

## MCT615 - Nonlinear Dynamical Systems

<b>Lecture Schedule</b>	See the time table	<b>Semester</b>	Fall 2018
<b>Credit Hours</b>	Three	<b>Pre-requisite</b>	Calculus, Differential Equations
<b>Instructor</b>	Dr. Sajid Iqbal	<b>Contact</b>	<a href="mailto:sajid.iqbal@uet.edu.pk">sajid.iqbal@uet.edu.pk</a>
<b>Office</b>	Faculty Room Department of Mechatronics & Control Engineering	<b>Office Hours</b>	11:00 am to 03:00 pm or by appointment
<b>Teaching Assistant</b>	None	<b>Lab Schedule</b>	N/A
<b>Course Description</b>	This course makes students acquire a systematic understanding of nonlinear phenomena in dynamical systems. It will also enhance students' research, inquiry and analytical thinking abilities. Many nonlinear ODEs do not have explicit solutions. Chaos theory shifts the focus from finding explicit solutions to discovering geometric properties of solutions. This course concentrates on simple models of dynamical systems, and their relevance to natural phenomena. Many open challenges in science and engineering involve dynamical systems that exhibit chaotic dynamics. The aperiodic nature of their dynamics makes them difficult to control, design, analyze, and predict.		
<b>CLOs</b>	<b>Description</b>	<b>PLOs, Level</b>	<b>Domain &amp; Level</b>
CLO1	Find and classify fixed points, 2-cycles, and bifurcations of maps and sets of ODEs.	PLO1, High	Cognitive, Two
CLO2	Evaluate the fractal, embedding, and topological dimensions of objects.	PLO2, Medium	Cognitive, Two
CLO3	Develop an understanding of geometric interpretation of dynamical systems and use nonlinear time-series analysis to determine the nature of the solutions of dynamical systems.	PLO3, Medium	Cognitive, Three
CLO4	Draft a review paper	PLO10, High	Cognitive, Six
<b>Textbooks</b>	<b>REQUIRED</b> 1. Paul S. Addison, <i>Fractals and Chaos: An illustrated course</i> . IOP. 1997 2. D. P. Feldman, <i>Chaos and Fractals An Elementary Introduction</i> 2012 3. Strogatz, S., <i>Nonlinear Dynamics And Chaos: With Applications To Physics, Biology, Chemistry, And Engineering</i> . 2 <sup>nd</sup> Edition. Westview Press, 2014. <b>OPTIONAL</b> • James Gleick, <i>Chaos Making a New Science</i> . 1987		
<b>Grading Policy</b>	<ul style="list-style-type: none"> <li>• Quizzes: 10% (CLO1, CLO3)</li> <li>• Term paper: 30% (CLO4)</li> <li>• Midterm: 25% (CLO1, CLO2)</li> <li>• Final: 35% (CLO1, CLO2, CLO3)</li> </ul>		

## Lecture Plan

Weeks	Lecture Plan	Ref.
1	Introduction to Nonlinear Dynamics and Chaos: Deterministic Chaos, Fractals, History of Dynamics, Examples.	Ch.1 (CLO 1)
2	Basic Concepts of Chaos Theory and Bifurcation Theory: Sensitive Dependence on Initial Conditions, The Butterfly Effect, Fixed points, Bifurcation, Stability, and the Feigenbaum constant.	Ch. 2 (CLO 1)
2	Fractals and Fractal dimension: The Similarity Dimension, Statistical Self-similarity, Koch curve, Sierpinski gasket and carpet, Examples.	Ch. 2 (CLO 2)
2	One-Dimensional maps: Population growth and the Verhulst model, The logistic map, Graphical method.	Ch. 5 (CLO 2)
	<b>M I D T E R M</b>	
2	Lorenz Model and Strange Attractors: Chaos in the Weather, Lorenz Equations and Lorenz Attractor, Examples.	Ch. 6 (CLO 2)
2	Quantification of Chaos: Visual Inspection, Frequency Spectra, Lyapunov Exponents, Correlation Dimension.	Ch. 7 (CLO 3)
2	Attractor reconstruction: Time-delay Embedding, Takens' embedding theorem, The Choice of Time-delay and Embedding Dimension, False Nearest Neighbor Algorithm, Autocorrelation Function, Estimation of Correlation Dimension and Largest Lyapunov Exponent.	Ch. 7 (CLO 3)
1	Design and implementation of chaos control systems, Introduction, Techniques for Chaos Control	Ch 3 (CLO3)
1	Writing of the term paper	(CLO 4)
	<b>Final Term</b>	

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