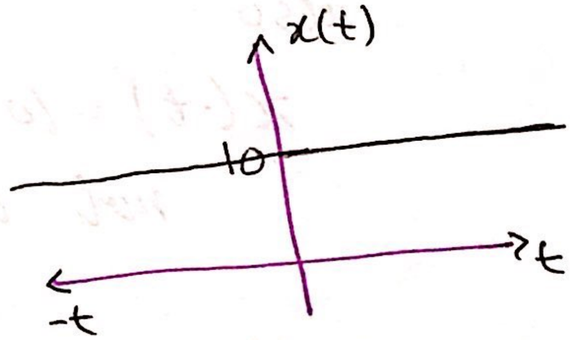
Properties of Even & Odd Signals

① DC value

$x(t) = 10 \quad \forall t$

then $x(t) = x(-t)$



⇒ DC value is an even signal.

⇒ when a CTS is purely Even then the Odd component $= 0$.

$$x_o(t) = 0 \Rightarrow \frac{1}{2}\left[x(t) - x(-t)\right] = \frac{1}{2}[10-10] = 0$$

$$x_e(t) = 10 \Rightarrow \frac{1}{2}\left(x(t) + x(-t)\right) = \frac{1}{2}(20) = 10$$

② Adding DC value to an Even signal

Ex: $\quad 10 + t^2 = x(t)$

$$x(-t) = 10 + (-t)^2 = 10 + t^2 = x(t)$$

DC value + Even $\longrightarrow$ Even

(Note Even + Even = Even)

3/ Adding Odd signal to DC value

Ex: $x(t) = 10 + t^3$

$x(-t) = 10 - t^3 \neq x(t)$

not even.

also

$x(-t) = 10 - t^3 \neq -x(t) = -10 - t^3$

not odd.

odd + DC → Neither even nor odd.

→ A conclusion is that

Even + Odd is neither even nor odd.

→ That is why we say that a general signal has even and odd components.

4/ Even Sig × Even Sig

Ex: $x(t) = t^2 \times t^4 = t^6 \rightarrow$ Even

$\because x(t) = x(-t)$

5/ Odd Sig. × Odd Sig. → Even Sig.

Ex: $x(t) = t^3 \times t^5 = t^8 \rightarrow$ Even

6/ Odd Sig. × Even Sig. → Odd

Ex: $x(t) = t^3 \times t^6 = t^9 \rightarrow$ Odd

7/ $\frac{d}{dt}$(Even Sig) = Odd signal

→ This ~~##~~ prop. is not valid for DC values $\because \frac{d}{dt} c = 0$

8/ $\frac{d}{dt}$(Odd Sig) = Even Signal

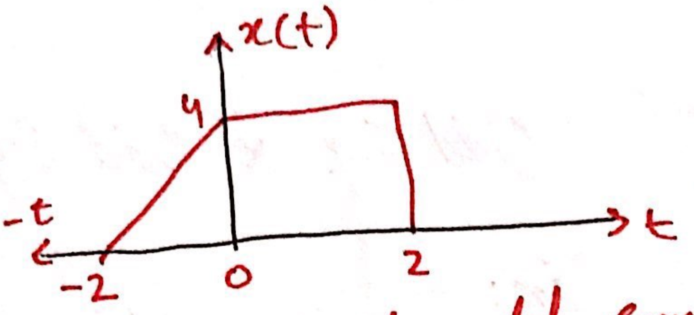
9/ $\int E$ = Odd Sig. $E \rightarrow$ Even signal

10/ $\int$ Odd Sig = E

11/ $\frac{1}{\text{Odd Sig}}$ = Odd. Sig

12 $\frac{1}{E} = E$

Q:→



Find the even and odd components?

Solution:

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

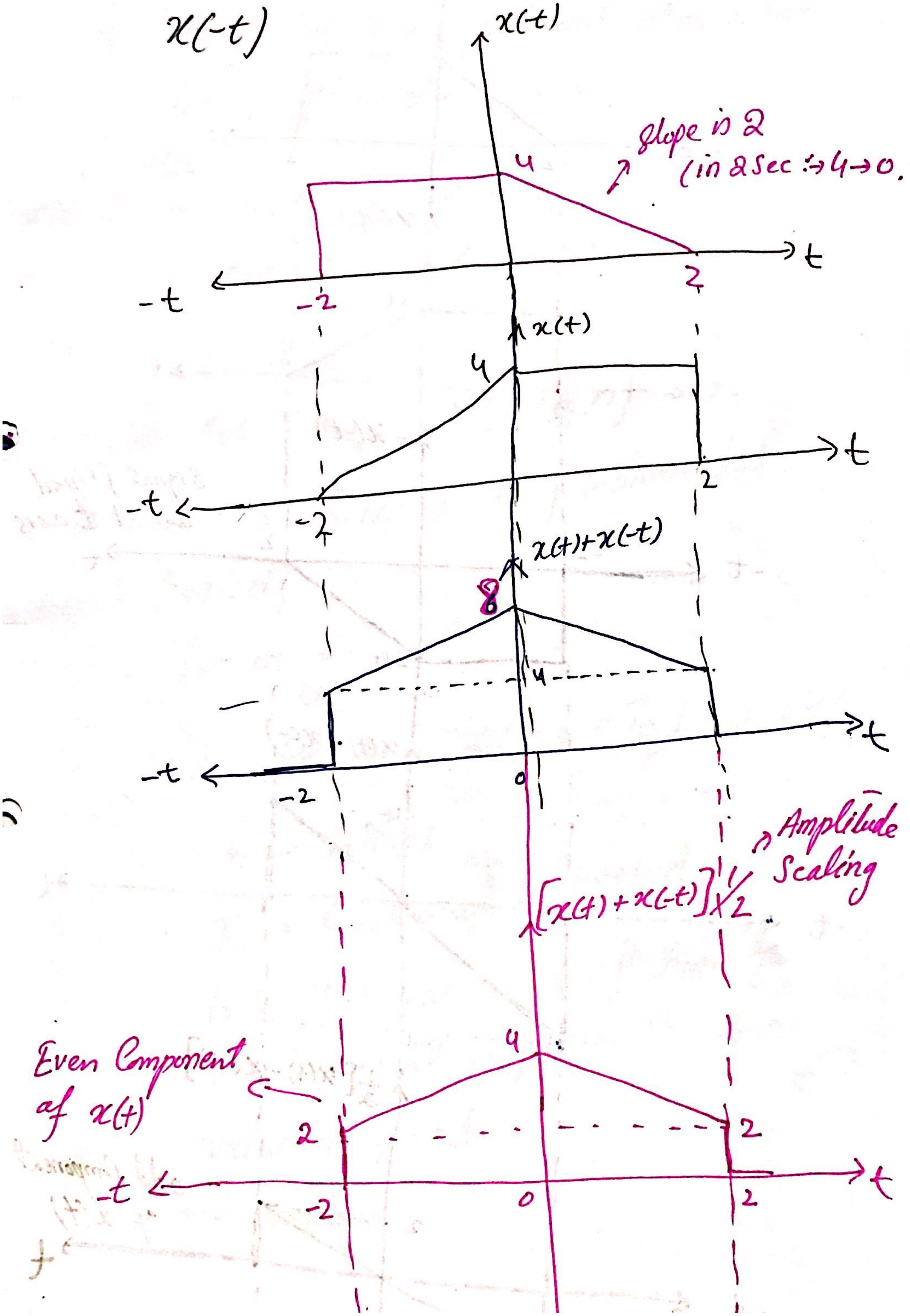
↳ In this instead of adding the time reversed signal (like in $x_e(t)$) we're subtracting the time reversed signal.

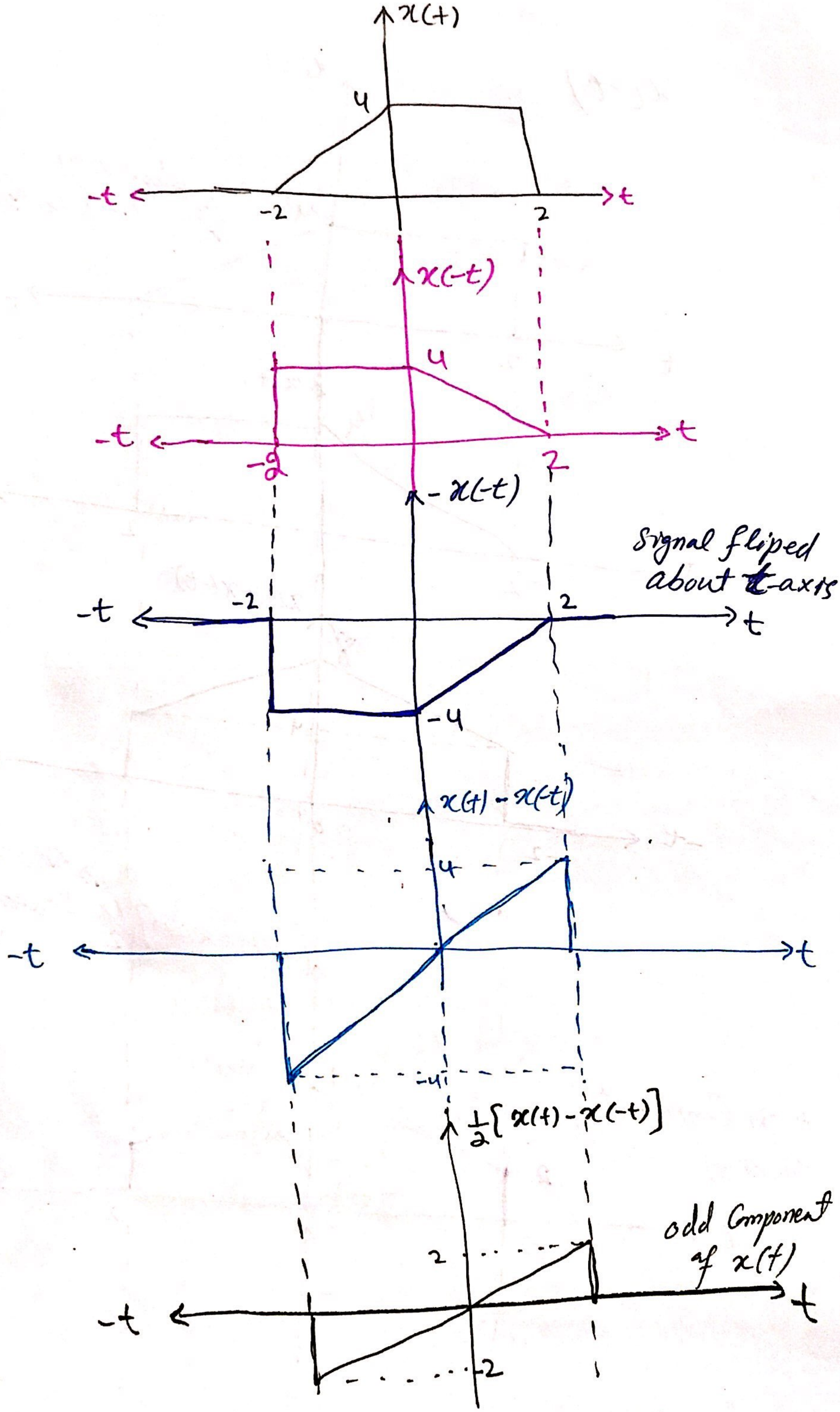
i.e

$$x_o(t) = \frac{1}{2}\left[x(t) \overset{A.R}{-} \overset{T.R}{x(-t)}\right]$$

→ Perform time reversal, then apply amplitude reversal & then add the obtained signal to $x(t)$ followed by a division by 2.

$x(-t)$


x(-t)
Slope is 2
(in 2 sec : 4→0.
4
t
-t
-2
2
x(t)
4
t
-t
-2
2
x(t)+x(-t)
8
6
4
t
-t
-2
0
Amplitude Scaling
[x(t)+x(-t)] 1/2
Even Component of x(t)
4
2
2
t
-t
-2
0
2


x(t)
4
-t
t
-2
2
x(-t)
4
-t
t
-2
2
-x(-t)
Signal fliped about t-axis
-t
-2
2
t
-4
x(t) - x(-t)
4
-t
t
-4
1/2[x(t) - x(-t)]
odd Component of x(t)
2
-t
t
-2

Ex: Let Time interval $\Delta t = 5$ sec

for a periodic signal.

then

$$x(t-5) = x(t) = x(t+5)$$

$$x(t-10) = x(t+5) = x(t+10)$$

$$\vdots \qquad\qquad \vdots$$

$$x(t-5(n)) = x(t+5(n-1)) = x(t+5n)$$

This 5 here is the fundamental period.

So in general

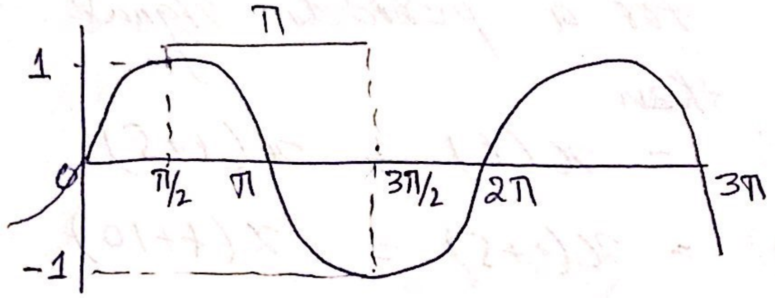
$$x(t) = x(t \pm nT_0) \quad \forall n \;\&\; \forall t$$

$n \rightarrow$ Intiger

$T_0 \rightarrow$ Fundamental period

↳ Smallest +ve value of time for which signal is periodic. This value is fixed.

Ex: To explain T_0 (Fundamental period) and to explain why it is smallest.



→ One may say that $T_0 = \pi$ ∵ $\sin 0 = \sin \pi = 0$

but it's not true ∵ it's not true for all the values of θ.

See the signal is not repeating itself

→ Before 0 it is $-ve$
while before π it is $+ve$.

→ So signal is not repeating itself when $T_0 = \pi$

→ This can also be varified by looking that $\sin(\pi/2) = 1$
& while $\sin\left(\frac{3\pi}{2}\right) = -1$

So the fundamental period is 2π

$$\sin\theta = \sin(\theta \pm n\,2\pi)$$

Let $n=2$

$$\sin\theta = \sin(\theta \pm 4\pi)$$

So $\sin\theta$ is also periodic for period 4π

$n=3$

$$\sin\theta = \sin(\theta + 6\pi)$$

So $\sin\theta$ is also periodic for 6π

→ The 4π and 6π .... are simply the periods of $\sin\theta$ but they are not the fundamental period. ∵ they are not the smallest

→ 2π is the smallest possible value.

Fundamental Frequency

Denoted by f_0

$$f_0 = \frac{1}{T_0} \text{ Cycles/Sec or Hz}$$

Fundamental Angular frequency

Denoted by ω_0

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \text{ rad/Sec}$$

Condition of Periodicity for DTS

$$x[n] = x[n + mN]$$

$m \to$ Intiger

$N \to$ Fundamental Period.

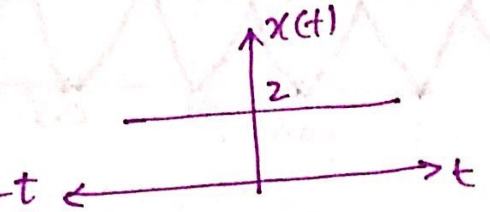
$\boxed{\text{N must be an intiger}}$

(Note that T_0 may or may not be intiger see CT case)

$$\text{Fundament freq} = F = \frac{1}{N} \text{ Hz}$$

Q:→ DC value is periodic?

Ans:→ Yes



Note that

$$x(t) = x(t + nT_0) \quad \forall n \text{ \& } \forall T_0$$

→ So a DC value is a periodic signal with an undefined T_0

then

$$f_0 = \frac{1}{T_0} = 0 \text{ cycles/sec (Hz)}$$

↓

0 cycles

or

$\boxed{\text{undefined } \infty \text{ \& } \frac{1}{\infty} = 0}$

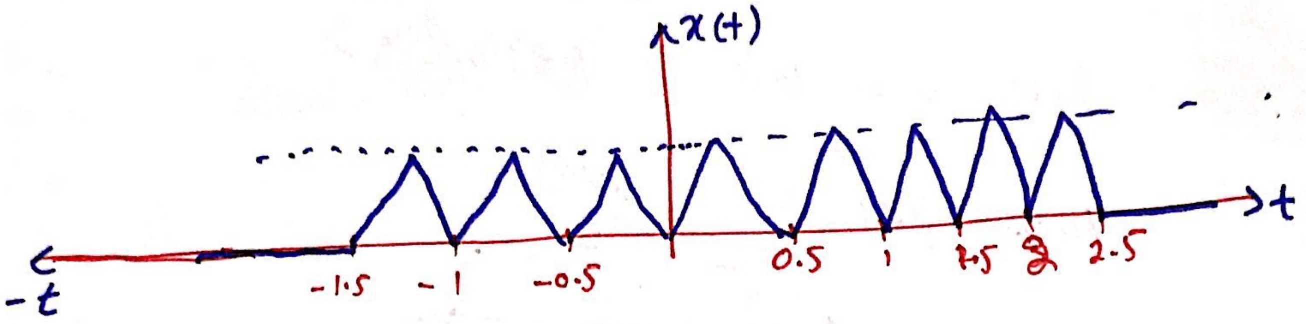
→ We can't use this relation as T_0 is undefined.

Properties of T_0

① $T_0 \neq 0$

② $T_0 \neq \infty$

③ $T_0 > 0$



Is $x(t)$ periodic?

<u>Solution</u> :→ According to the definition $x(t)$ is periodic if

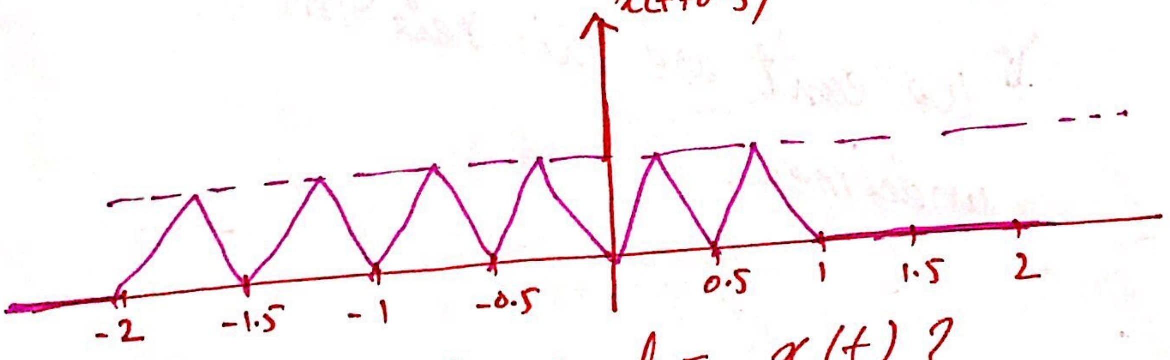
$$x(t) = x(t \pm nT_0)$$

Clearly it seems that $T_0 = 0.5$.

let $n = 1$

then $x(t) = x(t+0.5)$

i.e., ~~to~~ Left shift by 0.5



Q:→ Is this signal $= x(t)$?

No.

$x(t) \neq x(t+0.5)$ So $x(t)$ is non-periodic.

Calculations of T_0

Valid for non-composite signals.

The condition for periodicity is

$x(t) = x(t \pm nT_0)$

let $n=1$ and ignore minus sign.

$x(t) = x(t + T_0)$

let $x(t) = A_0 e^{Jw_0 t}$

then $x(t+T_0) = A_0 e^{Jw_0(t+T_0)}$

And

$$\cancel{A_0} e^{Jw_0 t} = \cancel{A_0} e^{Jw_0(t+T_0)}$$

$$\cancel{e^{Jw_0 t}} = \cancel{e^{Jw_0 t}} \times e^{Jw_0 T_0}$$

$$1 = e^{Jw_0 T_0}$$

From euler identity

$$\cos(w_0 T_0) + J\sin(w_0 T_0) = 1 + J0$$

$\cos(w_0 T_0) = 1$ & $\sin(w_0 T_0) = 0$

if $w_0 T_0 = 2\pi$ then this is true

$$\boxed{T_0 = \frac{2\pi}{w_0}}$$