Corner 2D Capacitance

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***Abstract*— The 2D geometric corner permeance (capacitance or conductance) was introduced for easier accurate calculation of magnetic and electric circuits containing regions with strong non uniform field. For infinitely long both arms of the corner and for infinitely long one arm and zero length of the other one, exact formulas for geometric corner permeance were derived and for short one arm and the length of the other one equal to zero or infinite good approximate formulas were proposed. In this paper the case of both short arms is studied by finite element method and a new approximate formula is proposed. Thus the problem of rectangular corner parameters is completely solved.**

Keywords*— *corner permeance, capacitance, conductance, FEM.

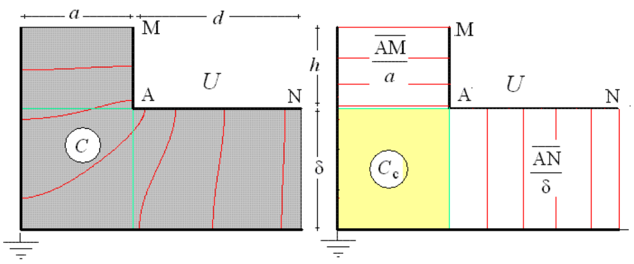
I. Introduction

For easier and more accurate calculation of the capacitance of corner region where the electric field is strong non uniform a "corner" capacitance can be used [1], (fig. 1). If the electric field lines issuing from M and N (fig. 1a) are straight, the geometric partial capacitance between MAN and grounded surface per unit depth can be evaluated as shown in fig. 1b:

$$C = \frac{\overline{\mathrm{AM}}}{a} + \frac{\overline{\mathrm{AN}}}{\delta} + C_\mathrm{c}(x, y, z) \qquad x = \frac{\delta}{a};\quad y = \frac{h}{a};\quad z = \frac{d}{\delta} \tag{1}$$

where C_c is the "corner" capacitance [1], [2]. It is clear that

$$C_\mathrm{c}(x, y, z) = C_\mathrm{c}(1/x, z, y) \tag{2}$$



a) b)

Fig. 1. Corner region

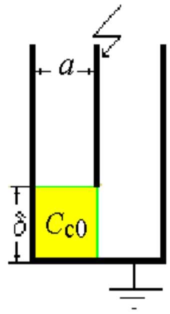


Fig. 2. Corner region for AN=d=z=0

II. Exact formulas for infinite or zero length arms

A. Case $y = z = \infty$

For $h = d = \infty$ (Fig. 1a) in [1] and [2] the following exact formula for 2D geometric corner capacitance was derived, using the conformal mapping:

$$C_\mathrm{c}(x, \infty, \infty) = C_\infty(x) = C_\infty\left(\frac{1}{x}\right)$$

$$C_\infty(x) = \frac{2}{\pi}\left[\frac{\arctan(x)}{x} + x\arctan\left(\frac{1}{x}\right) + \ln\frac{x^2+1}{4x}\right] \qquad x = \frac{\delta}{a};\quad y = z = \infty \tag{3}$$

The derivative of this expression, which can be useful for force evaluation, is the following

$$\frac{\mathrm{d}C_\infty}{\mathrm{d}x} = \frac{2}{\pi}\left[\arctan\left(\frac{1}{x}\right) - \frac{\arctan(x)}{x^2}\right] \tag{4}$$

The eq. (3) can be used in practice for $y > 1$ and $z > 1$.

B. Case $y = \infty$ and $z = 0$

For $h = \infty$ and $d = 0$ (Fig. 1a) also in [1] and [2] the following exact formula for 2D geometric corner capacitance of the region from Fig. 2 was derived, using the conformal mapping:

$$C_c(x,\infty,0) = C_0(x) = x + \frac{1}{\pi}\ln\frac{2}{\cosh(\pi x) - 1}$$
$$x = \frac{\delta}{a},\quad y = \frac{h}{a} = \infty,\quad z = \frac{d}{\delta} = 0 \tag{5}$$

with the corner capacitance derivative

$$C'_0(x) = \frac{dC_0}{dx} = 1 - \frac{\sinh(\pi x)}{\cosh(\pi x) - 1};\qquad x = \frac{\delta}{a} \tag{6}$$

For $\delta >> a$ (Fig. 2) the corner capacitance, when AN = d = 0 becomes

$$\lim_{\delta/a\to\infty} C_0 = \frac{2}{\pi}\ln 2 = 0.441,\quad y=\infty\quad z=0 \tag{7}$$

The values of the geometric corner capacitance given by the exact formulas (3) and (5) are given in Fig. 3. As can be seen, for small $x = \delta/a$, the values given by (3) and (5) approach and for large x the differences increase up to infinity:

$$\lim_{x\to 0}\frac{C_0}{C_\infty} = \lim_{x\to 0}\frac{\ln(2) - \ln(\pi x)}{1 - \ln(4x)} = \lim_{x\to 0}\frac{\ln(x) + 0.452}{\ln(x) + 0.386} = 1;$$
$$\lim_{x\to\infty}\frac{C_0}{C_\infty} = \lim_{x\to\infty}\frac{\ln(2)}{1 + \ln(x/4)} = 0 \tag{8}$$

The eq. (5) can be used enough exactly for $y > 1$ and $z < 0.001$.

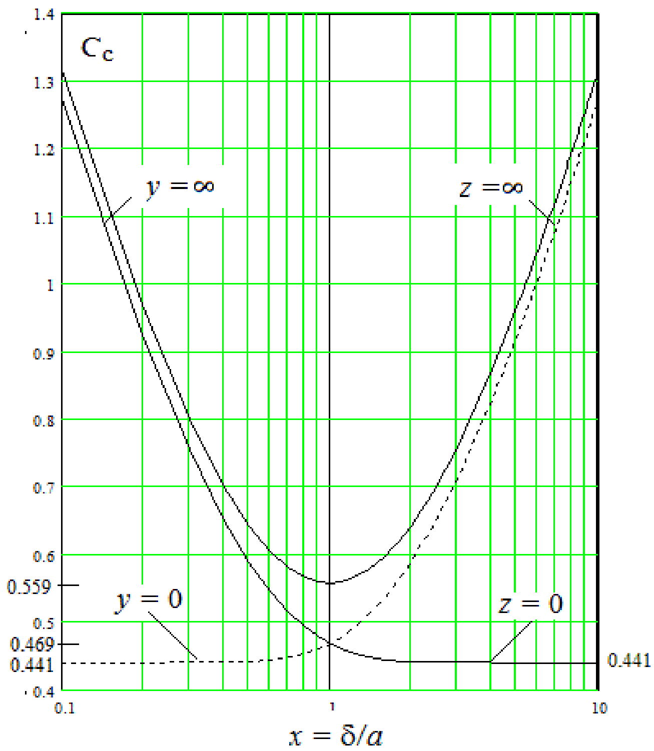


Fig. 3. Geometric corner capacitance for y and z equal to 0 and ∞, given by exact formulas (3) and (5),

III. Approximate Formulas for Short Arms

The proposed approximate formulas are based on the FEM results obtained using BELA package.

A. *Case $y = \infty$ and $z < 1$*

For $h > a$ and $0 < d < \delta$ the following approximate equation was previously proposed [3] (Fig. 4)

$$\boxed{\begin{aligned} &C_c\big|_{h>a}(x,z) \approx C_c(x,\infty,z) \approx \\ &C_\infty(x) - \left[C_\infty(x) - C_0(x)\right]\cdot \exp\left(\frac{-10z}{\left(1.6 - e^{-10z}\right)}\right)\end{aligned}} \tag{9}$$

(In the equation (51) from [3] there is a mistake: instead of 0.16 must be 1.6)

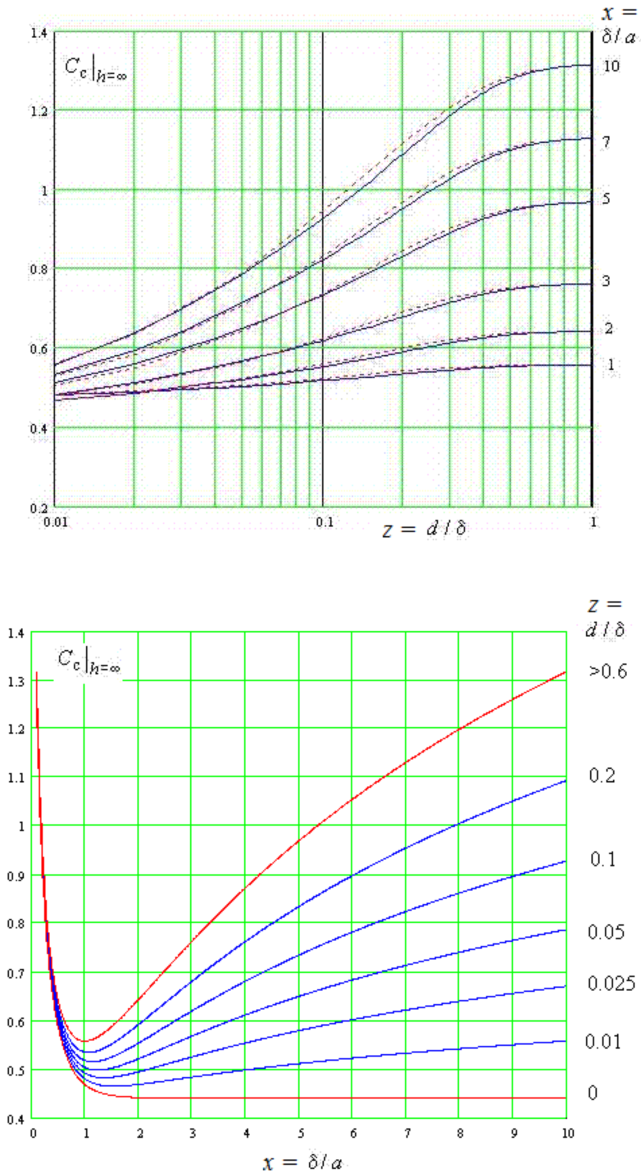


Fig. 4. Corner capacitance (9) for $h/a = \infty$ (> 1). (Dot line – FEM)

B. Case y = 0 and z < 1

Also in [3], for $h = 0$ and $0 < d < \delta$ the following approximate equation was proposed for geometric corner capacitance, on the basis of results obtained using FEM and conformal mapping (Fig. 5)

$$C_c\big|_{h=0}(x,z) \approx C_c(x,0,z) \approx$$
$$C_0(1/x) - [C_0(1/x) - 0.25] \cdot \exp\left(\frac{-10z}{\left(1.35 - e^{-10z}\right)}\right) \qquad (10)$$

Using (2) similar formula can be obtained for $z = 0$ and $y < 1$.

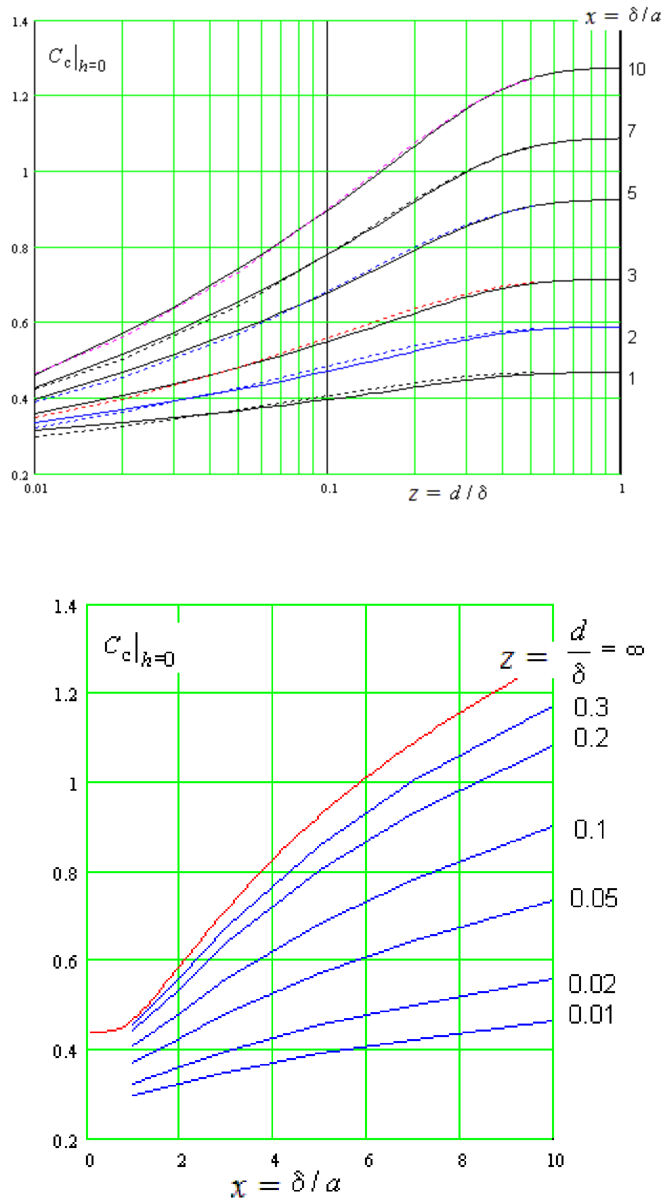


Fig. 5. Corner capacitance (10) for $h/a = 0$. (Dot line – FEM) [3]

In Fig. 6 the geometric corner capacitance is given for both cases *A* and *B* calculate with proposed eq (9) and (10)

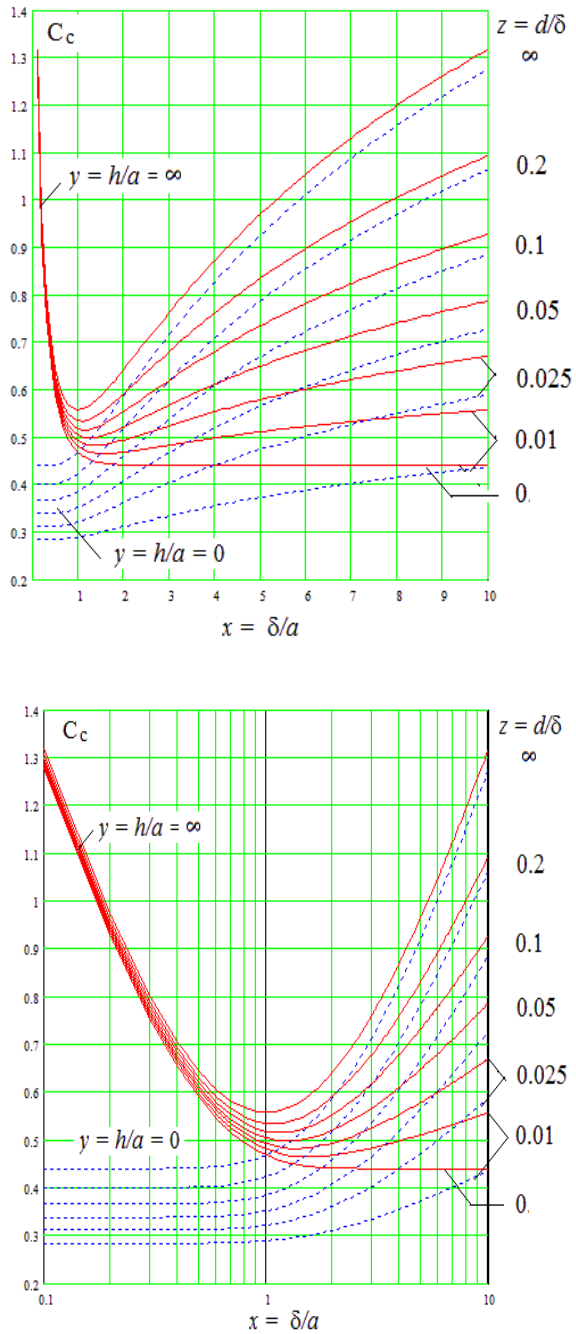


Fig. 6. Corner geometric capacitance (9) and (10) for $y = 0$ and ∞

C. Case y < 1 and z < 1

For both short arms, $h < a$ and $d < \delta$, the following new approximate formula is proposed for corner geometric capacitance, based on results obtained by FEM for $x > 1$:

$$C_c(x,y,z) \approx C_c|_{h>a}(x,z) - \left[C_c|_{h>a}(x,z) - C_c|_{h=0}(x,z)\right]\cdot \exp\left(\frac{-10\,y}{\left(1.35-e^{-10\,y}\right)}\right); \quad x=\frac{\delta}{a}>1 \tag{11}$$

For $x < 1$, using (2), from last equation results the following approximate formula for the case of both short arms:

$$C_c(x,y,z) \approx C_c|_{h>a}(1/x,y) - \left[C_c|_{h>a}(1/x,y) - C_c|_{h=0}(1/x,y)\right]\cdot \exp\left(\frac{-10\,z}{\left(1.35-e^{-10\,z}\right)}\right); \quad x<1 \tag{12}$$

The values of corner capacitance given by (11) for $x = 1$ are compared with FEM results in Fig. 7. The essential dependence of corner capacitance on y and z are shown in Fig. 8.

The values of corner capacitance given by (11) was also compared with the values obtained using the numerical conformal mapping of the domain in rectangle, offered by the Schwartz-Christoffel Toolbox from MATLAB. For values of x between 1 and 10 the discrepancies were up to 3.5 % (for $x = 7$, $y = 0.02$).

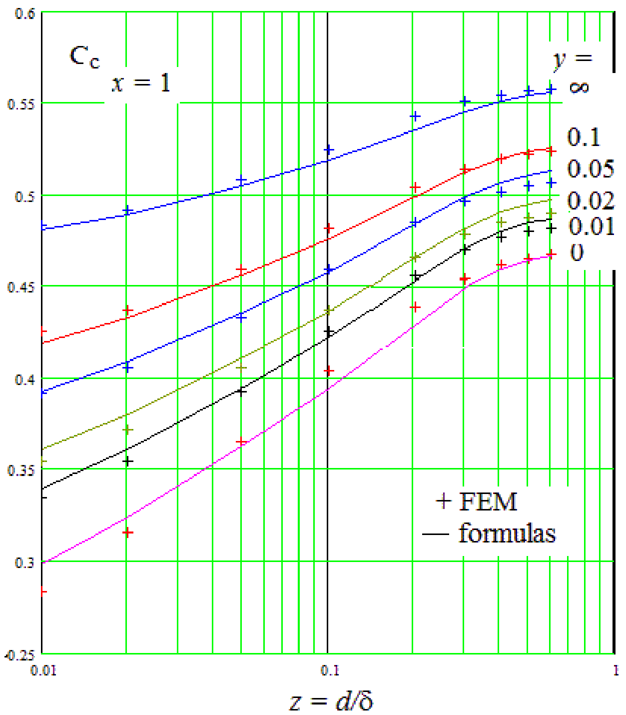


Fig. 7. Geometric corner capacitance for $x = 1$ versus z, y parameter

D. Case $x = 1$ and $y = z < 0.1$

For $x = 1$ ($h = a$) and very small $y = z$ (< 0.1) the domain can be approximated with the region between two coaxial cylinders and the geometric corner capacitance can be approximated with the simple equation (13):

$$C_c|_{\delta=a;h=d} \approx \frac{\pi}{2}\left[\frac{1}{\ln\left(1+\frac{\delta}{d}\right)} - \frac{d}{\delta}\right]; \quad h=d \ll \delta=a \tag{13}$$

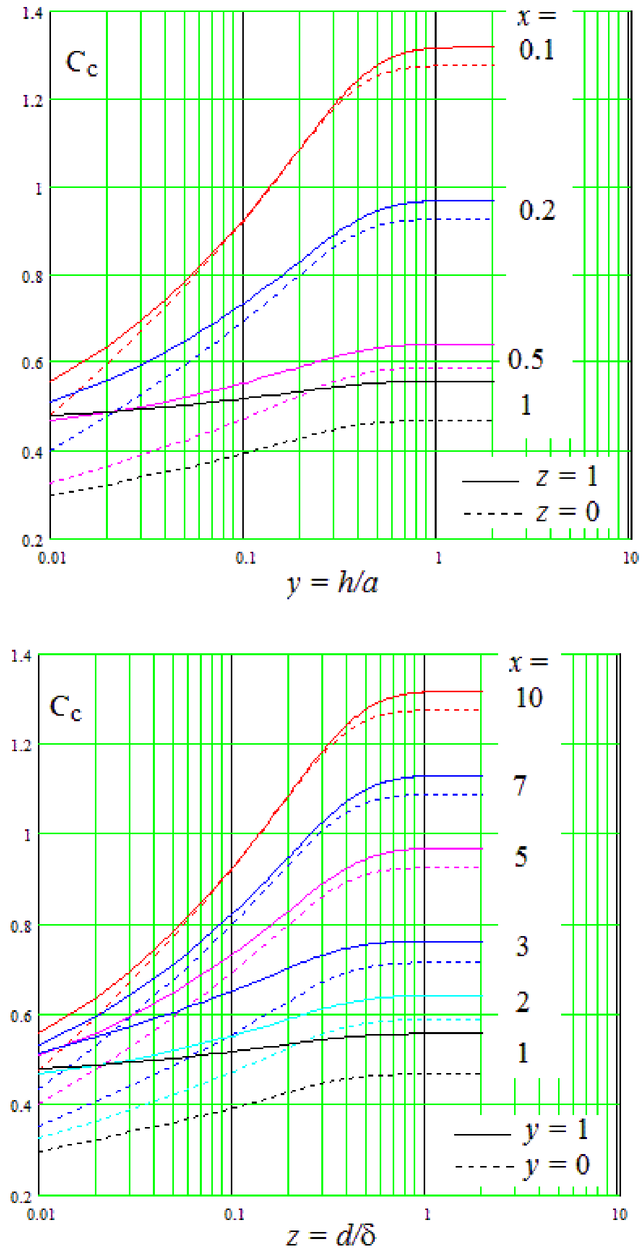


Fig. 8. Geometric corner capacitance for both short arms (12), (11) versus y and z, z parameter

In Fig. 9 the values given by (13) are compared with FEM results. It can be seen that for $y = z < 0.1$ the discrepancies are negligible.

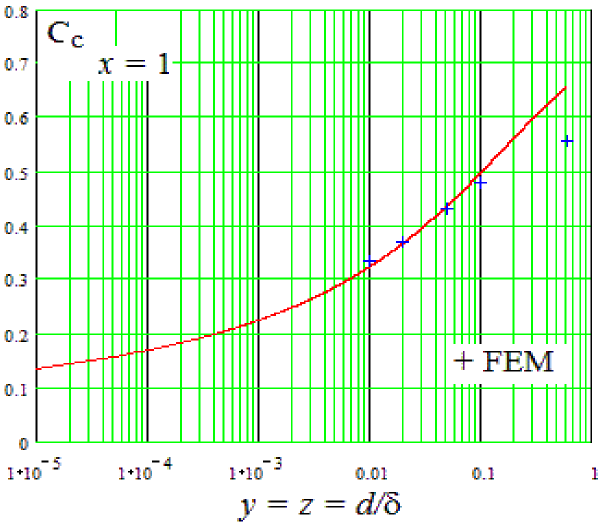
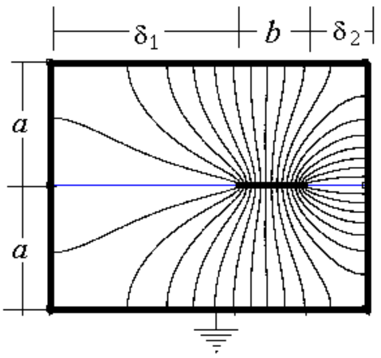


Fig. 9. Geometric corner capacitance for $c = 1$ and very small $y = z$

IV. Examples

The capacitance between the strip and grounded parallel rectangular envelop Fig. 10.



$a = 39$
$b = 21$
$\delta_1 = 60$
$\delta_2 = 19$

$x_1 = a/\delta_1 = 0.65$
$x_2 = a/\delta_2 = 2.053$
$y_1 = y_2 = 0$
$z \approx b/2/a = 0.269$

Fig. 10 The electric field between strip and envelop

Using the approximate eq. (10) the following values are obtained for the corner geometric capacitances:

$$C_c\big|_{h=0}(x_1, z) \approx 0.424 \quad C_c\big|_{h=0}(x_2, z) \approx 0.558 \tag{14}$$

Applying (1) the capacitance results:

$$C == 2\varepsilon\left(\frac{b}{a} + 0.424 + 0.558\right) = 3.041\varepsilon \ \ [\text{F/m}] \tag{15}$$

where ε is the absolute permittivity of the media. With FEM was obtained 3.072 ε or 1% more.

In [4] an unsuitable equation is used for this example and the result is 4.224 ε or 37.5 % larger than the FEM result.

If the slab thickness is 2 c = 20 and the envelope height 2(a+c), the values of y will change as follows

$$y_1 = \frac{c}{\delta_1} = 0.167 \qquad y_2 = \frac{c}{\delta_2} = 0.526 \tag{16}$$

and from the eq. (11) result the corner capacitances:

$$C_{c1}(x_1, y_1, z) \approx 0.546 \quad C_{c2}(x_2, y_2, z) \approx 0.613 \tag{17}$$

The capacitance of the 2 c thick slab will be:

$$C == 2\varepsilon\left(\frac{b}{a} + \frac{c}{\delta_1} + \frac{c}{\delta_2} + 0.546 + 0.613\right) = 4.782\,\varepsilon\,[\text{F/m}] \tag{18}$$

A 475000 nodes FEM result is 4.81ε, or 0.6 % more.

Other application of equation (3) can be found in [5] and [6].

V. Conclusions

1. The partial geometric capacitance (permeance or conductance) of the 2D domain from fig. 1a) is proposed to evaluate with equation (1), using the “corner” capacitance.

2. The corner capacitance can be easy evaluated with simple exact equations (3) and (5) for long arms, approximate equations (9), (10) for one short arm and (11), (12) for both short arms.

3. In practice, the values of y and z higher than 1 can be considered equal to infinity.

4. The 2D corner geometric capacitance can be considered also corner magnetic permeance or corner electric conductance.

5. With these formulas the problem of rectangular “corner” parameters is completely solved. The corner capacitance for arbitrary angle can be found in [7] and [8].

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