either

$a = +2$ or $a = -2$

Q5

(PART A)

Determine whether given set toghether with given operations is a vector space. The set of all positive real numbers U with $U \oplus V = UV$ and $c \odot U^c = U^c$

SOLUTION:-

let $V = (x, y)$

$U = (x_1, y_1)$ for all $x_1, y_1 \geq 0$

$V = (x_2, y_2)$ for all real numbers

$U \oplus V = UV = (x_1, y_1)(x_2, y_2) = (x_1 x_2, y_1 y_2)$

but here $x_1 x_2$ and $y_1 y_2$ are ≥ 0 here

~~not~~ closed under operation $\oplus$

$c \odot U^c = U^c$

$c \odot U = (x_1, y_1)^c = U^c$

but c here lies in the $\mathbb{R}$ real numbers + and -ve but x_1 and y_1 are only +ve numbers even when we take c as negative power it will still be greater than 0 which is a positive numbere hence closed under scalar mutiplication

"PART B"

$p(t) = a_2t^2 + a_1t + a_0 \quad a_0 = 2$

$p(t) = a_2t^2 + a_1t + 2$

$q(t) = a_2't^2 + a_1't + a_0'$

$p(t) \oplus q(t) = a_2t^2 + a_1t + a_0 + a_2't^2 + a_1't + a_0'$

$= (a_2 + a_2')t^2 + (a_1 + a_1')t + a_0 + a_0'$

$= (a_2 + a_2')t^2 + (a_1 + a_1')t + 4$

Not closed under the operation $\oplus$

$c \odot p(t) = c(a_2t^2 + a_1t + a_0)$

$= ca_2t^2 + ca_1t + ca_0$

$= ca_2t^2 + ca_1t + 2c$

$= ca_2t^2 + ca_1t + 2c$

Hence there are no extra coefficiets added or changed hence closed under $\odot$ - Hence not a subspace

"PART C"

Verify which of the following subsets of R^3 are subspaces of R^3

ci) $(a, b, 2)$

$U = (a_1, b_1, 2) \quad V = (a_2, b_2, 2)$

$U \oplus V = (a_1 + a_2, b_1 + b_2, 2 + 2)$

$U \oplus V = (a_1 + a_2, b_1 + b_2, 4)$ Not closed under $\oplus$

$c \odot U = c(a_1, b_1, 2) = (ca_1, cb_1, 2c)$

closed under $\odot$. Not a subspace

M T W T F S

(ii)

(a, b, c) where $c = a+b$

$u = (a_1, b_1, c_1)$ and $v = (a_2, b_2, c_2)$

$u \oplus v = (a_1+a_2, b_1+b_2, c_1+c_2)$

$u \oplus v = (a_1+a_2, b_1+b_2, a_1+a_2+b_1+b_2)$

Hence closed under operation $\oplus$

let

$c \odot u = c(a, b, c)$

$= (ca, cb, cc)$

$= (ca, cb, c(a+b))$

$= (ca, cb, ca+cb)$

Hence closed under opertion $\odot$

This is a subspace

(iii)

(a, b, c) where $c > 0$

$u = (a_1, b_1, c_1)$, $v = (a_2, b_2, c_2)$

$u \oplus v = (a_1+a_2, b_1+b_2, c_1+c_2)$

Hence closed under operation $\oplus$

$c \odot u = c(a_1, b_1, c_1)$

$= (ca_1, cb_1, cc_1)$

Hence it is closed under operation $\odot$

This is a subspace of $\mathbb{R}^3$

Q6

(a)

Discuss basis for a vector space.

Let $S = \{v_1, v_2, \ldots, v_n\}$ be the subset of Vector space V. The S is said to be a basis set of V iff.

(a) Space $S = V$ (b) S is linearly independent

Then vectors $v_1, v_2, \ldots, v_n$ will be called as basis of V

(b)

Does the following set of vectors form a basis for R^3

$\{(3,2,2),(-1,2,1),(0,1,0)\}$

by Formula

$c_1v_1 + c_2v_2 + c_3v_3 = V$

$(0,0,0) = c_1(3,2,2) + c_2(-1,2,1) + c_3(0,1,0)$

$(0,0,0) = (3c_1, 2c_1, 2c_1) + (-c_2, 2c_2, c_2) + (0, c_3, 0)$

$(0,0,0) = (3c_1 - c_2, 2c_1 + 2c_2 + c_3, 2c_1 + c_2)$

$3c_1 - c_2 = 0$

$2c_1 + 2c_2 + c_3 = 0$

$2c_1 + c_2 = 0$

$$\left[\begin{array}{ccc|c} 3 & -1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 2 & 2 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 3 & -1 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 3 & -1 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 2 & 2 & 1 & 0 \end{array}\right]$$

$R_3 - 2R_1, R_2 - 3R_1$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \frac{R_2}{-4}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad R_1 - R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Hence $c_1 = c_2 = c_3 = 0$

Linearly independent

Thus set of vectors ~~span~~ form inside the basis for $\mathbb{R}^3$

"Q7"

Find the characteristic polynomial eigen value and eigen vectors of matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ 3 & 2 & -2 \end{bmatrix}$$