# Area & average value of CTS

→ Area of $x(t) = \int_{-\infty}^{\infty} x(t)\,dt$

→ How if signal $x(t)$ exist only b/w $t_1$ & $t_2$ then


$$\boxed{\text{Area of } x(t) = \int_{t_1}^{t_2} x(t)\,dt .}$$

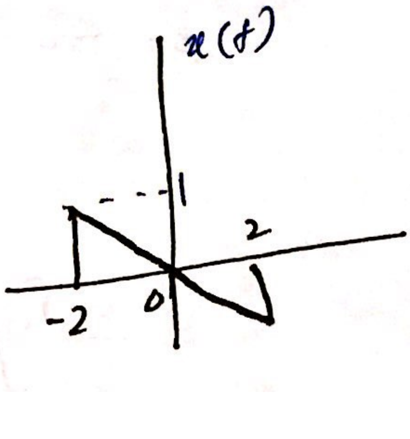
$\therefore$

$$\int_{-\infty}^{\infty} x(t)\,dt = \underbrace{\int_{-\infty}^{t_1} x(t)\,dt}_{0} + \int_{t_1}^{t_2} x(t)\,dt + \underbrace{\int_{t_2}^{\infty} x(t)\,dt}_{0}$$

So

$$\int_{-\infty}^{\infty} x(t)\,dt = \int_{t_1}^{t_2} x(t)\,dt$$

Ex



Ans

$$\text{Area of } x(t) = \int_{-2}^{2} x(t)\,dt$$

$$= \int_{-2}^{0} x(t)\,dt + \int_{0}^{2} x(t)\,dt$$

from $-2$ to $0$ $\quad x(t) = \underbrace{-0.5}_{\text{Slope of } -0.5} t$

and from $0$ to $2$ also $x(t) = -0.5t$

$$\text{area of } x(t) = \int_{-2}^{0} -0.5t\,dt + \int_{0}^{2} -0.5t\,dt$$

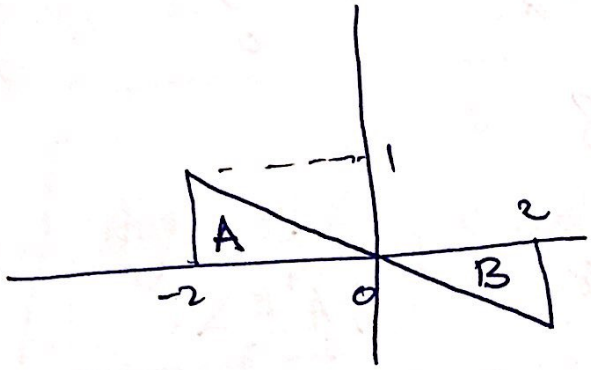
$$= -0.5\left[\frac{t^2}{2}\Big|_{-2}^{0} + \frac{t^2}{2}\Big|_{0}^{2}\right]$$

$$= \frac{-0.5}{2}\left[0 - 4 + 4 - 0\right] = 0$$

$$\boxed{\text{Area of } x(t) = 0}$$

Area of pure odd signals usually turn out to be 0

→ Also see it the other way



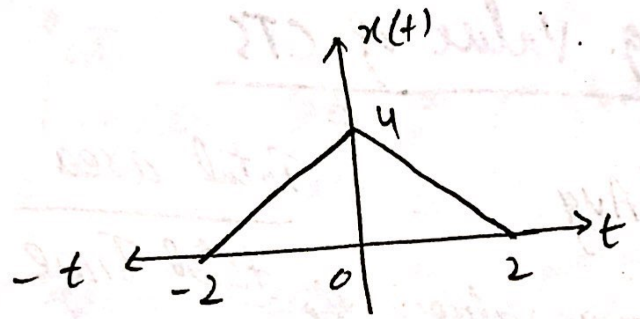

Area of Δ = $\frac{1}{2}$ × base × height.

we've Two triangle A & B. So

Area of $x(t)$ = Area of A & Area of B.

$$= \frac{1}{2}[2][1] + \frac{1}{2}[2][-1] = 0$$

**Ex:**




Area?

**Solution**

$$\text{Area of } x(t) = \int_{-2}^{0} x(t)\,dt + \int_{0}^{2} x(t)\,dt$$

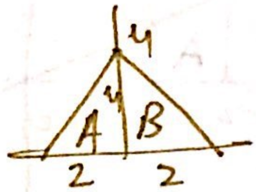
→ from $-2$ to $0$ $\quad x(t) = 2t + 4$ → Straight line with +ve slope

→ from $0$ to $2$ $\quad x(t) = -2t + 4$ → straight line with -ve slope.

$$\text{Area of } x(t) = \int_{-2}^{0} (2t+4)\,dt + \int_{0}^{2} (-2t+4)\,dt$$

$$= 8$$

Cross Check




$$\text{Area of } x(t) = \text{Area of } A + \text{Area of } B$$

$$= \frac{1}{\cancel{2}}[\cancel{2}][4] + \frac{1}{\cancel{2}}[\cancel{2}][4]$$

$$= 8$$

## Avg. Value of CTS

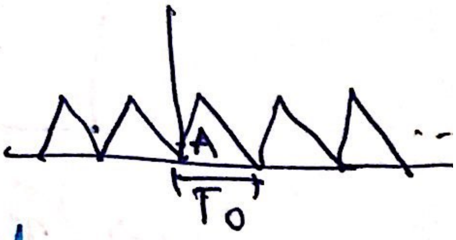
$$\text{Avg.} = \frac{\text{Total area}}{\text{total Time}} \equiv \frac{\int_{-\infty}^{\infty} x(t)\,dt}{T}$$

→ The average value is the DC value of a signal.

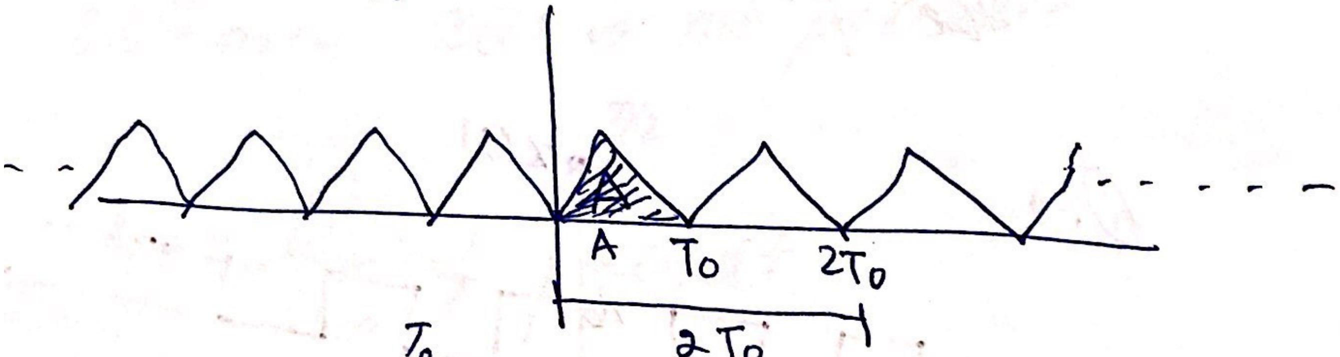
⇒ Recall the periodic signals. These signals repeat itself after $T_0$. So if we find the avg. of one chunk (for one $T_0$) this will be valid for the whole signal.

So Average value of a periodic sig is given by

$$\boxed{Avg = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt}$$




$T_0 \rightarrow$ Fundamental Period




$$A = \int_0^{T_0} x(t)\,dt \quad = \frac{A}{T_0}\ (Avg)$$

$$B = \int_0^{2T_0} x(t)\,dt = \frac{2A}{2T_0} = \frac{A}{T_0}\ (avg)$$

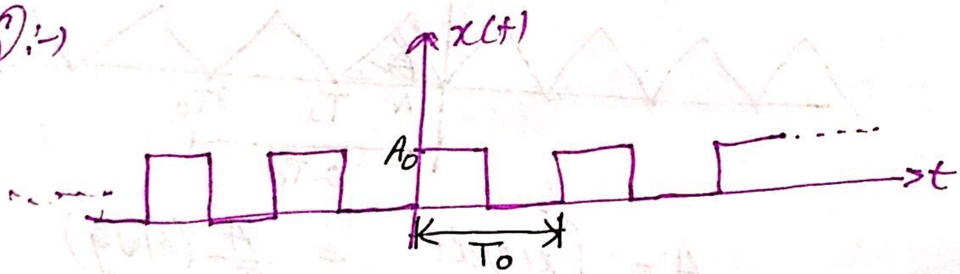
So we don't need to integrate from $-\infty$ to $\infty$. Integration over the fundamental period will be enough.

⇒ For non-periodic signal

$$\boxed{Avg = \lim_{t \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\,dt}$$

→ $x(t)$ is non-periodic so $T$ is not the period.

Q:-



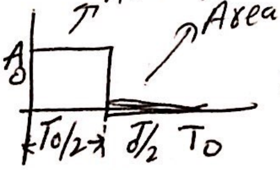

Find Avg (x(t))? or
Find the DC value of x(t)?

Solution ↳ First of all find whether a signal is periodic.

$$Avg = \frac{1}{T_0} \int_0^{T_0} x(t)\,dt \rightarrow ①$$

Area of Rectangle.

Area = 0

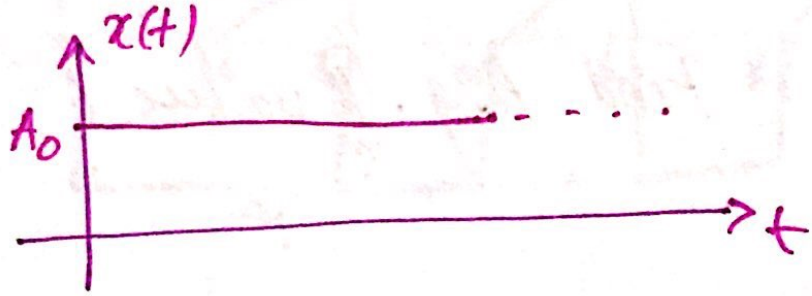



$$Area = A_0 \times \frac{T_0}{2} + 0$$

$$\int_0^{T_0/2} A_0\,dt = A_0 \int_0^{T_0/2} dt = A_0 \frac{T_0}{2} \text{ put in ①}$$

$$\text{Avg}[x(t)] = \frac{1}{\cancel{T_0}} A_0 \frac{\cancel{T_0}}{2} = \frac{A_0}{2}$$

Q :-




Solution :- $x(t)$ is non-periodic

$$\text{Avg} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\,dt$$

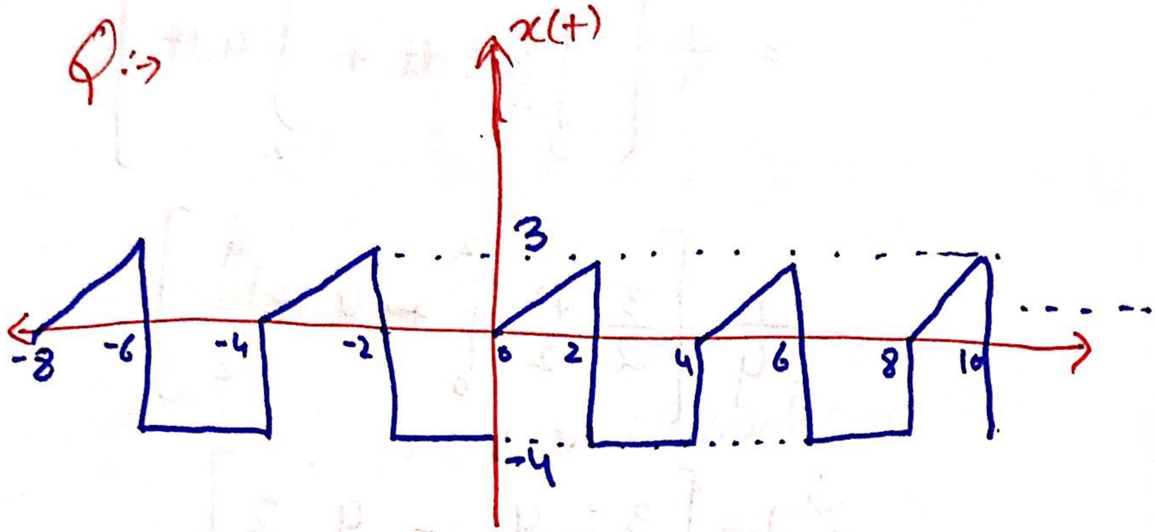
Note that $x(t) = 0 \quad \forall\, t < 0$

So

$$\text{Avg} = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T/2} A_0\,dt$$

$$= \lim_{T \to \infty} \frac{1}{\cancel{T}} A_0 \frac{\cancel{T}}{2} = \frac{A_0}{2}$$

Q:→




Average Value?

Solution:→ The $x(t)$ is periodic, so

$$\text{Avg value} = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$

$T_0 = 4$

But during one period $x(t)$ is a triangle followed by a rectangle.

From 0 to 2 : $x(t) = \frac{3}{2}t$

From 2 to 4 : $x(t) = -4$

$$\text{Avg. Value} = \frac{1}{4}\int_0^4 x(t)\,dt$$

$$= \frac{1}{4}\left[ \int_0^2 \frac{3}{2} t \, dt + \int_2^4 -4 \, dt \right]$$

$$= \frac{1}{4}\left[ \frac{3}{2} \frac{t^2}{2} \Big|_0^2 - 4 t \Big|_2^4 \right]$$

$$= \frac{1}{4}\left[ \frac{3}{2} \cdot \frac{4}{2} - 4 \cdot 2 \right]$$

$$= \frac{1}{4}\left[ 3 - 8 \right] = -\frac{5}{4}$$

$$\boxed{\text{Avg. value} = -\frac{5}{4}}$$

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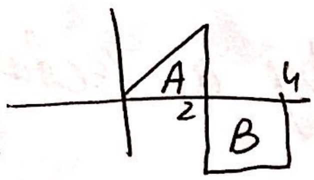
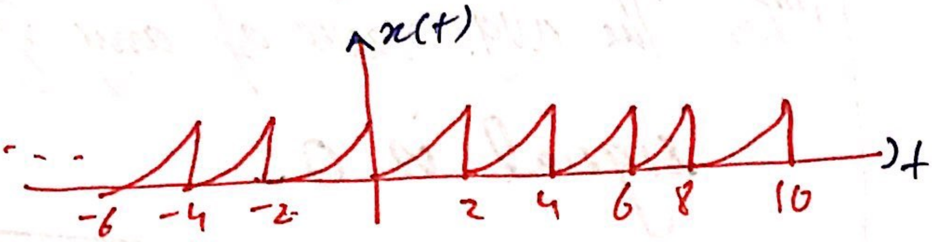
An-other way is that Find area A (triangle) and B (Rectangle) then add them and divide by $T_0 = 4$




$$\text{Avg. value} = \frac{1}{4}\left[ \frac{1}{2} \times 2 \times 3 + 2 \times (-4) \right]$$

$$= \frac{-5}{4}$$

Q




Avg. value?

Solution: $x(t)$ is periodic with

$$T_0 = 2$$

From 0 to 2 : $x(t) = t^2$ (Parabolic)

$$\text{Avg. Value} = \frac{1}{2}\int_0^2 t^2 dt$$

$$= \frac{1}{2}\,\frac{t^3}{3}\Big|_0^2 \Rightarrow \frac{1}{6}[8] = \frac{4}{3}$$

$$\boxed{\text{Avg. value} = \frac{4}{3}}$$

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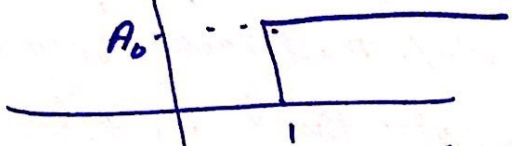
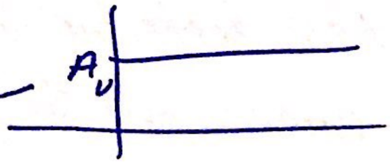
Imp : Time Shifting has no effect on avg. value.

avg. value $= \frac{A_0}{2}$ ←




compare with

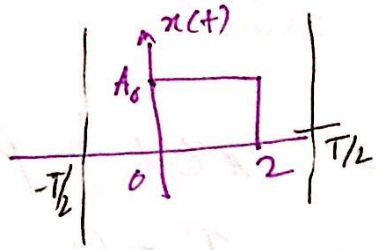
Avg value $= \frac{A_0}{2}$ ←




H.W: check whether Time Reversal has any effect on avg. value

Imp ⇒ The avg. value of any finite duration signal is 0.

Ex




Avg. Val = ?

Solution

$$\text{avg. val} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{2} A_0\,dt$$

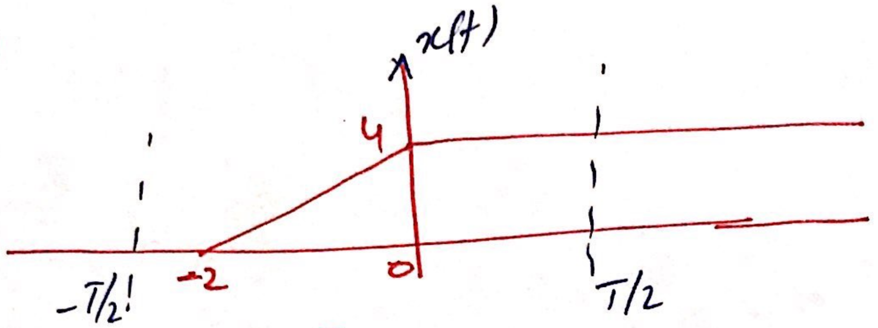
$$= \lim_{T\to\infty} \frac{1}{T} A_0 \, t \Big|_0^2$$

$$= \lim_{T\to\infty} \frac{1}{T} A_0 \times 2 = \frac{2A_0}{\infty} = 0$$

⇒ Look avg. val. is total area / total time (In case of finite duration signal area is very small compared to the total time $(-\infty, \infty)$ so very small value / very large value = 0.

→ We can also think an other way:
In avg. we're trying to distribute something equally. Now distributing the area from (0 – 2) over $-\infty$ to $\infty$ comes out to very small. Theoratically $= 0$

Q




Avg. Value ?

Ans ~~Aper~~ Aperiodic signal.

→ we usually take $-T/2$ and $T/2$ s.t it encloses all the transitions.

$$\text{avg. value} = \lim_{T \to \infty} \frac{1}{T}\left[\int_{-2}^{0}(2t+4)\,dt + \int_{0}^{T/2} 4\,dt\right]$$

$$= 2$$

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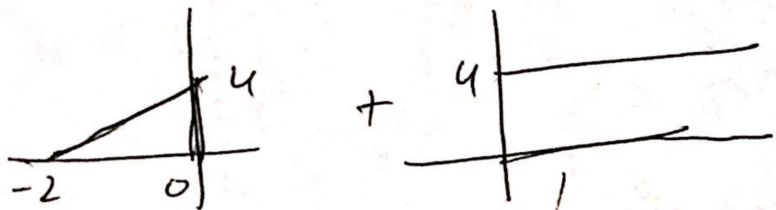
We can also find it an other way.

we can divide $x(t)$ in to 2 diff signals.




↓ Avg. value of this sig = 0 ∵ finite signal

↓ Avg. value = $\frac{4}{2}$

$0 + 2 = 2$ Avg. value of $x(t)$.