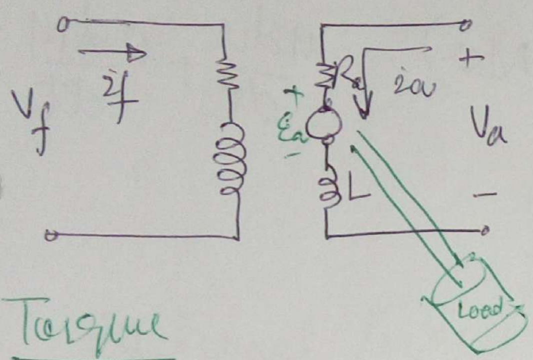
DC Motors

Types of DC Motors

1) Seperately Excited
2) Shunt type DC motor

Block Diagram



$$V_a = L\frac{di_a}{dt} + i_a R_a + E$$

$$V_a - E = L\frac{di_a}{dt} + i_a R_a$$

$$V_a(s) - E(s) = L[sI_a(s) - I(0)] + I_a(s)R_a$$

with zero initial condition.

$$V_a(s) - E(s) = LsI_a(s) + I_a(s)R_a$$

$$V_a(s) - E(s) = I_a(s)[sL + R_a]$$

$$I_a = \frac{V_a(s) - E(s)}{R_a + Ls}$$

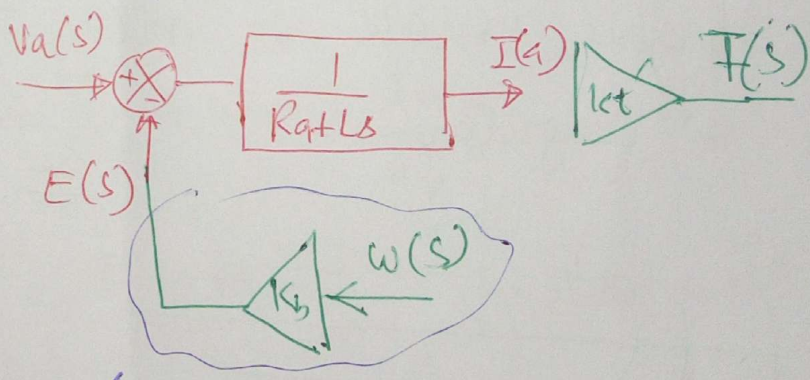
Torque

$$T_e = K_t I_a I_f$$

Assuming Constant flux operation

$$T_e = K_t' I_a$$

$$T_e(s) = K_t' I_a(s)$$



Back EMF

$$E_a \propto \omega$$

$$E_a \propto I_f \propto \text{flux}$$

$$E_a = K_b'' \phi_f \omega$$

and $\phi_f = K I_f$

$$E_a = K_b' I_f \omega$$

for constant flux

$$E_a(s) = K_b \omega(s)$$

$$\omega = \frac{d\theta}{dt}$$

$$\omega(s) = s\theta(s) - \theta(0)$$

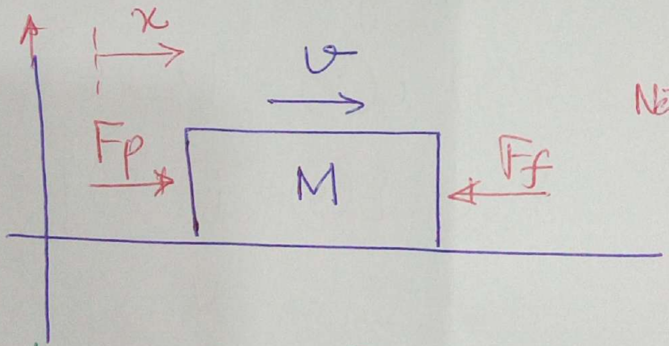
$$\omega(s) = s\theta(s)$$

Newton's Law state that

"The net force acting on a body of mass M equals to the rate of change of its mechanical momentum" which is equal to the product of its mass and its velocity in the direction of the net force.

$$F = \frac{d}{dt}(p) = \frac{d}{dt}(Mv) = M\frac{dv}{dt} + v\frac{dM}{dt}$$

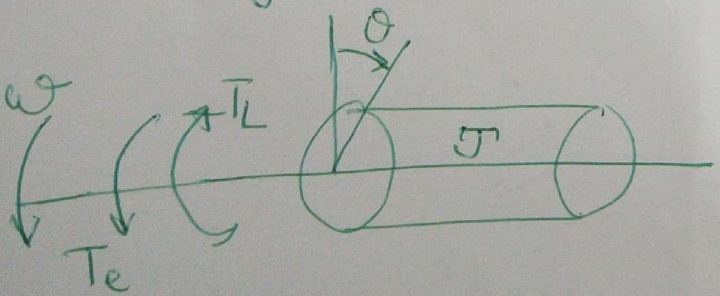
$$F = M\frac{dv}{dt} + v\frac{dM}{dt}$$



Net $F = F_p - F_f$

$$= M\frac{dv}{dt}$$

For rotational motion, the force is equivalent to torque, mass is moment of inertia and velocity is angular



So $$T = \frac{d}{dt}(J\omega)$$

$$= J\frac{d\omega}{dt} + \omega\frac{dJ}{dt}$$

Net Torque: $T_e - T_L$

$$T = J\frac{d\omega}{dt}$$

Now $\omega = \frac{d\theta}{dt}$ ($\theta \rightarrow$ angular displacement)

$$\Rightarrow T = J \frac{d^2\theta}{dt}$$

In rotating electrical machines the net torque is given by this expression.

$$T = T_e - T_L = J \frac{d\omega}{dt}$$

$T_e - T_L \Rightarrow T_e - T_L > 0 \rightarrow$ accelerating effect

$T_e - T_L < 0 \longrightarrow$ Braking

- Frictional torque is given as $B\omega$; if load is frictional in nature as then in most of the applications

$$T_e = J\frac{d\omega}{dt} + T_L = J\frac{d\omega}{dt} + B\omega$$

- for power balance

$$\omega_m T_e = \omega_m T_L + \omega_m J \frac{d\omega_m}{dt}$$

$$P_D = P_L + \omega_m J \frac{d\omega_m}{dt}$$

Driving power, load power, ? - change in K.E

- Integrating $\int_0^t P_D\, dt = \int_0^t P_L\, dt + \int_0^t \omega_m J \frac{d\omega_m}{dt}\, dt$

$$W_D = W_L + J \int_0^{\omega} \omega_m \cdot d\omega_m$$

$$\boxed{W_D = W_L + \frac{1}{2} J \omega_m^2}$$ — K.E. stored in the system

- analogous to energy stored in capacitor or inductor ($\frac{1}{2} L i^2$) or ($\frac{1}{2} C V^2$)

Considering the Mechanical torque equation again

$$T_e = J \frac{d\omega}{dt} + B\omega$$

$$T_e = J \frac{d^2\theta}{dt} + B \frac{d\theta}{dt}$$

$$J \frac{d^2\theta}{dt} + B \frac{d\theta}{dt} = T_e = K_t' I_a = K_t I_f I_a.$$

$$J\left[s^2 \theta(s) - s\cancel{\theta(0)}^{\approx 0} - \cancel{\theta'(0)}^{\approx 0}\right] + B\left[s\theta(s) - \cancel{\theta(0)}^{\approx 0}\right] = T_e$$

$$J s^2 \theta(s) + B s \theta(s) = T_e = K_t' I_a(s)$$

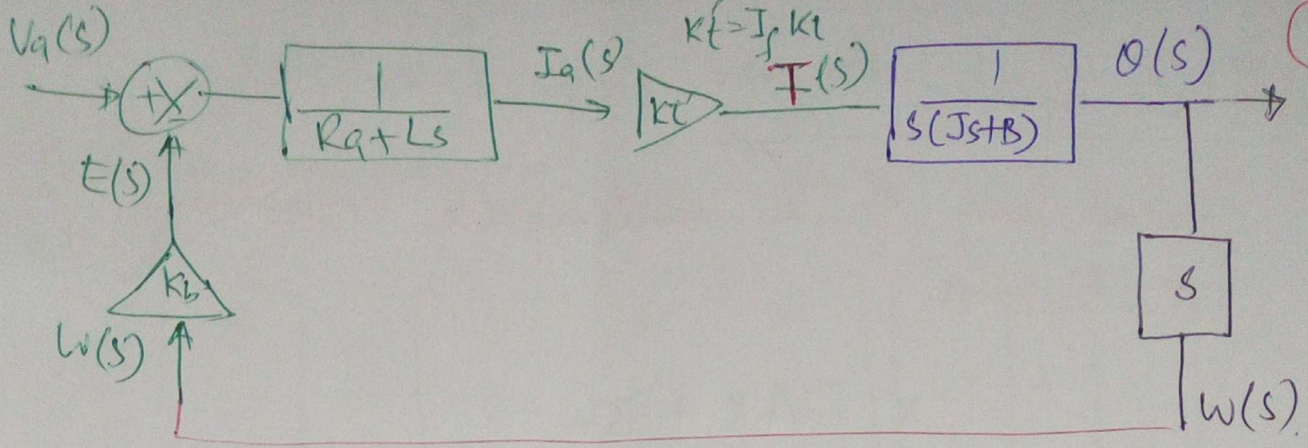
$$s.\theta(s)\left[Js + B\right] = T_e = K_t' I_a(s)$$

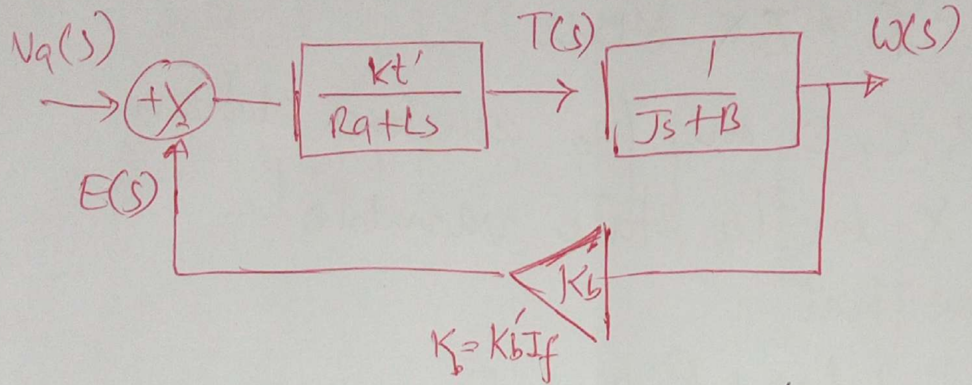
$$\frac{\theta(s)}{I_a(s)} = \frac{K_t'}{s(Js+B)} = \frac{K_t I_f}{s(Js+B)}$$

$$\boxed{\frac{\theta(s)}{I_a(s)} = \frac{K_t I_f}{s(Js+B)}}$$ — T(s) — (A)

Now updating the control transfer function we get:

$$\frac{\omega(s)}{I_a(s)} = \frac{K_t I_f}{Js+B} \Rightarrow \frac{\omega(s)}{T(s)} = \frac{1}{Js+B}$$





- For a DC motor $K_b = K_t' = K$.

so

$$\frac{\omega(s)}{V_a(s)} = \frac{\dfrac{K_t'}{(R_a+Ls)(Js+B)}}{1+\dfrac{K_t'}{(R_a+Ls)(Js+B)}\times K_b} = \frac{K}{(R_a+Ls)(Js+B)+K^2}$$

$$\boxed{\frac{\omega(s)}{V_a(s)} = \frac{K}{(R_a+Ls)(Js+B)+K^2}}$$

Second Order

Transfer function for position of the motor

$$\frac{\theta(s)}{V_a(s)} = \frac{K}{s\left[(R_a+Ls)(Js+B)+K^2\right]}$$

Third Order

State Space Model: (Speed Control)

General form of the state space equations for an LTI System:

$$\dot{X} = AX + Bu$$

$$Y = CX + Du$$

A, B, C, D are the constant matrices.

X is the state variable.

Now we know that

$$V_a = L\frac{di_a}{dt} + R_a i_a + E_a$$

$$\Longrightarrow \frac{di_a}{dt} = \frac{V_a}{L} - \frac{R_a}{L} i_a - \frac{E_a}{L} \quad E_a = K\omega \qquad \text{--- (1)}$$

Mechanical Equation:

$$J\frac{d\omega}{dt} = T_e - B\omega$$

$$\frac{d\omega}{dt} = -\frac{B}{J}\omega + \frac{T_e}{J}$$

$$\frac{d\omega}{dt} = -\frac{B}{J}\omega + \frac{K I_a}{J} \qquad \text{--- (2)}$$

Writing Eq. in state space form. (eq 1 & eq 2)

$$\left.\begin{aligned} \frac{d}{dt}\begin{bmatrix} \omega \\ i \end{bmatrix} &= \begin{bmatrix} -\frac{B}{J} & K/J \\ -K/L & -R_a/L \end{bmatrix}\begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V \\ Y &= \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \omega \\ i \end{bmatrix} \end{aligned}\right\} \text{--- (3)}$$

We can add more state variables in State space model eq.

$$\frac{d}{dt}\begin{bmatrix} \theta \\ \omega \\ i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -B/J & K/J \\ 0 & -K/L & -R/L \end{bmatrix}\begin{bmatrix} \theta \\ \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} V$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \omega \\ i \end{bmatrix}$$

Armature Equation $V_a = L\frac{di_a}{dt} + i_a R_a + E_a$

$$E_a = K_b \phi \omega$$

$$V_a = L\frac{di}{dt} + i_a R_a + K_b \phi \omega$$

In steady state $\frac{di}{dt} = 0$

So $\Rightarrow$ $V_a = i_a R_a + K_b \phi \omega$.

$\Rightarrow$ $\omega = \frac{V_a - i_a R_a}{K_b \phi}$

$\Rightarrow$ In general R_a is very small; and is intentionally kept very small to reduce Copper losses.

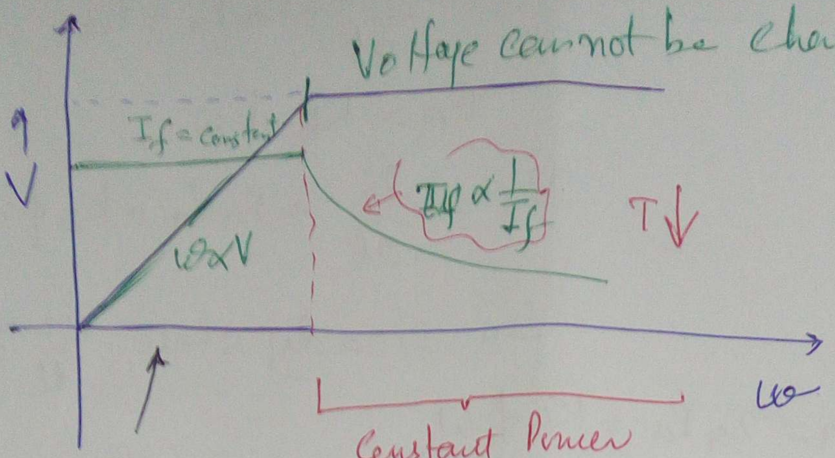
$\Rightarrow$ $\omega \approx \frac{V_a}{K_b \phi}$ → $\omega \propto V_a$; $\omega \propto \frac{1}{\phi}$

→ Flux is proportional to the current in field circuit 'I_f'

$$\phi = K I_f$$

$\Rightarrow$ $\boxed{\omega \propto \frac{1}{I_f}}$

- There are two methods to Control the Speed
  1. Voltage Control Method.
  2. Flux Control Method.



$T = K I_f I_a$

* Torque may be kept constant
→ called constant torque region

Constant Power Region V = constant
If is very small and does not change power.

Torque Speed X-tics

In steady state

$V = i_a R_a + E_a$

$V = i_a R_a + K\phi\omega$

$V = i_a R_a + K_b \omega$ —— (1)

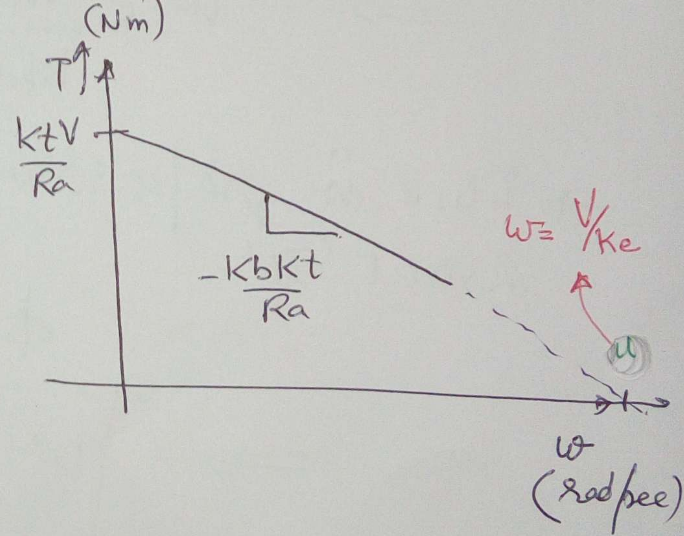
We know that

$T = K I_a \phi$

$T = K' I_f \cdot I_a$

$T = K_T \cdot I_a$ —— (2)

$\Rightarrow I_a = \frac{T}{K_t}$

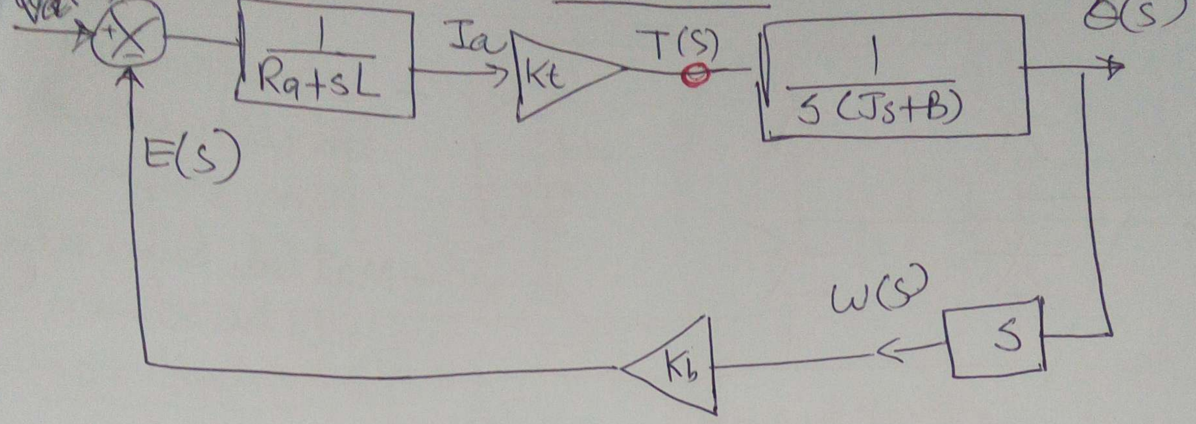


(1) $\Rightarrow V = \frac{T R_a}{K_t} + K_b \omega$

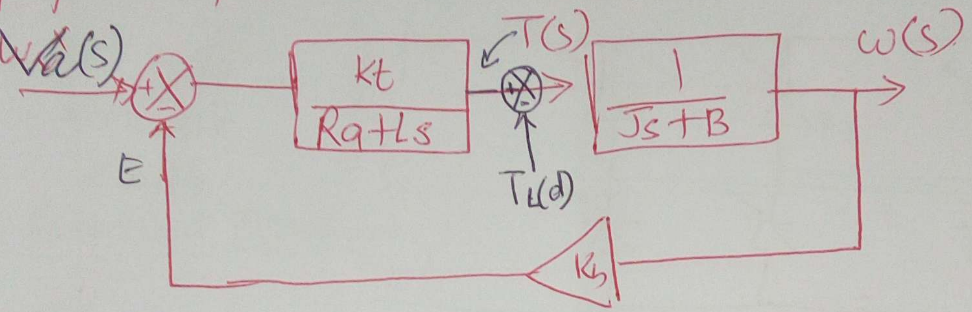
$\Rightarrow \frac{T R_a}{K_t} = V - K_b \omega$

$T = \frac{K_t}{R_a} \cdot V - \frac{K_b K_t \cdot \omega}{R_a} \Rightarrow \boxed{T = \frac{K_t V}{R_a} - \frac{K^2 \omega}{R_a}}$

Lecture 2



Simplified form

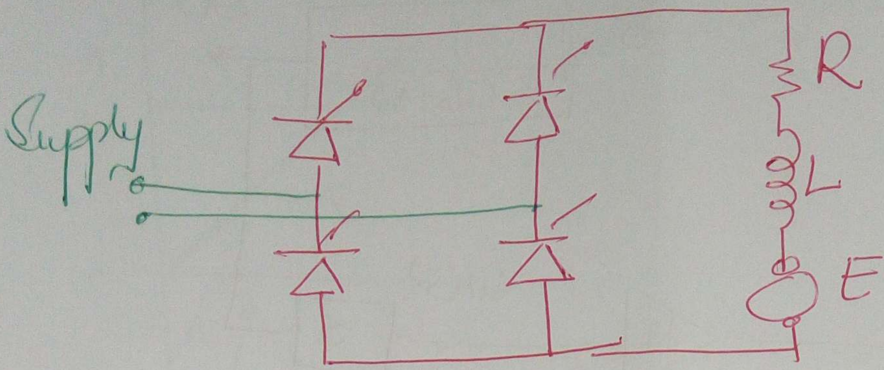


$$\frac{\omega(s)}{V(s)} = \frac{K_t}{(R_a+Ls)(Js+B)+K_tK_b}$$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{s[(R_a+Ls)(Js+B)+K_tK_b]}$$

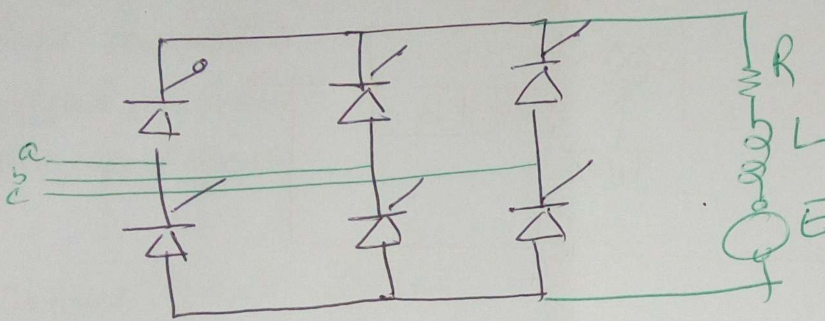
- We need to Control our Converter to adjust the voltage at the machine terminals.
  * Rectifier circuits.
  * Controlled Rectifier circuits
  * SMPS Concepts
    - → Buck Converter
    - → Boost Converter

1) Single Phase Controlled Rectifier



- waveforms to be discussed from slides
- Matlab Simulation is to be shown.

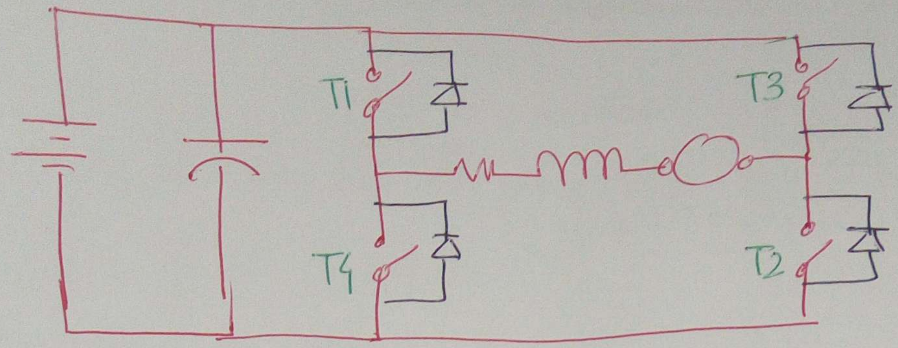
2) Three phase Controlled Rectifiers.

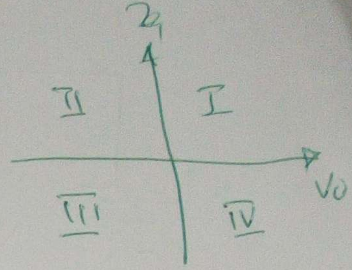


- may be discussed from slides

- See Power Electronics Slides

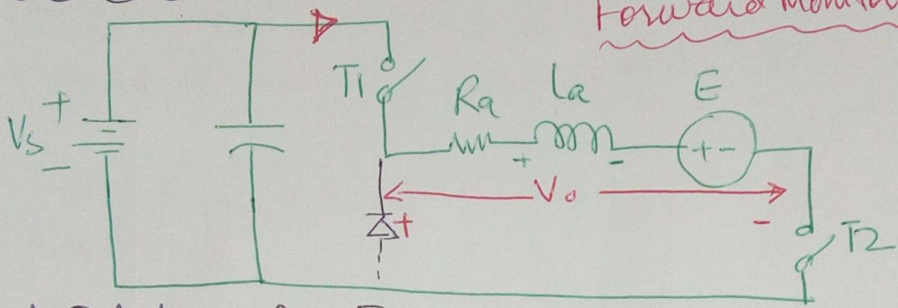
A four quadrant chopper circuit

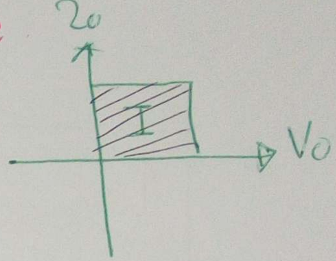




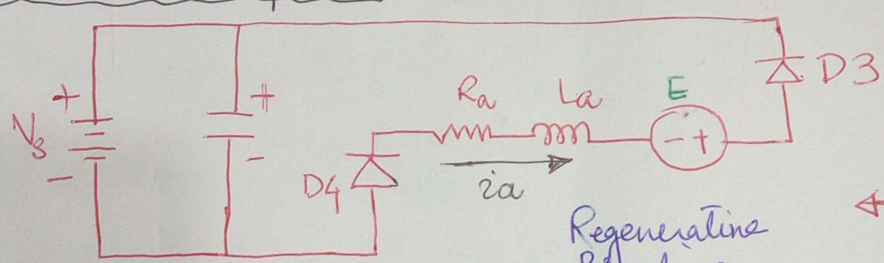
- First Quadrant Operation:

Forward Monitoring

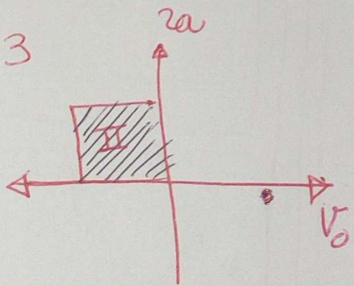




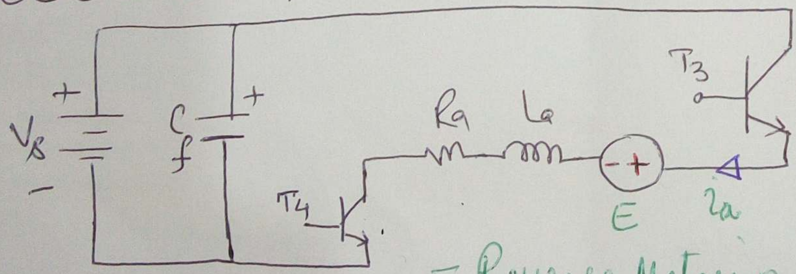
2nd Quadrant Operation:

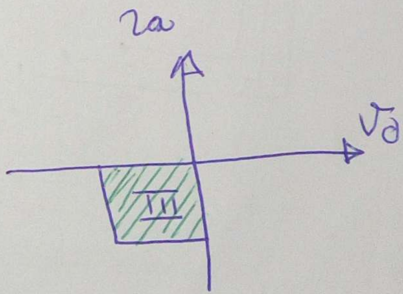


Regenerative Breaking

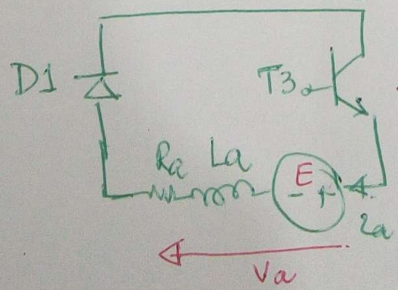


Third Quadrant Operation:



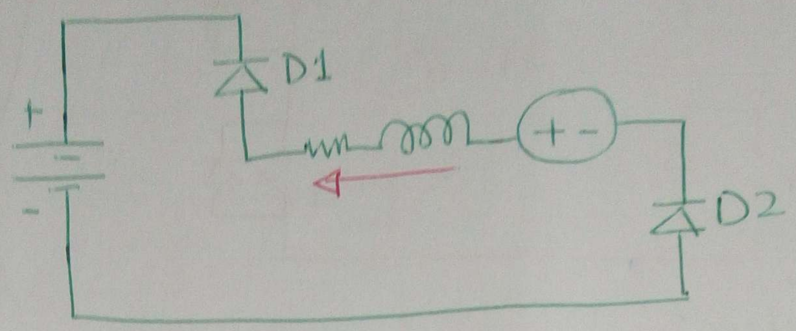


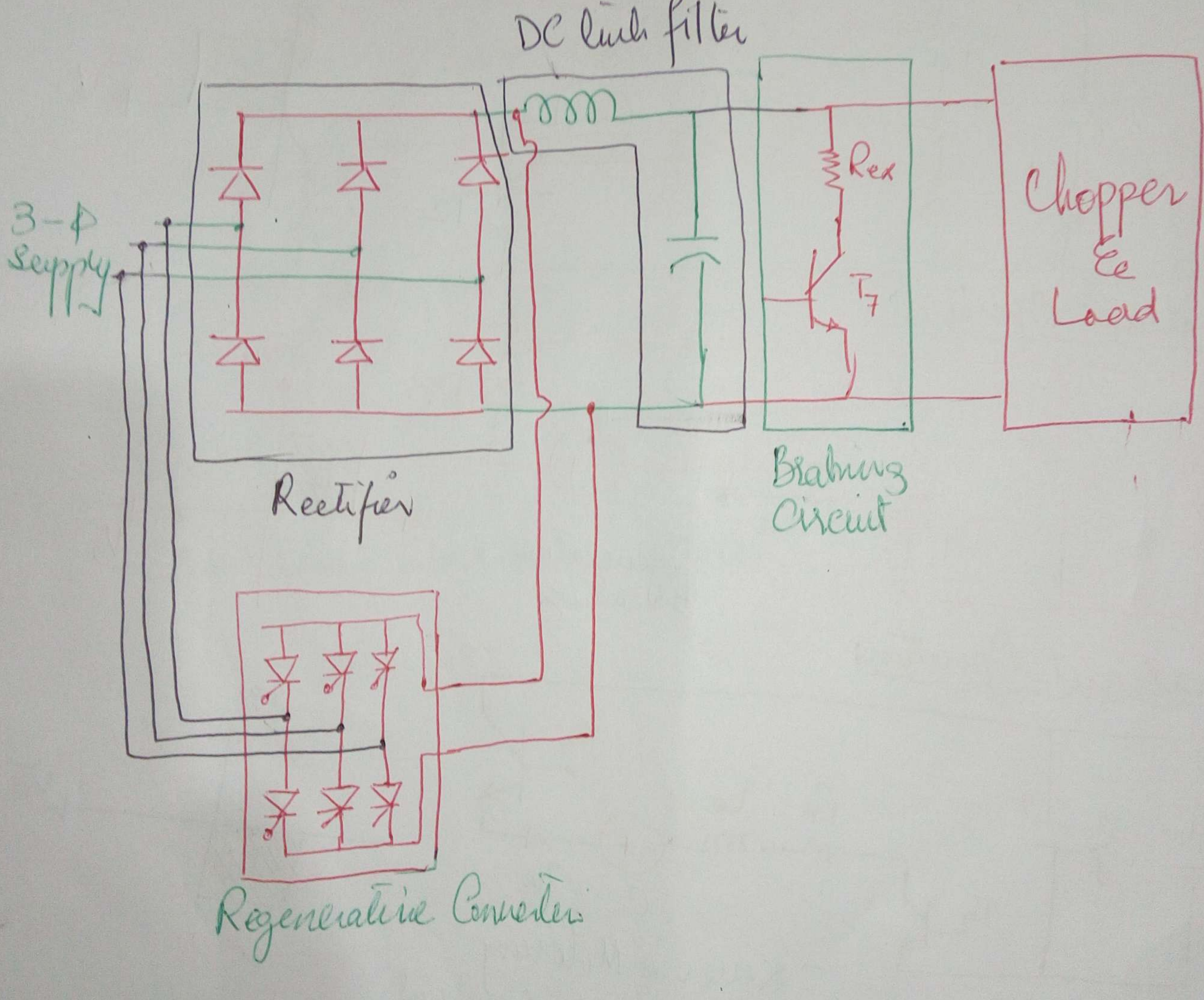
- Reverse Motoring
- Increasing load current



- Decreasing load current

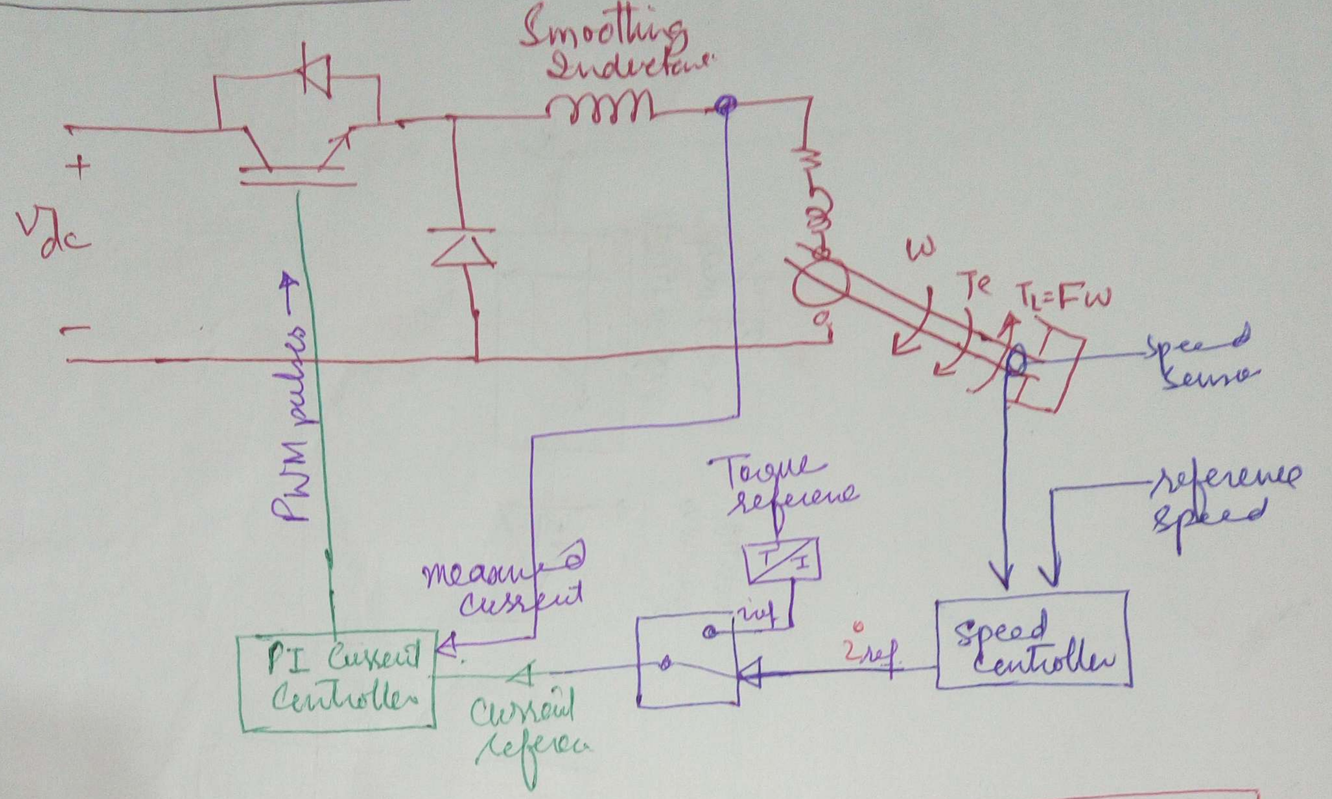
Fourth - Quadrant Operation



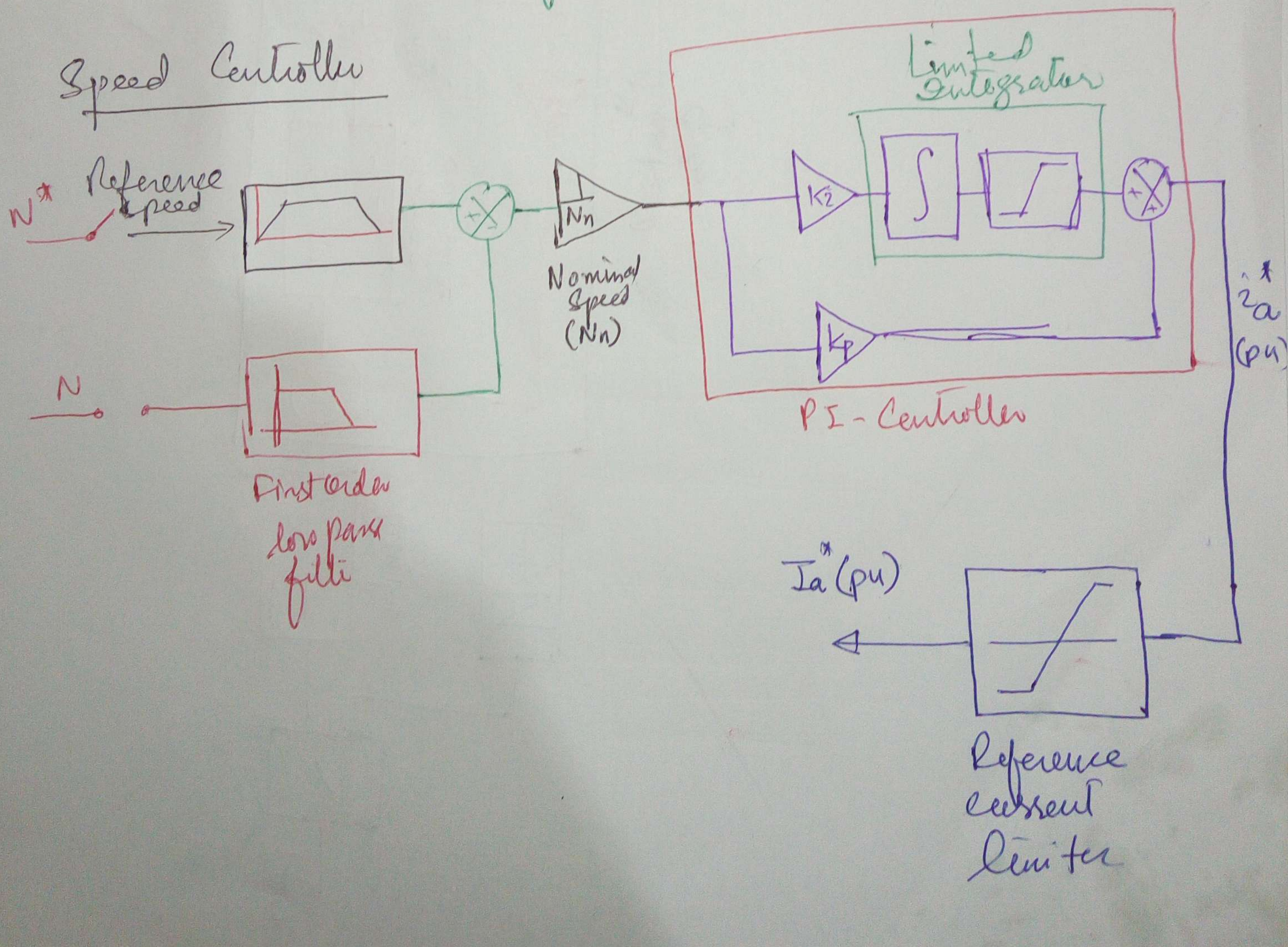


One Quadrant DC Motor Drive (Chopper Based)

Consider the circuit:



Speed Controller



Current Controller