# Applicaton of Linear system in Electrical circuits

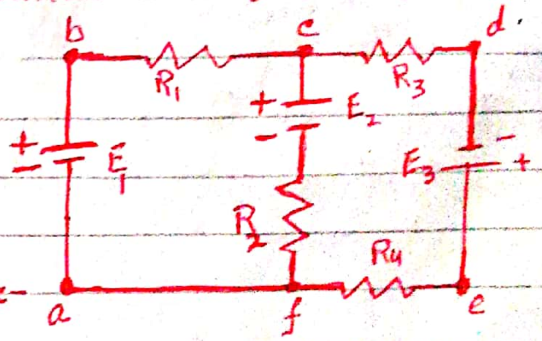
1) Elemements of Electrical circuits

i) Battery:- It is a source of direct current (or voltage) in the circuit. $\pm$ symbol

ii) Resister: It is a device, such as a lightbulb that reduces a current in a circuit by converting electrical energy into thermal energy. —/\/\/— symbol

iii) wire:- It is a conductor that allows a free flow of electrical current. ——— symbol.

Electrical circuit:- A simple electrical circuit is a closed connection of resistors, batteries and wires.




2) Physical quantities and theire units used in Electrical circuits:-

i) current denoted by I and is measured in Amperes (A)

ii) Resistance denoted by R and is measured in ohms ($\Omega$)

iii) Electrical Potential Difference denoted by E and is measured in Volts (V). E is +ve when measured from (-) to (+) and is -ve when measured from (+) to (-) terminal

3) Relations among these Physical quantities

i) one volt = (one Ampere) x (one ohm)

ii) $V = \pm IR$, called ohm's Law

+ve sign is used when E is measured across the resistor in the opposite direction of the current flow.

-ve sign is used when E is measured across the resistor in the direction of the current flow.

4) Paths and Points for conservation Laws in the circuit.

i) Voltage loop: A voltage loop is a closed connection within the circuit.

Exs: The given figure on Page-1, contains three loops

$a \to b \to c \to f \to a$

$c \to d \to e \to f \to c$

$a \to b \to c \to d \to e \to f \to a$

ii) Current node:- A current node is a Point where three or more segments of wire meet.

Exs: The given figure on Page-1, contains two current nodes at points c and f.

5) Conservation Laws in an Electrical circuit

i) Conservation of Energy (Kirchhoff's voltage Law):- Around any voltage loop, the total electrical potential difference is zero.

ii) Conservation of charge (Kirchhoff's current Law):- At any current node, the flow of all currents into the nodes equals the flow of all currents out of the node.

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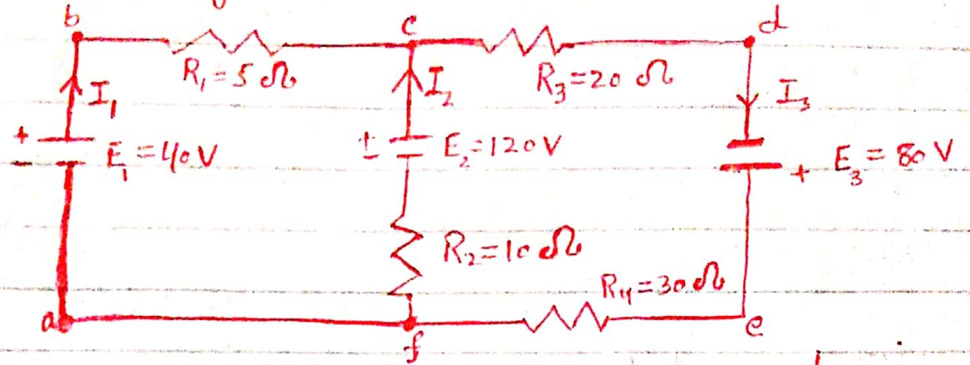
EX 1) Determine the unknowns in the given Electrical circuit.




③ i) Known quantities

$E_1 = 40V$, $E_2 = 120V$, $E_3 = 80V$

$R_1 = 5\,\Omega$, $R_2 = 10\,\Omega$, $R_3 = 20\,\Omega$, $R_4 = 30\,\Omega$

Unknown quantities

$I_1 = ?$, $I_2 = ?$, $I_3 = ?$

ii) There are two nodes at points c and f. Here we have only one useful equation from Kirchhoff's current law, the other equation is the linear combination of this equation/similar/dependent to this equation, as in general we find n-1 linearly independent equations by using Kirchhoff's current Law in an electrical circuit and one equation is the linear combination of the other n-1 equations

iii) There are three voltage loops which are as

$a \to b \to c \to f \to a$ ; $c \to d \to e \to f \to c$

$a \to b \to c \to d \to e \to f \to a$

we get two linearly independent equations from the first and second sub loops while the third equation from the above third loop (which is the larger outer loop) is the linear combination of these two equations, so we ignore it

Now by Kirchhoff's current Law at point c, we have

$$I_1 + I_2 = I_3$$

Also by Kirchhoff's current Law at point f, we have

$$I_3 = I_1 + I_2$$

From the above two equations being similar/dependent, we consider one equation in the following form

$$I_1 + I_2 - I_3 = 0 \quad \text{——— (1)}$$

By Kirchhoff's voltage Law around the closed loop $a \to b \to c \to f \to a$, we have the following equation:

$$(+E_1) + (-R_1 I_1) + (-E_2) + (R_2 I_2) = 0$$ by using ohms law in 2nd and last terms

$$\Rightarrow 40 - 5I_1 - 120 + 10 I_2 = 0$$

$$\Rightarrow I_1 - 2I_2 = -16 \quad \text{——— (2)}$$

Similarly by kirchhoff's voltage Law around the closed loop $c \to \;\; \to \;\; \to \;\; \to c$, we have

$$(-R_3 I_3) + (+E_3) + (-R_4 I_3) + (+E_2) + (-R_2 I_2) = 0$$

$$\Rightarrow -20 I_3 + 80 - 30 I_3 + 120 - 10 I_2 = 0$$

$$\Rightarrow 10 I_2 + 50 I_3 = 200$$

$$\Rightarrow I_2 + 5I_3 = 20 \quad \text{——— (3)}$$

Also from the closed loop $a \to b \to c \to d \to e \to f \to a$, by the Kirchhoff's voltage Law, we have

$$(+E_1) + (-R_1 I_1) + (-R_3 I_3) + (+E_3) + (-R_4 I_3) = 0$$

$$\Rightarrow 40 - 5I_1 - 20 I_3 + 80 - 30 I_3 = 0$$

$$\Rightarrow 5I_1 + 50 I_3 = 120 \Rightarrow I_1 + 10 I_3 = 24 \quad \text{— (4)}$$

But we ignore this as this is the linear combination of eq.(2) and eq.(3) i.e eq.(2) + 2 × eq.(3) = eq.(4)

Thus we have the following system of linearly independent equations

$$\left.\begin{aligned} I_1 + I_2 - I_3 &= 0 \quad \text{— (1)} \\ I_1 - 2I_2 &= -16 \quad \text{— (2)} \\ I_2 + 5I_3 &= 20 \quad \text{— (3)} \end{aligned}\right\} \longrightarrow (\mathrm{I})$$

In matrix form the above linear system (I) is as

$$\underbrace{\begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 0 \\ 0 & 1 & 5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{I} = \underbrace{\begin{bmatrix} 0 \\ -16 \\ 20 \end{bmatrix}}_{b} \Rightarrow AI = b \quad \text{—— (II)}$$

The augmented matrix of (II) is as

$$[A \vdots b] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & -2 & 0 & -16 \\ 0 & 1 & 5 & 20 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3 & 1 & -16 \\ 0 & 1 & 5 & 20 \end{array}\right]$$

$$\xrightarrow{R_2 \text{ by } R_1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 5 & 20 \\ 0 & -3 & 1 & -16 \end{array}\right] \xrightarrow[R_1 - R_2]{R_3 + 3R_2} \left[\begin{array}{ccc|c} 1 & 0 & -6 & -20 \\ 0 & 1 & 5 & 20 \\ 0 & 0 & 16 & 44 \end{array}\right]$$

$$\xrightarrow{\frac{1}{16}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -6 & -20 \\ 0 & 1 & 5 & 20 \\ 0 & 0 & 1 & 11/4 \end{array}\right] \xrightarrow[R_2 - 5R_3]{R_1 + 6R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -7/2 \\ 0 & 1 & 0 & \frac{25}{4} \\ 0 & 0 & 1 & 11/4 \end{array}\right]$$

$\Rightarrow I_1 = -7/2 \Rightarrow I_1 = -3.5\,A$. -ve sign indicates that the true direction is opposite to the given direction

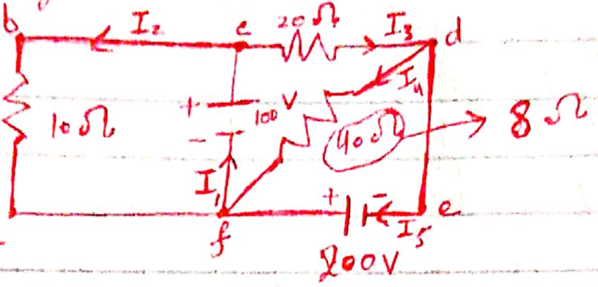
$I_2 = 25/4 \Rightarrow I_2 = 6.25\,A$

$I_3 = 11/4 \Rightarrow I_3 = 2.75\,A$

Note :— The total number of linear equations in the linear system obtained by using Kirchhoff's voltage and current laws in an electrical circuit is always equal to the number of different current assignments. Like in the above* the total number of different current assignments is 3 and the number of linear equations in the linear system (I) obtained is also 3.

## Exercise 2.4

Q3 Determine the unknown currents in the given circuit




(S) Here there are three node points which are at points c, d and f

By Kirchhoff's current law at node c, we have

$$I_1 = I_2 + I_3 \Rightarrow I_1 - I_2 - I_3 = 0 \quad \text{--- (1)}$$

By the above mentioned law at node d, we have

$$I_3 = I_4 + I_5 \Rightarrow I_3 - I_4 - I_5 = 0 \quad \text{--- (2)}$$

Also at node f by Kirchhoff's laws, we have

$$I_2 + I_4 + I_5 = I_1 \Rightarrow I_1 - I_2 - I_4 - I_5 = 0 \quad \text{--- (3)}$$

Equation (3) is the linear combination of eq. (1) and eq. (2), as Eq. (1) + Eq. (2) = Eq. (3), so we ignore equation (3)

There are three sub loops and two larger outer loops, which will be ignored because the equations obtained by Kirchhoff's voltage law from these large outer loops are the linear combinations of the equations obtained from the sub loops.

In loop $a \to b \to c \to f \to a$, by Kirchhoff's voltage loop, we have

$$+10I_2 - 100 = 0 \Rightarrow \boxed{I_2 = 10\ A} \quad \text{--- (4)}$$

In loop $c \to d \to f \to c$, by Kirchhoff's voltage loop, we have

$$-20I_3 - 40I_4 + 100 = 0$$

$$\Rightarrow I_3 + 2I_4 = 5 \quad \text{--- (5)}$$

In loop $d \to e \to f \to d$, by Kirchhoff's law,

$$-8I_5 + 200 + 8I_4 = 0$$

$$\Rightarrow \quad I_4 - I_5 = -25 \quad — (6)$$

(4) in (1) $\Rightarrow$ $I_1 - I_3 = 10 \quad — (7)$

Thus we have the following linear system

$$I_1 - I_3 = 10 \quad — (7) \qquad \text{from (1)}$$

$$I_3 - I_4 - I_5 = 0 \quad — (2)$$

$$I_3 + 2I_4 = 5 \quad — (5)$$

$$I_4 - I_5 = -25 \quad — (6)$$

$(2) - (6) \Rightarrow I_3 - 2I_4 = 25 \quad — (8)$

$(5) + (8) \Rightarrow 2I_3 = 30 \Rightarrow \boxed{I_3 = 15\,A}$

$(8) \Rightarrow 2I_4 = I_3 - 25 \Rightarrow I_4 = \frac{15-25}{2} \Rightarrow \boxed{I_4 = -5\,A}$

$(6) \Rightarrow I_5 = I_4 + 25 \Rightarrow I_5 = -5 + 25 \Rightarrow \boxed{I_5 = 20\,A}$

$(7) \Rightarrow I_1 = I_3 + 10 \Rightarrow I_1 = 15 + 10 \Rightarrow \boxed{I_1 = 25\,A}$

Hence we have the following solution

$I_1 = 25\,A$

$I_2 = 10\,A$

$I_3 = 15\,A$

$I_4 = -5\,A$, -ve sign indicates that true direction of $I_4$ is from f to d.

$I_5 = 20\,A$.