$z^2 - 3z = 0$
$z(z-3) = 0$
$z = 0, z = 3$

P.S 14.3

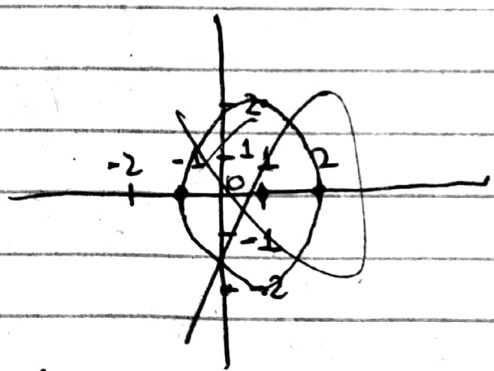
⑤ $\oint \frac{z+2}{z-2} dz$, $C: |z-1| = 2$

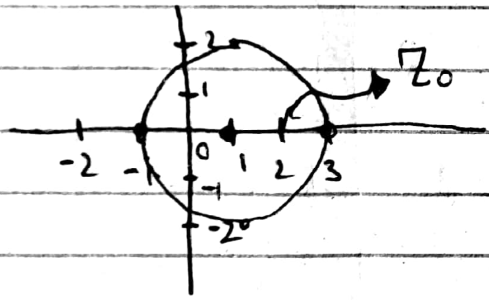
$f(z) = z+2$
$z_0 = 2$

$\boxed{f(z_0) = 4}$

By Using Cauchy-Integral Theorem.

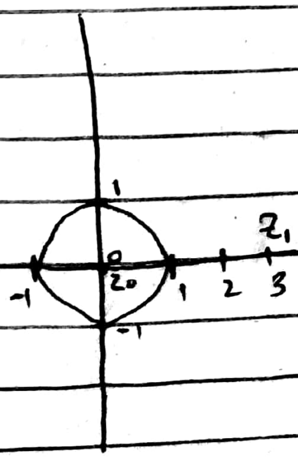
$\oint \frac{z+2}{z-2} dz = 2\pi i(4) = \boxed{8\pi i}$





⑦ $\oint \frac{\sinh \pi z}{z^2 - 3z} dz$, $C: |z| = 1$

Ans:-
$z^2 - 3z = 0$
$z(z-3) = 0$
$z_0 = 0, \ z_1 = 3$

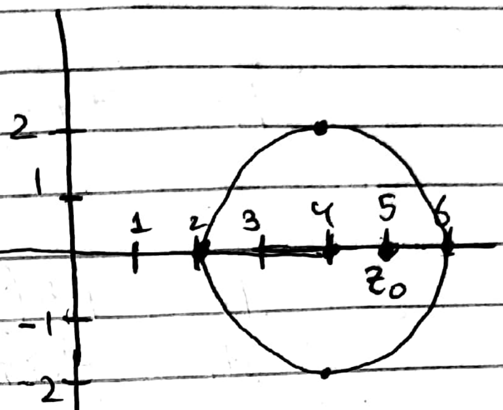


$\oint \frac{\sinh \pi z}{z^2 - 3z} = 2\pi i(\sinh 0)$

$" = 2\pi i(0) = \boxed{0}$

(15) $\oint \frac{\ln(z-1)}{(z-5)} dz \quad , \quad C: |z-4| = 2$

$z_0 = 5$



$$\boxed{\oint \frac{\ln(z-1)}{z-5} = 2\pi i(\ln(4))}$$

Derivatives Of An Analytic Function:-

$$\boxed{f^n(z_0) = \frac{n!}{2\pi i} \oint \frac{f(z)}{(z-z_0)^{n+1}} dz}$$

OR

$$\boxed{f^n(z_0) \cdot \frac{2\pi i}{n!} = \oint \frac{f(z)}{(z-z_0)^{n+1}} dz}$$

Example 1:-

$$\oint \frac{\cos z}{(z-\pi i)^2} = f'(\pi i) \cdot \frac{2\pi i}{1!}$$

$$\begin{aligned} // &= -\sin(\pi i) \cdot 2\pi i \\ // &= -i \sinh \pi \cdot 2\pi i \\ // &= -i^2 \sinh \pi \cdot 2\pi \\ // &= \boxed{\sinh \pi \cdot 2\pi} \end{aligned}$$

Example 2:-

$$\oint \frac{z^4 - 3z^2 + 6}{(z+i)^3} dz$$

$f'(z) = 4z^3 - 6z$
$f''(z) = 12z^2 - 6$

$$11 = \frac{2\pi i \cdot f''(-i)}{2!}$$

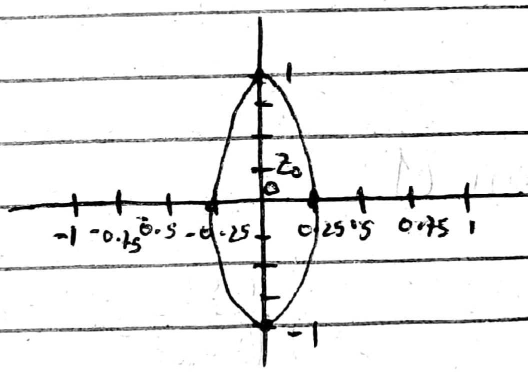
$$11 = \frac{\cancel{2}\pi i \cdot (12(-i)^2 - 6)}{\cancel{2}}$$

$$11 = \pi i(12(-1) - 6)$$
$$11 = \cancel{-12\pi i}\ \cancel{6\pi i}\ \boxed{-18\pi i}$$

PROBLEM SET 14.4

(11) $\frac{\tan \pi z}{z^2}$ $\quad C$: the ellipse $16x^2 + y^2 = 1$

Ans:-



$$\oint_C \frac{\tan \pi z}{(z-0)^2} = \frac{2\pi i\, f'(0)}{1!}$$

$$11 = 2\pi i(\tan(0)) = 2\pi i(0) = \boxed{0}$$

REVIEW EXERCISE

(16) $4z^3 + 2z$ from $-i$ to $2+i$ along any path.

Ans:- using 1st Evaluation Method

$$\int_c f(z)\,dz = \int_{-i}^{2+i} (4z^3 + 2z)\,dz$$

$$\text{"} = \left[\frac{\cancel{4}z^4}{\cancel{4}} + \frac{\cancel{2}z^2}{\cancel{2}}\right]_{-i}^{2+i}$$

$$\text{"} = \left[z^4 + z^2\right]_{-i}^{2+i}$$

$$\text{"} = \left[(2+i)^4 + (2+i)^2\right] - \left[(-i)^4 + (-i)^2\right]$$

$$\text{"} = (2+i)^2\{(2+i)^2 + 1\} - \left[i^4 + i^2\right]$$

$$\text{"} = (4 - 1 + 4i)\{4 - \cancel{1} + 4i + \cancel{1}\} - [\cancel{1} - \cancel{1}]$$

$$\text{"} = (3 + 4i)(4 + 4i)$$

$$\text{"} = 12 + 12i + 16i + 16i^2$$

$$\text{"} = 12 + 28i - 16 = -4 + 28i$$

(23) $$\int_c f(z)\,dz = \int_0^{2i} \cosh 4z\,dz$$

$$\text{"} = \left[\frac{\sinh 4z}{4}\right]_0^{2i}$$

$$\text{"} = \frac{\sinh 8i}{4} - \cancel{\frac{\sinh 0}{4}}^{\,0}$$

$$\text{"} = \boxed{\frac{i\sin 8}{4}}$$

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