Time Domain Analysis of Transmission Line Using FDTD Excited By Modulated Signal

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***Abstract*—In this paper, an unconditionally stable algorithm for time domain analysis of multiconductor transmission line using finite- difference time- domain (FDTD) is proposed. The results of the proposed algorithm are verified with the Leap-Frog algorithm. The complex phasor method is applied to multiconductor transmission line when the transmission line is extracted by modulated signal.**
The extracted equation is solved using the unconditionally stable formula. The results of the new equation are compared with the results of transmission line equation solved using Leap-Frog algorithm. Whereas the CPU time of the proposed method is very lower than the conventional method, the results of proposed method has a good agreement with the results of conventional method.

I. Introduction

The increasing demand of processing and transmitting more information at a faster rate leads the analog and digital electronic systems to operate at higher frequencies or higher clock speeds. The need for accurate device modeling are appreciated with the increasing operating frequency [1], [2]. The finite difference time domain method is used extremely in analysis of transmission lines. The accuracy of solved equation depend on the temporal and spatial step sizes [4]. The Leap-Frog algorithm due to symmetry and simplicity is used in FDTD method widely. However, the Leap-Frog algorithm must satisfy the Courant-Friedrich-Levy (CFL) stability condition, which means that time step cannot be chosen to be too large [5], [6]. The spatial step size must be chosen small enough to avoid discretization errors (i.e. smaller than the $\lambda_g/10$). On the other hand, the time step is restricted with the CFL stability condition. So the CPU time of the simulation increases extremely with increasing the operating frequency. One of the most method in EMC testing is transmission of information signals with bandwidth of several GHz which are modulated by a carrier signal with several KHz bandwidth.
In this cases, frequency of modulated signal which is much higher than the frequency of information signal, restricts the spatial step size.
Moreover, the behavior of system must be simulated in several period of the information signal. Therefore, the number of iteration in Leap-Frog algorithm are increased extremely, due to small time step size and large simulation time. But the CFL stability condition is removed by an unconditionally stable algorithm. The simulation time can be increased using unconditionally stable algorithm. However the number of sample points in the simulation must be at least two for each period of the modulated signal. When the transmission line is excited by a modulated signal which the frequency of carrier is much higher than the frequency of information signal, the unconditionally stable algorithm alone can not decrease simulation time.
In this paper, using the complex phasor the information signal is separated from the carrier signal. Then the unconditionally stable algorithm is used. The results of proposed method are confirmed with the results of Leap-Frog algorithm. While the CPU time of the proposed algorithm is very lower than the conventional algorithms.

II. Transmission Line Equation

The transmission line equations in time domain with quasi-TEM consumption given [3]:

$$\frac{\partial \mathbf{v}(z,t)}{\partial z} + \mathbf{R}\mathbf{i}(z,t) + \mathbf{L}\frac{\partial \mathbf{i}(z,t)}{\partial t} = 0$$
$$\frac{\partial \mathbf{i}(z,t)}{\partial z} + \mathbf{G}\mathbf{v}(z,t) + \mathbf{C}\frac{\partial \mathbf{v}(z,t)}{\partial t} = 0 \tag{1}$$

in which $\mathbf{i}, \mathbf{v}$ are the current and voltage vector respectively. The $\mathbf{R}, \mathbf{G}, \mathbf{L}$, and $\mathbf{C}$ are the per-unit-length parameters of the transmission line. Fig. 1 shows the discretization of a transmission line. The current of k^{th} spatial step is between the voltage of k^{th} and $k+1^{th}$ spatial step.
In the unconditionally stable algorithm, spatial derivative of voltage is approximated with forward-difference formula

$$\frac{\partial \mathbf{v}(z,t)}{\partial z} = \frac{\mathbf{v}_{k+1} - \mathbf{v}_k}{\Delta z} \tag{2}$$

and spatial derivative of current is approximated with backward-difference formula

$$\frac{\partial \mathbf{i}(z,t)}{\partial z} = \frac{\mathbf{i}_{k+1} - \mathbf{i}_k}{\Delta z} \tag{3}$$

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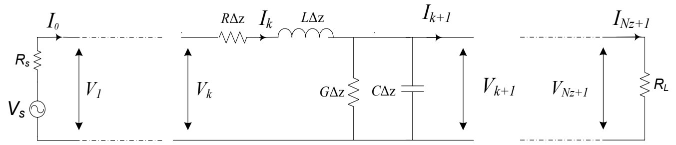

Fig. 1. Discretization of transmission line

Substituting these into (1) gives:

$$\frac{\mathbf{v}_{k+1}-\mathbf{v}_k}{\Delta z}+\mathbf{R}\mathbf{i}_k+\mathbf{L}\frac{\partial \mathbf{i}_1}{\partial t}=0 \tag{4}$$
$$\frac{\mathbf{i}_k-\mathbf{i}_{k-1}}{\Delta z}+\mathbf{G}\mathbf{v}_k+\mathbf{C}\frac{\partial \mathbf{v}_1}{\partial t}=0$$

Therefore:

$$\mathbf{v}_{k+1}=\mathbf{v}_k-\mathbf{R}\Delta z\mathbf{i}_k-\mathbf{L}\Delta z\frac{\partial \mathbf{i}_k}{\partial t} \qquad k=1,\cdots,N_z \tag{5}$$
$$\mathbf{i}_k=\mathbf{i}_{k-1}-\mathbf{G}\Delta z\mathbf{v}_k+\mathbf{C}\Delta z\frac{\partial \mathbf{v}_k}{\partial t} \qquad k=1,\cdots,N_z+1$$

In the beginning and the end of transmission line ($k=1$, $k=N_z+1$) must be considered the boundary conditions. With considering the boundary condition, Eq. 5 becomes:

$$\mathbf{i}_1=\mathbf{i}_0-\mathbf{G}\Delta z\mathbf{v}_1+\mathbf{C}\Delta z\frac{\partial \mathbf{v}_1}{\partial t} \tag{6}$$
$$\mathbf{i}_1+(\frac{\mathbf{G}\Delta z}{2}+\frac{1}{\mathbf{R}_s})\mathbf{v}_1+\frac{\mathbf{C}\Delta z}{2}\frac{\partial \mathbf{v}_1}{\partial t}=\frac{\mathbf{v}_s}{\mathbf{R}_s}$$

and

$$\mathbf{i}_{N_z+1}=\mathbf{i}_{N_z}-\mathbf{G}\Delta z\mathbf{v}_{N_z+1}+\mathbf{C}\Delta z\frac{\partial \mathbf{v}_{N_z+1}}{\partial t} \tag{7}$$
$$-\mathbf{i}_{N_z}+(\frac{\mathbf{G}\Delta z}{2}+\frac{1}{\mathbf{R}_L})\mathbf{v}_{N_z+1}+\frac{\mathbf{C}\Delta z}{2}\frac{\partial \mathbf{v}_{N_z+1}}{\partial t}=0$$

Therefore Equations (5), (6) , and (7) can be written as:

$$\mathbf{v}_{k+1}=\mathbf{v}_k-\mathbf{R}\Delta z\mathbf{i}_k-\mathbf{L}\Delta z\frac{\partial \mathbf{i}_k}{\partial t} \qquad k=1,\cdots,N_z$$
$$\mathbf{i}_1+(\frac{\mathbf{G}\Delta z}{2}+\frac{1}{\mathbf{R}_s})\mathbf{v}_1+\frac{\mathbf{C}\Delta z}{2}\frac{\partial \mathbf{v}_1}{\partial t}=\frac{\mathbf{v}_s}{\mathbf{R}_s} \tag{8}$$
$$\mathbf{i}_k=\mathbf{i}_{k-1}-\mathbf{G}\Delta z\mathbf{v}_k+\mathbf{C}\Delta z\frac{\partial \mathbf{v}_k}{\partial t} \qquad k=2,\cdots,N_z$$
$$-\mathbf{i}_{N_z}+(\frac{\mathbf{G}\Delta z}{2}+\frac{1}{\mathbf{R}_L})\mathbf{v}_{N_z+1}+\frac{\mathbf{C}\Delta z}{2}\frac{\partial \mathbf{v}_{N_z+1}}{\partial t}=0$$

Eq. 8 can be simplified as the following:

$$\mathbf{GV}+\mathbf{C}\frac{\partial \mathbf{V}}{\partial \mathbf{t}}+\mathbf{AI}=\mathbf{I}_s \tag{9}$$
$$\mathbf{RI}+\mathbf{L}\frac{\partial \mathbf{V}}{\partial \mathbf{t}}+\mathbf{BV}=\mathbf{0}$$

where $\mathbf{V}=[\mathbf{v}_1,\mathbf{v}_2,\cdots,\mathbf{v}_{N_z+1}]^T$, $\mathbf{I}=[\mathbf{i}_1,\mathbf{i}_2,\cdots,\mathbf{i}_{N_z}]^T$.
The temporal derivative of voltage and current are approximated with forward-difference formula. So Eq. 9 becomes:

$$(\frac{\mathbf{G}}{2}+\frac{\mathbf{C}}{\Delta t})\mathbf{V}^{n+1}+(\frac{\mathbf{G}}{2}-\frac{\mathbf{C}}{\Delta t})\mathbf{V}^{n}+\mathbf{A}\frac{\mathbf{I}^{n+1}+\mathbf{I}^{n}}{2}=\frac{\mathbf{I}_s^{n+1}+\mathbf{I}_s^{n}}{2} \tag{10}$$
$$(\frac{\mathbf{R}}{2}+\frac{\mathbf{L}}{\Delta t})\mathbf{I}^{n+1}+(\frac{\mathbf{R}}{2}-\frac{\mathbf{L}}{\Delta t})\mathbf{I}^{n}+\mathbf{B}\frac{\mathbf{V}^{n+1}+\mathbf{V}^{n}}{2}=0$$

with solving the matrix Equation (10), the voltage and current are given as following:

$$\mathbf{V}^{n+1}=(\frac{\mathbf{G}}{2}+\frac{\mathbf{C}}{\Delta t}-\frac{\mathbf{A}}{4}(\frac{\mathbf{R}}{2}+\frac{\mathbf{L}}{\Delta t})^{-1}\mathbf{B})^{-1}$$
$$(\frac{\mathbf{I}_s^{n}+\mathbf{I}_s^{n+1}}{2}-(\frac{\mathbf{G}}{2}-\frac{\mathbf{C}}{\Delta t}-\frac{\mathbf{A}}{4}(\frac{\mathbf{R}}{2}+\frac{\mathbf{L}}{\Delta t})^{-1}\mathbf{B})\mathbf{V}^{n}$$
$$-(\frac{\mathbf{A}}{2}+\frac{\mathbf{A}}{2}(\frac{\mathbf{R}}{2}+\frac{\mathbf{L}}{\Delta t})^{-1}(\frac{\mathbf{L}}{\Delta t}-\frac{\mathbf{R}}{2}))\mathbf{I}^{n}) \tag{11}$$

and

$$\mathbf{I}^{n+1}=(\frac{\mathbf{R}}{2}+\frac{\mathbf{L}}{\Delta t})^{-1}((\frac{\mathbf{L}}{\Delta t}-\frac{\mathbf{R}}{2})\mathbf{I}^{n}-\mathbf{B}\frac{\mathbf{V}^{n+1}+\mathbf{V}^{n}}{2}) \tag{12}$$

The bootstrapping method is used to solve these equations because of its simplicity and accuracy. In this approach, first the solutions start with an initially relaxed line having zero voltage and current values. Then, voltages along the electrodes of transistor are solved for a fixed time from Eq. (11) in terms of the previous solutions and then currents are solved from Eq. (12) in terms of these and previous values.

III. Complex Phasor Method

Any analog modulated signal (AM. FM, and PM) can be described by the following general expression [7]:

$$x(t)=X_1(t)\cos\omega_c t+X_2(t)\sin\omega_c t \tag{13}$$

where $X_1(t)$ and $X_2(t)$ are the components of the modulated signal and ω_c is the angular frequency of the carrier. Eq. (13) cab be written as:

$$x(t)=Re[(X_1(t)-jX_2(t))\exp(j\omega_c t)] \tag{14}$$

or as:

$$x(t)=|X(t)|Re[\exp(j\angle X(t))\exp(j\omega_c t)] \tag{15}$$

where $X(t) = X_1(t)-jX_2(t)$ therefore $\angle X(t) = \arctan(-X_2(t)/X_1(t))$ and $|X(t)|=\sqrt{X_1^2(t)+X_2^2(t)}$
Eq. (15) demonstrate that any modulated signal can be represented by a generalized phasor with time dependent magnitude and phase [8].
A new equation are extracted with applying the complex phasor method to transmission line equation. The new equation are solved using unconditionally stable algorithm.

IV. Simulation Results

The line considered here is a one conductor lossy transmission line whose parameters are $L=309nH, C=144pF, R=524\times10^{-3}\Omega, G=905\times10^{-9}S$. The total line length is $10cm$. The line is terminated at the beginning and end with 50Ω and 10Ω respectively. The wave velocity in the line is $v=1.499\times10^{8}m/s$. The spatial step size is considered $2.86mm$ to obtain a good accuracy. In Leap-Frog algorithm, the temporal step size must be smaller than $0.191pS$ for satisfying stability.
First, the transmission line is excited with a modulated signal which the information signal are two pulses $40MHz$ and

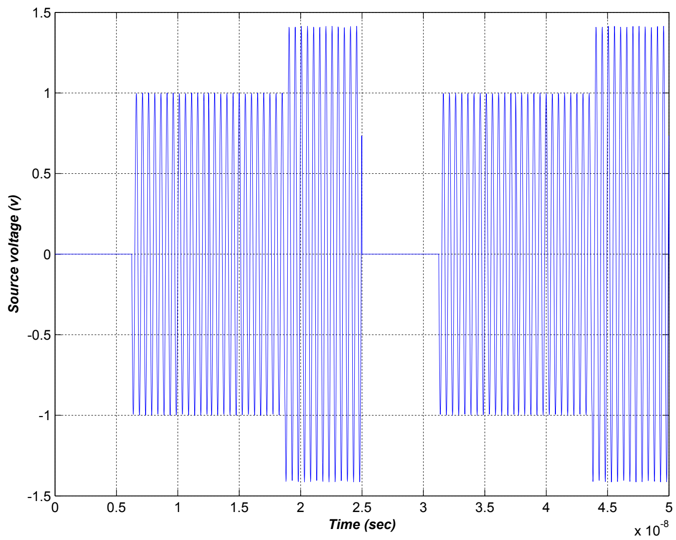

Fig. 2. Waveform of input signal

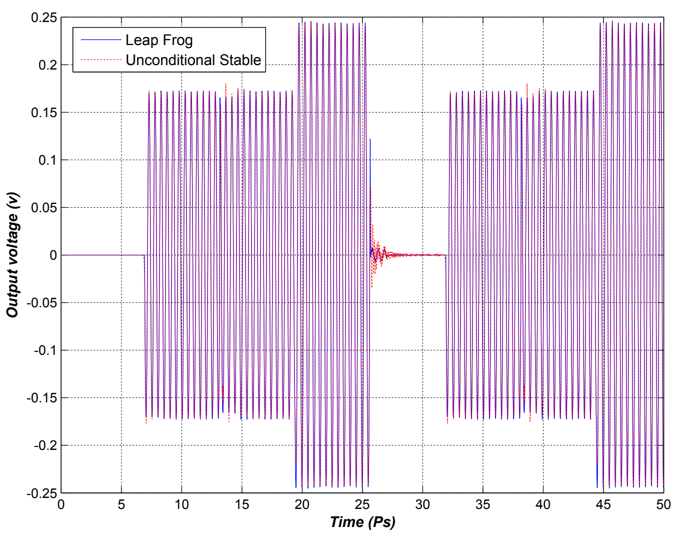

Fig. 3. Waveform of voltage at the end of line

$80MHz$ that modulated by a $2GHz$ signal. Fig. (2) shows the waveform of input signal. The waveform of voltage at the end of line using the Leap-Frog and unconditionally stable algorithms are depicted in Fig. (2). As seen in this figure, the results of proposed algorithm have a good agreement with the results of Leap-Frog algorithm. Now, we consider which the information signal are two pulses $10MHz$ and $2MHz$ that modulated by a $30GHz$ signal. The output signal is:

$$v_{out}(t) = v_{out1}(t)\cos\omega_c t + v_{out2}(t)\sin\omega_c t \tag{16}$$

the waveform of $v_{out1}(t)$ and $v_{out2}(t)$ for different temporal step size using the unconditionally stable algorithm are shown in Figs. (4), (5). When $\Delta t = 50\Delta t_{max}$ (Δt_{max} is the maximum allowable temporal step in Leap-Frog algorithm) the results of proposed algorithm have a good agreement with the results of Leap-Frog algorithm. Table. 1 shows the CPU time of Leap-Frog algorithm and unconditionally stable algorithms for several temporal step size.

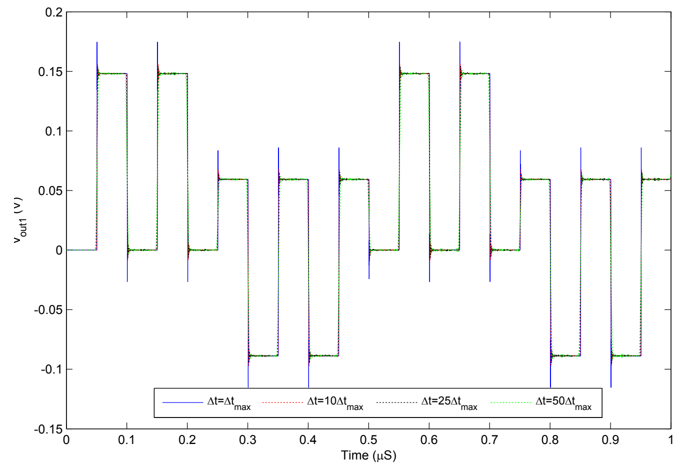

Fig. 4. Waveform of v_{out1}

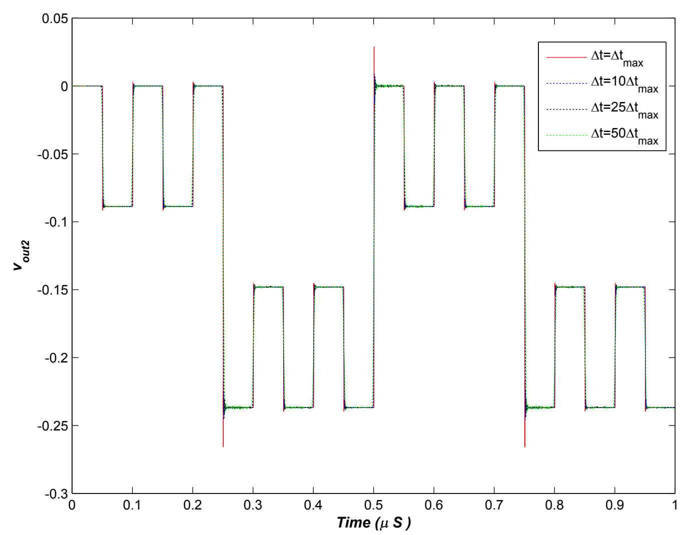

Fig. 5. Waveform of v_{out2}

TABLE I
CPU TIME OF DIFFERENT ALGORITHM FOR SEVERAL TEMPORAL STEP SIZES

Algorithm	Δt	CPU Time
Leap-Frog	$\Delta t = \Delta t_{max}$	1750.6 S
Unconditional stable	$\Delta t = \Delta t_{max}$	1775.5 S
Unconditional stable	$\Delta t = 10\Delta t_{max}$	5.3281 S
Unconditional stable	$\Delta t = 25\Delta t_{max}$	1.9531 S
Unconditional stable	$\Delta t = 50\Delta t_{max}$	0.75 S

V. CONCLUSION

An unconditionally stable algorithm has been proposed for time domain analysis of a transmission line using FDTD method. The presented algorithm cannot decrease the simulation time when the transmission line excited by modulated signal efficiently. To remove this problem, the carrier signal has been separated from the information signal and a new set of equations has been extracted. These equations have been solved by unconditional stable algorithm. The new technique improves time efficiency extremely.

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