

ERIGrid 2.0 LAB ACCESS APPLICATION FORM

USER PROJECT PROPOSAL

User Project Acronym	CYber Physical RESilient Distribution System (CYPRESS)
User Project Title	Cyber-Physical Resilient Distribution System
Main Scientific/Technical Field	Smart Grid/Resilience
Proposal resubmitted (Y/N)	N
Keywords (5 max., free text)	Cyber-physical, Distributed Energy Resources, Micro-Grid, Multi-agent system, Service restoration

PREFERRED HOST LABORATORY/RESEARCH INFRASTRUCTURE

Option 1	Smart Electricity Systems and Technologies Laboratory (AIT)
Option 2	Distributed Energy Resources Test Facility (RSE)
Option 3	SYSLAB and Energy Systems Simulation Lab (DTU)
Proposed starting date of the access	1.3.22
Expected access duration (in weeks)	2

LEADER OF THE PROPOSING USER GROUP

Name	Chaudhry Talha Hassan
Nationality	Pakistani
Gender	Male
Age below 35-year-old (Y/N)?	N
Email address	18060043@lums.edu.pk
Organization name	Lahore University of Management Sciences (LUMS)
Organization address	Sector U - Phase 5 DHA Lahore Pakistan
Organization website	www.lums.edu.pk
Organization activity type	UNI - University and other higher education organizations

MEMBERS OF THE PROPOSING USER GROUP (repeat for all Users)

Name	Muhammad Shamaas
Nationality	Pakistani
Gender	Male
Age below 35-year-old (Y/N)?	Y
Email address	18100217@lums.edu.pk
Organization name	Lahore University of Management Sciences (LUMS)
Organization address	Sector U - Phase 5 DHA Lahore Pakistan
Organization website	www.lums.edu.pk
Organization activity type	UNI - University and other higher education organizations

1. SUMMARY OF PROPOSED RESEARCH

Advancement in smart grid technologies has called for a higher reliability of service to the consumers. However, faults and outages are inevitable but service shall be restored back to normal in case of a blackout. Centralized approach for load restoration considers it as a classical optimization problem with an objective to maximize (critical) load pickup incorporating associated constraints. The distributed approach with Multi-agent system (MAS) is based on heuristics/expert systems and agent behavioral modeling. Lack of standardization has limited the scope of communication technologies in smart grid, especially service restoration process. However, with the increasing installation of smart switches, tie-lines, sectionizers and deployments of new Distributed Energy Resources (DERs) there is an open research area in exploring the role of communication layer in realizing cyber-physical distribution systems. Also there are new opportunities to consider rather obsolete Synchronous Generators or Back-up Generators (BUGs) as a source of providing blackstart capability to grid-tied non-dispatchable inverters. In this way, the utility of BUGs has increased to harness power from non-dispatchable inverters rather merely providing backup to local load. Therefore the primary objective of the research is to develop a distributed multi-level framework for **service restoration** in a distribution system with high penetration of DERs (including inverter based distributed generators, inverter interfaced Battery Energy Storage Systems (BESS) and Synchronous Generators) and remote control switches while maintaining grid stability and considering practical implications of communication among agents.

After a major blackout leaving multiple faults, the faulted areas are isolated from the upstream network and Micro-Grids (MGs) or intentional islands start to build-up around dispatchable DGs (like Synchronous Generators or Grid-Forming Inverter based DGs). These MGs grow gradually in time until all the critical loads are picked up. Constellation of faults and radial nature of the network determine the load pick-up order and boundaries of MGs. The overall restoration process is formulated as a Mixed Integer Second Order Cone Program (MISOCP) optimization problem solved in MATLAB/YALMIP with Gurobi Solver. The agent based modeling is carried out in Java Agent Development Environment (JADE) and the generated network traffic is simulated in NS-3. Power system stability analysis (including inter-switch delays) is performed in GridLab-D. The approach is validated over a modified IEEE-123 node test feeder and results are presented to demonstrate the efficacy of the framework for random multiple fault scenarios.

2. STATE-OF-THE-ART/RELATED WORK

Extreme events like natural disasters (including hurricanes, floods, earth quakes and lightning), man-made deliberate cyber/physical attacks and terrorism can cause severe damage to transmission and distribution system resulting in wide-spread blackout. However following a major outage, the distribution system can be sectionalized into several isolated MGs to pickup critical loads and bring back tripped non-dispatchable inverter based DGs by dispatchable DGs. Usually a MG contains atleast a blackstart DG or a grid-forming inverter based DG that can provide frequency and voltage reference within the Grid.

Service restoration is based on solving an optimization problem, usually a mixed Integer program with the objective of maximizing load pickups and minimizing switching operations. However several approaches are suggested based on stochastic nature of loads, intermittency of Renewables, physical limitations of switches and fluctuating frequency and voltage within the network. Modern Distribution Automation (DA) of the Distribution System (DS) incorporates remotely controlled smart switches in the network. These switches help network reconfiguration for an economic dispatch (in case of grid-connected mode) and service restoration in an event of blackout. This assumption of smart switches is valid as more utilities in the world are adopting smart switches for their distribution networks. In [1] the author described the usefulness and adoption of smart switches. An hourly network reconfiguration in presence of high penetration of Renewable energy resources is discussed in [2]. The aim is to reconfigure the network to assess thoroughly the potential loss reduction. The problem is formulated as Mixed Integer Conic Program. Another very insightful work is presented in [3] for distribution system reconfiguration under uncertainties of renewable resources and loads where the author tries to minimize the losses of a distribution system. Arif et al [4] considered types, capabilities and physical limits of different switching devices including

Sectionizers, Circuit Breakers, Load Breakers and Fuses for load restoration. Similarly in [5], the author proposed MG formation and switching sequence. It forms multiple MGs and includes optimally positioned mobile emergency resources to address potential risks in extreme conditions. Another interesting work is presented in [6] where the author considered network reconfiguration for load restoration in presence of Synchronous DGs. In [7], the author described several reconfiguration techniques used for service restoration in distribution systems under different practical considerations.

Above methods are centralized and may provide an optimal solution for small distribution systems. These centralized approaches have their own disadvantages especially with the expansion in distribution system scale and incorporation of intermittency of renewable sources and loads. Therefore MAS based distributed approaches are finding grounds in restoration problems. They have an inherent capability of flexibility, scalability, parallelism and robustness. An agent based restoration with support of distributed energy storage is presented in [8]. The author proposed different states of the switching device based on real-time measurements and communication. Large optimization problem for dynamic optimal power flow in a distribution system is solved distributively in a multi-agent environment in [9]. In [10] a stable MAS based load restoration algorithm is proposed for MGs based on the Average Consensus algorithm. Global information, needed for distributed load restoration is discovered by communication among neighboring agents. The proposed algorithm is stable and applicable to any network configuration (radial or mesh). In [11], the author presented a decentralized MAS approach for service restoration using DG islanding. It investigates the impact of Electric Vehicles (EV) as a controllable load and a storage resource to help in service restoration. Different components like DGs, Aggregators and loads are modelled as agents with specific behavior and interactions in a co-operative environment. In another work presented in [12], a Fault Location, Isolation and Service Restoration (FLISR) is implemented through cooperative MAS framework. The distribution system is sectionalized into zones and feeders for service restoration.

During a black start period, loads are picked up in steps where the amount of load at each step is defined based on the minimum allowable frequency (frequency nadir) of the system. MGs with a large proportion of inverter based DGs (IBDGs) tends to have a lower system inertia and damping. Therefore it is important to analyze the transient state of the DS with regards to frequency variation during load restoration. Zhang et al in [13] has considered the frequency dynamics constraint in optimization problem for load restoration in a twolevel scheme. In [8] the proposed scheme determines a proper switching sequence to ensure system dynamic performance in the restoration process.

Power flow analysis is a useful tool for a variety of power systems applications. Several models and algorithms exist in literature for power flow and optimal power flow analysis. The optimal power flow (OPF) problem is non-convex and approximations and relaxations have been developed to solve it. In [14] the author proposed a set of models including two semi-definite programming relaxations and a linear approximation of the power flow in unbalanced multi-phase radial network. Another exciting research work in the form a PHD dissertation is available in [15] where conic and quadratic optimization formulations are introduced for transmission systems planning and DS reconfiguration. The most appreciable work of the author includes introduction of Mixed-integer models with Quadratic, Conic and Semi-definite optimization programs for network reconfiguration and power loss analysis. Load flow problems of radially distributed networks can be modeled by convex optimization especially conic programs [16]. The proposed second-order-cone program (SOCP) is solved in MOSEK. In [17] the optimal power flow problem is formulated as a Quadratic program.

Although abundant literature is available on convex relaxation to OPF problems but a majority of research work formulates it as a Linear Program for the sake of simplicity compromising optimal solution. Few researchers have modeled power system transient analysis into the original optimization problem. The role of Synchronous Generator as a black-start capable DG has not been studied in detail for service restoration problems. Constraints like ramp-up/down time, inertial capacities of prime-movers and its excitation are important. The role of communication architecture within MAS for service restoration has not been considered in the past. The modeling of agents has mostly been behavioral based on rules/expert systems. Therefore research on Cyber-Physical realization of the service restoration problem is scarce.

3. DETAILED DESCRIPTION OF PROPOSED PROJECT: OBJECTIVES, EXPECTED OUTCOMES, AND FUNDAMENTAL SCIENTIFIC/TECHNICAL VALUE

Load restoration is a sequential multi-step process where initially black-start capable generators like Grid-forming inverters and BUGs start and pick-up certain amount of loads to form micro-grids around them. Afterwards, non-black start generators kick-in and participate in the restoration process to pickup more loads and the micro grid grows until all the (critical) load is restored. However IBDGs (with black-start capability/Grid-Forming type) are generally started in the initial stages because of their rapid response and fast ramp-up due to internal electronics. They provide the primary frequency control. At each step, the amount of load pick-up is determined by the frequency nadir of the power system. This ensures the stability of distribution system during load restoration.

A significant contribution of this work is towards a cyber-physical implementation of a resilient distribution system. The switching sequence and the load restoration decisions determined at the upper level (by solving the MISOCP optimization problem) are implemented over a distributed MAS framework. The network elements like DGs, switches and nodes are assigned corresponding agents. These intelligent self-sufficient agents operate in a cooperative manner with two-way negotiations to exchange information and control across the network. The MAS framework is implemented in Java Agent Development Environment (JADE) that provides the required communication framework and services. Based on this interaction, network traffic (payload) is generated at each stage. Finally the payload generated is simulated in NS-3 to determine latencies and Quality of Service (QOS) of the network. The resulting reconfigured power system having inter-switch operating delay is finally validated in Gridlab-D to determine the grid stability/frequency nadir. This provides the next stage load-step for the optimization problem. This approach provides a practical distributed implementation of the framework.

A modified IEEE 123-node radial distribution test feeder as described in [18] was selected as a test bed to simulate the results. Among the available test feeders, this feeder is the most comprehensive and popular distribution system. The availability of switches (among lines) makes it a popular choice for testing optimal network configuration and load restoration algorithms. The test feeder was modified to accommodate smart switches and DERs across the network. Multiple Faults were generated and restoration framework was tested on it. MG boundaries and load-ranking were determined based on fault occurrences and radial structure. Figure 3-1 depicts the energization ranking and MG formation for the modified IEEE-123 test feeder. The figure in (a) depicts the test feeder with smart switches. Not all the lines in a distribution system are switchable (due to cost implications). A group of load/buses under a switch is called bus block. Location and capacities of DGs (black-start & non-black start) and loads are pre-defined in the feeder. A reduced graph is shown in (b) for each bus-block being numbered. Faults are applied randomly. The figure in (c) shows the MG formation and bus block energization ranking maintaining a radial structure. A depth first iterative search algorithm was applied to determine energization hierarchy for bus-blocks. This approach can determine the ranking for any scale of distribution system. Bus-block numbering 2,4,11 and 14 contains black-start capable DG and MGs start to develop around them.

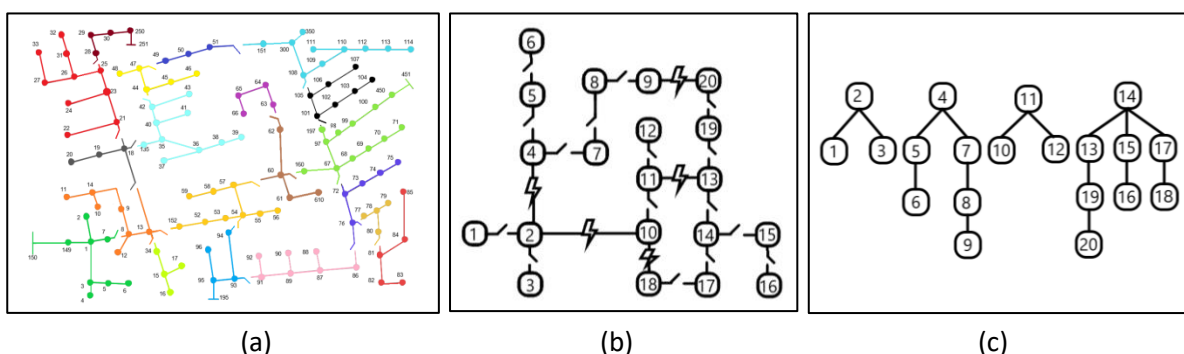


Figure 3-1: MG Formation

The optimization problem runs centrally with an objective to maximize the restored load. There are several constraints to the optimization problem including radial structure, power limits of transmission lines, ramp-up/down rates of synchronous generator, SOC constraints of BESS and AC-OPF. The problem is formulated in MATLAB using YALMIP and solved using Gurobi.

Figure 3-2 (a) illustrates the final stage of load restoration. Green nodes represent dispatchable DG where as yellow nodes represent non-blackstart DGs and the rest are loads. Four autonomous MGs with distinct boundaries are formed. Each MG has a black-start capable DG. Part (b) shows the total load restored

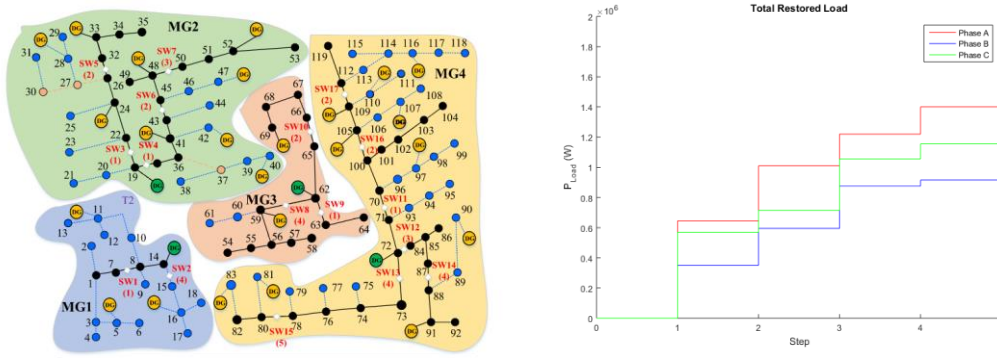


Figure 3-2: Final Restoration Stage

Figure 3-3 shows a dip/fall in frequency during load restoration without considering inter-switching delay. The switching is performed simultaneously. It takes 5 iterations for the algorithm to restore all load. The figure shows frequency profile for individual MG.

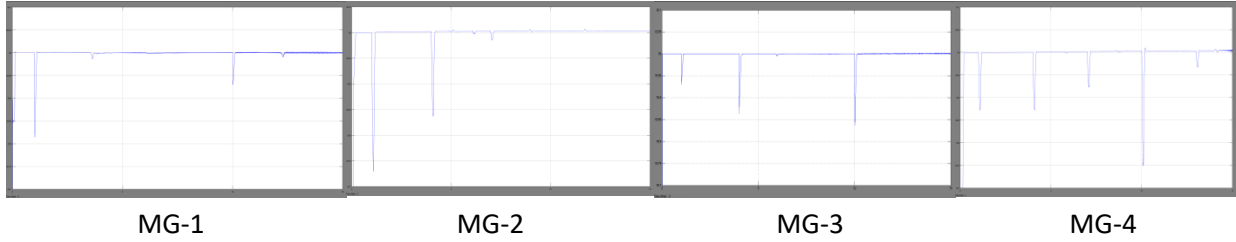


Figure 3-3: Frequency Deviation (without inter-switching delay)

Figure 3-4 illustrates the effect of inter-switching delays on frequency stability for each MG. It is clear from the study that incorporating communication delay among load and generating switches would cause the frequency to oscillate during each step.

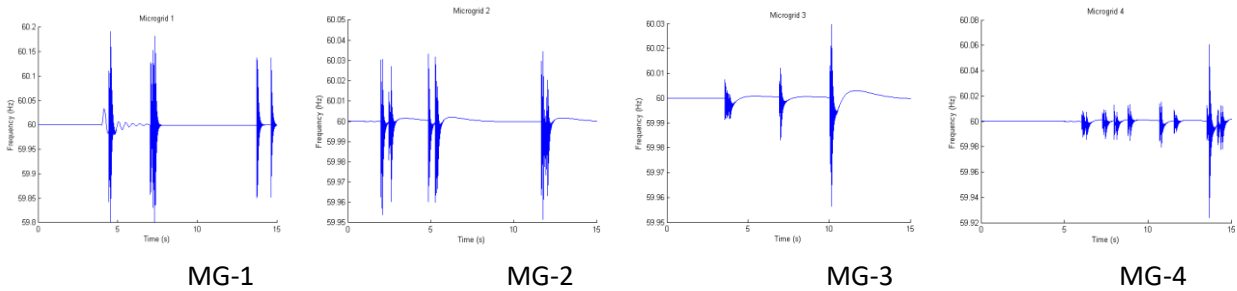


Figure 3-4: Frequency Deviation (with inter-switching delay)

To overcome the inter-switching latency issue simultaneous switching is recommended with a time step equal to $T + \delta t_{\max}$ where

$$\delta t_{\max} = \max\{\delta t_1, \delta t_2, \delta t_3, \dots, \delta t_n\} \text{ communication delays for individual switches}$$

The proposed work provides a valuable insight into the effects of communication delays among swithcing operations. It forms the basis of a Cyber-Physical resilient distribution system where fast communication is an essential part among interacting agents. Furthermore it provides an approach for harnessing power from non-dispatchable DGs utilizing BUGs.

The proposed framework is validated on IEEE-123 node test-bed with simulations performed in non-real-time simulators like GridLab-D, JADE and NS-3. However in order to validate the cyber-physical realization of this framework following aspects shall be considered.

- Time synchronization issues: The framework shall have a unified clock signal. Different interacting agents (DGs, Switches, load) and the real-time power system simulator shall have a unified clock. This is achievable with a GPS clock and PTP Master Server.
- Real-time Grid Simulator that can emulate the complete 123-node test bed.
- Real-time Network emulator to model communication latencies (like CORE or OPNET).
- Agents can be hardware controllers (like ARM-based Control boards) communicating over ethernet/LAN.
- There can be a fiber/Wimax link between the central station and each black-start DG agent.
- The framework can be tested against different types of loads (like Induction Motors or harmonic loads) realizing Power Hardware in the loop.
- Physical remote control MV sectionalizer switches along with RT-Lab in-built model switches can be considered.

ERIGrid 2.0 offers holistic and cyber-physical systems based validation approach for industrial and academic researchers. Among the participating labs, **AIT SmartEST** has the largest infrastructure for MV and LV distribution network testing. The broad array of available simulators (both for Power System and Communication Systems) would be useful for validating our proposed research. The lab has a history of conducting cyber-physical systems based validation and verification experiments. The experimental setup shown in Figure 3-5 can help us validate our framework.

- Multi-core OPAL-RT real-time simulator for modeling IEEE 123-node test feeder.
- Line impedance emulations for calculating line losses.
- Adjustable loads (to test the framework against exponential loads, thermostatically coupled loads and inductive loads).
- ICT simulator with IEC 61850 based communication.
- Utilizing available DERs (like PV inverters), Battery energy storage systems, Combined Heat & Power and Synchronous Generators.
- Suitable communication network simulator like OMNET++.
- GPS clock synchronization and PTP Master and Slave servers.

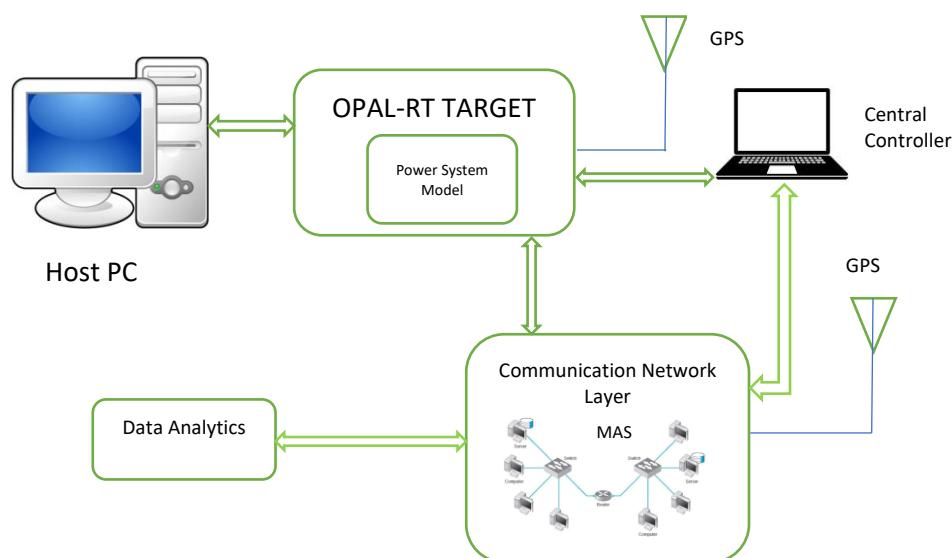


Figure 3-5: Experimental Setup

4. ORIGINALITY, INNOVATION AND IMPACT OF PROPOSED RESEARCH

This work proposes a novel framework for a cyber-physical resilient distribution network with high penetration of DERs. Although load restoration is a well-established field and plenty of research is available on it. However most of the research is based on passive distribution networks where the role of communication technologies is negligible and all nodes are electricity consumers. In passive networks the only consideration is reconfiguration of tie-line switches and sectionalizers to isolate faulted area from original feeder and restore load from adjacent feeder. The role of communication technologies has increased with the installation of automation equipment like remote control switches, smart meters, phasor measurement units (PMUs), power quality analyzers (PQA), step voltage regulators and DERs. The advancement in distribution system has opened new research in distribution management system (DMS), distribution automation (DA) and outage management system (OMS). This has increased resilience of distribution system towards outages. Now local resources can form MGs and pick-up critical loads. The role of real-time low latency and higher bandwidth communication has increased manifolds.

As discussed earlier, cyber-physical aspects of distribution system in service restoration were not considered in detail in the past. Our work has significant contributions towards the role of communication layer in load restoration. The role of synchronous generator has also been emphasized in our framework. Similarly MAS based distributive approach is considered. These intelligent agents cooperate among themselves to execute a common task. A very few researchers have considered the transient behaviour of distribution system during load restoration. Most of them used the power system simulation for only validating the sequence. However we have considered it not only for validating the stability but also the amount of load for next stage. Similarly we have considered a relaxed model for AC-OPF which guarantees an optimal solution. The load and available generation of renewable resources are also uncertain and it shall be modeled into the framework.

The type of load is another very important consideration in determining the restoration problem. We have initially modeled the framework with constant impedance, current and power type loads (ZIP). However a large percentage of residential loads can be thermostatically coupled (TCL) or cold-load pickups (CLPU) that can present a starting demand larger than pre-fault demand.

With the advancement in smart grid technologies and increasing penetration of DERs on the distribution side (MV and LV networks) the role of communication architecture is becoming inevitable in distribution networks. The distribution networks have turned from passive networks to active with consumers turning to prosumers with an increased interaction with the upstream grid. Our current work is a step towards self-healing which comes under the scope of DA. The MAS along with the support of PMUs installed across the network can help in realizing Fault Location, Isolation and Service Restoration (FLISR). FLISR or self healing is a salient feature of smart distribution systems enhancing resilience. The adoption of new controllable loads in the form of Electric Vehicle (EV) present several challenges and opportunities for distribution systems. However its integration with the existing framework can harness power from the mobile EV batteries and enhance grid resilience and reliability.

Major power outages have a significant impact on social and economic conditions of a country. Therefore supply security is of vital concern. Therefore the outage times should be as short as possible. Smart grid technologies like self-healing in conjunction with DERs penetrated across the network can help in reducing the outage duration to the customers.

Although we have considered the proposed framework for load restoration in an event of power outage but such events are rare and called High impact low probability (HILP). Therefore the framework can also be used for economic dispatch where stakeholders (like prosumers and utility) can benefit by hourly reconfiguration of the distribution system, demand response programs and frequency regulation.

5. SYNERGY WITH ONGOING RESEARCH

ERIGrid 2.0 offers holistic and cyber-physical systems based validation approach for industrial and academic researchers. There proposed work come under the scope of ERIGrid and finds relevance with its theme. Self-healing or FLISR is an active research area in power systems community and plethora of research is being conducted in this field. There are several good quality works that align with our proposed framework. A data-driven fault detection scheme was proposed by a user group under ERIGrid 1.0 program in summer 2017. The group from Florida State University (FSU) used PMU data stream to first make the model learn and then detect faults in the distribution system. If we consider the scope of FLISR then our work complements the FSU user group work to realize a cyber-physical FLISR problem.

Research on MAS and their role in distribution systems is finding grounds. The heterogeneous evolution of distribution system has encouraged the MAS approach where similar objects represent an agent. Therefore we have an agent for load, switches, DGs, SGs and BESS. MAS provides a distributed control of the network avoiding a single point of failure. The agents are intelligent and can learn from environment. MAS can realize several functions including FLISR, demand response, frequency regulation and economic dispatch.

6. PROPOSED HOST LAB/RESEARCH INFRASTRUCTURE – JUSTIFICATION

The research infrastructure available at AIT-smartEST is consistent to our experimental needs. The facility hosts a number of power system and communication simulators for the MV and LV networks. In addition to the co-simulation setup, distributed energy resources like PV, CHP and Synchronous Generators are also available. It has a configurable load base and a battery storage system. For the experimental setup we don't need any extra arrangements in the lab. There would be no specimen/Equipment Under Test (EUT) from our side. We have a simulation model that we would test on the real-time simulator. The practical setup would help us validate our model and refine it for cyber-physical system.

7. DISSEMINATION – EXPLOITATION OF RESULTS

We have gone through the lab access guide for Erigrid 2.0. We would publish a report for the conducted experiment. This work is a part of a PHD research requirement and efforts would be made to published the outcomes and results in IEEE transactions of Smart Grid, Power systems. A few publications are planned in the IET power systems journal. In addition to this, some conference papers may also be published.

8. TIME SCHEDULE

The user group comprises of two members. The group can start experiment from March 1st, 2022 (or even earlier provided they have been granted physical lab access). The estimated stay is for 2 weeks with the breakup as follows

- Real-time simulation (OPAL-RT) of proposed IEEE-123 test bed-----3 days
- Communication network emulation in CORE/OMNET-----4 days
- Integration and sets of experiments-----5 days
- Data analysis and processing-----2 days
- Possible refinements in the test setup-----2 days

*above tasks would be running in parallel by two members (so the time line would be 14 days)

9. DESCRIPTION OF THE PROPOSING TEAM

The user group comprises of two member namely Ch Talha Hassan and Muhammad Shamaas.

Ch Talha Hassan is a PHD student in the Electrical Engineering department of Lahore University of Management Sciences (LUMS). His work is being supervised by Dr. Tariq Jadoon. The proposed research work is part of his doctoral research at LUMS. Mr. Talha has over 15 years of experience in the research and development of smart energy meters and other grid equipments. He is well-versed with smart grid technologies, DLMS/COSEM protocols, SCADA systems and other grid protocols. In the proposed experimental setup he will look at the communication architecture and GPS clock synchronization.

Muhammad Shamaas is a research assistant in the Electrical Engineering department of LUMS. He is working with the same group. Mr. Shamaas has 2 years of experience in High Voltage DC substation and $\pm 660\text{kV}$ transmission line project. He will assist in real-time simulations of IEEE-123 node test feeder and MATLAB optimization code.

REFERENCES

- [1] Kevin Damron and Randy Spacek. Smart grid—smarter protection: Lessons learned.
- [2] Mohammad Reza Dorostkar-Ghamsari, Mahmud Fotuhi-Firuzabad, Matti Lehtonen, and Amir Safdarian. Value of distribution network reconfiguration in presence of renewable energy resources. *IEEE Transactions on Power Systems*, 31(3):1879–1888, 2015.
- [3] Hossein Haghighat and Bo Zeng. Distribution system reconfiguration under uncertain load and renewable generation. *IEEE Transactions on Power Systems*, 31(4):2666–2675, 2015.
- [4] Anmar Arif, Bai Cui, and Zhaoyu Wang. Switching device-cognizant sequential distribution system restoration. *arXiv preprint arXiv:2011.08236*, 2020.
- [5] Liang Che and Mohammad Shahidehpour. Adaptive formation of microgrids with mobile emergency resources for critical service restoration in extreme conditions. *IEEE Transactions on Power Systems*, 34(1):742–753, 2018.
- [6] Young-Jin Kim, Jianhui Wang, and Xiaonan Lu. A framework for load service restoration using dynamic change in boundaries of advanced microgrids with synchronous machine dgs. *IEEE Transactions on Smart Grid*, 9(4):3676–3690, 2016.
- [7] Ahmed EB Abu-Elanien, MMA Salama, and Khaled B Shaban. Modern network reconfiguration techniques for service restoration in distribution systems: A step to a smarter grid. *Alexandria engineering journal*, 57(4):3959–3967, 2018.
- [8] Cuong P Nguyen and Alexander J Flueck. Agent based restoration with distributed energy storage support in smart grids. *IEEE Transactions on Smart Grid*, 3(2):1029–1038, 2012.
- [9] Matt Kraning, Eric Chu, Javad Lavaei, Stephen P Boyd, et al. Dynamic network energy management via proximal message passing. *Citeseer*, 2014.
- [10] Yinliang Xu and Wenxin Liu. Novel multiagent based load restoration algorithm for microgrids. *IEEE Transactions on Smart Grid*, 2(1):152–161, 2011.
- [11] Anurag Sharma, Dipti Srinivasan, and Anupam Trivedi. A decentralized multiagent system approach for service restoration using dg islanding. *IEEE Transactions on Smart Grid*, 6(6):2784–2793, 2015.
- [12] Aboelsood Zidan and Ehab F El-Saadany. A cooperative multiagent framework for self-healing mechanisms in distribution systems. *IEEE transactions on smart grid*, 3(3):1525–1539, 2012.
- [13] Qianzhi Zhang, Zixiao Ma, Yongli Zhu, and Zhaoyu Wang. A two-level simulation-assisted sequential distribution system restoration model with frequency dynamics constraints. *IEEE Transactions on Smart Grid*, 2021.
- [14] Lingwen Gan and Steven H Low. Convex relaxations and linear approximation for optimal power flow in multiphase radial networks. In *2014 Power Systems Computation Conference*, pages 1–9. IEEE, 2014.
- [15] Joshua Adam Taylor. Conic optimization of electric power systems. PhD thesis, Massachusetts Institute of Technology, 2011.
- [16] Rabih A Jabr. Radial distribution load flow using conic programming. *IEEE transactions on power systems*, 21(3):1458–1459, 2006.
- [17] Brett A Robbins and Alejandro D Domínguez-García. Optimal reactive power dispatch for voltage regulation in unbalanced distribution systems. *IEEE Transactions on Power Systems*, 31(4):2903–2913, 2015.
- [18] William H Kersting. Radial distribution test feeders. *IEEE Transactions on Power Systems*, 6(3):975–985, 1991.