EXERCISE 2.1

Q1. Classify each of the following Statements as true or false.

(i)	Every whole number is a rational number.	(True)
(ii)	Every natural number is an irrational number.	(False)
(iii)	Every integer is a rational number.	(True)
(iv)	Every rational number is an integer.	(False)
(v)	Every real number is a rational number.	(False)
(vi)	I is closed w.r to Subtraction "–".	(False)
(vii)	Z is closed w.r to division "÷".	(False)
(viii)	J is closed w.r to division "÷".	(False)
(ix)	Division "÷" is commutative in I.	(False)
(x)	There exists an inverse element for "–" in I.	(False)

Q2. Name the following properties.

(i) Every real number is either a positive number, zero, or a negative number.

Ans: Trichotomy Property

(ii) Between any two real numbers a and b there exists a rational number.

Ans: Density Property

(iii) For any two positive real numbers a and b there exists a natural number n such that na > b if a < b.

Ans: Archimedean Property

(iv) $17 > 8 \Rightarrow 68 > 32$

Ans: Multiplication Property of inequality

(v) $a < b$ and $b < c \Rightarrow a < c \;\; \forall\, a, b, c \in R.$

Ans: Transitive Property of inequality

(vi) $a \,.\, b \in R, \;\; \forall\, a, b \in R.$

Ans: Closure Property w.r to Multiplication.

Q3. Show that

(i) $\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

Proof:

Taking L.H.S

$$\text{L.H.S} = \frac{1}{4} - \frac{1}{6} \Rightarrow = \left(\frac{1}{4}\right)\left(\frac{3}{3}\right) - \left(\frac{1}{6}\right)\left(\frac{2}{2}\right)$$

$$= \frac{3}{12} - \frac{2}{12} = \frac{3-2}{12} = \frac{1}{12} = \text{R.H.S}$$

(ii) $\frac{1}{8} \cdot \frac{4}{5} = \frac{1}{10}$

Proof:

Taking L.H.S

$$\text{L.H.S} = \frac{1}{8} \cdot \frac{4}{5} \Rightarrow = \left(\frac{1}{2 \times \cancel{4}}\right) \cdot \left(\frac{1 \times \cancel{4}}{5}\right) = \frac{1}{10} = \text{R.H.S}$$

(iii) $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

Proof:

Taking. L.H.S

$$\text{L.H.S} = \frac{1}{2}+\frac{1}{3} \Rightarrow = \left(\frac{1}{2}\right)\left(\frac{3}{3}\right)+\left(\frac{1}{3}\right)\left(\frac{2}{2}\right)$$
$$= \frac{3}{6}+\frac{2}{6}=\frac{3+2}{6}=\frac{5}{6}=\text{R.H.S}$$

(iv) $-3(3-4)=3$

Proof:

Taking L.H.S

$$\text{L.H.S} = -3(3-4)$$
$$= -3(-1)=3=\text{R.H.S}$$

(v) $\left(\frac{3}{4}+\frac{7}{12}\right)-\left(\frac{7}{12}-\frac{1}{4}\right)=1$

Proof:

Taking L.H.S

$$\text{L.H.S} = \left(\frac{3}{4}+\frac{7}{12}\right)-\left(\frac{7}{12}-\frac{1}{4}\right)$$
$$= \left\{\frac{3\times 3}{4\times 3}+\frac{7\times 1}{12}\right\}-\left\{\frac{7\times 1}{12}-\frac{1\times 3}{4\times 3}\right\}$$
$$= \left(\frac{9}{12}+\frac{7}{12}\right)-\left(\frac{7}{12}-\frac{3}{12}\right) \Rightarrow = \left(\frac{9+7}{12}\right)-\left(\frac{7-3}{12}\right)$$
$$= \frac{16}{12}-\frac{4}{12}=\frac{16-4}{12}=\frac{\cancel{12}^{1}}{\cancel{12}}=1=\text{R.H.S}$$

Q4. Justify each of the following statements by citing appropriate axioms.

(i) $x+5=2+5 \Rightarrow x=2$

Solution:

$x+5=2+5$

$(x+5)+(-5)=(2+5)+(-5)$ (Additive inverse law)

$x+\cancel{5}-\cancel{5}=2+\cancel{5}-\cancel{5}$ (left cancellation property for "+")

$x+0=2+0$ existence of additive identify)

$\boxed{x=2}$

(ii) $\frac{a}{2} = \frac{3}{4} \Rightarrow 4a = 2.3$

Solution:

$$\frac{a}{2} = \frac{3}{4} \Rightarrow {}^{4}\cancel{8} \times \frac{a}{\cancel{2}} = \frac{3}{\cancel{4}} \times \cancel{8}^{2} \text{ (by multiplicative property)}$$

$$\boxed{4a = 3.2}$$

(iii) $x + 4 = y$ **and** $y = 6 \Rightarrow x + 4 = 6$

Solution:

$x + 4 = y$ and $y = 6 \Rightarrow x + 4 = 6$

$\therefore\ a = b\ ,\ b = c \Rightarrow a = c$

(by Transitive law of equality)

(iv) $8 \, . \, 8^{-1} = 1$

Solution: $8 \, . \, 8^{-1} = 1$ (by multiplicative inverse law)

EXERCISE 2.2

Q1. Perform the indicated operations.

(i) $(7, -9) + (3,5)$

Solution:

$(7, -9) + (3,5) = (7 + 3, -9 + 5)$

$(7, -9) + (3, 5) = (10, -4)$ Ans.

(ii) $(7, -9) \, . \, (3\, , 5)$

Solution:

Using Formula

$$\boxed{(a\, , b) \, . \, (c, d) = (ac - bd, ad + bc)}$$

$(7, -9) \, . \, (3,5) = (7 \times 3 - (-9)(5)\, , 7 \times 5 + (-9)(3))$

$(7, -9) \, . \, (3,5) = (21 + 45, 35 - 27)$

$(7, -9) \, . \, (3,5) = (66, 8)$ Ans.

(iii) $(7, -9) - (3, 5)$

Solution:

$(7, -9) - (3, 5) = (7 - 3, -9 - 5)$

$(7, -9) - (3, 5) = (4\, , -14)$ Ans.

(iv) $(7, -9) + (3, 5)$

Solution:

Using Formula

Exercise No.=2.2

1.Perform the indicated operations.

(i)(7,-9)+(3,5),by using formula (a,b)+(c,d)= (a+c,b+d)(7+3,-9+5)=(10,-4) Ans

(ii) (7,-9).(3,5),by using formula (a,b).(c,d)=(ac-bd,bc+ad)

$=\{7\times3-(-9)(5)\},\{(-9)(3)+7\times5\}$ **=(21+45,-27+35)=(66,8)Ans**

(iii) (7,-9)-(3,5),by using formula (a,b)-(c,d)= (a-c,b-d)(7-3,-9-5)=(4,-14) Ans

(iv) $(7,-9)\div(3,5)$ **by using formula** $\frac{(a,b)}{(c,d)}=\left(\frac{ac+bd}{c^2+d^2},\frac{bc-ad}{c^2+d^2}\right)$

$=\left(\frac{7(3)+(-9)5}{3^2+5^2},\frac{-9(3)-7(5)}{3^2+5^2}\right)=\left(\frac{21-45}{9+25},\frac{-27-35}{9+25}\right)=\left(\frac{-24}{34},\frac{-62}{34}\right)=\left(\frac{-12}{17},\frac{-31}{17}\right)$ **Ans**

(v) $(2x,3y).(2x,-3y)$ **by using formula (a,b).(c,d)=(ac-bd,bc+ad)**

$=\{2x.2x-3y(-3y),2x(-3y)+3y(2x)\}=(4x^2+9y^2,-6xy+6xy)=(4x^2+9y^2,0)$ **Ans**

(vi) $(-2,1).(-\frac{2}{5},-\frac{1}{5})$ **.by using formula (a,b).(c,d)=(ac-bd,bc+ad)**

$=\{(-2)(-\frac{2}{5})-1(-\frac{1}{5}),(-2)(-\frac{1}{5})+1(-\frac{2}{5})\}=(\frac{4}{5}+\frac{1}{5},\frac{2}{5}-\frac{2}{5})=(1,0)$ **Ans**

2.Find the conjugate of the following complex numbers and also their modulus.

(i)-7+i,Conjugate=-7-i and $|-7,i|=\sqrt{x^2+y^2}=\sqrt{(-7)^2+1^2}=\sqrt{50}=\sqrt{25(2)}=5\sqrt{2}$

(ii)1+2i,Conjugate=1-2i and $|1,2|=\sqrt{x^2+y^2}=\sqrt{1^2+2^2}=\sqrt{1+4}=\sqrt{5}$.

(iii)(5,-2), Conjugate=5+2i and $|5,-2|=\sqrt{x^2+y^2}=\sqrt{5^2+(-2)^2}=\sqrt{25+4}=\sqrt{29}$

(iv)(2,1),Conjugate=2-i and $|2,1|=\sqrt{x^2+y^2}=\sqrt{2^2+1^2}=\sqrt{4+1}=\sqrt{5}$

3.Solve the following complex equations.

$(i)(x,y).(2,3)=(-4,7)=(2x-3y),(3x+2y)=(-4,7)$

$2x-3y=-4\rightarrow(1),3x+2y=7\rightarrow(2)$

Multiply eq (1) by 2 and eq : 2by 3

$4x-6y=-8$

$9x+6y=21$

$13x=13$ So $x=1$

by putting in (1)

$2(1)-3y=-4$

$-3y=-6$ So $y=2$

(ii) $(x+3i)^2=2yi$

Using $(a+b)^2=a^2+2ab+b^2$

$x^2+6xi-9=2yi\,,x^2-9+6xi=0+2yi$

$(x^2-9,6x)=(0+2y)$

$x^2-9=0,\ 6x=2y$

$x^2=9,x=\pm3\,,y=3x$

$y=3(3)=9,y=3(-3)=-9$

So $x=\pm3,y=\pm9$ **Ans**

$(iii)(x+2yi)^2=xi$

Using $(a+b)^2=a^2+2ab+b^2$

$x^2+2(x)(2yi)+(2yi)^2=xi\,,x^2+4xyi+4y^2i^2=xi$

$=x^2+4xyi+4y^2i(-1)=xi\ \therefore i^2=-1$

$=x^2+4xyi-4y^2=xi\,,x^2-4y^2+4xyi=0+xi$

$(x^2-4y^2,4xy)=(0,x)=x^2-4y^2=0,4xy=x$

$\sqrt{x^2}=\sqrt{4y^2},y=\frac{x}{4x}=\frac{1}{4}$

$x=\pm2y,y=\frac{1}{4},x=2y,x=-2y$

$x=2(\frac{1}{4}),x=-2(-\frac{1}{4}),x=\frac{1}{2},-\frac{1}{2}$

$x=\pm\frac{1}{2},y=\frac{1}{4}$ **Ans**

$(iv)(-x,3y),(2,0)$

$-x=2\,,3y=0$ **So** $x=-2,y=\frac{0}{3}=0$

4.Find the additive and multiplicative inverse of:

(i)(-3,8),Additive inverse =(-a,-b)=(3,-8)

Multiplicative inverse = $\left(\frac{a}{a^2+b^2},\frac{-b}{a^2+b^2}\right)=\left(\frac{-3}{(-3)^2+8^2},\frac{-8}{(-3)^2+8^2}\right)$

$=\left(\frac{-3}{9+64},\frac{-8}{9+64}\right)=\left(\frac{-3}{73},\frac{-8}{73}\right)$ **is the multiplicative inverse of (-3,8)**

(ii)(-3,5),Additive inverse =(-a,-b)=(3,-5)

Multiplicative inverse = $\left(\frac{a}{a^2+b^2},\frac{-b}{a^2+b^2}\right)=\left(\frac{-3}{(-3)^2+5^2},\frac{-5}{(-3)^2+5^2}\right)$

$=\left(\frac{-3}{9+25},\frac{-5}{9+25}\right)=\left(\frac{-3}{34},\frac{-5}{34}\right)$ **is the multiplicative inverse of (-3,5)**

(iii)(0,0),Additive inverse =(-a,-b)=(0,0).Multiplicative inverse of (0,0) does not exist.

(iv) $(\sqrt{3},\sqrt{\ }$ Additive inverse =(-a,-b)= $(-\sqrt{3},-\sqrt{2})$

Multipli ative inverse = $\left(\frac{a}{a^2+b^2},\frac{\ }{a^2+b^2}\right)\left(\frac{\sqrt{3}}{(\sqrt{3})^2+(\ )^2},\frac{-\sqrt{2}}{(\sqrt{3})^2+(\sqrt{2})^2}\right)$

$\left(\frac{\sqrt{3}}{3+2},\frac{-\sqrt{2}}{3+2}\right)=\left(\frac{\sqrt{3}}{5},\frac{-\sqrt{2}}{5}\right)$ is the multiplicative inverse of $(\sqrt{3},\sqrt{2})$.

5.If $z_1=1+i$ **and** $z_2=3-2i$ **than evaluate the following.**

(i) $|5z_1-4z_2|=|5(1+i)-4(3-2i)|=|5+5i-12+8i|=|5-12+5i+8i|=|-7+13i|=$ $\sqrt{(-7)^2+13^2}$

$=\sqrt{49+169}=\sqrt{218}$ Hence $|5z_1-4z_2|=\sqrt{218}$

(ii) $(\bar{z}_1)^2=(\overline{1+i})^2=(1-i)^2=1-2(1)(i)+i^2=1-2i-1=-2i$ Hence $(\bar{z}_1)^2=-2i$

(iii) $\frac{z_1}{z_2}=\frac{1+i}{3-2i}=\frac{(1,1)}{(3,-2)}=\left(\frac{ac+bd}{c^2+d^2},\frac{bc-ad}{c^2+d^2}\right)=\left(\frac{1(3)+1(-2)}{3^2+(-2)^2},\frac{1(3)-1(-2)}{3^2+(-2)^2}\right)=\left(\frac{3-2}{9+4},\frac{3+2}{9+4}\right)$

$=\left(\frac{1}{13},\frac{5}{13}\right)=\frac{1}{13}+\frac{5}{13}i$ Ans

6.(iv)Verify that 4i and – 4i are the roots of $x^2+16=0$.

Solution:- $x^2+16=0,\ x^2=-16,\ x=\sqrt{-16},\ \therefore x=\pm 4i$.

6(v)(2,2) and (2,-2) are the roots of $x^2-4x+8=0$.

Solution:-

(2,2) means 2+2i Now put 2+2i in the equation for x $(2+2i)^2-4(2+2i)+8=0$ $2^2+2(2)(2i)+(2i)^2-8-8i+8=0$ $4+8i+4i^2-8-8i+8=0 \quad \therefore i^2=-1$ 4+4(-1) = 0 => 4-4=0, 0=0	(2,-2) means 2-2i Now put 2-2i in the equation for x $2^2+2(2)(2i)+(2i)^2-8-8i+8=$ $4-8i+4i^2-8+8i+8=0$ 4+4(-1) = 0 , 4-4 = 0 , 0=0

Hence (2,2) and (2,-2) are the roots of equation $x^2-4x+8=0$.

7.Factorize in the set of complex numbers.

(i) $4a^2+9b^2$	(ii) $3m^2+8t^2$
$=(2a)^2+(3b)^2$	$=(\sqrt{3}m)^2+(\sqrt{8}t)^2=(\sqrt{3}m)^2-(-1)(\sqrt{8}t)^2$
$=(2a)^2-(-1)(3b)^2$	$=(\sqrt{3}m)^2-i^2(\sqrt{8}t)^2=(\sqrt{3}m)^2-(\sqrt{8}ti)^2$
$=(2a)^2-(i^2)(3b)^2$	$=(\sqrt{3}m)^2-(2\sqrt{2}ti)^2$
$=(2a)^2-(3bi)^2=(2a+3bi)(2a-3bi)$	$=(\sqrt{3}m+2\sqrt{2}ti)(\sqrt{3}m-2\sqrt{2}ti)$ *Ans*

8.Simplify

(i) $(2-i)^4$

$= (2-i)^2(2-i)^2$

$= \{2^2-2(2)(i)+i^2\}\{2^2-2(2)(i)+i^2\}$

$= \{4-4i-1\}\{4-4i-1\}$

$= (3-4i)(3-4i) = (3-4i)^2$

$= 9-2(3)(4i)+(4i)^2$

$= 9-24i+16(-1)$

$= 9-24i-16$

$= -7-24i$

(ii) $(1+i)^3 = (1+i)^2(1+i)$

$= (1^2+2(1)i+i^2)(1+i)$

$= (1+2i-1)(1+i)$

$= 2i(1+i)$

$= 2i+2i^2$

$= -2+2i$

(iii) $(-1+i\sqrt{3})^2$

$= (-1)^2+2(-1)i\sqrt{3}+(i\sqrt{3})^2$

$= 1-2i\sqrt{3}-3$

$= -2-2i\sqrt{3}$

(iv) $(-1-i\sqrt{3})^2$

$= (-1)^2-2(-1)(i\sqrt{3})+(i\sqrt{3})^2$

$= 1+2i\sqrt{3}-3$

$= -2+2i\sqrt{3}$

(v) $\dfrac{\sqrt{2}+i}{\sqrt{2}-i}$

$= \dfrac{\sqrt{2}+i}{\sqrt{2}-i} \times \dfrac{\sqrt{2}+i}{\sqrt{2}+i}$

$= \dfrac{2+i\sqrt{2}+i\sqrt{2}+i^2}{(\sqrt{2})^2-i^2}$

$= \dfrac{2+2i\sqrt{2}-1}{2+1} = \dfrac{1+2i\sqrt{2}}{3}$

$= \dfrac{1}{3}+\dfrac{2i\sqrt{2}}{3}$

(vi) $\dfrac{1+i}{1-i}\cdot\dfrac{2-i}{1-i}$

$= \dfrac{(1+i)(2-i)}{(1-i)^2}$

$= \dfrac{2-i+2i-i^2}{1-2i+i^2}$

$= \dfrac{2+i+1}{1-2i-1}$

$= \dfrac{3+i}{-2i} \times \dfrac{i}{i} = \dfrac{-1+3i}{2}$

(vii) $\dfrac{(2+i)^2}{3-4i} = \dfrac{2^2+2(2)i+i^2}{3-4i}$

$= \dfrac{4+4i-1}{3-4i} = \dfrac{3+4i}{3-4i} \times \dfrac{3+4i}{3+4i}$

$= \dfrac{9+12i+12i+16i^2}{9+12i-12i-16i^2}$

$= \dfrac{9+24i-16}{9+16}$

$= \dfrac{-7+24i}{25} = \dfrac{-7}{25}+\dfrac{24i}{25}$

9.Show that $z = 1 \pm i$ **satisfies the equation** $z^2-2z+2=0$.

$= (1+i)^2-2(1+i)+2=0$

$= (1+2(1)i+i^2)-2-2i+2=0$

$= 1+2i-1-2i = 0,0=0$

$= (1-i)^2-2(1-i)+2=0$

$= (1-2(1)i+i^2)-2+2i+2=0$

$= (1-2i-1)+2i = 0,0=0$

Do you know? Factors of (a + b) are $(\sqrt{a}+\sqrt{bi})(\sqrt{a}-\sqrt{bi})$.

11.Verify that.

(i) $(\sqrt{2}-i)+i(\sqrt{2}i-1) = -2i$

$= \sqrt{2}-i+\sqrt{2}i^2-i = -2i$

$= \sqrt{2}-i-\sqrt{2}-i = -2i$

$-2i = -2i$

(ii) $(1-i)^4 = -4$

$= (1-i)^2(1-i)^2 = -4$

$\{(1-2(1)i+i^2\}\{(1-2(1)i+i^2\} = -4$

$\{1-2i-1\}\{1-2i-1\} = -4$

$(-2i)(-2i) = -4\ ,\ 4i^2 = -4, -4 = -4.$

(iii) $i^3 = -i$ **and** $i^4 = 1$

$= (i^2)i = (-1)i = -i$

$= (i^2)(i^2) = (-1)(-1) = 1$

(iv) $\dfrac{1+2i}{3-4i}+\dfrac{2}{5} = \dfrac{i-2}{5i}$

$= \dfrac{1+2i}{3-4i} \times \dfrac{3+4i}{3+4i}+\dfrac{2}{5} = \dfrac{i-2}{5i}$

$= \dfrac{3+4i+6i+8i^2}{9+12i-12i-16i^2}+\dfrac{2}{5} = \dfrac{i-2}{5i}$

$= \dfrac{3+10i-8}{9+16}+\dfrac{2}{5} = \dfrac{i-2}{5i}$

$= \dfrac{-5+10i}{25}+\dfrac{2}{5} = \dfrac{i-2}{5i}$

$= \dfrac{-5(1-2i)}{25}+\dfrac{2}{5} = \dfrac{i-2}{5i}$

$= \dfrac{2i-1}{5}+\dfrac{2}{5} = \dfrac{i-2}{5i}$

$= \dfrac{2i-1+2}{5} = \dfrac{i-2}{5i}$

$= \dfrac{2i+1}{5} \times \dfrac{i}{i} = \dfrac{i-2}{5i}$

$= \dfrac{i-2}{5i} = \dfrac{i-2}{5i}$

Examples from the book.

1. *Let* $z = (4,-3)$ *then find* $|z|$ *and* $\bar{z}$ *and prove that* $|z| = |\bar{z}|$.

<u>*Solution.*</u> $|z| = \sqrt{4^2+(-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$. **Moreover**

$\bar{z} = (4,-(-3)) = (4,3)$. *So* $|\bar{z}| = 5$. *Hence* $|z| = |\bar{z}|$.

2. *Express* $x^2+y^2 = a^2$ *in terms of conjugate coordinates.*

<u>*Solution.*</u> **Using** $x = \dfrac{z+\bar{z}}{2}$ *and* $y = \dfrac{z-\bar{z}}{2i}$, *we get*

$x^2+y^2 = \left(\dfrac{z+\bar{z}}{2}\right)^2+\left(\dfrac{z-\bar{z}}{2i}\right)^2 = a^2$

$\Rightarrow (z^2+2z\bar{z}+\bar{z}^2)-(z^2-2z\bar{z}+\bar{z}^2) = 4a^2 \Rightarrow 4z|\bar{z}| = 4a^2$

$\Rightarrow z\bar{z} = a^2$

3. *Divide* $4-3i$ *by* $-1+2i$.

<u>*Solution.*</u> $\dfrac{4-3i}{-1+2i} = \dfrac{4-3i}{-1+2i} \times \dfrac{-1-2i}{-1-2i} = \dfrac{-10-5i}{5} = -2-i.$

QNo.(3)Find the real and imaginary parts of

(i) $i(3+2i) = 3i+2i^2 = 3i+2(-1) = 3i-2 = -2+3i$ So real part =-2 and imaginary part=3.

(ii) $\frac{2-i}{3i} = \frac{2-i}{3i} = \frac{2}{3i} - \frac{i}{3i} = \frac{2}{3i} - \frac{1}{3} = -\frac{1}{3} + \frac{2}{3}i$ So real part = $-\frac{1}{3}$ and imaginary part = $\frac{2}{3}$.

(iii) $\frac{3a+2bi}{a-bi}$

$= \frac{(3a+2bi)(a+bi)}{(a-bi)(a+bi)}$

$= \frac{(3a^2+3abi+2abi+2b^2i^2)}{(a^2-b^2i^2)}$

$= \frac{3a^2+5abi-2b^2}{a^2+b^2}$

$= \frac{3a^2-2b^2+5abi}{a^2+b^2}$

$= \frac{3a^2-2b^2}{a^2+b^2} + \frac{5abi}{a^2+b^2}$

Hence real part= $\frac{3a^2-2b^2}{a^2+b^2}$

And imaginary part= $\frac{5abi}{a^2+b^2}$

(iv) $\frac{\sqrt{3}+i}{\sqrt{3}-i}$

$= \frac{(\sqrt{3}+i)(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)}$

$= \frac{(\sqrt{3}+i)^2}{3-i^2} = \frac{(\sqrt{3})^2+2\sqrt{3}.i+i^2}{3-(-1)}$

$= \frac{3+2\sqrt{3}i-1}{3+1} = \frac{2+2\sqrt{3}i}{4}$

$= \frac{2}{4} + \frac{2\sqrt{3}i}{4} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

Hence real part= $\frac{1}{2}$

and imaginary part= $\frac{\sqrt{3}}{2}$

Qno(5)(iii).Is there any complex number whose additive and multiplicative inverse are equal.

Yes, it is (0,1).

Additive inverse = (-a,-b) = (0,-1).

Multiplicative inverse = $\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right) = \left(\frac{0}{0^2+1^2}, \frac{-1}{0^2+1^2}\right) = \left(\frac{0}{1}, \frac{-1}{1}\right) = (0,-1)$

Hence Additive inverse = Multiplicative inverse.

Exercise 2.3 , Qno. (03) Part v

$(a+3bi)^4 = \left\{(a+3bi)^2\right\}^2$

Using $(a+b)^2 = a^2+2ab+b^2$

$= \left\{(a+3bi)^2\right\}^2 = \left\{a^2+2(a)(3bi)+(3bi)^2\right\}^2$

$= \left\{a^2+6abi+9bi^2\right\}^2 ; = \left\{a^2+6abi-9b\right\}^2$

Now using $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$= \left\{a^2+6abi+(-9b)\right\}^2$

$= (a^2)^2+(6abi)^2+(-9b)^2+2(a^2)(6abi)+2(6abi)(-9b)+2(-9b)(a^2)$

$= a^4+36a^2b^2i^2+81b^2+12a^3bi-108ab^2i-18a^2b$

$= a^4-36a^2b^2+81b^2+12a^3bi-108ab^2i-18a^2b$

EXAMPLES FROM THE TEXT BOOK.

(4) $\overline{z_1+z_2} = \overline{(x_1,y_1)+(x_2,y_2)}$

$= \overline{(x_1+x_2, y_1+y_2)}$

$= (x_1+x_2, -y_1-y_2)$

$= (x_1,-y_1)+(x_2,-y_2)$

$= \overline{z_1}+\overline{z_2}$

(5) $\overline{z_1.z_2} = \overline{(x_1,y_1).(x_2,y_2)}$

$= \overline{(x_1x_2-y_1y_2, x_1y_2+x_2y_1)}$

$= (x_1x_2-y_1y_2, -x_1y_2-x_2y_1)$

$= (x_1,-y_1).(x_2,-y_2)$

$= \overline{z_1}.\overline{z_2}$

(6) $z.\bar{z} = (x,y).(x,-y)$

$= (x^2+y^2, -xy+xy)$

$= (x^2+y^2, 0) = (x^2+y^2)+i0$

$= |z|^2$

(7) $|z_1.z_2| = |(x_1,y_1).(x_2,y_2)|$

$= |(x_1x_2-y_1y_2, x_1y_2-x_2y_1)|$

$= \sqrt{x_1^2x_2^2+y_1^2y_2^2+x_1^2y_2^2+x_2^2y_1^2}$

$= \sqrt{(x_1^2+y_1^2).(x_2^2+y_2^2)}$

$= \sqrt{x_1^2+y_1^2}.\sqrt{x_2^2+y_2^2}$

$= |z_1|.|z_2|$

8. *Separate* $(2x-3yi)^4$ into *its real and imaginary parts.*

Solution. $(2x-3yi)^4$

$= (2x)^4+4(2x)^3(-3yi)+6(2x)^2(-3yi)^2+4(2x)(-3yi)^3+(-3yi)^4$

$= 16x^4-96x^3yi+216x^2y^2i^2-216xy^3i^3+81y^4i^4$

$= (16x^4-216x^2y^2+81y^4)+i(216xy^3-96x^3y)$

Real part $= 16x^4-216x^2y^2+81y^4$.

Imaginary part $= 216xy^3-96x^3y$.

Do you know?

Conjugate is also denoted by β.

POWERS OF IOTA (i).

$i=\sqrt{-1}$.

$i^2=-1$

$i^3=-i$

$i^4=1$ where $n\in Z$

$i^n+i^{n+1}+i^{n+2}+i^{n+3}=0, n\in Z$.