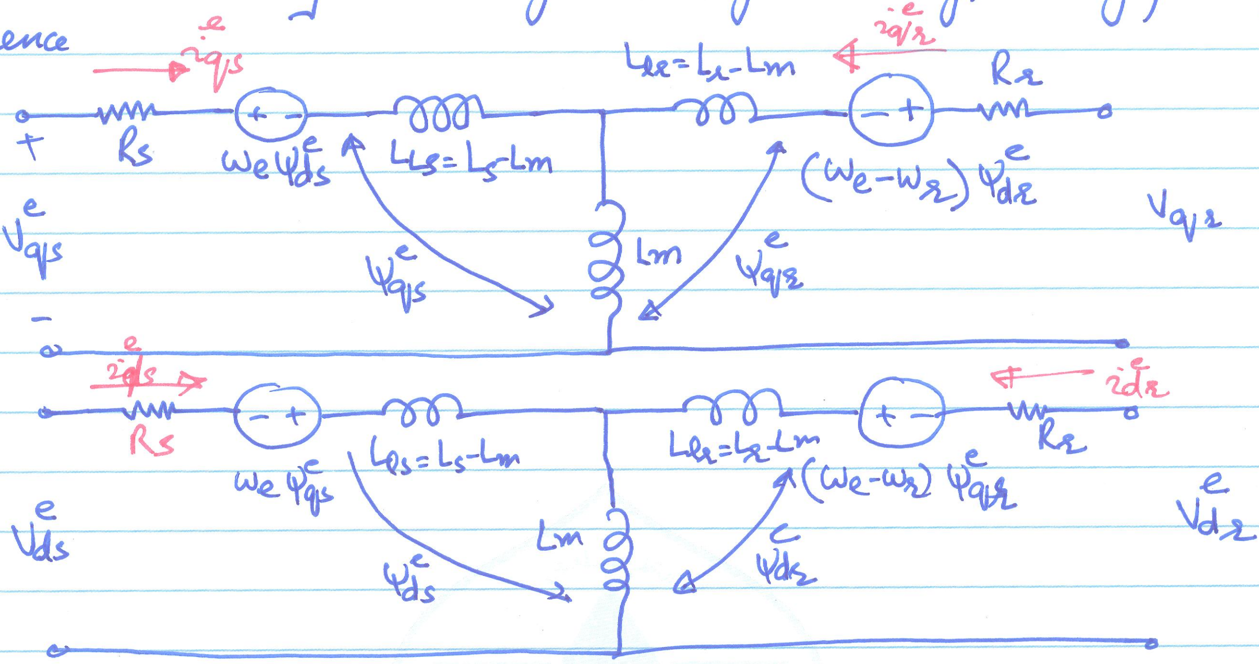
(*) Equivalent circuit of IM in dynamic (Synchronously) rotating frame of reference



(*) In indirect vector control only the motor's rotor dynamic is considered. From above circuit we can write the rotor equations as:

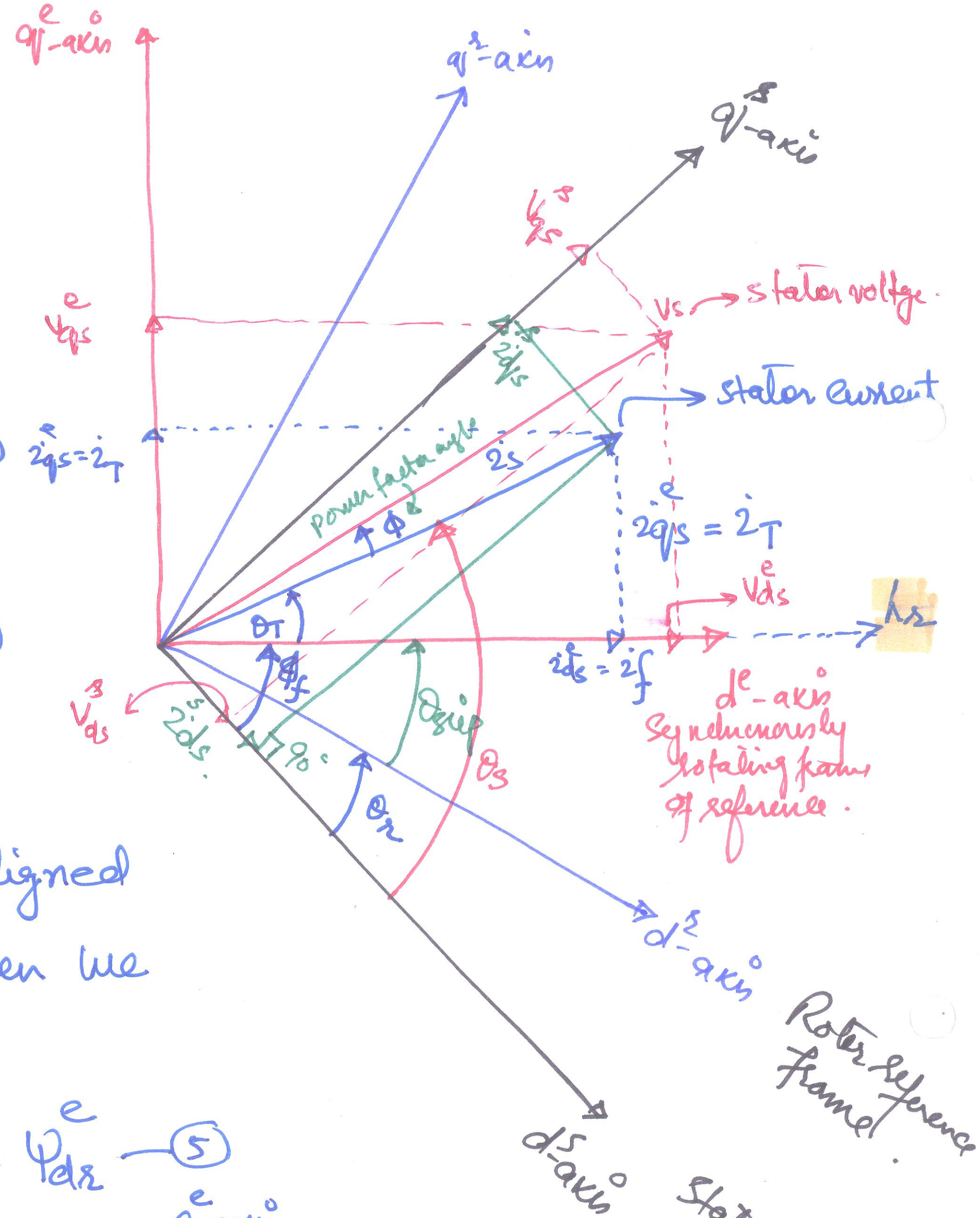
$$\left.\begin{aligned} i^e_{qr} R_r + (\omega_e - \omega_r)\psi^e_{dr} + \frac{d}{dt}\psi^e_{qr} &= V^e_{qr} = 0 \\ i^e_{dr} R_r - (\omega_e - \omega_r)\psi^e_{qr} + \frac{d}{dt}\psi^e_{dr} &= V^e_{dr} = 0 \end{aligned}\right\} \text{Squirrel cage IM} \quad (1)$$

Slip speed in induction motor is given as $\omega_{slip} = \omega_e - \omega_r$

$$\boxed{\omega_{slip} = \omega_e - \omega_r} \quad (2)$$

ω_r = Electrical rotor speed in rad/sec.
ω_e = Synchronous speed in rad/sec.

Consider the following phasor diagram presenting the concept of different reference frames.



From figure.

$$\boxed{\begin{aligned} \theta_s &= \theta_r + \theta_{slip} + \theta_T \\ \theta_s &= \theta_f + \theta_T. \end{aligned}} \quad \text{—(3)}$$

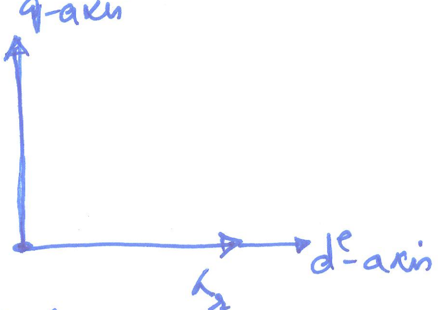
$$\boxed{\theta_f = \theta_{slip} + \theta_r} \quad \text{—(4)}$$

(*) If rotor flux is aligned with d^e-axis Then we can write as

$$\lambda_r = \lambda^e_{dr} = \psi^e_{dr} \quad \text{—(5)}$$

$$\& \quad \lambda^e_{qr} = 0 \quad \text{—(6)}$$

$$\frac{d\psi^e_{qr}}{dt} = 0 \quad \text{—(7)}$$



(*) This assumption will reduce a variable in analysis

* Using above assumption equation set 1 can be written as:

$$\left.\begin{aligned} i_{qr}^{e} R_r + (\omega_e - \omega_r)\psi_{dr}^{e} = 0 \\ \& \quad i_{dr}^{e} R_r + \frac{d}{dt}\psi_{dr}^{e} = 0 \end{aligned}\right\} \quad (8)$$

From equivalent circuit we can write the flux linkages expression as:

$$\psi_{qr}^{e} = (i_{qs}^{e} + i_{qr}^{e})L_m + L_{lr}\, i_{qr}^{e}$$

$$\psi_{qr}^{e} = (i_{qs}^{e} + i_{qr}^{e})L_m + (L_r - L_m)\, i_{qr}$$

$$\psi_{qr}^{e} = L_m i_{qs}^{e} + L_r i_{qr}^{e} \quad (9)$$

Similarly for d-axis we can write as:

$$\psi_{dr}^{e} = (i_{ds}^{e} + i_{dr}^{e})L_m + L_{lr}\, i_{dr}^{e}$$

$$= (i_{ds}^{e} + i_{dr}^{e})L_m + (L_r - L_m)\, i_{dr}^{e}$$

$$\psi_{dr}^{e} = L_m i_{ds}^{e} + L_r i_{dr}^{e} \quad (10)$$

So we have:

$$\boxed{\begin{aligned} \psi_{qr}^{e} &= L_m i_{qs}^{e} + L_r i_{qr}^{e} \\ \psi_{dr}^{e} &= L_m i_{ds}^{e} + L_r i_{dr}^{e} \end{aligned}} \quad (11)$$

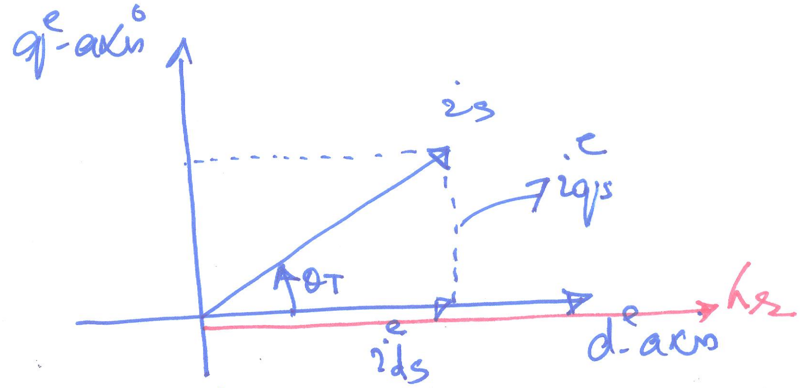
Using our assumption that rotor flux is aligned with d^e-axis $\psi_{qr}^{e} = 0$ and $\psi_{dr}^{e} = \lambda_r$ so the above eq can be written as:

$$\left.\begin{aligned} L_m i_{qs}^{e} + L_r i_{qr}^{e} &= 0 \\ L_m i_{ds}^{e} + L_r i_{dr}^{e} &= \lambda_r \end{aligned}\right\} \quad (12)$$

From eq (12) we can write as

$$\left.\begin{aligned} i_{qr}^{e} &= -\frac{L_m}{L_r} i_{qs}^{e} \\ \& \quad i_{dr}^{e} &= \frac{\lambda_r}{L_r} - \frac{L_m}{L_r} i_{ds}^{e} \end{aligned}\right\} \text{——} (13)$$

Now consider stator flux current from phasor diagram in dynamic frame of reference.



(*) i_{ds}^{e} is aligned with rotor flux and is called flux component of stator current and is represented onward as i_f.

(*) i_{qs}^{e} is along q^e-axis and is called torque component of stator current and is represented as i_T onward.

$$\boxed{\begin{aligned} i_{ds}^{e} &= i_f \\ i_{qs}^{e} &= i_T \end{aligned}} \text{——} (14)$$

Using (14) eq (13) can be written as:

$$\boxed{\begin{aligned} i^{e}_{qr} &= -\frac{L_m}{L_r}\, i_T \\ \text{Eq}\quad i^{e}_{dr} &= \frac{\lambda_r}{L_r} - \frac{L_m}{L_r}\, i_f \end{aligned}} \quad (15)$$

From eq (8) we can write as: $i^e_{qr} R_r + (\omega_e - \omega_r)\psi^e_{dr} = 0$

$$\Rightarrow R_r \underbrace{\left[-\frac{L_m}{L_r} i_T\right]}_{i^e_{qr}\text{ from eq 15}} + \omega_{slip} \underbrace{\lambda_r}_{\psi^e_{dr}} = 0$$

$$\Rightarrow \omega_{slip} = \frac{L_m R_r}{L_r} i_T \cdot \frac{1}{\lambda_r}$$

$$\Rightarrow \omega_{slip} = \frac{L_m i_T}{\left(L_r/R_r\right)\cdot \lambda_r} = \frac{L_m i_T}{T_r \cdot \lambda_r}.$$

where $\frac{L_r}{R_r} = T_r =$ Rotor time constant.

$$\boxed{\omega_{slip} = \frac{L_m i_T}{T_r \cdot \lambda_r}} \quad (16)$$

From eq (8) we have: $R_r i^e_{dr} + \frac{d}{dt}\psi_{dr} = 0$ using the i^e_{dr} from eq (15) we can write as.

$$R_r\left[\frac{\lambda_r}{L_r} - \frac{L_m}{L_r} i_f\right] + \frac{d}{dt}\lambda_r = 0$$

$$\Rightarrow \frac{\lambda_r}{L_r/R_r} - \frac{L_m}{L_r/R_r} i_f + \frac{d}{dt}\lambda_r = 0$$

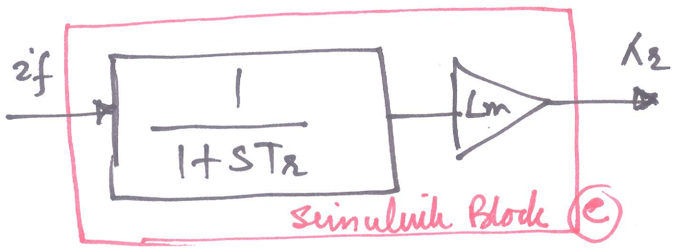
$$\Rightarrow \frac{\lambda_r}{T_r} - \frac{L_m}{T_r} i_f + s\lambda_r = 0$$

$$\Rightarrow \frac{L_m}{T_r} i_f = \frac{\lambda_r}{T_r} + s\lambda_r = \frac{\lambda_r}{T_r}(1+sT_r)$$

$$\boxed{i_f = \frac{\lambda_r}{L_m}(1+sT_r)} \quad \text{—(17)}$$

Or

$$\boxed{\lambda_r = L_m \frac{i_f}{(1+sT_r)}} \quad \text{—(18)}$$



Summary:

$$\lambda_r = \frac{L_m i_f}{(1+sT_r)} \qquad T_r = \frac{L_r}{R_r}$$

$$L_r = L_{ls} + L_m$$

Torque: Torque is given by the following expression

$$T_e = \frac{3}{2}\cdot\frac{P}{2}\left[\lambda_{ds}^{e} i_{qs}^{e} - \lambda_{qs}^{e} i_{ds}^{e}\right]$$

$$T_e = \frac{3}{2}\cdot\frac{P}{2}\frac{L_m}{L_r}\left[\lambda_{dr}^{e} i_{qs}^{e} - \cancel{\lambda_{qr}^{e}} i_{ds}^{e}\right]$$

$= 0$ (in vector control)

$$T_e = \frac{3}{2}\cdot\frac{P}{2}\cdot\frac{L_m}{L_r}\lambda_r \cdot i_T \quad \text{—(19a)}$$

$$\boxed{T_e = K_{te}\cdot\lambda_r\cdot i_T} \quad \text{—(19)}$$

where $\boxed{K_{te} = \frac{3}{2}\cdot\frac{P}{2}\frac{L_m}{L_r}}$ —(20)

(*) ω_m is the rotor mechanical speed in rad/sec. which can be related with ω_r (Electrical rotor speed in rad/sec) as:

$$\boxed{\omega_r = \frac{P}{2}\omega_m} \quad \text{—— (21)}$$

so, $\omega_{slip} = \omega_e - \omega_r = \omega_e - \frac{P}{2}\omega_m$

so

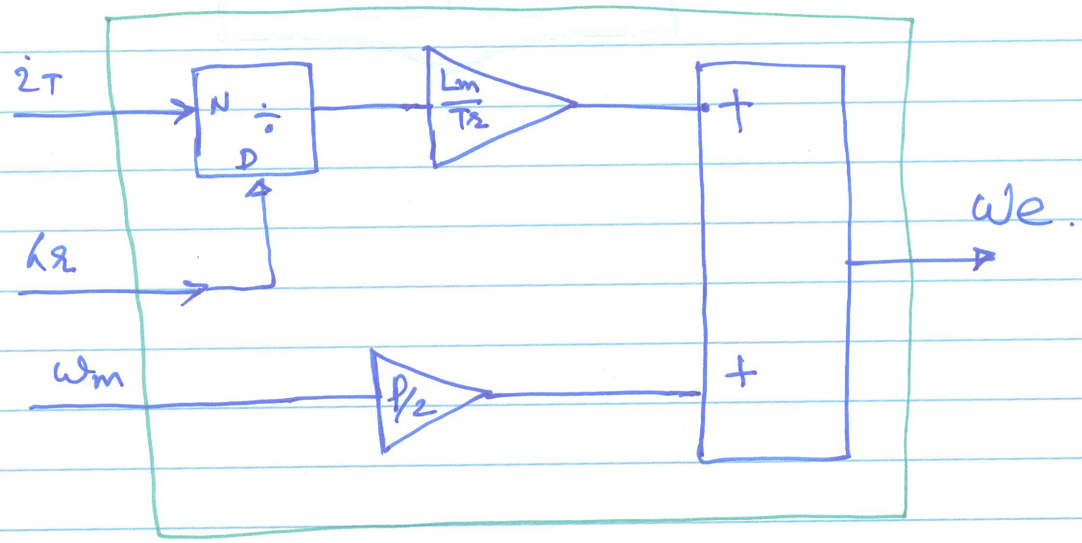
$$\omega_e = \omega_{slip} + \frac{P}{2}\omega_m$$

eq. 16

$$\boxed{\omega_e = \frac{L_m i_T}{T_r \lambda_r} + \frac{P}{2}\omega_m} \quad \text{—— (22)}$$

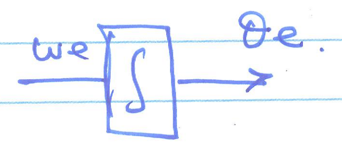
So the Simulink model:

Simulink - Block A

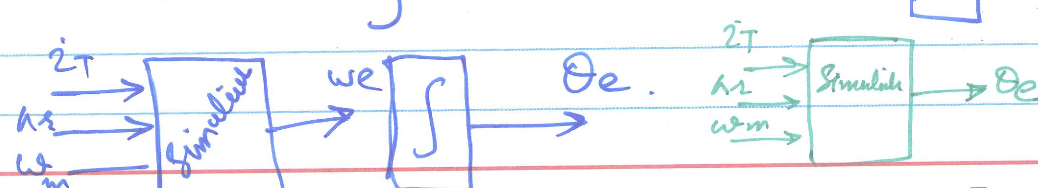


Now

$$\theta_e = \int \omega_e \, dt \quad \Longrightarrow$$

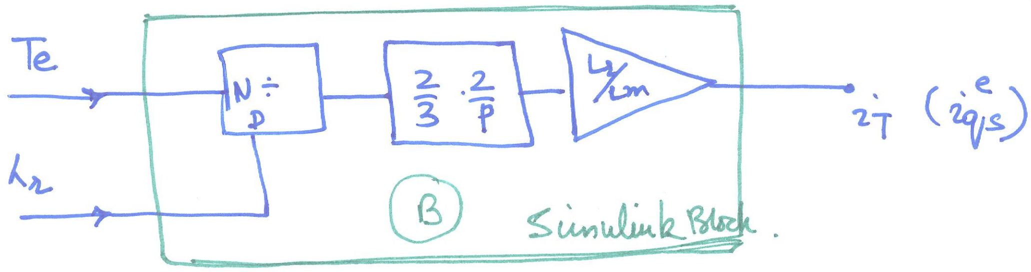


So

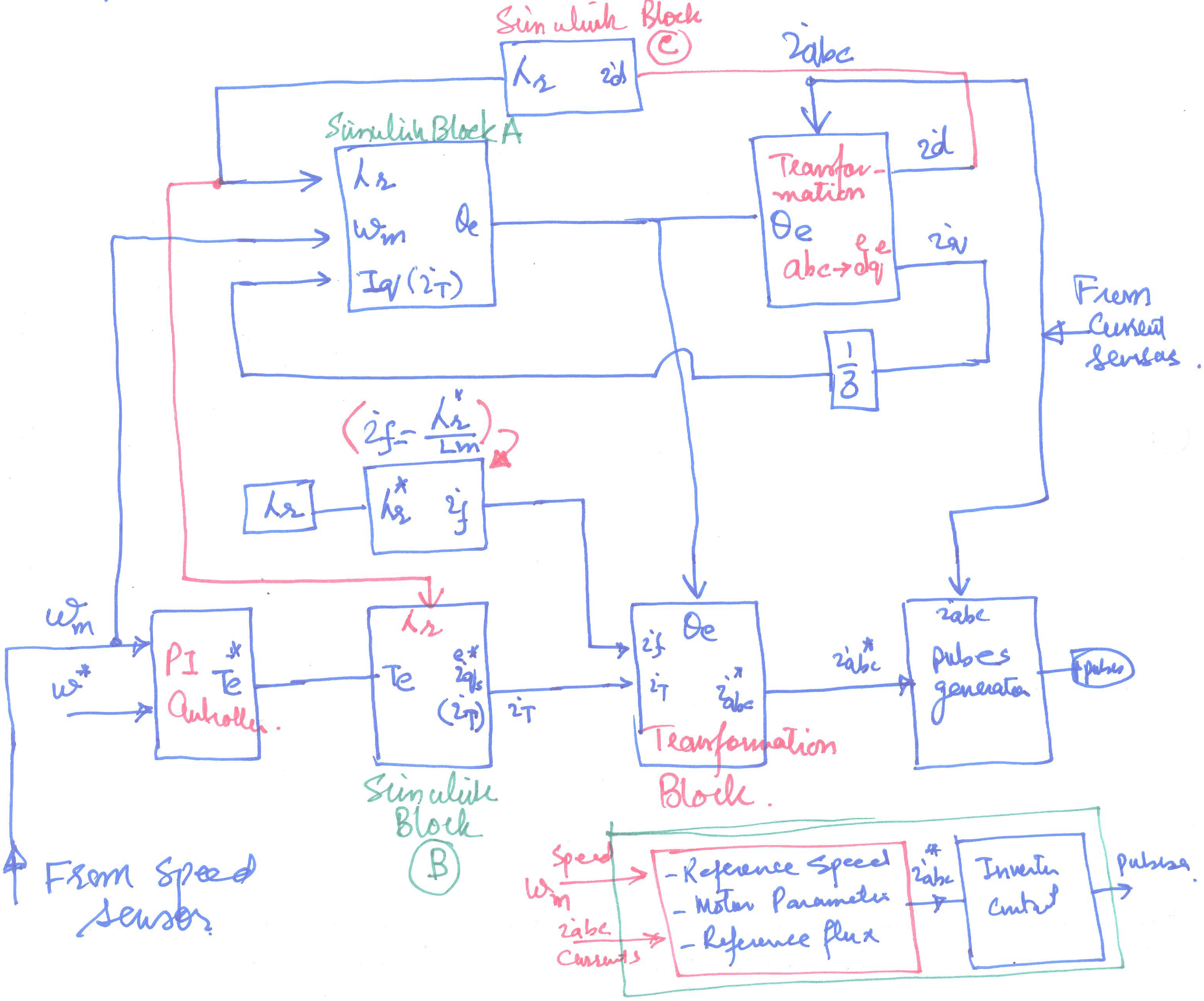


From (19a) we can write

$$T_e = \frac{3}{2}\cdot\frac{P}{2}\cdot\frac{L_m}{L_r}\cdot\lambda_r\cdot i_T \Rightarrow \boxed{i_T = \frac{2}{3}\cdot\frac{2}{P}\,\frac{L_r}{L_m}\cdot\frac{T_e}{\lambda_r}} \quad (23)$$



Complete Simulink Block



We have already discussed transformation blocks.

$$\begin{bmatrix} i_T \\ i_f \end{bmatrix} = \begin{bmatrix} i^e_{qs} \\ i^e_{ds} \end{bmatrix} = \frac{2}{3} \underbrace{\begin{bmatrix} \cos\theta_f & \cos(\theta_f - 2\pi/3) & \cos(\theta_f + 2\pi/3) \\ \sin\theta_f & \sin(\theta_f - 2\pi/3) & \sin(\theta_f + 2\pi/3) \end{bmatrix}}_{[T]} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \quad (24)$$

Inverter Control

$$|i_s| = \sqrt{(i_T)^2 + (i_f)^2} \quad (25)$$

$$[i^e_{qd}] = [T][i_{abc}]$$

$$[i^e_{qd}] = [i^e_{qs} \quad i^e_{ds}]^T$$

$$[i_{abc}] = [i_{as} \quad i_{bs} \quad i_{cs}]^T.$$

$$\theta_f = \theta_r + \theta_{slip} \quad \& \quad \theta_{slip} = \int \omega_{slip}\, dt.$$

From phasor diagram we can translate flux and torque components of current in stationary frame of reference using the following equation.

$$\begin{bmatrix} i^s_{qs} \\ i^s_{ds} \end{bmatrix} = \begin{bmatrix} \cos\theta_f & \sin\theta_f \\ -\sin\theta_f & \cos\theta_f \end{bmatrix} \begin{bmatrix} i_T \\ i_f \end{bmatrix} \quad (26)$$

In stationary frame of reference.

$$i^s_{qs} = |i_s| \sin\theta_s$$

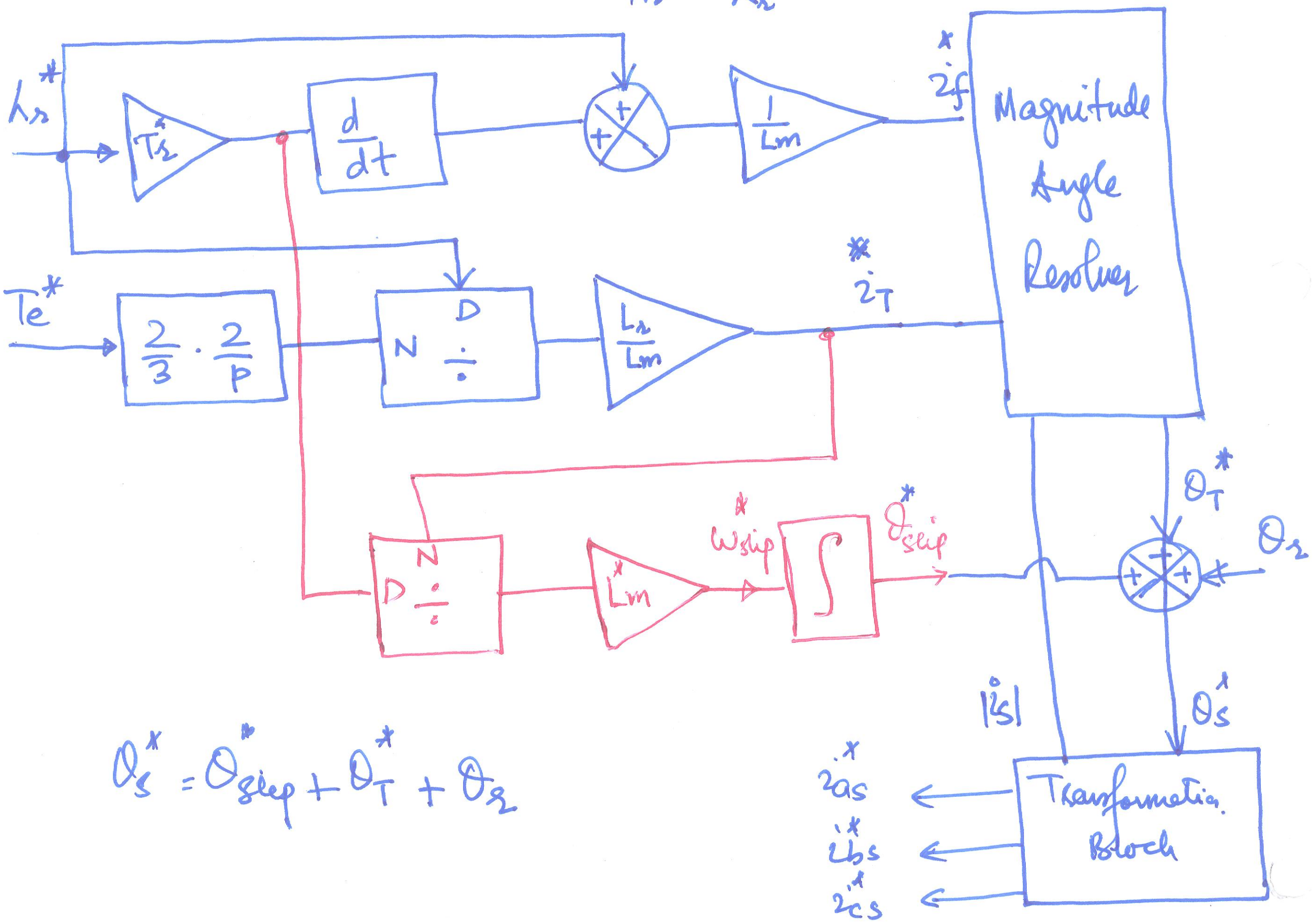
$$i^s_{ds} = |i_s| \cos\theta_s$$

Reference currents for inverter control can be generated using the following block diagram.

Using equation 19a $\Rightarrow i_T^* = \frac{2}{3} \cdot \frac{2}{P} \frac{L_r^*}{L_m^*} \frac{T_e^*}{\lambda_r^*}$

from equation 17 $\Rightarrow i_f^* = \frac{\lambda_r^*}{L_m^*}(1 + sT_r^*)$

from equation 16 $\Rightarrow \omega_{slip}^* = \frac{L_m^*}{T_r^*} \cdot \frac{i_T^*}{\lambda_r^*}$



$\theta_s^* = \theta_{slip}^* + \theta_T^* + \theta_r$

Stator equations in dynamic frame of reference are given as.

$$\left.\begin{aligned} V_{qs}^{e} &= (R_s + sL_s)\, i_{qs}^{e} + \omega_s L_s\, i_{ds}^{e} + L_m \frac{d}{dt} i_{qr}^{e} + \omega_s L_m\, i_{dr}^{e} \\ V_{ds}^{e} &= -\omega_s L_s\, i_{qs}^{e} + (R_s + sL_s)\, i_{ds}^{e} - \omega_s L_m\, i_{qr}^{e} + L_m s\, i_{dr}^{e} \end{aligned}\right\} \quad (27)$$

Using the concept of vector control

$$V_{qs}^{e} = (R_s + sL_s)\, i_{qs}^{e} + \omega_s L_s\, i_{ds}^{e} + L_m s\left(-\frac{L_m}{L_r} i_{qs}^{e}\right) + \omega_s L_m\left(\frac{\lambda_r}{L_r} - \frac{L_m}{L_r} i_{ds}^{e}\right)$$

$$V_{qs}^{e} = \left(R_s + sL_s - \frac{L_m^2}{L_r} s\right) i_{qs}^{e} + \left(\omega_s L_s - \frac{\omega_s L_m^2}{L_r}\right) i_{ds}^{e} + \omega_s \frac{L_m \lambda_r}{L_r}$$

$$V_{qs}^{e} = (R_s + \sigma L_s s) + \omega_s \sigma L_s\, i_{ds}^{e} + \omega_s \frac{L_m \lambda_r}{L_r} \quad (28)$$

where $\sigma = \left(1 - \frac{L_m^2}{L_s L_r}\right)$

Similarly for d-axis voltage from (27)

$$V_{ds}^{e} = -\omega_s L_s\, i_{qs}^{e} + (R_s + sL_s)\, i_{ds}^{e} - \omega_s L_m\left(-\frac{L_m}{L_r}\right) i_{qs}^{e} + L_m s\left(\frac{\lambda_r}{L_r} - \frac{L_m}{L_r} i_{ds}^{e}\right)$$

$$V_{ds}^{e} = \left(R_s + sL_s - \frac{L_m^2}{L_r} s\right) i_{ds}^{e} - \left(\omega_s L_s - \omega_s \frac{L_m^2}{L_r}\right) i_{qs}^{e} + L_m s \frac{\lambda_r}{L_r}$$

$$V_{ds}^{e} = (R_s + \sigma s L_s)\, i_{ds}^{e} - \sigma L_s \omega_s\, i_{qs}^{e} + \frac{L_m \lambda_r}{L_r} s \quad (29)$$

(*) Flux producing Component of stator current $= i_{ds}^{e} = i_f$.

(*) Torque producing Component of stator current $= i_{qs}^{e} = i_T$

so equation (28) gives.

$$V_{qs}^{e} = (R_s + \sigma L_s s)\, i_T + \sigma L_s \omega_s i_f + \omega_s \frac{L_m \lambda_r}{L_r}$$

$\lambda_r = L_m i_f$

$\left\{ \begin{array}{l} \text{Rotor flux linkages} : \lambda_r = L_m i_f \\ \text{and} \quad \sigma L_s = L_a \end{array} \right\}$

so $$V_{qs}^{e} = (R_s + L_a s)\, i_T + \omega_s L_a i_f + \frac{\omega_s L_m^2}{L_r} i_f .$$

$$\Rightarrow V_{qs}^{e} = (R_s + sL_a)\, i_T + \omega_s i_f \left(L_a + \frac{L_m^2}{L_r} \right)$$

$$\Rightarrow V_{qs}^{e} = (R_s + sL_a)\, i_T + \omega_s i_f \left(\sigma L_s + \frac{L_m^2}{L_r} \right)$$

$$\Rightarrow V_{qs}^{e} = (R_s + sL_a)\, i_T + \omega_s i_f \left(L_s - \frac{L_m^2}{L_r} + \frac{L_m^2}{L_r} \right)$$

$$\Rightarrow \boxed{V_{qs}^{e} = (R_s + sL_a)\, i_T + \omega_s i_f L_s} \quad (30)$$

We know that $\omega_{slip} = \frac{L_m}{T_r} \cdot \frac{i_T}{\lambda_r} = \frac{L_m i_T}{T_r (L_m i_f)} = \frac{i_T}{i_f} \cdot \frac{R_r}{L_r}$

and $\omega_s = \omega_r + \omega_{slip} = P/2 \cdot \omega_m + \omega_{slip}$

$$\Rightarrow V_{qs}^{e} = (R_s + sL_a)\, i_T + (\omega_r + \omega_{slip})\, i_f \cdot L_s$$

$$= (R_s + sL_a)\, i_T + \omega_r i_f \cdot L_s + \omega_{slip} i_f \cdot L_s$$

$$= (R_s + sL_a)\, i_T + \omega_r i_f \cdot L_s + \left(\frac{i_T}{i_f} \cdot \frac{R_r}{L_r} \right) \cdot i_f \cdot L_s$$

$$= (R_s + sL_a)\, i_T + \omega_r i_f \cdot L_s + i_T \cdot \frac{R_r L_s}{L_r}$$

$$V_{qs}^{e} = \left(R_s + sL_a + \frac{R_r L_s}{L_r} \right) i_T + \omega_r i_f \cdot L_s$$

$$V_{qs}^{e} - \omega_r L_s i_f = \left(R_s + \frac{R_r L_s}{L_r} + sL_a\right) i_T .$$

$$i_T = \frac{V_{qs}^{e} - \omega_r L_s i_f}{\left(R_s + \frac{R_r L_s}{L_r} + sL_a\right)} = \frac{V_{qs}^{e} - \omega_r L_s i_f}{(R_a + sL_a)}$$

where $\boxed{R_a = R_s + \frac{R_r L_s}{L_r}}$ — (31)

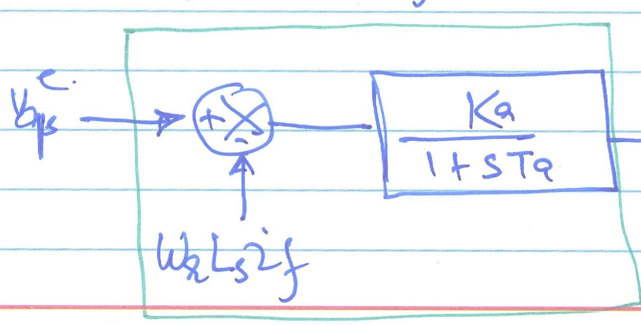
so $$i_T = \frac{1}{R_a} \cdot \frac{V_{qs}^{e} - \omega_r L_s i_f}{(1 + sL_a/R_a)} = \frac{1/R_a\,(V_{qs}^{e} - \omega_r L_s i_f)}{(1 + sT_a)}$$

where $\boxed{T_a = L_a/R_a}$ — (32)

so $$i_T = \frac{K_a}{1+sT_a}\left(V_{qs}^{e} - \omega_r L_s i_f\right).$$

$\boxed{K_a = 1/R_a}$ — (33)

$$\frac{i_T}{V_{qs}^{e} - \omega_r L_s i_f} = \frac{K_a}{1+sT_a} \quad \text{— (34)}$$



we know that

$$T_e = \frac{3}{2} \cdot \frac{P}{2} \cdot \frac{L_m}{L_r} \lambda_r \cdot i_T$$ for vector control.

if λ_r is kept constant then:

$$T_e = K_t \cdot i_T \qquad (34b)$$

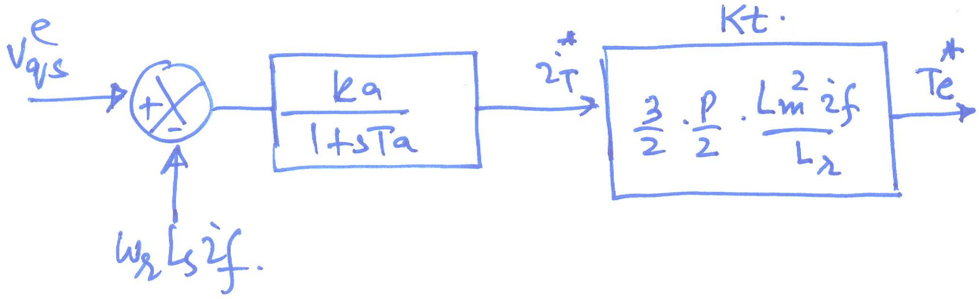
where

$$\boxed{K_t = \frac{3}{2} \cdot \frac{P}{2} \cdot \frac{L_m}{L_r} \cdot \lambda_r} \qquad (35)$$

$$K_t = 3/2 \cdot P/2 \cdot \frac{L_m (L_m i_f)}{L_r}$$

$$\boxed{K_t = 3/2 \cdot P/2 \cdot \frac{L_m^2 i_f}{L_r}} \qquad (36)$$

so



Mechanical Equation of machine is written as:

$$J \frac{d\omega_m}{dt} + B \omega_m = T_e - T_L. \qquad (37)$$

$$\frac{2}{P} J \frac{d\omega_r}{dt} + \frac{2}{P} B\omega_r = T_e - T_L \qquad (38)$$

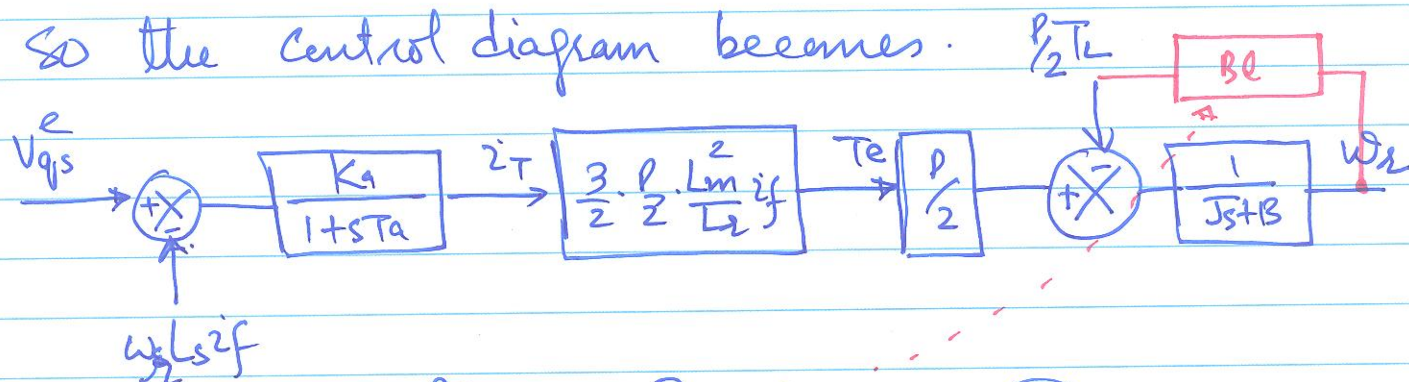
$$J\frac{d\omega_r}{dt} + B\omega_r = \frac{P}{2}(T_e - T_L)$$

$$J s\,\omega_r(s) + B\,\omega_r(s) = \frac{P}{2}(T_e - T_L) \quad \cancel{T_L = \tfrac{P}{2} T_\ell}$$

$$\omega_r(s)\left[Js + B\right] = \frac{P}{2}(T_e - T_L)$$

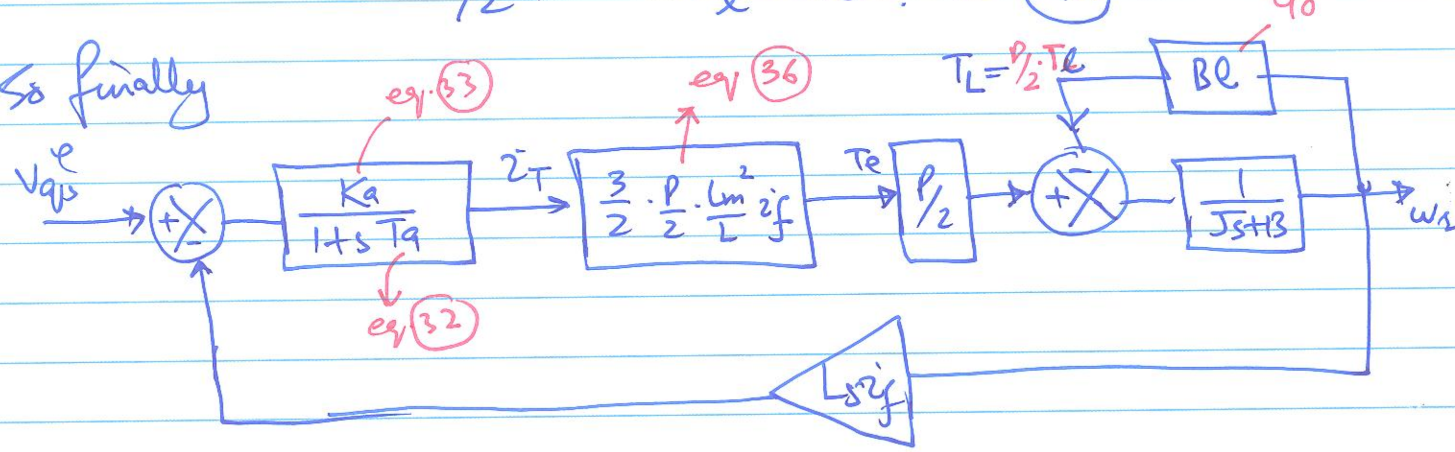
$$\frac{\omega_r(s)}{\frac{P}{2}(T_e - T_L)} = \frac{1}{Js+B} \quad \text{—— (39)}$$

So the control diagram becomes.



$$\frac{P}{2}T_L = B_\ell\cdot\omega_r \quad \text{—— (40)}$$

So finally



We know that

$$J\frac{d\omega_r}{dt} + B\omega_r = \frac{P}{2}\left[T_e - T_L\right] \quad \text{... eq. 34b}$$

$$J\frac{d\omega_r}{dt} + B\omega_r = \frac{P}{2}K_t i_T - \frac{P}{2}B_l \cdot \omega_r$$

$$= \frac{P}{2}K_t i_T - B_L \cdot \omega_r$$

$$J\frac{d\omega_r}{dt} + (B + B_L)\omega_r = \frac{P}{2}K_t i_T$$

$$J\frac{d\omega_r}{dt} + B_t\omega_r = \frac{P}{2} \cdot K_t \cdot i_T$$

$$\frac{J}{B_t}\frac{d\omega_r}{dt} + \omega_r = \frac{P}{2} \cdot \frac{K_t}{B_t} \cdot i_T$$

$$\boxed{\frac{J}{B_t} = T_m.} \quad (40) \qquad \boxed{K_m = \frac{P}{2} \cdot \frac{K_t}{B_t}} \quad (41) \qquad \boxed{B_t = B + B_L}$$

$$T_m s\,\omega_r(s) + \omega_r(s) = K_m i_T(s)$$

$$\omega_r(s)\left(1 + sT_m\right) = K_m i_T(s)$$

$$\boxed{\frac{\omega_r(s)}{i_T(s)} = \frac{K_m}{1 + sT_m}} \quad (42)$$

$$i_T \longrightarrow \boxed{\frac{K_m}{1 + sT_m}} \longrightarrow \omega_r.$$

Speed Controller:

Let us control speed using the PI Controller so the transfer function of PI Controller is given as

$$G_s(s) = \frac{K_s(1 + sT_s)}{sT_s}$$

Inverter: Let us assume that inverter has the following transfer function b/w its command i_{qs}^{e} and V_{qs}^{e} (O/P).

$$\frac{V_{qs}^{e}(s)}{i_{qs}^{e}(s)} = \frac{K_{in}}{1+sT_{in}}$$

where
K_{in} = gain of inverter
T_{in} = Inverter time response (constant)
$= 1/2f_s$

Current Feedback Transfer function: Very little filtering is common in current feed back.

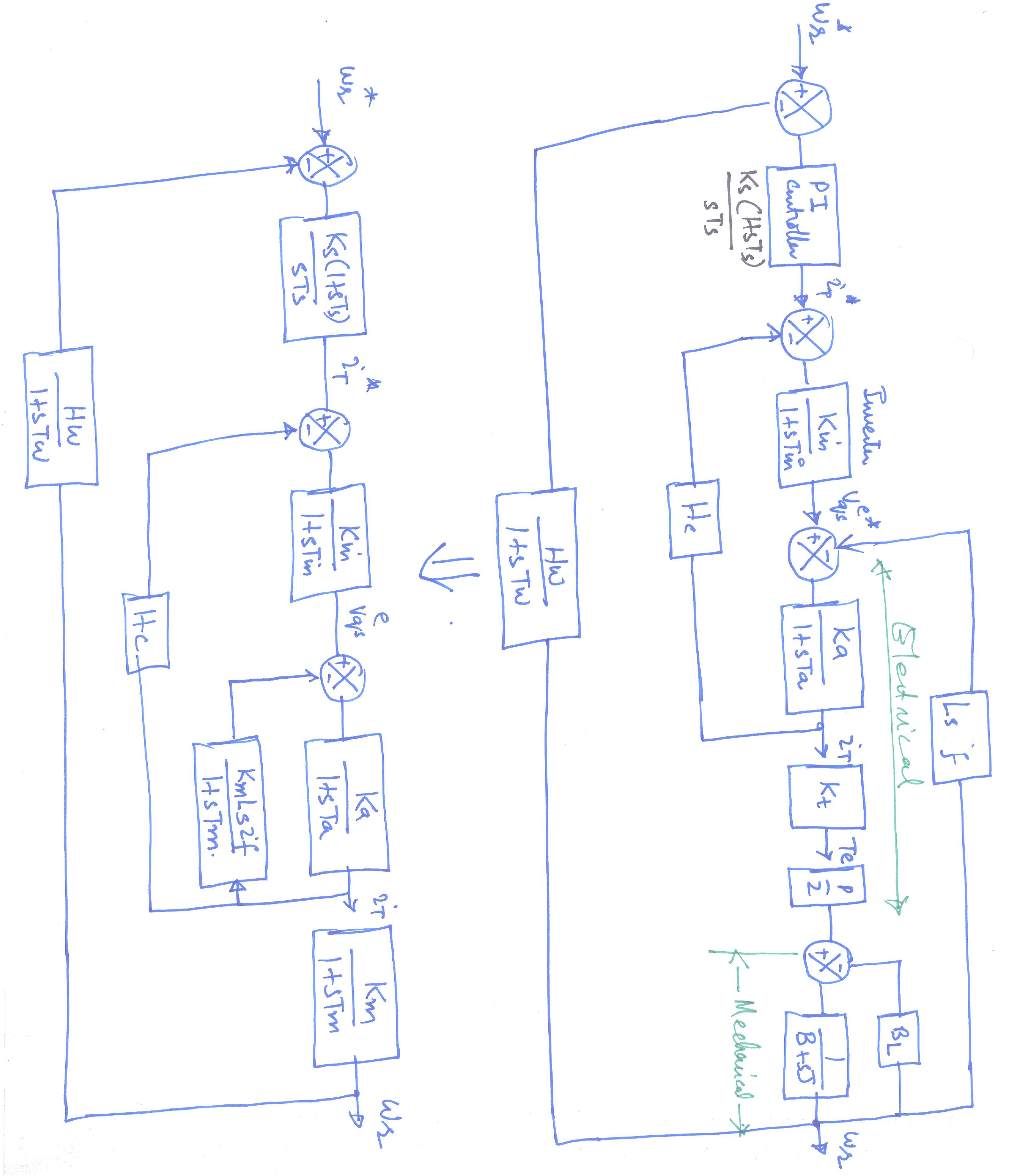
$$G_c(s) = H_c.$$

Speed Feedback

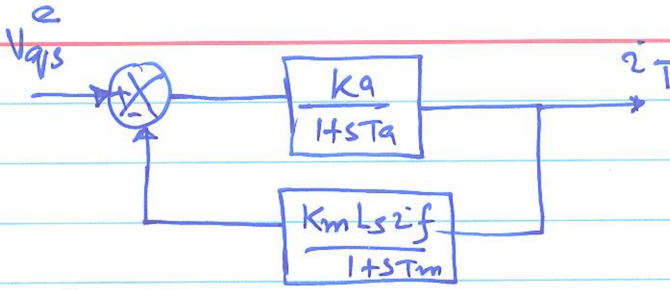
$$G_\omega(s) = \frac{\omega_{rm}(s)}{\omega_m(s)} = \frac{H_\omega}{1+sT_\omega}.$$

H_ω = gain
T_ω = Time constant of speed filter.

So the final control diagram:



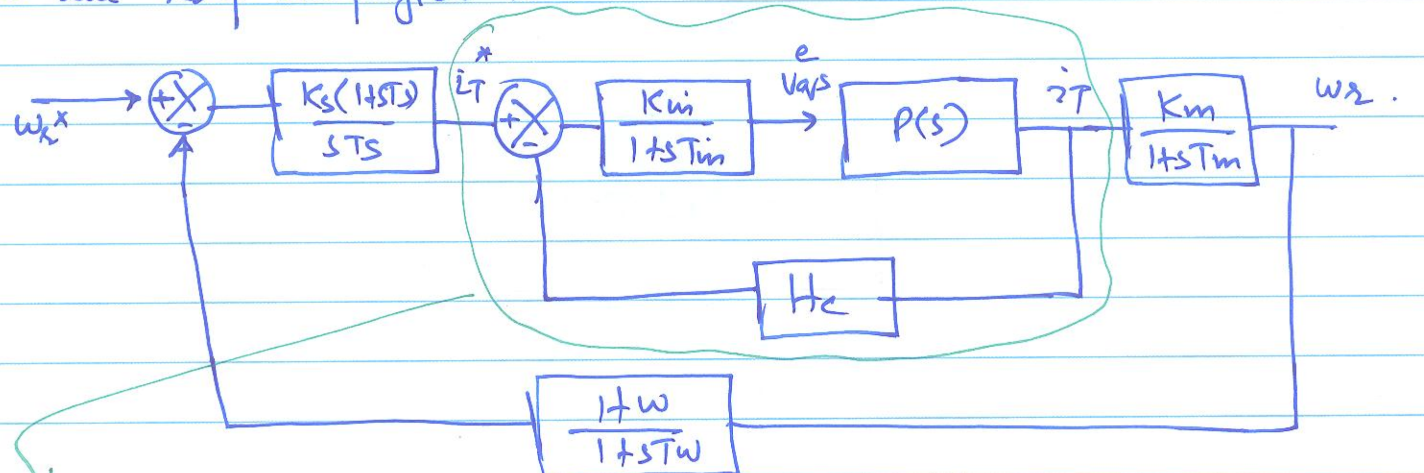
Consider the loop:



Closed loop Transfer function

$$P(s) = \frac{G}{1+GH} = \frac{K_a/(1+sT_a)}{1+\left(\frac{K_a}{1+sT_a}\right)\left(\frac{K_m L_s i_f}{1+sT_m}\right)} = \frac{K_a(1+sT_m)}{(1+sT_a)(1+sT_m)+K_a K_m L_s i_f}$$

So the loop simplifies to:



Consider the current loop now: Closed loop TF is given as.

$$G_{T_i}(s) = \frac{G}{1+GH} = \frac{\left(\frac{K_a(1+sT_m)K_{in}}{[(1+sT_a)(1+sT_m)+K_a K_m L_s i_f](1+sT_{in})}\right)}{\left[1+\frac{K_a(1+sT_m)K_{in}H_c}{[(1+sT_a)(1+sT_m)+K_a K_m L_s i_f](1+sT_{in})}\right]}$$

$$G_{T_i}(s) = \frac{K_a(1+sT_m)K_{in}}{[(1+sT_a)(1+sT_m)+K_a K_m L_s i_f](1+sT_{in})+K_a(1+sT_m)K_{in}H_c}$$

Now let

$$K_m i_f L_s = K_b$$

(*) Time constant of inverter is very small so

$$(1+sT_a)(1+sT_{in}) \simeq 1+s(T_a+T_{in})$$
$$\simeq 1+sT_{ar}.$$

or $(1+sT_{in}) \approx 1$.

(*) Using above approximations.

$$G_{i}(s) = \frac{K_a K_{in}(1+sT_m)}{\left[(1+sT_{ar})(1+sT_m)+K_a K_b \underbrace{(1+sT_{in})}_{\approx 1}\right] + H_c K_a K_{in}(1+sT_m)}$$

$$G_i(s) = \frac{K_a K_{in}(1+sT_m)}{\left[(1+sT_{ar})(1+sT_m)+K_a K_b\right] + H_c K_a K_{in}(1+sT_m)}$$

$$(1+sT_{ar})(1+sT_m) + K_a K_b + H_c K_a K_{in}(1+sT_m)$$
$$= 1+sT_m+sT_{ar}+s^2 T_{ar}T_m + K_a K_b + H_c K_a K_{in} + H_c K_a K_{in} T_m s$$
$$= s^2(T_{ar} T_m) + s(T_m + T_{ar} + H_c K_a K_i T_m) s +$$
$$(1+K_a K_b + H_c K_a K_i).$$
$$= T_{ar} T_m \left[s^2 + s\frac{(T_m + T_{ar} + H_c K_a K_i T_m)}{T_{ar} T_m} + \frac{1+K_a K_b + H_c K_a K_i}{T_{ar} T_m}\right]$$

$a = 1$
$b = (T_m + T_{ar} + H_c K_a K_i T_m)/T_{ar} T_m$
$c = (1+K_a K_b + H_c K_a K_i)\, T_{ar}/T_m.$

Now assuming

$$-\frac{1}{T_1}, \frac{-1}{T_2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \text{—— (A)}$$

$$\Rightarrow T_{ar}T_m\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right) = (1+sT_{ar})(1+sT_m)+K_aK_b+H_cK_aK_{in}(1+sT_m)$$

so

$$G_i(s) = \frac{K_a K_{in}(1+sT_m)}{T_{ar}T_m\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}$$

$$G_i(s) = \frac{K_a K_{in} T_1 T_2}{T_{ar} T_m} \frac{(1+sT_m)}{(1+sT_1)(1+sT_2)}$$

T_m is the mechanical time constant and very large as compared to T_1 & T_2. Similarly from (A) it can be observed that $T_1 < T_2$ so $T_1 < T_2 < T_m$

so

$$1+sT_m \simeq sT_m$$
$$1+sT_2 \simeq sT_2.$$

Using above approximations.

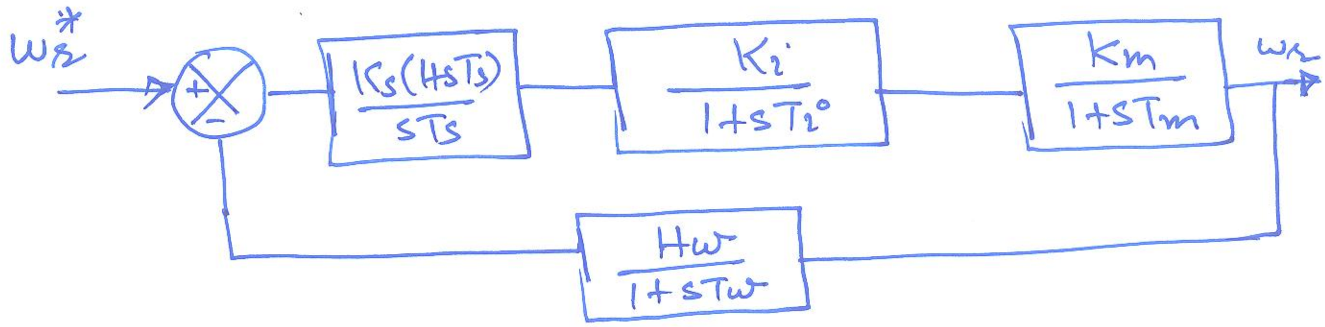
$$G_i(s) = \frac{K_a K_{in} T_1}{T_{ar}} \cdot \frac{1}{(1+sT_1)} = \frac{K_i}{1+sT_i}$$

where

$$K_i = \frac{K_a K_{in} T_1}{T_{ar}}$$

$$T_i = T_1$$

So the closed loop becomes:



Now
$$GH = \frac{K_s K_i K_m (1+sT_s) H_\omega}{sT_s(1+sT_i^0)(1+sT_m)(1+sT_\omega)}$$

Using approximation $(1+sT_m) \simeq sT_m$.

$$GH = \frac{K_i K_m H_\omega}{T_m} \cdot \frac{K_s}{T_s} \left[\frac{1+sT_s}{s^2 (1+sT_{\omega i}^0)} \right] \qquad T_{\omega i} = T_i + T_\omega.$$

$$GH = K_g \cdot \frac{K_s}{T_s} \cdot \frac{1+sT_s}{s^2(1+sT_{\omega i})}$$

Solving for $\frac{G}{1+GH}$ one can get

$$\frac{\omega_r}{\omega_r^*} = \frac{1+sT_s}{1+sT_s + \frac{T_s}{K_g K_s}s^2 + \frac{T_s T_{\omega i}}{K_g K_s}s^3}$$

Symmetric optimum function for $\zeta = 0.707$ is given as.

$$\frac{1+sT_s}{1+T_s s+\left(\frac{3}{8}T_s^2\right)s^2+\left(\frac{1}{16}T_s^3\right)s^3}$$

so proportional & Integral gains can be evaluated as

$$K_p = K_s = 4/9 \cdot \frac{1}{K_g T_{\omega i}}$$

$$K_i = \frac{K_s}{T_s} = \frac{2}{27} \frac{1}{K_g T_{\omega i}^2}$$