Lecture#

- **4.4.2 Space Vector Modulation?**

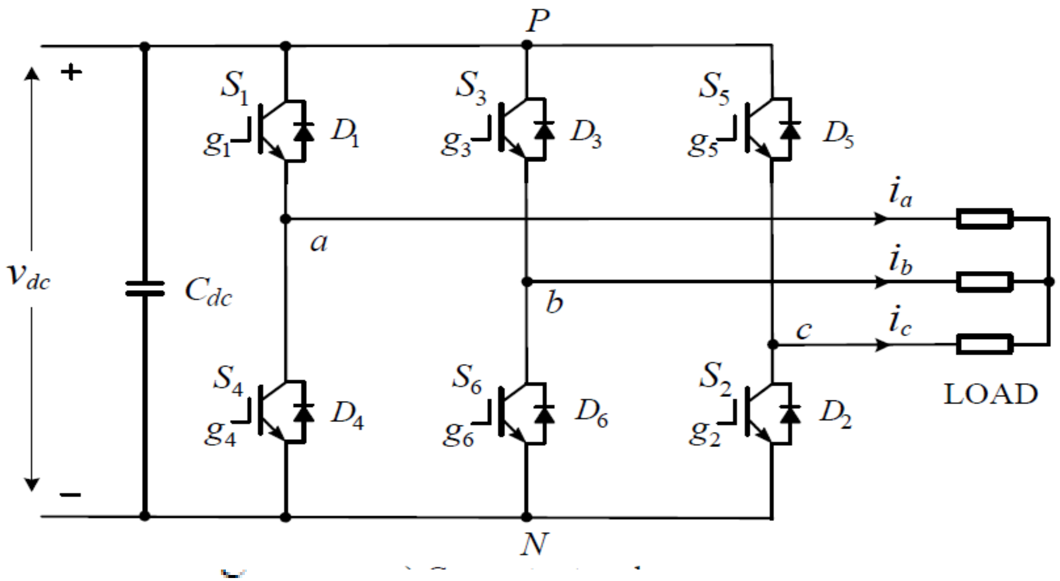
4.4.2 Space Vector Modulation

- Space vector modulation (SVM) is one of real-time modulation techniques.

- It is widely used for digital control of voltage source inverters.

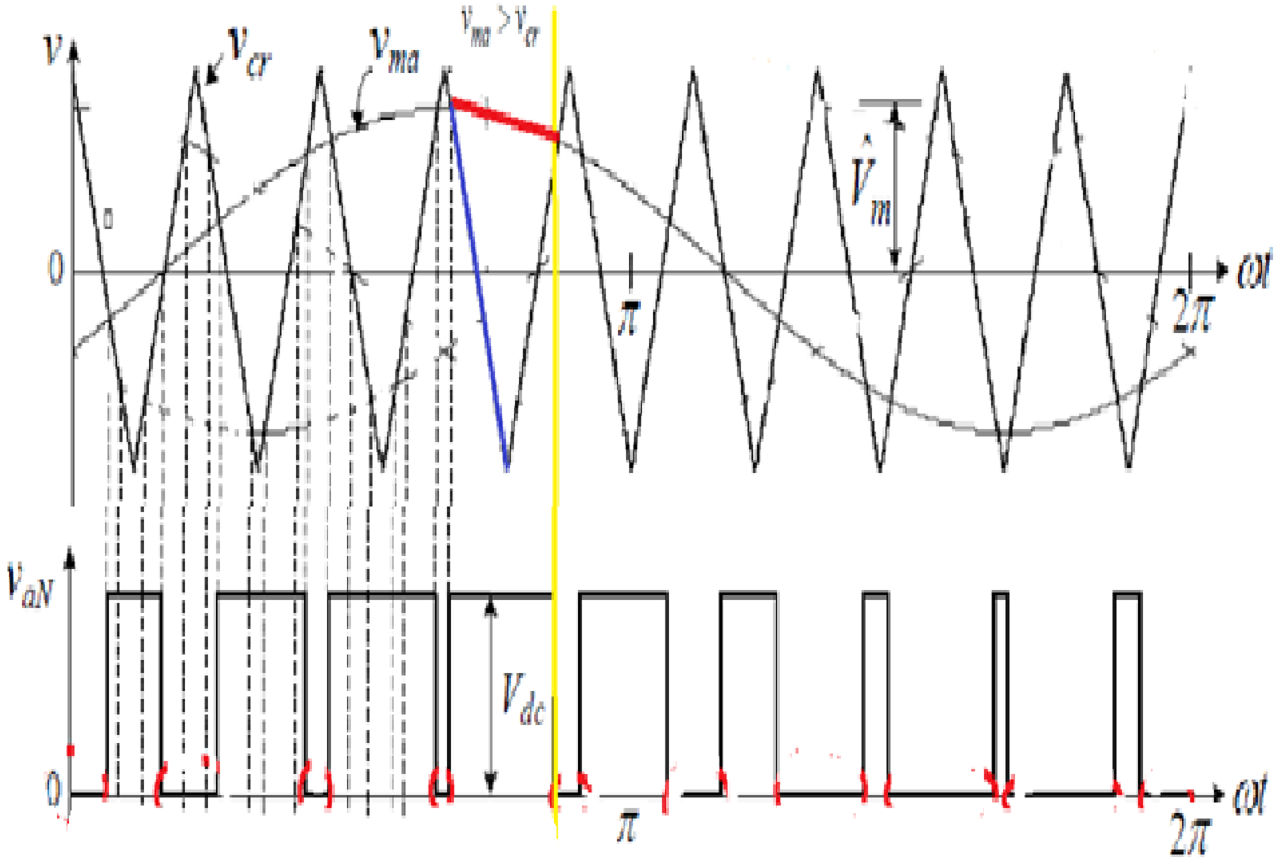
a) Switching States

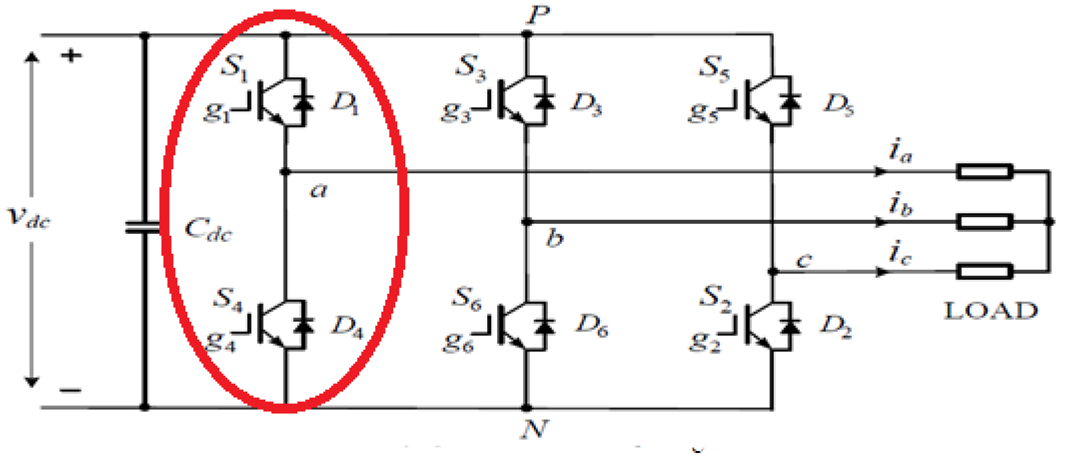
- Operating status of switches in 2-level inverter in Fig. can be represented by switching states.



Switching State	Leg *a*			Leg *b*			Leg *c*		
	S_1	S_4	v_{aN}	S_3	S_6	v_{bN}	S_5	S_2	v_{cN}
P	On	Off	V_{dc}	On	Off	V_{dc}	On	Off	V_{dc}
O	Off	On	0	Off	On	0	Off	On	0

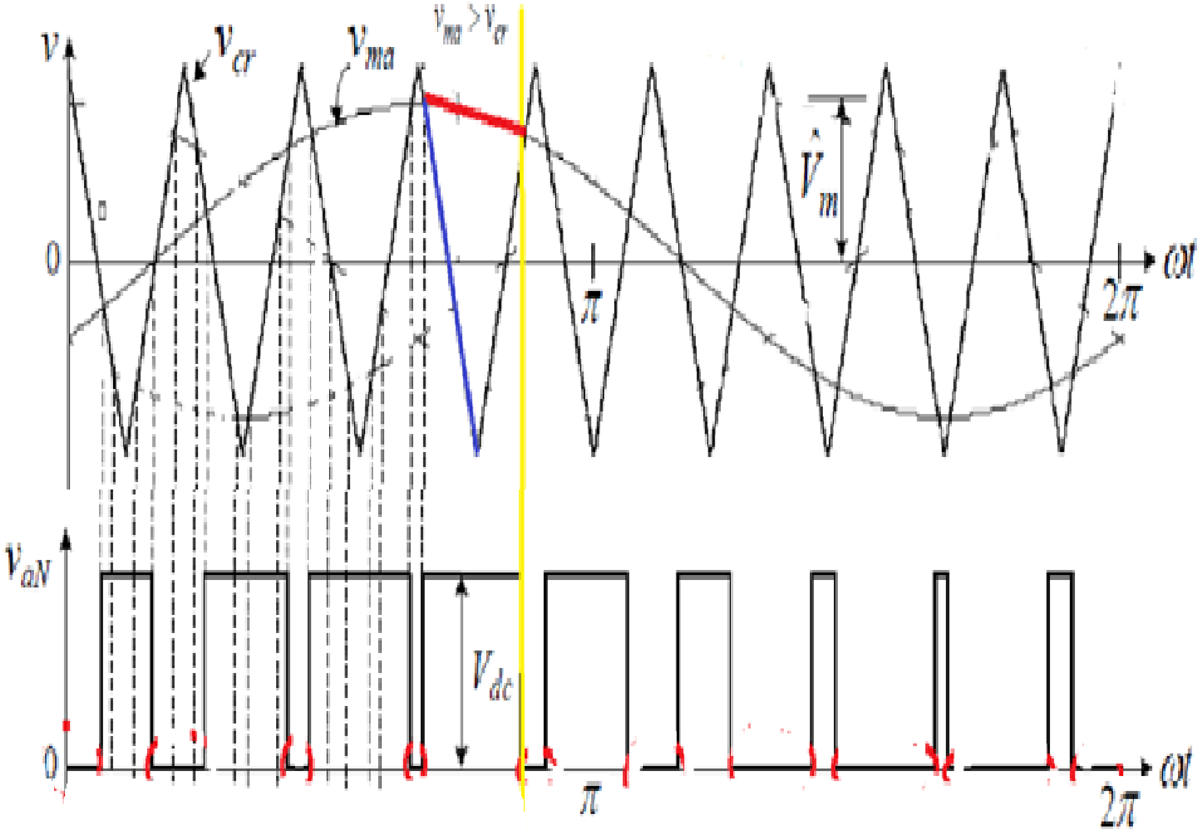
When $v_{ma} > v_{cr}$, upper switch $S1$ in inverter leg a is turned on. Lower switch $S4$ operates in a complementary manner & thus is switched off.

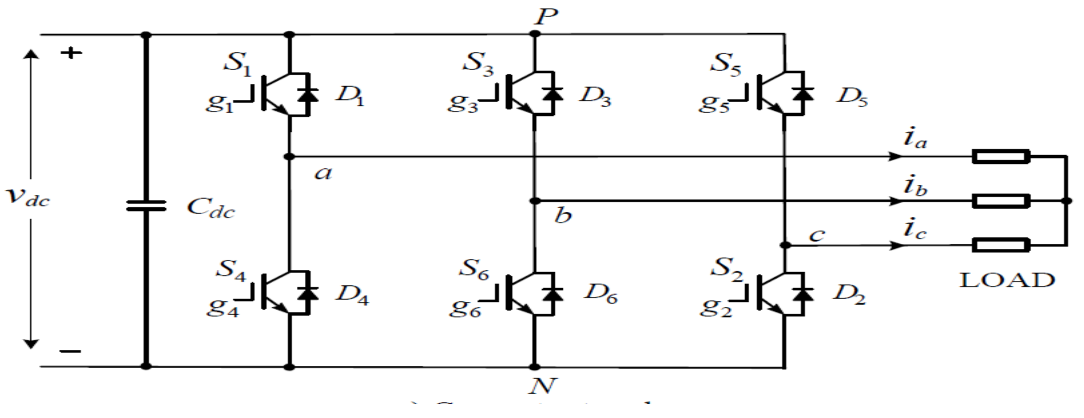




Switching State	Leg *a*			Leg *b*			Leg *c*		
	S_1	S_4	v_{aN}	S_3	S_6	v_{bN}	S_5	S_2	v_{cN}
P	On	Off	V_{dc}	On	Off	V_{dc}	On	Off	V_{dc}
0	Off	On	0	Off	On	0	Off	On	0

Resultant inverter terminal voltage *vaN* , which is voltage at phase-*a* terminal w.r.t -ve dc bus *N*, =dc voltage *Vdc*, *viz.* $v_{aN} = V_{dc}$



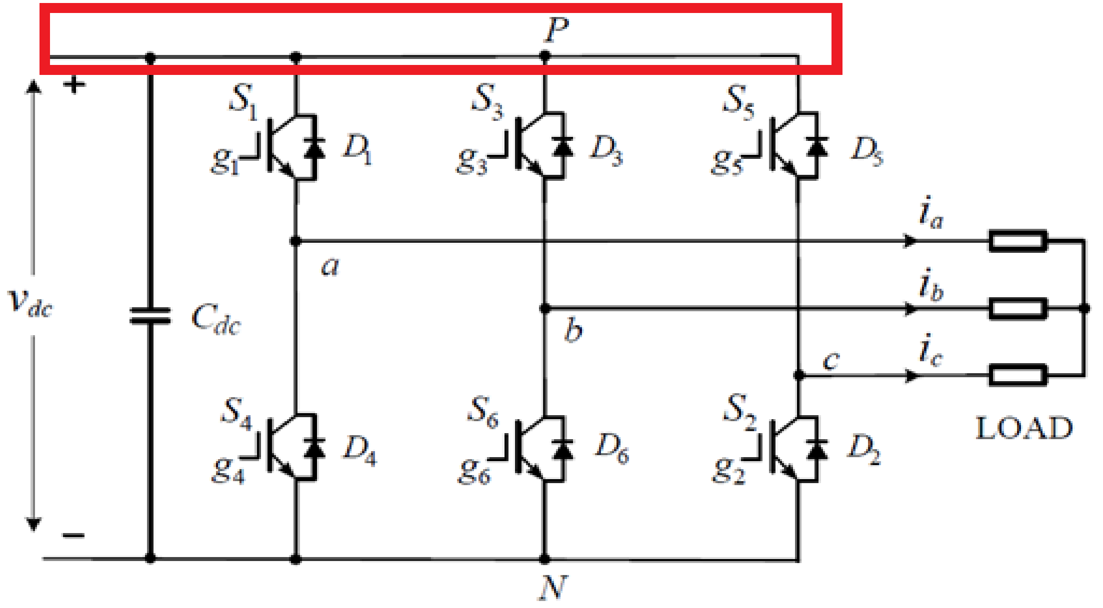


Switching State	Leg *a*			Leg *b*			Leg *c*		
	S_1	S_4	v_{aN}	S_3	S_6	v_{bN}	S_5	S_2	v_{cN}
P	On	Off	V_{dc}	On	Off	V_{dc}	On	Off	V_{dc}
O	Off	On	0	Off	On	0	Off	On	0

From table , switching state 'P' denotes that upper switches(S_1,S_3 & S_5) in an inverter leg is on

- Inverter terminal voltage (v_{aN} , v_{bN} or v_{cN}) is +ve ($+V_{dc}$)
- 'O' indicates that inverter terminal voltage is 0 due to conduction of lower switch.

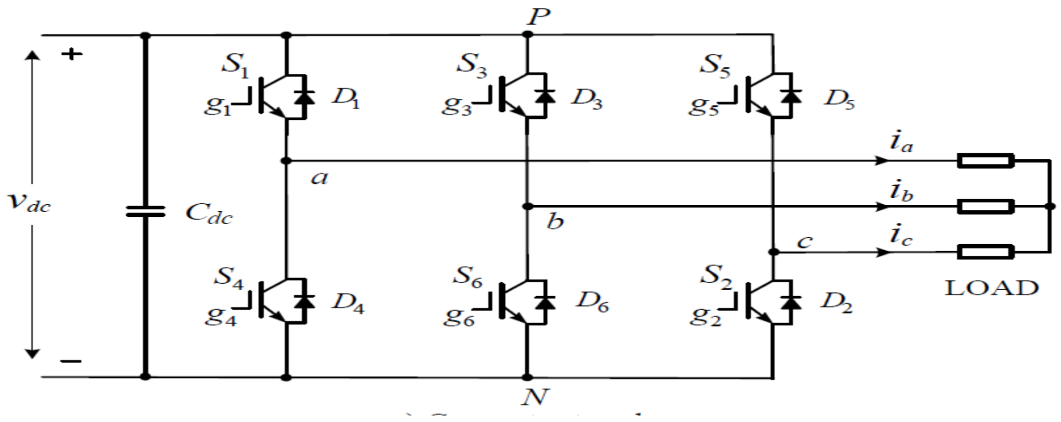
Switching State	Leg a			Leg b			Leg c		
	S_1	S_4	v_{aN}	S_3	S_6	v_{bN}	S_5	S_2	v_{cN}
P	On	Off	V_{dc}	On	Off	V_{dc}	On	Off	V_{dc}
0	Off	On	0	Off	On	0	Off	On	0

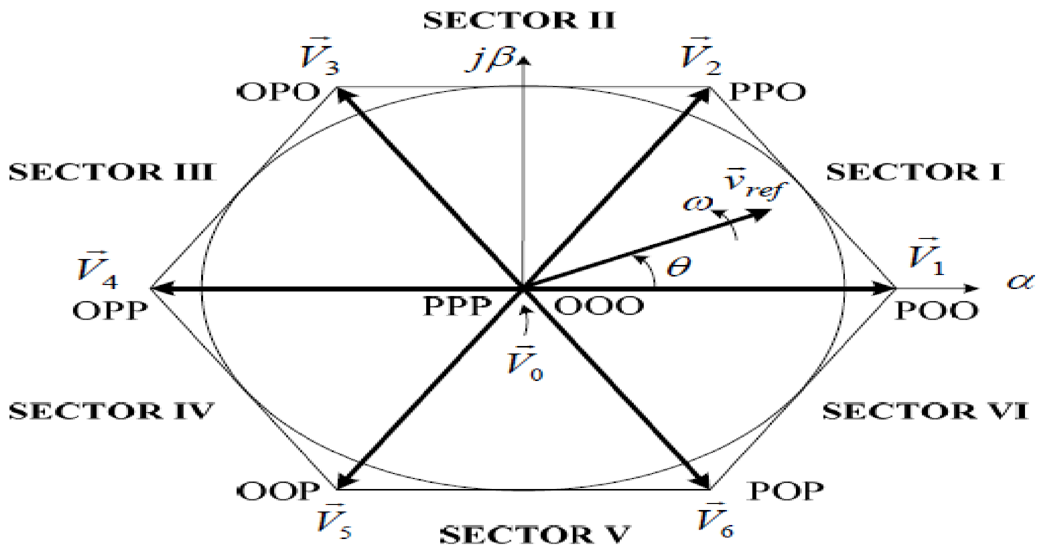


Q. Two-level inverter has 3 phases so how many possible combinations of switching states?

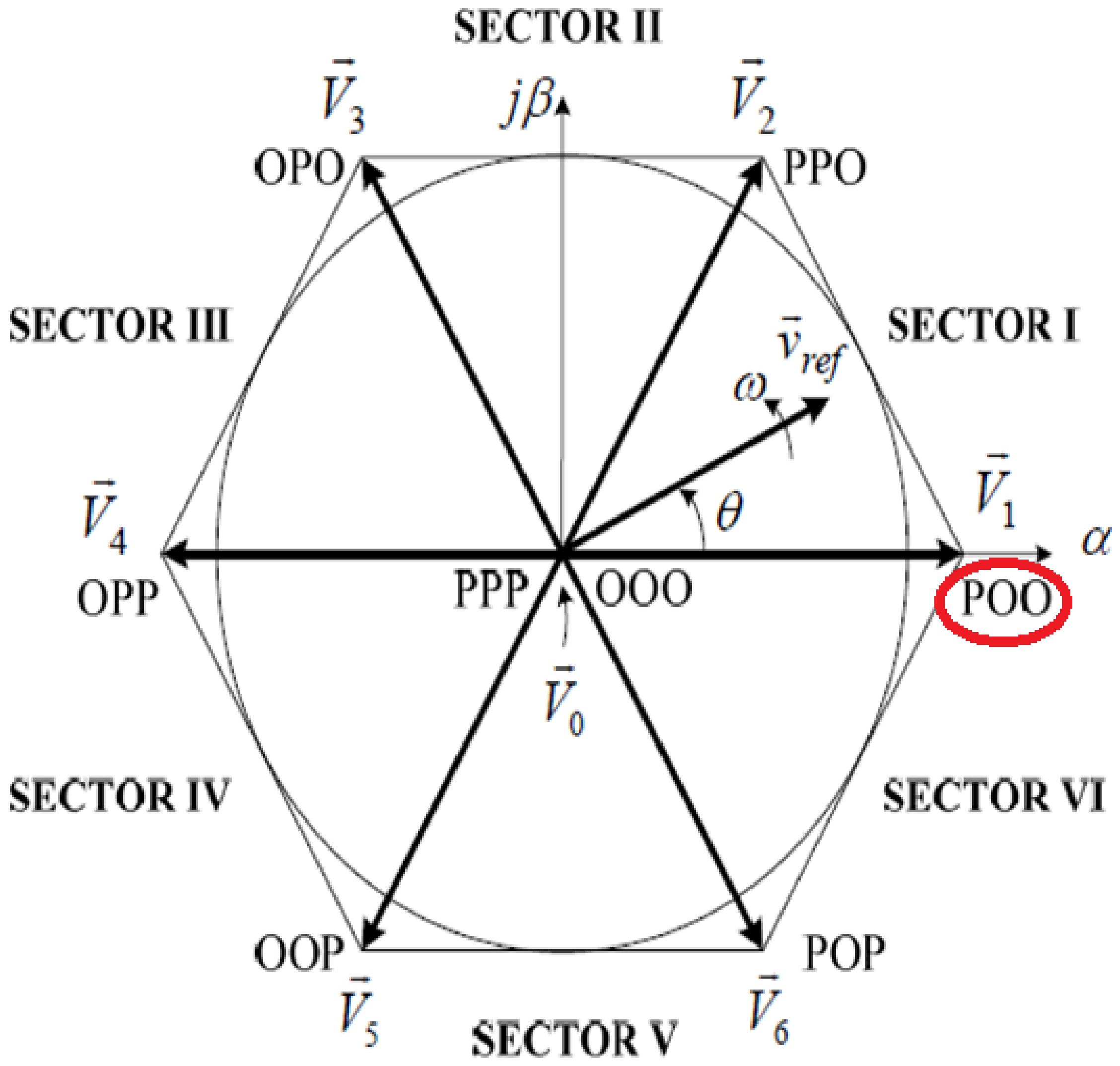
Answer. There can be 8 possible combinations(2^3) of switching states in 2-level inverter as listed in Table

Space Vector		Switching State (Three Phases)	On-state Switch	Vector Definition
Zero Vector	$\vec{V}_0$	[PPP]	S_1, S_3, S_5	$\vec{V}_0 = 0$
		[OOO]	S_4, S_6, S_2	
Active Vector	$\vec{V}_1$	[POO]	S_1, S_6, S_2	$\vec{V}_1 = \frac{2}{3} V_{dc} e^{j0}$
	$\vec{V}_2$	[PPO]	S_1, S_3, S_2	$\vec{V}_2 = \frac{2}{3} V_{dc} e^{j\frac{\pi}{3}}$
	$\vec{V}_3$	[OPO]	S_4, S_3, S_2	$\vec{V}_3 = \frac{2}{3} V_{dc} e^{j\frac{2\pi}{3}}$
	$\vec{V}_4$	[OPP]	S_4, S_3, S_5	$\vec{V}_4 = \frac{2}{3} V_{dc} e^{j\frac{3\pi}{3}}$
	$\vec{V}_5$	[OOP]	S_4, S_6, S_5	$\vec{V}_5 = \frac{2}{3} V_{dc} e^{j\frac{4\pi}{3}}$
	$\vec{V}_6$	[POP]	S_1, S_6, S_5	$\vec{V}_6 = \frac{2}{3} V_{dc} e^{j\frac{5\pi}{3}}$





Switching state [POO] corresponds to conduction of *S*1, *S*6 & *S*2 in inverter legs *a*, *b* and *c*, respectively.



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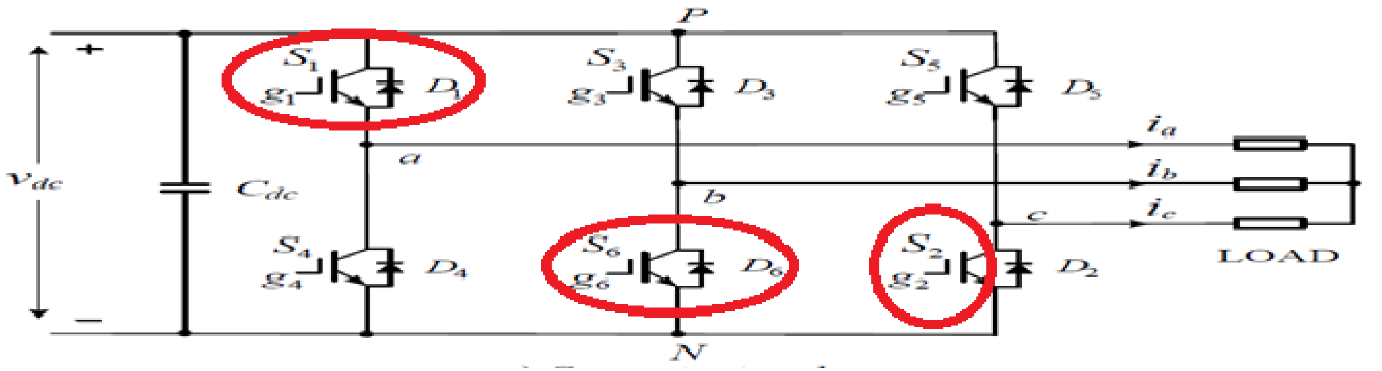
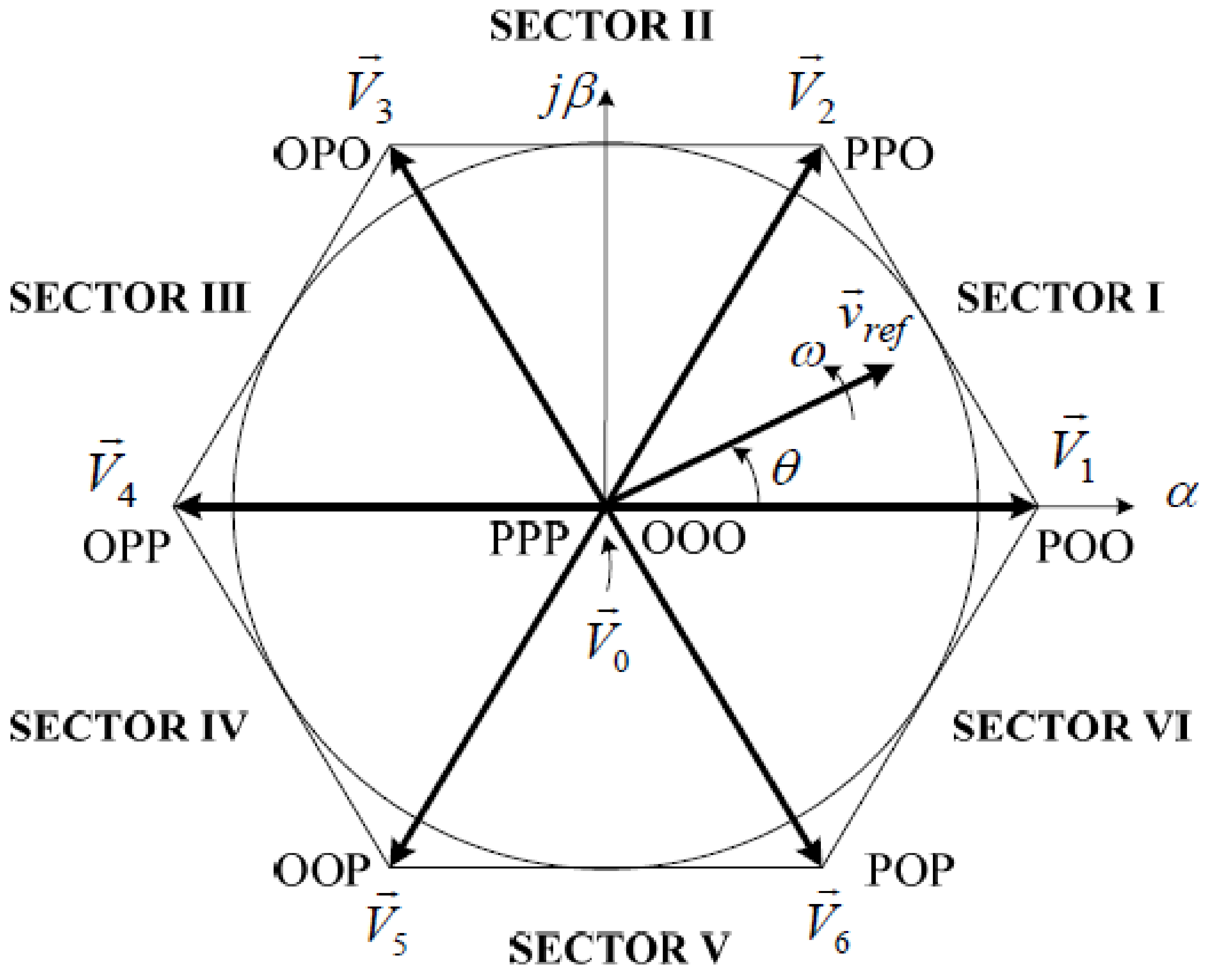
Active Vector	$\vec{V}_1$	[POO]	S_1, S_6, S_2	$\vec{V}_1 = \frac{2}{3} V_{dc}\, e^{j0}$
	$\vec{V}_2$	[PPO]	S_1, S_3, S_2	$\vec{V}_2 = \frac{2}{3} V_{dc}\, e^{j\frac{\pi}{3}}$
	$\vec{V}_3$	[OPO]	S_4, S_3, S_2	$\vec{V}_3 = \frac{2}{3} V_{dc}\, e^{j\frac{2\pi}{3}}$
	$\vec{V}_4$	[OPP]	S_4, S_3, S_5	$\vec{V}_4 = \frac{2}{3} V_{dc}\, e^{j\frac{3\pi}{3}}$
	$\vec{V}_5$	[OOP]	S_4, S_6, S_5	$\vec{V}_5 = \frac{2}{3} V_{dc}\, e^{j\frac{4\pi}{3}}$
	$\vec{V}_6$	[POP]	S_1, S_6, S_5	$\vec{V}_6 = \frac{2}{3} V_{dc}\, e^{j\frac{5\pi}{3}}$



Among 8 switching states, which states are zero states?



Space Vector		Switching State (Three Phases)	On-state Switch	Vector Definition
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		[OOO]	S_4, S_6, S_2	
Active Vector	$\vec{V}_1$	[POO]	S_1, S_6, S_2	$\vec{V}_1 = \frac{2}{3} V_{dc} e^{j0}$
	$\vec{V}_2$	[PPO]	S_1, S_3, S_2	$\vec{V}_2 = \frac{2}{3} V_{dc} e^{j\frac{\pi}{3}}$
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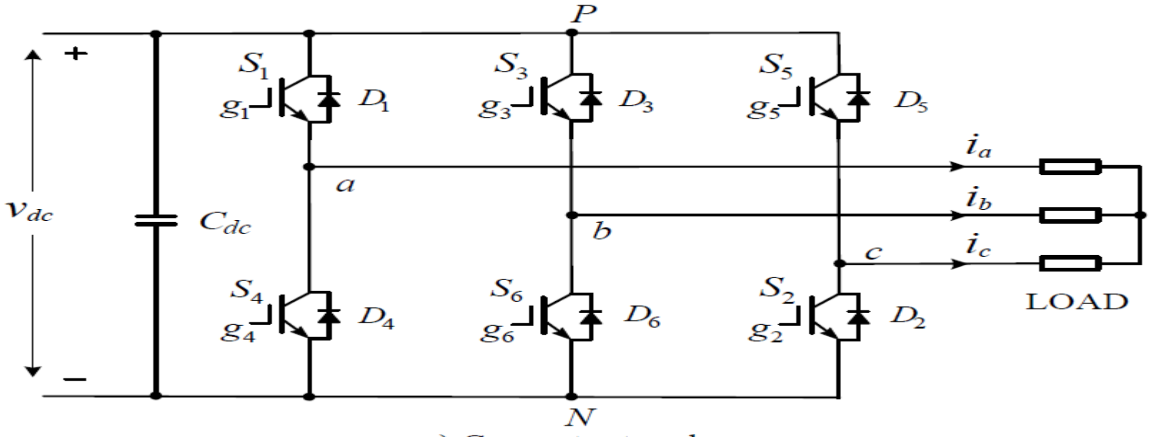
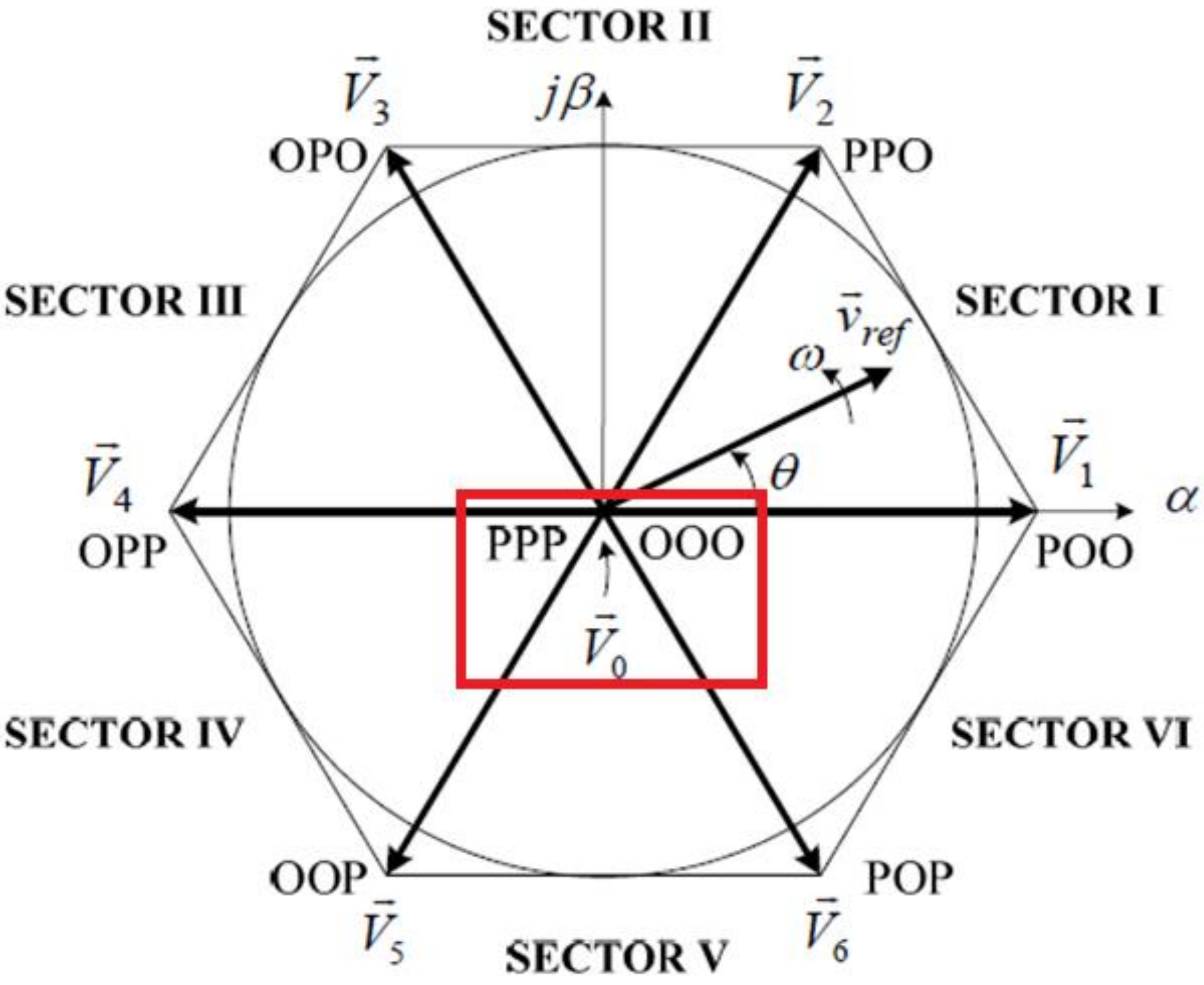
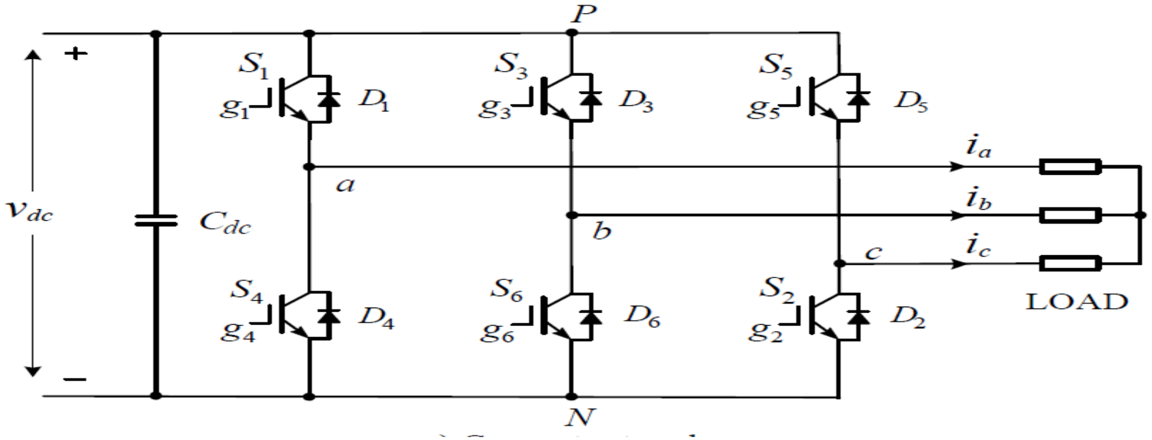
	$\vec{V}_6$	[POP]	S_1, S_6, S_5	$\vec{V}_6 = \frac{2}{3} V_{dc} e^{j\frac{5\pi}{3}}$



Among 8 switching states, [PPP] & [OOO] are 0 states(lies in center) & others are active states.



Space Vector		Switching State (Three Phases)	On-state Switch	Vector Definition
Zero Vector	$\vec{V}_0$	[PPP]	S_1, S_3, S_5	$\vec{V}_0 = 0$
		[OOO]	S_4, S_6, S_2	
Active Vector	$\vec{V}_1$	[POO]	S_1, S_6, S_2	$\vec{V}_1 = \frac{2}{3} V_{dc} e^{j0}$
	$\vec{V}_2$	[PPO]	S_1, S_3, S_2	$\vec{V}_2 = \frac{2}{3} V_{dc} e^{j\frac{\pi}{3}}$
	$\vec{V}_3$	[OPO]	S_4, S_3, S_2	$\vec{V}_3 = \frac{2}{3} V_{dc} e^{j\frac{2\pi}{3}}$
	$\vec{V}_4$	[OPP]	S_4, S_3, S_5	$\vec{V}_4 = \frac{2}{3} V_{dc} e^{j\frac{3\pi}{3}}$
	$\vec{V}_5$	[OOP]	S_4, S_6, S_5	$\vec{V}_5 = \frac{2}{3} V_{dc} e^{j\frac{4\pi}{3}}$
	$\vec{V}_6$	[POP]	S_1, S_6, S_5	$\vec{V}_6 = \frac{2}{3} V_{dc} e^{j\frac{5\pi}{3}}$

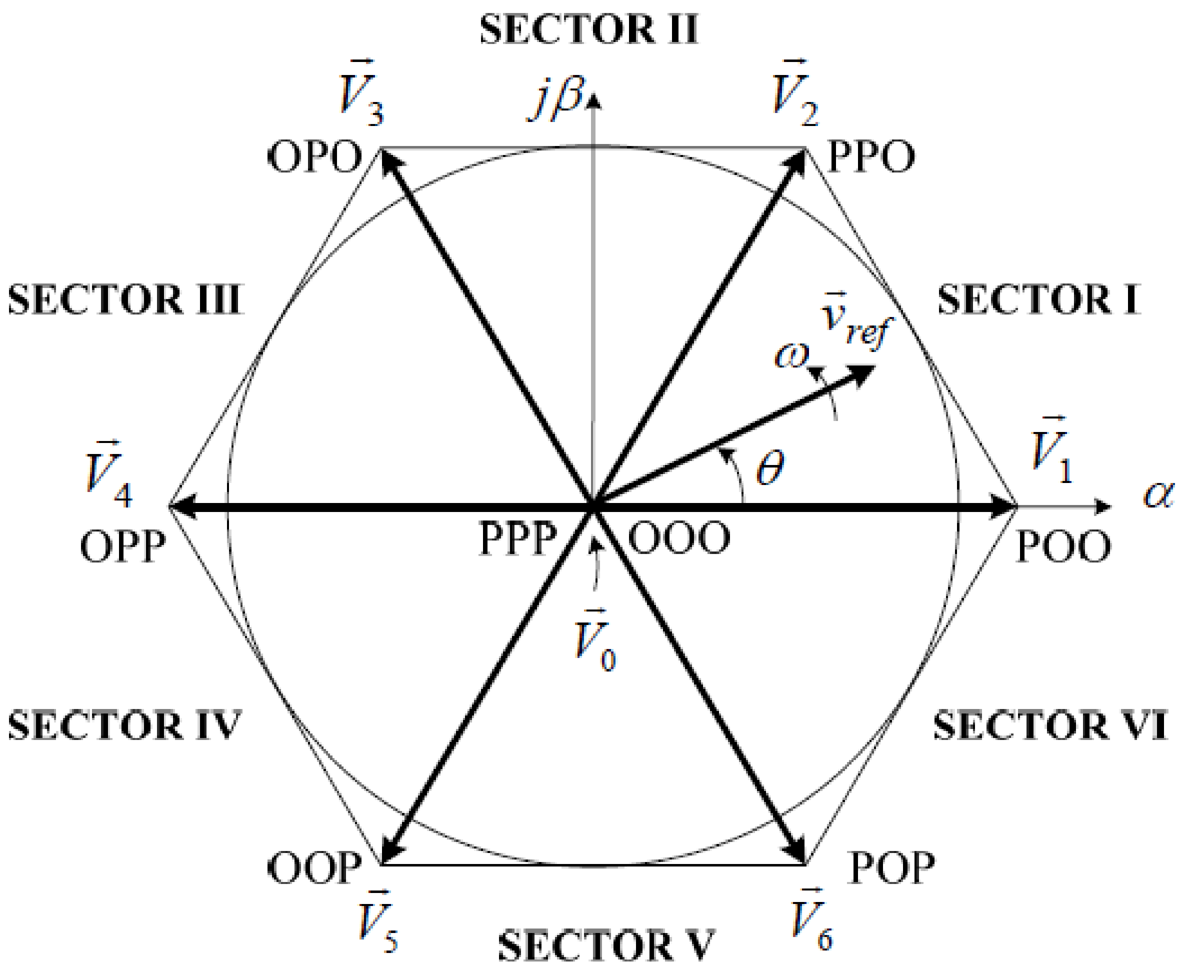


Space vectors, switching states & on-state switches

Space Vector		Switching State (Three Phases)	On-state Switch	Vector Definition
Zero Vector	$\vec{V}_0$	[PPP]	S_1, S_3, S_5	$\vec{V}_0 = 0$
		[OOO]	S_4, S_6, S_2	
Active Vector	$\vec{V}_1$	[POO]	S_1, S_6, S_2	$\vec{V}_1 = \frac{2}{3} V_{dc} e^{j0}$
	$\vec{V}_2$	[PPO]	S_1, S_3, S_2	$\vec{V}_2 = \frac{2}{3} V_{dc} e^{j\frac{\pi}{3}}$
	$\vec{V}_3$	[OPO]	S_4, S_3, S_2	$\vec{V}_3 = \frac{2}{3} V_{dc} e^{j\frac{2\pi}{3}}$
	$\vec{V}_4$	[OPP]	S_4, S_3, S_5	$\vec{V}_4 = \frac{2}{3} V_{dc} e^{j\frac{3\pi}{3}}$
	$\vec{V}_5$	[OOP]	S_4, S_6, S_5	$\vec{V}_5 = \frac{2}{3} V_{dc} e^{j\frac{4\pi}{3}}$
	$\vec{V}_6$	[POP]	S_1, S_6, S_5	$\vec{V}_6 = \frac{2}{3} V_{dc} e^{j\frac{5\pi}{3}}$

b) Space Vectors

- Active & 0 switching states can be represented by active & 0 space vectors, respectively.
- A typical space vector diagram for 2-level inverter is shown in Fig., where 6 active vectors V_1 to V_6 form a regular hexagon with 6 equal sectors (I to VI).
- 0 vector V_0 lies on the centre of the hexagon.

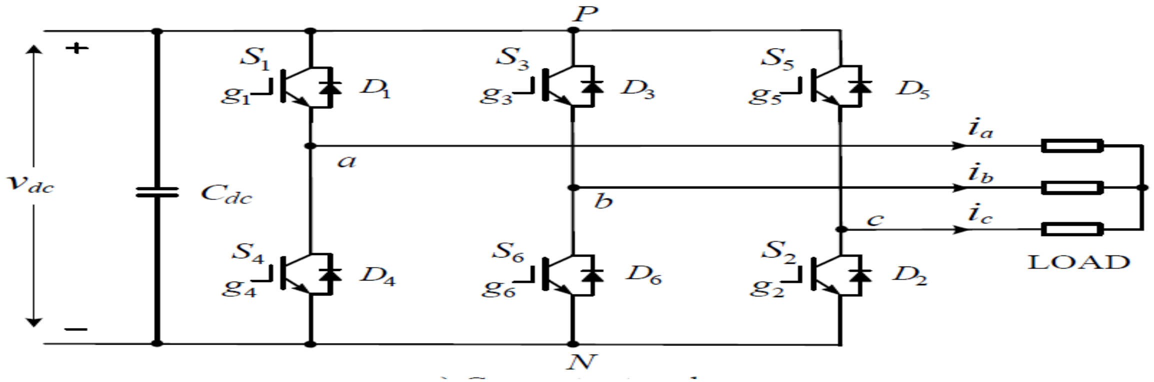


To derive relationship between space vectors & switching states, refer to 2-level inverter in Fig.

Assuming that operation of inverter is 3-phase balanced, we have

$$v_a(t) + v_b(t) + v_c(t) = 0$$

where *va* , *vb* & *vc* are instantaneous load phase voltages.



From mathematical point of view, one of phase voltages is redundant since given any 2 phase voltages, 3rd one can be readily calculated.

$$v_a(t) + v_b(t) + v_c(t) = 0$$

Therefore, it is possible to transform 3-phase variables to 2-phase variables through the *abc*/*αβ* transformation:

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}$$

A space vector can be generally expressed in terms of 2-phase voltages in *α*-*β* frame

$$\vec{v}(t) = v_\alpha(t) + j v_\beta(t)$$

From

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}$$

$$V_\alpha(\mathrm{t}) = \frac{2}{3}\left\{V_a(\mathrm{t}) - \frac{1}{2}V_b(\mathrm{t}) - \frac{1}{2}V_c(\mathrm{t})\right\}$$

$$V_\beta(\mathrm{t}) = \frac{2}{3}\left\{\frac{\sqrt{3}}{2}V_b(\mathrm{t}) - \frac{\sqrt{3}}{2}V_c(\mathrm{t})\right\}$$

Substituting

$$V_\alpha(\mathrm{t}) = \frac{2}{3}\left\{V_a(\mathrm{t}) - \frac{1}{2}V_b(\mathrm{t}) - \frac{1}{2}V_c(\mathrm{t})\right\}$$

$$V_\beta(\mathrm{t}) = \frac{2}{3}\left\{\frac{\sqrt{3}}{2}V_b(\mathrm{t}) - \frac{\sqrt{3}}{2}V_c(\mathrm{t})\right\}$$

into

$$\vec{v}(\mathrm{t}) = V_\alpha(\mathrm{t}) + jV_\beta(\mathrm{t})$$

Try to simplify:

$$V_\alpha(\mathrm{t}) + jV_\beta(\mathrm{t}) = \frac{2}{3}\left[V_a(\mathrm{t}) + (-\frac{1}{2} + \mathrm{j}\frac{\sqrt{3}}{2})V_b(\mathrm{t}) + (-\frac{1}{2} - \mathrm{j}\frac{\sqrt{3}}{2})V_c(\mathrm{t})\right]$$

$$a = (-\frac{1}{2} + \mathrm{j}\frac{\sqrt{3}}{2})$$

$$V_\alpha(\mathrm{t}) + jV_\beta(\mathrm{t}) = \frac{2}{3}\left[V_a(\mathrm{t}) + (-\frac{1}{2} + \mathrm{j}\frac{\sqrt{3}}{2})V_b(\mathrm{t}) + (-\frac{1}{2} - \mathrm{j}\frac{\sqrt{3}}{2})V_c(\mathrm{t}) \right]$$

$$a = (-\frac{1}{2} + \mathrm{j}\frac{\sqrt{3}}{2}) = e^{j\frac{2\pi}{3}}$$

$$a^2 = (-\frac{1}{2} - \mathrm{j}\frac{\sqrt{3}}{2}) = e^{j\frac{4\pi}{3}}$$

$$\vec{v}(\mathrm{t}) = \frac{2}{3}\left[V_a(\mathrm{t})e^{j0} + V_b(\mathrm{t})e^{j\frac{2\pi}{3}} + V_c(\mathrm{t})e^{j\frac{4\pi}{3}} \right]$$

For active switching state [POO], generated load phase voltages are $\frac{2}{3}V_{dc} = V_a(\mathrm{t})$

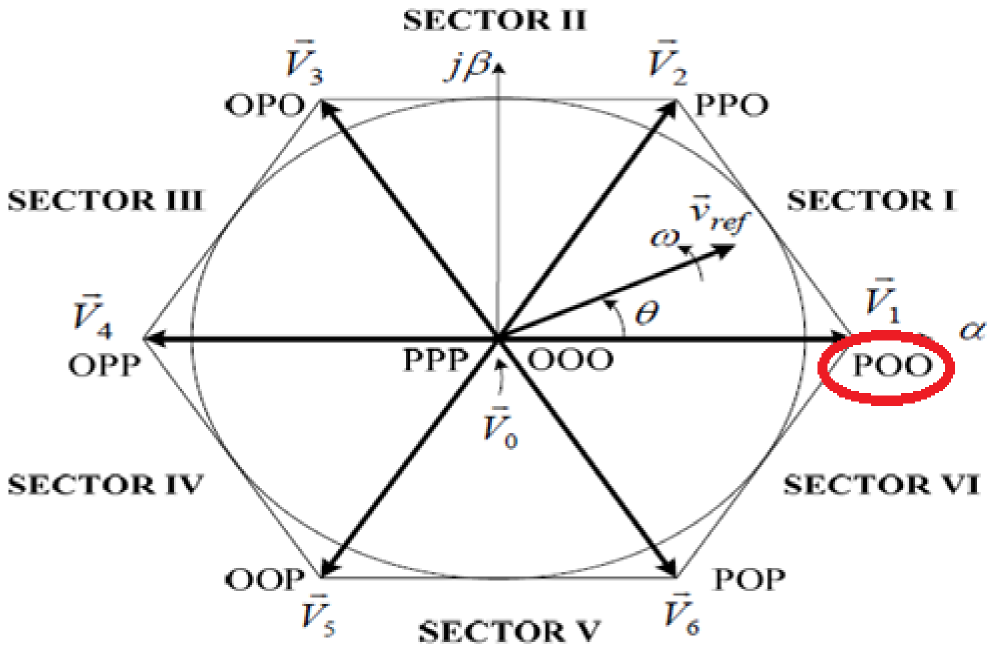
$$-\frac{1}{3}V_{dc} = V_b(\mathrm{t}) = V_c(\mathrm{t})$$

Corresponding space vector, denoted as $\vec{V}_1$ can be obtained by substituting these equations:

$$\vec{v}(\mathrm{t}) = \frac{2}{3}\left[V_a(\mathrm{t})e^{j0} + V_b(\mathrm{t})e^{j\frac{2\pi}{3}} + V_c(\mathrm{t})e^{j\frac{4\pi}{3}}\right]$$

$$\vec{v}(\mathrm{t}) = \frac{2}{3}\left[\frac{2}{3}V_{dc}e^{j0} - \frac{1}{3}V_{dc}e^{j\frac{2\pi}{3}} - \frac{1}{3}V_{dc}e^{j\frac{4\pi}{3}}\right]$$

$$\vec{v}(\mathrm{t}) = \frac{2}{3}\left[\frac{2}{3}V_{dc}e^{j0} - \frac{1}{3}V_{dc}(e^{j\frac{2\pi}{3}} + e^{j\frac{4\pi}{3}})\right]$$



$$a + a^2 + 1 = 0 \qquad a + a^2 = -1$$

$$\vec{v}(\mathrm{t}) = \frac{2}{3}\left[\frac{2}{3}V_{dc}e^{j0} - \frac{1}{3}V_{dc}(e^{j\frac{2\pi}{3}} + e^{j\frac{4\pi}{3}})\right]$$

$$a + a^2 = -1$$

$$\vec{v}(\mathrm{t}) = \frac{2}{3}\left[\frac{2}{3}V_{dc}e^{j0} + \frac{1}{3}V_{dc}\right]$$

$$\overrightarrow{v_1} = \left[\frac{2}{3}V_{dc}e^{j0}\right]$$

$$\overrightarrow{v_k} = \left[\frac{2}{3}V_{dc}e^{j(\mathrm{k}-1)\frac{\pi}{3}}\right] k = 1, 2, \ldots.6$$

Following the same procedure, all 6 active vectors can be derived $\vec{v}_k = \left[\frac{2}{3} V_{dc} e^{j(k-1)\frac{\pi}{3}}\right] k = 1, 2, \ldots.6$

$\vec{V}_1 = \frac{2}{3} V_{dc}\, e^{j0}$
$\vec{V}_2 = \frac{2}{3} V_{dc}\, e^{j\frac{\pi}{3}}$
$\vec{V}_3 = \frac{2}{3} V_{dc}\, e^{j\frac{2\pi}{3}}$
$\vec{V}_4 = \frac{2}{3} V_{dc}\, e^{j\frac{3\pi}{3}}$
$\vec{V}_5 = \frac{2}{3} V_{dc}\, e^{j\frac{4\pi}{3}}$
$\vec{V}_6 = \frac{2}{3} V_{dc}\, e^{j\frac{5\pi}{3}}$

Redundant vector

- 0 vector $\vec{V}_0$ has 2 switching states [PPP] & [OOO], one of which seems redundant.

- Redundant switching state can be utilized to minimize switching frequency of inverter or perform other useful functions.

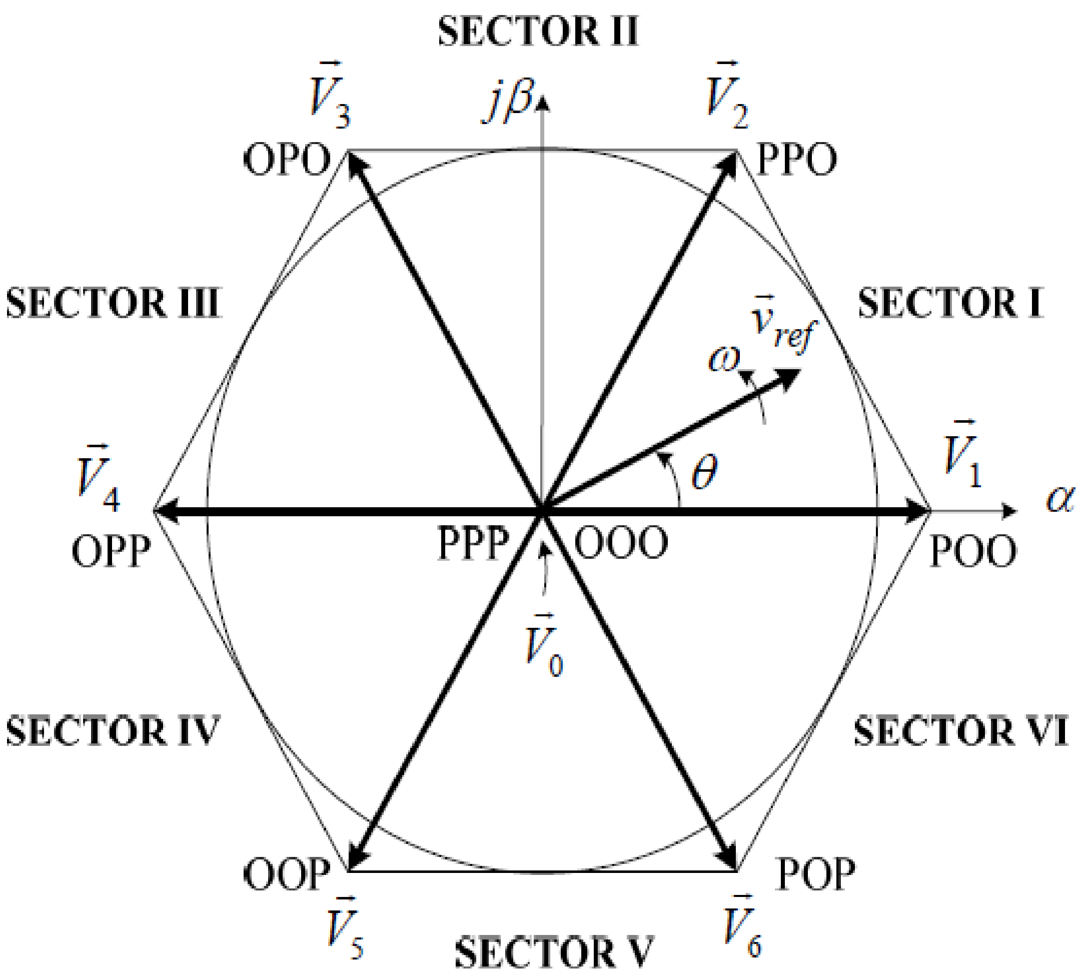
Relationship between space vectors & their corresponding switching states is given in Table

Space Vector		**Switching State (Three Phases)**	**On-state Switch**	**Vector Definition**
Zero Vector	$\vec{V}_0$	[PPP]	S_1, S_3, S_5	$\vec{V}_0 = 0$
		[OOO]	S_4, S_6, S_2	
Active Vector	$\vec{V}_1$	[POO]	S_1, S_6, S_2	$\vec{V}_1 = \frac{2}{3} V_{dc}\, e^{j0}$
	$\vec{V}_2$	[PPO]	S_1, S_3, S_2	$\vec{V}_2 = \frac{2}{3} V_{dc}\, e^{j\frac{\pi}{3}}$
	$\vec{V}_3$	[OPO]	S_4, S_3, S_2	$\vec{V}_3 = \frac{2}{3} V_{dc}\, e^{j\frac{2\pi}{3}}$
	$\vec{V}_4$	[OPP]	S_4, S_3, S_5	$\vec{V}_4 = \frac{2}{3} V_{dc}\, e^{j\frac{3\pi}{3}}$
	$\vec{V}_5$	[OOP]	S_4, S_6, S_5	$\vec{V}_5 = \frac{2}{3} V_{dc}\, e^{j\frac{4\pi}{3}}$
	$\vec{V}_6$	[POP]	S_1, S_6, S_5	$\vec{V}_6 = \frac{2}{3} V_{dc}\, e^{j\frac{5\pi}{3}}$

0 & active vectors do not move in space, & thus are referred to as stationary vectors.

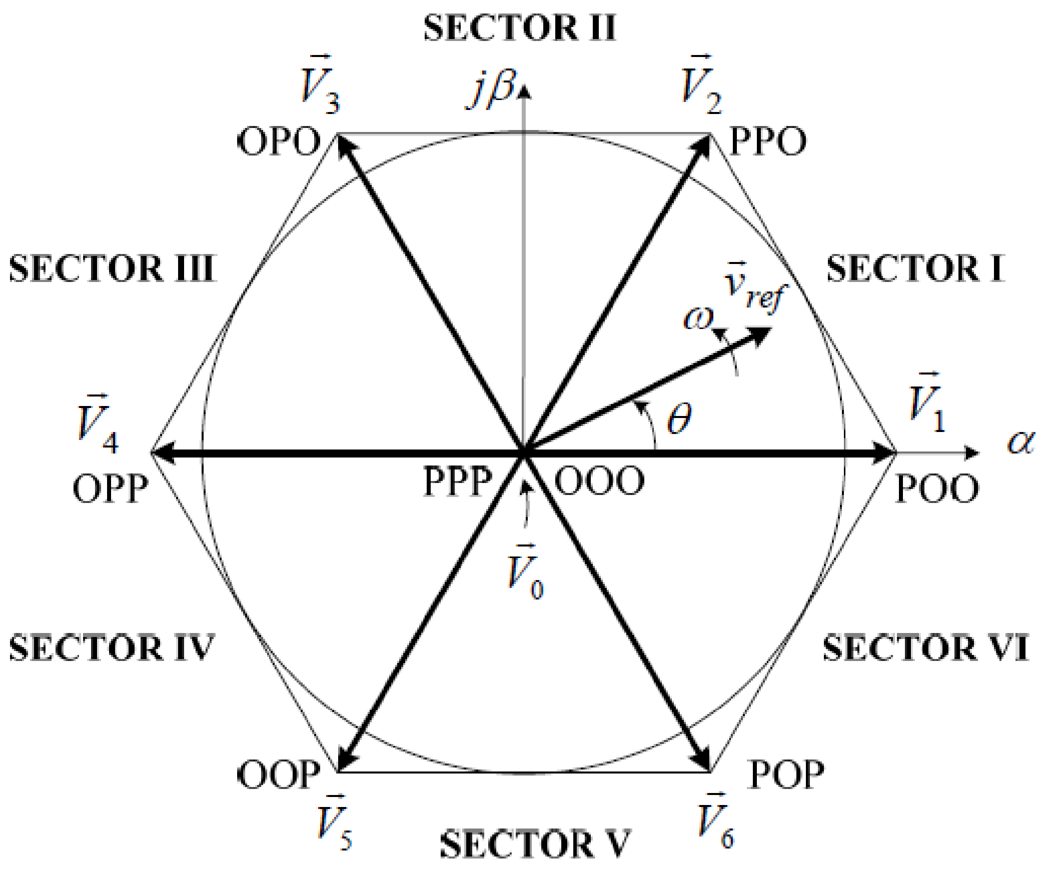
- On the contrary, reference vector $\vec{v}_{ref}$ rotates in space at an angular velocity $\omega = 2\pi f$
- where f is fundamental frequency of inverter output voltage.
- Angular displacement (θ) between $\vec{v}_{ref}$ & α-axis of α-β frame can be obtained:

$$\theta(t) = \int_0^t \omega(t)\, dt + \theta_0$$



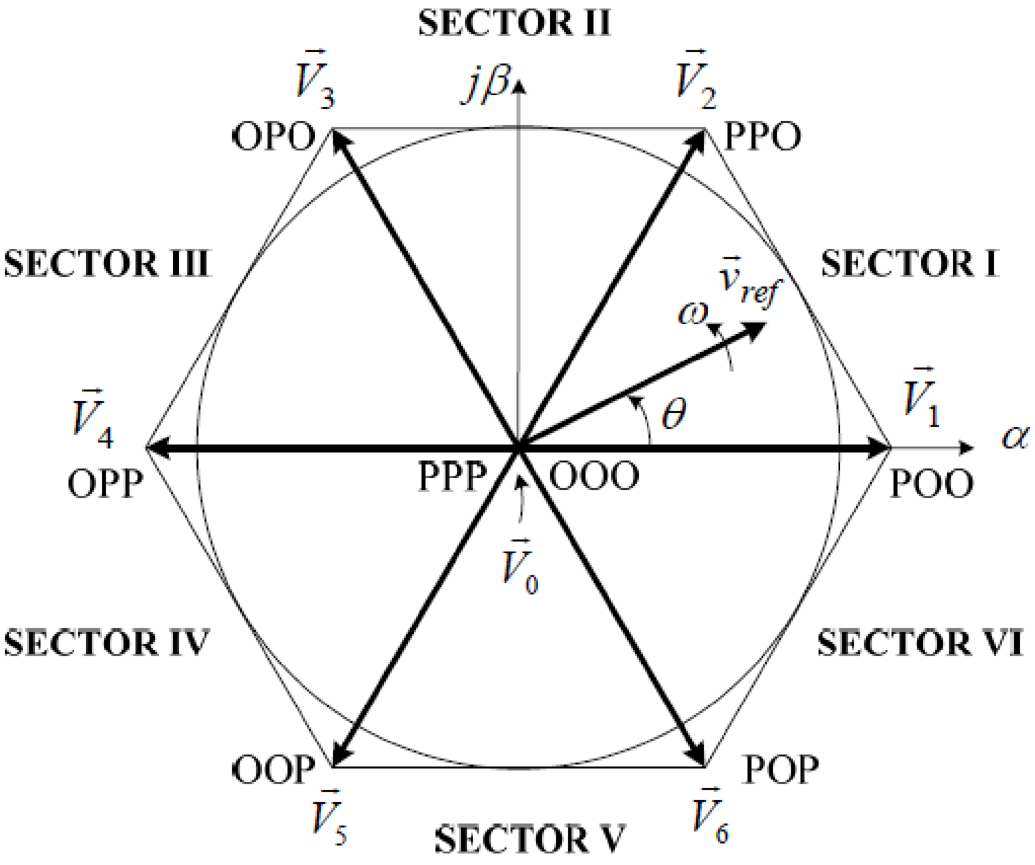
For a given magnitude (length) and position, $\vec{v}_{ref}$ can be synthesized by 3 nearby stationary vectors,

- Based on which switching states of inverter can be selected and
- Gate signals for active switches can be generated.



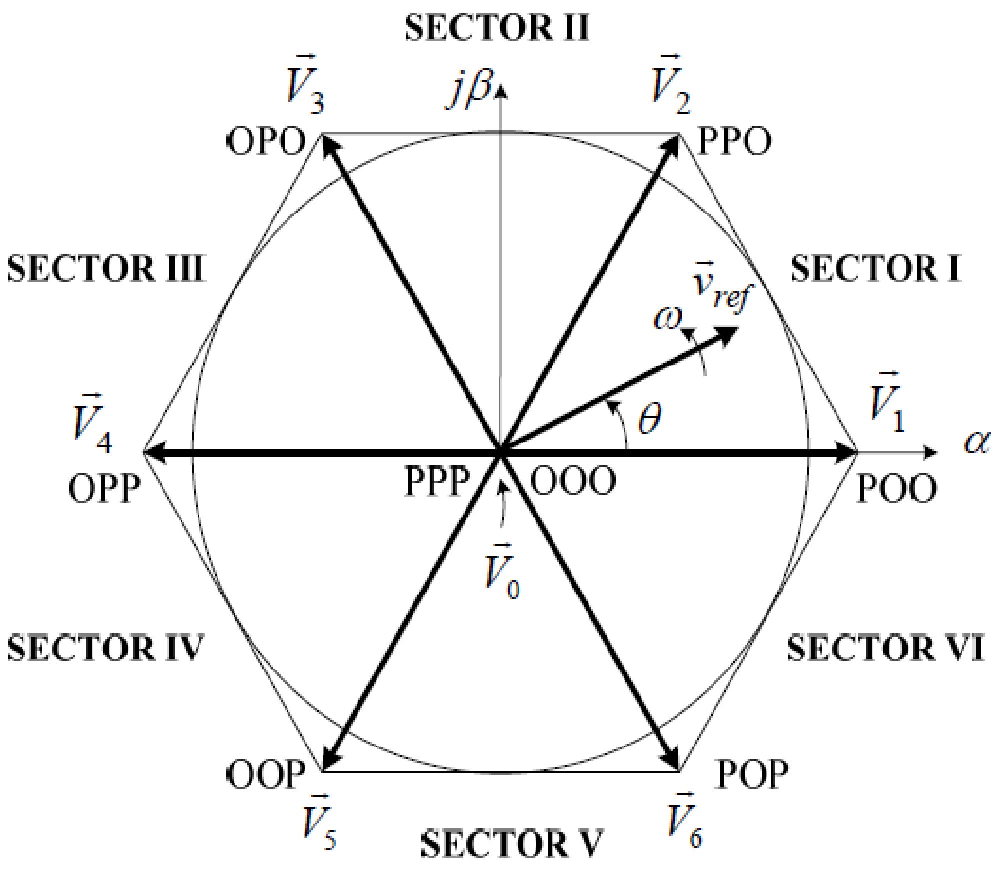
When $\vec{v}_{ref}$ passes through sectors one by one, different sets of switches will be turned on or off.

- As a result, when $\vec{v}_{ref}$ rotates 1 revolution in space,
- inverter output voltage varies 1 cycle over time.



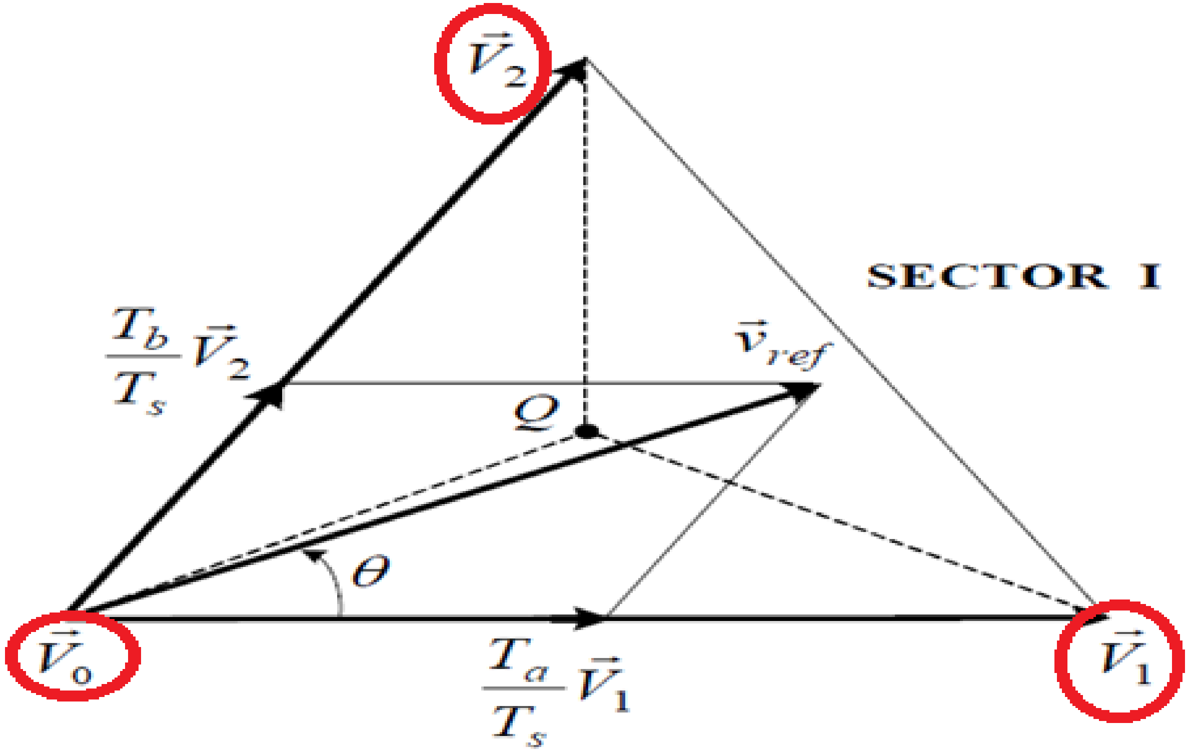
Inverter output frequency

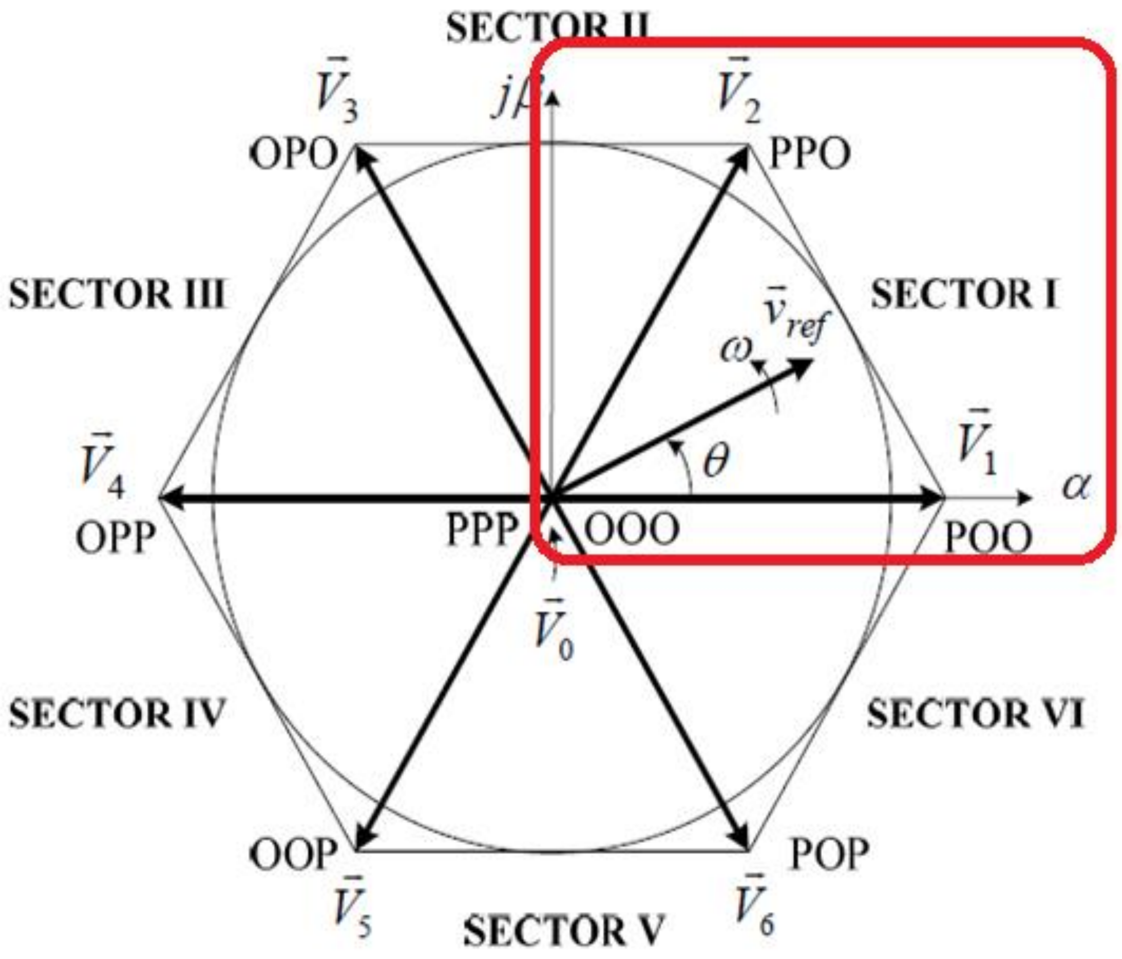
- Inverter output frequency corresponds to the rotating speed of $\vec{v}_{ref}$
- while its output voltage can be adjusted by the magnitude of $\vec{v}_{ref}$



c) Dwell Time Calculation

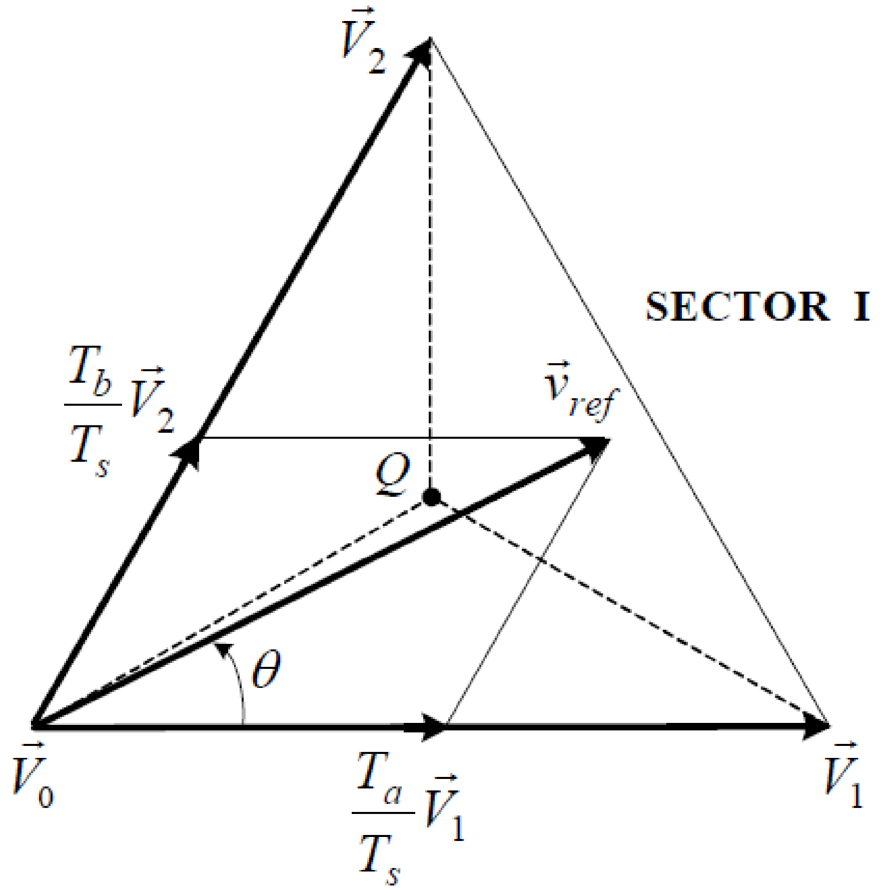
- Reference $\vec{v}_{ref}$ can be synthesized by 3 stationary vectors.





Dwell time for stationary vectors represents:

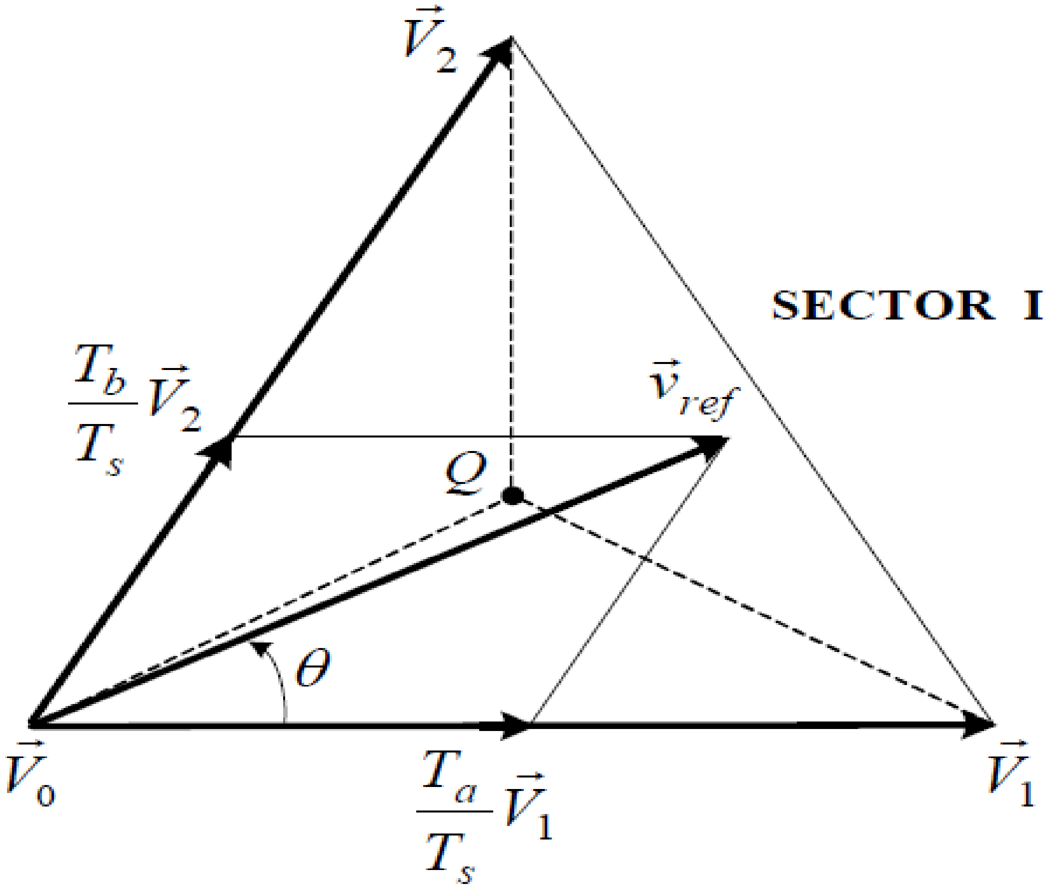
- Duty-cycle time (on-state or off-state time) of chosen switches during a sampling period *Ts* .



Dwell time calculation is based on **'volt-second balancing'** principle:

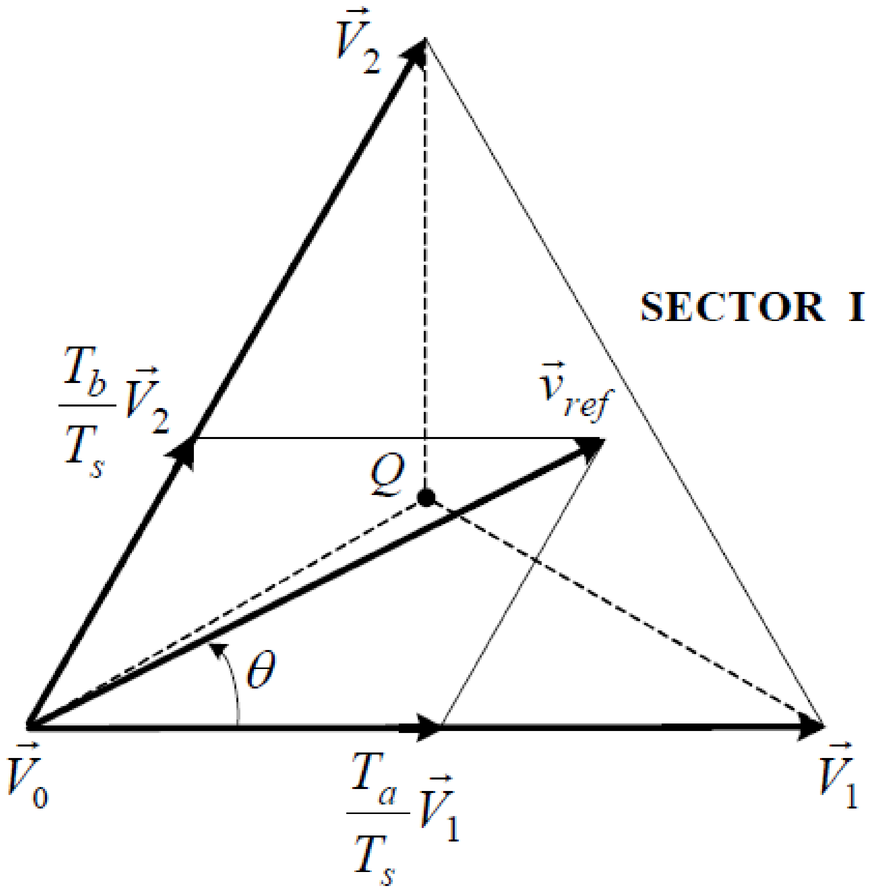
- i.e, product of reference voltage $\vec{v}_{ref}$ with sampling period *Ts* = Sum of voltage multiplied by time interval of chosen space vectors.

$$\begin{cases} \vec{v}_{ref}\, T_s = \vec{V}_1\, T_a + \vec{V}_2\, T_b + \vec{V}_0\, T_0 \\ T_s = T_a + T_b + T_0 \end{cases}$$



Assumption: Sampling period *Ts* is sufficiently small, reference vector $\vec{v}_{ref}$ can be considered constant during *Ts*.

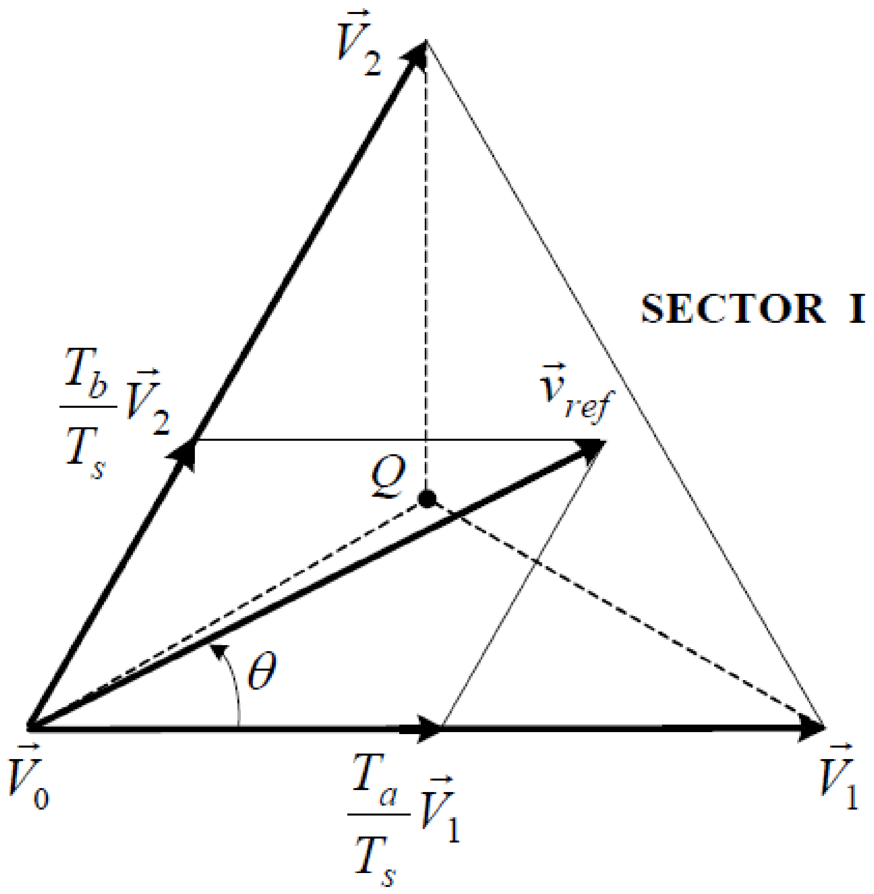
- $\vec{v}_{ref}$ can be approximated by 2 adjacent active vectors & one 0 vector.
- e.g, $\vec{v}_{ref}$ when falls into sector I as shown in Fig, it can be synthesized by $\vec{V}_1$, $\vec{V}_2$ and $\vec{V}_0$



Volt-second balancing equation is:

$$\begin{cases} \vec{v}_{ref}\,T_s = \vec{V}_1 T_a + \vec{V}_2 T_b + \vec{V}_0 T_0 \\ T_s = T_a + T_b + T_0 \end{cases}$$

where T_a , T_b and T_0 are dwell times for vectors



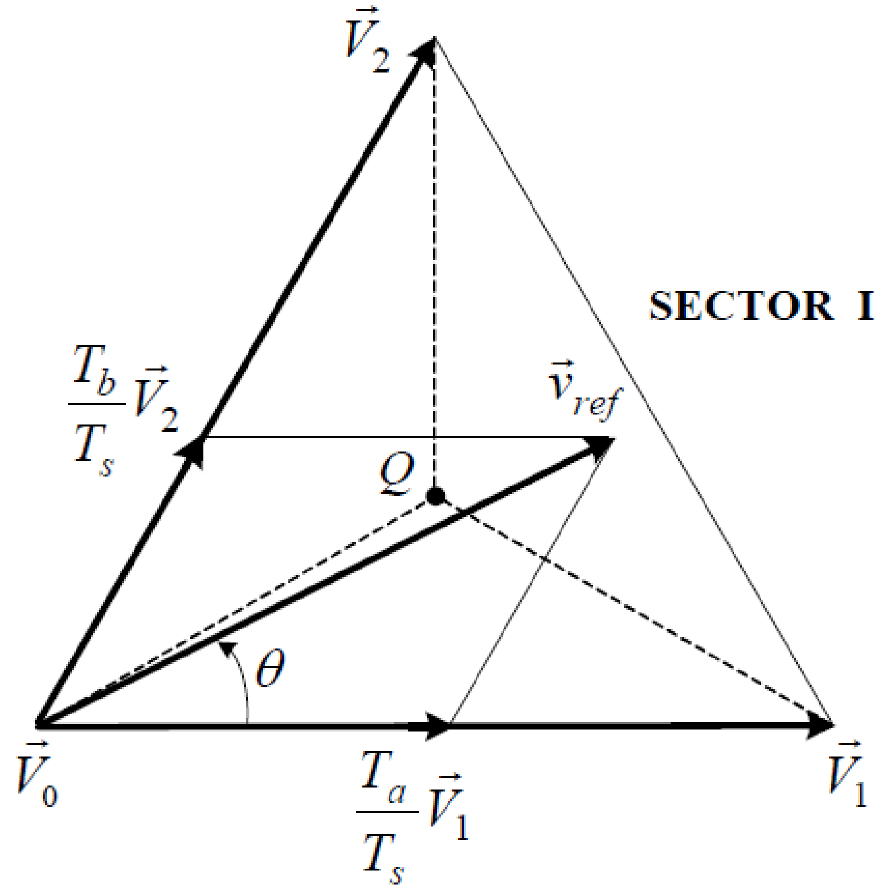
$\vec{V}_1$, $\vec{V}_2$ and $\vec{V}_0$, can be obtained by

$$\overrightarrow{v_k} = \left[\frac{2}{3} V_{dc} e^{j(k-1)\frac{\pi}{3}} \right] k = 1, 2, \ldots .6$$

$$\vec{V}_1 = \frac{2}{3} V_{dc} \, , \; \vec{V}_2 = \frac{2}{3} V_{dc} \, e^{j\frac{\pi}{3}} \quad \text{and} \quad \vec{V}_0 = 0$$

v_{ref} represents length of reference vector

$$\vec{v}_{ref} = v_{ref}\, e^{j\theta}$$



Substituting $\vec{v}_{ref} = v_{ref}\, e^{j\theta}$, $\vec{V}_1 = \frac{2}{3} V_{dc}$, $\vec{V}_2 = \frac{2}{3} V_{dc}\, e^{j\frac{\pi}{3}}$ and $\vec{V}_0 = 0$

into
$$\begin{cases} \vec{v}_{ref}\, T_s = \vec{V}_1\, T_a + \vec{V}_2\, T_b + \vec{V}_0\, T_0 \\ T_s = T_a + T_b + T_0 \end{cases}$$

& then splitting resultant equation into real (α-axis) & imaginary (β-axis) components in α-β frame, we have

$$\begin{cases} \text{Re}: \quad v_{ref}(\cos\theta) T_s = \frac{2}{3} V_{dc}\, T_a + \frac{1}{3} V_{dc}\, T_b \\ \text{Im}: \quad v_{ref}(\sin\theta)\, T_s = \frac{1}{\sqrt{3}} V_{dc}\, T_b \end{cases}$$

Solving
$$\begin{cases} \mathrm{Re}: \quad v_{ref}(\cos\theta)T_s = \frac{2}{3}V_{dc}T_a + \frac{1}{3}V_{dc}T_b \\ \mathrm{Im}: \quad v_{ref}(\sin\theta)\,T_s = \frac{1}{\sqrt{3}}V_{dc}T_b \end{cases}$$

together with $T_s = T_a + T_b + T_0$ yields

$$\begin{cases} T_a = \frac{\sqrt{3}\,T_s\,v_{ref}}{V_{dc}}\sin\left(\frac{\pi}{3} - \theta\right) \\ T_b = \frac{\sqrt{3}\,T_s\,v_{ref}}{V_{dc}}\sin\theta \\ T_0 = T_s - T_a - T_b \end{cases}$$

for $0 \le \theta < \pi/3$

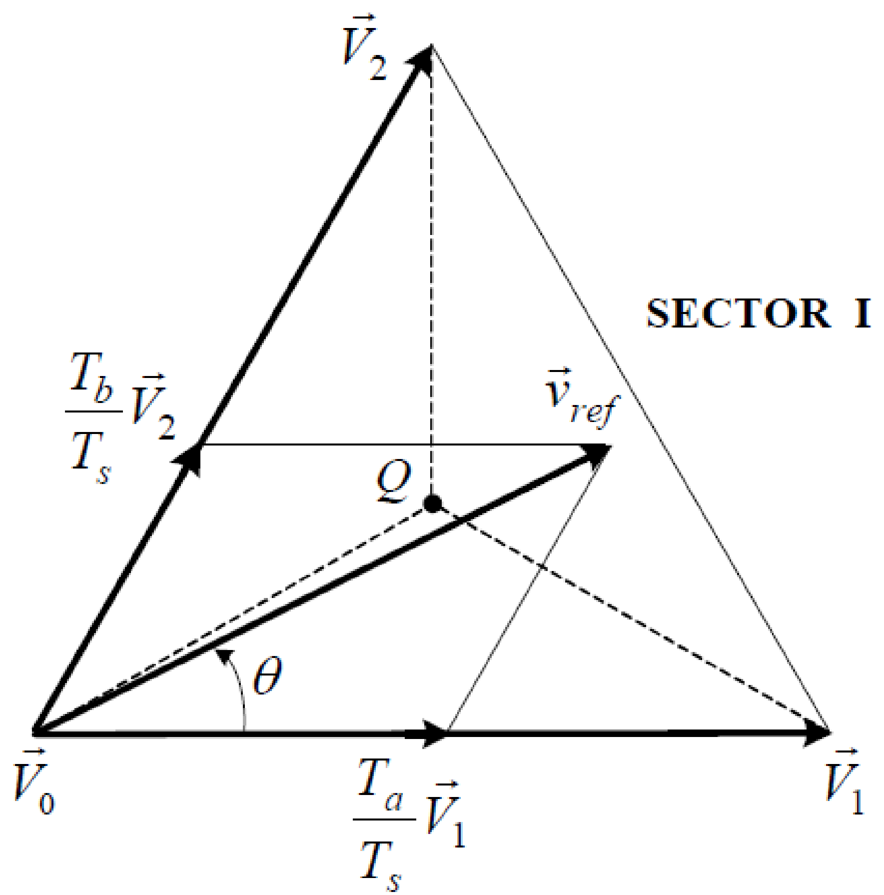
To visualize relationship between location of $\vec{v}_{ref}$ and dwell times, let us examine some special cases.

If $\vec{v}_{ref}$ lies exactly in middle between

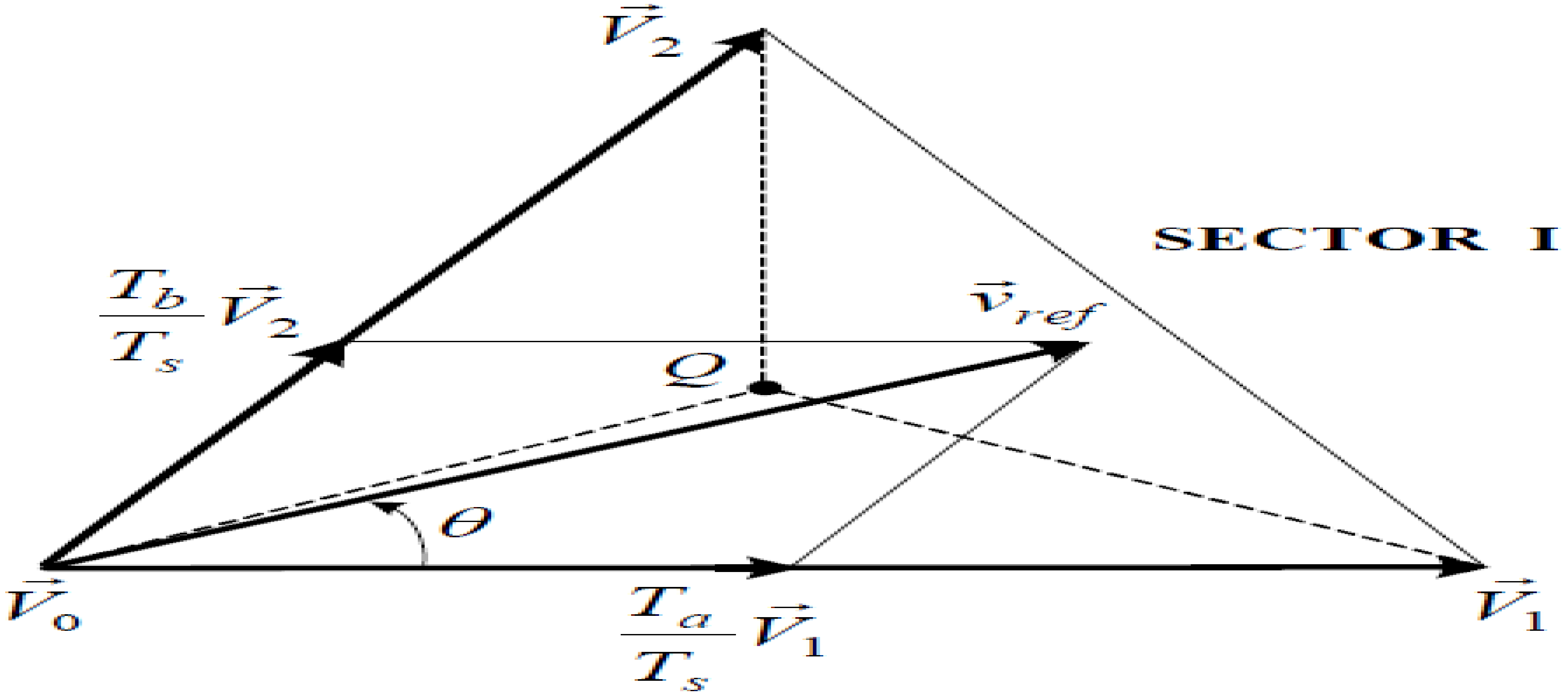
$\vec{V}_1$ and $\vec{V}_2$ (i.e., $\theta = \pi / 6$).

- Dwell time T_a of $\vec{V}_1$ will be equal to dwell time T_b of $\vec{V}_2$
- i.e *Ta=Tb*
- Because:

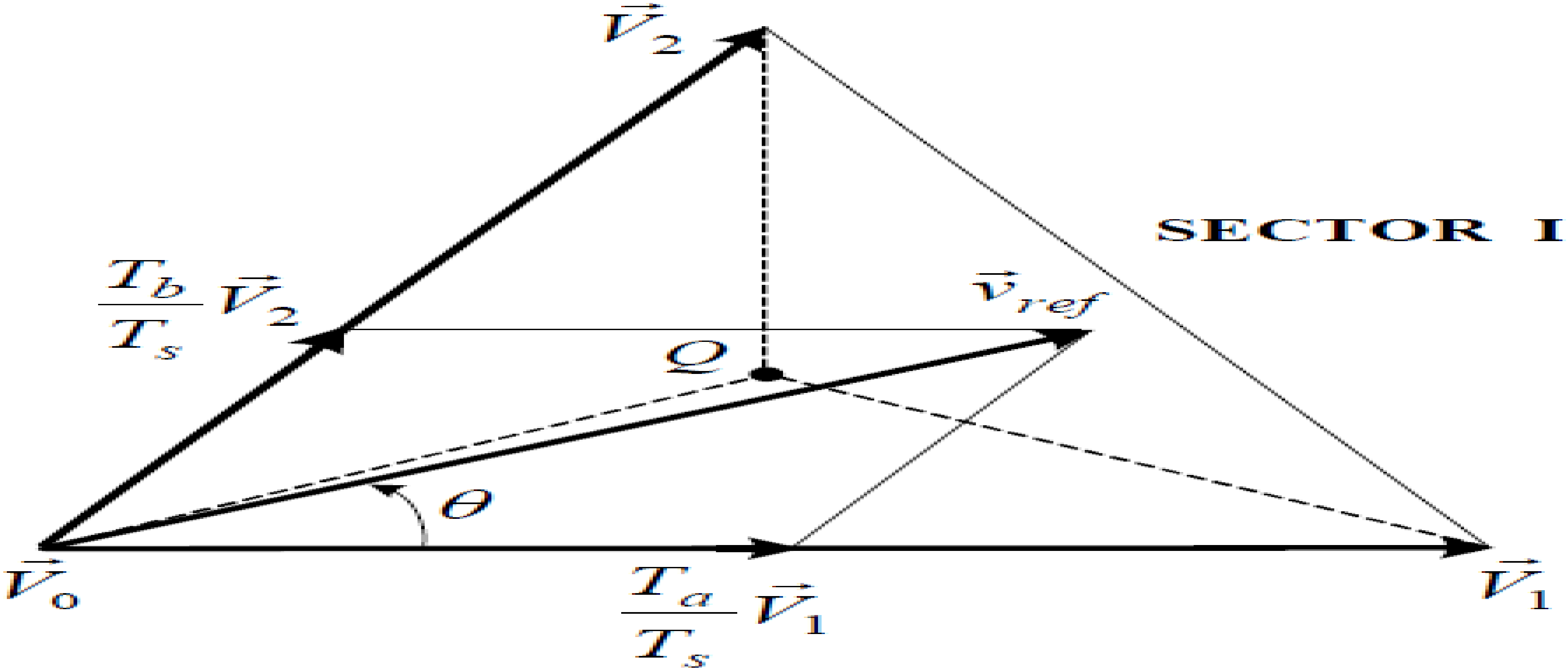
$$\frac{T_b}{\cancel{T_s}} \cancel{\vec{v}_2} = \frac{T_a}{\cancel{T_s}} \cancel{\vec{v}_1}$$



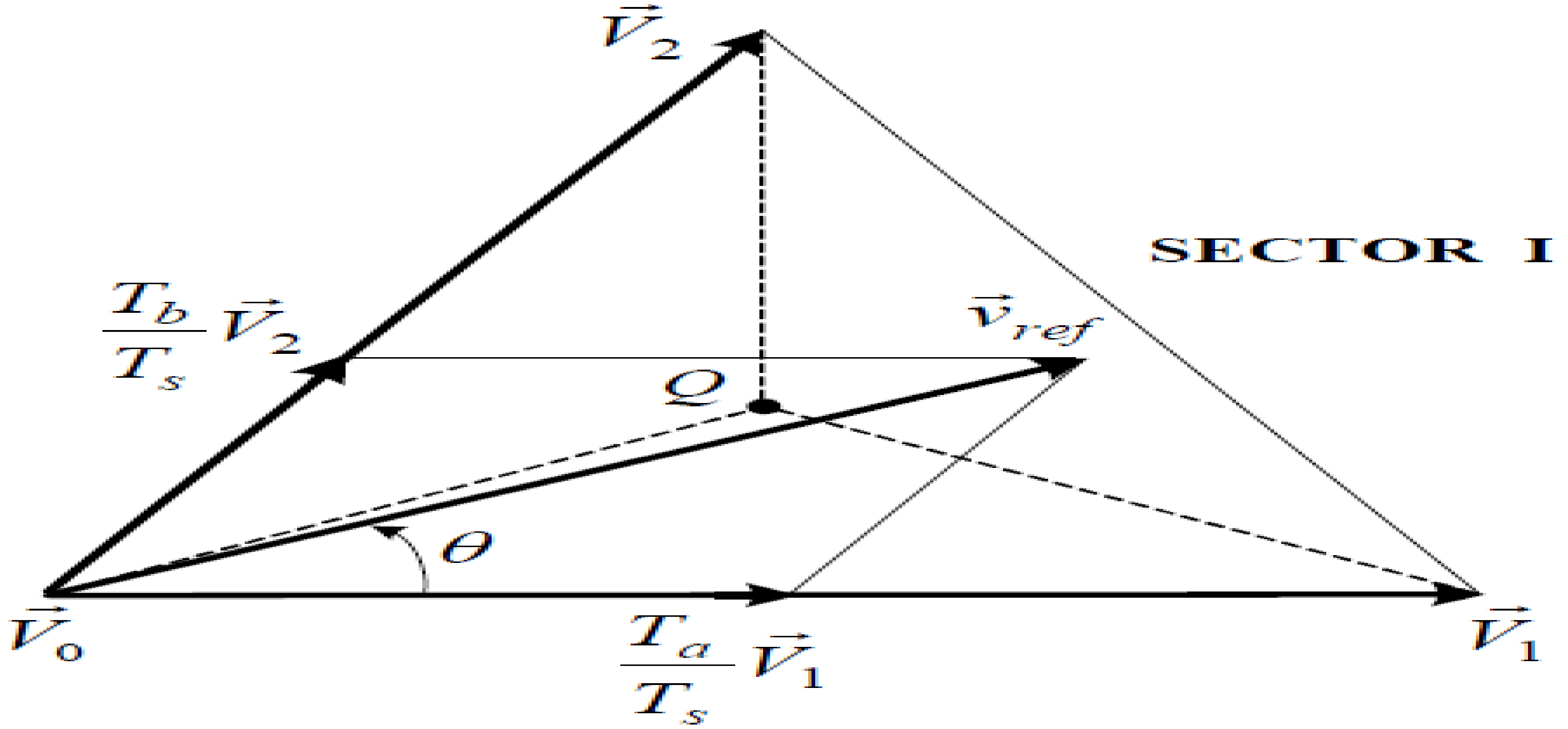
When $\vec{v}_{ref}$ is closer to $\vec{V}_2$, *Tb>Ta*



If $\vec{v}_{ref}$ is coincident with $\vec{V}_2$, *Ta=0*

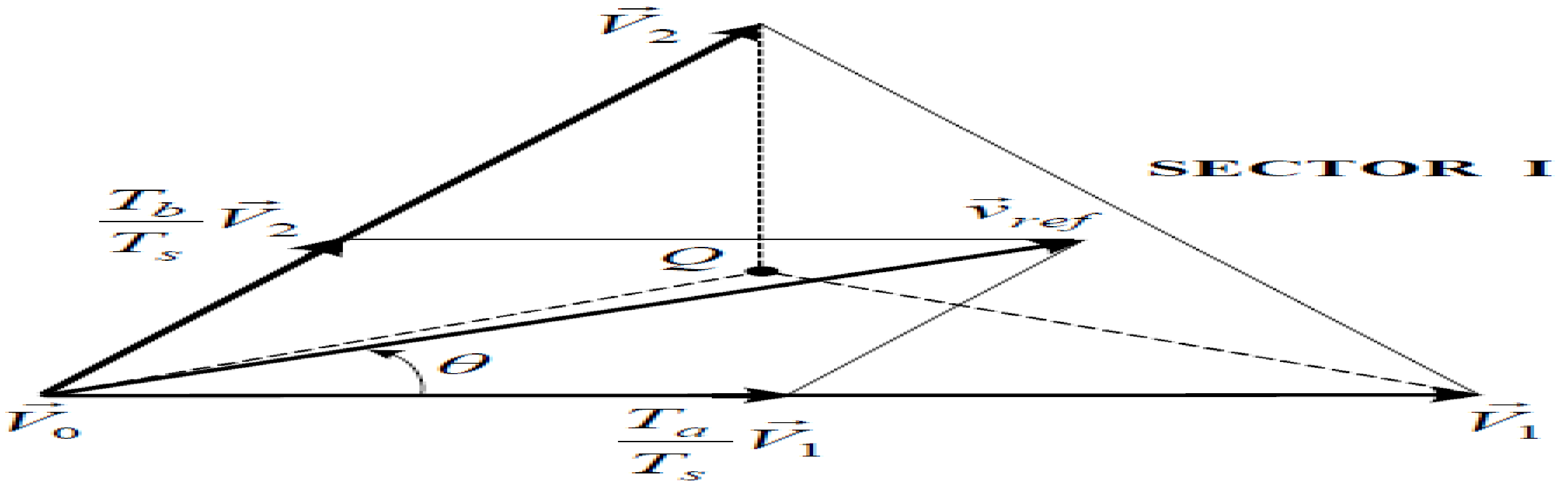


With head of $\vec{v}_{ref}$ located right on central point *Q*,
Ta = *Tb* = *T*0



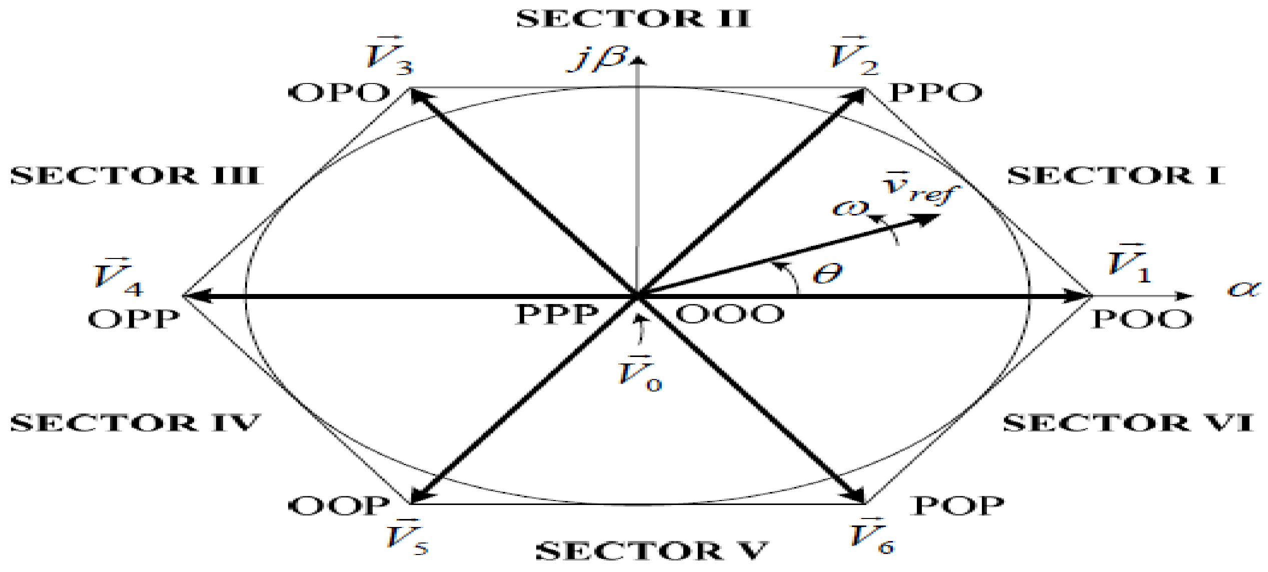
Relationship between $\vec{v}_{ref}$ location & dwell times is summarized in Table

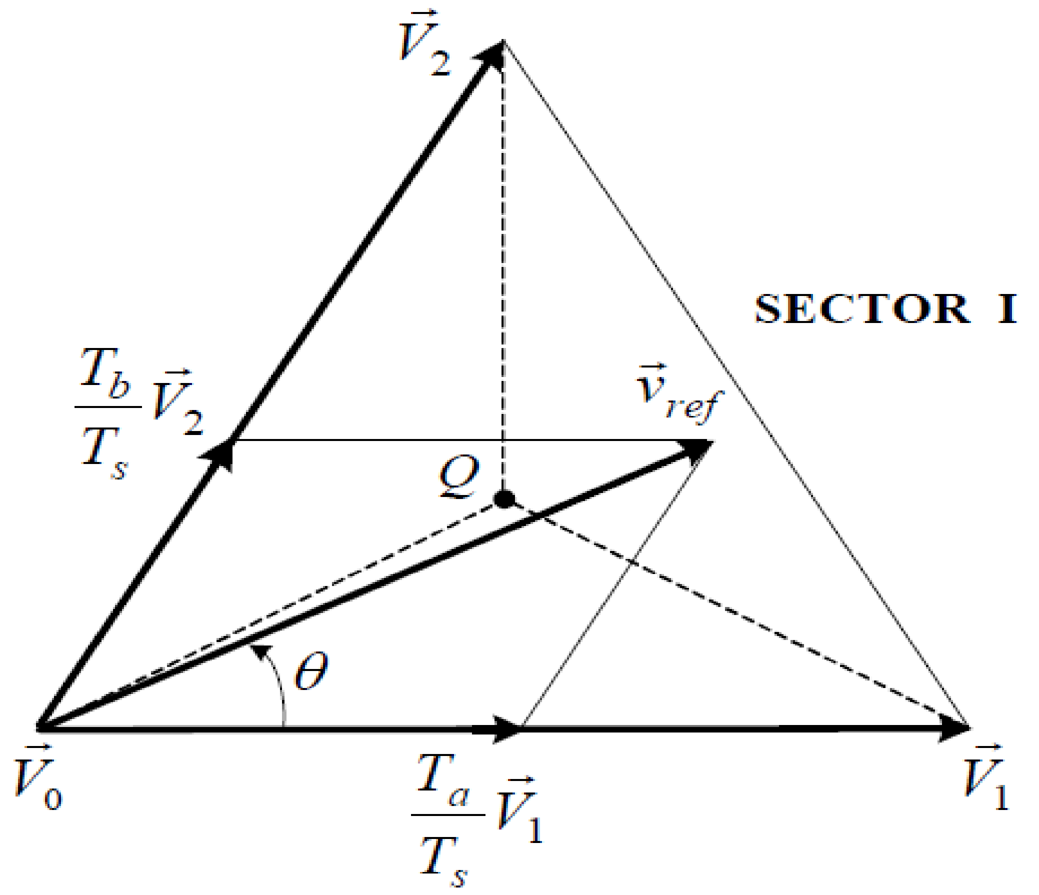
$\vec{v}_{ref}$ Location	$\theta = 0$	$0 < \theta < \frac{\pi}{6}$	$\theta = \frac{\pi}{6}$	$\frac{\pi}{6} < \theta < \frac{\pi}{3}$	$\theta = \frac{\pi}{3}$
Dwell Times	$T_a > 0$ $T_b = 0$	$T_a > T_b$	$T_a = T_b$	$T_a < T_b$	$T_a = 0$ $T_b > 0$



Although equation
$$\begin{cases} T_a = \dfrac{\sqrt{3}\,T_s\,v_{ref}}{V_{dc}} \sin\left(\dfrac{\pi}{3} - \theta\right) \\ T_b = \dfrac{\sqrt{3}\,T_s\,v_{ref}}{V_{dc}} \sin\theta \\ T_0 = T_s - T_a - T_b \end{cases} \qquad \text{for } 0 \le \theta < \pi/3$$

- is derived when $\vec{v}_{ref}$ is in sector I,
- it can also be used when $\vec{v}_{ref}$ is in other sectors?





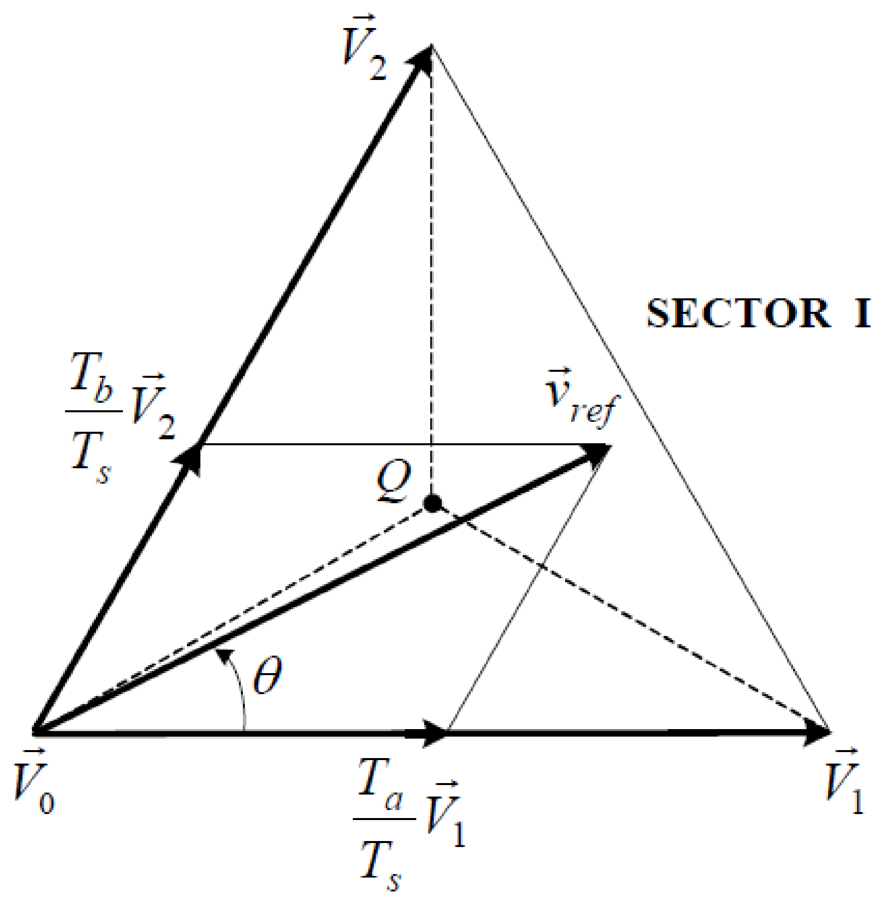
Answer: Yes

Although equation
$$\begin{cases} T_a = \dfrac{\sqrt{3}\,T_s\,v_{ref}}{V_{dc}} \sin\left(\dfrac{\pi}{3} - \theta\right) \\ T_b = \dfrac{\sqrt{3}\,T_s\,v_{ref}}{V_{dc}} \sin\theta \\ T_0 = T_s - T_a - T_b \end{cases} \qquad \text{for } 0 \le \theta < \pi/3$$

- is derived when $\vec{v}_{ref}$ is in sector I, it can also be used when $\vec{v}_{ref}$ is in other sectors provided that a multiple of π/3 is subtracted from actual angular displacement θ such that modified angle θ′ falls into range between 0 & π/3 for use in equation, i.e,

$$\theta' = \theta - (k-1)\pi/3 \qquad \text{for } 0 \le \theta' < \pi/3$$

- where *k* =1, 2, ..., 6 for sectors I, II, ..., VI, respectively.



d) Modulation Index

- Equation
$$\begin{cases} T_a = \dfrac{\sqrt{3}\,T_s\,v_{ref}}{V_{dc}}\sin\left(\dfrac{\pi}{3}-\theta\right) \\ T_b = \dfrac{\sqrt{3}\,T_s\,v_{ref}}{V_{dc}}\sin\theta \\ T_0 = T_s - T_a - T_b \end{cases}$$
$$\begin{cases} T_a = T_s m_a \sin\left(\dfrac{\pi}{3}-\theta\right) \\ T_b = T_s m_a \sin\theta \\ T_0 = T_s - T_a - T_b \end{cases}$$
- can also be expressed in terms of modulation index $m_a = \dfrac{\sqrt{3}\,v_{ref}}{V_{dc}}$

Length of reference vector $\vec{v}_{ref}$ represents peak value of fundamental-frequency component in inverter output phase voltage, i.e, $v_{ref} = \hat{V}_{a1} = \sqrt{2}V_{a1}$

where V_{a1} is rms value of fundamental component in inverter output phase (phase-a) voltage.

Relationship between *ma* & *Va*1

Substituting $v_{ref} = \hat{V}_{a1} = \sqrt{2} V_{a1}$ into $m_a = \dfrac{\sqrt{3}\, v_{ref}}{V_{dc}}$

$$m_a = \frac{\sqrt{3}\, v_{ref}}{V_{dc}} = \frac{\sqrt{6}\, V_{a1}}{V_{dc}}$$

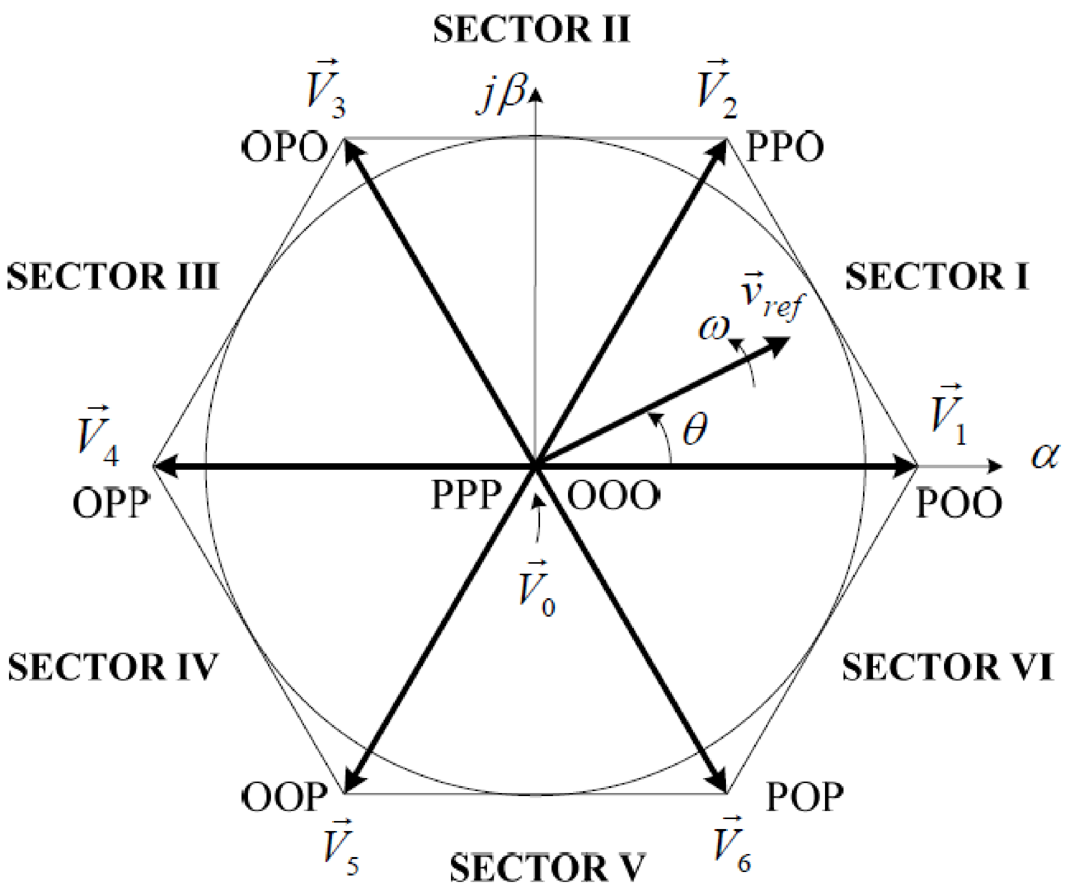
For a given dc voltage V_{dc}, inverter output voltage V_{a1} is proportional to modulation index *ma*.

$$V_{a1} \propto m_a$$

Maximum length of reference vector, $v_{ref,\max}$, corresponds to radius of largest circle that can be inscribed within hexagon as shown in Fig.

- Since hexagon is formed by 6 active vectors having a length of $2Vdc/3$, $v_{ref,\max}$,can be found from

$$v_{ref,\max} = \frac{2}{3}V_{dc} \times \frac{\sqrt{3}}{2} = \frac{V_{dc}}{\sqrt{3}}$$



Substituting $v_{ref,\max} = \dfrac{V_{dc}}{\sqrt{3}}$ into $m_a = \dfrac{\sqrt{3}\ v_{ref}}{V_{dc}}$

- Gives the maximum modulation index $m_{a,\max} = 1$

from which modulation index for SVM scheme is in range of

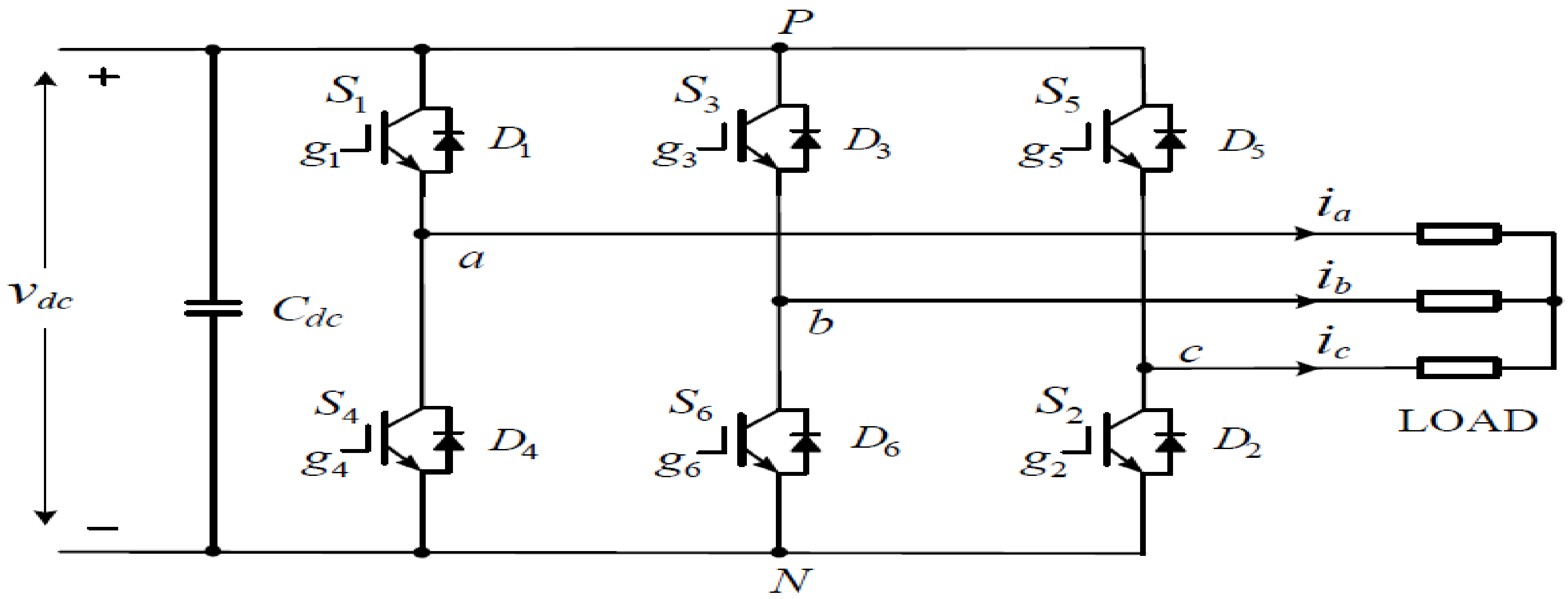
$$0 \le m_a \le 1$$

e) Switching Sequence

- With space vectors selected & their dwell times calculated,
- next step is to **arrange switching sequence.**
- In general, switching sequence design for a given $\vec{v}_{ref}$ is not unique,
- but it should satisfy following 2 requirements for minimization of device switching frequency:

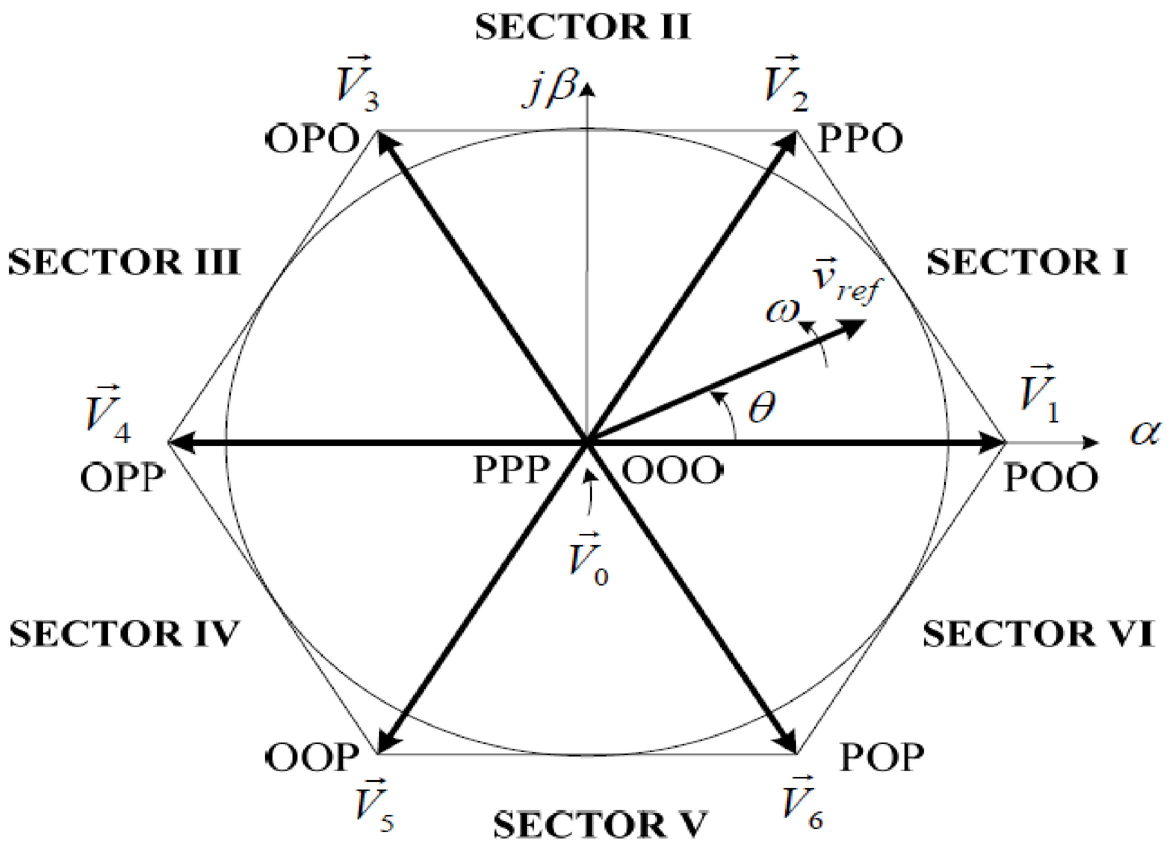
1st requirement for minimization of device switching frequency

a) Transition from one switching state to next involves only 2 switches in same inverter leg, one being switched on & other switched off

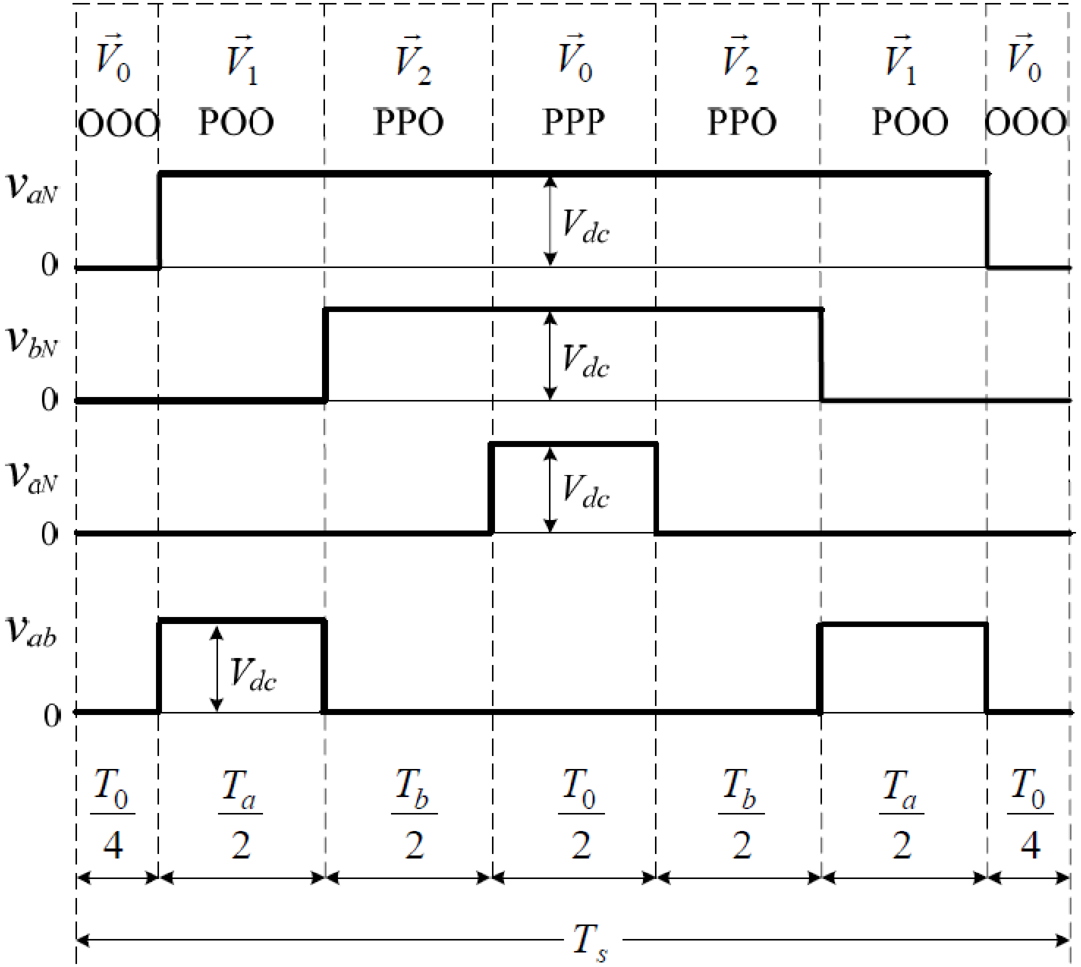


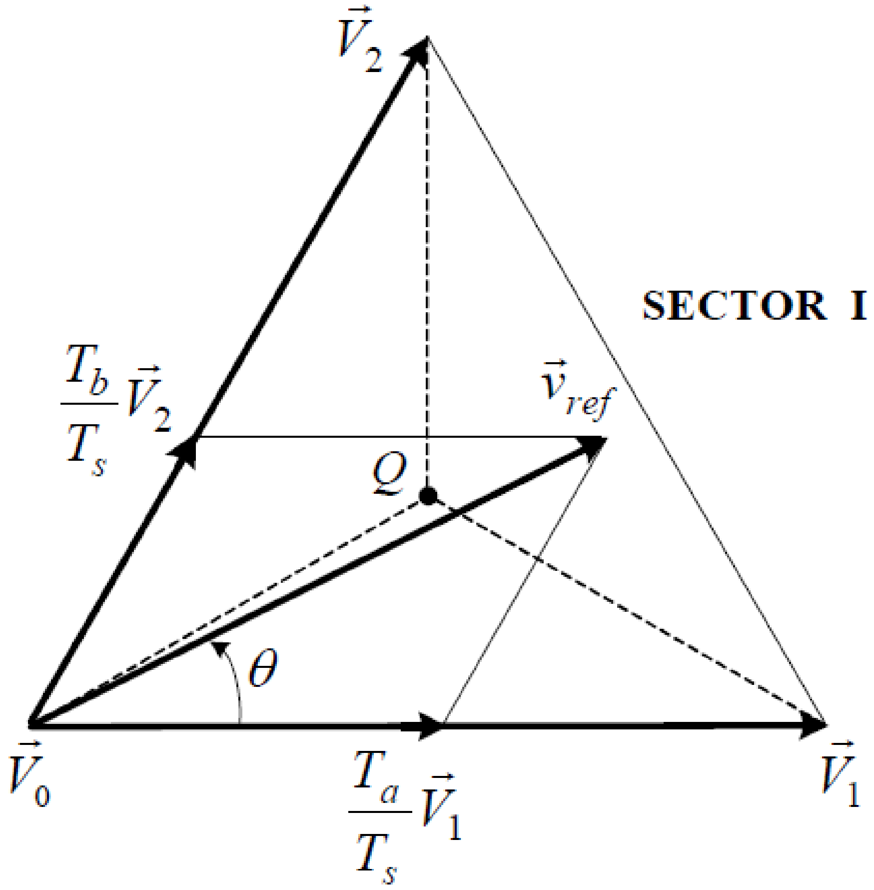
2nd requirement for minimization of device switching frequency

b) Transition for $\vec{v}_{ref}$ moving from 1 sector in space vector diagram to next requires no or minimum number of switchings.

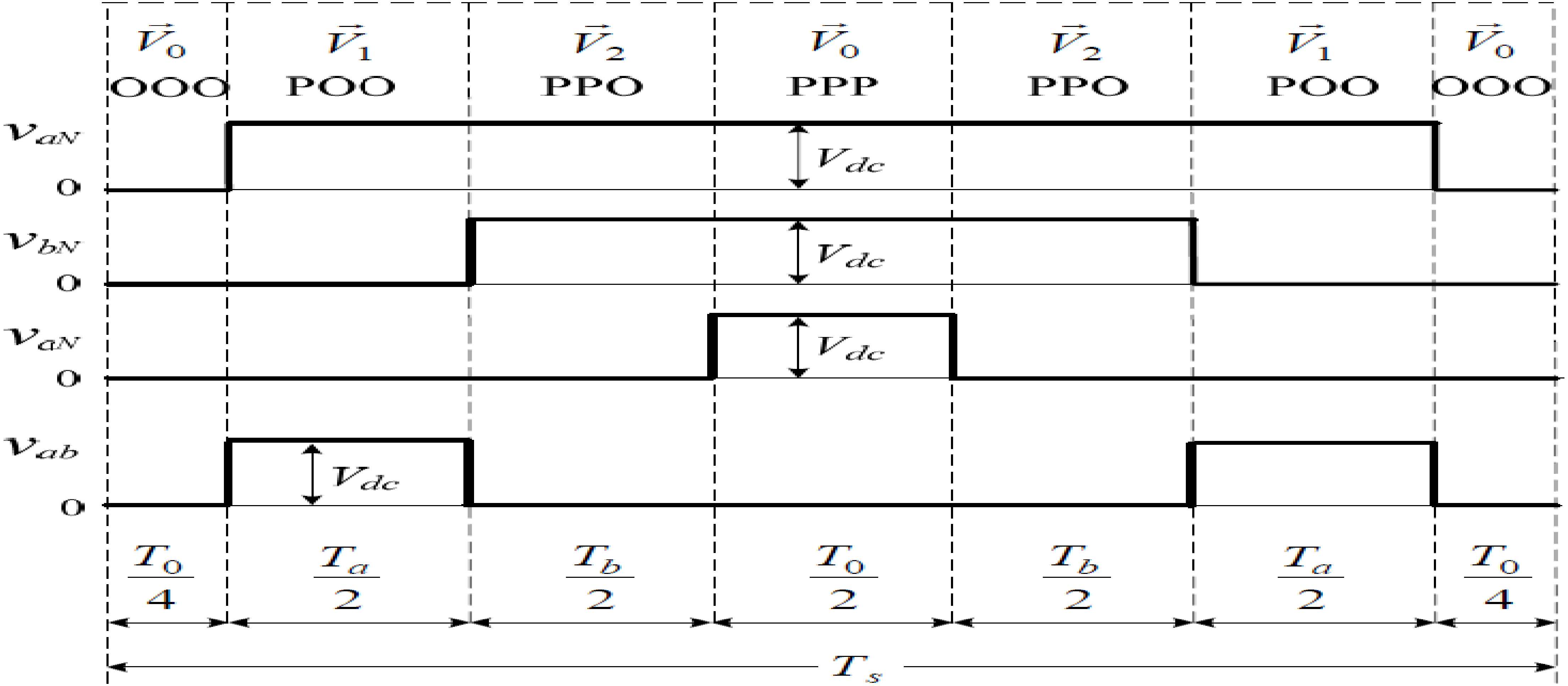


Seven-segment switching sequence & inverter output voltage waveforms for $\vec{v}_{ref}$ in sector I,where $\vec{v}_{ref}$s synthesized by $\vec{V}_1$, $\vec{V}_2$ and $\vec{V}_0$





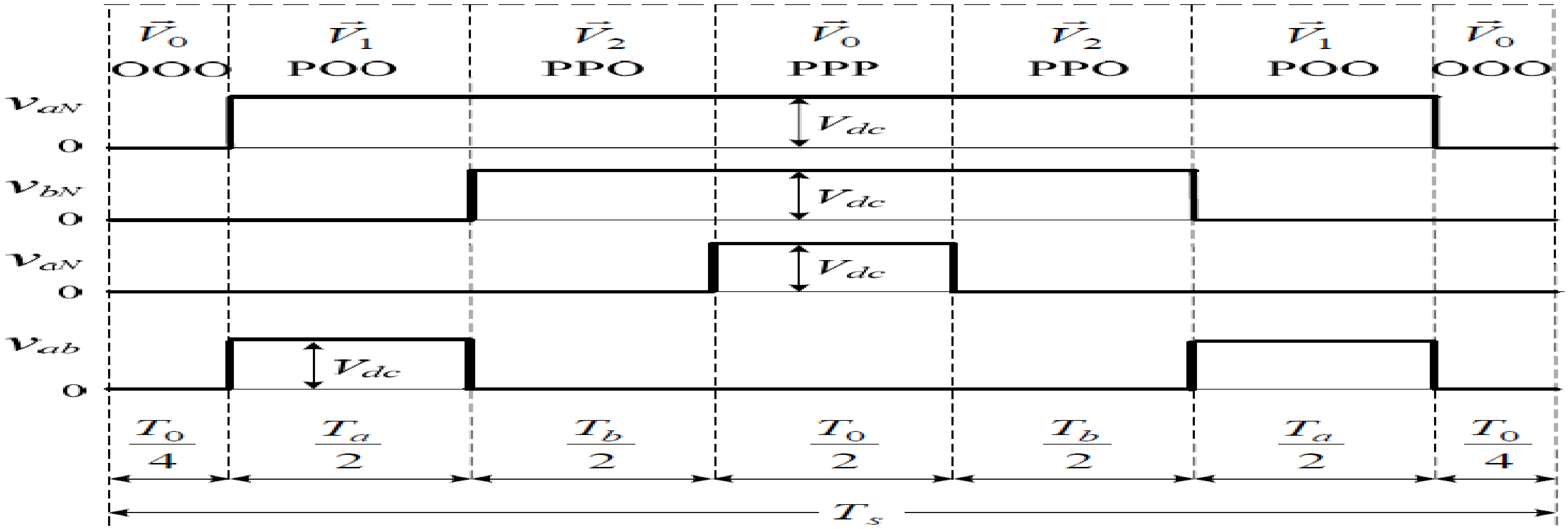
Sampling period *Ts* is divided into 7 segments for selected vectors.



It can be observed that:

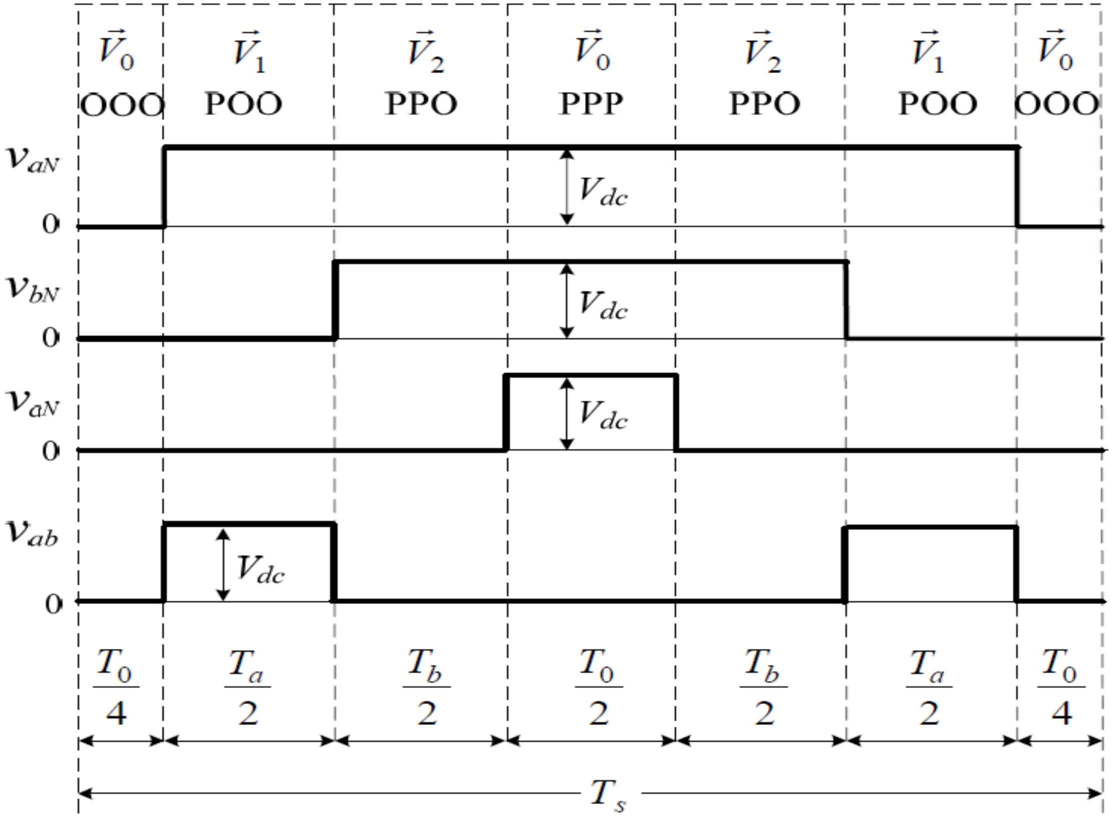
- Dwell times for 7 segments add up to the sampling period

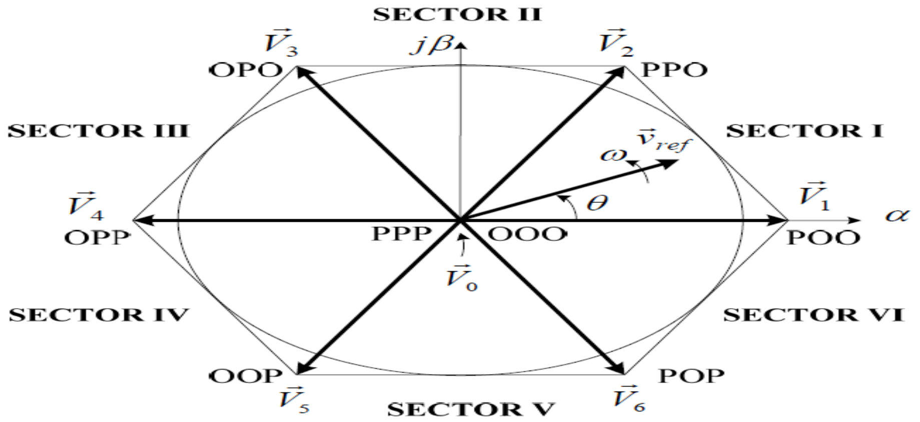
  ($Ts = Ta + Tb + T0$)

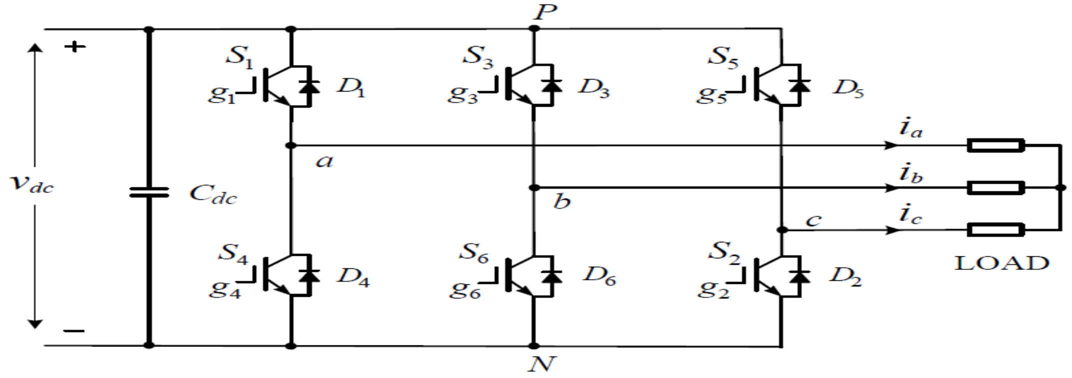


Design requirement a)is satisfied.

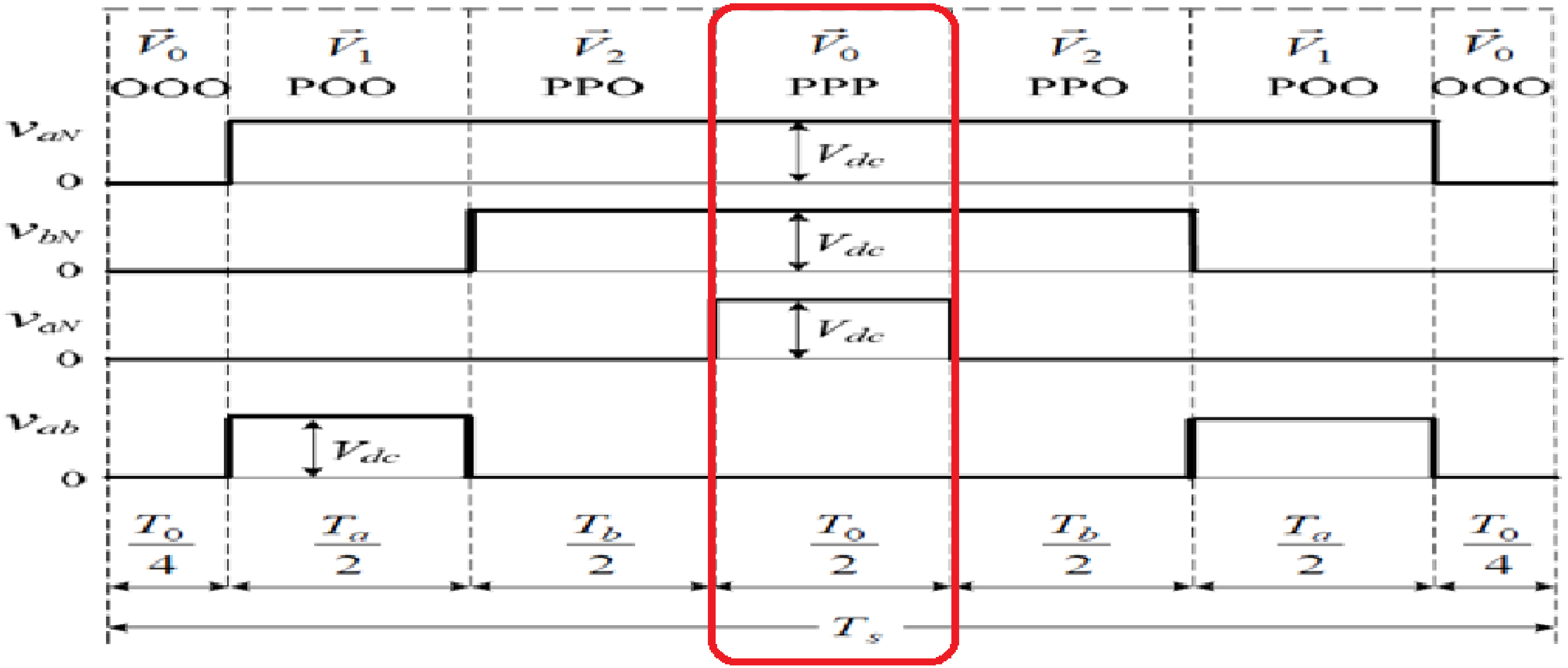
- For instance, transition from [OOO] to [POO] is accomplished by turning *S*1 on & *S*4 off, which involves only 2 switches;



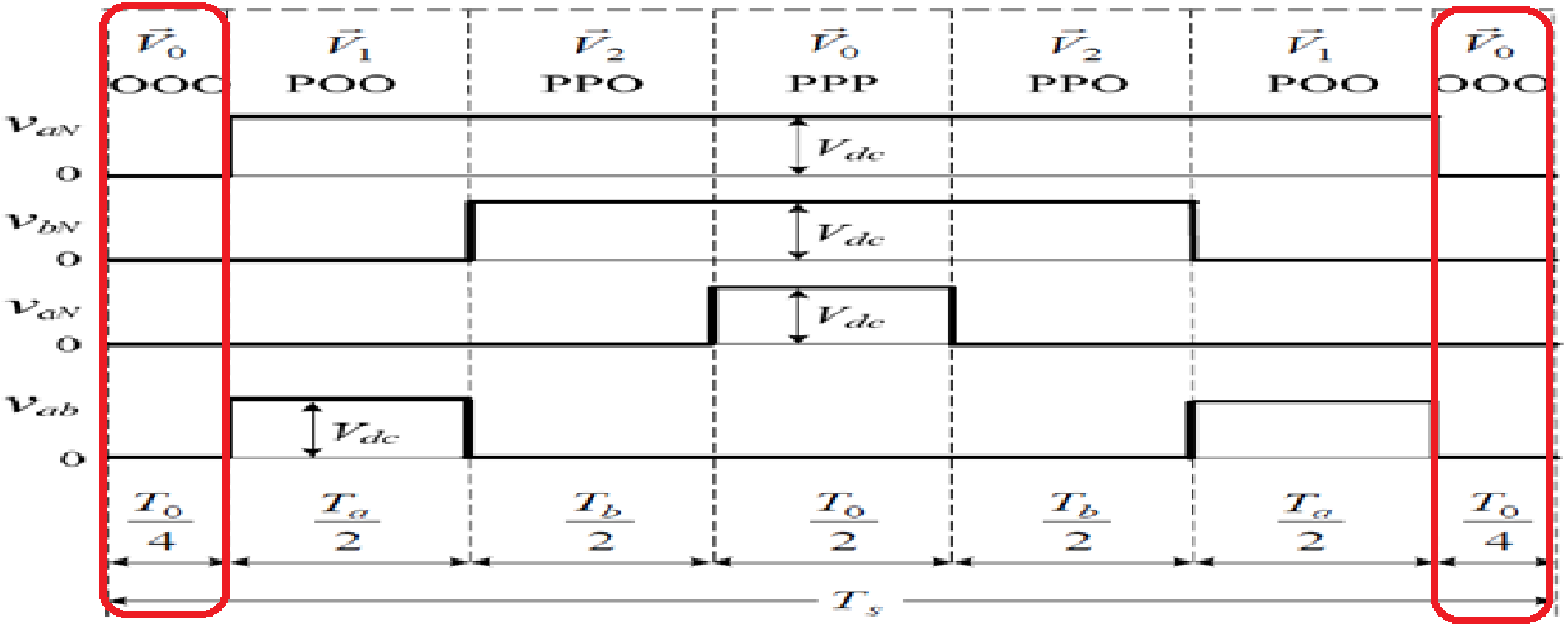




For $T0/2$ segment in centre of sampling period, switching state [PPP] is selected



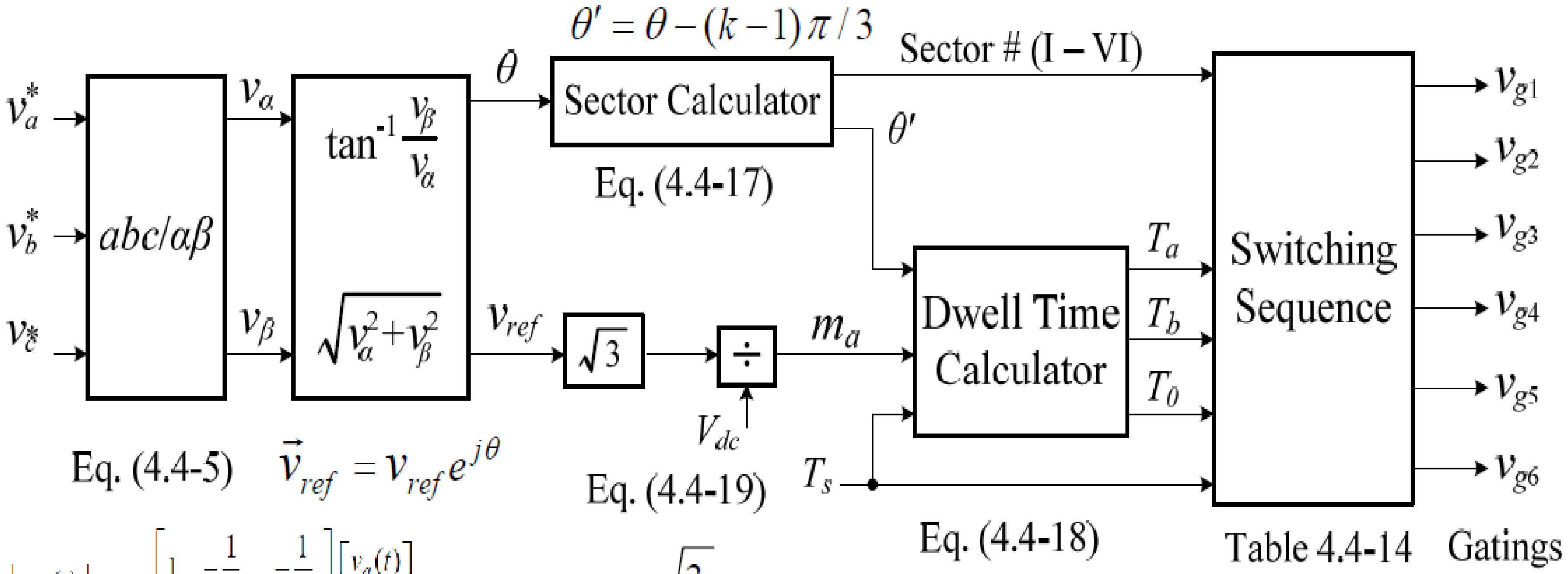
For *T*0 / 4 segments on both sides, state [OOO] is used



Case Study 4-3 Two-level Voltage Source Inverter with Space Vector Modulation

- In this case study, procedure for computer simulation & real-time digital implementation of SVM scheme is introduced.
- Simulation for 2-level voltage source inverter in performance, & harmonic performance of SVM scheme is analysed.

Block diagram for computer simulation & real-time digital implementation of SVM algorithm.

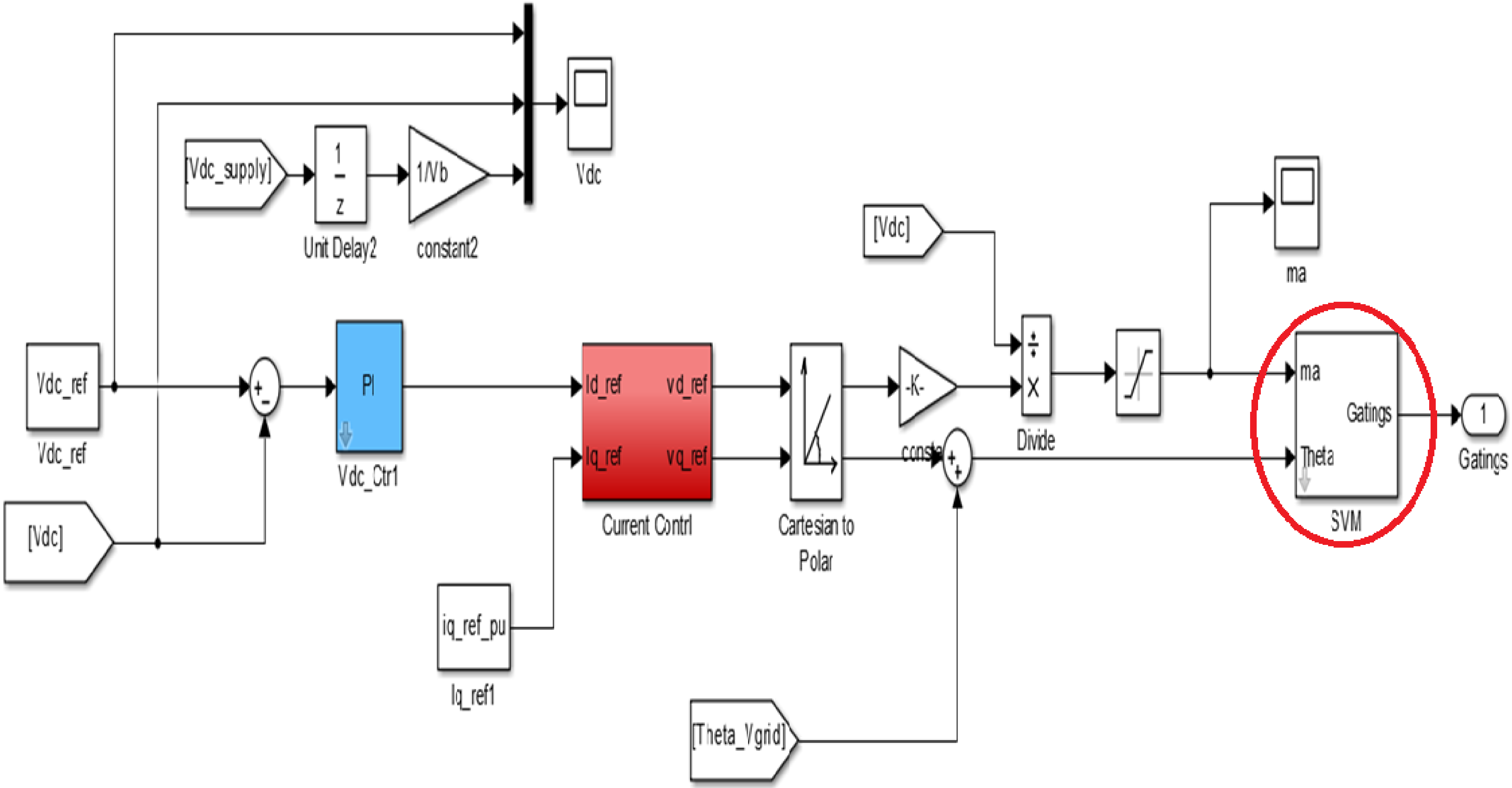


Sector	Switching Segment						
	1	2	3	4	5	6	7
I	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	PPO	PPP	PPO	POO	OOO
II	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	PPO	PPP	PPO	OPO	OOO
III	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	OPP	PPP	OPP	OPO	OOO
IV	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	OPP	PPP	OPP	OOP	OOO
V	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	POP	PPP	POP	OOP	OOO
VI	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	POP	PPP	POP	POO	OOO

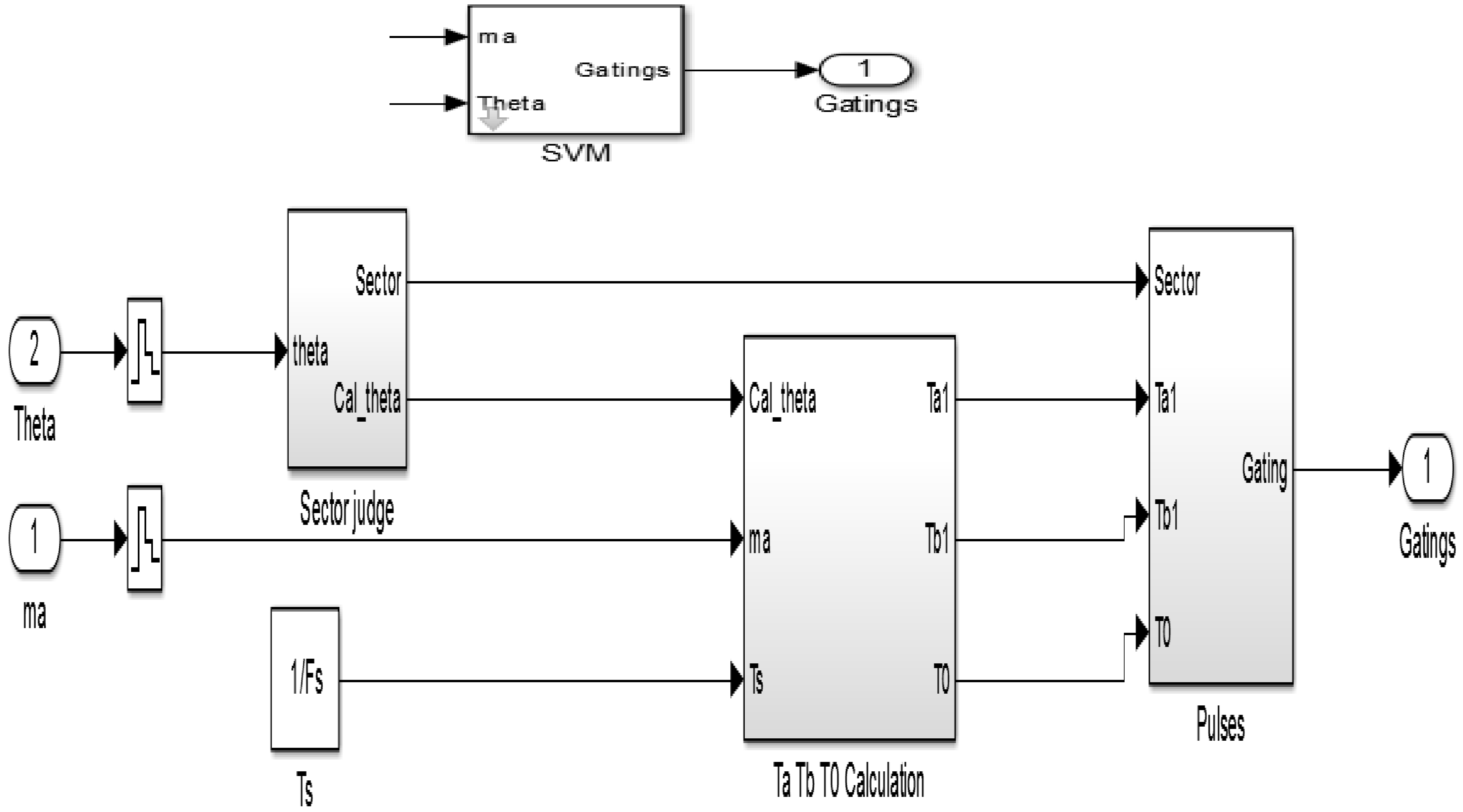
$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}$$

$$m_a = \frac{\sqrt{3}\, v_{ref}}{V_{dc}}$$

$$\begin{cases} T_a = T_s m_a \sin\left(\frac{\pi}{3} - \theta\right) \\ T_b = T_s m_a \sin\theta \\ T_0 = T_s - T_a - T_b \end{cases}$$



[Vdc_supply]
1
z
Unit Delay2
1/Vb
constant2
Vdc
[Vdc]
ma
Vdc_ref
Vdc_ref
PI
Vdc_Ctr1
Id_ref
Iq_ref
vd_ref
vq_ref
Current Contrl
Cartesian to
Polar
-K-
Divide
ma
Theta
Gatings
SVM
1
Gatings
[Vdc]
iq_ref_pu
Iq_ref1
[Theta_Vgrid]



ma
Theta
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SVM
1
Gatings
2
Theta
1
ma
theta
Sector
Cal_theta
Sector judge
1/Fs
Ts
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Ts
Ta1
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Ta Tb T0 Calculation
Sector
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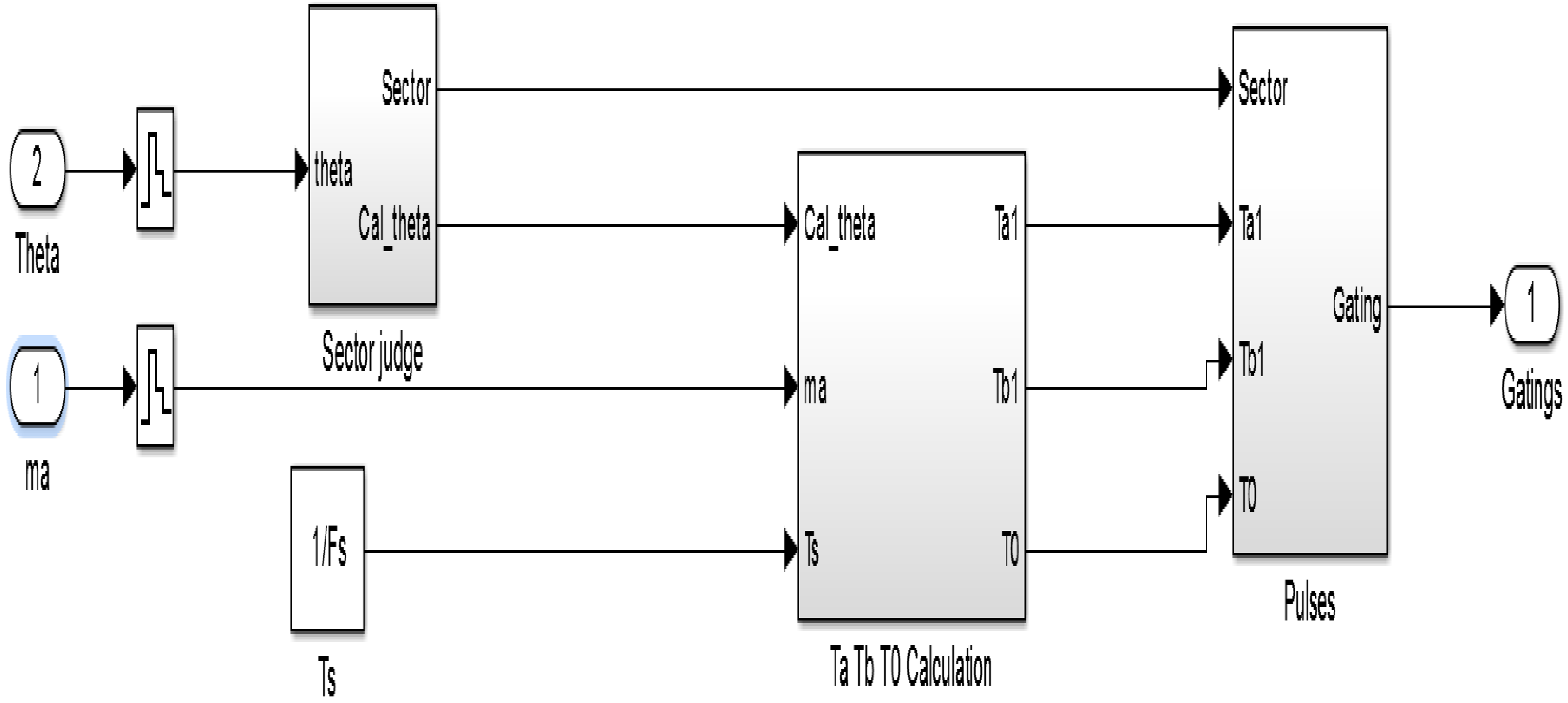
T0
Gating
Pulses
1
Gatings


2
Theta
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Cal_theta
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Ts
Cal_theta
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T0
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Gating
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Sector
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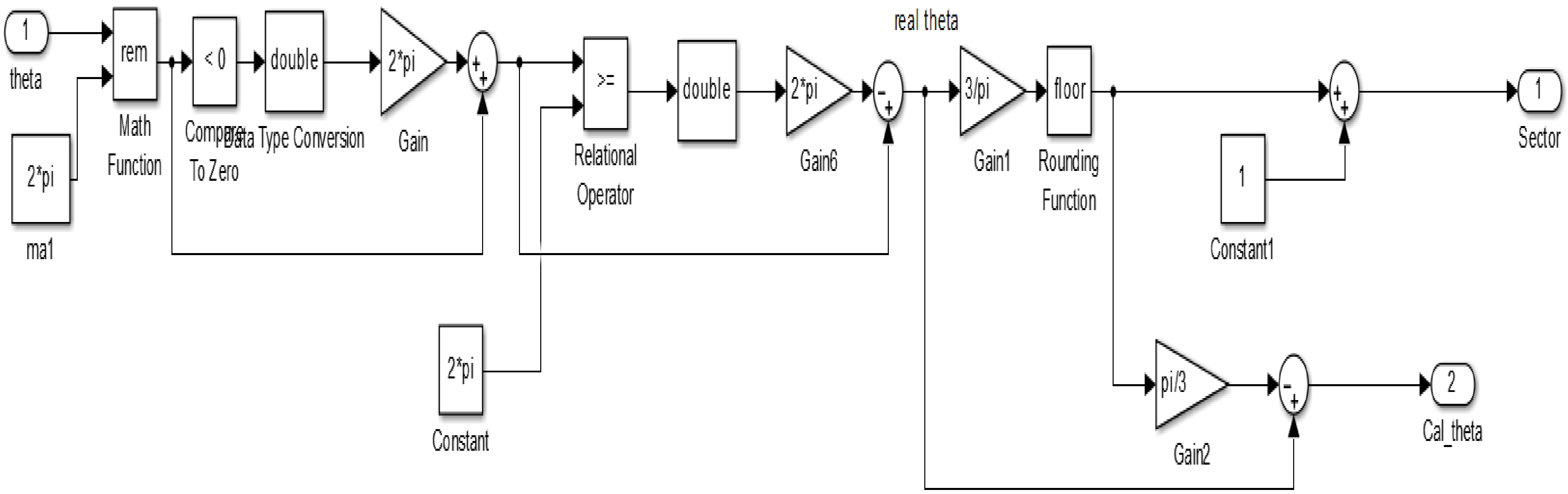
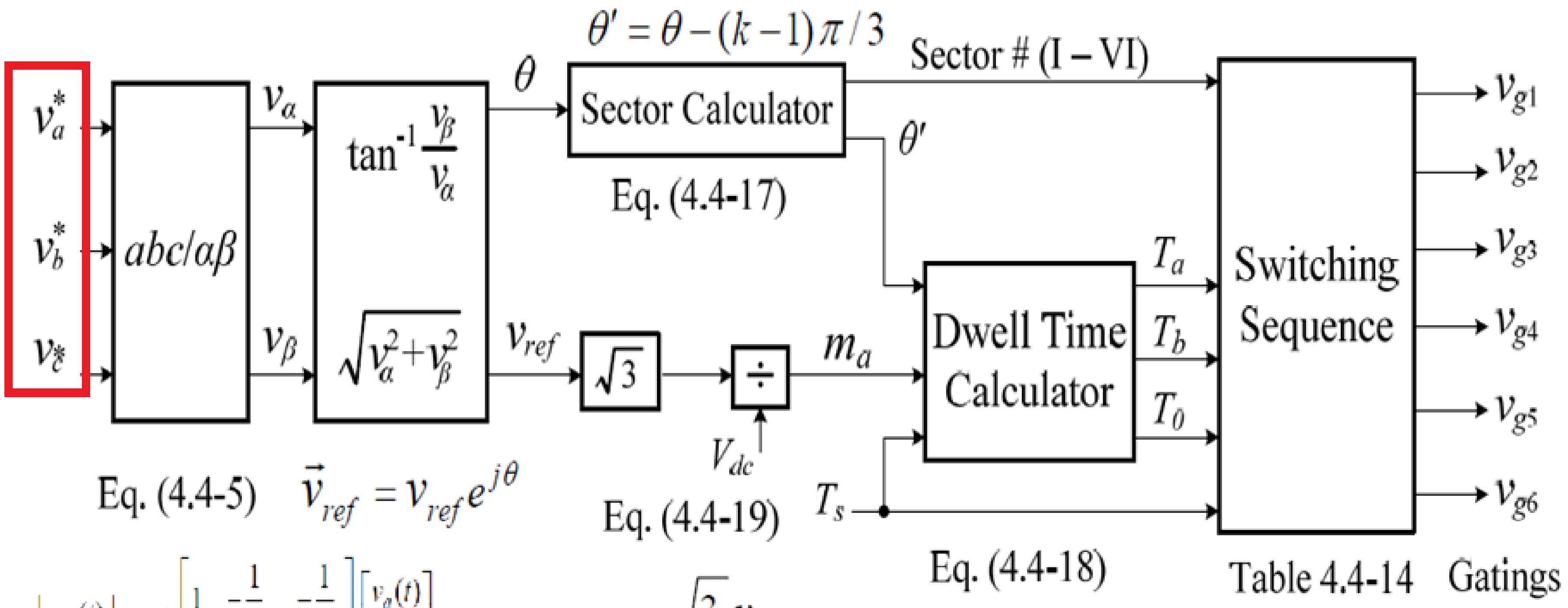

1
theta
2*pi
ma1
rem
Math Function
< 0
Compare To Zero
double
Data Type Conversion
2*pi
Gain
>=
Relational Operator
2*pi
Constant
double
2*pi
Gain6
real theta
3/pi
Gain1
floor
Rounding Function
1
Constant1
1
Sector
pi/3
Gain2
2
Cal_theta

Input variables, v_a^*, v_b^* & v_c^* are 3-phase reference voltages, which are also required output phase voltages of inverter



Sector	Switching Segment						
	1	2	3	4	5	6	7
I	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	PPO	PPP	PPO	POO	OOO
II	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	PPO	PPP	PPO	OPO	OOO
III	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	OPP	PPP	OPP	OPO	OOO
IV	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	OPP	PPP	OPP	OOP	OOO
V	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	POP	PPP	POP	OOP	OOO
VI	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	POP	PPP	POP	POO	OOO

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}$$

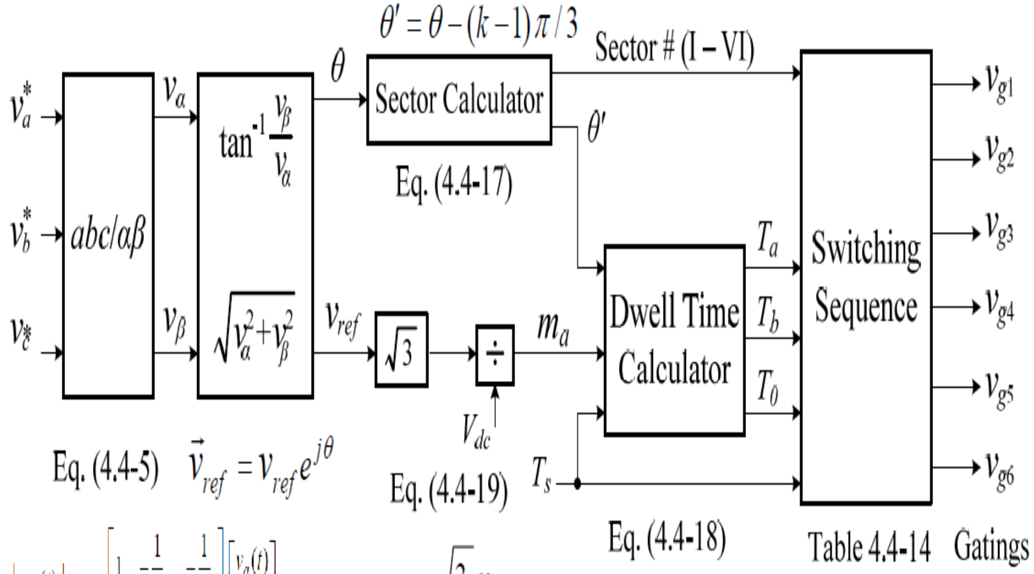
$$m_a = \frac{\sqrt{3}\, v_{ref}}{V_{dc}}$$

$$\begin{cases} T_a = T_s m_a \sin\left(\frac{\pi}{3} - \theta\right) \\ T_b = T_s m_a \sin\theta \\ T_0 = T_s - T_a - T_b \end{cases}$$

Reference voltages v_a^*, v_b^* & v_c^* are normally generated by controller in a wind energy conversion system.

- Through *abc/αβ* transformation, 3-phase reference voltages in *abc* stationary frame are transformed into 2-phase variables, v_α & v_β, in *α-β* stationary frame,
- from which reference vector for SVM scheme is established:

$$\vec{v}_{ref} = v_{ref} e^{j\theta}$$



Sector	Switching Segment						
	1	2	3	4	5	6	7
I	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	PPO	PPP	PPO	POO	OOO
II	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	PPO	PPP	PPO	OPO	OOO
III	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	OPP	PPP	OPP	OPO	OOO
IV	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	OPP	PPP	OPP	OOP	OOO
V	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	POP	PPP	POP	OOP	OOO
VI	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	POP	PPP	POP	POO	OOO

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}$$

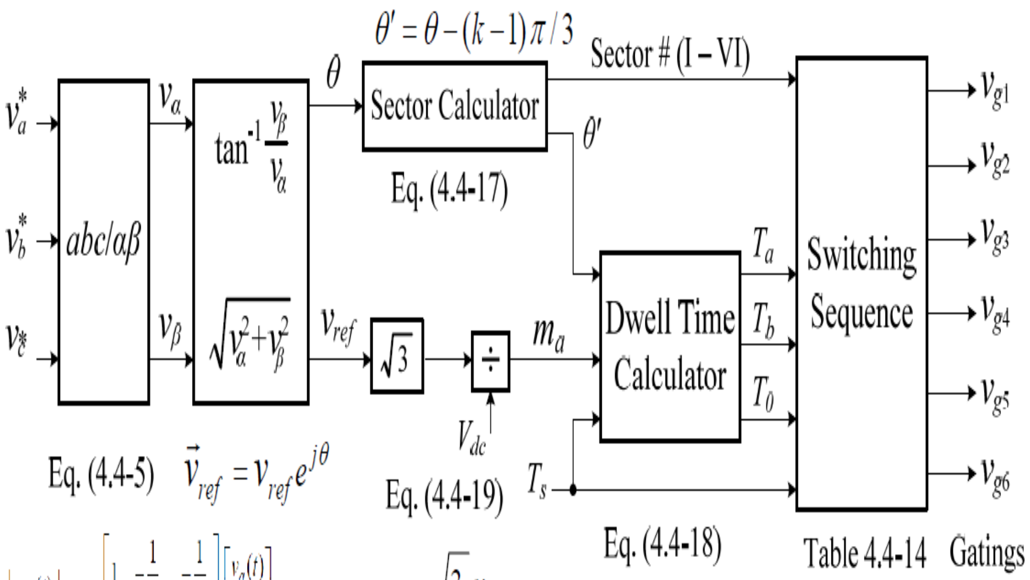
$$m_a = \frac{\sqrt{3}\, v_{ref}}{V_{dc}}$$

$$\begin{cases} T_a = T_s m_a \sin(\frac{\pi}{3} - \theta) \\ T_b = T_s m_a \sin\theta \\ T_0 = T_s - T_a - T_b \end{cases}$$

$$\begin{cases} v_{ref} = \sqrt{(v_\alpha)^2 + (v_\beta)^2} \\ \theta = \tan^{-1}\frac{v_\beta}{v_\alpha} \end{cases}$$

With reference vector in place, modulation index *ma* and sector number can be calculated by (4.4-19) and (4.4-17),

- Dwell times can be determined by (4.4-18), and switching sequence can be designed according to Table 4.4-4.
- Finally, gate signals for 6 switches in inverter can be generated.
- With SVM scheme, fundamental frequency and magnitude of inverter output voltages *va*, *vb*, and *vc* are equal to those of 3-phase reference voltage v_a^*, v_b^*, and v_c^*



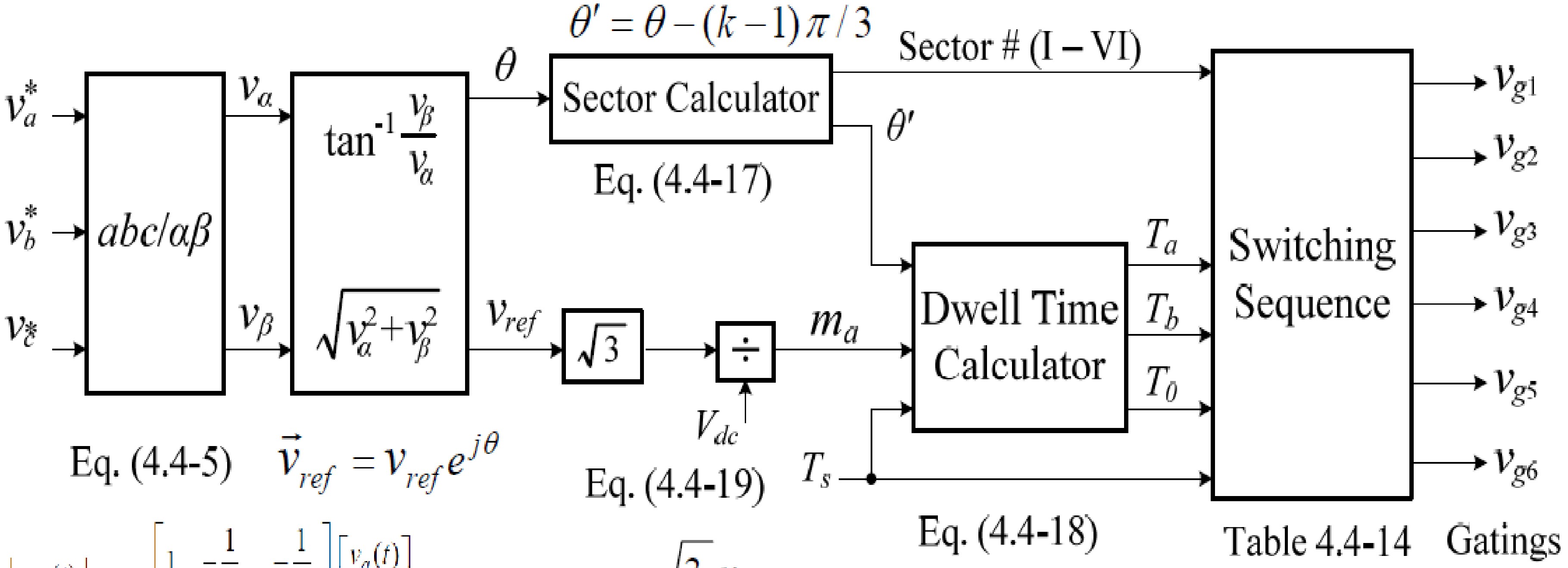
Sector	Switching Segment						
	1	2	3	4	5	6	7
I	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	PPO	PPP	PPO	POO	OOO
II	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	PPO	PPP	PPO	OPO	OOO
III	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	OPP	PPP	OPP	OPO	OOO
IV	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	OPP	PPP	OPP	OOP	OOO
V	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	POP	PPP	POP	OOP	OOO
VI	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	POP	PPP	POP	POO	OOO

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}$$

$$m_a = \frac{\sqrt{3}\, v_{ref}}{V_{dc}}$$

$$\begin{cases} T_a = T_s m_a \sin(\frac{\pi}{3}-\theta) \\ T_b = T_s m_a \sin\theta \\ T_0 = T_s - T_a - T_b \end{cases}$$

As result, inverter output voltage is fully controllable by its references



$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}$$

$$m_a = \frac{\sqrt{3}\, v_{ref}}{V_{dc}}$$

$$\begin{cases} T_a = T_s m_a \sin(\frac{\pi}{3} - \theta) \\ T_b = T_s m_a \sin\theta \\ T_0 = T_s - T_a - T_b \end{cases}$$

Sector	Switching Segment						
	1	2	3	4	5	6	7
I	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	PPO	PPP	PPO	POO	OOO
II	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	PPO	PPP	PPO	OPO	OOO
III	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	OPP	PPP	OPP	OPO	OOO
IV	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	OPP	PPP	OPP	OOP	OOO
V	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	POP	PPP	POP	OOP	OOO
VI	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	POP	PPP	POP	POO	OOO