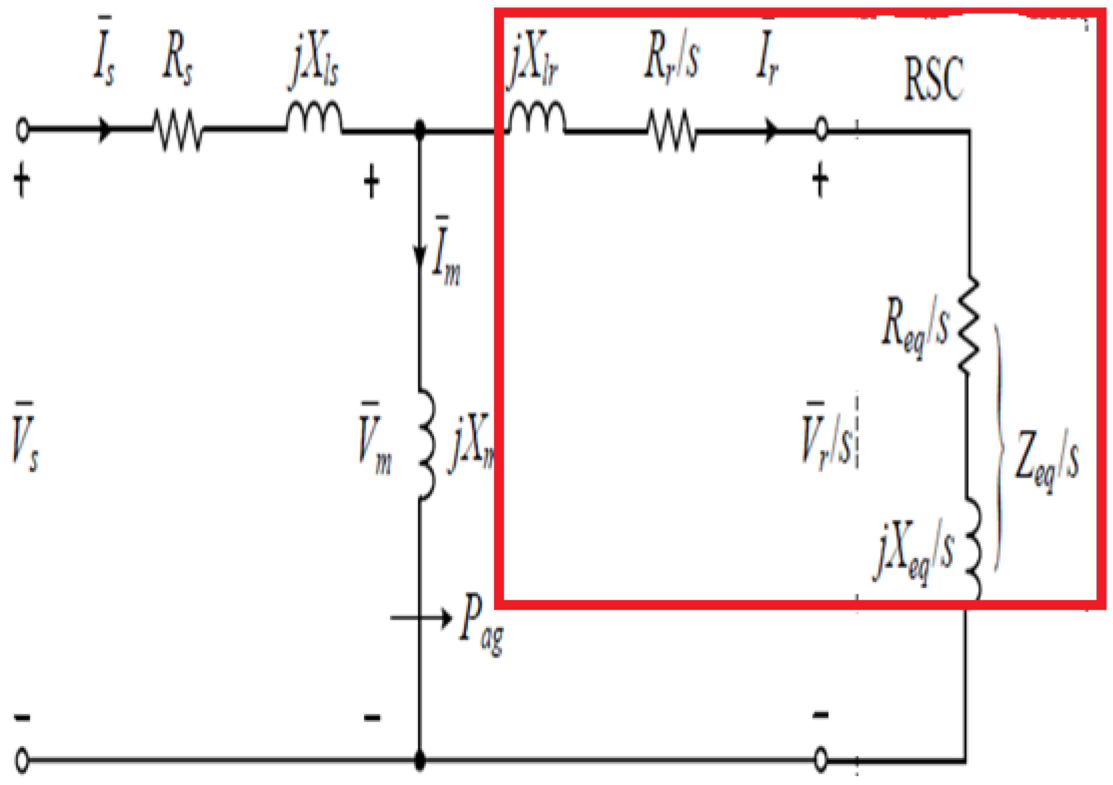
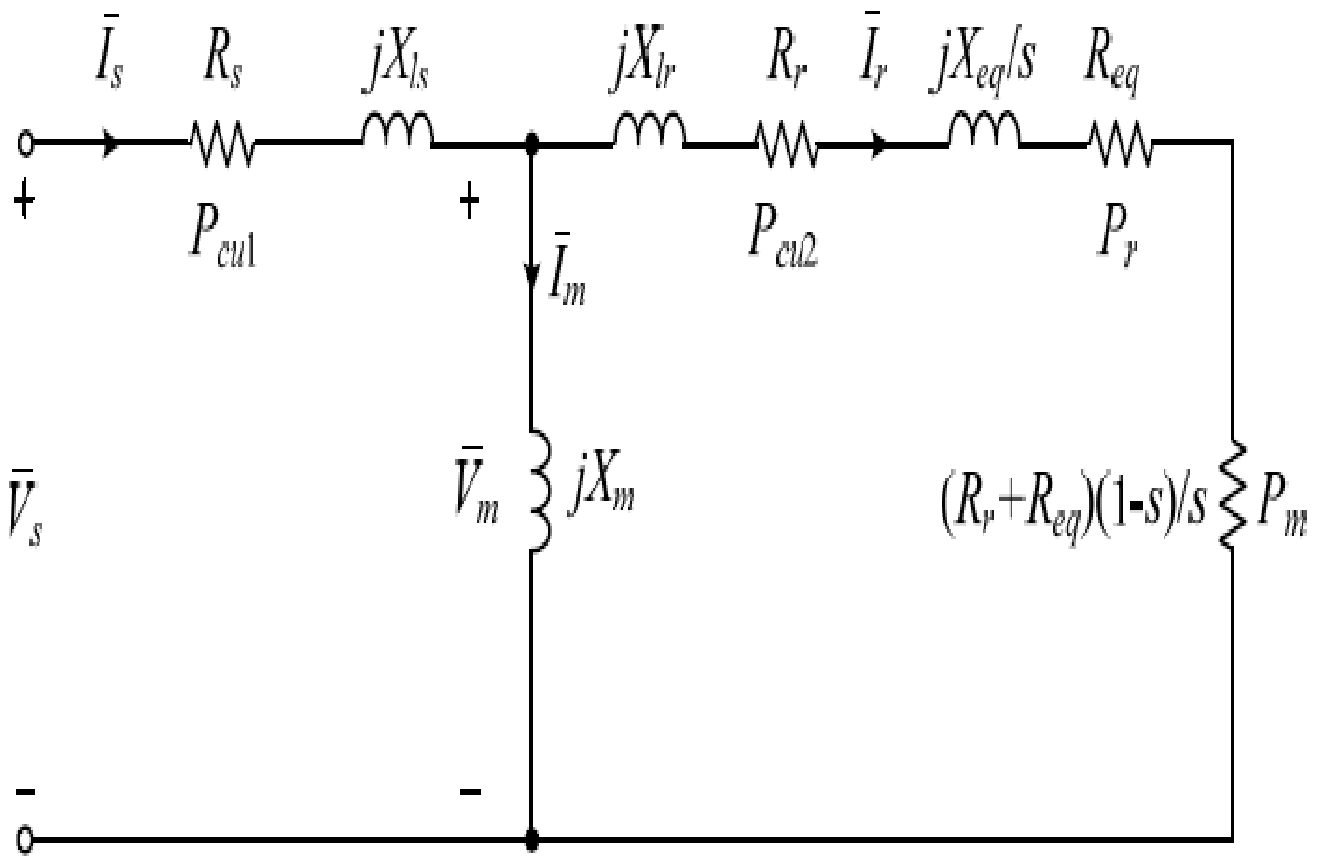
8.3.3 Steady-state Analysis of DFIG WECS with *PFs* = 1

For steady-state analysis of DFIG, equivalent circuit can be rearranged such that : (Rr+Req)+(Rr+Req)×(1-s)/s= (Rr/s+Req/s)





Mechanical power *Pm*, rotor power *Pr*, & stator & rotor winding losses, P_{cu1} and P_{cu2}, can be easily calculated by a general equation of $P = 3I^2R$.

- Rotor power *Pr* is power transferred from or to rotor-side converter.

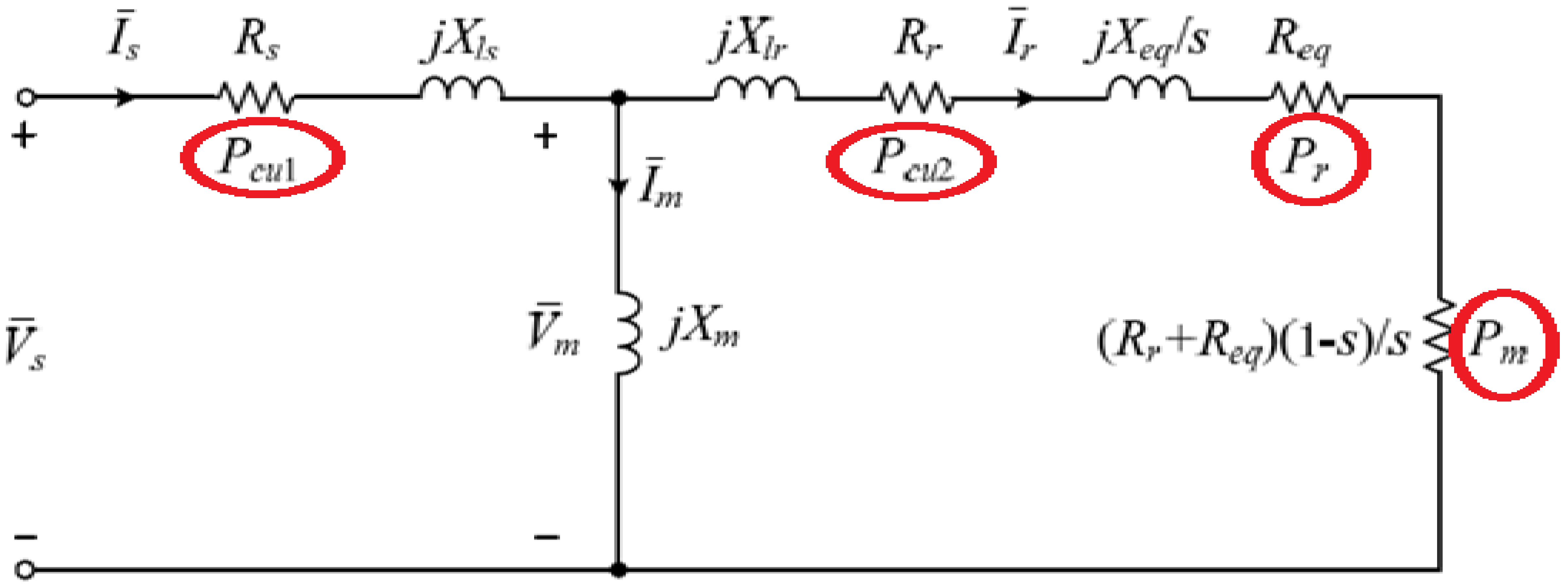
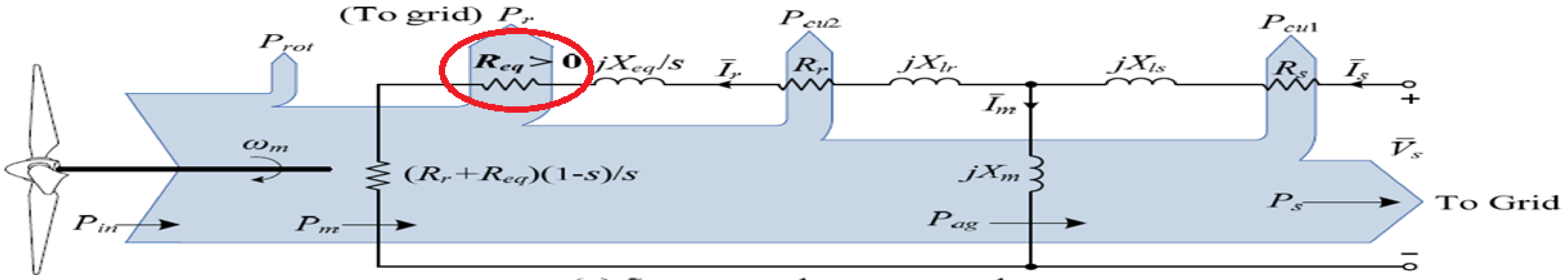
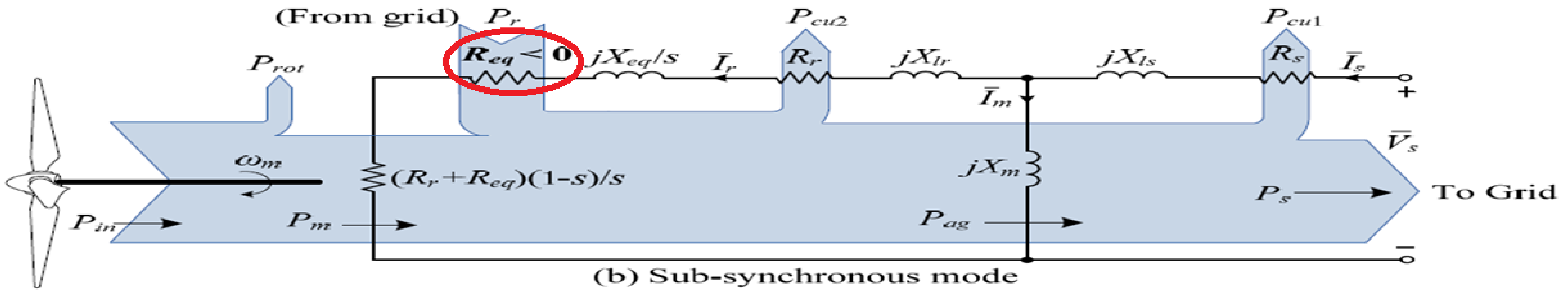


Fig. shows power flow of DFIG operating under super & sub-synchronous modes with rotor-side converter represented by Req & Xeq



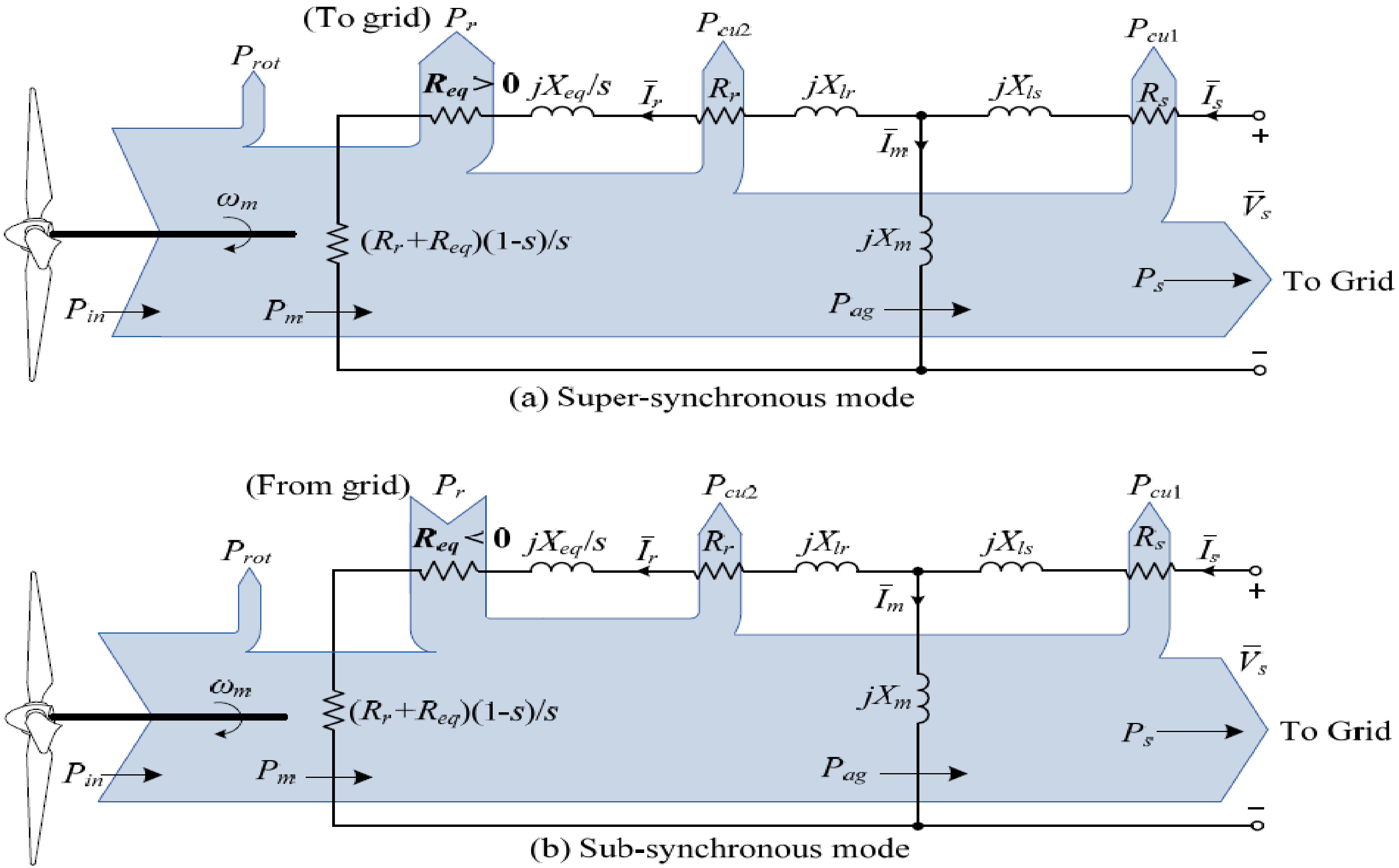
(a) Super-synchronous mode



(b) Sub-synchronous mode

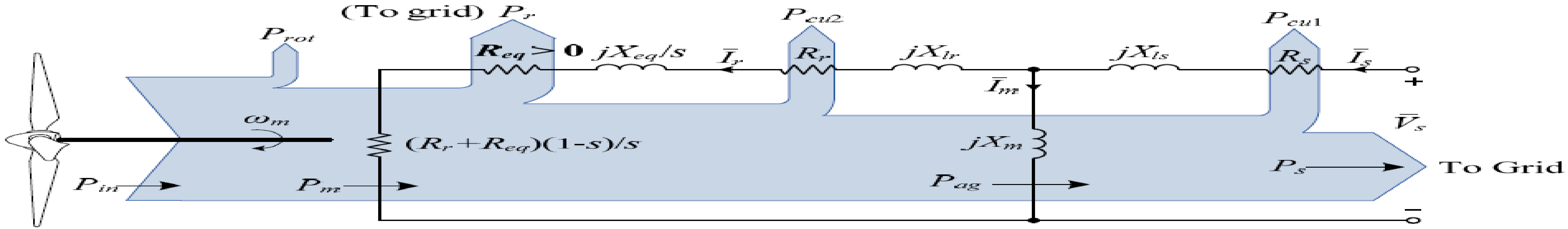
Neglecting rotational losses *Prot* of turbine, power transferred or dissipated in generator can be calculated by

$$
\begin{cases}
P_m = 3I_r^2(R_r + R_{eq})(1-s)/s \\
P_r = 3I_r^2 R_{eq} \\
P_{cu2} = 3I_r^2 R_r \\
P_{cu1} = 3I_s^2 R_s \\
P_s = 3V_s I_s \cos\varphi_s
\end{cases}
$$

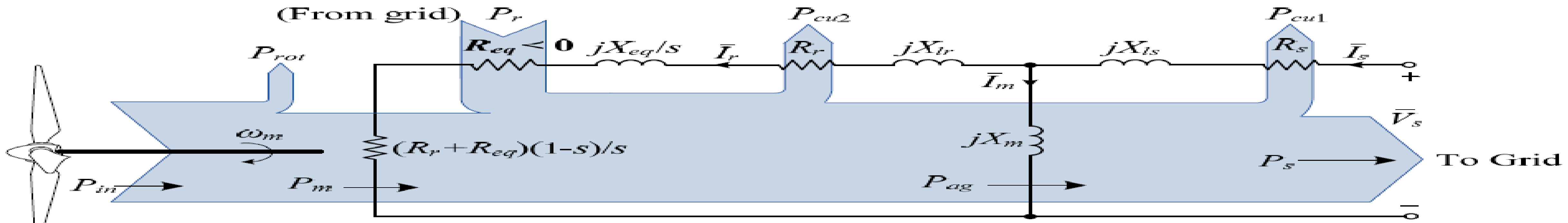


Power delivered to grid, *Pg*, is the sum of stator and rotor power, given by

$$|P_g| = \begin{cases} |P_s| + |P_r| & \text{for super - synchronou mode} \\ |P_s| - |P_r| & \text{for sub - synchronou mode} \end{cases}$$



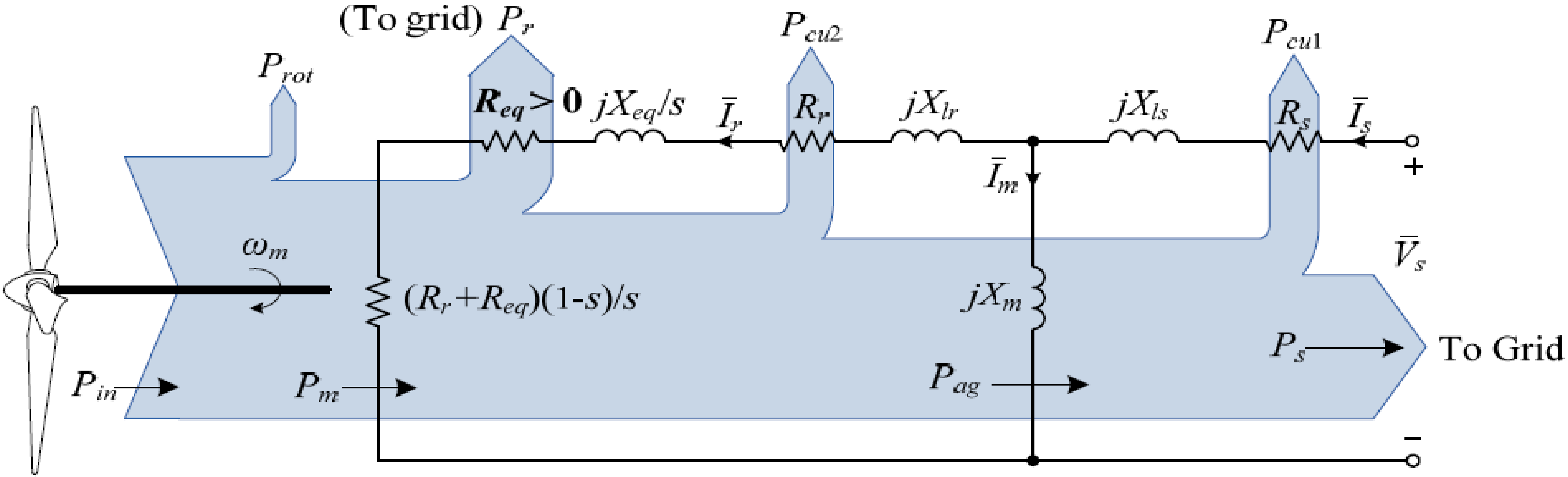
(a) Super-synchronous mode



(b) Sub-synchronous mode

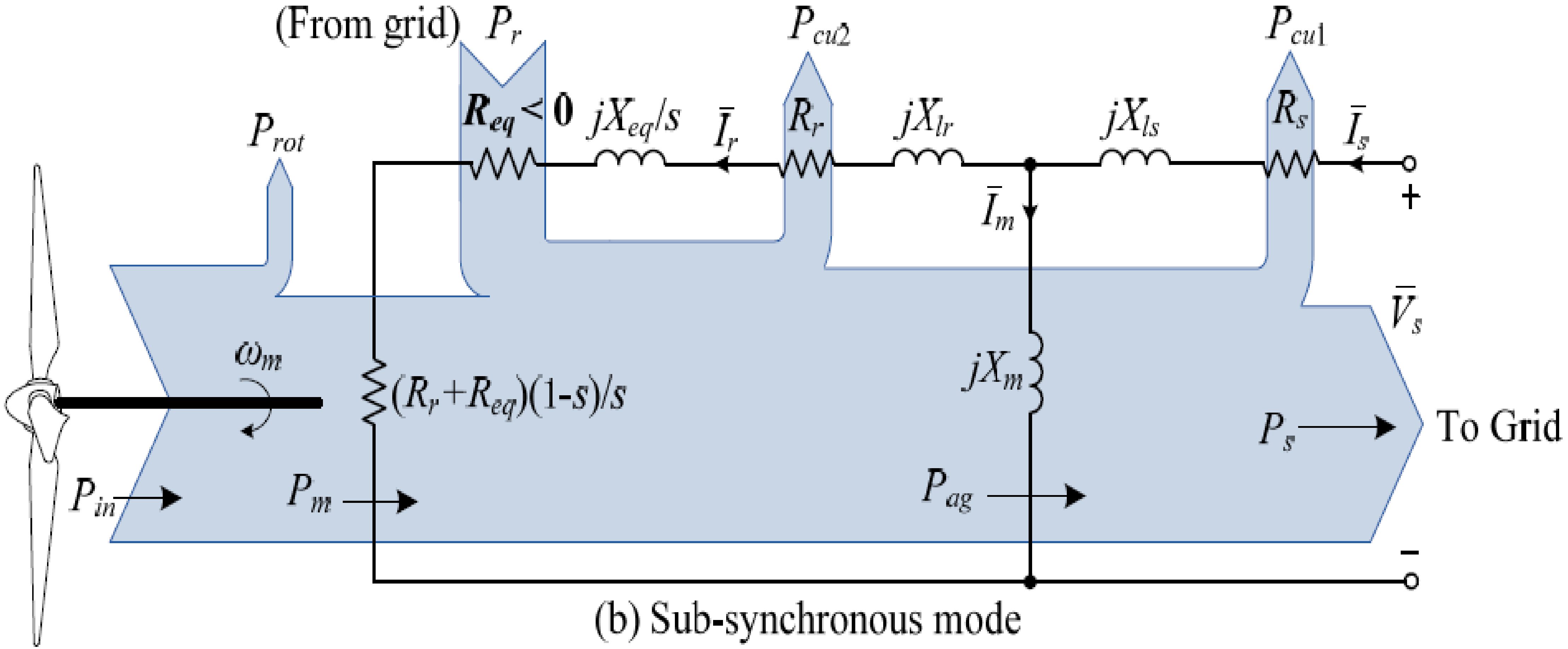
In super-synchronous operating mode, equivalent resistance *Req* of RSC has a +ve value & rotor power *Pr* is +ve. This implies that resistance *Req* consumes power similar to winding resistances *Rr* & *Rs*.

- In reality, rotor power *Pr* is not dissipated in *Req*, but transferred from rotor to grid through converters.



(a) Super-synchronous mode

In sub-synchronous mode, *Req* has a -ve value & rotor power *Pr* is also -ve. This indicates that rotor circuit receives power from grid through converters



(b) Sub-synchronous mode

Case Study 8-2 Steady-state Analysis of DFIG WECS with *PFs* = 1

- This case study is a continuation of Case Study 8-1, where equivalent impedance for rotor-side converter of a 1.5MW/690V DFIG wind energy system was developed.
- Steady-state operation of above system at super-synchronous, synchronous, & sub-synchronous speeds is analysed below.

i) DFIG Operation at Rotor Speed of 1750rpm (Super-synchronous Mode)

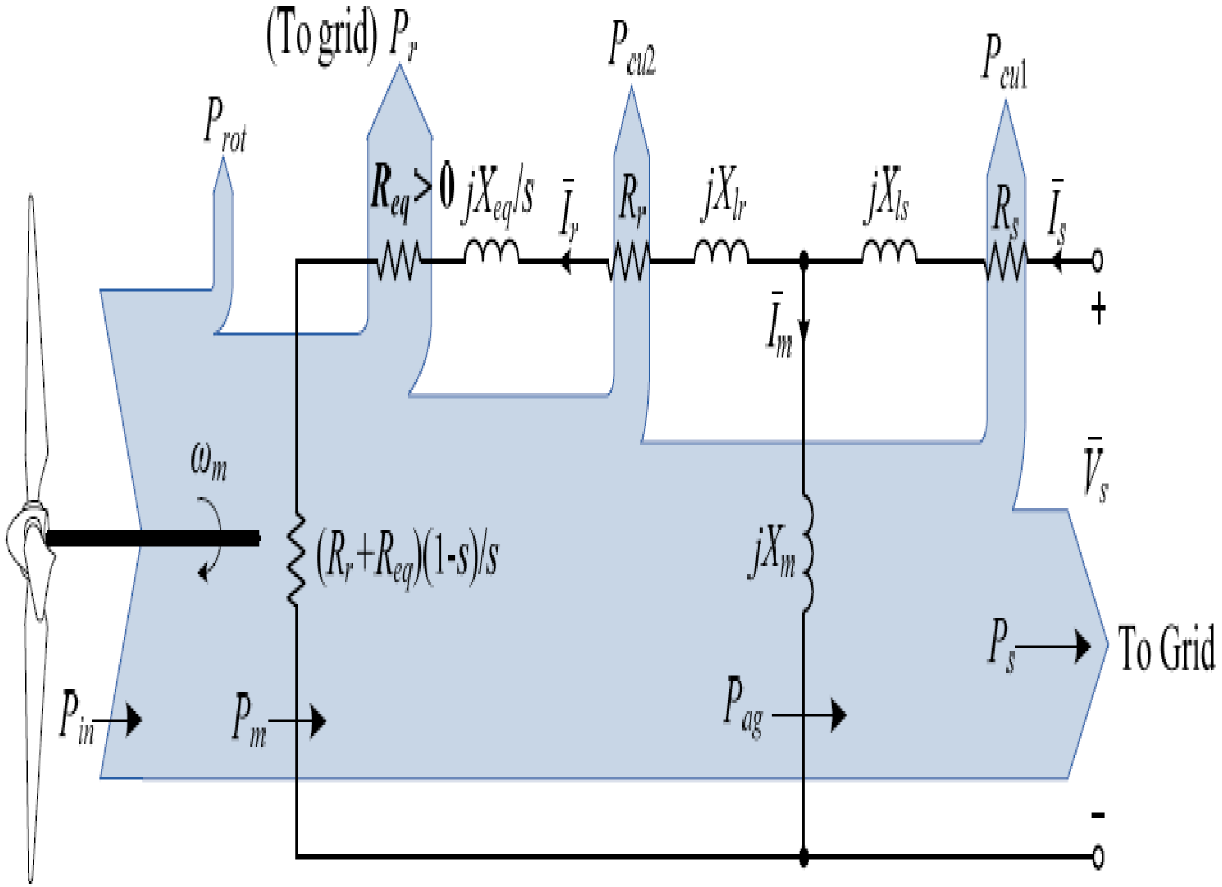
- At rated rotor speed of 1750 rpm, rotor current calculated in Case Study 8-1 is 1125.6 A, from which mechanical power of generator is calculated

$$P_m = 3I_r^2(R_{eq}+R_r)(1-s)/s$$

$$= 3\times 1125.6^2(0.05375+0.00263)(1+0.1667)/(-0.1667)$$

$$= -1500\ \text{kW}$$

or $P_m = T_m\omega_m = -8185.1\times 1750\times 2\pi/60 = -1500\ \text{kW}$



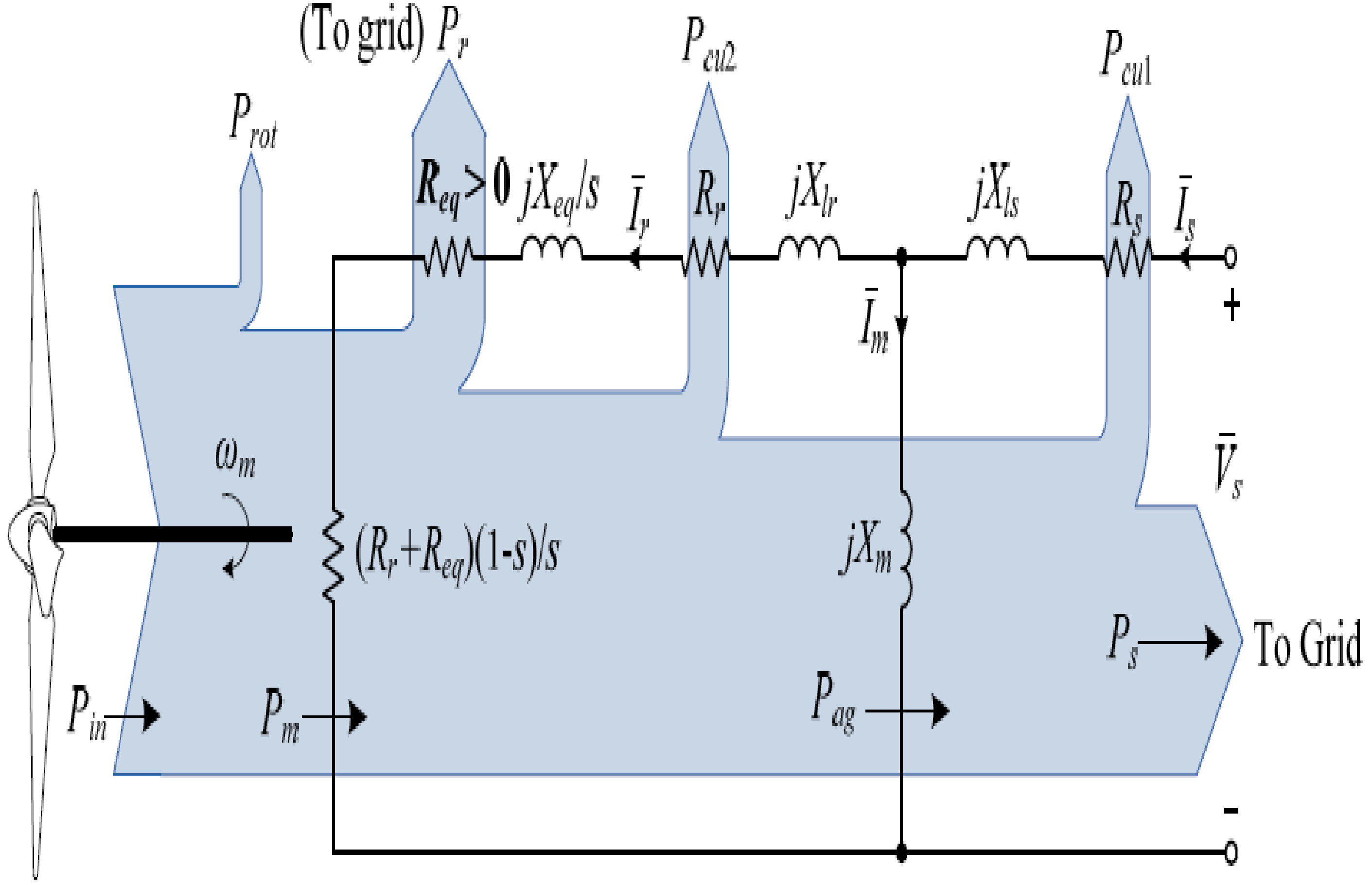
(a) Super-synchronous mode

Rotor power is $P_r = 3(I_r)^2 R_{eq} = 3\times1125.6\times0.05375 = 204.29\ \text{kW}$

- Rotor & stator winding losses are:

$$P_{cu2} = 3(I_r)^2 R_r = 10.0\ \text{kW},$$

$$P_{cu1} = 3(I_r)^2 R_s = 9.07\ \text{kW}$$



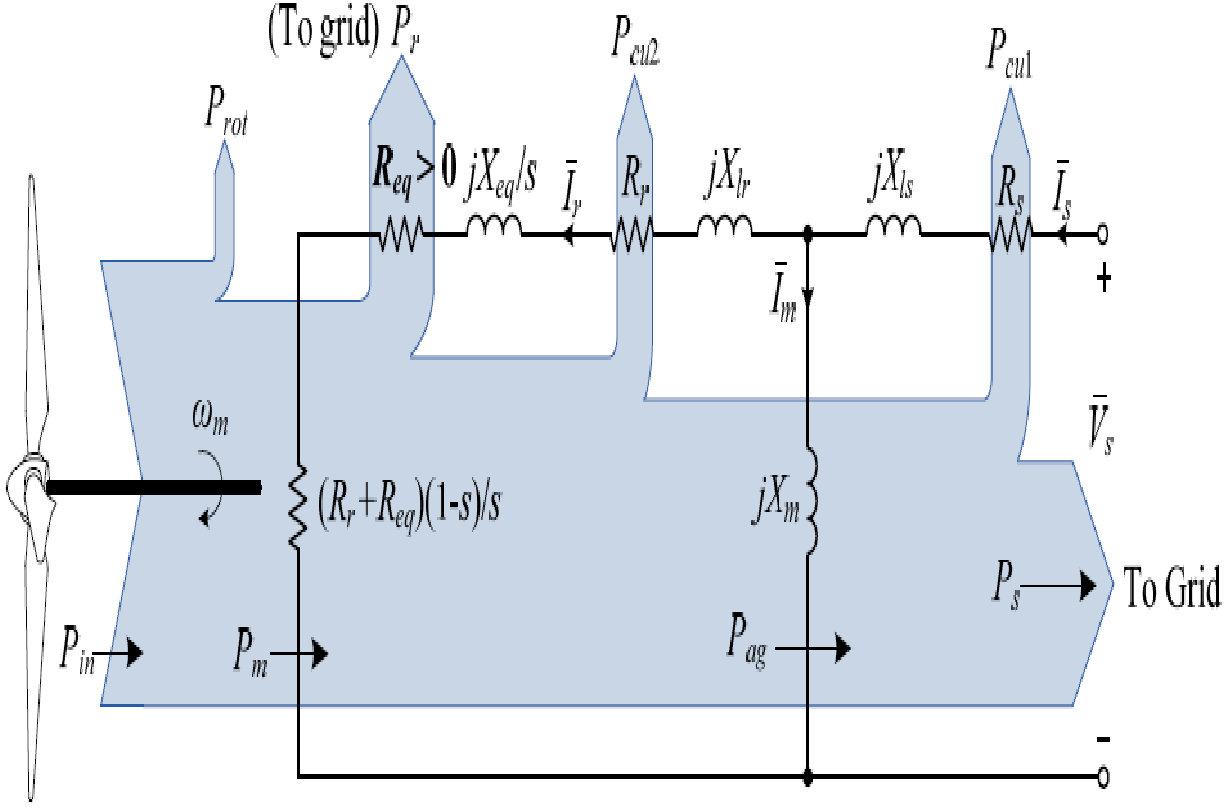
(a) Super-synchronous mode

Stator active powe

$$P_s = 3V_s I_s \cos\varphi_s = 690/\sqrt{3} \times 1068.2 \times \cos(180°) = -1276.64 \text{ kW}$$

where stator power factor angle $\varphi s = 180°$

- since DFIG operates in generating mode with a unity power factor.
- Total power delivered to grid is

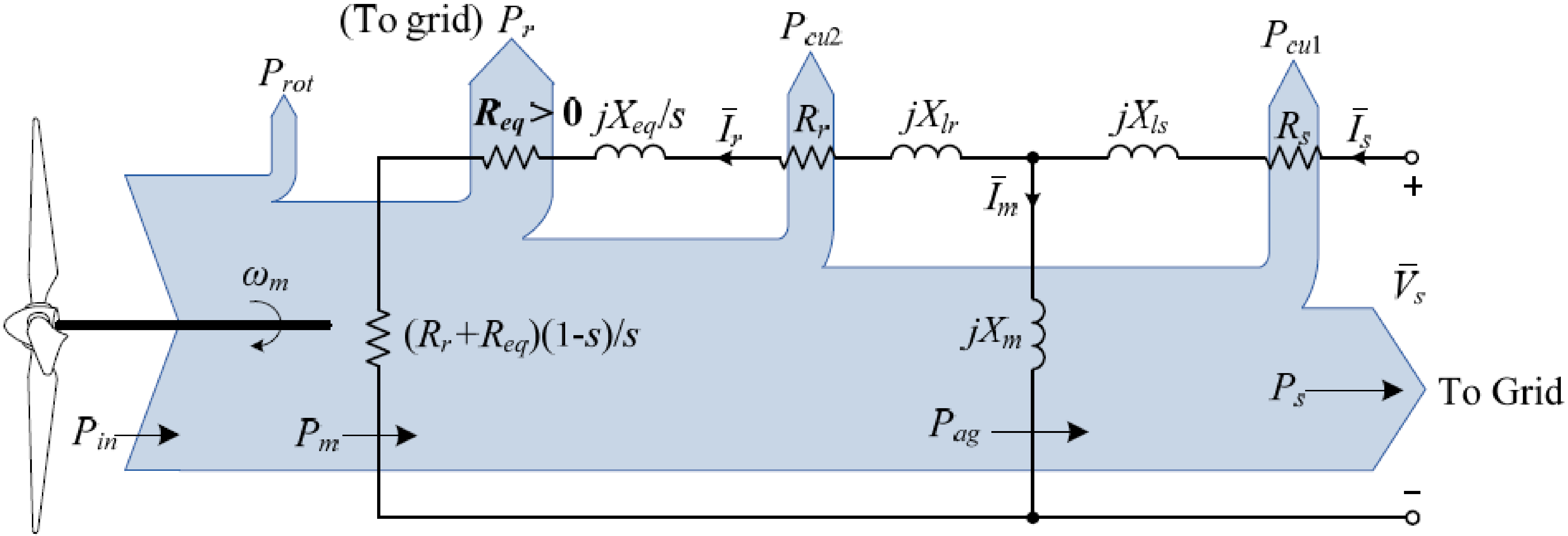


(a) Super-synchronous mode

$$|P_g| = |P_s| + |P_r| = 1276.64 + 204.29 = 1480.93\,\text{kW}$$

Difference between *Pm* and *Pg* is losses on stator and rotor windings:

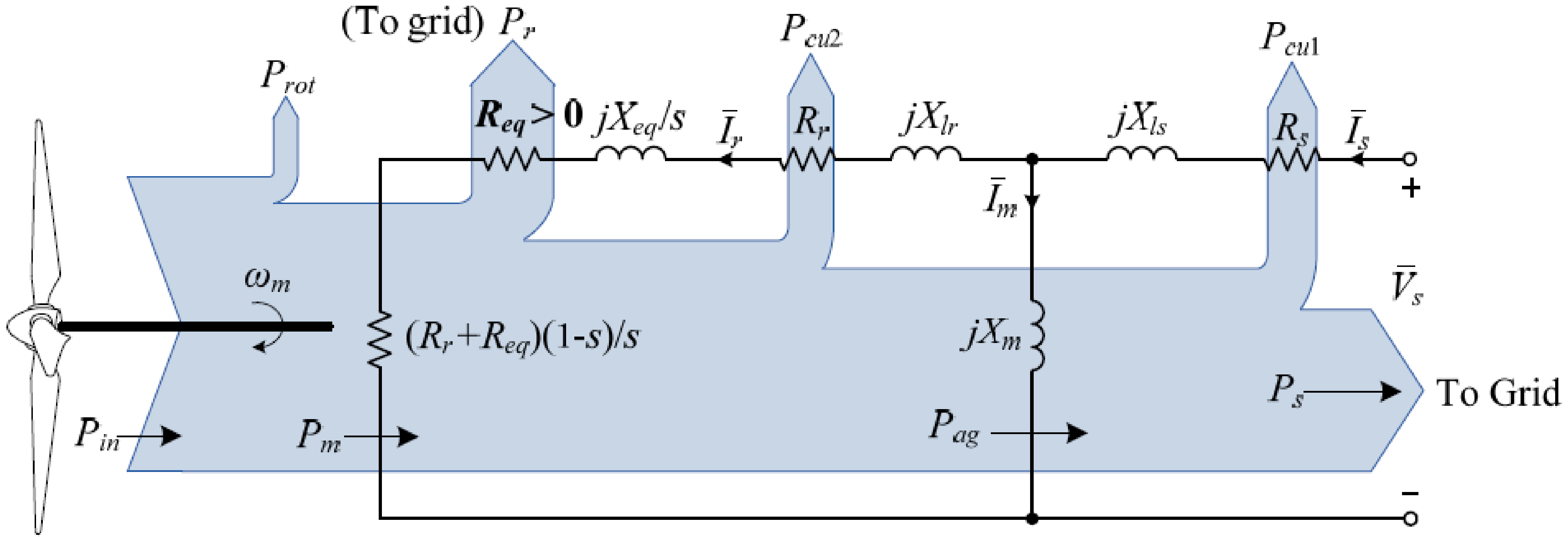
$$\left|P_m\right| - \left|P_g\right| = P_{cu2} + P_{cu1} = 19.07 \text{ kW}$$



(a) Super-synchronous mode

Efficiency of DFIG is then

$$\eta = P_g / P_m = 1480.93 / 1500 = 98.7\%$$



(a) Super-synchronous mode

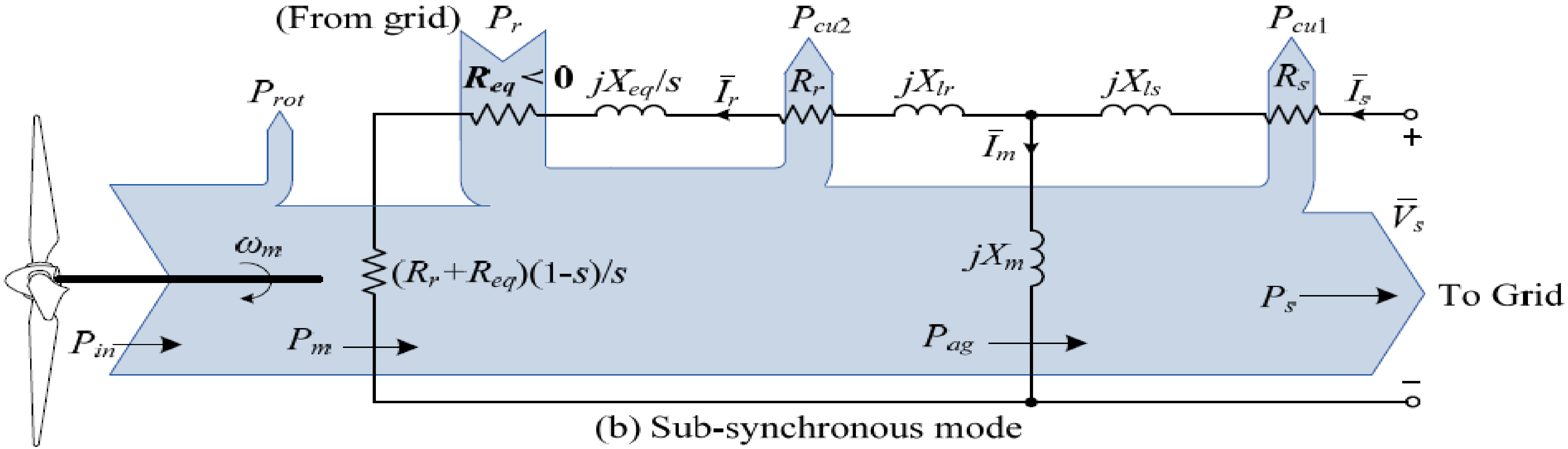
ii) DFIG Operation at Synchronous and Sub-synchronous Speeds

- Following same procedure , operation of DFIG in synchronous & sub-synchronous modes is tabulated.

Operating Mode		Sub-synchronous Operation	Synchronous Operation	Super-synchronous Operation
ω_m	[rpm]	1200	1500	1750 (rated)
s	Slip	0.2	0	-0.1667 (rated)
$\lvert T_m \rvert$	[kN.m]	3.849	6.014	8.1851
R_{eq}	[Ω]	-0.126989	-0.002630	0.053751
X_{eq}	[Ω]	-0.074293	0	0.027513
I_s	[A]	504.16	786.28	1068.22
I_r	[A]	569.29	843.28	1125.57
V_r	[V]	83.76	2.22	67.97
$\lvert P_m \rvert$	[kW]	483.64	944.61	1500.0
$\lvert P_r \rvert$	[kW]	123.47	5.61	204.29
P_{cu2}	[kW]	2.56	5.61	10.0
P_{cu1}	[kW]	2.02	4.92	9.07
$\lvert P_s \rvert$	[kW]	602.53	939.69	1276.64
$\lvert P_g \rvert$	[kW]	479.06	934.08	1480.93

It is noted that in sub-synchronous mode rotor circuit receives power from grid through converters. Therefore, power delivered to grid is $Pg = Ps - Pr$

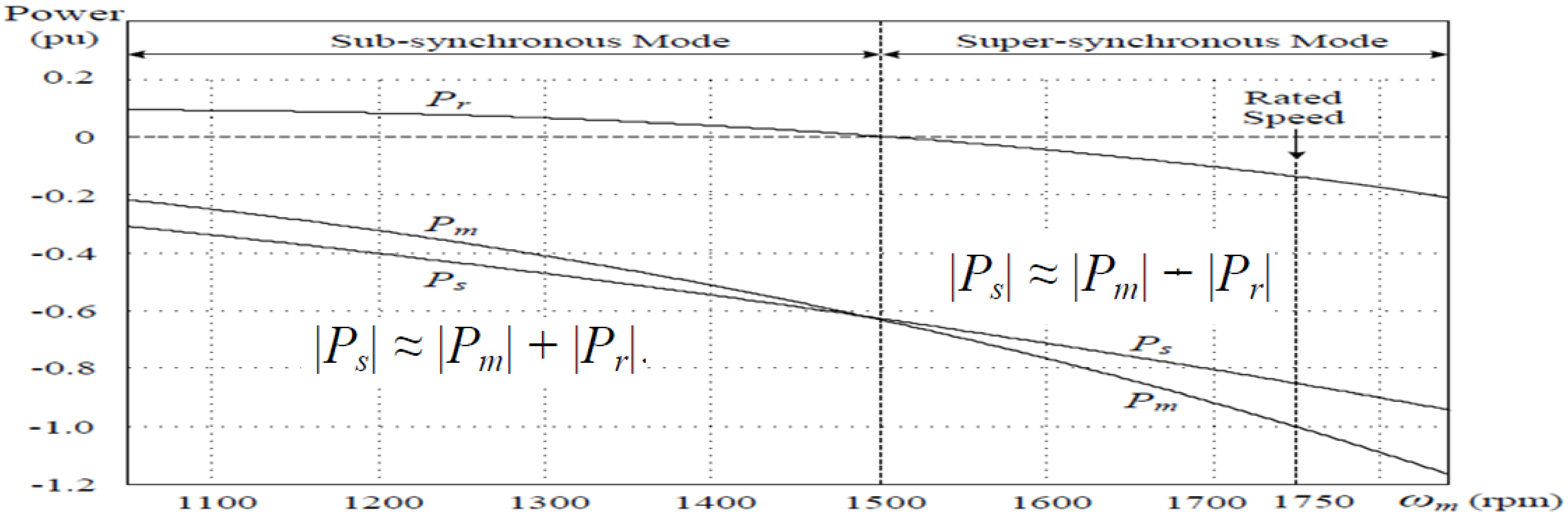
$$|P_g| = \begin{cases} |P_s| + |P_r| & \text{for super - synchronou mode} \\ |P_s| - |P_r| & \text{for sub - synchronou mode} \end{cases}$$



(b) Sub-synchronous mode

Fig. 8.3-7 shows relationship between stator, rotor and mechanical power of 1.5MW/690V DFIG WECS through rotor speed range.

- Stator power $|P_s| \approx |P_m|$ - $|P_r|$ in super-synchronous operating mode, whereas in sub-synchronous operating mode, $|P_s| \approx |P_m| + |P_r|$.



8.3.4 Simplified Calculations

- In large megawatt wind generators, stator resistance of generator is normally very small (< 0.01 pu).
- To simplify analysis, the stator resistance can be neglected.
- When generator operates with unity stator power factor, its air-gap power can be calculated by

$$P_{ag} = 3(V_s - I_s R_s) I_s \approx 3 V_s I_s$$

Stator current *Is* can be calculated by comparing equations

$$P_{ag} = 3V_s I_s \qquad P_{ag} = \frac{\omega_s T_m}{P}$$

$$I_s = \frac{T_m \omega_s / P}{3V_s}$$

With stator current *Is* known, equivalent impedance of rotor-side converter can be calculated, and steady-state performance of the DFIG can be analysed.

$$I_s = \frac{T_m \omega_s / P}{3V_s}$$

Comparsion with stator current given in (8.3-7), calculation of stator current by (8.3-48) is simpler.

$$I_s = \frac{V_s \pm \sqrt{V_s^2 - \frac{4R_s\omega_s T_m}{3P}}}{2R_s}$$

$$I_s = \frac{T_m \omega_s / P}{3V_s}$$

More importantly, this method can facilitate analysis of DFIG wind energy systems with non-unity stator power factor

Table 8.3-3 gives calculation results for 1.5MW/690V DFIG operating at 1200 rpm & 1750 rpm based on 2 methods:

Method 1 – accurate calculation of stator current *Is* based on (8.3-7),

$$I_s = \frac{V_s \pm \sqrt{V_s^2 - \dfrac{4R_s \omega_s T_m}{3P}}}{2R_s}$$

Method 2 – simplified calculation of *Is* using (8.3-48).

$$I_s = \frac{T_m \omega_s / P}{3V_s}$$

Results shown in Table 8.3-3 illustrate that errors generated by simplified method are minimal (less than 1.5%).

Operating Mode		Sub-synchronous Operation			Super-synchronous Operation		
ω_m	[rpm]	1200 rpm			1750 rpm		
		Method 1	Method 2	Error |%|	Method 1	Method 2	Error |%|
s	Slip	0.2	0.2	N/A	-0.1667	-0.1667	N/A
$\|T_m\|$	[kN.m]	3.849	3.849	N/A	8.185	8.185	N/A
R_{eq}	[Ω]	-0.12699	-0.12671	0.22	0.05375	0.05339	0.26
X_{eq}	[Ω]	-0.07429	-0.07398	0.31	0.02751	0.02735	0.57
I_s	[A]	504.2	505.8	0.32	1068.2	1075.8	0.71
I_r	[A]	569.3	570.9	0.28	1125.6	1133.2	0.68
V_r	[V]	83.6	83.7	0.12	67.9	68.0	0.15
$\|P_m\|$	[kW]	483.6	485.3	0.35	1500	1510.7	0.71
$\|P_r\|$	[kW]	123.5	123.9	0.34	204.3	205.7	0.68
P_{cu2}	[kW]	2.557	2.571	0.55	9.996	10.13	1.3
P_{cu1}	[kW]	2.020	2.034	0.69	9.071	9.201	1.4
$\|P_s\|$	[kW]	602.5	604.5	0.34	1276.6	1285.7	0.71
$\|P_g\|$	[kW]	479.0	480.6	0.33	1480.9	1491.4	0.77

8.4 Leading and Lagging Power Factor Operation

- When generator operates with a leading or lagging power factor, stator current can be calculated by

$$I_s = \frac{T_m \omega_s / P}{3 V_s \cos \varphi_s}$$

$$I_s = \frac{T_m \omega_s / P}{3 V_s}$$

where φs is power factor angle of stator.

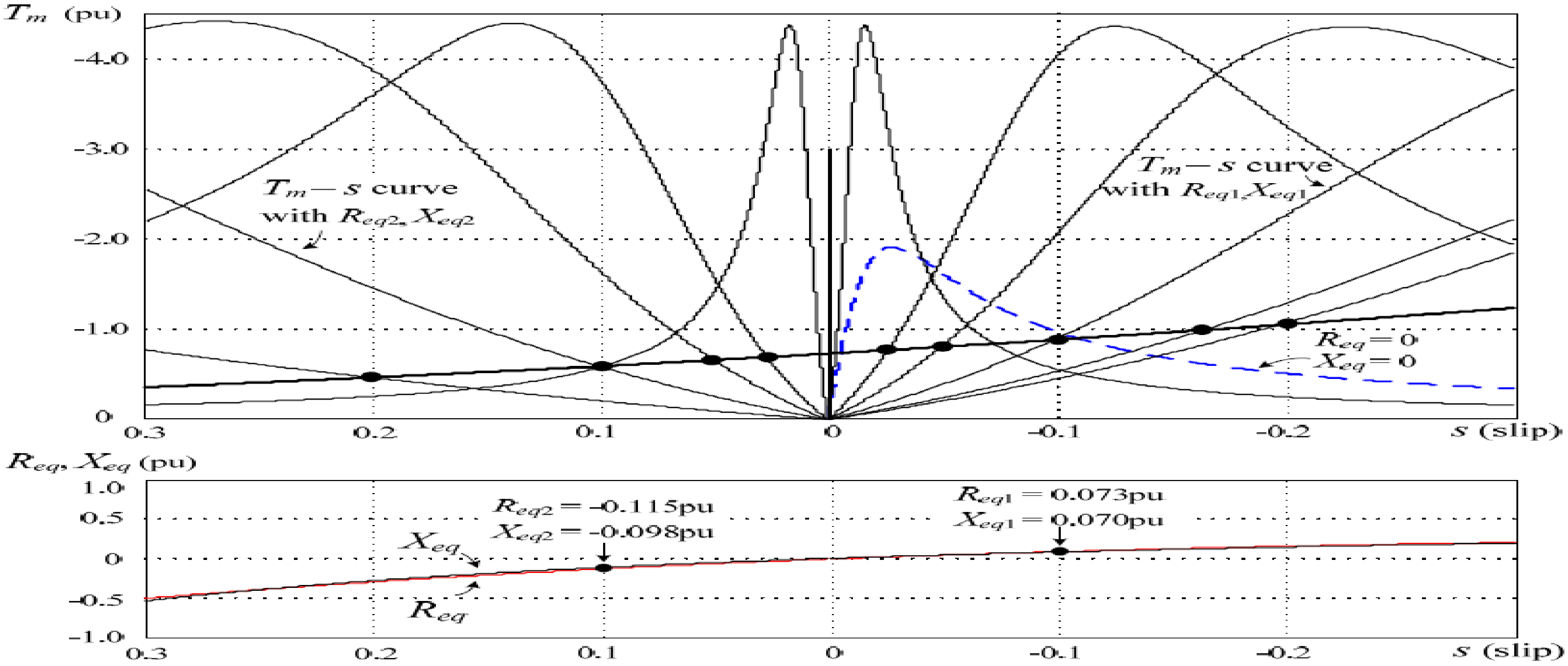
With stator current calculated, equivalent impedance of rotor-side converter can be obtained following same procedures given in Case Study 8-1.

Table 8.1-1 gives calculated converter equivalent impedance for DFIG operating under super- and sub-synchronous modes with 0.95 leading and lagging power factor.

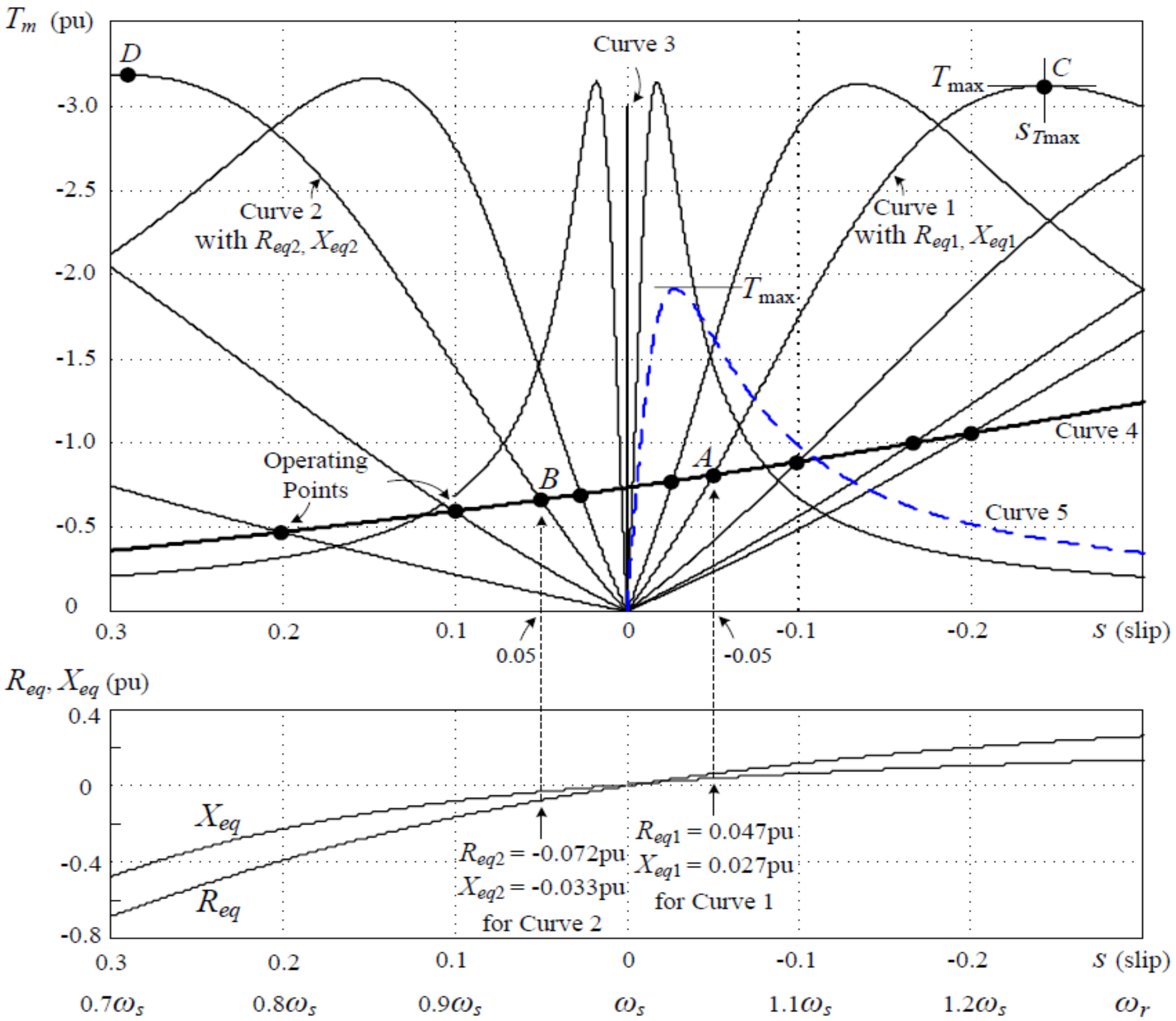
Stator Power Factor	ω_m (rpm)	1200	1350	1500	1650	1750
	s (Slip)	0.2	0.1	0	-0.1	-0.1667
Leading PF (0.95) $\varphi_s = -161.8°$	$\bar{V}_r$ [V]	$86.89\angle 5.7°$	$45.02\angle 6.3°$	$2.514\angle -31.6°$	$42.83\angle -165.7°$	$73.67\angle -165.3°$
	$\bar{I}_r$ [A]	$659.3\angle 142.2°$	$798.2\angle 145.7°$	$955.9\angle 148.4°$	$1131.9\angle 150.5°$	$1259.3\angle 151.7°$
	R_{eq} [Ω]	-0.0957	-0.0428	-0.00263	0.02731	0.04277
	X_{eq} [Ω]	-0.0906	-0.03667	0	0.02620	0.03992
Lagging PF (0.95) $\varphi_s = 161.8°$	$\bar{V}_r$ [V]	$80.65\angle 6.9°$	$41.16\angle 8.6°$	$2.146\angle 2.5°$	$36.59\angle -165.7°$	$62.30\angle -164.1°$
	$\bar{I}_r$ [A]	$525.2\angle 173.3°$	$660.5\angle 178.6°$	$815.9\angle -177.5°$	$990.5\angle -174.7°$	$1117.2\angle -173.3°$
	R_{eq} [Ω]	-0.1493	-0.0614	-0.00263	0.0365	0.0550
	X_{eq} [Ω]	-0.0360	-0.0108	0	0.0058	0.0089

- Once converter equivalent impedance is determined for a given operating condition, steady-state performance of DFIG wind energy system can be analysed using same procedure presented in previous section.

Fig. 8.4-1 shows calculated converter equivalent impedance as well as the torque-slip curves of DFIG with a leading power factor of 0.95.



Torque-slip curves are similar except that **maximum torque** values differ.



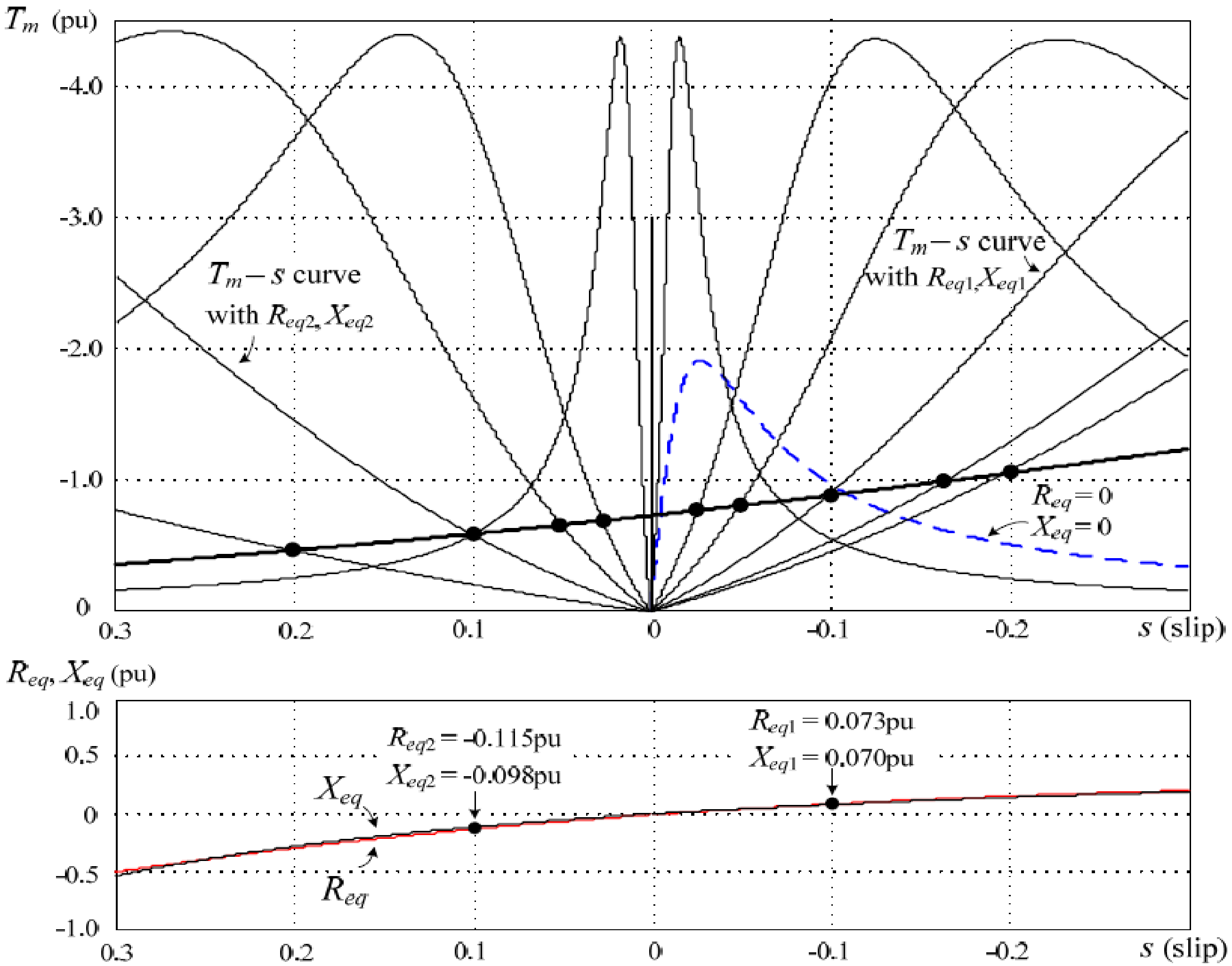
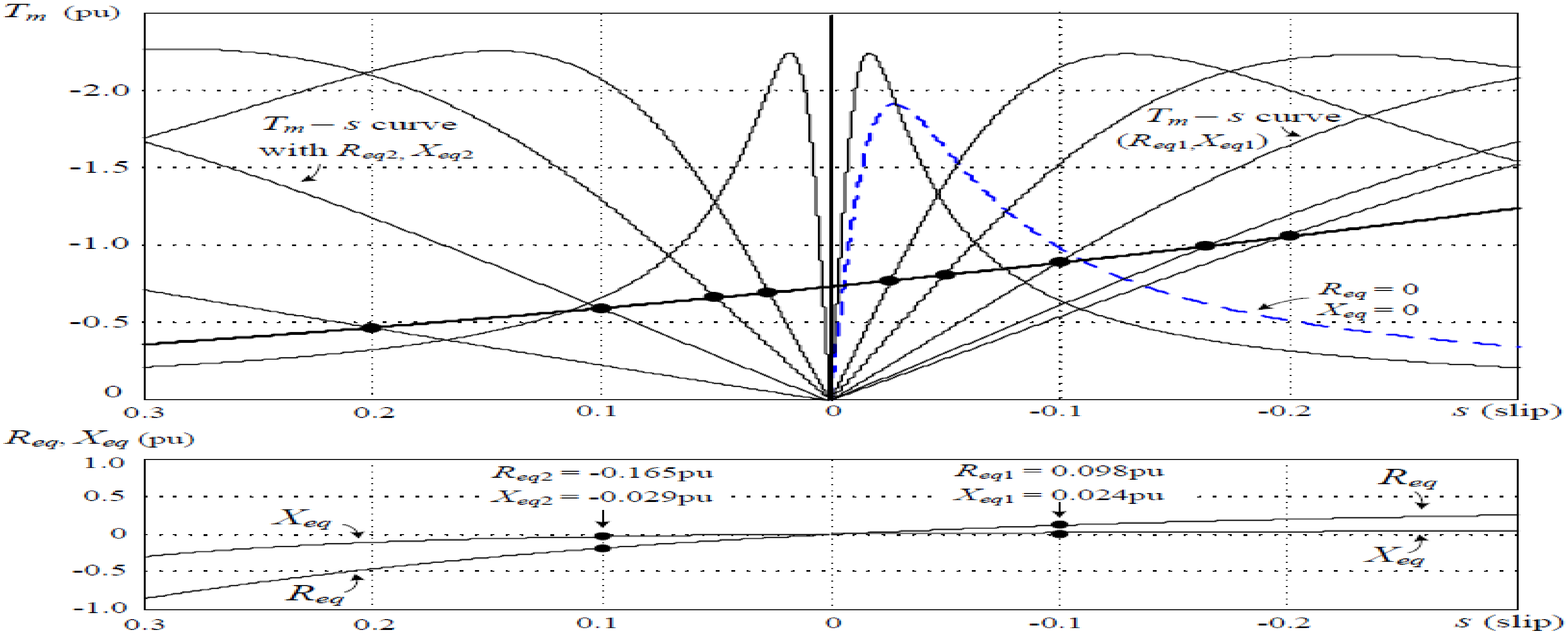


Fig. illustrates torque-slip curves of generator with a lagging power factor of 0.95.



Problems

Topic: Steady-state Analysis of DFIG WECS with *PFs*=1

8-7 (Solved Problem) A 5.0MW/950V/50Hz/1170rpm DFIG is employed in a variable-speed WECS. Parameters of generator are given in Table B-7 of Appendix B.DFIG WECS is connected to a grid (line-line voltage *VAB*=950V, 50Hz). Generator operates with an MPPT scheme & its stator power factor is unity. Corresponding equivalent resistance & reactance for RSC when DFIG operates at sub-synchronous speed of 670 rpm are -0.13059 Ω & -0.2624 Ω, respectively. Calculate following:

a) generator mechanical torque and power,

b) rms stator and rotor currents,

c) stator and rotor winding losses,

d) stator and rotor active powers,

e) net power delivered to the grid and efficiency of the DFIG, and

f) fundamental grid current.

Generator Type	DFIG, 5.0MW/950V/50Hz	
Rated Mechanical Power	5.0 MW	1.0 pu
Rated Stator Line-to-line Voltage	950 V (rms)	
Rated Stator Phase Voltage	548.48 V (rms)	1.0 pu
Rated Rotor Phase Voltage	381.05 V (rms)	0.6947 pu
Rated Stator Current	2578.4 A (rms)	0.8485 pu
Rated Rotor Current	3188.7 A (rms)	1.0494 pu
Rated Stator Frequency	50 Hz	1.0 pu
Rated Rotor Speed	1170 rpm	1.0 pu
Nominal Rotor Speed Range	670–1170 rpm	0.573–1.0pu
Rated Slip	-0.17	
Number of Pole Pairs	3	
Rated Mechanical Torque	40.809 kN.m	1.0 pu
Stator Winding Resistance R_s	1.552 mΩ	0.0086 pu
Rotor Winding Resistance R_r	1.446 mΩ	0.008 pu
Stator Leakage Inductance L_{ls}	1.2721 mH	2.2141 pu
Rotor Leakage Inductance L_{lr}	1.1194 mH	1.9483 pu
Magnetizing Inductance L_m	5.5182 mH	9.6044 pu
Base Current $I_B = 5\text{MW}/(\sqrt{3}\times 950\text{V})$	3038.7 A (rms)	1.0 pu
Base Flux Linkage Λ_B	1.7459 Wb (rms)	1.0 pu
Base Impedance Z_B	0.1805 Ω	1.0 pu
Base Inductance L_B	0.5746 mH	1.0 pu
Base Capacitance C_B	17634.9 μF	1.0 pu

Solution:

a) Rotor mechanical speed:

$$\omega_m = 670 \times (2\pi / 60) = 70.162 \text{ rad/sec}$$

Rotor electrical speed:

$$\omega_r = \omega_m \times P = 70.16 \times 3 = 210.48 \text{ rad/sec}$$

Rated rotor mechanical speed:

$$\omega_{m,R} = 1170 \times (2\pi / 60) = 122.52 \text{ rad/sec}$$

Stator frequency:

$$\omega_s = 2\pi \times 50 = 314.16 \text{ rad/sec}$$

The pu rotor speed

$$\omega_{m,\mathrm{pu}} = \omega_m \Big/ \omega_{m,R} = 70.162/122.52 = 0.5727\,\mathrm{pu}$$

Generator mechanical torque at 0.5727 pu rotor speed:

$$T_m = T_{m,R} \times (\omega_{m,\mathrm{pu}})^2 = -40809 \times \left(0.5727\right)^2 = -13382.4\ \mathrm{N.m}$$

Rated mechanical power:

$$P_{m,R} = \omega_{m,R} \times T_{m,R} = -122.52 \times 40809 = -5000 \times 10^3 \text{ W}$$

Generator mechanical power at 0.5727pu rotor speed:

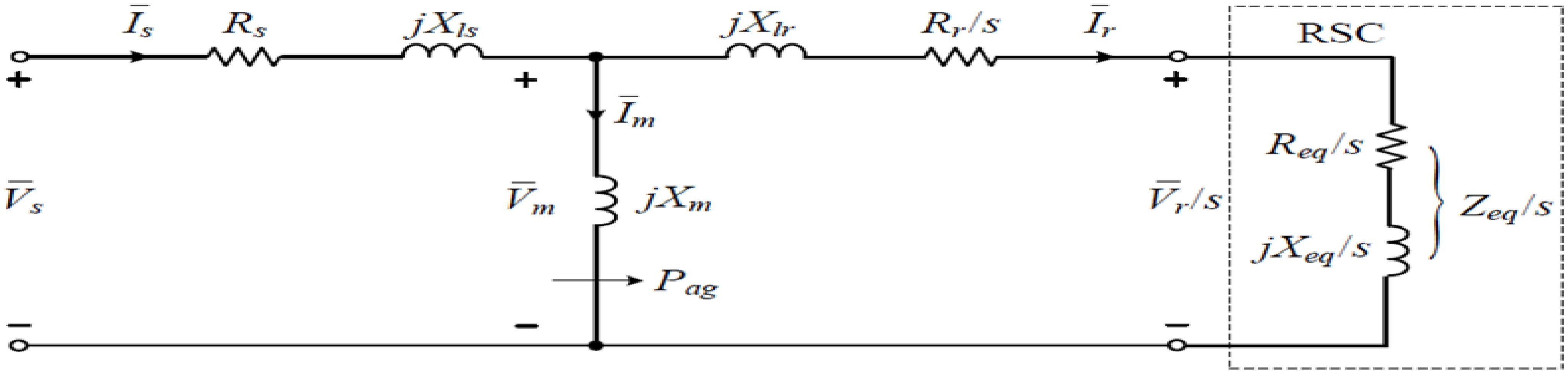
$$P_m = P_{m,R} \times (\omega_{m,\mathrm{pu}})^3 = -5000 \times 10^3 \times \left(0.5727\right)^3 = -938.94 \times 10^3 \text{ W}$$

b) Rms stator current using conventional expression:

$$I_s = \frac{V_s \pm \sqrt{V_s^2 - \dfrac{4R_s T_m \omega_s}{3P}}}{2R_s} = -849.64\ \text{A (rms)} \quad (I_s = 35.43 \times 10^4\ \text{A omitted})$$

where $V_s = 950/\sqrt{3}$ V, $T_m = -13382.4$ N.m, $\omega_s = 314.16$ rad/sec, $R_s = 1.552$ mΩ and $P = 3$

From the steady-state equivalent circuit of DFIG with the rotor-side converter as shown, rotor current can be calculated by



$$\bar{I}_r = \frac{jX_m \bar{I}_s}{jX_m + \left(\frac{R_r}{s} + jX_{lr}\right) + \left(\frac{R_{eq}}{s} + j\frac{X_{eq}}{s}\right)} = 1092.55\angle 163.13^\circ \text{ A (rms)}$$

where $s = (\omega_s - \omega_r)/\omega_s = (314.16 - 210.48)/314.16 = 0.33$

Alternatively, the rms rotor current can be found from the mechanical torque equation:

$$T_m = \frac{1}{\omega_s / P} \times 3 I_r'^2 \left(R_{eq} + R_r\right) / s \ \text{N.m}$$

from which

$$I_r = \sqrt{\frac{T_m \times (\omega_s / P)}{3(R_{eq} + R_r)/s}} = 1092.55 \ \text{A (rms)}$$

c) The stator and rotor winding losses:

$$P_{cu,s} = 3(I_s)^2 R_s = 3\times 849.64^2 \times 1.552\times 10^{-3} = 3.361\times 10^3 \text{ W}$$

$$P_{cu,r} = 3(I_r)^2 R_r = 3\times 1092.55^2 \times 1.446\times 10^{-3} = 5.178\times 10^3 \text{ W}$$

d) The stator and rotor active powers:

$$P_s = 3V_s I_s \cos\varphi_s = 3\times 950/\sqrt{3}\times 849.64\times\cos(180^\circ) = -1398\times 10^3 \text{ W}$$

$$P_r = 3(I_r)^2 R_{eq} = 3\times 1092\times -0.13059 = -467.64\times 10^3 \text{ W}$$

e) The net power delivered to the grid:

$$\left|P_g\right| = \left|P_s\right| - \left|P_r\right| = 1398\times10^3 - 467.64\times10^3 = 930.4\times10^3 \text{ W}$$

The difference between *Pm* & *Pg* is the losses on the stator and rotor windings, that is,

$$\left|P_m\right| - \left|P_g\right| = P_{cu,s} + P_{cu,r} = 8.539 \times 10^3 \ \text{W}$$

The efficiency of the DFIG neglecting rotational and core losses is then

$$\eta = P_g / \left| P_m \right| = 930.4 / 938.94 = 99.09\%$$

f) The fundamental grid current:

$$\left|I_g\right| = \frac{\left|P_g\right|}{3V_g} = \frac{930.4\times 10^3}{3\times 950/\sqrt{3}} = 565.44 \text{ A (rms)}$$

Cross Check:

$$T_m = \frac{1}{\omega_s / P} \times 3I_r^2 \left(R_{eq} + R_r\right) / s = -13382 \text{ N.m, verified.}$$

$$P_m = 3I_r^2 \left(R_{eq} + R_r\right)(1 - s) / s = -938.9 \times 10^3 \text{ W, verified.}$$

8-8 Repeat Problem 8-7 when DFIG operates at super-synchronous speed of 1050 rpm. Corresponding equivalent resistance and reactance for RSC are given 0.00718 Ω & 0.0349 Ω, respectively.

Answers:

a) $T_m = -32867\ \text{N.m}$, $P_m = -3613.9\times10^3\ \text{W}$

b) $I_s = -2079.5\ \text{A (rms)}$, $I_r = 2578.8\ \text{A (rms)}$

c) $P_{cu,s} = 20.134\times10^3\ \text{W}$, $P_{cu,r} = 28.848\times10^3\ \text{W}$

d) $P_s = -3421.71\times10^3\ \text{W}$, $P_r = 143.24\times10^3\ \text{W}$

e) $\left|P_g\right| = 3564.96\times10^3\ \text{W}$, $\eta = 98.65\%$

f) $\left|I_g\right| = 2166.6\ \text{A (rms)}$

8-9 Repeat Problem 8-7 when DFIG operates at super-synchronous speed of 1170 rpm (rated). Corresponding equivalent resistance & reactance for RSC are given 0.02237 Ω & 0.11739 Ω, respectively.

<u>Answers:</u>

a) $T_m = -40809\ \text{N.m}$, $P_m = -5000\times10^3\ \text{W}$

b) $I_s = -2578.4\ \text{A (rms)}$, $I_r = 3188.7\ \text{A (rms)}$

c) $P_{cu,s} = 30.95\times10^3\ \text{W}$, $P_{cu,r} = 44.109\times10^3\ \text{W}$

d) $P_s = -4242.5\times10^3\ \text{W}$, $P_r = 682.4\times10^3\ \text{W}$

e) $\left|P_g\right| = 4924.9\times10^3\ \text{W}$, $\eta = 98.5\%$

f) $\left|I_g\right| = 2993.1\ \text{A (rms)}$