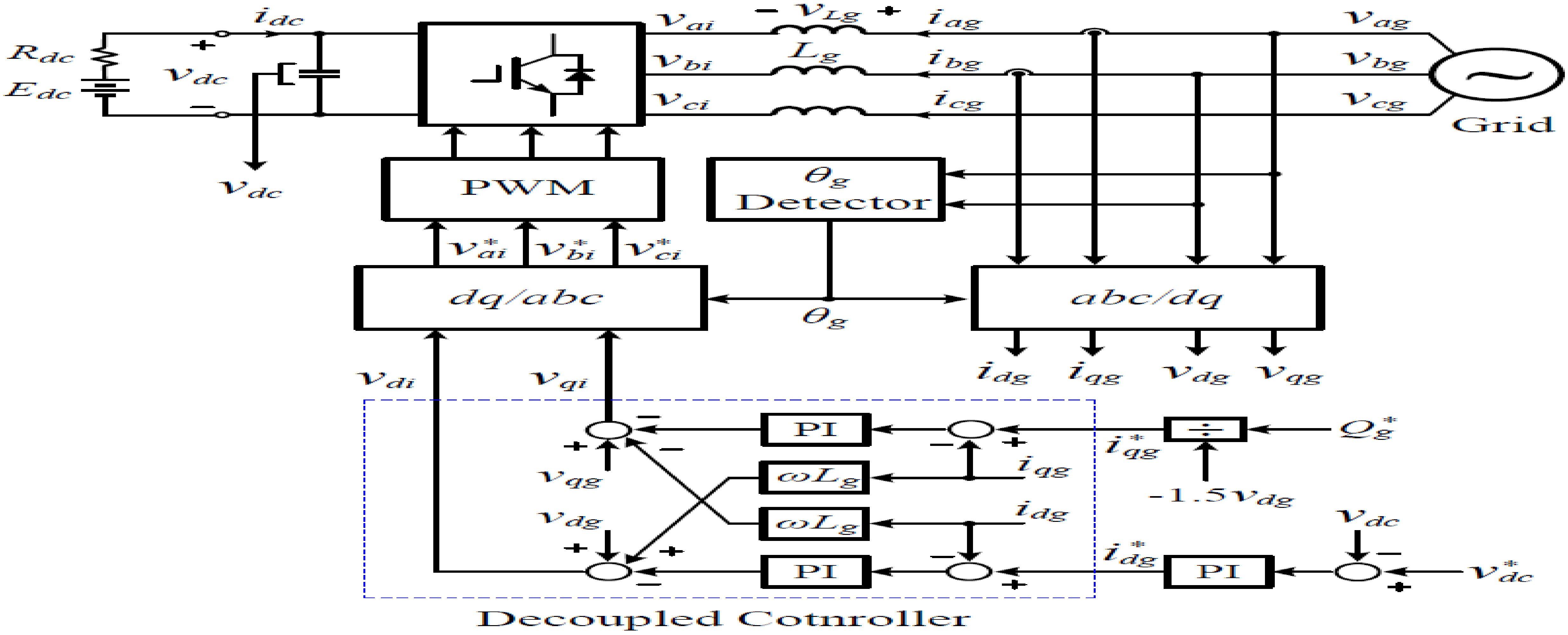
Lecture#

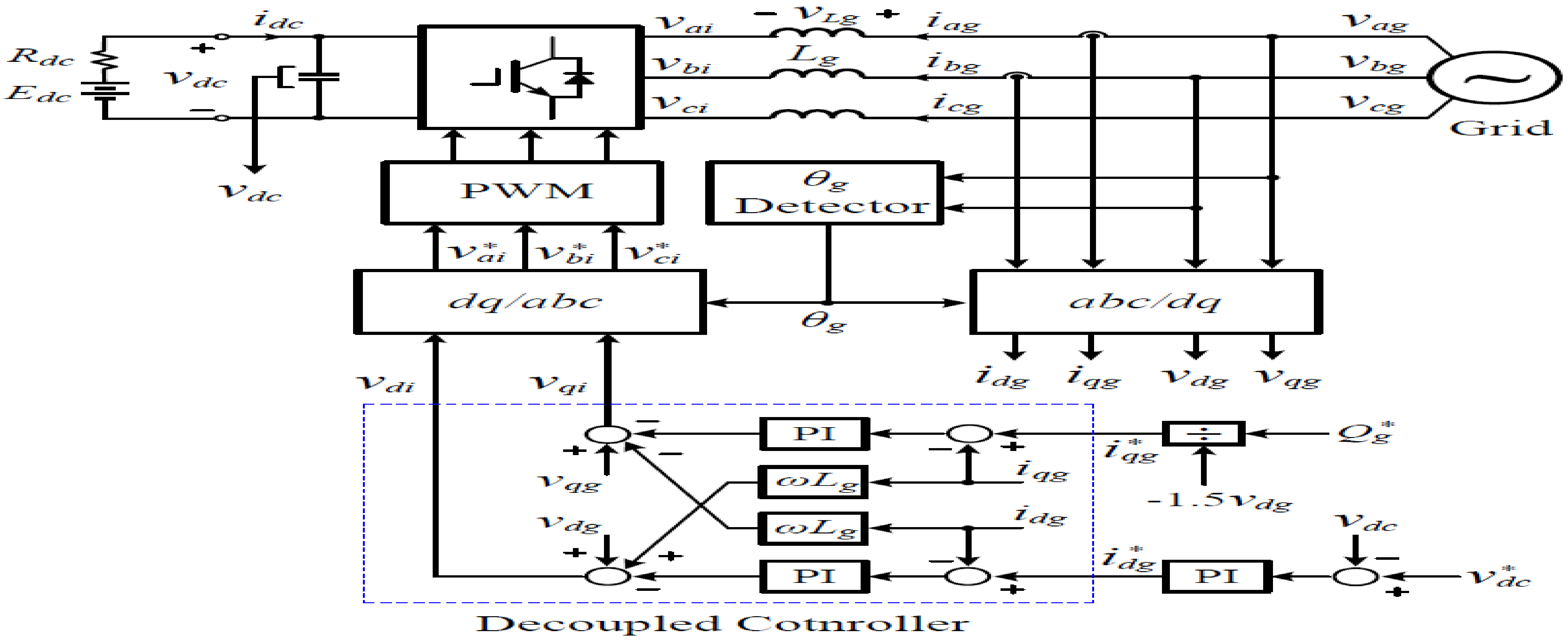
Numericals

- Topic: Control of Grid-Connected Converters

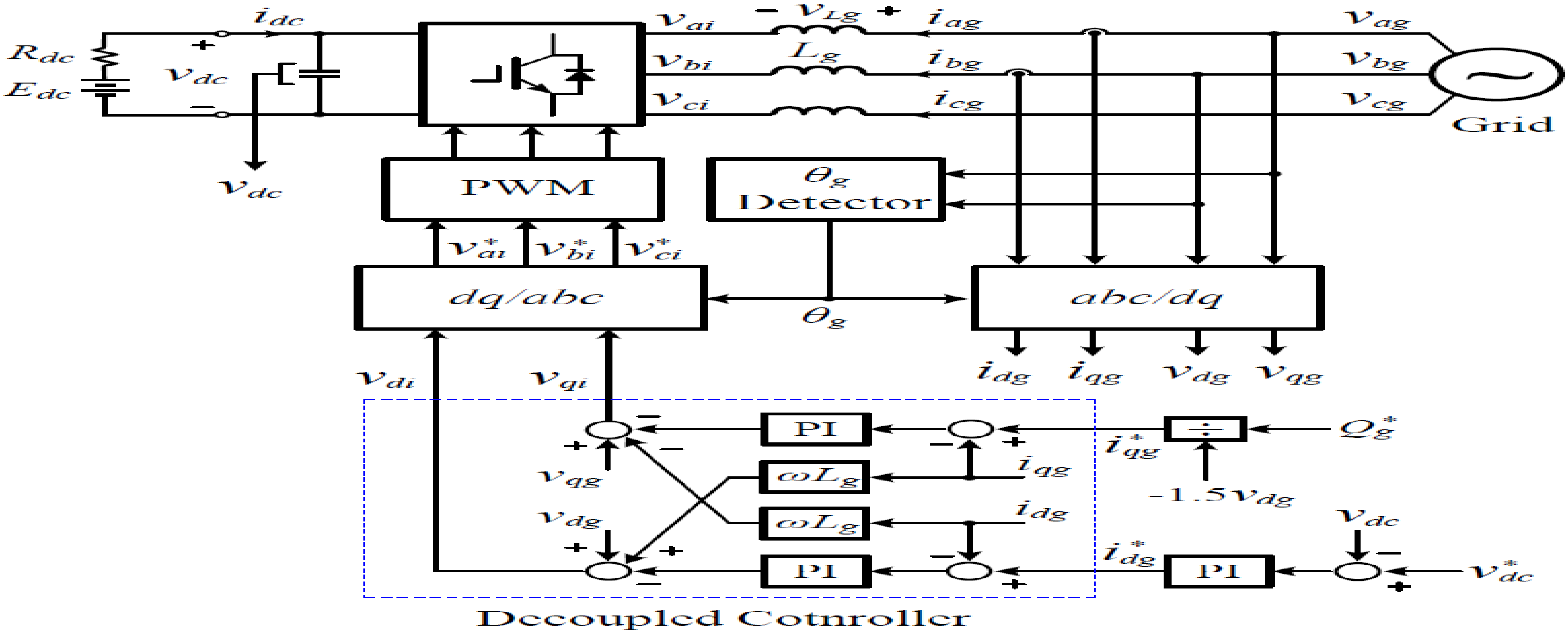
4-17 (Solved Problem.PP.272) Consider a grid-connected 2-level voltage source inverter with voltage oriented control (VOC) shown in Fig.



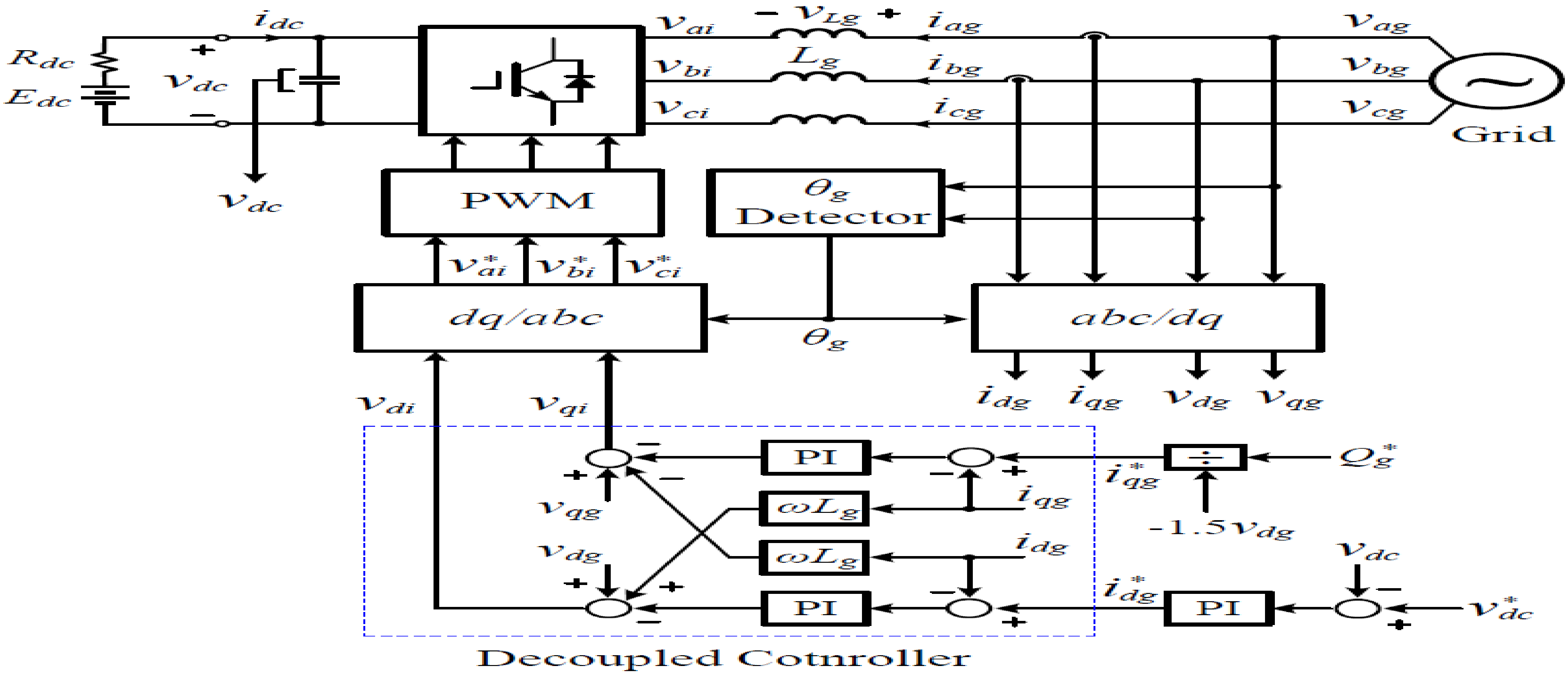
Inverter is connected to grid of 690V/50Hz & delivers 2.3MW to grid with unity power factor operation. Here *Vabi*1=690v is rms value of fundamental-frequency component of inverter line to line voltage.



Inverter is modulated by SVM scheme with modulation index of 0.8 & operates under steady state conditions. Line inductance *Lg* is 0.1098 mH.



- To simplify analysis, all harmonics produced by inverter are neglected. When grid voltage vector angle ϑg is -45º.



Determine following:

a) instantaneous 3-phase grid voltages & currents,

b) grid voltage angle,

c) *dq*-axis grid voltages & currents,

d) active & reactive powers delivered to grid (using *dq*-axis grid voltages & currents),

e) dc-link voltage & current, and

f) *dq*-axis & 3-phase reference voltages for PWM modulator.

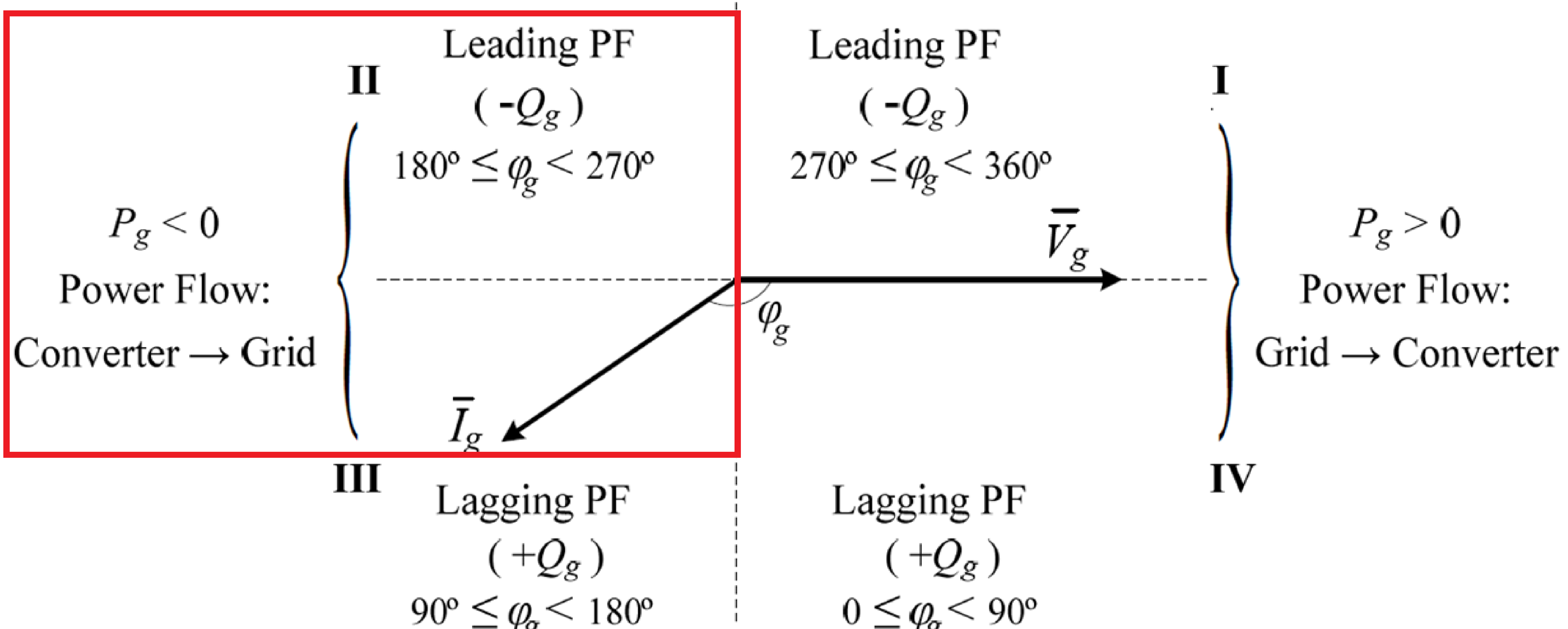
a) Grid voltage, current and frequency:

$$v_g = \frac{690}{\sqrt{3}} \times \sqrt{2} = 563.38 \;\; \text{V (peak)}$$

$$P_g = \frac{3}{2} v_g i_g$$

$$i_g = \frac{2P_g}{3v_g} = \frac{2 \times 2.3 \times 10^6}{3 \times 563.38} = 2721.6 \;\; \text{A (peak)}$$

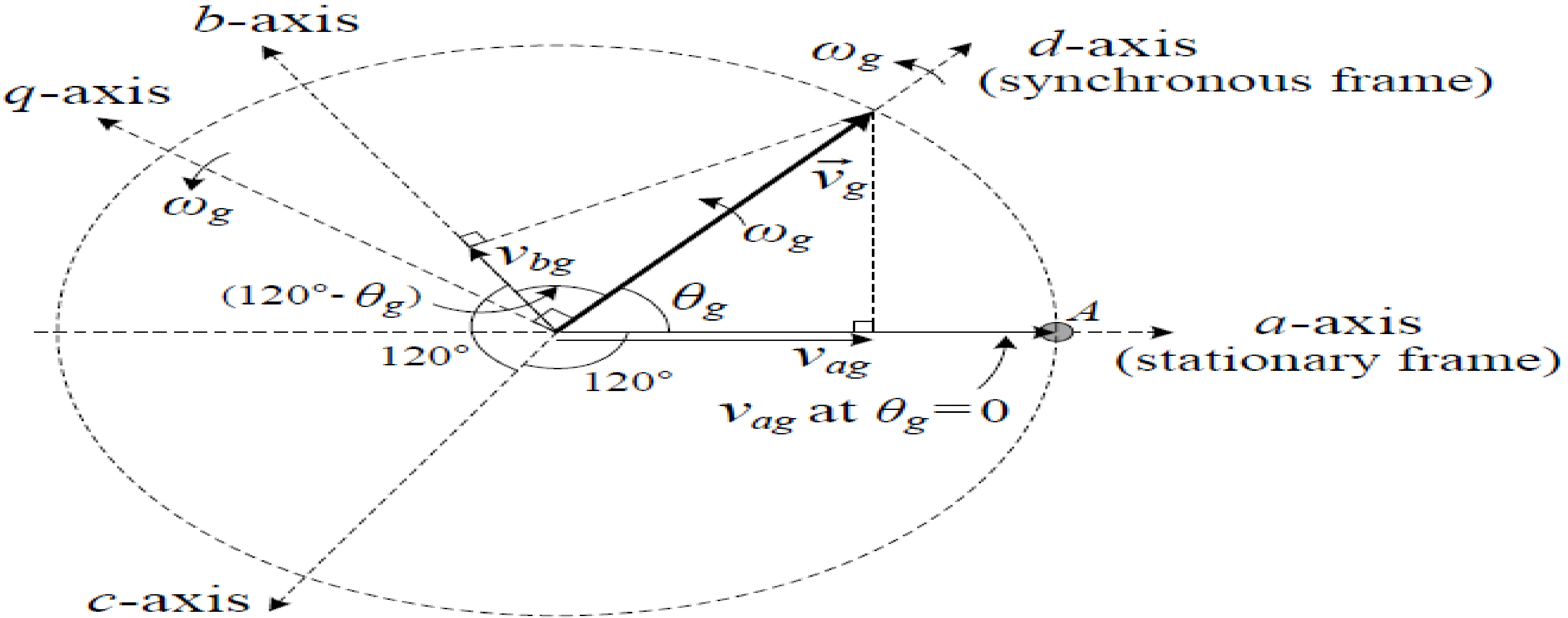
As inverter delivers 2.3MW to grid with unity power factor operation so $\varphi_g = 180^\circ = \pi$ rad (generating mode)



$$\omega_g = 2\pi \times 50 = 314.16 \text{ rad/sec}.$$

At $\theta_g = -45^{\circ}$, $\omega_g t = \theta_g = -\pi / 4$ rad

$$
\begin{cases}
v_{ag} = v_g \cos\theta_g = v_g \cos\omega_g t \\
v_{bg} = v_g \cos\left(\theta_g - 120^\circ\right) = v_g \cos\left(\omega_g t - 120^\circ\right) \\
v_{cg} = v_g \cos\left(\theta_g + 120^\circ\right) = v_g \cos\left(\omega_g t + 120^\circ\right)
\end{cases}
$$



Instantaneous 3-phase grid voltages:

$$
\begin{cases}
v_{ag} = v_g \cos\omega_g t = 563.38 \times \cos(-\pi/4) = 398.37 \text{ V (peak)} \\
v_{bg} = v_g \cos\left(\omega_g t - 2\pi/3\right) = 563.38 \times \cos(-\pi/4 - 2\pi/3) = -544.19 \text{ V (peak)} \\
v_{cg} = v_g \cos\left(\omega_g t + 2\pi/3\right) = 563.38 \times \cos(-\pi/4 + 2\pi/3) = 145.81 \text{ V (peak)}
\end{cases}
$$

Instantaneous 3-phase grid currents:

$$\begin{cases} i_{ag} = i_g \cos(\omega_g t - \varphi_g) = 2721.6 \times \cos(-\pi/4 - \pi) = -1924.5 \ \text{A (peak)} \\ i_{bg} = i_g \cos\left(\omega_g t - \varphi_g - 2\pi/3\right) = 2721.6 \times \cos(-\pi/4 - \pi - 2\pi/3) = 2628.9 \ \text{A (peak)} \\ i_{cg} = i_g \cos\left(\omega_g t - \varphi_g + 2\pi/3\right) = 2721.6 \times \cos(-\pi/4 - \pi + 2\pi/3) = -704.42 \ \text{A (peak)} \end{cases}$$

b) The α –β components of 3-phase grid voltages:

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}\begin{bmatrix} v_{ag} \\ v_{bg} \\ v_{cg} \end{bmatrix}$$

from which

$$v_\alpha = v_{ag} = 398.37 \ \text{V (peak)}$$

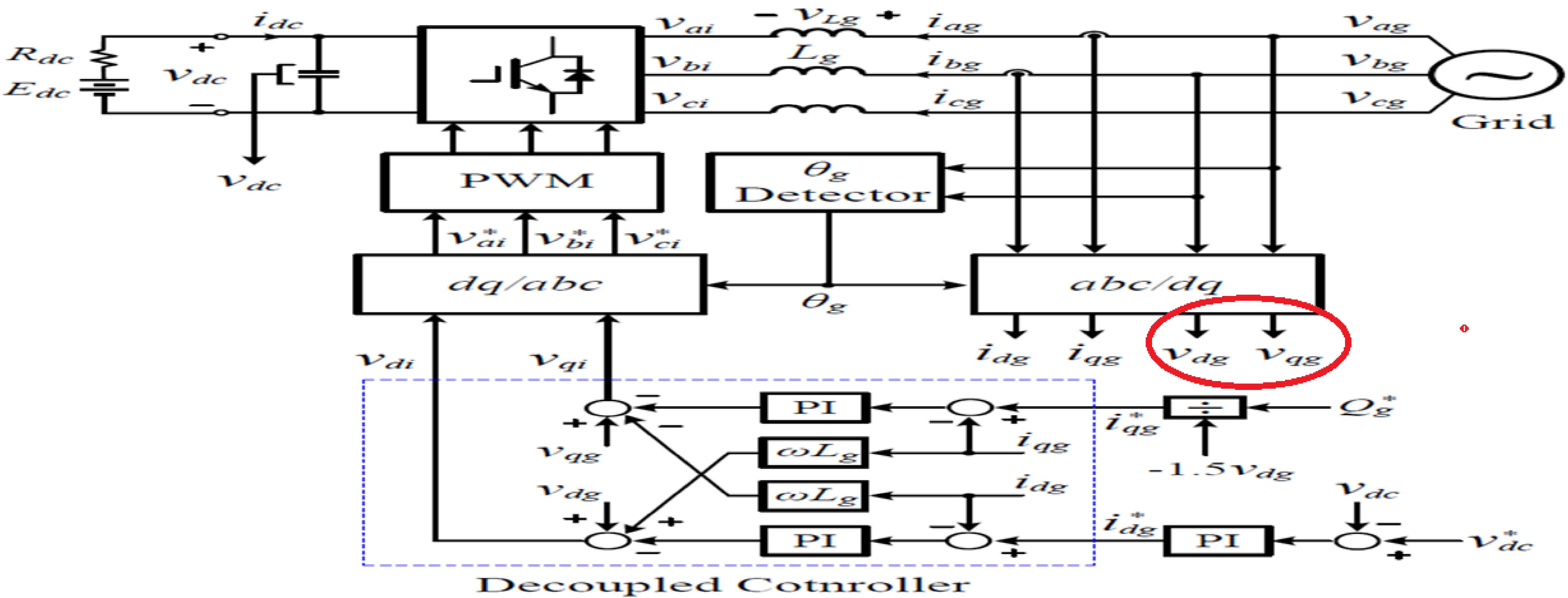
$$v_\beta = \frac{\sqrt{3}}{3}\left(v_{ag} + 2v_{bg}\right) = \frac{\sqrt{3}}{3}\left(398.37 + 2 \times -544.19\right) = -398.37 \ \text{V (peak)}$$

Verify the grid voltage vector angle:

$$\theta_g = \tan^{-1}\frac{v_\beta}{v_\alpha} = \tan^{-1}\frac{-398.37}{398.37} = -\pi/4 \ \text{rad}$$

c) The *dq/abc* transformation with voltage oriented control is given by

$$\begin{bmatrix} v_{dg} \\ v_{qg} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta_g & \cos(\theta_g - 2\pi/3) & \cos(\theta_g - 4\pi/3) \\ -\sin\theta_g & -\sin(\theta_g - 2\pi/3) & -\sin(\theta_g - 4\pi/3) \end{bmatrix} \cdot \begin{bmatrix} v_{ag} \\ v_{bg} \\ v_{cg} \end{bmatrix}$$



from which, *dq*-axis grid voltages can be calculated by

$$
\begin{aligned}
v_{dg} &= \frac{2}{3}\left(v_{ag}\cos\theta_g + v_{bg}\cos(\theta_g - 2\pi/3) + v_{cg}\cos(\theta_g - 4\pi/3)\right) \\
&= \frac{2}{3}\left(398.37\times\cos(-\pi/4) - 544.19\times\cos(-\pi/4 - 2\pi/3) + 145.81\times\cos(-\pi/4 - 4\pi/3)\right) \\
&= 563.38 \ \text{V (peak)}
\end{aligned}
$$

$$
\begin{aligned}
v_{qg} &= -\frac{2}{3}\left(v_{ag}\sin\theta_g + v_{bg}\sin(\theta_g - 2\pi/3) + v_{cg}\sin(\theta_g - 4\pi/3)\right) \\
&= -\frac{2}{3}\left(398.37\times\sin(-\pi/4) - 544.19\times\sin(-\pi/4 - 2\pi/3) + 145.81\times\sin(-\pi/4 - 4\pi/3)\right) \\
&= 0 \ \text{V}
\end{aligned}
$$

The dq-axis grid currents can be found in a similar way,

$$i_{dg} = i_{dg}^{*} = \frac{2}{3}\left(i_{ag}\cos\theta_g + i_{bg}\cos(\theta_g - 2\pi/3) + i_{cg}\cos(\theta_g - 4\pi/3)\right)$$

$$= \frac{2}{3}\left(-1924.5 \times \cos(-\pi/4) - 2628.9 \times \cos(-\pi/4 - 2\pi/3) - 704.42 \times \cos(-\pi/4 - 4\pi/3)\right)$$

$$= -2721.65 \text{ A (peak)}$$

$$i_{qg} = i_{qg}^{*} = -\frac{2}{3}\left(i_{ag}\sin\theta_g + i_{bg}\sin(\theta_g - 2\pi/3) + i_{cg}\sin(\theta_g - 4\pi/3)\right)$$

$$= -\frac{2}{3}\left(-1924.5 \times \sin(-\pi/4) - 2628.9 \times \sin(-\pi/4 - 2\pi/3) - 704.42 \times \sin(-\pi/4 - 4\pi/3)\right)$$

$$= 0 \text{ A (peak)}$$

d) Active & reactive powers delivered to grid:

$$\begin{cases} P_g = \frac{3}{2}\left(v_{dg} i_{dg} + v_{qg} i_{qg}\right) = \frac{3}{2}(563.38 \times -2721.6 + 0) = -2300 \times 10^3 \ \text{W} \ \ (-1.0\,\text{pu}) \\ Q_g = \frac{3}{2}\left(v_{qg} i_{dg} - v_{dg} i_{qg}\right) = -\frac{3}{2}(0 - 0) = 0 \ \text{VAR} \end{cases}$$

e) The dc-link voltage and current: with modulation index *ma* is 0.8, leaving 20% margin for adjustments.

$$V_{dc} = \frac{\sqrt{2}V_{abi1}}{m_a} = \frac{\sqrt{2} \times 690}{0.8} = 1220 \text{ V}$$

V_{abi1}=690v is rms value of fundamental-frequency component of inverter line to line voltage

Dc-link current:

$$I_{dc} = \frac{P_g}{V_{dc}} = \frac{-2300 \times 10^3}{1220} = -1885.6 \text{ A}$$

f) the *dq*-axis reference voltages for the PWM modulator:

$$\begin{cases} v_{di} = -(k_1 + k_2 / S)\left(i_{dg}^* - i_{dg}\right) + \omega_g L_g i_{qg} + v_{dg} \\ v_{qi} = -(k_1 + k_2 / S)\left(i_{qg}^* - i_{qg}\right) - \omega_g L_g i_{dg} + v_{qg} \end{cases}$$

In steady-state $i_{dg}^* = i_{dg}$ and $i_{qg}^* = i_{qg}$, thus

$$\begin{cases} v_{di} = \omega_g L_g i_{qg} + v_{dg} = 0 + 563.38 = 563.38 \text{ V (peak)} \\ v_{qi} = -\omega_g L_g i_{dg} + v_{qg} = -314.16 \times 0.1098 \times 10^{-3} \times -2721.65 + 0 = 93.88 \text{ V (peak)} \end{cases}$$

Note: Though the q-axis grid voltage v_{qg} is zero, its reference v_{qi} is not zero because of the $-\omega_g L_g i_{dg}$ term used in the decoupled controller.

3-phase reference voltages for PWM modulator can be obtained by *dq/abc* transformation:

$$\begin{bmatrix} x_{ai} \\ x_{bi} \\ x_{ci} \end{bmatrix} = \begin{bmatrix} \cos\theta_g & -\sin\theta_g \\ \cos(\theta_g - 2\pi/3) & -\sin(\theta_g - 2\pi/3) \\ \cos(\theta_g - 4\pi/3) & -\sin(\theta_g - 4\pi/3) \end{bmatrix} \cdot \begin{bmatrix} x_{di} \\ x_{qi} \end{bmatrix}$$

from which

$$\begin{aligned} v_{ai}^* &= v_{di}\cos\theta_g - v_{qi}\sin\theta_g \\ &= 563.38 \times \cos(-\pi/4) - 93.88 \times \sin(-\pi/4) = 464.76 \text{ V (peak)} \end{aligned}$$

$$\begin{aligned} v_{bi}^* &= v_{di}\cos(\theta_g - 2\pi/3) - v_{qi}\sin(\theta_g - 2\pi/3) \\ &= 563.38 \times \cos(-\pi/4 - 2\pi/3) - 93.88 \times \sin(-\pi/4 - 2\pi/3) = -519.89 \text{ V (peak)} \end{aligned}$$

$$\begin{aligned} v_{ci}^* &= v_{di}\cos(\theta_g - 4\pi/3) - v_{qi}\sin(\theta_g - 4\pi/3) \\ &= 563.38 \times \cos(-\pi/4 - 4\pi/3) - 93.88 \times \sin(-\pi/4 - 4\pi/3) = 55.13 \text{ V (peak)} \end{aligned}$$

Cross Check:

$$P_g = 3V_g I_g \cos\varphi_g = 3\times\frac{690}{\sqrt{3}}\times 1924.5\times\cos(180^\circ) = -2300\times 10^3\ \text{ W}$$

$$Q_g = 3V_g I_g \sin\varphi_g = 3\times\frac{690}{\sqrt{3}}\times 1924.5\times\sin(180^\circ) = 0\ \text{ VAR}$$

Answers:

a) $i_g = 1360.8$ A, $\varphi_g = 154.16^o$ (2.691 rad), $v_{ag} = -563.38$ V

$v_{bg} = 281.69$ V, $v_{cg} = 281.69$ V, $i_{ag} = -1224.7$ A, $i_{bg} = -98.67$ A, $i_{cg} = -1126.1$ A

b) $v_\alpha = -563.38$ V, $v_\beta = 0$ V, $\theta_g = -180^o$ ($-\pi$ rad)

c) $v_{dg} = 563.38$ V, $v_{qg} = 0$ V, $i_{dg} = -1224.7$ A, $i_{qg} = -593.171$ A

d) $P_g = -1035 \times 10^3$ W (−0.45 pu), $Q_g = 501.27 \times 10^3$ VAR (0.2179 pu)

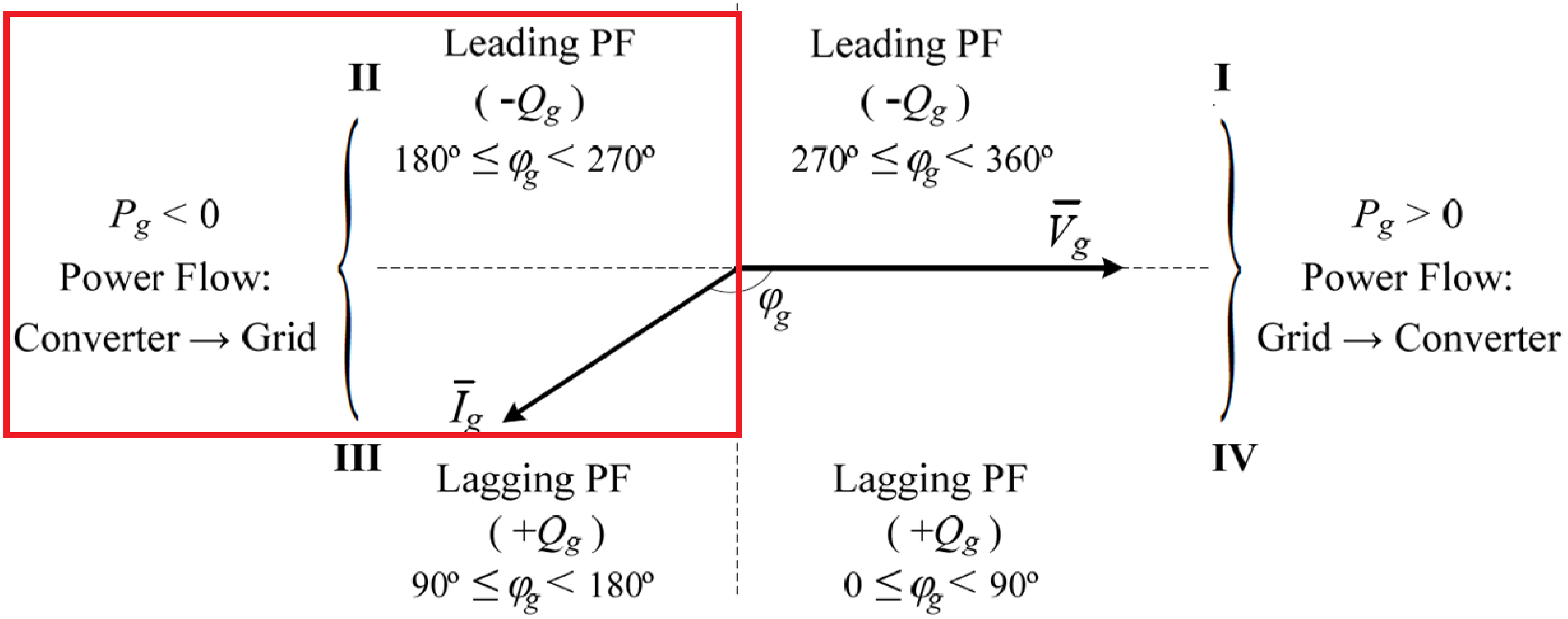
e) $V_{dc} = 1220$ V, $I_{dc} = -848.53$ A

f) $v_{di} = 542.92$ V, $v_{qi} = 42.25$ V, $v_{ai}^* = -542.92$ V, $v_{bi}^* = 234.87$ V, $v_{ci}^* = 308.05$ V

4-18 Repeat Problem 4-17 if the grid-side power factor is 0.8 leading. Perform calculations when grid voltage vector angle ϑ_g=90º.

As inverter delivers 2.3MW to grid with 0.8 leading power factor operation so Cos(216.87)=-0.8

$$\varphi_g = 216.87^{\circ} \quad (3.785 \text{ rad})$$
(generating mode)



a) Grid voltage, current and frequency:

$$v_g = \frac{690}{\sqrt{3}} \times \sqrt{2} = 563.38 \text{ V (peak)}$$

$$P_g = \frac{3}{2} v_g i_g$$

$$i_g = \frac{2P_g}{3v_g} = \frac{2 \times 2.3 \times 10^6}{3 \times 563.38} = 2721.6 \text{ A (peak)}$$

$$\omega_g = 2\pi \times 50 = 314.16 \text{ rad/sec}$$

At $\omega_g t = \theta_g = 90^{o} \quad (\pi / 2 \text{ rad})$

Instantaneous 3-phase grid voltages:

$$\begin{cases} v_{ag} = v_g \cos \omega_g t \\ v_{bg} = v_g \cos\left(\omega_g t - 2\pi / 3\right) \\ v_{cg} = v_g \cos\left(\omega_g t + 2\pi / 3\right) \end{cases}$$

$$\omega_g t = \theta_g = 90^0 \quad (\pi / 2 \text{ rad})$$

$$v_g = 563.38 \text{ V (peak)}$$

$$\begin{cases} i_{ag} = i_g \cos(\omega_g t - \varphi_g) \\ i_{bg} = i_g \cos\left(\omega_g t - \varphi_g - 2\pi/3\right) \\ i_{cg} = i_g \cos\left(\omega_g t - \varphi_g + 2\pi/3\right) \end{cases}$$

$i_{ag} = -1633\ \mathrm{A}$, $i_{bg} = -1069.1\ \mathrm{A}$, $i_{cg} = 2702.1\ \mathrm{A}$

Answers:

a) $\varphi_g = 216.87^{\circ}$ (3.785 rad), $v_{ag} = 0$ V, $v_{bg} = 487.9$ V, $v_{cg} = -487.9$ V

$i_{ag} = -1633$ A, $i_{bg} = -1069.1$ A, $i_{cg} = 2702.1$ A

b) $v_{\alpha} = 0$ V, $v_{\beta} = 562.38$ V, $\theta_g = 90^{\circ}$ ($\pi / 2$ rad)

c) $v_{dg} = 563.38$ V, $v_{qg} = 0$ V, $i_{dg} = -2177.3$ A, $i_{qg} = 1633$ A

d) $P_g = -1840 \times 10^3$ W (-0.8 pu), $Q_g = -1380 \times 10^3$ VAR (-0.6 pu)

e) $V_{dc} = 1220$ V, $I_{dc} = -1508.5$ A

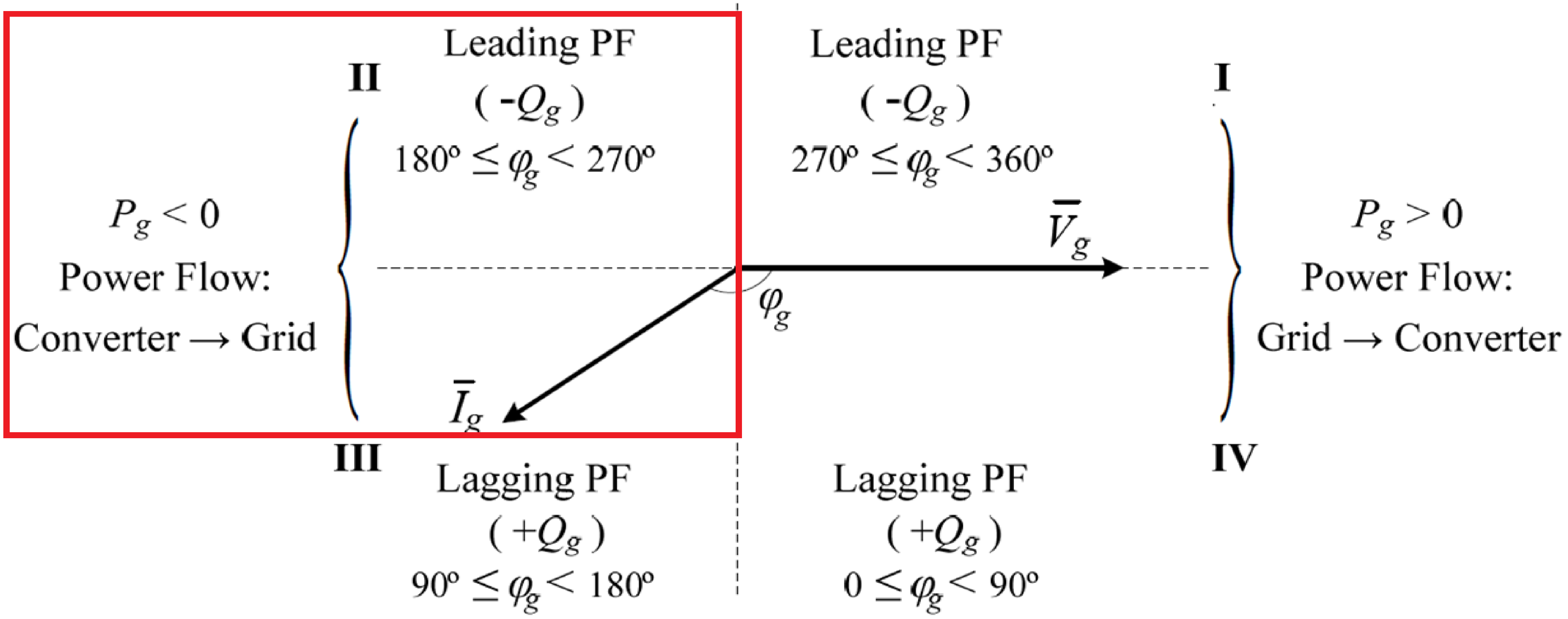
f) $v_{di} = 619.71$ V, $v_{qi} = 75.11$ V, $v_{ai}^* = -75.11$ V, $v_{bi}^* = 574.24$ V, $v_{ci}^* = -499.13$ V

4-19 Repeat Problem 4-17 if grid-side power factor is 0.9 lagging & inverter delivers 50% of rated power to grid. Perform calculations when grid voltage vector angle ϑg =180º.

As inverter delivers 2.3MW to grid with 0.9 lagging power factor operation so Cos(154.16)=-0.9

$$\varphi_g = 154.16^\circ \quad (2.691\,\text{rad})$$

(generating mode)



Answers:

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b) $v_{\alpha} = -563.38$ V, $v_{\beta} = 0$ V, $\theta_g = -180^{o}$ ($-\pi$ rad)

c) $v_{dg} = 563.38$ V, $v_{qg} = 0$ V, $i_{dg} = -1224.7$ A, $i_{qg} = -593.171$ A

d) $P_g = -1035 \times 10^3$ W (−0.45 pu), $Q_g = 501.27 \times 10^3$ VAR (0.2179 pu)

e) $V_{dc} = 1220$ V, $I_{dc} = -848.53$ A

f) $v_{di} = 542.92$ V, $v_{qi} = 42.25$ V, $v_{ai}^{*} = -542.92$ V, $v_{bi}^{*} = 234.87$ V, $v_{ci}^{*} = 308.05$ V