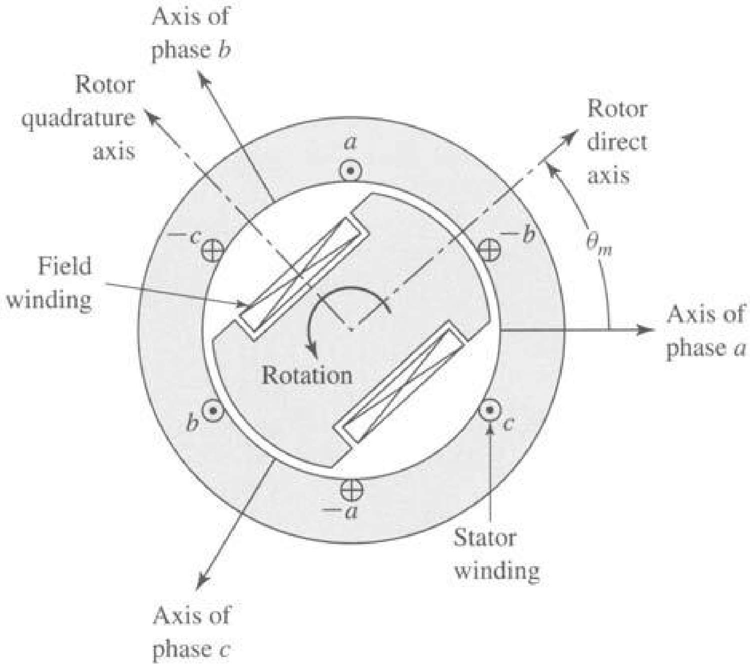
An ideal synchronous machine



dq0 transformation:

$$\begin{bmatrix} S_d \\ S_q \\ S_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta_{me}) & \cos(\theta_{me} - 120^\circ) & \cos(\theta_{me} + 120^\circ) \\ -\sin(\theta_{me}) & -\sin(\theta_{me} - 120^\circ) & -\sin(\theta_{me} + 120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix}$$

S is a stator quantity (voltage, current, or flux) to be transformed, and θ_{em} is the rotor electrical angle.

The inverse transformation is

$$\begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} = \begin{bmatrix} \cos(\theta_{me}) & -\sin(\theta_{me}) & 1 \\ \cos(\theta_{me} - 120^\circ) & -\sin(\theta_{me} - 120^\circ) & 1 \\ \cos(\theta_{me} + 120^\circ) & -\sin(\theta_{me} + 120^\circ) & 1 \end{bmatrix} \begin{bmatrix} S_d \\ S_q \\ S_0 \end{bmatrix}$$

Flux linkage:

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_f \end{bmatrix} = \begin{bmatrix} \mathcal{L}_{aa} & \mathcal{L}_{ab} & \mathcal{L}_{ac} & \mathcal{L}_{af} \\ \mathcal{L}_{ba} & \mathcal{L}_{bb} & \mathcal{L}_{bc} & \mathcal{L}_{bf} \\ \mathcal{L}_{ca} & \mathcal{L}_{cb} & \mathcal{L}_{cc} & \mathcal{L}_{cf} \\ \mathcal{L}_{fa} & \mathcal{L}_{fb} & \mathcal{L}_{fc} & \mathcal{L}_{ff} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix}$$

$$\mathcal{L}_{aa} = L_{aa0} + L_{al} + L_{g2} \cos 2\theta_{me}$$

$$\mathcal{L}_{bb} = L_{aa0} + L_{al} + L_{g2} \cos (2\theta_{me} + 120^\circ)$$

$$\mathcal{L}_{cc} = L_{aa0} + L_{al} + L_{g2} \cos (2\theta_{me} - 120^\circ)$$

$$\mathcal{L}_{ab} = \mathcal{L}_{ba} = -\frac{1}{2} L_{aa0} + L_{g2} \cos (2\theta_{me} - 120^\circ)$$

$$\mathcal{L}_{bc} = \mathcal{L}_{cb} = -\frac{1}{2} L_{aa0} + L_{g2} \cos 2\theta_{me}$$

$$\mathcal{L}_{ac} = \mathcal{L}_{ca} = -\frac{1}{2} L_{aa0} + L_{g2} \cos (2\theta_{me} + 120^\circ)$$

$$\mathcal{L}_{af} = \mathcal{L}_{fa} = L_{af} \cos \theta_{me}$$

$$\mathcal{L}_{bf} = \mathcal{L}_{fb} = L_{af} \cos (\theta_{me} - 120^\circ)$$

$$\mathcal{L}_{cf} = \mathcal{L}_{fc} = L_{af} \cos (\theta_{me} + 120^\circ)$$

$$\lambda_d = L_d i_d + L_{af} i_f$$

$$\lambda_q = L_q i_q$$

$$\lambda_f = \frac{3}{2} L_{af} i_d + L_{ff} i_f$$

$$\lambda_0 = L_0 i_0$$

$$L_d = L_{al} + \frac{3}{2}(L_{aa0} + L_{g2})$$

$$L_q = L_{al} + \frac{3}{2}(L_{aa0} - L_{g2})$$

$$L_0 = L_{al}$$

Transformations of the voltage equations:

$$v_a = R_a i_a + \frac{d\lambda_a}{dt}$$

$$v_b = R_a i_b + \frac{d\lambda_b}{dt}$$

$$v_c = R_a i_c + \frac{d\lambda_c}{dt}$$

$$v_f = R_f i_f + \frac{d\lambda_f}{dt}$$

$$v_d = R_a i_d + \frac{d\lambda_d}{dt} - \omega_{me}\lambda_q$$

$$v_q = R_a i_q + \frac{d\lambda_q}{dt} + \omega_{me}\lambda_d$$

$$v_f = R_f i_f + \frac{d\lambda_f}{dt}$$

$$v_0 = R_a i_0 + \frac{d\lambda_0}{dt}$$

$$p_s = v_a i_a + v_b i_b + v_c i_c$$

$$p_s = \frac{3}{2}(v_d i_d + v_q i_q + 2 v_0 i_0)$$

$$T_{mech} = \frac{3}{2}\left(\frac{\text{poles}}{2}\right)(\lambda_d i_q - \lambda_q i_d)$$