Impulse Response & Convolution Theorem

=> The term "IMPULSE RESPONSE" is valid for LTI systems only

=> The response (output) of an LTI system when UNIT IMPULSE is applied at its input is known as the IMPULSE RESPONSE

$$\delta(t) \longrightarrow \boxed{\begin{matrix}\text{LTI}\\ \text{Sys}\end{matrix}} \longrightarrow h(t)$$ [Impulse response]

=> The impulse response [h(t)] is fixed for an LTI system and we can use h(t) to find the output of the system for any INPUT.

$$h(t) \underset{L^{-1}}{\overset{L}{\rightleftarrows}} H(s)$$

Ex: $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$$\int dt$$

$$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

$h(t) = ?$

Solution :→ Taking Laplace T.

$$Y(s) = \frac{X(s)}{s}$$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{1}{s} = H(s)}$$

$$H(s) \xrightarrow{L^{-1}} h(t)$$

$$h(t) = L^{-1}\left[\frac{1}{s}\right] = u(t)$$

$$\boxed{h(t) = u(t)}$$

Method II

$$h(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

$$= u(t)$$

↳ it's simple in this case.

⇒ Often times in problems you are given the impulse Response and you are required to find the o/p of the sys.

$$h(t) \xrightarrow{L.T} H(s) = \frac{Y(s)}{X(s)}$$

$$\Rightarrow \quad Y(s) = H(s) \cdot X(s)$$

Multiplication in frequency domain becomes convolution in time domain and vice versa.

$$\Big\Downarrow L^{-1}$$

$$y(t) = h(t) * x(t)$$

where (*) represent the convolution

⇒ Convolution is a V.V.I.P Operator (tool).

⇒ " is Linear operator

⇒ Convolution operation is mainly used to calculate the o/p of a sys when the impulse Response is given.

⇒ So far we've studied the following operations.

① Shifting (time, amplitude)

② Scaling (" , ") [Reversal is its Special case]

③ Differentiation

[Modeling of continuous time signals]

④ Integration

Next we'll study

⑤ Convolution.

Convolution Operation

" A convolution is an integral that expresses the amount of overlap of one fun" when it is shifted over an other fun"."

$$\boxed{y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\,d\tau}$$

(A) the two signals are $x(\tau)$ and $h(\tau)$

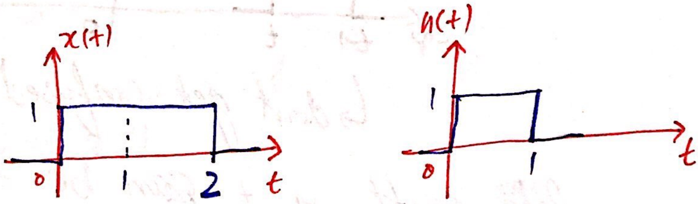
(B) One of the signals will be fixed e.g., $x(\tau)$ and the other signal will be shifted (moved) to calculate the overlap. e.g., $h(\tau)$

$$h(\tau) \xrightarrow{\text{T. Rev.}} h(-\tau) \xrightarrow{\text{T. shift}} h[-(\tau-t)] = h(t-\tau)$$

(C) $x(\tau)\cdot h(t-\tau)$

(d) $\int_{-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\,d\tau$

Ex

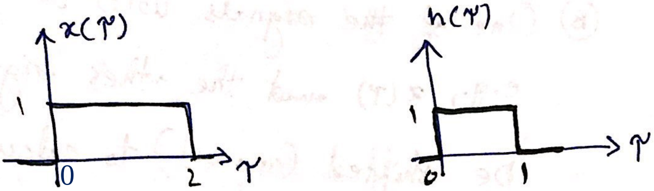


$y(t) = ?$

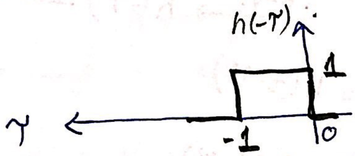
y(t) = x(t) * h(t) ?

Solution :-

(A) Replace t by τ so that we don't get confuse with the "t" [Shifting value]

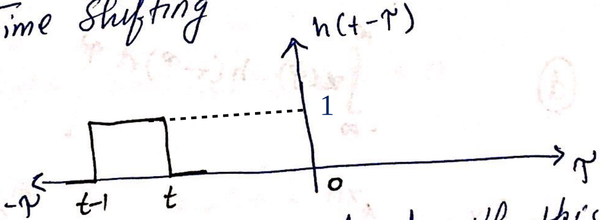


(B) Perform time reversal.



$h(-(\tau - t))$

(C) Time Shifting

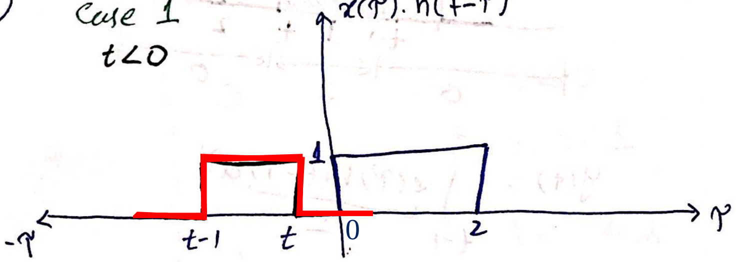


↳ don't get confused with this left shift as t can be +ve or -ve.

We'll start performing the integration by moving our signal from $-\infty$ to the right $(+\infty)$

⇒ The pts. of interest to us are the pts where the signal value is changing. e.g., at $\tau=0$ $x(t)$ changes from 0 to 1 and at $\tau=2$ $x(t)$ changes from 1 to 0

(d) Case 1
$t<0$

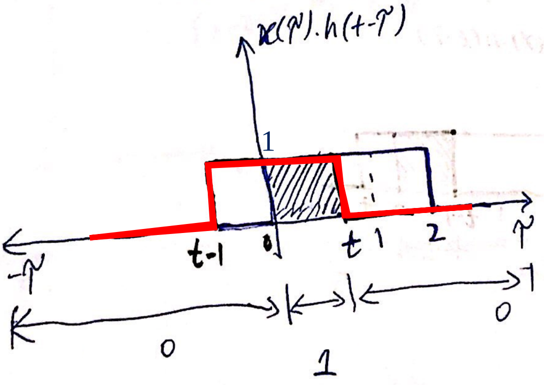


No overlap b/w two signals so

$$x(\tau)\cdot h(t-\tau)=0$$

$$y(t)=\int_{-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\,d\tau=0$$

Case 2: $0<t<1$

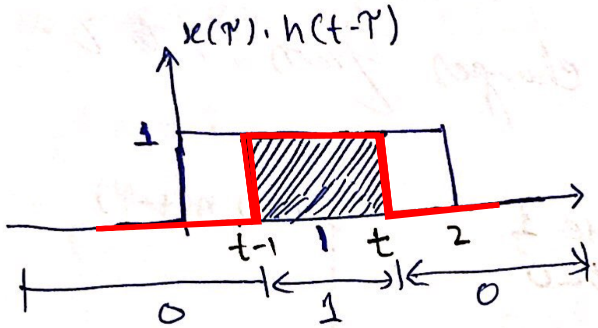


So we'll perform integration from 0 to t

$$y(t) = \int_0^t \underbrace{x(\tau)h(t-\tau)}_{1} d\tau$$

$$= \int_0^t d\tau = \quad t - 0 = t$$

<u>Case 3</u> $\quad 1 < t < 2$

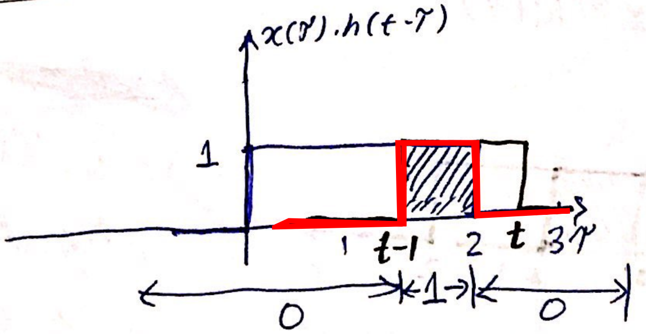


$$y(t) = \int_{t-1}^{t} \underbrace{x(\tau)h(t-\tau)}_{1} d\tau$$

$$= \int_{t-1}^{t} d\tau = t - t + 1 = 1$$

$$\boxed{y(t) = 1}$$

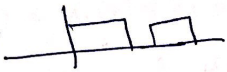
<u>Case 4</u> :→ $\quad 2 < t < 3$



$$y(t) = \int_{t-1}^{2} 1\, d\tau = 2 - t + 1 = 3 - t$$

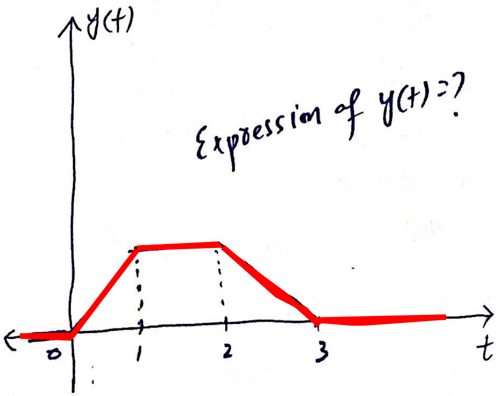
$$\boxed{y(t) = 3 - t}$$

Case 5 $\quad t > 3$

again no overleping 

So $\quad y(t) = 0$

$$y(t) = \begin{cases} 0 & t<0 \\ t & 0<t<1 \\ 1 & 1<t<2 \\ 3-t & 2<t<3 \\ 0 & t>3 \end{cases}$$



When we convolve two pulses having equal width then the resulting waveform will be triangular. However if the widths are un-equal then result will be a trapezoidal waveform

$$y(t) = 0 + r(t) - r(t-1) - r(t-2) + r(t-3)$$

Assignment :-> Create an animation of the convolution operation.

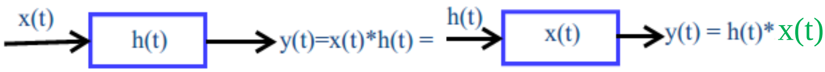
Properties of Convolution

The following outlined ten properties are very important ones

1) Commutative Property

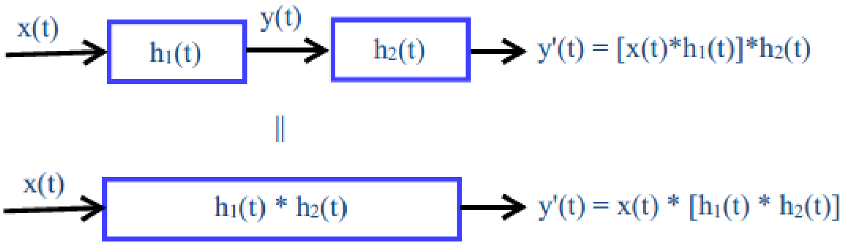
$$x_1(t)*x_2(t) = x_2(t)*x_1(t)$$

i.e., whichever of the signals we flip and move and whichever fix, doesn't matter



2) Associative Property

$$[x_1(t)*x_2(t)]*x_3(t) = x_1(t)*[x_2(t)*x_3(t)]$$

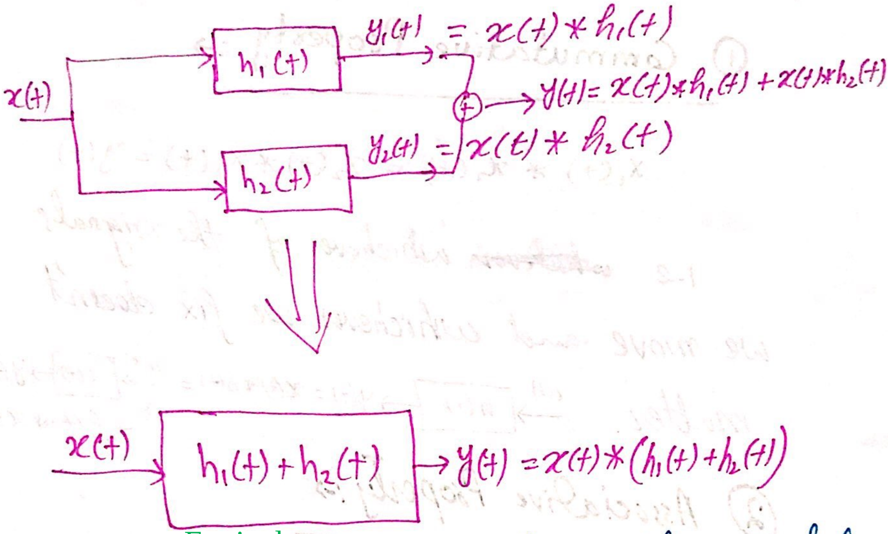


Equivalent System

IMPORTANT: Whenever we have two systems is cascade then the impulse response of the equivalent system will be the convolution of the impulse responses of the cascaded systems

③ Distributive Property :->

$$x_1(t) * (x_2(t) + x_3(t)) = [x_1(t) * x_2(t)] + [x_1(t) * x_3(t)]$$



Equivalent system

Imp: The impulse response of the equivalent system of parallel connected systems is the summation of the individual impulse responses.

④ Property of Delta Function :->

↳ Impulse signal / fun"

$$\boxed{x(t) * \delta(t - t_1) = x(t - t_1)}$$

For t

For $t_1 = 0$

$$\boxed{x(t) * \delta(t) = x(t)}$$

if weight of $\delta(t)$ is other than 1

then

$$\boxed{x(t) * A\delta(t-t_1) = Ax(t-t_1)}$$

"A" is constant

$$\boxed{x(t) * A\delta(t) = Ax(t)}$$

Ex 1 $x(t) = \gamma(t)$

then $\gamma(t) * \delta(t-2) = ?$

Solution

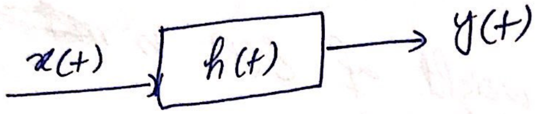
$$\gamma(t) * \delta(t-2) = \gamma(t-2)$$

Ex 2 $u(t+3) * \delta(t-1) = ?$

$$u(t+3) * \delta(t-1) = u(t-1+3) = u(t+2)$$

(5) Property of Derivative

Let we've a sys. with IR "$h(t)$"



then

$$y(t) = x(t) * h(t)$$

Now we want to take the first derivative of the o/p $y(t)$

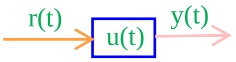
$$\frac{d}{dt} y(t) = ?$$

According to the prop.

$$\frac{d}{dt} y(t) = x(t) * \frac{d}{dt} h(t)$$

$$= \frac{d}{dt} x(t) * h(t)$$

Ex:→ $r(t)$ is the i/p to a system with IR, "$u(t)$" then find $\frac{dy(t)}{dt}$ where



$$y(t) = r(t) * u(t) \quad ?$$

Solution Method 1

$$\frac{d}{dt} y(t) = r(t) * \frac{d}{dt} u(t)$$

$= r(t) * \delta(t) = r(t)$

Method II

$$\frac{d}{dt} y(t) = \frac{d}{dt} r(t) * u(t)$$

$$= \underbrace{u(t) * u(t)}_{\text{why is this } r(t) \text{ will be}} = r(t)$$

discussed after property no. 6

(6) This property is related systems having $h(t) = u(t)$ [i.e., their IR is a unit step signal.

The prop. says.

$$y(t) = x(t) * u(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Proof:→

$$y(t) = x(t) * u(t)$$

$$\frac{dy(t)}{dt} = x(t) * \frac{d}{dt} u(t)$$

$$\frac{dy(t)}{dt} = \underbrace{x(t) * \delta(t)}_{\text{Prop: 4}} = x(t)$$

$$\int_{-\infty}^{t} \cancel{\frac{d}{d\tau}} y(\tau)\, d\tau = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$$\boxed{y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau = x(t) * u(t)}$$

↳ Proved.

Ex

$y(t) = r(t) * u(t)$?

Solution

$$y(t) = \int_{-\infty}^{t} r(\tau)\, d\tau$$

$= p(t) \rightarrow$ Parabolic signal

$$= \frac{t^2}{2} u(t)$$

Ex $y(t) = u(t) * u(t) = ?$

Soln $y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau = r(t)$

⑦ Property of Time Delay

$$x_1(t) \xrightarrow{\text{Delay of } t_1} \qquad * \qquad x_2(t) \xrightarrow{\text{Delay of } t_2} \qquad = \qquad y(t)$$

$$\boxed{x_1(t-t_1) \;*\; x_2(t-t_2) = y(t-[t_1+t_2])}$$

i.e., the delays in the convaluting signals gets added & become the delay of the resultant signal.

Ex: $u(t-1) * u(t-2)$?

Soln :- First we'll convalute signals when there is no delay.

$$\underbrace{u(t) * u(t) = \int_{-\infty}^{t} u(t)}_{\text{By prop: 6}} = r(t)$$

So $u(t-1) * u(t-2) = r(t-(1+2)) = r(t-3)$

Ex $r(t+2) * u(t-3) = ?$

$$r(t) * u(t) = \int_{-\infty}^{t} r(\tau)\,d\tau = \frac{t^2}{2}\,u(t)$$

$$\delta(t+2) * u(t-3) = \frac{(t-3+2)^2}{2} = \frac{(t-1)^2}{2} u(t-1)$$

(8) Prop. of Time Scaling

$$x_1(t) \quad * \quad x_2(t) = y(t)$$

$x_1(t)$ ↓ T. Scaling by a $\qquad$ $x_2(t)$ ↓ T. Scaling by a

$$\boxed{x_1(at) * x_2(at) = \frac{1}{|a|} y(at)}$$

$$a \neq 0$$

Ex let

$$(t^2+t) * t = y(t)$$

M =?
N = ?

the $M*N = \underbrace{\frac{1}{3} y(3t)}_{\text{from prop: we know } a=3}$

Solution $M(3t)*N(3t) = \frac{1}{3} y(3t)$

$M(t) = t^2+t \Rightarrow M(3t) = (3t)^2+3t$

$= 9t^2+3t$

$N(t) = t \Rightarrow N(3t) = 3t$

$$(9t^2+3t) * 3t = \frac{1}{3} y(3t)$$

⑨ Prop. of area

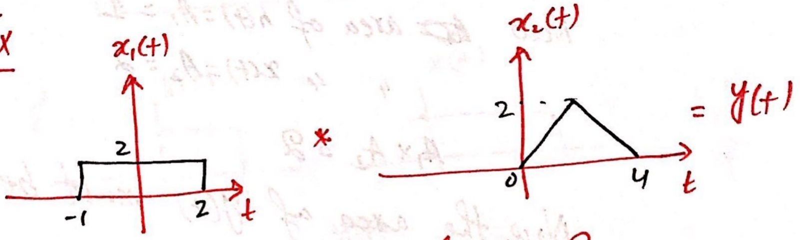
Let we convolve two signals x_1 & x_2 with areas A_1 & A_2 respectively

$$x_1 * x_2 = y$$

then the area of the resultant "$y(t)$" will be $A_1 \times A_2$

$$\boxed{\begin{array}{ccccc} x_1 & * & x_2 & = & y \\ \big\downarrow \text{area} & & \big\downarrow \text{Area} & & \big\downarrow \text{Area} \\ A_1 & \times & A_2 & = & A \end{array}}$$

Ex



What is the area of $y(t)$?

Solution :→ Area of $x_1 = A_1 = 3\times 2 = 6$

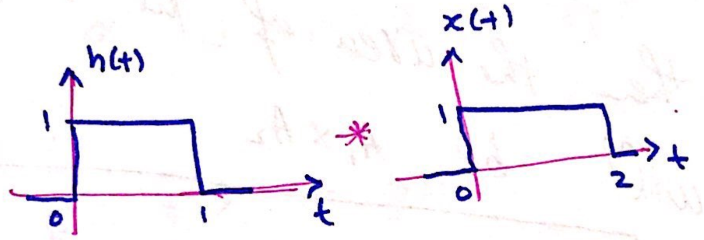
" " $x_2 = A_2 = \frac{1}{2}\times 4\times 2 = 4$

Area of $y(t) = A_y = A_1 \times A_2$

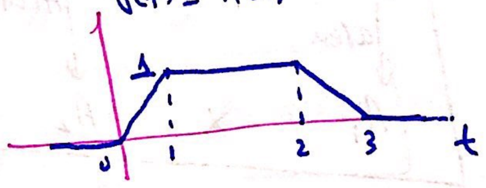
$= 6\times 4 = 24$

imp :→ Using this prop: We can cross check our answers/results.

e.g., Recall the very first example of Convolution.



= $y(t) = h(t) * x(t)$

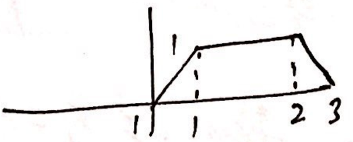


Now area of $h(t) = A_1 = 1$

" " $x(t) = A_2 = 2$

$A_1 \times A_2 = 2$

Now the area of $y(t)$ must be 2



$\frac{1}{2}(1 \times 1) + (1 \times 1) + \frac{1}{2}(1 \times 1)$

$\frac{1}{2} + 1 + \frac{1}{2} = 2$

(10) Prop. of Duration/Extension

let we've two signals $x_1(t)$ and $x_2(t)$

s.t

$x_1(t)$ is non-zero if $t_1 \leq t \leq t_2$

& $x_2(t)$ is non-zero if $t_3 \leq t \leq t_4$

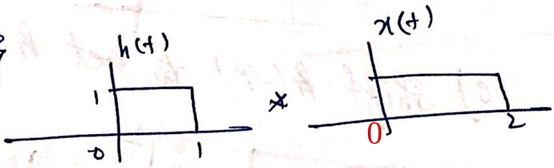
and let

$x_1(t) * x_2(t) = y(t)$

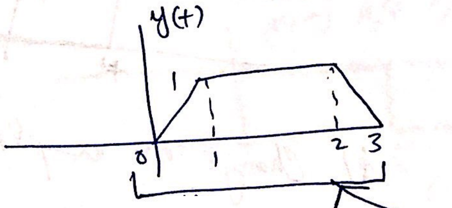
then $y(t)$ is non-zero if

$$\boxed{t_1 + t_3 \leq t \leq t_2 + t_4}$$

E.g



$t_1 = 0,\ t_2 = 1$
$t_3 = 0,\ t_4 = 2$
$t_1 + t_3 = 0$
$t_2 + t_4 = 3$

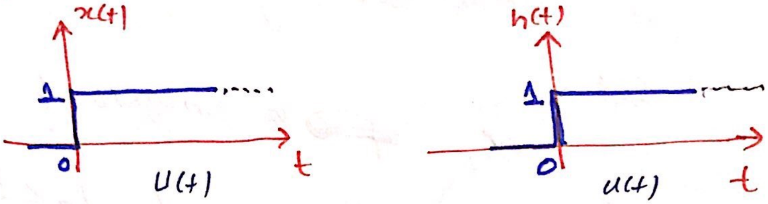


$t_1 = 0,\ t_2 = 1,\ t_3 = 0,\ t_4 = 2$

$0 + 0 \leq t \leq 1 + 2$

$0 \leq t \leq 3$

Q :-> Find the o/p y(t) of an LTI sys. for the i/p and IR (h(t)) given below?



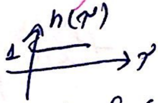
Solution

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = ?$$

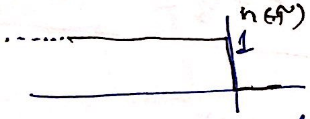
$$y(t) = x(t) * h(t)$$

Methode 1 :-> Conventional approach of convolution

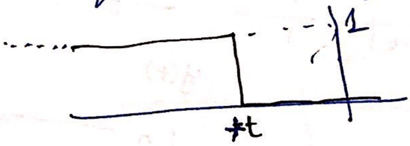
a) Replace t by τ



b) Reverse $h(\tau)$ to get $h(-\tau)$



c) Shift $h(-\tau)$ to get $h(t-\tau)$



d) Change in amplitude of $x(t)$ is at $t=0$

"Instants at which signal value changes are the most imp. ones"

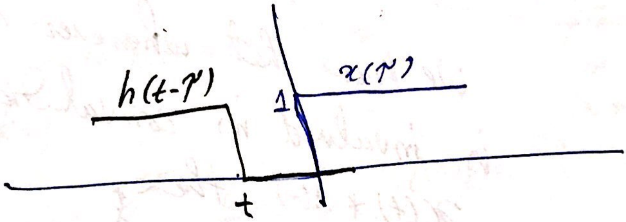
∵ we can't give infinite steps
however, we instead for steps before and beyond the instances of change.

⇒ In this case we need to check the overlap for two instances

(i) $t < 0$ ii) $t > 0$

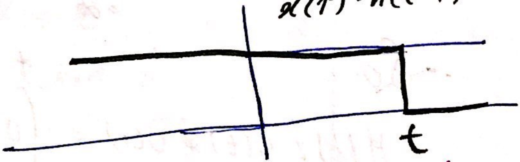
Case 1:→ For $t < 0$

$$x(\tau) \cdot h(\tau) = 0 = y(t)$$



Case 2:→ $t > 0$

$$x(\tau) \cdot h(t-\tau) = 1$$



$$y(t) = \int_{-\infty}^{\infty} 1 \, d\tau = \int_{-\infty}^{t} d\tau = \int_{0}^{t} d\tau = t$$

$$y(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases} \quad y(t) = r(t)$$

Method 2 :→ Using Laplace Transform.

$$y(t) = x(t) * h(t)$$

$$u(t) * u(t)$$

Taking L.T

$$Y(s) = \frac{1}{s} \cdot \frac{1}{s} = \frac{1}{s^2}$$

Now taking inverse Laplace Transform

$$y(t) = t = r(t)$$

Method 3 :→ Using Prop. of convolution.

We know that whenever step signal is involved in convolution i.e., $x(t) * u(t)$ then

$$\boxed{x(t) * u(t) = \int_{-\infty}^{t} x(\tau)\, d\tau}$$

So

$$y(t) = u(t) * u(t) = \int_{-\infty}^{t} u(\tau)\, d\tau = r(t)$$

Question

$$x(t+5) * \delta(t-7) = ?$$

Solution :-> Recall prop. 4

$$\boxed{x(t) * \delta(t-t_1) = x(t-t_1)}$$

Using this

$$x(t+5) * \delta(t-7) = x(t-7+5) =$$

$$= x(t-2)$$

Solving using L.T

$$y(t) = x(t+5) * \delta(t-7)$$

Taking L.T

$$Y(s) = X(s)\, e^{5s} \cdot 1\, e^{-7s} \quad \because L[\delta(t)] = 1$$

Convolution in t-domain is multiplication in s-domain.

$$= X(s)\, e^{-2s}$$

Now takin L^{-1}

$$\boxed{y(t) = x(t-2)}$$

Q :-> $x(-t) * \delta(-t-t_0) = ?$

a) $x(-t+t_0]$ b) $x(t-t_0)$

c) $x(t+t_0)$ d) $x(-t-t_0)$

Solution

$y(t) \quad x(-t) * \delta(-t-t_0)$

Taking L.T

$Y(s) = X(-s) \times 1 e^{-t_0 s}$

L^{-1}

$$\boxed{y(t) = x(-t-t_0)}$$

or use prop.

$y(t) = x(-t) * \delta(-t-t_0)$

$$\boxed{y(t) = x(-t-t_0)}$$

Q:→ The IR of a sys is $h(t) = \delta(t-0.5)$, if two such systems are cascaded, then the IR of the overall sys is ----?

Solution :→ We can solve it using properties as well as L.T.

We know $x(t) * \delta(t-t_1) = x(t-t_1)$

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = x(t) * h(t)$ ↗ given

$= x(t) * \delta(t-0.5)$

$= x(t-0.5)$

Now we cascade the other sys. having the same IR.

$$x(t) \longrightarrow \boxed{h(t)} \xrightarrow[x(t-0.5)]{y(t)} \boxed{h(t)} \longrightarrow y'(t)$$

$$y'(t) = y(t) * h(t)$$
$$= x(t-0.5) * \delta(t-0.5)$$
$$= x(t-0.5-0.5) = x(t-1)$$

$\downarrow$

o/p of the overall sys.. However we're asked for IR of the overall sys.

Let $h_o(t)$ is the IR of the overall sys. then we know

$$x(t) \longrightarrow \boxed{h_o(t)} \longrightarrow x(t-1)$$

$$x(t-1) = x(t) * h_o(t)$$

Taking L.T (keep in mind that L[IR] = t/f

$$X(s)\, e^{-s} = X(s) \cdot H_o(s)$$

$$H_o(s) = e^{-s}$$

Now taking ILT

$$\boxed{h_0(t) = \delta(t-1)} \text{ Ans.}$$

Method-2 Recall the result obtained from associative prop.

i.e., IR of equivalent sys of cascaded systems is the convolution of the IR of the individual systems.

$$h_0(t) = \delta(t-0.5) * \delta(t-0.5)$$

$$= \delta(t-0.5-0.5)$$

$$\boxed{h_0(t) = \delta(t-1)}$$

Q:→ For An LTI system with an IR $h(t)$

produces an o/p $y(t)$ when i/p $x(t)$ is applied. When the i/p $x(t-\tau)$ is applied to a system with IR $h(t-\tau)$, the o/p will be...?

Solution Recall the prop 7 (time delay)

which says

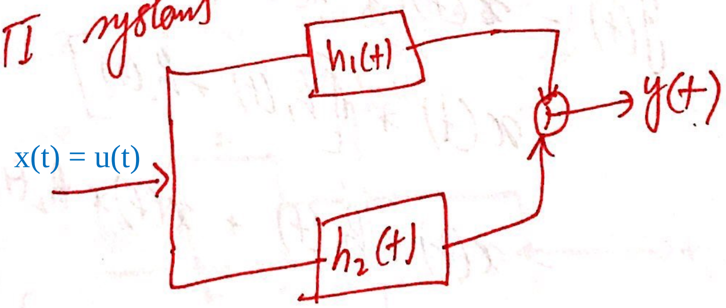
$$x_1(t-t_1) * x_2(t-t_2) = y(t-(t_1+t_2))$$

hence

$$x(t-\gamma) * h(t-\gamma) = y(t-(\gamma+\gamma))$$

$$\boxed{= y(t-2\gamma)}$$

Q→ Consider the // connection of two LTI systems



where $h_1(t) = 2\delta(t+2) - 3\delta(t+1)$

$h_2(t) = \delta(t-2)$

Find $y(t)$ and its total energy when the I/P $x(t)$ is a unit step?

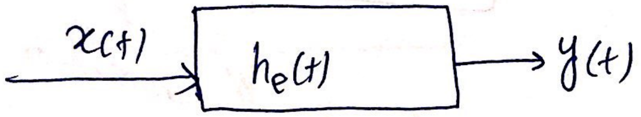
Solution :- Recall the result we've obtained from distributive prop.

i.e., IR of equivalent sys of parallel connected systems is the sum of the individual I. Responses.

i.e.,

$$h_e(t) = h_1(t) + h_2(t)$$

$\hookrightarrow$ equivalent



$$y(t) = x(t) * h_e(t)$$

$$= x(t) * [h_1(t) + h_2(t)]$$

$$= x(t) * h_1(t) + x(t) * h_2(t)$$

$$= x(t) * [2\delta(t+2) - 3\delta(t+1)]$$

$$+ x(t) * \delta(t-2)$$

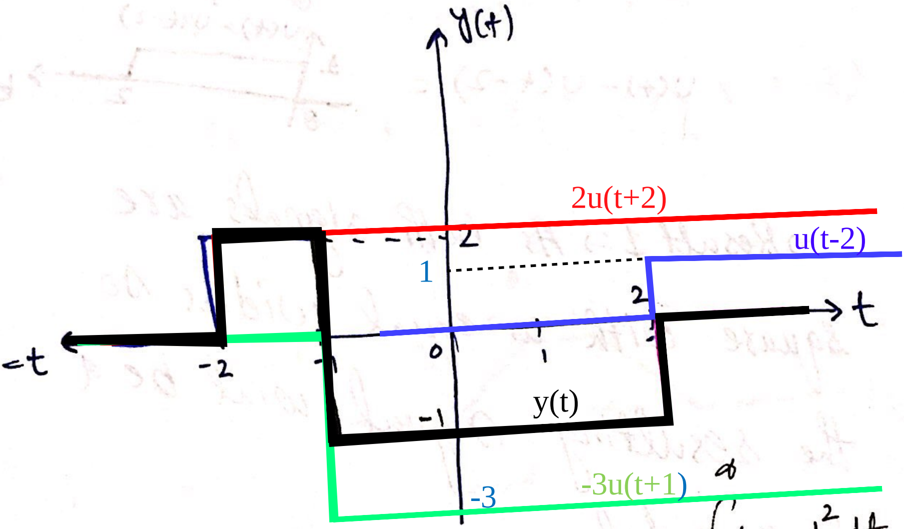
$$= 2\, x(t) * \delta(t+2) - 3\, x(t) * \delta(t+1)$$

$$+ x(t)\,\delta(t-2)$$

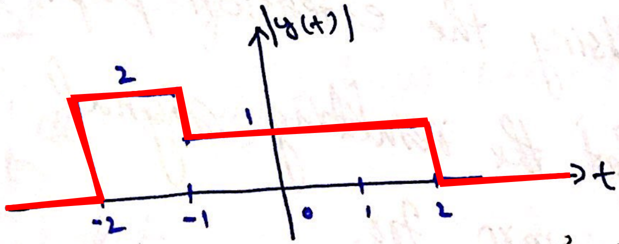
$= 2x(t+2) - 3x(t+1) + x(t-2)$

Now $x(t) = u(t)$

$= 2u(t+2) - 3u(t+1) + u(t-2)$



Now the total energy $= E = \int_{-\infty}^{\infty} |y(t)|^2 dt$



-2 to -1 $|y(t)| = 2$ and $|y(t)|^2 = 4$

-1 to 2 $|y(t)| = 1$ and $|y(t)|^2 = 1$

$$E = \int_{-\infty}^{\infty} |y(t)|^2 dt = \int_{-\infty}^{-2} 0\,dt + \int_{-2}^{-1} 4\,dt + \int_{-1}^{2} dt + \int_{2}^{\infty} 0\,dt$$

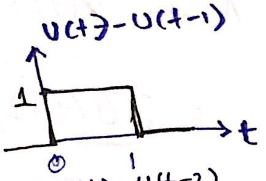
$$= 4(-1+2) + (2+1) = 4+3 = 7$$

Q :-> Plot

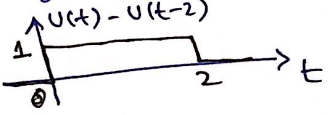
$$[u(t)-u(t-1)] * [u(t)-u(t-2)]$$

Solution

$u(t)-u(t-1) =$



$u(t)-u(t-2) =$



⇒ Result 1 :-> As both signals are square with unequal widths so the resulting signal will be a trapizoidal.

⇒ Using the extension prop. we know that the resulting signal will be non-zero for

$$0+0 \leq t \leq 1+2$$

$$0 \leq t \leq 3$$

Methode 1

$$[u(t)-u(t-1)] * [u(t)-u(t-2)]$$

$$= u(t) * [u(t)-u(t-2)] - u(t-1) * [u(t)-u(t-2)]$$

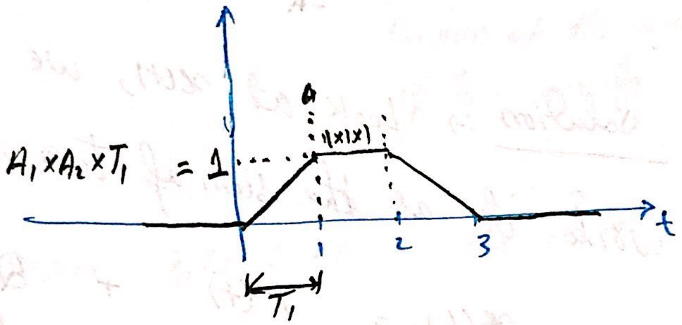
$$= u(t) * u(t) - u(t) * u(t-2) - u(t-1) * u(t)$$

$$+ u(t-1) * u(t-2)$$

$$= r(t) - r(t-2) - r(t-1) + e^{-s}\frac{1}{s} \cdot e^{-2s}\frac{1}{s}$$

$$= r(t) - r(t-1) - r(t-2) + e^{-3s}\frac{1}{s^2}$$

$$= r(t) - r(t-1) - r(t-2) + r(t-3)$$



<u>Method 2</u> Using Laplace transform

$$\left[u(t) - u(t-1)\right] * \left[u(t) - u(t-2)\right]$$

Taking L.T

$$= \left[\frac{1}{s} - \frac{1}{s}e^{-s}\right] \cdot \left[\frac{1}{s} - e^{-2s}\frac{1}{s}\right]$$

$$= \frac{1}{s^2}\left[1 - e^{-s}\right]\left[1 - e^{-2s}\right]$$

$$= \frac{1}{s^2}\left[1 - e^{-2s} - e^{-s} + e^{-3s}\right]$$

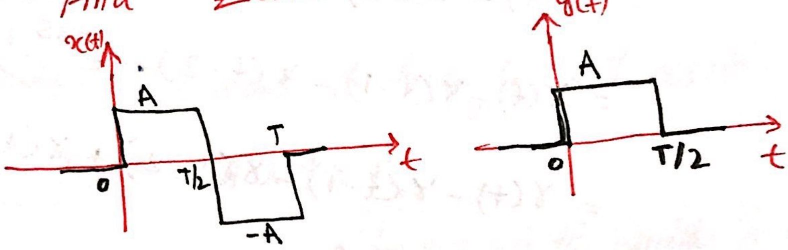
$$= \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-3s}}{s^2}$$

Now taking ILT

$= \delta(t) - \delta(t-1) - \delta(t-2) + \delta(t-3)$

Which is the same as previous.

Q :- Find $z(t) = x(t) * y(t)$



Solution :- Look at $x(t)$, we can write it as the sum of two signals

$x(t) = P(t) + Q(t)$

Also we can see that the only difference b/w $P(t)$ and $Q(t)$ is the -ve amplitude. i.e.,

$$P(t) = -Q(t)$$

or

$$Q(t) = -P(t)$$

$\Rightarrow$ In addition note that

$$y(t) = P(t) = -Q(t)$$

So

$$Z(t) = \left(P(t) + Q(t)\right) * y(t)$$

$$= P(t) * y(t) + \quad Q(t) * y(t) \rightarrow$$

$z_1 = P(t) * y(t) = ?$

widths are same so triangle will result.

max. peak $= A \times A \times \underbrace{T/2}$

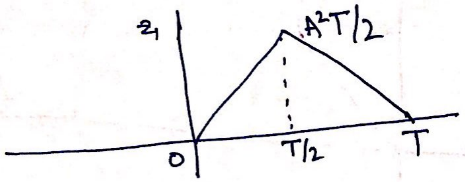
$\hookrightarrow$ min. of the widths of two signals.

$= A^2 T/2$

Moreover, $z_1(t) \neq 0$ in the range

$$0 + 0 \le t \le T/2 + T/2$$

$$0 \le t \le T$$



$z_2(t) = Q(t) * y(t) = ?$

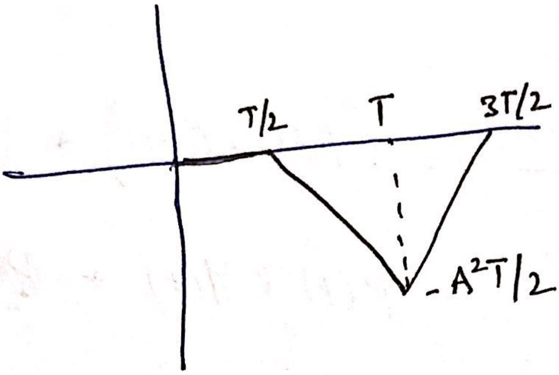
width $\rightarrow$ same so triangle.

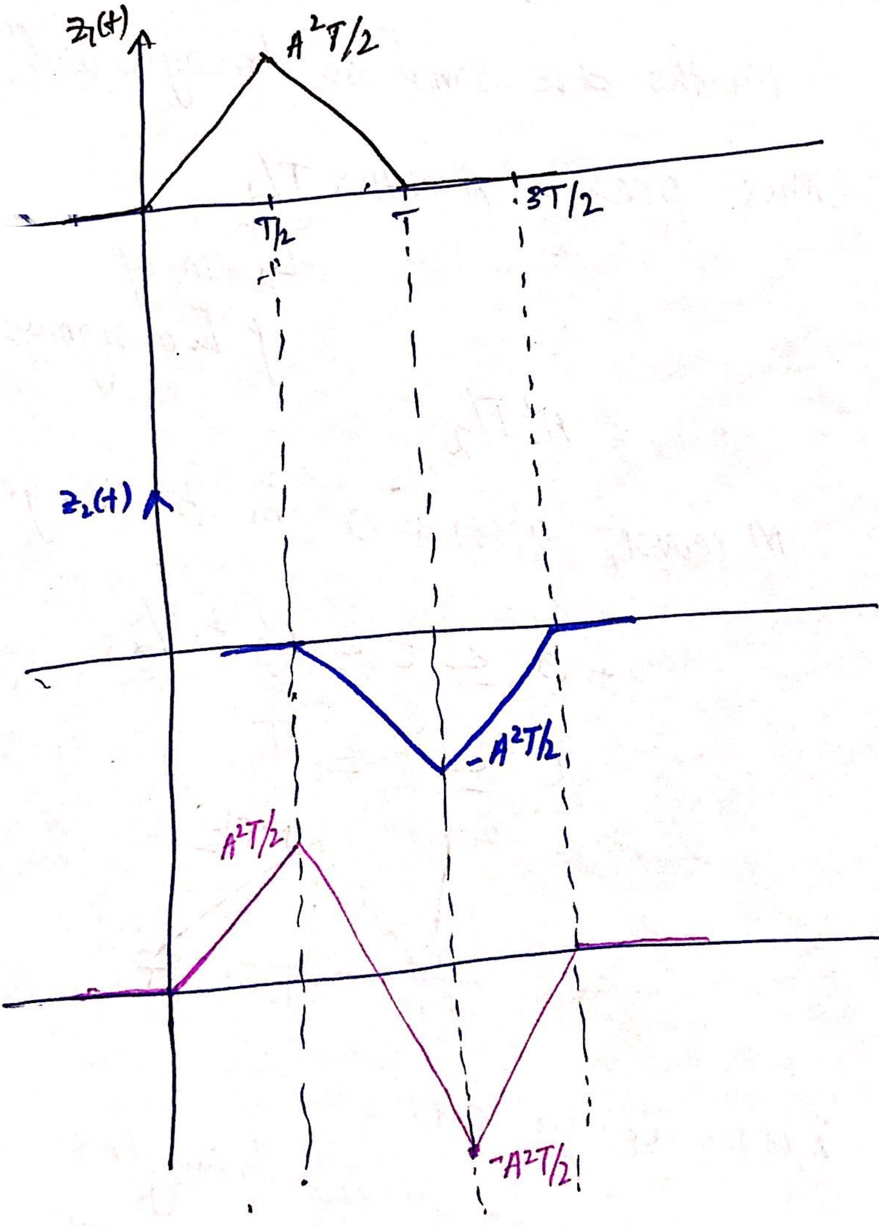
max. peak $= A \times (-A) \times T/2 = -A^2 T/2$

$z_2(t) \neq 0$ in the range

$$0 + T/2 \le t \le T + T/2$$

$$-T/2 \le t \le 3T/2$$


T/2
T
3T/2
-A²T/2


z₁(t)
A²T/2
T/2
T
3T/2
z₂(t)
-A²T/2
A²T/2
-A²T/2