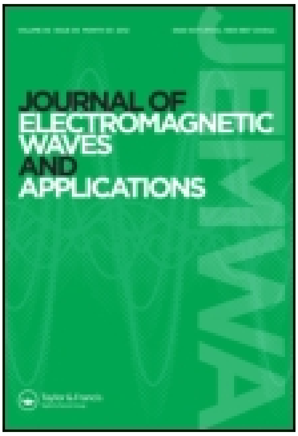

Journal of Electromagnetic Waves and Applications
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tewa20

Reformulation of magnetic transmission line theory using magnetic charges, magnetic currents, and magnetic voltages

J.A. Brandão Faria [a]

[a] Instituto de Telecomunicações, Instituto Superior Tecnico, Universidade de Lisboa , DEEC, Av. Rovisco Pais, Lisboa , 1049-001 , Portugal

Published online: 11 Sep 2013.

To cite this article: J.A. Brandão Faria (2013) Reformulation of magnetic transmission line theory using magnetic charges, magnetic currents, and magnetic voltages, Journal of Electromagnetic Waves and Applications, 27:17, 2145-2154, DOI: 10.1080/09205071.2013.833860

To link to this article: http://dx.doi.org/10.1080/09205071.2013.833860

Journal of Electromagnetic Waves and Applications, 2013
Vol. 27, No. 17, 2145–2154, http://dx.doi.org/10.1080/09205071.2013.833860

Reformulation of magnetic transmission line theory using magnetic charges, magnetic currents, and magnetic voltages

J.A. Brandão Faria*

Instituto de Telecomunicações, Instituto Superior Tecnico, Universidade de Lisboa, DEEC, Av. Rovisco Pais, Lisboa 1049-001, Portugal

(*Received 18 June 2013; accepted 7 August 2013*)

Transmission line structures are commonly used to support guided wave propagation. To that end, ordinary transmission lines utilize two parallel electric wires of very high conductivity. Very recently, another option has emerged, the so-called magnetic transmission line (MGTL), which utilizes two parallel magnetic wires of very high permeability. The theory of magnetic transmission lines is not completely settled; adjustments are still needed, and its own vocabulary requires some tuning as well. In this paper, new grounds are established for MGTL theory, encompassing two-wire and multiwire systems. For that purpose, the new concepts of magnetic charge, magnetic current, and magnetic voltage are introduced and discussed. These conceptual tools are made use not only to describe field equations in the transverse plane, but also field-wave equations in the propagation direction. With these new tools, the duality between electric and magnetic transmission line analysis becomes self-evident.

Keywords: transmission line theory; magnetic transmission line; wave propagation; electromagnetic fields; magnetic charge; magnetic current; magnetic voltage

1. Introduction

The Journal of Electromagnetic Waves and Applications has recently published a theoretical paper [1] on the analysis of the ideal transformer ($\mu \to \infty$), where the claim that stray magnetic field lines ought to exist was demonstrated. The analysis described in [1] utilized a coaxial geometry and used the two-wire magnetic transmission line theory firstly developed in [2], and later generalized to multiwire MGTLs in [3]. A novel contribution to the formulation (reformulation) of MGTL theory is presented here, where magnetic charges and magnetic currents are employed for the first time.

At this stage, readers should be put on guard about the meaning of «magnetic charges» and «magnetic currents». These are not real physical entities but only useful intellectual concepts. In fact, certain scientific terms require a historic background to be fully put in perspective. Everybody knows the meaning of «electric current»: in the so-called good conductors, such as silver, copper, gold, (and alkali metals as well), a single electron exists in the outermost shell of atoms. These electrons are not firmly attached to individual atoms, but are free to move randomly within the crystal lattice; when an electric field **E** is applied to the conducting medium, the free electrons acquire an

*Email: brandao.faria@ieee.org

average momentum antiparallel to **E**, and are set in motion (collisions included) forming an «electric current». Now, consider the seminal concept of «electric displacement current» $\partial(\varepsilon\mathbf{E})/\partial t$ introduced by Maxwell [4]. If the material medium is a vacuum ($\varepsilon = \varepsilon_0$), what is it that is being displaced and what is the charge that is set in motion to produce a current? Of course, these are just rhetorical questions to illustrate the idea that useful concepts are sometimes coined with inadequate designations. In short: do not interpret «magnetic charges» and «magnetic currents» as their names may suggest.

2. Two-wire transmission lines

Consider a general two-wire transmission line (TL) running parallel to the z-axis (the wave propagation direction). Here, the word «wire» is to be interpreted *sensu lato*, that is, it means a longitudinal piece of material with z-invariant cross-section, which can be made of an electric conducting medium of high conductivity σ_w (e.g., copper wire) or, alternatively, made of an electric insulating medium of high magnetic permeability μ_w (e.g. ferrite wire). In the first case, we say that we are dealing with an electric transmission line (ELTL) where longitudinal electric currents flow along the wires. In the second case, we are dealing with a magnetic transmission line where time-varying longitudinal magnetic induction fluxes flow along the wires.

Schematic longitudinal views of two-wire ELTL and two-wire MGTL are illustrated in Figure 1(a) and (b). TL cross-sectional views showing electric and magnetic field lines in the transverse plane are depicted in Figure 1(c) and (d) for the particular case of a coaxial geometry.

The analysis of electromagnetic wave propagation in ELTL and in MGTL is based on the Maxwell equations, whose differential and integral forms are presented in (1).

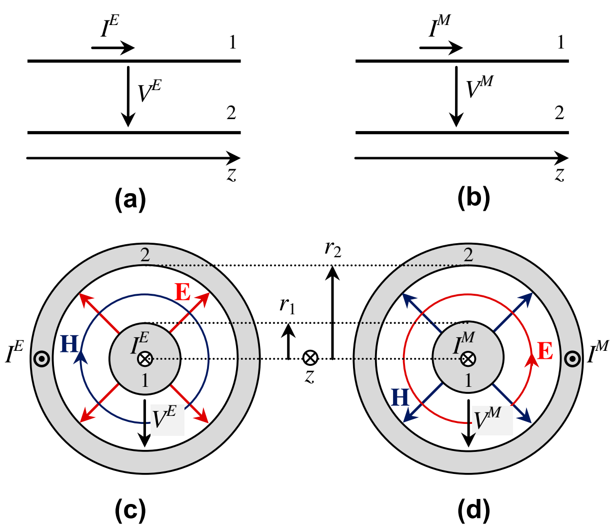

Figure 1. Two-wire transmission lines. Schematic longitudinal view: (a) ELTL, (b) MGTL. Cross-sectional view of coaxial TLs: (c) ELTL, and (d) MGTL.

$$curl\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \oint_{\mathbf{s}} \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt}\int_{S_s} \mathbf{B} \cdot \mathbf{n}_{\mathrm{s}}\ dS \tag{1a}$$

$$div\,\mathbf{D} = \rho_e, \quad \int_{S_V} \mathbf{D} \cdot \mathbf{n}_{\mathrm{o}}\ dS = \int_V \rho_e\, dV \tag{1b}$$

$$curl\,\mathbf{H} = \mathbf{J}_e + \frac{\partial \mathbf{D}}{\partial t}, \quad \oint_{\mathbf{s}} \mathbf{H} \cdot d\mathbf{s} = \int_{S_s} \mathbf{J}_e \cdot \mathbf{n}_{\mathrm{s}}\ dS + \frac{d}{dt}\int_{S_s} \mathbf{D} \cdot \mathbf{n}_{\mathrm{s}}\ dS \tag{1c}$$

$$div\,\mathbf{B} = 0, \quad \int_{S_V} \mathbf{B} \cdot \mathbf{n}_{\mathrm{o}}\, dS = 0 \tag{1d}$$

where the actual field sources are ρ_e and $\mathbf{J}_e$ $-$ the electric charge density (C/m^3), and the electric current density vector (A/m^2). As a parenthetical note it may be added that (1a) and (1d) are not entirely independent equations; in fact, by applying the divergence operator to (1a) yields $0 = \frac{\partial}{\partial t} div\,\mathbf{B}$, whose general solution is $div\,\mathbf{B} = k$. The time-invariant constant k is put to zero because magnetic monopoles do not exist in nature (according to experimental evidences).

The pair (1a) and (1c), which simultaneously involve time and space coordinates, is the key tool for wave propagation analysis. The differential form equations for TEM-guided wave propagation along z, in the dielectric medium (where $\rho_e = 0$, and $\mathbf{J}_e = 0$), are in cylindrical coordinates

$$\begin{cases} \dfrac{\partial E_\theta}{\partial z} = +\mu_0 \dfrac{\partial H_r}{\partial t} \\ \dfrac{\partial E_r}{\partial z} = -\mu_0 \dfrac{\partial H_\theta}{\partial t} \end{cases} \qquad \begin{cases} \dfrac{\partial H_\theta}{\partial z} = -\varepsilon \dfrac{\partial E_r}{\partial t} \\ \dfrac{\partial H_r}{\partial z} = +\varepsilon \dfrac{\partial E_\theta}{\partial t} \end{cases} \tag{2}$$

2.1. ELTL field and wave equations

In an ideal ELTL ($\sigma_w = \infty$), where the current density vector in the wires is axially directed $\mathbf{J}_e = \mathbf{J}_z = J_e\,\hat{z}$, the electromagnetic field is purely transverse: $\mathbf{E}\perp\hat{z}$, $\mathbf{H}\perp\hat{z}$.

The field structure in the transverse plane (z=constant, $\partial/\partial z = 0$) can be evaluated from (1a) and (1c) by considering a clockwise closed contour $\mathbf{s}$ with Stokes's normal $\mathbf{n}_{\mathrm{s}} = \hat{z}$. This leads to

$$\oint_{\mathbf{s}} \mathbf{E} \cdot d\mathbf{s} = 0, \quad curl\ \mathbf{E} = 0, \quad \mathbf{E} = -grad\ \Phi^E \tag{3}$$

$$\oint_{\mathbf{s}} \mathbf{H} \cdot d\mathbf{s} = \int_{S_s} \mathbf{J}_e \cdot \mathbf{n}_{\mathrm{s}}\ dS = \int_{S_s} J_e\ dS = I^E, \quad curl\,\mathbf{H} = \mathbf{J}_e \tag{4}$$

where Φ^E (V) is the electric scalar potential, and I^E (A) is the z-directed current intensity embraced by the contour $\mathbf{s}$. The above equations, together with (1b) and (1d), define the TEM field structure in the transverse plane.

$$\begin{cases} \mathbf{E} = -grad\ \Phi^E \\ div\ \mathbf{E} = \rho_e/\varepsilon \end{cases} \quad \begin{cases} curl\ \mathbf{H} = \mathbf{J}_e \\ div\ \mathbf{H} = 0 \end{cases} \tag{5}$$

The results in (5) are the familiar field equations for static/stationary regimes. The **E**-field is a gradient vector with open field lines starting and ending at the electrically charged wires 1 and 2. The **H**-field is a solenoidal vector with closed field lines encircling the current-carrying wires, field lines of **H** coinciding with equipotential lines of Φ^E – meaning that $\mathbf{E}\perp\mathbf{H}$.

The per unit length (pul) electric charge of the wires $Q^E = Q_1^E = -Q_2^E$ and the voltage V^E between wires 1 and 2 are related to each other by the pul transverse capacitance C_T (F/m), that is

$$Q^E = C_T V^E, \begin{cases} Q^E = \lim\limits_{\Delta z\to 0} \dfrac{\int_V \rho_e (dS\Delta z)}{\Delta z} = \varepsilon \lim\limits_{\Delta z\to 0} \dfrac{\int_S \mathbf{E}\cdot\mathbf{n}_o (ds\Delta z)}{\Delta z} \\ V^E = \int\limits_{\overrightarrow{12}} \mathbf{E}\cdot d\mathbf{s} = \Phi_1^E - \Phi_2^E \end{cases} \tag{6}$$

where $\mathbf{n}_o$ is the outward normal.

Besides C_T, another important parameter of ELTLs is the pul longitudinal inductance L_L (H/m). For its definition, we have to determine the magnetic induction flux ψ^M linked with a closed path accompanying the longitudinal wires and crossing the dielectric medium at the transverse planes z and $z+\Delta z$. The path orientation is the same as the one assigned to the wire current $I^E = I_1^E = -I_2^E$. Denoting by S_ψ the open surface bounded by the closed path, we can write

$$\psi^M = L_L I^E, \quad \begin{cases} \psi^M = \mu_0 \lim\limits_{\Delta z\to 0} \dfrac{\int_{S_\psi} \mathbf{H}\cdot\mathbf{n}_\psi (ds\Delta z)}{\Delta z} \\ I^E = \displaystyle\int_{S_1} \mathbf{J}_e\cdot\hat{z}\, dS_1 = \oint_{\mathbf{s}_1} \mathbf{H}\cdot d\mathbf{s}_1 \end{cases} \tag{7}$$

where $\mathbf{s}_1$ is a clockwise oriented path encircling the wire 1, and S_1 is the wire's cross-section. The parameters C_T and L_L depend only on the dielectric properties and on the geometry of the two-wire TL. For the case of homogeneous media, C_T and L_L are related through

$$L_L = \mu_0 g, \quad C_T = \varepsilon/g, \quad L_L C_T = \mu_0\varepsilon \tag{8}$$

where g is a dimensionless geometrical factor which, for coaxial geometries, is given by $g = \frac{1}{2\pi}\ln(r_2/r_1)$, [1,2].

Integral form equations for ELTL-guided waves are obtained from (2) using the concepts of electric voltage (6), electric current intensity (7), pul transverse capacitance (6), and pul longitudinal inductance (7). For the case of coaxial geometries, where the fundamental TEM mode is characterized by $\mathbf{E} = E\hat{r}$ and $\mathbf{H} = H\hat{\theta}$ we find

$$\frac{\partial E}{\partial z} = -\mu_0\frac{\partial H}{\partial t} \to \frac{\partial V^E}{\partial z} = -L_L\frac{\partial I^E}{\partial t} \quad \text{and} \quad \frac{\partial H}{\partial z} = -\varepsilon\frac{\partial E}{\partial t} \to \frac{\partial I^E}{\partial z} = -C_T\frac{\partial V^E}{\partial t} \tag{9}$$

2.2. MGTL field and wave equations

In an ideal MGTL ($\mu_w = \infty$), where the wires carry an axially directed magnetic induction vector $\mathbf{B} = \mathbf{B}_z = B_z\hat{z}$, the electromagnetic field is, as in 2.1, a purely transverse field, $\mathbf{E}\perp\hat{z}$, $\mathbf{H}\perp\hat{z}$. The great difference between ELTLs and MGTLs is that the latter possess neither electric charges nor electric currents ($\rho_e=0$, and $\mathbf{J}_e=0$).

The field structure in the transverse plane ($z=$constant, $\partial/\partial z = 0$) can be evaluated from (1c) and (1a) by considering an anticlockwise closed contour $\mathbf{s}$ with Stokes's normal $\mathbf{n}_s = -\hat{z}$. This leads to

$$\oint_{\mathbf{s}} \mathbf{H}\cdot d\mathbf{s} = 0, \quad curl\,\mathbf{H} = 0, \quad \mathbf{H} = -grad\,\Phi^M \tag{10}$$

$$\oint_{\mathbf{s}} \mathbf{E}\cdot d\mathbf{s} = -\frac{d}{dt}\int_{S_s} \mathbf{B}\cdot\mathbf{n}_s\,dS = \int_{S_s}\frac{\partial \mathbf{B}_z}{\partial t}\cdot\hat{z}\,dS = \frac{\partial \phi_z}{\partial t} \tag{11}$$

where Φ^M (A) is the magnetic scalar potential, and ϕ_z (Wb) is the z-directed magnetic induction flux embraced by the contour $\mathbf{s}$. Equations (10) and (3) are identical. In order to formally maintain the parallelism between (11) and (4), a magnetic current density $\mathbf{J}_m$ (V/m^2) flowing inside the wires can be defined through

$$\mathbf{J}_m = J_m\hat{z} = \frac{\partial \mathbf{B}_z}{\partial t} \tag{12}$$

from where Equation (11) can be rewritten as

$$\oint_{\mathbf{s}} \mathbf{E}\cdot d\mathbf{s} = -\int_{S_s}\mathbf{J}_m\cdot\mathbf{n}_s dS = \int_{S_s} J_m\,dS = I^M, \quad curl\,\mathbf{E} = -\mathbf{J}_m \tag{13}$$

where the magnetic current intensity I^M (V) carried by the wires is defined as

$$I^M = \frac{\partial \phi_z}{\partial t} \tag{14}$$

With regard to the divergence equations in (1b) and (1d), we have $div\,\mathbf{E} = 0$ (absence of electric charges) and $div\,\mathbf{B} = 0$. As shown in Figure 2 and if an infinitesimal section of length Δz of a magnetic wire is considered, we obtain

$$0 = div\,\mathbf{B} = div\,\mathbf{B}_z + \mu_0\,div\,\mathbf{H} \rightarrow div\,\mathbf{H} = \frac{\rho_m}{\mu_0}, \quad \rho_m = -div\,\mathbf{B}_z = -\frac{\partial B_z}{\partial z} \tag{15}$$

where ρ_m plays the role of a magnetic charge density (Wb/m^3). It may be noticed that the combination of (12) and (15) leads to $div\,\mathbf{J}_m + \partial\rho_m/\partial t = 0$ – a restatement of the well-known charge continuity equation, applied to magnetic quantities – see Appendix 1.

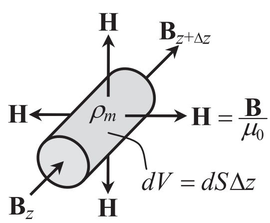

Figure 2. Application of $div\,\mathbf{B}=0$ to an infinitesimal volume of a magnetic wire.

Taking into account the results in (1b), (10), (13), and (15), the governing equations of the TEM field structure in the transverse plane of an MGTL can be summarized as follows:

$$\begin{cases} \mathbf{H} = -grad\ \Phi^M \\ div\ \mathbf{H} = \rho_m/\mu_0 \end{cases} \qquad \begin{cases} curl\ \mathbf{E} = -\mathbf{J}_m \\ div\ \mathbf{E} = 0 \end{cases} \tag{16}$$

The preceding MGTL results should be confronted with those in (5) for an ELTL. Now, the **H**-field is a gradient vector with open field lines starting and ending at the magnetically charged wires 1 and 2. The **E**-field is a solenoidal vector with closed field lines encircling the wires that carry longitudinal magnetic currents, field lines of **E** coinciding with equipotential lines of Φ^M – meaning that $\mathbf{E}\perp\mathbf{H}$.

The concepts of pul transverse inductance L_T (H/m) and pul longitudinal capacitance C_L (F/m) are utilized in MGTL theory,[2,3] and can be associated to magnetic and electric energy storage in the dielectric medium [1,3]; they are reinterpreted here based on magnetic charges and magnetic current intensities.

The per unit length (pul) magnetic charge of the wires $Q^M = Q_1^M = -Q_2^M$ and the magnetic voltage V^M between wires 1 and 2 are related to each other by the pul transverse inductance, that is

$$Q^M = L_T V^M, \qquad \begin{cases} Q^M = \lim\limits_{\Delta z \to 0} \dfrac{\int_V \rho_m (dS\Delta z)}{\Delta z} = \mu_0 \lim\limits_{\Delta z \to 0} \dfrac{\int_S \mathbf{H}\cdot\mathbf{n}_o (ds\Delta z)}{\Delta z} \\ V^M = \int\limits_{\overrightarrow{12}} \mathbf{H}\cdot d\mathbf{s} = \Phi_1^M - \Phi_2^M \end{cases} \tag{17}$$

The comparison established between (17) and (6) readily leads to the conclusion that, when geometrically identical MGTLs and ELTLs are considered, the following identity must be preserved:

$$\varepsilon L_T = \mu_0 C_T \tag{18}$$

For the definition of the pul longitudinal capacitance C_L, we have to determine the electric displacement flux ψ^E linked with a closed path, accompanying the longitudinal wires and crossing the dielectric medium at the transverse planes z and $z+\Delta z$. The path orientation is opposite to the one assigned to the magnetic current $I^M = I_1^M = -I_2^M$. Denoting by S_ψ the open surface bounded by the closed path, we can write

$$\psi^E = C_L I^M, \qquad \begin{cases} \psi^E = \varepsilon \lim\limits_{\Delta z \to 0} \dfrac{\int_{S_\psi} \mathbf{E}\cdot\mathbf{n}_\psi (ds\Delta z)}{\Delta z} \\ I^M = \displaystyle\int_{S_1} \mathbf{J}_m \cdot \hat{z}\ dS_1 = \oint_{\mathbf{s}_1} \mathbf{E}\cdot d\mathbf{s}_1 \end{cases} \tag{19}$$

where $\mathbf{s}_1$ is an anticlockwise oriented path encircling the wire 1, and S_1 is the wire's cross-section. The comparison established between (19) and (7) readily leads to the conclusion that, when geometrically identical MGTLs and ELTLs are considered, the following identity must be preserved:

$$\mu_0 C_L = \varepsilon L_L \tag{20}$$

As in the ELTL case, here also, the parameters L_T and C_L depend only on the dielectric properties and on the geometry of the two-wire TL. For a homogeneous dielectric, we obtain, from (20), (18), and (8),

$$L_T = \mu_0/g, \quad C_L = \varepsilon g, \quad L_T C_L = \mu_0 \varepsilon \tag{21}$$

Integral form equations for MGTL-guided waves are obtained from (2) using the concepts of magnetic voltage (17), magnetic current intensity (19), pul transverse inductance (17), and pul longitudinal capacitance (19). For the case of a coaxial geometry, where the fundamental TEM mode is characterized by $\mathbf{H} = H\hat{r}$ and $\mathbf{E} = -E\hat{\theta}$, we find

$$\frac{\partial E}{\partial z} = -\mu_0 \frac{\partial H}{\partial t} \rightarrow \frac{\partial I^M}{\partial z} = -L_T \frac{\partial V^M}{\partial t} \quad \text{and} \quad \frac{\partial H}{\partial z} = -\varepsilon \frac{\partial E}{\partial t} \rightarrow \frac{\partial V^M}{\partial z} = -C_L \frac{\partial I^M}{\partial t} \tag{22}$$

3. Frequency domain equations including dielectric inhomogeneity and TL losses

The results in 2 are summarized next, considering a time harmonic regime ($e^{j\omega t}$), and replacing time-varying variables by the corresponding complex amplitudes (phasors)

Electric transmission lines — Magnetic transmission lines

$$\begin{cases} \bar{\mathbf{E}} = -grad\ \bar{\Phi}^E \\ div\ \bar{\mathbf{E}} = \bar{\rho}_e/\varepsilon \end{cases}, \quad \begin{cases} curl\ \bar{\mathbf{H}} = \bar{\mathbf{J}}_e \\ div\ \bar{\mathbf{H}} = 0 \end{cases} \qquad \begin{cases} \bar{\mathbf{H}} = -grad\ \bar{\Phi}^M \\ div\ \bar{\mathbf{H}} = \bar{\rho}_m/\mu_0 \end{cases}, \quad \begin{cases} curl\ \bar{\mathbf{E}} = -\bar{\mathbf{J}}_m \\ div\ \bar{\mathbf{E}} = 0 \end{cases} \tag{23}$$

$$\begin{cases} \dfrac{d\bar{V}^E}{dz} = -j\omega L_L \bar{I}^E \\ \dfrac{d\bar{I}^E}{dz} = -j\omega C_T \bar{V}^E \end{cases} \qquad \begin{cases} \dfrac{d\bar{I}^M}{dz} = -j\omega L_T \bar{V}^M \\ \dfrac{d\bar{V}^M}{dz} = -j\omega C_L \bar{I}^M \end{cases} \tag{24}$$

where, from (12), (14), and (15): $\bar{\rho}_m = -\frac{\partial \bar{B}_z}{\partial z}$, $\bar{\mathbf{J}}_m = j\omega \bar{\mathbf{B}}_z$, and $\bar{I}^M = j\omega \bar{\phi}_z$.

In the case of TLs with coaxial geometry and where the dielectric medium is radially inhomogeneous, $\varepsilon = \varepsilon(r)$, an effective permittivity ε_{ef} can easily be defined. Consider the most familiar case of an ELTL where $D(r) = Q^E/(2\pi r)$ and $E(r) = D(r)/\varepsilon(r)$. The electric voltage between wires V^E and the corresponding pul transverse capacitance C_T are evaluated as

$$V^E = \int_{r_1}^{r_2} E dr = \frac{Q^E}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r\varepsilon(r)} \rightarrow C_T = \frac{Q^E}{V^E} = 2\pi \left(\int_{r_1}^{r_2} \frac{dr}{r\varepsilon(r)} \right)^{-1}$$

and, by making $C_T = \dfrac{2\pi\varepsilon_{ef}}{\ln(r_2/r_1)}$, we find

$$\varepsilon_{ef} = \left(\ln \frac{r_2}{r_1} \right) \left(\int_{r_1}^{r_2} \frac{dr}{r\varepsilon(r)} \right)^{-1} \tag{25}$$

For example, if $\varepsilon(r) \propto 1/r$, one will get $\varepsilon_{ef} = \varepsilon_{av} = \frac{1}{r_2 - r_1} \int_{r_1}^{r_2} \varepsilon(r) dr$.

Therefore, in the case of a coaxial MGTL with a radially inhomogeneous dielectric, the relationships in (18), (20), and (21) are maintained. Substituting ε_{ef} for ε yields

$$L_T = \frac{\mu_0 C_T}{\varepsilon_{ef}} = \frac{2\pi\mu_0}{\ln(r_2/r_1)}, \quad C_L = \frac{\varepsilon_{ef} L_L}{\mu_0} = \frac{\varepsilon_{ef}}{2\pi}\ln(r_2/r_1), \quad L_T C_L = \mu_0 \varepsilon_{ef} \tag{26}$$

Next, we address the different losses mechanisms that commonly affect the ideal behavior of TLs, especially at very high frequencies.

In ELTLs, nonideal electric wires are affected by the skin effect. In addition, the imperfect dielectric medium that separates the wires can suffer from conduction and polarization losses. Attempts to formulate these effects in the time domain have been advanced,[5] but the best way to deal with them is, undoubtedly, in the framework of frequency-domain analysis.

On the one hand, the pul skin-effect complex impedance $\bar{Z}^E_{skin}$ is added to $j\omega L_L$ in the $d\bar{V}^E/dz$ equation. On the other hand, dielectric losses can be account for by replacing the permittivity ε (or ε_{ef}) by a complex permittivity $\bar{\varepsilon} = \varepsilon(1 - j\tan\delta)$, where the loss angle δ can formally aggregate both conduction and polarization losses; in fact, when $j\omega\bar{\varepsilon}$ is evaluated, the result can be put in the form $j\omega\bar{\varepsilon} = \sigma_D + j\omega\varepsilon$, the apparent dielectric conductivity σ_D corresponding to $\varepsilon\omega\tan\delta$. The consideration of a complex permittivity leads to the substitution of the pul transverse capacitance by a complex $\bar{C}_T$ in the $d\bar{I}^E/dz$ equation.

In MGTLs, things are quite different as far as the inclusion of losses is concerned.[2,3] In nonideal magnetic wires, several unfamiliar aspects need to be paid attention. In the $d\bar{V}^M/dz$ equation, we have to account for the pul longitudinal magnetic voltage drop along the wires ($\bar{H}_z \neq 0$) due to the finitude of wires' permeability ($\mu_w \neq \infty$). This effect can be accounted by defining a pul magnetic reluctance R^M (H^{-1}/m), such that $\Delta\bar{V}^M_z = R^M\bar{\phi}_z = R^M\bar{I}^M/(j\omega)$. Moreover, MGTL wires are made of a mix of magnetic and dielectric materials, where magnetization losses, polarization losses, and conduction losses (eddy current phenomena) are present −these material imperfections being captured by a complex permeability $\bar{\mu}_w$ and complex permittivity $\bar{\varepsilon}_w$. The concomitant consideration of all of these aspects leads to the definition of a pul complex reluctance $\bar{R}^M$, whose computation can be made from frequency-domain Maxwell's equations,[6] following the same rationale used when dealing with the pul complex skin-effect impedance in electric wires. To summarize: in the $d\bar{V}^M/dz$ equation, the imperfection of magnetic wires is accounted by adding to $j\omega C_L$, a perturbation term given by $\bar{R}^M/(j\omega)$.

Finally, the imperfect dielectric medium that separates the two magnetic wires is treated as in the ELTL case: the permittivity ε is replaced by a complex permittivity $\bar{\varepsilon} = \varepsilon(1 - j\tan\delta_D)$, and the pul longitudinal capacitance C_L is replaced by a complex $\bar{C}_L$. It is worth emphasizing that the latter modification applies, again, to the $d\bar{V}^M/dz$ equation; meaning that the $d\bar{I}^M/dz$ equation is immune to MGTL losses.[2,3]

The frequency-domain equations in (24) are rewritten below for the case of nonideal lossy TLs

$$\begin{cases} \dfrac{d\bar{V}^E}{dz} = -\bar{Z}_L\bar{I}^E, \bar{Z}_L = \bar{Z}^E_{skin} + j\omega L_L \\ \dfrac{d\bar{I}^E}{dz} = -\bar{Y}_T\bar{V}^E, \ \bar{Y}_T = j\omega\bar{C}_T \end{cases} \quad \begin{cases} \dfrac{d\bar{I}^M}{dz} = -\bar{Z}_T\bar{V}^M, \ \bar{Z}_T = j\omega L_T \\ \dfrac{d\bar{V}^M}{dz} = -\bar{Y}_L\bar{I}^M, \ \bar{Y}_L = \dfrac{\bar{R}^M}{j\omega} + j\omega\bar{C}_L \end{cases} \tag{27}$$

The propagation parameters characterizing both TLs are evaluated as usually,

$$\text{ELTL}\begin{cases}\text{Propagation constant}: & \bar{\gamma}^E = \sqrt{\bar{Z}_L \bar{Y}_T} \quad (\text{m}^{-1})\\ \text{Characteristic impedance}: & \bar{Z}_W^E = \sqrt{\bar{Z}_L/\bar{Y}_T} \quad (\Omega)\end{cases}$$

$$\text{MGTL}\begin{cases}\text{Propagation constant}: & \bar{\gamma}^M = \sqrt{\bar{Z}_T \bar{Y}_L} \quad (\text{m}^{-1})\\ \text{Characteristic impedance}: & \bar{Z}_W^M = \sqrt{\bar{Z}_T/\bar{Y}_L} \quad (\Omega)\end{cases}$$

where, for matched lines, $\bar{Z}_W^E$ is the ratio $\dfrac{\bar{V}^E(z)}{\bar{I}^E(z)}$, and $\bar{Z}_W^M$ is the ratio $\dfrac{\bar{I}^M(z)}{\bar{V}^M(z)}$.

The generalization of (27) to multiwire systems, with $N+1$ wires, is achieved by substituting scalar variables by Nth order matrices,[3,6]

In ELTLs: $\bar{V}^E \rightarrow [\bar{V}^E]$, $\bar{I}^E \rightarrow [\bar{I}^E]$, $\bar{Z}_L \rightarrow [\bar{Z}_L]$, $\bar{Y}_T \rightarrow [\bar{Y}_T]$.
In MGTLs: $\bar{V}^M \rightarrow [\bar{V}^M]$, $\bar{I}^M \rightarrow [\bar{I}^M]$, $\bar{Z}_T \rightarrow [\bar{Z}_T]$, $\bar{Y}_L \rightarrow [\bar{Y}_L]$.

4. Conclusion

In this research, we reanalyzed, reworked, and reformulated an initial version of MGTL theory recently published. The novel development offered in this paper – which we believe is easier to understand and makes clear the dualism between ELTLs and MGTLs – was based on the new concepts of magnetic charges and magnetic currents, whose meaning was thoroughly discussed. Transverse field equations and wave propagation equations for ideal ELTLs and ideal MGTLs were time-domain formulated and, afterwards, the frequency-domain was utilized to account for system losses. The topic of dielectric medium inhomogeneity was also addressed considering the particular case of a coaxial geometry. Future work will focus on the numerical evaluation of the high-frequency propagation parameters (attenuation, wave velocity, and characteristic impedance) of lossy inhomogeneous coaxial MGTLs.

Acknowledgments

The Fundação para a Ciência e a Tecnologia (Project PEst-OE/EEI/LA0008/2013), and the Instituto de Telecomunicações (Project PC226), sponsored this work.

References

[1] Faria JA. A physical model of the ideal transformer based on magnetic transmission line theory. J. Electromagn. Waves Appl. 2013;27:365–373.
[2] Faria JA, Pires MP. Theory of magnetic transmission lines. IEEE Trans. Microwave Theory Tech. 2012;60:2941–2949.
[3] Faria JA. Formulation of multiwire magnetic transmission-line theory. Prog. Electromagn. Res. B. 2013;49:177–195.
[4] Maxwell JC. Treatise on electricity and magnetism. Oxford: Oxford Univ. Press; 1892.
[5] Svensson C, Dermer GE. Time-domain modeling of lossy interconnects. IEEE Trans. Adv. Pack. 2001;24:191–196.
[6] Faria JA. Multiconductor transmission-line structures. New York: Wiley; 1993.
[7] Polchinski J. Monopoles, duality, and string theory. Int. J. Modern Phys. A. 2004;19:145–156.
[8] Rajantie A. Introduction to magnetic monopoles. Contemp. Phys. 2012;53:195–211.

Appendix 1. Magnetic charge

The concept of magnetic charge density ρ_m appeared in (15) as a virtual means to describe the divergence of the stray magnetic field $\mathbf{H}$ in an MGTL, $\rho_m = -\partial B_z/\partial z$. There are other situations where ρ_m can also be used as a conceptual tool. For instance, consider a bulk magnetic medium where the permeability μ varies from point to point. If $div\ (\mu\mathbf{H})$ is evaluated at any point of the inhomogeneous medium, we will find

$$div\ (\mu\mathbf{H}) = 0 = \mu\ div\mathbf{H} + grad\mu \cdot \mathbf{H}$$

hence

$$div\ \mathbf{H} = \frac{\rho_m}{\mu}, \quad \text{where } \rho_m = -grad\mu \cdot \mathbf{H}.$$

Again, this shows that the concept of magnetic charge

$$q_m = \int_V \rho_m dV$$

has nothing to do with actual magnetic monopoles, whose existence in nature has never been conclusively evidenced. Nonetheless, the reader may wish to know that unified-field and string theories do foresee the existence of magnetic monopoles in the universe.[7,8]