# Home assignment

Obtain slip at maximum torque $s_{Tmax}$ by setting $dTm / ds = 0$?

$$T_m = \frac{1}{\omega_s / P} \times \frac{3V_s^2}{(R_s + R_r / s + R_{eq} / s)^2 + (X_{ls} + X_{lr} + X_{eq} / s)^2} \times \frac{R_r + R_{eq}}{s}$$

$$s_{T\max} = \pm \sqrt{\frac{(R_r + R_{eq})^2 + X_{eq}^2}{R_s^2 + (X_{ls} + X_{lr})^2}}$$

**8-4 (Solved Problem)** Consider a 1.0MW/575V/60Hz/2160rpm DFIG WECS. The parameters of the generator are given in Table B-6 of Appendix B. Generator operates with an MPPT scheme & its stator power factor is unity. At a given wind & generator speed, equivalent resistance *Req* & reactance *Xeq* for rotor side converter & maximum torque to mechanical torque ratio *T*max/*Tm* are found to be 0.03778Ω, 0.02165Ω & 4.4123 respectively.

Calculate following:

a)maximum torque & corresponding slip,

b) generator mechanical torque & power,

c) rotor mechanical & electrical speeds & slip,

d) rms stator current &

e) rms rotor current and voltage.

| **Generator Type** | **DFIG, 1.0MW/575V/60Hz** | |
|---|---|---|
| Rated Mechanical Power | 1.0 MW | 1.0 pu |
| Rated Stator Line-to-line Voltage | 575 V (rms) | |
| Rated Stator Phase Voltage | 331.98 V (rms) | 1.0 pu |
| Rated Rotor Phase Voltage | 67.97 V (rms) | 0.2047 pu |
| Rated Stator Current | 829.2 A (rms) | 0.8258 pu |
| Rated Rotor Current | 882.2 A (rms) | 0.8786 pu |
| Rated Stator Frequency | 60 Hz | 1.0 pu |
| Rated Rotor Speed | 2160 rpm | 1.0 pu |
| Nominal Rotor Speed Range | 1350–2160 rpm | 0.625–1.0pu |
| Rated Slip | -0.2 | |
| Number of Pole Pairs | 2 | |
| Rated Mechanical Torque | 4.421 kN.m | 1.0 pu |
| Stator Winding Resistance $R_s$ | 3.654 mΩ | 0.0111 pu |
| Rotor Winding Resistance $R_r$ | 3.569 mΩ | 0.0108 pu |
| Stator Leakage Inductance $L_{ls}$ | 0.1304 mH | 0.1487 pu |
| Rotor Leakage Inductance $L_{lr}$ | 0.1198 mH | 0.1366 pu |
| Magnetizing Inductance $L_m$ | 4.12 mH | 4.6978 pu |
| Base Current $I_B = 1\text{MW}/(\sqrt{3} \times 575\text{V})$ | 1004.1 A (rms) | 1.0 pu |
| Base Flux Linkage $\Lambda_B$ | 0.8806 Wb (rms) | 1.0 pu |
| Base Impedance $Z_B$ | 0.3306 Ω | 1.0 pu |
| Base Inductance $L_B$ | 0.877 mH | 1.0 pu |
| Base Capacitance $C_B$ | 8022.93 μF | 1.0 pu |

# Solution:

a) +ve values for *Req*= 0.03778Ω & *Xeq*= 0.02165Ω indicate super-synchronous mode of operation.

- -ve values indicate sub-synchronous operation.
- 0 value for *Xeq* indicate synchronous operation.

# Slip at which the maximum torque occurs can be obtained

| | |
|---|---|
| Stator Leakage Inductance $L_{ls}$ | 0.1304 mH |
| Rotor Leakage Inductance $L_{lr}$ | 0.1198 mH |

$Req = 0.03778\Omega$ & $Xeq = 0.02165\Omega$

$$s_{T\max} = \pm\sqrt{\frac{(R_r + R_{eq})^2 + X_{eq}^2}{R_s^2 + (X_{ls} + X_{lr})^2}} = -0.4944$$

($s = +0.4944$ is omitted because of the super-synchronous mode of operation)

# The maximum torque:

$$T_{\max} = \frac{1}{2\omega_s / P} \times \frac{3V_s^2}{R_s + \frac{(X_{ls} + X_{lr})X_{eq}}{R_r + R_{eq}} - \sqrt{\left((X_{ls} + X_{lr})^2 + R_s^2\right) \times \left(1 + \frac{X_{eq}^2}{(R_r + R_{eq})^2}\right)}} = -16391\,\text{N.m} \quad (3.708\ \text{pu})$$

# b) The generator mechanical torque:

$$T\text{max}/Tm = 4.4124\Omega$$

$$T_{\max} = -16391\ \text{N.m}$$

$$T_m = \frac{T_{\max}}{T_{\max}/T_m} = \frac{-16391}{4.4124} = -3714.8\ \text{N.m}$$

# Generator mechanical torque can be related to pu rotor speed as

$$T_m = T_{m,R} \times (\omega_{m,\text{pu}})^2 \text{ N.m}$$

From which pu rotor speed can be calculated by

$$\omega_{m,\text{pu}} = \sqrt{\frac{T_m}{T_{m,R}}}$$

$$T_m = -3714.8 \text{ N.m}$$

Rated Mechanical Torque=4.421kN.m

$$\omega_{m,\text{pu}} = \sqrt{\frac{-3714.8}{-4421}} = 0.9167$$

# Rated mechanical power:

Rated Mechanical Torque=4.421kN.m

$$P_{m,R} = \omega_{m,R} \times T_{m,R} = 2160(2\pi)/60 \times (-4421) = -1000 \times 10^3 \text{ W}$$

# Generator mechanical power at 0.91667 pu rotor speed:

$$P_m = P_{m,R} \times (\omega_{m,\text{pu}})^3 = -1000 \times 10^3 \times (0.9167)^3 = -770.26 \times 10^3 \text{ W}$$

# c) Rotor mechanical & electrical speeds:

$$\omega_m = \omega_{m,R} \times \omega_{m,\text{pu}} = 2160(2\pi)/60 \times 0.9167 = 207.35 \text{ rad/sec} \quad (1980 \text{ rpm})$$

$$\omega_r = \omega_m \times P = 207.35 \times 2 = 414.7 \text{ rad/sec}$$

The slip can be obtained as:

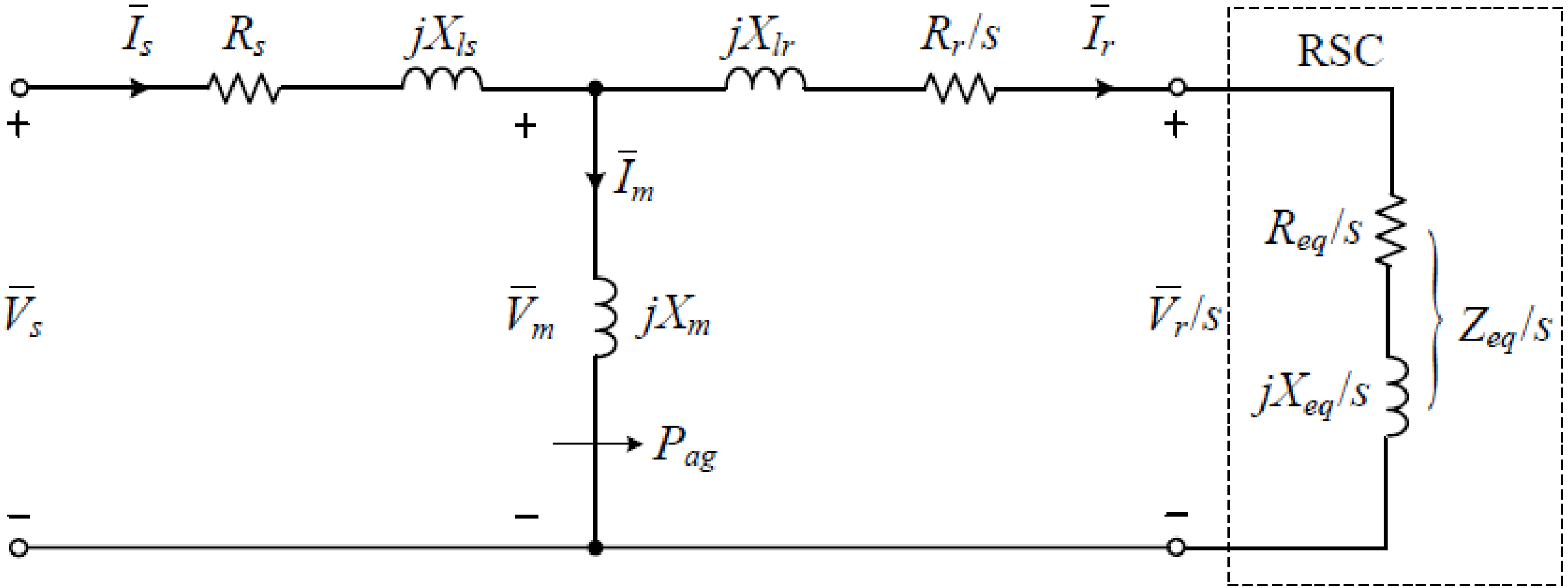
$$s = (\omega_s - \omega_r)/\omega_s = (376.99 - 414.7)/376.99 = -0.1$$

# d) The stator current:

$$I_s = \frac{V_s \pm \sqrt{V_s^2 - \dfrac{4R_s T_m \omega_s}{3P}}}{2R_s} = -697.74\,\text{A (rms)} \quad (I_s = 91.55\times 10^3\,\text{A omitted})$$

where $V_s = 575/\sqrt{3}\,\text{V}$, $T_m = -3714.8\,\text{N.m}$, $\omega_s = 376.99\,\text{rad/sec}$, $R_s = 3.654\,\text{m}\Omega$ and $P = 2$

# e) Find *Ir*?


$\bar{I}_s$
$R_s$
$jX_{ls}$
$jX_{lr}$
$R_r/s$
$\bar{I}_r$
RSC
+
+
+
$\bar{I}_m$
$R_{eq}/s$
$\bar{V}_s$
$\bar{V}_m$
$jX_m$
$\bar{V}_r/s$
$Z_{eq}/s$
$jX_{eq}/s$
$P_{ag}$
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−
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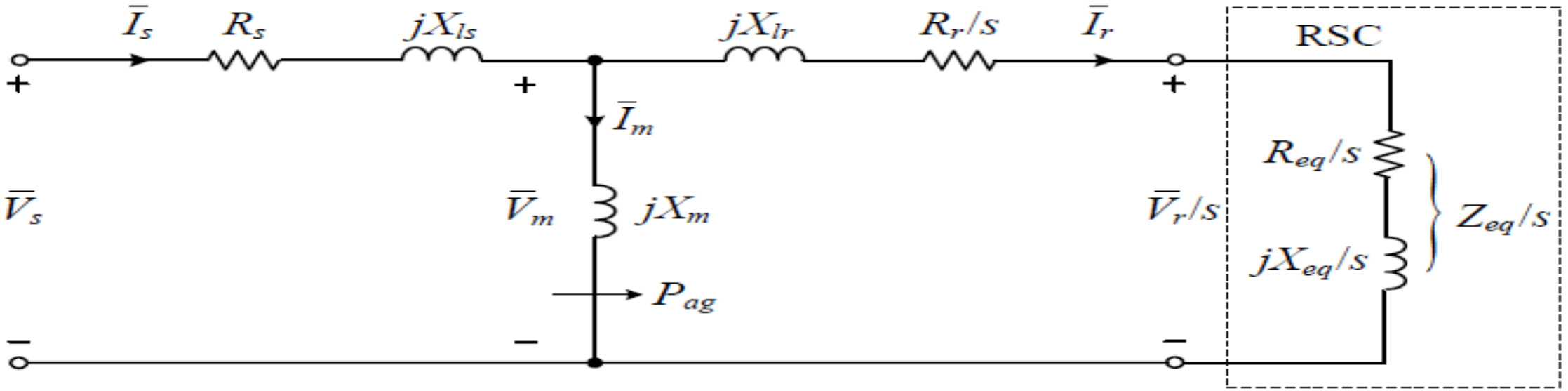
# e)Rotor current(*Ir*) can be calculated by




$$\bar{I}_r = \frac{jX_m \bar{I}_s}{jX_m + \left(\frac{R_r}{s} + jX_{lr}\right) + \left(\frac{R_{eq}}{s} + j\frac{X_{eq}}{s}\right)} = 751.35\angle 163.34^\circ \text{ A (rms)}$$

Alternatively, rms rotor current can be found from mechanical torque equation:

$$T_m = \frac{1}{\omega_s / P} \times 3 I_r^{\cdot 2} \left( R_{eq} + R_{r\cdot} \right) / s \ \ \text{N.m}$$

From above

$$I_r = \sqrt{\frac{T_m \times (\omega_s / P)}{3 (R_{eq} + R_r) / s}} = 751.35 \ \ \text{A (rms)}$$

# Equivalent impedance for the rotor side converter:

$$\bar{Z}_{eq} = \bar{V}_r / \bar{I}_r = 0.0378 + j0.02165 = 0.04354\angle 29.82^\circ\ \Omega\ \text{(given)}$$

The rotor voltage can be obtained f $\bar{Z}_{eq} = \bar{V}_r / \bar{I}_r$

$$\bar{V}_r = \bar{Z}_{eq} \times \bar{I}_r = 32.72\angle -166.84^\circ \text{ V (rms)}$$

# Cross Check:

$$T_m = \frac{1}{\omega_s / P} \times 3I_r^2 \left(R_{eq} + R_r\right)/s = \frac{1}{2\pi \times 60/2} \times 3 \times 751.35^2 \left(0.0378 + 3.569 \times 10^{-3}\right)/\left(-0.1\right) = -3714.87 \text{ N.m, verified.}$$

$$P_m = 3I_r^2 \left(R_{eq} + R_r\right)\left(1 - s\right)/s = 3 \times 751.35^2 \left(0.0378 + 3.569 \times 10^{-3}\right)\left(1 + 0.1\right)/\left(-0.1\right) = -770.26 \times 10^3 \text{ W, verified.}$$

**8-5** Repeat Problem 8-4 if the $R_{eq}$ , $X_{eq}$ and $T_{max}/T_m$ are –0.0295Ω, –0.0150Ω and 6.227, respectively.

**Answers:**

a) $s_{T\max} = 0.3175$, $T_{\max} = -17254$ N.m (3.903 pu)  b) $T_m = -2770.8$ N.m, $P_m = -496.18 \times 10^3$ W

c) $\omega_m = 179.07$ rad/sec (1710 rpm), $\omega_r = 358.14$ rad/sec, $s = 0.05$  d) $I_s = -521.43$ A (rms)

e) $\bar{I}_r = 579.9\angle 158.22°$ A (rms), $\bar{V}_r = 19.18\angle 5.17°$ V (rms)

**8-6** Repeat Problem 8-4 if the $R_{eq}$, $X_{eq}$ & $T_{max}/T_m$ are –0.0937Ω, –0.05887Ω & 8.322, respectively.

**Answers:**

a) $s_{T_{max}} = 1.14$, $T_{max} = -18459\,\text{N.m}$ (4.1754 pu)

b) $T_m = -2218\,\text{N.m}$, $P_m = -355.4 \times 10^3\,\text{W}$

c) $\omega_m = 160.22\,\text{rad/sec}$ (1530 rpm), $\omega_s = 320.44\,\text{rad/sec}$, $s = 0.15$

d) $I_s = -417.9\,\text{A (rms)}$

e) $\bar{I}_s = 481.64\angle 153.5^\circ\,\text{A (rms)}$, $\bar{V}_s = 53.28\angle 5.64^\circ\,\text{V (rms)}$