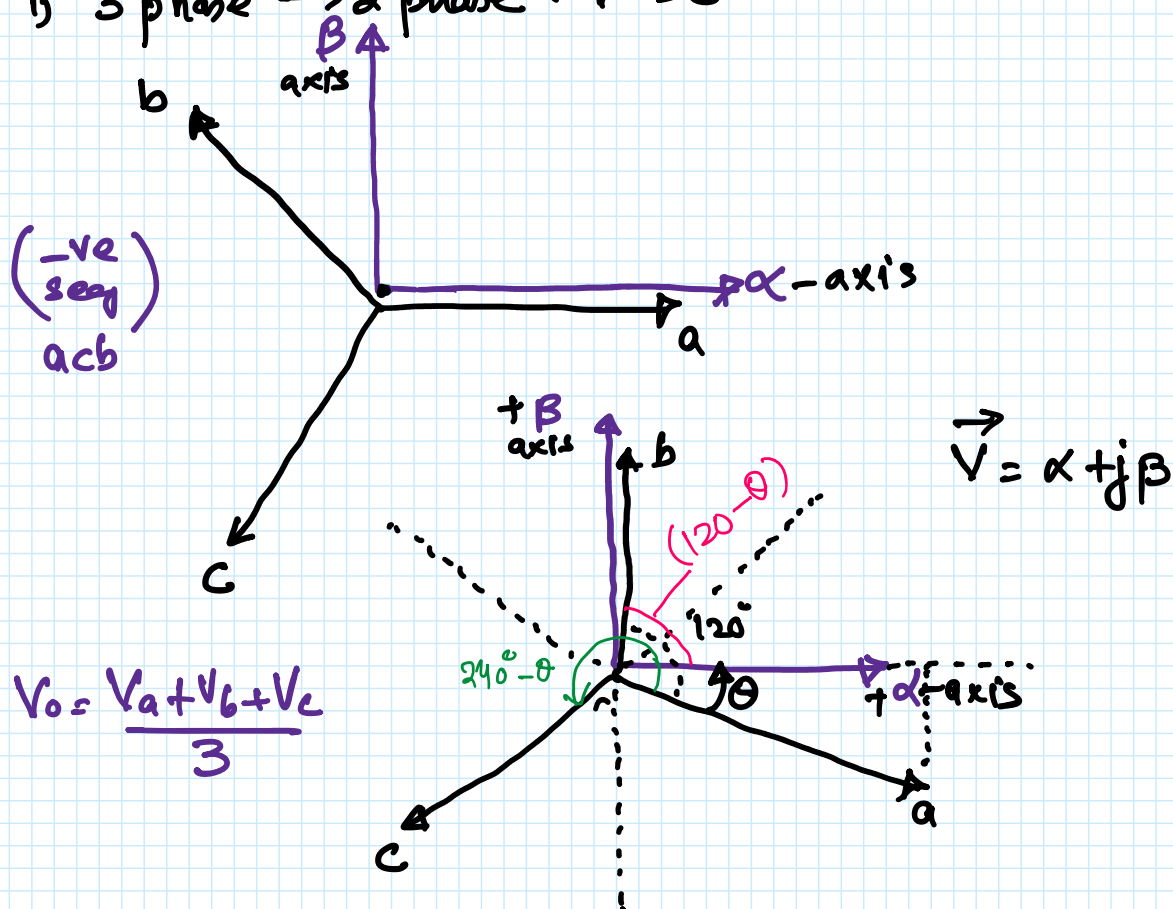


ij 3 phase  $\rightarrow$  2 phase  $\rightarrow$  DC



$$\alpha = a \cos \theta + b \cos (120^\circ - \theta) + c \cos (240^\circ - \theta)$$

$$\beta = a \sin \theta + b \sin (120^\circ - \theta) + c \sin (240^\circ - \theta)$$

$$0 = \frac{a}{3} + \frac{b}{3} + \frac{c}{3}$$

$$\begin{bmatrix} \alpha \\ \beta \\ 0 \end{bmatrix} = K \begin{bmatrix} \cos \theta & \cos (120^\circ - \theta) & \cos (240^\circ - \theta) \\ \sin \theta & \sin (120^\circ - \theta) & \sin (240^\circ - \theta) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

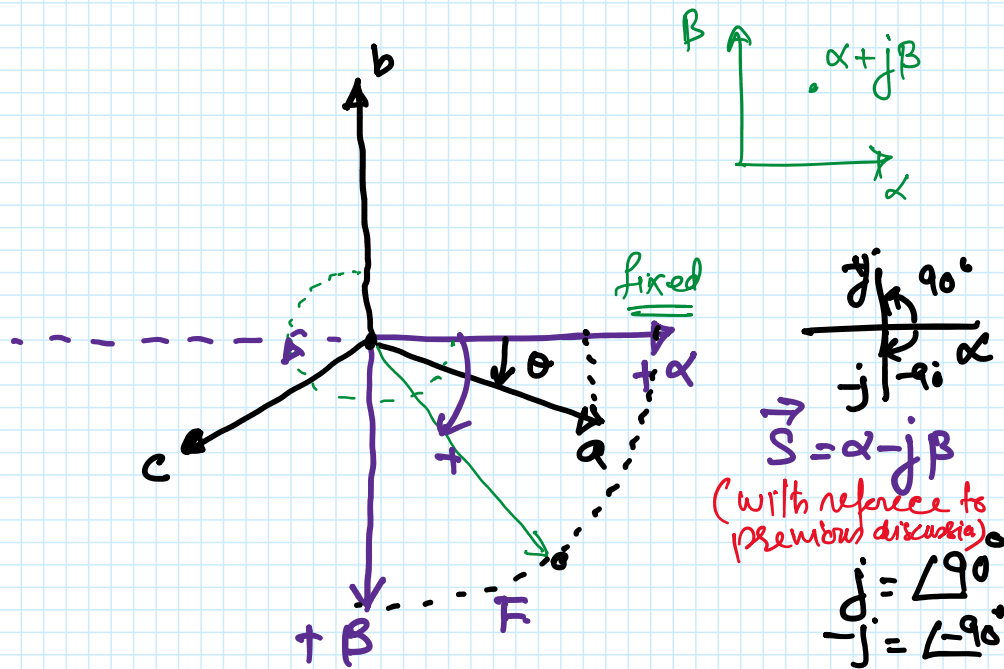
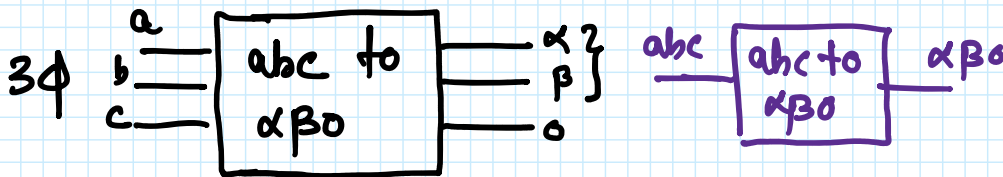
$K = 2/3$  [Tabc]

$$\begin{bmatrix} \alpha \\ \beta \\ 0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$[0] = \begin{bmatrix} \bar{1/2} & \bar{1/2} & \bar{1/2} \end{bmatrix} [c]$$

$$\begin{bmatrix} \alpha \\ \beta \\ 0 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ 0 & 1/\sqrt{3} & -1/\sqrt{3} \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Tabc | 0 = 0  $\Rightarrow$   $\alpha$ -axis is aligned with phase a



$$\alpha = a \cos \theta + b \cos(\theta + 240^\circ) + c \cos(\theta + 120^\circ)$$

$$\beta = a \sin \theta + b \sin(\theta + 240^\circ) + c \sin(\theta + 120^\circ)$$

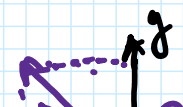
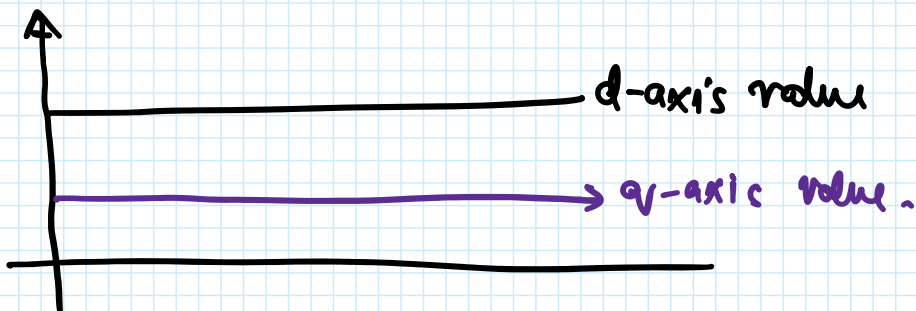
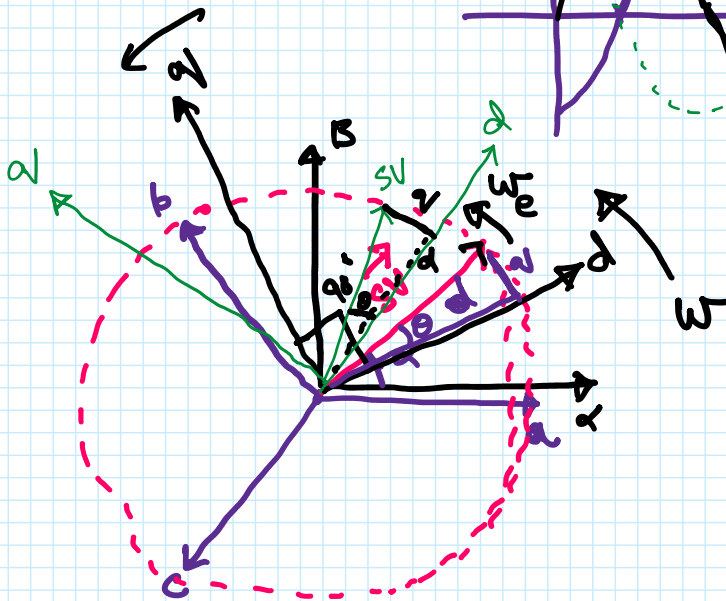
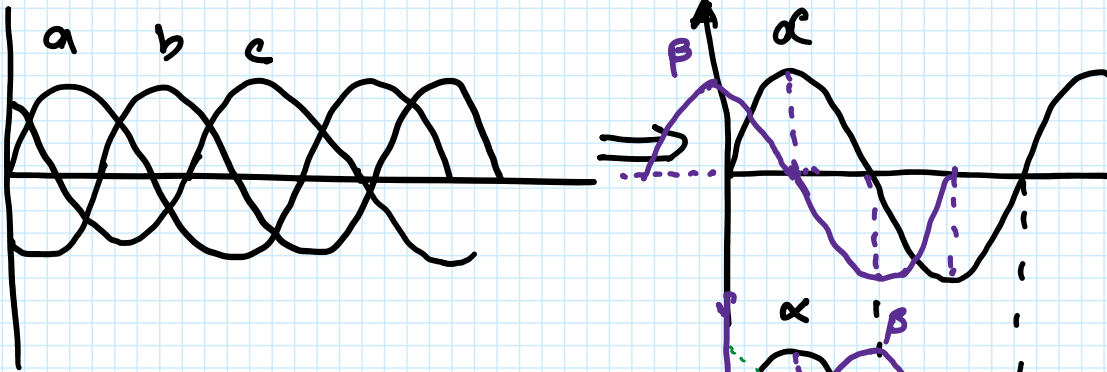
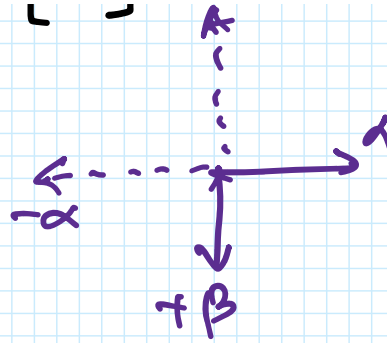
$$\begin{bmatrix} \alpha \\ \beta \\ 0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & -\sqrt{3}/2 & +\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$\alpha$   $\beta$

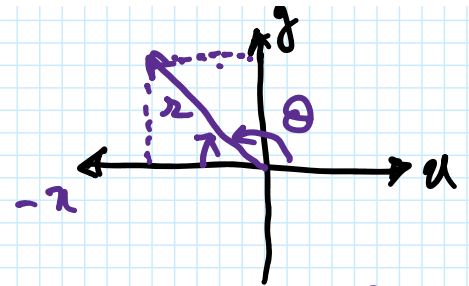
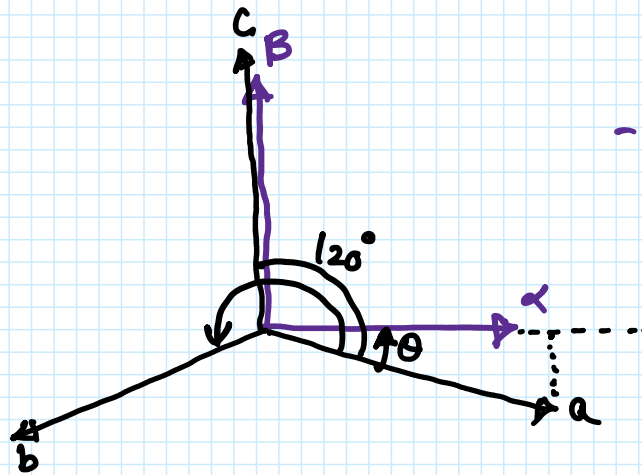
$$[T_{abc}]$$

$$[\alpha \beta 0] = [T_{abc}] [abc]$$

$$[abc] = [T_{abc}]^T [\alpha \beta 0]$$



[+ve seq  
abc]



$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \\ r &= -r \cos (180^\circ - \theta) \end{aligned}$$

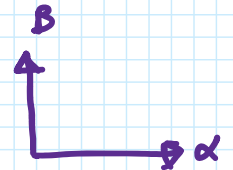
$$\alpha = a \cos \theta + b \cos (240^\circ - \theta) + c \cos (120^\circ - \theta)$$

$$\beta = a \sin \theta + b \sin (240^\circ - \theta) + c \sin (120^\circ - \theta)$$

$$\begin{bmatrix} \alpha \\ \beta \\ 0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos (240^\circ - \theta) & \cos (120^\circ - \theta) \\ \sin \theta & \sin (240^\circ - \theta) & \sin (120^\circ - \theta) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$\theta = 0 \Rightarrow \alpha$  axis &  $a$ -axis are aligned.

$$\begin{bmatrix} \alpha \\ \beta \\ 0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & -\sqrt{3}/2 & +\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$



+ve seq  
abc [Tabc]

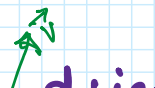
$$V_\alpha = \frac{2}{3} \left[ a - \frac{1}{2} b - \frac{1}{2} c \right] = \frac{2}{3} a - \frac{1}{3} b - \frac{1}{3} c$$

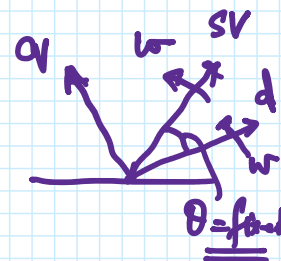
$$V_\beta = \frac{1}{\sqrt{3}} b + \frac{1}{\sqrt{3}} c = \frac{1}{\sqrt{3}} [c - b]$$

$$V_\beta = \frac{1}{\sqrt{3}} b - \frac{1}{\sqrt{3}} c = \frac{1}{\sqrt{3}} [b - c]$$

+ve seq  
abc

-ve seq  
abc





$$d = \alpha \sin \theta + \beta \sin \theta$$

$$q = -\alpha \ln\left(\frac{\pi}{2} - \theta\right) + \beta \ln \theta$$

$$q = -d \sin \theta + b \ln \theta$$

$\alpha \beta \rightarrow d q \nu$

$$d + jq = (\alpha \cos \theta + \beta \sin \theta) + j(-\alpha \sin \theta + \beta \cos \theta)$$

$$= \alpha (\cos \theta - j \sin \theta) + \beta (\sin \theta + j \cos \theta)$$

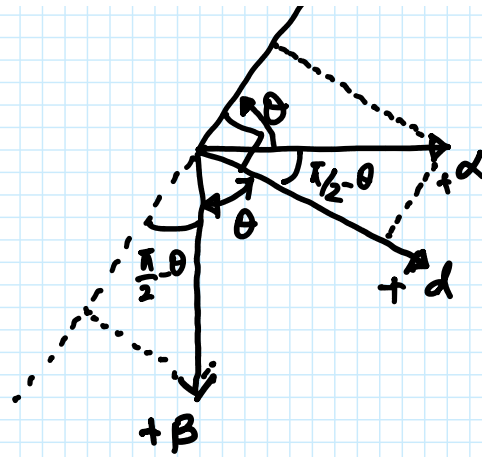
$$= \alpha (\cos \theta - j \sin \theta) + j \beta (\cos \theta - j \sin \theta)$$

$$= (\alpha + j\beta)(\cos\theta - j\sin\theta)$$

$$e^{-j\omega t}$$

$$e^{j\theta} = e^{j\omega t}$$





$$d = \alpha \sin \theta + \beta \cos \theta$$

$$q = \alpha \cos \theta - \beta \sin \theta$$

$$\begin{bmatrix} d \\ q \end{bmatrix} = \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & -\sin \theta \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\begin{aligned} d + jq &= \alpha (\sin \theta + j \cos \theta) + \beta (\cos \theta - j \sin \theta) \\ &= j\alpha (\cos \theta - j \sin \theta) + \beta (\cos \theta - j \sin \theta) \end{aligned}$$

$$\underbrace{d + jq}_{\text{R.F}} = \underbrace{(j\alpha + \beta)}_{\text{S.F}} e^{-j\theta}$$

Example:

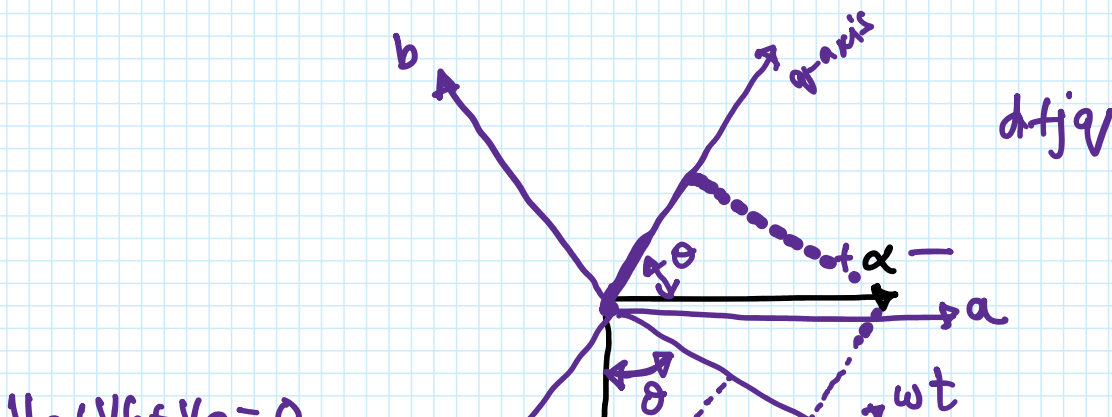
$$V_a = V_m \cos(\omega t + \phi)$$

$$V_b = V_m \cos(\omega t + \phi - 2\pi/3)$$

$$V_c = V_m \cos(\omega t + \phi + 2\pi/3)$$

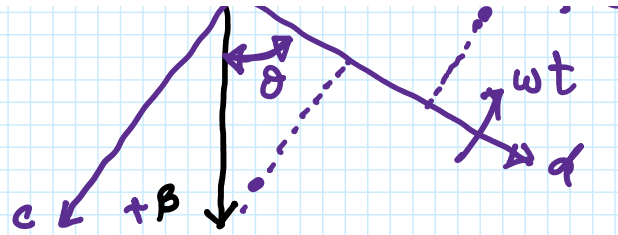
$$\phi = 30^\circ$$

$$\underline{\underline{\phi = 0^\circ}}$$



$$V_a + V_b + V_c = 0$$

$$V_b + V_c = -V_a$$



$$V_\alpha = \frac{2}{3} \left[ V_a - \frac{1}{2} V_b - \frac{1}{2} V_c \right] = V_a$$

$$= \frac{2}{3} \left[ V_a - \frac{1}{2} (V_b + V_c) \right] = \frac{2}{3} \left[ V_a - \frac{1}{2} (-V_a) \right]$$

$$= \frac{2}{3} \left[ V_a + \frac{V_a}{2} \right] = V_a$$

$$V_\alpha = V_m \cos(\omega t + \phi)$$

$$\cos A - \cos B = 2 \sin \frac{A+B}{2} \cdot \sin \frac{B-A}{2}$$

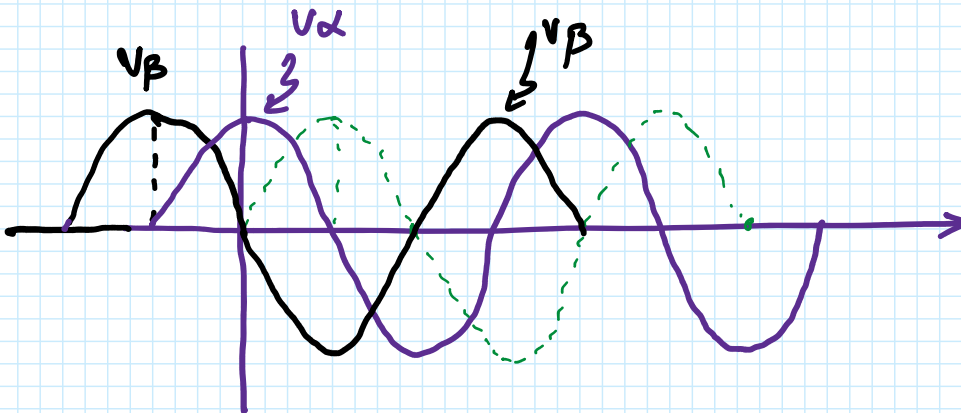
$$V_\beta = \frac{1}{\sqrt{3}} [V_c - V_b]$$

$$= \frac{1}{\sqrt{3}} \left[ V_m \cos(\omega t + \phi + \frac{2\pi}{3}) - V_m \cos(\omega t + \phi - \frac{2\pi}{3}) \right]$$

$$= \frac{1}{\sqrt{3}} \left[ 2 \sin(\omega t + \phi) \sin(-\frac{2\pi}{3}) \right]$$

$$= \frac{1}{\sqrt{3}} \left[ -\frac{\sqrt{3}}{2} \cdot 2 \sin(\omega t + \phi) \right]$$

$$V_\beta = -V_m \sin(\omega t + \phi)$$



$$V_\alpha - jV_\beta = V_m \cos(\omega t + \phi) + jV_m \sin(\omega t + \phi)$$

$$= V_m e^{j(\omega t + \phi)}$$

$$v_{\alpha} - jv_{\beta} = v_m \cos(\omega t + \phi) + jv_m \sin(\omega t + \phi) \\ = v_m e^{j(\omega t + \phi)}$$

$$v_{\alpha} - jv_{\beta} = v_m e^{j\omega t} \cdot e^{j\phi}$$

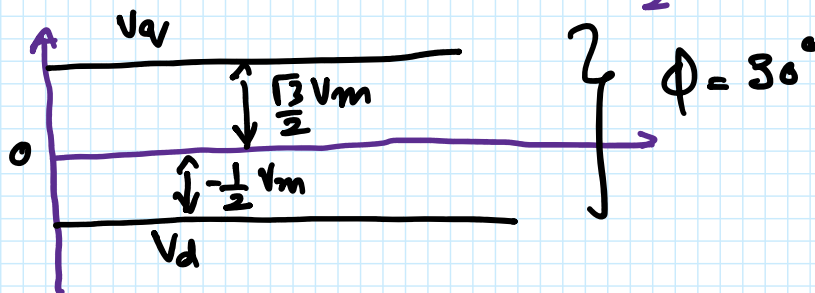
$$v_d + jv_q = v_m \cdot e^{j\omega t} \cdot e^{j\phi} \cdot e^{-j\omega t}$$

$$\begin{cases} v_q = v_{\alpha} \cos \theta - v_{\beta} \sin \theta \\ v_q = v_m \cos(\omega t + \phi) \cdot \cos \theta + v_m \sin(\omega t + \phi) \cdot \sin \theta \\ \boxed{v_q = v_m \cos \phi} \end{cases} \quad \omega t = \theta$$

$$\begin{cases} v_d = v_{\alpha} \sin \theta + v_{\beta} \cos \theta \\ = v_m \cos(\omega t + \phi) \sin \theta - v_m \sin(\omega t + \phi) \cdot \cos \theta \\ = v_m \sin(-\phi) \\ v_d = -v_m \sin \phi \end{cases}$$

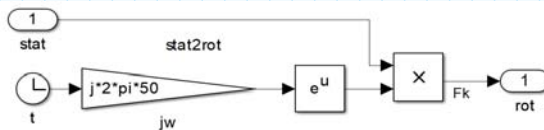
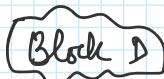
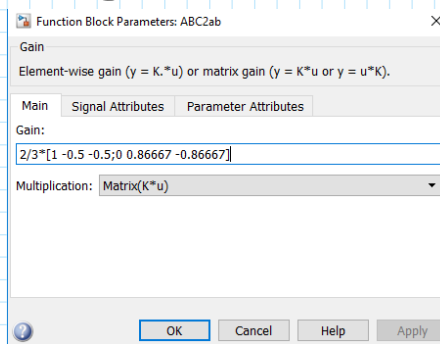
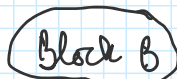
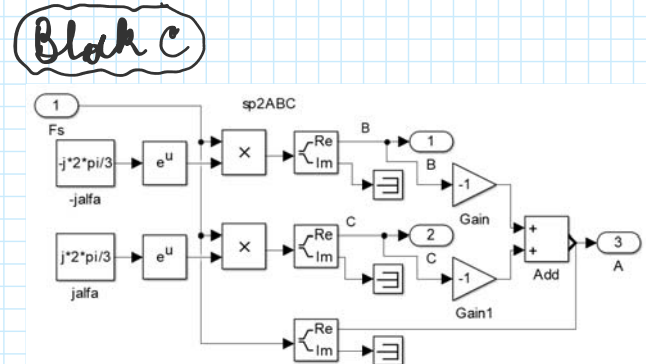
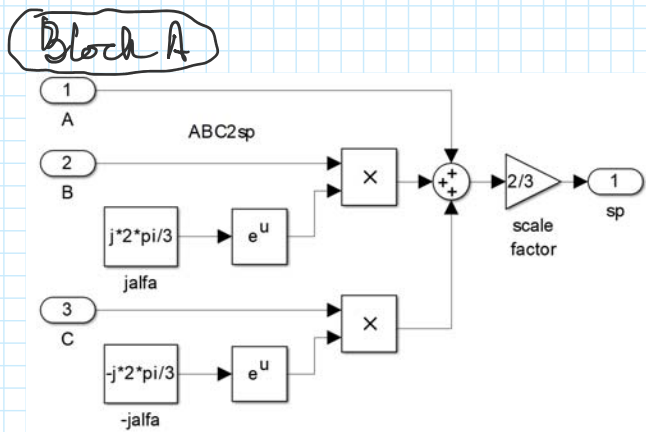
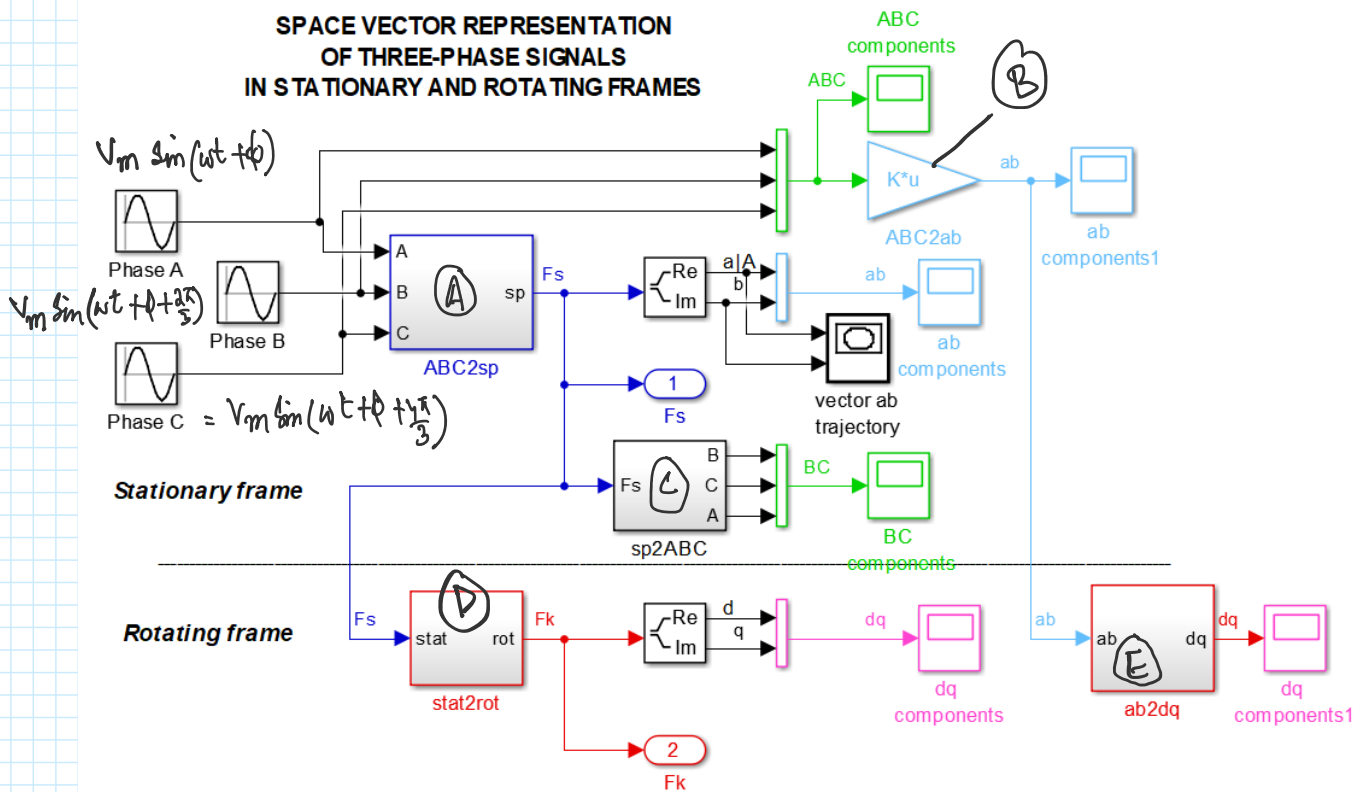
$$\phi = 0^\circ \quad \begin{aligned} v_q &= v_m \\ v_d &= 0 \end{aligned}$$

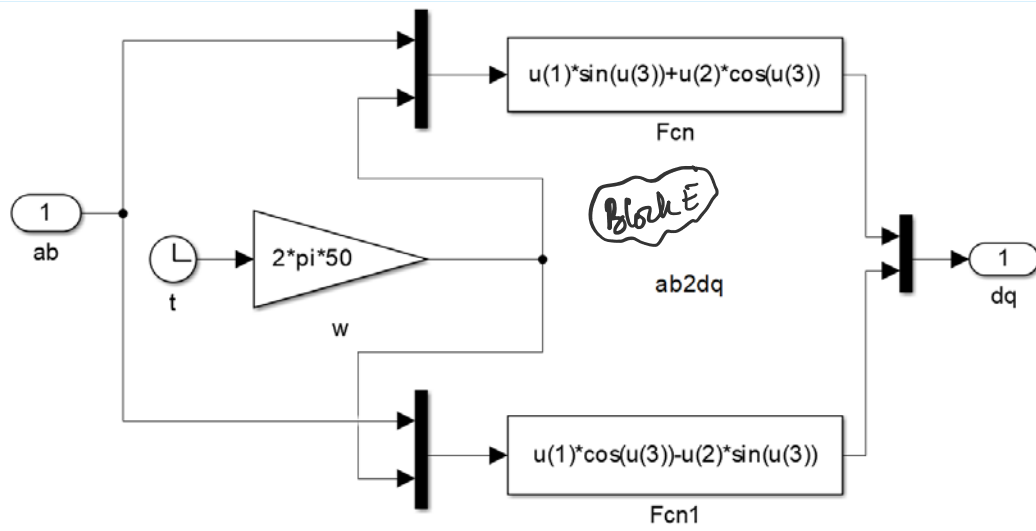
$$\phi = 30^\circ \quad \begin{aligned} v_q &= v_m \cos 30^\circ = v_m \frac{\sqrt{3}}{2} \\ v_d &= v_m \sin 30^\circ = -\frac{v_m}{2} \end{aligned}$$



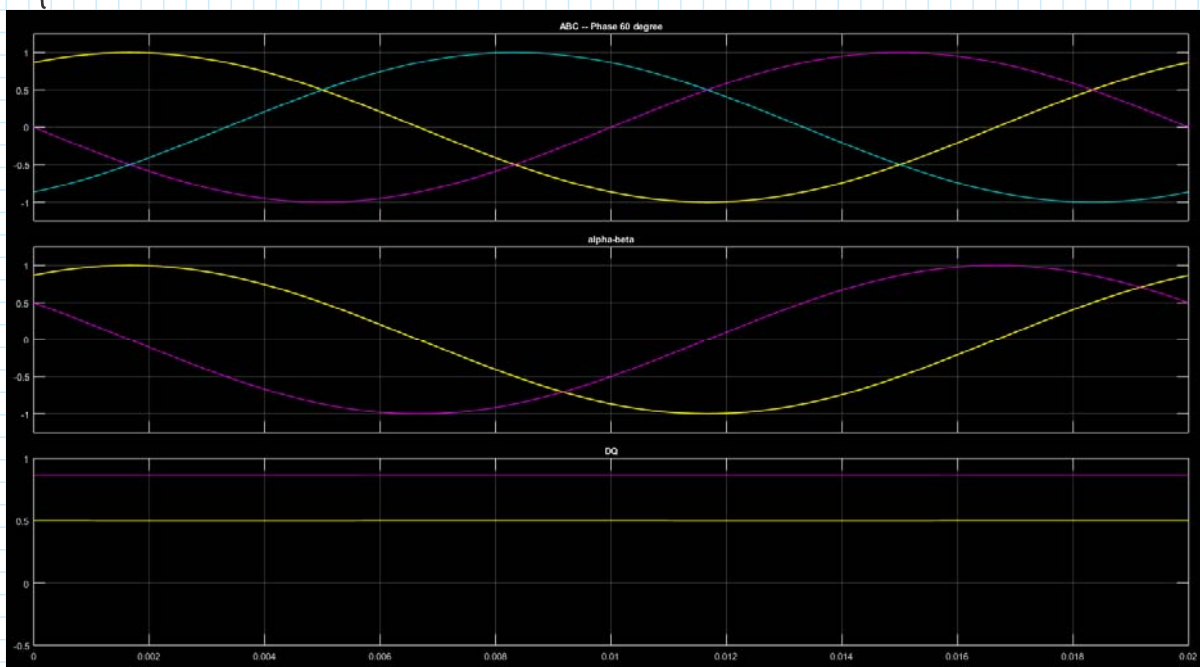


# SPACE VECTOR REPRESENTATION OF THREE-PHASE SIGNALS IN STATIONARY AND ROTATING FRAMES

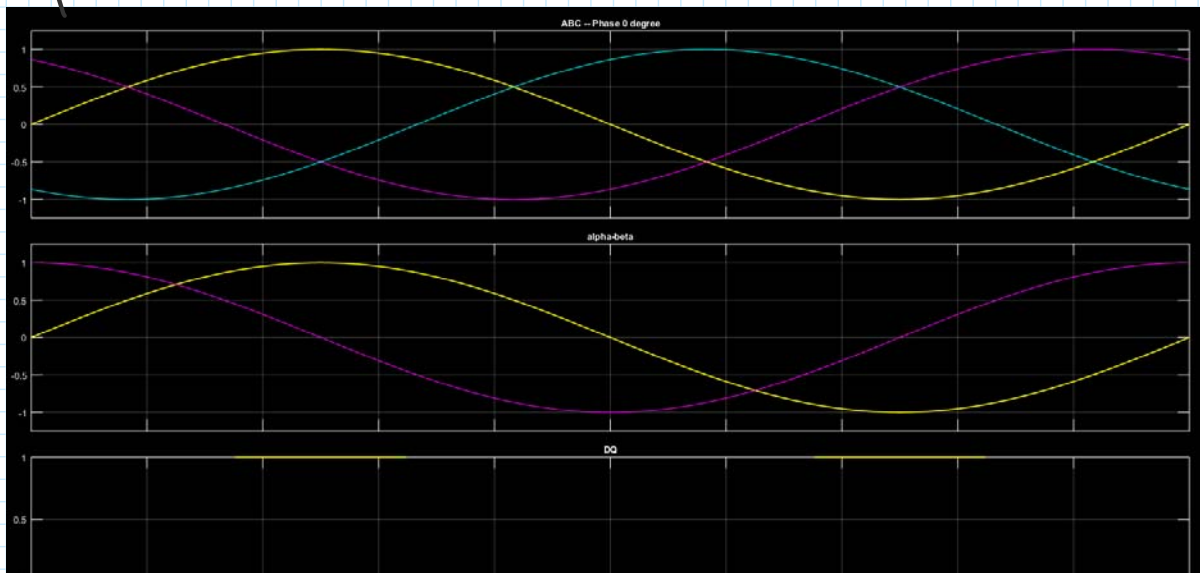


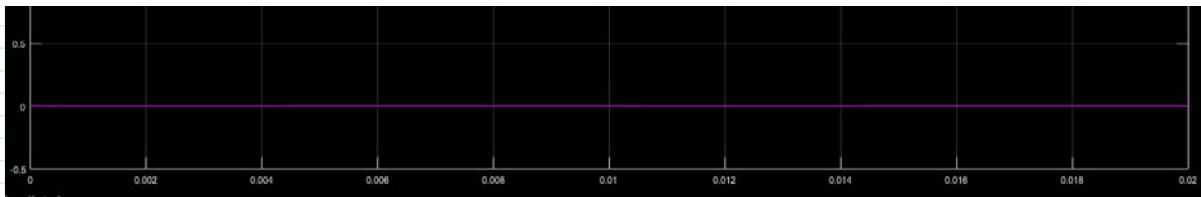


$\phi = 60^\circ$



$\phi = 0^\circ$





$$\phi = 90^\circ$$

