Linear system

Consider a linear system of m equations in n unknowns

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

Here m = Number of linear equations

n = Number of unknowns (variables)

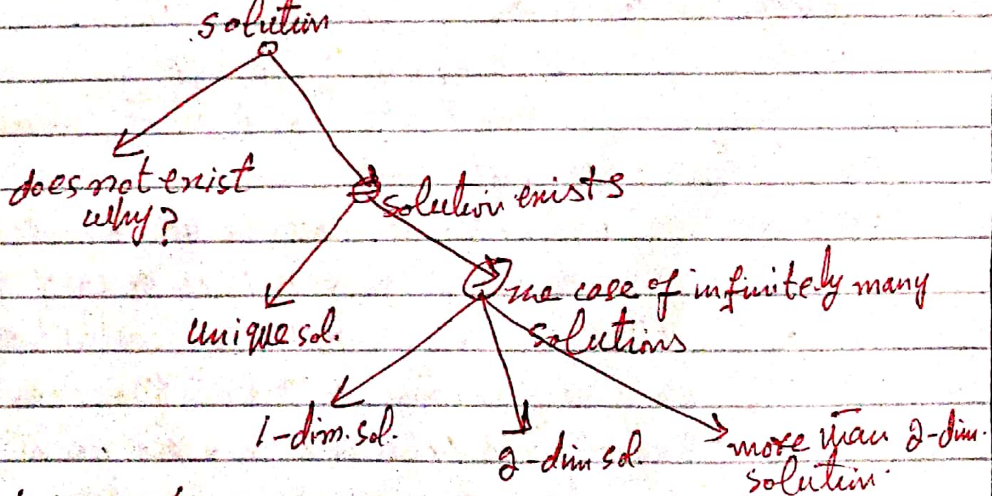
Three possibilities exist here

i) $m = n$

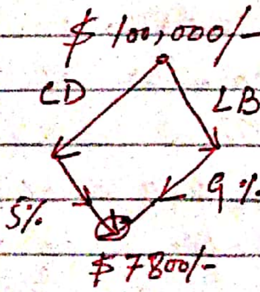
ii) $m < n$

iii) $m > n$

Some times, we need to construct a linear system from a given Problem and some times a linear system is given whose solution, we need to find/discuss.



P-2
EX1



Here $100,000/- is to be invested in two ways. one is by certificate deposit (CD) and the other is by Long Term Bond (LB). 5% of the amount in CD and 9% of the amount in LB yield after one year $7800/ as interest. Question is how much invested in CD and how much invested in LB.

③

Let x and y be the amount to be invested in CD and LB respectively, then

$$x + y = 100{,}000$$
$$5\% \text{ of } x + 9\% \text{ of } y = 7800$$

$$\Rightarrow \quad x + y = 100{,}000 \quad -① \qquad m = n$$
$$2 = 2$$
$$0.05x + 0.09y = 7800 \quad -②$$

$0.05 \times$ eqn.(1) $-$ eq.(2) $\Rightarrow$ $-0.04y = -2800$ $\Rightarrow y = 70{,}000/-$

① $\Rightarrow x = 30{,}000/-$. Thus $x = 30{,}000/-$, $y = 70{,}000/-$ is the desired solution which is unique.

P-3
Ex2: Consider the linear system

$$x - 3y = -7 \quad -① \qquad m = n$$
$$2x - 6y = 7 \quad -② \qquad 2 = 2$$

$2 \times$ eq.(1) $-$ eq.(2) $\Rightarrow$

$$\begin{array}{r} \cancel{2x} - \cancel{6y} = -14 \\ \pm \cancel{2x} \mp \cancel{6y} = \pm 7 \\ \hline 0 = -21 \end{array}$$

invalid case $\Rightarrow$

solution does not exis.

Reasons: i) As 0 can never be equal to -21

ii) The LHS of eq.(2) is twice the LHS of eq.(1) but the RHS of eq.(2) is not twice the RHS of eq.(1)

iii) Both the equations represent two parallel lines lying at some distance from one another i.e. they never intersect each other having no common points, means no solution exists as by solution, we mean the set of common points in the graphs of all equations.

$$y = m_1 x + c_1$$
$$y = m_2 x + c_2$$

(a) If $m_1 = m_2$ but $c_1 \neq c_2$, then lines are // but at some distance, hence the case of no solution.

b) If $m_1 = m_2$ and $c_1 = c_2$, then lines are // but lie on each other having many common points, so solution exists which is the case of infinitely many solution.

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p-3

Ex 3 Consider the linear system

$$x+2y+3z=6 \quad - (1)$$
$$2x-3y+2z=14 \quad - (2)$$
$$3x+y-z=-2 \quad - (3)$$

$m=n$
$3=3$

$2\times eq.(1)-eq.(2) \Rightarrow 7y+4z=-2$

$3\times eq.(1)-eq.(3) \Rightarrow 5y+10z=20 \Rightarrow y+z=4$

The reduced system is

$$x+2y+3z=6 \quad - (1)$$
$$y+2z=4 \quad - (4)$$
$$7y+4z=-2 \quad - (5)$$

$7\times eq.(4)-eq.(5) \Rightarrow 10z=30 \Rightarrow z=3$

Further reduced system is

$$x+2y+3z=6 \quad - (1)$$
$$y+2z=4 \quad - (4)$$
$$z=3$$

By back substitution, (4) $\Rightarrow y=-2$ and (1) $\Rightarrow x=1$

Thus $x=1,\ y=-2,\ z=3$, is the unique solution.

p-4

Ex 4) Consider the linear system

$$x+2y-3z=-4 \quad - (1)$$
$$2x+y-3z=4 \quad - (2)$$

$m<n$
$2<3$

$2\times eq.(1)-eq.(2) \Rightarrow 3y-3z=-12 \Rightarrow y-z=-4$

The reduced system is

$$x+2y-3z=-4 \quad - (1)$$
$$y-z=-4 \quad - (3)$$

(3) $\Rightarrow y=z-4 \Rightarrow y=r-4$, where $z=r\in\mathbb{R}$.

(1) $\Rightarrow$ ~~x+2r+8~~ $x+2(r-4)-3r=-4 \Rightarrow x=r+4$.

Thus $x=r+4,\ y=r-4,\ z=r,\ r\in\mathbb{R}$ is the desired solution which is the case of infinitely many solutions.

As the entire solution depends on only one parameter which is r here, so this is called 1-dimensional solution.

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P-4

Ex5/ Consider the linear system

$$x + 2y = 10 \quad \text{—(1)}$$
$$2x - 2y = -4 \quad \text{—(2)}$$
$$3x + 5y = 26 \quad \text{—(3)}$$

$m > n$

$3 > 2$

$2 \times eq.(1) - eq.(2) \Rightarrow 6y = 24 \Rightarrow y = 4$

$3 \times eq.(1) - eq.(3) \Rightarrow y = 4$

The reduced system is

$$x + 2y = 10 \quad \text{—(1)}$$
$$y = 4$$
$$y = 4$$

As y has both the same value, hence solution exists i.e.

(1) $\Rightarrow x + 2(4) = 10 \Rightarrow x = 2,\ y = 4$ is the unique sol.

P-5

Ex6) Consider the linear system

$$x + 2y = 10 \quad \text{—(1)}$$
$$2x - 2y = -4 \quad \text{—(2)}$$
$$3x + 5y = 20 \quad \text{—(3)}$$

$m > n$

$3 > 2$

$2 \times eq.(1) - eq.(2) \Rightarrow 6y = 24 \Rightarrow y = 4$

$3 \times eq.(1) - eq.(3) \Rightarrow y = 10$

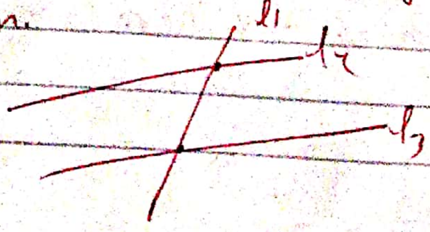
The reduced system is

$$x + 2y = 10 \quad \text{—(1)}$$
$$y = 4 \quad \text{—(3)}$$
$$y = 10 \quad \text{—(4)}$$

As y has two different values, hence solution does not exist.

Reason: From eq.(1) and eq.(2), we have $x = 2, y = 4$, which satisfy eq.(1) and eq.(2) but do not satisfy eq.(3), hence not considered to be the solution of the system.

From eq.(1) and eq.(3), we have $x = -10, y = 10$, which do not satisfy eq.(2), though eq.(1) and eq.(3) are satisfied, hence not considered to be the solution.



Exercise 1.1

Q_1 — Q_{14}: Similar to Ex2 — Ex6

Q_{15}, Given the linear system

$2x - y = 5$ — ①

$4x - 2y = t$ — ②

(a) determine a value of t so that the system has a solution

(b) determine a value of t so that the system has no solution

(c) how many different values of t can be selected in Part (b)?

Ⓢ $2 \times$ eq. (1) $-$ eq. (2) $\Rightarrow$

$$\begin{array}{r} 4x - 2y = 10 \\ \underline{\pm 4x \mp 2y = \pm t} \\ 0 = 10 - t \end{array} \quad \text{— ③}$$

(a) For $t = 10$, the given linear system has a solution

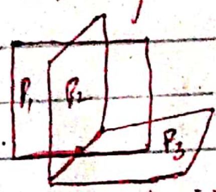
As for $t = 10$, (3) $=$ $0 = 0$ satisfied form.

(b) For $t = 5$, the given linear system has no solution

As for $t = 5$, (3) $\Rightarrow$ $0 = 10 - 5 \Rightarrow 0 = 5$ invalid case due to which the system is inconsistent i.e having no solution.

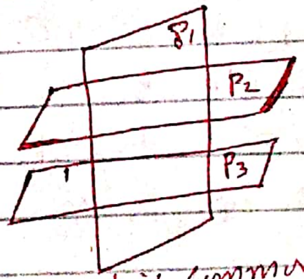
(c) All real numbers except $t = 10$, can be selected in Part (b).

In case of Planes, the following three possibilities exist



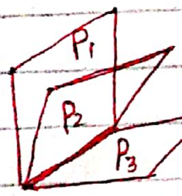
only one Point is common

unique solution



No Point is common

No solution exists



many Points are common

infinitely many solutions exist.

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(6-1)

since we must have $x_1 \geq 0, x_2 \geq 0$, and $x_3 \geq 0$. When $x_3 = 10$, we have

$$x_1 = 5, \quad x_2 = 10, \quad x_3 = 10$$

while

$$x_1 = \tfrac{13}{2}, \quad x_2 = 13, \quad x_3 = 7$$

when $x_3 = 7$. The reader should observe that one solution is just as good as the other. There is no best solution unless additional information or restrictions are given.

Key Terms

Linear equation
Unknowns
Solution to a linear equation
Linear system
Solution to a linear system
Method of elimination
Unique solution
No solution
Infinitely many solutions
Manipulations on a linear system

1.1 Exercises

In Exercises 1 through 14, solve the given linear system by the method of elimination.

1. $\begin{aligned} x + 2y &= 8 \\ 3x - 4y &= 4 \end{aligned}$

2. $\begin{aligned} 2x - 3y + 4z &= -12 \\ x - 2y + z &= -5 \\ 3x + y + 2z &= 1 \end{aligned}$

3. $\begin{aligned} 3x + 2y + z &= 2 \\ 4x + 2y + 2z &= 8 \\ x - y + z &= 4 \end{aligned}$

4. $\begin{aligned} x + y &= 5 \\ 3x + 3y &= 10 \end{aligned}$

5. $\begin{aligned} 2x + 4y + 6z &= -12 \\ 2x - 3y - 4z &= 15 \\ 3x + 4y + 5z &= -8 \end{aligned}$

6. $\begin{aligned} x + y - 2z &= 5 \\ 2x + 3y + 4z &= 2 \end{aligned}$

7. $\begin{aligned} x + 4y - z &= 12 \\ 3x + 8y - 2z &= 4 \end{aligned}$

8. $\begin{aligned} 3x + 4y - z &= 8 \\ 6x + 8y - 2z &= 3 \end{aligned}$

9. $\begin{aligned} x + y + 3z &= 12 \\ 2x + 2y + 6z &= 6 \end{aligned}$

10. $\begin{aligned} x + y &= 1 \\ 2x - y &= 5 \\ 3x + 4y &= 2 \end{aligned}$

11. $\begin{aligned} 2x + 3y &= 13 \\ x - 2y &= 3 \\ 5x + 2y &= 27 \end{aligned}$

12. $\begin{aligned} x - 5y &= 6 \\ 3x + 2y &= 1 \\ 5x + 2y &= 1 \end{aligned}$

13. $\begin{aligned} x + 3y &= -4 \\ 2x + 5y &= -8 \\ x + 3y &= -5 \end{aligned}$

14. $\begin{aligned} 2x + 3y - z &= 6 \\ 2x - y + 2z &= -8 \\ 3x - y + z &= -7 \end{aligned}$

15. Given the linear system

$$\begin{aligned} 2x - y &= 5 \\ 4x - 2y &= t, \end{aligned}$$

(a) determine a value of t so that the system has a solution.

(b) determine a value of t so that the system has no solution.

(c) how many different values of t can be selected in part (b)?

16. Given the linear system

$$\begin{aligned} 2x + 3y - z &= 0 \\ x - 4y + 5z &= 0, \end{aligned}$$

(a) verify that $x_1 = 1, y_1 = -1, z_1 = -1$ is a solution.

(b) verify that $x_2 = -2, y_2 = 2, z_2 = 2$ is a solution.

(c) is $x = x_1 + x_2 = -1, y = y_1 + y_2 = 1$, and $z = z_1 + z_2 = 1$ a solution to the linear system?

(d) is $3x, 3y, 3z$, where x, y, and z are as in part (c), a solution to the linear system?

17. Without using the method of elimination, solve the linear system

$$\begin{aligned} 2x + y - 2z &= -5 \\ 3y + z &= 7 \\ z &= 4. \end{aligned}$$

18. Without using the method of elimination, solve the linear system

$$\begin{aligned} 4x \qquad\qquad &= 8 \\ -2x + 3y \qquad &= -1 \\ 3x + 5y - 2z &= 11. \end{aligned}$$

19. Is there a value of r so that $x = 1, y = 2, z = r$ is a solution to the following linear system? If there is, find it.

$$\begin{aligned} 2x + 3y - z &= 11 \\ x - y + 2z &= -7 \\ 4x + y - 2z &= 12 \end{aligned}$$

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6-2

20. Is there a value of r so that $x = r$, $y = 2$, $z = 1$ is a solution to the following linear system? If there is, find it.

$$\begin{aligned} 3x \quad\quad - 2z &= 4 \\ x - 4y + z &= -5 \\ -2x + 3y + 2z &= 9 \end{aligned}$$

21. Describe the number of points that simultaneously lie in each of the three planes shown in each part of Figure 1.2.

22. Describe the number of points that simultaneously lie in each of the three planes shown in each part of Figure 1.3.

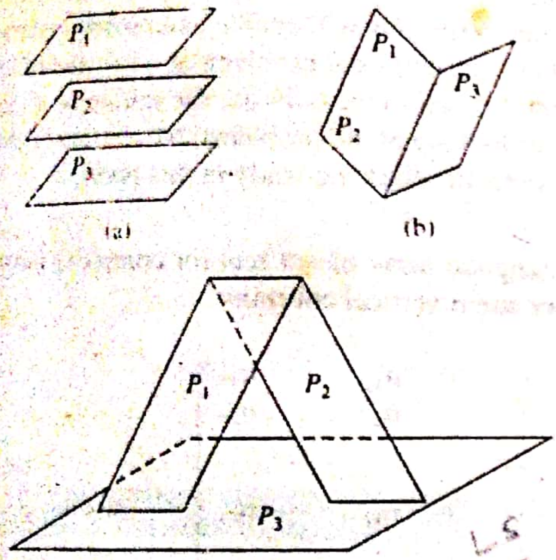

Figure 1.3 ▲

23. An oil refinery produces low-sulfur and high-sulfur fuel. Each ton of low-sulfur fuel requires 5 minutes in the blending plant and 4 minutes in the refining plant; each ton of high-sulfur fuel requires 4 minutes in the blending plant and 2 minutes in the refining plant. If the blending plant is available for 3 hours and the refining plant is available for 2 hours, how many tons of each type of fuel should be manufactured so that the plants are fully utilized?

24. A plastics manufacturer makes two types of plastic: regular and special. Each ton of regular plastic requires 2 hours in plant A and 5 hours in plant B; each ton of special plastic requires 2 hours in plant A and 3 hours in plant B. If plant A is available 8 hours per day and plant B is available 15 hours per day, how many tons of each type of plastic can be made daily so that the plants are fully utilized?

25. A dietician is preparing a meal consisting of foods A, B, and C. Each ounce of food A contains 2 units of protein, 3 units of fat, and 4 units of carbohydrate. Each ounce of food B contains 3 units of protein, 2 units of fat, and 1 unit of carbohydrate. Each ounce of food C contains 3 units of protein, 3 units of fat, and 2 units of carbohydrate. If the meal must provide exactly 25 units of protein, 24 units of fat, and 21 units of carbohydrate, how many ounces of each type of food should be used?

26. A manufacturer makes 2-minute, 6-minute, and 9-minute film developers. Each ton of 2-minute developer requires 6 minutes in plant A and 24 minutes in plant B. Each ton of 6-minute developer requires 12 minutes in plant A and 12 minutes in plant B. Each ton of 9-minute developer requires 12 minutes in plant A and 12 minutes in plant B. If plant A is available 10 hours per day and plant B is available 16 hours per day, how many tons of each type of developer can be produced so that the plants are fully utilized?

27. Suppose that the three points $(1, -5)$, $(-1, 1)$, and $(2, 7)$ lie on the parabola $p(x) = ax^2 + bx + c$.

(a) Determine a linear system of three equations in three unknowns that must be solved to find a, b, and c.

(b) Solve the linear system obtained in part (a) for a, b, and c.

28. An inheritance of \$24,000 is to be divided among three trusts, with the second trust receiving twice as much as the first trust. The three trusts pay interest at the rates of 9%, 10%, and 6% annually, respectively, and return a total in interest of \$2210 at the end of the first year. How much was invested in each trust?

Theoretical Exercises

T.1. Show that the linear system obtained by interchanging two equations in (2) has exactly the same solutions as (2).

T.2. Show that the linear system obtained by replacing an equation in (2) by a nonzero constant multiple of the equation has exactly the same solutions as (2).

T.3. Show that the linear system obtained by replacing an equation in (2) by itself plus a multiple of another equation in (2) has exactly the same solutions as (2).

T.4. Does the linear system

$$\begin{aligned} ax + by &= 0 \\ cx + dy &= 0 \end{aligned}$$

always have a solution for any values of a, b, c, and d?

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Q23) Let x and y be the desired number of tons of LSF and HSF to be manufactured respectively.

	LSF	HSF	
BP	$5x$	$4y$	$\rightarrow$ 3 hrs $= 3\times 60 = 180$ minutes
RP	$4x$	$2y$	$\rightarrow$ 2 hrs $= 2\times 60 = 120$ minutes

$5x + 4y = 180$ — ①

$4x + 2y = 120$ — ① $m=n$, $2=2$

eq.(1) $- 2\times$ eq.(2) $\Rightarrow -3x = -60 \Rightarrow x = 20$ tons

① $\Rightarrow 5(20) + 4y = 180 \Rightarrow y = 20$ tons.

Thus 20 tons of each fuel are manufactured

i.e $x = 20, y = 20$, is the unique solution.

Q24: Similar to Q20

Q25. Let x, y and z be the desired number of ounces of foods A, B and C to be used

	A	B	C	
Protein	$2x$	$3y$	$3z$	$\rightarrow 25$
Fat	$3x$	$2y$	$3z$	$\rightarrow 24$
carbohydrate	$4x$	y	$2z$	$\rightarrow 21$

$2x + 3y + 3z = 25$ — ① $m=n$

$3x + 2y + 3z = 24$ — ② $3=n$

$4x + y + 2z = 21$ — ③

solve by using Ex3 (P-3).

Q26: similar to Q25

Q27 $P(x) = ax^2 + bx + c$ — ①

Points on Parabola are $(1,-5)$, $(-1,1)$ and $(2,7)$

(a) For Point $(1,-5)$, (1) $\Rightarrow P(1) = -5 \Rightarrow a + b + c = -5$

For Point $(-1,1)$, (1) $\Rightarrow P(-1) = 1 \Rightarrow a - b + c = 1$

For Point $(2,7)$, (1) $\Rightarrow P(2) = 7 \Rightarrow 4a + 2b + c = 7$ is the desired system.

(b) Solve the above system for a, b and c by Ex3 (P-3) to get the values of a, b and c. By plugging back these values in (1), we get the desired Parabolas.

Q28: similar to Ex1.

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Matrix

Definition: An $m\times n$ matrix A is a rectangular array of mn real (complex) numbers arranged in m horizontal rows and n verticle columns.

i) If $m=n$, then A is a square matrix

ii) If all the entries in a square matrix off the main diagonal are zeros, then A is called diagonal matrix.

iii) If all the entries on the main diagonal of a diagonal matrix are the same i.e each is $r\in\mathbb{R}$, then A is called scalar matrix

Ex: $A=[a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$

size of A ← $m\times n$

① Total number of entries if $\times$ is ordinary multiplication

② m = stands for number of rows, n = stands for number of columns

→ m: the dimension of the range space, n = the dimension of the domain space if A is considered to be a transformation i.e $A(=T): \mathbb{R}^n \to \mathbb{R}^m$

Ex: $A=\begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}$ → scalar matrix

i) A is dilation transformation if $r>1$

ii) A represents contraction transformation if $0<r<1$

Operations on matrices: Addition / subtraction / scalar multiplication / Transpose / Matrix multiplication (product of matrices).

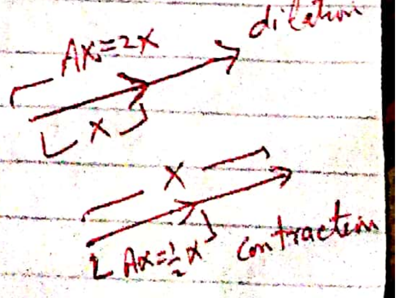
Two matrices are added or subtracted if they have the same sizes

If a scalar number say r is multiplied to a matrix A i.e rA, is called scalar multiplication

Ex: $2A = 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$.

Ex: $AX = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix} = 2\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2X$

$AX = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x/2 \\ y/2 \\ z/2 \end{bmatrix} = 1/2\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1/2 X$



Linear Combination: If $A_1, A_2, \ldots, A_k$ are all $m\times n$ matrices and $c_1, c_2, \ldots, c_k$ are real numbers, then an expression of the form

$c_1A_1 + c_2A_2 + \cdots + c_kA_k$

is called a linear combination of $A_1, A_2, \ldots, A_k$ and $c_1, c_2, \ldots, c_k$ are called coefficients.

Ex: $2A_1 - 3A_2 + \frac{1}{2}A_3 = 2\begin{bmatrix}1 & -1 & 0\\ 3 & 2 & 1\end{bmatrix} + (-3)\begin{bmatrix}2 & 1 & -1\\ 0 & 1 & 3\end{bmatrix} + \frac{1}{2}\begin{bmatrix}2 & 4 & 6\\ 4 & 8 & 10\end{bmatrix}$

$= \begin{bmatrix}2 & -2 & 0\\ 6 & 4 & 2\end{bmatrix} + \begin{bmatrix}-6 & -3 & 3\\ 0 & -3 & -9\end{bmatrix} + \begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\end{bmatrix} = \begin{bmatrix}-3 & -3 & 6\\ 8 & 5 & -2\end{bmatrix} = A$ is a linear combination of A_1, A_2 and A_3.

Bit: A bit is a binary digit, that is, either a 0 or 1.

Bit matrix: An $m\times n$ bit matrix, is a matrix all of whose entries are bits. That is, each entry is either a 0 or 1.

+	0	1
0	0	1
1	1	0

×	0	1
0	0	0
1	0	1

$0+0=0$

$1+0=0+1=1$

$1+1=0$

$0\times 0 = 0\times 1 = 1\times 0 = 0$

$1\times 1 = 1$

$A = \begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$, $B = ?$, $C = \begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$. find B if $A+B=C$

(S) $A+B=C \Rightarrow B = C-A = C+(-1)A = C+A$

$\Rightarrow B = \begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix} + \begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix} = \begin{bmatrix}1+1 & 0+1 & 1+0\\ 1+0 & 1+1 & 0+1\end{bmatrix} = \begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$.

Exercise 1.2

Q8: Is the matrix $\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}$ a linear combination of the matrices $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$? Justify your answer.

(S) Suppose $A = \begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}$ is a linear combination of $A_1 = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $A_2 = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$, then $c_1A_1 + c_2A_2 = A \Rightarrow c_1\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + c_2\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} = \begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}$

$\Rightarrow \begin{bmatrix}c_1+c_2 & 0\\ 0 & c_1\end{bmatrix} = \begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix} \Rightarrow \left.\begin{array}{l} c_1+c_2 = 3 \quad (1)\\ 0 = 0\\ c_1 = 2\end{array}\right\}$ (1) $\Rightarrow c_2 = 1$

As c_1 and c_2 exist, hence A is the linear combination of A_1 and A_2.

Q9: similar to Q8

Do also Q11 — Q13

10

Key Terms

Matrix
Rows
Columns
Size of a matrix
Square matrix
Main diagonal of a matrix
Element (or entry) of a matrix
ijth element
(i, j) entry
n-vector (or vector)
Diagonal matrix
Scalar matrix
$\mathbf{0}$, the zero vector
R^n, the set of all n-vectors
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Equal matrices
Matrix addition
Scalar multiplication
Scalar multiple of a matrix
Difference of matrices
Linear combination of matrices
Transpose of a matrix
Bit
Bit (or Boolean) matrix
Upper triangular matrix
Lower triangular matrix

1.2 Exercises

1. Let

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & -5 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 7 & 3 & 2 \\ -4 & 3 & 5 \\ 6 & 1 & -1 \end{bmatrix}.$$

(a) What is a_{12}, a_{22}, a_{23}?

(b) What is b_{11}, b_{31}?

(c) What is c_{13}, c_{31}, c_{33}?

2. If

$$\begin{bmatrix} a+b & c+d \\ c-d & a-b \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 2 \end{bmatrix},$$

find a, b, c, and d.

3. If

$$\begin{bmatrix} a+2b & 2a-b \\ 2c+d & c-2d \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -3 \end{bmatrix},$$

find a, b, c, and d.

In Exercises 4 through 7, let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix},$$

$$C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix},$$

$$E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix},$$

and $$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

4. If possible, compute the indicated linear combination.

(a) $C + E$ and $E + C$ (b) $A + B$

(c) $D - F$ (d) $-3C + 5O$

(e) $2C - 3E$ (f) $2B + F$

5. If possible, compute the indicated linear combination:

(a) $3D + 2F$

(b) $3(2A)$ and $6A$

(c) $3A + 2A$ and $5A$

(d) $2(D + F)$ and $2D + 2F$

(e) $(2 + 3)D$ and $2D + 3D$

(f) $3(B + D)$

6. If possible, compute:

(a) A^T and $(A^T)^T$

(b) $(C + E)^T$ and $C^T + E^T$

(c) $(2D + 3F)^T$

(d) $D - D^T$

(e) $2A^T + B$

(f) $(3D - 2F)^T$

7. If possible, compute:

(a) $(2A)^T$ (b) $(A - B)^T$

(c) $(3B^T - 2A)^T$

(d) $(3A^T - 5B^T)^T$

(e) $(-A)^T$ and $-(A^T)$

(f) $(C + E + F^T)^T$

8. Is the matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ a linear combination of the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$? Justify your answer.

9. Is the matrix $\begin{bmatrix} 4 & 1 \\ 0 & -3 \end{bmatrix}$ a linear combination of the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$? Justify your answer.

10. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & -2 & 3 \\ 5 & 2 & 4 \end{bmatrix} \quad \text{and} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

If λ is a real number, compute $\lambda I_3 - A$.

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Exercises 11 through 15 involve bit matrices.

11. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Compute each of the following.

(a) $A + B$ (b) $B + C$ (c) $A + B + C$
(d) $A + C^T$ (e) $B - C$

12. Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, and $D = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$. Compute each of the following.

(a) $A + B$ (b) $C + D$ (c) $A + B + (C + D)^T$
(d) $C - B$ (e) $A - B + C - D$

13. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

1^C

(a) Find B so that $A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

(b) Find C so that $A + C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

14. Let $\mathbf{u} = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}$. Find the bit 4-vector $\mathbf{v}$ so that $\mathbf{u} + \mathbf{v} = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}$.

15. Let $\mathbf{u} = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$. Find the bit 4-vector $\mathbf{v}$ so that $\mathbf{u} + \mathbf{v} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$.

Theoretical Exercises

T.1. Show that the sum and difference of two diagonal matrices is a diagonal matrix.

T.2. Show that the sum and difference of two scalar matrices is a scalar matrix.

T.3. Let

$$A = \begin{bmatrix} a & b & c \\ c & d & e \\ e & e & f \end{bmatrix}.$$

(a) Compute $A - A^T$.

(b) Compute $A + A^T$.

(c) Compute $(A + A^T)^T$.

T.4. Let O be the $n \times n$ matrix all of whose entries are zero. Show that if k is a real number and A is an $n \times n$ matrix such that $kA = O$, then $k = 0$ or $A = O$.

T.5. A matrix $A = [a_{ij}]$ is called **upper triangular** if $a_{ij} = 0$ for $i > j$. It is called **lower triangular** if $a_{ij} = 0$ for $i < j$.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & \cdots & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & \cdots & \cdots & a_{2n} \\ 0 & 0 & a_{33} & \cdots & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & a_{nn} \end{bmatrix}$$

Upper triangular matrix
(The elements below the main diagonal are zero.)

$$\begin{bmatrix} a_{11} & 0 & 0 & \cdots & \cdots & 0 \\ a_{21} & a_{22} & 0 & \cdots & \cdots & 0 \\ a_{31} & a_{32} & a_{33} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & 0 \\ a_{n1} & a_{n2} & a_{n3} & \cdots & \cdots & a_{nn} \end{bmatrix}$$

Lower triangular matrix
(The elements above the main diagonal are zero.)

(a) Show that the sum and difference of two upper triangular matrices is upper triangular.

(b) Show that the sum and difference of two lower triangular matrices is lower triangular.

(c) Show that if a matrix is both upper and lower triangular, then it is a diagonal matrix.

T.6. (a) Show that if A is an upper triangular matrix, then A^T is lower triangular.

(b) Show that if A is a lower triangular matrix, then A^T is upper triangular.

T.7. If A is an $n \times n$ matrix, what are the entries on the main diagonal of $A - A^T$? Justify your answer.

T.8. If $\mathbf{x}$ is an n-vector, show that $\mathbf{x} + \mathbf{0} = \mathbf{x}$.

Exercises T.9 through T.18 involve bit matrices.

T.9. Make a list of all possible bit 2-vectors. How many are there?

T.10. Make a list of all possible bit 3-vectors. How many are there?

T.11. Make a list of all possible bit 4-vectors. How many are there?

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Matrix Product / Product of two matrices:-
If $A_{m\times n}$ and $B_{n\times p}$, then the product of A and B is defined as $(AB)_{m\times p} = [col_1(AB)\ \ col_2(AB) \cdots col_p(AB)]$

where $col_j(AB) = A\, col_j(B)$, $j = 1, 2, \ldots p$ represent the linear combinations of the columns of A.

P-23
Ex4 Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix}_{2\times 3}$ and $B = \begin{bmatrix} -2 & 5 \\ 4 & -3 \\ 2 & 1 \end{bmatrix}_{3\times 2}$

S) $(AB)_{2\times 2} = [col_1(AB)\ \ col_2(AB)]$ — ①

$col_1(AB) = A\, col_1(B) = -2\begin{bmatrix}1\\3\end{bmatrix} + 4\begin{bmatrix}2\\1\end{bmatrix} + 2\begin{bmatrix}-1\\4\end{bmatrix}$

$= \begin{bmatrix}-2\\-6\end{bmatrix} + \begin{bmatrix}8\\4\end{bmatrix} + \begin{bmatrix}-2\\8\end{bmatrix} = \begin{bmatrix}4\\6\end{bmatrix}$

$col_2(AB) = A\, col_2(B) = 5\begin{bmatrix}1\\3\end{bmatrix} - 3\begin{bmatrix}2\\1\end{bmatrix} + 1\begin{bmatrix}-1\\4\end{bmatrix}$

$= \begin{bmatrix}5\\15\end{bmatrix} + \begin{bmatrix}-6\\-3\end{bmatrix} + \begin{bmatrix}-1\\4\end{bmatrix} = \begin{bmatrix}-2\\16\end{bmatrix}$

① ⇒ $AB = \begin{bmatrix} 4 & -2 \\ 6 & 16 \end{bmatrix}$, showing that each column of AB is a linear combination of the columns of matrix A.

Ex: $A_{60\times 50}$ and $B_{50\times 70}$, then $(AB)_{60\times 70}$

Exercise 1.3

Q12 — Q16 By expressing the columns of AB as the linear combinations of the columns of A.

13

1.3

In Exercises 7 and 8, let

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -1 & 5 \end{bmatrix},$$

$$C = \begin{bmatrix} 2 & 3 & 1 \\ 3 & -4 & 5 \\ 1 & -1 & -2 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & 0 & -3 \\ -2 & 1 & 5 \\ 3 & 4 & 2 \end{bmatrix}, \quad \textit{and} \quad F = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}.$$

7. If possible, compute:

(a) AB (b) BA (c) $CB + D$

(d) $AB + DF$ (e) $BA + FD$

8. If possible, compute:

(a) $A(BD)$ (b) $(AB)D$ (c) $A(C + E)$

(d) $AC + AE$ (e) $(D + F)A$

9 to 17

9. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$.

Compute the following entries of AB:

(a) The (1, 2) entry (b) The (2, 3) entry

(c) The (3, 1) entry (d) The (3, 3) entry

10. If $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$, compute DI_2 and I_2D.

11. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}.$$

Show that $AB \neq BA$.

12. If A is the matrix in Example 4 and O is the 3×2 matrix every one of whose entries is zero, compute AO.

In Exercises 13 and 14, let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 3 & -3 & 4 \\ 4 & 2 & 5 & 1 \end{bmatrix}.$$

13. Using the method in Example 12, compute the following columns of AB:

(a) The first column (b) The third column

14. Using the method in Example 12, compute the following columns of AB:

(a) The second column (b) The fourth column

15. Let

$$A = \begin{bmatrix} 2 & -3 & 4 \\ -1 & 2 & 3 \\ 5 & -1 & -2 \end{bmatrix} \quad \text{and} \quad \mathbf{c} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}.$$

Express $A\mathbf{c}$ as a linear combination of the columns of A.

16. Let

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 4 & 3 \\ 3 & 0 & -2 \end{bmatrix}_{3\times3} \quad \text{and} \quad B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 2 & 4 \end{bmatrix}_{3\times2}.$$

Express the columns of AB as linear combinations of the columns of A.

17. Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$.

(a) Verify that $AB = 3\mathbf{a}_1 + 5\mathbf{a}_2 + 2\mathbf{a}_3$, where $\mathbf{a}_j$ is the jth column of A for $j = 1, 2, 3$.

(b) Verify that $AB = \begin{bmatrix} (\text{row}_1(A))B \\ (\text{row}_2(A))B \end{bmatrix}$.

18. Write the linear combination

$$3\begin{bmatrix} -2 \\ 3 \end{bmatrix} + 4\begin{bmatrix} 2 \\ 5 \end{bmatrix} + 2\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

as a product of a 2×3 matrix and a 3-vector.

19. Consider the following linear system:

$$\begin{aligned} 2x + w &= 7 \\ 3x + 2y + 3z &= -2 \\ 2x + 3y - 4z &= 3 \\ x + 3z &= 5. \end{aligned}$$

(a) Find the coefficient matrix.

(b) Write the linear system in matrix form.

(c) Find the augmented matrix.

20. Write the linear system with augmented matrix

$$\left[\begin{array}{rrrr|r} -2 & -1 & 0 & 4 & 5 \\ -3 & 2 & 7 & 8 & 3 \\ 1 & 0 & 0 & 2 & 4 \\ 3 & 0 & 1 & 3 & 6 \end{array}\right].$$

21. Write the linear system with augmented matrix

$$\left[\begin{array}{rrr|r} 2 & 0 & -4 & 3 \\ 0 & 1 & 2 & 5 \\ 1 & 3 & 4 & -1 \end{array}\right].$$

22. Consider the following linear system:

$$\begin{aligned} 3x - y + 2z &= 4 \\ 2x + y &= 2 \\ y + 3z &= 7 \\ 4x - z &= 4. \end{aligned}$$

(a) Find the coefficient matrix.

(b) Write the linear system in matrix form.

(c) Find the augmented matrix.

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23. How are the linear systems whose augmented matrices are

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 2 & 3 & 6 & 2 \end{array}\right] \text{ and } \left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 2 & 3 & 6 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

related? both matrices are equivalent

24. Write each of the following as a linear system in matrix form.

(a) $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} + z\begin{bmatrix}0\\3\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$

(b) $x\begin{bmatrix}1\\1\\2\end{bmatrix} + y\begin{bmatrix}2\\1\\0\end{bmatrix} + z\begin{bmatrix}1\\2\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

25. Write each of the following linear systems as a linear combination of the columns of the coefficient matrix.

(a) $x + 2y = 3$
$2x - y = 5$

(b) $2x - 3y + 5z = -2$
$x + 4y - z = 3$

26. Let A be an $m \times n$ matrix and B an $n \times p$ matrix. What if anything can you say about the matrix product AB when:

(a) A has a column consisting entirely of zeros?

(b) B has a row consisting entirely of zeros?

27. (a) Find a value of r so that $AB^T = 0$, where

$$A = \begin{bmatrix} r & 1 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 & -1 \end{bmatrix}.$$

(b) Give an alternate way to write this product.

28. Find a value of r and a value of s so that $AB^T = 0$, where

$$A = \begin{bmatrix} 1 & r & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 2 & s \end{bmatrix}.$$

29. Formulate the method for adding partitioned matrices and verify your method by partitioning the matrices

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 2 & 1 \\ -2 & 3 & 1 \\ 4 & 1 & 5 \end{bmatrix}$$

in two different ways and finding their sum.

30. Let A and B be the following matrices:

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 & 2 \\ 1 & 2 & 3 & -1 & 4 \\ 2 & 3 & 2 & 1 & 4 \\ 5 & -1 & 3 & 2 & 6 \\ 3 & 1 & 2 & 4 & 6 \\ 2 & -1 & 3 & 5 & 7 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 & 1 \\ 2 & 1 & 3 & 2 & -1 \\ 1 & 5 & 4 & 2 & 3 \\ 2 & 1 & 3 & 5 & 7 \\ 3 & 2 & 4 & 6 & 1 \end{bmatrix}.$$

Find AB by partitioning A and B in two different ways.

31. *(Manufacturing Costs)* A furniture manufacturer makes chairs and tables, each of which must go through an assembly process and a finishing process. The times required for these processes are given (in hours) by the matrix

$$A = \begin{array}{c} \begin{array}{cc} \text{Assembly process} & \text{Finishing process} \end{array} \\ \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} \end{array} \begin{array}{l} \text{Chair} \\ \text{Table} \end{array}$$

The manufacturer has a plant in Salt Lake City and another in Chicago. The hourly rates for each of the processes are given (in dollars) by the matrix

$$B = \begin{array}{c} \begin{array}{cc} \text{Salt Lake City} & \text{Chicago} \end{array} \\ \begin{bmatrix} 9 & 10 \\ 10 & 12 \end{bmatrix} \end{array} \begin{array}{l} \text{Assembly process} \\ \text{Finishing process} \end{array}$$

What do the entries in the matrix product AB tell the manufacturer?

The study of how much the plants related to their simplicity BAT

32. *(Ecology-Pollution)* A manufacturer makes two kinds of products, P and Q, at each of two plants, X and Y. In making these products, the pollutants sulfur dioxide, nitric oxide, and particulate matter are produced. The amounts of pollutants produced are given (in kilograms) by the matrix

$$\underset{2\times 3}{A} = \begin{array}{c} \begin{array}{ccc} \text{Sulfur dioxide} & \text{Nitric oxide} & \text{Particulate matter} \end{array} \\ \begin{bmatrix} 300 & 100 & 150 \\ 200 & 250 & 400 \end{bmatrix} \end{array} \begin{array}{l} \text{Product } P \\ \text{Product } Q \end{array}$$

State and federal ordinances require that these pollutants be removed. The daily cost of removing each kilogram of pollutant is given (in dollars) by the matrix

$(AB)_{2\times 2}$

$$\underset{3\times 2}{B} = \begin{array}{c} \begin{array}{cc} \text{Plant } X & \text{Plant } Y \end{array} \\ \begin{bmatrix} 8 & 12 \\ 7 & 9 \\ 15 & 10 \end{bmatrix} \end{array} \begin{array}{l} \text{Sulfur dioxide} \\ \text{Nitric oxide} \\ \text{Particulate matter} \end{array}$$

What do the entries in the matrix product AB tell the manufacturer?

33. *(Medicine)* A diet research project consists of adults and children of both sexes. The composition of the participants in the project is given by the matrix

$$A = \begin{array}{c} \begin{array}{cc} \text{Adults} & \text{Children} \end{array} \\ \begin{bmatrix} 80 & 120 \\ 100 & 200 \end{bmatrix} \end{array} \begin{array}{l} \text{Male} \\ \text{Female} \end{array}$$

The number of daily grams of protein, fat, and carbohydrate consumed by each child and adult is given by the matrix

$$B = \begin{array}{c} \begin{array}{ccc} \text{Protein} & \text{Fat} & \text{Carbohydrate} \end{array} \\ \begin{bmatrix} 20 & 20 & 20 \\ 10 & 20 & 30 \end{bmatrix} \end{array} \begin{array}{l} \text{Adult} \\ \text{Child} \end{array}$$

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(a) How many grams of protein are consumed daily by the males in the project?

(b) How many grams of fat are consumed daily by the females in the project?

34. ***(Business)*** A photography business has a store in each of the following cities: New York, Denver, and Los Angeles. A particular make of camera is available in automatic, semiautomatic, and nonautomatic models. Moreover, each camera has a matched flash unit and a camera is usually sold together with the corresponding flash unit. The selling prices of the cameras and flash units are given (in dollars) by the matrix

$$\begin{array}{cccc} \text{Auto-matic} & \text{Semi-automatic} & \text{Non-automatic} & \end{array}$$
$$A = \begin{bmatrix} 200 & 150 & 120 \\ 50 & 40 & 25 \end{bmatrix} \begin{array}{l} \text{Camera} \\ \text{Flash unit} \end{array}$$

The number of sets (camera and flash unit) available at each store is given by the matrix

$$\begin{array}{ccc} \text{New York} & \text{Denver} & \text{Los Angeles} \end{array}$$
$$B = \begin{bmatrix} 220 & 180 & 100 \\ 300 & 250 & 120 \\ 120 & 320 & 250 \end{bmatrix} \begin{array}{l} \text{Automatic} \\ \text{Semiautomatic} \\ \text{Nonautomatic} \end{array}$$

(a) What is the total value of the cameras in New York?

(b) What is the total value of the flash units in Los Angeles?

35. Let $\mathbf{s}_1 = \begin{bmatrix} 18.95 & 14.75 & 8.98 \end{bmatrix}$ and $\mathbf{s}_2 = \begin{bmatrix} 17.80 & 13.50 & 10.79 \end{bmatrix}$ be 3-vectors denoting the current prices of three items at stores A and B, respectively.

(a) Obtain a 2×3 matrix representing the combined information about the prices of the three items at the two stores.

(b) Suppose that each store announces a sale so that the price of each item is reduced by 20%. Obtain a 2×3 matrix representing the sale prices at the two stores.

Exercises 36 through 41 involve bit matrices.

36. For bit vectors $\mathbf{a}$ and $\mathbf{b}$ compute $\mathbf{a} \cdot \mathbf{b}$.

(a) $\mathbf{a} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

(b) $\mathbf{a} = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

37. For bit vectors $\mathbf{a}$ and $\mathbf{b}$ compute $\mathbf{a} \cdot \mathbf{b}$.

(a) $\mathbf{a} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(b) $\mathbf{a} = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

38. Let $\mathbf{a} = \begin{bmatrix} 1 & x & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} x \\ 1 \\ 1 \end{bmatrix}$ be bit vectors. If $\mathbf{a} \cdot \mathbf{b} = 0$, find all possible values of x.

39. Let $A = \begin{bmatrix} 1 & 1 & x \\ 0 & y & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ be bit matrices. If $AB = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, find x and y.

40. For bit matrices

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

compute AB and BA.

41. For bit matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, determine a 2×2 bit matrix B so that $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Theoretical Exercises

T.1. Let $\mathbf{x}$ be an n-vector.

(a) Is it possible for $\mathbf{x} \cdot \mathbf{x}$ to be negative? Explain.

(b) If $\mathbf{x} \cdot \mathbf{x} = 0$, what is $\mathbf{x}$?

T.2. Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be n-vectors and let k be a real number.

(a) Show that $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$.

(b) Show that $(\mathbf{a} + \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot \mathbf{c} + \mathbf{b} \cdot \mathbf{c}$.

(c) Show that $(k\mathbf{a}) \cdot \mathbf{b} = \mathbf{a} \cdot (k\mathbf{b}) = k(\mathbf{a} \cdot \mathbf{b})$.

T.3. (a) Show that if A has a row of zeros, then AB has a row of zeros.

(b) Show that if B has a column of zeros, then AB has a column of zeros.

T.4. Show that the product of two diagonal matrices is a diagonal matrix.

T.5. Show that the product of two scalar matrices is a scalar matrix.

T.6. (a) Show that the product of two upper triangular matrices is upper triangular.

(b) Show that the product of two lower triangular matrices is lower triangular.

T.7. Let A and B be $n \times n$ diagonal matrices. Is $AB = BA$? Justify your answer.

T.8. (a) Let $\mathbf{a}$ be a $1 \times n$ matrix and B an $n \times p$ matrix. Show that the matrix product $\mathbf{a}B$ can be written as

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Q31

$$\underset{2\times 2}{A} = \begin{array}{c} \begin{array}{cc} \text{Assembly Process} & \text{Finishing Process} \end{array} \\ \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} \end{array} \begin{array}{l} \text{chair} \\ \text{Table} \end{array}$$

$$\underset{2\times 2}{B} = \begin{array}{c} \begin{array}{cc} \text{Salt lake city} & \text{Chicago} \end{array} \\ \begin{bmatrix} 9 & 10 \\ 10 & 12 \end{bmatrix} \end{array} \begin{array}{l} \text{Assembly Process} \\ \text{Finishing Process} \end{array}$$

As $A_{2\times 2}$ $B_{2\times 2}$ $\Rightarrow$ indicate rows in A → chair, table

indicate columns in B → salt lake city, chicago city

$$(AB)_{2\times 2} = \begin{bmatrix} col_1(AB) & col_2(AB) \end{bmatrix} \quad \text{--- ①}$$

$$col_1(AB) = A\,col_1(B) = 9\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 10\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 18 \\ 27 \end{bmatrix} + \begin{bmatrix} 20 \\ 40 \end{bmatrix} = \begin{bmatrix} 38 \\ 67 \end{bmatrix}$$

$$col_2(AB) = A\,col_2(B) = 10\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 12\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix} + \begin{bmatrix} 24 \\ 48 \end{bmatrix} = \begin{bmatrix} 44 \\ 78 \end{bmatrix}$$

$$① \Rightarrow (AB)_{2\times 2} = \begin{array}{c} \begin{array}{cc} S.C & C.C \end{array} \\ \begin{bmatrix} 38 & 44 \\ 67 & 78 \end{bmatrix} \end{array} \begin{array}{l} \text{chair} \\ \text{table} \end{array}$$

The entries in the matrix product AB tell the manufacturer the total cost of chairs and table in each city.

S.C → Salt lake city and C.C → chicago city.

Q32 ———— Q35 do yourself.

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