Assignment 4, PME, 4th Semester, Spring 2023

Deadline: Before the final exam paper of PME

Assignment should be hand written.
Write your name, registration No. and section; else your assignment may not be marked.
Copying is not allowed.
Properly staple your pages (binding is not required).

1. Let X be the number of heads obtained when a fair coin is flipped four times.
   a. Describe the underlying space S of this random experiment and specify the probabilities of its elementary events.
   b. Show the mapping from S to S_x, the range of X.
   c. Find the probabilities for the various values of X.

2. An urn contains nine Rs. 10 notes and one Rs. 50 note. Let the random variable X be the total amount that results when two notes are drawn from the urn without replacement.
   a. Describe the underlying space S of this random experiment and specify the probabilities of its elementary events.
   b. Show the mapping from S to S_x, the range of X.
   c. Find the probabilities for the various values of X.

3. Let X be a random variable with pmf $p_k = 0.6/k^2$ for $k = 1, 2, 3, \ldots$.
   a. Find $P[X > 4]$.
   b. Find $P[6 \leq X \leq 8]$.

4. Show that the $VAR[X]$ of a binomial random variable X is npq.

5. Find 1st moment, 2nd moment and variance of X in question 1 and 2.

6. $S_C = \{1, 2, 3, 4\}$ where C is a uniform random variable having four possible values of electric current. If W is a random variable and represents the corresponding power values such that $W = 3C^2$. Find the mean and variance of C and W. Also find $E[C^3]$.

7. The number N of customers arriving in t seconds at a restaurant is a Poisson random variable with $\alpha = \lambda t$ where λ is the average arrival rate in customers/second. Assume that the arrival rate is 6 customers per hour. Find the probability of the following events:
   a. more than 18 customers in 2 hours
   b. less than or equal to 12 customers in 2 hours.

8. Show that the probability generating function of a geometrically distributed random variable X with pmf is .

9. Find the mean of RV X if is .

10. Find the variance of RV X if is and is .

11. Find the mean of a geometric RV using generating function.

12. Let X be the number of heads obtained when a fair coin is flipped four times.
   a. Plot the cdf of X.
   b. Use properties of cdf to find $P[2 < X \leq 3]$, $P[0.7 \leq X \leq 1.3]$ and $P[1 \leq X < 3]$.