Design of a Very-Wide-Band Transformer

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Abstract—**Wide-band transformers are employed in many electronic circuits. They have been designed mainly by lumped parameter theory; however, for high-quality transformers in modern telecommunication systems and measuring equipment, distributed parameter design theory is necessary. This paper deals with distributed parameter design theory of a wide-band transformer.**

I. Introduction

TRANSFORMERS have been designed mainly by lumped parameter theory. However, with the development of communication systems and measuring equimpent, they are required to have very wide band characteristics, and distributed parameter design theory has become necessary.

Ruthroff reported some broad-band line type transformers and experimental data [1], and Pitzalis and Couse reported practical design information [2], [3]. However, their theories are concerned only with a balanced transmission mode and neglected the design consideration of characteristic impedances.

In this paper, a conventional isolation-type balance-to-unbalance transformer (balun transformer) is analyzed, and a very-wide-band balun transformer is developed by compensating the former transformer. Next, a design chart for characteristic impedance calculations is given. An experimental example of a very-wide-band transformer is shown. These results are also useful for the design of transformers as parts for microelectronic circuits.

II. Transformer Characteristics in Distributed Parameters

A. Isolation-Type Balun Transformer

The conventional form of this type is shown in Fig. 1(a), and the distributed parameter model is shown in Fig. 1(b). From the distributed parameter point of view, the transformer can be constructed with coupled lines as shown in Fig. 2. Along these lines, both balanced and unbalanced modes can be transmitted. The former is useful for the transformer, while the latter is unnecessary, but unavoidable in the low frequency range. The transformer response can be computed with balanced- and unbalanced-mode-transmission theory [3]–[5],

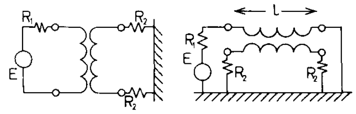

Fig. 1. Isolation-type transformer. (a) Conventional form. (b) Distributed parameter form.

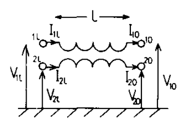

Fig. 2. Coupled lines.

for which fundamental equations are as follows:

$$\begin{bmatrix} V_{1l} \\ V_{2l} \\ I_{1l} \\ I_{2l} \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & b_0 & b_1 \\ a_1 & a_0 & b_1 & b_0 \\ c_0 & c_1 & a_0 & a_1 \\ c_1 & c_0 & a_1 & a_0 \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \\ I_{10} \\ I_{20} \end{bmatrix} \tag{1}$$

where

$$a_0 = (\cosh \gamma_u l + \cosh \gamma_b l)/2$$

$$b_0 = W_u \sinh \gamma_u l + W_b \sinh \gamma_b l/4$$

$$c_0 = \sinh \gamma_u l/4 W_u + \sinh \gamma_b l/W_b$$

$$a_1 = (\cosh \gamma_u l - \cosh \gamma_b l)/2$$

$$b_1 = W_u \sinh \gamma_u l - W_b \sinh \gamma_b l/4$$

$$c_1 = \sinh \gamma_u l/4 W_u - \sinh \gamma_b l/W_b.$$

$W_{b,u}$ is the characteristic impedance of the balanced or unbalanced mode, respectively, $\gamma_{b,u}$ is the propagation constant of each mode ($\gamma_{b,u} = \alpha_{b,u} + j\beta_{b,u}$), $\alpha_{b,u}$ and $\beta_{b,u}$ are the attenuation and phase constants of each mode, and l is the length of the line.

With (1) and the boundary conditions of Fig. 1(b), the input impedance Z_{in}, unbalanced attenuation U ($U = 10 \log P_B/P_u = 20 \log |(V_{2l} - V_{20})/(V_{2l} + V_{20})|$), and balanced effective attenuation B_E ($B_E = 10 \log P_0/P_B = 20 \log |E/2(V_{1l} -$

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$V_{2l})|$) of the transformer can be derived as

$$\frac{Z_{\text{in}}}{2R_2}=\frac{\{(K/m)^2+(1/m^2+m^2)/4\}A-(1-B)/2+K(C+D/m^2)}{(1/m^2+m^2)A/2+1+3B+(K+1/K)C+(m^2/K+K/m^2)D}$$

$$U=20\log\left|\frac{(m^2-1/m^2)A/2+K(C-D/m^2)-\cosh\gamma_u l+\cosh\gamma_b l-K(\sinh\gamma_u l/m^2-\sinh\gamma_b l)}{(m^2-1/m^2)A/2+K(C-D/m^2)+\cosh\gamma_u l-\cosh\gamma_b l+K(\sinh\gamma_u l/m^2-\sinh\gamma_b l)}\right|\ \text{(dB)}$$

$$B_E=20\log\left|\frac{\{(K/m)^2+3(1/m^2+m^2)/4\}A+(1+7B)/2+(2K+1/K)C+(m^2/K+2K/m^2)D}{(m^2-1/m^2)A/2+K(C-D/m^2)-\cosh\gamma_u l+\cosh\gamma_b l-K(\sinh\gamma_u l/m^2-\sinh\gamma_b l)}\right|\ \text{(dB)} \qquad (2)$$

where

$$A=\sinh\gamma_u l\sinh\gamma_b l$$

$$B=\cosh\gamma_u l\cosh\gamma_b l$$

$$C=\sinh\gamma_b l\cosh\gamma_u l$$

$$D=\sinh\gamma_u l\cosh\gamma_b l$$

$K = W_b/2R_2$ (matchig factor), and $m^2 = W_b/4W_u$ (leakage factor).

The response of a transformer of this type can be computed by using (2). In Fig. 3, the variation of unbalanced attenuation U and balanced effective attenuation B_E are shown for parameter m^2. It is assumed, for simplicity, that the attenuation constants α_b and α_u are negligible, and that the phase constants β_b and β_u are equal.

With this type of transformer it was found that in the low-frequency range, the balanced effective attenuation B_E is improved by reducing m^2, but in the high-frequency range, it is almost independent of m^2 and has a pronounced peak at $\beta l = 90°$. The unbalanced attenuation U is independent of m^2 and becomes very small as βl approaches 90°.

B. Very-Wide-Band Balun Transformer

The response of the conventional transformer described above is poor in the high-frequency range or as βl approaches 90°. It is thought that this is caused by the phase shift $\beta_b l$ of the winding length l.

Compensation can be accomplished by inserting the same amount of phase shift $\beta_b l$ before the other load R_2, as shown in Fig. 4.

In this circuit, the input impedance Z_{in}, unbalanced attenuation U, and balanced effective attenuation B_E are derived as

$$\frac{Z_{\text{in}}}{2R_2}=\frac{\{(K/m)^2+(1/m^2+m^2)/4\}A-(1-B)/2+(C+D/m^2)K}{(1/m^2+m^2)A/2+1+3B+(K+1/K)C+(m^2/K+K/m^2)D}$$

$$U=20\log\left|\frac{(m^2-1/m^2)A/2+K(C-D/m^2)-(\cosh\gamma_b l+\sinh\gamma_b l)\{\cosh\gamma_u l-\cosh\gamma_b l+K(\sinh\gamma_u l/m^2-\sinh\gamma_b l)\}}{(m^2-1/m^2)A/2+K(C-D/m^2)+(\cosh\gamma_b l+\sinh\gamma_b l)\{\cosh\gamma_u l-\cosh\gamma_b l+K(\sinh\gamma_u l/m^2-\sinh\gamma_b l)\}}\right|\ \text{(dB)}$$

$$B_E=20\log\left|\frac{\{(K/m)^2+3(1/m^2+m^2)/4\}A+(1+7B)/2+(2K+1/K)C+(m^2/K+2K/m^2)D}{(m^2-1/m^2)A/2+K(C-D/m^2)-(\cosh\gamma_b l+\sinh\gamma_b l)\{\cosh\gamma_u l-\cosh\gamma_b l+K(\sinh\gamma_u l/m^2-\sinh\gamma_b l)\}}\right|\ \text{(dB)}. \qquad (3)$$

The variation of U and B_E are illustrated for $K = 1/2$ ($W_b = R_2$) and the lossless case in Fig. 5. Here the B_E curves display no peaking at $\beta l = 90°$, and the U curves are improved.

III. Design Chart of Distributed Parameter

The conventional transformer congfigulation is shown in Fig. 6. A simple case is considered here whose turns ratio is 1:1, and whose primary and secondary windings are each in a single layer. Distributed inductances and capacitances per unit length of the line (transformer), namely,

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22}\end{bmatrix},\quad \begin{bmatrix} C_{11} & C_{12} \\ C_{12} & C_{22}\end{bmatrix}=\begin{bmatrix} C'_{11}+C'_{12} & -C'_{12} \\ -C'_{12} & C'_{22}+C'_{12}\end{bmatrix}$$

can be determined as follows:

$$L_{11}=4\pi^2a_1^2\eta^2\{\mu_0+(\mu-\mu_0)r^2/a_1^2\}\times10^{-7}$$

$$L_{12}=4\pi^2a_2^2\eta^2\{\mu_0+(\mu-\mu_0)r^2/a_2^2\}\times10^{-7}$$

$$2\sigma=2(L_{11}-L_{12})=2\pi\eta(a_1+a_2)\mu_0 \cdot\cosh^{-1}[(\sinh 2\pi r/p)/\sinh 2\pi d/p)]\times10^{-7}\ \text{(H/m)}$$

$$C'_{11}=\epsilon_0/138\cdot v_0\cdot\log_{10}D/a_1$$

$$C'_{22}=0$$

$$C'_{12}=\epsilon_0\pi(a_1+a_2)\eta/60\cdot v_0 \cdot\cosh^{-1}[(\sinh 2\pi r/p)/\sinh 2\pi d/p)]\ \text{(F/m)} \qquad (4)$$

where μ_0 = relative permeability of air, μ = that of the core, ϵ_0 = relative dielectric constant of air, v_0 = speed of light (3 $\times$ 10^8 m/s), and $\eta = 1/p$.

The capacitance C'_{12} can be derived by the following consideration. If a static field is applied between the conductors (primary and secondary windings), positive and negative charges are induced as in Fig. 7(a). The two charges on conductors are in the relation of a mirror image with respect to

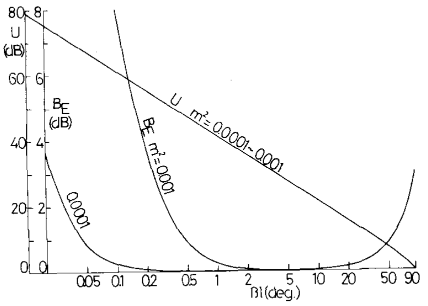


Fig. 3. Response of isolation-type transformer ($K = 1/2$).

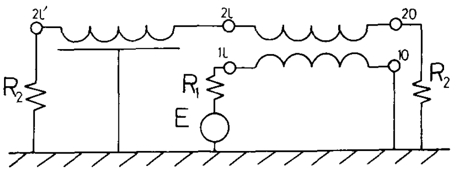
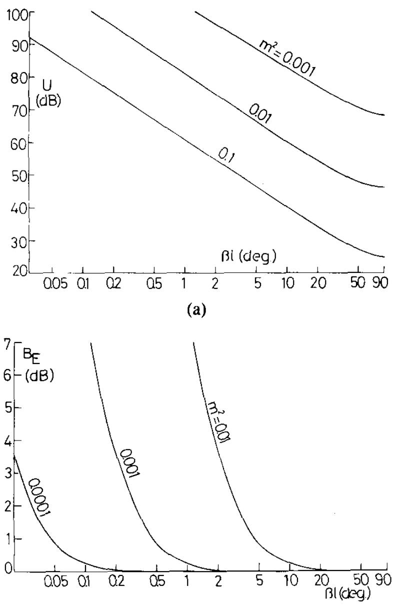


Fig. 4. Very-wide-band transformer.



(a)

(b)

Fig. 5. Response of a very-wide-band transformer. (a) Unbalanced attenuation U. (b) Balanced effective attenuation B_E.

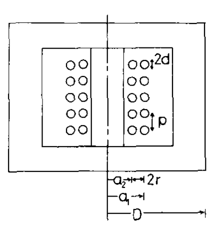


Fig. 6. The structure of a typical transformer.

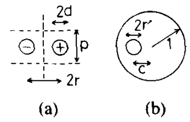


Fig. 7. Analysis of balanced mode parameters.

zero-potential surfaces situated in the middle, and they are in a periodic structure. One element can be mapped to a cross section of an eccentric coaxial line as shown in Fig. 7(b), by the function

$$c = \epsilon^{-2\pi r/p} \cdot \cosh 2\pi d/p$$

$$r' = \epsilon^{-2\pi r/p} \cdot \sinh 2\pi d/p \tag{5}$$

whose capacitance is known. The leakage inductance 2σ is given by C'_{12} and v_0.

Distributed parameters (secondary constants) are determined from

$$W_b = \sqrt{2\sigma/C'_{12}} = 120\sqrt{\mu_0/\epsilon_0} \cdot \cosh^{-1}[(\sinh 2\pi r/p)/(\sinh 2\pi d/p)] \ (\Omega)$$

$$\gamma_b = j\omega\sqrt{2\sigma C'_{12}} = j\omega\eta\pi(a_1 + a_2)\sqrt{\mu_0\epsilon_0}/v_0$$

$$W_u = \sqrt{L_{11}/C'_{11}} = 2\pi a_1\eta\sqrt{\mu_0/\epsilon_0} \cdot \sqrt{\{1 + (\mu/\mu_0 - 1)r^2/a_1^2\}} \cdot 1800 \ln D/a_1 \ (\Omega)$$

$$\gamma_u = j\omega\sqrt{L_{11}C'_{11}} = j\omega\sqrt{\mu_0\epsilon_0} \cdot \sqrt{2\pi^2 a_1^2\eta^2\{1 + (\mu/\mu_0 - 1)r^2/a_1^2\}/\ln D/a_1}/v_0. \tag{6}$$

With distributed parameter analysis on the wide-band transformer of Fig. 4, it is found that the B attenuation has no peaking at $\beta l = 90°$ for $K = 1/2$ (that is, the characteristic impedance W_b is mached to R_2). A design chart for W_b is shown in Fig. 8.

IV. Experimental Example

Fig. 9 shows a measurement method of U attenuation using a slotted line. Coaxial attenuators (20 dB, 50 Ω) are employed as pads and as loads ($R_2 = 50\ \Omega$). If the VSWR and potential minimum point x (displacement from the middle of the line) are measured on the slotted line, the U attenuation is defined by

$$U = 10 \log |(1 + r^2 - 2r\cos\theta)/(1 + r^2 + 2r\cos\theta)| \ \text{(dB)} \tag{7}$$

where

$$r = (\text{VSWR} + 1)/(\text{VSWR} - 1), \quad \theta = (\pi - 2\beta x),$$

$$V_{20}/V'_{2l} = r\epsilon^{j\theta}.$$

When the U attenuation is very large, the B_E attenuation can

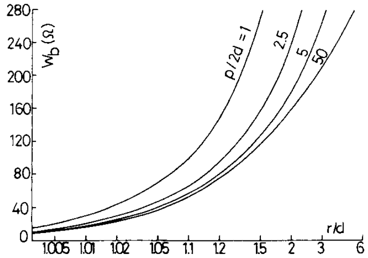


Fig. 8. Design chart for W_b.

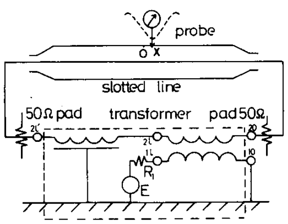


Fig. 9. U attenuation measurement.

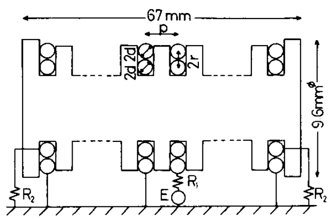


Fig. 10. Outline of designed wide-band transformer.

be obtained from measured VSWR (S_1) as

$$B_E = 10 \log \{(S_1+1)/2\sqrt{S_1}\} \quad \text{(dB)} \tag{8}$$

where S_1 is the VSWR measured at the input port when two loads R_2 are connected.

Fig. 10 shows the outline of the transformer used in the present experiment, where the left-half winding acts as a phase shifter. The balanced mode characteristic impedance W_b is 50 Ω (designed and measured). Enameled wire is wound helically in a bobbin screw engraved on a styrene bar, where $2d = 0.6$, $2r = 0.672$, and $p = 2.0$ mm, and $2N_1 = 2N_2 = 15$ turns. Measured U and B_E attenuations are shown in Fig. 11 together with the theoretical curve ($m^2 = 0.0014$).

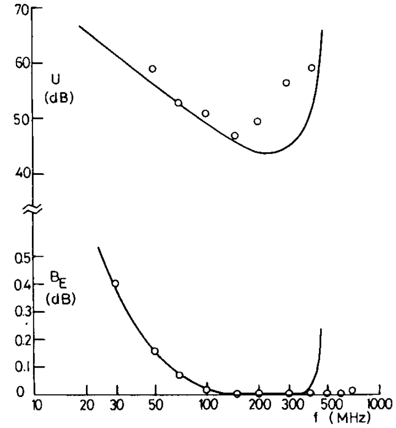


Fig. 11. U and B_E attenuations.

V. Conclusion

The responses of a conventional balun transformer are studied in terms of distributed parameter balanced- and unbalanced-mode transmission theory, and it is found that they are unsatisfactory as βl approaches 90°. Then a very-wide-band balun transformer is defined from a theoretical analysis, in which U and B_E responses are significantly improved.

Distributed parameters for the simplest and basic case whose turns ratio is 1:1 are studied. A design chart for W_b is diagrammed.

Very-wide-band responses are obtained experimentally.

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