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## **Preface to the Second Edition**

The first edition of this book was published in 1995. We are gratified with its high level of use by both the university and industrial-research communities. It is often the text in senior-year undergraduate and first-year graduate electrical engineering courses in computational electromagnetics, and is also frequently cited in refereed journal papers as a primary background reference for FDTD methods and applications.

We have had two primary goals in creating this second edition. First, we have worked to update the book's discussions of FDTD theory and applications to account for the continuing, rapid changes in these areas since 1995. This allows the professional engineer or scientist to have a convenient single-source reference concerning the latest FDTD techniques and research problems. Second, we have worked to enhance the educational content of the book from both a fundamental theoretical perspective and from the standpoint of the course instructor's ease of use.

### **Coverage of Advances in FDTD Theory and Numerical Algorithms**

Specifically, this second edition contains a large body of new material that discusses in detail the following recent advances in FDTD theory and numerical algorithms:

- Complex-wavenumber theory, which places our understanding of numerical dispersion on rigorous grounds and further provides a complete picture of numerical wave propagation and possible attenuation in the FDTD space lattice;
- Complex-frequency theory, an alternative rigorous approach to numerical stability, which yields key insights into the nature of unstable numerical modes in the space lattice;
- Pseudospectral time-domain algorithms, which permit the spatial discretization to approach the Nyquist limit of two points per wavelength;

- Alternating-direction implicit algorithms, which yield provable, unconditional numerical stability in three dimensions, regardless of the size of the time step;
- Electric and magnetic current-source theory, which yields key insights into the nature of the intrinsic capacitance and inductance of lattice space cells;
- Complementary-operator and concurrent complementary-operator absorbing boundary conditions, which increase the effectiveness of analytical absorbing boundary conditions by orders of magnitude;
- Uniaxial perfectly matched layer absorbing boundary conditions for terminating space lattices containing general materials, including lossy, dispersive, and inhomogeneous dielectrics;
- Piecewise-linear recursive-convolution and simplified auxiliary-differential equation techniques for modeling dispersive and nonlinear dispersive materials;
- Simplified, numerically stable subcell models of diagonal and curved perfect electric conductor surfaces in the space lattice;
- Theory and algorithms for the analysis of periodic structures;
- Padé approximations for simple, rapid, accurate calculation of the resonant frequencies and quality factors of high- $Q$  cavities and similar structures;
- Enhanced discussion of interfacing SPICE electronic circuits models with the FDTD space lattice, including Norton's and Thevenin's equivalent circuits for the lattice.

### **Coverage of Advances in FDTD Modeling Applications**

In addition to theoretical advances, this second edition contains significant new material that discusses in detail the following recent advances in FDTD modeling applications:

- Periodic structures, including antenna arrays, frequency-selective surfaces, and photonic bandgap structures;
- Antennas, including the standard-gain horn, whips (monopoles) mounted on cellphones, radome interactions, ground-penetrating radar, and use of photonic bandgap materials to realize all-dielectric reflectors;
- High-speed electronic circuits, including a multiplane, meshed printed-circuit board feeding power to a multichip module, and a 6-GHz MESFET amplifier analyzed for both linear and nonlinear performance;

- Microcavity optical resonators, including microrings and microdisks for wavelength-division multiplexing, vertical-cavity surface-emitting lasers, and lasers based upon photonic-bandgap structures.

In all, this book provides 57 wide-ranging examples of FDTD modeling applications that:

- Cover the electromagnetic spectrum from radio frequencies to optical frequencies;
- Include the most exciting contemporary applications of electromagnetic wave engineering, ranging from the analysis and design of modern GHz-regime computers and personal wireless communications systems, to advanced photonic devices such as the world's smallest lasers.

These 57 examples serve not only to illustrate the power and beauty of FDTD modeling, but also to inform and excite the reader about the *integral role* that electromagnetic wave phenomena play in the design and operation of our society's most advanced electronics and photonics technologies.

### Enhanced Educational Features

Finally, this second edition provides enhanced educational features including:

- Worked examples of FDTD modeling results contained within the text, which serve as answers to selected homework problems and projects;
- A CD-ROM containing MATLAB<sup>TM</sup> software for one-, two, and three-dimensional FDTD codes which readers can exercise to generate modeling examples of their own choosing. The two-dimensional code has a perfectly matched layer absorbing boundary condition. The CD-ROM also contains a mesh generator in executable form for conformal modeling of a two-dimensional perfectly conducting structure using the Dey-Mitra technique, and a separate FDTD solver incorporating this structure. Finally, the CD-ROM contains images of *all* of the figures and tables in the book as downloadable portable document format (PDF) files.

In our respective teaching experiences at Northwestern University and the University of Wisconsin–Madison, we have found that the second edition provides students with improved prospects for learning and eases the burden on their instructors. When used in a semester-length course (i.e., UW–Madison), there is sufficient time to cover in detail the first seven chapters plus the instructor's choice of an additional three chapters of interest in the remainder of the book. This includes time for the students to learn to write their own working FDTD software in one and two dimensions.



When used in a quarter-length course (i.e., Northwestern), there is sufficient time to cover the first seven chapters plus the instructor's choice of one additional chapter of interest in the remainder of the book. The entirety of the book can be covered in excellent detail in a two-semester course, leading to sufficient student background to begin Ph.D. research efforts in virtually the full range of current FDTD topics.

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