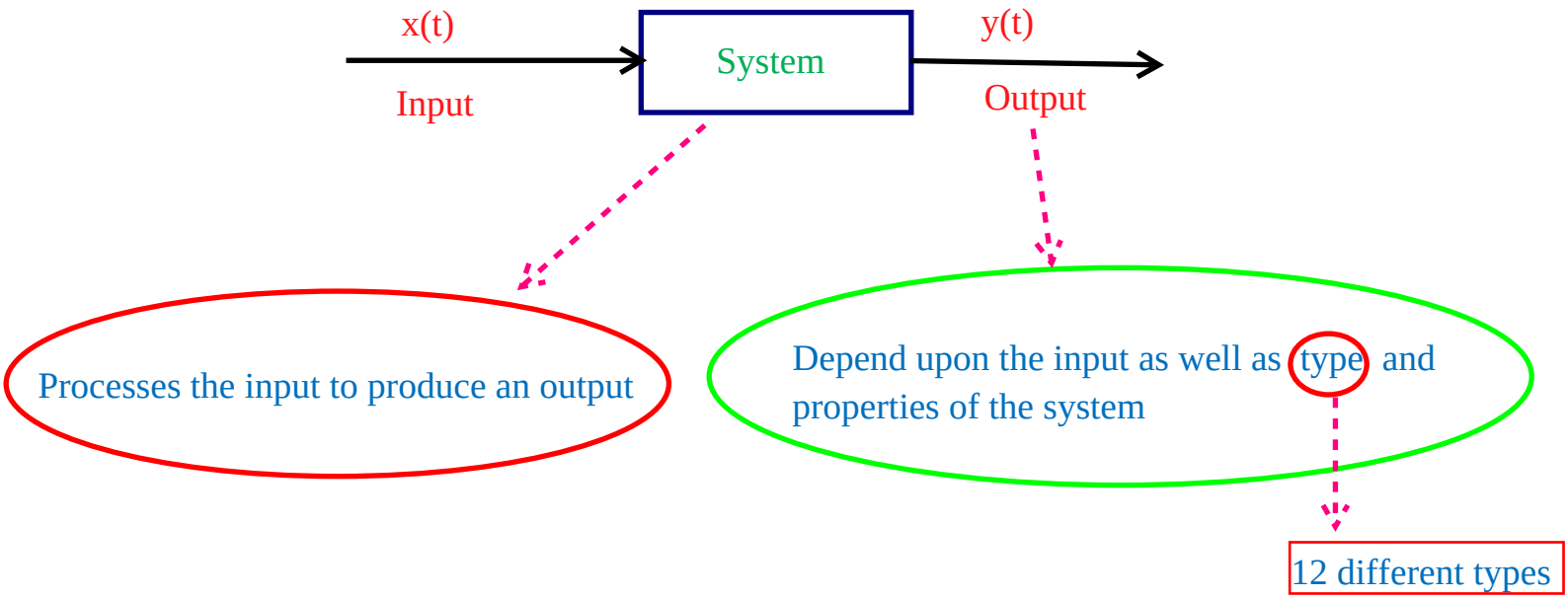
# Systems: Types and Properties

1. Static and Dynamic Systems
2. Causal and Non-Causal Systems
3. Time Varying and Time Invariant Systems
4. Linear and Non-Linear Systems
5. Invertible and Non-Invertible Systems
6. Stable and Unstable Systems




## Present, Past and Future Inputs

Determining at t = 0

1) y(t) = x(t)

↓

y(0) = x(0)

Output depend upon present value of the input

2) y(t) = x(t-1)

↓

y(0) = x(-1)

Output depend upon past value of the input

3) y(t) = x(t+1)

↓

y(0) = x(1)

Output depend upon future value of the input

# Static and Dynamic Systems


To understand the concept let us take the following 3 cases.

1) $y(t) = x(t - 1)$

2) $y(t) = x(t)$

3) $y(t) = x(t + 1)$

Notice in case 1 and 3 that the system has changed the input x(t) to x(t - 1) and x(t + 1) respectively [i.e., this change in x(t) is due to the system]

e.g., Let the input x(t) with the following values,

x(-2) = 1.5, x(-1) = 2, x(0) = 2.5, x(1) = 3

is fed to the system then

=> See case 1: $y(t) = x(t - 1)$

- At t = 0

  y(0) = x(-1) = 2; **instead of 2.5**

- At t = 1

  y(1) = x(0) = 2.5; **instead of 3**

## Static Systems:-

Output of the system depend only on present values of input e.g.,

1) $y(t) = 2x(t)$ 2) $y(t) = f(x(t))$

## Dynamic Systems:-

Output of the system depend upon past or future values of input at any instant of time. It can also depend upon present value of input. e.g.,

$$y(t) = x(t) + x(t - 1)$$

Example:

$$y(t) = x(t)e^{-(t+1)}$$

Find whether the system is static or dynamic?

Solution: Carefully note that $e^{-(t+1)}$ is the co-efficient i.e., y(t) depend upon x(t) [present input] while $e^{-(t+1)}$ is just a scaling factor. Thus the system is static.

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=> Static systems are also called memoryless while dynamic systems are known to have memory.

=> Static systems are memoryless because the dependence is only on x(t) [present input] while in dynamic systems the dependence is on x(t-1) [previous input] or x(t+1) [future input] which demands possession of memory.

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**Example:**

$$y(t) = x(2t)$$

Find whether the system is static or dynamic?

**Solution:**

At t = 0

$$y(0) = x(0)$$

However don't conclude immediately because the dependence on current input should hold for all t ($-\infty \leq t \leq \infty$ )

At t = 1

$$y(1) = x(2)$$

Now the dependence is on the future input.

At t = -1

$$y(-1) = x(-2)$$

Now the dependence is on past input

**So the system is dynamic**

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**Example:**

$$y(t+1) = x(t+1)$$

Find whether the system is static or dynamic?

**Solution:**

At t = 0

$$y(1) = x(1)$$

This system is static because it says that the future output depend upon the future inputs. A similar example is:

$$y(t+2) = x(t+2)$$

**Example:**

$$y(t) = x(-t)$$

Find whether the system is static or dynamic?

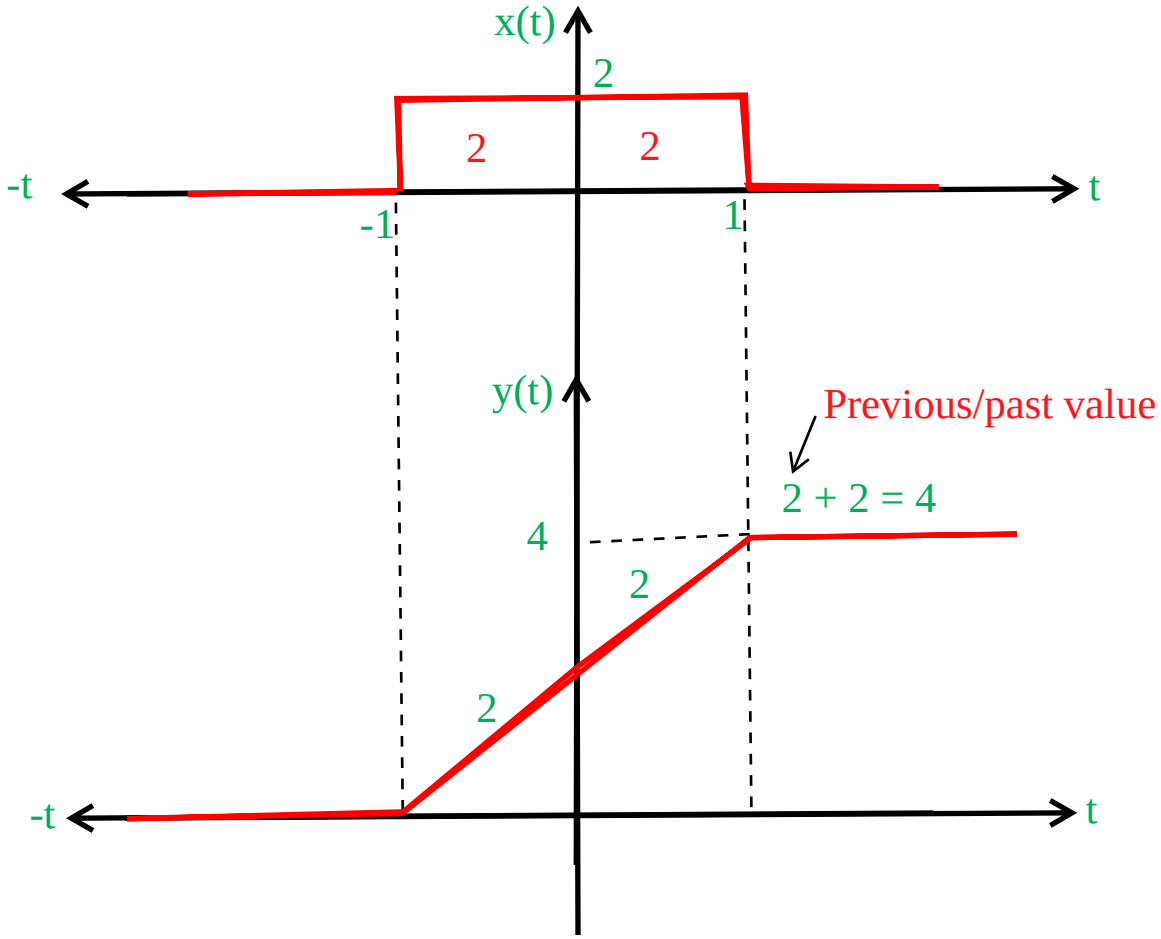
**Solution:** Here now we don't need to perform calculations. Whenever there is time scaling (time reversal is a special case of time scaling) then the system will be dynamic.

**Example:**

$$y(t) = \int_{-\infty}^{\infty} x(\tau)d\tau$$

Find whether the system is static or dynamic?

**Solution:** Recall the graphical integration where any present value depend upon past values/inputs.




So this is a dynamic system (Any system involving integration operation will be termed as dynamic)

**Example:**

$$y(t) = x(Sin(t))$$

Find whether the system is static or dynamic?

**Solution:**

At $t = 0$

$y(0) = x(0)$

However,

At $t = pi$

$y(pi) = y(3.14) = x(0)$ → Past input

**So this system is DYNAMIC**