Correcting the Core Flux of Dual Transformer Models using Negative Inductors in Multi-Winding Reversible Equivalent Circuits

2 authors:

Francisco De Leon
New York University

Digvijay Deswal
New York University

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Digvijay Deswal, *Student Member, IEEE*, and Francisco de León, *Fellow, IEEE*

***Abstract*—This paper presents a correction to existing transformer models to accurately represent the duality between electrical and magnetic domains. Existing models do not consider correctly the flux in the core/window but match the terminal behavior. The problem becomes prominent during short-circuit tests when the core flux is low. Finite element simulations and experimental tests are carried out to show the problem. A dual equivalent circuit, using negative inductors, is proposed to rectify the problem and create a reversible model.**

***Index Terms*—Duality, leakage inductance, short-circuit, reversible models, transformer modeling.**

I. Disclosure

After the research was finished and the paper was written (and even submitted to the IEEE for evaluation) we found a publication in German that about 60 years ago had already proposed the correct model [7]. Additionally, in [7] the negative inductance was eliminated elegantly using auto-transformers (which we do not use here). Note, however, that the nature of the negative inductance and its effects on the behavior of the overall transformer model still need to be properly explained. Our paper provides a clear explanation and formulas for the calculation of all model parameters. Reference [8] also uses the negative inductance model based on [7]. We would like to make this paper open source for the benefit of the research community as both [7] and [8] are not easily accessible and clear explanations are needed as made evident by the continuous publication of dual transformer models without the correction of negative inductances.

II. Introduction

Duality models for transformers have been used for a long time. The one-to-one correspondence between magnetic circuit and electrical circuit was introduced for transformers by Cherry in 1949 [1]. Slemon [2], [3] applied the principle of duality to most electrical machines including the effects of saturation. Dual models have been used for modeling low-frequency transformer transients, such as: inrush currents, GIC, ferroresonance, and short-circuit. The models are considered topologically correct and physically sound [4].

The authors are with the Department of Electrical and Computer Engineering at New York University, Five Metro-tech Center, Brooklyn, NY, 11201 (e-mails: dd1681@nyu.edu, fdeleon@nyu.edu).

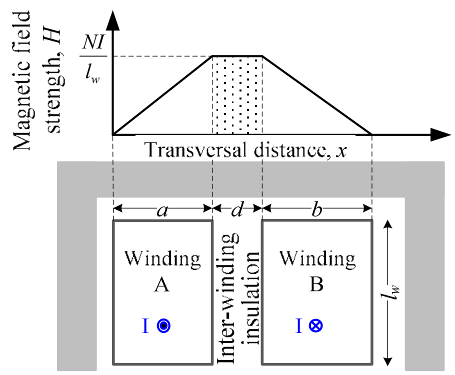

Fig. 1. Trapezoidal distribution of the magnetic field in a two-winding shell-type transformer during a short-circuit test.

Some are reversible, i.e. the parameters remain constant regardless of connection.

This paper investigates a phenomenon where existing leakage models, while giving the correct terminal behavior, do not properly represent the flux distribution in the transformer geometry; specifically during a short-circuit test. Theoretical derivations are presented to show the source of the problem. Finite element (FEM) simulations and laboratory tests are performed to confirm the existence of the discrepancy between the circuit model (π equivalent circuit) and reality. A correction method to eliminate the problem is presented.

III. Theory

The energy stored in the leakage field of a transformer is used to compute the leakage inductance as follows:

$$W = \iiint_V \overline{B} \bullet \overline{H} \, dV = \frac{1}{2} L I^2. \tag{1}$$

For a two-winding transformer, the magnetic field is commonly assumed to follow the trapezoidal distribution as shown in Fig. 1. Magnetic field strength on winding A is given by:

$$H(x) = \frac{NI}{a l_w} x \tag{2}$$

By integrating the energy stored in the field of Fig. 1 (including the two windings and the insulation between them) the following expression for the leakage inductance is obtained (each term represents a region in the transformer window) [2]:

$$L_{AB} = L_A + L_d + L_B = \frac{\mu N^2}{l_w} \left[\frac{a}{3} + d + \frac{b}{3} \right] \tag{3}$$

where a is winding A thickness, b is winding B thickness, d is the thickness of the inter-winding insulation, l_w is the height of the window (fringing flux is assumed negligible), N is the number of turns (assumed the same in both windings for simplicity). The leakage flux linked by winding A is

$$\lambda_{A,energy} = L_A I = \frac{\mu N^2 I}{l_w}\left(\frac{a}{3}\right) \tag{4}$$

Duality derived transformer models incorporate the leakage associated with a pair of windings using a single inductor (L_{AB}) as in (3). This leakage inductance is the one obtained from laboratory short-circuit tests and is matched by finite element simulations. The leakage flux crossing the physical space of winding A is computed by circuit simulators (such as EMTP-type programs) as:

$$\begin{aligned}\lambda_{A,dual} &= L_{A,dual} I = N\phi_A = N\int_0^a B\, dA = N\int_0^a \mu H(x)dx \\ &= N\int_0^a \mu \frac{NI}{a l_w} x dx = \frac{\mu N^2 I}{l_w}\left(\frac{a}{2}\right)\end{aligned} \tag{5}$$

This flux corresponds to the equivalent inductor that gives the correct (physical) dual behavior of the transformer. Note that circuit simulators do not consider that the number of linked turns in the width of the winding is also a function of x and therefore compute the wrong linkage flux. Kulkarni and Khaparde [5] have shown how to properly compute the linkage of leakage flux, given in (4), from the leakage flux using a linear distribution for the number of turns, $N(x) = Nx/a$, as:

$$\begin{aligned}\lambda_{A,linkage} &= \int_0^a N d\phi_A = \int_0^a N(x)B(x)dx \\ &= \int_0^a \frac{Nx}{a}\frac{\mu N I x}{a l_w} dx = \frac{\mu N^2 I}{l_w}\left(\frac{a}{3}\right)\end{aligned} \tag{6}$$

From (4), (5) and (6) one can clearly see that the terminal leakage inductance due to the stored energy (or leakage flux linked) in winding A is different from the dual (physical) space flux inductance necessary by a circuit simulator to compute the correct flux. Dual models using a single inductor give the wrong flux in the winding window but correct terminal inductance. Adjusting the value of the inductor to properly compute the flux yields a wrong terminal result.

Fig. 2 explains the difference between flux in the core and flux linkage (which induces voltage) in the windings. Note that the core and windings experience different flux concepts. The flux going in and out of the yoke is physical magnetic flux given in Webbers by equation (4). While the winding senses *flux linkage* (not physical flux) in Webber-turns by equation (5). Because the number of linked turns by the flux is a function of the transversal distance, according to (2), each layer of the winding links different flux. For example λ_1 is akin to ϕ_1, but λ_2 includes both ϕ_1 and ϕ_2, and so forth for the external layers of winding A.

Next section presents a model that can simultaneously compute the right terminal behavior and the correct leakage flux.

IV. Model Derivation

The difference in inductance between energy/linkage flux (4) and dual space flux (5) is given by:

$$L_{A,linkage} - L_{A,dual} = \frac{\mu N^2}{l_w}\left(\frac{a}{3}-\frac{a}{2}\right) = \frac{\mu N^2}{l_w}\left(-\frac{a}{6}\right). \tag{7}$$

The inductance of (7) is associated with the linkage of leakage flux in the winding width. This inductance is used to correct the duality model. Because this inductance corresponds to linkage, its flux needs not be accounted as part of the core as is only needed to correct the terminal behavior. More discussion on negative inductance can be found in [4], [6]. In [6], the same value for the negative inductance was obtained for three-winding transformers, but was applied to the center winding only, not for all windings as done in this paper. In this letter, inductance (7) applies to a single winding configuration and hence can be generalized to multi-winding transformers.

Fig. 3(a) presents the exiting duality model of a two-winding shell-type transformer used in the literature [4]. Dur-

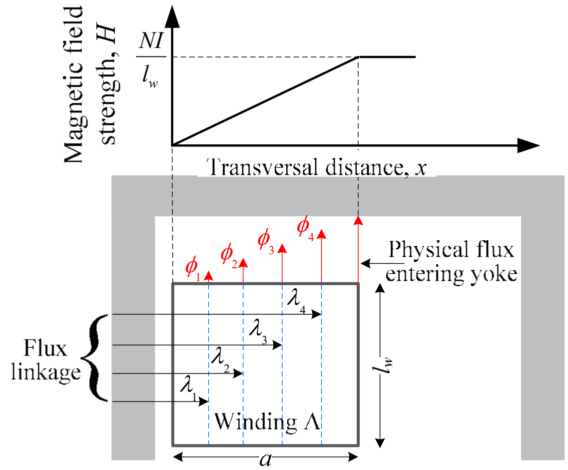

Fig. 2. Explaining the difference between physical flux and flux linkage in winding A of a two-winding shell-type transformer during a short-circuit test.

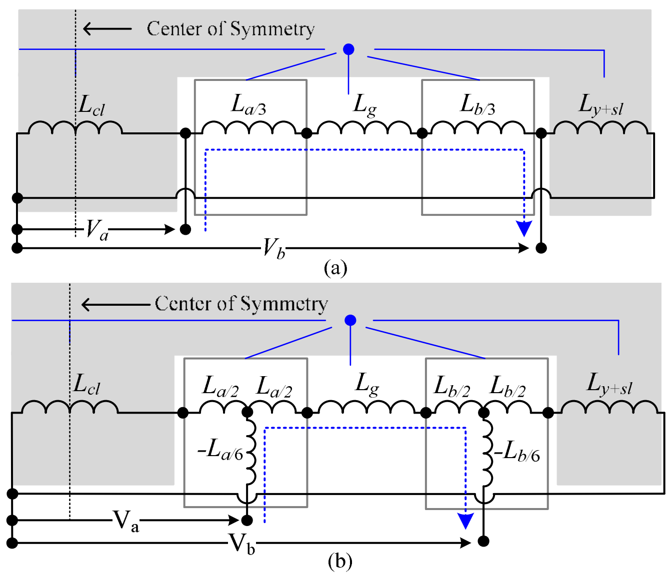

Fig. 3. Topological duality derived equivalent circuits for a two-winding shell-type transformer; (a) model based on leakage energy inductance; (b) model based on compensation of linkage flux by a negative inductance.

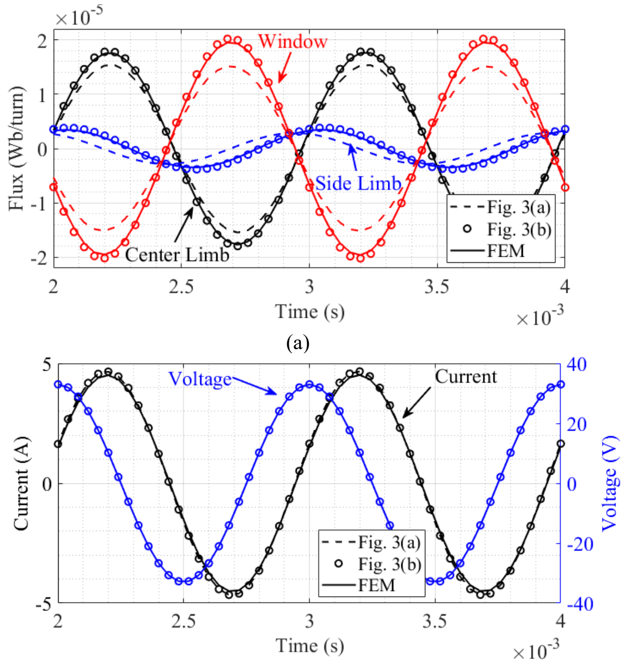

Fig. 4. Comparison between FEM and Fig. 3 models: (a) duality of center-limb, side-limb, and winding window flux; (b) terminal behavior at source for short-circuit applied at winding *B*.

ing short circuit, the terminal inductance seen for Fig. 3(a) is:

$$L_{AB,\text{terminal}} = (L_{a/3} + L_g + L_{b/3}) \tag{8}$$

$$L_{AB,dual,Fig.2(a)} = (L_{a/3} + L_g + L_{b/3}) \tag{9}$$

$$L_{AB,dual\_physical} = (L_{a/2} + L_g + L_{b/2}) \tag{10}$$

where

$$L_{i/k} = \frac{L_i}{k} = \frac{\mu N^2}{l_w}\left(\frac{i}{k}\right); \quad i = a, b; \quad k = 2, 3, 6$$

From (8) one can observe that the model of Fig. 3(a) gives the correct terminal leakage (3), but the flux distribution in the winding window is wrong as seen by comparing (9) and (10). This is so because the inductance corresponding to energy/linkage (9) produces the wrong flux in the winding physical space. The correct dual inductor is given in (10).

Similar analysis holds true for the open-circuit test when exciting winding *A*. In the open-circuit case, the winding contribution to the linkage flux is not accounted for during inner winding energization. The connection point of the ideal transformers is done in such a way that winding contribution is not properly considered as seen in Fig. 3(a). The error is not large, as the magnetizing flux is dominant and many times larger than the winding window flux (in normal operation). Therefore, no noticeable terminal error is observed. However, in saturation the contribution of the winding width needs to be considered, as the core inductance becomes comparable to winding inductance.

To correct the error in the calculation of flux in existing models, the physical inductance of the winding width is used. This gives the right flux, but to correct the terminal behavior a negative inductance (7) is necessary. The duality circuit for

TABLE I
TRANSFORMER PARAMETERS

Quantity	Value
Winding thickness (*a*, *b* and *c* when used)	9 mm
Conductivity	6×10^7 S/m
Frequency	1 kHz
Winding/ window height	84 mm
Winding depth	150 mm
Side-limb and yoke width	28 mm
Inter-winding insulation	3 mm
Insulation (between core and windings)	3 mm
Winding turns	161
Winding resistance	1.25 Ω
Core relative permeability	1000

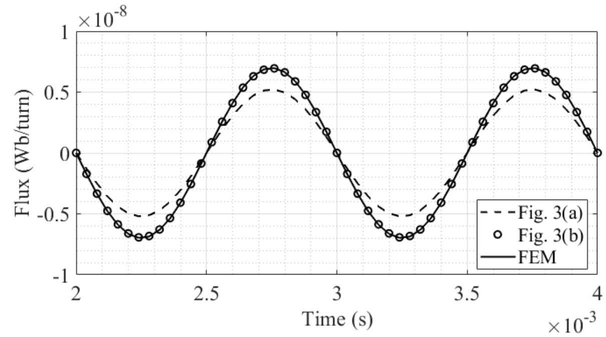

Fig. 5. Comparison between FEM and Fig. 3 model for winding window flux for open-circuit with excitation applied at winding *A*.

the transformer of Fig. 3(a) is presented in Fig. 3(b).

The negative inductance (7), corresponding to the linkage of leakage flux by the winding current distribution, is connected to the midpoint of the physical inductance representing the winding width. Since inductance (7) is not in the magnetic closed loop, it does not contribute with flux to the core.

As an example, the terminal inductance seen for Fig. 2(b) in short circuit (follow the dotted line) is:

$$\begin{aligned} L_{AB,\text{terminal}} &= (-L_{a/6} + L_{a/2} + L_g + L_{b/2} - L_{b/6}) \\ &= (L_{a/3} + L_g + L_{b/3}). \end{aligned} \tag{11}$$

Comparing (9) and (11) one can see that the terminal leakage inductance is correct. The flux contribution to the core is also correct as half of L_a and half of L_b are used in the leakage path to produce the right flux. If desired, the negative inductors can be transferred to the line side of the ideal transformers as they are not part of the magnetic-electric duality.

V. RESULTS

To illustrate the phenomenon and presented solution, FEM simulations are done in ANSYS Maxwell and compared with circuit models built in the EMTP-RV platform. Experimental demonstration of the phenomenon was done on a small laboratory transformer to verify the FEM simulations.

FEM and circuit simulations were performed on a two-winding transformer, dimensions are given in Table I (so that an interested reader can repeat our calculations). Fig. 4 presents the plot of magnetic flux in the center limb, winding window, and side limb during a short-circuit. It is apparent from Fig. 4 that the PI model of Fig. 3(a) is missing flux in the transformer core/window and the model of Fig. 3(b) can accu-

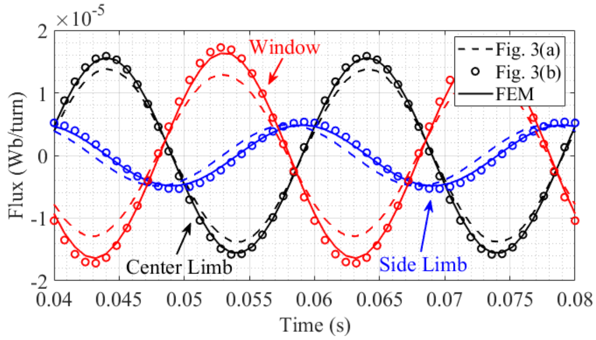

Fig. 6. Comparison between FEM and Fig. 3 models: duality of center-limb, side-limb, and winding window flux for short-circuit applied at winding *B*.

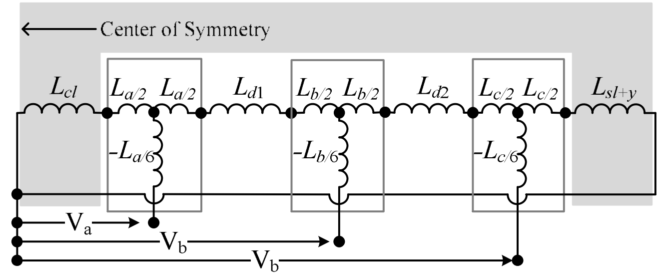

Fig. 7. Topological duality derived equivalent circuit (one-sided [4]) three-winding shell-type transformer based on proposed method.

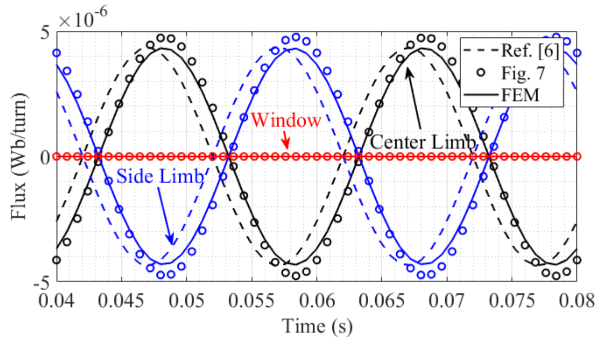

Fig. 8. Comparison between FEM, model of [6] and Fig. 7: duality of center-limb, side-limb, and winding window flux for short-circuit applied at paralleled combination of windings *A* and *C* and excitation placed at winding *B*.

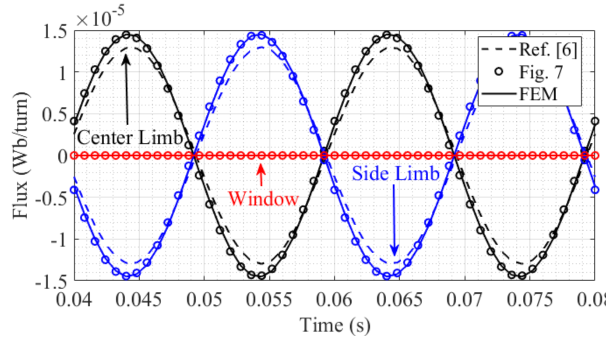

Fig. 9. Comparison between FEM, model of [6] and Fig. 7: duality of center-limb, side-limb, and winding window flux for short-circuit applied at winding *B* and excitation placed at parallel combination of windings *A* and *C*.

rately represent the flux and terminal behavior.

Fig. 5 presents the results for open-circuit test for inner winding energization. One can see that winding window flux is accurately computed with the model of Fig. 3(b) and incorrectly by the model of Fig 3(a). Center-limb, side-limb, and terminal measurements are almost identical in the two models (results not shown) since the magnetizing inductance is several orders of magnitude larger.

Simulations on this small transformer were performed at 1 kHz excitation to make inductances dominate, which is true for large power transformers, instead of the resistances of the small transformer. Results for a short-circuit test at power frequency (50 Hz) for a transformer with dimensions given in Table I are presented in Fig. 6. This shows that the phenomenon is independent of frequency. Fig. 6 transformer has winding resistance of 0.125. Terminal results are not shown as they match perfectly in all cases.

VI. Application to Three-winding Transformers

The proposed model is applied to the three-winding transformer shown in Fig. 7. The terminal short circuit values are:

$$\begin{aligned} L_{AB,\mathrm{terminal}} &= \left(-L_{a/6} + L_{a/2} + L_{d1} + L_{b/2} - L_{b/6}\right) \\ &= L_{a/3} + L_{d1} + L_{b/3} \end{aligned} \tag{11}$$

$$\begin{aligned} L_{AC,\mathrm{terminal}} &= \left(-L_{a/6} + L_{a/2} + L_{d1} + L_{b} + L_{d2} + L_{c/2} - L_{c/6}\right) \\ &= L_{a/3} + L_{d1} + L_{b} + L_{d2} + L_{c/3} \end{aligned} \tag{12}$$

$$\begin{aligned} L_{BC,\mathrm{terminal}} &= \left(-L_{b/6} + L_{b/2} + L_{d2} + L_{c/2} - L_{c/6}\right) \\ &= L_{b/3} + L_{d2} + L_{c/3} \end{aligned} \tag{13}$$

$$L_{AB,dual\_terminal} = L_{a/2} + L_{d1} + L_{b/2} \tag{14}$$

From (11), (12) and (13) one can see that proposed model can accurately predict the terminal leakage values, while still retaining the correct flux (14) in the transformer window. From this result one can expect that the model can be extended to transformers with a larger number of layer windings.

FEM and circuit simulations were performed on a three-winding transformer, dimensions are given in Table I (we added a third winding (C) with the same thickness and separation as the other two. Fig. 8 presents the plot of magnetic flux in the center limb, winding window, and side limb during a short-circuit (excitation is on winding *B* with windings *A* and *C* paralleled and short-circuited). It is apparent from Fig. 8 that the regular pi model of [6] has phase-shifted flux in the transformer core/window and the model of Fig. 7 can accurately represent the flux and terminal behavior. Terminal results are not shown here as they match perfectly.

Similar results for the case when excitation is placed on parallel combination of windings *A* and *C* with winding *B* short-circuited. Results are shown in Fig. 9.

VII. Conclusion

FEM simulations and experimental results demonstrate that many existing duality models do not consider the flux distribution correctly. The model presented in this letter provides the correct duality and terminal behavior. References [7] and [8] were been published before we re-discovered the inaccuracy of some duality-derived models. However, a proper explanation of the problem and where the inaccuracies show are given only in this paper.

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Digvijay Deswal (S’16) completed his B. Tech. and M. Tech. degrees in Electrical Engineering at the Indian Institute of Technology (IIT) Kharagpur, West Bengal, India in 2014. He obtained his PhD in Electrical Engineering at the New York University School of Engineering in 2019. He is currently working as an Electromagnetic Engineer for TechnipFMC.

His research interests include electromagnetic design and modeling, power electronics, and simulation and analysis of electrical machines.

Francisco de León (S’86–M’92–SM’02–F’15) received the B.Sc. and the M.Sc. (Hons.) degrees in electrical engineering from the National Polytechnic Institute, Mexico City, Mexico, in 1983 and 1986, respectively, and the Ph.D. degree in electrical engineering from the University of Toronto, Toronto, ON, Canada, in 1992.

He has held several academic positions in Mexico and has worked for the Canadian electric industry. Currently, he is a Professor with the Department of Electrical and Computer Engineering at New York University. His research interests include the analysis of power phenomena under non-sinusoidal conditions, the transient and steady state analyses of power systems, the thermal rating of cables and transformers, and the calculation of electromagnetic fields applied to machine design and modeling.

Prof. de León is the Editor-in-Chief of the IEEE TRANSACTIONS ON POWER DELIVERY and reviews papers for a large number of journals.