EE535: Control of Electrical Drive Systems

Assignment 1: DC Motor Drive

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DC Motor Model

A simulation model of the permanent-magnet DC motor was built. The armature voltage u_a and the load torque T_L are the inputs of the model. The armature current i_a and the angular rotor speed ωM are the outputs of the model. The DC Motor Dynamic Equations are

$$L_a \frac{di_a}{dt} = u_a - R_a i_a - k_f \omega_M$$

$$J \frac{d\omega_M}{dt} = k_f i_a - T_L$$

1. Per unit Current and per unit Speed

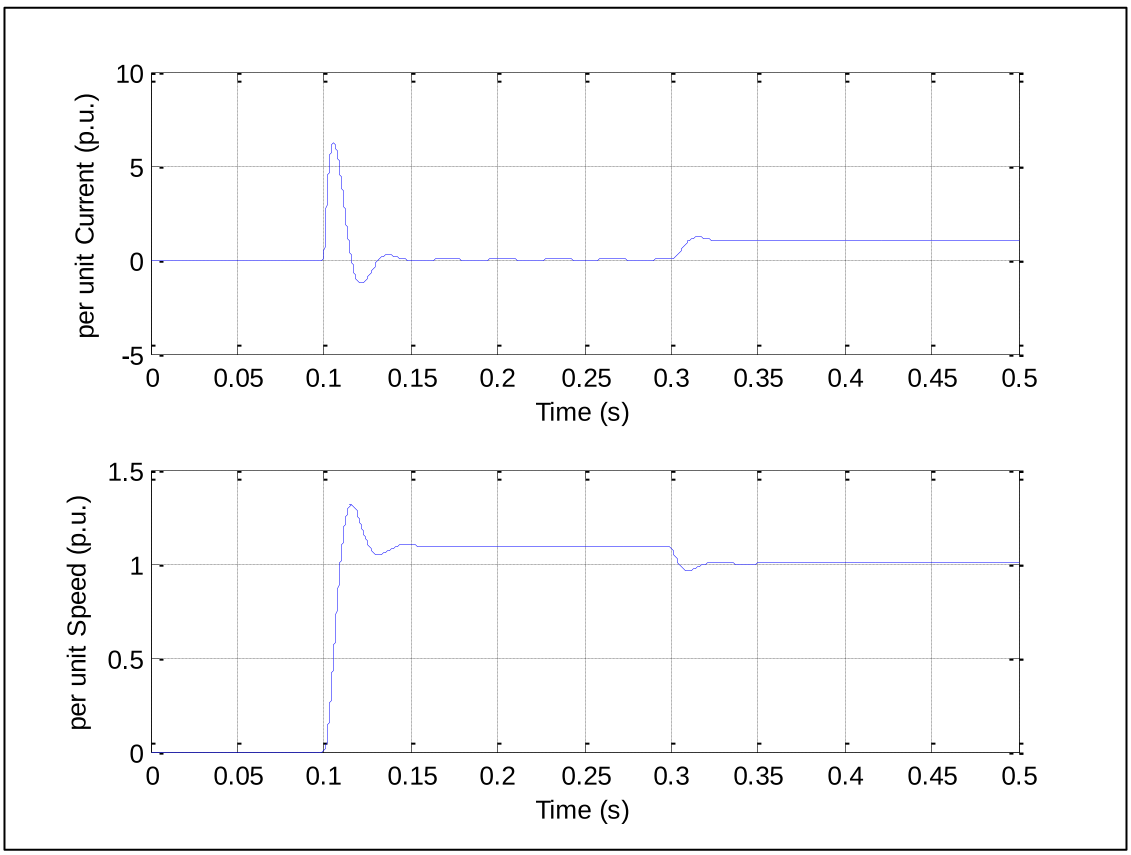

Figure 1: Per unit current and speed

DC Model Transfer Function

$$L_a s i_a(s) = u_a(s) - R_a - k_f \omega_M(s)$$

$$J s \omega_M(s) = k_f i_a(s) - T_L(s)$$

$$i_a(s) = \frac{s u_a(s)}{s^2 L_a + R_a s + \frac{k_f^2}{J}} + \frac{\frac{k_f}{J} T_L(s)}{s^2 L_a + R_a s + \frac{k_f^2}{J}}$$

$$\omega_M(s) = \frac{k_f i_a(s) - T_L(s)}{sJ}$$

u_a is a 120 V step input applied at t = 0.1 s.

$$u_a(t) = 120\, u(t - 0.1)$$

Substituting $u_a(s) = \frac{120.e^{-0.1s}}{s}$ in $i_a(s)$.

$$i_a(s) = \frac{120.\, e^{-0.1s}}{s^2 L_a + R_a s + \frac{k_f^2}{J}} = \frac{(-121.5i).\, e^{-0.1s}}{s - (-100 + 197.48i)} + \frac{(121.5i).\, e^{-0.1s}}{s - (-100 - 197.48i)}$$

Hence

$$i_a(t) = (-121.5i) e^{(-100+197.48i)(t-0.1)} + (121.5i) e^{(-100-197.48i)(t-0.1)}$$

$$i_a(t) = 243 e^{(-100)(t-0.1)} \sin\big(197.48(t - 0.1)\big).\, u(t - 0.1)$$

Which represents a decaying sinusoid starting at t = 0.1s. Hence the simulation results are correct and thus a peak is seen in i_a when u_a is applied.

2. Steady State per unit Current and per unit Speed

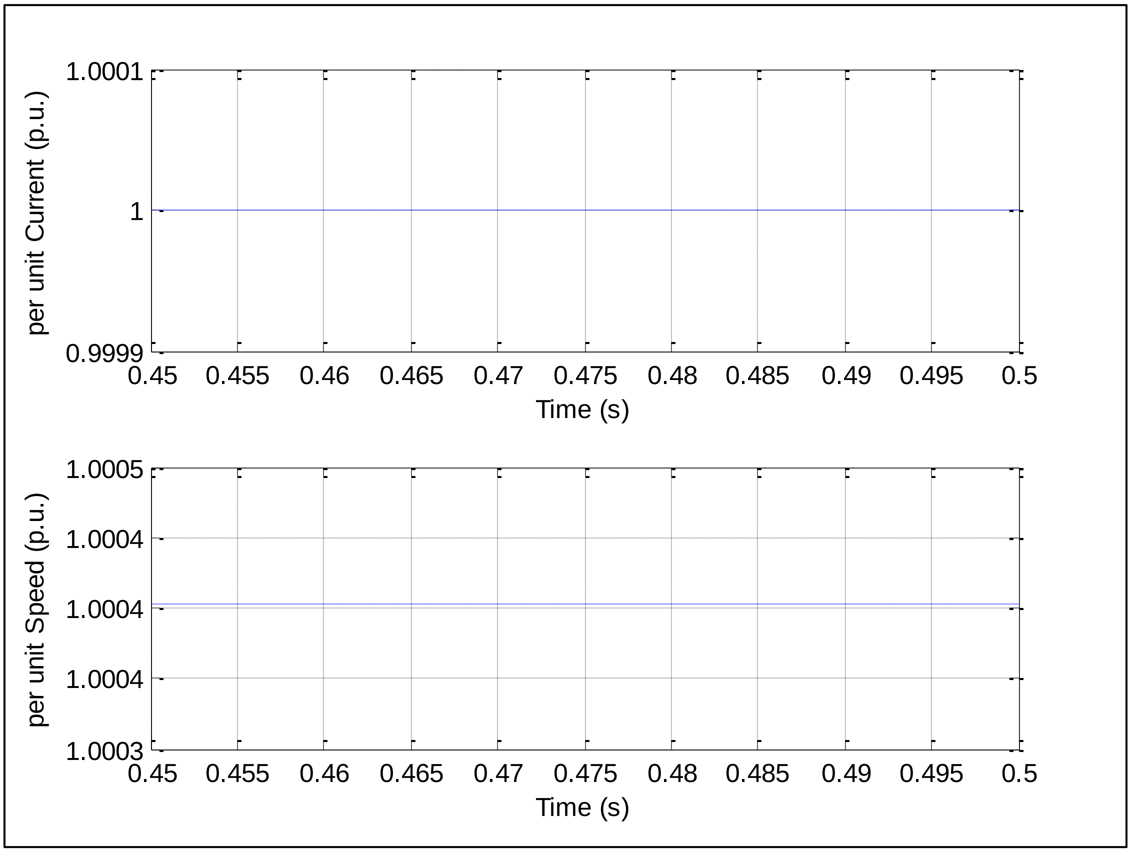

Figure 2: Steady state current and speed

<u>Steady State Values</u>

In steady state,

$$L_a \frac{di_a}{dt} = 0$$

$$J \frac{d\omega_M}{dt} = 0$$

$$i_{a,ss} = \frac{T_L}{k_f} = \frac{7}{0.35} = 20\ A\ (1\ p.u.)$$

$$\omega_{M,ss} = \frac{u_a - i_{a,ss}R_a}{k_f} = \frac{120 - (20)(0.5)}{0.35} = 100\pi \frac{rad}{s} = 3000\ \frac{rev}{min}(1.0\ p.u.)$$

The calculations match with the steady state simulation results in Figure 2.

3. Limiting Rising Rate of Voltage

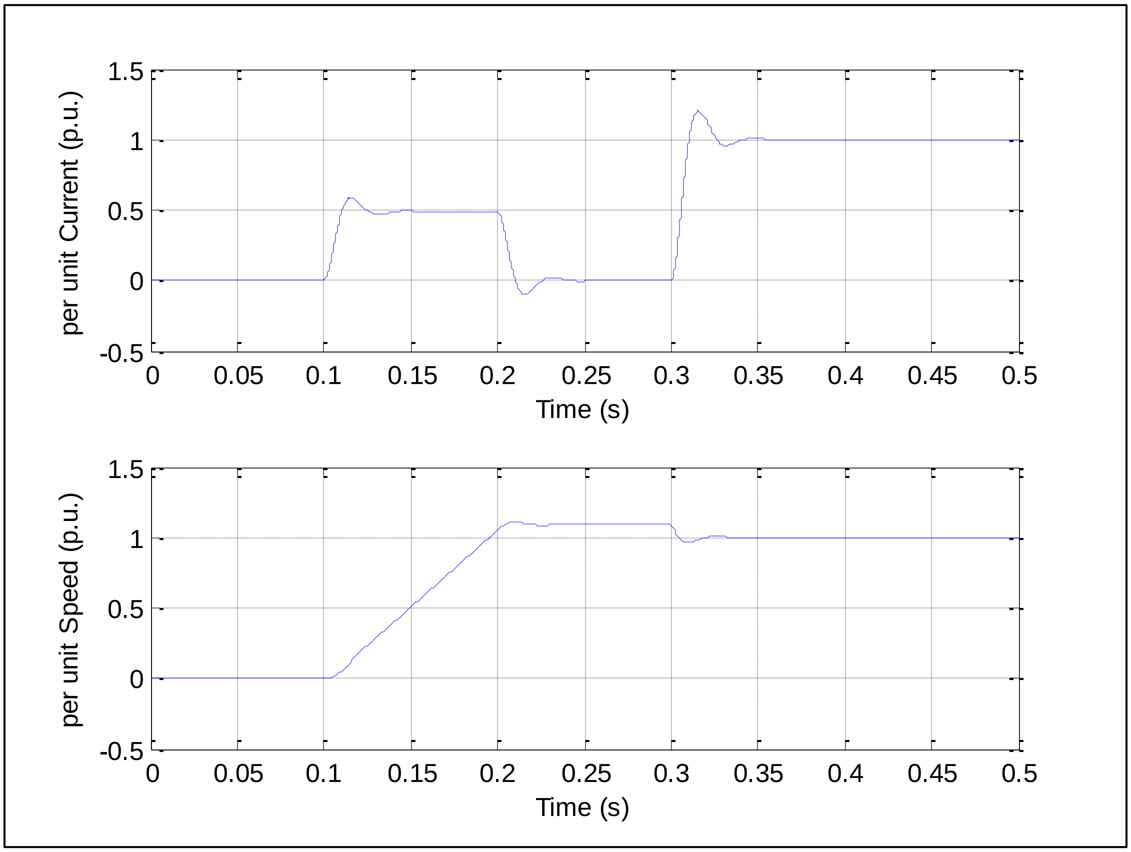

Figure 3: Per unit current and speed

<u>Effect of Rate Limiter</u>

$$i_a(s) = \frac{s u_a(s)}{s^2 L_a + R_a s + \frac{k_f^2}{J}} + \frac{\frac{k_f}{J} T_L(s)}{s^2 L_a + R_a s + \frac{k_f^2}{J}}$$

$$\omega_M(s) = \frac{k_f i_a(s) - T_L(s)}{sJ}$$

u_a is a 120 V / 0.1 s ramp starting at t = 0.1 s and ending at t = 0.2 s.

Hence, $u_a(s) = \frac{1200.e^{-0.1s}}{s^2} - \frac{1200.e^{-0.2s}}{s^2}$

$$i_a(s) = \frac{1200.e^{-0.1s}}{s(s^2 L_a + R_a s + \frac{k_f^2}{J})} - \frac{1200.e^{-0.2s}}{s(s^2 L_a + R_a s + \frac{k_f^2}{J})}$$

$$i_a(s) = \frac{9.7959.e^{-0.1s}}{s} + \frac{(-4.898 + 2.4802i).e^{-0.1s}}{s - (-100 + 197.48i)} + \frac{(-4.898 - 2.4802i).e^{-0.1s}}{s - (-100 - 197.48i)} - \frac{9.7959.e^{-0.2s}}{s} - \frac{(-4.898 + 2.4802i).e^{-0.2s}}{s - (-100 + 197.48i)} - \frac{(-4.898 - 2.4802i).e^{-0.2s}}{s - (-100 - 197.48i)}$$

$$\begin{aligned} i_a(t) = 9.796.u(t - 0.1) - 9.796.u(t - 0.2) \\ - 4.960e^{-100(t-0.1)}\sin(197.48(t - 0.1).u(t - 0.1) \\ - 9.796e^{-100(t-0.1)}\cos(197.48(t - 0.1).u(t - 0.1) \\ + 4.960e^{-100(t-0.2)}\sin(197.48(t - 0.2).u(t - 0.2) \\ + 9.796e^{-100(t-0.2)}\cos(197.48(t - 0.2).u(t - 0.2) \end{aligned}$$

Which represents a pulse starting at t = 0.1 s and ending at t = 0.2 s. The current also has an exponentially decaying sinusoid starting at t = 0.1s, an exponentially decaying cosine staring at t = 0.1 s, an exponentially decaying sinusoid starting at t = 0.2 s and an exponentially decaying cosine staring at t = 0.2 s. It can be verified using this equation that $i_a(t)$ = 0 A (0 p.u.) at t = 0.1 s and $i_a(t)$ = 9.796 A (0.4898 p.u.) at t = 0.2 s. This matches with the simulation result.

Since,

$$\omega_M(s) = \frac{k_f i_a(s)}{sJ}$$

$$\omega_M(t) = \frac{k_f}{J}\int_0^t i_a(s)dt$$

$$\begin{aligned} \omega_M(t) = 3428.5r(t - 0.1) - 3428.5r(t - 0.2) \\ + 14.0e^{-100(t-0.1)}\cos(197.48(t - 0.1).u(t - 0.1) \\ - 10.2e^{-100(t-0.1)}\sin(197.48(t - 0.1).u(t - 0.1) \\ + 14.0e^{-100(t-0.2)}\cos(197.48(t - 0.2).u(t - 0.2) \\ - 10.2e^{-100(t-0.2)}\sin(197.48(t - 0.2).u(t - 0.2) \end{aligned}$$

Hence, $\omega_M(t)$ is the scaled integral of current. It is a ramp function starting at t = 0.1 s and ending at t = 0.2 s; because current was a pulse function starting at t = 0.1 s and ending at t = 0.2 s. It also has an exponentially decaying cosine function starting at t = 0.1 s, an exponentially decaying sine function starting at t = 0.1 s, an exponentially decaying cosine function starting at t = 0.2 s and an exponentially decaying sine function starting at t = 0.2 s. These results also match with the simulation results.

DC-DC Converter and Unipolar PWM

4. Testing the Model

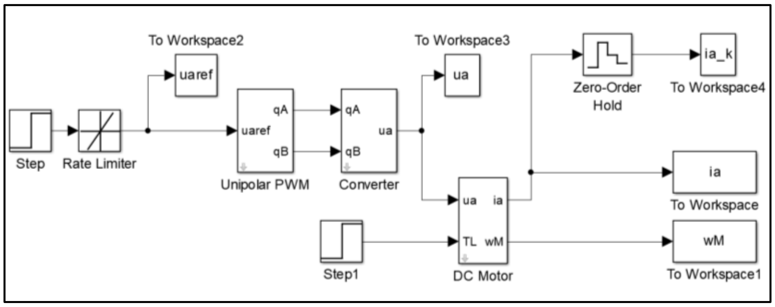

Figure 4: Voltage Control based Model

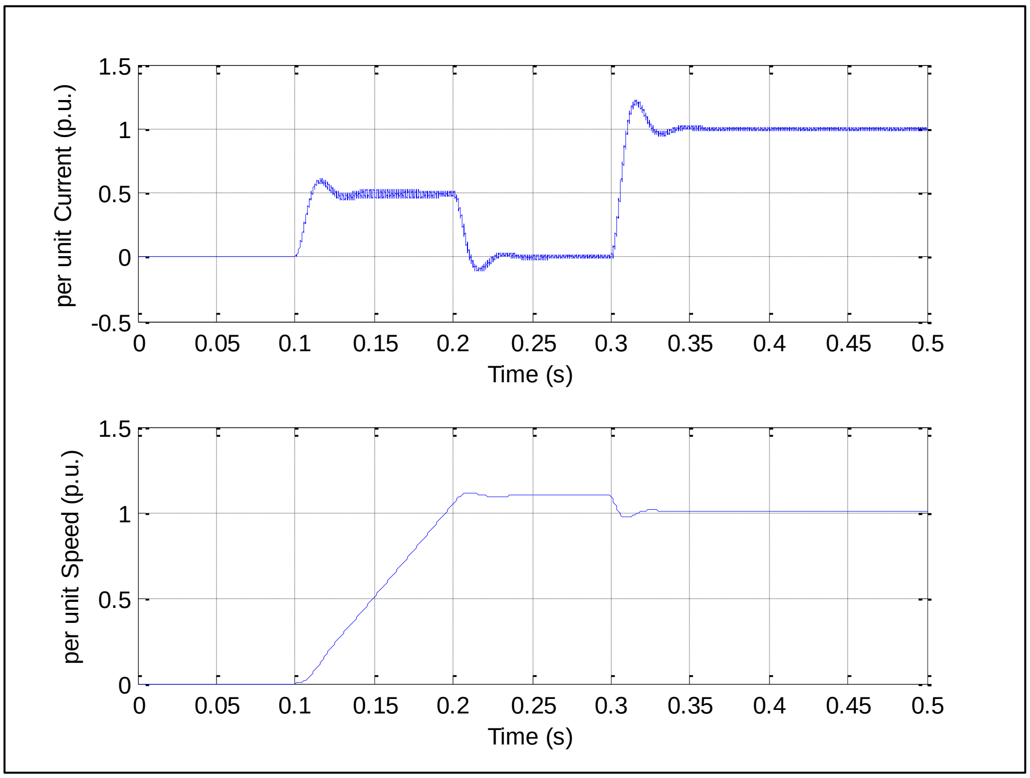

Figure 5: Per unit current and speed

In this simulation, u_a is supplied using PWM technique. The Average value of u_a is equal to the reference u_a. This is achieved by changing the duty cycles d_A and d_B. The results of current and speed are almost the same as the last section, where an ideal voltage source was used. The current has much more ripple around the mean value as compared to the last case. This is because the current graph has a much thicker line during transition periods. This is when unipolar PWM is adjusting the duty cycles to reach steady state current. The adjustments cause current to oscillate around the mean value. The Moment of Inertia is quite high hence the speed is unaffected by current oscillations. The speed graph is just like the earlier case.

5. Plotting Armature Voltage and Current

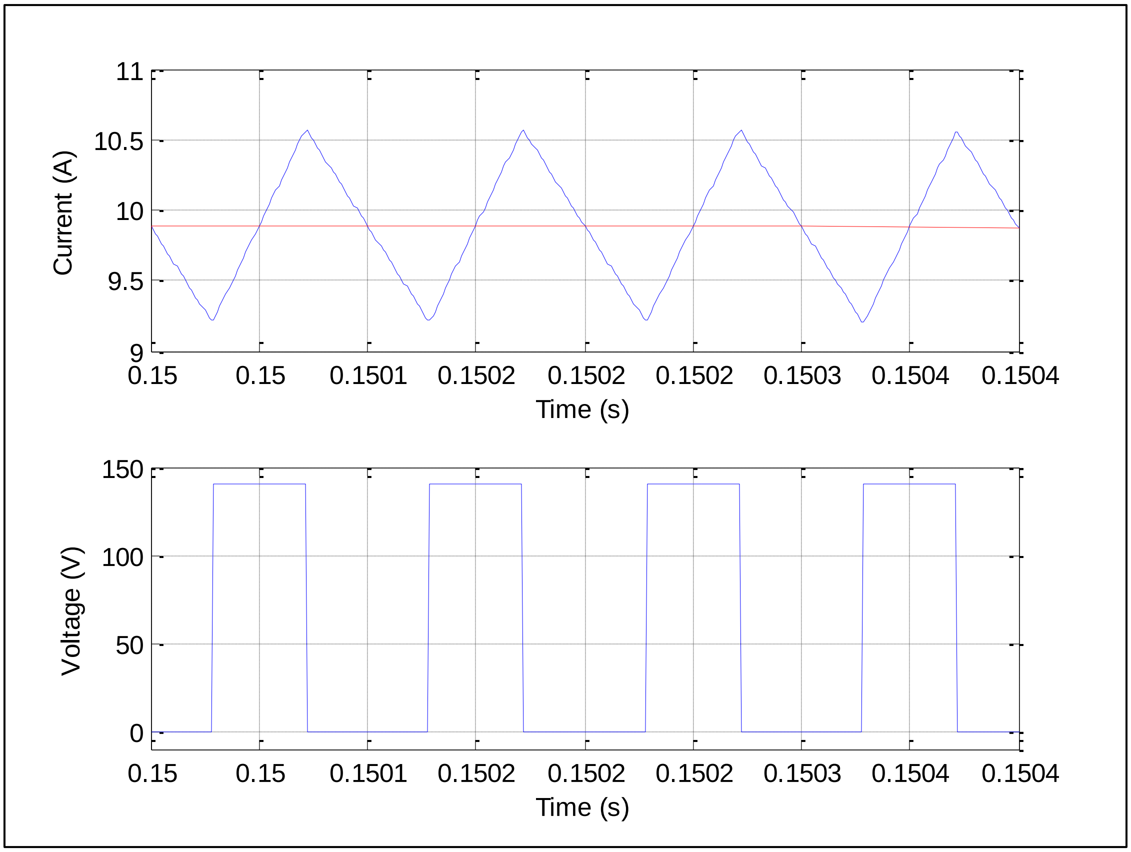

Figure 6: Steady state current and speed

The Armature current i_a (blue) is a triangular wave because L_a charges up (current rises) when converter output u_a is high; and L_a discharges (current deceases) when converter output becomes zero. The u_a on/off sequence is such that the current rises and drops by equal amounts in steady state. Hence the sampled average current i_{a_k} (red) seems constant.

Cascaded Control

6. Current Control

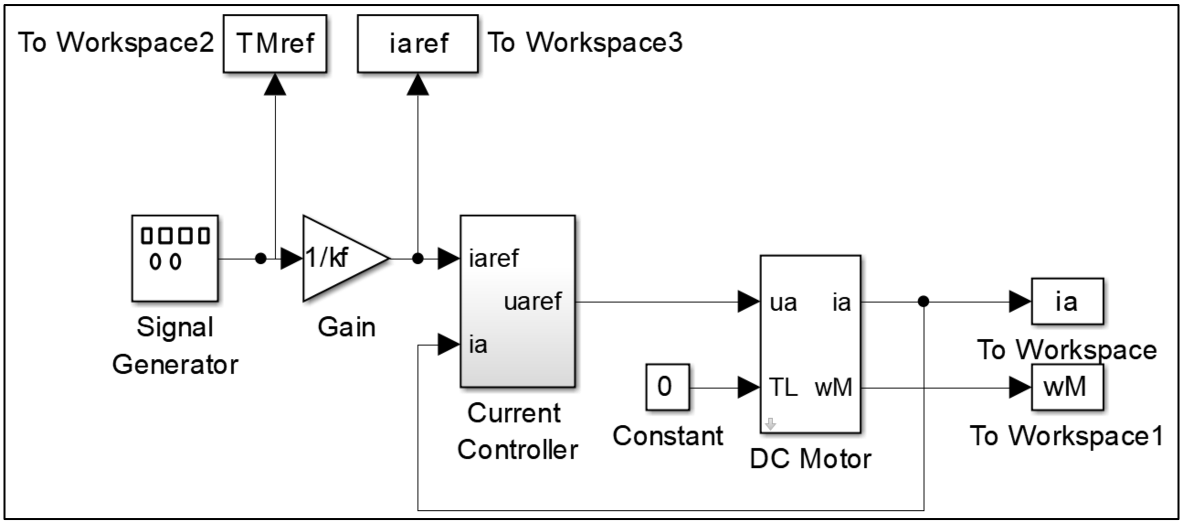

Figure 7: Current Control Loop Model

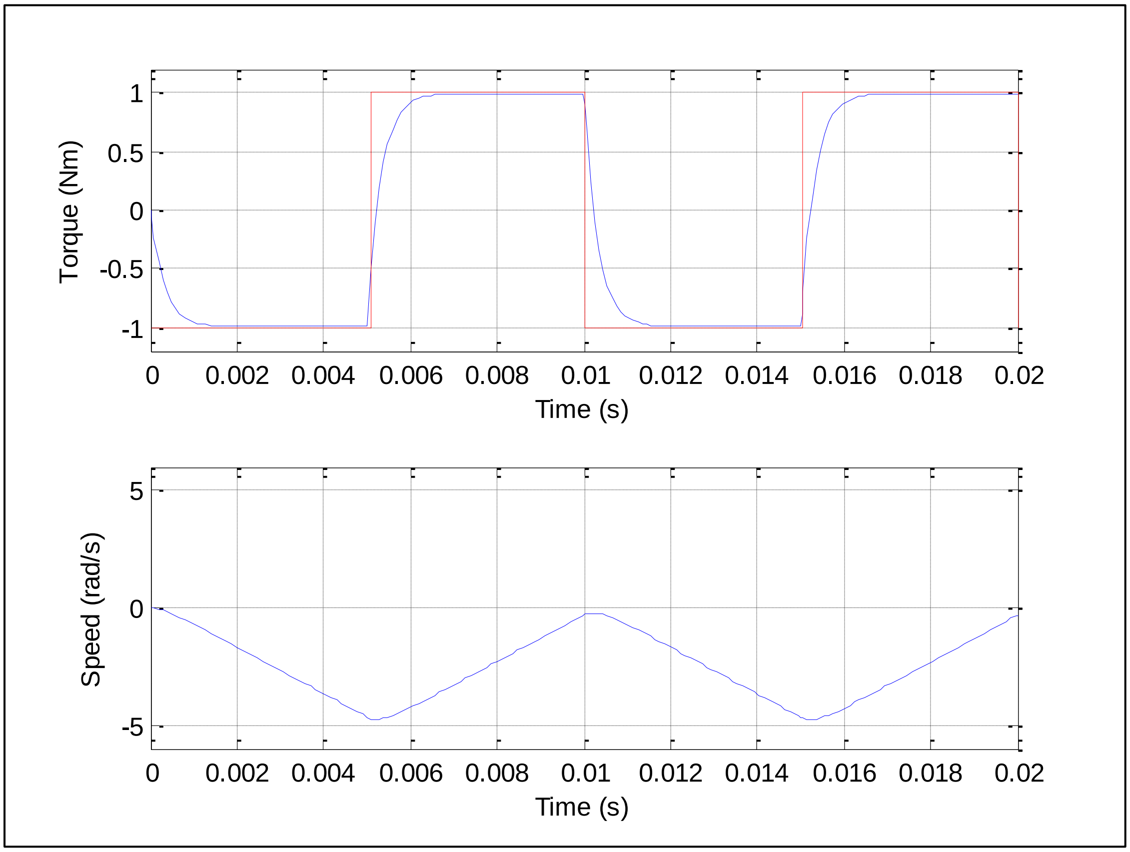

Figure 8: Torque and Speed Response

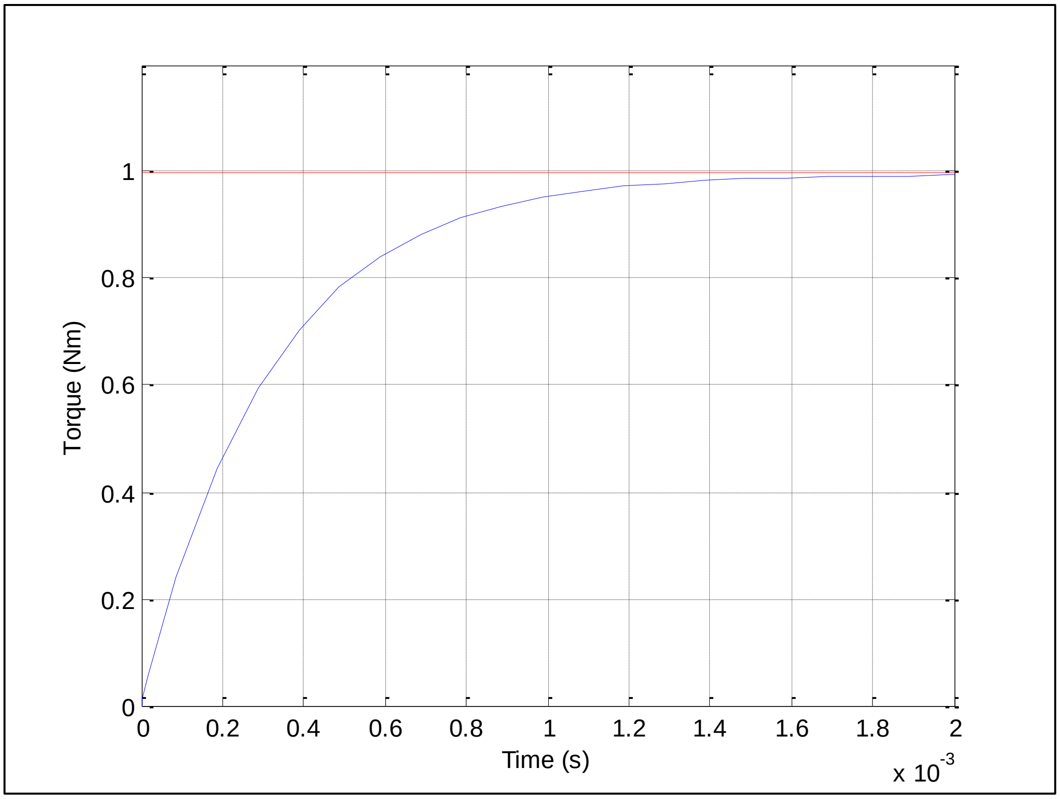

Figure 9: Torque Step Response

From the torque step response (Fig. 9), the rise time (from $T_M = 0.1$Nm to $T_M = 0.9$ Nm) is 0.709 ms.

$$H_{c,desired}(s) = \frac{i_a(s)}{i_{a,ref}(s)} = \frac{\alpha_c}{s + \alpha_c}$$

$$T_{M,ref}(s) = k_f i_{a,ref}(s)$$

$$T_M(s) = k_f i_a(s); \qquad T_L(s) = 0$$

$$\frac{T_M(s)}{T_{M,ref}(s)} = \frac{\alpha_c}{s + \alpha_c}$$

$$T_{M,ref}(s) = \frac{1}{s}$$

$$T_M(s) = \frac{\alpha_c}{s(s + \alpha_c)} = \frac{-1}{s + 1000\pi} + \frac{1}{s}$$

$$T_M(t) = -e^{-1000\pi t} + u(t)$$

The Time Constant of Torque is $\frac{1}{1000\pi}$. The rise time is $\frac{1}{1000\pi} * (\ln(0.9) - \ln(0.1)) = 0.6994$ ms. Hence the simulation result is correct and desired bandwidth has been achieved.

Cascaded Control

7. Transfer Function

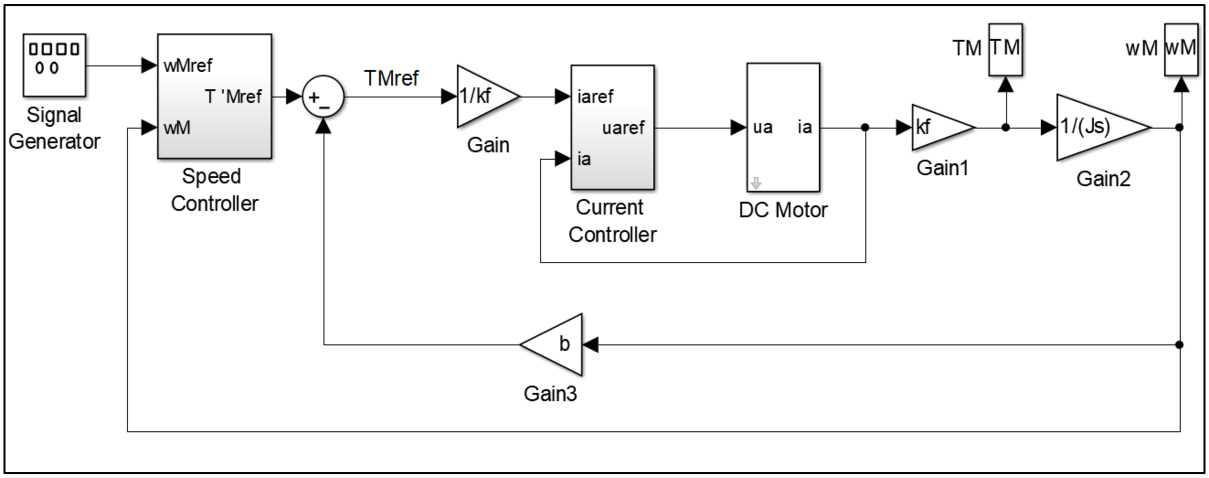

Figure 10: Simplified Overall System

The Current Control loop has the transfer function

$$H_{c,desired}(s) = \frac{i_a(s)}{i_{a,ref}(s)} = \frac{\alpha_c}{s+\alpha_c}$$

$$T_{M,ref} = k_f i_{a,ref}(s)$$

$$T_M(s) = k_f i_a(s); \qquad T_L(s) = 0$$

$$\frac{T_M(s)}{T_{M,ref}(s)} = \frac{\alpha_c}{s+\alpha_c}$$

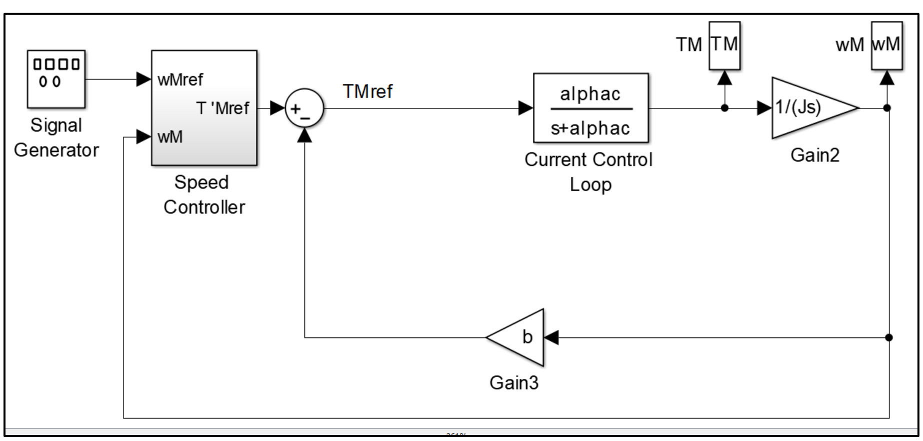

Figure 11: Simplified Overall System

Assuming ideal torque control,

$$T_M(s) = T_{M,ref}(s)$$

$$\frac{T_M(s)}{T_{M,ref}(s)} = 1$$

Since,

$$T_M(s) = Js\omega_M(s)$$

$$H(s) = \frac{\omega_M(s)}{T_{M,ref}(s)} = \frac{1}{Js}$$

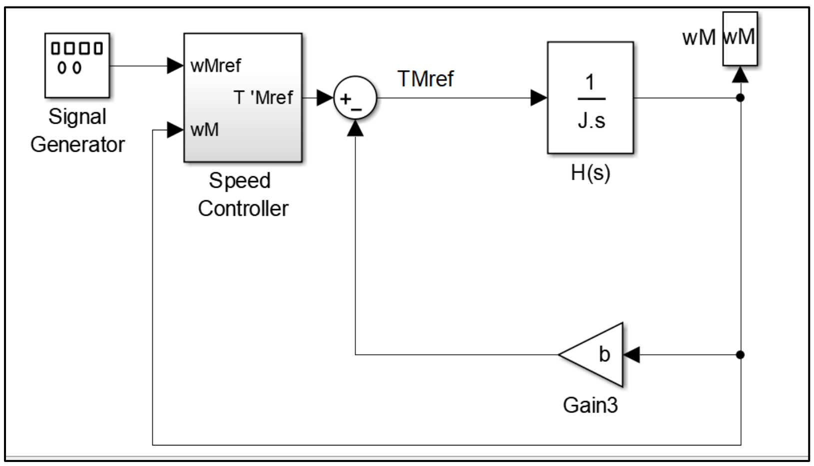

Figure 12: Simplified Overall System

Using the equation,

$$T'_{M,ref} = T_{M,ref}(s) + b\omega_M(s)$$

$$G'(s) = \frac{\omega_M(s)}{T'_{M,ref}} = \frac{H(s)}{1 + bH(s)} = \frac{1}{Js + b}$$

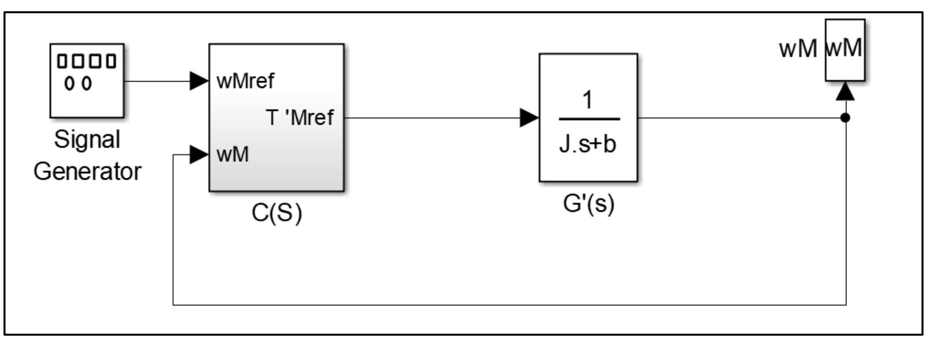

Figure 13:Simplified Overall System

The speed PI controller has desired transfer function C(s)= $\frac{T'_{M,ref}}{\omega_{M,ref}(s)}$. Hence the desired speed control loop transfer function is

$$H_{s,desired}(s) = \frac{\omega_M(s)}{\omega_{M,ref}(s)} = \frac{C(s)G'(s)}{1 + C(s)G'(s)} = \frac{\alpha_s}{s + \alpha_s}$$

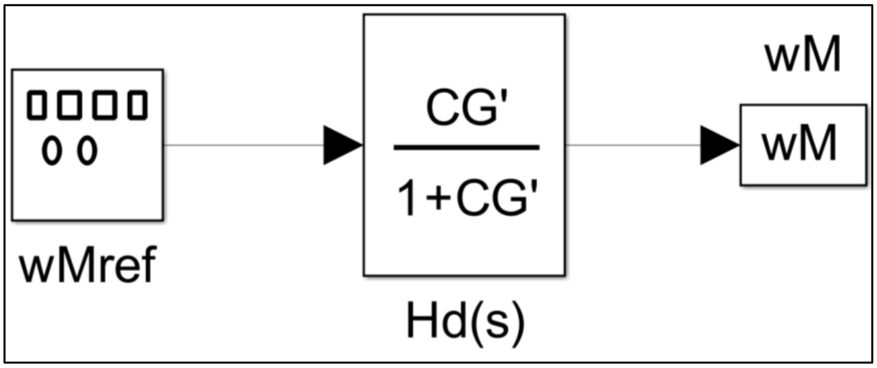

Figure 14:Simplified Overall System

Hence,

$$\frac{\alpha_s}{s} = C(s)G'(s)$$

$$C(s) = \frac{T'_{M,ref}}{\omega_{M,ref}(s)} = \frac{\alpha_s}{sG'(s)} = \frac{\alpha_s(Js + b)}{s} = \alpha_s J + \frac{b\alpha_s}{s} = K_{ps} + \frac{K_{is}}{s}$$

At $s = i\alpha_s$,

$$|C(i\alpha_s)G'(i\alpha_s)| = 1$$

$$|\frac{K_{ps} + \frac{K_{is}}{s}}{(Js + b)}| = |\frac{K_{ps}(i\alpha_s) + K_{is}}{(i\alpha_s)(J(i\alpha_s) + b)}| = 1$$

$$K_{is} = J\alpha_s^2, K_{ps} = J\alpha_s, b = J\alpha_s$$

8. Testing the Model

$$C(s) = \frac{T'_{M,ref}}{\omega_{M,ref}(s)} = \alpha_s J + \frac{b\alpha_s}{s}$$

$$\alpha_s = \frac{\alpha_c}{10} = 100\pi$$

$$b = \alpha_s J = 0.1\,\pi$$

$$K_{ps} = \alpha_s J = 0.1\,\pi$$

$$K_{is} = \alpha_s^2 J = 10\,\pi^2$$

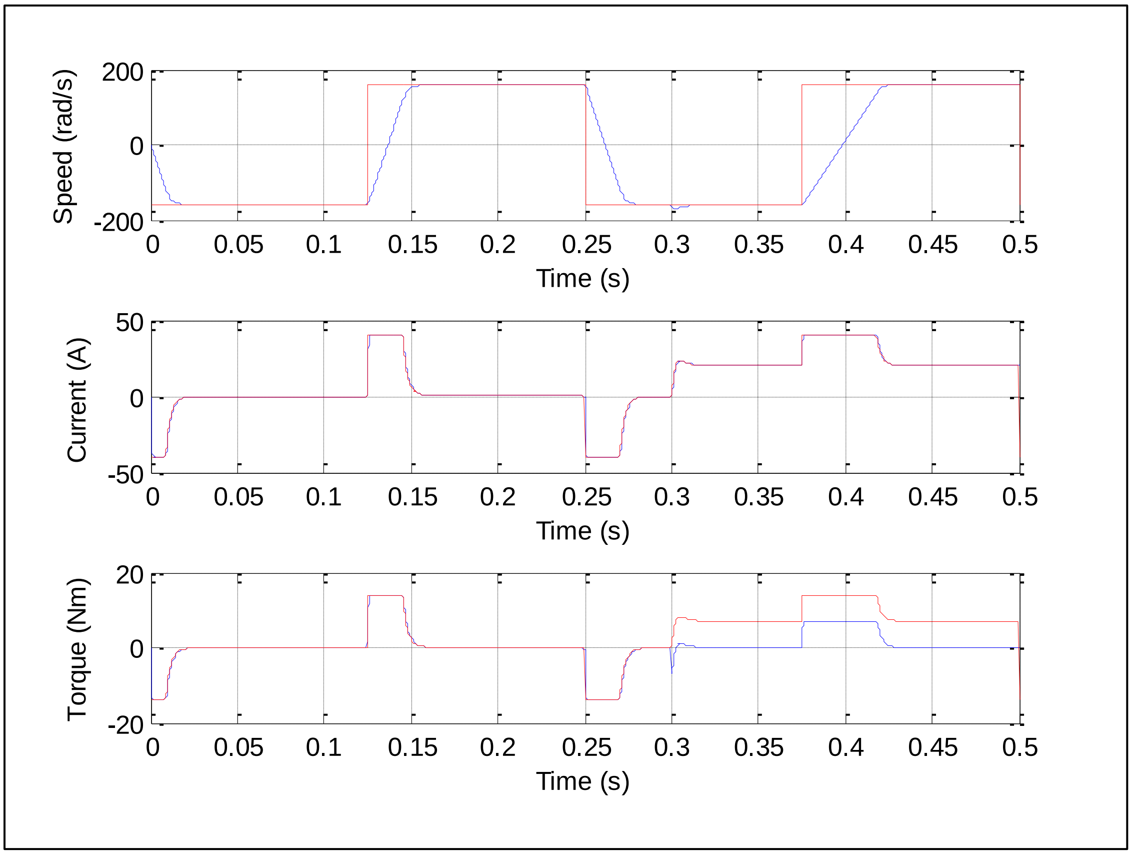

Figure 15: Speed, Current and Torque Response

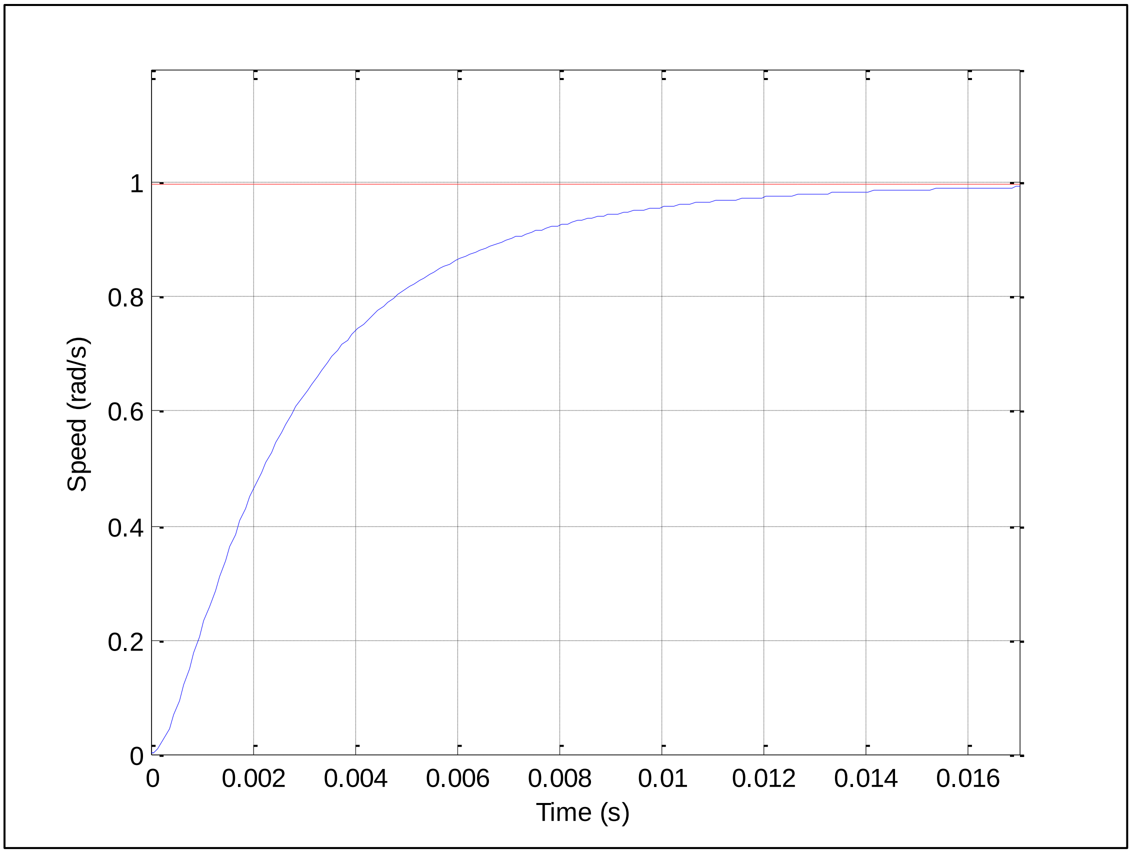

Figure 16: Speed Step Response

From the simulation results, the rise time (from ωM = 0.1 to rad/s = 0.9 rad/s) is 6.4 ms.

The theoretical rise time for ωM is:

$$\frac{1}{\alpha_s} * (\ln(0.9) - \ln(0.1)) = \frac{1}{100\pi} * (\ln(0.9) - \ln(0.1)) = \text{ 6.994 ms}$$

Hence the result matches with the simulation. The speed controller has the desired bandwidth.

9. Tolerances

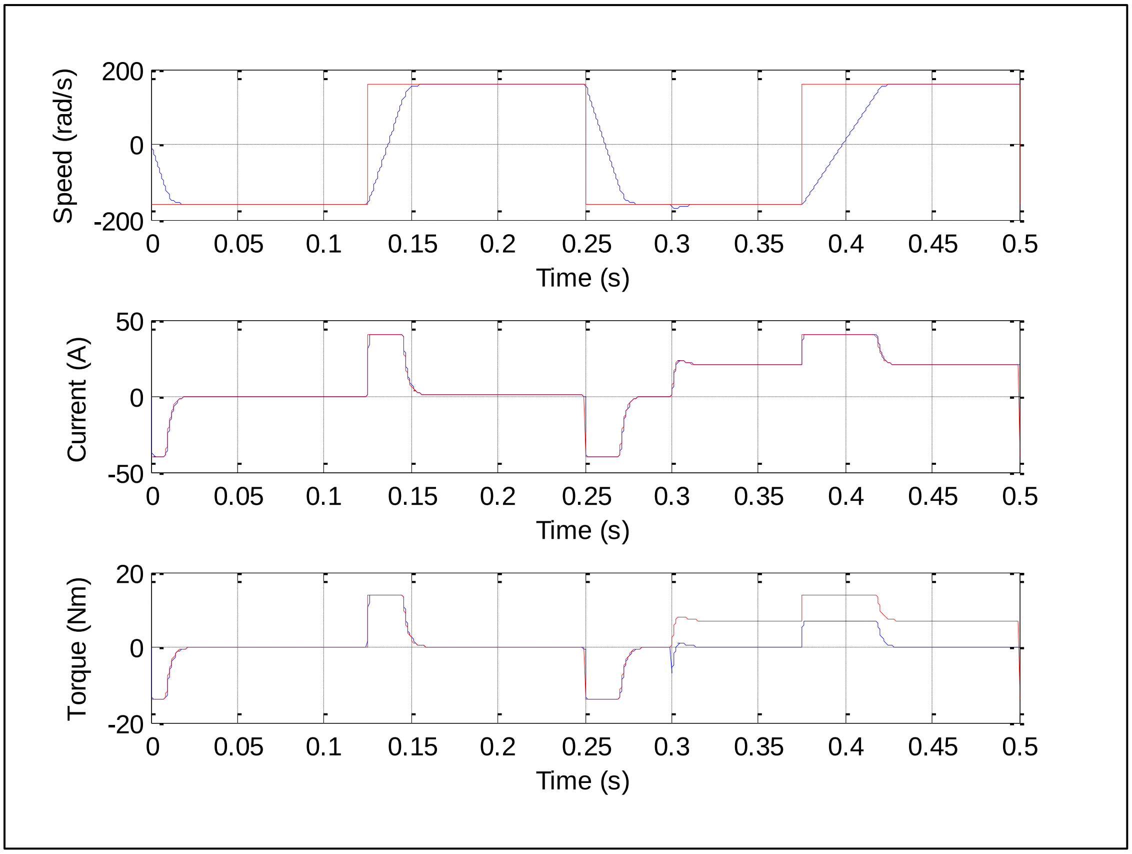

Figure 17: Speed, Current and Torque Response

The 2-DOF PI controller parameters were designed for the original values of Ra and La. The controllers provided desired closed loop bandwidth using perfect pole-zero cancellation.

$$K_{ic} = L_a \alpha_c^2, \qquad K_{pc} = L_a \alpha_c, \qquad r = L_a \alpha_c - R_a$$

With temperature changes, Ra increased; and magnetic saturation decreased La. However, the Speed, current and Torque responses are the same as before. This indicates that the controller is very robust against parameter errors. The PI speed and current controllers have been affected very little by the parameter errors. Although imperfect pole-zero cancellation occurs due to the 2-DOF PI controllers, they are still able to generate precise control commands to track speed, current and Torque references.

10. Removing Anti-windup from Speed Controller

The integral action in PI controller is an unstable mode if input error to the controller is large or the input error remains nonzero for a long time. This can cause saturation of controller leading to delayed response. The Anti-windup minimizes performance degradation by accounting for the saturation and reducing integrator input. This prevents the error from accumulating.

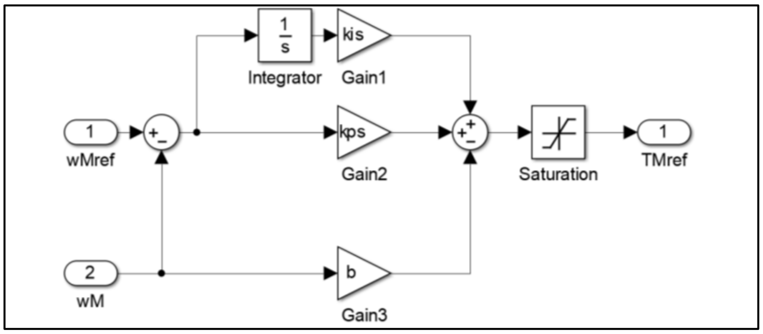

Figure 18: Speed Controller without Anti-windup

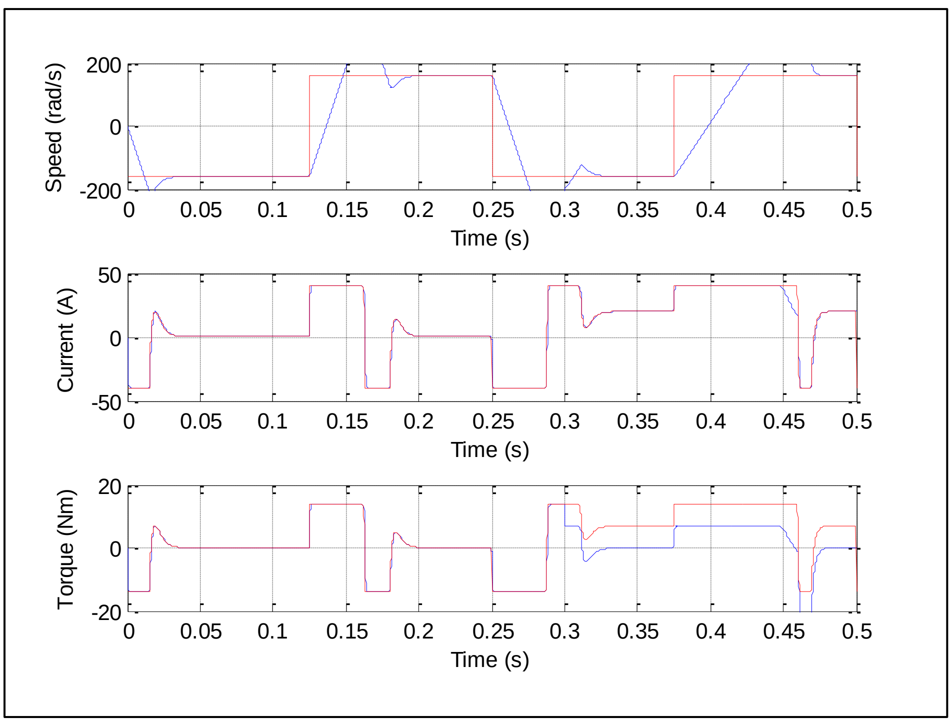

Figure 19: Speed, Current and Torque Response

When the anti-windup is removed, the integrator is not informed about saturating output signal. Hence integrator effect is not fully implemented in the system, because the output gets saturated. The integrator then keeps accumulating the error over a longer period resulting in a delayed response and overshoots in Torque and current references. High oscillations also occur in the speed, current and torque response due to accumulation of error in integrator. The resulting responses are much different from earlier case with anti-windup.