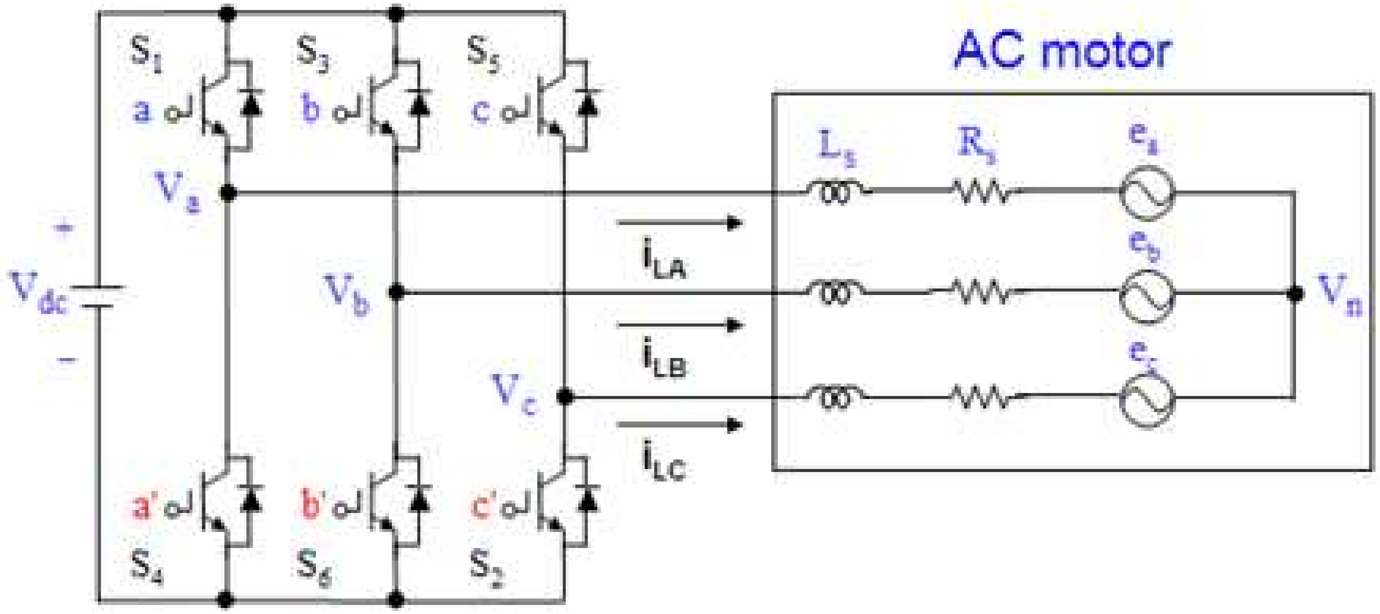
Space Vector PWM

➢ **Output voltages of three-phase inverter (1)**



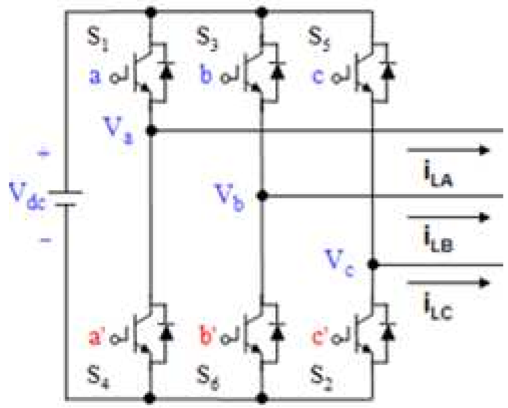
where, upper transistors: S_1, S_3, S_5

lower transistors: S_4, S_6, S_2

switching variable vector: a, b, c

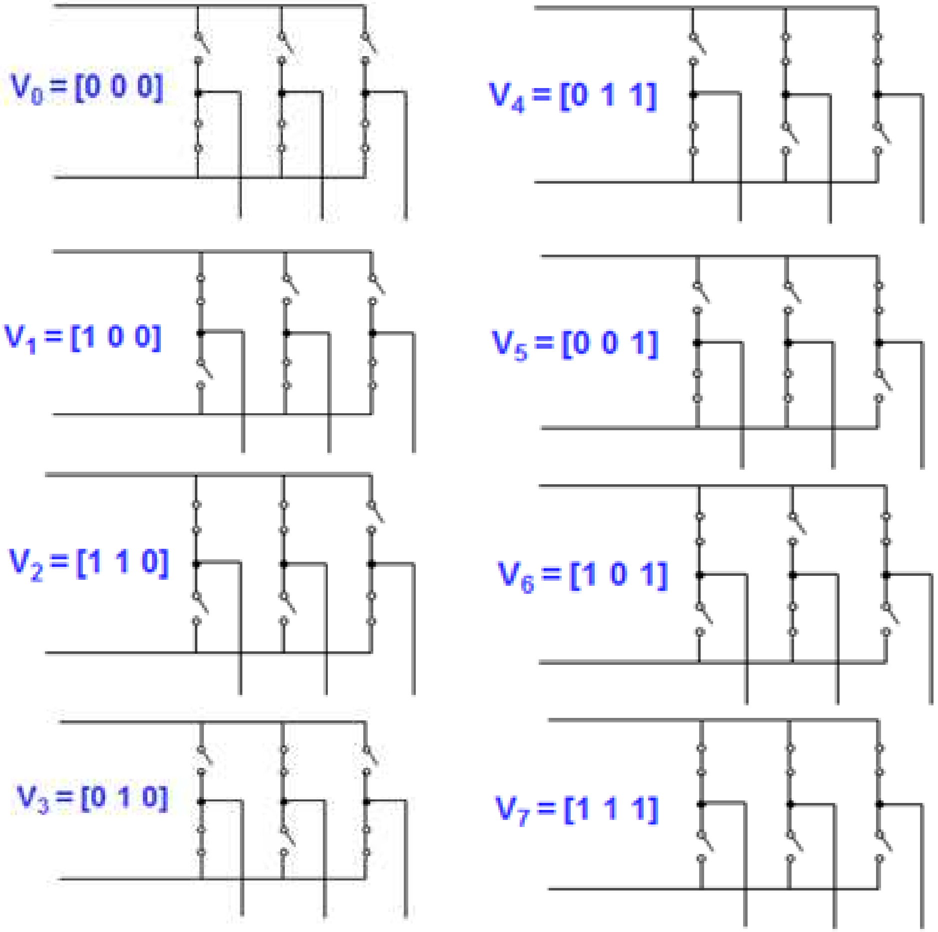
Space Vector PWM

Output voltages of three-phase inverter



- S_1 through S_6 are the six power transistors that shape the output voltage
- When an upper switch is turned on (i.e., a, b or c is "1"), the corresponding lower switch is turned off (i.e., a', b' or c' is "0")

Eight possible combinations of on and off patterns for the three upper transistors (S_1, S_3, S_5)



Voltage Vectors	Switching Vectors			Line to neutral voltage			Line to line voltage		
	a	b	c	V_{an}	V_{bn}	V_{cn}	V_{ab}	V_{bc}	V_{ca}
V_0	0	0	0	0	0	0	0	0	0
V_1	1	0	0	2/3	-1/3	-1/3	1	0	-1
V_2	1	1	0	1/3	1/3	-2/3	0	1	-1
V_3	0	1	0	-1/3	2/3	-1/3	-1	1	0
V_4	0	1	1	-2/3	1/3	1/3	-1	0	1
V_5	0	0	1	-1/3	-1/3	2/3	0	-1	1
V_6	1	0	1	1/3	-2/3	1/3	1	-1	0
V_7	1	1	1	0	0	0	0	0	0

(Note that the respective voltage should be multiplied by V_{dc})

SVPWM

Voltage Vectors	Switching Vectors			Line to neutral voltage			Line to line voltage		
	a	b	c	V_{an}	V_{bn}	V_{cn}	V_{ab}	V_{bc}	V_{ca}
V_0	0	0	0	0	0	0	0	0	0
V_1	1	0	0	2/3	-1/3	-1/3	1	0	-1
V_2	1	1	0	1/3	1/3	-2/3	0	1	-1
V_3	0	1	0	-1/3	2/3	-1/3	-1	1	0
V_4	0	1	1	-2/3	1/3	1/3	-1	0	1
V_5	0	0	1	-1/3	-1/3	2/3	0	-1	1
V_6	1	0	1	1/3	-2/3	1/3	1	-1	0
V_7	1	1	1	0	0	0	0	0	0

(Note that the respective voltage should be multiplied by V_{dc})

- **Line to neutral (phase) voltage vector $[V_{an}\ V_{bn}\ V_{cn}]^t$**

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \frac{1}{3} V_{dc} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

- **Line to line voltage vector $[V_{ab}\ V_{bc}\ V_{ca}]^t$**

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = V_{dc} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad \text{where switching variable vector } [a \quad b \quad c]^t$$

Space Vector PWM

➢ Principle of Space Vector PWM

- Treats the sinusoidal voltage as a constant amplitude vector rotating at constant frequency
- This PWM technique approximates the reference voltage V_{ref} by a combination of the eight switching patterns (V_0 to V_7)
- Coordinate Transformation (abc reference frame to the stationary d-q frame): A three-phase voltage vector is transformed into a vector in the stationary d-q coordinate frame which represents the spatial vector sum of the three-phase voltage
- The vectors (V_1 to V_6) divide the plane into six sectors (each sector: 60 degrees)
- V_{ref} is generated by two adjacent non-zero vectors and two zero vectors

Space Vector PWM

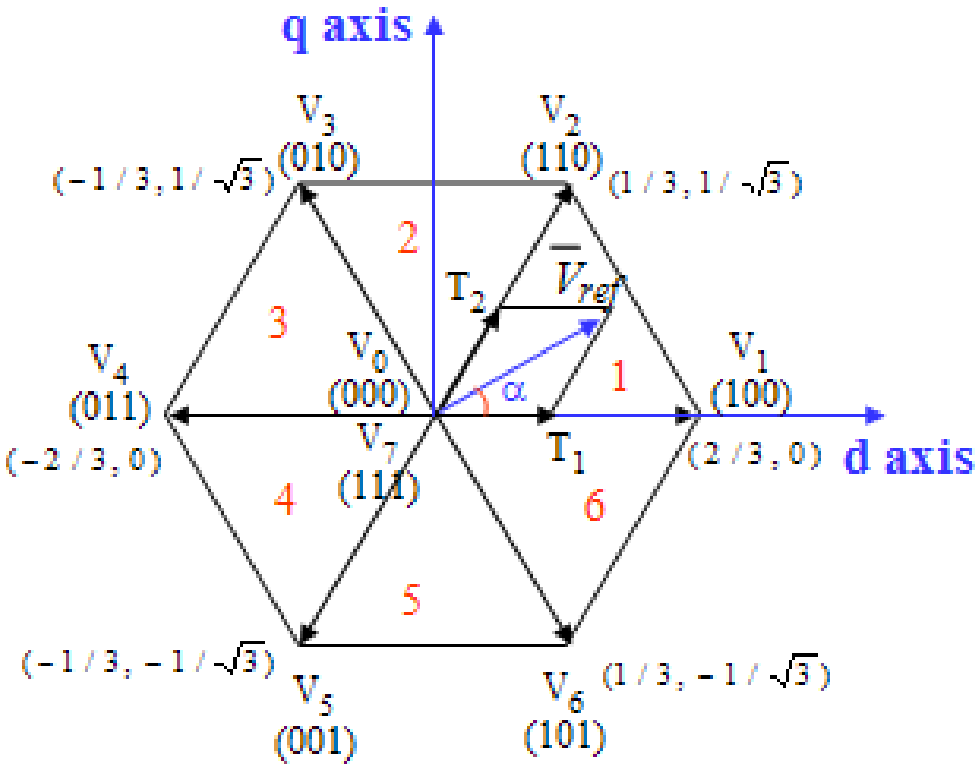
Basic switching vectors and Sectors

- 6 active vectors (V_1,V_2, V_3, V_4, V_5, V_6)

⇨ Axes of a hexagonal

⇨ DC link voltage is supplied to the load

⇨ Each sector (1 to 6): 60 degrees



- 2 zero vectors (V_0, V_7)

⇨ At origin

⇨ No voltage is supplied to the load

Space Vector PWM

Steps for implementation of Space Vector PWM

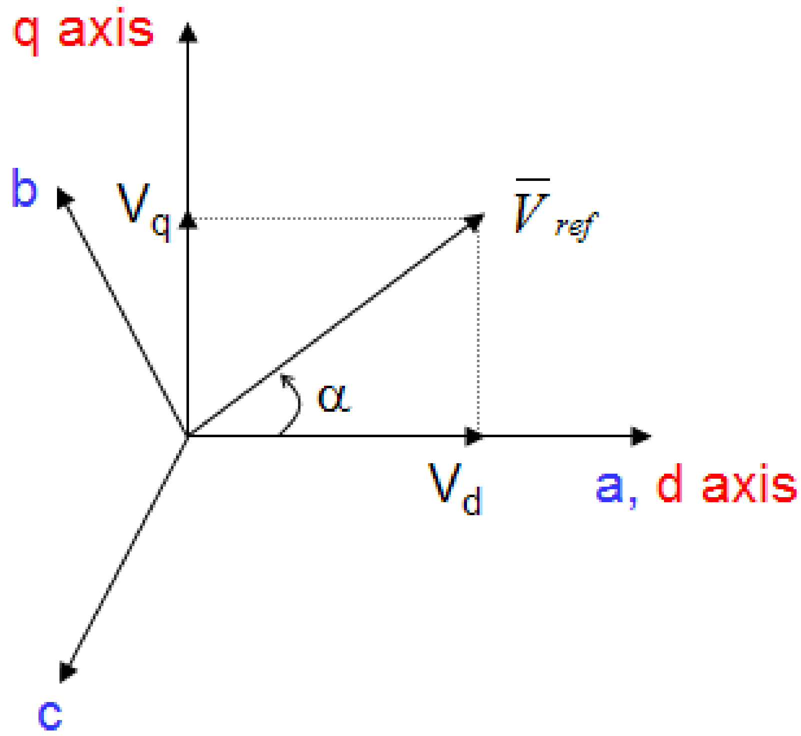
- **Step 1. Determine V_d, V_q, V_{ref}, and angle (α)**
- **Step 2. Determine time duration T_1, T_2, T_0**
- **Step 3. Determine the switching time of each transistor (S_1 to S_6)**

Space Vector PWM

➢ Step 1. Determine V_d, V_q, V_{ref}, and angle (α)

- **Coordinate transformation**

  : abc to dq



Voltage Space Vector and its components in (d, q).

$$V_d = V_{an} - V_{bn} \cdot \cos 60 - V_{cn} \cdot \cos 60$$
$$= V_{an} - \frac{1}{2} V_{bn} - \frac{1}{2} V_{cn}$$

$$V_q = 0 + V_{bn} \cdot \cos 30 - V_{cn} \cdot \cos 30$$
$$= V_{an} + \frac{\sqrt{3}}{2} V_{bn} - \frac{\sqrt{3}}{2} V_{cn}$$

$$\therefore \begin{bmatrix} V_d \\ V_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}$$

$$\left| \overline{V}_{ref} \right| = \sqrt{V_d^{\,2} + V_q^{\,2}}$$

$$\alpha = \tan^{-1}\left(\frac{V_q}{V_d}\right) = \omega_s t = 2\pi f_s t$$

(where f_s = fundamental frequency

Space Vector PWM

- **Step 2. Determine time duration T_1, T_2, T_0 (1)**

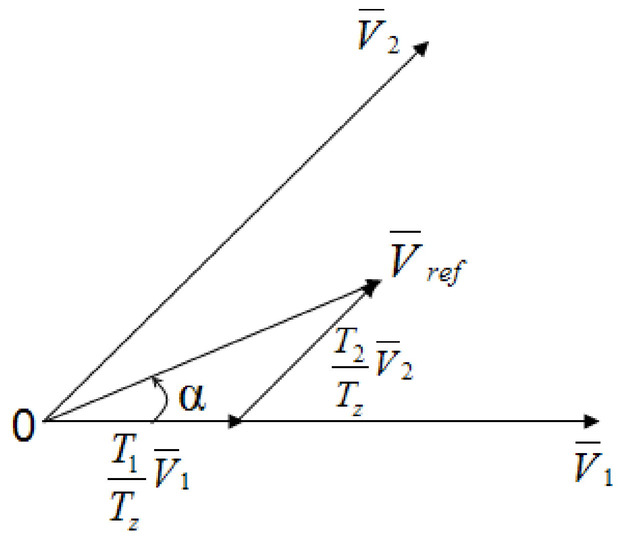


Fig. 14 Reference vector as a combination of adjacent vectors at sector 1.

Space Vector PWM

➢ **Step 2. Determine time duration T_1, T_2, T_0 (2)**

- **Switching time duration at Sector 1**

$$\int_0^{T_z} \overline{V}_{ref} = \int_0^{T_1} \overline{V}_1 dt + \int_{T1}^{T_1+T_2} \overline{V}_2 dt + \int_{T_1+T_2}^{T_z} \overline{V}_0$$

$$\therefore T_z \cdot \overline{V}_{ref} = (T_1 \cdot \overline{V}_1 + T_2 \cdot \overline{V}_2)$$

$$\Rightarrow T_z \cdot \left|\overline{V}_{ref}\right| \cdot \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} = T_1 \cdot \frac{2}{3} \cdot V_{dc} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + T_2 \cdot \frac{2}{3} \cdot V_{dc} \cdot \begin{bmatrix} \cos(\pi/3) \\ \sin(\pi/3) \end{bmatrix}$$

$$(\text{where}, 0 \le \alpha \le 60^\circ)$$

$$\therefore T_1 = T_z \cdot a \cdot \frac{\sin(\pi/3 - \alpha)}{\sin(\pi/3)}$$

$$\therefore T_2 = T_z \cdot a \cdot \frac{\sin(\alpha)}{\sin(\pi/3)}$$

$$\therefore T_0 = T_z - (T_1 + T_2), \quad \left(\text{where}, \; T_z = \frac{1}{f_s} \quad \text{and} \quad a = \frac{\left|\overline{V}_{ref}\right|}{\frac{2}{3} V_{dc}} \right)$$

Space Vector PWM

➢ Step 2. Determine time duration T_1, T_2, T_0 (3)

- **Switching time duration at any Sector**

$$\therefore T_1 = \frac{\sqrt{3}\cdot T_z \cdot \left|\overline{V}ref\right|}{V_{dc}}\left(\sin\left(\frac{\pi}{3} - \alpha + \frac{n-1}{3}\pi\right)\right)$$

$$= \frac{\sqrt{3}\cdot T_z \cdot \left|\overline{V}ref\right|}{V_{dc}}\left(\sin\frac{n}{3}\pi - \alpha\right)$$

$$= \frac{\sqrt{3}\cdot T_z \cdot \left|\overline{V}ref\right|}{V_{dc}}\left(\sin\frac{n}{3}\pi\cos\alpha - \cos\frac{n}{3}\pi\sin\alpha\right)$$

$$\therefore T_2 = \frac{\sqrt{3}\cdot T_z \cdot \left|\overline{V}ref\right|}{V_{dc}}\left(\sin\left(\alpha - \frac{n-1}{3}\pi\right)\right)$$

$$= \frac{\sqrt{3}\cdot T_z \left|\overline{V}ref\right|}{V_{dc}}\left(-\cos\alpha\cdot\sin\frac{n-1}{3}\pi + \sin\alpha\cdot\cos\frac{n-1}{3}\pi\right)$$

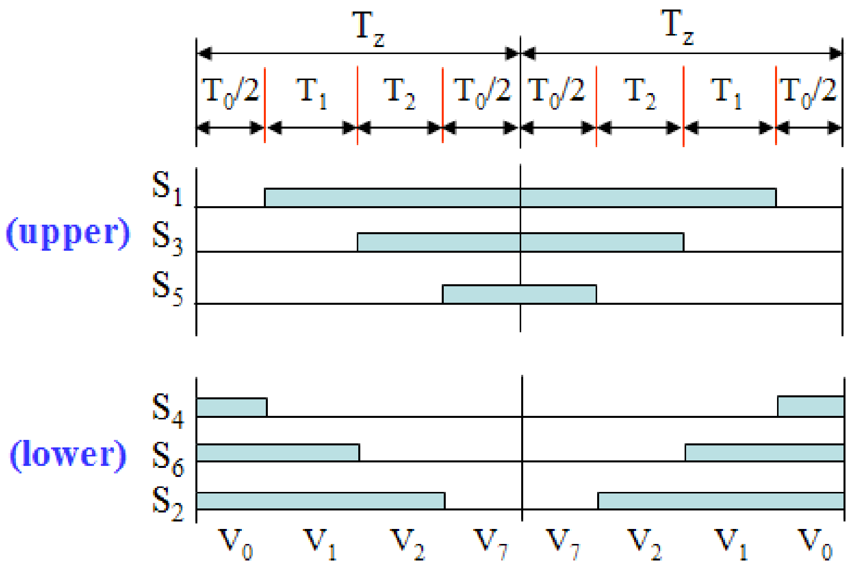
$$\therefore T_0 = T_z - T_1 - T_2, \quad \begin{pmatrix} \text{where, n} = 1 \text{ through } 6 \text{ (that is, Sector1 to 6)} \\ 0 \le \alpha \le 60^\circ \end{pmatrix}$$

Switching Sequence in a Sector

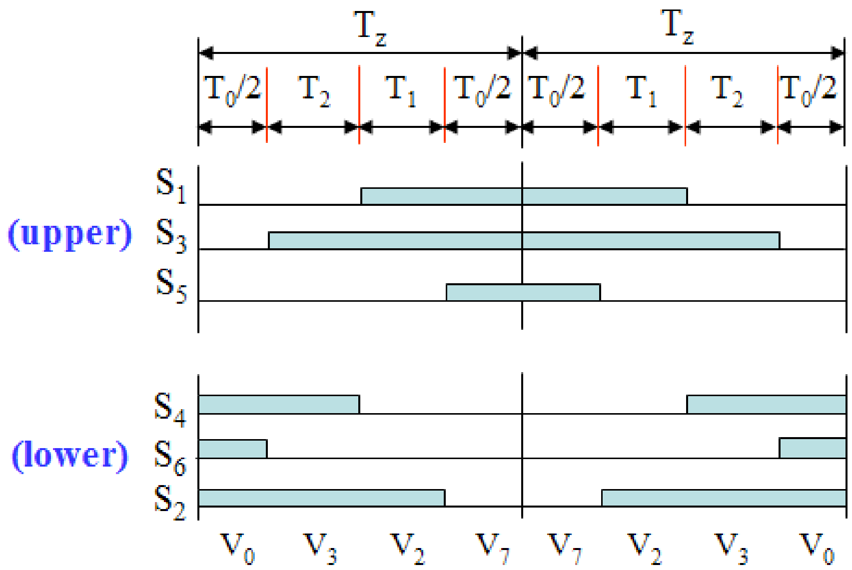
➢ **Step 3. Determine the switching time of each transistor**

Vector	State	Time
V1	100	T1
V2	110	T2
V0	000	To/2
V7	111	To/2

Vector	State	Time
V2	110	T1
V3	010	T2
V0	000	To/2
V7	111	To/2



(a) Sector 1.

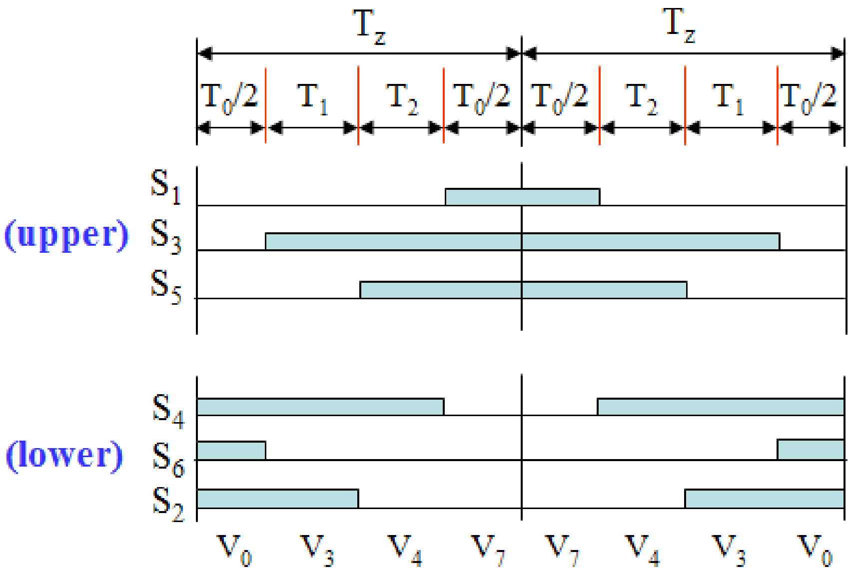


(b) Sector 2.

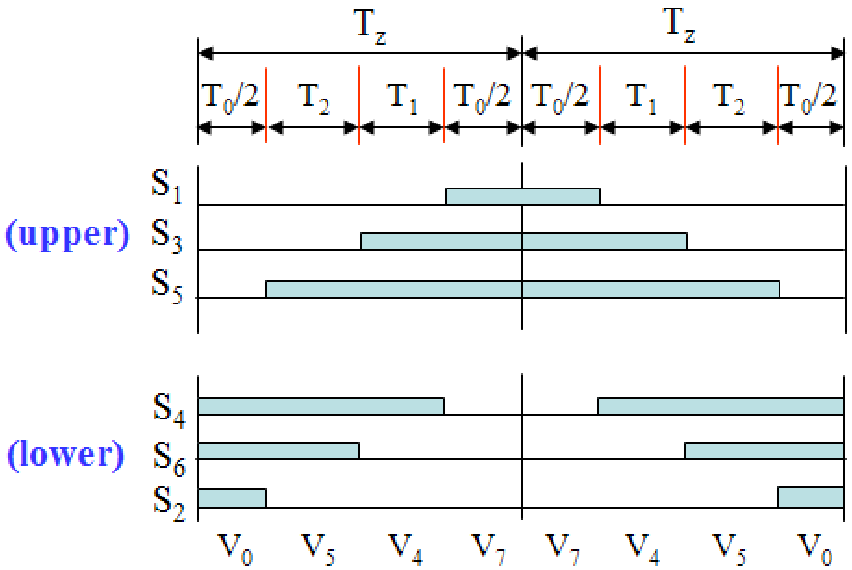
Switching Sequence in a Sector

Vector	State	Time
V3	010	T1
V4	011	T2
V0	000	To/2
V7	111	To/2

Vector	State	Time
V4	011	T1
V5	001	T2
V0	000	To/2
V7	111	To/2



(c) Sector 3.

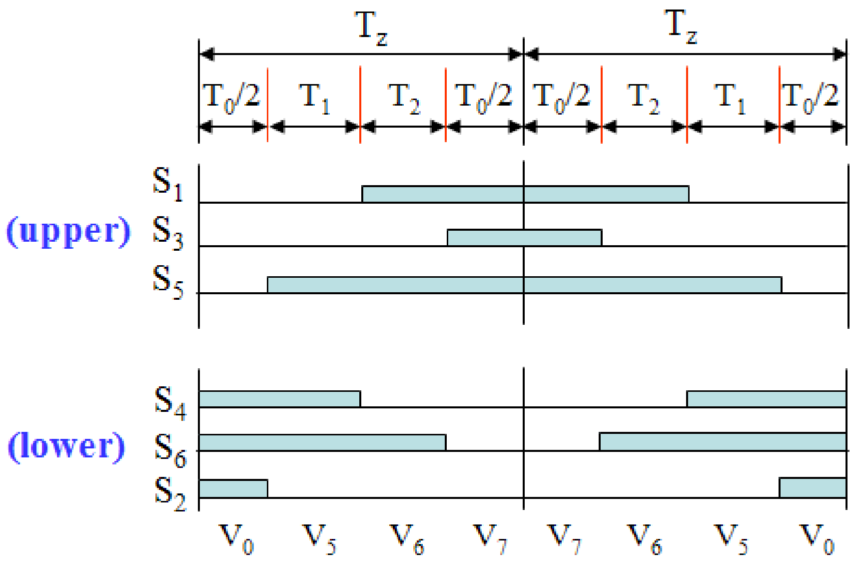


(d) Sector 4.

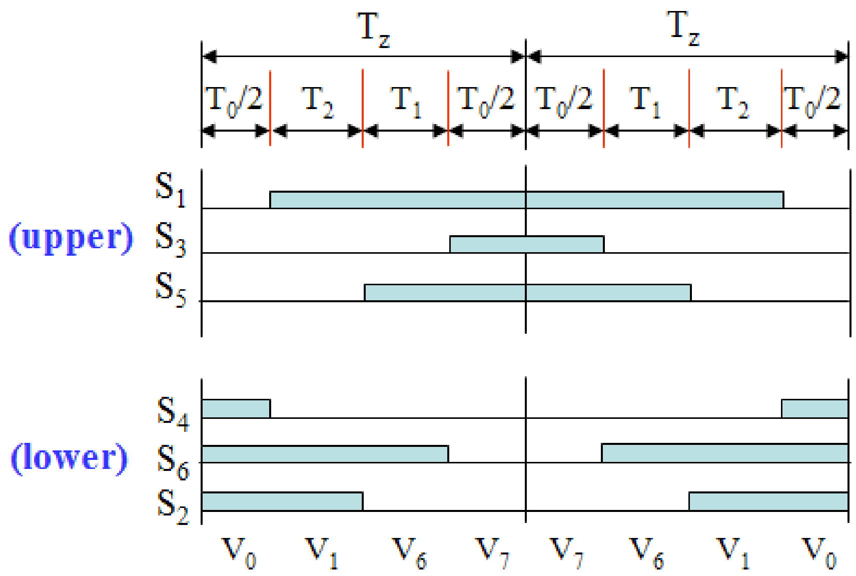
Switching Sequence in a Sector

Vector	State	Time
V5	001	T1
V6	101	T2
V0	000	To/2
V7	111	To/2

Vector	State	Time
V6	101	T1
V1	100	T2
V0	000	To/2
V7	111	To/2



(e) Sector 5.



(f) Sector 6.

Implementation of Space Vector PWM

Step 3. Determine the switching time of each transistor (S_1 to S_6)

Table 1. Switching Time Table at Each Sector

Sector	Upper Switches (S_1, S_3, S_5)	Lower Switches (S_4, S_6, S_2)
1	$S_1 = T_1 + T_2 + T_0/2$ $S_3 = T_2 + T_0/2$ $S_5 = T_0/2$	$S_4 = T_0/2$ $S_6 = T_1 + T_0/2$ $S_2 = T_1+T_2+T_0/2$
2	$S_1 = T_1 + T_0/2$ $S_3 = T_1 + T_2 + T_0/2$ $S_5 = T_0/2$	$S_4 = T_2 + T_0/2$ $S_6 = T_0/2$ $S_2 = T_1+T_2+T_0/2$
3	$S_1 = T_0/2$ $S_3 = T_1 + T_2 + T_0/2$ $S_5 = T_2 + T_0/2$	$S_4 = T_1+T_2+T_0/2$ $S_6 = T_0/2$ $S_2 = T_1 + T_0/2$
4	$S_1 = T_0/2$ $S_3 = T_1 + T_0/2$ $S_5 = T_1 + T_2 + T_0/2$	$S_4 = T_1+T_2+T_0/2$ $S_6 = T_2 + T_0/2$ $S_2 = T_0/2$
5	$S_1 = T_2 + T_0/2$ $S_3 = T_0/2$ $S_5 = T_1 + T_2 + T_0/2$	$S_4 = T_1 + T_0/2$ $S_6 = T_1+T_2+T_0/2$ $S_2 = T_0/2$
6	$S_1 = T_1 + T_2 + T_0/2$ $S_3 = T_0/2$ $S_5 = T_1 + T_0/2$	$S_4 = T_0/2$ $S_6 = T_1+T_2+T_0/2$ $S_2 = T_2 + T_0/2$

Space Vector PWM

- A voltage source inverter is supplied from a 620-V dc source and feeds a balance wye connected load. At a certain instant, the inverter is in state 3and the output currents in phase A and B are -72 and 67A, respectively. Neglect the voltage drops in the inverter and determine all the output voltages and input current.

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = V_{dc} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 620 \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -620 \\ 0 \\ 620 \end{bmatrix}$$

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \frac{620}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -414 \\ 207 \\ 207 \end{bmatrix}$$