On the Stability of FDTD-Based Numerical Codes to Evaluate Lightning-Induced Overvoltages in Overhead Transmission Lines

Massimo Brignone, *Member, IEEE*, Daniele Mestriner, *Student Member, IEEE*, Renato Procopio, *Senior Member, IEEE*, Alexandre Piantini, *Senior Member, IEEE*, and Farhad Rachidi, *Fellow, IEEE*

***Abstract*—One of the main issues that has to be faced when evaluating lightning-induced overvoltages on a realistic network configuration is to implement the algorithm that solves the field-to-line coupling problem into an electromagnetic simulator that is able to represent with a high level of detail all the power system components. On one hand, numerical stability conditions of the finite-difference time-domain schemes have been deeply investigated in the presence of boundary conditions known analytically; on the other hand, there is not so much literature on the situations in which such conditions are provided in a numerical way through an external electromagnetic simulator. The study reveals that the commonly used linear extrapolation of currents at the line extremities might rise to numerical instabilities. An efficient solution using the method of characteristics is presented and validated.**

***Index Terms*—Finite-difference time domain (FDTD), lightning electromagnetic pulse, lightning-induced effects and protection, numerical methods and modeling.**

I. Introduction

LIGHTNING discharges can provoke dangerous overvoltages in transmission and distribution grids. On one hand, transmission networks are mainly affected by direct strikes to line conductors; on the other hand, distribution systems, characterized by much lower insulation levels, can be seriously damaged even by indirect events, which, by the way, are much more frequent than direct ones. As a consequence, when designing lightning protection systems for distribution networks, it is mandatory to also take into account the induced overvoltages by indirect strikes, which is typically done in a statistical way assuming as random parameters both the lightning channel current and its point of impact.

This becomes a very complex task from a numerical point of view, as, for any randomly selected event, the solution of Maxwell's equations representing the field-to-line coupling process is required [1], [2]. The most commonly used transmission line-based model to describe lightning electromagnetic field coupling to overhead lines is the one proposed by Agrawal *et al.* in 1980 [3]. Since then, different implementations of this model have been proposed in the literature that can be basically divided into two categories: analytical [4]–[9] and numerical [10]–[17].

The numerical methods basically follow Agrawal *et al.* original idea to use a finite-difference time-domain (FDTD) scheme; such an approach requires the well-known Courant stability condition to be satisfied in order to prevent the numerical method from being unstable. For the first-order FDTD scheme and when boundary conditions are known analytically, if the space step is equal to the speed of light in vacuum times the time step (i.e., the so-called magic step), it can be shown that there is no discretization error, no matter the amplitude of the space step with respect to the line length (see, e.g., [18] for details).

However, when dealing with realistic network configurations, it is generally not possible to express analytically in the time domain the boundary conditions given by the relationship between voltage and current at the line extremities. For example, distribution networks are typically terminated on transformers with their surge protection devices. Expressing analytically the time-domain voltage–current relationship in this case is nearly impossible.

To cope with this problem, in many numerical approaches (e.g., lightning induced overvoltage code (LIOV) [10]–[14] and in a more recent code developed by the authors [15]–[17]), the algorithm that implements the FDTD solution of the Agrawal *et al.* equations is implemented into (or interfaced with) an electromagnetic simulator (electromagnetic transient program for LIOV and PSCAD-EMTDC for the other one), which is able to represent with a high level of detail the power components connected at the ends of the multiconductor transmission lines (MTLs) subject to the lightning discharge.

In both approaches, the interface between the field-to-line coupling code and the external simulator has been designed according to the following idea: the coupling code receives from the simulator the voltages at the line extremities (and at the line discontinuities, if any) at time step n and produces as output the currents at step $n+1$ in the same points.

Manuscript received August 14, 2018; revised November 28, 2018; accepted December 24, 2018. Date of publication January 8, 2019; date of current version February 13, 2020. *(Corresponding author: Daniele Mestriner.)*

M. Brignone, D. Mestriner, and R. Procopio are with the Naval and Electrical Engineering Department, University of Genoa, Genova I-16145, Italy (e-mail: massimo.brignone@unige.it; daniele.mestriner@edu.unige.it; renato.procopio@unige.it).

A. Piantini is with the Institute of Energy and Environment, University of São Paulo, São Paulo 05508-010, Brazil (e-mail: piantini@iee.usp.br).

F. Rachidi is with the EMC Laboratory, Swiss Federal Institute of Technology (EPFL), Lausanne CH-1015, Switzerland (e-mail: Farhad.Rachidi@epfl.ch).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TEMC.2018.2890043

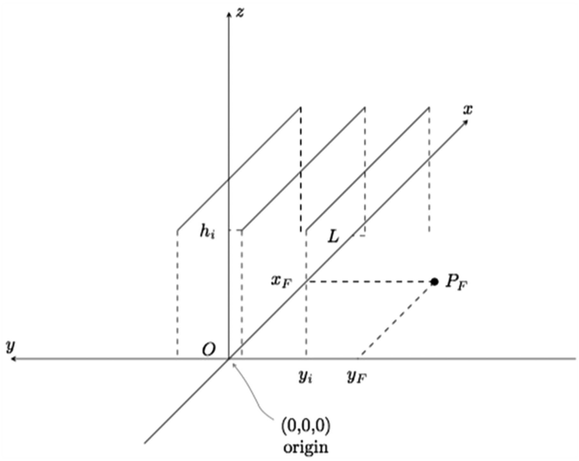

Fig. 1. Geometry of the problem.

As a consequence, it is necessary to provide a method to calculate the currents at the line extremities and discontinuities. None of the two telegraphers' equations can be used as they do not hold in the domain boundaries; for this reason, such currents are calculated by means of a linear extrapolation using the currents in the two closest points of the line. However, as will be detailed later on in this paper, this idea might originate numerical instabilities when the ratio between the FDTD space step and the line length increases. Moreover, the boundary limit is not well defined because it also depends on the line terminations values. This paper provides a possible solution to such problem by adopting the method of characteristics [18] in the first and in the last segments of the line (and in all the segments around a discontinuity) allowing in principle any kind of space discretization without resulting in numerical instability phenomena.

This paper is organized as follows. In Section II, the classical formulation and implementation of the field-to-line coupling problem is presented. The numerical stability issues of such implementation are investigated in Section III by means of dedicated simulations that allow us to understand their main causes. Section IV presents a numerically stable algorithm and applies it to the case of a single-conductor line. The effectiveness of the proposed approach is assessed by comparing its results with the available analytical solution. Section V aims at extending the approach for a more realistic case of an MTL system over a lossy ground and validates the developed approach against a numerical simulation performed with space (and time) steps ten times smaller (which would be too much time-consuming for realistic/statistical applications). Finally, in Section VI conclusions are drawn and some practical implications of this improvement are discussed.

II. FDTD-PSCAD Classical Formulation and Implementation of the Field-To-Line Coupling Problem

In Fig. 1(a), the MTL system is depicted consisting of M straight and parallel conductors of length L and diameter a_i, each of them placed at a height $h_i\ (i = 1, \ldots, M)$ above the ground. In order to define the position y_i of each conductor, a reference system Oxyz is placed with the x-axis parallel to the direction of the line conductors. Finally, let $P_F = (x_F, y_F, 0)$ be the lightning impact point.

In the case of indirect lightning, the induced voltages and currents can be computed according to the Agrawal *et al.* model [3] (whose validity has been extended in [18] to account for a finite ground conductivity σ_g)

$$\begin{cases} \frac{\partial V_i^s}{\partial x}(x,t) + \sum_{j=1}^{M} L_{ij} \frac{\partial I_j}{\partial t}(x,t) + V_i^g(x,t) = E_{x,i}^e(x,t) \\ \frac{\partial I_i}{\partial x}(x,t) + \sum_{j=1}^{M} C_{ij} \frac{\partial V_j^s}{\partial t}(x,t) = 0 \end{cases} \quad (1)$$

with

$$V_i^g(x,t) = \begin{cases} \int_0^t \xi_g^i(t-s) \frac{\partial I_i}{\partial s}(x,s)\, ds & \text{if } \sigma_g > 0 \\ 0 & \text{if } \sigma_g = \infty \end{cases} \quad (2)$$

where $V_i^s(x,t)$, $I_i(x,t)$, and $E_{x,i}^e(x,t)$ denote the scattered voltage, the current, and the tangential component of the exciting electric field on the ith conductor at a distance x from its left terminal and at time t, respectively. Moreover, L_{ij} and C_{ij} are the entries of the inductance and capacitance matrices [19] and ξ_i^g is the time-domain expression for the ground transient resistance (inverse Fourier transform of the ground impedance divided by $j\omega$) [20].

Finally, the total voltage V_i can be obtained with a good approximation starting from the scattered one as follows:

$$V_i(x,t) \approx V_i^s(x,t) - h_i E_z^e(r_i(x),t) \quad (3)$$

where $E_z^e(r_i(x),t)$ is the vertical component of the exciting electric field calculated at time t at ground level and distance $r_i(x) = \sqrt{(x-x_F)^2 + (y_i - y_F)^2}$ from the lightning strike impact point.

The solution of (1) is typically achieved by means of the FDTD technique [3]. In this way, a time step Δt and a spatial step Δx (such that $K_m = L/\Delta x$ is an integer number) have to be chosen in order to provide a discrete version of problem (1). In order to guarantee reliable results, the time and spatial steps have to satisfy the Courant condition, i.e., there exists $C < 1$ such that

$$c_0 \Delta t \le C \Delta x \quad (4)$$

c_0 being the speed of light in vacuum. Defining the time sequence

$$t_n = n\Delta t \quad n = 1, 2, \ldots \quad (5)$$

the spatial sequence

$$x_k = (k-1)\Delta x, \quad k = 1, \ldots, K_m \quad (6)$$

and the following quantities $V_{i,k}^{s,n} = V_i^s(x_k, t_n)$, $V_{i,k}^{g,n} = V_i^g(x_k, t_n)$, $I_{i,k}^n = I_i(x_k, t_n)$, and $E_{i,k}^{e,n} = E_{x,i}^e(x_k, t_n)$, the finite-difference solutions of (1) and (2) become (see [12] and

[15] for the algebraic manipulations)

$$V_{i,k}^{s,n+1} = V_{i,k}^{s,n} - c_0^2 \sum_{j=1}^{M} L_{i,j} \frac{I_{j,k+1}^{n} - I_{j,k-1}^{n}}{2\Delta x} \Delta t + c_0^2 \left(\frac{V_{i,k+1}^{s,n} - 2V_{i,k}^{s,n} + V_{i,k-1}^{s,n}}{\Delta x^2} + \frac{V_{i,k+1}^{g,n} - V_{i,k-1}^{g,n}}{2\Delta x} - \frac{E_{i,k+1}^{e,n} - E_{i,k-1}^{e,n}}{2\Delta x} \right) \frac{\Delta t^2}{2} \tag{7}$$

$$I_{i,k}^{n+1} = I_{i,k}^{s,n} - c_0^2 \sum_{j=1}^{M} C_{ij} \left(\frac{V_{j,k+1}^{s,n} - V_{j,k-1}^{s,n}}{2\Delta x} + V_{i,k}^{g,n} - E_{j,k}^{h,n} + \frac{V_{j,k}^{g,n} - V_{i,k}^{g,n-1}}{2} - \frac{E_{j,k}^{e,n} - E_{j,k}^{e,n-1}}{2x} \right) \Delta t + c_0^2 \left(\frac{I_{i,k+1}^{n} - 2I_{i,k}^{n} + I_{i,k-1}^{n}}{\Delta x^2} \right) \frac{\Delta t^2}{2}. \tag{8}$$

It is easy to see that neither (7) nor (8) can be used for updating the voltage or the current at both extremities of each conductor (and at the discontinuities, if they exist); moreover, such points are not governed by the original differential equations as they are the ones in which the boundary conditions should be imposed. To handle this situation, as proposed in [11], the currents in the first and in the last point can be updated by using the following linear extrapolation:

$$\begin{cases} I_{i,1}^{n+1} = 2I_{i,2}^{n+1} - I_{i,3}^{n+1} \\ I_{i,K_m}^{n+1} = 2I_{i,K_m-1}^{n+1} - I_{i,K_m-2}^{n+1}. \end{cases} \tag{9}$$

Such currents are the output of the coupling code and have to be passed to PSCAD that models the boundary conditions and produces the update of the voltage $V_{i,k}^{\mathrm{PSCAD},n+1}$ $k = \{1, K_m\}$ in the same two points, according to the specific lumped parameters network that represents the power system one wants to analyze. Exploiting (3), one obtains

$$V_{i,k}^{s,n+1} = V_{i,k}^{\mathrm{PSCAD},n+1} + h_i E_z^e \left(r_i(x_k), t_{n+1} \right). \tag{10}$$

From here on, the described numerical procedure will be labeled as FDTD-PSCAD solution to the field-to-line coupling problem.

III. Stability Problem of the Numerical Field-To-Line Coupling Codes

We will show in what follows that the FDTD-PSCAD approach could exhibit numerical instability, even though the Courant condition (4) is satisfied.

To highlight this issue, first a very simple geometry is considered that allows us to evaluate voltages and currents at the line extremities also in a (semi-)analytical way [18]. As shown in Fig. 2, a single-conductor line of length $L = 63$ m with diameter $a = 0.02$ m lays over a perfectly conducting ground at a height $h = 10$ m and is connected to ground with two equal resistances $R_0 = R_L = 10\,\Omega$ at its extremities.

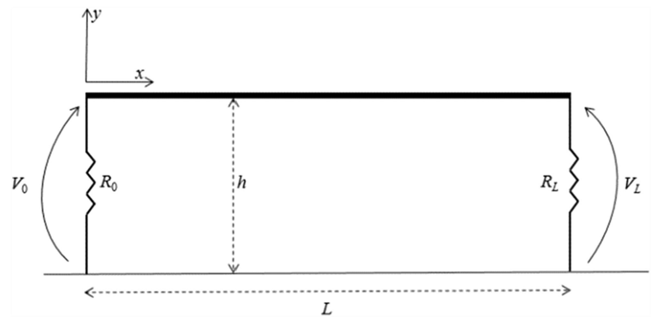

Fig. 2. Single-Conductor line.

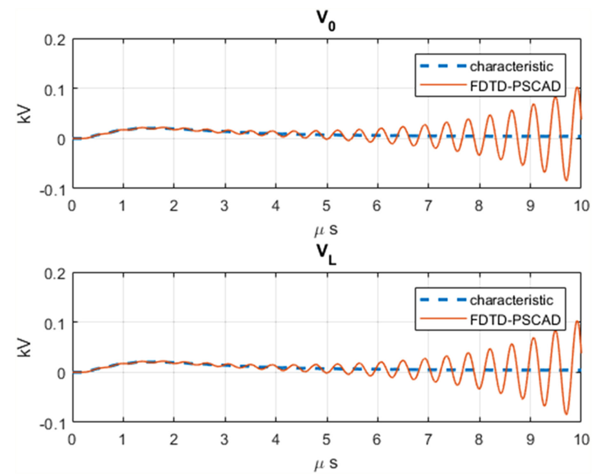

Fig. 3. Voltage at the beginning of the line (V_0) and at the end of the line (V_L) obtained with the FDTD-PSCAD approach (solid line) and via characteristic method (dashed line). Here, $\Delta t = 10$ ns and $\Delta x = 9$ m.

The line is illuminated by an electromagnetic field due to a lightning discharge striking the ground 30 m from the middle of the line, i.e., $P_F = (31.5,\ 30.0,\ 0.0)$. The lightning current propagation along the channel is described with the MTLE engineering model [21], [22], whereas the channel-base current is represented by a sum of two Heidler's functions [23], with a unitary peak value and a propagation velocity equal to $c_0/2$.

The time step $\Delta t = 10$ ns and the spatial step $\Delta x = 9$ m have been set so that condition (4) is satisfied (with $C = 1/3$).

In Fig. 3, the voltages at both extremities of the line are plotted as computed with the FDTD-PSCAD approach and via the characteristic method (see the Appendix for details); the exam of the figures highlights the unstable oscillations of the numerical approach, whereas the characteristic method appears to be stable.

Simulations have suggested that instability phenomena seem to be related to the ratio $L/\Delta x$; as a matter of fact, decreasing by ten times both the time and the space step ($\Delta t = 1$ ns and $\Delta x = 0.9$ m), it is possible to observe that analytical (method of characteristics) and numerical solutions are in perfect agreement (see Fig. 4).

A possible explanation of this behavior is that the ratio $L/\Delta x$ intrinsically influences the updating procedure (9) of the currents at the line extremities: the smaller is the spatial step, the more accurate is (9). To confirm this hypothesis, Fig. 5 plots the

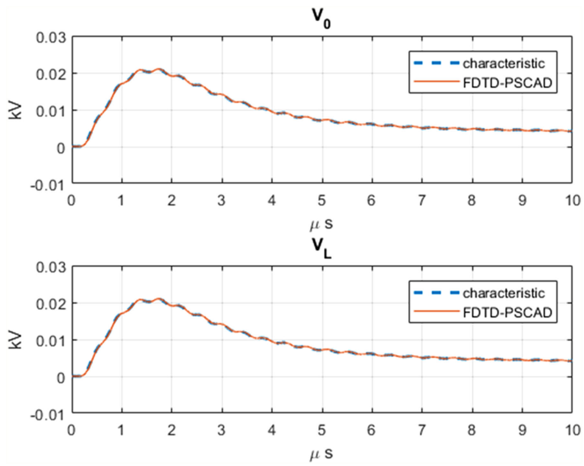

Fig. 4. Voltage at the beginning of the line (V_0) and at the end of the line (V_L) obtained by the FDTD-PSCAD approach (solid line) and via the method of characteristics (dashed line). Here, $\Delta t = 1$ ns and $\Delta x = 0.9$ m.

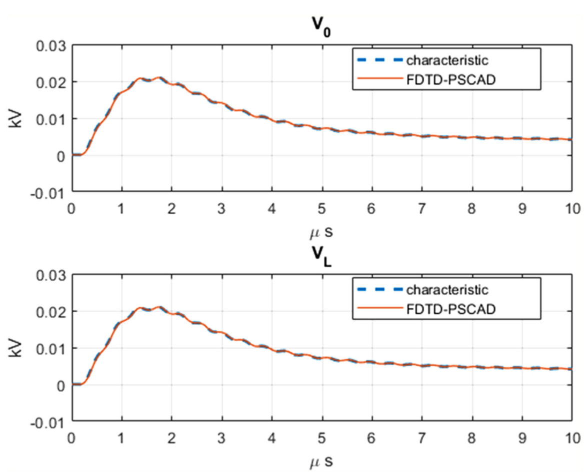

Fig. 5. Voltage at the beginning of the line (V_0) and at the end of the line (V_L) obtained by the FDTD-PSCAD approach (solid line) with analytical imposition of boundary conditions, and via characteristic method (dashed line). Here, $\Delta t =$ 10 ns and $\Delta x = 9$ m.

numerical solution of the problem with time step $\Delta t = 10$ ns and the spatial step $\Delta x = 9$ m but with analytical imposition of the boundary conditions (i.e., $V_1^n = -R_0 I_1^n$ and $V_{K_m}^n = R_L I_{K_m}^n$). In this case, substantially, the characteristic method, which requires the knowledge of the voltage–current law at the extremities, is used in order to evaluate the current at the extremities, which are then passed, for each time step, to the coupling code instead of those calculated with the extrapolation. The perfect agreement with the analytical solution allows us to state that the numerical instability problem occurs when (9) is used to update the currents at the line extremities and the ratio $L/\Delta x$ is not sufficiently small.

The presented analysis suggests that whenever it is not possible to impose boundary conditions in an analytical way, a very small time and space step has to be used in order to make the extrapolation (9) effective. However, this might require too high computational effort.

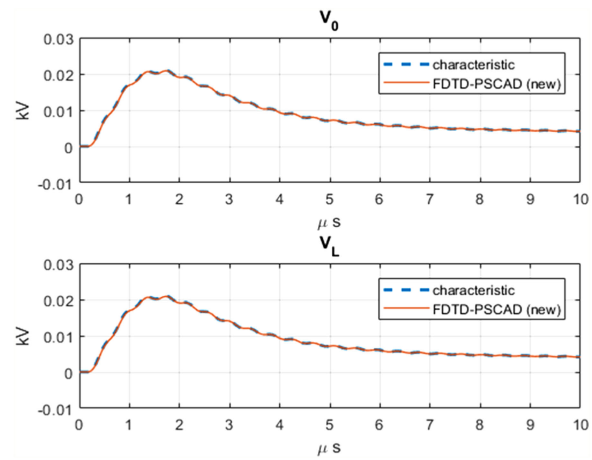

Fig. 6. Voltage at the beginning of the line (V_0) and at the end of the line (V_L) obtained by the FDTD-PSCAD (new) approach (solid line) and via characteristic method (dashed line). Here, $\Delta t = 10$ ns and $\Delta x = 9$ m.

TABLE I
GEOMETRY OF THE MTL SYSTEM

	Cond. 1	*Cond. 2*	*Cond. 3*
height from ground	8.0 m	8.0 m	8.6 m
distance from y axis	−1.2 m	1.2 m	0.0 m
conductor diameter	0.64 cm	0.64 cm	0.64 cm

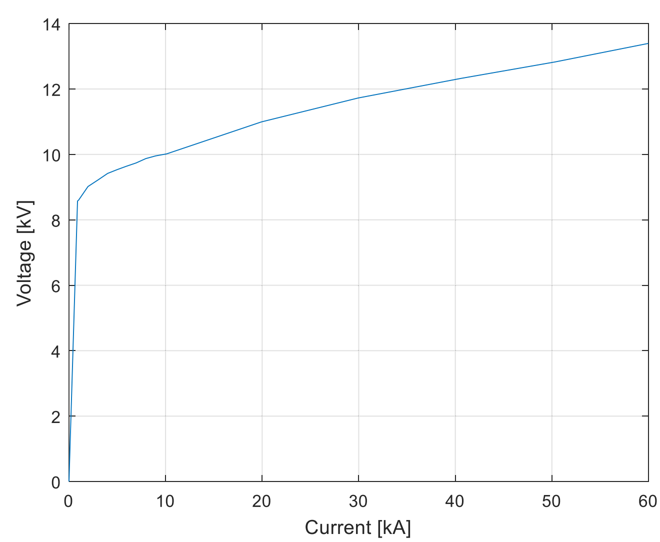

Fig. 7. Surge arrester *V–I* characteristic.

IV. PROPOSED SOLUTION FOR A SINGLE-CONDUCTOR LINE

As the numerical instability problem depends mostly on the use of extrapolation (9) when the space step is not sufficiently small with respect to the line length L, it is necessary to use another method to update the currents in the first and in the last point of the line (and in all the eventual discontinuities). The approach proposed in this paper provides the current updation applying the characteristics method [18] in the first and in the last line segments.

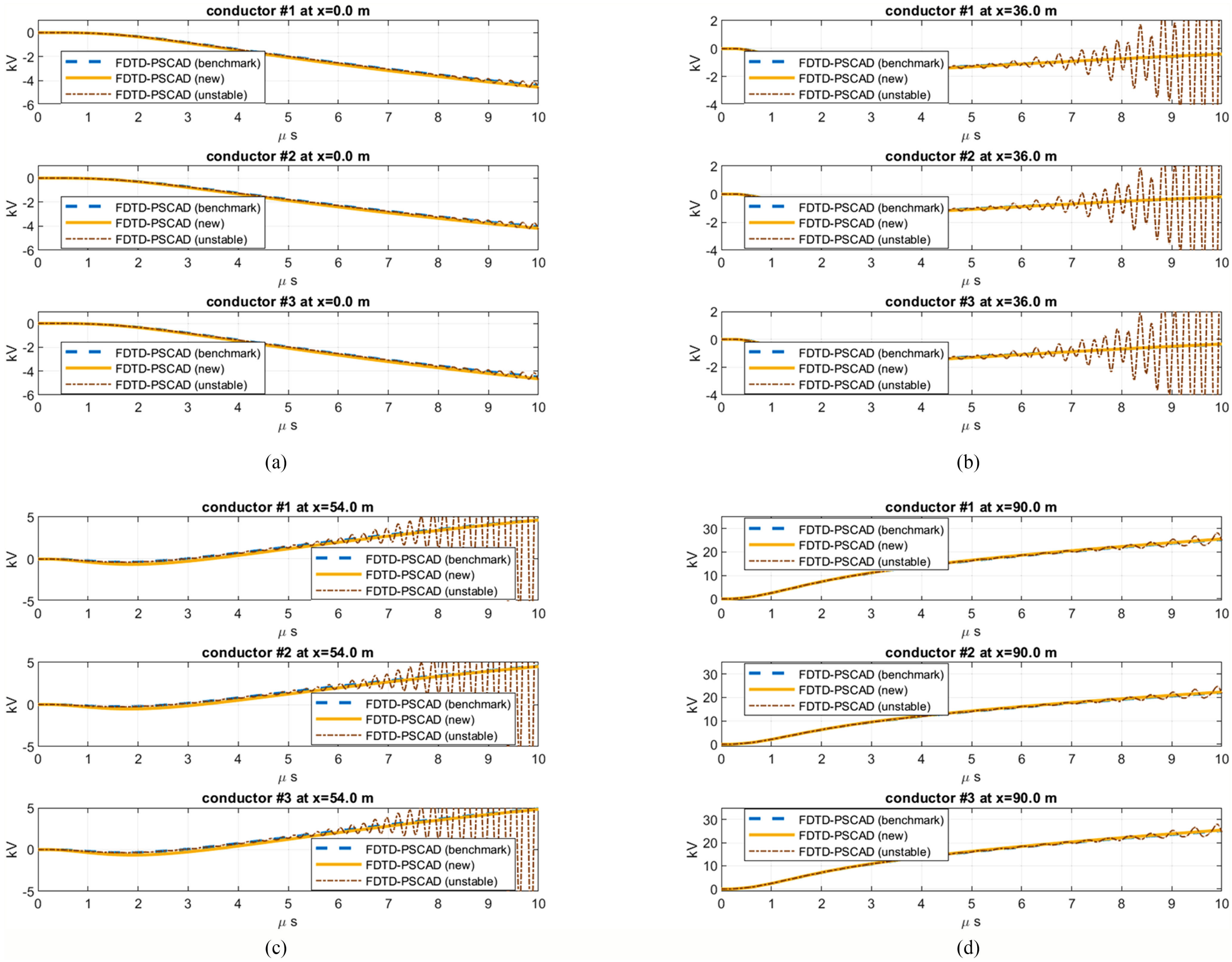

Fig. 8. (Case 1) Voltages for the three conductors at the distances of: (a) 0 m, (b) 36 m, (c) 54 m, and (d) 90 m from the beginning of the line. The solid lines are obtained by using the FDTD-PSCAD (new) approach with $\Delta t = 10$ ns and $\Delta x = 9$ m. The FDTD-PSCAD (unstable) plot (dash–dotted lines) and the FDTD-PSCAD (benchmark) one (dashed lines) have been both obtained using the FDTD-PSCAD approach in [15] with $\Delta t = 10$ ns and $\Delta x = 9$ m, and $\Delta t = 1$ ns and $\Delta x = 0.9$ m, respectively.

The main requirement is that there exists an integer N_T s.t.

$$N_T \Delta t = \frac{\Delta x}{c_0}. \tag{11}$$

In this way, the travel time T defined in (A3) is an integer multiple of the time step, so that applying (A1) and (A2) one obtains

$$V_{K_m}^{\mathrm{PSCAD},n+1} + Z_C I_{K_m}^{n+1} = \left[V_{K_m-1}^{n+1-N_T} + Z_C I_{K_m-1}^{n+1-N_T}\right] \cdot 1(n+1-N_T) + \Theta_{K_m-1,K_m;0}^{fw}(t_{n+1}) \tag{12}$$

$$V_1^{\mathrm{PSCAD},n+1} - Z_C I_1^{n+1} = \left[V_2^{n+1-N_T} - Z_C I_2^{n+1-N_T}\right] \cdot 1(n+1-N_T) + \Theta_{1,2;0}^{bw}(t_{n+1}). \tag{13}$$

As can be seen from (12) and (13), the method of characteristics basically defines two Thévenin equivalents of the analyzed line at its extremities. So, in order to know the voltage/current at such extremities, the functional relationship between them as dictated by the external circuit should be known, which is not possible when dealing with a numerical electromagnetic simulator. To solve this problem, one can suppose that

$$V_1^{\mathrm{PSCAD},n+1} \approx V_1^{\mathrm{PSCAD},n} \text{ and } V_{K_m}^{\mathrm{PSCAD},n+1} \approx V_{K_m}^{\mathrm{PSCAD},n}. \tag{14}$$

Inserting (14) into (12) and (13), it is possible to write the following updating relations for currents:

$$I_{K_m}^{n+1} = \frac{1}{Z_C}\left\{-V_{K_m}^{\mathrm{PSCAD},n} + \left[V_{K_m-1}^{n+1-N_T} + Z_C I_{K_m-1}^{n+1-N_T}\right] \cdot 1(n+1-N_T) + \Theta_{K_m-1,K_m;0}^{fw}(t_{n+1})\right\} \tag{15}$$

$$I_1^{n+1} = \frac{-1}{Z_C}\left\{-V_1^{\mathrm{PSCAD},n} + \left[V_2^{n+1-N_T} - Z_C I_2^{n+1-N_T}\right] \cdot 1(n+1-N_T) + \Theta_{1,2;0}^{bw}(t_{n+1})\right\}. \tag{16}$$

If one updates the currents with (15) and (16) instead of (9), even with $\Delta t = 10$ ns and $\Delta x = 9$ m, a perfect agreement between the proposed approach [labeled as FDTD-PSCAD (new) in the following] and the analytical solution is obtained for the case presented in Section III (see Fig. 6).

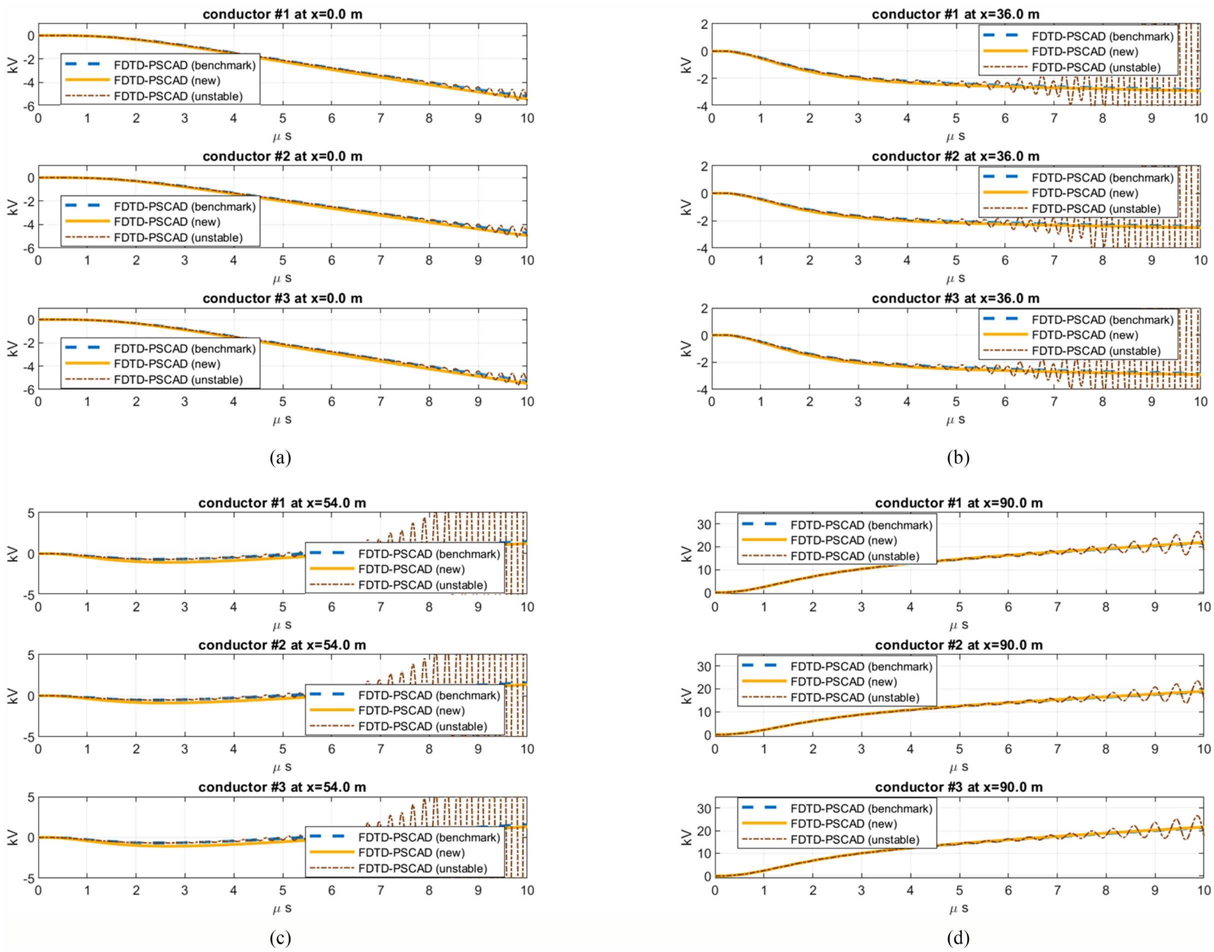

Fig. 9. (Case 2) Voltages for the three conductors at the distances of: (a) 0 m, (b) 36 m, (c) 54 m, and (d) 90 m from the beginning of the line. The solid lines are obtained by using the FDTD-PSCAD (new) approach with $\Delta t = 10$ ns and $\Delta x = 9$ m. The FDTD-PSCAD (unstable) plot (dash–dotted lines) and the FDTD-PSCAD (benchmark) one (dashed lines) have been both obtained using the FDTD-PSCAD approach in [15] with $\Delta t = 10$ ns and $\Delta x = 9$ m, and $\Delta t = 1$ ns and $\Delta x = 0.9$ m, respectively.

The reason why this approach is more effective than the previous one probably lies in the fact that the only condition to be satisfied is that the voltage does not change much from one time step to the subsequent one in order to make (14) hold. This hypothesis is typically satisfied with time steps normally adopted in these studies (about 10 ns necessary to correctly reproduce the waveform of the lightning electromagnetic fields).

V. Generalization to an MTL Over a Conducting Ground and Discontinuities

Although the method of characteristics is applicable only to a single-conductor line over a perfectly conducting ground, one can assume that the proposed strategy (12), (13) for updating currents is still valid also for an MTL system over a lossy ground. This hypothesis basically neglects the effect of the following:

1) the line ohmic;
2) the finite ground conductivity in the coupling equations;
3) the inductive and capacitive coupling among conductors (only in the first and in the last segments of the discretized line and, in case of discontinuities, in the two segments that include each discontinuity).

Notwithstanding the above, the hypothesis can be considered as reasonable since it applies only to a small portion of the whole system.

Starting from this assumption, the generalization of (12) and (13) is very simple: for the ith conductor one has

$$I_{i,K_m}^{n+1} = \frac{1}{Z_{C,i}}\Big\{-V_{i,K_m}^{\mathrm{PSCAD},n} + \Big[V_{i,K_m-1}^{n+1-N_T} + Z_{C,i}\, I_{i,K_m-1}^{n+1-N_T}\Big] \cdot 1(n+1-N_T) + \Theta^{fw}_{K_m-1,K_m;y_i}(t_{n+1})\Big\} \tag{17}$$

$$I_{i,1}^{n+1} = \frac{-1}{Z_{C,i}}\Big\{-V_{i,1}^{\mathrm{PSCAD},n} + \Big[V_{i,2}^{n+1-N_T} - Z_{C,i}\, I_{i,2}^{n+1-N_T}\Big] \cdot 1(n+1-N_T) + \Theta^{bw}_{1,2;y_i}(t_{n+1})\Big\} \tag{18}$$

$Z_{C,i}$ being the characteristic impedance of the ith conductor, evaluated for the case of a lossless wire.

In the case of discontinuities (i.e., points connected to the ground by means of suitable devices such as arresters or line junctions), the same procedure can be adopted, treating each of these discontinuity points in the same way as the terminations. Following [15], if one indicates with x_d the abscissa of the generic discontinuity point ($d = 1, \ldots, D$) and defines the corresponding index $k_d = x_d/\Delta x + 1$, it is possible to evaluate the left and right limits $I_{i,k_d}^{-,n+1}$ and $I_{i,k_d}^{+,n+1}$ of the current for x approaching x_d by means of (17) and (18). More precisely

$$I_{i,k_d}^{-,n+1} = \frac{1}{Z_{C,i}} \left\{ -V_{i,k_d}^{\mathrm{PSCAD},n} + \left[V_{i,k_d-1}^{n+1-N_T} + Z_{C,i} I_{i,k_d-1}^{n+1-N_T} \right] \cdot 1(n+1-N_T) + \Theta_{k_d-1,k_d;y_i}^{fw}(t_{n+1}) \right\} \quad (19)$$

$$I_{i,k_d}^{+,n+1} = \frac{-1}{Z_{C,i}} \left\{ -V_{i,k_d}^{\mathrm{PSCAD},n} + \left[V_{i,k_d+1}^{n+1-N_T} - Z_{C,i} I_{i,k_d+1}^{n+1-N_T} \right] \cdot 1(n+1-N_T) + \Theta_{k_d,k_d+1;y_i}^{bw}(t_{n+1}) \right\}. \quad (20)$$

Then, the Kirchhoff current law allows us to find out the update of the current flowing in the device connected to the line in the point x_d as

$$I_{i,k_d}^{n+1} = I_{i,k_d}^{-,n+1} - I_{i,k_d}^{+,n+1}. \quad (21)$$

The validation of the proposed approach has been performed on a realistic Italian distribution network MTL (see Fig. 1) consisting of three conductors of length $L = 189$ m, whose details are summarized in Table I.

A lossy ($\sigma_g = 0.005$ S/m) ground has been considered with a relative dielectric constant equal to 10. Regarding the terminations, two cases are presented: each conductor is terminated at both ends with a 10-Ω resistor and connected to ground every 63 m with a 20-Ω resistor; and each conductor is connected to ground every 63 m through a typical 10-kV surge arrester [24] (Fig. 7 shows its *V–I* characteristic), so that the line has four surge arresters: two at the terminations and two at the discontinuities

Such MTL is illuminated by an electromagnetic field due to a lightning return stroke that occurs 30 m far from the middle of the line, i.e., $P_F = (94.5,\, 30.0,\, 0.0)$ with the current distribution modeled as in Section IV.

For the numerical simulation of the FDTD-PSCAD (new) approach, again $\Delta t = 10$ ns and $\Delta x = 9$ m have been considered.

In order to test its performances, the FDTD-PSCAD with extrapolation presented in [15] has been used with $\Delta t = 1$ ns and $\Delta x = 0.9$ m for guaranteeing stable results, as proved at the beginning of this contribution in Fig. 4.

Figs. 8 and 9 present the voltages in four points on the first half of the line for cases 1 and 2, respectively. Note that the symmetry of the considered geometry allows us to state that the other half line exhibits the same behavior.

For both cases, a perfect agreement between the benchmark and the new approach can be observed, with a meaningful reduction in the computational effort. In particular, a simulation performed with the time and spatial steps here presented (i.e., 10 ns and 9 m) requires about 1/8 of the time required by a simulation performed with a time step of 1 ns and a spatial step of 1 m.

VI. Conclusion

This paper has investigated the numerical instability phenomenon that occurs when interfacing an FDTD solution of the field-to-line coupling equations with a power system electromagnetic simulator. Such instability takes place when the FDTD space step becomes a significant fraction of the overall line length and is basically due to the way in which the currents at the line extremities (or discontinuities) are updated before being transmitted to the power system simulator. An alternative approach has been proposed based on the use of the method of characteristics in the first and last line segment (and in all the segments around a discontinuity) that is shown to be stable over a wide variety of operating conditions. The proposed approach has been first applied to a single-conductor case in order to be compared with the available analytical solution and then on a more realistic situation in which an MTL lays over a lossy ground. Besides the solving of the instability problem, this new solution has a significant practical implication, as it allows us to simulate lines with discontinuities in principle as close as possible to each other. For this reason, future work will concern the possibility of analyzing the effects of surge arresters as protection devices for transformers investigating their optimal positions, being sure of the reliability of the simulation results.

Appendix

Let us consider a single-conductor line of extremities *A* and *B* and length L_{AB}, which lays over a perfectly conducting ground at height *h* and at distance *y* from the reference system. The method of characteristics [18] states that the voltages $V(\cdot, t)$ and the currents $I(\cdot, t)$ at the line extremities are related by

$$V(B,t) + Z_C I(B,t) = [V(A, t-T) + Z_C I(A, t-T)]\; 1(t-T) + \Theta_{A,B;y}^{fw}(t) \quad (A1)$$

$$V(A,t) - Z_C I(A,t) = [V(B, t-T) + - Z_C I(B, t-T)]\; 1(t-T) + \Theta_{A,B;y}^{bw}(t) \quad (A2)$$

where Z_C is the characteristic impedance of the line [18], and *T* is the travel time defined as

$$T = \frac{L_{AB}}{c_0} \quad (A3)$$

and finally, as usual, $1(t)$ is the Heaviside function, i.e.,

$$1(t) = \begin{cases} 1 & t > 0 \\ 0 & \text{otherwise.} \end{cases} \quad (A4)$$

The quantities $\Theta_{A,B;y}^{fw}$ and $\Theta_{A,B;y}^{bw}$ are completely defined by the incident electric field according to the following:

$$\Theta_{A,B;y}^{fw}(t) = \mathscr{E}_T(A, t-T) - \mathscr{E}_T(B, t) + \int_0^{L_{AB}} \mathscr{E}_{L,y}\left(x, t - T + \frac{x}{c_0}\right) dx \quad (A5)$$

$$\Theta_{A,B;y}^{bw}(t) = \mathscr{E}_T(B, t-T) - \mathscr{E}_T(A, t) - \int_0^{L_{AB}} \mathscr{E}_{L,y}\left(x, t - \frac{x}{c_0}\right) dx \quad (A6)$$

where

$$\mathscr{E}_{T,y}(x,t) \approx hE_z^e(r(x,y),t)1(t) \tag{A7}$$

and $\mathscr{E}_{L,y}$ is the projection onto the line of the radial component of the electric field, i.e.,

$$\mathscr{E}_{L,y}(x,t) = E_r(r(x,y),t)\frac{x-x_F}{r(x,y)}1(t). \tag{A8}$$

The presence of the function $1(t)$ accounts for the fact that the lightning is supposed to strike at $t = 0$. The function $r(x,y) = \sqrt{(x-x_F)^2 + (y-y_F)^2}$ is the distance of the generic point (x,y) belonging to the line from the lightning impact point $P_F = (x_F, y_F, 0)$. Notice that for a single conductor the x-axis of the reference system typically coincides with the line, so that $y = 0$. The presence of the distance y is important for the generalization to an MTL system.

REFERENCES

[1] C. A. Nucci and F. Rachidi, "Interaction of electromagnetic fields with electrical networks generated by lightning," in *The Lightning Flash*. London, U.K.: Inst. Eng. Technol., 2014, pp. 559–610.

[2] F. Rachidi, "A review of field-to-transmission line coupling models with special emphasis to lightning-induced voltages on overhead lines," *IEEE Trans. Electromagn. Compat.*, vol. 54, no. 4, pp. 898–911, Aug. 2012.

[3] A. K. Agrawal, H. J. Price, and S. H. Gurbaxani, "Transient response of multiconductor transmission lines excited by a nonuniform electromagnetic field," *IEEE Trans. Electromagn. Compat.*, vol. EMC-22, no. 2, pp. 119–129, May 1980.

[4] H. D. Hoidalen, "Lightning-Induced overvoltages in low-voltage systems," Ph.D. dissertation, Dept. Elect. Power Eng., Norwegian Univ. Sci. Technol., Trondheim, Norway, 1997.

[5] H. K. Hoidalen, "Calculation of lightning-induced overvoltages using MODELS," in *Proc. 4th Int. Conf. Power Syst. Transients*, Budapest, Hungary, 1999.

[6] H. K. Hoidalen, "Analyitical formulation of lightning-induced voltages on multiconductor overhead lines above lossy ground," *IEEE Trans. Electromagn. Compat.*, vol. 45, no. 1, pp. 92–100, Feb. 2003.

[7] H. K. Hoidalen, "Calculation of lightning-induced overvoltages in MODELS including lossy ground effects," in *Proc. Int. Conf. Power Syst. Transients*, New Orleans, LA, USA, 2003.

[8] A. Andreotti, A. Pierno, and V. A. Rakov, "A new tool for calculation of lightning induced voltages in power system—Part I: Development of circuit model," *IEEE Trans. Power Del.*, vol. 30, no. 1, pp. 326–333, Feb. 2015.

[9] A. Andreotti, A. Pierno, and V. A. Rakov, "A new tool for calculation of lightning-induced voltages in power systems—Part II: Validation study," *IEEE Trans. Power Del.*, vol. 30, no. 1, pp. 334–341, Feb. 2015.

[10] M. Paolone, C. A. Nucci, and F. Rachidi, "A new finite difference time domain scheme for the evaluation of lightning induced overvoltage on multiconductor overhead lines," in *Proc. Int. Conf. Power Syst. Transients*, Rio de Janeiro, Brazil, 2001.

[11] M. Paolone, "Modeling of lightning-induced voltages on distribution networks for the solution of power quality problems, and relevant implementation in a transient program," Ph.D. dissertation, Dept. Elect. Eng., Univ. Bologna, Bologna, Italy, 2002.

[12] A. Borghetti, J. A. Gutierrez, A. C. Nucci, M. Paolone, E. Petrache, and F. Rachidi, "Lightning-Induced voltages on complex distribution systems: Models, advanced software tools and experimental validation," *J. Electrostatics*, vol. 60, pp. 163–174, 2004.

[13] M. Paolone *et al.*, "Lightning electromagnetic field coupling to overhead lines: Theory, numerical simulations, and experimental validation," *IEEE Trans. Electromagn. Compat.*, vol. 51, no. 3, pp. 532–547, Aug. 2009.

[14] F. Napolitano, A. Borghetti, C. A. Nucci, and M. Paolone, "An advanced interface between the LIOV code and the EMPT-RV," in *Proc. 29th Int. Conf. Lightning Protection*, Uppala, Sweden, 2008.

[15] M. Brignone, F. Delfino, R. Procopio, M. Rossi, and F. Rachidi, "Evaluation of power system lightning performance, part I: Model and numerical solution using the PSCAD-EMTDC platform," *IEEE Trans. Electromagn. Compat.*, vol. 59, no. 1, pp. 137–145, Feb. 2017.

[16] M. Brignone, F. Delfino, R. Procopio, M. Rossi, and F. Rachidi, "Evaluation of power system lightning performance—Part II: Application to an overhead distribution network," *IEEE Trans. Electromagn. Compat.*, vol. 59, no. 1, pp. 146–153, Feb. 2017.

[17] M. Brignone, D. Mestriner, R. Procopio, A. Piantini, and F. Rachidi, "Evaluation of the mitigation effect of the shield wires on lightning induced overvoltages in MV distribution systems using statistical analysis," *IEEE Trans. Electromagn. Compat.*, vol. 60, no. 5, pp. 1400–1408, Oct. 2018.

[18] C. R. Paul, *Analysis of Multiconductor Transmission Lines*. Hoboken, NJ, USA: Wiley-IEEE Press, 2007.

[19] F. Rachidi, S. L. Loyka, C. A. Nucci, and M. Ianoz, "A new expression for the ground transient resistance matrix elements of multiconductor overhead transmission lines," *Elect. Power Syst. Res.*, vol. 65, pp. 41–46, Apr. 1, 2003.

[20] F. Delfino, R. Procopio, and M. Rossi, "Lightning return stroke current radiation in presence of a conducting ground: 1. Theory and numerical evaluation of the electromagnetic fields," *J. Geophys. Res., Atmos.*, vol. 113, 2008, Art. no. D05110.

[21] F. Rachidi and C. A. Nucci, "On the Master, Uman, Lin, Standler and the modified transmission line lightning return stroke current models," *J. Geophys. Res., Atmos.*, vol. 95, pp. 20389–20393, 1990.

[22] H. Heidler, "Analytische blitzstromfunktion zur LEMP-Berechnung," in *Proc. Int. Conf. Lightning Protection*, Munich, Germany, 1985.

[23] C. A. Nucci, "A survey on Cigré and IEEE procedures for the estimation of the lightning performance of overhead transmission and distribution lines," in *Proc. Asia-Pac. Int. Symp. Electromagn. Compat.*, 2010, pp. 1124–1133.

[24] ABB, "Overvoltage protection: Metal oxide surge arresters medium voltage systems," Application Guidelines, 2011.