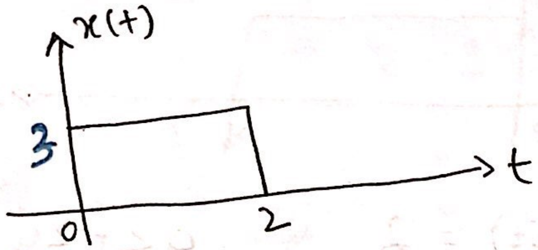
The concepts of signals

Time Scaling :→ The compression or expansion of signals in time.

$$x(t) \xrightarrow{T.S} y(t) = x(\alpha t) \quad \alpha \neq 0$$

Case 1 $\quad |\alpha| > 1$

i.e $\quad \alpha \in (-\infty, -1) \cup (1, \infty)$

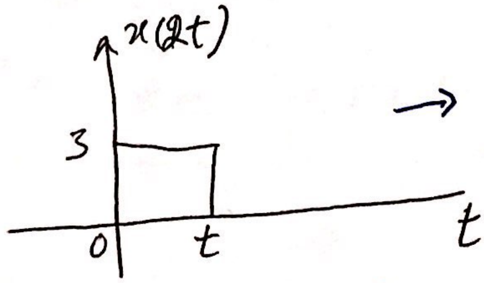


Q :→ find $x(2t)$? $\quad$ i.e., $\alpha = 2$

when $t = 0 \quad x(0) = 3$ and $x(2(0)) = x(0) = 3$

" $t = 1 \quad x(1) = 3$ and $x(2\times1) = x(2) = 3$

" $t = 2 \quad x(2) = 3$ and $x(2\times2) = x(4) = 0$

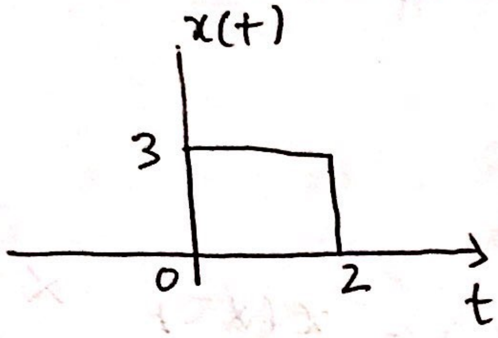


→ So $|\alpha| > 1$ corresponds to Compression

→ It leaves the amplitude unchanged.

Case II $\quad 0 < |\alpha| < 1$

$\alpha \in (-1, 0) \cup (0, 1)$



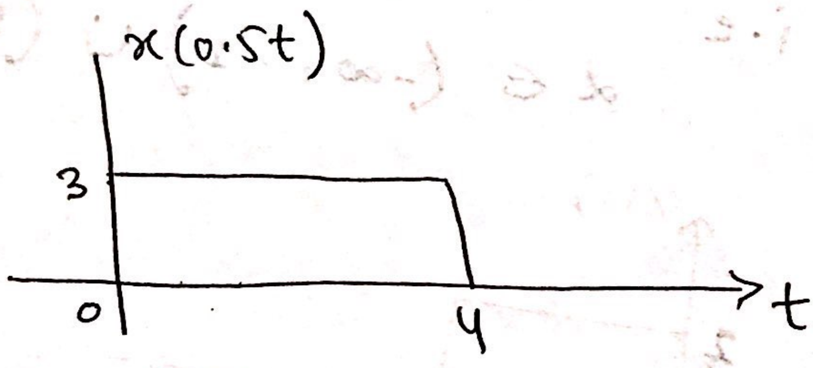
Q :-> Find $x(0.5t)$?

i.e., $\alpha = 0.5$

$\frac{2}{\alpha} = \frac{2}{0.5} = \frac{20}{5} = 4$

$\frac{0}{\alpha} = 0$

Keep Amplitude Same



$x(0.5t) = 3$ if $0 \leq t \leq 4$

Because we need to keep amplitude same

=> This case belongs to expansion of signals

Amplitude Scaling :→

Multiplying amplitude of a signal by a real number

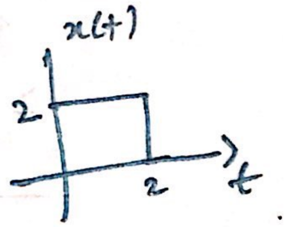
$$x(t) \xrightarrow[\beta]{A \cdot S} y(t) = \beta x(t)$$

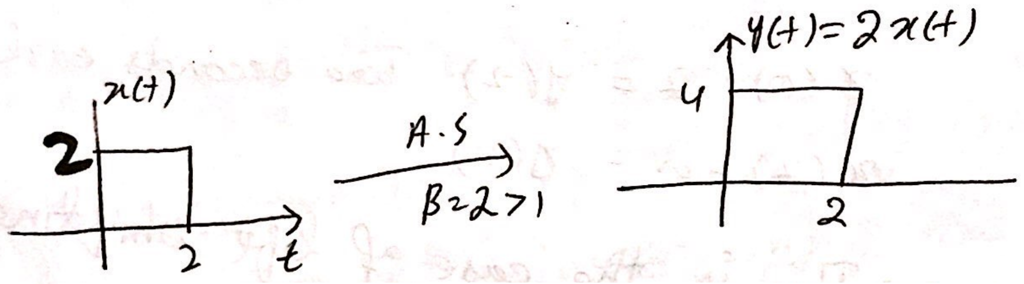
Case I :→ $|\beta| > 1$

i.e., $\beta \in (-\infty, -1) \cup (1, \infty)$

let

$$x(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \le t \le 2 \\ 0 & t > 2 \end{cases}$$



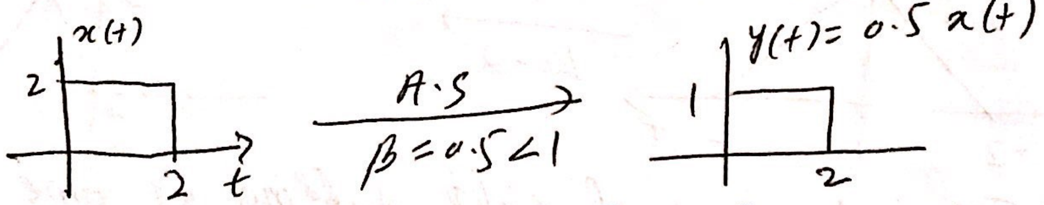


=> This is the case of amplification

=> There is no compression or expansion

Case II :→ $|\beta| < 1$

i.e., $\beta \in (-1, 0) \cup (0, 1)$



→ This is case of Reduction.

Shifting of CTS

① Time Shifting :→

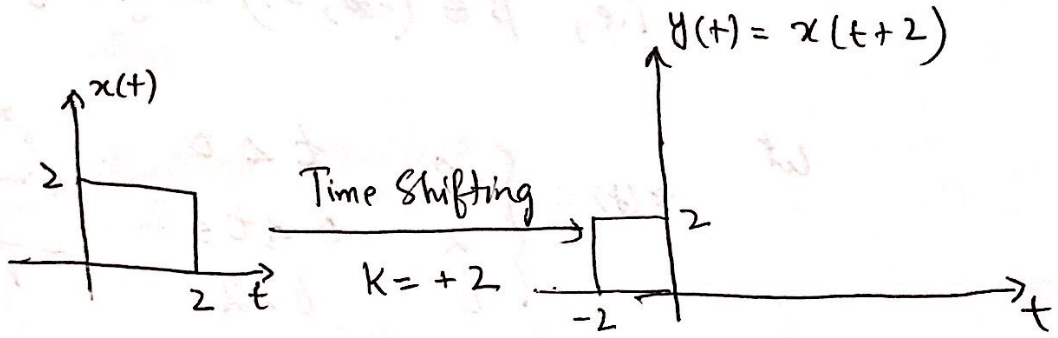
Adding ctt to the index

~~x(t)~~

$$x(t) \xrightarrow[k]{T.\text{ shifting}} y(t) = x(t+\underbrace{k}_{ctt})$$

Adding the time will make the event to occur earlier.

Case I :→ $K > 0$



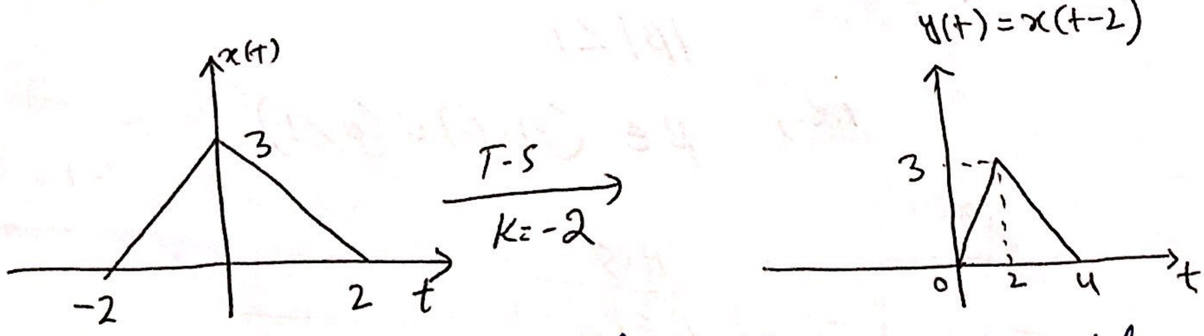
$x(0) = 2 = y(-2)$ Two seconds earlier

$x(2) = 2 = y(0)$

→ This is the case of left shifting or time advance.

Case II $K < 0$

everything is delayed by 2 sec



→ This is the case of right shifting or time delay.

⇒ In case of time shifting the shape of the waveform is unchanged. However, it is either shifted to the right ($K<0$, time delay) or to the left ($K>0$, time advance).

Amplitude Shifting

$$x(t) \xrightarrow[K]{\text{A-Shifting}} y(t) = x(t) + K$$

The difference b/w amplitude scaling ($y(t) = \beta x(t)$) and amplitude shifting ($y(t) = x(t) + \beta$) is that in case of amplitude shifting the original waveform will change however in the amplitude scaling the shape of the waveform doesn't change.

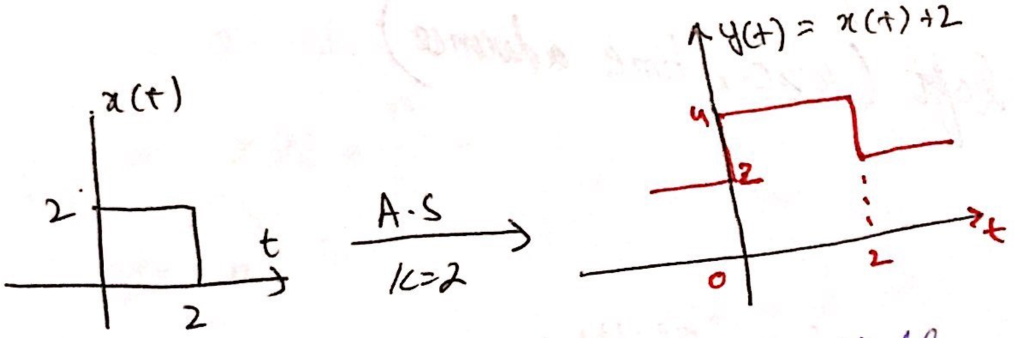
Case I

$K > 0$

let $K = +2$ and $x(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \le t \le 2 \\ 0 & t > 2 \end{cases}$

then

$$y(t) = x(t) + 2 = \begin{cases} 2 & t < 0 \\ 4 & 0 \le t \le 2 \\ 2 & t > 2 \end{cases}$$



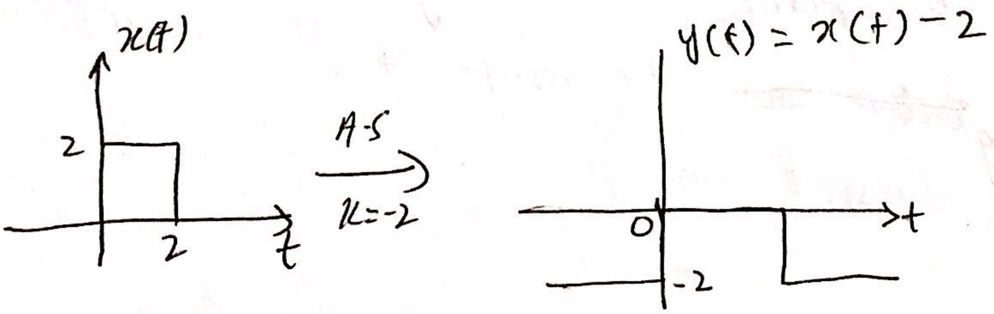
This is the case of upward. shifting

Case II

$K < 0$

let $K = -2$ then $y(t) = x(t) - 2$

$$y(t) = x(t) - 2 = \begin{cases} -2 & t < 0 \\ 0 & 0 \le t \le 2 \\ -2 & t > 0 \end{cases}$$



=> This is the case of downward shifting

Reversal of CTS

There are two types of reversals.

① Time Reversal ② Amplitude reversal

↓

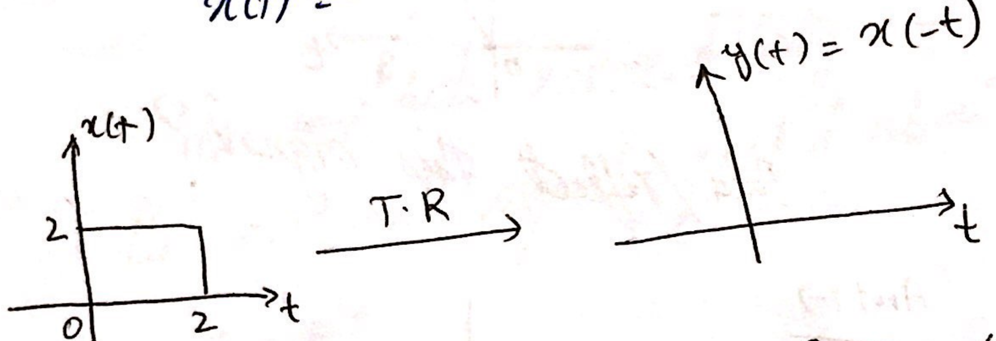
Special case of time scaling with

$\alpha = -1$

Time Scaling ← $x(t) \xrightarrow[\alpha]{T.S} y(t) = x(\alpha t)$

So in time reversal $\alpha = -1$

$$x(t) \xrightarrow{T.R} y(t) = x(-t)$$



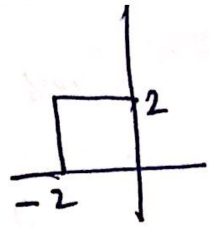
$$x(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \le t \le 2 \\ 0 & t > 0 \end{cases} \Rightarrow y(t) = \left.\begin{cases} 0 & -t < 0 \\ 2 & 0 \le -t \le 2 \\ 0 & -t > 0 \end{cases}\right\} \rightarrow ①$$

↙

note since it is time scaling with $\alpha = -1$ so the amplitude remains the same.

⇒ In ① dividing both sides of the inequality will flip the inequal sign.

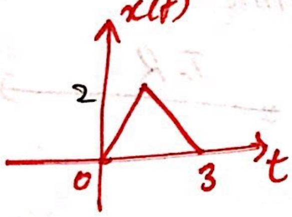
$$y(t) = \begin{cases} 0 & t > 0 \\ 2 & 0 \geq t \geq -2 \\ 0 & t < -2 \end{cases}$$



i.e., each time only fold the original sig about the y-axis.

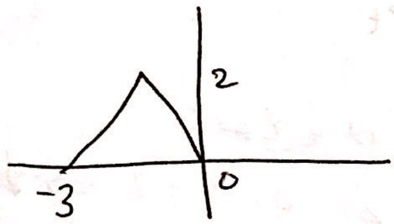
→ Time reversal is also known as folding or reflection.

Example



Fold / reflect this signal?

Ans :→



Amplitude Reversal :→ This is a special case of amplitude scaling with $\beta = -1$

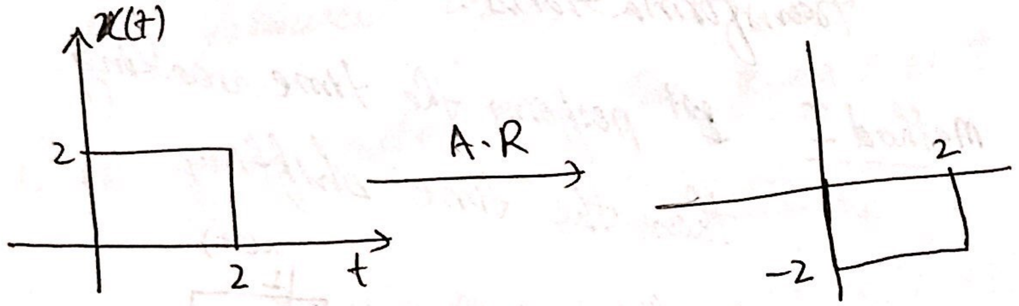
Amplitude Scaling $x(t) \xrightarrow{A \cdot S} y(t) = \beta x(t)$

Hence ~~magnitud~~ amplitude Reversal is

$$x(t) \xrightarrow{A \cdot R} y(t) = -x(t)$$

→ Recall that in case of amplitude scaling the time remains the same however, the amplitude change.

$$x(t) = \begin{cases} 0 & t<0 \\ 2 & 0 \le t \le 2 \\ 0 & t>2 \end{cases} \xrightarrow{A.R} y(t) = -x(t) = \begin{cases} 0 & t<0 \\ -2 & 0 \le t \le 2 \\ 0 & t>2 \end{cases}$$



i.e the amplitude reversed sig is the original signal folded about the x-axis.

Multiple transformations of CTS

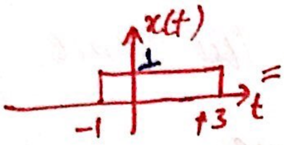
→ Scaling
→ Shifting
→ Reveral.

Priority order:

(1) Time Reveral
(2) Time shifting
(3) Time Scaling.

Amplitude scaling can be performed at any time

Example



$= x(t) \rightarrow$ System $\rightarrow y(t) = x(2t+3)$

Ans :-> $y(t) = x(2t + 3)$

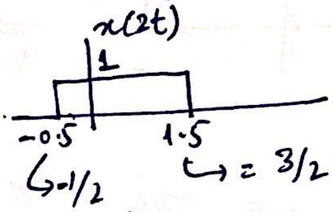
↓ Time scaling

↳ Time shifting

hence this is the case of multiple transformations.

<u>Method I</u> 1st perform the time scaling then the time shifting

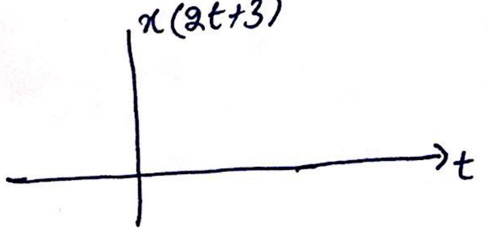
$x(t) \xrightarrow{\text{T. Scaling}} x(2t)$



↓ note compression.

Next we perform time shifting to $x(2t)$

$x(2t) \xrightarrow{\text{T. shifting}} x(2t+3)$

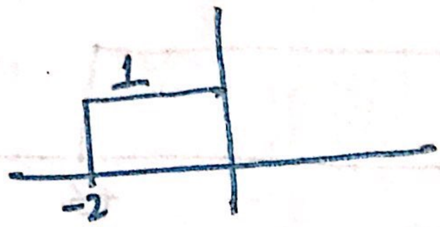


imp: here note that the x-axis is t not 2t and hence just shifting the $x(2t)$ sig 3 units to the left will give a wrong answer.

→ So we'll have to separate t

i.e.,

$$x(2t+3) = x(2(t+1.5))$$

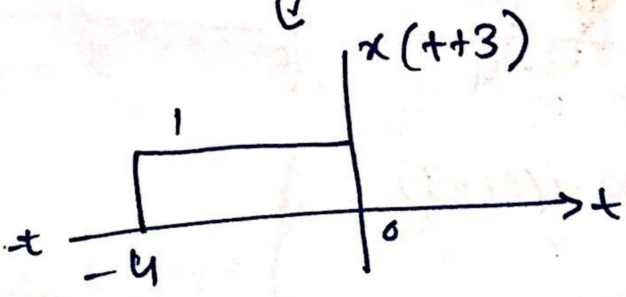


Answer.

Method II :→ Perform the time shifting 1st followed by time scaling.

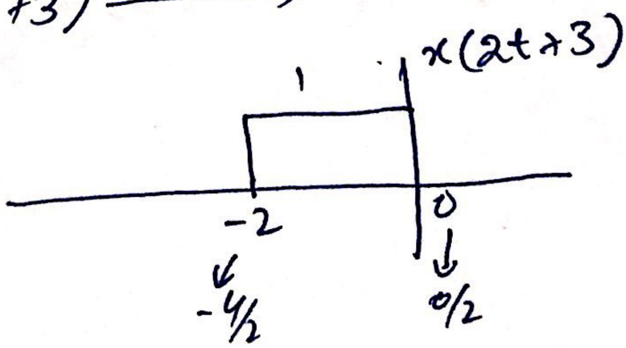
→ Once again note that the x-axis is t

so $x(t) \xrightarrow{\text{T. Shift}} x(t+3)$ not $x(2t+3)$

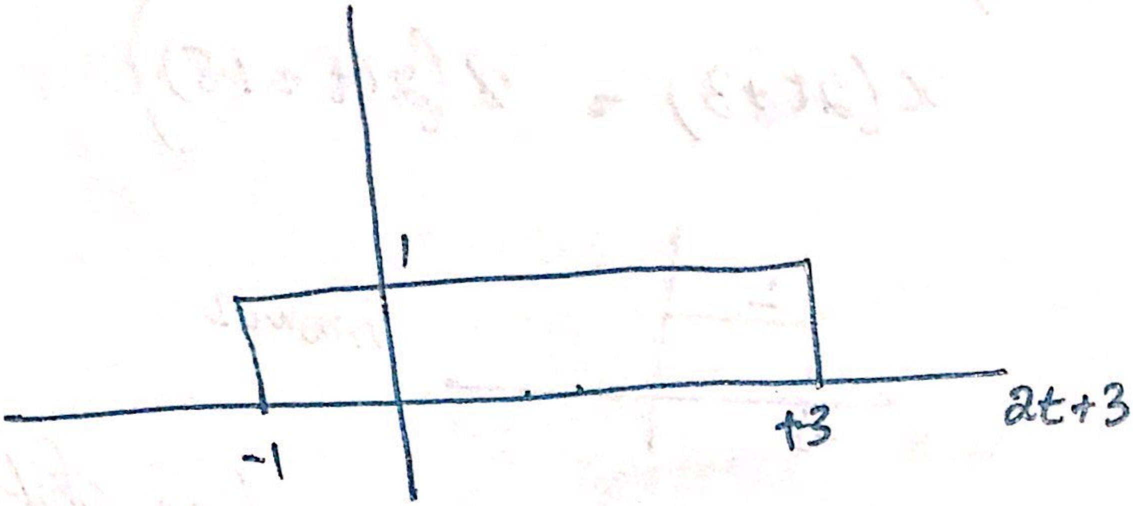


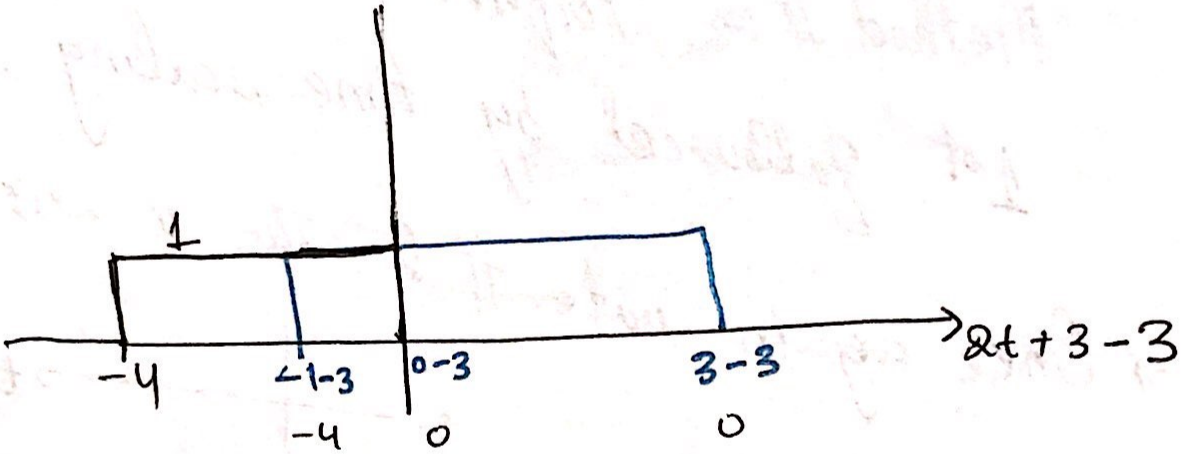
Now

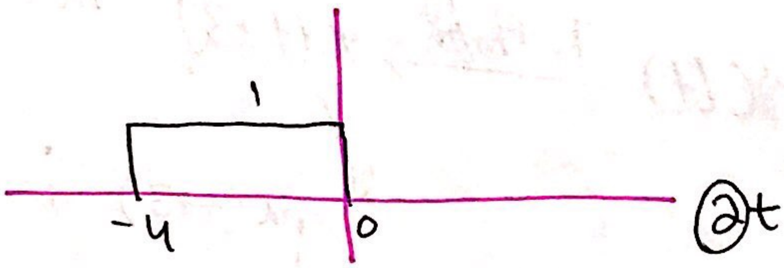
$x(t+3) \xrightarrow{\text{T. Scali}} x(2t+3)$

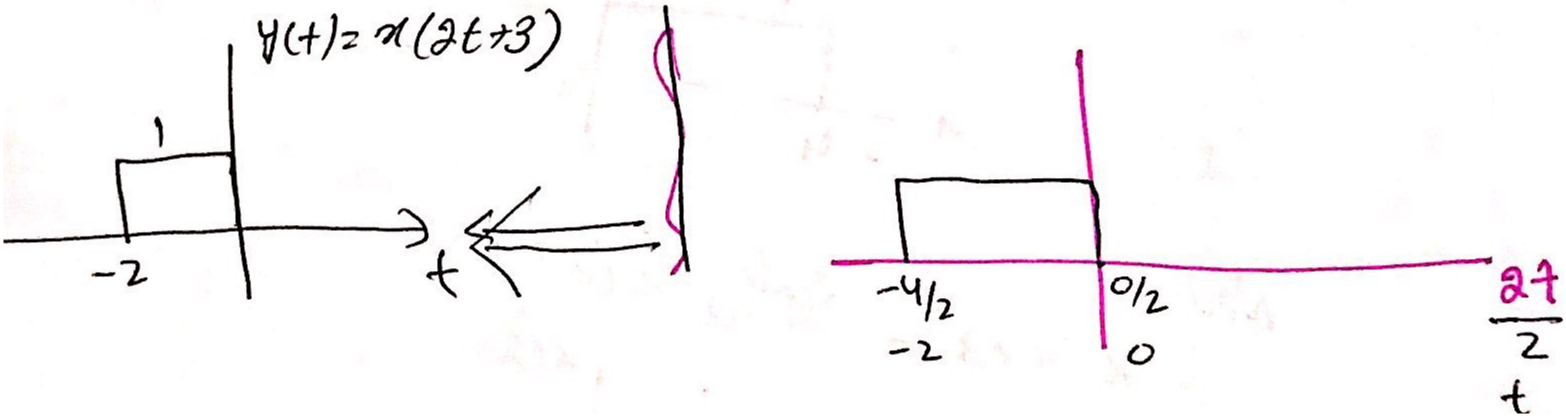


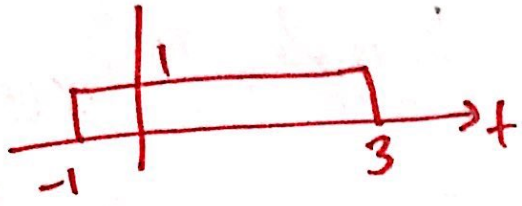
Method III Shortcut Method.









Q:→ if $x(t) =$ 

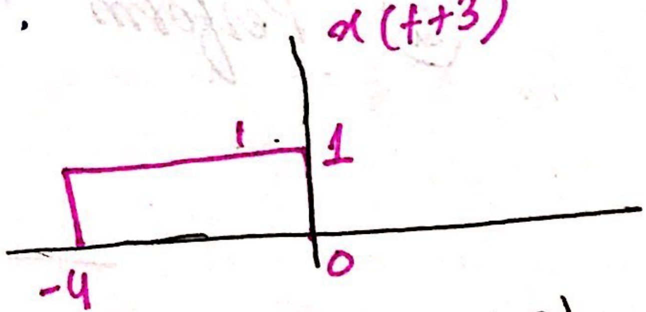
then plot $y(t) = 3\,x(2t+3)$

Ans Recall the priority order:

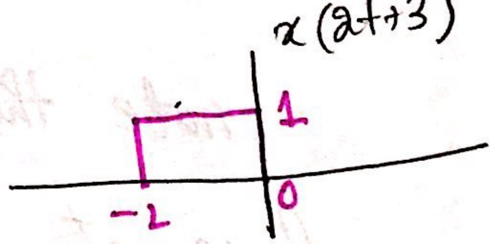
(1) Time Reversal
(2) " Shifting
(3) " Scaling

while amp. scaling can be performed at any time.

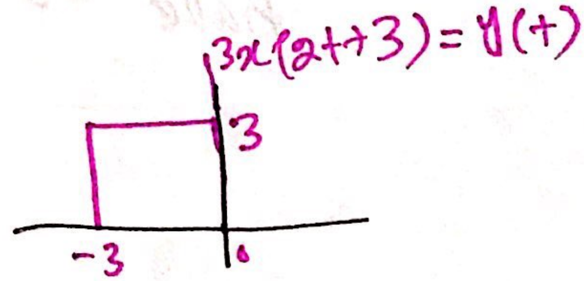
$x(t) \xrightarrow[k=3]{T.\,Shift} x(t+3)$



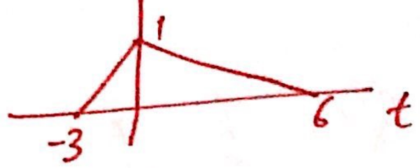
$x(t+3) \xrightarrow[\alpha=2]{T.\,Scal} x(2t+3)$



$x(2t+3) \xrightarrow[\beta=3]{Amp.\,Scal} 3x(2t+3)$



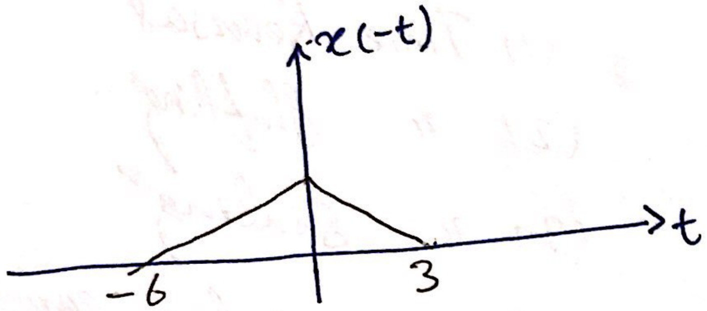
Example

$x(t) =$ 

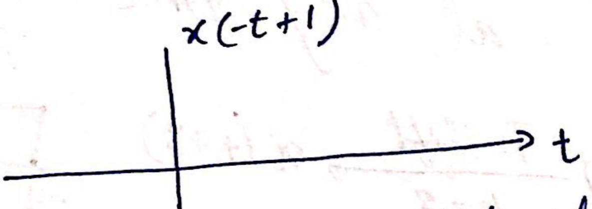
Plot $y(t) = x(-2t+1)$

Solution

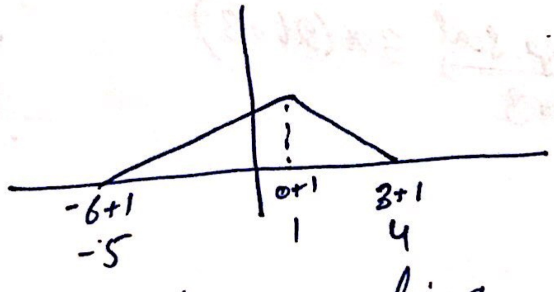
① Revers the signal



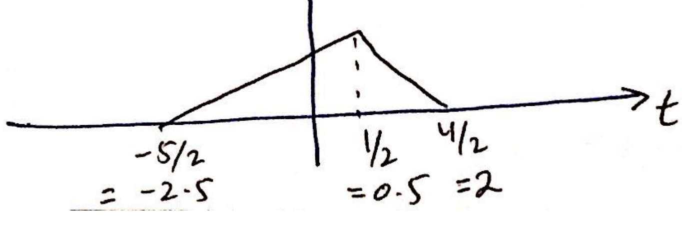
② Perform time shifting



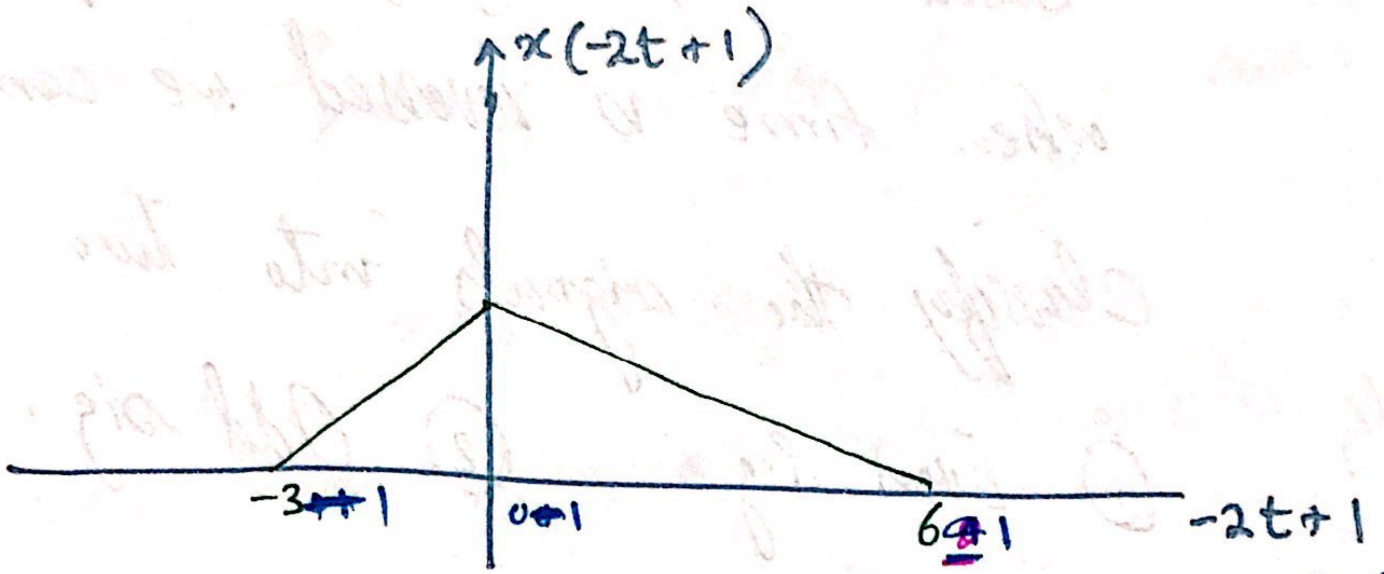
note that x-axis is t while above we have -t so $x(-(t-1))$



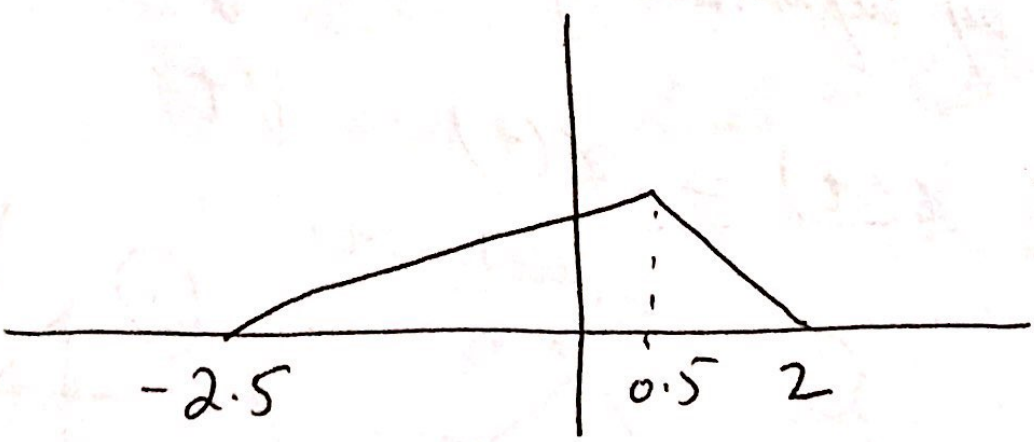
③ Perform time scaling.



=> The shortcut method.



-4	-1	5	+1 = 2t
$-4/2$	$-1/2$	$5/2$	$\frac{-2t}{2}$
-2	-0.5	2.5	= -t
-2(-1)	-0.5(-1)	2.5×(-1)	-1×(-t)
2	0.5	-2.5	



Even & Odd Signals

Based on symmetry of the signal when time is reversed we can classify the signals into two type

① Even sig ② Odd sig.

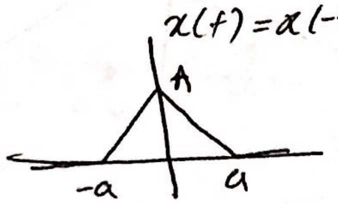
Even Signal Signal which remains identical (same) under folding/reflection/Time reversal.

$$x(t) \xrightarrow{T.R} x(-t)$$

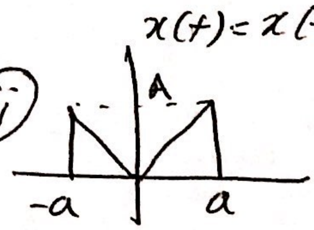
and by definition of even sig.

$$x(-t) = x(t) \quad \forall t$$

e.g. (i)

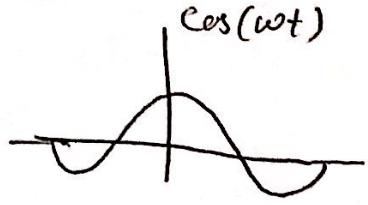


or (ii)



(iii) $\cos(\omega t) = x(t)$

$$x(-1) = \cos(-\omega t) = \cos(\omega t)$$



Odd Signals :→ Signals which doesn't remain identical / Same under folding operation.

i.e.,

$$x(-t) \neq x(t)$$

⇒ here $x(t)$ & $x(-t)$ are anti-symmetric about the y-axis.

$$x(t) = -x(-t)$$

⇒ Imp:

Note at $t=0$

$$x(0) = -x(-0)$$

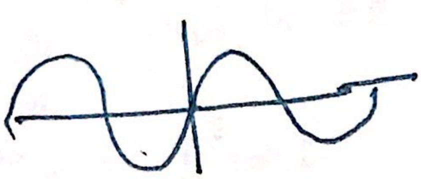
$$x(0) = -x(0) \rightarrow ①$$

eqn ① is true iff $x(0) = 0$

⟹ Therefore an odd signal must be zero at $t=0$. (An odd sig. will always pass through the origin)

⇒ Secondly the avg. or mean or the DC value of an odd signal is 0.

e.g. $x(t) = \sin \omega t$
then $x(-t) = -\sin \omega t$



→ Any signal can be decomposed into even and odd components.

i.e

$$x(t) = \underbrace{x_e(t)}_{\text{even}} + \underbrace{x_o(t)}_{\text{odd}}$$

Q :→ Find the even & odd components of signal $x(t)$ where

$$x(t) = \cos t + \sin t + \cos t \sin t$$

Solution Method-I

perform the Time reversal and find the components un-altered by it

$$x(-t) = \cos(-t) + \sin(-t) + \cos(-t)\sin(-t)$$

$$= \underbrace{\cos t}_{x_e(t)} - \sin t - \cos t \sin t$$

$$x_o(t) = \sin t\,(1 + \cos t)$$

Method II

Now use the formula

$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$

$$= \frac{1}{2}\left[\cos t + \cancel{\sin t} + \cancel{\sin t \cos t} + \cos t - \cancel{\sin t} - \cancel{\sin t \cos t}\right]$$

$$= \cos t$$

and

$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$

$$= \frac{1}{2}\left(\cancel{\cos t} + \sin t + \overset{\sin t \cos t}{\cancel{\sin t \cos t}} - \cancel{\cos t} + \sin t + \sin t \cos t\right)$$

$$= \sin t + \sin t \cos t = \sin t\,(1 + \cos t)$$

$\Rightarrow$ Imp:- ① Multiplying an even sig. by an odd sig. result in an odd sig.

$$E \times O \Rightarrow O$$

② Multiplying an even sig. by an even sig. will result in an even sig.

$$E \times E = E$$

③ Multiplying odd by odd sig. give even signal.

$$O \times O = E$$

④ $\frac{1}{\text{Even}} = E$ ⑤ $\frac{1}{\text{Odd}} = \text{Odd}$

⑥ Algebraic fun" having even power is even signal — e.g $t^2 = E$

While an algebraic expression having odd power is an odd sig.

e.g t^3 is odd.

Q :→ Find even and odd components of the following signal?

$$x(t) = \underbrace{t^2 \sin t}_{1} - \underbrace{\frac{t^3}{\sin^2 t}}_{2} + \underbrace{t^3 \cos t}_{3} - \underbrace{\frac{\cos^3 t}{t^2}}_{4} + \underbrace{\frac{t^5}{\sin^5 t}}_{5}$$

Solution

1 : Even × Odd = Odd

2 : $\frac{\text{Odd}}{\text{Odd} \times \text{Odd}} = \frac{\text{Odd}}{\text{Even}}$

↳ $\sin^2 t = (\sin t)^2 = \sin t \sin t$

$= \text{Odd} \times \frac{1}{\text{Even}} = \text{Odd} \times \text{Even} = \text{Odd}$

3 : Odd × Even = Odd.

4 : $\frac{\cos t \cos t \cos t}{t^2} = \frac{E \times E \times E}{E} = \frac{E}{E} = E \times \frac{1}{E}$

$= E \times E = \text{Even}$.

5 : $\frac{\text{Odd}}{\text{Odd} \times \text{Odd} \times \text{Odd} \times \text{Odd} \times \text{Odd}}$

$= \frac{\text{Odd}}{E \times E \times \text{Odd}} = \frac{\text{Odd}}{E \times \text{Odd}} = \frac{\text{Odd}}{\text{Odd}} = \text{Odd} \times \frac{1}{\text{Odd}}$

$= \text{Odd} \times \text{Odd} = \text{Even}$