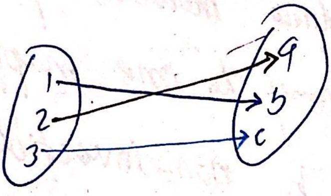
Invertible & Non-Invertible Systems.

For an invertible system there should be one to one mapping b/w the i/p and the o/p at each and every instant of time.

One-to-One Mapping :- For each distinct i/p the o/p is distinct

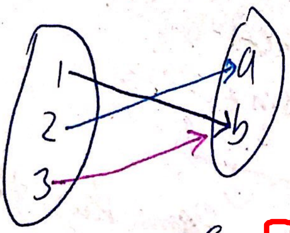
e.g.,



e.g., $y(t) = x(t)+2$

when $x(t) = 0$, $y(t) = 2$
for $x(t) = -4$, $y(t) = -2$

Many-to-One Mapping :- Distinct i/ps may produce same o/ps



1 & 3 and both produce b.

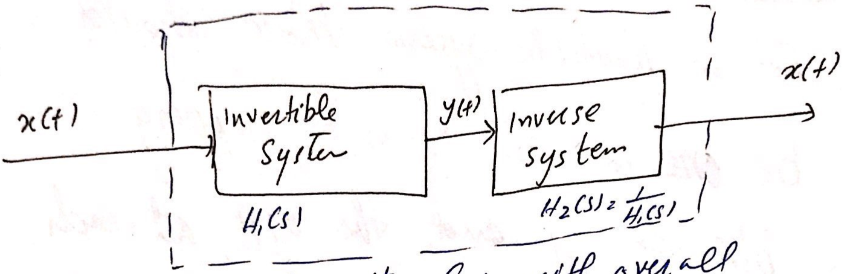
e.g., 1) $y(t) = (x(t))^2$

$y(t) = 4$ for $x = 2$ & $x = -2$

2) $y(t) = |x(t)|$

y(t) is the same for both positive and negative values of x(t)

Properties :->



Composite Sys with overall gain = $H(s) = 1$

Ex

⇒ ① $y(t) = |x(t)|$ invertible?

As we know modulus operation causes many to one mapping so this sys is non-invertible.

② $y(t) = \sin(t)\, x(t)$

	$x(t)$	$y(t)$
t=?	0	$\sin(t) \cdot 0 = 0$
t=?	1	$\sin(t)$
t=?	2	$2\sin t$

x(0) = 1 !!!!

From this the sys seem to be invertible but we can also have many to one mapping. e.g., at $t=0$, $x(0)=1$ then $y(t)=0$ etc. So we can't exactly conclude that whether the sys. is invertible or not

→ In such case the impulse fun $\delta(t)$ instead

$x(t)$	$y(t) = \sin t \; x(t)$
$\delta(t)$	0
$2\delta(t)$	0
	Many-to-One mapping

$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$

So. non-invertible.

Ex $y(t) = x(2t)$

Invertible ?

$x(t)$	$y(t) = x(2t)$
$U(t)$	$y(t) = U(2t)$
$-U(t)$	$y(t) = -U(2t)$

The system is performing time scaling to the input signal [x(t)]

Invertible.

H.W

Ex

① $y(t) = x(t^2)$

② $y(t) = \frac{d}{dt} x(t)$

Stable & Unstable Systems

BIBO Criteria :- For a stable sys. o/p should be bounded for bounded i/ps at each and every instant of time.

Some Examples of bounded signals.

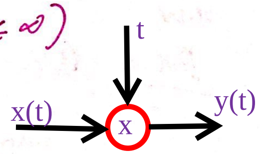
(1) DC value (Cst from $-\infty$ to ∞)
↳ finite

(2) $u(t)$ {0 from $-\infty$ to 0 and 1 from 0 to ∞)

(3) $\sin t$ (+1 to -1) $\forall -\infty \le t \le \infty$

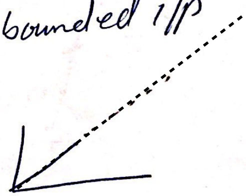
(4) $\cos t$ (+1 to -1 $\forall -\infty \le t \le \infty$)

Ex $\quad y(t) = t\,x(t)$



Is this system stable?

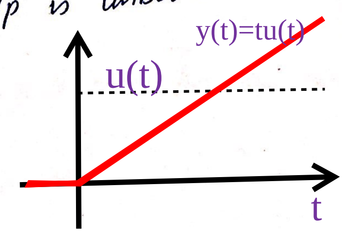
Solution :- Let us provide a bounded i/p say $x(t) = u(t)$.



$y(t) = t\,u(t) = r(t)$

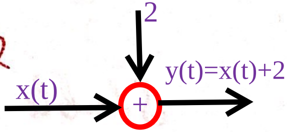
which means the o/p is unbounded for a bounded i/p

So Unstable.



Ex $y(t) = x(t) + 2$

stable?



Solution

let $x(t) = 4$ (DC value)

$y(t) = 6$

↳ ct which never gets unbounded so the sys is STABLE

H.W

Ex ① $y(t) = \sin(t)\, x(t)$

Ex $y(t) = \sin(x(t))$

Stable?

Solution $\to$ Whatever $x(t)$ is $\sin(x(t))$ is always b/w +1 and -1

So this system is stable.

Ex $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

Stable?

Solution $\to$ Let us choose $x(t)$ as a BOUNDED Signal

say $x(t) = \cos t$.

$$y(t) = \int_{-\infty}^{t} \cos\tau\, d\tau = \underbrace{\sin(\tau)\Big|_{-\infty}^{t}}$$

$$= \sin(t) - \sin(-\infty).$$

$$= \sin(t) + \underbrace{\sin(\infty)}_{\text{Some finite value. b/w -1 \& +1}}$$

-1 + Sin(t) <= y(t) <= 1 + Sin(t)

For $x(t) = \cos t$ Sys is stable.

Now let $x(t) = u(t)$

$$y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau = \underbrace{r(\tau)}_{\text{Ramp is unbounded.}} \Big|_{-\infty}^{t}$$

Sys is unstable for $u(t)$

⇒ So as if the sys is unstable for any one bounded signal then it is $\boxed{\text{unstable}}$

H.W
Ex :

$$y(t) = \int_{-\infty}^{t} \cos\tau \; x(\tau)\, d\tau \quad \text{stable?}$$

LTI Systems

Linear Time Invariant (LTI)

TI Sys :- ① There is no time scaling
② Co-efficients are cte
③ Any added/Subtracted term (excep i/p & o/p) must be cte or 0.

e.g., ① $y(t) = x(t) + 5$

①✓ , ②✓ , ③✓

TI sys.

Non-Linear Time Invariant System

but non-linear is of the added term.

② $y(t) = x(t^2)$

①× , ②✓ , ③✓

Linear Time Varying System

TV system. However it is linear.

③ $y(t) = \cos t \, x(t)$

①✓ , ②× , ③✓

TV system

However the sys is linear as linearity is independent of co-efficient.

(4) $y(t) = 2x(t)$

① ✓ ② ✓, ③ ✓

LTI sys.

LTI Systems [Laplace Transform]

Consider an LTI sys. in time domain

$$y(t) = x(t-1) + x(t+1)$$

→ To completely characterize this sys we need to solve the eqns or we need to find its impulse respons

Output of a system when input is an impulse is known as the "IMPULSE RESPONSE"

If we know the impulse response of an LTI system then we know everything about that system

$$\delta(t) \longrightarrow \boxed{Sys} \longrightarrow y(t)$$

↳ This $y(t)$ is now call the impulse response.

⇒ However, oftentime the calculations get seriously complex in time. So systems are often represented and solved in frequency domain

→ The tool to transform a sys from time domain to freq. domain is known as the "Laplace Transform".

$$f(t) \xrightarrow{L.T} L[f(t)] = F(s)$$

where $s = \sigma + j\omega \in \mathbb{C}$ Complex frequency

$$F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

→ However often a unilateral integral is used

e.g.,

$$F(s) = \int_{0}^{\infty} f(t)\, e^{-st}\, dt$$

e.g. $f(t) = 2$

$$F(s) = 2\int_{0}^{\infty} e^{-st} dt = 2\, \frac{1}{-s}\, e^{-st}\Big|_0^{\infty}$$

$$= -\frac{2}{s} \cdot [e^{-\infty} - e^{0}] = -\frac{2}{s}[0-1]$$

$$= \frac{2}{s}$$

⇒ Now once again consider the sys.

$$y(t) = x(t-1) + x(t+1)$$

$\Big\downarrow$ L.T

Let us consider
$L[y(t)] = Y(s)$
$L[x(t)] = X(s)$

$$Y(s) = X(s)\, e^{-s} + X(s)\, e^{s}$$

$$\boxed{\frac{Y(s)}{X(s)} = H(s) = e^{-s} + e^{s}}$$

H(s) is known as the "Transfer function".

<u>Transfer function</u> :→ Ratio of the transformed o/p to the transformed i/p assuming 0 initial conditions.

⇒ This H(s) is Very closely related to the impulse response

⇒ To move back to time domain we'll use inverse laplace transform.

$$\boxed{h(t) = \delta(t-1) + \delta(t+1)}$$

⇒ Transfer function or impulse response are valid for LTI systems only.

$L[h(t)] = H(s)$

$L^{-1}[H(s)] = h(t)$

h(t) = Impulse response

H(s) = Transfer function

LTI Systems

- Follow superposition (LOA + LOH)
- Any delay in the i/p will reflect in the o/p.

→ The most imp. parameter of an LTI sys. is its impulse response

$$\delta(t) \longrightarrow \boxed{\text{LTI System}} \longrightarrow y(t) = h(t)$$

(i.e., o/p of an LTI sys. when i/p is an impulse)

→ If we know the impulse response of an LTI sys. then we almost know each and everything about the sys. behavior.

⇒ $h(t) \xrightarrow{L.T} H(s) \rightarrow$ Transfer funⁿ

$\xleftarrow{ILT}$

T/f Calculations

Ex: 1

$$y(t) = x(t-1) + 5$$
$$H(s) = ?$$

Solution :-) Taking L.T

$$Y(s) = X(s)\, e^{-s} + \infty$$

$\hookrightarrow$ $\because$ L.T of a dc value or double sided power signal is ∞.

If instead we've

$$y(t) = x(t-1) + 5u(t)$$

then

$$Y(s) = X(s)\, e^{-s} + 5/s$$

Can we calculate $H(s) = \dfrac{Y(s)}{X(s)} = ?$

Note that $Y(s)$ & $X(s)$ are not separable.

So $H(s)$ is not possible $\because$ the sys $y(t) = x(t-1) + 5u(t)$ is non-linear.

Ex 2

$$y(t) = x(2t)$$

$$H(s) = ?$$

Solution

Taking L-T

$$Y(s) = \frac{1}{2} X(s/2)$$

The system is having "TIME SCALING" hence it is a time varying system.

Thus the transfer function calculation is not possible

Ex 3

$$y(t) = x(t-2) + x(t+2)$$

$$H(s) = ?$$

Solution

Taking L.T

$$Y(s) = X(s)\, e^{-2s} + X(s)\, e^{2s}$$

$$Y(s) = X(s)\left[e^{-2s} + e^{2s}\right]$$

$$H(s) = \frac{Y(s)}{X(s)} = e^{-2s} + e^{2s}$$

$$h(t) \xleftrightarrow{ILT} H(s)$$

$$h(t) = \delta(t-2) + \delta(t+2)$$

H.W

Ex:

$$Y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$H(s) = ?$ and $h(t) = ?$