(s)\left(\frac{30}{s}\right)$$

$$\therefore \quad H(s) = \frac{s(s + 3000)}{(s + 5000)(s + 8000)}$$

$$H(j6000) = \frac{(j6000)(3000 + j6000)}{(5000 + j6000)(8000 + j6000)} = 0.52\underline{/66.37^\circ}$$

$$\therefore \quad v_o(t) = 61.84\cos(6000t + 66.37^\circ)\ \text{V}$$

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P 13.81 Original charge on C_1; $q_1 = V_0C_1$

The charge transferred to C_2; $q_2 = V_0C_e = \dfrac{V_0C_1C_2}{C_1 + C_2}$

The charge remaining on C_1; $q_1' = q_1 - q_2 = \dfrac{V_0C_1^2}{C_1 + C_2}$

Therefore $V_2 = \dfrac{q_2}{C_2} = \dfrac{V_0C_1}{C_1 + C_2}$ and $V_1 = \dfrac{q_1'}{C_1} = \dfrac{V_0C_1}{C_1 + C_2}$

P 13.82 **[a]** After making a source transformation, the circuit is as shown. The impulse current will pass through the capacitive branch since it appears as a short circuit to the impulsive current,

Therefore $v_o(0^+) = 10^6 \displaystyle\int_{0^-}^{0^+} \left[\frac{\delta(t)}{1000}\right] dt = 1000\,\text{V}$

Therefore $w_C = (0.5)Cv^2 = 0.5\,\text{J}$

[b] $i_\text{L}(0^+) = 0$; therefore $w_\text{L} = 0\,\text{J}$

[c] $V_o(10^{-6})s + \dfrac{V_o}{250 + 0.05s} + \dfrac{V_o}{1000} = 10^{-3}$

Therefore

$$V_o = \frac{1000(s + 5000)}{s^2 + 6000s + 25 \times 10^6}$$

$$= \frac{K_1}{s + 3000 - j4000} + \frac{K_1^*}{s + 3000 + j4000}$$

$K_1 = 559.02\underline{/-26.57^\circ}$; $K_1^* = 559.02\underline{/26.57^\circ}$

$v_o = [1118.03e^{-3000t}\cos(4000t - 26.57^\circ)]u(t)\,\text{V}$

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[d] The s-domain circuit is

$$\frac{V_o s}{10^6} + \frac{V_o}{250 + 0.05s} + \frac{V_o}{1000} = 10^{-3}$$

Note that this equation is identical to that derived in part [c], therefore the solution for V_o will be the same.

P 13.83 **[a]**

$$V_o = \frac{20}{1000 + 0.4s} \cdot 0.32s$$

$$= \frac{16s}{s + 2500} = 16 - \frac{40{,}000}{s + 2500}$$

$$v_o(t) = 16\delta(t) - 40{,}000e^{-2500t}u(t)\,\text{V}$$

[b] At $t = 0$ the voltage impulse establishes a current in the inductors; thus

$$i_L(0) = \frac{10^3}{400}\int_{0^-}^{0^+} 20\delta(t)\,dt = 50\,\text{A}$$

It follows that since $i_L(0^-) = 0$,

$$\frac{di_L}{dt}(0) = 50\delta(t)$$

$$\therefore \quad v_o(0) = (0.32)(50\delta(t)) = 16\delta(t)$$

This agrees with our solution.

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At $t = 0^+$ our circuit is

$$\therefore \quad i_L(t) = 50e^{-t/\tau}\,\text{A}, \qquad t \geq 0^+$$

$$\tau = L/R = 0.4\,\text{ms}$$

$$\therefore \quad i_L(t) = 50e^{-2500t}\,\text{A}, \qquad t \geq 0^+$$

$$v_o(t) = 0.32\frac{di_L}{dt} = -40{,}000e^{-2500t}\,\text{V}, \qquad t \geq 0^+$$

which agrees with our solution.

P 13.84 **[a]** The s-domain circuit is

The node-voltage equation is $\dfrac{V}{sL_1} + \dfrac{V}{R} + \dfrac{V}{sL_2} = \dfrac{\rho}{s}$

Therefore $V = \dfrac{\rho R}{s + (R/L_e)}$ where $L_e = \dfrac{L_1 L_2}{L_1 + L_2}$

Therefore $v = \rho R e^{-(R/L_e)t} u(t)\,\text{V}$

[b] $I_1 = \dfrac{V}{R} + \dfrac{V}{sL_2} = \dfrac{\rho[s + (R/L_2)]}{s[s + (R/L_e)]} = \dfrac{K_0}{s} + \dfrac{K_1}{s + (R/L_e)}$

$$K_0 = \frac{\rho L_1}{L_1 + L_2}; \qquad K_1 = \frac{\rho L_2}{L_1 + L_2}$$

Thus we have $i_1 = \dfrac{\rho}{L_1 + L_2}[L_1 + L_2 e^{-(R/L_e)t}]u(t)\,\text{A}$

[c] $I_2 = \dfrac{V}{sL_2} = \dfrac{(\rho R/L_2)}{s[s + (R/L_e)]} = \dfrac{K_2}{s} + \dfrac{K_3}{s + (R/L_e)}$

$$K_2 = \frac{\rho L_1}{L_1 + L_2}; \qquad K_3 = \frac{-\rho L_1}{L_1 + L_2}$$

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Therefore $\quad i_2 = \dfrac{\rho L_1}{L_1 + L_2}[1 - e^{-(R/L_e)t}]u(t)$

[d] $\lambda(t) = L_1 i_1 + L_2 i_2 = \rho L_1$

P 13.85 **[a]** As $R \to \infty$, $v(t) \to \rho L_e \delta(t)$ since the area under the impulse generating function is ρL_e.

$$i_1(t) \to \frac{\rho L_1}{L_1 + L_2} u(t)\,\text{A} \quad \text{as} \quad R \to \infty$$

$$i_2(t) \to \frac{\rho L_1}{L_1 + L_2} u(t)\,\text{A} \quad \text{as} \quad R \to \infty$$

[b] The s-domain circuit is

$$\frac{V}{sL_1} + \frac{V}{sL_2} = \frac{\rho}{s}; \qquad \text{therefore} \quad V = \frac{\rho L_1 L_2}{L_1 + L_2} = \rho L_e$$

Therefore $\quad v(t) = \rho L_e \delta(t)$

$$I_1 = I_2 = \frac{V}{sL_2} = \left(\frac{\rho L_1}{L_1 + L_2}\right)\left(\frac{1}{s}\right)$$

Therefore $\quad i_1 = i_2 = \dfrac{\rho L_1}{L_1 + L_2} u(t)\,\text{A}$

P 13.86 **[a]** $Z_1 = \dfrac{1/C_1}{s + 1/R_1 C_1} = \dfrac{25 \times 10^{10}}{s + 20 \times 10^4}\Omega$

$$Z_2 = \frac{1/C_2}{s + 1/R_2 C_2} = \frac{6.25 \times 10^{10}}{s + 12{,}500}\Omega$$

$$\frac{V_0}{Z_2} + \frac{V_0 - 10/s}{Z_1} = 0$$

$$\frac{V_0(s + 12{,}500)}{6.25 \times 10^{10}} + \frac{V_0(s + 20 \times 10^4)}{25 \times 10^{10}} = \frac{10}{s}\,\frac{(s + 20 \times 10^4)}{25 \times 10^{10}}$$

$$V_0 = \frac{2(s + 200{,}000)}{s(s + 50{,}000)} = \frac{K_1}{s} + \frac{K_2}{s + 50{,}000}$$

$$K_1 = \frac{2(200{,}000)}{50{,}000} = 8$$

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$$K_2 = \frac{2(150{,}000)}{-50{,}000} = -6$$

$$\therefore \quad v_o = [8 - 6e^{-50{,}000t}]u(t)\,\text{V}$$

[b] $$I_0 = \frac{V_0}{Z_2} = \frac{2(s+200{,}000)(s+12{,}500)}{s(s+50{,}000)6.25\times 10^{10}}$$

$$= 32\times 10^{-12}\left[1 + \frac{162{,}500s + 25\times 10^8}{s(s+50{,}000)}\right]$$

$$= 32\times 10^{-12}\left[1 + \frac{K_1}{s} + \frac{K_2}{s+50{,}000}\right]$$

$$K_1 = 50{,}000; \qquad K_2 = 112{,}500$$

$$i_o = 32\delta(t) + [1.6\times 10^6 + 3.6\times 10^6 e^{-50{,}000t}]u(t)\ \text{pA}$$

[c] When $C_1 = 64\,\text{pF}$

$$Z_1 = \frac{156.25\times 10^8}{s+12{,}500}\Omega$$

$$\frac{V_0(s+12{,}500)}{625\times 10^8} + \frac{V_0(s+12{,}500)}{156.25\times 10^8} = \frac{10}{s}\,\frac{(s+12{,}500)}{156.25\times 10^8}$$

$$\therefore \quad V_0 + 4V_0 = \frac{40}{s}$$

$$V_0 = \frac{8}{s}$$

$$v_o = 8u(t)\,\text{V}$$

$$I_0 = \frac{V_0}{Z_2} = \frac{8}{s}\,\frac{(s+12{,}500)}{6.25\times 10^{10}} = 128\times 10^{-12}\left[1 + \frac{12{,}500}{s}\right]$$

$$i_o(t) = 128\delta(t) + 1.6\times 10^6 u(t)\,\text{pA}$$

P 13.87 Let $a = \dfrac{1}{R_1C_1} = \dfrac{1}{R_2C_2}$

Then $Z_1 = \dfrac{1}{C_1(s+a)}$ and $Z_2 = \dfrac{1}{C_2(s+a)}$

$$\frac{V_o}{Z_2} + \frac{V_o}{Z_1} = \frac{10/s}{Z_1}$$

$$V_oC_2(s+a) + V_0C_1(s+a) = (10/s)C_1(s+a)$$

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$$V_o = \frac{10}{s}\left(\frac{C_1}{C_1 + C_2}\right)$$

Thus, v_o is the input scaled by the factor $\dfrac{C_1}{C_1 + C_2}$.

P 13.88 **[a]** For $t < 0$:

$$R_{\text{eq}} = 0.8\,\text{k}\Omega \| 4\,\text{k}\Omega \| 16\,\text{k}\Omega = 0.64\,\text{k}\Omega; \qquad v = 5(640) = 3200\,\text{V}$$

$$i_1(0^-) = \frac{3200}{4000} = 0.8\,\text{A}; \qquad i_2(0^-) = \frac{3200}{1600} = 0.2\,\text{A}$$

[b] For $t > 0$:

$$i_1 + i_2 = 0$$

$$8(\Delta i_1) = 2(\Delta i_2)$$

$$i_1(0^-) + \Delta i_1 + i_2(0^-) + \Delta i_2 = 0; \qquad \text{therefore} \quad \Delta i_1 = -0.2\,\text{A}$$

$$\Delta i_2 = -0.8\,\text{A}; \qquad i_1(0^+) = 0.8 - 0.2 = 0.6\,\text{A}$$

[c] $i_2(0^-) = 0.2\,\text{A}$

[d] $i_2(0^+) = 0.2 - 0.8 = -0.6\,\text{A}$

[e] The s-domain equivalent circuit for $t > 0$ is

$$I_1 = \frac{0.006}{0.01s + 20{,}000} = \frac{0.6}{s + 2 \times 10^6}$$

$$i_1(t) = 0.6e^{-2\times 10^6 t}u(t)\,\text{A}$$

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[f] $i_2(t) = -i_1(t) = -0.6e^{-2\times10^6 t}u(t)\,\text{A}$

[g] $V = -0.0064 + (0.008s + 4000)I_1 = \dfrac{-0.0016(s + 6.5\times10^6)}{s + 2\times10^6}$

$$= -1.6\times10^{-3} - \frac{7200}{s + 2\times10^6}$$

$$v(t) = [-1.6\times10^{-3}\delta(t)] - [7200e^{-2\times10^6 t}u(t)]\,\text{V}$$

P 13.89 **[a]**

$$20 = sI_1 - 0.5sI_2$$

$$0 = -0.5sI_1 + \left(s + \frac{3}{s}\right)I_2$$

$$\Delta = \begin{vmatrix} s & -0.5s \\ -0.5s & (s + 3/s) \end{vmatrix} = s^2 + 3 - 0.25s^2 = 0.75(s^2 + 4)$$

$$N_1 = \begin{vmatrix} 20 & -0.5s \\ 0 & (s + 3/s) \end{vmatrix} = 20s + \frac{60}{s} = \frac{20s^2 + 60}{s} = \frac{20(s^2 + 3)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{20(s^2 + 3)}{s(0.75)(s^2 + 4)} = \frac{80}{3}\cdot\frac{s^2 + 3}{s(s^2 + 4)}$$

$$= \frac{K_0}{s} + \frac{K_1}{s - j2} + \frac{K_1^*}{s + j2}$$

$$K_0 = \frac{80}{3}\left(\frac{3}{4}\right) = 20; \qquad K_1 = \frac{80}{3}\left[\frac{-4 + 3}{(j2)(j4)}\right] = \frac{10}{3}\underline{/0^\circ}$$

$$\therefore\quad i_1 = \left[20 + \frac{20}{3}\cos 2t\right]u(t)\,\text{A}$$

[b] $N_2 = \begin{vmatrix} s & 20 \\ -0.5s & 0 \end{vmatrix} = 10s$

$$I_2 = \frac{N_2}{\Delta} = \frac{10s}{0.75(s^2 + 4)} = \frac{40}{3}\left(\frac{s}{s^2 + 4}\right) = \frac{K_1}{s - j2} + \frac{K_1^*}{s + j2}$$

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$$K_1 = \frac{40}{3}\left(\frac{j2}{j4}\right) = \frac{20}{3}\underline{/0^\circ}$$

$$i_2 = \left[\frac{40}{3}\cos 2t\right]u(t)\,\text{A}$$

[c] $$V_0 = \frac{3}{s}I_2 = \left(\frac{3}{s}\right)\frac{40}{3}\left(\frac{s}{s^2+4}\right) = \frac{40}{s^2+4} = \frac{K_1}{s-j2} + \frac{K_1^*}{s+j2}$$

$$K_1 = \frac{40}{j4} = -j10 = 10\underline{/90^\circ}$$

$$v_o = 20\cos(2t - 90^\circ) = 20\sin 2t$$

$$v_o = [20\sin 2t]u(t)\,\text{V}$$

[d] Let us begin by noting i_1 jumps from 0 to $(80/3)$ A between 0^- and 0^+ and in this same interval i_2 jumps from 0 to $(40/3)$ A. Therefore in the derivatives of i_1 and i_2 there will be impulses of $(80/3)\delta(t)$ and $(40/3)\delta(t)$, respectively. Thus

$$\frac{di_1}{dt} = \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t\ \text{A/s}$$

$$\frac{di_2}{dt} = \frac{40}{3}\delta(t) - \frac{80}{3}\sin 2t\ \text{A/s}$$

From the circuit diagram we have

$$\begin{aligned} 20\delta(t) &= 1\frac{di_1}{dt} - 0.5\frac{di_2}{dt} \\ &= \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t - \frac{20\delta(t)}{3} + \frac{40}{3}\sin 2t \\ &= 20\delta(t) \end{aligned}$$

Thus our solutions for i_1 and i_2 are in agreement with known circuit behavior.

Let us also note the impulsive voltage will impart energy into the circuit. Since there is no resistance in the circuit, the energy will not dissipate. Thus the fact that i_1, i_2, and v_o exist for all time is consistent with known circuit behavior.

Also note that although i_1 has a dc component, i_2 does not. This follows from known transformer behavior.

Finally we note the flux linkage prior to the appearance of the impulsive voltage is zero. Now since $v = d\lambda/dt$, the impulsive voltage source must be matched to an instantaneous change in flux linkage at $t = 0^+$ of 20. For the given polarity dots and reference directions of i_1 and i_2 we have

$$\lambda(0^+) = L_1 i_1(0^+) + M i_1(0^+) - L_2 i_2(0^+) - M i_2(0^+)$$

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$$\lambda(0^+) = 1\left(\frac{80}{3}\right) + 0.5\left(\frac{80}{3}\right) - 1\left(\frac{40}{3}\right) - 0.5\left(\frac{40}{3}\right)$$

$$= \frac{120}{3} - \frac{60}{3} = 20 \quad \text{(checks)}$$

P 13.90 **[a]** For $t < 0$, $0.5v_1 = 2v_2$; therefore $v_1 = 4v_2$

$v_1 + v_2 = 100$; therefore $v_1(0^-) = 80\,\text{V}$

[b] $v_2(0^-) = 20\,\text{V}$

[c] $v_3(0^-) = 0\,\text{V}$

[d] For $t > 0$:

$$I = \frac{100/s}{3.125/s} \times 10^{-6} = 32 \times 10^{-6}$$

$$i(t) = 32\delta(t)\,\mu\text{A}$$

[e] $v_1(0^+) = -\dfrac{10^6}{0.5}\displaystyle\int_{0^-}^{0^+} 32 \times 10^{-6}\delta(t)\,dt + 80 = -64 + 80 = 16\,\text{V}$

[f] $v_2(0^+) = -\dfrac{10^6}{2}\displaystyle\int_{0^-}^{0^+} 32 \times 10^{-6}\delta(t)\,dt + 20 = -16 + 20 = 4\,\text{V}$

[g] $V_3 = \dfrac{0.625 \times 10^6}{s} \cdot 32 \times 10^{-6} = \dfrac{20}{s}$

$v_3(t) = 20u(t)\,\text{V}$; $v_3(0^+) = 20\,\text{V}$

Check: $v_1(0^+) + v_2(0^+) = v_3(0^+)$

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P 13.91 **[a]**

$$V_o = \frac{0.5}{50{,}000 + 5 \times 10^6/s} \cdot \frac{10^6}{s}$$

$$\frac{500{,}000}{50{,}000s + 5 \times 10^6} = \frac{10}{s + 100}$$

$$v_o = 10e^{-100t}u(t)\,\text{V}$$

[b] At $t = 0$ the current in the $1\,\mu$F capacitor is $10\delta(t)\,\mu$A

$$\therefore \qquad v_o(0^+) = 10^6 \int_{0^-}^{0^+} 10 \times 10^{-6}\delta(t)\,dt = 10\,\text{V}$$

After the impulsive current has charged the $1\,\mu$F capacitor to 10 V it discharges through the 50 kΩ resistor.

$$C_e = \frac{C_1C_2}{C_1 + C_2} = \frac{0.25}{1.25} = 0.2\,\mu\text{F}$$

$$\tau = (50{,}000)(0.2 \times 10^{-6}) = 10^{-2}$$

$$\frac{1}{\tau} = 100 \text{ (checks)}$$

Note – after the impulsive current passes the circuit becomes

The solution for v_o in this circuit is also

$$v_o = 10e^{-100t}u(t)\,\text{V}$$

P 13.92 **[a]** The circuit parameters are

$$R_\text{a} = \frac{120^2}{1200} = 12\,\Omega \qquad R_\text{b} = \frac{120^2}{1800} = 8\,\Omega \qquad X_\text{a} = \frac{120^2}{350} = \frac{288}{7}\,\Omega$$

The branch currents are

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{12} = 10\underline{/0^\circ}\text{ A(rms)} \qquad \mathbf{I}_2 = \frac{120\underline{/0^\circ}}{j1440/35} = -j\frac{35}{12} = \frac{35}{12}\underline{/-90^\circ}\text{ A(rms)}$$

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$$\mathbf{I}_3 = \frac{120\underline{/0^\circ}}{8} = 15\underline{/0^\circ}\ \text{A(rms)}$$

$$\therefore\quad \mathbf{I}_o = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 25 - j\frac{35}{12} = 25.17\underline{/-6.65^\circ}\ \text{A(rms)}$$

Therefore,

$$i_2 = \left(\frac{35}{12}\right)\sqrt{2}\cos(\omega t - 90^\circ)\,\text{A} \qquad \text{and} \qquad i_o = 25.17\sqrt{2}\cos(\omega t - 6.65^\circ)\,\text{A}$$

Thus,

$$i_2(0^-) = i_2(0^+) = 0\,\text{A} \qquad \text{and} \qquad i_o(0^-) = i_o(0^+) = 25\sqrt{2}\,\text{A}$$

[b] Begin by using the s-domain circuit in Fig. 13.60 to solve for V_0 symbolically. Write a single node voltage equation:

$$\frac{V_0 - (V_g + L_\ell I_o)}{sL_\ell} + \frac{V_0}{R_a} + \frac{V_0}{sL_a} = 0$$

$$\therefore\quad V_0 = \frac{(R_a/L_\ell)V_g + I_o R_a}{s + [R_a(L_a + L_\ell)]/L_a L_\ell}$$

where $L_\ell = 1/120\pi$ H, $L_a = 12/35\pi$ H, $R_a = 12\,\Omega$, and $I_0 R_a = 300\sqrt{2}$ V. Thus,

$$V_0 = \frac{1440\pi(122.92\sqrt{2}s - 3000\pi\sqrt{2})}{(s + 1475\pi)(s^2 + 14{,}400\pi^2)} + \frac{300\sqrt{2}}{s + 1475\pi}$$

$$= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi} + \frac{300\sqrt{2}}{s + 1475\pi}$$

The coefficients are

$$K_1 = -121.18\sqrt{2}\,\text{V} \qquad K_2 = 61.03\sqrt{2}\underline{/6.85^\circ}\,\text{V} \qquad K_2^* = 61.03\sqrt{2}\underline{/-6.85^\circ}$$

Note that $K_1 + 300\sqrt{2} = 178.82\sqrt{2}$ V. Thus, the inverse transform of V_0 is

$$v_0 = 178.82\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)\,\text{V}$$

Initially,

$$v_0(0^+) = 178.82\sqrt{2} + 122.06\sqrt{2}\cos 6.85^\circ = 300\sqrt{2}\,\text{V}$$

Note that at $t = 0^+$ the initial value of i_L, which is $25\sqrt{2}$ A, exists in the $12\,\Omega$ resistor R_a. Thus, the initial value of V_0 is $(25\sqrt{2})(12) = 300\sqrt{2}$ V.

[c] The phasor domain equivalent circuit has a $j1\,\Omega$ inductive impedance in series with the parallel combination of a $12\,\Omega$ resistive impedance and a $j1440/35\,\Omega$ inductive impedance (remember that $\omega = 120\pi$ rad/s). Note that $\mathbf{V}_g = 120\underline{/0^\circ} + (25.17\underline{/-6.65^\circ})(j1) = 125.43\underline{/11.50^\circ}$ V(rms). The node voltage equation in the phasor domain circuit is

$$\frac{\mathbf{V}_0 - 125.43\underline{/11.50^\circ}}{j1} + \frac{\mathbf{V}_0}{12} + \frac{35\mathbf{V}_0}{1440} = 0$$

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$\therefore \quad \mathbf{V}_0 = 122.06\underline{/6.85^\circ}$ V(rms)

Therefore, $v_0 = 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)$ V, agreeing with the steady-state component of the result in part (b).

[d] A plot of v_0, generated in Excel, is shown below.

P 13.93 **[a]** At $t = 0^-$ the phasor domain equivalent circuit is

$$\mathbf{I}_1 = \frac{-j120}{12} = -j10 = 10\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{-j120(35)}{j1440} = -\frac{35}{12} = \frac{35}{12}\underline{/180^\circ}\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{-j120}{8} = -j15 = 15\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = -\frac{35}{12} - j25 = 25.17\underline{/-96.65^\circ}\text{A (rms)}$$

$$i_L = 25.17\sqrt{2}\cos(120\pi t - 96.65^\circ)\text{A}$$

$$i_L(0^-) = i_L(0^+) = -2.92\sqrt{2}\text{A}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t + 180^\circ)\text{A}$$

$$i_2(0^-) = i_2(0^+) = -\frac{35}{12}\sqrt{2} = -2.92\sqrt{2}\text{A}$$

$$\mathbf{V}_g = \mathbf{V}_o + j1\mathbf{I}_L$$

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$$\begin{aligned}
\mathbf{V}_g &= -j120 + 25 - j\frac{35}{12} \\
&= 25 - j122.92 = 125.43\underline{/-78.50^\circ}\text{V (rms)} \\
v_g &= 125.43\sqrt{2}\cos(120\pi t - 78.50^\circ)\text{V} \\
&= 125.43\sqrt{2}[\cos 120\pi t \cos 78.50^\circ + \sin 120\pi t \sin 78.50^\circ] \\
&= 25\sqrt{2}\cos 120\pi t + 122.92\sqrt{2}\sin 120\pi t
\end{aligned}$$

$$\therefore \quad V_g = \frac{25\sqrt{2}s + 122.92\sqrt{2}(120\pi)}{s^2 + (120\pi)^2}$$

s-domain circuit:

where

$$L_l = \frac{1}{120\pi}\text{ H}; \qquad L_a = \frac{12}{35\pi}\text{ H}; \qquad R_a = 12\,\Omega$$

$$i_L(0) = -2.92\sqrt{2}\text{ A}; \qquad i_2(0) = -2.92\sqrt{2}\text{ A}$$

The node voltage equation is

$$0 = \frac{V_o - (V_g + i_L(0)L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + i_2(0)L_a}{sL_a}$$

Solving for V_o yields

$$V_o = \frac{V_g R_a/L_l}{[s + R_a(L_l + L_a)/L_a L_l]} + \frac{R_a[i_L(0) - i_2(0)]}{[s + R_a(L_l + L_a)/L_l L_a]}$$

$$\frac{R_a}{L_l} = 1440\pi$$

$$\frac{R_a(L_l + L_a)}{L_l L_a} = \frac{12(\frac{1}{120\pi} + \frac{12}{35\pi})}{(\frac{12}{35\pi})(\frac{1}{120\pi})} = 1475\pi$$

$$i_L(0) - i_2(0) = -2.92\sqrt{2} + 2.92\sqrt{2} = 0$$

$$\begin{aligned}
\therefore \quad V_o &= \frac{1440\pi[25\sqrt{2}s + 122.92\sqrt{2}(120\pi)]}{(s + 1475\pi)[s^2 + (120\pi)^2]} \\
&= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}
\end{aligned}$$

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$K_1 = -14.55\sqrt{2}$ $\qquad$ $K_2 = 61.03\sqrt{2}\underline{/-83.15^\circ}$

$\therefore \quad v_o(t) = -14.55\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t - 83.15^\circ)\text{V}$

Check:

$v_o(0) = (-14.55 + 14.55)\sqrt{2} = 0$

[b]

PSpice schematic

PSpice output file

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**** 07/15/01 07:40:45 ************** PSpice Lite (Mar 2000) *****************

 ** Profile: "SCHEMATIC1-tran"  [ C:\shortcircuits\solutions\p9_76-SCHEMATIC1-tran.sim ]

 ****     CIRCUIT DESCRIPTION

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** Creating circuit file "p9_76-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

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* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
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.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_76-SCHEMATIC1.net"



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L_L2         0 VO  109.1348mH IC=0
R_R1         0 VO  12
R_R2         VO N01311  0.0001
R_R3         0 N01959  8
X_U2         VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg

**** RESUMING p9_76-SCHEMATIC1-tran.sim.cir ****
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[**c**] In Problem 13.92, the line-to-neutral voltage spikes at $300\sqrt{2}$ V. Here the line-to-neutral voltage has no spike. Thus the amount of voltage disturbance depends on what part of the cycle the sinusoidal steady-state voltage is switched.

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P 13.94 **[a]** First find V_g before R_b is disconnected. The phasor domain circuit is

$$\mathbf{I}_L = \frac{120\underline{/\theta^\circ}}{R_a} + \frac{120\underline{/\theta^\circ}}{R_b} + \frac{120\underline{/\theta^\circ}}{jX_a}$$
$$= \frac{120\underline{/\theta^\circ}}{R_aR_bX_a}[(R_a+R_b)X_a = jR_aR_b]$$

Since $X_l = 1\,\Omega$ we have

$$\mathbf{V}_g = 120\underline{/\theta^\circ} + \frac{120\underline{/\theta^\circ}}{R_aR_bX_a}[R_aR_b + j(R_a+R_b)X_a]$$

$$R_a = 12\,\Omega; \qquad R_b = 8\,\Omega; \qquad X_a = \frac{1440}{35}\,\Omega$$

$$\mathbf{V}_g = \frac{120\underline{/\theta^\circ}}{1400}(1475 + j300)$$
$$= \frac{25}{12}\underline{/\theta^\circ}(59 + j12) = 125.43\underline{/(\theta + 11.50)^\circ}$$

$$v_g = 125.43\sqrt{2}\cos(120\pi t + \theta + 11.50^\circ)\text{V}$$

Let $\beta = \theta + 11.50^\circ$. Then

$$v_g = 125.43\sqrt{2}(\cos 120\pi t\cos\beta - \sin 120\pi t\sin\beta)\text{V}$$

Therefore

$$V_g = \frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + (120\pi)^2}$$

The s-domain circuit becomes

where $\rho_1 = i_L(0^+)$ and $\rho_2 = i_2(0^+)$.
The s-domain node voltage equation is

$$\frac{V_o - (V_g + \rho_1 L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + \rho_2 L_a}{sL_a} = 0$$

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Solving for V_o yields

$$V_o = \frac{V_g R_a/L_l + (\rho_1 - \rho_2)R_a}{[s + \frac{(L_a+L_l)R_a}{L_a L_l}]}$$

Substituting the numerical values

$$L_l = \frac{1}{120\pi}\ \text{H}; \qquad L_a = \frac{12}{35\pi}\ \text{H}; \qquad R_a = 12\,\Omega; \qquad R_b = 8\,\Omega;$$

gives

$$V_o = \frac{1440\pi V_g + 12(\rho_1 - \rho_2)}{(s + 1475\pi)}$$

Now determine the values of ρ_1 and ρ_2.

$$\rho_1 = i_L(0^+) \quad \text{and} \quad \rho_2 = i_2(0^+)$$

$$\mathbf{I}_L = \frac{120\underline{/\theta^\circ}}{R_a R_b X_a}[(R_a + R_b)X_a - jR_a R_b]$$

$$= \frac{120\underline{/\theta^\circ}}{96(1440/35)}\left[\frac{(20)(1440)}{35} - j96\right]$$

$$= 25.17\underline{/(\theta - 6.65)^\circ}\text{A(rms)}$$

$$\therefore \quad i_L = 25.17\sqrt{2}\cos(120\pi t + \theta - 6.65^\circ)\text{A}$$

$$i_L(0^+) = \rho_1 = 25.17\sqrt{2}\cos(\theta - 6.65^\circ)\text{A}$$

$$\therefore \quad \rho_1 = 25\sqrt{2}\cos\theta + 2.92\sqrt{2}\sin\theta\text{A}$$

$$\mathbf{I}_2 = \frac{120\underline{/\theta^\circ}}{j(1440/35)} = \frac{35}{12}\underline{/(\theta - 90)^\circ}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t + \theta - 90^\circ)\text{A}$$

$$\rho_2 = i_2(0^+) = \frac{35}{12}\sqrt{2}\sin\theta = 2.92\sqrt{2}\sin\theta\text{A}$$

$$\therefore \quad \rho_1 = \rho_2 = 25\sqrt{2}\cos\theta$$

$$(\rho_1 - \rho_2)R_a = 300\sqrt{2}\cos\theta$$

$$\therefore \quad \text{V}_o = \frac{1440\pi}{s + 1475\pi}\cdot V_g + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{1440\pi}{s + 1475\pi}\left[\frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + 14{,}400\pi^2}\right] + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{K_1 + 300\sqrt{2}\cos\theta}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}$$

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Now

$$K_1 = \frac{(1440\pi)(125.43\sqrt{2})[-1475\pi\cos\beta - 120\pi\sin\beta]}{1475^2\pi^2 + 14{,}400\pi^2}$$

$$= \frac{-1440(125.43\sqrt{2})[1475\cos\beta + 120\sin\beta]}{1475^2 + 14{,}000}$$

Since $\beta = \theta + 11.50°$, K_1 reduces to

$$K_1 = -121.18\sqrt{2}\cos\theta + 14.55\sqrt{2}\sin\theta$$

From the partial fraction expansion for V_o we see $v_o(t)$ will go directly into steady state when $K_1 = -300\sqrt{2}\cos\theta$. It follow that

$$14.55\sqrt{2}\sin\theta = -178.82\sqrt{2}\cos\theta$$

or $\qquad \tan\theta = -12.29$

Therefore, $\qquad \theta = -85.35°$

[b] When $\theta = -85.35°$, $\beta = -73.85°$

$$\therefore \quad K_2 = \frac{1440\pi(125.43\sqrt{2})[-120\pi\sin(-73.85°) + j120\pi\cos(-73.85°)}{(1475\pi + j120\pi)(j240\pi)}$$

$$= \frac{720\sqrt{2}(120.48 + j34.88)}{-120 + j1475}$$

$$= 61.03\sqrt{2}\underline{/-78.50°}$$

$$\therefore \quad v_o = 122.06\sqrt{2}\cos(120\pi t - 78.50°)\,\text{V} \quad t > 0$$

$$= 172.61\cos(120\pi t - 78.50°)\,\text{V} \quad t > 0$$

[c] $v_{o1} = 169.71\cos(120\pi t - 85.35°)\text{V} \qquad t < 0$

$v_{o2} = 172.61\cos(120\pi t - 78.50°)\text{V} \qquad t > 0$

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PSpice schematic

PSpice output file

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* Local Libraries :
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L_L2          0 VO  109.1348mH IC=0
R_R1          0 VO  {Rb}
R_R2          VO N01311  0.0001
R_R3          0 N01959  8
X_U2          VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg
.PARAM  Rb=12

**** RESUMING p9_77-SCHEMATIC1-tran.sim.cir ****
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14

Introduction to Frequency-Selective Circuits

Assessment Problems

AP 14.1

$$f_c = 8\,\text{kHz}, \quad \omega_c = 2\pi f_c = 16\pi\,\text{krad/s}$$

$$\omega_c = \frac{1}{RC}; \qquad R = 10\,\text{k}\Omega;$$

$$\therefore \quad C = \frac{1}{\omega_c R} = \frac{1}{(16\pi \times 10^3)(10^4)} = 1.99\,\text{nF}$$

AP 14.2 **[a]** $\omega_c = 2\pi f_c = 2\pi(2000) = 4\pi\,\text{krad/s}$

$$L = \frac{R}{\omega_c} = \frac{5000}{4000\pi} = 0.40\,\text{H}$$

[b] $H(j\omega) = \dfrac{\omega_c}{\omega_c + j\omega} = \dfrac{4000\pi}{4000\pi + j\omega}$

When $\omega = 2\pi f = 2\pi(50{,}000) = 100{,}000\pi\,\text{rad/s}$

$$H(j100{,}000\pi) = \frac{4000\pi}{4000\pi + j100{,}000\pi} = \frac{1}{1 + j25} = 0.04\underline{/87.71^\circ}$$

$$\therefore \quad |H(j100{,}000\pi)| = 0.04$$

[c] $\therefore \quad \theta(100{,}000\pi) = -87.71^\circ$

AP 14.3

$$\omega_c = \frac{R}{L} = \frac{5000}{3.5 \times 10^{-3}} = 1.43\,\text{Mrad/s}$$

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AP 14.4 **[a]** $\omega_c = \dfrac{1}{RC} = \dfrac{10^6}{R} = \dfrac{10^6}{100} = 10\,\text{krad/s}$

[b] $\omega_c = \dfrac{10^6}{5000} = 200\,\text{rad/s}$

[c] $\omega_c = \dfrac{10^6}{3 \times 10^4} = 33.33\,\text{rad/s}$

AP 14.5 Let Z represent the parallel combination of $(1/SC)$ and R_L. Then

$$Z = \frac{R_L}{(R_L C s + 1)}$$

$$\text{Thus} \quad H(s) = \frac{Z}{R+Z} = \frac{R_L}{R(R_L C s + 1) + R_L}$$

$$= \frac{(1/RC)}{s + \frac{R+R_L}{R_L}\left(\frac{1}{RC}\right)} = \frac{(1/RC)}{s + \frac{1}{K}\left(\frac{1}{RC}\right)}$$

$$\text{where} \quad K = \frac{R_L}{R + R_L}$$

AP 14.6

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(24\pi \times 10^3)^2 (0.1 \times 10^{-6})} = 1.76\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{R/L} \quad \text{so} \quad R = \frac{\omega_o L}{Q} = \frac{(24\pi \times 10^3)(1.76 \times 10^{-3})}{6} = 22.10\,\Omega$$

AP 14.7

$$\omega_o = 2\pi(2000) = 4000\pi\,\text{rad/s};$$

$$\beta = 2\pi(500) = 1000\pi\,\text{rad/s}; \qquad R = 250\,\Omega$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad C = \frac{1}{\beta R} = \frac{1}{(1000\pi)(250)} = 1.27\,\mu\text{F}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(4000\pi)^2 (1.27)} = 4.97\,\text{mH}$$

AP 14.8

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(10^4\pi)^2 (0.2 \times 10^{-6})} = 5.07\,\text{mH}$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad R = \frac{1}{\beta C} = \frac{1}{400\pi(0.2 \times 10^{-6})} = 3.98\,\text{k}\Omega$$

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AP 14.9

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(400\pi)^2(0.2 \times 10^{-6})} = 31.66\,\text{mH}$$

$$Q = \frac{f_o}{\beta} = \frac{5 \times 10^3}{200} = 25 = \omega_o RC$$

$$\therefore \quad R = \frac{Q}{\omega_o C} = \frac{25}{(400\pi)(0.2 \times 10^{-6})} = 9.95\,\text{k}\Omega$$

AP 14.10

$$\omega_o = 8000\pi\,\text{rad/s}$$

$$C = 500\,\text{nF}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = 3.17\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$\therefore \quad R = \frac{1}{\omega_o CQ} = \frac{1}{(8000\pi)(500)(5 \times 10^{-9})} = 15.92\,\Omega$$

AP 14.11

$$\omega_o = 2\pi f_o = 2\pi(20{,}000) = 40\pi\,\text{krad/s}; \qquad R = 100\,\Omega; \qquad Q = 5$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{(R/L)} \quad \text{so} \quad L = \frac{QR}{\omega_o} = \frac{5(100)}{(40\pi \times 10^3)} = 3.98\,\text{mH}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad C = \frac{1}{\omega_o^2 L} = \frac{1}{(40\pi \times 10^3)^2(3.98 \times 10^{-3})} = 15.92\,\text{nF}$$

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Problems

P 14.1 **[a]** $\omega_c = \dfrac{R}{L} = \dfrac{1200}{50 \times 10^{-3}} = 24\,\text{krad/s}$

$$\therefore \quad f_c = \frac{\omega_c}{2\pi} = \frac{24{,}000}{2\pi} = 3819.72\,\text{Hz}$$

[b] $H(s) = \dfrac{\omega_c}{s + \omega_c} = \dfrac{24{,}000}{s + 24{,}000}$

$$H(j\omega) = \frac{24{,}000}{24{,}000 + j\omega}$$

$$H(j\omega_c) = \frac{24{,}000}{24{,}000 + j24{,}000} = 0.7071\underline{/-45^\circ}$$

$$H(j0.125\omega_c) = \frac{24{,}000}{24{,}000 + j3000} = 0.9923\underline{/-7.125^\circ}$$

$$H(j8\omega_c) = \frac{24{,}000}{24{,}000 + j192{,}000} = 0.124\underline{/-82.875^\circ}$$

[c] $v_o(t)|_{\omega_c} = 14.142\cos(24{,}000t - 45^\circ)\,\text{V}$

$$v_o(t)|_{0.125\omega_c} = 19.846\cos(3000t - 7.125^\circ)\,\text{V}$$

$$v_o(t)|_{8\omega_c} = 2.48\cos(192{,}000t - 82.875^\circ)\,\text{V}$$

P 14.2 **[a]** $\omega_c = \dfrac{1}{RC} = \dfrac{1}{(160)(5 \times 10^{-}6)} = 1250\,\text{rad/s}$

$$f_c = \frac{\omega_c}{2\pi} = 198.94\,\text{Hz}$$

[b] $H(j\omega) = \dfrac{\omega_c}{s + \omega_c} = \dfrac{1250}{s + 1250}$

$$H(j\omega) = \frac{1250}{1250 + j\omega}$$

$$H(j\omega_c) = \frac{1250}{1250 + j1250} = 0.7071\underline{/-45^\circ}$$

$$H(j0.1\omega_c) = \frac{1250}{1250 + j125} = 0.9950\underline{/-5.71^\circ}$$

$$H(j10\omega_c) = \frac{1250}{1250 + j12{,}500} = 0.0995\underline{/-84.29^\circ}$$

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[c] $v_o(t)|_{\omega_c} = 25(0.7071)\cos(1250t - 45°)$

$= 17.68\cos(1250t - 45°)\,\text{mV}$

$v_o(t)|_{0.1\omega_c} = 25(0.9950)\cos(125t - 5.71°)$

$= 24.88\cos(125t - 5.71°)\,\text{mV}$

$v_o(t)|_{10\omega_c} = 25(0.0995)\cos(12{,}500t - 84.29°)$

$= 2.49\cos(12{,}500t - 84.29°)\,\text{mV}$

P 14.3 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{sL + R + R_l} = \dfrac{(R/L)}{s + (R + R_l)/L}$

[b] $H(j\omega) = \dfrac{(R/L)}{\left(\frac{R+R_l}{L}\right) + j\omega}$

$|H(j\omega)| = \dfrac{(R/L)}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega^2}}$

$|H(j\omega)|_{\max}$ occurs when $\omega = 0$

[c] $|H(j\omega)|_{\max} = \dfrac{R}{R + R_l}$

[d] $|H(j\omega_c)| = \dfrac{R}{\sqrt{2}(R + R_l)} = \dfrac{R/L}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega_c^2}}$

$\therefore\quad \omega_c^2 = \left(\dfrac{R + R_l}{L}\right)^2; \qquad \therefore\quad \omega_c = (R + R_l)/L$

[e] $\omega_c = \dfrac{1200 + 300}{0.05} = 30{,}000\ \text{rad/s}$

$H(j\omega) = \dfrac{24{,}000}{30{,}000 + j\omega}$

$H(j0) = 0.8$

$H(j30{,}000) = \dfrac{0.8}{\sqrt{2}}\underline{/-45°} = 0.5657\underline{/-45°}$

$H(j6000) = \dfrac{24{,}000}{30{,}000 + j6000} = 0.7845\underline{/-11.31°}$

$H(j150{,}000) = \dfrac{24{,}000}{30{,}000 + j150{,}000} = 0.1569\underline{/-78.69°}$

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P 14.4 **[a]** Let $Z = \dfrac{R_L(1/sC)}{R_L + 1/sC} = \dfrac{R_L}{R_L Cs + 1}$

Then $H(s) = \dfrac{Z}{Z+R}$

$$= \frac{R_L}{RR_L Cs + R + R_L}$$

$$= \frac{(1/RC)}{s + \left(\dfrac{R+R_L}{RR_L C}\right)}$$

[b] $|H(j\omega)| = \dfrac{(1/RC)}{\sqrt{\omega^2 + [(R+R_L)/RR_L C]^2}}$

$|H(j\omega)|$ is maximum at $\omega = 0$.

[c] $|H(j\omega)|_{\max} = \dfrac{R_L}{R+R_L}$

[d] $|H(j\omega_c)| = \dfrac{R_L}{\sqrt{2}(R+R_L)} = \dfrac{(1/RC)}{\sqrt{\omega_C^2 + [(R+R_L)/RR_L C]^2}}$

$$\therefore \quad \omega_c = \frac{R+R_L}{RR_L C} = \frac{1}{RC}\left(1 + (R/R_L)\right)$$

[e] $\omega_c = \dfrac{1}{(160)(5\times 10^{-6})}[1 + (160/320)] = 1250(1+0.5) = 1875 \text{ rad/s}$

$$H(j0) = \frac{1250}{1875} = 0.667\underline{/0^\circ}$$

$$H(j\omega_c) = \frac{1250}{1875 + j1875} = 0.4714\underline{/-45^\circ}$$

$$H(j0.2\omega_c) = \frac{1250}{1875 + j375} = 0.6537\underline{/-11.31^\circ}$$

$$H(j5\omega_c) = \frac{1250}{1875 + j9375} = 0.1307\underline{/-78.69^\circ}$$

P 14.5 **[a]** $Z_L = j\omega L = j0L = 0$ so it is a short circuit.

At $\omega = 0$, $V_o = V_i$

[b] $Z_L = j\omega L = j\infty L = \infty$ so it is an open circuit.

At $\omega = \infty$, $V_o = 0$

[c] This is a low pass filter, with a gain of 1 at low frequencies and a gain of 0 at high frequencies.

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[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + sL} = \dfrac{R/L}{s + R/L}$

[e] $\omega_c = \dfrac{R}{L} = \dfrac{330}{0.01} = 33 \text{ krad/s}$

P 14.6 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{R\|R_L}{R\|R_L + sL} = \dfrac{\dfrac{R}{L}\left(\dfrac{R_L}{R + R_L}\right)}{s + \dfrac{R}{L}\left(\dfrac{R_L}{R + R_L}\right)}$

[b] $\omega_{c(UL)} = \dfrac{R}{L}; \qquad \omega_{c(L)} = \dfrac{R}{L}\left(\dfrac{R_L}{R + R_L}\right)$ so the cutoff frequencies are different.

$H(0)_{(UL)} = 1; \qquad H(0)_{(L)} = 1$ so the passband gains are the same.

[c] $\omega_{c(UL)} = 33{,}000 \text{ rad/s}$

$\omega_{c(L)} = 33{,}000 - 0.05(33{,}000) = 31{,}350 \text{ rad/s}$

$31{,}350 = \dfrac{330}{0.01}\left(\dfrac{R_L}{330 + R_L}\right)$ so $\dfrac{R_L}{330 + R_L} = 0.95$

$\therefore \quad 0.05R_L = 313.5$ so $R_L \geq 6270\,\Omega$

P 14.7 **[a]** $\dfrac{R}{L} = 10{,}000\pi \text{ rad/s}$

$R = (0.001)(10{,}000)(\pi) = 31.42\,\Omega$

[b] $R_e = 31.42\|68 = 21.49\,\Omega$

$\omega_{\text{loaded}} = \dfrac{R_e}{L} = 21{,}488.34 \text{ rad/s}$

$\therefore \quad f_{\text{loaded}} = 3419.98\,\text{Hz}$

[c] The $33\,\Omega$ resistor in Appendix H is closest to the desired value of $31.42\,\Omega$. Therefore,

$\omega_c = 33\,\text{krad/s}$ so $f_c = 5252.11\,\text{Hz}$

P 14.8 **[a]** $f_c = \dfrac{1600}{2\pi} = 254.65 \text{ Hz}$

[b] $1600 = \dfrac{R}{0.01}$ so $R = 1600(0.01) = 16\,\Omega$

[c] $\omega_c' = 1600 - 160 = 1440 \text{ rad/s}$

$R' = 1440(0.01) = 14.4\,\Omega$

$16||R_{\text{L}} = 14.4$ so $\dfrac{16R_{\text{L}}}{16 + R_{\text{L}}} = 14.4$

Solving, $R_{\text{L}} = 144\,\Omega$

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[d] The magnitude of $H(j\omega)$ when $\omega = 0$ is still 1. The resistive load affects the cutoff frequency but not the magnitude of this low-pass filter.

P 14.9 **[a]** $\omega_c = 2\pi(100) = 628.32$ rad/s

[b] $\omega_c = \dfrac{1}{RC}$ so $R = \dfrac{1}{\omega_c C} = \dfrac{1}{(628.32)(4.7 \times 10^{-6})} = 338.63\,\Omega$

[c]

[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{1/sC}{R + 1/sC} = \dfrac{1/RC}{s + 1/RC} = \dfrac{628.32}{s + 628.32}$

[e] $H(s) = \dfrac{V_o}{V_i} = \dfrac{(1/sC)\|R_L}{R + (1/sC)\|R_L} = \dfrac{1/RC}{s + \left(\dfrac{R + R_L}{R_L}\right) 1/RC} = \dfrac{628.32}{s + 2(628.32)}$

[f] $\omega_c = 2(628.32) = 1256.64$ rad/s

[g] $H(0) = 1/2$

P 14.10 **[a]** $f_c = \dfrac{\omega_c}{2\pi} = \dfrac{50{,}000}{2\pi} = \dfrac{50}{2\pi} \times 10^3 = 7957.75\,\text{Hz}$

[b] $\dfrac{1}{RC} = 50 \times 10^3$

$$R = \frac{1}{(50 \times 10^3)(0.5 \times 10^{-6})} = 40\,\Omega$$

[c] With a load resistor added in parallel with the capacitor the transfer function becomes

$$H(s) = \frac{R_L\|(1/sC)}{R + R_L\|(1/sC)} = \frac{R_L/sC}{R[R_L + (1/sC)] + R_L/sC}$$

$$= \frac{R_L}{RR_LsC + R + R_L} = \frac{1/RC}{s + [(R + R_L)/RR_LC]}$$

This transfer function is in the form of a low-pass filter, with a cutoff frequency equal to the quantity added to s in the denominator. Therefore,

$$\omega_c = \frac{R + R_L}{RR_LC} = \frac{1}{RC}\left(1 + \frac{R}{R_L}\right)$$

$$\therefore \quad \frac{R}{R_L} = 0.05 \qquad \therefore \quad R_L = 20R = 800\,\Omega$$

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[d] $H(j0) = \dfrac{R_L}{R + R_L} = \dfrac{800}{840} = 0.9524$

P 14.11 **[a]** $\dfrac{1}{RC} = \dfrac{1}{(20)(80 \times 10^{-6})} = 625\,\text{rad/s}$

$$f_c = \frac{625}{2\pi} = 99.47\,\text{Hz}$$

[b] $H(s) = \dfrac{s}{s + \omega_c} \quad \therefore \quad H(j\omega) = \dfrac{j\omega}{625 + j\omega}$

$$H(j\omega_c) = H(j6250) = \frac{j625}{625 + j625} = 0.7071\underline{/45^\circ}$$

$$H(j0.125\omega_c) = H(j78.125) = \frac{j78.125}{625 + j78.125} = 0.124\underline{/82.87^\circ}$$

$$H(j8\omega_c) = H(j5000\omega_c) = \frac{j5000}{625 + j5000} = 0.9923\underline{/7.125^\circ}$$

[c] $v_o(t)|_{\omega_c} = (0.7071)(75)\cos(625t + 45^\circ)$

$$= 53.03\cos(625t + 45^\circ)\,\text{mV}$$

$$v_o(t)|_{0.125\omega_c} = (0.124)(75)\cos(78.125t + 82.87^\circ)$$

$$= 9.3\cos(78.125t + 82.87^\circ)\,\text{mV}$$

$$v_o(t)|_{8\omega_c} = (0.9923)(75)\cos(5000t + 7.125^\circ)$$

$$= 74.42\cos(5000t + 7.125^\circ)\,\text{mV}$$

P 14.12 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + R_c + (1/sC)}$

$$= \frac{R}{R + R_c} \cdot \frac{s}{[s + (1/(R + R_c)C)]}$$

[b] $H(j\omega) = \dfrac{R}{R + R_c} \cdot \dfrac{j\omega}{j\omega + (1/(R + R_c)C)}$

$$|H(j\omega)| = \frac{R}{R + R_c} \cdot \frac{\omega}{\sqrt{\omega^2 + \frac{1}{(R+R_c)^2C^2}}}$$

The magnitude will be maximum when $\omega = \infty$.

[c] $|H(j\omega)|_{\max} = \dfrac{R}{R + R_c}$

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[d] $|H(j\omega_c)| = \dfrac{R\omega_c}{(R+R_c)\sqrt{\omega_c^2 + [1/(R+R_c)C]^2}}$

$$\therefore \quad |H(j\omega)| = \frac{R}{\sqrt{2}(R+R_c)} \qquad \text{when}$$

$$\therefore \quad \omega_c^2 = \frac{1}{(R+R_c)^2C^2}$$

$$\text{or } \omega_c = \frac{1}{(R+R_c)C}$$

[e] $\omega_c = \dfrac{1}{(25)(80\times 10^{-6})} = 500 \text{ rad/s}$

$$\frac{R}{R+R_c} = \frac{20}{25} = 0.8$$

$$\therefore \qquad H(j\omega) = \frac{0.8j\omega}{500+j\omega}$$

$$H(j\omega_c) = \frac{(0.8)j500}{500+j500} = 0.5657\underline{/45^\circ}$$

$$H(j0.125\omega_c) = \frac{(0.8)j62.5}{500+j62.5} = 0.099\underline{/82.87^\circ}$$

$$H(j8\omega_c) = \frac{(0.8)j4000}{500+j4000} = 0.7938\underline{/7.125^\circ}$$

P 14.13 **[a]** $\omega_c = \dfrac{1}{RC} = 2\pi(300) = 600\pi \text{ rad/s}$

$$\therefore \quad R = \frac{1}{\omega_c C} = \frac{1}{(600\pi)(100\times 10^{-9})} = 5305.16\,\Omega = 5.305\,\text{k}\Omega$$

[b] $R_e = 5305.16 \| 47{,}000 = 4767.08\,\Omega$

$$\omega_c = \frac{1}{R_e C} = \frac{1}{(4767.08)(100\times 10^{-9})} = 2097.7 \text{ rad/s}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{2097.7}{2\pi} = 333.86\,\text{Hz}$$

P 14.14 **[a]** For $\omega = 0$, the inductor behaves as a short circuit, so $V_o = 0$.
For $\omega = \infty$, the inductor behaves as an open circuit, so $V_o = V_i$.
Thus, the circuit is a high-pass filter.

[b] $H(s) = \dfrac{sL}{R+sL} = \dfrac{s}{s+R/L} = \dfrac{s}{s+15{,}000}$

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[c] $\omega_c = \dfrac{R}{L} = 15{,}000 \text{ rad/s}$

[d] $|H(jR/L)| = \left|\dfrac{jR/L}{jR/L + R/L}\right| = \left|\dfrac{j}{j+1}\right| = \dfrac{1}{\sqrt{2}}$

P 14.15 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{R_L \| sL}{R + R_L \| sL} = \dfrac{s\left(\dfrac{R_L}{R+R_L}\right)}{s + \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right)}$

$$= \frac{\frac{1}{2}s}{s + \frac{1}{2}(15{,}000)}$$

[b] $\omega_c = \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right) = \dfrac{1}{2}(15{,}000) = 7500 \text{ rad/s}$

[c] $\omega_{c(L)} = \dfrac{1}{2}\omega_{c(UL)}$

[d] The gain in the passband is also reduced by a factor of 1/2 for the loaded filter.

P 14.16 **[a]** $\omega_c = 2\pi(500) = 3141.59 \text{ rad/s}$

[b] $\omega_c = \dfrac{1}{RC}$ so $R = \dfrac{1}{\omega_c C} = \dfrac{1}{(3141.59)(220 \times 10^{-12})} = 1.45\,\text{M}\Omega$

[c]

[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + 1/sC} = \dfrac{s}{s + 1/RC} = \dfrac{s}{s + 3141.59}$

[e] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R \| R_L}{R \| R_L + (1/sC)} = \dfrac{s}{s + \left(\dfrac{R+R_L}{R_L}\right)1/RC} = \dfrac{s}{s + 2(3141.59)}$

[f] $\omega_c = 2(3141.59) = 6283.19 \text{ rad/s}$

[g] $H(\infty) = 1$

P 14.17 **[a]** $R = \omega_c L = (1500 \times 10^3)(100 \times 10^{-6}) = 150\,\Omega$ (a value from Appendix H)

[b] With a load resistor in parallel with the inductor, the transfer function becomes

$$H(s) = \frac{sL \| R_L}{R + sL \| R_L} = \frac{sLR_L}{R(sL + R_L) + sLR_L} = \frac{s[R_L/(R+R_L)]}{s + [RR_L/(R+R_L)]}$$

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This transfer function is in the form of a high-pass filter whose cutoff frequency is the quantity added to s in the denominator. Thus,

$$\omega_c = \frac{RR_L}{L(R+R_L)}$$

Substituting in the values of R and L from part (a), we can solve for the value of load resistance that gives a cutoff frequency of 1200 krad/s:

$$\frac{150R_L}{100\times 10^{-6}(150+R_L)} = 1200\times 10^3 \qquad \text{so} \qquad R_L = 600\,\Omega$$

The smallest resistor from Appendix H that is larger than $600\,\Omega$ is $680\,\Omega$.

P 14.18 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(10\times 10^{-3})(10\times 10^{-9})} = 10^{10}$

$\omega_o = 10^5 \text{ rad/s} = 100\,\text{krad/s}$

[b] $f_o = \dfrac{\omega_o}{2\pi} = \dfrac{10^5}{2\pi} = 15.9\,\text{kHz}$

[c] $Q = \omega_o RC = (100\times 10^3)(8000)(10\times 10^{-9}) = 8$

[d] $\omega_{c1} = \omega_o\left[-\dfrac{1}{2Q} + \sqrt{1+\left(\dfrac{1}{2Q}\right)^2}\right] = 10^5\left[-\dfrac{1}{16} + \sqrt{1+\dfrac{1}{256}}\right] = 93.95\,\text{krad/s}$

[e] $\therefore\quad f_{c1} = \dfrac{\omega_{c1}}{2\pi} = 14.95\,\text{kHz}$

[f] $\omega_{c2} = \omega_o\left[\dfrac{1}{2Q} + \sqrt{1+\left(\dfrac{1}{2Q}\right)^2}\right] = 10^5\left[\dfrac{1}{16} + \sqrt{1+\dfrac{1}{256}}\right] = 106.45\,\text{krad/s}$

[g] $\therefore\quad f_{c2} = \dfrac{\omega_{c2}}{2\pi} = 16.94\,\text{kHz}$

[h] $\beta = \dfrac{\omega_o}{Q} = \dfrac{10^5}{8} = 12.5\,\text{krad/s or } 1.99\,\text{kHz}$

P 14.19 $\omega_o = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{(121)(100)} = 110\,\text{krad/s}$

$f_o = \dfrac{\omega_o}{2\pi} = 17.51\,\text{kHz}$

$\beta = 121 - 100 = 21\,\text{krad/s} \quad \text{or} \quad 2.79\,\text{kHz}$

$Q = \dfrac{\omega_o}{\beta} = \dfrac{110}{21} = 5.24$

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P 14.20 $\beta = \dfrac{\omega_o}{Q} = \dfrac{50{,}000}{4} = 12.5\,\text{krad/s}; \qquad \dfrac{12{,}500}{2\pi} = 1.99\,\text{kHz}$

$$\omega_{c2} = 50{,}000\left[\frac{1}{8} + \sqrt{1 + \left(\frac{1}{8}\right)^2}\right] = 56.64\,\text{krad/s}$$

$$f_{c2} = \frac{56.64\,\text{k}}{2\pi} = 9.01\,\text{kHz}$$

$$\omega_{c1} = 50{,}000\left[-\frac{1}{8} + \sqrt{1 + \left(\frac{1}{8}\right)^2}\right] = 44.14\,\text{krad/s}$$

$$f_{c1} = \frac{44.14\,\text{k}}{2\pi} = 7.02\,\text{kHz}$$

P 14.21 **[a]** $\omega_o = \sqrt{1/LC} \quad \text{so} \quad L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(50{,}000)^2(0.01 \times 10^{-6})} = 40\,\text{mH}$

$$Q = \frac{\omega_o}{\beta} \quad \text{so} \quad \beta = \frac{\omega_o}{Q} = \frac{50{,}000}{8} = 6250 \text{ rad/s}$$

$$\beta = \frac{R}{L} \quad \text{so} \quad R = L\beta = (40 \times 10^{-3})(6250) = 250\,\Omega$$

[b] From part (a), $\beta = 6250$ rad/s. Then,

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm\frac{6250}{2} + \sqrt{\left(\frac{6250}{2}\right)^2 + 50{,}000^2} = \pm 3125 + 50{,}097.56$$

$$\omega_{c1} = 46{,}972.56 \text{ rad/s} \qquad \omega_{c2} = 53{,}222.56 \text{ rad/s}$$

P 14.22 $H(j\omega) = \dfrac{j\omega(6250)}{50{,}000^2 - \omega^2 + j\omega(6250)}$

[a] $H(j50{,}000) = \dfrac{j50{,}000(6250)}{50{,}000^2 - 50{,}000^2 + j(50{,}000)(6250)} = 1$

$$V_o = (1)V_i \quad \therefore \quad v_o(t) = 5\cos 50{,}000t\,\text{V}$$

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[b] $H(j46{,}972.56) = \dfrac{j46{,}972.56(6250)}{50{,}000^2 - 46{,}972.56^2 + j(46{,}972.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 3.54\cos(46{,}972.56t + 45^\circ)\,\text{V}$

[c] $H(j53{,}222.56) = \dfrac{j53{,}222.56(6250)}{50{,}000^2 - 53{,}222.56^2 + j(53{,}222.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 3.54\cos(53{,}222.56t - 45^\circ)\,\text{V}$

[d] $H(j5000) = \dfrac{j5000(6250)}{50{,}000^2 - 5000^2 + j(5000)(6250)} = 0.0126\underline{/89.3^\circ}$

$V_o = 0.0126\underline{/89.3^\circ}V_i \quad \therefore \quad v_o(t) = 63.1\cos(5000t + 89.3^\circ)\,\text{mV}$

[e] $H(j500{,}000) = \dfrac{j500{,}000(6250)}{50{,}000^2 - 500{,}000^2 + j(500{,}000)(6250)} = 0.0126\underline{/-89.3^\circ}$

$V_o = 0.0126\underline{/-89.3^\circ}V_i \quad \therefore \quad v_o(t) = 63.1\cos(500{,}000t - 89.3^\circ)\,\text{mV}$

P 14.23 $H(s) = 1 - \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \dfrac{s^2 + (1/LC)}{s^2 + (R/L)s + (1/LC)}$

$H(j\omega) = \dfrac{50{,}000^2 - \omega^2}{50{,}000^2 - \omega^2 + j\omega(6250)}$

[a] $H(j50{,}000) = \dfrac{50{,}000^2 - 50{,}000^2}{50{,}000^2 - 50{,}000^2 + j(50{,}000)(6250)} = 0$

$V_o = (0)V_i \quad \therefore \quad v_o(t) = 0\,\text{mV}$

[b] $H(j46{,}972.56) = \dfrac{50{,}000^2 - 46{,}972.56^2}{50{,}000^2 - 46{,}972.56^2 + j(46{,}972.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 3.54\cos(46{,}972.56t - 45^\circ)\,\text{V}$

[c] $H(j53{,}222.56) = \dfrac{50{,}000^2 - 53{,}222.56^2}{50{,}000^2 - 53{,}222.56^2 + j(53{,}222.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 3.54\cos(53{,}222.56t + 45^\circ)\,\text{V}$

[d] $H(j5000) = \dfrac{50{,}000^2 - 5000^2}{50{,}000^2 - 5000^2 + j(5000)(6250)} = 0.9999\underline{/-0.72^\circ}$

$V_o = 0.9999\underline{/-0.72^\circ}V_i \quad \therefore \quad v_o(t) = 4.9996\cos(5000t - 0.72^\circ)\,\text{V}$

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[e] $H(j500{,}000) = \dfrac{50{,}000^2 - 500{,}000^2}{50{,}000^2 - 500{,}000^2 + j(500{,}000)(6250)} = 0.9999\underline{/0.72^\circ}$

$$V_o = 0.9999\underline{/0.72^\circ}V_i \quad \therefore \quad v_o(t) = 4.9996\cos(500{,}000t + 0.72^\circ)\,\text{V}$$

P 14.24 By definition $Q = \omega_o/\beta$ therefore $\beta = \omega_o/Q$. Substituting this expression into Eqs. 14.34 and 14.35 yields

$$\omega_{c1} = -\frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

$$\omega_{c2} = \frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

Now factor ω_o out to get

$$\omega_{c1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{c2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

P 14.25 [a] $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(50\times10^{-9})(20\times10^3)^2} = 50\,\text{mH}$

$$R = \frac{Q}{\omega_o C} = \frac{5}{(20\times10^3)(50\times10^{-9})} = 5\,\text{k}\Omega$$

[b] $\omega_{c2} = \omega_o\left[\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2}\right] = 20{,}000\left[\dfrac{1}{10} + \sqrt{1 + \dfrac{1}{100}}\right]$

$$= 22.10\,\text{krad/s} \quad \therefore \quad f_{c2} = \frac{\omega_{c2}}{2\pi} = 3.52\,\text{kHz}$$

$$\omega_{c1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 20{,}000\left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$$

$$= 18.10\,\text{krad/s} \quad \therefore \quad f_{c1} = \frac{\omega_{c1}}{2\pi} = 2.88\,\text{kHz}$$

[c] $\beta = \dfrac{\omega_o}{Q} = \dfrac{20{,}000}{5} = 4000\ \text{rad/s} \quad \text{or} \quad 636.62\,\text{Hz}$

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P 14.26 **[a]** We need $\omega_c = 20{,}000$ rad/s. There are several possible approaches – this one starts by choosing $L = 1$ mH. Then,

$$C = \frac{1}{20{,}000^2(0.001)} = 2.5\,\mu\text{F}$$

Use the closest value from Appendix H, which is $2.2\,\mu$F to give

$$\omega_c = \sqrt{\frac{1}{(0.001)(2.2 \times 10^{-6})}} = 21{,}320 \text{ rad/s}$$

Then, $$R = \frac{Q}{\omega_o C} = \frac{5}{(21320)(2.2 \times 10^{-6})} = 106.6\,\Omega$$

Use the closest value from Appendix H, which is $100\,\Omega$ to give

$$Q = 100(21{,}320)(2.2 \times 10^{-6}) = 4.69$$

[b] $$\%\text{ error in } \omega_c = \frac{21{,}320 - 20{,}000}{20{,}000}(100) = 6.6\%$$

$$\%\text{ error in } Q = \frac{4.69 - 5}{5}(100) = -6.2\%$$

P 14.27 **[a]** $$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{[8000(2\pi)]^2(5 \times 10^{-9})} = 79.16\,\text{mH}$$

$$R = \frac{\omega_o L}{Q} = \frac{8000(2\pi)(79.16 \times 10^{-3})}{2} = 1.99\,\text{k}\Omega$$

[b] $$f_{c1} = 8000\left[-\frac{1}{4} + \sqrt{1 + \frac{1}{16}}\right] = 6.25\,\text{kHz}$$

[c] $$f_{c2} = 8000\left[\frac{1}{4} + \sqrt{1 + \frac{1}{16}}\right] = 10.25\,\text{kHz}$$

[d] $\beta = f_{c2} - f_{c1} = 4\,\text{kHz}$

or

$$\beta = \frac{f_o}{Q} = \frac{8000}{2} = 4\,\text{kHz}$$

P 14.28 **[a]** We need ω_c close to $2\pi(8000) = 50{,}265.48$ rad/s. There are several possible approaches – this one starts by choosing $L = 10$ mH. Then,

$$C = \frac{1}{[2\pi(8000)]^2(0.01)} = 39.58\,\text{nF}$$

Use the closest value from Appendix H, which is $0.047\,\mu$F to give

$$\omega_c = \sqrt{\frac{1}{(0.01)(47 \times 10^{-9})}} = 46{,}126.56 \text{ rad/s} \quad \text{or} \quad f_c = 7341.27\,\text{Hz}$$

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Then, $R = \dfrac{\omega_o L}{Q} = \dfrac{(46{,}126.56)(0.01)}{2} = 230\,\Omega$

Use the closest value from Appendix H, which is $220\,\Omega$ to give

$$Q = \frac{(46{,}126.56)(0.01)}{220} = 2.1$$

[b] $\%$ error in $f_c = \dfrac{7341.27 - 8000}{8000}(100) = -8.23\%$

$\%$ error in $Q = \dfrac{2.1 - 2}{2}(100) = 5\%$

P 14.29 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(40 \times 10^{-3})(40 \times 10^{-9})} = 625 \times 10^6$

$\omega_o = 25 \times 10^3 \text{ rad/s} = 25\,\text{krad/s}$

$f_o = \dfrac{25{,}000}{2\pi} = 3978.87\,\text{Hz}$

[b] $Q = \dfrac{\omega_o L}{R + R_i} = \dfrac{(25 \times 10^3)(40 \times 10^{-3})}{200} = 5$

[c] $\omega_{c1} = \omega_o \left[-\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2} \right] = 25{,}000 \left[-\dfrac{1}{10} + \sqrt{1 + \dfrac{1}{100}} \right]$

$= 22.625\,\text{krad/s} \quad \text{or} \quad 3.60\,\text{kHz}$

[d] $\omega_{c2} = \omega_o \left[\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2} \right] = 25{,}000 \left[\dfrac{1}{10} + \sqrt{1 + \dfrac{1}{100}} \right]$

$= 27.625\,\text{krad/s} \quad \text{or} \quad 4.4\,\text{kHz}$

[e] $\beta = \omega_{c2} - \omega_{c1} = 27.62 - 22.62 = 5\,\text{krad/s}$

or

$\beta = \dfrac{\omega_o}{Q} = \dfrac{25{,}000}{5} = 5\,\text{krad/s} \quad \text{or} \quad 795.77\,\text{Hz}$

P 14.30 **[a]** $H(s) = \dfrac{(R/L)s}{s^2 + \frac{(R+R_i)}{L}s + \frac{1}{LC}}$

For the numerical values in Problem 14.29 we have

$$H(s) = \frac{4500s}{s^2 + 5000s + 625 \times 10^6}$$

$$\therefore \quad H(j\omega) = \frac{4500j\omega}{(625 \times 10^6 - \omega^2) + j5000\omega}$$

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$$H(j\omega_o) = \frac{j4500(25 \times 10^3)}{j5000(25 \times 10^3)} = 0.9\underline{/0^\circ}$$

$$\therefore \quad v_o(t) = 10(0.9)\cos 25{,}000 = 9\cos 25{,}000t\,\text{V}$$

[b] From the solution to Problem 14.29,

$$\omega_{c1} = 22.625\,\text{krad/s}$$

$$H(j22.625\,\text{k}) = \frac{j4500(22.625 \times 10^3)}{(113.12 + j113.12) \times 10^6} = 0.6364\underline{/45^\circ}$$

$$\therefore \quad v_o(t) = 10(0.6364)\cos(22{,}625t + 45^\circ) = 6.364\cos(22{,}625t + 45^\circ)\,\text{V}$$

[c] From the solution to Problem 14.29,

$$\omega_{c2} = 27.625\ \text{krad/s}$$

$$H(j27.625\,\text{k}) = \frac{j4500(27.625 \times 10^3)}{(-138.12 + j138.12) \times 10^6} = 0.6364\underline{/-45^\circ}$$

$$\therefore \quad v_o(t) = 10(0.6364)\cos(27{,}625t - 45^\circ) = 6.364\cos(27{,}625t - 45^\circ)\,\text{V}$$

P 14.31 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(5 \times 10^{-3})(200 \times 10^{-12})} = 10^{12}$

$$\omega_o = 1\,\text{Mrad/s}$$

[b] $\beta = \dfrac{R + R_L}{R_L} \cdot \dfrac{1}{RC} = \left(\dfrac{500 \times 10^3}{400 \times 10^3}\right)\left(\dfrac{1}{(100 \times 10^3)(200 \times 10^{-12})}\right) = 62.5\,\text{krad/s}$

[c] $Q = \dfrac{\omega_o}{\beta} = \dfrac{10^6}{62.5 \times 10^3} = 16$

[d] $H(j\omega_o) = \dfrac{R_L}{R + R_L} = 0.8\underline{/0^\circ}$

$$\therefore \quad v_o(t) = 250(0.8)\cos(10^6 t) = 200\cos 10^6 t\,\text{mV}$$

[e] $\beta = \left(1 + \dfrac{R}{R_L}\right)\dfrac{1}{RC} = \left(1 + \dfrac{100}{R_L}\right)(50 \times 10^3)\ \text{rad/s}$

$$\omega_o = 10^6\ \text{rad/s}$$

$$Q = \frac{\omega_o}{\beta} = \frac{20}{1 + (100/R_L)} \qquad \text{where } R_L \text{ is in kilohms}$$

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[f]

P 14.32 **[a]**

[b] $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(50 \times 10^3)^2(20 \times 10^{-4})} = 20\,\text{mH}$

$$R = \frac{\omega_o L}{Q} = \frac{(50 \times 10^3)(20 \times 10^{-3})}{6.25} = 160\,\Omega$$

[c] $R_e = 160 \| 480 = 120\,\Omega$

$$R_e + R_i = 120 + 80 = 200\,\Omega$$

$$Q_{\text{system}} = \frac{\omega_o L}{R_e + R_i} = \frac{(50 \times 10^3)(20 \times 10^{-3})}{200} = 5$$

[d] $\beta_{\text{system}} = \dfrac{\omega_o}{Q_{\text{system}}} = \dfrac{50 \times 10^3}{5} = 10\,\text{krad/s}$

$$\beta_{\text{system}}(\text{Hz}) = \frac{10{,}000}{2\pi} = 1591.55\,\text{Hz}$$

P 14.33 **[a]** $\dfrac{V_o}{V_i} = \dfrac{Z}{Z + R}$ where $Z = \dfrac{1}{Y}$

and $Y = sC + \dfrac{1}{sL} + \dfrac{1}{R_L} = \dfrac{LCR_L s^2 + sL + R_L}{R_L L s}$

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$$H(s) = \frac{R_L L s}{R_L RLCs^2 + (R+R_L)Ls + RR_L}$$

$$= \frac{(1/RC)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{\left(\frac{R_L}{R+R_L}\right)\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{K\beta s}{s^2 + \beta s + \omega_o^2}, \qquad K = \frac{R_L}{R+R_L}, \qquad \beta = \frac{1}{(R\|R_L)C}$$

[b] $\beta = \left(\dfrac{R+R_L}{R_L}\right)\dfrac{1}{RC}$

[c] $\beta_U = \dfrac{1}{RC}$

$$\therefore \quad \beta_L = \left(\frac{R+R_L}{R_L}\right)\beta_U = \left(1 + \frac{R}{R_L}\right)\beta_U$$

[d] $Q = \dfrac{\omega_o}{\beta} = \dfrac{\omega_o RC}{\left(\frac{R+R_L}{R_L}\right)}$

[e] $Q_U = \omega_o RC$

$$\therefore \quad Q_L = \left(\frac{R_L}{R+R_L}\right)Q_U = \frac{1}{[1+(R/R_L)]}Q_U$$

[f] $H(j\omega) = \dfrac{Kj\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta}$

$H(j\omega_o) = K$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}} = \frac{K\omega_c\beta}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{\omega_c\beta}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

Squaring both sides leads to

$(\omega_o^2 - \omega_c^2)^2 = \omega_c^2\beta^2$ or $(\omega_o^2 - \omega_c^2) = \pm\omega_c\beta$

$\therefore \quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

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The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

where

$$\beta = \left(1 + \frac{R}{R_L}\right)\frac{1}{RC} \text{ and } \omega_o^2 = \frac{1}{LC}$$

P 14.34 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(2 \times 10^{-6})(50 \times 10^{-12})} = 10^{16}$

$$\omega_o = 100\,\text{Mrad/s}$$

$$Q_U = \omega_o RC = (100 \times 10^6)(2.4 \times 10^3)(50 \times 10^{-12}) = 12$$

$$\therefore \quad \left(\frac{R_L}{R + R_L}\right)12 = 7.5; \qquad \therefore \quad R_L = \frac{7.5}{4.5}R = 4\,\text{k}\Omega$$

P 14.35 $H(s) = \dfrac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$

[a] $\omega_o = \sqrt{\dfrac{1}{LC}} = \sqrt{\dfrac{1}{(0.15625)(100 \times 10^{-9})}} = 8000 \text{ rad/s}$

[b] $f_o = \dfrac{\omega_o}{2\pi} = 1273.24\,\text{Hz}$

[c] $Q = \sqrt{\dfrac{L}{R^2C}} = \sqrt{\dfrac{0.15625}{(1875)^2(100 \times 10^{-9})}} = \dfrac{2}{3}$

[d] $\beta = \dfrac{R}{L} = \dfrac{1875}{0.15625} = 12{,}000 \text{ rad/s}$

$$\beta \text{ (Hertz)} = \frac{12{,}000}{2\pi} = 1909.86\,\text{Hz}$$

[e] $\omega_{c1} = \dfrac{-\beta}{2} + \sqrt{\left(\dfrac{\beta}{2}\right)^2 + \omega_o^2}$

$$= \frac{-12{,}000}{2} + \sqrt{\left(\frac{12{,}000}{2}\right)^2 + 8000^2} = 4000 \text{ rad/s}$$

[f] $f_{c1} = \dfrac{4000}{2\pi} = 636.62\,\text{Hz}$

[g] $\omega_{c2} = \dfrac{\beta}{2} + \sqrt{\left(\dfrac{\beta}{2}\right)^2 + \omega_o^2}$

$$= \frac{12{,}000}{2} + \sqrt{\left(\frac{12{,}000}{2}\right)^2 + 8000^2} = 16{,}000 \text{ rad/s}$$

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[h] $f_{c2} = \dfrac{16{,}000}{2\pi} = 2546.48\,\text{Hz}$

P 14.36 **[a]** $H(j\omega) = \dfrac{\dfrac{1}{LC} - \omega^2}{\dfrac{1}{LC} - \omega^2 + j\dfrac{R}{L}\omega} = \dfrac{8000^2 - \omega^2}{8000^2 - \omega^2 + j12{,}000\omega}$

$\omega_o = 8000$ rad/s :

$$H(j\omega_o) = \frac{8000^2 - 8000^2}{8000^2 - 8000^2 + j12{,}000(8000)} = 0$$

$\omega_{c1} = 4000$ rad/s :

$$H(j\omega_{c1}) = \frac{8000^2 - 4000^2}{8000^2 - 4000^2 + j12{,}000(4000)} = 0.7071\underline{/-45^\circ}$$

$\omega_{c2} = 16{,}000$ rad/s :

$$H(j\omega_{c2}) = \frac{8000^2 - 16{,}000^2}{8000^2 - 16{,}000^2 + j12{,}000(16{,}000)} = 0.7071\underline{/45^\circ}$$

$0.1\omega_o = 800$ rad/s :

$$H(j0.1\omega_o) = \frac{8000^2 - 800^2}{8000^2 - 800^2 + j12{,}000(800)} = 0.9887\underline{/-8.62^\circ}$$

$10\omega_o = 80{,}000$ rad/s :

$$H(j10\omega_o) = \frac{8000^2 - 80{,}000^2}{8000^2 - 80{,}000^2 + j12{,}000(80{,}000)} = 0.9887\underline{/8.62^\circ}$$

[b] $\omega = \omega_o = 8000$ rad/s : $\quad V_o = H(j8000)V_i = 0V_i$

$v_o(t) = 0$

$\omega = \omega_{c1} = 4000$ rad/s :

$V_o = H(j4000)V_i = (0.7071\underline{/-45^\circ})(80) = 56.57\underline{/-45^\circ}$

$v_o(t) = 56.57\cos(4000t - 45^\circ)\,\text{V}$

$\omega = \omega_{c2} = 16{,}000$ rad/s :

$V_o = H(j16{,}000)V_i = (0.7071\underline{/45^\circ})(80) = 56.57\underline{/45^\circ}$

$v_o(t) = 56.57\cos(16{,}000t + 45^\circ)\,\text{V}$

$\omega = 0.1\omega_o = 800$ rad/s :

$V_o = H(j800)V_i = (0.9887\underline{/-8.62^\circ})(80) = 79.1\underline{/-8.62^\circ}$

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$$v_o(t) = 79.1\cos(800t - 8.62^\circ)\,\text{V}$$

$$\omega = 10\omega_o = 80{,}000 \text{ rad/s}:$$

$$V_o = H(j80{,}000)V_i = (0.9887\underline{/8.62^\circ})(80) = 79.1\underline{/8.62^\circ}$$

$$v_o(t) = 79.1\cos(80{,}000t + 8.62^\circ)\,\text{V}$$

P 14.37 **[a]** In analyzing the circuit qualitatively we visualize v_i as a sinusoidal voltage and we seek the steady-state nature of the output voltage v_o.
At zero frequency the inductor provides a direct connection between the input and the output, hence $v_o = v_i$ when $\omega = 0$.
At infinite frequency the capacitor provides the direct connection, hence $v_o = v_i$ when $\omega = \infty$.
At the resonant frequency of the parallel combination of L and C the impedance of the combination is infinite and hence the output voltage will be zero when $\omega = \omega_o$.
At frequencies on either side of ω_o the amplitude of the output voltage will be nonzero but less than the amplitude of the input voltage.
Thus the circuit behaves like a band-reject filter.

[b] Let Z represent the impedance of the parallel branches L and C, thus

$$Z = \frac{sL(1/sC)}{sL + 1/sC} = \frac{sL}{s^2LC + 1}$$

Then

$$H(s) = \frac{V_o}{V_i} = \frac{R}{Z + R} = \frac{R(s^2LC + 1)}{sL + R(s^2LC + 1)}$$

$$= \frac{[s^2 + (1/LC)]}{s^2 + \left(\frac{1}{RC}\right)s + \left(\frac{1}{LC}\right)}$$

$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}$$

[c] From part (b) we have

$$H(j\omega) = \frac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2 + j\omega\beta}$$

It follows that $H(j\omega) = 0$ when $\omega = \omega_o$.

$$\therefore \quad \omega_o = \frac{1}{\sqrt{LC}}$$

[d] $$|H(j\omega)| = \frac{\omega_o^2 - \omega^2}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{2}} \text{ when } \omega^2\beta^2 = (\omega_o^2 - \omega^2)^2$$

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or $\pm\,\omega\beta = \omega_o^2 - \omega^2$, thus

$\omega^2 \pm \beta\omega - \omega_o^2 = 0$

The two positive roots of this quadratic are

$$\omega_{c_1} = \frac{-\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

$$\omega_{c_2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

Also note that since $\beta = \omega_o/Q$

$$\omega_{c_1} = \omega_o\left[\frac{-1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{c_2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

[e] It follows from the equations derived in part (d) that

$\beta = \omega_{c_2} - \omega_{c_1} = 1/RC$

[f] By definition $Q = \omega_o/\beta = \omega_o RC$.

P 14.38 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(2.5\times10^{-3})(62.5\times10^{-9})} = 64\times10^8$

$\therefore\ \ \omega_o = 80$ krad/s

[b] $f_o = \dfrac{\omega_o}{2\pi} = 12.73\,\text{kHz}$

[c] $Q = \omega_o RC = (80{,}000)(3000)(62.5\times10^{-9}) = 15$

[d] $\omega_{c1} = \omega_o\left[-\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2}\right] = 80{,}000\left[-\dfrac{1}{30} + \sqrt{1 + \dfrac{1}{900}}\right]$

$= 77{,}377.77$ rad/s

[e] $f_{c1} = \dfrac{\omega_{c1}}{2\pi} = 12{,}315.05\,\text{Hz}$

[f] $\omega_{c2} = \omega_o\left[\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2}\right] = 80{,}000\left[\dfrac{1}{30} + \sqrt{1 + \dfrac{1}{900}}\right]$

$= 82{,}711.1$ rad/s

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[g] $f_{c2} = \dfrac{\omega_{c1}}{2\pi} = 13{,}163.88\,\text{Hz}$

[h] $\beta = f_{c2} - f_{c1} = 848.8\,\text{Hz}$

P 14.39 **[a]** $\omega_o = \sqrt{1/LC}$ so $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(25{,}000)^2(200 \times 10^{-9})} = 8\,\text{mH}$

$$Q = \frac{\omega_o}{\beta} \quad \text{so} \quad \beta = \frac{\omega_o}{Q} = \frac{25{,}000}{2.5} = 10{,}000 \text{ rad/s}$$

$$\beta = \frac{R}{L} \quad \text{so} \quad R = L\beta = (8 \times 10^{-3})(10{,}000) = 80\,\Omega$$

[b] From part (a), $\beta = 10{,}000$ rad/s.

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm\frac{10{,}000}{2} + \sqrt{\left(\frac{10{,}000}{2}\right)^2 + 25{,}000^2}$$

$$= \pm 5000 + 25{,}495.1$$

$$\omega_{c1} = 20{,}495.1 \text{ rad/s} \qquad \omega_{c2} = 30{,}495.1 \text{ rad/s}$$

P 14.40 $H(j\omega) = \dfrac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2 + j\omega\beta} = \dfrac{25{,}000^2 - \omega^2}{25{,}000^2 - \omega^2 + j\omega(10{,}000)}$

[a] $H(j25{,}000) = \dfrac{25{,}000^2 - 25{,}000^2}{25{,}000^2 - 25{,}000^2 + j(25{,}000)(10{,}000)} = 0$

$$V_o = (0)V_i \quad \therefore \quad v_o(t) = 0\,\text{mV}$$

[b] $H(j20{,}495.1) = \dfrac{25{,}000^2 - 20{,}495.1^2}{25{,}000^2 - 20{,}495.1^2 + j(20{,}495.1)(10{,}000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$$V_o = \frac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 7.071\cos(20{,}495.1t - 45^\circ)\,\text{V}$$

[c] $H(j30{,}495.1) = \dfrac{25{,}000^2 - 30{,}495.1^2}{25{,}000^2 - 30{,}495.1^2 + j(30{,}495.1)(10{,}000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$$V_o = \frac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 7.071\cos(30{,}495.1t + 45^\circ)\,\text{V}$$

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[d] $H(j3125) = \dfrac{25{,}000^2 - 3125^2}{25{,}000^2 - 3125^2 + j(3125)(10{,}000)} = 0.9987\underline{/-2.91^\circ}$

$V_o = 0.9987\underline{/-2.91^\circ}V_i \quad \therefore \quad v_o(t) = 9.987\cos(3125t - 2.91^\circ)\,\text{mV}$

[e] $H(j200{,}000) = \dfrac{25{,}000^2 - 200{,}000^2}{25{,}000^2 - 200{,}000^2 + j(200{,}000)(10{,}000)} = 0.9987\underline{/2.91^\circ}$

$V_o = 0.9987\underline{/2.91^\circ}V_i \quad \therefore \quad v_o(t) = 9.987\cos(200{,}000t + 2.91^\circ)\,\text{mV}$

P 14.41 $H(j\omega) = \dfrac{j\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta} = \dfrac{j\omega(10{,}000)}{25{,}000^2 - \omega^2 + j\omega(10{,}000)}$

[a] $H(j25{,}000) = \dfrac{j(25{,}000)(10{,}000)}{25{,}000^2 - 25{,}000^2 + j(25{,}000)(10{,}000)} = 1$

$V_o = (1)V_i \quad \therefore \quad v_o(t) = 10\cos 25{,}000t\,\text{V}$

[b] $H(j20{,}495.1) = \dfrac{j(20{,}495.1)(10{,}000)}{25{,}000^2 - 20{,}495.1^2 + j(20{,}495.1)(10{,}000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 7.071\cos(20{,}495.1t + 45^\circ)\,\text{V}$

[c] $H(j30{,}495.1) = \dfrac{j(30{,}495.1)(10{,}000)}{25{,}000^2 - 30{,}495.1^2 + j(30{,}495.1)(10{,}000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 7.071\cos(30{,}495.1t - 45^\circ)\,\text{V}$

[d] $H(j3125) = \dfrac{j(3125)(10{,}000)}{25{,}000^2 - 3125^2 + j(3125)(10{,}000)} = 0.051\underline{/87.09^\circ}$

$V_o = 0.051\underline{/87.09^\circ}V_i \quad \therefore \quad v_o(t) = 0.51\cos(3125t + 87.09^\circ)\,\text{V}$

[e] $H(j200{,}000) = \dfrac{j(200{,}000)(10{,}000)}{25{,}000^2 - 200{,}000^2 + j(200{,}000)(10{,}000)} = 0.051\underline{/-87.09^\circ}$

$V_o = 0.051\underline{/-87.09^\circ}V_i \quad \therefore \quad v_o(t) = 0.51\cos(200{,}000t - 87.09^\circ)\,\text{V}$

P 14.42 **[a]** $\omega_o = 2\pi f_o = 8\pi\,\text{krad/s}$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(8000\pi)^2(0.5\times 10^{-6})} = 3.17\,\text{mH}$$

$$R = \frac{Q}{\omega_o C} = \frac{5}{(8000\pi)(0.5\times 10^{-6})} = 397.89\,\Omega$$

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[b] $f_{c2} = f_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 4000\left[\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$

$= 4.42\,\text{kHz}$

$f_{c1} = f_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 4000\left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$

$= 3.62\,\text{kHz}$

[c] $\beta = f_{c2} - f_{c1} = 800\,\text{Hz}$

or

$\beta = \frac{f_o}{Q} = \frac{4000}{5} = 800\,\text{Hz}$

P 14.43 **[a]** $R_e = 397.89\|1000 = 284.63\,\Omega$

$Q = \omega_o R_e C = (8000\pi)(284.63)(0.5 \times 10^{-6}) = 3.58$

[b] $\beta = \frac{f_o}{Q} = \frac{4000}{3.58} = 1.12\,\text{kHz}$

[c] $f_{c2} = 4000\left[\frac{1}{7.16} + \sqrt{1 + \frac{1}{7.16^2}}\right] = 4.60\,\text{kHz}$

[d] $f_{c1} = 4000\left[-\frac{1}{7.16} + \sqrt{1 + \frac{1}{7.16^2}}\right] = 3.48\,\text{kHz}$

P 14.44 **[a]** We need $\omega_c = 2\pi(4000) = 25{,}132.74$ rad/s. There are several possible approaches – this one starts by choosing $L = 100\,\mu\text{H}$. Then,

$C = \frac{1}{[2\pi(4000)]^2(100 \times 10^{-6})} = 15.83\,\mu\text{F}$

Use the closest value from Appendix H, which is $22\,\mu\text{F}$, to give

$\omega_c = \sqrt{\frac{1}{100 \times 10^{-6})(22 \times 10^{-6})}} = 21{,}320.07 \text{ rad/s} \quad \text{so} \quad f_c = 3393.19\,\text{Hz}$

Then, $\quad R = \frac{Q}{\omega_o C} = \frac{5}{(21320.07)(22 \times 10^{-6})} = 10.66\,\Omega$

Use the closest value from Appendix H, which is $10\,\Omega$, to give

$Q = 10(21{,}320.07)(22 \times 10^{-6}) = 4.69$

[b] % error in $f_c = \frac{3393.19 - 4000}{4000}(100) = -15.2\%$

% error in $Q = \frac{4.69 - 5}{5}(100) = -6.2\%$

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P 14.45 **[a]** Let $Z = \dfrac{R_L(sL + (1/sC))}{R_L + sL + (1/sC)}$

$$Z = \frac{R_L(s^2LC + 1)}{s^2LC + R_LCs + 1}$$

Then $H(s) = \dfrac{V_o}{V_i} = \dfrac{s^2R_LCL + R_L}{(R + R_L)LCs^2 + RR_LCs + R + R_L}$

Therefore

$$H(s) = \left(\frac{R_L}{R + R_L}\right) \cdot \frac{[s^2 + (1/LC)]}{\left[s^2 + \left(\frac{RR_L}{R+R_L}\right)\frac{s}{L} + \frac{1}{LC}\right]}$$

$$= \frac{K(s^2 + \omega_o^2)}{s^2 + \beta s + \omega_o^2}$$

where $K = \dfrac{R_L}{R + R_L}$; $\omega_o^2 = \dfrac{1}{LC}$; $\beta = \left(\dfrac{RR_L}{R + R_L}\right)\dfrac{1}{L}$

[b] $\omega_o = \dfrac{1}{\sqrt{LC}}$

[c] $\beta = \left(\dfrac{RR_L}{R + R_L}\right)\dfrac{1}{L}$

[d] $Q = \dfrac{\omega_o}{\beta} = \dfrac{\omega_o L}{[RR_L/(R + R_L)]}$

[e] $H(j\omega) = \dfrac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$$H(j\omega_o) = 0$$

[f] $H(j0) = \dfrac{K\omega_o^2}{\omega_o^2} = K$

[g] $H(j\omega) = \dfrac{K\left[(\omega_o/\omega)^2 - 1\right]}{\left\{\left[(\omega_o/\omega)^2 - 1\right] + j\beta/\omega\right\}}$

$$H(j\infty) = \frac{-K}{-1} = K$$

[h] $H(j\omega) = \dfrac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$$H(j0) = H(j\infty) = K$$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}}$$

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$$\therefore \quad \frac{K}{\sqrt{2}} = \frac{K(\omega_o^2 - \omega_c^2)}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

Squaring both sides leads to

$$(\omega_o^2 - \omega_c^2)^2 = \omega_c^2\beta^2 \text{ or } (\omega_o^2 - \omega_c^2) = \pm\omega_c\beta$$

$$\therefore \quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

where

$$\beta = \frac{RR_L}{R + R_L} \cdot \frac{1}{L} \text{ and } \omega_o^2 = \frac{1}{LC}$$

P 14.46 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(10^{-6})(4 \times 10^{-12})} = 0.25 \times 10^{18} = 25 \times 10^{16}$

$$\omega_o = 5 \times 10^8 = 500\,\text{Mrad/s}$$

$$\beta = \frac{RR_L}{R + R_L} \cdot \frac{1}{L} = \frac{(30)(150)}{180} \cdot \frac{1}{10^{-6}} = 25\,\text{Mrad/s} = 3.98\,\text{MHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{500\,\text{M}}{25\,\text{M}} = 20$$

[b] $H(j0) = \dfrac{R_L}{R + R_L} = \dfrac{150}{180} = 0.8333$

$$H(j\infty) = \frac{R_L}{R + R_L} = 0.8333$$

[c] $f_{c2} = \dfrac{250}{\pi}\left[\dfrac{1}{40} + \sqrt{1 + \dfrac{1}{1600}}\right] = 81.59\,\text{MHz}$

$$f_{c1} = \frac{250}{\pi}\left[-\frac{1}{40} + \sqrt{1 + \frac{1}{1600}}\right] = 77.61\,\text{MHz}$$

Check: $\beta = f_{c2} - f_{c1} = 3.98\,\text{MHz}.$

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[d] $Q = \dfrac{\omega_o}{\beta} = \dfrac{500 \times 10^6}{\frac{RR_L}{R+R_L} \cdot \frac{1}{L}}$

$$= \frac{500(R + R_L)}{RR_L} = \frac{50}{3}\left(1 + \frac{30}{R_L}\right)$$

where R_L is in ohms.

[e]

P 14.47 **[a]** $\omega_o^2 = \dfrac{1}{LC} = 625 \times 10^6$

$$\therefore \quad L = \frac{1}{(625 \times 10^6)(25 \times 10^{-9})} = 64\,\text{mH}$$

$$\frac{R_L}{R + R_L} = 0.9; \qquad \therefore \quad 0.1R_L = 0.9R$$

$$\therefore \qquad R_L = 9R \quad \therefore \quad R = \frac{500}{9} = 55.6\,\Omega$$

[b] $\beta = \left(\dfrac{R_L}{R + R_L}\right) R \cdot \dfrac{1}{L} = 781.25 \text{ rad/s}$

$$Q = \frac{\omega_o}{\beta} = \frac{25{,}000}{781.25} = 32$$

P 14.48 **[a]** $|H(j\omega)| = \dfrac{25 \times 10^6}{\sqrt{(25 \times 10^6 - \omega^2)^2 + (1000\omega)^2}} = 1$

$$\therefore \qquad 0 = 625 \times 10^{12} = (25 \times 10^6 - \omega^2)^2 + (1000\omega)^2$$

$$= -50 \times 10^6\omega^2 + \omega^4 + 10^6\omega^2$$

From the above equation it is obvious that $\omega = 0$ is one solution. The other is as follows:

$\omega^2 = 49 \times 10^6 \quad$ so $\quad \omega = 7000\,\text{rad/s}$

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[b] From the equation for $|H(j\omega)|$ in part (a), the frequency for which the magnitude is maximum is the frequency for which the denominator is minimum. This is the frequency for which

$$(25 \times 10^6 - \omega^2)^2 = 0 \quad \text{so} \quad \omega = \sqrt{25 \times 10^6} = 5000\,\text{rad/s}$$

[c] $$|H(j5000)| = \frac{25 \times 10^6}{\sqrt{(25 \times 10^6 - 5000^2)^2 + [1000(5000)]^2}} = 5$$

P 14.49 **[a]** $$H(s) = \frac{sL}{R + sL + \frac{1}{sC}} = \frac{s^2LC}{RsC + s^2LC + 1} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

[b] When $s = j\omega$ is very small (think of ω approaching 0),

$$H(s) \approx \frac{s^2}{\frac{1}{LC}} = 0$$

[c] When $s = j\omega$ is very large (think of ω approaching ∞),

$$H(s) \approx \frac{s^2}{s^2} = 1$$

[d] The magnitude of $H(s)$ approaches 0 as the frequency approaches 0, and approaches 1 as the frequency approaches ∞. Therefore, this circuit is behaving like a high pass filter when the output is the voltage across the inductor.

[e] $$|H(j\omega_c)| = \frac{\omega_c^2}{\sqrt{(10^6 - \omega_c^2)^2 + (1500\omega_c)^2}} = \frac{1}{\sqrt{2}}$$

$$2\omega_c^4 = (10^6 - \omega_c^2)^2 + (1500\omega_c)^2 = 10^{12} - 2 \times 10^6\omega_c^2 + \omega_c^4 + 225 \times 10^4\omega_c^2$$

Simplifying, $\quad \omega_c^4 - 25 \times 10^4\omega_c^2 - 10^{12} = 0$

Solve for ω_c^2 and then ω_c:

$$\omega_c^2 = 1{,}132{,}782.22 \qquad \text{so} \qquad \omega_c = 1064.322 \text{ rad/s}$$

$$f = 169.4\,\text{Hz}$$

P 14.50 **[a]** $$H(s) = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}} = \frac{1}{RsC + s^2LC + 1} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

[b] When $s = j\omega$ is very small (think of ω approaching 0),

$$H(s) \approx \frac{\frac{1}{LC}}{\frac{1}{LC}} = 1$$

[c] When $s = j\omega$ is very large (think of ω approaching ∞),

$$H(s) \approx \frac{\frac{1}{LC}}{s^2} = 0$$

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[d] The magnitude of $H(s)$ approaches 1 as the frequency approaches 0, and approaches 0 as the frequency approaches ∞. Therefore, this circuit is behaving like a low pass filter when the output is the voltage across the capacitor.

[e] $|H(j\omega_c)| = \dfrac{10^6}{\sqrt{(10^6 - \omega_c^2)^2 + (1500\omega_c)^2}} = \dfrac{1}{\sqrt{2}}$

$$2 \times 10^{12} = (10^6 - \omega_c^2)^2 + (1500\omega_c)^2 = 10^{12} - 2 \times 10^6\omega_c^2 + \omega_c^4 + 225 \times 10^4\omega_c^2$$

Simplifying, $\quad \omega_c^4 + (1500^2 - 2 \times 10^6)\omega_c^2 - 10^{12} = 0$

Solve for ω_c^2 and then ω_c:

$\omega_c^2 = 882{,}782.22 \qquad$ so $\qquad \omega_c = 939.565$ rad/s

$f = 149.54\,\text{Hz}$

P 14.51 **[a]** Use the cutoff frequencies to calculate the bandwidth:

$\omega_{c1} = 2\pi(697) = 4379.38$ rad/s $\qquad \omega_{c2} = 2\pi(941) = 5912.48$ rad/s

Thus $\qquad \beta = \omega_{c2} - \omega_{c1} = 1533.10$ rad/s

Calculate inductance using Eq. (14.32) and capacitance using Eq. (14.31):

$$L = \frac{R}{\beta} = \frac{600}{1533.10} = 0.39\,\text{H}$$

$$C = \frac{1}{L\omega_{c1}\omega_{c2}} = \frac{1}{(0.39)(4379.38)(5912.48)} = 0.10\,\mu\text{F}$$

[b] At the outermost two frequencies in the low-frequency group (687 Hz and 941 Hz) the amplitudes are

$$|\text{V}_{697\text{Hz}}| = |V_{941\text{Hz}}| = \frac{|V_{\text{peak}}|}{\sqrt{2}} = 0.707|V_{\text{peak}}|$$

because these are cutoff frequencies. We calculate the amplitudes at the other two low frequencies using Eq. (14.32):

$$|V| = (|V_{\text{peak}}|)(|H(j\omega)|) = |V_{\text{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + (\omega\beta)^2}}$$

Therefore

$$|V_{770\text{Hz}}| = |V_{\text{peak}}| = \frac{(4838.05)(1533.10)}{\sqrt{(5088.52^2 - 4838.05^2)^2 + [(4838.05)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

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and

$$|V_{852\text{Hz}}| = |V_{\text{peak}}| = \frac{(5353.27)(1533.10)}{\sqrt{(5088.52^2 - 5353.27^2)^2 + [(5353.27)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

It is not a coincidence that these two magnitudes are the same. The frequencies in both bands of the DTMF system were carefully chosen to produce this type of predictable behavior with linear filters. In other words, the frequencies were chosen to be equally far apart with respect to the response produced by a linear filter. Most musical scales consist of tones designed with this dame property – note intervals are selected to place the notes equally far apart. That is why the DTMF tones remind us of musical notes! Unlike musical scales, DTMF frequencies were selected to be harmonically unrelated, to lower the risk of misidentifying a tone's frequency if the circuit elements are not perfectly linear.

[c] The high-band frequency closest to the low-frequency band is 1209 Hz. The amplitude of a tone with this frequency is

$$|V_{1209\text{Hz}}| = |V_{\text{peak}}| = \frac{(7596.37)(1533.10)}{\sqrt{(5088.52^2 - 7596.37^2)^2 + [(7596.37)(1533.10)]^2}}$$

$$= 0.344|V_{\text{peak}}|$$

This is less than one half the amplitude of the signals with the low-band cutoff frequencies, ensuring adequate separation of the bands.

P 14.52 The cutoff frequencies and bandwidth are

$$\omega_{c_1} = 2\pi(1209) = 7596\,\text{rad/s}$$

$$\omega_{c_2} = 2\pi(1633) = 10.26\,\text{krad/s}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = 2664\,\text{rad/s}$$

Telephone circuits always have $R = 600\,\Omega$. Therefore, the filters inductance and capacitance values are

$$L = \frac{R}{\beta} = \frac{600}{2664} = 0.225\,\text{H}$$

$$C = \frac{1}{\omega_{c_1}\omega_{c_2}L} = 0.057\,\mu\text{F}$$

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At the highest of the low-band frequencies, 941 Hz, the amplitude is

$$|V_\omega| = |V_{\text{peak}}| \frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

where $\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}}$. Thus,

$$|V_\omega| = \frac{|V_{\text{peak}}|(5912)(2664)}{\sqrt{[(8828)^2 - (5912)^2]^2 + [(5912)(2664)]^2}}$$

$$= 0.344\ |V_{\text{peak}}|$$

Again it is not coincidental that this result is the same as the response of the low-band filter to the lowest of the high-band frequencies.

P 14.53 From Problem 14.51 the response to the largest of the DTMF low-band tones is $0.948|V_{\text{peak}}|$. The response to the 20 Hz tone is

$$|V_{20\text{Hz}}| = \frac{|V_{\text{peak}}|(125.6)(1533)}{[(5089^2 - 125.6^2)^2 + [(125.6)(1533)]^2]^{1/2}}$$

$$= 0.00744|V_{\text{peak}}|$$

$$\therefore \quad \frac{|V_{20\text{Hz}}|}{|V_{770\text{Hz}}|} = \frac{|V_{20\text{Hz}}|}{|V_{852\text{Hz}}|} = \frac{0.00744|V_{\text{peak}}|}{0.948|V_{\text{peak}}|} = 0.5$$

$$\therefore \quad |V_{20\text{Hz}}| = 63.7|V_{770\text{Hz}}|$$

Thus, the 20Hz signal can be 63.7 times as large as the DTMF tones.

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15

Active Filter Circuits

Assessment Problems

AP 15.1

$$H(s) = \frac{-(R_2/R_1)s}{s + (1/R_1C)}$$

$$\frac{1}{R_1C} = 1\,\text{rad/s}; \qquad R_1 = 1\,\Omega, \quad \therefore \quad C = 1\,\text{F}$$

$$\frac{R_2}{R_1} = 1, \quad \therefore \quad R_2 = R_1 = 1\,\Omega$$

$$\therefore \qquad H_{\text{prototype}}(s) = \frac{-s}{s+1}$$

AP 15.2

$$H(s) = \frac{-(1/R_1C)}{s + (1/R_2C)} = \frac{-20{,}000}{s + 5000}$$

$$\frac{1}{R_1C} = 20{,}000; \quad C = 5\,\mu\text{F}$$

$$\therefore \quad R_1 = \frac{1}{(20{,}000)(5 \times 10^{-6})} = 10\,\Omega$$

$$\frac{1}{R_2C} = 5000$$

$$\therefore \quad R_2 = \frac{1}{(5000)(5 \times 10^{-6})} = 40\,\Omega$$

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AP 15.3

$$\omega_c = 2\pi f_c = 2\pi \times 10^4 = 20{,}000\pi \text{ rad/s}$$

$$\therefore \quad k_f = 20{,}000\pi = 62{,}831.85$$

$$C' = \frac{C}{k_f k_m} \qquad \therefore \qquad 0.5 \times 10^{-6} = \frac{1}{k_f k_m}$$

$$\therefore \quad k_m = \frac{1}{(0.5 \times 10^{-6})(62{,}831.85)} = 31.83$$

AP 15.4 For a 2nd order Butterworth high pass filter

$$H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

For the circuit in Fig. 15.25

$$H(s) = \frac{s^2}{s^2 + \left(\frac{2}{R_2 C}\right) s + \left(\frac{1}{R_1 R_2 C^2}\right)}$$

Equate the transfer functions. For $C = 1\text{F}$,

$$\frac{2}{R_2 C} = \sqrt{2}, \quad \therefore \quad R_2 = \sqrt{2} = 1.414\,\Omega$$

$$\frac{1}{R_1 R_2 C^2} = 1, \quad \therefore \quad R_1 = \frac{1}{\sqrt{2}} = 0.707\,\Omega$$

AP 15.5

$Q = 8, K = 5, \omega_o = 1000\,\text{rad/s}, C = 1\,\mu\text{F}$

For the circuit in Fig 15.26

$$H(s) = \frac{-\left(\dfrac{1}{R_1 C}\right) s}{s^2 + \left(\dfrac{2}{R_3 C}\right) s + \left(\dfrac{R_1 + R_2}{R_1 R_2 R_3 C^2}\right)}$$
$$= \frac{K\beta s}{s^2 + \beta s + \omega_o^2}$$

$$\beta = \frac{2}{R_3 C}, \quad \therefore \qquad R_3 = \frac{2}{\beta C}$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{8} = 125\,\text{rad/s}$$

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$$\therefore \quad R_3 = \frac{2 \times 10^6}{(125)(1)} = 16\,\text{k}\Omega$$

$$K\beta = \frac{1}{R_1 C}$$

$$\therefore \quad R_1 = \frac{1}{K\beta C} = \frac{1}{5(125)(1 \times 10^{-6})} = 1.6\,\text{k}\Omega$$

$$\omega_o^2 = \frac{R_1 + R_2}{R_1 R_2 R_3 C^2}$$

$$10^6 = \frac{(1600 + R_2)}{(1600)(R_2)(16{,}000)(10^{-6})^2}$$

Solving for R_2,

$$R_2 = \frac{(1600 + R_2)10^6}{256 \times 10^5}, \quad 246R_2 = 16{,}000, \quad R_2 = 65.04\,\Omega$$

AP 15.6

$$\omega_o = 1000\,\text{rad/s}; \qquad Q = 4;$$

$$C = 2\,\mu\text{F}$$

$$H(s) = \frac{s^2 + (1/R^2C^2)}{s^2 + \left[\dfrac{4(1-\sigma)}{RC}\right]s + \left(\dfrac{1}{R^2C^2}\right)}$$
$$= \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}; \qquad \omega_o = \frac{1}{RC}; \qquad \beta = \frac{4(1-\sigma)}{RC}$$

$$R = \frac{1}{\omega_o C} = \frac{1}{(1000)(2 \times 10^{-6})} = 500\,\Omega$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{4} = 250$$

$$\therefore \qquad \frac{4(1-\sigma)}{RC} = 250$$

$$4(1-\sigma) = 250RC = 250(500)(2 \times 10^{-6}) = 0.25$$

$$1 - \sigma = \frac{0.25}{4} = 0.0625; \quad \therefore \quad \sigma = 0.9375$$

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Problems

P 15.1 **[a]** $K = 10^{(10/20)} = 3.16 = \dfrac{R_2}{R_1}$

$$R_2 = \frac{1}{\omega_c C} = \frac{1}{(2\pi)(10^3)(750 \times 10^{-9})} = 212.21\,\Omega$$

$$R_1 = \frac{R_2}{K} = \frac{212.21}{3.16} = 67.16\,\Omega$$

[b]

P 15.2 **[a]** $\dfrac{1}{RC} = 2\pi(1000)$ so $RC = 1.5915 \times 10^{-4}$

There are several possible approaches. Here, choose $R_f = 150\,\Omega$. Then

$$C = \frac{1.5915 \times 10^{-4}}{150} = 1.06 \times 10^{-6}$$

Choose $C = 1\,\mu$F. This gives

$$\omega_c = \frac{1}{(150)(10^{-6})} = 6.67 \times 10^3 \text{ rad/s} \quad \text{so} \quad f_c = 1061\,\text{Hz}$$

To get a passband gain of 10 dB, choose

$$R_i = \frac{R_f}{3.16} = \frac{150}{3.16} = 47.47\,\Omega$$

Choose $R_i = 47\,\Omega$ to give $K = 20\log_{10}(150/47) = 10.08$ dB.

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The resulting circuit is

[b] % error in $f_c = \dfrac{1061 - 1000}{1000}(100) = 6.1\%$

% error in passband gain $= \dfrac{10.08 - 10}{10}(100) = 0.8\%$

P 15.3 **[a]** $\omega_c = \dfrac{1}{R_2 C}$ so $R_2 = \dfrac{1}{\omega_c C} = \dfrac{1}{2\pi(2500)(10 \times 10^{-9})} = 6366\,\Omega$

$K = \dfrac{R_2}{R_1}$ so $R_1 = \dfrac{R_2}{K} = \dfrac{6366}{5} = 1273\,\Omega$

[b] Both the cutoff frequency and the passband gain are changed.

P 15.4 **[a]** $5(3.5) = 17.5\,\text{V}$ so $V_{cc} \geq 17.5\,\text{V}$

[b] $H(j\omega) = \dfrac{-5(2\pi)(2500)}{j\omega + 2\pi(2500)}$

$$H(j5000\pi) = \frac{-5(5000\pi)}{5000\pi + j5000\pi} = -2.5 + j2.5 = \frac{5}{\sqrt{2}}\underline{/135^\circ}$$

$$V_o = \frac{17.5}{\sqrt{2}}\underline{/135^\circ}V_i \quad \text{so} \quad v_o(t) = 12.37\cos(5000\pi t + 135^\circ)\,\text{V}$$

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[c] $H(j625\pi) = \dfrac{-5(5000\pi)}{5000\pi + j625\pi} = 4.96\underline{/172.9^\circ}$

$V_o = 17.36\underline{/172.9^\circ}V_i \quad \text{so} \quad v_o(t) = 17.36\cos(625\pi t + 172.9^\circ)\,\text{V}$

[d] $H(j40{,}000\pi) = \dfrac{-5(5000\pi)}{5000\pi + j40{,}000\pi} = 0.62\underline{/97.1^\circ}$

$V_o = 2.2\underline{/97.1^\circ}V_i \quad \text{so} \quad v_o(t) = 2.2\cos(40{,}000\pi t + 97.1^\circ)\,\text{V}$

P 15.5 Summing the currents at the inverting input node yields

$$\frac{0 - V_i}{Z_i} + \frac{0 - V_o}{Z_f} = 0$$

$$\therefore \quad \frac{V_o}{Z_f} = -\frac{V_i}{Z_i}$$

$$\therefore \quad H(s) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i}$$

P 15.6 **[a]** $Z_f = \dfrac{R_2(1/sC_2)}{[R_2 + (1/sC_2)]} = \dfrac{R_2}{R_2C_2s + 1}$

$$= \frac{(1/C_2)}{s + (1/R_2C_2)}$$

Likewise

$$Z_i = \frac{(1/C_1)}{s + (1/R_1C_1)}$$

$$\therefore \quad H(s) = \frac{-(1/C_2)[s + (1/R_1C_1)]}{[s + (1/R_2C_2)](1/C_1)}$$

$$= -\frac{C_1}{C_2}\frac{[s + (1/R_1C_1)]}{[s + (1/R_2C_2)]}$$

[b] $H(j\omega) = \dfrac{-C_1}{C_2}\left[\dfrac{j\omega + (1/R_1C_1)}{j\omega + (1/R_2C_2)}\right]$

$$H(j0) = \frac{-C_1}{C_2}\left(\frac{R_2C_2}{R_1C_1}\right) = \frac{-R_2}{R_1}$$

[c] $H(j\infty) = -\dfrac{C_1}{C_2}\left(\dfrac{j}{j}\right) = \dfrac{-C_1}{C_2}$

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[d] As $\omega \to 0$ the two capacitor branches become open and the circuit reduces to a resistive inverting amplifier having a gain of $-R_2/R_1$.
As $\omega \to \infty$ the two capacitor branches approach a short circuit and in this case we encounter an indeterminate situation; namely $v_n \to v_i$ but $v_n = 0$ because of the ideal op amp. At the same time the gain of the ideal op amp is infinite so we have the indeterminate form $0 \cdot \infty$. Although $\omega = \infty$ is indeterminate we can reason that for finite large values of ω $H(j\omega)$ will approach $-C_1/C_2$ in value. In other words, the circuit approaches a purely capacitive inverting amplifier with a gain of $(-1/j\omega C_2)/(1/j\omega C_1)$ or $-C_1/C_2$.

P 15.7 **[a]** $Z_f = \dfrac{(1/C_2)}{s + (1/R_2C_2)}$

$$Z_i = R_1 + \frac{1}{sC_1} = \frac{R_1}{s}[s + (1/R_1C_1)]$$

$$H(s) = -\frac{(1/C_2)}{[s + (1/R_2C_2)]} \cdot \frac{s}{R_1[s + (1/R_1C_1)]}$$

$$= -\frac{1}{R_1C_2}\frac{s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

[b] $H(j\omega) = -\dfrac{1}{R_1C_2}\dfrac{j\omega}{\left(j\omega + \frac{1}{R_1C_1}\right)\left(j\omega + \frac{1}{R_2C_2}\right)}$

$$H(j0) = 0$$

[c] $H(j\infty) = 0$

[d] As $\omega \to 0$ the capacitor C_1 disconnects v_i from the circuit. Therefore $v_o = v_n = 0$.
As $\omega \to \infty$ the capacitor short circuits the feedback network, thus $Z_F = 0$ and therefore $v_o = 0$.

P 15.8 **[a]** $R_1 = \dfrac{1}{\omega_c C} = \dfrac{1}{(2\pi)(8 \times 10^3)(3.9 \times 10^{-9})} = 5.10\,\text{k}\Omega$

$$K = 10^{(14/20)} = 5.01 = \frac{R_2}{R_1}$$

$$\therefore \quad R_2 = 5.01R_1 = 25.55\,\text{k}\Omega$$

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[b]

P 15.9 **[a]** $\dfrac{1}{RC} = 2\pi(8000)$ so $RC = 19.89 \times 10^{-6}$

There are several possible approaches. Here, choose $C = 0.047\,\mu\text{F}$. Then

$$R_i = \frac{19.89 \times 10^{-6}}{0.047 \times 10^{-6}} = 423$$

Choose $R_i = 390\,\Omega$. This gives

$$\omega_c = \frac{1}{(0.047 \times 10^{-6})(390)} = 54.56 \text{ krad/s} \quad \text{so} \quad f_c = 8.68\,\text{kHz}$$

To get a passband gain of 14 dB, choose

$$R_f = 5R_i = 5(390) = 1950\,\Omega$$

Choose $R_f = 1.8\,\text{k}\Omega$ to give a passband gain of $20\log_{10}(1800/390) = 13.3$ dB. The resulting circuit is

[b] % error in $f_c = \dfrac{8683.76 - 8000}{8000}(100) = 8.5\%$

% error in passband gain $= \dfrac{13.3 - 14}{14}(100) = -5.1\%$

P 15.10 **[a]** $\omega_c = \dfrac{1}{R_1 C}$ so $R_1 = \dfrac{1}{\omega_c C} = \dfrac{1}{2\pi(4000)(250 \times 10^{-9})} = 159\,\Omega$

$$K = \frac{R_2}{R_1} \quad \text{so} \quad R_2 = KR_1 = (8)(159) = 1273\,\Omega$$

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[b] The passband gain changes but the cutoff frequency is unchanged.

P 15.11 **[a]** $8(2.5) = 20\,\text{V} \quad \text{so} \quad V_{cc} \geq 20\,\text{V}$

[b] $H(j\omega) = \dfrac{-8j\omega}{j\omega + 8000\pi}$

$$H(j8000\pi) = \frac{-8(j8000\pi)}{8000\pi + j8000\pi} = \frac{8}{\sqrt{2}}\underline{/-135^\circ}$$

$$V_o = 14.14\underline{/-135^\circ}V_i \quad \text{so} \quad v_o(t) = 14.14\cos(8000\pi t - 135^\circ)\,\text{V}$$

[c] $H(j1000\pi) = \dfrac{-8(j1000\pi)}{8000\pi + j1000\pi} = 0.99\underline{/-97.1^\circ}$

$$V_o = 2.48\underline{/-97.1^\circ}V_i \quad \text{so} \quad v_o(t) = 2.48\cos(1000\pi t - 97.1^\circ)\,\text{mV}$$

[d] $H(j64{,}000\pi) = \dfrac{-8(j64{,}000\pi)}{8000\pi + j64{,}000\pi} = 7.94\underline{/-172.9^\circ}$

$$V_o = 19.85\underline{/-172.9^\circ}V_i \quad \text{so} \quad v_o(t) = 19.85\cos(64{,}000\pi t - 172.9^\circ)\,\text{V}$$

P 15.12 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{s}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = \frac{RC}{k_f} = \frac{1}{k_f}; \qquad \frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{s}{s + (1/R'C')} = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

For the RL circuit

$$H(s) = \frac{s}{s + (R/L)}$$

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$$R' = k_m R; \qquad L' = \frac{k_m L}{k_f}$$

$$\frac{R'}{L'} = k_f\left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

P 15.13 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{(1/RC)}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = k_m R\frac{C}{k_m k_f} = \frac{1}{k_f}RC = \frac{1}{k_f}$$

$$\frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{(1/R'C')}{s + (1/R'C')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

For the RL circuit $\qquad H(s) = \dfrac{R/L}{s + R/L} \qquad$ so

$$R' = k_m R; \qquad L' = \frac{k_m}{k_f}L$$

$$\frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f}L} = k_f\left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{(R'/L')}{s + (R'/L')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

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P 15.14 $H(s) = \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \dfrac{\beta s}{s^2 + \beta s + \omega_o^2}$

For the prototype circuit $\omega_o = 1$ and $\beta = \omega_o/Q = 1/Q$.
For the scaled circuit

$$H'(s) = \frac{(R'/L')s}{s^2 + (R'/L')s + (1/L'C')}$$

where $R' = k_m R$; $L' = \dfrac{k_m}{k_f}L$; and $C' = \dfrac{C}{k_f k_m}$

$$\therefore \quad \frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f}L} = k_f\left(\frac{R}{L}\right) = k_f\beta$$

$$\frac{1}{L'C'} = \frac{k_f k_m}{\frac{k_m}{k_f}LC} = \frac{k_f^2}{LC} = k_f^2$$

$$Q' = \frac{\omega_o'}{\beta'} = \frac{k_f\omega_o}{k_f\beta} = Q$$

therefore the Q of the scaled circuit is the same as the Q of the unscaled circuit. Also note $\beta' = k_f\beta$.

$$\therefore \quad H'(s) = \frac{\left(\frac{k_f}{Q}\right)s}{s^2 + \left(\frac{k_f}{Q}\right)s + k_f^2}$$

$$H'(s) = \frac{\left(\frac{1}{Q}\right)\left(\frac{s}{k_f}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \frac{1}{Q}\left(\frac{s}{k_f}\right) + 1\right]}$$

P 15.15 **[a]** $L = 1\,\text{H}; \qquad C = 1\,\text{F}$

$$R = \frac{1}{Q} = \frac{1}{20} = 0.05\,\Omega$$

[b] $k_f = \dfrac{\omega_o'}{\omega_o} = 40{,}000; \qquad k_m = \dfrac{R'}{R} = \dfrac{5000}{0.05} = 100{,}000$

Thus,

$$R' = k_m R = (0.05)(100{,}000) = 5\,\text{k}\Omega$$

$$L' = \frac{k_m}{k_f}L = \frac{100{,}000}{40{,}000}(1) = 2.5\,\text{H}$$

$$C' = \frac{C}{k_m k_f} = \frac{1}{(40{,}000)(100{,}000)} = 250\,\text{pF}$$

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[c]

P 15.16 **[a]** Since $\omega_o^2 = 1/LC$ and $\omega_o = 1$ rad/s,

$$C = \frac{1}{L} = \frac{1}{Q}\,\text{F}$$

[b] $$H(s) = \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$$

$$H(s) = \frac{(1/Q)s}{s^2 + (1/Q)s + 1}$$

[c] In the prototype circuit

$$R = 1\,\Omega; \qquad L = 16\,\text{H}; \qquad C = \frac{1}{L} = 0.0625\,\text{F}$$

$$\therefore \quad k_m = \frac{R'}{R} = 10{,}000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 25{,}000$$

Thus

$$R' = k_m R = 10\,\text{k}\Omega$$

$$L' = \frac{k_m}{k_f}L = \frac{10{,}000}{25{,}000}(16) = 6.4\,\text{H}$$

$$C' = \frac{C}{k_m k_f} = \frac{0.0625}{(10{,}000)(25{,}000)} = 250\,\text{pF}$$

[d]

[e] $$H'(s) = \frac{\frac{1}{16}\left(\frac{s}{25{,}000}\right)}{\left(\frac{s}{25{,}000}\right)^2 + \frac{1}{16}\left(\frac{s}{25{,}000}\right) + 1}$$

$$H'(s) = \frac{1562.5s}{s^2 + 1562.5s + 625 \times 10^6}$$

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P 15.17 **[a]** Using the first prototype

$$\omega_o = 1 \text{ rad/s}; \qquad C = 1\,\text{F}; \qquad L = 1\,\text{H}; \qquad R = 25\,\Omega$$

$$k_m = \frac{R'}{R} = \frac{40{,}000}{25} = 1600; \qquad k_f = \frac{\omega_o'}{\omega_o} = 50{,}000$$

Thus,

$$R' = k_m R = 40\,\text{k}\Omega; \qquad L' = \frac{k_m}{k_f}L = \frac{1600}{50{,}000}(1) = 32\,\text{mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{1}{(1600)(50{,}000)} = 12.5\,\text{nF}$$

Using the second prototype

$$\omega_o = 1 \text{ rad/s}; \qquad C = 25\,\text{F}$$

$$L = \frac{1}{25} = 40\,\text{mH}; \qquad R = 1\,\Omega$$

$$k_m = \frac{R'}{R} = 40{,}000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 50{,}000$$

Thus,

$$R' = k_m R = 40\,\text{k}\Omega; \qquad L' = \frac{k_m}{k_f}L = \frac{40{,}000}{50{,}000}(0.04) = 32\,\text{mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{25}{(40{,}000)(50{,}000)} = 12.5\,\text{nF}$$

[b]

P 15.18 **[a]** For the circuit in Fig. P15.18(a)

$$H(s) = \frac{V_o}{V_i} = \frac{s + \dfrac{1}{s}}{\dfrac{1}{Q} + s + \dfrac{1}{s}} = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

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For the circuit in Fig. P15.18(b)

$$H(s) = \frac{V_o}{V_i} = \frac{Qs + \frac{Q}{s}}{1 + Qs + \frac{Q}{s}}$$

$$= \frac{Q(s^2+1)}{Qs^2 + s + Q}$$

$$H(s) = \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

[b] $$H'(s) = \frac{\left(\frac{s}{8000}\right)^2 + 1}{\left(\frac{s}{8000}\right)^2 + \frac{1}{10}\left(\frac{s}{8000}\right) + 1}$$

$$= \frac{s^2 + 64 \times 10^6}{s^2 + 800s + 64 \times 10^6}$$

P 15.19 For the scaled circuit

$$H'(s) = \frac{s^2 + \left(\frac{1}{L'C'}\right)}{s^2 + \left(\frac{R'}{L'}\right)s + \left(\frac{1}{L'C'}\right)}$$

$$L' = \frac{k_m}{k_f}L; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}; \qquad R' = k_m R$$

$$\therefore \quad \frac{R'}{L'} = k_f\left(\frac{R}{L}\right)$$

It follows then that

$$H'(s) = \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{R}{L}\right)k_f s + \frac{k_f^2}{LC}}$$

$$= \frac{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{LC}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \left(\frac{R}{L}\right)\left(\frac{s}{k_f}\right) + \left(\frac{1}{LC}\right)\right]}$$

$$= H(s)\big|_{s=s/k_f}$$

P 15.20 For the circuit in Fig. 15.31

$$H(s) = \frac{s^2 + \left(\frac{1}{LC}\right)}{s^2 + \frac{s}{RC} + \left(\frac{1}{LC}\right)}$$

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It follows that

$$H'(s) = \frac{s^2 + \frac{1}{L'C'}}{s^2 + \frac{s}{R'C'} + \frac{1}{L'C'}}$$

where $R' = k_m R; \qquad L' = \dfrac{k_m}{k_f} L;$

$$C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}$$

$$\frac{1}{R'C'} = \frac{k_f}{RC}$$

$$\begin{aligned} H'(s) &= \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{k_f}{RC}\right) s + \frac{k_f^2}{LC}} \\ &= \frac{\left(\frac{s}{k_f}\right)^2 + \frac{1}{LC}}{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{RC}\right)\left(\frac{s}{k_f}\right) + \frac{1}{LC}} \\ &= H(s)\big|_{s=s/k_f} \end{aligned}$$

P 15.21 For prototype circuit (a):

$$H(s) = \frac{V_o}{V_i} = \frac{Q}{Q + \frac{1}{s+\frac{1}{s}}} = \frac{Q}{Q + \frac{s}{s^2+1}}$$

$$H(s) = \frac{Q(s^2+1)}{Q(s^2+1) + s} = \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right) s + 1}$$

For prototype circuit (b):

$$\begin{aligned} H(s) &= \frac{V_o}{V_i} = \frac{1}{1 + \frac{(s/Q)}{(s^2+1)}} \\ &= \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right) s + 1} \end{aligned}$$

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P 15.22 **[a]** $k_m = \dfrac{80}{200} = 0.4$

$$\frac{k_m}{k_f}(0.4) = 0.02 \qquad \text{so} \qquad k_f = \frac{(0.4)(0.4)}{0.02} = 8$$

$$C' = \frac{20 \times 10^{-6}}{(0.4)(8)} = 6.25\,\mu\text{F}$$

[b] $Z_{\text{ab}} = \dfrac{1}{j\omega C} + R \| j\omega L = \dfrac{1}{j\omega C} + \dfrac{j\omega RL}{R + j\omega L}$

$$= \frac{(R + j\omega L - \omega^2 RLC)(-\omega^2 LC - j\omega RC)}{(-\omega^2 LC + j\omega RC)(-\omega^2 LC - j\omega RC)}$$

The denominator is purely real, so set the imaginary part of the numerator equal to 0 and solve for ω:

$$-\omega R^2 C - \omega^3 L^2 C^3_\omega R^2 LC^2 = 0$$

$$\therefore \qquad \omega^2 = \frac{R^2}{R^2 LC - L^2} = \frac{(80)^2}{(80)^2(0.02(6.25 \times 10^{-6}) - (0.02)^2} = 16 \times 10^6$$

Thus,

$$\omega = 4000 \text{ rad/s}$$

[c] In the original, unscaled circuit, the frequency at which the impedance Z_{ab} is purely real is

$$\omega^2_{\text{us}} = \frac{(200)^2}{(200)^2(0.4)(20 \times 10^{-6}) - (0.4)^2} = 250{,}000$$

Thus, $\qquad \omega_{\text{us}} = 500 \text{ rad/s}$

$$\frac{\omega_{\text{scaled}}}{\omega_{\text{us}}} = \frac{4000}{500} = 8 = k_f$$

P 15.23 **[a]** $k_f = \dfrac{10{,}000}{250} = 40$

$$C' = \frac{100 \times 10^{-6}}{40} = 2.5\,\mu\text{F}$$

$$L' = \frac{0.8}{40} = 20\,\text{mH}$$

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[b] $I_o = \dfrac{20 - j40}{120 + j160}(0.06) = 13.42\underline{/-116.57^\circ}\,\text{mA}$

$i_o = 13.42\cos(10{,}000t - 166.57^\circ)\,\text{mA}$

The magnitude and phase angle of the output current are the same as in the unscaled circuit.

P 15.24 From the solution to Problem 14.18, $\omega_o = 100$ krad/s and $\beta = 12.5$ krad/s. Compute the two scale factors:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(25 \times 10^3)}{100 \times 10^3} = \frac{\pi}{2}$$

$$L' = \frac{k_m L}{k_f} \qquad \text{so} \qquad k_m = \frac{L'}{L}k_f = \frac{25 \times 10^{-6}}{10 \times 10^{-3}}\left(\frac{\pi}{2}\right) = \frac{\pi}{800}$$

Thus,

$$R' = k_m R = \frac{8000\pi}{800} = 31.4\,\Omega \qquad C' = \frac{C}{k_m k_f} = \frac{10 \times 10^{-9}}{(\pi/800)(\pi/2)} = 1.62\,\mu\text{H}$$

Calculate the cutoff frequencies:

$$\omega_{c1}' = k_f\omega_{c1} = \frac{\pi}{2}(93.95 \times 10^3) = 147.576 \text{ krad/s}$$

$$\omega_{c2}' = k_f\omega_{c2} = \frac{\pi}{2}(106.45 \times 10^3) = 167.211 \text{ krad/s}$$

To check, calculate the bandwidth:

$$\beta' = \omega_{c2}' - \omega_{c1}' = 19.6 \text{ krad/s} = (\pi/2)\beta \text{ (checks!)}$$

P 15.25 From the solution to Problem 14.38, $\omega_o = 80$ krad/s and $\beta = 5.33$ krad/s. Calculate the scale factors:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{16 \times 10^3}{80 \times 10^3} = 0.2$$

$$k_m = \frac{C}{C'k_f} = \frac{62.5 \times 10^{-9}}{(50 \times 10^{-9})(0.2)} = 6.25$$

Thus,

$$R' = k_m R = (6.25)(3000) = 18.75\,\text{k}\Omega$$

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$$L' = \frac{k_m L}{k_f} = \frac{(6.25)(2.5 \times 10^{-3})}{0.2} = 78.125\,\text{mH}$$

Calculate the bandwidth:

$$\beta' = k_f \beta = (0.2)[5333.33] = 1066.67 \text{ rad/s}$$

To check, calculate the quality factor:

$$Q = \frac{\omega_o}{\beta} = \frac{80{,}000}{5333.33} = 15$$

$$Q' = \frac{\omega_o'}{\beta'} = \frac{16{,}000}{1066.67} = 15 \text{ (checks)}$$

P 15.26 **[a]** From Eq 15.1 we have

$$H(s) = \frac{-K\omega_c}{s + \omega_c}$$

where $K = \dfrac{R_2}{R_1}, \quad \omega_c = \dfrac{1}{R_2 C}$

$$\therefore \quad H'(s) = \frac{-K'\omega_c'}{s + \omega_c'}$$

where $K' = \dfrac{R_2'}{R_1'} \qquad \omega_c' = \dfrac{1}{R_2' C'}$

By hypothesis $R_1' = k_m R_1; \qquad R_2' = k_m R_2,$

and $C' = \dfrac{C}{k_f k_m}$. It follows that

$K' = K$ and $\omega_c' = k_f \omega_c$, therefore

$$H'(s) = \frac{-K k_f \omega_c}{s + k_f \omega_c} = \frac{-K\omega_c}{\left(\frac{s}{k_f}\right) + \omega_c}$$

[b] $H(s) = \dfrac{-K}{s+1}$

[c] $H'(s) = \dfrac{-K}{\left(\frac{s}{k_f}\right) + 1} = \dfrac{-K k_f}{s + k_f}$

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P 15.27 **[a]** From Eq. 15.4

$$H(s) = \frac{-Ks}{s+\omega_c} \text{ where } K = \frac{R_2}{R_1} \text{ and}$$

$$\omega_c = \frac{1}{R_1 C}$$

$$\therefore \quad H'(s) = \frac{-K's}{s+\omega_c'} \text{ where } K' = \frac{R_2'}{R_1'}$$

$$\text{and } \omega_c' = \frac{1}{R_1' C'}$$

By hypothesis

$$R_1' = k_m R_1; \qquad R_2' = k_m R_2; \qquad C' = \frac{C}{k_m k_f}$$

It follows that

$$K' = K \text{ and } \omega_c' = k_f \omega_c$$

$$\therefore \quad H'(s) = \frac{-Ks}{s + k_f\omega_c} = \frac{-K(s/k_f)}{\left(\frac{s}{k_f}\right) + \omega_c}$$

[b] $H(s) = \dfrac{-Ks}{s+1}$

[c] $H'(s) = \dfrac{-K(s/k_f)}{\left(\frac{s}{k_f}+1\right)} = \dfrac{-Ks}{s+k_f}$

P 15.28 **[a]** $H_{\text{hp}} = \dfrac{s}{s+1}; \qquad k_f = \dfrac{\omega_o'}{\omega} = \dfrac{200(2\pi)}{1} = 400\pi$

$$\therefore \quad H_{\text{hp}}' = \frac{s}{s+400\pi}$$

$$\frac{1}{R_H C_H} = 400\pi; \qquad \therefore \quad R_H = \frac{1}{(400\pi)(50\times 10^{-9})} = 15.915\,\text{k}\Omega$$

$$H_{\text{lp}} = \frac{1}{s+1}; \qquad k_f = \frac{\omega_o'}{\omega} = \frac{2000(2\pi)}{1} = 4000\pi$$

$$\therefore \quad H_{\text{lp}}' = \frac{4000\pi}{s+4000\pi}$$

$$\frac{1}{R_L C_L} = 4000\pi; \qquad \therefore \quad R_L = \frac{1}{(4000\pi)(50\times 10^{-9})} = 1591.5\,\Omega$$

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[b] $H'(s) = \dfrac{s}{s+400\pi} \cdot \dfrac{4000\pi}{s+4000\pi} \cdot 10$

$$= \frac{40{,}000\pi s}{(s+400\pi)(s+4000\pi)}$$

[c] $\omega_o = \sqrt{\omega_{c1}\omega_{c1}} = \sqrt{(400\pi)(4000\pi)} = 3973.8 \text{ rad/s}$

$$H'(j\omega_o) = \frac{(40{,}000\pi)(j3973.8)}{(400\pi + j3973.8)(4000\pi + j3973.8)}$$

$$= 9.1\underline{/0^\circ}$$

[d] $G = 20\log_{10}(9.1) = 19.2 \text{ dB}$

[e]

P 15.29 **[a]** For the high-pass section:

$$k_f = \frac{\omega_o'}{\omega} = \frac{800(2\pi)}{1} = 1600\pi$$

$$H'(s) = \frac{s}{s+1600\pi}$$

$$\therefore \quad \frac{1}{R_1(2\times 10^{-6})} = 1600\pi; \qquad R_1 = 99.5\,\Omega \quad \therefore \quad R_2 = 99.5\,\Omega$$

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For the low-pass section:

$$k_f = \frac{\omega_o'}{\omega} = \frac{80(2\pi)}{1} = 160\pi$$

$$H'(s) = \frac{160\pi}{s + 160\pi}$$

$$\therefore \qquad \frac{1}{R_2(2 \times 10^{-6})} = 160\pi; \qquad R_2 = 994.7\,\Omega \qquad \therefore \qquad R_1 = 994.7\,\Omega$$

0 dB gain corresponds to $K = 1$. In the summing amplifier we are free to choose R_f and R_i so long as $R_f/R_i = 1$. To keep from having many different resistance values in the circuit we opt for $R_f = R_i = 994.7\,\Omega$.

[b]

[c] $$H'(s) = \frac{s}{s + 1600\pi} + \frac{160\pi}{s + 160\pi}$$

$$= \frac{s^2 + 320\pi s + 256 \times 10^3 \pi^2}{(s + 160\pi)(s + 1600\pi)}$$

[d] $\omega_o = \sqrt{(1600\pi)(160\pi)} = 1589.5$

$$H'(j1589.5) = \frac{-(1589.5)^2 + 320\pi(j1589.5) + 256 \times 10^3\pi^2}{(160\pi + j1589.5)(1600\pi + j1589.5)}$$

$$= 0.1818\underline{/0^\circ}$$

[e] $G = 20\log_{10} 0.1818 = -14.81$ dB

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[f]

P 15.30 $\omega_o = 2\pi f_o = 400\pi \text{ rad/s}$

$\beta = 2\pi(1000) = 2000\pi \text{ rad/s}$

$\therefore \quad \omega_{c_2} - \omega_{c_1} = 2000\pi$

$\sqrt{\omega_{c_1}\omega_{c_2}} = \omega_o = 400\pi$

Solve for the cutoff frequencies:

$\omega_{c_1}\omega_{c_2} = 16 \times 10^4\pi^2$

$$\omega_{c_2} = \frac{16 \times 10^4\pi^2}{\omega_{c_1}}$$

$$\therefore \quad \frac{16 \times 10^4\pi^2}{\omega_{c_1}} - \omega_{c_1} = 2000\pi$$

or $\omega_{c_1}^2 + 2000\pi\omega_{c_1} - 16 \times 10^4\pi^2 = 0$

$\omega_{c_1} = -1000\pi \pm \sqrt{10^6\pi^2 + 0.16 \times 10^6\pi^2}$

$\omega_{c_1} = 1000\pi(-1 \pm \sqrt{1.16}) = 242.01 \text{ rad/s}$

$\therefore \quad \omega_{c_2} = 2000\pi + 242.01 = 6525.19 \text{ rad/s}$

Thus, $f_{c1} = 38.52$ Hz and $f_{c2} = 1038.52$ Hz

Check: $\beta = f_{c2} - f_{c1} = 1000\text{Hz}$

$$\omega_{c2} = \frac{1}{R_L C_L} = 6525.19$$

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$$R_L = \frac{1}{(6525.19)(5 \times 10^{-6})} = 30.65\,\Omega$$

$$\omega_{c1} = \frac{1}{R_H C_H} = 242.01$$

$$R_H = \frac{1}{(242.01)(5 \times 10^{-6})} = 826.43\,\Omega$$

P 15.31 $\omega_o = 1000\,\text{rad/s}; \qquad \text{GAIN} = 6$

$\beta = 4000\ \text{rad/s}; \qquad C = 0.2\,\mu\text{F}$

$$\beta = \omega_{c_2} - \omega_{c_1} = 4000$$

$$\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}} = 1000$$

Solve for the cutoff frequencies:

$$\therefore \quad \omega_{c_1}^2 + 4000\omega_{c_1} - 10^6 = 0$$

$$\omega_{c_1} = -2000 \pm 1000\sqrt{5} = 236.07\,\text{rad/s}$$

$$\omega_{c_2} = 4000 + \omega_{c_1} = 4236.07\,\text{rad/s}$$

Check: $\quad \beta = \omega_{c2} - \omega_{c1} = 4000\ \text{rad/s}$

$$\omega_{c_1} = \frac{1}{R_L C_L}$$

$$\therefore \quad R_L = \frac{1}{(0.2 \times 10^{-6})(236.07)} = 21.18\,\text{k}\Omega$$

$$\frac{1}{R_H C_H} = 4236.07$$

$$R_H = \frac{1}{(0.2 \times 10^{-6})(4236.07)} = 1.18\,\text{k}\Omega$$

$$\frac{R_f}{R_i} = 6$$

If $R_i = 1\,\text{k}\Omega \qquad R_f = 6R_i = 6\,\text{k}\Omega$

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P 15.32 $H(s) = \dfrac{V_o}{V_i} = \dfrac{-Z_f}{Z_i}$

$$Z_f = \frac{1}{sC_2}\|R_2 = \frac{(1/C_2)}{s + (1/R_2C_2)}; \qquad Z_i = R_1 + \frac{1}{sC_1} = \frac{sR_1C_1 + 1}{sC_1}$$

$$\therefore \quad H(s) = \frac{\dfrac{-1/C_2}{s + (1/R_2C_2)}}{\dfrac{s + (1/R_1C_1)}{s/R_1}} = \frac{-(1/R_1C_2)s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

$$= \frac{-K\beta s}{s^2 + \beta s + \omega_o^2}$$

[a] $H(s) = \dfrac{-250s}{(s+50)(s+20)} = \dfrac{-250s}{s^2 + 70s + 1000} = \dfrac{-3.57(70s)}{s^2 + 70s + (\sqrt{1000})^2}$

$\omega_o = \sqrt{1000} = 31.6$ rad/s

$\beta = 70$ rad/s

$K = -3.57$

[b] $Q = \dfrac{\omega_o}{\beta} = 0.45$

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2} = \pm 35 + \sqrt{35^2 + 1000} = \pm 35 + 47.17$$

$\omega_{c1} = 12.17$ rad/s $\qquad \omega_{c2} = 82.17$ rad/s

P 15.33 **[a]** $H(s) = \dfrac{(1/sC)}{R + (1/sC)} = \dfrac{(1/RC)}{s + (1/RC)}$

$$H(j\omega) = \frac{(1/RC)}{j\omega + (1/RC)}$$

$$|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + (1/RC)^2}}$$

$$|H(j\omega)|^2 = \frac{(1/RC)^2}{\omega^2 + (1/RC)^2}$$

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[b] Let V_a be the voltage across the capacitor, positive at the upper terminal. Then

$$\frac{V_a - V_{in}}{R_1} + sCV_a + \frac{V_a}{R_2 + sL} = 0$$

Solving for V_a yields

$$V_a = \frac{(R_2 + sL)V_{in}}{R_1LCs^2 + (R_1R_2C + L)s + (R_1 + R_2)}$$

But

$$v_o = \frac{sLV_a}{R_2 + sL}$$

Therefore

$$V_o = \frac{sLV_{in}}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(s) = \frac{sL}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(j\omega) = \frac{j\omega L}{[(R_1 + R_2) - R_1LC\omega^2] + j\omega(L + R_1R_2C)}$$

$$|H(j\omega)| = \frac{\omega L}{\sqrt{[R_1 + R_2 - R_1LC\omega^2]^2 + \omega^2(L + R_1R_2C)^2}}$$

$$|H(j\omega)|^2 = \frac{\omega^2L^2}{(R_1 + R_2 - R_1LC\omega^2)^2 + \omega^2(L + R_1R_2C)^2}$$

$$= \frac{\omega^2L^2}{R_1^2L^2C^2\omega^4 + (L^2 + R_1^2R_2^2C^2 - 2R_1^2LC)\omega^2 + (R_1 + R_2)^2}$$

[c] Let V_a be the voltage across R_2 positive at the upper terminal. Then

$$\frac{V_a - V_{in}}{R_1} + \frac{V_a}{R_2} + V_asC + V_asC = 0$$

$$(0 - V_a)sC + (0 - V_a)sC + \frac{0 - V_o}{R_3} = 0$$

$$\therefore \quad V_a = \frac{R_2V_{in}}{2R_1R_2Cs + R_1 + R_2}$$

and $V_a = -\dfrac{V_o}{2R_3Cs}$

It follows directly that

$$H(s) = \frac{V_o}{V_{in}} = \frac{-2R_2R_3Cs}{2R_1R_2Cs + (R_1 + R_2)}$$

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$$H(j\omega) = \frac{-2R_2R_3C(j\omega)}{(R_1+R_2)+j\omega(2R_1R_2C)}$$

$$|H(j\omega)| = \frac{2R_2R_3C\omega}{\sqrt{(R_1+R_2)^2+\omega^2 4R_1^2R_2^2C^2}}$$

$$|H(j\omega)|^2 = \frac{4R_2^2R_3^2C^2\omega^2}{(R_1+R_2)^2+4R_1^2R_2^2C^2\omega^2}$$

P 15.34 **[a]** $n \cong \dfrac{(-0.05)(-30)}{\log_{10}(7000/2000)} \cong 2.76$

$\therefore \quad n = 3$

[b] Gain $= 20\log_{10}\dfrac{1}{\sqrt{1+(7000/2000)^6}} = -32.65$ dB

P 15.35 For the scaled circuit

$$H'(s) = \frac{1/(R')^2C_1'C_2'}{s^2+\frac{2}{R'C_1'}s+\frac{1}{(R')^2C_1'C_2'}}$$

where

$$R' = k_mR; \qquad C_1' = C_1/k_fk_m; \qquad C_2' = C_2/k_fk_m$$

It follows that

$$\frac{1}{(R')^2C_1'C_2'} = \frac{k_f^2}{R^2C_1C_2}$$

$$\frac{2}{R'C_1'} = \frac{2k_f}{RC_1}$$

$$\therefore \quad H'(s) = \frac{k_f^2/RC_1C_2}{s^2+\frac{2k_f}{RC_1}s+\frac{k_f^2}{R^2C_1C_2}} = \frac{1/RC_1C_2}{\left(\frac{s}{k_f}\right)^2+\frac{2}{RC_1}\left(\frac{s}{k_f}\right)+\frac{1}{R^2C_1C_2}}$$

P 15.36 **[a]** $H(s) = \dfrac{1}{(s+1)(s^2+s+1)}$

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[b] $f_c = 2000\,\text{Hz}; \qquad \omega_c = 4000\pi \text{ rad/s}; \qquad k_f = 4000\pi$

$$H'(s) = \frac{1}{(\frac{s}{k_f}+1)[(\frac{s}{k_f})^2 + \frac{s}{k_f} + 1]}$$

$$= \frac{k_f^3}{(s+k_f)(s^2+k_f s + k_f^2)}$$

$$= \frac{(4000\pi)^3}{(s+4000\pi)[s^2+4000\pi s + (4000\pi)^2]}$$

[c] $H'(j14{,}000\pi) = \dfrac{64}{(4+j14)(-180+j52)}$

$$= 0.02332\underline{/-236.77^\circ}$$

$$\text{Gain } = 20\log_{10}(0.02332) = -32.65 \text{ dB}$$

P 15.37 **[a]** In the first-order circuit $R = 1\,\Omega$ and $C = 1$ F.

$$k_m = \frac{R'}{R} = \frac{1000}{1} = 1000; \qquad k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(2000)}{1} = 4000\pi$$

$$R' = k_m R = 1000\,\Omega; \qquad C' = \frac{C}{k_m k_f} = \frac{1}{(1000)(4000\pi)} = 79.58\,\text{nF}$$

In the second-order circuit $R = 1\,\Omega$, $2/C_1 = 1$ so $C_1 = 2$ F, and $C_2 = 1/C_1 = 0.5$ F. Therefore in the scaled second-order circuit

$$R' = k_m R = 1000\,\Omega; \qquad C_1' = \frac{C_1}{k_m k_f} = \frac{2}{(1000)(4000\pi)} = 159.15\,\text{nF}$$

$$C_2' = \frac{C_2}{k_m k_f} = \frac{0.5}{(1000)(4000\pi)} = 39.79\,\text{nF}$$

[b]

P 15.38 **[a]** $y = 20\log_{10}\dfrac{1}{\sqrt{1+\omega^{2n}}} = -10\log_{10}(1+\omega^{2n})$

From the laws of logarithms we have

$$y = \left(\frac{-10}{\ln 10}\right)\ln(1+\omega^{2n})$$

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Thus

$$\frac{dy}{d\omega} = \left(\frac{-10}{\ln 10}\right)\frac{2n\omega^{2n-1}}{(1+\omega^{2n})}$$

$$x = \log_{10}\omega = \frac{\ln\omega}{\ln 10}$$

$$\therefore \quad \ln\omega = x\ln 10$$

$$\frac{1}{\omega}\frac{d\omega}{dx} = \ln 10, \qquad \frac{d\omega}{dx} = \omega\ln 10$$

$$\frac{dy}{dx} = \left(\frac{dy}{d\omega}\right)\left(\frac{d\omega}{dx}\right) = \frac{-20n\omega^{2n}}{1+\omega^{2n}}\,\text{dB/decade}$$

at $\omega = \omega_c = 1\,\text{rad/s}$

$$\frac{dy}{dx} = -10n\,\text{dB/decade.}$$

[b] $y = 20\log_{10}\dfrac{1}{[\sqrt{1+\omega^2}]^n} = -10n\log_{10}(1+\omega^2)$

$$= \frac{-10n}{\ln 10}\ln(1+\omega^2)$$

$$\frac{dy}{d\omega} = \frac{-10n}{\ln 10}\left(\frac{1}{1+\omega^2}\right)2\omega = \frac{-20n\omega}{(\ln 10)(1+\omega^2)}$$

As before

$$\frac{d\omega}{dx} = \omega(\ln 10); \qquad \therefore \quad \frac{dy}{dx} = \frac{-20n\omega^2}{(1+\omega^2)}$$

At the corner $\omega_c = \sqrt{2^{1/n}-1}$ $\qquad \therefore \quad \omega_c^2 = 2^{1/n}-1$

$$\frac{dy}{dx} = \frac{-20n[2^{1/n}-1]}{2^{1/n}}\,\text{dB/decade.}$$

[c] For the Butterworth Filter

n	dy/dx (dB/decade)
1	-10
2	-20
3	-30
4	-40
∞	$-\infty$

For the cascade of identical sections

n	dy/dx (dB/decade)
1	-10
2	-11.72
3	-12.38
4	-12.73
∞	-13.86

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[d] It is apparent from the calculations in part (c) that as n increases the amplitude characteristic at the cutoff frequency decreases at a much faster rate for the Butterworth filter.
Hence the transition region of the Butterworth filter will be much narrower than that of the cascaded sections.

P 15.39 $n = 5$: $1 + (-1)^5 s^{10} = 0$; $\quad s^{10} = 1$

$s^{10} = 1\underline{/(0 + 360k)^\circ} \quad$ so $\quad s = 1\underline{/36k^\circ}$

k	s_{k+1}	k	s_{k+1}
0	$1\underline{/0^\circ}$	5	$1\underline{/180^\circ}$
1	$1\underline{/36^\circ}$	6	$1\underline{/216^\circ}$
2	$1\underline{/72^\circ}$	7	$1\underline{/252^\circ}$
3	$1\underline{/108^\circ}$	8	$1\underline{/288^\circ}$
4	$1\underline{/144^\circ}$	9	$1\underline{/324^\circ}$

Group by conjugate pairs to form denominator polynomial.

$$(s+1)[s - (\cos 108^\circ + j\sin 108^\circ)][(s - (\cos 252^\circ + j\sin 252^\circ)]$$

$$\cdot\,[(s - (\cos 144^\circ + j\sin 144^\circ)][(s - (\cos 216^\circ + j\sin 216^\circ)]$$

$$= (s+1)(s + 0.309 - j0.951)(s + 0.309 + j0.951)\cdot$$

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$$(s + 0.809 - j0.588)(s + 0.809 + j0.588)$$

which reduces to

$$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$$

$n = 6$: $1 + (-1)^6 s^{12} = 0 \qquad s^{12} = -1$

$s^{12} = 1\underline{/180^\circ + 360k}$

k	s_{k+1}	k	s_{k+1}
0	$1\underline{/15^\circ}$	6	$1\underline{/195^\circ}$
1	$1\underline{/45^\circ}$	7	$1\underline{/225^\circ}$
2	$1\underline{/75^\circ}$	8	$1\underline{/255^\circ}$
3	$1\underline{/105^\circ}$	9	$1\underline{/285^\circ}$
4	$1\underline{/135^\circ}$	10	$1\underline{/315^\circ}$
5	$1\underline{/165^\circ}$	11	$1\underline{/345^\circ}$

Grouping by conjugate pairs yields

$$(s + 0.2588 - j0.9659)(s + 0.2588 + j0.9659)\times$$

$$(s + 0.7071 - j0.7071)(s + 0.7071 + j0.7071)\times$$

$$(s + 0.9659 - j0.2588)(s + 0.9659 + j0.2588)$$

or $(s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.9318s + 1)$

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P 15.40 $$H'(s) = \frac{s^2}{s^2 + \dfrac{2}{k_m R_2 (C/k_m k_f)} s + \dfrac{1}{k_m R_1 k_m R_2 (C^2/k_m^2 k_f^2)}}$$

$$H'(s) = \frac{s^2}{s^2 + \dfrac{2k_f}{R_2 C} s + \dfrac{k_f^2}{R_1 R_2 C^2}}$$

$$= \frac{(s/k_f)^2}{(s/k_f)^2 + \dfrac{2}{R_2 C}\left(\dfrac{s}{k_f}\right) + \dfrac{1}{R_1 R_2 C^2}}$$

P 15.41 **[a]** $n = \dfrac{(-0.05)(-55)}{\log_{10}(2500/500)} = 3.93 \quad \therefore \quad n = 4$

From Table 15.1 the transfer function is

$$H(s) = \frac{s^2}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The capacitor values for the first stage prototype circuit are

$$\frac{2}{C_1} = 0.765 \quad \therefore \quad C_1 = 2.61\,\text{F}$$

$$C_2 = \frac{1}{C_1} = 0.38\,\text{F}$$

The values for the second stage prototype circuit are

$$\frac{2}{C_1} = 1.848 \quad \therefore \quad C_1 = 1.08\,\text{F}$$

$$C_2 = \frac{1}{C_1} = 0.92\,\text{F}$$

The scaling factors are

$$k_m = \frac{R'}{R} = 8000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 5000\pi$$

Therefore the scaled values for the components in the first stage are

$$R_1 = R_2 = R = 8000\,\Omega$$

$$C_1 = \frac{2.61}{(5000\pi)(8000)} = 20.8\,\text{nF}$$

$$C_2 = \frac{0.38}{(5000\pi)(8000)} = 3.04\,\text{nF}$$

The scaled values for the second stage are

$$R_1 = R_2 = R = 8000\,\Omega$$

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$$C_1 = \frac{1.08}{(5000\pi)(8000)} = 8.61\,\text{nF}$$

$$C_2 = \frac{0.92}{(5000\pi)(8000)} = 7.35\,\text{nF}$$

[b]

P 15.42 **[a]** $n = \dfrac{(-0.05)(-55)}{\log_{10}(200/40)} = 3.93 \quad \therefore \quad n = 4$

From Table 15.1 the transfer function of the first section is

$$H_1(s) = \frac{1}{s^2 + 0.765s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 0.765; \qquad R_2 = 2.61\,\Omega; \qquad R_1 = \frac{1}{R_2} = 0.383\,\Omega$$

The transfer function of the second section is

$$H_2(s) = \frac{1}{s^2 + 1.848s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 1.848; \qquad R_2 = 1.082\,\Omega; \qquad R_1 = \frac{1}{R_2} = 0.9240\,\Omega$$

The scaling factors are:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(40{,}000)}{1} = 80{,}000\pi$$

$$C' = \frac{C}{k_m k_f} \quad \therefore \quad 250 \times 10^{-9} = \frac{1}{80{,}000\pi k_m}$$

$$\therefore \quad k_m = \frac{1}{80{,}000\pi(250 \times 10^{-9})} = 15.9$$

Therefore in the first section

$$R_1' = k_m R_1 = 6.1\,\Omega; \qquad R_2' = k_m R_2 = 41.5\,\Omega$$

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In the second section

$R_1' = k_m R_1 = 14.7\,\Omega; \qquad R_2' = k_m R_2 = 17.2\,\Omega$

[b]

P 15.43 **[a]** The cascade connection is a bandpass filter.

[b] The cutoff frequencies are 2.5 kHz and 40 kHz.
The center frequency is $\sqrt{(2.5)(40)} = 10$ kHz.
The Q is $10/(40 - 2.5) = 0.267$

[c] For the high pass section $k_f = 5000\pi$. The prototype transfer function is

$$H_{\text{hp}}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

$$\therefore \qquad H_{\text{hp}}'(s) = \frac{(s/5000\pi)^4}{[(s/5000\pi)^2 + 0.765(s/5000\pi) + 1]} \cdot \frac{1}{[(s/5000\pi)^2 + 1.848(s/5000\pi) + 1]}$$

$$= \frac{s^4}{(s^2 + 3825\pi s + 25 \times 10^6\pi^2)(s^2 + 9240\pi s + 25 \times 10^6\pi^2)}$$

For the low pass section $k_f = 80{,}000\pi$

$$H_{\text{lp}}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

$$\therefore \qquad H_{\text{lp}}'(s) = \frac{1}{[(s/80{,}000\pi)^2 + 0.765(s/80{,}000\pi) + 1]} \cdot \frac{1}{[(s/80{,}000\pi)^2 + 1.848(s/80{,}000\pi) + 1]}$$

$$= \frac{(80{,}000\pi)^4}{([s^2 + 61{,}200\pi s + (80{,}000\pi)^2)][s^2 + 147{,}840\pi s + (80{,}000\pi)^2]}$$

The cascaded transfer function is

$$H'(s) = H_{\text{hp}}'(s)H_{\text{lp}}'(s)$$

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For convenience let

$$D_1 = s^2 + 3825\pi s + 25 \times 10^6\pi^2$$

$$D_2 = s^2 + 9240\pi s + 25 \times 10^6\pi^2$$

$$D_3 = s^2 + 61{,}200\pi s + 6400 \times 10^6\pi^2$$

$$D_4 = s^2 + 147{,}840\pi s + 6400 \times 10^6\pi^2$$

Then

$$H'(s) = \frac{4096 \times 10^{16}\pi^4 s^4}{D_1 D_2 D_3 D_4}$$

[d] $\omega_o = 2\pi(10{,}000) = 20{,}000\pi$ rad/s

$$s = j20{,}000\pi$$

$$s^4 = 16 \times 10^{16}\pi^4$$

$$D_1 = (25 \times 10^6\pi^2 - 400 \times 10^6\pi^2) + j(20{,}000\pi)(3825\pi)$$

$$= 10^6\pi^2(-375 - j76.5) = 10^6\pi^2(382.72\underline{/-168.47^\circ})$$

$$D_2 = (25 \times 10^6\pi^2 - 400 \times 10^6\pi^2) + j(20{,}000\pi)(9240\pi)$$

$$= 10^6\pi^2(-375 + j184.8) = 10^6\pi^2(418.06\underline{/153.77^\circ})$$

$$D_3 = (6400 \times 10^6\pi^2 - 400 \times 10^6\pi^2) + j(20{,}000\pi)(61{,}200\pi)$$

$$= 10^6\pi^2(6000 + j1224) = 10^6\pi^2(6123.58\underline{/11.53^\circ})$$

$$D_4 = (6400 \times 10^6\pi^2 - 400 \times 10^6\pi^2) + j(20{,}000\pi)(147{,}840\pi)$$

$$= 10^6\pi^2(6000 + j2956.8) = 10^6\pi^2(6689\underline{/26.23^\circ})$$

$$|H'(j\omega_o)| = \frac{(4096)(16)\pi^8 \times 10^{32}}{(\pi^8 \times 10^{24})[(382.72)(418.06)(6123.58)(6689)]}$$

$$= 0.999$$

P 15.44 **[a]** First we will design a unity gain filter and then provide the passband gain with an inverting amplifier. For the high pass section the cut-off frequency is 200 Hz. The order of the Butterworth is

$$n = \frac{(-0.05)(-40)}{\log_{10}(200/40)} = 2.86$$

$$\therefore \quad n = 3$$

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

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For the prototype first-order section

$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\text{F}$

For the prototype second-order section

$R_1 = 0.5\,\Omega, \quad R_2 = 2\,\Omega, \quad C = 1\,\text{F}$

The scaling factors are

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(200) = 400\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(10^{-6})(400\pi)} = 795.77$$

In the scaled first-order section

$R_1' = R_2' = k_m R_1 = 795.77\,\Omega$

$C' = 1\,\mu\text{F}$

In the scaled second-order section

$R_1' = 0.5k_m = 397.9\,\Omega$

$R_2' = 2k_m = 1591.5\,\Omega$

$C' = 1\,\mu\text{F}$

For the low-pass section the cut-off frequency is 2500 Hz. The order of the Butterworth filter is

$$n = \frac{(-0.05)(-40)}{\log_{10}(12{,}500/2500)} = 2.86; \qquad \therefore \quad n = 3$$

$$H_{\text{lp}}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\text{F}$

For the prototype second-order section

$R_1 = R_2 = 1\,\Omega; \qquad C_1 = 2\,\text{F}; \qquad C_2 = 0.5\,\text{F}$

The low-pass scaling factors are

$$k_m = \frac{R'}{R} = 2500; \qquad k_f = \frac{\omega_o'}{\omega_o} = (2500)(2\pi) = 5000\pi$$

For the scaled first-order section

$$R_1' = R_2' = 2.5\,\text{k}\Omega; \qquad C' = \frac{C}{k_f k_m} = \frac{1}{(5000\pi)(2500)} = 25.46\,\text{nF}$$

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For the scaled second-order section

$R_1' = R_2' = 2.5\,\text{k}\Omega$

$$C_1' = \frac{C_1}{k_f k_m} = \frac{2}{(5000\pi)(2500)} = 50.93\,\text{nF}$$

$$C_2' = \frac{C_2}{k_f k_m} = \frac{0.5}{(5000\pi)(2500)} = 12.73\,\text{nF}$$

GAIN AMPLIFIER

$20\log_{10} K = 40\text{ dB}, \qquad \therefore \quad K = 100$

Since we are using $2.5\,\text{k}\Omega$ resistors in the low-pass stage, we will use $R_f = 250\,\text{k}\Omega$ and $R_i = 2.5\,\text{k}\Omega$ in the inverting amplifier stage.

[b]

P 15.45 **[a]** Unscaled high-pass stage

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega_o'/\omega_o) = 400\pi$. Therefore the scaled transfer function is

$$H_{hp}'(s) = \frac{(s/400\pi)^3}{\left(\frac{s}{400\pi}+1\right)\left[\left(\frac{s}{400\pi}\right)^2 + \frac{s}{400\pi} + 1\right]}$$

$$= \frac{s^3}{(s+400\pi)[s^2 + 400\pi s + 160{,}000\pi^2]}$$

Unscaled low-pass stage

$$H_{lp}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

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The frequency scaling factor is $k_f = (\omega_o'/\omega_o) = 5000\pi$. Therefore the scaled transfer function is

$$H_{lp}'(s) = \frac{1}{\left(\frac{s}{5000\pi}+1\right)\left[\left(\frac{s}{5000\pi}\right)^2 + \left(\frac{s}{5000\pi}\right)+1\right]}$$
$$= \frac{(5000\pi)^3}{(s+5000\pi)(s^2+5000\pi s+25\times 10^6\pi^2)}$$

Thus the transfer function for the filter is

$$H'(s) = 100H_{hp}'(s)H_{lp}'(s) = \frac{125\times 10^{11}\pi^3 s^3}{D_1D_2D_3D_4}$$

where

$$D_1 = s+400\pi$$

$$D_2 = s+5000\pi$$

$$D_3 = s^2+400\pi s+160{,}000\pi^2$$

$$D_4 = s^2+5000\pi s+25\times 10^6\pi^2$$

[b] At 40 Hz $\qquad \omega = 80\pi$ rad/s

$$D_1(j80\pi) = \pi(400+j80)$$

$$D_2(j80\pi) = \pi(5000+j80)$$

$$D_3(j80\pi) = \pi^2(153{,}600+j32{,}000)$$

$$D_4(j80\pi) = \pi^2(24{,}993{,}600+j400{,}000)$$

Therefore

$$D_1D_2D_3D_4(j80\pi) = \pi^6 10^{18}(8\underline{/24.9^\circ})$$

$$H'(j80\pi) = \frac{(125\pi^3\times 10^{11})(512\times 10^3\pi^3)\underline{/-90^\circ}}{\pi^6\times 10^{18}(8\underline{/24.9^\circ})}$$

$$= 0.8\underline{/-114.9^\circ}$$

$$\therefore\quad 20\log_{10}|H'(j80\pi)| = 20\log_{10}(0.8) = -1.94 \text{ dB}$$

At $f = 1000$ Hz, $\qquad \omega = 2000\pi$ rad/s

Then

$$D_1(j2000\pi) = \pi(400+j2000)$$

$$D_2(j2000\pi) = \pi(5000+j2000)$$

$$D_3(j2000\pi) = \pi^2(-3{,}840{,}000+j800{,}000)$$

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$$D_4(j2000\pi) = \pi^2(21{,}000{,}000 + j10^7)$$

$$H'(j2000\pi) = \frac{(125 \times \pi^3 \times 10^{11})(8 \times 10^9\pi^3)\underline{/-90^\circ}}{10.02 \times 10^{20}\pi^6\underline{/-65.8^\circ}}$$

$$= 99.8\underline{/-24.2^\circ}$$

$$\therefore \quad 20\log_{10}|H'(j2000\pi)| = 39.98 \text{ dB}$$

[c] From the transfer function the gain is down $39.98 + 1.94$ or 41.9 dB at 40 Hz. Because the upper cut-off frequency is 25 times the lower cut-off frequency we would expect the high-pass stage of the filter to predict the loss in gain at 40 Hz. For a 3nd order Butterworth,

$$\text{GAIN} = 20\log_{10}\frac{1}{\sqrt{1 + (200/40)^6}} = -41.9 \text{ dB}.$$

1000 Hz is in the passband for this bandpass filter. Hence we expect the gain at 1000 Hz to nearly equal 40 dB as specified in Problem 15.44. Thus our scaled transfer function confirms that the filter meets the specifications.

P 15.46 **[a]** From Table 15.1

$$H_{lp}(s) = \frac{1}{(s+1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)}$$

$$H_{hp}(s) = \frac{1}{\left(\frac{1}{s} + 1\right)\left(\frac{1}{s^2} + 0.618\left(\frac{1}{s}\right) + 1\right)\left(\frac{1}{s^2} + 1.618\left(\frac{1}{s}\right) + 1\right)}$$

$$H_{hp}(s) = \frac{s^5}{(s+1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)}$$

P 15.47 **[a]** $k_f = 800$

$$H'_{\text{hp}}(s) = \frac{(s/800)^5}{[(s/800) + 1]}$$

$$\cdot \frac{1}{[(s/800)^2 + 0.618s/800 + 1][(s/800)^2 + 1.618s/800 + 1]}$$

$$= \frac{s^5}{(s + 800)(s^2 + 494.4s + 64 \times 10^4)}$$

$$\cdot \frac{1}{(s^2 + 1294.4s + 64 \times 10^4)}$$

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[b] $H'(j800) = \dfrac{(j800)^5}{[800 + j800][494.4(j800)][1294.4(j800)]}$

$$= \frac{j(800)^2}{-(1 + j1)(494.4)(1294.4)}$$

$$= 0.707\underline{/-135^\circ}$$

$$20\log_{10}|H'(j800)| = -3.01 \text{ dB}$$

P 15.48 From Eq 15.56 we can write

$$H(s) = \frac{-\left(\dfrac{2}{R_3C}\right)\left(\dfrac{R_3C}{2}\right)\left(\dfrac{1}{R_1C}\right)s}{s^2 + \dfrac{2}{R_3C}s + \dfrac{R_1 + R_2}{R_1R_2R_3C^2}}$$

or

$$H(s) = \frac{-\left(\dfrac{R_3}{2R_1}\right)\left(\dfrac{2}{R_3C}s\right)}{s^2 + \dfrac{2}{R_3C}s + \dfrac{R_1 + R_2}{R_1R_2R_3C^2}}$$

Therefore

$$\frac{2}{R_3C} = \beta = \frac{\omega_o}{Q}; \qquad \frac{R_1 + R_2}{R_1R_2R_3C^2} = \omega_o^2;$$

and $K = \dfrac{R_3}{2R_1}$

By hypothesis $C = 1\,\text{F}$ and $\omega_o = 1\,\text{rad/s}$

$$\therefore \quad \frac{2}{R_3} = \frac{1}{Q} \text{ or } R_3 = 2Q$$

$$R_1 = \frac{R_3}{2K} = \frac{Q}{K}$$

$$\frac{R_1 + R_2}{R_1R_2R_3} = 1$$

$$\frac{Q}{K} + R_2 = \left(\frac{Q}{K}\right)(2Q)R_2$$

$$\therefore \quad R_2 = \frac{Q}{2Q^2 - K}$$

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P 15.49 **[a]** From the statement of the problem, $K = 10$ ($= 20$ dB). Therefore for the prototype bandpass circuit

$$R_1 = \frac{Q}{K} = \frac{16}{10} = 1.6\,\Omega$$

$$R_2 = \frac{Q}{2Q^2 - K} = \frac{16}{502}\,\Omega$$

$$R_3 = 2Q = 32\,\Omega$$

The scaling factors are

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(6400) = 12{,}800\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(20 \times 10^{-9})(12{,}800\pi)} = 1243.40$$

Therefore,

$$R_1' = k_m R_1 = (1.6)(1243.30) = 1.99\,\text{k}\Omega$$

$$R_2' = k_m R_2 = (16/502)(1243.40) = 39.63\,\Omega$$

$$R_3' = k_m R_3 = 32(1243.40) = 39.79\,\text{k}\Omega$$

[b]

P 15.50 **[a]** At very low frequencies the two capacitor branches are open and because the op amp is ideal the current in R_3 is zero. Therefore at low frequencies the circuit behaves as an inverting amplifier with a gain of R_2/R_1. At very high frequencies the capacitor branches are short circuits and hence the output voltage is zero.

[b] Let the node where R_1, R_2, R_3, and C_2 join be denoted as a, then

$$(V_a - V_i)G_1 + V_a sC_2 + (V_a - V_o)G_2 + V_a G_3 = 0$$

$$-V_a G_3 - V_o sC_1 = 0$$

or

$$(G_1 + G_2 + G_3 + sC_2)V_a - G_2 V_o = G_1 V_i$$

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$$V_a = \frac{-sC_1}{G_3}V_o$$

Solving for V_o/V_i yields

$$H(s) = \frac{-G_1G_3}{(G_1+G_2+G_3+sC_2)sC_1+G_2G_3}$$

$$= \frac{-G_1G_3}{s^2C_1C_2+(G_1+G_2+G_3)C_1s+G_2G_3}$$

$$= \frac{-G_1G_3/C_1C_2}{s^2+\left[\frac{(G_1+G_2+G_3)}{C_2}\right]s+\frac{G_2G_3}{C_1C_2}}$$

$$= \frac{-\frac{G_1G_2G_3}{G_2C_1C_2}}{s^2+\left[\frac{(G_1+G_2+G_3)}{C_2}\right]s+\frac{G_2G_3}{C_1C_2}}$$

$$= \frac{-Kb_o}{s^2+b_1s+b_o}$$

where $K = \dfrac{G_1}{G_2}$; $\qquad b_o = \dfrac{G_2G_3}{C_1C_2}$

and $b_1 = \dfrac{G_1+G_2+G_3}{C_2}$

[c] Rearranging we see that

$$G_1 = KG_2$$

$$G_3 = \frac{b_oC_1C_2}{G_2} = \frac{b_oC_1}{G_2}$$

since by hypothesis $C_2 = 1\,\mathrm{F}$

$$b_1 = \frac{G_1+G_2+G_3}{C_2} = G_1+G_2+G_3$$

$$\therefore\ b_1 = KG_2+G_2+\frac{b_oC_1}{G_2}$$

$$b_1 = G_2(1+K)+\frac{b_oC_1}{G_2}$$

Solving this quadratic equation for G_2 we get

$$G_2 = \frac{b_1}{2(1+K)} \pm \sqrt{\frac{b_1^2-b_oC_14(1+K)}{4(1+K)^2}}$$

$$= \frac{b_1 \pm \sqrt{b_1^2-4b_o(1+K)C_1}}{2(1+K)}$$

For G_2 to be realizable

$$C_1 < \frac{b_1^2}{4b_o(1+K)}$$

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[d] 1. Select $C_2 = 1\,\text{F}$

2. Select C_1 such that $C_1 < \dfrac{b_1^2}{4b_o(1+K)}$

3. Calculate G_2 (R_2)

4. Calculate G_1 (R_1); $G_1 = KG_2$

5. Calculate G_3 (R_3); $G_3 = b_oC_1/G_2$

P 15.51 **[a]** In the second order section of a third order Butterworth filter $b_o = b_1 = 1$ Therefore,

$$C_1 \le \frac{b_1^2}{4b_o(1+K)} = \frac{1}{(4)(1)(5)} = 0.05\,\text{F}$$

$$\therefore \qquad C_1 = 0.05\,\text{F} \quad \text{(limiting value)}$$

[b] $G_2 = \dfrac{1}{2(1+4)} = 0.1\,\text{S}$

$$G_3 = \frac{1}{0.1}(0.05) = 0.5\,\text{S}$$

$$G_1 = 4(0.1) = 0.4\,\text{S}$$

Therefore,

$$R_1 = \frac{1}{G_1} = 2.5\,\Omega; \qquad R_2 = \frac{1}{G_2} = 10\,\Omega; \qquad R_3 = \frac{1}{G_3} = 2\,\Omega$$

[c] $k_f = \dfrac{\omega_o'}{\omega_o} = 2\pi(2500) = 5000\pi$

$$k_m = \frac{C_2}{C_2'k_f} = \frac{1}{(10\times 10^{-9})k_f} = 6366.2$$

$$C_1' = \frac{0.05}{k_fk_m} = 0.5\times 10^{-9} = 500\,\text{pF}$$

$$R_1' = (2.5)(6366.2) = 15.92\,\text{k}\Omega$$

$$R_2' = (10)(6366.2) = 63.66\,\text{k}\Omega$$

$$R_3' = (2)(6366.2) = 12.73\,\text{k}\Omega$$

[d] $R_1' = R_2' = (6366.2)(1) = 6.37\,\text{k}\Omega$

$$C' = \frac{C}{k_fk_m} = \frac{1}{10^8} = 10\,\text{nF}$$

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[e]

P 15.52 **[a]** By hypothesis the circuit becomes:

For very small frequencies the capacitors behave as open circuits and therefore v_o is zero. As the frequency increases, the capacitive branch impedances become small compared to the resistive branches. When this happens the circuit becomes an inverting amplifier with the capacitor C_2 dominating the feedback path. Hence the gain of the amplifier approaches $(1/j\omega C_2)/(1/j\omega C_1)$ or C_1/C_2. Therefore the circuit behaves like a high-pass filter with a passband gain of C_1/C_2.

[b] Summing the currents away from the upper terminal of R_2 yields

$$V_aG_2 + (V_a - V_i)sC_1 + (V_a - V_o)sC_2 + V_asC_3 = 0$$

or

$$V_a[G_2 + s(C_1 + C_2 + C_3)] - V_osC_2 = sC_1V_i$$

Summing the currents away from the inverting input terminal gives

$$(0 - V_a)sC_3 + (0 - V_o)G_1 = 0$$

or

$$sC_3V_a = -G_1V_o; \qquad V_a = \frac{-G_1V_o}{sC_3}$$

Therefore we can write

$$\frac{-G_1V_o}{sC_3}[G_2 + s(C_1 + C_2 + C_3)] - sC_2V_o = sC_1V_i$$

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Solving for V_o/V_i gives

$$H(s) = \frac{V_o}{V_i} = \frac{-C_1C_3s^2}{C_2C_3s^2 + G_1(C_1+C_2+C_3)s + G_1G_2]}$$

$$= \frac{\frac{-C_1}{C_2}s^2}{\left[s^2 + \frac{G_1}{C_2C_3}(C_1+C_2+C_3)s + \frac{G_1G_2}{C_2C_3}\right]}$$

$$= \frac{-Ks^2}{s^2 + b_1s + b_o}$$

Therefore the circuit implements a second-order high-pass filter with a passband gain of C_1/C_2.

[c] $C_1 = K$:

$$b_1 = \frac{G_1}{(1)(1)}(K+2) = G_1(K+2)$$

$$\therefore \quad G_1 = \frac{b_1}{K+2}; \qquad R_1 = \left(\frac{K+2}{b_1}\right)$$

$$b_o = \frac{G_1G_2}{(1)(1)} = G_1G_2$$

$$\therefore \quad G_2 = \frac{b_o}{G_1} = \frac{b_o}{b_1}(K+2)$$

$$\therefore \quad R_2 = \frac{b_1}{b_o(K+2)}$$

[d] From Table 15.1 the transfer function of the second-order section of a third-order high-pass Butterworth filter is

$$H(s) = \frac{Ks^2}{s^2+s+1}$$

Therefore $b_1 = b_o = 1$

Thus

$$C_1 = K = 8\,\text{F}$$

$$R_1 = \frac{8+2}{1} = 10\,\Omega$$

$$R_2 = \frac{1}{1(8+2)} = 0.1\,\Omega$$

P 15.53 **[a]** Low-pass filter:

$$n = \frac{(-0.05)(-24)}{\log_{10}(2000/1000)} = 3.99; \qquad \therefore \quad n = 4$$

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In the first prototype second-order section: $b_1 = 0.765$, $b_o = 1$, $C_2 = 1\,\text{F}$

$$C_1 \le \frac{b_1^2}{4b_o(1+K)} \le \frac{(0.765)^2}{(4)(2)} \le 0.0732$$

choose $C_1 = 0.03\,\text{F}$

$$G_2 = \frac{0.765 \pm \sqrt{(0.765)^2 - 4(2)(0.03)}}{4} = \frac{0.765 \pm 0.588}{4}$$

Arbitrarily select the larger value for G_2, then

$$G_2 = 0.338\ \text{S}; \quad \therefore \qquad R_2 = \frac{1}{G_2} = 2.96\,\Omega$$

$$G_1 = KG_2 = 0.338\ \text{S}; \quad \therefore \qquad R_1 = \frac{1}{G_1} = 2.96\,\Omega$$

$$G_3 = \frac{b_o C_1}{G_2} = \frac{(1)(0.03)}{0.338} = 0.089 \quad \therefore \qquad R_3 = 1/G_3 = 11.3\,\Omega$$

Therefore in the first second-order prototype circuit

$$R_1 = R_2 = 2.96\,\Omega; \qquad R_3 = 11.3\,\Omega$$

$$C_1 = 0.03\,\text{F}; \qquad C_2 = 1\,\text{F}$$

In the second second-order prototype circuit:
$b_1 = 1.848,\ b_0 = 1,\ C_2 = 1\,\text{F}$

$$\therefore \quad C_1 \le \frac{(1.848)^2}{8} \le 0.427$$

choose $C_1 = 0.30\,\text{F}$

$$G_2 = \frac{1.848 \pm \sqrt{(1.848)^2 - 8(0.3)}}{4} = \frac{1.848 \pm 1.008}{4}$$

Arbitrarily select the larger value, then

$$G_2 = 0.7139\ \text{S}; \quad \therefore \qquad R_2 = \frac{1}{G_2} = 1.4008\,\Omega$$

$$G_1 = KG_2 = 0.7139\ \text{S}; \quad \therefore \qquad R_1 = \frac{1}{G_1} = 1.4008\,\Omega$$

$$G_3 = \frac{b_o C_1}{G_2} = \frac{(1)(0.30)}{0.7139} = 0.4202\ \text{S} \quad \therefore \qquad R_3 = 1/G_3 = 2.3796\,\Omega$$

In the low-pass section of the filter

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(1000) = 2000\pi$$

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$$k_m = \frac{C}{C'k_f} = \frac{1}{(25 \times 10^{-9})k_f} = \frac{20{,}000}{\pi}$$

Therefore in the first scaled second-order section

$$R_1' = R_2' = 2.96k_m = 18.84\,\text{k}\Omega$$

$$R_3' = 11.3k_m = 71.94\,\text{k}\Omega$$

$$C_1' = \frac{0.03}{k_f k_m} = 750\,\text{pF}$$

$$C_2' = 25\,\text{nF}$$

In the second scaled second-order section

$$R_1' = R_2' = 1.4008k_m = 8.92\,\text{k}\Omega$$

$$R_3' = 2.38k_m = 15.15\,\text{k}\Omega$$

$$C_1' = \frac{0.3}{k_f k_m} = 7.5\,\text{nF}$$

$$C_2' = 25\,\text{nF}$$

High-pass filter section

$$n = \frac{(-0.05)(-24)}{\log_{10}(8000/4000)} = 3.99; \qquad n = 4$$

In the first prototype second-order section:
$b_1 = 0.765;\ b_o = 1;\ C_2 = C_3 = 1\,\text{F}$

$$C_1 = K = 1\,\text{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{0.765} = 3.92\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{0.765}{3} = 0.255\,\Omega$$

In the second prototype second-order section: $b_1 = 1.848$; $b_o = 1$;
$C_2 = C_3 = 1\,\text{F}$

$$C_1 = K = 1\,\text{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{1.848} = 1.623\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{1.848}{3} = 0.616\,\Omega$$

In the high-pass section of the filter

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(8000) = 16{,}000\pi$$

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$$k_m = \frac{C}{C'k_f} = \frac{1}{(25 \times 10^{-9})(16{,}000\pi)} = \frac{2500}{\pi}$$

In the first scaled second-order section

$R_1' = 3.92k_m = 3119.4\,\Omega$

$R_2' = 0.255k_m = 202.9\,\Omega$

$C_1' = C_2' = C_3' = 25\,\text{nF}$

In the second scaled second-order section

$R_1' = 1.623k_m = 1291.5\,\Omega$

$R_2' = 0.616k_m = 490.2\,\Omega$

$C_1' = C_2' = C_3' = 25\,\text{nF}$

In the gain section, let $R_i = 10\,\text{k}\Omega$ and $R_f = 100\,\text{k}\Omega$.

[b]

P 15.54 **[a]** The prototype low-pass transfer function is

$$H_{lp}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

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The low-pass frequency scaling factor is

$k_{f_{lp}} = 2\pi(1000) = 2000\pi$

The scaled transfer function for the low-pass filter is

$$H'_{lp}(s) = \frac{1}{\left[\left(\frac{s}{2000\pi}\right)^2 + \frac{0.765s}{2000\pi} + 1\right]\left[\left(\frac{s}{2000\pi}\right)^2 + \frac{1.848s}{2000\pi} + 1\right]}$$
$$= \frac{16 \times 10^{12}\pi^4}{[s^2 + 1530\pi s + (2000\pi)^2]\,[s^2 + 3696\pi s + (2000\pi)^2]}$$

The prototype high-pass transfer function is

$$H_{hp}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The high-pass frequency scaling factor is

$k_{f_{hp}} = 2\pi(8000) = 16{,}000\pi$

The scaled transfer function for the high-pass filter is

$$H'_{hp}(s) = \frac{(s/16{,}000\pi)^4}{\left[\left(\frac{s}{16{,}000\pi}\right)^2 + \frac{0.765s}{16{,}000\pi} + 1\right]\left[\left(\frac{s}{16{,}000\pi}\right)^2 + \frac{1.848s}{16{,}000\pi} + 1\right]}$$
$$= \frac{s^4}{[s^2 + 12{,}240\pi s + (16{,}000\pi)^2][s^2 + 29{,}568\pi s + (16{,}000\pi)^2]}$$

The transfer function for the filter is

$H'(s) = \left[H'_{lp}(s) + H'_{hp}(s)\right]$

[b] $f_o = \sqrt{f_{c1}f_{c2}} = \sqrt{(1000)(8000)} = 2000\sqrt{2}\,\text{Hz}$

$\omega_o = 2\pi f_o = 4000\sqrt{2}\pi\ \text{rad/s}$

$(j\omega_o)^2 = -32 \times 10^6\pi^2$

$(j\omega_o)^4 = 1024 \times 10^{12}\pi^4$

$$H'_{lp}(j\omega_o) = \frac{16 \times 10^{12}\pi^4}{[-28 \times 10^6\pi^2 + j1530(4000\sqrt{2}\pi^2)]} \times \frac{1}{[-28 \times 10^6\pi^2 + j3696(4000\sqrt{2}\pi^2)]}$$
$$= 0.0156\underline{/53.9^\circ}$$

$$H'_{hp}(j\omega_o) = \frac{1024 \times 10^{12}\pi^4}{[224 \times 10^6\pi^2 + j12{,}240(4000\sqrt{2}\pi^2)]} \frac{1}{[224 \times 10^6\pi^2 + j29{,}568(4000\sqrt{2}\pi^2)]}$$
$$= 0.0156\underline{/-53.9^\circ}$$

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$$\therefore \quad H'(j\omega_o) = 0.0156(1\underline{/53.9^\circ} + 1\underline{/-53.9^\circ})$$

$$= 0.0156(1.18\underline{/0^\circ}) = 0.0184\underline{/0^\circ}$$

$$G = 20\log_{10}|H'(j\omega_o)| = 20\log_{10}(0.0184) = -34.7 \text{ dB}$$

P 15.55 **[a]** At low frequencies the capacitor branches are open; $v_o = v_i$. At high frequencies the capacitor branches are short circuits and the output voltage is zero. Hence the circuit behaves like a unity-gain low-pass filter.

[b] Let v_a represent the voltage-to-ground at the right-hand terminal of R_1. Observe this will also be the voltage at the left-hand terminal of R_2. The s-domain equations are

$$(V_a - V_i)G_1 + (V_a - V_o)sC_1 = 0$$

$$(V_o - V_a)G_2 + sC_2V_o = 0$$

or

$$(G_1 + sC_1)V_a - sC_1V_o = G_1V_i$$

$$-G_2V_a + (G_2 + sC_2)V_o = 0$$

$$\therefore \quad V_a = \frac{G_2 + sC_2V_o}{G_2}$$

$$\therefore \quad \left[(G_1 + sC_1)\frac{(G_2 + sC_2)}{G_2} - sC_1\right]V_o = G_1V_i$$

$$\therefore \quad \frac{V_o}{V_i} = \frac{G_1G_2}{(G_1 + sC_1)(G_2 + sC_2) - C_1G_2s}$$

which reduces to

$$\frac{V_o}{V_i} = \frac{G_1G_2/C_1C_2}{s^2 + \frac{G_1}{C_1}s + \frac{G_1G_2}{C_1C_2}} = \frac{b_o}{s^2 + b_1s + b_o}$$

[c] There are four circuit components and two restraints imposed by $H(s)$; therefore there are two free choices.

[d] $b_1 = \dfrac{G_1}{C_1} \quad \therefore \quad G_1 = b_1C_1$

$$b_o = \frac{G_1G_2}{C_1C_2} \quad \therefore \quad G_2 = \frac{b_o}{b_1}C_2$$

[e] No, all physically realizeable capacitors will yield physically realizeable resistors.

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[f] From Table 15.1 we know the transfer function of the prototype 4th order Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

In the first section $b_o = 1, \quad b_1 = 0.765$

$\therefore \quad G_1 = (0.765)(1) = 0.765 \text{ S}$

$R_1 = 1/G_1 = 1.307\,\Omega$

$G_2 = \dfrac{1}{0.765}(1) = 1.307 \text{ S}$

$R_2 = 1/G_2 = 0.765\,\Omega$

In the second section $b_o = 1, \quad b_1 = 1.848$

$\therefore \quad G_1 = 1.848\,\text{S}$

$R_1 = 1/G_1 = 0.541\,\Omega$

$G_2 = \left(\dfrac{1}{1.848}\right)(1) = 0.541\,\text{S}$

$R_2 = 1/G_2 = 1.848\,\Omega$

P 15.56 **[a]** $k_f = \dfrac{\omega_o'}{\omega_o} = 2\pi(3000) = 6000\pi$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(4.7 \times 10^{-9})(6000\pi)} = \frac{10^6}{28.2\pi}$$

In the first section

$R_1' = 1.307k_m = 14.75\,\text{k}\Omega$

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$R_2' = 0.765k_m = 8.64\,\text{k}\Omega$

In the second section

$R_1' = 0.541k_m = 6.1\,\text{k}\Omega$

$R_2' = 1.848k_m = 20.86\,\text{k}\Omega$

[b]

P 15.57 **[a]** Interchanging the Rs and Cs yields the following circuit.

At low frequencies the capacitors appear as open circuits and hence the output voltage is zero. As the frequency increases the capacitor branches approach short circuits and $v_a = v_i = v_o$. Thus the circuit is a unity-gain, high-pass filter.

[b] The s-domain equations are

$$(V_a - V_i)sC_1 + (V_a - V_o)G_1 = 0$$

$$(V_o - V_a)sC_2 + V_oG_2 = 0$$

It follows that

$$V_a(G_1 + sC_1) - G_1V_o = sC_1V_i$$

and $V_a = \dfrac{(G_2 + sC_2)V_o}{sC_2}$

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Thus

$$\left\{\left[\frac{(G_2+sC_2)}{sC_2}\right](G_1+sC_1)-G_1\right\}V_o = sC_1V_i$$

$$V_o\{s^2C_1C_2+sC_1G_2+G_1G_2\} = s^2C_1C_2V_i$$

$$H(s)=\frac{V_o}{V_i}=\frac{s^2}{\left(s^2+\dfrac{G_2}{C_2}s+\dfrac{G_1G_2}{C_1C_2}\right)}$$

$$=\frac{V_o}{V_i}=\frac{s^2}{s^2+b_1s+b_o}$$

[c] There are 4 circuit components: R_1, R_2, C_1 and C_2.
There are two transfer function constraints: b_1 and b_o.
Therefore there are two free choices.

[d] $b_o=\dfrac{G_1G_2}{C_1C_2}; \qquad b_1=\dfrac{G_2}{C_2}$

$$\therefore \quad G_2=b_1C_2; \qquad R_2=\frac{1}{b_1C_2}$$

$$G_1=\frac{b_o}{b_1}C_1 \;\therefore\; R_1=\frac{b_1}{b_oC_1}$$

[e] No, all realizeable capacitors will produce realizeable resistors.

[f] The second-order section in a 3rd-order Butterworth high-pass filter is $s^2/(s^2+s+1)$. Therefore $b_o=b_1=1$ and

$$R_1=\frac{1}{(1)(1)}=1\,\Omega.$$

$$R_2=\frac{1}{(1)(1)}=1\,\Omega.$$

P 15.58 **[a]** $k_f=\dfrac{\omega_o'}{\omega_o}=1600\pi$

$$k_m=\frac{C}{C'k_f}=\frac{1}{(5\times10^{-6})(1600\pi)}=39.8$$

$$C_1'=C_2'=5\,\mu\text{F}; \qquad R_1'=R_2'=k_mR=39.8\,\Omega$$

[b] $R=39.8\,\Omega; \qquad C=5\,\mu\text{F}$

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[c]

[d] $H_{\text{hp}}(s) = \dfrac{s^3}{(s+1)(s^2+s+1)}$

$$H'_{\text{hp}}(s) = \frac{(s/1600\pi)^3}{[(s/1600\pi)+1][(s/1600\pi)^2+(s/1600\pi)+1]}$$

$$= \frac{s^3}{(s+1600\pi)(s^2+1600\pi s+256\times 10^4\pi^2)}$$

[e] $H'_{\text{hp}}(j1600\pi) = \dfrac{(j1600\pi)^3}{(j1600\pi+1600\pi)[(j1600\pi)^2+1600\pi(j1600\pi)+256\times 10^4\pi^2]}$

$$= 0.7071\underline{/135^\circ}$$

$$\therefore \qquad |H'_{\text{hp}}| = 0.7071 = -3.01 \text{ dB}$$

P 15.59 **[a]** It follows directly from Eqs 15.64 and 15.65 that

$$H(s) = \frac{s^2+1}{s^2+4(1-\sigma)s+1}$$

Now note from Eq 15.69 that $(1-\sigma)$ equals $1/4Q$, hence

$$H(s) = \frac{s^2+1}{s^2+\frac{1}{Q}s+1}$$

[b] For Example 15.13 $\omega_o = 5000\,\text{rad/s}$ and $Q = 5$. Therefore $k_f = 5000$ and

$$H'(s) = \frac{(s/5000)^2+1}{(s/5000)^2+\dfrac{1}{5}\left(\dfrac{s}{5000}\right)+1}$$

$$= \frac{s^2+25\times 10^6}{s^2+1000s+25\times 10^6}$$

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P 15.60 **[a]** $\omega_o = 8000\pi$ rad/s

$$\therefore \qquad k_f = \frac{\omega_o'}{\omega_o} = 8000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(0.5 \times 10^{-6})(8000\pi)} = \frac{250}{\pi}$$

$$R' = k_m R = 79.6\,\Omega \qquad \text{so} \qquad R'/2 = 39.8\,\Omega$$

$$\sigma = 1 - \frac{1}{4Q} = 1 - \frac{1}{4(10)} = 0.975$$

$$\sigma R' = 77.6\,\Omega; \qquad (1-\sigma)R' = 2\,\Omega$$

$$C' = 0.5\,\mu\text{F}$$

$$2C' = 1\,\mu\text{F}$$

[b]

[c] $k_f = 8000\pi$

$$H(s) = \frac{(s/8000\pi)^2 + 1}{(s/8000\pi)^2 + \frac{1}{10}(s/8000\pi) + 1}$$

$$= \frac{s^2 + 64 \times 10^6\pi^2}{s^2 + 800\pi s + 64 \times 10^6\pi^2}$$

P 15.61 To satisfy the gain specification of 14 dB at $\omega = 0$ and $\alpha = 1$ requires

$$\frac{R_1 + R_2}{R_1} = 5 \qquad \text{or} \qquad R_2 = 4R_1$$

Use the specified resistor of $10\,\text{k}\Omega$ for R_1 and a $50\,\text{k}\Omega$ potentiometer for R_2. Since $(R_1 + R_2)/R_1 \gg 1$ the value of C_1 is

$$C_1 = \frac{1}{2\pi(50)(50{,}000)} = 63.66 \text{ nF}$$

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Choose a capacitor value of 64 nF. Using the selected values of R_1 and R_2 the maximum gain for $\alpha = 1$ is

$$20\log_{10}\left(\frac{50}{10}\right)_{\alpha=1} = 13.98 \text{ dB}$$

When $C_1 = 64$ nF the frequency $1/R_2C_1$ is

$$\frac{1}{R_2C_1} = \frac{10^9}{50{,}000(64)} = 312.5 \text{ rad/s} = 49.7 \text{ Hz}$$

The magnitude of the transfer function at 312.5 rad/s is

$$|H(j312.5)|_{\alpha=1} = \left|\frac{50\times 10^3 + j312.5(10)(50)(64)10^{-3}}{10\times 10^3 + j312.5(10)(50)(64)10^{-3}}\right| = 3.61$$

Therefore the gain at 49.7 Hz is

$$20\log_{10}(3.61)_{\alpha=1} = 11.1 \text{ dB}$$

P 15.62 $20\log_{10}\left(\frac{R_1+R_2}{R_1}\right) = 20$

$$\therefore \quad \frac{R_1+R_2}{R_1} = 10; \qquad \therefore \quad R_2 = 9R_1$$

Choose $R_1 = 100\,\text{k}\Omega$. Then $R_2 = 900\,\text{k}\Omega$

$$\frac{1}{R_2C_1} = 150\pi \text{ rad/s}; \qquad \therefore \quad C_1 = \frac{1}{(150\pi)(900\times 10^3)} = 2.36\,\text{nF}$$

P 15.63 $|H(j0)| = \dfrac{R_1+\alpha R_2}{R_1+(1-\alpha)R_2} = \dfrac{10+\alpha(50)}{10+(1-\alpha)50}$

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P 15.64 **[a]** Combine the impedances of the capacitors in series in Fig. P15.64(b) to get

$$\frac{1}{sC_{\text{eq}}} = \frac{1-\alpha}{sC_1} + \frac{\alpha}{sC_1} = \frac{1}{sC_1}$$

which is identical to the impedance of the capacitor in Fig. P15.60(a).

[b]

$$V_x = \frac{\alpha/sC_1}{(1-\alpha)/sC_1 + \alpha/sC_1}V = \alpha V$$

$$V_y = \frac{\alpha R_2}{(1-\alpha)R_2 + \alpha R_2} = \alpha V = V_x$$

[c] Since x and y are both at the same potential, they can be shorted together, and the circuit in Fig. 15.34 can thus be drawn as shown in Fig. 15.53(c).

[d] The feedback path between V_o and V_s containing the resistance $R_4 + 2R_3$ has no effect on the ratio V_o/V_s, as this feedback path is not involved in the nodal equation that defines the voltage ratio. Thus, the circuit in Fig. P15.64(c) can be simplified into the form of Fig. 15.2, where the input impedance is the equivalent impedance of R_1 in series with the parallel combination of $(1-\alpha)/sC_1$ and $(1-\alpha)R_2$, and the feedback impedance is the equivalent impedance of R_1 in series with the parallel combination of α/sC_1 and αR_2:

$$Z_i = R_1 + \frac{\frac{(1-\alpha)}{sC_1}\cdot(1-\alpha)R_2}{(1-\alpha)R_2 + \frac{(1-\alpha)}{sC_1}}$$

$$= \frac{R_1 + (1-\alpha)R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

$$Z_f = R_1 + \frac{\frac{\alpha}{sC_1}\cdot\alpha R_2}{\alpha R_2 + \frac{\alpha}{sC_1}}$$

$$= \frac{R_1 + \alpha R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

P 15.65 As $\omega \to 0$

$$|H(j\omega)| \to \frac{2R_3 + R_4}{2R_3 + R_4} = 1$$

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Therefore the circuit would have no effect on low frequency signals. As $\omega \to \infty$

$$|H(j\omega)| \to \frac{[(1-\beta)R_4 + R_o](\beta R_4 + R_3)}{[(1-\beta)R_4 + R_3](\beta R_4 + R_o)}$$

When $\beta = 1$

$$|H(j\infty)|_{\beta=1} = \frac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=1} \cong \frac{R_o}{R_3} > 1$$

Thus, when $\beta = 1$ we have amplification or "boost". When $\beta = 0$

$$|H(j\infty)|_{\beta=0} = \frac{R_3(R_4 + R_o)}{R_o(R_4 + R_3)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=0} \cong \frac{R_3}{R_0} < 1$$

Thus, when $\beta = 0$ we have attenuation or "cut".
Also note that when $\beta = 0.5$

$$|H(j\omega)|_{\beta=0.5} = \frac{(0.5R_4 + R_o)(0.5R_4 + R_3)}{(0.5R_4 + R_3)(0.5R_4 + R_o)} = 1$$

Thus, the transition from amplification to attenuation occurs at $\beta = 0.5$. If $\beta > 0.5$ we have amplification, and if $\beta < 0.5$ we have attenuation.
Also note the amplification an attenuation are symmetric about $\beta = 0.5$. i.e.

$$|H(j\omega)|_{\beta=0.6} = \frac{1}{|H(j\omega)|_{\beta=0.4}}$$

Yes, the circuit can be used as a treble volume control because

- The circuit has no effect on low frequency signals
- Depending on β the circuit can either amplify ($\beta > 0.5$) or attenuate ($\beta < 0.5$) signals in the treble range
- The amplification (boost) and attenuation (cut) are symmetric around $\beta = 0.5$. When $\beta = 0.5$ the circuit has no effect on signals in the treble frequency range.

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P 15.66 **[a]** $|H(j\infty)|_{\beta=1} = \dfrac{R_o(R_4+R_3)}{R_3(R_4+R_o)} = \dfrac{(65.9)(505.9)}{(5.9)(565.9)} = 9.99$

$\therefore$ maximum boost $= 20\log_{10} 9.99 = 19.99$ dB

[b] $|H(j\infty)|_{\beta=0} = \dfrac{R_3(R_4+R_3)}{R_o(R_4+R_o)}$

$\therefore$ maximum cut $= -19.99$ dB

[c] $R_4 = 500\,\text{k}\Omega; \qquad R_o = R_1 + R_3 + 2R_2 = 65.9\,\text{k}\Omega$

$\therefore \quad R_4 = 7.59R_o$

Yes, R_4 is significantly greater than R_o.

[d] $$|H(j/R_3C_2)|_{\beta=1} = \left|\frac{(2R_3+R_4)+j\frac{R_o}{R_3}(R_4+R_3)}{(2R_3+R_4)+j(R_4+R_o)}\right|$$

$$= \left|\frac{511.8+j\frac{65.9}{5.9}(505.9)}{511.8+j565.9}\right|$$

$$= 7.44$$

$20\log_{10}|H(j/R_3C_2)|_{\beta=1} = 20\log_{10} 7.44 = 17.43$ dB

[e] When $\beta = 0$

$$|H(j/R_3C_2)|_{\beta=0} = \frac{(2R_3+R_4)+j(R_4+R_o)}{(2R_3+R_4)+j\dfrac{R_o}{R_3}(R_4+R_3)}$$

Note this is the reciprocal of $|H(j/R_3C_2)|_{\beta=1}$.

$\therefore \quad 20\log_{10}|H(j/R_3C_2)|_{\beta+0} = -17.43$ dB

[f] The frequency $1/R_3C_2$ is very nearly where the gain is 3 dB off from its maximum boost or cut. Therefore for frequencies higher than $1/R_3C_2$ the circuit designer knows that gain or cut will be within 3 dB of the maximum.

P 15.67 $$|H(j\infty)| = \frac{[(1-\beta)R_4+R_o][\beta R_4+R_3]}{[(1-\beta R_4+R_3][\beta R_4+R_o]}$$

$$= \frac{[(1-\beta)500+65.9][\beta 500+5.9]}{[(1-\beta)500+5.9][\beta 500+65.9]}$$

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16

Fourier Series

Assessment Problems

AP 16.1

$$a_v = \frac{1}{T}\int_0^{2T/3} V_m\,dt + \frac{1}{T}\int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\,dt = \frac{7}{9}V_m = 7\pi\,\text{V}$$

$$a_k = \frac{2}{T}\left[\int_0^{2T/3} V_m\cos\,k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\cos\,\text{k}\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\sin\left(\frac{4k\pi}{3}\right) = \left(\frac{6}{k}\right)\sin\left(\frac{4k\pi}{3}\right)$$

$$b_k = \frac{2}{T}\left[\int_0^{2T/3} V_m\sin k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\sin k\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\left[1-\cos\left(\frac{4k\pi}{3}\right)\right] = \left(\frac{6}{k}\right)\left[1-\cos\left(\frac{4k\pi}{3}\right)\right]$$

AP 16.2 **[a]** $a_v = 7\pi = 21.99\,\text{V}$

[b] $a_1 = -5.196 \quad a_2 = 2.598 \quad a_3 = 0 \quad a_4 = -1.299 \quad a_5 = 1.039$

$b_1 = 9 \quad b_2 = 4.5 \quad b_3 = 0 \quad b_4 = 2.25 \quad b_5 = 1.8$

[c] $w_0 = \left(\frac{2\pi}{T}\right) = 50\,\text{rad/s}$

[d] $f_3 = 3f_0 = 23.87\,\text{Hz}$

[e] $v(t) = 21.99 - 5.2\cos 50t + 9\sin 50t + 2.6\sin 100t + 4.5\cos 100t$

$-1.3\sin 200t + 2.25\cos 200t + 1.04\sin 250t + 1.8\cos 250t + \cdots\,\text{V}$

AP 16.3 Odd function with both half- and quarter-wave symmetry.

$$v_g(t) = \left(\frac{6V_m}{T}\right)t, \qquad 0 \le t \le T/6; \qquad a_v = 0, \qquad a_k = 0 \quad \text{for all } k$$

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$$b_k = 0 \quad \text{for } k \text{ even}$$

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt, \qquad k \text{ odd}$$

$$= \frac{8}{T}\int_0^{T/6}\left(\frac{6V_m}{T}\right)t\sin k\omega_0 t\,dt + \frac{8}{T}\int_{T/6}^{T/4} V_m \sin k\omega_0 t\,dt$$

$$= \left(\frac{12V_m}{k^2\pi^2}\right)\sin\left(\frac{k\pi}{3}\right)$$

$$v_g(t) = \frac{12V_m}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{3}\sin n\omega_0 t\ \text{V}$$

AP 16.4 **[a]** $A_1 = -5.2 - j9 = 10.4\underline{/-120^\circ}$; $\quad A_2 = 2.6 - j4.5 = 5.2\underline{/-60^\circ}$

$A_3 = 0$; $\quad A_4 = -1.3 - j2.25 = 2.6\underline{/-120^\circ}$

$A_5 = 1.04 - j1.8 = 2.1\underline{/-60^\circ}$

$\theta_1 = -120^\circ$; $\quad \theta_2 = -60^\circ$; $\quad \theta_3$ not defined;

$\theta_4 = -120^\circ$; $\quad \theta_5 = -60^\circ$

[b] $v(t) = 21.99 + 10.4\cos(50t - 120^\circ) + 5.2\cos(100t - 60^\circ)$
$+2.6\cos(200t - 120^\circ) + 2.1\cos(250t - 60^\circ) + \cdots \text{V}$

AP 16.5 The Fourier series for the input voltage is

$$v_i = \frac{8A}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{1}{n^2}\sin\frac{n\pi}{2}\right)\sin n\omega_0(t + T/4)$$

$$= \frac{8A}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{1}{n^2}\sin^2\frac{n\pi}{2}\right)\cos n\omega_0 t$$

$$= \frac{8A}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\cos n\omega_0 t$$

$$\frac{8A}{\pi^2} = \frac{8(281.25\pi^2)}{\pi^2} = 2250\,\text{mV}$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{200\pi}\times 10^3 = 10$$

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$$\therefore \qquad v_i = 2250 \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^2} \cos 10nt \text{ mV}$$

From the circuit we have

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + (1/j\omega C)} \cdot \frac{1}{j\omega C} = \frac{\mathbf{V}_i}{1 + j\omega RC}$$

$$\mathbf{V}_o = \frac{1/RC}{1/RC + j\omega}\mathbf{V}_i = \frac{100}{100 + j\omega}\mathbf{V}_i$$

$$\mathbf{V}_{i1} = 2250\underline{/0^\circ} \text{ mV}; \qquad \omega_0 = 10 \text{ rad/s}$$

$$\mathbf{V}_{i3} = \frac{2250}{9}\underline{/0^\circ} = 250\underline{/0^\circ} \text{ mV}; \qquad 3\omega_0 = 30 \text{ rad/s}$$

$$\mathbf{V}_{i5} = \frac{2250}{25}\underline{/0^\circ} = 90\underline{/0^\circ} \text{ mV}; \qquad 5\omega_0 = 50 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{100}{100 + j10}(2250\underline{/0^\circ}) = 2238.83\underline{/-5.71^\circ} \text{ mV}$$

$$\mathbf{V}_{o3} = \frac{100}{100 + j30}(250\underline{/0^\circ}) = 239.46\underline{/-16.70^\circ} \text{ mV}$$

$$\mathbf{V}_{o5} = \frac{100}{100 + j50}(90\underline{/0^\circ}) = 80.50\underline{/-26.57^\circ} \text{ mV}$$

$$\therefore \qquad v_o = 2238.33\cos(10t - 5.71^\circ) + 239.46\cos(30t - 16.70^\circ)$$

$$+80.50\cos(50t - 26.57^\circ) + \ldots \text{ mV}$$

AP 16.6 **[a]** $\omega_o = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.2\pi}(10^3) = 10^4 \text{ rad/s}$

$$v_g(t) = 840 \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin\frac{n\pi}{2} \cos n10{,}000t \text{ V}$$

$$= 840\cos 10{,}000t - 280\cos 30{,}000t + 168\cos 50{,}000t$$

$$- 120\cos 70{,}000t + \cdots \text{ V}$$

$$\mathbf{V}_{g1} = 840\underline{/0^\circ} \text{ V}; \qquad \mathbf{V}_{g3} = 280\underline{/180^\circ} \text{ V}$$

$$\mathbf{V}_{g5} = 168\underline{/0^\circ} \text{ V}; \qquad \mathbf{V}_{g7} = 120\underline{/180^\circ} \text{ V}$$

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$$H(s) = \frac{V_o}{V_g} = \frac{\beta s}{s^2 + \beta s + \omega_c^2}$$

$$\beta = \frac{1}{RC} = \frac{10^9}{10^4(20)} = 5000 \text{ rad/s}$$

$$\omega_c^2 = \frac{1}{LC} = \frac{(10^9)(10^3)}{400} = 25 \times 10^8$$

$$H(s) = \frac{5000s}{s^2 + 5000s + 25 \times 10^8}$$

$$H(j\omega) = \frac{j5000\omega}{25 \times 10^8 - \omega^2 + j5000\omega}$$

$$H_1 = \frac{j5 \times 10^7}{24 \times 10^8 + j5 \times 10^7} = 0.02\underline{/88.81^\circ}$$

$$H_3 = \frac{j15 \times 10^7}{16 \times 10^8 + j15 \times 10^7} = 0.09\underline{/84.64^\circ}$$

$$H_5 = \frac{j25 \times 10^7}{25 \times 10^7} = 1\underline{/0^\circ}$$

$$H_7 = \frac{j35 \times 10^7}{-24 \times 10^8 + j35 \times 10^7} = 0.14\underline{/-81.70^\circ}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{g1}H_1 = 17.50\underline{/88.81^\circ} \text{ V}$$

$$\mathbf{V}_{o3} = \mathbf{V}_{g3}H_3 = 26.14\underline{/-95.36^\circ} \text{ V}$$

$$\mathbf{V}_{o5} = \mathbf{V}_{g5}H_5 = 168\underline{/0^\circ} \text{ V}$$

$$\mathbf{V}_{o7} = \mathbf{V}_{g7}H_7 = 17.32\underline{/98.30^\circ} \text{ V}$$

$$v_o = 17.50\cos(10{,}000t + 88.81^\circ) + 26.14\cos(30{,}000t - 95.36^\circ)$$
$$+ 168\cos(50{,}000t) + 17.32\cos(70{,}000t + 98.30^\circ) + \cdots \text{ V}$$

[b] The 5th harmonic because the circuit is a passive bandpass filter with a Q of 10 and a center frequency of 50 krad/s.

AP 16.7

$$w_0 = \frac{2\pi \times 10^3}{2094.4} = 3 \text{ rad/s}$$

$$j\omega_0 k = j3k$$

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$$V_R = \frac{2}{2+s+1/s}(V_g) = \frac{2sV_g}{s^2+2s+1}$$

$$H(s) = \left(\frac{V_R}{V_g}\right) = \frac{2s}{s^2+2s+1}$$

$$H(j\omega_0 k) = H(j3k) = \frac{j6k}{(1-9k^2)+j6k}$$

$$v_{g_1} = 25.98\sin\omega_0 t\,\text{V}; \qquad V_{g_1} = 25.98\underline{/0^\circ}\,\text{V}$$

$$H(j3) = \frac{j6}{-8+j6} = 0.6\underline{/-53.13^\circ}; \qquad V_{R_1} = 15.588\underline{/-53.13^\circ}\,\text{V}$$

$$P_1 = \frac{(15.588/\sqrt{2})^2}{2} = 60.75\,\text{W}$$

$$v_{g_3} = 0, \quad \text{therefore} \quad P_3 = 0\,\text{W}$$

$$v_{g_5} = -1.04\sin 5\omega_0 t\,\text{V}; \qquad V_{g_5} = 1.04\underline{/180^\circ}$$

$$H(j15) = \frac{j30}{-224+j30} = 0.1327\underline{/-82.37^\circ}$$

$$V_{R_5} = (1.04\underline{/180^\circ})(0.1327\underline{/-82.37^\circ}) = 138\underline{/97.63^\circ}\,\text{mV}$$

$$P_5 = \frac{(0.1396/\sqrt{2})^2}{2} = 4.76\,\text{mW}; \qquad \text{therefore} \quad P \cong P_1 \cong 60.75\,\text{W}$$

AP 16.8 Odd function with half- and quarter-wave symmetry, therefore $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for k even; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/8} 2\sin\,\text{k}\omega_0 t\,dt + \frac{8}{T}\int_{T/8}^{T/4} 8\sin k\omega_0 t\,dt$$

$$= \left(\frac{8}{\pi k}\right)\left[1 + 3\cos\left(\frac{k\pi}{4}\right)\right], \quad k \text{ odd}$$

$$\text{Therefore} \quad C_n = \left(\frac{-j4}{n\pi}\right)\left[1 + 3\cos\left(\frac{n\pi}{4}\right)\right], \quad n \text{ odd}$$

AP 16.9 **[a]** $I_{\text{rms}} = \sqrt{\frac{2}{T}\left[(2)^2\left(\frac{T}{8}\right)(2) + (8)^2\left(\frac{3T}{8} - \frac{T}{8}\right)\right]} = \sqrt{34} = 5.7683\,\text{A}$

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[b] $C_1 = \dfrac{-j12.5}{\pi}; \quad C_3 = \dfrac{j1.5}{\pi}; \quad C_5 = \dfrac{j0.9}{\pi};$

$C_7 = \dfrac{-j1.8}{\pi}; \quad C_9 = \dfrac{-j1.4}{\pi}; \quad C_{11} = \dfrac{j0.4}{\pi}$

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5,\ldots}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}\left(12.5^2 + 1.5^2 + 0.9^2 + 1.8^2 + 1.4^2 + 0.4^2\right)}$$

$$\cong 5.777\,\text{A}$$

[c] $\%\ \text{Error} = \dfrac{5.777 - 5.831}{5.831} \times 100 = -0.93\%$

[d] Using just the terms $C_1 - C_9$,

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5,\ldots}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}\left(12.5^2 + 1.5^2 + 0.9^2 + 1.8^2 + 1.4^2\right)}$$

$$\cong 5.774\,\text{A}$$

$$\%\ \text{Error} = \frac{5.774 - 5.831}{5.831} \times 100 = -0.98\%$$

Thus, the % error is still less than 1%.

AP 16.10 $T = 32\,\text{ms}$, therefore 8 ms requires shifting the function $T/4$ to the right.

$$i = \sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} -j\frac{4}{n\pi}\left(1 + 3\cos\frac{n\pi}{4}\right)e^{jn\omega_0(t-T/4)}$$

$$= \frac{4}{\pi}\sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} \frac{1}{n}\left(1 + 3\cos\frac{n\pi}{4}\right)e^{-j(n+1)(\pi/2)}e^{jn\omega_0 t}$$

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Problems

P 16.1 **[a]** $\omega_{\text{oa}} = \dfrac{2\pi}{8 \times 10^{-3}} = 785.4 \text{ rad/s}$

$$\omega_{\text{ob}} = \frac{2\pi}{80 \times 10^{-3}} = 78.54 \text{ krad/s}$$

[b] $f_{\text{oa}} = \dfrac{1}{T} = \dfrac{1}{8 \times 10^{-3}} = 125\,\text{Hz}; \qquad f_{\text{ob}} = \dfrac{1}{80 \times 10^{-3}} = 12.5\,\text{Hz}$

[c] $a_{\text{va}} = \dfrac{50(4 \times 10^{-3})}{8 \times 10^{-3}} = 25\,\text{V}; \qquad a_{\text{vb}} = 0$

[d] The periodic function in Fig. P16.1(a):

$$a_{\text{va}} = 25\,\text{V}$$

$$a_{\text{ka}} = \frac{2}{T}\int_{-T/4}^{T/4} 50\cos\frac{2\pi kt}{T}\,dt$$

$$= \frac{100}{T}\frac{T}{2\pi k}\sin\frac{2\pi k}{T}t\,\Big|_{-T/4}^{T/4}$$

$$= \frac{100}{\pi k}\sin\frac{\pi k}{2}$$

$$b_{\text{ka}} = \frac{2}{T}\int_{-T/4}^{T/4} 50\sin\frac{2\pi kt}{T}\,dt$$

$$= \frac{-100}{T}\frac{T}{2\pi k}\cos\frac{2\pi k}{T}t\,\Big|_{-T/4}^{T/4}$$

$$= 0$$

The periodic function in Fig. P16.1(b):

$$a_{\text{vb}} = 0$$

$$a_{\text{kb}} = \frac{2}{T}\left[\int_0^{T/4} 90\cos\frac{2\pi kt}{T}\,dt + \int_{T/4}^{T/2} 30\cos\frac{2\pi kt}{T}\,dt\right]$$

$$-\frac{2}{T}\left[\int_{T/2}^{3T/4} 90\cos\frac{2\pi kt}{T}\,dt + \int_{3T/4}^{T} 30\cos\frac{2\pi kt}{T}\,dt\right]$$

$$= \frac{60}{T}\frac{T}{2\pi k}\left[3\sin\frac{2\pi kt}{T}\,\Big|_0^{T/4} + \sin\frac{2\pi kt}{T}\,\Big|_{T/4}^{T/2}\right]$$

$$-\frac{60}{T}\frac{T}{2\pi k}\left[3\sin\frac{2\pi kt}{T}\,\Big|_{T/2}^{3T/4} + \sin\frac{2\pi kt}{T}\,\Big|_{3T/4}^{T}\right]$$

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$$= \frac{30}{\pi k}\left[2\sin\frac{\pi k}{2} - 2\sin\frac{3\pi k}{2}\right] = \frac{120}{\pi k}\sin\frac{\pi k}{2}$$

Note that a_{kb} is 0 for even values of k.

$$b_{\text{kb}} = \frac{2}{T}\left[\int_0^{T/4} 90\sin\frac{2\pi kt}{T}\,dt + \int_{T/4}^{T/2} 30\sin\frac{2\pi kt}{T}\,dt\right]$$

$$-\frac{2}{T}\left[\int_{T/2}^{3T/4} 90\sin\frac{2\pi kt}{T}\,dt + \int_{3T/4}^{T} 30\sin\frac{2\pi kt}{T}\,dt\right]$$

$$= \frac{-60}{T}\frac{T}{2\pi k}\left[3\cos\frac{2\pi kt}{T}\Big|_0^{T/4} + \cos\frac{2\pi kt}{T}\Big|_{T/4}^{T/2}\right]$$

$$-\frac{-60}{T}\frac{T}{2\pi k}\left[3\cos\frac{2\pi kt}{T}\Big|_{T/2}^{3T/4} + \cos\frac{2\pi kt}{T}\Big|_{3T/4}^{T}\right]$$

$$= \frac{120}{\pi k}\left[1 - \cos(k\pi)\right]$$

Note that b_{kb} is 0 for even values of k and equal to $120(2)/k\pi$ for odd values of k.

[e] For the periodic function in Fig. P16.1(a),

$$v(t) = 25 + \frac{100}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{2}\cos n\omega_o t\right)\ \text{V}$$

For the periodic function in Fig. P16.1(b),

$$v(t) = \frac{120}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\left(\sin\frac{n\pi}{2}\cos n\omega_o t + 2\sin n\omega_o t\right)\ \text{V}$$

P 16.2 **[a]** Odd function with half- and quarter-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for even k; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/4} V_m \sin k\omega_0 t\,dt = \frac{4V_m}{k\pi}, \qquad k \text{ odd}$$

$$\text{and} \quad v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin n\omega_0 t\ \text{V}$$

[b] Even function: $b_k = 0$ for k

$$a_v = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{\pi}{T}t\,dt = \frac{2V_m}{\pi}$$

$$a_k = \frac{4}{T}\int_0^{T/2} V_m \sin\frac{\pi}{T}t\cos k\omega_0 t\,dt = \frac{2V_m}{\pi}\left(\frac{1}{1-2k} + \frac{1}{1+2k}\right)$$

$$= \frac{4V_m/\pi}{1-4k^2}$$

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$$\text{and} \quad v(t) = \frac{2V_m}{\pi}\left[1 + 2\sum_{n=1}^{\infty}\frac{1}{1-4n^2}\cos n\omega_0 t\right]\text{V}$$

[**c**] $a_v = \dfrac{1}{T}\displaystyle\int_0^{T/2} V_m \sin\left(\frac{2\pi}{T}\right)t\,dt = \frac{V_m}{\pi}$

$$a_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t\cos k\omega_0 t\,dt = \frac{V_m}{\pi}\left(\frac{1+\cos k\pi}{1-k^2}\right)$$

Note: $a_k = 0$ for k-odd, $a_k = \dfrac{2V_m}{\pi(1-k^2)}$ for k even,

$$b_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t\sin k\omega_0 t\,dt = 0 \quad \text{for} \quad k = 2, 3, 4, \ldots$$

For $k = 1$, we have $b_1 = \dfrac{V_m}{2}$; therefore

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin\omega_0 t + \frac{2V_m}{\pi}\sum_{n=2,4,6,\ldots}^{\infty}\frac{1}{1-n^2}\cos n\omega_0 t\,\text{V}$$

P 16.3 In studying the periodic function in Fig. P16.3 note that it can be visualized as the combination of two half-wave rectified sine waves, as shown in the figure below. Hence we can use the Fourier series for a half-wave rectified sine wave which is given as the answer to Problem 16.2(c).

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In using the previously derived Fourier series for the half-wave rectified sine wave we note $v_1(t)$ has been shifted $T/4$ units to the left and $v_2(t)$ has been shifted $T/4$ units to the right. Thus,

$$v_1(t) = \frac{50}{\pi} + 25\sin\omega_o(t+T/4) - \frac{100}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos n\omega_o(t+T/4)}{(1-n^2)}\,\text{V}$$

Now observe the following:

$$\sin\omega_o(t+T/4) = \sin(\omega_o t + \pi/2) = \cos\omega_o t$$

$$\cos n\omega_o(t+T/4) = \cos(n\omega_o t + n\pi/2) = \cos\frac{n\pi}{2}\cos n\omega_o t \quad \text{because } n \text{ is even.}$$

$$\therefore\quad v_1(t) = \frac{50}{\pi} + 25\cos\omega_o t - \frac{100}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos(n\pi/2)\cos(n\omega_o t)}{(1-n^2)}\,\text{V}$$

Also,

$$v_2(t) = \frac{25}{\pi} + 12.5\sin\omega_o(t-T/4) - \frac{50}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos n\omega_o(t-T/4)}{(1-n^2)}\,\text{V}$$

Again, observe the following:

$$\sin(\omega_o t - \pi/2) = -\cos\omega_o t$$

$$\cos(n\omega_o t - n\pi/2) = \cos(n\pi/2)\cos n\omega_o t \quad \text{because } n \text{ is even.}$$

$$\therefore\quad v_2(t) = \frac{25}{\pi} - 12.5\cos\omega_o t - \frac{50}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos(n\pi/2)\cos(n\omega_o t)}{(1-n^2)}\,\text{V}$$

Thus: $\quad v = v_1 + v_2$

$$\therefore\quad v(t) = \frac{75}{\pi} + 12.5\cos\omega_o t - \frac{150}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos(n\pi/2)\cos(n\omega_o t)}{(1-n^2)}\,\text{V}$$

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P 16.4 $a_{\mathrm{v}} = \dfrac{1}{T}\displaystyle\int_0^{T/4} V_m\,dt + \dfrac{1}{T}\int_{T/4}^{T} \dfrac{V_m}{2}\,dt = \dfrac{5}{8}V_m = 62.5\pi\ \mathrm{V}$

$$a_{\mathrm{k}} = \frac{2}{T}\left[\int_0^{T/4} V_m \cos k\omega_0 t\,dt + \int_{T/4}^{T} \frac{V_m}{2}\cos k\omega_0 t\,dt\right]$$

$$= \frac{V_m}{k\omega_0 T}\sin\frac{k\pi}{2} = \frac{50}{k}\sin\frac{k\pi}{2}$$

$$b_{\mathrm{k}} = \frac{2}{T}\left[\int_0^{T/4} V_m \sin k\omega_0 t\,dt + \int_{T/4}^{T} \frac{V_m}{2}\sin k\omega_0 t\,dt\right]$$

$$= \frac{V_m}{k\omega_0 T}\left[1-\cos\frac{k\pi}{2}\right] = \frac{50}{k}\left[1-\cos\frac{k\pi}{2}\right]$$

P 16.5 **[a]** $I_6 = \displaystyle\int_{t_o}^{t_o+T} \sin m\omega_0 t\,dt = -\frac{1}{m\omega_0}\cos m\omega_0 t\,\Big|_{t_o}^{t_o+T}$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0(t_o+T) - \cos m\omega_0 t_o]$$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0 t_o \cos m\omega_0 T - \sin m\omega_0 t_o \sin m\omega_0 T - \cos m\omega_0 t_o]$$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0 t_o - 0 - \cos m\omega_0 t_o] = 0 \quad \text{for all } m,$$

$$I_7 = \int_{t_o}^{t_o+T} \cos m\omega_0 t_o\,dt = \frac{1}{m\omega_0}[\sin m\omega_0 t]\,\Big|_{t_o}^{t_o+T}$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0(t_o+T) - \sin m\omega_0 t_o]$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0 t_o - \sin m\omega_0 t_o] = 0 \quad \text{for all } m$$

[b] $I_8 = \displaystyle\int_{t_o}^{t_o+T} \cos m\omega_0 t \sin n\omega_0 t\,dt = \frac{1}{2}\int_{t_o}^{t_o+T}[\sin(m+n)\omega_0 t - \sin(m-n)\omega_0 t]\,dt$

But $(m+n)$ and $(m-n)$ are integers, therefore from I_6 above, $I_8 = 0$ for all m, n.

[c] $I_9 = \displaystyle\int_{t_o}^{t_o+T} \sin m\omega_0 t \sin n\omega_0 t\,dt = \frac{1}{2}\int_{t_o}^{t_o+T}[\cos(m-n)\omega_0 t - \cos(m+n)\omega_0 t]\,dt$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we get

$$I_9 = \frac{1}{2}\int_{t_o}^{t_o+T} dt - \frac{1}{2}\int_{t_o}^{t_o+T}\cos 2m\omega_0 t\,dt = \frac{T}{2} - 0 = \frac{T}{2}$$

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[d] $I_{10} = \int_{t_o}^{t_o+T} \cos m\omega_0 t \cos n\omega_0 t \, dt$

$$= \frac{1}{2}\int_{t_o}^{t_o+T} [\cos(m-n)\omega_0 t + \cos(m+n)\omega_0 t] \, dt$$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we have

$$I_{10} = \frac{1}{2}\int_{t_o}^{t_o+T} dt + \frac{1}{2}\int_{t_o}^{t_o+T} \cos 2m\omega_0 t \, dt = \frac{T}{2} + 0 = \frac{T}{2}$$

P 16.6 $f(t)\sin k\omega_0 t = a_v \sin k\omega_0 t + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t \sin k\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \sin k\omega_0 t$

Now integrate both sides from t_o to $t_o + T$. All the integrals on the right-hand side reduce to zero except in the last summation when $n = k$, therefore we have

$$\int_{t_o}^{t_o+T} f(t)\sin k\omega_0 t \, dt = 0 + 0 + b_k\left(\frac{T}{2}\right) \quad \text{or} \quad b_k = \frac{2}{T}\int_{t_o}^{t_o+T} f(t)\sin k\omega_0 t \, dt$$

P 16.7 $a_v = \frac{1}{T}\int_{t_o}^{t_o+T} f(t)\,dt = \frac{1}{T}\left\{\int_{-T/2}^{0} f(t)\,dt + \int_{0}^{T/2} f(t)\,dt\right\}$

Let $t = -x, \quad dt = -dx, \quad x = \frac{T}{2}$ when $t = \frac{-T}{2}$

and $x = 0$ when $t = 0$

Therefore $\frac{1}{T}\int_{-T/2}^{0} f(t)\,dt = \frac{1}{T}\int_{T/2}^{0} f(-x)(-dx) = -\frac{1}{T}\int_{0}^{T/2} f(x)\,dx$

Therefore $a_v = -\frac{1}{T}\int_{0}^{T/2} f(t)\,dt + \frac{1}{T}\int_{0}^{T/2} f(t)\,dt = 0$

$$a_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t \, dt + \frac{2}{T}\int_{0}^{T/2} f(t)\cos k\omega_0 t \, dt$$

Again, let $t = -x$ in the first integral and we get

$$\frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t \, dt = -\frac{2}{T}\int_{0}^{T/2} f(x)\cos k\omega_0 x \, dx$$

Therefore $a_k = 0$ for all k.

$$b_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t \, dt + \frac{2}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t \, dt$$

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Using the substitution $t = -x$, the first integral becomes

$$\frac{2}{T}\int_0^{T/2} f(x)\sin k\omega_0 x\,dx$$

Therefore we have $\quad b_k = \dfrac{4}{T}\displaystyle\int_0^{T/2} f(t)\sin k\omega_0 t\,dt$

P 16.8 $\quad b_k = \dfrac{2}{T}\displaystyle\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt + \dfrac{2}{T}\int_0^{T/2} f(t)\sin k\omega_0 t\,dt$

Now let $t = x - T/2$ in the first integral, then $dt = dx$, $x = 0$ when $t = -T/2$ and $x = T/2$ when $t = 0$, also
$\sin k\omega_0(x - T/2) = \sin(k\omega_0 x - k\pi) = \sin k\omega_0 x\cos k\pi$. Therefore

$$\frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt = -\frac{2}{T}\int_0^{T/2} f(x)\sin k\omega_0 x\cos k\pi\,dx \quad \text{and}$$

$$b_k = \frac{2}{T}(1-\cos k\pi)\int_0^{T/2} f(x)\sin k\omega_0 t\,dt$$

Now note that $1 - \cos k\pi = 0$ when k is even, and $1 - \cos k\pi = 2$ when k is odd. Therefore $b_k = 0$ when k is even, and

$$b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin k\omega_0 t\,dt \quad \text{when } k \text{ is odd}$$

P 16.9 Because the function is even and has half-wave symmetry, we have $a_v = 0$, $a_k = 0$ for k even, $b_k = 0$ for all k and

$$a_k = \frac{4}{T}\int_0^{T/2} f(t)\cos k\omega_0 t\,dt, \qquad k \text{ odd}$$

The function also has quarter-wave symmetry;
therefore $f(t) = -f(T/2 - t)$ in the interval $T/4 \le t \le T/2$;
thus we write

$$a_k = \frac{4}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt + \frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\,dt$$

Now let $t = (T/2 - x)$ in the second integral, then $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$. Therefore we get

$$\frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\,dt = -\frac{4}{T}\int_0^{T/4} f(x)\cos k\pi\cos k\omega_0 x\,dx$$

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Therefore we have

$$a_k = \frac{4}{T}(1 - \cos k\pi) \int_0^{T/4} f(t) \cos k\omega_0 t \, dt$$

But k is odd, hence

$$a_k = \frac{8}{T} \int_0^{T/4} f(t) \cos k\omega_0 t \, dt, \qquad k \text{ odd}$$

P 16.10 Because the function is odd and has half-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, and $b_k = 0$ for k even. For k odd we have

$$b_k = \frac{4}{T} \int_0^{T/2} f(t) \sin k\omega_0 t \, dt$$

The function also has quarter-wave symmetry, therefore $f(t) = f(T/2 - t)$ in the interval $T/4 \le t \le T/2$. Thus we have

$$b_k = \frac{4}{T} \int_0^{T/4} f(t) \sin k\omega_0 t \, dt + \frac{4}{T} \int_{T/4}^{T/2} f(t) \sin k\omega_0 t \, dt$$

Now let $t = (T/2 - x)$ in the second integral and note that $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$, thus

$$\frac{4}{T} \int_{T/4}^{T/2} f(t) \sin k\omega_0 t \, dt = -\frac{4}{T} \cos k\pi \int_0^{T/4} f(x)(\sin k\omega_0 x) \, dx$$

But k is odd, therefore the expression for b_k becomes

$$b_k = \frac{8}{T} \int_0^{T/4} f(t) \sin k\omega_0 t \, dt$$

P 16.11 **[a]** $\omega_o = \dfrac{2\pi}{T} = \pi$ rad/s

[b] yes

[c] no

[d] yes

P 16.12 **[a]** $f = \dfrac{1}{T} = \dfrac{1}{40 \times 10^{-3}} = 25\,\text{Hz}$

[b] no

[c] yes

[d] yes

[e] yes

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[f] $a_v = 0,$ function is odd

$a_k = 0,$ for all k; the function is odd

$b_k = 0,$ for k even, the function has half-wave symmetry

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t, \qquad k \text{ odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8} -8t\sin k\omega_o t\,dt + \int_{T/8}^{T/4} -0.04\sin k\omega_o t\,dt\right\}$$

$$= \frac{8}{T}\{\text{Int1} + \text{Int2}\}$$

$$\text{Int1} = -8\int_0^{T/8} t\sin k\omega_o t\,dt$$

$$= -8\left[\frac{1}{k^2\omega_o^2}\sin k\omega_o t - \frac{t}{k\omega_o}\cos k\omega_o t\,\Big|_0^{T/8}\right]$$

$$= \frac{-8}{k^2\omega_o^2}\sin\frac{k\pi}{4} + \frac{T}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int2} = -0.04\int_{T/8}^{T/4}\sin k\omega_o t\,dt = \frac{0.04}{k\omega_o}\cos k\omega_o t\,\Big|_{T/8}^{T/4} = \frac{-0.04}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int1} + \text{Int2} = \frac{-8}{k^2\omega_o^2}\sin\frac{k\pi}{4} + \left(\frac{-0.04}{k\omega_o} + \frac{T}{k\omega_o}\right)\cos\frac{k\pi}{4}$$

$T = 0.04\,\text{s}$

$$\therefore \quad \text{Int1} + \text{Int2} = \frac{-8}{k^2\omega_o^2}\sin\frac{k\pi}{4}$$

$$b_k = \left[\frac{8}{T}\cdot\frac{-8}{4\pi^2k^2}\cdot T^2\right]\sin\frac{k\pi}{4} = \frac{-0.64}{\pi^2k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

$$i(t) = \frac{-640}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sin(n\pi/4)}{n^2}\sin n\omega_o t\,\text{mA}$$

P 16.13 **[a]** $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for all k, $a_k = 0$ for k-even; for odd k we have

$$a_k = \frac{8}{T}\int_0^{T/4} V_m\cos k\omega_0 t\,dt = \frac{4V_m}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

$$v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n}\sin\frac{n\pi}{2}\right]\cos n\omega_0 t\,\text{V}$$

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[b] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for k-even, $a_k = 0$ for all k; for k-odd we have

$$a_k = \frac{8}{T}\int_0^{T/4}\left(\frac{4V_p}{T}t - V_p\right)\cos k\omega_0 t\,dt = \frac{-8V_p}{\pi^2 k^2}$$

Therefore $\quad v(t) = \dfrac{-8V_p}{\pi^2}\displaystyle\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\cos n\omega_0 t\ \mathrm{V}$

P 16.14 **[a]**

[b] $a_v = 0$; $\quad a_k = 0$ for all k even; $\quad b_k = 0$ for all k

For k odd, $\quad a_k = \dfrac{8}{T}\displaystyle\int_0^{T/4} f(t)\cos k\omega_o t\,dt$

$$a_k = \frac{8}{T}\int_0^{T/8}\left(1 + \frac{16t}{T}\right)\cos k\omega_o t\,dt + \frac{8}{T}\int_{T/8}^{T/4}\left(6 - \frac{24t}{T}\right)\cos k\omega_o t\,dt$$

$$= \text{Int1} + \text{Int2}$$

$$\text{Int1} = \frac{8}{T}\int_0^{T/8}\cos k\omega_o t\,dt + \frac{128}{T^2}\int_0^{T/8} t\cos k\omega_o t\,dt$$

$$= \frac{8}{T}\frac{\sin k\omega_o t}{k\omega_o}\Big|_0^{T/8} + \frac{128}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t}{k\omega_o}\sin k\omega_o t\right]_0^{T/8}$$

$k\omega_o T = 2k\pi; \qquad (k\omega_o T)^2 = 4k^2\pi^2$

$$\text{Int1} = \frac{12}{k\pi}\sin\frac{k\pi}{4} + \frac{32}{k^2\pi^2}\left[\cos\left(\frac{k\pi}{4}\right) - 1\right] \qquad k \text{ odd}$$

$$\text{Int2} = \frac{48}{T}\int_{T/8}^{T/4}\cos k\omega_o t\,dt - \frac{192}{T^2}\int_{T/8}^{T/4} t\cos k\omega_o t\,dt$$

$$= \frac{48}{T}\frac{\sin k\omega_o t}{k\omega_o}\Big|_{T/8}^{T/4} - \frac{192}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t}{k\omega_o}\sin k\omega_o t\right]_{T/8}^{T/4}$$

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$$\text{Int2} = \frac{-12}{k\pi}\sin\frac{k\pi}{4} + \frac{48}{k^2\pi^2}\cos\frac{k\pi}{4} \qquad k \text{ odd}$$

$$\begin{aligned} a_k &= \text{Int1} + \text{Int2} \\ &= \frac{80}{k^2\pi^2}\cos\frac{k\pi}{4} - \frac{32}{k^2\pi^2} \end{aligned}$$

[c] $a_1 = \dfrac{80}{\pi^2}\cos\dfrac{\pi}{4} - \dfrac{32}{\pi^2} = 2.489$

$$a_3 = \frac{80}{9\pi^2}\cos\frac{3\pi}{4} - \frac{32}{9\pi^2} = -0.9971$$

$$a_5 = \frac{80}{25\pi^2}\cos\frac{5\pi}{4} - \frac{32}{25\pi^2} = -0.359$$

$$f(t) = 2.489\cos\omega_o t - 0.9971\cos 3\omega_o t - 0.359\cos 5\omega_o t - \cdots$$

[d] $f(T/8) = 2.489\cos(\pi/4) - 0.9971\cos(3\pi/4) - 0.359\cos(5\pi/4) = 2.719$

P 16.15 **[a]**

[b] Even, since $f(t) = f(-t)$

[c] Yes, since $f(t) = -f(T/2 - t)$ in the interval $0 < t < 10$.

[d] $a_v = 0, \quad a_k = 0, \quad$ for k even $\quad$ (half-wave symmetry)

$b_k = 0, \quad$ for all $k \quad$ (function is even)

Because of the quarter-wave symmetry, the expression for a_k is

$$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt, \quad k \text{ odd}$$

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$$= \frac{8}{20}\int_0^5 10t^2 \cos k\omega_0 t\, dt = 4\left[\frac{2t}{k^2\omega_0^2}\cos k\omega_0 t + \frac{k^2\omega_0^2 t^2 - 2}{k^3\omega_0^3}\sin k\omega_0 t\right]_0^5$$

$$k\omega_0(5) = k\left(\frac{2\pi}{20}\right)(5) = \frac{k\pi}{2}$$

$$\cos(k\pi/2) = 0, \quad \text{since } k \text{ is odd}$$

$$\therefore \qquad a_k = \frac{2}{5}\left[0 + \frac{(k^2\pi^2/4) - 2}{k^3\omega_0^3}\sin(k\pi/2)\right] = \frac{k^2\omega_0^2 - 8}{10k^3\omega_0^3}\sin(k\pi/2)$$

$$\omega_0 = \frac{2\pi}{20} = \frac{\pi}{10}; \qquad \omega_0^2 = \frac{\pi^2}{100}; \qquad \omega_0^3 = \frac{\pi^3}{1000}$$

$$a_k = \left(\frac{k^2\pi^2 - 800}{k^3\pi^3}\right)\sin(k\pi/2)$$

$$f(t) = \sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{n^2\pi^2 - 800}{\pi^3 n^3}\right]\sin(n\pi/2)\cos(n\omega_0 t)$$

[e] $\cos n\omega_0(t-5) = \cos(n\omega_0 t - n\pi/2) = \sin(n\pi/2)\sin n\omega_0 t$

$$f(t) = \sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{n^2\pi^2 - 800}{\pi^3 n^3}\right]\sin^2(n\pi/2)\sin(n\omega_0 t)$$

$$= \sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{n^2\pi^2 - 800}{\pi^3 n^3}\right]\sin(n\omega_0 t)$$

P 16.16 **[a]**

[b] Odd, since $f(-t) = -f(t)$

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[c] $f(t)$ has quarter-wave symmetry, since $f(T/2-t)=f(t)$ in the interval $0<t<4$.

[d] $a_v=0$, (half-wave symmetry); $a_k=0$, for all k (function is odd)

$b_k=0$, for k even (half-wave symmetry)

$$b_k=\frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt, \quad k \text{ odd}$$

$$=\frac{16}{20}\int_0^5 t^3\sin k\omega_0 t\,dt$$

$$=\frac{4}{5}\left[\frac{3t^2}{k^2\omega_0^2}\sin k\omega_0 t-\frac{6}{k^4\omega_0^4}\sin k\omega_0 t-\frac{t^3}{k\omega_0}\cos k\omega_0 t+\frac{6t}{k^3\omega_0^3}\cos k\omega_0 t\right]_0^5$$

$$k\omega_0(5)=k\left(\frac{2\pi}{20}\right)(5)=\frac{k\pi}{2}$$

$\cos(k\pi/2)=0$, since k is odd

$$\therefore \quad b_k=\left[\frac{60}{k^2\omega_0^2}\sin(k\pi/2)-\frac{4.8}{k^4\omega_0^4}\sin(k\pi/2)\right]$$

$$k\omega_0=k\left(\frac{2\pi}{20}\right)=\frac{k\pi}{10}; \qquad k^2\omega_0^2=\frac{k^2\pi^2}{100}; \qquad k^4\omega_0^4=\frac{k^4\pi^4}{10{,}000}$$

$$\therefore \quad b_k=\frac{6000}{\pi^2k^2}\left[1-\frac{8}{\pi^2k^2}\right]\sin(k\pi/2), \quad k \text{ odd}$$

$$f(t)=\frac{6000}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n^2}\left(1-\frac{8}{\pi^2n^2}\right)\sin(n\pi/2)\right]\sin n\omega_0 t$$

[e] $\sin n\omega_0(t-2)=\sin(n\omega_0 t-n\pi/2)=-\cos n\omega_0 t\sin(n\pi/2)$

$$f(t)=\frac{-6000}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n^2}\left(1-\frac{8}{\pi^2n^2}\right)\right]\cos n\omega_0 t$$

P 16.17 **[a]** $i(t)$ is odd, therefore $a_v=0$ and $b_k=0$ for all k.

$$f(t)=i(t)=I_m-\frac{2I_m}{T}t, \quad 0\le t\le T$$

$$b_k=\frac{4}{T}\int_0^{T/2} f(t)\sin k\omega_o t\,dt$$

$$=\frac{4}{T}\int_0^{T/2}\left(I_m-\frac{2I_m}{T}t\right)\sin k\omega_o t\,dt$$

$$=\frac{4I_m}{T}\left[\int_0^{T/2}\sin k\omega_0 t\,dt-\frac{2}{T}\int_0^{T/2} t\sin k\omega_0 t\,dt\right]$$

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$$= \frac{4I_m}{T}\left[\frac{-\cos k\omega_0 t}{k\omega_0}\Big|_0^{T/2} - \frac{2}{T}\left(\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t\cos k\omega_0 t}{k\omega_0}\right)\Big|_0^{T/2}\right]$$

$$= \frac{4I_m}{T}\left[\frac{1-\cos k\pi}{k\omega_0} + \frac{\cos k\pi}{k\omega_0}\right]$$

$$= \frac{4I_m}{k\omega_0 T} = \frac{2I_m}{k\pi}$$

$$\therefore \quad i(t) = \frac{2I_m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\omega_o t$$

[b] $i(t) = \dfrac{2I_m}{\pi}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin n\omega_o(t+T/2)$

$$= \frac{2I_m}{\pi}\sum_{n=1}^{\infty}\frac{\cos n\pi}{n}\sin n\omega_o t$$

P 16.18 $v_2(t+T/8)$ is even, so $b_k = 0$ for all k.

$$a_v = \frac{(V_m/2)(T/4)}{T} = \frac{V_m}{8}$$

$$a_k = \frac{4}{T}\int_0^{T/8}\frac{V_m}{2}\cos k\omega_0 t\,dt = \frac{V_m}{k\pi}\sin\frac{k\pi}{4}$$

Therefore, $\quad v_2(t+T/8) = \dfrac{V_m}{8} + \dfrac{V_m}{\pi}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi}{4}\cos n\omega_0 t$

so $\quad v_2(t) = \dfrac{V_m}{8} + \dfrac{V_m}{\pi}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi}{4}\cos n\omega_0(t-T/8)$

$$\therefore \quad v(t) = \frac{V_m}{2} + \frac{V_m}{8} + \frac{V_m}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{4}\cos\frac{n\pi}{4}\right)\cos n\omega_0 t + \left(\frac{1}{n}\sin^2\frac{n\pi}{4}\right)\sin n\omega_0 t$$

$$= \frac{5V_m}{8} + \frac{V_m}{2\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{2}\right)\cos n\omega_0 t + \left(1-\cos\frac{n\pi}{2}\right)\sin n\omega_0 t\ \text{V}$$

Thus, since $a_v = 5V_m/8 = 37.5\pi$ V,

$$a_k = \frac{V_m}{2\pi k}\sin\frac{k\pi}{2} = \frac{30}{k}\sin\frac{k\pi}{2}$$

and

$$b_k = \frac{V_m}{2\pi k}\left[1-\cos\frac{k\pi}{2}\right] = \frac{30}{k}\left[1-\cos\frac{k\pi}{2}\right]$$

These equations match the equations for a_v, a_k, and b_k derived in Problem 16.4.

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P 16.19 From Problem 16.1(a),

$$a_v = 25\,\text{V} = A_0$$

$$a_n = \frac{100}{n\pi}\sin\frac{n\pi}{2}$$

Therefore,

$$A_n = \frac{100}{n\pi}\sin\frac{n\pi}{2}$$

and

$$-\theta_n = 0^\circ$$

Thus, $v(t) = 25 + \dfrac{100}{\pi}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi}{2}\cos n\omega_o t\,\text{V}$

For the periodic function in Fig. P16.1(b):

$$a_n = \frac{120}{\pi n}\sin\frac{\pi n}{2} \qquad \text{and} \qquad b_n = \frac{240}{\pi n} \quad \text{for } n \text{ odd.}$$

$$A_n\underline{/-\theta_n} = a_n - jb_n = \frac{120}{\pi n}\sin\frac{\pi n}{2} - j\frac{240}{\pi n}, \qquad n \text{ odd}$$

Therefore,

$$A_n = \frac{120\sqrt{5}}{n\pi}, \qquad n \text{ odd}$$

and

$$\theta_n = \tan^{-1}(-240/120) = -63.43^\circ, \quad n = 1, 5, 9, ...$$

and

$$\theta_n = \tan^{-1}(-240/-120) = 63.43^\circ, \quad n = 3, 7, 11, ...$$

Thus, $v(t) = \dfrac{120\sqrt{5}}{\pi}\displaystyle\sum_{n=1,5,9,...}^{\infty}\frac{1}{n}\cos(n\omega_o t - 63.43^\circ)$

$$+ \frac{120\sqrt{5}}{\pi}\sum_{n=3,7,11,...}^{\infty}\frac{1}{n}\cos(n\omega_o t + 63.43^\circ)\,\text{V}$$

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P 16.20 The periodic function in Problem 16.12 is odd, so $a_v = 0$ and $a_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = 0 - jb_n = b_n\underline{/-90^\circ}$$

From Problem 16.12,

$$b_k = \frac{-0.64}{\pi^2 k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

Therefore,

$$A_n = \frac{-0.64}{\pi^2 k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

and

$$-\theta_n = -90^\circ, \qquad n \text{ odd}$$

Thus, $i(t) = \dfrac{640}{\pi^2}\displaystyle\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sin(n\pi/4)}{n^2}\cos(n\omega_o t + 90^\circ)\,\text{mA}$

P 16.21 The periodic function in Problem 16.15 is even, so $b_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = a_n = a_n\underline{/0^\circ}$$

From Problem 16.15,

$$a_v = 0 = A_0$$

$$a_n = \frac{n^2\pi^2 - 800}{\pi^3 n^3}\sin\frac{n\pi}{2}$$

Therefore,

$$A_n = \frac{n^2\pi^2 - 800}{\pi^3 n^3}\sin\frac{n\pi}{2}$$

and

$$-\theta_n = 0^\circ$$

Thus, $f(t) = \dfrac{1}{\pi^3}\displaystyle\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{n^2\pi^2 - 800}{n^3}\right)\sin\frac{n\pi}{2}\cos n\omega_o t$

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P 16.22 [**a**] The current has half-wave symmetry. Therefore,

$$a_v = 0; \qquad a_k = b_k = 0, \quad k \text{ even}$$

For k odd,

$$a_k = \frac{4}{T}\int_0^{T/2}\left(I_m - \frac{2I_m}{T}t\right)\cos k\omega_o t\,dt$$

$$= \frac{4}{T}\int_0^{T/2} I_m \cos k\omega_0 t\,dt - \frac{8I_m}{T^2}\int_0^{T/2} t\cos k\omega_0 t\,dt$$

$$= \frac{4I_m}{T}\frac{\sin k\omega_0 t}{k\omega_0}\Big|_0^{T/2} - \frac{8I_m}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_0^2} + \frac{t}{k\omega_0}\sin k\omega_0 t\right]_0^{T/2}$$

$$= 0 - \frac{8I_m}{T^2}\left[\frac{\cos k\pi}{k^2\omega_0^2} - \frac{1}{k^2\omega_0^2}\right]$$

$$= \left(\frac{8I_m}{T^2}\right)\left(\frac{1}{k^2\omega_0^2}\right)(1-\cos k\pi)$$

$$= \frac{4I_m}{\pi^2 k^2} = \frac{20}{k^2}, \quad \text{for } k \text{ odd}$$

$$b_k = \frac{4}{T}\int_0^{T/2}\left(I_m - \frac{2I_m}{T}t\right)\sin k\omega_o t\,dt$$

$$= \frac{4I_m}{T}\int_0^{T/2}\sin k\omega_0 t\,dt - \frac{8I_m}{T^2}\int_0^{T/2} t\sin k\omega_0 t\,dt$$

$$= \frac{4I_m}{T}\left[\frac{-\cos k\omega_0 t}{k\omega_0}\right]_0^{T/2} - \frac{8I_m}{T^2}\left[\frac{\sin k\omega_o t}{k^2\omega_0^2} - \frac{t}{k\omega_0}\cos k\omega_0 t\right]_0^{T/2}$$

$$= \frac{4I_m}{T}\left[\frac{1-\cos k\pi}{k\omega_0}\right] - \frac{8I_m}{T^2}\left[\frac{-T\cos k\pi}{2k\omega_0}\right]$$

$$= \frac{8I_m}{k\omega_0 T}\left[\frac{1}{2}\right]$$

$$= \frac{2I_m}{\pi k} = \frac{10\pi}{k}, \quad \text{for } k \text{ odd}$$

$$a_k - jb_k = \frac{20}{k^2} - j\frac{10\pi}{k} = \frac{10}{k}\left(\frac{2}{k} - j\pi\right) = \frac{10}{k^2}\sqrt{\pi^2k^2+4}\underline{/-\theta_k}$$

where $\quad \tan\theta_k = \dfrac{\pi k}{2}$

$$i(t) = 10\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sqrt{(n\pi)^2+4}}{n^2}\cos(n\omega_0 t - \theta_n), \qquad \theta_n = \tan^{-1}\frac{n\pi}{2}$$

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[b] $A_1 = 10\sqrt{4+\pi^2} \cong 37.24\,\text{A} \qquad \tan\theta_1 = \dfrac{\pi}{2} \qquad \theta_1 \cong 57.52^\circ$

$$A_3 = \frac{10}{9}\sqrt{4+9\pi^2} \cong 10.71\,\text{A} \qquad \tan\theta_3 = \frac{3\pi}{2} \qquad \theta_3 \cong 78.02^\circ$$

$$A_5 = \frac{10}{25}\sqrt{4+25\pi^2} \cong 6.33\,\text{A} \qquad \tan\theta_5 = \frac{5\pi}{2} \qquad \theta_5 \cong 82.74^\circ$$

$$A_7 = \frac{10}{49}\sqrt{4+49\pi^2} \cong 4.51\,\text{A} \qquad \tan\theta_7 = \frac{7\pi}{2} \qquad \theta_7 \cong 84.80^\circ$$

$$A_9 = \frac{10}{81}\sqrt{4+81\pi^2} \cong 3.50\,\text{A} \qquad \tan\theta_9 = \frac{9\pi}{2} \qquad \theta_9 \cong 85.95^\circ$$

$$\begin{aligned} i(t) \cong\ & 37.24\cos(\omega_o t - 57.52^\circ) + 10.71\cos(3\omega_o t - 78.02^\circ) \\ & +6.33\cos(5\omega_o t - 82.74^\circ) + 4.51\cos(7\omega_o t - 84.80^\circ) \\ & +3.50\cos(9\omega_o t - 85.95^\circ) + \ldots \end{aligned}$$

$$\begin{aligned} i(T/4) \cong\ & 37.24\cos(90 - 57.52^\circ) + 10.71\cos(270 - 78.02^\circ) \\ & +6.33\cos(450 - 82.74^\circ) + 4.51\cos(630 - 84.80^\circ) \\ & +3.50\cos(810 - 85.95^\circ) \cong 26.23\,\text{A} \end{aligned}$$

Actual value:

$$i\left(\frac{T}{4}\right) = \frac{1}{2}(5\pi^2) \cong 24.67\,\text{A}$$

P 16.23 The function has half-wave symmetry, thus $a_k = b_k = 0$ for k-even, $a_v = 0$; for k-odd

$$a_k = \frac{4}{T}\int_0^{T/2} V_m \cos k\omega_0 t\,dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC}\cos k\omega_0 t\,dt$$

where $\quad \rho = \left[1 + e^{-T/2RC}\right].$

Upon integrating we get

$$\begin{aligned} a_k =\ & \frac{4V_m}{T}\frac{\sin k\omega_0 t}{k\omega_0}\Bigg|_0^{T/2} \\ & -\frac{8V_m}{\rho T}\cdot\left\{\frac{e^{-t/RC}}{(1/RC)^2 + (k\omega_0)^2}\cdot\left[\frac{-\cos k\omega_0 t}{RC} + k\omega_0\sin k\omega_0 t\right]\Bigg|_0^{T/2}\right\} \\ =\ & \frac{-8V_m RC}{T[1 + (k\omega_0 RC)^2]} \end{aligned}$$

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$$b_k = \frac{4}{T}\int_0^{T/2} V_m \sin k\omega_0 t\,dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC}\sin k\omega_0 t\,dt$$

$$= -\frac{4V_m}{T}\frac{\cos k\omega_0 t}{k\omega_0}\Big|_0^{T/2}$$

$$-\frac{8V_m}{\rho T}\cdot\left\{\frac{-e^{-t/RC}}{(1/RC)^2+(k\omega_0)^2}\cdot\left[\frac{\sin k\omega_0 t}{RC}+k\omega_0\cos k\omega_0 t\right]\Big|_0^{T/2}\right\}$$

$$= \frac{4V_m}{\pi k} - \frac{8k\omega_0 V_m R^2C^2}{T[1+(k\omega_0 RC)^2]}$$

P 16.24 **[a]** $a_k^2 + b_k^2 = a_k^2 + \left(\dfrac{4V_m}{\pi k} + k\omega_0 RC a_k\right)^2$

$$= a_k^2\,[1+(k\omega_0 RC)^2] + \tfrac{8V_m}{\pi k}\left[\tfrac{2V_m}{\pi k} + k\omega_0 RC a_k\right]$$

But $\quad a_k = \left\{\dfrac{-8V_m RC}{T\,[1+(k\omega_0 RC)^2]}\right\}$

Therefore $\quad a_k^2 = \left\{\dfrac{64V_m^2R^2C^2}{T^2[1+(k\omega_0 RC)^2]^2}\right\}$, $\quad$ thus we have

$$a_k^2 + b_k^2 = \frac{64V_m^2R^2C^2}{T^2[1+(k\omega_0 RC)^2]} + \frac{16V_m^2}{\pi^2k^2} - \frac{64V_m^2k\omega_0R^2C^2}{\pi kT[1+(k\omega_0 RC)^2]}$$

Now let $\alpha = k\omega_0 RC$ and note that $T = 2\pi/\omega_0$, thus the expression for $a_k^2 + b_k^2$ reduces to $a_k^2 + b_k^2 = 16V_m^2/\pi^2k^2(1+\alpha^2)$. It follows that

$$\sqrt{a_k^2+b_k^2} = \frac{4V_m}{\pi k\sqrt{1+(k\omega_0 RC)^2}}$$

[b] $b_k = k\omega_0 RC a_k + \dfrac{4V_m}{\pi k}$

Thus $\quad \dfrac{b_k}{a_k} = k\omega_0 RC + \dfrac{4V_m}{\pi k a_k} = \alpha - \dfrac{1+\alpha^2}{\alpha} = -\dfrac{1}{\alpha}$

Therefore $\quad \dfrac{a_k}{b_k} = -\alpha = -k\omega_0 RC$

P 16.25 Since $a_v = 0$ (half-wave symmetry), Eq. 16.38 gives us

$$v_o(t) = \sum_{1,3,5,\ldots}^{\infty}\frac{4V_m}{n\pi}\frac{1}{\sqrt{1+(n\omega_0 RC)^2}}\cos(n\omega_0 t - \theta_n) \quad \text{where} \quad \tan\theta_n = \frac{b_n}{a_n}$$

But from Eq. 16.57, we have $\tan\beta_k = k\omega_0 RC$. It follows from Eq. 16.72 that $\tan\beta_k = -a_k/b_k$ or $\tan\theta_n = -\cot\beta_n$. Therefore $\theta_n = 90^\circ + \beta_n$ and

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$\cos(n\omega_0 t - \theta_n) = \cos(n\omega_0 t - \beta_n - 90°) = \sin(n\omega_0 t - \beta_n)$, thus our expression for v_o becomes

$$v_o = \frac{4V_m}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{\sin(n\omega_0 t - \beta_n)}{n\sqrt{1 + (n\omega_0 RC)^2}}$$

P 16.26 **[a]** $e^{-x} \cong 1 - x$ for small x; therefore

$$e^{-t/RC} \cong \left(1 - \frac{t}{RC}\right) \quad \text{and} \quad e^{-T/2RC} \cong \left(1 - \frac{T}{2RC}\right)$$

$$v_o \cong V_m - \frac{2V_m[1 - (t/RC)]}{2 - (T/2RC)} = \left(\frac{V_m}{RC}\right)\left[\frac{2t - (T/2)}{2 - (T/2RC)}\right]$$

$$\cong \left(\frac{V_m}{RC}\right)\left(t - \frac{T}{4}\right) = \left(\frac{V_m}{RC}\right)t - \frac{V_m T}{4RC} \qquad \text{for} \quad 0 \le t \le \frac{T}{2}$$

[b] $a_k = \left(\frac{-8}{\pi^2 k^2}\right)V_p = \left(\frac{-8}{\pi^2 k^2}\right)\left(\frac{V_m T}{4RC}\right) = \frac{-4V_m}{\pi\omega_0 RCk^2}$

P 16.27 **[a]** Express v_g as a constant plus a symmetrical square wave. The constant is $V_m/2$ and the square wave has an amplitude of $V_m/2$, is odd, and has half- and quarter-wave symmetry. Therefore the Fourier series for v_g is

$$v_g = \frac{V_m}{2} + \frac{2V_m}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin n\omega_0 t$$

The dc component of the current is $V_m/2R$ and the kth harmonic phase current is

$$\mathbf{I}_k = \frac{2V_m/k\pi}{R + jk\omega_0 L} = \frac{2V_m}{k\pi\sqrt{R^2 + (k\omega_0 L)^2}}\underline{/-\theta_k}$$

$$\text{where} \quad \theta_k = \tan^{-1}\left(\frac{k\omega_0 L}{R}\right)$$

Thus the Fourier series for the steady-state current is

$$i = \frac{V_m}{2R} + \frac{2V_m}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{\sin(n\omega_0 t - \theta_n)}{n\sqrt{R^2 + (n\omega_0 L)^2}} \text{ A}$$

[b]

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The steady-state current will alternate between I_1 and I_2 in exponential traces as shown. Assuming $t = 0$ at the instant i increases toward (V_m/R), we have

$$i = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right) e^{-t/\tau} \quad \text{for} \quad 0 \le t \le \frac{T}{2}$$

and $i = I_2 e^{-[t-(T/2)]/\tau}$ for $T/2 \le t \le T$, where $\tau = L/R$. Now we solve for I_1 and I_2 by noting that

$$I_1 = I_2 e^{-T/2\tau} \quad \text{and} \quad I_2 = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right) e^{-T/2\tau}$$

These two equations are now solved for I_1. Letting $x = T/2\tau$, we get

$$I_1 = \frac{(V_m/R)e^{-x}}{1 + e^{-x}}$$

Therefore the equations for i become

$$i = \frac{V_m}{R} - \left[\frac{V_m}{R(1 + e^{-x})}\right] e^{-t/\tau} \qquad \text{for} \quad 0 \le t \le \frac{T}{2} \quad \text{and}$$

$$i = \left[\frac{V_m}{R(1 + e^{-x})}\right] e^{-[t-(T/2)]/\tau} \qquad \text{for} \quad \frac{T}{2} \le t \le T$$

A check on the validity of these expressions shows they yield an average value of $(V_m/2R)$:

$$\begin{aligned} I_{\text{avg}} &= \frac{1}{T}\left\{\int_0^{T/2}\left[\frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-t/\tau}\right]dt + \int_{T/2}^{T} I_2 e^{-[t-(T/2)]/\tau}\,dt\right\} \\ &= \frac{1}{T}\left\{\frac{V_m T}{2R} + \tau(1 - e^{-x})\left(I_1 - \frac{V_m}{R} + I_2\right)\right\} \\ &= \frac{V_m}{2R} \quad \text{since} \quad I_1 + I_2 = \frac{V_m}{R} \end{aligned}$$

P 16.28 **[a]** From the solution to Problem 16.13(a) the Fourier series for the input voltage is

$$v_g = \frac{4V_m}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n}\sin\frac{n\pi}{2}\right]\cos n\omega_0 t\ \text{V}$$

Since $V_m = 0.5\pi$ V and $T = 10\pi$ ms, we can write the input voltage as

$$\begin{aligned} v_g &= 2\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\right]\cos 200nt\ \text{V} \\ &= 2\cos 200t - \frac{2}{3}\cos 600t + \frac{2}{5}\cos 1000t - \frac{2}{7}\cos 1400t + \cdots \end{aligned}$$

We can phasor transform this Fourier series to get

$$\mathbf{V}_{g1} = 2\underline{/0^\circ} \qquad \omega_0 = 200\ \text{rad/s}$$

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$\mathbf{V}_{g3} = 0.667\underline{/180^\circ} \qquad 3\omega_0 = 600 \text{ rad/s}$

$\mathbf{V}_{g5} = 0.4\underline{/0^\circ} \qquad 5\omega_0 = 1000 \text{ rad/s}$

$\mathbf{V}_{g7} = 0.286\underline{/180^\circ} \qquad 7\omega_0 = 1400 \text{ rad/s}$

From the circuit in Fig. P16.28 we have

$$\frac{V_o}{R} + \frac{V_o - V_g}{sL} + (V_o - V_g)sC = 0$$

$$\therefore \qquad \frac{V_o}{V_g} = H(s) = \frac{s^2 + 1/LC}{s^2 + (s/RC) + (1/LC)}$$

Substituting in the numerical values gives

$$H(s) = \frac{s^2 + 10^6}{s^2 + 40s + 10^6}$$

$$H(j200) = \frac{96}{96 + j0.8} = 0.99997\underline{/-0.48^\circ}$$

$$H(j600) = \frac{64}{64 + j2.4} = 0.9993\underline{/-2.15^\circ}$$

$$H(j1000) = 0$$

$$H(j1400) = \frac{-96}{-96 + j5.6} = 0.9983\underline{/3.34^\circ}$$

$$\mathbf{V}_{o1} = (2\underline{/0^\circ})(0.99997\underline{/-0.48^\circ}) = 1.9999\underline{/-0.48^\circ}\,\text{V}$$

$$\mathbf{V}_{o3} = (0.667\underline{/180^\circ})(0.9993\underline{/-2.15^\circ}) = 0.6662\underline{/177.85^\circ}\,\text{V}$$

$$\mathbf{V}_{o5} = 0\,\text{V}$$

$$\mathbf{V}_{o7} = (0.286\underline{/180^\circ})(0.9983\underline{/3.34^\circ}) = 0.286\underline{/-176.66^\circ}\,\text{V}$$

$$v_o = 1.9999\cos(200t - 0.48^\circ) + 0.6662\cos(600t + 177.85^\circ)$$
$$+0.286\cos(1400t - 176.66^\circ) + \ldots \text{ V}$$

[b] The 5th harmonic at the frequency $\sqrt{1/LC} = 1000$ rad/s has been eliminated from the output voltage by the circuit, which is a band reject filter with a center frequency of 1000 rad/s.

P 16.29 $$v_i = \frac{4A}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin n\omega_0(t + T/4)$$

$$= \frac{4A}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{2}\right)\cos n\omega_0 t$$

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$$\omega_0 = \frac{2\pi}{4\pi} \times 10^6 = 10{,}000 \text{ rad/s}; \qquad \frac{4A}{\pi} = 120$$

$$v_i = 120 \sum_{n=1,3,5,\ldots}^{\infty} \left(\frac{1}{n} \sin \frac{n\pi}{2}\right) \cos 10{,}000nt \text{ V}$$

From the circuit

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + j\omega L} \cdot j\omega L = \frac{j\omega}{R/L + j\omega}\mathbf{V}_i = \frac{j\omega}{30{,}000 + j\omega}\mathbf{V}_i$$

$$\mathbf{V}_{i1} = 120\underline{/0^\circ} \text{ V}; \qquad \omega = 10{,}000 \text{ rad/s}$$

$$\mathbf{V}_{i3} = -40\underline{/0^\circ} = 40\underline{/180^\circ} \text{ V}; \qquad 3\omega = 30{,}000 \text{ rad/s}$$

$$\mathbf{V}_{i5} = 24\underline{/0^\circ} \text{ V}; \qquad 5\omega = 50{,}000 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{j10{,}000}{30{,}000 + j10{,}000}(120\underline{/0^\circ}) = 37.95\underline{/71.57^\circ} \text{ V}$$

$$\mathbf{V}_{o3} = \frac{j30{,}000}{30{,}000 + j30{,}000}(40\underline{/180^\circ}) = 28.28\underline{/-135^\circ} \text{ V}$$

$$\mathbf{V}_{o5} = \frac{j50{,}000}{30{,}000 + j50{,}000}(24\underline{/0^\circ}) = 20.58\underline{/30.96^\circ} \text{ V}$$

$$\therefore \qquad v_o = 37.95\cos(10{,}000t + 71.57^\circ) + 28.28\cos(30{,}000t - 135^\circ)$$

$$+20.58\cos(50{,}000t + 30.96^\circ) + \ldots \text{ V}$$

P 16.30 **[a]** $\dfrac{V_0 - V_g}{16s} + V_0(12.5 \times 10^{-6}s) + \dfrac{V_0}{1000} = 0$

$$V_0\left[\frac{1}{16s} + 12.6 \times 10^{-6}s + \frac{1}{1000}\right] = \frac{V_g}{16s}$$

$$V_0(1000 + 0.2s^2 + 16s) = 1000V_g$$

$$V_0 = \frac{5000V_g}{s^2 + 80s + 5000}$$

$$I_0 = \frac{V_0}{1000} = \frac{5V_g}{s^2 + 80s + 5000}$$

$$H(s) = \frac{I_0}{V_g} = \frac{5}{s^2 + 80s + 5000}$$

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$$H(nj\omega_0) = \frac{5}{(5000 - n^2\omega_0^2) + j80n\omega_0}$$

$$\omega_0 = \frac{2\pi}{T} = 240\pi; \qquad \omega_0^2 = 57{,}600\pi^2; \qquad 80\omega_0 = 19{,}200\pi$$

$$H(jn\omega_0) = \frac{5}{(5000 - 57{,}600\pi^2 n^2) + j19{,}200\pi n}$$

$$H(0) = 10^{-3}$$

$$H(j\omega_0) = 8.82 \times 10^{-6}\underline{/-173.89^\circ}$$

$$H(j2\omega_0) = 2.20 \times 10^{-6}\underline{/-176.96^\circ}$$

$$H(j3\omega_0) = 9.78 \times 10^{-7}\underline{/-177.97^\circ}$$

$$H(j4\omega_0) = 5.5 \times 10^{-7}\underline{/-178.48^\circ}$$

$$v_g = \frac{680}{\pi} - \frac{1360}{\pi}\left[\frac{1}{3}\cos\omega_0 t + \frac{1}{15}\cos 2\omega_0 t + \frac{1}{35}\cos 3\omega_0 t + \frac{1}{63}\cos 4\omega_0 t + \ldots\right]$$

$$\begin{aligned}
i_0 = {} & \frac{680}{\pi} \times 10^{-3} - \frac{1360}{3\pi}(8.82 \times 10^{-6})\cos(\omega_0 t - 173.89^\circ) \\
& -\frac{1360}{15\pi}(2.20 \times 10^{-6})\cos(2\omega_0 t - 176.96^\circ) \\
& -\frac{1360}{35\pi}(9.78 \times 10^{-7})\cos(3\omega_0 t - 177.97^\circ) \\
& -\frac{1360}{63\pi}(5.5 \times 10^{-7})\cos(4\omega_0 t - 178.48^\circ) - \ldots \\
& = 216.45 \times 10^{-3} + 1.27 \times 10^{-3}\cos(240\pi t + 6.11^\circ) \\
& +6.35 \times 10^{-5}\cos(480\pi t + 3.04^\circ) \\
& +1.21 \times 10^{-5}\cos(720\pi t + 2.03^\circ) \\
& +3.8 \times 10^{-6}\cos(960\pi t + 1.11^\circ) - \ldots
\end{aligned}$$

$$i_0 \cong 216.45 + 1.27\cos(240\pi t + 6.11^\circ)\,\text{mA}$$

Note that the sinusoidal component is very small compared to the dc component, so

$$i_0 \cong 216.45\,\text{mA} \quad \text{(a dc current)}$$

[b] The circuit is a low pass filter, so the harmonic terms are greatly reduced in the output.

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P 16.31 The function is odd with half-wave and quarter-wave symmetry. Therefore,

$$a_k = 0, \qquad \text{for all } k\text{; the function is odd}$$

$$b_k = 0, \qquad \text{for } k \text{ even, the function has half-wave symmetry}$$

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t, \qquad k \text{ odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/10} 100t\sin k\omega_o t\,dt + \int_{T/10}^{T/4} 5\sin k\omega_o t\,dt\right\}$$

$$= \frac{8}{T}\{\text{Int1} + \text{Int2}\}$$

$$\text{Int1} = 100\int_0^{T/10} t\sin k\omega_o t\,dt$$

$$= 100\left[\frac{1}{k^2\omega_o^2}\sin k\omega_o t - \frac{t}{k\omega_o}\cos k\omega_o t\,\Big|_0^{T/10}\right]$$

$$= \frac{100}{k^2\omega_o^2}\sin\frac{k\pi}{5} - \frac{10T}{k\omega_o}\cos\frac{k\pi}{5}$$

$$\text{Int2} = \int_{T/10}^{T/4} 5\sin k\omega_o t\,dt = \frac{-5}{k\omega_o}\cos k\omega_o t\,\Big|_{T/10}^{T/4} = \frac{5}{k\omega_o}\cos\frac{k\pi}{5}$$

$$\text{Int1} + \text{Int2} = \frac{100}{k^2\omega_o^2}\sin\frac{k\pi}{5} + \left(\frac{5}{k\omega_o} - \frac{10T}{k\omega_o}\right)\cos\frac{k\pi}{5}$$

$$10T = 10(0.5) = 5$$

$$\therefore \quad \text{Int1} + \text{Int2} = \frac{100}{k^2\omega_o^2}\sin\frac{k\pi}{5}$$

$$b_k = \left[\frac{8}{T}\cdot\frac{100}{4\pi^2k^2}\cdot T^2\right]\sin\frac{k\pi}{5} = \frac{100}{\pi^2k^2}\sin\frac{k\pi}{5}, \qquad k \text{ odd}$$

$$i(t) = \frac{100}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sin(n\pi/5)}{n^2}\sin n\omega_o t\ \text{A}$$

From the circuit,

$$H(s) = \frac{V_o}{I_g} = Z_{\text{eq}}$$

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$$Y_{\text{eq}} = \frac{1}{R_1} + \frac{1}{R_2 + sL} + sC$$

$$Z_{\text{eq}} = \frac{1/C(s + R_2/L)}{s^2 + s(R_1R_2C + L)/R_1LC + (R_1 + R_2)/R_1LC}$$

Therefore,

$$H(s) = \frac{20{,}000(s + 400)}{s^2 + 10{,}400s + 450 \times 10^4}$$

We want the output for the third harmonic:

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi; \qquad 3\omega_0 = 12\pi$$

$$I_{g3} = \frac{100}{\pi^2}\frac{1}{9}\sin\frac{3\pi}{5} = 1.07\underline{/0^\circ}$$

$$H(j12\pi) = \frac{20{,}000(j12\pi + 400)}{(j12\pi)^2 + 10{,}400(j12\pi) + 450 \times 10^4} = 1.78\underline{/0.403^\circ}$$

Therefore,

$$V_{o3} = H(j12\pi)I_{g3} = (1.78\underline{/0.403^\circ})(1.07\underline{/0^\circ}) = 1.9\underline{/0.403^\circ}\ \text{V}$$

$$v_{o3} = 1.9\sin(12\pi t + 0.403^\circ)\ \text{V}$$

P 16.32 $\omega_o = \dfrac{2\pi}{T} = \dfrac{2\pi}{10\pi} \times 10^6 = 200\ \text{krad/s}$

$$\therefore \quad n = \frac{3 \times 10^6}{0.2 \times 10^6} = 15; \qquad n = \frac{5 \times 10^6}{0.2 \times 10^6} = 25$$

$$H(s) = \frac{V_o}{V_g} = \frac{(1/RC)s}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{RC} = \frac{10^{12}}{(250 \times 10^3)(4)} = 10^6; \qquad \frac{1}{LC} = \frac{(10^3)(10^{12})}{10)(4)} = 25 \times 10^{12}$$

$$H(s) = \frac{10^6 s}{s^2 + 10^6 s + 25 \times 10^{12}}$$

$$H(j\omega) = \frac{j\omega \times 10^6}{(25 \times 10^{12} - \omega^2) + j10^6\omega}$$

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15th harmonic input:

$$v_{g15} = (150)(1/15)\sin(15\pi/2)\cos 15\omega_o t = -10\cos 3\times 10^6 t\,\text{V}$$

$$\therefore\quad \mathbf{V}_{g15} = 10\underline{/-180^\circ}\,\text{V}$$

$$H(j3\times 10^6) = \frac{j3}{16+j3} = 0.1843\underline{/79.38^\circ}$$

$$\mathbf{V}_{o15} = (10)(0.1843)\underline{/-100.62^\circ}\,\text{V}$$

$$v_{o15} = 1.84\cos(3\times 10^6 t - 100.62^\circ)\,\text{V}$$

25th harmonic input:

$$v_{g25} = (150)(1/25)\sin(25\pi/2)\cos 5\times 10^6 t = 6\cos 5\times 10^6 t\,\text{V}$$

$$\therefore\quad \mathbf{V}_{g25} = 6\underline{/0^\circ}\,\text{V}$$

$$H(j5\times 10^6) = \frac{j5}{0+j5} = 1\underline{/0^\circ}$$

$$\mathbf{V}_{o25} = 6\underline{/0^\circ}\,\text{V}$$

$$v_{o25} = 6\cos 5\times 10^6 t\,\text{V}$$

P 16.33 **[a]** $a_v = \dfrac{1}{T}\left[\dfrac{1}{2}\left(\dfrac{T}{2}\right)I_m + \dfrac{T}{2}I_m\right] = \dfrac{3I_m}{4}$

$$i(t) = \frac{2I_m}{T}t, \qquad 0 \le t \le T/2$$

$$i(t) = I_m, \qquad T/2 \le t \le T$$

$$a_k = \frac{2}{T}\int_0^{T/2}\frac{2I_m}{T}t\cos k\omega_o t\,dt + \frac{2}{T}\int_{T/2}^{T} I_m\cos k\omega_o t\,dt$$

$$= \frac{I_m}{\pi^2 k^2}(\cos k\pi - 1)$$

$$b_k = \frac{2}{T}\int_0^{T/2}\frac{2I_m}{T}t\sin k\omega_o t\,dt + \frac{2}{T}\int_{T/2}^{T} I_m\sin k\omega_o t\,dt$$

$$= -\frac{I_m}{\pi k}$$

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$$a_v = \frac{3I_m}{4}, \qquad a_1 = \frac{-2I_m}{\pi^2}, \quad a_2 = 0,$$

$$b_1 = -\frac{I_m}{\pi}, \quad b_2 = -\frac{I_m}{2\pi}$$

$$\therefore \quad I_{\text{rms}} = I_m\sqrt{\frac{9}{16} + \frac{2}{\pi^4} + \frac{1}{2\pi^2} + \frac{1}{8\pi^2}} = 0.8040I_m$$

$$I_{\text{rms}} = 4.02\,\text{A}$$

$$P = (4.02)^2(2500) = 40.4\,\text{kW}$$

[b] Area under i^2:

$$A = \int_0^{T/2} \frac{4I_m^2}{T^2}t^2\,dt + I_m^2\frac{T}{2}$$

$$= \frac{4I_m^2}{T^2}\frac{t^3}{3}\bigg|_0^{T/2} + I_m^2\frac{T}{2}$$

$$= I_m^2 T\left[\frac{1}{6} + \frac{3}{6}\right] = \frac{2}{3}TI_m^2$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\cdot\frac{2}{3}TI_m^2} = \sqrt{\frac{2}{3}}I_m = 4.0825\,\text{A}$$

$$P = (4.0825)^2(2500) = 41.67\,\text{kW}$$

[c] Error $= \left(\dfrac{40.4}{41.67} - 1\right)100 = -3.05\%$

P 16.34 **[a]** $a_v = \dfrac{2\left(\frac{1}{2}\frac{T}{4}V_m\right)}{T} = \dfrac{V_m}{4}$

$$a_k = \frac{4}{T}\int_0^{T/4}\left[V_m - \frac{4V_m}{T}t\right]\cos k\omega_o t\,dt$$

$$= \frac{4V_m}{\pi^2k^2}\left[1 - \cos\frac{k\pi}{2}\right]$$

$$b_k = 0, \quad \text{all } k$$

$$a_v = \frac{200}{4} = 50\,\text{V}$$

$$a_1 = \frac{800}{\pi^2}$$

$$a_2 = \frac{800}{4\pi^2}(1 - \cos\pi) = \frac{400}{\pi^2}$$

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$$V_{\text{rms}} = \sqrt{(50)^2 + \frac{1}{2}\left[\left(\frac{800}{\pi^2}\right)^2 + \left(\frac{400}{\pi^2}\right)^2\right]} = 81.28\,\text{V}$$

$$P = \frac{(81.28)^2}{400} = 16.516\,\text{W}$$

[b] Area under v^2; $\quad 0 \le t \le T/4$

$$v^2 = 40{,}000 - \frac{320{,}000}{T}t + \frac{640{,}000}{T^2}t^2$$

$$A = 2\int_0^{T/4}\left[40{,}000 - \frac{320{,}000}{T}t + \frac{640{,}000}{T^2}t^2\right]dt = 6666.67T$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}6666.67T} = \sqrt{6666.67} = 81.65\,\text{V}$$

$$P = \sqrt{6666.67}^2/400 = 16.667\,\text{W}$$

[c] $\text{Error} = \left(\frac{16.516}{16.667} - 1\right)100 = -0.904\%$

P 16.35 $v_g = 10 - \frac{80}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\cos n\omega_o t\,\text{V}$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{4\pi}\times 10^3 = 500\ \text{rad/s}$$

$$v_g = 10 - \frac{80}{\pi^2}\cos 500t - \frac{80}{9\pi^2}\cos 1500t + \ldots$$

$$\frac{V_o - V_g}{sL} + sCV_o + \frac{V_o}{R} = 0$$

$$V_o(RLCs^2 + Ls + R) = RV_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{1/LC}{s^2 + s/RC + 1/LC}$$

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$$\frac{1}{LC} = \frac{10^6}{(0.1)(10)} = 10^6$$

$$\frac{1}{RC} = \frac{10^6}{(50\sqrt{2})(10)} = 1000\sqrt{2}$$

$$H(s) = \frac{10^6}{s^2 + 1000\sqrt{2}s + 10^6}$$

$$H(j\omega) = \frac{10^6}{10^6 - \omega^2 + j1000\omega\sqrt{2}}$$

$$H(j0) = 1$$

$$H(j500) = 0.9701\underline{/-43.31^\circ}$$

$$H(j1500) = 0.4061\underline{/-120.51^\circ}$$

$$v_o = 10(1) + \frac{80}{\pi^2}(0.9701)\cos(500t - 43.31^\circ)$$

$$+\frac{80}{9\pi^2}(0.4061)\cos(1500t - 120.51^\circ) + \ldots$$

$$v_o = 10 + 7.86\cos(500t - 43.31^\circ) + 0.3658\cos(1500t - 120.51^\circ) + \ldots$$

$$V_{\text{rms}} \cong \sqrt{10^2 + \left(\frac{7.86}{\sqrt{2}}\right)^2 + \left(\frac{0.3658}{\sqrt{2}}\right)^2} = 11.44\,\text{V}$$

$$P \cong \frac{V_{\text{rms}}^2}{50\sqrt{2}} = 1.85\,\text{W}$$

Note – the higher harmonics are severely attenuated and can be ignored. For example, the 5th harmonic component of v_o is

$$v_{o5} = (0.1580)\left(\frac{80}{25\pi^2}\right)\cos(2500t - 146.04^\circ) = 0.0512\cos(2500t - 146.04^\circ)\,\text{V}$$

P 16.36 **[a]** $v = 30 + 60\cos 2000t + 20\cos(8000t - 90^\circ)\,\text{V}$

$$i = 3 + 4\cos(2000t - 25^\circ) + \cos(8000t - 45^\circ)\,\text{A}$$

$$P = (30)(3) + \frac{1}{2}(60)(4)\cos(25^\circ) + \frac{1}{2}(20)(1)\cos(-45^\circ) = 205.83\,\text{W}$$

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[**b**] $V_{\text{rms}} = \sqrt{(30)^2 + \left(\frac{60}{\sqrt{2}}\right)^2 + \left(\frac{20}{\sqrt{2}}\right)^2} = 53.85\,\text{V}$

[**c**] $I_{\text{rms}} = \sqrt{(3)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 4.18\,\text{A}$

P 16.37 [**a**] Area under $v^2 = A = 4\int_0^{T/6} \frac{36V_m^2}{T^2} t^2\,dt + 2V_m^2\left(\frac{T}{3} - \frac{T}{6}\right)$

$$= \frac{2V_m^2 T}{9} + \frac{V_m^2 T}{3}$$

Therefore $V_{\text{rms}} = \sqrt{\frac{1}{T}\left(\frac{2V_m^2 T}{9} + \frac{V_m^2 T}{3}\right)} = V_m\sqrt{\frac{2}{9} + \frac{1}{3}} = 74.5356\,\text{V}$

[**b**] From Asssessment Problem 16.3,

$$v_g = 105.30\sin\omega_0 t - 4.21\sin 5\omega_0 t + 2.15\sin 7\omega_0 t + \cdots\,\text{V}$$

Therefore $V_{\text{rms}} \cong \sqrt{\frac{(105.30)^2 + (4.21)^2 + (2.15)^2}{2}} = 74.5306\,\text{V}$

P 16.38 [**a**] v_g has half-wave symmetry, quarter-wave symmetry, and is odd

$\therefore\ a_v = 0,\ a_k = 0$ all $k,\ b_k = 0$ k-even

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t\,dt, \quad k\text{-odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8} V_m \sin k\omega_o t\,dt + \int_{T/8}^{T/4} \frac{V_m}{2}\sin k\omega_o t\,dt\right\}$$

$$= \frac{8V_m}{T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\right]_0^{T/8} + \frac{8V_m}{2T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\right]_{T/8}^{T/4}$$

$$= \frac{8V_m}{k\omega_o T}\left[1 - \cos\frac{k\pi}{4}\right] + \frac{8V_m}{2Tk\omega_o}\left[\cos\frac{k\pi}{4} - 0\right]$$

$$= \frac{8V_m}{k\omega_o T}\left\{1 - \cos\frac{k\pi}{4} + \frac{1}{2}\cos\frac{k\pi}{4}\right\}$$

$$= \frac{4V_m}{\pi k}\left\{1 - 0.5\cos\frac{k\pi}{4}\right\} = \frac{1}{k}[16 - 8\cos(k\pi/4)]$$

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$$b_1 = 16 - 8\cos(\pi/4) = 10.34$$
$$b_3 = \frac{1}{3}[16 - 8\cos(3\pi/4)] = 7.22$$
$$b_5 = \frac{1}{5}[16 - 8\cos(5\pi/4)] = 4.33$$

$$V_g(\text{rms}) \approx \mathbf{V}_m\sqrt{\frac{10.34^2 + 7.22^2 + 4.33^2}{2}} = 9.43$$

[b] $\text{Area} = 2\left[2(4\pi)^2\left(\frac{T}{8}\right) + (2\pi)^2\left(\frac{T}{4}\right)\right] = 10\pi^2 T$

$$V_g(\text{rms}) = \sqrt{\frac{1}{T}(10\pi^2)T} = \sqrt{10}\pi = 9.935$$

[c] $\%\ \text{Error} = \left(\frac{9.43}{9.935} - 1\right)(100) = -5.08\%$

P 16.39 **[a]** $v(t) = \frac{480}{\pi}\{\sin\omega_o t + \frac{1}{3}\sin 3\omega_o t + \frac{1}{5}\sin 5\omega_o t + \frac{1}{7}\sin 7\omega_o t + \frac{1}{9}\sin 9\omega_o t + \cdots$

$$\begin{aligned} V_{\text{rms}} &= \frac{480}{\pi}\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{5\sqrt{2}}\right)^2 + \left(\frac{1}{7\sqrt{2}}\right)^2 + \left(\frac{1}{9\sqrt{2}}\right)^2} \\ &= \frac{480}{\pi\sqrt{2}}\sqrt{1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81}} \\ &= 117.55\,\text{V} \end{aligned}$$

[b] $\%\ \text{error} = \left(\frac{117.55}{120} - 1\right)(100) = -2.04\%$

[c] $v(t) = \frac{960}{\pi^2}\left\{\sin\omega_o t + \frac{1}{9}\sin 3\omega_o t + \frac{1}{25}\sin 5\omega_o t\right.$

$$\left. - \frac{1}{49}\sin 7\omega_o t + \frac{1}{81}\sin 9\omega_o t - \cdots\right\}$$

$$V_{\text{rms}} \cong \frac{960}{\pi^2\sqrt{2}}\sqrt{1 + \frac{1}{81} + \frac{1}{625} + \frac{1}{2401} + \frac{1}{6561}}$$

$$\cong 69.2765\,\text{V}$$

$$V_{\text{rms}} = \frac{120}{\sqrt{3}} = 69.2820\,\text{V}$$

$$\%\ \text{error} = \left(\frac{69.2765}{69.2820} - 1\right)(100) = -0.0081\%$$

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P 16.40 **[a]** $v(t) \approx \dfrac{340}{\pi} - \dfrac{680}{\pi}\left\{\dfrac{1}{3}\cos\omega_o t + \dfrac{1}{15}\cos 2\omega_o t + \cdots\right\}$

$$V_{\text{rms}} \approx \sqrt{\left(\frac{340}{\pi}\right)^2 + \left(\frac{680}{\pi}\right)^2\left[\left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{15\sqrt{2}}\right)^2\right]}$$

$$= \frac{340}{\pi}\sqrt{1 + 4\left(\frac{1}{18} + \frac{1}{450}\right)} = 120.0819\,\text{V}$$

[b] $V_{\text{rms}} = \dfrac{170}{\sqrt{2}} = 120.2082$

$$\%\ \text{error} = \left(\frac{120.0819}{120.2082} - 1\right)(100) = -0.11\%$$

[c] $v(t) \approx \dfrac{170}{\pi} + 85\sin\omega_o t - \dfrac{340}{3\pi}\cos 2\omega_o t$

$$V_{\text{rms}} \approx \sqrt{\left(\frac{170}{\pi}\right)^2 + \left(\frac{85}{\sqrt{2}}\right)^2 + \left(\frac{340}{3\sqrt{2}\pi}\right)^2} \approx 84.8021\,\text{V}$$

$$V_{\text{rms}} = \frac{170}{2} = 85\,\text{V}$$

$$\%\ \text{error} = -0.23\%$$

P 16.41 **[a]**

Area under i^2:

$$\text{A} = 4\left[\int_0^{T/8}\left(1 + \frac{16}{T}t\right)^2 dt + \int_{T/8}^{T/4}\left(6 - \frac{24}{T}t\right)^2 dt\right]$$

$$= 4\left[\frac{T}{8} + \frac{T}{4} + \frac{T}{6} + 9T - 4.5T - 9T + +2.25T + 3T - 0.0375T\right]$$

$$= \frac{11T}{3}$$

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$$I_{\text{rms}} = \sqrt{\frac{1}{T}\left(\frac{11T}{3}\right)} = \sqrt{\frac{11}{3}} = 1.915$$

[b] $P = I_{\text{rms}}^2(100) = 366.7\,\text{W}$

[c] From Problem 16.14:

$$a_1 = 2.489\text{A}$$

$$i_g \approx 2.489\cos\omega_o t\,\text{A}$$

$$P = \left(\frac{2.489}{\sqrt{2}}\right)^2(100) = 309.76\,\text{W}$$

[d] $\%\text{ error } = \left(\frac{308.76}{366.7} - 1\right) = -15.52\%$

P 16.42 **[a]** Half-wave symmetry $a_v = 0$, $a_k = b_k = 0$, even k

$$a_k = \frac{4}{T}\int_0^{T/4}\frac{4I_m}{T}t\cos k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4} t\cos k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\cos k\omega_0 t}{k^2\omega_0^2} + \frac{t}{k\omega_0}\sin k\omega_0 t\,\Big|_0^{T/4}\right\}$$

$$= \frac{16I_m}{T^2}\left\{0 + \frac{T}{4k\omega_0}\sin\frac{k\pi}{2} - \frac{1}{k^2\omega_0^2}\right\}$$

$$a_k = \frac{2I_m}{\pi k}\left[\sin\left(\frac{k\pi}{2}\right) - \frac{2}{\pi k}\right], \qquad k\text{—odd}$$

$$b_k = \frac{4}{T}\int_0^{T/4}\frac{4I_m}{T}t\sin k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4} t\sin k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t}{k\omega_0}\cos k\omega_0 t\,\Big|_0^{T/4}\right\} = \frac{4I_m}{\pi^2k^2}\sin\left(\frac{k\pi}{2}\right)$$

[b] $a_k - jb_k = \frac{2I_m}{\pi k}\left\{\left[\sin\left(\frac{k\pi}{2}\right) - \frac{2}{\pi k}\right] - \left[j\frac{2}{\pi k}\sin\left(\frac{k\pi}{2}\right)\right]\right\}$

$$a_1 - jb_1 = \frac{2I_m}{\pi}\left\{\left(1 - \frac{2}{\pi}\right) - j\frac{2}{\pi}\right\} = 0.47I_m\underline{/-60.28^\circ}$$

$$a_3 - jb_3 = \frac{2I_m}{3\pi}\left\{\left(-1 - \frac{2}{3\pi}\right) + j\left(\frac{2}{3\pi}\right)\right\} = 0.26I_m\underline{/170.07^\circ}$$

$$a_5 - jb_5 = \frac{2I_m}{5\pi}\left\{\left(1 - \frac{2}{5\pi}\right) - j\left(\frac{2}{5\pi}\right)\right\} = 0.11I_m\underline{/-8.30^\circ}$$

$$a_7 - jb_7 = \frac{2I_m}{7\pi}\left\{\left(-1 - \frac{2}{7\pi}\right) + j\left(\frac{2}{7\pi}\right)\right\} = 0.10I_m\underline{/175.23^\circ}$$

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$$i_g = 0.47I_m\cos(\omega_0 t - 60.28^\circ) + 0.26I_m\cos(3\omega_0 t + 170.07^\circ)$$
$$+0.11I_m\cos(5\omega_0 t - 8.30^\circ) + 0.10I_m\cos(7\omega_0 t + 175.23^\circ) + \cdots$$

[c] $I_g = \sqrt{\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{A_n^2}{2}\right)}$

$$\cong I_m\sqrt{\frac{(0.47)^2 + (0.26)^2 + (0.11)^2 + (0.10)^2}{2}} = 0.39I_m$$

[d] $\text{Area} = 2\int_0^{T/4}\left(\frac{4I_m}{T}t\right)^2 dt = \left(\frac{32I_m^2}{T^2}\right)\left(\frac{t^3}{3}\right)\Big|_0^{T/4} = \frac{I_m^2 T}{6}$

$$I_g = \sqrt{\frac{1}{T}\left(\frac{I_m^2 T}{6}\right)} = \frac{I_m}{\sqrt{6}} = 0.41I_m$$

[e] $\%\ \text{error} = \left(\frac{\text{estimated}}{\text{exact}} - 1\right)100 = \left(\frac{0.3927I_m}{(I_m/\sqrt{6})} - 1\right)100 = -3.8\%$

P 16.43 Figure P16.43(b): $t_a = 0.2s$; $t_b = 0.6s$

$v = 50t \quad 0 \le t \le 0.2$

$v = -50t + 20 \quad 0.2 \le t \le 0.6$

$v = 25t - 25 \quad 0.6 \le t \le 1.0$

$$\text{Area 1} = A_1 = \int_0^{0.2} 2500t^2\,dt = \frac{20}{3}$$

$$\text{Area 2} = A_2 = \int_{0.2}^{0.6} 100(4 - 20t + 25t^2)\,dt = \frac{40}{3}$$

$$\text{Area 3} = A_3 = \int_{0.6}^{1.0} 625(t^2 - 2t + 1)\,dt = \frac{40}{3}$$

$$A_1 + A_2 + A_3 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

Figure P16.43(c): $t_a = t_b = 0.4s$

$v(t) = 25t \quad 0 \le t \le 0.4$

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$$v(t) = \frac{50}{3}(t-1) \quad 0.4 \le t \le 1$$

$$A_1 = \int_0^{0.4} 625t^2\,dt = \frac{40}{3}$$

$$A_2 = \int_{0.4}^{1.0} \frac{2500}{9}(t^2 - 2t + 1)\,dt = \frac{60}{3}$$

$$A_1 + A_2 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}(A_1 + A_2)} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

Figure P16.43(d): $t_a = t_b = 1$

$$v = 10t \quad 0 \le t \le 1$$

$$A_1 = \int_0^1 100t^2\,dt = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

P 16.44 $C_o = A_v = \dfrac{V_m T}{2} \cdot \dfrac{1}{T} = \dfrac{V_m}{2}$

$$C_n = \frac{1}{T}\int_0^T \frac{V_m}{T} t e^{-jn\omega_o t}\,dt$$

$$= \frac{V_m}{T^2}\left[\frac{e^{-jn\omega_0 t}}{-n^2\omega_0^2}(-jn\omega_0 t - 1)\right]_0^T$$

$$= \frac{V_m}{T^2}\left[\frac{e^{-jn2\pi T/T}}{-n^2\omega_0^2}\left(-jn\frac{2\pi}{T}T - 1\right) - \frac{1}{-n^2\omega_0^2}(-1)\right]$$

$$= \frac{V_m}{T^2}\left[\frac{1}{n^2\omega_0^2}(1 + jn2\pi) - \frac{1}{n^2\omega_0^2}\right]$$

$$= j\frac{V_m}{2n\pi}, \quad n = \pm1, \pm2, \pm3, \ldots$$

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P 16.45 **[a]** $V_{\text{rms}} = \sqrt{\dfrac{1}{T}\displaystyle\int_0^T v^2\,dt} = \sqrt{\dfrac{1}{T}\displaystyle\int_0^T \left(\dfrac{V_m}{T}\right)^2 t^2\,dt}$

$$= \sqrt{\frac{V_m^2}{T^3}\frac{t^3}{3}\bigg|_0^T}$$

$$= \sqrt{\frac{V_m^2}{3}} = \frac{V_m}{\sqrt{3}}$$

$$P = \frac{(150/\sqrt{3})^2}{25} = 300\,\text{W}$$

[b] From the solution to Problem 16.44

$$C_0 = \frac{150}{2} = 75\,\text{V};$$

$$C_1 = j\frac{150}{2\pi} = j\frac{75}{\pi}; \qquad C_2 = j\frac{150}{4\pi} = j\frac{37.5}{\pi}$$

$$C_3 = j\frac{150}{6\pi} = j\frac{25}{\pi}; \qquad C_4 = j\frac{150}{8\pi} = j\frac{18.75}{\pi}$$

$$V_{\text{rms}} = \sqrt{C_o^2 + 2\sum_{n=1}^{\infty}|C_n|^2}$$

$$= \sqrt{75^2 + \tfrac{2}{\pi^2}(75^2 + 37.5^2 + 25^2 + 18.75^2)}$$

$$= 85.13\,\text{V}$$

[c] $P = \dfrac{(85.13)^2}{25} = 289.88\,\text{W}$

$$\%\ \text{error} = \left(\frac{289.88}{300} - 1\right)(100) = -3.37\%$$

P 16.46 $C_n = \dfrac{1}{T}\displaystyle\int_0^{T/4} V_m e^{-jn\omega_o t}\,dt = \dfrac{V_m}{T}\left[\dfrac{e^{-jn\omega_o t}}{-jn\omega_o}\bigg|_0^{T/4}\right]$

$$= \frac{V_m}{Tn\omega_o}[j(e^{-jn\pi/2} - 1)] = \frac{V_m}{2\pi n}\sin\frac{n\pi}{2} + j\frac{V_m}{2\pi n}\left(\cos\frac{n\pi}{2} - 1\right)$$

$$= \frac{V_m}{2\pi n}\left[\sin\frac{n\pi}{2} - j\left(1 - \cos\frac{n\pi}{2}\right)\right]$$

$$v(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_o t}$$

$$C_o = A_v = \frac{1}{T}\int_0^{T/4} V_m\,dt = \frac{V_m}{4}$$

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or

$$C_o = \frac{V_m}{2\pi}\lim_{n\to 0}\left[\frac{\sin(n\pi/2)}{n} - j\frac{1-\cos(n\pi/2)}{n}\right]$$

$$= \frac{V_m}{2\pi}\lim_{n\to 0}\left[\frac{(\pi/2)\cos(n\pi/2)}{1} - j\frac{(\pi/2)\sin(n\pi/2)}{1}\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{\pi}{2} - j0\right] = \frac{V_m}{4}$$

Note it is much easier to use $C_o = A_v$ than to use L'Hopital's rule to find the limit of 0/0.

P 16.47 **[a]** $C_o = a_v = \dfrac{(1/2)(T/2)V_m}{T} = \dfrac{V_m}{4}$

$$C_n = \frac{1}{T}\int_0^{T/2}\frac{2V_m}{T}te^{-jn\omega_o t}\,dt$$

$$= \frac{2V_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^t}(-jn\omega_o t - 1)\right]_0^{T/2}$$

$$= \frac{V_m}{2n^2\pi^2}[e^{-jn\pi}(-jn\pi + 1) - 1]$$

Since $e^{-jn\pi} = \cos n\pi$ we can write

$$C_n = \frac{V_m}{2\pi^2 n^2}(\cos n\pi - 1) + j\frac{V_m}{2n\pi}\cos n\pi$$

[b] $C_o = \dfrac{54}{4} = 13.5\,\text{V}$

$$C_{-1} = \frac{-54}{\pi^2} + j\frac{27}{\pi} = 10.19\underline{/122.48^\circ}\,\text{V}$$

$$C_1 = 10.19\underline{/-122.48^\circ}\,\text{V}$$

$$C_{-2} = -j\frac{13.5}{\pi} = 4.30\underline{/-90^\circ}\,\text{V}$$

$$C_2 = 4.30\underline{/90^\circ}\,\text{V}$$

$$C_{-3} = \frac{-6}{\pi^2} + j\frac{9}{\pi} = 2.93\underline{/101.98^\circ}\,\text{V}$$

$$C_3 = 2.93\underline{/-101.98^\circ}\,\text{V}$$

$$C_{-4} = -j\frac{6.75}{\pi} = 2.15\underline{/-90^\circ}\,\text{V}$$

$$C_4 = 2.15\underline{/90^\circ}\,\text{V}$$

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[c]

$$\frac{V_o}{250} + \frac{V_o}{sL} + V_o sC + \frac{V_o - V_g}{62.5} = 0$$

$$\therefore \qquad (250LCs^2 + 5sL + 250)V_o = 4sLV_g$$

$$\frac{V_o}{V_g} = H(s) = \frac{(4/250C)s}{s^2 + 1/50C + 1/LC}$$

$$H(s) = \frac{16{,}000s}{s^2 + 2 \times 10^4 s + 4 \times 10^{10}}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{10\pi} \times 10^6 = 2 \times 10^5 \text{ rad/s}$$

$$H(j0) = 0$$

$$H(j2 \times 10^5 k) = \frac{j8k}{100(1 - k^2) + j10k}$$

Therefore,

$$H_{-1} = 0.8\underline{/0^\circ}; \qquad H_1 = 0.8\underline{/0^\circ}$$

$$H_{-2} = \frac{-j16}{-300 - j20} = 0.0532\underline{/86.19^\circ}; \qquad H_2 = 0.0532\underline{/-86.19^\circ}$$

$$H_{-3} = \frac{-j24}{-800 - j30} = 0.0300\underline{/87.85^\circ}; \qquad H_2 = 0.0300\underline{/-87.85^\circ}$$

$$H_{-4} = \frac{-j32}{-1500 - j40} = 0.0213\underline{/88.47^\circ}; \qquad H_2 = 0.0213\underline{/-88.47^\circ}$$

The output voltage coefficients:

$$C_0 = 0$$

$$C_{-1} = (10.19\underline{/122.48^\circ})(0.8\underline{/0^\circ}) = 8.15\underline{/122.48^\circ} \text{ V}$$

$$C_1 = 8.15\underline{/-122.48^\circ} \text{ V}$$

$$C_{-2} = (4.30\underline{/-90^\circ})(0.05\underline{/86.19^\circ}) = 0.2287\underline{/-3.81^\circ} \text{ V}$$

$$C_2 = 0.2287\underline{/3.81^\circ} \text{ V}$$

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$$C_{-3} = (2.93\underline{/101.98^\circ})(0.03\underline{/87.85^\circ}) = 0.0878\underline{/-170.17^\circ}\,\text{V}$$

$$C_3 = 0.0878\underline{/170.17^\circ}\,\text{V}$$

$$C_{-4} = (2.15\underline{/-90^\circ})(0.02\underline{/88.47^\circ}) = 0.0458\underline{/-1.53^\circ}\,\text{V}$$

$$C_4 = 0.0458\underline{/1.53^\circ}\,\text{V}$$

[d] $$V_{\text{rms}} \cong \sqrt{C_o^2 + 2\sum_{n=1}^{4}|C_n|^2} \cong \sqrt{2\sum_{n=1}^{4}|C_n|^2}$$

$$\cong \sqrt{2(8.15^2 + 0.2287^2 + 0.0878^2 + 0.0458^2} \cong 11.53\,\text{V}$$

$$P = \frac{(11.53)^2}{250} = 531.95\,\text{mW}$$

P 16.48 **[a]** $$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{T/2}\left(\frac{2V_m}{T}t\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\left[\frac{4V_m^2}{T^2}\frac{t^3}{3}\right]_0^{T/2}}$$

$$= \sqrt{\frac{4V_m^2}{(3)(8)}} = \frac{V_m}{\sqrt{6}}$$

$$V_{\text{rms}} = \frac{54}{\sqrt{6}} = 22.05\,\text{V}$$

[b] From the solution to Problem 16.47

$$C_0 = 13.5; \qquad |C_3| = 2.93$$

$$|C_1| = 10.19; \qquad |C_4| = 2.15$$

$$|C_2| = 4.30$$

$$V_g(\text{rms}) \cong \sqrt{13.5^2 + 2(10.19^2 + 4.30^2 + 2.93^2 + 2.15^2)} \cong 21.29\,\text{V}$$

[c] $$\%\ \text{Error} = \left(\frac{21.29}{22.05} - 1\right)(100) = -3.44\%$$

P 16.49 **[a]** From Example 16.3 we have:

$$a_v = \frac{40}{4} = 10\,\text{V}, \qquad a_k = \frac{40}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

$$b_k = \frac{40}{\pi k}\left[1 - \cos\left(\frac{k\pi}{2}\right)\right], \qquad A_k\underline{/-\theta_k^\circ} = a_k - jb_k$$

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$A_1 = 18.01\,\text{V} \qquad \theta_1 = 45°, \qquad A_2 = 12.73\,\text{V}, \qquad \theta_2 = 90°$

$A_3 = 6\,\text{V}, \qquad \theta_3 = 135°, \qquad A_4 = 0, \qquad A_5 = 3.6\,\text{V}, \qquad \theta_5 = 45°$

$A_6 = 4.24\,\text{V}, \qquad \theta_6 = 90°, \qquad A_7 = 2.57\,\text{V}, \qquad \theta_7 = 135°; \qquad A_8 = 0$

[b] $C_n = \dfrac{a_n - jb_n}{2}, \qquad C_{-n} = \dfrac{a_n + jb_n}{2} = C_n^*$

$C_0 = a_v = 10\,\text{V} \qquad C_3 = 3\underline{/135°}\,\text{V} \qquad C_6 = 2.12\underline{/90°}\,\text{V}$

$C_1 = 9\underline{/45°}\,\text{V} \qquad C_{-3} = 3\underline{/-135°}\,\text{V} \qquad C_{-6} = 2.12\underline{/-90°}\,\text{V}$

$C_{-1} = 9\underline{/-45°}\,\text{V} \qquad C_4 = C_{-4} = 0 \qquad C_7 = 1.29\underline{/135°}\,\text{V}$

$C_2 = 6.37\underline{/90°}\,\text{V} \qquad C_5 = 1.8\underline{/45°}\,\text{V} \qquad C_{-7} = 1.29\underline{/-135°}\,\text{V}$

$C_{-2} = 6.37\underline{/-90°}\,\text{V} \quad C_{-5} = 1.8\underline{/-45°}\,\text{V}$

P 16.50 **[a]** From the solution to Problem 16.33 we have

$$A_k = a_k - jb_k = \frac{I_m}{\pi^2 k^2}(\cos k\pi - 1) + j\frac{I_m}{\pi k}$$

$$A_0 = 0.75 I_m = 3.75\,\text{A}$$

$$A_1 = \frac{5}{\pi^2}(-2) + j\frac{5}{\pi} = 1.89\underline{/122.48°}\,\text{A}$$

$$A_2 = j\frac{5}{2\pi} = 0.796\underline{/90°}\,\text{A}$$

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$$A_3 = \frac{5}{9\pi^2}(-2) + j\frac{5}{3\pi} = 0.542\underline{/101.98^\circ}\,\text{A}$$

$$A_4 = j\frac{5}{4\pi} = 0.398\underline{/90^\circ}\,\text{A}$$

$$A_5 = \frac{5}{25\pi^2}(-2) + j\frac{5}{5\pi} = 0.321\underline{/97.26^\circ}\,\text{A}$$

$$A_6 = j\frac{5}{6\pi} = 0.265\underline{/90^\circ}\,\text{A}$$

[b] $C_0 = A_0 = 3.75\,\text{mA}$

$$C_1 = \frac{1}{2}A_1\underline{/\theta_1} = 0.945\underline{/122.48^\circ}\,\text{A}$$

$$C_{-1} = 0.945\underline{/-122.48^\circ}\,\text{A}$$

$$C_2 = \frac{1}{2}A_2\underline{/\theta_2} = 0.398\underline{/90^\circ}\,\text{A}$$

$$C_{-2} = 0.398\underline{/-90^\circ}\,\text{A}$$

$$C_3 = \frac{1}{2}A_3\underline{/\theta_3} = 0.271\underline{/101.98^\circ}\,\text{A}$$

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$$C_{-3} = 0.271\underline{/-101.98^\circ}\ \text{A}$$

$$C_4 = \frac{1}{2}A_4\underline{/\theta_4} = 0.199\underline{/90^\circ}\ \text{A}$$

$$C_{-4} = 0.199\underline{/-90^\circ}\ \text{A}$$

$$C_5 = \frac{1}{2}A_5\underline{/\theta_5} = 0.161\underline{/97.26^\circ}\ \text{A}$$

$$C_{-5} = 0.161\underline{/-97.26^\circ}\ \text{A}$$

$$C_6 = \frac{1}{2}A_6\underline{/\theta_6} = 0.1325\underline{/90^\circ}\ \text{A}$$

$$C_{-6} = 0.1325\underline{/-90^\circ}\ \text{A}$$

P 16.51 **[a]** $v = A_1\cos(\omega_o t - 90^\circ) + A_3\cos(3\omega_o t + 90^\circ)$

$$+A_5\cos(5\omega_o t - 90^\circ) + A_7\cos(7\omega_o t + 90^\circ)$$

$$v = -A_1\sin\omega_o t + A_3\sin 3\omega_o t - A_5\sin 5\omega_o t + A_7\sin 7\omega_o t$$

[b] $v(-t) = -A_1\sin\omega_o t + A_3\sin 3\omega_o t - A_5\sin 5\omega_o t + A_7\sin 7\omega_o t$

$$\therefore\quad v(-t) = -v(t); \qquad \text{odd function}$$

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[c] $v(t-T/2) = A_1\sin(\omega_o t-\pi) - A_3\sin(3\omega_o t-3\pi)$

$$+A_5\sin(5\omega_o t-5\pi) - A_7\sin(7\omega_o t-7\pi)$$

$$= -A_1\sin\omega_o t + A_3\sin 3\omega_o t - A_5\sin 5\omega_o t + A_7\sin 7\omega_o t$$

$\therefore\ v(t-T/2) = -v(t)$, yes, the function has half-wave symmetry

[d] Since the function is odd, with hws, we test to see if

$f(T/2-t) = f(t)$

$$f(T/2-t) = A_1\sin(\pi-\omega_o t) - A_3\sin(3\pi-3\omega_o t)$$

$$+A_5\sin(5\pi-5\omega_o t) - A_7\sin(7\pi-7\omega_o t)$$

$$= A_1\sin\omega_o t - A_3\sin 3\omega_o t + A_5\sin 5\omega_o t - A_7\sin 7\omega_o t$$

$\therefore\ f(T/2-t) = f(t)$ and the voltage has quarter-wave symmetry

P 16.52 **[a]** $i = 8.82\cos(250t+90^\circ) + 0.98\cos(500t-90^\circ) + 0.353\cos(750t+90^\circ)$

$$+0.18\cos(1000t-90^\circ)\text{ A}$$

$$= -8.82\sin 250t + 0.98\sin 500t - 0.353\sin 750t + 0.18\sin 1000t\text{ A}$$

[b] $i(t) = -i(-t)$, Function is odd

[c] Yes, $A_0 = 0$, $A_n = 0$ for n even

[d] $I_{\text{rms}} = \sqrt{\dfrac{8.82^2+0.98^2+0.353^2+0.18^2}{2}} = 6.28\text{ A}$

[e] $C_{-1} = 4.41\underline{/-90^\circ}\text{ A}$; $C_1 = 4.41\underline{/90^\circ}\text{ A}$

$C_{-3} = 0.49\underline{/90^\circ}\text{ A}$; $C_3 = 0.49\underline{/-90^\circ}\text{ A}$

$C_{-5} = 0.177\underline{/-90^\circ}\text{ A}$; $C_5 = 0.177\underline{/90^\circ}\text{ A}$

$C_{-7} = 0.09\underline{/90^\circ}\text{ A}$; $C_7 = 0.09\underline{/-90^\circ}\text{ A}$

$$i = j0.09e^{-j1000t} - j0.177e^{-j750t} + j0.49e^{-j500t}$$

$$-j4.41e^{-j250t} + j4.41e^{j250t} - j0.49e^{j500t}$$

$$+j0.177e^{j750t} - j0.09e^{j1000t}\text{ A}$$

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[f]

P 16.53 From Table 15.1 we have

$$H(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

After scaling we get

$$H'(s) = \frac{s^3}{(s+2500)(s^2+2500s+625\times 10^4)}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{400\pi}\times 10^6 = 5000 \text{ rad/s}$$

$$\therefore \quad H'(jn\omega_o) = \frac{-j8n^3}{(1+j2n)[(1-4n^2)+j2n]}$$

It follows that

$$H(j0) = 0$$

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$$H(j\omega_o) = \frac{-j8}{(1+j2)(-3+j2)} = 0.992\underline{/60.255^\circ}$$

$$H(j2\omega_o) = \frac{-j64}{(1+j4)(-15+j4)} = 0.9999\underline{/28.97^\circ}$$

$$v_g(t) = \frac{A}{\pi} + \frac{A}{2}\sin\omega_o t - \frac{2A}{\pi}\sum_{n=2,4,6,}^{\infty}\frac{\cos n\omega_o t}{n^2-1}$$

$$= 270 + 135\pi\sin\omega_o t - 180\cos 2\omega_o t - \cdots \text{ V}$$

$$\therefore \quad v_o = 0 + 420.84\sin(5000t + 60.255^\circ) - 179.98\cos(10{,}000t + 28.97^\circ) - \cdots \text{ V}$$

P 16.54 Using the technique outlined in Problem 16.18 we can derive the Fourier series for $v_g(t)$. We get

$$v_g(t) = 100 + \frac{800}{\pi^2}\sum_{n=1,3,5,}^{\infty}\frac{1}{n^2}\cos n\omega_o t$$

The transfer function of the prototype second-order low pass Butterworth filter is

$$H(s) = \frac{1}{s^2+\sqrt{2}s+1}, \qquad \text{where } \omega_c = 1 \text{ rad/s}$$

Now frequency scale using $k_f = 2000$ to get $\omega_c = 2$ krad/s:

$$H(s) = \frac{4\times 10^6}{s^2 + 2000\sqrt{2}s + 4\times 10^6}$$

$$H(j0) = 1$$

$$H(j5000) = \frac{4\times 10^6}{(j5000)^2 + 2000\sqrt{2}(j5000)^2 + 4\times 10^6} = 0.1580\underline{/-146.04^\circ}$$

$$H(j15{,}000) = \frac{4\times 10^6}{(j15{,}000)^2 + 2000\sqrt{2}(j15{,}000)^2 + 4\times 10^6} = 0.0178\underline{/-169.13^\circ}$$

$$\mathbf{V}_{\text{dc}} = 100\,\text{V}$$

$$\mathbf{V}_{g1} = \frac{800}{n^2}\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_{g3} = \frac{800}{9\pi^2}\underline{/0^\circ}\,\text{V}$$

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$$V_{odc} = 100(1) = 100\,\text{V}$$

$$\mathbf{V}_{o1} = \frac{800}{\pi^2}(0.1580\underline{/-146.04^\circ}) = 12.81\underline{/-146.04^\circ}\,\text{V}$$

$$\mathbf{V}_{o3} = \frac{800}{9\pi^2}(0.0178\underline{/-169.13^\circ}) = 0.16\underline{/-169.13^\circ}\,\text{V}$$

$$v_o(t) = 100 + 12.81\cos(5000t - 146.04^\circ) + 0.16\cos(15{,}000t - 169.13^\circ) + \cdots\ \text{V}$$

P 16.55 $$v_g = \frac{2(2.5\pi)}{\pi} - \frac{4(2.5\pi)}{\pi}\frac{\cos 5000t}{4-1} = 5 - (10/3)\cos 5000t - \cdots\ \text{V}$$

$$H(j0) = 1$$

$$H(j5000) = \frac{10^6}{(10^6 - 25\times 10^6) + j5\sqrt{2}\times 10^6} = 0.04\underline{/-163.58^\circ}$$

$$\therefore\quad v_o(t) = 5 - 0.1332\cos(5000t - 163.58^\circ) - \cdots\ \text{V}$$

P 16.56 **[a]** Let V_a represent the node voltage across R_2, then the node-voltage equations are

$$\frac{V_a - V_g}{R_1} + \frac{V_a}{R_2} + V_a sC_2 + (V_a - V_o)sC_1 = 0$$

$$(0 - V_a)sC_2 + \frac{0 - V_o}{R_3} = 0$$

Solving for V_o in terms of V_g yields

$$\frac{V_o}{V_g} = H(s) = \frac{\frac{-1}{R_1C_1}s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

It follows that

$$\omega_o^2 = \frac{R_1 + R_2}{R_1R_2R_3C_1C_2}$$

$$\beta = \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$K_o = \frac{R_3}{R_1}\left(\frac{C_2}{C_1 + C_2}\right)$$

Note that

$$H(s) = \frac{-\frac{R_3}{R_1}\left(\frac{C_2}{C_1+C_2}\right)\frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \left(\frac{R_1+R_2}{R_1R_2R_3C_1C_2}\right)}$$

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[b] For the given values of R_1, R_2, R_3, C_1, and C_2 we have

$$H(s) = \frac{-400s}{s^2 + 400s + 10^8}$$

$$v_g = \frac{(8)(2.25\pi^2)}{\pi^2} \sum_{n=1,3,5,}^{\infty} \frac{1}{n^2} \cos n\omega_o t$$

$$= 18\left[\cos\omega_o t + \frac{1}{9}\cos 3\omega_o t + \frac{1}{25}\cos 5\omega_o t + \cdots\right] \text{ mV}$$

$$= [18\cos\omega_o t + 2\cos 3\omega_o t + 0.72\cos 5\omega_o t + \cdots] \text{ mV}$$

$$\omega_o = \frac{2\pi}{0.2\pi} \times 10^3 = 10^4 \text{ rad/s}$$

$$H(jk10^4) = \frac{-400jk10^4}{10^8 - k^2 10^8 + j400k10^4} = \frac{-jk}{25(1-k^2) + jk}$$

$$H_1 = -1 = 1\underline{/180^\circ}$$

$$H_3 = \frac{-j3}{-200 + j3} = 0.015\underline{/90.86^\circ}$$

$$H_5 = \frac{-j5}{-600 + j5} = 0.0083\underline{/90.48^\circ}$$

$$v_o = -18\cos\omega_o t + 0.03\cos(3\omega_o t + 90.86^\circ)$$

$$+ 0.006\cos(5\omega_o t + 90.48^\circ) + \cdots \text{ mV}$$

[c] The fundamental frequency component dominates the output, so we expect the quality factor Q to be quite high.

[d] $\omega_o = 10^4$ rad/s and $\beta = 400$ rad/s. Therefore, $Q = 10{,}000/400 = 25$. We expect the output voltage to be dominated by the fundamental frequency component since the bandpass filter is tuned to this frequency!

P 16.57 **[a]** Using the equations derived in Problem 16.56(a),

$$K_o = \frac{R_3}{R_1}\left(\frac{C_2}{C_1 + C_2}\right) = \frac{400}{313}$$

$$\beta = \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 2000 \text{ rad/s}$$

$$\omega_o^2 = \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2} = 16 \times 10^8$$

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[b] $$H(jn\omega_o) = \frac{-(400/313)(2000)jn\omega_o}{16\times 10^8 - n^2\omega_o^2 + j2000n\omega_o}$$

$$= \frac{-j(20/313)n}{(1-n^2)+j0.05n}$$

$$H(j\omega_o) = \frac{-j(20/313)}{j(0.050)} = -\frac{400}{313} = -1.28$$

$$H(j3\omega_o) = \frac{-j(20/313)(3)}{-8+j0.15} = 0.0240\underline{/91.07^\circ}$$

$$H(j5\omega_o) = \frac{-j(100/313)}{-24+j0.25} = 0.0133\underline{/90.60^\circ}$$

$$v_g(t) = \frac{4A}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin(n\pi/2)\cos n\omega_o t$$

$$A = 15.65\pi \text{ V}$$

$$v_g(t) = 62.60\cos\omega_o t - 20.87\cos 3\omega_o t + 12.52\cos 5\omega_o t - \cdots$$

$$v_o(t) = -80\cos\omega_o t - 0.50\cos(3\omega_o t + 91.07^\circ)$$
$$+ 0.17\cos(5\omega_o t + 90.60^\circ) - \cdots \text{ V}$$

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17

The Fourier Transform

Assessment Problems

AP 17.1 **[a]** $F(\omega) = \int_{-\tau/2}^{0} (-Ae^{-j\omega t})\,dt + \int_{0}^{\tau/2} Ae^{-j\omega t}\,dt$

$$= \frac{A}{j\omega}[2 - e^{j\omega\tau/2} - e^{-j\omega\tau/2}]$$

$$= \frac{2A}{j\omega}\left[1 - \frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2}\right]$$

$$= \frac{-j2A}{\omega}[1 - \cos(\omega\tau/2)]$$

[b] $F(\omega) = \int_0^\infty te^{-at}e^{-j\omega t}\,dt = \int_0^\infty te^{-(a+j\omega)t}\,dt = \frac{1}{(a+j\omega)^2}$

AP 17.2

$$f(t) = \frac{1}{2\pi}\left\{\int_{-3}^{-2} 4e^{jt\omega}\,d\omega + \int_{-2}^{2} e^{jt\omega}\,d\omega + \int_{2}^{3} 4e^{jt\omega}\,d\omega\right\}$$

$$= \frac{1}{j2\pi t}\left\{4e^{-j2t} - 4e^{-j3t} + e^{j2t} - e^{-j2t} + 4e^{j3t} - 4e^{j2t}\right\}$$

$$= \frac{1}{\pi t}\left[\frac{3e^{-j2t} - 3e^{j2t}}{j2} + \frac{4e^{j3t} - 4e^{-j3t}}{j2}\right]$$

$$= \frac{1}{\pi t}(4\sin 3t - 3\sin 2t)$$

AP 17.3 **[a]** $F(\omega) = F(s)\,|_{s=j\omega} = \mathcal{L}\{e^{-at}sin\omega_0 t\}_{s=j\omega}$

$$= \frac{\omega_0}{(s+a)^2 + \omega_0^2}\bigg|_{s=j\omega} = \frac{\omega_0}{(a+j\omega)^2 + \omega_0^2}$$

[b] $F(\omega) = \mathcal{L}\{f^-(t)\}_{s=-j\omega} = \left[\frac{1}{(s+a)^2}\right]_{s=-j\omega} = \frac{1}{(a-j\omega)^2}$

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[c] $f^+(t) = te^{-at}, \qquad f^-(t) = -te^{-at}$

$$\mathcal{L}\{f^+(t)\} = \frac{1}{(s+a)^2}, \quad \mathcal{L}\{f^-(t)\} = \frac{-1}{(s+a)^2}$$

Therefore $\quad F(\omega) = \dfrac{1}{(a+j\omega)^2} - \dfrac{1}{(a-j\omega)^2} = \dfrac{-j4a\omega}{(a^2+\omega^2)^2}$

AP 17.4 **[a]** $f'(t) = \dfrac{2A}{\tau}, \quad \dfrac{-\tau}{2} < t < 0; \qquad f'(t) = \dfrac{-2A}{\tau}, \quad 0 < t < \dfrac{\tau}{2}$

$$\therefore \qquad f'(t) = \frac{2A}{\tau}[u(t+\tau/2) - u(t)] - \frac{2A}{\tau}[u(t) - u(t-\tau/2)]$$

$$= \frac{2A}{\tau}u(t+\tau/2) - \frac{4A}{\tau}u(t) + \frac{2A}{\tau}u(t-\tau/2)$$

$$\therefore \qquad f''(t) = \frac{2A}{\tau}\delta\left(t+\frac{\tau}{2}\right) - \frac{4A}{\tau} + \frac{2A}{\tau}\delta\left(t-\frac{\tau}{2}\right)$$

[b] $\mathcal{F}\{f''(t)\} = \left[\dfrac{2A}{\tau}e^{j\omega\tau/2} - \dfrac{4A}{\tau} + \dfrac{2A}{\tau}e^{-j\omega\tau/2}\right]$

$$= \frac{4A}{\tau}\left[\frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2} - 1\right] = \frac{4A}{\tau}\left[\cos\left(\frac{\omega\tau}{2}\right) - 1\right]$$

[c] $\mathcal{F}\{f''(t)\} = (j\omega)^2 F(\omega) = -\omega^2 F(\omega); \qquad$ therefore $\quad F(\omega) = -\dfrac{1}{\omega^2}\mathcal{F}\{f''(t)\}$

Thus we have $\quad F(\omega) = -\dfrac{1}{\omega^2}\left\{\dfrac{4A}{\tau}\left[\cos\left(\dfrac{\omega\tau}{2}\right) - 1\right]\right\}$

AP 17.5

$$v(t) = V_m\left[u\left(t+\frac{\tau}{2}\right) - u\left(t-\frac{\tau}{2}\right)\right]$$

$$\mathcal{F}\left\{u\left(t+\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{j\omega\tau/2}$$

$$\mathcal{F}\left\{u\left(t-\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{-j\omega\tau/2}$$

Therefore $\quad V(\omega) = V_m\left[\pi\delta(\omega) + \dfrac{1}{j\omega}\right]\left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$

$$= j2V_m\pi\delta(\omega)\sin\left(\frac{\omega\tau}{2}\right) + \frac{2V_m}{\omega}\sin\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{(V_m\tau)\sin(\omega\tau/2)}{\omega\tau/2}$$

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AP 17.6 **[a]** $I_g(\omega) = \mathcal{F}\{10\text{sgn}\, t\} = \dfrac{20}{j\omega}$

[b] $H(s) = \dfrac{V_o}{I_g}$

Using current division and Ohm's law,

$$V_o = -I_2 s = -\left[\frac{4}{4+1+s}\right](-I_g)s = \frac{4s}{5+s}I_g$$

$$H(s) = \frac{4s}{s+5}, \qquad H(j\omega) = \frac{j4\omega}{5+j\omega}$$

[c] $V_o(\omega) = H(j\omega)\cdot I_g(\omega) = \left(\dfrac{j4\omega}{5+j\omega}\right)\left(\dfrac{20}{j\omega}\right) = \dfrac{80}{5+j\omega}$

[d] $v_o(t) = 80e^{-5t}u(t)\,\text{V}$

[e] Using current division,

$$i_1(0^-) = \frac{1}{5}i_g = \frac{1}{5}(-10) = -2\,\text{A}$$

[f] $i_1(0^+) = i_g + i_2(0^+) = 10 + i_2(0^-) = 10 + 8 = 18\,\text{A}$

[g] Using current division,

$$i_2(0^-) = \frac{4}{5}(10) = 8\,\text{A}$$

[h] Since the current in an inductor must be continuous,

$$i_2(0^+) = i_2(0^-) = 8\,\text{A}$$

[i] Since the inductor behaves as a short circuit for $t < 0$,

$$v_o(0^-) = 0\,\text{V}$$

[j] $v_o(0^+) = 1i_2(0^+) + 4i_1(0^+) = 80\,\text{V}$

AP 17.7 **[a]** $V_g(\omega) = \dfrac{1}{1-j\omega} + \pi\delta(\omega) + \dfrac{1}{j\omega}$

$$H(s) = \frac{V_a}{V_g} = \frac{0.5\|(1/s)}{1+0.5\|(1/s)} = \frac{1}{s+3}, \qquad H(j\omega) = \frac{1}{3+j\omega}$$

$$\begin{aligned}
V_a(\omega) &= H(j\omega)V_g(j\omega) \\
&= \frac{1}{(1-j\omega)(3+j\omega)} + \frac{1}{j\omega(3+j\omega)} + \frac{\pi\delta(\omega)}{3+j\omega} \\
&= \frac{1/4}{1-j\omega} + \frac{1/4}{3+j\omega} + \frac{1/3}{j\omega} - \frac{1/3}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \\
&= \frac{1/4}{1-j\omega} + \frac{1/3}{j\omega} - \frac{1/12}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega}
\end{aligned}$$

Therefore $\quad v_a(t) = \left[\dfrac{1}{4}e^{t}u(-t) + \dfrac{1}{6}\text{sgn}\, t - \dfrac{1}{12}e^{-3t}u(t) + \dfrac{1}{6}\right]\text{V}$

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[b] $v_a(0^-) = \frac{1}{4} - \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{4}\,\text{V}$

$$v_a(0^+) = 0 + \frac{1}{6} - \frac{1}{12} + \frac{1}{6} = \frac{1}{4}\,\text{V}$$

$$v_a(\infty) = 0 + \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{3}\,\text{V}$$

AP 17.8

$$v(t) = 4te^{-t}u(t); \qquad V(\omega) = \frac{4}{(1+j\omega)^2}$$

Therefore $\quad |V(\omega)| = \dfrac{4}{1+\omega^2}$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{\sqrt{3}} \left[\frac{4}{(1+\omega^2)}\right]^2 d\omega$$

$$= \frac{16}{\pi}\left\{\frac{1}{2}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\sqrt{3}}\right\}$$

$$= 16\left[\frac{\sqrt{3}}{8\pi} + \frac{1}{6}\right] = 3.769\,\text{J}$$

$$W_{1\Omega}(\text{total}) = \frac{8}{\pi}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\infty} = \frac{8}{\pi}\left[0 + \frac{\pi}{2}\right] = 4\,\text{J}$$

Therefore $\quad \% = \dfrac{3.769}{4}(100) = 94.23\%$

AP 17.9

$$|V(\omega)| = 6 - \left(\frac{6}{2000\pi}\right)\omega, \qquad 0 \le \omega \le 2000\pi$$

$$|V(\omega)|^2 = 36 - \left(\frac{72}{2000\pi}\right)\omega + \left(\frac{36}{4\pi^2 \times 10^6}\right)\omega^2$$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{2000\pi}\left[36 - \frac{72\omega}{2000\pi} + \frac{36\times 10^{-6}}{4\pi^2}\omega^2\right]d\omega$$

$$= \frac{1}{\pi}\left[36\omega - \frac{72\omega^2}{4000\pi} + \frac{36\times 10^{-6}\omega^3}{12\pi^2}\right]_0^{2000\pi}$$

$$= \frac{1}{\pi}\left[36(2000\pi) - \frac{72}{4000\pi}(2000\pi)^2 + \frac{36\times 10^{-6}(2000\pi)^3}{12\pi^2}\right]$$

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$$= 36(2000) - \frac{72(2000)^2}{4000} + \frac{36 \times 10^{-6}(2000)^3}{12}$$

$$= 24\,\mathrm{kJ}$$

$$W_{6\mathrm{k}\Omega} = \frac{24 \times 10^3}{6 \times 10^3} = 4\,\mathrm{J}$$

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Problems

P 17.1 **[a]** $F(\omega) = \int_{-\tau/2}^{\tau/2} \frac{2A}{\tau} t e^{-j\omega t}\, dt$

$$= \frac{2A}{\tau}\left[\frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{2A}{\omega^2\tau}\left[e^{-j\omega\tau/2}\left(\frac{j\omega\tau}{2}+1\right) - e^{j\omega\tau/2}\left(\frac{-j\omega\tau}{2}+1\right)\right]$$

$$F(\omega) = \frac{2A}{\omega^2\tau}\left[e^{-j\omega\tau/2} - e^{j\omega\tau/2} + j\frac{\omega\tau}{2}\left(e^{-j\omega\tau/2} + e^{j\omega\tau/2}\right)\right]$$

$$F(\omega) = j\frac{2A}{\tau}\left[\frac{\omega\tau\cos(\omega\tau/2) - 2\sin(\omega\tau/2)}{\omega^2}\right]$$

[b] Using L'Hopital's rule,

$$F(0) = \lim_{\omega\to 0} 2A\left[\frac{\omega\tau(\tau/2)[-\sin(\omega\tau/2)] + \tau\cos(\omega\tau/2) - 2(\tau/2)\cos(\omega\tau/2)}{2\omega\tau}\right]$$

$$= \lim_{\omega\to 0} 2A\left[\frac{-\omega\tau(\tau/2)\sin(\omega\tau/2)}{2\omega\tau}\right]$$

$$= \lim_{\omega\to 0} 2A\left[\frac{-\tau\sin(\omega\tau/2)}{4}\right] = 0$$

$$\therefore \qquad F(0) = 0$$

[c] When $A = 10$ and $\tau = 0.1$

$$F(\omega) = j200\left[\frac{0.1\omega\cos(\omega/20) - 2\sin(\omega/20)}{\omega^2}\right]$$

$$|F(\omega)| = \left|\frac{20\omega\cos(\omega/20) - 400\sin(\omega/20)}{\omega^2}\right|$$

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P 17.2 **[a]** $F(\omega) = A + \dfrac{2A}{\omega_o}\omega, \quad -\omega_o/2 \le \omega \le 0$

$$F(\omega) = A - \frac{2A}{\omega_o}\omega, \quad 0 \le \omega \le \omega_o/2$$

$$F(\omega) = 0 \qquad\qquad \text{elsewhere}$$

$$f(t) = \frac{1}{2\pi}\int_{-\omega_o/2}^{0}\left(A + \frac{2A}{\omega_o}\omega\right)e^{jt\omega}\,d\omega$$

$$+\frac{1}{2\pi}\int_{0}^{\omega_o/2}\left(A - \frac{2A}{\omega_o}\omega\right)e^{jt\omega}\,d\omega$$

$$f(t) = \frac{1}{2\pi}\left[\int_{-\omega_o/2}^{0} Ae^{jt\omega}\,d\omega + \int_{-\omega_o/2}^{0}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega\right.$$

$$\left.+\int_{0}^{\omega_o/2} Ae^{jt\omega}\,d\omega - \int_{0}^{\omega_o/2}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega\right]$$

$$= \frac{1}{2\pi}[\ \text{Int1} + \text{Int2} + \text{Int3} - \text{Int4}\ ]$$

$$\text{Int1} = \int_{-\omega_o/2}^{0} Ae^{jt\omega}\,d\omega = \frac{A}{jt}(1 - e^{-jt\omega_o/2})$$

$$\text{Int2} = \int_{-\omega_o/2}^{0}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega = \frac{2A}{\omega_o t^2}(1 - j\frac{t\omega_o}{2}e^{-jt\omega_o/2} - e^{-jt\omega_o/2})$$

$$\text{Int3} = \int_{0}^{\omega_o/2} Ae^{jt\omega}\,d\omega = \frac{A}{jt}(e^{jt\omega_o/2} - 1)$$

$$\text{Int4} = \int_{0}^{\omega_o/2}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega = \frac{2A}{\omega_o t^2}(-j\frac{t\omega_o}{2}e^{jt\omega_o/2} + e^{jt\omega_o/2} - 1)$$

$$\text{Int1} + \text{Int3} = \frac{2A}{t}\sin(\omega_o t/2)$$

$$\text{Int2} - \text{Int4} = \frac{4A}{\omega_o t^2}[1 - \cos(\omega_o t/2)] - \frac{2A}{t}\sin(\omega_o t/2)$$

$$\therefore \qquad f(t) = \frac{1}{2\pi}\left[\frac{4A}{\omega_o t^2}\left(1 - \cos(\omega_o t/2)\right)\right]$$

$$= \frac{2A}{\pi\omega_o t^2}\left[2\sin^2(\omega_o t/4)\right]$$

$$= \frac{4\omega_o A}{\pi\omega_o^2 t^2}\sin^2(\omega_o t/4)$$

$$= \frac{\omega_o A}{4\pi}\left[\frac{\sin(\omega_o t/4)}{(\omega_o t/4)}\right]^2$$

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[b] $f(0) = \dfrac{\omega_o A}{4\pi}(1)^2 = 79.58 \times 10^{-3}\omega_o A$

[c] $A = 5\pi; \qquad \omega_o = 100 \text{ rad/s}$

$$f(t) = 125\left[\frac{\sin(t/25)}{(t/25)}\right]^2$$

P 17.3 **[a]** $F(\omega) = \displaystyle\int_{-2}^{2}\left[A\sin\left(\frac{\pi}{2}\right)t\right]e^{-j\omega t}\,dt = \frac{-j4\pi A}{\pi^2 - 4\omega^2}\sin 2\omega$

[b] $F(\omega) = \displaystyle\int_{-\tau/2}^{0}\left(\frac{2A}{\tau}t + A\right)e^{-j\omega t}\,dt + \int_{0}^{\tau/2}\left(\frac{-2A}{\tau}t + A\right)e^{-j\omega t}\,dt$

$$= \frac{4A}{\omega^2\tau}\left[1 - \cos\left(\frac{\omega\tau}{2}\right)\right]$$

P 17.4 $\mathcal{F}\{\sin\omega_0 t\} = \mathcal{F}\left\{\dfrac{e^{j\omega_0 t}}{2j}\right\} - \mathcal{F}\left\{\dfrac{e^{-j\omega_0 t}}{2j}\right\}$

$$= \frac{1}{2j}[2\pi\delta(\omega - \omega_0) - 2\pi\delta(\omega + \omega_0)]$$

$$= j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

P 17.5 **[a]** $F(s) = \mathcal{L}\{te^{-at}\} = \dfrac{1}{(s+a)^2}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \left[\frac{1}{(a+j\omega)^2}\right] + \left[\frac{1}{(a-j\omega)^2}\right]$$

$$= \frac{2(a^2-\omega^2)}{(a^2-\omega^2)^2 + 4a^2\omega^2} = \frac{2(a^2-\omega^2)}{(a^2+\omega^2)^2}$$

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[b] $F(s) = \mathcal{L}\{t^3 e^{-at}\} = \dfrac{6}{(s+a)^4}$

$$F(\omega) = F(s)\Big|_{s=j\omega} - F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{6}{(a+j\omega)^4} + \frac{6}{(a-j\omega)^4} = -j48a\omega\frac{a^2-\omega^2}{(a^2+\omega^2)^4}$$

[c] $F(s) = \mathcal{L}\{e^{-at}\cos\omega_0 t\} = \dfrac{s+a}{(s+a)^2+\omega_0^2} = \dfrac{0.5}{(s+a)-j\omega_0} + \dfrac{0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$\begin{aligned} F(\omega) &= \frac{0.5}{(a+j\omega)-j\omega_0} + \frac{0.5}{(a+j\omega)+j\omega_0} \\ &\quad + \frac{0.5}{(a-j\omega)-j\omega_0} + \frac{0.5}{(a-j\omega)+j\omega_0} \\ &= \frac{a}{a^2+(\omega-\omega_0)^2} + \frac{a}{a^2+(\omega+\omega_0)^2} \end{aligned}$$

[d] $F(s) = \mathcal{L}\{e^{-at}\sin\omega_0 t\} = \dfrac{\omega_0}{(s+a)^2+\omega_0^2} = \dfrac{-j0.5}{(s+a)-j\omega_0} + \dfrac{j0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} - F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{-ja}{a^2+(\omega-\omega_0)^2} + \frac{ja}{a^2+(\omega+\omega_0)^2}$$

[e] $F(\omega) = \displaystyle\int_{-\infty}^{\infty} \delta(t-t_o)e^{-j\omega t}\,dt = e^{-j\omega t_o}$

(Use the sifting property of the Dirac delta function.)

P 17.6 $$\begin{aligned} f(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)+jB(\omega)][\cos t\omega + j\sin t\omega]\,d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega - B(\omega)\sin t\omega]\,d\omega \\ &\quad + \frac{j}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\sin t\omega + B(\omega)\cos t\omega]\,d\omega \end{aligned}$$

But $f(t)$ is real, therefore the second integral in the sum is zero.

P 17.7 By hypothesis, $f(t) = -f(-t)$. From Problem 17.6, we have

$$f(-t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega + B(\omega)\sin t\omega]\,d\omega$$

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For $f(t) = -f(-t)$, the integral $\int_{-\infty}^{\infty} A(\omega)\cos t\omega\, d\omega$ must be zero. Therefore, if $f(t)$ is real and odd, we have

$$f(t) = \frac{-1}{2\pi}\int_{-\infty}^{\infty} B(\omega)\sin t\omega\, d\omega$$

P 17.8 $F(\omega) = \dfrac{-j2}{\omega}$; therefore $B(\omega) = \dfrac{-2}{\omega}$; thus we have

$$f(t) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{-2}{\omega}\right)\sin t\omega\, d\omega = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin t\omega}{\omega}\, d\omega$$

But $\dfrac{\sin t\omega}{\omega}$ is even; therefore $f(t) = \dfrac{2}{\pi}\displaystyle\int_{0}^{\infty}\frac{\sin t\omega}{\omega}\, d\omega$

Therefore,

$$\left.\begin{aligned} f(t) &= \frac{2}{\pi}\cdot\frac{\pi}{2} = 1 \qquad t > 0 \\ f(t) &= \frac{2}{\pi}\cdot\left(\frac{-\pi}{2}\right) = -1 \quad t < 0 \end{aligned}\right\}\text{from a table of definite integrals}$$

Therefore $f(t) = \operatorname{sgn} t$

P 17.9 From Problem 17.5[c] we have

$$F(\omega) = \frac{\epsilon}{\epsilon^2 + (\omega - \omega_0)^2} + \frac{\epsilon}{\epsilon^2 + (\omega + \omega_0)^2}$$

Note that as $\epsilon \to 0$, $F(\omega) \to 0$ everywhere except at $\omega = \pm\omega_0$. At $\omega = \pm\omega_0$, $F(\omega) = 1/\epsilon$, therefore $F(\omega) \to \infty$ at $\omega = \pm\omega_0$ as $\epsilon \to 0$. The area under each bell-shaped curve is independent of ϵ, that is

$$\int_{-\infty}^{\infty}\frac{\epsilon d\omega}{\epsilon^2 + (\omega - \omega_0)^2} = \int_{-\infty}^{\infty}\frac{\epsilon d\omega}{\epsilon^2 + (\omega + \omega_0)^2} = \pi$$

Therefore as $\epsilon \to 0$, $F(\omega) \to \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$

P 17.10 $A(\omega) = \displaystyle\int_{-\infty}^{0} f(t)\cos\omega t\, dt + \int_{0}^{\infty} f(t)\cos\omega t\, dt = 0$

since $f(t)\cos\omega t$ is an odd function.

$B(\omega) = -2\displaystyle\int_{0}^{\infty} f(t)\sin\omega t\, dt$, since $f(t)\sin\omega t$ is an even function.

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P 17.11 $A(\omega) = \int_{-\infty}^{\infty} f(t)\cos\omega t\,dt$

$$= \int_{-\infty}^{0} f(t)\cos\omega t\,dt + \int_{0}^{\infty} f(t)\cos\omega t\,dt$$

$$= 2\int_{0}^{\infty} f(t)\cos\omega t\,dt, \quad \text{since } f(t)\cos\omega t \text{ is also even.}$$

$B(\omega) = 0$, since $f(t)\sin\omega t$ is an odd function and

$$\int_{-\infty}^{0} f(t)\sin\omega t\,dt = -\int_{0}^{\infty} f(t)\sin\omega t\,dt$$

P 17.12 **[a]** $\mathcal{F}\left\{\dfrac{df(t)}{dt}\right\} = \int_{-\infty}^{\infty} \dfrac{df(t)}{dt} e^{-j\omega t}\,dt$

Let $u = e^{-j\omega t}$, then $du = -j\omega e^{-j\omega t}\,dt$; let $dv = [df(t)/dt]\,dt$, then $v = f(t)$.

Therefore $\mathcal{F}\left\{\dfrac{df(t)}{dt}\right\} = f(t)e^{-j\omega t}\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(t)[-j\omega e^{-j\omega t}\,dt]$

$$= 0 + j\omega F(\omega)$$

[b] Fourier transform of $f(t)$ exists, i.e., $f(\infty) = f(-\infty) = 0$.

[c] To find $\mathcal{F}\left\{\dfrac{d^2 f(t)}{dt^2}\right\}$, let $g(t) = \dfrac{df(t)}{dt}$

Then $\mathcal{F}\left\{\dfrac{d^2 f(t)}{dt^2}\right\} = \mathcal{F}\left\{\dfrac{dg(t)}{dt}\right\} = j\omega G(\omega)$

But $G(\omega) = \mathcal{F}\left\{\dfrac{df(t)}{dt}\right\} = j\omega F(\omega)$

Therefore we have $\mathcal{F}\left\{\dfrac{d^2 f(t)}{dt^2}\right\} = (j\omega)^2 F(\omega)$

Repeated application of this thought process gives

$$\mathcal{F}\left\{\frac{d^n f(t)}{dt^n}\right\} = (j\omega)^n F(\omega)$$

P 17.13 **[a]** $\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{t} f(x)\,dx\right] e^{-j\omega t}\,dt$

Now let $u = \int_{-\infty}^{t} f(x)\,dx$, then $du = f(t)dt$

Let $dv = e^{-j\omega t}\,dt$, then $v = \dfrac{e^{-j\omega t}}{-j\omega}$

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Therefore,

$$\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \frac{e^{-j\omega t}}{-j\omega}\int_{-\infty}^{t} f(x)\,dx\,\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\left[\frac{e^{-j\omega t}}{-j\omega}\right] f(t)\,dt$$

$$= 0 + \frac{F(\omega)}{j\omega}$$

[b] We require $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 0$

[c] No, because $\displaystyle\int_{-\infty}^{\infty} e^{-ax}u(x)\,dx = \frac{1}{a} \neq 0$

P 17.14 **[a]** $\displaystyle\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(at)e^{-j\omega t}\,dt$

Let $u = at, \quad du = adt, \quad u = \pm\infty \quad$ when $\quad t = \pm\infty$

Therefore,

$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega u/a}\left(\frac{du}{a}\right) = \frac{1}{a}F\left(\frac{\omega}{a}\right), \qquad a > 0$$

[b] $\displaystyle\mathcal{F}\{e^{-|t|}\} = \frac{1}{1+j\omega} + \frac{1}{1-j\omega} = \frac{2}{1+\omega^2}$

Therefore $\displaystyle\mathcal{F}\{e^{-a|t|}\} = \frac{(1/a)2}{(\omega/a)^2+1}$

Therefore $\displaystyle\mathcal{F}\{e^{-0.5|t|}\} = \frac{4}{4\omega^2+1}, \qquad \mathcal{F}\{e^{-|t|}\} = \frac{2}{\omega^2+1}$

$\mathcal{F}\{e^{-2|t|}\} = 1/[0.25\omega^2+1]$, yes as "$a$" increases, the sketches show that $f(t)$ approaches zero faster and $F(\omega)$ flattens out over the frequency spectrum.

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P 17.15 **[a]** $\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(t-a)e^{-j\omega t}\,dt$

Let $u = t - a$, then $du = dt$, $t = u + a$, and $u = \pm\infty$ when $t = \pm\infty$. Therefore,

$$\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega(u+a)}\,du$$

$$= e^{-j\omega a}\int_{-\infty}^{\infty} f(u)e^{-j\omega u}\,du = e^{-j\omega a}F(\omega)$$

[b] $\mathcal{F}\{e^{j\omega_0 t}f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_0)t}\,dt = F(\omega-\omega_0)$

[c] $\mathcal{F}\{f(t)\cos\omega_0 t\} = \mathcal{F}\left\{f(t)\left[\dfrac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right]\right\}$

$$= \frac{1}{2}F(\omega-\omega_0) + \frac{1}{2}F(\omega+\omega_0)$$

P 17.16 $Y(\omega) = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda\right]e^{-j\omega t}\,dt$

$$= \int_{-\infty}^{\infty} x(\lambda)\left[\int_{-\infty}^{\infty} h(t-\lambda)e^{-j\omega t}\,dt\right]d\lambda$$

Let $u = t - \lambda$, $du = dt$, and $u = \pm\infty$, when $t = \pm\infty$.

Therefore $Y(\omega) = \int_{-\infty}^{\infty} x(\lambda)\left[\int_{-\infty}^{\infty} h(u)e^{-j\omega(u+\lambda)}\,du\right]d\lambda$

$$= \int_{-\infty}^{\infty} x(\lambda)\left[e^{-j\omega\lambda}\int_{-\infty}^{\infty} h(u)e^{-j\omega u}\,du\right]d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda)e^{-j\omega\lambda}H(\omega)\,d\lambda = H(\omega)X(\omega)$$

P 17.17 $\mathcal{F}\{f_1(t)f_2(t)\} = \int_{-\infty}^{\infty}\left[\dfrac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)e^{jtu}du\right]f_2(t)e^{-j\omega t}\,dt$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} F_1(u)f_2(t)e^{-j\omega t}e^{jtu}\,du\right]dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[F_1(u)\int_{-\infty}^{\infty} f_2(t)e^{-j(\omega-u)t}\,dt\right]du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega-u)\,du$$

P 17.18 **[a]** $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$

$$\frac{dF}{d\omega} = \int_{-\infty}^{\infty}\frac{d}{d\omega}\left[f(t)e^{-j\omega t}\,dt\right] = -j\int_{-\infty}^{\infty} tf(t)e^{-j\omega t}\,dt = -j\mathcal{F}\{tf(t)\}$$

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Therefore $\quad j\dfrac{dF(\omega)}{d\omega} = \mathcal{F}\{tf(t)\}$

$$\frac{d^2F(\omega)}{d\omega^2} = \int_{-\infty}^{\infty}(-jt)(-jt)f(t)e^{-j\omega t}\,dt = (-j)^2\mathcal{F}\{t^2f(t)\}$$

Note that $\quad (-j)^n = \dfrac{1}{j^n}$

Thus we have $\quad j^n\left[\dfrac{d^nF(\omega)}{d\omega^n}\right] = \mathcal{F}\{t^nf(t)\}$

[b] (i) $\quad \mathcal{F}\{e^{-at}u(t)\} = \dfrac{1}{a+j\omega} = F(\omega); \qquad \dfrac{dF(\omega)}{d\omega} = \dfrac{-j}{(a+j\omega)^2}$

Therefore $\quad j\left[\dfrac{dF(\omega)}{d\omega}\right] = \dfrac{1}{(a+j\omega)^2}$

Therefore $\quad \mathcal{F}\{te^{-at}u(t)\} = \dfrac{1}{(a+j\omega)^2}$

(ii) $\quad \mathcal{F}\{|t|e^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} - \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} - j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} + \frac{1}{(a-j\omega)^2}$$

(iii) $\quad \mathcal{F}\{te^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} + \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} + j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} - \frac{1}{(a-j\omega)^2}$$

P 17.19 **[a]** $f_1(t) = \cos\omega_0 t, \qquad F_1(u) = \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]$

$f_2(t) = 1, \quad -\tau/2 < t < \tau/2, \qquad$ and $f_2(t) = 0$ elsewhere

Thus $\quad F_2(u) = \dfrac{\tau\sin(u\tau/2)}{u\tau/2}$

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Using convolution,

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega - u)\,du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]\tau\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\int_{-\infty}^{\infty} \delta(u+\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$+ \frac{\tau}{2}\int_{-\infty}^{\infty} \delta(u-\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\cdot\frac{\sin[(\omega+\omega_0)\tau/2]}{(\omega+\omega_0)(\tau/2)} + \frac{\tau}{2}\cdot\frac{\sin[(\omega-\omega_0)\tau/2]}{(\omega-\omega_0)\tau/2}$$

[b] As τ increases, the amplitude of $F(\omega)$ increases at $\omega = \pm\omega_0$ and at the same time the width of the frequency band of $F(\omega)$ approaches zero as ω deviates from $\pm\omega_0$.

The area under the $[\sin x]/x$ function is independent of τ, that is

$$\frac{\tau}{2}\int_{-\infty}^{\infty}\frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}\,d\omega = \int_{-\infty}^{\infty}\frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}[(\tau/2)\,d\omega] = \pi$$

Therefore as $t \to \infty$,

$$f_1(t)f_2(t) \to \cos\omega_0 t \quad \text{and} \quad F(\omega) \to \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$$

P 17.20 **[a]** Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$v_{\text{Th}} = \frac{5}{6}v_g; \qquad R_{\text{Th}} = 480\|2400 = 400\,\text{k}\Omega$$

$$I_o = \frac{V_{\text{Th}}}{400 + 10^9/625s} = \frac{625sV_{\text{Th}}}{250{,}000s + 10^9}$$

$$H(s) = \frac{I_o}{V_{\text{Th}}} = \frac{25\times10^{-4}s}{s+4000}; \qquad H(j\omega) = \frac{25\times10^{-4}j\omega}{j\omega+4000}$$

$$v_{\text{Th}} = \frac{5}{6}v_g = 50\text{sgn}(t); \qquad V_{\text{Th}} = \frac{100}{j\omega}$$

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$$I_o = H(j\omega)V_{\text{Th}}(j\omega) = \left(\frac{100}{j\omega}\right)\left(\frac{25\times 10^{-4}j\omega}{j\omega+4000}\right) = \frac{0.25}{j\omega+4000}$$

$$i_o(t) = 250e^{-4000t}u(t)\,\text{mA}$$

[b] At $t = 0^-$ the circuit is

At $t = 0^+$ the circuit is

$$i_g(0^+) = \frac{60+50}{480} = 229.167\,\text{mA}$$

$$i_{2.4k}(0^+) = \frac{50}{2400} = 20.83\,\text{mA}$$

$$i_o(0^+) = 229.167 + 20.83 = 250\,\text{mA}$$

which agrees with our solution.
We also know $i_o(\infty) = 0$, which agrees with our solution.
The time constant with respect to the terminals of the capacitor is $R_{\text{Th}}C$
Thus,

$$\tau = (400)(625\times 10^{-9}) = 0.25\,\text{ms}; \qquad \therefore\ \frac{1}{\tau} = 4000,$$

which also agrees with our solution.
Thus our solution makes sense in terms of known circuit behavior.

P 17.21 **[a]** From the solution of Problem 17.20 we have

$$V_o = \frac{V_{\text{Th}}}{400 + (10^9/625s)}\cdot\frac{10^9}{625s}$$

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$$H(s) = \frac{V_o}{V_{\text{Th}}} = \frac{4000}{s + 4000}$$

$$H(j\omega) = \frac{4000}{j\omega + 4000}$$

$$V_{\text{Th}}(\omega) = \frac{100}{j\omega}$$

$$V_o(\omega) = H(j\omega)V_{\text{Th}}(\omega) = \left(\frac{100}{j\omega}\right)\frac{4000}{j\omega + 4000}$$

$$= \frac{400{,}000}{(j\omega)(j\omega + 4000)} = \frac{100}{j\omega} - \frac{100}{j\omega + 4000}$$

$$v_o(t) = 50\text{sgn}(t) - 100e^{-4000t}u(t)\,\text{V}$$

[b] $v_o(0^-) = -50\,\text{V}$

$$v_o(0^+) = 50 - 100 = -50\,\text{V}$$

This makes sense because there cannot be an instantaneous change in the voltage across a capacitor.

$$v_o(\infty) = 50\,\text{V}$$

This agrees with $v_{\text{Th}}(\infty) = 50$ V.
As in Problem 17.21 we know the time constant is 0.25 ms.

P 17.22 **[a]** $v_g = 50u(t)$

$$V_g(\omega) = 50\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$$

$$H(s) = \frac{400}{2s + 400} = \frac{200}{s + 200}$$

$$H(\omega) = \frac{200}{j\omega + 200}$$

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{10{,}000\pi\delta(\omega)}{j\omega + 200} + \frac{10{,}000}{j\omega(j\omega + 200)}$$

$$= V_1(\omega) + V_2(\omega)$$

$$v_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{10{,}000\pi e^{jt\omega}}{j\omega + 200}\delta(\omega)\,d\omega = \frac{1}{2\pi}\left(\frac{10{,}000\pi}{200}\right) = 25 \text{ (sifting property)}$$

$$V_2(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{j\omega + 200} = \frac{50}{j\omega} - \frac{50}{j\omega + 200}$$

$$v_2(t) = 25\text{sgn}(t) - 50e^{-200t}u(t)$$

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$$v_o(t) = v_1(t) + v_2(t) = 25 + 25\text{sgn}(t) - 50e^{-200t}u(t)$$
$$= 50u(t) - 50e^{-200t}u(t)$$
$$v_o(t) = 50(1 - e^{-200t})u(t)\,\text{V}$$

[b]

P 17.23 **[a]** From the solution to Problem 17.22

$$H(\omega) = \frac{200}{j\omega + 200}$$

Now,

$$V_g(\omega) = \frac{50}{j\omega}$$

Then,

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{10{,}000}{j\omega(j\omega + 200)} = \frac{K_1}{j\omega} + \frac{K_2}{j\omega + 200} = \frac{50}{j\omega} - \frac{50}{j\omega + 200}$$

$$\therefore \qquad v_o(t) = 25\text{sgn}(t) - 50e^{-200t}u(t)\,\text{V}$$

[b]

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P 17.24 **[a]** $I_o = \dfrac{I_g R}{R + 1/sC} = \dfrac{RCsI_g}{RCs + 1}; \qquad H(s) = \dfrac{I_o}{I_g} = \dfrac{s}{s + 1/RC}$

$$\frac{1}{RC} = 1000; \qquad H(j\omega) = \frac{j\omega}{j\omega + 1000}$$

$$i_g = 40\text{sgn}(t)\,\text{mA}; \qquad I_g = (40 \times 10^{-3})\left(\frac{2}{j\omega}\right) = \frac{80 \times 10^{-3}}{j\omega}$$

$$I_o = I_g[H(j\omega)] = \frac{80 \times 10^{-3}}{j\omega} \cdot \frac{j\omega}{j\omega + 1000} = \frac{80 \times 10^{-3}}{j\omega + 1000}$$

$$i_o(t) = 80e^{-1000t}u(t)\,\text{mA}$$

[b] Yes, at the time the source current jumps from $-40\,\text{mA}$ to $+40\,\text{mA}$ the capacitor is charged to $(1250)(0.04) = 50$ V, positive at the lower terminal. The circuit at $t = 0^-$ is

At $t = 0^+$ the circuit is

The time constant is $(1250)(0.8 \times 10^{-3}) = 1$ ms.

$$\therefore \quad \frac{1}{\tau} = 1000 \quad \therefore \quad \text{for } t > 0, \quad i_o = 80e^{-1000t}\,\text{mA}$$

P 17.25 **[a]** $V_o = \dfrac{I_g R(1/sC)}{R + (1/sC)} = \dfrac{I_g R}{RCs + 1}$

$$H(s) = \frac{V_o}{I_g} = \frac{1/C}{s + (1/RC)} = \frac{125 \times 10^4}{s + 1000}$$

$$H(j\omega) = \frac{125 \times 10^4}{1000 + j\omega}; \qquad I_g(\omega) = \frac{80 \times 10^{-3}}{j\omega}$$

$$V_o(\omega) = H(j\omega)I_g(\omega) = \left(\frac{80 \times 10^{-3}}{j\omega}\right)\left(\frac{125 \times 10^4}{1000 + j\omega}\right)$$

$$= \frac{10^5}{j\omega(1000 + j\omega)} = \frac{100}{j\omega} - \frac{100}{1000 + j\omega}$$

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$v_o(t) = 50\text{sgn}(t) - 100e^{-1000t}u(t)\,\text{V}$

[b] Yes, at the time the current source jumps from -40 to $+40\,\text{mA}$ the capacitor is charged to -50 V. That is, at $t = 0^-$,
$v_o(0^-) = (1250)(-40 \times 10^{-3}) = -50$ V.
At $t = \infty$ the capacitor will be charged to $+50$ V. That is,
$v_o(\infty) = (1250)(40 \times 10^{-3}) = 50$ V
The time constant of the circuit is $(1250)(0.8 \times 10^{-3}) = 1$ ms, so $1/\tau = 1000$. The function $v_o(t)$ is plotted below:

P 17.26 **[a]** $\dfrac{V_o}{V_g} = H(s) = \dfrac{16{,}000/s}{100 + 0.1s + 16{,}000/s}$

$$H(s) = \frac{160{,}000}{s^2 + 1000s + 160{,}000} = \frac{160{,}000}{(s+200)(s+800)}$$

$$H(j\omega) = \frac{160{,}000}{(j\omega + 200)(j\omega + 800)}$$

$$V_g(\omega) = \frac{16}{j\omega}$$

$$V_o(\omega) = V_g(\omega)H(j\omega) = \frac{256 \times 10^4}{j\omega(j\omega + 200)(j\omega + 800)}$$

$$V_o(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{j\omega + 200} + \frac{K_3}{j\omega + 800}$$

$$K_1 = \frac{256 \times 10^4}{16 \times 10^4} = 16; \qquad K_2 = \frac{256 \times 10^4}{(-200)(600)} = -21.33$$

$$K_3 = \frac{256 \times 10^4}{(-800)(-600)} = 5.33$$

$$V_o(\omega) = \frac{16}{j\omega} - \frac{21.33}{j\omega + 200} + \frac{5.33}{j\omega + 800}$$

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$$v_o(t) = 8\text{sgn}(t) - 21.33e^{-200t}u(t) + 5.33e^{-800t}u(t)\,\text{V}$$

[b] $v_o(0^-) = -8\,\text{V}$

[c] $v_o(0^+) = 8 - 21.33 + 5.33 = -8\,\text{V}$

[d] For $t \geq 0^+$:

$$\frac{V_o - 8/s}{100 + 0.1s} + \frac{(V_o + 8/s)s}{16{,}000} = 0$$

$$V_o\left[\frac{10}{s+1000} + \frac{s}{16{,}000}\right] = \frac{80}{s(s+1000)} - \frac{8}{16{,}000}$$

$$V_o = \frac{-8s^2 - 8000s + 80(16{,}000)}{s(s+200)(s+800)} = \frac{K_1}{s} + \frac{K_2}{s+200} + \frac{K_3}{s+800}$$

$$K_1 = \frac{80(16{,}000)}{(200)(800)} = 8; \qquad K_2 = \frac{80(16{,}000) - 320{,}000 + 16 \times 10^5}{(-200)(600)} = -21.33$$

$$K_3 = \frac{80(16{,}000) - 512 \times 10^4 + 64 \times 10^5}{(-800)(-600)} = 5.33$$

$$v_o(t) = (8 - 21.33e^{-200t} + 5.33e^{-800t})u(t)\,\text{V}$$

[e] Yes.

P 17.27 **[a]** $I_o = \dfrac{V_g}{100 + 0.1s + 16{,}000/s}$

$$H(s) = \frac{I_o}{V_g} = \frac{10s}{s^2 + 1000s + 160{,}000} = \frac{10s}{(s+200)(s+800)}$$

$$H(j\omega) = \frac{10(j\omega)}{(j\omega + 200)(j\omega + 800)}$$

$$V_g(\omega) = \frac{16}{j\omega}$$

$$I_o(\omega) = H(j\omega)V_g(\omega) = \frac{160}{(j\omega + 200)(j\omega + 800)}$$

$$= \frac{0.267}{j\omega + 200} - \frac{0.267}{j\omega + 800}$$

$$i_o(t) = (0.267e^{-200t} - 0.267e^{-800t})u(t)\,\text{A}$$

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[b] $i_o(0^-) = 0$

[c] $i_o(0^+) = 0$

[d]

$$I_o = \frac{16/s}{100 + 0.1s + 16{,}000/s} = \frac{160}{s^2 + 1000s + 160{,}000}$$

$$= \frac{160}{(s+200)(s+800)} = \frac{0.267}{s+200} - \frac{0.267}{s+800}$$

$$i_o(t) = (0.267e^{-200t} - 0.267e^{-800t})u(t)\,\text{A}$$

[e] Yes.

P 17.28 **[a]** $i_g = 2e^{-100|t|}$

$$\therefore \qquad I_g(\omega) = \frac{2}{j\omega + 100} + \frac{2}{-j\omega + 100} = \frac{400}{(j\omega + 100)(-j\omega + 100)}$$

$$\frac{V_o}{500} + 10^{-4}sV_o = I_g$$

$$\therefore \qquad \frac{V_o}{I_g} = H(s) = \frac{10^4}{s+20}; \qquad H(\omega) = \frac{10^4}{j\omega + 20}$$

$$V_o(\omega) = I_g(\omega)H(\omega) = \frac{4 \times 10^6}{(j\omega + 20)(j\omega + 100)(-j\omega + 100)}$$

$$= \frac{K_1}{j\omega + 20} + \frac{K_2}{j\omega + 100} + \frac{K_3}{-j\omega + 100}$$

$$K_1 = \frac{4 \times 10^6}{(120)(80)} = 416.67$$

$$K_2 = \frac{4 \times 10^6}{(-80)(200)} = -250$$

$$K_3 = \frac{4 \times 10^6}{(120)(200)} = 166.67$$

$$V_o(\omega) = \frac{416.67}{j\omega + 20} - \frac{250}{j\omega + 100} + \frac{166.67}{-j\omega + 100}$$

$$v_o(t) = [416.67e^{-20t} - 250e^{-100t}]u(t) + 166.67e^{100t}u(-t)\,\text{V}$$

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[b] $v_o(0^-) = 166.67\,\text{V}$

[c] $v_o(0^+) = 416.67 - 250 = 166.67\,\text{V}$

[d] $i_g = 2e^{-100t}u(t), \quad t \geq 0^+$

$$I_g = \frac{2}{s+100}; \qquad H(s) = \frac{10^4}{s+20}$$

$$v_o(0^+) = 166.67\,\text{V}; \qquad \gamma C = 0.0167$$

$$\frac{V_o}{500} + \frac{V_o s}{10^4} = I_g + 0.0167$$

$$\frac{V_o(s+20)}{10^4} = \frac{2}{s+100} + 0.0167$$

$$V_o = \frac{36{,}666.67 + 166.67s}{(s+20)(s+100)} = \frac{416.687}{s+20} - \frac{250}{s+100}$$

$$\therefore \qquad v_o(t) = (416.67e^{-20t} - 250e^{-100t})u(t)\,\text{V}$$

[e] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$v_o(t) = (416.67e^{-20t} - 250e^{-100t})u(t)\,\text{V}$$

P 17.29 **[a]** $I_o = \dfrac{500}{500 + 10^4/s} I_g = \dfrac{500s}{500s + 10^4} I_g = \dfrac{s}{s+20} I_g$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s+20}$$

$$H(j\omega) = \frac{j\omega}{j\omega + 20}$$

$$I_g(\omega) = \frac{400}{(j\omega+100)(-j\omega+100)} = \frac{2}{-j\omega+100} + \frac{2}{j\omega+100}$$

$$I_o(\omega) = H(j\omega)I_g(j\omega) = \frac{j\omega}{j\omega+20}\left[\frac{2}{-j\omega+100} + \frac{2}{j\omega+100}\right]$$

$$= \frac{2j\omega}{(j\omega+20)(-j\omega+100)} + \frac{2j\omega}{(j\omega+20)(j\omega+100)}$$

$$= \frac{K_1}{j\omega+20} + \frac{K_2}{-j\omega+100} + \frac{K_3}{j\omega+20} + \frac{K_4}{j\omega+100}$$

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$$K_1 = \frac{2(-20)}{120} = -0.33; \qquad K_2 = \frac{2(100)}{120} = 1.67$$

$$K_3 = \frac{2(-20)}{80} = -0.5; \qquad K_4 = \frac{2(-100)}{-80} = 2.5$$

$$\therefore \quad I_o(\omega) = \frac{-0.833}{j\omega + 20} + \frac{1.67}{-j\omega + 100} + \frac{2.5}{j\omega + 100}$$

$$i_o(t) = 1.67e^{100t}u(-t) + [-0.833e^{-20t} + 2.5e^{-100t}]u(t)\,\text{A}$$

[b] $i_o(0^-) = 1.67\,\text{V}$

[c] $i_o(0^+) = 1.67\,\text{V}$

[d] Note – since $i_o(0^+) = 1.67$ A, $v_o(0^+) = 1000 - 833.33 = 166.67$ V.

$$I_o = \frac{V_g - (166.67/s)}{500 + (10^4/s)} = \frac{sV_g - 166.67}{500s + 10^4}; \qquad V_g = \frac{1000}{s + 100}$$

$$\therefore \quad I_o = \frac{1.67s - 33.33}{(s+20)(s+100)} = \frac{-0.833}{s+20} + \frac{2.5}{s+100}$$

$$i_o(t) = (-0.833e^{-20t} + 2.5e^{-100t})u(t)\,\text{A}$$

[e] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$i_o(t) = (-0.833e^{-20t} + 2.5e^{-100t})u(t)\,\text{A}$$

P 17.30

$$\frac{V_o - V_g}{2s} + \frac{100V_o}{s} + \frac{V_o s}{100s + 125 \times 10^4} = 0$$

$$\therefore \quad V_o = \frac{s(100s + 125 \times 10^4)V_g}{125(s^2 + 12{,}000s + 25 \times 10^6)}$$

$$I_o = \frac{sV_o}{100s + 125 \times 10^4}$$

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$$H(s) = \frac{I_o}{V_g} = \frac{s^2}{125(s^2 + 12{,}000s + 25 \times 10^6)}$$

$$H(j\omega) = \frac{-8 \times 10^{-3}\omega^2}{(25 \times 10^6 - \omega^2) + j12{,}000\omega}$$

$$V_g(\omega) = 300\pi[\delta(\omega + 5000) + \delta(\omega - 5000)]$$

$$I_o(\omega) = H(j\omega)V_g(\omega) = \frac{-2.4\pi\omega^2[\delta(\omega + 5000) + \delta(\omega - 5000)]}{(25 \times 10^6 - \omega^2) + j12{,}000\omega}$$

$$i_o(t) = \frac{-2.4\pi}{2\pi}\int_{-\infty}^{\infty} \frac{\omega^2[\delta(\omega + 5000) + \delta(\omega - 5000)]}{(25 \times 10^6 - \omega^2) + j12{,}000\omega} e^{jt\omega}\, d\omega$$

$$= -1.2\left\{\frac{25 \times 10^6 e^{-j5000t}}{-j(12{,}000)(5000)} + \frac{25 \times 10^6 e^{j5000t}}{j(12{,}000)(5000)}\right\}$$

$$= \frac{6}{12}\left\{\frac{e^{-j5000t}}{-j} + \frac{e^{j5000t}}{j}\right\}$$

$$= 0.5[e^{-j(5000t+90^\circ)} + e^{j(5000t+90^\circ)}]$$

$$i_o(t) = 1\cos(5000t + 90^\circ)\ \text{A}$$

P 17.31 **[a]**

$$\frac{V_o - V_g}{sL_1} + \frac{V_o}{sL_2} + \frac{V_o}{R} = 0$$

$$\therefore\quad V_o = \frac{RV_g}{L_1\left[s + R\left(\frac{1}{L1} + \frac{1}{L_2}\right)\right]}$$

$$I_o = \frac{V_o}{sL_2}$$

$$\therefore\quad \frac{I_o}{V_g} = H(s) = \frac{R/L_1L_2}{s(s + R[(1/L_1) + (1/L_2)])}$$

$$\frac{R}{L_1L_2} = 12 \times 10^5$$

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$$R\left(\frac{1}{L_1}+\frac{1}{L_2}\right)=3\times 10^4$$

$$\therefore\quad H(s)=\frac{12\times 10^5}{s(s+3\times 10^4)}$$

$$H(j\omega)=\frac{12\times 10^5}{j\omega(j\omega+3\times 10^4)}$$

$$V_g(\omega)=125\pi[\delta(\omega+4\times 10^4)+\delta(\omega-4\times 10^4)]$$

$$I_o(\omega)=H(j\omega)V_g(\omega)=\frac{1500\pi\times 10^5[\delta(\omega+4\times 10^4)+\delta(\omega-4\times 10^4)]}{j\omega(j\omega+3\times 10^4)}$$

$$i_o(t)=\frac{1500\pi\times 10^5}{2\pi}\int_{-\infty}^{\infty}\frac{[\delta(\omega+4\times 10^4)+\delta(\omega-4\times 10^4)]e^{jt\omega}}{j\omega(j\omega+3\times 10^4)}\,d\omega$$

$$i_o(t)=750\times 10^5\left\{\frac{e^{-j40{,}000t}}{-j40{,}000(30{,}000-j40{,}000)}\right.$$

$$\left.+\frac{e^{j40{,}000t}}{j40{,}000(30{,}000+j40{,}000)}\right\}$$

$$=\frac{75\times 10^6}{4\times 10^8}\left\{\frac{e^{-j40{,}000t}}{-j(3+j4)}+\frac{e^{j40{,}000t}}{j(3+j4)}\right\}$$

$$=\frac{75}{400}\left\{\frac{e^{-j40{,}000t}}{5\underline{/-143.13^\circ}}+\frac{e^{j40{,}000t}}{5\underline{/143.13^\circ}}\right\}$$

$$=0.075\cos(40{,}000t-143.13^\circ)\text{ A}$$

$$i_o(t)=75\cos(40{,}000t-143.13^\circ)\text{ mA}$$

[b] In the phasor domain:

$$\frac{\mathbf{V}_o-125}{j200}+\frac{\mathbf{V}_o}{j800}+\frac{\mathbf{V}_o}{120}=0$$

$$12\mathbf{V}_o-1500+3\mathbf{V}_o+j20\mathbf{V}_o=0$$

$$\mathbf{V}_o=\frac{1500}{15+j20}=60\underline{/-53.13^\circ}\text{ V}$$

$$\mathbf{I}_o=\frac{\mathbf{V}_o}{j800}=75\times 10^{-3}\underline{/-143.13^\circ}\text{ A}$$

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$$i_o(t) = 75\cos(40{,}000t - 143.13^\circ)\,\text{mA}$$

P 17.32 **[a]**

$$\frac{(V_o - V_g)s}{10^6} + \frac{V_o}{4s} + \frac{V_o}{800} = 0$$

$$\therefore \quad V_o = \frac{s^2 V_g}{s^2 + 1250s + 25 \times 10^4}$$

$$\frac{V_o}{V_g} = H(s) = \frac{s^2}{(s+250)(s+1000)}$$

$$H(j\omega) = \frac{(j\omega)^2}{(j\omega+250)(j\omega+1000)}$$

$$v_g = 45e^{-500|t|}; \qquad V_g(\omega) = \frac{45{,}000}{(j\omega+500)(-j\omega+500)}$$

$$\therefore \quad V_o(\omega) = H(j\omega)V_g(\omega) = \frac{45{,}000(j\omega)^2}{(j\omega+250)(j\omega+500)(j\omega+1000)(-j\omega+500)}$$

$$= \frac{K_1}{j\omega+250} + \frac{K_2}{j\omega+500} + \frac{K_3}{j\omega+1000} + \frac{K_4}{-j\omega+500}$$

$$K_1 = \frac{45{,}000(-250)^2}{(250)(750)(750)} = 20$$

$$K_2 = \frac{45{,}000(-500)^2}{(-250)(500)(1000)} = -90$$

$$K_3 = \frac{45{,}000(-1000)^2}{(-750)(-500)(1500)} = 80$$

$$K_4 = \frac{45{,}000(500)^2}{(750)(1000)(1500)} = 10$$

$$\therefore \quad v_o(t) = [20e^{-250t} - 90e^{-500t} + 80e^{-1000t}]u(t) + 10e^{500t}u(-t)\,\text{V}$$

[b] $v_o(0^-) = 10\,\text{V}; \qquad V_o(0^+) = 20 - 90 + 80 = 10\,\text{V}$

$$v_o(\infty) = 0\,\text{V}$$

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[c] $I_L = \dfrac{V_o}{4s} = \dfrac{0.25sV_g}{(s+250)(s+1000)}$

$$H(s) = \frac{I_L}{V_o} = \frac{0.25s}{(s+250)(s+1000)}$$

$$H(j\omega) = \frac{0.25(j\omega)}{(j\omega+250)(j\omega+1000)}$$

$$I_L(\omega) = \frac{0.25(j\omega)(45{,}000)}{(j\omega+250)(j\omega+500)(j\omega+1000)(-j\omega+500)}$$

$$= \frac{K_1}{j\omega+250} + \frac{K_2}{j\omega+500} + \frac{K_3}{j\omega+1000} + \frac{K_4}{-j\omega+500}$$

$$K_4 = \frac{(0.25)(500)(45{,}000)}{(750)(1000)(1500)} = 5\,\text{mA}$$

$$i_L(t) = 5e^{500t}u(-t); \qquad \therefore \quad i_L(0^-) = 5\,\text{mA}$$

$$K_1 = \frac{(0.25)(-250)(45{,}000)}{(250)(750)(750)} = -20\,\text{mA}$$

$$K_2 = \frac{(0.25)(-500)(45{,}000)}{(-250)(500)(1000)} = 45\,\text{mA}$$

$$K_3 = \frac{(0.25)(-1000)(45{,}000)}{(-750)(-500)(1500)} = -20\,\text{mA}$$

$$\therefore \quad i_L(0^+) = K_1 + K_2 + K_3 = -20 + 45 - 20 = 5\,\text{mA}$$

Checks, i.e., $\quad i_L(0^+) = i_L(0^-) = 5\,\text{mA}$
At $t = 0^-$:

$$v_C(0^-) = 45 - 10 = 35\,\text{V}$$

At $t = 0^+$:

$$v_C(0^+) = 45 - 10 = 35\,\text{V}$$

[d] We can check the correctness of out solution for $t \geq 0^+$ by using the Laplace transform. Our circuit becomes

$$\frac{V_o}{800} + \frac{V_o}{4s} + \frac{(V_o - V_g)s}{10^6} + 35 \times 10^{-6} + \frac{5 \times 10^{-3}}{s} = 0$$

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$$\therefore \quad (s^2 + 1250s + 24 \times 10^4)V_o = s^2V_g - (35s + 5000)$$

$$v_g(t) = 45e^{-500t}u(t)\,\text{V}; \qquad V_g = \frac{45}{s + 500}$$

$$\therefore \quad (s + 250)(s + 1000)V_o = \frac{45s^2 - (35s + 5000)(s + 500)}{(s + 500)}$$

$$\therefore \quad V_o = \frac{10s^2 - 22{,}500s - 250 \times 10^4}{(s + 250)(s + 500)(s + 1000)}$$

$$= \frac{20}{s + 250} - \frac{90}{s + 500} + \frac{80}{s + 1000}$$

$$\therefore \quad v_o(t) = [20e^{-250t} - 90e^{-500t} + 80e^{-1000t}]u(t)\,\text{V}$$

This agrees with our solution for $v_o(t)$ for $t \geq 0^+$.

P 17.33 **[a]**

From the plot of v_g note that v_g is -50 V for an infinitely long time before $t = 0$. Therefore

$$\therefore \qquad v_o(0^-) = -50\,\text{V}$$

There cannot be an instantaneous change in the voltage across a capacitor, so

$$v_o(0^+) = -50\,\text{V}$$

[b] $i_o(0^-) = 0\,\text{A}$

At $t = 0^+$ the circuit is

$$i_o(0^+) = \frac{150 - (-50)}{25} = \frac{200}{25} = 8\,\text{A}$$

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[**c**] The s-domain circuit is

$$V_o = \left[\frac{V_g}{25 + (5000/s)}\right]\left(\frac{5000}{s}\right) = \frac{200V_g}{s + 200}$$

$$\frac{V_o}{V_g} = H(s) = \frac{200}{s + 200}$$

$$H(j\omega) = \frac{200}{j\omega + 200}$$

$$V_g(\omega) = 25\left(\frac{2}{j\omega}\right) - 25[2\pi\delta(\omega)] + \frac{150}{j\omega + 100} = \frac{50}{j\omega} - 50\pi\delta(\omega) + \frac{150}{j\omega + 100}$$

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{200}{j\omega + 200}\left[\frac{50}{j\omega} - 50\pi\delta(\omega) + \frac{150}{j\omega + 100}\right]$$

$$= \frac{10{,}000}{j\omega(j\omega + 200)} - \frac{10{,}000\pi\delta(\omega)}{j\omega + 200} + \frac{30{,}000}{(j\omega + 200)(j\omega + 100)}$$

$$= \frac{K_0}{j\omega} + \frac{K_1}{j\omega + 200} + \frac{K_2}{j\omega + 200} + \frac{K_3}{j\omega + 100} - \frac{10{,}000\pi\delta(\omega)}{j\omega + 200}$$

$$K_0 = \frac{10{,}000}{200} = 50; \quad K_1 = \frac{10{,}000}{-200} = -50;$$

$$K_2 = \frac{30{,}000}{-100} = -300; \quad K_3 = \frac{30{,}000}{100} = 300$$

$$V_o(\omega) = \frac{50}{j\omega} - \frac{350}{j\omega + 200} + \frac{300}{j\omega + 100} - \frac{10{,}000\pi\delta(\omega)}{j\omega + 200}$$

$$v_o(t) = 25\text{sgn}(t) + [300e^{-100t} - 350e^{-200t}]u(t) - 25\,\text{V}$$

P 17.34 [**a**]

$$V_g(\omega) = \frac{36}{4 - j\omega} - \frac{36}{4 + j\omega} = \frac{72j\omega}{(4 - j\omega)(4 + j\omega)}$$

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$$V_o(s) = \frac{(16/s)}{10 + s + (16/s)} V_g(s)$$

$$H(s) = \frac{V_o(s)}{V_g(s)} = \frac{16}{s^2 + 10s + 16} = \frac{16}{(s+2)(s+8)}$$

$$H(j\omega) = \frac{16}{(j\omega + 2)(j\omega + 8)}$$

$$V_o(j\omega) = H(j\omega) \cdot V_g(\omega) = \frac{1152j\omega}{(4 - j\omega)(4 + j\omega)(2 + j\omega)(8 + j\omega)}$$

$$= \frac{K_1}{4 - j\omega} + \frac{K_2}{4 + j\omega} + \frac{K_3}{2 + j\omega} + \frac{K_4}{8 + j\omega}$$

$$K_1 = \frac{1152(4)}{(8)(6)(12)} = 8$$

$$K_2 = \frac{1152(-4)}{(8)(-2)(4)} = 72$$

$$K_3 = \frac{1152(-2)}{(6)(2)(6)} = -32$$

$$K_4 = \frac{1152(-8)}{(12)(-4)(-6)} = -32$$

$$\therefore \quad V_o(j\omega) = \frac{8}{4 - j\omega} + \frac{72}{4 + j\omega} - \frac{32}{2 + j\omega} - \frac{32}{8 + j\omega}$$

$$\therefore \quad v_o(t) = 8e^{4t}u(-t) + [72e^{-4t} - 32e^{-2t} - 32e^{-8t}]u(t)\text{V}$$

[b] $v_o(0^-) = 8\text{V}$

[c] $v_o(0^+) = 72 - 32 - 32 = 8\text{V}$

The voltages at 0^- and 0^+ must be the same since the voltage cannot change instantaneously across a capacitor.

P 17.35 **[a]**

$$V_o = \frac{V_g s}{25 + (100/s) + s} = \frac{V_g s^2}{s^2 + 25s + 100}$$

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$$H(s) = \frac{V_o}{V_g} = \frac{s^2}{(s+5)(s+20)}; \qquad H(j\omega) = \frac{(j\omega)^2}{(j\omega+5)(j\omega+20)}$$

$$v_g = 25i_g = 450e^{10t}u(-t) - 450e^{-10t}u(t)\,\text{V}$$

$$V_g = \frac{450}{-j\omega+10} - \frac{450}{j\omega+10}$$

$$V_o(\omega) = H(j\omega)V_g = \frac{450(j\omega)^2}{(-j\omega+10)(j\omega+5)(j\omega+20)}$$

$$+\frac{-450(j\omega)^2}{(j\omega+10)(j\omega+5)(j\omega+20)}$$

$$= \frac{K_1}{-j\omega+10} + \frac{K_2}{j\omega+5} + \frac{K_3}{j\omega+20} + \frac{K_4}{j\omega+5} + \frac{K_5}{j\omega+10} + \frac{K_6}{j\omega+20}$$

$$K_1 = \frac{450(100)}{(15)(30)} = 100 \qquad K_4 = \frac{-450(25)}{(5)(15)} = -150$$

$$K_2 = \frac{450(25)}{(15)(15)} = 50 \qquad K_5 = \frac{-450(100)}{(-5)(10)} = 900$$

$$K_3 = \frac{450(400)}{(30)(-15)} = -400 \qquad K_6 = \frac{-450(400)}{(-15)(-10)} = -1200$$

$$V_o(\omega) = \frac{100}{-j\omega+10} + \frac{-100}{j\omega+5} + \frac{-1600}{j\omega+20} + \frac{900}{j\omega+10}$$

$$v_o = 100e^{10t}u(-t) + [900e^{-10t} - 100e^{-5t} - 1600e^{-20t}]u(t)\,\text{V}$$

[b] $v_o(0^-) = 100\,\text{V}$

[c] $v_o(0^+) = 900 - 100 - 1600 = -800\,\text{V}$

[d] At $t = 0^-$ the circuit is

Therefore, the solution predicts $v_1(0^-)$ will be 350 V.
Now $v_1(0^+) = v_1(0^-)$ because the inductor will not let the current in the $25\,\Omega$ resistor change instantaneously, and the capacitor will not let the voltage across the 0.01 F capacitor change instantaneously.

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At $t = 0^+$ the circuit is

From the circuit at $t = 0^+$ we see that v_o must be -800 V, which is consistent with the solution for v_o obtained in part (a).
It is informative to solve for either the current in the circuit or the voltage across the capacitor and note the solutions for i_o and v_C are consistent with the solution for v_o

The solutions are

$$i_o = 10e^{10t}u(-t) + [20e^{-5t} + 80e^{-20t} - 90e^{-10t}]u(t)\,\text{A}$$

$$v_C = 100e^{10t}u(-t) + [900e^{-10t} - 400e^{-5t} - 400e^{-20t}]u(t)\,\text{V}$$

P 17.36 $$V_o(s) = \frac{40}{s} + \frac{60}{s+100} - \frac{100}{s+300} = \frac{24{,}000(s+50)}{s(s+100)(s+300)}$$

$$V_o(s) = H(s)\cdot\frac{20}{s}$$

$$\therefore \qquad H(s) = \frac{1200(s+50)}{(s+100)(s+300)}$$

$$\therefore \qquad H(\omega) = \frac{1200(j\omega+50)}{(j\omega+100)(j\omega+300)}$$

$$\therefore \qquad V_o(\omega) = \frac{40}{j\omega}\cdot\frac{1200(j\omega+50)}{(j\omega+100)(j\omega+300)} = \frac{48{,}000(j\omega+50)}{j\omega(j\omega+100)(j\omega+300)}$$

$$V_o(\omega) = \frac{80}{j\omega} + \frac{120}{j\omega+100} - \frac{200}{j\omega+300}$$

$$v_o(t) = 160\text{sgn}(t) + [120e^{-100t} - 200e^{-300t}]u(t)\,\text{V}$$

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P 17.37 **[a]** $f(t) = \dfrac{1}{2\pi}\left\{\displaystyle\int_{-\infty}^{0} e^{\omega}e^{jt\omega}\,d\omega + \int_{0}^{\infty} e^{-\omega}e^{jt\omega}\,d\omega\right\} = \dfrac{1/\pi}{1+t^2}$

[b] $W = 2\displaystyle\int_0^\infty \frac{(1/\pi)^2}{(1+t^2)^2}\,dt = \frac{2}{\pi^2}\int_0^\infty \frac{dt}{(1+t^2)^2} = \frac{1}{2\pi}\,\text{J}$

[c] $W = \dfrac{1}{\pi}\displaystyle\int_0^\infty e^{-2\omega}\,d\omega = \frac{1}{\pi}\frac{e^{-2\omega}}{-2}\bigg|_0^\infty = \frac{1}{2\pi}\,\text{J}$

[d] $\dfrac{1}{\pi}\displaystyle\int_0^{\omega_1} e^{-2\omega}\,d\omega = \frac{0.9}{2\pi}, \qquad 1 - e^{-2\omega_1} = 0.9, \qquad e^{2\omega_1} = 10$

$\omega_1 = (1/2)\ln 10 \cong 1.15\,\text{rad/s}$

P 17.38 $I_o = \dfrac{0.5sI_g}{0.5s+25} = \dfrac{sI_g}{s+50}$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s+50}$$

$$H(j\omega) = \frac{j\omega}{j\omega+50}$$

$$I(\omega) = \frac{12}{j\omega+10}$$

$$I_o(\omega) = H(j\omega)I(\omega) = \frac{12(j\omega)}{(j\omega+10)(j\omega+50)}$$

$$|I_o(\omega)| = \frac{12\omega}{\sqrt{(\omega^2+100)(\omega^2+2500)}}$$

$$|I_o(\omega)|^2 = \frac{144\omega^2}{(\omega^2+100)(\omega^2+2500)}$$

$$= \frac{-6}{\omega^2+100} + \frac{150}{\omega^2+2500}$$

$$W_o(\text{total}) = \frac{1}{\pi}\int_0^\infty \frac{150d\omega}{\omega^2+2500} - \frac{1}{\pi}\int_0^\infty \frac{6d\omega}{\omega^2+100}$$

$$= \frac{3}{\pi}\tan^{-1}\left(\frac{\omega}{50}\bigg|_0^\infty\right) - \frac{0.6}{\pi}\tan^{-1}\left(\frac{\omega}{10}\bigg|_0^\infty\right)$$

$$= 1.5 - 0.3 = 1.2\,\text{J}$$

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$$W_o(0 — 100 \text{ rad/s}) = \frac{3}{\pi}\tan^{-1}(2) - \frac{0.6}{\pi}\tan^{-1}(10)$$

$$= 1.06 - 0.28 = 0.78\,\text{J}$$

Therefore, the percent between 0 and 100 rad/s is

$$\frac{0.78}{1.2}(100) = 64.69\%$$

P 17.39

$$I_o = \frac{I_g R}{R + (1/sC)} = \frac{RCsI_g}{RCs + 1}$$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s + (1/RC)}$$

$$RC = (2000)(2.5 \times 10^{-6}) = 0.005; \qquad \frac{1}{RC} = \frac{1}{0.005} = 200$$

$$H(s) = \frac{s}{s + 200}; \qquad H(j\omega) = \frac{j\omega}{j\omega + 200}$$

$$I_g(\omega) = \frac{0.01}{j\omega + 50}$$

$$I_o(\omega) = H(j\omega)I_g(\omega) = \frac{0.01j\omega}{(j\omega + 50)(j\omega + 200)}$$

$$|I_o(\omega)| = \frac{\omega(0.01)}{(\sqrt{\omega^2 + 50^2})(\sqrt{\omega^2 + 200^2})}$$

$$|I_o(\omega)|^2 = \frac{10^-4\omega^2}{(\omega^2 + 50^2)(\omega^2 + 200^2)} = \frac{K_1}{\omega^2 + 2500} + \frac{K_2}{\omega^2 + 4 \times 10^4}$$

$$K_1 = \frac{(10^{-4})(-2500)}{(37{,}500)} = -6.67 \times 10^{-6}$$

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$$K_2 = \frac{(10^{-4})(-4\times 10^4)}{(-37{,}500)} = 106.67\times 10^{-6}$$

$$|I_o(\omega)|^2 = \frac{106.67\times 10^{-6}}{\omega^2+4\times 10^4} - \frac{6.67\times 10^{-6}}{\omega^2+2500}$$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^\infty |I_o(\omega)|^2\,d\omega = \frac{106.67\times 10^{-6}}{\pi}\int_0^\infty \frac{d\omega}{\omega^2+4\times 10^4} - \frac{6.67\times 10^{-6}}{\pi}\int_0^\infty \frac{d\omega}{\omega^2+2500}$$

$$= \frac{0.533\times 10^{-6}}{\pi}\tan^{-1}\frac{\omega}{200}\bigg|_0^\infty - \frac{0.133\times 10^{-6}}{\pi}\tan^{-1}\frac{\omega}{50}\bigg|_0^\infty$$

$$= \left(\frac{0.533}{\pi}\cdot\frac{\pi}{2} - \frac{0.133}{\pi}\cdot\frac{\pi}{2}\right)\times 10^{-6} = 0.2\times 10^{-6} = 200\,\text{nJ}$$

Between 0 and 100 rad/s

$$W_{1\Omega} = \left[\frac{0.533}{\pi}\tan^{-1}\frac{1}{2} - \frac{0.133}{\pi}\tan^{-1}2\right]\times 10^{-6} = 31.79\,\text{nJ}$$

$$\% = \frac{31.79}{200}(100) = 15.9\%$$

P 17.40 **[a]** $V_g(\omega) = \dfrac{60}{(j\omega+1)(-j\omega+1)}$

$$H(s) = \frac{V_o}{V_g} = \frac{0.4}{s+0.5}; \qquad H(\omega) = \frac{0.4}{(j\omega+0.5)}$$

$$V_o(\omega) = \frac{24}{(j\omega+1)(j\omega+0.5)(-j\omega+1)}$$

$$V_o(\omega) = \frac{-24}{j\omega+1} + \frac{32}{j\omega+0.5} + \frac{8}{-j\omega+1}$$

$$v_o(t) = [-24e^{-t} + 32e^{-t/2}]u(t) + 8e^t u(-t)\,\text{V}$$

[b] $|V_g(\omega)| = \dfrac{60}{(\omega^2+1)}$

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[c] $|V_o(\omega)| = \dfrac{24}{(\omega^2+1)\sqrt{\omega^2+0.25}}$

[d] $W_i = 2\displaystyle\int_0^\infty 900e^{-2t}\,dt = 1800\left.\frac{e^{-2t}}{-2}\right|_0^\infty = 900\,\text{J}$

[e] $W_o = \displaystyle\int_{-\infty}^0 64e^{2t}\,dt + \int_0^\infty (-24e^{-t}+32e^{-t/2})^2\,dt$

$$= 32 + \textstyle\int_0^\infty [576e^{-2t} - 1536e^{-3t/2} + 1024e^{-t}]\,dt$$

$$= 32 + 288 - 1024 + 1024 = 320\,\text{J}$$

[f] $|V_g(\omega)| = \dfrac{60}{\omega^2+1}, \quad |V_g^2(\omega)| = \dfrac{3600}{(\omega^2+1)^2}$

$$W_g = \frac{3600}{\pi}\int_0^2 \frac{d\omega}{(\omega^2+1)^2}$$

$$= \frac{3600}{\pi}\left\{\frac{1}{2}\left(\frac{\omega}{\omega^2+1} + \tan^{-1}\omega\right)\Big|_0^2\right\}$$

$$= \frac{1800}{\pi}\left(\frac{2}{5} + \tan^{-1}2\right) = 863.53\,\text{J}$$

$$\therefore \quad \% = \left(\frac{863.53}{900}\right)\times 100 = 95.95\%$$

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[g] $|V_o(\omega)|^2 = \dfrac{576}{(\omega^2+1)^2(\omega^2+0.25)}$

$$= \frac{1024}{\omega^2+0.25} - \frac{768}{(\omega^2+1)^2} - \frac{1024}{(\omega^2+1)}$$

$$W_o = \frac{1}{\pi}\left\{1024\cdot 2\cdot \tan^{-1} 2\omega \,\Big|_0^2 - 768\left(\frac{1}{2}\right)\left(\frac{\omega}{\omega^2+1} + \tan^{-1}\omega\right)_0^2\right.$$

$$\left. -1024\tan^{-1}\omega \,\Big|_0^2\right\}$$

$$= \frac{2048}{\pi}\tan^{-1} 4 - \frac{384}{\pi}\left(\frac{2}{5} + \tan^{-1} 2\right) - \frac{1024}{\pi}\tan^{-1} 2$$

$$= 319.2\,\text{J}$$

$$\% = \frac{319.2}{320}\times 100 = 99.75\%$$

P 17.41 **[a]** $|V_i(\omega)|^2 = \dfrac{4\times 10^4}{\omega^2}$; $\quad |V_i(100)|^2 = \dfrac{4\times 10^4}{100^2} = 4$; $\quad |V_i(200)|^2 = \dfrac{4\times 10^4}{200^2} = 1$

[b] $V_o = \dfrac{V_i R}{R + (1/sC)} = \dfrac{sRCV_i}{RCs+1}$

$$H(s) = \frac{V_o}{V_i} = \frac{s}{s+(1/RC)}; \qquad \frac{1}{RC} = \frac{10^6 10^{-3}}{(0.5)(20)} = \frac{1000}{10} = 100$$

$$H(j\omega) = \frac{j\omega}{j\omega + 100}$$

$$|V_o(\omega)| = \frac{200}{|\omega|}\cdot\frac{|\omega|}{\sqrt{\omega^2+10^4}} = \frac{200}{\sqrt{\omega^2+10^4}}$$

$$|V_o(\omega)|^2 = \frac{4\times 10^4}{\omega^2+10^4}, \quad 100 \le \omega \le 200 \text{ rad/s}; \quad |V_o(\omega)|^2 = 0, \quad \text{elsewhere}$$

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$$|V_o(100)|^2 = \frac{4\times 10^4}{10^4+10^4} = 2; \qquad |V_o(200)|^2 = \frac{4\times 10^4}{5\times 10^4} = 0.8$$

[c] $$W_{1\Omega} = \frac{1}{\pi}\int_{100}^{200} \frac{4\times 10^4}{\omega^2}\,d\omega = \frac{4\times 10^4}{\pi}\left[-\frac{1}{\omega}\right]_{100}^{200}$$

$$= \frac{4\times 10^4}{\pi}\left[\frac{1}{100} - \frac{1}{200}\right] = \frac{200}{\pi} \cong 63.66\,\text{J}$$

[d] $$W_{1\Omega} = \frac{1}{\pi}\int_{100}^{200} \frac{4\times 10^4}{\omega^2+10^4}\,d\omega = \frac{4\times 10^4}{\pi}\cdot \tan^{-1}\frac{\omega}{100}\Big|_{100}^{200}$$

$$= \frac{400}{\pi}[\tan^{-1} 2 - \tan^{-1} 1] \cong 40.97\,\text{J}$$

P 17.42 **[a]** $$V_i(\omega) = \frac{A}{a+j\omega}; \qquad |V_i(\omega)| = \frac{A}{\sqrt{a^2+\omega^2}}$$

$$H(s) = \frac{s}{s+\alpha}; \qquad H(j\omega) = \frac{j\omega}{\alpha+j\omega}; \qquad |H(\omega)| = \frac{\omega}{\sqrt{\alpha^2+\omega^2}}$$

Therefore $$|V_o(\omega)| = \frac{\omega A}{\sqrt{(a^2+\omega^2)(\alpha^2+\omega^2)}}$$

Therefore $$|V_o(\omega)|^2 = \frac{\omega^2 A^2}{(a^2+\omega^2)(\alpha^2+\omega^2)}$$

$$W_{\text{IN}} = \int_0^\infty A^2 e^{-2at}\,dt = \frac{A^2}{2a}; \qquad \text{when } \alpha = a \text{ we have}$$

$$W_{\text{OUT}} = \frac{A^2}{\pi}\int_0^a \frac{\omega^2\,d\omega}{(\omega^2+a^2)^2} = \frac{A^2}{\pi}\left\{\int_0^a \frac{d\omega}{a^2+\omega^2} - \int_0^a \frac{a^2\,d\omega}{(a^2+\omega^2)^2}\right\}$$

$$= \frac{A^2}{4a\pi}\left(\frac{\pi}{2} - 1\right)$$

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$$W_{\text{OUT}}(\text{total}) = \frac{A^2}{\pi}\int_0^\infty \left[\frac{\omega^2}{(a^2+\omega^2)^2}\right] d\omega = \frac{A^2}{4a}$$

Therefore $\dfrac{W_{\text{OUT}}(a)}{W_{\text{OUT}}(\text{total})} = 0.5 - \dfrac{1}{\pi} = 0.1817$ or 18.17%

[b] When $\alpha \neq a$ we have

$$W_{\text{OUT}}(\alpha) = \frac{1}{\pi}\int_0^\alpha \frac{\omega^2 A^2 d\omega}{(a^2+\omega^2)(\alpha^2+\omega^2)}$$

$$= \frac{A^2}{\pi}\left\{\int_0^\alpha \left[\frac{K_1}{a^2+\omega^2} + \frac{K_2}{\alpha^2+\omega^2}\right] d\omega\right\}$$

where $K_1 = \dfrac{a^2}{a^2-\alpha^2}$ and $K_2 = \dfrac{-\alpha^2}{a^2-\alpha^2}$

Therefore

$$W_{\text{OUT}}(\alpha) = \frac{A^2}{\pi(a^2-\alpha^2)}\left[a\tan^{-1}\left(\frac{\alpha}{a}\right) - \frac{\alpha\pi}{4}\right]$$

$$W_{\text{OUT}}(\text{total}) = \frac{A^2}{\pi(a^2-\alpha^2)}\left[a\frac{\pi}{2} - \alpha\frac{\pi}{2}\right] = \frac{A^2}{2(a+\alpha)}$$

Therefore $\dfrac{W_{\text{OUT}}(\alpha)}{W_{\text{OUT}}(\text{total})} = \dfrac{2}{\pi(a-\alpha)} \cdot \left[a\tan^{-1}\left(\dfrac{\alpha}{a}\right) - \dfrac{\alpha\pi}{4}\right]$

For $\alpha = a\sqrt{3}$, this ratio is 0.2723, or 27.23% of the output energy lies in the frequency band between 0 and $a\sqrt{3}$.

[c] For $\alpha = a/\sqrt{3}$, the ratio is 0.1057, or 10.57% of the output energy lies in the frequency band between 0 and $a/\sqrt{3}$.

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18

Two-Port Circuits

Assessment Problems

AP 18.1 With port 2 short-circuited, we have

$$I_1 = \frac{V_1}{20} + \frac{V_1}{5}; \qquad \frac{I_1}{V_1} = y_{11} = 0.25\,\text{S}; \qquad I_2 = \left(\frac{-20}{25}\right) I_1 = -0.8I_1$$

When $V_2 = 0$, we have $\quad I_1 = y_{11}V_1 \quad$ and $\quad I_2 = y_{21}V_1$

Therefore $\quad I_2 = -0.8(y_{11}V_1) = -0.8y_{11}V_1$

Thus $\quad y_{21} = -0.8y_{11} = -0.2\,\text{S}$

With port 1 short-circuited, we have

$$I_2 = \frac{V_2}{15} + \frac{V_2}{5}; \qquad \frac{I_2}{V_2} = y_{22} = \left(\frac{4}{15}\right)\text{ S}$$

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$$I_1 = \left(\frac{-15}{20}\right) I_2 = -0.75 I_2 = -0.75 y_{22} V_2$$

Therefore $\quad y_{12} = (-0.75)\dfrac{4}{15} = -0.2\,\mathrm{S}$

AP 18.2

$$h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\mathrm{S}$$

$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\mathrm{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

AP 18.3

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} = \frac{5\times10^{-6}}{50\times10^{-3}} = 0.1\,\mathrm{mS}$$

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{200\times10^{-3}}{50\times10^{-3}} = 4$$

$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0} = \frac{2\times10^{-6}}{0.5\times10^{-6}} = 4$$

$$g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0} = \frac{10\times10^{-3}}{0.5\times10^{-6}} = 20\,\mathrm{k}\Omega$$

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AP 18.4 First calculate the b-parameters:

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} = \frac{15}{10} = 1.5\,\Omega; \qquad b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} = \frac{30}{10} = 3\,\text{S}$$

$$b_{12} = \frac{-V_2}{I_1}\bigg|_{V_1=0} = \frac{-10}{-5} = 2\,\Omega; \qquad b_{22} = \frac{-I_2}{I_1}\bigg|_{V_1=0} = \frac{-4}{-5} = 0.8$$

Now the z-parameters are calculated:

$$z_{11} = \frac{b_{22}}{b_{21}} = \frac{0.8}{3} = \frac{4}{15}\,\Omega; \qquad z_{12} = \frac{1}{b_{21}} = \frac{1}{3}\,\Omega$$

$$z_{21} = \frac{\Delta b}{b_{21}} = \frac{(1.5)(0.8) - 6}{3} = -1.6\,\Omega; \qquad z_{22} = \frac{b_{11}}{b_{21}} = \frac{1.5}{3} = \frac{1}{2}\,\Omega$$

AP 18.5

$z_{11} = z_{22}, \quad z_{12} = z_{21}, \quad 95 = z_{11}(5) + z_{12}(0)$

Therefore, $\quad z_{11} = z_{22} = 95/5 = 19\,\Omega$

$$11.52 = 19I_1 - z_{12}(2.72)$$

$$0 = z_{12}I_1 - 19(2.72)$$

Solving these simultaneous equations for z_{12} yields the quadratic equation

$$z_{12}^2 + \left(\frac{72}{17}\right) z_{12} - \frac{6137}{17} = 0$$

For a purely resistive network, it follows that $z_{12} = z_{21} = 17\,\Omega$.

AP 18.6 **[a]** $I_2 = \dfrac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_gZ_L + a_{22}Z_g}$

$$= \frac{-50 \times 10^{-3}}{(5 \times 10^{-4})(5 \times 10^3) + 10 + (10^{-6})(100)(5 \times 10^3) + (-3 \times 10^{-2})(100)}$$

$$= \frac{-50 \times 10^{-3}}{10} = -5\,\text{mA}$$

$$P_L = \frac{1}{2}(5 \times 10^{-3})^2(5 \times 10^3) = 62.5\,\text{mW}$$

[b] $Z_{\text{Th}} = \dfrac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \dfrac{10 + (-3 \times 10^{-2})(100)}{5 \times 10^{-4} + (10^{-6})(100)}$

$$= \frac{7}{6 \times 10^{-4}} = \frac{70}{6}\,\text{k}\Omega$$

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[c] $V_{\text{Th}} = \dfrac{V_g}{a_{11} + a_{21}Z_g} = \dfrac{50 \times 10^{-3}}{6 \times 10^{-4}} = \dfrac{500}{6}\,\text{V}$

Therefore $\quad V_2 = \dfrac{250}{6}\,\text{V}; \qquad P_{\max} = \dfrac{(1/2)(250/6)^2}{(70/6) \times 10^3} = 74.4\,\text{mW}$

AP 18.7 **[a]** For the given bridged-tee circuit, we have

$$a'_{11} = a'_{22} = 1.25, \qquad a'_{21} = \frac{1}{20}\,\text{S}, \qquad a'_{12} = 11.25\,\Omega$$

The a-parameters of the cascaded networks are

$$a_{11} = (1.25)^2 + (11.25)(0.05) = 2.125$$

$$a_{12} = (1.25)(11.25) + (11.25)(1.25) = 28.125\,\Omega$$

$$a_{21} = (0.05)(1.25) + (1.25)(0.05) = 0.125\,\text{S}$$

$$a_{22} = a_{11} = 2.125, \qquad R_{\text{Th}} = (45.125/3.125) = 14.44\,\Omega$$

[b] $V_t = \dfrac{100}{3.125} = 32\,\text{V}; \qquad$ therefore $\quad V_2 = 16\,\text{V}$

[c] $P = \dfrac{16^2}{14.44} = 17.73\,\text{W}$

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Problems

P 18.1 $h_{11} = \left(\dfrac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\text{S}$$

$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\text{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

P 18.2

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 1 + 12 = 13\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 12\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 4 + 12 = 16\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 12\,\Omega$$

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P 18.3 $\Delta z = (13)(16) - (12)(12) = 64$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{16}{64} = 0.25\,\text{S}$$

$$y_{12} = \frac{-z_{12}}{\Delta z} = \frac{-12}{64} = -0.1875\,\text{S}$$

$$y_{21} = \frac{-z_{21}}{\Delta z} = \frac{-12}{64} = -0.1875\,\text{S}$$

$$y_{22} = \frac{-z_{11}}{\Delta z} = \frac{13}{64} = 0.203125\,\text{S}$$

P 18.4 $y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$; $\qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$

$$\frac{V-1}{20} + \frac{V}{10} + \frac{V}{4} = 0; \quad \text{so} \quad V = 0.125\,\text{V}$$

$$\therefore \quad I_1 = \frac{1-0.125}{20} + \frac{1-0}{8} = 168.75\,\text{mA}; \qquad I_2 = \frac{0-0.125}{4} + \frac{0-1}{8} = -156.25\,\text{mA}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 168.75\,\text{mS}; \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -156.25\,\text{mS}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}; \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$\frac{V}{20} + \frac{V}{10} + \frac{V-1}{4} = 0; \quad \text{so} \quad V = 0.625\,\text{V}$$

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$$\therefore \quad I_1 = \frac{0-0.625}{20} + \frac{0-1}{8} = -156.25\,\text{mA};$$

$$I_2 = \frac{1-0.625}{4} + \frac{1-0}{8} = 218.75\,\text{mA}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -156.25\,\text{mS}; \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = 218.75\,\text{mS}$$

Summary:

$$y_{11} = 168.75\,\text{mS} \quad y_{12} = -156.25\,\text{mS} \quad y_{21} = -156.25\,\text{mS} \quad y_{22} = 218.75\,\text{mS}$$

P 18.5 $V_2 = b_{11}V_1 - b_{12}I_1$

$$I_2 = b_{21}V_1 - b_{22}I_1$$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0}; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$

$$5\|15 = (15/4)\,\Omega; \qquad 10\|20 = (20/3)\,\Omega$$

$$I_2 = \frac{V_2}{(15/4)+(20/3)} = \frac{12V_2}{125}; \qquad I_1 = I_\text{b} - I_\text{a}$$

$$I_\text{a} = \frac{15}{20}I_2; \qquad I_\text{b} = \frac{20}{30}I_2$$

$$I_1 = \left(\frac{20}{30} - \frac{15}{20}\right)I_2 = \frac{-5}{60}I_2 = \frac{-1}{12}I_2$$

$$b_{22} = \frac{-I_2}{I_1} = 12$$

$$b_{12} = \frac{-V_2}{I_1} = \frac{-V_2}{I_2}\left(\frac{I_2}{I_1}\right) = \frac{125}{12}(12) = 125\,\Omega$$

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$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$

$$V_1 = V_\mathrm{a} - V_\mathrm{b}; \quad V_\mathrm{a} = \frac{10}{15}V_2; \quad V_\mathrm{b} = \frac{20}{35}V_2$$

$$V_1 = \frac{10}{15}V_2 - \frac{20}{35}V_2 = \frac{2}{21}V_2$$

$$b_{11} = \frac{V_2}{V_1} = \frac{21}{2} = 10.5$$

$$V_2 = (10+5)\|(20+15)I_2 = 10.5I_2$$

$$b_{21} = \frac{I_2}{V_1} = \left(\frac{I_2}{V_2}\right)\left(\frac{V_2}{V_1}\right) = \left(\frac{1}{10.5}\right)(10.5) = 1\,\mathrm{S}$$

P 18.6 $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$

$$\frac{V_1}{I_1} = 80\|[10+20\|20] = 80\|20 = 16\,\Omega \qquad \therefore \quad h_{11} = 16\,\Omega$$

$$I_6 = \frac{80}{80+20}I_1 = 0.8I_1$$

$$I_2 = \frac{-20}{20+20}I_6 = -0.5I_6 = -0.5(0.8)I_1 = -0.4I_1 \qquad \therefore \quad h_{21} = -0.4$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

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$$\frac{V_2}{I_2} = 80\|[20 + 20\|90] = 25\,\Omega \qquad \therefore \quad h_{22} = \frac{1}{25} = 40 \text{ mS}$$

$$V_x = \frac{20\|90}{20 + 20\|90} V_2$$

$$V_1 = \frac{80}{80 + 10} V_x = \frac{80(20\|90)}{90(20 + 20\|90)} V_2 = 0.4 V_2$$

$$\therefore \qquad h_{12} = 0.4$$

Summary:

$$h_{11} = 16\,\Omega; \quad h_{12} = 0.4; \quad h_{21} = -0.4; \quad h_{22} = 40 \text{ mS}$$

P 18.7 $$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = R_1\|R_2 = 4 \quad \therefore \quad \frac{R_1 R_2}{R_1 + R_2} = 4$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = \frac{-R_2}{R_1 + R_2} = -0.8$$

$$\therefore \qquad R_2 = 0.8R_1 + 0.8R_2 \quad \text{so} \quad R_1 = \frac{R_2}{4}$$

Substituting,

$$\frac{(R_2/4)R_2}{(R_2/4) + R_2} = 4 \quad \text{so} \quad R_2 = 20\,\Omega \quad \text{and} \quad R_1 = 5\,\Omega$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{1}{R_3\|(R_1 + R_2)} = \frac{1}{R_3\|25} = 0.14$$

$$\therefore \qquad R_3 = 10$$

Summary:

$$R_1 = 5\,\Omega; \qquad R_2 = 20\,\Omega; \qquad R_3 = 10\,\Omega$$

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P 18.8 For $V_2 = 0$:

$$V_1 = (400 + 1200)I_1$$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = \frac{1600}{1} = 1600\,\Omega$$

$$V_p = 1200(1\,\text{A}) = 1200\,\text{V} = V_n$$

At V_n,

$$\frac{1200}{500} + \frac{1200 - V_o}{1000} = 0 \quad \text{so} \quad V_o = 3600\,\text{V}$$

$$\therefore \qquad I_2 = -\frac{3600}{200} = -18\,\text{A}$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{I_1=0} = \frac{-18}{1} = -18$$

For $I_1 = 0$:

$$V_1 = 0$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = \frac{0}{1} = 0$$

At V_n,

$$\frac{V_n}{500} + \frac{V_n - V_o}{100} = 0$$

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But $V_n = V_p = 0$ so $V_o = 0$; therefore,

$$I_2 = \frac{1\,\text{V}}{200\,\Omega} = 5\,\text{mS}$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{5\,\text{m}}{1} = 5\,\text{mS}$$

P 18.9 For $I_2 = 0$:

$$30I_1 - 25I_2 = 1$$

$$-25I_1 + 175I_2 - 13(25)(I_1 - I_2) = 0 \quad \text{so} \quad -350I_1 + 500I_2 = 0$$

Solving,

$$I_1 = 80\,\text{mA}; \qquad I_2 = 56\,\text{mA}$$

$$V_2 = 50I_2 - 13(25)(I_1 - I_2) = -5\,\text{V}$$

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} = \frac{80\,\text{m}}{1} = 80\,\text{mS}$$

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{-5}{1} = -5$$

For $V_1 = 0$:

$$30I_1 - 25I_2 = 0$$

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$$-25I_1 + 175I_2 + 50 - 13(25)(I_1 - I_2) = 0 \quad \text{so} \quad -350I_1 + 500I_2 = -50$$

Solving,

$$I_1 = -200\,\text{mA}; \qquad I_2 = -240\,\text{mA}$$

$$V_2 = 50(I_2 + 1) - 13(25)(I_1 - I_2) = 25\,\text{V}$$

$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0} = \frac{-200\,\text{m}}{1} = -0.2$$

$$g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0} = \frac{25}{1} = 25\,\Omega$$

P 18.10 $V_1 = a_{11}V_2 - a_{12}I_2$

$$I_1 = a_{21}V_2 - a_{22}I_2$$

$$a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0}; \qquad a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0}$$

$$V_1 = 10^3 I_1 + 10^{-4}V_2 = 10^3(-0.5 \times 10^{-6})V_2 + 10^{-4}V_2$$

$$\therefore \quad a_{11} = -5 \times 10^{-4} + 10^{-4} = -4 \times 10^{-4}$$

$$V_2 = -(50I_1)(40 \times 10^3); \qquad \therefore \quad a_{21} = -\frac{1}{2 \times 10^6} = -0.5\,\mu\text{S}$$

$$a_{12} = \frac{-V_1}{I_2}\bigg|_{V_2=0}; \qquad a_{22} = \frac{-I_1}{I_2}\bigg|_{V_2=0}$$

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$$I_2 = 50I_1; \qquad \therefore \quad a_{22} = -\frac{I_1}{I_2} = -\frac{1}{50}$$

$$V_1 = 1000I_1; \qquad \therefore \quad a_{12} = -\frac{V_1}{I_2} = -\frac{V_1}{I_1}\frac{I_1}{I_2} = -(1000)(1/50) = -20\,\Omega$$

Summary

$$a_{11} = -4 \times 10^{-4}; \quad a_{12} = -20\,\Omega; \quad a_{21} = -0.5\,\mu\text{S}; \quad a_{22} = -0.02$$

P 18.11 $g_{11} = \dfrac{a_{21}}{a_{11}} = \dfrac{-0.5 \times 10^{-6}}{-4 \times 10^{-4}} = 1.25\,\text{mS}$

$$g_{12} = \frac{-\Delta a}{a_{11}} = \frac{-(-4 \times 10^{-4})(-1/50) - (-0.5 \times 10^{-6})(-20)}{-4 \times 10^{-4}} = -0.005$$

$$g_{21} = \frac{1}{a_{11}} = \frac{1}{-4 \times 10^{-4}} = -2500$$

$$g_{22} = \frac{a_{12}}{a_{11}} = \frac{(-20)}{-400 \times 10^{-6}} = 5 \times 10^4\,\Omega$$

P 18.12 For $V_2 = 0$:

$$I_a = \frac{50I_2}{200} = \frac{1}{4}I_2 = -I_2; \quad \therefore \qquad I_2 = 0$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = 0$$

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$$V_1 = (10 + j20)I_1 \quad \therefore \quad h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = 10 + j20\,\Omega$$

For $I_1 = 0$:

$$V_1 = 50I_2; \qquad I_2 = \frac{V_2}{-j100} + \frac{V_2 - 50I_2}{200}$$

$$200I_2 = j2V_2 + V_2 - 50i_2$$

$$250I_2 = V_2(1 + j2)$$

$$50I_2 = V_2\left(\frac{1 + j2}{5}\right) = (0.2 + j0.4)V_2$$

$$\therefore \quad V_1 = (0.2 + j0.4)V_2$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = 0.2 + j0.4$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{1 + j2}{250} = 4 + j8 \text{ mS}$$

Summary:

$$h_{11} = 10 + j20\,\Omega; \quad h_{12} = 0.2 + j0.4; \quad h_{21} = 0; \quad h_{22} = 4 + j8\,\text{mS}$$

P 18.13 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_2 = g_{21}V_1 + g_{22}I_2$

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} = \frac{0.25 \times 10^{-6}}{20 \times 10^{-3}} = 12.5 \times 10^{-6} = 12.5\,\mu\text{S}$$

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{-5}{20} \times 10^3 = -250$$

$$0 = -250(10) + g_{22}(50 \times 10^{-6})$$

$$g_{22} = \frac{2500}{50 \times 10^{-6}} = 50\,\text{M}\Omega$$

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$$200 \times 10^{-6} = 12.5 \times 10^{-6}(10) + g_{12}(50 \times 10^{-6})$$

$$(200 - 125)10^{-6} = g_{12}(50 \times 10^{-6})$$

$$g_{12} = \frac{75}{50} = 1.5$$

P 18.14 **[a]** $I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$

$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = \frac{50 \times 10^{-6}}{10} = 5\,\mu\text{S}$$

$$0 = y_{21}(20 \times 10^{-3}) + y_{22}(-5)$$

$$\therefore \qquad y_{22} = \frac{1}{5}y_{21}(20 \times 10^{-3}) = 20\,\text{nS}$$

$$200 \times 10^{-6} = y_{11}(10) \quad \text{so} \quad y_{11} = 20\,\mu\text{S}$$

$$0.25 \times 10^{-6} = 20 \times 10^{-6}(20 \times 10^{-3}) + y_{12}(-5)$$

$$y_{12} = \frac{0.25 \times 10^{-6} - 0.4 \times 10^{-6}}{-5} = 30\,\text{nS}$$

Summary:

$$y_{11} = 20\,\mu\text{S}; \quad y_{12} = 30\,\text{nS}; \quad y_{21} = 5\,\mu\text{S}; \quad y_{22} = 20\,\text{nS}$$

[b] $y_{11} = \dfrac{\Delta g}{g_{22}}; \quad y_{12} = \dfrac{g_{12}}{g_{22}}; \quad y_{21} = \dfrac{-g_{21}}{g_{22}}; \quad y_{22} = \dfrac{1}{g_{22}}$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21} = (12.5 \times 10^{-6})(50 \times 10^{6}) - 1.5(-250)$$

$$= 625 + 375 = 1000$$

$$y_{11} = \frac{1000}{50 \times 10^{6}} = 20\,\mu\text{S}; \qquad y_{21} = \frac{250}{5 \times 10^{6}} = 5\,\mu\text{S}$$

$$y_{12} = \frac{1.5}{50 \times 10^{6}} = 30\,\text{nS}; \qquad y_{22} = \frac{1}{5 \times 10^{6}} = 20\,\text{nS}$$

These values are the same as those in part (a).

P 18.15 $V_1 = h_{11}I_1 + h_{12}V_2; \qquad I_2 = h_{21}I_1 + h_{22}V_2$

Solve the first equation for I_1 and the second equation for V_2:

$$I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}}V_2; \qquad V_2 = \frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}}I_1$$

Work with the I_1 equation, substituting in the expression for V_2:

$$I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}}\left[\frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}}I_1\right]$$

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$$I_1\left[1-\frac{h_{12}h_{21}}{h_{11}h_{22}}\right]=\frac{V_1}{h_{11}}-\frac{h_{12}}{h_{11}h_{22}}I_2$$

Thus,

$$g_{11}=\frac{1/h_{11}}{1-(h_{12}h_{21})/(h_{11}h_{22})}=\frac{h_{22}}{h_{11}h_{22}-h_{12}h_{21}}=\frac{h_{22}}{\Delta h}$$

$$g_{12}=\frac{-h_{12}/(h_{11}h_{22})}{1-(h_{12}h_{21})/(h_{11}h_{22})}=\frac{-h_{12}}{h_{11}h_{22}-h_{12}h_{21}}=\frac{-h_{12}}{\Delta h}$$

Now work with the V_2 equation, substituting in the expression for I_1:

$$V_2=\frac{I_2}{h_{22}}-\frac{h_{21}}{h_{22}}\left[\frac{V_1}{h_{11}}-\frac{h_{12}}{h_{11}}V_2\right]$$

$$V_2\left[1-\frac{h_{12}h_{21}}{h_{11}h_{22}}\right]=\frac{I_2}{h_{22}}-\frac{h_{21}}{h_{11}h_{22}}V_1$$

Thus,

$$g_{21}=\frac{-h_{21}/(h_{11}h_{22})}{1-(h_{12}h_{21})/(h_{11}h_{22})}=\frac{-h_{21}}{h_{11}h_{22}-h_{12}h_{21}}=\frac{-h_{21}}{\Delta h}$$

$$g_{22}=\frac{1/h_{22}}{1-(h_{12}h_{21})/(h_{11}h_{22})}=\frac{h_{11}}{h_{11}h_{22}-h_{12}h_{21}}=\frac{h_{11}}{\Delta h}$$

P 18.16 $V_1=a_{11}V_2-a_{12}I_2;\qquad I_1=a_{21}V_2-a_{22}I_2$

$$I_2=\frac{a_{21}}{a_{22}}V_2-\frac{1}{a_{22}}I_1$$

$$\therefore\qquad h_{21}=\frac{-1}{a_{22}}\quad\text{and}\quad h_{22}=\frac{a_{21}}{a_{22}}$$

$$V_1=a_{11}V_2-a_{12}\left(\frac{a_{21}}{a_{22}}V_2-\frac{1}{a_{22}}I_1\right)=\left(a_{11}-\frac{a_{12}a_{21}}{a_{22}}\right)V_2+\frac{a_{12}}{a_{22}}I_1$$

$$\therefore\qquad h_{11}=\frac{a_{12}}{a_{22}}\quad\text{and}\quad h_{12}=a_{11}-\frac{a_{12}a_{21}}{a_{22}}=\frac{\Delta a}{a_{22}}$$

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P 18.17 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_2 = g_{21}V_1 + g_{22}I_2$

$$I_2 = \frac{1}{g_{22}}V_2 - \frac{g_{21}}{g_{22}}V_1$$

$$\therefore \qquad y_{21} = \frac{-g_{21}}{g_{22}} \quad \text{and} \quad y_{22} = \frac{1}{g_{22}}$$

$$I_1 = g_{11}V_1 + g_{12}\left(\frac{1}{g_{22}}V_2 - \frac{g_{21}}{g_{22}}V_1\right) = \left(g_{11} - \frac{g_{12}g_{21}}{g_{22}}\right)V_1 + \frac{g_{12}}{g_{22}}V_2$$

$$\therefore \qquad y_{11} = g_{11} - \frac{g_{12}g_{21}}{g_{22}} = \frac{\Delta g}{g_{22}} \quad \text{and} \quad y_{12} = \frac{g_{12}}{g_{22}}$$

P 18.18

For $I_2 = 0$:

$$V_1 = \left(\frac{2}{s} + 10\right)I_1 \qquad \text{so} \qquad z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = \frac{10s+2}{s}\,\Omega$$

$$V_2 = 10I_1 \qquad \text{so} \qquad z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = 10\,\Omega$$

For $I_1 = 0$:

$$V_1 = 10I_2 \qquad \text{so} \qquad z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = 10\,\Omega$$

$$V_2 = (0.2s + 10)I_2 \qquad \text{so} \qquad z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} = 0.2s + 10\,\Omega$$

P 18.19 $I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$

Since the circuit is symmetric and reciprocal, $y_{11} = y_{22}$ and $y_{12} = y_{21}$.

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$$I_1 = \frac{V_1}{(1/s)} + \frac{V_1}{s + (s/s+1)}$$

$$\frac{I_1}{V_1} = s + \frac{s+1}{s^2+2s} = \frac{s^3+2s^2+s+1}{s(s+2)}$$

$$y_{11} = y_{22} = \frac{s^3+2s^2+s+1}{s(s+2)}$$

$$I_a = V_1 s; \qquad I_b = \frac{V_1(s+1)}{s(s+2)} \cdot \frac{1}{(s+1)} = \frac{V_1}{s(s+2)}$$

$$I_2 = -(I_a + I_b) = -\left[V_1 s + \frac{V_1}{s(s+2)}\right]$$

$$\frac{I_2}{V_1} = -\left[s + \frac{1}{s(s+2)}\right] = -\frac{s^3+2s^2+1}{s(s+2)}$$

$$y_{12} = y_{21} = -\frac{s^3+2s^2+1}{s(s+2)}$$

P 18.20 **[a]** $a_{12} = \dfrac{-V_1}{I_2}\bigg|_{V_2=0}; \qquad a_{22} = \dfrac{-I_1}{I_2}\bigg|_{V_2=0}$

$$V_1 = (R + sL)I_1 - sMI_2$$

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$$0 = -sMI_1 + (R + sL)I_2$$

$$\Delta = \begin{vmatrix} (R+sL) & -sM \\ -sM & (R+sL) \end{vmatrix} = (R+sL)^2 - s^2M^2$$

$$N_2 = \begin{vmatrix} R+sL & V_1 \\ -sM & 0 \end{vmatrix} = sMV_1$$

$$I_2 = \frac{N_2}{\Delta} = \frac{sMV_1}{(R+sL)^2 - s^2M^2}; \qquad a_{12} = -\frac{V_1}{I_2} = \frac{(sM)^2 - (R+sL)^2}{sM}$$

$$0 = -sMI_1 + (R+sL)I_2; \qquad \therefore \quad a_{22} = \frac{-I_1}{I_2} = \frac{-(R+sL)}{sM}$$

$$a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0}; \qquad a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0}$$

$$V_2 = -sMI_1; \qquad I_1 = \frac{V_1}{R+sL}$$

$$V_2 = \frac{-sMV_1}{R+sL}; \qquad a_{11} = \frac{V_1}{V_2} = \frac{R+sL}{-sM}$$

$$a_{21} = \frac{I_1}{V_2} = \frac{-1}{sM}$$

[b] $a_{11} = a_{22}$ (symmetric, reciprocal)

$$a_{11}a_{22} - a_{12}a_{21} = 1 \quad \text{(reciprocal)}$$

$$a_{11} = \frac{R+sL}{-sM}; \qquad a_{22} = \frac{R+sL}{-sM} \quad \text{(checks)}$$

$$a_{11}a_{22} - a_{12}a_{21} = \frac{(R+sL)^2}{(sM)^2} + \frac{(sM)^2 - (R+sL)^2}{sM} \cdot \frac{1}{sM}$$

$$= \frac{(R+sL)^2 + (sM)^2 - (R+sL)^2}{(sM)^2} = 1 \quad \text{(checks)}$$

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P 18.21 For $I_2 = 0$:

$$\frac{V_2 - V_1}{20} + 0.025V_1 + \frac{V_2}{40} = 0$$

$$2V_2 - 2V_1 + V_1 + V_2 = 0 \quad \text{so} \quad 3V_2 = V_1$$

$$\therefore \qquad a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0} = 3$$

$$I_1 = \frac{V_1}{-j50} + \frac{V_1 - 5V_2}{100} + \frac{V_1 - V_2}{20}$$

$$= V_1\left[\frac{j}{50} + \frac{1}{100} + \frac{1}{20}\right] - V_2\left[\frac{5}{100} + \frac{1}{20}\right]$$

$$= V_1\left[\frac{6 + j2}{100}\right] - V_2\left[\frac{1}{10}\right]$$

But $V_1 = 3V_2$ so

$$I_1 = \left[\frac{18 + j6 - 10}{100}\right]V_2 = (0.08 + j0.06)V_2$$

$$a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0} = 0.08 + j0.06\,\text{S} = 80 + j60\,\text{mS}$$

For $V_2 = 0$:

$$I_1 = \frac{V_1}{-j50} + \frac{V_1}{100} + \frac{V_1}{20} = V_1\frac{(6 + j2)}{100}$$

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$$I_2 = 0.025V_1 - \frac{V_1}{20} = -0.025V_1$$

$$a_{12} = -\frac{V_1}{I_2}\bigg|_{V_2=0} = \frac{1}{0.025} = 40\,\Omega$$

$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0} = \frac{-2V_1(3+j1)}{100(-0.025)V_1} = 2.4 + j0.8$$

Summary:

$$a_{11} = 3; \quad a_{12} = 40\,\Omega; \quad a_{21} = 80 + j60\,\text{mS}; \quad a_{22} = 2.4 + j0.8$$

P 18.22 $h_{11} = \dfrac{a_{12}}{a_{22}} = \dfrac{40}{(0.8)(3+j1)} = 15 - j5\,\Omega$

$$h_{12} = \frac{\Delta a}{a_{22}}$$

$$\Delta a = 3(2.4 + j0.8) - 40(0.08 + j0.06) = 7.2 + j2.4 - 3.2 - j2.4 = 4$$

$$h_{12} = \frac{4}{(0.8)(3+j1)} = 1.5 - j0.50$$

$$h_{21} = -\frac{1}{a_{22}} = \frac{-1}{(0.8)(3+j1)} = -0.375 + j0.125$$

$$h_{22} = \frac{a_{21}}{a_{22}} = \frac{0.08 + j0.06}{(0.8)(3+j1)} = 0.0375 + j0.0125 = 37.5 + j12.5\,\text{mS}$$

P 18.23 First we note that

$$z_{11} = \frac{(Z_\text{b} + Z_\text{c})(Z_\text{a} + Z_\text{b})}{Z_\text{a} + 2Z_\text{b} + Z_\text{c}} \quad \text{and} \quad z_{22} = \frac{(Z_\text{a} + Z_\text{b})(Z_\text{b} + Z_\text{c})}{Z_\text{a} + 2Z_\text{b} + Z_\text{c}}$$

Therefore $z_{11} = z_{22}$.

$z_{12} = \dfrac{V_1}{I_2}\bigg|_{I_1=0}$; Use the circuit below:

$$V_1 = Z_bI_x - Z_cI_y = Z_bI_x - Z_c(I_2 - I_x) = (Z_b + Z_c)I_x - Z_cI_2$$

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$$I_x = \frac{Z_b + Z_c}{Z_a + 2Z_b + Z_c} I_2 \qquad \text{so} \qquad V_1 = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} I_2 - Z_c I_2$$

$$\therefore \quad Z_{12} = \frac{V_1}{I_2} = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} - Z_c = \frac{Z_b^2 - Z_a Z_c}{Z_a + 2Z_b + Z_c}$$

$z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0}$; Use the circuit below:

$$V_2 = Z_b I_x - Z_c I_y = Z_b I_x - Z_c(I_1 - I_x) = (Z_b + Z_c) I_x - Z_c I_1$$

$$I_x = \frac{Z_b + Z_c}{Z_a + 2Z_b + Z_c} I_1 \qquad \text{so} \qquad V_2 = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} I_1 - Z_c I_1$$

$$\therefore \quad z_{21} = \frac{V_2}{I_1} = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} - Z_c = \frac{Z_b^2 - Z_a Z_c}{Z_a + 2Z_b + Z_c} = z_{12}$$

Thus the network is symmetrical and reciprocal.

P 18.24 $V_1 = a_{11} V_2 - a_{12} I_2; \qquad V_1 = V_g - Z_g I_1$

$I_1 = a_{21} V_2 - a_{22} I_2; \qquad V_2 = -Z_L I_2$

$$V_1 = a_{11} V_2 - a_{12} \frac{V_2}{-Z_L} = \left(a_{11} + \frac{a_{12}}{Z_L}\right) V_2 = \frac{a_{11} Z_L + a_{12}}{Z_L} V_2$$

$$\frac{V_2}{V_1} = \frac{Z_L}{a_{12} + a_{11} Z_L}$$

P 18.25

$I_1 = y_{11} V_1 + y_{12} V_2; \qquad V_1 = V_g - I_1 Z_g$

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$$I_2 = y_{21}V_1 + y_{22}V_2; \qquad V_2 = -Z_L I_2$$

$$V_2 = -Z_L(y_{21}V_1 + y_{22}V_2)$$

$$V_2(1 + Z_L y_{22}) = -Z_L y_{21} V_1$$

$$V_2 = \frac{-Z_L y_{21}}{1 + Z_L y_{22}} V_1$$

$$I_1 = y_{11}V_1 - \frac{y_{12}y_{21}Z_L}{1 + Z_L y_{22}} V_1$$

$$\frac{I_1}{V_1} = y_{11} - \frac{y_{12}y_{21}Z_L}{1 + Z_L y_{22}}$$

P 18.26 $V_1 = h_{11}I_1 + h_{12}V_2; \qquad V_1 = V_g - I_1 Z_g$

$$I_2 = h_{21}I_1 + h_{22}V_2; \qquad V_2 = -Z_L I_2$$

$$I_2 = h_{21}I_1 + h_{22}(-Z_L I_2)$$

$$(1 + h_{22}Z_L)I_2 = h_{21}I_1$$

$$\frac{I_2}{I_1} = \frac{h_{21}}{1 + h_{22}Z_L}$$

P 18.27 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_1 = V_g - I_1 Z_g$

$$V_2 = g_{21}V_1 + g_{22}I_2; \qquad V_2 = -Z_L I_2$$

$$V_{\text{Th}} = V_2 \Big|_{I_2=0}:$$

$$V_2 = g_{21}V_1; \qquad V_1 = \frac{I_1}{g_{11}} = \frac{V_g - V_1}{Z_g g_{11}}$$

$$\therefore \qquad V_1(1 + Z_g g_{11}) = V_g \qquad \text{so} \qquad V_1 = \frac{V_g}{1 + Z_g g_{11}}$$

Thus,

$$V_2 = V_{\text{Th}} = \frac{g_{21}V_g}{1 + Z_g g_{11}}$$

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$$Z_{\text{Th}} = \frac{V_2}{I_2}\bigg|_{V_g=0}:$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$V_1 = \frac{I_1 - g_{12}I_2}{g_{11}} = \frac{-(V_1/Z_g) - g_{12}I_2}{g_{11}} = \frac{-V_1 - g_{12}Z_gI_2}{Z_gg_{11}}$$

$$(1 + Z_gg_{11})V_1 = -g_{12}Z_gI_2$$

$$\therefore \qquad V_2 = g_{21}\left(\frac{-g_{12}Z_gI_2}{1 + Z_gg_{11}}\right) + g_{22}I_2 = \left(g_{22} - \frac{g_{12}g_{21}Z_g}{1 + Z_gg_{11}}\right)I_2$$

Thus,

$$Z_{\text{Th}} = \frac{V_2}{I_2} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + Z_gg_{11}}$$

P 18.28 $V_2 = b_{11}V_1 - b_{12}I_1; \qquad V_1 = V_g - I_1Z_g$

$$I_2 = b_{21}V_1 - b_{22}I_1; \qquad V_2 = -Z_LI_2$$

$$V_g - Z_gI_1 = \frac{1}{b_{11}}V_2 + \frac{b_{12}}{b_{11}}I_1$$

$$V_g = \frac{1}{b_{11}}V_2 + \left(\frac{b_{12}}{b_{11}} + Z_g\right)I_1$$

$$I_1 = \frac{V_g - V_2/b_{11}}{Z_g + (b_{12}/b_{11})} = \frac{b_{11}V_g - V_2}{Z_gb_{11} + b_{12}}$$

$$\frac{-V_2}{Z_L} = b_{21}V_1 - b_{22}I_1 = b_{21}(V_g - Z_gI_1) - b_{22}I_1$$

$$= b_{21}V_g - (Z_gb_{21} + b_{22})I_1 = b_{21}V_g - (Z_gb_{21} + b_{22})\left[\frac{b_{11}V_g - V_2}{Z_gb_{11} + b_{12}}\right]$$

$$V_2\left(-\frac{1}{Z_L} - \frac{Z_gb_{21} + b_{22}}{Z_gb_{11} + b_{12}}\right) = \left(b_{21} - \frac{(Z_gb_{21} + b_{22})b_{11}}{Z_gb_{11} + b_{12}}\right)V_g$$

$$V_2\left(\frac{Z_gb_{11} + b_{12} + Z_gZ_Lb_{21} + b_{22}Z_L}{Z_L(Z_gb_{11} + b_{12})}\right) = \left(\frac{Z_gb_{11}b_{21} + b_{22}b_{11} - Z_gb_{11}b_{21} - b_{12}b_{21}}{Z_gb_{11} + b_{12}}\right)V_g$$

$$\frac{V_2}{V_g} = \frac{Z_L(b_{11}b_{22} - b_{12}b_{21})}{b_{12} + Z_gb_{11} + Z_Lb_{22} + Z_gZ_Lb_{21}} = \frac{Z_L\Delta b}{b_{12} + Z_gb_{11} + Z_Lb_{22} + Z_gZ_Lb_{21}}$$

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P 18.29 $I_1 = g_{11}V_1 + g_{12}I_2$

$V_2 = g_{21}V_1 + g_{22}I_2$

From the first measurement:

$$g_{11} = \frac{I_1}{V_1} = \frac{100 \times 10^{-6}}{0.1} = 1\,\text{mS}$$

$$g_{21} = \frac{V_2}{V_1} = \frac{200}{0.1} = 2000$$

From the second measurement:

$$g_{12} = \frac{I_1}{I_2} = \frac{-25 \times 10^{-6}}{5 \times 10^{-3}} = -0.005$$

$$g_{22} = \frac{V_2}{I_2} = \frac{200}{5 \times 10^{-3}} = 40\,\text{k}\Omega$$

Summary:

$g_{11} = 1\,\text{mS}; \quad g_{12} = -0.005; \quad g_{21} = 2000; \quad g_{22} = 40\,\text{k}\Omega$

From the circuit,

$Z_g = 1\,\text{k}\Omega; \qquad V_g = 4.5\,\text{mV}$

$$Z_{\text{Th}} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g} = 40{,}000 + \frac{10(1000)}{1+1} = 45{,}000$$

$$V_{\text{Th}} = \frac{g_{21}V_g}{1 + g_{11}Z_g} = \frac{2000(0.0045)}{1+1} = 4.5\,\text{V}$$

$$i = \frac{4.5}{90{,}000} = 50\,\mu\text{A}$$

$$P = (50 \times 10^{-6})^2(45{,}000) = 112.5\,\mu\text{W}$$

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P 18.30 **[a]** $Z_{\text{Th}} = g_{22} - \dfrac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$

$$g_{12}g_{21} = \left(-\frac{1}{2} + j\frac{1}{2}\right)\left(\frac{1}{2} - j\frac{1}{2}\right) = j\frac{1}{2}$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$\therefore \quad Z_{\text{Th}} = 1.5 + j2.5 - \frac{j3}{2 - j1} = 2.1 + j1.3\,\Omega$$

$$\therefore \quad Z_L = 2.1 - j1.3\,\Omega$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$g_{21}Z_L = \left(\frac{1}{2} - j\frac{1}{2}\right)(2.1 - j1.3) = 0.4 - j1.7$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$g_{22} + Z_L = 1.5 + j2.5 + 2.1 - j1.3 = 3.6 + j1.2$$

$$g_{12}g_{21}Z_g = j3$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{0.4 - j1.7}{(2 - j1)(3.6 + j1.2) - j3} = \frac{0.4 - j1.7}{8.4 - j4.2}$$

$$\mathbf{V}_2 = \frac{0.4 - j1.7}{8.4 - j4.2}(42\underline{/0^\circ}) = 5 - j6\,\text{V(rms)} = 7.81\underline{/-50.19^\circ}\,\text{V(rms)}$$

The rms value of V_2 is 7.81 V.

[b] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{Z_L} = \dfrac{-5 + j6}{2.1 - j1.3} = -3 + j1\,\text{A(rms)}$

$$P = |\mathbf{I}_2|^2(2.1) = 21\,\text{W}$$

[c] $\dfrac{\mathbf{I}_2}{\mathbf{I}_1} = \dfrac{-g_{21}}{g_{11}Z_L + \Delta g}$

$$\Delta g = \left(\frac{1}{6} - j\frac{1}{6}\right)\left(\frac{3}{2} + j\frac{5}{2}\right) - \left(\frac{1}{2} - j\frac{1}{2}\right)\left(-\frac{1}{2} + j\frac{1}{2}\right)$$

$$= \frac{3}{12} + j\frac{5}{12} - j\frac{3}{12} + \frac{5}{12} - j\frac{1}{2} = \frac{2}{3} - j\frac{1}{3}$$

$$g_{11}Z_L = \left(\frac{1}{6} - j\frac{1}{6}\right)(2.1 - j1.3) = \frac{0.8}{6} - j\frac{3.4}{6}$$

$$\therefore \quad g_{11}Z_L + \Delta g = \frac{0.8}{6} - j\frac{3.4}{6} + \frac{4}{6} - j\frac{2}{6} = 0.8 - j0.9$$

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$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-[(1/2) - j(1/2)]}{0.8 - j0.9}$$

$$\therefore \quad \mathbf{I}_1 = \frac{(0.8 - j0.9)\mathbf{I}_2}{-0.5 + j0.5} = \left(\frac{1.6 - j1.8}{-1 + j1}\right)\mathbf{I}_2$$

$$= (-1.7 + j0.1)(-3 + j1) = 5 - j2\,\text{A (rms)}$$

$$\therefore \quad P_g(\text{developed}) = (42)(5) = 210\,\text{W}$$

$$\%\ \text{delivered} = \frac{21}{210}(100) = 10\%$$

P 18.31 **[a]** $$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(70{,}000) = -35$$

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(70 \times 10^3) = -2.5$$

$$y_{21}Z_L = (100 \times 10^{-3})(70 \times 10^3) = 7000$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{7000}{-35 - (6)(-2.5)} = \frac{7000}{-20} = -350$$

$$\mathbf{V}_2 = -350\mathbf{V}_g = -350(80) \times 10^{-3} = -28\,\text{V(rms)}$$

$$V_2 = 28\,\text{V(rms)}$$

[b] $$P = \frac{|\mathbf{V}_2|^2}{70{,}000} = 11.2 \times 10^{-3} = 11.20\,\text{mW}$$

[c] $$\mathbf{I}_2 = \frac{-28\underline{/180^\circ}}{70{,}000} = -0.4 \times 10^{-3}\underline{/180^\circ} = 400\underline{/0^\circ}\,\mu\text{A}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

$$\Delta y = (2 \times 10^{-3})(-50 \times 10^{-6}) - (-2 \times 10^{-6})(100 \times 10^{-3})$$

$$= 100 \times 10^{-9}$$

$$\Delta y Z_L = (100)(70) \times 10^3 \times 10^{-9} = 7 \times 10^{-3}$$

$$y_{11} + \Delta y Z_L = 2 \times 10^{-3} + 7 \times 10^{-3} = 9 \times 10^{-3}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{9 \times 10^{-3}} = \frac{100}{9}$$

$$\therefore \quad 100\mathbf{I}_1 = 9\mathbf{I}_2; \qquad \mathbf{I}_1 = \frac{9(400 \times 10^{-6})}{100} = 36\,\mu\text{A(rms)}$$

$$P_g = (80)10^{-3}(36) \times 10^{-6} = 2.88\,\mu\text{W}$$

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P 18.32 **[a]** $Z_{\text{Th}} = \dfrac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$

From the solution to Problem 18.31

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$y_{22} + \Delta y Z_g = -50 \times 10^{-6} + 10^{-7}(2500) = 200 \times 10^{-6}$$

$$Z_{\text{Th}} = \frac{6}{200} \times 10^6 = 30{,}000\,\Omega$$

$$Z_L = Z_{\text{Th}}^* = 30{,}000\,\Omega$$

[b] $y_{21}Z_L = (100 \times 10^{-3})(30{,}000) = 3000$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(30{,}000) = -15$$

$$1 + y_{11}Z_g = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(30 \times 10^3) = -0.5$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{3000}{-15 - 6(-0.5)} = \frac{3000}{-12} = -250$$

$$\mathbf{V}_2 = -250(80 \times 10^{-3}) = -20 = 20\underline{/180^\circ}\ \text{V(rms)}$$

$$P = \frac{|\mathbf{V}_2|^2}{30{,}000} = \frac{400}{30} \times 10^{-3} = 13.33\,\text{mW}$$

[c] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{30{,}000} = \dfrac{20\underline{/0^\circ}}{30{,}000} = \dfrac{2}{3}\,\text{mA}$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{2 \times 10^{-3} + 10^{-7}(30{,}000)} = \frac{100 \times 10^{-3}}{5 \times 10^{-3}} = 20$$

$$\mathbf{I}_1 = \frac{\mathbf{I}_2}{20} = \frac{2 \times 10^{-3}}{3(20)} = \frac{1}{30}\,\text{mA(rms)}$$

$$P_g(\text{developed}) = (80 \times 10^{-3})\left(\frac{1}{30} \times 10^{-3}\right) = \frac{8}{3}\,\mu\text{W}$$

P 18.33 $\dfrac{V_2}{V_g} = \dfrac{\Delta b Z_{\text{L}}}{b_{12} + b_{11}Z_g + b_{22}Z_{\text{L}} + b_{21}Z_gZ_{\text{L}}}$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21} = (25)(-40) - (1000)(-1.25) = 250$$

$$\therefore \quad \frac{V_2}{V_g} = \frac{250(100)}{1000 + 25(20) - 40(100) - 1.25(2000)} = -5$$

$$V_2 = -5(120\underline{/0^\circ}) = 600\underline{/180^\circ}\ \text{V(rms)}$$

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$$I_2 = \frac{-V_2}{100} = \frac{-600\underline{/180^\circ}}{100} = 6\,\text{A(rms)}$$

$$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_\text{L}} = \frac{-250}{25 - 1.25(100)} = 2.5$$

$$\therefore \qquad I_1 = \frac{I_2}{2.5} = \frac{6}{2.5} = 2.4\,\text{A(rms)}$$

$$\therefore \qquad P_g = (120)(2.4) = 288\,\text{W}; \qquad P_o = 36(100) = 3600\,\text{W}$$

$$\therefore \qquad \frac{P_o}{P_g} = \frac{3600}{288} = 12.5$$

P 18.34 **[a]** For $I_2 = 0$:

$$V_2 = -j30I_1 = -j30\frac{V_1}{10 + j10} = \frac{-j3V_1}{1 + j1}$$

$$a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0} = \frac{1 + j1}{-j3} = \frac{-1 + j1}{3}$$

$$a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0} = \frac{1}{-j30} = \frac{j}{30}\,\text{S}$$

For $V_2 = 0$:

$$V_1 = (10 + j10)I_1 - j30I_2$$

$$0 = -j30I_1 + (80 + j160)I_2$$

$$\Delta = \begin{vmatrix} 10 + j10 & -j30 \\ -j30 & 80 + j160 \end{vmatrix} = 100(1 + j24)$$

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$$N_2 = \begin{vmatrix} 10 + j10 & V_1 \\ -j30 & 0 \end{vmatrix} = j30V_1$$

$$I_2 = \frac{N_2}{\Delta} = \frac{j30V_1}{100(1 + j24)}$$

$$a_{12} = \frac{-V_1}{I_2}\bigg|_{V_2=0} = -80 + j\frac{10}{3}\,\Omega$$

$$j30I_1 = (80 + j160)I_2$$

$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0} = -\frac{8}{3}(2 - j1)$$

[b] $$V_{\text{Th}} = \frac{V_g}{a_{11} + a_{21}Z_g} = \frac{80\underline{/0^\circ}}{(-1 + j1)/3 + j5/30} = \frac{(80\underline{/0^\circ})30}{-10 + j10 + j5} = \frac{2400\underline{/0^\circ}}{-10 + j15}$$

$$= 133.13\underline{/-123.69^\circ}\,\text{V}$$

$$Z_{\text{Th}} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{[-(10/3)(24 - j1)] + [(-8/3)(2 - j1)(5)]}{[(-1 + j1)/3] + [(j/30)(5)]}$$

$$= 121.54 + j132.31\,\Omega$$

[c] $$V_2 = \frac{200}{121.54 + j132.31}(133.13\underline{/-123.69^\circ}) = 132.87\underline{/-124.16^\circ}$$

$$v_2(t) = 132.87\cos(400t - 124.16^\circ)\,\text{V}$$

P 18.35 When $V_2 = 0$

$$V_1 = 20\,\text{V}, \qquad I_1 = 1\,\text{A}, \qquad I_2 = -1\,\text{A}$$

When $I_1 = 0$

$$V_2 = 80\,\text{V}, \qquad V_1 = 400\,\text{V}, \qquad I_2 = 3\,\text{A}$$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = \frac{20}{1} = 20\,\Omega$$

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$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = \frac{400}{80} = 5$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = \frac{-1}{1} = -1$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{3}{80} = 37.5\,\text{mS}$$

$$Z_{\text{Th}} = \frac{Z_g + h_{11}}{h_{22}Z_g + \Delta h} = 10\,\Omega$$

Source-transform the current source and parallel resistance to get $V_g = 240$ V. Then,

$$I_2 = \frac{h_{21}V_g}{(1 + h_{22}Z_{\text{L}})(h_{11} + Z_g) - h_{12}h_{21}Z_{\text{L}}} = -1.5\,\text{A}$$

$$P = (-1.5)^2(10) = 22.5\,\text{W}$$

P 18.36 **[a]** $y_{11} = \dfrac{I_1}{V_1}\bigg|_{V_2=0}$; $\qquad y_{21} = \dfrac{I_2}{V_1}\bigg|_{V_2=0}$

$$V_1 = \left[s + \left(\frac{1}{s}\|s\right)\right] I_1 = \frac{s(s^2+1)+s}{s^2+1} I_1$$

$$\therefore \qquad y_{11} = \frac{I_1}{V_1} = \frac{s^2+1}{s(s^2+2)}$$

$$I_2 = \frac{-(1/s)}{s + (1/s)} I_1 = \frac{-1}{s^2+1} \cdot \frac{s^2+1}{s(s^2+2)} V_1 = \frac{-1}{s(s^2+2)} V_1$$

$$\therefore \qquad y_{21} = \frac{-1}{s(s^2+2)}$$

Because the two-port circuit is symmetric,

$$y_{12} = y_{21} = \frac{-1}{s(s^2+2)} \qquad \text{and} \qquad y_{22} = y_{11} = \frac{s^2+1}{s(s^2+2)}$$

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[b] $$\frac{V_2}{V_g} = \frac{y_{21}Z_g}{y_{12}y_{21}Z_gZ_L - (1+y_{11}Z_g)(1+y_{22}Z_L)}$$

$$= \frac{y_{21}}{y_{12}y_{21} - (1+y_{11})(1+y_{22})}$$

$$= \frac{\dfrac{-1}{s(s^2+2)}}{\dfrac{1}{s^2(s^2+2)^2} - \left(1+\dfrac{s^2+1}{s(s^2+2)}\right)\left(1+\dfrac{s^2+1}{s(s^2+2)}\right)}$$

$$= \frac{-s(s^2+2)}{1-(s^3+s^2+2s+1)^2}$$

$$= \frac{1}{s^3+2s^2+3s+2}$$

$$= \frac{1}{(s+1)(s^2+s+2)}$$

$$\therefore \quad V_2 = \frac{10}{(s+1)(s+2)(s^2+s+2)}$$

$$s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

$$V_2 = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{K_3^*}{s+\frac{1}{2}+j\frac{\sqrt{7}}{2}}$$

$$K_1 = 5; \qquad K_2 = -2.5; \qquad K_3 = 1.34\underline{/159.3^\circ}$$

$$\therefore \quad v_2(t) = [5e^{-t} - 2.5e^{-2t} + 2.67e^{-0.5t}\cos(1.32t + 159.3^\circ)]u(t)\,\text{V}$$

P 18.37 **[a]** $g_{11} = \dfrac{I_1}{V_1}\Big|_{I_2=0}$; $\qquad g_{21} = \dfrac{V_2}{V_1}\Big|_{I_2=0}$

$$\frac{V_1}{I_1} = \frac{1}{sC}\Big\|\left(sL + \frac{1}{sC}\right) = \frac{[sL+(1/sC)](1/sC)}{sL+(2/sC)}$$

$$= \frac{sL+(1/sC)}{s^2LC+2} = \frac{s^2LC+1}{sC(s^2LC+2)} = \frac{(1/C)[s^2+(1/LC)]}{s[s^2+(2/LC)]}$$

$$\therefore \qquad g_{11} = \frac{Cs[s^2+(2/LC)])}{s^2+(1/LC)}$$

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$$V_2 = \frac{(1/sC)}{sL + (1/sC)} V_1 \qquad \text{so} \qquad \frac{V_2}{V_1} = \frac{(1/sC)}{sL + (1/sC)} = \frac{1}{s^2LC + 1} = \frac{(1/LC)}{s^2 + (1/LC)}$$

$$\therefore \qquad g_{21} = \frac{(1/LC)}{s^2 + (1/LC)}$$

$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0}; \qquad g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0}$$

$$I_1 = \frac{-(1/sC)}{sL + (1/sC)} I_2 \qquad \text{so} \qquad g_{12} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$g_{22} = sL \| (1/sC) = \frac{sL/sC}{sL + (1/sC)} = \frac{sL}{s^2LC + 1} = \frac{(1/C)s}{s^2 + (1/LC)}$$

Summary:

$$g_{11} = \frac{Cs[s^2 + (2/LC)])}{s^2 + (1/LC)}; \qquad g_{12} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$g_{21} = \frac{(1/LC)}{s^2 + (1/LC)}; \qquad g_{22} = \frac{(1/C)s}{s^2 + (1/LC)}$$

[b] $$\frac{1}{LC} = \frac{10^9}{(0.05)(32)} = 625 \times 10^6$$

$$g_{11} = \frac{32 \times 10^{-9} s(s^2 + 1250 \times 10^6)}{s^2 + 625 \times 10^6}$$

$$g_{12} = \frac{-625 \times 10^6}{s^2 + 625 \times 10^6}$$

$$g_{21} = \frac{625 \times 10^6}{s^2 + 625 \times 10^6}$$

$$g_{22} = \frac{3125 \times 10^4 s}{s^2 + 625 \times 10^6}$$

$$\frac{V_2}{V_1} = \frac{g_{21} Z_L}{g_{22} + Z_L} = \frac{\left(\frac{625 \times 10^6}{s^2 + 625 \times 10^6}\right) 500}{\frac{3125 \times 10^4 s}{(s^2 + 625 \times 10^6)} + 500}$$

$$\frac{V_2}{V_1} = \frac{625 \times 10^6}{s^2 + 62{,}500s + 625 \times 10^6} = \frac{625 \times 10^6}{(s + 12{,}500)(s + 50{,}000)}$$

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$$V_1 = \frac{4}{s}$$

$$V_2 = \frac{2500 \times 10^6}{s(s+12{,}500)(s+50{,}000)} = \frac{4}{s} - \frac{5.33}{s+12{,}500} + \frac{1.33}{s+50{,}000}$$

$$v_2 = [4 - 5.33e^{-12{,}500t} + 1.33e^{-50{,}000t}]u(t) \quad \text{V}$$

P 18.38 $a'_{11} = \dfrac{z_{11}}{z_{21}} = \dfrac{35/3}{4000/3} = 8.75 \times 10^{-3}\,\Omega$

$$a'_{12} = \frac{\Delta z}{z_{21}} = \frac{25 \times 10^4/3}{4000/3} = 62.5\,\Omega$$

$$a'_{21} = \frac{1}{z_{21}} = \frac{1}{4000/3} = 0.75 \times 10^{-3}\,\Omega$$

$$a'_{22} = \frac{z_{22}}{z_{21}} = \frac{10{,}000/3}{4000/3} = 2.5\,\Omega$$

$$a''_{11} = \frac{-y_{22}}{y_{21}} = \frac{-40 \times 10^{-6}}{-800 \times 10^{-6}} = 0.05\,\text{S}$$

$$a''_{12} = \frac{-1}{y_{21}} = \frac{-1}{-800 \times 10^{-6}} = 1250\,\text{S}$$

$$a''_{21} = \frac{-\Delta y}{y_{21}} = \frac{-4 \times 10^{-8}}{-800 \times 10^{-6}} = 50 \times 10^{-6}\,\text{S}$$

$$a''_{22} = \frac{-y_{11}}{y_{21}} = \frac{-200 \times 10^{-6}}{-800 \times 10^{-6}} = 0.25\,\text{S}$$

$$a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = (8.75 \times 10^{-3})(0.05) + (62.5)(50 \times 10^{-6}) = 3.5625 \times 10^{-3}$$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (8.75 \times 10^{-3})(1250) + (62.5)(0.25) = 26.5625$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (0.75 \times 10^{-3})(0.05) + (2.5)(50 \times 10^{-6}) = 162.5 \times 10^{-6}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (0.75 \times 10^{-3})(1250) + (2.5)(0.25) = 1.5625$$

$$V_2 = \frac{Z_L V_g}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$= \frac{(15{,}000)(0.03)}{[3.5625 \times 10^{-3} + (162.5 \times 10^{-6})(10)](15{,}000) + 26.5625 + (1.5625)(10)} = 3.75\,\text{V}$$

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P 18.39 The a parameters of the first two port are

$$a'_{11} = \frac{-\Delta h}{h_{21}} = \frac{-5 \times 10^{-3}}{40} = -125 \times 10^{-6}$$

$$a'_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{40} = -25\,\Omega$$

$$a'_{21} = \frac{-h_{22}}{h_{21}} = \frac{-25}{40} \times 10^{-6} = -625 \times 10^{-9}\,\text{S}$$

$$a'_{22} = \frac{-1}{h_{21}} = \frac{-1}{40} = -25 \times 10^{-3}$$

The a parameters of the second two port are

$$a''_{11} = \frac{5}{4}; \quad a''_{12} = \frac{3R}{4}; \quad a''_{21} = \frac{3}{4R}; \quad a''_{22} = \frac{5}{4}$$

or $a''_{11} = 1.25; \quad a''_{12} = 54\,\text{k}\Omega; \quad a''_{21} = \dfrac{1}{96}\,\text{mS}; \quad a''_{22} = 1.25$

The a parameters of the cascade connection are

$$a_{11} = -125 \times 10^{-6}(1.25) + (-25)(10^{-3}/96) = \frac{-10^{-2}}{24}$$

$$a_{12} = -125 \times 10^{-6}(54 \times 10^{3}) + (-25)(1.25) = -38\,\Omega$$

$$a_{21} = -625 \times 10^{-9}(1.25) + (-25 \times 10^{-3})(10^{-3}/96) = \frac{-10^{-4}}{96}\,\text{S}$$

$$a_{22} = -625 \times 10^{-9}(54 \times 10^{3}) + (-25 \times 10^{-3})(1.25) = -65 \times 10^{-3}$$

$$\frac{V_o}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$a_{21}Z_g = \frac{-10^{-4}}{96}(800) = \frac{-10^{-2}}{12}$$

$$a_{11} + a_{21}Z_g = \frac{-10^{-2}}{24} + \frac{-10^{-2}}{12} = \frac{-10^{-2}}{8}$$

$$(a_{11} + a_{21}Z_g)Z_L = \frac{-10^{-2}}{8}(72{,}000) = -90$$

$$a_{22}Z_g = -65 \times 10^{-3}(800) = -52$$

$$\frac{V_o}{V_g} = \frac{72{,}000}{-90 - 38 - 52} = -400$$

$$v_o = V_o = -400V_g = -3.6\,\text{V}$$

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P 18.40 **[a]** From reciprocity and symmetry

$$a'_{11} = a'_{22}, \quad \Delta a' = 1; \quad \therefore \quad 5^2 - 24a'_{21} = 1, \quad a'_{21} = 1\,\text{S}$$

For network B

$$a''_{11} = \frac{\mathbf{V}_1}{\mathbf{V}_2}\bigg|_{I_2=0}$$

$$\mathbf{V}_1 = (5 + j15 - j10)\mathbf{I}_1 = (5 + j5)\mathbf{I}_1$$

$$\mathbf{V}_2 = (-j10 + j5)\mathbf{I}_1 = -j5\mathbf{I}_1$$

$$a''_{11} = \frac{5 + j5}{-j5} = -1 + j1$$

$$a''_{21} = \frac{\mathbf{I}_1}{\mathbf{V}_2}\bigg|_{I_2=0} = \frac{1}{-j5} = j0.2\,\text{S}$$

$$a''_{22} = a''_{11} = -1 + j1$$

$$\Delta a'' = 1 = (-1 + j1)(-1 + j1) - j0.2a''_{12}$$

$$\therefore \quad a''_{12} = -10 + j5$$

Summary:

$a'_{11} = 5$ $\qquad a''_{11} = -1 + j1$

$a'_{12} = 24\,\Omega$ $\qquad a''_{12} = -10 + j5\,\Omega$

$a'_{21} = 1\,\text{S}$ $\qquad a''_{21} = j0.2\,\text{S}$

$a'_{22} = 5$ $\qquad a''_{22} = -1 + j1$

[b] $a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = -5 + j9.8$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = -74 + j49\,\Omega$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = -1 + j2\,\text{S}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = -15 + j10$$

$$\mathbf{I}_2 = \frac{-V_g}{a_{11}Z_\text{L} + a_{12} + a_{21}Z_gZ_\text{L}} = 0.295 + j0.279\,\text{A}$$

$$\mathbf{V}_2 = -10I_2 = -2.95 - j2.79\,\text{V}$$

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P 18.41 **[a]** At the input port: $V_1 = h_{11}I_1 + h_{12}V_2$;
At the output port: $I_2 = h_{21}I_1 + h_{22}V_2$

[b] $\dfrac{V_2}{10^4} + (100 \times 10^{-6}V_2) + 100I_1 = 0$

therefore $\quad I_1 = -2 \times 10^{-6}V_2$

$V_2' = 1000I_1 + 15 \times 10^{-4}V_2 = -5 \times 10^{-4}V_2$

$100I_1' + 10^{-4}V_2' + (-2 \times 10^{-6})V_2 = 0$

therefore $\quad I_1' = 205 \times 10^{-10}V_2$

$V_g = 1500I_1' + 15 \times 10^{-4}V_2' = 3000 \times 10^{-8}V_2$

$$\frac{V_2}{V_g} = \frac{10^5}{3} = 33{,}333$$

P 18.42 **[a]** $V_1 = I_2(z_{12} - z_{21}) + I_1(z_{11} - z_{21}) + z_{21}(I_1 + I_2)$

$= I_2z_{12} - I_2z_{21} + I_1z_{11} - I_1z_{21} + z_{21}I_1 + z_{21}I_2 = z_{11}I_1 + z_{12}I_2$

$V_2 = I_2(z_{22} - z_{21}) + z_{21}(I_1 + I_2) = z_{21}I_1 + z_{22}I_2$

[b] Short circuit V_g and apply a test current source to port 2 as shown. Note that $I_T = I_2$. We have

$$\frac{V}{z_{21}} - I_T + \frac{V + I_T(z_{12} - z_{21})}{Z_g + z_{11} - z_{21}} = 0$$

Therefore

$$V = \left[\frac{z_{21}(Z_g + z_{11} - z_{12})}{Z_g + z_{11}}\right] I_T \quad \text{and} \quad V_T = V + I_T(z_{22} - z_{21})$$

Thus $\quad \dfrac{V_T}{I_T} = Z_{\text{Th}} = z_{22} - \left(\dfrac{z_{12}z_{21}}{Z_g + z_{11}}\right)$

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For V_{Th} note that $V_{\text{oc}} = \dfrac{z_{21}}{Z_g + z_{11}} V_g$ since $I_2 = 0$.

P 18.43 **[a]** $V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2) = z_{11}I_1 + z_{12}I_2$

$V_2 = (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_2 + I_1) = z_{21}I_1 + z_{22}I_2$

[b] With port 2 terminated in an impedance Z_L, the two mesh equations are

$V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2)$

$0 = Z_L I_2 + (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_1 + I_2)$

Solving for I_1:

$$I_1 = \frac{V_1(z_{22} + Z_L)}{z_{11}(Z_L + z_{22}) - z_{12}z_{21}}$$

Therefore

$$Z_{\text{in}} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

P 18.44 **[a]** $I_1 = y_{11}V_1 + y_{21}V_2 + (y_{12} - y_{21})V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$

$I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{12}V_1 + y_{22}V_2 + (y_{21} - y_{12})V_1$

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[b] Using the second circuit derived in part [a], we have

where $y_a = (y_{11} + y_{12})$ and $y_b = (y_{22} + y_{12})$

At the input port we have

$$I_1 = y_a V_1 - y_{12}(V_1 - V_2) = y_{11}V_1 + y_{12}V_2$$

At the output port we have

$$\frac{V_2}{Z_L} + (y_{21} - y_{12})V_1 + y_b V_2 - y_{12}(V_2 - V_1) = 0$$

Solving for V_1 gives

$$V_1 = \left(\frac{1 + y_{22}Z_L}{-y_{21}Z_L}\right) V_2$$

Substituting Eq. (18.2) into (18.1) and at the same time using $V_2 = -Z_L I_2$, we get

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

P 18.45 **[a]** The g-parameter equations are $I_1 = g_{11}V_1 + g_{12}I_2$ and $V_2 = g_{21}V_1 + g_{22}I_2$. These equations are satisfied by the following circuit:

[b] The g parameters for the first two port in Fig P 18.39(a) are

$$g_{11} = \frac{h_{22}}{\Delta h} = \frac{25 \times 10^{-6}}{5 \times 10^{-3}} = 5 \times 10^{-3}\ \text{S}$$

$$g_{12} = \frac{-h_{12}}{\Delta h} = \frac{-5 \times 10^{-4}}{5 \times 10^{-3}} = -0.10$$

$$g_{21} = \frac{-h_{21}}{\Delta h} = \frac{-40}{5 \times 10^{-3}} = -8000$$

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$$g_{22} = \frac{h_{11}}{\Delta h} = \frac{1000}{5 \times 10^{-3}} = 200\,\text{k}\Omega$$

From Problem 3.67 $R_{ef} = 72\,\text{k}\Omega$, hence our circuit reduces to

$$V_2 = \frac{-8000V_1(72)}{272}$$

$$I_2 = \frac{-V_2}{72{,}000} = \frac{8V_1}{272}$$

$$v_g = 9\,\text{mV}$$

$$\therefore \quad \frac{V_1 - 9 \times 10^{-3}}{800} + V_1(5 \times 10^{-3}) - 0.1\frac{8V_1}{272} = 0$$

$$V_1 - 9 \times 10^{-3} + 4V_1 - \frac{80V_1}{34} = 0$$

$$\therefore \quad V_1 = 3.4 \times 10^{-3}$$

$$V_2 = \frac{-8000(72)}{272} \times 3.4 \times 10^{-3} = -7.2\,\text{V}$$

From Problem 3.67

$$\frac{V_o}{V_2} = 0.5; \qquad \therefore \quad V_o = -3.6\,\text{V}$$

This result matches the solution to Problem 18.39.

P 18.46 **[a]** To determine b_{11} and b_{21} create an open circuit at port 1. Apply a voltage at port 2 and measure the voltage at port 1 and the current at port 2. To determine b_{12} and b_{22} create a short circuit at port 1. Apply a voltage at port 2 and measure the currents at ports 1 and 2.

[b] The equivalent b-parameters for the black-box amplifier can be calculated as follows:

$$b_{11} = \frac{1}{h_{12}} = \frac{1}{10^{-3}} = 1000$$

$$b_{12} = \frac{h_{11}}{h_{12}} = \frac{500}{10^{-3}} = 500\,\text{k}\Omega$$

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$$b_{21} = \frac{h_{22}}{h_{12}} = \frac{0.05}{10^{-3}} = 50\,\text{S}$$

$$b_{22} = \frac{\Delta h}{h_{12}} = \frac{23.5}{10^{-3}} = 23{,}500$$

Create an open circuit a port 1. Apply 1 V at port 2. Then,

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} = \frac{1}{V_1} = 1000 \quad \text{so} \quad V_1 = 1\,\text{mV measured}$$

$$b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} = \frac{I_2}{10^{-3}} = 50\,\text{S} \quad \text{so} \quad I_2 = 50\,\text{mA measured}$$

Create a short circuit a port 1. Apply 1 V at port 2. Then,

$$b_{12} = -\frac{V_2}{I_1}\bigg|_{V_1=0} = \frac{-1}{I_1} = 500\,\text{k}\Omega \quad \text{so} \quad I_1 = -2\,\mu\text{A measured}$$

$$b_{22} = -\frac{I_2}{I_1}\bigg|_{V_1=0} = \frac{-I_2}{-2\times 10^{-6}} = 23{,}500 \quad \text{so} \quad I_2 = 47\,\text{mA measured}$$

P 18.47 **[a]** To determine y_{11} and y_{21} create a short circuit at port 2. Apply a voltage at port 1 and measure the currents at ports 1 and 2. To determine y_{12} and y_{22} create a short circuit at port 1. Apply a voltage at port 2 and measure the currents at ports 1 and 2.

[b] The equivalent y-parameters for the black-box amplifier can be calculated as follows:

$$y_{11} = \frac{1}{h_{11}} = \frac{1}{500} = 2\ \text{mS}$$

$$y_{12} = \frac{-h_{12}}{h_{11}} = \frac{-10^{-3}}{500} = -2\,\mu\text{S}$$

$$y_{21} = \frac{h_{21}}{h_{11}} = \frac{1500}{500} = 3\ \text{S}$$

$$y_{22} = \frac{\Delta h}{h_{11}} = \frac{23.5}{500} = 47\ \text{mS}$$

Create a short circuit at port 2. Apply 1 V at port 1. Then,

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{I_1}{1} = 2\ \text{mS} \quad \text{so} \quad I_1 = 2\,\text{mA measured}$$

$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = \frac{I_2}{1} = 3\ \text{S} \quad \text{so} \quad I_2 = 3\,\text{A measured}$$

Create a short circuit at port 1. Apply 1 V at port 2. Then,

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = \frac{I_1}{1} = -2\,\mu\text{S} \quad \text{so} \quad I_1 = -2\,\mu\text{A measured}$$

$$y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} = \frac{I_2}{1} = 47\ \text{mS} \quad \text{so} \quad I_2 = 47\,\text{mA measured}$$

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