USEFUL FORMULAE

Set Theory:

(1) Commutative Property of Union

$A \cup B = B \cup A$

(2) Commutative Property of Intersection

$A \cap B = B \cap A$

(3) Associative Property of Union

$A \cup (B \cup C) = (A \cup B) \cup C$

(4) Associative Property of Intersection

$A \cap (B \cap C) = (A \cap B) \cap C$

(5) Distributive Property of Union Over Intersection

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(6) Distributive Property of Intersection Over Union

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(7) De Morgan's Laws.

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

(8) Total Number of Subsets = 2^n

Where $n \longrightarrow$ Number of Elements in Set

(9) Distributive property of Cartesian product over Union

If A, B and C are any three Sets then

(a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(LEFT DISTRIBUTIVITY)

(b) $(B \cup C) \times A = (B \times A) \cup (C \times A)$

(RIGHT DISTRIBUTIVITY)

(10) Distributive property of Cartesian product over Intersection:

(a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(LEFT DISTRIBUTIVITY)

(b) $(B \cap C) \times A = (B \times A) \cap (C \times A)$

(RIGHT DISTRIBUTIVITY)

(11) Distributive property of the Cartesian product over Complement:

If A, B and C are any three Sets, then

(a) $(A - B) \times C = (A \times C) - (B \times C)$

(b) $C \times (A - B) = (C \times A) - (C \times B)$

Important Laws:

for any set A

(i) $A \cup A = A$ (ii) $A \cap A = A$

Identity Laws:

for any Set A

(i) $A \cup A' = U$ (ii) $A \cap A' = \phi$

(iii) $(A')' = A$ (iv) $U' = \phi$ and $\phi' = U$

Standard Fundamental Algebraic Formulae:

(1) $(a + b)^2 = a^2 + 2ab + b^2$

(2) $(a - b)^2 = a^2 - 2ab + b^2$

(3) $a^2 - b^2 = (a - b)(a + b)$

(4) $(a + b)^2 - (a - b)^2 = 4ab$

(5) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(6) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

(7) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= a^3 + b^3 + 3ab(a + b)$

(8) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= a^3 - b^3 - 3ab(a - b)$

(9) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(10) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(11) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

(12) $a^2 + b^2 + c^2 - ab - bc - ac = \left(\frac{a-b}{\sqrt{2}}\right)^2 + \left(\frac{b-c}{\sqrt{2}}\right)^2 + \left(\frac{c-a}{\sqrt{2}}\right)^2$

Real And Complex Number: let $Z = a + ib$ be a complex number. Complex number can be written in the form of an order pair $Z = (a, b)$

Properties of Complex number:

(i) **Equality:**

$(a, b) = (c, d)$ iff $a = c, b = d$

(ii) **Addition:**

$(a, b) + (c, d) = (a + c, b + d)$

(iii) **Subtraction:**

$(a, b) - (c, d) = (a - c, b - d)$

(iv) **Multiplication:**

$(a, b).(c, d) = (ac - bd, ad + bc)$

(v) **Division:**

$$\frac{(a, b)}{(c, d)} = \left(\frac{ac + bd}{c^2 + d^2}, \frac{bc - ad}{c^2 + d^2}\right)$$

(vi) **Multiplicative Inverse of (a, b):**

$$(a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2}\right)$$

Modulus and Conjugate of Complex Number:

If $Z = x + iy$ be a Complex number then $\overline{Z} = x - iy$ be a conjugate of Complex number and $|Z| = \sqrt{x^2 + y^2}$ be a modulus of a complex number.

Properties of Modulus and Conjugate of Complex Numbers:

(1) $\overline{Z_1 + Z_2} = \overline{Z_1} + \overline{Z_2}$

(2) $\overline{Z_1 \cdot Z_2} = \overline{Z_1} \cdot \overline{Z_2}$

(3) $Z \cdot \overline{Z} = |Z|^2$

(4) $|Z_1 \cdot Z_2| = |Z_1| \cdot |Z_2|$

(5) $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$

(6) $\overline{(\overline{Z})} = Z$

* $Z + \overline{Z}$ is purely real.

* $Z - \overline{Z}$ is purely imaginary

* $x = \frac{Z + \overline{Z}}{2}$ and $y = \frac{Z - \overline{Z}}{2i}$ are called conjugates coordinates.

The triangle Inequality:

$|Z_1| - |Z_2| \leq |Z_1 + Z_2| \leq |Z_1| + |Z_2|$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Cube Roots of Unity:

(i) $i = \sqrt{-1} \Rightarrow i^2 = -1$ (ii) $1 + \omega + \omega^2 = 0$ (iii) $\omega^3 = 1$

Nature of the Roots of a Quadratic Equation:

(1) If $D = 0$ then the roots are equal

(2) If $D > 0$ then the roots are real and unequal

(3) If $D < 0$ then the roots are complex and unequal

(4) If D is a perfect square then the roots are rational and unequal.

Where $D = b^2 - 4ac$ is called **DISCRIMINANT**.

Roots of a Quadratic Equation:

general form of a quadratic equation.

$ax^2 + bx + c = 0$

(1) Sum of Roots $= \alpha + \beta = -\frac{b}{a}$

(2) Product of Roots = $\alpha\beta = \frac{c}{a}$

To form a quadratic equation when its roots are given:

$x^2 -$ (sum of the roots) x + (products of roots) = 0

Properties of Matrix Operation:

for any three matrices A, B and C

(1) Commutative property w.r to Addition $A + B = B + A$

(2) Associative property w. r to Addition

$A + (B + C) = (A + B) + C$

(3) Distributive property of multiplication of matrices over Addition. $A(B + C) = AB + AC$

(4) Distributive property of multiplication of matrices over Subtraction. $A(B - C) = AB - AC$

(5) Associative property of matrices w.r to multiplication

$A(BC) = (AB)C$

(6) $AI = IA = A$ where A and I are of the Same Order.

(7) $K(AB) = (KA)B = A(KB)$, where K is a Scalar.

Properties of Transposed Matrix:

If two matrices A and B are Conformable for addition and multiplication then,

(1) $(A \pm B)^t = A^t \pm B^t$

(2) $(KA)^t = KA^t$; where K is a Scalar

(3) $(AB)^t = B^t A^t$

(4) $(A^t)^t = A$

Inverse of Matrix A:

$$A^{-1} = \frac{1}{|A|} \text{Adj of } A.$$

Sequence and Series:

(1) **nth term of Arithmetic Sequence:**

$$\boxed{T_n = a + (n - 1)d}$$

(2) **Arithmetic Series:**

(a) $S_n = \frac{n}{2}\{2a + (n - 1)d\}$ (b) $S_n = \frac{n}{2}(a + \ell)$

(3) **Arithmetic Mean:**

$\boxed{A.M = \frac{a + b}{2}}$ (When two numbers are given).

n – Arithmetic means b/w a & b:

$\boxed{A_1 = a + d}$

$\boxed{A_2 = a + 2d}$

$\boxed{A_3 = a + 3d}$

$\boxed{A_n = a + nd}$

Where $d = \frac{b - a}{n + 1}$

a = first term

b = Second term

n = Number of A.M's

d = Common difference

(4) **n^{th} term of Geometric Sequence:**

$\boxed{T_n = ar^{n-1}}$

(5) **Geometric Series:**

(a) $S_n = \frac{a(1 - r^n)}{1 - r}$; $r < 1$ (b) $S_n = \frac{a(r^n - 1)}{r - 1}$; $r > 1$

(c) $S_n = \frac{a - r\ell}{1 - r}$; $r < 1$ (d) $S_n = \frac{r\ell - a}{r - 1}$; $r > 1$

(we use this formula when first and last terms are given)

(6) **Geometric Mean:**

$\boxed{G.M = \pm\sqrt{ab}}$ (when two numbers are given)

n - Geometric means b/w a & b:

a = first term, b = last term.

$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ (Common ratio)

$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$

$G_3 = ar^3 = a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}$

$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

(7) **Infinite Geometric Series:**

$\boxed{S_\infty = \frac{a}{1-r}}$; $r < 1$

(8) **Harmonic Sequence:**

(a) $T_n = \frac{ab}{b + (n-1)(a-b)}$ (b) $\begin{vmatrix} \frac{1}{x} & \frac{1}{y} & \frac{1}{z} \\ P & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0$

where $T_p = x$, $T_q = y$, $T_r = z$

(9) **Harmonic Mean:**

$\boxed{H.M = \frac{2ab}{a+b}}$

n – Harmonic means b/w a & b:

$H_1 = \frac{(n+1)\,ab}{a+nb}$, $H_2 = \frac{(n+1)\,ab}{2a+(n-1)\,b}$

$H_3 = \frac{(n+1)\,ab}{3a+(n-2)\,b}$, $H_4 = \frac{(n+1)\,ab}{4a+(n-3)\,b}$

(10) **Arithmetico – Geometric Series:**

$T_n = \{\, a + (n-1)\,d \,\}\, r^{n-1}$

(11) **Relations between Arithmetic, Geometric And Harmonic Means:**

(a) $A.M > G.M > H.M$

(b) $A.M \times H.M = (G.M)^2$

Combination and Permutation:

(1) **Factorial Notation:**

$n! = n \times (n-1) \times (n-2) \times \ldots\ldots \times 3 \times 2 \times 1$

$o! = 1$ and $1! = 1$

(2) **Permutation of "n" different Objects taken "r" at a time:**

${}^nP_r = \frac{n!}{(n-r)!}$

(3) Permutation when repetition is allowed:

${}^nP_r = n^r$

(4) **Permutation of "n" Objects when they are not all different:**

$P = \frac{n!}{n_1! \times n_2! \times \ldots\ldots \times n_K!}$

(5) **Circular Permutation:**

$P = (n-1)\,!$

(6) **Combination:**

${}^nC_r = \frac{n!}{(n-r)!\,r!}$

(11) **Complementary Combination:**

(i) ${}^nC_r = {}^nC_{n-r}$

(ii) ${}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r$ (Pascal's Rule)

(iii) ${}^nC_r \times r! = {}^nP_r$ (iv) ${}^nC_n = 1$

(v) ${}^nC_o = 1$ (vi) ${}^nC_1 = n$

(vii) ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$ (viii) ${}^nC_{n-1} = n$

(12) **Division into Section or parcels:**

${}^{r+s}C_r = \frac{(r+s)!}{r!\,s!}$

Probability Theory:

(1) **Probability of an event A:**

$\boxed{P(A) = \frac{n(A)}{n(S)}}$

where $0 \le p(A) \le 1$ also $p(S) = 1$ and $p(\phi) = 0$

(2) **Probability of the Complement of an event A:**

$\boxed{P(A') = 1 - P(A)}$

(3) **Addition law for Mutually exclusive events:**

$$\boxed{P(A\cup B) = P(A) + P(B)}$$

(4) **Addition law for Not-Mutually exclusive events:**

$$\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

(5) **Multiplication law of Independent events:**

$$\boxed{P(A\cap B) = P(A).\,P(B)}$$

(6) **Multiplication law for Dependent events:**

$$\boxed{P(A\cap B) = P(A).\,P(B/A)}$$

(7) **Conditional Probability of A given B:**

$$\boxed{P(A/B) = \frac{p(A\cap B)}{P(B)}}$$

Mathematical Induction:

(1) **The sum of the first n Natural numbers:**

$$1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}\ ;\ \forall n \in N$$

(or) $\boxed{\Sigma n = \frac{n(n+1)}{2}}$

(2) **The sum of the squares of the first n Natural numbers:**

$$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}\ ;\ n \in N$$

(or) $\boxed{\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}}$

(3) **The sum of the cubes of the first n Natural numbers:**

$$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \frac{n^2(n+1)^2}{4}\ ;\ n \in N$$

(or) $\boxed{\Sigma n^3 = \left[\frac{n(n+1)}{2}\right]^2}$

Binomial Theorem:

(1) $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \ldots\ldots + b^n$

(2) **General term in the expansion of $(a+b)^n$:**

$T_{r+1} = {}^nC_r\, a^{n-r} b^r$

(3) **The Middle term in the expansion of $(a+b)^n$:**

Case # 1 (If n = even)

Middle term $= \left(\frac{n+2}{2}\right)$ th term

Case # 2 (If n = odd)

Middle term $= \left(\frac{n+1}{2}\right)$ th term

Middle term $= \left(\frac{n+3}{2}\right)$ th term

(4) **The first Negative term in the expansion of $(1+x)^n$:**

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{r!}x^r$$

Trigonometry:

(Trigonometric Ratios)

(1) $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$	(4) $\text{cosec}\theta = \frac{\text{Hypotenuse}}{\text{perpendicular}}$
(2) $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$	(5) $\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}}$
(3) $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$	(6) $\cot\theta = \frac{\text{Base}}{\text{Perpendicular}}$

Reciprocal Relations Between The Trigonometric Ratios:

(1) $\sin\theta = \frac{1}{\text{Cosec}\theta}$	(5) $\tan\theta = \frac{1}{\cot\theta}$
(2) $\text{Cosec}\theta = \frac{1}{\sin\theta}$	(6) $\cot\theta = \frac{1}{\tan\theta}$

(3) $\text{Cos}\theta = \dfrac{1}{\text{Sec}\theta}$

(4) $\text{Sec}\theta = \dfrac{1}{\text{Cos}\theta}$

(7) $\tan\theta = \dfrac{\text{Sin}\theta}{\text{Cos}\theta}$

(8) $\text{Cot}\theta = \dfrac{\text{Cos}\theta}{\text{Sin}\theta}$

Relation Between ARC Length and Central Angle:

$\boxed{S = r\theta}$

Conversion From Degree To Radian:

$\overset{\circ}{1} = \dfrac{\pi}{180}$ radians ≈ 0.0174 rad

Conversion From Radian To Degree:

$1 \text{ rad} = \dfrac{180}{\pi}$ degree $\approx 57.\overset{\circ}{3}$

Values of Trigonometric Ratios:

θ°	0°	30°	45°	60°	90°	120°	180°	270°	360°
Rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{3\pi}{2}$	2π
Sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0	−1	0
Cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	−1	0	1
Tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	0	∞	0
Cosecθ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	∞	−1	∞
Secθ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	−2	−1	∞	1
Cotθ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	∞	0	∞

Trigonometric Identities:

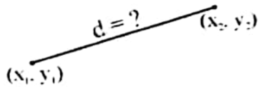
(1) $\text{Sin}^2\theta + \text{Cos}^2\theta = 1$

(2) $1 + \tan^2\theta = \text{Sec}^2\theta$

(3) $1 + \text{Cot}^2\theta = \text{Cosec}^2\theta$

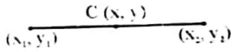
Distance between given two points:

$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$



Mid-point Formula:

$\boxed{C(x, y) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)}$



Fundamental Law:

$\boxed{\text{Cos}(\alpha - \beta) = \text{Cos}\alpha\,\text{Cos}\beta + \text{Sin}\alpha\,\text{Sin}\beta}$

Deduction From the Fundamental Law:

(1) $\text{Cos}(\alpha + \beta) = \text{Cos}\,\alpha\,\text{Cos}\beta - \text{Sin}\alpha\,\text{Sin}\beta$

(2) $\text{Sin}(\alpha + \beta) = \text{Sin}\alpha\,\text{Cos}\beta + \text{Cos}\alpha\,\text{Sin}\beta$

(3) $\text{Sin}(\alpha - \beta) = \text{Sin}\alpha\,\text{Cos}\beta - \text{Cos}\alpha\,\text{Sin}\beta$

(4) $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

(5) $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

(6) $\text{Cot}(\alpha + \beta) = \dfrac{\text{Cot}\,\alpha\,\text{Cot}\beta - 1}{\text{Cot}\alpha + \text{Cot}\beta}$

(7) $\text{Cot}(\alpha - \beta) = \dfrac{\text{Cot}\alpha\,\text{Cot}\beta + 1}{\text{Cot}\alpha - \text{Cot}\beta}$

Product to Sum Formulae:

(1) $\text{Sin}\alpha\,\text{Cos}\beta = \dfrac{1}{2}[\,\text{Sin}(\alpha + \beta) + \text{Sin}(\alpha - \beta)\,]$

(2) $\text{Cos}\alpha\,\text{Sin}\beta = \dfrac{1}{2}[\,\text{Sin}(\alpha + \beta) - \text{Sin}(\alpha - \beta)\,]$

(3) $\text{Cos}\alpha\,\text{Cos}\beta = \dfrac{1}{2}[\text{Cos}(\alpha + \beta) + \text{Cos}(\alpha - \beta)\,]$

(4) $\text{Sin}\alpha\,\text{Sin}\beta = \dfrac{-1}{2}[\,\text{Cos}(\alpha + \beta) - \text{Cos}(\alpha - \beta)\,]$

Sum to Product Formulae:

(1) $\text{Sin}U + \text{Sin}V = 2\,\text{Sin}\left(\frac{U+V}{2}\right)\text{Cos}\left(\frac{U-V}{2}\right)$

(2) $\text{Sin}U - \text{Sin}V = 2\,\text{Cos}\left(\frac{U+V}{2}\right)\text{Sin}\left(\frac{U-V}{2}\right)$

(3) $\text{Cos}U + \text{Cos}V = 2\text{Cos}\left(\frac{U+V}{2}\right)\text{Cos}\left(\frac{U-V}{2}\right)$

(4) $\text{Cos}U - \text{Cos}V = -2\text{Sin}\left(\frac{U+V}{2}\right)\text{Sin}\left(\frac{U-V}{2}\right)$

Trigonometric Functions With Negative Angle:

(1) $\text{Sin}(-\theta) = -\text{Sin}\,\theta$

(2) $\text{Cos}(-\theta) = \text{Cos}\theta$

(3) $\tan(-\theta) = -\tan\theta$

(4) $\text{Cot}(-\theta) = -\text{Cot}\theta$

Trigonometric Ratios of Allied Angles:

The angles associated with basic angles of measure θ to a right angle or its multiple are called allied angles. So the angles of measures $90° \pm \theta$, $180° \pm \theta$, $270° \pm \theta$, $360° \pm \theta$ are known as allied angles.

(1) $\text{Sin}(90° + \theta) = \text{Cos}\theta$	(1) $\text{Sin}(180° + \theta) = -\text{Sin}\theta$
(2) $\text{Sin}(90° - \theta) = \text{Cos}\theta$	(2) $\text{Sin}(180° - \theta) = \text{Sin}\theta$
(3) $\text{Cos}(90° + \theta) = -\text{Sin}\theta$	(3) $\text{Cos}(180° + \theta) = -\text{Cos}\theta$
(4) $\text{Cos}(90° - \theta) = \text{Sin}\theta$	(4) $\text{Cos}(180° - \theta) = -\text{Cos}\theta$
(5) $\tan(90° + \theta) = -\text{Cot}\theta$	(5) $\tan(180° + \theta) = \tan\theta$
(6) $\tan(90° - \theta) = \text{Cot}\theta$	(6) $\tan(180° - \theta) = -\tan\theta$
(7) $\text{Cot}(90° + \theta) = -\tan\theta$	(7) $\text{Cot}(180° + \theta) = \text{Cot}\theta$
(8) $\text{Cot}(90° - \theta) = \tan\theta$	(8) $\text{Cot}(180° - \theta) = -\text{Cot}\theta$

(1) $\text{Sin}(270° + \theta) = -\text{Cos}\theta$	(1) $\text{Sin}(360° + \theta) = \text{Sin}\theta$
(2) $\text{Sin}(270° - \theta) = -\text{Cos}\theta$	(2) $\text{Sin}(360° - \theta) = -\text{Sin}\theta$
(3) $\text{Cos}(270° + \theta) = \text{Sin}\theta$	(3) $\text{Cos}(360° + \theta) = \text{Cos}\theta$
(4) $\text{Cos}(270° - \theta) = -\text{Sin}\theta$	(4) $\text{Cos}(360° - \theta) = \text{Cos}\theta$
(5) $\tan(270° + \theta) = -\text{Cot}\theta$	(5) $\tan(360° + \theta) = \tan\theta$
(6) $\tan(270° - \theta) = \text{Cot}\theta$	(6) $\tan(360° - \theta) = -\tan\theta$
(7) $\text{Cot}(270° + \theta) = -\tan\theta$	(7) $\text{Cot}(360° + \theta) = \text{Cot}\theta$
(8) $\text{Cot}(270° - \theta) = \tan\theta$	(8) $\text{Cot}(360° - \theta) = -\text{Cot}\theta$

Double Angle Formulae:

(1) $\text{Sin}2\theta = 2\text{Sin}\theta\,\text{Cos}\theta$

(2) $\text{Cos}2\theta = \text{Cos}^2\theta - \text{Sin}^2\theta$

$\text{Cos}2\theta = 2\text{Cos}^2\theta - 1$

$\text{Cos}2\theta = 1 - 2\,\text{Sin}^2\theta$

$\text{Cos}2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

(3) $\text{Sin}2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

(4) $\tan2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Half Angle Formulae:

(1) $\text{Sin}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\text{Cos}\alpha}{2}}$ (2) $\text{Cos}\frac{\alpha}{2} = \pm\sqrt{\frac{1+\text{Cos}\alpha}{2}}$

(3) $\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\text{Cos}\alpha}{1+\text{Cos}\alpha}}$

Triple Angle Formulae:

(1) $\text{Sin}3\theta = 3\text{Sin}\theta - 4\text{Sin}^3\theta$

(2) $\text{Cos}3\theta = 4\text{Cos}^3\theta - 3\text{Cos}\theta$

(3) $\tan3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Sine Law: $\boxed{\frac{a}{\text{Sin}\alpha} = \frac{b}{\text{Sin}\beta} = \frac{c}{\text{Sin}\gamma}}$

Cosine Law:

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos\alpha \\ b^2 &= a^2 + c^2 - 2ac\cos\beta \\ c^2 &= a^2 + b^2 - 2ab\cos\gamma \end{aligned}}$$

Tangent Law:

$$\frac{\tan\frac{\alpha-\beta}{2}}{\tan\frac{\alpha+\beta}{2}}=\frac{a-b}{a+b}$$

$$\frac{\tan\frac{\beta-\gamma}{2}}{\tan\frac{\beta+\gamma}{2}}=\frac{b-c}{b+c}$$

$$\frac{\tan\frac{\gamma-\alpha}{2}}{\tan\frac{\gamma+\alpha}{2}}=\frac{c-a}{c+a}$$

Half Angle Formulae:

(a) The Sine of Half the angle in terms of the Sides:

In any triangle ABC

(1) $\sin\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$ where $S=\frac{a+b+c}{2}$

$S \rightarrow$ Semi-perimeter of triangle

(2) $\sin\frac{\beta}{2}=\sqrt{\frac{(s-c)(s-a)}{ac}}$

(3) $\sin\frac{\gamma}{2}=\sqrt{\frac{(s-a)(s-b)}{ab}}$

(b) The Cosine of Half the angle in terms of the sides:

In any triangle ABC

(1) $\cos\frac{\alpha}{2}=\sqrt{\frac{s(s-a)}{bc}}$

(2) $\cos\frac{\beta}{2}=\sqrt{\frac{s(s-b)}{ac}}$

(3) $\cos\frac{\gamma}{2}=\sqrt{\frac{s(s-c)}{ab}}$

(c) The tangent of Half the angle in term of the sides:

(a) $\tan\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ (d) $\cot\frac{\alpha}{2}=\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$

(b) $\tan\frac{\beta}{2}=\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$ (e) $\cot\frac{\beta}{2}=\sqrt{\frac{s(s-b)}{(s-c)(s-a)}}$

(c) $\tan\frac{\gamma}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$ (f) $\cot\frac{\gamma}{2}=\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

Area of Triangle:

Case # 1 Area of triangle in terms of the measures of two sides and their included angle:

(1) $\blacktriangle ABC=\frac{1}{2}bc\sin\alpha$

(2) $\blacktriangle ABC=\frac{1}{2}ac\sin\beta$

(3) $\blacktriangle ABC=\frac{1}{2}ab\sin\gamma$

Case # 2 Area of triangle in terms of the measures of one side and two angles:

(1) $\blacktriangle ABC=\frac{1}{2}a^2\frac{\sin\gamma\sin\beta}{\sin\alpha}$

(2) $\blacktriangle ABC=\frac{1}{2}b^2\frac{\sin\alpha\sin\gamma}{\sin\beta}$

(3) $\blacktriangle ABC=\frac{1}{2}c^2\frac{\sin\alpha\sin\beta}{\sin\gamma}$

Case # 3 Area of triangle in terms of the measures of its sides:

$\blacktriangle=\sqrt{s(s-a)(s-b)(s-c)}$ $\therefore S=\frac{a+b+c}{2}$

Half Angle Formulae:

(We use these formulae when three sides of triangle are given and we have to find the three angles.)

(1) $\tan\frac{\alpha}{2}=\frac{r}{s-a}$

(2) $\tan\frac{\beta}{2}=\frac{r}{s-b}$

(3) $\tan\frac{\gamma}{2}=\frac{r}{s-c}$

where $r^2=\frac{(S-a)(S-b)(S-c)}{S}$

$S=\frac{a+b+c}{2}$

Circles Connected with a Given Triangles:

(1) **Circum Radius**

$$R = \frac{abc}{4\blacktriangle}$$

(2) **In – Radius:**

$$r = \frac{\blacktriangle}{s}$$

(3) **Radii of e-circle:**

$$r_1 = \frac{\blacktriangle}{s-a}$$

$$r_2 = \frac{\blacktriangle}{s-b}$$

$$r_3 = \frac{\blacktriangle}{s-c}$$

Inverse Trigonometric Functions Formulae:

(1) $\tan^{-1}a + \tan^{-1}b = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$

(2) $\tan^{-1}a - \tan^{-1}b = \tan^{-1}\left(\frac{a-b}{1+ab}\right)$

Note: If in any Δ $a = b = c$ then it is called an equilateral Δ. The area is $\frac{\sqrt{3}}{4}(\text{side})^2$.

Note: (1) $\text{Sin}\theta = 2\,\text{Sin}\frac{\theta}{2}\text{Cos}\frac{\theta}{2}$

(2) $1 - \text{Cos}\,\theta = 2\,\text{Sin}^2\frac{\theta}{2}$

(3) $1 + \text{Cos}\theta = 2\,\text{Cos}^2\frac{\theta}{2}$