A Time Domain Algorithm for Spinwave Resonance in Magnetic Media

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***Abstract*—This paper presents a time domain scheme that predicts spinwave resonances in magnetic materials. The proposed scheme incorporates the Landau–Lifschitz equation of motion (LLE) including losses and exhange effects with the traditional Yee scheme. With uniform rf mode excitation, spinwave resonances for a simple structure are observed.**

***Index Terms*—FDTD methods, Ferrimagnetic materials, Magnetic resonance.**

I. Introduction

Because traditional ferrite devices (i.e., circulators, phase-shifters, resonators, etc) typically operate in the dominant mode, rf wavelength is the limiting factor for the dimension of these devices. To overcome this problem, ferrite devices can operate utilizing spinwave resonances, which have been observed in permalloy thin-film [1], cobalt thin-film [2], and other magnetic materials. These microscopic resonances generally exhibit much shorter wavelengths than traditional rf devices, and can lead to significant reductions in device size while still operating at microwave frequencies. With this goal in mind, the next generation of microwave devices could be integrated on chip in connection with active components, such as amplifiers, oscillators, etc.

The design of such devices would be greatly enhanced by an effective simulation tool, where multiple spinwave resonances may be observed over a wide range of frequencies. A finite-difference time-domain (FDTD) scheme would be the best approach to handle this task, because of its broad bandwidth response in a single simulation.

Our proposed 2D FDTD scheme incorporates the Landau-Lifschitz equation (LLE) directly, including losses and exchange effects, to predict spinwave resonances in ferrite materials. Using uniform rf excitation as suggested in [3], a parallel-plate waveguide with thin film ferrite loading and perfect electric conductor (PEC) termination is used as the simulating structure. Our results have shown direct observation of spinwave resonances.

II. Governing Equations

In free space, the electromagnetic wave is governed by the Maxwell equations. In point form, the curl equations can be written as:

$$\nabla \times \mathbf{H} = \epsilon \frac{\partial \mathbf{E}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

In most cases the ferrite is a dielectric (i.e., there is scalar polarization $\mathbf{D} = \epsilon_o \epsilon_r \mathbf{E}$ and no current $J = 0$ flows on the surface). The constitutive equation relating $\mathbf{B}$ to $\mathbf{H}$ remains in the form $\mathbf{B} = \mu_\circ(\mathbf{H} + \mathbf{M})$.

To relate $\mathbf{M}$ and $\mathbf{H}$ in the ferrite media, this scheme employs the equation of motion in the Landau-Lifschitz form [4]. This particular form has proven effective for late-time stability and accuracy as demonstrated in [5]. The continuous-time LLE is shown in Eqn. (3).

$$\begin{aligned}\frac{\partial \mathbf{M}}{\partial t} =& -\mu_o|\gamma|\mathbf{M} \times \mathbf{H} - \mu_o|\gamma|\frac{\alpha}{|\mathbf{M}|}\mathbf{M} \times (\mathbf{M} \times \mathbf{H}) \\ & -\mu_o|\gamma|\frac{H_{ex}a^2}{|\mathbf{M}|}\mathbf{M} \times \nabla^2 \mathbf{M}\end{aligned} \tag{3}$$

where $\gamma = 1.759 \times 10^{11}$ C/kg is the gyromagnetic ratio, α is the phenomenological loss term, H_{ex} is the effective exchange field, and a is the lattice constant.

In consideration of the 4-point averaging approach on the magnetization equation, the TM mode is chosen for the proposed scheme. When the ferrite is saturated, the dc components of $\mathbf{M}$ and $\mathbf{H}$ point in the direction of bias ($\hat{u}$). They can be denoted as $|\mathbf{M}_{dc}| = M_s \hat{u}$ for saturation magnetization and $|\mathbf{H}_{dc}| = H_o \hat{u}$ for effective internal field, respectively. Assuming small-signal (i.e., $|\mathbf{M}_{dc}|, |\mathbf{H}_{dc}| \gg |\mathbf{M}_{ac}|, |\mathbf{H}_{ac}|$) and uniform $\hat{z}$-bias (i.e., $\frac{\partial}{\partial z} = 0$ and $\nabla^2 \hat{u} = 0$), the Landau-Lifschitz equation (LLE) can be approximated to

$$\begin{aligned}\frac{\partial \mathbf{M}}{\partial t} =& -\omega_m \hat{a}_z \times \mathbf{H}_{ac} + \omega_o \hat{a}_z \times \mathbf{M}_{ac} \\ & + \alpha \hat{a}_z \times (-\omega_m \hat{a}_z \times \mathbf{H}_{ac}) + \alpha \omega_o \hat{a}_z \times (\hat{a}_z \times \mathbf{M}_{ac}) \\ & - \omega_{ex} a^2 \hat{a}_z \times \nabla^2 \mathbf{M}_{ac}\end{aligned} \tag{4}$$

where $\omega_m = \mu_o|\gamma|M_s$ is the magnetization frequency, $\omega_o = \mu_o|\gamma|H_o$ is the Larmour precession frequency, and $\omega_{ex} = \mu_o|\gamma|H_{ex}$ is the exchange frequency.

III. Numerical Results

Eqn. (1), (2), (4), and the magnetic constitutive relation are discretized in space as shown in Fig. 1. In this way, the

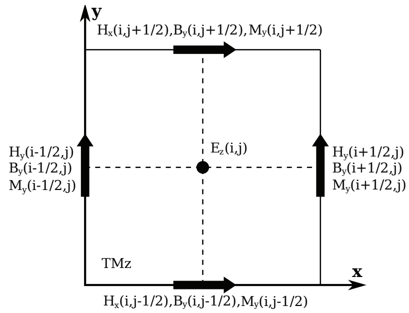

Fig. 1. The 2D FDTD Grid in TM_z with location of field components. The components of $\mathbf{H}$, $\mathbf{M}$ and $\mathbf{B}$ fields are co-located on the edge of the cell, and the $\mathbf{E}$ field is located at the center of the cell directed out of the plane

components of $\mathbf{M}$ are colocated in time and space with the respective components of $\mathbf{B}$ and $\mathbf{H}$. Fig. 2 shows a layout of the parallel-plate waveguide loaded with a ferrite thin-film with a thickness of 300 nm near the PEC termination. A dc

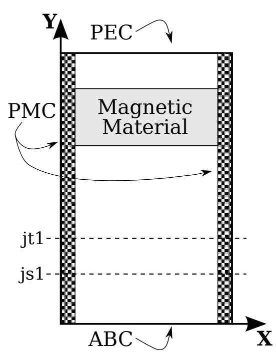

Fig. 2. The 2D parallel-plated waveguide with PEC termination at the end. Using a port detection method in scattered field mode, the source line is placed at $js1$, and the terminal line is at $jt1$. Also, the port is terminated with a 1^{st}-order Liao absorbing boundary condition (ABC) to minimize reflection at the truncated computing boundary.

bias field of 4000 Oe is applied in the z-direction (i.e., out of plane). The magnetic and atomic structure parameters of the ferrite film are: $4\pi M_s = 4000$ Gauss, $\Delta H = 300$ Oe, $\epsilon_r = 12.4$, exchange frequency is $\omega_{ex} = 117.24 \times 10^{12}$ rad/s, and lattice constant is $a = 0.355 \times 10^{-9}$ m [2]. Note that the conductivity and uniaxial anisotropy field are neglected.

The simulation domain is 300 cells in the x-direction and 100 cells in the y-direction. To assure the observation of spinwave resonances, the spatial-step is set to $\delta_x = \delta_y = 10$ nm. This leads to a time-step of $\delta_t = 1.179 \times 10^{-17}$ s. A modulated Gaussian pulse is used to form a plane wave propagating along the guide. In order to provide adequate resolution in the frequency domain, the field data are re-sampled every 800 time steps over the 1.76 ns simulation time. The total number of time steps for the simulation is 150,000,000 steps.

To validate the FDTD result, the dominant resonance of this structure is compared with a common frequency-domian solver (i.e., HFSS). Note that this commercial solver does not incorporate spinwave effects in the solution. Fig. 3 shows a comparison of the magnitude of S11 in dB as measured at

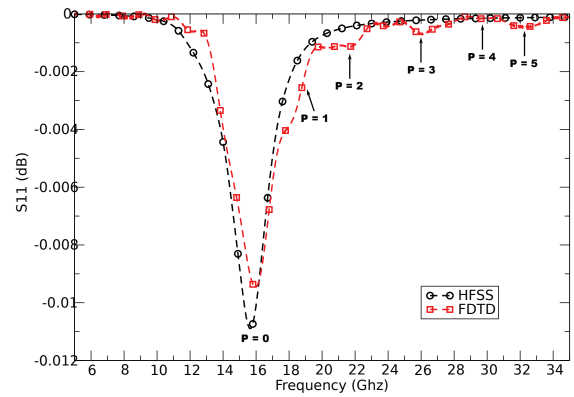

Fig. 3. The magnitude of S11 in dB with FDTD and HFSS. The dominant resonance occurs at 15.8 GHz for FDTD and 15.5 GHz for HFSS. And, the spinwave resonances occur at 19.5 GHz (P=1), 21.5 GHz (P=2), 25.8 GHz (P=3), 29.2 GHz (P=4), and 32.4 GHz (P=5).

the port using both FDTD and HFSS. The extra peaks shown in the FDTD scheme represent the different spinwave modes. Also, these can be seen with the Fourier transform of the time-domain magnetization data on Fig. 4. Here we can directly observe the magnetic resonances in frequency.

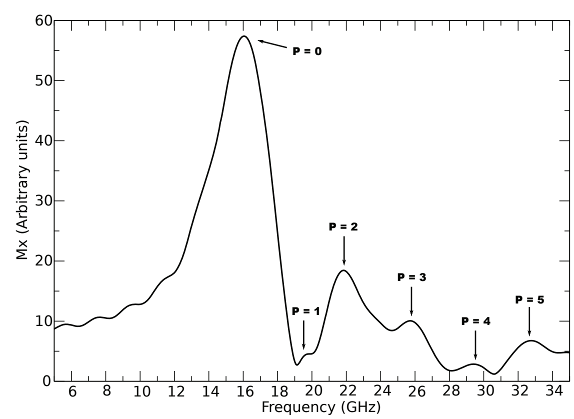

Fig. 4. The Fourier transformed data of Mx. The dominant mode occurs at 15.8 GHz (P=0), and the first few spinwave modes at 19.5 GHz (P=1), 21.5 GHz (P=2), 25.8 GHz (P=3), 29.2 GHz (P=4), and 32.4 GHz (P=5).

IV. Acknowledgement

This material is based upon work supported by the National Science Foundation under Grant No. 1028472.

References

[1] J. M. H. Seavey and P. E. Tannenwald, "Direct observation of spin-wave resonance," *Physical Review Letters*, vol. 1, no. 5, pp. 168–169, 1958.
[2] P. E. Tannenwald and R. Weber, "Exchange integral in cobalt from spin-wave resonance," *Physical Review*, vol. 121, no. 3, p. 715, 1961.
[3] C. Kittel, "Excitation of spin-wave in a ferromagnet by a uniform rf field," *Physical Review*, vol. 110, no. 6, pp. 1295–1297, 1958.
[4] B. Lax and K. J. Button, *Microwave Ferrites and Ferrimagnetics*. New York, NY: McGraw–Hill, 1962.
[5] R. S. Adams and T. K. Lai, "A second-order accurate time-stepping algorithm for dominant mode propagation in ferrite media," *IEEE Trans. Ant. Propagat.*, vol. 60, no. 6, June 2012.