Lecture #4

In last lecture we discussed 180° conduction (Square wave) inverter

State #	Switching States	V_{ab}	V_{bc}	V_{ca}	V_{an}	V_{bn}	V_{cn}	Space Vector Line voltage	Phase voltages
1	1 0 0	V_s	0	$-V_s$	$\frac{2}{3}V_{dc}$	$-\frac{1}{3}V_{dc}$	$-\frac{1}{3}V_{dc}$	$\frac{2}{\sqrt{3}}\angle 30^\circ$	$2/3\angle 0^\circ$
2	1 1 0	0	$+V_s$	$-V_s$	$\frac{1}{3}V_{dc}$	$\frac{1}{3}V_{dc}$	$-\frac{2}{3}V_{dc}$	$2/\sqrt{3}\angle 90^\circ$	$2/3\angle 60^\circ$
3	0 1 0	$-V_s$	V_s	0	$-1/3$	$2/3$	$-1/3$	$2/\sqrt{3}\angle 150^\circ$	$2/3\angle 120^\circ$
4	0 1 1	$-V_s$	0	V_s	$-2/3$	$1/3$	$1/3$	$2/\sqrt{3}\angle 210$	$2/3\angle 180^\circ$
5	0 0 1	0	$-V_s$	V_s	$-1/3$	$-1/3$	$2/3$	$2/\sqrt{3}\angle 270$	$2/3\angle 240$
6	1 0 1	V_s	$-V_s$	0	$1/3$	$-2/3$	$1/3$	$2/\sqrt{3}\angle 330^\circ$	$2/3\angle 300^\circ$
7	1 1 1	0	0	0	0	0	0	0	0
8	0 0 0	0	0	0	0	0	0	0	0.

State 1: (1 0 0)

$$Q_s = \frac{2}{3}\left[1 + 0 \cancel{+} 1 e^{j2\pi/3}\right]$$

$$= \frac{2}{3}\left[\sqrt{3}\angle 30^\circ\right] = \frac{2}{\sqrt{3}}\angle 30^\circ$$

State 2: (1 1 0)

$$Q_s = \frac{2}{3}\left[\cancel{1} + e^{j2\pi/3} \cancel{+} e^{j4\pi/3}\right]$$

$$Q_s = \frac{2}{3}\left[\sqrt{3}\angle 90^\circ\right] = \frac{2}{\sqrt{3}}\angle 90^\circ$$

State 3: (0 1 0)

$$Q_s = \frac{2}{3}\left[-1 + e^{j2\pi/3}\right]$$

$$= \frac{2}{3}\left[\sqrt{3}\angle 150^\circ\right] = \frac{2}{\sqrt{3}}\angle 150^\circ$$

State 4: (0 1 1)

$$Q_s = \frac{2}{\sqrt{3}}\angle 210$$

State 5:

$$Q_s = \frac{2}{\sqrt{3}}\angle 270$$

State 6

$$Q_s = \frac{2}{3}\angle 330^\circ$$

State 1:

$$Q_s = \frac{2}{3}\left[\frac{2}{3} - \frac{1}{3}e^{j2\pi/3} - \frac{1}{3}e^{j4\pi/3}\right]$$

$$Q_s = \frac{2}{3}\angle 0^\circ$$

State 2

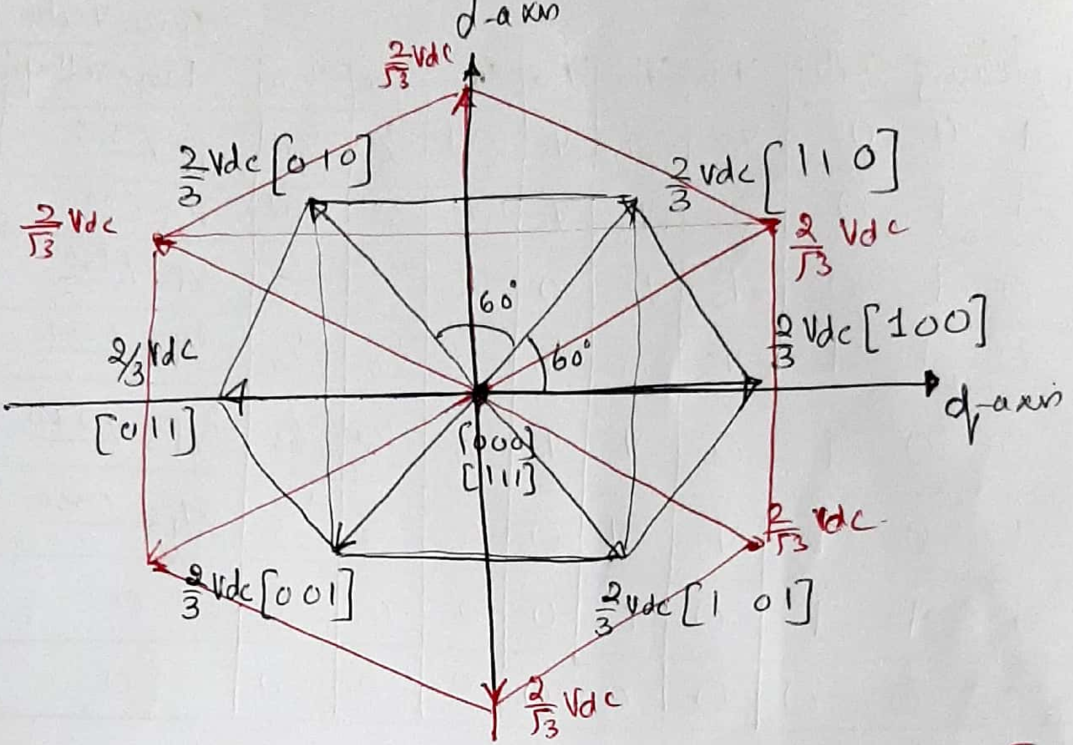
$$Q_s = \frac{2}{3}\left[\frac{1}{3} + \frac{1}{3}e^{j2\pi/3} - \frac{2}{3}e^{j4\pi/3}\right]$$

$$Q_s = \frac{2}{3}\angle 60^\circ$$

In general

$$V_k = \begin{cases} \frac{2}{3}e^{j(v-1)\pi/3} & v = 1, 2, 3, \dots 6 \\ 0 & v = 0, 7 \end{cases}$$

- So the space vector representation for phase voltages



- Red colour shows the position of space vector of line voltage. (higher magnitude && phase difference of 30° with respect to phase voltage space vector)

- Space vector is based upon balancing of volt second product of reference vector with adjacent vectors of the sectors.

ime Calculation

Consider the reference vector in sector 1 as shown in the figure

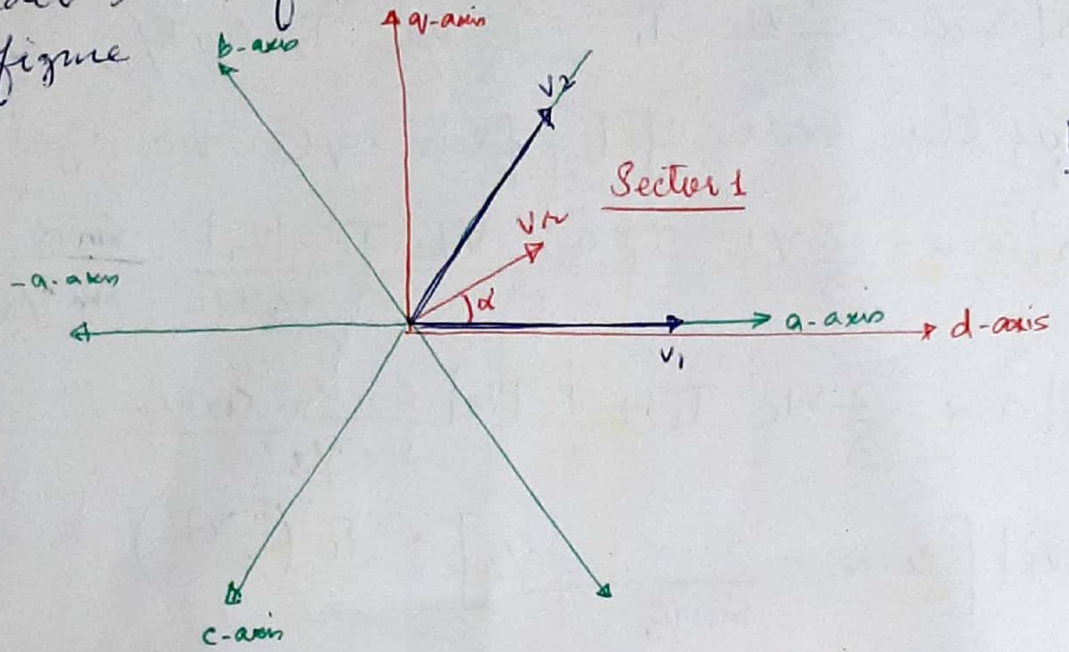


Figure A

- Volt second product in sector 1 can be written as

$$\vec{V}_r \times T_s = \vec{V}_1 \times T_1 + \vec{V}_2 \times T_2 + \vec{V}_z \times T_0 \quad \text{—} (1)$$

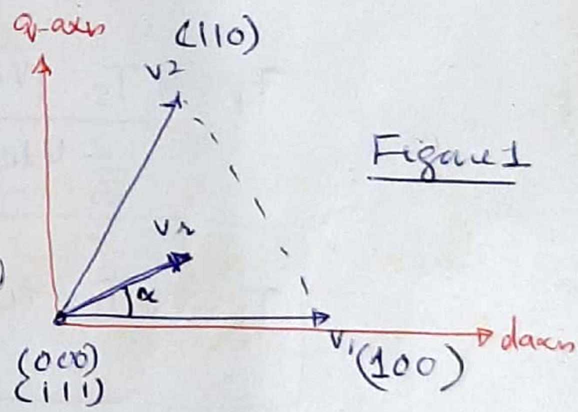


Figure 1

$V_r = |V_r|\cos\alpha + j\,|V_r|\sin\alpha$

$V_1 = \frac{2}{3}V_{dc} + j(0)$

$V_2 = \frac{2}{3}V_{dc}\cos\pi/3 + j\frac{2}{3}V_{dc}\sin\pi/3$

- equation (1) can be written as

$$T_s|V_r|\begin{bmatrix}\cos\alpha \\ \sin\alpha\end{bmatrix} = T_1\left(\frac{2}{3}V_{dc}\right)\begin{bmatrix}1 \\ 0\end{bmatrix} + T_2\left(\frac{2}{3}V_{dc}\right)\begin{bmatrix}\cos\pi/3 \\ \sin\pi/3\end{bmatrix} + 0\times T_0 \quad \text{—} (2)$$

from eq (2)

$$T_s|V_r|\sin\alpha = T_2\left(\frac{2}{3}V_{dc}\right)\sin\pi/3$$

$$T_2 = \frac{|V_r|}{\left[\frac{2}{3}V_{dc}\right]}\cdot T_s\cdot\frac{\sin\alpha}{\sin\pi/3}$$

$$\boxed{T_2 = T_s\cdot a\,\frac{\sin\alpha}{\sin\pi/3}} \quad \text{—} (3) \qquad \text{where } a = \frac{|V_r|}{\left[\frac{2}{3}V_{dc}\right]}$$

from eq (2)

$$T_s |V_s| \cos\alpha = \frac{2}{3}V_{dc} \cdot T_1 + \frac{2}{3}V_{dc} \cdot T_2 \cos \pi/3$$

placing the value of T_2 from eq (3) we get

$$T_s |V_s| \cos\alpha = \frac{2}{3}V_{dc} \cdot T_1 + \frac{2}{3} V_{dc} \cdot T_s \cdot \frac{|V_s|}{2/3 V_{dc}} \cdot \frac{\sin\alpha}{\sin \pi/3} \cos \pi/3$$

$$T_s |V_s| \cos\alpha = \frac{2}{3}V_{dc} \cdot T_1 + T_s |V_s| \frac{\sin\alpha}{\sin \pi/3} \cdot \cos \pi/3$$

$$T_s |V_s| \left[\cos\alpha - \frac{\sin\alpha}{\sin\pi/3} \cdot \cos \pi/3 \right] = T_1 \cdot \left(\frac{2}{3}V_{dc}\right)$$

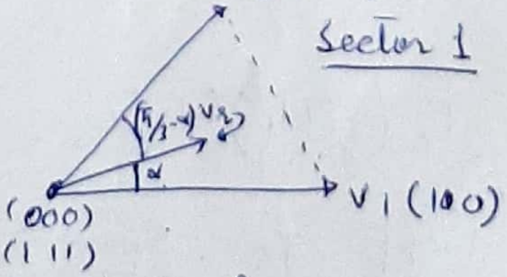
$$T_1 = \frac{T_s \cdot |V_s|}{\frac{2}{3}V_{dc}} \left[\frac{\cos\alpha \sin\pi/3 - \sin\alpha \cos\pi/3}{\sin \pi/3} \right]$$

$$\boxed{T_1 = T_s \cdot a \left[\frac{\sin(\pi/3 - \alpha)}{\sin \pi/3} \right]} \quad \text{— (4)}$$

Now

$$\boxed{T_0 = T_s - (T_1 + T_2)} \quad \text{— (5)}$$

* Looking at figure in sector 1 we can generalize the



expression of T_1 & T_2 for any sector 'n'.

→ For second sector

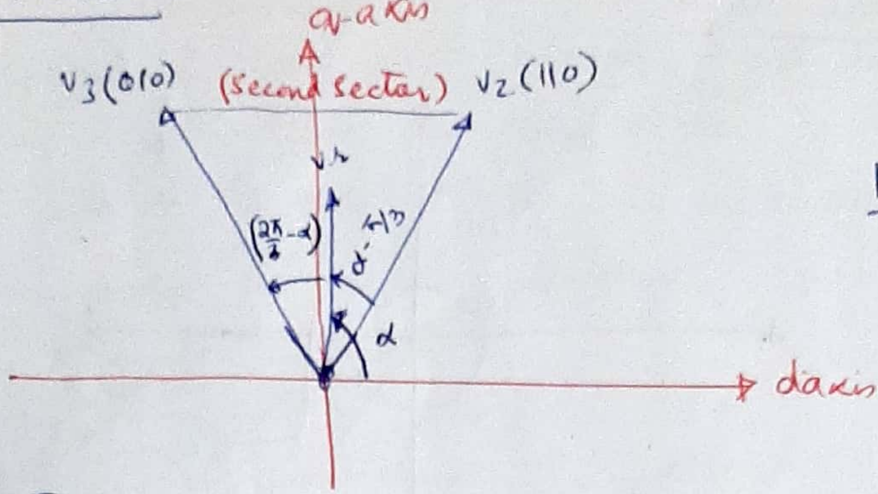


Figure 2

→ For third sector

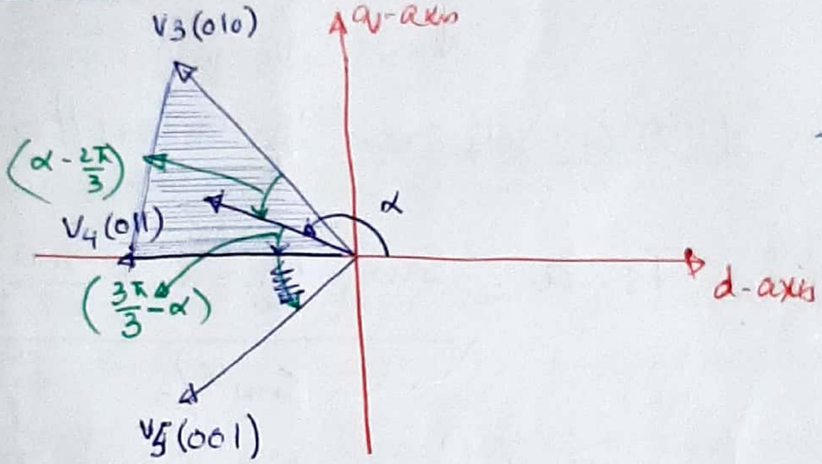


Figure 3

→ For sector no. 4

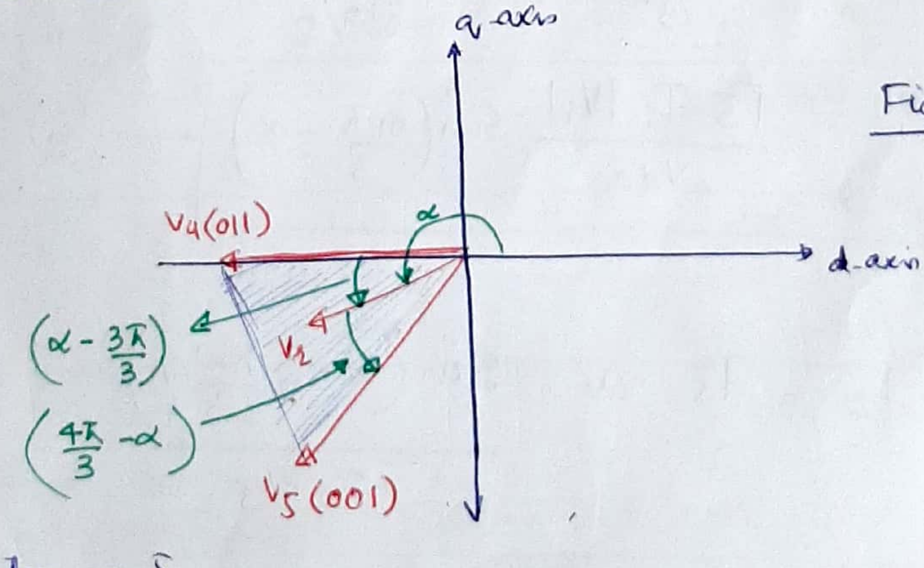


Figure 4

→ For sector no. 5

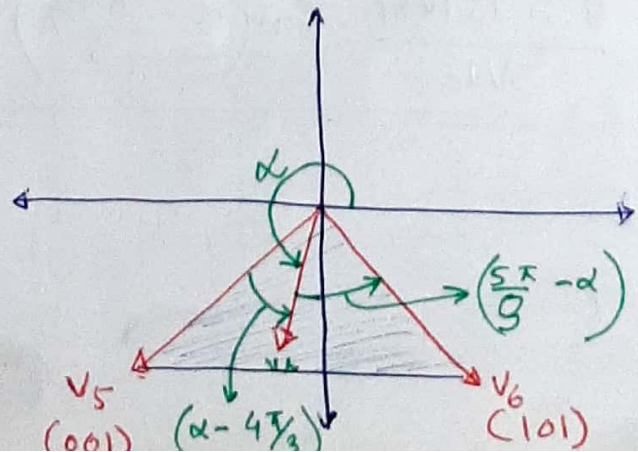
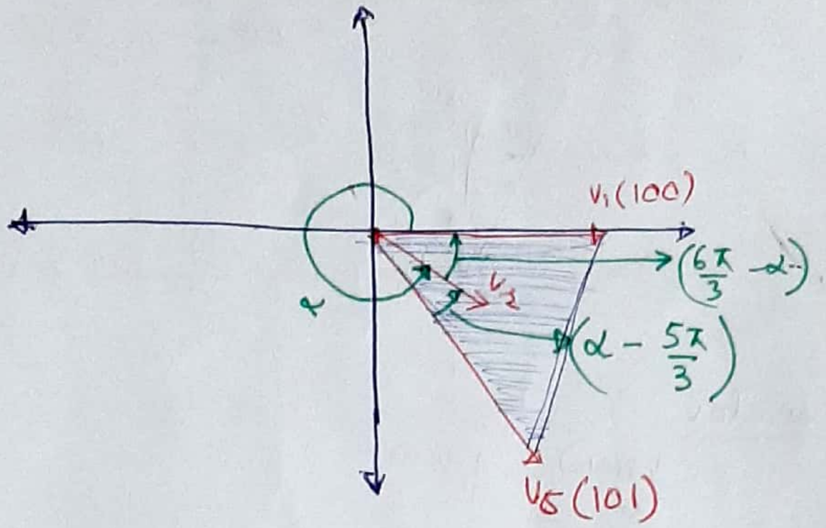


Figure 5

For sector no. 6



From figure 1 to 6 it can be written as

$$T_1 = T_s \cdot a \cdot \frac{\sin\left(\pi/3 - \alpha + \frac{n-1}{3}\pi\right)}{\sin \pi/3}$$

$$T_1 = T_s \cdot \frac{|V_s|}{\frac{2}{3}V_{dc}} \cdot \frac{\sin\left(\pi/3 - \alpha + \frac{n\pi}{3} - \pi/3\right)}{\sqrt{3}/2}$$

$$\boxed{T_1 = \frac{\sqrt{3}\, T_s\, |V_s|}{V_{dc}} \sin\left(\frac{n\pi}{3} - \alpha\right)} \quad \text{— (6)}$$

Similarly

$$T_2 = T_s \cdot a \cdot \frac{\sin\left(\alpha - \frac{n-1}{3}\pi\right)}{\sin \pi/3}$$

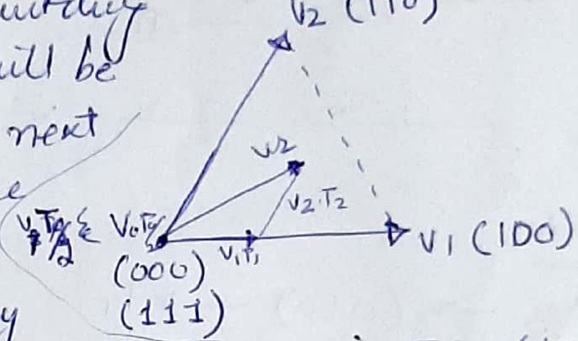
$$\boxed{T_2 = \frac{\sqrt{3}\, T_s\, |V_s|}{V_{dc}} \sin\left(\alpha - \frac{n-1}{3}\pi\right)} \quad \text{— (7)}$$

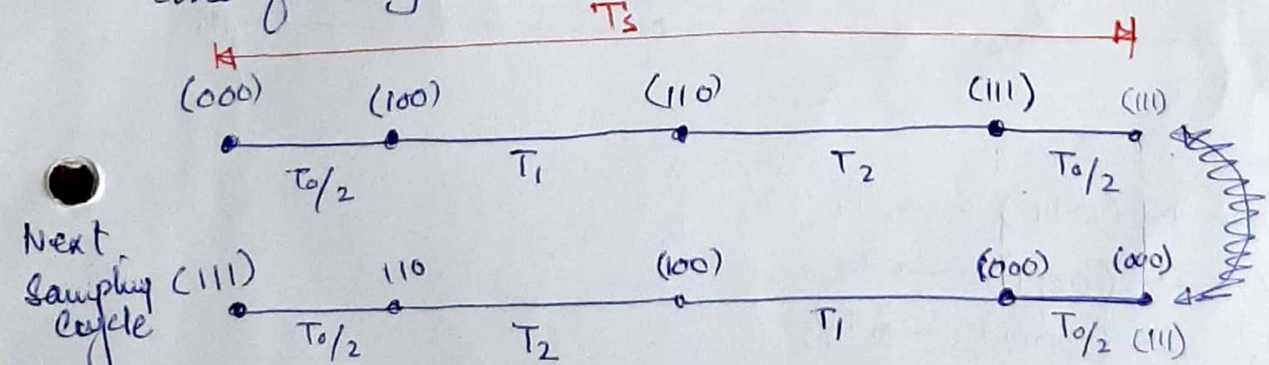
Switching Diagrams & Sequences

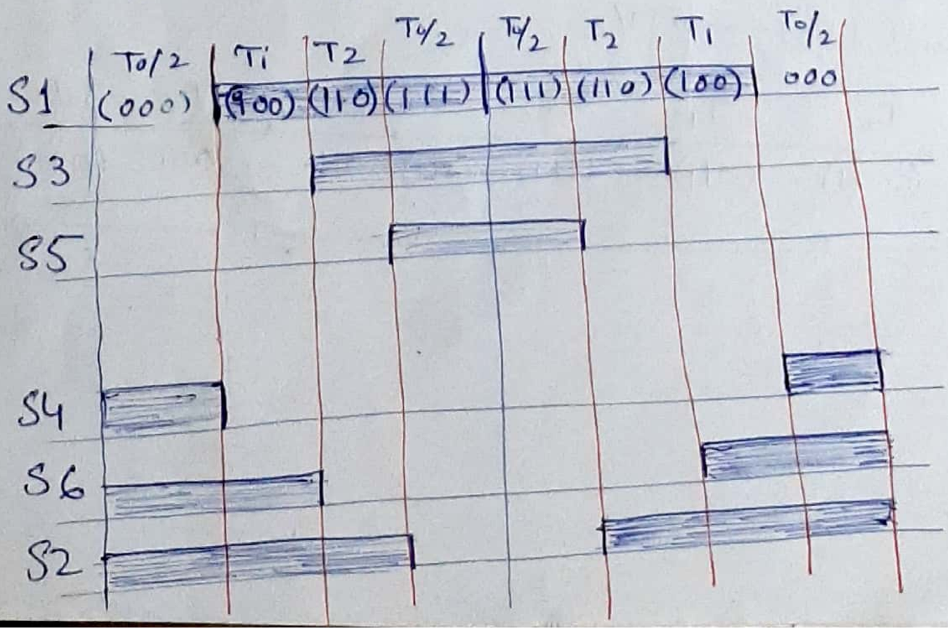
* V_0 & V_8 represent the zero vectors and are the part of every sector. Time for zero vector 'T_0' can be divided in two parts each of '$T_0/2$' to include both of the zero states in a sector switching

→ Consider the <u>sector 1</u>

Let us start with the switching combination (000). This will be applied for time '$T_0/2$'. Then next three/two switching combination are there i.e (111) (100) & (110) but we switch to (100) as only one switch goes through transition. Combination (100) represents the voltage vector V_1 & applied for time 'T_1'. At the end of 'T_1' we have possible combinations of (110) & (111) but (110) is switched for time 'T_2' and finally (111) is switched for '$T_0/2$'







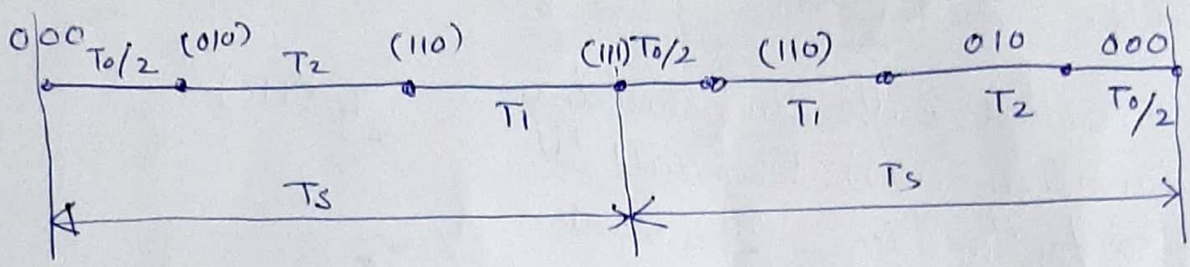
Sector 2

$V_2 \rightarrow (110) \rightarrow T_1$

$V_3 \rightarrow (010) \rightarrow T_2$

$V_0 \rightarrow (000) \rightarrow T_0/2$

$V_7 \rightarrow (111) \rightarrow T_0/2$



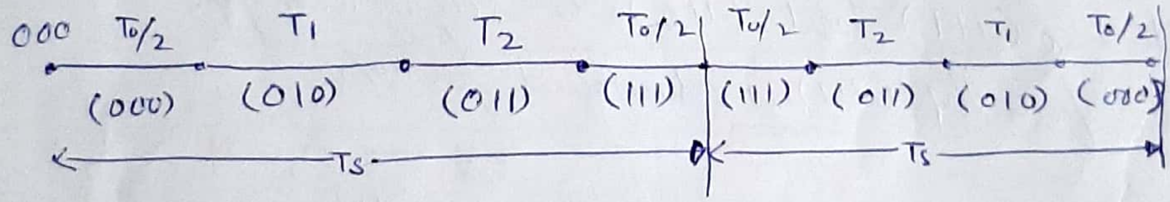
Sector 3

$V_3 \rightarrow (010) \rightarrow T_1$

$V_4 \rightarrow (011) \rightarrow T_2$

$V_0 \rightarrow (000) \rightarrow T_0/2$

$V_7 \rightarrow (111) \rightarrow T_0/2$



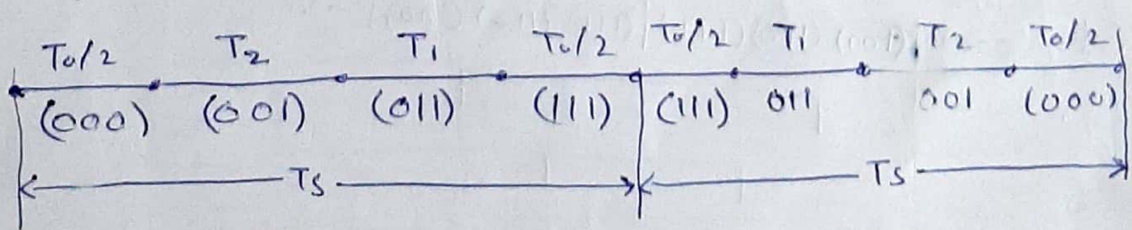
Sector 4

$V_4 \rightarrow (011) \rightarrow T_1$

$V_5 \rightarrow (001) \rightarrow T_2$

$V_0 \rightarrow (000) \rightarrow T_0/2$

$V_7 \rightarrow (111) \rightarrow T_0/2$



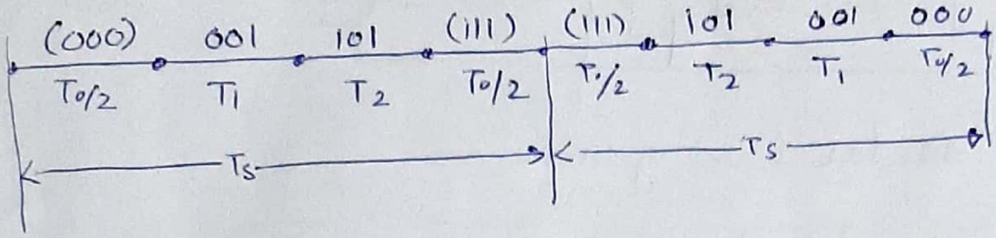
Sector 5

$V_5 \rightarrow (001) \rightarrow T_1$

$V_6 \rightarrow (101) \rightarrow T_2$

$V_0 \rightarrow (000) \rightarrow T_0/2$

$V_7 \rightarrow (111) \rightarrow T_0/2$



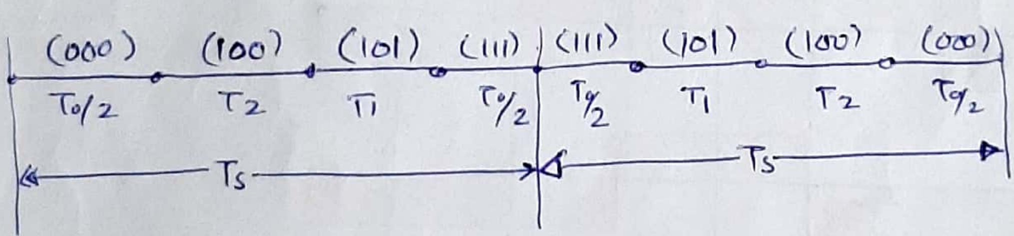
Sector 6

$V_6 \rightarrow (101) \rightarrow T_1$

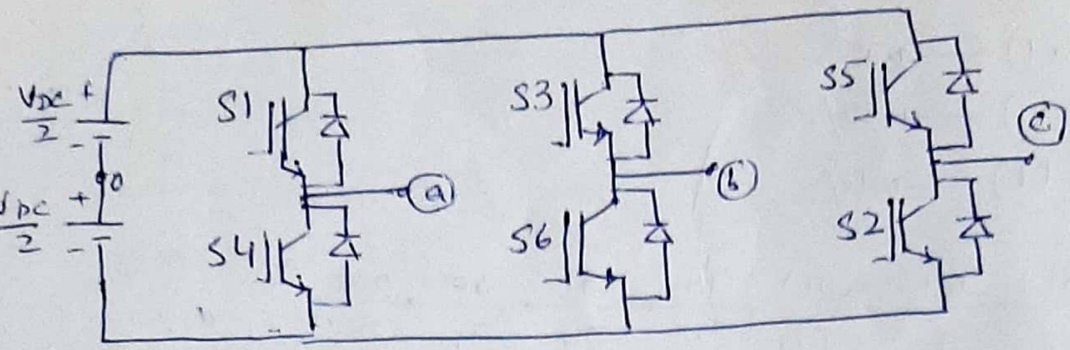
$V_1 \rightarrow (100) \rightarrow T_2$

$V_0 \rightarrow (000) \rightarrow T_0/2$

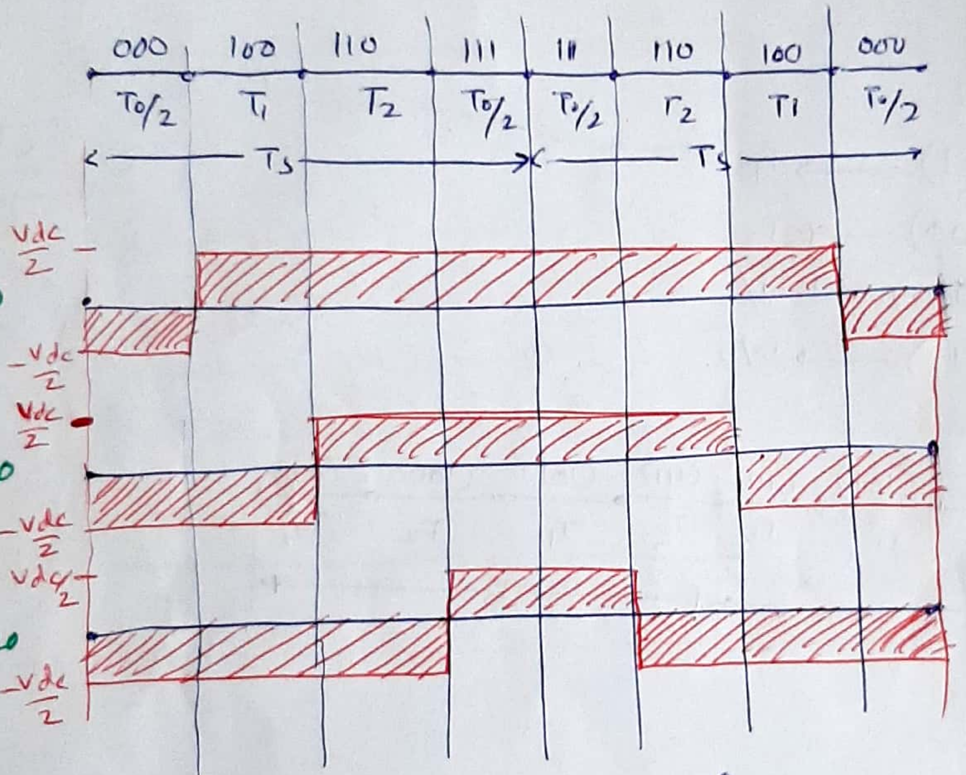
$V_7 \rightarrow (111) \rightarrow T_0/2$



Average Pole voltages



Sequence for sector 1 is given as



* Similarly we can plot for every sector

→ Looking at the above waveforms

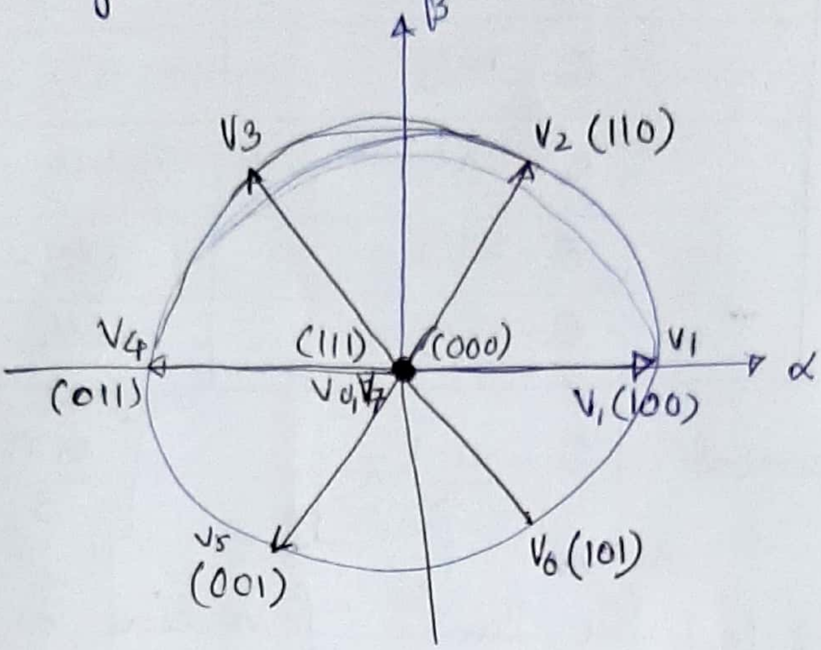
$$V_{ao}(avg) = \frac{V_{DC}/2}{T_s}\left[-\frac{T_0}{2} + T_1 + T_2 + \frac{T_0}{2}\right] = \frac{V_{DC}/2}{T_s}\left[T_1 + T_2\right]$$

$$V_{bo}(avg) = \frac{V_{DC}/2}{T_s}\left[-\frac{T_0}{2} - T_1 + T_2 + \frac{T_0}{2}\right] = \frac{V_{DC}/2}{T_s}\left[-T_1 + T_2\right]$$

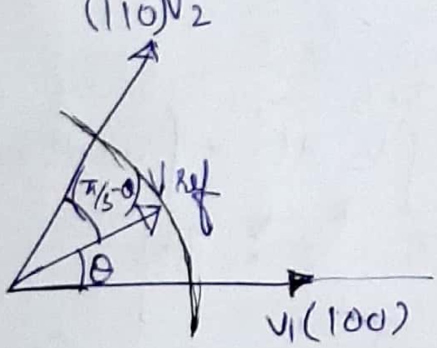
$$V_{co}(avg) = \frac{V_{DC}/2}{T_s}\left[-\frac{T_0}{2} - T_1 - T_2 + \frac{T_0}{2}\right] = \frac{V_{DC}/2}{T_s}\left[-T_1 - T_2\right]$$

Control of Electrical Machine Drives

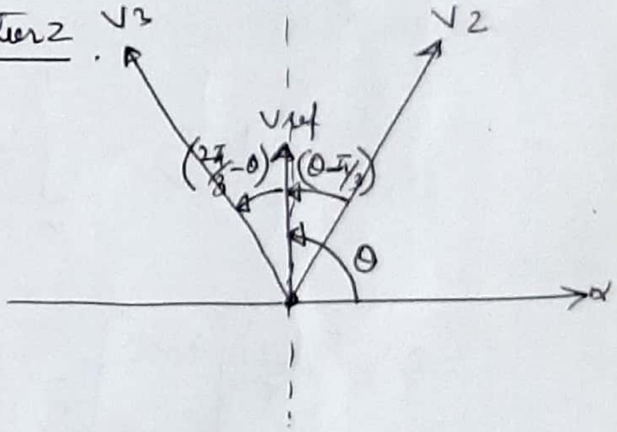
Space vector plane of a three phase inverter is shown in Figure that we have discussed in last lecture.



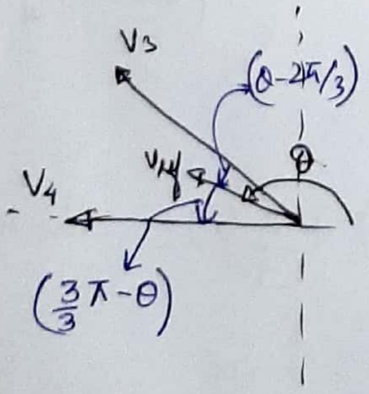
Sector 1:



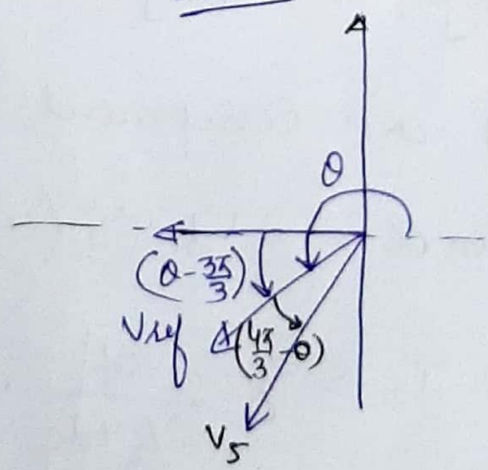
Sector 2



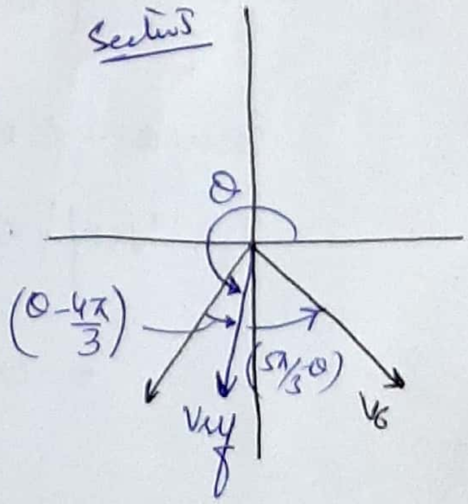
Sector 3:



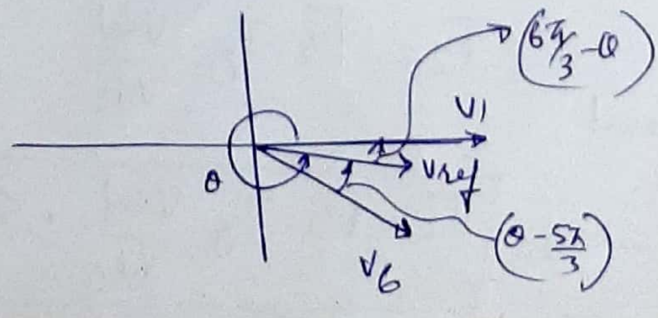
Sector 4



Sector 5



Sector 6



Sector	Angle b/w starting vector & ref	Angle b/w Ref & Ending vector
1	θ	$\pi/3 - \theta$
2	$\theta - \pi/3$	$2\pi/3 - \theta$
3	$\theta - 2\pi/3$	$3\pi/3 - \theta$
4	$\theta - 3\pi/3$	$4\pi/3 - \theta$
5	$\theta - 4\pi/3$	$5\pi/3 - \theta$
6	$\theta - 5\pi/3$	$6\pi/3 - \theta$

In general $\left[\theta - \frac{n-1}{3}\pi\right]$ $\left[\frac{n\pi}{3} - \theta\right]$

We calculated the times of vectors in last lecture using

$$\vec{V}_{ref} \cdot T_z = \int_0^{T_1} V_1\, dt + \int_{T_1}^{T_1+T_2} V_2\, dt + \int_{T_1+T_2}^{T_1+T_2+T_0/2} V_0\, dt + \int_{T_1+T_2+T_0/2}^{T_1+T_2+T_0} V_7\, dt$$

$$\Rightarrow |V_{ref}| \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} T_z = \frac{2}{3} V_{dc} \begin{bmatrix} 1 \\ 0 \end{bmatrix} T_1 + \frac{2}{3} V_{dc} \begin{bmatrix} \cos\pi/3 \\ \sin\pi/3 \end{bmatrix} T_2$$

Comparing 'β' axis components.

$$T_z \cdot |V_{ref}| \sin\alpha = \frac{2}{3} V_{dc} \cdot T_1 (0) + \frac{2}{3} V_{dc} \cdot \sin\pi/3 \cdot T_2$$

$$\Rightarrow T_2 = T_z \cdot \frac{|V_{ref}|}{2/3\, V_{dc}} \cdot \frac{\sin\alpha}{\sin\pi/3}$$

$$T_2 = \sqrt{3} \cdot T_z \frac{V_{ref}}{V_{dc}} \cdot \sin\alpha$$

In general.

$$\boxed{T_2 = \sqrt{3} \cdot T_z \frac{V_{ref}}{V_{dc}} \cdot \sin\left(\alpha - \frac{n-1}{3}\pi\right)}$$

Comparing 'α' axis components we get

$$T_Z . |V_{ref}| \cos\alpha = \frac{2}{3} V_{dc} . T_1 + \frac{2}{3} V_{dc} \cos\pi/3 . T_2$$

using the value of T_2 and simplifying above expression one can get

$$T_1 = \sqrt{3} . T_Z \frac{V_{ref}}{V_{dc}} \sin(\pi/3 - \alpha)$$

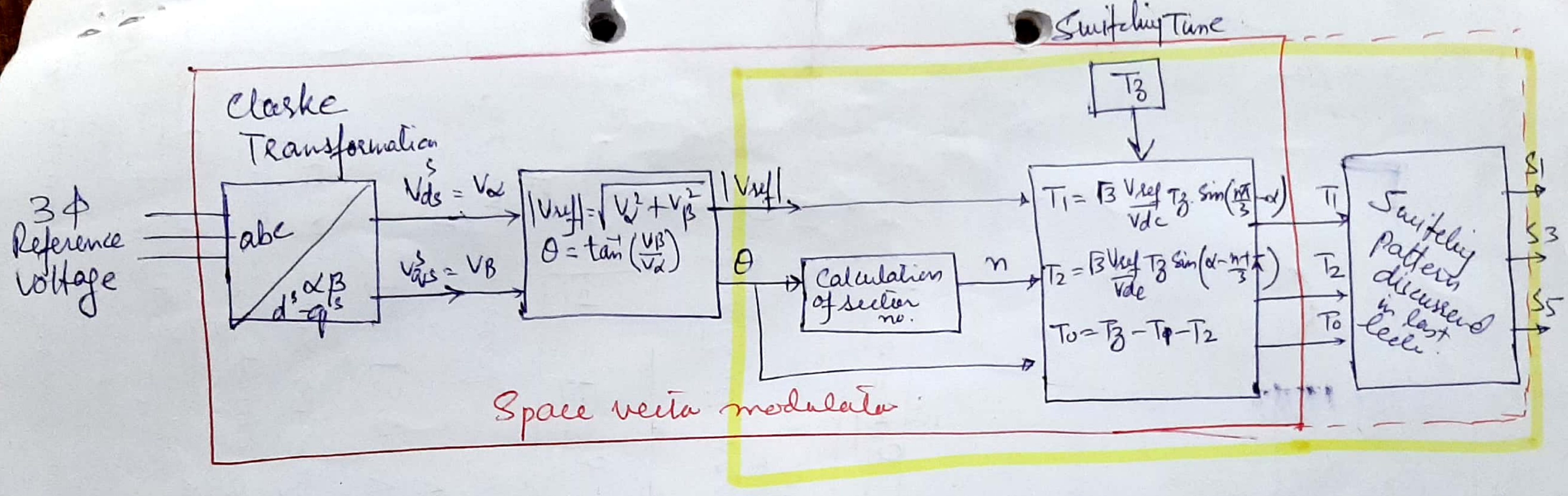
and in general

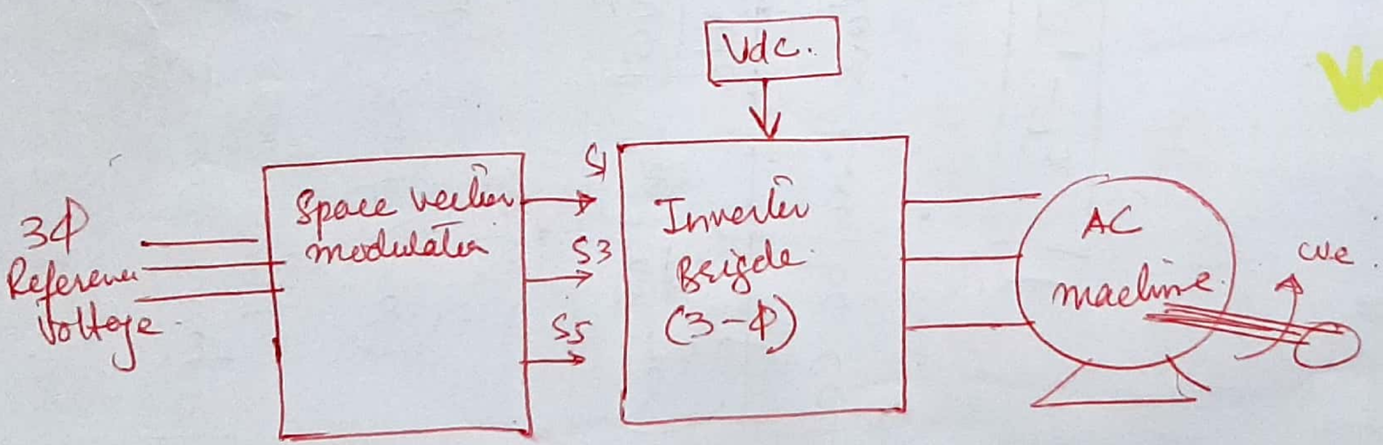
$$\boxed{T_1 = \sqrt{3}\, T_Z \frac{V_{ref}}{V_{dc}} \sin\left(\frac{n\pi}{3} - \alpha\right)}$$

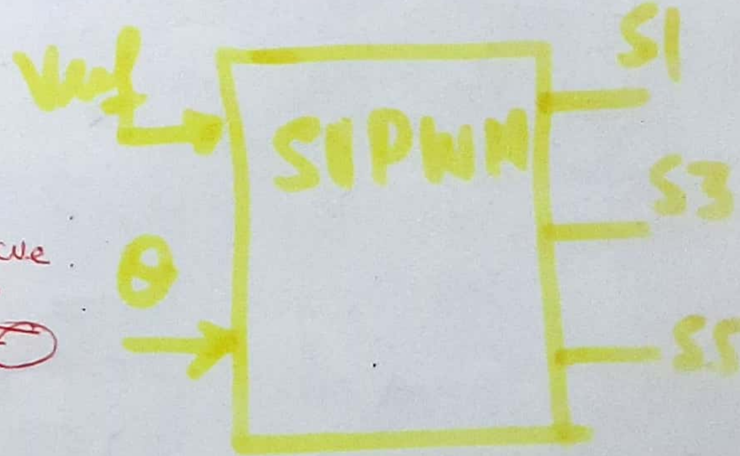
Time of zero/null vectors can be calculated as

$$\boxed{T_0 = T_Z - T_1 - T_2}$$

Control Diagram







Same time this block is used.