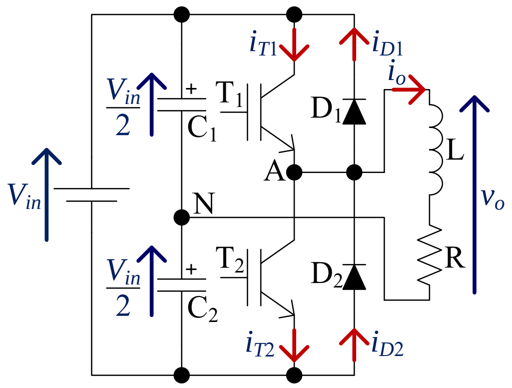
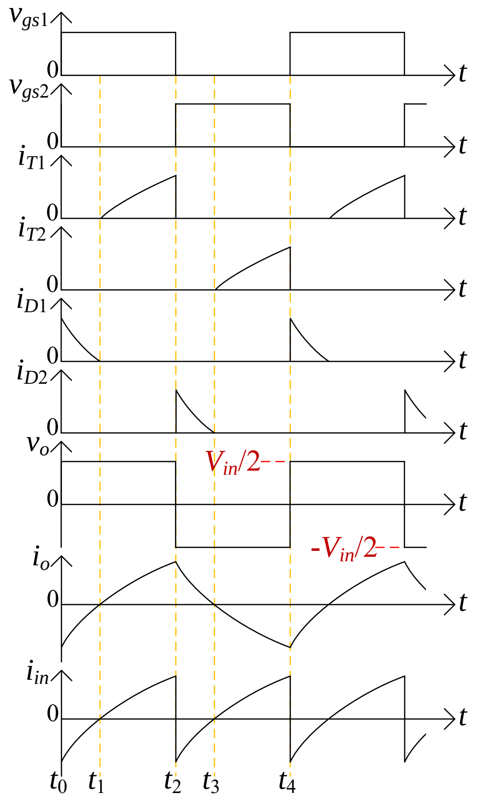
TUTORIAL OF POWER INVERTERS

(1) A single-phase half-bridge inverter is connected to a L-R load. The input voltage of the converter is 300VDC. The duty ratio of the gate signals of each transistor is 0.5. The inductance and the resistance of the L-R load is 10mH and 20Ω. The switching frequency is 50Hz. Calculate the rms output voltage and the rms output current of the inverter.





The peak output voltage is:

$$V_{o\_pk} = \frac{V_{in}}{2} = 150V$$

Since the output voltage is a purely AC square, the rms output voltage of the inverter is:

$$V_{o(rms)} = \frac{V_{in}}{2} = 150V$$

When the output voltage is positive, the equation of the output current and voltage is:

$$\frac{V_{in}}{2} = L\frac{di_o(t)}{dt} + Ri_o(t)$$

Solving the equation by Laplace Transform,

$$\frac{V_{in}}{2s} = sLI_o - LI_0 + RI_o \qquad (1)$$

where I_0 is the value of $i_o(t)$ when t is t_0.

$$I_o = \frac{LI_0}{(sL+R)} + \frac{V_{in}}{2s(sL+R)}$$

By partial faction technique, let

$$\frac{1}{s(sL+R)} = \frac{A}{s} + \frac{Bs+C}{sL+R}$$

$$Bs^2 + (AL+C)s + RA = 1$$

Hence,

$$A = \frac{1}{R},\ B = 0,\ C = -AL = -\frac{L}{R}$$

(1) is rewritten as:

$$I_o = \frac{LI_0}{sL+R} + \frac{V_{in}}{2sR} - \left(\frac{LV_{in}}{2R}\right)\frac{1}{sL+R}$$

$$I_o = \left(I_0 - \frac{V_{in}}{2R}\right)\frac{1}{s+\frac{R}{L}} + \frac{V_{in}}{s2R}$$

Solving the above equation:

$$i_o(t-t_0) = \left(I_0 - \frac{V_{in}}{2R}\right)e^{-\frac{R}{L}(t-t_0)} + \frac{V_{in}}{2R} \qquad (2)$$

When the output voltage is negative,

$$-\frac{V_{in}}{2} = L\frac{di_o(t-t_2)}{dt} + Ri_o(t-t_2)$$

$$-\frac{V_{in}}{2s} = sLI_o - LI_2 + RI_o$$

Solving the equation by Laplace Transform,

$$I_o = \left(I_2 + \frac{V_{in}}{2R} \right) \frac{1}{s + \frac{R}{L}} - \frac{V_{in}}{s2R}$$

where I_2 is the value of $i_o(t)$ when t is t_2.
Solving the above equation:

$$i_o(t - t_2) = \left(I_2 + \frac{V_{in}}{2R} \right) e^{-\frac{R}{L}(t-t_2)} - \frac{V_{in}}{2R} \qquad (3)$$

From (2), let T_S as the switching period, when $t = t_2$,

$$I_2 = \left(I_0 - \frac{V_{in}}{2R} \right) e^{-\frac{RT_S}{2L}} + \frac{V_{in}}{2R}$$

From (3), when $t = t_4 = t_0$,

$$I_4 = I_0 = \left(I_2 + \frac{V_{in}}{2R} \right) e^{-\frac{RT_S}{2L}} - \frac{V_{in}}{2R}$$

Hence,

$$I_0 + I_2 = 0 \qquad (4) \qquad \text{and} \qquad I_2 - I_0 = \frac{V_{in}}{R} \left(\frac{1 - e^{-\frac{RT_S}{2L}}}{1 + e^{-\frac{RT_S}{2L}}} \right) \qquad (5)$$

(4) – (5),

$$I_0 = -\frac{V_{in}}{2R} \left(\frac{1 - e^{-\frac{RT_S}{2L}}}{1 + e^{-\frac{RT_S}{2L}}} \right) \qquad (6) \qquad \text{and} \qquad I_2 = \frac{V_{in}}{2R} \left(\frac{1 - e^{-\frac{RT_S}{2L}}}{1 + e^{-\frac{RT_S}{2L}}} \right) \qquad (7)$$

Substituting (6) and (7) to (2) and (3), respectively, the equation of the output current when the output voltage is positive and negative are:

$$i_o(t - t_0) = -\frac{V_{in}}{2R} \left[\left(\frac{1 - e^{-\frac{RT_S}{2L}}}{1 + e^{-\frac{RT_S}{2L}}} + 1 \right) e^{-\frac{R}{L}(t-t_0)} - 1 \right] \qquad (8)$$

$$i_o(t - t_2) = \frac{V_{in}}{2R} \left[\left(\frac{1 - e^{-\frac{RT_S}{2L}}}{1 + e^{-\frac{RT_S}{2L}}} + 1 \right) e^{-\frac{R}{L}(t-t_2)} - 1 \right] \qquad (9)$$

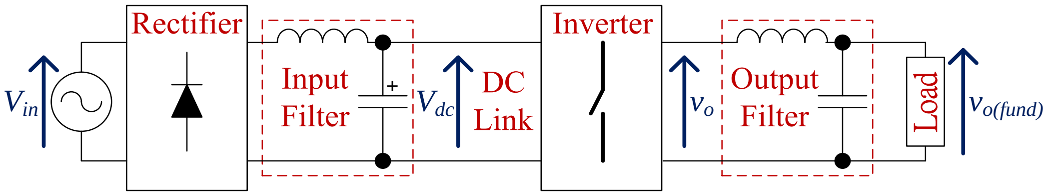
The rms value of the output current from (9) is:

$$I_{o(rms)}{}^2 = -\frac{1}{2T_S} \int_0^{T_S} i_o(t - t_2) d(t - t_2) = \frac{V_{in}}{2RT_S} \int_0^{T_S} \left[\left(\frac{1 - e^{-\frac{RT_S}{2L}}}{1 + e^{-\frac{RT_S}{2L}}} + 1 \right) e^{-\frac{R}{L}(t-t_2)} - 1 \right] d(t - t_2)$$

$$I_{o(rms)}{}^2 = -\frac{50}{(2)(20)} \int_0^{\frac{1}{50}} \left[\left(\frac{1 - e^{-\frac{(20)}{2(50)(10m)}}}{1 + e^{-\frac{(20)}{2(50)(10m)}}} + 1 \right) e^{-\frac{20}{10m}(t-t_2)} - 1 \right] d(t - t_2)$$

$$\underline{I_{o(rms)} = \frac{\sqrt{202}}{10} A = 1.421A}$$

(2) The DC link of a single-phase full-bridge inverter is connected to an output low-pass LC filter and an L-R load. The transistors of the inverter are controlled by a set of SPWM signals with the modulation index equal to 0.8. Modulation frequency of the SPWM is 50Hz. The DC link voltage is supplied by a $220V_{rms}$ 50Hz AC voltage source with a full-bridge rectifier and a low-pass LC filter. The inductance and the resistance of the L-R load is 0.1H and 20Ω. The switching frequency is 50Hz. Assuming all components are ideal and without power loss, the low-pass L-C filters on both the DC link and the output of the inverter are large enough to filter all harmonics, and the DC link voltage is constant, calculate the root-mean-square voltage, the root-mean-square output current and the power factor of the load.



Since the output LC filter has filtered all frequency harmonics, the output voltage is a purely sine wave. Power factor of the load is:

$$PF = \cos\theta = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

where $\omega = 2\pi f_M$

$$PF = \frac{20}{\sqrt{20^2 + [(2\pi)(50)(0.1)]^2}} = 0.537$$

The DC link voltage of the inverter is:

$$V_{dc} = \frac{2\hat{V}_{in}}{\pi} = \frac{2\sqrt{2}V_{in}}{\pi} = 198.07V$$

The peak output voltage of the load is:

$$\hat{V}_o = MV_{DC} = (0.8)(198.07) = 158.456V$$

The rms output voltage of the load is:

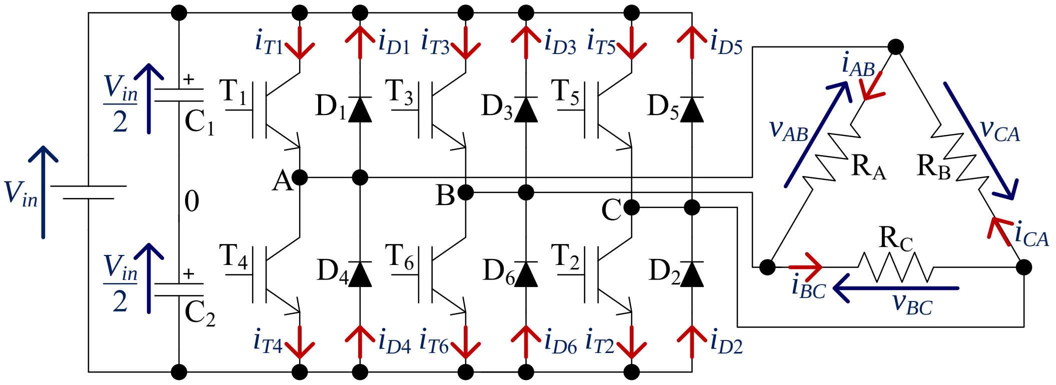
$$V_{o(rms)} = \frac{\hat{V}_o}{\sqrt{2}} = \frac{158.456}{\sqrt{2}} = 112.045V$$

The rms output current of the load is:

$$I_{o(rms)} = \frac{V_{o(rms)}}{Z_{load}} = \frac{V_{o(rms)}}{\sqrt{R^2 + (\omega L)^2}}$$

$$I_{o(rms)} = \frac{112.045}{\sqrt{20^2 + [(2\pi)(50)(0.1)]^2}} = 3.009A$$

(3) Prove the elimination of 3rd harmonic on line voltage of a 3-phase SPWM inverter.



Assuming the 3-phase load is balanced, the equations of the phase voltage with 3rd harmonic are:

$$v_{A0} = \hat{V}_1 \sin(\omega t) + \hat{V}_3 \sin(3\omega t)$$

$$v_{B0} = \hat{V}_1 \sin\left(\omega t - \frac{2\pi}{3}\right) + \hat{V}_3 \sin\left(3\omega t - \frac{2\pi}{3}\right)$$

$$v_{C0} = \hat{V}_1 \sin\left(\omega t + \frac{2\pi}{3}\right) + \hat{V}_3 \sin\left(3\omega t + \frac{2\pi}{3}\right)$$

The equation of the line voltage, v_{AB}, of the 3-phase inverter is:

$$v_{AB} = v_{A0} - v_{B0}$$

$$v_{AB} = \hat{V}_1 \sin(\omega t) + \hat{V}_3 \sin(3\omega t) - \hat{V}_1 \sin\left(\omega t - \frac{2\pi}{3}\right) - \hat{V}_3 \sin\left(3\omega t - \frac{2\pi}{3}\right)$$

$$v_{AB} = \hat{V}_1\left[\sin(\omega t) - \sin\left(\omega t - \frac{2\pi}{3}\right)\right] + \hat{V}_3\left[\sin(3\omega t) - \sin\left(3\omega t - \frac{2\pi}{3}\right)\right]$$

For 3rd harmonic, one per:iod of the harmonic voltage is $\frac{2\pi}{3}$, so that

$$\sin\left(3\omega t - \frac{2\pi}{3}\right) = \sin(3\omega t)$$

Hence,

$$v_{AB} = \hat{V}_1\left[\sin(\omega t) - \sin\left(\omega t - \frac{2\pi}{3}\right)\right]$$

$$v_{AB} = \sqrt{3}\hat{V}_1 \sin\left(\omega t + \frac{\pi}{6}\right)$$

This show that the 3rd harmonic in the line voltage is eliminated even if the phase voltage has 3rd harmonic.

(4) A single-phase half-bridge SPWM inverter with modulation index equal to 0.9. The input voltage of the inverter is 300VDC. The output of the inverter is connected to an inductor in series with a resistor. The inductance and the resistance of the L-R load is 10mH and 2Ω, respectively. The carrier frequency of the SPWM is 20kHz. The modulation frequency is 60Hz. The inverter is assumed as no power loss. Calculate the rms output voltage and the rms output current of the inverter.

Peak output voltage of the inverter is:

$$\hat{V}_o = \frac{MV_{in}}{2}$$

$$\hat{V}_o = \frac{(0.9)(300)}{2} = 135\text{V}$$

The rms output voltage of the inverter is:

$$V_{o(rms)} = \frac{\hat{V}_o}{\sqrt{2}}$$

$$V_{o(rms)} = \frac{135}{\sqrt{2}} = 95.459V$$

The total impedance of the L-R load is:

$$Z_o = \sqrt{(\omega L)^2 + R^2}$$

The rms output current of the inverter is:

$$I_{o(rms)} = \frac{V_{o(rms)}}{R} = \frac{V_{o(rms)}}{\sqrt{(\omega L)^2 + R^2}}$$

$$I_{o(rms)} = \frac{V_{o(rms)}}{R} = \frac{95.459}{\sqrt{[(2\pi)(60)(10m)]^2 + (2)^2}}$$

$$I_{o(rms)} = 22.368A$$