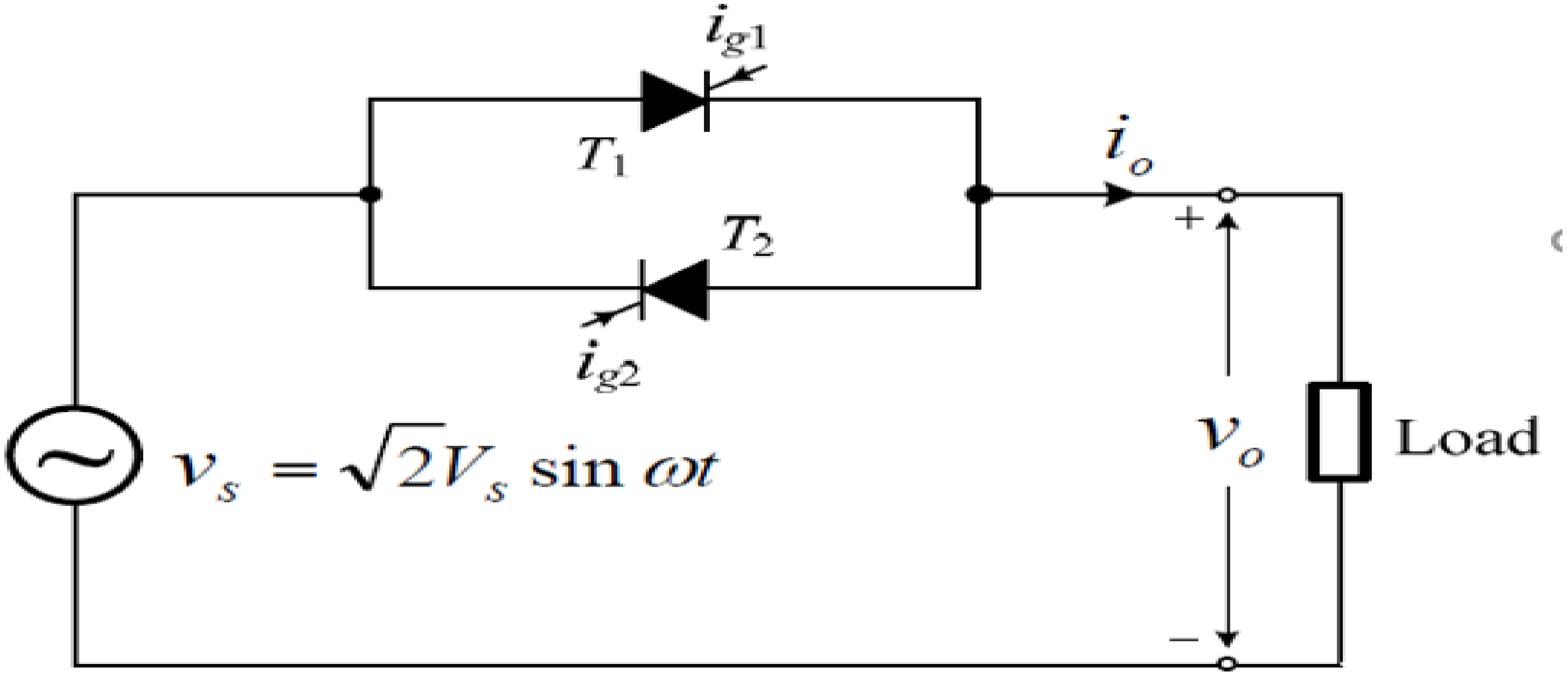
# Lecture#

- 3-phase AC Voltage Controller with R load
- Numericals:4-2 to 4-5

Q. Please design 3-phase AC Voltage Controller using 1-phase AC Voltage Controller?



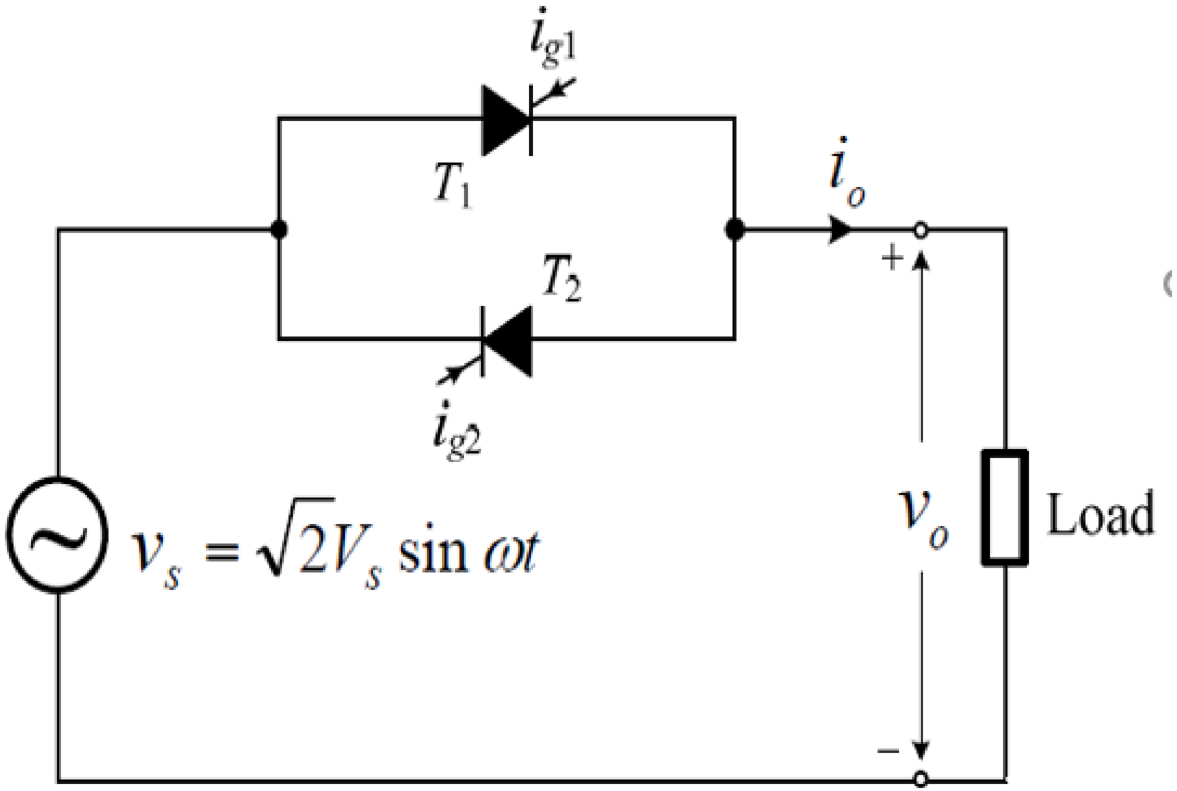
Q. Also mention that where should we connect 3-phase AC Voltage Controller?

# Answer:

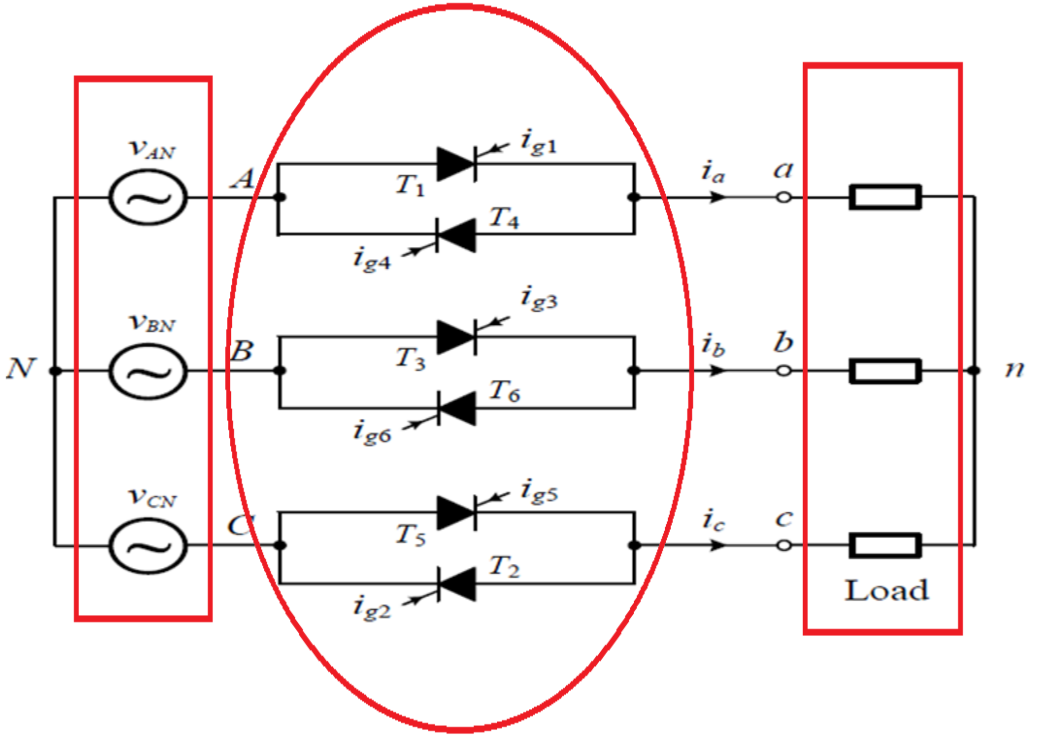
- Between the 3-phase power supply and the load.

# 3-phase AC Voltage Controller

**1-phase AC Voltage Controller**




**3-phase AC Voltage Controller**

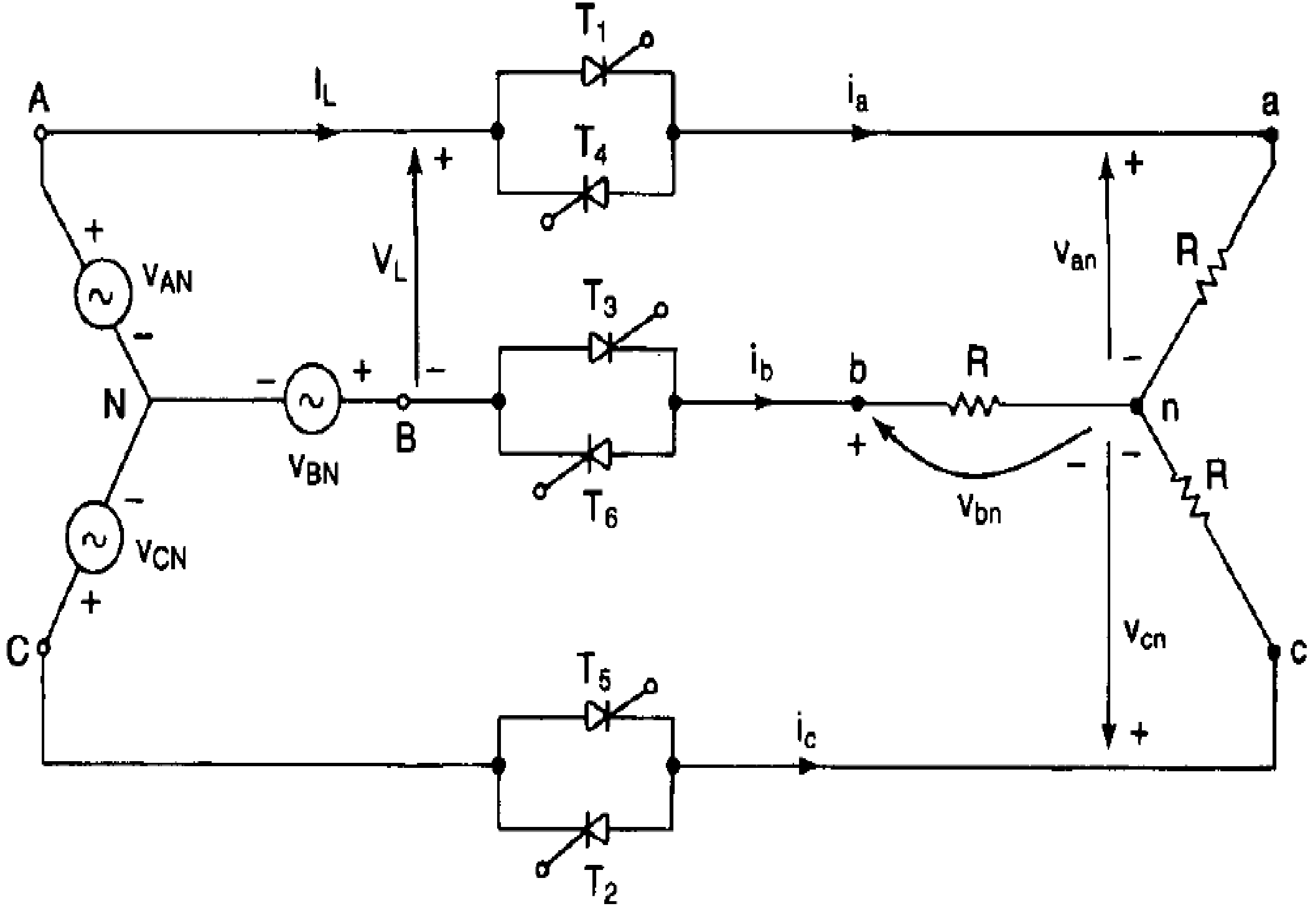


# If we define instantaneous input phase voltages as:

$$v_{AN} = \sqrt{2}V_s \sin \omega t$$

$$v_{BN} = \sqrt{2}V_s \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{CN} = \sqrt{2}V_s \sin\left(\omega t - \frac{4\pi}{3}\right)$$

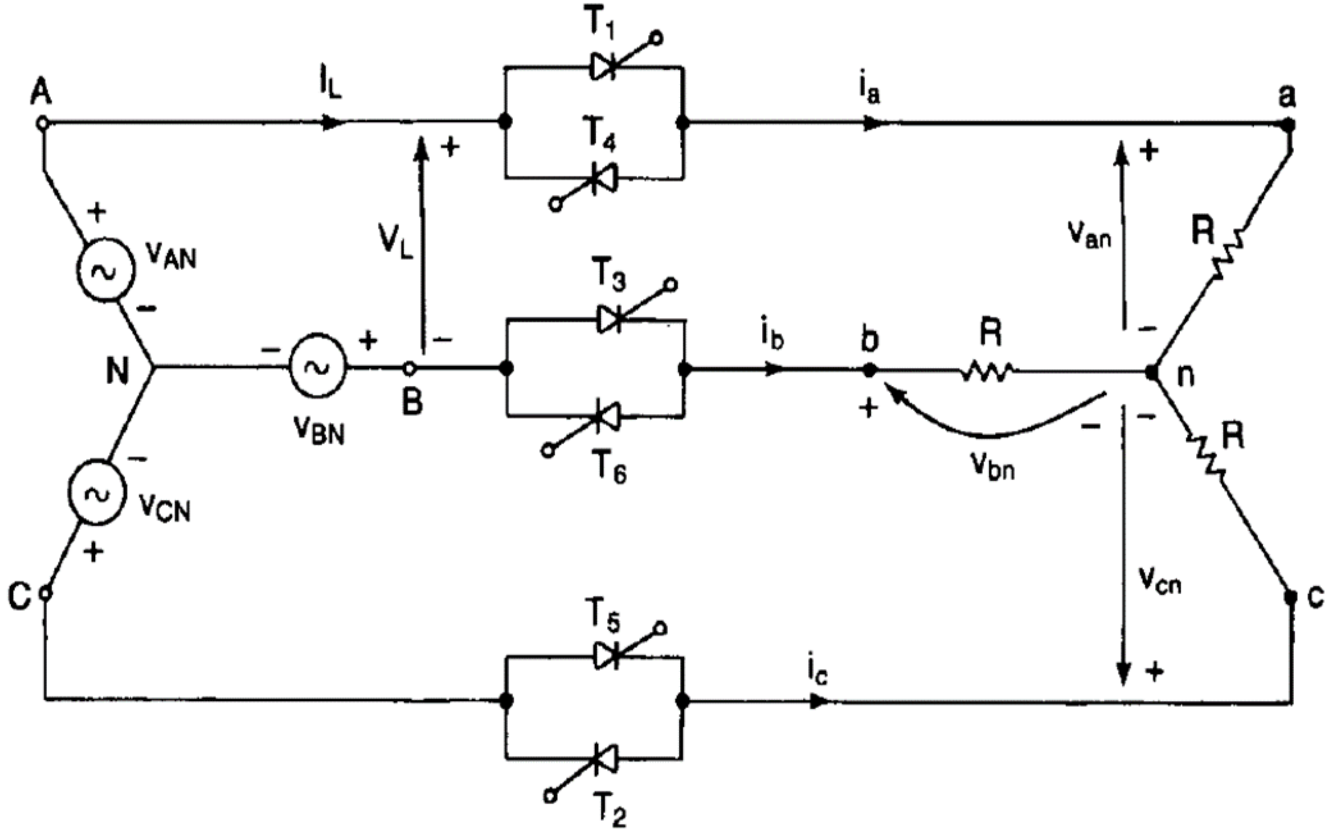


# Q. What would be the values of?

$$V_{AB} =$$

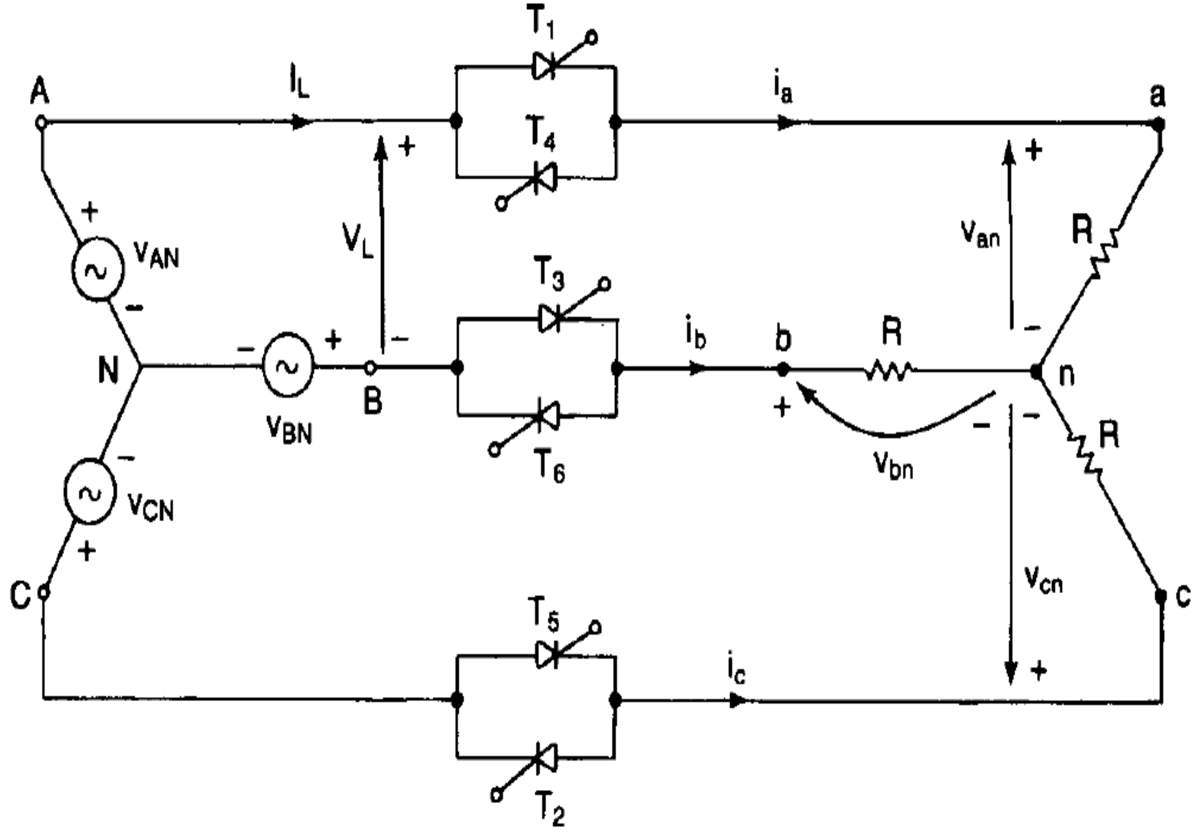
$$V_{BC} =$$

$$V_{CA} =$$



# By applying KVL

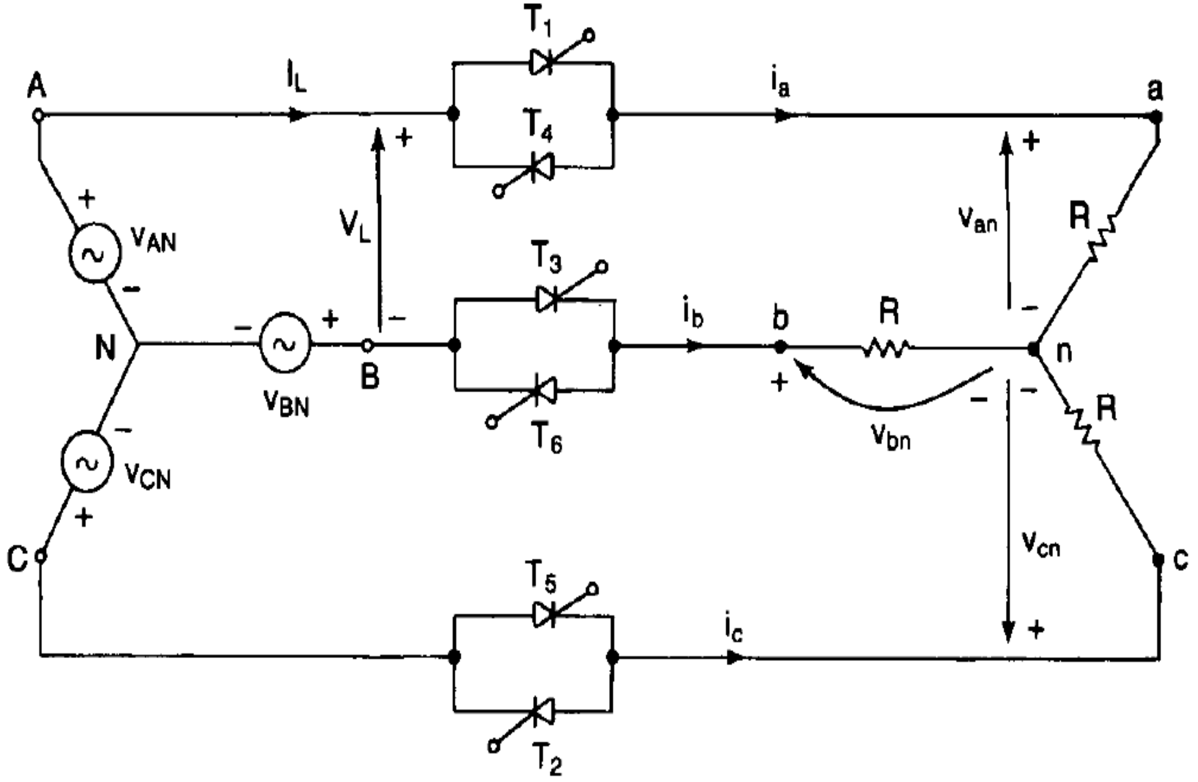
$$V_{AB} = V_{AN} - V_{BN}$$



# Similarly

$$V_{BC} = V_{BN} - V_{CN}$$

$$V_{CA} = V_{CN} - V_{AN}$$



# Please evaluate?

$$V_{AB} = V_{AN} - V_{BN}$$

where

$$v_{AN} = \sqrt{2}V_s \sin \omega t$$

$$v_{BN} = \sqrt{2}V_s \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$V_{AB} = V_{AN} + (-V_{BN})$$

$$V_{AB} = \sqrt{2}V_s[\mathrm{Sin}(\omega t) - \mathrm{Sin}(\omega t - \frac{2\pi}{3})]$$

$$\mathrm{Sin(a)} - \mathrm{Sin(b)} = 2\,\mathrm{Cos}(\frac{a+b}{2})\mathrm{Sin}(\frac{a-b}{2})$$

$$\mathrm{a} = \omega t,$$

$$\mathrm{b} = (\omega t - \frac{2\pi}{3})$$

$$(\frac{a+b}{2}) = (\omega t - \frac{\pi}{3}), \qquad (\frac{a-b}{2}) = \frac{\pi}{3}$$

$$V_{AB} = \sqrt{2}V_s[2Cos(\omega t - \frac{\pi}{3})\mathrm{Sin}(\frac{\pi}{3})]$$

$$V_{AB} = \sqrt{6}V_s Cos(\omega t - \frac{\pi}{3})$$

$$V_{AB} = \sqrt{6}V_s\,\mathrm{Sin}(\omega t - \frac{\pi}{3} + \frac{\pi}{2})$$

$$V_{AB} = \sqrt{6}V_s\,\mathrm{Sin}(\omega t + \frac{\pi}{6})$$

# Similarly

$$V_{AB} = \sqrt{6}V_s \operatorname{Sin}(\omega t + \frac{\pi}{6})$$

$$v_{BC} = \sqrt{6}\, V_s \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{CA} = \sqrt{6}\, V_s \sin\left(\omega t - \frac{7\pi}{6}\right)$$

Draw Waveforms for the supply voltages($V_{AN}$,$V_{BN}$,$V_{CN}$) [4pi/3-2pi/3-0=2pi/3=120º i.e all are 120º apart from each other.]

$$v_{AN} = \sqrt{2}V_s \sin \omega t$$

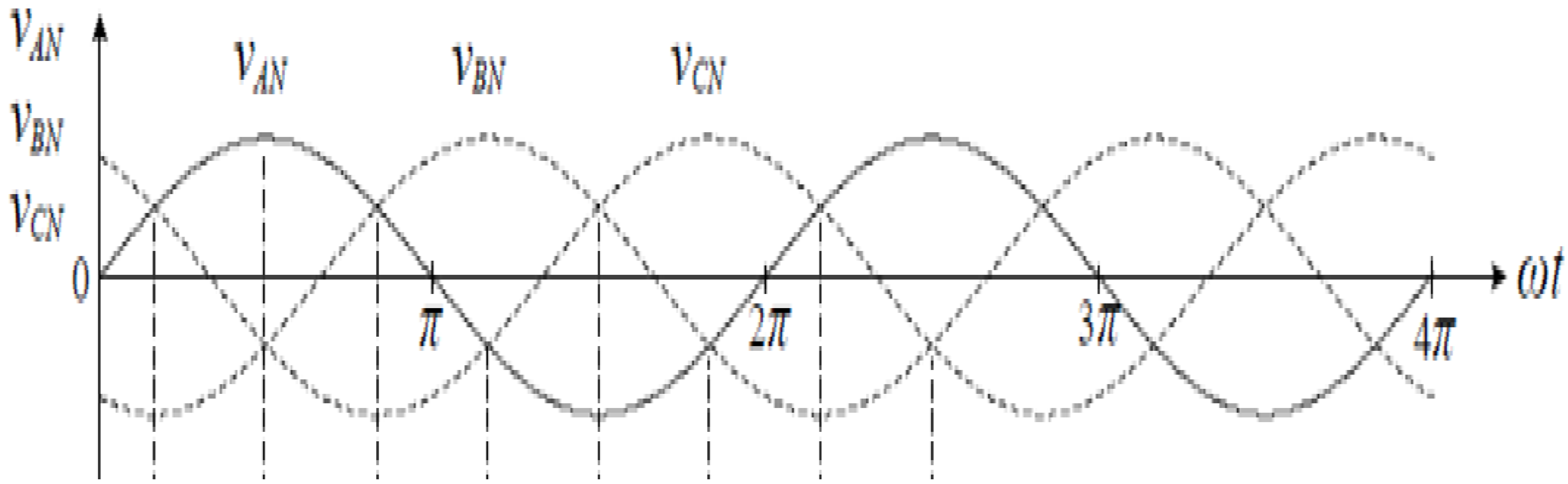
$$v_{BN} = \sqrt{2}V_s \sin \left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{CN} = \sqrt{2}V_s \sin \left(\omega t - \frac{4\pi}{3}\right)$$

$$v_{AN} = \sqrt{2}V_s \sin \omega t$$

$$v_{BN} = \sqrt{2}V_s \sin \left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{CN} = \sqrt{2}V_s \sin \left(\omega t - \frac{4\pi}{3}\right)$$



# Line-line voltages

$$V_{AB} = \sqrt{6}V_s \, \mathrm{Sin}(\omega t + \frac{\pi}{6})$$

$$v_{BC} = \sqrt{6}\, V_s \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{CA} = \sqrt{6}\, V_s \sin\left(\omega t - \frac{7\pi}{6}\right)$$

Can we change the angle of $V_{CA}$?

$$v_{CA} = \sqrt{6}\, V_s \sin\left(\omega t - \frac{7\pi}{6}\right)$$

# Angle of $V_{CA}$ can be changed by adding π

$$V_{CA} = \sqrt{6}V_s Sin(\omega t - \frac{7\pi}{6})$$

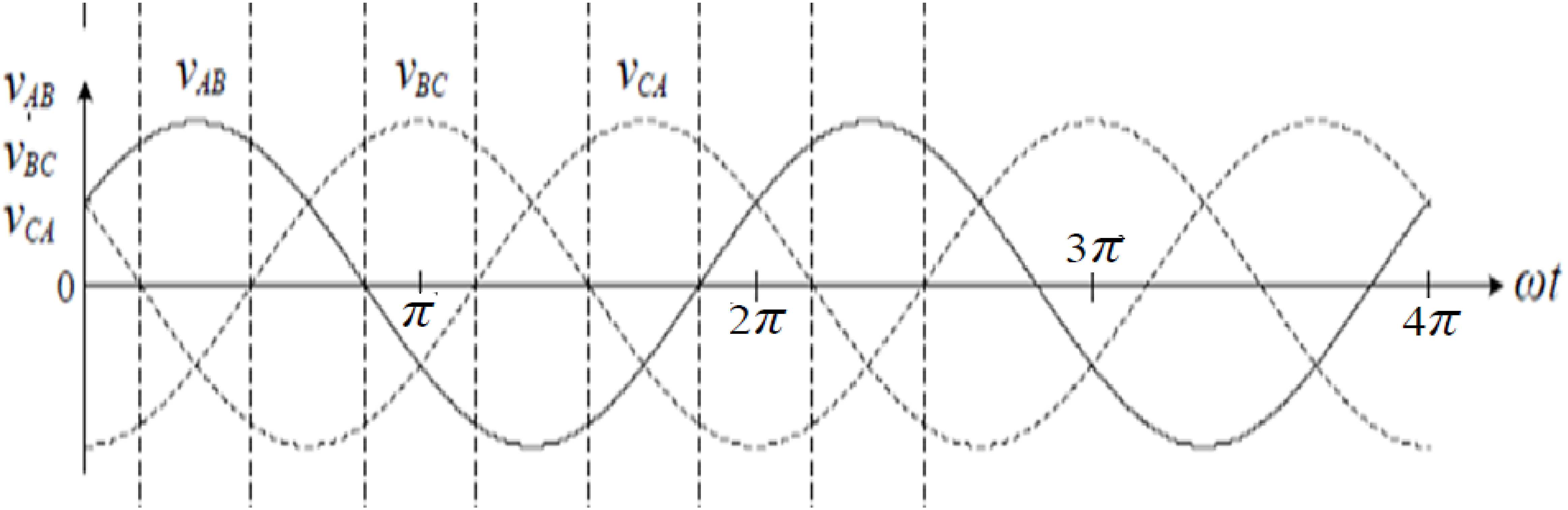
$$V_{AC} = \sqrt{6}V_s Sin(\omega t - \frac{7\pi}{6} + \pi)$$

$$V_{AC} = \sqrt{6}V_s Sin(\omega t - \frac{\pi}{6})$$

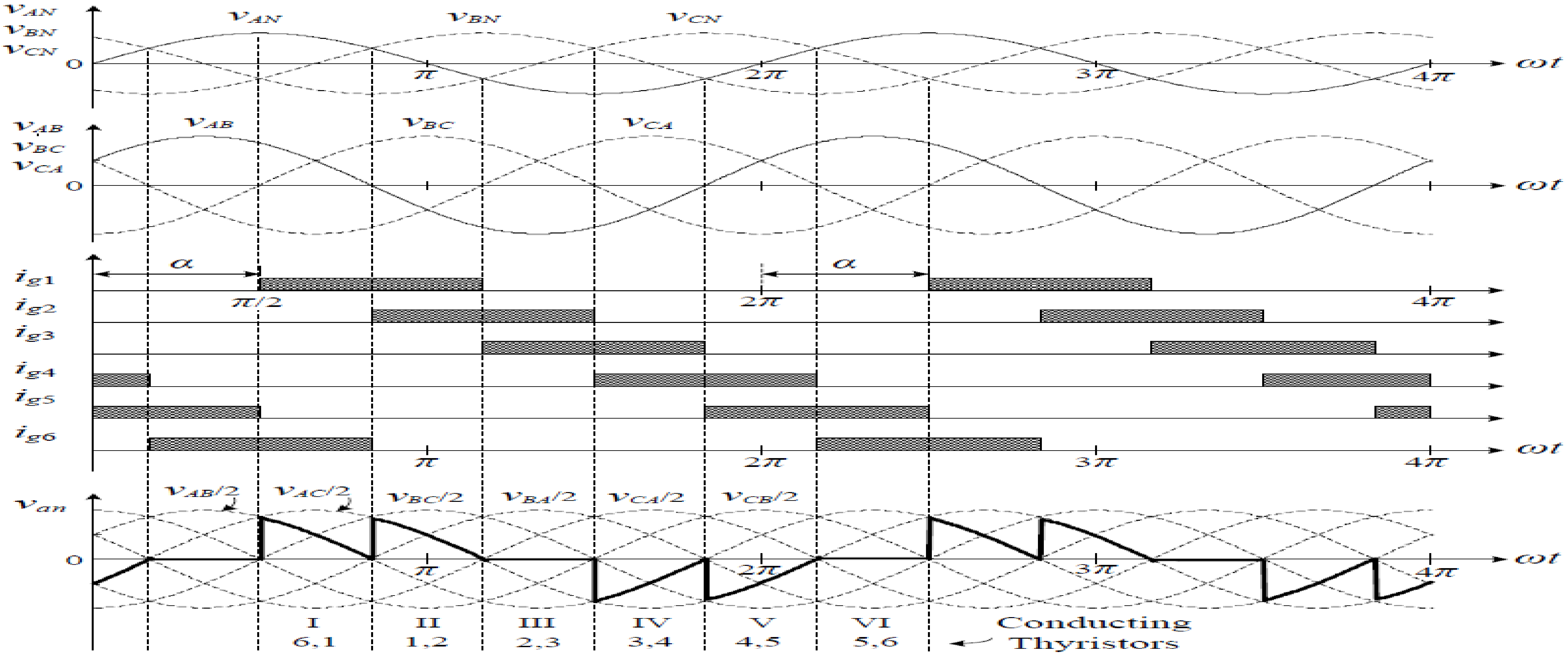
$$V_{AB} = \sqrt{6}V_s Sin(\omega t + \frac{\pi}{6})$$

$$V_{BC} = \sqrt{6}V_s Sin(\omega t - \frac{\pi}{2})$$

$$V_{AC} = \sqrt{6}V_s Sin(\omega t - \frac{\pi}{6})$$

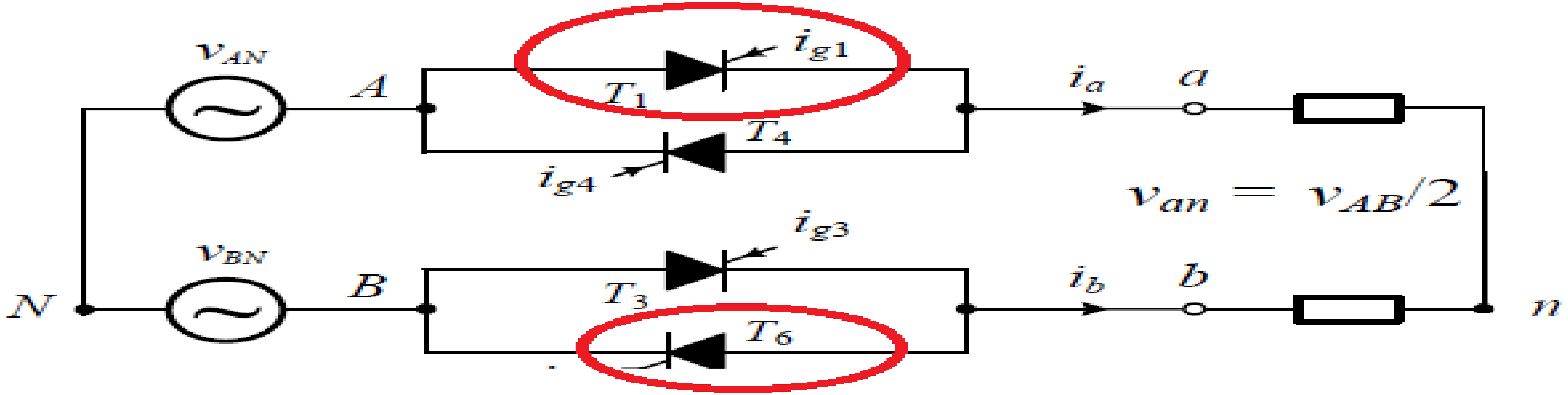


Waveforms for supply voltages ($V_{AN}$,$V_{BN}$,$V_{CN}$), thyristor gating currents($i_{g1}$ *to* $i_{g6}$), & phase-*a* load voltage $v_{an}$



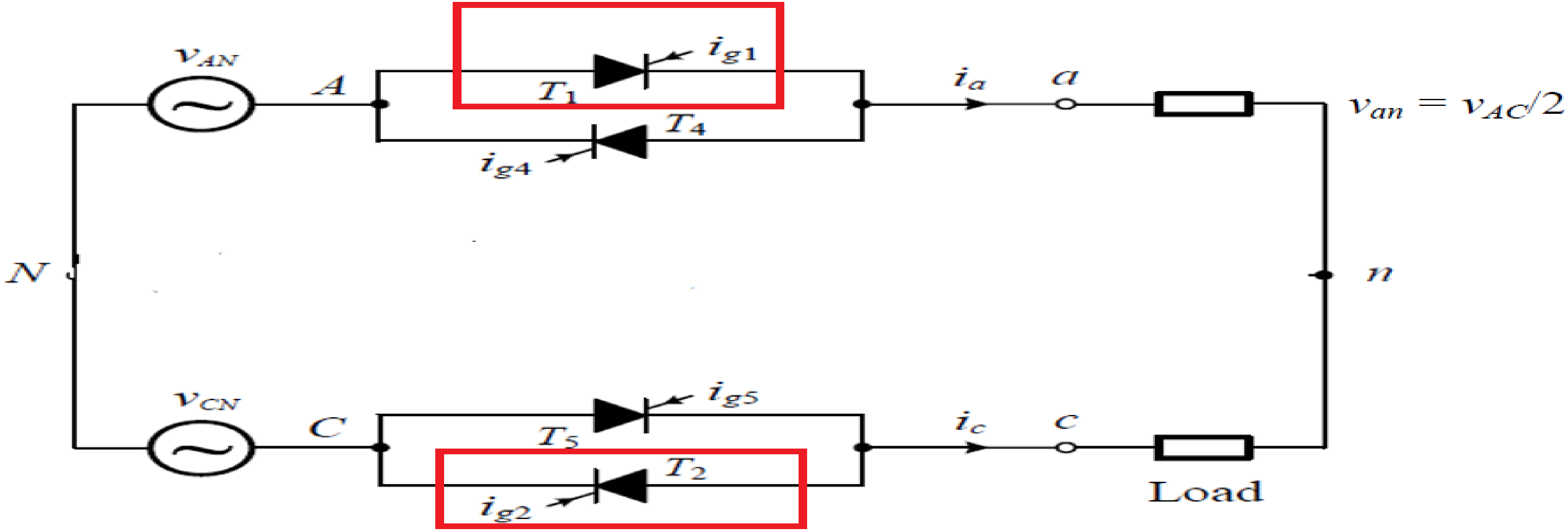
# During period I: $v_{an} = v_{AB}/2$

- Thyristors $T_6$ & $T_1$ are turned ON.
- Line-to-line supply voltage $v_{AB}$ is applied to phase-*a* & *b* load resistors.
- Since T5 &T2 in phase-c are both off, phase-a load voltage $v_{an} = v_{AB}/2$



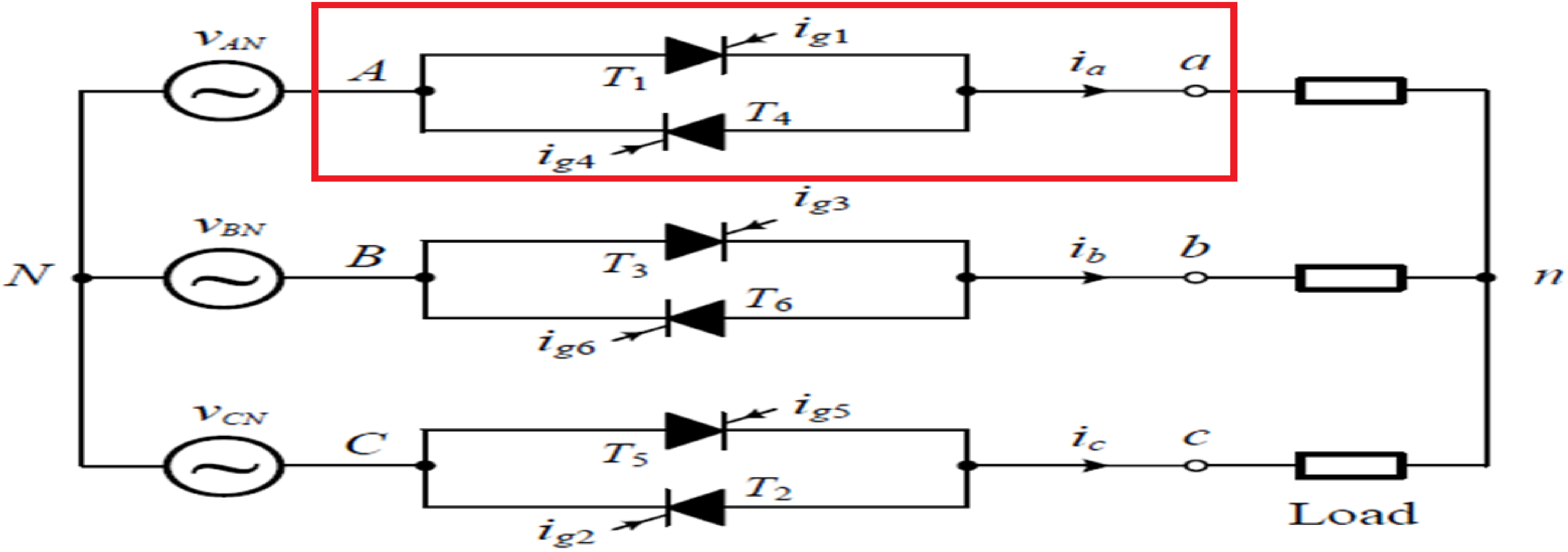
# Period II: $v_{an} = v_{AC}/2$

- Thyristors $T1$ & $T2$ conduct, leading to $v_{an} = v_{AC}/2$

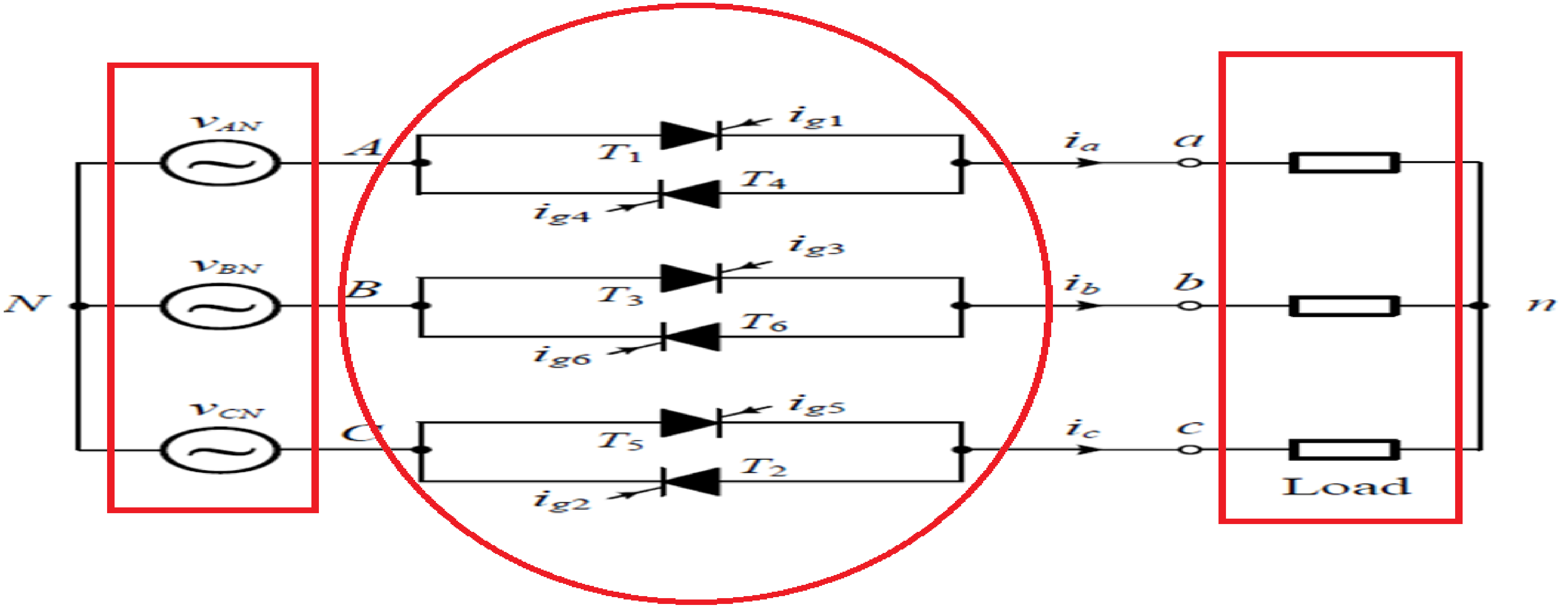


# Period-III(Thyristors $T2$ & $T3$ are turned on)

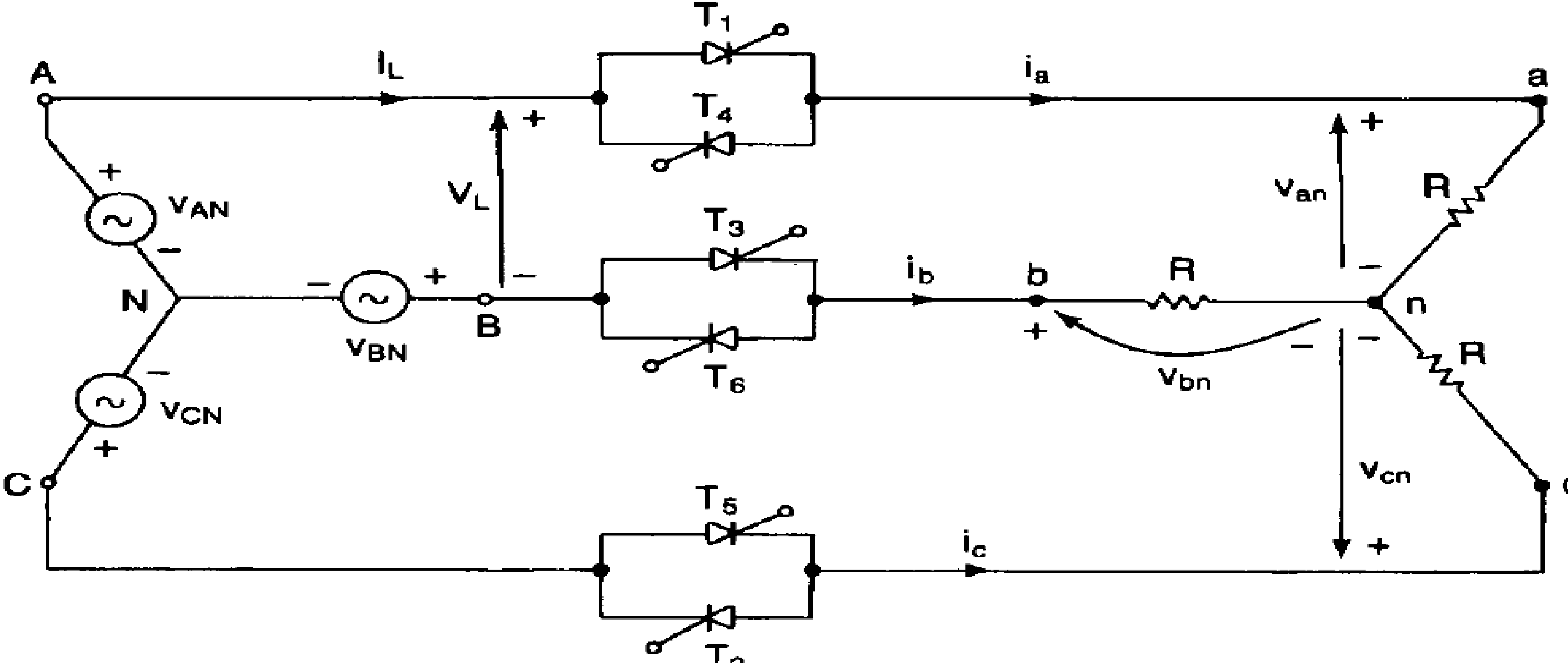
- None of the phase-*a* thyristors(T1 & T4) is ON, *so* $van = 0$.



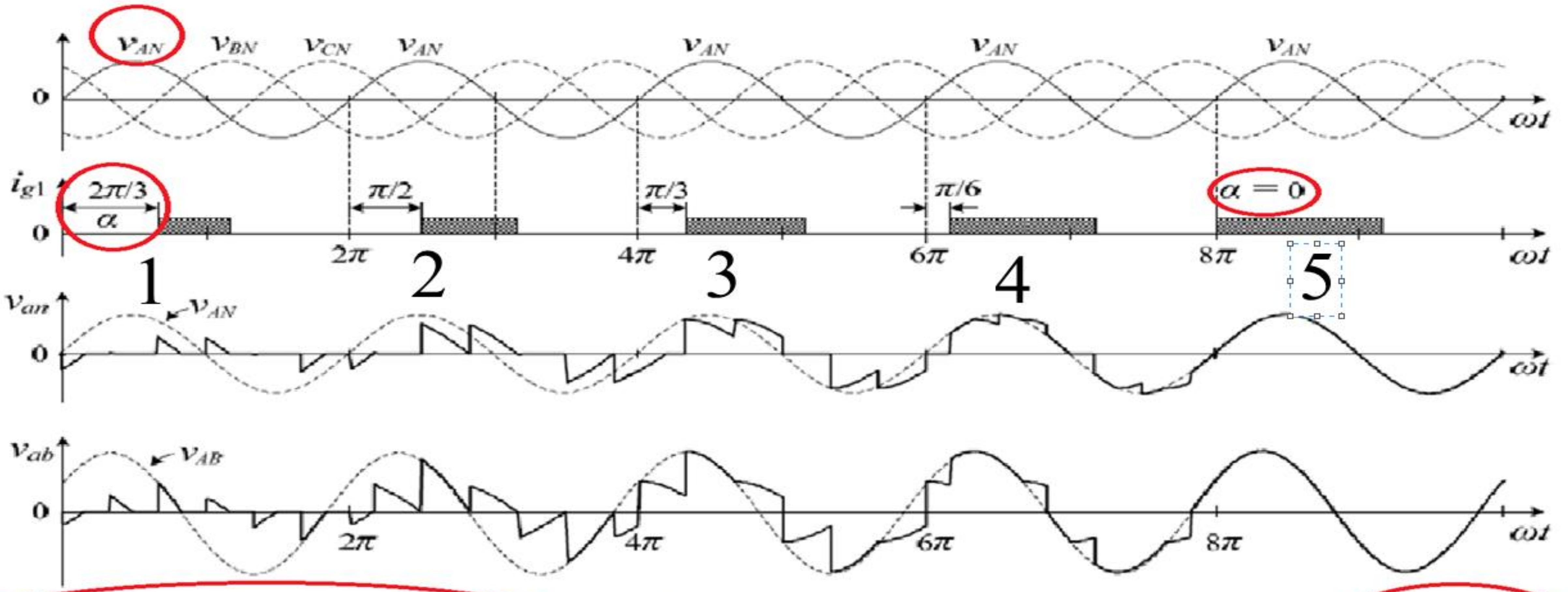
# The operating principle of the controller



Waveforms for load line-to-line voltages (*vab*, *vbc*, *vca*): *vab* = (*van* – *vbn*) , *vbc* = (*vbn* – *vcn*) , and *vca* = (*vcn* – *van*)



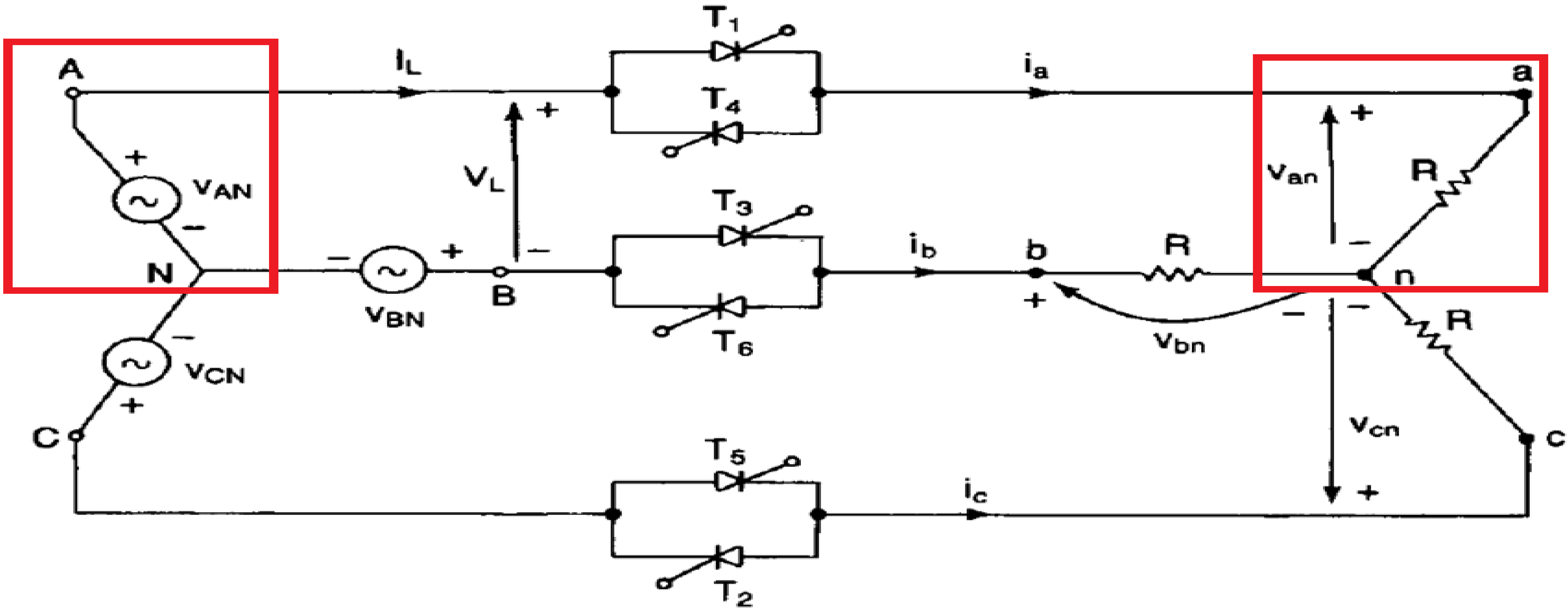
Waveforms for *van* & *vab* with delay angle α changes from $2\pi/3$=120º to 0 in steps. [(π/2-2 π /3)= (π /2- π /3)= (π /3- π /6)= (π /6-0)= π /6=30º]



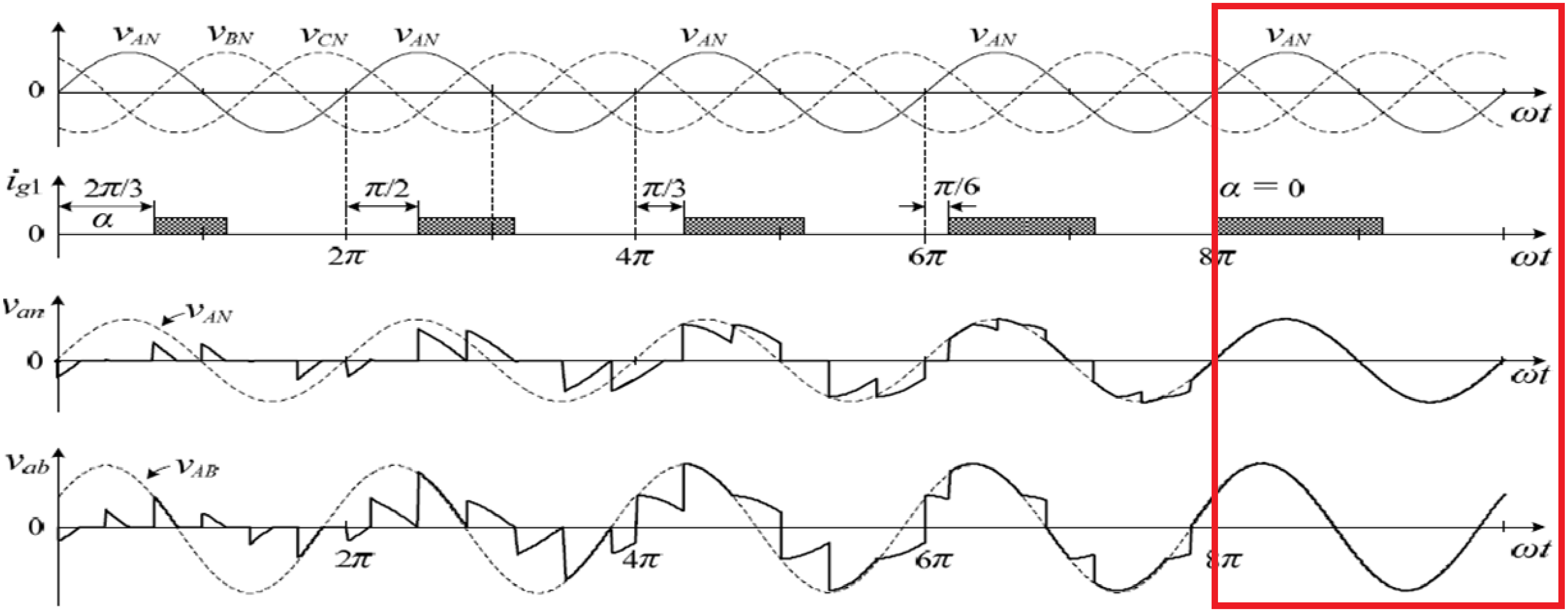

Delay angle: $\alpha = 2\pi/3$ (1st cycle), $\pi/2$ (2nd cycle), $\pi/3$ (3rd cycle), $\pi/6$ (4th cycle) and 0 (5th cycle)

At what value of $\alpha$ : Supply phase voltage =Load phase voltage

i.e $van = v_{AN}$ & $v_{ab} = v_{AB}$ ($\alpha$=0º,30º,60º,90º,120º=?)



By decreasing delay angle α, load phase voltage *van* & line-to-line voltage $v_{ab}$ increase accordingly. So at $\alpha = 0$ Supply phase voltage =Load phase voltage i.e $v_{an}=v_{AN}$ and $v_{ab}= v_{AB}$




Delay angle: $\alpha = 2\pi/3$ (1st cycle), $\pi/2$ (2nd cycle), $\pi/3$ (3rd cycle), $\pi/6$ (4th cycle) and 0 (5th cycle)

Depending on delay angle $\alpha$, operation of 3-phase AC voltage controller can be classified into 3 operating modes.

1. Mode I for $\boldsymbol{\pi/2 \leq \alpha < 5\pi/6}$, during which there are periods when none or 2 thyristors in each phase conduct;
2. Mode II for $\boldsymbol{\pi/3 \leq \alpha < \pi/2}$, during which 2 thyristors in each phase are turned on &
3. Mode III for $\boldsymbol{0 \leq \alpha < \pi/3}$, during which 3 thyristors or 2 thyristors conduct simultaneously .

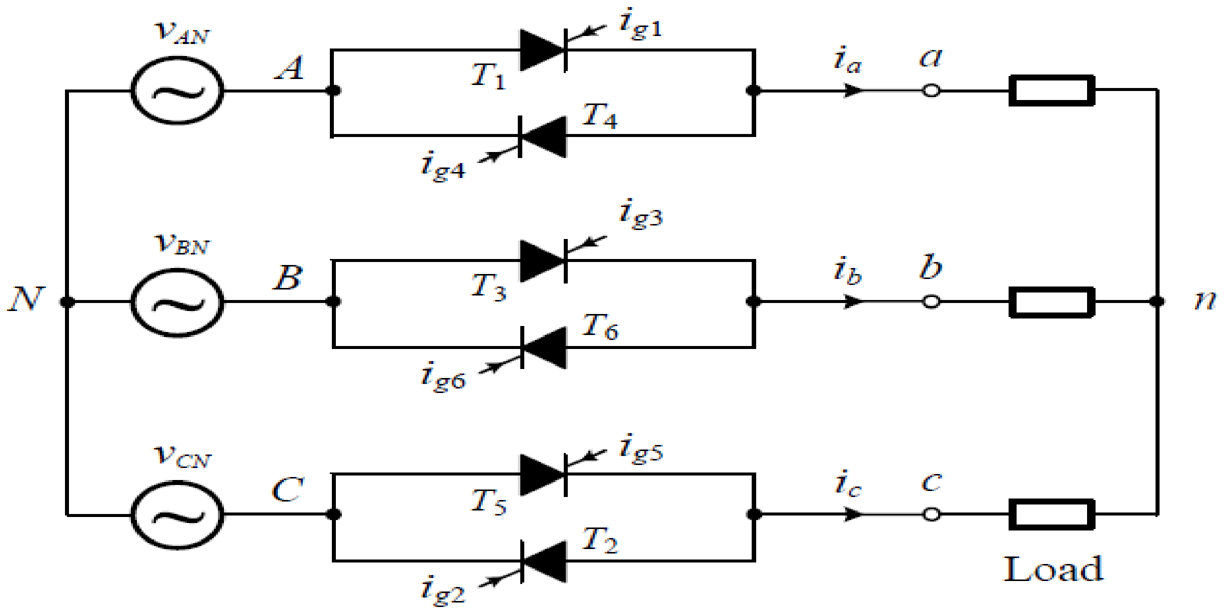
# No need to extend delay angle($\alpha$) beyond $5\pi/6$ ($150^{o}$)?

- Delay angle **$\boldsymbol{\alpha}$ is in range of 0 to $\boldsymbol{\pi}$**, in 1-phase AC voltage controller.

- Delay angle $\alpha$ is in range of 0 to $5\pi/6$ ($150^{o}$) in 3-phase AC voltage controller

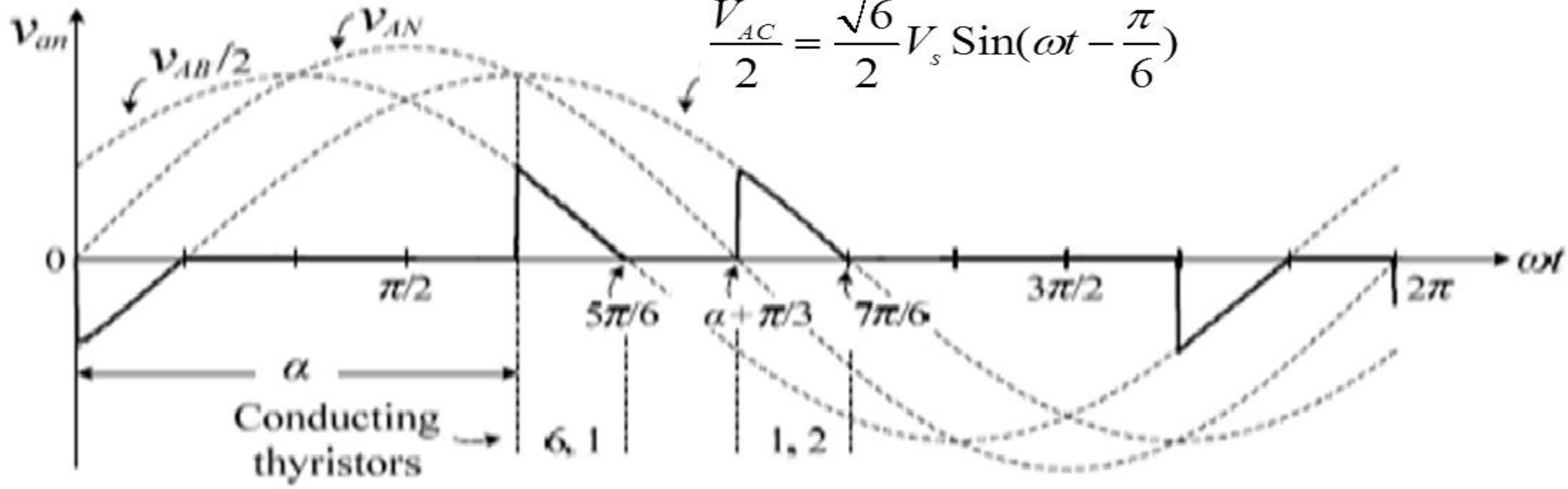
- Beyond which ($5\pi/6 < \alpha \leq \pi$) output voltage of controller is kept 0.

# Mode I for $\pi/2 \le \alpha < 5\pi/6$



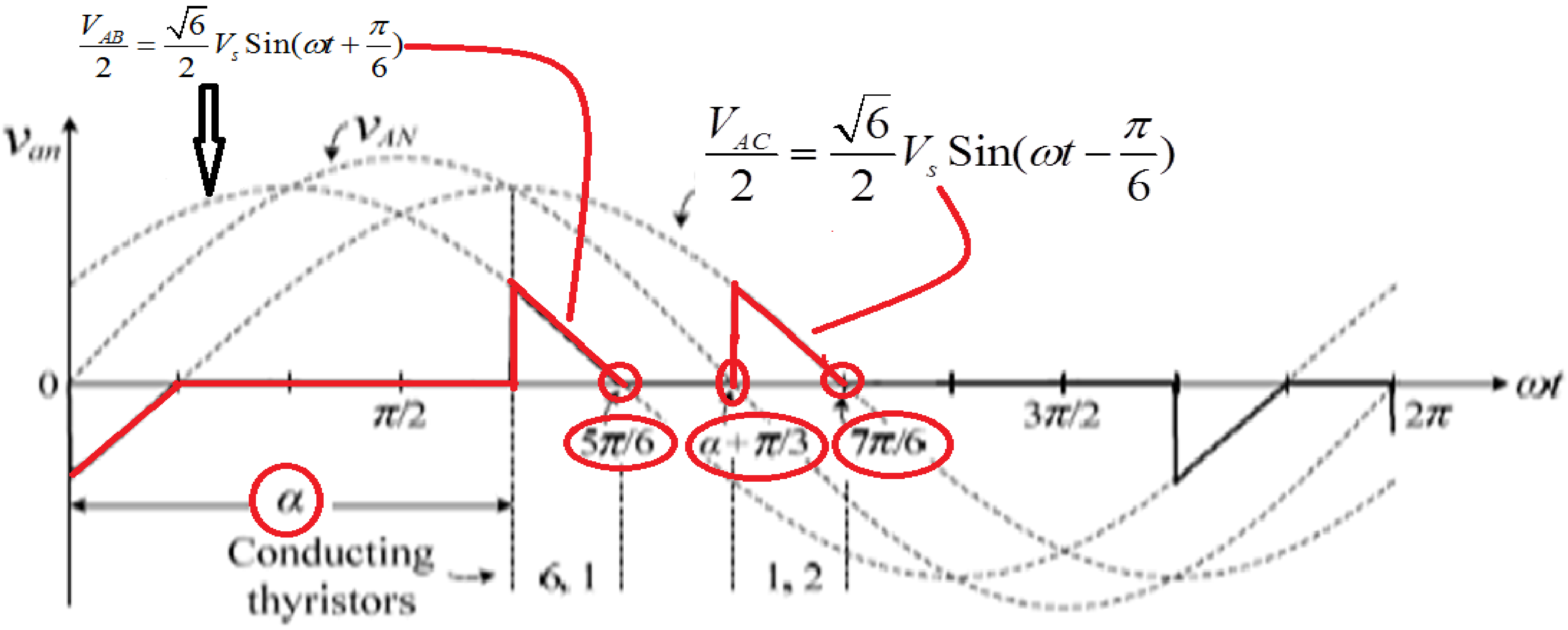

$$\frac{V_{AB}}{2} = \frac{\sqrt{6}}{2} V_s \,\mathrm{Sin}(\omega t + \frac{\pi}{6})$$

$$\frac{V_{AC}}{2} = \frac{\sqrt{6}}{2} V_s \,\mathrm{Sin}(\omega t - \frac{\pi}{6})$$

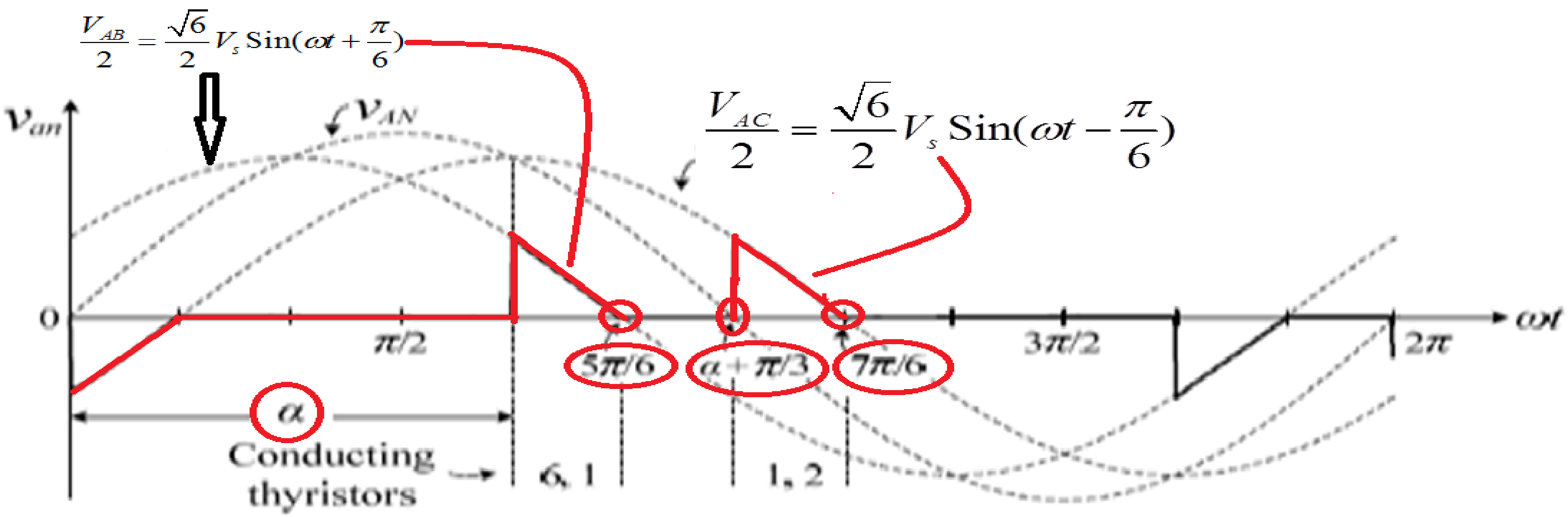



(a) $\alpha = 2\pi/3$ (120°, Mode I)

# Find *Van*=?




(a) $\alpha = 2\pi/3$ (120°, Mode I)



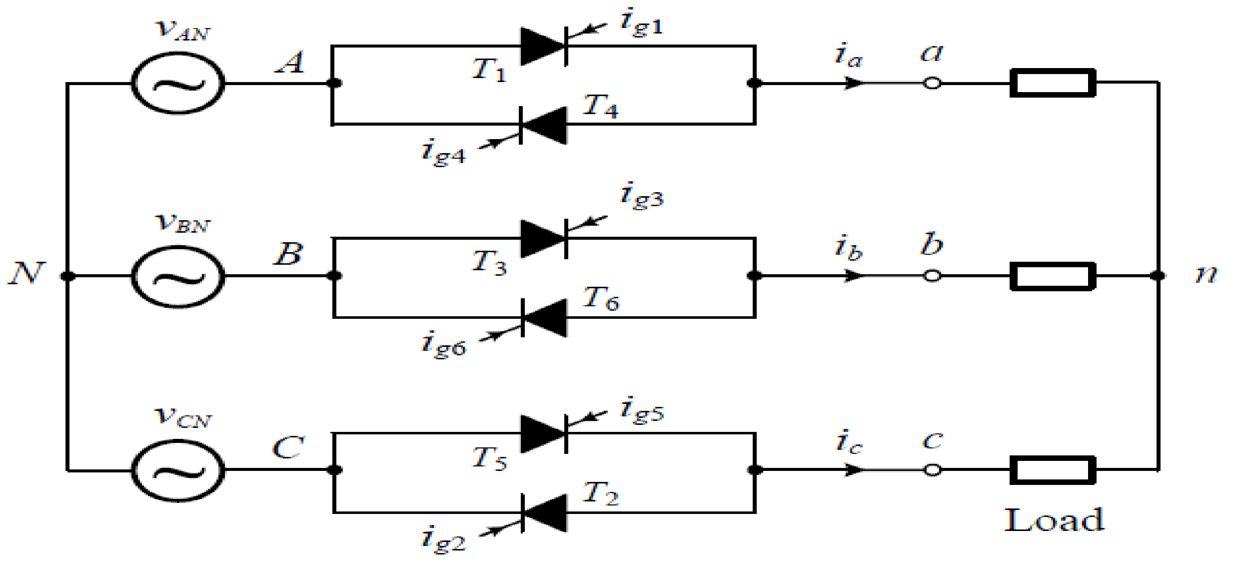

(a) $\alpha = 2\pi/3$ (120°, Mode I)

$$V_{an} = \left( \frac{1}{\pi} \left( \int_{\alpha}^{5\pi/6} \left( \frac{\sqrt{6}}{2} V_s \sin(\omega t + \pi/6) \right)^2 d(\omega t) + \int_{\alpha+\pi/3}^{7\pi/6} \left( \frac{\sqrt{6}}{2} V_s \sin(\omega t - \pi/6) \right)^2 d(\omega t) \right) \right)^{1/2}$$

$$= V_s \left( \frac{5}{4} - \frac{3\alpha}{2\pi} + \frac{3\sin\left(2\alpha + \pi/3\right)}{4\pi} \right)^{1/2} \qquad \text{for Mode I } (\pi/2 \le \alpha < 5\pi/6)$$

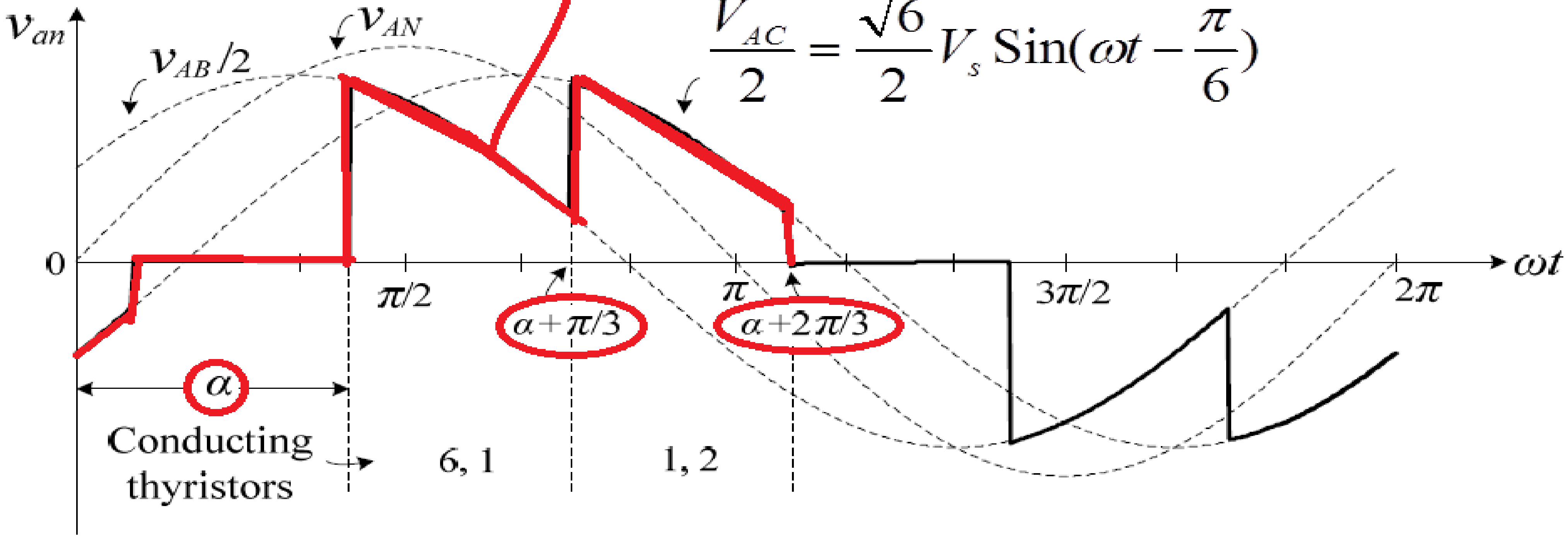
# Mode II for $\pi/3 \leq \alpha < \pi/2$

# Mode II for $\pi/3 \le \alpha < \pi/2$

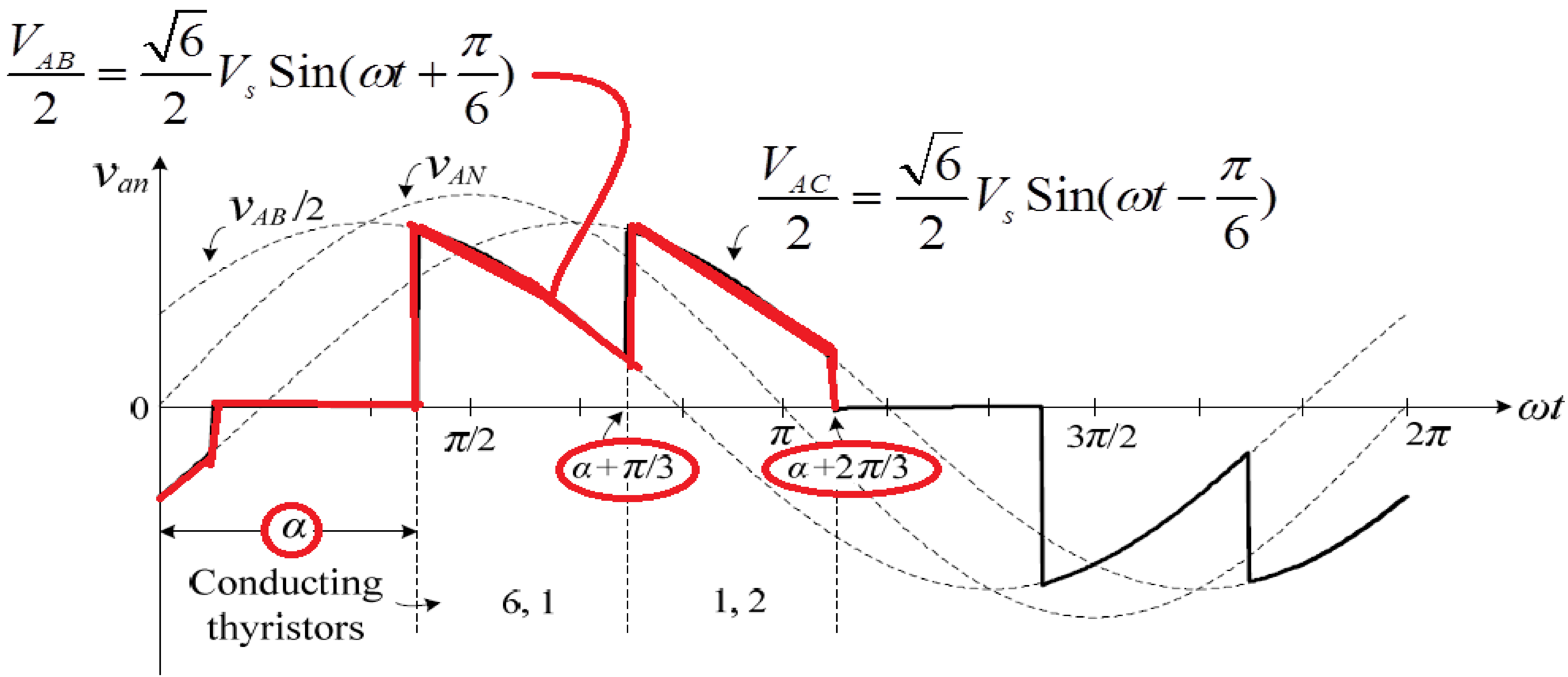



$$\frac{V_{AB}}{2} = \frac{\sqrt{6}}{2} V_s \operatorname{Sin}(\omega t + \frac{\pi}{6})$$

$$\frac{V_{AC}}{2} = \frac{\sqrt{6}}{2} V_s \operatorname{Sin}(\omega t - \frac{\pi}{6})$$



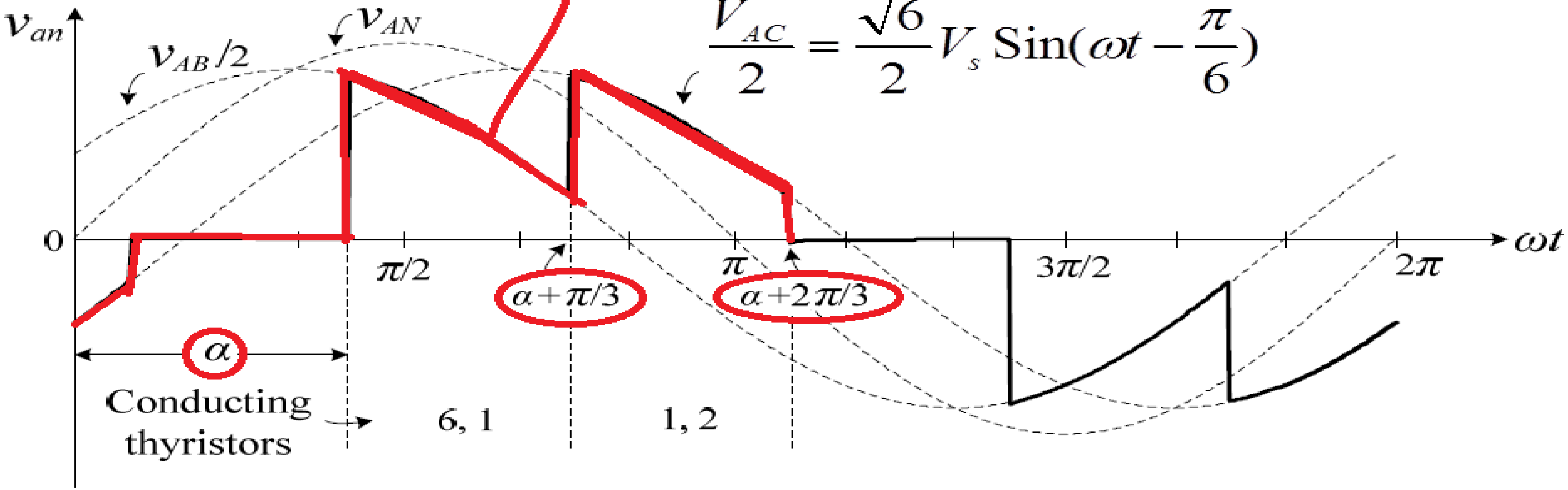
# Find *Van*=?



$\frac{V_{AB}}{2} = \frac{\sqrt{6}}{2} V_s \operatorname{Sin}(\omega t + \frac{\pi}{6})$
$v_{an}$
$v_{AN}$
$v_{AB}/2$
$\frac{V_{AC}}{2} = \frac{\sqrt{6}}{2} V_s \operatorname{Sin}(\omega t - \frac{\pi}{6})$
0
$\pi/2$
$\alpha+\pi/3$
$\pi$
$\alpha+2\pi/3$
$3\pi/2$
$2\pi$
$\omega t$
$\alpha$
Conducting thyristors
6, 1
1, 2

$$\frac{V_{AB}}{2} = \frac{\sqrt{6}}{2} V_s \operatorname{Sin}(\omega t + \frac{\pi}{6})$$

$$\frac{V_{AC}}{2} = \frac{\sqrt{6}}{2} V_s \operatorname{Sin}(\omega t - \frac{\pi}{6})$$



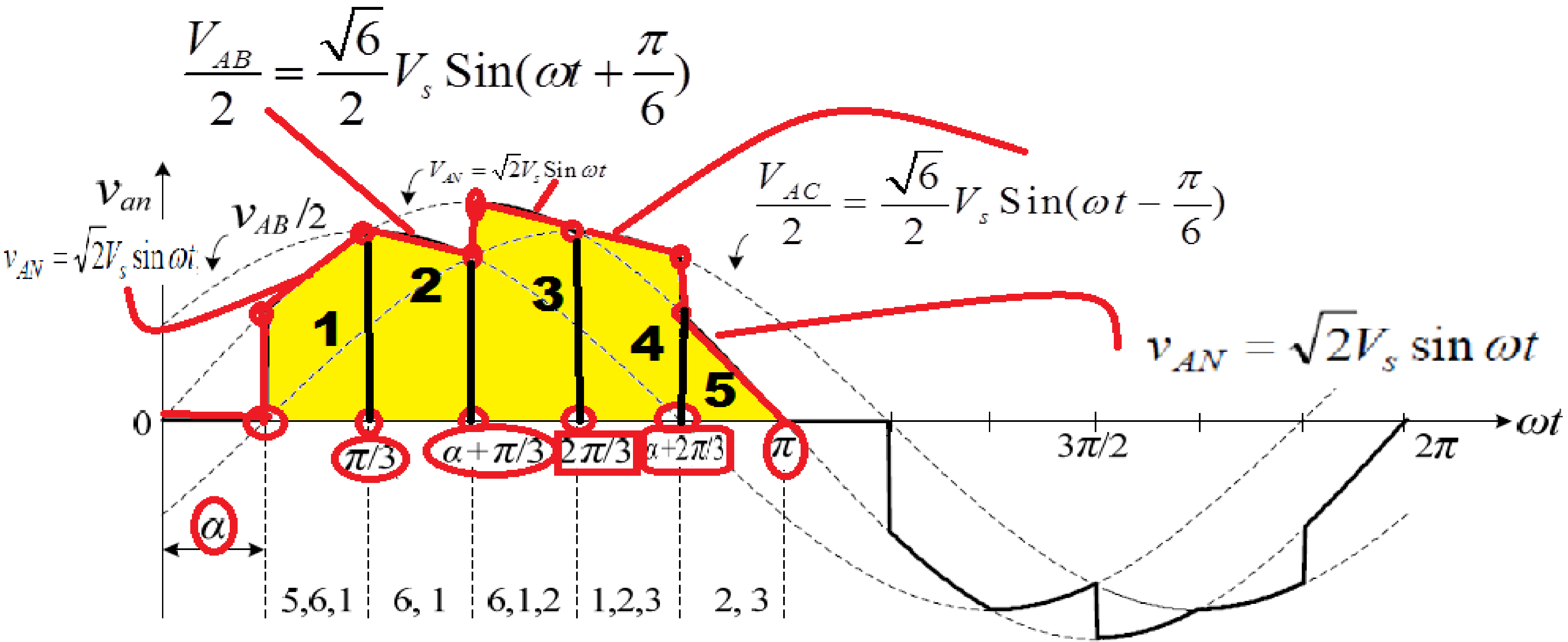

$$V_{an} = \left( \frac{1}{\pi} \left( \int_{\alpha}^{\alpha+\pi/3} \left( \frac{\sqrt{6}}{2} V_s \sin(\omega t + \pi/6) \right)^2 d(\omega t) + \int_{\alpha+\pi/3}^{\alpha+2\pi/3} \left( \frac{\sqrt{6}}{2} V_s \sin(\omega t - \pi/6) \right)^2 d(\omega t) \right) \right)^{1/2}$$

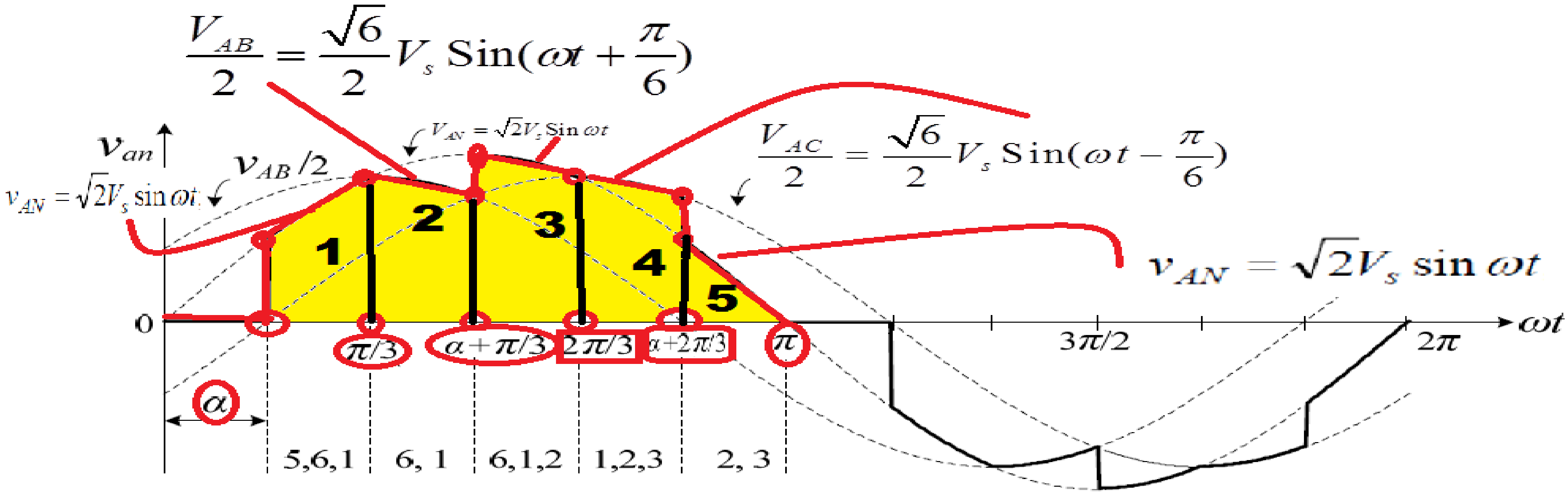
$$= V_s \left( \frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \sin(2\alpha + \pi/6) \right)^{1/2}$$

for Mode II $(\pi/3 \le \alpha < \pi/2)$

# Mode III for $0 \leq \alpha < \pi/3$

# Find *van* for in Mode III, where delay angle $\pi/6$=(30º)?






$$V_{an} = \left( \frac{1}{\pi} \left( \overset{1}{\int_{\alpha}^{\pi/3} (\sqrt{2} V_s \sin \omega t)^2 d(\omega t)} + \overset{2}{\int_{\pi/3}^{\alpha+\pi/3} \left( \frac{\sqrt{6}}{2} V_s \sin(\omega t + \pi/6) \right)^2 d(\omega t)} + \overset{3}{\int_{\alpha+\pi/3}^{2\pi/3} (\sqrt{2} V_s \sin \omega t)^2 d(\omega t)} + \right.\right.$$

$$\left.\left. \overset{4}{\int_{2\pi/3}^{\alpha+2\pi/3} \left( \frac{\sqrt{6}}{2} V_s \sin(\omega t - \pi/6) \right)^2 d(\omega t)} + \overset{5}{\int_{\alpha+2\pi/3}^{\pi} (\sqrt{2} V_s \sin \omega t)^2 d(\omega t)} \right) \right)^{1/2}$$

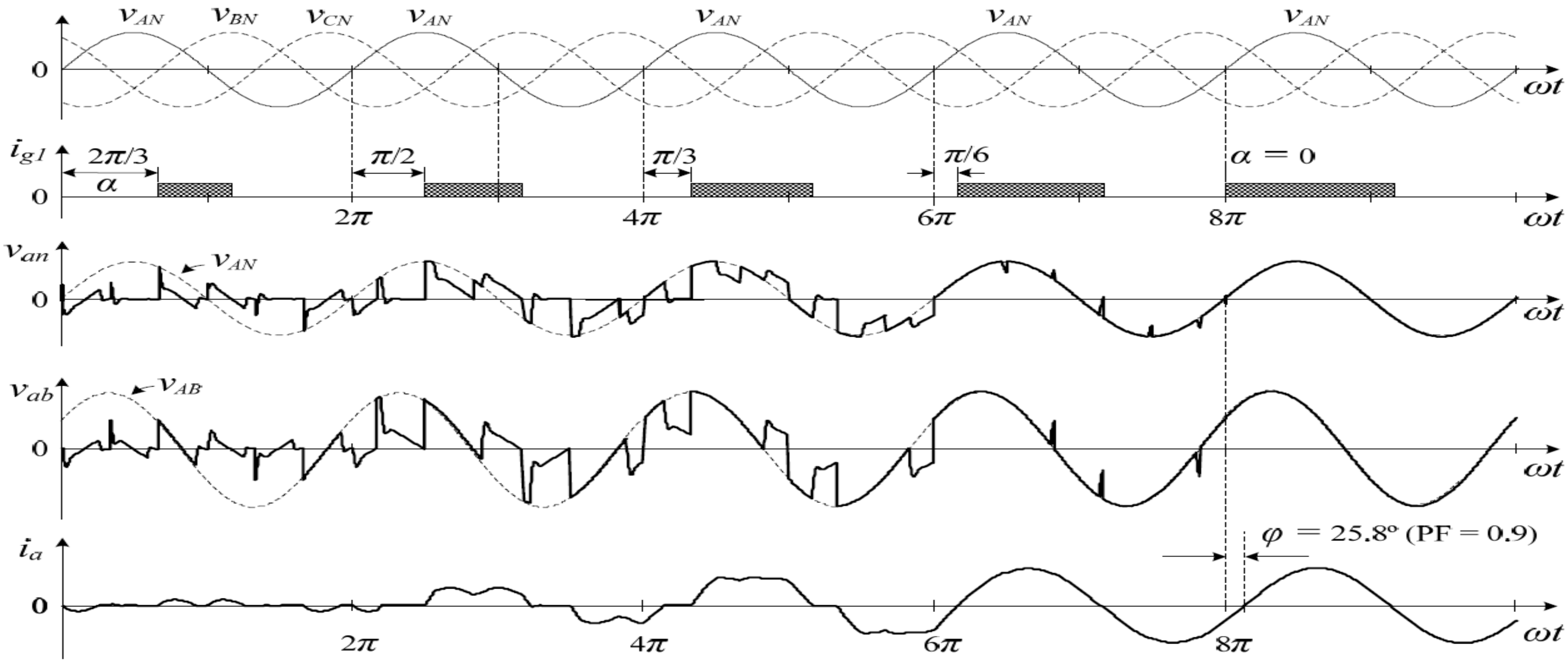
# *van* for  in Mode III

$$V_{an} = V_s\left(1 - \frac{3\alpha}{2\pi} + \frac{3\sin 2\alpha}{4\pi}\right)^{1/2} \qquad \text{for Mode III } (0 \le \alpha < \pi/3)$$

# 03-phase AC voltage controller with inductive load

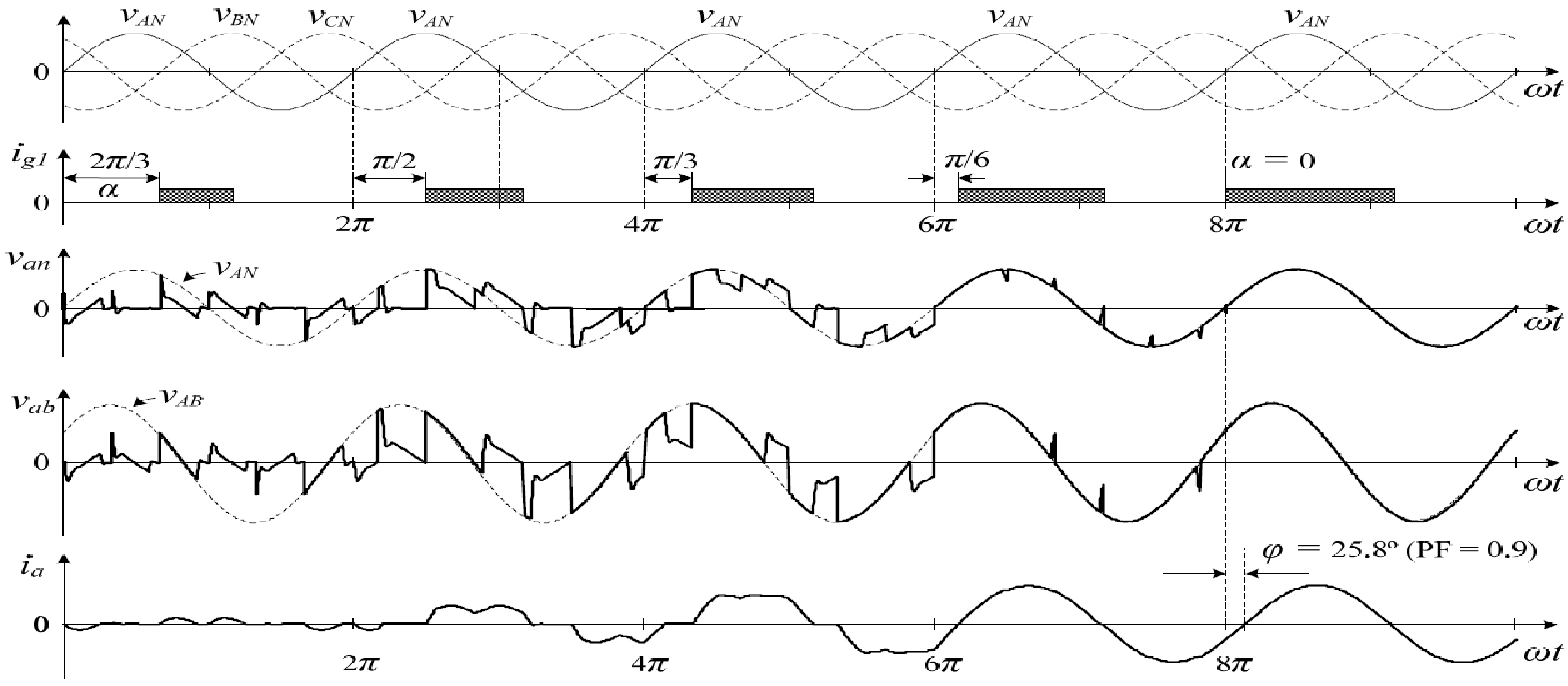
- Analysis of 3-phase AC voltage controller with inductive load is quite complex since thyristors do not cease conducting when supply voltage falls down to 0 & becomes -ve, the same phenomenon as discussed in single-phase AC voltage controller.
- Computer simulation provides an effective means of obtaining load voltage & current waveforms.

Fig. shows simulated waveforms for phase-*a* load voltage *van* , line-to-line voltage *vab* & load current *ia* when voltage controller operates with a 3-phase Y-connected RL load having a power factor of 0.9 at different delay angles.




$\alpha = 2\pi/3$ (1st cycle), $\pi/2$ (2nd cycle), $\pi/3$ (3rd cycle), $\pi/6$ (4th cycle), and 0 (5th cycle)

Waveform for phase-*a* load current *ia* is much smoother than its phase voltage *van* due to filtering effect of load inductance. Load power factor angle $\varphi$=25.8º as indicated in figure.

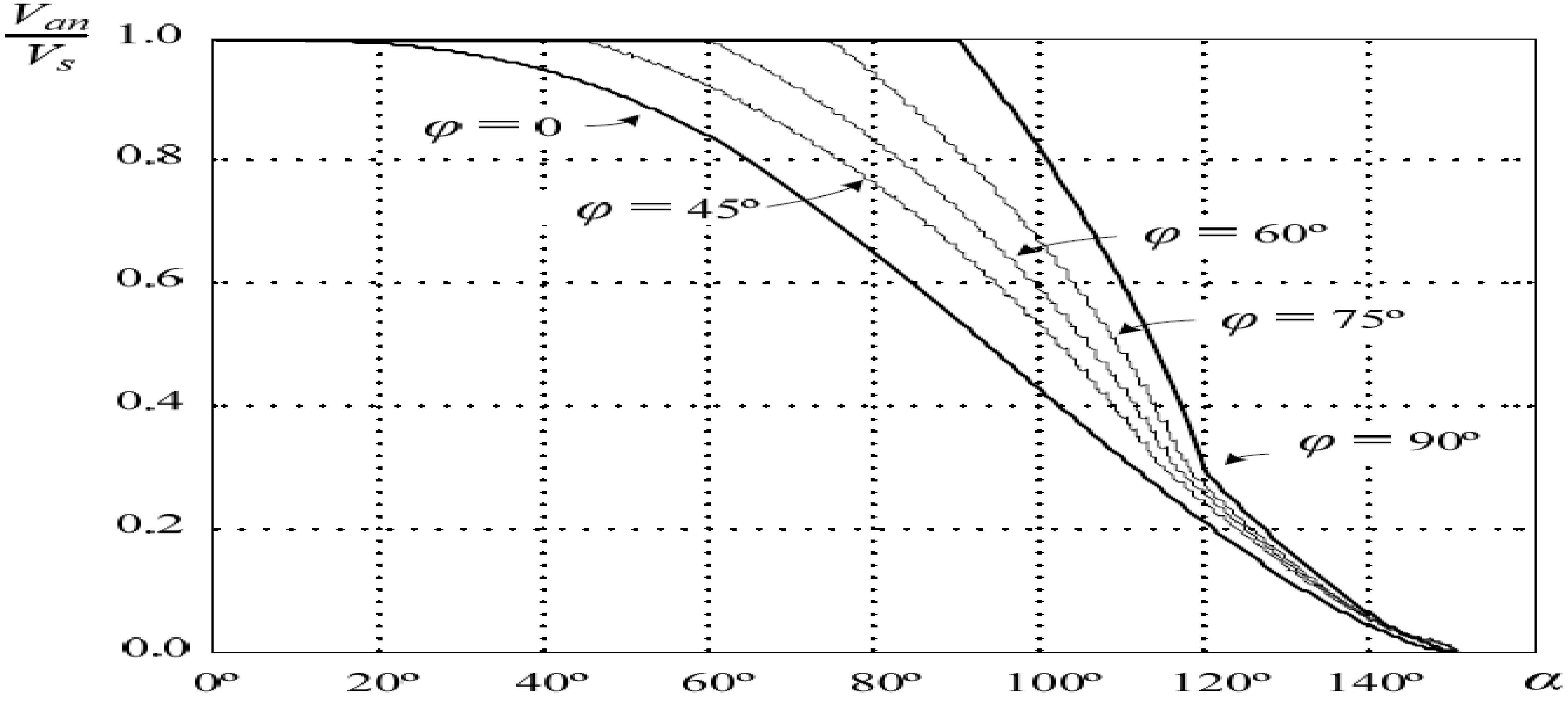



$\alpha = 2\pi/3$ (1st cycle), $\pi/2$ (2nd cycle), $\pi/3$ (3rd cycle), $\pi/6$ (4th cycle), and 0 (5th cycle)

With pure inductive load ($\varphi = \pi/2$) rms value of load voltage *van* of 3-phase AC voltage controller is given by

$$V_{an} = \begin{cases} V_s & \text{for } 0 \le \alpha < \pi/2 \\ V_s\left(\dfrac{5}{2} - \dfrac{3\alpha}{\pi} + \dfrac{3\sin(2\alpha)}{2\pi}\right)^{1/2} & \text{for } \pi/2 \le \alpha < 2\pi/3 \\ V_s\left(\dfrac{5}{2} - \dfrac{3\alpha}{\pi} + \dfrac{3\sin(2\alpha + \pi/3)}{2\pi}\right)^{1/2} & \text{for } 2\pi/3 \le \alpha < 5\pi/6 \end{cases}$$

# Load voltage to supply voltage ratio *Van*/*Vs* versus delay angle $\alpha$ for three-phase AC voltage controller



# It is noted that when delay angle $\alpha$ <load power factor angle $\varphi$

- Load voltage *Van* is equal to supply voltage *Vs i.e Van= Vs*
- So it is no longer adjustable
- Same phenomenon as discussed in 1-phase AC voltage controller.

# VIDEO 3-PHASE FULL WAVE AC VOLTAGE CONTROLLER (BIDIRECTIONAL CONTROLLER) WITH RL LOAD

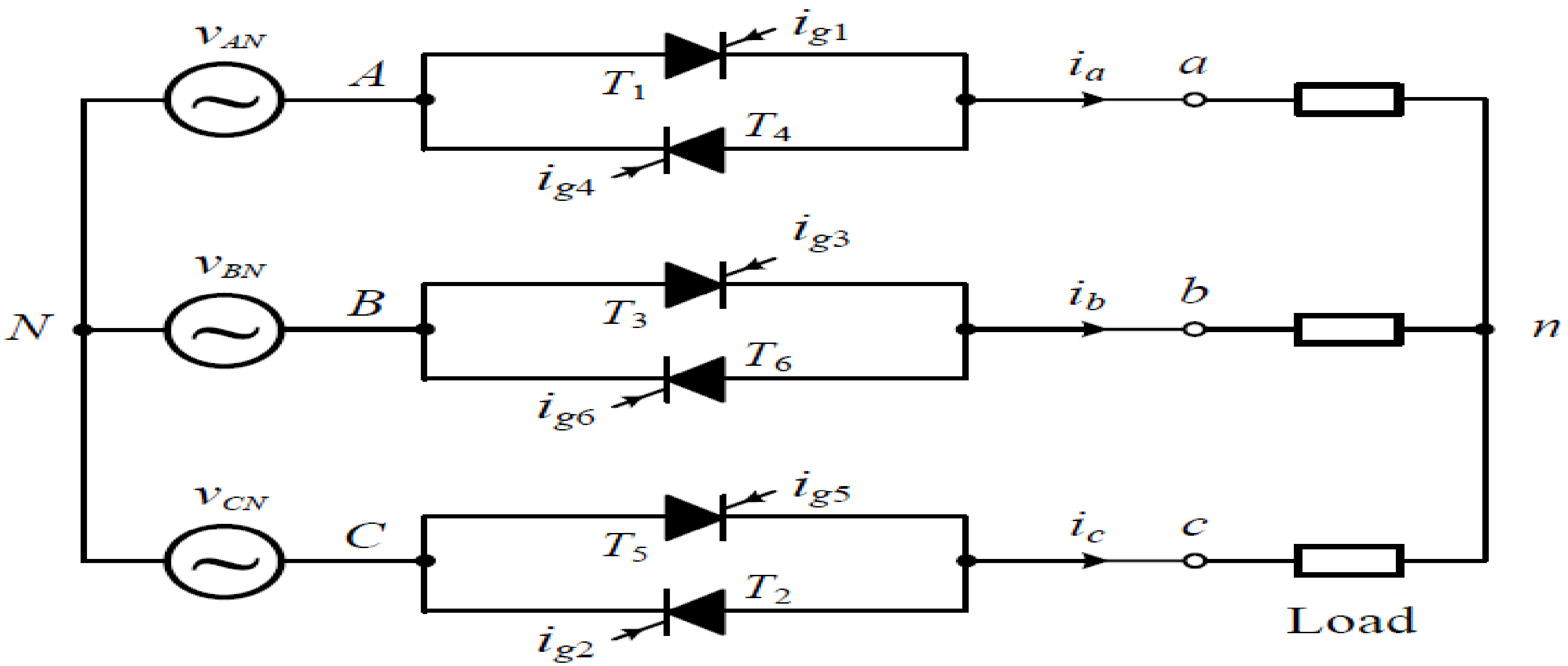
THREE PHASE FULL CONVERTER WITH RL LOAD

LEARN AND GROW

LEARN AND GROW

# Numericals

**4-3 (Solved Problem)** A 3-phase 690V/2.3MVA AC voltage controller is loaded with a star-connected R load of 1.0 pu per phase. The controller is supplied by a 3-phase utility voltage of 690V& 50Hz & operates at a firing angle of 120°



Assuming that power converter is ideal, calculate/answer following:

a) load resistance value,

b) rms load phase voltage and line current

c) 3-phase apparent, active and reactive powers of the load,

d) rms input line current,

e) 3-phase input apparent, active, reactive powers and power factor. State the reason why the input power factor is not unity even though the controller is loaded with a pure resistive load, and

f) using template given in Fig. P4-3, draw waveform for phase-*a* load voltage *van*.

# Solution:

a) Load resistance value:

$$S_B = \frac{2300\times10^3}{3} = 766.67\times10^3 \;\text{ VA (1.0 pu)},$$

$$V_s = V_B = \frac{690}{\sqrt{3}} = 398.38 \text{ V (1.0 pu)}$$

$$I_B = \frac{S_B}{V_B} = \frac{766.67\times10^3}{398.38} = 1924.5 \text{ A (1.0 pu)},$$

$$Z_B = \frac{V_B}{I_B} = \frac{398.38}{1924.5} = 0.207\,\Omega \;\text{ (1.0 pu)}$$

$$R = Z_B \times R_{pu} = 0.207\times1.0 = 0.207\,\Omega$$

# b) The load phase voltage:

$$V_{an} = V_s\left(\frac{5}{4} - \frac{3\alpha}{2\pi} + \frac{3\sin\left(2\alpha + \pi/3\right)}{4\pi}\right)^{1/2}$$

$$= 398.37\times\left(\frac{5}{4} - \frac{3\times(2\pi/3)}{2\pi} + \frac{3\sin\left(2\times(2\pi/3) + \pi/3\right)}{4\pi}\right)^{1/2} = 82.85\ \text{V (rms)}$$

($\alpha$ is in radians)

The load line current: $I_a = \dfrac{V_{an}}{R} = \dfrac{82.85}{0.207} = 400.24\ \text{A (rms)}$

c) 3-phase apparent power of load:

$$S_o = 3 \times V_{an} \times I_a = 3 \times 82.85 \times 400.24 = 99.48 \times 10^3 \text{ VA}$$

3-phase active power consumed by load:

$$P_o = 3 \times I_a^2 \times R = 3 \times 400.24^2 \times 0.207 = 99.48 \times 10^3 \text{ W}$$

3-phase reactive power consumed by load:

$$Q_o = \sqrt{S_o^2 - P_o^2} = \sqrt{99478^2 - 99478^2} = 0 \text{ VAR}$$

d) The input line current:

$$I_A = I_a = 400.24 \text{ A (rms)}$$

e) 3-phase input apparent power:

$$S_S = 3 \times V_S \times I_A = 3 \times 398.38 \times 400.24 = 478.33 \times 10^3 \text{ VA}$$

The input active power:

$$P_s = S_s \times \cos\varphi_s = 3 \times V_s \times I_A \times \cos\varphi_s = 3 \times 398.38 \times 400.24 \times 0.208 = 99.48 \times 10^3 \text{ W}$$

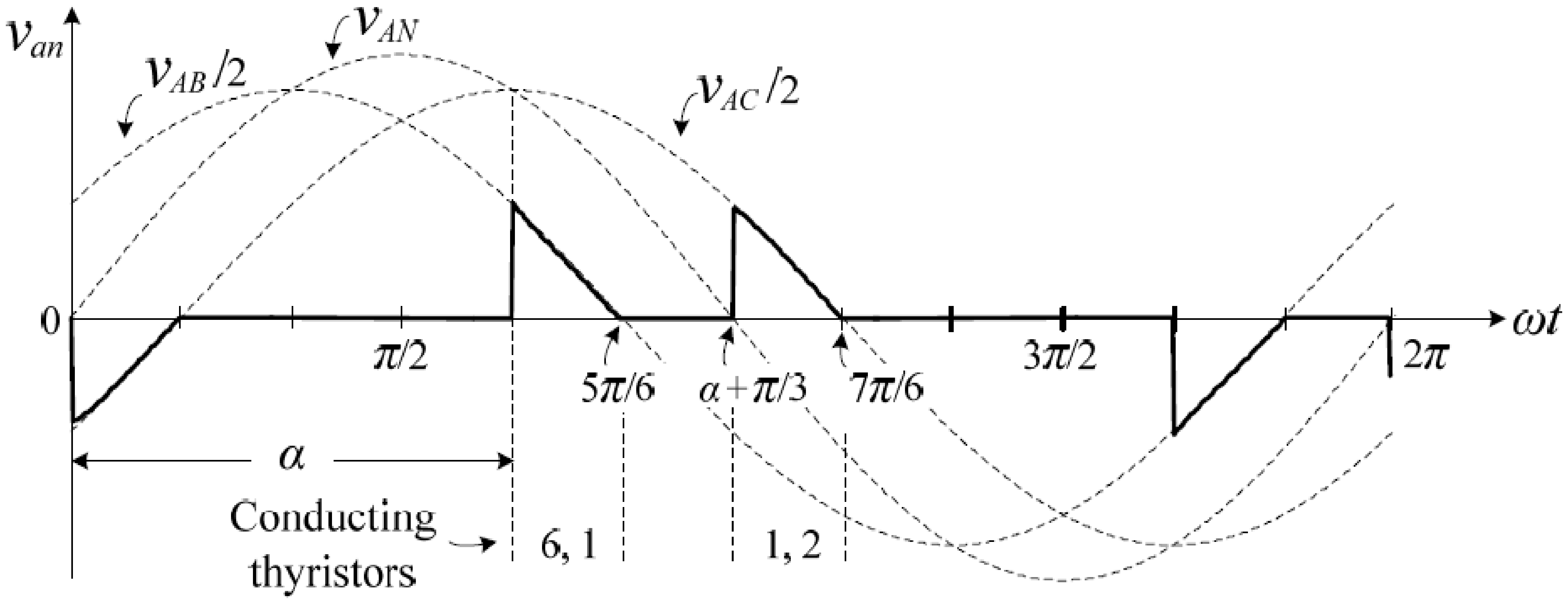
The input reactive power:

$$Q_s = \sqrt{S_s^2 - P_s^2} = \sqrt{478330^2 - 99478^2} = 467.87 \times 10^3 \text{ VAR}$$

# The input power factor:

$$PF_s = \frac{P_s}{S_s} = \frac{99.48 \times 10^3}{478.33 \times 10^3} = 0.208$$

# f) Waveforms




(a) $\alpha = 2\pi/3$ (120°, Mode I)

**4-4** Repeat the Problem 4-3 with a firing angle of 75º. Compare the input power factor and explain why the input power factor is improved?

**Answers:**

a) $R = 0.207\ \Omega$ b) $V_{an} = 281.69\ \text{V}$, $I_a = 1360.8\ \text{A}$

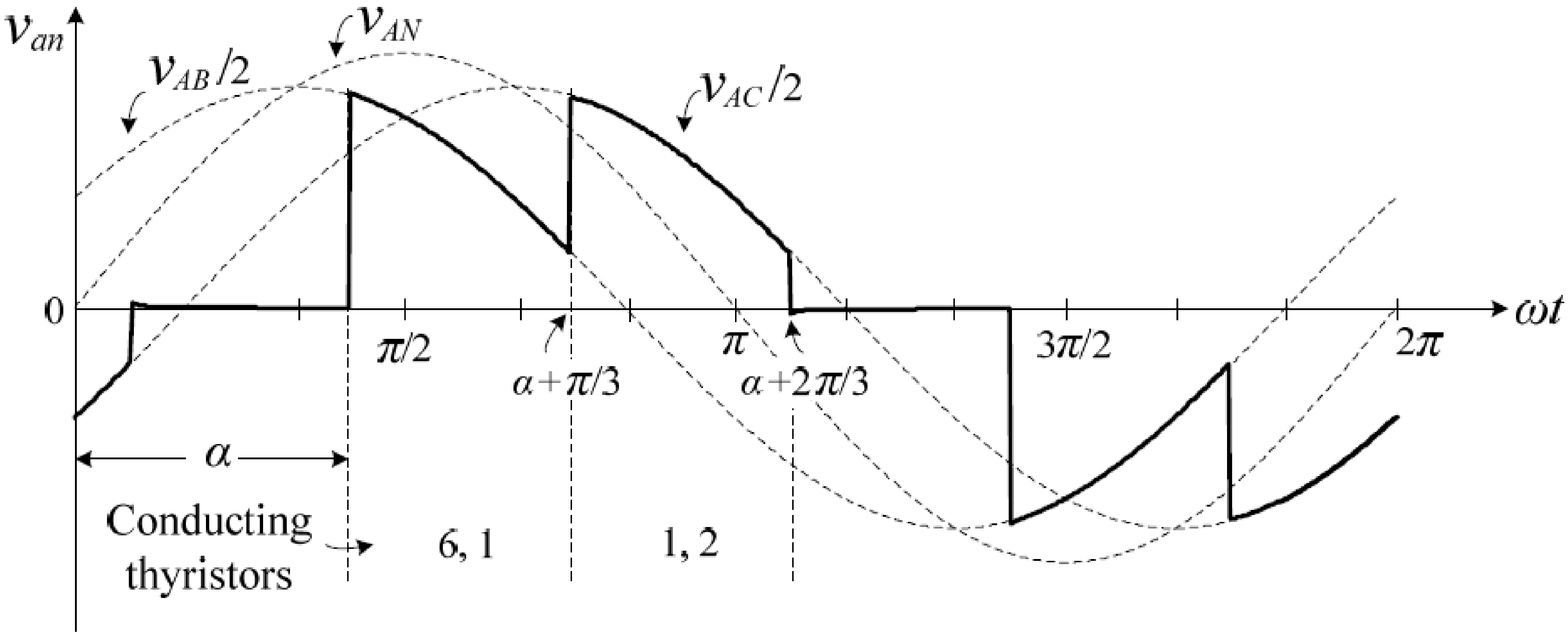
c) $S_o = 1150\times10^3\ \text{VA}$, $P_o = 1150\times10^3\ \text{W}$, $Q_o = 0\ \text{VAR}$

d) $I_A = 1360.8\ \text{A}$

e) $S_s = 1626.3\times10^3\ \text{VA}$, $P_s = 1150\times10^3\ \text{W}$, $Q_s = 1150\times10^3\ \text{VAR}$, $PF_s = 0.7071$

f) Waveforms: See Fig. 4.2-8b.

# f) Waveforms




(b) $\alpha = 5\pi/12$ (75°, Mode II)

**4-5** Repeat the Problem 4-3 with a firing angle of 30° and compare the input power factor and explain why the input power factor is improved?

**Answers:**

a) $R = 0.207\ \Omega$ b) $V_{an} = 389.7\ \text{V}$, $I_a = 1882.4\ \text{A}$

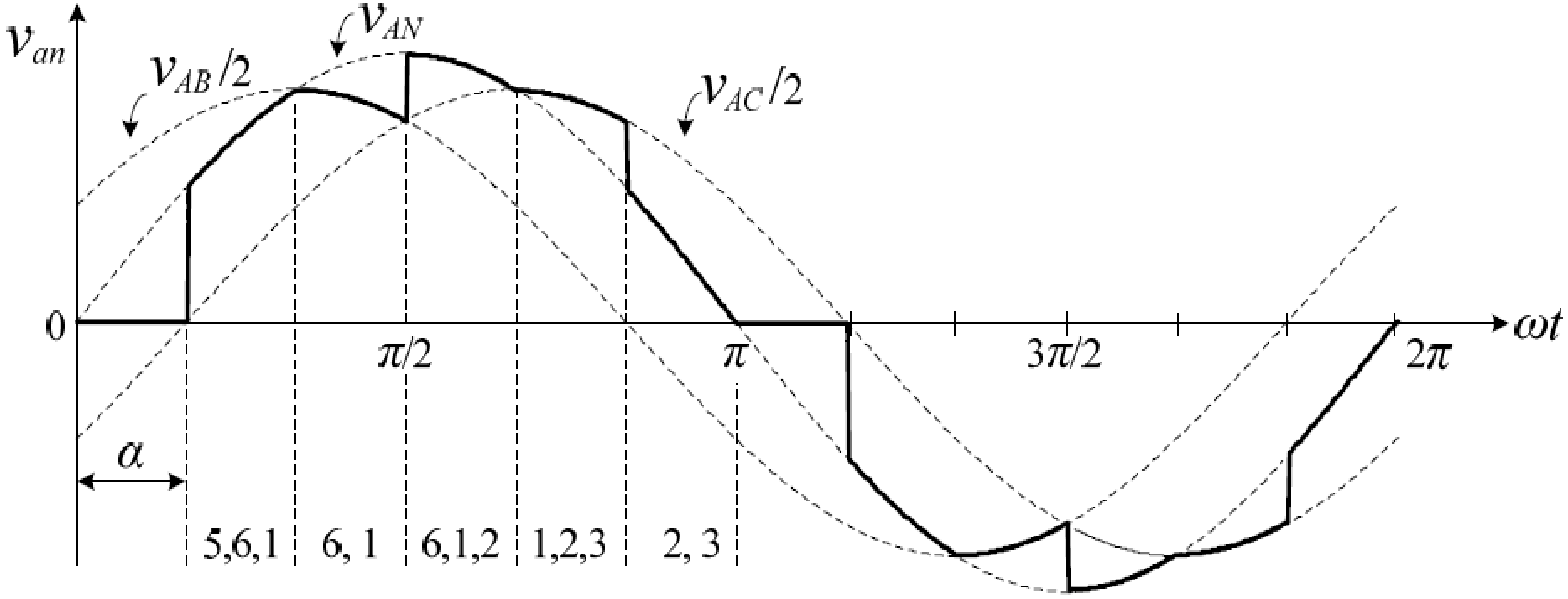
c) $S_o = 2200.5\times10^3\ \text{VA}$, $P_o = 2200.5\times10^3\ \text{W}$, $Q_o = 0\ \text{VAR}$

d) $I_A = 1882.4\ \text{A}$

e) $S_s = 2249.7\times10^3\ \text{VA}$, $P_s = 2200.5\times10^3\ \text{W}$, $Q_s = 467.8\times10^3\ \text{VAR}$, $PF_s = 0.978$

f) Waveforms: See Fig. 4.2-8c.

# f) Waveforms




(c) $\alpha = \pi/6$ (30º, Mode III)