Estimation of the Effective Permeability of Stacking Dispersive Conductor Magnetic Layers

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***Abstract*— Based on the works of Sarabandi and Mosallaei related to the model of "eddy currents", we are introducing the concept of resistive permeability μ_R in order to determine the effective permeability of multi-layer stacking for dispersive conductor magnetic layers of LLG type. The results show that the numerical results agree well with the analytic solution with an error less than for the extrinsic permeability $|\mu_{ext}|<5\%$.**

I. Introduction

For the last few years, the search for innovative materials has been successful, particularly in regards to using ferromagnetic thin layers. These magnetic-conducting materials allow high permeability (until 15GHz) for a thickness of less than 1µm. Commonly, the effective permeability μ_{eff} of multi-layers depends on the extrinsic permeability μext for each material used. The model based on "eddy currents" offers a good description of the physical phenomena from Maxwell equations and allows for the determination of the extrinsic permeability μ_{ext} of magnetic materials [1]. However, this model can not be directly used for dispersive materials, notably if the permeability is negative (for example, Landau Lifschitz Gilbert – LLG dispersive permeability). In this paper, we rely on an eddy current model to determine the effective medium for stacking LLG dispersive conductor magnetic materials.

II. Eddy Current Model

Sarabandi and Mosallaei propose a simple formula which allows calculation of the effective permeability μ_{eff} for a heterogeneous double layer magnetic structure (see Fig. 1), by using the following equation [2]:

$$\mu_{eff} = \mu_{ext,1}\frac{d_1}{d_1+d_2} + \mu_{ext,2}\frac{d_2}{d_1+d_2} \tag{1}$$

with $\mu_{ext,1}$ and d_1, the permeability and the thickness of material 1 (respectively $\mu_{ext,2}$ and d_2).

To take into account the conductivity of magnetic layers, we have shown in [7] that the effective permeability μ_{eff} have a dispersive behavior similar to a Debye model (Fig. 2), and can be expressed as :

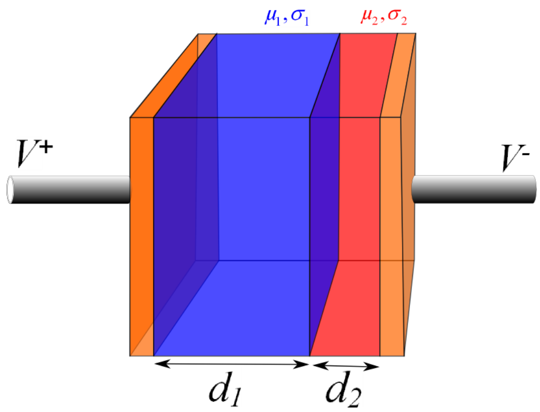


Fig. 1 Description of stacking of two magnetic layers

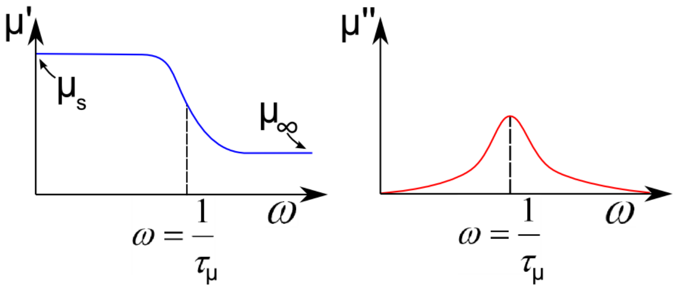


Fig. 2 Description of the Debye model

$$\mu_{eff} \approx \mu_\infty + \frac{\mu_s - \mu_\infty}{1 - j\omega\tau_\mu} \tag{2}$$

Where : $$\mu_s = \mu_1\frac{d_1}{d_1+d_2} + \mu_2\frac{d_2}{d_1+d_2} \tag{3}$$

$$\mu_\infty = \left\{ \begin{array}{l} 0 \; si \;\; \sigma_1,\sigma_2 = 0 \\ \dfrac{\mu_1 d_1}{d_1+d_2} si \;\; \sigma_1 = 0 \\ \dfrac{\mu_2 d_2}{d_1+d_2} si \;\; \sigma_2 = 0 \end{array} \right\} \tag{4}$$

In a general case, the estimation of the time-constant (also called relaxation time-constant) τ_μ of heterogeneous material, is complex. However, by considering that only one of the two layers is a conductor ($\sigma_1 \neq 0$ and $\sigma_2 = 0$), the time-constant τ_μ can be obtained by this formulas:

$$\tau_\mu \approx K d_1^2 \mu_1 \sigma_1 \tag{5}$$

with a constant $K \approx 8\times10^{-7}$ and d_1, the thickness of the conductor layer.

From (1), the proposed Debye model can be related to the model of "eddy currents" [1] by calculating the extrinsic permeability μ_{ext} for each magnetic material with:

$$\mu_{ext} = \mu_{\text{int}} F_{CF} \tag{6}$$

$$\mu_{\text{int}} = \mu'_{\text{int}} - j\mu''_{\text{int}} \tag{7}$$

while δ and d represent the skin depth and material thickness, respectively where:

$$F_{CF} = \left[\frac{2\delta}{(1+j)d}\right]\tanh\left[\frac{(1+j)d}{2\delta}\right] \tag{8}$$

The variable F_{CF} behaves like a coefficient range from 0 to 1 on the intrinsic permeability μ_{int} of material. We can note that the ratio d/δ increases when the extrinsic permeability μ_{ext} of material decreases.

III. Estimation of the Extrinsic Permeability

Before to study the case of dispersive intrinsic permeability, it is important to note, when the conductor's magnetic material has a non-dispersive intrinsic permeability $\mu_{int} = \mu_{DC}$, the skin depth δ is calculated by the equation:

$$\delta = \sqrt{\frac{\rho}{\pi f \mu_0 \mu_{DC}}} \tag{9}$$

The real part and the imaginary part of F_{CF} (8) are obtained with the following equations:

$$\Re e[F_{CF}] = \frac{\tanh\left(\frac{d}{2\delta}\right)\left(1+\tan^2\left(\frac{d}{2\delta}\right)\right)+\tan\left(\frac{d}{2\delta}\right)\left(1-\tanh^2\left(\frac{d}{2\delta}\right)\right)}{x\left(1+\tanh^2\left(\frac{d}{2\delta}\right)\tan^2\left(\frac{d}{2\delta}\right)\right)} \tag{10}$$

$$\Im m[F_{CF}] = \frac{\tanh\left(\frac{d}{2\delta}\right)\left(1+\tan^2\left(\frac{d}{2\delta}\right)\right)-\tan\left(\frac{d}{2\delta}\right)\left(1-\tanh^2\left(\frac{d}{2\delta}\right)\right)}{x\left(1+\tanh^2\left(\frac{d}{2\delta}\right)\tan^2\left(\frac{d}{2\delta}\right)\right)} \tag{11}$$

From (10) and (11), we can rewrite the extrinsic permeability μ_{ext} (2) as:

$$\Re e[\mu_{ext}] = \mu'_{\text{int}}\Re e[F_{CF}] - \mu''_{\text{int}}\Im m[F_{CF}] \tag{12}$$

$$\Im m[\mu_{ext}] = \mu''_{\text{int}}\Re e[F_{CF}] + \mu'_{\text{int}}\Im m[F_{CF}] \tag{13}$$

Then, the relaxation time-constant $\tau_{\mu,CF}$ is evaluated by resolving the equation:

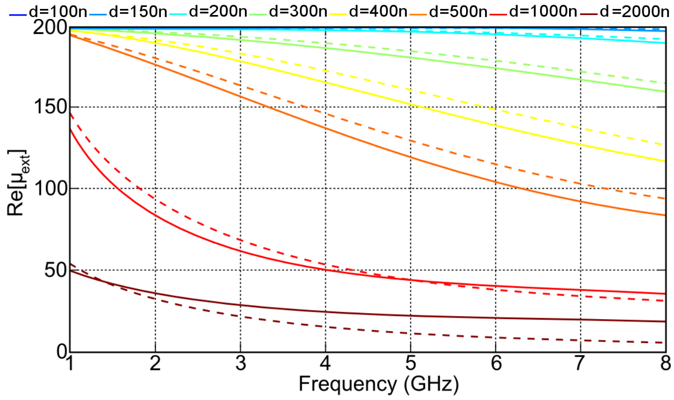
$$\frac{\partial\, \Im m[F_{CF}]}{\partial f} = 0 \tag{14}$$

So, the solution of the equation (14) is given by:

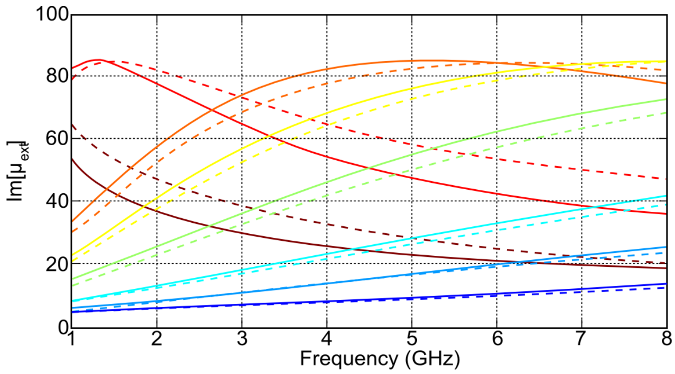
$$\tau_{\mu,CF} = K d^2 \mu_{DC} \sigma \tag{15}$$

with a constant $K \approx \frac{\mu_0}{10}$, d_1 and σ_1, the thickness and the conductivity of the conductor layer.

To estimate the extrinsic permeability μ_{ext} of a material, we applied the Nicolson-Ross-Weir method using the HFSS numerical method [3 and 4]. The numerical results (full line) are then compared with the magnetic model (8) and (9) (dot line) with the various thicknesses of the conductor's magnetic materials *{d:100nm to 2000nm}* (Fig. 3).



(a)



(b)

Fig. 3 Evolution of extrinsic permeability (a) $\text{Re}[\mu_{ext}]$ and (b) $\text{Im}[\mu_{ext}]$ of a non-dispersive conductor's magnetic layer with various thicknesses d

To generalize the magnetic model being presented, we analyzed the behavior of a material with a static permeability $\mu_{DC}>1$, a conductivity $\sigma \neq 0$ and a LLG dispersive behavior. The spectrum of permeability μ_{LLG} is given by the equation:

$$\mu_{LLG} = \frac{\gamma 4\pi Ms\left(\gamma H_{eff} + \gamma 4\pi Ms + j\omega\alpha\right)}{\left(\gamma H_{eff} + j\omega\alpha\right)\left(\gamma 4\pi Ms + j\omega\alpha\right) - \omega^2} \tag{16}$$

with $4\pi Ms$, the magnetization, H_{eff}, the magnetic field, α, the factor attenuation and ω, the pulsation. The intrinsic characteristics of the materials studied are: $4\pi Ms = 24$kG, $\gamma = 2.8$MHz/Oe, $H_{eff} = 80$Oe, $\alpha = 0.01$ and $\sigma = 5$M.S^{-1} [4] (Fig. 4).

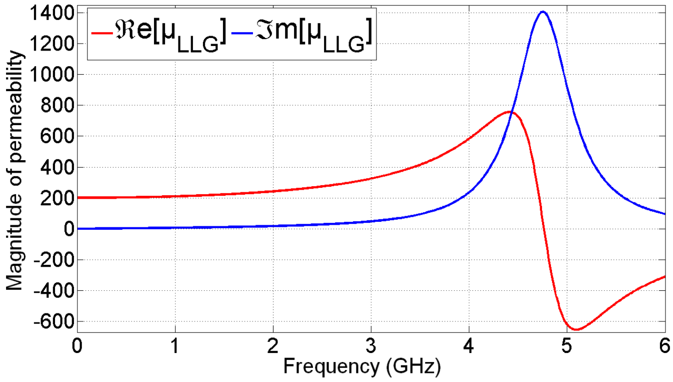


Fig. 4 Spectrum of the LLG dispersive permeability

We can see in Fig. 4 that the studied magnetic materials present a high absorption peak at the ferromagnetic resonance frequency close to 5GHz.

When the conductor's magnetic material has a LLG dispersive permeability, the skin depth δ (known as "abnormal skin depth") is calculated by the equation [1]:

$$\delta = \sqrt{\frac{\rho}{\pi f \mu_0 \left(\sqrt{\mu'^2 + \mu''^2} + \mu''\right)}} \tag{17}$$

For a conductor magnetic material thicknesses of *d={150 and 500}nm*, we compared the magnetic model, (12) and (13), with the numerical results obtained with HFSS (Fig. 5). We noticed that the magnetic model is not valid for a particular range of a LLG dispersive permeability. To solve this problem, we introduced the concept of resistive permeability μ_R into the magnetic model [6]. Thus, from the eddy current model (given by the Maxwell equations), we have:

$$k\left(s_{Laplace}\right) = \sqrt{s\mu\sigma} \Rightarrow k(\omega) = \sqrt{j\omega\mu\sigma} \tag{18}$$

$$k \propto \sqrt{j\left(\mu'_{\text{int}} - j\mu''_{\text{int}}\right)} = \frac{1}{\sqrt{2}}\left(\sqrt{\mu_L} - j\sqrt{\mu_R}\right) \tag{19}$$

with

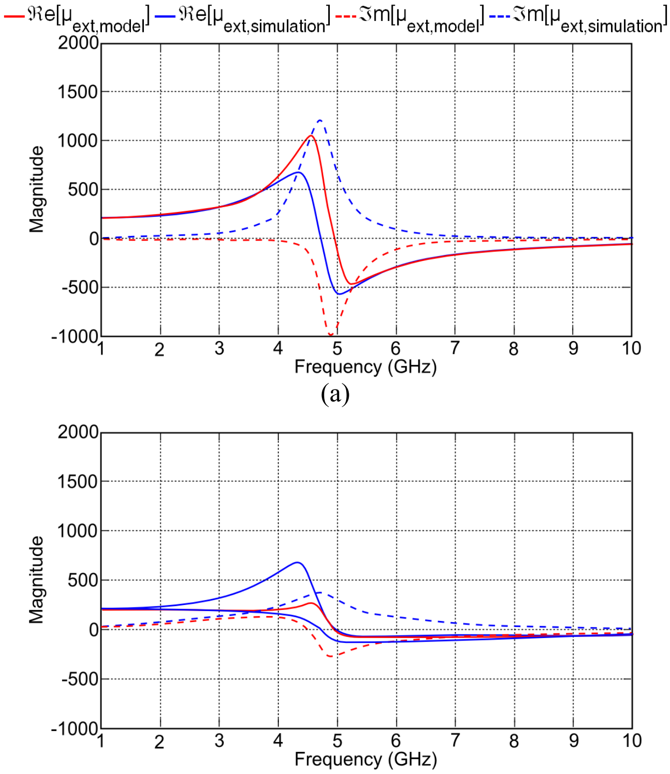
$$\mu_R = \sqrt{\mu'^2_{\text{int}} + \mu''^2_{\text{int}}} + \mu''_{\text{int}} \tag{20}$$

$$\mu_L = signe(\mu')\left[\sqrt{\mu'^2_{\text{int}} + \mu''^2_{\text{int}}} - \mu''_{\text{int}}\right] \tag{21}$$

With the concept of resistive permeability μ_R incorporated into the magnetic model, we are able to evaluate the extrinsic permeability of an LLG dispersive conductor's magnetic material by the equation:

$$\mu_{\text{ext,modified}} = \mu'_{\text{int}}\Re e\left[F_{CF}\right] - j\mu_R \Im m\left[F_{CF}\right] \tag{22}$$

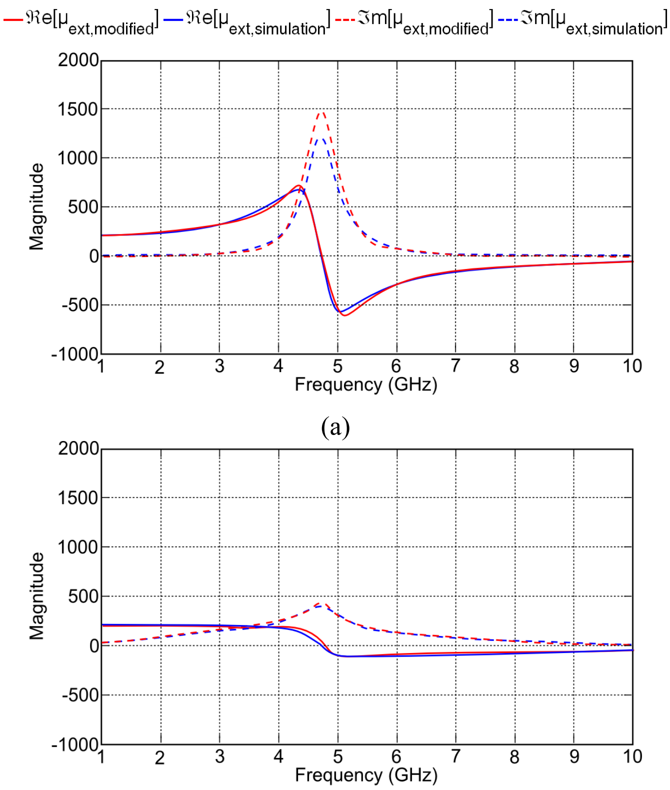
For a conductor with magnetic material thickness of *d={150 and 500}nm*, we compared the numerical results obtained with HFSS with modified analytical model (22) (Fig. 6). The analytical model without correction differs from numerical results whereas the numerical results agree well with the modified analytic solution with an error rate of less than $|\mu_{ext}|$<5%.



(a)

(b)

Fig. 5 Comparison (model/simulation) of the extrinsic permeability μ_{ext} of a conductor's magnetic layer for (a) d=150nm and (b) d=500nm



(a)

(b)

Fig. 6 Comparison (modified model/simulation) of the extrinsic permeability μ_{ext} of a conductor's magnetic layer for (a) d=150nm and (b) d=500nm

IV. MODELING OF 'N' MAGNETIC LAYERS

The proposed magnetic model (22) can be extended to N-multi-layer magnetic structure as shown in the Fig. 7.

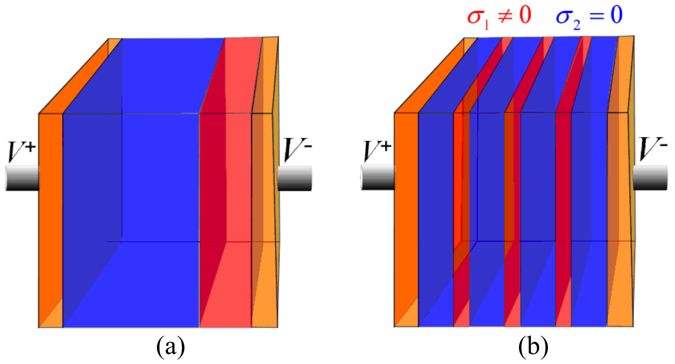


Fig. 7 Description of stacking of magnetic layers (a) N=2 et (b) N=7

In static, equations (1) can easily be extended to order 'N' with the following equation:

$$\mu_s = \frac{\sum_{i=1}^{N} \mu_i d_i}{\sum_{i=1}^{N} d_i} \tag{23}$$

In dynamic, these relations are more difficult to obtain by using simple analytical formulas because we have several relaxation frequencies possible. However, if magnetic stacking is alternated with conductor/insulator material, the previous models may be extended by recurrence with a stacking of N layers by using the following expression:

$$\mu_{eff\,(1..N)} = \frac{\mu_{1..N-1} d_{1..N-1} + \mu_N d_N}{d_{1..N-1} + d_N} \tag{24}$$

When a multi-layer magnetic structure is alternated conductor/insulator materials with a N-period, using $\sigma_1 \neq 0$ and $\sigma_2 = 0$, the time-constant is found by using:

$$\tau_\mu \approx K d_1^2 N^2 \mu_1 \sigma_1 \tag{25}$$

Thereafter, we equitably subdivided a magnetic conductor material within N-layers with {N: 2 to 20}. Arbitrary values were selected for layer 1 $\{\mu_{r,1}=2,\ \sigma=10^3$ and $10^5 \mathrm{S.m^{-1}}\}$ and layer 2 $\{\mu_{r,2}=5,\ \sigma_2=0 \mathrm{S.m^{-1}}\}$. The magnetic model (22 and 24) was then compared with the numerical results obtained from HFSS (Fig. 8).

We note that the numerical results agree with the magnetic model presented previously with an error rate of less than $|\mu_{ext}|<5\%$.

V. CONCLUSION

In this article, we showed that Sarabandi and Mosallaei works associated with the eddy current model allow for determination of the extrinsic characteristics of stacking non-dispersive conductor magnetic layers. The analytical results were validated by numerical results (HFSS). Based on the eddy current model, we extended the magnetic model to LLG dispersive materials with the concept of resistive permeability μ_R. Then, the magnetic model has been extended to N-multi-layer magnetic structure. The results showed that the numerical results agree well with the analytic solution with an error rate of less than $|\mu_{ext}|<5\%$.

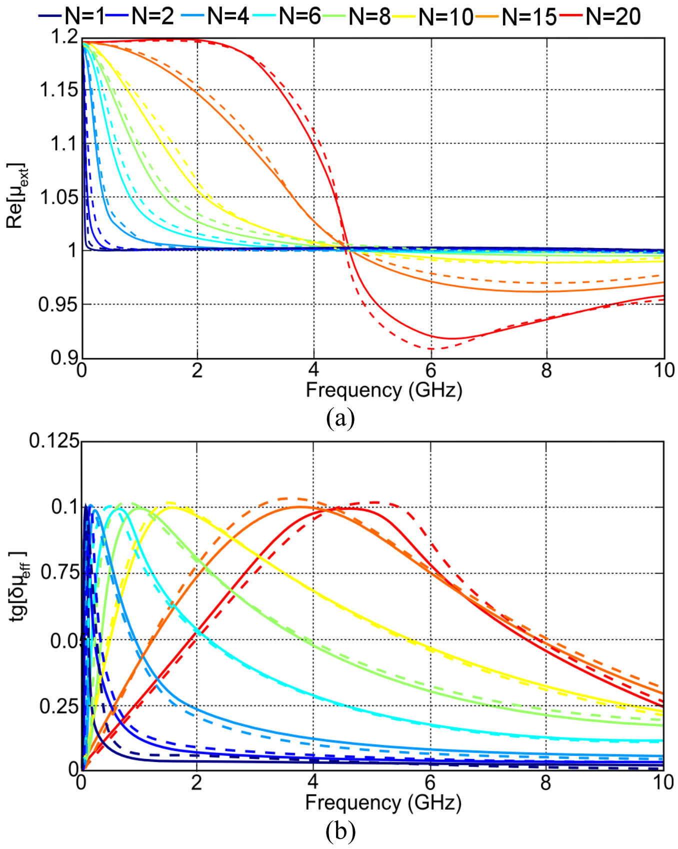


Fig. 8 Comparison of magnetic stack with a N-periods (stacked layers HFSS with dotted-line / equivalent model with full-line): (a) Re[μ_{eff}] and (b) Im[μ_{eff}]

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