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A theoretical justification of the two-conductor HF transmission-line model for indoor single-phase Low Voltage triplex cables

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Abstract

Transfer properties of single-phase mains tricels in the High Frequency band are investigated from a theoretical point of view. Assuming transverse-electromagnetic wave propagation, transmission line formed of the phase and neutral wires is due to cables' symmetry found absolutely unaffected by the proximal presence of ground conductor. Resulting distributed parameters of such three-wire assembly lead moreover to complete conformance with the isolated two-conductor transmission-line model. Reference to relevant experimental results from bibliography is also provided, as well as hints of practical value.

1. Introduction

Characterisation of the power electric network in relation to HF signal transmission is of prime significance as to achieving reliable and efficient Power-Line Communications applications. With regard to the indoor Low Voltage (LV) grid, implementation of modern Local Area Networks (LAN's) is the major terminus. Mostly single-phase networks are found both in domestic and premises installations, and respective PLC channels exhibit intensively fading properties. Modelling of the indoor LV cabling as transfer medium in the HF band is of major importance in order for reliable channel-analysing tools and simulating methods to be developed.

The cable type cross-section of which is depicted in fig. 1, is widely used in Europe for electrical installations in buildings. Such a configuration comprises a phase conductor, the neutral and the ground one. Each conductor, either of one ply or stranded, is vested by its dielectric jacket, and an additional external insulating sheath encloses all three wires, which have identical radii and are often self-twisted as a whole.

Antecedent experimental evaluations have shown good compliance to the two-conductor uniform transmission-line model [1-4]. The work presented in [4] involves a thorough study on the cable type considered and transfer function measurements over numerous sample cables verify remarkable precision of the model proposed therein. However, no theoretical support of the two-conductor assumption has up to now been provided

In this paper, such a justification is introduced employing analytical reasoning based on transverse-electromagnetic (TEM) wave principles. Supposing transmission between the phase wire and the neutral, elimination of the ground conductor is initially demonstrated in Section 2. Parameter compliance with the isolated two-conductor model ensues in Section 3, and a concluding summation closes the paper in Section 4.

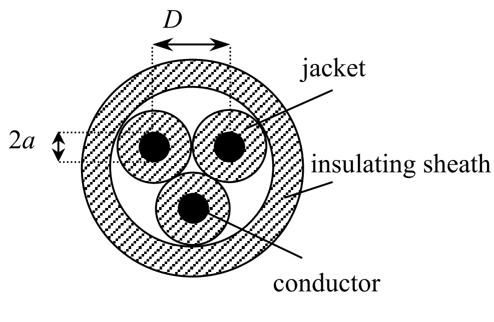

Fig. 1. Cable's cross-section

2. Transmission-Line Decoupling

2.1. Three-Conductor Transmission Line Model

Let the three-conductor transmission-line model of fig. 2 be assumed [5], representing the cable structure defined in fig. 1. Adopting neutral as reference conductor, subscript “0” characterises the latter, “G” the phase conductor (Generator), and “R” the ground one (Receptor), whereas “m” indicates mutual coupling parameters between generator and receptor. Per-unit-length inductance, resistance, capacitance and conductance are denoted respectively by l, r, c and g.

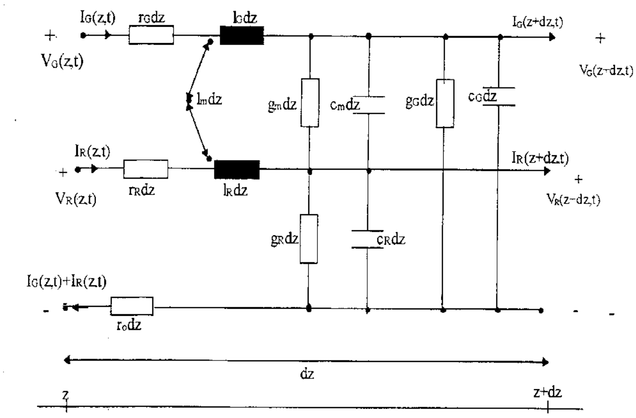

Fig. 2. Three-conductor transmission-line model

In line with the fundamental TEM field assumption, from which deviation is negligibly low for good conductors below the GHz range [5], per-unit-length parameter matrices have as

$$\mathbf{R} = \begin{bmatrix} r_G + r_0 & r_0 \\ r_0 & r_R + r_0 \end{bmatrix}, \tag{1a}$$

$$\mathbf{L} = \begin{bmatrix} l_G & l_m \\ l_m & l_R \end{bmatrix}, \tag{1b}$$

$$\mathbf{G} = \begin{bmatrix} g_G + g_m & -g_m \\ -g_m & g_R + g_m \end{bmatrix}, \tag{1c}$$

and

$$\mathbf{G} = \begin{bmatrix} c_G + c_m & -c_m \\ -c_m & c_R + c_m \end{bmatrix}. \tag{1d}$$

The voltage vector is

$$\mathbf{V}(z,t) = \begin{bmatrix} V_G(z,t) \\ V_R(z,t) \end{bmatrix}, \tag{2a}$$

and considering analysis in the frequency domain so that the sinusoidal steady state is assumed, the phasor voltage vector is

$$\hat{\mathbf{V}}(z) = \begin{bmatrix} \hat{V}_G(z) \\ \hat{V}_R(z) \end{bmatrix} \tag{2b}$$

defined by

$$\hat{\mathbf{V}}(z,t) = \mathrm{Re}\left\{\hat{\mathbf{V}}(z)e^{j\omega t}\right\} \tag{2c}$$

Current vector is

$$\mathbf{I}(z,t) = \begin{bmatrix} I_G(z,t) \\ I_R(z,t) \end{bmatrix}, \tag{3a}$$

and the relevant phasor current vector

$$\hat{\mathbf{I}}(z) = \begin{bmatrix} \hat{I}_G(z) \\ \hat{I}_R(z) \end{bmatrix} \tag{3b}$$

is defined by

$$\hat{\mathbf{I}}(z,t) = \mathrm{Re}\left\{\hat{\mathbf{I}}(z)e^{j\omega t}\right\}. \tag{3c}$$

Matrices

$$\hat{\mathbf{Z}} = \mathbf{R} + j\omega\mathbf{L} \quad \text{and} \tag{5a}$$

$$\hat{\mathbf{Y}} = \mathbf{G} + j\omega\mathbf{C} \tag{5b}$$

are respectively designated as the per-unit-length impedance and resistance matrix. For the phasor voltage and current vectors it is from the second-order Telegrapher's equations obtained

$$\frac{d^2\hat{\mathbf{V}}(z)}{dz^2} = \hat{\mathbf{Z}}\,\hat{\mathbf{Y}}\,\hat{\mathbf{V}}(z) \tag{6a}$$

and

$$\frac{d^2\hat{\mathbf{I}}(z)}{dz^2} = \hat{\mathbf{Y}}\hat{\mathbf{Z}}\,\hat{\mathbf{I}}(z)\,. \tag{6b}$$

2.2. Magnetic-Field Study

Per-unit-length resistance is intrinsic of the conductor, defined in the HF band as the real part of the internal impedance attributed to the skin-effect field penetration [6]. Having thus identical conductors gives

$$r_G = r_R = r_0 = r\,, \tag{7}$$

so that per-unit-length resistance matrix becomes

$$\mathbf{R} = r\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}. \tag{8}$$

The imaginary part of the skin-effect internal impedance forms conductor's internal inductance, which compared to external self-inductances is for the configuration studied sufficiently small to be ignored [5]. Parameters l_G and l_R refer hence correspondingly to the external self-inductances of generator's and receptor's circuits. By virtue of the TEM assumption, instant magnetic field structure is same as a static one [5], i.e. as if constant currents I_G and I_0 flow. Denoting by ψ_G and ψ_R the per-unit-length magnetic fluxes that penetrate respectively the circuits of generator and receptor, per-unit-length external inductance matrix is related as

$$\boldsymbol{\psi} = \mathbf{LI}\,, \tag{9a}$$

$$\boldsymbol{\psi} = \begin{bmatrix} \psi_G \\ \psi_R \end{bmatrix}, \tag{9b}$$

$$\mathbf{I} = \begin{bmatrix} I_G \\ I_R \end{bmatrix}. \tag{9c}$$

Having three identical parallel conductors at equal distances in between, $l_G = l_R$ comes of, and per-unit-length mutual and external self inductances are determined setting $I_R = 0$, as

$$l_G = l_R = \left.\frac{\psi_G}{I_G}\right|_{I_R=0} \tag{10a}$$

and

$$l_m = \left.\frac{\psi_R}{I_G}\right|_{I_R=0} \tag{10b}$$

Such structure is specified in fig. 3. For wire's proximity within the cable, current is not uniformly distributed around the conductor periphery; direction of magnetic flux has yet as illustrated. From cable's geometrical symmetry

$$\psi_G = 2\psi_R \tag{11}$$

is apparently deduced, so that

$$l_G = l_R = 2l_m = 2l\,, \tag{12}$$

and per-unit-length inductance matrix becomes

$$\mathbf{L} = l\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}. \tag{13}$$

Per-unit-length impedance matrix results by (5a), (8) and (12) in

$$\mathbf{Z} = (r + j\omega l)\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{14}$$

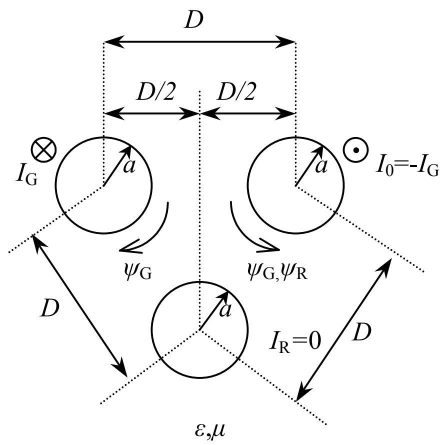

Fig. 3. Cable's static magnetic field structure

2.3. Electric-Field Study

After the static field assumption justified above, instant electric field is same as if generator and receptor were raised to constant potentials V_G and V_R, provided that reference conductor is kept grounded. Denoting by q_G and q_R the per-unit-length charges on generator and receptor respectively, per-unit-length capacitance matrix is related as

$$\mathbf{q} = \mathbf{CV}\,, \tag{15a}$$

$$\mathbf{q} = \begin{bmatrix} q_G \\ q_R \end{bmatrix}, \tag{15b}$$

$$\mathbf{V} = \begin{bmatrix} V_G \\ V_R \end{bmatrix}. \tag{15c}$$

Having three identical parallel conductors at equal distances in between, $c_G = c_R$ comes of, and per-unit-length mutual and self capacitances are determined setting $V_R = 0$, as

$$c_G = c_R = \left.\frac{q_G}{V_G}\right|_{V_R=0} \tag{16a}$$

and

$$c_m = \left.\frac{q_R}{V_G}\right|_{V_R=0} \tag{16b}$$

Such configuration is specified in fig. 4. For wire's proximity inside the cable, charge is not uniformly distributed around the conductor periphery; direction of electric flux has yet as illustrated. From cable's geometrical symmetry, $q_G = 2q_R$ is apparently deduced, as due to symmetry equal portions of the overall electric flux originating from generator end at the receptor and reference conductors. Distributed capacitances are thus

$$c_G = c_R = c_m = c\,, \tag{17}$$

and per-unit-length capacitance matrix becomes

$$\mathbf{C} = c\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}. \tag{18}$$

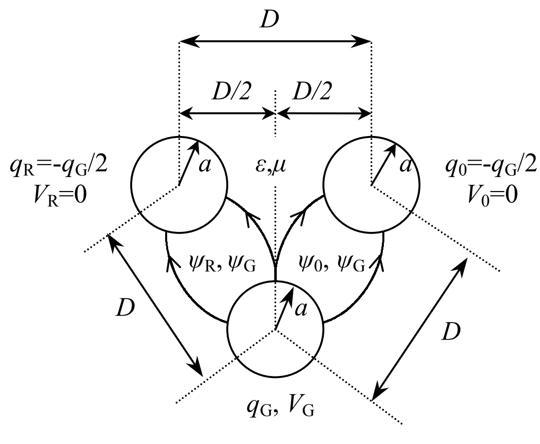

Fig. 4. Cable's static electric field structure

Per-unit-length conductance parameters are by definition commensurate with respective capacitances as [6]

$$g_x = \omega c_x \tan\delta, \quad x = G, R, m \tag{19}$$

resulting in

$$g_G = g_R = g_m = g\,, \tag{20}$$

and distributed-conductance parameter matrix is thereby

$$\mathbf{G} = g\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}. \tag{21}$$

The $\tan\delta$ parameter is dielectric's dissipation factor, defined as [6]

$$\tan\delta = \frac{\sigma_{\text{diel}}}{\omega\varepsilon}\,, \tag{22}$$

where σ_{diel} is conductivity of the lossy medium and ε the real part of its permittivity. Per-unit-length admittance matrix gets from (5b), (18) and (21):

$$\hat{\mathbf{Y}} = (g + j\omega c)\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \tag{23}$$

2.4 Decoupling

From (14) and (23)

$$\hat{\mathbf{Z}}\,\hat{\mathbf{Y}} = \hat{\mathbf{Y}}\,\hat{\mathbf{Z}} = (r + j\omega l)(g + j\omega c)\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \tag{24}$$

comes of, and combination with (6) gives

$$\frac{d^2\hat{\mathbf{V}}(z)}{dz^2} = (r + j\omega l)(g + j\omega c)\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\hat{\mathbf{V}}(z) \tag{25a}$$

and

$$\frac{d^2\hat{\mathbf{I}}(z)}{dz^2} = (r + j\omega l)(g + j\omega c)\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\hat{\mathbf{I}}(z)\,, \tag{25b}$$

wherefrom it is profoundly inferred that the lines of generator and receptor carry each one a single TEM mode non-interfering with each other, both having identical transmission constants γ_{III} with

$$\gamma_{\text{III}}^{\ 2} = 3(r + j\omega l)(g + j\omega c)\,. \tag{26}$$

Being conductors not perfect, i.e. $r \neq 0$, yields a small longitudinal electric-field component due to the line currents flowing through the conductors, but such losses are for common good conductors well approximated in the TEM transmission-line formulation [7].

Regarding hence from above stated phase wire as generator, neutral as reference and the ground one as receptor, it has heretofore been demonstrated that HF communication signalling transmitted over single-phase three-wire mains cabling between the phase and neutral conductors is absolutely unaffected by vicinal presence of the ground conductor. Such deduction arises from TEM field concepts, justifiably adopted with absolute validity for common cable types, conductors and dielectrics, and frequencies up to the GHz range [5-7]. Setting reference at the neutral instead of ground is well acceptable, as selection of the zero-potential level may consequently be arbitrary.

3. Two-Conductor approach

Let at here the uniform two-conductor distributed transmission-line model be considered as fig. 5 illustrates, where R, L, C, and G denote respectively per unit length series resistance, series inductance, shunt capacitance and shunt conductance. Transmission constant of such a configuration is extracted as

$$\gamma_{\mathrm{II}}{}^{2} = (R + j\omega L)(G + j\omega C). \tag{27}$$

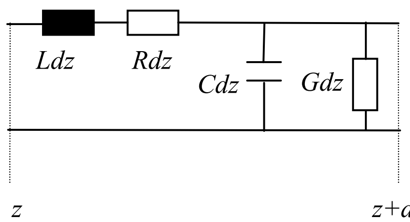

Fig. 5. Two-conductor transmission line model

Supposing two parallel conductors inside homogeneous dielectric, transmission constant is expressed as

$$\gamma_{\mathrm{II}}{}^{2} = \gamma^{2}_{\mathrm{II,TEM}} + \gamma^{2}_{\mathrm{II,LONG}}, \tag{28a}$$

where

$$\gamma^{2}_{\mathrm{II,TEM}} = j\omega L(G + j\omega C) \tag{28b}$$

and

$$\gamma^{2}_{\mathrm{II,LONG}} = R(G + j\omega C). \tag{28c}$$

The term $\gamma^{2}_{\mathrm{II,TEM}}$ represents line's pure TEM wave mode, whereas $\gamma^{2}_{\mathrm{II,LONG}}$ incorporates additional longitudinal electric field components arisen on account of conductors' imperfection. Usually

$$R << \omega L, \tag{29}$$

so that longitudinal electric field is small enough to be ignored and the TEM assumption is a fairly good approximation of the field structure. Impact of R on the attenuation coefficient coming of (27) is though of considerable range. Transmission constant of the three-conductor assembly, given in (26) above, may thereupon be equivalently expressed as

$$\gamma_{\mathrm{III}}{}^{2} = \gamma^{2}_{\mathrm{III,TEM}} + \gamma^{2}_{\mathrm{III,LONG}}, \tag{30a}$$

with

$$\gamma^{2}_{\mathrm{III,TEM}} = j3\omega l(g + j\omega c) \tag{30b}$$

and

$$\gamma^{2}_{\mathrm{III,LONG}} = 3r(g + j\omega c). \tag{30c}$$

Applying to any uniform two-conductor line inside homogeneous lossy medium, for the purely TEM portion of transmission constant by definition it stands [7]

$$\gamma^{2}_{\mathrm{TEM}} = j\omega\mu(\sigma + j\omega\varepsilon), \tag{31}$$

where μ the permeability. From (26)-(28), (30) and (31)

$$j3\omega l(g + j\omega c) = j\omega L(G + j\omega C)$$

$$\Rightarrow 3lg + j3\omega lc = LG + j\omega LC \tag{32}$$

is obtained, wherefrom substitution from (19) and (22) gives

$$3lc = LC. \tag{33}$$

In order for distributed inductance l of the three-conductor configuration to be determined in relation with respective two-conductor parameter L, field structure of fig. 3 is considered. Absence of receptor obviously amounts to a two-conductor line, whereat

$$\psi_G = LI_G \Rightarrow L = \frac{\psi_G}{I_G}. \tag{34}$$

Introducing receptor causes no alteration upon the overall magnetic flux ψ_G, as both conductors' material – normally a diamagnetic or paramagnetic one like copper or aluminium – and dielectric medium are characterised by permeability sufficiently close to the vacuum value, i.e. $\mu \approx \mu_0$, and no current flows along the former. Combination of (10)-(12) and (34) yields

$$l = \frac{L}{2}. \tag{35}$$

Substituting (35) in (33) results in

$$c = \frac{2C}{3}, \tag{36}$$

and consequently

$$g = \frac{2G}{3} \tag{37}$$

Parameter r refers besides to a single conductor, whereas distributed resistance R incorporates resistive losses on both conductors of the line, providing

$$r = \frac{R}{2}. \tag{38}$$

From (26) and (35)-(38) it is finally deduced

$$\gamma_{\mathrm{III}}^2 = 3\left(\frac{R}{2} + j\omega\frac{L}{2}\right)\left(\frac{2G}{3} + j\omega\frac{2C}{3}\right)$$

$$\Rightarrow \gamma_{\mathrm{III}}^2 = (R + j\omega L)(G + j\omega C) = \gamma_{\mathrm{II}}^2, \tag{39}$$

i.e. propagation properties of the generator-reference transmission circuit are same as if the two conductors formed an isolated uniform transmission line at equal distance in between. All reasoning presented has been developed on the basis of TEM field adoption, well valid for common good conductors and dielectrics in the HF band

Experimental investigation that has up to now been performed verifies the rationale developed herein. The results presented in [4] are of remarkable accuracy, as no deviation above 20% is observed on linear attenuation scale. Applicability thus to HF signalling over indoor symmetric triplex cabling is well inferred, as many common cable types fall under the case studied.

4. Conclusion

In this paper, theoretical HF transmission-line analysis has been introduced concerning indoor single-phase tricels assembled of identical adjacent wires. By the TEM propagation mode assumption and the resultant static-field concept, transfer properties of a conductor pair within the cable have been shown to follow the isolated uniform two-conductor model. The latter in practice means that PLC signalling between phase and neutral is unaffected by the vicinal ground wire. The theoretical investigation presented justifies adoption of the two-conductor transmission line model for characterisation purposes of corresponding PLC channels. Moreover, symmetric triplex cable configurations are hereby strongly suggested as for interference with ground wire to be avoided.

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