Practice 15 [Recursion]

Solve all problems with recursion:

1. Find GCD of two numbers, the recursive definition is:

$$\gcd(x, y) = \left\{ \begin{array}{c} \gcd(y, x)\ if\ x < y \\ y\ if\ x\%y = 0 \\ \gcd(y, x\%y) \end{array} \right\}$$

2. Find square root of a number (x) using following steps:
   - Let square root sq = 1
   - Send number and assumed square root to recursive function
   - Check if absolute difference of number divided by square root with square root is very smaller say less than 0.00001, return square root i.e **| x / sq - sq | < 0.000001**
   - Otherwise call function again with new guess of square root i.e.
     - $\boldsymbol{sq} = \frac{(\boldsymbol{sq} + \frac{\boldsymbol{x}}{\boldsymbol{sq}})}{\mathbf{2}}$

Note you can vary the number in comparison of absolute difference to reduce/ increase accuracy of your result

3. Find binary string, for any given integer, the recursive definition is:

$$\text{binary\_string}(x) = \left\{ \begin{array}{c} '0'if\ x = 0 \\ '1'if\ x = 1 \\ \text{binary string}\left(int\left(\frac{\text{x}}{2}\right)\right) + \text{x}\%2 \end{array} \right\}$$

4. Find function to convert binary to decimal, the recursive definition is:

$$\text{binary\_to\_decimal}(x) = \left\{ \begin{array}{c} \text{binary\_to\_decimal}\big(\text{int}(\text{x})\big)if\ type(x) = str \\ 0\ if\ x = 0 \\ (\text{x}\%2)\text{x}(2^p) + \text{binary\_to\_decimal}\left(int(\frac{\text{x}}{10}), \text{p} + 1\right) \end{array} \right\}$$

5. Find maximum number from list

6. Find index of maximum number in list

7. Print all the possible x, y pairs of the list, where x, y may exist anywhere in the list? See example:

```
Consider list = [3, 4, 8, 2]
Required Output:
(3, 4) (3, 8) (3, 2)
(4, 8) (4, 2)
(8, 2)
```

8. Find maximum pair sum, where pair of elements need not to be adjacent in the list, see examples:

```
Consider list = [3, 4, 8, 2]
Required Output:11
Consider list = [3, 4, 8, 9]
Required Output:17
Consider list = [3, 7, 8, 2]
Required Output:15
Consider list = [8, 7, 3, 9]
Required Output:17
```