

Lecture#

4.3 Interleaved Boost Converters

4.3.1 Single-channel Boost Converter

- Numericals:

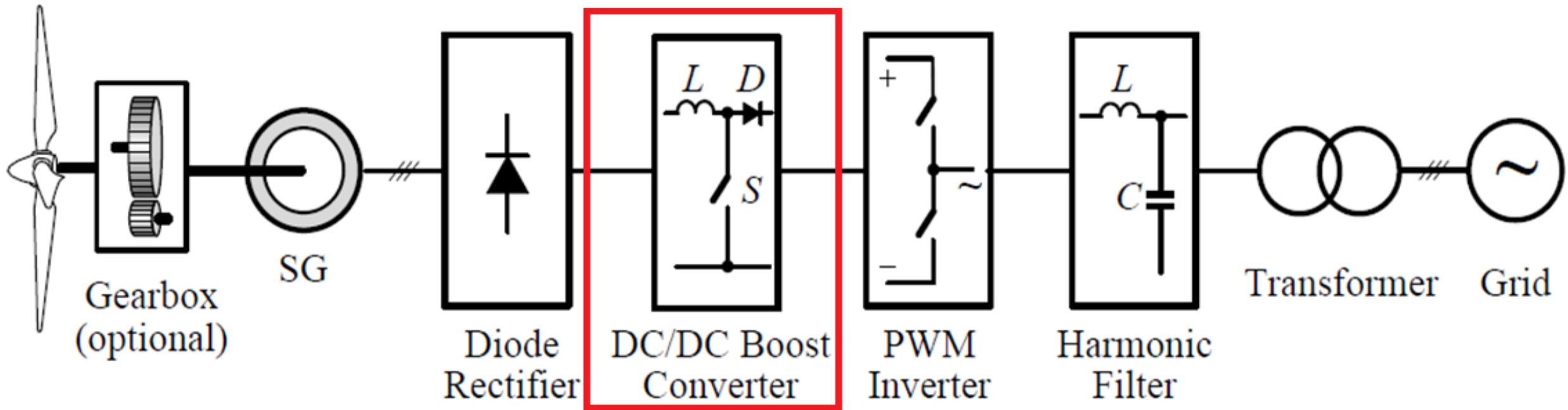
4.3 Interleaved Boost Converters

- What is the meaning of **Interleaved**?

Interleaved means inserted or sandwiched

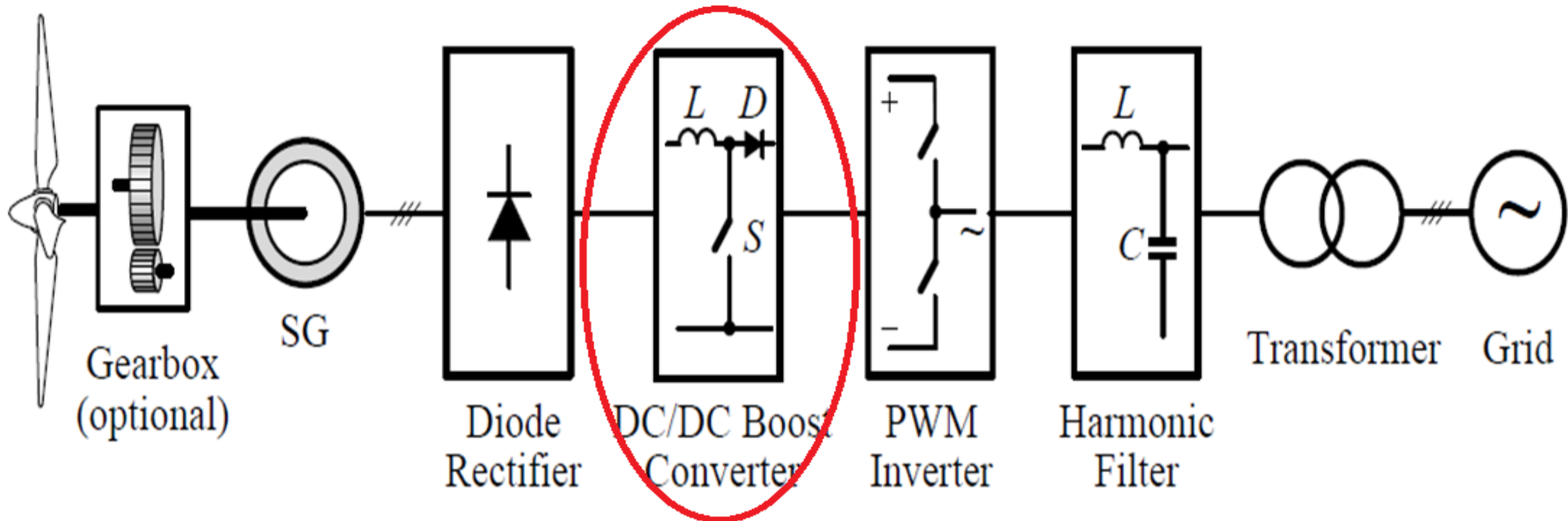
Interleaved Boost Converters

- DC/DC boost converter is often used in synchronous generator (SG) based WECS.



(c) Variable-speed WECS with DC/DC boost converter

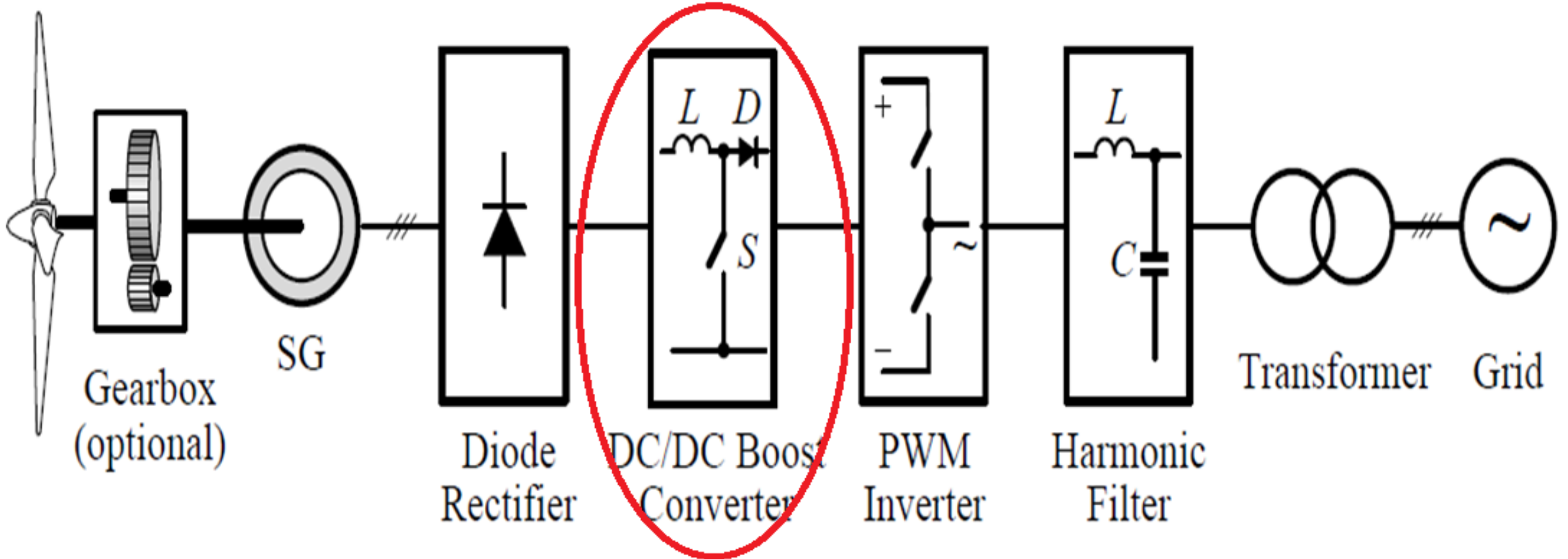
Converter is placed between diode rectifier & inverter of power conversion system.



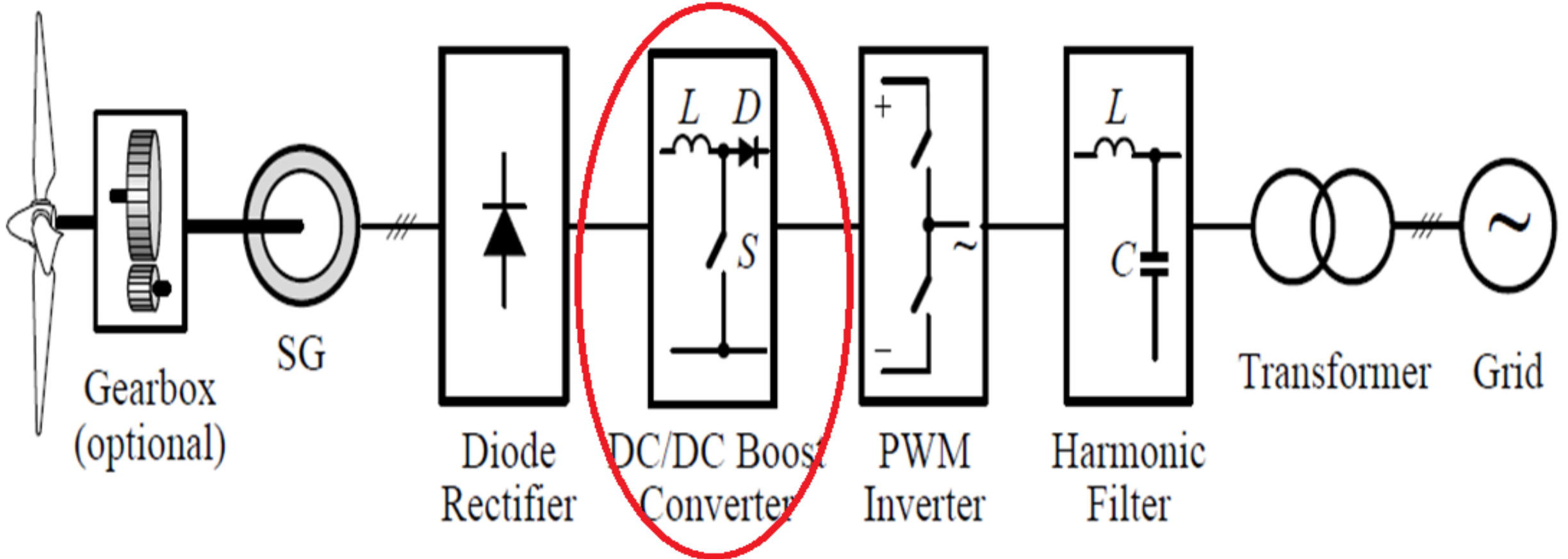
(c) Variable-speed WECS with DC/DC boost converter

What are the 02 main functions of boost converter?

1. Tracking maximum power from wind & boosting dc voltage to an appropriate value for inverter.

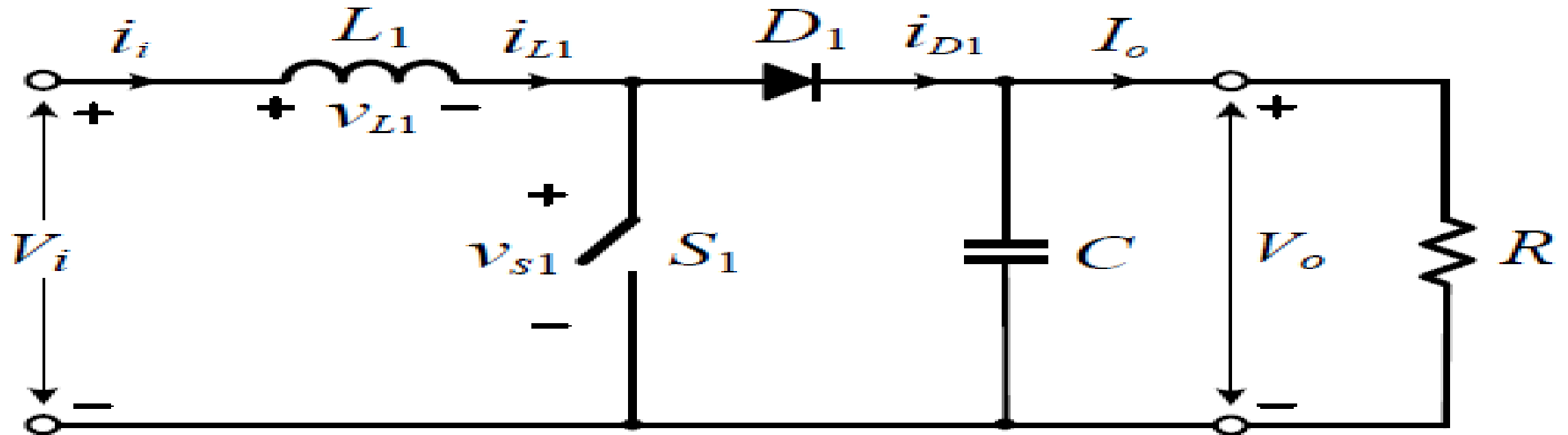


2. Capture of maximum power from wind at all wind speeds.



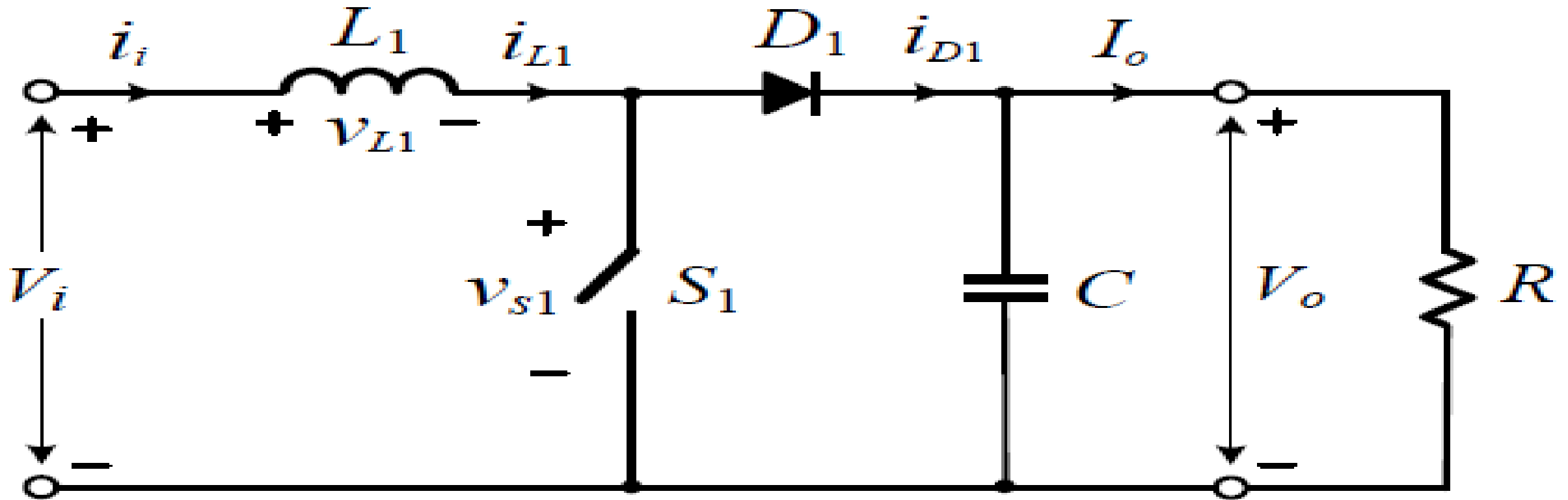
A single-channel boost converter is often used for:

- Low & medium-power wind energy systems of a few kW to hundred kW.



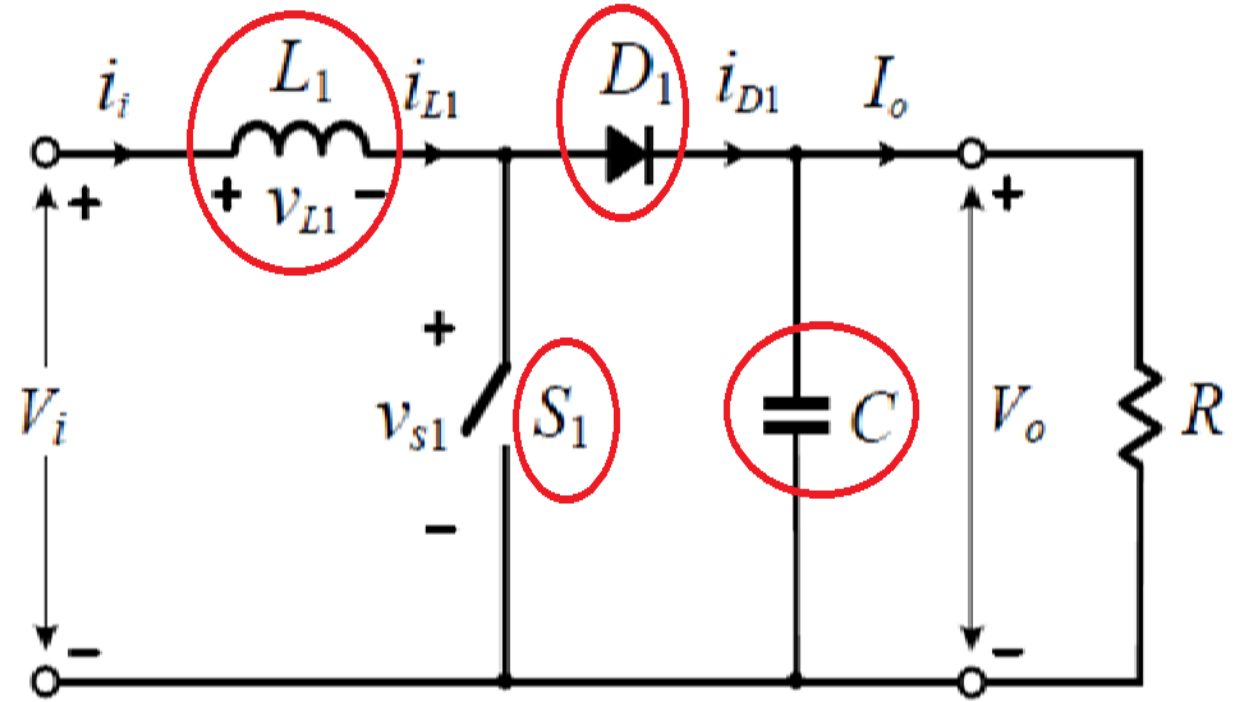
4.3.1 Single-channel Boost Converter

- It is a power converter with an output dc voltage greater than its input dc voltage. i.e $V_o > V_i$

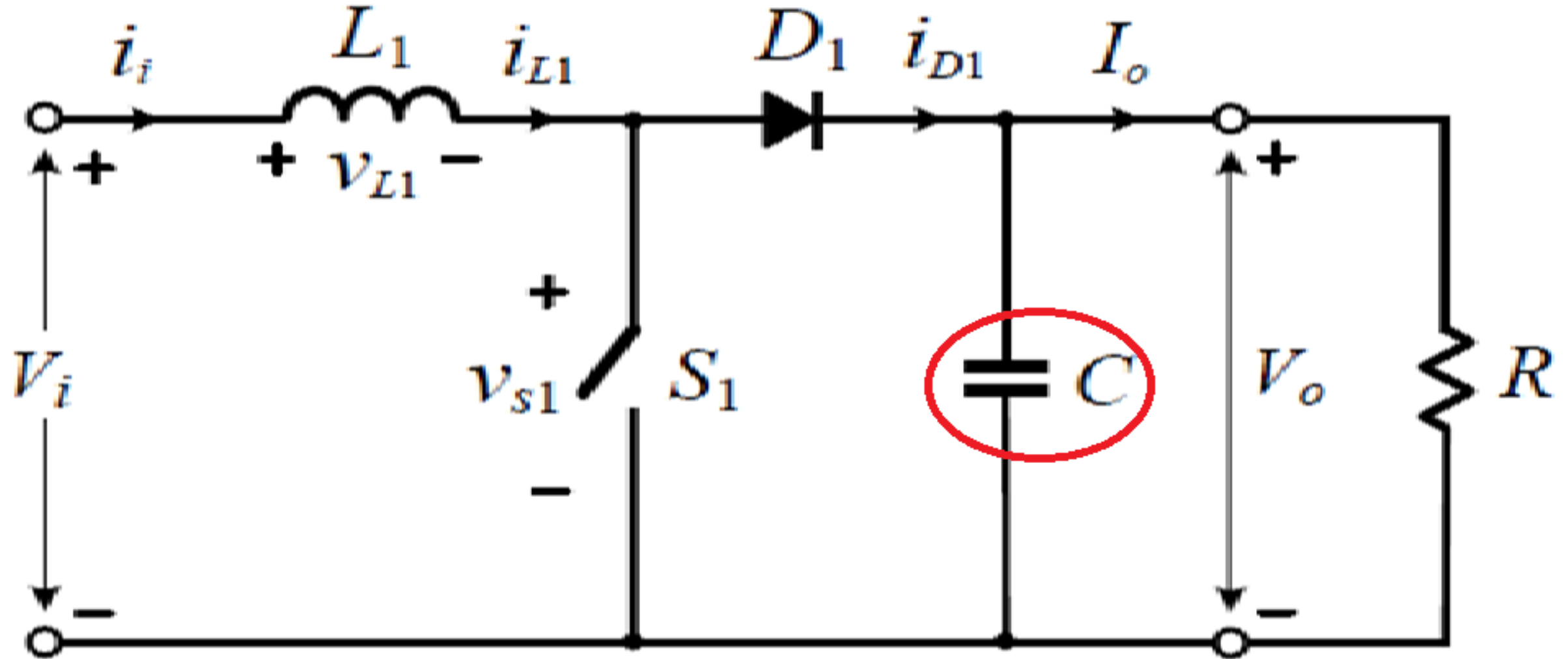


Composition

1. Switch S_1
2. Diode D_1
3. Dc inductor L_1
4. Filter capacitor C

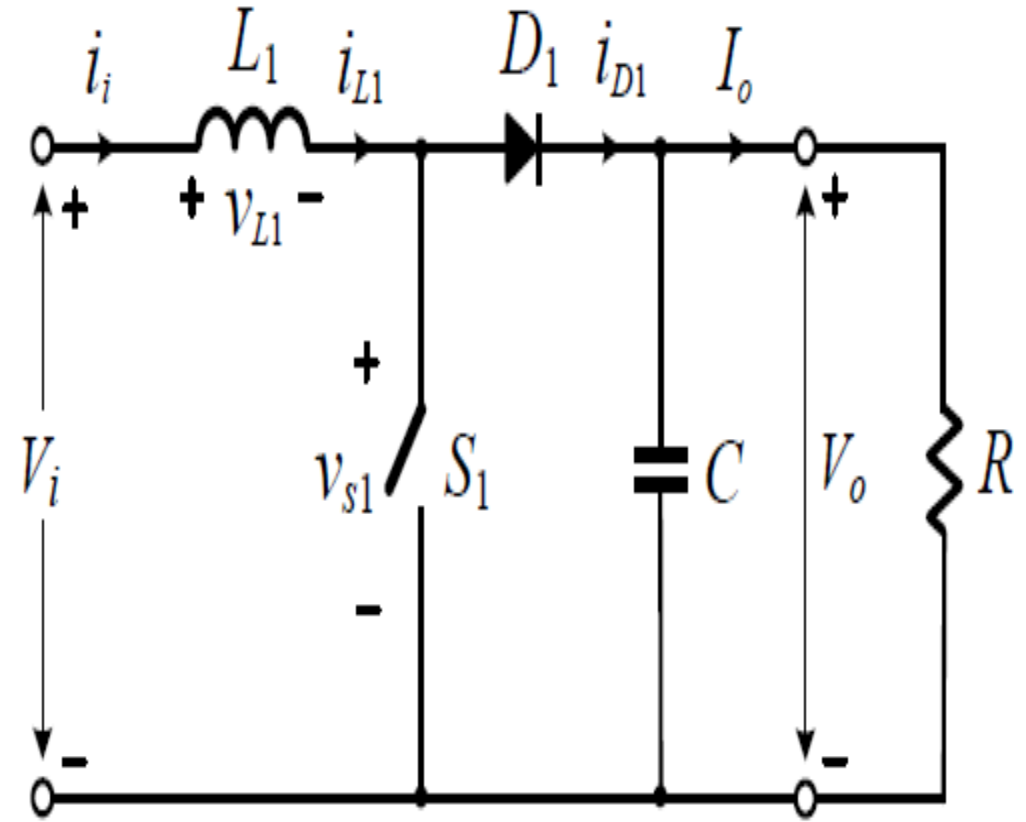


Q. What happened if I do not use C in circuit?



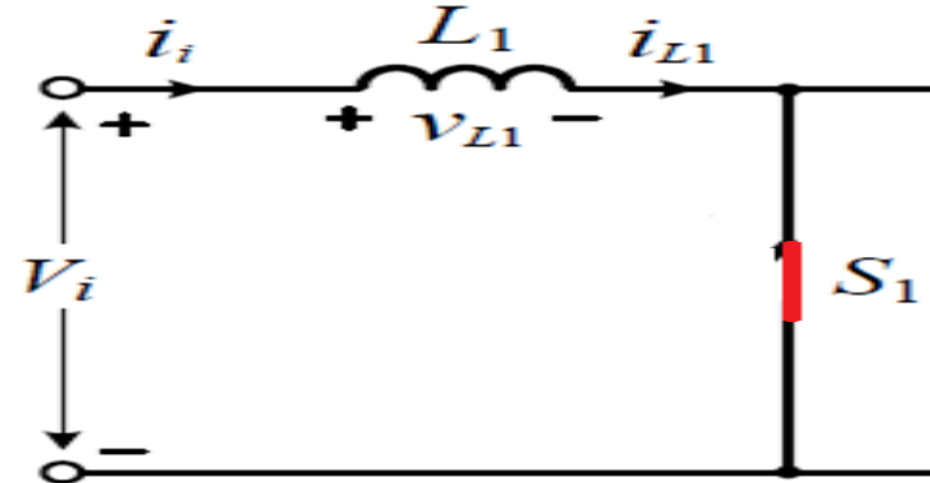
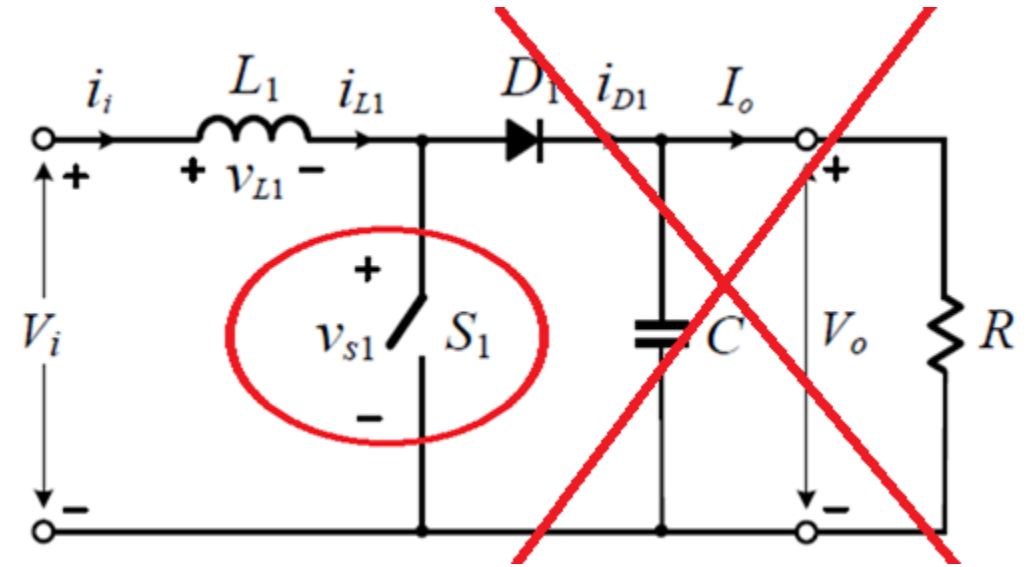
Assumptions

- i) All components in converter are ideal (no power or voltage losses)
- ii) Output filter capacitor C is very large hence output voltage of converter is ripple free.



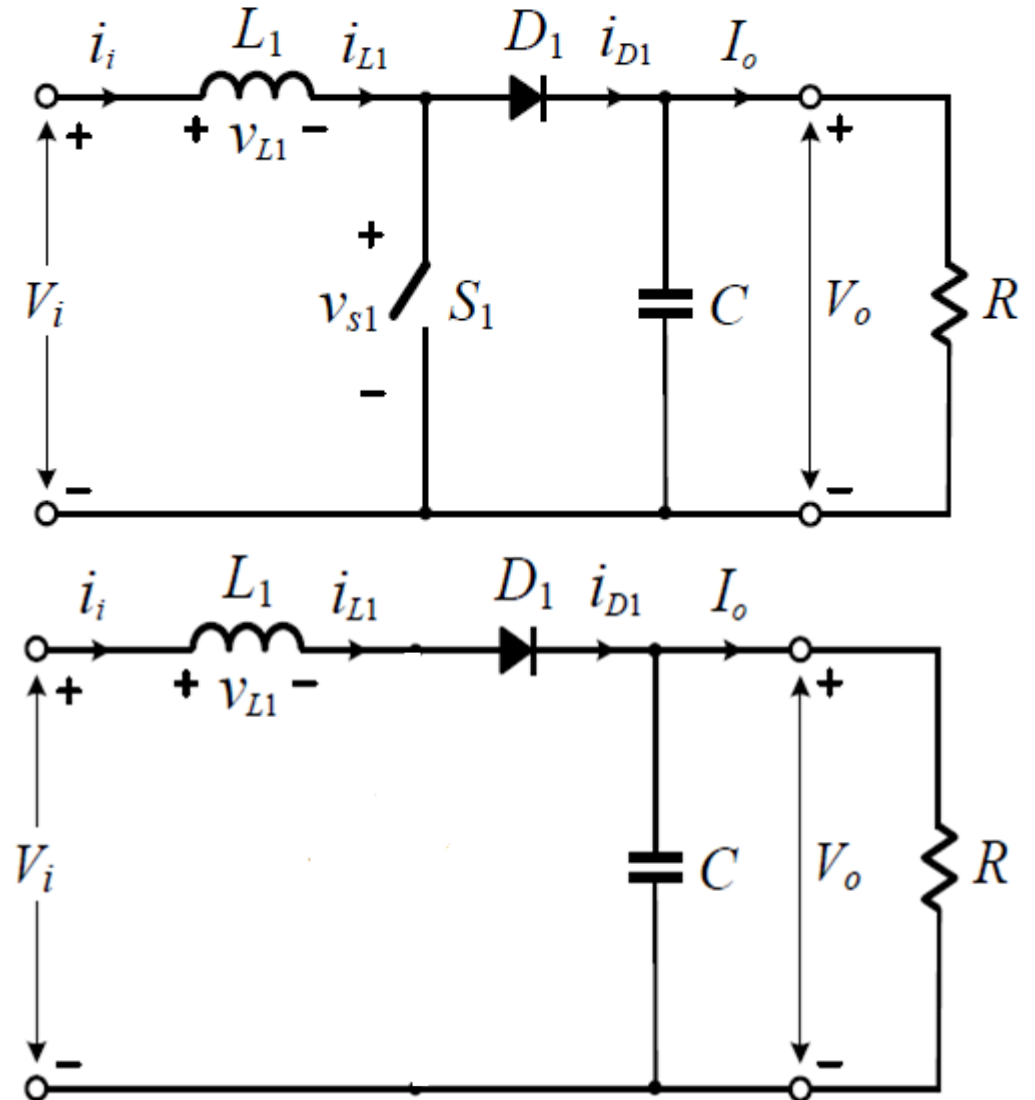
Case-I: When switch S_1 is turned ON

- Diode D_1 is reverse biased.
- Output (V_o) is isolated from input (V_i).
- Input (V_i) supplies energy to inductor L_1 .

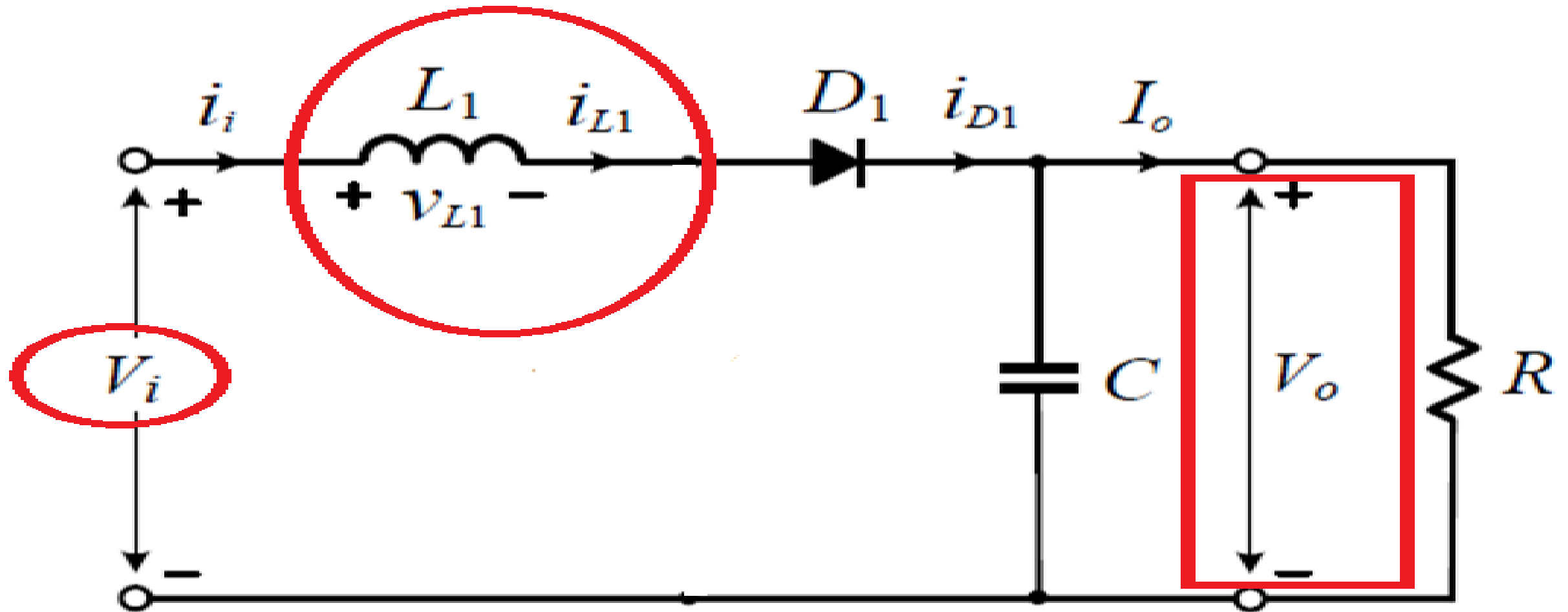


Case-II: When the switch is turned off

- Diode D_1 is forward biased
- Energy stored in L_1 is released to load(R) through diode.

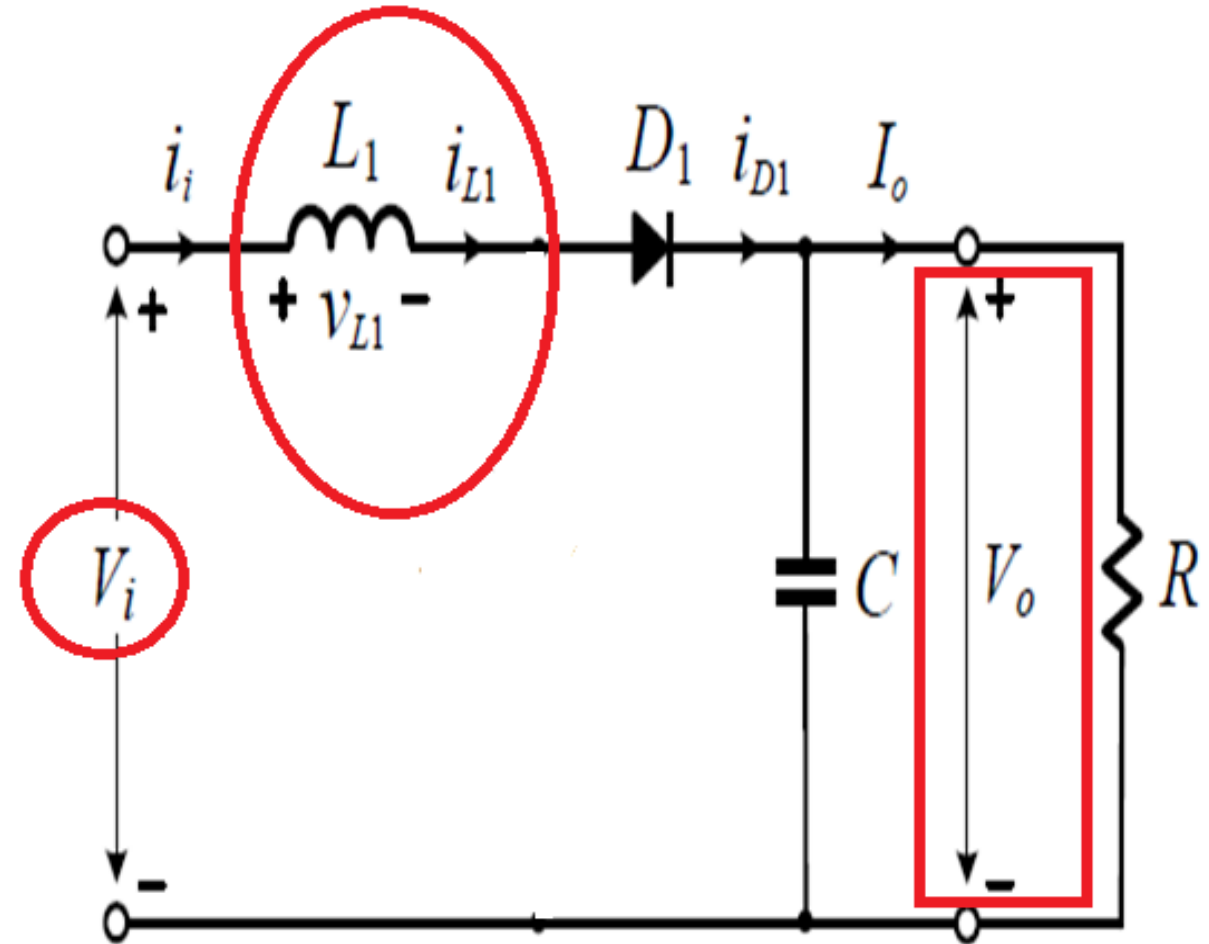


How converter output voltage V_o > than its input voltage V_i ?



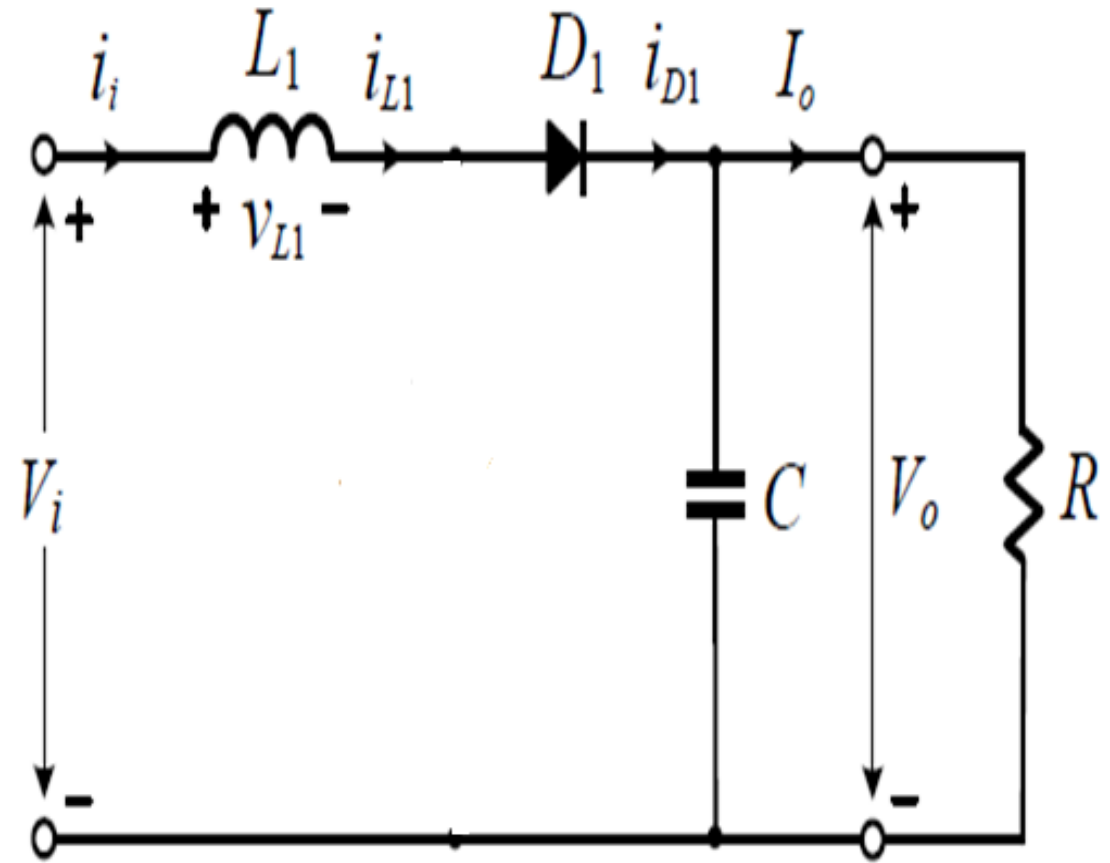
How converter output voltage V_o
>than its input voltage V_i ?

- Output voltage V_o is sum of input voltage V_i & inductor voltage v_{L1} i.e
- $V_o = V_i + V_{L1}$



Continuous current mode (CCM) & discontinuous current mode (DCM).

- Depending on continuity of dc inductor current i_{L1}
- Operation of converter can be divided to 2 operating modes:
 1. Continuous current mode (CCM)
 2. Discontinuous current mode (DCM).



Continuous current mode (CCM)

- When a boost converter operates in CCM, inductor current i_{L1} never falls to 0.

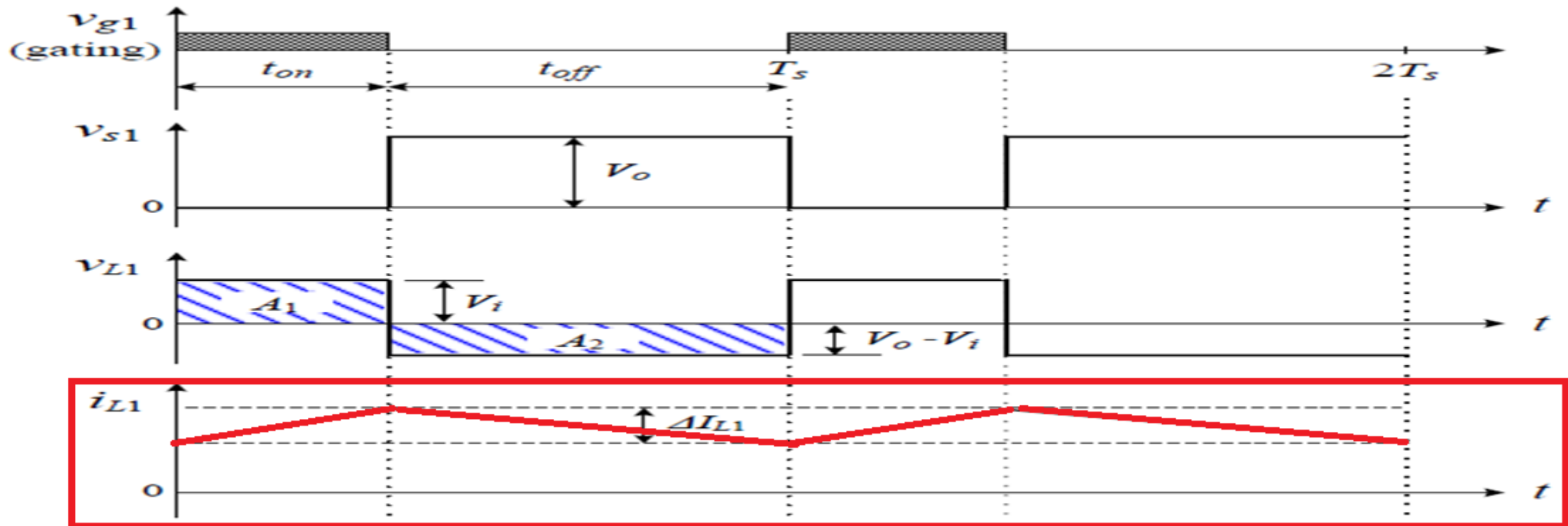
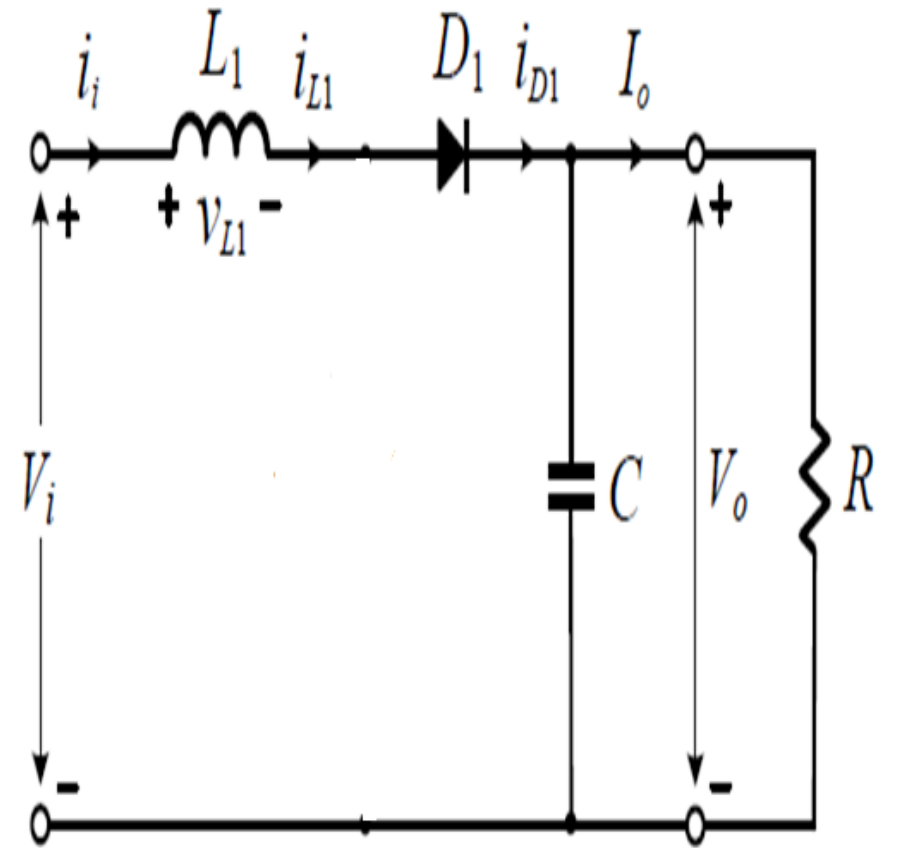
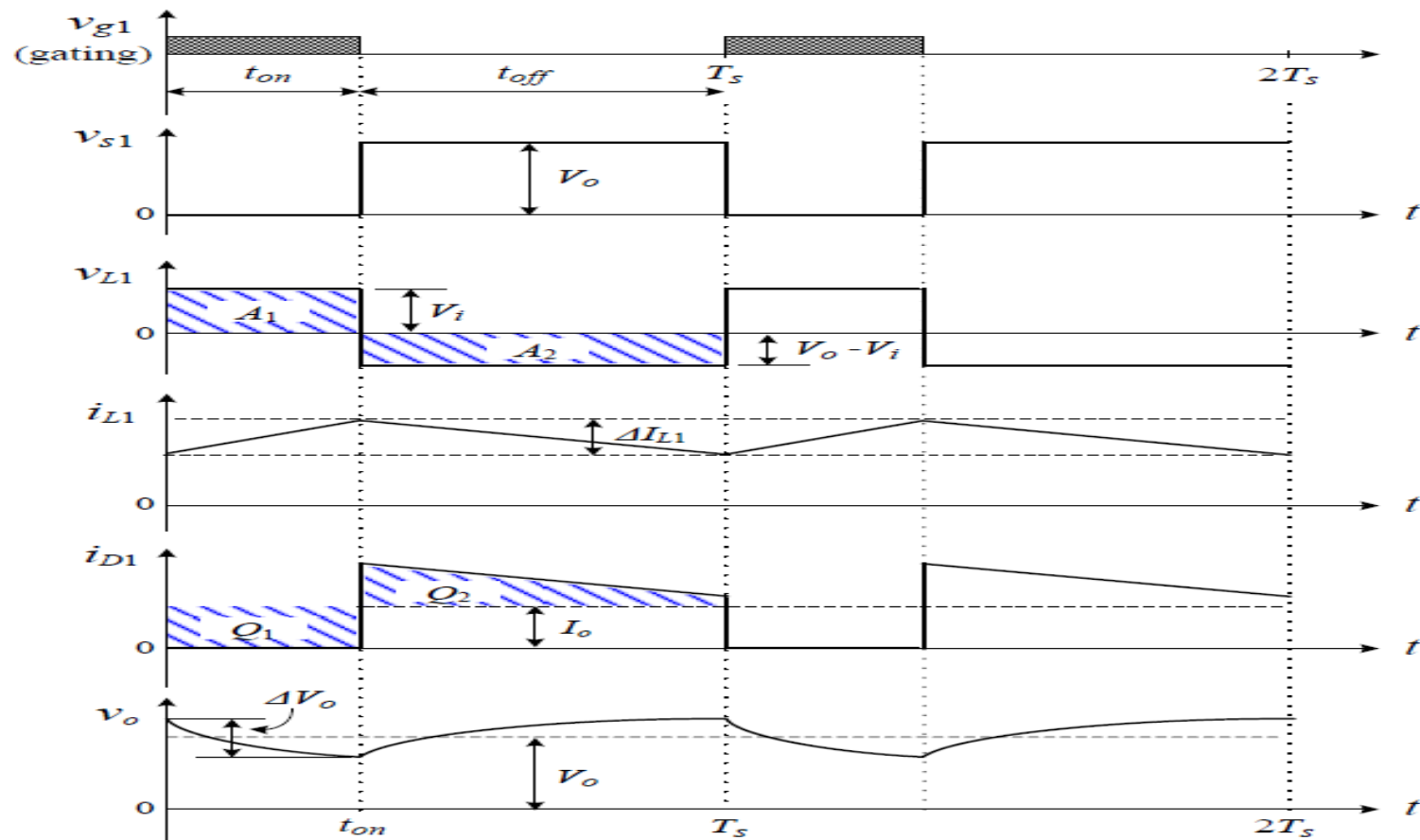
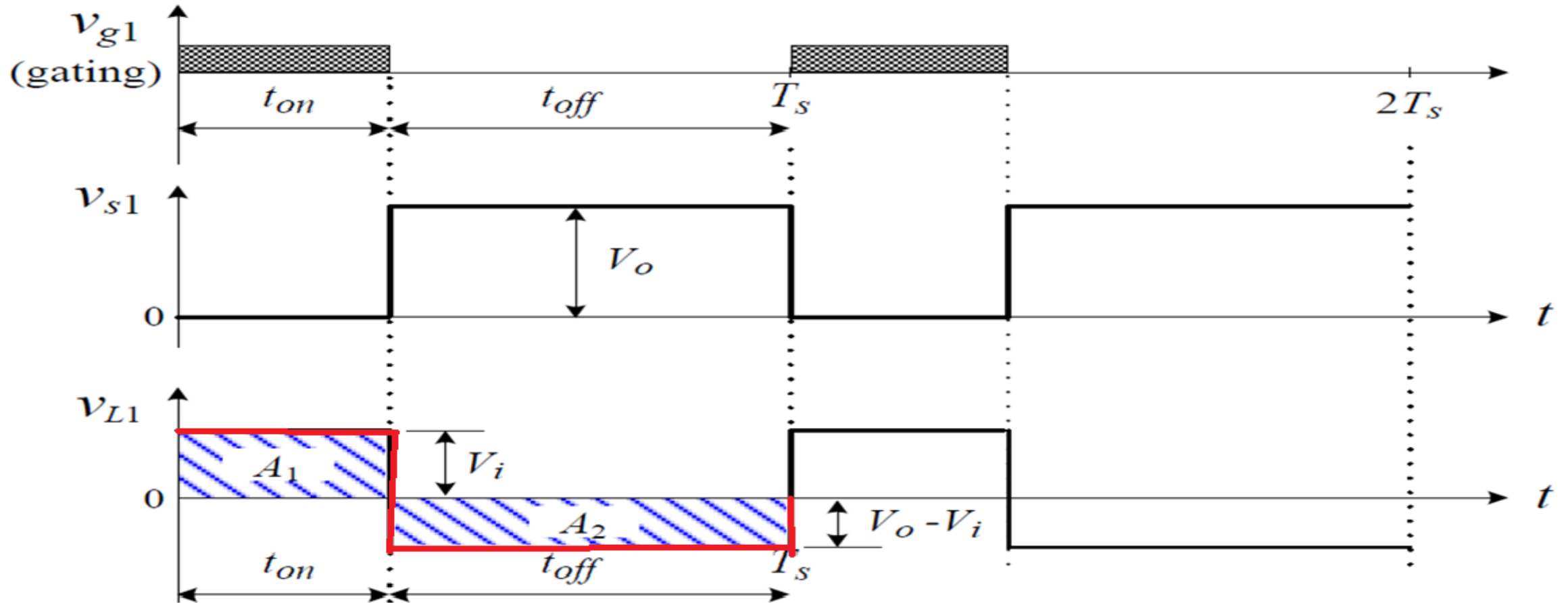


Fig. shows the typical waveforms of currents & voltages in boost converter operating in CCM mode.

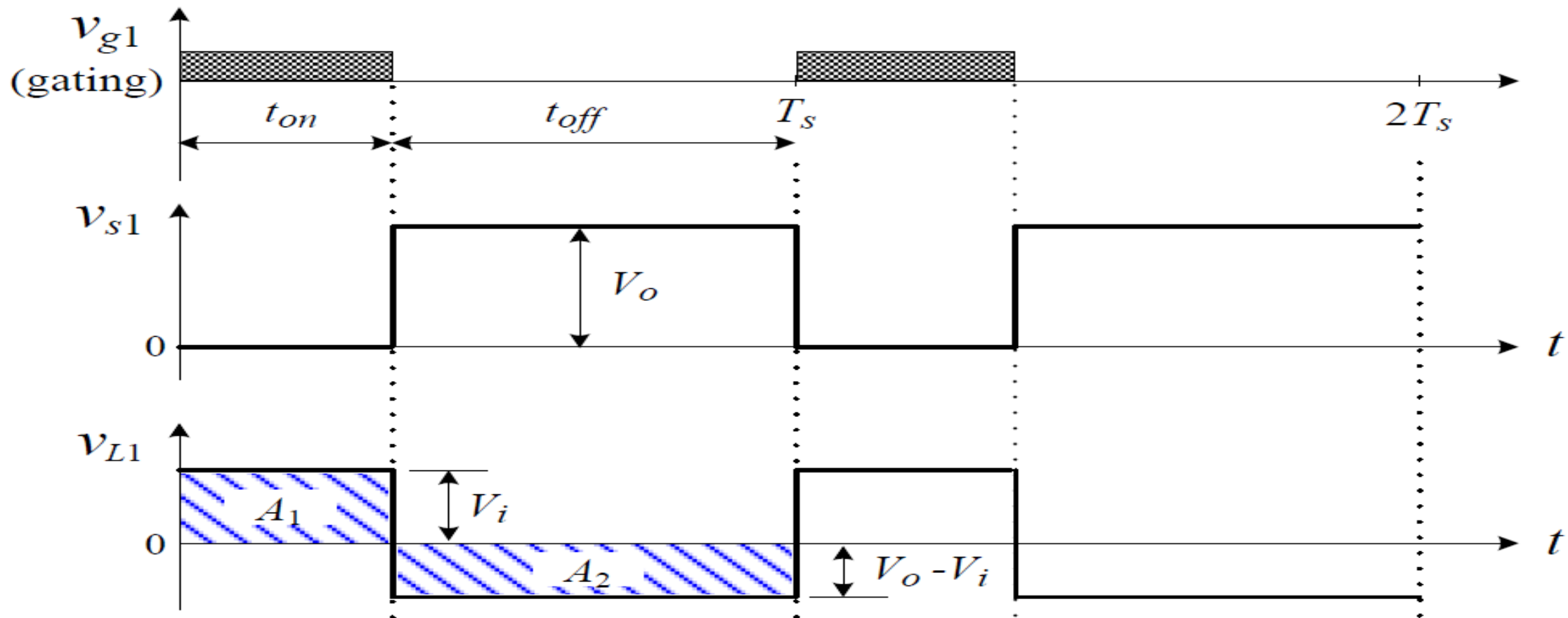


Steady state operation of converter

Average voltage across inductor $L1$ over $T_s = ?$

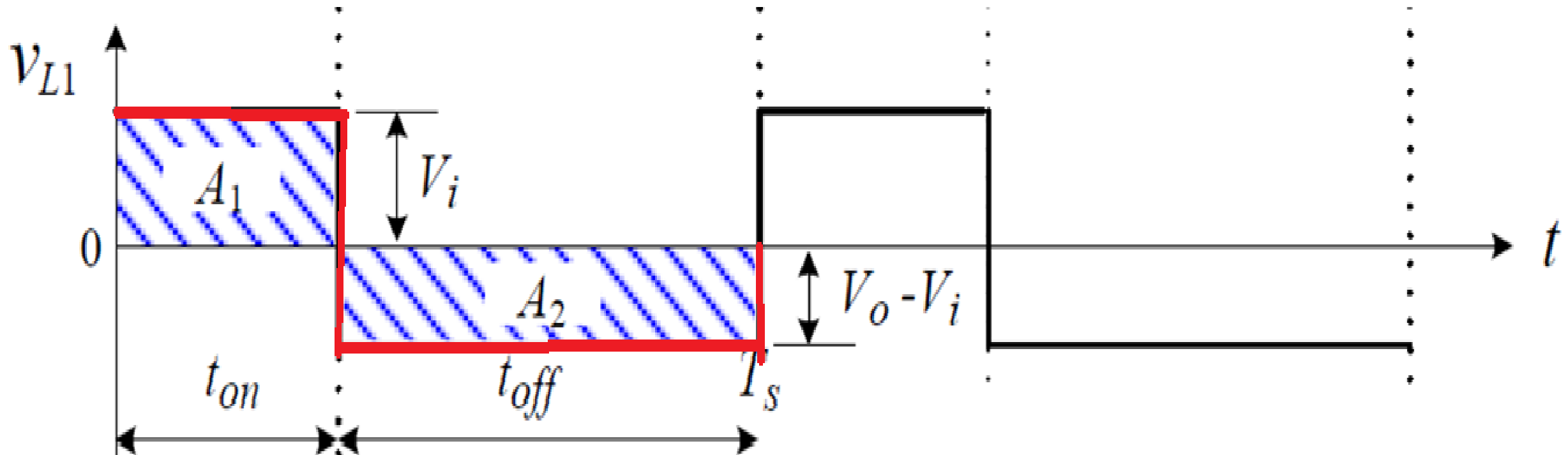


Integral of inductor voltage v_{L1} over time period T_s must be 0. This implies that average voltage across inductor $L1$ over $T_s=0$.



Graphical interpretation says that area $A_1 = A_2$, *i.e.*,

$$V_i t_{on} = (V_o - V_i) t_{off}$$

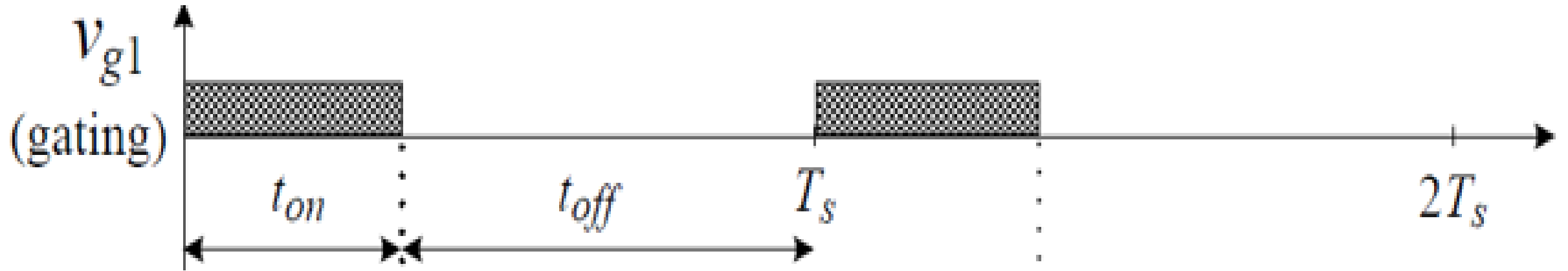


Please obtain $V_o/V_i = ?$

$$V_i t_{on} = (V_o - V_i) t_{off}$$

$$V_i t_{on} + V_i t_{off} = V_o t_{off}$$

T_s is switching period t_{on} & t_{off} are turn-on & turn-off times of switch S

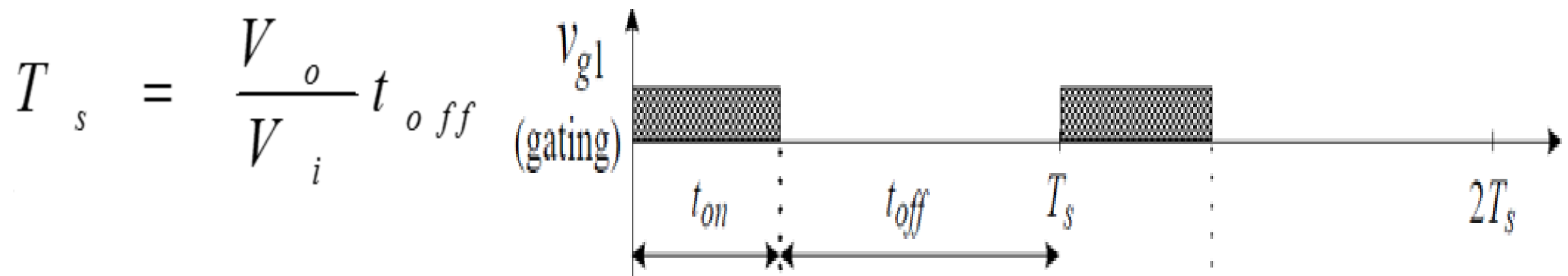


$$V_i t_{on} + V_i t_{off} = V_o t_{off}$$

$$t_{on} + t_{off} = \frac{V_o}{V_i} t_{off}$$

$$T_s = t_{on} + t_{off}$$

$$T_s = \frac{V_o}{V_i} t_{off}$$



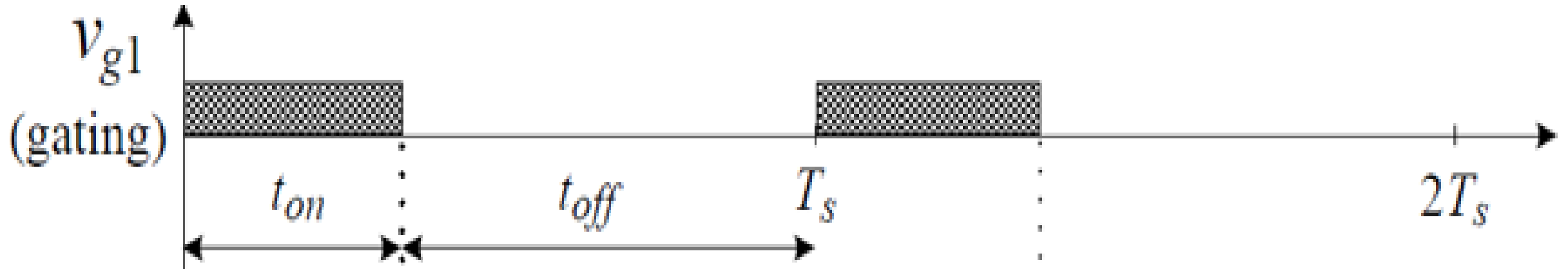
$$\frac{T_s}{t_{off}} = \frac{V_o}{V_i}$$

$$\frac{V_o}{V_i} = \frac{T_s}{T_s - t_{on}}$$

$$\frac{V_o}{V_i} = \frac{1}{1 - \frac{t_{on}}{T_s}}$$

Define *Duty* cycle of converter= D ?

Duty cycle of converter=D



$$\frac{V_o}{V_i} = \frac{1}{1 - \frac{t_{on}}{T_s}}$$

$$\frac{V_o}{V_i} = \frac{1}{1 - D}$$

$$D = \frac{t_{on}}{T_s}$$

What information we can extract from this expression?

$$\frac{V_o}{V_i} = \frac{1}{1 - D}$$

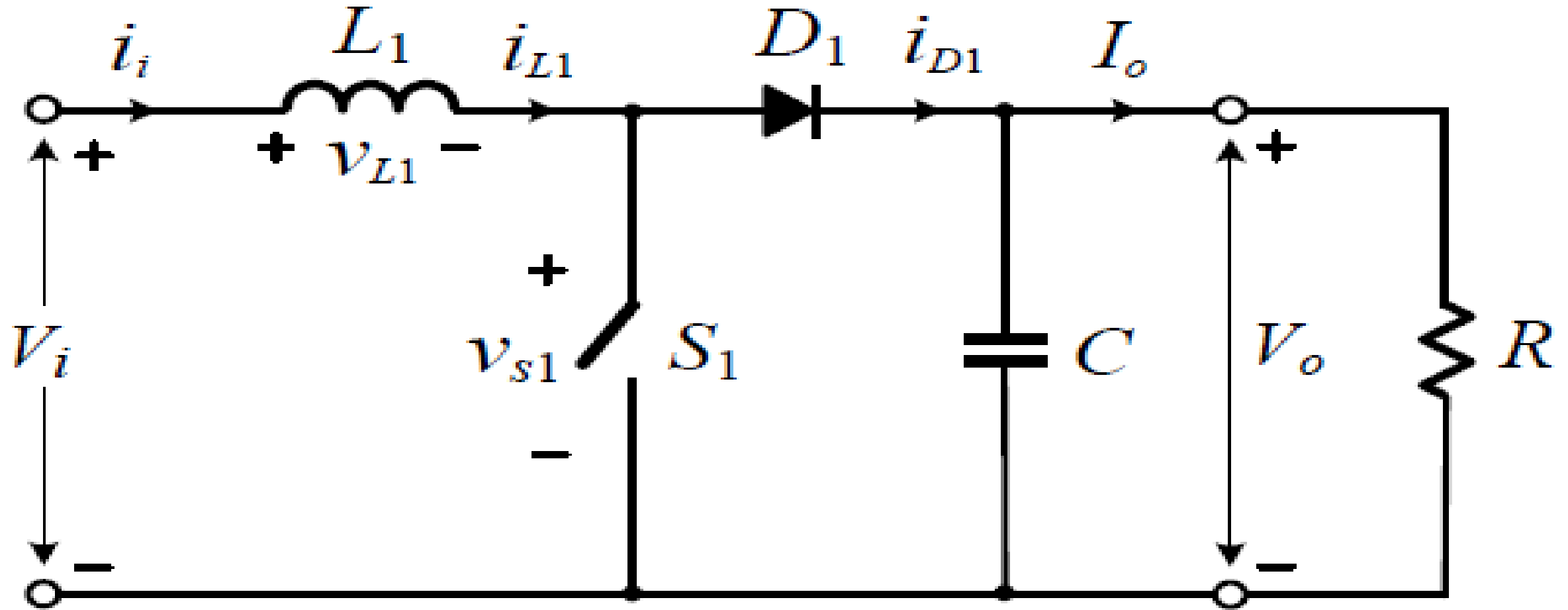
If we put $D=1/2$ then $V_o=2V_i$

$$\frac{V_o}{V_i} = \frac{1}{1-D}$$

Expression indicates that output voltage of converter V_o is always $>$ its input voltage V_i .

$$\frac{V_o}{V_i} = \frac{1}{1-D}$$

Which expression should be use to derive relationship between converter input current I_i & output current I_o ?

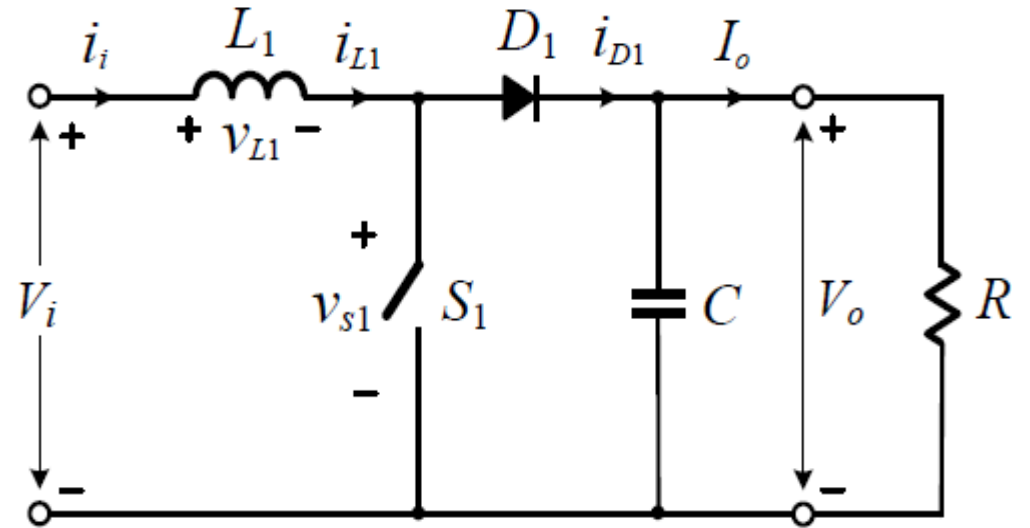


Relationship between converter input current I_i & output current I_o can be derived from: **input power=Output power**

$$V_i I_i = V_o I_o$$

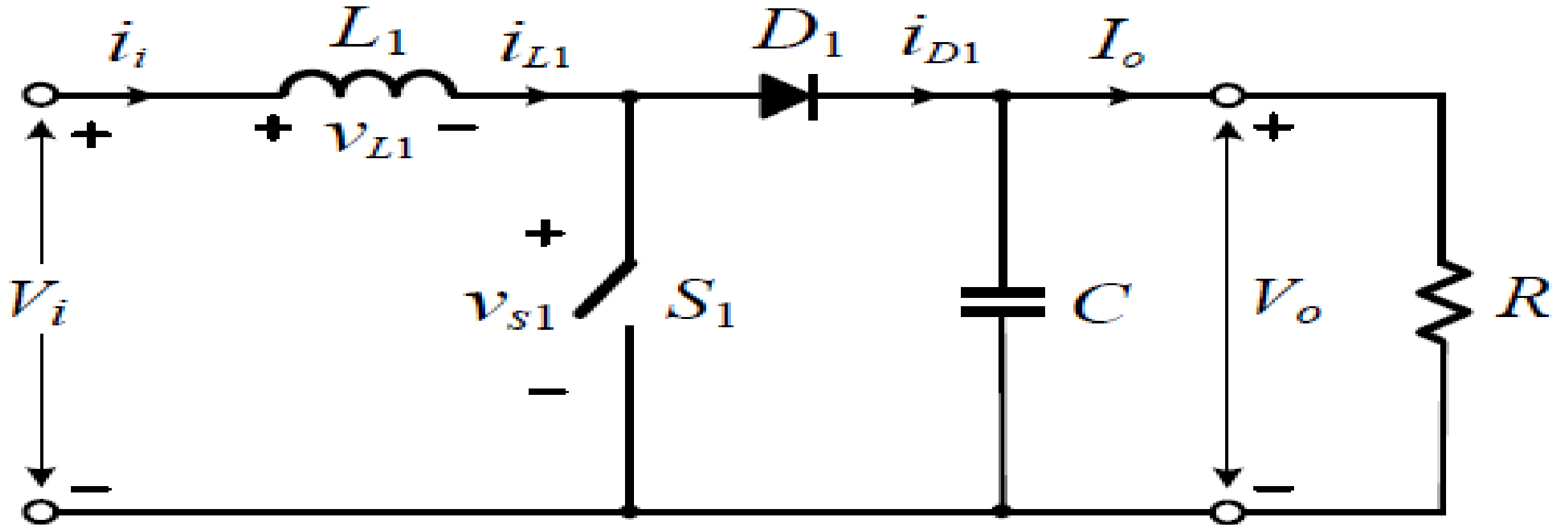
$$\frac{I_o}{I_i} = \frac{V_i}{V_o}$$

$$\frac{V_o}{V_i} = \frac{1}{1-D}$$

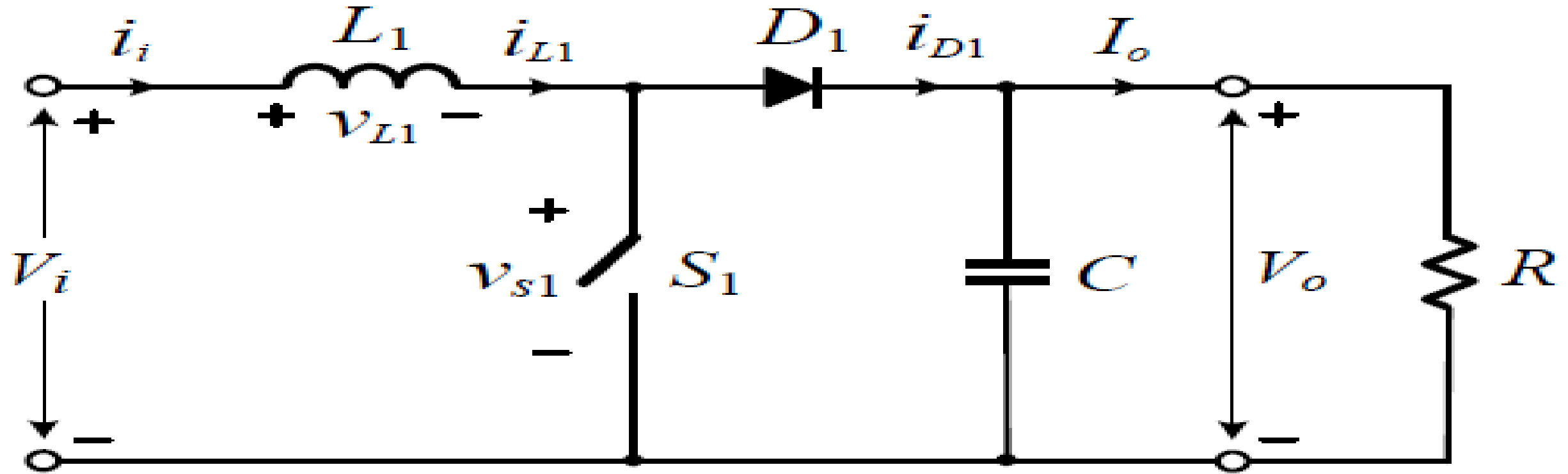


$$\frac{I_o}{I_i} = 1-D \quad \text{for } 0 \leq D < 1$$

Write an expression to calculate ripple current in inductor?



An expression to calculate ripple current in inductor



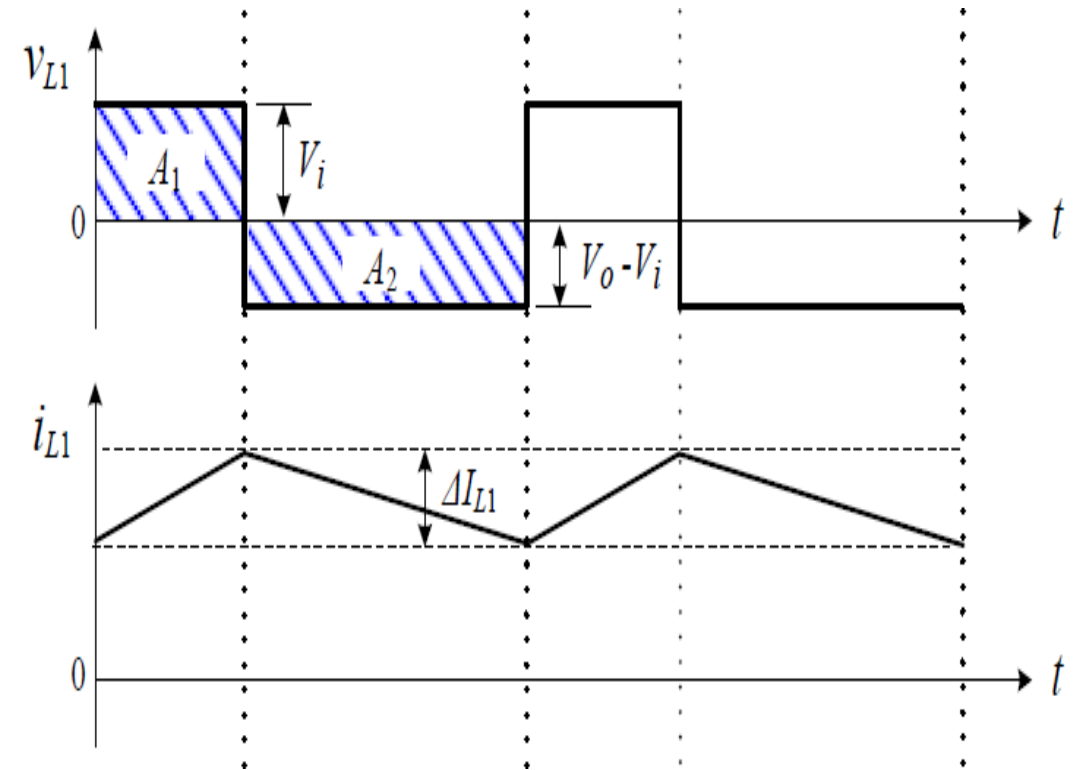
$$v_{L1} = L_1 (di_{L1} / dt)$$

Since inductor current i_{L1} changes linearly with time

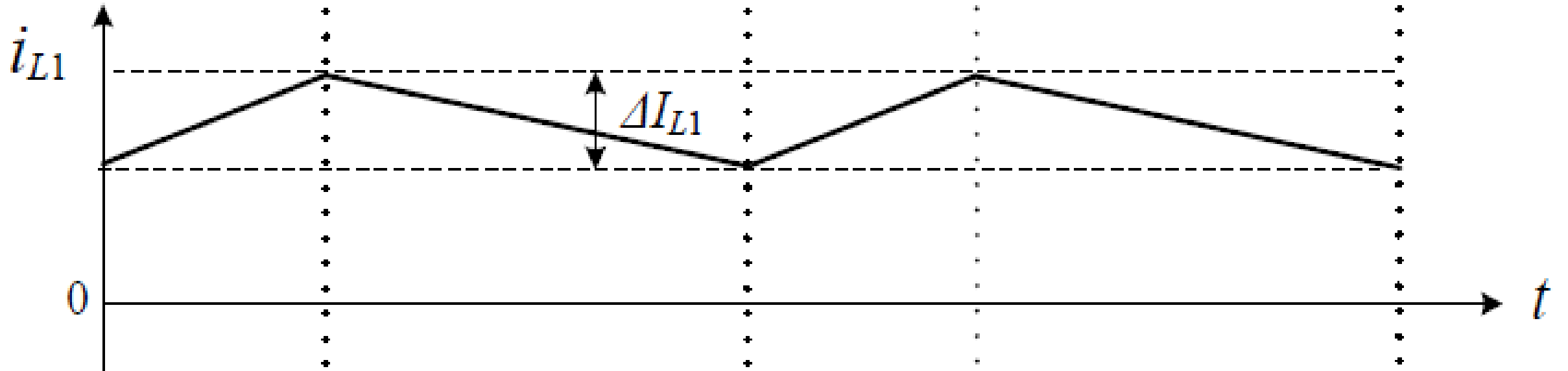
$$v_{L1} = L_1(di_{L1} / dt)$$

So above equation can be replaced by:

$$\Delta v_{L1} = L_1(\Delta i_{L1} / \Delta t)$$



Obtain inductor current from $\Delta v_{L1} = L_1(\Delta i_{L1} / \Delta t)$



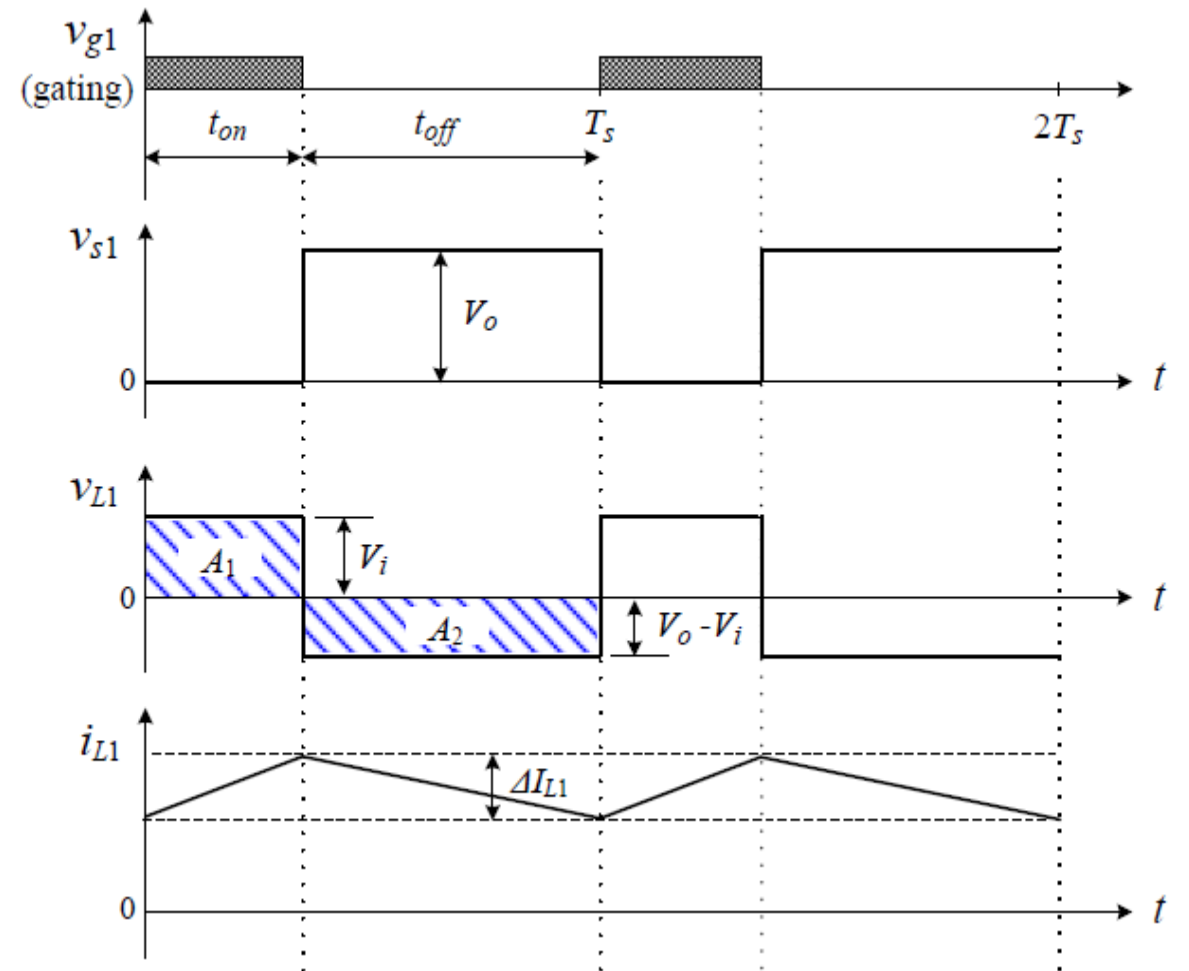
$$\Delta i_{L1} = \frac{\Delta v_{L1}}{L_1} \Delta t$$

$$\Delta i_{L1} = \frac{(V_o - V_i)}{L_1} t_{off}$$

In order to obtain inductor ripple current Δi_{L1}

- We need to replace $(V_o - V_i)$ & t_{off}

$$\Delta i_{L1} = \frac{\Delta v_{L1}}{L_1} \Delta t = \frac{(V_o - V_i)}{L_1} t_{off}$$



Please drive the equation?

$$\Delta i_{L1} = D(1-D) \frac{V_o T_s}{L_1}$$

From

$$\Delta i_{L1} = \frac{\Delta v_{L1}}{L_1} \Delta t = \frac{(V_o - V_i)}{L_1} t_{off}$$

From
$$\frac{V_o}{V_i} = \frac{1}{1 - D}$$

$$DV_o = (V_o - V_i)$$

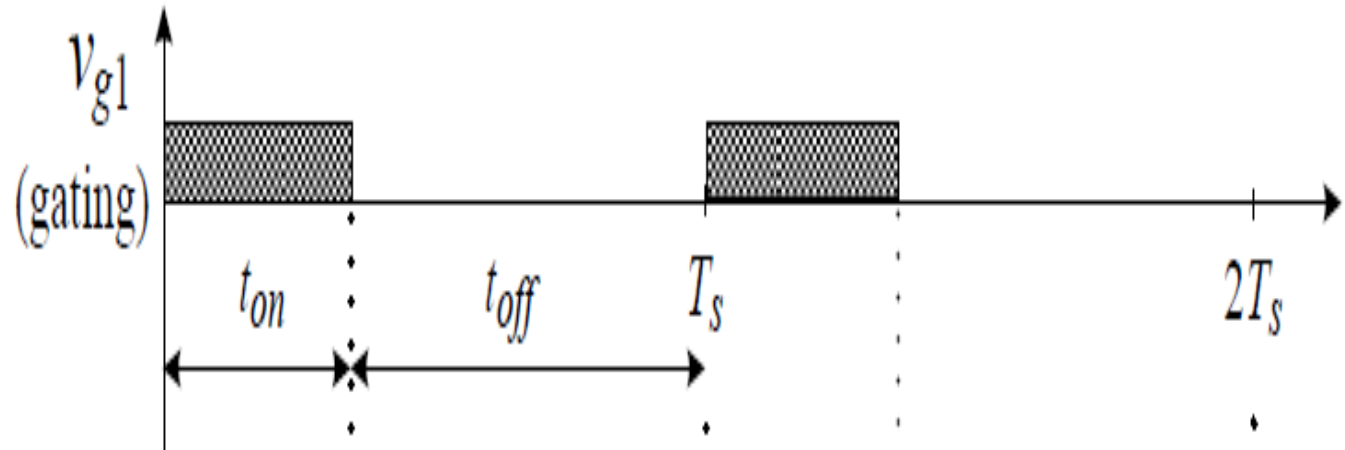
$$t_{off} = (T_s - t_{on})$$

$$D = \frac{t_{on}}{T_s}$$

$$t_{on} = D T_s$$

$$t_{off} = (T_s - D T_s)$$

$$t_{off} = (1 - D) T_s$$



After substitution: $DV_o = (V_o - V_i)$ &

$$t_{off} = (1 - D)T_s$$

In

$$\Delta i_{L1} = \frac{(V_o - V_i)}{L_1} t_{off}$$

$$\Delta i_{L1} = D(1 - D) \frac{V_o T_s}{L_1}$$

Calculate Maximum current ripple?

$$\Delta i_{L_1} = D(1 - D) \frac{V_o T_s}{L_1}$$

For Maximum current ripple take derivative on both sides & then equate to 0

$$\Delta i_{L1} = D(1-D) \frac{V_o T_s}{L_1}$$

$$0 = K(1-2D)$$

$$D = 1/2$$

Maximum current ripple $\Delta I_{L1,\max}$

- For single-channel boost converter when duty cycle $D=0.5$ then we can obtain maximum current ripple:

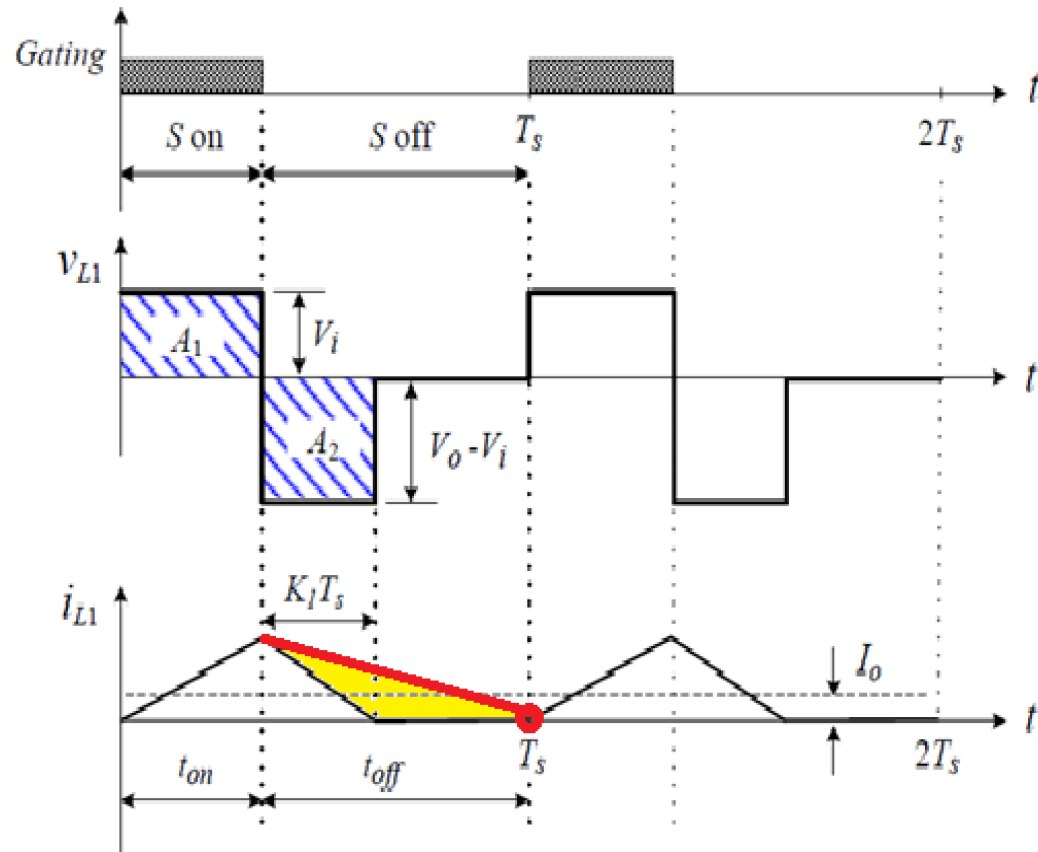
$$\Delta i_{L1} = D(1-D) \frac{V_o T_s}{L_1}$$

$$\Delta I_{L1,\max} = \frac{V_o T_s}{4L_1}$$

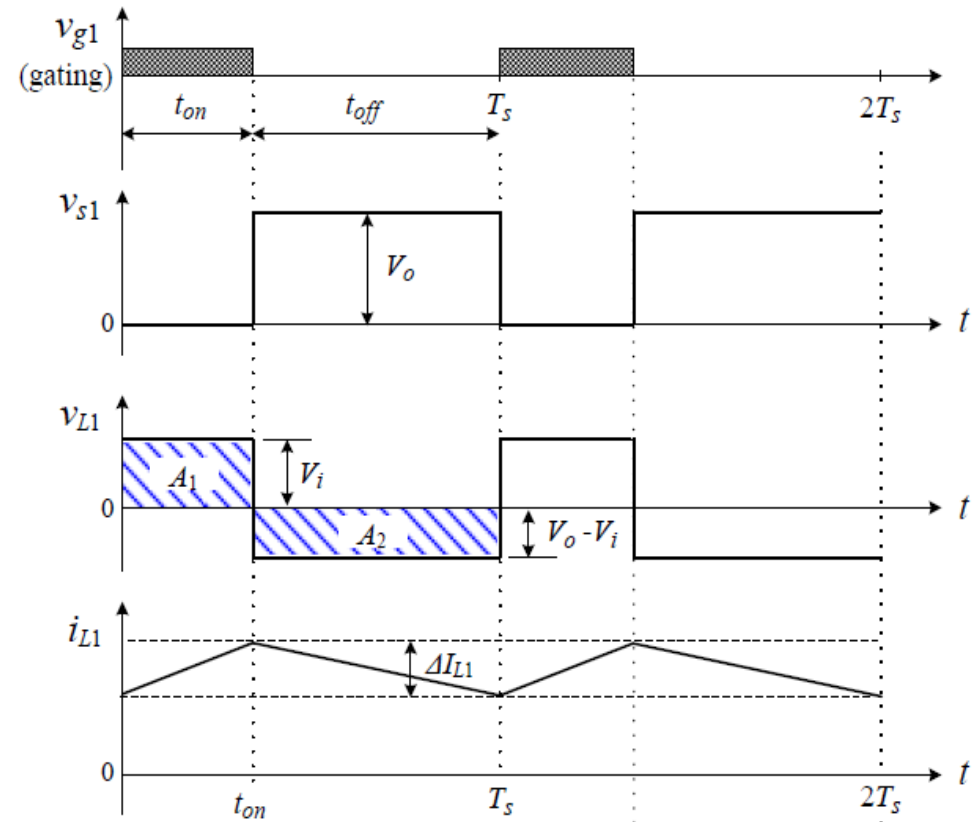
Discontinuous current mode(DCM)

What is difference between these waveforms?

Discontinuous current mode(DCM)

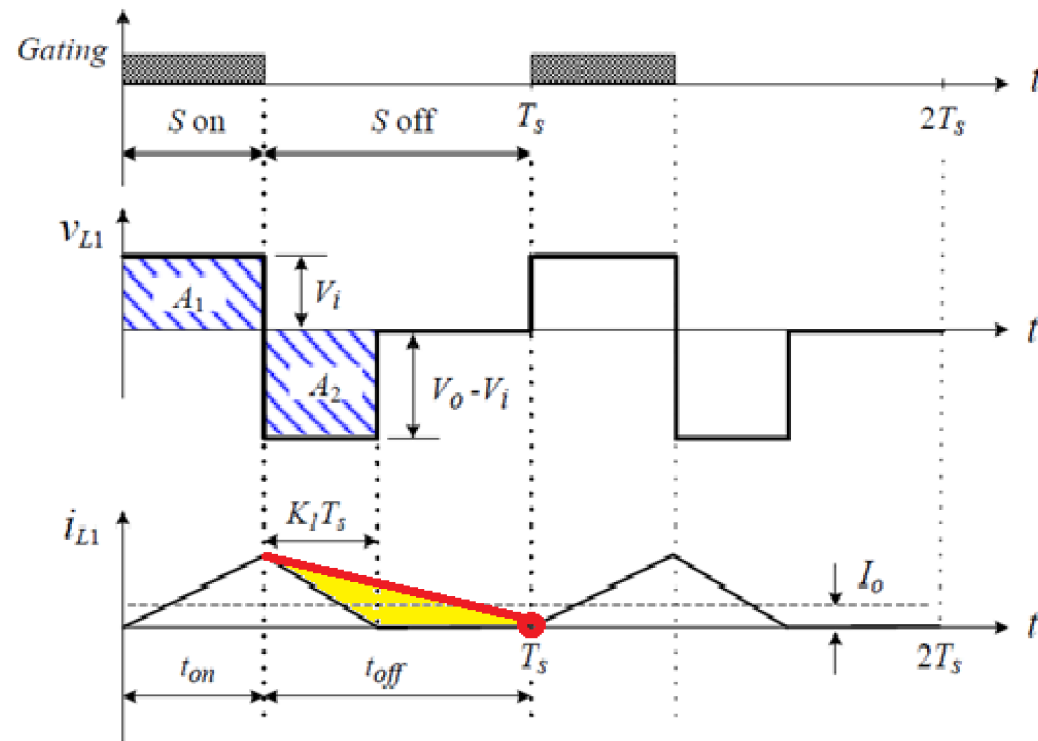


Continuous current mode(CM)

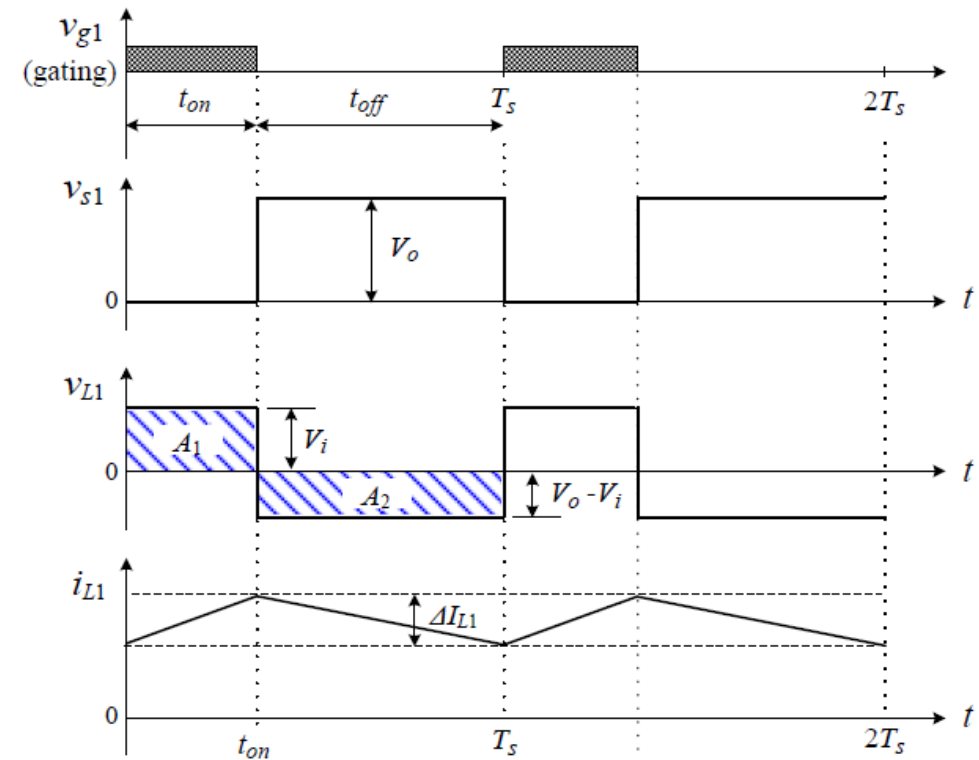


Inductor current i_{L1} reaches 0 before end of t_{off} period & therefore becomes discontinuous.

Discontinuous current mode(DCM)



Continuous current mode(CM)



Why Inductor current i_{L1} reaches 0 before end of toff period?

Answer:

- When converter operated under light load then load current is low or we can say that current in inductor $L1$ (I_{L1}) is low.
- Energy stored in inductor during t_{on} period may not be sufficient to maintain its current during t_{off} period.

Consequently, inductor current i_{L1} reaches 0 before end of t_{off} period & therefore becomes discontinuous.

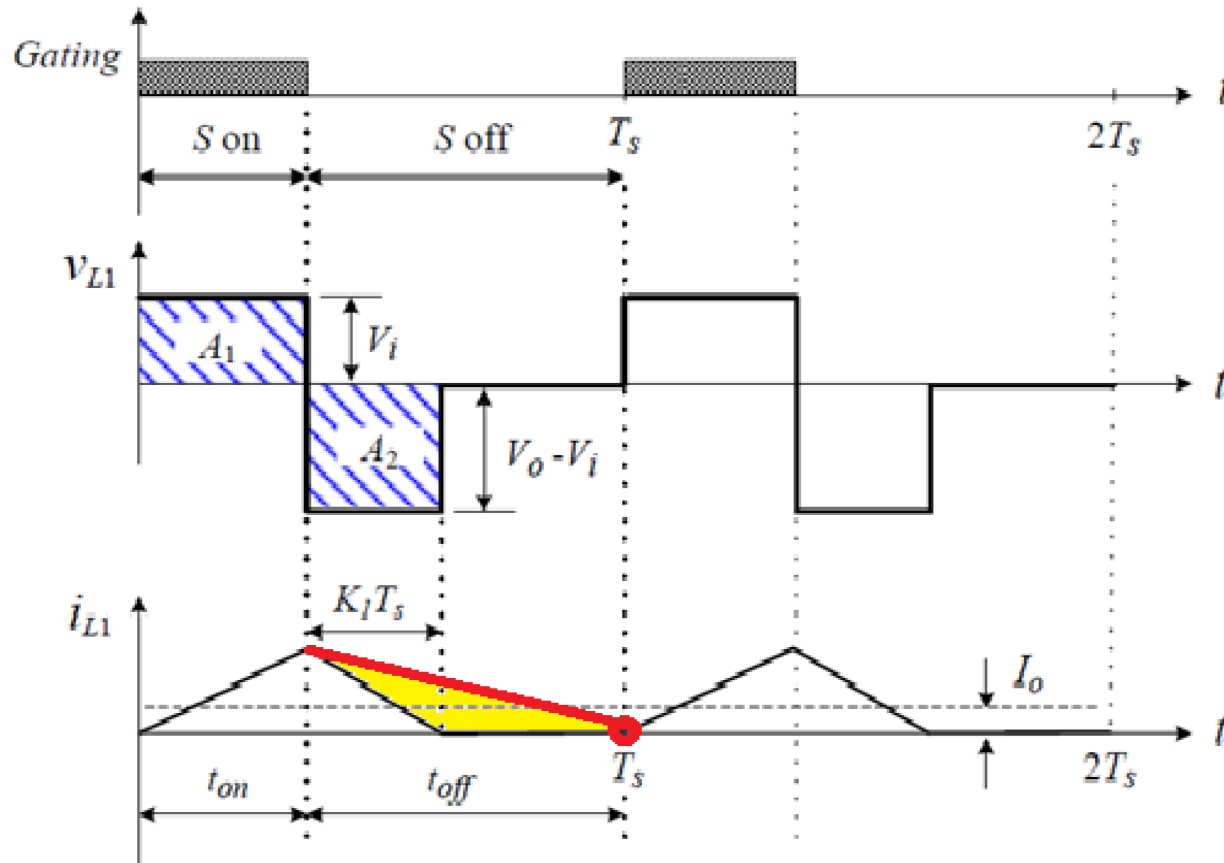
- Converter thus operates in discontinuous current mode.

$$\Delta i_{L1} = D(1 - D) \frac{V_o T_s}{L_1}$$

$$I_{LB} = \boxed{D(1 - D) \frac{V_o T_s}{L_1}} \cdot \frac{1}{2} = \frac{\Delta i_{L1}}{2}$$

Inductor current at boundary(I_{LB}) between CCM & DCM is given by

Discontinuous current mode(DCM)



$$\Delta i_{L1} = D(1 - D) \frac{V_o T_s}{L_1}$$

$$I_{LB} = D(1 - D) \frac{V_o T_s}{L_1} \frac{1}{2} = \frac{\Delta i_{L1}}{2}$$

1/2 Indicates that Inductor current i_{L1} reaches 0 before end of toff period

Maximum inductor boundary current $I_{LB,\max}$ occurs
at

$D = 0.5$, which can be calculated by $I_{LB} = \frac{\Delta i_{L1}}{2}$

$$I_{LB,\max} = \Delta I_{L1,\max} \div 2$$

$$I_{LB,\max} = \frac{V_o T_s}{4L_1} \div 2$$

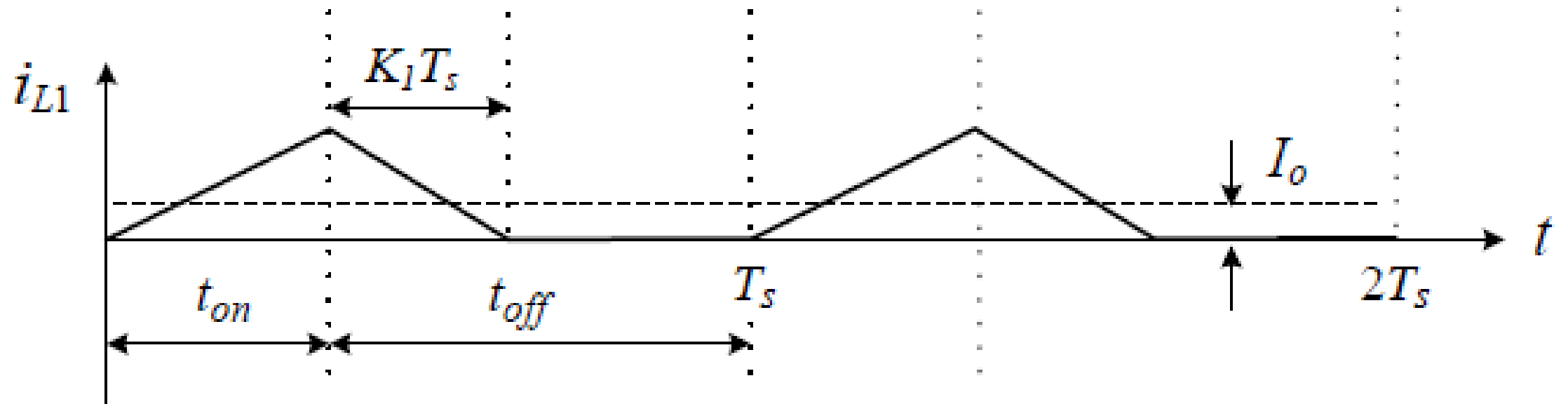
$$I_{LB,\max} = \frac{V_o T_s}{8L_1}$$

Boundary output current(I_{OB}) can be found from

$$I_{LB} = D(1-D) \frac{V_o T_s}{L_1} \times \frac{(1-D)}{2}$$

$$\Delta i_{L1} = D(1-D) \frac{V_o T_s}{L_1}$$

$$I_{oB} = (1-D) \frac{\Delta i_{L1}}{2}$$



Boundary output current I_{OB} can be deduced :

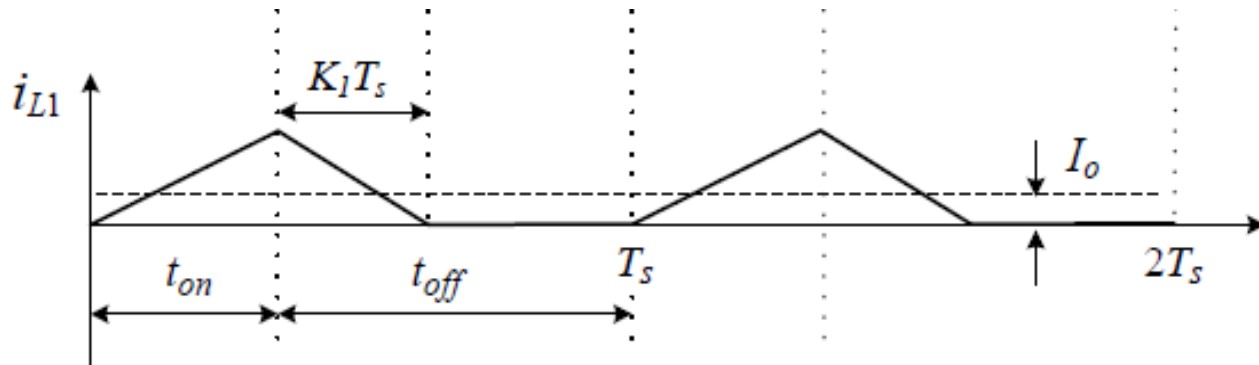
$$I_{OB} = (1-D) \times I_{LB}$$

$$I_{LB} = \frac{\Delta i_{L1}}{2}$$

$$I_{LB} = D(1-D) \frac{V_o T_s}{2L_1}$$

$$I_{OB} = (1-D) \times D(1-D) \frac{V_o T_s}{2L_1}$$

$$\Delta i_{L1} = D(1-D) \frac{V_o T_s}{L_1}$$



Boundary output current I_{OB} would be maximum $I_{OB,max}$ at $D = ?$

$$I_{OB} = (1 - D) \times D(1 - D) \frac{V_o T_s}{2L_1}$$

$$I_{OB} = (1 - D) \times D(1 - D) \frac{V_o T_s}{2L_1}$$

$$I_{OB} = D(1 - D)^2 \times K$$

$$I_{OB} = D - D^3 - 2D^2$$

$$0 = 3D^2 - 4D + 1$$

$$a = , b = -4, c = 1$$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D \neq 1$$

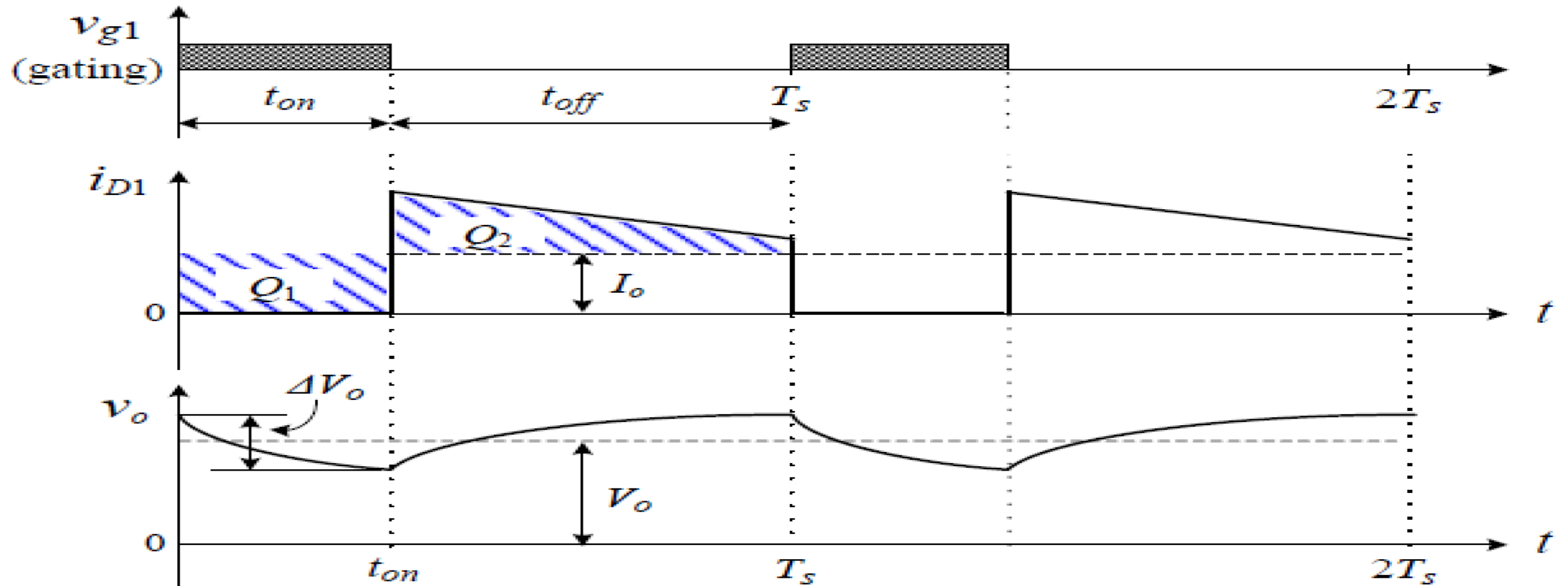
$$D = 1/3$$

Boundary output current I_{OB} would be maximum at $D = 1/3$

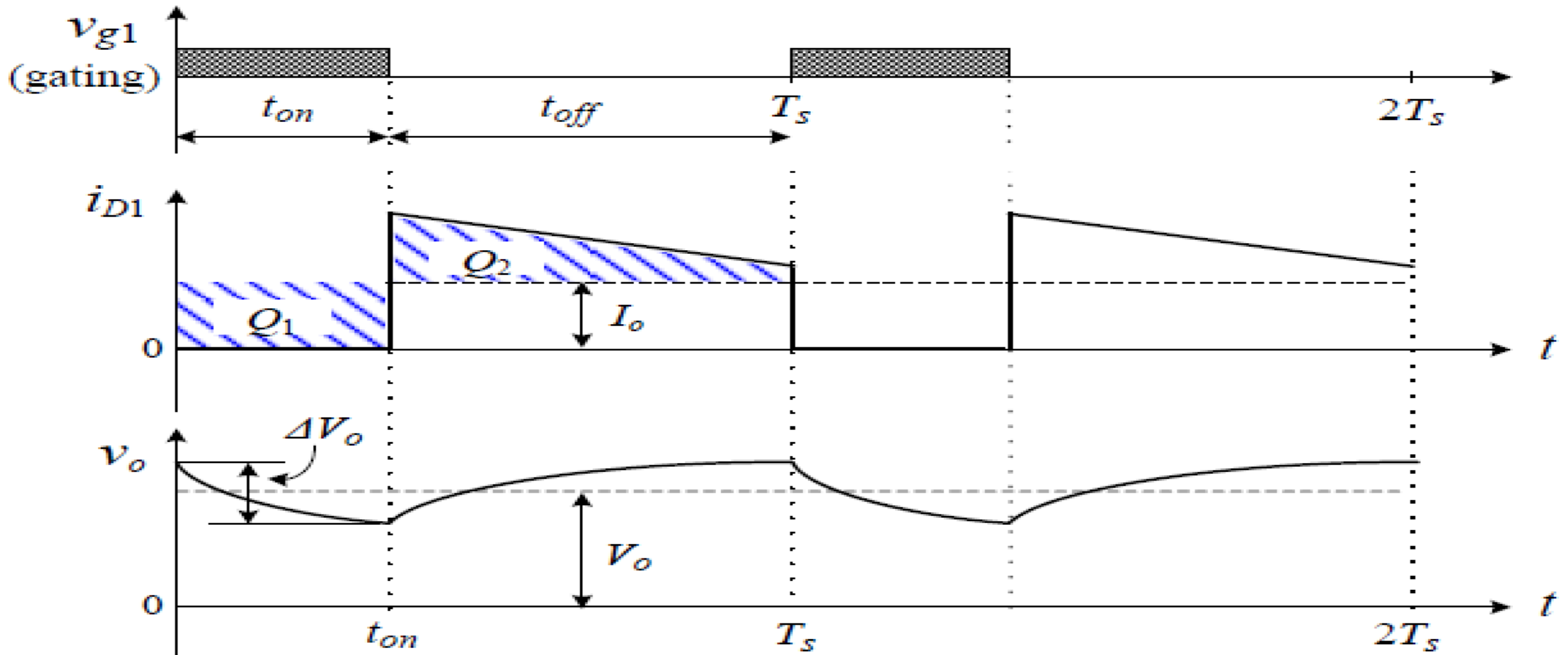
$$I_{OB} = (1 - D) \times D(1 - D) \frac{V_o T_s}{2L_1}$$

$$I_{OB,max} = \frac{2}{27} \frac{V_o T_s}{L_1}$$

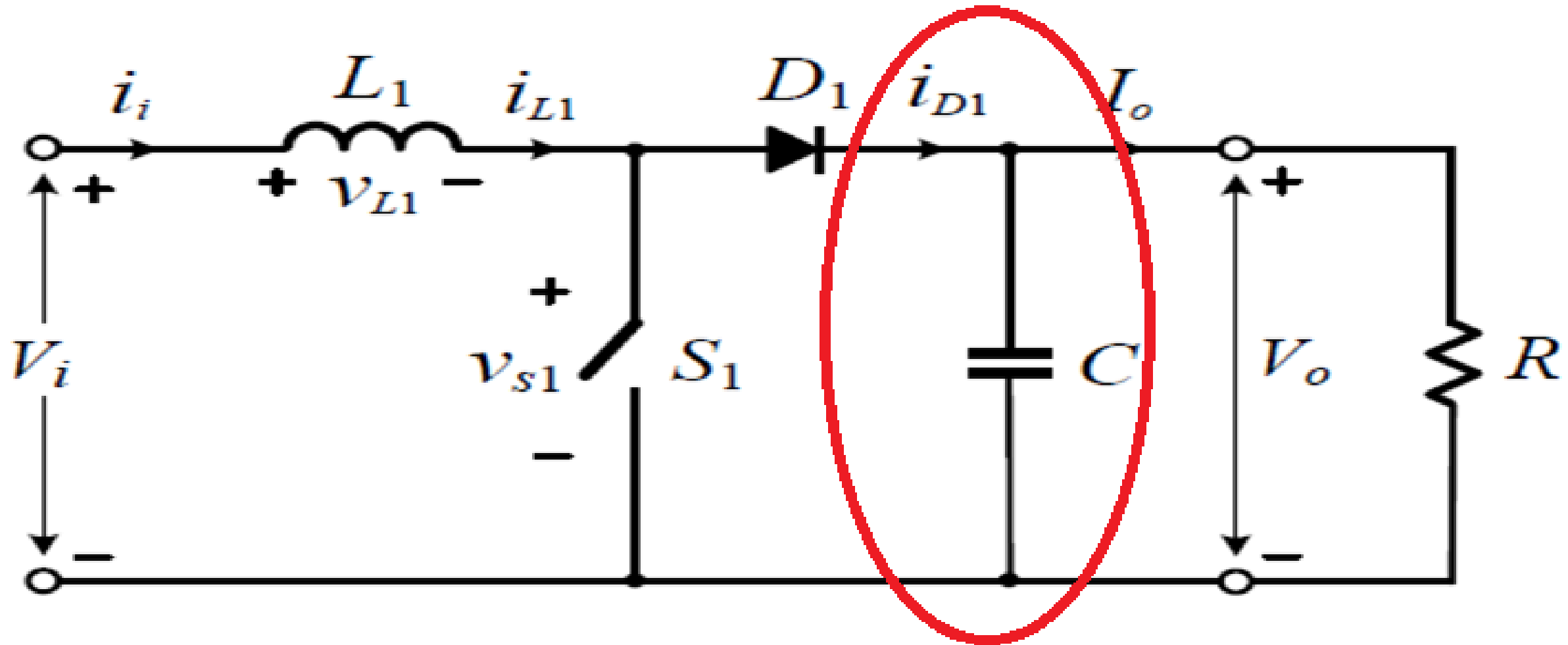
To calculate output voltage ripple(v_o) in single-channel boost converter considered waveform of current i_{D1} in diode $D1$



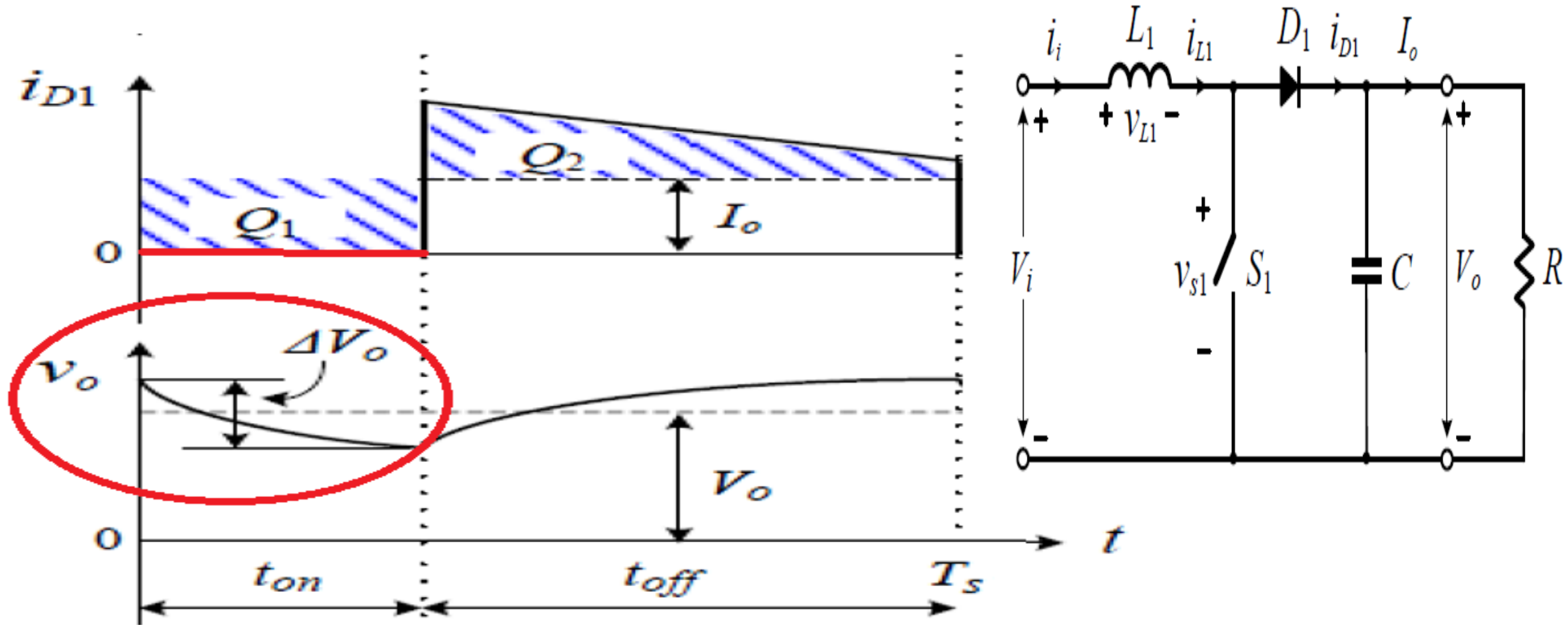
Assuming that all ripple current component in $D1$ is absorbed by large output capacitor C



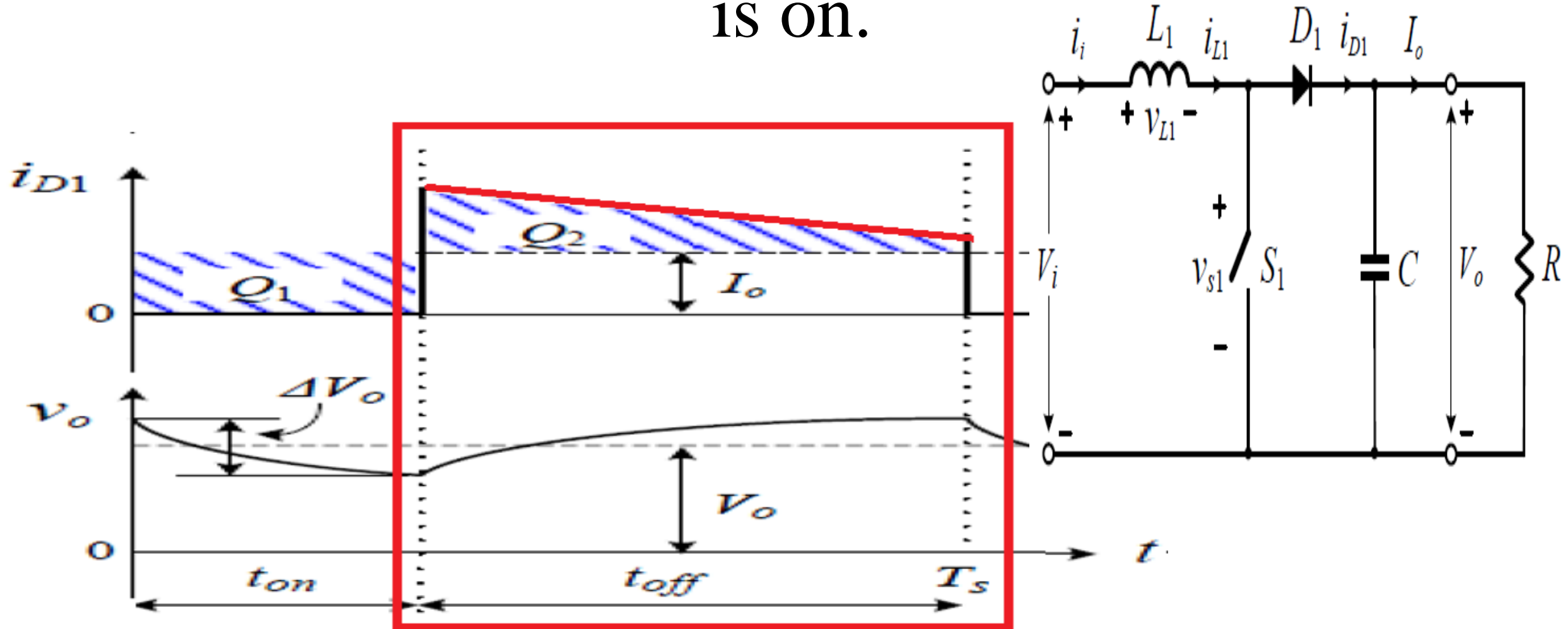
Behavior of Capacitor C during t_{on} & t_{off} period



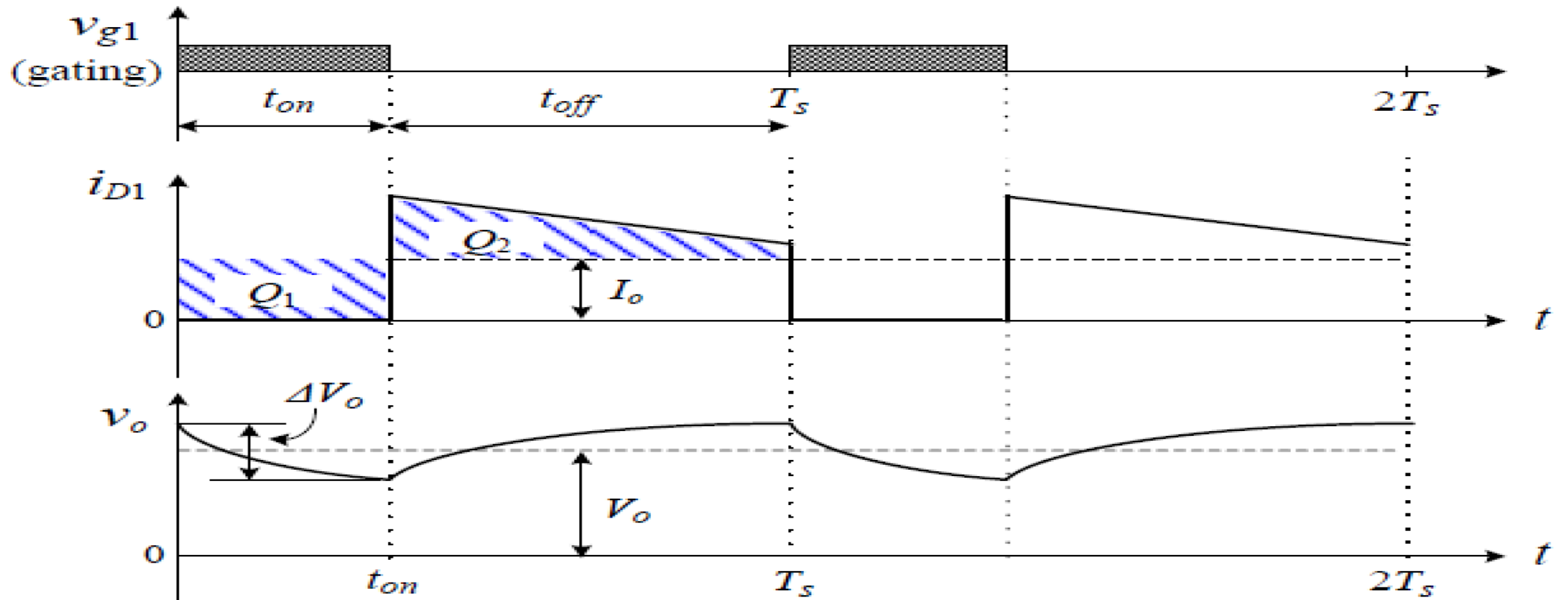
Capacitor C is discharged to load during t_{on} period when diode D_1 is turned off



Capacitor C is charged during t_{off} period when $D1$ is on.



Amount of charges, Q_1 during t_{on} & Q_2 during t_{off} , represented by shaded areas should be equal.



Peak-to-peak ripple voltage ΔV_o

$$Q = CV$$

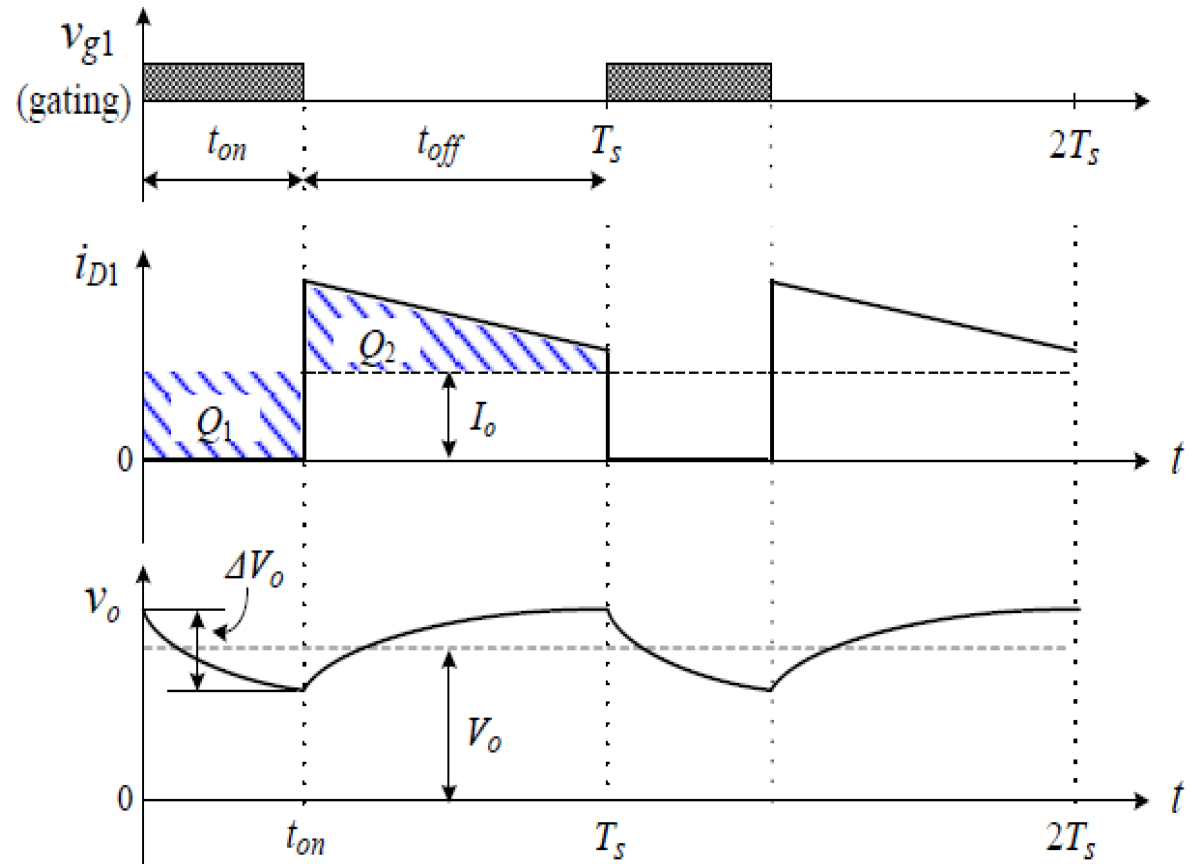
$$Q_1 = C\Delta V_o$$

$$\Delta V_o = \frac{Q_1}{C}$$

$$\Delta V_o = \frac{I_o t_{ON}}{C}$$

$$\Delta V_o = \frac{V_o DT_s}{RC}$$

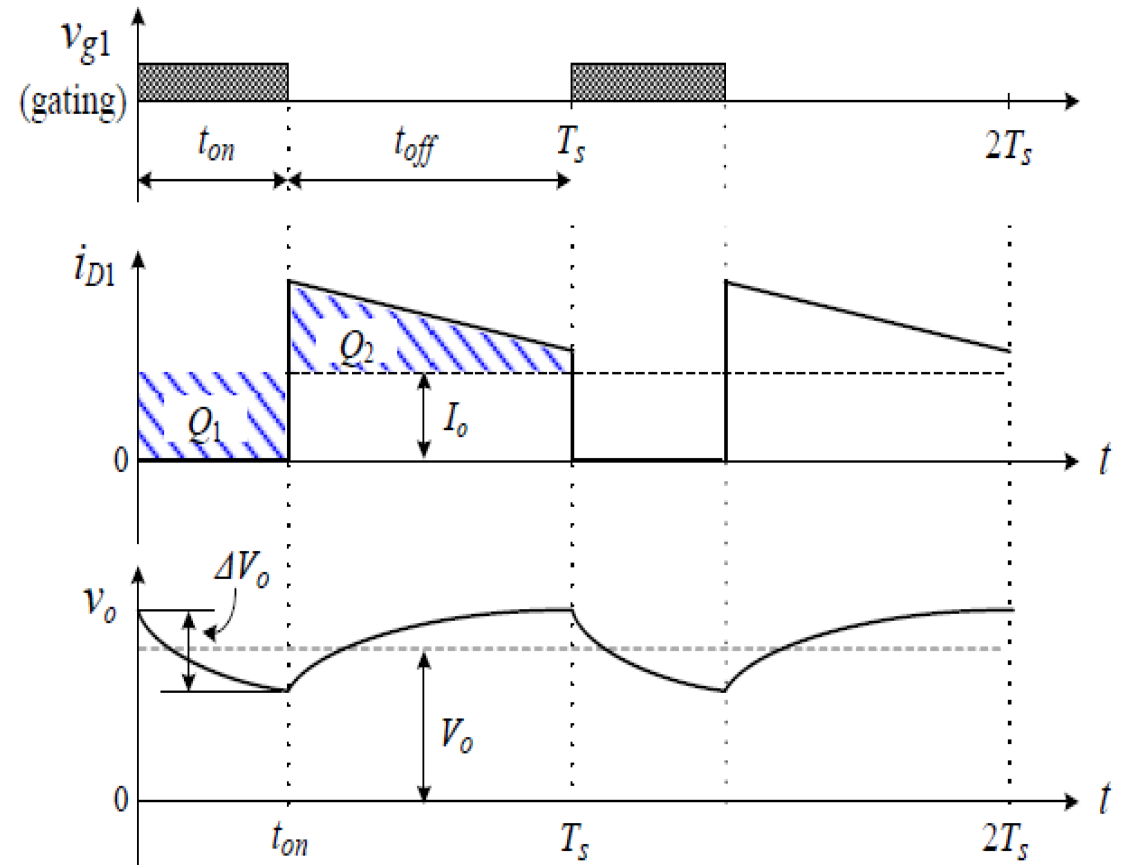
$$I_o = \frac{V_o}{R} \quad \& \quad t_{ON} = DT_s$$



For load resistance R & filter capacitor C , ripple voltage ΔV_o increases with duty cycle D .

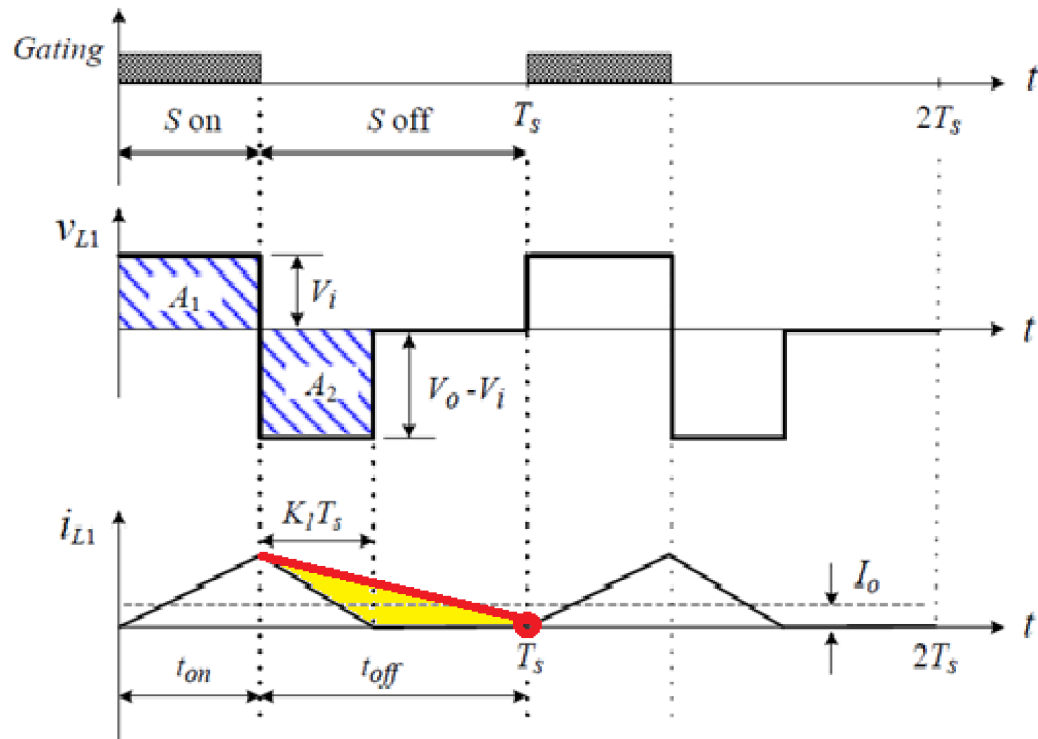
$$\Delta V_o = \frac{V_o D T_s}{RC}$$

$$\frac{\Delta V_o}{V_o} = \frac{D T_s}{RC}$$

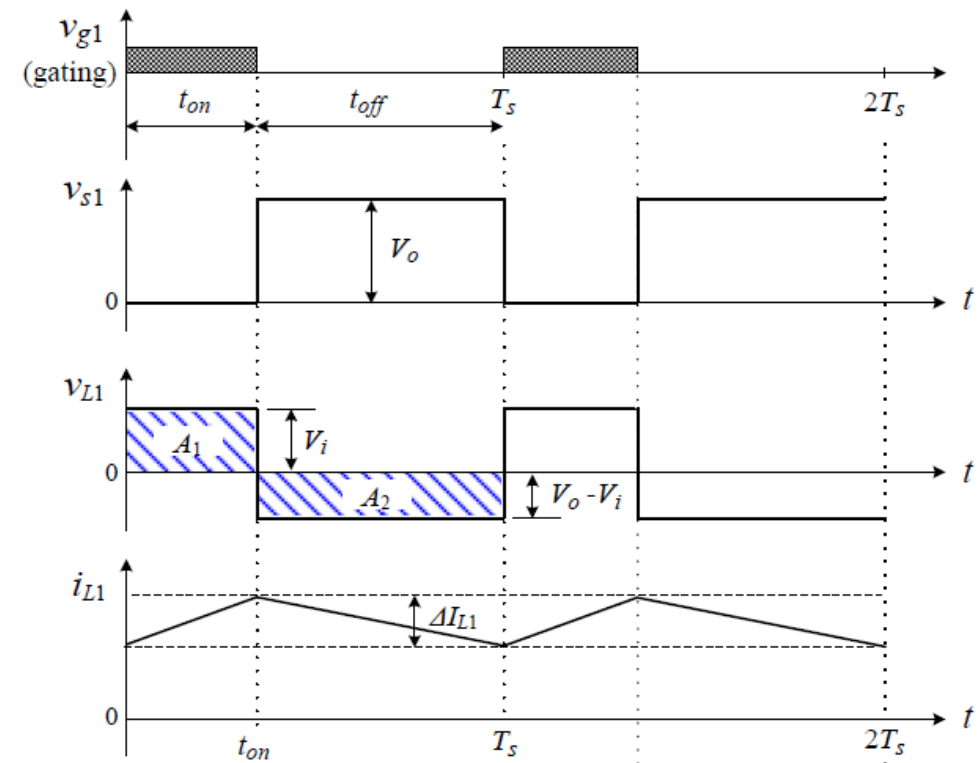


Operation of converter in Discontinuous current mode(DCM) & Continuous current mode(CM) is same (True or False)

Discontinuous current mode(DCM)

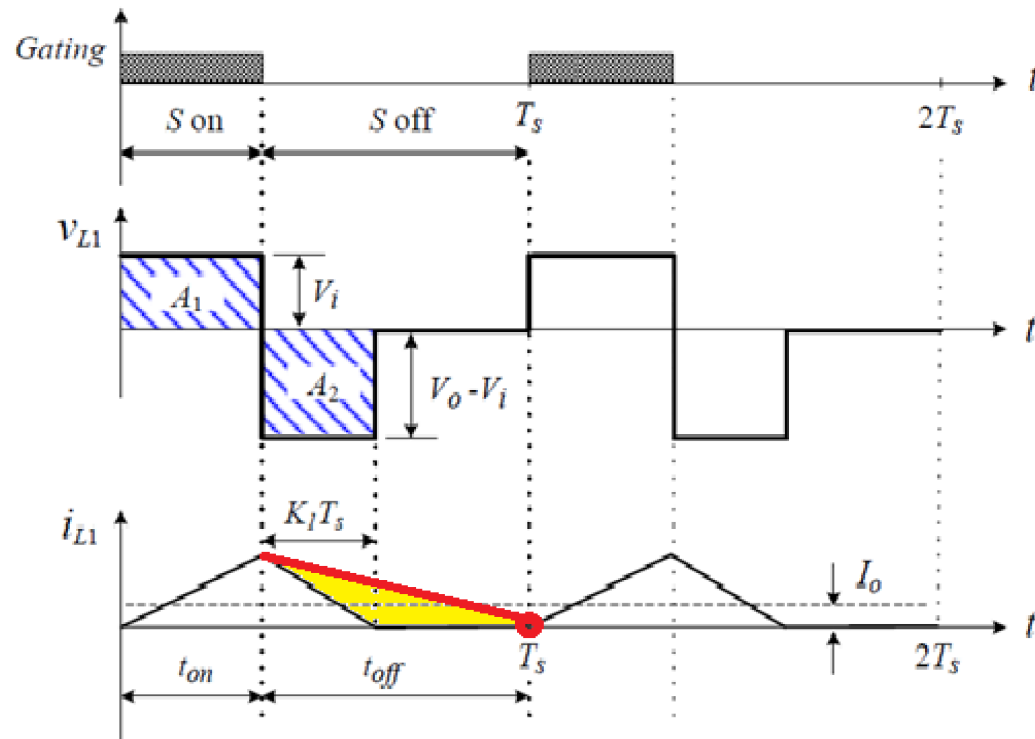


Continuous current mode(CM)

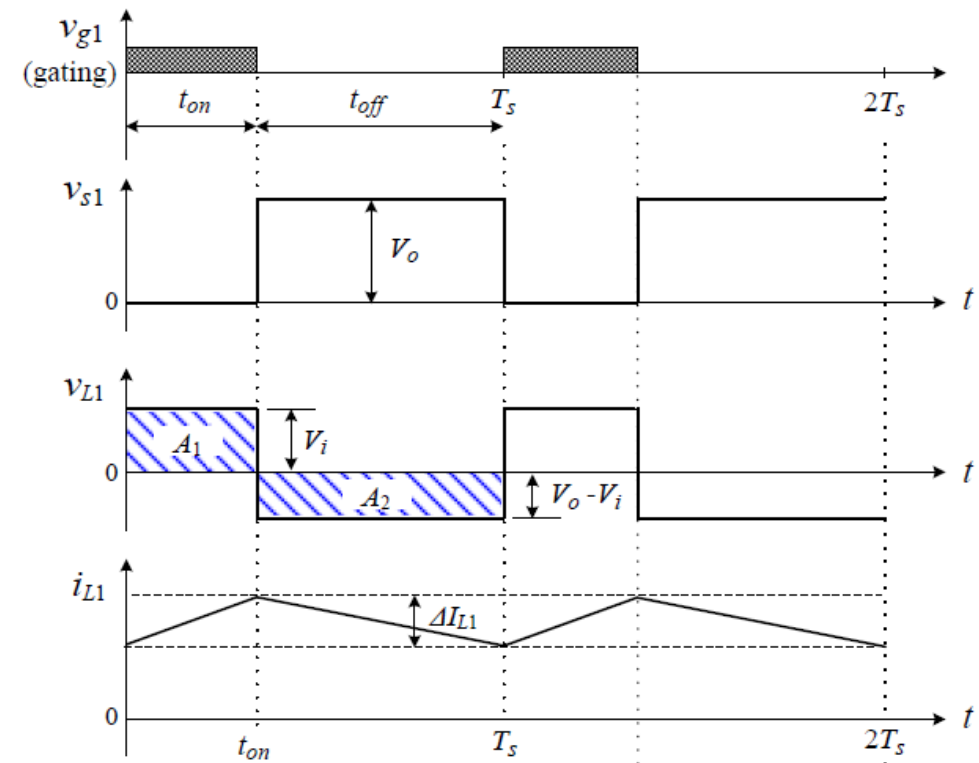


During t_{on} period, operation of converter in DCM is same as that in CCM.

Discontinuous current mode(DCM)

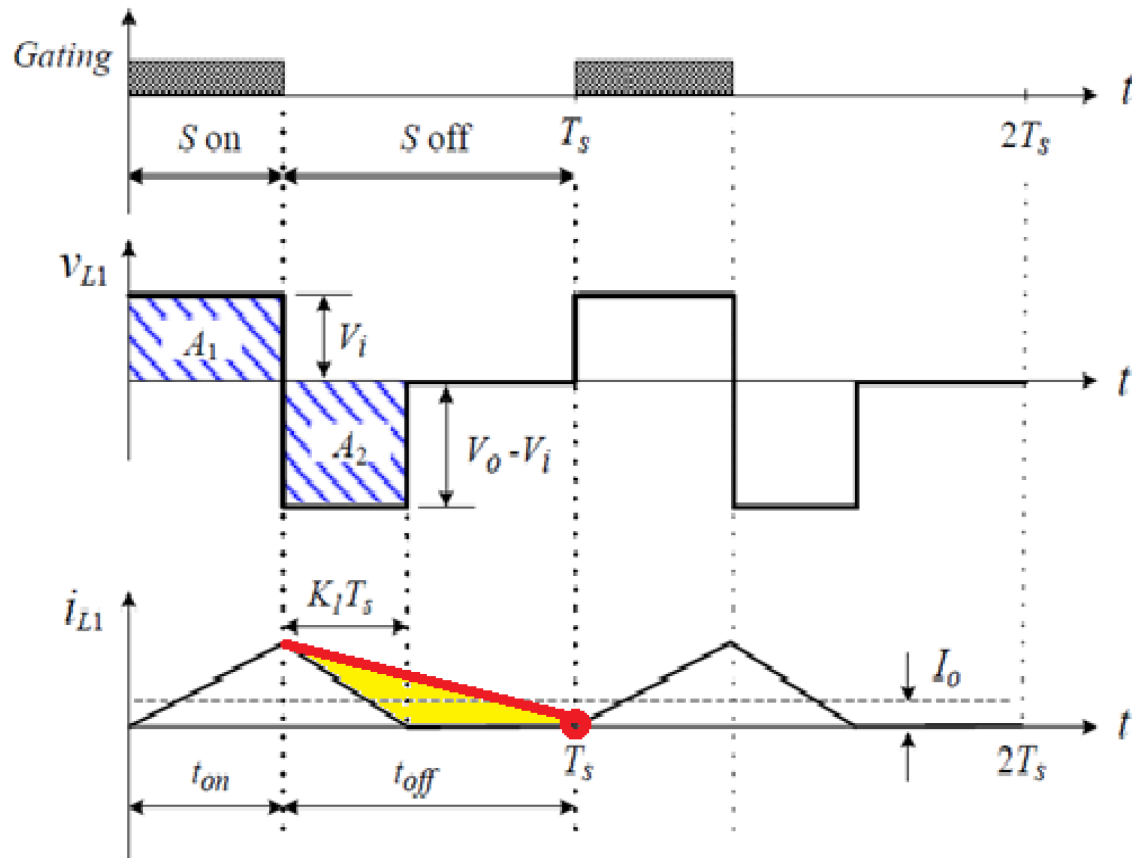


Continuous current mode(CM)

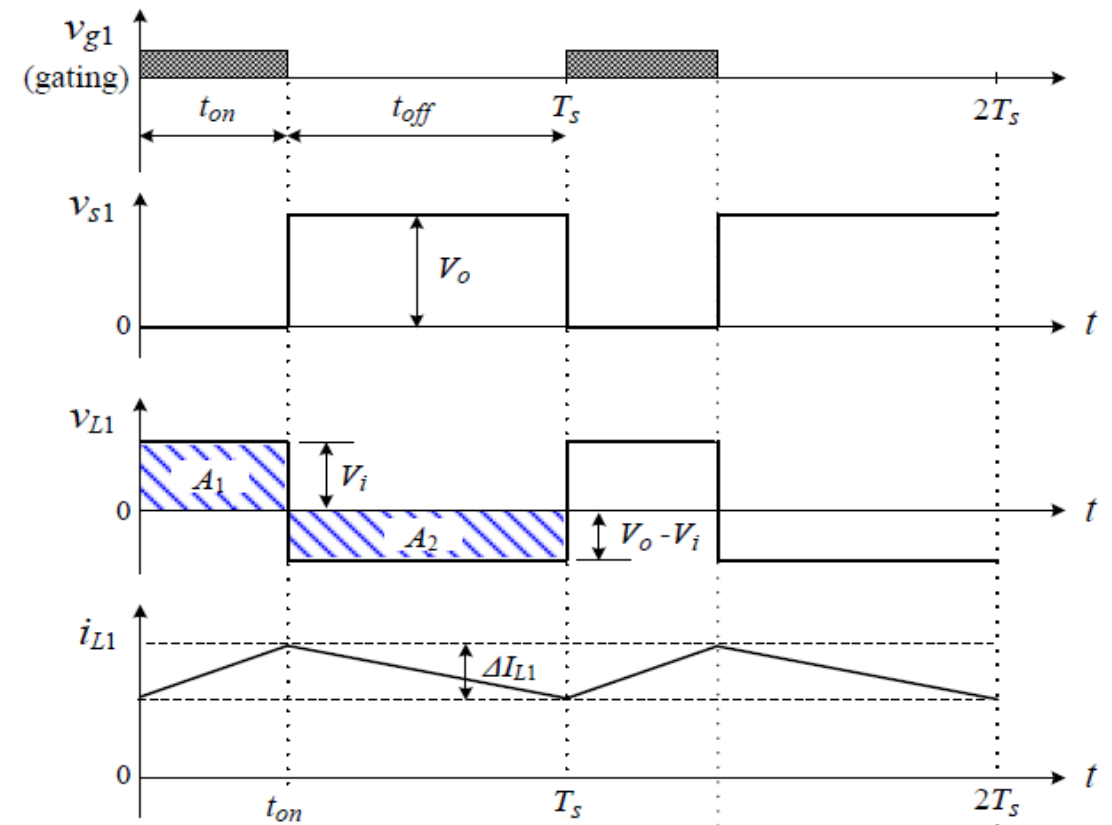


Current i_{L1} increases over time, & energy is stored in $L1$. During t_{off} period, inductor current i_{L1} falls to 0 at end of $K1T_s$ period, at which all energy stored in $L1$ during t_{on} period is completely released.

Discontinuous current mode(DCM)

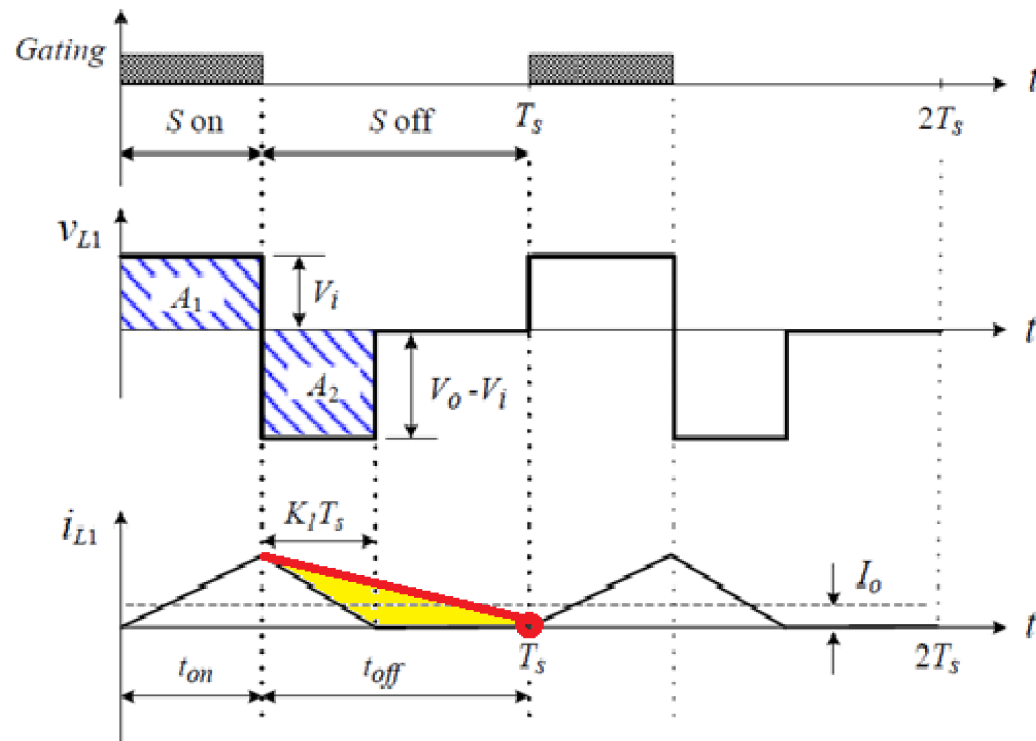


Continuous current mode(CM)

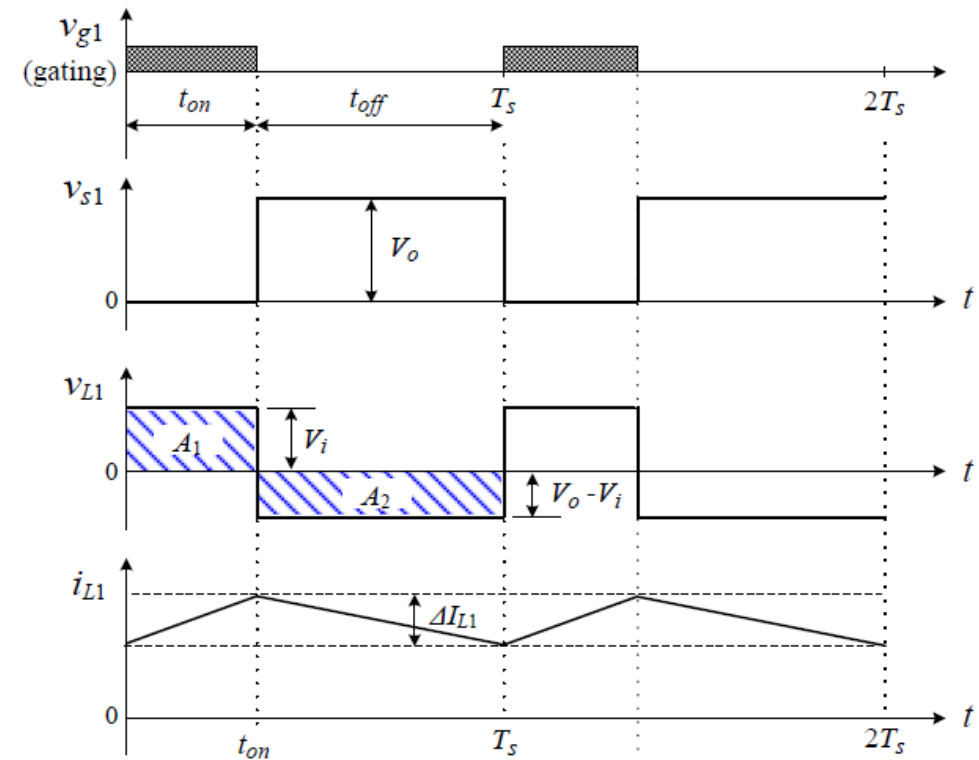


Since the average voltage across the inductor over switching period T_s is equal to 0, area $A_1=A_2$, that is, $V_i t_{on} = (V_o - V_i) K_1 T_s$

Discontinuous current mode(DCM)



Continuous current mode(CM)



As

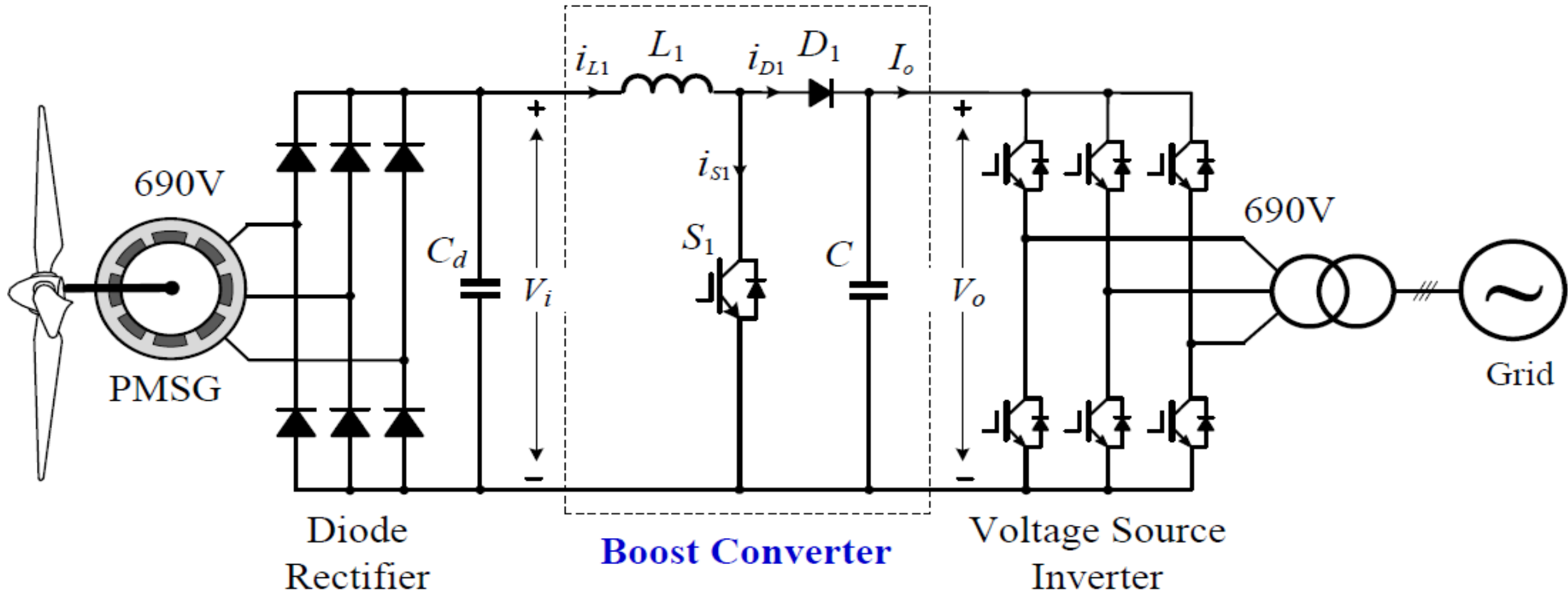
$$V_i t_{on} = (V_o - V_i) K_1 T_s$$

$$\frac{V_o}{V_i} = \frac{K_1 + D}{K_1} \quad \text{for } 0 \leq D < 1$$

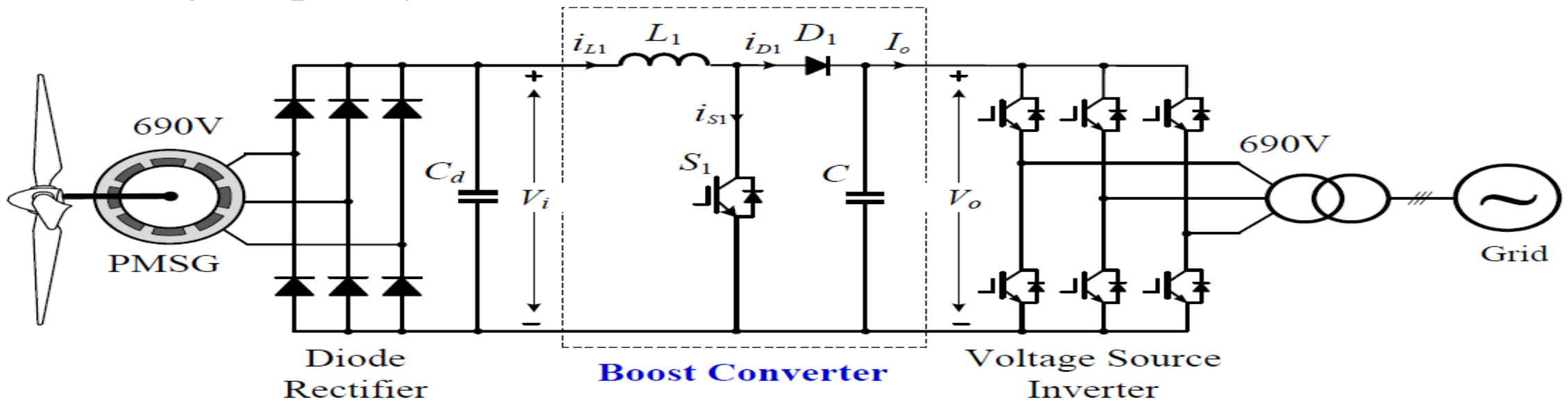
Problem Solving Session

Topic: Interleaved Boost Converters

4-6 (Solved Problem) A single-channel boost converter is used in a 600kW/690V/50Hz PMSG wind energy conversion system as shown in Fig.



- Boost converter transfers power from generator to grid of 690V/50Hz via a diode rectifier & 2--level voltage source inverter. Line-to-line stator voltages of generator operating at minimum & rated wind speeds are 80 V & 690 V, respectively.
- Output voltage of boost converter, v_o , is set by inverter with a modulation index ma of 0.8, which leaves 20% margin for adjustments.
- Switching harmonics generated by inverter are neglected, & thus inverter ac-side fundamental-frequency voltage is equal to grid voltage of 690V. Switching frequency of boost converter is 2000 Hz.



Calculate/answer following:

- a) Minimum & maximum input voltages & output voltage of boost converter
- b) Minimum & maximum duty ratios
- c) Average output current & maximum inductor current when generator delivers rated power to grid

d) Derive expression for calculation of maximum boundary inductor current $I_{LB,\max}$, boundary output current I_{OB} & maximum boundary output current $I_{OB,\max}$

e) Minimum value of inductance to operate boost converter in CCM mode

f) Minimum output capacitor value assuming that maximum allowable output voltage ripple is 8% of average output voltage.

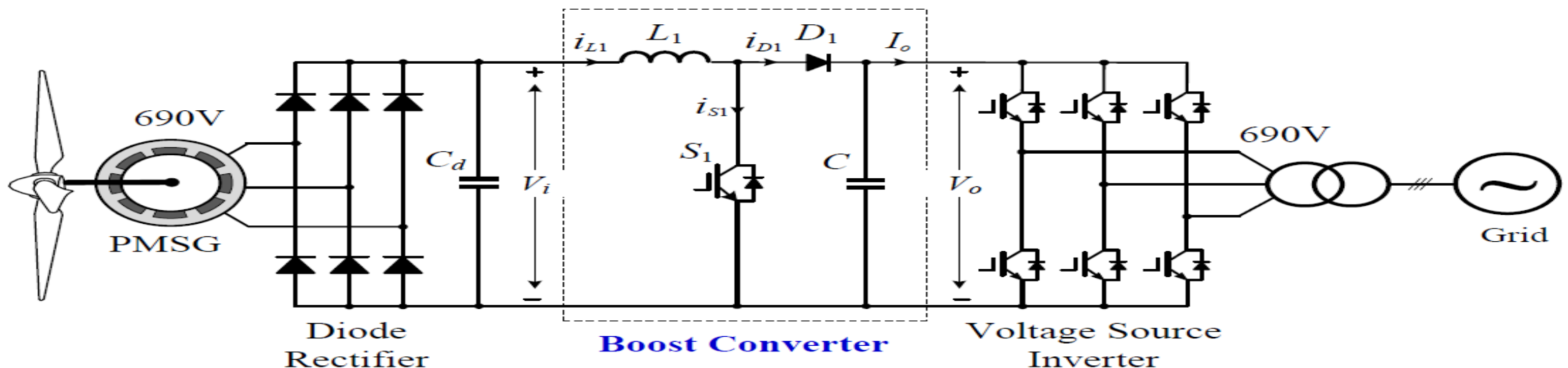
Solution:

a) Minimum & maximum input voltages for boost converter can be calculated by

$$V_{i(\min)} = \frac{3\sqrt{2}}{\pi} \times V_{LL,\min} \approx 1.35 \times 80 = 108 \text{ V}$$

$$V_{i(\max)} = \frac{3\sqrt{2}}{\pi} \times V_{LL,\max} \approx 1.35 \times 690 = 931.5 \text{ V}$$

where $3\sqrt{2} / \pi$ is the voltage gain of the diode rectifier.



Output voltage of boost converter is set by inverter:

$$V_o = \frac{\sqrt{2}V_{grid}}{m_a} = \frac{\sqrt{2} \times 690}{0.8} \approx 1220 \text{ V}$$

b) Minimum & maximum duty ratios:

$$D_{\min} = 1 - \frac{V_{i(\max)}}{V_o} = 1 - \frac{931.5}{1220} = 0.2365$$

$$D_{\max} = 1 - \frac{V_{i(\min)}}{V_o} = 1 - \frac{108}{1220} = 0.9115$$

c) Average output & maximum inductor current:

$$I_o = \frac{P_o}{V_o} = \frac{P_i}{V_o} = \frac{600 \times 10^3}{1220} = 491.8 \text{ A}$$

$$I_{L1(\max)} = \frac{I_o}{1 - D_{\max}} = \frac{491.8}{1 - 0.9115} = 5556 \text{ A}$$

d) Boundary inductor current:

$$I_{LB} = D(1-D) \frac{V_o T_s}{2L_1}$$

$$\frac{dI_{LB}}{dD} = 0 \rightarrow D = 0.5; \quad I_{LB, \max} = \frac{V_o T_s}{8L_1} \quad \text{for } D = 0.5$$

$$I_o = (1-D)I_{L1}, \quad I_{oB} = (1-D)I_{LB} = D(1-D)^2 \times \frac{V_o T_s}{2L_1}$$

$$\frac{dI_{oB}}{dD} = 0 \rightarrow D = 0.333; \quad I_{oB, \max} = 0.074 \frac{V_o T_s}{L_1} \quad \text{for } D = 0.333$$

e) CCM mode of operation can be achieved when $I_o > I_{oB,\max}$

The maximum boundary output current $I_{oB,\max}$ occurs at $D = 0.333$.

The minimum value of the inductance to operate boost converter at the boundary of the CCM can be obtained by equating I_o to $I_{oB,\max}$, from which

$$L_{1,\min} = 0.074 \frac{V_o T_s}{I_o} = 0.074 \frac{1220 \times 0.5 \times 10^{-3}}{491.8} = 91.785 \mu\text{H}$$

f) Minimum output capacitor value:

$$C_{\min} = \frac{I_o D_{\max} T_s}{\Delta V_o} = \frac{491.8 \times 0.9115 \times 0.5 \times 10^{-3}}{0.08 \times 1200} = 2296.45 \mu\text{F}$$

4-7 Repeat Problem 4-6 with following changes: a 500kW/575V/60Hz PMSG is connected to grid of 575V/60Hz via power converters, its line-to-line stator voltages at minimum and rated wind speeds are 70 V & 575 V & switching frequency of boost converter is 2200 Hz.

Answers:

a) $V_{i(\min)} = 94.5 \text{ V}$, $V_{i(\max)} = 776.2 \text{ V}$, $V_o \approx 1020 \text{ V}$

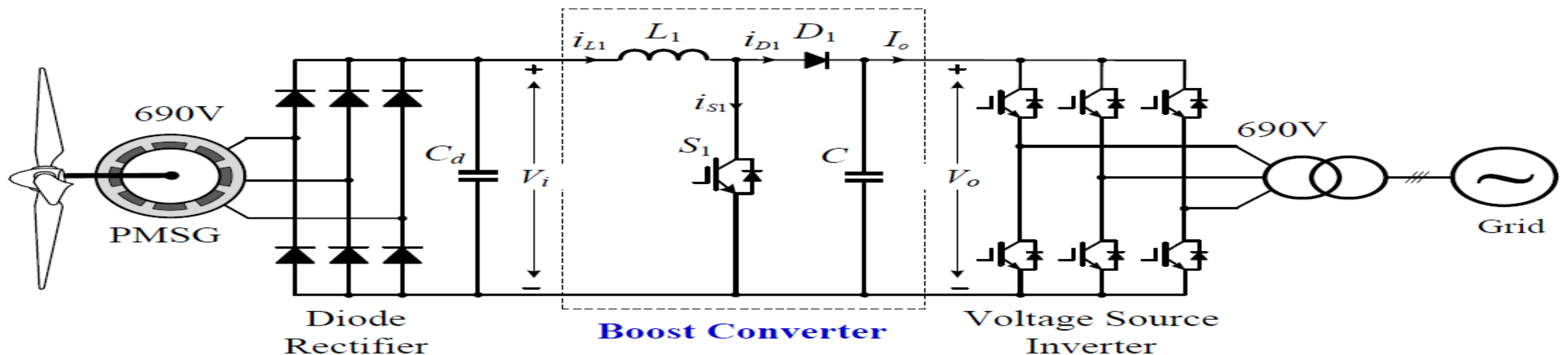
b) $D_{\min} = 0.239$, $D_{\max} = 0.9074$

c) $I_o = 490.2 \text{ A}$, $I_{L1(\max)} = 5291 \text{ A}$

e) $L_{1,\min} = 70 \mu\text{H}$

f) $C_{\min} = 2477.6 \mu\text{F}$

4-8 (Solved Problem) A single-channel boost converter is used in a 600kW/690V/50Hz PMSG wind energy conversion system as shown in Fig. P4-6. Boost converter transfers power from generator to a grid of 690V/50Hz via a diode rectifier & 2-level voltage source inverter. Inductance L_1 & capacitance C of boost converter are $270\ \mu\text{H}$ & $2300\ \mu\text{F}$, respectively. Switching frequency of boost converter is 2000 Hz. Output voltage v_o of boost converter is set by inverter to 1220 V. Generator operates with an MPPT scheme & its stator active power is proportional to cube of rotor speed. At a given wind speed, PMSG is operating at 0.9 pu rotor speed & its line-to-line stator voltage is 640 V.



Calculate/answer the following:

- a) the input voltage and power to the boost converter,
- b) the boundary inductor and output currents, operating mode and duty cycle,
- c) the peak-to-peak and average inductor currents,
- d) the percentage inductor current ripple,
- e) the percentage output voltage ripple, and
- f) draw the waveforms for gate signals, i_{L1} , v_{L1} , i_{S1} , v_{S1} , i_{D1} and v_{D1} , assuming the output filter C is very large and the output voltage is ripple free. Indicate values on the waveforms.

Solution:

a) Input voltage to boost converter:

$$V_i = \frac{3\sqrt{2}}{\pi} \times V_{LL} \approx 1.35 \times 640 = 864 \text{ V}$$

Input power to boost converter at 0.9 pu rotor speed:

$$P_i = P_{m,R} \times (\omega_{m,\text{pu}})^3 = 600 \times 10^3 \times (0.9)^3 = 437.4 \times 10^3 \text{ W}$$

b) Assume that converter is operating in continuous conduction mode (CCM). Based on this assumption, duty cycle can be calculated by

$$D = 1 - \frac{V_i}{V_o} = 1 - \frac{864}{1220} = 0.2918$$

Boundary inductor current:

$$I_{LB} = D(1-D)\frac{V_o T_s}{2L_1} = 0.2918 \times 0.7082 \times \frac{1220 \times 0.5 \times 10^{-3}}{2 \times 270 \times 10^{-6}} = 233.44 \text{ A}$$

Boundary & average output currents:

$$I_{oB} = (1-D)I_{LB} = (0.7082) \times 233.44 = 165.32 \text{ A}$$

$$I_o = \frac{P_o}{V_o} = \frac{P_i}{V_o} = \frac{437.4 \times 10^3}{1220} = 358.52 \text{ A}$$

Since $I_o > I_{oB}$, the converter operates in the CCM. Therefore, the assumption is valid.

The duty cycle thus is $D = 0.2918$

c) Peak-to-peak inductor current:

$$\Delta I_{L1} = D(1-D) \frac{V_o T_s}{L_1} = 0.2918 \times 0.7082 \times \frac{1220 \times 0.5 \times 10^{-3}}{270 \times 10^{-6}} = 466.89 \text{ A}$$

Alternatively,

$$\Delta I_{L1} = D \frac{V_i T_s}{L_1} = 0.2918 \times \frac{864 \times 0.5 \times 10^{-3}}{270 \times 10^{-6}} = 466.89 \text{ A}$$

or

$$\Delta I_{L1} = (1-D) \frac{(V_o - V_i) T_s}{L_1} = 0.7082 \times \frac{(1220 - 864) \times 0.5 \times 10^{-3}}{270 \times 10^{-6}} = 466.89 \text{ A}$$

Average inductor current:

$$I_{L1} = \frac{I_o}{1-D} = \frac{358.52}{1-0.2918} = 506.25 \text{ A}$$

d) Percentage inductor current ripple:

$$\frac{\Delta I_{L1}}{I_{L1}} = \frac{466.89}{506.25} \times 100 = 92.22\%$$

e) Output voltage ripple:

$$\Delta V_o = D \frac{I_o T_s}{C} = 0.2918 \times \frac{358.52 \times 0.5 \times 10^{-3}}{300 \times 10^{-6}} = 22.74 \text{ V}$$

Percentage output ripple voltage:

$$\frac{\Delta V_o}{V_o} = \frac{22.74}{1220} \times 100 = 1.86 \%$$

4-9 Repeat Problem 4-8 when PMSG operates at 0.5 pu rotor speed with its line-to-line voltage of 150 V.

Answers:

a) $V_i = 202.5 \text{ V}$, $P_i = 75 \times 10^3 \text{ W}$

b) $D = 0.834$ $I_{LB} = 156.38 \text{ A}$, $I_{oB} = 25.95 \text{ A}$ $I_o = 61.48 \text{ A}$ CCM

c) $\Delta I_{L1} = 312.76 \text{ A}$

d) $I_{L1} = 370.37 \text{ A}$, $\frac{\Delta I_{L1}}{I_{L1}} = 84.44\%$

e) $\frac{\Delta V_o}{V_o} = 0.914\%$