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$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad \text{--- (i)}$$

$$\sin t = \frac{1}{2i}(e^{it} - e^{-it}) \quad , \quad \cos t = \frac{1}{2}(e^{it} + e^{-it})$$

Let $t = nx$

$$\sin nx = -\frac{i}{2}(e^{inx} - e^{-inx}) \quad , \quad \cos t = \frac{1}{2}(e^{inx} + e^{-inx})$$

$$a_n \cos nx + b_n \cdot \sin nx = b_n\left[-\frac{i}{2}(e^{inx} - e^{-inx})\right] + a_n\left[\frac{1}{2}(e^{inx} + e^{-inx})\right]$$

$$\text{''} \quad = -\frac{i}{2} b_n \cdot e^{inx} + \frac{i}{2} b_n e^{-inx} + \frac{a_n}{2} \cdot e^{inx} + \frac{a_n}{2} e^{-inx}$$

$$\text{''} \quad = e^{inx}\left(\frac{1}{2}a_n - \frac{ib_n}{2}\right) + e^{-inx}\left(\frac{1}{2}a_n + \frac{i b_n}{2}\right)$$

$$a_n \cos nx + b_n \sin nx = \frac{1}{2}(a_n - ib_n)e^{inx} + \frac{1}{2}(a_n + ib_n)e^{-inx}$$

Put in (i)

$$f(x) = a_0 + \sum^{\infty} \left(\frac{1}{2}(a_n - ib_n)e^{inx} + \frac{1}{2}(a_n + ib_n)e^{-inx}\right)$$

Let $a_0 = c_0$, $\frac{1}{2}(a_n - ib_n) = c_n$

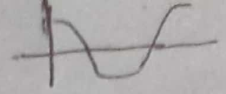
$\frac{1}{2}(a_n + ib_n) = k_n$

~~$f(x) = c_0 + \sum_{n=1}^{\infty}$~~

$$\therefore f(x) = c_0 + \sum_{n=1}^{\infty} \left(c_n e^{inx} + k_n e^{-inx} \right)$$

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, e^{-inx}\, dx$$

$$k_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, e^{inx}\, dx$$



Example ①:-

0 for all n

$$e^{\pm in\pi} = \cos n\pi \pm i\,\cancel{\sin n\pi}$$

$$\cos n\pi = -1 \quad \text{for odd } n$$
$$\cos n\pi = 1 \quad \text{for even } n$$

$$\therefore \cos n\pi = (-1)^n \rightarrow \boxed{e^{\pm in\pi} = (-1)^n}$$

Now

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^x \cdot e^{-inx}\,dx$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{x-inx}\,dx$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{x(1-in)}\,dx$$

$$C_n = \frac{1}{2\pi}\cdot \left.\frac{e^{x(1-in)}}{1-in}\right|_{-\pi}^{\pi}$$

$$C_n = \frac{1}{2\pi}\cdot\frac{1}{1-in}\cdot\left[e^{\pi(1-in)} - e^{-\pi(1-in)}\right]$$

$$C_n = \frac{1}{2\pi}\cdot\frac{1}{1-in}\cdot\left[e^{\pi-i\pi n} - \frac{1}{e^{\pi-n\pi i}}\right]$$

$(-1)^n$ $\quad$ $(-1)^n$

$$C_n = \frac{1}{2\pi}\cdot\frac{1}{1-in}\cdot\left[e^{\pi}\cdot e^{-n\pi i} - \frac{e^{n\pi i}}{e^{\pi}\cdot}\right]$$

$$C_n = \frac{1}{2\pi}\cdot\frac{1}{1-in}\left[e^{\pi}(-1)^n - e^{-\pi}\cdot(-1)^n\right]$$

$$\sinh \pi = \frac{e^{\pi} - e^{-\pi}}{2}$$

$P = 2L$

$\cancel{2\pi} = \cancel{2L}$

$$C_n = \frac{1}{2\pi} \frac{1}{1-in} (-1)^n \left[e^{\pi} - e^{-\pi} \right]$$

$$C_n = \frac{1}{2\pi} \frac{1}{1-in} \times \frac{1+in}{1+in} \left[2 \sinh \pi \right]$$

$$C_n = \frac{1}{2\pi} \cdot \frac{1+in}{(1)^2 - (in)^2} \cdot 2 \sinh \pi$$

$$C_n = \frac{1}{2\pi} \cdot \frac{1+in}{1+n^2} \cdot 2 \sinh \pi$$

Now $$f(x) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{\frac{in\pi x}{L}}$$

$L = \pi$

$$\therefore e^x = \sum_{n=-\infty}^{\infty} \left(\frac{\cancel{2} \sinh \pi}{\cancel{2}\pi} \cdot \frac{1+in}{1+n^2} \right) e^{\frac{in\cancel{\pi}x}{\cancel{\pi}}}$$

$$e^x = \sum_{n=-\infty}^{\infty} \left(\frac{\sinh \pi}{\pi} \cdot \frac{1+in}{1+n^2} \right) e^{inx}$$

$$e^x = \frac{\sinh \pi}{\pi} \cdot \sum_{n=-\infty}^{\infty} \frac{(1+in)\, e^{inx}}{1+n^2}$$

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(7) $f(x) = -1 \quad -\pi < x < 0$
$f(x) = 1 \quad 0 < x < \pi$

Ans:-

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx} \quad \text{—(i)}$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, e^{-inx}\, dx$$

$$C_n = \frac{1}{2\pi}\left(\int_{-\pi}^{0} -e^{-inx}\, dx + \int_{0}^{\pi} e^{-inx}\, dx\right)$$

$$C_n = \frac{1}{2\pi}\left(-\int_{-\pi}^{0} e^{-inx}\, dx + \int_{0}^{\pi} e^{-inx}\, dx\right)$$

$$C_n = \frac{1}{2\pi}\left(-\left.\frac{e^{-inx}}{-in}\right|_{-\pi}^{0} + \left.\frac{e^{-inx}}{-in}\right|_{0}^{\pi}\right)$$

$$C_n = \frac{1}{2\pi}\left(\left.\frac{e^{-inx}}{in}\right|_{-\pi}^{0} - \left.\frac{e^{-inx}}{in}\right|_{0}^{\pi}\right)$$

$$C_n = \frac{1}{2\pi}\left(\left[\frac{e^{0}}{in} - \frac{e^{n\pi i}}{in}\right] - \left[\frac{e^{-n\pi i}}{in} - \frac{e^{0}}{in}\right]\right)$$

$$C_n = \frac{1}{2\pi}\left(\frac{1}{in} - \frac{e^{n\pi i}}{in} - \frac{e^{-n\pi i}}{in} + \frac{1}{in}\right)$$

$$C_n = \frac{1}{2\pi}\left(\frac{2}{in} - \frac{(-1)^n}{in} - \frac{(-1)^n}{in}\right)$$

$$C_n = \frac{1}{2\pi}\left(\frac{2}{in} - \frac{2(-1)^n}{in}\right)$$

$$C_n = \frac{1}{\cancel{2}\pi}\left(\frac{\cancel{2}(1-(-1)^n)}{in}\right)$$

$$C_n = \frac{1-(-1)^n}{in\pi} = \frac{(-1+(-1)^n)i}{n\pi} \quad \left(\because \frac{1}{i} = -i\right)$$

$$C_n = \begin{cases} \frac{-2i}{n\pi} & \text{if } n = \pm 1, \pm 3, \pm 5, \text{----} \\ 0 & \text{if } n = 0, \pm 2, \pm 4, \text{----} \end{cases}$$

(i) $\Rightarrow$ $$f(x) = \sum_{n=-\infty}^{\infty} \frac{i}{n\pi}\left[-1+(-1)^n\right] e^{inx}$$

$$'' = \sum_{n=-\infty}^{\infty} \frac{i}{(2n+1)\pi} \cdot \left[-1+(-1)^{2n+1}\right] e^{(2n+1)ix}$$

$$'' = \sum_{n=-\infty}^{\infty} \frac{i}{(2n+1)\pi}\left[-1-1\right] e^{(2n+1)ix}$$

$$'' = \frac{-2i}{\pi} \sum_{n=-\infty}^{\infty} \frac{e^{i(2n+1)x}}{(2n+1)}$$

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$$C_n = \frac{1}{2\pi}\left[-\frac{e^{-inx}}{in} + \frac{e^{-inx}}{n^2}\right]_{-\pi}^{\pi}$$

$$C_n = \frac{1}{2\pi}\left[\left(-\frac{e^{-n\pi i}}{ni} + \frac{e^{-n\pi i}}{n^2}\right) - \left(-\frac{e^{n\pi i}}{ni} + \frac{e^{n\pi i}}{n^2}\right)\right]$$

$$C_n = \frac{1}{2\pi}\left[-\frac{e^{-n\pi i}}{ni} + \frac{e^{-n\pi i}}{n^2} + \frac{e^{n\pi i}}{ni} - \frac{e^{n\pi i}}{n^2}\right]$$

$$C_n = \frac{1}{2\pi}\left[\frac{-e^{-n\pi i} + e^{n\pi i}}{ni} + \frac{e^{-n\pi i} - e^{n\pi i}}{n^2}\right]$$

$$C_n = \frac{1}{2\pi}\left[\frac{-(-1)^n + (-1)^n}{ni} + \frac{(-1)^n - (-1)^n}{n^2}\right]$$

---

(13) $f(x) = x, \quad 0 < x < 2\pi$?

$p = 2\pi$

$2L = 2\pi$

$L = \pi$

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx}$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} (x)\, e^{-inx}\, dx$$

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$$C_n = \frac{1}{2\pi}\left[\frac{-x e^{-inx}}{in} + \frac{e^{-inx}}{n^2}\right]_{-\pi}^{\pi}$$

$$C_n = \frac{1}{2\pi}\left[\left(\frac{-\pi e^{-n\pi i}}{in} + \frac{e^{-n\pi i}}{n^2}\right) - \left(\frac{\pi \cdot e^{n\pi i}}{in} + \frac{e^{n\pi i}}{n^2}\right)\right]$$

$$C_n = \frac{1}{2\pi}\left[\frac{-\pi(-1)^n}{in} + \cancel{\frac{(-1)^n}{n^2}} - \frac{\pi \cdot (-1)^n}{in} - \cancel{\frac{(-1)^n}{n^2}}\right]$$

$$C_n = \frac{1}{\cancel{2}\pi}\left[\frac{-\cancel{2}\pi(-1)^n}{in}\right]$$

$$C_n = \frac{-\pi(-1)^n}{in} = \frac{\pi i(-1)^n}{n}$$

$$\therefore \quad f(x) = \sum_{n=-\infty}^{\infty} \frac{\pi i(-1)^n}{n} \cdot e^{inx}$$

$$f(x) = \pi i \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{n} \cdot e^{inx}$$

P.S 11.7

(1) $\int_0^\infty \frac{\cos x\omega + \omega \sin x\omega}{1+\omega^2} d\omega = \begin{cases} 0 & \text{if } x<0 \\ \pi/2 & \text{if } x=0 \\ \pi e^{-x} & \text{if } x>0 \end{cases}$

Let $f(x) = \begin{cases} 0 & \text{if } x<0 \\ \pi e^{-x} & \text{if } x>0 \end{cases}$

$$f(x) = \int_0^\infty \left(A(\omega) \cdot \cos(\omega x) + B(\omega) \cdot \sin \omega x \right) d\omega$$

$$A(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(v) \cdot \cos(\omega v)\, dv$$

$$= \frac{1}{\pi} \int_0^\infty \pi \cdot e^{-v} \cdot \cos \omega v\, dv$$

$$= \frac{\pi}{\pi} \int_0^\infty e^{-v} \cdot \cos \omega v\, dv$$

$$A(\omega) = \lim_{b \to \infty} \int_0^b e^{-v} \cdot \cos \omega v\, dv$$

Integration by parts give

$$A(\omega) = \lim_{b \to \infty} \left[\frac{1}{1+\omega^2} \left(e^{-v} (\sin \omega v \cdot \omega - \cos \omega v) \right) \right]_0^b$$

$$A(\omega) = \lim_{b \to \infty} \frac{1}{1+\omega^2} \left[e^{-b} (\sin \omega b \cdot \omega - \cos \omega b) - \right.$$

$$e^{0}\left(\sin 0 . \omega - \cos 0\right)\Big]$$

$$A(\omega) = \lim_{b \to \infty} \frac{1}{1+\omega^2}\Big[e^{-b}\left(\sin\omega b . \omega - \cos\omega b\right) - 1(0-$$

$$A(\omega) = \lim_{b \to \infty} \frac{1}{1+\omega^2}\Big[e^{-b}\left(\sin\omega b . \omega - \cos\omega b\right) + 1$$

$$A(\omega) = \frac{1}{1+\omega^2}\Big[\frac{1}{e^{\infty}}\left(\sin(\omega \times \infty) . \omega - \cos(\omega \times \infty$$

$$A(\omega) = \frac{1}{1+\omega^2}\Big[\frac{1}{\infty}\left(\quad " \quad \right) + 1$$

$$A(\omega) = \frac{1}{1+\omega^2}\left[0 + 1\right] = \frac{1}{1+\omega^2}$$

$$B(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(v) . \sin(\omega v)\, dv$$

$$B(\omega) = \frac{1}{\pi}\int_{0}^{\infty} \pi . e^{-v} . \sin(\omega v)\, dv$$

$$B(\omega) = \lim_{b \to \infty}\left[\frac{-\omega . e^{-v} . \cos(\omega v) - e^{-v}\sin(\omega v)}{\omega^2+1}\right]_0^b$$

$$B(\omega) = \lim_{b \to \infty}\Big[\frac{-\omega e^{-b} . \cos(b\omega) - e^{-b} . \sin(\omega b)}{\omega^2 + \ldots} -$$

$$\left. - \frac{-\omega e^{0} \cdot \cos(0) - e^{0}(\sin(0))}{\omega^2+1} \right]$$

$$B(\omega) = \lim_{b \to \infty} \left[\frac{-\omega \cdot e^{-\infty} \cdot \cos(\infty\omega) - e^{-\infty} \sin(\infty b)}{\omega^2+1} - \frac{-\omega - 0}{\omega^2+1} \right]$$

$$B(\omega) = \left[\cancel{0} - \frac{-\omega}{\omega^2+1} \right) = \frac{\omega}{\omega^2+1}$$

$$\therefore \quad f(x) = \int_0^\infty \left[\left(\frac{1}{1+\omega^2} \right) \cdot \cos(\omega x) + \left(\frac{\omega}{\omega^2+1} \right) \cdot \sin(\omega x) \right] d\omega$$

$$f(x) = \int_0^\infty \left[\frac{\cos(\omega x) + \omega \cdot \sin(\omega x)}{\omega^2+1} \right] d\omega$$

(2) $$\int_0^\infty \frac{\sin\omega - \omega\cos\omega}{\omega^2} \cdot \sin x\omega \, d\omega = \begin{cases} \frac{1}{2}\pi x & 0 < x < 1 \\ \pi/4 & x = 1 \\ 0 & x > 1 \end{cases}$$

Ans: The integrand is odd

for $0 < x < 1$, $\therefore$ $\boxed{A(\omega) = 0}$

$$B(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(v).\sin\omega v\,dv$$

$$'' = \frac{2}{\pi}\int_{0}^{\infty} f(v).\sin\omega v\,dv$$

$$'' = \frac{2}{\pi}\left[\int_{0}^{1} f(v).\sin\omega v\,dv + \int_{1}^{\infty} 0.\sin\omega v\,dv\right]$$

$$'' = \frac{2}{\pi}\left[\int_{0}^{1} \frac{1}{2}\pi v.\sin(\omega v)\,dv + 0\right]$$

$$'' = \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\cancel{\pi}}{\cancel{2}}\int_{0}^{1} v.\sin(\omega v)\,dv\right]$$

$$B(\omega) = \int_{0}^{1} v.\sin(\omega v)\,dv$$

Solving by integration by parts.

$$B(\omega) = \left[\frac{-\omega.\cos(\omega v).v + \sin(\omega v)}{\omega^2}\right]_0^1$$

$$B(\omega) = \frac{-\omega.\cos(\omega).1 + \sin(\omega)}{\omega^2} - \frac{-\omega.\cos(0).0 + \sin(0)}{\omega^2}$$

$$B(\omega) = \frac{-\omega.\cos(\omega) + \sin(\omega)}{\omega^2} - 0$$

Now $$f(x) = \int_0^\infty \left[A(\omega)\cdot\cos(\omega x) + B(\omega).\sin(\omega x)\right] d\omega$$

$$f(x) = \int_0^\infty \left[0 . \cancel{\cos(\omega x)} + \left(\frac{-\omega.\cos(\omega) + \sin(\omega)}{\omega^2}\right).\sin(\omega x)\right] d\omega$$

$$f(x) = \int_0^\infty \left(\frac{-\omega.\cos(\omega) + \sin(\omega)}{\omega^2}\right)\sin(\omega x)\, d\omega$$

$$f(x) = \int_0^\infty \left(\frac{\sin(\omega) - \omega.\cos(\omega)}{\omega^2}\right)\sin(\omega x)\, d\omega$$

(7) $$f(x) = \begin{cases} 1, & 0 < x < a \\ 0, & x > a \end{cases}$$

$$f(x) = \int_0^\infty A(\omega)\cos(\omega x)\, d\omega$$

$$A(\omega) = \frac{2}{\pi}\int_0^\infty f(v).\cos(\omega v)\, dv$$

$$A(\omega) = \frac{2}{\pi}\left[\int_0^a (1).\cos(\omega v) + \int_a^\infty 0 .\cos(\omega v) dv\right]$$

$$A(\omega) = \frac{2}{\pi}\left[\int_0^a \cos(\omega v)\, dv + 0\right]$$

$$A(\omega) = \frac{2}{\pi} . \left.\frac{\sin \omega v}{\omega}\right|_0^a$$

$$A(\omega) = \frac{2}{\pi}\left[\frac{\sin(\omega a)}{\omega} - \frac{\sin(0)}{\omega}\right]$$

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$$A(\omega) = \frac{2}{\pi}\left[\frac{\sin(\omega a)}{\omega} - 0\right]$$

$$A(\omega) = \frac{2}{\pi} \cdot \frac{\sin(\omega a)}{\omega}$$

$$\therefore \quad f(x) = \int_0^\infty \frac{2}{\pi} \frac{\sin(\omega a)}{\omega} \cos(\omega x)\, d\omega$$

$$f(x) = \frac{2}{\pi} \int_0^\infty \frac{\sin(\omega a)}{\omega} \cdot \cos(\omega x)\, d\omega$$

(9) $f(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$

Similar to Q7 just
Replace a by 1 in Q7.

we get

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{\sin(a)}{\omega} \cos(\omega x)\, d\omega$$

(11) $f(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & x > \pi \end{cases}$

$$A(\omega) = \frac{2}{\pi}\int_0^\infty f(v) \cdot \cos(\omega v)\, dv$$

$$A(\omega) = \frac{2}{\pi}\left[\int_0^\pi \sin(v) \cdot \cos(\omega v)\, dv + \int_\pi^\infty (0) \cdot \cos(\omega v)\, dv\right]$$

$$A(\omega) = \frac{2}{\pi}\left[\int_0^{\pi} \sin(v)\cdot\cos(\omega v) + 0\right]$$

We know that

$$\sin A.\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

Here $A = v$, $B = \omega v$

$$\therefore \quad A(\omega) = \frac{\cancel{2}}{\pi}\int_0^{\pi}\left\{\frac{1}{\cancel{2}}\sin(v+\omega v) + \sin(v-\omega v)\right]$$

$$A(\omega) = \frac{1}{\pi}\int_0^{\pi}\left[\sin v(\omega+1) + \sin v(1-\omega)\right] dv$$

$$A(\omega) = \frac{1}{\pi}\left[\int_0^{\pi}\sin v(\omega+1) + \int_0^{\pi}\sin v(1-\omega)\right]$$

$$A(\omega) = \frac{1}{\pi}\left\{\frac{-\cos v(\omega+1)}{\omega+1}\Bigg|_0^{\pi} + \frac{-\cos v(1-\omega)}{1-\omega}\Bigg|_0^{\pi}\right]$$

$$A(\omega) = \frac{1}{\pi}\left[\left\{\left(\frac{-\cos\pi(\omega+1)}{\omega+1}\right) - \left(\frac{-\cos 0}{\omega+1}\right)\right\} + \left\{\left(\frac{-\cos\pi(1-\omega)}{1-\omega}\right) - \left(\frac{-\cos 0}{1-\omega}\right)\right\}\right]$$

$$A(\omega) = \frac{1}{\pi}\left[\left\{\frac{-\cos\pi(\omega+1)}{\omega+1} - \frac{-1}{\omega+1}\right\} + \left\{\frac{-\cos\pi(1-\omega)}{1-\omega} - \frac{-1}{1-\omega}\right\}\right]$$

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$$A(\omega) = \frac{1}{\pi}\left[\frac{-\cos \pi(\omega+1) + 1}{\omega+1} + \frac{-\cos(\pi(1-\omega)) + 1}{1-\omega}\right]$$

$$A(\omega) = \frac{1}{\pi}\left[\frac{(1-\omega)\left(-\cos(\pi(1+\omega)) + 1\right) + (1+\omega)\left(-\cos(\pi(1-\omega)) + 1\right)}{1-\omega^2}\right]$$

$$A(\omega) = \frac{1}{\pi}\left[\frac{-\cos(\pi(1+\omega)) + 1 + \omega\cos(\pi(1+\omega)) - \cancel{\omega} - \cos(\pi(1-\omega)) + 1 - \omega\cdot\cos(\pi(1-\omega)) + \cancel{\omega}}{1-\omega^2}\right]$$

$$A(\omega) = \frac{1}{\pi}\left[\frac{-\cos(\pi(1+\omega)) - \cos(\pi(1-\omega)) + \omega\cdot\cos(\pi(1+\omega)) - \omega\cdot\cos(\pi(1-\omega)) + 2}{1-\omega^2}\right]$$

$$A(\omega) = \frac{1}{\pi}\left[\frac{-(-\cos(\pi\omega)) - (-\cos(\pi\omega)) + \omega(-\cos(\pi\omega)) - \omega\cdot(-\cos(\pi\omega)) + 2}{1-\omega^2}\right]$$

$$A(\omega) = \frac{1}{\pi}\left[\frac{\cos\pi\omega + \cos\pi\omega - \cancel{\omega\cdot\cos(\pi\omega)} + \cancel{\omega\cos(\pi\omega)} + 2}{1-\omega^2}\right]$$

$$A(\omega) = \frac{1}{\pi}\left[\frac{2\cdot\cos\pi\omega + 2}{1-\omega^2}\right] = \frac{2(\cos\pi\omega + 1)}{\pi(1-\omega^2)}$$

$$\therefore f(x) = \int_0^\infty A(\omega)\cdot\cos\omega x \, d\omega$$

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$$f(x) = \frac{1}{\pi}\int_0^\infty \frac{2(1+\cos(\omega\pi))}{1-\omega^2}\, d\omega$$

(17) $f(x) = \begin{cases} 1 & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$

$$f(x) = \int_0^\infty B(\omega) \cdot \sin \omega x \, d\omega$$

$$B(\omega) = \frac{2}{\pi}\int_0^\infty f(v) \cdot \sin \omega v \, dv$$

$$B(\omega) = \frac{2}{\pi}\left[\int_0^a 1 \cdot \sin \omega v \, dv + \int_a^\infty 0 \cdot \sin \omega v \, dv\right]$$

$$B(\omega) = \frac{2}{\pi}\left[\int_0^a \sin \omega v \, dv + 0\right]$$

$$B(\omega) = \frac{2}{\pi}\left(\left. \frac{-\cos \omega v}{\omega} \right|_0^a\right)$$

$$B(\omega) = \frac{2}{\pi}\left(\frac{-\cos(a\omega)}{\omega} - \frac{-\cos(0)}{\omega}\right)$$

$$B(\omega) = \frac{2}{\pi}\left[\frac{-\cos(a\omega)}{\omega} + \frac{1}{\omega}\right]$$

$$B(\omega) = \frac{2}{\pi}\left[\frac{1-\cos(a\omega)}{\omega}\right]$$

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$$\therefore f(x) = \int_0^{\infty} \frac{2}{\pi} \left[\frac{1 - \cos(a\omega)}{\omega} \right] . \sin \omega x \, d\omega$$

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \left[\frac{1 - \cos(a\omega)}{\omega} \right] . \sin \omega x \, d\omega$$

17. $f(x) = \begin{cases} \pi - x, & 0 < x < \pi \\ 0, & x > 0 \end{cases}$?

$$f(x) = \int_0^{\infty} B(\omega) . \sin \omega x \, d\omega$$

$$B(\omega) = \frac{2}{\pi} \int_0^{\infty} f(v) . \sin \omega v \, dv$$

$$B(\omega) = \frac{2}{\pi} \left[\int_0^{\pi} (\pi - v) \sin \omega v \, dv + \int_{\pi}^{\infty} (0) \sin \omega v \, dv \right]$$

$$B(\omega) = \frac{2}{\pi} \left[\int_0^{\pi} (\pi . \sin \omega v - v . \sin \omega v \, dv) + 0 \right]$$

$$B(\omega) = \frac{2}{\pi} \left[\pi . \int_0^{\pi} \sin \omega v - \int_0^{\pi} v . \sin \omega v \, dv \right]$$

$$B(\omega) = \frac{2}{\pi} \left[\pi \left(- \frac{\cos \omega v}{\omega} \Big|_0^{\pi} \right) - \left(\frac{\sin(\omega v) - \omega . v . \cos(\omega v)}{\omega^2} \right) \Big|_0^{\pi} \right]$$