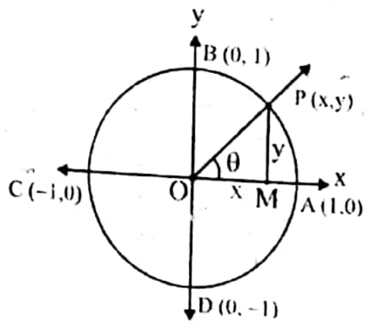
CHAPTER 11 Graphs of Trigonometric Functions

Related Definitions and Derivations of Formulae

Introduction: Let us first domains and ranges of trigonometric functions before drawing their graphs.

Domains and Ranges of Sine and Cosine Functions: We have already defined trigonometric functions $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$ and $\cot\theta$. We know that if P (x, y) is any point on unit circle with centre at the origin O such that $\angle XOP = \theta$ is standard position, then



$$\cos\theta = x \quad \text{and} \quad \sin\theta = y$$

$\Rightarrow$ for any real number θ there is one and only one value of each x and y i.e. of each $\cos\theta$ and $\sin\theta$

Hence $\sin\theta$ and $\cos\theta$ are the functions of θ and their domain is R, a set of real numbers.

Since P (x, y) is a point on the unit circle with centre at the origin O.

$\therefore \quad -1 \le x \le 1 \quad$ and $\quad -1 \le y \le 1$

$\Rightarrow \quad -1 \le \cos\theta \le 1 \quad$ and $\quad -1 \le \sin\theta \le 1$

Thus the range of both the sine and cosine function is $[-1, 1]$

Domains and Ranges of Tangent and Cotangent Functions:

Above Figure:

(i) $\tan\theta = \dfrac{y}{x}, \qquad x \neq 0$

$\Rightarrow$ terminal side $\overrightarrow{OP}$ should not coincide with OY or OY′. (i.e., Y–axis)

$\Rightarrow \theta \neq \pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}, \pm\dfrac{5\pi}{2}, \ldots\ldots$

$\Rightarrow \theta \neq (2n+1)\dfrac{\pi}{2}$, where $n \in Z$

$\therefore$ domain of tangent function $= R - \{ x \mid x = (2n+1)\frac{\pi}{2}, n \in Z\}$ and Range. of tangent function $= R =$ set of real numbers.

(ii) From Above figure

$\cot\theta = \frac{x}{y}$, $y \neq 0$

$\Rightarrow$ terminal side $\overrightarrow{OP}$ should not coincide with OX or OX' (i.e., X–axis)

$\Rightarrow$ $\theta \neq 0, \pm\pi, \pm 2\pi, \ldots$

$\Rightarrow$ $\theta \neq n\pi$, where $n \in Z$

$\therefore$ Domain of cotangent function $= R - \{ x \mid x = n\pi, n \in Z\}$

and Range of cotangent function = R = set of real numbers.

Domain and Range of Secant Function:

From Above Figure

$\sec\theta = \frac{1}{x}$, $x \neq 0$

$\Rightarrow$ terminal side $\overrightarrow{OP}$ should not coincide with OY or OY' (i.e. Y–axis)

$\Rightarrow$ $\theta \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots\ldots\ldots\ldots$

$\Rightarrow$ $\theta \neq (2n+1)\frac{\pi}{2}$, where $n \in Z$

$\therefore$ Domain of secant function $= R - \{ x \mid x = (2n+1)\frac{\pi}{2}, n \in Z\}$

As $\sec\theta$ attain all real values except those between -1 and 1

$\therefore$ Range of Secant function $= R - \{ x \mid -1 < x < 1 \}$

Domain and Range of Cosecant Function:

From Above Figure.

$\csc\theta = \frac{1}{y}$; $y \neq 0$

$\Rightarrow$ terminal side $\overrightarrow{OP}$ should not coincide with OX or OX' (i.e., X–axis)

$\Rightarrow$ $\theta \neq 0 \pm \pi, +2, \ldots\ldots\ldots$ where $n \in Z$

$\therefore$ Domain of cosecant function $= R - \{ x \mid x = n\pi, n \in Z\}$

As $\csc\theta$ attains all values except those between -1 and 1

$\therefore$ Range of cosecant function $= R - \{ x \mid -1 < x < 1 \}$

The following table summarizes the domains and ranges of the trigonometric functions.

Function	Domain	Range
$y = \sin x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$
$y = \cos x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$
$y = \tan x$	$-\infty < x < +\infty,\ x \neq \frac{(2n+1)\pi}{2}$, $n \in Z$	$-\infty < y < +\infty$
$y = \cot x$	$-\infty < x < +\infty,\ x \neq n\pi$, $n \in Z$	$-\infty < y < +\infty$
$y = \sec x$	$-\infty < x < +\infty,\ x \neq \frac{(2n+1)\pi}{2}$, $n \in Z$	$y \geq 1$ or $y \leq -1$
$y = \operatorname{cosec} x$	$-\infty < x < +\infty,\ x \neq n\pi$, $n \in Z$	$y \geq 1$ or $y \leq -1$

Period of Trigonometric Functions:

All the six trigonometric functions repeat their values for each increase or decrease of 2π in θ i – e, the value of trigonometric functions for θ and $\theta \pm 2n\pi$ where $\theta \in R$ and $n \in Z$, are the same this behaviour of trigonometric functions is called **periodicity.**

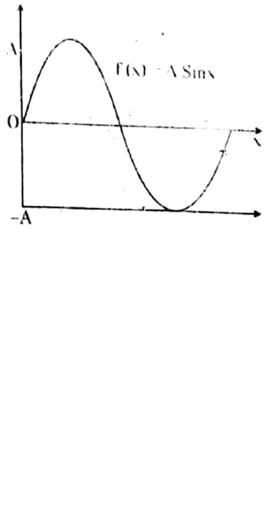
Period of a trigonometric function is the smallest +ve number which, when added to the original circular measure of the angle, gives the same value of the function.

A function f(x) is said to be the periodic function if there exists a smallest positive number P such that f (x + p) = f (x) for all x in the domain of f, and P is said to be the period of f.

S.No	Function	Period	S.No.	Function	Period
1	Sinx	2π	1	Sinbx	$2\pi/\|b\|$
2	Cosx	2π	2	Cosbx	$2\pi/\|b\|$
3	tanx	π	3	tanbx	$\pi/\|b\|$

Period of Sinbx, Cosbx or tanbx is "$\frac{1}{b}$" times the period of Sinx, Cosx or tanx.

Amplitude of Sinx and Cosx:



The graphs of f(x) = Asinx. The number |A| is the amplitude.

Theorem: Sine is a periodic function and its period is 2π.

Proof: Suppose P is the period of sine function such that

$\text{Sin}(\theta + P) = \text{Sin}\theta$ ———— (1) $\forall\, \theta \in R$

Now put $\theta = 0$ we have

$\text{Sin}(0 + P) = \text{Sin}0$

$\Rightarrow \quad \text{Sin}P = 0$

$\Rightarrow \quad P = \text{Sin}^{-1}(0)$

$\Rightarrow \quad P = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots\ldots\ldots$

(i) If $P = \pi$ then from (1)

$\text{Sin}(\theta + \pi) = \text{Sin}\theta$ (not true)

$\boxed{\therefore \text{Sin}(\theta + \pi) = -\text{Sin}\theta}$

$\therefore$ π is not the period of $\text{Sin}\theta$.

(ii) If $P = 2\pi$ then from (1)

$\text{Sin}(\theta + 2\pi) = \text{Sin}\theta$ which is true

As 2π is the smallest + ve real number for which

$\text{Sin}(\theta + 2\pi) = \text{Sin}\theta$

$\therefore$ 2π is the period of $\text{Sin}\theta$

Theorem: Tangent is a periodic function and its period is π.

Proof: Suppose P is the period of tangent function such that

$\tan(\theta + P) = \tan\theta$ ———— (1) $\forall\, \theta \in R$

Now put $\theta = 0$ we have

$\tan(0 + P) = \tan 0$

$\Rightarrow \quad \tan P = 0$

$\Rightarrow \quad P = \tan^{-1}(0)$

$\therefore \quad P = 0, \pi, 2\pi, 3\pi, \ldots\ldots.$

(i) If $P = \pi$ then from (1)

$\tan(\theta + \pi) = \tan\theta$ which is true

As π is the smallest +ve number for which $\tan(\theta + \pi) = \tan\theta$

$\therefore$ π is the period of $\tan\theta$

Example: Find the periods of (i) Sin2x (ii) $\tan\frac{x}{3}$

Solution: (i) We know that the period of Sine is 2π.

$f(x + p) = f(x)$

$\therefore \quad \text{Sin}(2x + 2\pi) = \text{Sin}2x$

$\Rightarrow \quad \text{Sin}2(x + \pi) = \text{Sin}2x$

It means that the value of Sin2x repeats when x is increased by π.

Hence π is the period of Sin2x.

(ii) We know that the period of tangent is π

$\therefore \quad \tan\left(\frac{x}{3} + \pi\right) = \tan\frac{x}{3}$

$\Rightarrow \quad \tan\frac{1}{3}(x + 3\pi) = \tan\frac{x}{3}$

it means that the value of $\tan\frac{x}{3}$ repeats when x is increased by 3π.

Hence the period of $\tan\frac{x}{3}$ is 3π.

Variations of the trigonometric functions in the four quadrants:

Variations of the trigonometric functions in the four quadrants are also depicted by their graphs. They are summarized in the following table.

	I = Increase		D = Decrease	
Quad	**1st**	**2nd**	**3rd**	**4th**
Sinx	I : 0 to 1	D : 1 to − 1	D : 0 to − 1	I: −1 to 0
Cosx	D : 1 to 0	D : 0 to − 1	I : −1 to 0	I: 0 to 1
tanx	I : 0 to ∞	I : − ∞ to 0	I : 0 to ∞	I: −∞ to 0
Cotx	D : ∞ to 0	D : ∞ to 0	D : ∞ to 0	D: 0 to −∞
Secx	I : 1 to ∞	I : −∞ to −1	D : −1 to −∞	D: ∞ to 1
Cosecx	D : ∞ to 1	I : 1 to ∞	I: −∞ to −1	D: 1 to −∞

Graphs of Trigonometric functions: We shall now learn the method of drawing the graphs of all the Six trigonometric functions.

The following procedure is adopted to draw the graphs of the trigonometric functions.

(i) table of ordered pairs (x, y) is constructed, when x is the measure of the angle and y is the value of the trigonometric ratio for the angle of measure x.

(ii) The measures of the angles are taken along the x−axis.

(iii) The values of the trigonometric functions are taken along the y−axis.

(iv) The points corresponding to the ordered pairs are plotted on the graph paper.

(v) These points are Joined with the help of Smooth Curves.

Note: As we shall see that the graphs of trigonometric functions will be smooth curves and none of them will be line segments or will have sharp corners or breaks within their domains. This behaviour of the curve is called **continuity**. It means that the trigonometric functions are continuous, wherever they are defined. Moreover, as the trigonometric functions are periodic so their curves repeat after a fixed intervals.

Graph of y = sinx from -2π to 2π

We know that the period of sine function is 2π so, we will first draw the graph for the interval from 0° to 360° i.e. from 0 to 2π.

To graph the sine function, first recall that $-1 \leq \sin x \leq 1$ for all $x \in \mathbb{R}$.

i.e., the range of the sine function is $[-1, 1]$, so the graph will be between the horizontal lines $y = +1$ and $y = -1$.

The table of the ordered pairs satisfying $y = \sin x$ is as follows:

x	0 or $0°$	$\frac{\pi}{6}$ or $30°$	$\frac{\pi}{3}$ or $60°$	$\frac{\pi}{2}$ or $90°$	$\frac{2\pi}{3}$ or $120°$	$\frac{5\pi}{6}$ or $150°$	π or $180°$	$\frac{7\pi}{6}$ or $210°$	$\frac{4\pi}{3}$ or $240°$	$\frac{3\pi}{2}$ or $270°$	$\frac{5\pi}{3}$ or $300°$	$\frac{11\pi}{6}$ or $330°$	2π or $360°$
Sin x	0	0.5	0.87	1	0.87	0.5	0	–0.5	–0.87	–1	–0.87	–0.5	0

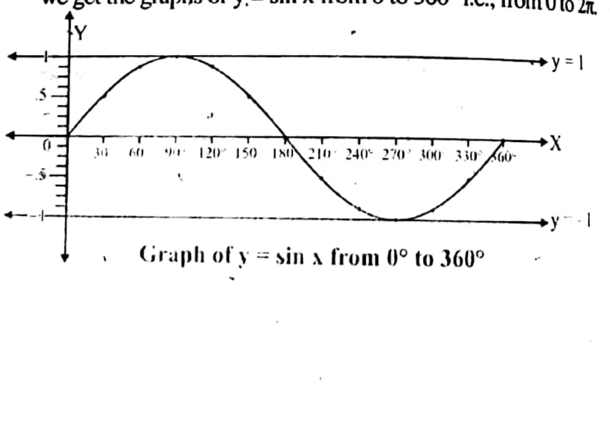
To draw the graphs:

(i) Take a convenient scale $\begin{cases} \text{1 side of small square on the x–axis} = 10° \\ \text{1 side of big square on the y–axis} = 1 \text{ unit} \end{cases}$

(ii) Draw the coordinate axes.

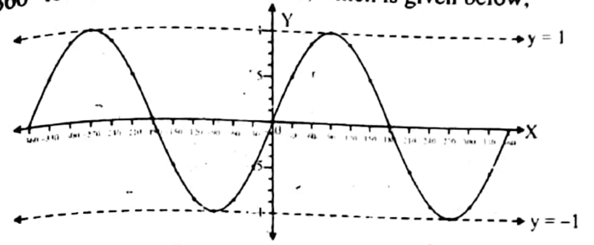
(iii) Plot the points corresponding to the ordered pairs in the table above i.e., (0,0), (30°, 0.5), (60°, 0.87) and so on.

(iv) Join the points with the help of a smooth curve as shown so we get the graphs of y = sin x from 0 to 360° i.e., from 0 to 2π.



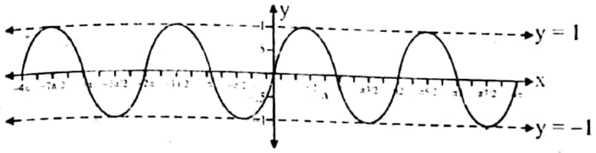
Graph of y = sin x from 0° to 360°

In a similar way, we can draw the graph for the interval from 0° to –360°. This will complete the graph of y = sin x from –360° to 360° i.e. from -2π to 2π, which is given below;



Graph of y = sinx from –360° to 360°

The graph in the interval $[0, 2\pi]$ is called a cycle. Since the period of sine function is 2π, so the sine graph can be extended on both sides of x–axis through every interval of 2π (360°) as shown below;



Graph of y = cos x from -2π to 2π:

We know that the period of cosine function is 2π so, we will first draw the graph for the interval from 0° to 360° i.e., from 0 to 2π.

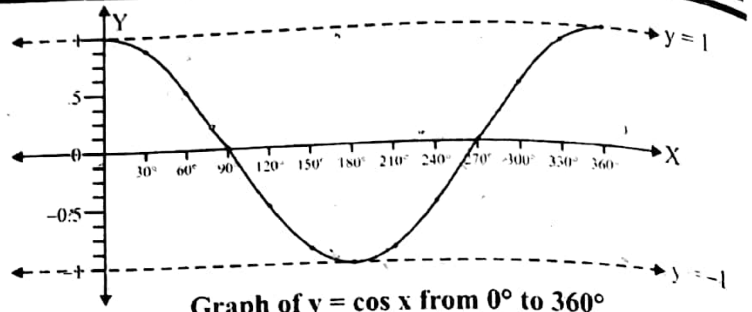
To graph the cosine function, first, recall that $-1 \leq \text{Sin } x \leq 1$ for all $x \in \mathbb{R}$

i.e., the range of the cosine function is $[-1,1]$, so the graph will be between the horizontal lines $y = +1$ and $y = -1$

The table or the ordered pairs satisfying $y = \cos x$ is as follows;

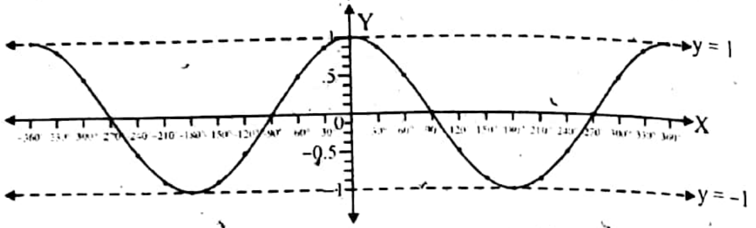
x	0 or $0°$	$\frac{\pi}{6}$ or $30°$	$\frac{\pi}{3}$ or $60°$	$\frac{\pi}{2}$ or $90°$	$\frac{2\pi}{3}$ or $120°$	$\frac{5\pi}{6}$ or $150°$	π or $180°$	$\frac{7\pi}{6}$ or $210°$	$\frac{4\pi}{3}$ or $240°$	$\frac{3\pi}{2}$ or $270°$	$\frac{5\pi}{3}$ or $300°$	$\frac{11\pi}{6}$ or $330°$	2π or $360°$
Cos x	1	0.87	0.5	0	–0.5	–0.87	–1	–0.87	–0.5	0	0.5	0.87	1

The graphs of y = cos x from 0° to 360° is given below:



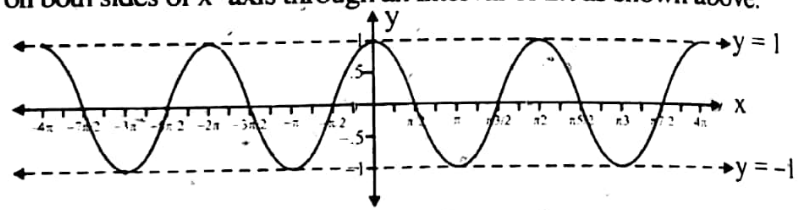
Graph of y = cos x from 0° to 360°

In a similar way, we can draw the graph for the interval from 0° to −360°. This will complete the graph of y = cos x from −360° to 360° i.e. from −2π to 2π, which is given below:



Graph of y = cos x from −360° to 360°

As in the case of sine graph, the cosine graph is also extended on both sides of x–axis through an interval of 2π as shown above:



Graph of y = sin x from - 4 π to 4 π

<u>Graph of y = tan x from − π to π:</u>

We know that tan (−x) = − tan x and tan (π − x) = −tan x, so the values of tan x for x = 0°, 30°, 45°, 60° can help us in making the table.

Also we know that tan x is undefined at x = ±90°, when

(i) x approaches $\frac{\pi}{2}$ from left i.e., $x \to \frac{\pi}{2} - 0$, tanx increases indefinitely in I Quard.

(ii) x approaches $\frac{\pi}{2}$ from right i.e., $x \to \frac{\pi}{2} + 0$, tanx increases indefinitely in IV Quard.

(iii) x approaches $-\frac{\pi}{2}$ from left i.e., $x \to -\frac{\pi}{2} - 0$, tanx increases indefinitely in II Quard.

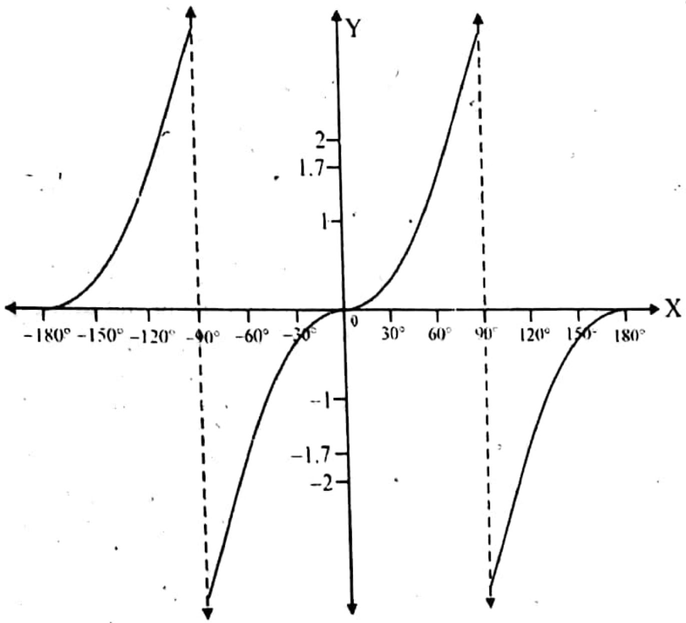
(iv) x approaches $-\frac{\pi}{2}$ from right i.e. $x \to -\frac{\pi}{2} + 0$, tan x increases indefinitely in III Quard.

We know that the period of tangent is π, so we shall first draw the graph for the interval from −π to π i.e., from −180° to 180°.

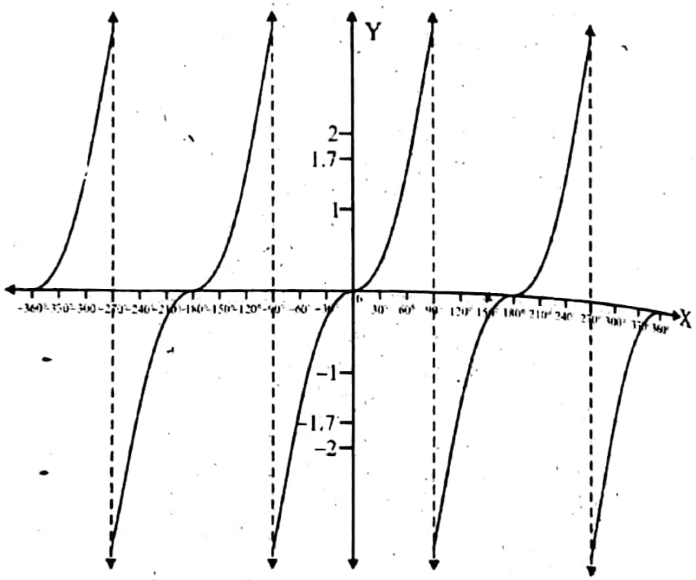
∴ The table of ordered pairs satisfying y = tanx is given below:

x	$-\pi$ or −180°	$-\frac{5\pi}{6}$ or −150°	$-\frac{2\pi}{3}$ or −120°	$-\frac{\pi}{2}-0$ or −90°−0	$-\frac{\pi}{2}+0$ or −90 + 0	$-\frac{\pi}{3}$ or −60°	$-\frac{\pi}{6}$ or −30°	0 or 0	$\frac{\pi}{6}$ or 30°	$\frac{\pi}{3}$ or 60°	$\frac{\pi}{2}-0$ or 90° − 0	$\frac{\pi}{2}+0$ or 90° + 0	$\frac{2\pi}{3}$ or 120°	$\frac{5\pi}{6}$ or 150°	π or 180°
Tan x	0	0.58	1.73	+∞	−∞	−1.73	−0.58	0	0.58	1.73	+∞	−∞	−1.73	−0.58	0

Graph of y = tan x from −180° to 180°



We know that the period of the tangent function is π. The graphs is extended on both sides of x–axis through an interval of π in the same pattern and so we obtain the graph of $y = \tan x$ from $-360°$ to $360°$ as shown below:



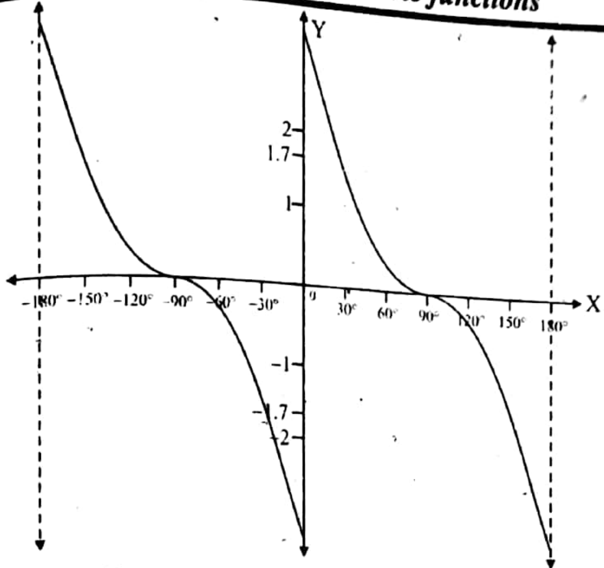
Graph of $y = \tan x$ from $-360°$ to $360°$

Graph of y = cotx From -2π to π:

We know that $\cot(-x) = -\cot x$ and $\cot(\pi - x) = -\cot x$, so the values of cot x for $x = 0°, 30°, 45°, 60°, 90°$ can help us in making the table.

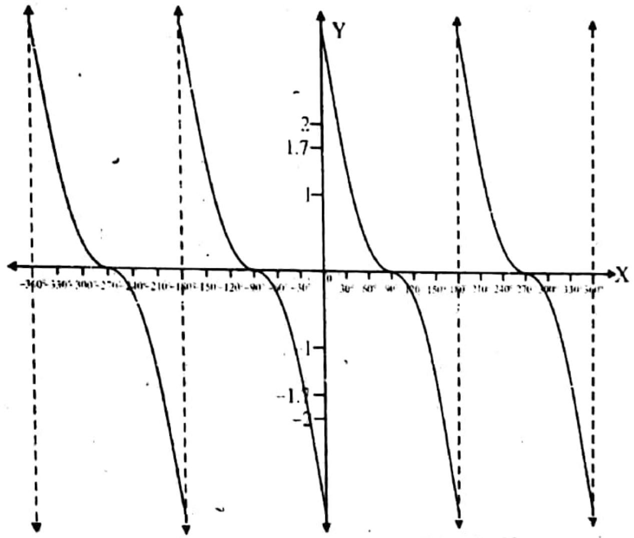
The period of the cotangent function is also π. So its graph is drawn in a similar way of tangent graph using the table given below for the interval from $-180°$ to $180°$.

x	$-\pi$ or $-180°$	$-\frac{5\pi}{6}$ or $-150°$	$-\frac{2\pi}{3}$ or $-120°$	$-\frac{\pi}{2}-0$ or $-90°-0$	$-\frac{\pi}{2}+0$ or $-90+0$	$-\frac{\pi}{3}$ or $-60°$	$-\frac{\pi}{6}$ or $-30°$	0 or	$\frac{\pi}{6}$ or $30°$	$\frac{\pi}{3}$ or $60°$	$\frac{\pi}{2}-0$ or $90°-0$	$\frac{\pi}{2}+0$ or $90°+0$	$\frac{2\pi}{3}$ or $120°$	$\frac{5\pi}{6}$ or $150°$	π or $180°$
Cot x	$\pm\infty$	1.73	0.58	$+\infty$	$-\infty$	-0.58	-1.73	$+\infty$	1.73	0.58	$+\infty$	$-\infty$	-0.58	-1.73	$\pm\infty$



Graph of $y = \cot x$ from $-180°$ to $180°$

We know that the period of the cotangent function is π. The graph is extended on both sides of x–axis through an interval of π in the same pattern and so we obtain the graph of $y = \cot x$ from $-360°$ to $360°$ as shown below.



Graph of $y = \text{Cot } x$ from $-360°$ to $360°$

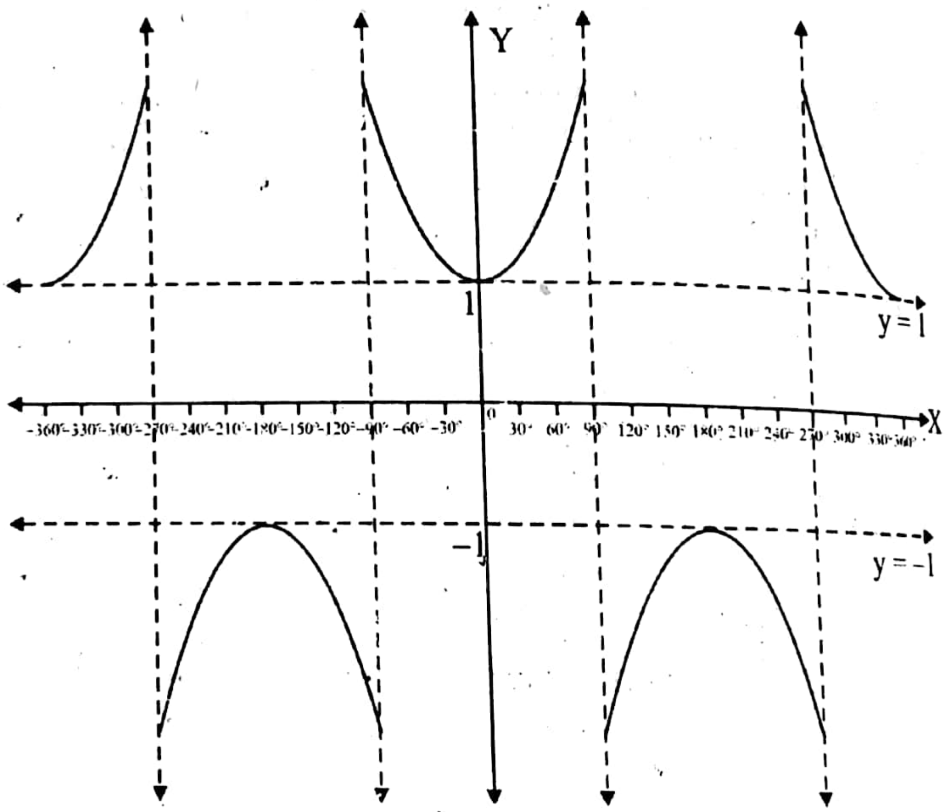
Graph of y = secπ from −2π to 2π:

We know that: sec (−x) = secx and sec(π − x) = −sex x,

So the values of sec x for x = 0°, 30°, 45°, 60°, can help us in making the following table of the ordered pairs for drawing the graph of y = secx for the interval 0° to 360°:

x	0 or 0	$\frac{\pi}{6}$ or 30°	$\frac{\pi}{3}$ or 60°	$\frac{\pi}{2}-0$ or $90-0$	$\frac{\pi}{2}+0$ or $90+0$	$\frac{2\pi}{8}$ or 120°	$\frac{5\pi}{6}$ or 150°	π or 180°	$\frac{7\pi}{6}$ or 210°	$\frac{4\pi}{3}$ or 240°	$\frac{3\pi}{2}-0$ or $270-0$	$\frac{3\pi}{2}+0$ or $270+0$	$\frac{5\pi}{3}$ or 300°	$\frac{5\pi}{6}$ or 330°	2π or 360°
Sec x	1	1.15	2	∞	$-\infty$	−2	−1.15	−1	−1.15	−2	$-\infty$	$+\infty$	2	1.15	1

Since the period of sec x is also 2π, so we have the following graph of y = sec x from − 360° to 360° i.e., from −2π to 2π:



Graph of y = sec x from −360° to 360°

Graph of y = csc x from −2π to 2π

We know that: csc (−x) = −csc x and csc (π − x) = csc x

So the values of csc x from x = 0°, 30°, 45°, 60°, can help as in making the following table of the ordered pairs for drawing the graph of y = csc x for the interval 0° to 360°.

Exercise No.= 11.1

Q1.Show that $\tan\theta$ is a periodic function of π.

Let X and Y be the subsets of real numbers.A function $f: X \to Y$ is called a periodic function of period 'p' if $f(x+p)=f(x)$, for all X, and p is the smallest such positive number.

Now $\tan(\theta+\pi)=\dfrac{\tan\theta+\tan\pi}{1-\tan\theta\tan\pi}=\dfrac{\tan\theta+0}{1-\tan\theta(0)}=\dfrac{\tan\theta}{1-0}=\tan\theta$ **Hence $\tan\theta$ is a periodic function of π.**

Q2.Find the periods of the following functions.

(i) $Cosx+Sinx$

Let $f(x)=Cosx+Sinx$

$\therefore f(x+p)=Cos(x+p)+Sin(x+p)$

$=CosxCosp-SinxSinp+SinxCosp+CosxSinp$

$=CosxCosp+SinxCosp+CosxSinp-SinxSinp$

$=Cosp(Cosx+Sinx)+Sinp(Cosx-Sinx)$

If $p=2\pi$ **then**

$f(x+2\pi)=Cos2\pi(Cosx+Sinx)+Sin2\pi(Cosx-Sinx)$

But $Cos2\pi=1$ **and** $Sin2\pi=0$

$\therefore f(x+2\pi)=Cosx+Sinx=f(x)$

Hence period of function is 2π.

$(ii)\ \tan 3x$ Let $f(x)=\tan 3x$

$\therefore f(x+p)=\tan 3(x+p)=\tan(3x+3p)$

$=\dfrac{\tan 3x+\tan 3p}{1-\tan 3x.\tan 3p}$ $Let\ p=\dfrac{\pi}{3}$

$f(x+\dfrac{\pi}{3})=\dfrac{\tan 3x+\tan\left(3\times\dfrac{\pi}{3}\right)}{1-\tan 3x.\tan\left(3\times\dfrac{\pi}{3}\right)}$

$=\dfrac{\tan 3x+\tan\pi}{1-\tan 3x.\tan\pi}=\dfrac{\tan 3x+0}{1-\tan 3x\times 0}$

$\therefore f(x+\dfrac{\pi}{3})=\dfrac{\tan 3x}{1}=\tan 3x\ \therefore p=\dfrac{\pi}{3}$

Hence period of function is $\dfrac{\pi}{3}$.

Q3.What are the greatest and least value of $Sin(-\theta)$?

We know that $Sin(-\theta)=-Sin\theta$ and we also know that if $f(\theta)=Sin\theta$, then $-1\le Sin\theta\le 1$

$\therefore Sin\theta=\pm 1$

Hence $-Sin\theta=-(\pm 1)$ **Hence the greatest and least value of $Sin(-\theta)$ are +1 and -1.**

Q2(iii) $Sin\dfrac{x}{2}+Cot\dfrac{x}{4}$

$Let\ f(x)=Sin\dfrac{x}{2}+Cot\dfrac{x}{4}$

$\therefore f(x+p)=Sin\dfrac{x+p}{2}+Cot\dfrac{x+p}{4}$

$=Sin\left(\dfrac{x}{2}+\dfrac{p}{2}\right)+Cot\left(\dfrac{x}{4}+\dfrac{p}{4}\right)$

$=Sin\dfrac{x}{2}Cos\dfrac{p}{2}+Cos\dfrac{x}{2}\sin\dfrac{p}{2}+\dfrac{Cos\left(\dfrac{x}{4}+\dfrac{p}{4}\right)}{Sin\left(\dfrac{x}{4}+\dfrac{p}{4}\right)}$

$=Sin\dfrac{x}{2}Cos\dfrac{p}{2}+Cos\dfrac{x}{2}\sin\dfrac{p}{2}+\dfrac{Cos\dfrac{x}{4}Cos\dfrac{p}{4}-Sin\dfrac{x}{4}Sin\dfrac{p}{4}}{Sin\dfrac{x}{4}Cos\dfrac{p}{4}+Cos\dfrac{x}{4}\sin\dfrac{p}{4}}$

If p=4π then

$f(x+4\pi)=Sin\dfrac{x}{2}Cos\dfrac{p}{2}+Cos\dfrac{x}{2}\sin\dfrac{p}{2}+\dfrac{Cos\dfrac{x}{4}Cos\dfrac{p}{4}-Sin\dfrac{x}{4}Sin\dfrac{p}{4}}{Sin\dfrac{x}{4}Cos\dfrac{p}{4}+Cos\dfrac{x}{4}\sin\dfrac{p}{4}}$

$=Sin\dfrac{x}{2}Cos2\pi+Cos\dfrac{x}{2}\sin 2\pi+\dfrac{Cos\dfrac{x}{4}Cos\pi-Sin\dfrac{x}{4}Sin\pi}{Sin\dfrac{x}{4}Cos\pi+Cos\dfrac{x}{4}\sin\pi}$

$Since\ \ Cos2\pi=1.Sin2\pi=0, Cos\pi=-1, Sin\pi=0$

$=Sin\dfrac{x}{2}(1)+Cos\dfrac{x}{2}(0)+\dfrac{Cos\dfrac{x}{4}(-1)-Sin\dfrac{x}{4}(0)}{Sin\dfrac{x}{4}(-1)+Cos\dfrac{x}{4}(0)}$

$=Sin\dfrac{x}{2}+\left(\dfrac{-Cos\dfrac{x}{4}}{-Sin\dfrac{x}{4}}\right)$ **thus** $f(x+4\pi)=Sin\dfrac{x}{2}+Cot\dfrac{x}{2}$ **thus** $p=4\pi$ **Hence the period of function is 4π.**

Q2(iv) $\dfrac{1}{2}Sin2x$ $Let\ f(x)=\dfrac{1}{2}Sin2x$

$\therefore f(x+p)=\dfrac{1}{2}Sin2(x+p)=\dfrac{1}{2}Sin(2x+2p)$

$=\dfrac{1}{2}(Sin2xCos2p+Cos2xSin2p)$

put $Cos2p=1; 2p=\cos^{-1}(1)=2\pi; p=\pi$

$\therefore f(x+\pi)=\dfrac{1}{2}(Sin2xCos2\pi+Cos2xSin2\pi)$

$=\dfrac{1}{2}(Sin2x\times 1+Cos2x\times 0)$

$=\dfrac{1}{2}Sin2x$

thus $f(x+\pi)=\dfrac{1}{2}Sin2x$

So p=π.Hence period of function is π.

Q4: Find the greatest and least value of Cos $(-\theta)$?Ans.Range of Cos $(-\theta)$ is $-1\le Cos(-\theta)\le 1$.Greatest value of Cos$(-\theta)$ is 1. Least value of Cos$(-\theta)$ is -1.

Q3. What are the greatest and least values of sin $(-\theta)$?

Solution:

We know that $\sin(-\theta) = -\sin\theta$, and we also know that if $f(\theta) = \sin\theta$, then $-1 \le \sin\theta \le 1$

i.e $\sin\theta = \pm 1$

Hence $\sin(-\theta) = -(\mp 1) = \pm 1$

$\therefore -1 \le \sin(-\theta) \le 1$

So the required greatest and least value of $\sin(-\theta)$ are $+1$ & -1 Ans.

Q4. Find the greatest and least values of cos $(-\theta)$?

Solution:

We know that $\cos(-\theta) = \cos\theta$, and we also know that if $f(\theta) = \cos\theta$, then $-1 \le \cos\theta \le 1$

i.e. $\cos\theta = \pm 1$

Hence $\cos(-\theta) = \cos\theta = \pm 1$

Hence the greatest and least value of $\cos(-\theta)$ are $+1$ and -1

Ans.

Q5. Would the definition of the trigonometric functions be changed in any way if the radius of the circle of reference were permitted to change?

Solution: No, the definitions of the trigonometric functions be changed in any way because they do not depend upon the radius of reference circle.

We can prove this by taking an example.

if $r = 5$ then $x = r\cos\theta = 5\cos\theta$

$y = r\sin\theta = 5\sin\theta$

Using $x^2 + y^2 = r^2$

$(5\cos\theta)^2 + (5\sin\theta)^2 = (5)^2$

$25\cos^2\theta + 25\sin^2\theta = 25$

$\div$ b.s by 25

$\boxed{\cos^2\theta + \sin^2\theta = 1}$ Proved.

EXERCISE 11.2

Q1. Draw the graph of $\sin\theta$, where $-\pi \le \theta \le \pi$, from the graph find the value of sin 130°.

Solution:

Let $Y = \sin\theta$

$\theta = X$	0°	±30°	±60°	±90°	±120°	±150°	±180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$Y = \sin\theta$	0	± 0.5	± 0.86	± 1	±0.86	±0.5	0

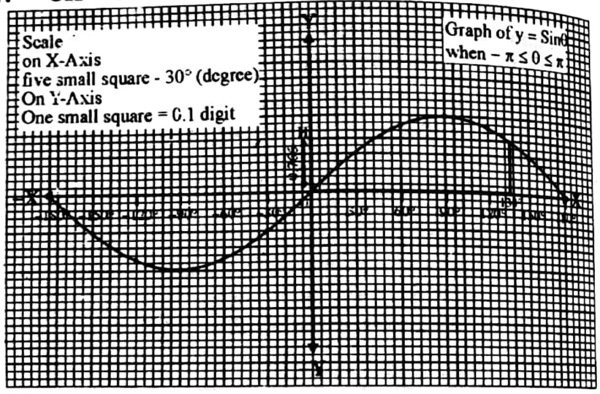
Construction:

Let $\overleftrightarrow{OX}$ & $\overleftrightarrow{OY}$ be x – axis & y – axis respectively. Plot 0° ± 30°, ±60°, ±90°, ±120°, ±150°, ±180 on x–axis by taking 5 small squares = 30°and 1 small box on y–axis = 0.1 digit.

Plot the points (0°, 0) (±30°, ±0.5), (60°, ±0.9), (±90°, ±1) (±120, ±0.9), (±150°, ±0.5), (±180°, 0) on the graph paper. By joining all the plotted points we get a curve. This curve is the graph of $y = \sin\theta, -\pi \le \theta \le \pi$.

for 130° on x – axis draw a line ⊥'er on x – axis intersecting the curve at D. from D draw DH ⊥ y – axis. & read OH on y – axis.

∴ OH = 0.766 ∴ sin130° = 0.766



Q2. Draw the graph of cosθ, where $-\pi \le \theta \le \pi$ From the graph find the value of cos 70°.

Solution:

Let Y = cosθ

$\theta = X$	0°	±30°	±60°	±90°	±120°	±150°	±180°
	0°	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$Y = \cos\theta$	1	±0.87	±0.5	0	0.5	±0.87	±1

Construction:

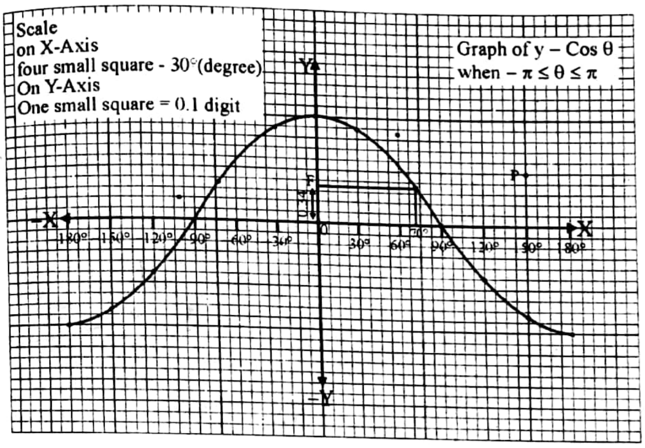
Let $\overleftrightarrow{OX}$ & $\overleftrightarrow{OY}$ be x – axis & y – axis respectively. Plot 0° ±30°, ±90°, ±120°, ±150°, ±180° on x–axis by taking 5 small squares = 30°.

Plot the points (0°, 1), (±30°, ±0.9), (60°, ±0.5), (± 90°, ±1) (±120, ±0.5), (±150°,±0.87), (±180°, ±1) on the graph paper by taking 1 small square on y – axis = 0.1 and 5 small box = 30°.

By joining all the plotted points we get a curve. This curve is the graph of $y = \cos\theta, -\pi \le \theta \le \pi$.

for 70° on x–axis draw a line kP ⊥'er on x–axis intersecting the curve at P. from P draw PH ⊥ y–axis & read OF

∴ OF = 0.342 ∴ Cos70° = 0.342



Q3. Draw the graph of tanθ. Find the value of tan80° from the graph.

Solution:

Let Y = tanθ Take $-\pi \le \theta \le \pi$

$\theta = X$	0°	±30°	±45°	±60°	±90°
	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$Y = \tan\theta$	0	±0.577	±1	±1.73	±∞

Construction:

Draw $\overleftrightarrow{OX}$ & $\overleftrightarrow{OY}$ be x–axis & y – axis respectively. Plot 0°, ± 30°, ±90° on x–axis by taking 5 small squares = 30°.

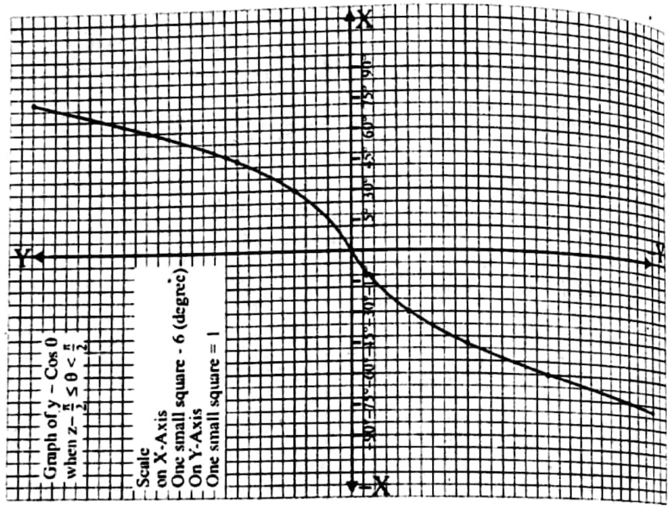
Plot the points (0°, 0), (± 30°, ±0.6), (±45°, ±1), (±60°, ±1.73) (±90°, ±∞) on the graphs paper by taking 1 small square on y – axis = 0.1.

By joining all the plotted points we get a curve. This curve is the graph of $y = \tan\theta, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

for 80° on x–axis draw a line HN ⊥'er on x–axis intersecting the curve at N. from N draw NT ⊥'er on y–axis.

Read OT

∴ OT = 5.671 ∴ tan80° = 5.671



Q4. Draw the graph of sin (–θ) where 0 ≤ θ ≤ 2π.

Solution:

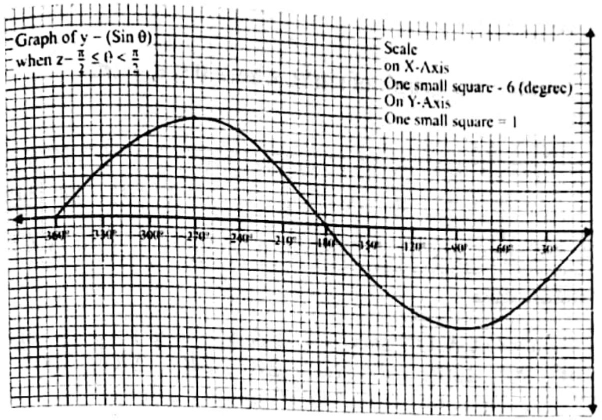
Let Y = Sin (–θ)

θ = X	0°	–30°	–60°	–90°	–120°	–150°	–180°	–210°	–240°	–270°	–300°	–330°	–360°
	0°	$-\frac{\pi}{6}$	$-\frac{\pi}{3}$	$-\frac{\pi}{2}$	$-\frac{2\pi}{3}$	$-\frac{5\pi}{6}$	$-\pi$	$-\frac{7\pi}{6}$	$-\frac{4\pi}{3}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{3}$	$-\frac{11\pi}{3}$	-2π
Y = sin(–θ)	0	–0.5	–0.866	–1	–0.866	–0.5	0	0.5	0.9	1	0.866	0.5	0

Construction:

Draw $\overleftrightarrow{OX}$ & $\overleftrightarrow{OY}$ as x – & y – axis respectively. Plot 0°, –30°, –90°, –120°, –150°, –180°, –210°, –240°, –270°, –330°, –360° on x–axis by taking 5 small squares = 30°.

Plot the points (0°, 0), (–30°, –0.5), (60°, –0.9), (–90°, –1) (–120, –0.9), (–150°, –0.5), (–180°,o), (–210°, 0.5), (–240°, 0.9), (–270°, 1), (–300°, 0.9) (–330°, 0.9) (–330°, 0.5), (–360°,0) on the graph paper, Join all the plotted points. We get the curve line which is the graph of y = sin (–θ).



Q5. Draw the graph of cos 2θ, where –π ≤ θ ≤ π.

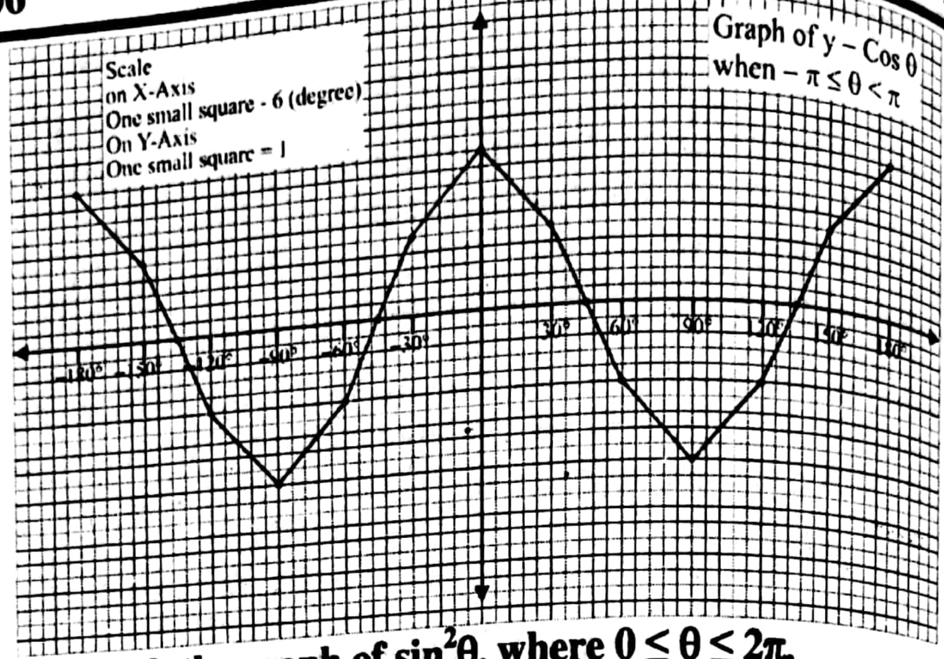
Solution: Let y = Cos 2θ

θ	0°	±30°	±60°	±90°	±120°	±150°	±180°
	0°	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$2\frac{\pi}{3}$	$\frac{5\pi}{6}$	π
Cos2θ	1	0.5	–0.5	–1	–0.5	0.5	1

Construction:

Draw $\overleftrightarrow{OX}$ & $\overleftrightarrow{OY}$ as x – & y – axis respectively. Plot 0° ± 30°, ± 60°, ±90°, ±120, ±150°, ±180°, along x–axis by taking 5 small squares = 30°.

Plot the points (0°, 1) (±30°, 0:5), (±60°, –0.5), (±90°, –1) (±120°, –0.5) (±150°, 0.5), (±180°,1) on the graph paper by taking 1 small square = 0.1. Join all the plotted pints. By joining all the plotted points we get the graph of y = cosθ, – π ≤ θ ≤ π.



Q6. Sketch the graph of $\sin^2\theta$, where $0 \le \theta \le 2\pi$.

Solution:

Let $Y = \sin 2\theta$

$\theta = X$	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
	0°	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{3}$	-2π
$\sin^2\theta$	0	0.25	0.75	1	0.75	0.25	0	0.25	0.75	0	0.75	0.25	0

Construction:

Draw $\overleftrightarrow{OX}$ & $\overleftrightarrow{OY}$ as x – axis & y – axis respectively. Plot 0°, 30°, 90°, 120°, 150°, 180°, 210°, 240°, 270°, 300°, 330°, 360°, on x – axis by taking 5 small squares = 30°.

Plot the points (0°, 0), (30°, 0.25), (60°, 0.75), (90°, 1) (120, 0.75), (150°, 0.25), (180°, 0), (210°, 0.25), (240°, 0.75), (270°, 0), (300°, 0.75), (330°, 0.25), (360°,0) on the graph paper. Join all the plotted points. Joint all the we get the graph of y = $\sin 2\theta$, $0 \le \theta \le 2\pi$.