(13)

$$f(x) = \begin{cases} 0, & -\pi < x < -\pi/2 \\ 1, & -\pi/2 < x \; \pi/2 \\ 0, & \pi/2 < x < \pi \end{cases}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad \text{—(i)}$$

$$a_0 = \frac{1}{2\pi}\left(\int_{-\pi}^{-\pi/2} 0\,dx + \int_{-\pi/2}^{\pi/2} 1\,dx + \int_{\pi/2}^{\pi} 0\,dx\right)$$

$$a_0 = \frac{1}{2\pi}\left(0 + x\Big|_{-\pi/2}^{\pi/2} + 0\right)$$

$$a_0 = \frac{1}{2\pi}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)$$

$$a_0 = \frac{1}{2\pi}\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \frac{1}{2\pi}(\pi) = \boxed{\frac{1}{2}}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{-\pi/2} \cancelto{0}{(0)\cos nx\,dx} + \int_{-\pi/2}^{\pi/2} (1)\cos nx\,dx + \int_{\pi/2}^{\pi} \cancelto{0}{(0)\cos nx\,dx}\right].$$

$$a_n = \frac{1}{\pi}\left[\frac{\sin nx}{n}\Big|_{-\pi/2}^{\pi/2}\right] = \frac{1}{\pi}\left[\frac{\sin n\pi/2}{n} - \frac{\sin (n\text{–}}{n}\right.$$

$$\cancel{a_n = \frac{1}{\pi}\left[\frac{2}{\pi}\cdot\sin\left(\frac{n\pi}{2}\right) - \frac{2}{\pi}\sin\left(\frac{n\pi}{2}\right)\right]}$$

$$a_n = \frac{1}{n\pi}\left[\sin\frac{n\pi}{2} + \sin\frac{n\pi}{2}\right]$$

$$a_n = \frac{2\cdot\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$

$$b_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \sin nx \, dx = \boxed{0} \quad \left(\because \sin nx \text{ is odd ftion}\right)$$

$$\therefore \quad f(x) = \frac{1}{x} + \sum_{n=1}^{\infty}\left(\frac{2\cdot\sin\left(\frac{n\pi}{2}\right)}{n\pi}\cdot\cos n\pi + (0)\sin nx\right)$$

$$f(x) = \frac{1}{x} + \sum_{n=1}^{\infty}\left(\frac{2\cdot\sin\left(\frac{n\pi}{2}\right)}{n\pi}\cdot\cos(n\pi)\right)$$

22 $f(x) = x^2 \quad 0 < x < 2\pi$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n \sin nx\right) \quad \text{—(1)}$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x^2\,dx = \frac{1}{2\pi}\cdot\left.\frac{x^3}{3}\right|_0^{2\pi}$$

$$\quad\quad = \frac{1}{2\pi}\left[\frac{8\pi^3}{3} - 0\right]$$

$$\boxed{a_0 = \frac{4\pi^2}{3}}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x^2 . \cos nx \, dx$$

Integratsion by parts give

$$a_n = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{x^2 . \sin nx}{n} + \frac{2x . \cos nx}{n^2} - \frac{2 \sin nx}{n^3}\right)$$

$$a_n = \frac{1}{\pi}\left[\left(\frac{(2\pi)^2 . \sin n(2\pi)}{n} + \frac{2(2\pi) . \cos n(2\pi)}{n^2} - \frac{2 \sin(2n\pi)}{n^3}\right) - \left(\frac{(0) . \sin n(0)}{n} + \frac{2(0) . \cos(n(0))}{n^2} - \frac{2 \sin(0)}{n^3}\right)\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{4\pi^2 . (0)}{n} + \frac{4\pi(1)}{n^2} - \frac{2(0)}{n^3}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{4\pi}{n^2}\right] = \frac{4}{n^2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x^2 . \sin nx \, dx$$

$$b_n = \frac{-4\pi}{n}$$

(1) $\Rightarrow$ $f(x) = \frac{4\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{4}{n^2}\cos nx - \frac{4\pi}{n}\sin nx\right)$

P.S 11.2

Q 1 $f(x) = \begin{cases} -1 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases}$

$P = 2L$

$4 = 2L$

$L = 2$

Now

$a_0 = \frac{1}{2(2)}\int_{-2}^{2} f(x)\,dx$

$a_0 = \frac{1}{4}\left(\int_{-2}^{0} -1\,dx + \int_{0}^{2} 1\,dx\right)$

$a_0 = \frac{1}{4}\left[\left(-x\Big|_{-2}^{0}\right) + \left(x\Big|_{0}^{2}\right)\right]$

$a_0 = \frac{1}{4}\left[(-(0)) - (-(-2)) + (2-0)\right]$

$a_0 = \frac{1}{4}\left[-2+2\right] = \boxed{0}$

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$$a_n = \frac{1}{2}\int_{-2}^{2} (f(x)) \cos nx \, dx$$

$$a_n = \frac{1}{2}\left[\int_{-2}^{0} (-1) \cdot \cos nx \, dx + \int_{0}^{2} (1) \cos nx \, dx \right]$$

$$a_n = \frac{1}{2}\left[- \frac{\sin nx}{n}\Big|_{-2}^{0} + \frac{\sin nx}{n}\Big|_{0}^{2} \right]$$

$$a_n = \frac{1}{2}\left[- \frac{\sin(0)}{n} - \left(- \frac{\sin(-2n)}{n} \right) + \frac{\sin 2n}{n} + \frac{\sin(0)}{n} \right]$$

$$a_n = \frac{1}{2}\left[\frac{\sin(-2n)}{n} + \frac{\sin 2n}{n} + 0 \right]$$

$$a_n = \frac{1}{2}\left[- \cancel{\frac{\sin(2n)}{n}} + \cancel{\frac{\sin(2n)}{n}} \right]$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{2}\int_{-2}^{2} f(x) \cdot \sin nx \, dx$$

$$b_n = \frac{1}{2}\left[\int_{-2}^{0} - \sin nx \, dx + \int_{0}^{2} \sin nx \, dx \right]$$

$$b_n = \frac{1}{2}\left[\frac{\cos nx}{n}\Big|_{-2}^{0} + \left(- \frac{\cos nx}{n}\Big|_{0}^{2} \right) \right]$$

$$b_n = \frac{1}{2}\left[\frac{\cos 0}{n} - \frac{\cos(-2n)}{n} + \left(-\frac{\cos 2n}{n} - \left(-\frac{\cos 0}{n}\right)\right)\right]$$

$$b_n = \frac{1}{2}\left[\frac{1}{n} - \frac{\cos 2n}{n} - \frac{\cos 2n}{n} + \frac{1}{n}\right]$$

$$b_n = \frac{1}{2}\left[\frac{2}{n} - \frac{2\cos 2n}{n}\right]$$

$$b_n = \frac{1}{\cancel{2}} \cdot \cancel{2}\left(\frac{1}{n} - \frac{\cos 2n}{n}\right)$$

$$b_n = \frac{1-\cos 2n}{n}$$

$$\therefore \quad f(x) = 0 + \sum_{n=0}^{\infty}\left((0)\cos nx + \left(\frac{1-\cos 2n}{n}\right)\cdot \sin nx\right)$$

$$f(x) = \sum_{n=0}^{\infty}\left(\frac{1-\cos 2n}{n}\right)\cdot \sin nx$$

PS. 11.3

(11) $f(x) = \pi - |x| \quad -\pi < x < \pi$

f is even

$\therefore b_n = 0$

$P = 2\pi$

$L = \pi$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} (\pi - |x|)\,dx$$

$$a_0 = \frac{1}{\cancel{2}\pi}\cdot\cancel{2}\int_{0}^{\pi} (\pi - |x|)\,dx \quad (\because f(x) \text{ is even})$$

$$a_0 = \frac{1}{\pi}\left[\int_0^{\pi} \pi\,dx - \int_0^{\pi}(+x)\,dx\right]$$

$$a_0 = \frac{1}{\pi}\left[\pi x\Big|_0^{\pi} - \frac{x^2}{2}\Big|_0^{\pi}\right]$$

$$a_0 = \frac{1}{\pi}\left[\frac{\pi^2}{1} - \frac{\pi^2}{2}\right]$$

$$a_0 = \frac{1}{\pi}\left[\frac{2\pi^2 - \pi^2}{2}\right] \Rightarrow a_0 = \frac{1}{\cancel{\pi}}\cdot\frac{\pi^{\cancel{2}}}{2} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(\pi - |x|)(\cos nx)\,dx$$

Given integrand is also even
b/c even × even = even.

$$\therefore a_n = \frac{2}{\pi}\int_0^{\pi} (\pi - |x|) \cdot \cos nx \, dx$$

$$a_n = \frac{2}{\pi}\left[\int_0^{\pi} \pi \cdot \cos nx - \int_0^{\pi} x \cdot \cos nx \, dx\right]$$

$$a_n = \frac{2}{\pi}\left[\pi \cdot \frac{\sin nx}{n}\Big|_0^{\pi} - \left(\frac{x \cdot \sin nx}{n} + \frac{\cos nx}{n^2}\right)\Big|_0^{\pi}\right]$$

$$a_n = \frac{2}{\pi}\left[\frac{\pi \cdot \sin n\pi}{n} - \frac{\cancel{\pi \cdot \sin(0)}}{n}^{0} - \left(\left(\frac{\pi \cdot \sin n\pi}{n} + \frac{\cos n\pi}{n^2}\right) - \left(\frac{\cancel{0 \cdot \sin(0)}}{n}^{0} + \frac{\cos(0)}{n^2}\right)\right)\right]$$

$$a_n = \frac{2}{\pi}\left[\frac{\pi \cdot \sin n\pi}{n} - \left(\frac{\pi \cdot \sin n\pi}{n} + \frac{\cos n\pi}{n^2} + \frac{1}{n^2}\right)\right]$$

$$a_n = \frac{2}{\pi}\left[\cancel{\frac{\pi \cdot \sin n\pi}{n}} - \cancel{\frac{\pi \cdot \sin n\pi}{n}} - \frac{\cos n\pi}{n^2} - \frac{1}{n^2}\right]$$

$$a_n = \frac{2}{\pi}\left[-\frac{\cos n\pi}{n^2} - \frac{1}{n^2}\right]$$

(24) $f(x) = x^2 \quad 0 < x < L$

(a) $a_0 = \frac{1}{2L} \cdot \int_{-L}^{L} f(x)\, dx$

$a_0 = \frac{1}{2L} \cdot 2 \int_0^L x^2\, dx$

$a_0 = \frac{1}{L} \cdot \frac{x^3}{3} \Big|_0^L = \frac{1}{L} \cdot \frac{L^3}{3} = \boxed{\frac{L^2}{3}}$

$a_n = \frac{1}{L} \int_{-L}^{L} f(x) . \cos n x\, dx$

$a_n = \frac{2}{L} . \int_0^L x^2 . \cos n x\, dx$

$a_n = \frac{2}{L} \left(\frac{x^2 . \sin n x}{n} + \frac{2x . \cos n x}{n^2} - \frac{2 \sin n x}{n^3} \right) \Big|_0^L$

$a_n = \frac{2}{L} \left[\left(\frac{L^2 . \sin(nL)}{n} + \frac{2L . \cos(nL)}{n^2} - \frac{2 \sin(nL)}{n^3} \right) - \right.$

$$\left(0 + 0 - 2\frac{\sin(0)}{n^3}\right)\Big]$$

$$a_n = \frac{2}{L}\left[L^2 \cdot \frac{\sin(nL)}{n} + 2L \cdot \frac{\cos(nL)}{n^2} - \frac{2\sin(nL)}{n^3} \right]$$

$$a_n = \frac{2}{L}\left[\frac{L^2 \cdot \sin(nL) \cdot n^2 - 2 \cdot \sin(nL)}{n^3} + \frac{2L \cdot \cos(nL)}{n^2} \right]$$

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