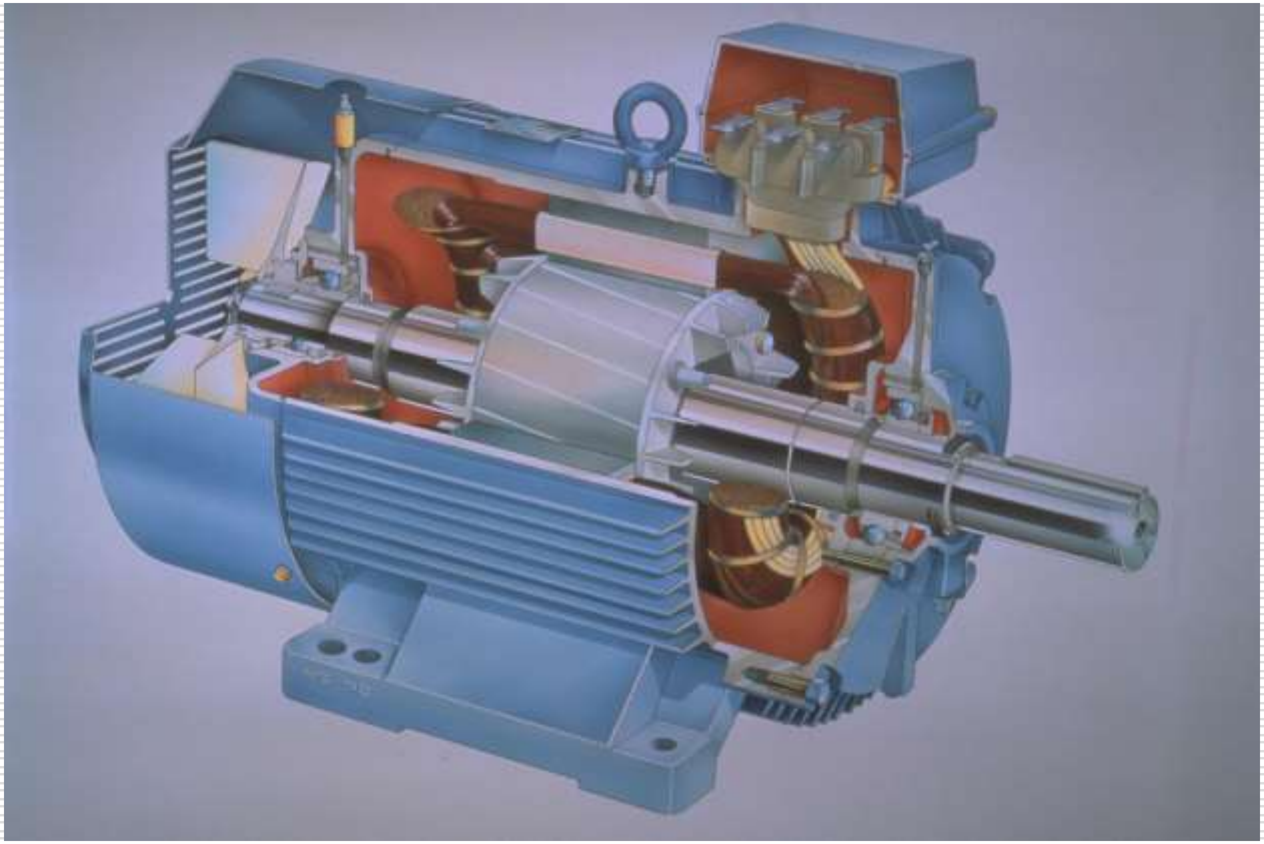
Induction Motors

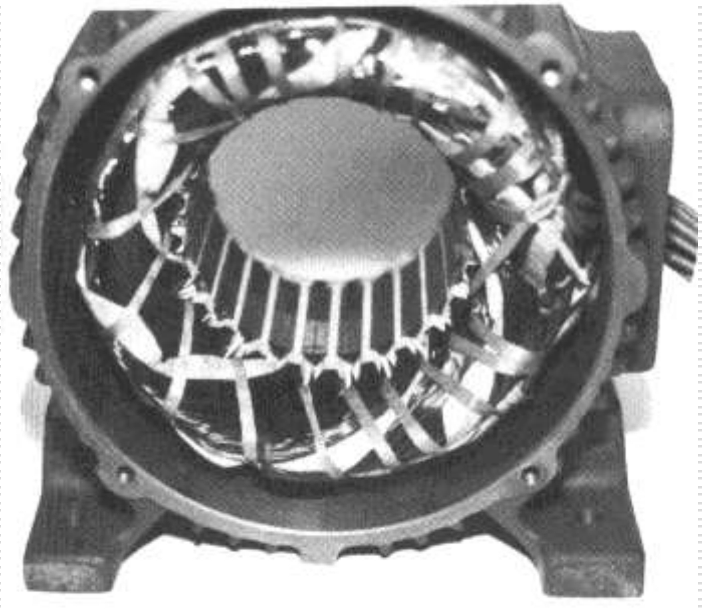


Introduction

- Three-phase induction motors are the most common and frequently encountered machines in industry
  - simple design, rugged, low-price, easy maintenance
  - wide range of power ratings: fractional horsepower to 10 MW
  - run essentially as constant speed from no-load to full load
  - Its speed depends on the frequency of the power source
    - not easy to have variable speed control
    - requires a variable-frequency power-electronic drive for optimal speed control

Construction

- An induction motor has two main parts
  - a stationary stator
    - consisting of a steel frame that supports a hollow, cylindrical core
    - core, constructed from stacked laminations (why?), having a number of evenly spaced slots, providing the space for the stator winding



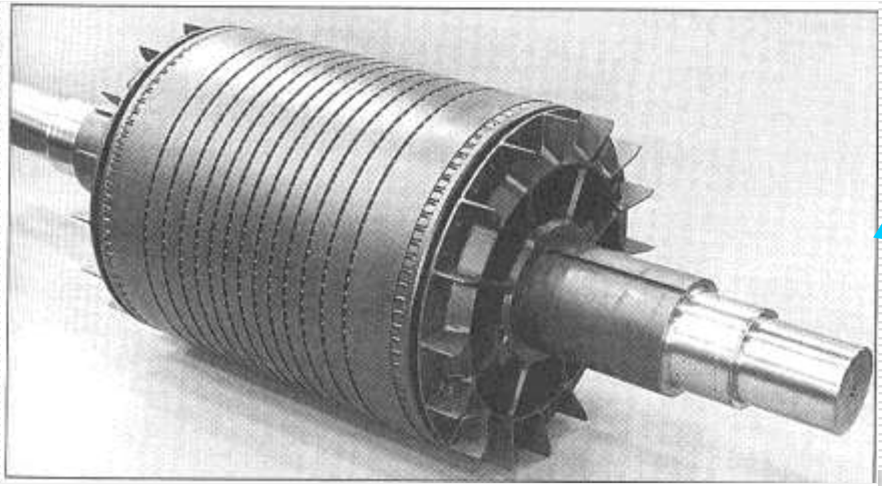
Stator of IM

Construction

- a revolving rotor
  - composed of punched laminations, stacked to create a series of rotor slots, providing space for the rotor winding
  - one of two types of rotor windings
  - conventional 3-phase windings made of insulated wire (wound-rotor) » similar to the winding on the stator
  - aluminum bus bars shorted together at the ends by two aluminum rings, forming a squirrel-cage shaped circuit (squirrel-cage)

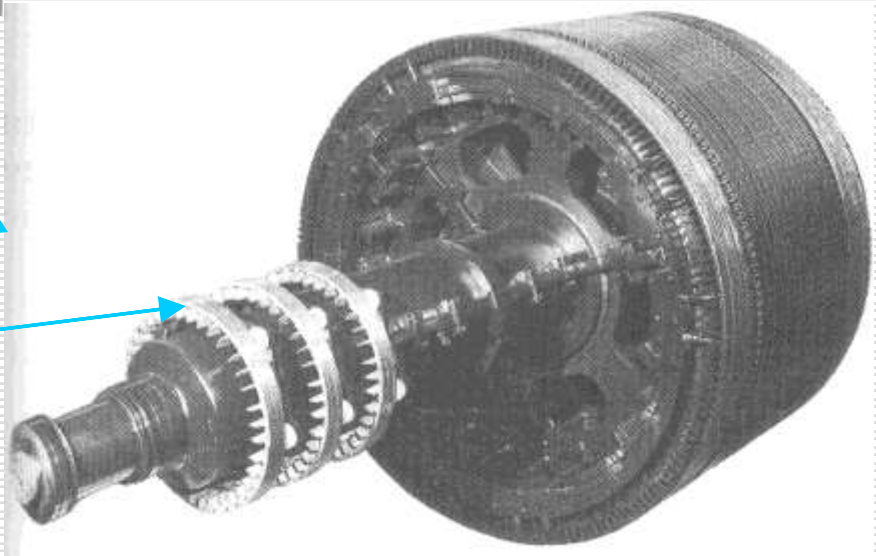
- Two basic design types depending on the rotor design
  - squirrel-cage: conducting bars laid into slots and shorted at both ends by shorting rings.
  - wound-rotor: complete set of three-phase windings exactly as the stator. Usually Y-connected, the ends of the three rotor wires are connected to 3 slip rings on the rotor shaft. In this way, the rotor circuit is accessible.

Construction



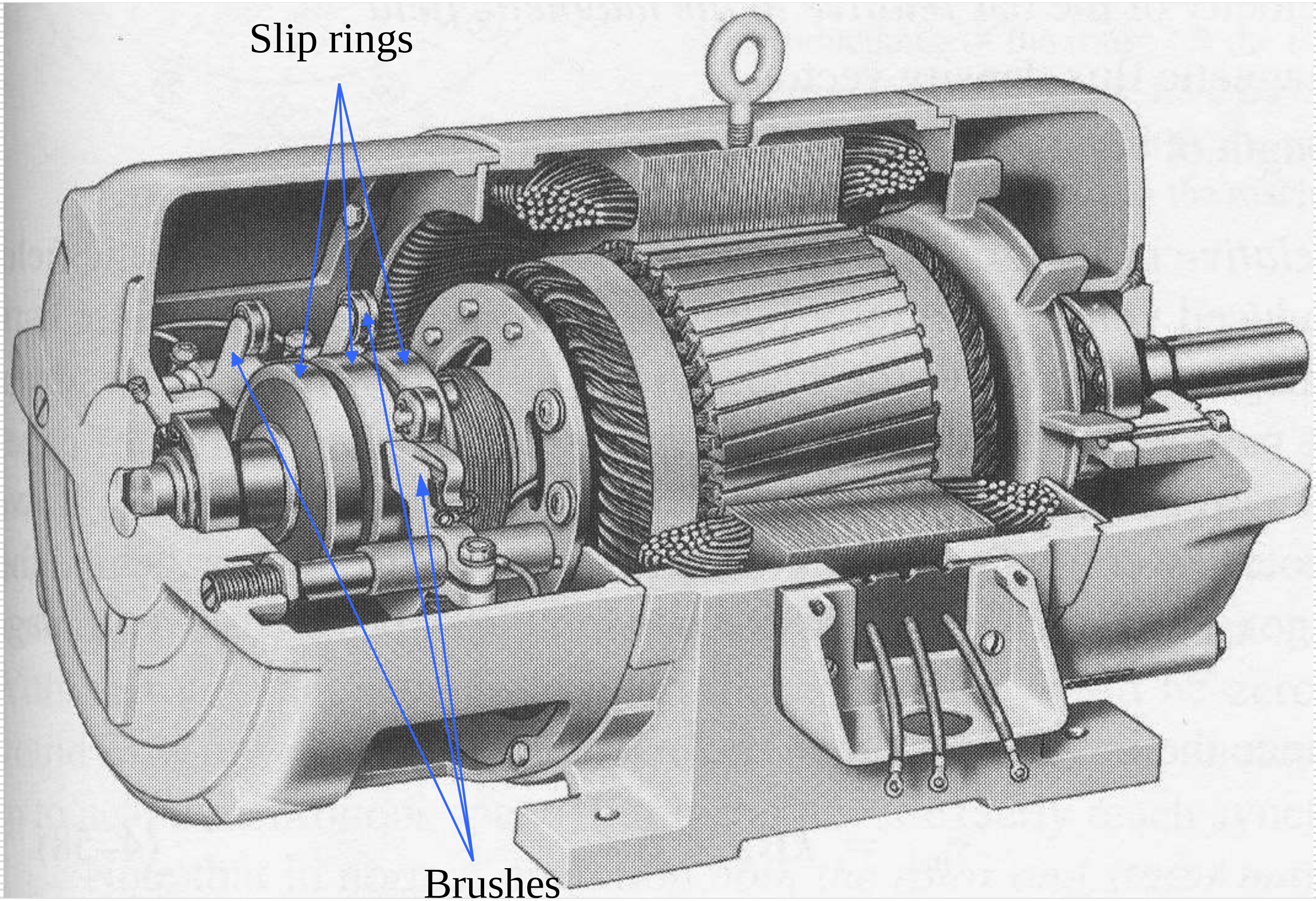
Squirrel cage rotor

Wound rotor



Notice the
slip rings

Construction

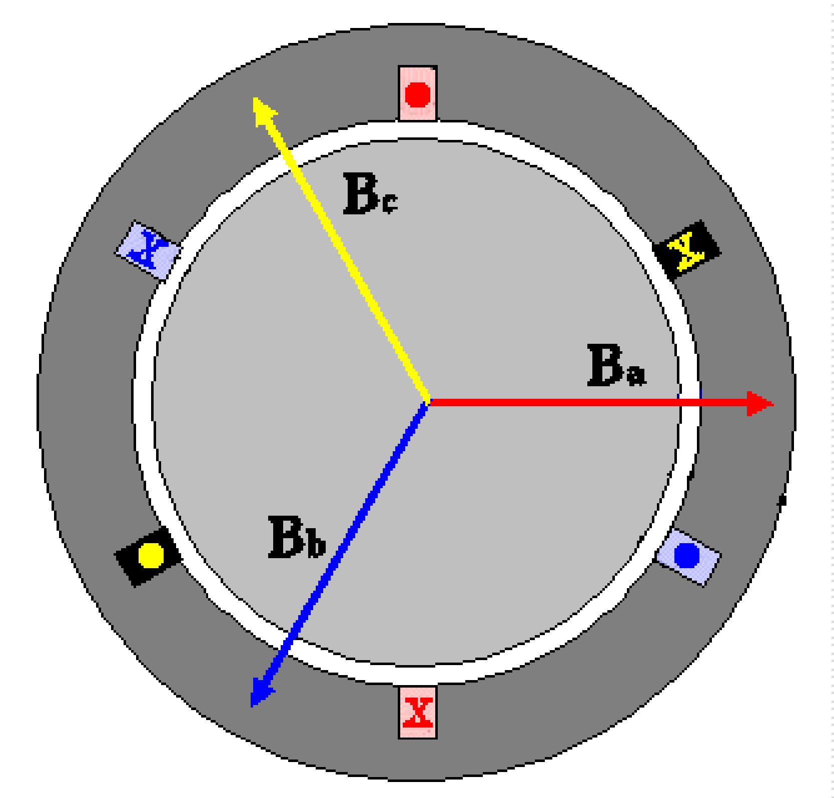


Cutaway in a typical wound-rotor IM. Notice the brushes and the slip rings

Rotating Magnetic Field

- Balanced three phase windings, i.e. mechanically displaced 120 degrees form each other, fed by balanced three phase source
- A rotating magnetic field with constant magnitude is produced, rotating with a speed

$$n_{sync} = \frac{120 f_e}{P} \quad rpm$$

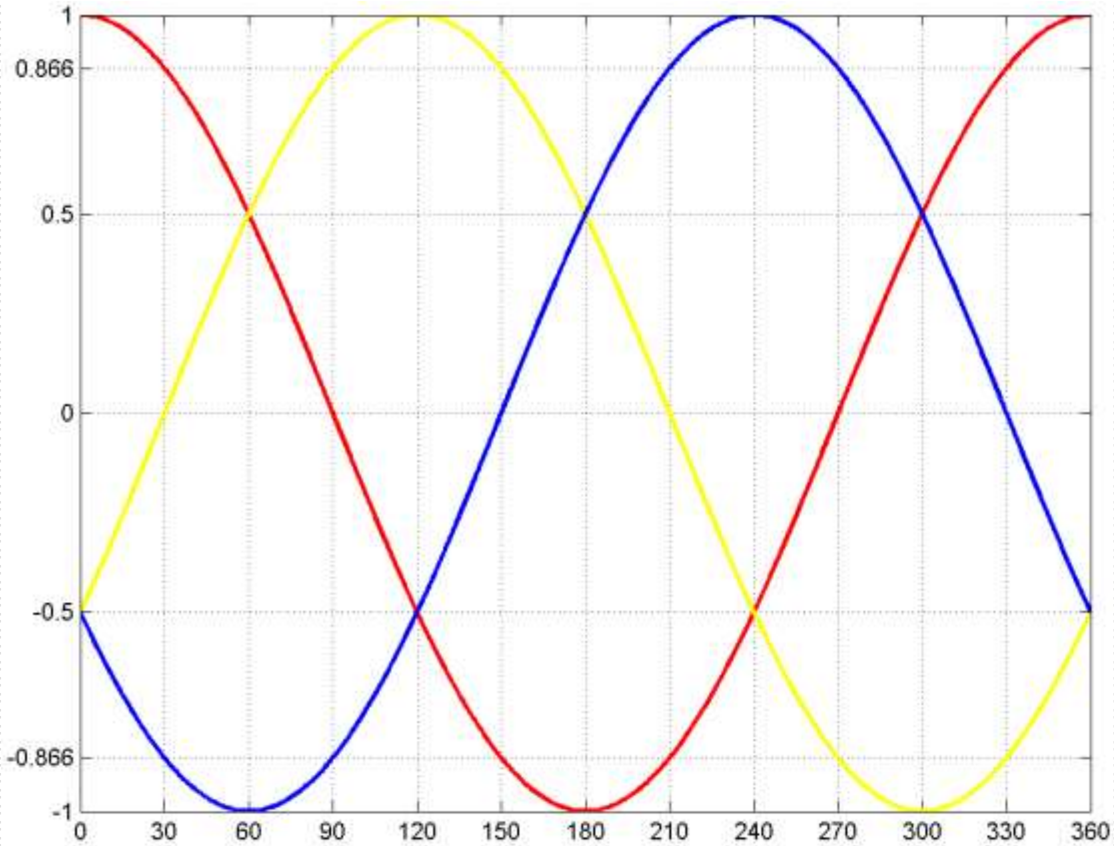


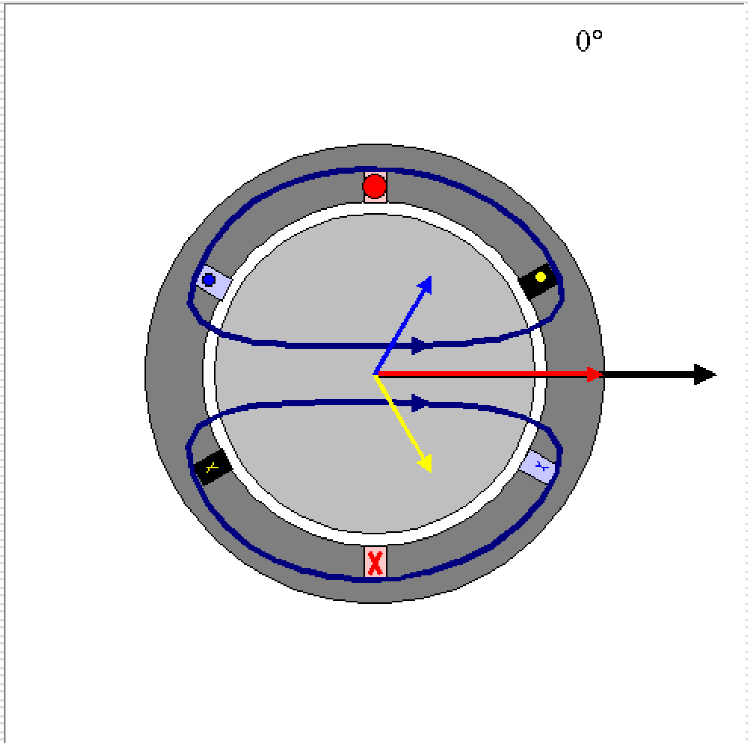
Where f_e is the supply frequency and P is the no. of poles and n_{sync} is called the synchronous speed in *rpm* (revolutions per minute)

Synchronous speed

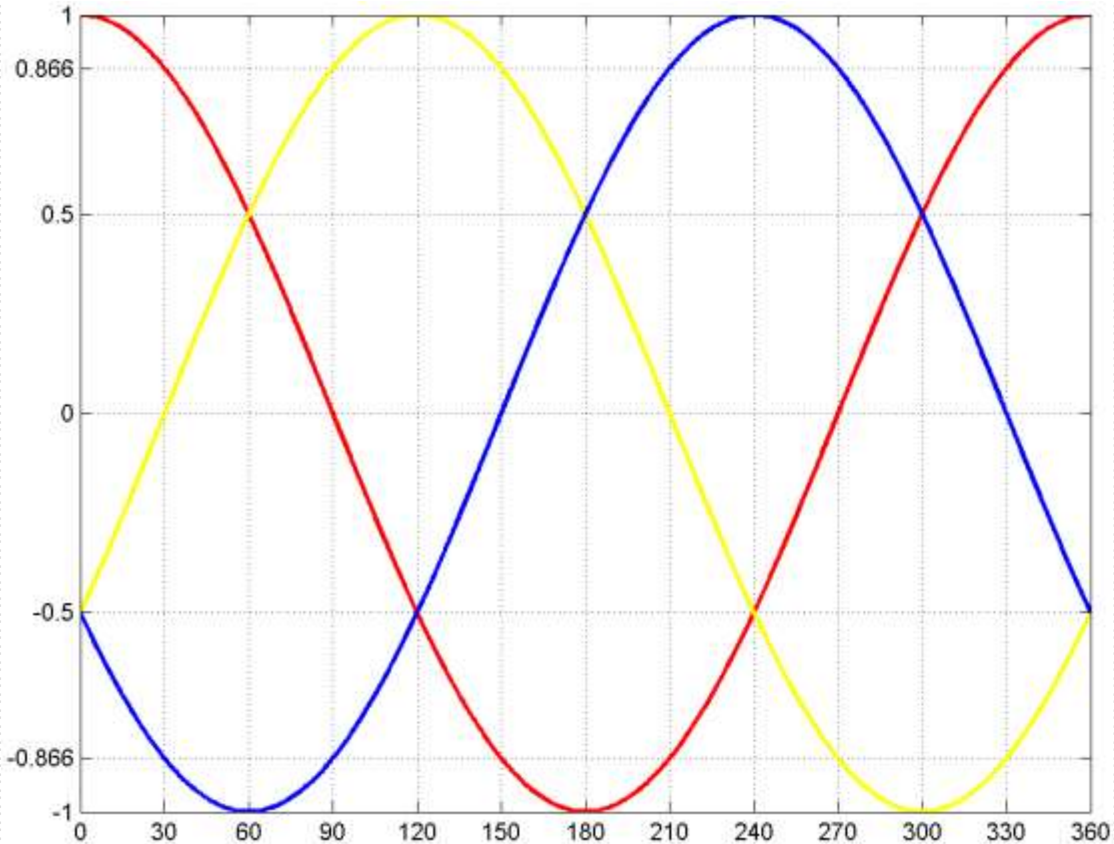
P	50 Hz	60 Hz
2	3000	3600
4	1500	1800
6	1000	1200
8	750	900
10	600	720
12	500	600

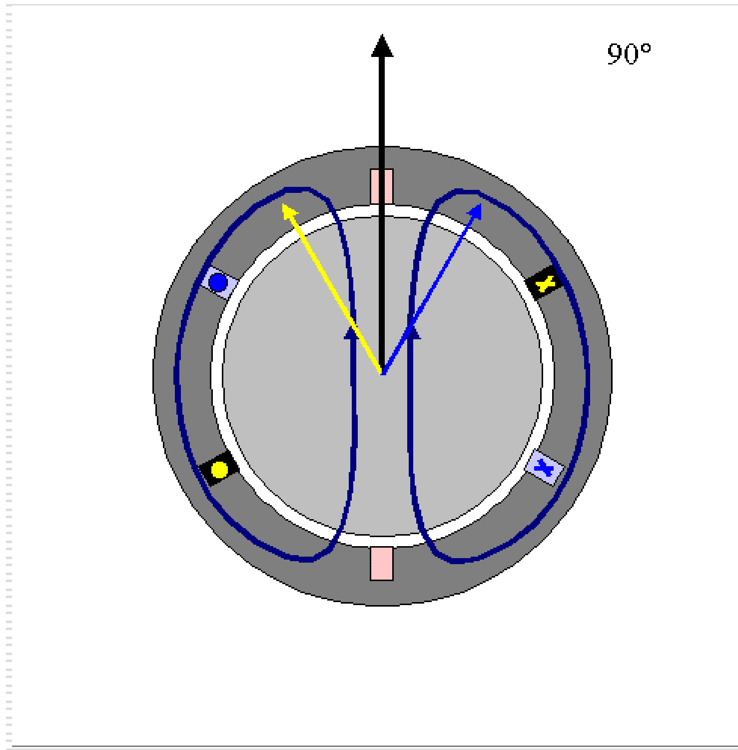
Rotating Magnetic Field





Rotating Magnetic Field





Rotating Magnetic Field

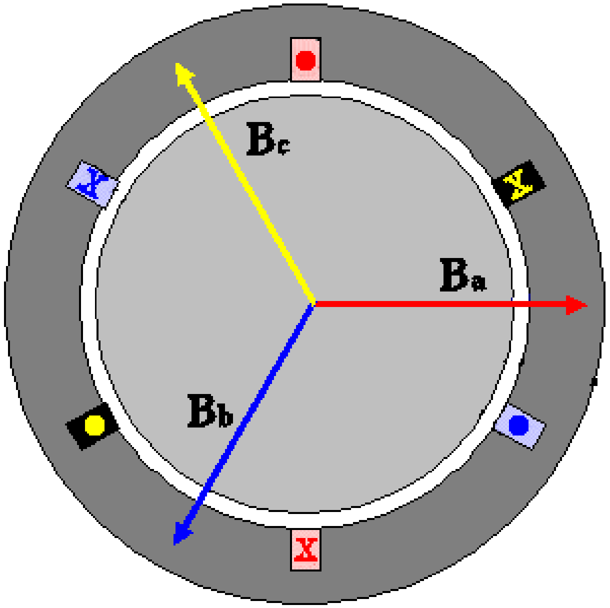
$$B_{net}(t) = B_a(t) + B_b(t) + B_c(t)$$

$$= B_M \sin(\omega t)\angle 0° + B_M \sin(\omega t - 120°)\angle 120° + B_M \sin(\omega t - 240)\angle 240°$$

$$= B_M \sin(\omega t)\hat{\mathbf{x}}$$

$$-[0.5B_M \sin(\omega t - 120°)]\hat{\mathbf{x}} - [\frac{\sqrt{3}}{2} B_M \sin(\omega t - 120°)]\hat{\mathbf{y}}$$

$$-[0.5B_M \sin(\omega t - 240°)]\hat{\mathbf{x}} + [\frac{\sqrt{3}}{2} B_M \sin(\omega t - 240°)]\hat{\mathbf{y}}$$



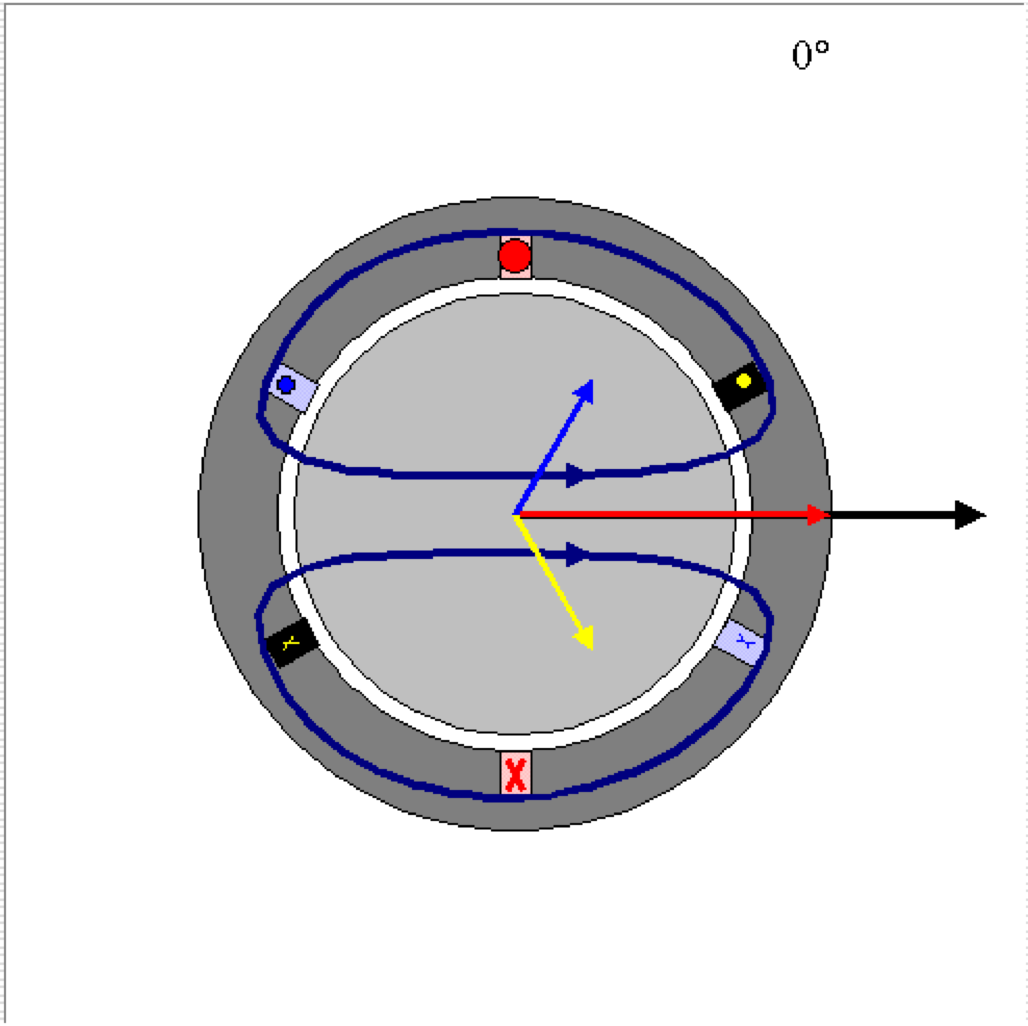
Rotating Magnetic Field

$$B_{net}(t) = [B_M \sin(\omega t) + \frac{1}{4} B_M \sin(\omega t) + \frac{\sqrt{3}}{4} B_M \cos(\omega t) + \frac{1}{4} B_M \sin(\omega t) - \frac{\sqrt{3}}{4} B_M \cos(\omega t)]\hat{\mathbf{x}}$$

$$+[-\frac{\sqrt{3}}{4} B_M \sin(\omega t) - \frac{3}{4} B_M \cos(\omega t) + \frac{\sqrt{3}}{4} B_M \sin(\omega t) - \frac{3}{4} B_M \cos(\omega t)]\hat{\mathbf{y}}$$

$$= [1.5 B_M \sin(\omega t)]\hat{\mathbf{x}} - [1.5 B_M \cos(\omega t)]\hat{\mathbf{y}}$$

Rotating Magnetic Field



Principle of operation

- This rotating magnetic field cuts the rotor windings and produces an induced voltage in the rotor windings
- Due to the fact that the rotor windings are short circuited, for both squirrel cage and wound-rotor, and induced current flows in the rotor windings
- The rotor current produces another magnetic field
- A torque is produced as a result of the interaction of those two magnetic fields

$$\tau_{ind} = kB_R \times B_s$$

Where τ_{ind} is the induced torque and B_R and B_S are the magnetic flux densities of the rotor and the stator respectively

Induction motor speed

- At what speed will the IM run?
  - Can the IM run at the synchronous speed, why?
  - If rotor runs at the synchronous speed, which is the same speed of the rotating magnetic field, then the rotor will appear stationary to the rotating magnetic field and the rotating magnetic field will not cut the rotor. So, no induced current will flow in the rotor and no rotor magnetic flux will be produced so no torque is generated and the rotor speed will fall below the synchronous speed
  - When the speed falls, the rotating magnetic field will cut the rotor windings and a torque is produced

Induction motor speed

- So, the IM will always run at a speed lower than the synchronous speed
- The difference between the motor speed and the synchronous speed is called the *Slip*

$$\boxed{n_{slip} = n_{sync} - n_m}$$

Where n_{slip}= slip speed

n_{sync}= speed of the magnetic field

n_m = mechanical shaft speed of the motor

The Slip

$$\boxed{s = \frac{n_{sync} - n_m}{n_{sync}}}$$

Where s is the *slip*

Notice that : if the rotor runs at synchronous speed

$$s = 0$$

if the rotor is stationary

$$s = 1$$

Slip may be expressed as a percentage by multiplying the above eq. by 100, notice that the slip is a ratio and doesn't have units

Induction Motors and Transformers

- Both IM and transformer works on the principle of induced voltage
  - Transformer: voltage applied to the primary windings produce an induced voltage in the secondary windings
  - Induction motor: voltage applied to the stator windings produce an induced voltage in the rotor windings
  - The difference is that, in the case of the induction motor, the secondary windings can move
  - Due to the rotation of the rotor (the secondary winding of the IM), the induced voltage in it does not have the same frequency of the stator (the primary) voltage

Frequency

- The frequency of the voltage induced in the rotor is given by

$$f_r = \frac{P \times n}{120}$$

Where f_r = the rotor frequency (Hz)

P = number of stator poles

n = slip speed (rpm)

$$f_r = \frac{P \times (n_s - n_m)}{120}$$

$$= \frac{P \times sn_s}{120} = sf_e$$

Frequency

- What would be the frequency of the rotor's induced voltage at any speed n_m?

$$\boxed{f_r = s\, f_e}$$

- When the rotor is blocked (s=1) , the frequency of the induced voltage is equal to the supply frequency
- On the other hand, if the rotor runs at synchronous speed (s = 0), the frequency will be zero

Torque

- While the input to the induction motor is electrical power, its output is mechanical power and for that we should know some terms and quantities related to mechanical power
- Any mechanical load applied to the motor shaft will introduce a Torque on the motor shaft. This torque is related to the motor output power and the rotor speed

$$\boxed{\tau_{load} = \frac{P_{out}}{\omega_m} \quad N.m} \quad \text{and} \quad \boxed{\omega_m = \frac{2\pi n_m}{60} \quad rad/s}$$

Horse power

- Another unit used to measure mechanical power is the horse power
- It is used to refer to the mechanical output power of the motor
- Since we, as an electrical engineers, deal with watts as a unit to measure electrical power, there is a relation between horse power and watts

$$\boxed{hp = 746\ watts}$$

Example

A 208-V, 10hp, four pole, 60 Hz, Y-connected induction motor has a full-load slip of 5 percent

1. What is the synchronous speed of this motor?
2. What is the rotor speed of this motor at rated load?
3. What is the rotor frequency of this motor at rated load?
4. What is the shaft torque of this motor at rated load?

Solution

1. $$n_{sync} = \frac{120 f_e}{P} = \frac{120(60)}{4} = 1800\ rpm$$

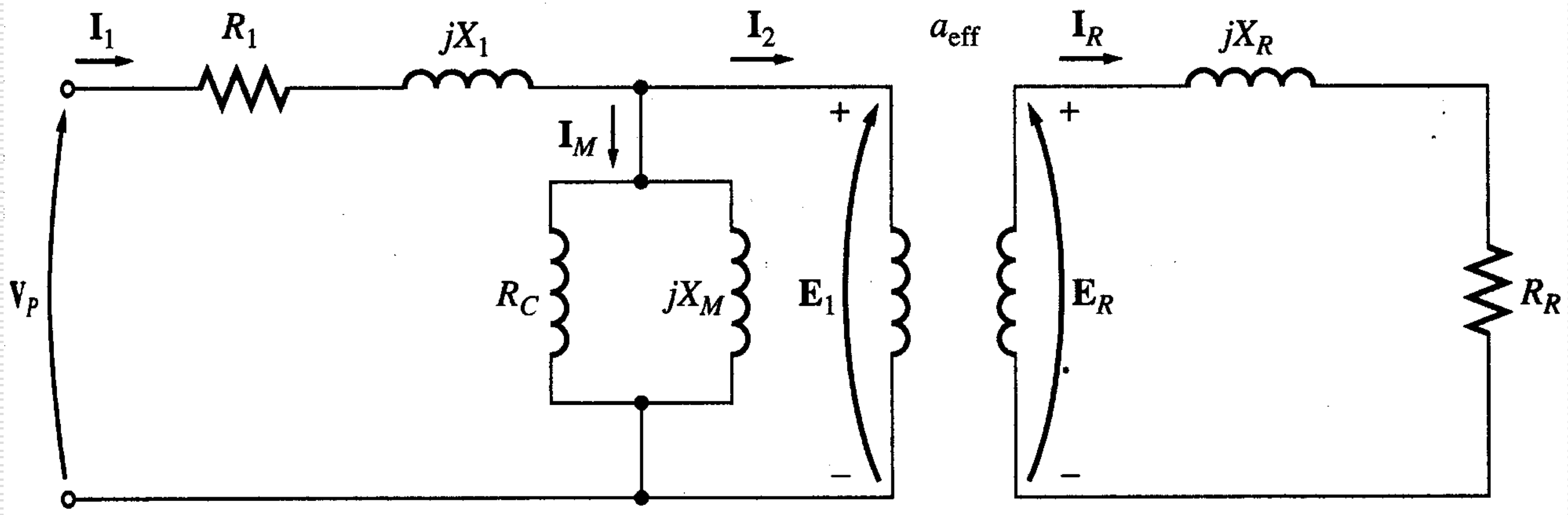
2. $$n_m = (1-s)n_s$$
$$= (1-0.05)\times 1800 = 1710\ rpm$$

3. $$f_r = sf_e = 0.05\times 60 = 3Hz$$

4. $$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{P_{out}}{2\pi \frac{n_m}{60}}$$
$$= \frac{10\,hp \times 746\,watt/hp}{1710\times 2\pi \times (1/60)} = 41.7\ N.m$$

Equivalent Circuit

- The induction motor is similar to the transformer with the exception that its secondary windings are free to rotate



As we noticed in the transformer, it is easier if we can combine these two circuits in one circuit but there are some difficulties

Equivalent Circuit

- When the rotor is locked (or blocked), i.e. s =1, the largest voltage and rotor frequency are induced in the rotor, Why?
- On the other side, if the rotor rotates at synchronous speed, i.e. s = 0, the induced voltage and frequency in the rotor will be equal to zero, Why?

$$\boxed{E_R = sE_{R0}}$$

Where E_{R0} is the largest value of the rotor's induced voltage obtained at s = 1(loacked rotor)

Equivalent Circuit

- The same is true for the frequency, i.e.

$$\boxed{f_r = s\ f_e}$$

- It is known that

$$\boxed{X = \omega L = 2\pi f\ L}$$

- So, as the frequency of the induced voltage in the rotor changes, the reactance of the rotor circuit also changes

$$\boxed{\begin{aligned} X_r &= \omega_r L_r = 2\pi f_r\ L_r \\ &= 2\pi s f_e L_r \\ &= s X_{r0} \end{aligned}}$$

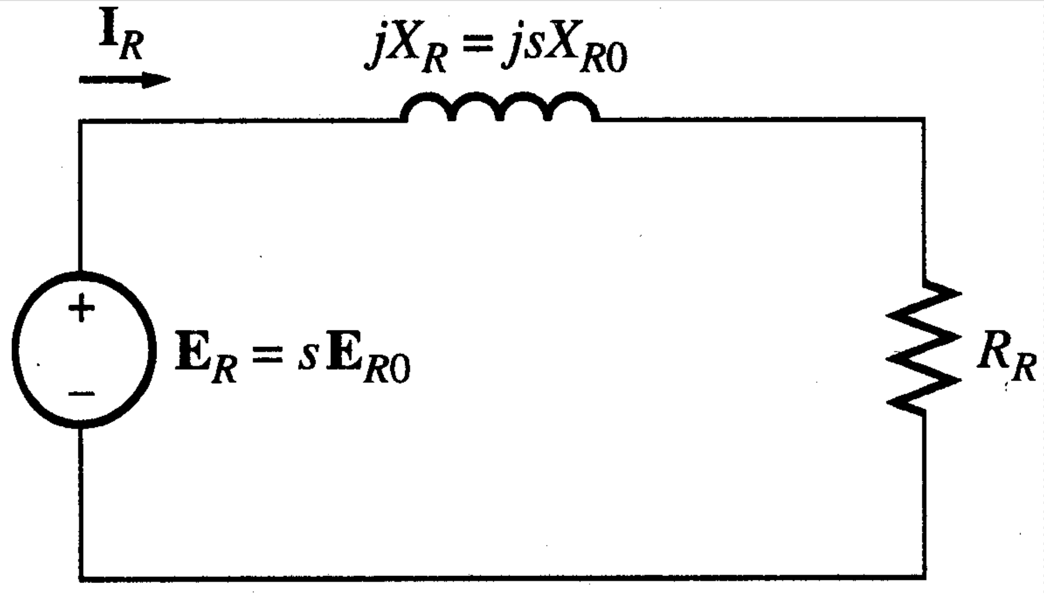
Where X_{r0} is the rotor reactance

at the supply frequency

(at blocked rotor)

Equivalent Circuit

- Then, we can draw the rotor equivalent circuit as follows



Where E_R is the induced voltage in the rotor and R_R is the rotor resistance

Equivalent Circuit

- Now we can calculate the rotor current as

$$I_R = \frac{E_R}{(R_R + jX_R)}$$

$$= \frac{sE_{R0}}{(R_R + jsX_{R0})}$$

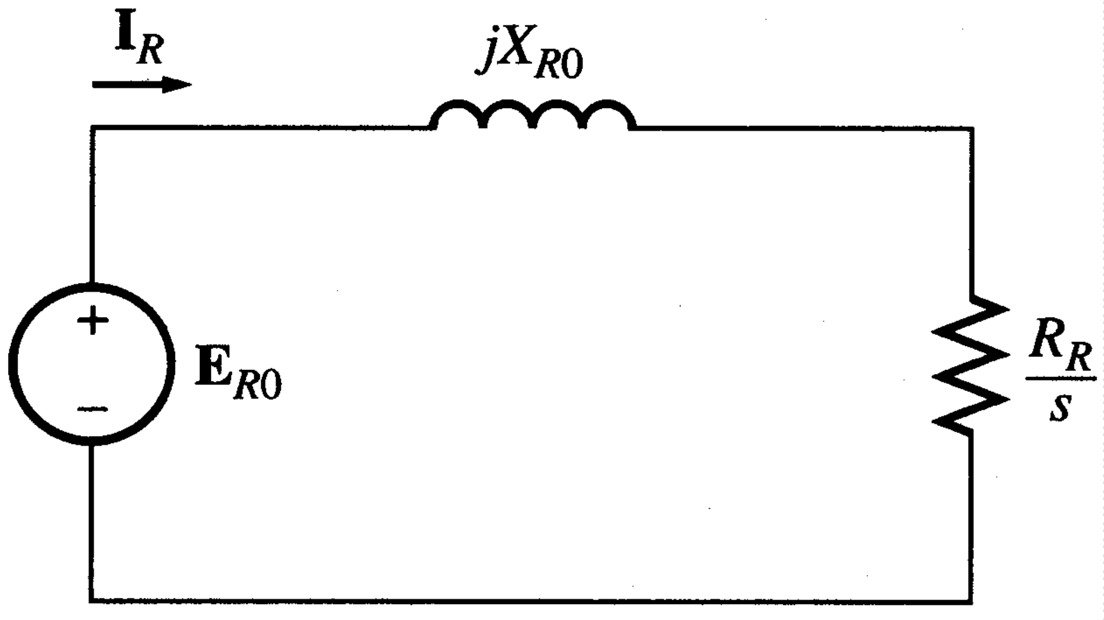
- Dividing both the numerator and denominator by *s* so nothing changes we get

$$\boxed{I_R = \frac{E_{R0}}{(\frac{R_R}{s} + jX_{R0})}}$$

Where E_{R0} is the induced voltage and X_{R0} is the rotor reactance at blocked rotor condition ($s = 1$)

Equivalent Circuit

- Now we can have the rotor equivalent circuit



Equivalent Circuit

- Now as we managed to solve the induced voltage and different frequency problems, we can combine the stator and rotor circuits in one equivalent circuit

Where

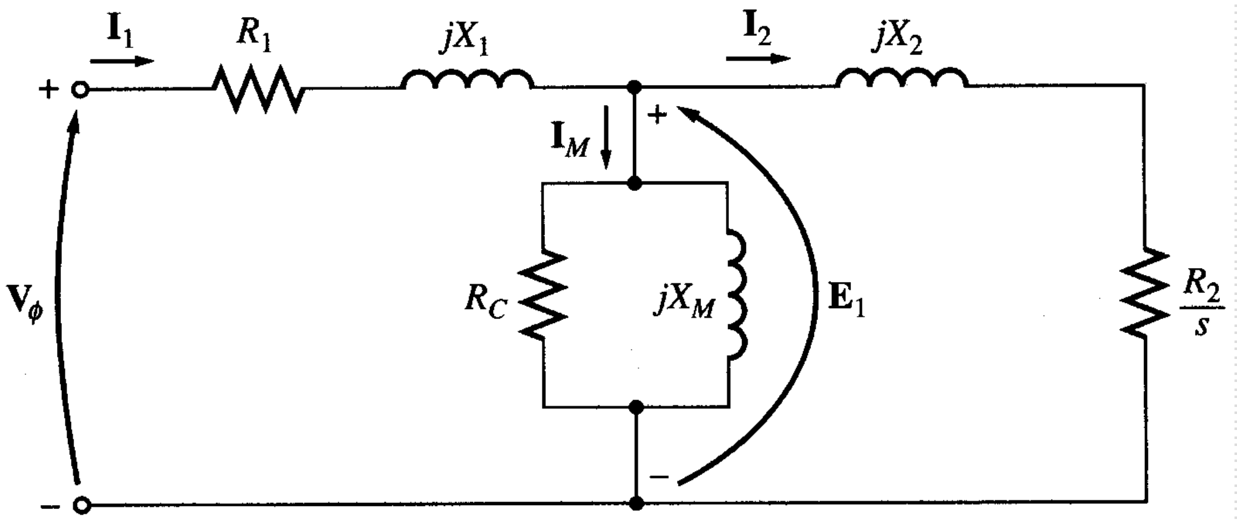
$$X_2 = a_{eff}^2 X_{R0}$$

$$R_2 = a_{eff}^2 R_R$$

$$I_2 = \frac{I_R}{a_{eff}}$$

$$E_1 = a_{eff} E_{R0}$$

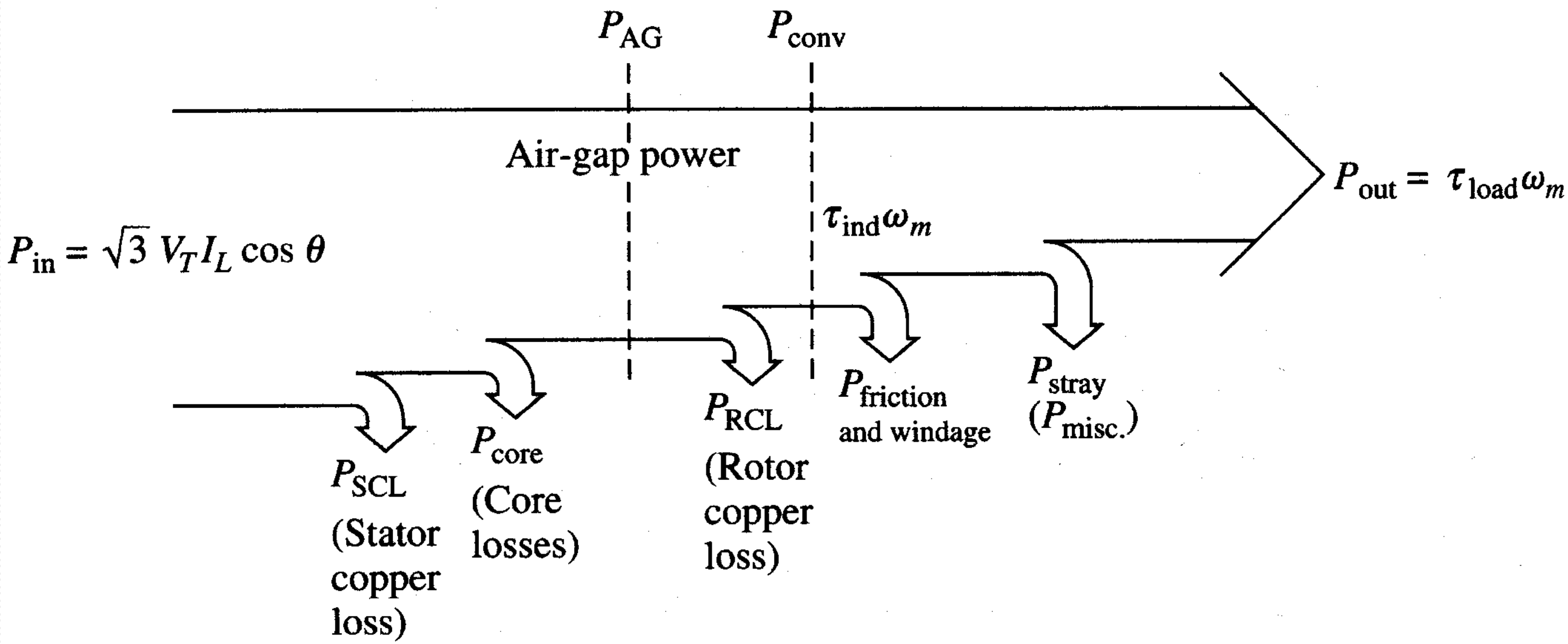
$$a_{eff} = \frac{N_S}{N_R}$$



Power losses in Induction machines

- Copper losses
  - Copper loss in the stator (P_{SCL}) = $I_1^2R_1$
  - Copper loss in the rotor (P_{RCL}) = $I_2^2R_2$
- Core loss (P_{core})
- Mechanical power loss due to friction and windage
- How this power flow in the motor?

Power flow in induction motor



Power relations

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3 V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core})$$

$$P_{RCL} = 3 I_2^2 R_2$$

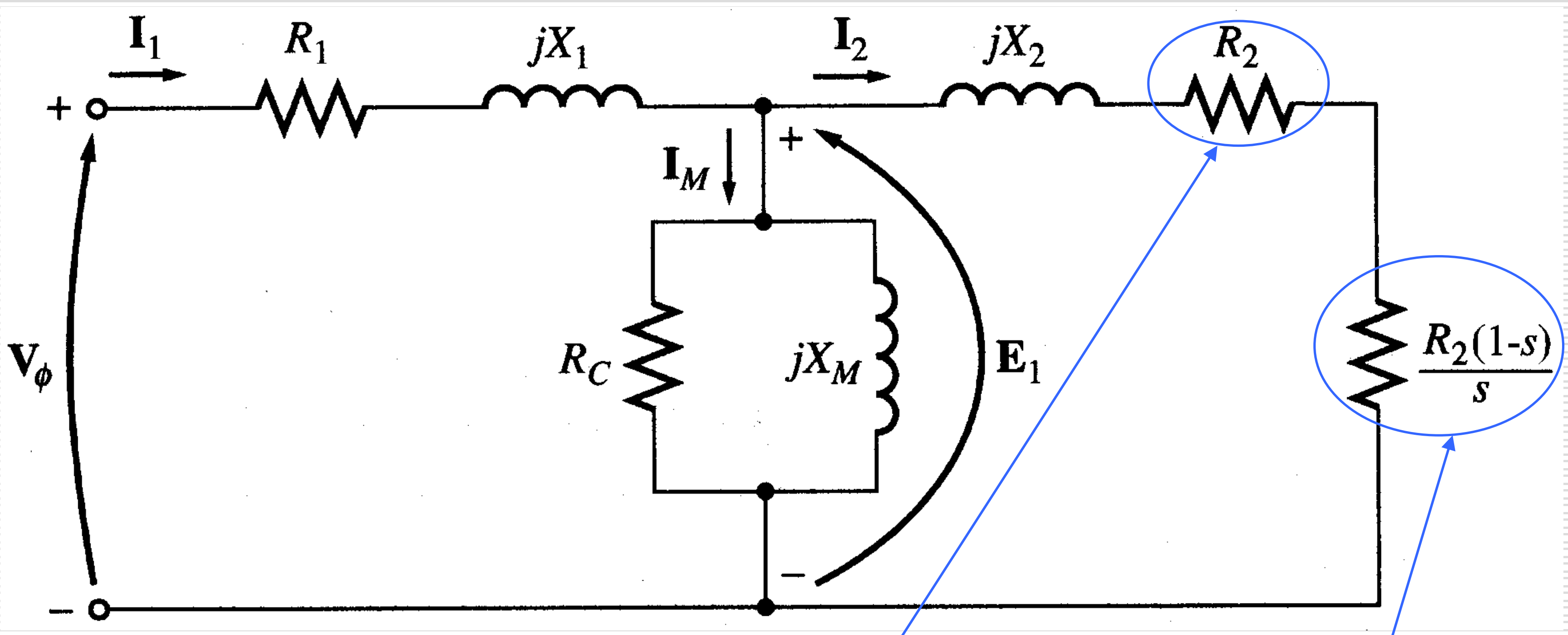
$$P_{conv} = P_{AG} - P_{RCL}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray})$$

$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$

Equivalent Circuit

- We can rearrange the equivalent circuit as follows



Actual rotor resistance

Resistance equivalent to mechanical load

Power relations

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3 V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core}) \quad = P_{conv} + P_{RCL} \quad = 3I_2^2 \frac{R_2}{s} \quad = \frac{P_{RCL}}{s}$$

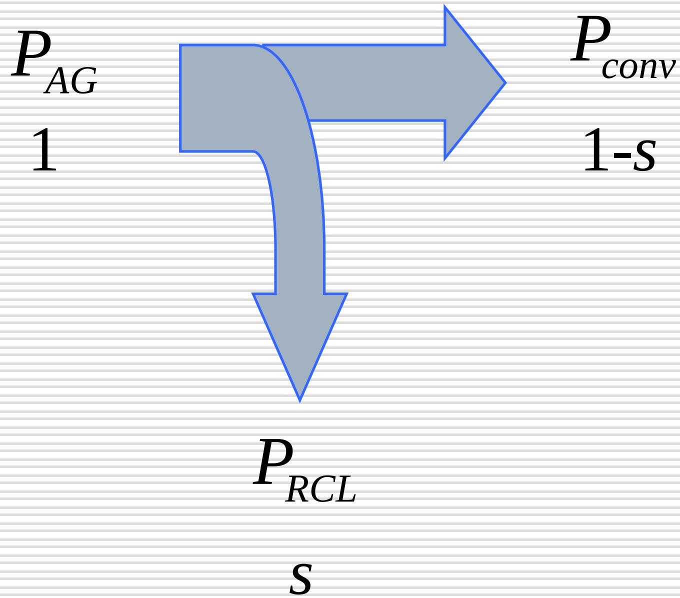
$$P_{RCL} = 3I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL} \quad = 3I_2^2 \frac{R_2(1-s)}{s} \quad = \frac{P_{RCL}(1-s)}{s}$$

$$P_{conv} = (1-s)P_{AG}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray}) \qquad \tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s)P_{AG}}{(1-s)\omega_s}$$

Power relations



$$\boxed{\begin{array}{ccccc} P_{AG} & : & P_{RCL} & : & P_{conv} \\ 1 & : & s & : & 1-s \end{array}}$$

Example

A 480-V, 60 Hz, 50-hp, three phase induction motor is drawing 60A at 0.85 PF lagging. The stator copper losses are 2 kW, and the rotor copper losses are 700 W. The friction and windage losses are 600 W, the core losses are 1800 W, and the stray losses are negligible. Find the following quantities:

1. The air-gap power $P_{AG.}$
2. The power converted P_{conv}.
3. The output power P_{out}.
4. The efficiency of the motor.

Solution

1. $$P_{in} = \sqrt{3} V_L I_L \cos\theta$$

$$= \sqrt{3} \times 480 \times 60 \times 0.85 = 42.4 \text{ kW}$$

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$

$$= 42.4 - 2 - 1.8 = 38.6 \text{ kW}$$

2. $$P_{conv} = P_{AG} - P_{RCL}$$

$$= 38.6 - \frac{700}{1000} = 37.9 \text{ kW}$$

3. $$P_{out} = P_{conv} - P_{F\&W}$$

$$= 37.9 - \frac{600}{1000} = 37.3 \text{ kW}$$

Solution

$$P_{out} = \frac{37.3}{0.746} = 50 \text{ hp}$$

4. $$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$= \frac{37.3}{42.4} \times 100 = 88\%$$

Example

A 460-V, 25-hp, 60 Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$R_1 = 0.641\Omega$ $R_2 = 0.332\Omega$

$X_1 = 1.106\ \Omega$ $X_2 = 0.464\ \Omega$ $X_M = 26.3\ \Omega$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

1. Speed
2. Stator current
3. Power factor
4. P_{conv} and P_{out}
5. τ_{ind} and τ_{load}
6. Efficiency

Solution

1. $$n_{sync} = \frac{120 f_e}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

   $$n_m = (1 - s) n_{sync} = (1 - 0.022) \times 1800 = 1760 \text{ rpm}$$

2. $$Z_2 = \frac{R_2}{s} + jX_2 = \frac{0.332}{0.022} + j0.464$$

   $$= 15.09 + j0.464 = 15.1\angle 1.76^\circ\ \Omega$$

   $$Z_f = \frac{1}{1/jX_M + 1/Z_2} = \frac{1}{-j0.038 + 0.0662\angle -1.76^\circ}$$

   $$= \frac{1}{0.0773\angle -31.1^\circ} = 12.94\angle 31.1^\circ\ \Omega$$

Solution

$$Z_{tot} = Z_{stat} + Z_f$$

$$= 0.641 + j1.106 + 12.94\angle 31.1°\ \Omega$$

$$= 11.72 + j7.79 = 14.07\angle 33.6°\ \Omega$$

$$I_1 = \frac{V_\phi}{Z_{tot}} = \frac{\frac{460\angle 0°}{\sqrt{3}}}{14.07\angle 33.6°} = 18.88\angle -33.6°\ \text{A}$$

3. $PF = \cos 33.6° = 0.833 \quad \text{lagging}$
4. $P_{in} = \sqrt{3} V_L I_L \cos\theta = \sqrt{3} \times 460 \times 18.88 \times 0.833 = 12530\ \text{W}$

$$P_{SCL} = 3 I_1^2 R_1 = 3(18.88)^2 \times 0.641 = 685\ \text{W}$$

$$P_{AG} = P_{in} - P_{SCL} = 12530 - 685 = 11845\ \text{W}$$

Solution

$$P_{conv} = (1-s)P_{AG} = (1-0.022)(11845) = 11585 \text{ W}$$

$$P_{out} = P_{conv} - P_{F\&W} = 11585 - 1100 = 10485 \text{ W}$$

$$= \frac{10485}{746} = 14.1 \text{ hp}$$

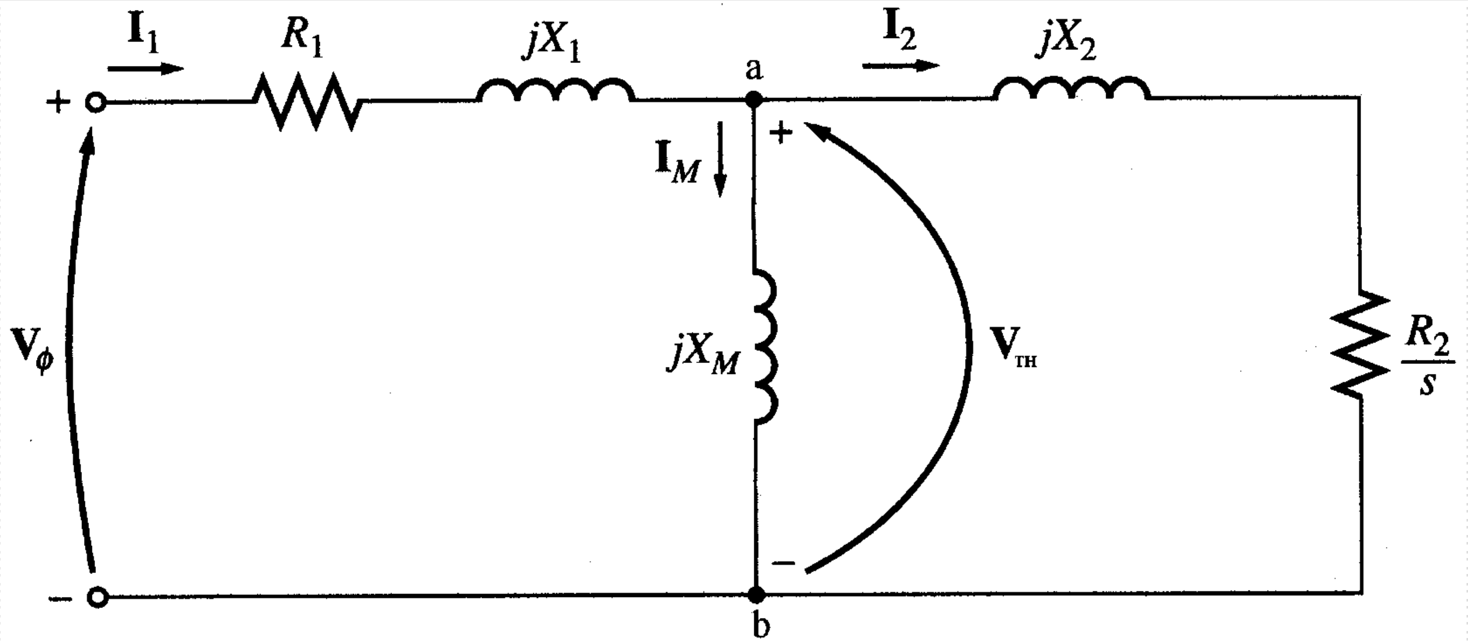
5. $$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{11845}{2\pi \times 1800/60} = 62.8 \text{ N.m}$$

$$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{10485}{2\pi \times 1760/60} = 56.9 \text{ N.m}$$

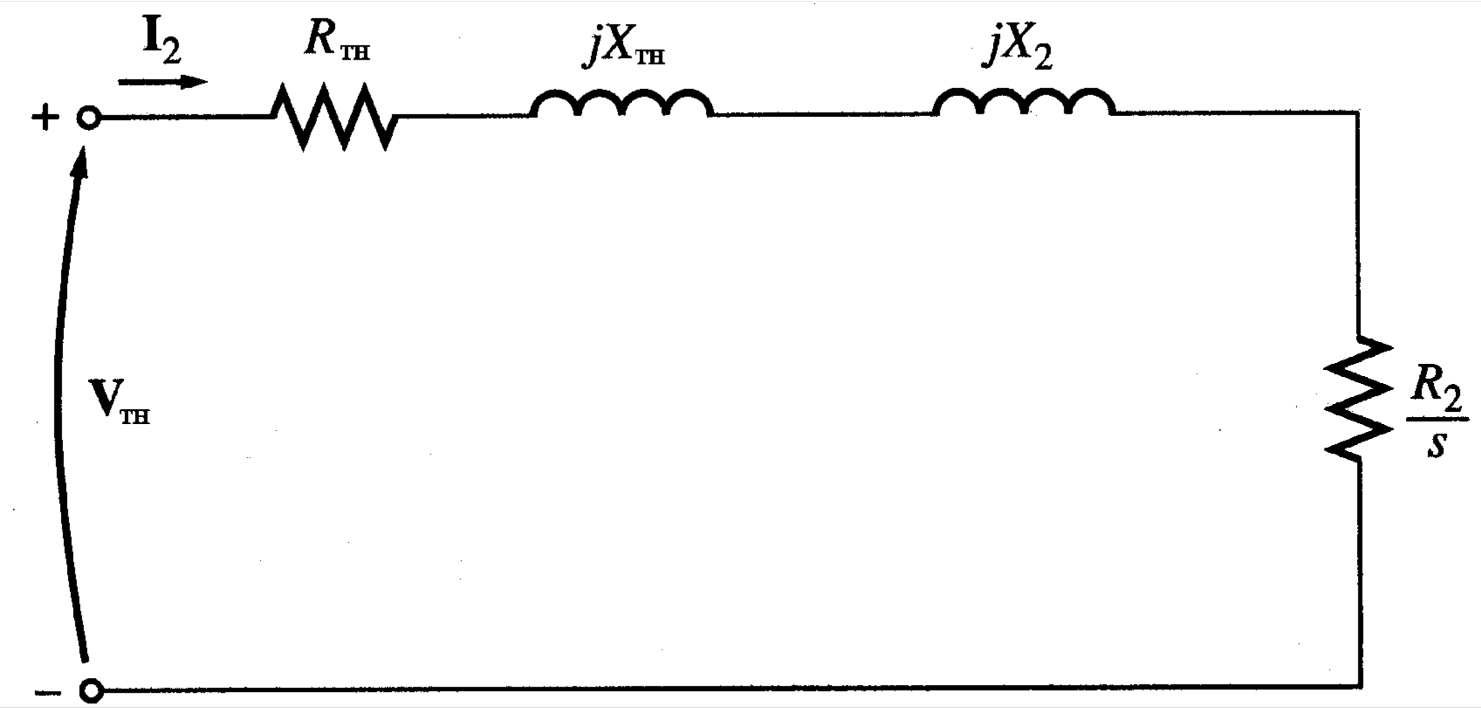
6. $$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{10485}{12530} \times 100 = 83.7\%$$

Torque, power and Thevenin's Theorem

- Thevenin's theorem can be used to transform the network to the left of points 'a' and 'b' into an equivalent voltage source V_{TH} in series with equivalent impedance $R_{TH}+jX_{TH}$



Torque, power and Thevenin's Theorem



$$V_{TH} = V_\phi \frac{jX_M}{R_1 + j(X_1 + X_M)} \qquad |V_{TH}| = |V_\phi| \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

$$R_{TH} + jX_{TH} = (R_1 + jX_1) // jX_M$$

Torque, power and Thevenin's Theorem

- Since $X_M >> X_1$ and $X_M >> R_1$

$$\boxed{V_{TH} \approx V_\phi \frac{X_M}{X_1 + X_M}}$$

- Because $X_M >> X_1$ and $X_M + X_1 >> R_1$

$$\boxed{\begin{gathered} R_{TH} \approx R_1 \left(\frac{X_M}{X_1 + X_M} \right)^2 \\ X_{TH} \approx X_1 \end{gathered}}$$

Torque, power and Thevenin's Theorem

$$I_2 = \frac{V_{TH}}{Z_T} = \frac{V_{TH}}{\sqrt{\left(R_{TH} + \frac{R_2}{s}\right)^2 + (X_{TH} + X_2)^2}}$$

Then the power converted to mechanical (P_{conv})

$$P_{conv} = 3I_2^2 \frac{R_2(1-s)}{s}$$

And the internal mechanical torque (T_{conv})

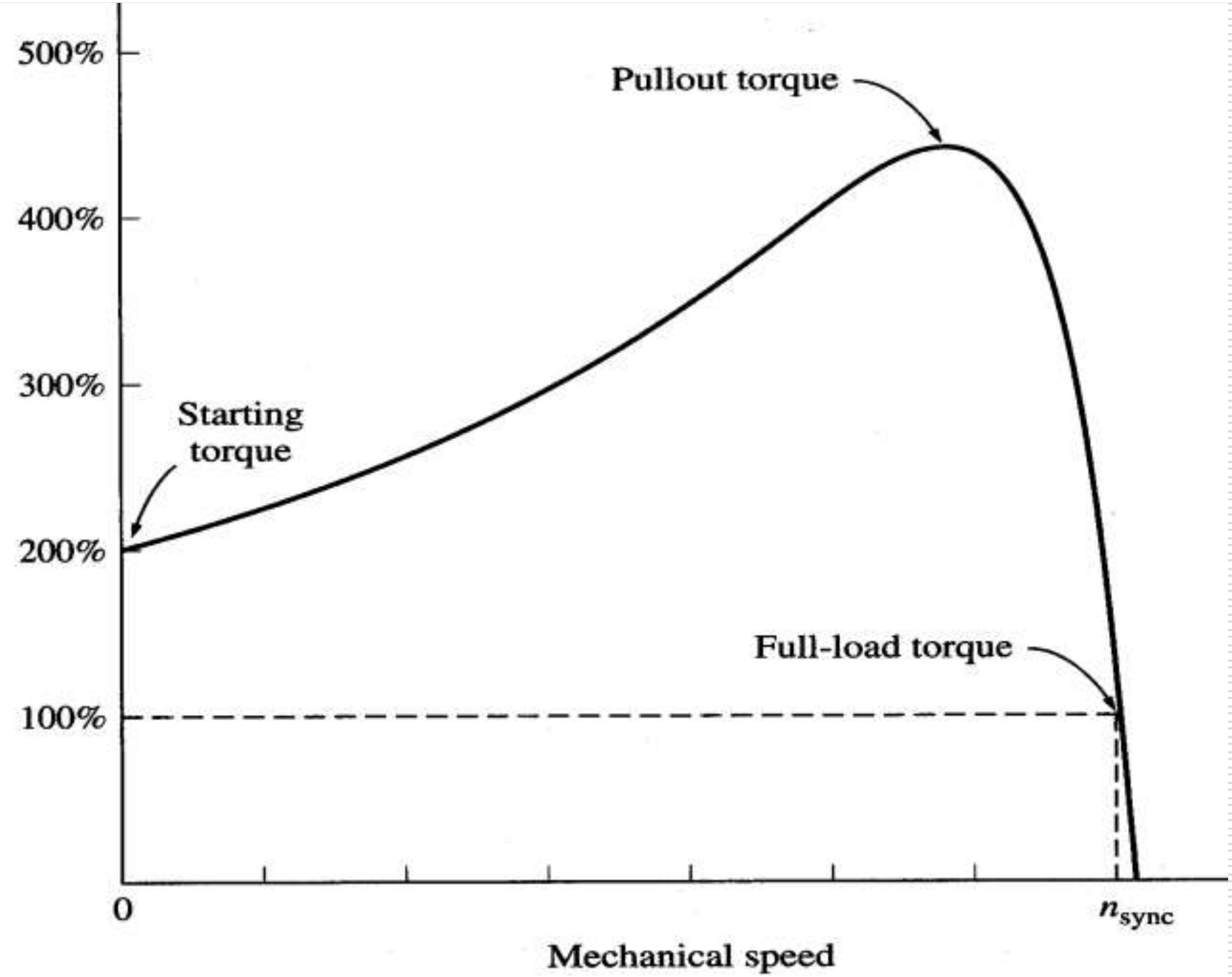
$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{P_{conv}}{(1-s)\omega_s} = \frac{3I_2^2 \frac{R_2}{s}}{\omega_s} = \frac{P_{AG}}{\omega_s}$$

Torque, power and Thevenin's Theorem

$$\tau_{ind} = \frac{3}{\omega_s}\left(\frac{V_{TH}}{\sqrt{\left(R_{TH}+\frac{R_2}{s}\right)^2+(X_{TH}+X_2)^2}}\right)^2\left(\frac{R_2}{s}\right)$$

$$\boxed{\tau_{ind} = \frac{1}{\omega_s}\frac{3V_{TH}^2\left(\frac{R_2}{s}\right)}{\left(R_{TH}+\frac{R_2}{s}\right)^2+(X_{TH}+X_2)^2}}$$

Torque-speed characteristics



Typical torque-speed characteristics of induction motor

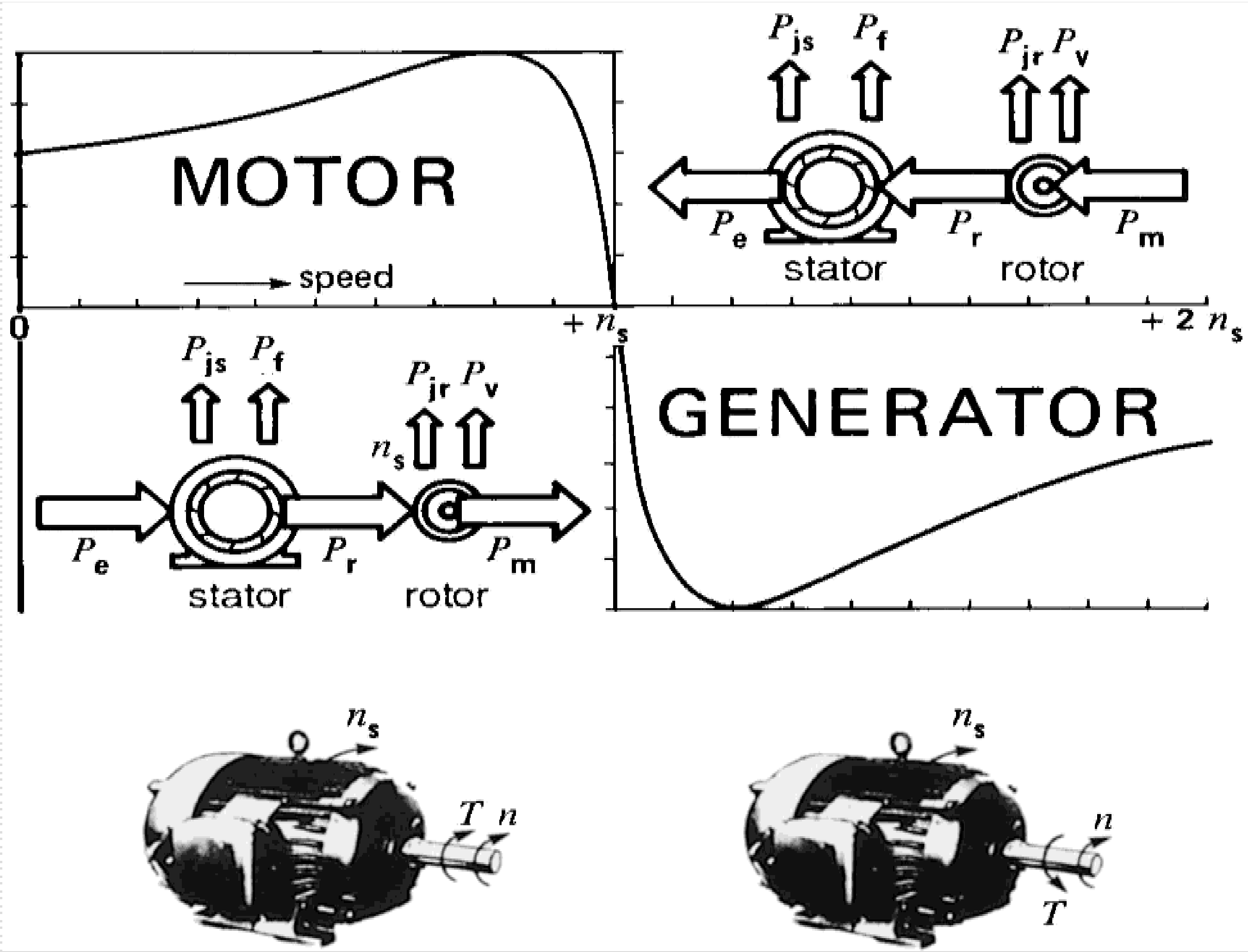
Comments

1. The induced torque is zero at synchronous speed. Discussed earlier.
2. The curve is nearly linear between no-load and full load. In this range, the rotor resistance is much greater than the reactance, so the rotor current, torque increase linearly with the slip.
3. There is a maximum possible torque that can’t be exceeded. This torque is called *pullout torque* and is 2 to 3 times the rated full-load torque.

Comments

4. The starting torque of the motor is slightly higher than its full-load torque, so the motor will start carrying any load it can supply at full load.
5. The torque of the motor for a given slip varies as the square of the applied voltage.
6. If the rotor is driven faster than synchronous speed it will run as a generator, converting mechanical power to electric power.

Complete Speed-torque c/c



Maximum torque

- Maximum torque occurs when the power transferred to R_2/s is maximum.
- This condition occurs when R_2/s equals the magnitude of the impedance $R_{TH} + j\,(X_{TH} + X_2)$

$$\frac{R_2}{s_{T_{\max}}} = \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}$$

$$\boxed{s_{T_{\max}} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}}$$

Maximum torque

- The corresponding maximum torque of an induction motor equals

$$\boxed{\tau_{\max} = \frac{1}{2\omega_s}\left(\frac{3V_{TH}^2}{R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}\right)}$$

The slip at maximum torque is directly proportional to the rotor resistance R_2

The maximum torque is independent of R_2

Maximum torque

- Rotor resistance can be increased by inserting external resistance in the rotor of a wound-rotor induction motor.

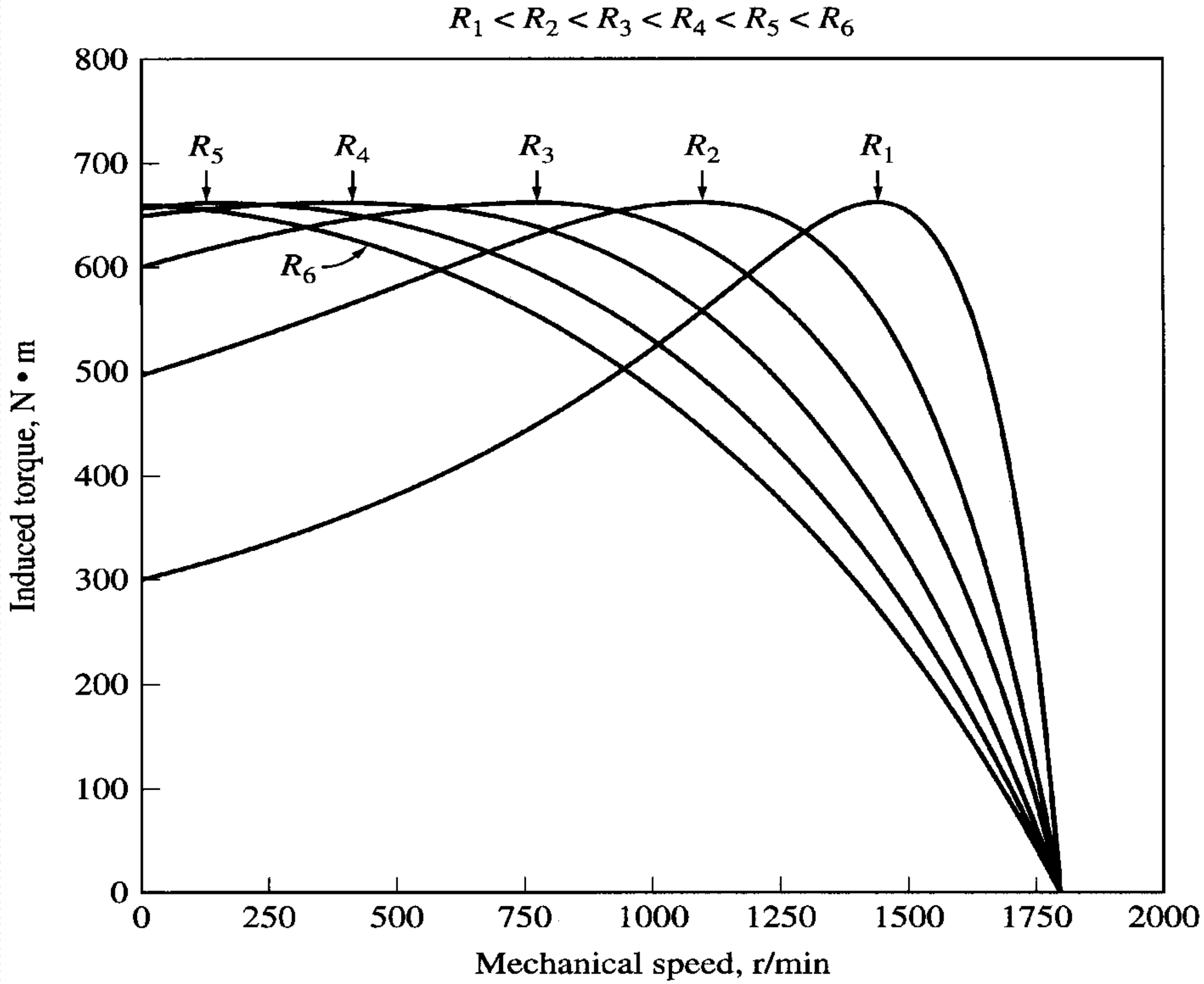
The

value of the maximum torque remains unaffected

but

the speed at which it occurs can be controlled.

Maximum torque



Effect of rotor resistance on torque-speed characteristic

Example

A two-pole, 50-Hz induction motor supplies 15kW to a load at a speed of 2950 rpm.

1. What is the motor's slip?
2. What is the induced torque in the motor in N.m under these conditions?
3. What will be the operating speed of the motor if its torque is doubled?
4. How much power will be supplied by the motor when the torque is doubled?

Solution

1. $$n_{sync} = \frac{120 f_e}{P} = \frac{120 \times 50}{2} = 3000 \text{ rpm}$$

$$s = \frac{n_{sync} - n_m}{n_{sync}} = \frac{3000 - 2950}{3000} = 0.0167 \text{ or } 1.67\%$$

2. $\because$ no P_{f+W} given

$\therefore$ assume $P_{conv} = P_{load}$ and $\tau_{ind} = \tau_{load}$

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{15 \times 10^3}{2950 \times \frac{2\pi}{60}} = 48.6 \text{ N.m}$$

Solution

3. In the low-slip region, the torque-speed curve is linear and the induced torque is direct proportional to slip. So, if the torque is doubled the new slip will be 3.33% and the motor speed will be

$$n_m = (1-s)n_{sync} = (1-0.0333)\times 3000 = 2900 \text{ rpm}$$

4. $P_{conv} = \tau_{ind}\omega_m$

$$= (2\times 48.6)\times(2900\times\frac{2\pi}{60}) = 29.5 \text{ kW}$$

Example

A 460-V, 25-hp, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit

$R_1 = 0.641\Omega$ $R_2 = 0.332\Omega$

$X_1 = 1.106\ \Omega$ $X_2 = 0.464\ \Omega$ $X_M = 26.3\ \Omega$

1. What is the maximum torque of this motor? At what speed and slip does it occur?
2. What is the starting torque of this motor?
3. If the rotor resistance is doubled, what is the speed at which the maximum torque now occur? What is the new starting torque of the motor?
4. Calculate and plot the *T*-*s* c/c for both cases.

Solution

$$V_{TH} = V_{\phi} \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

$$= \frac{\frac{460}{\sqrt{3}} \times 26.3}{\sqrt{(0.641)^2 + (1.106 + 26.3)^2}} = 255.2 \text{ V}$$

$$R_{TH} \approx R_1 \left(\frac{X_M}{X_1 + X_M} \right)^2$$

$$\approx (0.641) \left(\frac{26.3}{1.106 + 26.3} \right)^2 = 0.590\Omega$$

$$X_{TH} \approx X_1 = 1.106\Omega$$

Solution

1. $$s_{T_{\max}} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$= \frac{0.332}{\sqrt{(0.590)^2 + (1.106 + 0.464)^2}} = 0.198$$

The corresponding speed is

$$n_m = (1 - s)n_{sync} = (1 - 0.198) \times 1800 = 1444 \text{ rpm}$$

Solution

The torque at this speed is

$$\tau_{\max} = \frac{1}{2\omega_s}\left(\frac{3V_{TH}^2}{R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}\right)$$

$$= \frac{3\times(255.2)^2}{2\times(1800\times\frac{2\pi}{60})[0.590+\sqrt{(0.590)^2+(1.106+0.464)^2}\,]}$$

$$= 229 \text{ N.m}$$

Solution

2. The starting torque can be found from the torque eqn. by substituting $s = 1$

$$\tau_{start} = \tau_{ind}\Big|_{s=1} = \frac{1}{\omega_s} \left. \frac{3V_{TH}^2 \left(\dfrac{R_2}{s}\right)}{\left(R_{TH} + \dfrac{R_2}{s}\right)^2 + (X_{TH} + X_2)^2} \right|_{s=1}$$

$$= \frac{3V_{TH}^2 R_2}{\omega_s[\left(R_{TH} + R_2\right)^2 + (X_{TH} + X_2)^2]}$$

$$= \frac{3 \times (255.2)^2 \times (0.332)}{1800 \times \dfrac{2\pi}{60} \times [(0.590 + 0.332)^2 + (1.106 + 0.464)^2]}$$

$$= 104 \text{ N.m}$$

Solution

3. If the rotor resistance is doubled, then the slip at maximum torque doubles too

$$s_{T_{\max}} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} = 0.396$$

The corresponding speed is

$$n_m = (1 - s)n_{sync} = (1 - 0.396) \times 1800 = 1087 \text{ rpm}$$

The maximum torque is still

τ_{max} = 229 N.m

Solution

The starting torque is now

$$\tau_{start} = \frac{3\times(255.2)^2\times(0.664)}{1800\times\frac{2\pi}{60}\times[(0.590+0.664)^2+(1.106+0.464)^2]}$$

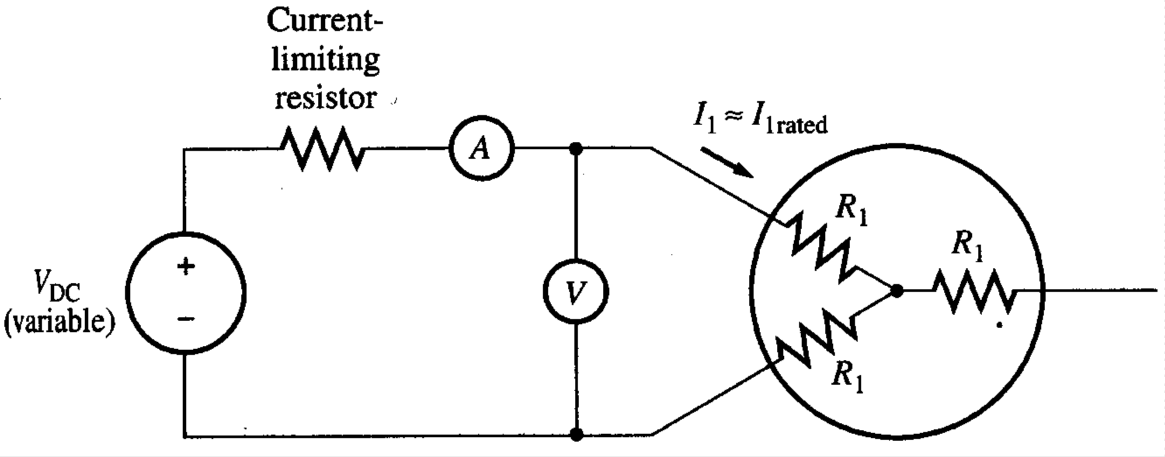
$$=170 \text{ N.m}$$

Determination of motor parameters

- Due to the similarity between the induction motor equivalent circuit and the transformer equivalent circuit, same tests are used to determine the values of the motor parameters.
  - DC test: determine the stator resistance R_1
  - No-load test: determine the rotational losses and magnetization current (similar to no-load test in Transformers).
  - Locked-rotor test: determine the rotor and stator impedances (similar to short-circuit test in Transformers).

DC test

- The purpose of the DC test is to determine R_1. A variable DC voltage source is connected between two stator terminals.
- The DC source is adjusted to provide approximately rated stator current, and the resistance between the two stator leads is determined from the voltmeter and ammeter readings.



DC test

- then

$$R_{DC} = \frac{V_{DC}}{I_{DC}}$$

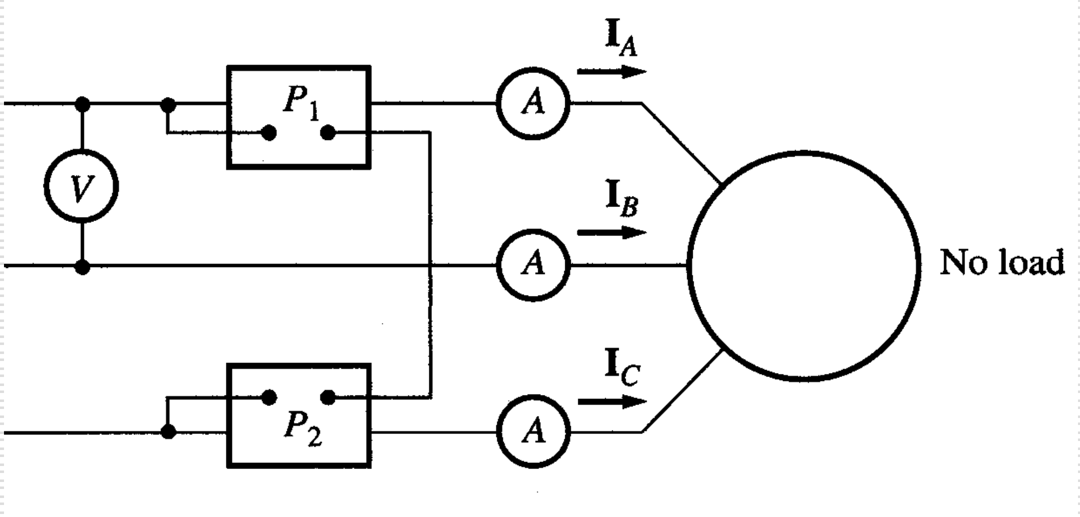
- If the stator is Y-connected, the per phase stator resistance is

$$\boxed{R_1 = \frac{R_{DC}}{2}}$$

- If the stator is delta-connected, the per phase stator resistance is

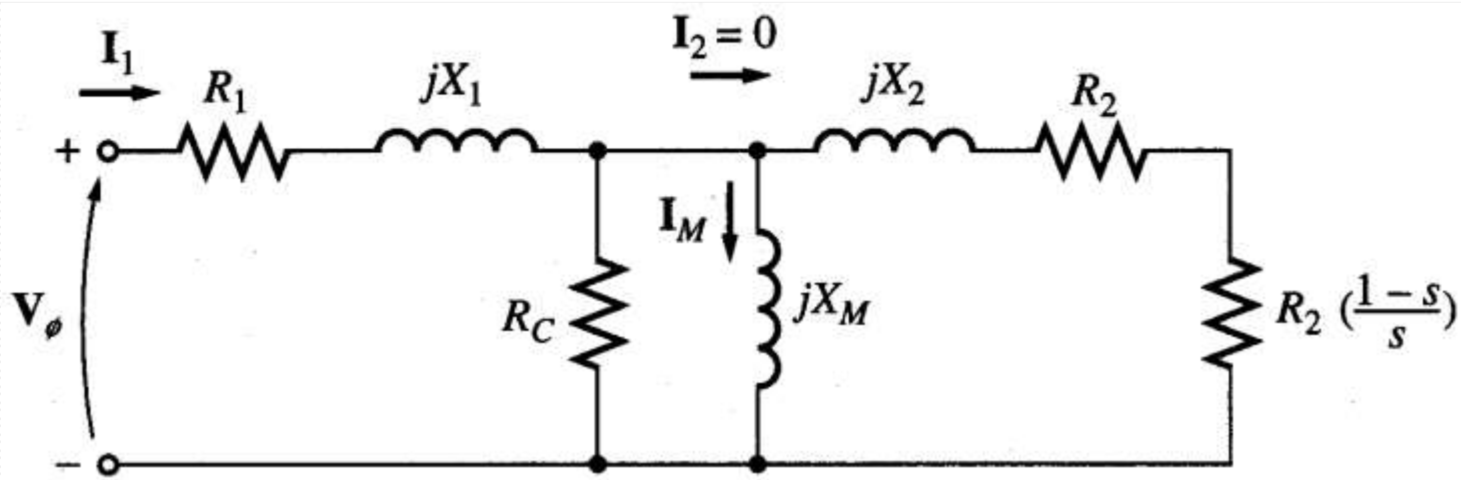
$$\boxed{R_1 = \frac{3}{2} R_{DC}}$$

No-load test



1. The motor is allowed to spin freely
2. The only load on the motor is the friction and windage losses, so all P_{conv} is consumed by mechanical losses
3. The slip is very small

No-load test

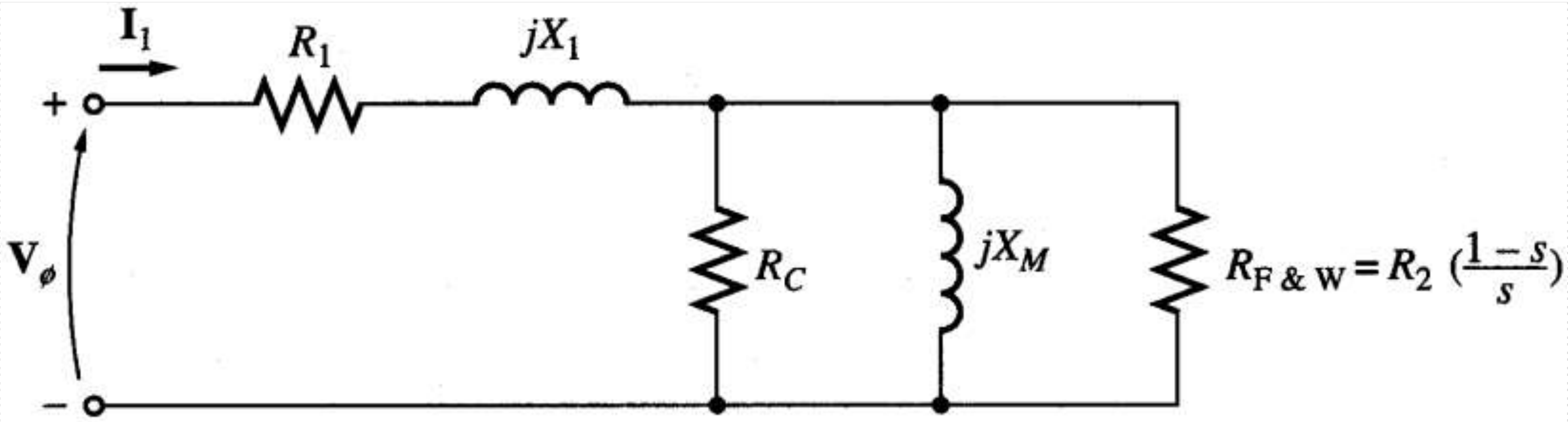


4. At this small slip

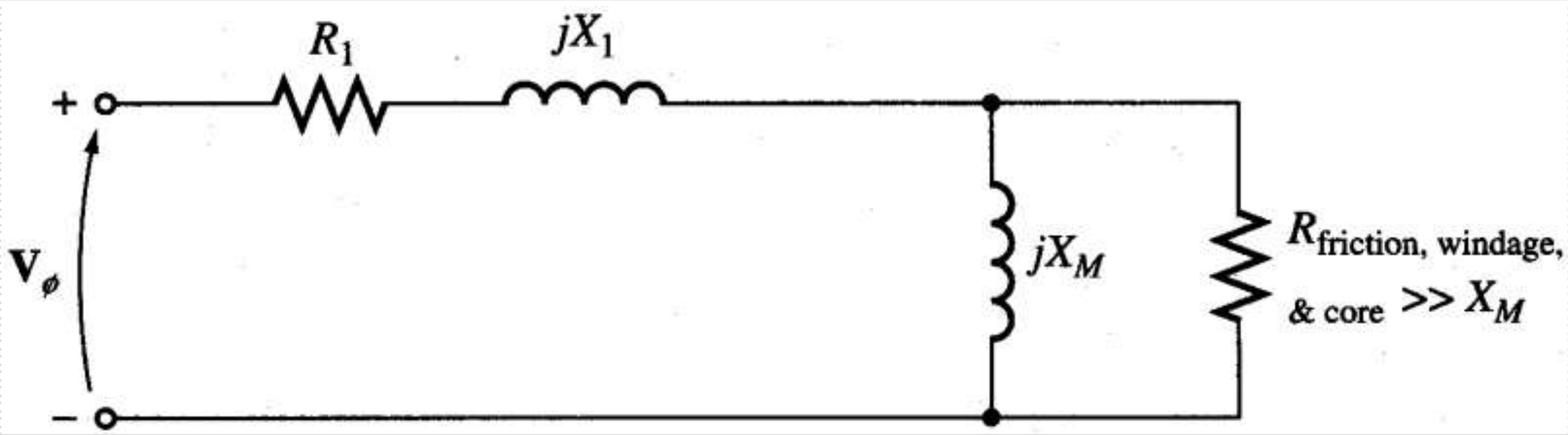
$$\frac{R_2(1-s)}{s} \gg R_2 \qquad \& \qquad \frac{R_2(1-s)}{s} \gg X_2$$

The equivalent circuit reduces to…

No-load test



5. Combining R_c & R_{F+W} we get……



No-load test

6. At the no-load conditions, the input power measured by meters must equal the losses in the motor.
7. The P_{RCL} is negligible because I_2 is extremely small because $R_2(1-s)/s$ is very large.
8. The input power equals

$$\boxed{\begin{aligned} P_{in} &= P_{SCL} + P_{core} + P_{F\&W} \\ &= 3I_1^2R_1 + P_{rot} \end{aligned}}$$

Where

$$P_{rot} = P_{core} + P_{F\&W}$$

No-load test

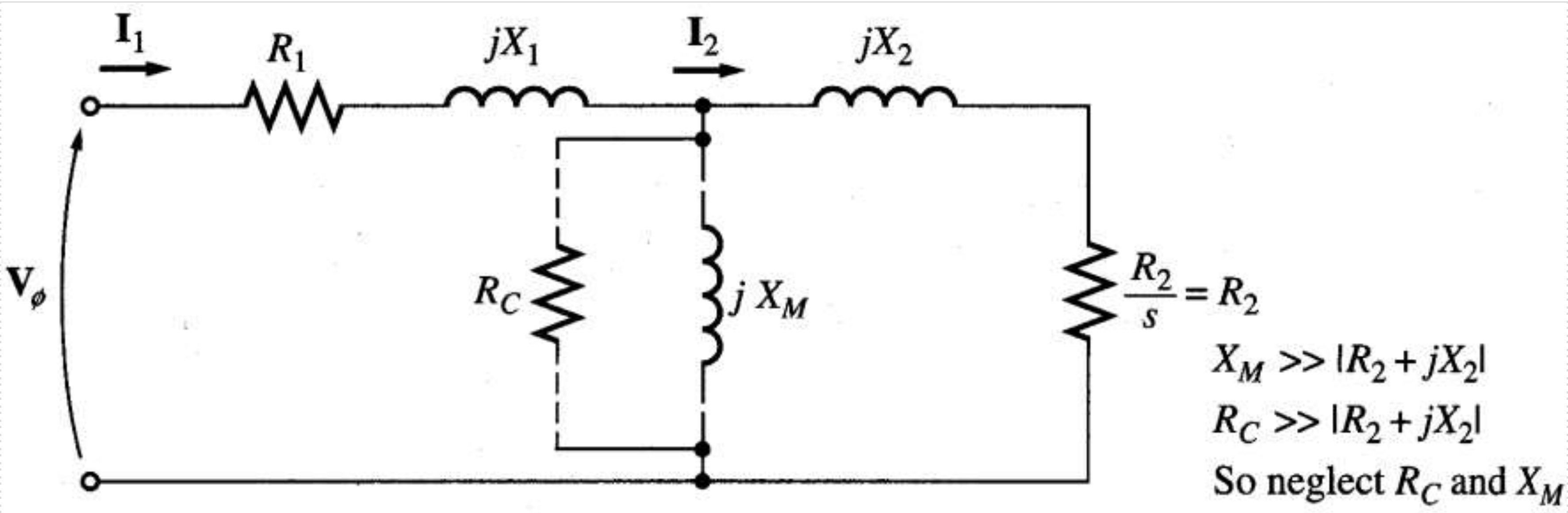
9. The equivalent input impedance is thus approximately

$$\left|Z_{eq}\right| = \frac{V_\phi}{I_{1,nl}} \approx X_1 + X_M$$

If X_1 can be found, in some other fashion, the magnetizing impedance X_M will be known

Blocked-rotor test

- In this test, the rotor is locked or blocked so that it cannot move, a voltage is applied to the motor, and the resulting voltage, current and power are measured.



Blocked-rotor test

- The AC voltage applied to the stator is adjusted so that the current flow is approximately full-load value.
- The locked-rotor power factor can be found as

$$\boxed{PF = \cos\theta = \frac{P_{in}}{\sqrt{3}V_l I_l}}$$

- The magnitude of the total impedance

$$\boxed{\left|Z_{LR}\right| = \frac{V_\phi}{I}}$$

Blocked-rotor test

$$|Z_{LR}| = R_{LR} + jX'_{LR}$$

$$= |Z_{LR}|\cos\theta + j|Z_{LR}|\sin\theta$$

$$R_{LR} = R_1 + R_2$$

$$X'_{LR} = X'_1 + X'_2$$

Where X'_1 and X'_2 are the stator and rotor reactances at the test frequency respectively

$$R_2 = R_{LR} - R_1$$

$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = X_1 + X_2$$

Blocked-rotor test

	X_1 and X_2 as function of X_{LR}	
Rotor Design	X_1	X_2
Wound rotor	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design A	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design B	$0.4\ X_{LR}$	$0.6\ X_{LR}$
Design C	$0.3\ X_{LR}$	$0.7\ X_{LR}$
Design D	$0.5\ X_{LR}$	$0.5\ X_{LR}$

Example

The following test data were taken on a 7.5-hp, four-pole, 208-V, 60-Hz, design A, Y-connected IM having a rated current of 28 A.

<u>DC Test</u>:

$V_{DC} = 13.6$ V $I_{DC} = 28.0$ A

<u>No-load Test</u>:

$V_l = 208$ V $f = 60$ Hz

$I = 8.17$ A $P_{in} = 420$ W

<u>Locked-rotor Test</u>:

$V_l = 25$ V $f = 15$ Hz

$I = 27.9$ A $P_{in} = 920$ W

(a) Sketch the per-phase equivalent circuit of this motor.

(b) Find the slip at pull-out torque, and find the value of the pull-out torque.