Question 1

(a) Linear transformation:-

A Linear transformation L of R^n into R^m is a function assigning a unique vector $L(u)$ in R^m to each u in R^n and every scalar k.

(a) $L(u+v) = L(u) + L(v)$
(b) $L(ku) = k[L(u)]$

Linear operator.
(1) Reflection with respect to the X-axis.
(2) Projection into the xy-plane.
(3) Dilation

(b) Sketch Image

Counterclock wise

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Rotation point $P(-1,3)$

Let $\theta = 90$

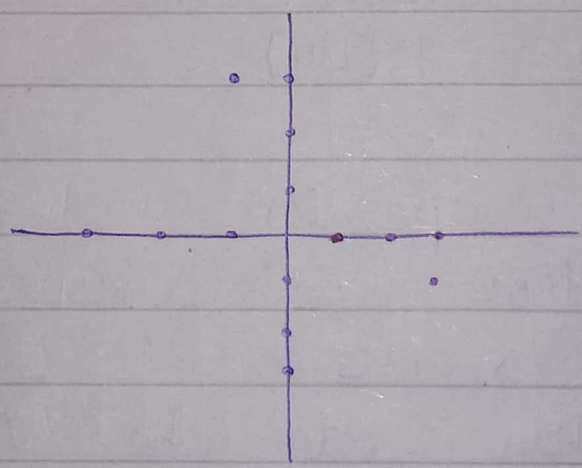
$$L = \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix} \qquad U = (-1, 3)$$

$$L = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$LU = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 - (-1 \times 3) \\ (1 \times -1) - 0 \end{bmatrix}$$

$$L(U) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$



Question 2

(a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

Code the message

SEND HIM MONEY

$\underbrace{19\ 5\ 14}_{x_1}\ \underbrace{4\ 8\ 9}_{x_2}\ \underbrace{13\ 13\ 15}_{x_3}\ \underbrace{14\ 5\ 25}_{x_4}$

$$\Rightarrow Ax_1 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix}$$

$$Ax_1 = \begin{bmatrix} 19+10+42 \\ 19+5+28 \\ 0+19+28 \end{bmatrix}$$

$$Ax_1 = \begin{bmatrix} 71 \\ 52 \\ 37 \end{bmatrix}$$

$$\Rightarrow Ax_2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} 4+16+27 \\ 4+8+18 \\ 0+8+18 \end{bmatrix} \Rightarrow \begin{bmatrix} 47 \\ 30 \\ 26 \end{bmatrix}$$

$$\Rightarrow Ax_3 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix}$$

$$= \begin{bmatrix} 13+26+45 \\ +3+13+30 \\ 0+13+30 \end{bmatrix} \Rightarrow \begin{bmatrix} 64 \\ 56 \\ 43 \end{bmatrix}$$

$$\Rightarrow AX_4 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 14 \\ 5 \\ 25 \end{bmatrix}$$

$$= \begin{bmatrix} 14+10+75 \\ 14+5+50 \\ 0+5+50 \end{bmatrix} \Rightarrow \begin{bmatrix} 99 \\ 69 \\ 55 \end{bmatrix}$$

Thus the message coded is as

71 52 37 47 30 26 64 56 43 99 69 55

(b) Decode the message

$\underbrace{64\ 44\ 41}_{V_1}$ $\underbrace{49\ 39\ 19}_{V_2}$ $\underbrace{113\ 76\ 62}_{V_3}$ $\underbrace{104\ 69\ 55}_{V_4}$

$L(x) = AX = V$

$AX = V$

$X = A^{-1}V$

$$\Rightarrow X_1 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 67 \\ 44 \\ 41 \end{bmatrix}$$

$$= \begin{bmatrix} 0+44-41 \\ 134-88-41 \\ -67+44+41 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ 5 \\ 18 \end{bmatrix}$$

$$\Rightarrow X_2 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 49 \\ 39 \\ 19 \end{bmatrix}$$

$$= \begin{bmatrix} 0+39-19 \\ 98-78-19 \\ -49+39+19 \end{bmatrix} \Rightarrow \begin{bmatrix} 20 \\ 1 \\ 9 \end{bmatrix}$$

$$\Rightarrow X_3 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 113 \\ 76 \\ 62 \end{bmatrix}$$

$$= \begin{bmatrix} 0+76-62 \\ 226-152-62 \\ -113+76+62 \end{bmatrix} \Rightarrow \begin{bmatrix} 14 \\ 12 \\ 25 \end{bmatrix}$$

$$\Rightarrow X_4 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 104 \\ 69 \\ 55 \end{bmatrix}$$

$$= \begin{bmatrix} 0+69-55 \\ 208-138-55 \\ -104+69+55 \end{bmatrix} \Rightarrow \begin{bmatrix} 14 \\ 15 \\ 20 \end{bmatrix}$$

Hence the message decoded

13

3 5 18 20 1 9 14 12 25

14 15 20

CERTAINLY NOT

Question 3

(a) Point $(2,4,-3)$ and is parallel $-2x+4y-5z+6=0$

Sol

$x_0=2$, $y_0=4$, $z_0=-3$

$a=-2$, $b=4$, $c=-5$, $d=6$

We know that

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$-2(x-2)+4(y-4)-5(z+3)=0$

$-2x+4+4y-16-5z-15=0$

$-2x+4y-5z-27=0$

(b) Find parametric equation $P(-2,3,4)$ and perpendicular to the line passing through the points $(3,-2,4)$ and $(0,3,4)$

Sol

$x=x_0+at$

$y=y_0+bt$

$z=z_0+ct$

$\overrightarrow{P_1P_2}=(0-3,3+2,4-4)$

$\overrightarrow{P_1P_2}=(-3,5,0)$

$U=\overrightarrow{P_1P_2}=(-3,5,0)$ $a=-3, b=5, c=0$

$P_0(x_0,y_0,z_0)=(-2,3,4)$ $x_0=-2, y_0=3, z_0=4$

$x=-2-3t$

$y=3+5t$

$z=4$

Q4:- (a) Let $L: R^3 \longrightarrow R^3$ be the linear transformation defined by

$$L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Is $W = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ in range L.

Sol

$$L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -x+2y \\ x+y+z \\ 2x-y+z \end{bmatrix}$$

$$L(v) = W$$

$$\begin{bmatrix} -x+2y \\ x+y+z \\ 2x-y+z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$-x+2y = 1 \quad \text{—— (i)}$$
$$x+y+z = 2 \quad \text{—— (ii)}$$
$$2x-y+z = -1 \quad \text{—— (iii)}$$

Add eq (ii) and eq (iii)

$$x+\cancel{y}+z = 2$$
$$2x-\cancel{y}+z = -1$$
$$\underline{+ \quad - \quad +}$$
$$3x+2z = 1 \quad \text{—— (iv)}$$

Multiply (2) eq (iii) and Add eq (i)

$$-x+\cancel{2y} = 1$$
$$4x-\cancel{2y}+2z = -2$$
$$3x+2z = -1 \quad \text{—— (v)}$$

Subtract eq (iv) from eq (v)

$$\cancel{3x} + \cancel{2z} = -1$$
$$\cancel{3x} + \cancel{2z} = 1$$
$$0 = -2$$

w is not in range at
L.

Question No 5

(a) Determine whether the given set together with the given operation $U \oplus V = UV$ and $c \odot U = U^c$

Sol: (α) $U \oplus V = UV$

(a) $U \oplus V = UV$ — (i)

$V \oplus U = VU$ — (ii)

$\boxed{L.H.S = R.H.S}$

(b) $U \oplus (V \oplus W) = U \oplus (VW)$

$U \oplus (V \oplus W) = UVW$ — (i)

$(U \oplus V) \oplus W = (UV) \oplus W$

$(U \oplus V) \oplus W = UVW$ — (ii)

$\boxed{L.H.S = R.H.S}$

(c) $U \oplus 0 = U$ — (i)

$0 \oplus U = U$ — (ii)

$U \oplus 0 = 0 \oplus U = U$

$\boxed{L.H.S = R.H.S}$

(d) $U \oplus -U = \cancel{U} - \cancel{U}$

$U \oplus -U = 0$ — (i)

$0 = 0$ — (ii)

$\boxed{L.H.S = R.H.S}$

Scanned with CamScanner

(e) $c \odot u = u^c$

(e) $c \odot (u \oplus v) = c \odot (uv)$

$c \odot (u \oplus v) = (uv)^c$ — (i)

$c \odot u \oplus c \odot v = u^c \oplus v^c$

$c \odot u \oplus c \odot v = (uv)^c$ — (ii)

$\boxed{L.H.S = R.H.S}$

(f) $(c \oplus d) \odot u = u^{c \oplus d}$ — (i)

$c \odot u \oplus d \odot u = u^c \oplus u^d$

$c \odot u \oplus d \odot u = u^{cd}$ — (ii)

$\boxed{L.H.S = R.H.S}$

(g) $c \odot (d \odot u) = c \odot (u^d)$

~~$c \odot (d \odot u) = u^{cd}$ — (i)~~

$c \odot (d \odot u) = (u^{cd})$ — (i)

$cd \odot u = u^{cd}$ — (ii)

$\boxed{L.H.S = R.H.S}$

(h) $1 \odot u = u^1$ — (i)

$u = u$ — (ii)

$\boxed{L.H.S = R.H.S}$

(b) Determine whether the given Subset of is a subspace.
$a_2t^2 + a_1t + a_0$ where $a_0 = 2$

Sol:- let $w = \{ a_2t^2 + a_1t + a_0 \,/\, a_2, a_1, a_0 \in \mathbb{R} \}$
but $a_0 = 2$

$u(t)$ and $v(t)$ be in w

where $a_0 = 2$

$a_0' = 2$

$u(t) = a_2t^2 + a_1t + a_0$

$v(t) = a_2't^2 + a_1't + a_0'$

$u(t) \oplus v(t) = (a_2+a_2', a_1+a_1', a_0+a_0')$

$= (a_2+a_2', a_1+a_1', 2+2)$

$u(t) \oplus v(t) = (a_2+a_2', a_1+a_1', 4) \notin W$

$a_0 = 2$

$a_0 + a_0' \neq 2$

w is not closed under p.

w is not subspace of p.

(c) Verify which of the following subsets of R^3 are subspace of R^3

(i) $(a, b, 2)$

Let $w = \{a, b, 2 \mid a, b \in \mathbb{R}\}$

$u = [a, b, 2]$

$v = [a', b', 2]$

$u \oplus v = [a+a', b+b', 4] \notin w$

So w is not vector space.

(ii) (a, b, c) where $c = a+b$

Let $w = \{a, b, c \mid a, b, c \in \mathbb{R}\}$ but $c = a+b$

$u = \{ a, b, c]$ $\quad c = a + b$

$v = [a', b', c']$ $\quad c' = a' + b'$

Cond 1:- $u \oplus v = [a + a', b + b', c + c']$

$u \oplus v = [a + a', b + b', a + a' + b + b'] \in W$

$$c = a + b$$

$$c + c' = a + b + a' + b'$$

$$c + c' = a + b + a' + b'$$

$$c + c' = c + c'$$

Cond 2:- $k \odot u = (ka, kb, kc)$

$k \odot u = [ka, kb, k(a + b)] \in W$

$$kc = k(a + b)$$

So W is a subspace of V.

(iii) (a, b, c) where $c > 0$

$W = \{ a, b, c \mid a, b, c \in \mathbb{R} \}$ but $c > 0$

$u = [a, b, c]$ $\quad c' > 0$

$v = [a', b', c']$

cond 1:- $u \oplus v = [a + a', b + b', c + c'] \in W$

As $c_1 + c_2 > 0$