Time Domain Analysis of Transmission Line Based on WLP-FDTD

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***Abstract* — In this paper, a novel solution based on weighted Laguerre polynomials finite-difference time-domain (WLP-FDTD) method is proposed for time domain analysis of interconnect modeled as a transmission line. Compared with finite-difference time-domain (FDTD) method, which is an explicit scheme and limited by Courant condition, the WLP-FDTD method is unconditionally stable since the solution is implicit. Numerical results show that the proposed WLP-FDTD solution is as accurate as but much faster than conventional FDTD-based solutions.**

***Index Terms* — FDTD, Courant condition, unconditionally stable, WLP-FDTD, transmission line.**

I. Introduction

Time domain analysis of CMOS gates driving interconnects is essential for the design and application of CMOS digital integrated circuits and much research effort has been made in this field. Usually, interconnects are modeled as transmission lines, and CMOS driver and loader are simplified as a resister and a capacitor, respectively. Then finite-different time-domain (FDTD) method [1] [2] can be adopted to analyze this linear transmission line circuit. Recently, FDTD method has been expanded to analyze transmission lines with nonlinear terminals to include short-channel effects on CMOS gates [3] [4]. Numerical results show the above FDTD-based solutions are as accurate as but much faster than SPICE tool for analysis of transmission lines. However, the FDTD method, as an explicit time-marching technique, is limited by the well-known Courant stability condition, that is, the time step must be no greater than the propagation time over each cell. Therefore, the FDTD method has low efficiency for small cell step. To avoid the stability limitation of the conventional FDTD, alternating-direction-implicit FDTD (ADI-FDTD) method was proposed in [5], in which big time step was chosen at the cost of large dispersion error [6]. Moreover, weighted Laguerre polynomials FDTD (WLP-FDTD) method, as an unconditionally stable method, was developed by eliminating time step [7]. Therefore, it has no dispersion error and is much faster than the conventional FDTD scheme for multiscale simulation [7]. Until now, the WLP-FDTD method has been applied to calculate electromagnetic field in circuit packages [8] and periodic structures with oblique incident wave [9].

In this paper, the WLP-FDTD method is firstly applied to time-domain analysis of CMOS gate driving an interconnect line. Numerical results show that the proposed WLP-FDTD-based solution is as accurate as but much faster than conventional FDTD solution. The rest of the paper is presented as follows. Section II introduces the WLP-FDTD method to analyze COMS gate driving a transmission line. Section III gives numerical results to show the accuracy and efficiency of the proposed solution. Finally, the conclusion is made in Section IV.

II. The WLP-FDTD Formulation

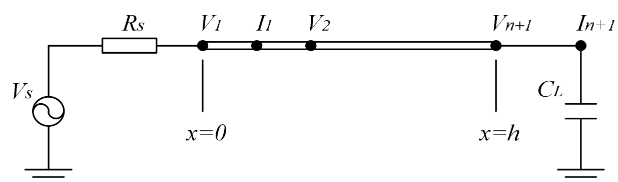

Fig.1 Illustration of the discrete transmission line and its terminals.

An interconnect line with CMOS driver and loader is illustrated in Fig. 1, where the CMOS driver and loader are simplified as a resister R_s and a capacitor C_L [10], respectively. The interconnect line is modeled by a transmission line, which can be described by telegraph equations:

$$\frac{\partial V(x,t)}{\partial x}+l\frac{\partial I(x,t)}{\partial t}+rI(x,t)=V_F(x,t) \qquad (1)$$

$$\frac{\partial I(x,t)}{\partial x}+c\frac{\partial V(x,t)}{\partial t}+gV(x,t)=I_F(x,t) \qquad (2)$$

where r, l, g, and c are the resistance, inductance, conductance and capacitance per unit length of the transmission line, respectively. $V(x,t)$ and $I(x,t)$ are the voltage and current at the position x and time t, respectively. $V_F(x,t)$ and $I_F(x,t)$ are the voltage and current sources at the position x and time t, respectively.

$V(x,t)$ and $I(x,t)$ can be expanded with weighted Laguerre polynomial basis functions, shown as

$$V(x,t)=\sum_{p=0}^{m-1}V^p(x)\varphi_p(\bar{t}) \qquad (3)$$

$$I(x,t)=\sum_{p=0}^{m-1}I^p(x)\varphi_p(\bar{t}) \qquad (4)$$

where $\bar{t}=s\bullet t$ is the scaled time, s is the scaled factor, and m is the number of the weighted Laguerre polynomial basis functions. The p order of the weighted Laguerre polynomial basis function is defined as

$$\varphi_p(\bar{t}) = e^{-\bar{t}/2} L_p(\bar{t}) \quad (5)$$

where $L_p(\bar{t})$ is the p order of the Laguerre polynomial [7] . $V^p(x)$ and $I^p(x)$ are the corresponding coefficients. Since the real time scale is quite small, in order to use the above basis functions properly, one should transform the real time scale using an appropriate scale factor s [7] .

From (3) and (4), the first derivative of $V(x,t)$ and $I(x,t)$ with respect to time t is

$$\frac{\partial V(x,t)}{\partial t} = s\sum_{p=0}^{m-1}(0.5V^p(x) + \sum_{k=0,p>0}^{p-1} V^k(x))\varphi_p(\bar{t}) \quad (6)$$

$$\frac{\partial I(x,t)}{\partial t} = s\sum_{p=0}^{m-1}(0.5I^p(x) + \sum_{k=0,p>0}^{p-1} I^k(x))\varphi_p(\bar{t}) \quad (7)$$

And the first derivative of $V(x,t)$ and $I(x,t)$ with respect to position x is

$$\frac{\partial V(x,t)}{\partial x} = \sum_{p=0}^{m-1}\frac{\partial V^p(x)}{\partial x}\varphi_p(\bar{t}) \quad (8)$$

$$\frac{\partial I(x,t)}{\partial x} = \sum_{p=0}^{m-1}\frac{\partial I^p(x)}{\partial x}\varphi_p(\bar{t}) \quad (9)$$

Substituting (3)-(4) and (6)-(9) into (1)-(2) yields

$$\begin{aligned}&\sum_{p=0}^{m-1}\frac{\partial V^p(x)}{\partial x}\varphi_p(\bar{t}) + ls\sum_{p=0}^{m-1}(0.5I^p(x) + \sum_{k=0,p>0}^{p-1} I^k(x))\varphi_p(\bar{t})\\ &+r\sum_{p=0}^{m-1} I^p(x)\varphi_p(\bar{t}) = V_F(x,t)\end{aligned} \quad (10)$$

$$\begin{aligned}&\sum_{p=0}^{m-1}\frac{\partial I^p(x)}{\partial x}\varphi_p(\bar{t}) + cs\sum_{p=0}^{m-1}(0.5V^p(x) + \sum_{k=0,p>0}^{p-1} V^k(x))\varphi_p(\bar{t})\\ &+g\sum_{p=0}^{m-1} V^p(x)\varphi_p(\bar{t}) = I_F(x,t)\end{aligned} \quad (11)$$

To eliminate the time-dependent terms $\varphi_p(\bar{t})$, the orthogonal property of the weighted Laguerre polynomial basis functions will be used. Multiplying both sides of (10) and (11) by $\varphi_q(\bar{t})$ and integrating them over $\bar{t} = [0,\infty)$ yields

$$\frac{\partial V^q(x)}{\partial x} + ls(0.5I^q(x) + \sum_{k=0,q>0}^{q-1} I^k(x)) + rI^q(x) = J_V^q(x) \quad (12)$$

$$\frac{\partial I^q(x)}{\partial x} + cs(0.5V^q(x) + \sum_{k=0,q>0}^{q-1} V^k(x)) + gV^q(x) = J_I^q(x) \quad (13)$$

where

$$J_V^q(x) = \int_0^{T_f} V_F(x,t)\varphi^q(\bar{t})d\bar{t} \quad (14)$$

$$J_I^q(x) = \int_0^{T_f} I_F(x,t)\varphi^q(\bar{t})d\bar{t} \quad (15)$$

The upper limit of infinity is replaced by a finite time interval T_f in (14) and (15).This interval is chosen in such a way that the waveforms of interest have practically decayed to zero.

The transmission line of length h is divided into n sections and the length of each section is Δx . The discretized voltage nodes and current nodes are $V_1, V_2, \cdots, V_{n+1}$ and $I_1, I_2, \cdots, I_{n+1}$, respectively, denoted as

$$V_j \equiv V((j-1)\Delta x, t) \quad (16)$$

$$I_j \equiv I((j-\frac{1}{2})\Delta x, t) \quad (17)$$

where $j = 1, 2, \cdots, n+1$. The spatial discretization of (12) and (13) are

$$\begin{aligned}&I_j^q + \frac{2}{(ls+2r)\Delta x}V_{j+1}^q - \frac{2}{(ls+2r)\Delta x}V_j^q =\\ &\frac{2}{ls+2r}J_{V|j}^q - \frac{2ls}{ls+2r}\sum_{k=0,q>0}^{q-1} I_j^k\end{aligned} \quad (18)$$

for $j = 1, 2, \cdots, n$, and

$$\begin{aligned}&V_j^q + \frac{2}{(cs+2g)\Delta x}I_j^q - \frac{2}{(cs+2g)\Delta x}I_{j-1}^q =\\ &\frac{2}{cs+2g}J_{I|j}^q - \frac{2cs}{cs+2g}\sum_{k=0,q>0}^{q-1} V_j^k\end{aligned} \quad (19)$$

for $j = 1, 2, \cdots, n+1$. Note that $I_0^q = 0$ for $q = 0, 1, \cdots, m-1$

Referring to Fig.1, the voltage source V_s in series with the source resistance R_s can be replaced by a current source $I_s = V_s / R_s$ in parallel with a source resistance R_s according to Norton equivalent principle. Then at the terminal x=0,

$$g = 1/(R_s\Delta x) \quad (20)$$

$$I_F = V_s/(R_s\Delta x) \quad (21)$$

$$V_F = 0 \quad (22)$$

And c should be replaced by c/2 only at the two terminals j=1 and j=n+1 in (19). Except at the driving point, there is no source so

$$J_{V|j}^{q} = V_F = 0 \tag{23}$$

for $j = 2, 3, \cdots, n$.

$$J_{I|j}^{q} = I_F = 0 \tag{24}$$

for $j = 2, 3, \cdots, n+1$.

At the right terminal $x = h$,

$$C_L \frac{\partial V(h,t)}{\partial t} - I(h,t) = 0 \tag{25}$$

Expanding $V(h,t)$ and $I(h,t)$ with the weighted Laguerre polynomial basis functions and then computing the first derivative of $V(h,t)$ with respect to time t yields

$$C_L s \sum_{p=0}^{m-1} (0.5V^p(h) + \sum_{k=0,p>0}^{p-1} V^k(h)) - \sum_{p=0}^{m-1} I^p(h)\varphi_p(\bar{t}) = 0 \tag{26}$$

Using the orthogonal property of the weighted Laguerre polynomial functions, multiplying both sides of (26) by $\varphi_q(\bar{t})$ and integrating them over $\bar{t} = [0,\infty)$ yields

$$C_L s(0.5V^q(h) + \sum_{k=0,q>0}^{q-1} V^k(h)) - I^q(h) = 0 \tag{27}$$

By making the spatial discretization of (27), we have

$$V_{n+1}^q - \frac{2}{C_L s} I_{n+1}^q = -2 \sum_{k=0,q>0}^{q-1} V_{n+1}^k \tag{28}$$

For q=0, there are 2n+2 unknown coefficients $V_1^0, V_2^0, \cdots, V_{n+1}^0$ and $I_1^0, I_2^0, \cdots, I_{n+1}^0$, which can be solved from 2n+2 equations (18) (19) and (28) with matrix inversion. Then using iterative calculation, we can get $V_1^q, V_2^q, \cdots, V_{n+1}^q$ and $I_1^q, I_2^q, \cdots, I_{n+1}^q$ for all q from 1 to m-1 from (18) (19) and (28) with the same matrix inversion. Finally, $V(x,t)$ and $I(x,t)$ can be obtained from (3) and (4).

The conventional FDTD method utilizes the customary explicit leapfrog time scheme and hence has stability limitation described by Courant condition. In contrast, the proposed method does not make time difference and is no longer affected by the size of time step which is only used to calculate the time integration in (14) and (15).

III. Numerical Examples

A transmission line with the parameters h=1mm, r=24mΩ/μm, l=1.2pH/μm, c=350aF/μm, and g =0 is used in this section. Its voltage source is a 200MHz periodic trapezoidal waveform with 50% duty cycle and rise/fall times τ= 10ps, the source resistance R_s =400Ω, and the load capacitance is 50fF. These give velocities in the lossless case of $v = 1/\sqrt{lc} = 4.9 \times 10^7$ m/s. The spatial discretization for the FDTD method is chosen to be Δx = 0.1mm so that each cell is $\lambda/10$ (one tenth of wavelength) at the break frequency $1/\pi\tau$ of the pulse spectrum.

150 weighted Laguerre polynomial basis functions are chosen due to the smallest error at time t=0 [8]. The timescale factor is set to be $s = 10^{11}$. The proposed WLP-FDTD solution has the same cell step as the FDTD solution [1]. In comparison, the time step is 10ps in the proposed solution and 0.5ps in the FDTD solution. Fig.2 shows the time domain responses of the transmission line analyzed by the FDTD method and the WLP-FDTD method. It is shown that the waveform from the proposed solution closely matches the FDTD solution. In addition, the runtime of the FDTD-based solution is 0.04s but that of the WLP-FDTD solution is only 0.004s. Therefore, the proposed WLP-FDTD solution is much faster than the FDTD solution.

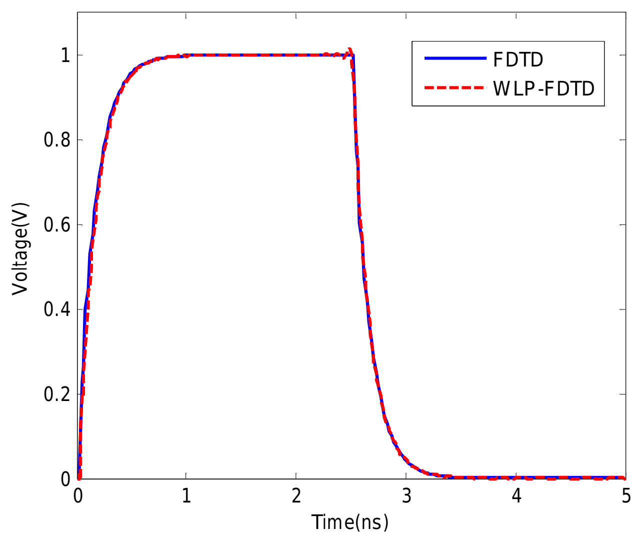

Fig.2 Time domain responses of the transmission line from the FDTD solution and the proposed WLP-FDTD solution.

IV. Conclusion

This paper firstly applies the WLP-FDTD method to analyze time domain responses of CMOS gate driving interconnect modeled by a transmission line. Since it uses weighted Laguerre polynomial basis functions for expansion and makes time integral instead of time difference, the WLP-FDTD method is unconditionally stable and not limited by the Courant condition. The number of iterations in the proposed WLP-FDTD-based

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solution is much less than that in the conventional FDTD-based solution. Numerical results show that the proposed WLP-FDTD solution matches the conventional FDTD solution very well with higher efficiency.

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REFERENCES

[1] C. R. Paul, "Incorporation of terminal constraints in the FDTD analysis of transmission lines," *IEEE Transactions on Electromagnetic Compatibility*, vol. 36, no.2, pp. 85-91, May 1994.

[2] E.-X. Liu, E. P. Li, and Z. X. Shen, "Finite-difference time-domain macromodel for simulation of electromagnetic interference at high-speed interconnects," *IEEE Trans. Magnetics,* vol. 41, no. 1, pp. 65–71, Jan. 2005.

[3] X. Li, J. Mao, and M. Swaminathan,"Accurate analysis of CMOS inverter driving transmission line based on FDTD," in *Proc. IEEE MTT-S Int. Microwave Symp. Dig.*, Jun. 2009, pp. 1573–157.

[4] Xiao-Chun Li, Jun-Fa Mao, Swaminathan M, "Transient Analysis of CMOS-Gate-Driven RLGC Interconnects Based on FDTD" *IEEE Transactions on Computer-Aided Desigh of Integrated Circuits and Systems, vol. 30,* no. 4, Apr. 2011, pp. 574-583.

[5] T. Namiki, "A new FDTD algorithm based on alternating-direction implicit method" *IEEE Trans. Microwave Theory Tech*, vol. 47, pp.2003–2007, Oct. 1999.

[6] An Ping Zhao, "Analysis of the Numerical Dispersion of the 2-D Alternating-Direction Implicit FDTD Method" *IEEE Transactions on Microwave Theory and Techniques,* vol. 50, pp.1156–1164, Apr. 2002.

[7] Y. Chung, T. K. Sarkar, B. H. Jung, and M. Salazar-Palma, "An unconditionally stable scheme for the finite-difference time-domain method," *IEEE Trans. Microwave Theory Tech.*, vol. 51, no. 3, pp. 697–704, Mar.2003.

[8] Myunghyun Ha, Srinivasan, K., Swaminathan, M," Transient Chip-Package Cosimulation of Multiscale Structures Using the Laguerre-FDTD Scheme " *IEEE Transactions on Advanced Packaging, vol. 32, no. 4, pp. 816-830, Nov. 2009.*

[9] Zhao-Yang Cai, Bin Chen, Qin Yin, Run Xiong, "The WLP-FDTD Method for Periodic Structures With Oblique Incident Wave" *IEEE Transactions on Antennas and Propagation,* vol. 59, pp.3780–3785, Oct. 2011.

[10] J. Qian, S. Pullela, and L. Pillage, "Modeling the 'effective capacitance'for the RC interconnect of CMOS gates," *IEEE Trans. Comput.-Aided Des*, vol. 13, no. 12, pp. 1526–1535, Dec. 1994.