Dispersive Finite-Difference Time-Domain Simulation of Electromagnetic Cloaking Structures

Yan Zhao*, Christos Argyropoulos, and Yang Hao
Electronic Engineering, Queen Mary, University of London
Mile End Road, London E1 4NS, UK
E-mail: yan.zhao@elec.qmul.ac.uk

Introduction

Recently the development of electromagnetic cloaking structures has received considerable attention. Based on the coordinate transformation technique, Pendry *et al.* [1] proposed an electromagnetic material through which electromagnetic fields can be excluded from a penetrable object without perturbing the exterior fields, thereby rendering the interior effectively "invisible" to the outside. The proposed cloaking structure in [1] is composed of anisotropic media with each permittivity and permeability element independently controlled, and these media can be realised using metamaterials. The modelling of cloaking structures has been performed by using both the analytical approach [1,2] and frequency-domain numerical techniques [3]. In this paper, we propose a new dispersive FDTD method to deal with metamaterials for the cloaking structures which are anisotropic and possess radial dependent permittivity and permeability.

Dispersive FDTD Modelling of the Cloaking Structure

Ideally, a full set of material parameters of the cloaking structure is given by [1]:

$$\varepsilon_r = \mu_r = \frac{r - R_1}{r}, \quad \varepsilon_\phi = \mu_\phi = \frac{r}{r - R_1}, \quad \varepsilon_z = \mu_z = \left(\frac{R_2}{R_2 - R_1}\right)^2 \frac{r - R_1}{r}, \qquad (1)$$

where R_1 and R_2 are the inner and outer radius of the cloaking shell, respectively as shown in Fig. 1(a). It can be easily identified from (1) that the ranges

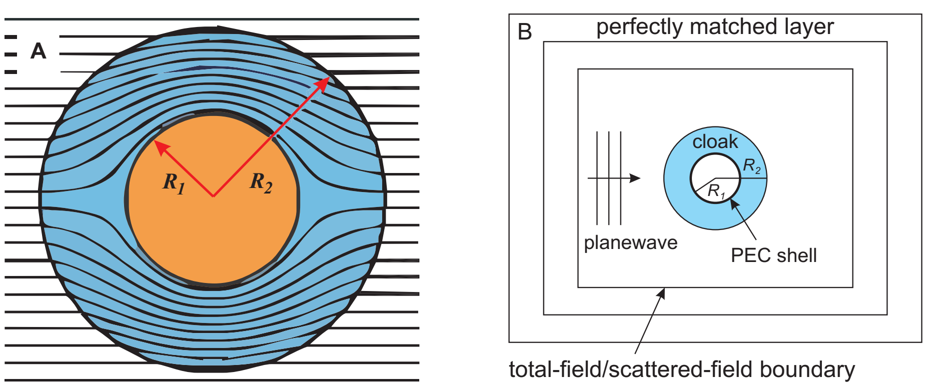

Figure 1: (a) A two-dimensional (2-D) cross section of rays striking the cloaking shell proposed in [1]. (b) 2-D FDTD simulation domain of the cloaking shell for the case of plane-wave incidence.

of permittivity and permeability within the shell are $\varepsilon_r, \mu_r \in [0, (R_2 - R_1)/R_2]$, $\varepsilon_\phi, \mu_\phi \in [R_2/(R_2 - R_1), \infty]$ and $\varepsilon_z, \mu_z \in [0, R_2/(R_2 - R_1)]$. Since the values of ε_r, μ_r, ε_z and μ_z are less than one, like the left-handed materials (LHMs), such media

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can not be found in nature and in the FDTD modeling, we need map the material parameters using e.g. Drude dispersion model

$$\varepsilon_r(\omega) = 1 - \frac{\omega_{pe}^2}{\omega^2 - j\omega\gamma_e}, \qquad \mu_z(\omega) = 1 - \frac{\omega_{pm}^2}{\omega^2 - j\omega\gamma_m} \tag{2}$$

where ω_{pe} and ω_{pm} are the plasma frequencies and γ_e and γ_m are the collision frequencies of the material. By varying the plasma frequencies, the radial dependent material parameters (1) can be modelled. In this paper, we have implemented the dispersive FDTD method for a 2-D transverse electric (TE) case therefore only three field components are non-zero: E_x, E_y and H_z (hence only ε_r, ε_ϕ and μ_z come into play, but note that in the following, the derivation is only given for ε_r, and the updating equation for μ_z can be derived following the same procedure by choosing appropriate plasma frequencies ω_{pm}). For the conventional Cartesian FDTD mesh, the coordinate transformation [3]

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} \varepsilon_r \cos^2\phi + \varepsilon_\phi \sin^2\phi & (\varepsilon_r - \varepsilon_\phi)\sin\phi\cos\phi \\ (\varepsilon_r - \varepsilon_\phi)\sin\phi\cos\phi & \varepsilon_r \sin^2\phi + \varepsilon_\phi \cos^2\phi \end{bmatrix} \tag{3}$$

is used since the material parameters given in (1) are in cylindrical coordinates. The tensor form of the constitutive relation is given by

$$\varepsilon_0 \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} D_x \\ D_y \end{bmatrix} \quad \Leftrightarrow \quad \varepsilon_0 \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix}^{-1} \begin{bmatrix} D_x \\ D_y \end{bmatrix}. \tag{4}$$

Substituting (3) into (4) and taking the inverse of the permittivity tensor give

$$\begin{cases} \varepsilon_r\varepsilon_\phi\varepsilon_0 E_x = \left(\varepsilon_r \sin^2\phi + \varepsilon_\phi \cos^2\phi\right) D_x + (\varepsilon_\phi - \varepsilon_r)\sin\phi\cos\phi D_y \\ \varepsilon_r\varepsilon_\phi\varepsilon_0 E_y = \left(\varepsilon_r \cos^2\phi + \varepsilon_\phi \sin^2\phi\right) D_y + (\varepsilon_\phi - \varepsilon_r)\sin\phi\cos\phi D_x \end{cases}, \tag{5}$$

Express ε_r in the Drude form of (1), the first equation of (5) can be written as

$$\begin{aligned} \varepsilon_0\varepsilon_\phi\left(\omega^2 - j\omega\gamma - \omega_p^2\right) E_x \;=\; & \left[\left(\omega^2 - j\omega\gamma - \omega_p^2\right)\sin^2\phi + \varepsilon_\phi\left(\omega^2 - j\omega\gamma\right)\cos^2\phi\right] D_x \\ & + \left[\varepsilon_\phi\left(\omega^2 - j\omega\gamma\right) - \left(\omega^2 - j\omega\gamma - \omega_p^2\right)\right]\sin\phi\cos\phi D_y. \end{aligned} \tag{6}$$

Notice that the ε_ϕ remains in (6) because its value is always greater than one (except at the inner surface of the shell) and can be directly used in conventional FDTD updating equations [4]. Using the inverse Fourier transformation i.e. $j\omega \to \partial/\partial t$, $\omega^2 \to -\partial^2/\partial t^2$, and following the standard discretisation procedure [4], the updating equation in FDTD can be obtained as

$$E_x^{n+1} = \left[aD_x^{n+1} + bD_x^n + cD_x^{n-1} + d\overline{D_y}^{n+1} + e\overline{D_y}^{n} + f\overline{D_y}^{n-1} - \left(gE_x^n + hE_x^{n-1}\right)\right]/l. \tag{7}$$

where the coefficients a to l are given by

$$a = \sin^2\phi\left[\frac{1}{(\Delta t)^2} + \frac{\gamma}{2\Delta t} + \frac{\omega_p^2}{4}\right] + \varepsilon_\phi\cos^2\phi\left[\frac{1}{(\Delta t)^2} + \frac{\gamma}{2\Delta t}\right],$$

$$b = \sin^2\phi\left[-\frac{2}{(\Delta t)^2} + \frac{\omega_p^2}{2}\right] - \varepsilon_\phi\cos^2\phi\frac{2}{(\Delta t)^2}, \quad e = \left\{\varepsilon_\phi\left[-\frac{2}{(\Delta t)^2}\right] - \left[-\frac{2}{(\Delta t)^2} + \frac{\omega_p^2}{2}\right]\right\}\sin\phi\cos\phi,$$

$$c = \sin^2\phi\left[\frac{1}{(\Delta t)^2} - \frac{\gamma}{2\Delta t} + \frac{\omega_p^2}{4}\right] + \varepsilon_\phi\cos^2\phi\left[\frac{1}{(\Delta t)^2} - \frac{\gamma}{2\Delta t}\right], \quad g = \varepsilon_\phi\left[-\frac{2}{(\Delta t)^2} + \frac{\omega_p^2}{2}\right],$$

$$d = \left\{\varepsilon_\phi\left[\frac{1}{(\Delta t)^2} + \frac{\gamma}{2\Delta t}\right] - \left[\frac{1}{(\Delta t)^2} + \frac{\gamma}{2\Delta t} + \frac{\omega_p^2}{4}\right]\right\}\sin\phi\cos\phi, \quad h = \varepsilon_\phi\left[\frac{1}{(\Delta t)^2} - \frac{\gamma}{2\Delta t} + \frac{\omega_p^2}{4}\right],$$

$$f = \left\{\varepsilon_\phi\left[\frac{1}{(\Delta t)^2} - \frac{\gamma}{2\Delta t}\right] - \left[\frac{1}{(\Delta t)^2} - \frac{\gamma}{2\Delta t} + \frac{\omega_p^2}{4}\right]\right\}\sin\phi\cos\phi, \quad l = \varepsilon_\phi\left[\frac{1}{(\Delta t)^2} + \frac{\gamma}{2\Delta t} + \frac{\omega_p^2}{4}\right].$$

where Δt is the discrete time step in the FDTD domain. Note that the field quantities $\overline{D_y}$ in (7) are locally averaged values of D_y since the x and y-components of the field are in different locations in the FDTD domain. The averaged value can be calculated using [5]

$$\overline{D_y}(i,j) = [D_y(i,j) + D_y(i+1,j) + D_y(i,j-1) + D_y(i+1,j-1)]/4. \tag{8}$$

where (i,j) is the coordinate of the field component.

Following the same procedure, the updating equations for the second equation of (5) and for the magnetic field component H_z can be obtained. The updating equation from **H** to **D** and **E** to **H** remain the same as the standard FDTD method [4].

Numerical Results and Discussion

We have implemented the above proposed dispersive FDTD method for the 2-D TE case. The simulation domain is shown in Fig. 1(b). The following parameters are used in simulations: FDTD cell size is $\Delta x = \Delta y = \lambda/75$ where λ is the wavelength at the operating frequency $f = 2.0$ GHz; the time step is $\Delta t = \Delta x/\sqrt{2}c$, chosen according to the Courant stability criterion [4]. In this paper, we assume the ideal lossless case i.e. the collision frequency in (2) is equal to zero ($\gamma = 0$), therefore the radial dependent plasma frequency can be calculated using $\omega_p = \omega\sqrt{1-\varepsilon_r}$ with a given value of ε_r calculated from (1). The radii of the shell are: $R_1 = 0.2$ m and $R_2 = 0.4$ m. The case of plane-wave incidence is considered in our simulations, where the total-field/scattered-field approach [4] is used to generate a plane-wave source from the left side of the computation domain.

Figure 2(a) shows the distribution of normalised magnetic field from the dispersive FDTD simulations of lossless cloaking shell for the case of plane-wave incidence.

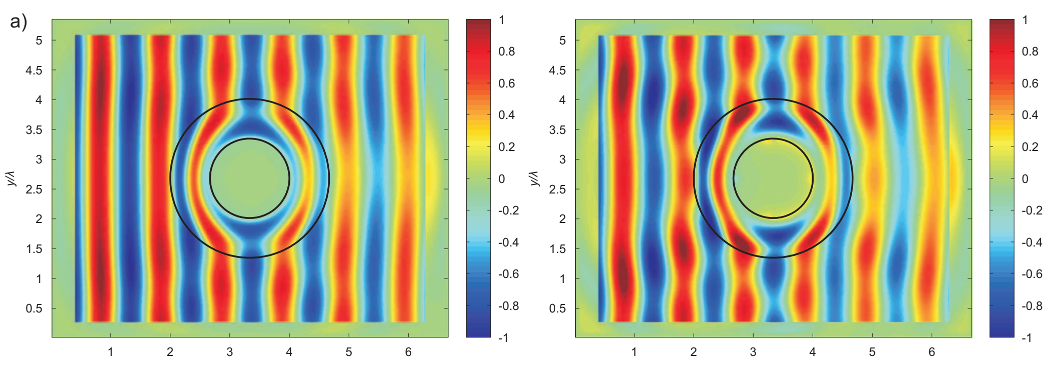

Figure 2: Normalised magnetic field distributions from dispersive FDTD simulations of the cloaking shell with (a) the complete set and (b) the reduced set of material parameters.

It can be seen in Fig. 2(a) that a plane-wave is generated from the left hand side of the total-field/scattered-field boundary and propagates towards the right direction. Within the cloaking shell, the wave is bent to propagate around the shell; behind the shell, the distribution remains plane-wave like, as if there was no object blocking the direction of wave propagation. Therefore the objects inside the cloaking shell become "invisible" to external electromagnetic waves. Notice that for the ideal case, due to the zero fields inside the cloaking structure, the object placed inside does not disturb the performance of such cloaking structure, in contrast to the cloaking structures composed of LHMs [6]. However the disturbance of the cloaking structure to the plane-wave as seen on the right side of the shell in Fig. 2(a) is caused by the staircase approximation of the curved surfaces in the Cartesian FDTD mesh used in our simulations. The accuracy of simulation can be improved using a conformal scheme in addition to the dispersive FDTD method as it was done for the case of homogeneous dispersive materials [7].

The complete set of material parameters given by (1) (all permittivity and permeability are radial dependent) provides perfect cloaking. However in reality, it is difficult to realise such structures. Therefore it has been proposed in [8] that while keeping the same wave trajectory, a reduced set of material parameters also allows constructing cloaking structures. The reduced set of material parameters is given by (for the 2-D TE case) [8]: $\varepsilon_r = \left(\frac{R_2}{R_2 - R_1}\right)^2 \left(\frac{r - R_1}{R_1}\right)^2, \varepsilon_\phi = \left(\frac{R_2}{R_2 - R_1}\right)^2, \mu_z = 1$. Following the same procedure, we have also modified the above proposed dispersive FDTD method to model the reduced set of material parameters and analysed its cloaking performance. The simulation result is shown in Fig. 2(b). It can be seen that since the impedance matching at the outer boundary of the shell is not anymore satisfied, considerable reflections disturb the field distribution both in front of and behind the cloaking shell, in accordance with [8].

Conclusion

In this paper, we have proposed a dispersive FDTD method to analyse cloaking structures. The Drude dispersion model is used to describe the radial dependent material parameters and the auxiliary differential equation method is applied to take into account the material frequency dispersion in FDTD. Numerical simulations validate the theoretical predication of the ideal cloaking shell.

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