A Full-Wave Two-Dimensional Finite-Difference Frequency-Domain Analysis of Ferrite-Loaded Structures

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***Abstract*--** A full-wave two-dimensional (2-D) finite-difference frequency-domain (FDFD) method is presented for analyzing the dispersion characteristics of transversely magnetized ferrite loaded structures. Results for a number of different ferrite loaded structures are presented. The accuracy of the FDFD formulation is verified by comparing the numerical results with analytical results and those obtained using the FDTD method, and very good agreement is observed.

Index Terms: Finite Difference, Ferrite, Waveguides, Numerical techniques.

I. Introduction

Ferrite materials are widely used in the manufacturing of microwave devices, such as isolators, circulators, and phase shifters, and the study of the electromagnetic behavior of these materials is a crucial issue in the design of such devices. In general, the study of ferrite-loaded transmission lines and waveguides has been mainly performed either in frequency domain (spectral domain method, mode matching method, finite element method), or the time domain (finite-difference time-domain method). Recently, the use of finite-difference time-domain (FDTD) method for the dispersion analysis of ferrite-loaded structures has been the subject of many papers [1-5].

Very recently, a compact two-dimensional (2-D) finite-difference frequency-domain (FDFD) method has been proposed for the dispersion analysis of guiding structures containing isotropic materials [6-9]. In this method, based on Yee's cell, Maxwell's equations are discretized in the frequency domain, as opposed to time domain in FDTD method. The 2D-FDFD method has proven to be an efficient and powerful technique in the dispersion analysis of general guiding structures [6-9]. In contrast to the 2D-FDTD method, the phase constant in the 2D-FDFD method is found for a given frequency, which, practically, is more convenient. In [10], the 2D-FDFD method has been extended to analyze a rectangular waveguide that is completely filled with a ferrite medium. The four transverse field components were involved in the analysis. Moreover, full details of the formulation and the approach used to enforce the boundary conditions on the conducting walls were not provided in [10].

In this paper, a two-dimensional (2-D) FDFD formulation is used to analyze ferrite-loaded structures. Six field components are used in the analysis. FDFD results of various structures loaded with ferrite are compared to analytical results and those from previous work, and a very good agreement is obtained.

II. Analysis

Since ferrite is a magnetically anisotropic material, Maxwell curl equations in the frequency domain will be in the form [11]:

$$\nabla \times \vec{E} = -j\omega[\mu]\vec{H} \tag{1}$$

$$\nabla \times \vec{H} = j\omega\varepsilon_0[\varepsilon_r]\vec{E} \tag{2}$$

For simplicity, the analysis is performed for the case of electrically anisotropic media with diagonal dielectric constant tensor $[\varepsilon_r]$. Ferrite regions are characterized by the Polder permeability tensor $[\mu]$ that is related to the direction of the external magnetic field. In the case of y-directed magnetization, $[\mu]$ is expressed as [11]:

$$[\mu]=\begin{bmatrix} \mu & 0 & j\kappa \\ 0 & \mu_0 & 0 \\ -j\kappa & 0 & \mu \end{bmatrix} \tag{3}$$

where,

$$\mu=\mu_0\left(1+\frac{\omega_m\omega_0}{\omega_0^2-\omega^2}\right) \tag{4}$$

$$\kappa=\mu_0\left(\frac{\omega\,\omega_m}{\omega_0^2-\omega^2}\right) \tag{5}$$

$$\omega_m=\gamma M_S \tag{6}$$

$$\omega_0=\gamma H_0 \tag{7}$$

and γ is the gyromagnetic ratio, γ = -2.21042$\times 10^5$ (rad/s)/(A/m). In the above equations, H_0 and M_s are the DC magnetic bias and the magnetic saturation, respectively.

It is assumed that the guided-wave structure is uniform along the z-axis and the wave propagates in the positive z-direction. Thus, the fields in a general guided-wave structure can be expressed as

$$\vec{E}(x,y,z)=[E_x(x,y)\hat{x}+E_y(x,y)\hat{y} + E_z(x,y)\hat{z}]\,e^{-j\beta z} \tag{8}$$

$$\vec{H}(x,y,z)=[H_x(x,y)\hat{x}+H_y(x,y)\hat{y} + H_z(x,y)\hat{z}]\,e^{-j\beta z} \tag{9}$$

where β is the phase constant. With the z dependence as given by (8) and (9), the derivatives $\partial/\partial z$ in (1) and (2) are replaced by $-j\beta$.

Then, $[\mu]\vec{H}$ in (1) is found to be:

$$\vec{B}=[\mu]\vec{H}=\begin{bmatrix} \mu H_x+j\kappa H_z \\ \mu_0 H_y \\ -j\kappa H_x+\mu H_z \end{bmatrix} \tag{10}$$

As the analysis is performed in two-dimensions only, the 3-D Yee's grid system is reduced to a compact 2-D mesh as shown in Fig. 1.

Using (10) and Yee's grid system shown in Fig. 1, the discretization of (1) and (2) yields the following expressions:

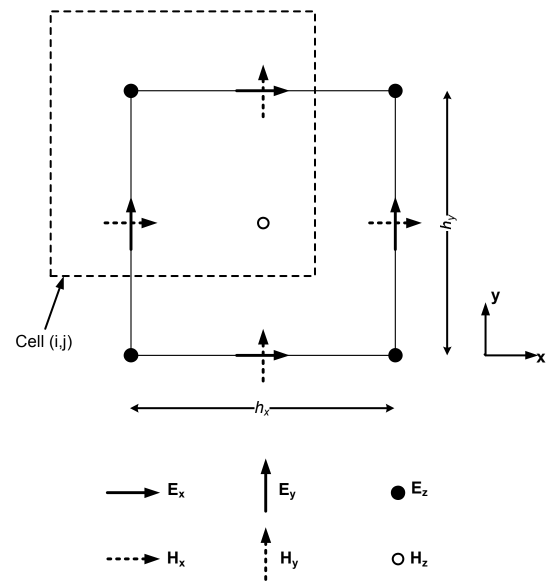

Figure 1: Compact 2-D mesh.

$$\mu H_x(i,j)+j\frac{\kappa}{2}\left[H_z(i,j)+H_z(i-1,j)\right] = -\frac{\beta}{\omega}E_y(i,j)+j\frac{1}{\omega h_y}\left[E_z(i,j+1)-E_z(i,j)\right] \tag{11}$$

$$\mu_0 H_y(i,j)=\frac{\beta}{\omega}E_x(i,j) - j\frac{1}{\omega h_x}\left[E_z(i+1,j)-E_z(i,j)\right] \tag{12}$$

$$\mu H_z(i,j)-j\frac{\kappa}{2}\left[H_x(i,j)+H_x(i+1,j)\right] = j\frac{1}{\omega h_x}\left[E_y(i+1,j)-E_y(i,j)\right] -j\frac{1}{\omega h_y}\left[E_x(i,j+1)-E_x(i,j)\right] \tag{13}$$

$$\varepsilon_{xx}E_x(i,j)=\frac{\beta}{\omega}H_y(i,j) - j\frac{1}{\omega h_y}[H_z(i,j)-H_z(i,j-1)] \tag{14}$$

$$\varepsilon_{yy} E_y(i,j) = -\frac{\beta}{\omega} H_x(i,j) + j\frac{1}{\omega h_x}\left[H_z(i,j) - H_z(i-1,j) \right] \tag{15}$$

$$\varepsilon_{zz} E_z(i,j) = j\frac{1}{\omega h_y}\left[H_x(i,j) - H_x(i,j-1)\right] - j\frac{1}{\omega h_x}\left[H_y(i,j) - H_y(i-1,j)\right] \tag{16}$$

where $\varepsilon_{xx} = \varepsilon_0 \varepsilon_{rx}$, $\varepsilon_{yy} = \varepsilon_0 \varepsilon_{ry}$, $\varepsilon_{zz} = \varepsilon_0 \varepsilon_{rz}$, and h_x and h_y are the grid size in the x and y directions, respectively.

It should be noted that since the components H_x and H_z are coupled, due to the application of a DC magnetic field in the y direction, interpolation is used in (11) and (13) as an approximation to discretize the two components at the same points of the space, as follows:

$$B_x(i,j) = \mu H_x(i,j) + j\kappa H_z\left(i - \frac{1}{2}, j\right) = \mu H_x(i,j) + j\frac{\kappa}{2}\left[H_z(i,j) + H_z(i-1,j)\right] \tag{17}$$

$$B_z(i,j) = \mu H_z(i,j) - j\kappa H_x\left(i + \frac{1}{2}, j\right) = \mu H_z(i,j) - j\frac{\kappa}{2}\left[H_x(i,j) + H_x(i+1,j)\right] \tag{18}$$

Reordering (11), (12), (14) and (15) gives:

$$\beta E_x(i,j) = j\frac{1}{h_x}\left[E_z(i+1,j) - E_z(i,j) \right] + \omega\mu_0 H_y(i,j) \tag{19}$$

$$\beta E_y(i,j) = j\frac{1}{h_y}\left[E_z(i,j+1) - E_z(i,j) \right] - j\omega\frac{\kappa}{2}\left[H_z(i,j) + H_z(i-1,j)\right] - \omega\mu H_x(i,j) \tag{20}$$

$$\beta H_x(i,j) = -\omega\varepsilon_{yy} E_y(i,j) + j\frac{1}{h_x}\left[H_z(i,j) - H_z(i-1,j)\right] \tag{21}$$

$$\beta H_y(i,j) = \omega\varepsilon_{xx} E_x(i,j) + j\frac{1}{h_y}\left[H_z(i,j) - H_z(i,j-1) \right] \tag{22}$$

Now, multiplying (16) by β, and substituting by (21) and (22) gives:

$$\beta E_z(i,j) = -j\frac{\varepsilon_{xx}}{h_x \varepsilon_{zz}}\left[E_x(i,j) - E_x(i-1,j) \right] - j\frac{\varepsilon_{yy}}{h_y \varepsilon_{zz}}\left[E_y(i,j) - E_y(i,j-1)\right] \tag{23}$$

Multiplying (13) by β, dividing by μ and reordering yields to:

$$\beta H_z(i,j) = j\frac{1}{\omega\mu h_x}\left[\beta E_y(i+1,j) - \beta E_y(i,j) \right] - j\frac{1}{\omega\mu h_y}\left[\beta E_x(i,j+1) - \beta E_x(i,j) \right] + j\frac{\kappa}{2\mu}\left[\beta H_x(i,j) + \beta H_x(i+1,j) \right] \tag{24}$$

Now, using (19), (20) and (21) in (24) gives:

$$\beta H_z(i,j) = -j\frac{1}{h_x}\left[H_x(i+1,j) - H_x(i,j) \right] - j\frac{1}{h_y}\left[H_y(i,j+1) - H_y(i,j) \right] - j\frac{\omega\kappa\varepsilon_{yy}}{2\mu}\left[E_y(i,j) + E_y(i+1,j)\right] \tag{25}$$

It should be noted that setting the gyromagnetic ratio γ to zero reduces equations (19)-(23) and (25) to the relations of a non-magnetic medium.

The boundary conditions on the perfectly conducting walls of the waveguide are met by setting the tangential electric field components and the normal magnetic flux density components on these walls to zero. In this case, extrapolation is used in the approximation of $H_z(i-1/2)$ at the left wall and $H_z(i+1/2)$ at the right wall as follows:

$$H_z\left(i-\frac{1}{2},j\right)=\frac{1}{2}\left[3H_z(i,j)-H_z(i+1,j)\right] \quad (26)$$

$$H_z\left(i+\frac{1}{2},j\right)=\frac{1}{2}\left[3H_z(i,j)-H_z(i-1,j)\right] \quad (27)$$

It should be noted that care must be taken when setting the equations at the upper and lower walls and the corners of waveguides and at the edges of conductors of the transmission lines to meet the necessary boundary conditions.

Finally, (19)-(23), (25), and the other boundary equations are cast into the following eigen-equation:

$$[A]\cdot\{x\}=\beta\cdot\{x\} \quad (28)$$

where $\{x\}$ is the vector containing the four or six field components in all of the Yee cells in the system, and $[A]$ is a sparse matrix containing the coefficients in (19)-(23) and (25).

The eigenvalue, β, is found by solving the following equation:

$$\det\{[A]-\beta\,[I]\}=0 \quad (29)$$

where $[I]$ is the identity matrix.

III. Numerical Results

To show the validity of the 2-D FDFD method proposed above, three examples are given below. The first example is a rectangular waveguide that is completely filled with a lossy ferrite material that has a dielectric constant ε_r = 9, damping factor α = 0.02, H_0 = 15.9155 KA/m and M_s = 159.155 KA/m, as shown in Fig. 2. The waveguide dimensions are a = 22.86 mm and b = 10.16 mm. In this case, where loss is taken into account, ω_0 in (7) is expressed as $\omega_0=\gamma H_0+j\alpha\omega$ and the resultant β will be complex with its real part representing the phase constant, and its imaginary part representing the attenuation constant. Fig. 3 shows the phase constant and the attenuation constant of the dominant TE_{10} mode. The FDFD results agree very well with those obtained analytically.

It should be noted that since the ferrite is lossy in this case, β_{m0} will be complex. The real part of β_{m0} will represent the phase constant while the imaginary part will represent the attenuation constant.

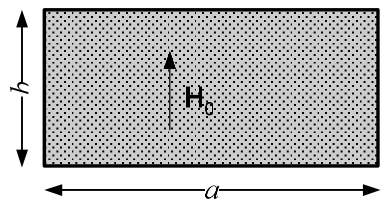

Figure 2: Cross section of a rectangular waveguide that is completely filled with ferrite.

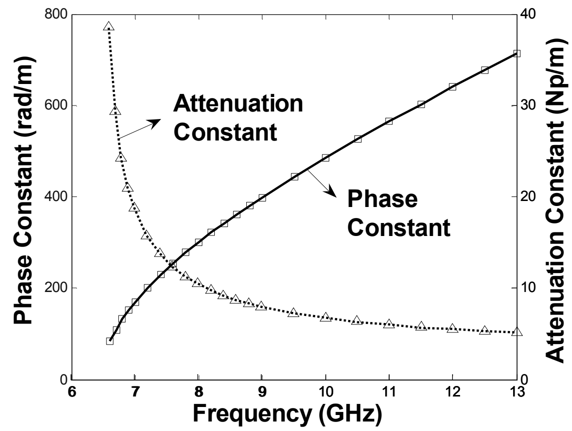

Figure 3: Dispersion characteristics of a rectangular waveguide that is completely filled with lossy ferrite. Solid Line: Phase Constant (FDFD); Squares: Phase Constant (Exact); Dashed Line: Attenuation Constant (FDFD); Triangles: Attenuation Constant (Exact).

The second example is a partially filled rectangular waveguide with an asymmetrically localized ferrite slab, as shown in Fig. 4. This structure allows the propagation of forward and backward waves with different propagation constants. The waveguide is of width a = 22.86 mm and height b = 10.16 mm. The ferrite has w = a/3, dielectric constant ε_r = 9, H_0 = 15.915 KA/m and M_s = 159.15 KA/m. Good agreement was achieved, as shown in Fig. 5, between the FDFD method results and the analytical results obtained by solving the transcendental equation for the propagation constant reported in [11].

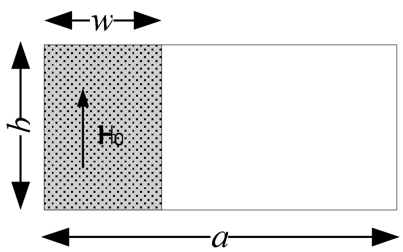

Figure 4: Cross section of a rectangular waveguide that is partially filled with an asymmetric localized ferrite slab.

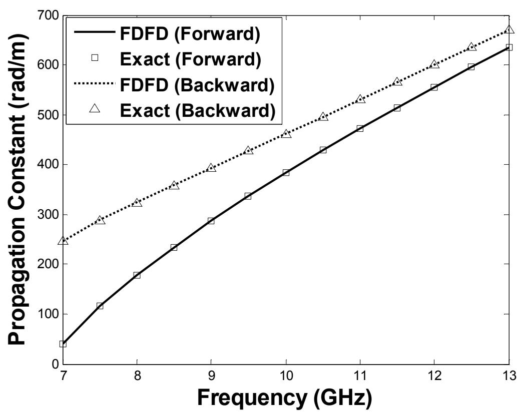

Figure 5: Dispersion characteristics of a rectangular waveguide partially filled with an asymmetric localized ferrite slab.

The third example is a finline with a composite dielectric-ferrite substrate as shown in Fig. 6. The structure in this example has the dimensions l_1 = 1.625 mm, d_f = 0.5 mm, d_d = 0.25 mm, l_2 = 2.375 mm, b = 2.392 mm, s = 1.196 mm. The ferrite has ε_f = 12.5, H_0 = 39.7887 KA/m, M_s = 397.887 KA/m. The dielectric has ε_r = 9.6 or 16. Fig. 7 shows the dispersion characteristics of the dominant mode of the forward and backward waves. The results are compared with those obtained using FDTD method [12], and they show good agreement.

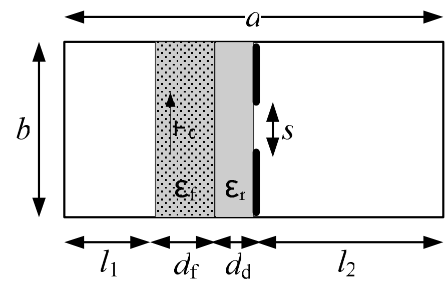

Figure 6: Cross section of a finline with a composite dielectric-ferrite substrate.

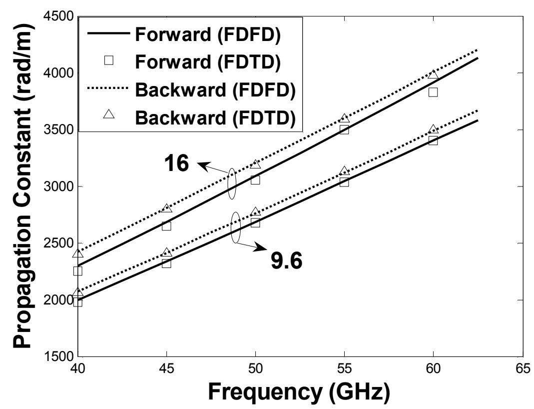

Figure 7: Dispersion characteristics of a finline with a composite dielectric-ferrite substrate.

IV. Conclusions

In this paper, a 2-D FDFD method was formulated for the analysis of dispersion characteristics of transversely magnetized ferrite loaded structures. The method can be also used to analyze magnetically isotropic structures simply by setting the gyromagnetic ratio γ to zero. This method finds the propagation constant β for a given frequency of operation, and the electric and magnetic field distributions.

Several structures were analyzed to test the validity of the method and the results were compared with those obtained analytically or from previous work using FDTD method. The results showed very good agreement. More results will be shown in the presentation.

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