Topic: Leading and Lagging Power Factor Operation

8-13 (Solved Problem) A 1.0MW/575V/60Hz/2160rpm DFIG is used in wind energy conversion system. The parameters of the generator are given in Table B-6 of Appendix B. Generator operates with an MPPT scheme and its stator power factor is 0.95 leading. Determine following when DFIG operates at super-synchronous speed of 2160 rpm:

a) generator mechanical torque and power,

b) rms stator current using simplified expression,

c) rms magnetizing voltage and current,

d) rms rotor current and voltage,

e) equivalent resistance and reactance for the rotor side converter, and

f) maximum torque and the corresponding slip.

Generator Type	DFIG, 1.0MW/575V/60Hz	
Rated Mechanical Power	1.0 MW	1.0 pu
Rated Stator Line-to-line Voltage	575 V (rms)	
Rated Stator Phase Voltage	331.98 V (rms)	1.0 pu
Rated Rotor Phase Voltage	67.97 V (rms)	0.2047 pu
Rated Stator Current	829.2 A (rms)	0.8258 pu
Rated Rotor Current	882.2 A (rms)	0.8786 pu
Rated Stator Frequency	60 Hz	1.0 pu
Rated Rotor Speed	2160 rpm	1.0 pu
Nominal Rotor Speed Range	1350–2160 rpm	0.625–1.0pu
Rated Slip	-0.2	
Number of Pole Pairs	2	
Rated Mechanical Torque	4.421 kN.m	1.0 pu
Stator Winding Resistance R_s	3.654 mΩ	0.0111 pu
Rotor Winding Resistance R_r	3.569 mΩ	0.0108 pu
Stator Leakage Inductance L_{ls}	0.1304 mH	0.1487 pu
Rotor Leakage Inductance L_{lr}	0.1198 mH	0.1366 pu
Magnetizing Inductance L_m	4.12 mH	4.6978 pu
Base Current $I_B = 1\text{MW}/(\sqrt{3}\times 575\text{V})$	1004.1 A (rms)	1.0 pu
Base Flux Linkage Λ_B	0.8806 Wb (rms)	1.0 pu
Base Impedance Z_B	0.3306 Ω	1.0 pu
Base Inductance L_B	0.877 mH	1.0 pu
Base Capacitance C_B	8022.93 μF	1.0 pu

Solution:
a) The rotor mechanical speed:

$$\omega_m = 2160 \times (2\pi / 60) = 226.19 \text{ rad/sec}$$

The rotor electrical speed:

$$\omega_r = \omega_m \times P = 226.195 \times 2 = 452.39 \text{ rad/sec}$$

The rated rotor mechanical speed:

$$\omega_{m,R} = 2160 \times (2\pi / 60) = 226.19 \text{ rad/sec}$$

The stator frequency:

$$\omega_s = 2\pi \times 60 = 376.99 \text{ rad/sec}$$

The pu rotor speed

$$\omega_{m,\text{pu}} = \omega_m \big/ \omega_{m,R} = 226.19/226.19 = 1.0\,\text{pu}$$

The generator mechanical torque at 1.0 pu rotor speed:

$$T_m = T_{m,R} \times (\omega_{m,\mathrm{pu}})^2 = -4421 \times (1.0)^2 = -4421\,\mathrm{N.m}$$

The rated mechanical power:

$$P_{m,R} = \omega_{m,R} \times T_{m,R} = -226.19 \times 4421 = -1000 \times 10^3 \text{ W}$$

The generator mechanical power at 1.0 pu rotor speed:

$$P_m = P_{m,R} \times (\omega_{m,\text{pu}})^3 = -1000 \times 10^3 \times (1.0)^3 = -1000 \times 10^3 \text{ W}$$

b) The rms stator current using simplified expression:

$$I_s = \frac{|T_m|\omega_s / P}{3V_s \cos\varphi_s} = \frac{4421 \times 2\pi \times 60 / 2}{3 \times (575/\sqrt{3}) \times 0.95} = 880.78\ \mathrm{A\ (rms)}$$

When DFIG operates with a leading power factor, its stator power factor angle is in the range of

$$180° \le \varphi_s \le 270°$$

stator power factor angle for 0.95 leading power factor can then be calculated by

$$\varphi_s = 180° + \cos^{-1}(0.95) = 180° + 18.2° = 198.2°$$

from which the stator current phasor is

$$\bar{I}_s = I_s \angle -\varphi_s = 880.78 \angle -198.2° \text{ A (rms)}$$

c) The magnetizing branch voltage:

$$\begin{aligned}\overline{V}_m &= \overline{V}_s - \overline{I}_s\left(R_s + j\omega_s L_{ls}\right)\\ &= 575/\sqrt{3}\angle 0° - 880.78\angle -198.2° \times (3.654\times 10^{-3} + j120\pi \times 0.1304\times 10^{-3})\\ &= 350.86\angle 6.57° \ \ \text{V (rms)}\end{aligned}$$

Magnetizing current can be calculated by

$$\bar{I}_m = \frac{\bar{V}_m}{j\omega_s L_m} = 225.9\angle -83.43^\circ \ \ \text{A (rms)}$$

d) The rotor current:

$$\bar{I}_r = \bar{I}_s - \bar{I}_m = 880.78\angle -198.2° - 225.9\angle -83.43° = 996.74\angle 149.93° \text{ A (rms)}$$

The rotor voltage:

$$\overline{V}_r = s\overline{V}_m - \bar{I}_r\left(R_r + js\omega_s L_{lr}\right) = 73.29\angle -166.1^\circ \text{ V (rms)}$$

where $s = (\omega_s - \omega_r)/\omega_s = (376.99 - 452.39)/376.99 = -0.2$

e) The equivalent impedance for the rotor side converter is given by

$$\bar{Z}_{eq} = \bar{V}_r / \bar{I}_r = 0.05292 + j0.05105 \ \Omega$$

from which $R_{eq} = 0.05292 \ \Omega$ & $X_{eq} = 0.05105 \ \Omega$

f) The slip at which the maximum torque occurs can be obtained

$$s_{T\max} = \pm\sqrt{\frac{(R_r + R_{eq})^2 + X_{eq}^2}{R_s^2 + (X_{ls} + X_{lr})^2}} = -0.8066$$

($s = +0.8066$ is omitted because of the super-synchronous mode of operation)

The maximum torque:

$$T_{\max} = \frac{1}{2\omega_s / P} \times \frac{3V_s^2}{R_s + \frac{(X_{ls} + X_{lr})X_{eq}}{R_r + R_{eq}} - \sqrt{\left((X_{ls} + X_{lr})^2 + R_s^2\right) \times \left(1 + \frac{X_{eq}^2}{(R_r + R_{eq})^2}\right)}} = -22876\,\mathrm{N.m} \quad (5.17\ \mathrm{pu})$$

Cross Check:

$$T_m = \frac{1}{\omega_s / P} \times 3I_r^2 \left(R_{eq} + R_r\right)/s = \frac{1}{2\pi \times 60/2} \times 3 \times 996.74^2 \left(0.05292 + 3.569 \times 10^{-3}\right)/\left(-0.2\right) = -4466.12 \text{ N.m}$$

$$P_m = 3I_r^2 \left(R_{eq} + R_r\right)\left(1-s\right)/s = 3 \times 996.74^2 \left(0.05292 + 3.569 \times 10^{-3}\right)\left(1+0.2\right)/\left(-0.2\right) = -1010.2 \times 10^3 \text{ W}$$

8-14 Repeat Problem 8-13 when the DFIG operates at sub-synchronous speed of 1350 rpm and stator power factor of 0.95 lagging.

Answers:

a) $T_m = -1726.9\ \text{N.m}$, $P_m = -244.1\times10^3\ \text{W}$

b) $\bar{I}_s = 344.1\angle -161.8°\ \text{A (rms)}$

c) $\bar{V}_m = 328.3\angle 2.87°\ \text{V (rms)}$, $\bar{I}_m = 211.37\angle -87.13°\ \text{A (rms)}$

d) $\bar{I}_r = 353.02\angle 162.92°\ \text{A (rms)}$, $\bar{V}_r = 84.69\angle 5.12°\ \text{V (rms)}$

e) $R_{eq} = -0.2221\ \Omega$, $X_{eq} = -0.0906\ \Omega$

f) $s_{T\max} = 2.5064$, $T_{\max} = -14759\ \text{N.m}\ (3.338\ \text{pu})$

8-15 Repeat Problem 8-13 when the DFIG operates at sub-synchronous speed of 1780 rpm and stator power factor of 0.9 lagging.

<u>**Answers:**</u>

a) $T_m = -3002.3$ N.m, $P_m = -559.6 \times 10^3$ W

b) $\bar{I}_s = 631.37\angle -154.16°$ A (rms)

c) $\bar{V}_m = 321.83\angle 5.2°$ V (rms), $\bar{I}_m = 207.2\angle -84.84°$ A (rms)

d) $\bar{I}_r = 590.89\angle -173.3°$ A (rms), $\bar{V}_r = 5.69\angle 8.72°$ V (rms)

e) $R_{eq} = -0.00962\ \Omega$, $X_{eq} = -0.00034\ \Omega$

f) $s_{T\max} = 0.0642$, $T_{\max} = -10247$ N.m (2.3178 pu)

0.95 lagging power factor operation for the DFIG WECS

8-16 (Solved Problem) Consider a 1.5MW/690V/50Hz/1750 rpm DFIG WECS. Parameters of the generator are given in Table B-5 of Appendix B. The generator operates with an MPPT scheme. At a given wind and generator speed, the stator active power *Ps* is found to be –1008.616 kW. Calculate the following assuming **0.95 lagging power factor** operation for the DFIG WECS:

a) the rms stator current,

b) the generator mechanical torque and power,

c) the rotor mechanical and electrical speeds and slip,

d) the rms magnetizing voltage and current,

e) the rms rotor current and voltage, and

f) the equivalent resistance and reactance for the rotor side converter.

Generator Type	DFIG, 1.5MW/690V/50Hz	
Rated Mechanical Power	1.5 MW	1.0 pu
Rated Stator Line-to-line Voltage	690 V (rms)	
Rated Stator Phase Voltage	398.4 V (rms)	1.0 pu
Rated Rotor Phase Voltage	67.97 V (rms)	0.1706 pu
Rated Stator Current	1068.2 A (rms)	0.8511 pu
Rated Rotor Current	1125.6 A (rms)	0.8968 pu
Rated Stator Frequency	50 Hz	1.0 pu
Rated Rotor Speed	1750 rpm	1.0pu
Nominal Rotor Speed Range	1200–1750 rpm	0.686–1.0pu
Rated Slip	-0.1667	
Number of Pole Pairs	2	
Rated Mechanical Torque	8.185 kN.m	1.0 pu
Stator Winding Resistance R_s	2.65 mΩ	0.0084 pu
Rotor Winding Resistance R_r	2.63 mΩ	0.0083 pu
Stator Leakage Inductance L_{ls}	0.1687 mH	0.167 pu
Rotor Leakage Inductance L_{lr}	0.1337 mH	0.1323 pu
Magnetizing Inductance L_m	5.4749 mH	5.419 pu
Base Current $I_B = 1.5\text{MW}/(\sqrt{3}\times 690\text{V})$	1255.1 A (rms)	1.0 pu
Base Flux Linkage Λ_B	1.2681 Wb (rms)	1.0 pu
Base Impedance Z_B	0.3174 Ω	1.0 pu
Base Inductance L_B	1.0103 mH	1.0 pu
Base Capacitance C_B	10028.7 μF	1.0 pu

Solution:

a) The rms stator can be calculated by

$$I_s = \left|P_s\right| \Big/ (3V_s \cos\varphi_s) = 1008.6 \times 10^3 \Big/ (3 \times 690/\sqrt{3} \times 0.95) = 888.37 \text{ A (rms)}$$

When the DFIG operates with a lagging power factor, its stator power factor angle is in the range of

$$90^\circ \le \varphi_s \le 180^\circ$$

stator power factor angle for 0.95 lagging power factor can then be calculated by

$$\varphi_s = 180^\circ - \cos^{-1}(0.95) = 180^\circ - 18.2^\circ = 161.8^\circ$$

from which the stator current phasor is

$$\bar{I}_s = I_s \angle -\varphi_s = 888.37\angle -161.8° \text{ A (rms)}$$

b) The generator air-gap power can be given by

$$\frac{\omega_s T_m}{P} = 3V_s I_s \cos\varphi_s = P_s$$

from which generator mechanical torque can be obtained as

$$T_m = P_s \times (P / \omega_s) = -6421.05 \text{ N.m}$$

The generator mechanical torque can be related to pu rotor speed as

$$T_m = T_{m,R} \times (\omega_{m,\mathrm{pu}})^2 \ \mathrm{N.m}$$

from which pu rotor speed can be calculated by

$$\omega_{m,\mathrm{pu}} = \sqrt{\frac{T_m}{T_{m,R}}} = \sqrt{\frac{-6421.05}{-8185}} = 0.8857$$

The rated mechanical power:

$$P_{m,R} = \omega_{m,R} \times T_{m,R} = 1750(2\pi)/60 \times (-8185) = -1500 \times 10^{3} \text{ W}$$

The generator mechanical power at 0.8857 pu rotor speed:

$$P_m = P_{m,R} \times (\omega_{m,\mathrm{pu}})^3 = -1500 \times 10^3 \times (0.8857)^3 = -1042.24 \times 10^3 \text{ W}$$

c) The rotor mechanical and electrical speeds:

$$\omega_m = \omega_{m,R} \times \omega_{m,\text{pu}} = 1750(2\pi)/60 \times 0.8857 = 162.32 \text{ rad/sec} \quad (1550 \text{ rpm})$$

$$\omega_r = \omega_m \times P = 162.32 \times 2 = 324.64 \text{ rad/sec}$$

The slip can be calculates as:

$$s = (\omega_s - \omega_r)/\omega_s = (314.16 - 324.64)/314.16 = -0.0333$$

d) The magnetizing branch voltage:

$$\overline{V}_m = \overline{V}_s - \bar{I}_s\left(R_s + j\omega_s L_{ls}\right)$$
$$= 690/\sqrt{3}\angle 0° - 888.37\angle -161.8° \times (2.65\times 10^{-3} + j100\pi \times 0.1687\times 10^{-3})$$
$$= 388.58\angle 6.72° \text{ V (rms)}$$

The magnetizing current can be calculated by

$$\bar{I}_m = \frac{\bar{V}_m}{j\omega_s L_m} = 225.92\angle -83.28^\circ \text{ A (rms)}$$

e) The rotor current:

$$\bar{I}_r = \bar{I}_s - \bar{I}_m = 888.37\angle -161.8^\circ - 225.92\angle -83.28^\circ = 871.99\angle -176.5^\circ \text{ A (rms)}$$

The rotor voltage:

$$\overline{V}_r = s\overline{V}_m - \overline{I}_r\left(R_r + js\omega_s L_{lr}\right) = 10.82\angle -166.1^\circ \text{ V (rms)}$$

f) The equivalent impedance for the rotor side converter is given by

$$\overline{Z}_{eq} = \overline{V}_r / \overline{I}_r = 0.01220 + j0.00224 \ \Omega$$

from which $R_{eq} = 0.01220 \ \Omega$ & $X_{eq} = 0.00224 \ \Omega$

Cross Check:

$$T_m = \frac{1}{\omega_s / P} \times 3 I_r^2 \left(R_{eq} + R_r\right) / s = \frac{1}{2\pi \times 50 / 2} \times 3 \times 871.99^2 \left(0.01220 + 2.63 \times 10^{-3}\right) / (-0.0333) = -6461 \text{ N.m}$$

$$P_m = 3 I_r^2 \left(R_{eq} + R_r\right)(1 - s)/s = 3 \times 871.99^2 \left(0.01220 + 2.63 \times 10^{-3}\right)(1 + 0.0333)/(-0.0333) = -1048.72 \times 10^3 \text{ W}$$

$$P_s = 3 V_s I_s \cos\varphi_s = 3 \times 690 / \sqrt{3} \times 888.37 \times \cos(161.8^{\circ}) = -1008.616 \times 10^3 \text{ W, verified.}$$

8-17 Repeat Problem 8-16 if the stator active power *Ps* is –1285.7 kW. Assume stator power factor as 0.95 leading.

<u>**Answers:**</u>

a) $\bar{I}_s = 1132.4\angle -198.2^\circ$ A (rms)

b) $T_m = -8185$ N.m , $P_m = -1500\times 10^3$ W

c) $\omega_m = 183.26$ rad/sec (1750 rpm), $\omega_r = 366.52$ rad/sec , $s = -0.1667$

d) $\bar{V}_m = 423.7\angle 7.61^\circ$ V (rms) , $\bar{I}_m = 246.33\angle -82.4^\circ$ A (rms)

e) $\bar{I}_r = 1259.3\angle 151.7^\circ$ A (rms) , $\bar{V}_r = 73.67\angle -165.3^\circ$ V (rms)

f) $R_{eq} = 0.04277\ \Omega$, $X_{eq} = 0.03992\ \Omega$

8-18 Repeat problem 8-16 if the stator active power *Ps* is –604.54 kW. Assume stator power factor as 0.8 leading.

Answers:

a) $\bar{I}_s = 632.3\angle -216.87°$ A (rms)

b) $T_m = -3848.6$ N.m , $P_m = -483.63 \times 10^3$ W

c) $\omega_m = 125.66$ rad/sec (1200 rpm), $\omega_r = 251.32$ rad/sec , $s = 0.2$

d) $\bar{V}_m = 420.61\angle 3.52°$ V (rms) , $\bar{I}_m = 244.54\angle -86.48°$ A (rms)

e) $\bar{I}_r = 812.4\angle 129.88°$ A (rms) , $\bar{V}_r = 90.92\angle 4.98°$ V (rms)

f) $R_{eq} = -0.06402\ \Omega$, $X_{eq} = -0.09179\ \Omega$