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A FDTD method for Nonuniform Transmission Line Analysis Using Yee's-lattice and Wavelet Expansion

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Abstract **—A 1D finite-difference time-domain (FDTD) method for nonuniform transmission line analysis by using wavelet expansion is presented. The approach are based on scaling functions only or on a combination of scaling functions and wavelets leading to a variable mesh griding. The proposed schemes compared to the conventional Yee's FDTD scheme shows a good capability to approximate the exact solution with negligible error for sampling rates approaching the Nyquist limit. A linear tapered transmission line that is one of models of interconnect is analyzed in order to illustrate the application of this method and to demonstrate the advantages over Yee's FDTD scheme with respect to memory requirements and execution time. And to show the stableness, eye diagram analysis for lossy uniform transmission line is shown.**

Index Terms **— nonuniform transmission line, wavelet expansion, FDTD method, time-domain analysis**

I. Introduction

Distortion, delay, and crosstalk on interconnect cannot be disregarded as the speed-up and the densification of the circuits advance. That is, analysis of high speed signal propagation on substrates and in chips is an important problem of the signal integrity. It is necessary that the transient analysis method for signals in interconnect is highly accurate, high speed and high stability. Interconnect is modeled by uniform or nonuniform transmission lines. Therefore, the FDTD method has been applied so far as the method of obtaining the time-domain characteristic of the transmission line[1][2] and wavelet expansion[3] is used to obtain an efficient calculation[4]-[6]. In this paper, the FDTD method for transmission line analysis by using wavelet expansion is presented. In this method, scaling functions only or on a combination of scaling functions and wavelets are used to express the partial differentiation of the voltage and the current in the nonuniform trans-mission line equations. The advantage of this method is that a global approximation is achieved by wavelet expansion, and the number of lattices that calculate the voltages and the currents are less than the conventional FDTD method. Moreover, the simultaneous equations need not be solved like multiresolution time-domain (MRTD) schemes. To show the effectiveness of this method, a linear tapered transmission line that is one of models of interconnect and eye diagram of lossy

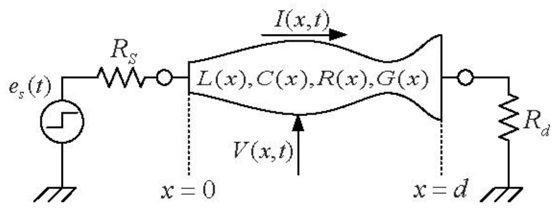

Fig. 1. A terminated lossy nonuniform transmission line.

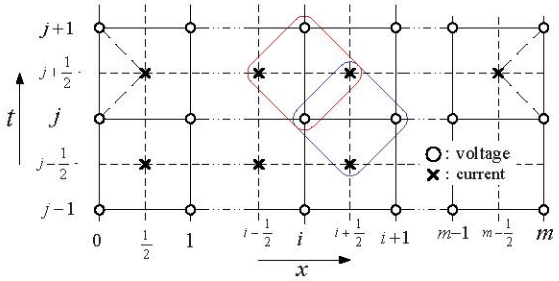

Fig. 2. $x-t$ plane.

uniform transmission line are analyzed.

II. UPDATE EQUATIONS

The equations of the nonuniform transmission line shown in Fig. 1. are

$$-\frac{\partial V(x,t)}{\partial x} = L(x)\frac{\partial I(x,t)}{\partial t} + R(x)I(x,t) \tag{1}$$

$$-\frac{\partial I(x,t)}{\partial x} = C(x)\frac{\partial V(x,t)}{\partial t} + G(x)V(x,t) \tag{2}$$

And boundary conditions at both ends are

$$e_s(t) = V(0,t) + R_s I(0,t) \tag{3}$$

$$0 = V(d,t) + R_d I(d,t) \tag{4}$$

where, $V(x,t)$ and $I(x,t)$ are voltage and current at coordinates x along trans-mission line and time t. $e_s(t)$ is the driving voltage source and $L(x)$, $C(x)$, $R(x)$, and $G(x)$

are per unit length inductance, capacitance, resistance and conductance at x . And, R_s and R_d are terminating resistances. The calculation lattice point of $V(x,t)$ and $I(x,t)$ are staggered at coordinates on the transmission line x and time t as shown in Fig. 2. When the approximation functions $V^h(x,t)$, $I^h(x,t)$ of true solutions $V(x,t)$, $I(x,t)$ are shown by the linear combination of the scaling function $\varphi_k(x)$, the approximation functions are

$$\begin{aligned} V^h(x,t) &= V(x_0,t)\varphi_0(x) + V(x_1,t)\varphi_1(x) + \cdots \\ &\quad + V(x_{m-1},t)\varphi_{m-1}(x) + V(x_m,t)\varphi_m(x) \\ &= \sum_{k=0}^{m} V(x_k,t)\varphi_k(x) \\ &(k = 0,1,2,\cdots m-1,m) \end{aligned} \tag{5}$$

$$\begin{aligned} I^h(x,t) &= I(x_{\frac{1}{2}},t)\varphi_{\frac{1}{2}}(x) + I(x_{1+\frac{1}{2}},t)\varphi_{1+\frac{1}{2}}(x) + \cdots \\ &\quad + I(x_{m-\frac{3}{2}},t)\varphi_{m-\frac{3}{2}}(x) + I(x_{m-\frac{1}{2}},t)\varphi_{m-\frac{1}{2}}(x) \\ &= \sum_{k=0}^{m} I(x_{k+\frac{1}{2}},t)\varphi_{k+\frac{1}{2}}(x) \\ &(k = 0,1,2,\cdots m-2,m-1) \end{aligned} \tag{6}$$

where, m is a number of discretization section of transmission line. The central difference approximations at $(x_{i+1/2},t_j)$ and $(x_i,t_{j+1/2})$ of expression (1) and (2) are

$$-\frac{\partial V^j_{i+\frac{1}{2}}}{\partial x} = L_{i+\frac{1}{2}} \frac{I^{j+\frac{1}{2}}_{i+\frac{1}{2}} - I^{j-\frac{1}{2}}_{i+\frac{1}{2}}}{\Delta t} + R_{i+\frac{1}{2}} \frac{I^{j+\frac{1}{2}}_{i+\frac{1}{2}} + I^{j-\frac{1}{2}}_{i+\frac{1}{2}}}{2} \tag{7}$$

$$-\frac{\partial I^{j+\frac{1}{2}}_i}{\partial x} = C_i \frac{V^{j+1}_i - V^j_i}{\Delta t} + G_i \frac{V^{j+1}_i + V^j_i}{2} \tag{8}$$

where, $V(x_i,t_j)$, $I(x_i,t_j)$, $R(x_i)$, $L(x_i)$, $G(x_i)$, $C(x_i)$ are abbreviated with V^j_i, I^j_i, R_i, L_i, G_i, C_i. Then, the update equations are expressed as follows.

$$I^{j+\frac{1}{2}}_{i+\frac{1}{2}} = \frac{1 - \dfrac{\Delta t R_{i+\frac{1}{2}}}{2L_{i+\frac{1}{2}}}}{1 + \dfrac{\Delta t R_{i+\frac{1}{2}}}{2L_{i+\frac{1}{2}}}} I^{j-\frac{1}{2}}_{i+\frac{1}{2}} - \frac{\dfrac{\Delta t}{L_{i+\frac{1}{2}}}}{1 + \dfrac{\Delta t R_{i+\frac{1}{2}}}{2L_{i+\frac{1}{2}}}} \frac{\partial V^j_{i+\frac{1}{2}}}{\partial x} \tag{9}$$

$$V^{j+1}_i = \frac{1 - \dfrac{\Delta t G_i}{2C_i}}{1 + \dfrac{\Delta t G_i}{2C_i}} V^j_i - \frac{\dfrac{\Delta t}{C_i}}{1 + \dfrac{\Delta t G_i}{2C_i}} \frac{\partial I^{j+\frac{1}{2}}_i}{\partial x} \tag{10}$$

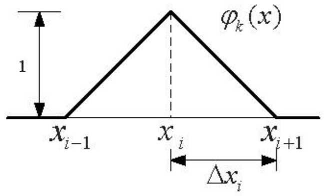

Fig. 3 second order cardinal B spline.

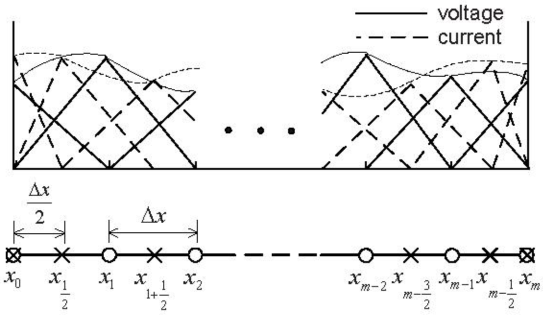

Fig. 4 the calculation lattice points and the functions.

The partial differential terms are represented as follows by using the scaling function $\varphi_k(x)$ of wavelet expansion.

$$\frac{\partial V^j_{i+\frac{1}{2}}}{\partial x} = \sum_{k=0}^{m} V^j_k \frac{\partial \varphi_k(x_{i+\frac{1}{2}})}{\partial x} \quad (i = 0,\cdots m-1) \tag{11}$$

$$\frac{\partial I^{j+\frac{1}{2}}_i}{\partial x} = \sum_{k=0}^{m-1} I^{j+\frac{1}{2}}_{k+\frac{1}{2}} \frac{\partial \varphi_{k+\frac{1}{2}}(x_i)}{\partial x} \quad (i = 1,\cdots m-1) \tag{12}$$

Expression (11) and (12) can be calculated first because $\varphi_k(x)$ are already-known functions. As an example, second order cardinal B spline shown in Fig. 3. is used as a scaling function, the calculation lattice points and the functions for voltages and currents are staggered and shifted as shown in Fig. 4. In this case, the partial differential terms are derived as follows.

$$\frac{\partial \varphi_k(x_{i+\frac{1}{2}})}{\partial x} = \begin{cases} -1/\Delta x_i & (k = i) \\ 1/\Delta x_i & (k = i+1) \\ 0 & (\text{others}) \end{cases} \tag{13}$$

$$\frac{\partial \varphi_{k+\frac{1}{2}}(x_i)}{\partial x} = \begin{cases} -1/\Delta x_{i-\frac{1}{2}} & (k = i-1) \\ 1/\Delta x_{i-\frac{1}{2}} & (k = i) \\ 0 & (\text{others}) \end{cases} \tag{14}$$

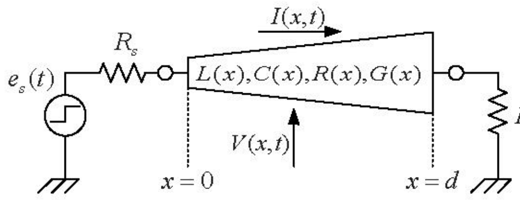

Fig. 5 Linear tapered transmission line.

They lead to a variable mesh griding.

III. NUMERICAL EXAMPLE

A. Step responses

A linear tapered transmission line shown in Fig. 5. is analyzed. Table 1. shows the transmission line parameters. The transmission line is driven by a sine square step wave instead of a simple unit step wave to suppress high frequency components.

$$e_s(t) = \begin{cases} 0 & (t < t_1) \\ \sin^2\left(\dfrac{\pi}{2}\dfrac{t-t_1}{t_2-t_1}\right) & (t_1 < t < t_2) \\ 1 & (t_2 < t) \end{cases} \tag{15}$$

Here $t_1 = 1\text{ns}, t_2 = 1.5\text{ns}$. Because the accuracy of the analysis by the FDTD method has already been confirmed, the result when the number of partitions is taken enough by the FDTD method is assumed to be a true value. Fig. 6. and Fig. 7. show the step responses of a linear tapered transmission line. Fig. 6 shows the comparison with conventional FDTD method, and a good approximation is obtained when time step Δt is 9 times longer than the conventional FDTD method. Fig. 7 shows the comparison when the calculation point is arranged at the same position, and a good approximation is obtained when the calculation point is staggered. The computational complexity in-creases in proportion to $m/\Delta t$. Therefore, it is understood that the proposal method saves the computational complexity from Fig. 6. and 7.

B. Eye diagram analysis

Eye diagram analysis needs the long analysis time. Fig. 8. shows the eye diagram in same parameters as [7].

Table 1: Parameter of linear tapered transmission line.

$R_s = 50\Omega$	$R_d = 1000\Omega$	$d = 8cm$
$m = \dfrac{d}{\Delta x}$	$L_0 = 360nH/m$	$C_0 = 100pF/m$
$L(x) = L_0 f(x)$	$C(x) = C_0 f(x)^{-1}$	$R_0 = 100\Omega/m$
$G_0 = 0.01S/m$	$R(x) = R_0 f(x)$	$G(x) = G_0 f(x)^{-1}$
$Z(x) = \sqrt{\dfrac{L(x)}{C(x)}}$	$v_p = \dfrac{1}{\sqrt{L_0 C_0}}$	$\tau = \dfrac{d}{v_p}$
$\nu = 0.9$	$Z(0) = 60\Omega$	$Z(d) = 120\Omega$
$\Delta t = \dfrac{\tau}{m} \times \nu$	$f(x) = (1 + \varepsilon x)$	

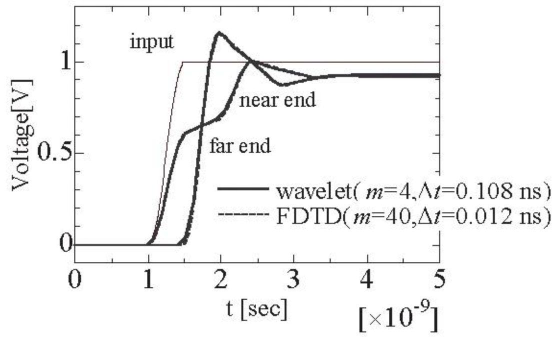

Fig. 6. Step responses of linear tapered line.

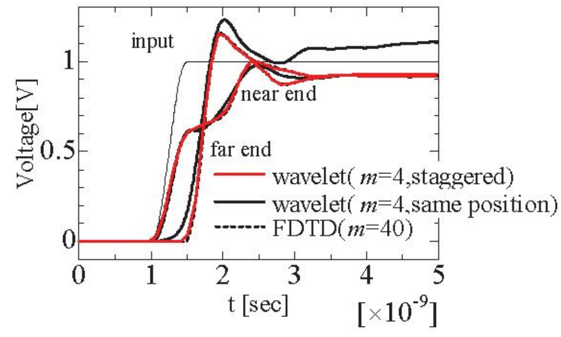

Fig. 7. Step responses of linear tapered line.

It is shown to calculate in high accuracy with stability. Because eye diagram is one line when the transmission line satisfies no distortion condition. The jitter exists as shown in (a) though transmission line is matched at both end, while it disappears as shown in (b) when

$G(x) = 1.965$ S/m is added(distortionless). However, the amplitude of the far end voltage becomes small. Then, the amplitude can be enlarged as shown in (c) by enlarging R_d and reducing R_s. When the loss of the transmission line is a little, jitter occurs because of mismatch at transmission line ends, while the loss of the transmission line is large, the cause of jitter is the multiple reflection in the transmission line. The multiple reflection can be prevented when the transmission line is made a no distortion.

IV. CONCLUSION

As the Yee lattice FDTD method has used the central difference approximation by staggered calculation lattice points of voltage and current on space and time, a good approximation has been obtained. In this paper, Yee-lattice FDTD method by using wavelet expansion for the direction of the space is proposed. And, the effectiveness of this method was shown by the step response analysis and the eye diagram analysis.

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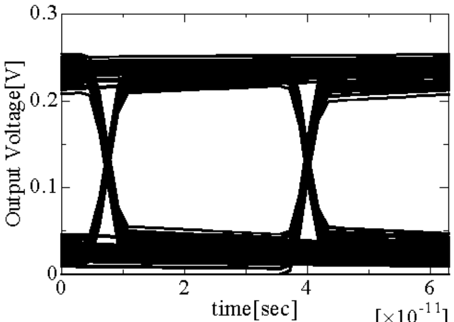
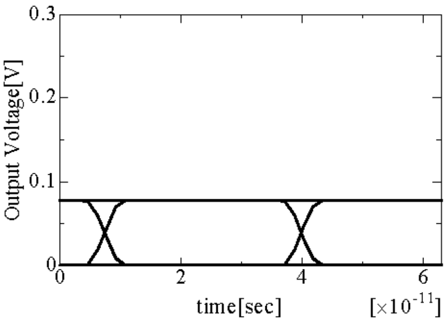
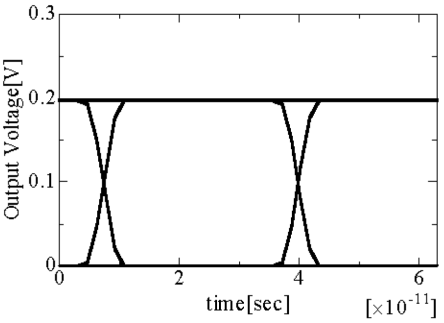


(a) matched at both end.
($G(x) = 0, R_S = R_d = Z_0 = 47.33\Omega$)



(b) matched at both end and satisfied non-distortion cond .
($G(x) = 1.965\text{S/m}, R_S = R_d = 47.33\Omega$)



(c) not matched at both end and satisfied non-distortion cond.($G(x) = 1.965\text{S/m}, R_S = 20\Omega, R_d = 1\text{M}\Omega$)

Fig. 8. Eye diagram at far end($\Delta t = 1.55$ ps).