UNIVERSITY OF ENGINEERING AND TECHNOLOGY LAHORE

Assignment # 2

Economic Dispatch with Linear Programming

Course Title: Advanced Power System Operation and Control

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Economic Dispatch with Linear Programming

Problem Statement

This study aims to determine the economic operating point for three generator units when delivering a total of 850 MW. The operating cost of each unit is specified as:

$$\boldsymbol{Unit\ 1}:\ F_1(P_{gen1}) = 561 + 7.92P_{gen1} + 0.001562P_{gen1}^2\ \$/h$$

$$\boldsymbol{Unit\ 2}:\ F_2(P_{gen2}) = 310 + 7.85P_{gen2} + 0.00194P_{gen2}^2\ \$/h$$

$$\boldsymbol{Unit\ 3}:\ F_3(P_{gen3}) = 78 + 7.97P_{gen3} + 0.00482P_{gen3}^2\ \$/h$$

The operating limits of the units are:

$$\boldsymbol{Unit\ 1}:\ 150\ MW \leq P_{gen1} \leq 600\ MW$$

$$\boldsymbol{Unit\ 2}:\ 100\ MW \leq P_{gen2} \leq 400\ MW$$

$$\boldsymbol{Unit\ 3}:\ 50\ MW \leq P_{gen3} \leq 200\ MW$$

The incremental cost rates of the units are:

$$\boldsymbol{Unit\ 1}:\ \frac{dF_1(P_{gen1})}{P_{gen1}} = 7.92 + 0.003124P_{gen1}\ \$/MWh$$

$$\boldsymbol{Unit\ 2}:\ \frac{dF_2(P_{gen2})}{P_{gen2}} = 7.85 + 0.00388P_{gen2}\ \$/MWh$$

$$\boldsymbol{Unit\ 3}:\ \frac{dF_3(P_{gen3})}{P_{gen3}} = 7.97 + 0.00964P_{gen3}\ \$/MWh$$

We must minimize the total operating cost:

$$F_T = F_1(P_1) + F_2(P_2) + F_3(P_3)$$

$$subject\ to\ \emptyset = P_{total} - \sum_{i=1}^{3} P_i = 0$$

$$P_{total} = 850\ MW$$

The Lagrange Function is:

$$L = F_T + \lambda\,\emptyset$$

At the economic operating point,

$$\frac{dL}{dP_i} = \frac{dF_i(P_i)}{dP_i} - \lambda = 0$$

Using the incremental cost rates of the three units,

$$\frac{dF_1(P_{gen1})}{P_{gen1}} = 7.92 + 0.003124P_{gen1} = \lambda$$

$$\frac{dF_2(P_{gen2})}{P_{gen2}} = 7.85 + 0.00388P_{gen2} = \lambda$$

$$\frac{dF_3(P_{gen3})}{P_{gen3}} = 7.97 + 0.00964P_{gen3} = \lambda$$

The exact solution is:

$$\lambda = 9.148\ \$/MWh$$

$$P_{gen1} = 393.2\ MW$$

$$P_{gen2} = 334.6\ MW$$

$$P_{gen1} = 122.2\ MW$$

The solution will now be presented using iterative method of Linear Programing. This method uses piecewise linear cost functions for the three units. A sample case of 2 linear segments is presented to illustrate the method of lambda search using Linear Programming.

Case Study: 2 Segments

The cost functions are partitioned into two linear segments as follows:

$$\boldsymbol{Unit\ 1} : F_{gen1}(P_{gen1}) = P_{gen1min} + s_{11}P_{gen11} + s_{12}P_{gen12}$$

$$\boldsymbol{Unit\ 2} : F_{gen2}(P_{gen2}) = P_{gen2min} + s_{21}P_{gen21} + s_{22}P_{gen22}$$

$$\boldsymbol{Unit\ 3} : F_{gen3}(P_{gen3}) = P_{gen3min} + s_{31}P_{gen31} + s_{32}P_{gen32}$$

The conditions for the segments are:

$$P_{gen11}, P_{gen12} \epsilon \left[0, \frac{1}{2}(P_{gen1max} - P_{gen1min})\right] = [0,225]$$

$$P_{gen21}, P_{gen22} \epsilon \left[0, \frac{1}{2}(P_{gen2max} - P_{gen2min})\right] = [0,150]$$

$$P_{gen31}, P_{gen32} \epsilon \left[0, \frac{1}{2}(P_{gen3max} - P_{gen3min})\right] = [0,75]$$

The corresponding incremental operating costs are

$$s_{11} = \frac{F_{11,max} - F_{11,min}}{P_{11,max} - P_{11,min}} = \frac{3750.7 - 1784.1}{375 - 150} = 8.7401$$

$$s_{12} = \frac{F_{12,max} - F_{12,min}}{P_{12,max} - P_{12,min}} = \frac{5875.3 - 3750.7}{600 - 375} = 9.4429$$

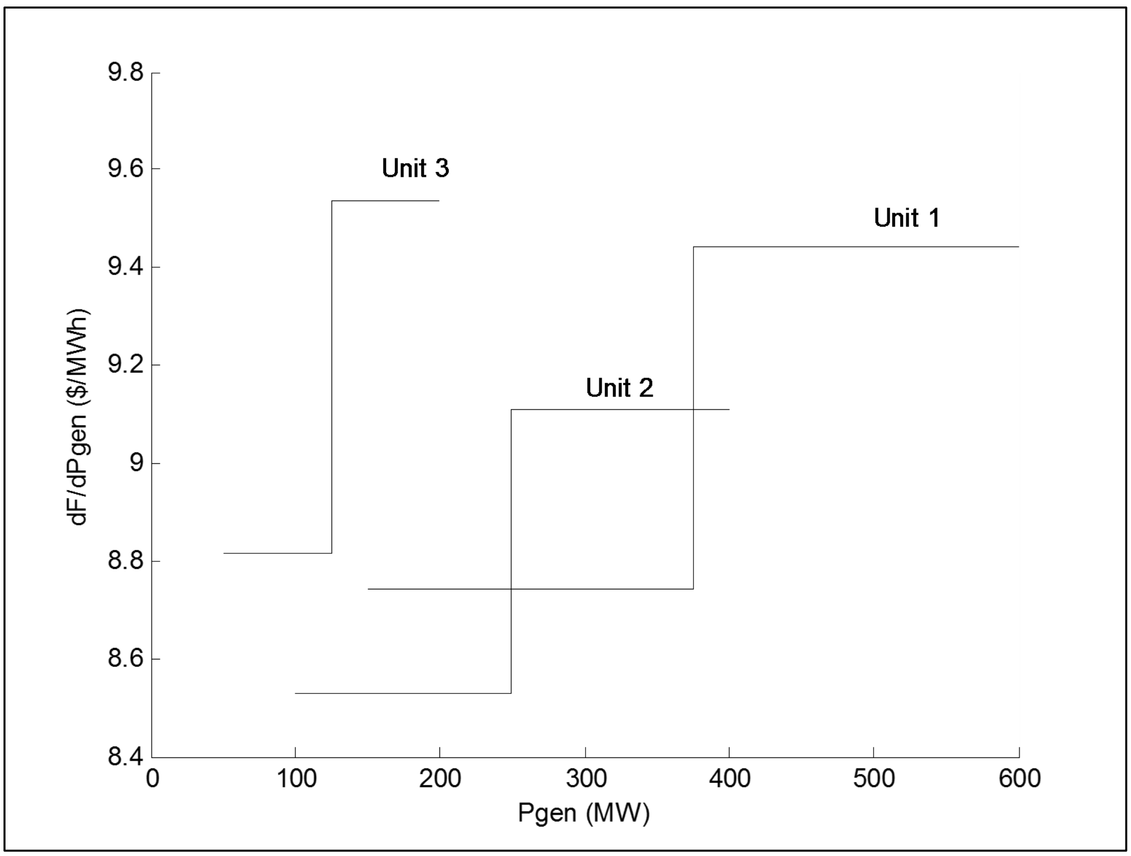
$$s_{21} = \frac{F_{21,max} - F_{21,min}}{P_{21,max} - P_{21,min}} = \frac{2393.7 - 1114.4}{250 - 100} = 8.5290$$

$$s_{22} = \frac{F_{22,max} - F_{22,min}}{P_{22,max} - P_{22,min}} = \frac{3760.4 - 2393.7}{400 - 250} = 9.1110$$

$$s_{31} = \frac{F_{31,max} - F_{31,min}}{P_{31,max} - P_{31,min}} = \frac{1149.6 - 488.5}{125 - 50} = 8.8135$$

$$s_{32} = \frac{F_{32,max} - F_{32,min}}{P_{32,max} - P_{32,min}} = \frac{1864.8 - 1149.6}{200 - 125} = 9.5365$$

The piecewise linear cost functions are shown in the figure below.



Iteration # 1

$$\lambda^{(1)} = s_{21} = 8.529$$

$$P_{gen1} = P_{gen11,min} = 150\ MW$$

$$P_{gen3} = P_{gen31,min} = 50\ MW$$

$$P_{gen2} = \min\left(P_{gen21,max}, P_{load} - P_{gen1} - P_{gen3}\right) = \min(250, 850 - 150 - 50) = 250 MW$$

$$P_{gen1} + P_{gen2} + P_{gen3} = 150 + 250 + 50 = 450\ MW$$

This is not a valid solution.

Iteration # 2

$$\lambda^{(2)} = s_{11} = 8.7401$$

$$P_{gen2} = P_{gen21,max} = 250\ MW$$

$$P_{gen3} = P_{gen31,min} = 50\ MW$$

$$P_{gen1} = min\left(P_{gen11,max}, P_{load} - P_{gen2} - P_{gen3}\right) = \min(375,850 - 250 - 50) = 375\ MW$$

$$P_{gen1} + P_{gen2} + P_{gen3} = 375 + 250 + 50 = 675\ MW$$

This is not a valid solution.

Iteration # 3

$$\lambda^{(3)} = s_{31} = 8.8135$$

$$P_{gen1} = P_{gen11,max} = 375\ MW$$

$$P_{gen2} = P_{gen21,max} = 250\ MW$$

$$P_{gen3} = min\left(P_{gen31,max}, P_{load} - P_{gen1} - P_{gen2}\right) = \min(125,850 - 375 - 250) = 125\ MW$$

$$P_{gen1} + P_{gen2} + P_{gen3} = 375 + 250 + 125 = 750\ MW$$

This is not a valid solution.

Iteration # 4

$$\lambda^{(4)} = s_{22} = 9.111$$

$$P_{gen1} = P_{gen11,max} = 375\ MW$$

$$P_{gen3} = P_{gen31,max} = 125\ MW$$

$$P_{gen2} = min\left(P_{gen22,max}, P_{load} - P_{gen1} - P_{gen3}\right) = \min(400,850 - 375 - 125) = 350\ MW$$

$$P_{gen1} + P_{gen2} + P_{gen3} = 375 + 350 + 125 = 850\ MW$$

This is a valid solution.

Iteration # 5

$$\lambda^{(5)} = s_{12} = 9.4429$$

$$P_{gen2} = P_{gen22,max} = 400\ MW$$

$$P_{gen3} = P_{gen31,max} = 125\ MW$$

$$P_{gen1} = \min\left(P_{gen12,max}, P_{load} - P_{gen2} - P_{gen3}\right) = \min(600{,}850 - 400 - 125) = 325\ MW$$

$$P_{gen1} = max\left(P_{gen12,min}, P_{load} - P_{gen2} - P_{gen3}\right) = \max(375{,}850 - 400 - 125) = 375\ MW$$

$$P_{gen1} + P_{gen2} + P_{gen3} = 375 + 400 + 125 = 900\ MW$$

This is not a valid solution.

Iteration # 6

$$\lambda^{(6)} = s_{32} = 9.5365$$

$$P_{gen1} = P_{gen12,max} = 600\ MW$$

$$P_{gen2} = P_{gen22,max} = 400\ MW$$

$$P_{gen3} = \min\left(P_{gen32,max}, P_{load} - P_{gen1} - P_{gen2}\right) = \min(200{,}850 - 600 - 400) = -150\ MW$$

$$P_{gen3} = max\left(P_{gen32,min}, P_{load} - P_{gen1} - P_{gen2}\right) = \max(125{,}850 - 600 - 400) = 125\ MW$$

$$P_{gen1} + P_{gen2} + P_{gen3} = 600 + 400 + 125 = 1125\ MW$$

This is not a valid solution.

Solution

$$\lambda = 9.111\ \$/MWh$$

$$P_{gen1} = 375\ MW$$

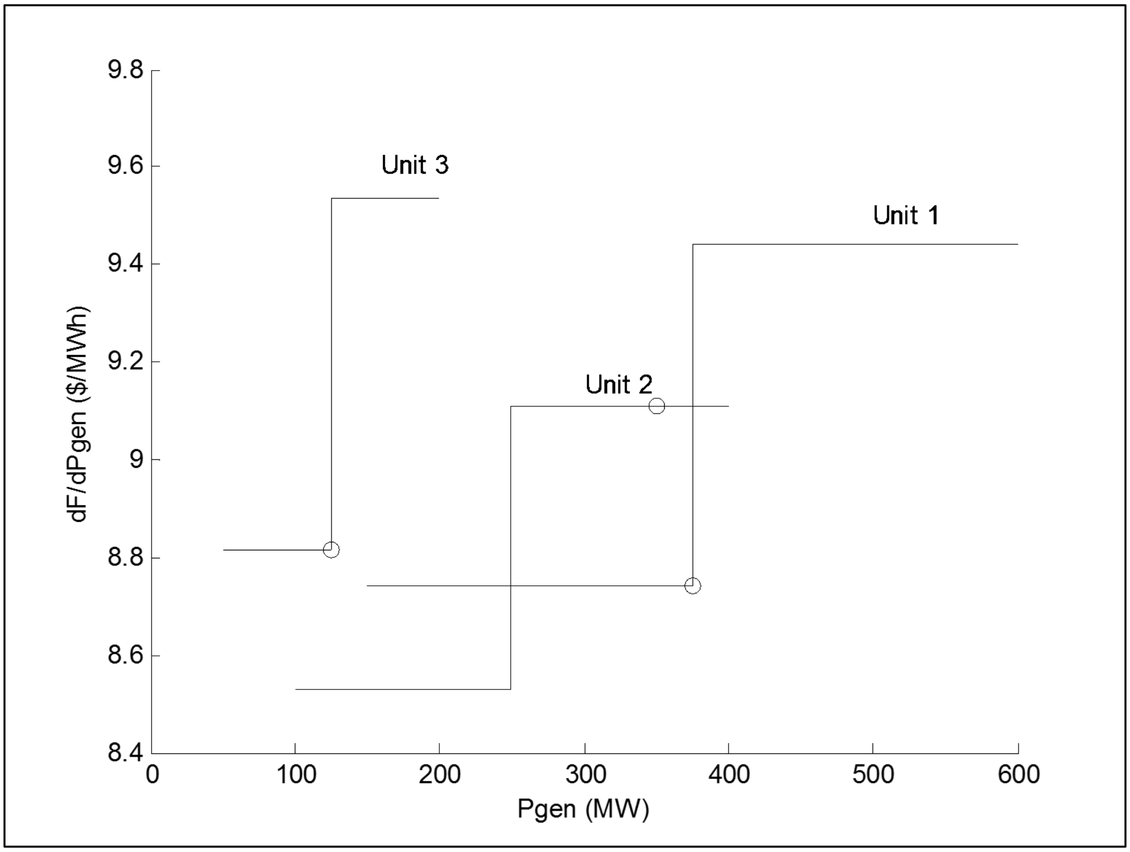
$$P_{gen2} = 350\ MW$$

$$P_{gen1} = 125\ MW$$

The corresponding operating cost is:

$$F_T = F_1(375) + F_2(350) + F_3(125) = \$\,8195.4$$

The operating points are shown in the figure below using circles.



This algorithm was implemented in MATLAB. The results are presented here for the cases of 1, 2, 3, 5, 10 and 50 segments.

Number of Segments	Generator 1 (MW)	Generator 2 (MW)	Generator 3 (MW)	Total Cost ($/h)	Lambda ($/MWh)
1	400	400	50	8227.870	9.0915
2	375	350	125	8195.369	9.1110
3	450	300	100	8204.105	9.0915
5	400	340	110	8195.206	9.0915
10	385	340	125	8194.554	9.1618
50	393	335	122	8194.357	9.1576
Standard solution with lambda search	393.2	334.6	122.2	8194.356	9.1480

MATLAB Code

```
clc;clear all;

%%
Ngen=3;
Pmax=[600 400 200];
Pmin=[150 100 050];
Pload=850;
divisions=50;

%%
for i=1:Ngen
    range(i)=Pmax(i)-Pmin(i);
    dP(i)=range(i)/divisions;
end

%%
for i=1:Ngen
    for k=1:divisions
        sPmin(i,k)=Pmin(i)+(k-1)*dP(i);
        sPmax(i,k)=Pmin(i)+k    *dP(i);
        sFmin(i,k)=F(i,sPmin(i,k));
        sFmax(i,k)=F(i,sPmax(i,k));
        s    (i,k)=(sFmax(i,k)-sFmin(i,k))/dP(i);
    end
end
ordered_s=sort(transpose(reshape(s,[],1)));

%%
Pgen=[0 0 0;0 0 0];
Pgen=[Pmin(1) Pmin(2) Pmin(3);Pmax(1) Pmax(2) Pmax(3)];
eps1=Pload;
eps2=Pload;
threshold=1;
iter=1;
maxiter=Ngen*divisions;
TotalCost1=0;
TotalCost2=0;
found=0;
Ans=[];
OldCostMin=1e5;
Oldeps=Pload;
lamdaOld=0;

for n=1:maxiter
    lamda=ordered_s(n);

    for rep=1:2
        for i=1:Ngen
            for k=1:divisions

                if (s(i,k)<lamda)
```

```
    if (k<divisions)
        if (s(i,k+1)>lamda)
            Pgen(2,i)=sPmax(i,k);
        end
    else
        Pgen(2,i)=sPmax(i,k);
    end
    if (divisions==1)
        Pgen(1,i)=sPmax(i,k);
    end
end

if (s(i,k)>lamda)
    if (k>1)
        if (s(i,k-1)<lamda)
            Pgen(1,i)=sPmin(i,k);
        end
    else
        Pgen(1,i)=sPmin(i,k);
    end
    if (divisions==1)
        Pgen(2,i)=sPmin(i,k);
    end
end

if (s(i,k)==lamda)
    Pgen(1,i)=sPmin(i,k);
    Pgen(2,i)=sPmax(i,k);
    t1=Pload-sum(Pgen(1,:))+Pgen(1,i);
    t2=Pload-sum(Pgen(2,:))+Pgen(2,i);
    if ((t1<=sPmax(i,k))&&(t1>=sPmin(i,k)))
        Pgen(1,i)=t1;
    end
    if ((t2<=sPmax(i,k))&&(t2>=sPmin(i,k)))
        Pgen(2,i)=t2;
    end
end

Pgen;
eps1=abs(sum(Pgen(1,:))-Pload);
eps2=abs(sum(Pgen(2,:))-Pload);
eps_his(n*i*k)=eps;
TotalCost1=0;
TotalCost2=0;
for m=1:Ngen
    TotalCost1=TotalCost1+F(m,Pgen(1,m));
    TotalCost2=TotalCost2+F(m,Pgen(2,m));
end

if(eps1==0)&&(TotalCost1<OldCostMin)
    Ans=Pgen;
    TotalCost1;
    Oldeps=eps1;
    OldCostMin=TotalCost1;
    lamdaOld=lamda;
end
```

```
                if(eps2==0)&&(TotalCost2<OldCostMin)
                    Ans=Pgen;
                    TotalCost2;
                    Oldeps=eps2;
                    OldCostMin=TotalCost2;
                    lamdaOld=lamda;
                end

            end
        end
    end
    lamda;
    Pgen;
    %plot(Pgen,lamda,'o')
    Pgen=[Pmin(1) Pmin(2) Pmin(3);Pmax(1) Pmax(2) Pmax(3)];

end



Pgen=Ans(2,:);
lamda=lamdaOld;

%     hold on
%     stairs([sPmin(1,1) sPmax(1,1)],[s(1,1),s(1,1)],'r')
%     stairs([sPmin(1,2) sPmax(1,1)],[s(1,1),s(1,2)],'r')
%     stairs([sPmin(1,2) sPmax(1,2)],[s(1,2),s(1,2)],'r')
%
%     stairs([sPmin(2,1) sPmax(2,1)],[s(2,1),s(2,1)],'b')
%     stairs([sPmin(2,2) sPmax(2,1)],[s(2,1),s(2,2)],'b')
%     stairs([sPmin(2,2) sPmax(2,2)],[s(2,2),s(2,2)],'b')
%
%     stairs([sPmin(3,1) sPmax(3,1)],[s(3,1),s(3,1)],'g')
%     stairs([sPmin(3,2) sPmax(3,1)],[s(3,1),s(3,2)],'g')
%     stairs([sPmin(3,2) sPmax(3,2)],[s(3,2),s(3,2)],'g')
%     xlabel('Pgen (MW)');
%     ylabel('dF/dPgen ($/MWh)');
%     text([500],[9.5],'Unit 1');
%     text([300],[9.15],'Unit 2');
%     text([160],[9.6],'Unit 3');
%     plot([50,600],[lamda,lamda])
%     plot(Pgen,lamda,'o')
%     plot(Pgen(1,:),[s(1,1) lamdaOld s(3,1)],'o')

TotalCost=0;
for i=1:Ngen
    TotalCost=TotalCost+F(i,Pgen(1,i));
end

Pgen
TotalCost
lamda
```