Chapter 2

Dynamic Modeling of Induction Machine

2.1. Introduction

Induction motor dynamic model was implemented in Simulink Matlab. The steady-state model as given in references [2]-[5],[22] does not work properly especially when very accurate control is required as it uses a number of approximations, and parameters (voltage, current and torque) are coupled with each other. In implementing the control of machines i.e., adjustable-speed drives and soft starters, the machine constitutes an element with feedback loop which makes it essential to observe the transient behavior of the electric machine [23]-[24]. Vector or field-oriented control uses the dynamic d-q model which provides dc machine like analogy for the analysis of induction machine by decoupling the torque and flux using direct-axis and quadrature-axis components. Park's transformations equations formulates the changes in variables by replacing the variables associated with stator winding with variables associated with fictitious winding rotating with the rotor at synchronous speed. This transformation eliminates the time-varying inductances and magnetic reluctances [5]. Stanely [6] eliminated the time-varying inductances by transforming rotor variables to synchronously rotating reference frame. G. Kron proposed a transformation of both the stator and rotor variables to a synchronously rotating reference that moves with the rotating field [4].

2.2. d-q Model of Induction Motor

Figure 2.1(a) shows the three phase symmetrical induction machine with stationary as-bs-cs axes, at $2\pi/3$- radians apart. First the three phase stationary frame (as-bs-cs) variables will be transformed into two phase stationary reference frame (d^s-q^s) variables and then two phase stationary reference frame variables will be transformed to synchronously rotating reference frame (d^e-q^e) and vice versa [25]. The complete transformation block was implemented in Matlab Simulink to carry out the simulations under different conditions using references [26] -[27].

2.2.1. Axes Transformation in Stationary Reference Frame

Assume that (d^s-q^s) axes are oriented at angle θ with respect to (as-bs-cs) axes as shown in Figure 2.1(a). The voltages V_{ds}^s and V_{qs}^s can be resolved into as-bs-cs components and can be represented as:

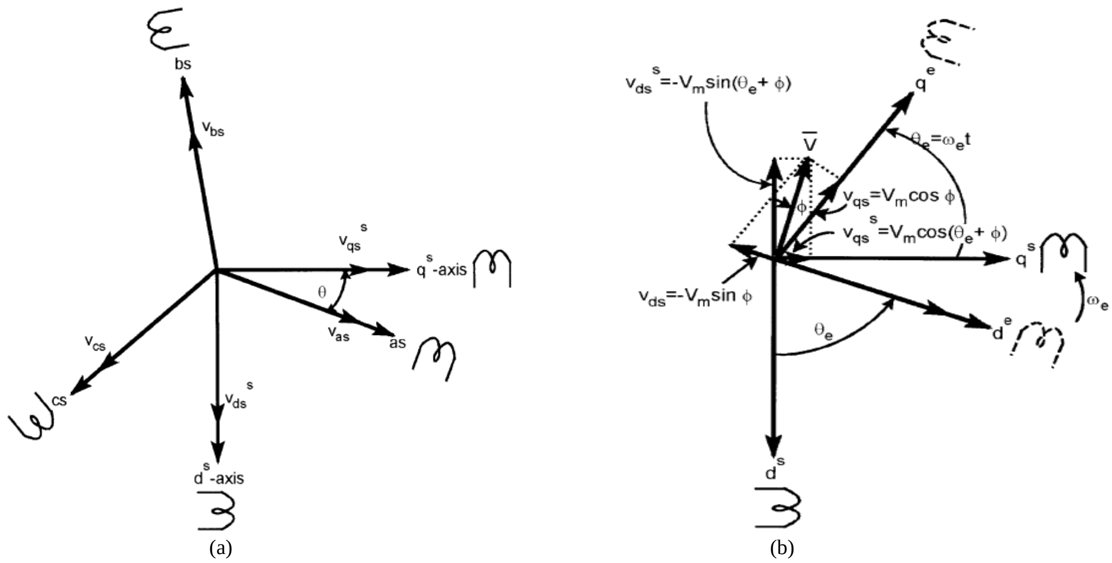


Figure 2.1. (a) Stationary frame a-b-s to d^s-q^s axis transformation (b) Stationary frame d^s-q^s to synchronously rotating frame d^e-q^e transformation.

$$\begin{aligned}
V_{as} &= V_{qs}^s \cos\theta + V_{ds}^s \cos(\frac{\pi}{2} - \theta) + V_{os}^s \\
V_{bs} &= V_{qs}^s \cos(\frac{2\pi}{3} - \theta) + V_{ds}^s \cos(\frac{2\pi}{3} + \frac{\pi}{2} - \theta) + V_{os}^s \\
V_{cs} &= V_{qs}^s \cos(\frac{2\pi}{3} + \theta) + V_{ds}^s \cos(\frac{2\pi}{3} - \frac{\pi}{2} - \theta) + V_{os}^s
\end{aligned} \tag{2.1}$$

where V_{os}^s is the zero sequence component. The equations (2.1) can be written as:

$$\begin{aligned}
V_{as} &= V_{qs}^s \cos\theta + V_{ds}^s \sin\theta + V_{os}^s \\
V_{bs} &= V_{qs}^s \cos(\theta - \frac{2\pi}{3}) + V_{ds}^s \sin(\theta - \frac{2\pi}{3}) + V_{os}^s \\
V_{cs} &= V_{qs}^s \cos(\theta + \frac{2\pi}{3}) + V_{ds}^s \sin(\theta + \frac{2\pi}{3}) + V_{os}^s
\end{aligned} \tag{2.2}$$

In matrix form,

$$\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 1 \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 1 \end{bmatrix} \begin{bmatrix} V_{qs}^s \\ V_{ds}^s \\ V_{os}^s \end{bmatrix} \tag{2.3}$$

Corresponding inverse equations can be written as:

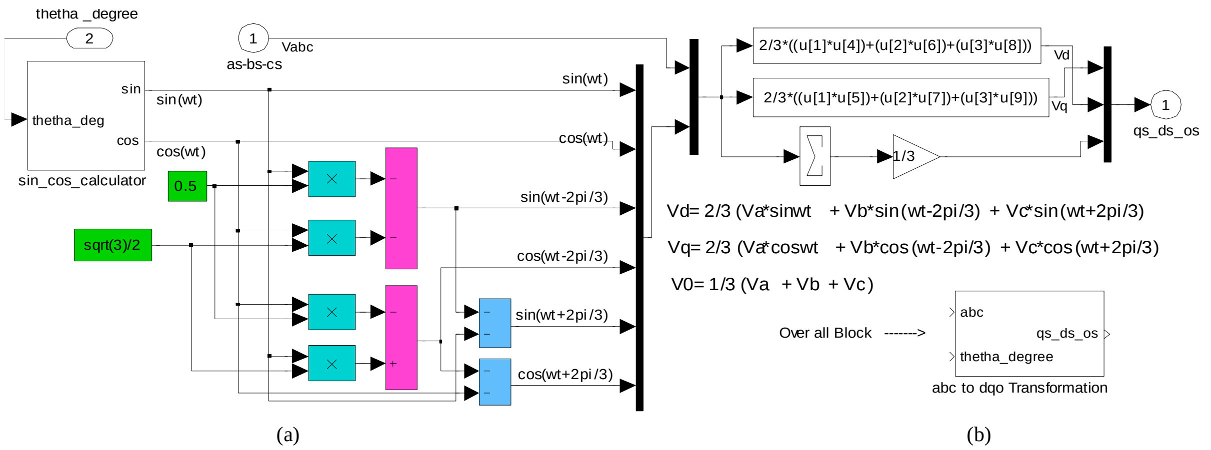


Figure 2.2 (a) Internal Matlab Simulink block diagram which transforms as-bs-cs axis values to q-d-o axis values in stationary reference frame of axis (b) Overall block diagram for transformation in stationary reference frame.

$$\begin{bmatrix} V_{qs}^{s} \\ V_{ds}^{s} \\ V_{os}^{s} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} \tag{2.4}$$

Equations (2.3) and (2.4) give the transformations, which are equally valid for currents and fluxes as well. The vector-control techniques use dynamic equivalent circuit of induction machine. There are at least three fluxes i.e., rotor, air gap and stator fluxes and three currents or mmfs (in stator and rotor, and magnetizing mmf) in an induction machine. For the dynamic response, the interaction between currents, fluxes and speed must be taken into account: this is essential for obtaining the dynamic model of the motor and to determine the control strategies.

All fluxes, currents and mmfs rotate at synchronous speed. Vector control aligns axes of an mmf and flux orthogonal at all the times. It is easier to align the stator current mmf orthogonally to the rotor flux. Figure 2.2 shows the implemented Matlab Simulink model for equations (2.4) which take θ and stator voltages along as-bs-cs axes as the input and give the voltages along q-d-o axes. The block, in Figure 2.2(b), gives the overall transformation in stationary reference frame.

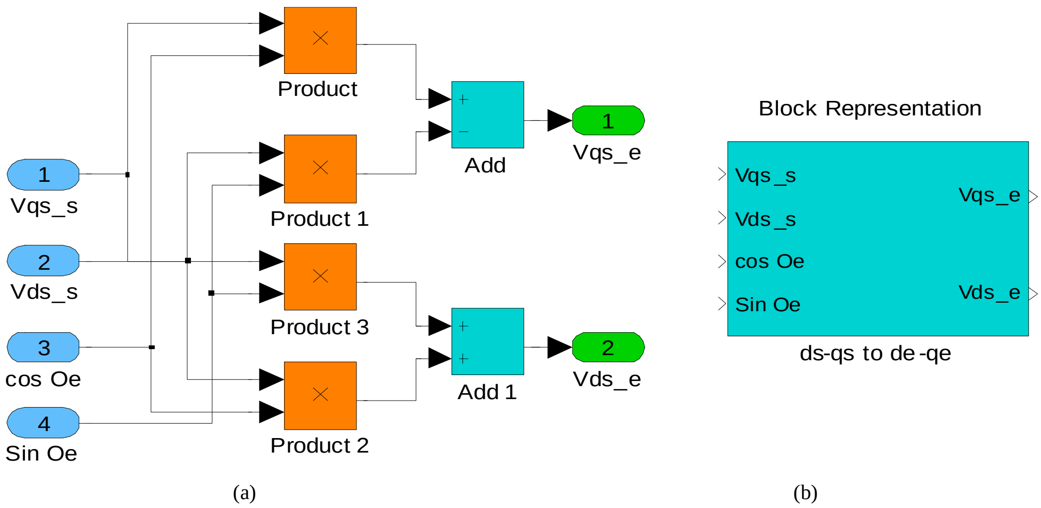


Figure 2.3. (a) Simulink block for equations (2.7) which transform values from stationary reference frame to synchronously rotating reference frame. (b) Over all block diagram for transformation.

2.2.2. Axes Transformation in Synchronously Rotating Frame of Reference

Synchronously rotating frame of reference model is shown in Figure 2.3. Synchronously rotating (d^e-q^e) axes rotate at synchronous speed ω_e with respect to (d^s-q^s) axes. The angle between stationary reference frame and synchronously rotating frame of reference is θ_e which is given as $\theta_e = \omega_e t$. Voltages on (d^s-q^s) axes can be resolved into (d^e-q^e) frame as follows:

From Figure 2.1(b)

$$V_{qs}^e = V_{qs}^s \cos\theta_e + V_{ds}^s \cos(\frac{\pi}{2} + \theta_e)$$
$$V_{qs}^e = V_{qs}^s \cos\theta_e - V_{ds}^s \sin(\theta_e) \tag{2.5}$$

and

$$V_{ds}^e = V_{qs}^s \cos(\frac{\pi}{2} - \theta_e) + V_{ds}^s \cos\theta_e$$
$$V_{ds}^e = V_{qs}^s \sin\theta_e + V_{ds}^s \cos\theta_e \tag{2.6}$$

Above equations can be written in matrix form as

$$\begin{bmatrix} V_{qs}^e \\ V_{ds}^e \end{bmatrix} = \begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix} \begin{bmatrix} V_{qs}^s \\ V_{ds}^s \end{bmatrix} \tag{2.7}$$

Figure 2.3 shows the implemented Matlab Simulink model which takes in q and d axis stator voltages in stationary frame and gives the same in dynamic frame of reference. The rotating reference frame parameters can be resolved into the stationary frame of reference from Figure 2.1(b) as:

$$\begin{aligned} V_{qs}^{s} &= V_{qs}^{e}\cos\theta_e + V_{ds}^{e}\cos(90-\theta_e) \\ V_{qs}^{s} &= V_{qs}^{e}\cos\theta_e + V_{ds}^{e}\sin\theta_e \end{aligned} \tag{2.8}$$

and

$$\begin{aligned} V_{ds}^{s} &= -V_{qs}^{e}\cos(90-\theta_e) + V_{ds}^{e}\cos\theta_e \\ V_{ds}^{s} &= -V_{qs}^{e}\sin\theta_e + V_{ds}^{e}\cos\theta_e \end{aligned} \tag{2.9}$$

Above equations can be expressed in matrix form as:

$$\begin{bmatrix} V_{qs}^{s} \\ V_{ds}^{s} \end{bmatrix} = \begin{bmatrix} \cos\theta_e & \sin\theta_e \\ -\sin\theta_e & \cos\theta_e \end{bmatrix} \begin{bmatrix} V_{qs}^{e} \\ V_{ds}^{e} \end{bmatrix} \tag{2.10}$$

2.2.3. Synchronously Rotating Reference Frame ---Dynamic Model

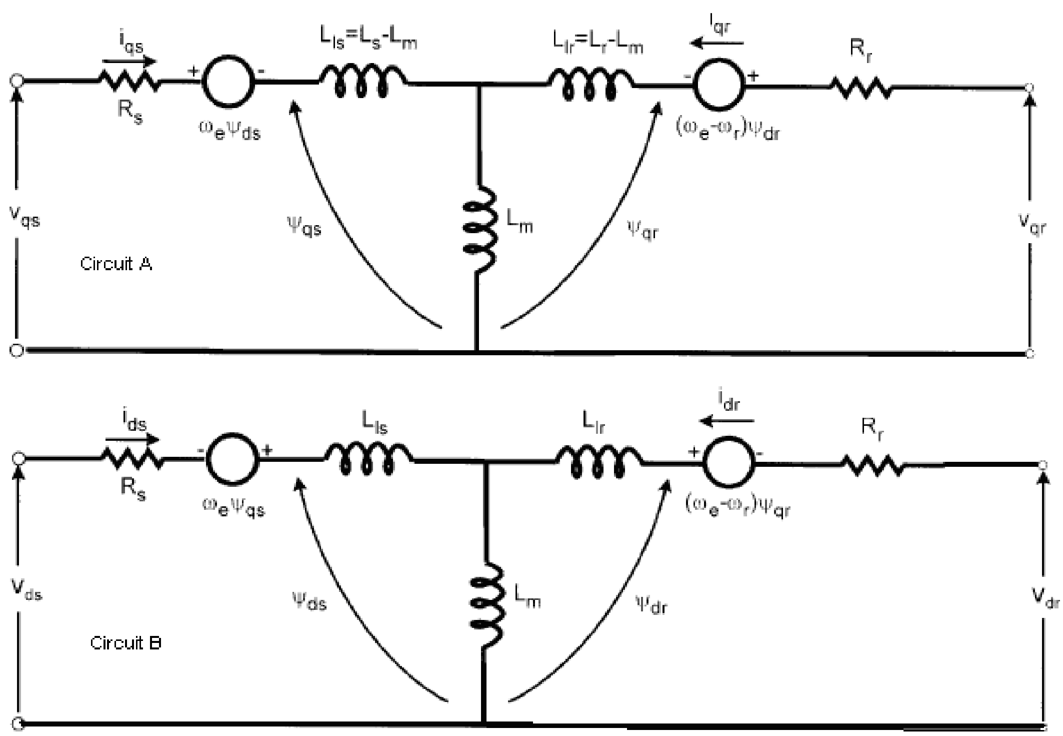


Figure 2.4. Dynamic d_e-q_e equivalent circuit of induction machine (a) q_e-axis circuit (b) d_e-axis circuit

Figure 2.4 gives the equivalent circuit of the induction machine in dynamic frame of reference [25]. Figure 2.4(a) gives q-axis circuit while Figure 2.4(b) gives d-axis equivalent circuit.

2.2.3.1. Stator Flux Linkages

From Figures 2.4(a) and 2.4(b) q-and d-axis stator flux linkages can be written as:

$$\psi_{qs} = L_{ls}i_{qs} + L_m(i_{qs} + i_{qr}) = (L_s - L_m)i_{qs} + L_m(i_{qs} + i_{qr}) = L_s i_{qs} + L_m i_{qr} \tag{2.11}$$

$$\psi_{ds} = L_{ls}i_{ds} + L_m(i_{ds} + i_{dr}) = (L_s - L_m)i_{ds} + L_m(i_{ds} + i_{dr}) = L_s i_{ds} + L_m i_{dr} \tag{2.12}$$

Total flux Linkages of stator side= $\psi_s = \sqrt{\psi_{ds}^2 + \psi_{qs}^2}$ (2.13)

2.2.3.2. Rotor Flux Linkages

From Figures 2.4(a) and 2.4(b) q-and d-axis rotor flux linkages can be written as:

$$\psi_{qr} = L_{lr}i_{qr} + L_m(i_{qs} + i_{qr}) = (L_r - L_m)i_{qr} + L_m(i_{qs} + i_{qr}) = L_r i_{qr} + L_m i_{qs} \tag{2.14}$$

$$\psi_{dr} = L_{lr}i_{dr} + L_m(i_{ds} + i_{dr}) = (L_r - L_m)i_{dr} + L_m(i_{ds} + i_{dr}) = L_r i_{dr} + L_m i_{ds} \tag{2.15}$$

Total flux Linkages of rotor side= $\psi_r = \sqrt{\psi_{dr}^2 + \psi_{qr}^2}$ (2.16)

2.2.3.3. Air Gap Flux Linkages

From Figures 2.4(a) and 2.4(b) q-and d-axis air gap flux linkages can be written as:

$$\psi_{qm} = L_m(i_{qs} + i_{qr}) \tag{2.17}$$

$$\psi_{dm} = L_m(i_{ds} + i_{dr}) \tag{2.18}$$

Total flux Linkages of air gap = $\psi_m = \sqrt{\psi_{dm}^2 + \psi_{qm}^2}$ (2.19)

2.2.3.4 q- and d-axis Voltages

From Figure 2.4a, we can write the equation for q-axis stator voltages as:

$$V_{qs} = R_s i_{qs} + \frac{d}{dt}\psi_{qs} + \omega_e \psi_{ds} = R_s i_{qs} + \frac{d}{dt}[L_s i_{qs} + L_m i_{qr}] + \omega_e [L_s i_{ds} + L_m i_{dr}]$$

$$V_{qs} = (R_s + sL_s)i_{qs} + \omega_e L_s i_{ds} + sL_m i_{qr} + \omega_e L_m i_{dr} \tag{2.20}$$

Similarly from Figure 2.4(b), d-axis stator voltages can be expressed as:

$$V_{ds} = R_s i_{ds} + \frac{d}{dt}\psi_{ds} - \omega_e \psi_{qs} = R_s i_{ds} + \frac{d}{dt}[L_s i_{ds} + L_m i_{dr}] + \omega_e [L_s i_{qs} + L_m i_{qr}]$$

$$V_{ds} = -\omega_e L_s i_{qs} + (R_s + sL_s)i_{ds} - \omega_e L_m i_{qr} + sL_m i_{dr} \tag{2.21}$$

Equation for q-axis rotor voltages, from Figure 2.4(a), can be written as:

$$V_{qr} = R_r i_{qr} + \frac{d}{dt}\psi_{qr} + (\omega_e - \omega_r)\psi_{dr} = R_r i_{qr} + \frac{d}{dt}[L_r i_{qr} + L_m i_{qs}] + (\omega_e - \omega_r)[L_r i_{dr} + L_m i_{ds}]$$

$$V_{qr} = sL_m i_{qs} + (\omega_e - \omega_r)L_m i_{ds} + (R_r + sL_r)i_{qr} + (\omega_e - \omega_r)L_r i_{dr} \tag{2.22}$$

Similarly from Figure 2.4(b) d-axis rotor voltages can be expressed as:

$$V_{dr} = R_r i_{dr} + \frac{d}{dt}\psi_{dr} - (\omega_e - \omega_r)\psi_{qr} = R_r i_{dr} + \frac{d}{dt}[L_r i_{dr} + L_m i_{ds}] - (\omega_e - \omega_r)[L_r i_{qr} + L_m i_{qs}]$$

$$V_{dr} = -(\omega_e - \omega_r)L_m i_{qs} + sL_m i_{ds} - (\omega_e - \omega_r)L_r i_{qr} + (R_r + sL_r)i_{dr} \tag{2.23}$$

Equations 2.20-2.23 can be written in matrix form as:

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{qr} \\ V_{dr} \end{bmatrix} = \begin{bmatrix} (R_s + sL_s) & \omega_e L_s & sL_m & \omega_e L_m \\ -\omega_e L_s & (R_s + sL_s) & -\omega_e L_m & sL_m \\ sL_m & (\omega_e - \omega_r)L_m & (R_r + sL_r) & (\omega_e - \omega_r)L_r \\ -(\omega_e - \omega_r)L_m & sL_m & -(\omega_e - \omega_r)L_r & (R_r + sL_r) \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} \tag{2.24}$$

2.2.4. Simulink Modeling

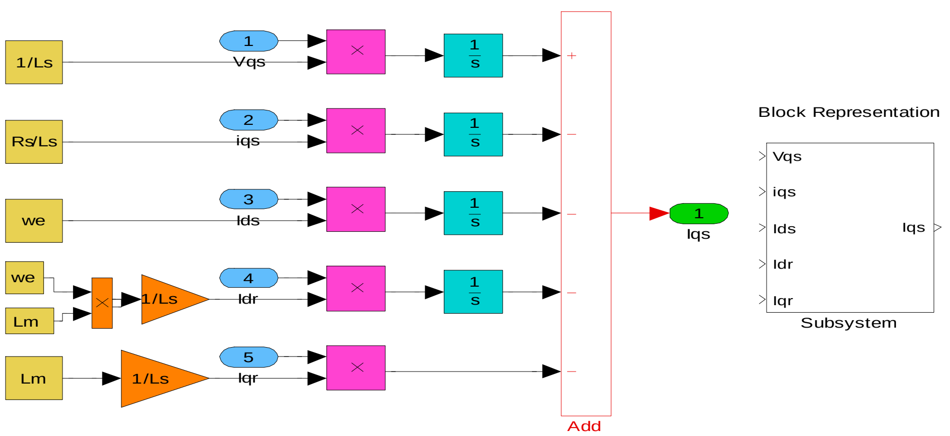
Using the derived equations in section 2.2.3 Matlab Simulink models were developed which are shown below. Equation (2.20) can be written as:

$$V_{qs} = R_s i_{qs} + L_s \frac{d}{dt} i_{qs} + \omega_e L_s i_{ds} + L_m \frac{d}{dt} i_{qr} + \omega_e L_m i_{dr}$$

$$L_s \frac{d}{dt} i_{qs} = V_{qs} - R_s i_{qs} - \omega_e L_s i_{ds} - L_m \frac{d}{dt} i_{qr} - \omega_e L_m i_{dr}$$

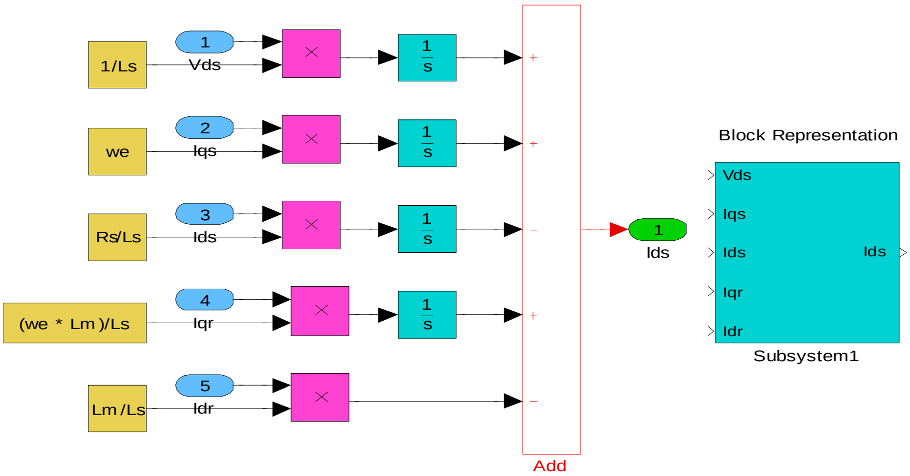
$$i_{qs} = \frac{1}{L_s}\int V_{qs}dt - \frac{R_s}{L_s}\int i_{qs}dt - \omega_e \int i_{ds}dt - \frac{L_m}{L_s} i_{qr} - \frac{\omega_e L_m}{L_s}\int i_{dr}dt \tag{2.25}$$

Figure 2.5 shows the Matlab Simulink block for the above equation



(a) (b)

Figure 2.5. (a) Simulink block for equation (2.25) (b) overall Simulink block for equation (2.25).



(a) (b)

Figure 2.6. (a) Simulink block for equations (2.26) (b) overall Simulink block for equations (2.26).

Equation (2.21) can be written as:

$$\begin{aligned} V_{ds} &= -\omega_e L_s i_{qs} + R_s i_{ds} + L_s \frac{d}{dt} i_{ds} - \omega_e L_m i_{qr} + L_m \frac{d}{dt} i_{dr} \\ L_s \frac{d}{dt} i_{ds} &= V_{ds} + \omega_e L_s i_{qs} - R_s i_{ds} + \omega_e L_m i_{qr} - L_m \frac{d}{dt} i_{dr} \\ i_{ds} &= \frac{1}{L_s} \int V_{ds} dt + \omega_e \int i_{qs} dt - \frac{R_s}{L_s} \int i_{ds} dt + \frac{\omega_e L_m}{L_s} \int i_{qr} dt - \frac{L_m}{L_s} i_{dr} \end{aligned} \tag{2.26}$$

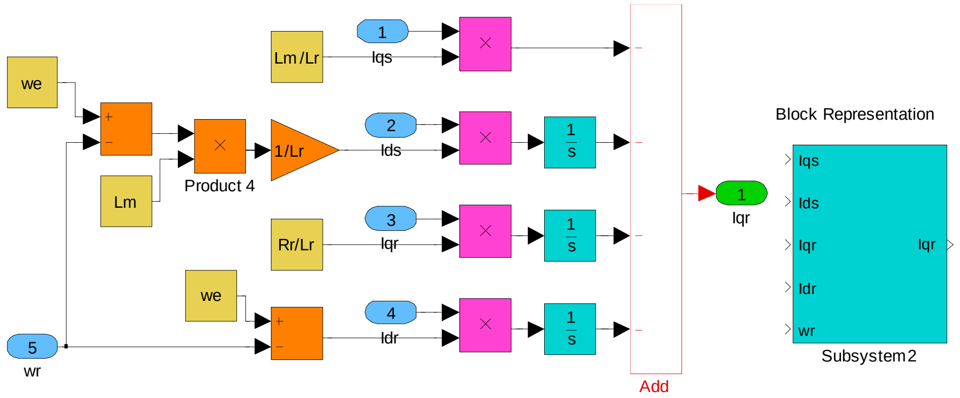
Figure 2.6 shows the Matlab Simulink block which calculate d-axis stator current with given parameters using the equations (2.26). Equation (2.22) can be written as:

$$\begin{aligned} V_{qr} &= L_m \frac{d}{dt} i_{qs} + (\omega_e - \omega_r) L_m i_{ds} + R_r i_{qr} + L_r \frac{d}{dt} i_{qr} + (\omega_e - \omega_r) L_r i_{dr} \\ L_r \frac{d}{dt} i_{qr} &= V_{qr} - L_m \frac{d}{dt} i_{qs} - (\omega_e - \omega_r) L_m i_{ds} - R_r i_{qr} - (\omega_e - \omega_r) L_r i_{dr} \\ i_{qr} &= \frac{1}{L_r} \int V_{qr} dt - \frac{L_m}{L_r} i_{qs} - \frac{(\omega_e - \omega_r) L_m}{L_r} \int i_{ds} dt - \frac{R_r}{L_r} \int i_{qr} dt - (\omega_e - \omega_r) \int i_{dr} dt \end{aligned} \tag{2.27}$$

For squirrel-cage motor V_{qr} =0 so the above equation becomes:

$$i_{qr} = -\frac{L_m}{L_r} i_{qs} - \frac{(\omega_e - \omega_r) L_m}{L_r} \int i_{ds} dt - \frac{R_r}{L_r} \int i_{qr} dt - (\omega_e - \omega_r) \int i_{dr} dt \tag{2.28}$$

Figure 2.7 gives the Matlab Simulink block for equation (2.28).



(a) (b)

Figure 2.7. (a) Simulink block for equation (2.28) (b) overall Simulink block for equation (2.28).

From equation (2.23) it can be written as:

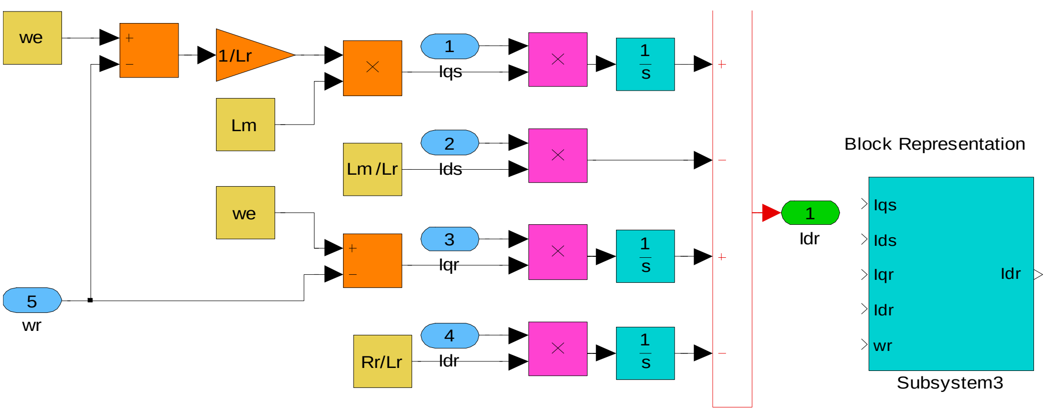
$$V_{dr} = -(\omega_e - \omega_r) L_m i_{qs} + L_m \frac{d}{dt} i_{ds} - (\omega_e - \omega_r) L_r i_{qr} + R_r i_{dr} + L_r \frac{d}{dt} i_{dr}$$

$$L_r \frac{d}{dt} i_{dr} = V_{dr} + (\omega_e - \omega_r) L_m i_{qs} - L_m \frac{d}{dt} i_{ds} + (\omega_e - \omega_r) L_r i_{qr} - R_r i_{dr}$$

$$i_{dr} = \frac{1}{L_r} \int V_{dr} dt + \frac{(\omega_e - \omega_r) L_m}{L_r} \int i_{qs} dt - \frac{L_m}{L_r} i_{ds} + (\omega_e - \omega_r) \int i_{qr} dt - \frac{R_r}{L_r} \int i_{dr} dt \tag{2.29}$$

For squirrel-cage motor V_{dr} =0 so the above equation becomes:

$$i_{dr} = \frac{(\omega_e - \omega_r) L_m}{L_r} \int i_{qs} dt - \frac{L_m}{L_r} i_{ds} + (\omega_e - \omega_r) \int i_{qr} dt - \frac{R_r}{L_r} \int i_{dr} dt \tag{2.30}$$



(a) (b)

Figure 2.8. (a) Simulink block for equation (2.30) (b) overall Simulink block for equation (2.30).

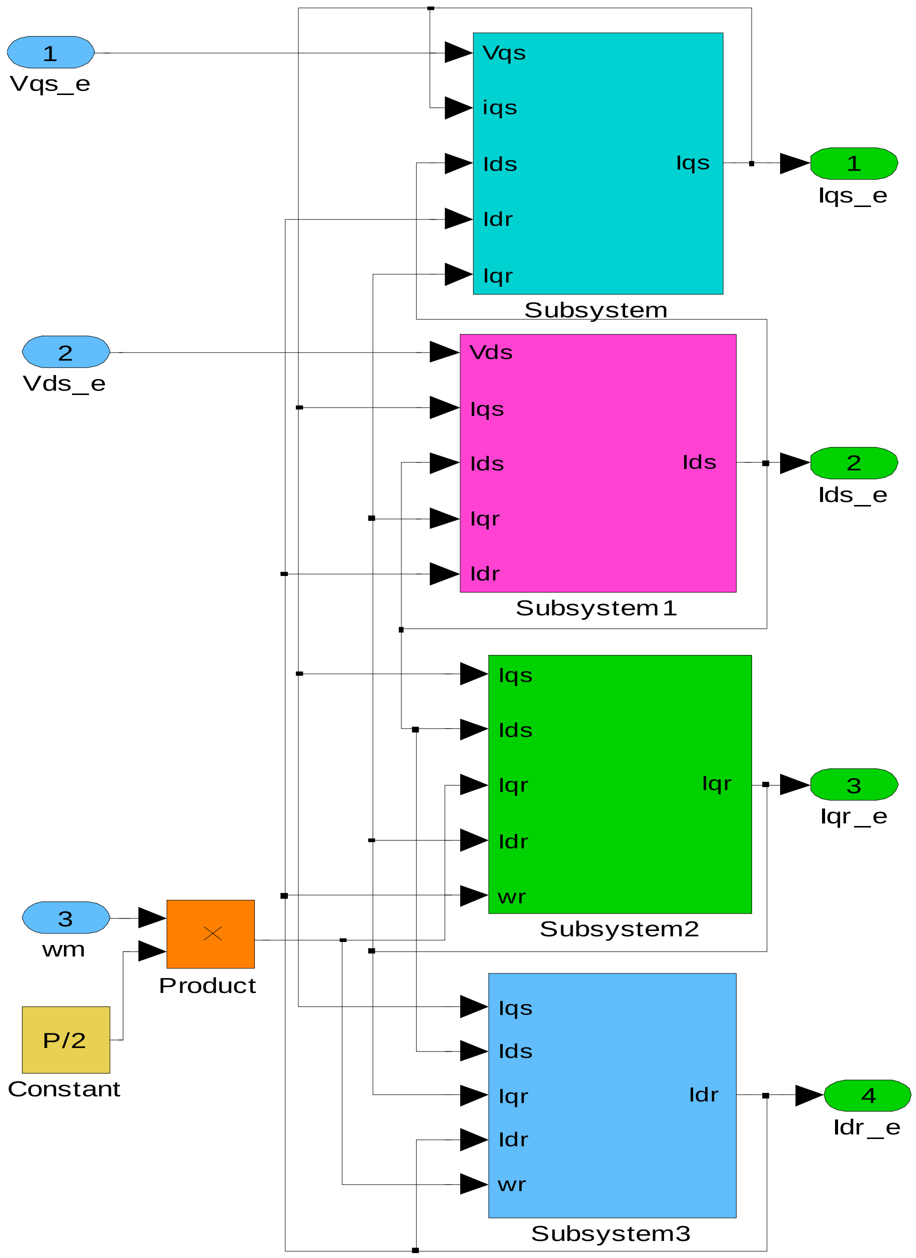


Figure 2.9. All the Simulink blocks connected together, conversion of q-and d-axis voltage into q-and d-axis currents of rotor and stator.

Figure 2.8 shows the Simulink block for equations (2.30) while Figure 2.9 shows all the developed Simulink blocks connected together to get d-and q-axis stator and rotor currents from the d-and q- axis stator voltages in synchronously rotating frame of reference.

2.2.5. Modeling of Mechanical System

If Tm is mechanical torque, J is rotor inertia, F is friction constant and w_m is mechanical speed then the mechanical system of the induction machine can be modeled using the following differential equation:

$$\frac{d}{dt}\omega_m = \frac{1}{2H}(T_e - F\omega_m - T_m) \Rightarrow \omega_m = \frac{1}{2H}\int (T_e - F\omega_m - T_m)dt$$
$$\Rightarrow \omega_m = \frac{1}{J}\int (T_e - F\omega_m - T_m)dt \quad (2.31)$$

Figure 2.10 shows the Matlab Simulink block implemented for the mechanical system of the machine using equation (2.31).

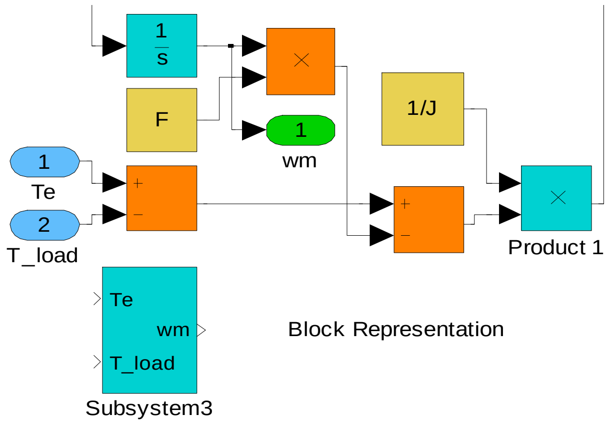


Figure 2.10. Simulink block for mechanical system of induction machine.

2.2.6. Developed Torque

Developed torque can be calculated in a number of ways using the parameters in synchronously rotating frame of reference as:

$$T_e = (\frac{3}{2})(\frac{P}{2})(\psi_{dm} i_{qr} - \psi_{qm} i_{dr}) \quad (2.32)$$

$$T_e = (\frac{3}{2})(\frac{P}{2})(\psi_{dm} i_{qs} - \psi_{qm} i_{ds}) \quad (2.33)$$

$$T_e = (\frac{3}{2})(\frac{P}{2})(\psi_{ds} i_{qr} - \psi_{qs} i_{dr}) \quad (2.34)$$

$$T_e = (\frac{3}{2})(\frac{P}{2})(\psi_{dr} i_{qr} - \psi_{qr} i_{dr}) \quad (2.35)$$

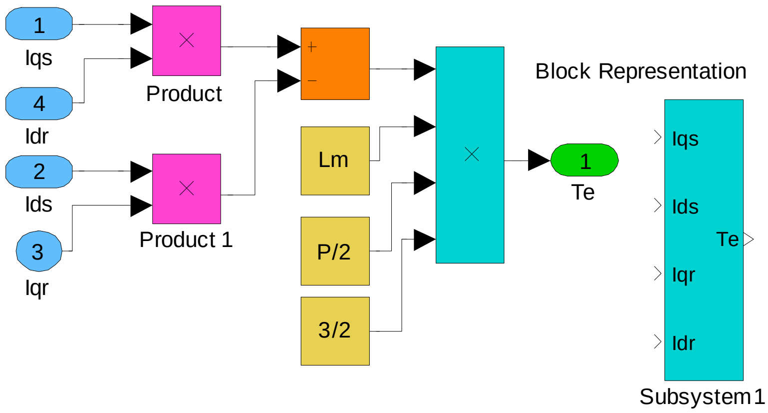


Figure 2.11. Simulink block for the developed toque calculation of induction machine using equation (2.36)

$$T_e = (\frac{3}{2})(\frac{P}{2})L_m(i_{qs}i_{dr} - i_{ds}i_{qr}) \tag{2.36}$$

Figure 2.11 shows the Matlab Simulink block to calculate the developed torque using equation (2.36). Similarly Matlab blocks can be implemented using any of the equation from (2.32) to (2.35).

2.2.7. Overall Model

Figure 2.12 shows the dynamic model of induction machine developed by integrating the above Simulink modules shown in Figures 2.2-2.11

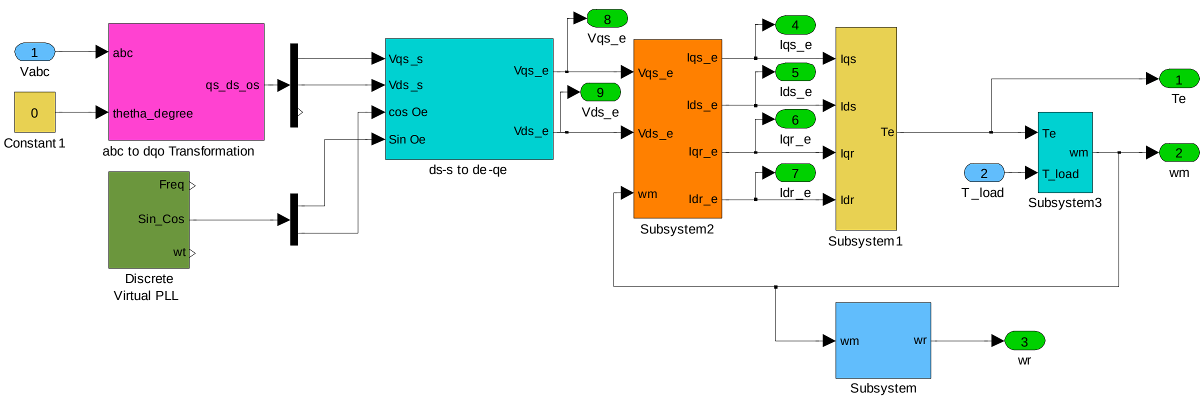


Figure 2.12. Simulink block combined together to implement the dynamic model of induction machine.