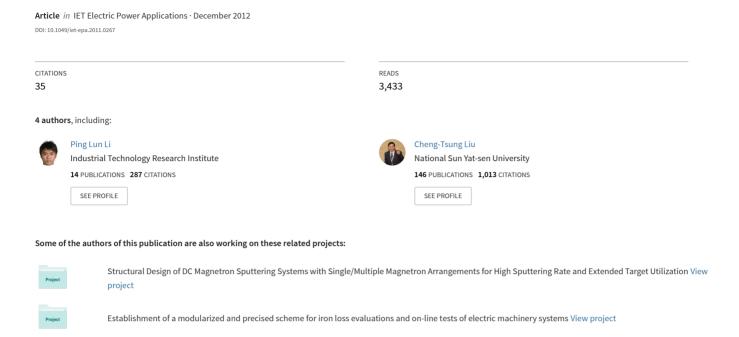
## Design and analysis of a brushless DC motor for applications in robotics



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# Design and analysis of a brushless DC motor for applications in robotics

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**Abstract:** This study presents the design of a three-phase slotless brushless DC (BLDC) motor for use as an electromagnetic actuator in robotics. To achieve the high torque-to-inertia and torque-to-weight ratios required for fast response in robotic applications, genetic algorithms (GAs) are used to optimise the initial motor design within a reasonable geometry. We examine the machine performance using 3-D finite element analysis (FEA) and validate them by experimental measurement.

#### 1 Introduction

Brushless DC (BLDC) motors have attracted significant interest for applications in robotics because of their large torque-producing capability, high reliability and low maintenance [1–5]. To satisfy the miniaturisation requirement for applications in robotics, a motor with a slotless stator and a rotor equipped with high-energy permanent magnets (PMs), such as neodymium iron boron (NdFeB) or samarium—cobalt (SmCo) magnets, is one of the most common configurations for use as an electromagnetic actuator in current robotics.

Two common winding arrangements employed in miniature slotless BLDC motors are axial and non-axial windings. Faulhaber, rhombic and zigzag windings are examples of non-axial windings [6–8]. One important advantage of these windings is a short-end connection. However, they have been patented [9, 10] and the tooling and manufacturing processes needed for their construction are not any easier. In this study, single-layer short-pitch windings were employed. Since the stator consists of a simple cylinder without any slots, the pre-wound axial windings could easily be attached and fixed to the inner surface of the cylinder using glue or epoxy. This enables easier manufacturing of this kind of miniaturised machines [11, 12].

The two-pole rotor and three-phase slotless stator discussed above, with a single-layer short-pitch windings motor was, designed based on the well-established computerised design procedures [3–5, 13]. Moreover, to achieve the high torque-to-inertia and torque-to-weight ratios required for fast response in robotic applications, genetic algorithms (GAs) were utilised for the design optimisation of the motor within a reasonable geometry [14–16]. Finally, a prototype motor was fabricated and tested to confirm the

computational results which were revealed to be in good agreement.

#### 2 Model of the motor

The initial design of a three-phase, 12 V, 5 W, 6000 rpm, two-pole PM motor is shown in Fig. 1. The motor consists of a slotless stator that accommodates the single-layer shortpitch windings, and a sintered two-pole NdFeB magnet rotor with parallel magnetisation. The stator laminations are made of non-oriented silicon steel with a saturated flux density  $(B_s)$  of 1.7 T. The rotor consists of a unique cylindrical magnet with a remanence  $(B_r)$  of 1.445 T and a relative recoil permeability  $(\mu_r)$  of 1.05, and a ferromagnetic shaft. The initial design of the machine parameters is given in Table 1. Since the rotor length-todiameter is less than 3, no retaining sleeve is required around the magnets for the desired application [11]. The conductors are spread in the winding segment as shown in Fig. 1. Each phase covers 120 electrical degrees with the bobbin angle  $\beta$  between the positive (e.g. A+) and negative (e.g. A-) winding segments. The angle is introduced for the phase winding wounding on a bobbin before it is inserted into the stator. The angle size does not affect the performance of the motor because the amount of flux linkage that can be generated in the region is very small, especially for a low number of poles [12, 17]. In this study, the angle  $\beta$  was fixed at 5 electrical degrees.

#### 3 Genetic algorithms

Any optimisation procedure involves finding a vector  $X = (x_1, x_2, \ldots, x_n)$ , representing a set of n design variables, each of them bounded by  $x_{im} \le x_i \le x_{in}$ , i,  $i = 1, 2, \ldots, n$ , so that the objective function F(X) is

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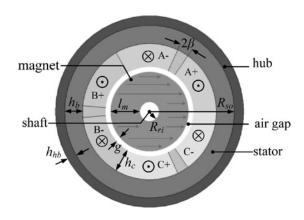


Fig. 1 Initial design of the motor

Table 1 Machine parameters

Parameter	Symbol	Value
copper wire mass density	$ ho_{\sf sw}$	8900 kg/m <sup>3</sup>
core mass density	$ ho_{ extsf{sc}}$	7650 kg/m <sup>3</sup>
magnet mass density	$ ho_{\sf pm}$	7400 kg/m <sup>3</sup>
shaft mass density	$ ho_{sh}$	7800 kg/m <sup>3</sup>
hub mass density	$ ho_{hb}$	7930 kg/m <sup>3</sup>
hub outer radius	$R_{ m hb}$	11 mm
rotor outer radius	$R_{\rm ro}$	4.5 mm
rotor inner radius	$R_{ m ri}$	1.5 mm
stator outer radius	$R_{ m so}$	10 mm
stator stack length	$L_{s}$	21 mm
shaft length	$L_{sh}$	30 mm
rotor length	$L_{r}$	20 mm
hub axial length	$L_{hb}$	25 mm
hub thickness	$h_{hb}$	1 mm
radial winding thickness	$h_{\mathrm{c}}$	2.8 mm
stator back iron width	$h_{b}$	2.2 mm
mechanical air gap length	G	0.5 mm
number of turns per phase	$N_{t}$	94

maximised (or minimised), while satisfying a set of k constraints  $G_j(X) \leq 0$ , j = 1, 2, ..., k [14–16]. In this paper, GA optimisation was applied to the design of a slotless BLDC motor to maximise the torque-to-inertia and torque-to-weight ratios.

#### 3.1 Objective function

The rotor inertia and motor weight are taken as the objective functions to be minimised for a rated torque not less than the specified value. The rotor inertia (J) is defined as

$$J = F_1(x_i) = \frac{\pi}{2} [L_{\rm s} \rho_{\rm pm} (R_{\rm ri} + l_{\rm m})^4 + L_{\rm sh} \rho_{\rm sh} R_{\rm ri}^4]$$
 (1)

where  $\rho_{\rm pm}$  and  $\rho_{\rm sh}$  are the specific mass densities of magnet and shaft, respectively,  $L_{\rm s}$  and  $L_{\rm sh}$  are the axial stack lengths of stator and shaft, respectively,  $R_{\rm ri}$  is the radius of shaft and  $l_{\rm m}$  is the magnet thickness. The motor weight (M) is the sum of the stator, rotor, magnet and winding weights and is given by

$$M = F_2(x_i) = [R_{so}^2 - (R_{so} - h_b)^2] \rho_{sc} \pi L_s + R_{ri}^2 \rho_{sh} \pi L_{sh}$$
$$+ \left[ 2L_s + \frac{\pi^2 (R_{ri} + l_m + g + h_c/2)}{3} \right] N_{ph} N_t A_w \rho_{sw}$$

$$+ [(l_{\rm m} + R_{\rm ri})^2 - R_{\rm ri}^2] \rho_{\rm pm} \pi L_{\rm r}$$

$$+ [(R_{\rm so} + h_{\rm hb})^2 - (R_{\rm so})^2] \rho_{\rm hb} \pi L_{\rm s}$$

$$+ 2[(R_{\rm so} + h_{\rm hb})^2 - R_{\rm ri}^2] \rho_{\rm hb} \pi h_{\rm hb}$$
(2)

where  $\rho_{\rm sc}$  and  $\rho_{\rm sw}$  are the specific mass densities of the stator core and copper, respectively,  $h_{\rm c}$  is the winding segment thickness,  $A_{\rm w}$  is the cross-sectional area of a conductor,  $N_{\rm sh}$  is the number of parallel paths,  $N_{\rm t}$  is the number of turns in series per phase and  $N_{\rm ph}$  is the number of phases.

#### 3.2 Variables and their limits

In this study, six geometrical variables were defined as design variables,  $x_1 = l_{\rm m}$ ,  $x_2 = h_{\rm b}$ ,  $x_3 = h_{\rm c}$ ,  $x_4 = R_{\rm ri}$ ,  $x_5 = g$  and  $x_6 = k_{\rm f}$ . Here,  $k_{\rm f}$  is the winding filling factor. The remaining design parameters can be expressed in terms of the above variables or assigned fixed values.

According to the geometry of the motor and the materials used, the bounds of the design variables can be expressed as

$$2 \le x_1 \le 3.5$$
;  $1.8 \le x_2 \le 2.5$ ;  $2.5 \le x_3 \le 3.5$   
 $1 \le x_4 \le 2$ ;  $0.5 \le x_5 \le 0.8$ ;  $0.5 \le x_5 \le 0.8$ ;  $0.5 \le x_6 \le 0.9$  (4)

Maintaining the flux density through the stator back iron below the core saturation of 1.7 T, gives

$$\frac{\pi k_{\rm l} \alpha B_{\rm r} l_{\rm m}}{2 h_{\rm b} \ln \left( (R_{\rm ri} + l_{\rm m} + g + h_{\rm c}) / (R_{\rm ri}) \right)} \le 1.7 \tag{5}$$

where  $\alpha$  is the pole arc to pole pitch ratio and  $k_l$  is a leakage factor which can be approximated by [18]

$$k_{\rm l} = 1 - \frac{1}{1 + 0.9[(R_{\rm ri} + l_{\rm m})/(\alpha(g + h_{\rm c}))]^2}$$
 (6)

Considering the demagnetisation curve of the NdFeB magnet as a straight line in the second quadrant with relative recoil permeability  $\mu_{\rm r}$ ,  $B_{\rm g}$  can be calculated as [18]

$$B_{\rm g} = \frac{k_{\rm l} B_{\rm r} l_{\rm m}}{(R_{\rm ri} + l_{\rm m} + g) \ln \left( (R_{\rm ri} + l_{\rm m} + g + h_{\rm c}) / (R_{\rm ri}) \right)}$$
(7)

The number of turns per phase  $N_t$  can be expressed as

$$N_{\rm t} = \frac{A_{\rm sp}}{N_{\rm sh}A_{\rm w}} k_{\rm f} \tag{8}$$

where  $A_{\rm sp}$  is the cross-sectional area occupied by one winding segment.

If the converter is supplied by an ideal direct voltage source *V*, then at any instant the following equation should be satisfied

$$V \ge E_{\rm L-L} + 2I_{\rm ph}R_{\rm ph} \tag{9}$$

where  $I_{\rm ph}$  is the phase current and the line-to-line back-EMF  $(E_{\rm L-L})$  is given by [13]

$$E_{\rm L-L} = \sqrt{3} k_{\rm w} N_{\rm sp} B_{\rm g} N_{\rm t} (R_{\rm ri} + l_{\rm m} + g + h_{\rm c}) L_{\rm s} \omega_{\rm m}$$
 (10)

where  $\omega_{\rm m}$  is the rotor speed, and the resistance per phase  $(R_{\rm ph})$  is given by

$$R_{\rm ph} = \rho_{\rm R} N_{\rm WS} \left[ 2L_{\rm s} + \frac{\pi}{3} \left( R_{\rm ri} + l_{\rm m} + g + \frac{h_{\rm c}}{2} \right) \right] / (N_{\rm sh} N_{\rm t} A_{\rm w})$$
(11)

where  $\rho_R$  is the resistivity of conductor and  $N_{WS}$  is the number of winding segment pairs per phase.

Another constraint is that the motor torque (T) is not less than 8 mN m and is given as follows

$$T = \frac{3}{\sqrt{2}} \frac{\pi (l_{\rm m} + R_{\rm ri} + g) L_{\rm s} B_{\rm g} k_{\rm w} N_{\rm t} I_{\rm ph}}{2} \ge 0.008$$
 (12)

where  $k_{\rm w}$  is the winding factor.

The last constraint is that the outer radius of stator  $(R_{so})$  is not greater than 10 mm,

$$10 \ge R_{\rm so} = R_{\rm ri} + l_{\rm m} + h_{\rm b} + h_{\rm c} + g \tag{13}$$

#### 4 Result and discussions

In this study, a program based on GAs was constructed and results were obtained after 36 iterations when setting the population size to be 200 for each parameter, and the crossover and mutation rates to be 0.95 and 0.1, respectively. The machine performance characteristics were obtained from 3-D finite element analysis (FEA) using Magsoft software [19]. Fig. 2 shows the photographs of the prototype motor. Fig. 3 shows the magnetic flux distribution within the motor under the no load condition. It shows that there is no flux above the level of saturation within the motor. Fig. 4 shows the simulated result of the air gap flux density under no load. As anticipated, with parallel magnetisation, the flux density distribution in the air gap is nearly sinusoidal. The back-EMF is an important parameter in robotics because of the power limitations. The analytical solution of line-to-line back-EMF was found to be 4.214 V (RMS) at 6000 rpm by employing (10). The simulated and measured waveforms of back-EMFs at 6000 rpm have similar sinusoidal waveforms as shown in Fig. 5. They are in agreement with each other. Moreover, the measured waveform of back-EMF has a peak of 6.08 V that meets the specifications.

The design parameters and the performances of the initial design and the GA-optimised design are compared in Table 2. It shows that the torque-to-inertia ratio and the torque-to-weight ratio increase from  $8.6339 \times 10^4 \, \text{N/kg}$  m and  $0.2132 \, \text{N}$  m/kg in the initial design to

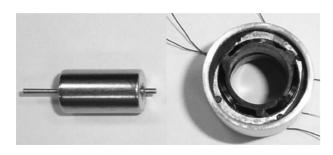


Fig. 2 Photographs of the realised prototype

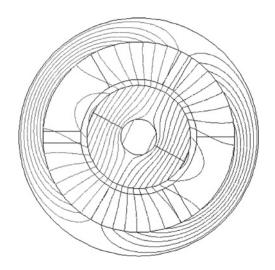


Fig. 3 Magnetic flux distribution

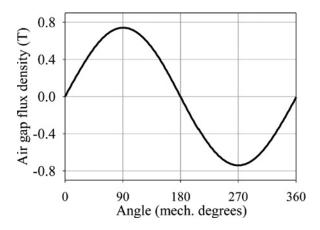


Fig. 4 Air gap flux density under no load

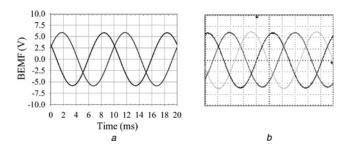


Fig. 5 Line-to-line back-EMF waveforms at 6000 rpm

a Simulated back-EMF

b Measured back-EMF

Both traces have 2.5 V/division and 2 ms/division

 Table 2
 Comparison of design parameters

Parameter	Initial	GAs 3.2508	
I <sub>m</sub> , mm	3		
h <sub>b</sub> , mm	2.2	1.9567	
h <sub>c</sub> , mm	2.8	2.5022	
R <sub>ri</sub> , mm	1.5	1.0006	
g, mm	0.5	0.5	
$k_{f}$	0.5639	0.6864	
$N_{\rm t}$ , turns	94	96	
<i>T</i> , N m	$8.345 \times 10^{-3}$	$8.684 \times 10^{-3}$	
<i>J</i> , kg m <sup>2</sup>	$9.6654 \times 10^{-8}$	$7.6192 \times 10^{-8}$	

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Table 3 Comparison of volume, weight and costs

Item	Initial		GAs		Price, USD/kg
	Volume, cm <sup>3</sup>	Weight, g	Volume, cm <sup>3</sup>	Weight, g	
magnet	1.131	8.3694	1.072	7.9382	531.3661
copper	1.2179	10.8389	1.2159	10.8211	13.2203
iron	2.5898	19.7639	2.0241	16.2588	1.7288
shaft	0.2121	1.6541	0.0629	0.736	5.0847
hub	2.8932	22.1501	2.4779	19.6521	4.2373
total	8.044	62.7764	6.8528	55.4062	_
cost	4.7269 USD	_	4.4763 USD	_	_

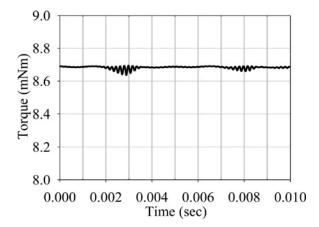


Fig. 6 Torque waveform

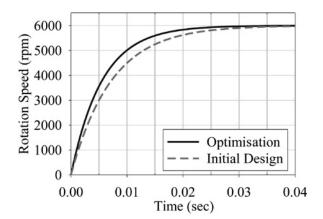


Fig. 7 Speed against time response

 $11.3975 \times 10^4$  N/kg m and 0.25 N m/kg in the GAs, respectively. These results confirm that the optimisation is of benefit to the motor to speed up masses of robotics. In addition, we can also compare the active materials' volume, weight and cost between the initial design and the GAs from the simulation as shown in Table 3. As the table shows, the volume, weight and cost reduce from the initial design of  $8.044 \, \mathrm{cm}^3$ ,  $62.7764 \, \mathrm{g}$  and  $4.7269 \, \mathrm{USD}$  to the GAs of  $6.8528 \, \mathrm{cm}^3$ ,  $55.4062 \, \mathrm{g}$  and  $4.4763 \, \mathrm{USD}$ . The volume and weight are reduced by  $14.8 \, \mathrm{and} \, 11.74\%$ , respectively. Again, this illustrates that the motor size and manufacturing costs through the GAs are reduced by the use of the GAs.

It is worth investigating the variation of current with torque to compute the torque constant that couples the electrical circuit equations with the mechanical equations. The torque constant was found to be 10.098 mN m/A. We can also address the instantaneous torque from the simulation. Fig. 6 shows the torque waveform. The average torque is found to be 8.684 mN m with a 0.63% torque ripple, which meets

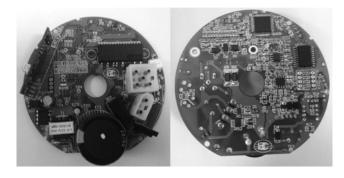


Fig. 8 Prototyped controller using Infineon XC886CM

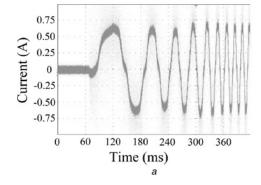
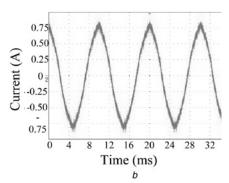


Fig. 9 Phase current (6000 rpm)

a At motor starting stage

b At steady running stage



the specifications. The speed against time for the motor startup is shown in Fig. 7. From this figure, the mechanical time constant is found to decrease from 7.1823 ms in the initial design to 5.521 ms in the GAs. This value can result in a suitable motor startup performance.

#### 5 Experimental tests

Owing to the rapid progress of semiconductor manufacturability, the state-of-art field oriented control (FOC) algorithm can be implemented using a low-cost 8-bit MCU such as the Infineon XC886/888 series. The FOC-based controller also exhibits higher efficiency, better dynamic response and fewer torque harmonics and it perfectly matches with the motor design having the sinusoidal back-EMF. In this paper, the prototype controller (see Fig. 8) has been tested with the designed motor.

The phase current waveforms for motor startup and for steady running at 6000 rpm were observed. Fig. 9a shows the successful startup of the motor after a few cycles. Fig. 9b shows the near sinusoidal waveform of phase current. The slight distortion of this current waveform is acceptable on account of it having less motor noise and vibration compared with the square waveform.

#### 6 Conclusions

The design of a three-phase slotless BLDC motor drive for use as an electromagnetic actuator in robotics is presented. The GAs are utilised for the design optimisation to achieve the high torque-to-inertia and torque-to-weight ratios required for fast response in robotic applications. The prototype motor drive has been constructed and tested, validating the theoretical design and analysis.

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