

# **Probability and Statistics**

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# Textbooks

❑ **Probability & Statistics for Engineers & Scientists**, Ninth Edition, Ronald E. Walpole, Raymond H. Myer

❑ **Elementary Statistics: Picturing the World**, 6<sup>th</sup> Edition, Ron Larson and Betsy Farber

❑ **Elementary Statistics**, 13<sup>th</sup> Edition, Mario F. Triola

# Reference books

- ❑ **Probability Demystified**, Allan G. Bluman
- ❑ **Schaum's Outline of Probability and Statistics**
- ❑ **MATLAB Primer**, Seventh Edition
- ❑ **MATLAB Demystified** by McMahan, David

# References

Readings for these lecture notes:

❑ **Probability & Statistics for Engineers & Scientists**,  
Ninth edition, Ronald E. Walpole, Raymond H. Myer

❑ **Probability Demystified**, Allan G. Bluman

❑ [https://en.wikipedia.org/wiki/Law\\_of\\_large\\_numbers](https://en.wikipedia.org/wiki/Law_of_large_numbers)

These notes contain material from the above three resources.

**“There is only one thing that makes a dream impossible to achieve: the fear of failure.”**

— Paulo Coelho, *The Alchemist*

# Basic concepts [1]

❑ **Probability** can be defined as the mathematics of chance.

❑ Statisticians use the word **experiment** to describe any process that **generates a set of data**.

OR

❑ A **probability experiment** is a chance process that leads to well defined outcomes or results. **For example**, tossing a coin can be considered a probability experiment since there are two well-defined outcomes—heads and tails.

# Basic concepts [2]

- ❑ An **outcome** of a probability experiment is the result of a single trial of a probability experiment.
- ❑ A **trial** means flipping a coin once, or drawing a single card from a deck. A trial could also mean rolling two dice at once, tossing three coins at once, or drawing five cards from a deck at once.

# Basic concepts [3]

- ❑ The set of all possible outcomes of a statistical experiment is called the **sample space** and is represented by the symbol **S**.

OR

- ❑ The set of all outcomes of a probability experiment is called a **sample space**. Some sample spaces for various probability experiments are shown here.

Experiment	Sample space
Toss one coin	H, T
Roll a die	1, 2, 3, 4, 5, 6
Toss two coins	HH, HT, TH, TT



# Basic concepts [4]

- ❑ Each outcome in a sample space is called an **element** or a **member** of the sample space, or simply a **sample point**.
- ❑ Each outcome of a probability experiment occurs at **random**.
- ❑ Each outcome of the experiment is **equally likely** unless otherwise stated.

# Basic concepts [5]

❑ An **event** then usually consists of one or more outcomes of the sample space.

OR

❑ An **event** is a subset of a sample space.

❑ An event with one outcome is called a **simple event**.

❑ An event consists of two or more outcomes, it is called a **compound event**.

# Example

A single die is rolled. List the outcomes in each event:

- a. Getting an odd number
- b. Getting a number greater than four
- c. Getting less than one

## Example cont.

### Solution:

$$S = \{1, 2, 3, 4, 5, 6\}$$

- a. Let **A** be the event contains the outcomes 1, 3, and 5.

$$A = \{1, 3, 5\}, n(A) = 3$$

- b. Let **B** be the event contains the outcomes 5, and 6.

$$B = \{5, 6\}, n(B) = 2$$

- c. Let **C** be the event that contains a number less than one

$$C = \{\}$$

# Basic concepts [6]

## Classical Probability:

The formula for determining the probability of an event **E** is

$$P(E) = \frac{n(E)}{n(S)}$$

OR

$$P(E) = \frac{\text{Number of outcomes contained in the event E}}{\text{Total number of outcomes in the sample space}}$$

## Example:

Two coins are tossed; find the probability that both coins land heads up.

## Solution:

$$S = \{HH, HT, TH, \text{ and } TT\}$$

$$n(S) = 4$$

Let **A** be the event of getting a both heads

$$A = \{HH\}$$

$$n(A) = 1$$

$$P(A) = \frac{1}{4} = \mathbf{0.25 \text{ (or 25 \%)}}$$

# Example:

A die is tossed; find the probability of each event:

- a. Getting a two
- b. Getting an even number
- c. Getting a number less than 5



## Example cont.

### Solution:

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$n(S) = 6$$

$$P(E) = \frac{\text{Number of outcomes contained in the event } E}{\text{Total number of outcomes in the sample space}}$$

**a.** Let A be the event of getting a “two”

$$A = \{2\}$$

$$n(A) = 1$$

$$P(A) = \frac{1}{6} = 0.1667 \text{ (or 16.67\%)}$$

## Example cont.

b. a. Let **B** be the event of getting a “even number”

$$A = \{2, 4, 6\}$$

$$n(A) = 3$$

$$P(B) = \frac{3}{6} = \frac{1}{2} = 0.5 \text{ (or 50\%)}$$

c. a. Let **C** be the event of getting a “less than 5”

$$C = \{1, 2, 3, 4\}$$

$$n(C) = 4$$

$$P(C) = \frac{4}{6} = \frac{2}{3} = 0.6666 \text{ (or 66.67\%)}$$

# Basic concepts [7]

**Rule 1:** The probability of any event will always be a number from **zero to one**. Probabilities cannot be **negative** nor can they **be greater than one**.

**Rule 2:** When an event cannot occur, the probability will be **zero**.

**Example:** A die is rolled; find the probability of getting a 7.

# Basic concepts [8]

**Rule 3:** When an event is certain to occur, the probability is **1**.

**Example:** A die is rolled; find the probability of getting a number less than 7.

**Rule 4:** The sum of the probabilities of all of the outcomes in the **sample space** is 1.

**Example:**  $P(H) = 1/2$ ,  $P(T) = 1/2$ ,  $P(H) + P(T) = 1$ .

# Basic concepts [9]

**Complement** : The **complement** of an event  $A$  with respect to  $S$  is the subset of all elements of  $S$  that are not in  $A$ . We denote the complement of  $A$  by the symbol  **$A'$  or  $\bar{A}$  or  $A^c$**

**Rule 5:** The probability that an event will not occur is equal to 1 minus the probability that the event will occur.

**Example:**  $P(H) = 1/2$ ,  $P(T) = 1 - P(H) = 1/2$

# Basic concepts

The **probability** of an event  $A$  is the sum of the weights of all **sample points** in  $A$ .

Therefore,

I.  $0 \leq P(A) \leq 1$

II.  $P(\varphi) = 0$

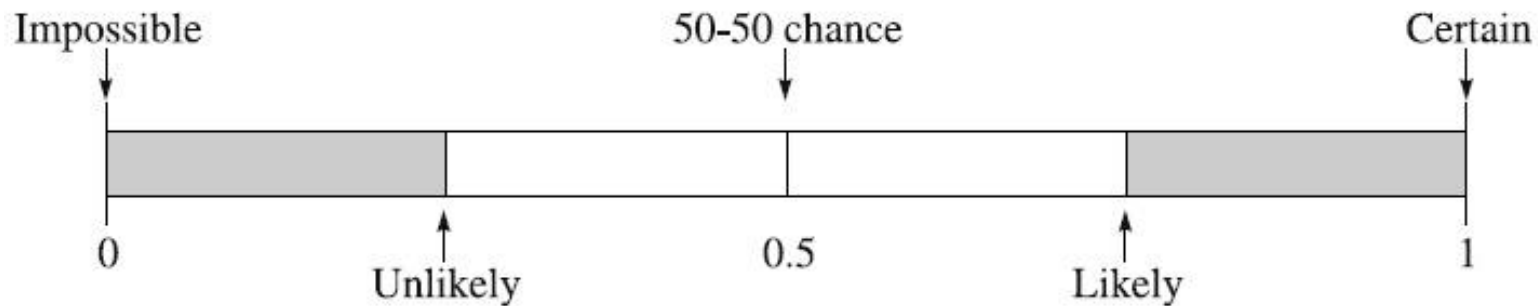
III.  $P(S) = 1.$

# Basic concepts

- ❑ When the probability of an event is close to **zero**, the occurrence of the event is relatively **unlikely**. For example, if the chances that you will win a certain lottery are **0.001** or one in one thousand, you probably won't win, unless of course, you are very **"lucky."**
- ❑ When the probability of an event is **0.5** or  $\frac{1}{2}$ , there is a **50–50 chance** that the event will happen—the same.

# Basic concepts

When the probability of an event is close to one, the event is almost sure to occur. For example, if the chance of it snowing tomorrow is **90%**, more than **likely**, you'll see some snow.





# Empirical Probability [1]

Probabilities can be computed for situations that do not use sample spaces. In such cases, frequency distributions are used and the probability is called **empirical probability**.

Rank	Frequency
Freshmen	4
Sophomores	6
Juniors	8
Seniors	7
TOTAL	25

# Empirical Probability [2]

$$P(E) = \frac{\text{Frequency of E}}{\text{Sum of the frequencies}}$$

$$P(\text{Freshmen}) = 4/25$$

Empirical probability is sometimes called **relative frequency probability**.

# Law of large numbers

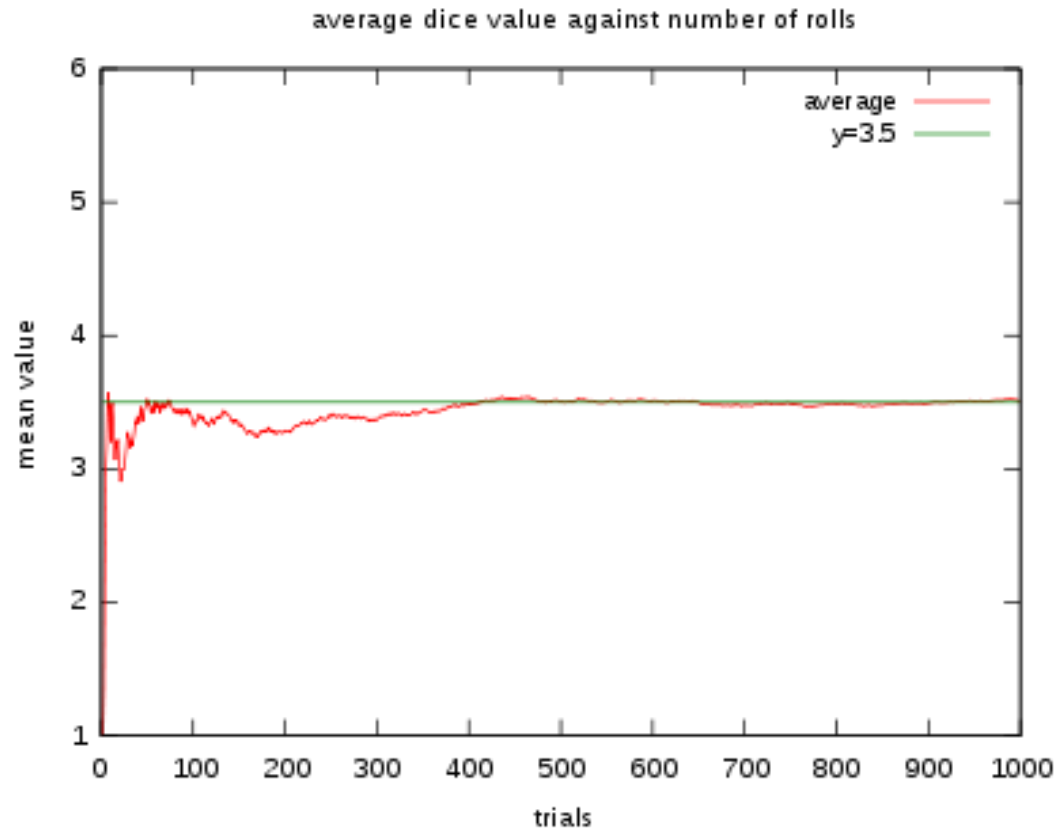
- ❑ In probability theory, the **law of large numbers (LLN)** is a theorem that describes the **result** of performing the **same experiment a large number of times**.
- ❑ According to the law, the average of the results obtained from a large number of trials should be close to the **expected value**, and will tend to become **closer** as **more trials** are performed.

# Law of large numbers

❑ The LLN is important because it **"guarantees" stable long-term** results for the averages of some random events.

❑ **For example**, while a casino may lose money in a single spin of the **roulette** wheel, its earnings will tend towards a **predictable percentage** over a large number of spins.

❑ Mean of rolls of a die =  $\frac{1+2+3+4+5+6}{6} = 3.5$



An illustration of the law of **large numbers** using a particular **run of rolls of a single die**. As the number of rolls in this run increases, the average of the values of all the results approaches **3.5**.

# Law of Large Numbers

## Questions:

What happens if we toss the coin **100 times** ? Will we get **50** heads?

What will happen if we toss a coin **1000 times**? Will we get exactly **500** heads?

# Law of Large Numbers

❑ **Solution:** Probably not.

❑ However, as the number of tosses increases, the ratio of the number of heads to the total number of tosses will get closer to  $1/2$ .

❑ This phenomenon is known as the **law of large numbers**.

# Suggested Readings

**Probability & Statistics for Engineers & Scientists,**  
Ninth Edition, Ronald E. Walpole, Raymond H. Myer

2.1 Sample space

2.2 Events

2.3 Counting Sample Points

2.4 Probability of an Event