Chapter 1

Magnetic Circuits

1.1 Introduction

Practically all transformers and electric machinery use magnetic material for shaping and directing the magnetic fields which act as the medium for transferring and converting energy. Thus it is important to analyze and describe magnetic field quantities for understanding these devices. Magnetic materials play a big role in determining the properties of a piece of electromagnetic equipment or the electric machine and affect its size and efficiency.

In electrical machines, ferromagnetic materials may form the magnetic circuits only (as in transformers) or by ferromagnetic materials in conjunction with an air medium (as in rotating machines). In most electrical machines, except permanent magnet machines, the magnetic field (or flux) is produced by passing an electrical current through coils wound on ferromagnetic materials.

This chapter will develop some basic tools for the analysis of magnetic field systems and will provide a brief introduction to the properties of practical magnetic materials. These results will then be applied to the analysis of transformers and rotating machines. So a carfull study for this chapter is recommended to fully understand the next chapters.

1.2 Magnetic Field Intensity, H And Flux Density, B

When a conductor carries current a magnetic field is produced around it, as shown in Fig.1.1. The direction of flux lines or magnetic field intensity *H* (A/m) can be determined by what is known as the *thumb rule.*

thumb rule

"If the conductor is held with the right hand with the thumb indicating the direction of current in the conductor then, the fingertips will indicate the direction of magnetic field intensity ".

Fig.1.1 can explains Thumb rule

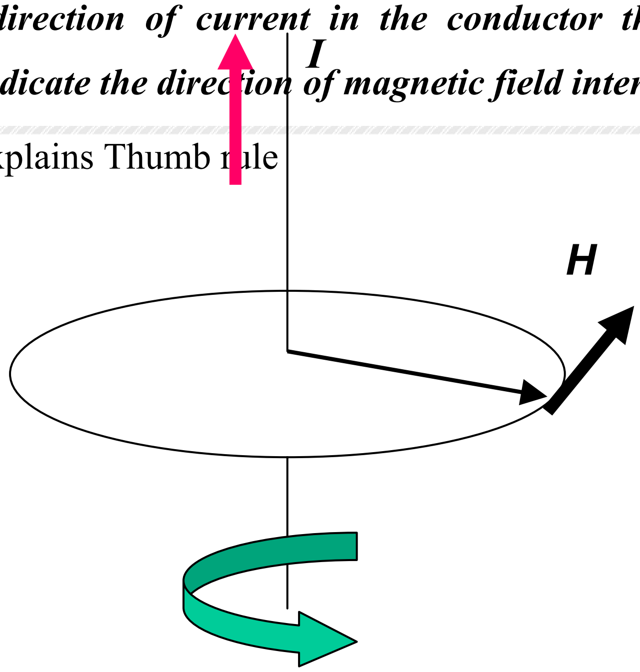


Fig.1.1 Field around an infinitely long, straight conductor carrying a current.

Ampere's law:

The magnetic field intensity H around a closed contour C is equal to the total current passing through any surface S linking that contour which is known as *Ampere's law* as shown in equation (1.1)

$$\oint H.dl = \sum i \tag{1.1}$$

where H is the magnetic field intensity at a point on the contour and dl is the incremental length at that point.

Suppose that the field strength at point C distant r meters from the center of the conductor is H. Then it means that if a unit N-pole is placed at C, it will experience a force of H Newton. The direction of this force would be tangential to the circular line of force passing through C. If the unit N-pole is moved once round the conductor against this force, then work done, this work can be obtained from the following releation:

$$Work = Force * dis\tan ce = H * 2\pi * r \tag{1.2}$$

The relationship between the magnetic field intensity H and the magnetic flux density B is a property of the material in which the field exists which is known as the permeability of the material; Thus,

$$B = uH \tag{1.3}$$

where u is the *permeability.*

In SI units B is in *webers per square meter,* known as *tesla* (T), and

In SI units the permeability of free space, Vacume or nonmagnetic materials is $u_0 = 4\pi * 10^{-7}$. The permeability of ferromagnetic material can be expressed in terms of its value relative to that of free space, or $u = u_0 * u_r$. Where u_r is known as ***relative permeability*** of the material. Typical values of u_r range from 2000 to 80,000 for feromagnetic materials used in transformers and rotating machines. For the present we assume that u_r is a known constant for specific material, although it actually varies appreciably with the magnitude of the magnetic flux density.

Fig.1.2 shows a simple magnetic circuit having a ring-shaped magnetic core, called toroid, and a coil that extends around it. When current i flows through the coil of N turns, magnetic flux is mostly confined in the core material. The flux outside the toroid, called *leakage flux*, is so small that for all practical purposes it can be neglected. Consider a path at main radius r. The magnetic intensity on this path is H and, from Ampere's circuit law, the following relation can be obtained:

$$\oint H.dl = Ni \qquad (1.4)$$

Then $Hl = H * 2\pi r = Ni \qquad (1.5)$

Where $r = \frac{1}{2} * \left(\frac{ID + OD}{2}\right)$

Where as shown in Fig.1.2 ID and OD are inner and outer diameter of the core of the triode.

The quantity Ni is called the *magnetomotive force* (*mmf*), and its unit is Ampere-turn (*At*).

$$H = \frac{N}{l} i \ At/m \qquad (1.6)$$

From Eqs. (1.3) And (1.6)

$$B = \frac{uNi}{l} \ Tesla \qquad (1.7)$$

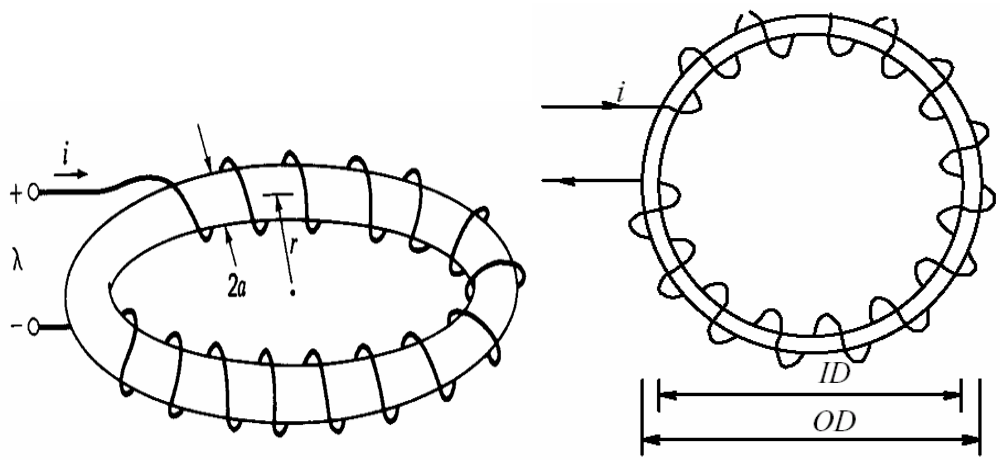


Fig.1.2 Toroid magnetic circuit.

If we assume that all the fluxes are confined in the toroid, that is, there is no magnetic leakage, the flux crossing the cross section of the toroid is:

$$\phi = \int B\,dA \tag{1.8}$$

Then $\phi = B\,A$ Web. (1.9)

Where B is the average flux density in the core and A is the area of cross section of the toroid. The average flux density may correspond to the path at the mean radius of the toroid. If H is the magnetic intensity for this path, then from Eqs. (1.7) and (1.9),

$$\phi = \frac{uNi}{l} A = \frac{Ni}{l/uA} = \frac{Ni}{\Re} = \frac{mmf}{\Re} \tag{1.10}$$

Where $\Re = \dfrac{l}{uA}$ (1.11)

$\Re$ is called the *reluctance* of the magnetic. Equation (1.10) suggests that the driving force in the magnetic circuit of Fig.1.2 is the magnetomotive force *mmf*, which produces a flux ϕ against a magnetic reluctance $\Re$. The magnetic circuit of the toroid can therefore be represented by a magnetic equivalent circuit as shown in Fig.1.3. Also note that Equation (1.10) has the form of *Ohm's law* for an electric circuit (*i* = *E*/*R*). The analogous electrical circuit is shown in Fig.1.3. A magnetic circuit is often looked upon as analogous to an electric circuit. The analogy is illustrated in Table 1.1.

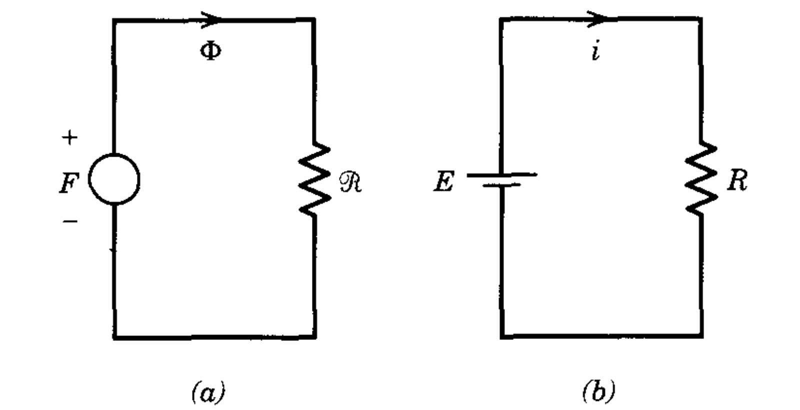


Fig.1.3 Analogy between (a) magnetic circuit and (b) electric circuit

Table 1.1 Magnetic versus Electrical circuits.

Items	Magnetic circuit	Electric circuit
Driving force	*mmf (F=Ni)*	*Emf*=E
Produces	Flux $\left(\phi = \frac{F}{\Re}\right)$	Current $\left(i = \frac{E}{R}\right)$
Limited By	Reluctance $\left(\Re = \frac{l}{uA}\right)$	Resistance $R = \frac{l}{\sigma A}$

Where, u is the permeability and σ is the conductivity.

1.3 Magnetization Curve

A typical ferromagnetic material is silicon steel, which is widely used for the cores of transformers and rotating machines. When such a material is magnetized by slowly increasing the applied magnetizing force H, the resulting flux density B follows a curve of the form shown in Fig.1.4. This is known as *the magnetization curve* for the material.

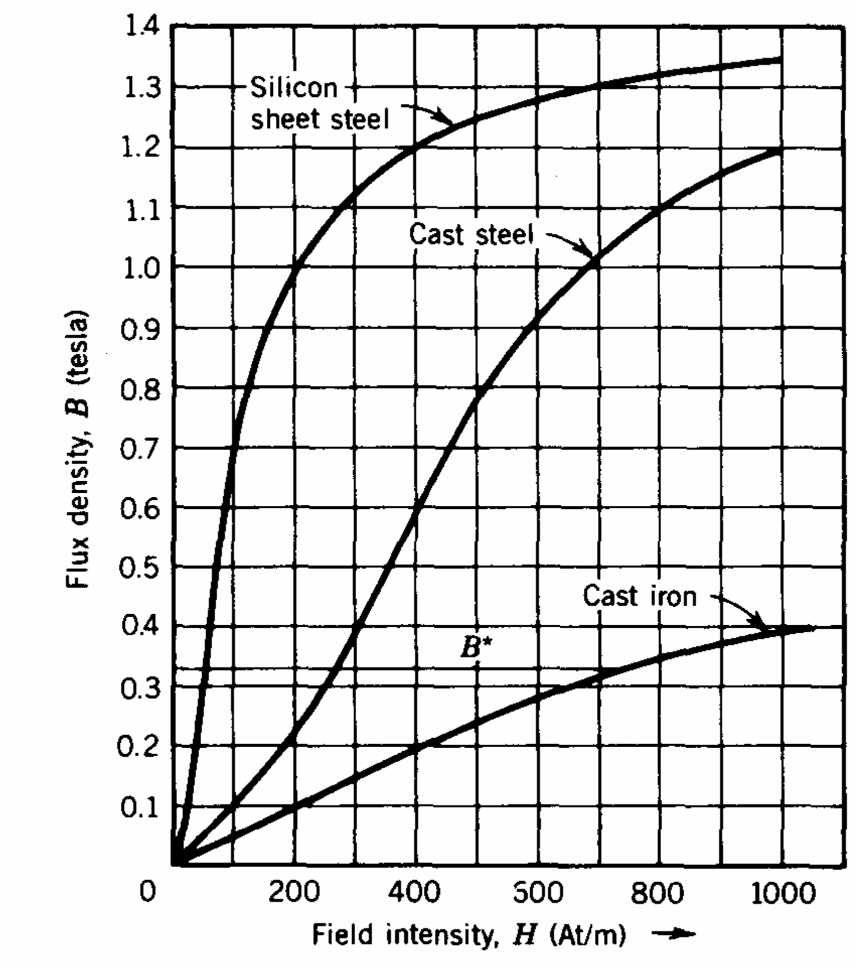


Fig.1.4 Magnetization curves for different magnetic materials.

The part of the magnetization curve where the slope begins to change rapidly is termed the ***knee***. Below the knee it is often possible to use a linear approximation to the actual characteristic, with a corresponding constant value for the relative permeability. But the onset of saturation above the knee marks a dramatic change in the properties of the material, which must be recognized in the design and analysis of magnetic structures.

Transformers are wound on closed cores like that of Fig.1.5. Energy conversion devices which incorporate a moving element must have air gaps in their magnetic circuits. A magnetic circuit with an air gap is shown in Fig.1.5. When the air gap length g is much smaller than the dimensions of the adjacent core faces, the magnetic flux ϕ is constrained essentially to reside in the core and the air gap and is continuous throughout the magnetic circuit. Thus, the configuration of Fig.1.5 can be analyzed as a magnetic circuit with two series components:

- a magnetic core of permeability $u_o * u_r$ and mean length l, and
- an air gap of permeability u_0, cross-sectional area A_g, and length g.

In the core the flux density is uniform, and the cross-sectional area is A_c thus in the core,

$$B_c = \frac{\phi}{A_c} \tag{1.12}$$

$$B_g = \frac{\phi}{A_g} \tag{1.13}$$

The magnetic field lines bulge outward somewhat as they cross the air gap. This phenomena known as ***fringing of magnetic field***.

The effect of the fringing fields is to increase the effective cross-sectional area A_c of the air gap as illustrated in Fig.1.6. Various empirical methods have been developed to account for this effect. A correction for such fringing fields in short air gaps can be made by adding the gap length to each of the two dimensions making up its cross-sectional area. If fringing is neglected,

$$A_g = A_c \text{ and } B_g = B_c = \frac{\phi}{A_c} \tag{1.14}$$

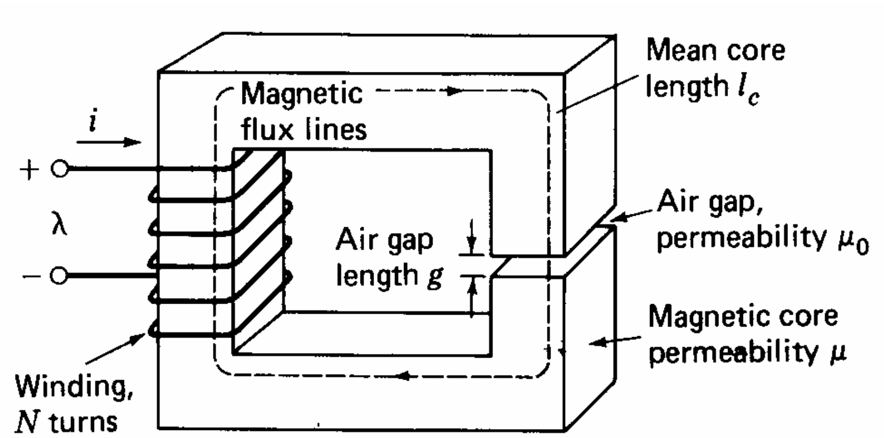


Fig.1.5 Magnetic circuit with air gap.

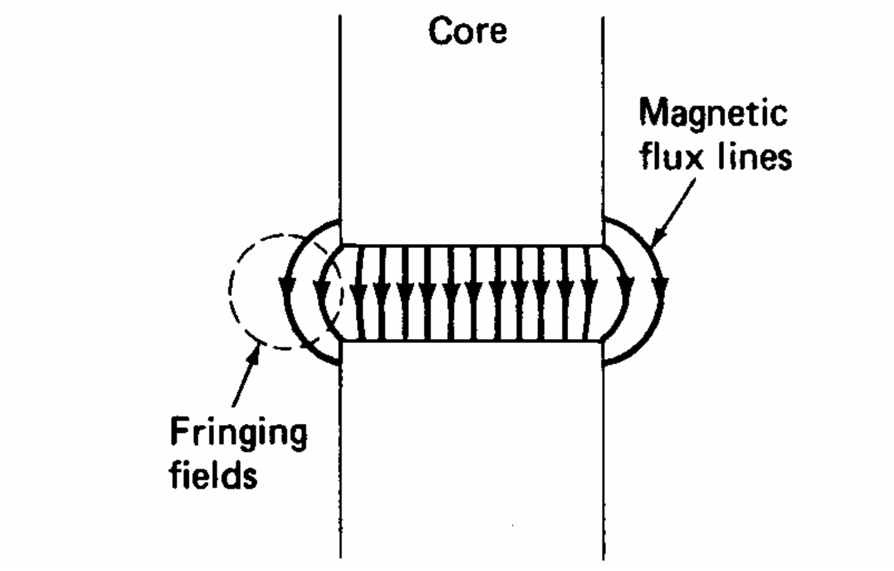


Fig.1.6 Fringing flux.

Example 1.1 *In the magnetic system of Fig.1.7 two sides are thicker than the other two sides. The depth of the core is 10 cm, the relative permeability of the core,* $u_r = 2000$*, the number of turns* $N = 500$*, and the current flowing through the coil is* $i = 1\ A$*.*

(a) *Determine the flux in the core.*

(b) *Determine the flux densities in the parts of the core.*

(c) *Find the current i in the coil to produce a flux (*ϕ*=0.012Wb).*

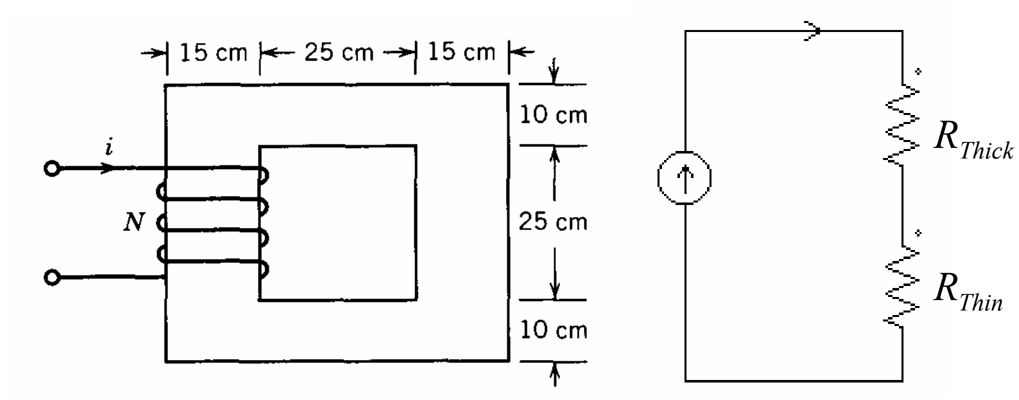


Fig.1.7

Solution:

(a)

$$R_{Thick} = \frac{70*10^{-2}}{2000*4*\pi*10^{-7}*15*10*10^{-4}} = 18568.03\,At/Web$$

$$R_{Thin} = \frac{80*10^{-2}}{2000*4*\pi*10^{-7}*10*10*10^{-4}} = 31830.91\,At/Web$$

Then, $R_{Thick} + R_{Thin} = 50398.94\,At/Web$

Then, $\phi = \dfrac{500*1}{50398.94} = 0.009921 Wb$

(b) $B_{Thick} = \dfrac{0.009921}{150*10^{-4}} = 0.6614\ T$

$$B_{Thin} = \frac{0.009921}{100*10^{-4}} = 0.9921\ T$$

(c) $B_{Thick} = \dfrac{0.012}{15*10*10^{-4}} = 0.8\ T$

$$B_{Thin} = \frac{0.012}{10*10*10^{-4}} = 1.2\ T$$

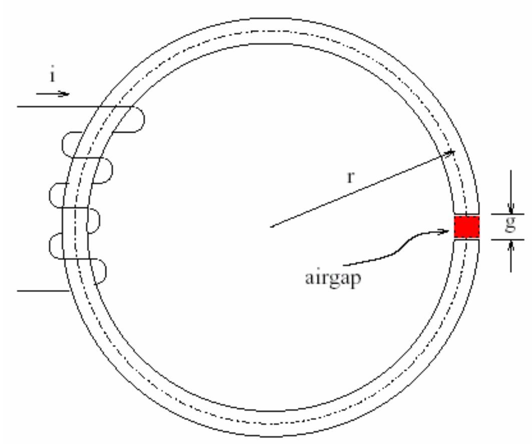
$$H_{Thick} = \frac{0.8}{2000*4\pi*10^{-7}} = 318.31\ At/m$$

$$H_{Thin} = \frac{1.2}{2000*4\pi*10^{-7}} = 477.46\ At/m$$

$$\mathrm{F} = 318.31*2*35*10^{-2} + 477.46*2*40*10^{-2} = 604.79At$$

Then, $i = \dfrac{604.79}{500} = 1.2096\ A$

Example 1.2 *A circular ring of magnetic material has a mean length of 1.0 m and a cross-sectional area of 0.001 m^2. A saw cut of 5 mm width is made in the ring. Calculate the magnetizing current required to produce a flux of 1.0 mWb in the air-gap if the ring is wound uniformly with a coil of 200 turns. Take relative permeability of the ring material =500 and neglect leakage and fringing.*



Solution:

$$B = 1 * 10^{-3} / 0.001 = 1 Wb / m^2$$

The ampere-turn for air gap is:

$$AT_a = \frac{B}{\mu_0} * l = \frac{1}{4\pi * 10^{-7}} * 5 * 10^{-3} = 3.978 AT$$

The ampere-turn for core material is:

$$AT_c = \frac{B}{\mu_0 \mu_r} * l = \frac{1}{4\pi * 10^{-7} * 500} * 1 = 1.591 AT$$

Total AT=3,978 + 1,591= 5,569 At;

Exciting current =5,569/200=27.35 A

Another Solution of Example 1.2

Assume $\Re_g$ and $\Re_c$ are the reluctance of air gab and core respectively. So;

$$\Re_g = \frac{\lg}{u_o A_g} = \frac{5 * 10^{-3}}{4\pi * 10^{-7} * 0.001} = 3.98 * 10^6 \; At / Web.$$

$$\Re_c = \frac{lc}{u_r u_o A_g} = \frac{1}{500*4\pi*10^{-7}*0.001} = 1.59*10^6\ At/Web.$$

The total reluctance is

$$\Re = \Re_g + \Re_c = 3.98*10^6 + 1.59*10^6 = 5.57*10^6 \text{ At/Web.}$$

Then Amper turn required is $\phi * \Re = .001*5.57*10^6 = 5.570$ At

Then exciting current is: $\dfrac{Total\ At}{nomber\ of\ turns} = \dfrac{5570}{200} = 27.35$ A

Example 1.3. *A ring of mean diameter 21 cm and cross-section 10 cm^2 is made up of semi-circular sections of cast steel and cast iron. If each joint has reluctance equal to an air gap of 0.2 mm as shown in Fig.1.8, find the Amp. turn required to produce a flux of* $5*10^{-4}$ *weber in the magnetic circuit. Take* u_r, *for steel and iron as 825 and 165 respectively: Neglect leakage and fringing.*

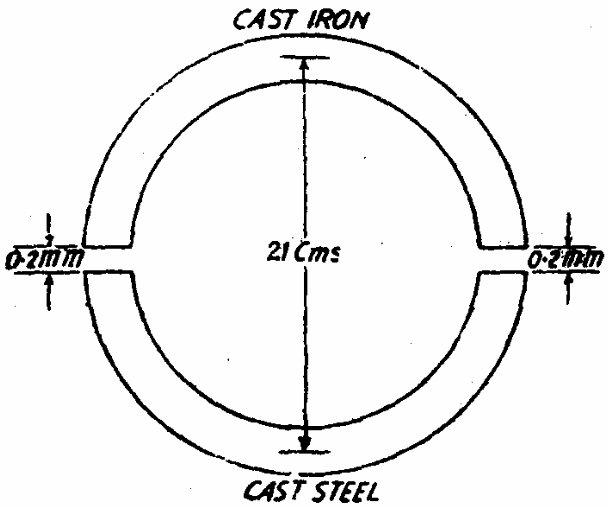


Fig.1.8

Solution:

ϕ=5 x 10^{-4} *Wb* ; *A*= 10 cm^2=10^{-3} m^2

Then, *B*=5 * $10^{-4}/10^{-3}$=0.5 *Wb/m²*

$$H = \frac{B}{u_o} = \frac{0.5}{4\pi * 10^{-7}} = 3.977 * 10^5 \ At/m$$

Air gap length is $= 0.2 * 2 = 0.4mm = 4 * 10^{-4} \ m$

Then, Ampere-turn required $= 3.977 * 10^5 * 4 * 10^{-4} = 159 At$

Cast steel path $H = \frac{B}{u_o u_r} = \frac{0.5}{4\pi * 10^{-7} * 825} = 482 At/m$

Cast steel path length $= \pi * D/2 cm$

$$= 21\pi/2 = 33cm = 0.33m$$

$$AT \ required = Hl = 482 * 0.33 = 159 At$$

Cast iron path $H = 0.5/4\pi * 10^{-7} * 165 = 2411 At/m$

Cast iron path length is:

$\pi * D/2 = \pi * 21/2 = 0.33m$

Ampere turn required =2411*0.33=795.6 *At*

Then the total Ampere turn required =159+159+795.6=1113.6 *At*

Example 1.4 *Two coils are wound on a toroidal core as shown in Fig. 1.9. The core is made of silicon sheet steel and has a square cross section. The coil currents are* $i_1 = 0.28$ *A and* $i_2 = 0.56$ *A.*

(a) Determine the flux density at the mean radius of the core.

(b) Assuming constant flux density (same as at the mean radius) over the cross section of the core, determine the flux in the core.

(c) Determine the relative permeability,

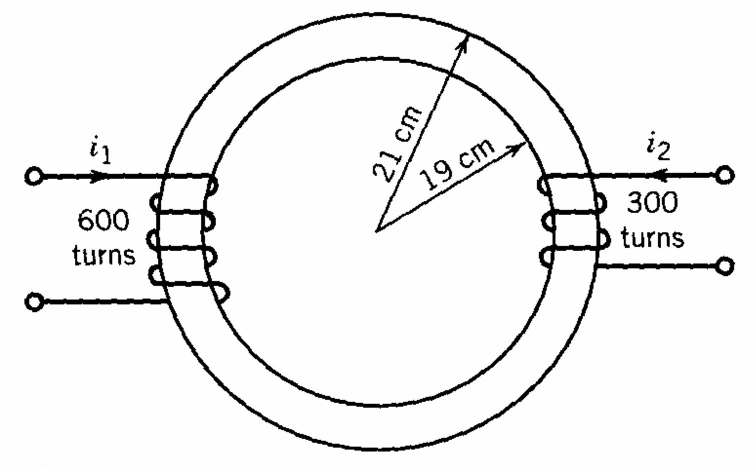


Fig.1.9

Solution:

The Two *mmf*s aid each other. Then,

mmf=600*0.28+300*0.56=336 *At*

$$H = \frac{336}{2\pi * 20 * 10^{-2}} = 267.38\ At/m$$

B=1.14 T From the curve shown in Fig.1.4

(b) ϕ=1.14*2*2*10^{-4}=0.000456 *Wb*

(c) $u_r = \dfrac{B}{u_0 H} = \dfrac{1.14}{4\pi * 10^{-7} * 267.38} = 3393$

Example 1.5 *The magnetic circuit of Fig. 1.10 provides flux in the two air gaps. The coils ($N_1 = 700$, $N_2 = 200$) are connected in series and carry a current of 0.5 ampere. Neglect leakage flux, reluctance of the iron (i.e., infinite permeability), and fringing at the air gaps. Determine the flux and flux density in the air gaps*

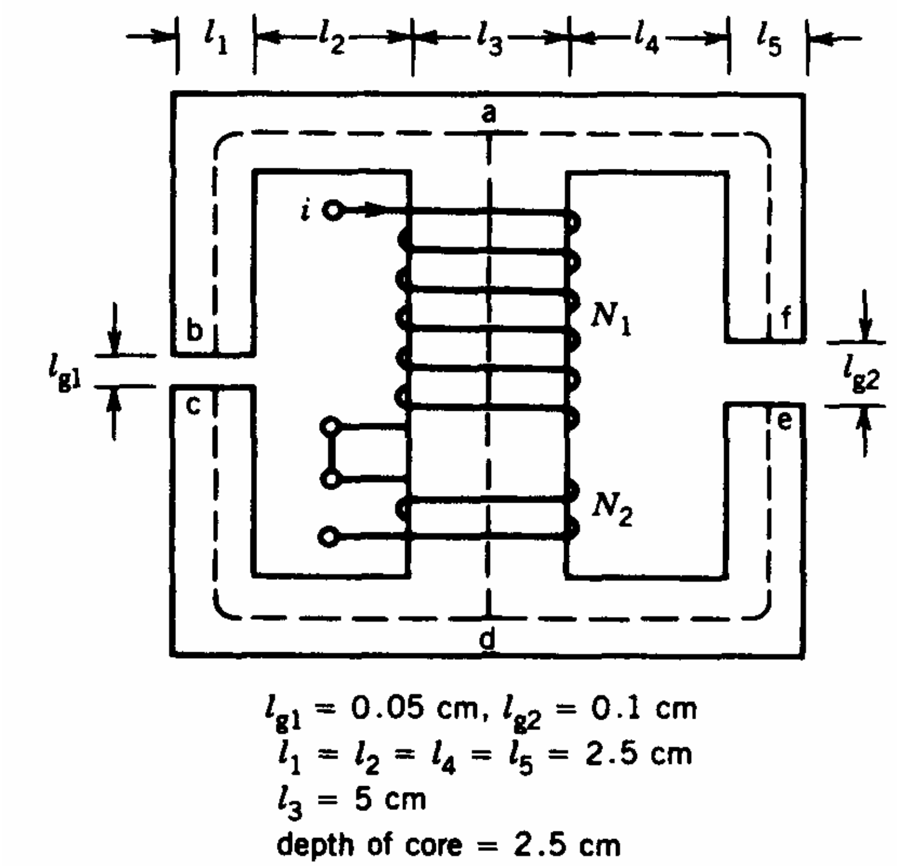


Fig.1.10

Solution:

*mmf*s of the two coils oppose each other

Then,

$$Ag_1 = Ag_2 = 2.5*2.5*10^{-4} = 6.25*10^{-4}\,m^2$$

$$R_{g1} = \frac{0.05*10^{-2}}{4\pi*10^{-7}*6.25*10^{-4}} = 0.637*10^{6}\ At/Wb$$

$$R_{g2} = \frac{0.1*10^{-2}}{4\pi*10^{-7}*6.25*10^{-4}} = 1.274*10^{6}\ At/Wb$$

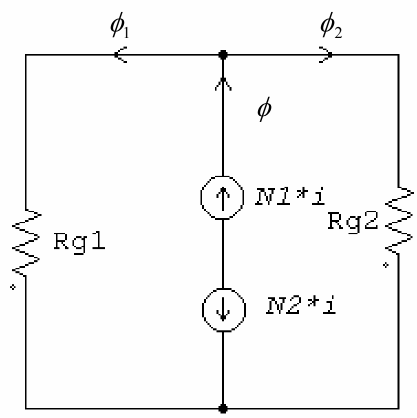
$$\phi_1 = \frac{(700-200)*0.5}{0.637*10^{6}} = 0.392*10^{-3}\ Wb$$

$$\phi_2 = \frac{500*0.5}{1.274*10^{6}} = 0.196*10^{-3}\ Wb$$

$$\phi = \phi_1 + \phi_2 = 0.588*10^{-3}\ Wb$$

Then, $B_{g1} = \dfrac{0.392*10^{-3}}{6.25*10^{-4}} = 0.627\ Wb/m^2$

And, $B_{g2} = \dfrac{0.196*10^{-3}}{6.25*10^{-4}} = 0.3135\ Wb/m^2$



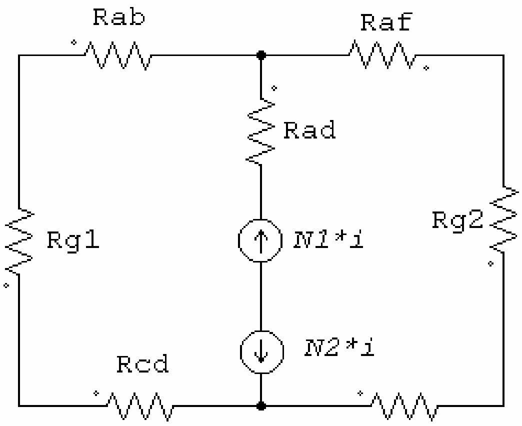


Fig.1.11 Fig.1.12

Example 1.6 *The electromagnet shown in Fig. 1.13 can be used to lift a length of steel strip. The coil has 500 turns and can carry a current of 20 amps without overheating. The magnetic material has negligible reluctance at flux densities up to 1.4 tesla. Determine the maximum air gap for which a flux density of 1.4 tesla can be established with a coil current of 20 amps. Neglect magnetic leakage and fringing of flux at the air gap.*

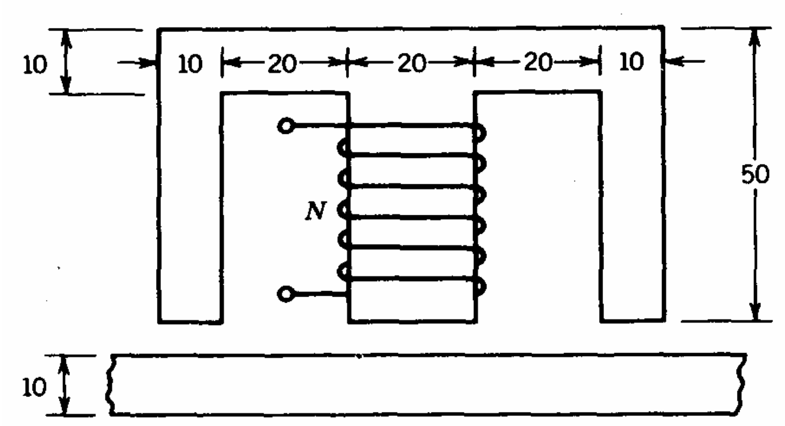


Fig.1.13

Solution:

B=1.4 T Throughout $H_c = 0$

$$Ni = H_{g1} * g_1 + H_{g2} * g_2$$

$$H_{g1} = H_{g2} = \frac{B}{u_0}$$

$$g_1 = g_2 = g$$

$$Ni = 2 * \frac{B}{u_0} g$$

$$g = \frac{u_0 Ni}{2B} = \frac{4\pi * 10^{-7} * 500 * 20}{2 * 1.4} = 4.5\ mm$$

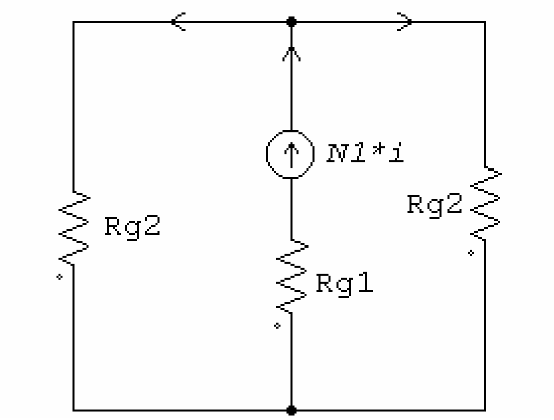


Fig.1.14

For the magnetic circuit shown in Fig.1.15 all dimensions are in cm and all the air-gaps are 0.5 mm wide. Net thickness of the core is 3.75 cm throughout. The turns are arranged on the center limb as shown. Calculate the mmf required to produce a flux of 1.7 mWb in the center limb. Neglect the leakage and fringing. The magnetization data for the material is as follows

H (At/m) :	400	440	500	600	800
B (Wb/m2) :	0.8	0.9	1.0	1.1	1.2

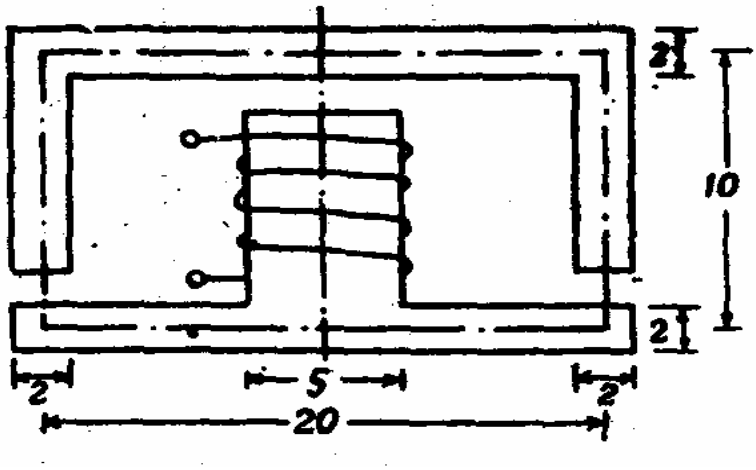
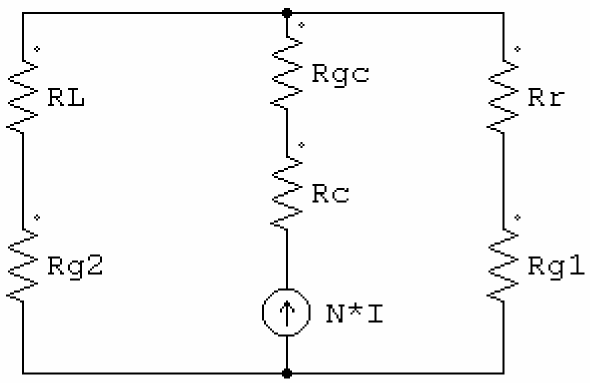


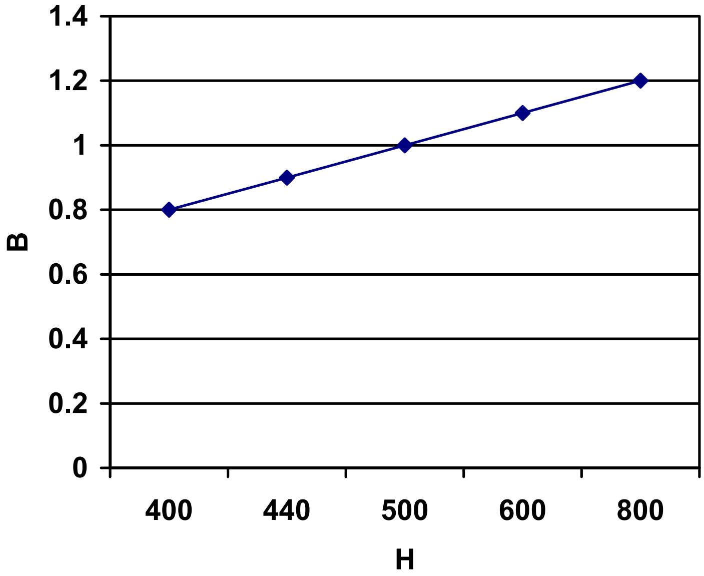
Fig.1.15



From the symetery of the magnetic circuit we can say that

$R_r = R_L$ and $R_{g1} = R_{g2}$

Also from the The magnetization data we can draw the following figure:-



The average value of $u_r u_o \cong \frac{1}{500} = 0.002$

$$R_r = \frac{30*10^{-2}}{0.002*2*3.75*10^{-4}} = 200000 At/Web..$$

$$R_{g1} = \frac{0.5*10^{-3}}{4\pi*10^{-7}*2*3.75*10^{-4}} = 530516.5 At/Web.$$

$$R_c = \frac{10*10^{-2}}{0.002*5*3.75*10^{-4}} = 26666 At/Web.$$

$$R_{gc} = \frac{0.5*10^{-3}}{4\pi*10^{-7}*2*3.75*10^{-4}} = 212206 At/Web.$$

$$NI = \phi * \left[R_c + R_{gc} + \frac{R_r + R_{g1}}{2} \right]$$

$$NI = 1.7 * 10^{-3} * \left[26666 + 212206 + \frac{2100000 + 530516.5}{2} \right] = 576\,\mathrm{At}$$

1.4 Electromagnetic Induction

In 1820 *Oersted* discovered the magnetic effect of an electric current, and the first primitive electric motor was built in the following year. *Faraday's* discovery of electromagnetic induction in 1831 completed the foundations of electromagnetism, and the principles were vigorously exploited in the rapidly growing field of electrical engineering. By 1890 the main types of rotating electrical machine had been invented, and the next forty years saw the development of many ingenious variations, along with refinement of the basic types. This was the golden age of machine development. Many machines are now obsolete which were once made in large numbers. Thus the cross-field DC machines, or rotary amplifiers, have been replaced by solid-state power amplifiers; while the Schrage motor and other ingenious variable-speed AC machines have given way to the thyristorcontrolled DC motor and the inverter-fed induction motor.

When a conductor moves in a magnetic field, an *EMF* is generated; when it caries current in a magnetic field, a force is produced. Both of these effects may be deduced from one of the most fundamental principles of electromagnetism, and they provide the basis for a number of devices in which conductors move freely in a magnetic field. It has already been mentioned that most electrical machines employ a different form of construction.

Faraday summed up the above facts into two laws known as *Faraday's* Laws of Electromagnetic Induction, it revealed a fundamental relationship between the voltage and flux in a circuit.

First Law states: -

Whenever the magnetic flux linked with a circuit changes, an EMF is always induced in it. Whenever a conductor cuts magnetic flux, an EMF is induced in that conductor.

Second Law states: -

The magnitude of the induced EMF is equal to the rate of change of flux-linkages.

Explanation. Suppose a coil has N turns and flux through it changes from an initial value of ϕ_1 webers to the final value of ϕ_2, webers in time t seconds. Then remembering that by flux-linkages is meant the product of number of turns by the flux linked with the coil, we have the following relation:

Initial flux linkages = $N\phi_1$. And final flux linkages = $N\phi_2$

Then the induced *EMF* is

$$e = \frac{N\phi_2 - N\phi_1}{t} = N\frac{\phi_2 - \phi_1}{t} = N\frac{d\phi}{dt}\ Volts$$

Usually a minus sign is given to the right-hand side expression to signify the fact that the induced *EMF* sets up current in such a direction that magnetic effect produced by it opposes the very cause producing it.

$$e = -N\frac{d\phi}{dt}\ Volts$$

Example 1.7 *The field coils of a 6-pole DC generator each having 500 turns, are connected in series. When the field is excited, there is magnetic flax of 0.02 Wb/pole. If the field circuit is opened in 0.02 second and the residual magnetism is 0.002 Wb/pole, calculate the average voltage that is induced across the field terminals. In which direction is this voltage directed relative to the direction of the current.*

Solution

Total number of turns, *N*=6*500=3000 turns

Total initial flux =6 *0.02=0.12 *Wb*;

Total residual flux= 6 * 0.002=0.012 *Wb,* Change in flux, $d\phi$=0.12-0.012=0.108 Wb;

Time of opening the circuit, dt = 0.02 second

Then the induced $EMF = \frac{d\phi}{dt} = 3000 * \frac{0.108}{0.02} = 16200\ V$

Example 1.8 *A coil of resistance 100 Ω is placed in a magnetic field of 1 mWb. The coil has 100 turns and a galvanometer of 400 Ω resistance is connected in series with it: Find the average EMF and the current if the coil is moved in 1/10th second from the given field to a field of 0.2 mWb.*

Solution:

Induced *Emf,* $e = \frac{d\phi}{dt}\ Volts$ volt

Here $d\phi$=1-0.2=0.8 mWb =8* 10^{-3} Wb

dt= 1/ 10=0.1 second ; N=100

$e = 100 * 0.8 * 10^{-3} / 0.1 = 0.8V$

Total circuit resistance =100+400=500 Ω

Current induced =0.8/500=1.6* 10^{-3} =1.6 mA

1.5 Direction Of Induced EMF And Current

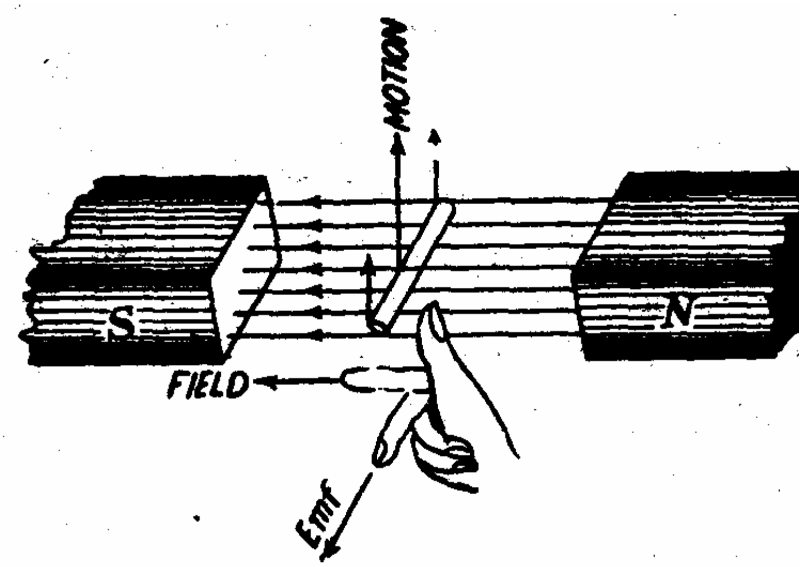
There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either *Fleming's Right-hand Rule* or

Lenz's Law. *Fleming's rule* is used where induced *EMF* is due to, flux cutting (i.e. dynamically induced. *EMF*) and *Lenz's* when it is due to change by flux linkages (i.e. statically induced *Emf*).

Fleming's Right-Hand Rule

"Hold out your right hand with forefinger, second finngure, and thumb at right angles to one another. If the forefinger represents the direction of the field, and the thumb represents the direction of the motion then, the second finger represents the direction of the induced emf in the coil".

Fleming's Right-hand Rule can be explained as shown in Figure



Lenz's Law

The direction of the induced current may also be found by this law which was formulated by Lenz.1835.

Lenz Law states, in effect, that electromagnetically induced current always flows in such a direction that the action of the magnetic field set up by it tends to oppose the very cause, which produces it.

This statement will be clarified with reference to Figs.1.15 and Fig.1.16. It is found that when *N*-pole of the bar magnet approaches the coil, the induced current setup by the induced *EMF* flaws in the anti-clockwise direction in the coil as seen from the magnet side. The result is that the face of the coil becomes a *N*-pole and so tends to retard the onward approach of the N pole' of the magnet (tike poles repel each other). The mechanical energy spent in overcoming this repulsive force is converted into electrical energy, which appears in the coil.

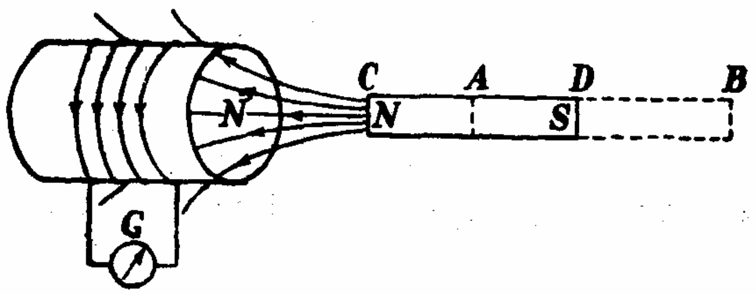


Fig.1.15

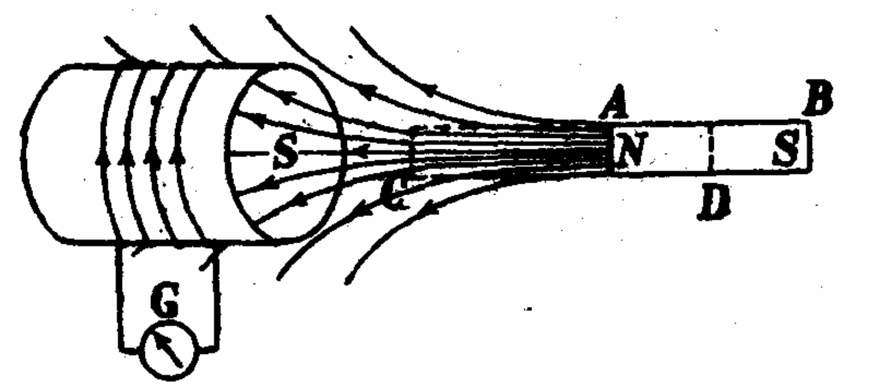


Fig.1.16

When the magnet is withdrawn as in Fig.1.16, the induced current flows in the clockwise direction, thus making the face of the coil (facing the magnet) a *S*-pole. Therefore, the *N*-pole of the magnet has to be withdrawn against the attractive force of the S-pole of the coil. Again the mechanical energy required to overcome this force of attraction is converted into electric energy.

It can be shown that the *Lenz's law* is a direct consequence of *law of conservation of energy*. Imagine for a moment that when N pole of the magnet (Fig.1.16) approaches the coil, induced current flows, in such a direction as to make the coil face a *S*-pole. Then due to inherent attraction between unlike poles, the magnet would be automatically pulled towards the coil without the expenditure of any mechanical energy. It means that we would be able to create electric energy out of nothing, which is denied by the inviolable

Law of Conservation of Energy. In fact, to maintain the sanctity of this law, it is imperative for the induced current to flow in such a direction that the magnetic effect produced by it tends to, oppose the very cause, which produces it. In the present case it is the relative motion of the magnet with respect to the coil which is the cause of the production of the induced current. Hence, the induced current always flows in such a direct as to oppose this relative motion (i.e., the approach or withdrawal of the magnet).

Electromagnetic Force

The basic principle of motor action is the so called *electromagnetic force or Lorentz force* production.

Lorentz force states that "when a current carrying conductor is placed in a magnetic field, it is subject to a force which we call Lorentz force ".

The magnitude of the force depends upon the orientation of the conductor with respect to the direction of the field. The force is greatest when the conductor is perpendicular to the field and zero when it is parallel to it. Between these two extremes, the force has intermediate values.

The maximum force acting on a straight conductor is given by

$F = Bli$

Where F : Is the force acting on the conductor (N),

B : Is the flux density of the field (T), and,

l : Is the length of the conductor facing the magnetic field(m).

i: the current in the conductor (A).

The direction of the magnetic force can be determined by using *Felming left hand rule*. Before going to show this rule, it is better to explain the physical meaning of the *lorentz force*. This can be easly explaind by with the help of the following two figures (Fig. And Fig.). For a current flowing into the page of this book, the circular lines of force have the direction shown in Figure 2.32a. The same figure shows the magnetic field created between the N, S poles of a powerful permanent magnet.

The magnetic field does not, of course, have the shape shown in the figure because lines of force never cross each other. What, then, is the shape of the resulting field?. To answer the question, we observe that the lines of force created respectively by the conductor and the permanent magnet act in the same direction above the conductor and in opposite directions below it. Consequently, the number of lines above the conductor must be greater than the number below. The resulting magnetic field therefore has the shape given in Figure 2.32b.

Recalling that lines of flux act like stretched elastic bands, it is easy to visualize that a force acts upon the conductor, tending to push it downward.

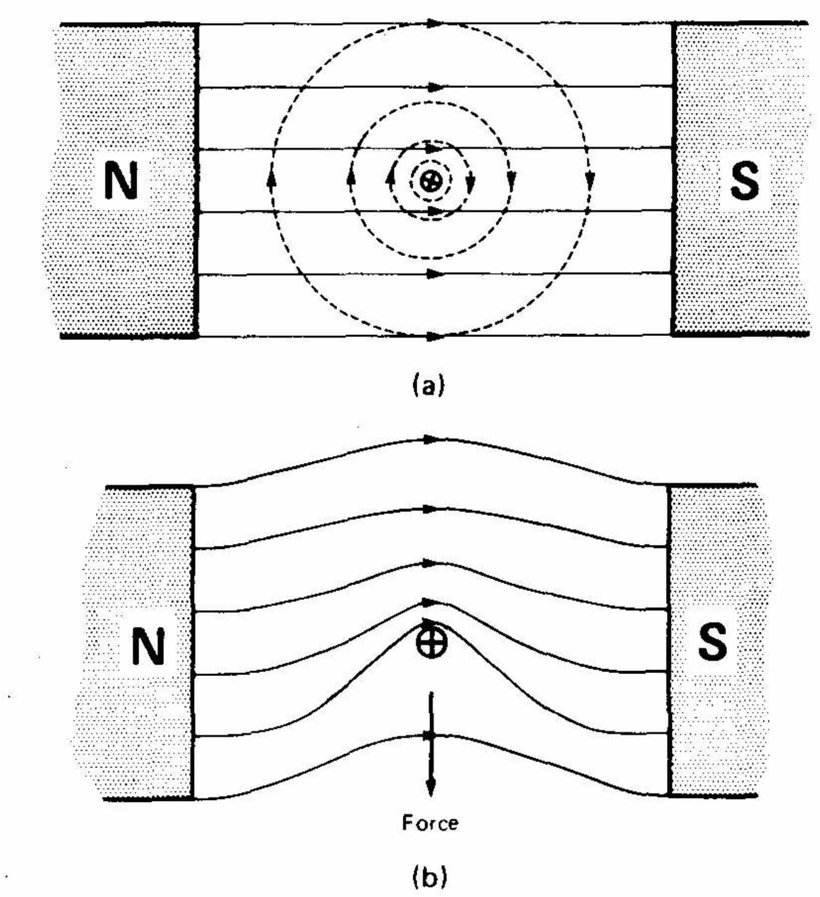
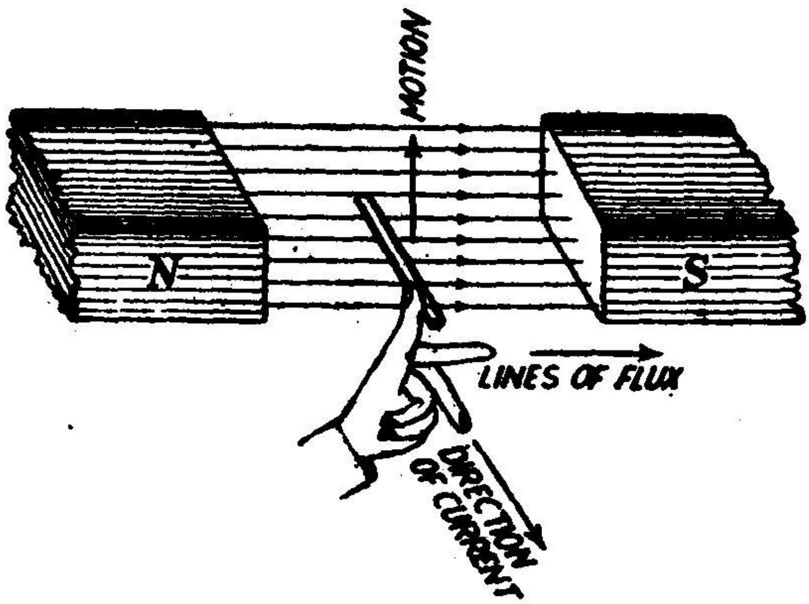


Figure 2.32
a. Magnetic field due to magnet and conductor.
b. Resulting magnetic field pushes the conductor downward.

Now let us Define *Felmeng left hand rule* It is illustrated in Fig. 6-9.

Felmeng left hand rule:

"*Hold out your left hand with forefinger, second finger and thumb at right angles to one another. If the forefinger represents the direction of the field, and the second finger that of the current, then thumb gives the direction of the motion or force*."



The direction of the force can also be determined by using the *right-hand screw rule*, illustrated in Fig.2.2(b).

Turn the current vector i toward the flux vector B. If a screw is turned in the same way, the direction in which the screw will move represents the direction of the force f.

Note that in both cases (i.e., determining the polarity of the induced voltage and determining the direction of the force) the moving quantities (v and i) are turned toward B to obtain the screw movement.

Equations (2.1) and (2.2) can be used to determine the induced voltage and the electromagnetic force or torque in an electric machine. There are, of course, other methods by which these quantities *(e* and *f)* can be determined.

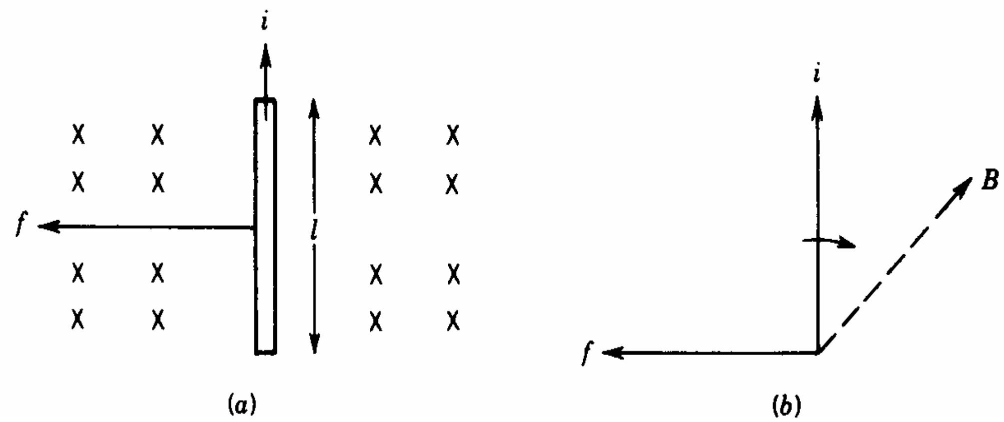


Fig.2.2 Electromagnetic force. (a) Current-carrying conductor moving in a magnetic field. (b) Force direction.

1.6 Coefficient of Self-Induction (L)

It may be defined in any one of the three ways given below

First Method for L The coefficient of self-induction of a coil is defined as *the weber turns per ampere in the coil*.

By weber-turns is meant *the product of flux in webers and the number of turns with which the flux is linked*. In other words, *it is the flux-linkages of the coil*.

Consider a solenoid having N turns and carrying a current of 1 ampere. If the flux produced is (ϕ webers, then weber-turns are $N * \phi$. Hence, weber-turns per ampere are $N\phi / I$. Then by definition, $L = N\phi / I$ Henry (1.14)

Hence, a coil is said to have a self-inductance of one Henry if a current of 1 ampere when flowing through it produces flux linkages of l Wb turn in it.

Example 1.9. *The field winding of a DC electromagnet is wound' with 960 turns and has resistance of 50Ω. When the exciting voltage is 230 V, the magnetic flux linking the coil is 0.005 Wb. Calculate the self-inductance of the coil and the energy stored in the magnetic field.*

Solution:

Current through the coil is 230/50=4.6 A

$$L = \frac{N\phi}{I} = \frac{960*0.005}{4.6} = 1.0435H$$

Energy stored is $\frac{1}{2}LI^2 = \frac{1}{2}*1.0435*4.6^2 = 11.84 Jouls$

In ***second method*** *for L as we know before from equation*

$$\phi = \frac{uNi}{l}A = \frac{Ni}{l/uA}$$

Then $\frac{\phi}{i} = \frac{N}{l/uA}$, but,

$$L = N*\frac{\phi}{I} = 960*\frac{0.005}{4.6} = 1.0435\ H$$

$$= N\frac{N}{l/uA} = \frac{N^2}{l/uA} = \frac{N^2}{\Re} \qquad (1.15)$$

Example 1.10 An *air-cored solenoid 1 cm in diameter and 1 meter long has an inductance of 0.1 mH. Find the number of effective turns on the coil.*

Solution:

$$A = \pi * r^2 = \pi * 0.005^2 = 7.85*10^{-5}\ m^2$$

$$\because L = \frac{N^2}{l/uA} \tag{1.16}$$

Then

$$N = \sqrt{L * l / uA} = \sqrt{\frac{0.1*10^{-3}*1}{(4\pi*10^{-7}*7.85*10^{-5})}} = 1007 \; turns$$

Third method for L,

As we know before that $L = \frac{N\phi}{I}$ then $LI = N\phi$

Differentiating both sides, we get:

$$-N\frac{d}{dt}\phi = -L\frac{d}{dt}I$$

But we know from Faraday law that $-N\frac{d}{dt}\phi =$ the induced *EMF*. Then if $\frac{dI}{dt} = 1 \; Ampere/\sec ond$, and *EMF=1*Voltthen *L*=1H.

Then, $e = L\frac{dI}{dt}$ (1.17)

Hence, a coil has a self-inductance of one hennery if one volt is induced in it when current through it changes at the rate of one ampere/second.

1.7 Mutual Inductance

L_{12} MUTUAL INDUCTANCE [H]

The mutual inductance between two coils.

$$L_{12} = \frac{N_2 \Lambda_{12}}{I_1} = \frac{N_2 N_1 \Psi_{12}}{I_1}$$

Neumann formula:

$$L_{12} = \frac{\mu_0 N_1 N_2}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{|\mathbf{r} - \mathbf{r}'|}$$

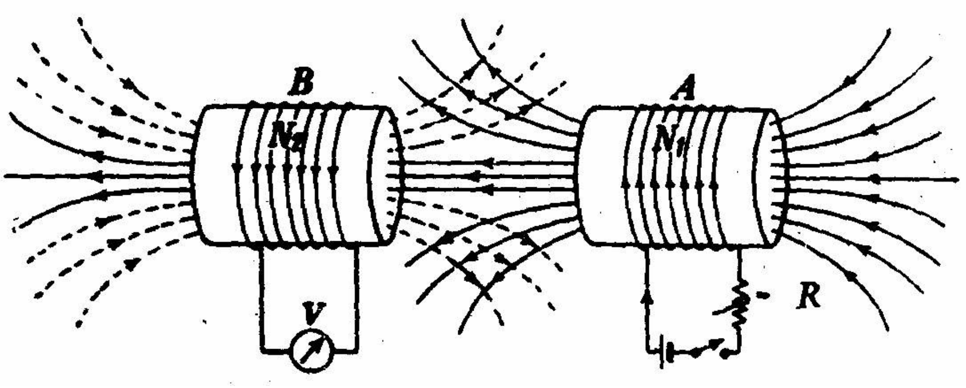
N = number of turns of the coil
Λ = flux linkage [Wb]
I = current [A]
Ψ = magnetic flux [Wb]
$\mathbf{r}$ = vector to the point of observation
$\mathbf{r}'$ = vector to source

Mutual inductance may be defined as *the ability of one coil (or circuit) to produce an EMF in a nearby coil by induction when the current in the first coil changes. This action being reciprocal, the second coil can also induce an EMF in the first when current in the second coil changes. This ability of reciprocal induction is measured in terms of the coefficient of mutual induction M.*

Mutual inductnce can also be defined in two ways as given below:

<u>First method for M</u>

Let there be two magnetically-coupled coils having N_1 and N_2 turns as shown in Fig.7.6. Coefficient of mutual inductance between two coils is defined as the Weber turns in one coil due to one Ampere current in the other.



Let a current of I_1 ampere when flowing in the first coil produces a flux ϕ_1 webers in it. It is supposed that whole of this flux links with the turns of the second coil. Then flux linkages in the second coil for unit current in the first coil are $N_2\phi_1 / I_1$. Hence by definition

$$M = \frac{N_2\phi_1}{I_1}$$

If Weber-turns in second coil due to one ampere current in the first coil i.e. $\frac{N_2\phi_1}{I_1} = 1$, then, as seen from above, M=1 H

Hence, two coils are said to have a mutual inductance of I henry if one ampere current flowing in one coil produces flux linkages of one Weber-turn in the other coil.

Example 7-13. Two indentical coils X and Y of 1,000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. If a current of 5 A flowing in X produces a flux of 0.5 mWb in it, find the mutual inductance between X and Y

Solution. Formula used $M = \frac{N_2 \phi_1}{I_1}$

Flux produced in X=0.05mWb=$0.05*10^{-3}$Wb

Flux linked witrh Y=$0.05*10^{-3}*0.8=0.04*10^{-3}$

Then,

$$M = \frac{1000 * 0.04 * 10^{-3}}{5} = 8 * 10^{-3} H = 8mH$$

Second Method for M

We will now deduce an expression for the coefficient of mutual inductance in terms of the dimensions of the two coils.

Flux in the first coil $\phi_1 = \frac{N_1 I_1}{l / u_o u_r A} Wb$

Flux per ampere=$\frac{\phi_1}{I_1} = \frac{N_1}{l / u_o u_r A}$

Assuming that whole of this flux (it usually is some percent of it) is linked with the other coil having N_2 turns, then Weber-turns in it due to the flux/ampere in the first coil is:

$$M = \frac{N_2 \phi_1}{I_1} = \frac{N_2 * N_1}{l / u_o u_r A}$$

Then $M = \frac{u_o u_r A N_1 N_2}{l} \ H$

Also, $M = \frac{N_1 N_2}{l / u_o u_r A} \ H = \frac{N_1 N_2}{\mathrm{Re}\,luc\tan ce}$

Example 7-15. Calculate the mutual inductance between two toroidal windings which are closely wound on an iron core of $u_r = 1000$. The mean radius of the toroid is 8 cm and the radius of the troid is 8 cm and the radius of its cross-section is 1 cm. Each winding has 1000 turns.

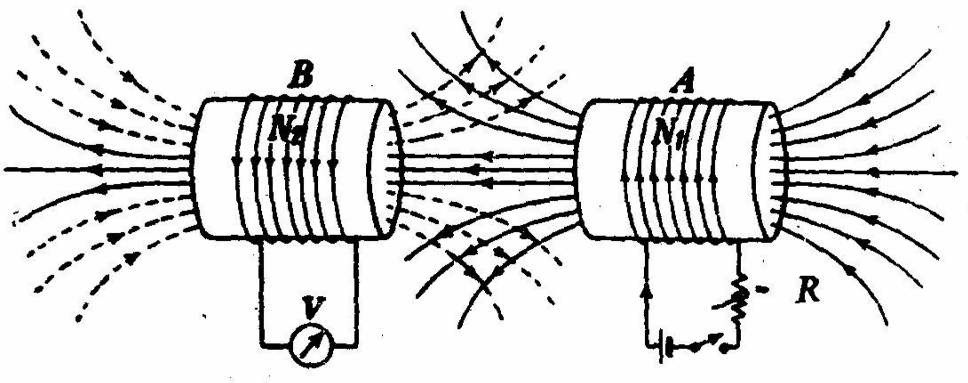
Solution

$Area = \pi * 1^2 = \pi cm^2 = \pi * 10^{-4} m^2$

$M = \frac{N_1 N_2}{l / u_o u_r A} \ H = \frac{N_1 N_2}{\mathrm{Re}\,luc\tan ce}$

$l = \pi * mean \ \ diameter = \pi * 16 \ \ cm = 16\pi * 10^{-2} \ m$

Then, $M = \frac{4\pi * 10^{-7} * 1000 * \pi * 10^{-4} * 1000^2}{16\pi * 10^{-2}} = 0.7855H$



1.8 Hysteresis loss

When a magnetic material is taken through a cycle of magnetization, energy is dissipated in the material in the form of heat. This is known as the *hysteresis loss.*

Transformers and most electric motors operate on alternating current. In such devices the flux in the iron changes continuously both in value and direction. The magnetic domains are therefore oriented first in one direction, then the other, at a rate that depends upon the frequency. Thus, if the flux has a frequency of 50 Hz, the domains describe a complete cycle every 1/50 of a second, passing successively through peak flux densities $+B_m$ and $-B_m$ as the peak magnetic field intensity alternates between $+H_m$ and $-H_m$. If we plot the flux density B as a function of H, we obtain a closed curve called *hysteresis loop* (Fig.1.18). The residual induction B_r and coercive force H_c have the same significance as before.

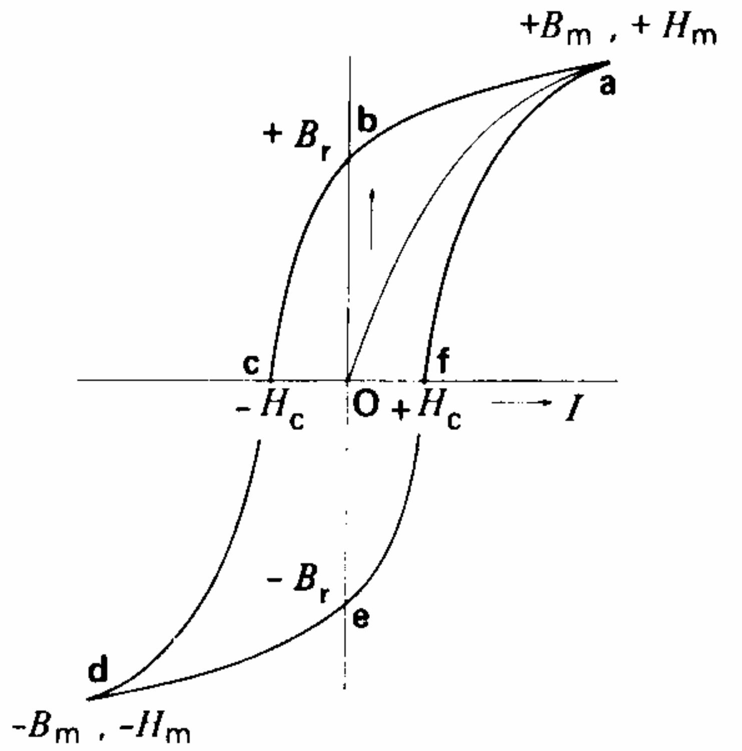


Figure 1.18 Hysteresis loop. If B is expressed in tesla and H in amperes per meter, the area of the loop is the energy dissipated per cycle, in joules per kilogram.

In describing a hysteresis loop, the flux moves successively from $+B_m$, $+B_r$ 0, $-B_m$, $-B_r$, 0, and $+B_m$, corresponding respectively to points a, b, c, d, e, f, and a, of Fig.1.18. The magnetic material absorbs energy during each cycle and this energy is dissipated as heat. We can prove that the amount of heat released per cycle (expressed in J/m^3) is equal to the area (in T-A/m) of the hysteresis loop. To reduce hysteresis losses, we select magnetic materials that have a narrow hysteresis loop, such as the grain-oriented silicon steel used in the cores of alternating current transformers.

So the net energy losses/cycle/m^3= (hystrisi loop area) Jule

Scale factors of B and H should be taken into considration while calculating the actual loop area. For example if the scale are 1 cm=x AT/m for H and 1cm=y Wb/m^2 for B Then,

$$W_h = xy * (area\ of\ \ BH\ \ loop)\ Joule / m^3 / cycle$$

It may be shown that the energy loss per unit volume for each cycle of magnetization is equal to the area of the hysteresis loop. The area of the loop will depend on the nature of the material and the value of B_{max} (Fig.1.18), and an approximate empirical relationship discovered by Steinmetz is:

$$W_h = \lambda_h B_{\max}^n \ \ Jouls / m^3 \qquad (1.18)$$

In this expression W_h is the loss per unit volume for each cycle of magnetization; the index n has a value of about 1.6 to 1.8 for many materials; and the coefficient λ_h is a property of the material, with typical values of 500 for 4 percent silicon steel and 3000 for cast iron.

When the material is subjected to an alternating magnetic field of constant amplitude there will be a constant energy loss per cycle, and the power absorbed is therefore proportional to the frequency.

Assuming the Steinmetz law, we have the following expression for the hysteresis loss per unit volume

$$P_h = \lambda_h B_{\max}^{1.6} \; f \quad watts/m^3 \qquad (1.19)$$

Where f is the frequency in Hertz.

Example The hysteresis loop of a sample of sheet steel subjected to a maximum flux density of 1.3 Wb/m^2 has an area of 93 cm^2, the scales being 1cm = 0.1 Wb/m^2 and 1 Cm =50 AT/m. Calculate the hysteresis loss in watts when 1500 cm^3 of the same material is subjected to an alternating flux density of 1.3 Wb/m^2 peak valve at a frequency of 65 Hz.

Solutyion:

$loss = xy * (areaofloop) J/m^3/cycle = 0.1*50*93 = 465 J/m^3/cycle$

Then volume=1500cm3

=1.5*10-3 m3

Then, wh=465*1.510-3*65j/s=45.3W

<u>Example 8-5 seraga</u> In a transformer core of volume 016 m^3 the total iron loss was found to be 2170 W at 50 Hz. The hysteresis loop of the core material, taken to the same maximum flux density, had an area of 9 cm^2 when drawn to scales of 1 cm=0.1 Wb/m^2 and 1 cm=250 AT/m. Calculate the total iron loss to the transformer

core if it is energised to the same maximum flux density, but at a frequency of 60 Hz. ,

Solution. W_h=xy * (area of hysteresis loop)

where x and y are the scale factors.

W_h=9 x 0.1 x250=225 J/m^3/cycle

At 50 Hz:

Hystrisis loss=225*0.16*50=1800W

Eddy current loss=2170-1800=370 W

At 60 Hz

Hystris loss=1800*60/50=2160W

Eddy current loss=370*(60/50)^2 =533W

Then total loss=2160+533=2693W

Example 8-7. The area of the hysteresis loop obtained with a m certain specimen of iron was 9.3 cm^2. The coordinates were such that 1 cm=1000 AT/m and 1cm=0.2 Wb/m^2, calculate (a)Hysteresis loss per m^3 per cycle and (b) the hysteresis loss per m^3 at a frequency of 50 Hz if the maximum flux density were 1.5 Wb/m^2 (c) calculate the hysteresis loss per m^3 for a maximum flux density of 1.2 wb/m2 and a frequency of 30 Hz, assuming the loss to be proportional to $B_{\max}^{1.8}$

Solution:

(a) $W_h = xy*(area\, of\; BH\; loop)$

$=1000*0.2*9.3=1860\,J/m^3\;/cycle$

(b) $W_h = 1860*50J/s/m^3 = 93000\;W/m^3$

(C) $W_h = \eta\, B_{\max}^{1.8} fV\;\; W$

For given specimen, $W_h \alpha\, B_{ma}^{1.} f$

In (b) above , $93000\alpha 1.5^{1.8} *50$ and $W_h \alpha 1.2^{1.8} *30$

Then, $\dfrac{W_h}{93000} = \left(\dfrac{1.2}{1.5}\right)^{1.8} * \dfrac{30}{50}$,Wh

=93000*0.669*0.6=37360W

Example 8.8(seraga). Calculate the loss of energy caused by hysteresis in one hour in 50 kg of iron if the peak flux density reached is 1.3 Wb/m^2 and the frequency is 25 Hz. Assume Steinmetz iCient as 628 J/m^3 and density of iron as 7.8*10^3 kg/m^3. What will be the area of BH curve or this specimen if 1 cm =12.5 AT/m and 1 cm=0.1 Wb/m^2.

Solution:

$W_h = \eta\, B_{\max}^{1.6} f\;\; V\;\; W$

$Volume\;\; V = \dfrac{50}{7.8*10^3} = 6.41*10^{-3}\,m^3$

Then, $W_h = 628 * 1.3^{1.6} * 25 * 6.41 * 10^{-3} = 153 J/s$

Loss in iron in one hour=153*3600=551200J

As per Steinmetz law, hystrisi loss=$\eta\ B_{\max}^{1.6}\ J/m^3 cycle$

Also, Hystrisi loss=xy*(area of BH loop)

Equating the two we get:

$628 * 1.3^{1.6} = 12.5 * 0.1 * loop\ \ area$

Then $loop\ \ area = 764.3\, cm^2$

1.10 Eddy current losses

If a closed loop of wire is placed in an alternating magnetic field, the induced *EMF* will circulate a current round the loop. A solid block of metal will likewise have circulating currents induced in it by an alternating field, as shown in Fig.1.19. These are termed *eddy currents*, and they are a source of energy loss in the metal. Eddy current losses occur whenever conducting material is placed in a changing magnetic field; the magnitude of the loss is dependent on the properties of the material, its dimensions and the frequency of the alternating field.

Magnetic structures carrying alternating magnetic flux are usually made from a stack of thin plates or laminations, separated from one another by a layer of insulation (Fig.1.20). This construction breaks up the eddy current paths, with a consequent reduction in the loss; qualitatively, the effect may be explained as follows. With solid metal (Fig.1.19) the currents would flow in approximately square paths; these paths enclose a large area for a given perimeter, and the induced *EMF* is high for a path of given resistance. When the metal is divided into laminations (Fig.1.20), the current paths are long narrow rectangles; the area enclosed by a given perimeter is much smaller, and the induced *EMF* is smaller, giving lower currents and reduced losses.

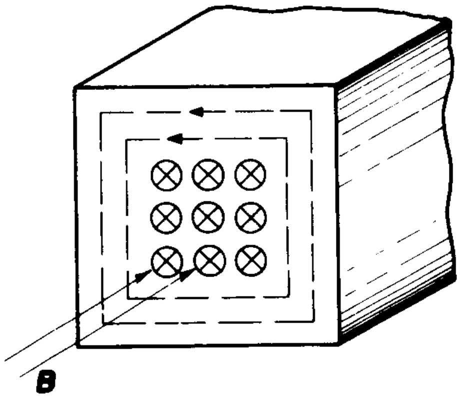


Fig.1.19

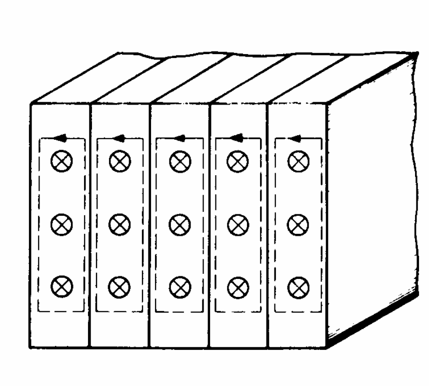


Fig.1.20

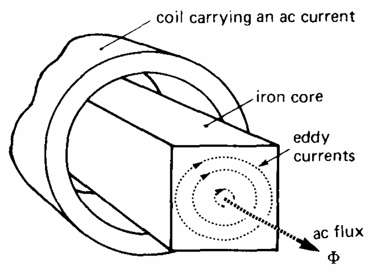


Figure 2.39a
Solid iron core carrying an ac flux.

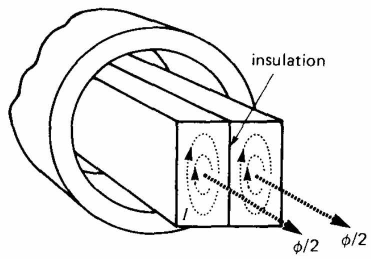


Figure 2.39b
Eddy currents are reduced by splitting the core in half.

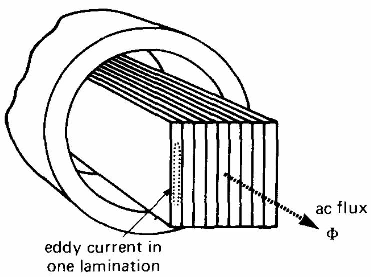


Figure 2.39c
Core built up of thin, insulated laminations.

An approximate analysis shows that in plates of thickness t (where t is much smaller than the width or length) the eddy current loss per unit volume is given by

$$p_e = \frac{\pi^2 B_{\max}^2 f^2 t^2}{6\rho} \qquad (1.20)$$

where the flux density is an alternating quantity of the form,

$$B = B_{\max} \sin 2\pi f t \tag{1.21}$$

and ρ is the resistivity of the material. Thus if the lamination thickness is reduced by a factor x, the loss is reduced by a factor x^2. As might be expected, the loss varies inversely with the resistivity ρ. The addition of 3-4 percent of silicon to iron increases the resistivity by about four times, as well as reducing the hysteresis loss; this is the main reason for the widespread use of silicon steel in electrical machines. The thickness of the laminations is typically 0.3-0.5 mm, which ensures that the eddy current loss will be less than the hysteresis loss at a frequency of 50 Hz.

1.11 Skin Effect

The eddy currents in a bar such as the one shown in Fig.1.19 will produce a magnetic field within the bar, which by *Lenz's law* will oppose the applied field. Thus the magnetic flux density will fall from a value B_o at the surface to some lower value in the interior. The effect depends on the properties of the material, the frequency of the alternating field and the dimensions of the bar. It is possible for the magnitude of the flux density to fall very rapidly in the interior of the bar, so that most of the flux is confined to a thin layer or skin near the surface. The phenomenon is termed *skin* *effect*, and it implies very inefficient use of the magnetic material (quite apart from any eddy current losses). A similar effect occurs in conductors carrying alternating current, where the current density falls from some value J_O at the surface to a lower value in the interior.

The phenomenon of *skin effect* gives a second reason for using laminated magnetic circuits. If the thickness of a plate is much more than twice the depth of penetration b, the central region will carry very little flux. The material will be fully utilized if it is divided into laminations less than b in thickness, for the flux density will then be fairly uniform across the lamination. The depth of penetration in silicon steel is about 1 mm at a frequency of 50 Hz, so the typical lamination thickness of 0.5 mm ensures that skin effect will not be significant.

Permanent magnet materials

Permanent magnets find wide application in electrical measuring instruments, magnetos,; magnetic chucks and moving-coil loudspeakers etc. . In permanent magnets, high retentivity as welt as high coercivity are most desirable in order to resist demagnetisation. In fact, the product B,H, is the best criterion for the merit of a permanent magnet. The material commonly used for such purposes is carbon-free iron-nickel-aluminium-copper-cobalt alloys which are made anisotropic by heating to a very high temperature and then cooling in a strong magnetic field. This alloy possesses BrFIa value of about 40,000 J/m' as compared with 2,500 J/m' far chromium-steel.

Problems

١- *An iron ring of mean length 10 cm has as air gap of 1 mm and a winding of 200 turns. If the relative permeability of iron is 300 when a current of 1A Rows through the coil, find the flux density.*

٢- *Steel ring has the following particulars Mean diameter of the ring =30 cm; flux density of I Wb/m' is produced by 4000 At/m. The cross-section of the ring is 10 cm^2, number of turns wound =600 of If a gap of 1 cm is cut in the ring and the flux density of 1 Web/m2 is maintained in the air gap, determine the inductance.*

٣- *The magnetic circuit made of wrought iron is arranged as shown in Fig.1.21 below. The central limb has a cross-sectional area of 8 cm^2 and each of the side limb has a cross-sectional area of 5 cm^2. Calculate the ampere-turns required to produce a flux of 1 milli-weber in the central limb, neglecting magnetic leakage and fringing. The magnetization of wrought iron is given by*

B-flux density (Wb/m^2)	100	1.25	
H-in At/m		200	500

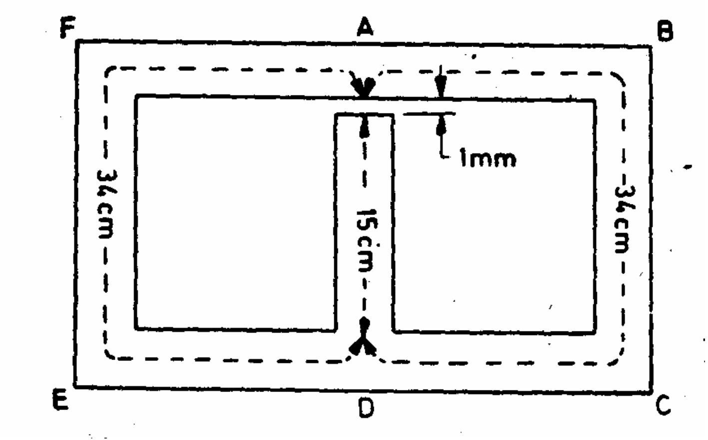


Fig.1.21

٤- *A cast-steed DC electromagnet shown in Fig.1.22 has a coil of 1000 turns on its central limb. Determine the current that the coil should carry to produce a flux of 2.5 mWb in the. air-gap. Neglect leakage. Dimensions are given in cm: The magnetization curve for cast steel is as under*

Flux density (Wb/my) :~	*0.2*	*0.5*	*0.7*	*l.0*	*1.2*
Amp-turns/meter :	*300*	*540*	*650*	*900*	*1150*

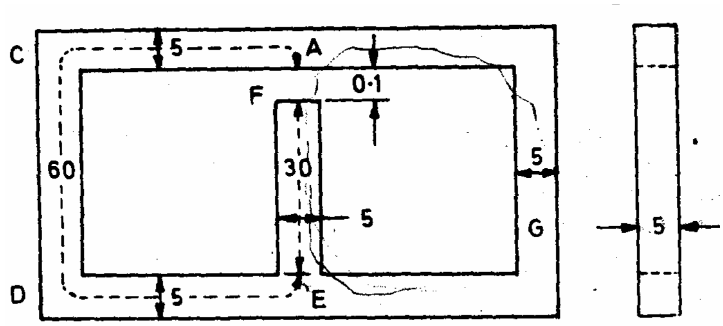


Fig.1.22

6- A cast-steel magnetic structure made of a bar of section 2 cm^2 cm is shown in Fig.1.23. Determine the current that the 500-turn magnetizing coil on the left limb should carry so that a flux of 2 mWb is produced in the right limb. Take u_r=600 and neglect leakage.

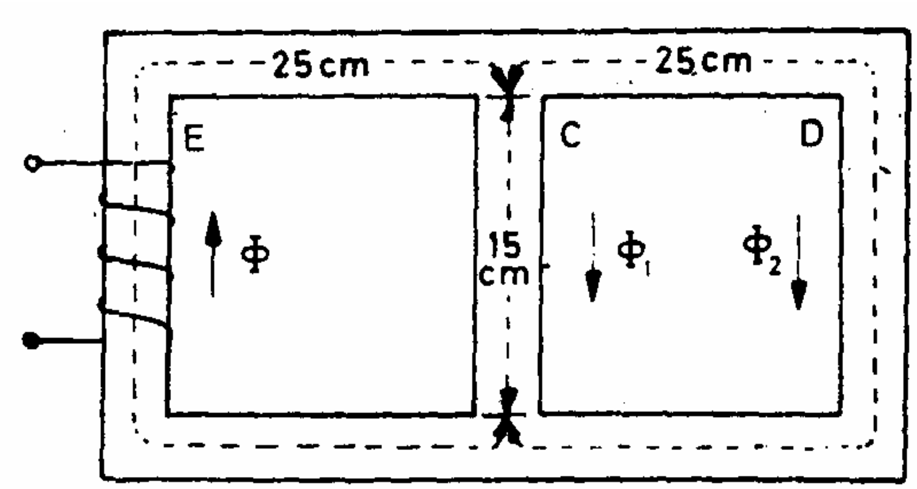


Fig.1.23

7- A magnetic circuit consists of an iron ring of mean circumference 80 cm with cross-sectional area 12 cm^2. A current of 2 A in the magnetizing coil of 200 turns produces a total flux of 12 mWb in the iron. Calculate :-

(a) the flux density in the ion

(b) the absolute and relative permeability of iron.

(c) the reluctance of the circuit.

٨- A series magnetic circuit has an iron path of length 50 cm and an air-gap of length 1 mm. The cross-sectional area of the iron is 6 cm^2 and the exciting coil has 400 turns. Determine the current required to produce a flux of 0.9 mWb in the circuit. The following points are taken from the magnetization characteristic

Flux density (Wb/m^2)	1.2	1.35	1.45	1.55
Magnetizing force (At/m)	500	1,000	2,000	4,500

9- The magnetic circuit of Fig.1.24 has a core of relative permeability u_r = 2000. The depth of the core is 5 cm. The coil has 400 turns and carries a current of 1.5 A.

(a) Draw the magnetic equivalent circuit. (b) Find the flux and the flux density in the core. (c) Determine the inductance of the coil.

(d) Repeat (a) ,(b) and (c) for a 1.0cm wide air gap in the core. Assume a 10% increase in the effective cross-sectional area of the air gap due to fringing in the air gap.

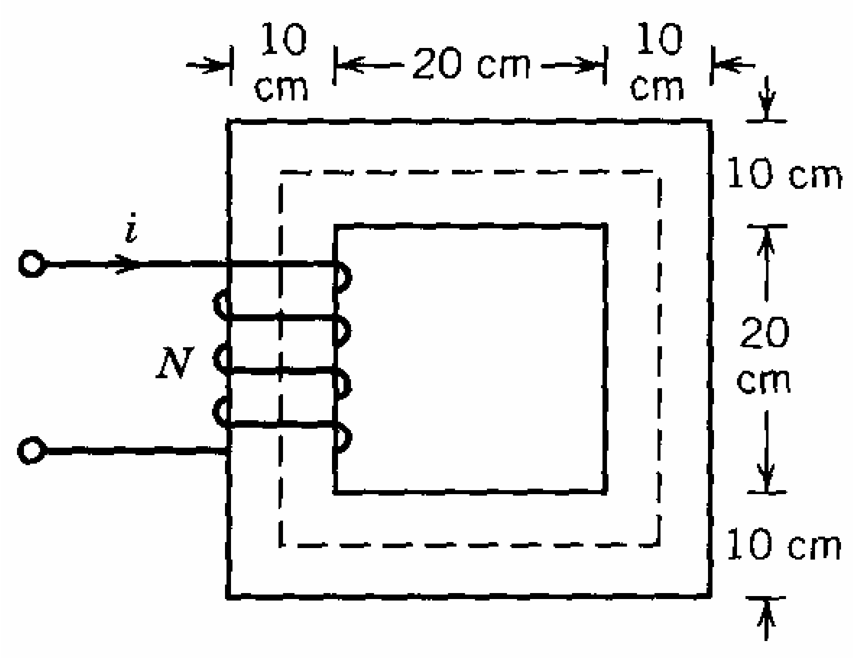


Fig.1.24

10 For the magnetic circuit shown in Fig.1.25 all dimensions are in cm and all the air-gaps are 0.5 mm wide. Net thickness of the core is 375 cm throughout. The turns are arranged on the center limb as shown.

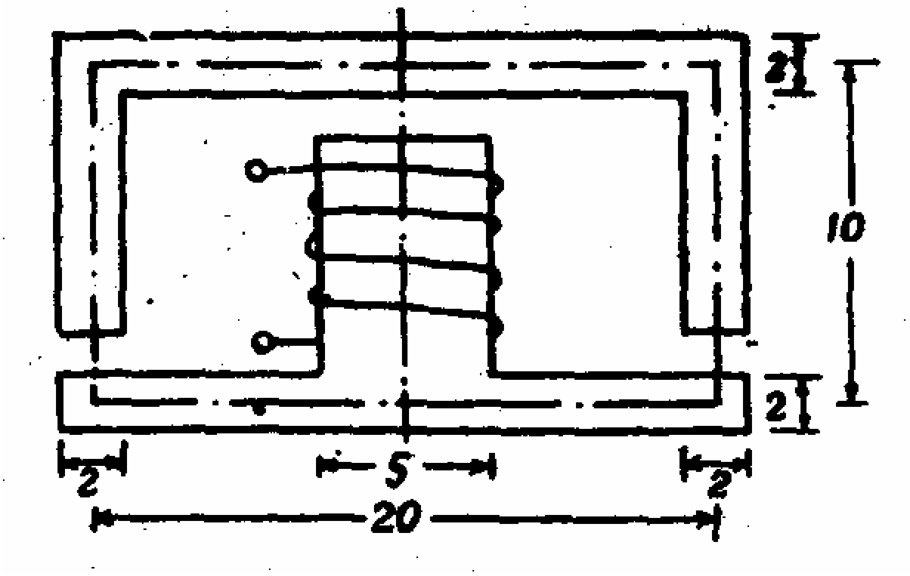


Fig.1.25

Calculate the mmf required to produce a flux of 1.7 mWb in the center limb. Neglect the leakage and fringing. The magnetization data for the material is as follows

H (At/m) :	*400*	*440*	*500*	*600*	*800*
B (Wb/m) :*	*0.8*	*0.9*	*1.0*	*I.I*	*1.2*