Signal :→ A signal is $\boxed{\text{something}}^{*}$ that $\boxed{\text{varies}}^{**}$ with some other $\boxed{\text{thing}}^{***}$.

→ Mathematicians call it a fun^n^

where
* is the fun^n^ itself.
*** " " independent variable
** Variations, which carry info., is the dependant variable.

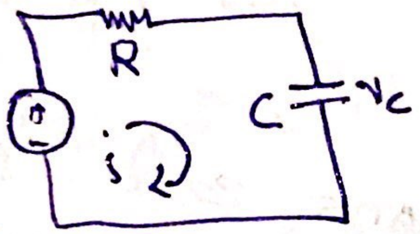
→ For example:
Voltage, current, Sound
$x(t)$, heart beat, temp, vibrations

Signal :→ "A fun^n^ of $\boxed{\text{one}}^{*}$ or $\boxed{\text{more}}^{**}$ variables that convey info. on the nature of a physical phenomenon".

* Signal depending upon one independent variable are known as one-dimensional and there are two or more independant variables then the signal is called multidimensional.

→ We'll primarily be studying signals that are funⁿ of time only.

<u>Example:→</u> Signal in an electrical ckt

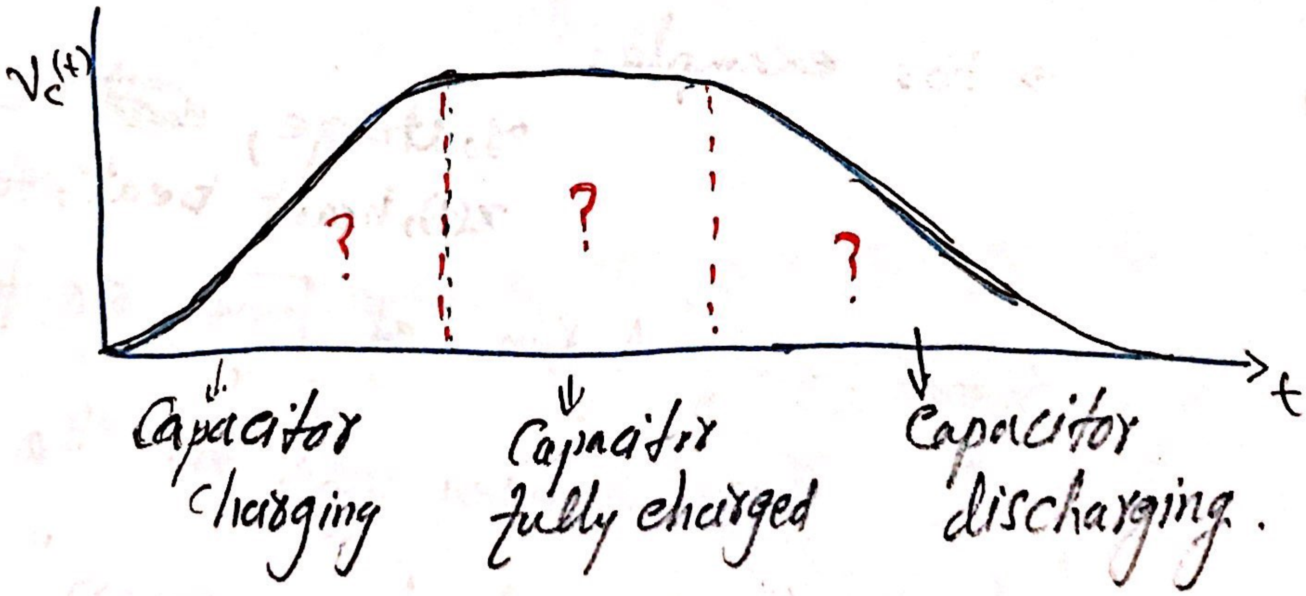


$$i(t) = \frac{v_s(t) - v_c(t)}{R}$$

$$i(t) = C\,\frac{dv_c(t)}{dt}$$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_s(t)$$

→ Signals $v_c(t)$ and $v_s(t)$ are patterns of variation over time.



System :→ A meaningfull interconnection of components or objects to perform a task.

e.g., an electric motor (water pump)

Q:→ Does the water pump do what it's meant to do by itself?

Ans:→ No it requires some i/p.

Q:→ What do we expect from a water pump?

Ans:→ To output some water.

→ Thus we have an other definition of system.

"System process i/p sig. to produce o/p signals." "A sys. takes an i/p sig & transform it to an o/p sig."

I/P $\xrightarrow{x(t)}$ $\boxed{\text{System}}$ $\xrightarrow{y(t)}$ O/P

Relation b/w $y(t)$ and $x(t)$

⇒ This is what we'll study throughout this subject.

The Concept of CT & DT

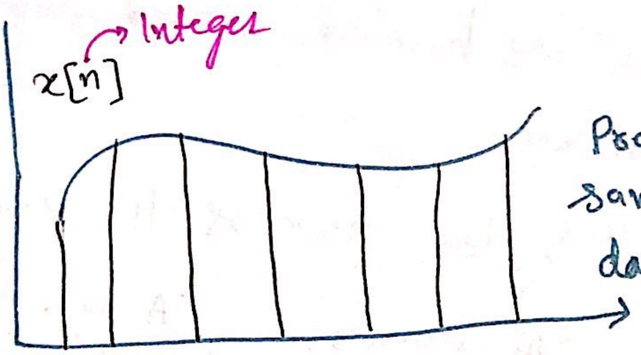
CT :-> Signal is ~~available~~ defined at any instant of time.

* -> Signal specified for every value of time

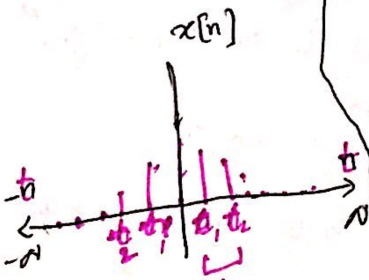
-> The advent of $\boxed{\text{digital}}$ computers and its growing use.

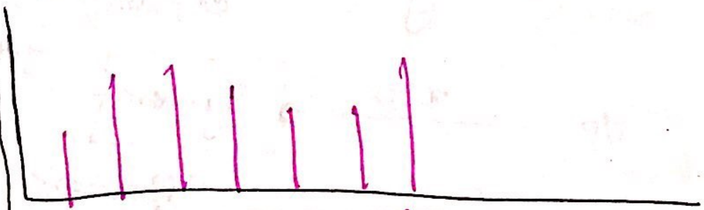
-> Limited processing capabilities

** -> Specified at discrete time intervals



Process sampled data.





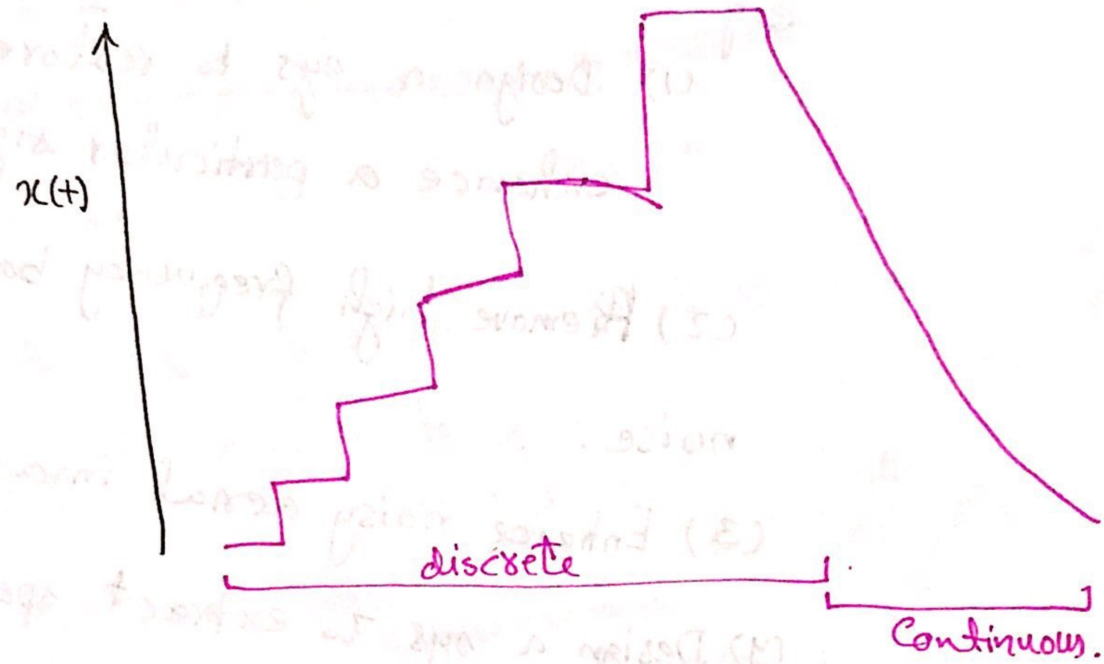
DT signal

not defined b/w this interval

if $t_1 - t_0 = t_2 - t_1 = \dots \, t_n - t_{n-1} = \Delta t$ then the ~~signal~~ DTS is said to be uniformly sampled ($\Delta t = T$

Q:→ Is time discrete?

Ans:→ No, its us who look at discrete instants af ~~of~~ time.



⇒ So signals & their study involve both CTS & DTS.

CTS:→ CT signals take on real or Complex values as a fun' of an independent variable that ranges over the real nos. & are denoted by $x(t)$. *

DTS:→ DT signals take on real or complex values as a fun' of an independent variable that ranges over the integers and is denoted by $x[n]$. **

Q:→ What does the subject of SSS do?

Ans:→ To design systems s.t a signal is processed in particular ways.

e.g

(1) Design a sys to restore or enhance a particular signal.

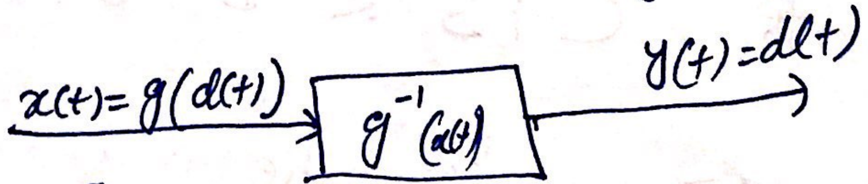
(2) Remove high frequency background noise.

(3) Enhance noisy aerial images.

(4) Design a sys to extract specific pieces of info. from a signal.

e.g

(5) Estimate the heart beat rate from an electrocardiogram



(6) To design a (dynamic) sys. to modify or control another (dynamic) sys.

Classification of Signals

(1) Periodic & non-periodic signals.

(2) Even & odd signals.

(3) Deterministic & random signals.

① Periodic Signal :→ A signal is periodic if it repeats itself after a fixed period T. (imp: This repetition should be valid from $-\infty$ to ∞ i.e., $\forall t$

See more Explanation in

mathematically.

$$x(t) = x(t+T) \quad \forall t,$$

e.g $\sin(t)$

→ The smallest value of T that satisfies this definition is called the period of the signal.

② Even & Odd Signal :→ A signal is even if

$$x(-t) = x(t), \quad x[-n] = x[n]$$

e.g., $\cos(t)$

→ A signal is odd if

$$x(-t) = -x(t), \quad x[-n] = -x[n]$$

Let $x_e(t)$ represent even signal

and $x_o(t)$ " odd "

→ Then any signal can be represented as a sum of unique $x_e(t)$ and $x_o(t)$.

i.e

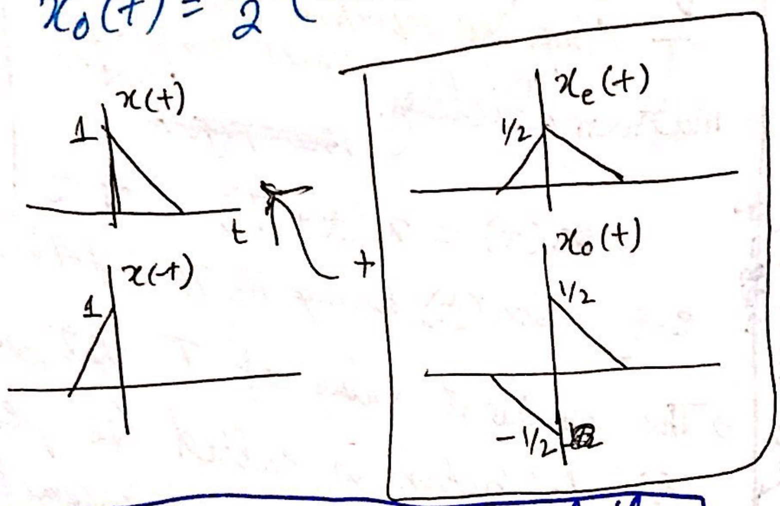
$$x(t) = x_e(t) + x_o(t)$$

where

$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$

and

$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$



See S&S-3 also for further details.

③ Deterministic & Random Signals

Deterministic Signal :→ A signal is deterministic if

(a) its behavior is predictable w-r-t time

(b) They can be expressed mathematically while having no uncertainity

e.g $\sin(t)$, $\cos(3t)$ etc.

Random Signals :-> A signal is random in nature if

(a) if its behavior is not predictable w-r-t time.

(b) These signals can't have an exact (certain) mathematical representation.

e.g Noise

Some Basic Signals.

1. Sinusoidal
2. Step
3. Ramp
4. Impulse
5. Exponential.

Classification of Systems

- System
  - Linear
    - Time Varying
    - Time Invariant
  - non-linear
    - Time Varying
    - Time Invariant.

→ A further division is that a sys can be static or dynamic.

→ Moreover, in all the above categories a sys can be stable or un-stable, causal or non-causal.

Causal & non-Causal systems

Causal System: → A sys is causal if its present o/p depends upon only on the present i/p and/or past values of the i/p

e.g

$$y(t) = x(t) + \frac{1}{2}x(t-1)$$

or

$$y[n] = x[n] + \frac{1}{2}x[n-1]$$

For a causal sys the response doesn't begin before the application of $x(t)$

i.e

$y(t_0)$ depends on i/p $x(t)$ $\forall\, t \le t_0$

→ Usually most physical systems are causal.

→ Causal sys. are physically realizable ∵ of no dependence on future values.

e.g (a) Memoryless system

$$y(t) = 1 - x(t) \cos \omega t$$

(b) Autoregressive filter

$$y(t) = \int_0^\infty x(t-\tau)\, e^{-\beta\tau}\, d\tau$$

Non-causal or Anti-causal System

A system is non-causal if its current o/p has some dependence on the future values of the i/p in addition to its present and past values.

However if the current o/p depends solely on future values and/or present values of the i/p ~~then~~ but NOT the past values is called anticausal.

e.g

(a) Non-causal.

① $y(t) = \int_0^{\infty} \sin(t+\tau)\, x(\tau)\, d\tau$

② central moving average.

$$y[n] = \frac{1}{2} x[n+1] + \frac{1}{2} x[n-1]$$

(b) Anticausal

① $y(t) = \int_0^{\infty} x(t+\tau)\, d\tau$

② Look ahead

$$y[n] = x[n+1]$$

→ Non-causal or anticausal sys are not physically implementable.

→ however, if the i/p signal is available in advance then such sys. may be implemented.

Examples ① Population Growth.

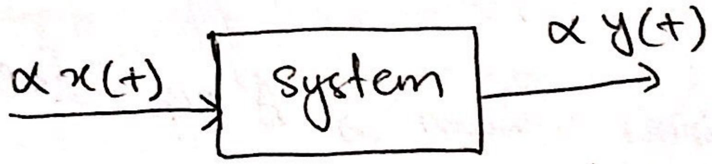
② Weather forecasting

③ Planning Commission etc.

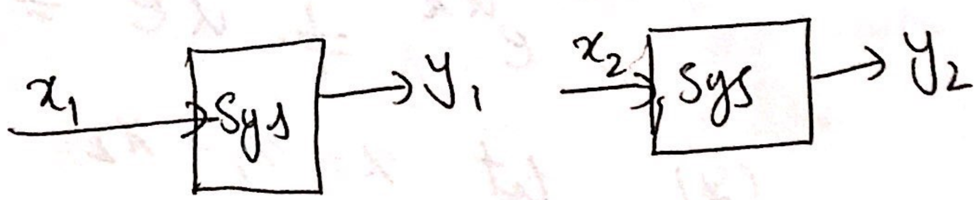
Linear Systems

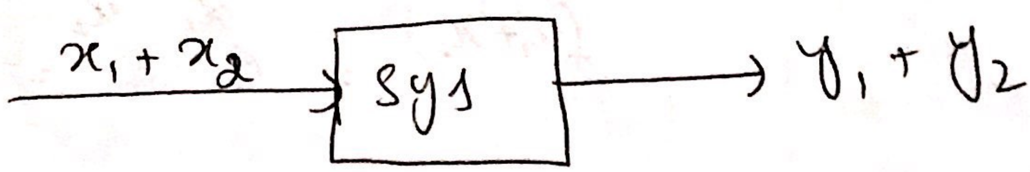
Systems satisfying homogeneity and additivity
i.e systems obeying superposition are termed as linear systems.

→ Homogeneity (scale rule) means scaled inputs will produce o/ps scaled with the same amount



→ Additivity means that the overall o/p of the system is the sum of the o/ps resulting from individual i/ps





→ So in general a linear system can be represented as

$$\alpha x_1(t)+\beta x_2(t) \longrightarrow \boxed{\text{System}} \longrightarrow \alpha y_1(t)+\beta y_2(t)$$

or

$$\sum_{i=-\infty}^{\infty} c_i x_i(t) \longrightarrow \boxed{\text{System}} \longrightarrow \sum_{i=-\infty}^{\infty} c_i y_i(t)$$

e.g $\quad y(t) = \frac{1}{2} x(t)$

<u>Non-linear System</u> :→ A sys. doesn't obeying superposition principle is non-linear

e.g $\quad y = e^{x}$

(1) Scaling x by α

$$e^{\alpha x} \neq \alpha e^{x} = \alpha y$$

(2) Let $x = x_1 + x_2$

$$e^{(x_1+x_2)} = e^{x_1} e^{x_2} \neq e^{x_1} + e^{x_2} = y_1 + y_2$$

Time-Invariant Systems

A sys. is time invariant if a time shift in the i/p sig causes an identical time shift in the the o/p.

$$x(t-t_0) \longrightarrow \boxed{\text{Time-Invariant system}} \longrightarrow y(t-t_0)$$

Example

① $y(t) = \cos(x(t))$

determine time invariance?

Sol :-> let $x_2(t) = x_1(t-t_0)$

then $y_2(t) = \cos(x_2(t))$

$= \cos(x_1(t-t_0))$

$= y_1(t-t_0)$ hence time-invariant.

② $y(t) = x(t)\cos t$

let $x_2(t) = x_1(t-t_0)$

$y_2(t) = x_2(t)\cos(t)$

$= x_1(t-t_0)\cos t$

$y_1(t-t_0)$

$= x_1(t-t_0)$

$\cos(t-t_0)$

$y_1 \neq y_2$ hence time varying

→ In other words

" A system is said to be time-invariant if the i/p, o/p xtics do not change with time ".

Examples from DTS.

① $y[n] = x[n] - x[n-2]$

Time-Invariant?

Solution :→

Step 1 :→ Delay the i/p by K samples and denote the o/p by $y[n,k]$

therefore

$$y[n,k] = x[n-k] - x[n-2-k]$$

Step 2 :→ Replace n by $n-k$ throughout the given eqn.

$$y[n-k] = x[n-k] - x[n-2-k]$$

Step 3 :→ Compare $y[n,k]$ and $y[n-k]$

if $y[n,k] = y[n-k]$ then time-invariant.

(2) $y[n] = x[n] + n\,x[n-2]$

Time invariant?

Solution

Step 1

$$y[n,k] = x[n-k] + n\,x[n-2-k]$$

Step 2

$$y[n-k] = x[n-k] + (n-k)\,x[n-2-k]$$

Step 3

$$y[n,k] \neq y[n-k]$$

So time varying system.

The main focus of this course will be Linear systems that are time invariant, generally known as Linear Time-Invariant (LTI) systems

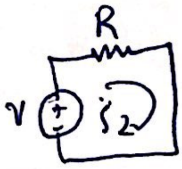
Static Systems :→

A sys. not having any energy storing elements are static systems.

Such systems are memoryless and hence depend only on the current values of the i/p.

e.g.,

$$i(t) = \frac{v(t)}{R}$$



Dynamic Systems.

" System with memory : A sys. is said to possess memory if its o/p depends on past &/or present values of i/p.

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau$$

"

$$y[n] = x[n] + x[n-1]$$

Systems having storage devices are termed as dynamic. Such systems possesses memory.