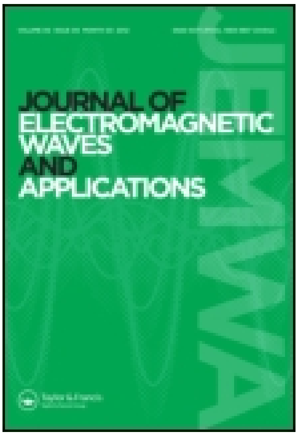
A physical model of the ideal transformer based on magnetic transmission line theory

Article *in* Journal of Electromagnetic Waves and Applications · March 2013

DOI: 10.1080/09205071.2013.746632

1 author:

Jose A Brandao Faria
University of Lisbon



Journal of Electromagnetic Waves and Applications
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tewa20

A physical model of the ideal transformer based on magnetic transmission line theory

J.A. Brandão Faria [a]
[a] Instituto de Telecomunicações, Instituto Superior Técnico, Technical University of Lisbon, Av. Rovisco Pais, Lisboa, 1049-001, Portugal
Version of record first published: 20 Nov 2012.

Journal of Electromagnetic Waves and Applications, 2013
Vol. 27, No. 3, 365–373, http://dx.doi.org/10.1080/09205071.2013.746632

A physical model of the ideal transformer based on magnetic transmission line theory

J.A. Brandão Faria*

Instituto de Telecomunicações, Instituto Superior Técnico, Technical University of Lisbon, Av. Rovisco Pais, Lisboa 1049-001, Portugal

(*Received 6 September 2012; accepted 8 October 2012*)

The ideal transformer is a simple device whose analysis is covered in most electrical engineering textbooks, encompassing power systems, circuit analysis, and basic electromagnetics. Ordinary analysis of ideal transformer functioning poses several delicate physical questions whose answer is given in this paper, enabling an understanding of the whole picture. On one hand, it is shown that if the magnetic stray flux is absent (as usually is assumed), then power flow between primary and secondary windings cannot occur. On the other hand, a physically correct perspective to deal with ideal transformers is developed by resorting to frequency-domain transmission line theory. This approach, based on Maxwell's equations and Poynting's vector, applies indistinctly to low and high frequency regimes. For analysis purposes, the ideal transformer is modeled by means of an ideal magnetic transmission line. Transverse electric and magnetic fields are obtained considering the particular example of a coaxial geometry. Ordinary results concerning the ideal transformer, i.e. voltage, current, and impedance ratios, are thoroughly discussed.

1. Introduction

Transformer modeling, transmission line theory, and power flow issues related to the Poynting vector are important electrical engineering subjects [1–11].

Albeit nonexistent, ideal transmission lines and ideal transformers are very useful concepts in the framework of a zeroth order approach. Additional inclusion of actual loss perturbations allows for the analysis of real transmission lines and real transformers.

In this paper, we address the properties of ideal transformers, and match them with those pertaining to ideal transmission lines. A key issue under discussion is the misconception that magnetic stray flux is absent in ideal transformers (transformers whose magnetic core has infinite permeability).

The propagation of signals between the far ends of an electric transmission line (ELTL) is achieved by employing a pair of parallel good conductors immersed in a nonmagnetic insulating dielectric medium (the air, will be considered).

In an ideal ELTL, with perfect conductors, the time domain telegraphers' equations read as [12,13],

*Email: brandao.faria@ist.utl.pt

$$\frac{\partial}{\partial z}\left\{\begin{array}{c} v(z,t) \\ i(z,t) \end{array}\right\} + \left\{\begin{array}{c} L \\ C \end{array}\right\} \times \frac{\partial}{\partial t}\left\{\begin{array}{c} i(z,t) \\ v(z,t) \end{array}\right\} = \left\{\begin{array}{c} 0 \\ 0 \end{array}\right\}, \tag{1}$$

where v and i are the line voltage and line current. L and C are the inductance and capacitance per unit length of the line, and z is the longitudinal coordinate along which signal propagation and energy flow take place.

For an air dielectric medium, where L and C are easily correlated, the result in (1) transforms into the well-known wave equation [12,13],

$$\left[\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial (ct)^2}\right]\left\{\begin{array}{c} v(z,t) \\ i(z,t) \end{array}\right\} = \left\{\begin{array}{c} 0 \\ 0 \end{array}\right\}, \tag{2}$$

where $c = 1/\sqrt{LC} = 1/\sqrt{\varepsilon_0 \mu_0}$ is the light speed in a vacuum.

Consider a time harmonic regime ($e^{j\omega t}$). The complex power $\bar{P}$ carried by the electromagnetic field wave, measured at a *transverse plane z* of the ELTL, is obtained through [13],

$$\bar{P} = P + jQ = \frac{1}{2}\bar{V}\bar{I}^* = \int_{\text{Trans. Plane}} \left(\frac{1}{2}\bar{\mathbf{E}} \times \bar{\mathbf{H}}^*\right) \cdot \vec{e}_z \, \mathrm{d}S, \tag{3}$$

where P and Q are the active and reactive power, respectively, and $\bar{\mathbf{E}} = -\nabla\bar{\Phi}_E$, $\nabla \cdot \bar{\mathbf{H}} = 0$, where $\bar{\Phi}_E$ is the electric scalar potential. $\bar{\mathbf{E}}$ is a gradient field with open field lines between line conductors and $\bar{\mathbf{H}}$ is a solenoidal field encircling the line conductors. The external product between wave fields leads to the complex Poynting vector, $\bar{\mathbf{S}} = \frac{1}{2}\bar{\mathbf{E}} \times \bar{\mathbf{H}}^*$ – a local vector that quantifies power flow per unit area.

The technology and applications of magnetic transmission lines (MGTL) are, at this stage, a new open area of research [14–16]. The concept of an MGTL can be explained in quite simple terms: an MGTL is just the dual counterpart of an ELTL. An MGTL can be seen as a magnetic circuit of longitudinal length l whose pieces carry a time-varying magnetic flux, described by its complex amplitude $\bar{\phi}$, and where a transverse magnetic voltage $\bar{U}_{\mathrm{m}}$ exists between pieces.

The ideal MGTL is constituted by two parallel longitudinal pieces made of a perfect magnetic material ($\mu \to \infty$) immersed in a nonmagnetic insulating dielectric medium (air). In the *transverse plane z* of an MGTL, $\bar{\mathbf{H}}$ is a gradient field vector with open field lines between pieces, and $\bar{\mathbf{E}}$ is a solenoidal field encircling the same pieces.

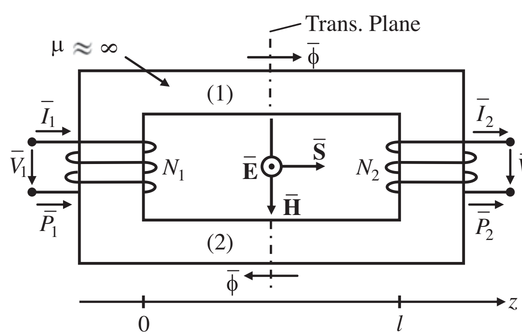


Figure 1. Ideal transformer. The flux of power conveyed by the Poynting vector requires the presence of stray field lines of **H** between pieces #1 and #2.

2. Flow of power in an ideal transformer

Consider the magnetic circuit depicted in Figure 1, where the permeability μ of the core tends to infinity. Primary and secondary windings are characterized by N_1 and N_2 turns.

A standard assumption in high-permeability transformer analysis is the neglecting of magnetic stray flux, that is, all field lines of **B** are assumed to be confined within the core, so that the magnetic flux ϕ across the primary and secondary windings is the same. The preceding assumption is taken for granted if $\mu \to \infty$.

Also, according to standard teaching of ideal transformer functioning, the voltages and currents at the primary and secondary windings are only dependent on the number of turns [13]:

$$N_1\bar{I} - N_2\bar{I}_2 = 0 \to \frac{\bar{I}_1}{\bar{I}_2} = \frac{N_2}{N_1} \tag{4}$$

$$\bar{V}_1 = j\omega N_1\bar{\phi}; \quad \bar{V}_2 = j\omega N_2\bar{\phi} \to \frac{\bar{V}_1}{\bar{V}_2} = \frac{N_1}{N_2} \tag{5}$$

Note: According to the ideal model of the transformer, the internal resistances of the primary and secondary windings were ignored in (5); likewise, the leakage fluxes pertaining to the primary and secondary windings were also ignored in (5).

From (4) and (5), it comes without surprise that the complex powers referred to the primary and secondary windings are equal.

$$\frac{\bar{P}_1}{\bar{P}_2} = \frac{\frac{1}{2}\bar{V}_1\bar{I}_1^*}{\frac{1}{2}\bar{V}_2\bar{I}_2^*} = 1 \to \bar{P}_1 = \bar{P}_2 \tag{6a}$$

Also, the primary impedance $\bar{Z}_1 = \bar{V}_1/\bar{I}_1$ and the secondary load impedance $\bar{Z}_2 = \bar{V}_2/\bar{I}_2$ are related through

$$\bar{Z}_1/\bar{Z}_2 = (N_1/N_2)^2 \tag{6b}$$

The ideal transformer behaves as a "transparent object" – the complex power delivered to the primary winding terminals is totally transferred to the secondary winding terminals.

At this point, a crucial physical question arises: How is energy transferred between the terminals of the ideal transformer?

The flux of electromagnetic energy from one place to another is conveyed by the complex Poynting vector $\bar{\mathbf{S}} = \frac{1}{2}\bar{\mathbf{E}} \times \bar{\mathbf{H}}^*$ and therefore, requires the simultaneous presence of the electric and magnetic fields. But $\bar{\mathbf{H}}$ is zero inside the core of an ideal transformer and, according to usual assumptions, it is also zero in the dielectric medium, because the magnetic stray flux is assumed to be absent. Hence, we end up with $\bar{\mathbf{S}} = 0$ and $\bar{\mathbf{P}} = 0$ – which cannot be true.

Moreover, according to $\nabla \times \bar{\mathbf{E}} = -j\omega\bar{\mathbf{B}}$, the magnetic induction field $\bar{\mathbf{B}}$ carried by the transformer pieces, gives rise to a solenoidal electric induction field $\bar{\mathbf{E}}$ encircling the core pieces #1 and #2. Such an **E**-field provides electric energy W_e storage in the dielectric medium. By applying the complex Poynting theorem to the whole volume of the ideal transformer (core and dielectric), and taking (6a) into account, we would find

$$0 = \bar{P}_1 - \bar{P}_2 = j(Q_1 - Q_2) = 2j\omega((W_\mathrm{m})_\mathrm{av} - (W_\mathrm{e})_\mathrm{av}) \tag{7}$$

Questions are: where is the magnetic energy stored necessary to cancel the electric energy of the electric induction field so that $\bar{P}_1 = \bar{P}_2$? Are $\bar{P}_1$ and $\bar{P}_2$ really equal? And, if they are, what are the required conditions?

Since the transformer core is unable to store any magnetic energy ($\mathbf{H} = \mathbf{B}/\mu = 0$), there is only one answer: The magnetic energy must reside in the dielectric medium ($\mu_0 \neq 0$). Open field lines of $\mathbf{H}$, of gradient type, between pieces #1 and #2 of the magnetic circuit must be present. Magnetic stray flux must exist, even in an ideal transformer ($\mu \to \infty$).

A magnetic voltage $\bar{U}_m$ between pieces #1 and #2 is therefore definable

$$\bar{U}_m = \int_{\overrightarrow{12}} \bar{\mathbf{H}} \cdot dl = (\bar{\Phi}_H)_1 - (\bar{\Phi}_H)_2 \tag{8}$$

where $\overrightarrow{12}$ is an integration path belonging to the transverse plane, and $\bar{\Phi}_H$ is the magnetic scalar potential.

In any transverse plane, the electric and magnetic fields in the dielectric medium surrounding the transformer obey Maxwell's equations,

$$\nabla \times \bar{\mathbf{E}} = -j\omega\bar{\mathbf{B}} \quad \text{and} \quad \nabla \times \bar{\mathbf{H}} = 0 \to \bar{\mathbf{H}} = -\nabla\bar{\Phi}_H \tag{9}$$

Hence, the complex Poynting vector is determined through $\bar{\mathbf{S}} = \frac{1}{2}(\nabla\bar{\Phi}_H^*) \times \bar{\mathbf{E}}$.

As it was shown in a recent paper [17], the z-directed flux of $\bar{\mathbf{S}}$ across a transverse plane of the transformer permits the calculation of the transmitted complex power

$$\bar{P} = \int_{\text{Trans Plane}} \bar{\mathbf{S}} \cdot \vec{\mathrm{e}}_z \, \mathrm{d}S = \frac{j\omega\bar{\phi}}{2}\bar{U}_m^* \tag{10}$$

At $z = 0$, we have:

$$(\bar{U}_m)_1 = \bar{U}_m(z=0) = N_1\bar{I}_1; \quad \bar{V}_1 = j\omega N_1\bar{\phi}_1; \quad \bar{\phi}_1 = \bar{\phi}(z=0) \tag{11a}$$

At $z = l$, we have:

$$(\bar{U}_m)_2 = \bar{U}_m(z=l) = N_2\bar{I}_2; \quad \bar{V}_2 = j\omega N_2\bar{\phi}_2; \quad \bar{\phi}_2 = \bar{\phi}(z=l) \tag{11b}$$

The preceding results being substituted into (10) yield the complex powers at the primary and secondary windings

$$\bar{P}_1 = \frac{1}{2}\bar{V}_1\bar{I}_1^*, \quad \bar{P}_2 = \frac{1}{2}\bar{V}_2\bar{I}_2^* \tag{12}$$

In Sections 3 to 6, the equality $\bar{P}_1 = \bar{P}_2$ is discussed.

3. Fields in the dielectric of a coaxial cable

Consider a coaxial transformer with magnetically perfect pieces ($\mu \to \infty$), where piece #2 completely encircles the inner piece #1. The cylindrical core pieces are characterized by radii r_1 and r_2. See Figure 2.

In an electric coaxial cable, the gradient **E**-field and the solenoidal **H**-field are given by [13],

$$\bar{\mathbf{E}} = \bar{E}\vec{e}_{\mathrm{r}}, \quad \bar{E} = \frac{\bar{V}}{r\ln(r_2/r_1)} \tag{13a}$$

$$\bar{\mathbf{H}} = \bar{H}\vec{e}_{\theta}, \quad \bar{H} = \frac{\bar{I}}{2\pi r} \tag{13b}$$

In a magnetic coaxial transformer, owing to duality, the solenoidal **E**-field and the gradient **H**-field are given by (see field lines in Figure 2),

$$\bar{\mathbf{H}} = \bar{H}\vec{\mathrm{e}}_{r}, \quad \bar{H} = \frac{\bar{U}_m}{r\ln(r_2/r_1)} \tag{14a}$$

$$\bar{\mathbf{E}} = -\bar{E}\vec{\mathrm{e}}_{\theta}, \quad \bar{E} = \frac{j\omega\bar{\phi}}{2\pi r} \tag{14b}$$

Note that the results in (13) and (14) are valid for $r_1<r<r_2$; for $r>r_2$, the fields are zero. From (14), the complex Poynting's vector is easily evaluated

$$\bar{\mathbf{S}} = \frac{1}{2}\bar{\mathbf{E}} \times \bar{\mathbf{H}}^* = \bar{S}\vec{\mathrm{e}}_z, \quad \bar{S} = \frac{j\omega\bar{\phi}\bar{U}_m^*}{4\pi r^2\ln(r_2/r_1)} \tag{15}$$

The flux of $\bar{\mathbf{S}}$ through a cross-sectional plane, from r_1 to r_2, gives

$$\int_{\text{Trans.Plane}} \bar{\mathbf{S}} \cdot \vec{\mathrm{e}}_z \,\mathrm{d}S \;=\; \int_{r_1}^{r_2} \bar{S}\; \underbrace{(2\pi\ r\ \mathrm{d}r)}_{\mathrm{d}S} = \frac{j\omega\bar{\phi}}{2}\bar{U}_m^*, \tag{16}$$

which confirms the general result in (10).

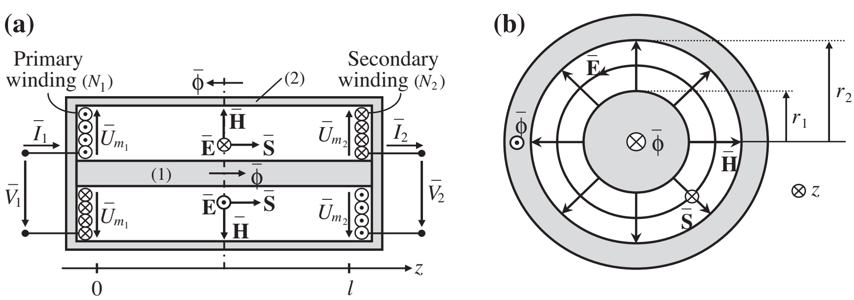


Figure 2. Coaxial magnetic transmission line with radial **H**-field and azimuthal **E**-field. (a) Longitudinal cross section. (b) Transverse cross section.

4. Matched coaxial transformer

The wave equation in (2) also applies to ideal magnetic transmission lines, provided that v is substituted by $\varphi = \partial\phi/\partial t$ (in volt), and i is substituted by u_m (in ampere) [16].

In a matched ideal MGTL, both $\bar{\varphi}(z)$ and $\bar{U}_m(z)$ have constant amplitude, but varying phase [16], that is

$$\bar{\varphi}(z) = j\omega\bar{\phi}(z) = j\omega\bar{\phi}(z=0) \times \mathrm{e}^{-j\beta z} \tag{17a}$$

$$\bar{U}_m(z) = \bar{U}_m(z=0) \times \mathrm{e}^{-j\beta z} \tag{17b}$$

the corresponding rms values φ_{rms} and $(U_m)_{\text{rms}}$ remaining constant along z. In (17), β is the phase constant $\beta = \omega/c$.

The average values of the magnetic and electric energies stored inside the coaxial transformer can be evaluated through

$$(W_m)_{\text{av}} = \int_V \frac{1}{2}\mu_0 H_{\text{rms}}^2 \mathrm{d}V, \quad (W_e)_{\text{av}} = \int_V \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 \mathrm{d}V, \tag{18}$$

where V is the volume of the dielectric medium, corresponding to $r_1<r<r_2$ and $0<z<l$.

Substituting (14) into (18) yields

$$(W_m)_{\text{av}} = \frac{\mu_0 \pi l}{\ln(r_2/r_1)}(U_m)_{rms}^2 \tag{19}$$

and

$$(W_e)_{\text{av}} = \frac{\varepsilon_0 l}{4\pi}\ln(r_2/r_1) \times (\varphi_{\text{rms}})^2 \tag{20}$$

From (7), if the identity $\bar{P}_1 = \bar{P}_2$ is to be verified at any frequency, then $(W_m)_{\text{av}}$ and $(W_e)_{\text{av}}$ have to be the same (resonance situation). By equating (19) and (20), the following condition must be enforced

$$\frac{\varphi_{\text{rms}}}{(U_m)_{\text{rms}}} = \frac{\omega\phi_{\text{rms}}}{(U_m)_{\text{rms}}} = Z_W^M = \frac{2\pi}{\ln(r_2/r_1)}\sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{21}$$

The parameter Z_W^M is the intrinsic characteristic wave impedance of the magnetic coaxial transformer (independent of the windings' number of turns). Since ϕ_{rms} and $(U_m)_{\text{rms}}$ are the same at $z=0$ and at $z=l$, we conclude that in an ideal transformer (core with infinite permeability), *taking into account the presence of magnetic stray flux and the presence of electric induction field*, the complex powers $\bar{P}_1$ and $\bar{P}_2$ turn out to be equal if the secondary winding is loaded with an impedance $\bar{Z}_2 = \bar{V}_2/\bar{I}_2 = N_2^2 Z_W^M$, and in that case, the primary winding impedance is $\bar{Z}_1 = \bar{V}_1/\bar{I}_1 = N_1^2 Z_W^M$. In this special situation, the ordinary results in (6) are satisfied.

Note that the wave impedance in (21), for the magnetic structure, is different from the wave impedance of an electric coaxial cable, which is given by [13],

$$Z_W^E = \frac{\ln(r_2/r_1)}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

both wave impedances are related through $Z_W^M Z_W^E = \mu_0/\varepsilon_0$.

5. Nonmatched coaxial transformer

In a nonmatched ideal MGTL, the solution for $\bar{\varphi}(z)$ and $\bar{U}_m(z)$ involves the superposition of incident and reflected waves [16]. Similar to the ELTL, the solution is written as

$$\begin{cases} \bar{\varphi}(z) = \bar{\varphi}_{inc} \mathrm{e}^{j\beta(l-z)} \left(1 + \bar{\Gamma} \mathrm{e}^{-j2\beta(l-z)}\right) \\ \bar{U}_m(z) = \dfrac{\bar{\varphi}_{inc.}}{Z_W^M} e^{j\beta(l-z)} \left(1 - \bar{\Gamma} e^{-j2\beta(l-z)}\right) \end{cases}, \tag{22}$$

where $\bar{\varphi}_{inc}$ is the complex amplitude of the incident φ-wave at $z = l$, and $\bar{\Gamma}$ is the reflection factor measured at $z = l$,

$$\bar{\Gamma} = \frac{\bar{Z}_2 - N_2^2 Z_W^M}{\bar{Z}_2 + N_2^2 Z_W^M} \tag{23a}$$

$$\bar{\varphi}_{inc} = \frac{\bar{V}_2}{N_2(1 + \bar{\Gamma})}, \tag{23b}$$

where $\bar{Z}_2 = \bar{V}_2 / \bar{I}_2$ is the secondary load impedance.

Check, from (23), that for a matched load (Section 4) $\bar{\Gamma} = 0$ is obtained and, in such a case, $\bar{V}_2 = N_2 \bar{\varphi}_{inc} = j\omega N_2 \bar{\phi}_2$.

For an arbitrary frequency ω, arbitrary load $\bar{Z}_2$, and arbitrary length l, the equality $\bar{P}_1 = \bar{P}_2$ is not obeyed, since, in general $(W_e)_{av} \neq (W_m)_{av}$.

6. Low frequency approach

Next, the case of low frequencies is examined, where $\beta l \ll 1$. In this limiting case, by making $e^{j\beta(l-z)} = 1$, the dependence on z expressed in (22) disappears, leading to

$$\begin{cases} \bar{\varphi}(z) = \bar{\phi}_{inc}(1 + \bar{\Gamma}) \\ \bar{U}_m(z) = \dfrac{\bar{\varphi}_{inc}}{Z_W^M}(1 - \bar{\Gamma}) \end{cases} \rightarrow \begin{cases} \bar{\varphi}(z) = \dfrac{\bar{V}_2}{N_2} \\ \bar{U}_m(z) = \dfrac{\bar{V}_2}{N_2 Z_W^M} \dfrac{1 - \bar{\Gamma}}{1 + \bar{\Gamma}} \end{cases} \tag{24}$$

Further, recalling that $\bar{\varphi} = j\omega\bar{\phi}$, using (23a), Equation (24) transforms to

$$\begin{cases} \bar{\phi}(z) = \dfrac{\bar{V}_2}{j\omega N_2} \\ \bar{U}_m(z) = N_2 \bar{I}_2 \end{cases} \tag{25}$$

Therefore, for low frequency situations, the magnetic flux and the magnetic voltage in the ideal transformer are independent of z. From (25) and (11), the following results are obtained

$$\bar{\phi}(z = 0) = \bar{\phi}_1 = \frac{\bar{V}_1}{j\omega N_1} = \bar{\phi}(z = l) = \bar{\phi}_2 = \frac{\bar{V}_2}{j\omega N_2} \tag{26}$$

$$\bar{U}_m(z = 0) = (\bar{U}_m)_1 = N_1 \bar{I}_1 = \bar{U}_m(z = l) = (\bar{U}_m)_2 = N_2 \bar{I}_2 \tag{27}$$

The results in (26)–(27) agree with those in (4)–(6), and allow for the conclusion that $\bar{P}_1 = \bar{P}_2$. However, contrary to the case examined in Section 4, this identity does not mean that the average values of the electric and magnetic energies stored in the dielectric medium are equal. The interpretation is, from (7),

$$\lim_{\omega\to 0} (\bar{P}_1 - \bar{P}_2) = \lim_{\omega\to 0} (2j\omega((W_m)_{\text{av}} - (W_e)_{\text{av}})) \approx 0 \tag{28}$$

Even for low frequency regimes, transverse fields **E** and **H** must simultaneously exist in the dielectric; otherwise, the Poynting vector would be zero, and power would not be transferred from primary to secondary windings.

7. Real transformer

The preceding analysis applies to ideal transformers. In the case of a real transformer, a set of modifications would be needed. Namely:

(a) The expressions for the terminal voltages $\bar{V}_1 = j\omega N_1\bar{\phi}_1$ and $\bar{V}_2 = j\omega N_2\bar{\phi}_2$ in (11), change to $\bar{V}_1 = j\omega N_1\bar{\phi}_1 + (r_{11} + j\omega\lambda_{11})\bar{I}_1$ and $\bar{V}_2 + (r_{22} + j\omega\lambda_{22})\bar{I}_2 = j\omega N_2\bar{\phi}_2$, where r_{11}, r_{22}, λ_{11}, and λ_{22}, respectively denote the primary and secondary internal resistances, and the primary and secondary leakage inductances.

(b) The wave propagation equations of the ideal MGTL [16], are given by (29)

$$\begin{cases} \frac{d}{dz}\bar{U}_m(z) = -j\omega\varepsilon_0 g\bar{\varphi}(z) \\ \frac{d}{dz}\bar{\varphi}(z) = -\dfrac{j\omega\mu_0}{g}\bar{U}_m(z) \end{cases}, \tag{29}$$

where g is a geometrical factor, given by $g = \ln(r_2/r_1)/(2\pi)$ for a coaxial geometry.

For a nonideal MGTL, where the magnetic material is characterized by a complex magnetic permeability $\bar{\mu}(\omega)\neq\infty$, one would have to take into account the nonzero complex magnetic reluctance of both pieces #1 and #2 (an example of the computation of the magnetic reluctance $\bar{R}_m$ can be found in [18]).

Whenever $\bar{R}_m\neq 0$, the result in (29) changes to [16],

$$\begin{cases} \dfrac{d}{dz}\bar{U}_m(z) = -\left(j\omega\varepsilon_0 g + \dfrac{\bar{R}_m}{j\omega}\right)\bar{\varphi}(z) \\ \dfrac{d}{dz}\bar{\varphi}(z) = -\dfrac{j\omega\mu_0}{g}\bar{U}_m(z) \end{cases} \tag{30}$$

8. Conclusion

Power transmission between the primary and secondary windings of an ideal transformer is only possible if magnetic stray flux is taken into account. The behavior of the ideal transformer was analyzed by employing a model based on an ideal magnetic transmission line, whose key variables are the magnetic voltage between transformer pieces and the time-derivative of the magnetic flux carried by the same pieces.

An application example consisting of a transformer with coaxial geometry was analyzed, showing that the gradient **H**-field is radial and that the solenoidal **E**-field is azimuthal inside the dielectric medium that separates the transformer pieces. It was also shown that, in the framework of a general frequency-domain transmission line theory, the well-known voltage, current, and impedance ratios of the ideal transformer are correct when matched conditions are enforced (at any frequency), or when low frequency regimes are considered.

Acknowledgment

FCT – the Portuguese Foundation for Science and Technology sponsored this work (Project PTDC/EEI-TEL/1448/2012).

References

[1] Newcomb WA. Where is the Poynting vector in an ideal transformer? Am. J. Phys. 1984;52:723–724.
[2] Herrmann F, Schmid GB. The Poynting vector field and the energy flow within a transformer. Am. J. Phys. 1986;54:528–532.
[3] Vaessen PT. Transformer model for high frequencies. IEEE Trans. Power Del. 1988;3:1761–1768.
[4] Hasman T. Reflection and transmission of traveling waves on power transformers. IEEE Trans. Power Del. 1997;12:1684–1689.
[5] Edwards J, Saha TK. Power flow in transformers via the Poynting vector. Proceedings of the Australasian Universities Power Engineering Conference, AUPEC 2000; 2000 Sep; Brisbane.
[6] Semlyen A. Some frequency domain aspects of wave propagation on nonuniform lines. IEEE Trans. Power Del. 2003;18:315–322.
[7] An S, Gedra TW. UPFC ideal transformer model. Proc. North Amer. Power Symp; 2003 Oct; Moscow, Idaho. p. 46–50.
[8] Gustavsen B. Wide band modeling of power transformers. IEEE Trans. Power Del. 2004;19:414–422.
[9] Czarneki LS. Could power properties of three-phase systems be described in terms of the Poynting vector? IEEE Trans. Power Del. 2006;21:339–344.
[10] Antonini G. A general framework for the analysis of metamaterial transmission lines. Prog. Electromagn. Res. B. 2010;20:353–373.
[11] Shamaileh KA, Qaroot AM, Dib NI. Non-uniform transmission line transformers and their application in the design of compact multi-band bagley power dividers with harmonics suppression. Prog. Electromagn. Res. 2011;113:269–284.
[12] Magnusson P, Alexander G, Tripathi V, Weisshaar A. Transmission lines and wave propagation. 4th ed. Boca Raton (FL): CRC Press; 2001.
[13] Faria J. Electromagnetic foundations of electrical engineering. Chichester: Wiley; 2008.
[14] Kerns QA. Transient-suppressing magnetic transmission line. Patent US 3376523. 1968 Apr 2.
[15] Faria J. Dispositivo formado por uma linha magnética de transmissão para uso em circuitos integrados para aplicações na tecnologia terahertz [Magnetic transmission line device for terahertz integrated circuits]. Patent PT 106056. 2011 Dec 12.
[16] Faria J, Pires MP. Theory of magnetic transmission lines. IEEE Trans. Microwave Theory Tech. 2012;60:2941–2949.
[17] Faria J. Poynting vector flow analysis for contactless energy transfer in magnetic systems. IEEE Trans. Power Electronics. 2012;27:4292–4300.
[18] Faria J. Complex reluctance of inhomogeneous Euler-Cauchy tubular ferrites taking into account frequency-dependent complex permeability. Prog. Electromagn. Res. M. 2012;25:71–85.