Wideband Two-Section Impedance Transformer With Flat Real-to-Real Impedance Matching

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***Abstract*—This letter presents the design of an impedance transformer with wideband, maximally flat real-to-real impedance matching. The design formulas for two-section quarter-wave transformer are presented and exact solutions for transmission lines' parameters are derived in explicit form for any impedance transformation ratio. The results of this study are useful for a number of practical design problems, especially power amplification circuits. To validate the design formulas, three impedance transformers terminated in a fixed impedance of 50 Ω and three target impedances of 100, 150, and 200 Ω are fabricated and measured. Measurements show a good agreement with theory and simulations.**

***Index Terms*—Impedance transformer, power amplifier, quarter-wave transmission line, two-section, wideband matching.**

I. Introduction

IMPEDANCE transformers are one of the most important components in many microwave circuits, such as power dividers/combiners, antennas, and power amplifiers (PAs) [1]–[7]. In PA circuits, an impedance transformer is useful for matching an output load, R_L, to the optimal impedance of the device, $R_{\rm opt}$. The Doherty PA, for instance, is a popular power amplification topology that requires the use of impedance transformers to operate [5]–[7].

In principle, to obtain broadband PA operation, the impedance transformation, R_L-to-$R_{\rm opt}$, should be maintained over a desired frequency bandwidth. For this aim, multiple-section quarter-wave (e.g., Binomial or Chebyshev), continuously tapered, or stepped transformers [8]–[11] may be adopted. The limitation of these well-known techniques, however, is that they cannot keep the real part of the input impedance of the transformer, real$(Z_{\rm in})$, equal to $R_{\rm opt}$ as the frequency of operation changes, which is critical for PA operation. In fact, at frequencies where real$\{Z_{\rm in}\} < R_{\rm opt}$, the dc-to-RF conversion efficiency of the PA is compromised. Indeed, considering that the delivered RF power of the PA $(P_{\rm RF})$, the fundamental output current (I_1), and real$\{Z_{\rm in}\}$ are related by: $P_{\rm RF} = (1/2)(I_1)^2\text{real}\{Z_{\rm in}\}$, it is clear that the drop of real$\{Z_{\rm in}\}$ below $R_{\rm opt}$ induces a loss of output power, and consequently, reduced efficiency. The reduction in RF performance can be demonstrated to be proportional to real$\{Z_{\rm in}\}/R_{\rm opt}$, which is lower than unity.

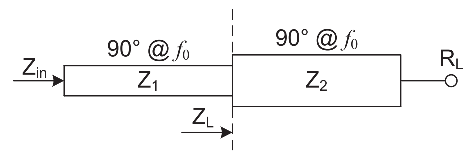

Fig. 1. Two-section quarter-wave transformer.

Conversely, at frequencies where real$\{Z_{\rm in}\} > R_{\rm opt}$, the linearity of the PA is degraded. Indeed, because the device sees impedance greater than $R_{\rm opt}$, its load line enters the knee region, distorting the drain current waveform when the voltage swings below the knee voltage. This gives rise to a bifurcated current waveform with reduced fundamental component that manifests as compression in gain versus power characteristic of the PA [12]. Device operation into hard compression impacts the linearity of the PA and compromises its reliability.

This letter presents an approach to design quarter-wave transformers with maximally flat real-to-real impedance transformation over wide frequency band. As explained, this feature is greatly useful for broadband PA applications. This work targets two-section transformers since they can be optimized to provide a good trade-off between physical size and bandwidth performance. In Section II, we demonstrate that the proposed two-section transformer provides a closed-form solution for maximally flat real-to-real impedance matching under all impedance transformation ratios. Numerical and experimental validations of the proposed solution are presented in Sections III and IV.

II. Solution for Maximally Flat Impedance Transformation

At a given frequency f, the input impedance $Z_{\rm in}$ of the two-section impedance transformer shown in Fig. 1 is given by

$$Z_{\rm in}(f) = Z_1 \frac{Z_L(f) + jZ_1\tan(\pi/2\cdot f/f_0)}{Z_1 + jZ_L(f)\tan(\pi/2\cdot f/f_0)} \tag{1}$$

where

$$Z_L(f) = Z_2 \frac{R_L + jZ_2\tan(\pi/2\cdot f/f_0)}{Z_2 + jR_L\tan(\pi/2\cdot f/f_0)} \tag{2}$$

and Z_1 and Z_2 designate the characteristic impedances of the quarter-wavelength transmission lines forming the two-section transformer. f_0 is the center frequency at which electrical length of the two transmission lines Z_1 and Z_2 is equal to 90°. R_L is the load impedance and Z_L is the impedance seen at the input of the transmission line (Z_2) that is connected to R_L.

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TABLE I
IMPEDANCE TRANSFORMER DESIGN

$k = Z_0/R_L$	Z_1/Z_0	Z_2/Z_0
0.25	1.7321	3.4641
0.5	1.3066	1.8478
1	1.0000	1.0000
2	0.7769	0.5493
3	0.6748	0.3896
4	0.6124	0.3062
6	0.5362	0.2189

Substituting Z_L in (1) with its expression (2) gives

$$Z_{\text{in}}(f)=Z_1\frac{Z_2R_L-Z_1R_L\tan^2(\sigma f)+j\left(Z_2^2+Z_1Z_2\right)\tan(\sigma f)}{Z_1Z_2-Z_2^2\tan^2(\sigma f)+j(Z_1R_L+Z_2R_L)\tan(\sigma f)} \tag{3}$$

where $\sigma = \pi/(2f_0)$.

First, we want the input impedance Z_{in} to be equal to Z_0 at the center frequency f_0. Equating the expression of Z_{in} in (3) to Z_0 at f_0 leads to

$$Z_0 = \frac{Z_1^2 R_L}{Z_2^2}. \tag{4}$$

In addition, to achieve the maximally flat impedance transformation, the second derivative of real$\{Z_{\text{in}}\}$ should be equal to zero as f approaches f_0. This condition is mathematically

$$\lim_{f\to f_0} \frac{d^2\text{real}\left\{Z_{\text{in}}(f)\right\}}{df^2} = 0. \tag{5}$$

With the help of a computer algebra system (e.g., Maple software), (5) can be calculated and arranged as

$$2Z_2^4 + 2Z_1Z_2^3 - R_L^2Z_2^2 - 2R_L^2Z_1Z_2 - R_L^2Z_1^2 = 0. \tag{6}$$

Solving (6) for an explicit expression of Z_1 while accounting for (4) gives

$$Z_1 = \frac{Z_0}{\sqrt{2}}\frac{1+\frac{1}{\sqrt{k}}}{\sqrt{1+\sqrt{k}}} \tag{7}$$

and Z_2 can be obtained from (4) as

$$Z_2 = \frac{Z_1}{\sqrt{k}} \tag{8}$$

where k $(k = Z_0/R_L)$ is the impedance transformation ratio.

This completes the derivation of the design equations of the two-section transformer for maximally flat real-to-real impedance matching.

III. NUMERICAL VALIDATION

Exact design results can be found via direct computation of (7) and (8). The results of such calculations are listed in Table I, which gives the transmission line impedances for various values of k.

Fig. 2 shows real (Z_{in}) of the proposed transformer for $k = 2$ with $R_L = 50\ \Omega$. The result is compared with two-section Binomial and Chebyshev transformers [8]. As expected, the proposed design solution allows the fulfillment of the maximally flat condition for real-to-real impedance transformation versus frequency.

Direct computations of (1), (2), (7), and (8) have been made to calculate the return loss, S_{11}, with respect to the targeted Z_0.

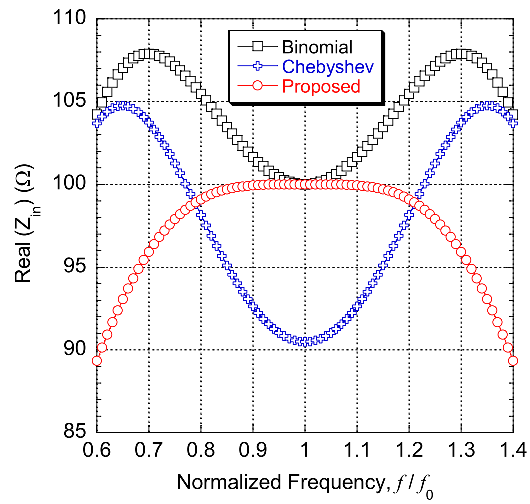

Fig. 2. Real part of the input impedance of Binomial, Chebychev, and the proposed transformer for $k = 2$ and $R_L = 50\ \Omega$.

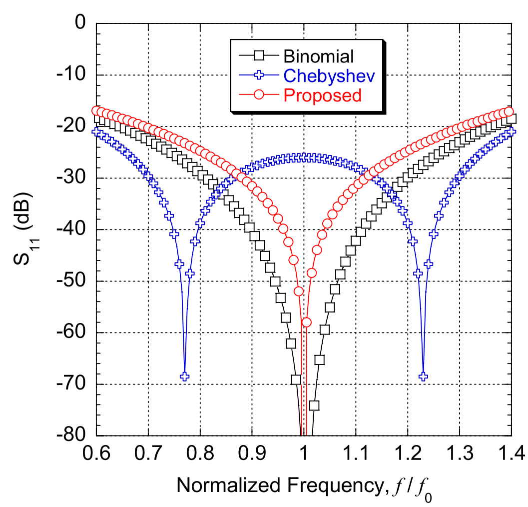

Fig. 3. Return loss of Binomial, Chebychev, and the proposed transformer for $k = 2$ and $R_L = 50\ \Omega$.

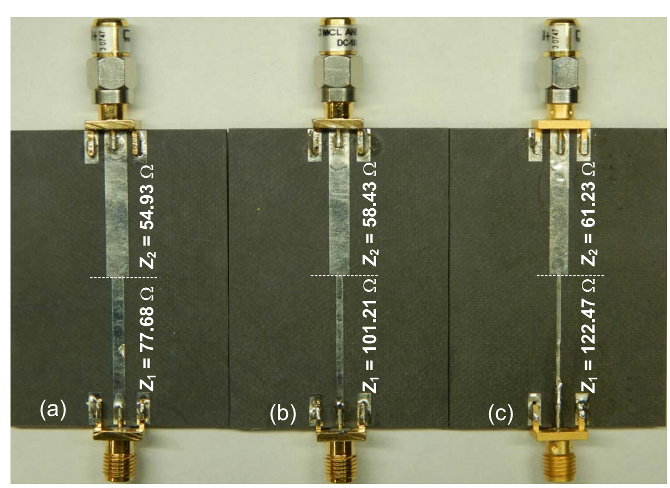

Fig. 4. Fabricated impedance transformers with $R_L = 50\ \Omega$. (a) $Z_0 = 100\ \Omega$. (b) $Z_0 = 150\ \Omega$. (c) $Z_0 = 200\ \Omega$.

Fig. 3 reports the return loss for the proposed, Binomial, and Chebychev transformers with $k = 2$ and $R_L = 50\ \Omega$. It can be inferred that the proposed transformer exhibits slightly narrower bandwidth in terms of return loss.

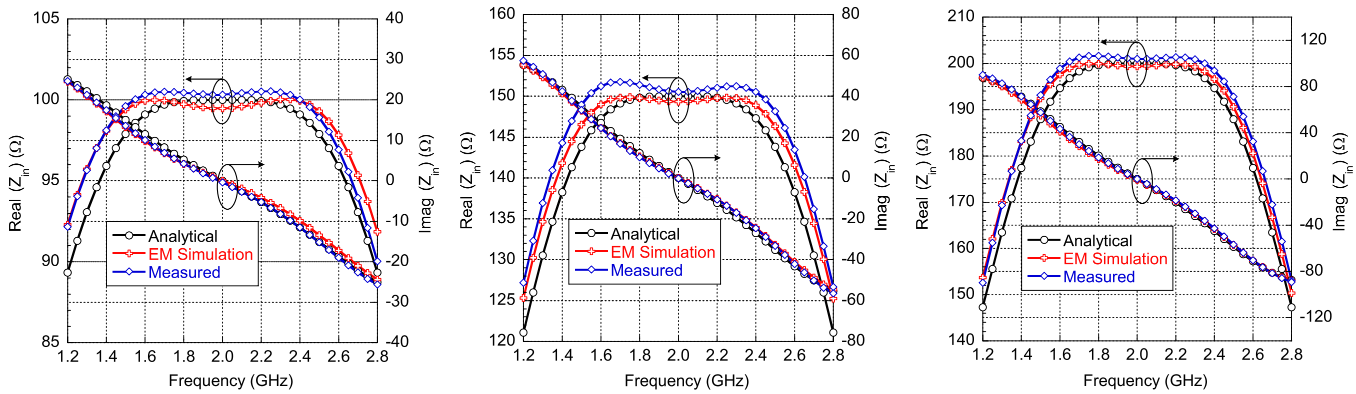

Fig. 5. Analytical, EM simulated, and measured Z_{in} with $R_L = 50\ \Omega$. (Left) $Z_0 = 100\ \Omega$. (Middle) $Z_0 = 150\ \Omega$. (Right) $Z_0 = 200\ \Omega$.

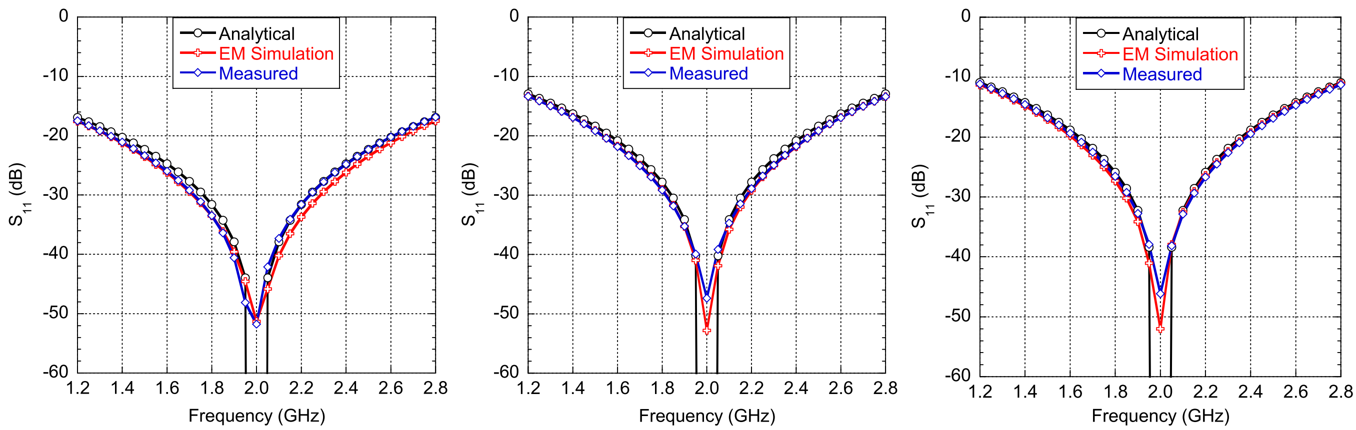

Fig. 6. Analytical, EM simulated, and measured $|S_{11}|$ with $R_L = 50\ \Omega$. (Left) $Z_0 = 100\ \Omega$. (Middle) $Z_0 = 150\ \Omega$. (Right) $Z_0 = 200\ \Omega$.

IV. Experimental Validation

For practical validation, the proposed methodology is applied to design three impedance transformers terminated in $R_L = 50\ \Omega$ and $Z_0 = 100$, 150, and 200 Ω at a center frequency of 2 GHz. The transformers are implemented on a Rogers' substrate, RO5880 with $\varepsilon_r = 2.2$, $H = 62$ mil and $\tan\delta = 0.0009$. The fabricated circuits are depicted in Fig. 4 where the targeted impedances of $Z_0 =$ 100, 150, and 200 Ω are indicated in Fig. 4(a)–(c), respectively. The three transformers are loaded with 50 Ω terminations as shown in Fig. 4.

Measured Z_{in} is reported in Fig. 5 along with analytical and EM simulations results that are obtained using Advanced Design Systems (ADS) software. In general, there is a good agreement between measured and simulated data. Besides, the scattering parameters of $|S_{11}|$ are better than -40 dB around the design center frequency of 2 GHz as illustrated in Fig. 6.

V. Conclusion

A novel two-section impedance transformer has been proposed for achieving maximally flat real-to-real impedance matching over wide frequency bandwidth, which is vital for achieving optimum PA efficiency and linearity performance. Closed-form design equations have been presented and validated numerically for several impedance transformation ratios. Three impedance transformers were designed following the proposed equations to match $R_L = 50\ \Omega$ to 100, 150, and 200 Ω. Measurement results confirmed the wideband operation of the designed transformers from 1.2 to 2.8 GHz.

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