(3) $\displaystyle\sum_{n=0}^{\infty} \frac{n+5i}{(2n)!} (z-i)^n$

Center $= z_0 = i$

Using theorem-2 (Cauchy Hadamard formula)

Sol.

$$\sum_{n=0}^{\infty} \frac{n+5i}{(2n)!}$$

Expand $\dfrac{n+5i}{(2n)!} = \dfrac{n}{(2n)!} + \dfrac{5i}{(2n)!}$.

$$\sum_{n=0}^{\infty} \frac{n+5i}{(2n)!} = \sum_{n=0}^{\infty} \frac{n}{(2n)!} + \sum_{n=0}^{\infty} \frac{5i}{(2n)!}$$

$$\sum_{n=0}^{\infty} \frac{n}{(2n)!}$$

Apply Cauchy Hadamard formula

$$a_n = \frac{n}{(2n)!} \qquad a_{n+1} = \frac{n+1}{\{2(n+1)\}!}$$

$$R = \frac{1}{L^*} = \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right|$$

$$= \lim_{n\to\infty} \frac{\frac{n+1}{(2n+2)!}}{\frac{n}{(2n)!}}$$

$$= \lim_{n\to\infty} \frac{(n+1)(2n)!}{n(2n+2)(2n+1)2n!}$$

$$= \lim_{n\to\infty} \frac{n+1}{n(2n+2)(2n+1)}$$

$\boxed{R = \infty}$ R.S Converge for all z

$$\sum_{n=0}^{\infty} \frac{5i}{(2n)!}$$

$$a_n = \frac{5i}{(2n)!} \qquad a_{n+1} = \frac{5i}{(2n+2)!}$$

Using Cauchy Hadamard formula

$$R = \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right|$$

$$R = \lim_{n\to\infty} \left| \frac{\frac{5i}{(2n)!}}{\frac{5i}{(2n+2)!}} \right|$$

$$R = \lim_{n\to\infty} \frac{(\cancel{5i})(2n+2)(2n+1)\cancel{(2n)!}}{\cancel{5i}\,\cancel{(2n)!}}$$

$$R = \lim_{n\to\infty} (2n+2)(2n+1)$$

$\boxed{R = \infty}$ Converges for all values of z.

$$\sum_{n=0}^{\infty} \frac{n+5i}{(2n)!} = \sum_{n=0}^{\infty} \frac{n}{(2n)!} + \sum_{n=0}^{\infty} \frac{5i}{(2n)!}$$

$=$ Converges + Converges

$$\sum_{n=0}^{\infty} \frac{n+5i}{(2n)!} = \text{Converges}$$

$R = \infty$

Problem Set 15.2

(3) $\sum_{n=1}^{\infty} \frac{(z+i)^n}{n^2}$

Power Series General form

$\sum_{n=0}^{\infty} a_n (z-z_0)$

$z_0 =$ Center

So Here $z_0 = -i$

$a_n = \frac{1}{n^2}$

Now according to Cauchy-Hadamard formula

$$R = \frac{1}{L^*} = \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right|$$

Here $a_n = \frac{1}{n^2}$

$a_{n+1} = \frac{1}{(n+1)^2}$

$$R = \lim_{n\to\infty} \left| \frac{\frac{1}{n^2}}{1/(n+1)^2} \right|$$

$$R = \lim_{n\to\infty} \frac{(n+1)^2}{n^2}$$

$$= \lim_{n\to\infty} \frac{n^2+1+2n}{n^2}$$

$$= \lim_{n\to\infty} \frac{n^2\left(1+\frac{1}{n^2}+\frac{2}{n}\right)}{n^2}$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n^2}+\frac{2}{n}\right)$$

$$= 1$$

$\because$ Center $= -i$ Radius $= 1$

(5) $\sum_{n=0}^{\infty} \frac{n!}{n^n}(z+1)^n$

Sol: Here $z_0 = -1$, $a_n = \frac{n!}{n^n}$

Now according to Cauchy-Hadamard formula

$$R = \frac{1}{L^+} = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|$$

$$a_n = \frac{n!}{n^n}$$

$$a_{n+1} = \frac{(n+1)!}{(n+1)^{n+1}}$$

$$R = \lim_{n\to\infty} \left| \frac{\frac{n!}{n^n}}{\frac{(n+1)!}{(n+1)^{n+1}}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)^{n+1}\, n!}{(n+1)!\, n^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)^n \cdot \cancel{(n+1)^1}\, \cancel{n!}}{\cancel{(n+1)}\cancel{n!}\, n^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)^n}{n^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{n}\left(1+\frac{1}{n}\right)^n}{\cancel{n^n}} \right|$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$$

$$\boxed{R = e}$$

So Center $= -1$ and $R = e$

⑥ $\sum_{n=0}^{\infty} \frac{2^{100n}}{n!} z^n$

Sol:

Here $z_0 = 0$ $\quad a_n = \frac{2^{100n}}{n!}$

(By Cauchy-Hadamard formula)

$$R = \frac{1}{L^*} = \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right|$$

$$a_n = \frac{2^{100n}}{n!}$$

$$a_{n+1} = \frac{2^{100(n+1)}}{(n+1)!}$$

$$R = \lim_{n\to\infty} \left| \frac{\frac{2^{100n}}{n!}}{\frac{2^{100(n+1)}}{(n+1)!}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{2^{100n}(n+1)!}{2^{100n+100}\, n!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{2^{100n}(n+1)n!}{2^{100n} \cdot 2^{100}\, n!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n+1}{2^{100}} \right|$$

$\boxed{R = \infty}$ Converges for all values of z

(8) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2} z^{2n}$

Center: $z_0 = 0$

Sol: By Cauchy Hadamard formula

$$R = \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right|$$

Here

$$a_n = \frac{(-1)^n}{2^{2n}(n!)^2}$$

$$a_{n+1} = \frac{(-1)^{n+1}}{2^{2(n+1)}[(n+1)!]^2}$$

$$R = \lim_{n\to\infty} \left| \frac{\dfrac{(-1)^n}{2^{2n}(n!)^2}}{\dfrac{(-1)^{n+1}}{2^{2(n+1)}[(n+1)!]^2}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)^n\, 2^{2n}\cdot 2^2 [(n+1)!]^2}{(-1)^{n+1}\, 2^{2n}(n!)^2} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{(-1)^n}\, \cancel{2^{2n}}\cdot 2^2\, [(n+1)!]^2}{\cancel{(-1)^n}(-1)^1\, \cancel{2^{2n}}(n!)^2} \right|$$

$$= \lim_{n\to\infty} \frac{4}{-1} \left[\frac{(n+1)!}{n!} \right]^2$$

$$= \lim_{n\to\infty} \left| \frac{4}{-1} \left[\frac{(n+1)\cancel{n!}}{\cancel{n!}} \right]^2 \right|$$

$$= \lim_{n\to\infty} \left| \frac{4(n+1)^2}{-1} \right|$$

$$= \infty$$

The ~~re~~ series converges for all value of z.

Hence the center is 0 and radius is ∞