Basic System Properties

1) Static & Dynamic Systems

(2) Causal & Non-Causal "

(3) Time Varying & Time invariant "

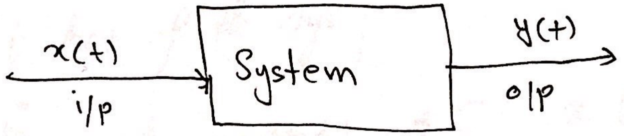
(4) Linear & Non-linear "

(5) Invertible & non-invertible "

(6) Stable & un-stable systems.

→ 12 different types of systems.

⇒



y(t) depends upon the i/p and the type and properties of the sys.

⇒ Sys. processes the i/p to produce some o/p.

Present, Future and Past i/ps

Determining at $t=0$

(1) $y(t) = x(t)$ o/p dependent upon Present i/p

$\Downarrow$

$y(0) = x(0)$

(2) $y(t) = x(t-1)$ o/p dependent upon Past i/p

$\Downarrow$

$y(0) = x(-1)$

(3) $y(t) = x(t+1)$ o/p dependent upon Future i/p

$\Downarrow$

$y(0) = x(1)$

Static & Dynamic Systems

$$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

Let us take

(1) $y(t) = x(t-1)$

(2) $y(t) = x(t)$

(3) $y(t) = x(t+1)$

⇒ Notice in the case 1 and 3, the system has changed the i/p $x(t)$ to $x(t-1)$ and $x(t+1)$ respectively. [i.e., this change in $x(t)$ is bcz of the sys]

E.G., let $x(-2) = 1.5$, $x(-1) = 2$, $x(0) = 2.5$, $x(1) = 3$

let we are feeding this i/p. to a sys.

⇒ See case 1:

P-T-O

$y(t) = x(t-1)$

at $t=0$

$y(0) = x(-1) = 2.0$ instead of 2.5

at $t=1$

$y(1) = x(0) = 2.5$ instead of 3.0

Static Systems:-

Output of the system depend only on present values of input e.g.,

1) $y(t) = 2x(t)$
2) $y(t) = f(x(t))$

Dynamic Systems:-

Output of the system depend upon past or future values of input at any instant of time. It can also depend upon present value of input. e.g.,

$$y(t) = x(t) + x(t-1)$$

Ex:

$$y(t) = x(t)\, e^{-(t+1)}$$

Static or dynamic ?

Solution :-> Carefully note that $e^{-(t+1)}$ is the co-efficient. The o/p depends on $x(t)$ while $e^{-(t+1)}$ is just a scaling factor

=> ~~So~~ the system is static.

=> Static Systems are also known as memoryless Systems while dynamic systems are known to have memory.

=> Static are memoryless bcz the dependence is only on $x(t)$ [Present values].

=> While in case of dynamic sys. the dependance is on past or future values of ~~x~~ i/p, demands possession of memory.
↓
which

Ex: $y(t) = x(2t)$

Static or dynamic ?

Solution

at $t=0$

$y(0) = x(0)$

→ However, don't conclude immediately as the dependence on current i/p should be from $-\infty \leq t \leq \infty$.

at $t=1$

$y(1) = x(2)$

now the dependence is on the future i/p.

or at $t=-1$

y(-1) = x(-2)

Now the dependence is on the past input.

Thus the system is dynamic

Ex:

$$y(t+1) = x(t+1)$$

Static or dynamic?

Solution :->

at $t=0$

$$y(1) = x(1)$$

=> This system is static as it says that future o/p depends upon future i/p

-> Similar example is

$y(t-2) = x(t-2)$ Static.

Ex:-> $y(t) = x(-t)$

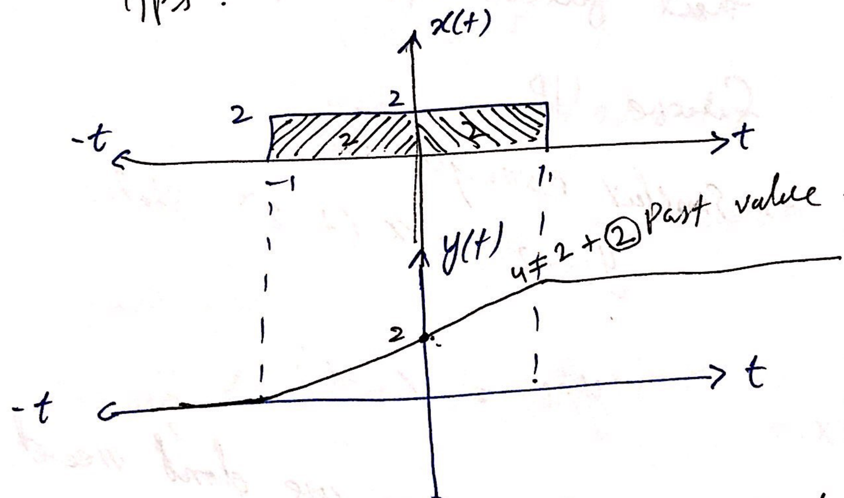
Solution :-> Here now we don't need to perform the calculations. Whenever, there is time scaling (time reversal is a special case of time scaling) the system will be dynamic.

Ex

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, d\tau$$

Static or dynamic?

Solution :-> Recall the graphical integration where any present value was dependent upon past i/ps.



So this is dynamic system.

Ex: $y(t) = x(\sin(t))$

Static or dynamic ?

Solution At $t=0$

$y(0) = x(0)$

at $t = \pi$

$y(\pi) = x(\sin \pi)$

$y(\pi) = x(0)$

$\underbrace{y(3.14) = x(0)}_{\text{depend upon}}$

past i/p

→ So the sys. is dynamic in nature.

Exaples (1) $y(t) = x(t+5)+6$

(2) $y(t) = x(t)\sin(2t)$

(3) $y(t) = e^{-2x(t)}$

Conclusion

① Whenever, the o/p of a sys. is dependent upon the time scaled or time shifted i/p then the sys will be dynamic in nature

② Whenever the o/p is dependendent upon the integral of the i/p the sys. will be dynamic

Ex: $y(t) = Re\,(x(t))$

Static or dynamic?

Solution :- let us first find out the nature of the system producing this o/p.

let $x(t) = a + jb \rightarrow ①$

this is what we want.

$$\overline{x(t)} = a - jb \rightarrow ②$$

① + ②

$$x(t) + \overline{x(t)} = 2a$$

$$a = \frac{x(t) + \overline{x(t)}}{2} = Re(x(t))$$

So

$$y(t) = \frac{x(t) + \overline{x(t)}}{2}$$

and so $y(t)$ is dependent only on the present i/p. So the system is static.

Ex $\quad y(t) = E[x(t)]$

even component of $x(t)$

static or dynamic?

Solution

$$E[x(t)] = \frac{\overset{\text{Prent i/p}}{x(t)} + \overset{\text{Future i/p }(t<0)\text{, Past i/p }(t\geq 0)}{x(-t)}}{2}$$

So in this case the dependance is on both present & past so the sys. is dynamic.

Ex $y(t) = x(t^3)$

static or dynamic?

Solution at $t=0$

$y(0) = x(0)$

at $t=1$

$y(1) = x(1)$

at $t=2$

$y(2) = \underbrace{x(8)}_{\text{future}}$

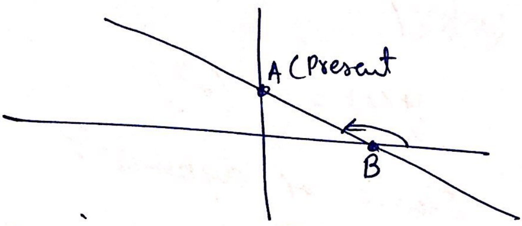
So dynamic.

Whenever, there is any operator on time the system is going to be dynamic e.g $x(t^2)$

Ex. $y(t) = \frac{d}{dt} x(t) \rightarrow$ Slope of $x(t)$

Static or dynamic?

Solution :- To calculate the slope we need two different pts.



So in combination to a present value we need a past or a future value to calculate the slope.

→ So any sys. with a differentiated o/p is dynamic in nature.

Examples

(1) $y(t) = x(\sqrt{t})$

(2) $y(t) = \text{Im}\left[x(t)\right]$

(3) $y(t) = \int_{t-4}^{t+4} x(\tau)\, d\tau$

(4) $\frac{d}{dt} y(t) + 3y(t) = 2x(t)$

Static or dynamic?