# Computational Electrodynamics

## The Finite-Difference Time-Domain Method

**Third Edition**

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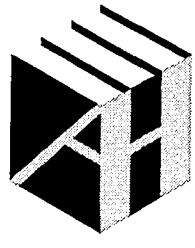

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