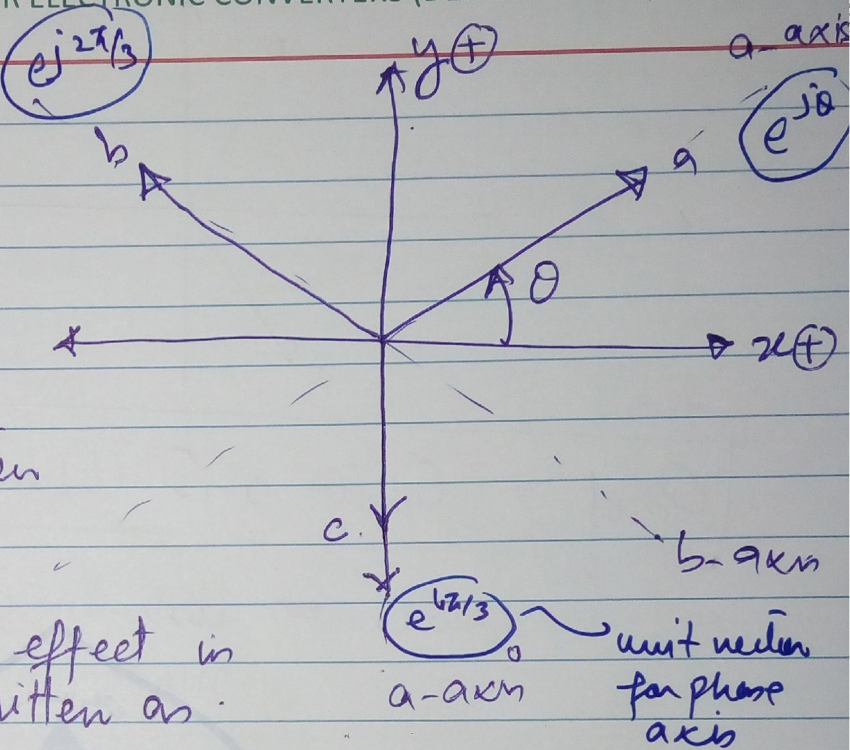
Consider the diagram.



$$\begin{cases} Q_a = Q\sin(\omega t) \\ Q_b = Q\sin(\omega t + 2\pi/3) \\ Q_c = Q\sin(\omega t + 4\pi/3) \end{cases}$$

* All these will vary along their own axis with time.

* At any instant the net effect in x-y plane can be written as.

$$Q_{net} = Q\sin(\omega t)\left[\cos\theta + j\sin\theta\right] + Q\sin(\omega t + 2\pi/3)\left[\cos(\theta + 2\pi/3) + j\sin(\theta + 2\pi/3)\right] + Q\sin(\omega t + 4\pi/3)\left[\cos(\theta + 4\pi/3) + j\sin(\theta + 4\pi/3)\right]$$

$$Q_{net} = Q_a(t)\, e^{j\theta} + Q_b(t)\, e^{j(\theta + 2\pi/3)} + Q_c(t)\, e^{j(\theta + 4\pi/3)}$$

$$Q_{net} = e^{j\theta}\left[Q_a(t) + Q_b(t)\underbrace{e^{j2\pi/3}}_{=a} + Q_c(t)\underbrace{e^{j4\pi/3}}_{=a^2}\right]$$

$$Q_{net} = e^{j\theta}\left[Q_a(t) + a\,Q_b(t) + a^2 Q_c(t)\right]$$

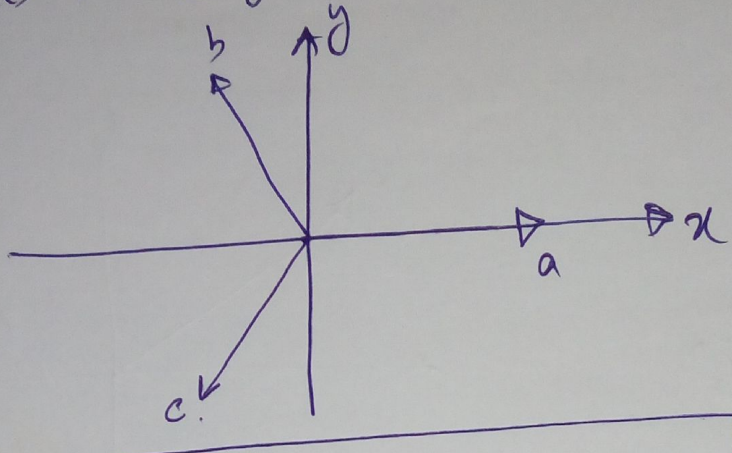
θ = Angle in space / (x-y) plane b/w phase a & real axis

θ = Displacement angle.

$a = e^{j2\pi/3}$ = unit vector of phase B

$a^2 = e^{j4\pi/3}$ = unit vector of phase C

If $Qa(t)$ is aligned with x-axis



$$\boxed{Qnet = Qa(t) + a\,Qb(t) + Qc(t).}$$

$\theta = 0 \Rightarrow$ Usually it is considered to make analysis simple.

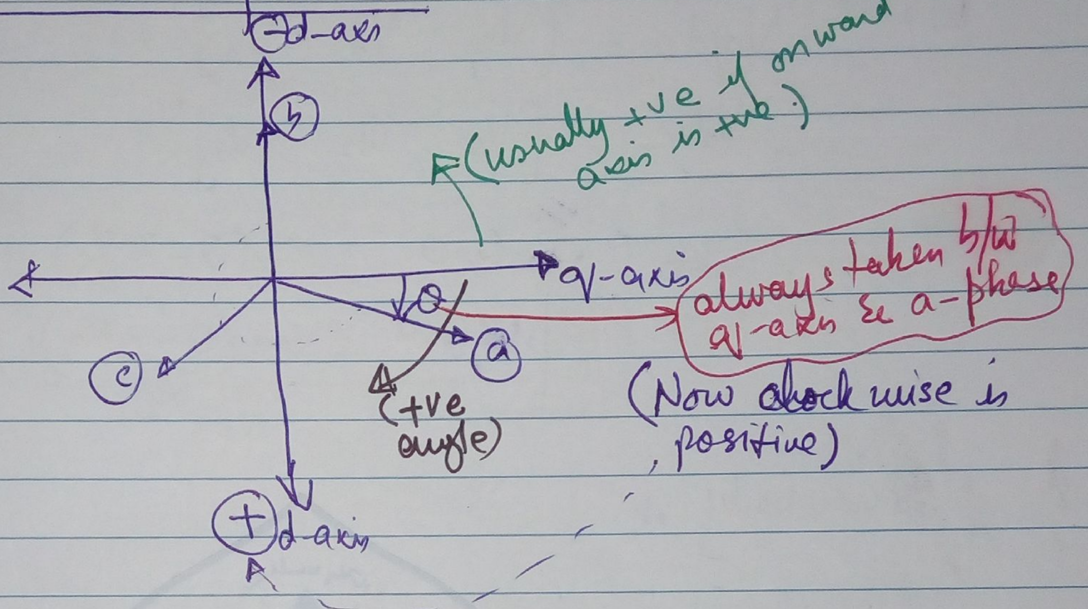
The generalize space vector

$$Qnet = k\left[Qa(t) + aQb(t) + a^2 Qc(t)\right]$$

$K = 2/3$ in order to keep magnitude of phase quantity equal to magnitude of space vector.

(*) (Show Matlab figures)

3-phase to 2-phase Transformation.



$$V_q = V_a\cos\theta + V_b\cos(\theta+240) + V_c\cos(\theta+120)$$
$$V_q = V_a\cos\theta + V_b\cos(\theta-120^\circ) + V_c\cos(\theta-240^\circ) \quad \text{— ①} \quad (\times 2/3) = k$$

Similarly

$$V_d = V_a\sin\theta + V_b\sin(\theta+240^\circ) + V_c\sin(\theta+120^\circ)$$
$$V_q = V_a\sin\theta + V_b\sin(\theta-120^\circ) + V_c\sin(\theta-240^\circ) \quad \text{— ②} \quad (\times 2/3) = k$$

Defining zero sequence Component

$$V_0 = \frac{V_a + V_b + V_c}{3} = \frac{V_a}{3} + \frac{V_b}{3} + \frac{V_c}{3} \quad \text{— ③}$$

So from ① ② & ③

$$\begin{bmatrix} V_q \\ V_d \\ V_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta-120^\circ) & \cos(\theta-240^\circ) \\ \sin\theta & \sin(\theta-120^\circ) & \sin(\theta-240^\circ) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\Longrightarrow [V_{qdo}] = \frac{2}{3}[T_{abc}][V_{abc}]$$

To make amplitude equal to phase voltage.

where $V_{qdo} = [V_q \quad V_d \quad V_0]^T$.

$V_{abc} = [V_a \quad V_b \quad V_c]^T$.

$$T_{abc} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta - 4\pi/3) \\ \sin\theta & \sin(\theta - 2\pi/3) & \sin(\theta - 4\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

Similarly

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = [T_{abc}]^{-1}\begin{bmatrix} V_q \\ V_d \\ V_0 \end{bmatrix} \Longrightarrow [V_{abc}] = [T_{abc}]^{-1}[V_{qdo}]$$

q – is real axis
d – is imaginary lagging behind by $\pi/2$.

$[V_{abc}] = [V_a \quad V_b \quad V_c]^T$.

$[V_{qdo}] = [V_q \quad V_d \quad V_0]^T$.

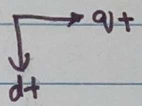
$$[T_{abc}]^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta - 2\pi/3) & \sin(\theta - 2\pi/3) & 1 \\ \cos(\theta - 4\pi/3) & \sin(\theta - 4\pi/3) & 1 \end{bmatrix}$$

$$[\vec{V}_{qdo}] = V_q - jV_d.$$ Think!!!

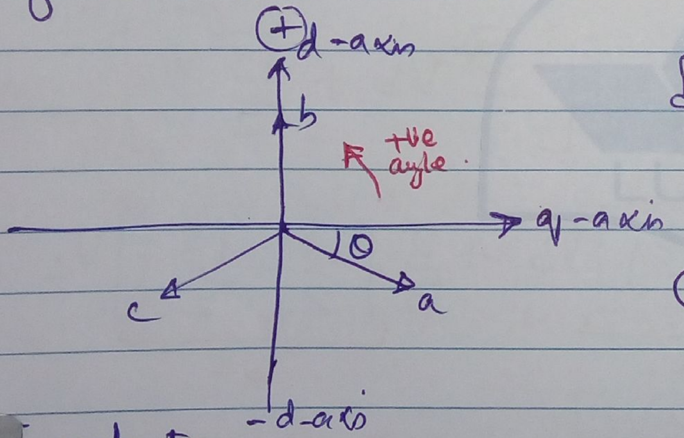
$$= V_\alpha - jV_\beta \Longrightarrow \begin{Bmatrix} V_q = V_\alpha \\ V_d = V_\beta \end{Bmatrix}$$!!!

Suppose the phase-a-axis is aligned with q-axis (real axis) then $\theta = 0°$.
so

$$\begin{bmatrix} V_q \\ V_d \\ V_0 \end{bmatrix} = \frac{2}{3}\underbrace{\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & -\sqrt{3}/2 & \sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}}_{T_{abc}|_{\theta=0°}} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \text{— (A)}$$



(*) Extra Work:
If we consider



$$\begin{cases} V_q = V_a\cos\theta + V_b\cos(120-\theta) + V_c\cos(240-\theta) \\ V_d = V_a\sin\theta + V_b\sin(120-\theta) + V_c\sin(240-\theta) \\ V_0 = \dfrac{V_a + V_b + V_c}{3} \end{cases}$$

(*) $V_{qdo} = V_q + jV_d$.

Important:

(*) If supply is balanced then $V_a + V_b + V_c = 0$ at any instance

$$V_q = \frac{2}{3}\left[V_a + \left(-\tfrac{1}{2}V_b\right) + \left(\tfrac{-1}{2}V_c\right)\right] = \frac{2}{3}\left[V_a - \frac{1}{2}(V_b + V_c)\right] = \frac{2}{3}\left[V_a - \frac{1}{2}(-V_a)\right]$$

$$\boxed{V_q = V_a}$$

Similarly $V_d = \frac{2}{3}\left[-\frac{\sqrt{3}}{2}V_b + \frac{\sqrt{3}}{2}V_c\right] = \frac{V_c}{\sqrt{3}} - \frac{V_b}{\sqrt{3}} = \frac{1}{\sqrt{3}}\left[V_c - V_b\right]$

and $V_0 = 0$.

Note:

Consider real axis (q-axis) to be α-axis

Consider imag axis (d-axis) to be β-axis

$$V_\alpha = \frac{2}{3}\left[V_a - \frac{1}{2}V_b - \frac{1}{2}V_c\right] = V_a$$

$$V_\beta = \frac{2}{3}\left[0 - \frac{\sqrt{3}}{2}V_b + \frac{\sqrt{3}}{2}V_c\right] = \frac{1}{\sqrt{3}}\left[V_c - V_b\right]$$

So the space vector is given as

$$\vec{V} = V_\alpha - jV_B$$

$$\vec{V} = \left[\frac{2}{3}V_a - \frac{1}{3}V_b - \frac{1}{3}V_c\right] - j\frac{2}{3}\left[-\frac{\sqrt{3}}{2}V_b + \frac{\sqrt{3}}{2}V_c\right]$$

$$\vec{V} = \frac{2}{3}V_a + \frac{2}{3}\underbrace{\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}j\right]}_{a}V_b + \frac{2}{3}\underbrace{\left[-\frac{1}{2} - \frac{\sqrt{3}}{2}j\right]}_{a^2}V_c$$

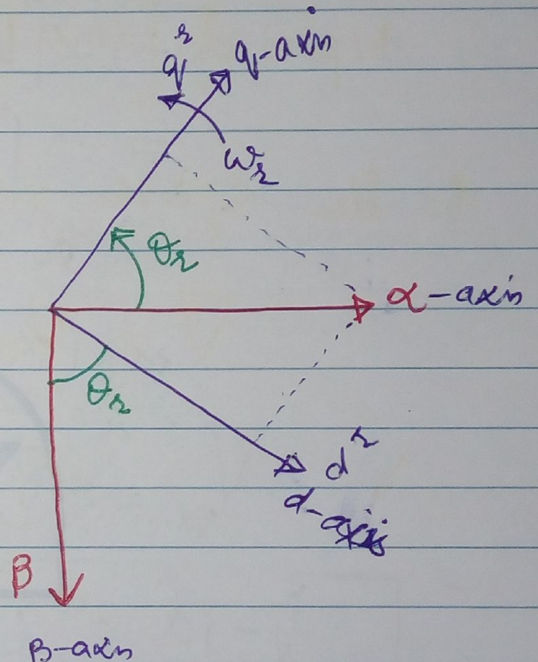
$$\vec{V} = \frac{2}{3}\left[V_a + aV_b + a^2V_c\right]$$

$$\boxed{\begin{aligned} a &= -\frac{1}{2} + \frac{\sqrt{3}}{2}j \\ a &= e^{j2\pi/3} \end{aligned}} \qquad \boxed{\begin{aligned} a^2 &= -\frac{1}{2} - \frac{\sqrt{3}}{2}j \\ a^2 &= e^{j4\pi/3} = e^{-j2\pi/3}. \end{aligned}}$$

a and a^2 are the unit vectors aligned with b-axis and c-axis of the machine and reference axis is a-axis.

Stationary to Rotating frame of Reference

We have considered α-β frame of reference that was fixed. Now we consider d^r-q^r frame of reference rotating with speed ω_r.



So the q^r-axis component from α-β components can be evaluated as.

$$q^r = \alpha \cos\theta_r + \beta \cos(\pi/2 + \theta_r)$$
$$q^r = \alpha \cos\theta_r - \beta \sin\theta_r$$

Similarly

$$d^r = \alpha \cos(\pi/2 - \theta_r) + \beta \cos\theta_r$$
$$d^r = \alpha \sin\theta_r + \beta \cos\theta_r$$

- Using above equation we can write in matrix form as

$$\begin{bmatrix} q^r \\ d^r \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

If q-d are rotating with synchronous speed $\omega_e \Rightarrow \theta_e = \omega_e t$ then frame of reference is said to be synchronously rotating frame of reference and q^e-d^e can be written as.

$$\begin{bmatrix} q^e \\ d^e \end{bmatrix} = \begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

So the vector in rotating frame of reference can be reprented as.

$$V^e_{qd} = V^e_q - jV^e_d = q^e - jd^e \text{ (general form)}$$

$$= (\alpha \cos\theta_e - \beta \sin\theta_e) - j(\alpha \sin\theta_e + \beta \cos\theta_e)$$

$$= \alpha[\cos\theta_e - j\sin\theta_e] - \beta[\sin\theta_e + j\cos\theta_e]$$

$$= \alpha[\cos\theta_e - j\sin\theta_e] - j\beta[\cos\theta_e - j\sin\theta_e]$$

$$= \alpha e^{-j\theta_e} - j\beta e^{-j\theta_e}$$

$$= (\alpha - j\beta) e^{-j\theta_e}$$

$$\vec{V}_{qd} = \vec{V}_{\alpha\beta} e^{-j\theta}$$

$$\boxed{q^e - jd^e = (\alpha - j\beta) e^{-j\theta_e}} = r e^{-j\theta}$$

$$r = \sqrt{\alpha^2 + \beta^2}$$

and inversely.

$$\underbrace{(\alpha - j\beta)}_{\text{Stationary frame of reference}} = \underbrace{(q^e - jd^e)}_{\text{rotating frame of reference}} e^{j\theta_e}$$

$$\text{Stationary frame} \xrightarrow{e^{-j\theta_e}} \text{Rotating frame}$$

$$\text{Rotating frame} \xrightarrow{e^{j\theta_e}} \text{Stationary fram}$$

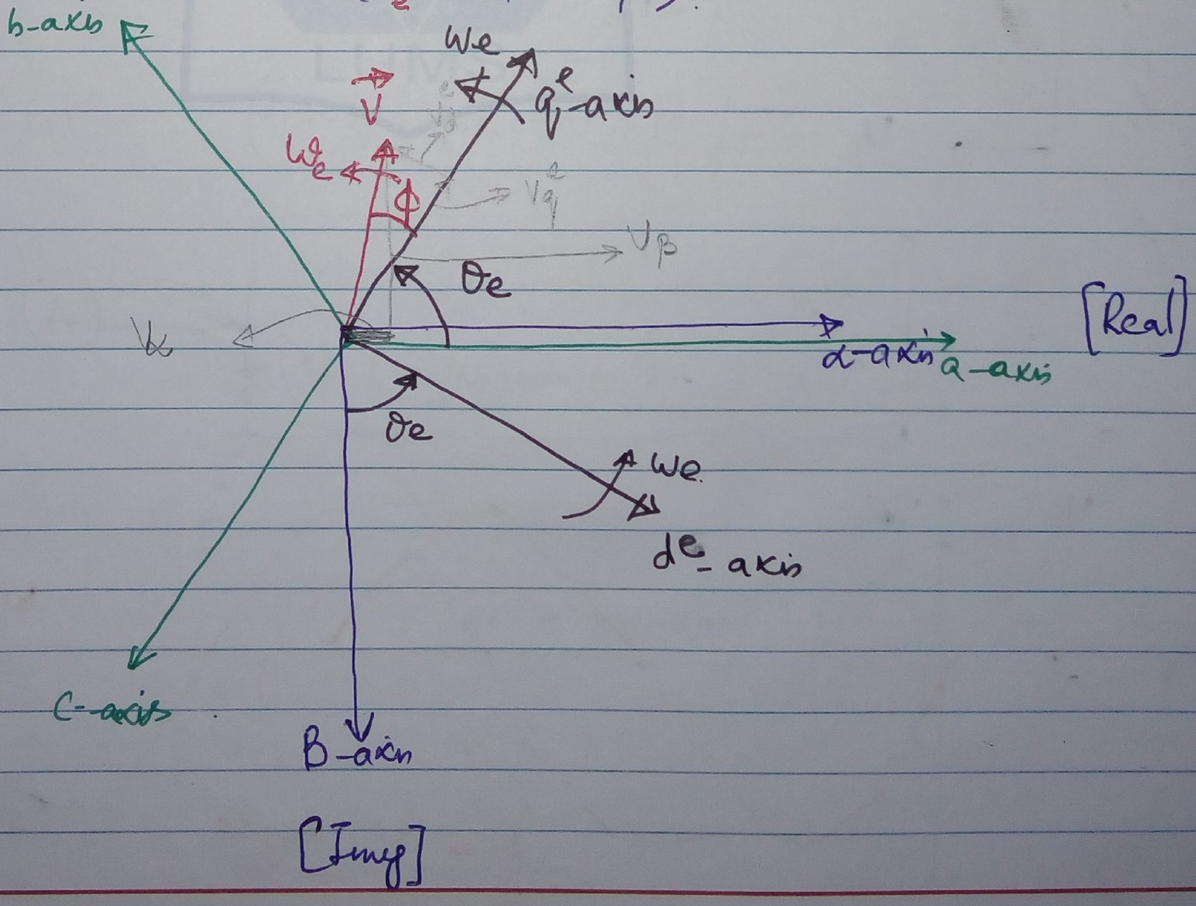
$\theta_e = w_e t$ where w_e is the speed of rotating frame.

(*) $e^{j\theta_e}$ is the vector rotational operator
It converts rotating frame variable into stationary frame

(*) $e^{-j\theta_e}$ is the inverse vector rotator that converts stationary frame variable into rotating frame variables.

Example: Consider a three phase system defined as

$$V_a = V_m \sin(\omega_e t + \phi)$$
$$V_b = V_m \sin(\omega_e t + \phi - 2\pi/3)$$
$$V_c = V_m \sin(\omega_e t + \phi + 2\pi/3).$$



abc → αβ

$$V_\alpha = V_a \longrightarrow V_\alpha = V_m \cos(\omega_e t + \phi)$$

$$V_\beta = \frac{1}{\sqrt{3}}(V_c - V_b)$$

$$V_\beta = \frac{1}{\sqrt{3}}\left[V_m \cos(\omega_e t + \phi + 2\pi/3) - V_m \cos(\omega_e t + \phi - \tfrac{2\pi}{3})\right]$$

Math:

$$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{B-A}{2}\right)$$

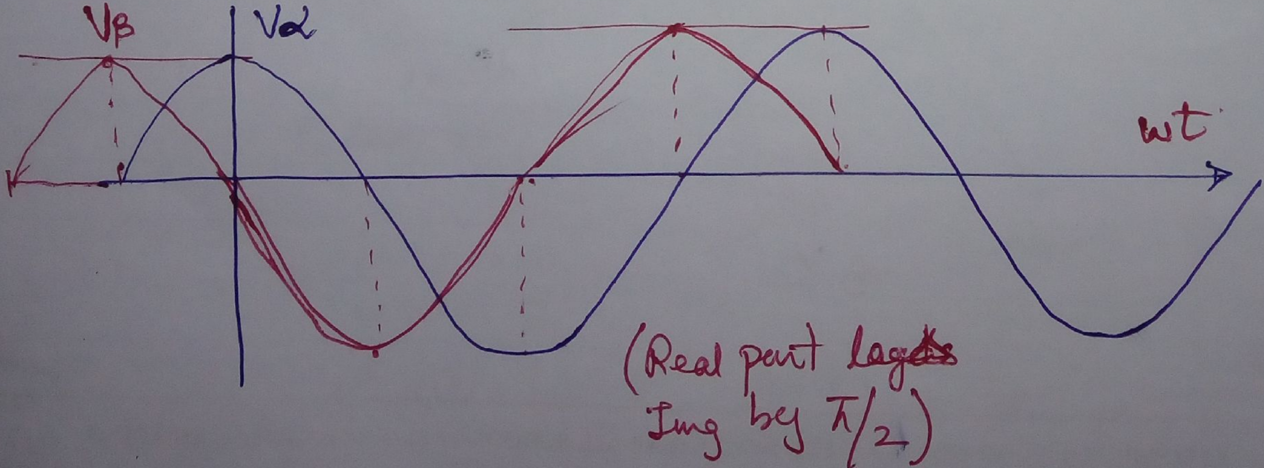
$$= 2\sin\left(\frac{\omega_e t + \phi + 2\pi/3 + \omega_e t + \phi - 2\pi/3}{2}\right) \cdot \sin\left(\frac{\omega_e t + \phi - 2\pi/3 - \omega_e t - \phi - 2\pi/3}{2}\right)$$

$$= +2\sin(\omega_e t + \phi)\,\sin\left(-2\pi/3\right)$$

$$= -\sqrt{3}\sin(\omega_e t + \phi)$$

$$V_\beta = \frac{-1}{\sqrt{3}}\left[\sqrt{3}\,V_m \sin(\omega_e t + \phi)\right] = -V_m \sin(\omega_e t + \phi).$$

$$\boxed{\begin{aligned} V_\alpha &= V_m \cos(\omega_e t + \phi) \\ V_\beta &= -V_m \sin(\omega_e t + \phi) \end{aligned}} \quad \text{— (A)}$$



- The above figure shows we have balanced two phase voltages of equal magnitude and V_β leads V_α by the angle of $\pi/2$.
- As space vector rotates there appears two sinusoidals along α-axis and β-axis.
- Space vector from equation (A) can be written as

$$V_\alpha - jV_\beta = V_m \cos(\omega_e t + \phi) + jV_m \sin(\omega_e t + \phi)$$

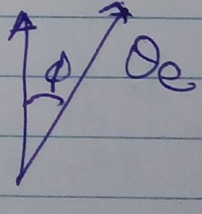
$$= V_m\left[\cos(\omega_e t + \phi) + j\sin(\omega_e t + \phi)\right]$$

$$= V_m\, e^{j(\omega_e t + \phi)}$$

$$= V_m \cdot e^{j\omega_e t} \cdot e^{j\phi}$$

$$= V_m \cdot e^{j\phi}\; e^{j\omega_e t}$$

Constant   speed → rotating counter clock wise.



- <u>Converting from α-β to rotating d-q:</u>

We know that $V_q = V_\alpha \cos\theta_e + V_\beta \sin\theta_e$

$$\Rightarrow V_q^e = V_m \cos(\omega_e t + \phi)\cdot\cos\theta_e + V_m \sin(\omega_e t + \phi)\cdot\sin\theta_e$$

$$= V_m\left[\cos(\theta_e + \phi)\cos\theta_e + \sin(\theta_e + \phi)\sin\theta_e\right]$$

$$V_q^e = V_m\left[\cos(\theta_e + \phi - \theta_e\right] = V_m \cos\phi.$$

$$\boxed{V_q = V_m \cos\phi}$$

Now

$$V_d^e = V_\alpha \sin\theta_e + V_\beta \cos\theta_e$$

$$V_d^e = V_m \cos(\theta_e+\phi)\sin\theta_e - V_m \sin(\theta_e+\phi)\cos\theta_e$$

$$= V_m\left[\cos(\theta_e+\phi)\sin\theta_e - \sin(\theta_e+\phi)\cos\theta_e\right]$$

$$= V_m \sin(\theta_e-\theta_e-\phi)$$

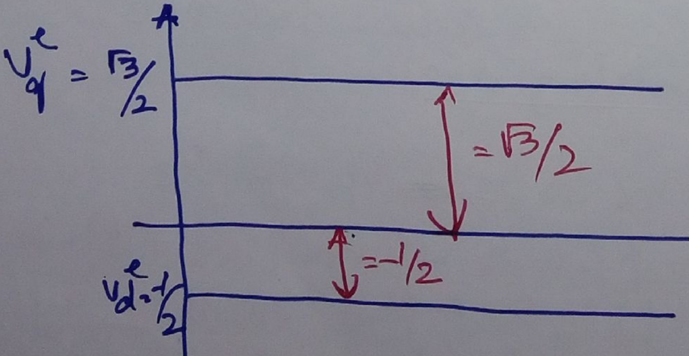
$$= V_m \sin(-\phi)$$

$$V_d^e = -V_m \sin\phi$$

$$\boxed{\begin{aligned} V_q^e &= V_m \cos\phi \\ V_d^e &= -V_m \sin\phi \end{aligned}} \qquad \alpha\text{-}\beta \rightarrow q^e\text{-}d^e$$

* with respect to $\alpha-\beta$, ϕ is constant so V_q^e & V_d^e are dc quantities in synchronously rotating frame of reference.

* for $\phi = 30$ & $V_m = 1$ $\quad V_q^e = \sqrt{3}/2$ & $V_d^e = -1/2$.



in rotating frame of reference.

Summary:

$$\begin{bmatrix} q^e \\ d^e \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix}}_{T^s_{qd} = T_{\alpha\beta}} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad \text{——} \quad (1)$$

$$[V^e_{qd}] = [q^e \quad d^e]^T \qquad T_{qd} \text{ or } T_{\alpha\beta} = \begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix}$$

$$[\alpha\beta] = [\alpha \quad \beta]^T$$

$$[qd] = [T^s_{qd}][\alpha\beta] \quad \text{or} \quad [qd] = [T_{\alpha\beta}][\alpha\beta]$$

From above equation (1) we can write.

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix}^{-1} \begin{bmatrix} q^e \\ d^e \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta_e & \sin\theta_e \\ -\sin\theta_e & \cos\theta_e \end{bmatrix}}_{\hat{T}_{qd} = T_{qd}.} \begin{bmatrix} q^e \\ d^e \end{bmatrix}$$

$$[\alpha\beta] = [T_{qd}]\,[qd]$$