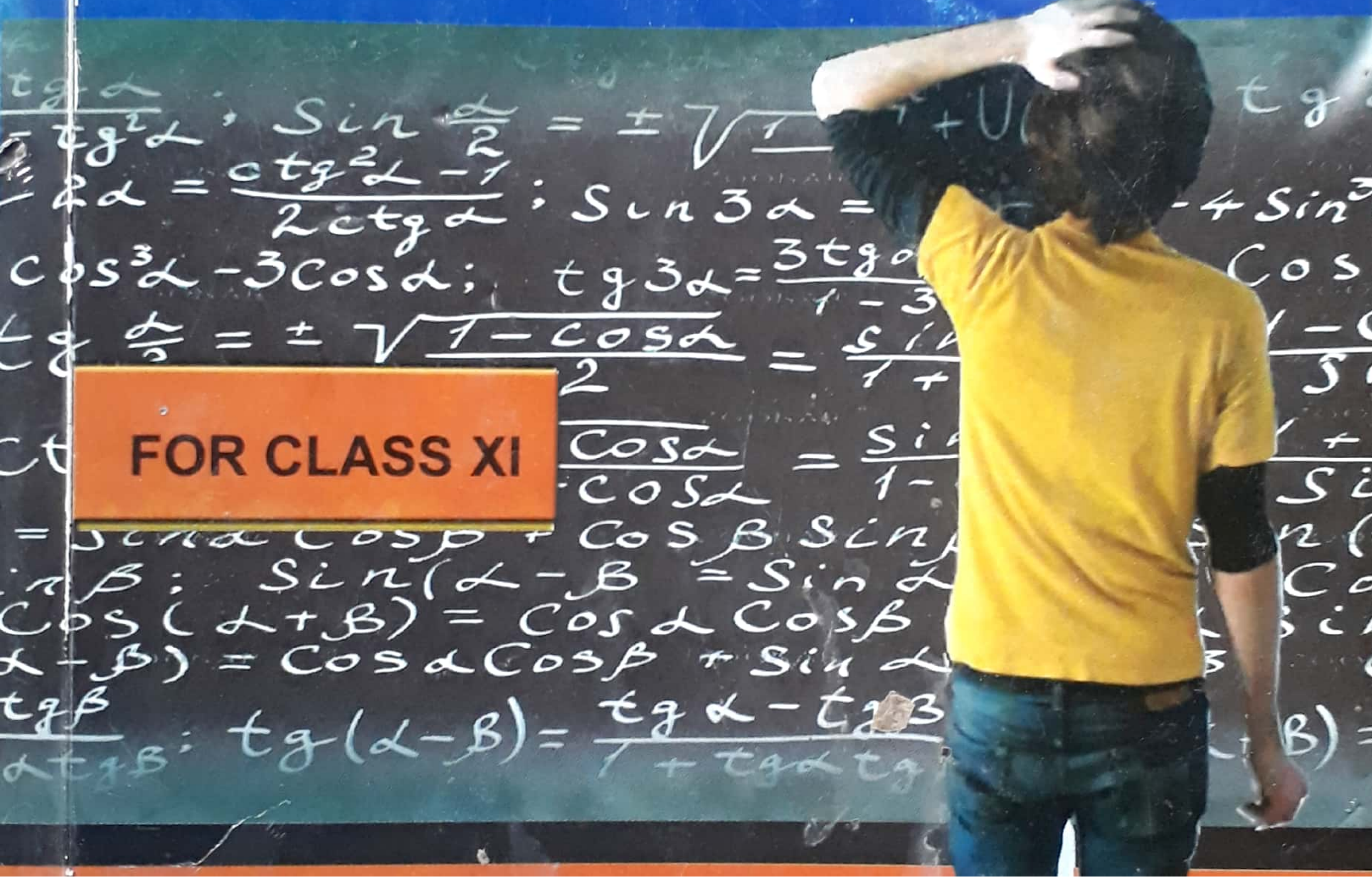
Revised Edition

# COLLEGE MATHEMATICS

## COMPLETE SOLUTION



**FOR CLASS XI**

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# EXERCISE NO. = 1.1

**SET** Collection of well defined distinct objects is called set.

**Ways of describing a set** There are three ways to describe a set.

| Way of description | Definition | Example |
|---|---|---|
| 1.The descriptive Method | In this method the set can be described in words. | The set of natural numbers. |
| 2.The tabular Method | In this method the set is described by listing its elements with in brackets. | Set of natural numbers ={1,2,3,......} |
| 3.Set Builder Method | In this method a symbol or leter is used for an arbitrary member of the set and starting the property common to all the members of the set. | Set of natural numbers $=\{x / x \in N\}$ |

## ORDER OF A SET.

The number of elements present in a set is called order of the set.

## TABLE OF SETS

| Name | Definition |
|---|---|
| Set of natural numbers or Counting number. | $N=\{1,2,3......\}$ |
| Set of whole numbers | $W=\{0,1,2,3......\}$ |
| Set of integers or directed numbers | $Z=\{0,\pm1,\pm2,\pm3,.......\}$ |
| Set of positive integers | $Z^{+}=\{1,2,3,.....\}$ |
| Set of negative integers | $Z^{-}=\{-1,-2,-3,.....\}$ |
| Set of odd numbers | $O=\{1,3,5,.....\}$ |
| Set of even numbers | $E=\{0,2,4,.....\}$ |
| Set of prime numbers | $P=\{2,3,5,7,11,13,17,19,....\}$ |
| Set of rational numbers | $Q=\left\{x / x=\frac{p}{q}.p,q \in z.q \neq 0\right\}$ |
| Set of irrational numbers | $Q'=\left\{x / x \neq \frac{p}{q}.p,q \in z.q \neq 0\right\}$ |
| Set of real numbers | $R=Q \cup Q'$ |

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## TYPES OF SETS (W.R.T THE ORDER).

| Name of the set | Definition | Example |
|---|---|---|
| 1.Finite set | If a set has definite number of elements | $\{1,2,3,....10\}$ |
| 2.Infinite set | If a set has indefinite number of elements present in it. | $N=\{1,2,3.....\}$<br>$Z=\{0,\pm 1,\pm 2,\pm 3,....\}$ |
| 3.Null set or Empty set or void set | A set having no element.<br>It is denoted by $\phi$. | $\phi=\{\ \}$ |
| 4.Singleton set | A set having only one element. | $\{7\}$ |
| 5.Equal sets | Two sets A and B are equal if they have same elements.Order of the elements of the set does not matter. | If $A=\{c,a,t\}$ & $B=\{a,c,t\}$ then A=B. |
| 6:Equivalent or similar sets | Two sets A and B are equivalent if a one to one correspondence exists between the elements of the sets.It is denoted by $A\sim B$ or $A\equiv B$. | If $A=\{1,2,3,4,5\}$ & $B=\{a,e,i,o,u\}$ then $A\sim B$ or $A\equiv B$. |
| 7.Subset | If every element of a set A is an element of a set B, then A is a subset of B written as $A\subseteq B$. | Let $A=\{1,2,3\}$ & $B=\{1,2,3,.....10\}$ then $A\subseteq B$.If a set has n elements then it has $2^n$ subsets. |
| 8.Superset | If $A\subseteq B$ then B is called superset of A.It is denoted by $B\supset A$. | Let $A=\{1,2,3\}$ & $B=\{1,2,3,.....10\}$ then $B\supset A$. |
| 9.Proper subset. | If $A\subseteq B$ and B contains at least one element which is not an element of the set A then A is proper subset of B. | Let $A=\{1,2,3\}$ & $B=\{1,2,3,.....10\}$ then $A\subset B$. |
| 10.Improper subset | Every set itself and the empty set are the subsets of every set. They are calles improper subsets of a set. | Note: The empty and singleton set has no proper subsets. |
| 11.Power set | The collection of all proper and improper subsets of a set A is called power set of A. Denoted by P(A). | If A={1,2} then<br>$P(A)=\{\phi,\{1\},\{2\},\{1,2\}\}$<br>If a set has n-elements then there are $2^n$ elements in its power set. |

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| Name of the set | Definition | Example |
|---|---|---|
| 12.Universal set or Universe of Discourse | The super set of all the sets under a particular discussion. | It is denoted by $U, E$ or $X$. |
| 13.Disjoint sets or Exclusive sets | For two sets A and B If $A\cap B=\phi$. | Let A={1,2}&B={3,4} So $A\cap B=\phi$. |
| 14.Overlapping sets | For two sets A&B if $A\cap B\neq\phi$ but neither $A\subseteq B$ nor $B\subseteq A$. | Let A={1,2}& B={2,3} So A&B are overlapping sets. |
| 15.Cells | If $A\cup B=U$ and $A\cap B=\phi$ then A&B are cells. | Let U={1,2,3} A={1,2},B={3}. |
| 16.Exhaustive sets | If $A\cup B=U$ and $A\cap B\neq\phi$ Then A&B are exhaustive sets. | Let U={1,2,3} A={1,2},B={2,3}. |

| | |
|---|---|
| De Morgan's laws | $(i)(A\cup B)'=A'\cap B'$<br>$(ii)(A\cap B)'=A'\cup B'$ |

## SOME PROPERTIES OF THE SETS.

$A\cup B=B\cup A$ (Commutative property for the union of two sets.)

$A\cap B=B\cap A$ (Commutative property for the intersection of two sets.)

$(A\cup B)\cup C=A\cup(B\cup C)$ (Associative property of union.)

$(A\cap B)\cap C=A\cap(B\cap C)$ (Associative property of intersection.)

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ (Distributive property of union over intersection.)

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$ (Distributive property of intersection over union.)

## EXAMPLE FROM THE TEXT BOOK.

**1.Verify De Morgan's laws when A={1,2},B={2,3} and U={1,2,3,4}.**

**Solution: (a)**

$(A\cup B)=\{1,2,3\}$ *So* $(A\cup B)'=\{4\}$.*Now* $A'=\{3,4\}$ *and* $B'=\{1,4\}$.*So* $A'\cap B'=\{4\}$.

*Hence* $(A\cup B)'=A'\cap B'$. Now $(A\cap B)'=A'\cup B'$. $(A\cap B)=\{2\}$ *So* $(A\cap B)'=\{1,3,4\}$.

*Now* $A'=\{3,4\}$ *and* $B'=\{1,4\}$.*So* $A'\cup B'=\{1,3,4\}$. *Hence* $(A\cap B)'=A'\cup B'$.

# Exercise No. = 1.1

**1.Distinguish between the sets $\phi, \{0\}$ and $\{\phi\}$.**

Solution=The set $\phi$ contains no element so it is the null set.

The set {0} contains one element, the number 0.

The set $\{\phi\}$ contains one element $\phi$, the null set.

**2.Decide which sets are proper subsets of the others.**

(i)T={t|t is a rectangle}(ii)P={p|p is a parallelogram}

(iii)S={s|s is a square} (iv)X={$x/x$ is a quadrilateral}

Solution.(i)Since every rectangle is a quadrilateral :. $T \subset X$ or,T is a proper subset of X.

(ii)Since every parallelogram is a quadrilateral.: $P \subset X$, or P is a proper subset of X.

(iii)Since every square is a rectangle,:. $S \subset T$ or S is a proper subset of T.

It is also a parallelogram.So, $S \subset P$ or S is a proper subset of P.

It is also a quadrilateral So, $S \subset X$, or S is a proper subset of X.

(iv)Since every rectangle is also a parallelogram :. $T \subset P$.

Hence $T \subset P, T \subset X, P \subset X, S \subset T, S \subset P$ and $S \subset X$.

**3.Write down all the subsets of $\{x,y,z,t\}$ and find the power set of $S=\{4,8,12\}$.**

All the subsets of $\{x,y,z,t\}$ are $\phi,\{x\},\{y\},\{z\},\{t\},\{x,y\},\{x,z\},\{x,t\},\{y,z\},\{y,t\},$

$\{z,t\},\{x,y,z\},\{x,y,t\},\{x,z,t\},\{y,z,t\},\{x,y,z,t\}$

The power set of $S=\{4,8,12\}$. is $[\{\phi,\{4\},\{8\},\{12\},\{4,8\},\{4,12\},\{8,12\},\{4,8,12$

**4.Show that N={1,2,3,....},E={2,4,6,....} and O={1,3,5,....} are equivalent sets.**

In sets N and E,the elements of N can be placed in one-to-one correspondence with E.Thus N ~ E.Similarly in set,E and O the element of E can be placed in one-to-one correspondence with O.Thus set E is equivalent to O or E~O ... E~O.





**5.Let S={1,2,10,20} and T={a,b,c,d}.Then show that S~T but S≠T.**

The elements of S and T are in one-to-one correspondence thus S~T.But the elements of S are not equal to the elements of T.Therefore S≠T.

**6.Which of the following sets are null sets?**

(i)A={$x/x^2=16$ and $2x=4$} (ii)B={$x/x+6=6$}.

(i)As there is no number such that its square is 16 and twice the number is 4,so A=∅ (ii)If B={$x/x+6=6$} then B={0} So B is not a null set.

**7.If A={a,b,d} ,B={b,c,d},$\cup=\{a,b,c,d,e\}$ .Find**

(i) $A'$=? $A'=\cup-A=\{a,b,c,d,e\}-\{a,b,d\}=\{c,e\}$

(ii) $B'=\cup-B=\{a,b,c,d,e\}-\{b,c,d\}=\{a,e\}$

(iii) $(A')'=\cup-A'=\{a,b,c,d,e\}-\{c,e\}=\{a,b,d\}$ So $(A')'=A$

(iv) $(A\cup B)'=?$ $(A\cup B)=\{a,b,d\}\cup\{b,c,d\}=\{a,b,c,d\}$,
$(A\cup B)'=\cup-(A\cup B)=\{a,b,c,d,e\}-\{a,b,c,d\}=\{e\}$

(v) $(A\cap A')'=?$ $(A\cap A')=\{a,b,d\}\cap\{c,e\}=\phi$,
$(A\cap A')'=U-(A\cap A')=\{a,b,c,d,e\}-\phi=\{a,b,c,d,e\}$

(vi) $B'-A'=\{a,c\}-\{c,e\}=\{a\}$

**8. Let U be the set of all students of an intermediate science college. (Pre – Medical & Pre - Enginnering)**

**A the set of all students of first year class.**

**B the set of all students of second year pre – medical class.**

**C the set of all students of pre – medical class.**

**D the set of all students who play cricket.**

**E the set of all students who put on glasses.**

**Find:**

(i)Set of all students of Pre – Engineering class.

(ii)Set of all students of first year pre – medical class.

(iii)Set of all students of first year pre – engineering class.

(iv)Set of all students of second year pre – engineering class who play cricket but do not wear glasses.



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Solution:

| (i)U-C | (ii)C-B | (iii)A-(C-B) | (iv)$(C'-P)\cap(D\cap E')$ where $P=A-(C-B)$ |
|---|---|---|---|

**9.If** $U=\{x/x\in\mathbb{Z},-1<x<5\}, A=\{y/y\in W,1\le y\le 3\}$ and $B=\{z/z\in\mathbb{N},2<z\le 4\}$

**Then verify (i)De Morgan's Laws (ii)** $B-A=B\cap A'$.

**Solution:-** We have U = {0,1,2,3,4}, A = {1,2,3}, B = {3,4}

| | |
|---|---|
| $(a)(A\cup B)'=A'\cap B'$ | $(b)(A\cap B)'=A'\cup B'$ |
| *Taking L.H.S* | **Taking L.H.S** |
| $A\cup B=\{1,2,3,4\}$ | $A\cap B=\{1,2,3\}\cap\{3,4\}, \quad A\cap B=\{3\}$ |
| $(A\cup B)'=U-(A\cup B)$ | $(A\cap B)'=U-(A\cap B)$ |
| $(A\cup B)'=\{0,1,2,3,4\}-\{1,2,3,4\}$ | $(A\cap B)'=\{0,1,2,3,4\}-\{3\}$ |
| $(A\cup B)'=\{0\}$ | $(A\cap B)'=\{0,1,2,4\}$ |
| **Now Taking R.H.S** | **Now taking R.H.S** |
| $A'=U-A=\{0,1,2,3,4\}-\{1,2,3\}, A'=\{0,4\}$ | $R.H.S=A'\cup B'$ |
| $B'=U-B=\{0,1,2,3,4\}-\{3,4\}, B'=\{0,1,2\}$ | $A'\cup B'=\{0,4\}\cup\{0,1,2\}$ |
| *Now* $A'\cap B'=\{0\}$ | $A'\cup B'=\{0,1,2,4\}$ |
| **Hence L.H.S = R.H.S {0} = {0}** | **Hence L.H.S = R.H.S . Verified** |

**11.(i)** $A\cup\phi=A$ **(ii)** $A\cap\phi=\phi$ **(iii)** $A\cap A=A$ **(iv)** $A\cup U=U$ **(v)** $\phi\cap\phi=\phi$ **(vi)** $A\cup A=A$

**(vii)** $(A')'=A$ **(viii)** $A\cap A'=\phi$ **(ix)** $U'=\phi$

**10.Let** $B=\{2,3\},C=\{x,y,z\}$ **.Find** $B\times C, C\times B, B\times B$ **and show that** $B\times C\neq C\times B$.

$B\times C=\{2,3\}\times\{x,y,z\}=B\times C=\{(2,x),(2,y),(2,z),(3,x),(3,y),(3,z)\}$ *and*

$C\times B=\{(x,2),(x,3),(y,2),(y,3),(z,2),(z,3)\}$ *Since* $(2,x)\neq(x,2)$. *No ordered pair of* $B\times C$ *is equal to any ordered pair of* $C\times B$ *So* $B\times C\neq C\times B$.

$B\times B=\{2,3\}\times\{2,3\}=\{(2,2),(2,3),(3,2),(3,3)\}$.

**12. Determine the conditions under which**

$(i)A\cup B=A\ (ii)A\cup B=B\ (iii)A'\cap U=U\ \ (iv)A\cap B=A\ \ (v)B'\subseteq A'$.

**(i)If** $A\cup B=A$, **it implies that** $B\subseteq A$ **i.e B is a proper subset of A.**





**(ii)If** $A\cup B=B$, **it implies that** $A\subseteq B$ **i.e A is a proper subset of B.**

**(iii)If** $A'\cap U=U$, **it implies that** $A'=U$ **and if** $A'=U$, **then** $A=\phi$.

**(iv)If** $A\cap B=A$, **it implies that** $A\subseteq B$.

**(v)If** $B'\subseteq A'$ **then** $(U-B)\subseteq(U-A)$ **or** $A\subseteq B$ **i.e A is a proper subset of B.**

**13.Prove that** $(i)A\cup B=A\Rightarrow A'\cap B'=A'\ \ (ii)A\cap B=A\Rightarrow A'\cup B'=A'$

**(i)By De Morgan's law** $A'\cap B'=(A\cup B)'$ **but** $A\cup B=A$ **So if** $A\cup B=A$ **then** $A'\cap B'=A'$

**(ii) By De Morgan's law** $A'\cup B'=(A\cap B)'$ **but** $A\cap B=A$ **So if** $A\cap B=A$ **then** $A'\cup B'=A'$.

**14.Let A={2,3},B={3,4},C={c,f} and U={2,3,4,c,f}. Find**

$(i)A\times(B\cup C)=\{2,3\}\times[\{3,4\}\cup\{e,f\}]=\{2,3\}\times\{3,4,e,f\}$ **thus**

$A\times(B\cup C)=\{(2,3),(2,4),(2,e),(2,f),(3,3),(3,4),(3,e),(3,f)\}$

$(ii)A\times(B\cap C)=\{3,4\}\times[\{3,4\}\cap\{e,f\}]=\{3,4\}\times\phi$ **thus** $A\times(B\cap C)=\phi$.

$(iii)(A\times B)\cup(A\times C)=\{2,3\}\times\{3,4\}\cup\{2,3\}\times\{e,f\}$
$=\{(2,3),(2,4),(3,3),(3,4)\}\cup\{(2,e),(2,f),(3,e),(3,f)\}$ **thus**
$(A\times B)\cup(A\times C)=\{(2,3),(2,4),(3,3),(3,4),(2,e),(2,f),(3,e),(3,f)\}$

$(iv)(A\times B)\cap(A\times C)=\{(2,3),(2,4),(3,3),(3,4)\}\cap\{(2,e),(2,f),(3,e),(3,f)\}$ **thus**

$(A\times B)\cap(A\times C)=\phi$

**15.For the sets A={2,3},B={3,4},C={4,5} and U={2,3,4,5} , verify that**

$(i)(A-B)\times C=(A\times C)-(B\times C)$

$L.H.S=(A-B)\times C=[\{2,3\}-\{3,4\}]\times\{4,5\}=\{2\}\times\{4,5\}=\{(2,4),(2,5)\}$

$R.H.S=(A\times C)-(B\times C)=[\{2,3\}\times\{4,5\}]-[\{3,4\}\times\{4,5\}]$
$=[(2,4),(2,5),(3,4),(3,5)]-[(3,4),(3,5),(4,4),(4,5)]$ **thus** $(A\times C)-(B\times C)=\{(2,4),(2,5)\}$



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(ii) $C\times(A-B)=(C\times A)-(C\times B)$

$L.H.S=C\times(A-B)=\{4,5\}\times\{2\}=\{(4,2),(5,2)\}\quad\therefore(A-B)=\{2\}$

$R.H.S=(C\times A)-(C\times B)=[\{4,5\}\times\{2,3\}]-[\{4,5\}\times\{3,4\}]$

$=\{(4,2),(4,3),(5,2),(5,3)\}-\{(4,3),(4,4),(5,3),(5,4)\}=\{(4,2),(5,2)\}=L.H.S$

**Thus** $C\times(A-B)=(C\times A)-(C\times B)$

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(iii) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

$L.H.S=A\times(B\cap C)=\{2,3\}\times[\{3,4\}\cap\{4,5\}]$ **thus** $A\times(B\cap C)=\{2,3\}\times\{4\}=\{(2,4),(3,4)\}$

$R.H.S=(A\times B)\cap(A\times C)=[\{2,3\}\times\{3,4\}]\cap[\{2,3\}\times\{4,5\}]$
$=\{(2,3),(2,4),(3,3),(3,4)\}\cap\{(2,4),(2,5),(3,4),(3,5)\}=\{(2,4),(3,4)\}=L.H.S$

**Thus** $A\times(B\cap C)=(A\times B)\cap(A\times C)$.

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(iv) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

$L.H.S=A\times(B\cup C)=\{2,3\}\times[\{3,4\}\cup\{4,5\}]=\{2,3\}\times\{3,4,5\}$

**So** $A\times(B\cup C)=\{(2,3),(2,4),(2,5),(3,3),(3,4),(3,5)\}$

$R.H.S=(A\times B)\cup(A\times C)=\{(2,3),(2,4),(3,3),(3,4)\}\cup\{(2,4),(2,5),(3,4),(3,5)\}$

$=\{(2,3),(2,4),(2,5),(3,3),(3,4),(3,5)\}=L.H.S$ **Hence** $A\times(B\cup C)=(A\times B)\cup(A\times C)$

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(v) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

$L.H.S=A\cup(B\cap C)=\{2,3\}\cup[\{3,4\}\cap\{4,5\}]=\{2,3\}\cup\{4\}=\{2,3,4\}$
$R.H.S=(A\cup B)\cap(A\cup C)=[\{2,3\}\cup\{3,4\}]\cap[\{2,3\}\cup\{4,5\}]$
$=\{2,3,4\}\cap\{2,3,4,5\}=\{2,3,4\}=L.H.S$ **thus** $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

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(vi) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

$L.H.S=A\cap(B\cup C)=\{2,3\}\cap[\{3,4\}\cup\{4,5\}]=\{2,3\}\cap\{3,4,5\}=\{3\}$

$R.H.S=(A\cap B)\cup(A\cap C)=[\{2,3\}\cap\{3,4\}]\cup[\{2,3\}\cap\{4,5\}]=\{3\}\cup\phi=\{3\}$

**Thus** $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

---

(vii) $(A-B)\cap B=\phi$

$=[\{2,3\}-\{3,4\}]\cap\{3,4\}=\{2\}\cap\{3,4\}=\phi$ **thus** $(A-B)\cap B=\phi$





(viii) $A-B=A\cap B'$ , $L.H.S=A-B=\{2,3\}-\{3,4\}=\{2\}$ ,

$R.H.S=A\cap B'=A\cap(U-B)=\{2,3\}\cap[\{2,3,4,5\}-\{3,4\}]=\{2,3\}\cap\{2,5\}=\{2\}$

**thus** $A-B=A\cap B'$

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(ix) $A\cap B\subset A\subset A\cup B$

$A\cap B=\{2,3\}\cap\{3,4\}=\{3\}\subset\{2,3\}$ **i.e** $A\cap B\subset A$

**And** $A\cup B=\{2,3\}\cup\{3,4\}=\{2,3,4\}\supset\{2,3\}$ **thus** $A\cup B\supset A$ **or** $A\subset A\cup B$

**Hence** $A\cap B\subset A\subset A\cup B$

---

**16. If A and B be subsets of a set U, then prove that**

(i) $A\cup B=A\cup(A'\cap B)$.

**Taking R.H.S** $=A\cup(A'\cap B)=(A\cup A')\cap(A\cup B)\quad\therefore A\cup A'=U$

**So** $=U\cap(A\cup B)=A\cup B=L.H.S$

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(ii) $B=(A\cap B)\cup(A'\cap B)$

$R.H.S=(A\cap B)\cup(A'\cap B)=B\cap(A\cup A')\quad\therefore A\cup A'=U$ **So** $B\cap U=B=L.H.S$



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