Theoretical Study of Gilbert Damping Constants in Magnetic Multilayer Films

Dai Ozaki, Daisuke Miura, and Akimasa Sakuma

Department of Applied Physics, Tohoku University, Sendai 980-8579, Japan

A theory is presented, describing Gilbert damping of a ferromagnetic layer by treating the outflow of the spin angular momentum to lattice and nonmagnetic systems on the same footing. The Mori formalism is applied to the entire system, including at the interface between different bulk (or film) systems. The Gilbert damping constant is determined from both Kamersky's spin-orbit-related torque correlation (TC) term and the hopping-related TC term. In addition, we confirm that the hopping-related TC term corresponds to conventional Gilbert damping enhanced by spin pumping.

Index Terms— **Gilbert damping, Landau–Lifshitz–Gilbert (LLG) equation, spin dynamics.**

I. Introduction

CONTROLLING the dynamics of magnetization is a central problem in the development of high-speed magnetic devices. Magnetization $\boldsymbol{M}$ obeys the Landau–Lifshitz–Gilbert (LLG) equation

$$\frac{d\boldsymbol{M}}{dt} = -\gamma\,\boldsymbol{M} \times (\mu_0 \boldsymbol{H}_{\text{eff}}) + \alpha \frac{\boldsymbol{M}}{M} \times \frac{d\boldsymbol{M}}{dt} \tag{1}$$

where γ, μ_0, and $\boldsymbol{H}_{\text{eff}}$ represent, respectively, the gyromagnetic ratio, the vacuum permeability, and the effective magnetic field acting on $\boldsymbol{M}$. The second term on the right-hand side of the equation represents magnetization relaxation, the strength of which is represented by a phenomenological parameter α, called the Gilbert damping constant, which dominates the rate of switching of $\boldsymbol{M}$. There is much motivation for evaluating this important parameter quantitatively and to understand its microscopic origin.

Kamberský [1] and Gilmore *et al.* [2] proposed a theoretical model in 2007, denoted "torque correlation (TC) model," to describe Gilbert damping in bulk systems. The Gilbert damping constant of ferromagnetic (FM) metals was there computed from first principles. The TC model assumes that Gilbert damping arises from spin consumption resulting from the atomic spin–orbit interaction (SOI). Furthermore, the TC model applies to disordered systems. We examined α in chemical- and spin-disordered alloys [3], [4] using the tight-binding linear muffin-tin orbital method combined with the coherent-potential approximation. Meanwhile, Ebert *et al.* [5] adopted a different approach to express α for disordered transition metal alloys using the linear-response theory. In their approach, α can be expressed as the imaginary part of a spin-correlation (SC) function. We hereafter refer to this model as the "SC model" [6], [7]. Furthermore, the equivalence between the TC and SC models was confirmed by Garate and MacDonald [8] and Sakuma [9].

Although the above-mentioned magnetization damping relates to bulk systems, Mizukami *et al.* [10] experimentally observed that Gilbert damping depends on magnetic structures and showed that it is controllable by manipulating each element or/and, in the case of magnetic multilayers, the layer thickness. This effect is explained by the spin-pumping theory, which is itself based on the scattering theory [11]. Spin-pumping theory describes nonequilibrium spin currents flowing between FM and nonmagnetic (NM) layers induced by magnetization precession. The contribution to magnetization damping, in addition to the bulk-value effect, observed by Mizukami *et al.* [10] can be understood as originating from nonequilibrium spin currents. First-principles calculations based on the spin-pumping theory were also performed to evaluate α quantitatively in ordered [12] and disordered alloys [13]. At first glance, although the spin-pumping theory seems to differ from the TC (or SC) model approach, their equivalence was confirmed by Brataas *et al.* [14].

As stated earlier, different explanations have been proposed for Gilbert damping, depending on its occurrence in bulk or multilayer systems, i.e., respectively, by the TC or SC model or by the spin-pumping model. This paper sought to develop a unified theory of Gilbert damping based on Mori's general relaxation formalism and to reveal the relationship between the earlier theories systematically. By assuming magnetic multilayers consisting of FM and NM layers, we theoretically investigated Gilbert damping within the FM layer. To apply our theory and calculate α, we considered a simple tight-binding Hamiltonian with the Rashba-type SOI (RSOI). Consequently, we show that α is expressed not only by the spin torque originated from the SOI (henceforth denoted as "spin-orbit-related TC term") that appears in the TC or SC model but also by the hopping-related TC term that corresponds to spin pumping.

II. Model and Formulation

We describe Gilbert damping within FM layers in a magnetic multilayer system as follows. Denoting $\boldsymbol{M}_{\text{FM}}$ as the magnetization within a FM layer and applying a transverse

Manuscript received November 8, 2018; revised December 21, 2018; accepted December 27, 2018. Date of publication February 21, 2019; date of current version June 20, 2019. Corresponding author: D. Ozaki (e-mail: ozaki@solid.apph.tohoku.ac.jp).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TMAG.2019.2893370

magnetic field rotating at frequency ω to $\boldsymbol{M}_{\rm FM}$, the LLG equation (1) predicts its transverse dynamical susceptibility as

$$\chi^{-+}_{\rm LLG}(\omega) = \frac{-2\mu_0\mu_B|\boldsymbol{M}_{\rm FM}|}{\hbar\omega - \hbar\omega_0 + i\alpha\hbar\omega} \tag{2}$$

where ω_0 is the resonance frequency, and μ_B is the Bohr magneton. In the same situation, the Kubo formula [15] specifies the microscopic expression as

$$\chi^{-+}_{\rm Kubo}(\omega) = \frac{{\rm i}\mu_0 V_{\rm FM}}{2\hbar}\int_0^\infty dt e^{{\rm i}(\omega+{\rm i}\delta)t}\langle[\hat{M}^-_{\rm FM}(t), \hat{M}^+_{\rm FM}]\rangle \tag{3}$$

where $V_{\rm FM}$ is the total volume of the FM layers, $\hat{M}^\pm_{\rm FM} := \hat{M}^x_{\rm FM} \pm {\rm i}\hat{M}^y_{\rm FM}$, δ is a positive infinitesimal number, a hat symbol denotes an operator, and $\langle\ldots\rangle$ represents a statistical average. Applying the Mori formalism [16], $\chi^{-+}_{\rm Kubo}(\omega)$ can be reduced to

$$\chi^{-+}_{\rm Kubo}(\omega) = \frac{-2\mu_0\mu_B\langle\hat{M}^z_{\rm FM}\rangle}{\hbar\omega - \Omega_{\rm FM}(\hbar\omega+{\rm i}\delta) + \Gamma_{\rm FM}(\hbar\omega+{\rm i}\delta)} \tag{4}$$

where $\Omega_{\rm FM}(\hbar\omega + i\delta)$ is a real quantity, and $\Gamma(z)$ is given by

$$\Gamma_{\rm FM}(z) = \frac{\hbar^2 V_{\rm FM}}{4\mu_{\rm B}\langle\hat{M}^z_{\rm FM}\rangle}\frac{\rm i}{\hbar}\int_0^\infty dt {\rm e}^{{\rm i}zt}\langle[\hat{T}^-_{\rm FM}(t), \hat{T}^+_{\rm FM}]\rangle \tag{5}$$

where $\hat{T}^\pm_{\rm FM}$ is a torque operator corresponding to $\hat{M}^\pm_{\rm FM}$. Its actual form is given later, as it depends on the system Hamiltonian. Comparing both spin susceptibilities $\chi^{-+}_{\rm LLG}(\omega)$ and $\chi^{-+}_{\rm Kubo}(\omega)$, the microscopic expression for α in a FM layer in the magnetic multilayer system is given by

$$\alpha_{\rm FM} := \alpha = \lim_{\omega\to 0}\frac{1}{\hbar\omega}\Im\Gamma_{\rm FM}(\hbar\omega + i\delta). \tag{6}$$

This treatment is clearly the same as Kambersky's for the case of a single FM layer. However, both analyses yield different results for the case of multilayers, where local damping within the FM layers is displayed. In other words, in the present theory, the layers outside the FM layers serve as sinks for the spins flowing out from the FM layers. From this perspective, our model is expected to also include an effect analogous to spin pumping; furthermore, the information relative to the spin sinks appears explicitly within the Hamiltonian.

We now consider Gilbert damping within the FM layer in a bilayer system that consists of FM and NM layers as shown in Fig. 1. The following simple Hamiltonian is adopted:

$$\hat{\mathcal{H}} := \hat{\mathcal{H}}_{\rm hop} + \hat{\mathcal{H}}_{\rm ex} + \hat{\mathcal{H}}_{\rm SOI} \tag{7}$$

where the hopping term is given by

$$\hat{\mathcal{H}}_{\rm hop} := \sum_{\boldsymbol{k}}\sum_{m=1}^{N_\perp}\epsilon_{\boldsymbol{k}}c^\dagger_{\boldsymbol{k},m}c_{\boldsymbol{k},m} - T_{\rm hop}\sum_{\boldsymbol{k}}\sum_{m=1}^{N_\perp-1}\left(c^\dagger_{\boldsymbol{k},m}c_{\boldsymbol{k},m+1} + c^\dagger_{\boldsymbol{k},m+1}c_{\boldsymbol{k},m}\right) \tag{8}$$

where $c_{\boldsymbol{k},m}$ is the destruction operator for an electron with an in-layer wave vector $\boldsymbol{k}$ in the mth layer, $N_\perp$ is the total stack number, $\epsilon_{\boldsymbol{k}} = -2T_{\rm hop}(\cos k_x a + \cos k_y a)$ is the kinetic energy corresponding to the hopping integral $-T_{\rm hop}$ and a square-lattice constant a, and the hopping energy between the

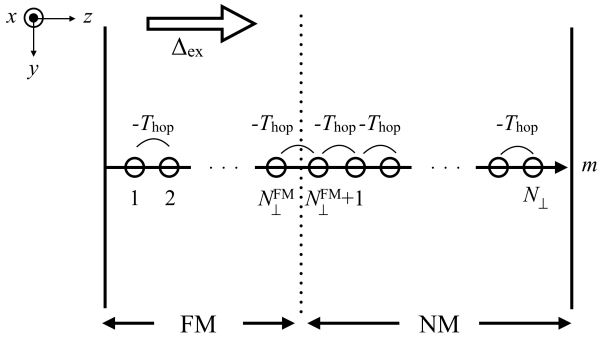


Fig. 1. Model calculation framework for a magnetic bilayer with $N_\perp$ monolayers stacked along the z-direction. $\Delta_{\rm ex}$ is the strength of the exchange field applied within the FM layer ($N_\perp^{\rm FM}$ monolayers), oriented along the z-direction. The crystal structure in the xy plane is a periodic square lattice.

mth and $(m+1)$st layers is described by the second term. The exchange term is given by

$$\hat{\mathcal{H}}_{\rm ex} := \Delta_{\rm ex}\frac{V_{\rm FM}}{\mu_{\rm B}}\hat{M}^z_{\rm FM} \tag{9}$$

$$\hat{\boldsymbol{M}}_{\rm FM} := -\frac{\mu_B}{V_{\rm FM}}\sum_{\boldsymbol{k}}\sum_{m=1}^{N_\perp^{\rm FM}}c^\dagger_{\boldsymbol{k},m}\boldsymbol{\sigma}c_{\boldsymbol{k},m} \tag{10}$$

where $\Delta_{\rm ex}$ is the strength of the exchange coupling, $\boldsymbol{\sigma}$ denotes the Pauli matrices, and the summation over m is limited to the FM-layer sites from $m = 1$ to $m = N_\perp^{\rm FM}$. The SOI is described by

$$\mathcal{H}_{\rm SOI} := \lambda_{\rm FM}\sum_{\boldsymbol{k}}\sum_{m=1}^{N_\perp^{\rm FM}}c^\dagger_{\boldsymbol{k},m}(\sigma^y\sin k_x a - \sigma^x\sin k_y a)c_{\boldsymbol{k},m} + \lambda_{\rm NM}\sum_{\boldsymbol{k}}\sum_{m=N_\perp^{\rm FM}+1}^{N_\perp^{\rm NM}}c^\dagger_{\boldsymbol{k},m}(\sigma^y\sin k_x a - \sigma^x\sin k_y a)c_{\boldsymbol{k},m} \tag{11}$$

where $\lambda_{\rm FM}$ and $\lambda_{\rm NM}$ are the strengths of the RSOI [17], [18] within the FM and NM layers, respectively.

Before calculating $\hat{T}^\pm_{\rm FM}$ associated with (7), we note that the contribution from $\hat{\mathcal{H}}_{\rm ex}$ should be excluded from the calculation, given its origin. The fact that $\hat{\mathcal{H}}_{\rm ex}$ originates from the electron–electron Coulomb interaction energy, $\hat{\mathcal{H}}_{\rm ee}$, is reminiscent of the reaction torque between $\hat{\mathcal{H}}_{\rm ex}$ and an electron. Because the action and reaction torques cancel, the contribution from $\hat{\mathcal{H}}_{\rm ee}$ vanishes. We can, thus, conclude that the contribution from $\hat{\mathcal{H}}_{\rm ex}$ is essentially absent.

Based on these considerations, we define

$$\hat{T}^\pm_{\rm FM} := \frac{\rm i}{\hbar}[\hat{\mathcal{H}}_{\rm hop} + \hat{\mathcal{H}}_{\rm SOI}, \hat{M}^\pm_{\rm FM}] \tag{12}$$

$$\equiv \hat{T}'^\pm_{\rm FM} + \hat{T}''^\pm_{\rm FM} \tag{13}$$

where

$$\hat{T}'^-_{\rm FM} = \frac{\rm i}{\hbar}[\mathcal{H}_{\rm SOI}, \hat{M}^-_{\rm FM}] \tag{14}$$

$$= \frac{2\mu_{\rm B}}{V_{\rm FM}}\lambda_{\rm FM}\frac{\rm i}{\hbar}\sum_{\boldsymbol{k}}\sum_{m=1}^{N_\parallel^{\rm FM}}\left(c^\dagger_{\boldsymbol{k},m,\uparrow}c_{\boldsymbol{k},m,\uparrow} - c^\dagger_{\boldsymbol{k},m,\downarrow}c_{\boldsymbol{k},m,\downarrow}\right) \times(i\sin k_x a + \sin k_y a) \tag{15}$$

which corresponds to Kambersky's spin-orbit-related torque arising from the intrinsic SOI. Note that $\hat{T}^{+}_{\mathrm{FM}} = (\hat{T}^{-}_{\mathrm{FM}})^{\dagger}$. In addition, in the case of multilayers, the commutator between the FM magnetization and the inter-layer hopping term is

$$\hat{T}^{-\prime\prime}_{\mathrm{FM}} \equiv \frac{\mathrm{i}}{\hbar}\left[\mathcal{H}_{\mathrm{hop}}, \hat{M}^{-}_{\mathrm{FM}}\right] \tag{16}$$

$$= \frac{2\mu_B}{V_{\mathrm{FM}}} T_{\mathrm{hop}} \frac{\mathrm{i}}{\hbar} \sum_{\boldsymbol{k}} \left(c^{\dagger}_{\boldsymbol{k},N^{\mathrm{FM}}_{\perp}+1,\downarrow} c_{\boldsymbol{k},N^{\mathrm{FM}}_{\perp},\uparrow} - c^{\dagger}_{\boldsymbol{k},N^{\mathrm{FM}}_{\perp},\downarrow} c_{\boldsymbol{k},N^{\mathrm{FM}}_{\perp}+1,\uparrow}\right). \tag{17}$$

This result recalls the spin current $\boldsymbol{J}_s$

$$\frac{\partial \boldsymbol{M}}{\partial t} = -\gamma\, \mathrm{div}\boldsymbol{J}_s + \boldsymbol{\tau}_{\mathrm{SOI}} \tag{18}$$

where $\boldsymbol{\tau}_{\mathrm{SOI}}$ is a torque originating from the SOI. The first term of (18) can readily be shown to vanish when the operator $\hat{M}^{-}_{\mathrm{FM}}$ is replaced by $\hat{M}^{-}_{\mathrm{total}}$ to include both the FM and NM layers. This corresponds to Kambersky's treatment.

Consequently, by using the eigenstates satisfying $\mathcal{H}|\boldsymbol{k}, i\rangle = E_{\boldsymbol{k},i}|\boldsymbol{k}, i\rangle$, α can be expressed as

$$\alpha = -\frac{\pi}{\langle s^z\rangle} \sum_{\boldsymbol{k}} \sum_{i,j} \left| \sum_{m=1}^{N^{\mathrm{FM}}_{\perp}} \left(T^{i,j,-}_{\boldsymbol{k},m} + J^{i,j,-}_{\boldsymbol{k},\mathrm{int}}\right) \right|^2 \times \delta(\epsilon_{\mathrm{F}} - \epsilon_{i,\boldsymbol{k}})\delta(\epsilon_{\mathrm{F}} - \epsilon_{j,\boldsymbol{k}}) \tag{19}$$

where ϵ_{F} is the Fermi level and

$$T^{i,j,-}_{\boldsymbol{k},m} := \lambda_{\mathrm{FM}}(\langle \boldsymbol{k}, i|\boldsymbol{k}, m, \uparrow\rangle\langle \boldsymbol{k}, m, \uparrow|\boldsymbol{k}, j\rangle - \langle \boldsymbol{k}, i|\boldsymbol{k}, m, \downarrow\rangle \times \langle \boldsymbol{k}, m, \downarrow|\boldsymbol{k}, j\rangle)(i\sin k_x a + \sin k_y a) \tag{20}$$

and

$$J^{i,j,-}_{\boldsymbol{k},\mathrm{int}} := T_{\mathrm{hop}}\left(\left\langle \boldsymbol{k}, i\middle|\boldsymbol{k}, N^{\mathrm{FM}}_{\perp}+1, \downarrow\right\rangle\left\langle \boldsymbol{k}, N^{\mathrm{FM}}_{\perp}, \uparrow\middle|\boldsymbol{k}, j\right\rangle - \left\langle \boldsymbol{k}, i\middle|\boldsymbol{k}, N^{\mathrm{FM}}_{\perp}, \downarrow\right\rangle\left\langle \boldsymbol{k}, N^{\mathrm{FM}}_{\perp}+1, \uparrow\middle|\boldsymbol{k}, j\right\rangle\right). \tag{21}$$

Kambersky also expressed the delta function as $\delta(\epsilon) = -(1/\pi)\mathrm{Im}\{1/(\epsilon + i\delta)\}$ and replaced δ with $\hbar/\tau$, where τ represents the electron lifetime. The expression for α apparently consists of three terms: a spin-orbit-related TC term

$$\alpha_T = -\frac{\pi}{\langle s^z\rangle} \sum_{\boldsymbol{k}} \sum_{i,j} \sum_{m=1}^{N^{\mathrm{FM}}_{\perp}} \left|T^{i,j,-}_{\boldsymbol{k},m}\right|^2 \delta(\epsilon_{\mathrm{F}} - \epsilon_{i,\boldsymbol{k}})\delta(\epsilon_{\mathrm{F}} - \epsilon_{j,\boldsymbol{k}}) \tag{22}$$

a hopping-related TC term

$$\alpha_J = -\frac{\pi}{\langle s^z\rangle} \sum_{\boldsymbol{k}} \sum_{i,j} \left|J^{i,j,-}_{\boldsymbol{k},\mathrm{int}}\right|^2 \delta(\epsilon_{\mathrm{F}} - \epsilon_{i,\boldsymbol{k}})\delta(\epsilon_{\mathrm{F}} - \epsilon_{j,\boldsymbol{k}}) \tag{23}$$

and a hybrid term

$$\alpha_{T-J} = -\frac{\pi}{\langle s^z\rangle} \sum_{\boldsymbol{k}} \sum_{i,j} \sum_{m=1}^{N^{\mathrm{FM}}_{\perp}} \left(T^{i,j,-}_{\boldsymbol{k},m} J^{i,j,+}_{\boldsymbol{k},\mathrm{int}} + J^{i,j,-}_{\boldsymbol{k},\mathrm{int}} T^{i,j,+}_{\boldsymbol{k},m}\right) \times \delta(\epsilon_{\mathrm{F}} - \epsilon_{i,\boldsymbol{k}})\delta(\epsilon_{\mathrm{F}} - \epsilon_{j,\boldsymbol{k}}) \tag{24}$$

that need not vanish in general but which does vanish in our numerical calculations for the present system.

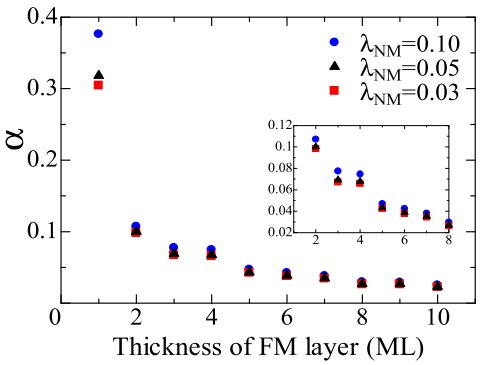
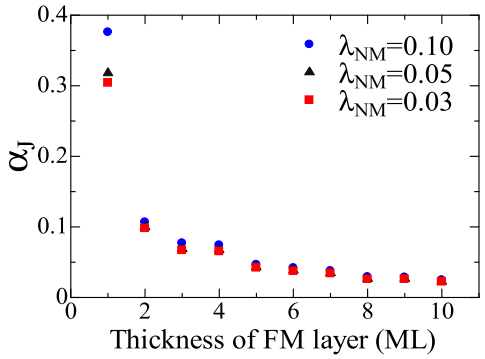
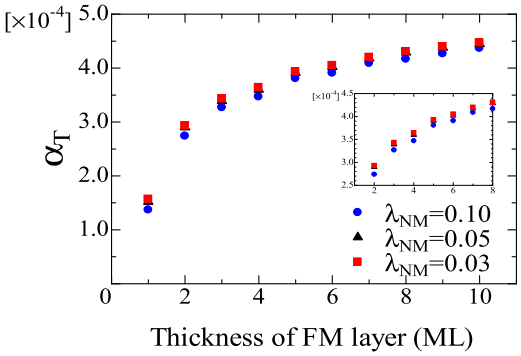


Fig. 2. FM-layer thickness dependence of α. The units of T_{hop} are normalized by setting $\Delta_{\mathrm{ex}} = -1.0$, $\lambda_{\mathrm{FM}} = 0.01$, and $\hbar/\tau = 0.1$. The electron concentration is $n = 0.20$.



(a)



(b)

Fig. 3. FM-layer thickness dependences of α_J and α_T. The units of T_{hop} are normalized by setting $\Delta_{\mathrm{ex}} = -1.0$, $\lambda_{\mathrm{FM}} = 0.01$, and $\hbar/\tau = 0.1$. The electron concentration is $n = 0.20$. (a) FM-layer thickness dependence of α_J originating from the in-layer hopping term. (b) FM-layer thickness dependence of α_T originating from the RSOI.

III. Results and Discussion

The following calculation normalizes the units of T_{hop} by setting $\Delta_{\mathrm{ex}} = -1.0$, $\lambda_{\mathrm{FM}} = 0.01$, and $\hbar/\tau = 0.1$, which are typical values for FM metals. The electron concentration is, thus, fixed to $n = 0.20$ to guarantee magnetism [18].

According to (19), we examined the FM-layer thickness dependence of α, as shown in Fig. 2. Several values of λ_{NM} were chosen according to the experiments of Mizukami *et al.* [10]. By increasing the FM-layer thickness, α decreases drastically. Moreover, the strength of α is proportional to λ_{NM}. Thus, we can confirm that our numerical result is qualitatively equivalent to their results.

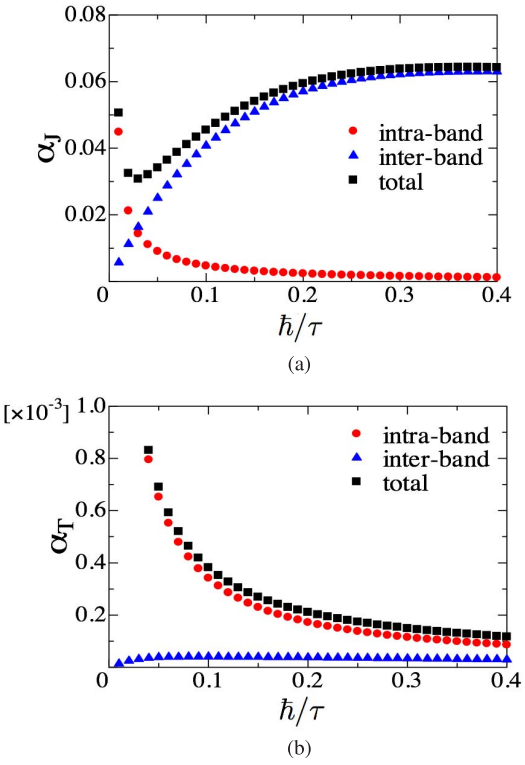


Fig. 4. Dependences of α_J and α_T on the electron scattering rate $\hbar/\tau$. Units of $T_{\rm hop}$ are normalized by setting $\Delta_{\rm ex} = -1.0$, $\lambda_{\rm FM} = 0.01$, and $\lambda_{\rm NM} = 0.05$. The electron concentration is $n = 0.20$. (a) Calculated α_J, originating from the inter-layer hopping term, as a function of the electron scattering rate $\hbar/\tau$. (b) Calculated α_T, originating from the RSOI, as a function of the electron scattering rate $\hbar/\tau$.

Equation (19) can be decomposed into α_J, originating from the interlayer hopping term, and α_T, originating from the RSOI. Each result is plotted in Fig. 3. Since α_J and α_T are proportional to $T_{\rm hop}^2$ and $\lambda_{\rm FM}^2$, respectively, the ratio of their respective α values seems to be reasonable. In Fig. 3(a), α_J decreases as the FM-layer thickness increases and shows a significant enhancement. From this behavior and from (18), the hopping-related TC term α_J turns out to be conventional Gilbert damping enhanced by spin pumping, resulting from the conduction of spin-carrying electrons from the FM to the NM layers. In Fig. 3(b), α_T increases as the FM-layer thickness increases and asymptotically approaches a constant value. Barati *et al.* [19] confirmed the enhancement of α with increasing FM-layer thickness, using Kambersky's torque-correlation model. We confirm this tendency based on first principles [20]. This enhancement is caused by the modification of the electronic structure at the NM–FM layer interface. However, the behavior in Fig. 3(b) differs from the result obtained by Barati *et al.* [19] The reason is that our calculated α is a *local* value within FM layers. Furthermore, our expression clearly distinguishes between the scenarios of spin pumping and of the SOI.

Fig. 4 plots the dependences of α_J and α_T on the electron scattering rate $\hbar/\tau$. The form of (19) allows a separation of the intra- and inter-band components. Their behavior is consistent with that in [1]. In Fig. 4, although the inter-band component appears to be small, these behaviors depend on the parameter of the model Hamiltonian.

We confirmed the validity of (19) for multilayered films. It combines, on the same footing, the outflow of the spin angular momentum to the lattice and NM systems. In Kambersky's formula, generalized spin torques are defined in the entire system; consequently, the hopping-related TC term does not appear because of spin conservation. However, in a realistic system, there should be an outflow of spin current to the environment that interacts with the system. This idea is similar to that proposed by Tserkovnyak *et al.* [11], which considered a magnetic film connected to the leads. The same idea also features in the torque-correlation model, i.e., the orbital angular momentum L is not considered as contributing a magnetic moment because L imparts angular momentum to the environment. Moreover, if we consider the NM layers as constituting the environment in the concrete form of the interaction of the interference between the FM and NM layers, such a hybrid orbital is imposed on the outflow calculation of the spin current without requiring an additional phenomenological parameter.

IV. Conclusion

We have constructed a theory, based on the Mori formalism, to describe Gilbert damping in bulk and multilayer systems. The Gilbert damping constant α is expressed as a correlation function between generalized spin torques, comprising a hopping-related TC term α_J that originates from the inter-layer hopping term of the conduction electrons, and a spin-orbit TC term α_T derived by Kambersky. We have confirmed that the hopping-related TC term corresponds to conventional Gilbert damping enhanced by spin pumping.

Acknowledgment

This work was supported in part by JSPS KAKENHI under Grant 16K06702, Grant 16H02390, Grant 16H04322, and Grant 17K14800 in Japan.

References

[1] V. Kamberský, "Spin-orbital Gilbert damping in common magnetic metals," *Phys. Rev. B, Condens. Matter*, vol. 76, no. 13, Oct. 2007, Art. no. 134416, doi: 10.1103/PhysRevB.76.134416.

[2] K. Gilmore, Y. U. Idzerda, and M. D. Stiles, "Identification of the dominant precession-damping mechanism in Fe, Co, and Ni by first-principles calculations," *Phys. Rev. Lett.*, vol. 99, no. 2, Jul. 2007, Art. no. 027204, doi: 10.1103/PhysRevLett.99.027204.

[3] A. Sakuma, "First-principles study on the Gilbert damping constants of transition metal alloys, Fe–Ni and Fe–Pt systems," *J. Phys. Soc. Jpn.*, vol. 81, no. 8, Jul. 2012, Art. no. 084701, doi: 10.1143/JPSJ.81.084701.

[4] D. Ozaki, D. Miura, and A. Sakuma, "First-principles study on the magnetic damping of transition metals in the presence of spin fluctuation," *IEEE Trans. Magn.*, vol. 53, no. 11, Nov. 2017, Art. no. 1300604.

[5] H. Ebert, S. Mankovsky, D. Ködderitzsch, and P. J. Kelly, "*Ab initio* calculation of the Gilbert damping parameter via the linear response formalism," *Phys. Rev. Lett.*, vol. 107, no. 6, Aug. 2011, Art. no. 066603, doi: 10.1103/PhysRevLett.107.066603.

[6] E. Šimánek and B. Heinrich, "Gilbert damping in magnetic multilayers," *Phys. Rev. B, Condens. Matter*, vol. 67, no. 14, Apr. 2003, Art. no. 144418, doi: 10.1103/PhysRevB.67.144418.

[7] N. Umetsu, D. Miura, and A. Sakuma, "Spin-wave-induced spin torque in Rashba ferromagnets," *Phys. Rev. B, Condens. Matter*, vol. 91, no. 17, May 2015, Art. no. 174440, doi: 10.1103/PhysRevB.91.174440.

[8] I. Garate and A. MacDonald, "Gilbert damping in conducting ferromagnets. II. Model tests of the torque-correlation formula," *Phys. Rev. B, Condens. Matter*, vol. 79, no. 6, Feb. 2009, Art. no. 064404, doi: 10.1103/PhysRevB.79.064404.

[9] A. Sakuma, "Theoretical investigation on the relationship between the torque correlation and spin correlation models for the Gilbert damping constant," *J. Appl. Phys.*, vol. 117, no. 1, Jan. 2015, Art. no. 013912, doi: 10.1063/1.4905429.

[10] S. Mizukami, Y. Ando, and T. Miyazaki, "The study on ferromagnetic resonance linewidth for NM/80nife/NM (NM=Cu, Ta, Pd and Pt) films," *Jpn. J. Appl. Phys.*, vol. 40, no. 2, p. 580, Feb. 2001, doi: 10.1143/JJAP.40.580/meta.

[11] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, "Enhanced Gilbert damping in thin ferromagnetic films," *Phys. Rev. Lett.*, vol. 88, Mar. 2002, Art. no. 117601, doi: 10.1103/PhysRevLett.88.117601.

[12] M. Zwierzycki, Y. Tserkovnyak, P. J. Kelly, A. Brataas, and G. E. W. Bauer, "First-principles study of magnetization relaxation enhancement and spin transfer in thin magnetic films," *Phys. Rev. B, Condens. Matter*, vol. 71, Feb. 2005, Art. no. 064420, doi: 10.1103/PhysRevB.71.064420.

[13] A. A. Starikov, P. J. Kelly, A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, "Unified first-principles study of Gilbert damping, spin-flip diffusion, and resistivity in transition metal alloys," *Phys. Rev. Lett.*, vol. 105, no. 23, Dec. 2010, Art. no. 236601, doi: 10.1103/PhysRevLett.105.236601.

[14] A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, "Scattering theory of Gilbert damping," *Phys. Rev. Lett.*, vol. 101, no. 3, Jul. 2008, Art. no. 037207, doi: 10.1103/PhysRevLett.101.037207.

[15] R. Kubo, "Statistical-mechanical theory of irreversible processes. I. General theory and simple applications to magnetic and conduction problems," *J. Phys. Soc. Jpn.*, vol. 12, no. 6, pp. 570–586, Mar. 1957, doi: 10.1143/JPSJ.12.570.

[16] H. Mori, "Transport, collective motion, and Brownian motion," *Prog. Theor. Phys.*, vol. 33, no. 3, pp. 423–455, Mar. 1965. [Online]. Available: https://academic.oup.com/ptp/article/33/3/423/1925580

[17] E. I. Rashba, "Properties of semiconductors with an extremum loop. 1. Cyclotron and combinational resonance in a magnetic field perpendicular to the plane of the loop," *Sov. Phys. Solid State*, vol. 2, p. 1109, Jan. 1960.

[18] A. Sakuma, "First-principles study on the non-collinear magnetic structures of disordered alloys," *J. Phys. Soc. Jpn.*, vol. 69, no. 9, pp. 3072–3083, May 2000, doi: 10.1143/JPSJ.69.3072.

[19] E. Barati, M. Cinal, D. M. Edwards, and A. Umerski, "Gilbert damping in magnetic layered systems," *Phys. Rev. B, Condens. Matter*, vol. 90, no. 1, Jul. 2014. Art. no. 014420, doi: 10.1103/PhysRevB.90.014420.

[20] R. Hiramatsu, D. Miura, and A. Sakuma, "First principles calculation for Gilbert damping constants in ferromagnetic/non-magnetic junctions," *AIP Adv.*, vol. 8, no. 5, Dec. 2017, Art. no. 056016, doi: 10.1063/1.5007255.