

Matrices and Determinants

Related Definitions and Formulae

The Concept of a matrix: It is often desirable to present a set of numbers (or other elements) in a rectangular array of rows and columns. The table of values of trigonometric functions is an example of such an arrangement; in it the column have the heading sine, cosine, tangent and cotangent, and the rows are designated by angles, expressed in degrees. It is conventional to call the vertical lines columns and the horizontal lines rows.

e.g#1 Here is a bowling analysis in cricket.

| 4.0 | Overs | Maidens | Runs | Wickets |
|---------------|-------|---------|------|---------|
| Wasim Akram | 15 | 7 | 70 | 5 |
| Waqar Younis | 18 | 6 | 55 | 4 |
| Shoaib Akther | 10 | 3 | 21 | 1 |

e.g#2 Here is an example of simultaneous linear equations

$$2x - 3y = 7$$

 $\frac{1}{2}x + 5y = 9$ These may be set down as below

| Cofficient of x | Coefficient of y | Constant term |
|------------------|------------------|---------------|
| .2 | -3 | 7 . |
| 1 | 5 | 9 |
| $\overline{2}$. | | |

Tables are a concise method of presenting a mass of information. When we construct a table from a collection of data, we generally arrange the data in rows and columns. We extract the information form the table by reading the entry corresponding to a row and column intersection. Any table is a matrix.

Applications of Matrices: Matrices and determinants are important mathematical tools which have wide applications in certain branches of physics, chemistry, statistics, biology, economics, psychology and various branches of engineering.

Tons of applications including.

Solving System of linear equations.

Computer graphics, Image processing

- Models within many areas of computational science of Engineering
- Quantum Mechanics, Quantum Computing.
- Many, many more

Meanings:

- Matrix is a latin word which means a place in which something develops or orginates.
- A common device for summarizing and displaying numbers (or) data.

DEVELOPERS: English Mathematicians

(1) James Joseph Sylvester (1814 — 1897)

(2) Arther Cayley (1821 — 1895)

Notations: (1) [] (2) () (3) |

Basic Purpose to use: To reduce the amount of writing.

Matrix: A matrix is a rectangular array of numbers enclosed in square brackets (or parentheses)

or

"A Matrix is a rectangular array in shape, whose elements are written within square bracket in a definite order, in rows and columns".

Matrix can never be expanded.

It is always represented by Capital letters. (A, B, C.,.... etc)

Row: The horizontal arrangement of numbers is called row.

Column: The vertical arrangement of numbers is called column.

Entries: The numbers used in rows or columns are said to be the entries or elements of the matrix.

Order (or) Dimension of a Matrix: If a matrix A has m rows and n columns, then the oder of the matrix is $m \times n$.

Types of Matrices:

(1) Row Matrix (or) Row Vector: A matrix having only one row is called row matrix (or) Row vector.

e.g# A =
$$[\sqrt{5} \ 1]$$
 B = $[2 \ 0 \ 1]$

(2) Column Matrix (or) Column Vector: A matrix having only one column is called column matrix (or) column vector.

e.g#
$$C = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$
; $D = \begin{bmatrix} \sqrt{2} \\ \sqrt{5} \end{bmatrix}$

Exercise No.=4.1

Qno.(1)Specify the type of each of the following matrices.

$$\text{(i)} \begin{pmatrix} 3 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \sqrt{5} \end{pmatrix} \text{(ii)} \begin{pmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{pmatrix} \text{ (iii)} \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix} \text{(iv)} \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \text{(v)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(i)Diagonal Matrix.(ii)Scalar Matrix.(iii)Scalar Matrix.(iv)Diagonal(v)Unit Matrix.

Qno.(2)Write down the matrices of the coefficents of x,y and z in the following system of linear equations.

(i)
$$3x+2y+8z-5=0$$
, $\begin{bmatrix} 3 & 2 & 8 \\ 5 & 8 & -4 \end{bmatrix}$ (ii) $8x+5y+2z-3=0$
 $5x+8y-4z+2=0$ $\begin{bmatrix} 5 & 8 & -4 \end{bmatrix}$ $6x+4y+3z+2=0$

7x-3y+5z-9=0

$$\begin{pmatrix} 8 & 5 & 2 \\ 6 & 4 & 3 \\ 7 & -3 & 5 \end{pmatrix}$$
Ans

Qno.(3)Write down in tabular form.

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{bmatrix} (ii)Y = \begin{bmatrix} y_{1k} \end{bmatrix}_{(4,2)}$$

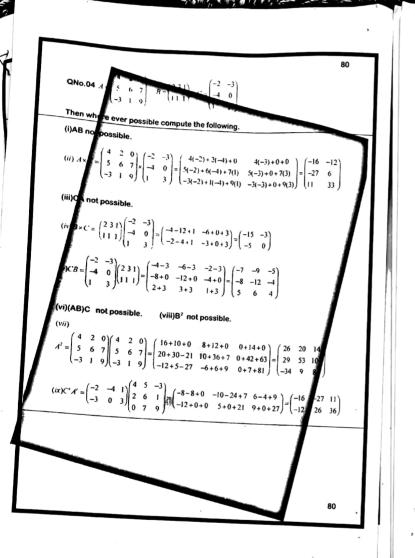
$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \\ y_{31} & y_{32} \\ y_{41} & y_{42} \end{bmatrix}$$

Number of rows=3

Number of columns=4

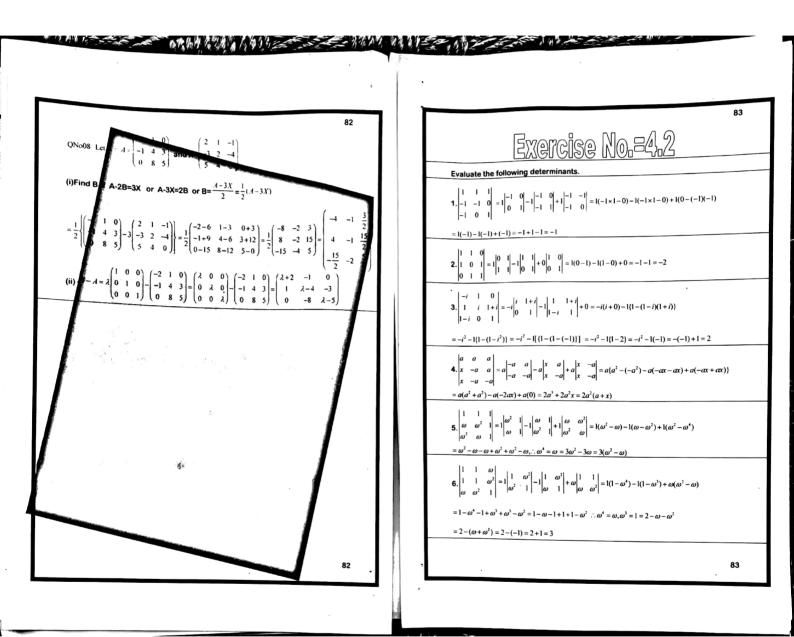
Number of rows=4

Number of columns=2



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QN_0.05 \ IF \ A = \begin{pmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -3 & -1 \end{pmatrix} : B = \begin{pmatrix} 2 & 1 & -1 \\ 3 & -2 & -1 \\ 2 & -5 & -1 \end{pmatrix} : C = \begin{pmatrix} 1 & 4 & 1 \\ 2 & 1 & 1 \\ 1 & -2 & 1 \end{pmatrix}  then show that AB=AC.

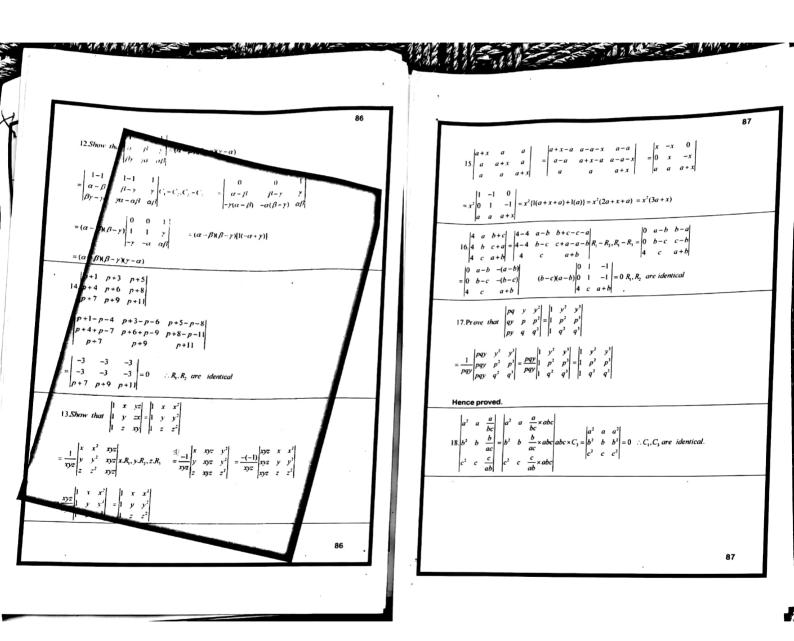
A \times B = \begin{pmatrix} 2-9+4 & 1+6-10 & -1+3-2 \\ 4+3-6 & 2-2+15 & -2-1+3 \\ 8-9-2 & 4+6+5 & -4+3+1 \end{pmatrix} = \begin{pmatrix} -3 & -3 & 0 \\ 3 & 15 & 0 \end{pmatrix}
A \times C = \begin{pmatrix} 1-6+2 & 4-3-4 & 1-3+2 \\ 2+2-3 & 8+1+6 & 2+1-3 \\ 4-6-1 & 16-3+2 & 4-3-1 \end{pmatrix} = \begin{pmatrix} -3 & -3 & 0 \\ 1 & 15 & 0 \\ -3 & 15 & 0 \end{pmatrix}  Proved
QN_0.06 \begin{pmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta & 0 \end{pmatrix} \begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\begin{pmatrix} \sin\theta \sin\theta + \cos\theta \cos\theta & \sin\theta \cos\theta - \cos\theta \sin\theta \\ \cos\theta \sin\theta - \sin\theta \cos\theta & \cos\theta \cos\theta + \sin\theta \sin\theta \end{pmatrix} = \begin{pmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
If A+B matrices; then explain why in general.
QN_07(i) (A+B)^2 \times A^2 + 2AB+B^2 : (A+B)(A+B) = A^2 + AB+BA+B^2
In \ general \ AB \times BA : (A+B)^2 \times A^2 + 2AB+B^2
QN_07(ii) \ (A-B)^2 \times A^2 - 2AB + B^2 : (A+B)(A-B) = A^2 - AB - BA + B^2
In \ general \ AB \times BA : (A-B)^2 \times A^2 - 2AB+B^2
QN_07(iii) \ (A+B)(A-B) \times A^2 - B^2 : (A+B)(A-B) = A^2 - AB + BA - B^2
In \ general \ AB \times BA : (A+B)^2 \times A^2 - 2AB+B^2
QN_07(iii) \ (A+B)(A-B) \times A^2 - B^2 : (A+B)(A-B) = A^2 - AB + BA - B^2
In \ general \ AB \times BA : (A+B)(A-B) \times A^2 - B^2
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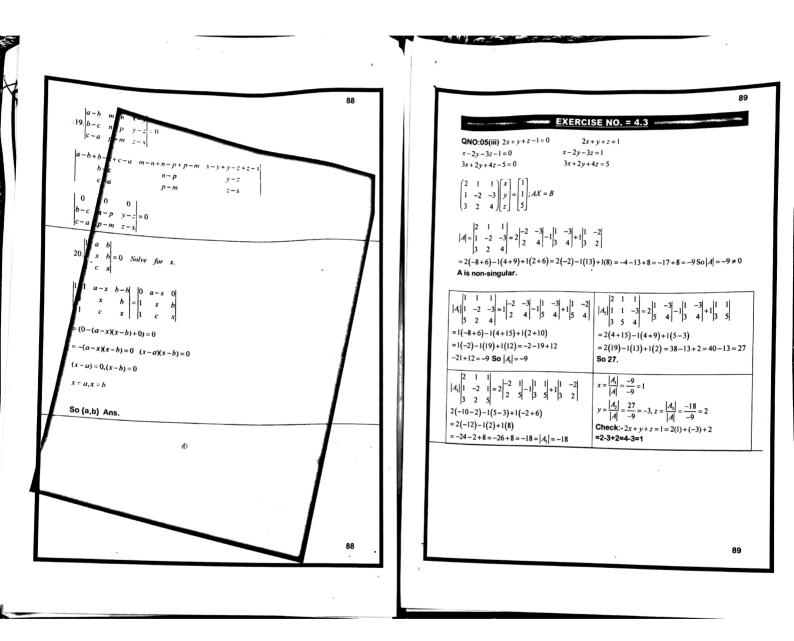


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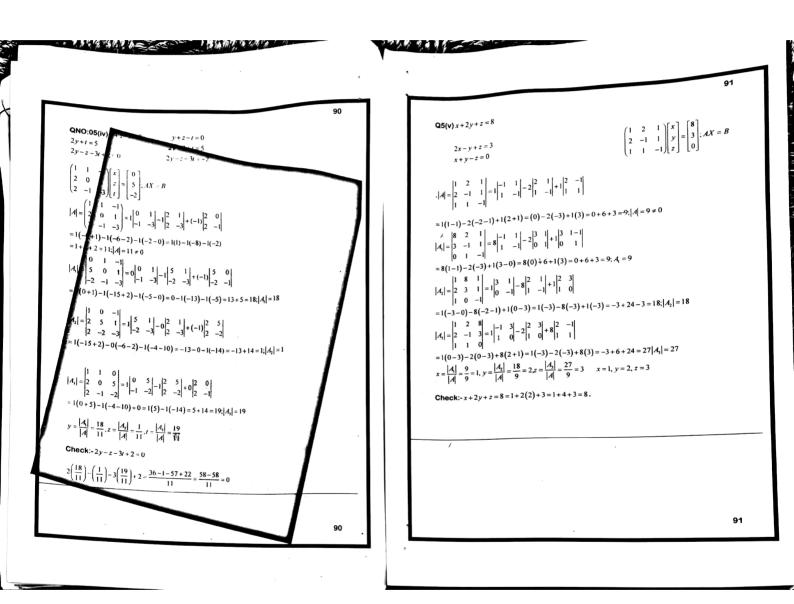
\begin{aligned}
\mathbf{9} & \mid \begin{array}{c} x^{2} \\ y^{2} \\ y^{2} \\ y^{2} \\ y^{2} \\ y \\ z^{2} \\ y \\ y \end{aligned} = \begin{vmatrix} 1-1 & x^{2}-y^{2} & x-y \\ -1-1 & y^{2}-z^{2} & y-z \\ -1 & z^{2} & z \\ \end{vmatrix} = \begin{pmatrix} 0 & (x-y)(x+y) & x-y \\ 0 & (y-z)(y+z) & y-z \\ -1 & z^{2} & z \\ \end{vmatrix} = (x-y)(y-z)\{0 & x+y & 1 \\ 0 & (y-z)(y+z) & y-z \\ -1 & z^{2} & z \\ \end{vmatrix} = (x-y)(y-z)\{0 - (x+z)(0-1)+0\}, & = (x-y)(y-z)\{-(x-z)(-1)\} \\ = (x-y)(y-z)\{0 - (x+z)(0-1)+0\}, & = (x-y)(y-z)\{-(x-z)(-1)\} \\ = (x-y)(y-z)\{0 - (x+z)(0-1)+0\}, & = (x-y)(y-z)\{-(x-z)(-1)\} \\ = (x-y)(y-z)(x-z) & = -(x-y)(y-z)(z-x) \end{aligned}

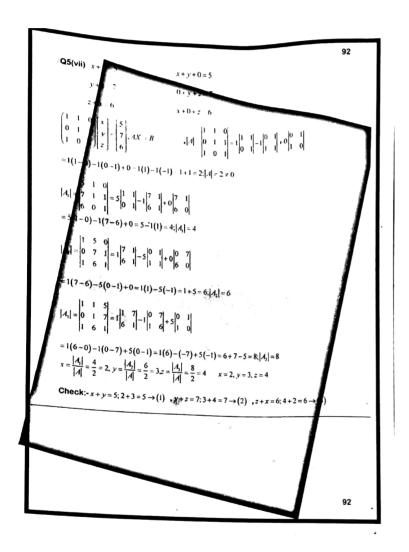
\begin{aligned}
\mathbf{10} & \begin{pmatrix} x+y & 1 \\ 0 & (y-z)(y-z) \\ -(x-z)(-1) \\ -(x-y)(y-z)(x-z) & = -(x-y)(y-z)(z-x) \\ -(x-
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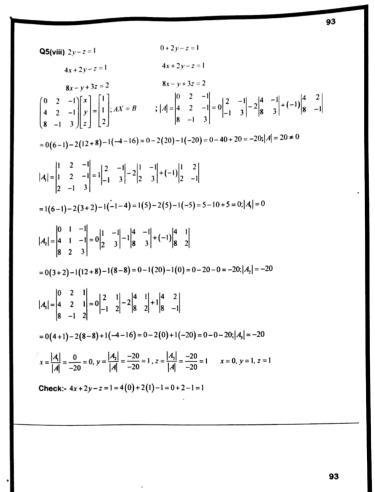




https://stbmathematics.blogspot.com/







$$\begin{aligned} \mathbf{QS(x)} & \times + (1+d)y + z = 2d \\ & \times + y + (1+d)z = 0 \end{aligned} \qquad \vdots \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+d & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{vmatrix} = \begin{bmatrix} d \\ 2d \\ 0 \end{bmatrix} : AX = B \end{aligned}$$

$$\begin{aligned} |A| &= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+d & 1 \\ 1 & 1 & 1+d \end{vmatrix} = 1 \begin{vmatrix} 1+d & 1 \\ 1 & 1+d \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & 1+d \end{vmatrix} + 1 \begin{vmatrix} 1 & 1+d \\ 1 & 1 \end{vmatrix} = 1 \end{vmatrix} = 1 \begin{vmatrix} 1+d & 1 \\ 1 & 1+d \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & 1+d \end{vmatrix} + 1 \begin{vmatrix} 1 & 1+d \\ 1 & 1 \end{vmatrix} = 1 \end{vmatrix}$$

$$= 1\{(1+d)^2 - 1\} - 1(1+d-1) + 1\{(1-(1+d))\} = 1\{(1+2d+d^2-1) - 1(d) + 1(1-1-d)\}$$

$$= 2d + d^2 - 1(d) + 1(-d) = 2d + d^2 - d - d = 2d + d^2 - 2d = d^2; |A| = d^2 \end{aligned}$$

$$|A_1| &= \begin{vmatrix} d & 1 & 1 \\ 2d & 1+d & 1 \\ 0 & 1 & 1+d \end{vmatrix} = d \begin{vmatrix} 1+d & 1 \\ 1 & 1+d \end{vmatrix} - 1 \begin{vmatrix} 2d & 1 \\ 0 & 1+d \end{vmatrix} + 1 \begin{vmatrix} 2d & 1+d \\ 0 & 1 \end{vmatrix} = d \end{aligned}$$

$$= d\{(1+d)^2 - 1\} - 1\{2d(1+d) - 0\} + 1\{2d - 0\} = d\{1 + 2d + d^2 - 1\} - 1\{2d + 2d^2\} + 1(2d)$$

$$= d\{2d + d^2 - 2d\} = d\{2d + d^2 - 2d\} = d(2^2) = d^3$$

$$|A_2| &= \begin{vmatrix} 1 & d & 1 \\ 1 & 2d & 1 \\ 1 & 0 & 1+d \end{vmatrix} = 1 \begin{vmatrix} 2d & 1 \\ 0 & 1+d \end{vmatrix} - d \begin{vmatrix} 1 & 1 \\ 1 & 1+d \end{vmatrix} + 1 \begin{vmatrix} 1 & 2d \\ 1 & 0 \end{vmatrix}$$

$$= 1\{(2d(1+d) - 0) - d(1+d-1) + 1(0-2d) = 1\{2d + 2d^2\} - d(d) + 1(-2d) = 2d + 2d^2 - d^2 - 2d$$

$$= d^2 & : |A_1| = d^2$$

$$|A_1| &= \begin{vmatrix} 1 & 1 & d \\ 1 & 1 & 0 \end{vmatrix} = 1 \begin{vmatrix} 1+d & 2d \\ 1 & 0 \end{vmatrix} - 1 \begin{vmatrix} 1 & 2d \\ 1 & 0 \end{vmatrix} + d \begin{vmatrix} 1 & 1+d \\ 1 & 1 \end{vmatrix}$$

$$= 1(0-2d) - 1(0-2d) + d\{1 - (1+d)\} = -2d + 2d + d(-d) = -d^2$$

$$|A_1| &= \frac{d^3}{4}, \quad y = \frac{|A_1|}{|A|} = \frac{d^2}{d^2} = 1, \quad z = \frac{|A_1|}{|A|} = \frac{-d^2}{d^2} = -1$$

$$\mathbf{Check:} \cdot x + (1+d)y + z = 2d \rightarrow (2) = d + (1+d)(1) + (-1) = d + 1 + d - 1 = 2d$$

$$x + y + (1+d)z = 0 = d + 1 + (1+d)(-1) = d + 1 - 1 - d = 0$$

