

8.3.2 Torque-slip Characteristics of DFIG WECS

- To find torque-slip characteristics of DFIG, we can follow same procedure as that in Chapter 3, except that total rotor circuit resistance= R_r+R_{eq} , & total reactance= $X_{lr}+X_{eq}$.

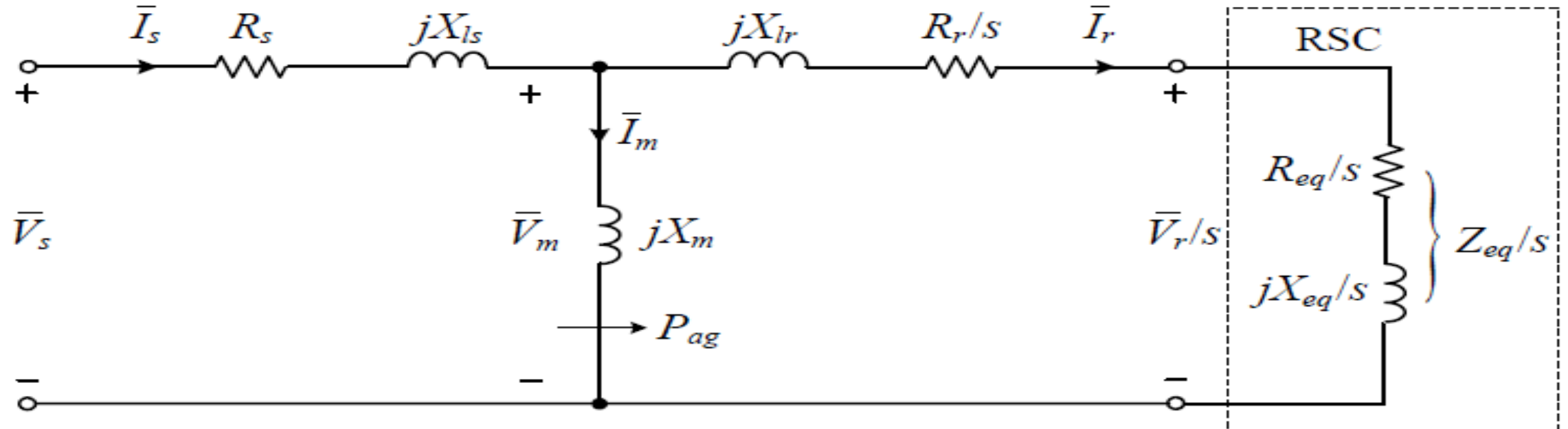
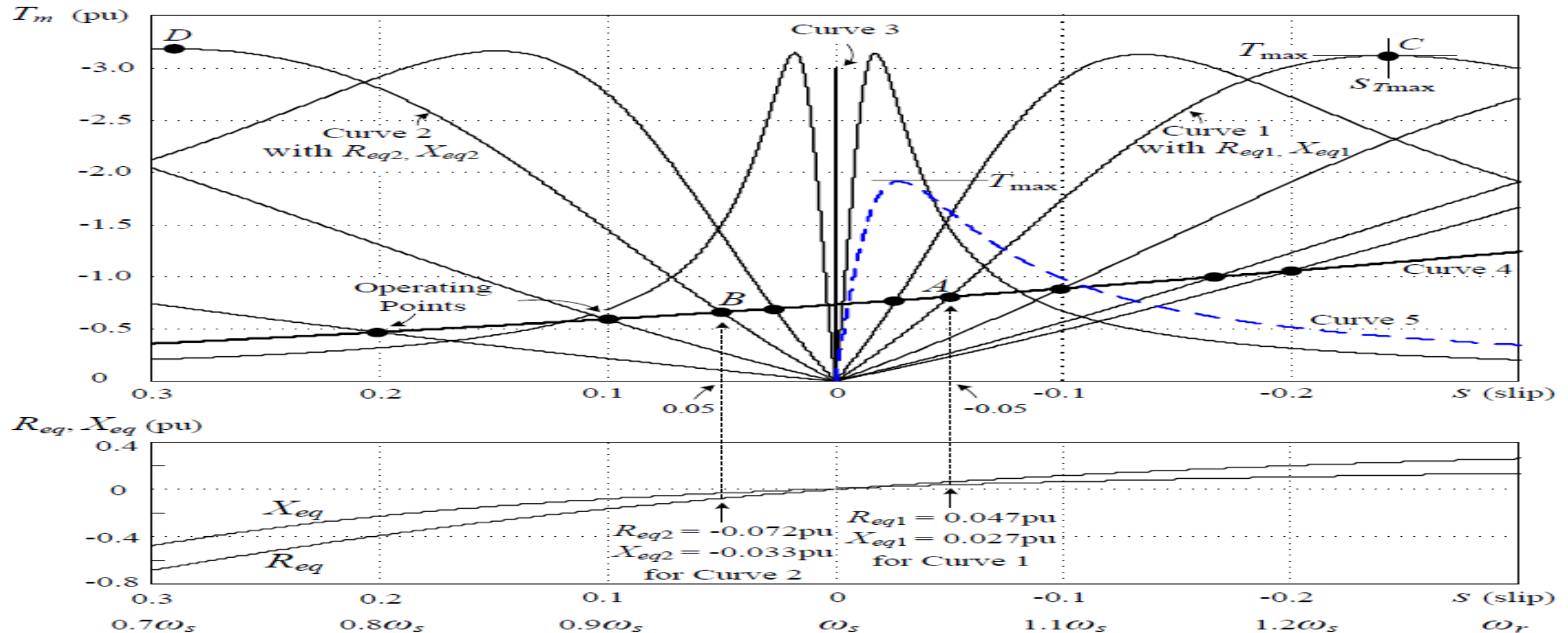
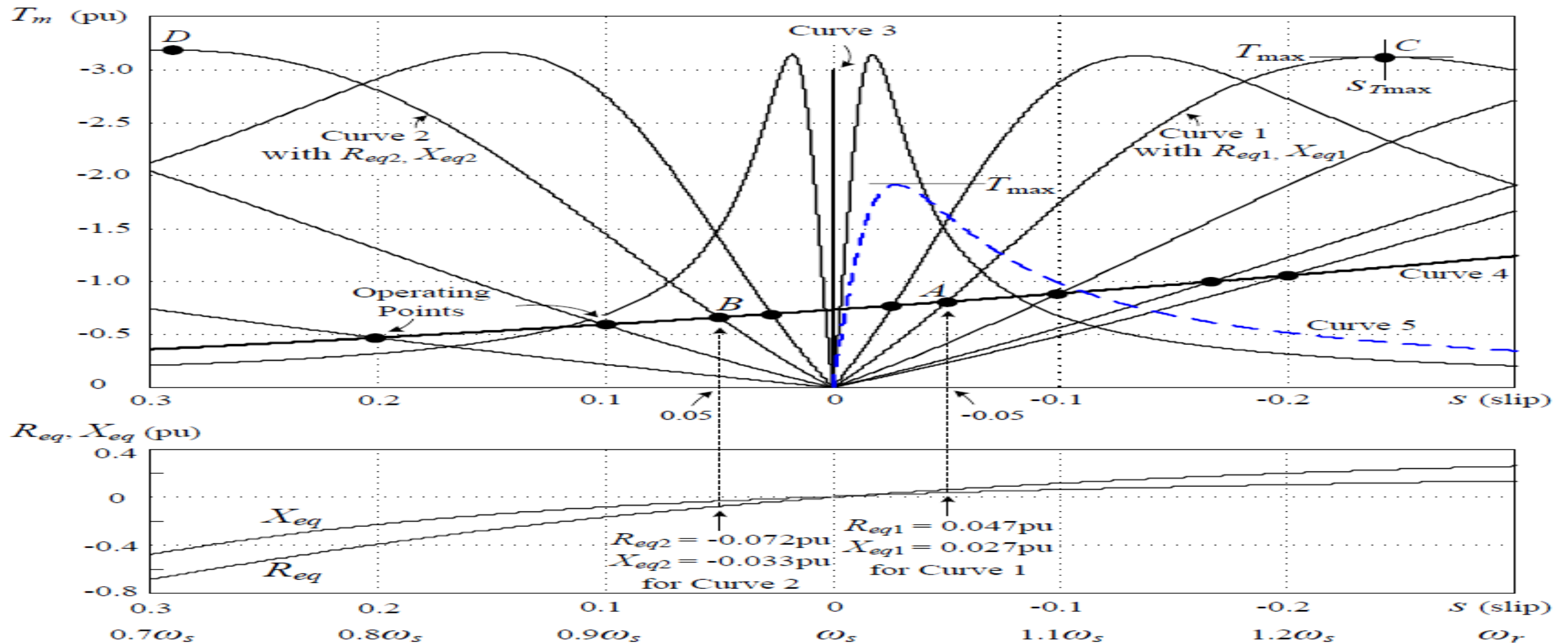


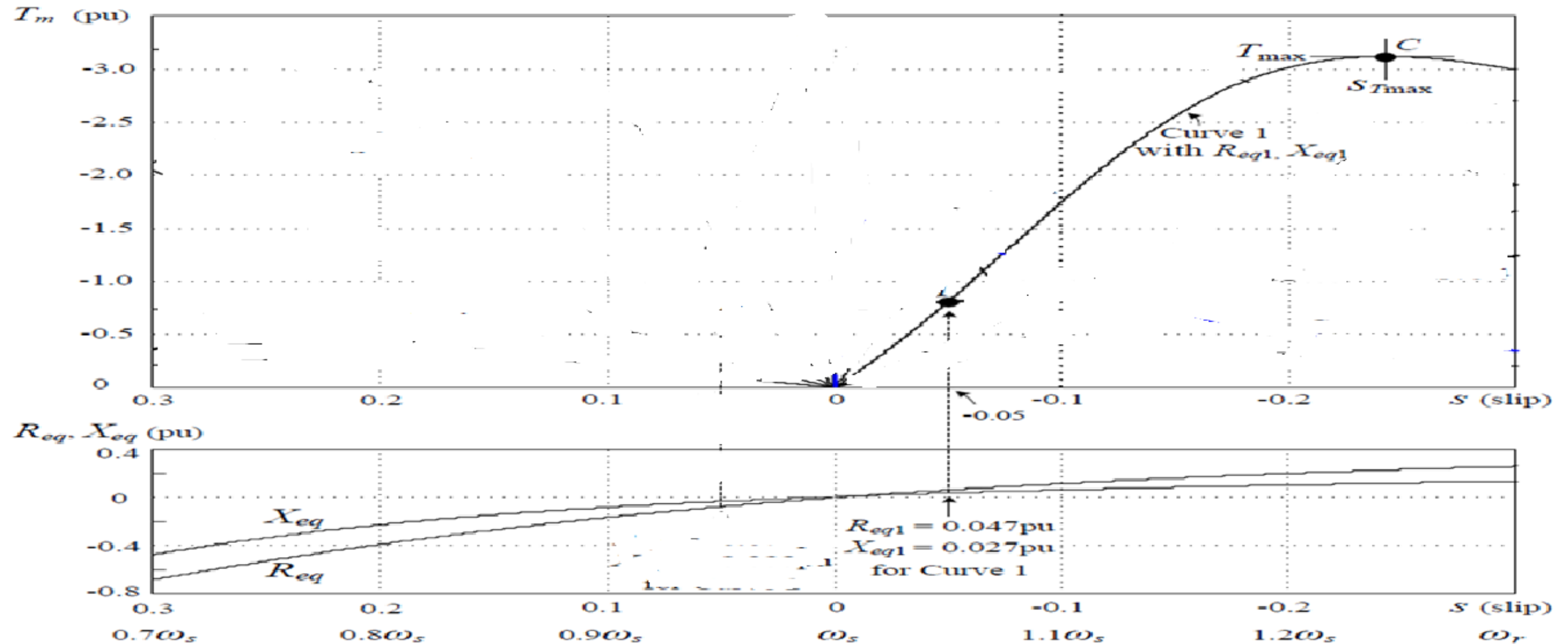
Fig. shows set of torque-slip curves of DFIG operating at super-synchronous synchronous & sub-synchronous speeds.



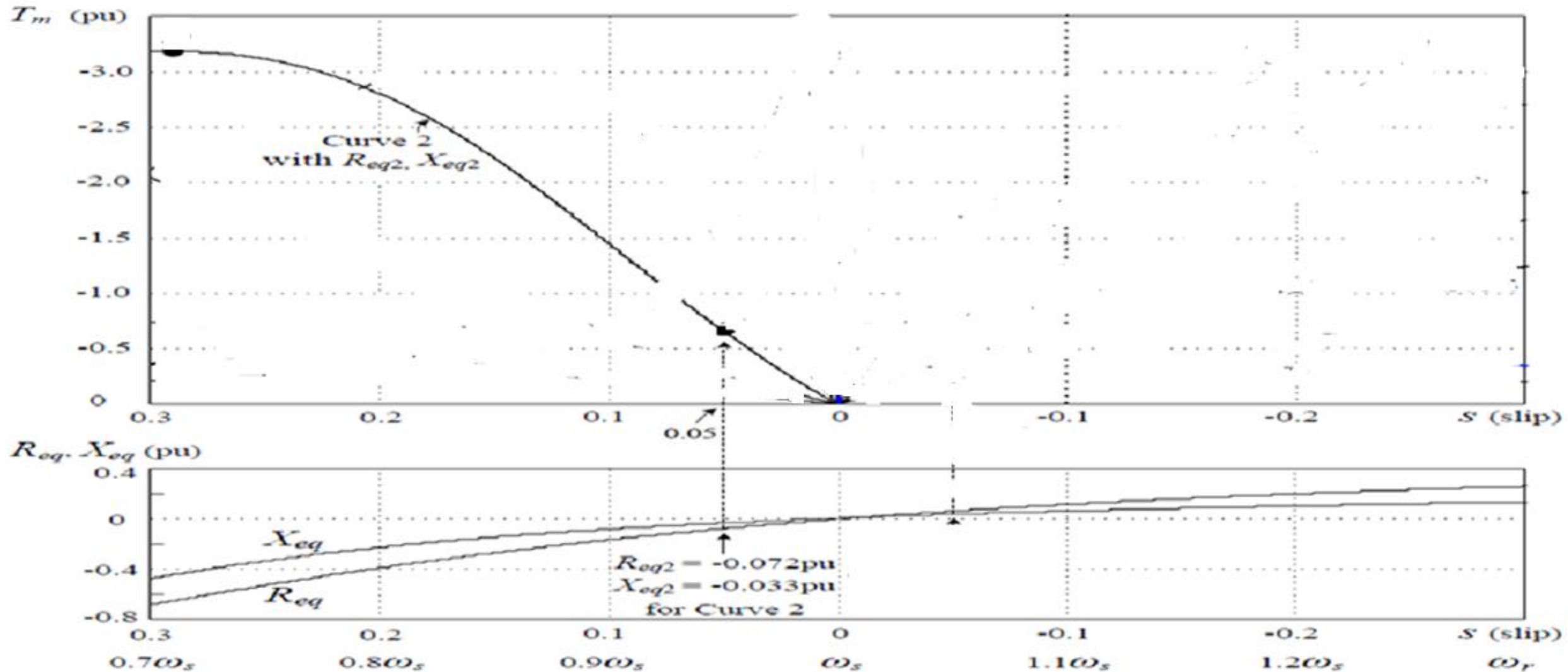
With different values of R_{eq} & X_{eq} , different torque-slip characteristics can be obtained.



With $Req_1=+0.047\text{pu}$ & $Xeq_1=+0.027\text{pu}$ for 1.5MW/690V DFIG, torque-slip Curve 1 is obtained,

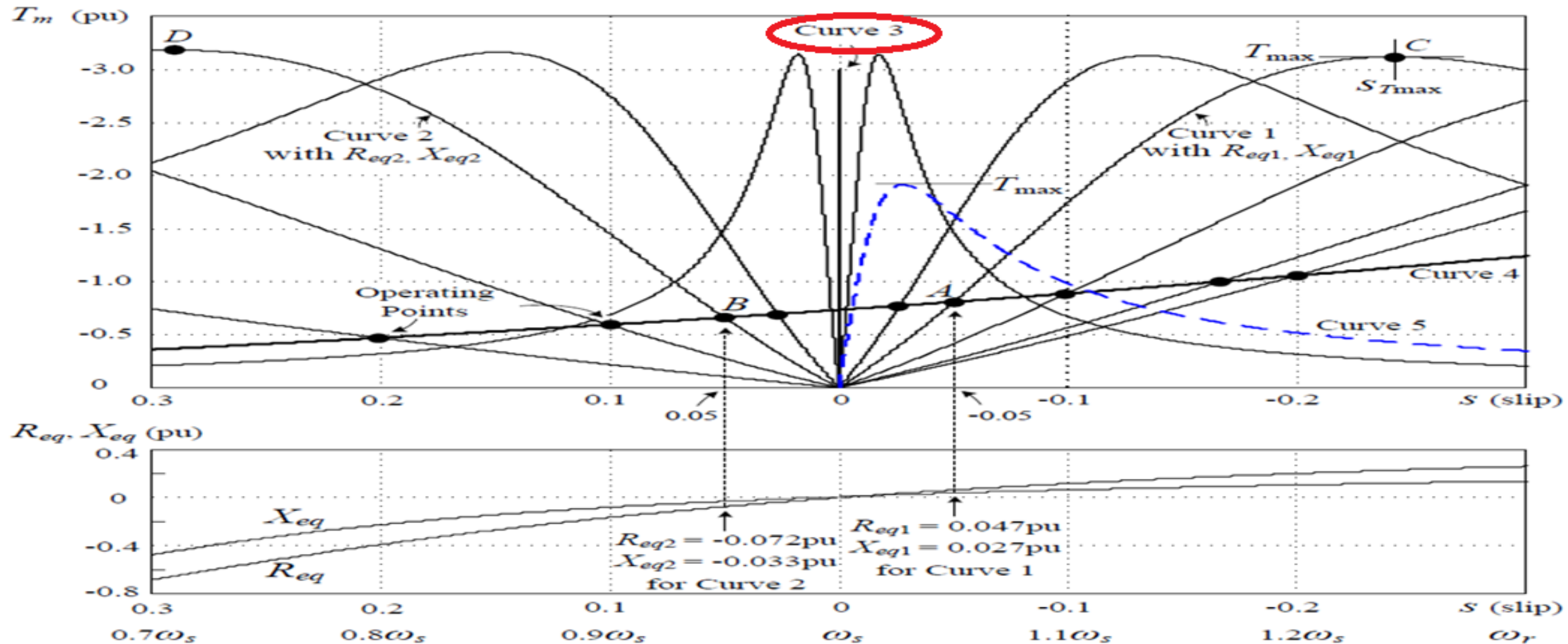


Curve 2 corresponds to $Req_2 = -0.072$ pu & $Xeq_2 = -0.033$ pu.



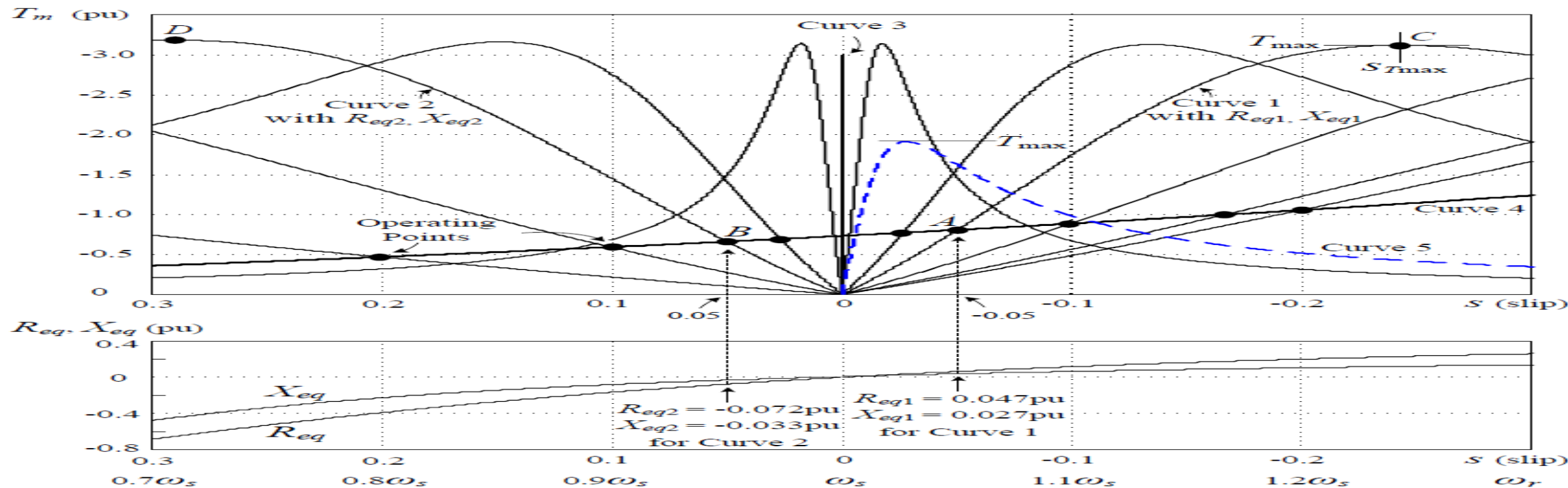
At synchronous speed, what should be the shape of torque-slip curve?

At synchronous speed, torque-slip curve is a straight line (Curve 3).

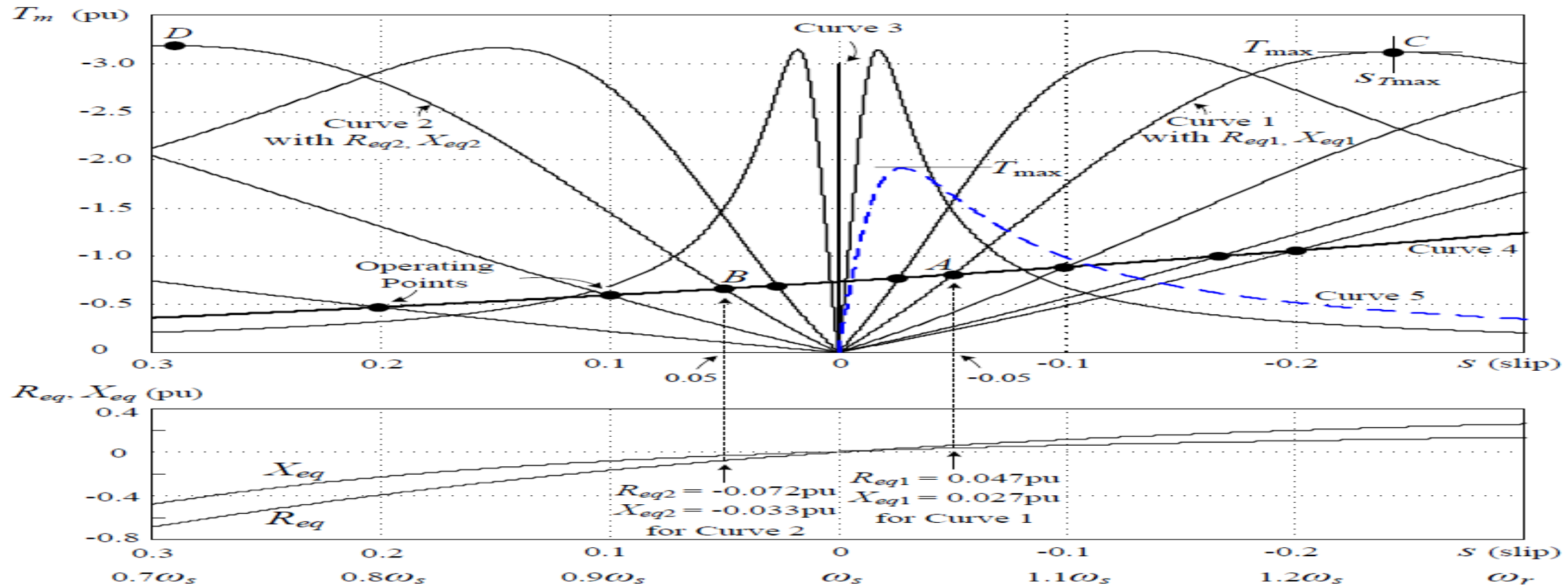


In an MPPT controlled wind energy system, mechanical torque is proportional to square of generator speed
 $(T_m \propto \omega_r^2)$

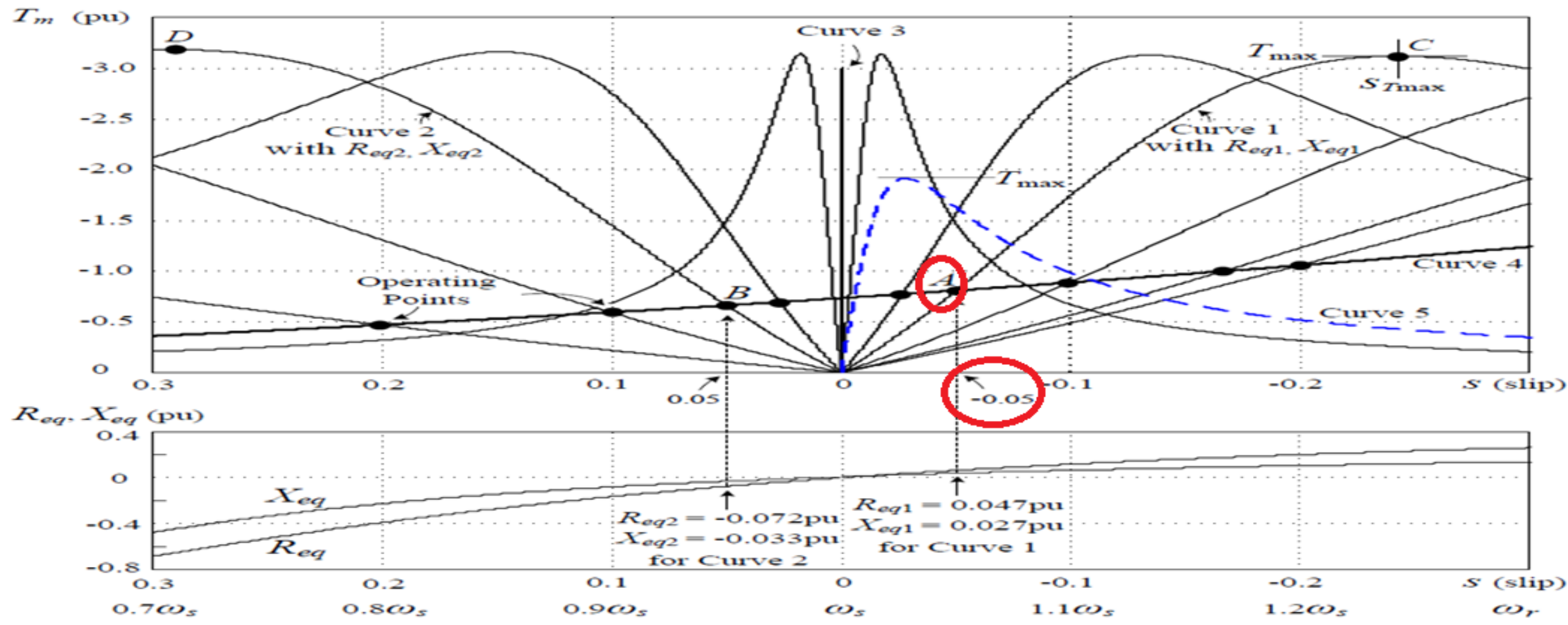
- This relationship is illustrated by Curve 4 . Intersections of this curve with other torque-slip curves are steady-state operating points of generator.



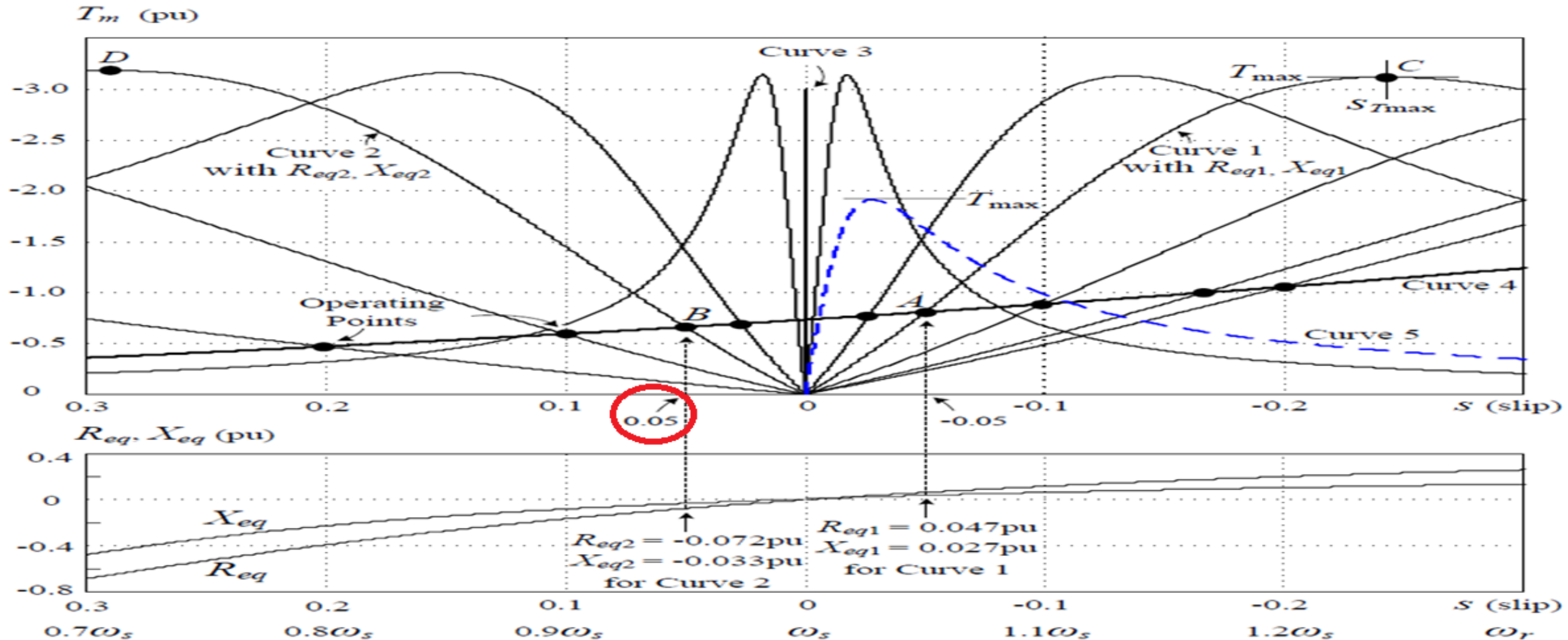
At higher wind speed, generator operates at point?



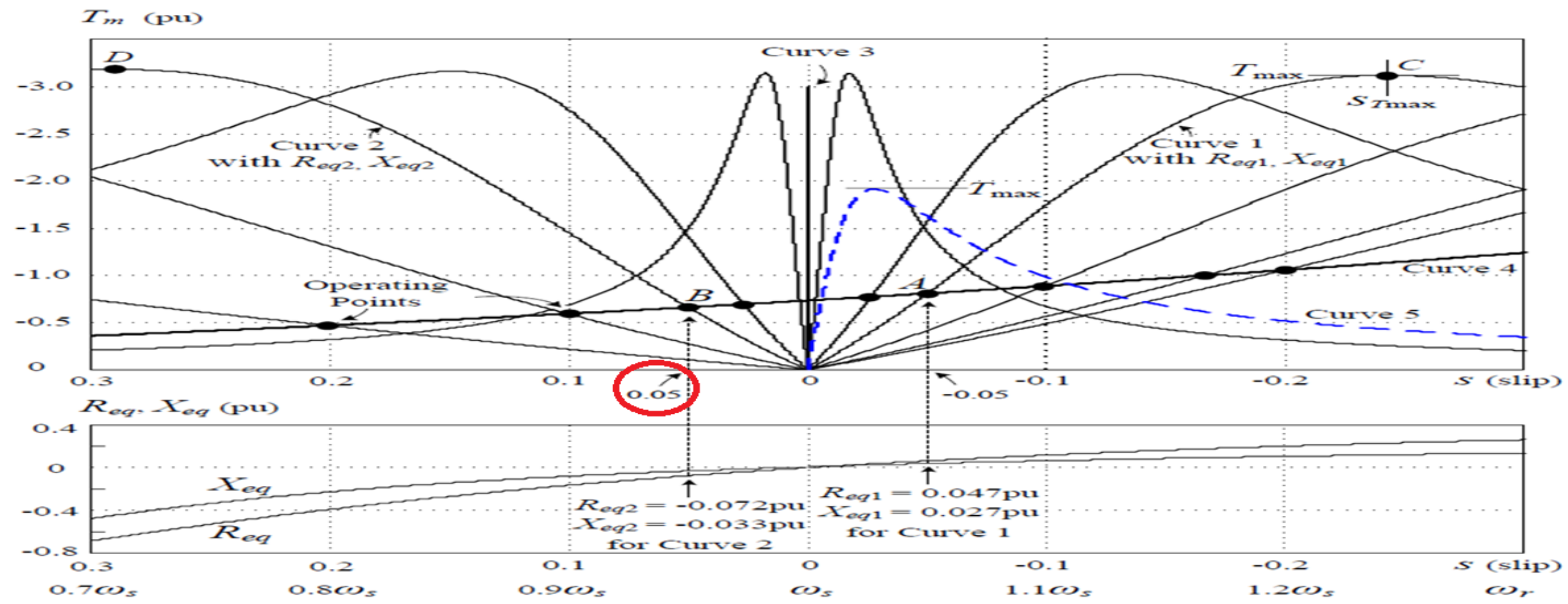
At higher wind speed ,generator operates at point A where Curve 4 intersects with Curve 1, generator operates in super-synchronous mode with a slip=-0.05.



At a lower wind speed, generator operates at point?

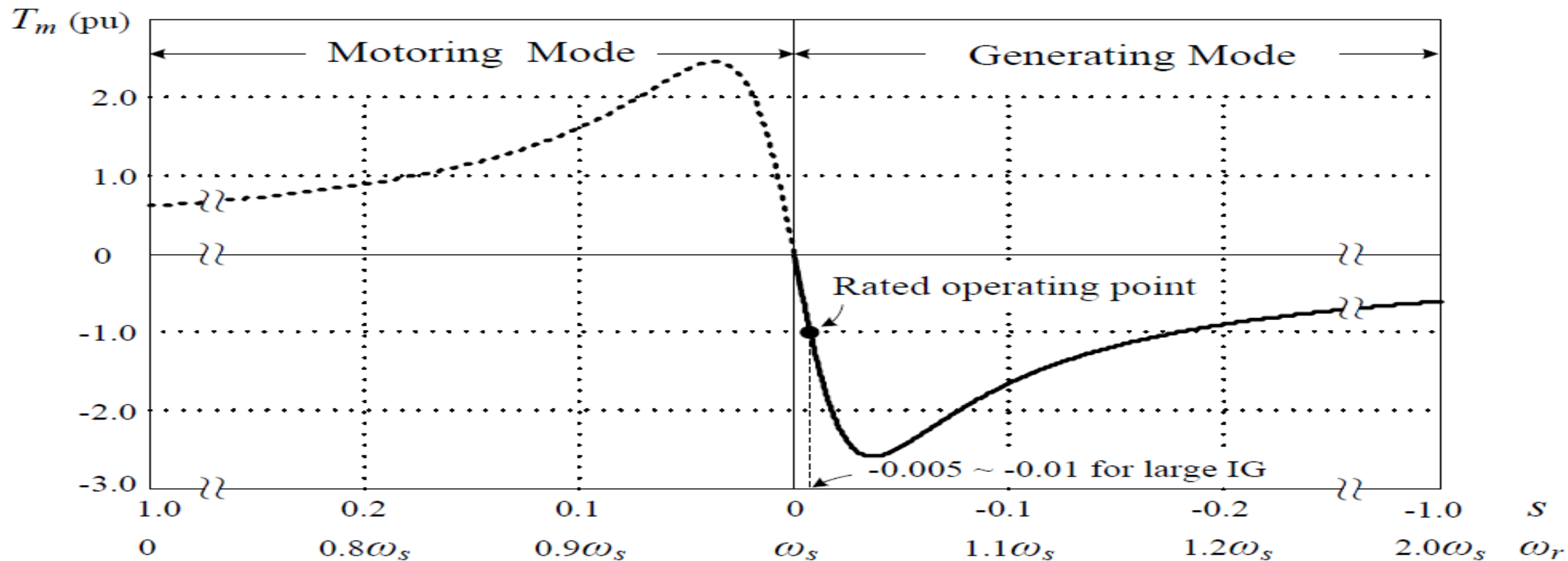


At a lower wind speed, generator operates at Point B, where Curve 4 intersects with Curve 2 & generator is in sub-synchronous operating mode with a slip of +0.05.

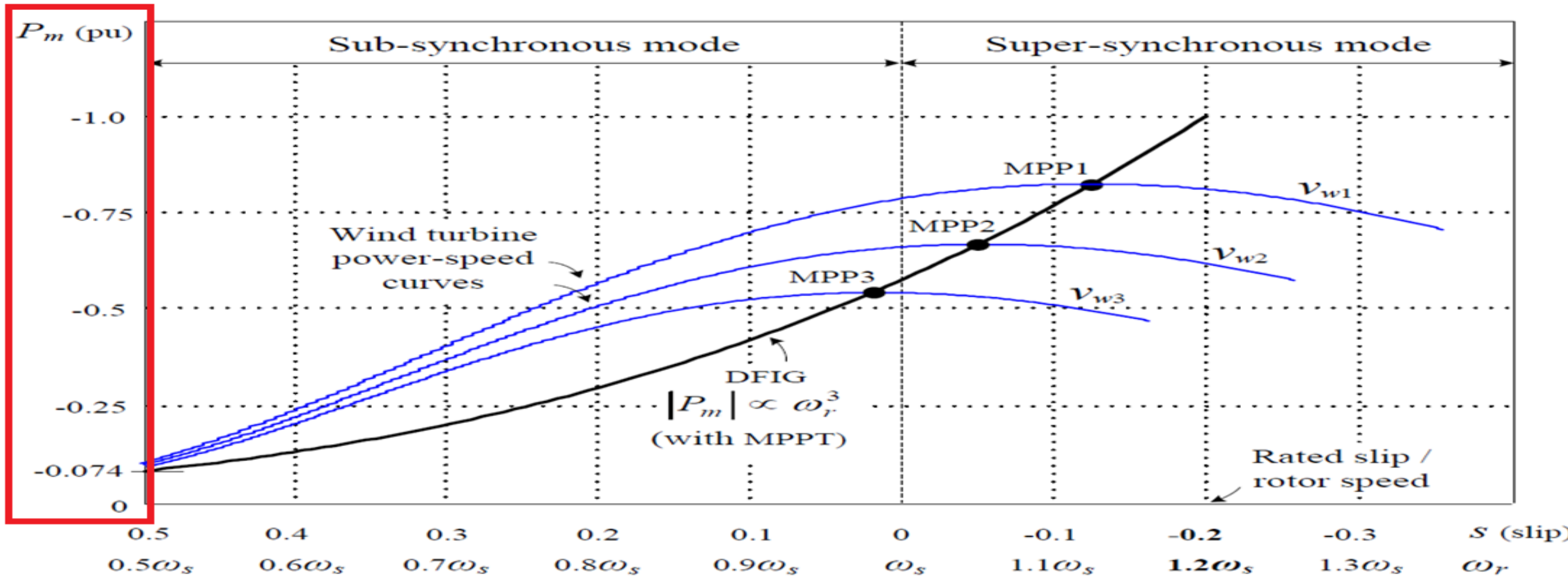


Operation of SCIG vs DFIG

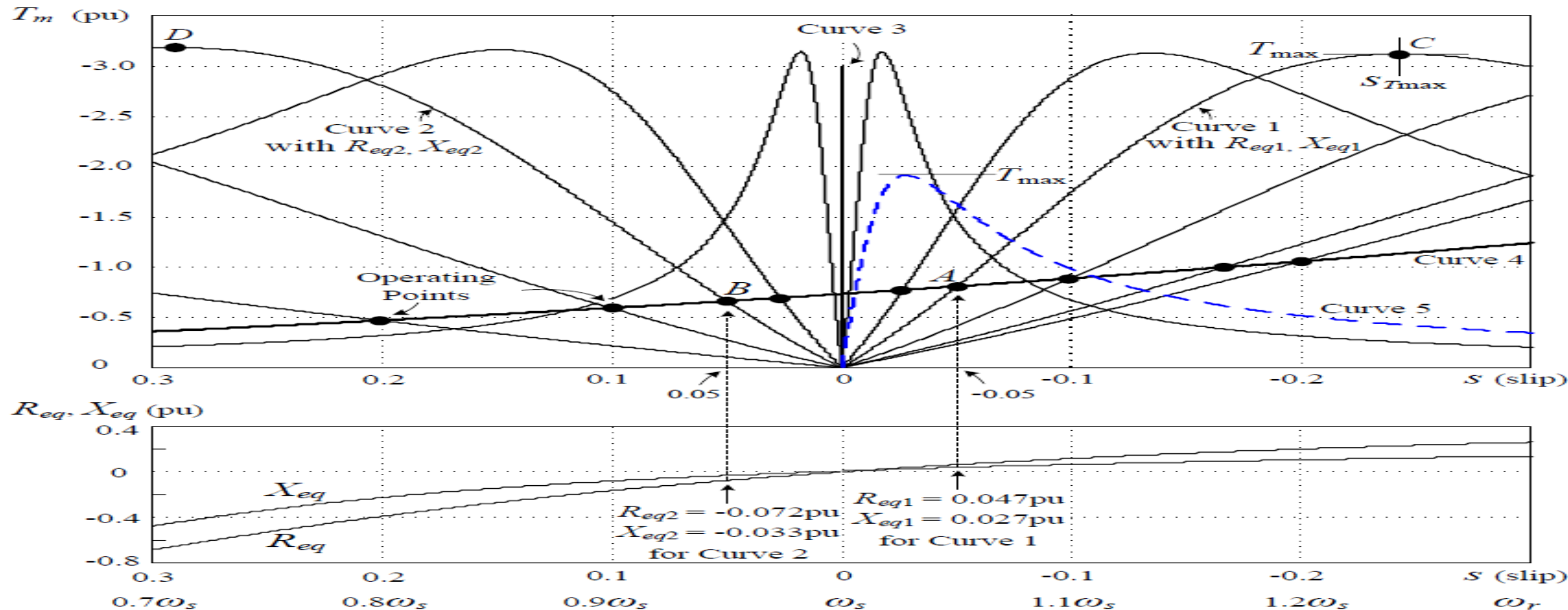
In SCIG, generator is in motoring mode when slip is +ve, while in generating mode with a -ve slip.



Whether slip is +ve or -ve, DFIG is always in generating mode & delivers its mechanical power $|P_m|$ to grid.



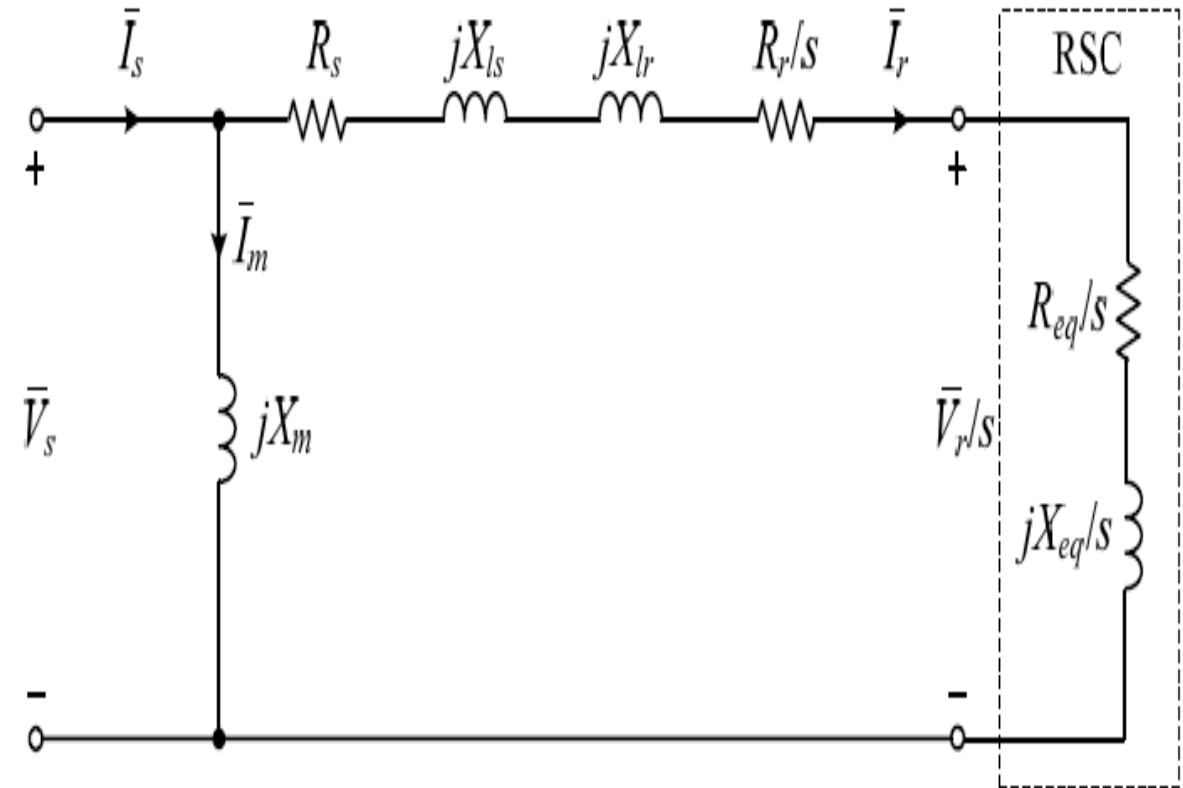
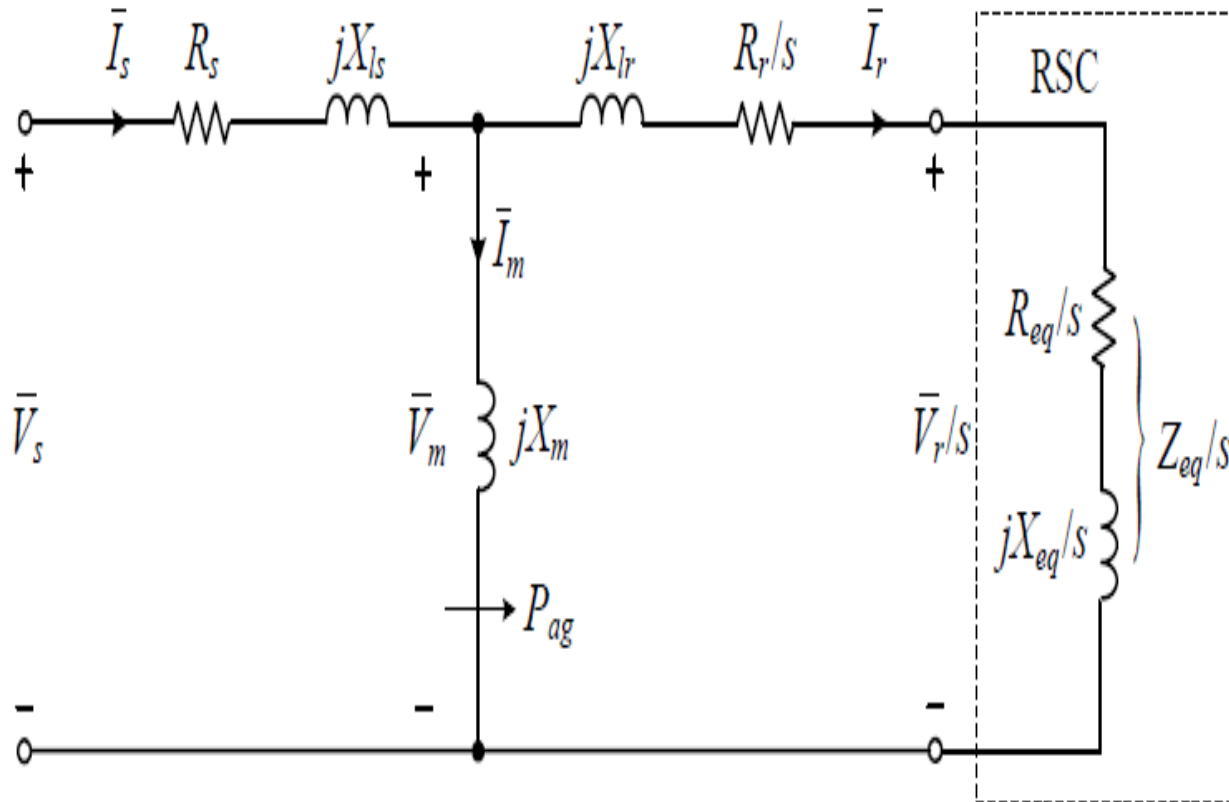
For comparison purposes, torque-slip characteristics of DFIG with rotor circuit shorted ($R_{eq} = X_{eq} = 0$) is given by Curve 5, whose maximum torque T_{max} is much lower than that of torque-slip curves with R_{eq} & X_{eq} taken into account.



To find maximum torque of DFIG, a simplified steady-state equivalent circuit can be used, where magnetizing branch is moved to left of stator circuit

Steady-state equivalent circuit of DFIG with rotor-side converter represented by R_{eq} and X_{eq} .

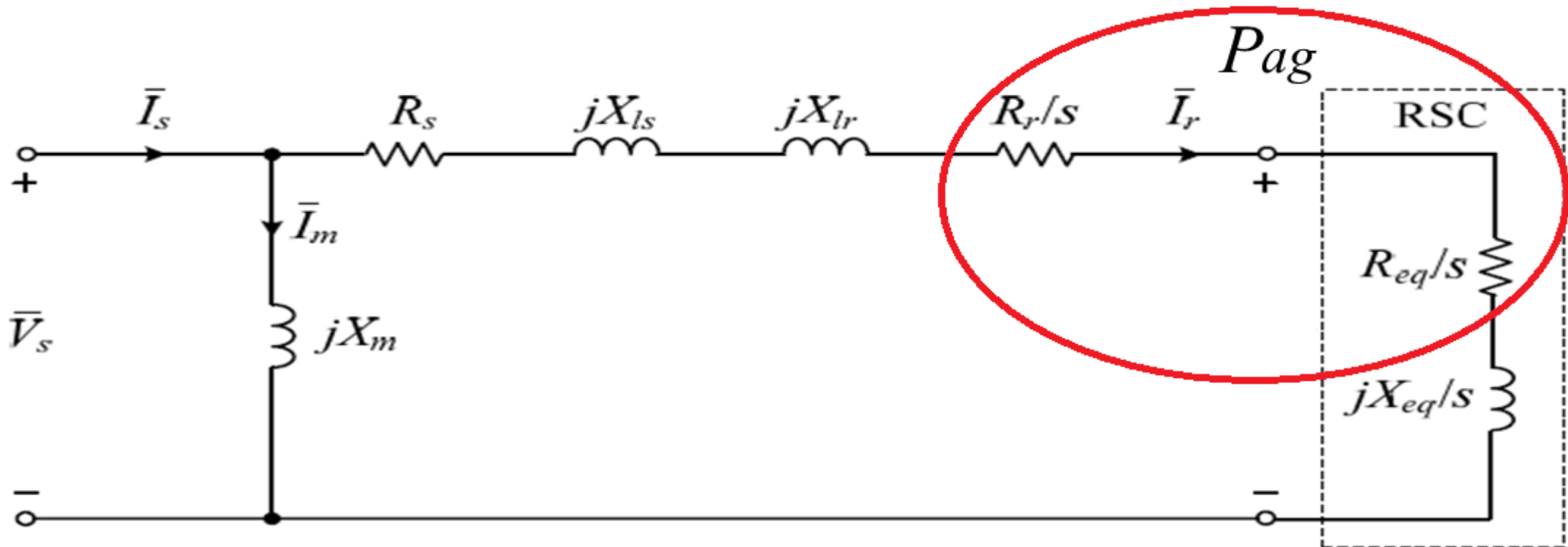
Simplified DFIG steady-state equivalent circuit for calculation of T_{max} .



Maximum torque of DFIG with simplified steady-state equivalent circuit

$$T_m = \frac{P_{ag}}{\omega_s / P}$$

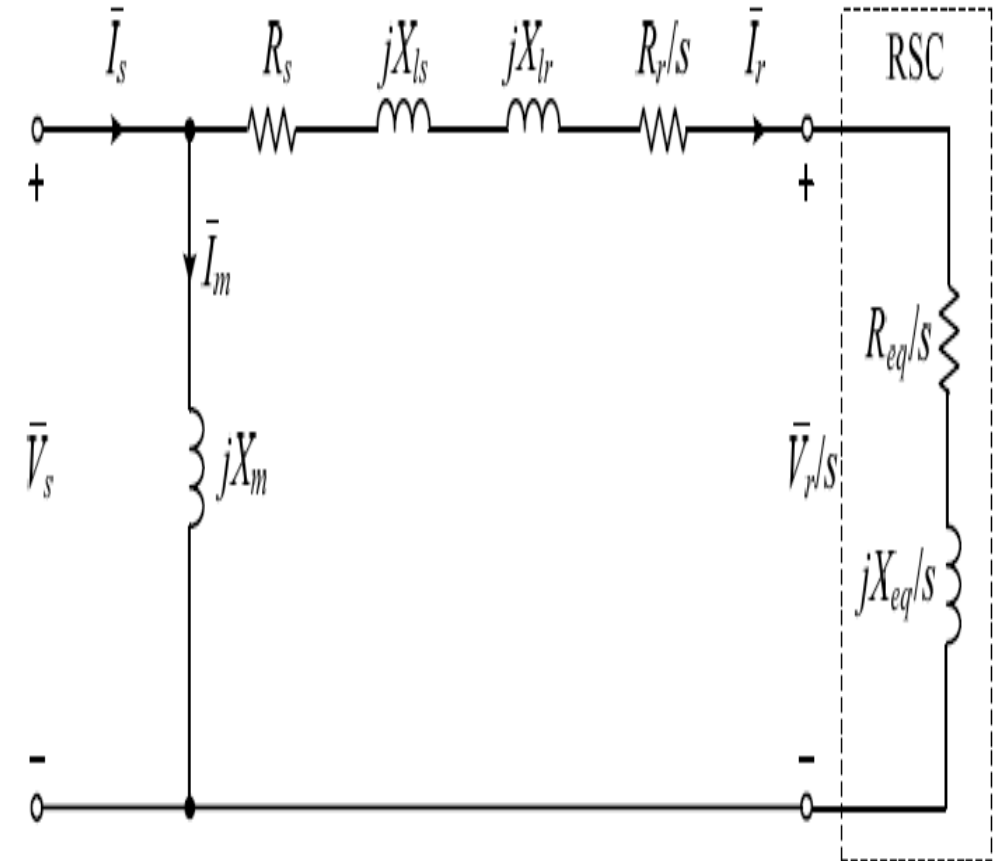
$$P_{ag} = \left(3I_r^2\right) \frac{R_r + R_{eq}}{s}$$



Maximum torque of DFIG with simplified steady-state equivalent circuit

$$T_m = \frac{1}{\omega_s / P} \times (3I_r^2) \frac{R_r + R_{eq}}{s}$$

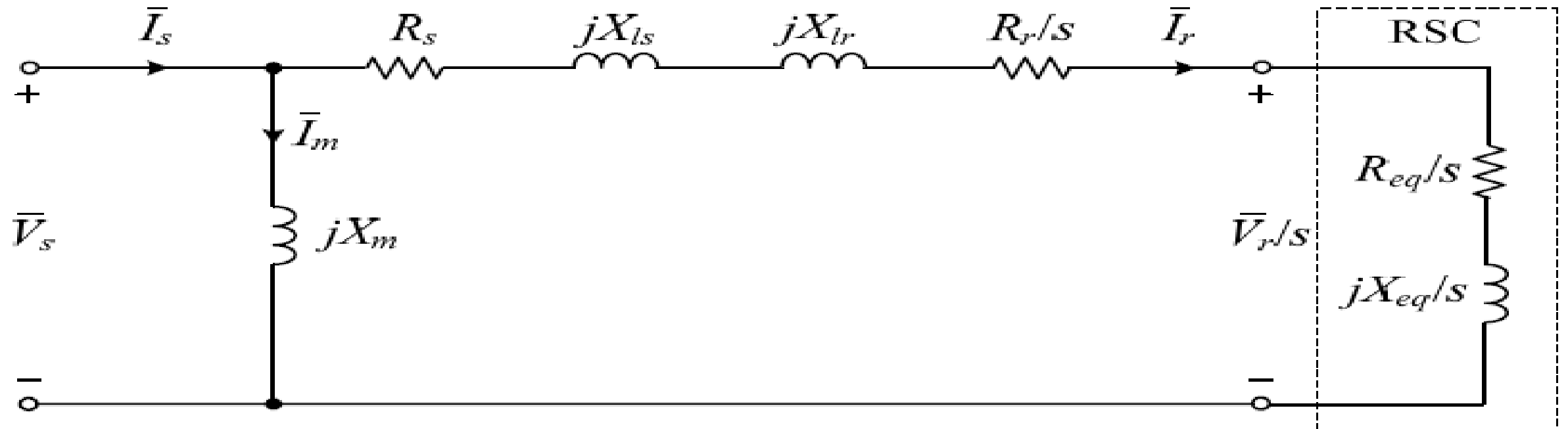
$$I_r = \frac{V_s}{(R_s + R_r / s + R_{eq} / s) + j(X_{ls} + X_{lr} + X_{eq} / s)}$$



$$T_m = \frac{1}{\omega_s / P} \times (3I_r^2) \frac{R_r + R_{eq}}{s}$$

$$T_m = \frac{1}{\omega_s / P} \times \frac{3V_s^2}{(R_s + R_r / s + R_{eq} / s)^2 + (X_{ls} + X_{lr} + X_{eq} / s)^2} \times \frac{R_r + R_{eq}}{s}$$

$$I_r = \frac{V_s}{(R_s + R_r / s + R_{eq} / s) + j(X_{ls} + X_{lr} + X_{eq} / s)}$$



Maximum torque T_{\max} & slip at maximum torque $s_{T\max}$ can be obtained by setting $dT_m / ds = 0$, from which

$$T_m = \frac{1}{\omega_s / P} \times \frac{3V_s^2}{(R_s + R_r / s + R_{eq} / s)^2 + (X_{ls} + X_{lr} + X_{eq} / s)^2} \times \frac{R_r + R_{eq}}{s}$$

$$s_{T\max} = \pm \sqrt{\frac{(R_r + R_{eq})^2 + X_{eq}^2}{R_s^2 + (X_{ls} + X_{lr})^2}}$$

- Plus & minus signs in above equation signify the sub- and super-synchronous modes of operation.

Home assignment

Obtain slip at maximum torque $s_{T\max}$ by setting $dT_m / ds = 0$?

$$T_m = \frac{1}{\omega_s / P} \times \frac{3V_s^2}{(R_s + R_r / s + R_{eq} / s)^2 + (X_{ls} + X_{lr} + X_{eq} / s)^2} \times \frac{R_r + R_{eq}}{s}$$

$$s_{T\max} = \pm \sqrt{\frac{(R_r + R_{eq})^2 + X_{eq}^2}{R_s^2 + (X_{ls} + X_{lr})^2}}$$

Maximum torque can be found by substituting slip at maximum torque

into

$$s_{T \max} = \pm \sqrt{\frac{(R_r + R_{eq})^2 + X_{eq}^2}{R_s^2 + (X_{ls} + X_{lr})^2}}$$

$$T_m = \frac{1}{\omega_s / P} \times \frac{3V_s^2}{(R_s + R_r / s + R_{eq} / s)^2 + (X_{ls} + X_{lr} + X_{eq} / s)^2} \times \frac{R_r + R_{eq}}{s}$$

We get

$$T_{\max} = \frac{1}{2\omega_s / P} \times \frac{3V_s^2}{R_s + \frac{(X_{ls} + X_{lr})X_{eq}}{R_r + R_{eq}} - \sqrt{\left((X_{ls} + X_{lr})^2 + R_s^2\right) \times \left(1 + \frac{X_{eq}^2}{(R_r + R_{eq})^2}\right)}}$$

Above equation is valid for both super- & sub-synchronous modes of operation

Problems

Topic: Steady-state Equivalent Circuit of DFIG with Rotor-side Converter

8-1 (Solved Problem) A 1.0MW/575V/60Hz/2160rpm DFIG is used in a variable-speed wind energy conversion system. Parameters of generator are given in Table . Generator operates with an MPPT scheme & its stator power factor is unity. Assuming that the DFIG operates at a super-synchronous speed of 2160 rpm, determine following:

- a) generator mechanical torque & power,
- b) rms stator current,
- c) rms magnetizing voltage and current,
- d) rms rotor current and voltage,
- e) equivalent resistance and reactance for rotor side converter, and
- f) maximum torque & corresponding slip.

Table B-6 1.0MW/575V/60Hz DFIG Parameters

Generator Type	DFIG, 1.0MW/575V/60Hz	
Rated Mechanical Power	1.0 MW	1.0 pu
Rated Stator Line-to-line Voltage	575 V (rms)	
Rated Stator Phase Voltage	331.98 V (rms)	1.0 pu
Rated Rotor Phase Voltage	67.97 V (rms)	0.2047 pu
Rated Stator Current	829.2 A (rms)	0.8258 pu
Rated Rotor Current	882.2 A (rms)	0.8786 pu
Rated Stator Frequency	60 Hz	1.0 pu
Rated Rotor Speed	2160 rpm	1.0 pu
Nominal Rotor Speed Range	1350–2160 rpm	0.625–1.0pu
Rated Slip	-0.2	
Number of Pole Pairs	2	
Rated Mechanical Torque	4.421 kN.m	1.0 pu
Stator Winding Resistance R_s	3.654 m Ω	0.0111 pu
Rotor Winding Resistance R_r	3.569 m Ω	0.0108 pu
Stator Leakage Inductance L_{ls}	0.1304 mH	0.1487 pu
Rotor Leakage Inductance L_{lr}	0.1198 mH	0.1366 pu
Magnetizing Inductance L_m	4.12 mH	4.6978 pu
Base Current $I_B = 1\text{MW}/(\sqrt{3} \times 575\text{V})$	1004.1 A (rms)	1.0 pu
Base Flux Linkage λ_B	0.8806 Wb (rms)	1.0 pu
Base Impedance Z_B	0.3306 Ω	1.0 pu
Base Inductance L_B	0.877 mH	1.0 pu
Base Capacitance C_B	8022.93 μF	1.0 pu

Solution:

a) Rotor mechanical speed in rad/sec:

Super-synchronous speed=2160 rpm

$$\omega_m = 2160 \times (2\pi / 60) = 226.19 \text{ rad/sec}$$

Rotor electrical speed:

$$\omega_r = \omega_m \times P = 226.19 \times 2 = 452.39 \text{ rad/sec}$$

Rated rotor mechanical speed in rad/sec:

Rated Rotor Speed	2160 rpm
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$$\omega_{m,R} = 2160 \times (2\pi / 60) = 226.19 \text{ rad/sec}$$

Stator frequency:

$$\omega_s = 2\pi \times 60 = 376.99 \text{ rad/sec}$$

The pu rotor speed

$$\omega_{m,\text{pu}} = \omega_m / \omega_{m,R} = 226.19 / 226.19 = 1.0 \text{ pu}$$

Generator mechanical torque at 1.0 pu rotor speed:

Rated Mechanical Torque	4.421 kN.m
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$$T_m = T_{m,R} \times (\omega_{m,\text{pu}})^2 = -4421 \times (1.0)^2 = -4421 \text{ N.m}$$

Rated mechanical power:

$$P_{m,R} = \omega_{m,R} \times T_{m,R} = 226.19 \times (-4421) = -1000 \times 10^3 \text{ W}$$

Generator mechanical power at 1.0 pu rotor speed:

Rated Mechanical Power	1.0 MW
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$$P_m = P_{m,R} \times (\omega_{m,\text{pu}})^3 = -1000 \times 10^3 \times (1.0)^3 = -1000 \times 10^3 \text{ W}$$

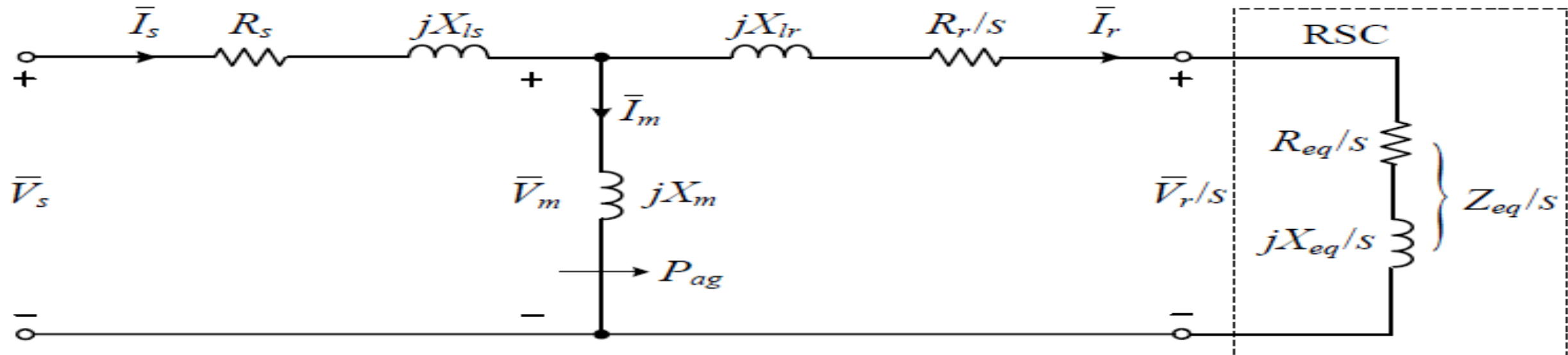
b) Stator current

$$I_s = \frac{V_s \pm \sqrt{V_s^2 - \frac{4R_s T_m \omega_s}{3P}}}{2R_s} = -829.18 \text{ A (rms)} \quad (I_s = 91.682 \times 10^3 \text{ A omitted})$$

where $V_s = 575/\sqrt{3}$ V, $T_m = -4421$ N.m, $\omega_s = 376.99$ rad/sec, $R_s = 3.654$ m Ω and $P = 2$

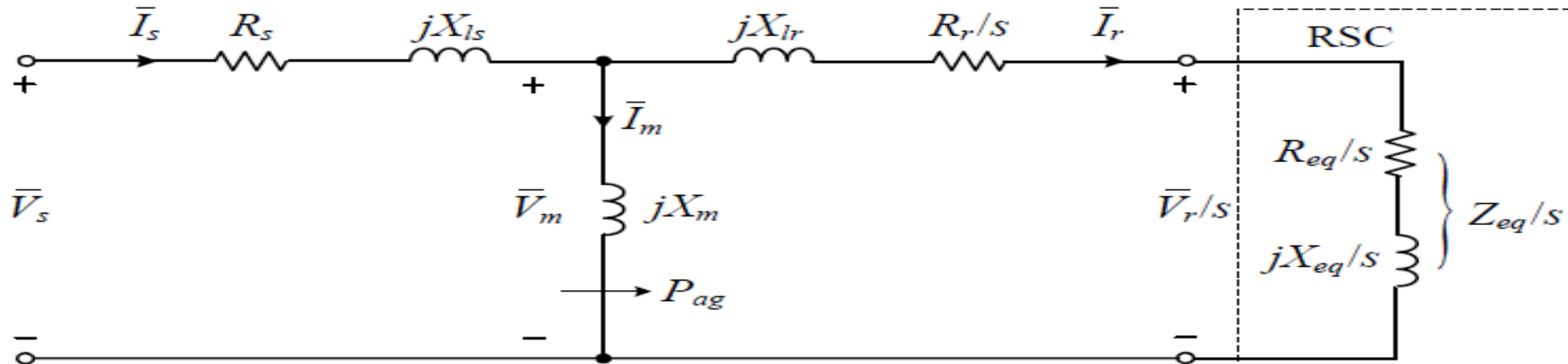
c) Magnetizing branch voltage:

$$\begin{aligned}
 \bar{V}_m &= \bar{V}_s - \bar{I}_s (R_s + j\omega_s L_{ls}) \\
 &= 575 / \sqrt{3} \angle 0^\circ - 829.18 \angle 180^\circ \times (3.654 \times 10^{-3} + j120\pi \times 0.1304 \times 10^{-3}) \\
 &= 337.48 \angle 6.94^\circ \text{ V (rms)}
 \end{aligned}$$



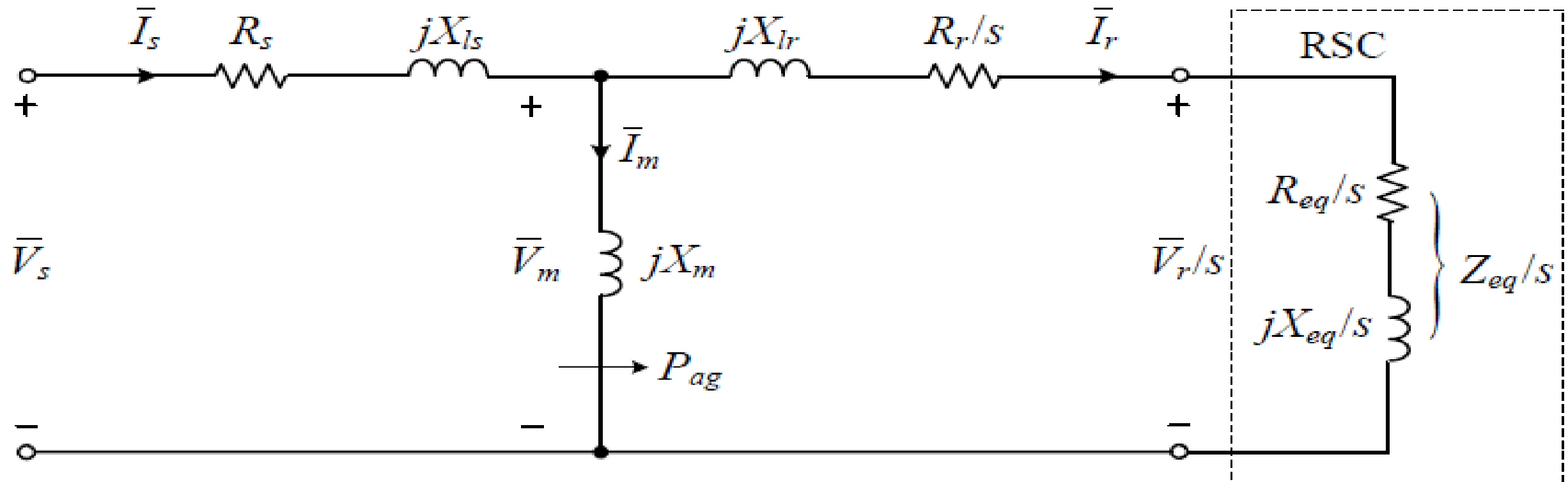
The magnetizing current can be calculated by

$$\bar{I}_m = \frac{\bar{V}_m}{j\omega_s L_m} = 217.28 \angle -83.1^\circ \text{ A (rms)}$$



d) The rotor current:

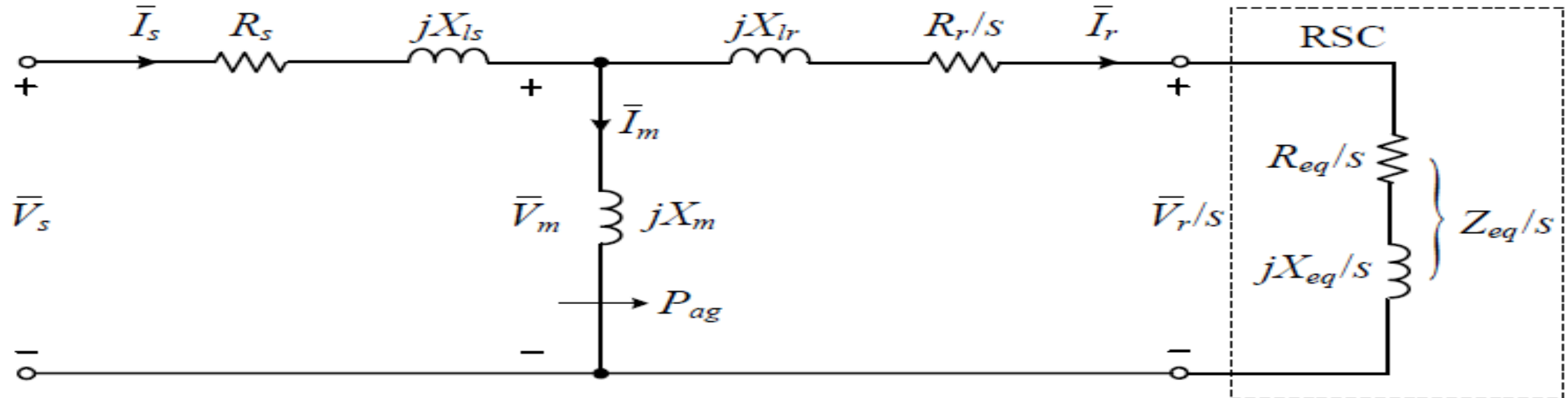
$$\bar{I}_r = \bar{I}_s - \bar{I}_m = 829.18 \angle 180^\circ - 217.28 \angle -83.1^\circ = 882.19 \angle 165.85^\circ \text{ A (rms)}$$



Rotor voltage:

$$\bar{V}_r = s \bar{V}_m - \bar{I}_r (R_r + js\omega_s L_{lr}) = 67.97 \angle -165.83^\circ \text{ V (rms)}$$

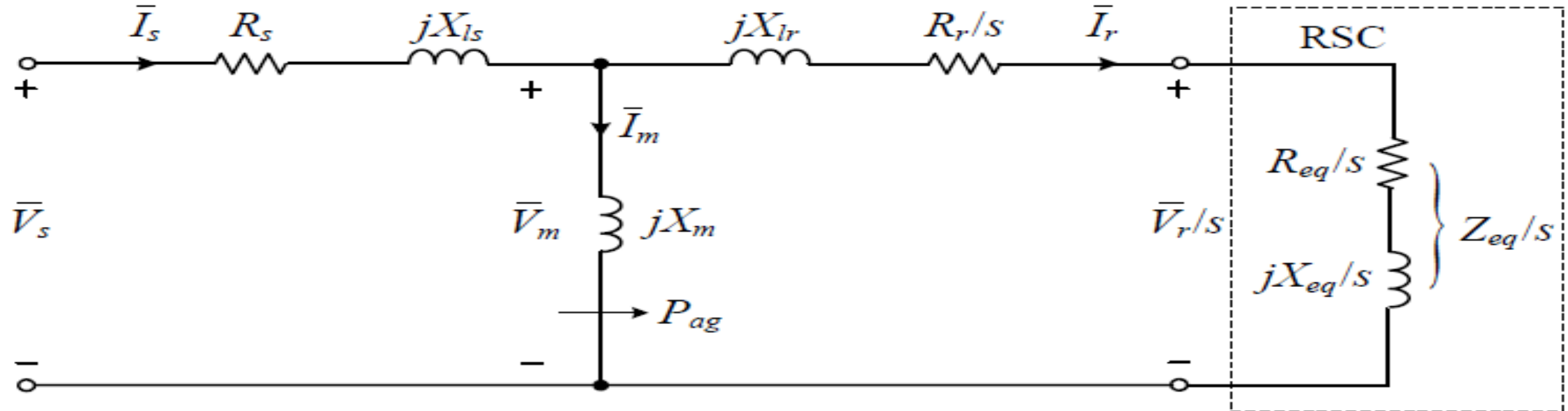
where $s = (\omega_s - \omega_r) / \omega_s = (376.99 - 452.39) / 376.99 = -0.2$



e) Equivalent impedance for rotor side converter is given by

$$\bar{Z}_{eq} = \bar{V}_r / \bar{I}_r = 0.06782 + j0.03656 \ \Omega$$

from which $R_{eq} = 0.06782 \ \Omega$ and $X_{eq} = 0.03656 \ \Omega$



f) The slip at which the maximum torque occurs can be obtained

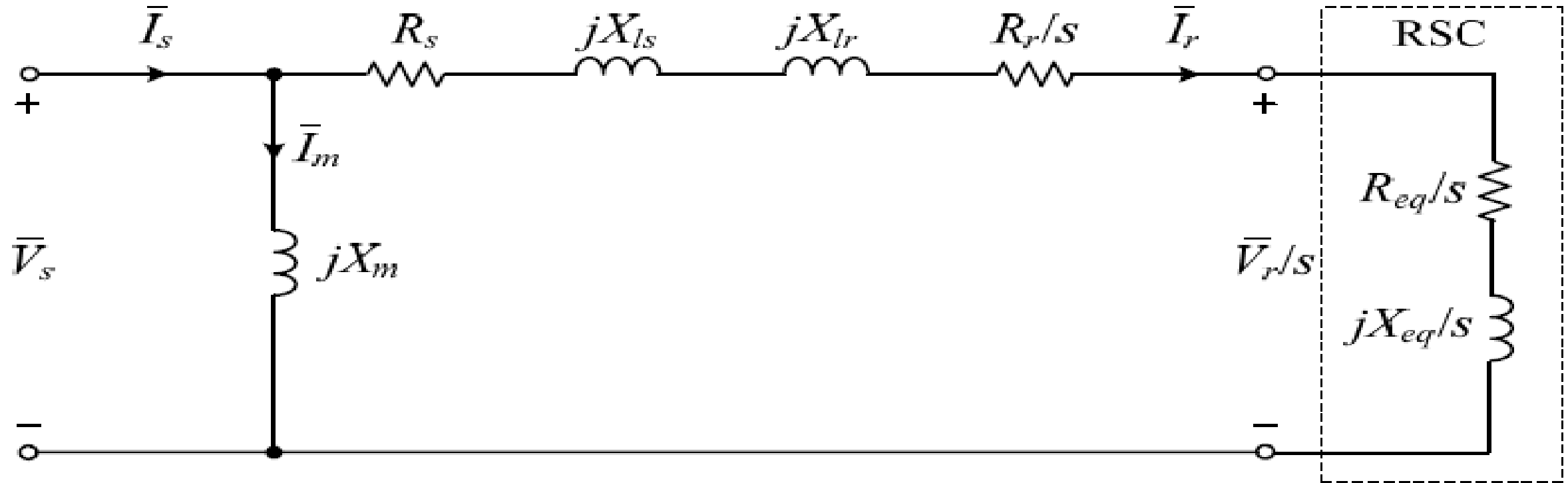
$$s_{T_{\max}} = \pm \sqrt{\frac{(R_r + R_{eq})^2 + X_{eq}^2}{R_s^2 + (X_{ls} + X_{lr})^2}} = -0.8497 \quad (s = +0.8497 \text{ is omitted because of the super-synchronous mode of operation})$$

The maximum torque:

$$T_{\max} = \frac{1}{2\omega_s / P} \times \frac{3V_s^2}{R_s + \frac{(X_{ls} + X_{lr})X_{eq}}{R_r + R_{eq}} - \sqrt{\left((X_{ls} + X_{lr})^2 + R_s^2\right) \times \left(1 + \frac{X_{eq}^2}{(R_r + R_{eq})^2}\right)}} = -16214 \text{ N.m} \quad (3.6675 \text{ pu})$$

Cross Check:

Maximum torque of DFIG with simplified steady-state equivalent circuit



$$T_m = \frac{1}{\omega_s / P} \times \left(3 I_r^2 \right) \frac{R_r + R_{eq}}{s}$$

Cross Check:

$$T_m = \frac{1}{\omega_s / P} \times 3I_r^2 (R_{eq} + R_r) / s = \frac{1}{2\pi \times 60 / 2} \times 3 \times 882.19^2 (0.06782 + 3.569 \times 10^{-3}) / (-0.2) = -4421 \text{ N.m, verified.}$$

$$P_m = 3I_r^2 (R_{eq} + R_r) (1 - s) / s = 3 \times 882.19^2 (0.06782 + 3.569 \times 10^{-3}) (1 + 0.2) / (-0.2) = -1000 \times 10^3 \text{ W, verified.}$$

8-2 Repeat Problem 8-1 when DFIG operates at synchronous speed of 1800 rpm.

Answers:

a) $T_m = -3070.1 \text{ N.m}$, $P_m = -578.71 \times 10^3 \text{ W}$

b) $I_s = -577.14 \text{ A (rms)}$

c) $\bar{V}_m = 335.29 \angle 4.86^\circ \text{ V (rms)}$,

$\bar{I}_m = 215.87 \angle -85.14^\circ \text{ A (rms)}$

d) $\bar{I}_r = 633.32 \angle 160.15^\circ \text{ A (rms)}$, $\bar{V}_r = 2.26 \angle -19.85^\circ \text{ V (rms)}$

e) $R_{eq} = -0.00357 \Omega$, $X_{eq} = 0 \Omega$

f) $s_{T_{\max}} = 0$, $T_{\max} = -6752 \text{ N.m (1.5272 pu)}$

8-3 Repeat Problem 8-1 when DFIG operates at sub-synchronous speed of 1350 rpm.

Answers:

- a) $T_m = -1726.9 \text{ N.m}$, $P_m = -244.14 \times 10^3 \text{ W}$ b) $I_s = -325.69 \text{ A (rms)}$ c) $\bar{V}_m = 333.55 \angle 2.75^\circ \text{ V (rms)}$,
 $\bar{I}_m = 214.75 \angle -87.25^\circ \text{ A (rms)}$ d) $\bar{I}_r = 398.63 \angle 147.45^\circ \text{ A (rms)}$, $\bar{V}_r = 87.2 \angle 4.63^\circ \text{ V (rms)}$
e) $R_{eq} = -0.17428 \Omega$, $X_{eq} = -0.13219 \Omega$ f) $s_{T_{\max}} = 2.2873$, $T_{\max} = -20542 \text{ N.m (4.6465 pu)}$