Calculations of the Dispersive Characteristics of Microstrips by the Time-Domain Finite Difference Method

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Abstract —The dispersive characteristics of microstrips have been investigated by many authors [1]–[4] using various numerical and empirical methods. Those results showed a lack of agreement with each other, and the true dispersive characteristics of microstrips still need to be identified. In this paper, a direct time-domain finite difference method is used to recharacterize the microstrip. Maxwell's equations are discretized both in time and space and a Gaussian pulse is used to excite the microstrip. The frequency-domain design data are obtained from the Fourier transform of the calculated time-domain field values. Since this method is completely independent of all the above-mentioned investigations, the new results can be considered as an impartial verification of the published results.

The comparison of the time-domain results and those from the frequency-domain methods has shown the integrity of the time-domain computations. Since this method is very general and can be applied to model many other microwave components, its success in the microstrip problem is an important step toward its general application.

I. Introduction

THE APPLICATION OF the time-domain method to model microstrip problems is not new [7]–[11]. However, it is more frequently used to obtain qualitative results that graphically illustrate the field propagation and the transport of power than to obtain design data. It is interesting to note that when the time-domain method is used to obtain frequency-domain data, the investigators often go back to sinusoidal excitations [7]–[10]. This compromises the advantage of the time-domain approach, which is believed to be capable of producing wide-band frequency-domain results with a single computation process.

In the process of this investigation, it is found that the Fourier transform of the time-domain results are very sensitive to numerical errors, notably those resulting from the imperfect treatment of the absorbing boundary conditions used to truncate the numerical computations of that of an open structure. Thus, even though the time-domain results may be reasonably accurate, the frequency-domain results obtained from their Fourier transform may not be acceptable as useful data. That may explain, at least in part, why many investigators mysteriously reverted to sinusoidal signals while it seems obvious that a pulse in the time domain could have been used to advantage, and others turned to solve only the closed structures [11].

To improve the absorbing boundary conditions, we have used an open-circuit, short-circuit technique to cancel the reflections from the terminations of the finite difference meshes. The improvements in the resulting computations enabled us to get reliable frequency-domain data, i.e., the effective dielectric constant and the characteristic impedance, through Fourier transforms, and to compare with various numerical results of the microstrip, presently available in the open literature.

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II. Finite Difference Method and Absorbing Boundary Conditions

The finite difference method was first introduced by K. S. Yee [12] to solve electromagnetic scattering problems. In using this method, Maxwell's equations are discretized both in time and space. Knowing the initial, boundary and excitation conditions, the fields on the nodal points of the space–time mesh can be calculated in a leapfrog time marching manner. For more details of the method, [11]–[15] could be consulted.

Since the finite difference equations cannot be applied to the boundary nodes, an absorbing type of boundary condition must be used at the mesh boundary where an open space is to be simulated. There are several publications discussing the absorbing boundary conditions [16]–[17]. Most of the methods are quite good to terminate the finite difference equations, and they have been applied to the scattering problems [13]–[15]. But, they are not good enough for the microstrip problems, since the Fourier transform of the time-domain results is very sensitive to the imperfect boundary treatment. Minute errors in the time domain may produce fairly large errors in the frequency domain. To minimize the reflection error, we have chosen to use the open- and short-circuit boundary conditions [16]. In this method, the problem is solved twice for each absorbing boundary, once with vanishing tangential electric field (termination with electrically conducting wall) and once with vanishing tangential magnetic field (termination with magnetically conducting wall). The results of the two solutions are averaged to give the desired solution. The rationale of this method is quite simple. Because the reflection coefficient of a short-circuit boundary is -1, while that of an open circuit is $+1$, the

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sum of the two results should have the reflected fields completely canceled out. However, caution must be exercised for computations of long duration, because the multiple reflections may not be canceled out. So, it is necessary to consider the size of the finite difference mesh domain to accommodate the anticipated duration of computations.

Using the open-circuit, short-circuit boundary conditions, we are able to reduce the reflection error to an acceptable level such that the corresponding frequency domain results are smooth and accurate.

III. Numerical Results

Using the time-domain method discussed above, a typical microstrip on a gallium arsenide substrate has been investigated. Unlike other investigations, which look for the propagation velocity of the field only [1], [3], the time-domain approach actually finds the space–time distributions of the fields everywhere within the finite difference space–time mesh. In many cases this provides clear pictures and illuminating details of the field variations. At the same time, the frequency-domain design data can also be easily obtained through Fourier transform of the time-domain fields.

A. Parameters Used in Computations

The microstrip structure used in our calculations is shown in Fig. 1. (Because of the symmetry of the problem, only half of the structure is considered.) The parameters used for most of our computations are as follows:

thickness of the substrate:	$H = 0.1$ mm
width of the metal strip:	1) $W = 0.075$ mm ($W/H = 0.75$)
	2) $W = 0.15$ mm ($W/H = 1.5$)
dielectric constant of the substrate:	$\varepsilon_r = 13.0$
thickness of the metal strip:	0.

To accommodate the structural details of the microstrip, we have used the mesh parameters as follows:

space interval $dh = 0.1 \text{ mm}/16 = 6.25 \cdot 10^{-3}$ mm (for $W/H = 0.75$)
$dh = 0.1 \text{ mm}/8 = 1.25 \cdot 10^{-2}$ mm (for $W/H = 1.5$)
$n_1 = 30,\ n_2 = 55,\ n_3 = 160$

time step $dt = k \cdot dh/c$ sec.

where c is the velocity of light in air and k is the constant restricted by stability criterion [13] ($k = 0.515$ in our calculation). The most often used values for r_1, r_2, r_3 are $r_1 = r_2 = r_3 = 1.0$. But sometimes other choices of r_1, r_2, r_3 (with $r_1, r_2, r_3 \geq 1$) are used to ease the problem with a particular W/H.

A Gaussian pulse excitation is used at the front surface, uniform under the strip (in plane "abcd" of Fig. 1) with only E_x component:

$$E_x(t) = \exp\left[-\frac{(t-t_0)^2}{T^2}\right] \tag{1}$$

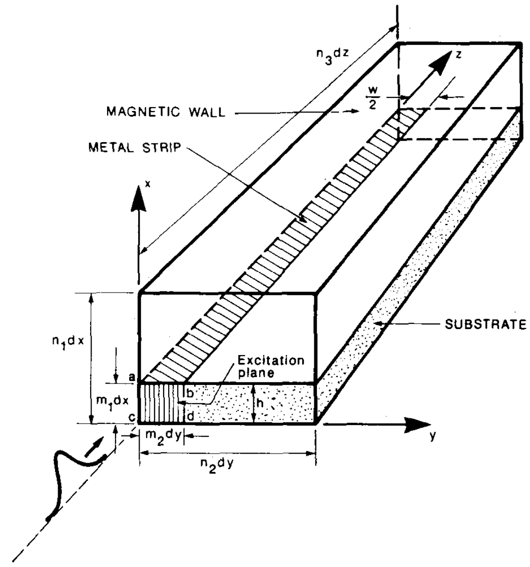

Fig. 1. Microstrip structure.

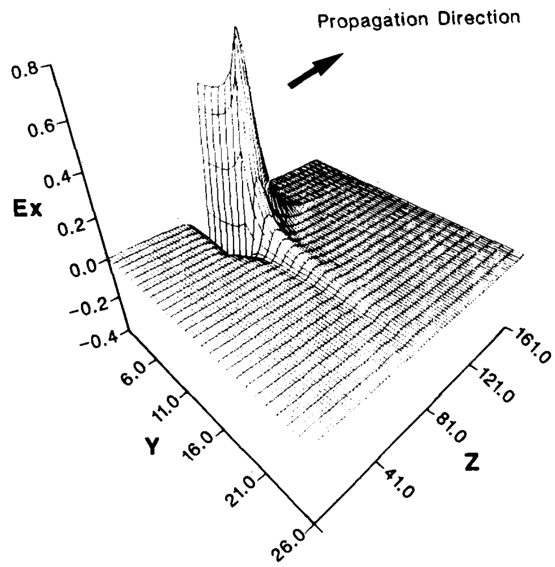

Fig. 2. Gaussian pulse propagation in microstrip (E_x component just underneath the air dielectric interface).

where $t_0 = 140dt$ and $T = 140dt$; elsewhere on the front surface, we set $E_x = E_y = 0$. The pulse width in space is about $20dh$, which is wide enough to obtain good resolution. The frequency spectrum of the pulse is from dc to about 700 GHz.

B. Time-Domain Fields

Fig. 2 shows the propagation of a Gaussian pulse along the microstrip. This is a plot of the space distributions of the vertical electric field on a surface just underneath the microstrip. Fig. 3 shows the time variations of vertical electric field at different positions along the propagation direction. The dispersive properties of the microstrip are

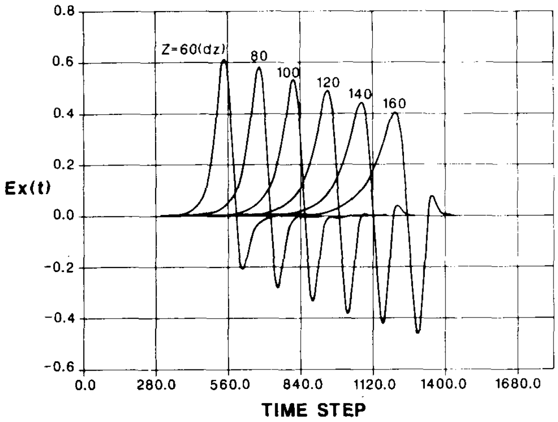

Fig. 3. Time variation of E_x at different positions along the direction of propagation.

quite obvious from the distortion of the pulse as it travels away from the feeding point. Indeed, the result (Fig. 3) is very similar to that calculated by Veghte and Balanis [6], who reconstructed the time-domain waveforms using the frequency-domain empirical formulas of effective dielectric constant.

C. Effective Dielectric Constant

The effective dielectric constant $\varepsilon_{reff}(\omega)$ can be used to account for the dispersive characteristics of the microstrip. It is calculated as follows:

Take the Fourier transforms of $E_x(t)$ at two different positions (underneath the center of the strip), with a separation of L, along the propagation direction:

$$E_x(\omega, z=0) = \int_{-\infty}^{\infty} E_x(t, z=0) e^{-j\omega t}\, dt \tag{2}$$

$$E_x(\omega, z=L) = \int_{-\infty}^{\infty} E_x(t, z=L) e^{-j\omega t}\, dt. \tag{3}$$

Taking the ratio of (2) and (3), we can get the transfer function of this section of microstrip, which is

$$e^{-\gamma(\omega)L} = \frac{Ex(\omega, z=L)}{Ex(\omega, z=0)} \tag{4}$$

where

$$\gamma(\omega) = \alpha(\omega) + j\beta(\omega). \tag{5}$$

The constant $\varepsilon_{reff}(\omega)$ is defined through $\beta(\omega)$ as

$$\beta(\omega) = \omega\sqrt{u_0 \varepsilon_0 \varepsilon_{reff}(\omega)} \tag{6}$$

or

$$\varepsilon_{reff}(\omega) = \frac{\beta^2(\omega)}{\omega^2 \varepsilon_0 \mu_0}. \tag{7}$$

The theoretical study of the effective dielectric constant has been done by many authors. Fig. 4 shows a set of results of different investigators which were gathered together by Kuester and Chang [1]. These results were obtained by using different numerical techniques and it can

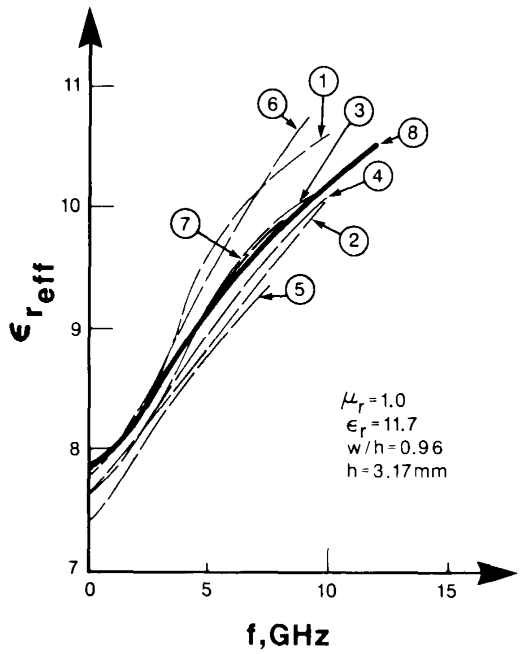

Fig. 4. Comparison of effective dielectric constant ε_{reff} as computed by different authors [1) Farrar and Adam, 2) Itoh and Mittra, 3) Van de Capelle and Luypaert, 4) Denlinger, 5) Schmitt and Sarges ($\varepsilon_r = 11.2$), 6) Chang and Kuester, 7) Pregla and Kowalaki, 8) time-domain result in this paper]. All of the results except the last one are from Kuester and Chang's paper [1].

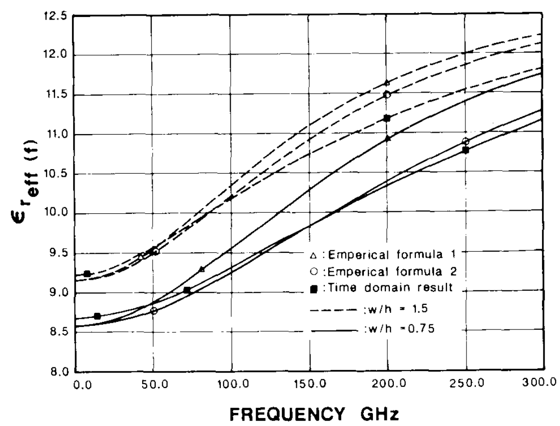

Fig. 5. Effective dielectric constant ($\varepsilon_r = 13.0$). Comparison of the results from time-domain method with two empirical formulas: 1) Edward and Owen [5], 2) Pramanick and Bhartia [6], [4].

be observed that they do not agree with one another very well. Our result is plotted on the same figure.

Fig. 5 shows the comparisons of our result with the empirical formulas of Edward and Owen [5] and that of Pramanick and Bhartia ([4], [6]). For $W/H < 1$, our result is closer to that of Pramanick and Bhartia than to Edward and Owen's. For $W/H > 1$, both empirical formulas are off from our result in the frequency range shown. It is noticed that for both $W/H < 1$ and $W/H > 1$, the time-domain results provide fairly accurate results at dc (the dc value

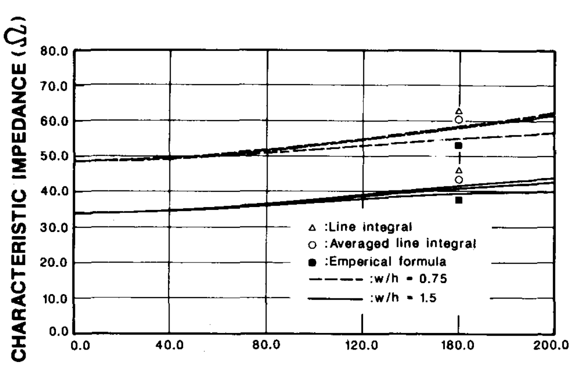

Fig. 6. Characteristic impedance ($\epsilon_r = 13.0$). Comparison of the results from time-domain method with empirical formula (Bianco *et al.* [5]).

used in these empirical formulas has 1 to 2 percent error, as claimed in [5]), and that the accuracy of the finite difference method, unlike the empirical formulas, is not affected by the ratio of W/H. In view of our agreement with Pramanick and Bhartia at $W/H<1$, it is not an unrealistic assumption to say that the time-domain results are more credible than those of the empirical formulas in the range $W/H>1$.

D. Characteristic Impedance

The variation of the characteristic impedance with frequency is obtained through the ratio of $V(\omega)/I(\omega)$. Here $I(\omega)$ is the Fourier transform of the current defined as the loop integral of the magnetic field around the metal strip. For $V(\omega)$, two kinds of definitions are used, one is the line integral of vertical electric field under the center of the strip, the other is the averaged line integral of vertical electric field under the whole strip.

Fig. 6 shows the comparison of our results with the result from the empirical formulas of Bianco *et al.* [5]. For a fairly large frequency range starting from dc, it turns out that the results of center line integral and averaged line integral are very close to each other, indicating that the voltage uniqueness is well satisfied in that frequency range.

IV. Conclusions

It has been demonstrated that the time-domain finite difference method is a credible numerical method capable of producing frequency-domain design data through Fourier transform. It has been a frequent practice to use a sinusoidal source in the time-domain approach to obtain frequency-domain results. Our experience leads us to believe that investigators use monochromatic sources because the Fourier transform of a time-domain result is very sensitive to time-domain errors. But the monochromatic approach is expensive, especially at low frequencies. This is because, using the explicit time-domain finite difference method to model the microstrip problems, will lead to an "oversampled" time mesh. That is, we are using a much smaller time step than that required by the Nyquist criterion. This situation is brought about by the Courant condition requiring $c \cdot dt < dh/\sqrt{3}$, (dh = smallest dimension of the mesh), and dh is kept small by the requirement of representing the small dimension of the microstrip. The above sequence of conditions limits the size of dt in the computation and results in a seemingly paradoxical situation in that it requires a longer time to calculate the response of a broad pulse than that of a sharp pulse. Consequently, it is much more economic to use a sharp Gaussian pulse source to obtain wide-band frequency results than to do it one frequency at a time. Indeed, it takes much more time to caluclate the response of a single sinusoidal wave than to calculate that of a sharp Gaussian pulse.

Many of the theoretical analyses of the microstrip are solely for microstrips, but the time-domain finite difference method is very general. With minor modifications this method has already been used by the authors to investigate many interesting microstrip-related microwave components and the results are to be published. This investigation paves the way for accurate modeling of passive microwave components at high frequencies and in mm-MMIC's.

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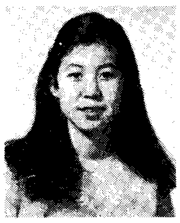

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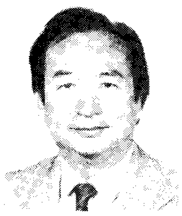

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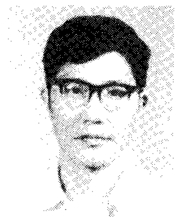

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