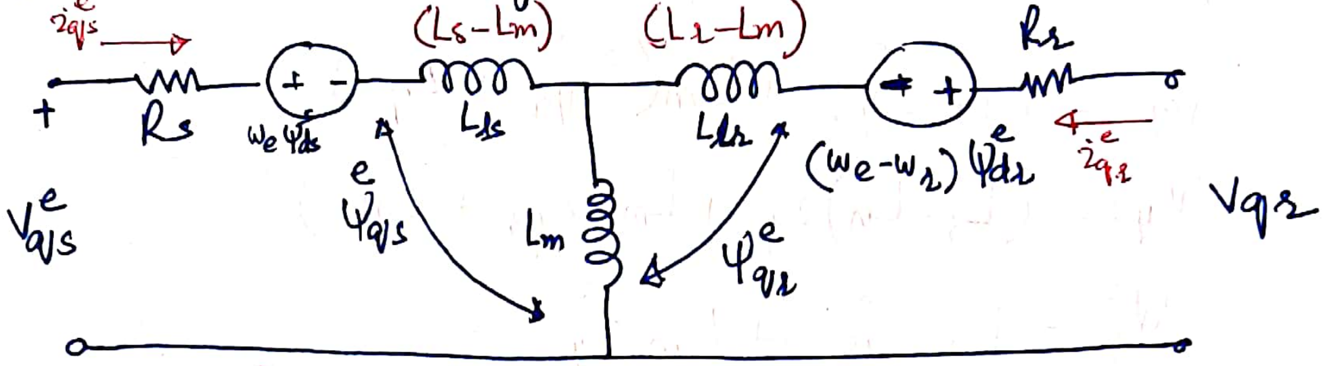
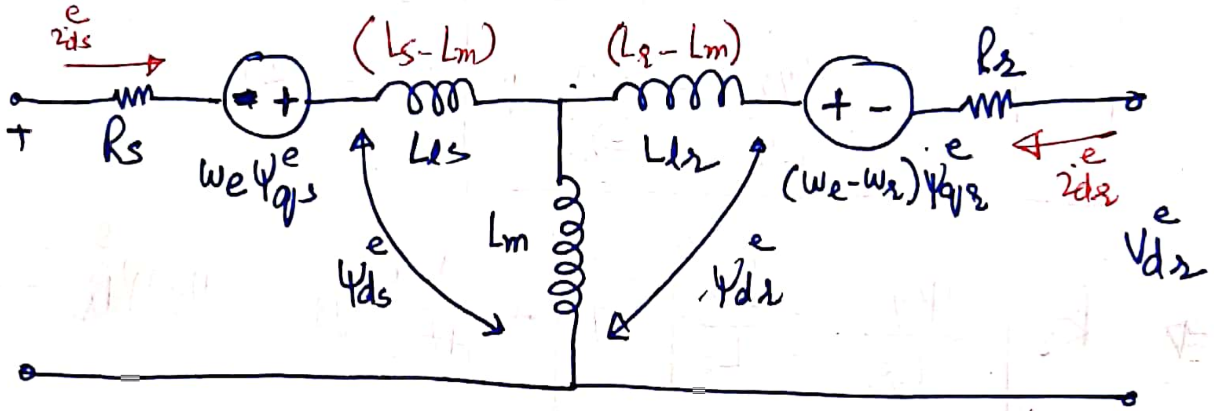
Lecture #5:

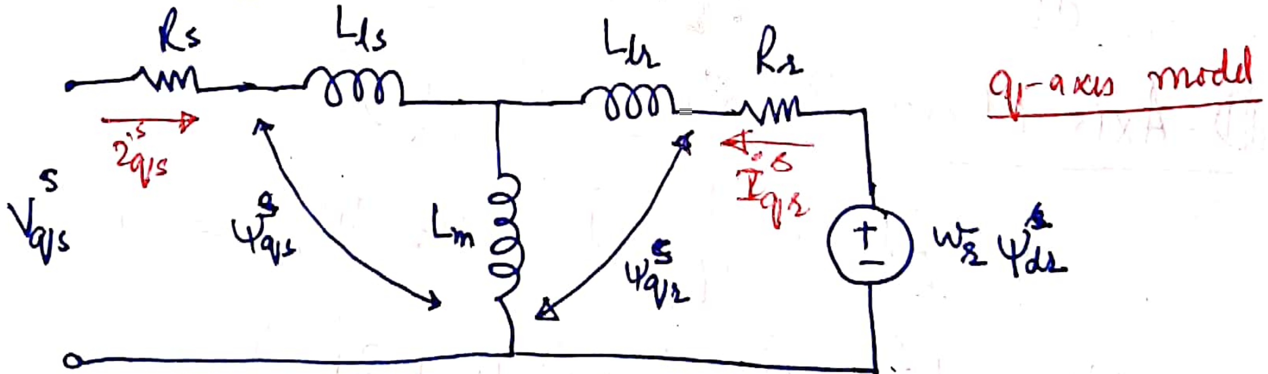
1. State Space Model of Induction Machine

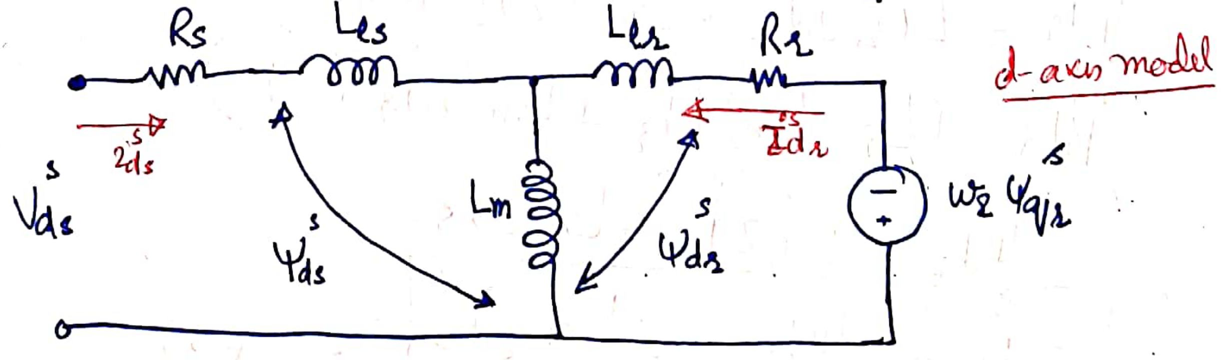
* We know in arbitrary fram of reference the equivalent circuit of induction motor is given as.





* In stationary frame of reference $\omega_e = 0$ and considering the squirrel cage machine.





Q-axis model Equation (rotor side)

$$R_r i_{qr}^s + \frac{d}{dt}\psi_{qr}^s - \omega_r \psi_{dr}^s = 0 \quad \text{—— (1)}$$

$$\psi_{qr}^s = L_{lr} i_{qr}^s + L_m(i_{qr}^s + i_{qs}^s)$$

$$\psi_{qr}^s = (L_r - L_m) i_{qr}^s + L_m(i_{qr}^s + i_{qs}^s)$$

$$\psi_{qr}^s = L_r i_{qr}^s + L_m i_{qs}^s$$

$$\boxed{i_{qr}^s = \frac{\psi_{qr}^s}{L_r} - \frac{L_m}{L_r} i_{qs}^s} \quad \text{—— (2)}$$

$$(1) \Rightarrow R_r\left[\frac{\psi_{qr}^s}{L_r} - \frac{L_m}{L_r} i_{qs}^s\right] + \frac{d}{dt}\psi_{qr}^s - \omega_r \psi_{dr}^s = 0$$

$$\frac{d}{dt}\psi_{qr}^s = \frac{R_r L_m}{L_r} i_{qs}^s - \frac{R_r}{L_r}\psi_{qr}^s + \omega_r \psi_{dr}^s \quad \text{—— (3)}$$

D-Axis Model Equation (rotor side)

$$R_r i_{dr}^s + \frac{d}{dt}\psi_{dr}^s + \omega_r \psi_{qr}^s = 0 \quad \text{—— (4)}$$

$$\psi_{dr} = L_{lr} i_{dr}^s + L_m(i_{dr}^s + i_{ds}^s)$$

$$\psi_{dr} = (L_r - L_m) i_{dr}^s + L_m(i_{dr} + i_{ds}^s)$$

$$\psi_{dr}^s = L_r i_{dr}^s + L_m i_{ds}^s$$

$$\boxed{i_{dr}^s = \frac{\psi_{dr}^s}{L_r} - \frac{L_m}{L_r} i_{ds}^s} \quad \text{—— (5)}$$

$$④ \Rightarrow R_r\left[\frac{\psi^s_{dr}}{L_r} - \frac{L_m}{L_r} i^s_{ds}\right] + \frac{d}{dt}\psi^s_{dr} + \omega_r \psi^s_{qr} = 0$$

$$\Rightarrow \dot{\psi}^s_{dr} = \frac{L_m R_r}{L_r} i^s_{ds} - \frac{R_r}{L_r}\psi^s_{dr} - \omega_r \psi^s_{qr} \quad \text{—} ⑥$$

Q-model Equation (stator side)

$$V^s_{qs} = R_s i^s_{qs} + \frac{d}{dt}\psi^s_{qs} = R_s i^s_{qs} + \frac{d}{dt}\left(L_{ls} i^s_{qs} + L_m\left(i^s_{qs} + i^s_{qr}\right)\right)$$

$$V^s_{qs} = R_s i^s_{qs} + \frac{d}{dt}\left[(L_s - L_m) i^s_{qs} + L_m\left(i^s_{qs} + i^s_{qr}\right)\right]$$

$$V^s_{qs} = R_s i^s_{qs} + L_s \frac{d}{dt} i^s_{qs} + L_m \frac{d}{dt} i^s_{qr} \quad \text{—} ⑦$$

using the expression of i^s_{qr} from ② we can write The above equation as.

$$V^s_{qs} = R_s i^s_{qs} + L_s \frac{d}{dt} i^s_{qs} + L_m \frac{d}{dt}\left[\frac{\psi_{qr}}{L_r} - \frac{L_m}{L_r} i^s_{qs}\right]$$

$$V^s_{qs} = R_s i^s_{qs} + L_s \frac{d}{dt} i^s_{qs} + \frac{L_m}{L_r}\frac{d}{dt}\psi^s_{qr} - \frac{L_m^2}{L_r}\frac{d}{dt} i^s_{qs}$$

$$V^s_{qs} = R_s i^s_{qs} + \left(L_s - \frac{L_m^2}{L_r}\right)\frac{d}{dt} i^s_{qs} + \frac{L_m}{L_r}\underbrace{\frac{d}{dt}\psi^s_{qr}}$$

Now using the expression for $\frac{d}{dt}\psi^s_{qr}$ from eq ③ we can write as.

$$V^s_{qs} = R_s i^s_{qs} + \left(L_s - \frac{L_m^2}{L_r}\right)\frac{d}{dt} i^s_{qs} + \frac{L_m}{L_r}\left[\frac{R_r L_m}{L_r} i^s_{qs} - \frac{R_r}{L_r}\psi^s_{qr} + \omega_r \psi^s_{dr}\right]$$

$$V^s_{qs} = R_s i^s_{qs} + L_s\left(1 - \frac{L_m^2}{L_s L_r}\right)\frac{d}{dt} i^s_{qs} + \frac{L_m^2 R_r}{L_r^2} i^s_{qs} - \frac{L_m R_r}{L_r^2}\psi^s_{qr} + \frac{L_m}{L_r}\omega_r \psi^s_{dr}$$

Now Let $\sigma = 1 - \frac{L_m^2}{L_r L_s}$

Now

$$V_{qs}^{s} = \left[R_s + \frac{L_m^2 R_r}{L_r^2}\right] i_{qs}^{s} - \frac{L_m R_r}{L_r^2}\psi_{qr}^{s} + \frac{L_m}{L_r}\omega_r \psi_{dr}^{s} + L_s\sigma \frac{d}{dt} i_{qs}^{s}$$

Rearranging for $d/dt\,(i_{qs}^{s})$ we get.

$$\frac{d}{dt} i_{qs}^{s} = \frac{V_{qs}^{s}}{L_s\sigma} - \left[\frac{R_s L_r^2 + L_m^2 R_r}{L_r^2}\right]\frac{i_{qs}^{s}}{L_s\sigma} + \frac{L_m R_r}{L_r^2 L_s \sigma}\psi_{qr}^{s} - \frac{L_m}{L_r}\cdot\frac{\omega_r \psi_{dr}^{s}}{\sigma L_s} \quad \longleftarrow (8)$$

D-model Equation for stator side

Step 1: $V_{ds}^{s} = R_s i_{ds}^{s} + \frac{d}{dt}\psi_{ds}^{s}$

$$= R_s i_{ds}^{s} + d/dt\left[L_{ls} i_{ds}^{s} + L_m\left(i_{ds}^{s} + i_{dr}^{s}\right)\right]$$

$$= R_s i_{ds}^{s} + \frac{d}{dt}\left[(L_s - L_m) i_{ds}^{s} + L_m\left(i_{ds}^{s} + i_{dr}^{s}\right)\right]$$

$$= R_s i_{ds}^{s} + \frac{d}{dt} L_s i_{ds}^{s} + \underbrace{\frac{d}{dt} L_m i_{dr}^{s}}$$

Step 2: Using the value of i_{dr}^{s} from equation (5)

$$V_{ds}^{s} = R_s i_{ds}^{s} + \frac{d}{dt} L_s i_{ds}^{s} + \frac{d}{dt} L_m\left[\frac{\psi_{dr}^{s}}{L_r} - \frac{L_m}{L_r} i_{ds}^{s}\right]$$

$$V_{ds}^{s} = R_s \cancel{i_{ds}^{s}} + L_s\left(1 - \frac{L_m^2}{L_r L_s}\right)\frac{d}{dt} i_{qs}^{s} + \frac{L_m}{L_r}\underbrace{d/dt\ \psi_{dr}^{s}}$$

Step 3: Using the value of $d/dt\,(\psi_{dr}^{s})$ from (6) and simplifying above equation we get.

$$\frac{d}{dt} i_{ds}^{s} = \frac{V_{ds}^{s}}{L_s\sigma} - \left[\frac{R_s L_r^2 + L_m^2 R_r}{L_r^2}\right]\frac{i_{ds}^{s}}{L_s\sigma} + \frac{L_m R_r}{L_r^2 L_s\sigma}\psi_{dr}^{s} + \frac{L_m}{L_r}\frac{\omega_r}{\sigma L_s}\psi_{qr}^{s}$$

Now writing the equation in state space form as (3)

$$\underbrace{\frac{d}{dt}\begin{bmatrix} i_{ds}^{s} \\ i_{qs}^{s} \\ \psi_{dr}^{s} \\ \psi_{qr}^{s} \end{bmatrix}}_{\dot{X}} = \underbrace{\begin{bmatrix} -\left(\frac{L_m^2 R_r + L_r^2 R_s}{\sigma L_s L_r}\right) & 0 & \frac{L_m R_r}{\sigma L_s L_r^2} & \frac{L_m W_r}{\sigma L_s L_r} \\ 0 & -\left(\frac{L_m^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2}\right) & \frac{-L_m W_r}{\sigma L_s L_r} & \frac{L_m R_r}{\sigma L_s L_r^2} \\ \frac{L_m R_r}{L_r} & 0 & -R_r/L_r & -W_r \\ 0 & \frac{L_m R_r}{L_r} & W_r & \frac{R_r}{L_r} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_{ds}^{s} \\ i_{qs}^{s} \\ \psi_{dr}^{s} \\ \psi_{qr}^{s} \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} \frac{1}{L_s\sigma} & 0 \\ 0 & \frac{1}{L_s\sigma} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} V_{ds}^{s} \\ V_{qs}^{s} \end{bmatrix}}_{u}$$

$$\dot{X} = AX + BU$$

$$Y = CX.$$

$$X = \begin{bmatrix} i_{ds}^{s} & i_{qs}^{s} & \psi_{dr}^{s} & \psi_{qr}^{s} \end{bmatrix}^T$$

$$U = \begin{bmatrix} V_{ds}^{s} & V_{qs}^{s} \end{bmatrix}^T$$

So

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* See page 5b (second side)

Rotor flux estimation

Stator Flux :-

(*) From equivalent circuit of Induction machine in stationary frame of reference we know that

$$V_{ds}^s = R_s i_{ds}^s + \frac{d}{dt}\psi_{ds}^s \Rightarrow \psi_{ds}^s = \int (V_{ds}^s - R_s i_{ds}^s)\,dt \quad (1)$$

$$V_{qs}^s = R_s i_{qs}^s + \frac{d}{dt}\psi_{qs}^s \Rightarrow \psi_{qs}^s = \int (V_{qs}^s - R_s i_{qs}^s)\,dt \quad (2)$$

$$\psi_s = \sqrt{\psi_{ds}^{2} + \psi_{qs}^{s}} \quad (3)$$

(*) Magnetizing flux can be written as

$$\psi_{dm}^s = \psi_{ds}^s - L_{ls} i_{ds}^s = L_m (i_{ds}^s + i_{dr}^s) \quad (4)$$

$$\psi_{qm}^s = \psi_{qs}^s - L_{ls} i_{qs}^s = L_m (i_{qs}^s + i_{qr}^s) \quad (5)$$

(*) Rotor flux:

$$\psi_{dr}^s = L_m i_{ds}^s + L_r i_{dr}^s \quad (6)$$ [see eq 1 of last section]

$$\psi_{qr}^s = L_m i_{qs}^s + L_r i_{qr}^s \quad (7)$$

from (4) & (5) we can find out i_{dr}^s & i_{qr}^s as

$$\psi_{dm}^s = L_m i_{ds}^s + L_m i_{dr}^s$$
$$L_m i_{dr}^s = \psi_{dm}^s - L_m i_{ds}^s$$
$$\boxed{i_{dr}^s = \frac{\psi_{dm}^s}{L_m} - i_{ds}^s}$$

$$\psi_{qm}^s = L_m i_{qs}^s + L_m i_{qr}^s$$
$$\cancel{\psi_{qm}^s =} L_m i_{qr}^s = \psi_{qm}^s - L_m i_{qs}^s$$
$$\boxed{i_{qr}^s = \frac{\psi_{qm}^s}{L_m} - i_{qs}^s}$$

Now equation 6 & 7 can be written using the above expression as

$$\psi_{dr}^s = L_m i_{ds}^s + L_r\left[\frac{\psi_{dm}^s}{L_m} - i_{ds}^s\right]$$
$$\psi_{dr}^s = L_m i_{ds}^s + \frac{L_r}{L_m}\psi_{dm}^s - L_r i_{ds}^s.$$
$$\psi_{dr}^s = \frac{L_r}{L_m}\psi_{dm}^s - (L_r - L_m)\, i_{ds}^s$$
$$\psi_{dr}^s = \frac{L_r}{L_m}\psi_{dm}^s - L_{lr}\, i_{ds}^s$$

$$\psi_{qr}^s = L_m i_{qs}^s + L_r\left[\frac{\psi_{qm}^s}{L_m} - i_{qs}^s\right]$$
$$\psi_{qr}^s = L_m i_{qs}^s + \frac{L_r}{L_m}\psi_{qm}^s - L_r i_{qs}^s$$
$$\psi_{qr}^s = \frac{L_r}{L_m}\psi_{qm}^s - (L_r - L_m)\, i_{qs}^s$$
$$\psi_{qr}^s = \frac{L_r}{L_m}\psi_{qm}^s - L_{lr}\, i_{qs}^s$$

So $$\psi^s_{dr} = \frac{L_r}{L_m}\psi^s_{dm} - L_{lr} i^s_{ds} \quad \text{—— (8)}$$

$$\psi^s_{qr} = \frac{L_r}{L_m}\psi^s_{qm} - L_{lr} i^s_{qs} \quad \text{—— (9)}$$

Altenative Representation.

$$\psi^s_{dr} = L_m i^s_{ds} + L_r i^s_{dr}$$

Using the expression of i^s_{dr}

$$\psi^s_{dr} = L_m i^s_{ds} + L_r\left[\frac{\psi^s_{dm}}{L_m} - i^s_{ds}\right]$$

$$= L_m i^s_{ds} + \frac{L_r}{L_m}\left[\psi^s_{ds} - L_{ls} i^s_{ds}\right] - L_r i^s_{ds}$$

$$= L_m i^s_{ds} + \frac{L_r}{L_m}\psi^s_{ds} - \frac{L_r}{L_m}L_{ls} i^s_{ds} - L_r i^s_{ds}$$

$$= L_m i^s_{ds} + \frac{L_r}{L_m}\psi^s_{ds} - \frac{L_r}{L_m} i_{ds}\left(L_{ls} + \frac{L_m \cdot L_r}{\cancel{L_r}}\right)$$

$$= L_m i^s_{ds} + \frac{L_r}{L_m}\psi^s_{ds} - \frac{L_r}{L_m} i_{ds} L_s$$

$$= \frac{L_r}{L_m}\psi^s_{ds} + \frac{L_r}{L_m}\left[\frac{L_m}{L_r}L_m i_{ds} - L_s i_{ds}\right]$$

$$= \frac{L_r}{L_m}\psi^s_{ds} + \frac{L_r}{L_m}\left[\frac{L_m^2}{L_r} i_{ds} - L_s i_{ds}\right]$$

$$= \frac{L_r}{L_m}\psi^s_{ds} + \frac{L_r}{L_m}\left[\frac{L_m^2}{L_s L_r} - 1\right] i_{ds}\cdot L_s$$

$$= \frac{L_r}{L_m}\psi^s_{ds} - \frac{L_r}{L_m}L_s i_{ds}\underbrace{\left[1 - \frac{L_m^2}{L_s L_r}\right]}_{\sigma}$$

$$\boxed{\psi_{dr} = \frac{L_r}{L_m}\left[\psi^s_{ds} - \sigma L_s i^s_{ds}\right]}$$

$$\psi^s_{qr} = L_m i^s_{qs} + L_r i^s_{qr}$$

Using the expression of i^s_{qr}

$$\psi^s_{qr} = L_m i^s_{qs} + L_r\left[\frac{\psi^s_{qm}}{L_m} - i^s_{qs}\right]$$

$$\psi^s_{qr} = L_m i^s_{qs} + \frac{L_r}{L_m}\psi^s_{qm} - L_r i^s_{qs}$$

$$\psi^s_{qr} = L_m i^s_{qs} + \frac{L_r}{L_m}\left[\psi^s_{qs} - L_{ls} i^s_{qs}\right] - L_r i^s_{qs}$$

$$\psi^s_{qr} = L_m i^s_{qs} + \frac{L_r}{L_m}\psi^s_{qs} - \frac{L_r}{L_m} i^s_{qs}\left(L_{ls} + \frac{L_m}{L_r}L_r\right)$$

$$\psi^s_{qr} = L_m i^s_{qs} + \frac{L_r}{L_m}\psi^s_{qs} - \frac{L_r}{L_m} i^s_{qs} L_s$$

$$\psi^s_{qr} = \frac{L_r}{L_m}\psi^s_{qs} + \frac{L_r}{L_m}\left[\frac{L_m}{L_r}L_m i_{qs} - L_s i^s_{qs}\right]$$

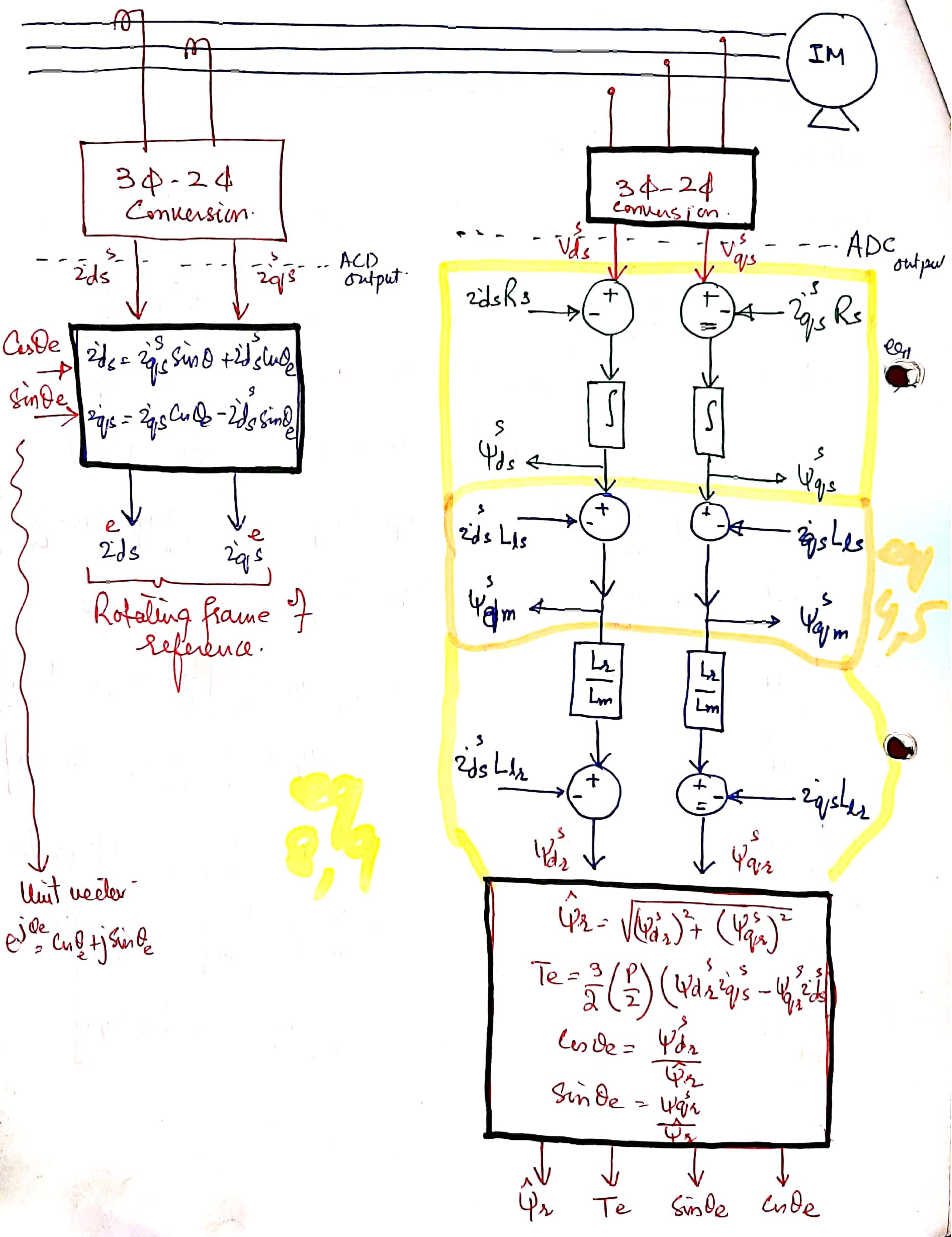
$$= \frac{L_r}{L_m}\psi^s_{qs} + \frac{L_r}{L_m}\left[\frac{L_m^2}{L_r} i_{qs} - L_s i^s_{qs}\right]$$

$$= \frac{L_r}{L_m}\psi^s_{qs} + \frac{L_r}{L_m}\left[\frac{L_m^2}{L_s L_r} - 1\right] i_{qs}\cdot L_s$$

$$= \frac{L_r}{L_m}\psi^2_{qs} - \frac{L_r}{L_m}\left[1 - \frac{L_m^2}{L_s L_r}\right] L_s \cdot i_{qs}$$

$$= \frac{L_r}{L_m}\left[\psi^s_{qs} - \sigma L_s i_{qs}\right]$$

Flow chart:


IM
3φ-2φ Conversion
ACD output
ADC output
Rotating frame of reference.
Unit vector
L_r/L_m
Te
sinθe
cosθe

From the discussion of previous chapters we know that

$$T_e = \frac{3}{2}\frac{P}{2}\left(\psi_{dm}\, i_{qs} - \psi_{qm}\, i_{ds}\right)$$

we know that

$$\psi_{qr}^{s} = \frac{L_r}{L_m}\psi_{qm}^{s} - L_{lr} i_{qs}^{s} \Rightarrow \boxed{\psi_{qm}^{s} = \frac{\psi_{qr}^{s} + L_{lr} i_{qs}^{s}}{(L_r/L_m)}}$$

$$\psi_{dr}^{s} = \frac{L_r}{L_m}\psi_{dm}^{s} - L_{lr} i_{ds}^{s} \Rightarrow \boxed{\psi_{dm}^{s} = \frac{\psi_{dr}^{s} + L_{lr} i_{ds}^{s}}{(L_r/L_m)}}$$

so the torque expression

$$T_e = \frac{3}{2}\left(\frac{P}{2}\right)\left[\left(\frac{\psi_{dr}^{s} + L_{lr} i_{ds}^{s}}{(L_r/L_m)}\right) i_{qs}^{s} - \left(\frac{\psi_{qr}^{s} + L_{lr} i_{qs}^{s}}{(L_r/L_m)}\right) i_{ds}^{s}\right]$$

$$\boxed{T_e = \frac{3}{2}\left(\frac{P}{2}\right)\frac{L_m}{L_r}\left[\psi_{dr}^{s} i_{qs}^{s} - \psi_{qr}^{s} i_{ds}^{s}\right]}$$

Limitations:

1) Near zero speed V_{ds}^{s} & V_{qs}^{s} are very low and ADC does not give the correct output.
2) Ideal integration becomes difficult because DC offset tends to build up at the integrator O/P.
3) Parameter variation effect of resistance R_s and inductance L_{ls}, L_{lr}, and L_m tend to reduce accuracy of the estimated signal. Temperature based R_s variation becomes more dominant. However compensation of R_s is easier and will be discussed later on.
4) Close to zero speed this estimation is not valid/accurate.

Relating Stator and Rotor flux linkages

We know that.

$$\psi_{qs}^s = L_{ls} i_{qs}^s + L_m (i_{qs}^s + i_{qr}^s)$$

$$\psi_{qs}^s = L_{ls} i_{qs}^s + L_m \left(i_{qs}^s + \frac{\psi_{qr}^s}{L_r} - \frac{L_m}{L_r} i_{qs}^s\right)$$

$$\psi_{qs}^s = (L_{ls} + L_m) i_{qs}^s + L_m \left(\frac{\psi_{qr}^s}{L_r} - \frac{L_m}{L_r} i_{qs}^s\right)$$

$$\psi_{qs}^s = L_s \, i_{qs}^s + \frac{L_m^2}{L_r} i_{qs}^s + \frac{L_m}{L_r} \psi_{qr}^s$$

$$\psi_{qs}^s = L_s \left(1 - \frac{L_m^2}{L_r L_s}\right) i_{qs}^s + \frac{L_m}{L_r} \psi_{qr}^s$$

$$\Rightarrow \boxed{\psi_{qs}^s = L_s \sigma \, i_{qs}^s + \frac{L_m}{L_r} \psi_{qr}^s}$$

Re-arranging we get

$$\boxed{\psi_{qr}^s = \frac{L_r}{L_m} \left(\psi_{qs}^s - L_s \sigma \, i_{qs}^s\right)}$$

Now

$$\psi_{ds}^s = L_{ls} i_{ds}^s + L_m (i_{ds}^s + i_{dr}^s)$$

$$= L_{ls} i_{ds}^s + L_m i_{ds}^s + L_m \left(\frac{\psi_{dr}^s}{L_r} - \frac{L_m}{L_r} i_{ds}^s\right)$$

$$= L_s i_{ds}^s \mp \frac{L_m^2}{L_r} i_{ds}^s + \frac{L_m}{L_r} \psi_{dr}^s$$

$$= L_s \left(1 - \frac{L_m^2}{L_s L_m}\right) i_{ds}^s + \frac{L_m}{L_r} \psi_{dr}^s$$

Using the Same procedure used above.

$$\boxed{\psi_{ds}^s = \frac{L_m}{L_r} \psi_{dr}^s + L_s \sigma \, i_{ds}^s}$$

$$\Rightarrow \boxed{\psi_{dr}^s = \frac{L_r}{L_m} \left[\psi_{ds}^s - \sigma L_s \, i_{ds}^s\right]}$$

So in the matrix form.

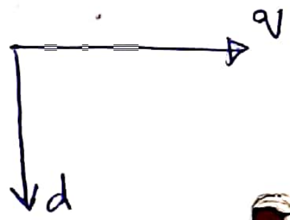
$$\begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ \psi_{ds}^s \\ \psi_{qs}^s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \sigma L_s & 0 & \frac{L_m}{L_r} & 0 \\ 0 & \sigma L_s & 0 & \frac{L_m}{L_r} \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ \psi_{dr}^s \\ \psi_{qr}^s \end{bmatrix}$$

Electromagnetic Torque of Induction Machine

$$T_{em} = \frac{3}{2} \cdot \frac{P}{2} \left[\vec{\psi}_s \times \vec{I}_r \right]$$

$$T_{em} = \frac{3}{2} \cdot \frac{P}{2} \left[\left(\psi^s_{qs}\, \hat{i} - \hat{j}\, \psi^s_{ds} \right) \times \left(i^s_{qs}\, \hat{i} - i^s_{ds}\, \hat{j} \right) \right]$$

$$= \frac{3}{2} \cdot \frac{P}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \psi^s_{qs} & -\psi^s_{ds} & 0 \\ i^s_{qs} & -i^s_{ds} & 0 \end{vmatrix}$$



$$T_{em} = \frac{3}{2} \cdot \frac{P}{2} \cdot \hat{k} \left(\psi^s_{ds}\, i^s_{qs} - \psi^s_{qs}\, i^s_{ds} \right)$$

$$\boxed{ |T_{em}| = \frac{3}{2} \cdot \frac{P}{2} \left(\psi^s_{ds}\, i^s_{qs} - \psi^s_{qs}\, i^s_{ds} \right) }$$

Mechanical Equation:

$$J \frac{d\omega_m}{dt} + B\omega_m = T_{em} - T_L$$

or $$T_L = T_{em} - J \frac{d\omega_m}{dt} - B\omega_m .$$

Current Model:

From the equivalent of induction machine in stationary frame of reference (squirrel cage) we can write

$$\left.\begin{aligned} \frac{d\psi^s_{dr}}{dt} + R_r i^s_{dr} + \omega_r \psi^s_{qr} &= 0 \\ \frac{d\psi^s_{qr}}{dt} + R_r i^s_{qr} - \omega_r \psi^s_{dr} &= 0 \end{aligned}\right\} \quad (1)$$

- Adding $\frac{L_m R_r}{L_r} i^s_{ds}$ and $\frac{L_m R_r}{L_r} i^s_{qs}$ respectively on both sides of the above equations, we get.

$$\left.\begin{aligned} \frac{d\psi^s_{dr}}{dt} + \frac{R_r}{L_r}\left(L_m i^s_{ds} + L_r i^s_{dr}\right) + \omega_r \psi^s_{qr} &= \frac{L_m R_r}{L_r} i^s_{ds} \\ \frac{d\psi^s_{qr}}{dt} + \frac{R_r}{L_r}\left(L_m i^s_{qs} + L_r i^s_{qr}\right) - \omega_r \psi^s_{dr} &= \frac{L_m R_r}{L_r} i^s_{qs} \end{aligned}\right\} \quad (2)$$

- We know that

$$\left.\begin{aligned} \psi^s_{dr} &= L_m i^s_{ds} + L_r i^s_{dr} \\ \psi^s_{qr} &= L_m i^s_{qs} + L_r i^s_{qr} \end{aligned}\right\} \quad (3)$$

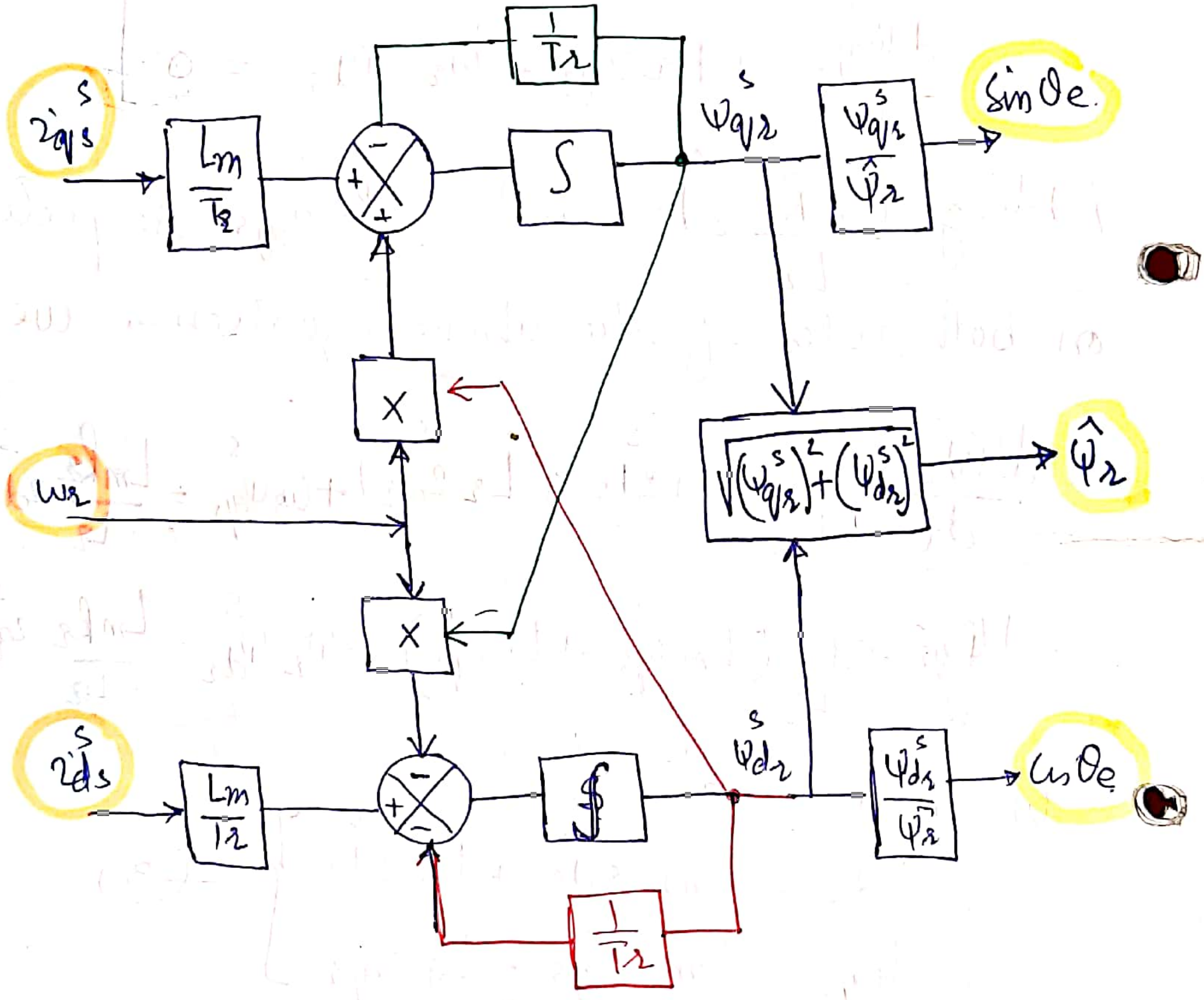
Using (3) in (2) we can write

$$\frac{d\psi^s_{dr}}{dt} = \frac{L_m}{T_r} i^s_{ds} - \omega_r \psi^s_{qr} - \frac{R_r}{L_r} \psi^s_{qr}$$

$$\left.\begin{aligned} \frac{d\psi^s_{dr}}{dt} &= \frac{L_m}{T_r} i^s_{d} - \omega_r \psi^s_{qr} - \frac{1}{T_r} \psi^s_{qr} \\ \frac{d\psi^s_{qr}}{dt} &= \frac{L_m}{T_r} i^s_{qs} + \omega_r \psi^s_{dr} - \frac{1}{T_r} \psi^s_{qr} \end{aligned}\right\} \quad (4)$$

where $T_r = \frac{L_r}{R}$ = Rotor circuit Time constant.

The block diagram to estimate the rotor flux from current signals canbe drawn using the current model equations as



- W_r: Speed encoder is required for the current model.
- Estimation is effected by variation of machine parameters
- Rotor resistance variation may be upto 50% because of temperature and skin effect.
- Compensation is difficult for R_r as it is inaccessable.